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Design and Analysis of a Four-Wire Probe for Incompressible Flows

by

Reza Sadr

B.Sc. (Mechanical Engineering)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of

Master of Engineering

in

Aerospace Engineering

Ottawa-Carleton Institute for Mechanical
and Aerospace Engineering

Department of Mechanical and Aerospace Engineering
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**Design and Analysis of a Four-Wire
Probe for Incompressible Flows**

submitted by

Reza Sadr, B.Sc.

in partial fulfilment of the requirements for
the degree of Master of Engineering

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Abstract

A four-wire hot-wire probe was designed and analyzed for the measurement of velocity in three-dimensional, incompressible flows. The probe consists of four tungsten hot-wires of $5\text{ }\mu\text{m}$, 1 mm length slanted at about 45° with respect to the probe axis and arranged in a pyramid configuration. The ends of the tungsten wires are copper plated such that the sensitive part of the hot-wires form a measuring volume of approximately 3.3 mm in diameter and 0.9 mm in height.

Two methods of calibration and velocity measurements are considered for the four-wire probe, namely the method of directional sensitivity coefficients and the method of directional sensitivity function. Detailed measurements were undertaken to examine the accuracy of these methods. The results showed that the method of directional sensitivity coefficients does not provide satisfactory accuracy in the measurement of the velocity vector. The uncertainty associated with the empirical coefficients utilized in this method were found to account for the observed inaccuracy. The method of directional sensitivity function, on the other hand, displayed less sensitivity to the experimental uncertainties than the method of directional sensitivity coefficients, consequently providing satisfactory accuracy in the measurement of the velocity vector.

The performance of the four-wire probe was compared with a seven-hole pressure probe for the measurement of the velocity vector in steady flows. The result indicated that both probes measure the velocity magnitude and direction with comparable accuracy within their calibration range.

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to my mother

Table of Contents

Abstract	iii
Acknowledgements	iv
Table of Contents	vi
List of Tables	xi
List of Figures	xii
Nomenclature	xix
1 Introduction	1
1.1 Background	1
1.2 Research Objectives	3

2	Literature Review	4
2.1	Introduction	4
2.2	Hot-Wire Terminology	5
2.3	Heat Transfer from a Hot Wire	7
2.4	Aerodynamic Interferences	16
3	Four-Wire Probe Design	19
3.1	Hot-Wire Sensor	20
3.2	Hot-Wire Sensor Plating	26
3.2.1	The Electroplating Process	28
3.3	Prongs	31
3.3.1	Four-Wire Prong Configurations	33
3.4	Probe Stem	36
4	Apparatus	42
4.1	Calibration-Jet	42
4.2	Data Acquisition Method	46
4.2.1	Pressure Transducers	46
4.2.2	Pressure-Probe Measurement System	47

4.2.3	Constant-Temperature Anemometer	49
4.3	Probe Rotation System	53
4.4	Probe Alignment	53
5	Calibration and Measurement Procedures for a Four-Wire Probe	59
5.1	The Method of Directional Sensitivity Coefficients	60
5.1.1	Calibration	61
5.1.2	Uncertainty Analysis	73
5.1.3	Velocity Measurement	82
5.2	The Method of Directional Sensitivity Function	99
5.2.1	Calibration	99
5.2.2	Velocity Measurement	101
6	Comparison of Four-Wire Probe and Seven-Hole Pressure Probe Characteristics	108
6.1	Seven-Hole Pressure Probe	108
6.2	Comparison of the Seven-Hole Probe with the Four-Wire Probe	110
6.2.1	Uncertainty in the Measurement of Velocity	111
6.2.2	Effects of Reynolds Number on the Seven-Hole Probe Measurements	112

7 Conclusions and Recommendations	120
7.1 Conclusions	120
7.2 Recommendations	122
References	123
Appendices	131
A Theory of Hot-Wire Anemometry	132
A.1 Heat Transfer	132
A.1.1 The Static Response of a Hot-Wire	136
A.1.2 The Dynamic Response of a Hot-Wire	138
A.2 Constant-Temperature Anemometry	141
A.2.1 Basic Circuitry	142
A.2.2 Frequency Response of the Anemometer	146
A.2.3 Frequency Response Test	149
B Plating Procedure	153
C Soldering Procedure	158
D Birational Spline Fitting	162

E Seven-Hole Probe Calibration Procedure	165
E.1 Low Angle Regime	166
E.2 High Angle Regime	169

List of Tables

3.1	Some properties of common hot-wire materials: adapted from Finger son (1994)	23
3.2	Comparison of several existing multi-wire probe geometries.	38
5.1	Typical calibration coefficients for the four-wire probe.	72
5.2	Typical uncertainties associated with the coefficients in King's law and the resulting uncertainty in the effective cooling velocity.	77
5.3	Typical uncertainties associated with the pressure transducer and the resulting uncertainty in the calibration-jet velocity for $ \vec{V}_{jet} =20 \text{ m/s}$	78

List of Figures

2.1	A typical single-wire probe.	6
2.2	a. Cross-wire probe, b. Three-wire probe.	7
2.3	Effect of spatial velocity gradients on hot-wire response; adapted from Gessner and Moller (1971)	10
2.4	Velocity components in a wire frame of reference.	11
2.5	Variation of k and h with pitch and yaw angles; reproduced from Jorgensen (1971)	14
2.6	Effect of hot-wire contamination on the velocity measurement; adapted from Martinez <i>et al.</i> (1982)	15
2.7	Probe configurations used by Comte-Bellot <i>et al.</i> (1971).	17

2.8	Effect of pitch angle on the extracted mean velocity; reproduced from Comte-Bellot <i>et al.</i> (1971)	18
2.9	a. Perturbations resulting from different parts of probe D; b. Mean and fluctuating velocity measured with the reference probe in the presence of probe D; reproduced from Comte-Bellot <i>et al.</i> (1971).	18
3.1	Temperature distribution along a platinum wire; adapted from Champagne <i>et al.</i> (1967)	21
3.2	Resistance-temperature diagram	25
3.3	Schematic of a plating set-up.	29
3.4	Partial view of a typical plated wire.	31
3.5	A three-wire probe with mutually perpendicular wires; reproduced from Yeung and Squire (1993)	33
3.6	Possible wire configurations for four-wire probes; a. Samet and Einav (1987), b. Döbbeling <i>et al.</i> (1990a).	34
3.7	Measurement volume of the four-wire probe.	35
3.8	Method of setting the relative lengths of the prongs during manufacturing.	36

3.9	Geometry of the present four-wire probe.	39
3.10	Schematic of the present four-wire probe	41
4.1	Calibration-jet setup.	43
4.2	Calibration-jet velocity profile two diameters downstream of the nozzle exit.	45
4.3	Rosemount (serial No. 557) pressure transducer calibration curve. . .	47
4.4	Pressure coefficient versus time after stepping the scanning valve. . .	49
4.5	Anemometer output with the hot-wire sensor exposed to still air. . .	50
4.6	Square-wave test of the anemometer, $f_{cut-off} = \frac{1}{1.3\tau}$	51
4.7	Sinusoidal-wave test of the anemometer.	52
4.8	Schematic of the probe rotation system	54
4.9	Four-hole pressure probe geometry.	56
4.10	Pitch, yaw and roll angles in the alignment of the four-hole probe. . .	57
4.11	Variation of pitch and yaw pressure differentials for roll angles of 0° and 180°.	58

5.1	Wire frame of reference.	62
5.2	Typical calibration curve for one of the wires of the four-wire probe .	63
5.3	Hot-wire rotation to obtain a. k_i and b. h_i	65
5.4	Variation of the yaw sensitivity coefficient with yaw misalignment. . .	66
5.5	Variation of the pitch sensitivity coefficient with pitch misalignment..	66
5.6	Wire orientations on the four-wire probe.	67
5.7	The effect of uncertainty in the wire orientation on k	68
5.8	The effect of uncertainty in the wire orientation on h	68
5.9	The effect of the velocity magnitude on k	70
5.10	The effect of the velocity magnitude on h	70
5.11	Calibration curves for the four wires of the probe.	72
5.12	Typical variation of uncertainties associated with k	80
5.13	Typical variation of uncertainties associated with h	80
5.14	Uncertainty band in k	81
5.15	Uncertainty band in h	81

5.16	The four-wire probe frame of reference.	83
5.17	Location of the tails of the velocity vectors yielding the same effective cooling velocity for an infinitely long hot-wire.	86
5.18	Location of the tails of the velocity vectors yielding the same effective cooling velocity for a finite hot-wire.	86
5.19	The intersection of the three orthogonal cylinders each defined by Equation 5.38.	87
5.20	The validity domain for a triple orthogonal wire probe.	87
5.21	The intersections of three ellipsoids represent the velocity vectors which satisfy Jorgensen's equation for three hot-wires of the present probe	89
5.22	The intersections of four ellipsoids represent the velocity vectors which satisfy Jorgensen's equation for hot-wires of the present probe	90
5.23	Error in calculated pitch angle; $k_i = k_i(\alpha)$; $h_i = h_i(\beta)$	95
5.24	Error in calculated yaw angle; $k_i = k_i(\alpha)$; $h_i = h_i(\beta)$	96
5.25	Error in calculated velocity magnitude; $k_i = k_i(\alpha)$; $h_i = h_i(\beta)$	97
5.26	Typical calibration curve for one of the wires of the four-wire probe. .	101

5.27	Directional sensitivity functions for the wires of the four-wire probe. .	102
5.28	Error in computed θ due to an assumed error in E_1	106
5.29	Error in computed φ due to an assumed error in E_1	106
5.30	Error in computed $ \vec{V} $ due to an assumed error in E_1	107
6.1	Seven-hole pressure probe geometry.	109
6.2	Tangential and polar co-ordinate systems for the seven-hole pressure probe.	112
6.3	Effect of Reynolds number on the measured flow misalignment in the low-angle regime, $\alpha = 15^\circ$	115
6.4	Effect of Reynolds number on the measured velocity magnitude in the low-angle regime, $\alpha = 15^\circ$	116
6.5	Effect of Reynolds number on the measured flow misalignment in the high-angle regime	117
6.6	Effect of Reynolds number on the measured flow misalignment in the high-angle regime (2)	118
6.7	Effect of Reynolds number on the measured velocity magnitude in the high-angle regime	119

A.1	Differential element of a hot-wire.	134
A.2	Schematic of a constant-temperature anemometer circuit with provision for a test signal e_t ; adapted from Blackwelder (1981).	143
A.3	Typical frequency response of a constant-temperature anemometer to velocity fluctuations; adapted from Blackwelder (1981).	148
A.4	Typical response of a constant-temperature anemometer to a square-wave signal.	150
A.5	Typical response of a constant-temperature anemometer to a sine-wave signal; adapted from Weidman and Browand (1975).	152
B.1	Schematic of the holding fork.	154
B.2	Schematic of the plating apparatus.	156
C.1	Schematic of the soldering setup	160
C.2	Relative position of the plated wire, prongs and soldering needle in the soldering process.	161
D.1	Domain R and corresponding values of G_{ij} given on it.	163
D.2	The effect of the tension factor on the smoothness of the spline functions.	164

E.1	Seven-hole pressure probe port numbering scheme.	166
E.2	Tangential co-ordinate system used for low angle regime, reproduced from De Cecco (1995).	168
E.3	Sector allocation criteria, adapted from Everett <i>et. al.</i> (1983).	170
E.4	Polar co-ordinate system used for high angle regime, reproduced from De Cecco (1995).	171

Nomenclature

English Symbols

a_r	over heat ratio
A	area
A_0	DC gain of the amplifier (Eqn. A.24)
A, B, n	empirical coefficients in King's law (Eqn. A.17)
c	specific heat
c_p	specific heat at constant pressure
C_0	stagnation pressure coefficient (Eqns. E.3 & E.7)
C_p	pressure coefficient $\left(\equiv \frac{P - P_{ref}}{\frac{1}{2} \rho V_{jet}^2} \right)$
C_q	dynamic pressure coefficient (Eqns. E.3 & E.7)
C_t	transducer coefficient (Eqn. 4.2)
d	hot-wire diameter

D	four-wire probe stem diameter
$\mathcal{D}$	thermal diffusivity $\left(\equiv \frac{k}{\rho c_p} \right)$
e	voltage
E	anemometer/transducer output voltage
f	frequency
g	directional sensitivity function of the four-wire probe
$\hat{g}$	normalized directional sensitivity function $\left(\equiv \frac{g(\theta, \varphi)}{g(0, 0)} \right)$
Gr	Grashof number $\left(\equiv \frac{g\beta(T_w - T_\infty)d^3}{\nu^2} \right)$
h	pitch sensitivity coefficient of a hot-wire; convective heat transfer coefficient
i, I	current
l	hot-wire length
L	prong length
k	yaw directional sensitivity coefficient of a hot-wire; thermal conductivity
K_0	electronic circuit gain parameter (Eqn. A.31)
m	mass
M	hot-wire time constant (Eqn. A.19)
Nu	Nusselt number $\left(\equiv \frac{hd}{k_f} \right)$
p	spline tension factor
P	pressure

P_d	transducer differential pressure
q	dynamic pressure $\left(\equiv \frac{1}{2}\rho V^2\right)$
Q	rate of heat transfer
r	radius
R	electrical resistance
R_0	electrical resistance of a hot-wire at ambient temperature
Re	Reynolds number based on the hot-wire diameter $\left(\equiv \frac{V_{eff}d}{\nu_f}\right)$
S	correction vector (Eqn. 5.52): mean-velocity gradient parameter $\left(\equiv \frac{\Delta \bar{V}/\bar{V}_c}{l/d}\right)$
t	time
T	temperature
T_f	film temperature $\left(\equiv \frac{1}{2}(T_{surface} + T_\infty)\right)$
u'	streamwise velocity fluctuation
u, v, w	velocity components in the wire frame of reference
U, V, W	velocity components in the probe frame of reference
$\bar{V}_c$	mean velocity at the centre of the hot-wire length
V_{eff}	effective cooling velocity

Greek Symbols

α_r	temperature coefficient of resistivity
α, β	yaw and pitch misalignment in the wire frame of reference (Fig. 5.1); tangential coordinate system for the seven-hole pressure probe (Fig. 6.2)
β	volume coefficient of expansion
δ	unbalance parameter for the Wheatstone bridge (Eqn. A.25)
ϵ	emissivity
ε	error (Eqn. 5.44)
$\mathcal{E}_m$	non-dimensional residual (Eqns. 5.43 & 5.50)
ζ	damping coefficient (Eqn. A.32)
θ, φ	yaw and pitch misalignment relative to the probe axis (Fig. 5.16)
θ, ϕ	pitch and roll misalignment of the seven-hole pressure probe (Fig. 6.2)
μ	amplifier time constant (Eqn. A.24)
ν	kinematic viscosity
ρ	density
ρ_r	electrical resistivity (Eqn. A.3)
σ	Stefan-Boltzmann constant ($56.7 \times 10^{-9} \frac{W}{m^2 K^4}$); tensile strength
ψ, γ	orientation of the hot-wires on the probe (Fig. 5.6)
τ	time constant
ω_0	characteristic frequency (Eqn. A.30)

Subscripts

<i>atm</i>	ambient value
<i>core</i>	core region of the calibration jet
<i>f</i>	film value
<i>i</i>	indices for identifying the hot-wires
<i>jet</i>	calibration jet
<i>p</i>	probe
<i>ref</i>	reference
<i>settling</i>	settling chamber
<i>t</i>	test signal, transducer
<i>w</i>	wire
∞	free stream

Superscripts

<i>m</i>	number of iterations
----------	----------------------

Chapter 1

Introduction

1.1 Background

Design of fluid machinery and flight vehicles with the aid of computational fluid mechanics has been receiving increasing attention. Supported by rapid development of high performance computers, numerical simulation of fluid flows is now an accepted design tool. However, for many flows, accuracy of the solution strongly depends on the accuracy of the turbulence model used. Current turbulence models do not apply universally, and thus are often "tuned" for specific flow problems based on experimentally obtained data.

Many instruments and measuring techniques are available for the measurement of time independent flow quantities. Relatively few are suitable for the measurement of temporal variations in fluid-flow properties. Hinze (1959) summarized the basic

requirements for a sensor and the associated circuitry for reliable measurement of small velocity fluctuations in a three-dimensional turbulent flow. These requirements are as follows:

- The sensor must be sufficiently small such that it introduces negligible disturbances on the flow pattern.
- The sensor must be sufficiently small to resolve the smallest spatial variations in the velocity field.
- The frequency response of the circuitry must be high enough to respond to the most rapid temporal variations in the flow.
- The instrument must be sensitive enough to detect small amplitude fluctuations which may be a very small percentage of the mean value.
- The instrument must be stable with no noticeable drift in the calibration parameters during at least one test run.
- The instrument should be rigid enough to withstand aerodynamic loads without vibrating.

The two most commonly used measurement methods which reasonably satisfy all these mentioned requirements are hot-wire anemometry and laser-Doppler anemometry.

Prior to the introduction of the laser anemometry in the 1970s, the hot-wire anemometer had been the nearly exclusive measurement instrument in turbulence

research. While laser anemometry can provide non-intrusive measurements in certain unique situations such as separated flows, very high temperature flows and flows highly sensitive to disturbances, the hot-wire anemometer remains a popular tool in fluid mechanics research. The major reasons for this are its relatively low cost and the fact that the performance and reliability of the hot-wire anemometry technique is well-established.

1.2 Research Objectives

The work reported in this thesis had three main objectives:

- design and construction of a hot-wire probe tailored for the measurement of three-dimensional, unsteady, turbulent flow inside an existing curved diffuser,
- establishment of proper calibration and measurement procedures for the hot-wire probe,
- comparison of the hot-wire probe performance with that of an existing seven-hole probe, which, so far, has been the primary instrument utilized in the aforementioned curved-diffuser work.

Chapter 2

Literature Review

2.1 Introduction

The exact origin of hot-wire anemometry is difficult to ascertain, however, according to Freymuth (1980), it has been investigated and used since the last two decades of the nineteenth century. The paper by King (1914) is generally considered to be the first complete work on hot-wire anemometry. Hot-wire anemometry has since undergone vigorous development. Efforts are scattered over many applications, journals and reports. The entire body of literature on hot-wire anemometry and related subjects published before 1980 consists of about 1400 papers (Freymuth, 1980), including the excellent reviews by Corrsin (1963), Bradshaw (1971), Vagt (1979), and more recently by Blackwelder (1981), Perry (1982), Lomas (1986) and Bruun (1995).

The content of the review in this chapter is organized to give the reader a general background on hot-wire anemometry to assist her/him in following the design, calibration and data reduction procedures of a four-wire probe, described in this thesis.

Appendix A provides a more detailed review of the theory of hot-wire anemometry.

2.2 Hot-Wire Terminology

Hot-wire anemometry is defined as the measurement of velocity in a fluid flow using the variation in heat transfer from an electrically-heated thin wire exposed to the flow.

The electrical resistance of a hot-wire is a non-linear function of its temperature (Hinze, 1959). However, for a typical hot-wire sensor it can be approximated as:

$$R_w = R_0 [1 + \alpha_r(T_w - T_\infty)] \quad (2.1)$$

where α_r is the temperature coefficient of resistivity, T_∞ is the free stream temperature, T_w is the wire temperature, and R_0 is the electrical resistance of the wire at T_∞ . The overheat ratio of a hot-wire is defined as

$$a_r = \frac{R_w - R_0}{R_0} \quad (2.2)$$

Using Equation 2.1 it is observed that the overheat ratio is proportional to the temperature difference between the hot-wire and the surrounding fluid, i.e.,

$$a_r = \alpha_r(T_w - T_\infty) .$$

On hot-wire probes the hot-wire is held in place by prongs (Figure 2.1). The diameter of the prongs' tip is large enough so that its temperature is equal to the

Figure 2.1: A typical single-wire probe.

ambient fluid temperature. The ends of the hot-wire are often plated with a metal, e.g. copper, mainly to reduce the aerodynamic effects of the prongs on the hot-wire sensor. The effects of the prongs and of the plated part on the hot-wire sensor and the factors influencing the choice of the diameter of the plated part are discussed in Chapter 3.

Depending on the application, a hot-wire probe may be constructed with more than one hot-wire sensor. A probe with two wires, usually perpendicular to each other, is called a cross-wire probe (Figure 2.2a). This type of probe is commonly used in two-dimensional flow measurements. Probes with three or four hot-wire sensors are used for measurements in three-dimensional flows. Figure 2.2b shows a typical three-wire probe with mutually perpendicular wires. Hot-wire probes with more than four wires have been used for specific applications. For example, a probe with nine hot-wire sensors was used by Vukoslavčević *et al.* (1989) to simultaneously measure the velocity and vorticity vectors in a three-dimensional turbulent flow.

Figure 2.2: a. Cross-wire probe, b. Three-wire probe.

2.3 Heat Transfer from a Hot-Wire

King (1914) was first to derive an expression for the rate of forced heat convection from an infinitely long cylinder. The following expression is equivalent of that of king:

$$Nu = A_o + B_o Re^{0.5} \quad (2.3)$$

where A_o and B_o are empirical constants, Re is the Reynolds number based on the wire diameter

$$Re \equiv \frac{Vd}{\nu},$$

with ν as the kinematic viscosity, and Nu is the Nusselt number

$$Nu \equiv \frac{hd}{k}.$$

with k as the thermal conductivity. Collis and Williams (1959) studied the heat transfer from a long hot-wire by placing it perpendicular to a uniform air stream

with Reynolds numbers ranging from 0.01 to 140. Based on their experimental results, they modified Equation 2.3:

$$Nu \left(\frac{T_f}{T_\infty} \right)^{0.17} = A_1 + B_1 Re^n \quad (2.4)$$

where A_1 , B_1 and n are empirical constants. In Equations 2.3 and 2.4, fluid properties used in computing Nu and Re are evaluated at the film temperature,

$$T_f = \frac{1}{2}(T_w + T_\infty)$$

where T_w is the wire temperature and T_∞ is the free stream temperature. Collis and Williams (1959) also concluded that free convection may be neglected when $Gr < Re^{\frac{1}{3}}$, where Gr is the Grashof number

$$Gr \equiv \frac{g\beta(T_w - T_\infty)d^3}{\nu^2}$$

with g as the gravitational acceleration and β as the volume coefficient of expansion.

Because the diameter of the prongs is significantly larger than that of the hot-wire sensor, the prongs do not overheat as the control circuitry passes current through them. Consequently, the temperature of the prongs is essentially the same as that of the surrounding fluid. The resultant temperature difference between the hot-wire and the prongs causes large temperature gradients along the length of the hot-wire, close to the prongs. This temperature non-uniformity, and the aerodynamic interference of the prongs, are the main causes for the differing behaviour of the finite-length hot-wire and the infinitely long cylinder examined above. According to the experimental results of Champagne *et al.* (1967), for a platinum wire with an overheat ratio of 0.8 and air velocity of about 6 m/sec, heat conduction loss from

the wire is about 4% of the total heat loss for a wire aspect ratio (l/d) of 400, 8% for $l/d = 200$ and up to 20% for $l/d = 99$. They also observed that these percentages decrease as the air velocity increases.

In the presence of a mean velocity gradient the hot-wire would have a skewed temperature distribution along its length. Gessner and Mollier (1971) performed detailed analytical and experimental investigations measuring the effect of shear flow on the hot-wire response. Based on a shear parameter defined as $S = (\Delta \bar{V}/\bar{V}_c)/(l/d)$, where $\Delta \bar{V}$ is the mean velocity increment across the wire, $\bar{V}_c$ is the velocity magnitude at the centre of the wire length and l/d is the wire aspect ratio, their results are summarized in Figure 2.3. $Nu_{s \neq 0}$ is the overall Nusselt number of the wire evaluated in shear flow and $Nu_{s=0}$ is the overall Nusselt number evaluated in uniform flow where the overall Nusselt number is defined as:

$$Nu = \frac{R_w I^2}{\pi k_f l (T_w - T_\infty)} ,$$

with I being the current passing through the hot-wire. From Figure 2.3 it is observed that at a given Re_∞ , $Nu_{s \neq 0}$ is less than $Nu_{s=0}$. However, the difference is quite small, i.e. less than 1% for $l/d=200$.

As the response of a hot-wire depends on the rate of heat convection from the wire to the surrounding fluid, the relative fluid temperature has a significant effect on the response of a hot-wire. If a probe is used at a fluid temperature different from that during calibration, an error will be introduced in the extracted mean velocity and turbulence correlations. Tavoularis (1983) showed that for a platinum plated tungsten wire with an aspect ratio of 250 and an overheat ratio of 0.8, an

Figure 2.3: Effect of spatial velocity gradients on hot-wire response; adapted from Gessner and Moller (1971); Re =Reynolds number based on wire diameter and $\bar{V}_c$,
 — $a_r=0.8$, - - $a_r=0.4$.

increase of $2.4^\circ C$ in ambient temperature, if not accounted for, could cause errors up to 65% in a measured mean velocity of 8.2 m/s . In such a case, while mean velocity is always underestimated, no particular trend appears to exist for turbulent properties. Bearman (1971) presented a simple method for correcting for the effect of ambient temperature drift in incompressible flows. In this method, it is assumed that the change in ambient temperature is small so that the properties of the fluid adjacent to the wire are assumed to remain unchanged.

The Reynolds number in equations 2.3 and 2.4 is based on an effective cooling velocity, V_{eff} . For an *infinitely* long wire, this velocity is defined as:

$$V_{eff} = |\vec{V}| \cos \alpha \quad (2.5)$$

Figure 2.4: Velocity components in a wire frame of reference.

where $|\vec{V}|$ is the velocity magnitude and α is the yaw angle of the velocity vector relative to the hot-wire as shown in Figure 2.4.

For a finite-length hot-wire several analytical expressions have been proposed to relate the velocity vector to the cooling velocity. Hinze (1959) introduced the cosine law in which the velocity vector is decomposed into components normal and tangential to the wire:

$$V_{eff} = |\vec{V}| \sqrt{\cos^2 \alpha + k^2 \sin^2 \alpha} . \quad (2.6)$$

Several alternative expressions are:

$$V_{eff} = |\vec{V}| \cos^m \alpha \quad (2.7)$$

$$V_{eff} = |\vec{V}| [1 - b(1 - \sqrt{\cos \alpha})]^2 \quad (2.8)$$

$$V_{eff} = |\vec{V}| \cos \alpha_r \quad (2.9)$$

which were proposed by Davies and Bruun (1968), Friehe and Schwarz (1968) and

Bradshaw (1971), respectively. k , m and α_r are obtained through calibration. All of the above equations assume that there is no change in the effective cooling velocity if the velocity vector is rotated around the hot-wire. Due to the aerodynamic interference of the prongs and of the probe body this assumption is not correct. To account for these interferences, Jorgensen (1971) introduced another constant, h , into Equation 2.6:

$$V_{eff} = \sqrt{u^2 + k^2 v^2 + h^2 w^2} \quad (2.10)$$

where u , v and w are the mutually perpendicular velocity components in the wire frame of reference (Figure 2.4).

It has been suggested that the values of k and h are not constant, but depend on the wire aspect ratio, flow direction relative to the wire and the Reynolds number. Champagne *et al.* (1967) studied the effect of yaw misalignment and wire aspect ratio (l/d) on the value of k . They found that k varies from 0.2, for $l/d=200$, to essentially zero for $l/d=600$. They also concluded that the change of k with yaw misalignment is due to aerodynamic interference of the prongs. The results of Bruun (1972) confirm the effect of wire aspect ratio on k observed by Champagne *et al.* (1967); his work also shows a slight effect of Reynolds number on the value of k . The physical explanation for the dependency of k on yaw misalignment and the Reynolds number is that the flow distortion around the prongs is altered with changes in yaw misalignment and the Reynolds number, affecting the convective heat transfer from the hot-wire. If the wire aspect ratio is large enough, this has little effect on the overall heat transfer.

Jorgensen (1971) showed that the values of k and h change with yaw and pitch

misalignment, respectively. He found that plating the ends of the wire reduces the values of k and h and, furthermore, decreases their sensitivity to flow misalignment (Figure 2.5). This is due to the diminished effect of the prongs on the aerodynamics/heat transfer over the sensitive part of the wire when the wire ends are coated.

Bruun *et al.* (1990) showed that, for a cross-wire probe, assuming A_1 , B_1 and n in Equation 2.4 to be functions of yaw angle and setting k to zero in Equation 2.6 improves the accuracy of the measured velocity vector.

Laboratory air may be contaminated by dust, lint, flakes of dried wax from floors, or small droplets of oil from nearby machinery. If a hot-wire is used in such an environment, some degree of probe fouling will reduce the sensitivity of the hot-wire to velocity variations. The effects of sensor contamination on mean velocity and turbulence measurements was studied by Martinez *et al.* (1982) using a hot-wire in a mixing layer of a two dimensional jet in quiescent air. Figure 2.6 shows the result of their measurements where U and u' are the local mean and fluctuating velocities and U_o is the mean jet-core velocity. It is seen that typical contamination does not have a significant effect on the mean-velocity measurement. However, substantial error in the measured turbulence intensity is evident. This behaviour was explained by the low-pass filtering effect of the dirt layer on the sensor. A hot-wire probe can be cleaned by gently swirling it in a container of solvent. Commonly used solvents are organic dissolvents (Martinez *et al.* 1982), acetone and carbon tetrachloride (Lomas 1986). After cleaning, the probe is rinsed in distilled water and dried.

Figure 2.5: Variation of k and h with pitch and yaw angles; reproduced from Jorgensen (1971); a. Gold-plated tungsten-wire, b. Unplated tungsten-wire.

Figure 2.6: Effect of sensor contamination on a. mean velocity measurement, b. turbulence intensity measurement; adapted from Martinez *et al.* (1982); $\times$ ≡ old (after 10 hours operating), Δ ≡ old after cleaning, $\circ$ ≡ new.

2.4 Aerodynamic Interferences

A hot-wire sensor itself has a very small diameter, generally between 1 to 10 microns, and thus introduces negligible aerodynamic disturbance. However, a hot-wire probe contains additional parts, namely the prongs and the stem. The disturbances resulting from the various parts of a single hot-wire probe were investigated by Comte-Bellot *et al.* (1971). The results obtained from a reference probe were compared with four other probes with differing geometries (Figure 2.7).

The results in Figure 2.8 show that the probe geometry has a significant effect on the pitch sensitivity. For example, while the reference probe shows less than 5% variations in the mean velocity reading within $\pm 90^\circ$, probe C gives about 20% variations in the same range. This results in a significant difference between the two probes' pitch sensitivity coefficient, h , where the pitch sensitivity coefficient of the reference probe would be almost one.

According to the results in Figure 2.9a, the disturbance due to the prongs and stem is minimum at $\alpha \approx 45^\circ$. At $\alpha \approx 0^\circ$ the blockage effect of the stem and the prongs would be at its maximum whereas at $\alpha \approx 90^\circ$ the flow acceleration between the prongs would reach its peak value. At $\alpha \approx 45^\circ$ these two effects appear to offset each other. Figure 2.9b shows the result obtained in a flow with high turbulence intensity ($I = \sqrt{u_o'^2}/U_o = 19\%$) where u' is the streamwise velocity fluctuation and subscript "o" indicates the corresponding quantity measured by the reference probe in the absence of the test probe (probe D).

Figure 2.7: Probe configurations used by Comte-Bellot *et al.* (1971); dimensions in millimeters unless specified otherwise.

Figure 2.8: Effect of pitch angle on the extracted mean velocity from the probes illustrated in Figure 2.7; reproduced from Comte-Bellot *et al.* (1971).

Figure 2.9: a. Perturbations resulting from different parts of probe D; b. Mean and fluctuating velocity measured with the reference probe in the presence of probe D; reproduced from Comte-Bellot *et al.* (1971).

Chapter 3

Four-Wire Probe Design

Hot-wire techniques for the measurement of mean velocity and turbulence may be classified in three groups based on the number of hot-wires used in the measurements, i.e. single-wire probes, cross-wire probes and three or four-wire probes. As mentioned by Lofdahl (1986), in some situations the use of single-wire and cross-wire methods is not practical, as both methods may require rotation of the probe to determine the local mean velocity vector and turbulence correlations. To overcome this difficulty three or four-wire probes are used. However, these probes have their own limitations. Multi-wire probes are usually larger than simpler ones, leading to higher local disturbance and reduced spatial resolution.

As was mentioned in the introduction, the present work aims to develop and analyze a hot-wire probe to be used in the measurement of velocity and turbulence correlations in a large-scale curved diffuser. The following sections examine various aspects of the hot-wire probe geometry ultimately leading to a four-wire probe design that provides a reasonable trade-off in terms of manufacturing cost, simplicity of data reduction and accuracy.

3.1 Hot-Wire Sensor

The hot-wire aspect ratio, defined as the ratio of the sensor length to its diameter, is an important design parameter in hot-wire anemometry. The choice of the wire aspect ratio is based on a number of conflicting design specifications. A small aspect ratio reduces wire vibration and minimizes stresses due to aerodynamic loads thus reducing the likelihood of wire breakage related to these factors. On the other hand, increasing the wire aspect ratio reduces the aerodynamic and heat-transfer interferences of the prongs on the hot-wire. Champagne *et al.* (1967) measured the temperature distribution along hot-wires with different aspect ratios. Figure 3.1 shows the result of two such measurements for wire aspect ratios of 99 and 200. For a hot-wire with an aspect ratio of 99, there appears to be no uniform temperature region along the wire, whereas the wire with an aspect ratio of 200 has a uniform temperature distribution along 40% of its length. As discussed in Appendix A, uniform temperature distribution along the length of a hot-wire is an assumption leading to significant simplification in the dynamic-response analysis of the hot-wire.

Kassab (1991) and Azad (1993) observed that hot-wires with aspect ratios less than about 200 underestimate the value of the local turbulence intensity. These observations are in agreement with Corrsin's (1963) physical reasoning regarding the effects of heat conduction to the prongs on the transient response of a hot-wire. According to Corrsin (1963), for a small wire aspect ratio which would have a nonzero temperature gradient along the whole length of the wire, in passing from one steady state to another of different velocity, the local wire temperature at intermediate

Figure 3.1: Temperature distribution along a platinum wire; adapted from Champagne *et al.* (1967) $a_r=0.8$; a. $l/d=200$, b. $l/d=99$, o normal wire, ▽ 55° slanted wire.

times would not pass through a simple sequence of steady state distributions; readjustment involves the propagation of a thermal wave along the wire. This effect, which is commonly called the "Bechov effect", can be neglected under either of the following conditions. If the relaxation time for conduction along the wire is much smaller than the smallest time interval to be resolved in the measured turbulence, the complete wire length fluctuates about its mean temperature in phase and a "quasi-steady" situation exists. Alternatively, if the relaxation time is much larger than the largest time scale to be resolved, each infinitesimal segment of the wire length fluctuates independently about its own mean temperature, with virtually no effect from the neighbouring segment, and again the "quasi-steady" situation exists for each segment. Corrsin (1963) showed that for hot-wires with small wire aspect ratios, neither of these assumptions are valid. Hence, for such hot-wires the measured turbulence quantities can be erroneous.

In addition to the wire aspect ratio, the absolute values of the wire length and diameter are important design parameters as well. It is generally desirable to have the hot-wire as short as possible to increase the spatial resolution of the probe. Ligrani *et al.* (1987) showed that the hot-wire response diminishes significantly as the sensor length exceeds 1 mm. This is due to the spatial eddy averaging that occurs along the wire length. Azad (1993) observed a similar trend and concluded that a hot-wire with an aspect ratio of about 200 and a length of about 1 mm has the best performance in measuring the turbulence energy spectrum in the high frequency range. In decreasing the wire length, the wire diameter has to be decreased as well so as to maintain the wire aspect ratio. From Equation A.19 it is observed

			Tungsten	Platinum	80% Platinum, 20% Iridium
Temperature coefficient of resistivity	α_r	$^{\circ}\text{C}^{-1}$	0.0045	0.0039	0.0008
Electrical resistivity	ρ_r	$\Omega \cdot \text{cm}$	5.5×10^{-6}	10×10^{-6}	31×10^{-6}
Ultimate tensile strength	σ	kg/m^2	420	24.6	100
Thermal conductivity	k_u	$\text{cal}/\text{cm}^{\circ}\text{C}$	0.47	0.166 ^d	0.042

Table 3.1: Some properties of common hot-wire materials; adapted from Fingerson (1994)

that decreasing the wire diameter improves the frequency response of a hot-wire. However, a thin wire cannot withstand high hydrodynamic loads. Furthermore, the effect of wire contamination on the calibration characteristics increases with decreasing wire diameter.

Two important specifications of a hot-wire material are its tensile strength and its temperature coefficient of resistivity. The wire material should have high tensile strength to withstand against breakage at the smallest possible diameters. Tungsten, platinum and platinum-iridium are commonly used materials for hot-wires as they offer reasonable trade offs in terms of these parameters.

Platinum is available in very small diameters, down to $0.25 \mu\text{m}$, has a high tem-

perature coefficient of resistivity and does not oxidize at operating temperatures up to 1100 °C. However, its tensile strength, specially at high operating temperatures, is rather low and therefore even aerodynamic drag alone can cause wire breakage. Platinum can be soldered or spot welded to the prongs. Platinum-iridium has a higher tensile strength, but has a considerably lower temperature coefficient of resistivity which yields to a relatively large wire time constant. Platinum-iridium is commonly used for measurements in high temperature flows.

The most commonly used material for hot-wire sensors is tungsten. It offers a high temperature coefficient of resistivity, high tensile strength and a high melting point. Lomas (1986) mentioned that tungsten can be used in *non-oxidizing* gases at hot-wire temperatures up to 2000 K. In air flows, the primary limitation of tungsten is its rather low oxidation temperature, the temperature limit being at about 300 °C (Fingerson 1994). Some commercial tungsten wires are plated with a thin layer of platinum or gold to get around the oxidization problem. Tungsten wires are commercially available in diameters down to 2.5 μm . Tungsten cannot be soldered to the prongs, and thus is generally attached to the prongs by spot welding. A summary of the material properties for tungsten, platinum and platinum-iridium is given in Table 3.1.

Tungsten wires with 5 μm diameter were chosen for the present probe. The sensitive length of the hot-wires was chosen to be about 1 mm, yielding an aspect ratio of 200. The temperature coefficient of resistivity, α_r , is given in the literature, e.g. Table 3.1, as well as by the suppliers. However, it is recommended that for

Figure 3.2: Resistance-temperature diagram

each wire roll bought, the value of α be obtained by testing. Figure 3.2 shows the variation of the wire resistance with temperature for the $5\mu\text{m}$ tungsten wire used on the present probe. This curve was obtained by placing the wire in the core region of a pipe flow. The temperature of the flow was changed by means of an electrical heater upstream of the flow and was measured with a thermocouple with an accuracy of $\pm 0.2\text{ }^{\circ}\text{C}$. The temperature coefficient of resistivity, i.e. the slope of the temperature versus resistance curve, was found to be $\alpha_r = 0.0035\text{ }^{\circ}\text{C}^{-1}$.

3.2 Hot-Wire Sensor Plating

The ends of tungsten hot-wires are often plated to:

- reduce the effect of aerodynamic disturbances caused by the prongs;
- provide the possibility of soft soldering of tungsten wires to the prongs;
- increase the prong spacing which reduces the localized blockage caused by the prongs.

Common materials used for plating are gold, silver and copper. Gold-plated wire sensors are commercially available. High electric conductivity of gold makes it a desirable material for plating hot-wires. However, the gold plating process needs relatively expensive plating equipment. Silver-coated tungsten wires are called Wollaston wires and are commercially available. The Wollaston wire is attached to the prongs and the silver plating is then etched away from the sensitive part by placing a droplet of dilute nitric acid on the wire. Many researchers prefer to plate hot-wires themselves using electroplating methods due to economic advantages and the flexibility in choosing the proper dimensions for the plated part of the hot-wire. Availability of chemical solutions and ease of the electroplating process makes copper a widely used material for electroplating tungsten wires. Copper plating of the wire-ends was chosen for the present probe.

There are two limiting factors for the diameter of the plating. In the case of large plating diameters, the plating acts in the same manner as the prongs by intro-

ducing comparable aerodynamic disturbances on the flow over the sensitive portion of the hot-wire. On the other hand, the minimum diameter of the plated portion is dictated by the following factors. First, the diameter of the plated portion cannot be reduced below a certain level where the resistance of the plated portion becomes significant compared with the hot-wire, as the plated portion would become sensitive to the velocity fluctuations. Second, the diameter of the plated portion should be large enough so that the heat conduction from the hot-wire does not increase the temperature of the plated portion. If the temperature of the plated portion becomes significantly more than the fluid temperature, the plated portion would become sensitive to the velocity fluctuations. The dynamic response of the plated part, however, would not be the same as that of the hot-wire. As was discussed in the previous section, Corrsin (1963) showed that this would lead to erroneous turbulence measurements. For a $5\text{ }\mu\text{m}$ tungsten wire, the resistance of the plated portion is negligible if the diameter of the plated part is more than $10\text{ }\mu\text{m}$. According to Perry (1982), for a minimum plating diameter of about $25\text{ }\mu\text{m}$ over a $5\text{ }\mu\text{m}$ wire, the plating temperature remains at about the fluid temperature.

A plating diameter of $25\text{ }\mu\text{m}$ was chosen for the present probe. This diameter is in the same range as those noted in the available literature (Vagt 1979, DANTEC gold-plated wire).

3.2.1 The Electroplating Process

The important factors in the electroplating process are the positive (the anode) and negative (the cathode) electrodes, the electrolyte, the plating voltage and the plating time (Figure 3.3). The object to be plated is chosen as the cathode and the anode is made of the metal to be deposited on the cathode.

The setup shown in Figure 3.3 was used to plate the ends of hot-wires of the present probe. In this setup a $5\text{ }\mu\text{m}$ tungsten wire, which is spot welded on a holding fork, is partially dipped into the electrolyte such that one end of the wire to be plated is submerged and the wire is normal to the free surface of the electrolyte. The plating process is then started. Subsequently, the fork is turned 180° around its axis and the other end of the wire is dipped into the electrolyte. The plating process is then repeated.

An acid sulphate bath was used to electroplate copper on the tungsten wire. Copper sulphate and sulphuric acid are the primary constituents of this electrolyte. The metal ions are provided by the copper sulphate. Sulphuric acid increases the electrolyte's conductivity and the anode's corrosion, and helps to prevent the formation of unwanted salts. The quality of the plating, such as smoothness and uniformity, are optimized by varying the composition of the electrolyte. The optimization process for the present experiment is described below. For more detailed information about the specifications of copper sulphate electrolytes the reader is referred to Weisenberger (1989).

Figure 3.3: Schematic of a plating set-up.

A set of experiments was performed to determine the chemical composition of the electrolyte that yields plating with desirable smoothness and uniformity. In the preparation of the optimum electrolyte, a solution of 20 *gr* of copper sulphate crystal, $CuSO_4$, was dissolved in 100 *ml* distilled water. 20 *ml* of a solution of 29 *ml* sulphuric acid, H_2SO_4 , in 100 *ml* distilled water was then added to this $CuSO_4$ solution. Finally, two droplets of hydrochloric acid, HCL , were added to avoid streaky plating.

The plating voltage has to be optimized to obtain a smooth plated surface. Low voltage causes the plating process to be too slow whereas high voltage causes a nonuniform deposition. For the electrolyte composition described, the proper voltage was found to be about 0.20 *Volts*. The current passing through the solution is also monitored. For a fixed voltage and electrolyte composition the current depends on the surface area of the cathode and the anode exposed to the solution. For the present configuration, the current was found to be about 8 *mA*.

The plating time depends on the bath composition, plating voltage, anode and fork geometry and is optimized experimentally to give the proper plating thickness. With the present setup, a plating time of about 35 seconds yielded a plated wire diameter about 25 μm . Further details of the plating procedure are presented in Appendix B.

In the present work the plated diameter was set to $25 \pm 5 \mu m$. This diameter can be changed by altering the plating time or by a slight change of the plating voltage. The non-plated (sensitive) wire length was chosen $1 \pm 0.1 mm$. For a 5 μm wire this

Figure 3.4: Partial view of a typical plated wire.

length corresponds to an aspect ratio of about 200. Figure 3.4 shows a portion of a typical plated hot-wire.

The finished cost of an in-house plated wire was less than 10% of the commercially available DANTEC gold-plated hot-wires. Considering the fact that hot-wires are very fragile and thus break frequently, the resulting savings are significant.

3.3 Prongs

Hot-wires are held in place by prongs. Prongs have to be strong enough to withstand aerodynamic loads and not vibrate, be as thin as possible to minimize the associated aerodynamic disturbances and have small electrical resistance.

Commonly used materials for prongs are stainless steel, phospho-bronze and

constantan (Lomas 1986). One of the best choices is ordinary sewing needles which are made from hardened steel, plated to prevent corrosion and tapered at the tip. Jewellers broaches have been used as prongs as well (e.g. Samet and Einav, 1987).

Primarily two factors influence the choice of the prong diameter: for a given prong material, a large prong diameter increases mechanical strength. On the other hand, a large prong diameter introduces increased aerodynamic disturbances on the sensor. As far as prong spacing is concerned, the results of Comte-Bellot *et al.* (1971) showed that, for a given prong diameter, the effect of the prongs on the measured mean velocity and turbulence decreases as the prong spacing is increased. For probes with prong spacing to prong diameter ratio in excess of 5 the error in the mean velocity and turbulence measurement is less than 2.5%. Plating the ends of the hot-wire sensor and tapering the prongs toward their tips are known to further reduce these effects of the prongs on the hot-wire sensor. Finally, as far as the length of the prongs is concerned they have to be long enough to minimize the aerodynamic disturbance of the probe body on the flow in the vicinity of the sensor, but excessive prong lengths may result in unacceptable vibration of the prongs under aerodynamic loading.

The number of hot-wire sensors on a probe dictates the total number of the prongs, hence the overall extent of the blockage and aerodynamic disturbance. In this context three-wire probes are preferable over four-wire probes. In the present work, initially several three-wire probe designs were considered. However, the data reduction procedure for these probes requires the wires to be mutually perpendicular

Figure 3.5: A three-wire probe with mutually perpendicular wires; DISA Model 55P91; reproduced from Yeung and Squire (1993).

and relatively precisely oriented. This results in relatively complex prong geometries as shown in Figure 3.5 for a typical commercial probe. Use of such prong geometries in the present work would have resulted in probe designs that would have been more expensive to manufacture in-house than purchasing commercially available three-wire probes. Consequently, a four-wire probe configuration, which allowed the use of simple straight prongs, was chosen for the present work.

3.3.1 Four-Wire Prong Configurations

Four-wire probes have been used for velocity measurements by Pailhas and Cousteix (1986), Samet and Einav (1987), Döbbeling *et al.* (1990b) and Pompeo and Thourmann (1993) among others. Pailhas and Cousteix (1986) studied the effect of the wires' inclination relative to the probe axis on the response of a four-wire probe. They considered the case with all four-wires having the same inclination. Their results showed that

Figure 3.6: Possible wire configurations for four-wire probes; a. Samet and Einav (1987), b. Döbbeling *et al.* (1990a).

as the angle of the wires' inclination increases the validity domain¹ of the probe increases as well. On the other hand, the greatest velocity sensitivity of the probe is obtained for 0° inclination relative to the probe axis. Thus, they concluded that a four-wire probe with a wire inclination of about 45° would yield a reasonable balance between the size of the validity domain and the sensitivity to the velocity magnitude.

Two alternatives for a four-wire probe are shown in Figure 3.6. Samet and Einav (1987) showed that for the probe in Figure 3.6a the range of flow misalignments for which the probe can be used is essentially the same as the angle of the wires relative to the probe axis. The limiting factor is the unacceptable levels of aerodynamic interference of the prongs for larger flow misalignments.

¹The concept of validity domain of a multi-wire probe is discussed in Chapter 5.

Figure 3.7: Measurement volume of the four-wire probe.

Döbbeling *et al.* (1990a) showed that the probe in Figure 3.6b for which the wires are inclined at 45° relative to the probe axis, can be used for misalignments up to 60° relative to the probe axis. Furthermore, this prong arrangement simplifies the soldering of the hot-wire sensors onto the prongs considerably. The prong configuration in Figure 3.6b was chosen for the present four-wire probe.

Eight sewing needles each of 0.45 mm diameter were used as prongs on the present four-wire probe and hot-wire sensors were soft soldered to the tapered tip of the prongs. The needle tips were slightly filed until a tip diameter of 0.2 mm was reached so as to provide a sufficient base for soldering the sensors onto the prongs. The lengths of the central and outer prongs were chosen to be 13 mm and 11.2 mm , respectively. These prong lengths and a prong spacing of 1.8 mm between each pair yielded wire inclinations of about 45° relative to the probe axis, with the ratio of prong spacing to prong diameter being about 4. The measurement volume on this four-wire probe consists of a cylinder of 3.3 mm diameter and 0.83 mm height (Figure 3.7). This measurement volume is somewhat large for turbulence measurements. However, the probe was designed with the primary objective of

Figure 3.8: Method of setting the relative lengths of the prongs during manufacturing.

measuring unsteady flows in a curved diffuser for which the limited spatial resolution of the probe is not as crucial.

During manufacturing, a brass alignment jig placed over the tip of the prongs was used to set the relative lengths of the prongs. A sketch of this setup is shown in Figure 3.8. The center of the jig was drilled to a diameter to accommodate the four inner prongs, and a depth that was equal to the difference in the length of the two sets of prongs. The tips of the eight prongs were placed against the jig with the four central prongs located inside the hole. Then the prongs were permanently glued to the probe stem.

3.4 Probe Stem

The aerodynamic disturbance induced by the probe stem on the hot-wire sensors is dominantly influenced by the diameter of the stem and the distance between the

hot-wires and the stem, i.e. the prong length. Comte-Bellot *et al.* (1971) showed that the extent of these aerodynamic disturbances varies approximately inversely with the square of the ratio of the prong length to the stem diameter. Table 3.2 compares several multi-wire hot-wire probes in terms of these geometric parameters. The prong length-to-stem diameter ratio for the present four-wire probe was chosen to be $L/D=2$ which falls within the range covered in this table.

The probe stem of the present probe consists of three components: a dielectric tip, a stainless steel coupling and a brass section (Figure 3.9). The dielectric tip of the probe supports the prongs. On the present probe an epoxy-fibreglass composite known as G10 was used as the dielectric material. This material has high strength and good machining properties. The main difficulty in manufacturing this dielectric tip was the drilling of the small diameter holes for the prongs through considerable length during which the drill bit tended to deviate from its intended path. Machining the dielectric tip as two shorter parts reduced this difficulty considerably: the passages were drilled through both parts then the prongs were passed through the parts. Once axial alignment of the prongs was ensured through visual inspection, and the relative prong lengths were set using the previously-described jig, the two parts were glued to each other and to the prongs.

The dielectric tip is attached to the main body of the probe through a stainless steel coupling. The stainless steel coupling consists of two mating sections. The two sections are concentric about the probe axis and can be rotated relative to each other. Their relative angular alignment can be set to 0° , 60° and -60° within ± 0.4

	No. of wires on the probe	Body diameter, D [mm]	Prong length, L [mm]	L/D [-]	Wire plating
Gorton <i>et al.</i> (1976)	3	1.5	2	1.4	Copper
Fabris (1978)	4	7.9	25	3.2	Silver
Butler (1983)	3	4.7	7	1.5	—
Mathioudakis (1985)	3	4.6	9.5	2.06	—
Samet & Einav (1987)	4	2.5	14.5	5.8	—
Döbbeling <i>et.al</i> (1990a)	4	2	3	1.5	—
DISA <i>type 55P91</i>	3	3	10	3	Gold
Present Probe	4	6.3	13	2.0	Copper

Table 3.2: Comparison of several existing multi-wire probe geometries.

wire #	1	2	3	4
ψ [± 1 deg]	50	52	43	47
γ [± 1 deg]	1	88	175	257

Figure 3.9: Geometry of the present four-wire probe.

degrees using a set of alignment holes. Three alignment holes, separated by an angular distance of 60° , are located on one of the two stainless-steel sections. A pin is used to align one of these holes with the single alignment hole on the remaining section. Once aligned, a set-screw is used to maintain the relative position of the two sections (Figure 3.10). This design feature of the coupling allows precisely controlled rotation of the probe tip about the probe axis. Such rotations would be necessary for turbulence measurements with the sensors of the probe being sampled sequentially.

The four-wire probe main body is made out of a hollow brass tube of square cross section. The main body outer geometry is identical to that of seven-hole probes extensively used in fluid dynamics research within the department. Thus, the same probe holders can be used for both types of probes. The length of the main body was chosen such that it provides considerable room for adjustment of the probe tip location by sliding the probe within the probe holder. The fine electric wires connected to the prongs pass through the brass stem.

Once the manufacturing of the probe stem was completed, the previously-described plated tungsten wires were soft soldered to the prongs of the four-wire probe. Details of the soldering procedure are provided in Appendix C.

The four-wire probe was constructed at the Science and Technology Center, Carleton University, at a cost of about 14% of the current purchase price of a DANTEC triple-wire probe model 55P91.

Figure 3.10: Schematic of the present four-wire probe (all dimensions in millimeters).

Chapter 4

Apparatus

This chapter describes the experimental setup used for the current study. Specifically, the chapter contains detailed descriptions of a calibration-jet, data acquisition systems, a constant-temperature anemometer, and a probe rotation system.

4.1 Calibration-Jet

For the present work an existing set up located in room 2268ME was used to generate a uniform air jet with relatively low turbulence intensity. The set up consists of a Roots blower connected to a settling chamber which feeds a converging nozzle. Figure 4.1 shows a schematic of this setup.

A Roots Whispair blower Model 2506J driven by a 7.5 *HP* AC motor takes in atmospheric air and delivers the air to a settling chamber. The blower is capable of delivering a maximum flow rate of $0.106 \text{ m}^3/\text{s}$ at a total pressure rise of 41.2 kPa . In the present work the blower was operated with a flow rate of $0.092 \text{ m}^3/\text{s}$ and

Figure 4.1: Calibration-jet setup.

Figure 5.3: Hot-wire rotation to obtain a. k_i and b. h_i .

From Figure 5.4 it is seen that the yaw sensitivity coefficient, k_i , displays notable variation with yaw misalignment. This trend for k_i agrees well with those in the literature for similar wire specifications, e.g. Chew *et al.* (1988) and Jorgensen (1971). In Figure 5.5 it is observed that the pitch sensitivity coefficient, h_i , is not nearly as sensitive to the pitch angle as k_i is to yaw misalignment. Once again, this is also in agreement with the literature, e.g. Chew *et al.* (1988) and Jorgensen (1971), although the magnitude of h_i is somewhat higher than those reported for typical commercial hot-wire probes. This may be due to the four centre prongs of the present probe which aerodynamically act like one prong with a large diameter. This would generate relatively larger acceleration, hence yield larger values of h_i .

Through computer simulations Buresti and Talamelli (1992) showed that any error in the wire orientation on a single wire probe causes significant error in the calculated directional sensitivity coefficients. In order to evaluate the dependence

Figure 4.2: Calibration-jet velocity profile two diameters downstream of the nozzle exit.

There are two basic aerodynamic requirements for the core region of the calibration jet; coaxial flow, uniform mean velocity distribution and low turbulence intensity. The required extent of the core region is dictated by the size of the probe to be calibrated. Figure 4.2 shows the mean velocity profile of the jet two nozzle diameters downstream of the nozzle exit obtained with a pitot probe. It is observed that at this streamwise position the cross-stream extent of the core is sufficient for the calibration of the four-wire probe designed and analyzed in the present work. Turbulence intensity ($\sqrt{u'^2}/V$) in the core region of the calibration-jet was measured with a DISA single hot-wire probe Model 55P01. For the velocity range used in the present work, the turbulence intensity of the inviscid core of the calibration-jet was $2.9 \pm 0.4\%$.

4.2 Data Acquisition Method

Different methods of data acquisition were used for hot-wire measurements and pressure-probe measurements. In hot-wire measurements the outputs of a constant-temperature anemometer and a pressure transducer were displayed on digital voltmeters and were recorded manually. The data acquisition system for pressure-probe calibration and measurements consisted of two pressure transducers, a Model 901 CPU and serial Interface¹, a Model 81 Electronic Measurement system¹ and a microcomputer.

4.2.1 Pressure Transducers

Three RosemountTM variable capacitance differential pressure transducers were used to convert the settling chamber and transducer pressure signals into analog signals. The transducers have a pressure range of $\pm 3.4 \text{ kPa}$ and a full scale output range of $\pm 2.5 \text{ Volts DC}$. All pressure transducers were calibrated against inclined manometers. Figure 4.3 shows a typical transducer calibration curve. It is observed that the calibration curve can be well approximated by a straight line expressed as

$$P_d = C_t(E_t - E_{t_0}) \quad (4.2)$$

where P_d is the differential pressure applied to the two ports of the transducer, C_t is the transducer calibration coefficient and E_t is the transducer output. E_{t_0} is the transducer output at $P_d = 0$ which is recorded before each set of measurements.

¹Manufactured by Sciometric Instruments Inc.

Figure 4.3: Rosemount (serial No. 557) pressure transducer calibration curve.

The uncertainty in the transducer calibration coefficient, δC_t , is estimated to be $\delta C_t = 0.002 \text{ Pa/mV}$ which was obtained by studying several sets of calibration data for one of the transducers.

4.2.2 Pressure-Probe Measurement System

Multi-hole pressure probes are used for the measurement of mean flow velocity and total pressure². In multi-hole pressure probe measurements, the analog signals from the pressure transducers are digitized by the M81 measurement system which uses a 12-bit-plus-sign analog-to-digital converter. This provides a resolution of 0.024%

²Detailed description of a seven-hole probe and its calibration procedure are given in Chapter 6 and Appendix E.

of the input range (i.e. 1.22 mV for ± 5 Volts input range). The communication between the M81 and the microcomputer is provided by the M901 serial interface unit. The data acquired during one sampling period is temporarily stored in the 24 Kbyte random access memory available on the M901. This enables a sampling rate of five samples per channels from each of the two transducers.

A 48-port scanning valve was employed for multi-hole pressure probe measurements. The scanning valve is controlled by the microcomputer. When the scanning valve switches ports, a pressure transient is set up in the tubing system as a result of the step change in the measurement pressure. Hence, the data acquisition system delays sampling until pressure transients dissipate. Figure 4.4 shows a typical variation of the measured C_p with time when the scanning valve connected to a multi-hole pressure probe switches ports. In the present work, the transient time was determined to be less than 40 sec. This transient time is affected by the internal volume in the scanning valve, transducer and the pressure tubes connecting these instruments.

All the probe pressures were non-dimensionalized in the form of pressure coefficients defined as:

$$C_p = \frac{P - P_{\text{settling}}}{P_{\text{settling}} - P_{\text{atm}}} \quad (4.3)$$

where the denominator is essentially the dynamic pressure of the calibration-jet. Since the data acquisition system described above was not able to sample the probe pressure and the settling-chamber pressure simultaneously, they were sampled sequentially for a period of 25 sec per port. Once sampling is completed, the data

Figure 4.4: Pressure coefficient versus time after stepping the scanning valve.

is transferred from the M901 to the computer in the form of measurement cycles consisting of "settling-chamber pressure, probe pressure, settling-chamber pressure". Each probe pressure is then non-dimensionalized by the average of the dynamic pressures acquired in the same cycle. Finally, the individual non-dimensional samples (C_p 's) are averaged. According to Yaras (1990), this method, as opposed to forming C_p 's based on average pressures, provides improved repeatability for the data.

4.2.3 Constant-Temperature Anemometer

A TSI (Thermo Systems Inc.) constant-temperature anemometer (Model 1053B) in conjunction with a variable decade (Model 1056) and a power supply (Model 1051 6) was used with the hot-wire probes. The DC and root-mean-square voltage signals

Figure 4.5: Anemometer output with the hot-wire sensor exposed to still air.

from the anemometer were measured with a Fluke 45 Dual-Display Multimeter. The anemometer has a low impedance output with a cut-off frequency, $f_{\text{cut-off}}$, of about 10 kHz. Figure 4.5 shows the noise level in the anemometer output for a 5 μm tungsten wire operating at an over-heat ratio of $a_r=0.7$ in still air. Figures 4.6 and 4.7 show the frequency response of the anemometer obtained through square-wave and sinusoidal-wave tests, respectively. In the sinusoidal test the amplitude of the anemometer output is non-dimensionalized by that at 5 Hz. The cut-off frequency obtained from these tests agree well with each other, which demonstrates the internal consistency of the tests.

Figure 4.6: Square-wave test of the anemometer, $f_{cut-off} = \frac{1}{1.3\tau}$.

Figure 4.7: Sinusoidal-wave test of the anemometer.

4.3 Probe Rotation System

An existing motorized calibration gear shown in Figure 4.8 was used for calibration of the hot-wire and pressure probes. On this gear the probe is mounted on a two-axis turn-table system which provides variations in probe orientation within two mutually perpendicular planes (hereafter referred to as yaw and pitch orientations) while maintaining the probe-tip location within the core of the calibration-jet. 4-phase stepping motors (Superior Electronics, type M061-FD02) provide rotation in the pitch and yaw directions. The motors are used in full stepping mode yielding one motor shaft revolution per 200 steps. This shaft rotation is translated into probe rotation in the yaw and the pitch directions through high-precision worm gears. The 180:1 translation rate of the worm gears, yield 0.01 degrees of probe rotation per motor step.

The motors are controlled by a four-axis stepper driver (LAB 40.5), which is in turn controlled by a LAB 40 bus installed in the microcomputer ³.

4.4 Probe Alignment

Accurate angular calibration of the multi-wire hot-wire and multi-hole pressure probes requires precise referencing of the probe orientation with respect to the calibration-jet direction. A four-hole pressure probe shown in Figure 4.9 was used

³Both LAB 40 and 40.5 units are products of Computer Continuum

Figure 4.8: Schematic of the probe rotation system; 1. Yaw motor, 2. Pitch motor, 3. Yaw gear, 4. Pitch gear, 5. Probe holder, 6. Probe, 7. Calibration-jet.

for the alignment procedure. In the alignment procedure the four-hole probe axis, and hence the probe holder (see Figure 4.8), is aligned with the calibration-jet direction. Once aligned, the four-hole probe is removed from the probe holder and is replaced by the probe to be calibrated.

To align the four-hole probe axis with the jet-flow direction, first the probe is placed onto the probe holder, mounted on the rotation system such that the probe tip is located at the centre of rotation of the yaw and pitch gears and the probe is visually aligned with the estimated direction of the jet-flow. Probe pressures are then acquired for a series of yaw and pitch angles. The probe is then rotated by 180° around its axis and a second set of measurements is taken (Figure 4.10). The results are then plotted as shown in Figure 4.11. The intersection of the lines fitted to each set of data reveals the probe misalignment with respect to the oncoming flow. The alignment of the probe is then adjusted accordingly. The uncertainty in the yaw and pitch alignment based on this procedure is conservatively estimated to be 0.3° .

Figure 4.9: Four-hole pressure probe geometry.

Figure 4.10: Pitch, yaw and roll angles in the alignment of the four-hole probe.

Figure 4.11: Variation of pitch and yaw pressure differentials for roll angles of 0° and 180°.

Chapter 5

Calibration and Measurement

Procedures for a Four-Wire Probe

In the calibration of the four-wire probe the flow velocity vector is correlated to the outputs of the anemometers regulating the temperature of the hot-wires. Generally, calibration is the tedious part of working with hot-wire probes. Rapid wire contamination and wire breakage require relatively frequent calibration of these probes.

Once a four-wire probe is calibrated, it can be used to measure the mean velocity and turbulence correlations in a three-dimensional flow. Two methods of data acquisition are possible for the measurement of these quantities. In one method the outputs of the anemometers are recorded sequentially. In this instance recording of the anemometer outputs involves measurement of both the mean and fluctuating components. These quantities are then related to the mean and fluctuating components of the local velocity. However, this method involves relatively complex mathematical manipulations to extract the turbulence correlations. Furthermore, according to Hinze (1959) due to linearization of Jorgensen's equation in this

method, in a flow with turbulence intensities higher than about 20%, errors up to 10% can be expected in the measured turbulence correlations. In the second method, the anemometer outputs are measured simultaneously with a fast data acquisition system, yielding a record of the instantaneous velocity vector as a function of time. This method needs relatively expensive data acquisition instruments, and a large amount of data is to be processed digitally at each measuring location. However, unlike the first method, it provides the means of extracting all the information a hot-wire signal contains about the flow field with high accuracy. The data reduction procedures examined in the present work are based on this second method.

Two approaches of determining the instantaneous velocity vector are described in this chapter. The method of directional sensitivity *coefficients* makes explicit use of King's law and Jorgensen's equation for a hot-wire to establish a relationship between the anemometer outputs and the local velocity vector. In the second approach, the method of directional sensitivity *function*, the anemometer outputs are related to the local velocity vector through empirical functions obtained through calibration, (Döbbeling *et al.* 1990a).

5.1 The Method of Directional Sensitivity Coefficients

In this method, King's law (Equation A.17).

$$E^2 = A + BV_{eff}^n$$

and Jorgensen's equation (Equation 2.10),

$$V_{eff} = \sqrt{u^2 + k^2 v^2 + h^2 w^2}$$

are used for each wire to form a system of equations with three mutually perpendicular components of the instantaneous velocity vector as the unknowns. Calibration of the probe yields the values of k , h , A , B and n for each wire of the probe.

5.1.1 Calibration

Figure 5.1 shows the i th wire of the four-wire probe and the wire frame of reference associated with it. It should be noted that the wire is in the plane of the prongs, but it is not perpendicular to the prongs. α_i and β_i are called the yaw and pitch angles, respectively, in the wire frame of reference. The output of the anemometer connected to this wire relates to the effective cooling velocity, $V_{eff,i}$, through King's law,

$$E_i^2 = A_i + B_i V_{eff,i}^n \quad (5.1)$$

u_i , v_i and w_i , the components of the local velocity vector, $\vec{V}$, in the wire frame of reference,

$$\begin{aligned} u_i &= |\vec{V}| \cos \alpha_i \cos \beta_i, \\ v_i &= |\vec{V}| \sin \alpha_i, \\ w_i &= |\vec{V}| \cos \alpha_i \sin \beta_i, \end{aligned} \quad (5.2)$$

relate to $V_{eff,i}$ through Jorgensen's equation:

$$V_{eff,i} = \sqrt{u_i^2 + k_i^2 v_i^2 + h_i^2 w_i^2} \quad (5.3)$$

Figure 5.1: Wire frame of reference.

where k_i and h_i are referred to as directional sensitivity coefficients.

The first step of the calibration involves evaluation of the constants A_i , B_i and n_i in Equation 5.1. To obtain these constants the probe is oriented in such a way that the calibration velocity vector, $\vec{V}_{jet}$, is aligned with the x_i axis of the wire, i.e. $\alpha_i = 0$ and $\beta_i = 0$. In this configuration Equations 5.2 yield:

$$u_i = |\vec{V}_{jet}|,$$

$$v_i = 0,$$

$$w_i = 0.$$

Substituting these into Equation 5.3 one obtains:

$$V_{effi} = |\vec{V}_{jet}|. \quad (5.4)$$

In this configuration, evaluation of A_i , B_i and n_i involves recording a set of values for E_i versus V_{jet} and performing a least-square fit to the data. Figure 5.2 shows typical results obtained for one of the wires.

Figure 5.2: Typical calibration curve for one of the wires of the four wire probe; $\circ$ measured, — least- square curve fit.

Variation of the Directional Sensitivity Coefficients with Pitch and Yaw Angles

In order to find the yaw sensitivity coefficient, k_i , $|\vec{V}_{jet}|$ was set to a constant value on the calibration apparatus and the output of the anemometer was recorded while β_i was kept at zero degrees and α_i was varied from 10° to 90° in 5° increments (Figure 5.3a). The effective cooling velocity was then calculated using Equation 5.1:

$$V_{eff_i} = \left(\frac{E_i^2 - A_i}{B_i} \right)^{\frac{1}{n_i}} \quad (5.5)$$

Finally, Equation 5.3 was solved for k_i :

$$k_i = \left[\frac{\left(\frac{V_{eff_i}}{|\vec{V}_{jet}|} \right)^2 - \cos^2 \alpha_i}{\sin^2 \alpha_i} \right]^{0.5} \quad (5.6)$$

To find the pitch sensitivity coefficient, h_i , α_i was kept at zero degrees while β_i was varied from 10° to 90° in 5° increments, (Figure 5.3b). Equation 5.3 was then solved for h_i :

$$h_i = \left[\frac{\left(\frac{V_{eff_i}}{|\vec{V}_{jet}|} \right)^2 - \cos^2 \beta_i}{\sin^2 \beta_i} \right]^{0.5} \quad (5.7)$$

This procedure was repeated for each wire at a calibration-jet velocity of $|\vec{V}_{jet}| = 20 \text{ m/s}$ and a hot-wire over-heat ratio of $a_r = 0.7$.

Figure 5.3: Hot-wire rotation to obtain a. k_t and b. h_t .

From Figure 5.4 it is seen that the yaw sensitivity coefficient, k_t , displays notable variation with yaw misalignment. This trend for k_t agrees well with those in the literature for similar wire specifications, e.g. Chew *et al.* (1988) and Jorgensen (1971). In Figure 5.5 it is observed that the pitch sensitivity coefficient, h_t , is not nearly as sensitive to the pitch angle as k_t is to yaw misalignment. Once again, this is also in agreement with the literature, e.g. Chew *et al.* (1988) and Jorgensen (1971), although the magnitude of h_t is somewhat higher than those reported for typical commercial hot-wire probes. This may be due to the four centre prongs of the present probe which aerodynamically act like one prong with a large diameter. This would generate relatively larger acceleration, hence yield larger values of h_t .

Through computer simulations Buresti and Talamelli (1992) showed that any error in the wire orientation on a single wire probe causes significant error in the calculated directional sensitivity coefficients. In order to evaluate the dependence

Figure 5.4: Variation of the yaw sensitivity coefficient with yaw misalignment.

Figure 5.5: Variation of the pitch sensitivity coefficient with pitch misalignment.

Figure 5.6: Wire orientations on the four-wire probe.

of k_i and h_i on errors in ψ_i and γ_i on the present probe (Figure 5.6), the measured ψ_i and γ_i for one of the wires were artificially modified in small increments. The yaw and pitch traverses were then repeated to determine k_i and h_i for this wire, on the basis of the artificially modified wire orientation. The results are shown in Figures 5.7 and 5.8. From Figure 5.7 it is observed that k_i is quite sensitive to errors in ψ_i , particularly at small α_i , but it is considerably less sensitive to similar errors in γ_i . This can be explained by the fact that uncertainty in the specification of ψ_i dominantly influences the corresponding α_i , whereas the influence of errors in γ_i on the value of α_i is secondary. Figure 5.8 shows that h_i is sensitive to errors in γ_i , but not to errors in ψ_i . This is caused by β_i being much more sensitive to errors in γ_i than to errors in ψ_i . The errors, for both k_i and h_i , are higher at small flow misalignments. This is due to the relatively high sensitivity of k_i and h_i to yaw and pitch angles, respectively, at small misalignment angles.

Figure 5.7: The effect of uncertainty in the wire orientation on k .

Figure 5.8: The effect of uncertainty in the wire orientation on h .

The Effect of Velocity Magnitude on the Directional Sensitivity Coefficients

The effect of velocity magnitude on the values of k_i and h_i was studied by repeating the evaluation of these coefficients at different calibration-jet velocities. Figure 5.9 and 5.10 show typical results obtained for one of the wires of the probe. It is observed that, for the velocity range considered, the dependence of the directional sensitivity coefficients on the velocity magnitude is quite small.

Evaluation of A_i , B_i and n_i

In the usual method of obtaining A_i , B_i and n_i appearing in Equation 5.1, the calibration-jet is aligned with the x axis in the wire frame of reference and the anemometer output is recorded for a series of flow velocities. On a multi wire probe this procedure would normally be repeated for each of the wires. This is, in fact, the only possible approach prior to evaluation of k_i and h_i for the wires. However, once k_i and h_i are quantified, A_i , B_i and n_i for all wires of the probe can be evaluated simultaneously. For this purpose the probe would be placed in the calibration-jet with the jet velocity vector aligned with the X axis of the probe frame of reference (Figure 5.6). In this configuration the pitch and yaw angles of the velocity vector in the wire frame of reference are:

$$\begin{aligned}\alpha_i &= -\psi_i, \\ \beta_i &= 0.\end{aligned}\tag{5.8}$$

Figure 5.9: The effect of the velocity magnitude on k .

Figure 5.10: The effect of the velocity magnitude on h .

Substituting these into Equation 5.2, the velocity components in the wire frame of reference are obtained as:

$$\begin{aligned} u_i &= |\vec{V}_{jet}|(\cos \psi_i) , \\ v_i &= |\vec{V}_{jet}|(-\sin \psi_i) , \\ w_i &= 0 . \end{aligned} \quad (5.9)$$

Thus, with k_i obtained from Figure 5.4, the effective cooling velocity can be determined as

$$V_{eff,i} = |\vec{V}_{jet}| \sqrt{\cos^2 \psi_i + k_i^2 \sin^2 \psi_i} . \quad (5.10)$$

In this configuration, the anemometer outputs, E_i ($i=1, \dots, 4$), are recorded for a series of calibration-jet velocities. Then, the corresponding effective cooling velocity for each wire, $V_{eff,i}$, is obtained using Equation 5.10. Finally A_i , B_i and $\dots$, appearing in King's law (Equation 5.1), are determined through a curve-fit to the E_i versus $V_{eff,i}$ data. A typical set of results for the four wires of the probe is shown in Table 5.1 and Figure 5.11.

Figure 5.12: Typical variation of uncertainties associated with k .

Figure 5.13: Typical variation of uncertainties associated with h .

	Wire 1	Wire 2	Wire 3	Wire 4
A	5.02	5.35	3.47	5.07
B	2.79	2.68	3.31	2.88
n	0.409	0.401	0.386	0.407

Table 5.1: Typical calibration coefficients for the four-wire probe.

Figure 5.11: Calibration curves for the four wires of the probe.

5.1.2 Uncertainty Analysis

Since no measurement is perfectly accurate, means for describing inaccuracies are needed. "It is generally agreed that the appropriate concept for expressing inaccuracies is an *uncertainty* and that the value should be provided by an *uncertainty analysis*" (Kline 1985).

Moffat (1982) defines levels of replication (i.e., how much of the experiment is allowed to change) as the basis for classifying the uncertainty terms. The first order replication accounts for the stochastic uncertainty due to unsteadiness. This uncertainty must be expected when several independent observations are made with the apparatus at hand. This is the principal quantitative way of deciding when the experiment is sufficiently repeatable, that is, well controlled. The most general replication level is called *Nth* order replication. It accounts for the uncertainty of the calibration of the instrument used as well as the stochastic uncertainty due to unsteadiness. The *Nth* order calculations allow results from one experiment to be compared with those from another ostensibly similar one, or with the *true* values. In this section the *Nth* order replication level is invoked to estimate the uncertainty associated with the directional sensitivity coefficients.

Description of Methodology

Let R be a quantity computed from m measurands, $x_1, x_2, \dots, x_m$;

$$R = R(x_1, x_2, \dots, x_m) . \quad (5.11)$$

In Moffat's N th order replication level, the uncertainty in R is estimated by Constant Odds Combination.

$$\delta R = \sqrt{(\delta R_{x_1})^2 + (\delta R_{x_2})^2 + \dots + (\delta R_{x_m})^2} \quad (5.12)$$

where δR_{x_i} denotes the uncertainty in R due to δx_i , the uncertainty in x_i . For most functions of engineering importance and for small δx_i , δR_{x_i} can be approximated by

$$\delta R_{x_i} = \frac{\partial R}{\partial x_i} \delta x_i \quad (5.13)$$

However in cases where the analytical derivation of R is difficult, R is not a well-behaved function, or δx_i is large, δR_{x_i} is obtained directly by computing the variation in R due to introduction of an uncertainty in x_i , i.e.,

$$\delta R_{x_i} = R_{x_i + \delta x_i} - R_{x_i} \quad (5.14)$$

Uncertainty in the Calculated Directional Sensitivity Coefficients

k_i and h_i , the directional sensitivity coefficients associated with the i th wire, were previously calculated using,

$$k_i = \left[\frac{\left(\frac{V_{eff_i}}{|V_{ref}|} \right)^2 - \cos^2 \alpha_i}{\sin^2 \alpha_i} \right]^{0.5} \quad (5.15)$$

$$h_i = \left[\frac{\left(\frac{V_{eff_i}}{|V_{ref}|} \right)^2 - \cos^2 \beta_i}{\sin^2 \beta_i} \right]^{0.5} \quad (5.16)$$

Using Equation 5.12, the uncertainties in k_i and h_i can be calculated from

$$\delta k_i = \sqrt{(\delta k_{\alpha_i})^2 + (\delta k_{V_{eff,i}})^2 + (\delta k_{|\vec{V}_{jet}|})^2} . \quad (5.17)$$

$$\delta h_i = \sqrt{(\delta h_{\beta_i})^2 + (\delta h_{V_{eff,i}})^2 + (\delta h_{|\vec{V}_{jet}|})^2} . \quad (5.18)$$

where, for example, $\delta k_{V_{eff,i}}$ is the uncertainty in the calculated yaw sensitivity coefficient due to the uncertainty in the effective cooling velocity. Due to the relatively large values of second derivatives associated with Equations 5.15 and 5.16 at small misalignment angles, the values of δk_{α_i} and δh_{β_i} are evaluated using Equation 5.14. Based on observations discussed in Chapter 3, an uncertainty of 1° is considered for both α_i and β_i . The remainder of the uncertainties on the right-hand side of Equations 5.17 and 5.18 are evaluated using Equation 5.13 as follows:

$$\delta k_{V_{eff,i}} = \frac{1}{k_i \sin^2 \alpha_i} \left(\frac{V_{eff,i}}{|\vec{V}_{jet}|} \right)^2 \frac{\delta V_{eff,i}}{V_{eff,i}} , \quad (5.19)$$

$$\delta h_{V_{eff,i}} = \frac{1}{h_i \sin^2 \beta_i} \left(\frac{V_{eff,i}}{|\vec{V}_{jet}|} \right)^2 \frac{\delta V_{eff,i}}{V_{eff,i}} , \quad (5.20)$$

$$\delta k_{|\vec{V}_{jet}|} = \frac{1}{k_i \sin^2 \alpha_i} \left(\frac{V_{eff,i}}{|\vec{V}_{jet}|} \right)^2 \frac{\delta |\vec{V}_{jet}|}{|\vec{V}_{jet}|} , \quad (5.21)$$

$$\delta h_{|\vec{V}_{jet}|} = \frac{1}{h_i \sin^2 \beta_i} \left(\frac{V_{eff,i}}{|\vec{V}_{jet}|} \right)^2 \frac{\delta |\vec{V}_{jet}|}{|\vec{V}_{jet}|} . \quad (5.22)$$

Using Equation 5.12, the uncertainty in $V_{eff,i}$, $\delta V_{eff,i}$, is approximated by:

$$\delta V_{eff} = \sqrt{(\delta V_{eff})_A^2 + (\delta V_{eff})_B^2 + (\delta V_{eff})_n^2 + (\delta V_{eff})_E^2} \quad (5.23)$$

where $(\delta V_{eff})_A$, $(\delta V_{eff})_B$ and $(\delta V_{eff})_E$ are found by applying Equation 5.13 to

$$V_{eff} = \left(\frac{E_i^2 - A_i}{B_i} \right)^{\frac{1}{n_i}} \quad (5.24)$$

yielding,

$$(\delta V_{eff})_A = V_{eff} \frac{1}{n_i B_i V_{eff}^{n_i}} \delta A_i, \quad (5.25)$$

$$(\delta V_{eff})_B = V_{eff} \frac{1}{n_i B_i} \delta B_i, \quad (5.26)$$

$$(\delta V_{eff})_E = V_{eff} \frac{2E_i}{n_i B_i V_{eff}^{n_i}} \delta E_i. \quad (5.27)$$

Applying Equation 5.13 to Equation 5.24, $(\delta V_{eff})_n$ is obtained as:

$$\frac{(\delta V_{eff})_n}{V_{eff}} = -\ln(V_{eff}) \frac{\delta n_i}{n_i}.$$

It is observed that the right-hand side of this expression is not dimensionless due to the $\ln(V_{eff})$ term. Therefore, $(\delta V_{eff})_n$ was determined using Equation 5.14 by calculating the difference between the values of V_{eff} , obtained using n_i and $n_i + \delta n_i$.

Substituting Equations 5.25, 5.26 and 5.27 into Equation 5.23, one obtains:

$$\frac{\delta V_{eff,i}}{V_{eff,i}} = \sqrt{\left(\frac{\delta A_i}{n_i B_i V_{eff,i}^{n_i}}\right)^2 + \left(\frac{\delta B_i}{n_i B_i}\right)^2 + \left(\frac{\delta(V_{eff,i})_{n_i}}{V_{eff,i}}\right)^2 + \left(\frac{2E_i}{n_i B_i V_{eff,i}^{n_i}} \delta E_i\right)^2}. \quad (5.28)$$

The uncertainties in A_i , B_i and n_i were obtained by studying several sets of calibration data for the four-wire probe acquired before and after obtaining each set of data for the calculation of k_i and h_i . The uncertainty in the outputs of the anemometers, δE_i , was obtained by monitoring the fluctuations in E_i at randomly chosen misalignment angles. These uncertainties in A_i , B_i , n_i , E_i and the resultant uncertainty in V_{eff} are summarized in Table 5.2.

As was discussed in Chapter 4, the calibration velocity, $|\vec{V}_{jet}|$, was obtained through Equation 4.1 as

$$|\vec{V}_{jet}| = \sqrt{2 \frac{P_d}{\rho_{air}}} \quad (5.29)$$

where P_d is the differential transducer pressure, $P_d = P_o - P_{atm}$, obtained from Equation 4.2,

$$P_d = C_t(E_t - E_{t_0}) \quad (5.30)$$

and C_t and E_t are the pressure transducer calibration coefficient and the transducer output, respectively. Substituting Equation 5.30 into Equation 5.29 and applying Equation 5.12, the uncertainty in $|\vec{V}_{jet}|$ is obtained as

$\frac{\delta A_i}{A_i}$	$\frac{\delta B_i}{B_i}$	$\frac{\delta n_i}{n_i}$	$\frac{\delta E_i}{E_i}$	$\frac{\delta V_{eff,i}}{V_{eff,i}}$
0.6%	0.2%	1%	0.17%	2%

Table 5.2: Typical uncertainties associated with the coefficients in King's law and the resulting uncertainty in the effective cooling velocity.

Figure 5.16: The four-wire probe frame of reference.

Substituting the expressions for u_i , v_i , w_i in Equation 5.32 into Jorgensen's equation and simplifying yields the effective cooling velocity for the i th wire as:

$$V_{eff,i}^2 = C_1 U^2 + C_2 V^2 + C_3 W^2 + C_4 UV + C_5 UW + C_6 VW \quad (5.33)$$

where C_1 , to C_6 , are given as:

$$\begin{aligned} C_1 &= R_{11}^2 + k_i^2 R_{21}^2 + h_i^2 R_{31}^2, \\ C_2 &= R_{12}^2 + k_i^2 R_{22}^2 + h_i^2 R_{32}^2, \\ C_3 &= R_{13}^2 + k_i^2 R_{23}^2 + h_i^2 R_{33}^2, \\ C_4 &= R_{11} R_{12} + k_i^2 R_{21} R_{22} + h_i^2 R_{31} R_{32}, \\ C_5 &= R_{11} R_{13} + k_i^2 R_{21} R_{23} + h_i^2 R_{31} R_{33}, \\ C_6 &= R_{12} R_{13} + k_i^2 R_{22} R_{23} + h_i^2 R_{32} R_{33}, \end{aligned} \quad (5.34)$$

with the indices of R indicating the row and columns of matrix R .

Defining θ and φ as the yaw and pitch angles of the local velocity vector in the

and Chew *et al.* (1988). However, the relative magnitudes of the uncertainties quoted herein are somewhat larger than those reported by these authors. Based on the information provided by these authors, this discrepancy may be explained by the difference in the replication levels used in the uncertainty analyses. Jorgensen (1971) and Chew *et al.* (1988) deduced the uncertainty levels based on repeatability tests only, which is equivalent to a first order replication, whereas in the present work an uncertainty analysis based on N th order replication was invoked.

Figure 5.12: Typical variation of uncertainties associated with k .

Figure 5.13: Typical variation of uncertainties associated with h .

Figure 5.14: Uncertainty band in k .

Figure 5.15: Uncertainty band in h .

5.1.3 Velocity Measurement

Once the four-wire probe is calibrated, it can be used for the measurement the instantaneous velocity vector in a three-dimensional flow. In this section, the procedure for extracting this quantity from simultaneously measured outputs of the anemometers (E_i , $i = 1, \dots, 4$) is described.

General Formulation

For the i th wire, the velocity components in the wire frame of reference are related to the velocity components in the probe frame of reference, U , V and W , through

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = [R]_i \cdot \begin{bmatrix} U \\ V \\ W \end{bmatrix} \quad (5.32)$$

where

$$[R]_i = \begin{bmatrix} \cos \psi_i & \sin \psi_i \cos \gamma_i & \sin \psi_i \sin \gamma_i \\ -\sin \psi_i & \cos \psi_i \cos \gamma_i & \cos \psi_i \sin \gamma_i \\ 0 & -\sin \gamma_i & \cos \gamma_i \end{bmatrix}$$

is the rotation matrix from the probe frame of reference to the frame of reference for the i th wire (Figure 5.16). As was discussed earlier, Jorgensen's equation relates u_i , v_i and w_i to V_{eff} :

$$V_{eff,i} = \sqrt{u_i^2 + k_i^2 v_i^2 + h_i^2 w_i^2}.$$

Figure 5.16: The four-wire probe frame of reference.

Substituting the expressions for u_i , v_i , w_i in Equation 5.32 into Jorgensen's equation and simplifying yields the effective cooling velocity for the i th wire as:

$$V_{eff,i}^2 = C_1 U^2 + C_2 V^2 + C_3 W^2 + C_4 UV + C_5 UW + C_6 VW \quad (5.33)$$

where C_1 , to C_6 , are given as:

$$\begin{aligned} C_1 &= R_{11,i}^2 + k_i^2 R_{21,i}^2 + h_i^2 R_{31,i}^2, \\ C_2 &= R_{12,i}^2 + k_i^2 R_{22,i}^2 + h_i^2 R_{32,i}^2, \\ C_3 &= R_{13,i}^2 + k_i^2 R_{23,i}^2 + h_i^2 R_{33,i}^2, \\ C_4 &= R_{11,i} R_{12,i} + k_i^2 R_{21,i} R_{22,i} + h_i^2 R_{31,i} R_{32,i}, \\ C_5 &= R_{11,i} R_{13,i} + k_i^2 R_{21,i} R_{23,i} + h_i^2 R_{31,i} R_{33,i}, \\ C_6 &= R_{12,i} R_{13,i} + k_i^2 R_{22,i} R_{23,i} + h_i^2 R_{32,i} R_{33,i}, \end{aligned} \quad (5.34)$$

with the indices of R indicating the row and columns of matrix R .

Defining θ and φ as the yaw and pitch angles of the local velocity vector in the

probe frame of reference (Figure 5.16), one can write

$$\begin{aligned} U &= |\vec{V}| \cos \theta \cos \varphi, \\ V &= |\vec{V}| \sin \theta \cos \varphi, \\ W &= |\vec{V}| \sin \varphi \end{aligned} \quad (5.35)$$

Substituting these into Equation 5.33 and simplifying yields,

$$V_{eff}^2 = |\vec{V}|^2 g_i(\theta, \varphi) \quad (5.36)$$

where V_{eff} is related to the measured anemometer output, E , through King's law, and $g_i(\theta, \varphi)$, the directional sensitivity function of the i th wire, is

$$\begin{aligned} g_i(\theta, \varphi) &= C_1 \cos^2 \theta \cos^2 \varphi + C_2 \sin^2 \theta \cos^2 \varphi + C_3 \sin^2 \varphi \\ &+ C_4 \sin 2\theta \cos^2 \varphi + C_5 \cos \theta \sin 2\varphi + C_6 \sin \theta \sin 2\varphi \end{aligned} \quad (5.37)$$

Uniqueness of the Solution

Willmarth (1985) used a graphical representation of the angular sensitivity of an infinitely long hot-wire to visualize how the individual sensor response and the geometrical arrangement of the sensors are related to the measurements made with a multi-wire probe. For an infinitely long hot-wire Jorgensen's equation becomes:

$$V_{eff}^2 = u^2 + w^2. \quad (5.38)$$

If the tips of all the velocity vectors which give the same effective cooling velocity are placed at a fixed point on the hot-wire, the tails of these vectors will lie on a cylinder of radius V_{eff} , centred on the hot-wire (Figure 5.17).

For a hot-wire of finite length, Jorgensen's equation can be written as:

$$\frac{u^2}{D_1^2} + \frac{v^2}{D_2^2} + \frac{w^2}{D_3^2} = 1 \quad (5.39)$$

where

$$\begin{aligned} D_1 &= V_{eff} , \\ D_2 &= \frac{V_{eff}}{k} , \\ D_3 &= \frac{V_{eff}}{h} . \end{aligned}$$

This is graphically represented by an affine contraction of a rotational ellipsoid as shown in Figure 5.18.

As was mentioned in Chapter 2, a common probe configuration for turbulence measurements in three-dimensional flows is a triple-wire probe with the wires oriented in a mutually perpendicular configuration. For this configuration, the intersection of the three cylinders corresponding to the three wires represent the location of the tail of the velocity vector which yields the correct effective cooling velocity for each of the three wires. Figure 5.19 shows that there are eight intersection points for this wire configuration, A, B, C, D, and four others hidden directly beneath these four points. These eight points are located symmetrically in the wires' frame of reference. In the case of three orthogonal hot-wires including the effect of the prongs, the intersection of three ellipsoids once again yields eight intersection points. These points are again symmetrically located with respect to the wires' frame of reference.

The validity domain is defined as the range of flow misalignments in the probe frame of reference for which the anemometer outputs correspond to a unique velocity

Figure 5.17: Location of the tails of the velocity vectors yielding the same effective cooling velocity for an infinitely long hot-wire.

Figure 5.18: Location of the tails of the velocity vectors yielding the same effective cooling velocity for a finite hot-wire.

Figure 5.19: The intersection of the three orthogonal cylinders each defined by Equation 5.38.

Figure 5.20: The validity domain for a triple orthogonal wire probe.

vector. Thus, the probe can be used to extract the velocity magnitude and direction as long as the local velocity vector lies within this domain. Figure 5.20 shows that for an orthogonal triple-wire probe, the validity domain is defined by three planes, each plane containing two of the three wires. In practice, the validity domain for such a probe is conservatively considered to be a cone of 35.26° half angle which is tangent to the three planes.

Maciejewski and Moffat (1994) noted that in a flow with turbulence intensities more than about 30%, difficulties arise in the interpretation of the data from an orthogonal triple-wire probe due to the problem described as rectification. The instantaneous velocity vector may be rectified due to the inability of the probe to distinguish the direction of the velocity vector. Essentially, rectification errors occur when the instantaneous velocity vector is out of the validity domain.

If the wires are not orthogonal, not only the intersection points of the three ellipsoids may not be symmetrically located, but they may also be located very close to each other. This means that the system of equations (Equation 5.33) may have solutions close to each other which cannot be easily separated based on geometric considerations. For example, Figure 5.21 shows three ellipsoids corresponding to three wires of the present four-wire probe. It is observed that two intersections of the three ellipsoids, points *A* and *F*, are quite close to each other compared to the proximity for the intersections of a triple orthogonal probe (Figure 5.19).

A method of circumventing this problem of multiple solutions and the resulting restrictions on the validity domain, is the use of a fourth wire. From Figure 5.22 it

Figure 5.21: The intersections of three ellipsoids represent the velocity vectors which satisfy Equation 5.36 for hot-wires no. 1, 2 and 3.

Figure 5.22: The intersections of four ellipsoids represent the velocity vectors which satisfy Equation 5.36 for four-wires.

is observed that the four ellipsoids associated with the wires of the four-wire probe intersect at only two points, point *A*, visible on the top view, and another one on the opposite side, hidden by the ellipsoids. This implies that in principle a four-wire probe can resolve the local velocity vector within a validity domain consisting of a hemisphere. It should be noted that points *B* and *C* in Figure 5.22 are intersections of only three of the ellipsoids thus do not represent possible solutions for the velocity vector.

Velocity Measurement Procedure

Equation 5.36 for $i = 1$ to 4 represents a system of four equations with three unknowns, $|\vec{V}|$, θ and φ , which collectively define the unknown velocity vector. For an analytical solution only three equations are needed. The wire which has the maximum effective cooling velocity would be the one facing the unknown velocity vector. Initially, this wire and the two neighbouring wires would be chosen to form the three equations needed for the analytical solution of Equation 5.36. Hereafter these three wires will be referred to as wires *I*, *II* and *III*, with wire *II* being the wire with the maximum effective cooling velocity. Respective summation of the left- and right-hand sides of Equation 5.36 for these three wires yields:

$$\sum_{i=I}^{III} V_{eff,i}^2 = (|\vec{V}|)^2 \sum_{i=I}^{III} g_i(\theta, \varphi). \quad (5.40)$$

Dividing Equation 5.36 for wires *I* and *III* by Equation 5.40 and rearranging,

$$\left(\sum_{i=I}^{III} V_{eff,i}^2 \right) g_I(\theta, \varphi) - (V_{eff,I}^2) \sum_{i=I}^{III} g_i(\theta, \varphi) = 0 . \quad (5.41)$$

$$\left(\sum_{i=I}^{III} V_{eff,i}^2 \right) g_{III}(\theta, \varphi) - (V_{eff,III}^2) \sum_{i=I}^{III} g_i(\theta, \varphi) = 0 .$$

With the cooling effective velocities obtained from the anemometer outputs through King's law, the system of Equations 5.41 are solved for the unknowns, θ and φ . Subsequently, $|\vec{V}|$ is found by substituting the calculated θ and φ into Equation 5.36 for wire *II*:

$$|\vec{V}| = \sqrt{\left(\frac{V_{eff,II}^2}{g_{II}(\theta, \varphi)} \right)} . \quad (5.42)$$

Several combinations of $|\vec{V}|$, θ and φ would satisfy this solution procedure based on only three wires. This can be observed in Figure 5.21. In this figure the illustrated ellipsoid sizes imply that wire 2 is facing the local velocity vector (i.e. wire 1 $\equiv$ wire *I*, wire 2 $\equiv$ wire *II* and wire 3 $\equiv$ wire *III*). The velocity vector associated with point *A* as well as point *F* would satisfy the set of equations based on these wires. There are also several other possibilities that are not visible on this figure. The results of numerical experiments suggest that for velocity vectors with misalignment angles up to 40 degrees, one would predict an incorrect velocity vector about 40 percent of the time.

The fourth wire is used to remove this ambiguity. The velocity vector obtained from the three wires facing the local velocity vector (i.e. wires *I*, *II* and *III* in this case), would be used to calculate a residual defined as:

$$\mathcal{E} = \frac{\sum_{i=1}^4 \left[\left| (|\vec{V}|)^2 \cdot g_i(\theta, \varphi) - (V_{eff_i})^2 \right| \right]}{\sum_{i=1}^4 (V_{eff_i})^2} \quad (5.43)$$

If this residual is not below a preselected minimum, $\mathcal{E}_m$, the whole solution procedure would be repeated with different initial values for θ and φ , and this would be continued until an $\mathcal{E}$ value less than $\mathcal{E}_m$ is obtained. In numerical experiments, the value of $\mathcal{E}_m$ can be set close to the machine accuracy. However, for experimental data the value of $\mathcal{E}_m$ is dictated primarily by experimental uncertainties.

Several sets of experiments were performed to verify this velocity measurement procedure. In the first set, velocity measurements were performed while the velocity magnitude was varied with the probe remaining aligned with the flow whereas in the second set of experiments the velocity magnitude was kept constant while the misalignment angle was varied. In this latter set, several different assumptions were made about the values of k_i and h_i and the results were compared.

In the first experiment, the probe was aligned with the calibration-jet. Then the outputs of the anemometers, E_i , were recorded for a series of calibration-jet velocity magnitudes ranging from 7 m/s to 35 m/sec. Calculated velocity vectors, with k_i and h_i extracted from Figures 5.4 and 5.5, agreed with the experimental ones with less than 2 degrees error in angles and 2% error in the velocity magnitude.

In the second set of experiments, the outputs of the anemometers were recorded while θ and φ were varied from -30° to 30° in 5° increments with a fixed calibration-jet velocity of 20 m/sec. At first, constant values of k_i and h_i obtained from Fig-

ures 5.4 and 5.5 at $\alpha_i = 45^\circ$ and $\beta_i = 45^\circ$, respectively, were used to calculate the velocity vectors. The results indicated large errors in the calculated velocity magnitude and direction. Subsequently the approximation of k_i and h_i was improved by considering them as functions of α_i and β_i , respectively. For this purpose third order polynomials were fitted to the k_i and h_i data in Figures 5.4 and 5.5. Assuming initial values for θ and φ , an iterative method was used to find the velocity vector, using appropriate values of k_i and h_i extracted from the fitted polynomials at each step of the iteration. The errors in the calculated velocity magnitude and direction (Figures 5.23 to 5.25) were found to be similar to the case with constant k_i and h_i . In these figures the errors ε_θ , ε_φ and ε_V are defined as

$$\begin{aligned}\varepsilon_\theta &= \theta - \theta_{\text{calculated}} , \\ \varepsilon_\varphi &= \varphi - \varphi_{\text{calculated}} , \\ \varepsilon_V &= \frac{|\vec{V}| - |\vec{V}|_{\text{calculated}}}{|\vec{V}|} .\end{aligned}\tag{5.44}$$

Apart from the generally large magnitude of these errors, at several flow misalignments abrupt changes in the error levels are observed between two neighbouring points. For example, for the neighbouring points labeled as a and b:

$$\begin{aligned}\varepsilon_{\theta_a} &= 0.5 \quad , \quad \varepsilon_{\theta_b} = -14 \text{ [deg]} \\ \varepsilon_{\varphi_a} &= 0.5 \quad , \quad \varepsilon_{\varphi_b} = 11 \text{ [deg]} \\ \varepsilon_{V_a} &= 0.5 \quad , \quad \varepsilon_{V_b} = -10 \text{ [%]}\end{aligned}$$

These abrupt changes and the resulting overall variation of the errors with flow misalignment can be explained as follows. For any given set of four effective cooling velocities, the data reduction procedure searches for an intersection point for the four

Figure 5.23: Error in calculated pitch angle; $k_i = k_i(\alpha)$; $h_i = h_i(\beta)$.

Figure 5.24: Error in calculated yaw angle; $k_i = k_i(\alpha)$; $h_i = h_i(\beta)$.

Figure 5.25: Error in calculated velocity magnitude; $k_i = k_i(\alpha)$; $h_i = h_i(\beta)$.

ellipsoids, such as point *A* in Figure 5.22. If one of the ellipsoids were to deviate slightly from its illustrated shape, point *A* would no longer be a common intersection of all four ellipsoids. For example, if the ellipsoid of wire 1 were to expand slightly, point *B* would move towards point *C* and eventually they would overlap. This point would then be the intersection of the four ellipsoids. Such a convergence to a wrong "solution" would yield sudden increase in errors as observed in Figures 5.23 to 5.25.

Recalling that the dimensions of each of these ellipsoids (D_1 , D_2 and D_3 in Figure 5.18) are functions of V_{eff} , k and h (Equation 5.39), any uncertainty in these quantities would imply uncertainty in the dimensions of the corresponding ellipsoids. Figures 5.14 and 5.15 show that the uncertainty in the values of k and h can be up to 80%. Numerical experiments showed that the abrupt change between points *a* and *b* can be eliminated by varying k and h within their respective uncertainty ranges. It should be noted that there is yet another source of uncertainty in the values of k and h , not accounted for in Figures 5.14 and 5.15. This source of uncertainty is the assumption that $k=k(\alpha)$ and $h=h(\beta)$, i.e., possible dependency of k on β and of h on α has not been accounted for. The resultant inaccuracy in k and h may be a contributing factor leading to the error distributions observed in Figures 5.23 to 5.25.

It is evident that, for the present four-wire probe, the method of directional sensitivity coefficients does not provide satisfactory accuracy for velocity measurements. The following section describes an alternative method that overcomes these difficulties associated with the method of directional sensitivity coefficients.

5.2 The Method of Directional Sensitivity Function

In the preceding section it was shown that for the present four-wire probe configuration, the method of directional sensitivity coefficients does not provide the desired accuracy for velocity measurements. The method of directional sensitivity function, described in this section, uses birational spline surfaces for $g_i(\theta, \varphi)$ instead of Jorgensen's equation. This eliminates the need for accurate determination of wire orientations, or assumptions regarding the functional forms of k and h .

5.2.1 Calibration

As for the previous method, the starting point for the calibration procedure is Equation 5.36:

$$V_{eff,i}^2 = |\vec{V}|^2 g_i(\theta, \varphi),$$

normalizing this equation,

$$\frac{V_{eff,i}^2}{g_i(0,0)} = |\vec{V}|^2 \hat{g}_i(\theta, \varphi) \quad (5.45)$$

where $\hat{g}_i(\theta, \varphi)$ is the normalized directional sensitivity function:

$$\hat{g}_i(\theta, \varphi) = \frac{g_i(\theta, \varphi)}{g_i(0,0)}. \quad (5.46)$$

Taking $V_{eff,i}/\sqrt{g_i(0,0)}$ as the quantity to be correlated to the anemometer output, E_i , yields:

$$E_i^2 = A_i + C_i \left(\frac{V_{eff,i}}{\sqrt{g_i(0,0)}} \right)^{n_i} \quad (5.47)$$

where C_i is related to the constant B_i which has previously been used in King's law, through,

$$C_i = B_i \left[\sqrt{g_i(0,0)} \right]^{n_i}.$$

The values of A_i , C_i and n_i are obtained through calibration. For this purpose, the probe axis is aligned with the calibration-jet velocity. In this configuration $V_{eff,i}/\sqrt{g_i(0,0)}$ in Equation 5.47 can be replaced by $|\vec{V}_{jet}|$. Thus, acquisition of a series of E_i versus $|\vec{V}_{jet}|$ values allows quantification of A_i , C_i and n_i . Figure 5.26 shows a typical result obtained from such a calibration test.

Once $V_{eff,i}/\sqrt{g_i(0,0)}$ is correlated against E_i , the remainder of the calibration procedure involves evaluation of $\hat{g}_i(\theta, \varphi)$ in Equation 5.45. For this purpose, the anemometer outputs E_i ($i=1, \dots, 4$) are recorded at a series of (θ, φ) combinations for a constant value of $|\vec{V}_{jet}|$. For each data point, $\hat{g}_i(\theta, \varphi)$ is calculated using

$$\hat{g}_i(\theta, \varphi) = \frac{1}{|\vec{V}_{jet}|^2} \left(\frac{E_i^2 - A_i}{C_i} \right)^{\frac{2}{n_i}}. \quad (5.48)$$

In the present experiments θ and φ were varied in the range of -30° to 30° at 5° increments for a jet velocity of 20 m/s. Based on the discrete set of values for $\hat{g}_i(\theta, \varphi)$, a continuous function was then obtained through a birational spline surface fit. Details of the spline fitting procedure, adapted from Späth (1974), are given in

Figure 5.26: Typical calibration curve for one of the wires of the four-wire probe.

Appendix D. Figure 5.27 shows the resultant directional sensitivity functions for all wires of the four-wire probe.

5.2.2 Velocity Measurement

In extracting the velocity vector from the four-wire probe data, first $V_{eff,i}/\sqrt{g_i(0,0)}$ is obtained from the anemometer outputs (E_i , $i = 1, \dots, 4$) through King's law (Equation 5.47):

$$\frac{V_{eff,i}}{\sqrt{g_i(0,0)}} = \left(\frac{E_i^2 - A_i}{C_i} \right)^{\frac{1}{n_i}} \quad (5.49)$$

Figure 5.27: Directional sensitivity functions for the wires of the four-wire probe.

For an arbitrary choice of $|\vec{V}|$, θ and φ , Equation 5.45 would generally not be satisfied, yielding a nonzero value for the residual defined as:

$$\mathcal{E}_i = \left[\frac{1}{|\vec{V}|^2} \left(\frac{V_{eff,i}}{\sqrt{g_i(0,0)}} \right)^2 - \hat{g}_i(\theta, \varphi) \right]^2. \quad (5.50)$$

The sum of this residual for all four wires,

$$\mathcal{E}_{sum} = \sum_{i=1}^4 \left[\frac{1}{|\vec{V}|^2} \left(\frac{V_{eff,i}}{\sqrt{g_i(0,0)}} \right)^2 - \hat{g}_i(\theta, \varphi) \right]^2, \quad (5.51)$$

is a function of θ , φ and $|\vec{V}|$. Based on the discussion on the validity domain for the present four-wire probe, for a given set of E_i , and thus $V_{eff,i}/\sqrt{g_i(0,0)}$ ($i=1,\dots,4$), this function would reach a minimum for a combination of $|\vec{V}|$, θ and φ which represents the local velocity vector. The following section describes the algorithm used in the search of $|\vec{V}|$, θ and φ that minimize $\mathcal{E}_{sum}$ for a given set of $V_{eff,i}/\sqrt{g_i(0,0)}$.

Residual Minimization Algorithm

In this algorithm, one seeks the minimum $\mathcal{E}_{sum}(\theta^*, \varphi^*, |\vec{V}^*|)$ of the function $\mathcal{E}_{sum}(\theta, \varphi, |\vec{V}|)$ such that

$$\mathcal{E}_{sum}(\theta^*, \varphi^*, |\vec{V}^*|) \leq \mathcal{E}_{sum}(\theta, \varphi, |\vec{V}|)$$

for all θ , φ and $|\vec{V}|$ in the calibration range. Almost all numerical methods for determining the minimum of a given non-linear function are iterative and start from an initial estimate. The initial estimate is improved by generating a sequence of new estimates using the minimization procedure,

$$\begin{pmatrix} \theta^{m+1} \\ \varphi^{m+1} \\ |\vec{V}|^{m+1} \end{pmatrix} = \begin{pmatrix} \theta^m \\ \varphi^m \\ |\vec{V}|^m \end{pmatrix} + \underbrace{\begin{pmatrix} \Delta\theta^m \\ \Delta\varphi^m \\ \Delta|\vec{V}|^m \end{pmatrix}}_{S^m} \quad (5.52)$$

where superscripts indicate the iteration level and S is the correction vector. More and Wright (1993) suggest that S^m be found using Newton's method as

$$\nabla^2 \mathcal{E}_{sum}^m \cdot S^m = -\nabla \mathcal{E}_{sum}^m \quad (5.53)$$

where $\nabla \mathcal{E}_{sum}^m$ is the gradient vector,

$$\nabla \mathcal{E}_{sum}^m = \begin{bmatrix} \frac{\partial \mathcal{E}_{sum}}{\partial \theta} \\ \frac{\partial \mathcal{E}_{sum}}{\partial \varphi} \\ \frac{\partial \mathcal{E}_{sum}}{\partial |\vec{V}|} \end{bmatrix}^m \quad (5.54)$$

and $\nabla^2 \mathcal{E}_{sum}^m$ is the Hessian matrix,

$$\nabla^2 \mathcal{E}_{sum}^m = \begin{bmatrix} \frac{\partial^2 \mathcal{E}_{sum}}{\partial \theta^2} & \frac{\partial^2 \mathcal{E}_{sum}}{\partial \theta \partial \varphi} & \frac{\partial^2 \mathcal{E}_{sum}}{\partial \theta \partial |\vec{V}|} \\ \frac{\partial^2 \mathcal{E}_{sum}}{\partial \varphi \partial \theta} & \frac{\partial^2 \mathcal{E}_{sum}}{\partial \varphi^2} & \frac{\partial^2 \mathcal{E}_{sum}}{\partial \varphi \partial |\vec{V}|} \\ \frac{\partial^2 \mathcal{E}_{sum}}{\partial |\vec{V}| \partial \theta} & \frac{\partial^2 \mathcal{E}_{sum}}{\partial |\vec{V}| \partial \varphi} & \frac{\partial^2 \mathcal{E}_{sum}}{\partial |\vec{V}|^2} \end{bmatrix}^m \quad (5.55)$$

Experimental Validation

In order to test this velocity measurement method, the calibration jet velocity was set to a constant value and the outputs of the anemometers were recorded while θ and φ were varied from -30° to 30° in 5° increments. Within this range, the method of directional sensitivity function was able to extract the flow direction with less than 2° error and the velocity magnitude within 2% of its true value. Numerical experiments with several tension factors ($p = 0$ to $p = 8$) for the birational spline fits to $\hat{g}_i(\theta, \varphi)$ showed no noticeable effect of p on the calculated velocity vector.

Since this data reduction procedure involves many iterations and uses table look-up algorithms, it is impractical to perform an analytical uncertainty analysis. Alternatively, the uncertainty can be quantified by computing the variation in the calculated velocity vector due to an uncertainty in E_i , δE_i ¹. Several numerical experiments were performed to study the effect of a range of uncertainty values, δE_i , on the resultant θ , φ and $|\vec{V}|$. Figures 5.28 to 5.30 show typical uncertainties in the computed θ , φ and $|\vec{V}|$ for an uncertainty of $\delta E_1 = 6 \text{ mV}$ in E_1 ($\delta E_1/E_1$ of about 0.2%) using both the method of directional sensitivity coefficients and the method of directional sensitivity function.

From these figures it is observed that the method of directional sensitivity function is considerably less sensitive to uncertainties in E_i than the method of directional sensitivity coefficients. This difference was observed to be even more pronounced for larger values of δE_i .

¹This procedure is essentially the use of Equation 5.14 to calculate the uncertainty.

Figure 5.28: Error in computed θ due to an assumed error in E_1 ; a. the method of directional sensitivity function, b. the method of directional sensitivity coefficients; $\circ \delta E = 6$ mV, $\bullet \delta E = -6$ mV, $\theta = 25^\circ$, $|\vec{V}_{jet}| = 30$ m/s.

Figure 5.29: Error in computed ϕ due to an assumed error in E_1 ; a. the method of directional sensitivity function, b. the method of directional sensitivity coefficients; $\circ \delta E = 6$ mV, $\bullet \delta E = -6$ mV, $\theta = 25^\circ$, $|\vec{V}_{jet}| = 30$ m/s.

Figure 5.30: Error in computed $|\vec{V}|$ due to an assumed error in E_1 ; a. the method of directional sensitivity function, b. the method of directional sensitivity coefficients; $\circ \delta E = 6$ mV, $\bullet \delta E = -6$ mV, $\theta = 25^\circ$, $|\vec{V}_{jet}| = 30$ m/s.

Chapter 6

Comparison of Four-Wire Probe and Seven-Hole Pressure Probe Characteristics

The mean velocity vector in a three-dimensional steady flow can be measured with multi-wire hot-wire probes as well as multi-hole pressure probes. In this chapter the mean velocity measurement accuracy of the four-wire probe designed and constructed in the present work is compared with that of an existing seven-hole pressure probe.

6.1 Seven-Hole Pressure Probe

The overall dimensions of the seven-hole pressure probe are given in Figure 6.1. The calibration method first proposed by Gerner and Maurer (1981) was used to calibrate the probe in the calibration-jet described in Chapter 4 for flow misalignments up to 50° relative to the probe axis.

Figure 6.1: Seven-hole pressure probe geometry.

The calibration was split into two regimes: a low-angle regime for which flow misalignments with respect to the probe axis remained within 20° and a high-angle regime with flow misalignments greater than 20° . In the low-angle regime the flow misalignment is expressed in a tangential co-ordinate system whereas in the high-angle regime it is more convenient to use a polar co-ordinate system (Figure 6.2). The calibration in the high-angle regime is performed on the basis of *sectors* with each sector being centred at one of the six side-holes of the probe. Details on these sectors and the overall calibration procedure are presented in Appendix E.

6.2 Comparison of the Seven-Hole Probe with the Four-Wire Probe

The main advantage of hot-wire probes over pressure probes is their ability to measure velocity fields experiencing temporal variations. As is discussed in Chapter A, the upper frequency limit of velocity fluctuations which can be measured with a hot-wire probe depends on the hot-wire sensor and the control circuit. In the present work the four hot-wires of the four-wire probe were successfully tested up to a cut-off frequency of 10 kHz , with the control circuitry being the primary limiting factor.

Pressure probes are preferred over hot-wire probes mainly due to their ability to measure total pressure as well as velocity. Once a pressure probe is calibrated, it can be used for lengthy periods without a need for recalibration whereas a hot-wire probe has to be recalibrated practically before and after each set of measurements

due to relatively rapid contamination and slight drifts in the bridge output voltage. Simplicity, ruggedness and absence of expensive control circuitry are other factors that make a pressure probe a preferable instrument for the measurement of mean-flow properties.

6.2.1 Uncertainty in the Measurement of Velocity

As was discussed in Chapter 5, the uncertainty associated with the four-wire probe measurements is estimated at $\pm 2^\circ$ for the flow direction, and $\pm 2\%$ for the local velocity magnitude at flow misalignments of up to 30° relative to the probe axis. Döbbeling *et al.* (1990a) showed that the uncertainty in the measured flow direction for a similar four-wire probe configuration can be as high as about $\pm 6^\circ$ at flow misalignments of about 60° relative to the probe axis. This increase in uncertainty was attributed to the increased aerodynamic interference of the prongs on the hot-wire sensors.

The uncertainty in the flow direction and velocity magnitude measured by a seven-hole pressure probe were conservatively estimated at $\pm 1.5^\circ$ and $\pm 2\%$, respectively (Yaras, 1990). In this respect, within 30° flow misalignment, the seven-hole probe and the four-wire probe appear to measure the local velocity vector with comparable accuracy. However, as discussed in the next section, Reynolds number effects tend to increase the uncertainty associated with the seven-hole probe measurements.

Figure 6.2: Tangential and polar co-ordinate systems for the seven-hole pressure probe.

6.2.2 Effects of Reynolds Number on the Seven-Hole Probe Measurements

Effects of Reynolds number on the probe calibration are accounted for in the calibration of multi-wire probes whereas multi-hole pressure probes are typically calibrated at a single probe Reynolds number. In order to study the effects of Reynolds number on the seven-hole pressure probe calibration characteristics, the probe was calibrated at probe Reynolds numbers of 2130, 3330, 4550 and 5590 where the probe Reynolds number is defined as

$$Re_p = \frac{|\vec{V}_{jet}| d}{\nu}$$

with d being the probe tip diameter and ν the kinematic viscosity of the fluid. In the low-angle regime, up to 20° of misalignment, calibration data were acquired for all combinations of yaw and pitch misalignments with 5° increments. Complete

calibration of the low angle regime revealed the extent of the symmetry of the probe tip. Hence, only Sector 5 (centred at pressure port No. 5) of the six high angle sectors was considered for the high angle regime. Calibration of this sector involved measurements within 20° to 50° of misalignment at 5° increments in the pitch and roll directions (Figure 6.2).

Figures 6.3 and 6.4 show the typical effects of Reynolds number on the measured flow misalignment and velocity magnitude in the low-angle regime. From Figure 6.3 it is observed that in this misalignment range the effect of Reynolds number on the measured flow direction is negligible, whereas Figure 6.4 illustrates a reduction of about 3% in the measured velocity magnitude as the Reynolds number is reduced from 5590 to 2130.

The effects of Reynolds number on velocity magnitude and flow direction in the high-angle regime are presented in Figures 6.5 to 6.7. In this angle range a monotonic change in the effect of Reynolds number was observed as the pitch angle was varied from $\theta = 25^\circ$ to 50° . Therefore, for conciseness, only the data at these extremes are presented. The effect of Reynolds number on the measured pitch angle illustrated in Figure 6.5, is seen to become more pronounced with increasing flow misalignment. At $\theta = 50^\circ$ variation of Reynolds number within the range of 2130 to 5590 can lead to an error up to 2° in the measured pitch angle. From Figure 6.6 negligible influence of Reynolds number on the measured roll angle is observed. Finally, Figure 6.7 shows that, for the range of Reynolds number investigated, the measured velocity magnitude in the high-angle regime can be underestimated by up

to about 5%.

Dominy and Hodson (1993) studied the effect of Reynolds number on the calibration of five-hole pressure probes for Reynolds numbers ranging from 7×10^3 to 80×10^3 . In that work, the five-hole probe was calibrated for flow misalignments up to 20° using a calibration method similar to that of the seven-hole probe in the low-angle regime. The results showed effects of Reynolds number on the measured velocity magnitude and flow direction that are similar to those in the present work.

In view of the present results, the uncertainty in the flow direction and velocity magnitude measured by the seven-hole pressure probe is estimated at 2.5° and 5.5%, respectively. This leads to the conclusion that the overall uncertainty associated with the seven-hole probe and the four-wire probe is similar, with the hot-wire probe providing a slightly better accuracy in the measurement of velocity magnitude at large flow misalignments.

Figure 6.3: Effect of Reynolds number on the measured flow misalignment in the low-angle regime, $\alpha = 15^\circ$.

Figure 6.4: Effect of Reynolds number on the measured velocity magnitude in the low-angle regime, $\alpha = 15^\circ$.

Figure 6.5: Effect of Reynolds number on the measured flow misalignment (polar co-ordinate system) in the high-angle regime; a. $\theta = 25^\circ$, b. $\theta = 50^\circ$.

Figure 6.6: Effect of Reynolds number on the measured flow misalignment (polar co-ordinate system) in the high-angle regime; a. $\theta = 25^\circ$, b. $\theta = 50^\circ$.

Figure 6.7: Effect of Reynolds number on the measured velocity magnitude in the high-angle regime; a. $\theta = 25^\circ$, b. $\theta = 50^\circ$.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

A four-wire hot-wire probe for the measurement instantaneous velocity vector in three-dimensional flows was designed and constructed. Subsequently, a suitable calibration and measurement scheme for this probe was established.

The four-wire probe consists of four tungsten wires arranged in the form of a pyramid at about 45° angle relative to the probe axis. The ends of the wires are copper plated, yielding a cylindrical measuring volume of approximately 3.3 *mm* in diameter and 0.9 *mm* in height. The probe is intended to be used in the measurement of the unsteady, three-dimensional flow in a large-scale curved diffuser of strong curvature. In-house construction of the probe has provided the flexibility of tailoring the probe characteristics for this particular application. In-house development efforts have also yielded considerable savings: the probe was constructed at a cost of about 14% of the current purchase price of a DANTEC triple-wire probe

Model 55P91. In-house copper plating of the wire ends' resulted in a finished cost for the hot-wire sensors that is less than 10% of the cost of commercially available gold-plated hot-wire sensors.

Two methods of calibration/data reduction were considered for the probe: the method of directional sensitivity coefficients and the method of directional sensitivity function. In the method of directional sensitivity coefficients the values of yaw and pitch sensitivity coefficients for this probe were found to be dominant functions of yaw and pitch angle, respectively; negligible effect of velocity magnitude on these coefficients was observed. The yaw sensitivity coefficient was found to be considerably influenced by errors in wire inclination particularly at small yaw angles. The effect of errors in wire orientations on the pitch sensitivity coefficient was considerably less pronounced.

The uncertainty in the evaluated yaw and pitch sensitivity coefficients was shown to reach values as high as 80% at low flow misalignment. Such large uncertainties associated with the directional sensitivity coefficients resulted in the method of directional sensitivity coefficients not providing satisfactory accuracy for velocity measurements with the present four-wire probe.

The method of directional sensitivity function was found to provide accurate velocity measurements with relatively small sensitivity to experimental uncertainties. This method eliminates the need for accurate determination of wire orientations, or assumptions regarding the functional forms of directional sensitivity coefficients.

Comparison of seven-hole pressure probe and four-wire probe characteristics showed that in steady flows the probes measure the velocity magnitude and direction with comparable accuracy within their calibration range.

7.2 Recommendations

In the present study, the calibration methods were evaluated for measurement of the mean velocity in a steady flow by manually recording the output signal of each wire. The performance of the four-wire probe for the measurement of turbulence quantities remains to be examined. In particular, the effects of the spatial resolution of the probe and the blockage introduced by the prongs on turbulence measurements should be investigated. For the data reduction methods reviewed in this thesis, measurement of turbulence quantities will require high-speed, simultaneous recording of the output signals of the four hot-wires.

The wire plating setup was originally designed for plating one wire at a time. In order to increase the time efficiency of the plating process, a larger plating setup should be designed to accommodate several wires during each plating process.

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Appendices

Appendix A

Theory of Hot-Wire Anemometry

Hot-wire anemometry is used to measure mean velocity and turbulence. To obtain accurate measurements over a given frequency band of temporal variations, one must fully understand the operation of the anemometer circuit and the aerodynamics/heat transfer of the particular probe being used. In this chapter these basics of hot-wire anemometry are reviewed.

A.1 Heat Transfer

When a wire is exposed to a fluid stream of lower temperature, heat is removed from the wire by convection. Heat is generated in the wire by passing an electrical current through it. The heat balance relationship for such a wire is:

$$R_w I^2 - mc_w \frac{dT_w}{dt} - Q = 0 \quad (\text{A.1})$$

where R_w , m and c_w are the electrical resistance, mass and specific heat of the wire, respectively. T_w is the wire temperature, I is the current passing through the wire

and Q is the rate of heat loss from the hot-wire to the fluid. The electrical resistance of the hot-wire is obtained from:

$$R_w = \frac{\rho_r l}{A} \quad (\text{A.2})$$

where ρ_r is the electrical resistivity of the wire material, l is the length and A is the cross section area of the wire. As mentioned in Chapter 2, for a given wire diameter and length, R_w varies linearly with the wire temperature (Equation 2.1). This is due to variation of ρ_r with temperature as:

$$\rho_r|_{T_w} = \rho_r|_{T_0} [1 + \alpha_r(T_w - T_0)] \quad (\text{A.3})$$

with T_0 being an arbitrary reference temperature.

The heat transfer from the hot-wire, Q , is the sum of conduction, radiation and convection terms:

$$Q = Q_{\text{conduction}} + Q_{\text{radiation}} + Q_{\text{convection}} \quad (\text{A.4})$$

Heat conduction in a hot-wire occurs in both radial and axial directions. The radial temperature gradient within the wire is negligible compared to the axial temperature gradient¹. Hence, the radial heat conduction within the wire can be neglected in comparison to the axial conduction.

The net rate at which heat is conducted out of a hot-wire element shown in Figure A.1, $Q_{\text{conduction}}$, is given as:

$$Q_{\text{conduction}} = -A \frac{\partial}{\partial x} \left(k_w \frac{\partial T_w}{\partial x} \right) dx \quad (\text{A.5})$$

¹Champagne *et al.* (1967) quoted the radial temperature gradient for a typical hot-wire to be less than 2% of the axial temperature gradient.

Figure A.1: Differential element of a hot-wire.

where k_w is the thermal conductivity of the hot-wire. The net rate of heat transfer out of the element by radiation is:

$$Q_{\text{radiation}} = \pi d \sigma \epsilon (T_w^4 - T_\infty^4) dx \quad (\text{A.6})$$

where d is the hot-wire diameter, σ is the Stefan-Boltzmann constant, ϵ is the emissivity of the sensor material and T_∞ is the surrounding fluid temperature. Finally, heat convection from the element, $Q_{\text{convection}}$, is obtained from:

$$Q_{\text{convection}} = \pi d h (T_w - T_\infty) dx \quad (\text{A.7})$$

where h is the coefficient of convective heat transfer.

Substituting Equations A.2, A.4, A.5, A.6, and A.7 into Equation A.1 yields the following differential equation for the heat balance in a hot-wire:

$$\frac{I^2 \rho_r}{A} - \rho_w c_w A \frac{\partial T_w}{\partial t} + A \frac{\partial}{\partial x} \left(k_w \frac{\partial T_w}{\partial x} \right) - \pi d h (T_w - T_\infty) - \pi d \sigma \epsilon (T_w^4 - T_\infty^4) = 0 \quad (\text{A.8})$$

where ρ_w is the density of the hot-wire. For a typical hot-wire, Equation A.8 can be simplified considerably.

Heat conduction loss from a hot-wire to its prongs occurs due to the temperature gradient along the wire close to the prongs. As was noted in Chapter 2, for a typical hot-wire with an aspect ratio of about 200 or more, heat conduction to the prongs can be considered negligible in comparison to the heat convection from the wire. Furthermore, radiation heat transfer from a hot-wire, compared with the convection term, is usually negligible. With these two assumptions Equation A.8 reduces to:

$$\rho c_w A \frac{dT_w}{dt} = \frac{I^2 \rho_r}{A} - \pi dh(T_w - T_\infty) . \quad (\text{A.9})$$

Heat convection from a hot-wire includes both free and forced convection regimes. As was noted in Chapter 2, free convection can be neglected if

$$Re > Gr^{\frac{1}{3}} ,$$

where Re is the Reynolds number based on fluid velocity and wire diameter and Gr is the Grashof number. Based on this criterion, Blackwelder (1981) showed that for a typical hot-wire operating in air, free convection can be neglected for velocities greater than about 0.2 m/sec. Forced heat convection from a hot-wire of finite length can be approximated by Equation 2.4,

$$Nu \left(\frac{T_f}{T_\infty} \right)^{0.17} = A_1 + B_1 Re^n \quad (\text{A.10})$$

where A_1 , B_1 , n are calibration constants and T_f is the film temperature. The Nusselt number is defined as:

$$Nu = \frac{hd}{k_f} \quad (\text{A.11})$$

where k_f is the thermal conductivity of the fluid evaluated at the film temperature.

The Reynolds number is calculated as:

$$Re = \frac{V_{eff}d}{\nu_f} \quad (A.12)$$

where V_{eff} is effective cooling velocity and ν_f is the kinematic viscosity at the film temperature. As was discussed in Chapter 2, for a hot-wire of finite length, V_{eff} is related to the normal, tangential and binormal components of the velocity vector through Jorgensen's equation,

$$V_{eff} = \sqrt{u^2 + k^2v^2 + h^2w^2}$$

where k and h are empirical directional sensitivity coefficients.

Using the expressions for the coefficient of convective heat transfer from Equation A.10, and for ρ_r from Equation A.3, Equation A.9 can be written as:

$$\frac{dT_w}{dt} = \mathcal{F}(I, V_{eff}, T_w) . \quad (A.13)$$

Equation A.13 shows that at a given operating condition, the fluctuation in the wire temperature is due to fluctuations in either the current or the effective cooling velocity. Thus, to be able to solve Equation A.13 for V_{eff} , an additional constraint must be imposed. The additional constraint is provided by maintaining either a constant current through the hot-wire or a constant wire temperature.

A.1.1 The Static Response of a Hot-Wire

In constant-temperature hot-wire anemometry, the wire temperature is kept constant through an electronic circuitry, i.e. the anemometer. For a wire of finite

length l , Equation A.9 can be written as:

$$R_w I^2 = \pi l d h (T_w - T_\infty) .$$

Substituting I from Ohm's law,

$$I = \frac{E}{R_w} ,$$

and solving for h yields

$$h = \frac{E^2}{\pi R_w l d (T_w - T_\infty)} , \quad (\text{A.14})$$

where E is the voltage drop across the hot-wire. The Nusselt number is calculated from Equation A.11 as:

$$Nu = \frac{E^2}{\pi R_w l k_f (T_w - T_\infty)} . \quad (\text{A.15})$$

Substituting Equations A.15 and A.12 into Equation A.10 yields:

$$\frac{E^2}{R_w (T_w - T_\infty)} \left(\frac{T_f}{T_\infty} \right)^{0.17} = \pi l k_f (A_1 + B_1 \left(\frac{d}{\nu_f} \right)^n V_{eff}^n) . \quad (\text{A.16})$$

Since the temperature of the hot-wire is assumed constant, all terms in Equation A.16 are constant except E and V_{eff} . Hence, Equation A.16 can be written as:

$$E^2 = A + B V_{eff}^n \quad (\text{A.17})$$

where A , B and n are dimensional constants to be evaluated through calibration. Equation A.17, referred to as King's law, is widely used for the calibration of hot-wire probes operating in the constant-temperature mode.

In the derivation of Equation A.17, it was assumed that natural convection is negligible. In some applications, where the local velocity magnitude is small, this may introduce significant error. Wu and Bose (1994) examined alternatives for Equation A.17 and concluded that for velocities below 2 *m/sec*,

$$E^2 = A + B V_{eff}^n + C V_{eff}^{2n}$$

provides better accuracy than King's law.

A.1.2 The Dynamic Response of a Hot-Wire

When contemplating the use of a measuring system in a turbulent flow, it is essential to consider the dynamic response of the measuring system as well as its static behaviour. The speed at which a system responds to velocity fluctuations determines the highest frequency of velocity fluctuations it can measure. In constant-temperature anemometry this depends on two factors. The frequency response of the electronic circuitry which maintains the sensor at constant-temperature, and the dynamic response of the wire when there is no electronic compensation. The dynamic response of the wire is discussed in this section and the overall frequency response of the system will be discussed later.

Dynamic analysis of a heated wire without some simplifying assumptions is very difficult. Fortunately, for a typical size and operating condition of a hot-wire, the instantaneous rate of convective heat loss at a given velocity is well approximated by the steady-state rate of heat loss at the same velocity. This *quasi-steady* ap

proximation can be justified by comparing the various time scales relevant to the unsteadiness of the flow around the wire. Corrsin (1963) presented these time scales as follows:

$$\tau_1 \equiv \frac{d}{\bar{V}} \quad [\approx 2.5 \times 10^{-7} \text{ sec}]$$

is the time for a fluid particle to sweep past the wire, where $\bar{V}$ is the mean flow velocity.

$$\tau_2 \equiv \frac{d^2}{\nu} \quad [\approx 4 \times 10^{-7} \text{ sec}]$$

and

$$\tau_3 \equiv \frac{d^2}{\mathcal{D}} \quad [\approx 2.8 \times 10^{-6} \text{ sec}]$$

are the times for a shear disturbance and a thermal disturbance, respectively, to diffuse through the viscous layer around the wire. $\mathcal{D}$ is the thermal diffusivity of the fluid defined as:

$$\mathcal{D} = \frac{k_f}{\rho_f c_p}$$

where c_p is the specific heat at constant pressure for the fluid. Finally,

$$\tau_4 \equiv \frac{d^2}{\mathcal{D}'} \quad [\approx 2.5 \times 10^{-6} \text{ sec}]$$

is the time for radial conduction heat transfer in the wire, where $\mathcal{D}'$ is thermal diffusivity of the wire material. For the quasi-steady assumption to be valid, all four time scales should be less than the smallest time scale encountered by the hot-wire. In a turbulent flow, the smallest characteristic time imposed by the flow on the wire

would be the passage time of the Kolmogorov microscale²

$$\tau_k \equiv \bar{V}^{-1} \left(\frac{\nu^3}{\bar{\varepsilon}} \right)^{\frac{1}{4}} \quad [\approx 1 \times 10^{-5} \text{ sec}]$$

where $\bar{\varepsilon}$ is the mean viscous-dissipation rate. The numbers in brackets are given by Comte-Bellot (1976) for a $2.5 \mu m$ platinum wire with $\bar{V}=10 \text{ m/s}$. From these numbers, it is observed that τ_k is considerably larger than τ_1 , τ_2 , τ_3 and τ_4 , hence, the quasi-steady assumption is valid. Therefore, hot-wire calibration under steady conditions is sufficient for the study of the dynamic response of a hot-wire.

With the quasi-steady assumption, Equation A.13 can be used to derive an expression for the dynamic response of a hot-wire. This is done by decomposing each parameter in Equation A.13 into a mean value, denoted by an over bar, and a fluctuating part, denoted by prefix Δ .

$$T_w = \bar{T}_w + \Delta T_w, \quad V_{eff} = \bar{V}_{eff} + \Delta V_{eff}, \quad I = \bar{I}_w + \Delta I.$$

Expanding Equation A.13 into a Taylor series and neglecting second order and higher terms, Blackwelder (1981) derived the dynamic response of a hot-wire as:

$$M \frac{d}{dt} \Delta T_w + \Delta T_w = a \Delta I + b \Delta V_{eff} \quad (\text{A.18})$$

where a and b are constants. M is the time constant of the wire,

$$M = \left(\frac{c_w \rho_w}{\rho_r \alpha_r} \right) \left(\frac{1}{4} \pi d^2 \right)^2 \left(\frac{a_r}{\bar{T}^2} \right) \quad (\text{A.19})$$

²The Kolmogorov microscale is a measure of the distance over which the viscosity of the fluid is effective in dissipating the velocity fluctuations

with a_r being the hot-wire overheat ratio defined in Section 2.2 .

In Equation A.19, the first, second and third group of parameters on the right hand side represent the effects of wire material, wire geometry and operating condition on the time constant of the hot-wire, respectively. It is observed that the time constant depends strongly upon the wire diameter. However, it should be emphasized that some of these parameters are implicitly related to each other. For example, for a fixed overheat ratio, the diameter cannot be decreased without decreasing the current. Using King's heat transfer relation for heat convection, (Equation 2.3), Corrsin (1963) showed that at high Ve_{eff} , the time constant, M , varies with $d^{1.5}$.

From a practical point of view, Equation A.19 cannot be used to accurately calculate the time constant of a hot-wire because of the simplifying assumptions made to obtain this equation. Nevertheless, it provides insight into the effects of different parameters on the time constant of a hot-wire. Experimentation appears to be the best way of quantifying the time constant of a hot-wire.

A.2 Constant-Temperature Anemometry

In constant-temperature anemometry the resistance of a hot-wire is kept virtually constant through electronic circuitry. For a given wire, the wire resistance is only a function of wire temperature (Equation 2.1),

$$R_w = R_0 [1 + \alpha_r (T_w - T_\infty)] \quad (A.20)$$

Thus keeping the resistance constant yields a constant wire temperature.

In constant-temperature anemometry, in principle, Equation A.9 would reduce to an algebraic equation since $dT/dt = 0$. Hence the problem caused by thermal lag of the wire would be suppressed. This implies that there would be no limit for the frequency response of the system. However, in reality, the constant-temperature anemometer does not hold the wire temperature exactly constant. The wire resistance, which is directly related to the wire temperature, has to vary by a small amount to provide the necessary feedback signal to maintain the operation of the anemometer. Furthermore, time constant of the feed-back circuitry, although much less than the time constant of the wire, affects the overall frequency response of a constant-temperature anemometer.

In the following section, the basic components and the frequency response of a constant-temperature anemometer are reviewed.

A.2.1 Basic Circuitry

Figure A.2 shows a typical constant-temperature anemometer circuit in schematic form where the hot-wire is placed on one leg of a Wheatstone bridge. The total current passing through the bridge, I , is divided into i_1 and i_2 , each passing through one leg of the bridge, where

$$i_1 = \frac{E}{R_1 + R} , \quad i_2 = \frac{E}{R_2 + R_3} .$$

Figure A.2: Schematic of a constant-temperature anemometer circuit with provision for a test signal e_t ; adapted from Blackwelder (1981).

Dividing i_1 by i_2 yields

$$\frac{i_1}{i_2} = \frac{R_2 + R_3}{R_1 + R} . \quad (\text{A.21})$$

The Wheatstone bridge is balanced if

$$e_1 = e_2 \quad (\text{A.22})$$

where e_1 and e_2 are the voltage drop across the hot-wire and resistance R_3 , respectively. Using Ohm's law, Equation A.22 yields

$$\frac{i_1}{i_2} = \frac{R_3}{R} .$$

Substituting i_1/i_2 from Equation A.21 into this equation and rearranging yields

$$R R_2 = R_1 R_3 . \quad (\text{A.23})$$

The hot-wire overheat ratio is defined in Equation 2.2 as

$$a_r = \frac{R}{R_0} - 1$$

where R_0 is the resistance of the hot-wire at ambient temperature. Substituting R from Equation A.23 yields

$$a_r = \frac{R_3}{R_2/R_1} \frac{1}{R_0} - 1.$$

From this equation it is observed that, for a fixed bridge ratio (R_2/R_1), the overheat ratio of the hot-wire is increased by increasing resistance R_3 .

The amplifier is an operational amplifier which amplifies the voltage difference between the two input terminals identified by the (+) and (-) sign³. The dynamic response of the amplifier is modeled (Weidman and Browand 1975) as:

$$\mu \frac{d}{dt} \Delta E + \Delta E = -A_0 \Delta(e_2 - e_1) \quad (\text{A.24})$$

where A_0 is the DC gain of the amplifier and μ is its time constant. The basic response of an amplifier is expressed by its gain-bandwidth product defined by $A_0/2\pi\mu$, which is typically 10^6 to 10^7 Hz.

The frequency response of the anemometer can be measured by means of a suitable electric test signal e_t which is fed into the bridge through resistance R_t . To isolate the bridge from the test signal generator, R_t is chosen to be large compared with the bridge resistances. Square-wave and sinusoidal test signals are commonly used for this purpose.

³The (+) and (-) signs do not mean that one terminal has to be kept positive with respect to the other, they just show the relative phase of the output

For a hot-wire operating on a balanced bridge, any instantaneous change in the fluid velocity, V , would tend to change the hot-wire temperature and hence the wire resistance, R . This change in R would disturb the bridge balance causing a differential voltage $e_1 - e_2 \neq 0$ input to the amplifier. This would result in a change in the amplifier's output that is inversely proportional to the change in R . The altered output of the amplifier, I_0 , feeding back into the top of the bridge would then restore the hot-wire's resistance, and thus its temperature to its original value.

For a balanced Wheatstone bridge, the output of the amplifier becomes zero, hence I_0 would be zero and no operating point would be established. Thus, an offset voltage must be provided to establish a mean current through the hot-wire, as indicated in Figure A.2. Blackwelder (1981) showed that the offset voltage can be considered as an additional input to the amplifier as

$$\epsilon_{offset} = \frac{\delta R_2 R}{(R + R_1)(R_2 + R_3)} E$$

where $\delta \ll 1$ is an *unbalance* parameter. Although the bridge is assumed to operate in a balanced condition, the effect of the offset voltage is similar to maintaining the bridge with a slight unbalance given by

$$\delta = \frac{R_2 R - R_1 R_3}{R_2 R} . \quad (A.25)$$

The presence and magnitude of the offset voltage has further implications in terms of the frequency response of the anemometer which is reviewed in the next section.

A.2.2 Frequency Response of the Anemometer

Over the years the frequency response of the constant-temperature anemometer has been analyzed at varying degrees of complexity. Perry and Morrison (1971) studied the effects of amplifier offset voltage, bridge unbalance and finite amplifier gain on the stability and frequency response of the system through a zero order response for the amplifier. Weidman and Browand (1975) and Blackwelder (1981) used a first order response for the amplifier to obtain the general differential equation governing the output response of the anemometer. Freymuth (1977) accounted for second order responses in the electronic amplifier which yields a third order differential equation for the general response of a constant-temperature anemometer. The following derivation of the frequency response of a constant-temperature anemometer has been adapted from Blackwelder (1981).

The voltage difference at the amplifier input is

$$(e_1 - e_2) = \frac{R_2 R - R_1 R_3}{(R + R_1)(R_2 + R_3)} E + e_{offset} + g_t e_t \quad (\text{A.26})$$

where g_t is the attenuation due to R_t . Decomposing the voltages and the wire resistance into mean and fluctuating components, Equation A.26 becomes

$$\Delta(e_1 - e_2) = \frac{\bar{E} R_1}{(\bar{R} + R_1)^2} \Delta R + \frac{\delta \bar{R} R_1}{(\bar{R} + R_1)^2} \Delta E + g_t \Delta e_t. \quad (\text{A.27})$$

Recalling the dynamic response equation for a hot-wire (Equation A.18)

$$M \frac{d}{dt} \Delta T_w + \Delta T_w = a \Delta I + b \Delta V_{eff},$$

and relating the wire temperature to wire resistance (Equation A.20), the dynamic response of the wire is obtained as

$$M \frac{d}{dt} \Delta R + \Delta R = \alpha_r (a \Delta i_1 + b \Delta V_{eff}) . \quad (\text{A.28})$$

where α_r is the temperature coefficient of resistivity. Equations A.24, A.27 and A.28 are combined to yield the relationship between the output of the anemometer E and the input variables V_{eff} and e_t .

$$\begin{aligned} \frac{d^2}{dt^2} \Delta E + 2\zeta\omega_0 \frac{d}{dt} \Delta E + \omega_0^2 \Delta E \\ = \omega_0^2 \left\{ S_u \Delta V_{eff} + S_t \left[M \frac{d}{dt} \Delta e_t + \left(1 + \frac{2a_r \bar{R}}{\bar{R} + R_1} \right) \Delta e_t \right] \right\} \end{aligned} \quad (\text{A.29})$$

where ω_0 is the characteristic frequency, ζ is the damping coefficient, S_u and S_t are velocity - and test signal - sensitivity coefficients, respectively. The characteristic frequency is obtained from

$$\omega_0 = \left(\frac{2a_r K_0}{\mu M} \right)^{\frac{1}{2}} \quad (\text{A.30})$$

where

$$K_0 = \frac{A_0 \bar{R} R_1}{(\bar{R} + R_1)^2} . \quad (\text{A.31})$$

It is observed that the frequency response of the anemometer increases with decreasing M and increasing K_0/μ , where M is the time constant of the hot-wire and K_0/μ is proportional to the gain-bandwidth product of the amplifier. As previously noted, decreasing the diameter of the hot-wire is an effective way of reducing M . The diameter of the hot-wire, however, cannot be reduced below some certain value due to the wire breakage problem. Hence, to increase the frequency response of the anemometer, amplifiers with large gain-bandwidth products are required.

Figure A.3: Typical frequency response of a constant-temperature anemometer to velocity fluctuations; adapted from Blackwelder (1981).

For a typical hot-wire anemometer, the time constant of the amplifier is substantially lower than the time constant of the wire. In that case, the damping coefficient can be written as

$$2\zeta = (1 + K_0\delta) \left(\frac{M}{2K_0\mu a_r} \right)^{\frac{1}{2}}. \quad (\text{A.32})$$

From this equation it is observed that for a given amplifier and hot-wire, the response of the system depends upon the value of the unbalance parameter. Figure A.3 shows the effect of ζ on the response of a constant-temperature anemometer where ΔE_{ref} is the output of the anemometer when $f \rightarrow 0$. From this figure it is observed that a flat frequency response with maximum bandwidth dictates that ζ , and hence δ , be

set to a critically damped position. As was shown before, the unbalance parameter can be altered by changing the offset voltage. The proper value for e_{offset} is found by frequency response testing of the anemometer.

A.2.3 Frequency Response Test

The frequency response of an anemometer can be analyzed by means of the test signal e_t . The electric test is based on the assumption that heating and cooling of the hot-wire by varying the fluid velocity is thermodynamically identical to heating and cooling of the sensor by varying the heating current. Square-wave and sinusoidal signal are commonly used to study the frequency response of an anemometer (Frey-muth (1967, 1977) and Weidman and Browand (1975)).

In the square-wave test, a square wave is fed into the anemometer and the response of the system to the front or back step of the signal is observed on an oscilloscope. Such a signal adds a small-amplitude square wave to the sensor heating current. When the square wave raises the current through the wire, the feedback amplifier decreases its output current to maintain the bridge balance. A sudden decrease in the fluid velocity would have the same effect. Alternatively, when the square wave reduces the current through the wire, the feedback amplifier increases its output current to bring the sensor resistance back to its original value. Figure A.4 shows the typical output of a constant-temperature anemometer during a square wave test. Using the dynamic response equation of the anemometer, Freymuth (1977) showed that the cut-off frequency of the anemometer can be calculated

Figure A.4: Typical response of a constant-temperature anemometer to a square-wave signal.

from,

$$f_{\text{cut-off}} = \frac{1}{1.3\tau} \quad (\text{A.33})$$

where τ is defined in Figure A.4.

Alternatively, a sinusoidal wave can be used to investigate the frequency response of an anemometer. In this method a sinusoidal wave is fed into the anemometer and the root mean square value of the anemometer output is recorded while the frequency of the sinusoidal test signal is varied. Although more time consuming than the square-wave test, this method offers additional information about the system operation since the entire frequency range of interest is explored. Figure A.5 shows the typical response of a critically damped constant-temperature anemometer to a sinusoidal test signal and velocity fluctuations. These trends were obtained from

Equation A.29.

The response of an anemometer to a sinusoidal test signal (Figure A.5a) is initially flat but begins to rise at a frequency which corresponds to the time constant of the hot-wire, M , (Weidman and Browand 1975). The response peaks at the cut-off frequency of the anemometer and ultimately asymptotes to -20 dB/decade . Figure A.5b shows the frequency response of the anemometer to a change in the effective cooling velocity, ΔV_{eff} . As was discussed before, for a critically damped situation, $\zeta=1$, the response is flat at low frequencies and for frequencies larger than the cut-off frequency of the anemometer ($f_{cut-off} = 1/(2\pi\omega_0)$) the output falls off at 40 dB/decade . According to Equation A.29 the anemometer responds to velocity fluctuations, whereas during electronic testing the anemometer responds both to the test signal e_t and its time derivative. This explains the difference in the response of an anemometer to the electrical test signal and a velocity perturbation.

Figure A.5: Typical response of a constant-temperature anemometer to a sine-wave signal; adapted from Weidman and Browand (1975).

Appendix B

Plating Procedure

The plating setup was designed and constructed to provide an accurate means for electroplating the tungsten wire. Figure B.2 shows the schematic of the plating set up which consists of an electrolyte bath, an anode, a holding fork, a traversing device and a control panel. The electrolyte was described in Chapter 3. The anode is formed from a piece of 2 *mm* thick copper wire which is rounded at 32 *mm* diameter and placed at the bottom of the electrolyte bath.

The holding fork is constructed by passing two sewing needles through a cylindrical plexiglass body (Figure B.1). Tungsten wire is spot welded on the needles using a DISA 55A12 welding apparatus. The tungsten wire should not be under tension when the welding is performed. This can cause wire breakage during the soldering process or later wire operation.

The traversing device contains a micrometer which is attached to a holding bar. The micrometer provides fine vertical movement of the tungsten wire in the electrolyte bath. The control panel contains a set of connectors and a potentiometer.

Figure B.1: Schematic of the holding fork.

The connectors are connected to a power supply, a voltmeters, an ammeter, the anode and the holding fork. The potentiometer provides fine adjustment of the plating voltage and current which are monitored on the voltmeters and ammeter respectively.

The plating process is carried out as follows:

1. The tungsten wire must be carefully cleaned to remove any grease or other possible debris (experience shows that plating on a wire which is not properly cleaned results in a poor plating). This is done by first using a chemical cleaner that do not leave any deposits such as those used to clean printed circuit boards, and then rinsing the wire in a stream of warm water to clean off the chemical cleaner.
2. The container is filled to about 3 cm height with the electrolyte solution.
3. The holding fork is inserted inside the holding bar such that the axis of the

tungsten wire is aligned with the axis of the holding bar. The fork and the anode are then connected to the control panel (power off).

4. The micrometer is used to submerge the holding fork into the bath for the desired length of plating on one end of the wire. In this position the wire should be perpendicular to the liquid surface.
5. The power supply is turned on with the plating voltage set to about 0.20 volts. For the specified electrolyte solution, the proper plating time was found to be about 35 seconds.
6. The power is turned off after the plating of one end is completed. The holding fork is then removed from the bath and turned 180 degrees to plate the other end of the wire.
7. The holding fork is placed back in the bath and submerged inside the bath until the lower edge of the plating on the other end of the wire touches the liquid surface. The use of a magnifier is suggested for accurate observation of this adjustment.
8. The wire is then pulled out of the bath using the micrometer for 1 *mm*.
9. The plating process, step 5, is then repeated for this end of the wire.
10. The power is turned off, the fork is removed and the wire is washed with warm water.
11. The diameter of the plated part and the length of the hot-wire is controlled visually using a Laborlux microscope model 12ME.

Figure B.2: Schematic of the plating apparatus.

After every three to five platings the needles on the holding fork should be replaced. This is due to the chemical reaction between the needle material and the acid in the electrolyte.

Appendix C

Soldering Procedure

An extension bar is added to DISA 55A13 welding apparatus to adapt this apparatus to use for soldering the plated tungsten wire onto the prongs of the four-wire probe. One end of the extension bar is attached to the body of the welding apparatus using a small screw. The probe holder is then attached to the other end of the extension bar. The four-wire probe is then placed into the probe holder. Figure C.1 shows the schematic of the soldering setup. Due to the small size of the wire, the soldering process is observed with a magnifier. A sewing needle is used as the heat source for the soldering process since the soldering iron is too bulky for the size of the probe tip.

The soldering process is carried out as follows:

1. A small amount of solder is placed onto the tip of the prongs
using a soldering iron.
2. The holding fork is secured by a rubber band over the two small clamps of the

DISA welding apparatus such that the wire is horizontal.

3. The probe is then placed in the probe holder such that the two prongs to which the wire is to be soldered are positioned parallel to the coated wire.
4. The coated wire is placed onto the tip of the prongs such that the unplatted part of the wire is located midway between the prongs.
5. The soldering iron is brought into contact with the soldering needle about 1 *cm* from the tip of the soldering needle.
6. The tip of the soldering needle is then briefly placed at the junction of the wire and the prong in order to melt the solder on the tip of the prong (Figure C.2).
7. The same procedure is repeated for the other prong.
8. The holding fork is disconnected from the plated wire after the wire is soldered to both prongs using the blade of a cutter.

Figure C.1: Schematic of the soldering setup; 1. soldering iron, 2. soldering needle, 3. holding fork, 4. four-wire probe, 5. probe holder, 6. DISA welding apparatus, 7. extension bar.

Figure C.2: Relative position of the plated wire, prongs and soldering needle in the soldering process.

Appendix D

Birational Spline Fitting

The following is a summary of the procedure described by Späth (1974).

In a cartesian co-ordinate system, let

$$G_{ij} \quad (i = 1, \dots, n; j = 1, \dots, m) \quad (\text{D.1})$$

be given at the nodes of a rectangular grid in domain R ,

$$R = \{(\theta, \varphi) : \theta_{\min} \leq \theta \leq \theta_{\max}, \varphi_{\min} \leq \varphi \leq \varphi_{\max}\},$$

as shown in Figure D.1. The objective is to obtain a smooth surface $G(\theta, \varphi)$ interpolating the ordinates G_{ij} in the domain R . Considering the rectangle R_{ij}

$$R_{ij} = \{(\theta, \varphi) : \theta_i \leq \theta \leq \theta_{i+1}, \varphi_j \leq \varphi \leq \varphi_{j+1}\}, \quad (\text{D.2})$$

function $\mathcal{G}_{ij}(\theta, \varphi)$ is defined in R_{ij} as

$$\begin{aligned} \mathcal{G}_{ij}(\theta, \varphi) &= \sum_{k,l=1}^4 a_{ijkl} f_{ik}(\theta) f_{jl}(\varphi) \\ &\{i = 1, \dots, (n-1); j = 1, \dots, (m-1)\} \end{aligned} \quad (\text{D.3})$$

where

$$f_{i1}(\theta) = u, \quad f_{i2}(\theta) = t, \quad f_{i3}(\theta) = \frac{u^3}{pt+1}, \quad f_{i4}(\theta) = \frac{t^3}{pu+1}, \quad (\text{D.4})$$

Figure D.1: Domain R and corresponding values of G_{ij} , given on it.

u and t are non-dimensional variables defined as:

$$t = \frac{\theta - \theta_i}{\Delta\theta_i}, \quad u = 1 - t. \quad (\text{D.5})$$

and p ($0 \leq p \leq \infty$) is called the tension factor. $f_{ik}(\theta)$ is defined in the same way as $f_{jl}(\varphi)$. G_{ij} is formed by connecting $G_{ij}(\theta, \varphi)$ on each rectangle R_{ij} such that the resulting function is twice differentiable with respect to θ and φ .

For $p = 0$ the birational spline function becomes a bicubic spline function, and for $p \rightarrow \infty$ it becomes a linear interpolating function. Figure D.2 shows a cross section of a birational function using different values of p .

Figure D.2: The effect of the tension factor on the smoothness of the spline functions.

To determine the $16(n-1)(m-1)$ coefficients a_{ijkl} in Equation D.3 such that $G(\theta, \varphi)$ is twice differentiable everywhere, certain values of the derivatives of $G(\theta, \varphi)$ on the boundary of domain R must be provided in addition to the values of G_{ij} . It is sufficient if the following values are provided:

$$\begin{aligned} \frac{\partial}{\partial \theta} G(\theta, \varphi) \Big|_{\theta_i, \varphi_j} & \quad i = 1; j = 1, \dots, m \\ \frac{\partial}{\partial \varphi} G(\theta, \varphi) \Big|_{\theta_i, \varphi_j} & \quad i = 1; j = 1, \dots, m \\ \frac{\partial^2}{\partial \theta \partial \varphi} G(\theta, \varphi) \Big|_{\theta_i, \varphi_j} & \quad i = 1, m; j = 1, n \end{aligned}$$

These derivatives may be approximated using forward and backward differencing. Having these derivatives, the value of G_{ij} at the nodes, and implementing the condition of continuous first and second derivatives at the nodes, a system of $16(n-1)(m-1)$ linear equations is obtained to be solved for a_{ijkl} .

Appendix E

Seven-Hole Probe Calibration Procedure

The calibration procedure described herein was first proposed by Gerner and Maurer (1981). The present calibration procedure is only valid for incompressible flows; for extension to compressible flows the reader is referred to Gerner and Maurer (1981) and Everett *et al.* (1983).

The calibration procedure is divided into low angle and high angle regimes. At low relative flow angles to the probe axis the flow over the probe is expected to remain attached, hence, all seven port pressures are used. Whereas at high relative flow angles the flow tends to separate over the leeward side of the probe. Therefore, only four pressures on the windward side of the probe are used to reduce the sensitivity of the probe to viscous effects.

Figure E.1: Seven-hole pressure probe port numbering scheme.

E.1 Low Angle Regime

Since the flow remains attached to the probe tip, all seven pressure port readings (see Figure E.1) are used to define the following non-dimensional parameters:

$$\begin{aligned} C_{\alpha_1} &= \frac{P_4 - P_1}{P_7 - P_M} \\ C_{\alpha_2} &= \frac{P_6 - P_3}{P_7 - P_M} \\ C_{\alpha_3} &= \frac{P_5 - P_2}{P_7 - P_M} \end{aligned} \quad (\text{E.1})$$

where P_M is the average of the six surrounding port pressures. These non-dimensional parameters are respectively sensitive to flow angle variations in three separate planes, each 60° apart, passing through the probe tip axis. The denominator in the above expressions, referred to as the *pseudo-dynamic pressure*, approximates the local dynamic pressure since P_7 approximates the local total pressure and P_M approximates the local static pressure. From the three coefficients, only two are required to uniquely define the flow direction. To use all the information in the three coefficients, equal weighting of the three solutions from the $C_{\alpha_1} - C_{\alpha_2} - C_{\alpha_3}$ reference system are

resolved into a single ordered pair, C_α and C_β in the tangential coordinate system (Figure E.2) as follows:

$$\begin{aligned} C_\alpha &= \frac{1}{3}(C_{\alpha_1} - 0.5C_{\alpha_2} + 0.5C_{\alpha_3}) \\ C_\beta &= \frac{1}{2}(C_{\alpha_2} + C_{\alpha_3}) . \end{aligned} \quad (\text{E.2})$$

These two variable can then be related to the local pitch angle, α , yaw angle, β , total pressure coefficient, C_0 , and dynamic pressure coefficient, C_q , through the following relations:

$$\begin{aligned} \alpha &= f_1(C_\alpha, C_\beta) \\ \beta &= f_2(C_\alpha, C_\beta) \\ C_0 &= f_3(C_\alpha, C_\beta) \\ C_q &= f_4(C_\alpha, C_\beta) \end{aligned} \quad (\text{E.3})$$

where f_1 , f_2 , f_3 and f_4 are determined by calibrating the probe in a known flow. The total and dynamic pressure coefficients are defined as follows:

$$\begin{aligned} C_0 &= \frac{P_T - P_0}{P_T - P_M} \\ C_q &= \frac{P_T - P_M}{P_0 - P} \end{aligned} \quad (\text{E.4})$$

where P_0 and P are the local stagnation and static pressures, respectively. The calibration procedure outlined above is used as long as the centre port reads the highest pressure among the seven ports. For the seven-hole probe used in this study, this is typical of flow misalignment angle of less than 20° .

Figure E.2: Tangential co-ordinate system used for low angle regime, reproduced from De Cecco (1995).

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PRECISIONSM RESOLUTION TARGETS

E.2 High Angle Regime

In this case the pressure coefficients are redefined so that they contain the port pressures in the attached flow region. For this purpose the six side ports of the probe are grouped into six 60° wedge-shaped sectors each containing the central port and three of the side ports (Figure E.3). At large misalignment angles it is more appropriate to use polar coordinate system, shown in Figure E.4, to define the non-dimensional calibration coefficients:

$$\begin{aligned} C_{\theta_n} &= \frac{P_n - P_r}{P_n - 0.5(P_{n^-} + P_{n^+})} \\ C_{\phi_n} &= \frac{P_{n^-} - P_{n^+}}{P_n - 0.5(P_{n^-} + P_{n^+})} \end{aligned} \quad (\text{E.5})$$

where the subscript n indicates the side port number which reads the highest pressure, and n^+ and n^- are the adjacent ports clockwise and counterclockwise to port n , respectively. These non-dimensional coefficients can be related to the local pitch angle, θ , roll angle, ϕ , total pressure coefficient, and dynamic pressure coefficient, through the following relations:

$$\begin{aligned} \theta_p &= f_1(C_{\theta_n}, C_{\phi_n}) \\ \phi_p &= f_2(C_{\theta_n}, C_{\phi_n}) \\ C_0 &= f_3(C_{\theta_n}, C_{\phi_n}) \\ C_q &= f_4(C_{\theta_n}, C_{\phi_n}) \end{aligned} \quad (\text{E.6})$$

where total and dynamic pressure coefficients are defined by:

$$\begin{aligned} C_0 &= \frac{P_n - P_0}{P_n - 0.5(P_{n^-} + P_{n^+})} \\ C_q &= \frac{P_n - 0.5(P_{n^-} + P_{n^+})}{P_0 - P} \end{aligned} \quad (\text{E.7})$$

Figure E.3: Sector allocation criteria, adapted from Everett *et. al.* (1983).

Figure E.4: Polar co-ordinate system used for high angle regime, reproduced from De Cecco (1995).

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