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FINITE ELEMENT MODELLING OF TRUSS PLATE CONNECTIONS

by

Domenic Fagotto, B. Eng.

A thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment of
the requirements for the Degree of

MASTER OF ENGINEERING

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Carleton University
Ottawa, Ontario

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The undersigned recommend to the Faculty of
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ABSTRACT

The finite element method is used as a means of analyzing and simulating truss plate connections. Constant strain triangles with orthotropic elastic properties and isotropic elastic properties are used to model the wood and truss plate respectively. To take into account the non-linear character of the load-deformation relationship, a triangular "bond" element with linear displacement expansion is used to simulate the assumed "continuous" contact surface between the wood and the truss plate. An incremental method is used to model the nonlinearity of the truss plate connection. The connector properties are derived from those for a single tooth.

Several truss plate connections were studied and the finite element models exhibited a good ability in predicting their behaviour. The modelled truss plate connections' overall joint characteristics and respective failure modes are in concurrence with observed behaviour. The simulation of a structural system comprising of several truss plates predicted results within 15% of the empirical results.

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LIST OF SYMBOLS

Symbol	Definition
[A]	Matrix which relates the nodal displacement vector $\{\delta\}$ with the generalized coordinate vector $\{a\}$
[B]	Matrix which relates the strain vector $\{\epsilon\}$ with the displacement vector $\{\delta\}$, for an element.
[D]	Constitutive relation matrix.
E_n	Elastic moduli ($n = L, T$).
E	Elastic modulus of steel.
$E\epsilon$	Virtual external work
E_i	Virtual internal work.
$\{F\}$	Load vector.
$F()$	Load per tooth.
ΔF	Nodal forces on bond element.
G_{xy}	Shear modulus.
G_i	Slip modulus ($i = x, y$) depending on direction.
i, j, m	Nodal points.
[K]	Global stiffness matrix.
$[K']_e$	Element stiffness matrix for wood with respect to the local coordinate axis.
$[K]_e$	Element stiffness matrix for wood with respect to the global coordinate axes.

LIST OF SYMBOLS

Symbol	Definition
$[K_p]_e$	Element stiffness matrix for the truss plate.
$[K_I]_e$	Element stiffness matrix for the bond element.
m_0	Stiffness modulus at large slips.
m_1	Intercept of the asymptote with slope m_0 .
t	Thickness.
T	Transformation matrix.
u, v u_x, v_y	Deformations in coordinate directions.
x', y'	Local coordinates of elements.
X, Y	Global coordinates.
Z	Boolean transformation matrix.
α_i	Polynomial coefficients ($i=1,2,3,\dots$)
$\{\alpha\}$	Polynomial coefficient vector.
β	Density of teeth per unit area of truss plate.
$\{\delta\}$	Nodal displacement vector.
$\{\Delta\delta\}$	Relative nodal displacement vector.
$\{\delta^*\}$	Virtual displacement vector.
$\{\epsilon\}$	Strain vector.
ν_i	Poisson's ratio ($i=LT, TL, XY, YX$) depending on direction.
$\{\sigma\}$	Stress vector for an element.
σ_i	Axial component of stress* ($i= X, Y, L, T$)

LIST OF SYMBOLS

Symbol	Definition
$\Delta u_x, \Delta u_y$	Relative displacement.
$\Delta u, \Delta v$ Δu^*	Virtual relative displacement.
τ	Shear stress.
θ	Angle between plate major axis and the X direction.
ω	Angle between the grain direction and the X direction.
$P1, P2, P3,$ $P4, PX, PY,$ PA, PB	Truss plate properties depending on direction.

CHAPTER 1

INTRODUCTION: Scope and Objectives

Since the introduction of the metal plate (or "truss plate") connector in 1955, it has been utilized in the construction of roof and floor support systems in lightframe construction. The metal plate roof truss has proven to be an economical system of covering long, clear spans with the advantages of efficiency and quality control of factory assembly. These prefabricated trusses represent the most important market to the wood products industry in North America (McCracken 1979). Other than its economic advantages, this structural system is one of the most versatile on the market. Along with residential roof construction, there are innumerable combinations of commercial, industrial, and farm structures which account for a countless variety of roof designs. In addition to roof truss assemblies, prefabricated metal plate floor trusses are replacing conventional floor framing.

Metal plate connected trusses are indeterminate structures with joints having a nonlinear load-displacement behaviour; that is, they allow relative movement between the joined members in the plane of the truss. In a commercial environment, the design and analysis of these structural systems is based on pinned analysis, prototype testing, and engineering experience. The pinned analysis usually produces an overdesigned structure; however, due to this simple analysis improper detailing of joints may result.

The truss plate connections are complex and full scale tests of trusses show that a major cause of failure is due to incorrect design of a joint (MHBH 1980). Designers usually encounter difficulties in trying to implement an analysis of a truss plate connection.

More comprehensive methods of analysing these complex structures are available (Misra 1968, Reardon 1971, Suddarth 1972, Beineke 1974, Foschi 1977). Most of these analytical techniques require properties of the specific joint configuration which is to be modelled. The required data to evaluate these properties usually involves a costly testing program.

Prediction of load-deformation characteristics of joints can contribute to the reduction of some of the testing required to obtain data for the analysis of complete structural systems. The investigation of stress distribution of joints can also reduce the number of prototype tests to yield a proper design.

This study proposes the use of the finite element method as a means of analysing and simulating truss plate connections. The application of the finite element method in the study of joints was initiated in solving geotechnical engineering problems; however, the manner in which the method is being utilized in this thesis has not been previously reported.

The primary objectives of this work can be stated as follows:

- i) To develop a finite element program which is capable of simulating and analysing truss plate connections, and incorporating the nonlinear load-slip characteristics of the connection.
- ii) To develop a predictive capability of truss plate connection behaviour.
- iii) To investigate the distribution of stress in wood, plate and the contact area between the wood and plate.

The load-displacement characteristics of several types of joints are investigated, along with the behaviour of a structural system where these joints are encountered. The results are compared with empirical data and results from other theoretical methods. The stress distributions of several types of truss plate connections are examined and compared with observed behaviour.

1.1 Literature Survey

This section discusses earlier attempts at modelling truss plate connections, the use of finite element techniques in modelling the behaviour of wood, and a brief survey on the use of interface elements.

1.1.1 Modelling of Truss Plate Connections

The first use of truss plate connections was in the construction of residential roof trusses in 1957 (Meeks 1979). These trusses were readily regarded as highly

complex indeterminate frames. These structures have loads applied to members between joints. As well as exhibiting a nonlinear load-deformation relationship, as sketched in Figure 1.1, the joints are eccentric rather than being junctions of centroids and load paths. And also, wood is a variable, anisotropic, viscoelastic material.

Early methods of indeterminate analysis applied to wood trusses assumed the members to be either pinned or rigid (Suddarth, 1961, 1963). Suddarth (1961) used pinned and rigid joints to solve for member forces and then applied a rotation-moment formula derived and tested for nailed joints with wood gussset plates by Perkins (1962) to solve for the force distribution in the truss plate connection.

An analytical method is presented by Rodda (1965) which included semirigid joints to model the truss plate connection. The moment-rotation relationship developed by Perkins (1962) is used to determine a "joint rigidity factor" (Rodda 1965) which is included in a modified slope deflection analysis. An iterative solution is used to implement the nonlinearity of the moment rotation function (Rodda 1965). Comparisons between results from Rodda's theoretical model and experimental results showed "quite good agreement" (Beineke 1974) for the behaviour of a simple triangular truss.

Porteous (1965) employed the stiffness method to analyse roof trusses. The joints were assumed to be semi-rigid. Two procedures for modelling the truss plate connections were proposed. The first method is the use of

FIGURE 1.1

NON-LINEAR LOAD-DEFORMATION RELATIONSHIP OF A TRUSS PLATE CONNECTION UNDER A TENSILE FORCE.

fictitious members in the theoretical analog of the real structure. The fictitious members are given assumed dimensions and mechanical properties to simulate the stiffness characteristics of a specific truss plate connection. The second method proposed is the use of dimensionless springs, (i.e., concentrated stiffnesses), located at the member ends. The linear spring stiffness is calculated from load-slip and moment-rotation curves of specific truss plate connections.

The second method is incorporated into an existing computer program which uses the stiffness method of analysis. Computed deflections from the theoretical model were 80% of the actual deflection of the experimental truss test (Porteous 1965).

Brynildsen and Booth (1967) developed a computer program based on the stiffness method to analyse statically indeterminate structures. Porteous's method of using dimensionless springs is also implemented into the program. The springs are assumed to act at the centre of gravity of the truss plate connections and on the centre line of the wood members. Unlike Porteous, a nonlinear analysis is incorporated by the use of an incremental method. The external loads are applied in increments and the spring stiffness varies from one load step to another.

The results of two full scale trusses are compared with the theoretical analysis. There was good agreement between results up to 2.5 times design load (Brynildsen and Booth 1967).

Reardon (1971) also developed a computer program based on the stiffness method to analyse plane frame structures with semirigid joints. In the same manner as Porteous (1965), and Brynildsen and Booth (1967), the truss plate connection is modelled by dimensionless springs which take into account both axial and rotational deformation. Figure 1.2 shows a member used in the computer analog. The proper linear spring constants are given to the dimensionless springs to simulate the behaviour of the truss plate connection.

A comparison between experimental and computed deflections of two timber trusses showed close agreement (Reardon 1971).

Suddarth (1972) developed a computer program, Purdue Plane Structures Analyser, based on the stiffness method where the joints of the analog structure are either pinned or rigid. The truss plate connections are simulated by fictitious members as described by Porteous (1965).

An important note is that in all analytical methods previously discussed, the wood members are assumed to be homogeneous and linearly elastic.

The analytical methods just described, except for Suddarth (1961, 1963), are concerned with the calculation of gross displacements of truss plate connections and member forces in an idealized structure. Misra and Esmay (1966) proposed two methods to analyse the stress distribution in a truss plate connection under uniaxial tension. The first

FIGURE 1.2

THE TRUSS PLATE CONNECTIONS ARE
SIMULATED BY DIMENSIONLESS SPRINGS
AT THE MEMBER ENDS.

method used finite difference equations while the other used the principle of minimum complementary energy. Both analyses assume linear elastic material behaviour and a rigid connection between the truss plate and the wood. The results of the theoretical analyses are shown in Figure 1.3 and show that both methods show agreement with each other.

Attempts to verify the theoretical analyses by using a photostress technique and strain gauges to measure the distribution of stress in the truss plate were made. The photostress analysis showed stress patterns too complicated, due to the holes and stress concentrations in the plate, to help validate the theory. The strain gauges placed along the truss plate showed a gradual increase of stress from the free end to the joint centre, but too few points were monitored along the truss plate to be compared with the theory.

Beineke (1974) presents a thorough review of previously developed classical mechanics analysis of both mechanical fastener and adhesive joints. Results of this review indicated that all of the different approaches exhibited nearly the same distribution of forces within a joint. Figure 1.4 shows the results of the theories analysing a tension joint. The graph shows "that no matter what choice of assumptions is made, whether the derivation considers discrete or continuous fastener systems, the predicted stress distribution will be nearly the same" (Beineke and Suddarth 1979).

FIGURE 1.3
COMPARISON OF STRESS DISTRIBUTIONS IN THE METAL PLATE CONNECTOR OBTAINED FROM THE ANALYSIS OF THE MINIMUM COMPLEMENTARY ENERGY AND THE DIFFERENCE EQUATION.

FIGURE 1.4 COMPARISON OF SEVERAL THEORETICAL LOAD DISTRIBUTIONS IN THE TRUSS PLATE.

Beineke (1974) develops a model based on the concept of a fictitious adhesive which simulates the behaviour of the truss plate connection. The work is based on Demarkles' theory that shearing displacements of the adhesive layer can be used to simulate slip displacements in a truss plate connection (Demarkles 1955). The method incorporates axial and bending displacement models which are modified to allow the prediction of gross displacement of the truss plate connections. Experimental verification of the model showed that this model "exhibited only moderate abilities in fitting the experimental data" (Beineke 1974). This model is based on the same classical mechanics as other studies, and therefore "one would expect that they would exhibit the same deficiency" (Beineke and Suddarth 1979).

Most of the previously discussed modelling techniques simulate the behaviour of truss plate connections by means of equivalent springs or fictitious members. To obtain the proper properties which are assigned to the springs or fictitious members, tests on the actual connections must be performed. Foschi (1977) presents an analysis of truss plate connections without resorting to equivalent springs or fictitious members. The connection between truss plate and wood is considered to be continuous. The mathematical model includes the actual shape of the connected area, gaps between members, buckling capacity of the plate and its yielding in tension and shear. The nonlinear load-slip behaviour of the truss plate connection is implemented in the analysis. The principle of virtual work is used in formulating the stiffness of the contact area between truss plate and wood.

The nonlinear tooth load-slip curve is assumed to be in the following form:

$$F(|\Delta u|) = (m_0 + m_1 |\Delta u|) \left[1 - e^{\frac{-k |\Delta u|}{m_0}} \right] \quad (1.0)$$

where

k = initial stiffness

m_1 = the stiffness at large slips

m_0 = the intercept of the asymptote with slope m_1

F = force required to produce slip

$|\Delta u|$ = magnitude of slip

Equation (1.0) is illustrated in Figure 1.5.

The parameters k , m_0 , m_1 are to be derived from single tooth tests. These tests are usually carried out on a plate containing many teeth and the tooth load-slip parameters are taken to be the mean value (Foschi 1977).

The analysis has been implemented in a computer program, Structural Analysis of Diaphragms and Trusses (SADT). The nonlinear analysis is solved by an iterative technique and uses a convergence acceleration technique.

To verify the theory, a truss plate connection eccentrically loaded in tension was tested. Good agreement is obtained between test and computed results (Foschi 1977).

Figure 1.5 Tooth load-slip relationship.

1.1.2 Finite Element Modelling of Wood

Wood is a biological material which can be described as anisotropic, hygroscopic, viscoelastic and variable. The behaviour of wood under load is affected by its nonhomogeneous nature, and by its growth characteristics such as knots and grain distortions. In most theoretical and experimental studies wood is considered to be an orthotropic elastic material and these elastic parameters transform on rotation of axes according to tensor transformation law (Goodman and Bodig 1978). An extensive experimental investigation and theoretical analysis of the orthotropic behaviour of wood was conducted by Goodman and Bodig (1970). The application of the tensor transformation law for orthotropic elasticity was compared with empirical results. The results showed reasonably good agreement "between assumed theory and measured values of longitudinal strains for loadings within a principal orthotropic plane" (Goodman and Bodig 1970), while for general loadings there was little agreement (Goodman and Bodig 1970).

The elastic parameters required for the use of orthotropic elasticity are not readily available. Bodig and Goodman (1973) provide a comprehensive listing of elastic parameters for a large number of wood species. From this study a series of predictive equations for the elastic parameters are formulated.

Investigations considering the nonhomogeneous nature of wood require more extensive knowledge of material properties such as the mechanical properties of knots. To apply the

finite element method to problems such as cross-grain behaviour of wood, proper transformation of the orthotropic properties is needed. The use of the tensor transformation law is the only method available currently (Goodman and Bodig 1978).

The following literature survey presents studies where the finite element method has been used in the modelling of wood.

One of the first attempts of modelling wood material is by McGlenn and Hartz (1968) in the finite element analysis of plywood plates. The finite element used is a rectangular plate element with twelve degrees of freedom as shown in Figure 1.6. The plywood is assumed to be a plate formed of homogeneous orthotropic layers. The moduli of elasticity along the two principal axes are determined experimentally while handbook values are used for the remaining elastic constants (McGlenn and Hartz 1968).

Al-Dabbagh, Goodman and Bodig (1972) apply the finite element method to analyze wood as a nonhomogeneous material; that is to include knots and checks as well as cross-grain. A three dimensional "brick" element with three degrees of freedom at each corner node, as shown in Figure 1.7, is used for this study. The wood properties within a discrete element are considered to be those of a perfectly elastic, homogeneous material with orthotropic behaviour. The computer program developed is capable of assigning specific stiffnesses to elements which allowed the treatment of wood as a layered system. The values of the elastic parameters are taken from a previous study by Bodig and Goodman (1970).

FIGURE 1.6 RECTANGULAR PLATE ELEMENT.

FIGURE 1.7 "BRICK" OR PARALLELEPIPED ELEMENT.

Barrett and Schniewind (1973) presents a finite element model of cylindrical wood fibres. The fibres are considered as concentric, multilayered, orthotropic cylinders. A quadrilateral, linear isoparametric element is used to model the cylinders. Material properties can be specified for each individual element which allows the modelling of the layered structure.

Fernandez and Polensek (1979) present a model for predicting strength and stiffness of wood studs. The procedure incorporates a Monte Carlo-type simulation, finite element method, and a "linear step-by-step technique" (Fernandez and Polensek 1979) and accounts for nonlinearity, local material variability, and defects in studs.

Two types of elements are used to model geometric shapes such as knots and grain distortions. The first type is a stress hybrid element (Cook 1974), a rectangular element with eight degrees of freedom; i.e., two orthogonal translations at each node. The element is illustrated in Figure 1.8. The stress varies linearly along the element. The second type is a constant stress triangle to model knots in the studs.

The wood is assumed to behave as an elastic orthotropic material. These material properties are constant throughout each element, but vary from one element to another. Knots are simulated in tension and compression with elastic properties which differ from those of clear wood. The load-deflection relationship of a stud is nonlinear and hence the stress-strain relation is assumed nonlinear. In this study

the nonlinear relations are approximated by three straight lines. A "linear step-by-step procedure" is used for the completion of the analysis (Fernandez and Polensek 1979).

Comparison of theoretical results and experimental values showed "reasonably good agreement" (Fernandez and Polensek 1979). A sensitivity study was also performed to investigate the effect of local material properties on the theoretical deflection-load curves. It was concluded that "local stiffness and strength of clear wood, grain angle and knots greatly affect stud stiffness" (Fernandez and Polensek 1979).

Foschi and Barrett (1980) developed a model to predict the strength of glued laminated beams taking into account the effects of beam size, lamination thickness, beam lay-up, E-rating or proof loading, and the influence of end-joints. The model is used to estimate population statistics for similar beams and to derive data to be used in the estimation of design values.

A finite element simulation is used for the analysis. A five-node element, shown in Figure 1.9, is used in the simulation. The internal node is eliminated during construction of the element stiffness matrix. The properties for each element can be adjusted for different densities and for the influence of knots.

Investigation of radial stresses in curved glulam beams using the finite element analysis is performed by Gopu (1980). A trapezoidal finite element is utilized to analyse the beams.

FIGURE 1.8 HYBRID STRESS ELEMENT.

FIGURE 1.9 FIVE NODE ELEMENT.

The finite element method has also been used to simulate structural systems. Polensek (1976) used this method to analyse and simulate the behaviour of wood-stud walls. The analytical method developed allows for partial composite action, load sharing, variable material and joint properties, and the nonlinear behaviour of studs and is able to predict deflections, stresses and slips at service loads and overloads, and the ultimate load of wood-stud walls (Polensek 1976). Beam and plate elements are used to simulate the structural system. A linear incremental method is used to approximate the nonlinear behaviour of the studs.

A published finite element computer program, (FEAFLO: Finite Element Analysis of FLOors), for the analysis of layered wood systems was developed by Thompson, Vanderbilt and Goodman (1977). Examples of layered wood systems are wood joist and sheathing construction commonly used in housing for floors, and wall and roof structures. The method includes the effects of slip between layers due to fastener deformations, orthotropic material properties and gaps in layers.

1.1.3 Interface Elements

Many researchers have developed interface elements to model joints and material interfaces in the finite element analysis. A brief summary of interface elements is presented in this section.

Early studies which used this type of element required a model to simulate slip behaviour. Studies in the analysis of reinforced concrete and in rock mechanics initialized the development of interface elements.

There are three main types of finite element models to simulate discontinuities, joints or bond. The first attempt in this modelling was the use of isolated springs. Ngo and Scordelis (1967) analysed singly reinforced concrete beams by the finite element method. A one-dimensional linkage element, shown in Figure 1.10, was developed to simulate the bond conditions between the reinforcing steel and the surrounding concrete. The linkage element is dimensionless and can be oriented at any arbitrary angle with the global system of axes. The element consists of two springs (one normal, the other tangential to the reinforcement), whose mechanical behaviour can be fully described by two spring constants.

The second type is an infinite spring (or "contact", "interface", or "bond") element. Goddman, Taylor and Brekke (1968) developed a one-dimensional interface element in a model for the analysis of jointed rock. The rock was idealized by a system of constant strain triangular elements.

FIGURE 1.10 LINKAGE ELEMENT.

FIGURE 1.11 JOINT ELEMENT.

To account for the cracks between boundaries a four-nodal point element with finite length and zero width was formulated. A schematic of the element is shown in Figure 1.11. There are two degrees of freedom at each node, i.e., two orthogonal translations. The properties of this element are related to the unit joint stiffness in the normal and tangential directions. The strain energy of the finite element is expressed in terms of the relative displacements and the corresponding relative forces between element faces.

Scafer (1975) expanded on this first work by Goodman, et al. (1968) to develop "bond elements", (Scafer 1975), to account for load-slip behaviour between contact surfaces. He develops a flat triangular bond element with linear, Figure 1.12, and parabolic displacement expansion. He also formulates a one-dimensional bond element with linear and parabolic expansion to describe the bond between the reinforcement and the surrounding concrete. The bond element is shown in Figure 1.13. The use of this type of bond element has the advantage of having compatibility between nodal points which does not occur when springs are used as linkage elements.

The third approach is the use of isoparametric elements. In this method the experimental unidirectional force gross displacement relationships are replaced by equivalent stress-strain relationships (Anis, et al. 1980). Zienkiewicz, Best, Dullage, and Stagg (1970) used this type of element for the analysis of jointed rock. Both four noded rectangular and curved elements with variable thickness, Figure 1.14, were utilized. The strain

FIGURE 1.12 ONE DIMENSIONAL BOND ELEMENT.

FIGURE 1.13 TRIANGULAR CONTACT ELEMENT WITH LINEAR DISPLACEMENT EXPANSION.

Figure 1.14(a) A simple joint element of rectangular shape. Points 1-4 and 2-3 assumed to have same co-ordinates.

Figure 1.14(b) A curved, variable thickness, cubic joint element.

components varied linearly along and perpendicular to the longitudinal axes of the joint for the rectangular joint elements. Higher order shape functions for the curved isoparametric elements allow the strain components to vary parabolically or cubically.

Depending on the problem, the complexity and completeness of the model, further development has been accomplished in the three basic methods just discussed.

CHAPTER 2

DEVELOPMENT OF FINITE ELEMENT MODEL

Joints and interface between dissimilar materials are common features in geotechnical and structural engineering. The behaviour of rock masses is greatly influenced by the presence of joints and "pre-existing weak planes of potential discontinuity" (Faruque 1980). Material interfaces are important in soil-structure interaction; e.g., soil retaining wall, soil footing, and soil culvert systems (Hermann 1978). On the other hand, the behaviour of joints in a structure greatly influences the performance of the building. In concrete construction, joints between reinforced elements are usually filled with mortar or bonded with adhesive (Schafer 1975). Such a system is the panelized concrete wall-floor joint (Axis, Burnett and McNiece 1980). All of the above examples of joints and interface can be characterized by some form of load-slip behaviour.

In wood construction all structural joints can be described by load-slip curves of varying degrees of non-linearity. The structural joint being made of any number of connector types ranging from nails to truss plate connectors.

The early methods of analysis of truss plate connections was to simulate their behaviour by means of equivalent springs (e.g., Brynildsen and Booth 1967, Reardon 1971), or by the use of fictitious members (e.g., Porteous 1965, Suddarth 1972). Mathematical models (e.g., Misra and Esmay 1968, Beineke 1974) have been proposed for the analysis of truss plate connections. A very complete analysis is presented by Foschi (1977) using a finite element approach to formulate the model. A method of analysis based on assumptions similar to those forwarded by Foschi is presented. The truss plate connection is regarded as a contact problem between two bodies; the truss plate and the wood member. The truss plate connection has a nonlinear load-slip relationship which may be reformulated in terms of a relation of shear stress and relative deformation.

The wood and truss plate are modelled by the finite element method. The load-slip relationship, or bond behaviour, is modelled using an interface (or "bond") element. A schematic of the analog is illustrated in Figure 2.1.

As described in the previous sections, various types of finite elements have been employed in the finite element analysis of wood (e.g., Al-Dabbagh, et al., 1972, Polensek 1979, Gopu 1980, etc.). The choice of element should be such that the original body to be analysed is simulated as accurately as possible. Consideration of factors such as geometry, loading, material properties, bandwidth, etc., must be taken into account.

FIGURE 2.1

PROPOSED ANALOG OF A TRUSS
PLATE CONNECTION.

In this study, a constant strain triangular element having orthotropic elastic properties is used to simulate the wood. To model the truss plate, the same constant strain triangular element having isotropic elastic properties is used.

The interface (or "bond") element chosen is also a three node triangular element with relative nodal displacements as independent degrees-of-freedom so that there is compatibility with the continuums on either side of the element.

The reasons for the choice of the three nodal triangular elements are:

- (1) The triangular element is the simplest of all two-dimensional elements, and is advantageous in approximating any boundary shape.
- (2) The bandwidth is considerably less than the bandwidth using higher order elements.

The nonlinear load-slip behaviour is simulated by an incremental load technique.

In the subsequent sections of this chapter the development of the constant strain triangles for the modelling of wood and truss plate connection is presented. The development of the interface element and the incremental method used to simulate the nonlinear behaviour of the truss plate connection are described.

2.1 Assumptions

The following assumptions have been made in the formulation of the finite element model:

- (1) A typical truss plate connection is shown in Figure 2.1. The connection consists of two punched metal plates pressed into the wood. The contact surface between the truss plate and the wood is considered continuous.
- (2) The wood is assumed to be a homogeneous orthotropic linearly elastic material. Natural defects such as knots are not included in the model. Tensor transformation is used to modify the elastic properties.
- (3) Stiffness and strength properties and grain and ring angles are constant within each element.
- (4) The truss plate is assumed to be a homogeneous continuous isotropic linearly elastic material. The effect of the punched holes in the actual truss plate is neglected in calculating stresses, but the reduction of cross sectional area due to their presence is taken into account by an equal decrease of the cross sectional area in the model.

- (5) The physical behaviour of truss plate connections is nonlinear. The load-slip relationship for a tooth is taken to be as presented by Foschi (1977) as shown in Figure 1.4 and in the mathematical form of equation (1.0). The strength and stiffness properties are dependent on the direction of loading with respect to the truss plate-wood orientation. Hankinson's formula is used for calculating stiffness values for different orientations of truss plate and grain of wood (Foschi 1977).
- (6) The analysis is restricted to short term loading. Creep and relaxation of wood, and time effects on the truss plate connection are not considered in this study.

2.2 Development of Constant Strain Triangle with Orthotropic Elastic Properties

Wood can be described as an anisotropic, hygroscopic, viscoelastic and variable material. However, for analysis purposes wood is considered to be orthotropic, with the main principal directions corresponding to the longitudinal, radial and tangential axis as shown in Figure 2.2. These orthotropic elastic properties are used in the formulation of the element stiffness matrix (Fernandez and Polensek 1979), Goodman and Bodig 1978).

FIGURE 2.2 PROPOSED ORTHOTROPIC DIRECTIONS.

In previous studies various elements, such as eight-noded rectangular elements, three-noded triangular elements to three-dimensional brick elements have been employed in the finite element analysis of wood. In this work a constant strain triangular element is used.

The derivation of a constant strain triangle can be found in most introductory texts on the finite element method (e.g., Bowes 1975). The following formulation takes into consideration the orthotropic elastic property of wood.

2.2.1 Derivation of Constant Strain Triangular Element with Orthotropic Elastic Properties

The constant strain triangular element is shown in Figure 2.3, with nodes i, j, m numbered in a counter-clockwise order. At each node there are two degrees of freedom; two orthogonal translations. The unknown nodal displacements for the triangular element can be expressed in the form:

$$\{\delta'\} = \begin{Bmatrix} u'_i \\ u'_j \\ u'_m \\ v'_i \\ v'_j \\ v'_m \end{Bmatrix} \quad (2.0)$$

FIGURE 2.3 CONSTANT STRAIN TRIANGLE.

The general displacements function (u' , v') required to describe the behaviour of the elements can be expressed by two first order polynomials in x' and y' .

$$u' = \alpha_1 + \alpha_2 x' + \alpha_3 y' \quad (2.1)$$

$$v' = \alpha_4 + \alpha_5 x' + \alpha_6 y' \quad (2.2)$$

where α_i ($i=1, \dots, 6$) are the undetermined coefficients. The values of these coefficients are obtained by evaluating the functions u' and v' at the nodal points i , j and m . This procedure leads to six linear equations which are shown in matrix form:

$$\begin{bmatrix} 1 & x'_i & y'_i & 0 & 0 & 0 \\ 1 & x'_j & y'_j & 0 & 0 & 0 \\ 1 & x'_m & y'_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x'_i & y'_i \\ 0 & 0 & 0 & 1 & x'_j & y'_j \\ 0 & 0 & 0 & 1 & x'_m & y'_m \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} = \begin{Bmatrix} u'_i \\ u'_j \\ u'_m \\ v'_i \\ v'_j \\ v'_m \end{Bmatrix} \quad (2.3)$$

$$[A] \{\alpha\} = \{\delta\} \quad (2.4)$$

The coefficients α_i 's ($i=1, \dots, 6$) can now be expressed in the form:

$$\{\alpha\} = [A^{-1}] \{\delta'\} = \begin{bmatrix} A^{-1} & 0 \\ 0 & A^{-1} \end{bmatrix} \{\delta'\} \quad (2.5)$$

where:

$$[A] = \begin{bmatrix} 1 & x'_i & y'_i \\ 1 & x'_j & y'_j \\ 1 & x'_m & y'_m \end{bmatrix} \quad (2.6)$$

The strain components can be written as:

$$\begin{aligned} \epsilon_{x'} &= \frac{\partial u'}{\partial x'} \\ \epsilon_{y'} &= \frac{\partial v'}{\partial y'} \\ \epsilon_{x'y'} &= \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \end{aligned} \quad (2.7)$$

where:

$\epsilon_{x'}$ = strain in the x' direction

$\epsilon_{y'}$ = strain in the y' direction

$\epsilon_{x'y'}$ = shearing strain in the x' - y' plane

For a homogeneous, orthotropic linearly elastic material the stress-strain relationships can be expressed in the form:

$$\{\sigma'\} = [D] \{\epsilon'\} \quad (2.8)$$

where:

$$\{\sigma'\} = \begin{Bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \sigma_{x'y'} \end{Bmatrix} \quad (2.9)$$

and

$$[D] = \begin{bmatrix} \frac{E_{x'}}{1-\nu_{x'y'}\nu_{y'x'}} & \frac{E_{y'}\nu_{x'y'}}{1-\nu_{x'y'}\nu_{y'x'}} & 0 \\ \frac{E_{x'}\nu_{y'x'}}{1-\nu_{x'y'}\nu_{y'x'}} & \frac{E_{y'}}{1-\nu_{x'y'}\nu_{y'x'}} & 0 \\ 0 & 0 & G_{x'y'} \end{bmatrix} \quad (2.10)$$

where: $E_{x'}$ = modulus of elasticity along the x' axis

$E_{y'}$ = modulus of elasticity along the y' axis

$G_{x'y'}$ = shear modulus in the x' - y' plane

ν_{ij} = Poisson ratio = $\frac{\text{strain on the } j \text{ axis}}{\text{strain on the } i \text{ axis}}$

(For a uniform normal stress in the i axis direction)

The x' and y' axes may correspond to any two of the principal axes. The three principal axes being longitudinal, radial and tangential as shown in Figure 2.2.

The stiffness matrix of the element is formulated using the equilibrium method (Bowes 1975). The required element stiffness matrix $[K']_e$ for the $x'y'$ axes is expressed in the matrix form

$$[K']_e = [A^{-1}] [B]^T [D'] [A^{-1}] \text{ vol} \quad (2.11)$$

The nodal forces are calculated using

$$\{F'\} = [K']_e \delta' \quad (2.12)$$

Equation (2.11) defines the stiffness of the element and relates displacement and force components parallel to the x' and y' axes. The analysis of the truss plate connection is made with reference to the global axes, X and Y . The element stiffness must be transformed so that it relates the displacement and force components parallel to the global axes. By transformation of forces and displacements parallel to x' and y' axes, forces and displacements parallel to the global axes can be calculated. This procedure is shown in matrix form as:

$$\{F\} = [T] \{F'\} \quad (2.13)$$

and

$$\{\delta\} = [T] \{\delta'\} \quad (2.14)$$

where

$$[T] = \begin{bmatrix} \cos \xi & 0 & -\sin \xi & 0 \\ 0 & \cos \xi & 0 & -\sin \xi \\ \sin \xi & 0 & \cos \xi & 0 \\ 0 & \sin \xi & 0 & \cos \xi \end{bmatrix} \quad (2.15)$$

Hence

$$\{F\} = [T] [A^{-1}]^T [B]^T [D'] [A^{-1}] [T]^T \text{vol } \{\delta\} \quad (2.16)$$

$$\text{or} \quad \{F\} = [T] [K']_e [T]^T \{\delta\} \quad (2.17)$$

$$\text{and} \quad [K]_e = [T] [K']_e [T]^T \quad (2.18)$$

The element stiffness matrix with respect to the global axis is $[K]_e$.

After the element stiffness matrices are obtained, these are then assembled to form the global matrix $[K]$.

2.2.2 Numerical Accuracy

A computer program was developed which incorporates the constant strain triangle with orthotropic elastic properties. This program is based on the work of (Bowes 1975). To check the accuracy of the new version, the following problem has been solved and the computed results are compared with the exact solution.

The problem considered is that of distribution of stresses in an orthotropic plate with a circular hole under uniform stress conditions. The problem has been investigated by Zienkiewicz (1971) and is repeated here using the same element mesh used by Zienkiewicz as shown in Figure 2.4.

Figure 2.5 shows the comparison between the finite element results and the elasticity solution. The graph shows a high level of agreement.

FIGURE 2.4 A CIRCULAR HOLE IN A UNIFORM STRESS FIELD; $E_x = E_1 = 1$, $E_y = E_2 = 3$, $\nu = 0.1$, $\nu_2 = 0$, $G_{xy} = 0.42$

FIGURE 2.5

COMPARISON OF RESULTS OBTAINED
FROM THEORETICAL AND FINITE
ELEMENT SOLUTIONS.

2.3 Development of Constant Strain Triangular Element with Isotropic Elastic Properties

Modelling a plate with in-plane loading may be accomplished by using any two dimensional plane stress element (Zienkiewicz 1977). The three-noded constant strain triangle shown in Figure 2.6 is used in this study.

The derivation of the constant strain triangle with isotropic elastic properties follows the same steps used in the formulation of the constant strain triangle with orthotropic properties. Since the elastic properties are isotropic, the element stiffness can be formulated using the global axes X and Y.

2.3.1 Derivation of Element Stiffness Matrix

Following the same procedures as the formulation of the constant strain triangle shown in Section 2.2.1, the element stiffness matrix can be expressed in the form

$$[K_p]_e = [A^{-1}] [B] [D] [A^{-1}] \text{ vol} \quad (2.19)$$

with respect to the global axes;

where

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (2.20)$$

FIGURE 2.6

TRIANGULAR CONSTANT STRAIN ELEMENT
WITH ISOTROPIC ELASTIC PROPERTIES.

where E = modulus of elasticity
 ν = Poisson's ratio

$$\text{Then } \{F\} = [K_p]_e \{\delta\} \quad (2..21)$$

The consistent load vector for a constant strain triangle has been discussed in Section 2.2.1.

2.3.2 Numerical Accuracy

The theory discussed in the previous section is implemented in a computer program described by (Bowes 1975). The program used here is a modified version of the one mentioned above. To check the accuracy of this program, the problem of a thin disc with interval pressure investigated by Bowes (1975) was repeated using the same data. The computed results are exactly the same as those reported.

2.4 Development of the Interface Element

The wood and the truss plate have been modelled by constant strain triangular elements. The load-slip behaviour of the connection could be simulated by positioning springs between nodes of the two adjacent finite elements. An alternate method has been developed by Goodman (1968) by introducing connecting elements for the slip layers in rock. Scaffer (1977) expanded on this work and developed "bond elements" for various types of contact problems.

A flat triangular interface element with linear expansion formulated by Scaffer (1977) is used in this study. A more complete formulation of this interface element, taking into account this specific application, is presented here.

2.4.1 Derivation of Interface Element Stiffness Matrix

The three-noded triangular interface element with local axes at its centroid is shown in Figure 2.7. The nodes i, j, m are numbered in counter-clockwise order. The relative displacements Δu and Δv can be expressed by the following first order polynomials:

$$\Delta u = \alpha_1 + \alpha_2 x + \alpha_3 y \quad (2.22)$$

$$\Delta v = \alpha_4 + \alpha_5 x + \alpha_6 y \quad (2.23)$$

where α_i ($i=1, \dots, 6$) are the undetermined coefficients which can be calculated by evaluating the function Δu and Δv at the nodal points i, j, m . The relative displacements at the three nodes can be expressed as:

$$\begin{bmatrix} 1 & x_i & y_i & 0 & 0 & 0 \\ 1 & x_j & y_j & 0 & 0 & 0 \\ 1 & x_m & y_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_i & y_i \\ 0 & 0 & 0 & 1 & x_j & y_j \\ 0 & 0 & 0 & 1 & x_m & y_m \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} = \begin{Bmatrix} \Delta u_i \\ \Delta u_j \\ \Delta u_m \\ \Delta v_i \\ \Delta v_j \\ \Delta v_m \end{Bmatrix} \quad (2.24)$$

FIGURE 2.7

BOND ELEMENT WITH LINEAR DISPLACEMENT EXPANSION.

or

$$\{\Delta\delta\} = [A] \{a\} \quad (2.25)$$

The unknown coefficients a_i [$i=1, \dots, 6$] can be obtained in the following manner:

$$\{a\} = [A^{-1}] \{\Delta\delta\} \quad (2.26)$$

The slip can now be expressed by the nodal degrees of freedom

$$\{\Delta u\} = [P] [A^{-1}] \{\Delta\delta\} \quad (2.27)$$

The stiffness matrix is formulated by equating virtual external work, E_E , to the virtual work of shear stresses, which is dissipated into thermal energy by means of virtual slip.

The virtual external work corresponding to the nodal forces $\{\Delta F\}$ and virtual nodal deformations $\{\Delta\delta^*\}$ is given by

$$E_E = \{\Delta\delta^*\}^T \{\Delta F\} \quad (2.28)$$

The virtual work of the shear stresses, E_i is

$$E_i = \int_{\text{surface}} \tau (\{\Delta u\}) (\{\Delta u^*\}) dx dy \quad (2.29)$$

During virtual deformation the relative displacement is given by

$$\{\Delta u^*\} = [P] [A^{-1}] \{\Delta \delta^*\} \quad (2.30)$$

The load-slip relationship for a single tooth is given by

$$F(\Delta u) = (m_0 + m_1 \Delta u) \left(1 - e^{-\frac{k \Delta u}{m_0}}\right) \quad (2.31)$$

where F is in load per tooth. The $\tau - u$ relation may be obtained by the following expression:

$$\tau = \beta \cdot F(\Delta u) \quad (2.32)$$

where β is the density of teeth (number of teeth per unit area of plate). Hence the units for τ is N/m^2 .

The relationship between τ and Δu is given by

$$d\tau = G(\Delta u) d\Delta u \quad (2.33)$$

where G is defined on the slip modulus and has the dimension N/m^3 .

Orthogonal anisotropy of the τ - Δu relation is assumed whose principal axes are taken to coincide with the coordinate direction. This can be expressed in the following matrix form:

$$\begin{Bmatrix} \tau_x \\ \tau_y \end{Bmatrix} = \begin{bmatrix} G_x(\Delta u) & 0 \\ 0 & G_y(\Delta v) \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} \quad (2.34)$$

$$\{\tau\} = [G] \{\Delta u\} \quad (2.35)$$

Substituting equation (2.30) and equation (2.35) into equation (2.29), the expression for internal virtual work

$$E_i = \{\Delta \delta^*\}^T [A^{-1}]^T \left(\int_s [P]^T [G] [P] dx dy \right) [A^{-1}] \{\Delta \delta\} \quad (2.36)$$

From the virtual work condition, $E_E = E_i$, the equilibrium of the element follows:

$$\{\Delta F\} = [K_I]_e \{\Delta \delta\} \quad (2.37)$$

The element stiffness is given by:

$$[K_I]_e = [A^{-1}]^T \left(\int_s [P]^T [G] [P] dx dy \right) [A^{-1}] \quad (2.38)$$

The integral contains high order variables. Since the origin is at the centroid, this integration is simple.

The connection between relative nodal displacements and nodal forces of the adjacent finite elements is accomplished by incidence matrices. For the two adjacent elements A and B, as shown in Figure 2.7, the relationship is:

$$\begin{Bmatrix} \Delta u_i \\ \Delta u_j \\ \Delta u_m \\ \Delta v_i \\ \Delta v_j \\ \Delta v_m \end{Bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_{iA} \\ u_{jA} \\ u_{mA} \\ v_{iA} \\ v_{jA} \\ v_{mA} \\ u_{iB} \\ u_{jB} \\ u_{mB} \\ v_{iB} \\ v_{jB} \\ v_{mB} \end{Bmatrix} \quad (2.39)$$

or

$$\{\Delta \delta\} = [Z] \{\delta\} \quad (2.40)$$

This formulation holds for the nodal forces as shown in the following expression

$$\{\Delta F\} = [Z] \{F\} \quad (2.41)$$

The element stiffness matrices are assembled in the whole global stiffness matrix by

$$[K_I] = [Z]^T [K_I]_e [Z] \quad (2.42)$$

2.4.2 Nonlinearity of the Interface Element

The behaviour of a truss plate connection can be described by a nonlinear load-slip curve. Figure 1.1 shows a typical load-slip curve for a tension test and has been idealized by Foschi (1977) for a single tooth illustrated in Figure 1.5 and in mathematical form expressed in equation (1.0). These nonlinear properties are derived from four tests which are combinations of applied force with respect to the plate major axis (i.e., plate axis is perpendicular to the longest edge of the tooth) and with respect to the grain orientation. The test used in this study is the tests required by CSA 086 (via CSA S347) and therefore by the National Building Code of Canada to obtain truss plate design values. The four tests are:

- (1) the force is applied parallel to the plate major axis and parallel to the grain;

- (2) the force is applied parallel to the plate major axis and perpendicular to the grain;
- (3) the load is applied perpendicular to the plate major axis and parallel to grain;
- (4) the force is applied perpendicular to the plate major axis and also perpendicular to the grain.

Using these four tests and the use of Hankinson's Formula, (Foschi 1977), the values required for the solution in the global coordinate system are obtained.

The parameters required are those for a single tooth; however, the tests just described are performed on a truss plate containing many teeth. Therefore, the single tooth values are taken to be the mean value (Foschi 1977). The displacement measured in this test includes the deformation of the metal, the wood, and the relative displacement between the truss plate and the wood. This leads to further modifications in the parameters to obtain a relationship for the load-slip (i.e., relative displacement) curve. These modifications are explained in Appendix A.

Letting P_1 , P_2 , P_3 and P_4 represent a property obtained from the four respective tests. Consider Figure 2.8 in which θ is the angle between the plate major axis and the X direction, measured counterclockwise. The angle between the grain direction and the X axis is ω . Using Hankinson's Formula the properties in the direction A and B are

FIGURE 2.8 GEOMETRY FOR GENERAL LOAD-SLIP RELATIONSHIP.

$$P_A = \frac{P_1 P_2}{P_1 \sin^2 (\theta - \omega) + P_2 \cos^2 (\theta - \omega)} \quad (2.43)$$

and

$$P_B = \frac{P_3 P_4}{P_3 \sin^2 (\theta - \omega) + P_4 \cos^2 (\theta - \omega)} \quad (2.44)$$

For the values corresponding to forces applied in the X and in the Y direction, these may be obtained from

$$P_x = \frac{P_A P_B}{P_A \sin^2 \theta + P_B \cos^2 \omega} \quad (2.45)$$

and

$$P_y = \frac{P_A P_B}{P_A \cos^2 \theta + P_B \sin^2 \omega} \quad (2.46)$$

2.5 A Finite Element Computer Program to Analyze Truss Plate Connections

The constant strain triangle elements with orthotropic and isotropic linearly elastic properties and the interface element with nonlinear load-slip characteristics have been implemented in a computer program. The nonlinear properties of the interface element is simulated using an incremental technique.

2.5.1 Nonlinear Solution Technique

An incremental method is implemented to simulate the nonlinearity of the interface properties. The total load is subdivided into increments which can be of different magnitude. The load is applied one increment at a time and for each subdivision the equations are assumed to be linear. For each step of load the stiffness matrix, $[K]$, is recalculated and the solution is obtained as an increment of the displacements. These displacement increments are added to give the total displacement at the particular stage of loading. This process is continued until the total load has been reached.

2.5.2 Accuracy of Solution

An important requirement of the finite element method is that the solution converge within the limitations of the techniques used, to the exact solution of the problem. To fill this requirement, two cases must be investigated:

- (1) Convergence of stresses as the mesh is progressively subdivided;
- (2) Convergence of displacements as the number of loading increments is increased.

Increasing the mesh divisions leads to a rise in the computer running time and in the storage demands. Since the stiffness matrix has to be recalculated and solved for each

load increment, run time, hence cost, is dependent on the number of load increments. A suitable mesh subdivision and number of load increments to achieve a solution with sufficient accuracy are required.

A finite element solution of a truss plate connection under uniform tension is investigated using different mesh sizes and progressively increasing the number of load increments. The mesh divisions are illustrated in Figure 2.9.

The results from the three meshes are shown in Figure 2.10. It can be seen that the mesh configuration in Figure 2.7(b) yields suitable results. Figure 2.11 shows that 26 load increments are required for a satisfactory description of the load-slip behaviour.

2.5.3 Limitations

The finite element program developed in this study to analyze truss plate connections has the following limitations:

- i) The program is limited to truss plate connections under tension, bending and small compression loads. Wood to wood contact on compression joints has not been implemented. Also for compression and large bending in a joint, plate buckling must be taken into consideration.

FIGURE 2.9 MESH USED IN CONVERGENCE TESTS.

FIGURE 2.10 CONVERGENCE TEST RESULTS.

FIGURE 2.11 NONLINEAR CONVERGENCE.

- ii) Only short term loading cases can be analyzed since creep and relaxation of the connection and the wood were not incorporated in the formulation of the model.
- iii) The nonlinearity of the wood properties along with local material variability and defects are not taken into account in the model.

The failure criteria for the truss plate connection is defined by the behaviour of the components which comprise the truss plate. The truss plate can fail in shear, tension, or by buckling, all of which are dependent on the material properties of the plate and the truss plate connection configuration. The wood can fail in shear, tension and compression and is dependent on the mechanical properties of the wood. The truss plate can also fail by tooth pull-out. This type of failure depends on the joint detail and the load application and the strength properties of the plate. Failure can also be defined by excessive gross displacement of the joint or the amount of slip between the wood and the truss plate.

In this study plate buckling and compression failure in the wood due to contact between the wood members have not been modelled. The failure of the contact surface is defined when the slip between the wood and the plate is 0.76 mm. A displacement criteria is used because the load-deformation curve used enables the contact surface to transfer a load after the ultimate strength has been reached.

CHAPTER 3

ANALYSIS OF TRUSS PLATE CONNECTIONS

3.1 General

The previous chapter presented the theoretical method by which truss plate connections can be analyzed and simulated. In this chapter several configurations of truss plate connections are analyzed which will enable us to study the stress distributions in the wood, the plate and the assumed contact medium between the wood and plate. The load-displacement curves describing the overall behaviour of the joint are compared with results and observations of previous studies.

The last part of the chapter deals with the predictive ability of the method with respect to the behaviour of a structural system. A built-up wood joist composed of two pieces of small dimension lumber joined together by means of toothed metal plates.

3.2 Analysis of Tension Joints

The tension joints analyzed are shown in Figure 3.1 and Figure 3.2. Both of these truss plate connections are used in tests carried out in accordance with CSA Standard S347, Method of Test for Evaluation of Truss Plates Used in Lumber Joints to determine truss plate mechanical properties.

FIGURE 3.1

TENSION JOINT: LOAD IS APPLIED
PARALLEL TO THE GRAIN AND
PARALLEL TO THE PLATE.

FIGURE 3.2

TENSION JOINT: LOAD IS APPLIED
PERPENDICULAR TO THE GRAIN AND
PARALLEL TO THE PLATE.

The tension joint shown in Figure 3.1 is of a truss plate with teeth embedded perpendicular to the longitudinal axis of the wood. This type of joint is similar to splice joints in the bottom chord of trusses. In this study only an axial force is applied, neglecting any moment on the joint.

The uniaxial tension joint has been used in previous theoretical studies involving discrete and continuous fasteners as a means of developing theories which describe the stress distribution along the joint (Misra, Goodman, Suddarth, Landos, Isuyomov, etc.). These stress distributions can easily be plotted from the results using the F.E.M. The stress distributions along the sections shown in Figure 3.3 are plotted. The finite element mesh used in simulating the truss plate connection is shown in Figure 3.4.

The load-displacement curve for the overall behaviour of the joint from the analysis is plotted in Figure 3.5. As a comparison, the assumed load-displacement curve is also shown.

Using the mechanical properties of the truss plate used in this analysis, the equation developed by Misra and Esmay (1968) can be compared with the results of this study. The two curves are shown in Figure 3.6.

The stress distribution along various cross sections is of interest. The longitudinal stress at two load levels cross sections 1, 2, and 3 are shown in Figure 3.7.

FIGURE 3.3 TENSION JOINT ANALOG.

A) WOOD

B) TRUSS PLATE

C) CONTACT AREA (BOND ELEMENTS)

FIGURE 3.4

ELEMENT SUBDIVISION

FIGURE 3.5 LOAD VERSUS GROSS DISPLACEMENT

FIGURE 3.6 COMPARISON OF LOAD DISTRIBUTION ALONG TRUSS PLATE.

FIGURE 3.7 LONGITUDINAL STRESS DISTRIBUTION IN THE WOOD ALONG SECTIONS 1, 2, AND 3 OF THE TENSION JOINT.

All these graphs show a fairly constant stress distribution along the cross-section. A more revealing description of the stresses in the wood, the plate and the bond elements can be seen by plotting the longitudinal stress along cross section 4. The stress profile in the wood is shown in Figure 3.8. The distribution is constant just before the edge of the plate, then a slight increase near the edge of the plate followed by a decrease in stress to the cut edge of the wood. The stress profile in the plate is shown in Figure 3.9. The distribution shows an increase from the edges of the plate to a maximum at the centre, this profile being the opposite of that in the wood. Both figures show the profiles at two load-levels. The higher load affects the stresses as shown by a greater rate towards the maximum or minimum stress levels.

The bond stress is relatively constant as shown by Figure 3.10. There are slight increases in stress towards the extremities of the bonded areas, which has been described by Cramer and Lantos, and others.

The second type of tension joint is shown in Figure 3.2 where the load is applied parallel to both the plate and the wood grain in one member and is applied parallel to the plate and perpendicular to the woodgrain in the other member. This configuration is used in tests to obtain the properties of a truss plate where the load is applied parallel to the plate and perpendicular to the wood grain.

FIGURE 3.8 LONGITUDINAL STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 4.

FIGURE 3.9 LONGITUDINAL STRESS DISTRIBUTION IN THE TRUSS PLATE ALONG SECTION 4.

FIGURE 3.10 LONGITUDINAL STRESS DISTRIBUTION IN THE BOND ELEMENTS ALONG SECTION 4.

This type of loading condition is found on the bottom chord of joints on girders as described by McAlpine 1979, and is a simplification of web members in a truss. Both McAlpine (1979) and Egerup (1979) discuss the problem in such joints of the stresses perpendicular to grain. As stated by Egerup (1979), "Most plate design rules do not recognize the mode of failure." It is therefore of interest to examine the stresses perpendicular to grain in such a joint.

The joint is discretized as shown in Figure 3.11. The load is applied in increments to a level greater than the capacity of the truss plate connection. The joint is analyzed with two boundary conditions, as shown in Figure 3.12, to show the possible difference in the distribution of the stress perpendicular to the grain with a variation in position of load application.

The stress perpendicular to the grain along the bottom chord is plotted in Figure 3.13 for Case A and in Figure 3.14 in Case B. Both cases show a rapid increase of the stress perpendicular to the grain near the truss plate location. Case B shows a slightly greater maximum stress and greater gradient towards this maximum. The load level at which these stresses occur is at the service condition as defined by CSA Standard S347 as the ultimate load divided by 2.5. The stresses at this level are fairly high and should be taken into consideration as expressed by Egerup (1979) and McAlpine (1979).

A) WOOD

ELEMENT SUBDIVISION FOR THE TENSION JOINT WITH THE LOAD
APPLIED PARALLEL TO THE TRUSS PLATE AND PERPENDICULAR
TO THE GRAIN OF THE WOOD.

FIGURE 3.11

2

B) TRUSS PLATE

C) BOND ELEMENTS

FIGURE 3.11 ELEMENT SUBDIVISION FOR THE TENSION JOINT WITH THE LOAD APPLIED PARALLEL TO THE TRUSS PLATE AND PERPENDICULAR TO THE GRAIN OF THE WOOD.

FIGURE 3.12 TENSION JOINT CONFIGURATIONS

FIGURE 3.13 CASE A: STRESS PERPENDICULAR TO GRAIN PLOTTED ALONG THE BOTTOM WOOD MEMBER.

FIGURE 3.14 CASE B: STRESS PERPENDICULAR TO GRAIN PLOTTED ALONG THE BOTTOM WOOD MEMBER.

The stress distribution along the truss plate is shown in Figure 3.15 at service and ultimate load levels. Figure 3.16 shows these same stress distributions in a nondimensionalized form by dividing the stress at each point along the plate by the maximum stress in the plate. Both stress distributions show approximately the same curve. As stated in the first tension joint studied, the stress increases from the edges of the truss plate to a maximum value at the cross section of the truss plate at the juncture of the two wood members.

The shear stress versus relative displacement for the bond elements is plotted in Figure 3.17. Curve A1 and Curve B1 are the curves obtained from the analysis. Curve A1 is of a bond element located between the truss plate and vertical member where the applied load is parallel to both the truss plate and the wood grain. Hence this curve is similar to the load-displacement curve for the load applied parallel to the truss plate and parallel to the grain. Curve B1 is of a bond element located between the truss plate and the horizontal member where the applied load is parallel to the truss plate and perpendicular to the grain. Similarly this curve is in good agreement with the load-displacement curve for the load applied parallel to the truss plate and perpendicular to the grain. A comparison between these two curves shows the complexity of the behaviour of the joint. Each of the connected areas of the truss plate behaves in a different manner as indicated by the two load-slip curves for the bond elements. The total behaviour of the connection is affected by the strength and stiffness of the areas connected. The configuration shown

FIGURE 3.15 STRESS IN THE y -DIRECTION IN THE TRUSS PLATE ALONG SECTION 1.

FIGURE 3.16

STRESS DISTRIBUTION IN THE TRUSS
PLATE ALONG SECTION 1 AS OBTAINED
FROM THE FINITE ELEMENT SOLUTION.

FIGURE 3.17 COMPARISON OF THE SHEAR STRESS - RELATIVE DISPLACEMENT CURVES FOR THE TWO CONTACT AREAS.

FIGURE 3.18 LOAD - GROSS DISPLACEMENT CURVE AS OBTAINED FROM THE FINITE ELEMENT SOLUTION.

in Figure 3.2 is used to obtain properties of the truss plate connection where the load is applied parallel to the plate and perpendicular to the grain. This is not strictly correct as shown by the analysis. The test actually incorporates the strength and stiffness of two different systems.

The load-displacement curve for the overall behaviour of the truss plate connection is shown in Figure 3.18. A comparison with the assumed curve shows fairly good agreement in terms of the stiffness of the joint but the analysis indicates a slightly higher ultimate strength.

3.3 Analysis of Heel Joints in Long Span Trusses

In 1981, the firm of Morrison, Hershfield, Burgess and Huggins, Limited (MHBH), conducted an "Investigation of Structural Safety and Economics of Long Span Light Wood Trusses Connected with Truss Plates" (MHBH 1981). The study of the structural safety of long-span light wood trusses was conducted over the concern of the size effect phenomenon in the strength of wood elements as related specifically to long span trusses. Concern "that while the engineering principles traditionally applied to the design of short-span trusses have been proven adequate, the same principles and methods may not be adequate for long-span trusses" (MHBH 1981). A series of short-span and long-span trusses were fabricated and then loaded to failure. The part of the study which is of relevant interest is the modes of failure of the long-span trusses.

3.3.1 Heel Joint: Long Span, Large Chord Truss

The truss geometry chosen in the investigation was the common 4/12 gable truss with the "Fink" or "W" type web configuration. There were two types of long span trusses tested. One truss was made up of S-P-F No. 1 38x286's (2x12) for both the top chord and bottom chord with Construction 38x89 (2x4) and 38x114's (2x5) as web members. The other was constructed of S-P-F No. 1 38x184's (2x8) for the top and bottom chord and S-P-F No. 2 38x89's (2x4) for the web members. A span of 21.3 m. (70 ft.) was chosen to be representative of a long span. Under a balanced load condition, ten of each of the above mentioned trusses were loaded to failure.

The predominant failure mode for the long-span, large chord trusses was failure in shear in the top chord while the remainder failed in bottom chord tension. "The top chord shear failures occurred over the truss reaction points with the wood separation following the line of the uppermost row of teeth in the heel connector plates" (MHBH 1981). Figure 3.19 shows the heel joint detail in which the truss bottom chord is scarfed and the truss plate is oriented with its long axis centred over the scarf cut.

The heel joint detail is to be analyzed using the analysis technique described in the previous chapters. The subdivided heel joint and boundary conditions are shown in Figure 3.20. The applied loads are obtained from two types of analysis; one method assumes pinned joints, the other assumes rigid joints. The stresses and observations are therefore based on these two types of member loading

FIGURE 3.19

HEEL JOINT ANALOG USED TO SIMULATE THE HEEL JOINT DETAIL IN LONG SPAN LARGE CHORD TRUSSES

FIGURE 3.20 ELEMENT SUBDIVISION OF THE HEEL JOINT ANALOG:
a) WOOD, b) TRUSS PLATE, c) CONTACT AREA.

FIGURE 3.20 (cont'd) ELEMENT SUBDIVISION OF THE HEEL JOINT
ANALOG: a) WOOD, b) TRUSS PLATE, and
c) CONTACT AREA.

FIGURE 3.20 (cont'd) ELEMENT SUBDIVISION OF THE HEEL JOINT ANALOG:
a) WOOD, b) TRUSS PLATE, and c) CONTACT AREA.

conditions. For both types of member loads, the stresses are plotted at the design load and at the average failure load. The cross-sections of interest are indicated in Figure 3.19.

The stress distributions across Sections 1 to 4 of the wood members are plotted in Figure 3.21 to Figure 3.28. In most cases, the shape of the stress distributions using loads from the pinned analysis and rigid analysis are similar with variation in the magnitude of the stress. Section 1 is located near the plate, while Section 2 is taken at the mid-length of the plate. The longitudinal stress distribution indicates a fairly high moment and axial tension force. The presence of tension perpendicular to the grain is noted with the stress being greater along Section 2. The shear stress distribution is dissimilar at the two sections in contrast to the stress distribution of the stress parallel to the grain and the stress perpendicular to grain. Along Section 2, the shear stress seems to show a change in slope at a point which coincides with the outer edge of the truss plate. The influence of the truss plate on the stresses is only observed in the shear stresses across Section 2.

The stress distribution across Section 3, as shown in Figures 3.25 to 3.26, indicates a high compressive flexural stress, tension perpendicular to grain and fairly high shear stress. The stress distributions across Section 4 are shown in Figures 3.27 to 3.28. The stress parallel to grain is less than the respective stress across Section 3. The stress perpendicular to the grain is present with its magnitude approximately doubling in value. The most noticeable variation is in the shear stress which increases

FIGURE 3.21 STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 1, RIGID ANALYSIS

FIGURE 3.22 STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 1, PINNED ANALYSIS

FIGURE 3.23 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 2, RIGID ANALYSIS.

FIGURE 3.24 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 2; PINNED ANALYSIS.

FIGURE 3.25 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 3;
RIGID ANALYSIS.

FIGURE 3.26 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 3; PINNED ANALYSIS.

FIGURE 3.27 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 4;
RIGID ANALYSIS.

FIGURE 3.28 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 4; PINNED ANALYSIS.

Figure 3.29 Line of maximum shear stress in the top chord.

FIGURE 3.30 STRESS PERPENDICULAR TO GRAIN ALONG THE TOP CHORD.

from a maximum stress of 0.39 Mpa (56psi) to over 1.28 MPa (200 psi). The shape of the shear stress distribution does not seem to be affected by the presence of the truss plate as noticed in the bottom chord member. However, the shear stress is the only stress which shows significant difference from Section 3 to Section 4.

A comparison of allowable stresses to the stress levels obtained from the analysis indicates that the shear stress in the top chord is the most critical. There is interaction between the shear stress and tension perpendicular to grain. However this interaction has not been considered in this study.

The location of the points of highest shear stress in the top chord are shown in Figure 3.29. The line connecting these points follows the upper edge of the truss plate which conforms with the description of the majority of the failures in long-span, large-chord trusses tested by MHBH (1980).

A point of interest is the presence of tension stress perpendicular to grain in both the top and bottom chords. The maximum stress perpendicular to grain along the top chord is shown in Figure 3.30.

The stress distributions along Sections 5 to 7 of the truss plate are shown in Figures 3.31 to 3.36. The effect of the contact area defined by the bond elements is noticeable in the stress distributions across the various sections in the truss plate.

Section 5 is located near the support location of the truss. The stress in the x-direction indicates a bending

FIGURE 3.31 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 5: RIGID ANALYSIS.

FIGURE 3.32 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 5; PINNED ANALYSIS.

FIGURE 3.33 STRESS DISTRIBUTION IN THE TRUSS PLATE ALONG SECTION 6;
RIGID ANALYSIS.

FIGURE 3.34 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 6; PINNED ANALYSIS.

FIGURE 3.35 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 7;
RIGID ANALYSIS.

FIGURE 3.36 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 7;
PINNED ANALYSIS.

moment due to the compression force of the top chord and tension force in the bottom chord to be resisted by the truss plate. The stresses in the x-direction at Sections 5 to 7 are similar. There is a slight difference in the slope at the midpoint of the plate where the intersection of the top chord and bottom chord occurs. As defined by the bond elements, there is no contact between the truss plate and the wood members so that the transfer of force must take place through the truss plate.

The stress in the y-direction is an indication of the possible load transfer. At Section 5, nearest the truss support, the stress distribution shows a compression stress with a maximum at the mid-height of the truss plate at the juncture of the two chords. There is tension stress in the area in contact with the top chord. In Section 6, the truss plate in contact with the top chord is in compression while the area in contact with the bottom chord is in tension. The maximum stresses are at the extremities of the truss plate, with a discontinuity at the centre of the truss plate. The stress distribution across Section 7, at the top of the plate, shows the plate in tension, increasing from the centre to maximum at the top and bottom edge of the truss plate. These stress distributions in the truss plate conform with the tension stress found perpendicular to grain in the top and bottom chord wood members. At the bottom end of the truss plate near the reaction force from the truss support, the truss plate is in compression and then is in tension away from the support. The bottom chord and top chord are displacing relatively to each other and to obtain

compatibility of displacement the truss plate acts as a tension member in this case. That is, the bottom chord displacement is greater than the top chord displacement and therefore to restrain the bottom chord member with respect to the top chord member the truss plate is in tension. This mechanism induces tensile stresses in both the top chord and bottom chord.

The stress distributions plotted in Figures 3.37 to 3.42 of Sections 8 to 10 of the bond elements represent the load transfer mechanism between the truss plate and the wood members. The cross-sections to be studied were chosen to correspond with the cross-sections studied in the truss plate. Stresses along the global x and y axis are shown. In all cases, the magnitude of the shear stresses at the average failure load are below the ultimate load for teeth failure and teeth pull out. There are two areas of contact: the truss plate to the top chord and the truss plate to the bottom chord. The chord members are under load mechanisms and the truss plate is restraining both members. The contact areas show two different types of stress distributions, which correspond to the respective load mechanisms being transferred from the two wood members to the truss plate.

The stress in the global x-direction along the top chord contact area remains relatively constant from Section 8 to Section 10. The stresses are well below the ultimate shear strength of the truss plate connection. The stresses in the global y-direction along the top chord are more complex. The stress distributions from Section 8 to Section 10 show a stress reversal which corresponds to a stress

FIGURE 3.37 STRESS DISTRIBUTIONS IN THE BOND ELEMENTS ALONG SECTION 8; RIGID ANALYSIS.

FIGURE 3.38 STRESS DISTRIBUTIONS IN THE BOND ELEMENTS ALONG SECTION 8; PINNED ANALYSIS.

FIGURE 3.39 STRESS DISTRIBUTION IN THE BOND ELEMENTS ALONG SECTION 9; RIGID ANALYSIS.

FIGURE 3.40 STRESS DISTRIBUTION IN THE BOND ELEMENTS ALONG SECTION 9; PINNED ANALYSIS.

FIGURE 3.41 STRESS DISTRIBUTIONS IN THE BOND ELEMENTS ALONG SECTION 10; RIGID ANALYSIS.

FIGURE 3.42 STRESS DISTRIBUTIONS IN THE BOND ELEMENTS ALONG SECTION 10; PINNED ANALYSIS.

reversal in the stress in the global y-direction in the truss plate. The contact stresses vary from +0.41 MPa (60.0 psi) to -0.41 MPa (-60.0 psi) which are well below the ultimate shear stress.

The stress in the global x-direction along the bottom chord contact area also remains constant from Section 9 to Section 10. While at Section 8, near the support, the contact shear stress shows a reversal along the section. The stresses are also below the ultimate shear stress at the average failure load. The contact shear stress in the global y-direction is at a maximum at Section 8 near the support and decreases to Section 9 at the mid length of the truss plate and remains at this level to Section 10. The magnitude of the stresses is below the ultimate shear stress at the failure load.

3.3.2 Heel Joint: Long Span Small Chord Truss

The second type of heel joint detail of interest is typically used for long span small chord trusses. All the trusses tested by MHBH in this series failed by tension in the bottom chord with the majority of failures located at the heel connection. Figure 3.43 shows a typical detail of the joint in which the truss bottom chord is scarfed and the truss plate is installed with its major axis horizontal. "These typical failures generally appeared to be initiated at the inboard end of the truss plate connection" (MHBH, 1981) within the scarfed portion of the bottom chord.

FIGURE 3.43 A) HEEL JOINT ANALOG USED TO SIMULATE THE HEEL JOINT DETAIL IN LONG SPAN SMALL CHORD TRUSSES.

TRUSS PLATE

CONTACT AREA

FIGURE 3.43 B) HEEL JOINT ANALOG USED TO SIMULATE THE HEEL JOINT DETAIL IN LONG SPAN SMALL CHORD TRUSSES.

FIGURE 3.44 ELEMENT SUBDIVISION OF THE HEEL JOINT ANALOG

B) TRUSS PLATE

C) BOND ELEMENTS

FIGURE 3.44 ELEMENT SUBDIVISION OF THE HEEL
JOINT ANALOG.

The mesh used to simulate the heel joint detail as well as the boundary conditions is shown in Figure 3.44. As in the previous heel joint detail studied, the loads applied to the heel joint are obtained by pinned and rigid analyses. The stresses obtained from the two different member loads are considered to be possible bounds for the stresses in the respective mediums. The stresses of locations along the heel joint, as shown in Figure 3.43, are plotted for the two load cases. Again for both member loading conditions, the stresses are plotted at the design load and at the average failure load.

The stress distribution across the specified cross-sections of the top and bottom chord are shown in Figures 3.45 to 3.57. Graphs of the stress parallel to grain, perpendicular to grain, and shear stress, are plotted.

The stress distributions across Section 1 to 6 of the timber members are plotted in Figures 3.45 to 3.56. Sections 1 to 3 are located at points of interest along the bottom chord. Section 1 is away from the truss plate and through the full section of the bottom chord. Section 2 is located at the edge of the truss plate where the cross section of the bottom chord is reduced due to the scarf and Section 3 is at the mid-length of the truss plate, through the scarf end of the bottom chord.

FIGURE 3.45 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION I;
RIGID ANALYSIS.

FIGURE 3.46 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 1; PINNED ANALYSIS.

FIGURE 3.47 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 2;
RIGID ANALYSIS.

FIGURE 3.48 STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 2; PINNED ANALYSIS.

FIGURE 3.49 STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 3;
RIGID ANALYSIS.

FIGURE 3.50 STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 3, PINNED ANALYSIS.

FIGURE 3.51 STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 4, RIGID ANALYSIS.

FIGURE 3.52 STRESS DISTRIBUTION IN THE WOOD ALONG SECTION 4, PINNED ANALYSIS.

FIGURE 3.53 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 5, RIGID ANALYSIS.

FIGURE 3.54 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 5, PINNED ANALYSIS.

FIGURE 3.55 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 6, RIGID ANALYSIS.

FIGURE 3.56 STRESS DISTRIBUTIONS IN THE WOOD ALONG SECTION 6, PINNED ANALYSIS.

The stress parallel to grain increases from Section 1 to Section 2 and then decreases from Section 2 to Section 3. Across Section 1, the stress parallel to grain indicates the presence of an axial tension force and a moment which produces tension in the bottom and compression at the top of the section. While at Section 2, the stress parallel to the grain indicates an increase in the tensile stress and the flexural stress has reversed; hence the moment, since there is now tension at the top of the section and compression at the bottom of the section. The magnitude of the stresses has also increased. At Section 3, the stresses parallel to the grain indicate flexural stresses due to a moment and a reduction in tensile stresses. The flexural stresses shown in Figures 3.49 and 3.50 indicate tension at the top of the section and compression at the bottom.

The stresses perpendicular to the grain also reverse from being in a state of compression along Section 1 to a condition of tension along Section 3, while across Section 2 the stresses vary from compression in the top half of the section to tension in the bottom half. The analysis produces shear stresses as shown by the distributions which vary in direction along the cross-section. The magnitude of the shear stresses increases from Section 1 to Section 3 and also reverses from Section 1 to Section 2. The shear stresses are above the allowable shear stresses at Sections 2 and 3.

Sections 4 to 6 in the top chord are located at points similar to the sections in the bottom chord. The stress parallel to the grain decreases from Section 4 to Section 6.

The axial tension stress remains relatively constant from Section 4 to Section 5 and from the flexural stress distributions the flexural stress decreases from Section 4 to Section 5. The stress distribution of the stress parallel to the grain across Section 6 is bilinear, with the point of intersection of the two lines being near the upper edge of the truss plate. The stress distribution is constant from the top of the section to a point just above the top edge of the truss plate, then decreases linearly to the bottom of the chord.

The stress perpendicular to the grain shows tension stresses across Sections 4 to 6 with the maximum at Section 4, then decreasing to Sections 5 and 6.

The shear stress distributions from the analysis are as shown in Figures 3.51 to 3.56. The shear stress is above the allowable stress, and increases from Section 4 to Section 6. The shear stress reverses at Section 5 which is located at the edge of the truss plate. The effect of the truss plate is not as noticeable on the shear distributions on this heel joint compared to the heel joint used on the long span large chord truss.

Stress distributions along cross-sections at each extremity of the truss plate and at its mid-length are plotted in Figures 3.57 to 3.62.

The stress in the global x-axis direction along Section 7 can be defined in terms of an axial tensile stress and flexural stress produced by a tension force and moment

FIGURE 3.57 STRESS DISTRIBUTIONS IN TRUSS PLATE ALONG SECTION 7, RIGID ANALYSIS.

FIGURE 3.58 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 7, PINNED ANALYSIS.

FIGURE 3.59 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 8, RIGID ANALYSIS.

FIGURE 3.60 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 8, PINNED ANALYSIS.

FIGURE 3.61 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 9, RIGID ANALYSIS.

FIGURE 3.62 STRESS DISTRIBUTIONS IN THE TRUSS PLATE ALONG SECTION 9, PINNED ANALYSIS.

causing a tensile stress at the top of the section and a compressive stress at the bottom of the section. The stress distribution of the stress in the global x-direction along Section 8 is bilinear with the point of intersection located near the juncture of the top chord and bottom chord timber members. The two areas of the steel plate are connected to two different members, both of which apply different forces on the truss plate. The stress distribution along Section 9, as shown in Figures 3.61 and 3.62, is more complex. The stress is constant at the bottom of the truss plate, then abruptly increases to a maximum near the juncture of the top and bottom chord and then the stress reverses and decreases toward the top of the plate. The three stress distributions show clearly the complexity of the load application on the truss plate.

The stresses in the global y-direction along Sections 7 and 8 are compressive with the stress distribution along Section 8 as bilinear, increasing to a maximum at the scarf point, then decreasing to the top of the plate. The stress distribution along Section 9 indicates a small tensile stress.

The shear stresses reverse from Section 7 to Section 9. Along Section 7 the shear stresses increase from the top of the truss plate to the bottom of the section. For Section 8 and 9 the maximum shear stress occurs approximately at the scarf joint and decreases toward the edges of the truss plate. In all three sections the maximum shear stress occurs at the scarf joint.

The stress in the y-direction along the scarf joint is shown in Figure 3.63. The stress distribution from the application of member forces obtained from a pinned analysis shows that the plate is in compression from the support location to a maximum and then decreases and is in tension away from the support. The stress distribution from the application of member forces obtained from the rigid analysis differs from the pinned analysis stress distribution. The stress distribution shows the plate is in tension near the support location and is in compression away from the support.

The contact areas are defined as shown by the finite element mesh shown in Figure 3.44. The contact shear stress for Sections 11 to 13, which correspond to Sections 7 to 9 in the truss plate, is shown in Figure 3.64. Section 11 is through the contact area between the top chord member and the truss plate. The contact shear stress in the global x-direction is maximum at the top of the area and decreases toward the bottom of the area at the juncture of the two chords. Section 12 is through the two contact areas: the top area between the top chord and the truss plate, and the bottom area between the bottom chord and the truss plate. The contact shear stress in the top area is similar to the stress distribution along Section 11. The contact shear stress is a maximum at the top of the area and decreases to a minimum at the bottom of the contact area which corresponds to the location of the juncture between the top chord and the bottom chord. The contact shear stress in the bottom area is more complex, as shown in Figure 3.64.

FIGURE 3.63 STRESS IN THE GLOBAL Y-DIRECTION IN THE TRUSS PLATE ALONG SECTION 10 AT THE AVERAGE FAILURE LOAD.

FIGURE 3.64 STRESS IN THE GLOBAL X-DIRECTION IN THE BOND ELEMENTS ALONG SECTIONS 11, 12 AND 13.

FIGURE 3.65 AREAS OF OVERSTRESS IN THE CONTACT AREA.

The contact shear stress changes direction which indicates a shearing action within the area. Section 13 is through both contact areas, the top area is narrowing while the bottom area is increasing to conform to the scarf joint. The contact shear stresses in the top area are at a maximum at the top of the area and decrease towards the bottom of the area. The contact shear stresses are at a maximum at the top of the area which corresponds to the location of the scarf cut and decreases towards the bottom and then changes direction near the edge of the contact area. In general, the contact shear stresses are increasing from the contact area near the support to the contact area at the opposite end of the areas around the scarf cut. The maximum shear stresses at the upper right end are above the ultimate shear strength of the truss plate connection. Excessive slips occur at this area and load redistribution over the rest of the plate would be occurring. Figure 3.65 indicates the areas of overstress of the truss plate connection at the average failure load. The analysis also shows that the truss plate connection is overstressed at the design load level.

From the analysis the critical concerns of this heel joint detail is the excessive longitudinal stress and shear stress in the bottom chord and the overstress of the truss plate connection. As in the heel joint detail for the long span, large chord trusses, tension stress perpendicular to grain has been indicated in this analysis.

3.4 Splice Joints with Extended Truss Plate Connections

A certain amount of interest has led to a study of possible means to strengthen the bottom chord splices in trusses. A limited study was conducted at the University of Toronto in 1977 (Hayward 1977). Two possible alternatives were studied; one method is to extend the width of the plate past the edge of the timber chord member such that the increase in net section will also increase the strength of the joint; the second method, a modification of the first, is to embed the extended portion of the truss plate into a block of wood adjacent to the timber member. The second alternative is shown in Figure 3.66. The empirical study examines the effectiveness of the two alternatives with various increments of extension of the truss plate past the chord. The plates were extended past the chord's edge in approximately 13mm (0.5 inch) increments to a maximum extended width of 95mm (3.75 inches) for joints with blocking. The joint was detailed as to prevent tooth withdrawal failure before plate failure. The truss plate connection was loaded in tension.

The failure mode of the joint with the extended plate embedded in block was similar to the failures of the extended plates without blocking. The "plate failures were initiated on the unextended side and progressed along the abutting member's interface toward the extended section" .. and "the plate extension fractured along the same line .. fracturing the blocking simultaneously." For plates 178mm (7 inches) or greater, the blocking and the extended plate did not break, as secondary fracture occurred along the border between the blocking and the member's edge as a shear

FIGURE 3.66

TYPICAL TENSION JOINT WITH THE TRUSS PLATE
EXTENDED INTO THE WOOD BLOCKING.

FIGURE 3.67

TYPICAL FAILURE MODE OF TENSION JOINTS WITH
TRUSS PLATE EXTENDED INTO WOOD BLOCKING.

failure (Hayward 1977). The failure mode is shown in Figure 3.67.

A finite element analysis of a splice joint with the truss plate extensions embedded in blocking under a tensile force is examined. Truss plates with extensions of 37, 67, and 83mm (1.45, 2.65 and 3.25 inches) are used to study the influence of the extended plate with blocking on the strength of the joint. The finite element mesh for the three splice joint details is shown in Figures 3.68 to 3.70.

The maximum stress was found to occur at the midlength of the plate where the two timber members butt. The distributions of the stress in the global x-direction along this cross-section for the three splice joints are shown in Figure 3.71.

The tensile axial load is applied in increments to enable monitoring of the stresses in the truss plate and to evaluate the load level at which the truss plate reaches the yield stress. As shown by the stress distributions, the maximum stress occurs at the unextended edge of the truss plate and decreases towards the edge of the extended portion of the truss plate. Due to the eccentric load condition, a bending moment is induced in the truss plate as indicated by the stress distribution. In the three cases studied, the yield stress is reached at the same load level and the stress distributions are the same within the common portions of the truss plate. The analysis indicates that there is no noticeable increase in the splice joint strength with increase in the truss plate extension.

FIGURE 3.68 ELEMENT SUBDIVISION FOR mm TRUSS PLATE EXTENSION,
a) WOOD, b) TRUSS PLATE, and c) BOND ELEMENTS.

b) Truss Plate

c) Bond Elements

FIGURE 3.68 (cont'd)

ELEMENT SUBDIVISION FOR mm TRUSS
PLATE EXTENSION; a) WOOD, b) TRUSS
PLATE, AND c) BOND ELEMENTS.

FIGURE 3.69 ELEMENT SUBDIVISION FOR mm TRUSS PLATE EXTENSION;
a) WOOD, b) TRUSS PLATE, AND c) BOND ELEMENTS.

b) Truss Plate

c) Bond Elements

FIGURE 3.69 (cont'd)

ELEMENT SUBDIVISION FOR mm TRUSS
PLATE EXTENSION; a) WOOD, b) TRUSS
PLATE, AND c) BOND ELEMENT.

FIGURE 3.70 ELEMENT SUBDIVISION FOR mm TRUSS PLATE EXTENSION;
a) WOOD, b) TRUSS PLATE, and c) BOND ELEMENTS.

b) Truss Plate

c) Bond Elements

FIGURE 3.70 (cont'd)

ELEMENT SUBDIVISION FOR mm TRUSS
PLATE EXTENSION; a) WOOD, b) TRUSS
PLATE, and c) BOND ELEMENTS.

FIGURE 3.71

COMPARISON OF STRESS DISTRIBUTIONS IN THE TRUSS PLATE WITH VARIOUS PLATE EXTENSIONS.

FIGURE 3.72

SHEAR STRESS DISTRIBUTION ALONG THE TRUSS
PLATE WITH VARIOUS PLATE EXTENSIONS.

FIGURE 3.73

LINES OF MAXIMUM TENSILE STRESS AND SHEAR STRESS.

The shear stress distribution across the section, as shown in Figure 3.72, indicates the maximum shear stress occurs at the intersection of the blocking and the top edge of the chord members. The lines A-A' and B-B' in Figure 3.73 show the lines of maximum axial stress and maximum shear stress respectively. The analysis conforms to the description of the failure modes of the splice joint with plate extension embedded in blocking, as described by Hayward 1977.

3.5 Simulation of a Built-Up Joist

While the previous case studies are involved with the analysis and simulation of specific truss plates, the behaviour of a structural system comprised of more than one truss plate connection is simulated in this section.

"Built-up wood joists are fabricated by joining together two pieces of small dimension lumber to form a larger sized unit. Toothed metal plates are pressed on both sides of the joist to provide discrete means of shear transfer and achieve some degree of composite action" - Salinas (1980). In a previous study by Williams (1979), the effect of plate spacing on the stiffness of built-up wood joists was investigated. Figure 3.74 shows a typical arrangement for these tests where the load was applied by a 400,000 lb. Tinius Olsen Universal Testing Machine. The metal plates were 100x110 mm, 20 gauge, with a tooth density of 1.24 teeth per square centimetre. Spruce-Pine-Fir, Number 1 grade lumber at 12% moisture content was used in all tests. A series of tests were conducted with truss plate spacings of 100, 280 and 600 mm centre to centre. The test results

FIGURE 3.74 TYPICAL BUILT-UP JOIST TEST SETUP.

FIGURE 3.75 BUILT-UP JOIST ANALOG.

FIGURE 3.76 ELEMENT SUBDIVISION OF THE BUILT-UP JOIST
ANALOG: a) WOOD, b) TRUSS PLATE, AND c) CONTACT AREA.

from the built-up joist comprised of two 38x140's (2x6's) joined together by truss plates spaced at 600mm centre to centre are to be used in this study.

Figure 3.75 shows the configuration of the built-up joist, along with loading and boundary conditions, to be used to simulate the actual behaviour of the built-up joist. The discretized form of the built-up joist is shown in Figure 3.76. In the finite element model, the elastic parameters for the wood are obtained by using the probabilistic formulas, as proposed by Suddarth. The appropriate elastic properties for the truss plate behaviour are used as discussed previously.

Williams tested the built-up wood joist by loading up to a level that would produce approximately a 13mm (0.5 inch) deflection at midspan. The load-deflection curve from Williams' study is shown in Figure 3.77, and, as shown, a linear response from the structural system was observed. Also shown in Figure 3.77 is the load-deflection computed from the finite element method. At the load levels from the test, the finite element method results also produce a linear load deflection curve. The stiffness obtained from the analytical process is slightly higher as indicated by the ratio of the computed stiffness to the actual stiffness being 1.15.

The analytical model assumes that there is no bearing or frictional interaction between the two wood members, so that the forces are transferred completely through the truss

FIGURE 3.77

COMPARISON OF EMPIRICAL AND FINITE ELEMENT RESULTS OF LOAD VERSUS MID-SPAN DEFLECTION CURVES.

plate. The two wood elements that make up the built-up joist are in contact to some extent, so that bearing and frictional forces will contribute to the composite action of the structural system and increase the stiffness of the built-up joist. Also, as stated previously, the wood material is assumed as a continuous homogeneous medium, therefore increasing the predicted analytical stiffness. Considering the assumptions made in the assigning of the elastic parameters and the above arguments, the results from the analytical simulation seem reasonable.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

This study proposes a finite element method to analyze truss plate connections. A finite element computer program which is capable of analyzing and simulating truss plate connections, incorporating the orthotropic nature of wood and the nonlinear load-slip characteristics of the connection has been developed. This enables both the study of the overall joint behaviour and of the specific components which comprise the truss plate connection.

As shown in the previous chapter, the models of the truss plate connections studied exhibit a good ability in predicting the behaviour of the actual truss plate connections. Taking into account the deterministic nature of the study and the means by which material properties are obtained further substantiates the qualifying of the results. The modelled truss plate connections' overall joint characteristics and respective failure modes are in concurrence with observed behaviour.

The inherent nature of the finite element method allows the investigation of stresses in the wood, the plate and the contact area between the wood and the plate. This enables the study of truss plate connections to an unprecedented level, as shown in all the models of such connections presented in the thesis.

The simulation of a structural system comprising several truss plates predicted results within 15% of the actual behaviour.

Both the modelling of discrete truss plate connections and of the built-up joist showed some capacity for predictions of behaviour. Inherent to the finite element analysis, precise material properties are required for an accurate analysis. This was not possible due to the nature of the materials, especially wood, and the difficulty and expense in obtaining these exact mechanical properties. The mechanical properties of the wood are obtained from predictive equations given by Goodman 1972. However, even if this method is not exact, the results obtained conform to the behaviour of the truss plate connections and their failure modes.

4.2 Recommendations

The study has attempted to investigate the behaviour of selected discrete truss plate connections and the overall behaviour of a structural system comprising of truss plates. The accuracy of the method depends on the completeness of the model and the accuracy of the mechanical properties of the various media. The study has indicated the need for improvement of the model and the possible areas which the analysis can contribute.

The model can be improved in the following areas:

- 1) In this study, the wood is assumed to be a linear, orthotropic, homogeneous material. The effects of defects, such as knots, should be included in the model.
- 2) The model presented does not incorporate the contact between two wood members. Compatibility of deformation at the contact surfaces and transfer of forces through bearing at such contact areas should be included in the model.
- 3) The non-linear load slip behaviour as shown in Figure 1.0 is used in this analysis. At large slips, the bond element still transfers a shear equal to the ultimate value; however, when tooth withdrawal occurs a load redistribution occurs, which the model should simulate.
- 4) The computational costs of the analysis is presently high and the mesh size in certain circumstances is restricted by the computational procedure. A frontal technique of solving should be implemented to reduce computational costs and increase the accuracy of the solution through a decrease in element size.

Further studies incorporating this method are:

- 1) A parameter study should be conducted to investigate the possible effects of variations in material properties on the strength and stiffness of truss plate connections.
- 2) An extension to the above study would be the use of probability distributions for local material properties of the wood and defects to predict the strength and stiffness of its truss plate connections. An appropriate application is to predict strength and stiffness of truss plate connections as defined by CSA to evaluate the properties of truss plates.

REFERENCES

- Al-Dabbagh, A.M.A., Goodman, J.R., and Bodig, J., 1972. "Finite Element Methods for Wood Mechanics." Journal of the Structural Division, ASCE. 98 (ST3): 569-586.
- Arbek, T., 1979. "The Effect of Time on the Strength of Truss Plate Joints." Unpublished Report. Department of Engineering, Carleton University, Ottawa, Ontario, Canada.
- Aris, A.H., Burnett, E.F.P. and McNiece, G.M., 1980. "Finite Element Modelling of Jointed Structures." Canadian Journal of Civil Engineering, Vol. 7, pp 540-546, 1980.
- Barrett, J.O., and Schniewind, A.P., 1973. "Three-Dimensional Finite-Element Models of Cylindrical Wood Fibers." Wood and Fiber, Vol. 5, No. 3, Fall 1973, pp 215-225.
- Beineke, L.A., 1974. "Structural Analysis of Wood Plane Frames Connected with Light Gauge Metal Plates." Ph.D. Thesis, Purdue University.
- Beineke, L.A., and Suddarth, S.K., 1979. "Modelling Joints made with Light-Gauge Metal Connector Plates." Forest Product Journal 29(8), pp
- Bodig, J., and Goodman, J.R., 1973. "Prediction of Elastic Parameters for Wood." Wood Science 5(4): 249-264.
- Bowes, W.H., and Russel, L.T., (1975). "Stress Analysis by the Finite Element Method for Practising Engineers." Lexington Books, Massachusetts.
- Brynildsen, O.A., and Booth, L.G., "Structural Analysis of Timber Trusses with Semi-Rigid Joints." Paper presented at meeting of IUFRO Section 41, Munich, September, 1967.
- Cook, R.O., 1974. "Improved Two-Dimensional Finite Element." Journal of the Structural Division, ASCE. 100 (ST9): 1851-1863.
- Cook, R.O., and Al-Abdulla, J.K., 1969. "Some Plane Quadrilateral 'Hybrid' Finite Elements." Journal American Institute of Aeronautics, 7(11): 2184-2185.

REFERENCES
(cont'd)

Demarkles, L.R., 1955. "Investigation of the use of a Rubber Analog in the Study of Stress Distribution in Riveted and Cemented Joints." National Advisory Committee for Aeronautics Tech. Note. No. 3413.

Egerup, A.R., 1979. "European Practice and Future Development in the Design of Metal Plate Connected Wood Trusses." pp 117-122. F.P.R.S. Proceedings, p-79-28.

Faruque, M.O., 1980. "The Role of Interface Elements in Finite Element Analysis of Geotechnical Engineering Problems." M.Eng. Thesis, Carleton University, 1980.

Fernandez, V.A., and Polensek, A., 1979. "Model for Predicting Strength and Stiffness of Wood Studs." Wood Science, Vol. 12, No. 2, pp 65-72.

Foschi, R.O., "Analysis of Wood Diaphragms and Trusses. Part II: Truss-Plate Connections." Canadian Journal of Civil Engineering, Vol. 4, No. 3, 1977.

Foschi, R.O., and Barrett, S.P., 1980. "Glued-Laminated Beam Strength: A Model." Journal of the Structural Division, ASCE. Vol. 106 (ST8): 1735-1754.

Goodman, J.R., and Bodig, J., 1970. "Orthotropic Elastic Properties of Wood." Journal of the Structural Division, ASCE 96 (ST11): 2301-2319.

Goodman, J.R., and Bodig, J., 1971. "Orthotropic Strength of Wood in Compression." Wood Science 4(2): 83-94.

Goodman, J.R., and Bodig, J., 1978. "Contemporary Methods for Modeling the Elastic Behaviour of Wood." General Constitutive Relations for Wood and Wood-Based Materials, pp 17-39. Report of a workshop July 1978, sponsored by The National Science Foundation Engineering Mechanics Section Solid Mechanics Program.

Goodman, R.E., Taylor, R.L., and Brekke, T.L., 1968. "A Model for the Mechanics of Jointed Rock." Journal of Soil Mechanics and Foundation Division, ASCE, Vol. 94, 1968, p 637.

REFERENCES
(cont'd)

Gopu, V.K.A., 1980. "Radial Stresses in Curved Glulam Beams." Journal of the Structural Division, ASCE, Vol. 106 (ST11): 2143-2150.

Herrmann, L.R., 1978. "Finite Element Analysis of Contact Problems." Journal of the Engineering Mechanics Division, ASCE, Vol. 104, No. EM5, October, 1978.

McAlpine, W.R., and Grossthanner, O.A., 1979. "Proposed Design Methods for Three Typical Details." F.P.R.S. Proceedings, P-79-28, pp 227-231.

McCracken, P.N., 1979. "Lumber Industry Profile." Metal Plate Wood Truss Conference, 1979. FPRS Proceedings, P-79-28, pp 8-9.

McGlenn, J., and Hartz, B.J., 1968. "Finite Element Analysis of Plywood Plates." Journal of the Structural Division, ASCE. (ST2): 551-563.

Meeks, John E., 1979. "Industrial Profile of the Metal Plate Connected Wood Truss Industry." Metal Plate Wood Truss Conference, 1979. FPRS Proceedings, P-79-28, p 3.

Misra, R.D., and Esmay, M.L. "Stress Distribution in the Punched Plate of a Timber Joint." Transaction of the American Society of Agricultural Engineers, Paper No. 65-907, 1968.

Morris, E.M., 1967. "A Modern Concept in Timber Design." Civil Engineering - ASCE 37(3): 52-53.

Ngo, O., and Scordelis, A.C., 1967. "Finite Element Analysis of Reinforced Concrete Beams." ACI Journal, Proceedings, V. 64, No. 3, Mar., pp 162-168.

Perkins, R.H., Suddarth, S.K., and Dale, A.C., 1962. "Rotational Resistance of Three-Membered Nail Joints Subjected to Bending Moments." Purdue Agric. Expt. Sta. Res. Bul. 753.

REFERENCES
(cont'd)

Pdlensek, A., 1976. "Finite Element Analysis of Wood-Stud Walls." Journal of the Structural Division, ASCE, Vol. 102 (ST7): 1317-1335.

Porteous, A., "Methods of Analyzing Timber Roof Trusses, Taking Into Account Consideration Flexibility of Joints." M.Sc. Thesis, Imperial College, 1965.

Reardon, G.F., 1971. "A Structural Analysis of Frames with Semi-Rigid Joints (including a description of computer program)." Division of Forest Products Technological Paper, No. 59, CSIRO, Australia, 1971.

Rodda, E.D., 1967. Analysis of Indeterminate Nailed Structures. Forest Product Journal 14(11): 528, June 1967.

Salinas, J.J., "Built-Up Joists, An Economic Alternative in Residential Construction." Proceedings Fifth Symposium, Engineering Applications of Mechanics, University of Ottawa, pp 21-24.

Scafer, H. 1975. "A Contribution to the Solution of Contact Problems with the Aid of Bond Elements." Computer Methods in Applied Mechanics and Engineering 6(1975), pp 335-354.

Suddarth, S.K., 1961. "Determination of Member Stresses in Wood Trusses with Rigid Joints." Purdue University Agr. Exp. Sta. Res. Bull. No. 714, 1961.

Suddarth, S.K., 1963. "A Detailed Study of a W-Truss made with Metal Gusset Plates." Purdue University Agr. Exp. Sta. Res. Pro. Report No. 5, 1963.

Suddarth, S.K., "The Engineering Design of Mechanically Fastened Trusses: A Review." Wood Science, 1(4), 1969.

Suddarth, S.K., 1972. "A Computerized Wood Engineering System: Purdue Plane Structures Analyzer." USDA Forest Service, Research Paper, FPL 168, 1972.

REFERENCES

(cont'd)

Thompson, E.G., Vanderbilt, M.D., and Goodman, J.R., 1977.
"FEAFLO: A Program for the Analysis of Layered Wood
Systems." Computers & Structures, Vol. 7, pp 237-248.

Williams, F., 1979. "The Structural Stiffness of Built-Up
Timber Joists." Report. Department of Civil Engineering,
Carleton University, Ottawa, Ontario, Canada, 41 pp.

Zienkiewicz, O.C., (1971). "The Finite Element Method in
Engineering Science." McGraw-Hill, New York.

Zienkiewicz, O.C., Best, B., Dullage, C., and Stagg, K.G.
"Analysis of Nonlinear Problems in Rock Mechanics with
Particular Reference to Jointed Rock Systems." Proc. 2nd
Cong., Int. Society for Rock Mechanics (ISRM), 1970, V. 3,
pp 501-509.

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Appendix A

Truss Plate Property Selection and Modification for Use in the Joint Models

The use of available data regarding the truss plate was thought to be sufficient, so that an expensive and time consuming data gathering study would not be required. This available data is obtained from standard tests required and specified by the Truss Plate Institute of Canada (TPIC). Four configurations of truss plate connections are tested:

- 1) load parallel to grain, plate length parallel to load;
- 2) load parallel to grain, plate length perpendicular to load;
- 3) load perpendicular to grain, plate length parallel to load, and
- 4) load perpendicular to grain, plate length perpendicular to load.

For each of these tests, the ultimate loads and the loads at 0.762 mm (0.030 in.) are measured and recorded. The mean values of the test results are used to obtain design values for the specific truss plate.

The properties obtained from these tests were modified in the following manner to be utilized in the load-slip relationship formulated in Equations 1.0 and 2.31.

$$F(\Delta u) = (m_0 + m_1 \Delta u) \left(1 - e^{\frac{-k\Delta u}{m_0}} \right)$$

where k = initial stiffness

m_1 = the stiffness at large slips

m_0 = the intercept of the asymptote with m_1

F = force required to produce slip

Δu = magnitude of slip

As previously mentioned, the load at 0.762 mm (0.030 in.) slip is recorded. The slope of the line passing through the origin and the point defined by the load at 0.762 mm (0.030 in.) is defined by dividing the load by 0.762 mm. Data from tensile tests on truss plate connections by Arbek (1979) shows that the initial slope is (about) on the average three times the slope at 0.762 mm (0.030 in.). The slope from the standard test data is therefore multiplied by three to obtain the initial slope, k .

The stiffness matrix at large slips, m_1 , is taken as an arbitrarily small number, usually $k/200$. The intercept of the asymptote with m_1 is taken as the average ultimate load.

Depending on the truss plate design, and the applicable test, the truss plate will have a certain number of teeth, 'n'. Dividing the above properties by the number of teeth, the properties will be average tooth properties. Also for a specific truss plate design, the number of teeth per unit area, β , is unique to that type. Hence, by multiplying the above properties by β , the properties are now based on the area of the plate.

The analog of the truss plate connection is such that the truss plate is in contact with the wood members on one side only. Therefore all the above properties are doubled for use in this study.

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