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**FREQUENCY HOPPING IN THE PRESENCE
OF PARTIAL-BAND JAMMING
WITH AND WITHOUT RAYLEIGH FADING**

by

Robert Scott Myers

**A thesis submitted to the Faculty of Graduate Studies
and Research in partial fulfillment of the requirements
for the degree of
Master of Engineering**

**Department of Electronics
Faculty of Engineering
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Canada**

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FREQUENCY HOPPING IN THE PRESENCE
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submitted by Robert Scott Myers
in partial fulfillment of the requirements
for the degree of
Master of Engineering.

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April 1987

ABSTRACT

The performance of frequency hopped (one to five hops per symbol) coherent and incoherent MFSK, and coherent MPSK has been analysed by computer simulation for three different receiver structures in the presence of optimal partial-band jamming with and without Rayleigh fading.

The first type of receiver used linear combining; the second used majority voting; the third used hard limiters with majority voting.

The objective was to establish a relationship between bit error rate, and the energy per bit to average power spectral density of the jammer, E_b/N_J , under worst-case jamming for the given receiver type, channel condition, modulation scheme, and number of hops per symbol.

The SNR required under jam-free (noise only) conditions to produce a bit error rate of 10^{-4} has been established by simulation. This SNR has then been held constant and jamming introduced into the system. Plots have been generated showing the variation of BER with E_b/N_J given the fixed SNR without jamming, for all receiver types and conditions.

It has been found that it is advantageous to increase the alphabet size M , under any condition, except for MPSK. The Linear Combining Receiver has been found to be a poor choice for multi-hop communication in a hostile environment, and increasing the number of hops per symbol worsens the performance. The Majority Vote Receiver has been found to perform better. The most promising receiver structure studied has been found to be the Hard Limiter Receiver. In fading channels, increasing the number of hops per symbol to three or more, always provided some diversity improvement.

For Michael and Sonia

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LIST OF PRINCIPAL SYMBOLS AND ABBREVIATIONS

AWGN	Additive white gaussian noise
α_n	Attenuation factor of path n
BER	Bit error rate
β	Signal power to CW tone power
B_c	Correlation bandwidth
BW_{ss}	Spread spectrum bandwidth
BW_{info}	Information bandwidth
E	Energy of $s(t)$ (symbol energy)
E_b	Energy per bit
E_2	$2 \times E / L_{hops}$
f_c	Carrier frequency
G_p	Processing gain
γ	Fraction of BW_{ss} that is jammed
γ_{opt}	Optimum fraction of BW_{ss} to jam
ISI	Intersymbol interference
k	Number of bits per symbol
L_{hops}	Number of hops per symbol
M	Level of the modulation scheme
M_J	Jamming margin
N_0	Thermal noise power spectral density
N_J	Jammer's average noise power spectral density
N_M	Number of M-ary clusters in BW_{ss}
N_s	Number of single frequencies in BW_{ss}
$\Phi_{zz}(f)$	Low-pass noise power spectral density
$\phi_{zz}(\tau)$	Autocorrelation function of random process $z(t)$

P_b	Bit error rate
P_J	Total jammer power
P_M	Symbol error rate
Q	Number of CW jamming tones
q	Number of CW tones in M-ary cluster
$r(t)$	Received signal
R_{hop}	Hop rate
$s(t)$	Transmitted signal
$S(f)$	Fourier transform of $s(t)$
σ^2	Noise power
$T_g(\omega)$	Group delay
T_{hop}	Hop time
T_j	Jitter time
T_m	Time spread
T_n	Time delay of the n th path
$T_p(\omega)$	Phase delay
T_{sym}	Symbol signalling interval (total time)
$\dot{\tau}$	Group delay slope
$\theta(\omega)$	Phase at ω ($2\pi T_p(\omega)$)
$u(t)$	Low-pass equivalent signal
$U(f)$	Fourier transform of $u(t)$
U_m	Decision variable for symbol m
x_1, x_2	Two normally distributed unity variance numbers
$z(t)$	Low-pass equivalent noise random process
Z_{thresh}, R_{thresh}	Hard limiter thresholds

Chapter 1

INTRODUCTION

1.1 Why Spread the Spectrum

Spread spectrum communication systems have been in existence since the 1950's. Originally they were developed exclusively for military use [SCH 82]. Today spread spectrum techniques are finding increased use in commercial applications, though military applications still receive the most attention.

A spread spectrum system provides some protection against:

- (i) Intentional interference.
- (ii) Unintentional interference.
- (iii) Multipath, or self interference.

Intentional interference would most likely occur in a military environment. Here a foe may wish to disrupt the communication ability by injecting high powered signals into the the communication channel. The only purpose for these interference signals is to disrupt communication.

Unintentional interference can occur in almost any environment where there are other signal sources. These other sources do not wish to hinder the communication, and originate from other users trying to send information to each other. For example various users of the 27 MHz "Citizens Band" within the same city, must share 40 voice band channels. Inevitably there is an interference event whenever two or more people are transmitting on the same frequency at the same time.

A spread spectrum signal has the ability to discriminate between signals arriving at the receiver at slightly different times. When a signal arrives at a receiver via multiple paths separated in time, this can cause a distorted version of the transmitted signal to appear at the receiver. By discriminating between these paths

(through high time resolution), the spread spectrum radio system can eliminate this form of self interference

A spread spectrum system allows :

- (i) Low power spectral density.
- (ii) Low probability of being intercepted or detected.
- (iii) Selective addressing environment of multiple users on the same frequency band. This is referred to as code division multiple access or CDMA.
- (iv) Gradual degradation in system performance as other users make use of the system [CNG 79].

Since the power of the transmitter is fixed, the wider the bandwidth the signal occupies (averaged over time), the lower the power spectral density (or energy spectral density, if we consider the signal to be of finite duration).

Because of the lower energy density the signal becomes less noticeable. A signal that is not detected, can't be intercepted. The lower the spectral density, the lower will be the probability of detection.

In CDMA, different codes are assigned to different users. These codes have low cross-correlation and thus can be used to generate channel waveforms of low cross correlation values. Signals with a high degree of dissimilarity in time, frequency, or phase tend to interfere less with each other. A CDMA receiver observes the cross-correlation of unwanted signals, as well as the peak of the auto-correlation of the desired signal. Thus, a large number of users can be operating within the same channel provided they have low cross-correlation peaks.

As opposed to single channel carrier systems, a gradual degradation of the system occurs in CDMA. A blocked, or severely disturbed link will occur if more users than possible frequency assignments are desired in a fixed frequency system (such as FM mobile radio). In such a case, all the interference is forced on a few individual users. In a CDMA system, the extra interference is essentially shared between all users.

By far the most important applications for spreading the spectrum are anti-jamming capabilities (A/J) , and low probability of intercept communication (LPI) [C C 85], [P W 85]. These are of interest to any organization associated with secrecy, especially the military establishment, embassies, law enforcement agencies, etc.

1.2 Definition of Spread Spectrum

A "spread spectrum" communication system is one where the transmitted signal occupies a bandwidth several times greater than the information bandwidth. The spectral spreading must be accomplished by the use of a special code that is independent of the information. Correct reception of the signal requires knowledge of the special code used to spread the spectrum. Using this code, the system is "de-spread", and the data recovered in the usual way. All of the above must occur in a true spread spectrum system.

Notice that although modulation systems such as PCM, and FM result in transmitted bandwidths wider than the information bandwidths, these do not qualify as spread spectrum signals since there is no independent spreading code involved.

1.3 Spreading by Direct Sequence

One method of spreading the spectrum involves multiplication of the digital signal by a much higher rate binary code in the form of a pseudo-random sequence. That is, what would be an ordinary unspread signal is phase shifted by 180 degrees pseudo-randomly. This is referred to as "direct sequence" or DS spread spectrum.

Multiplication in the time domain is a convolution in the frequency domain. Hence the resultant signal has a bandwidth greater than that of the high rate spreading code. With fixed power and wider bandwidth, a lower power spectral density is achieved. This is shown in Figure 1.1. The spectrum is instantaneously spread out over a wide band at all times. A different form of spread spectrum

will be explained later whereby the frequency is spread out by occupying various frequencies at different times.

The lower spectral density may be required so as not to interfere with other narrow band users within the same frequency band. A spectrum regulatory agency usually decides on the limits for the power spectral density for a particular set of frequencies.

At the receiver, the signal is multiplied by an identical replica of the spreading code. This results in removal of all phase shifts that were due to the spreading code. What is left is the narrow band original information signal. The basic transmitter and receiver block diagrams are shown in Figure 1.2. Notice that while the original narrow band signal is recovered at the receiver, wide band signals are created out of other signals that were within the spread bandwidth. These other signals could be from other friendly users or they could be due to intentional interference (jamming). The point is that after the de-spreading operation at the receiver, these signals cannot pass through the narrow band output filter, and are either rejected, or at least their influence on the desired signal is greatly reduced.

In much the same way, other spread spectrum users are also rejected. For other users coded differently, spectral de-spreading does not result after multiplication with the code. Thus they too cannot pass through the information bandwidth filter, or only a small portion of their energy can pass through.

There could be a case of the correct de-spreading code being used at the wrong time. That is, out of phase with the received spreading sequence. This could result when a signal arrives at the receiver two or more times due to multipath. If the receiver's PN code is correctly aligned for reception of one of the rays (usually the strongest ray), the others are rejected. Thus intersymbol interference due to multipath can be reduced. The ability to reject these other signals is best when the peak of the auto-correlation function of the spreading code is large relative to any side lobes (occurring at any time shift not equal to zero). Constructing families of codes with large auto-correlation peaks, with small (and few) side lobes,

Figure 1.1 Multiplication of the data by a high rate PN code.

Figure 1.2 Block diagram of the basic direct-sequence spread spectrum transmitter and receiver.

and whose set of cross-correlation peaks between members is small, is the subject of much research today. Code construction however is not dealt with here, and it is assumed that such codes do exist [MCG 83], [WHB 81], [HOL 82], [GOL 67].

It should be noted that under jamming conditions there is considerably more interfering power than under conditions of multipath or interference from other users such as SSB or FM voice. What is worse, the jammer may choose to distribute his power in a way to cause maximum degradation of the BER.

A desire to reduce power spectral density will require a much larger bandwidth, so the chip width of the PN sequence must be reduced. But there are physical limits to the speed of the electronic components necessary for this. If jammers are still having a damaging effect on the signal (due to the lack of uncertainty in the position of the signal in the frequency domain), the signal bandwidth can be even further spread out by using a technique known as frequency hopping.

1.4 Spreading by Frequency Hopping

In a frequency hopped (FH) system the carrier frequency of what would be an ordinary narrow band signal is changed in a pseudo-random pattern. That is, instead of changing the phase as in a DS system, the frequency is changed. A PN code is used to select one frequency out of a set of frequencies which all together make up the spread spectrum bandwidth. A frequency hopper then consists of a PN generator and a frequency synthesizer which can respond to this code. To select one of 2^n frequencies requires the synthesizer to have n binary inputs. These n inputs can be the last n outputs from a PN generator. Figure 1.3 shows the block diagram of a basic FH transmitter and receiver.

As with DS systems, the construction of hopping patterns with good CDMA characteristics is the subject of much research [HEA 85], [NIY 82], [ORR 81].

Frequency hoppers are said to be "slow" or "fast" depending on how long the carrier dwells at each frequency before jumping to the next one.

Figure 1.3 Block diagram of the basic frequency-hopped spread spectrum transmitter and receiver.

1.4.1 Slow Frequency Hopping

In a slow FH system, the carrier does not change for perhaps several symbol intervals, so several symbols can be transmitted at each frequency. The receiver does not have to reconstruct, or combine energy from many frequencies before making a decision on which symbol was sent. Thus its implementation is simpler than that of a fast hopper.

Unfortunately the slowness may allow a jammer time to scan the entire spread bandwidth, locate the current transmitter frequency, and jam with all its power concentrated on this frequency. If the jammer is computer controlled (or "smart"), it can repeat the process continuously and simply follow the hopper from one frequency to another.

1.4.2 Fast Frequency Hopping

Fast frequency hopping differs from slow hopping in that the dwell time at each frequency is short. This is used to defeat the smart type of jammer mentioned earlier. If the receiver receives the transmitted signal and jumps to the next frequency before the jammer has time to locate the transmitting frequency and propagate its interference to the receiver, the jammer will be defeated. Obviously the closer the jammer to the receiver, the higher the hop rate must be.

Not only is the dwell time reduced in a fast frequency hopping system, but also only fractions of a symbol are transmitted at each hopping frequency. Some method must be used to decide, after several hops, which symbol was sent. There must be an algorithm for dealing with the partial information gathered on each hop about the symbol being transmitted. Due to this fact, fast hoppers are more complex than slow hoppers, and the timing requirements are more stringent.

In conjunction with the combining algorithm, powerful error correcting codes could also be used to allow unjammed hops to aid in the correction of jammed hops. Error correcting codes are usually necessary in frequency hopping systems

[MIL 85], [P S 85], [STA 82]. However, this topic is out of the scope of this thesis.

1.5 Processing Gain

By first spreading then despreading its bandwidth, a spread spectrum system obtains a "processing gain". This is the gain that the system has over other users, or jammers, relative to an ordinary unspread narrowband system. It should be noted that this "gain" is a bit fictitious if not thought of clearly. After transmission, a signal of fixed power has a wider bandwidth. Thus there is more noise of all types within the signal bandwidth and hence the signal-to-noise ratio must decrease. This loss in SNR can be recovered (except for receiver losses) at the receiver. At the receiver, as was said before, other forms of interference are spread out in the frequency domain, the desired signal collapsed in the frequency domain, and a narrow band filter employed to allow the information signal to pass without the undesired signals. The SNR can thus be increased at the receiver by the ratio of the spread bandwidth to the output information filter bandwidth.

This gain is approximately equal to:

$$\begin{aligned}\text{Processing Gain} &= \frac{BW_{ss}}{BW_{info}} \\ &= G_p\end{aligned}\tag{1.5.1}$$

where BW_{ss} is the spread spectrum bandwidth, and BW_{info} is the ordinary bandwidth of the information signal. It is exactly equal to the above only if the noise within BW_{ss} is white and independent of the spread spectrum signal.

Well what does this mean? How does this gain help us? This gain is only helpful when we consider either other narrow band users or intentional jamming. It does not alleviate in any way thermal noise. In fact it worsens it due to the complexity of the receiver. It *does* show us how much better off we are relative to a system that has chosen to operate narrow band on the same frequency as another narrow band signal (provided this other signal is the only disruption that would occur within BW_{ss}). If the entire spread bandwidth is packed tightly with other users, the value

of G_p remains mathematically the same but its effect may be useless. That is to say, a narrow band transmission in the first place may have the same error rate as its spread spectrum counterpart due to the fact that increasing the bandwidth would only cause an increase in the interfering power in the band (as is the case with thermal noise, this increase in interference power is then reduced by G_p at the receiver but nothing is gained).

In the case of jamming the situation is quite different. Here the jammer is assumed to have a finite fixed power available. Increasing the spread bandwidth then does not cause an increase in the jamming power within the band. The same can be said if there are only a few narrow band users in operation at one time (with frequency assignments not being known). As the spread spectrum signal's bandwidth is increased, the increase in interference power is not directly proportional, hence the processing gain becomes useful. Notice a jammer who spreads his signal out over the entire BW_{ss} can be thought of as being like additional AWGN (additive white gaussian noise), but now G_p is effective against this. The more the signal is spread, the more uncertain the jammer is of where the signal is, hence it must spread out too. At the receiver output filter, the more spread out the fixed power jammer, the lower its spectral power density, and hence the lower its output power.

But does the jammer have to spread its power out at all? In general, yes. It must do so in some way. If it did not, a narrow band notch filter could be simply added at the receiver at the jammers' frequency. This could cut out virtually 100% of the jammers power, but would only cut out a fraction of the spread spectrum signal power. And this would occur even if the jammer increased his power. Of course this assumes that there are many frequencies to hop to, or that in the case of DS systems that G_p is large. For if this were not the case, cutting out a fraction of the desired signal would be unacceptable as it would be cutting out a large percentage of the signal power. For example, in a BFSK frequency hopping system with only 100 slots in the frequency domain to jump to, adding a notch filter at any frequency would immediately, and without considering any other noise, cause a

BER of one in 200 (assuming one hop per bit, and a uniform probability of jumping to any frequency).

The only thing for certain is that the processing gain shows an increase in the SNR at the output of the receiver relative to the input SNR. And this is true in all cases.

In the case of DS systems, the processing gain is achieved by multiplication at the receiver of the received signals with the PN code. With FH systems, G_p is obtained by avoidance of the interference signal; the more frequencies to choose from, the more time spent avoiding the interference, hence higher the processing gain. For frequency hopper then, G_p is approximately equal to:

$$\begin{aligned} G_p &= \frac{BW_{ss}}{BW_{info}} \\ &= N \times \frac{BW_{info}}{BW_{info}} \\ &= N \end{aligned} \quad (1.5.2)$$

where N is the number of frequency hopping positions available.

1.6 Jamming Margin

The processing gain does not tell us the maximum power a jammer can have before it has a detrimental effect on the performance of the system. It simply tells us by how much we can reduce the effective power density of the jammer. The amount of jamming we can tolerate before the system BER deteriorates to an unacceptable level is referred to as the "jamming margin".

For example, if we require an SNR of 8 dB at the output of a receiver, and if the receiver losses are 3dB, with a processing gain of 30 dB we could operate with up to 19 dB SNR degradation due to jamming.

Mathematically then:

$$\begin{aligned}\text{Jamming Margin} &= G_p - (L + SNR_{\text{output}}) \\ &= M_J\end{aligned}\tag{1.6.1}$$

where L are all other losses, and SNR_{output} is the SNR required for the maximum allowable (worst) BER.

The beauty of spread spectrum processing is that it allows for rejection of unwanted signals without having to know anything (although more knowledge would help) about the interfering signal. It is not necessary to know the direction of the interference source. But with a large antenna, one would be able to reduce its effects even more by trying to null the interference and increase the desired signal's strength.

1.7 Synchronization and Tracking

Synchronization plays a crucial role in spread spectrum systems. In order for any spread spectrum system to operate, there must be a synchronization of the signals involved. In order to despread the signal we require complete knowledge of the spreading code, or pattern. Apart from all the other timing requirements of an unspread digital system, there is the added problem of synchronizing the PN code generated at the receiver (whether in FH or DS systems) with that of the received signal.

This is particularly important in DS systems where timing of the received, and receiver generated PN codes must agree to within a small fraction of a chip period. Since there are many chips per symbol in a DS system (to give a large processing gain), and since the symbol rate is generally not slow (in order to transmit a significant amount of information in a given time), there are many chips per second. Hence the small allowable timing error in terms of chip fraction, implies the *absolute* timing error involved must be very small as well.

In a frequency hopping system the codes (or frequencies effectively) must be synchronized to within fractions of the dwell time at these frequencies. Since the

processing gain in a FH system is obtained by the number of frequency hopping choices, the dwell times can be larger than the equivalent (in terms of processing gain) DS system chip widths. Thus code synchronization is usually easier with frequency hopping systems.

Before a code can be followed closely it first must be initially aligned. This initial attempt at synchronizing the codes is referred to as acquisition. It is often the case that the best codes to counteract jamming, or codes used for security purposes, are the most difficult to acquire lock. Some trade off always has to be made between making the PN code (or the hopping pattern) as noise like as possible, and assuring lock can be acquired within a reasonable amount of time.

Tracking and synchronization of spread spectrum systems are discussed in references [DAV 84], [NIE 75], [POL 81], [BOU 81], [HAR 74]. These topics are not dealt with in this thesis. Here it is assumed that hopping patterns are in perfect synchronization at all times, and carrier phases (if necessary) are known precisely. While these assumptions may seem at times unreasonable (for example in frequency hopped MPSK) from today's technological point of view, with further developments, the material presented in this thesis will provide some guidelines in system design.

1.8 Thesis Objectives and Organization

Communication in the HF (3-30 MHz) band is of great importance to civilians and the military. Civilian users consist of international broadcast services, radio telegraph, ship communication, and transoceanic air traffic communication. The military has always relied on HF radio for both long distance, and short distance links. The advantage of the HF channel for communication is its low cost for beyond the line of sight communication, and its relatively high availability (given the cost). One can always find a band of frequencies within the HF band suitable for communication between military bases and outposts around the world. This channel is virtually indestructible and is always usable to some degree. This cannot

be said of microwave relay tower communication, nor satellite communication; both being easily subject to physical destruction in the event of war or near war. There are many disadvantages to HF links however: limited bandwidths, variations in the ionosphere from night to day, or even hour to hour at times, and multipath interference.

Spread spectrum can be used to advantage here. Slow data rate communication can be used with spectral spreading to allow anti-jamming communication. This signalling technique can often resolve the multipath modes, and therefore reduce the effects of fading, hence the circuit reliability can be increased, or the transmitter power decreased. Thus there exists an important need for investigation of spread spectrum radio over the HF channel.

Here we consider frequency hopping as opposed to direct sequence spread spectrum. Timing requirements are less stringent for FH systems. FH systems are easier to synchronize, can provide larger processing gain, can be discontinuous in the frequency domain (thereby easily avoiding known problem interference spots), and can, with careful selection of hopping codes, accomodate group delay slope in the channel.

The objective of the thesis is to determine the effectiveness of spread spectrum radio on the HF channel (and HF like channels), with the possibility of fading, no fading, and worst-case (optimum) partial-band jamming being present. The results will be of interest to radio users as well as to radio jammers; the optimum fraction of the band to jam being of interest to the latter.

Three modulation schemes (used in conjunction with frequency hopping) under the above conditions were investigated :

- (i) MFSK coherently detected.
- (ii) MFSK envelope squared (incoherently) detected.
- (iii) MPSK coherently detected.

Three levels for each modulation scheme were considered; $M = 2, 4$, and 8 .

Three different receiver structures under all of the above modulation schemes, levels, and channel conditions were considered—the number of frequency hops per each data symbol being 1, 2, 3, 4, and 5. These structures will be described in detail in Chapter 4. The three types are :

- (i) A linear combining receiver.
- (ii) A majority voting receiver (which divides the symbol energy).
- (iii) A hard limited, majority voting receiver.

Comparison between receiver structures and modulation schemes could be based on signal-to-noise ratios at various points in the system, however the BER versus E_b/N (energy per bit to noise power spectral density—effectively the SNR per bit) is considered to be the best measure of comparison between the systems. This criterion has been used as a metric of comparison. Also comparisons can be made in terms of the fraction of the band jammed required to optimize BER degradation.

The analysis was carried out by direct simulation on a digital computer. In many text books, curves of BER versus SNR per bit are plotted, and closed form relations are sometimes available. Many times approximations are made—especially for M-ary systems with M greater than two, and for high signal-to-noise ratios. In a jammed environment, a high SNR may not be present all of the time. In order to calculate the BER in a partial-band jammed case though, we must know the effect that a low SNR has on the system—for any value of M. The possibility of fading further complicates things, as does multiple hops per symbol. Often equations (if and when developed) to take into account all variables (including different receiver structures) are so massive and confusing that one can scarcely understand them. Even if they can be understood after long study (or at least accepted), their actual evaluation is another task. Double integrals from zero to infinity of powers of the error function, or bessel functions are common. Well, how can one evaluate them? Especially if those functions are not available on the computer. The error function itself involves an infinite integral.

By “building” the system on the digital computer, and by actually “sending”

symbols through, one can simplify the understanding of the system, make it easier to modify, and avoid approximations of the usual sort (the union bound for example). Of course approximations arise in the form of only being able to send a finite number symbols through before stopping, counting the errors, and hence estimating the error rate. And one must be able to generate gaussian like noise numbers whose experimental density function only may approximate the known closed form mathematical density function—even if a very large number of samples are taken. So there are two problems: a finite sample, and the tendency of the samples (finite or infinite) to reflect the true samples aimed for (gaussian, Rayleigh, non-central chi-square or whatever you have). Since the gaussian model for noise is only a model of what we may find in the system, it in itself is already an approximation. The random number generator on the Electronics department's VAX/750 computer was utilized and mathematically altered to generate all numbers of particular distributions. Notice that evaluation (where possible) of some mathematically "elegant" solutions border on simulation themselves, as limits are made finite, and various criteria are used to divide and sum areas etc.

But is the union bound so bad after all? Yes. The use of the this approximation in the case of M-ary systems in partial-band noise jamming can introduce "substantial" errors [LEE 85]. Error performance in fact can go contrary to what is to be expected. The union bound was applied to the investigation of frequency hopped MFSK on the partial-band jammed channel. It was found that increasing M made the BER degrade. This goes against what one would expect from the experience in the gaussian channel [LEE 85]. Indeed in [LEE 85] two receiver structures were analysed by closed form solution. It was found by this exact analysis that the BER in fact *improved* with increasing M in the presence of jamming. See Figure 1.4, which illustrates this for a receiver with "clippers" or soft limiters.

Chapter 2 describes the channel model, noise, and fading—and what causes these effects. In Chapter 3, different modulation schemes, and their reception is discussed. Chapter 4 describes the receiver structures and the associated decision

Figure 1.4 Limitation of the use of the union bound [LEE 85].

algorithms. Chapter 5 shows the effect that partial-band, and CW jamming has on the received signal. Chapter 6 presents and discusses the simulation results, and describes the simulation programs. Finally, Chapter 7 gives the conclusions and recommendations for future work.

Chapter 2

THE CHANNEL

2.1 Introduction

The channel is the media through which the electromagnetic wave carrying the information is sent. The channels we are concerned with here are those that exhibit fading (flat, or frequency selective). Also those which are not faded but are only corrupted by gaussian noise.

Multipath is a phenomenon whereby a signal may arrive at a receiver via more than one path. When this happens, a delayed signal can act as an interference signal with respect to the principal signal, producing intersymbol interference (ISI).

Fading is a phenomenon which can be caused either by destructive interference of the multipath signals, or by a change in the physical make up of the channel. Sometimes the density of ions causing the effective reflection (enabling long range communication) may change; the absorption of the radio wave in the atmosphere may change; or the height of the ionized layer may change—these events can occur along with multipath. Not only that, but their effect can vary with frequency as well, although they affect a wider band of frequencies, than fading due to multipath.

These characteristics are common to many real communication channels in use today. The 3 to 30 MHz (HF) band (ionospheric communication) exhibits this, where the signal arrives via multiple sky paths. Also, beyond the horizon (tropospheric) communication from 300 to 3000 MHz (UHF), and 3000 to 30000 MHz (SHF), as well as the UHF mobile radio channel, suffer from fading. In the mobile radio environment, this often happens due to reflections of the signal off many office buildings.

If a short pulse is transmitted over a multipath channel, the received signal consists of a series of pulses arriving at different times. If an identical signal is transmitted again at a later time, we may get a completely different series of pulses at the receiver. This time the number of pulses and their amplitudes may be different. Thus two things can occur: varying time spreads, and varying amplitudes of the individual paths. Since these effects are random, or unpredictable they are characterized in a statistical sense. This is shown in Figure 2.1.

Assume the transmitted signal is given by:

$$s(t) = \text{Re}[u(t) \exp(j2\pi f_c t)], \quad (2.1.1)$$

where $u(t)$ is the low-pass equivalent waveform, and f_c is the carrier frequency.

Under our assumption, we expect a different attenuation factor and time delay for each path. All paths are independent of each other. The attenuation factors and time delays vary with time because of the changing physical structure of the medium.

The signal seen at the receiver is then:

$$x(t) = \sum_n \alpha_n(t) s[t - T_n(t)], \quad (2.1.2)$$

where $\alpha_n(t)$ is the attenuation factor of the n th path, and $T_n(t)$ is the time delay of the n th path.

Substituting for $s(t)$ from (2.1.1) into (2.1.2) shows $x(t)$ as:

$$x(t) = \text{Re} \left(\left\{ \sum_n \alpha_n(t) \exp[-j2\pi f_c T_n(t)] \times u[t - T_n(t)] \right\} \exp[j2\pi f_c t] \right) \quad (2.1.3)$$

Thus the equivalent low-pass received signal is:

$$r(t) = \sum_n \alpha_n(t) \exp[-j2\pi f_c T_n(t)] u[t - T_n(t)] \quad (2.1.4)$$

Figure 2.1 Typical response of a time variant multipath channel.

Now $r(t)$ is the response of an equivalent low-pass channel to an equivalent low-pass signal $u(t)$. The time varying impulse response of the channel is found by replacing $u(t)$ with $\delta(t)$ and is given by:

$$c(T; t) = \sum_n \alpha_n(t) \exp[-j2\pi f_c T_n(t)] \delta[t - T_n(t)], \quad (2.1.5)$$

where $c(T; t)$ is the response of the channel at time t due to an impulse given at time $t - T$.

If we consider the frequency response of the channel by transmitting an unmodulated carrier at frequency f_c (with $u(t) = 1$) then the received signal becomes:

$$r(t) = \sum_n \alpha_n(t) \exp[-j2\pi f_c T_n(t)] \quad (2.1.6)$$

The received signal is the sum of a number of phasors with various amplitudes and phases. Notice that large changes in the medium are required for $\alpha(t)$ to change. But its easy for the signal to be affected drastically by small changes in the delays of the various rays. Notice the phase of a path will change by 2π whenever $T_n(t)$ changes by $1/f_c$. Since $1/f_c$ is a small number, the phase can change appreciably with small changes in path length (motion of the ionosphere for example). The delay of each path may change at different rates as well (randomly). With a large number of paths the central limit theorem can be applied. Then $r(t)$ can be modelled as a complex gaussian process.

The fading phenomenon (at a particular frequency) is mostly a result of the interfering phases of the various paths. Sometimes the vectors add to cancel each other resulting in almost no signal. At other times they may add in phase to produce a strong signal. Therefore, signal fading is a result of the time-variant characteristics of the channel.

Under the above conditions, the envelope of the signal observed at any time t , has a Rayleigh distribution. Rayleigh fading has often been encountered on the HF channel, and on VHF, UHF channels [PRO 83]. This model has been adopted for use in this thesis.

2.2 White Gaussian Noise

All communication channels are corrupted by noise of some kind. The noise generated within the transmitter and receiver is usually modelled, within the band of interest, as being gaussian and white. That is, the power spectral density (frequency domain) is flat, and that when viewed in the time domain at some arbitrary time t , the noise sample will take on a value consistent with the gaussian probability density.

A bandpass filter wide enough to pass the most of the signal waveform energy is used at the receiver. Noise also passes through this filter. After passing through this filter the noise is called bandpass white noise. The low-pass version of this noise has a power spectral density:

$$\begin{aligned}\Phi_{xx}(f) &= N_0 & |f| < B/2 \\ &= 0 & \text{elsewhere}\end{aligned}\tag{2.2.1}$$

The auto correlation function:

$$\phi_{xx}(\tau) = \frac{N_0 \times \sin(\pi B\tau)}{\pi\tau}\tag{2.2.2}$$

As B gets larger and larger this tends to:

$$\phi_{xx}(\tau) = N_0\delta(\tau)\tag{2.2.3}$$

2.3 Rayleigh Fading

The Rayleigh density has the form:

$$p(R) = \frac{R}{\sigma^2} \exp\left(\frac{-R^2}{2\sigma^2}\right) \quad R \geq 0\tag{2.3.1}$$

This represents the amplitude distribution that a single sine wave would have at any frequency viewed at some random time; or the independent amplitude R_1 and R_2 , of two such sine waves separated in frequency by the coherence bandwidth

of the channel (but viewed at the same time); or the independent amplitude distribution on any number of sine waves viewed (on the same frequency) at time separation greater than the coherence time of the channel. The above assumes that the coherence time does not vary with frequency, and the coherence bandwidth does not vary with time.

This probability density applies also to each isolated mode where no multipath is present. i.e. this is the density associated with $\alpha(t)$.

2.4 Coherence Bandwidth and Time Spread

If we transmit a wideband signal, the cross-correlation of the signal with a replica delayed by τ will give the average power output as a function of the delay in the channel (the power profile). One can then determine the time spread, T_m , of the channel which is defined as the value of delay for which the profile goes to 0.5.

The channel time spread provides the frequency domain characteristics of the channel. The Fourier transform of this yields a function of frequency difference, and is defined as the spaced frequency correlation function. It can be measured directly by transmitting two sine waves at frequency f_1 , and f_2 , and cross-correlating the two received signals. One then can obtain a plot of frequency coherence of the channel. The value of Δf where the plot goes to 0.5 is the coherence bandwidth, B_c , of the channel. Frequencies separated by more than B_c , when transmitted over the channel, produce uncorrelated sinusoids.

T_m and the coherence bandwidth B_c are related as follows:

$$B_c = \frac{1}{T_m} \quad (2.4.1)$$

If a signal is transmitted whose bandwidth is much narrower than the coherence bandwidth (or its baud interval is much longer than T_m) then all the components will fade together and there will be very little intersymbol interference (ISI). This is referred to as flat fading.

If the transmitted signal has a bandwidth larger than the coherence bandwidth, then severe distortion (ISI) can occur at the receiver. The fading then is said to be frequency selective. That is the time spread is long in comparison to the baud interval, so several symbols may overlap in time and interfere with each other.

2.5 Coherence Time and Frequency Spread

In a channel that has variations in its characteristics with time, there will be corresponding changes in the frequency domain. The faster the channel characteristics change, the more spread out in frequency a single sine wave will become. This is referred to as the doppler spread, B_d . The inverse of this is the coherence time, T_c , of the channel. A slowly changing channel has a large coherence time and a small doppler spread. On the HF channel the frequency spread is relatively unimportant due to long coherence times of five to ten seconds or sometimes even more.

2.6 Phase and Group Delay

Group delay is defined as:

$$T_g(\omega) = \frac{d\theta(\omega)}{d\omega} \quad (2.6.1)$$

Phase delay is defined as:

$$T_p(\omega) = \frac{\theta(\omega)}{\omega}, \quad (2.6.2)$$

where $\theta(\omega)$ is the phase that a steady single sine wave would have if transmitted at frequency ω .

If we send a signal whose bandwidth is relatively narrow compared to the carrier frequency ω_c , it can be shown that the envelope is delayed by the group delay evaluated at ω_c , and the carrier is delayed by the phase delay. (The above statement can be easily proven by using the first two terms of the Taylor series expansion to describe the phase versus frequency characteristic of the channel).

For an ideal channel with no phase distortion, the phase versus frequency relationship is linear. $T_p = T_g$ for all frequencies.

The real channel may not have this linear relationship. The phase and group delay may vary with frequency. For incoherent receivers, only the group delay is important since they operate only on the envelope of the signal. On the other hand, for coherent receivers both the phase delay and group delay information is critical.

In a frequency hopped system, the differences in group delay values at different frequencies can lead to timing jitter in the arrival of the individual pulses. Maximum jitter occurs when two consecutive hops are located at the extremities of the band. Thus the group delay slope over a band may deteriorate the performance of a FH system.

Neither the problems of group delay, nor phase delay variations were considered in the simulations. But for incoherent systems (such as likely to be used on the HF channel), it is the group delay slope that may determine the available hopping bandwidth.

2.7 Fast Fading

In what is known as fast fading, the channel characteristics vary significantly over the duration of one baud. This results in a distorted received signal. It also means that certain channel parameters cannot be tracked.

2.8 Slow Fading

Slow fading implies that the channel transfer function is essentially constant over the signalling interval. When it is constant over many bauds the changing parameters can be measured and accounted for (or equalized).

2.9 The Ionosphere

Long range HF communication is due to reflection of the signal off various ionized layers in the atmosphere. These layers make up the ionosphere, and consist of oxygen, nitrogen, helium, and hydrogen. Ultra-violet and X-ray radiation from the sun, break these molecules into ions and free electrons.

Free electrons, and their density, have a large effect on the propagation of the radio wave, resulting in refraction. The number of free electrons depends on the density of the gasses at a particular height, and the intensity of the radiation. Generally, radiation is more intense at higher altitudes, whereas gas density decreases. The more intense the radiation the greater the tendency to form ions, which favors high altitudes. At the same time, the denser the gas, the more molecules there are available to be ionized, which favors lower altitudes. Thus, there is some optimum height at which there will be a maximum ion density (and corresponding free electron density).

If the sun's radiation consisted of only one frequency, and there was only one gas in the atmosphere, there would be only one optimum height at which a maximum density of free electrons would occur. But the sun's radiation is made up of many frequencies, and there are several gasses in the atmosphere. Thus there may be several layers of peak ionization formed, where the intensity of the radiation is traded off against the number of particular gas molecules available to be ionized.

The electrons produced ~~will~~ eventually recombine with other positive ions, or diffuse naturally. The equilibrium state between production and recombination of electrons depends on the variation of the radiation, and the density of gasses. In the lower parts of the atmosphere where gas density is high, and recombination can take place quickly, and hence the reflective layer may be a strong function of current radiation intensity. At higher elevations (300-400 Km) the gasses are less dense, and recombination time can be long (hours). Thus the electron density here does not drop off suddenly when the radiation is reduced or cut-off [SAL 84].

2.10 Layers in the Ionosphere

The highest of the ionized layers is called the F layer. Its maximum electron density is at a height of about 300-500 Km. In the daytime and in the summer, this layer splits into two parts. A lower layer forms at about 200 Km. This is called the F1 layer. The layer above this is called the F2 layer. The F1 layer is most intense at local noon (that is, during the most intense time of solar radiation). At sunset, it rises to merge with the F2 layer.

Another important layer present during the day is called the E layer. It exists at an altitude of about 90-140 Km.

Below the E layer is the D layer at 65-85 Km above the earth. The electron density here is small, but in HF communications this tends to be a zone of high absorption of RF energy as the free electrons collide with neutral molecules.

In the summer time, sometimes a very strong reflective layer can develop quickly at around E layer height. It tends to be geographically quite localized, and lasts briefly (about an hour). This is called Sporadic E. It is thought to be due to wind shears within the E layer, resulting in the sudden production of high density electron clouds. The reflective ability of the Sporadic E layer tends to vary with frequency. These clouds sometimes return a much higher maximum frequency than that of the E or F layer. They can completely block the signal, obstructing the higher layers; and at other times the signal can get through what appear to be gaps in the clouds. This can cause fades at a rate of 0.01 to 0.1 Hz [M S 82].

As was said, the electron density within the layers varies with radiation intensity. This intensity varies seasonally, diurnally, and depends on the sunspot activity as well. Because of this, the E and D layers (when present) tend to develop with sunrise, and disappear as the sun goes down. The F1 layer also disappears at sunset—rising to merge with the F2 layer at the same time.

The F2 layer tends to be the most stable layer, and it may exist (due to its altitude) even as the sun disappears. The height of the F2 layer is lowest at sunrise,

and rises as noon or evening approaches due to the effect of solar heat. The F₂ layer is the main layer used for long range HF communication, so the quality of HF communication can be as variable as the character of this layer.

Like Sporadic E, there is an anomaly that sometimes develops in the F layer. This is called Spread F. In this phenomenon, a wide range of irregularities exist in the F region. Multiple and diffusing-like RF mirrors develop at various heights. It can be seen on an ionogram as a broken or diffuse structure. The multipath due to Spread F causes rapid fading at a rate of about 10-100 Hz [M S 82]. This could cause serious degradation in spread spectrum usage due to problems of acquiring, and tracking a single mode. The Spread F phenomenon occurs more often at night [SAL 84].

2.11 Multipath Due To The Ionosphere

At HF, propagation due to ionized layers can be treated like a bending or reflection of the waves. Ignoring the effects of the earth's magnetic field, we can say that since reflection of frequencies depends both on electron density, and the angle of incidence, two rays can be reflected from a "single layer" due to different angles of incidence and electron densities at slightly different heights [SAL 84]. These two rays are called the "high ray" and the "low ray". At the maximum usable frequency between two points on earth, these rays merge into one, and the signal strength enhanced. There will usually be a time difference in the arrival times of these two rays, the high ray always arriving later. What is worse, there can be multiple reflections between layers and the ground—each of these having their own high and low rays for the next reflecting layer.

The many layers allow the possibility of having more than one reflective route between two points on earth. Combinations like single hop F with double hop E (1F + 2E), or (1F + 1E) may occur. A sharp Sporadic E layer may also suddenly develop causing another reflective route.

Plots made of the received signal pulse time delays versus frequency are called

ionograms. A typical ionogram, reference [M S 82], for the HF channel is shown in Figure 2.2. Croft [CRO 114] did a study of a large number of ionosphere models, and concluded that the minimum time delay between pulses of different modes was about $100 \mu s$. In Figure 2.2 we can see that some modes disappear at higher frequencies. Also at higher frequencies though, the group time delay slope can be seen to increase.

To avoid unwanted modes it is desirable to operate close to the highest usable frequency, however this is an area of high group delay slope which can potentially cause timing jitter. Moving lower in operating frequencies lowers the group delay slope, but one may have to increase the symbol rate to avoid the now present multi-mode phenomenon. The combination of the above situations may limit the number of frequency hopping choices.

2.12 The Earth's Magnetic Field

The effect of the earth's magnetic field is to add yet another multipath component. Two waves, known as the ordinary ("o") wave, and the extraordinary ("x") wave, are generated when a linearly polarized wave is incident on the ionosphere. These two waves travel independently through the ionosphere and exit as two separate waves. In general, both waves exit from the ionosphere each elliptically polarized, but in opposite senses. The result, seen at the antenna, is an elliptically polarized wave. A linearly polarized antenna will intercept a portion of the power according to the tilt of the ellipse.

Since the ionosphere is dispersive, and the x and o waves travel independently, the relative phases between the two waves will change with frequency. This will tend to rotate the tilt of the ellipse with frequency. For a linearly polarized antenna then, the field resolvable along it will vary with frequency—this effect produces nulls in the frequency domain, and position of the nulls can change with time due to the changing ionosphere.

Figure 2.2 Idealized oblique incidence ionogram [M S 82].

2.13 Stability

Variations in time delays between modes will cause fades with time. Since phase variations are a function of frequency, the rate of fading depends on the frequency. The higher the frequency the more rapid the fading due to this.

Movements of the electrons within the layer, height changes due to heat, or increased solar radiation, can cause path attenuation or path length to change. Variations in the phase path cause a change in the frequency of the received signal. If the time delay variations were constant, only a frequency shift would be seen at the receiver. Since the rate of change of the time delay is variable, it is a frequency *spread* that is seen at the receiver.

Stability of the group time delay for a single mode is important. The group time delay must be tracked in a spread spectrum system attempting to isolate a single mode. Stability of the group time delay has been found to be typically 50 ns/s [M S 82].

2.14 Frequency Hopping on the HF channel

In the case of an incoherent frequency hopped system on the HF channel, the carrier at each hop does not have to be phased with the receiver's local oscillator. Only the envelope of the received signal is dealt with. Any group delay slope will result in arrival time jitter of the individual hops. Maximum jitter will occur when two consecutive hops are located at the extremities of the band and the hop rate is high—but there may be a minimum hop rate required to avoid the unwanted nodes.

To avoid the effects of fading, we would like (ideally) to lock onto the strongest mode present, and switch over to the next strongest that comes along if this fades. If we are hopping outside the coherence bandwidth of the single mode things become more complicated. Then we must acquire, and track the strongest mode at each

frequency. Or if this is too complex, we can perhaps stay locked onto one mode although it will enter into fades at times (at some frequencies). Using multiple hops per symbol will provide some diversity protection against these fades—although to lock on the strongest mode at all times results in a more reliable system.

Using slower hop times may make the multipath unresolvable, but all is not lost. If the symbol intervals are long enough the ISI thus introduced may be small. Also the coherence bandwidth when multiple modes are present is smaller than that of the single mode. Thus jumping to the next frequency would likely result in a position outside the coherence bandwidth, which would result in independent Rayleigh fading on each hop. For multiple hops per symbol, this results in a form of diversity gain. The timing requirements are of course less stringent in this case as well.

2.15 Jitter

If we restrict the maximum jitter to one tenth of a symbol interval, we then set a limit on the number of available hop frequencies:

$$T_j = BW_{ss} \times \dot{\tau} \quad (2.15.1)$$

where T_j is the maximum timing jitter, and $\dot{\tau}$ is the group delay slope. That is,

$$\dot{\tau} = \frac{dT_g(\omega)}{d\omega} \quad (2.15.2)$$

Restricting T_j ,

$$T_j = BW_{ss} \times \dot{\tau} \leq 0.1 \times T_{hop}, \quad (2.15.3)$$

where T_{hop} is the hopping dwell time defined by the need to avoid unwanted modes.

Therefore, BW_{ss} is limited,

$$BW_{ss} \leq 0.1 \times \frac{T_{hop}}{\dot{\tau}} \quad (2.15.4)$$

For example, if incoherently detected 2-FSK is considered, BW_{hop} for the two "mark" and "space" frequencies together is $2/T_{hop}$. One can now determine the maximum number of frequency hopping slots available, N .

$$\begin{aligned}
 N &= \frac{BW_{ss}}{BW_{hop}} \\
 &= \frac{BW_{ss}}{2/T_{hop}} \\
 &= \frac{T_{hop} \times BW_{ss}}{2} \\
 &= \frac{0.1 \times (T_{hop})^2}{2\dot{\tau}} \\
 &= \frac{0.1}{2\dot{\tau}(R_{hop})^2},
 \end{aligned} \tag{2.15.5}$$

where BW_{hop} is the bandwidth of one hop $= 2/T_{hop}$,

and R_{hop} is the hop rate $= 1/T_{hop}$.

For example, if the minimum mode difference is $250 \mu s$, and if $\dot{\tau}$ is $100 \mu s/MHz$ (suppose) then,

$$T_{hop} = 250 \mu s \quad (R_{hop} = 5000 \text{ hops/s}),$$

and $N = 31$.

This is not very good. It shows that, on the HF channel at least, we may have to limit the spreading according to the group delay slope. It is possible that the bandwidth could actually be spread further (N increased) if the PN code were checked to ensure it would not cause a jump over two consecutive frequencies liable to cause this jitter. That is, the hopper could *gradually* hop over a very large bandwidth even with a bad group delay slope.

If we take logs of both sides of (2.15.5) we get the maximum value for N (the log) as:

$$\log(N) = \log(0.05) - 2 \times \log(R_{hop}) - \log(\dot{\tau}) \tag{2.15.6}$$

A plot of $\log_{10}(N)$ versus the hopping rate R_{hop} , with τ as a parameter is shown in Figure 2.3. This applies in general to any multipath channel with group delay slope for which we are attempting to avoid unwanted multipath.

Figure 2.3 Number of hop positions versus the symbol hop rate for square-law detected 2-FSK with various group delay slopes.

Chapter 3

MODULATION SCHEMES

3.1 Introduction

The probability of error (on a per hop basis) for three modulation schemes is derived in this Chapter. This does not yield an equation for the overall error rate performance of the various systems of different receiver structures, nor does it include the effects of jamming. However the derivations here are necessary as they are the basic building blocks from which the overall error rates of the complete system are generated by simulation.

It is assumed that matched filters are used at the receiver, and that the real channel contains additive white gaussian noise of power spectral density N_0 W/Hz.

Three modulation schemes were considered. Coherent and incoherent M-ary frequency shift keying (MFSK), and M-ary phase shift keying (MPSK). In M-ary keying schemes, one of M possible waveforms is sent representing a symbol. This symbol represents binary information, hence each symbol describes a particular group of $k = \log_2(M)$ bits. Which symbols correspond to which group of k bits depend on the mapping used.

To eliminate the need to express the carrier frequency in all derivations, equivalent low-pass waveforms were used [PRO 83].

3.2 Bandpass Real Signals

A real signal at a carrier frequency f_c , occupying a bandwidth much smaller than f_c can be expressed as:

$$s(t) = a(t) \cos[2\pi f_c t + \theta(t)] , \quad (3.2.1)$$

where $a(t)$ is the amplitude of $s(t)$, and $\theta(t)$ is the phase of $s(t)$.

Expanding the cosine term we get:

$$s(t) = a(t) \cos \theta(t) \cos 2\pi f_c t - a(t) \sin \theta(t) \sin 2\pi f_c t \quad (3.2.2)$$

Letting

$$x(t) = a(t) \cos \theta(t) \quad (3.2.3)$$

and

$$y(t) = a(t) \sin \theta(t) \quad (3.2.4)$$

and

$$u(t) = x(t) + jy(t) \quad (3.2.5)$$

Then $s(t)$ can be expressed as:

$$s(t) = \text{Re} [u(t) e^{j2\pi f_c t}] , \quad (3.2.6)$$

where $u(t)$ is referred to as the low-pass equivalent waveform.

The functional form that $u(t)$ takes on for the various modulation schemes will be described later. It is mathematically convenient to deal with these low-pass waveforms as the actual carrier frequency does not enter into calculations, and the BER results are general to any carrier.

Taking the Fourier transform of $s(t)$ we obtain:

$$\begin{aligned} S(f) &= \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt \\ &= \int_{-\infty}^{\infty} \text{Re} [u(t) e^{j2\pi f_c t}] e^{-j2\pi f t} dt \\ &= \frac{1}{2} \int_{-\infty}^{\infty} [u(t) e^{j2\pi f_c t} + u^*(t) e^{-j2\pi f_c t}] e^{-j2\pi f t} dt \end{aligned} \quad (3.2.7)$$

So,

$$S(f) = \frac{1}{2} [U(f - f_c) + U^*(-f - f_c)] \quad (3.2.8)$$

Where $U(f)$ is the Fourier transform of $u(t)$.

We know that the frequencies in the real signal are all centred around the carrier frequency f_c . So the last equation tells us that $U(f)$ must be centred around zero (DC). This is why $u(t)$ is called a "low-pass" equivalent signal.

If we look at the energy in $s(t)$ now, we get:

$$\begin{aligned}
 E &= \int_{-\infty}^{+\infty} s^2(t) dt \\
 &= \int_{-\infty}^{+\infty} \left(\operatorname{Re} \left[u(t) e^{j2\pi f_c t} \right] \right)^2 dt \\
 &= \frac{1}{2} \int_{-\infty}^{+\infty} |u(t)|^2 dt + \frac{1}{2} \int_{-\infty}^{+\infty} |u(t)|^2 \cos [4\pi f_c t + 2\theta(t)] dt \quad (3.2.9)
 \end{aligned}$$

But since $s(t)$ is narrow band, its envelope varies slowly compared to its carrier frequency. So there are many cycles of the carrier that occur during very small changes in the envelope.

Thus the term

$$\frac{1}{2} \int_{-\infty}^{+\infty} |u(t)|^2 \cos [4\pi f_c t + 2\theta(t)] dt, \quad (3.2.10)$$

will integrate out to be very near to zero, at least much smaller than the term

$$\frac{1}{2} \int_{-\infty}^{+\infty} |u(t)|^2 dt$$

Thus,

$$E \approx \frac{1}{2} \int_{-\infty}^{+\infty} |u(t)|^2 dt \quad (3.2.11)$$

For example, consider just a sine wave of amplitude A which lasts from 0 to T

seconds. Then we obtain the familiar equation for E as:

$$\begin{aligned}
 E &= \frac{1}{2} \int_{-\infty}^{+\infty} |A^2| dt \\
 &= \frac{1}{2} \int_0^T A^2 dt \\
 &= \frac{A^2 T}{2}
 \end{aligned} \tag{3.2.12}$$

3.3 The Low-Pass Received Signal

We assume to have isolated a single mode therefore no intersymbol interference exists. Then the transmitted waveform is attenuated by some factor, and delayed in time as it travels from the transmitter to the receiver. It is also corrupted by additive gaussian noise. Thus the equivalent low-pass received signal is given by:

$$r(t) = \alpha u_m(t - t_d) e^{-j2\pi f_c t_d} + z(t), \tag{3.3.1}$$

where t_d is the time delay, α is the attenuation factor and $z(t)$ is the equivalent low-pass additive noise, which is assumed to be a zero mean gaussian random process.

Assuming symbol synchronization between transmitter and receiver, the equation can be simplified. To be in symbol synchronization, the delay t_d must be known to within a small fraction of T_{sym} , the symbol duration. Notice that the carrier phase may still be in large error due to the large value of f_c . That is, $\phi = 2\pi f_c t_d$ can vary appreciably with a small variation in t_d . Even if the variations, or the error in the estimate of t_d is still good enough for symbol synchronization, the carrier estimation may be completely wrong, or reliable. Because of this, the carrier phase is left in the next equation.

$$r(t) = \alpha e^{-j\phi} u_m(t) + z(t) \tag{3.3.2}$$

When the carrier phase is not estimated, the demodulation of the received signal is an incoherent process. When the phase is estimated and used in the demodulation, it is referred to as coherent demodulation.

3.4 Coherent MFSK

In MFSK, one of M frequencies is transmitted representing k bits of binary data. The M frequencies are chosen so that they are orthogonal to each other. That is, the frequencies are separated by $1/(2 \times T_{sym})$. In a hopped system, T_{sym} would be replaced by T_{hop} to maintain orthogonality over the hop time.

The ordinary MFSK system could be implemented by using M separate oscillators. As the binary data comes in to the modulator it is divided into groups of k bits. One oscillator is then selected according to the k bits.

The equivalent low-pass waveform thus becomes:

$$u(t) = \sum_{n=0}^{\infty} e^{j\pi \Delta f t I_n} g(t - n \times T_{sym}), \quad (3.4.1)$$

where

$g(t)$ is a rectangular pulse of unit amplitude and duration of T_{sym} seconds,

Δf is the frequency separation between the M frequencies ($\Delta f = 1/2T_{sym}$),
and

$I_n = \pm 1, \pm 3, \dots, \pm(M-1)$ which corresponds to the mapping of the k bits in one of M frequencies.

The optimum receiver (in terms of minimizing symbol errors) in a channel containing additive white gaussian noise (AWGN), for equal energy symbols, is one that calculates the decision variables U_m as follows [PRO 83]:

$$U_m = \text{Re} \left[e^{j\phi} \int_0^T r(t) u_m^*(t) dt \right] \quad m = 1, 2, 3 \dots M \quad (3.4.2)$$

The receiver then decides in favor of the symbol for which U_m is largest. These decision variables are generated by M matched filters.

The probability of error can be derived assuming $s_1(t)$ was sent. Then the equivalent low-pass received signal is given by:

$$r(t) = \alpha e^{-j\omega_c t} u_1(t) + z(t) \quad (3.4.3)$$

Substituting $r(t)$ into equation (3.4.2) we get:

$$U_1 = 2\alpha E + N_{1r} \quad (3.4.4)$$

As the transmitted frequencies are orthogonal, the output of the other $(M-1)$ matched filters consist of noise only

$$U_{m \neq 1} = N_{mr} \quad (3.4.5)$$

with

$$N_{mr} = \text{Re} \left[\int_0^T z(t) u_m^*(t) dt \right] \quad (3.4.6)$$

These noise components N_{mr} are zero mean gaussian random numbers with variance [PRO 83]:

$$\sigma^2 = 2EN_0 \quad (3.4.7)$$

Since the signals are orthogonal, the noise terms are uncorrelated. They are also gaussian distributed and hence statistically independent. Therefore the probability densities are as follows:

$$p(U_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(U_1 - 2\alpha E)^2 / 2\sigma^2} \quad (3.4.8)$$

$$p(U_m) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{U_m^2}{2\sigma^2}} \quad m \neq 1 \quad (3.4.9)$$

Now we can derive the probability of a correct decision (on this hop), for the Linear Combining, and Majority Vote type receiver.

$$\begin{aligned} P_{\text{correct}} &= \text{Prob} (U_1 > \text{all } U_{m \neq 1}) \\ &= \int_{-\infty}^{+\infty} \text{Prob} (U_2 < U_1, U_3 < U_1, \dots, U_M < U_1 | U_1) p(U_1) dU_1 \end{aligned} \quad (3.4.10)$$

The conditional probability on U_1 can be factored into a product of many conditional probabilities due to the statistical independence of the noise.

Looking at just one term of this product we get:

$$\begin{aligned} \text{Prob} (U_m < U_1 | U_1) &= \int_{-\infty}^{U_1} p(U_m) dU_m \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{U_1/\sqrt{2EN_0}} e^{-x^2/2} dx \end{aligned} \quad (3.4.11)$$

But these terms are the same for all $m = 2$ to M . Therefore the probability of a correct symbol decision is given by:

$$P_{\text{correct}} = \int_{-\infty}^{+\infty} \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{U_1}{\sqrt{2EN_0}}} e^{-\frac{x^2}{2}} dx \right)^{M-1} \times p(U_1) dU_1 \quad (3.4.12)$$

The probability of a symbol error is then

$$P_M = 1 - P_{\text{correct}} \quad (3.4.13)$$

Substituting for $p(U_1)$ we get

$$P_M = 1 - \int_{-\infty}^{+\infty} \left\{ \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{y}{\sqrt{2EN_0}}} e^{-\frac{x^2}{2}} dx \right)^{M-1} \times \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-2\alpha E)^2}{2\sigma^2}} \right\} dy \quad (3.4.14)$$

Since it is equally likely that any of the M symbols were transmitted, and since they are all orthogonal, this result is general even though derived assuming $s_1(t)$ was transmitted. For this reason, U_1 was replaced by dummy variable y in the above equation.

All comparisons in this thesis were done on the basis of bit error rate, the probability that a received decoded bit is in error. For MFSK (coherently and incoherently detected) it is simple to convert between symbol error rate (just derived) and the bit error rate. Since all symbols are equally likely to be transmitted, and since they are all orthogonal, then given a symbol error, it is equally likely that any particular *wrong* symbol was chosen (all types of symbol errors are equally likely to occur). They occur with probability,

$$\frac{P_M}{M-1} = \frac{P_M}{2^k-1} \quad (3.4.15)$$

There are $\binom{k}{n}$ ways in which n bits out of k bits can be in error ($n=1$ to k) any of these will constitute a symbol error. Therefore the average number of bits in error in a k -bit symbol is,

$$\sum_{n=1}^k n \binom{k}{n} \frac{P_M}{2^k-1} = k \left(\frac{2^{k-1}}{2^k-1} \right) P_M \quad (3.4.16)$$

Dividing by k , the number of bits in a symbol, yields the rate of bit errors.

$$\begin{aligned} P_b = \text{BER} &= \frac{2^{k-1}}{2^k-1} P_M \\ &= \left(\frac{M}{2} \right) \frac{P_M}{(M-1)} \end{aligned} \quad (3.4.17)$$

Signal-to-noise ratios will also always be expressed in terms of the SNR per bit. The SNR per bit is the SNR per symbol divided by k , the number of bits per symbol.

$$SNR_{\text{symbol}} = \alpha^2 E / N_0 \quad (3.4.18)$$

$$SNR_{bit} = \frac{\alpha^2 E}{k N_0} \quad (3.4.19)$$

$$\text{Noting that the energy per bit } E_b = \frac{E}{k} \quad (3.4.20)$$

then

$$SNR_{bit} = \alpha^2 \frac{E_b}{N_0} \quad (3.4.21)$$

All the receiver structures considered generate these decision variables $U_1, U_2, U_3, \dots, U_M$ in their processing on each hop at some point. What these receivers do with these "incomplete" decision variables to generate some final decision variables after all hopping is done, varies depending on the receiver. That is, these decision variables are processed and combined in some way after all hopping is done. The result is then used to make a decision on which symbol was transmitted. This will be discussed in the next Chapter which describes receiver structures.

3.5 Incoherent MFSK

Again one of M frequencies is transmitted corresponding to the mapping of k bits. Therefore the equivalent low-pass waveform is similar to the case of coherent MFSK, and physical generation of the waveforms can be done similarly. There is a difference, however, in the minimum frequency separation required. With incoherent detection, the frequency spacing Δf , must be at least $1/T_{sym}$ (or $1/T_{hop}$ for multihop per symbol cases) or integral multiples thereof.

At the receiver however, unlike coherent MFSK, no use is made of the carrier phase. Only the envelope of the carrier wave is detected. This is a great simplification in terms of timing requirements, and is often used in practice—especially on fading channels.

Assuming equal energy symbol waveforms, the optimum receiver [PRO 83] computes the decision variables U_m as follows:

$$U_m = \left| \int_0^T r(t) u_m^*(t) dt \right| \quad (3.5.1)$$

The receiver then selects the m corresponding to the largest U_m .

Sometimes a square-law detector rather than an envelope detector is used. This yields a set of decision variables U_m^2 , $m = 1, 2, 3, 4 \dots M$. The two detection methods give almost identical BER results. In the case of one hop per symbol frequency hopping, the results are identical. For the case of multiple hops per symbol, the difference in performance is small [PRO 83]. So the square-law device can approximate quite accurately the envelope (optimum) detector. The square-law detector is also easier to analyze mathematically, and faster to simulate. This detection method was used in the simulation.

For the square-law detector the decision variables generated on each hop are,

$$U_m = \left| \int_0^T r(t) u_m^*(t) dt \right|^2 \quad m = 1, 2, 3 \dots M \quad (3.5.2)$$

This can be thought of as correlating the signal using two correlators, each in quadrature, and squaring and adding their outputs.

Again assuming $s_1(t)$ was sent, the decision variables formed on each hop will be:

$$U_1 = |2\alpha E + N_1|^2 \quad (3.5.3)$$

and

$$U_m = |N_m|^2 \quad \text{for } m \neq 1, \quad (3.5.4)$$

where

$$N_m = N_R + jN_I \quad (3.5.5)$$

The N 's are zero mean, complex gaussian random noise variables with variance [PRO 83] given by:

$$\begin{aligned}\sigma_N^2 &= \frac{1}{2} E[|N_m|^2] \\ &= 2EN_0\end{aligned}\quad (3.5.6)$$

Because of the gaussian noise, U_1 is then a non-central chi-square random variable with non-centrality parameter s given by:

$$s^2 = (2\alpha E)^2 \quad (3.5.7)$$

Hence the probability density of U_1 is given as [PRO 83]:

$$p(U_1) = \frac{1}{4EN_0} \left(\frac{U_1}{s^2}\right) \exp\left[-\frac{(s^2 + U_1)}{4EN_0}\right] I_0\left(\frac{s\sqrt{U_1}}{2EN_0}\right), \quad (3.5.8)$$

where I_0 is the modified Bessel function of the first kind order zero.

The other decision variables U_m , $m = 2, 3, \dots, M$ are given as:

$$p(U_m) = \frac{1}{4EN_0} U_m \cdot e^{-\frac{U_m}{4EN_0}} \quad (3.5.9)$$

These are central chi-square distributed since they are made up of the sum of two zero mean gaussian variables. They are also statistically independent.

Now the probability of a symbol error P_M on a single hop can be derived for the Linear Combining and Majority Vote receivers.

$$P_M = 1 - P_{\text{correct}}$$

Again, the above can be factored due to statistical independence. Hence P_M can be rewritten as:

$$P_M = 1 - \int_0^\infty [\text{Prob}(U_2 < U_1 | U_1)]^{M-1} p(U_1) dU_1 \quad (3.5.10)$$

and

$$\text{Prob}(U_2 < U_1 | U_1) = 1 - e^{\frac{-U_1}{4EN_0}} \quad (3.5.11)$$

So

$$P_M = 1 - \int_0^\infty \left[1 - e^{\frac{-U_1}{4EN_0}} \right]^{M-1} p(U_1) d(U_1) \quad (3.5.12)$$

Then the same conversion from symbol error rate to bit error rate follows.

$$P_b = \left(\frac{M}{2} \right) \frac{P_M}{(M-1)}$$

Again this is only for one hop; and the receiver using hard limiters does not calculate errors like this. Chapter 4, on receiver structures, and Chapter 6, which describes the simulation, explain this in detail.

3.6 Coherent MPSK

In MPSK it is the phase of the carrier that is modulated. One of M phases is selected according to the k bits grouped at the receiver. The equivalent low-pass wave form is thus:

$$u(t) = \sum_{n=0}^{\infty} e^{j\theta_n} g(t - nT), \quad (3.6.1)$$

where θ_n corresponds to the mapping of k bits into one of M phases, $\frac{2\pi(m-1)}{M}$ for $m = 1, 2, 3 \dots M$.

The optimum receiver (minimizing symbol errors) [PRO 83] generates the decision variables (per hop) in the following manner. The decision variables come out of a single cross-correlator which calculates the vector,

$$V = e^{j\phi} \int_0^T r(t) u^*(t) dt \quad (3.6.2)$$

This vector is then projected onto M unit vectors, each at the M possible angles. This means the dot product is taken between the received signal and the M unit

vectors. The largest (most positive) of the resulting M numbers is used to determine which of the M phases was most likely sent. Thus the receiver makes a decision based on which reference angle the received signal is closest to.

Expressing V as,

$$V = X + jY \quad (3.6.3)$$

Then the decision variables U_m are expressed as:

$$U_m = 2EX \cos\left(\frac{2\pi(m-1)}{M}\right) + 2EY \sin\left(\frac{2\pi(m-1)}{M}\right) \quad (3.6.4)$$

The set

$$\left(\cos \frac{2\pi(m-1)}{M}, \sin \frac{2\pi(m-1)}{M} \right)$$

for $m = 1, 2, 3 \dots M$, are the set of reference vectors with which V forms the dot product.

Assume $s_1(t)$ was transmitted, that is, symbol one, corresponding to a zero shift in phase. The received signal, with carrier phase known, is then:

$$r(t) = \alpha e^{-j\phi} u(t) + z(t) \quad (3.6.5)$$

Hence V is given by:

$$V = 2\alpha E + N, \quad (3.6.6)$$

where N is a complex gaussian random variable with zero mean, and variance given by:

$$\frac{1}{2} E[|N|^2] = 2EN_0 \quad (3.6.7)$$

Since $V = X + jY$ then

$$X = 2\alpha E + \text{Re}(N) \quad (3.6.8)$$

and

$$Y = \text{Im}(N) \quad (3.6.9)$$

$\text{Re}(N)$, and $\text{Im}(N)$ are real numbers. They are gaussian, independent, and each has a variance of:

$$\sigma^2 = 2EN_0 \quad (3.6.10)$$

For a single hop, considering only the Linear Combining and the Majority Vote receivers, we can derive the probability of error.

X and Y are jointly gaussian with joint probability density function [PRO 83] given by:

$$p(X, Y) = \frac{1}{2\pi\sigma^2} \exp \left[\frac{-(X - 2\alpha E)^2 - Y^2}{2\sigma^2} \right], \quad (3.6.11)$$

with

$$\sigma^2 = 2EN_0 \quad (3.6.12)$$

The phase of V is $\theta = \tan(Y/X)$. The reference phase that this is closest to is decided on as being the symbol sent. We can get the probability density function for θ by a change of variables.

Letting

$$R = \sqrt{X^2 + Y^2} \quad (3.6.13)$$

and

$$\theta = \tan^{-1} \left(\frac{Y}{X} \right) \quad (3.6.14)$$

Then,

$$p(r, \theta) = \frac{r}{2\pi\sigma^2} e^{-(r^2 + 4\alpha^2 E^2 - 4\alpha E r \cos \theta)/2\sigma^2} \quad (3.6.15)$$

Now $p(\theta)$ is found by integrating r out of the equation:

$$p(\theta) = \int_0^\infty p(r, \theta) dr$$

An error is caused in the selection of a symbol (assuming $s_1(t)$ was sent) if the noise causes the phase (of V) to fall outside the range $\frac{-\pi}{M}$ to $\frac{+\pi}{M}$.

After integrating out r , we obtain the probability density of θ as [PRO 83]:

$$p(\theta) = \frac{r}{2\pi} e^{-\gamma} \left(1 + \sqrt{4\pi\gamma} \cos \theta e^{\gamma \cos^2 \theta} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{2\gamma} \cos \theta} e^{-\frac{x^2}{2}} dx \right), \quad (3.6.16)$$

where $\gamma = \frac{\alpha^2 E}{N_0}$ is the SNR per symbol.

Then

$$P_M = 1 - \int_{-\pi/M}^{\pi/M} p(\theta) d\theta \quad (3.6.17)$$

This is a general result since it is equally likely that any signal would be transmitted. But it is general only so far as symbol errors are concerned. Deriving a BER from the symbol error rate is not as easy as in the case of MFSK. It depends on the particular mapping of bits into symbols. Here Gray coding, where adjacent symbols differ by only one bit, has been assumed. This is an acceptable coding scheme, since symbol errors at high SNR's, are most likely to occur between neighboring symbols. Even with Gray coding, care has to be taken when calculating how many bits are in error given that a symbol is in error. So, one must average the effect of an incorrect phase decision (or difference from the correct phase) over all possible correct absolute phases.

For $M=2$, the BER is identical to the symbol error rate.

For $M=4$, the average effect of deciding on the symbol $+\pi/2$ or $-\pi/2$ degrees offset to the correct symbol phase, is that one bit will be in error. Being in error by an offset π will cause two bits to be in error.

For $M=8$, averaging over all symbols assumed sent, the average effect of deciding on the symbol $+\pi/4$ or $-\pi/4$ degrees offset to the correct symbol phase, is that one bit will be in error. Being in error by any other offset will cause two bits to be in error. In the simulations M was never more than 8.

Chapter 4

RECEIVER STRUCTURES

4.1 Introduction

For each of the modulation schemes, three different receiver structures have been considered:

- (i) A linear combining receiver.
- (ii) A receiver which makes a decision on each hop then takes a majority vote to decide on the symbol transmitted.
- (iii) A receiver with hard limiters.

4.1.1 The Linear Combining Receiver

The M decision variables, appropriate to the modulation scheme in use, were generated for each hop. These were generated as described in Chapter 3. After all hopping was done, a new set of decision variables were formed—each of these being made up of the linear sum of the corresponding incremental decision variables for each symbol formed on each hop. A decision was then based on which of these final summed decision variables was largest.

4.1.2 The Majority Vote Receiver

The M decision variables were generated on each hop as with the Linear Combining receiver. At this time, the largest of these variables was erased and assigned a value of 1. All other decision variables calculated on this hop were then erased and assigned a value of 0.

After all hopping was completed, the 1's and 0's which were assigned to each of the decision variables generated on each hop were summed. This formed a new set

of M decision variables. A decision was then made based on which of the summed set of decision variables was largest.

This algorithm in fact, makes a final decision in favor of the symbol most often decided upon on a per hop basis. Thus a majority vote is taken. If there was a tie vote, a random selection was made among the tied symbols.

4.1.3 Hard Limiter Combining

Again the M decision variables were generated on each hop. Each decision variable was then passed through a hard limiter and changed to a value of 1 or 0 depending on whether it was above or below a certain threshold. Unlike the Majority Vote receiver, in the case of the receiver with hard limiters, the assignment of a 1 or a 0 to a hop decision variable was made independently of the other decision variable values. Thus, it was possible with this type of receiver, to have more than one decision variable (on any hop) assigned a value of 1.

After all hopping was completed, algorithm from this point on was the same as that of the Majority Vote receiver.

4.2 Hard Limiter Thresholds

Choosing the threshold for the hard limiter was a fundamental problem. A method had to be found to find a threshold value that would minimize the probability of error. The method used to determine this value of the threshold will now be described.

Let us begin by defining two hypotheses:

H_0 : that a symbol we are testing for is not present. (i.e., this symbol's decision variable is being considered but this symbol was not sent, another one was.)

H_1 : that a symbol we are testing for is present. (i.e., this symbol's decision variable is being considered and this symbol was sent.)

We are considering now the ordinary decision variables generated for the appropriate modulation scheme on each hop as described in Chapter 3. The question is: what magnitude should these exceed for us to say "this symbol was sent"? That is, choose H_1 (or not sent perhaps, in which case choose H_0). What rule can be used to ensure that we are correct in our decisions as often as possible?

Let z be the observation variable. In this case z is the output of a particular matched filter, or a decision variable. And now consider two conditional probabilities:

$P(H_0|z)$ – the probability of H_0 being true given z .

$P(H_1|z)$ – the probability of H_1 being true given z .

The rule used to form the thresholds is based on the idea that, naturally, given an observed value of z , one should pick as being correct, or "true", the hypothesis with the largest probability given that observed value of z .

That is, given the observed value of z :

$$\text{If } P(H_0|z) > P(H_1|z) \quad \text{choose } H_0 \text{ as the truth} \quad (4.2.1)$$

$$\text{If } P(H_1|z) > P(H_0|z) \quad \text{choose } H_1 \text{ as the truth} \quad (4.2.2)$$

This is the Maximum A posteriori Probability criterion (MAP). It is difficult to find the above conditional probabilities, but they can be derived from other conditional probabilities as follows:

Assume H_0 and find $p(z|H_0)$ (note that this is a density function and not a probability). Next, assume H_1 and find $p(z|H_1)$.

Now use Bayes' theorem to obtain the probabilities we want:

$$P(H_0|z) = \frac{p(z|H_0)P(H_0)}{p(z)}, \quad (4.2.3)$$

$$P(H_1|z) = \frac{p(z|H_1)P(H_1)}{p(z)}, \quad (4.2.4)$$

where $p(z)$ is the probability density of z ,

and $P(H_0)$ and $P(H_1)$ are the apriori probabilities that H_0 or H_1 is true.

Substituting the above into the MAP criterion formulas (4.2.1) and (4.2.2) and cancelling the common $p(z)$, we obtain the following rule:

$$\text{For values of } z \text{ which satisfy } p(z|H_0)P(H_0) > p(z|H_1)P(H_1) \quad \text{choose } H_0 \quad (4.2.5)$$

$$\text{For values of } z \text{ which satisfy } p(z|H_1)P(H_1) > p(z|H_0)P(H_0) \quad \text{choose } H_1 \quad (4.2.6)$$

To find the threshold value of z , the inequality can be replaced by an equality. We can then solve (when possible) for the value of z at the border line of the decision. This value of z is the threshold Z_{thresh} . It is this Z_{thresh} to which we can then compare observed values of z to decide which hypothesis to choose as being correct.

In an M-ary system, the apriori probabilities $P(H_0)$ and $P(H_1)$ are:

$$P(H_0) = \frac{M-1}{M} \quad (4.2.7)$$

$$P(H_1) = \frac{1}{M} \quad (4.2.8)$$

Substituting the above apriori probabilities into (4.2.5) and (4.2.6) and cancelling the common M in the denominators, the threshold is obtained by finding the value of z satisfying the following equation:

$$(M-1)p(z|H_0) = p(z|H_1) \quad (4.2.9)$$

Essentially the threshold value is found where the two conditional probability densities cross on a graph, noting that the density associated with the absence of a symbol must first be weighted by a factor of $(M-1)$. With this in mind, and having found Z_{thresh} , it is easy to see that:

If z is greater than Z_{thresh} choose H_1 (hard limit to 1).

If z is less than Z_{thresh} choose H_0 (hard limit to 0).

Of course, to find Z_{thresh} one must manipulate the probability densities associated with the various levels and modulation schemes. When the signal is transmitted over a fading channel it can complicate things as well. Sometimes the threshold value cannot be solved in closed form, in which case numerical analysis must be done to find the value of z which satisfies the equality.

In derivations to follow on hard limiter thresholds, it should be noted that since these thresholds involve observations on each hop, the signal energy E , used in the formulas from Chapter 3, is now replaced with $E/(L_{\text{hops}})$ (the total symbol energy divided by the number of hops per symbol, L_{hops}).

Since the term $2 \times E/(L_{\text{hops}})$ appears often, for the sake of reducing symbols and improving readability, it is replaced by the term $E2$. This term, $E2$, appears often in the actual FORTRAN code as well.

Also, in the simulation, N_0 (the noise power spectral density) was set to unity. Since performance was specified in terms of SNR (not signal power alone) this resulted in mathematical simplification without any loss in generality of the results. Bearing this in mind, in the derivations to follow, the variance of the zero mean complex gaussian noise random variable $N = N_R + jN_I$, is given by:

$$\begin{aligned}\sigma_N^2 &= \sigma_{N_R}^2 = \sigma_{N_I}^2 = 2 \times E/(L_{\text{hops}}) \times N_0 \\ &= E2 \quad (\text{for } N_0 = 1)\end{aligned}\tag{4.2.10}$$

4.2.1 Threshold For Coherent MFSK Under No Fading

Consider the output of the M matched filter branches as shown in Figure 4.1. Here the two weighted conditional densities $p(z|H_0)P(H_0)$ and $p(z|H_1)P(H_1)$ are shown.

Figure 4.1 Weighted conditional probability densities used to find the hard limiter threshold value for coherently detected MFSK with no fading.

Assuming H_1 , the symbol was sent:

$$\begin{aligned} z &= 2E/L_{hops} + N_R \\ &= (E2) + N_R \end{aligned} \quad (4.2.11)$$

Thus the conditional density is:

$$p(z|H_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z-E2}{\sigma}\right)^2} \quad (\text{Gaussian}) \quad (4.2.12)$$

Assuming H_0 , the signal was not sent:

$$z = N_R \quad (4.2.13)$$

Thus

$$p(z|H_0) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2} \quad (\text{Gaussian}) \quad (4.2.14)$$

Now using $(M-1)p(z|H_0) = p(z|H_1)$ we can solve for Z_{thresh} . Rearranging terms we obtain,

$$\begin{aligned} \frac{e^{-\frac{1}{2}\left(\frac{z-E2}{\sigma}\right)^2}}{e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2}} &= M-1 \\ e^{-\frac{1}{2}\left[\frac{z^2-2z(E2)+(E2)^2-z^2}{\sigma^2}\right]} &= M-1 \end{aligned}$$

Taking logs of both sides, z (thus Z_{thresh}) can be solved for:

$$\begin{aligned} Z_{thresh} &= \frac{\sigma^2 \ln(M-1) + 0.5(E2)^2}{E2} \\ &= N_0 \ln(M-1) + \frac{1}{2}(E2) \\ &= \ln(M-1) + \frac{1}{2}(E2) \quad (\text{for } N_0 = 1) \end{aligned} \quad (4.2.15)$$

4.2.2 Threshold For Incoherent MFSK Under No Fading

To evaluate the threshold required for square-law detected MFSK the calculations are done based on the envelope (r) of the signal, although r^2 is the output of the receiver. When the threshold was found (based on the envelope) it was simply squared to produce the correct threshold for a square-law detection system. This allowed a smaller range of search to be used for locating the threshold.

First consider low to medium SNR's. The envelope for conditions of H_0 are shown in Figure 4.2. Assuming H_1 , the symbol is present, we know:

$$\begin{aligned} z^2 &= (2E/L_{hops} + N_R)^2 + N_I^2 \\ &= (E2 + N_R)^2 + N_I^2 \end{aligned} \quad (4.2.16)$$

z^2 is made up of a squared non-zero mean gaussian variable, and a zero mean gaussian random variable squared. Thus it has a non-central chi-squared distribution with 2 degrees of freedom.

We can change this to something more familiar by a change of variables.

Let

$$r = \sqrt{z^2} \quad (4.2.17)$$

Then r will have a Ricean p.d.f.

$$p(r|H_1) = \frac{r}{\sigma^2} e^{-\frac{(r^2+s^2)}{2\sigma^2}} I_0\left(\frac{rs}{\sigma^2}\right) \quad r \geq 0 \quad (4.2.18)$$

where I_0 is the modified Bessel Function of the first kind and order zero, and s^2 is the non-centrality parameter given by:

$$\begin{aligned} s^2 &= \sum_{\text{degrees}} (\text{means})^2 \\ &= (2E/L_{hops})^2 + 0^2 \\ &= (E2)^2 \end{aligned} \quad (4.2.19)$$

Figure 4.2 Weighted conditional probability densities used to find the hard limiter threshold value for square-law detected MFSK at low SNR with no fading.

Recalling that $\sigma^2 = E2$ for $N_0 = 1$, we can write that given H_1

$$p(r|H_1) = \left(\frac{r}{E2}\right) e^{-\frac{(r^2 + E2^2)}{2 \times E2}} \times I_0(r) \quad r \geq 0 \quad (4.2.20)$$

Now assuming H_0 , signal absent:

$$z^2 = N_R^2 + N_I^2 \quad (\text{noise only}) \quad (4.2.21)$$

and

$$\begin{aligned} p(r|H_0) &= \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \\ &= \left(\frac{r}{E2}\right) e^{-\frac{r^2}{2 \times E2}} \quad (\text{Rayleigh}) \end{aligned} \quad (4.2.22)$$

Now using $(M-1)p(r|H_0) = p(r|H_1)$ to solve for r , then

$$\left(\frac{r}{E2}\right) e^{-\frac{r^2}{2 \times E2}} (M-1) = \frac{r}{E2} e^{-\frac{(r^2 + E2^2)}{2 \times E2}} \times I_0(r) \quad (4.2.23)$$

A program was written to solve this for r , thus finding R_{thresh} . The square of this value was then used as the threshold in the square-law detector.

Higher SNR's:

At higher SNR's, things become somewhat simpler. Then $p(r|H_1)$ can be described as Gaussian instead of Ricean. Thus,

$$p(r|H_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{r-E2}{\sigma}\right)^2} \quad (\text{Gaussian}) \quad (4.2.24)$$

$$p(r|H_0) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad (\text{Rayleigh, as before}) \quad (4.2.25)$$

These weighted densities are shown in Figure 4.3. Solving for the threshold then,

$$(M-1) \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{r-E2}{\sigma}\right)^2}$$

Figure 4.3 Weighted conditional probability densities used to find the hard limiter threshold value for square-law detected MFSK at high SNR with no fading.

$$\ln \left[\frac{\sqrt{2\pi}(M-1)r}{\sigma} \right] - \frac{r^2}{2\sigma^2} = \frac{-1}{2} \left(\frac{r - E2}{\sigma} \right)^2$$

With $\sigma^2 = E2$, and solving for r ,

$$r = \frac{E2}{2} + \ln \left[\frac{(M-1)\sqrt{2\pi}r}{\sqrt{E2}} \right] \quad (4.2.26)$$

A program was written to find the value of $r = R_{thresh}$, satisfying the equation by searching over a range of values.

4.2.3 Threshold For Coherent MPSK Under No Fading

As described in Chapter 3, the dot product of the vector V with the M unit vectors is formed. Let us divide the complex calculated vector V into the real and imaginary parts, X and Y as shown in Figure 4.4.

Assume symbol J_s was sent ($s = 1, 2, 3, \dots, M$), the output vector is $V = X + jY$ with,

$$X = (E2) \cos \frac{2\pi(J_s - 1)}{M} + N_R, \quad (4.2.27)$$

and

$$Y = (E2) \sin \frac{2\pi(J_s - 1)}{M} + N_I \quad (4.2.28)$$

The dot products (for $J=1,2,3,\dots,M$) are formed with the set

$$(XD_J, YD_J)$$

With

$$XD_J = \cos \frac{2\pi(J - 1)}{M}, \quad (4.2.29)$$

and

$$YD_J = \sin \frac{2\pi(J - 1)}{M} \quad (4.2.30)$$

Figure 4.4 The dot product is formed between the vector V and the M unit vectors (shown for 8-PSK).

The decision variable $U_{J|J_s}$ is thus,

$$U_{J|J_s} = X \cdot XD_J + Y \cdot YD_J \quad (4.2.31)$$

with

$$X \cdot XD_J = (E2) \cos \frac{2\pi(J_s - 1)}{M} \cdot \cos \frac{2\pi(J - 1)}{M} + N_R \cos \frac{2\pi(J - 1)}{M}$$

and

$$Y \cdot YD_J = (E2) \sin \frac{2\pi(J_s - 1)}{M} \cdot \sin \frac{2\pi(J - 1)}{M} + N_I \sin \frac{2\pi(J - 1)}{M}$$

Adding the above,

$$U_{J|J_s} = E2 (\cos A \cos B + \sin A \sin B) + N_R \cos B + N_I \sin B, \quad (4.2.32)$$

where

$$A = \frac{2\pi(J_s - 1)}{M}$$

and

$$B = \frac{2\pi(J - 1)}{M}$$

$N_R \cos B + N_I \sin B$ is a sum of independent gaussian variables. It can be replaced with one random number of equal total variance. Therefore it is replaced with N_R for convenience.

Now using $\cos(A - B) = \cos A \cos B + \sin A \sin B$, we obtain:

$$U_{J|J_s} = E2 \cos \frac{2\pi(J_s - J)}{M} + N_R \quad (4.2.33)$$

It is important to note that this conditional decision variable that is obtained when J_s was sent, and we are testing for symbol J , is only a function of the difference between J_s and J .

We can now form the conditional densities for the decision variables when testing for the presence of a particular symbol.

Assuming H_1 , that is $J = J_s$ (the symbol we are testing for was sent):

$$p(z|H_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z-E2}{\sigma}\right)^2} \quad (4.2.34)$$

Assuming H_0 , the symbol being tested for is not the one that was sent:

$$p(z|H_0) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{\Delta J} e^{-\frac{1}{2}\left(\frac{z-m_{\Delta J}}{\sigma}\right)^2} \times \frac{1}{(M-1)}, \quad (4.2.35)$$

where $\Delta J = J_s - J$, $J_s \neq J$,

and $m_{\Delta J} = (E2) \cos \frac{2\pi\Delta J}{M}$.

Figure 4.5 shows these weighted densities for $M = 2, 4$, and 8.

As can be seen $p(z|H_0)$ is made up of a sum of gaussian densities with different mean values, while $p(z|H_1)$ is a gaussian density with a mean value of $E2$. Now the weighted densities are equated to find the threshold. Using $(M-1)p(z|H_0) = p(z|H_1)$ and cancelling the common $1/\sigma\sqrt{2\pi}$ term, and letting $\sigma^2 = E2$, we obtain:

$$\sum_{\Delta J} e^{-\frac{1}{2}\left(\frac{z-m_{\Delta J}}{E2}\right)^2} = e^{-\frac{1}{2}\left(\frac{z-E2}{E2}\right)^2} \quad (4.2.36)$$

A program was written to solve this equality for Z_{thresh} . Note at high SNR's, the sum on the left hand side was only taken over $\Delta J = 1$ and $(M-1)$. That is, only the effect of the nearest neighbor was included. This is because the tails of other densities with means farther away would contribute almost nothing to the sum in the vicinity of where Z_{thresh} would be. Also trying to calculate on the computer sums of numbers so small caused arithmetic underflow problems.

4.3 Thresholds In a Fading Environment

If the thresholds calculated for use in a non-fading environment are used in fading, the bit error rate tails off to an irreducible error even at high SNR's. What happens in this case is that the noise is not contributing to the error rate, rather

Figure 4.5 Weighted conditional probability densities used to find the hard limiter threshold value for coherently detected 2-PSK, 4-PSK, and 8-PSK with no fading.

it is the result of the fading. Fading causes the signal to drop below the threshold when it should not, whereas noise has a negligible effect. This suggests that a good method for calculating hard limiter thresholds in fading might be to consider the probability density associated with signal fading only (and not noise) in the conditional p.d.f's of equation (4.2.9). That is, given the signal is not present, noise comes through. But given the signal is present, noise is neglected and the statistics due to fading only are considered.

Based on the above, thresholds were found for each modulation scheme, under the assumption of Rayleigh Fading.

4.3.1 Threshold For Coherent MFSK Under Rayleigh Fading

We again have the same problem of finding the point where two weighted conditional densities cross. With no signal present, we have zero mean gaussian noise; with the signal present we assume a Rayleigh conditional density only. The no signal case is $(M - 1)$ times more likely to occur than the case of the signal present so it is assigned this weighting before finding the intersection point. Figure 4.6 shows these two densities.

Assuming H_0 , symbol not present:

$$z = N_R, \quad (4.3.1)$$

with

$$\begin{aligned} \sigma_{N_R}^2 &= (2E/L_{hops})N_0 \\ &= E^2 \quad \text{for } N_0 = 1 \end{aligned}$$

The conditional density, given H_0 is then,

$$\begin{aligned} p(z|H_0) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2} \\ &= \frac{1}{\sqrt{2\pi}E^2} e^{-\frac{1}{2}\left(\frac{z^2}{E^2}\right)} \end{aligned} \quad (4.3.2)$$

Assuming H_1 , symbol sent:

$$\begin{aligned} z &= 2\alpha E/L_{hops} \\ &= \alpha E^2 \end{aligned} \quad (4.3.3)$$

Figure 4.6 Weighted conditional probability densities used to find the hard limiter threshold value for coherently detected MFSK with Rayleigh fading.

The p.d.f. of α is Rayleigh and given by:

$$p_{\alpha}(\alpha) = \frac{\alpha}{\sigma_{\alpha}^2} e^{-\frac{\alpha^2}{2\sigma_{\alpha}^2}} \quad \alpha \geq 0 \quad (4.3.4)$$

In the simulations for mathematical ease we have:

$$\sigma_{\alpha}^2 = \frac{1}{2}$$

Hence the variance of α is given by:

$$\text{Var}[\alpha] = 2\sigma_{\alpha}^2 = 1 \quad (4.3.5)$$

Therefore,

$$p_{\alpha}(\alpha) = 2\alpha e^{-\alpha^2} \quad (4.3.6)$$

We must now find the p.d.f. of z . We can calculate this in the following manner:

Where a and b are constants, and x a random variable of probability density $p_x(x)$, we know [PAP 65] that for some random variable y given by:

$$y = ax + b, \quad (4.3.7)$$

that

$$p_y(y) = \frac{1}{|a|} p_x\left(\frac{y-b}{a}\right) \quad (4.3.8)$$

This is related to $p(z|H_1)$ by letting

$$x = \alpha,$$

$$a = E2,$$

and

$$b = 0,$$

and noting that α has Rayleigh statistics.

Hence the p.d.f associated with (4.3.3) is given by:

$$\begin{aligned} p(z|H_1) &= \frac{1}{E2} p_\alpha \left(\frac{z}{E2} \right) \\ &= \frac{1}{E2} \left(2 \times \frac{z}{E2} e^{-\left(\frac{z}{E2}\right)^2} \right) \end{aligned} \quad (4.3.9)$$

Now we can equate the weighted densities to obtain the threshold value:

$$\frac{1}{\sqrt{2\pi E2}} e^{-\frac{1}{2}\left(\frac{z}{E2}\right)^2} (M-1) = \frac{2}{(E2)^2} z e^{-\left(\frac{z}{E2}\right)^2}$$

Rearranging and taking the natural logs of both sides of the last equation we obtain:

$$-\frac{1}{2} \frac{z^2}{(E2)} + \left(\frac{z}{E2} \right)^2 = \ln \left[\frac{2\sqrt{2\pi} z}{(E2)^{3/2} (M-1)} \right] \quad (4.3.10)$$

A program was written to solve this equation for $z = Z_{thresh}$.

4.3.2 Threshold For Incoherent MFSK Under Rayleigh Fading

In this case, with no signal present we have Rayleigh p.d.f. due to the noise alone. With the signal present (examining the envelope) we have another Rayleigh p.d.f. but this time due to fading only. We weight the no signal case by $M-1$ as usual, then equate the weighted conditional densities and find the intersection point to locate the threshold. Figure 4.7 shows the two weighted conditional densities involved.

Assuming H_0 , symbol not present:

$$r = \sqrt{N_R^2 + N_I^2}, \quad (4.3.11)$$

with

$$\begin{aligned} \sigma_{N_R}^2 &= \sigma_{N_I}^2 \\ &= E2 \quad \text{for } N_0 = 1 \end{aligned}$$

The conditional density, given H_0 is then,

$$\begin{aligned} p(z|H_0) &= \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \\ &= \frac{r}{(E2)} e^{-\frac{r^2}{2(E2)}} \end{aligned} \quad (4.3.12)$$

Figure 4.7 Weighted conditional probability densities used to find the hard limiter threshold value for square-law detected MFSK with Rayleigh fading.

Assuming H_1 , symbol sent:

$$\begin{aligned} r &= 2\alpha E / L_{\text{hops}} \\ &= \alpha E 2 \end{aligned} \quad (4.3.13)$$

Now, similar to the case of coherent MFSK under fading, we obtain:

$$p(r|H_1) = \frac{2}{(E2)^2} r e^{-\left(\frac{r}{E2}\right)^2} \quad (4.3.14)$$

Equating these densities, and weighting $p(r|H_0)$ by $M-1$ we obtain:

$$\frac{e^{-\frac{r^2}{2(E2)^2}}}{e^{-\left(\frac{r}{E2}\right)^2}} = \frac{2}{(E2)(M-1)}$$

Taking logs of both sides and solving for r^2 we get:

$$r^2 = \left[\frac{-1}{2(E2)} + \frac{1}{(E2)^2} \right]^{-1} \ln \left[\frac{2}{(E2)(M-1)} \right] \quad (4.3.15)$$

This is a closed form solution for the threshold r^2 usable with the square-law detector in fading.

4.3.3 Threshold For Coherent MPSK Under Rayleigh Fading

In this case, for $M = 2$, the threshold is still located at zero. This is obvious when the conditional densities shown in Figure 4.8 are considered. The effect of fading in MPSK is to increase or decrease the spacing between the means of the gaussian densities.

In the case of $M = 4$, we can calculate the threshold based on the sum of two gaussian densities (zero mean) intersecting with the Rayleigh fading statistics associated with the signal present conditional density.

The case of $M = 8$ was not considered due to its complexity. When Rayleigh statistics (only) are applied to the case of the signal present conditional density, the gaussian means are not fixed, but move in accordance with the particular Rayleigh sample. Then the method we have been using for locating the threshold is thus not applicable to $M = 8$.

Figure 4.8 Weighted conditional probability densities used to find the hard limiter threshold value for coherently detected 2-PSK and 4-PSK with Rayleigh fading.

We can derive a threshold value for $M = 4$, however; we do this by assuming a fixed gaussian density of zero mean (made up of the sum of two gaussian curves) as shown in Figure 4.8. It is also assumed that the single gaussian conditional density with a negative mean does not contribute to the sum of the two zero mean curves—that is its tail is very small near the threshold value. Thus we eliminate the problems of this negative mean density moving around during fading by ignoring it.

Thus for $M = 4$, assuming H_0 (symbol not present):

We have a zero mean gaussian density which will be weighted by a factor of 2. Thus,

$$\begin{aligned} p(z|H_0) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^2} \\ &= \frac{1}{\sqrt{2\pi}E2} e^{-\frac{1}{2}\left(\frac{z^2}{E^2}\right)} \end{aligned} \quad (4.3.16)$$

Assuming H_1 , symbol present:

$$\begin{aligned} z &= 2\alpha E / L_{\text{hope}} \\ &= \alpha E2 \end{aligned} \quad (4.3.17)$$

As before, since α has Rayleigh statistics,

$$p(z) = \frac{2z}{(E2)^2} e^{-\left(\frac{z}{E2}\right)^2} \quad (4.3.18)$$

Equating the above two densities and weighting the zero mean density twice as much as the other density we get,

$$\frac{1}{\sqrt{2\pi}E2} e^{-\frac{1}{2}\left(\frac{z^2}{E^2}\right)} \times 2 = \frac{2z}{(E2)^2} e^{-\left(\frac{z}{E2}\right)^2}$$

Taking the natural logs of both sides of the equation we obtain:

$$-\frac{1}{2} \left(\frac{z^2}{E2} \right) + \left(\frac{z}{E2} \right)^2 = \ln \left[\frac{z \sqrt{2\pi E2}}{(E2)^2} \right] \quad (4.3.19)$$

A program was written to solve this for $z = Z_{thresh}$.

Chapter 5

JAMMING

5.1 Partial-Band Jamming

The partial-band jammer is modelled as additional gaussian noise in the channel. The noise is assumed to be spread over a bandwidth of $\gamma \cdot BW_{ss}$. That is the fraction of the spread spectrum being jammed is γ , and the jammer's power is spread uniformly over this fraction of the band. The jammer's total power spectral density averaged over the entire spread bandwidth is N_J . Therefore the actual power spectral density within the jammed bandwidth is N_J/γ . This is illustrated in Figure 5.1.

It is further assumed that given a hop is jammed, all symbol frequencies (in the case of MFSK) are affected. N_J is related to the given values of E_b/N_0 , and E_b/N_J as follows:

$$\frac{\left(\frac{E_b}{N_0}\right)}{\left(\frac{E_b}{N_J}\right)} = \left(\frac{N_J}{N_0}\right) \quad (5.1.1)$$

$$= N_J \quad \text{if} \quad N_0 = 1$$

Or, if E_b/N_0 and E_b/N_J are given in dB, N_J is found as:

$$\begin{aligned} 10 \log_{10} \left[\frac{E_b/N_0}{E_b/N_J} \right] &= 10 \log_{10} \left(\frac{E_b}{N_0} \right) - 10 \log_{10} \left(\frac{E_b}{N_J} \right) \\ &= 10 \log N_J \\ &= \left(\frac{E_b}{N_0} \right)_{dB} - \left(\frac{E_b}{N_J} \right)_{dB} \\ &= (N_J)_{dB} \quad \text{for } N_0 = 1 \end{aligned} \quad (5.1.2)$$

Therefore,

$$N_J = 10^{\left[\frac{\left(\frac{E_b}{N_0} \right)_{dB} - \left(\frac{E_b}{N_J} \right)_{dB}}{10} \right]}$$

Figure 5.1 Power spectral density over the spread spectrum bandwidth with partial-band jamming.

For a given system in operation, there is some fraction γ , γ_{opt} , that causes the most degradation in performance.

5.1.1 The Effect of Jamming On The Equations of Chapter 3.

In Chapter 3, the noise density in the channel was assumed to be N_0 W/Hz. It will remain so as long as the receiver is not operating within the band covered by the jammer. If it is at a frequency covered by the jammer, the effective noise power spectral density is modified to become:

$$N' = N_0 + N_J \quad (5.1.3)$$

Thus N' replaces N_0 in the formulas of Chapter 3 if it is given that this hop is jammed.

5.2 Continuous Wave (CW) Jamming

In this type of jamming, the jammer divides his total power into Q CW tones spaced throughout the spread bandwidth. One orderly method is to put exactly q tones in each jammed M -ary cluster (instead of randomly distributing them across BW_{ss}) thus jamming Q/q clusters. Levitt suggests that using $q = 1$ is the optimum (for BER degradation) way of distributing these Q CW tones [LEV 85].

Suppose Q frequencies are jammed, and that the jammer has a total power of P_J Watts. Each CW tone will have a power of P_J/Q Watts. If there are N_s possible single frequencies¹ available to be potentially jammed across BW_{ss} , then

¹ Be carefull! What is referred to here is what the jammer can observe in the spectrum without knowing the modulation scheme or the level, M . i.e., a "single frequency" has a bandwidth of $1/T_{hop}$.

the fraction² of available single frequencies jammed is given as:

$$\gamma_s = \frac{Q}{N_s} \quad (5.2.1)$$

The number of single frequencies in BW_{ss} available is:

$$\begin{aligned} N_s &= \frac{BW_{ss}}{1/T_{hop}} \\ &= BW_{ss} \cdot \left(\frac{T_{sym}}{L_{hop}} \right) \end{aligned} \quad (5.2.2)$$

Let the ratio of the signal power S , to CW power per each jamming tone be β . Then,

$$\beta = S \cdot \left(\frac{Q}{P_J} \right) \quad (5.2.3)$$

Now,

$$\begin{aligned} Q &= \left(\frac{Q}{N_s} \right) \cdot N_s \\ &= \left(\frac{Q}{N_s} \right) \cdot BW_{ss} \cdot \left(\frac{T_{sym}}{L_{hop}} \right) \end{aligned} \quad (5.2.4)$$

Substituting from (5.2.4) into (5.2.3) yields,

$$\begin{aligned} \beta &= \frac{S}{P_J} \cdot \left(\frac{Q}{N_s} \right) \cdot BW_{ss} \cdot \left(\frac{T_{sym}}{L_{hop}} \right) \\ &= (S \cdot T_{sym}) \cdot \left(\frac{BW_{ss}}{P_J} \right) \left(\frac{Q}{N_s} \right) \frac{1}{L_{hop}} \\ &= \left(\frac{E}{N_J} \right) \cdot \gamma_s \cdot \frac{1}{L_{hop}} \\ &= \left(\frac{E_b}{N_J} \right) \cdot k \gamma_s \cdot \frac{1}{L_{hop}} \end{aligned} \quad (5.2.5)$$

One can also talk about the fraction of M-ary cluster slots (instead of frequencies) to jam³ and relate this to β . If we call this fraction just γ , and we call N_M

² To jam q out of M frequencies in the M-ary clusters requires $\gamma_s \leq 2q/(M+1)$ for coherent MFSK; $\gamma_s \leq q/M$ for incoherent MFSK, or else there will be more sets of q CW jamming signals than available M-ary clusters—[LEV 85] suggest $q=1$ is the best choice. (Only restriction for coherent MPSK is $\gamma_s \leq 1$).

³ A coherent MPSK slot bandwidth on a hop is $1/T_{hop}$; coherent MFSK is $(M+1)/2$ times that; and incoherent MFSK is M times $1/T_{hop}$ —these are the bandwidths of the full M-ary clusters.

2

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the total number of *slots* in BW_{ss} (which will depend on the modulation scheme and level) then another relation involving β , but this time γ , can be derived. The number of *slots* jammed is Q/q (Q is still the total number of frequencies jammed; q cw tones in each jammed slot). Therefore the fraction of jammed slots is given by:

$$\gamma = \frac{Q}{q \cdot N_M} \quad (5.2.6)$$

And

$$N_M = \frac{BW_{ss}}{(1/T_{hop}) \cdot X} \quad (5.2.7)$$

Now following same procedure as in the derivation of (5.2.5):

$$\begin{aligned} \beta &= \frac{S}{P_J} \cdot \left(\frac{Q}{q N_M} \right) \cdot BW_{ss} \cdot \left(\frac{T_{sym}}{L_{hops}} \right) \cdot \frac{q}{X} \\ &= \left(\frac{E_b}{N_J} \right) \cdot k \gamma \cdot \frac{q}{X \cdot L_{hops}}, \end{aligned} \quad (5.2.8)$$

where X (a result of cluster bandwidth dependence on modulation scheme) is $(M+1)/2$ for coherent MFSK; M for incoherent MFSK; and 1 for MPSK, and q can be one of 1,2,3... M (except for MPSK, where it can only be 1).

In a multiple hop per symbol system, we have two cases that can arise under jamming: either we are being jammed on this hop, or we are not. Also, given that we are being jammed ("hit") on this hop, in the case of MFSK, we may be jamming the symbol waveform which was sent, or we may be hitting one of the other $M-1$ frequencies.

Taking $q = 1$, the effect (on a per hop basis) of CW jamming on the decision variables generated on each hop will now be derived for each modulation scheme.

5.2.1 MFSK Coherently Detected Under CW Jamming

Let us assume that the jammer has hit the frequency of the signal that was sent. Figure 5.2 shows a CW interference signal at some random phase ϕ with respect to the desired signal. Under these conditions, the decision variable that will

Figure 5.2 Model of CW interference on coherently detected MFSK.

be generated by the receiver is given by:

$$\begin{aligned}
 U_m &= \sqrt{2S} \left[\alpha \sqrt{2S} + \sqrt{\frac{2S}{\beta}} \cos \phi \right] T_{hop} + N_R \\
 &= 2ST_{hop} \left[\alpha + \frac{\cos \phi}{\sqrt{\beta}} \right] + N_R \\
 &= (E2) \left[\alpha + \frac{\cos \phi}{\sqrt{\beta}} \right] + N_R
 \end{aligned} \tag{5.2.9}$$

If the interference occurs on a frequency not of the sent signal, the effect can be derived by letting α go to zero in the last equation. Then,

$$U_m = (E2) \left(\frac{\cos \phi}{\sqrt{\beta}} \right) + N_R \tag{5.2.10}$$

Under fading α has Rayleigh statistics, in non-fading environments, it is set to unity.

5.2.2 MFSK Square-Law Detected Under CW Jamming

Assume again that the jammer has hit the frequency of the signal that was sent. Figure 5.3 shows a CW interference signal at some random phase ϕ with respect to the desired signal. Under these conditions, the decision variable that will be generated by the receiver is,

$$U_m = X^2 + Y^2,$$

$$\begin{aligned}
 X &= \sqrt{2S} \left[\sqrt{2S} \sqrt{\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \cos^2 \theta} \right] T_{hop} + N_R \\
 &= 2ST_{hop} \sqrt{\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \cos^2 \theta} + N_R
 \end{aligned} \tag{5.2.11}$$

$$\begin{aligned}
 Y &= \sqrt{2S} \left[\sqrt{2S} \sqrt{\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \sin^2 \theta} \right] T_{hop} + N_I \\
 &= 2ST_{hop} \sqrt{\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \sin^2 \theta} + N_I
 \end{aligned} \tag{5.2.12}$$

Figure 5.3 Model of CW interference on square-law detected MFSK.

So,

$$X^2 = (E2)^2 \left(\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \right) \cos^2 \theta + 2N_R(E2) \sqrt{\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta}} \cos \theta + N_R^2 \quad (5.2.130)$$

$$Y^2 = (E2)^2 \left(\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \right) \sin^2 \theta + 2N_I(E2) \left(\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \right) \sin \theta + N_I^2 \quad (5.2.14)$$

Adding X^2 and Y^2 ,

$$\begin{aligned} X^2 + Y^2 &= (E2)^2 \left(\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta} \right) \\ &\quad + 2(E2) \sqrt{\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta}} (N_R \cos \theta + N_I \sin \theta) \\ &\quad + N_I^2 + N_R^2 \\ &= \left[(E2) \sqrt{\alpha^2 + \frac{2\alpha \cos \phi}{\sqrt{\beta}} + \frac{1}{\beta}} + N_R \right]^2 + N_I^2 \end{aligned} \quad (5.2.15)$$

If the interference does not occur in the transmitted symbol's frequency, then replacing α with zero will give the decision variable generated as:

$$X^2 + Y^2 = \left[\frac{(E2)}{\sqrt{\beta}} + N_R \right]^2 + N_I^2 \quad (5.2.16)$$

Under fading α has Rayleigh statistics. In non-fading environments it is set to unity.

5.2.3 MPSK Coherently Detected Under CW Jamming

Figure 5.4 shows what happens to an MPSK signal when it is interfered with by a CW signal. The interfering signal is at a random phase ϕ with respect to the desired signal.

Figure 5.4 Model of CW interference on coherently detected MPSK.

Recalling Chapter 3, taking the complex integral, and breaking the vector V into its real and imaginary parts we obtain:

$$\begin{aligned}
 \text{Re}(V) &= \sqrt{2S} \left[\alpha \sqrt{2S} + \sqrt{\frac{2S}{\beta}} \cos \phi \right] T_{hop} + N_R \\
 &= 2ST_{hop} \left[\alpha + \frac{\cos \phi}{\sqrt{\beta}} \right] + N_R \\
 &= (E2) \left[\alpha + \frac{\cos \phi}{\sqrt{\beta}} \right] + N_R
 \end{aligned} \tag{5.2.17}$$

and

$$\begin{aligned}
 \text{Im}(V) &= \sqrt{2S} \left[\sqrt{\frac{2S}{\beta}} \sin \phi \right] T_{hop} + N_I \\
 &= 2ST_{hop} \frac{\sin \phi}{\sqrt{\beta}} + N_I \\
 &= (E2) \frac{\sin \phi}{\sqrt{\beta}} + N_I
 \end{aligned} \tag{5.2.18}$$

Under no CW interference (this hop not hit) then as usual,

$$\text{Real}(V) = \alpha(E2) + N_R \tag{5.2.19}$$

and

$$\text{Im}(V) = N_I \tag{5.2.20}$$

In fading, again, α is given Rayleigh statistics. In non-fading environments it is set to unity.

Chapter 6

SIMULATION

6.1 Introduction

This chapter gives a general description of the programs used to calculate the BER under various conditions, and presents the results of the simulation. The generation of the random numbers of a gaussian, or Rayleigh distribution is also discussed.

The calculations were done using the Electronics department's VAX 11/750 computer. All programs were written in FORTRAN. Typical running times for these programs was very high, on the order of one or two days CPU time. To reduce the disruption to other users, the programs were run at the lowest priority setting while in batch mode. After running, special graphics data files were transferred to the METHEUS computer and plotted on the HP-7580 drafting plotter. Due to the large number of plots generated, it was decided to code these figures in a manner that would make them quickly identifiable without reading through long, similar looking titles on each graph, describing all the associated parameters. The convention for coding these figures is described later.

There are two main divisions of the results:

1. The first, plots of BER versus E_b/N_0 , were used to determine the value of E_b/N_0 required to yield a 10^{-4} BER, and are shown in Appendices A, B, and C for the Linear Combining, the Majority Voting, and the Hard Limiting receivers respectively. Values for E_b/N_0 were obtained by viewing the plots on the METHEUS graphics terminal and using the moveable cross hairs to locate the point where various curves crossed the 10^{-4} BER axis. These values are shown in Tables 6.1(a,b,c), 6.2(a,b,c), and 6.3(a,b,c) for the Linear Combining receiver, the Majority Voting receiver, and the Hard Limiter receivers respectively. The a,b,c

Table 6.1a E_b/N_0 (dB) for 10^{-4} BER: Linear Receiver, Coherent MFSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	11.49	11.49	11.49	11.49	11.49
	4	8.49	8.49	8.49	8.49	8.49
	8	7.44	7.44	7.44	7.44	7.44
Faded	2	36.41	23.58	18.72	17.30	16.11
	4	35.56	20.41	17.81	15.15	13.40
	8	35.22	21.15	15.61	14.14	12.10

Table 6.1b E_b/N_0 (dB) for 10^{-4} BER: Linear Receiver, Square-Law MFSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	12.08	12.96	13.13	13.74	13.65
	4	9.63	10.27	10.60	11.01	11.01
	8	8.28	8.85	9.13	9.42	9.63
Faded	2	40.09	26.07	20.75	19.56	18.89
	4	38.67	24.14	18.55	16.34	15.44
	8	38.22	21.15	17.59	15.55	14.36

Table 6.1c E_b/N_0 (dB) for 10^{-4} BER: Linear Receiver, Coherent MPSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	8.29	8.29	8.29	8.29	8.29
	4	8.29	8.29	8.29	8.29	8.29
	8	11.77	11.77	11.77	11.77	11.77
Faded	2	32.51	20.07	16.17	14.25	13.06
	4	34.78	20.36	16.39	13.63	13.29
	8	36.92	22.56	19.45	17.19	16.57

Table 6.2a E_b/N_0 (dB) for 10^{-4} BER: Majority Receiver, Coherent MFSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	11.40	14.39	12.68	13.82	13.14
	4	8.69	11.61	10.65	11.47	11.22
	8	7.44	10.36	9.55	9.47	10.08
Faded	2	37.67	41.00	25.13	25.30	20.00
	4	38.76	41.88	23.34	22.81	18.71
	8	35.29	38.22	22.45	20.31	17.55

Table 6.2b E_b/N_0 (dB) for 10^{-4} BER: Majority Receiver, Square-Law MFSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	12.11	15.11	14.18	15.50	14.23
	4	9.55	12.54	12.00	12.90	12.50
	8	8.26	11.22	10.54	10.90	11.47
Faded	2	39.65	43.39	27.00	27.89	23.25
	4	41.34	43.75	25.48	25.48	21.65
	8	38.22	40.63	24.77	22.81	19.87

Table 6.2c E_b/N_0 (dB) for 10^{-4} BER: Majority Receiver, Coherent MPSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	8.44	11.33	10.22	11.15	10.12
	4	8.37	11.36	9.93	11.15	10.58
	8	11.76	14.68	13.54	14.46	13.93
Faded	2	34.39	36.18	20.85	23.16	16.39
	4	34.48	38.04	23.08	21.83	17.73
	8	36.98	40.09	25.75	25.48	21.03

Table 6.3a E_b/N_0 (dB) for 10^{-4} BER: Hard Receiver, Coherent MFSK

		Hops per Symbol.				
	M	1	2	3	4	5
Unfaded	2	14.40	13.69	13.76	13.58	13.51
	4	11.62	11.34	11.23	11.05	10.98
	8	10.30	10.43	10.05	10.16	10.14
Faded	2	47.92	29.43	24.22	21.79	19.82
	4	47.29	26.43	21.45	18.74	17.66
	8	44.69	25.75	20.32	17.66	16.20

Table 6.3b E_b/N_0 (dB) for 10^{-4} BER: Hard Receiver, Square-Law MFSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	15.00	15.15	15.15	15.29	15.33
	4	12.19	12.34	12.48	12.51	12.44
	8	10.73	11.02	10.80	11.20	11.52
Faded	2	48.59	31.01	25.41	23.21	21.62
	4	48.03	27.56	22.13	20.55	20.00
	8	45.31	26.54	21.45	19.08	17.72

Table 6.3c E_b/N_0 (dB) for 10^{-4} BER: Hard Receiver, Coherent MPSK

		Hops per Symbol				
	M	1	2	3	4	5
Unfaded	2	8.52	11.37	10.05	11.30	10.09
	4	11.84	10.91	10.70	10.70	10.70
	8	20.39	19.78	19.21	18.59	18.47
Faded	2	33.67	36.83	20.72	22.70	16.37
	4	45.43	27.62	20.39	18.00	16.45

table letters refer to MFSK coherent, MFSK square-law, and MPSK coherent modulation schemes operating with the given receiver structures. Once the required values of E_b/N_0 were obtained, they were entered into batch files to be read by the programs performing the jamming BER calculations.

2. The second, plots of BER versus E_b/N_J (BER in optimum partial-band jamming), used the fixed value of E_b/N_0 and are shown in Appendices D, E, and F for the Linear Combining, the Majority Voting, and the Hard Limiting receivers respectively. As well, some typical plots of $\log_{10} \gamma_{opt}$ ($\log_{10}$ of the optimum fraction of the band to jam) versus E_b/N_J , are shown in Appendix G.

6.2 Gaussian and Rayleigh Distributed Numbers

In Chapters 3 and 4 a description was given for decision variables calculated on a per hop basis. In these equations there were random noise variables present each time a calculation for a particular symbol occurred. These variables had a gaussian distribution under our assumption. In the simulation, therefore, it was necessary to obtain these number samples (of the correct variance) from some set having a gaussian distribution. The Box and Muller technique [LEW 75] was used to generate this set of gaussian numbers using the uniform random number generator available in the FORTRAN library on the VAX. The method is described below.

Let u_1 and u_2 be independent uniformly distributed numbers between 0 and 1. Two normally distributed random numbers, x_1 and x_2 , can then be generated using the transformation given by:

$$x_1 = \sqrt{-2 \log_e u_1} \cos 2\pi u_2 \quad (6.2.1)$$

$$x_2 = \sqrt{-2 \log_e u_1} \sin 2\pi u_2 \quad (6.2.2)$$

Provided u_1 and u_2 are independent, the above can be derived by looking at the joint density, $p(x_1, x_2)$, of the independent gaussian variables.

$$\begin{aligned}
 p(x_1, x_2) &= p(x_1)p(x_2) \\
 &= \frac{1}{\sqrt{2\pi}} e^{-\frac{x_1^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x_2^2}{2}}
 \end{aligned} \tag{6.2.3}$$

Letting

$$x_1 = r \cos \theta \tag{6.2.4}$$

and

$$x_2 = r \sin \theta \tag{6.2.5}$$

$p(x_1, x_2)$ can be transformed into $p(r, \theta)$. And,

$$\begin{aligned}
 p(r, \theta) \cdot dr \cdot d\theta &= \frac{1}{\sqrt{2\pi}} e^{-\frac{r^2}{2}} \cdot r \cdot dr \cdot d\theta \\
 &= e^{-\left(\frac{r^2}{2}\right)} d\left(\frac{r^2}{2}\right) \cdot \frac{1}{\sqrt{2\pi}} d\theta \\
 &= p_1(r) p_2(\theta)
 \end{aligned} \tag{6.2.6}$$

Thus, r^2 and θ are independently distributed and can be sampled from $p_1(r)$ and $p_2(\theta)$. These particular samples, r_0 and θ_0 , can then be used to find x_1 and x_2 :

$$\begin{aligned}
 u_1 &= \int_0^{r_0} p_1(z) dz \\
 &= \int_0^{r_0} e^{-\frac{z^2}{2}} d\left(\frac{z^2}{2}\right) \\
 &= -e^{-\frac{z^2}{2}} \Big|_0^{r_0} \\
 &= 1 - e^{-\frac{r_0^2}{2}}
 \end{aligned} \tag{6.2.7}$$

Since u_1 is uniformly distributed between 0 and 1, $1 - u_1$ and u_1 yield the same result (they are statistically equivalent). Therefore one can use $1 - u_1$ as it simplifies things.

Then,

$$r_0 = \sqrt{-2 \log_e u_1} \quad (6.2.8)$$

Next, for u_2 :

$$\begin{aligned} u_2 &= \int_0^{\theta_0} \frac{1}{2\pi} dz \\ &= \frac{\theta_0}{2\pi}, \end{aligned} \quad (6.2.9)$$

hence,

$$\theta_0 = 2\pi u_2 \quad (6.2.10)$$

Finally, substituting r_0 from (6.2.8), and θ_0 from (6.2.10), back into (6.2.4) and (6.2.5), we obtain equations (6.2.1) and (6.2.2) giving x_1 and x_2 .

One million of these numbers were computed using the uniform random number generator (RANDU) on the VAX. These numbers were read into a one million element array and accessed sequentially from the array when needed during the many loops of the simulation. This saved multiple calculations of square roots, cosines, sines, and logs. When the end of the array was reached, its index was set back to the beginning. It was found that by saving these numbers, and recycling them, computation time was speeded up by a factor of roughly five. Histograms are shown in Figures 6.1, 6.2, and 6.3 for 1000, 10000, and 100000 samples of the computer generated gaussian numbers respectively. It can be seen that the curve looks more gaussian as the number of samples is increased.

Depending on the number of symbol errors encountered under a given set of conditions, the number of symbols examined varied. At very low SNRs perhaps only 1000 symbols would be sent before calculating the BER. At higher SNRs perhaps 50000 or 100000 symbols would need to be tried before a suitable number of errors were found. The criteria for deciding the number of symbols to be sent is discussed in Sections 6.3 and 6.4.

Figure 6.1 Histogram of 1,000 samples of computer generated gaussian distributed numbers.

Figure 6.2 Histogram of 10,000 samples of computer generated gaussian distributed numbers.

Figure 6.3 Histogram of 100,000 samples of computer generated gaussian distributed numbers.

Rayleigh distributed variables were also needed to provide the fading attenuation factor α . This variable was computed by first generating two gaussian random numbers of unity variance, x_1 and x_2 , by the method just described. Each gaussian number was then multiplied $\sqrt{\frac{1}{2}}$ (thus giving two gaussian numbers of variance $\frac{1}{2}$ each). The square root of the sum of the squares was then taken and the result assigned to α . Thus,

$$\alpha = \sqrt{\left(\frac{x_1}{\sqrt{2}}\right)^2 + \left(\frac{x_2}{\sqrt{2}}\right)^2} \quad (6.2.11)$$

The expected value of α^2 was then $\frac{1}{2} + \frac{1}{2} = 1$. This was mathematically convenient as it meant that the average SNR per bit in fading, which is given by $E[\alpha^2 \cdot \frac{E_b}{N_0}]$, was just $\frac{E_b}{N_0}$. A histogram made up of 500000 samples of the computer generated Rayleigh distributed variable is shown in Figure 6.4.

6.3 BER versus E_b/N_0

This section describes in general the method used to generate plots of BER versus E_b/N_0 . There were separate programs written for each receiver and modulation scheme. There were many similarities between these programs, the basic common steps are described below:

1. Fill the 1,000,000 element array with the computer generated unit variance gaussian numbers.
2. Set the seed (four bytes) of the uniform random number generator RANDU.
3. Set the numerical value of the AWGN power spectral density, N_0 to unity for mathematical simplification.
4. Read ILOW and IBITMAX from the batch file input.
 - a) ILOW defines the minimum number of symbols to be tested before counting the number of errors to determine the BER. This would normally be the number of symbols checked at low SNRs.

Figure 6.4 Histogram of 500,000 samples of computer generated Rayleigh distributed numbers.

- b) IBITMAX defines the absolute possible maximum symbols that will be tested before calculating the BER under a certain set of conditions. The number of symbols tested will always be somewhere between ILOW and IBITMAX.
5. Read M, the M-ary level for the modulation under consideration, and the range of E_b/N_0 for which the BER calculations are to be made. If M is greater than 256 stop the program (this step is the outermost loop of the program).
 6. DO loop for number of hops per symbol (L_{hops} going from 1 to 5).
 - a) Set IEND, the initial number of symbols to test, to ILOW.
 7. DO loop for the range of E_b/N_0 .
 - a) Reset the symbol error counter to zero (SERC=0).
 - b) Calculate the energy per symbol given M, and E_b/N_0 , and the noise variance associated with the matched filter.
 - c) Set the index of the gaussian array to the beginning.
 8. DO loop for sending IEND symbols through.

For the Linear Combining receiver

- (i) Form the linear sum of the intermediate decision variables calculated over hops 1 to L_{hops} . The specific formula used on each hop depends on the modulation scheme as described in Chapter 3. The energy detected on each hop is the total symbol energy E_s , divided by L_{hops} .
- (ii) Next form linear sum of all the intermediate calculations for the other symbols (2 to M).
- (iii) Compare the M linear sums. If any of the sums for symbols 2 to M is greater than the sum calculated for symbol 1 (assumed sent) then increment the symbol error counter SERC.

For the Majority Voting receiver

- (i) Fill the M element vote array (which keeps track of the number of votes

received by each symbol over the L_{hops}) with zeros.

- (ii) Over each and all hops from 1 to L_{hops} , decide which symbol's decision variable is largest and increment that element of the M element vote array.
- (iii) Compare the totals contained in the vote array. If the first element (representing the vote total for symbol 1) is less than any other of the M elements then a symbol decision error was made. Increment the symbol error counter and proceed with sending through the next of IEND symbols.
- (iv) If the first total is the same as some other total in the array, it means there is a tie vote between the correct symbol and some incorrect symbols. In this case a random choice is made, and the symbol error counter incremented appropriately.

For the Hard Limiter receiver

- (i) Fill the M element vote array (which keeps track of the number of votes received by each symbol over the L_{hops}) with zeros.
- (ii) Over each and all hops from 1 to L_{hops} , and for each symbol, decide if the symbol's decision variable calculated on that hop is greater than the threshold value THRESH, and increment the vote array element associated with that symbol if that is the case. Note that the value of THRESH depends on the energy per hop. Values of THRESH for all combinations of E_b/N_0 from -5 to 50 dB, and values of L_{hops} from 1 to 5, were calculated as described in Chapter 4 by a separate program. A two dimensional array based on these two parameters (which effectively define the symbol energy per hop for a given M-ary level) was then filled from disk at the start of the program.
- (iii) Compare values contained in the vote array. If the first element (representing the vote total for symbol 1) is less than any other of the M elements then a symbol decision error was made. Increment the symbol error counter and proceed with sending through the next of IEND symbols.

- (iv) If the first element is the same as some other elements, it means there is a tie vote between the correct symbol and some incorrect symbols. In this case a random choice is made, and the symbol error counter incremented appropriately.

Note that for all of the above receivers operating with MPSK modulation, when a symbol error is made it is important *which* symbol is in error. This is the only way to keep track of the true BER in this case. For this reason there is only a bit error counter, BEREC, that is incremented according to the number of bits in error for a symbol decided on. Also, for the Majority Voting, and Hard Limiter receiver, there must be a random guess made between a set of *wrong* symbols should some symbols be tied for votes and have more votes than symbol 1.

9. End of the IEND symbols DO loop.
10. After sending IEND symbols through the system, now check for a reasonable amount of errors. If we have counted less than 10 symbol errors, and IEND is less than IBITMAX, IEND is multiplied by 10, the symbol error counter SERC reset to zero, and the IEND loop started again (Step 8).
11. Determine the BER by dividing the average number of bits in error by the number of bits sent, and write this to disk along with E_b/N_0 , the symbol error rate, and the number of symbols tested.
12. If no symbol errors were found, don't do any more calculations at higher values of E_b/N_0 . Instead leave this loop and go on to the next value for L_{hops} (go to Step 14).
13. End of the E_b/N_0 DO loop.
14. End of the L_{hops} DO loop.
15. Go back to Step 5 and read the next value of M to consider.

6.3.1 Code Convention for Plots of BER versus E_b/N_0

These plots are described by a series of hyphenated letters. These codes describe the receiver type, the modulation scheme, and detection method, as well as whether fading was included. The code is the last item in the figure description, and in general has the following form:

$$R - M_1 M_2 M_3 M_4 - D_1 D_2 D_3 - F$$

The first letter, R , describes the receiver type and has three possible assignments:

- (i) **L** -referring to the Linear receiver.
- (ii) **M** -referring to the Majority Vote receiver.
- (iii) **H** -referring to the Hard Limiter receiver.

The second set, $M_1 M_2 M_3 M_4$, describes the modulation scheme, and has five possible assignments:

- (i) **2FSK** or,
- (ii) **4FSK** or,
- (iii) **8FSK**.

In the above cases the plots will have the number of hops per symbol, L_{hops} , as a parameter ranging from 1 to 5. Thus, these figures will each consist of 5 plotted curves.

- (iv) **MFSK** or,
- (v) **MPSK**.

In the last two cases, M (the level of the modulation scheme) is the varying parameter on the plots ($M=2, 4$, and 8). Thus there will be three curves on each of these plots.

The third set, $D_1 D_2 D_3$, describes the detection technique:

- (i) **COH** -refers to coherent detection.

(ii) **SQR** -refers to square-law detection.

The last letter, **F**, may or may not be present.

(i) **F** -when present, refers to a plot in which Rayleigh fading was included in the simulation. When this last letter is missing, it means no fading was accounted for in the simulation.

For example, a code of **L-8PSK-COH-F**, refers to a plot of the BER versus E_b/N_0 for the Linear Combining receiver, using coherently detected 8-PSK with Rayleigh fading.

6.4 BER versus E_b/N_f

Many parts of these programs were similar to their corresponding jam-free versions. The main differences will be described here:

1. The range of $\log_{10} \gamma$ is read in. This always started at -4 and went to 0 in increments of 0.25 in its DO loop. That is, the fraction of the band under jamming went from 10^{-4} , to completely full band.

It should be noted that the absolute minimum possible portion of the band that can be jammed (under the assumption of all symbol frequencies are hit in the case of MFSK) varies with M and L_{hops} ; in the case of MPSK it depends on L_{hops} only. This is because of our assumption that BW_{ss} is fixed for all cases to be compared. Given the fixed spread spectrum bandwidth, BW_{ss} , the minimum possible fraction of the band to jam occurs for MPSK at 1 hop per symbol. If we want to say the minimum γ (in general) is the smallest γ that will allow for the full M -ary bandwidth to be jammed on a hop, then relative to 1 hop MPSK, this new minimum will be L_{hops} times as large—and for coherent MFSK this will mean a minimum (relative to 1 hop MPSK) that is increased by a factor of $\frac{(M+1)}{2} \times L_{hops}$, while the increase for incoherent MFSK is $M \times L_{hops}$. The MPSK minimum can be determined from the symbol time T_{sym} , and the actual value of BW_{ss} . Then all the other real minimum values of γ under the

other cases can be determined.

How can we start γ at 10^{-4} in all cases without considering the modulation scheme and L_{hops} ? We can do this if we assume a large number of frequencies to hop over. Then the differences between the various true absolute possible minimum values of γ , under all combinations of M and L_{hops} , become small. Or we could simply say that these simulations apply to the case of a jammer who will only jam the band at a minimum fraction of 10^{-4} —which, although perhaps not known to the jammer, may not be the minimum possible γ for the modulation scheme and L_{hops} combination in use.

2. Read ILOW and ITOP. ILOW is the minimum value for IEND to assume when beginning the IEND DO loop. ITOP is assigned to IBITMAX upon the first scan through the range of γ . If the maximum number of symbol errors encountered (for an assumed number of hops jammed at a particular value of γ) is not large enough, IEND is increased to a maximum of IBITMAX.
3. M , as well as five values of E_b/N_0 are read near the beginning of the program. These five values of E_b/N_0 correspond to the requirements for a 10^{-4} BER (as determined by the first set of programs) for L_{hops} taking on the values 1 through 5.
4. Instead of sweeping E_b/N_0 over a range, it is now E_b/N_J which is varied. It is swept usually from 0 to 42 dB in steps of 7 dB.
5. Between Steps 7 and 8 of the previously described unjammed program Steps, two new loops occur:
 DO loop for sweeping $\log_{10} \gamma$ from -4 to 0 in steps of 0.25.
 - a) Set IBITMAX to ITOP.
 - b) Calculate the variance of the noise in jamming, VARJAM, and the variance of the noise when jamming not present, VARFREE.
 - c) Set the symbol error rate counter to 0.
6. DO loop for L_{jammed} ; L_{jammed} goes from 0 to L_{hops} .

L_{jammed} is the number of hops assumed jammed (out of L_{hops}).

- a) Set IEND to ILOW.
- b) Set the *conditional* symbol error counter, SECGL, to zero. SECGL is the symbol error count given L_{jammed} out of L_{hops} hops are jammed.
- c) Calculate the probability of this many hops being jammed (given γ and L_{hops}). If the probability of this happening is less than 5×10^{-7} don't bother counting the symbol errors under these conditions as it will not contribute enough to the overall BER (recall that the BER must be greater than or equal to 10^{-4}).

7. DO loop for sending IEND symbols through given L_{jammed} .

This loop is similar to the unjammed case except it contains two sections. The first goes through L_{jammed} hops (which are jammed). The next goes through $L_{hops} - L_{jammed}$ hops (which are not jammed). The noise variance was set according to whether we are on one of the assumed jammed hops. The processing for each type of receiver is the same as was described in the previous section.

In the case of CW jamming (one frequency jammed in each M-ary cluster given that it is jammed), 1 of the M frequencies would be chosen at random to jam. The CW phase would also be chosen at random.

8. End of the IEND DO loop

9. Check if enough conditional symbol errors have occurred.

- a) If SECGL is less than 5, set IEND to be $IENTD \times 10$.
- b) If IEND is now greater than IBITMAX, set it equal to IBITMAX.
- c) Reset SECGL to 0.
- d) Go to the beginning of the IEND DO loop (Step 5).

10 Calculate the bit error rate under this condition (L_{jammed}), and continue adding this to a running sum (over all L_{jammed}) with a weighting that reflects the probability of the condition occurring. This weighting is,

$$W(\gamma, L_{\text{jammed}}, L_{\text{hops}}) = \binom{L_{\text{hops}}}{L_{\text{jammed}}} \gamma^{L_{\text{jammed}}} (1 - \gamma)^{(L_{\text{hops}} - L_{\text{jammed}})} \quad (6.4.1)$$

11. End of the L_{jammed} DO loop

13. We now have the BER at a particular value of γ . That is we have formed the BER given γ as,

$$\text{BER given } \gamma, L_{\text{hops}} = \sum_{L_{\text{jammed}}=0}^{L_{\text{hops}}} W(\gamma, L_{\text{jammed}}, L_{\text{hops}}) \times (\text{BER} | L_{\text{jammed}}, \gamma) \quad (6.4.2)$$

a) Keep track of the largest BER, and the corresponding γ , γ_{opt} .

14. End of $\log_{10} \gamma$ loop

15. If the BER maximum is less than 2×10^{-3} , and we have only searched once through this range of γ 's, then set IBITMAX to ITOP $\times$ 5 and run through the γ loop once more (Step 3a).

16. The rest continues similar to the unjammed case.

6.4.1 Codes for Plots of BER, and $\log_{10} \gamma_{\text{optimum}}$, versus E_b/N_J

These codes refer to plots done in which optimum partial-band jamming is present. The convention here is nearly the same as for the plots in the case of no jamming. The first three groups of letters described the same thing. $R - M_1 M_2 M_3 M_4 - D_1 D_2 D_3$ all have the same meaning as before. However these letters are followed by the following codes $F_1 F_2 F_3 F_4 - Y_1 Y_2 Y_3$ which will be described below.

$F_1 F_2 F_3 F_4$ has two possible assignments:

- (i) FJAM -refers to jamming as well as Rayleigh fading.
- (ii) JAM -refers to jamming with no fading.

$Y_1 Y_2 Y_3$ takes on two possible assignments:

- (i) BER -meaning that the Y-axis is the BER.
- (ii) GAM -meaning that the Y-axis is $\log_{10} \gamma_{opt}$.

For example, a code of H-4FSK-SQR-FJAM-BER, refers to a plot of the BER versus E_b/N_J for the Hard Limiter receiver, using square-law detected 4-FSK under optimum partial-band jamming with Rayleigh fading.

6.5 Results

6.5.1 Linear Combining Receiver

Appendix A contains all the plots of the BER versus E_b/N_0 for the Linear Combining Receiver in an unfaded, and Rayleigh faded channel.

BER Without Fading or Jamming

Figures A.2 to A.4 show BER versus E_b/N_0 in for the unfaded case of square-law detected MFSK. The BER curves are worse for increasing values of L_{hops} . This isn't surprising since with this incoherent demodulation scheme, the noise terms have a greater effect due to the squaring process. The curves are all grouped to within about 2 dB of each other when L_{hops} is the parameter. Figure A.1, MFSK coherently detected, has curves shown for $M=2, 4$, and 8 — L_{hops} is *not* a parameter here. This is because the E_b/N_0 result is valid for all number of hops per symbol. The same can be said for Figure A.5, which shows the case of MPSK coherently detected. Since the random noise is added linearly, jumping over any number of hops simply results in a noise sample that has the same variance as single noise sample taken over a single hop. This is because the variance of a sum of independent gaussian variables is equal to the sum of the individual variances.

Figure A.1 shows coherent MFSK: increasing M improves the BER. This is not surprising. Figure A.5 shows MPSK coherently detected: $M=2$, and $M=4$ produce the same BER while increasing M to 8 worsens things by approximately 3 dB. This is in keeping with theoretical calculations.

Figures A.2 to A.4 show the results for 2,4, and 8-FSK square-law detected. For all M it is advantageous to hop at 1 hop per symbol. The BER is worst at 5 hops, and worsens steadily as the number of hops per symbol goes from 1 to 5. All curves for any given value of M are within about 2dB of each other. It can be seen that going from $M=2$ to $M=4$ improves all curves by roughly 2.5 dB. Going from $M=4$ to 8 improves all curves (shifts the group left) by approximately 1.7 dB.

BER Including Fading But Without Jamming

Figures A.6 to A.8 show BER results for coherent MFSK in Rayleigh fading. In these it can be seen that using 1 hop per symbol results in the worst performance this time. This is due to the lack of any diversity to counter the fading. Even when only 2 hops per symbol is used, there is an improvement for all M of about 15 db relative to the 1 hop per symbol case. There is about 18-19 db difference between 1 and 3 hops per symbol. In all cases 5 hops per symbol is best, and 4 hops per symbol is next best—worse by approximately 1.2 to 2 dB. Moving from $M=2$ to $M=4$ improves things by about 3db; going to $M=8$ further improves results by about 2 dB.

Square-law detected MFSK (Figures A.9 to A.11) is about 3 dB worse than coherent MFSK at 1 hop per symbol. As L_{hops} is increased to 5, this changes to become about 2 dB worse. There is 3 dB improvement going from $M=2$ to $M=4$. Going from $M=4$ to $M=8$ provides an average improvement of about 1 dB, except to the case of 2 hops per symbol, which is improved by 3 dB.

Figures A.12 to A.14 shows MPSK in fading. The diversity improvement can clearly be seen as L_{hops} goes from 1 to 5. Curves for $M=2$ and $M=4$ are virtually the same which is expected since they are the same in a unfaded environment. These curves are about 6 dB better than $M=2$ square-law detected MFSK, due to elimination of quadrature noise and the fact that a single rather than double matched filter can be used in the detector. When M is increased to 8, the curves are 3 dB worse than $M=2$ and $M=4$. This is due to the fact that the spacing between angles with $M=8$ is half that of $M=4$, but the noise power is 1.5 times as

much (for the same E_b/N_0).

BER With Jamming But Without Fading

Appendix D, showing Figures D.1 to D.9 show plots of the BER versus E_b/N_J under worst-case partial-band jamming without fading.

Figures D.1 to D.3 describe coherent MFSK. For all values of M it can be seen that there is a linear decline in the BER with increasing E_b/N_J . There would appear to be no advantage to increasing the number of hops per symbol beyond 1. The 2 dB advantage that appears to occur for $M=8$ is not reliable due to the fact that the irreducible BER at $E_b/N_J=45$ dB is about 0.33 times that of $M=2$ and $M=4$. That is, the E_b/N_0 settings were not chosen accurately enough to cause all the BER tails to be the same.

Figures D.4 to D.6 describe square-law detected MFSK. For all M results worsen as L_{hops} goes from 1 to 5. This is likely due to the fact that the noise terms are squared before being added hence never have a tendency to cancel each other due to opposite signs. There is 1 dB separation between the various curves as L_{hops} goes from 1 to 5. Increasing M from 2 to 4 is advantageous, providing a small improvement of 1 dB. Going to $M=8$ has no improvement over $M=4$ —although it must be kept in mind that $M=8$ has been simulated with E_b/N_0 that is about 4 dB less (for all L_{hops}) than $M=2$, and about 1.5 dB less than $M=4$. This holds the tails of the BER plots constant but had all E_b/N_0 been held constant for the three graphs, $M=8$ would have been cited as giving the best results. This reversal of results would not have occurred between the various curves on each graph (with the hops per symbol as the parameter) had all E_b/N_0 been held constant however. This is because as the graphs stand now it can be seen that increasing L_{hops} requires an increase in E_b/N_0 , and yet the BER worsens with increasing L_{hops} . This worsening would be even more pronounced with for a constant E_b/N_0 (for 10^{-4} BER at 1 hop per symbol).

Figures D.7 to D.9 describe coherently detected MPSK. On the graphs for $M=2$, 4, and 8, it can be seen that there is no advantage to increasing the number of hops

per symbol. All these curves overlay each other (and this time E_b/N_0 is a constant between values of L_{hops} for a given M). The reason for this is the fact that the independent noise samples, when added linearly, simply add on a power basis—the resultant noise sample therefore is the the same for any number of hops. It is interesting that this process does not seem to be affected when the fraction of the band jammed, γ , is optimized.

$M=2$ and 4 provide virtually the same performance, while $M=8$ is about 1.5 dB worse (even while its E_b/N_0 is greater than that for $M=2$ or 4 as opposed to the situation with square-law MFSK).

BER With Jamming and Fading

Appendix D, showing Figures D.10 to D.18 show plots of the BER versus E_b/N_J under worst-case partial-band jamming in the presence of fading.

Figures D.10 to D.12 describe jammed MFSK coherently detected in Rayleigh fading. In these three Figures it can be seen that 1 hop per symbol is the worst choice and 5 hops per symbol appears to be the best choice. Even when the 5 hops per symbol tail BER is greater than the others (implying a choice for E_b/N_0 that was a bit too low), it still outperforms other values of L_{hops} . Also 5 hops per symbol requires the lowest E_b/N_0 (for a 10^{-4} BER without jamming). Thus a strong case can be made for using $L_{hops}=5$. There is a 1 dB improvement going from $M=2$ to $M=4$. There is about a 0.5 dB improvement going to $M=8$.

Figures D.13 to D.15 describe jammed square-law MFSK in Rayleigh fading. All these curves are approximately 6 dB worse than the corresponding coherently detected cases just described. Normally one might expect there to be only 3 dB difference. The effect of optimal jamming is to increase this. The best choice for hops per symbol, L_{hops} , is 2 for all $M=2, 4$, and 8. For $M=4$ and 8, $L_{hops}=5$ is the worst choice. It is the worst choice for $M=2$ at values of E_b/N_J less than 20 dB, and is second worse (to $L_{hops}=1$) at higher values of E_b/N_J . For all M the curves for various L_{hops} are within 2.5 dB of each other for the most part. Even after calling 5 hops per symbol the worst choice, it has to be noted that this choice has been

evaluated about 7 dB down from the E_b/N_0 required for the case of $L_{hops}=2$. Certainly without jamming, high values of L_{hops} provide diversity gain over Rayleigh fading, but in optimal jamming it would appear that the combining this with the squaring of the noise terms before summing causes the diversity improvement to be not as noticeable. This is probably due to the fact that a large noise power if hit just once is bad, and *squaring* it is worse. So with 5 hops per symbol one might to have this occurring with a higher probability.

Figures D.16 to D.18 describe jammed MPSK coherently detected in Rayleigh fading. $M=2$ and $M=4$ are identical. $M=8$ provides an improvement of about 2.5 dB. In all cases 5 is the best, and 1 is the worst choice for number of hops per symbol. Curves for $L_{hops}=2, 3, 4$, and 5 are however very close (within 0.7 dB) of each other—1 hop per symbol has a curve 3 dB worse than these. There is an improvement of 19, 7, and 4.5 dB for $M=2, 4$, and 8 respectively of this set of curves over the corresponding curves for square-law detected MFSK.

It should be noted that for the coherent cases described here, coherence may be very difficult to maintain from hop to hop—especially in a Rayleigh fading channel. In all likelihood an incoherent detection method would have to be used.

6.5.2 Majority Vote Receiver

Appendix B contains all the BER versus E_b/N_0 plots for the Majority Vote Receiver, and Appendix E contains all the BER versus E_b/N_J plots.

BER Without Fading or Jamming

Figures B.1 to B.9 show the results for the unfaded case.

Figures B.1 to B.3 show plots for coherent MFSK. In all of these plots it can be seen that the best results are obtained for 1 hop per symbol. Also the 1 hop per symbol curves are identical to the 1 hop per symbol curves for the corresponding Linear Combining Receiver graphs. This is reassuring since with 1 hop, the algorithm for both receivers is the same. The worst results for all M occur for $L_{hops}=2$. This case is always 3 dB worse than the case of $L_{hops}=1$. This is due to the fact

that taking a majority vote over 2 symbols does not help in any way—the only time no errors are guaranteed not to occur is when both hops are registered as the same symbol. This in effect makes it the same as a system operating with half the signal energy (and theoretical analysis shows this shift is exactly 3 dB for $M=2$ so it's nice to see it occur in the simulation as well).

Still 2 hops per symbol were considered since this may have a beneficial effect in a fading channel. Three hops per symbol is the optimum choice for L_{hops} greater than 1, for all values of M .

Figures B.4 to B.6 show the results for square-law detected MFSK. Again the case of 1 hop per symbol gives the same results as the Linear Receiver for $M=2$ to 8. Increasing M from 2 to 4 improves things by about 3 dB; from 4 to 8 by about 1 dB. For L_{hops} greater than 1, the best choice is $L_{hops}=3$. This choice is best when it is the 10^{-4} BER that is of interest. At higher SNRs and lower BERs the case for $L_{hops}=5$ would probably be strong. It appears to have a higher downward slope as higher SNRs are approached. At higher SNRs the individual hops would be more reliable hence the effectiveness of majority voting quite high.

Figures B.7 to B.9 describe the BER for MPSK. As for the past two modulation schemes $L_{hops}=1$ is the best choice, with $L_{hops}=3$ or 5 roughly tied for best choice for L_{hops} greater than 1. $L_{hops}=2$ can be seen to be the worst choice.

BER Including Fading But Without Jamming

Figures B.10 to B.18 describe the BER in a Rayleigh fading environment.

Figure B.10 to B.12 show the BER versus E_b/N_0 curves for coherently detected MFSK. $L_{hops}=2$ provides worst performance as usual for all graphs. But now instead of being the best choice as in the unfaded cases, $L_{hops}=1$ is second worst—18 dB away (for all M) from the best choice of $L_{hops}=5$. This shows that not only the Linearly Combining Receiver, but the Majority Voting algorithm receiver as well, can provide effective diversity (for L_{hops} greater than 2) in Rayleigh fading. It should be noted that even the best curves here are 4 to 5 dB worse than the corresponding Linear Combining Receiver curves. For $L_{hops}=5$, going from $M=2$

to 4 improves the performance by about 1.3 dB. Increasing M to 8 provides almost no improvement for the best curve.

Figure B.13 to B.15 show the BER versus E_b/N_0 curves for square-law detected MFSK. $M=2$ and $M=4$ curves are about 3 dB worse than the previous corresponding set of curves for this receiver. The $M=8$ curves are about 1.5 dB worse than the coherently detected curves. Again it's easy to see the diversity improvement as a result of increasing the number of hops per symbol.

Figures B.16 to B.18 show the effect of fading on coherently detected MPSK. Results are similar to the past cases. $L_{hops}=2$ is worst, then $L_{hops}=1$. The best choice, for all M , is $L_{hops}=5$ —with $L_{hops}=3$ and 4 almost the same for $M=4$ and 8, and within about 2.5 dB for $M=2$. $M=2$ and 4 provide almost the same curves; $M=8$ is about 3 dB worse. For $M=2$, the best curve is 3 dB worse than the corresponding curve for the Linear Combining Receiver—this difference is about 4 dB for $M=4$ and 8.

BER With Jamming But Without Fading

Appendix E contains plots of the BER versus E_b/N_J for this receiver in jamming.

Figures E.1 to E.9 contain are the plots of the BER in the presence of jamming. In all the Figures $L_{hops}=2$ is the worst choice. $L_{hops}=1$ is the second worst choice. Notice this contrasts with the unjammed plots without fading. There, 1 hop per symbol was always the best choice. This demonstrates that the Majority Voting Receiver is aiding in the relief against the jammer. The jammed hops, unlike with the Linear Combining Receiver, do not get weighted in proportion to the jammers power—rather the harmful effects of being jammed on a particular hop can be *cancelled* simply by making a correct decision on some other hop. As the number of hops per symbol is increased, this effect should be even more noticeable. Unfortunately there were no simulations done for L_{hops} greater than 5.

Figures E.1 to E.3 show results for MFSK coherently detected. Looking at $M=2$ (Figure E.1) it is interesting to note that at a 3×10^{-4} BER, $L_{hops}=5$ provides a 7

dB advantage over $L_{hops}=1$, even though it requires only a E_b/N_0 2.75 dB greater than $L_{hops}=1$. This is the same gain that $L_{hops}=5$ has over the Linear Combining Receiver's corresponding jamming curve. As M is increased from 2 to 4 then to 8 there is an improvement in the performance. The $L_{hops}=5$ curve flattens out to its tail BER (10^{-4}) faster as M increases. Further, $M=8$ requires the least E_b/N_0 to achieve this 10^{-4} BER. Still at high jamming powers, where E_b/N_J is less than 7 dB, $L_{hops}=1$ is the best choice, and $L_{hops}=5$ becomes the second best choice—although at this point the BERs are all very close.

Figures E.4 to E.6 describe BER versus E_b/N_J for square-law detected MFSK. For $M=2$ it would appear that for E_b/N_J greater than 11 dB, 3 hops per symbol is the best choice. This may not quite be correct though. If the tails at E_b/N_J are looked at, it can be seen that the $L_{hops}=5$ curve finishes off near 3×10^{-4} while the other curves finish closer to 10^{-4} as they should. Again a crossover effect can be observed whereby at low values of E_b/N_J , $L_{hops}=1$ gives better performance than the other curves, and $L_{hops}=3$ gives the second best performance. This is because at high jamming strengths, the voting process tends to be ineffective due to a randomizing of the received signal as its energy is divided smaller and smaller to accomodate more hops per symbol. Figure E.5 ($M=4$) is particularly interesting as it shows a complete reversal in terms of the ordering of L_{hops} from best to worst—different crossover points occurring at various points. Ignoring the case of $L_{hops}=2$ for the moment, then at $E_b/N_J=5$ dB, for example the best choice is $L_{hops}=1$, followed by 3, 4, and worst 5. But when E_b/N_J reaches 30 dB, the order has reversed. $L_{hops}=5$ is now best, followed by $L_{hops}=4, 3$, and 1. This crossing over effect was not observed in the Linear Combining Receiver. In Figure E.6, showing the case for $M=8$, it can be seen that the curve for $L_{hops}=5$ flattens out faster to its tail than the case for $M=2$ or 4.

Figures E.7 to E.9 show the results for MPSK. It is difficult to judge Figure E.7 as the tails seem to flatten out by 40 dB E_b/N_J , but they do not align at a BER of 10^{-4} —indicating that the E_b/N_0 was not chosen quite correctly. However, for all

It can still be seen that $L_{hops}=1$, and 2 are both bad choices. Crossover points between best values of L_{hops} can still be observed. Figures E.8 and E.9 ($M=4$, and 8) would indicate that most likely $L_{hops}=5$ is the best choice.

BER With Jamming and Fading

Figures E.10 to E.19 give the BER performance in jamming with Rayleigh fading.

Figures E.10 to E.12 gives BER results for MFSK coherently detected. As usual it can be seen that $L_{hops}=2$ yields the worst results for all values of M . Compared to the same curves for the Linear Combining Receiver, there is more separation between the 5 curves on each Figure. $L_{hops}=5$ gives the best results for all M for this receiver. In addition, for all cases the E_b/N_0 required for $L_{hops}=5$ is about 18 to 19 dB less than that required for $L_{hops}=1$. Increasing M to 4 from 2 provides an improvement, for $L_{hops}=3, 4$, and 5, of a little over 3 dB at a BER of 3×10^{-4} . There is a slight improvement as well as M is increased to 8 from 4.

Figures E.13 to E.15 describe square-law detected MFSK. At E_b/N_J greater than 12 dB, $L_{hops}=5$ is the best choice for all M —this is not the case for the Linear Combining Receiver where the corresponding curves had $L_{hops}=5$ as almost always the worst choice. There is an improvement of about 3 dB at a BER of 3×10^{-4} as M is increased from 2 to 4—a very small improvement can be seen as M increases from 4 to 8. For $L_{hops}=5$, there is a between a 1 and 2 dB improvement for these curves ($M=2, 4, 8$) compared to the corresponding curves for the coherently detected MFSK for this same receiver. At values of E_b/N_J less than about 7 dB, $L_{hops}=1$ becomes the best choice.

Figures E.16 to E.18 show the results for coherent MPSK. $L_{hops}=5$ is the best choice. Although for all M it appears that $L_{hops}=3$, or 4 is better in the tail regions, this is probably due to a bad choice for E_b/N_0 . The curves are the same for $M=2$ and 4. Looking at $M=4$ and $M=8$, $L_{hops}=5$ curves are separated by about 2.5 dB with $M=8$ a poorer choice than $M=4$. Compared to the best curves of the Linear Combining Receiver for $M=2$, there is a 5 dB improvement using $L_{hops}=5$ with the

Majority Vote Receiver.

6.5.3 Hard Limiter Receiver

Appendix C contains the plots for the Hard Limiter Receiver without jamming. Appendix F show similar plots but with fading included.

BER Without Fading or Jamming

Looking first at Figures C.1 to C.3 which describe coherent MFSK, we can see that $L_{\text{hops}} = 5$ is the best choice for $M=2$ and 4—for $M=8$, in the vicinity of 10^{-4} BER, 1, 2, 3, 4, and 5 are equally good choices for the number of hops per symbol. The value of increasing M can be easily seen. Moving from $M=2$ to $M=4$ improves the BER curve (shifts it left) by about 3 dB. From $M=4$ to 8 the worst curve ($L_{\text{hops}}=1$) is improved by 1.5 dB, while $L_{\text{hops}}=5$ is improved by about 0.75 dB. Roughly the same E_b/N_0 is required when $L_{\text{hops}}=5$ for this receiver and the Majority Vote Receiver.

Square-law detected MFSK is described in Figures C.4 to C.6. Again increasing M helps. There is a 3 dB improvement from $M=2$ to 4 and a smaller improvement of about 1.2 dB going from $M=4$ to 8. As the SNR increases it $L_{\text{hops}}=5$ seems to become the best eventually. At low SNRs, when the number of hops per symbol is increased performance is degraded.

Hard limited MPSK is shown in Figures C.7 to C.9. Here, increasing M degrades performance, especially for $M=8$ which has its best curve ($L_{\text{hops}}=5$) 10 dB worse than the best curve for $M=2$ ($L_{\text{hops}}=1$). $L_{\text{hops}}=1$ is the best choice for $M=2$, $L_{\text{hops}}=5$ the best for $M=4$ and $M=8$. The E_b/N_0 required when $M=2$ is almost the same as that of the Majority Vote Receiver. 8-PSK is much worse when hard limited than when used with the Majority Voting Receiver—the Hard Limiter Receiver providing performance 6 dB worse when best curves are compared.

It is interesting to note that with this receiver, $L_{\text{hops}}=2$ usually is very close to, or better than $L_{\text{hops}}=1$, in terms of the required E_b/N_0 . This was not the case for the Majority Vote Receiver—where $L_{\text{hops}}=2$ was always a poorer choice than

$L_{hops}=1$. Further, $L_{hops}=2$ with the Hard Limiter Receiver required less E_b/N_0 than $L_{hops}=2$ for the Majority Vote Receiver, except for 8-PSK.

BER Including Fading But Without Jamming

Figures C.10 to C.17 show the performance of the Hard Limiter Receiver in a Rayleigh fading channel.

Coherently detected MFSK is shown in Figures C.10 to C.12. All of these show 1 hop per symbol to be by far the worst choice. In order of improving performance, the number of hops per symbol is 1,2,3,4 then 5. It can be seen that $L_{hops}=2$ is around 18.5 dB better than $L_{hops}=1$ for $M=2$; 21 dB better for $M=4$; and 19 dB for $M=8$. Thus there is a large diversity improvement just by going to 2 hops per symbol. There is about a 3 dB advantage in going to $M=4$ from 2 for $L_{hops}=2,3$, and 4, and a 2 dB advantage for $L_{hops}=5$ —there is no advantage for $L_{hops}=1$. At $M=8$ there is a 3.5 to 4 dB advantage over all the curves for $M=2$. For all M , the $L_{hops}=5$ curves are about 4 dB worse than the corresponding curves for the Linear Combining Receiver. Comparing this receiver to the Majority Vote Receiver, we can see that for L_{hops} greater than 1, and with $M=2$, the Hard Limiter Receiver is better than, or equal in performance. The difference is even more pronounced when $M=4$ or 8. When $L_{hops}=1$ however, performance with the Hard Limiter Receiver is far worse than that of the Majority Vote Receiver. It should be noted that 1 hop per symbol with a Majority Voting Receiver is the same thing as a Linear Combining Receiver operating at 1 hop per symbol.

For square-law detected MFSK (Figures C.13 to C.15) similar curves can be seen. The best choice is $L_{hops}=5$, the worst, $L_{hops}=1$. The $L_{hops}=4$, and 5 curves are about 1.5 to 2 dB worse, respectively, than the corresponding curves for coherently detected MFSK. Going from $M=2$ to 4 is advantageous, providing 3.5 dB improvement at $L_{hops}=2$, to 1.6 dB improvement between the best curves (at $L_{hops}=5$). Increasing M from 4 to 8 causes a further improvement of about 2 to 3 dB. For square-law detected MFSK, it can be seen that the Hard Limiter Receiver, for L_{hops} greater than 1, outperforms the Majority Vote Receiver. The biggest dif-

ference in performance occurring when there are 2 hops per symbol—the smallest at 5 hops per symbol. However the Hard Limiter Receiver performs worse than the Majority Vote Receiver when $L_{hops}=1$.

MPSK coherently detected is shown in Figures C.16 to C.17. Notice only $M=2$ and $M=4$ was considered since the threshold for $M=8$ was not calculated. At $L_{hops}=5$ (the best curve) $M=2$ and $M=4$ provide the same results. At $L_{hops}=3$ the results are almost the same as well, with $M=4$ better by about 0.5 dB. $L_{hops}=1$ is actually 12 dB worse at $M=4$ than $M=2$ —this worsening is normally what one expects when comparing 4-PSK to 2-PSK. When $L_{hops}=2$, $M=2$ is 11 dB worse than $M=4$. So for all L_{hops} greater than 1 it is advantageous to increase M to 4. Comparing the 2-PSK curves to those of the Majority Vote Receiver, we see that the performance is the same essentially. For $M=4$, the Hard Limiter Receiver again has a clear advantage over the Majority Vote Receiver. This advantage, as in the past cases, is greatest for $L_{hops}=2$, and smallest for $L_{hops}=5$. Since the curves have a different slopes (from one receiver to another), it would appear that at higher SNRs (or lower desired BERs) the differences would be even greater.

BER With Jamming But Without Fading

Appendix F, with Figures F.1 to F.9 show the BER performance for the Hard Limiter Receiver in jamming. Figure F.1 to F.3 show the results for coherent MFSK. It appears that some E_b/N_0 values were not chosen correctly resulting in tail BERs that are not exactly 10^{-4} . Certainly however $L_{hops}=1$ is the worst choice for all M , followed by $L_{hops}=2$ and 3. Figure F.2 show the results for $M=4$. Here $L_{hops}=3, 4$, and 5 produce almost the same curves. Had E_b/N_0 been chosen more accurately for $M=2$ and $M=4$ the result may have been similar there. In any case, in the presence of high strength jamming (E_b/N_J) less than 15 dB $L_{hops}=4$ or 5 is the best choice. There is a 3 dB improvement in all curves when M is increased from 2 to 4, and a similar improvement going from $M=4$ to 8 when $L_{hops}=4$ or 5. These curves are not substantially different from those of the Majority Vote Receiver. Except that at high jamming strengths (BERs of 10^{-1} to 10^{-2}) the Majority Vote Receiver is

best at 1 or 5 hops per symbol—whereas at all jamming strengths the Hard Limiter Receiver is better at 5 hops per symbol.

Figures F.4 to F.6 describes the performance of square-law detected MFSK. $L_{hops}=1$ is the worst choice for E_b/N_J greater than 10 dB. Results are worse than for the coherent case, but as before, $L_{hops}=4$ and 5 are very closely grouped together. There is about 2 dB to be gained by increasing M from 2 to 4 at moderate jamming levels. There is almost no improvement when M is increased from 4 to 8. The curves are similar to the Majority Vote Receiver curves for L_{hops} greater than 3. Especially at high jamming levels.

Figures F.7 to F.9 show plots for MPSK in jamming. $L_{hops}=1$ or 2 are the worst choices for all M . For values of L_{hops} greater than 2, $M=2$ and 4 provide almost the same results. Results were worst for $M=8$. Here, at 15 dB E_b/N_J , the best BER was ten times worse than the same curve for $M=2$. In strong jamming, 4-PSK with hard limiters outperforms 4-PSK with Majority Voting. However it requires a higher E_b/N_0 . 2-PSK with this receiver produces the same jamming curves as the Majority Vote Receiver. 8-PSK with Hard Limiting requires a higher E_b/N_0 , and produces worse jamming performance curves than the Majority Vote Receiver.

BER With Jamming and Fading

Figures F.10 to F.17 show the performance of the Hard Limiter Receiver in fading with jamming.

One hop per symbol was always bad and completely unusable except for 2-PSK as shown in Figure F.16. This is because as soon as the jammer exceeds the threshold and limits to 1, the probability of making a symbol error is 0.5—or worse for larger M . This BER was observed for 1 hop per symbol for values of E_b/N_J less than 27 dB for all of these Figures except Figure F.16. Figure F.16 shows the results for 2-PSK. This is a special case since the threshold, when both fading and noise are present, is zero—exactly, without any approximations. In all the other cases, the thresholds were calculated assuming no noise (on the transmitted symbol)—this produced less than optimum thresholds under real conditions.

On all of the figures it can be seen that 5 hops per symbol is the best choice. Ignoring the 2-PSK case for the moment, for all other figures it can be seen that in order of improving performance, the value of L_{hops} is 1, 2, 3, 4 and 5. Figures F.10 to F.17 have the most noticeable spread between the curves (for various L_{hops}) than any other set of BER curves in jamming.

Figures F.10 to F.12 shows coherently detected MFSK. Increasing M helps—most notably between $M=2$ and $M=4$ which increases performance by about 3dB. Increasing M to 8 causes a negligible improvement. When $L_{hops}=5$, the performance is the almost the same as the Majority Vote Receiver.

Square-law detected MFSK shown in Figures F.13 to F.15. $M=2$ produced a BER about 1 order of magnitude worse than the coherent case at moderate jamming levels ($E_b/N_J=20$ dB). Increasing M to 4 improved things. The best curve, $L_{hops}=5$, was shifted left by about 5 dB at E_b/N_J of near 20 dB. Increasing M to 8 caused a further, but almost unnoticeable improvement. At high jamming strengths, the Majority Vote Receiver outperforms the Hard Limiter Receiver when $L_{hops}=5$ and $M=2$ or 4, by 3dB. For $M=8$ it is outperformed by 2 dB.

Looking at coherent 2, and 4-PSK in Figures F.16 and F.17, it can be seen that $M=2$ is much better than square-law detected MFSK at $L_{hops}=5$. When $E_b/N_J=15$ dB for example, with 2-PSK the BER is about 10^{-4} , and 10^{-1} for 2-FSK. There is no advantage in going to $M=4$ however. The 2-PSK jamming curves for both this receiver and the Majority Vote Receiver are the same. The 4-PSK performance for $L_{hops}=5$ is the same; but the Hard Limiter Receiver performs worse for L_{hops} less than 5.

6.5.4 Typical plots of $\log_{10} \gamma_{opt}$ versus E_b/N_J

Appendix G show some selected plots $\log_{10}$ of the optimum fraction of the band to jam, versus E_b/N_J .

Figure G.1 shows that for the Linear Combining Receiver in an unfaded channel, there is a tendency for γ_{opt} decrease as E_b/N_J is increased. Further, as the number

of hops per symbol increases the γ_{opt} line is shifted down. This example is for 4-FSK coherently detected without fading.

Figure G.2 shows 2-FSK coherently detected, this time with fading. All the curves are shifted up somewhat, demonstrating that a slightly larger portion of the band should be jammed when fading is present. Although as L_{hops} is increased, the same downward trend (in γ_{opt}) remains. For $L_{hops}=1$, γ_{opt} is greater than 0.1 all of the time and flattens out as E_b/N_J is increased. There is a sudden rise at $E_b/N_J=40$ dB with $L_{hops}=4$. This is not significant since the BER produced at the low jamming level is almost unaffected no matter what portion of the band is jammed—it is approximately 10^{-4} .

The behavior of γ_{opt} for the Majority Vote Receiver is shown in Figure G.3 (without fading) and Figure G.4 (with fading). Compared to the Linear Receiver there is a reversal of the order of the lines when fading is not present. $L_{hops}=5$ requires the widest portion of the band to jam, while $L_{hops}=1$ and 2 require the least. With the Majority Vote Receiver, the greater the number of hops per symbol, the greater the optimum fraction of the band to jam. In Figure G.4, which shows 2-PSK in fading, it is again apparent that γ_{opt} is greater in fading. This time almost all curves (except for a sudden dip for $L_{hops}=2$, and 4) indicate that γ_{opt} should be greater than 0.1.

The γ_{opt} variations for the Hard Limiter Receiver is shown in Figure G.5 (without fading) and G.6 (with fading). Again both curves show γ_{opt} increasing as L_{hops} is increased. In fading, Figure G.6, the optimum fraction to jam is unity for all L_{hops} when E_b/N_J less than 21 dB. γ_{opt} for $L_{hops}=5$ remains at full band for all E_b/N_J —which is the same for the case of $L_{hops}=1$. The others drop slightly.

Chapter 7

CONCLUSIONS

7.1 Conclusions

BER performance for three different receiver structures has been evaluated by computer simulation. Various channel conditions were examined: non-faded, Rayleigh faded, jammed and unjammed. Three M -ary ($M=2,4,8$) modulation schemes, coherent MFSK, square-law detected MFSK, and coherent MPSK were tested for 1 to 5 hops per symbol.

It was found that, except for MPSK, it is advantageous to increase the alphabet size M , under any combination of the above conditions—the advantage usually was more pronounced between $M=2$ and $M=4$ than between $M=4$ and $M=8$. This was true for all receiver types studied: the Linear Combining Receiver, Majority Vote Receiver, and the Hard Limiter Receiver. Increasing the number of hops per symbol improved performance for the Hard Limiter (at high SNRs) and Majority Vote Receivers under all conditions—for all receivers this was especially true in the presence of Rayleigh fading. There is nothing to be gained by increasing the number of hops per symbol with the Linear receiver when fading is not present however—in fact the performance was seen to degrade as L_{hops} was increased to 5 hops per symbol, in the presence of jamming, for square-law detected MFSK.

The Linear Combining Receiver in jamming is affected more by smaller and smaller band jamming fractions as the number of hops per symbol is increased—whether in fading or not. The Linear Receiver is therefore a poor choice for multihop per symbol digital communication when there is the possibility of narrow band interference and when the number of hops per symbol is high. This is contrary to the basic idea in spread spectrum. The result is due to the fact that being hit even

once during a set of symbol hops by a jammer much stronger than the signal, requires *many* unjammed hops to occur to cancel this effect.

The Majority Vote Receiver is less and less affected by smaller jamming bandwidths as the number of hops per symbol is increased. As the hops per symbol is increased, the jammer must spread his power out more.

The Hard Limiter Receiver has characteristics not unlike that of the Majority Vote Receiver. The Hard Limiter Receiver, when L_{hops} is greater than 3, provides better performance in most cases however. The reason for this is that the Hard Limiter has the ability to recover from jammed hops easier than the Majority Vote Receiver. It does this by essentially making a jammed hop "not count" in the process of hopping about. For the Hard Limiter Receiver operating with correctly chosen thresholds, a correct decision will be made provided at least one of the hops was not jammed (ignoring thermal noise for the moment)—all jammed hops being neutralized since the correct decision will *also* have been registered on these hops (recall that more than one decision variable can be hard limited to 1 on an individual hop with this receiver). The Majority Voting Receiver however, must have an unjammed hop to counteract the effect of each and every jammed hop in order to make a correct decision (this is being pessimistic since it assumes the jammed hops will all align on the same wrong symbol frequency). The Hard Limiter Receiver requires the widest band jamming in fading to degrade its performance. It is expected that this receiver would also provide good performance in the presence of CW jamming. For these reasons the Hard Limiter Receiver seems to be the best choice for communicating in a hostile environment. However in high jamming powers combined with fading, the results are not as good, due to the fact that noise was assumed to be absent in fading with this receiver.

The thresholds for the Hard Limiter in fading were calculated based on the absence of noise on the received signal (given that the signal was sent). The effect of Rayleigh fading only was included in these cases. This is not completely accurate (although the results were better than assuming noise only and disregarding the

fading), and given that this receiver still performed well in fading and jamming, it is expected that with more analysis and simulations, the thresholds could be chosen in such a way to make this receiver look even more attractive—especially at a higher number of hops per symbol.

Finally, the value of simulating a Majority Vote Receiver at 1 and 2 hops per symbol may be questioned. It is useful however. At 1 hop per symbol the results should be the same as the Linear Combining Receiver (they were). Thus this can be used as a check of the confidence in the accuracy of the programs. $L_{hops}=2$ is not effective with this receiver but it still can be used for comparison purposes between other receivers operating at 2 hops per symbol where it is sometimes useful (like in fading).

7.2 Recommendations for Future Work

It is hoped that some more work will be done on evaluating the performance of the Hard Limiter Receiver. It is unfortunate that simulations in this thesis were limited to 5 hops per symbol at most. More insight into the value of this receiver could be gained by looking into the performance at much higher values of L_{hops} . Perhaps 10, 20 and 50 hops per symbol could be looked into—with special processing done to speed up the CPU time.

A different algorithm for determining the optimum fraction of the band to jam also deserves some work. An algorithm which searches more quickly than fixed delta stepping should be used. This would also encourage the use of more hops per symbol as more CPU time would be available. Perhaps by determining, through simulation, the optimum ratio of intercepted jammer energy to symbol energy which maximizes the BER on a hop, the search procedure for γ (sweeping γ) would not be necessary. This could be done by combining the aforementioned simulation with the formula for the probability of being jammed given the number of hops per symbol and γ etc. If these two formulas could be combined and maximized once, it may not be necessary to do so many search calculations for each change in M , L_{hops} ,

or E_b/N_j (which is now the case). The resulting reduction in CPU time could be reassigned to other interesting problems such as the evaluation of the performance of a receiver with soft limiters, or the evaluation of a receiver whose limiter transfer function may be of an unusual form.

It is also hoped that the simulations for CW jamming will be run sometime—perhaps only for square-law detected MFSK to save time. It may be interesting to see the results of plain envelope detected MFSK as well. Results should be close the results for square-law detected MFSK.

Some simulation of intersymbol interference (due to frequency selective fading) could also be looked into. It is difficult to decide on the type of ISI, or amount to include however. If this is randomized, it will require even more symbols to be sent through. If it is not randomized, but fixed in some way, the results may not be general enough.

Examining the effect of a lack of independence between hops would be useful as well. For all hops (within a symbol signalling interval) may not always be separated by the coherence bandwidth.

Finally, for non-fading conditions, it might be wise to evaluate the performance of the receivers by fixing the E_b/N_0 to some constant value (probably for a 10^{-4} BER at 1 hop per symbol), and holding this constant even as L_{hops} increases. This can't really be done in fading cases though, due to the large diversity gains at multiple hops per symbol. In that case, as was done in this thesis for both faded and unfaded cases, it would most probably be better to let E_b/N_0 vary with L_{hops} , and hold the BER (without jamming) to 10^{-4} , or some other value.

It would also be a good idea to smooth the BER curves using a spline, but this is not a problem of great concern.

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APPENDIX A

Plots of BER versus E_b/N_0 for the Linear Combining Receiver

Figure A.1 BER versus $\frac{E_b}{N_0}$ for M=2, 4, and 8. L-MFSK-COH

MFSK ENVSQ (L), M=2 HOPS/SYMBOL VARIED (file. fske2)

Figure A.2 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. L-2FSK-SQR

Figure A.3 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. L-4FSK-SQR

Figure A.4 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. L-8FSK-SQR

MPSK COHERENTLY DETECTED, M VARIED (file: pskcohd)

Figure A.5 BER versus $\frac{E_b}{N_0}$ for M=2, 4, and 8. L-MPSK-COH

Figure A.6 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-2FSK-COH-F

Figure A.7 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.

L-4FSK-COH-F

Figure A.8. BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-8FSK-COH-F

Figure A.9 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-2FSK-SQR-F

Figure A.10 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-4FSK-SQR-F

Figure A.11 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-8FSK-SQR-F

Figure A.12 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-2PSK-COH-F

Figure A.13 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-4PSK-COH-F

Figure A.14 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
L-8PSK-COH-F

APPENDIX B

Plots of BER versus E_b/N_0 for the Majority Vote Receiver

Figure B.1 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-2FSK-COH

Figure B.2 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-4FSK-COH

Figure B.3 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-8FSK-COH

Figure B.4 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-2FSK-SQR

Figure B.5 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-4FSK-SQR

Figure B.6 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-8FSK-SQR

Figure B.7 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-2PSK-COH

Figure B.8 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-4PSK-COH

Figure B.9 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. M-8PSK-COH

Figure B.10 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
M-2FSK-COH-F

Figure B.11 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
M-4FSK-COH-F

Figure B.12 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
M-8FSK-COH-F

Figure B.13 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
M-2FSK-SQR-F

Figure B.14 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
M-4FSK-SQR-F

Figure B.15 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
M-8FSK-SQR-F

Figure B.16 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading, M-2PSK-COH-F

Figure B.17 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.

M-PSK-COH-F

Figure B.18 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
M-8PSK-COH-F

APPENDIX C

Plots of BER versus E_b/N_0 for the Hard Limiter Receiver

Figure C.1 BER versus $\frac{E_s}{N_0}$ for 1 to 5 hops per symbol. H-2FSK-COH

Figure C.2 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-4FSK-COH

Figure C.3 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-8FSK-COH

Figure C.4 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-2FSK-SQR

Figure C.5 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-4FSK-SQR

Figure C.6 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-8FSK-SQR

Figure C.7 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-2PSK-COH

Figure C.8 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-4PSK-COH

Figure C.9 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol. H-8PSK-COH

Figure C.10 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
H-2FSK-COH-F

Figure C.11 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.

H-4FSK-COH-F

Figure C.12 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
H-8FSK-COH-F

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Figure C.13 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
H-2FSK-SQR-F

Figure C.14 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
H-4FSK-SQR-F

Figure C.15 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
H-8FSK-SQR-F

Figure C.16 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
H-2PSK-COH-F

Figure C.17 BER versus $\frac{E_b}{N_0}$ for 1 to 5 hops per symbol with Rayleigh fading.
H-4PSK-COH-F

APPENDIX D

Plots of BER versus E_b/N_f for the Linear Combining Receiver

Figure D.1 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-2FSK-COH-JAM-BER

Figure D.2 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-4FSK-COH-JAM-BER

Figure D.3 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-8FSK-COH-JAM-BER

Figure D.4 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-2FSK-SR-JAM-BER

Figure D.5 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-4FSK-SQR-JAM-BER

Figure D.6 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-8FSK-SQR-JAM-BER

Figure D.7 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-2PSK-COH-JAM-BER

Figure D.8 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-4PSK-COH-JAM-BER

Figure D.9 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-8PSK-COH-JAM-BER

Figure D.10 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-2FSK-COH-FJAM-BER

Figure D.11 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-4FSK-COH-FJAM-BER

Figure D.12 BER versus $\frac{E_b}{N_0}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-8FSK-COH-FJAM-BER

Figure D.13. BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-2FSK-SQR-FJAM-BER

Figure D.14 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-4FSK-SQR-FJAM-BER

Figure D.15 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-8FSK-SQR-FJAM-BER

Figure D.16 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-2PSK-COH-FJAM-BER

Figure D.17 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
L-4PSK-COH-FJAM-BER

Figure D.18 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 L-8PSK-COH-FJAM-BER

APPENDIX E

Plots of BER versus E_b/N_J for the Majority Vote Receiver

Figure E.1 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-2FSK-COH-JAM-BER

Figure E.2 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-4FSK-COH-JAM-BER

Figure E.3 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-8FSK-COH-JAM-BER

Figure E.4 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-2FSK-SQR-JAM-BER

Figure E.5 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-4FSK-SQR-JAM-BER

Figure E.6 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-8FSK-SQR-JAM-BER

Figure E.7 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-2PSK-COH-JAM-BER

Figure E.8 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-4PSK-COH-JAM-BER

Figure E.9 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-8PSK-COH-JAM-BER

Figure E.10 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 M-2FSK-COH-FJAM-BER

Figure E.11 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 M-4FSK-COH-FJAM-BER

Figure E.12 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
M-8FSK-COH-FJAM-BER

Figure E.13 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 M-2FSK-SQR-FJAM-BER

Figure E.14 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 M-4FSK-SQR-FJAM-BER

Figure E.15 BER versus $\frac{E_b}{N_J}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 M-8FSK-SQR-FJAM-BER

Figure E.16 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
M-2PSK-COH-FJAM-BER

Figure E.17 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

M-4PSK-COH-FJAM-BER

Figure E.18 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

M-8PSK-COH-FJAM-BER

APPENDIX F

Plots of BER versus E_b/N_J for the Hard Limiter Receiver

Figure F.1 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-2FSK-COH-JAM-BER

Figure F.2 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-4FSK-COH-JAM-BER

Figure F.3 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-8FSK-COH-JAM-BER

Figure F.4 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-2FSK-SQR-JAM-BER

Figure F.5 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-4FSK-SQR-JAM-BER

Figure F.6 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-8FSK-SQR-JAM-BER

Figure F.7 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-2PSK-COH-JAM-BER

Figure F.8 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-4PSK-COH-JAM-BER

Figure F.9 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-8PSK-COH-JAM-BER

Figure F.10 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 H-2FSK-COH-FJAM-BER

Figure F.11 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

H-4FSK-COH-FJAM-BER

Figure F.12 BER versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

H-8FSK-COH-FJAM-BER

Figure F.13 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

H-2FSK-SQR-FJAM-BER

Figure F.14 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

H-4FSK-SQR-FJAM-BER

Figure F.15 BER versus $\frac{E_b}{N_0}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.
 H-8FSK-SQR-FJAM-BER

Figure F.16 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

H-2PSK-COH-FJAM-BER

Figure F.17 BER versus $\frac{E_b}{N_f}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming with Rayleigh fading.

H-4PSK-COH-FJAM-BER

APPENDIX G

Some Typical Plots of $\log_{10} \gamma_{opt}$ versus E_b/N_J

Figure G.1 $\log_{10} \gamma_{opt}$ versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-4FSK-COH-JAM-GAM

Figure G.2 $\log_{10} \gamma_{opt}$ versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

L-2FSK-COH-FJAM-GAM

Figure G.3 $\log_{10} \gamma_{opt}$ versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-2FSK-COH-JAM-G.M

Figure G.4 $\log_{10} \gamma_{opt}$ versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

M-2PSK-COH-FJAM-GAM

Figure G.5 $\log_{10} \gamma_{opt}$ versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-4FSK-SQR-JAM-GAM

Figure G.6 $\log_{10} \gamma_{opt}$ versus $\frac{E_b}{N_j}$ ($\frac{E_b}{N_0}$ fixed for 10^{-4} BER without jamming) for 1 to 5 hops per symbol under worst-case partial-band jamming.

H-2FSK-SQR-FJAM-GAM

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