Modelling and Synthesis of High Speed Craft Acceleration Profiles

by

Mohamed El Tayeby

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Submitted by **Mohamed El Tayeby**
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Dr. Robert Langlois, Supervisor

Dr. Fred Afagh, Co-Supervisor

Dr. Metin Yaras, Department Chair

Carleton University

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Abstract

High Speed Craft (HSC) occupants are subjected to a harsh environment characterized by vibration and repeated shock. Significant research has therefore been invested into HSC shock mitigation. The objective of this work was to determine standard shock acceleration profiles for HSC based on sea trial data. The knowledge of real-life sea inputs is important for in-lab testing of HSC shock mitigation seats. Sea trial vertical acceleration data of a Rigid-Hull Inflatable Boat was analyzed with the aim of characterizing shock acceleration pulses experienced by the boat. Five common types of shock acceleration profiles were identified. These profiles were represented by a combination of modified Gaussian curves. The statistical characteristics of the observed pulses were obtained by curve fitting the data. An algorithm employing copulas and non-parametric density estimation was developed for generating synthetic time histories which have the same statistical characteristics as the observed data.

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Table of Contents

Abstract	iii
Acknowledgments	iv
Table of Contents	v
List of Tables	viii
List of Figures	x
List of Acronyms	xiii
1 Introduction	1
1.1 Literature Review	3
1.2 Thesis Objectives	6
1.3 Thesis Outline	7
2 Sea Trial Data Analysis and Standard Acceleration Profiles	8
2.1 Data Processing	9
2.1.1 Accelerometer Gravity Offset	10

2.1.2	Filtering	11
2.1.3	Impact Pulse Extraction	12
2.2	Acceleration Profiles	13
2.2.1	Type 1: Half-Sine	14
2.2.2	Type 2: Half-Sine with Single Shoulder	16
2.2.3	Type 3: Half-Sine with Multiple Shoulders	18
2.2.4	Type 4: Delayed Peak	21
2.2.5	Type 5: Double peak	23
2.3	Curve Fitting Algorithm	26
2.3.1	Pulse Type Detection	27
2.4	Reduction of Sea Trial Data	28
2.5	Discussion	42
2.5.1	On the Curve Fitting Process	42
2.5.2	On the Results	43
2.6	Conclusion	44
3	Synthetic Time History Generation	45
3.1	Synthetic Pulse Generation Methods	46
3.1.1	Simulating Dependent Random Variables using Copulas	46
3.1.2	Simulating Dependent Random Variables by Non-parametric Density Estimation	48
3.2	Inter-Pulse Characteristics	50
3.3	Results	56
3.4	Discussion	68

3.4.1	On Synthetic Pulses	68
3.4.2	On Inter-Pulse Characteristics	69
3.5	Conclusion	69
4	Conclusion	70
	References	72
	Appendix A Reduction of Additional Data Set	74
	Appendix B Data Pre-processing Code	79
	Appendix C Curve Fitting Algorithm	84
	Appendix D Classification of Fitted Pulses into the Different Profiles	
	Code	92
	Appendix E Synthetic Data Generation Code	114
	Appendix F Code for Obtaining Parameter Mean Values	170

List of Tables

2.1	Observed Type 5 variations.	24
2.2	Pulse detection rate.	28
2.3	Day 1 – bow parameter mean values for Types 1 to 4 profiles.	29
2.4	Day 1 – bow parameter mean values for Type 5 Profiles.	30
2.5	Day 2 – bow parameter mean values Types 1 to 4 Profiles.	31
2.6	Day 2 – bow parameter mean values for Type 5 Profiles.	32
2.7	Day 3 – bow parameters mean values Types 1 to 4 Profiles.	33
2.8	Day 3 – bow parameter mean values for Type 5 Profiles.	34
2.9	Day 4 – bow parameter mean values for Types 1 to 4 Profiles.	35
2.10	Day 4 – bow parameter mean values for Type 5 Profiles.	36
3.1	Parameter mean values comparison for Types 1 through 4 – copula method.	63
3.2	Parameter mean values comparison for Types 1 through 4 – density estimation method.	64
3.3	Parameter mean values comparison for Type 5 profiles – copula method.	65
3.4	Parameter mean values comparison for Type 5 profiles – density esti- mation method.	66

3.5	Prescribed vs detected profiles.	67
A.1	Forward position – parameter mean values for Types 1 to 4 profiles. .	77
A.2	Forward position – parameter mean values for Type 5 profiles. . . .	78

List of Figures

1.1	Examples of HSC.	3
1.2	Sample of RHIB vertical acceleration time history.	5
1.3	Algorithm for analyzing and simulating HSC acceleration data.	7
2.1	Wave height and period.	10
2.2	Power spectral density of unfiltered vertical acceleration of the midships portion of a RHIB.	11
2.3	Vertical acceleration time history of the midships portion of a RHIB.	12
2.4	Typical normalized acceleration pulse.	13
2.5	Type 1 profile.	16
2.6	Type 2 profile.	18
2.7	Type 3.I profile.	19
2.8	Type 3.II profile.	20
2.9	Type 3.III profile.	20
2.10	Type 4.I profile.	22
2.11	Type 4.II profile.	22
2.12	Type 4.III profile.	23
2.13	Type 1-2 profile.	25

2.14	Type 2-2 profile.	25
2.15	Type 2-3.I profile.	26
2.16	Day 1 profile distributions.	37
2.17	Distribution of Type 5 profiles for Day 1 – bow.	37
2.18	Day 2 profile distributions.	38
2.19	Distribution of Type 5 profiles for Day 2 – bow.	38
2.20	Day 3 profile distributions.	39
2.21	Distribution of Type 5 profiles for Day 3 – bow.	39
2.22	Day 4 profile distributions.	40
2.23	Distribution of Type 5 profiles for Day 4 – bow.	40
2.24	Profiles at parameter mean values of Day 1 at the bow.	41
3.1	Illustration of space discretization for two variables.	50
3.2	Example of a motion cycle.	54
3.3	Example of drift correction.	55
3.4	Synthetic time history.	57
3.5	Matrix plot of the original parameters for Type 2 (in blue) and the simulated parameters using copulas (in green).	58
3.6	Matrix plot of the original parameters for Type 2 (in blue) and simu- lated parameters using density estimation (in green).	59
3.7	Distribution of the original parameters for Type 2.	60
3.8	Distribution of the simulated parameters using copulas for Type 2. . .	61
3.9	Distribution of the simulated parameters using density estimation for Type 2.	62
A.1	Profile distribution – forward position.	75

A.2	Distribution of Type 5 profiles – forward position.	76
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List of Acronyms

Acronyms	Definition
HSC	high speed craft
SMS	shock mitigation seat
RHIB	rigid-hull inflatable boat
DRDC	Defence Research and Development Canada
ICI	impact count index

Chapter 1

Introduction

A research project investigating the dynamics of High Speed Craft (HSC) Shock Mitigation Seats (SMSs) was recently conducted by Carleton University's Applied Dynamics Lab in collaboration with Defence Research and Development Canada (DRDC) – Atlantic. The research presented in this thesis addresses one of the overall project objectives which was to determine HSC standard shock acceleration profiles based on the analysis of sea trial acceleration data. The remaining objectives of the project were to:

1. Develop 3-D dynamic models of sample SMSs that were provided by DRDC;
2. Characterize dynamic properties of seats and their components such as springs and dampers using both a drop tower and a suspension component test frame; and
3. Design and build a test rig which allows testing of seats under simulated sea-like conditions.

The common goal of the project was to develop tools which would contribute to making an informed selection of SMSs based on the prevailing operational environment for the Canadian Forces. The SMSs which were modelled and tested are used on Rigid-Hull Inflatable Boats (RHIBs).

A HSC is generally defined as a vessel which can achieve speeds higher than 30 knots [1]. Faltinsen classifies HSC into three categories: submerged-hull-supported, foil-supported, and air-cushion-supported vessels. Furthermore, submerged-hull-supported vessels are divided into semi-displacement and planing craft [1]. The work presented in this thesis pertains mostly to planing craft such as small patrol and rescue boats used by navies and coast guards (Figure 1.1). Occupants of HSC are subjected to high acceleration impacts and repeated shock which often can lead to injury [2–4]. Research into HSC shock mitigation has been undertaken in order to reduce occupant injury and discomfort during high speed transits. Three areas of study and their mutual interaction are involved in HSC shock mitigation:

1. HSC dynamics;
2. Seat dynamics; and
3. Human body dynamics.

The topic of this thesis is concerned with HSC dynamics. More specifically, the focus is on the shock motion of HSC. The overall goal of the research is to characterize HSC vertical impact acceleration pulses and to establish standard acceleration profiles based on sea trial boat data. The importance of this investigation lies in the fact that knowledge of real-life seat inputs is necessary for meaningful in-lab seat testing

(a) DAMEN Interceptor high speed planing craft [5] (b) Titan Boats Rigid Inflatable Boat [6]

Figure 1.1: Examples of HSC.

and simulation of seat motion. This research also opens the door for establishing correlations between boat characteristics, sea parameters, and impact profiles.

In the remainder of this chapter, Section 1.1 presents background information on HSC shock motion, biased towards planing craft motion. In Section 1.2, the thesis objectives and approach are explained. Finally, the thesis outline is presented in Section 1.3.

1.1 Literature Review

Much of the research on HSC dynamics has been experimental in nature. An extensive review of experimental and numerical methods for the analysis of HSC motion can be found in [7]. While the manoeuvring and seakeeping performance of low speed craft are well understood, the prediction of HSC performance, particularly of planing craft, is a challenge due to the occurrence of numerous wave slamming/impact events during high speed transits [4, 8] (Figure 1.2). This type of shock loading

increases occupant injury risk and therefore is a major concern in the study of HSC motion. Wave slamming occurs when the forward portion of the craft impacts with oncoming waves while the boat pitches and heaves about a rotation point close to the aft portion of the craft [3]. Wave slamming can also result from the entire boat leaving the water and re-entering [3].

Experiments have been conducted to study hull vertical water entry. Generally these experiments consisted of dropping wedge shaped objects into water. Numerical methods, such as CFD and smoothed particle hydrodynamics, have been used to simulate wedge water entry and have been validated against experimental test data [1,9]. The wedge water entry problem is a starting point for understanding hull water entry in real-life sea conditions.

Empirical formulae are available for the determination of the average impact acceleration of particular types of planing hulls in prescribed sea conditions [10]. These formulae do not, however, provide information about impact pulse shape. Besides empirical correlations, statistical measures have been developed to characterize HSC shock motion. For example, the Impact Count Index (ICI) is a measure of the number of impacts in a given vertical acceleration time history which have a maximum amplitude below a certain acceleration value. By using the ICI, a criterion can be defined which sets a shock exposure limit [11]. Another example of statistical methods applied to HSC motion is a technique which has been developed to synthesize vertical acceleration time histories for planing craft [12]. The generalized Pareto distribution was fitted to impact peak values from recorded time histories. Random numbers were then generated and converted to peak values using the Pareto distribution in

order to produce time histories with the same peak value characteristics. An additional step was taken in which the mean of the Pareto distribution was set to the average impact acceleration predicted by the aforementioned empirical correlations, such that time histories can be obtained which are representative of different types of planing craft in different sea conditions. In this study, the impact pulses of the produced time histories were assumed to have a half-sine shape.

Statistical methods have focused so far on the peak values of HSC impact pulses. However, the pulse shape characteristics are also important since they contain information about the energy of impact. Therefore, for the purpose of testing SMSs under sea-like conditions, it is necessary to have knowledge of the impact pulse shape characteristics of HSC.

Figure 1.2: Sample of RHIB vertical acceleration time history (data provided by DRDC).

1.2 Thesis Objectives

The main objective of this work was to characterize HSC vertical impact acceleration pulses. The reason for considering vertical impact accelerations is that they are responsible for spine and knee injury which are the most common injury locations. The characterization of acceleration pulses was achieved through the analysis of RHIB sea trial data. While RHIBs are one type of HSC, the presented work could potentially apply to other types of HSC. A set of equations was developed to fit the observed acceleration profiles. Based on the sea trial data, five types of acceleration profiles were distinguished. The second objective of this work was to produce acceleration time histories having the same pulse shape characteristics as those of the observed data. Two methods were implemented in order to produce correlated random variables. The first employs mathematical functions known as copulas; the second relies on a non-parameteric estimate of the joint probability density function of the variables. These methods were used to generate equation parameters and in turn to synthesize acceleration time histories.

The flow chart provided in Figure 1.3 illustrates the workings of the produced algorithm. The sea trial data is first processed in order to remove the accelerometer gravity offset and noise from the acceleration signal. The acceleration pulses are then extracted from the available time histories. These pulses are then curve-fitted with the equation form proposed in Chapter 2, using a built-in Matlab function (`lsqcurvefit`). From the curve fitting process, the distribution of acceleration profiles can be determined as well as equation parameters. From the acquired statistics, synthetic time histories can then be produced which have the same pulse shape

characteristics as the original sea trial data.

Figure 1.3: Algorithm for analyzing and simulating HSC acceleration data.

1.3 Thesis Outline

The remainder of this thesis is organized as follows. Chapter 2 provides a detailed description of how the original raw boat data was processed. The curve fitting equations are presented and the curve fitting process is explained. The statistical characteristics of the observed pulses, in the sea trial data provided by DRDC, are presented. A discussion follows in which comments are made on the curve fitting procedure and the results. Chapter 3 provides an explanation of the time history generation process. The statistical characteristics of a sample synthesized time history are presented. A discussion is then provided in which generated time histories are compared to the observed data. Chapter 4 summarizes the contributions of this work and provides recommendations for future work.

Chapter 2

Sea Trial Data Analysis and Standard Acceleration Profiles

Sea trial RHIB acceleration data, provided by DRDC, was processed and analysed in order to understand the characteristics of HSC impact pulses. From a thorough inspection of the data it was postulated that impact pulses could be classified into several distinct types. An equation was sought to represent each one of those types. Ultimately, the observed pulses were classified into five main categories after an iterative process which required re-examining the data and evaluating various equation forms. A set of equations consisting of a combination of modified Gaussian curves was found to be an ideal fit for the data.

The processing of the raw data is explained in Section 2.1. The algorithm used for the curve fitting process is described in Section 2.3. The different types of acceleration pulses are presented, along with their representative equations, in Section 2.2. Results of the analysis are presented in Section 2.4. The results are discussed in Section 2.5. Concluding remarks are presented in Section 2.6.

2.1 Data Processing

Data collected from a RHIB sea trial spanning five days (June 11th – 15th, 2007) was provided to the research team. The variation of wave height and modal period over the duration of the trial are shown in Figure 2.1. Two data acquisition packages were mounted on the longitudinal centreline of the RHIB with one at midships and one at the bow of the boat. Each package contained:

- a Crossbow CXL-HF Series ± 10 g accelerometer (sampling rate of 2000 Hz);
- a Crossbow CXL-HF Series ± 100 g accelerometer (sampling rate of 2000 Hz); and
- a Crossbow NAV420C A-100 GPS-aided MEMS inertial system (sampling rate of 100 Hz).

The Nav420 inertial system provided acceleration and angular orientation data. Fluctuations were found in the time increments at which accelerations were recorded by the CXL accelerometers. For this reason, it was not possible to synchronize the orientation data of the NAV420 with the acceleration data of the CXL accelerometers. As a result, only the Nav420 data was considered in the analysis described in the remainder of this chapter. Furthermore, some data was missing from the first day of sea trial; therefore the last four days were considered in the analysis (June 12th–15th, 2007).

The processing of the data involved three operations: accelerometer gravity offset removal, filtering, and impact pulse extraction and organisation.

Figure 2.1: Wave height and period [provided by DRDC].

2.1.1 Accelerometer Gravity Offset

A tri-axial accelerometer measures specific force [13]:

$$\vec{f} = \vec{a} - \vec{g} \quad (2.1)$$

where $\vec{f}$ is the specific force, $\vec{a}$ is the inertial acceleration, and $\vec{g}$ is the gravitational vector. Therefore, an accelerometer at rest will indicate $-\vec{g}$. This gravity offset was removed from the data by finding the components of $\vec{g}$ in the local frame of the boat and then adding these components to the accelerometer outputs. The xyz Euler angle sequence was used to determine $\vec{g}_{local}$:

$$\vec{g}_{local} = -9.81 \begin{pmatrix} \sin \theta_x \sin \theta_z - \cos \theta_x \sin \theta_y \cos \theta_z \\ \sin \theta_x \cos \theta_z + \cos \theta_x \sin \theta_y \sin \theta_z \\ \cos \theta_x \cos \theta_y \end{pmatrix} \quad (2.2)$$

where θ_x is the roll angle, θ_y is the pitch angle, and θ_z the yaw angle in a right-handed coordinate system where the x-axis points towards the bow of the boat and the z-axis points towards the sky.

2.1.2 Filtering

Generally, the frequency range of rigid body motion of a HSC does not exceed 10 Hz (Figure 2.2). A 10 Hz low-pass fourth-order Butterworth filter was applied to the data in order to eliminate high-frequency acceleration spikes resulting from structural vibration of the boat deck. Furthermore, a 0.05 Hz high-pass fourth-order Butterworth filter was applied in order to remove any offset caused by accelerometer bias.

Figure 2.2: Power spectral density of unfiltered vertical acceleration of the midships portion of a RHIB.

2.1.3 Impact Pulse Extraction

There is no universally-accepted value defining the maximum acceleration of a slam event. Peak values observed in the data were up to 3.8 g (Figure 2.3). For the analysis presented in this thesis, an acceleration of 2 g was chosen as the minimum acceleration peak for an impact event. 2 g is approximately the median of the acceleration values. An algorithm was written to extract impact pulses based on the 2 g threshold. This threshold, however, can be changed in the developed algorithm and corresponding computer implementation, to a different value if a more precise threshold is determined as a result of future advancements into HSC impact motion research. The pulses were then normalized such that their peak values and durations were both equal to one. Linear interpolation was used at the extremities such that the curves pass through (0, 0) and (1, 0) (Figure 2.4).

Figure 2.3: Vertical acceleration time history of the midships portion of a RHIB.

Figure 2.4: Typical normalized acceleration pulse.

2.2 Acceleration Profiles

After a thorough inspection of the acceleration curves, five general types of pulses were identified. The equations which were used to curve fit the data consist of a combination of modified Gaussian terms:

$$F(x) = \sum_{i=1}^n x(x-1)a_i e^{-\left(\frac{x-m_i}{\sigma_i}\right)^2} \quad (2.3)$$

where x is normalized time and $F(x)$ is normalized acceleration; also associated with each term is a scaling parameter a_i , a location parameter m_i and a shape parameter

σ_i . Initially a different equation form was used:

$$F(x) = (1 + k \sin(\frac{\pi x}{c + \omega x})^j (e^{-zx} + a)) \sum_{i=1}^n A_i \sin(\pi x) e^{-u_i x} \text{ where } j = 1, 2 \text{ or } 3 \quad (2.4)$$

which is the product of a modified decaying sinusoid and a sum of modified half-sine curves.

While Equation 2.4 was found to be suitable for fitting the pulses, it was not well suited for time history generation (Chapter 3). The reason for this is that the parameters of Equation 2.4 are highly correlated. Efforts to rewrite and rearrange the equation were made so as to “de-correlate” the parameters were not successful.

The number of terms in Equation 2.3 varies from 1 to 5 depending on the type of pulse and its characteristics. In the following sections, the different types are presented. In all subsequent derivations x is normalized time and $F(x)$ is normalized acceleration.

2.2.1 Type 1: Half-Sine

Shown in Figure 2.5 is a Type 1 profile. This curve resembles a half-sine which can be represented by:

$$F(x) = x(x - 1)a_1 e^{-(\frac{x-m_1}{\sigma_1})^2} \quad (2.5)$$

The location parameter m_1 can be written in terms of the peak time x_p as follows:

$$\begin{aligned} \frac{dF(x_p)}{dx} &= ((2x_p - 1) - \frac{2}{\sigma_1^2} x_p (x_p - m_1)) a_1 e^{-(\frac{x-m_1}{\sigma_1})^2} = 0 \\ \Rightarrow m_1 &= \frac{-(2x_p - 1)\sigma_1^2}{2x_p(x_p - 1)} + x_p \end{aligned} \quad (2.6)$$

Furthermore, a_1 is written as follows:

$$\begin{aligned} F(x_p) &= x_p(x_p - 1)a_1 e^{-\left(\frac{x_p - m_1}{\sigma_1}\right)^2} = 1 \\ \Rightarrow a_1 &= \frac{1}{x_p(x_p - 1)e^{-\left(\frac{x_p - m_1}{\sigma_1}\right)^2}} \end{aligned} \quad (2.7)$$

The final equation is then:

$$F(x) = \frac{x(x - 1)e^{-\left(\frac{x - m_1}{\sigma_1}\right)^2}}{x_p(x_p - 1)e^{-\left(\frac{x_p - m_1}{\sigma_1}\right)^2}} \quad (2.8)$$

The equation parameters are the peak time x_p and shape parameter σ_1 which controls the sharpness of the pulse. Due to the fact that the data was sampled at a relatively low rate (100 Hz), the peak time obtained from a given curve might be slightly offset from the peak time of a perfectly-smooth version of this curve. For this reason x_p was treated as a variable in the curve fitting process. However, x_p was constrained to be close to the observed peak time. This constraint was applied by setting appropriate lower and upper bounds for x_p in the built-in Matlab function presented in Section 2.3. The specific values of the upper and lower bounds for each parameter are included in the code provided in Appendix C.

Figure 2.5: Type 1 profile.

2.2.2 Type 2: Half-Sine with Single Shoulder

Shown in Figure 2.6 is a Type 2 profile. This pulse consists of a half-sine followed by a shoulder or bump. The equation for this pulse is:

$$F(x) = \frac{x(x-1)a_1 e^{-\left(\frac{x-m_1}{\sigma_1}\right)^2}}{x_1(x_1-1)e^{-\left(\frac{x_1-m_1}{\sigma_1}\right)^2}} + \frac{x(x-1)a_2 e^{-\left(\frac{x-m_2}{\sigma_2}\right)^2}}{x_2(x_2-1)e^{-\left(\frac{x_2-m_1}{\sigma_1}\right)^2}} \quad (2.9)$$

where m_1 and m_2 are functions of x_1 and x_2 respectively:

$$m_1 = \frac{-(2x_1-1)\sigma_1^2}{2x_1(x_1-1)} + x_1, \quad m_2 = \frac{-(2x_2-1)\sigma_2^2}{2x_2(x_2-1)} + x_2 \quad (2.10)$$

and a_1 and a_2 are the peak values of the first and second terms respectively. The expressions for m_1 and m_2 were obtained in the same fashion as for Type 1 (Equation 2.6). In addition, a shape parameter σ is associated with each term. Parameter a_1 was constrained to be equal to approximately 1; as a result, it can be assumed that x_1 coincides with x_p . Using this assumption it is possible to express a_2 in terms of h_2 which is the value of the curve at the peak time of the second term:

$$F(x_2) = \frac{x_2(x_2 - 1)a_1 e^{-\left(\frac{x_2 - m_1}{\sigma_1}\right)^2}}{x_1(x_1 - 1)e^{-\left(\frac{x_1 - m_1}{\sigma_1}\right)^2}} + a_2 = h_2$$

$$a_2 = h_2 - \frac{x_2(x_2 - 1)a_1 e^{-\left(\frac{x_2 - m_1}{\sigma_1}\right)^2}}{x_1(x_1 - 1)e^{-\left(\frac{x_1 - m_1}{\sigma_1}\right)^2}} \quad (2.11)$$

However, for the curve fitting process, the variable a_2 was used instead of h_2 . The reason for this is that h_2 is close to but not necessarily equal to the amplitude of a known characteristic point on the curve such as an inflection point or a local minimum. Parameter a_2 was therefore used in order to reduce computational time.

Figure 2.6: Type 2 profile.

2.2.3 Type 3: Half-Sine with Multiple Shoulders

Type 3 profiles are characterized by a half-sine shape followed by 2, 3, or 4 shoulders as shown in Figures 2.7 through 2.9, respectively. Type 3 pulses were classified according to the number of shoulders following the main peak:

- Type 3.I: half-sine followed by 2 shoulders;
- Type 3.II: half-sine followed by 3 shoulders; and
- Type 3.III: half-sine followed by 4 shoulders.

Three equations are therefore necessary:

$$F_i(x) = \sum_{j=1}^{i+1} a_j \frac{x(x-1)e^{-\left(\frac{x-m_j}{\sigma_j}\right)^2}}{x_j(x_j-1)e^{-\left(\frac{x_j-m_j}{\sigma_j}\right)^2}} \quad i = 2, 3, \text{ and } 4 \quad (2.12)$$

Figure 2.7: Type 3.I profile.

where the index i is the number of shoulders. The equation parameters are treated similarly as those of Types 1 and 2:

$$m_i = \frac{-(2x_i - 1)\sigma_i^2}{2x_i(x_i - 1)} + x_i \quad (2.13)$$

Figure 2.8: Type 3.II profile.

Figure 2.9: Type 3.III profile.

2.2.4 Type 4: Delayed Peak

In Type 4 profiles the peak time is generally more delayed and the main peak is preceded by a shoulder as shown in Figures 2.10 through 2.12. Similarly to Type 3, Type 4 includes three variations, each one with a particular number of underlying curves:

- Type 4.I: 1 shoulder, followed by main peak, followed by 1 shoulder;
- Type 4.II: 1 shoulder, followed by main peak, followed by 2 shoulders; and
- Type 4.III: 1 shoulder, followed by main peak, followed by 3 shoulders.

The representative equation is the same as for Type 3:

$$F_i(x) = \sum_{j=1}^{i+1} a_j \frac{x(x-1)e^{-\left(\frac{x-m_j}{\sigma_j}\right)^2}}{x_j(x_j-1)e^{-\left(\frac{x_j-m_j}{\sigma_j}\right)^2}} \quad (2.14)$$

where i is the number of shoulders and, as opposed to the previous types, $x_2 < x_1$.

Similarly to the previous types:

$$m_i = \frac{-(2x_i - 1)\sigma_i^2}{2x_i(x_i - 1)} + x_i \quad (2.15)$$

Figure 2.10: Type 4.I profile.

Figure 2.11: Type 4.II profile.

Figure 2.12: Type 4.III profile.

2.2.5 Type 5: Double peak

Type 5 profiles consist of two successive impacts. In Type 5 profiles the acceleration does not die out after the first peak but picks up again as shown in Figures 2.13 through 2.15. The equation for Type 5 profiles is:

$$F_i(x) = \sum_{j=1}^i a_j \frac{x(x-1)e^{-\left(\frac{x-m_j}{\sigma_j}\right)^2}}{x_j(x_j-1)e^{-\left(\frac{x_j-m_j}{\sigma_j}\right)^2}} \quad (2.16)$$

where i can vary from 3 to 5 depending on how many parameters are necessary to model a particular Type 5 pulse.

A number of different variations can occur in this type of profile. The following numbering convention has been chosen in order to refer to the different variations:

Table 2.1: Observed Type 5 variations.

Number of Gaussian Terms	Observed Variations
3	1-2, 2-1, 1-2 ^r
4	1-3.I, 1-4.I, 2-2, 3.I-1, 4.I-1, 1-3.I ^r , 2-2 ^r
5	1-3.II, 1-4.II, 2-3.I, 2-4.I, 3.I-2, 1-3.II ^r , 1-4.II ^r

- a double peak profile consisting of a Type x followed by a Type y, where the maximum amplitude is reached during the first profile, is denoted as Type x-y; and
- a double peak profile consisting of a Type x followed by a Type y, where the maximum amplitude is reached during the second profile, is denoted as Type x-y^r (where *r* stands for reverse). For example, a Type 3.I followed by a Type 2, where the maximum amplitude is reached during the second profile, is denoted as Type 3.I-2^r. The observed variations are outlined in Table 2.1.

To detect Type 5 pulses the minimum point between the two peaks (x_{min}, a_{min}), shown in Figure 2.13, is first identified. If the difference in amplitude between this minimum point and the second highest peak (Δa) is larger than a certain threshold then the pulse is considered of Type 5. A threshold of 0.2 was determined for Δa based on the inspection of double peak pulses. Curve fitting this type of pulse as a whole is an alternative to considering the two peaks as separate events and fitting each one individually.

Figure 2.13: Type 1-2 profile.

Figure 2.14: Type 2-2 profile.

Figure 2.15: Type 2-3.I profile.

2.3 Curve Fitting Algorithm

A built-in Matlab function (`lsqcurvefit`) was used for curve fitting the acceleration pulses. Initial guesses of the parameter values are input to the function as well as parameter constraints (upper and lower bounds). The output is a set of parameter values which minimizes the sum of the squares of the residuals (SR) between the original data and the fitted curve. The algorithm is sensitive to initial guesses and constraints; for this reason different starting points and bounds were used in curve fitting the data. A maximum SR below which a fitted curve is considered acceptable was set based on experience. The threshold SR is not however a fixed value but is proportional to the number of data points and to an average residual at each data

point:

$$SR_{\text{threshold}} = (\text{Average Residual Threshold})^2 \times \text{number of data points} \quad (2.17)$$

The average residual threshold was determined as 0.01 after a thorough inspection of the fitted curves.

2.3.1 Pulse Type Detection

A two-step procedure was followed in order to detect the different types of profiles from the observed pulses automatically. For each pulse, the first step was to determine the number of equation terms which are necessary to obtain a matching fitted curve according to the established $SR_{\text{threshold}}$. This was performed by successive curve fitting with increasing number of terms until the obtained SR was lower than the $SR_{\text{threshold}}$. Sequential fitting was used in order to obtain the best possible fit with the least number of equation parameters/terms. The second step was to classify the detected pulse, from the first step, into the different types. For Types 1 and 2, the second step was not necessary since these profiles are not characterized by any variations. The remaining types were detected by a two-step elimination process. First it is determined whether or not the pulse is a Type 5 by the procedure described in Section 2.2.5. Second, if the pulse is not a Type 5, then it is a Type 4 if one of the underlying curves has a peak time smaller than the peak time of the overall pulse. If the previous criteria are not met then a Type 3 is identified. It was observed that in some cases, the second highest peak of a Type 5 profile precedes the highest peak. For this reason, Type 5 takes precedence over Type 4 in the elimination process.

2.4 Reduction of Sea Trial Data

Four pairs of time histories that were collected (by DRDC) over four consecutive days were analyzed. Each pair consists of the time histories of the forward portion of the boat and the centre portion as mentioned in Section 2.1. In Table 2.2, the pulse detection rate for each time history is shown. In Figures 2.16 through 2.23, the profile distribution for each time history is presented. The parameter mean values for each type are shown in Tables 2.3 through 2.10. The last two columns in each table correspond to the mean peak acceleration and pulse duration, respectively. Types 1 through 4 can be visualized in Figure 2.24.

An additional data set, also provided by DRDC, has been reduced. The results are presented in Appendix A.

Table 2.2: Pulse detection rate.

Day	Position on Boat	Number of Pulses	Detection Rate (%)
1	Midships	17	100
	Bow	712	92
2	Midships	232	86
	Bow	1949	92
3	Midships	46	83
	Bow	1196	93
4	Midships	192	88
	Bow	2305	90

Table 2.3: Day 1 – bow parameter mean values for Types 1 to 4 profiles.

	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
Type 1	1	0.42	0.53													2.4	133
Type 2	1	0.29	0.28	0.18	0.69	0.24										2.5	248
Type 3	I	1	0.18	0.26	0.41	0.13	0.14	0.66	0.29							2.7	446
	II	1	0.13	0.29	0.28	0.08	0.21	0.44	0.16	0.14	0.69	0.33				2.7	610
	III	1	0.11	0.32	0.23	0.06	0.26	0.35	0.11	0.19	0.53	0.21	0.12	0.77	0.29	2.6	725
Type 4	I	1	0.37	0.23	0.19	0.12	0.22	0.74	0.19							2.1	353
	II	1	0.30	0.20	0.09	0.10	0.24	0.50	0.12	0.16	0.71	0.25				2.3	529
	III	1	0.28	0.21	0.08	0.11	0.28	0.45	0.08	0.25	0.57	0.13	0.14	0.76	0.27	2.4	565

Table 2.4: Day 1 – bow parameter mean values for Type 5 Profiles.

[illegible]

Table 2.5: Day 2 – bow parameter mean values Types 1 to 4 Profiles.

	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
Type 1	1	0.42	0.50													2.5	132
Type 2	1	0.29	0.30	0.19	0.70	0.26										2.8	231
Type 3	I	0.17	0.17	0.27	0.41	0.13	0.15	0.66	0.30							2.9	413
	II	0.12	0.13	0.30	0.29	0.09	0.23	0.46	0.16	0.13	0.72	0.30				2.9	561
	III	0.11	0.11	0.32	0.24	0.07	0.28	0.37	0.12	0.19	0.57	0.18	0.10	0.83	0.23	2.8	658
Type 4	I	0.41	0.23	0.27	0.13	0.15	0.21	0.75	0.18							2.5	347
	II	0.33	0.14	0.25	0.09	0.10	0.29	0.52	0.11	0.17	0.73	0.22				2.5	490
	III	0.30	0.12	0.24	0.08	0.10	0.30	0.45	0.08	0.25	0.57	0.13	0.13	0.77	0.24	2.6	579

Table 2.6: Day 2 – bow parameter mean values for Type 5 Profiles.

Type	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
1-2	1	0.12	0.13	0.56	0.44	0.15	0.27	0.67	0.39							2.5	459
2-1	1	0.13	0.14	0.15	0.31	0.10	0.42	0.66	0.20							2.4	460
1-2 ^r																	
1-3.I	1	0.11	0.11	0.54	0.32	0.11	0.47	0.46	0.16	0.15	0.73	0.31				2.5	539
1-4.I	1	0.11	0.12	0.22	0.33	0.11	0.47	0.50	0.13	0.25	0.70	0.27				2.6	521
2-2	1	0.11	0.11	0.23	0.27	0.08	0.30	0.59	0.12	0.26	0.72	0.28				2.8	603
3.I-1	1	0.09	0.09	0.26	0.22	0.08	0.08	0.45	0.23	0.39	0.84	0.13				3.5	666
4.I-1																	
1-3.I ^r																	
2-2 ^r																	
1-3.II	1	0.10	0.11	0.58	0.40	0.14	0.21	0.56	0.08	0.21	0.71	0.14	0.07	0.87	0.23	3.0	585
1-4.II	1	0.11	0.13	0.23	0.34	0.13	0.22	0.45	0.07	0.38	0.54	0.13	0.20	0.72	0.28	2.2	648
2-3.I	1	0.09	0.10	0.26	0.24	0.07	0.39	0.46	0.09	0.35	0.60	0.15	0.14	0.82	0.20	2.4	620
2-4.I	1	0.12	0.10	0.22	0.22	0.08	0.22	0.55	0.17	0.30	0.70	0.10	0.14	0.86	0.13	2.6	640
3.I-2	1	0.10	0.11	0.27	0.26	0.08	0.18	0.40	0.14	0.31	0.73	0.16	0.21	0.86	0.18	2.5	670
1-3.II ^r	1	0.35	0.11	0.68	0.09	0.10	0.30	0.50	0.09	0.25	0.63	0.11	0.14	0.79	0.19	2.4	562
1-4.II ^r	1	0.66	0.11	0.91	0.10	0.10	0.42	0.39	0.09	0.04	0.76	0.06	0.19	0.88	0.14	2.4	458

Table 2.7: Day 3 – bow parameters mean values Types 1 to 4 Profiles.

	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
Type 1	1	0.41	0.47													2.4	152
Type 2	1	0.28	0.28	0.19	0.69	0.27										2.6	250
Type 3	I	1	0.18	0.26	0.41	0.13	0.16	0.66	0.29							2.7	432
	II	1	0.13	0.29	0.29	0.09	0.22	0.44	0.16	0.14	0.69	0.32				2.7	595
	III	1	0.10	0.29	0.23	0.07	0.25	0.35	0.12	0.18	0.57	0.16	0.12	0.78	0.29	2.6	751
Type 4	I	1	0.40	0.23	0.14	0.14	0.23	0.76	0.18							2.3	334
	II	1	0.31	0.25	0.09	0.10	0.24	0.50	0.11	0.15	0.75	0.22				2.4	504
	III	1	0.29	0.24	0.08	0.08	0.25	0.45	0.08	0.22	0.57	0.13	0.15	0.77	0.24	2.3	600

Table 2.9: Day 4 – bow parameter mean values for Types 1 to 4 Profiles.

	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
Type 1	1	0.40	0.53													2.4	125
Type 2	1	0.29	0.30	0.20	0.70	0.25										2.7	235
Type 3	I	1	0.17	0.29	0.41	0.13	0.16	0.67	0.31							2.8	412
	II	1	0.13	0.33	0.30	0.09	0.25	0.46	0.16	0.14	0.72	0.31				2.8	551
	III	1	0.11	0.33	0.25	0.07	0.28	0.37	0.12	0.22	0.59	0.19	0.10	0.83	0.24	2.7	658
Type 4	I	1	0.40	0.17	0.13	0.18	0.18	0.76	0.19							2.3	330
	II	1	0.31	0.23	0.10	0.10	0.29	0.53	0.11	0.17	0.74	0.24				2.4	468
	III	1	0.28	0.30	0.08	0.09	0.29	0.46	0.09	0.25	0.60	0.14	0.12	0.80	0.21	2.5	568

Table 2.10: Day 4 – bow parameter mean values for Type 5 Profiles.

Type	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
1-2	1	0.15	0.14	0.49	0.46	0.14	0.28	0.68	0.27							2.4	392
2-1	1	0.12	0.12	0.22	0.29	0.09	0.39	0.68	0.22							2.6	516
1-2 $''$	1	0.43	0.13	0.51	0.11	0.13	0.38	0.66	0.14							2.8	398
1-3.I	1	0.09	0.10	0.41	0.31	0.09	0.34	0.44	0.16	0.16	0.71	0.35				2.7	604
1-4.I	1	0.09	0.10	0.22	0.28	0.08	0.56	0.43	0.17	0.20	0.71	0.37				2.3	548
2-2	1	0.11	0.11	0.26	0.28	0.08	0.35	0.57	0.16	0.19	0.76	0.34				2.7	574
3.I-1	1	0.10	0.11	0.26	0.25	0.08	0.13	0.40	0.13	0.37	0.70	0.18				2.2	603
4.I-1																	
1-3.I $''$	1	0.31	0.14	0.86	0.08	0.09	0.33	0.51	0.11	0.17	0.71	0.25				2.7	587
2-2 $''$																	
1-3.II	1	0.10	0.10	0.58	0.29	0.08	0.45	0.42	0.12	0.24	0.68	0.17	0.10	0.87	0.25	2.5	692
1-4.II	1	0.08	0.09	0.26	0.27	0.09	0.55	0.43	0.13	0.33	0.61	0.15	0.14	0.80	0.32	2.3	665
2-3.I	1	0.10	0.10	0.21	0.22	0.07	0.43	0.49	0.11	0.24	0.64	0.17	0.14	0.80	0.31	2.5	647
2-4.I	1	0.10	0.09	0.25	0.24	0.07	0.16	0.46	0.10	0.30	0.66	0.17	0.14	0.80	0.24	2.3	713
3.I-2	1	0.08	0.09	0.38	0.22	0.07	0.21	0.37	0.12	0.35	0.72	0.10	0.16	0.85	0.35	2.4	755
1-3.II $''$	1	0.35	0.11	0.61	0.09	0.10	0.31	0.49	0.07	0.27	0.60	0.12	0.13	0.79	0.22	2.5	604
1-4.II $''$	1	0.66	0.09	0.46	0.15	0.08	0.17	0.45	0.08	0.30	0.77	0.06	0.32	0.85	0.10	2.0	720

Figure 2.16: Day 1 profile distributions.

Figure 2.17: Distribution of Type 5 profiles for Day 1 – bow.

Figure 2.18: Day 2 profile distributions.

Figure 2.19: Distribution of Type 5 profiles for Day 2 – bow.

Figure 2.20: Day 3 profile distributions.

Figure 2.21: Distribution of Type 5 profiles for Day 3 – bow.

Figure 2.22: Day 4 profile distributions.

Figure 2.23: Distribution of Type 5 profiles for Day 4 – bow.

Figure 2.24: Profiles at parameter mean values of Day 1 at the bow.

2.5 Discussion

In the first part of this discussion comments are made on the curve fitting process.

In the second part, the results are discussed.

2.5.1 On the Curve Fitting Process

The peak detection rate varies between 83% and 100% which shows that the chosen equation form fits the observed acceleration pulses fairly well. Inspection of undetected pulses shows that for the majority of these pulses a different set of initial parameters or parameter constraints (which are input to the curve fitting algorithm) can lead to a solution. Other curves, however, have shapes which could not be captured with a sum of Gaussian curves.

The amplitude of the first Gaussian curve in the sum was constrained to be equal to approximately 1 such that its peak coincides (approximately) with the peak of the pulse. The reason why it cannot be set to exactly 1 is that the equation terms cannot be equal to 0 on $]0,1[$. It is possible to obtain a fit in which the underlying curves overlap at the peak time. However, this overlapping is not favourable because the peak time would not appear “naturally” in the equation.

It was decided to work with the amplitudes of the Gaussian curves a_n rather than the corresponding pulse amplitudes h_n . Since h_n are close to but not exactly equal to amplitudes at characteristic points (e.g., inflection points), it was decided to reduce the number of computations by working with a_n instead. However, as it was shown in Section 2.2.2, a_n can be expressed in terms of h_n .

2.5.2 On the Results

The profile distribution for Types 1 through 4 for each of the four days is almost identical which is not surprising since the variations in wave height and period were small during the course of the sea trial (Figure 2.1). Some profiles are not present in the time histories collected at the centre portion of the boat which indicate that the characteristics of the motion at the midships portion of the boat is different than at the bow. This is expected since the motion of the bow is more pronounced than the midships portion of the boat. As shown in Table 2.2, the midships portion experiences much fewer impacts than the bow of the boat.

More variability is observed in the profile distributions of Type 5 profiles. This is expected since only a small portion of pulses are of Type 5. For the majority of the Type 5 profiles, the acceleration peak value occurs during the first pulse. Overall, the most common variation is Type 2-2 and the least common variations are Type 3.I-1 and 4.I-1.

Type 5 profiles were classified differently than the other types because they consist of a combination of profiles. As a consequence, a classification according to the number of underlying Gaussian curves is not appropriate. For instance, in a combination of 4 Gaussian term, the third Gaussian curve could correspond to either the third shoulder of the first pulse or the second highest peak or the first shoulder of the second pulse. Therefore, the distribution of the equation parameters would be multimodal and the mean values of the parameters would not be an adequate description. Subdividing Type 5 variations into the categories shown in the previous section resolves this issue.

When comparing the parameter mean values of each type (1-4) for different days it

can be seen that the values are close to each other. This suggests that the pulse shapes are similar in the analyzed time histories. The duration and maximum accelerations of the pulses differ, however. It can be seen that the accelerations increase with increasing wave height. While pulse durations for each type are within the same range, it seems that there is no correlation between pulse duration, wave height, and period. Types 3.I and 3.II were shown to have consistently the highest accelerations. These profiles are also the most frequent covering more than 60% of detected pulses. The parameter mean values for Type 5 are within the same range when comparing different days. However, once again the small sample size of Type 5 profiles results in values which are more variable than for Types 1 through 4.

2.6 Conclusion

The observed acceleration pulses were classified and characterized. The curve fitting equations and algorithm that were developed are reliable as evidenced by the high detection rate. The different profiles were characterized by their parameter mean values. These values suggest a close similarity between the pulse shapes observed in different sea conditions. In the next chapter, the distributions of the parameters are exploited in order to generate time histories with the same statistical characteristics as the ones from the sea trial data.

Chapter 3

Synthetic Time History Generation

The generation of synthetic acceleration time histories consisting of the profiles seen in Chapter 2 requires knowledge of the distributions of the equation parameters and the dependencies between these parameters. Formally, given an m -by- n matrix $X = [X_1 \ X_2 \dots X_n]$ where m is the number of observations and n is the number of variables, it is required to find an l -by- n matrix Y whose entries are not identical to the entries in X , and such that Y_i has the same distribution as X_i and the inter-dependencies between the Y_i 's are the same as those between the X_i 's. In here, a matrix X is associated with each type of pulse and is obtained by curve fitting the original time histories. The objective behind simulating equation parameters is to extend a given time history or to synthesize a new one. Put differently, the simulation procedure serves as a way to interpolate between the original parameter values.

Two methods are presented in this chapter for simulating dependent random variables. The first method is based on the theory of mathematical functions called copulas. The second method consists of obtaining a non-parameteric estimate of the joint probability density function of X and using it to generate synthetic data. These

two methods were used to generate synthetic pulses.

In order to create transitions between impacts, cam profiles were fitted in between consecutive pulses. The properties of these profiles were adjusted so as to minimize drift in both velocity and displacement. The two methods are presented in Section 3.1. Section 3.2 presents the inter-pulse characteristics. Simulation results are presented in Section 3.3. The results are discussed in Section 3.4. Concluding remarks are made in Section 3.5.

3.1 Synthetic Pulse Generation Methods

The procedure for simulating dependent random variables using copulas is outlined in the first part of this section. The method for estimating the joint probability density of the variables and its application to data simulation is presented in the second part.

3.1.1 Simulating Dependent Random Variables using Copulas

A copula is a function which joins a multivariate distribution function to its one-dimensional marginal distribution functions [14]. This is formally stated by Sklar's Theorem: Let H be a two-dimensional distribution function with marginal distribution functions F_1 and F_2 . Then there exists a copula C such that $H(x_1, x_2) = C(F_1(x_1), F_2(x_2))$. Conversely, for any distribution functions F_1 and F_2 and any copula C , the function H defined above is a two-dimensional distribution function with marginals F_1 and F_2 . Furthermore, if F_1 and F_2 are continuous, C is

unique.

The terms F_1 , F_2 , and H denote cumulative probability distributions, e.g., $F(x_1) = P(X_1 \leq x_1)$. The copula C is the joint distribution function for the random variables $U_1 = F_1(X_1)$ and $U_2 = F_2(X_2)$. Note that $F(X_1)$ and $G(X_2)$ are uniformly distributed on $[0,1]$. In essence, a copula contains the dependence structure information of a data set $X = [X_1 \ X_2 \dots X_n]$. Knowing the copula of a data set X and the individual marginals F_i , it is then possible to simulate a similar data set Y in a two step procedure [15]. Random variables $U = [U_1 \ U_2 \dots U_n]$ are first drawn from the distribution C . A transformation is applied to each U_i in order to obtain simulated variables Y_i . From probability theory, if X_1 is a continuous random variable with distribution function F_1 , then the random variable $U_1 = F_1(X_1)$ has a uniform distribution on $[0,1]$. Conversely, applying the inverse transformation of a distribution F_1 to a uniformly distributed random variable U_1 results in a random variable with the distribution F_1 . Therefore $Y_i = F_i^{-1}(U_i)$.

While the simulation procedure is straightforward, it remains to find C and all the F_i . Constructing the empirical copula of a given data set is possible [16]. However, it was found that the Gaussian copula is an adequate model for the dependence structure of the observed data. A Gaussian copula is parameterized by a correlation matrix. A suitable parameteric model (e.g., normal, beta, lognormal) for the individual variables X_i was not found. For this reason, the empirical distribution of each variable was used instead. The complete simulation procedure for each type of pulse can be summarized by the following steps:

1. Compute the correlation matrix of X ;

2. Draw random numbers U from the Gaussian copula with correlation matrix computed from Step 1;
3. Determine the empirical distribution of each variable X_i ; and
4. Invert each U_i to obtain the simulated variables Y .

This procedure is repeated for each type of pulse. The matrix X is obtained from the curve fitting process described in Chapter 2. The simulated parameters in Y are then plugged into the representative equations in order to obtain synthetic pulses.

3.1.2 Simulating Dependent Random Variables by Non-parametric Density Estimation

The probability density function f of a random vector X_i is a function that describes the distribution of X_i and allows for calculating probabilities associated with R :

$$P(a < X_i < b) = \int_a^b f(x)dx \text{ for all } a < b \quad (3.1)$$

Density estimation is the construction of an estimate of f from observed data [17]. The most basic estimator is the histogram. For $X = [X_1 \ X_2 \dots X_n]$ with m observations, the variable's space is subdivided into hyper-rectangles with size $h_1 \times h_2 \times \dots \times h_n$ (Figure 3.1). The joint probability density function at a point with coordinates $\mathbf{x}$ in the hyper-rectangle B_k , is then estimated as [18]:

$$\hat{f}_{Hist}(\mathbf{x}) = \frac{\# \text{ of data points in } B_k}{mh_1h_2\dots h_n}; \ \mathbf{x} \text{ in } B_k \quad (3.2)$$

The probability that a point falls into B_k is then:

$$P(\mathbf{x} \in B_k) = \frac{\# \text{ of data points in } B_k}{m} \quad (3.3)$$

When the data is normally distributed, the optimal bin size is given by Scott's normal reference rule [18]:

$$h_i \approx 3.5\sigma_i m^{\frac{-1}{2+n}}; \quad i = 1, \dots, n \quad (3.4)$$

where σ_i is the standard deviation of the i^{th} variable, m is the number of observations, and n is the number of variables.

However, this rule yields large h_i for skewed data, which is the case with the parameters at hand. Therefore, the normal reference rule was used as a starting point for determining bin size. Fine tuning of h_i was performed iteratively and by visual inspection. The following estimate was ultimately used:

$$h_i = \frac{\max(X_i) - \min(X_i)}{15}; \quad i = 1, \dots, n \quad (3.5)$$

The probability associated with each hyper-rectangle is then input into a random number generator to produce a synthetic data set Y . Synthetic pulses are then produced by inputting the parameters in Y into the representative equations as described in the previous section.

Figure 3.1: Illustration of space discretization for two variables.

3.2 Inter-Pulse Characteristics

Synthetic acceleration pulses were generated using the methods described in Sections 3.1.1 and 3.1.2. Cam profiles were fitted in between consecutive pulses:

$$y_{cam}(x) = \begin{cases} k_1 \sin\left(\frac{\pi x}{2t_1}\right) & 0 \leq x < t_1, \\ k_1 & t_1 \leq x < t_1 + t_2, \\ (1 - k_2) \left(\sin\left(\frac{\pi(x + \frac{t_3}{2} - t_1 - t_2)}{t_3}\right) - 1 \right) - k_1 & t_1 + t_2 \leq x < t_1 + t_2 + t_3, \\ \cos\left(\frac{\pi(x - t_1 - t_2 - t_3)}{2t_4}\right) & t_1 + t_2 + t_3 \leq x \leq t_1 + t_2 + t_3 + t_4. \end{cases} \quad (3.6)$$

where k_1 is the amplitude of the first portion of the profile, $k_2 + k_1 = -1$ which is the total amplitude, and $t_1 + t_2 + t_3 + t_4$ is the total duration.

Figure 3.2 shows an example of a motion cycle. The parameters of the cam profiles were adjusted at each cycle such as to ensure continuous jerk and to correct drift in both velocity and displacement. To ensure compatibility of slopes between the ends of a cam profile and its two adjoining pulses, parameters t_1 and t_4 were written as

follows:

$$\begin{aligned} \frac{d \left(k_1 \sin \left(\frac{\pi x}{2t_1} \right) \right)}{dx} &= s_1 \text{ for } x = 0, \text{ and } \frac{d \left(\cos \left(\frac{\pi(x-t_1-t_2-t_3)}{2t_4} \right) \right)}{dx} = s_2 \text{ for } x = t_1 + t_2 + t_3 + t_4 \\ \Rightarrow t_1 &= \frac{k_1 \pi}{2s_1}, \text{ and } t_4 = -\frac{\pi}{2s_2} \end{aligned} \quad (3.7)$$

where s_1 is the jerk at the end of the impact portion of a cycle and s_2 is the jerk at the beginning of the following cycle.

It was assumed that following the impact part of the cycle, the displacement should return as close as possible to its value at the beginning of the cycle. Therefore, the target velocity at the beginning of any cycle should satisfy:

$$\begin{aligned} \int_{\text{impact}} (v + c) dt &= \Delta d + cT = 0 \\ \Rightarrow c &= -\frac{\Delta d}{T} \end{aligned} \quad (3.8)$$

where v is the velocity, d is the displacement, T is the duration of the impact portion of the cycle, and c is the corrective offset that needs to be added to v at the impact portion of a cycle.

The transition part of a cycle should return the displacement as close as possible to its baseline which was arbitrarily chosen as zero. An additional constraint was imposed that the displacement should be near the baseline at t_1+t_2 , which is before the boat, or portion of it, starts to free fall. This constraint is required in order to pull up the displacement if it starts to drift downwards (Figure 3.3). In summary,

the parameters of a cam profile, k_1 , k_2 , and t_2 , should minimize:

$$\delta_1 = d_{end\ of\ cycle} \quad (3.9)$$

$$\delta_2 = v_{end\ of\ cycle} - v_{target} \quad (3.10)$$

$$\delta_3 = d_{t_1+t_2} \quad (3.11)$$

The acceleration pulses at hand correspond to wave heights of up to 2 metres (Chapter 2). Based on this wave height, a threshold of ± 0.5 metres was chosen for δ_1 and δ_3 , and a threshold of 0.5 m/s was chosen for δ_2 . If at any given cycle, the solution for the parameters of y_{cam} results in δ higher than the prescribed limits, then a different cam profile is used:

$$y_{cam2}(x) = \begin{cases} k \sin\left(\frac{\pi x}{2t_1}\right) & 0 \leq x < t_1, \\ k & t_1 \leq x < t_1 + t_2, \\ k \cos\left(\frac{\pi(x-t_1-t_2)}{2t_3}\right) & t_1 + t_2 \leq x \leq t_1 + t_2 + t_3. \end{cases} \quad (3.12)$$

where k is the amplitude and $t_1+t_2+t_3$ is the total duration of the profile.

Similarly to the first profile, the parameters t_1 and t_2 are expressed in terms of s_1 and s_2 :

$$t_1 = \frac{k\pi}{2s_1}, \text{ and } t_2 = -\frac{k\pi}{2s_2} \quad (3.13)$$

In some cases, the required change in velocity is lower than what can be achieved with y_{cam} . In other cases, a larger downwards acceleration is required. For these reasons, the amplitude k for y_{cam2} was not constrained as opposed to the other profile which has a fixed amplitude of -1. The two unknowns of y_{cam2} , which are

k and t_2 , are solved for so as to minimize δ_1 and δ_2 . In the case where suitable parameters are not found, then the errors are corrected at the next cycle by the first cam profile, as shown in Figure 3.3.

The representative equation of each type was integrated analytically using the Symbolic Toolbox in Matlab. The output expression for Type 1 can be found in Appendix E. A non-linear equation solver (fsolve in Matlab) was used for each cycle to determine the parameters of the cam profiles.

Figure 3.2: Example of a motion cycle.

Figure 3.3: Example of drift correction.

3.3 Results

A synthetic time history consisting of 1785 pulses was generated using the pulse characteristics of the Day 2-forward position time history (Figure 3.4). Type 2 pulses were chosen to demonstrate the pulse simulation methods. Figures 3.5 and 3.6 show the relationships between the parameters of the Type 2 profile as compared with the simulated parameters by both the copula and density estimation methods. Further, the individual parameter distributions of the original parameters and the simulated ones are shown in Figures 3.7 through 3.9. The generated pulses were fed back into the curve fitting algorithm. The detection rate was found to be 95%. The parameter mean values are compared in Tables 3.1 through 3.4. Finally, Table 3.5 shows a comparison between the number of types prescribed by the random number generator and the number of detected pulses by the curve fitting algorithm.

Figure 3.4: Synthetic time history.

Figure 3.5: Matrix plot of the original parameters for Type 2 (in blue) and the simulated parameters using copulas (in green).

Figure 3.6: Matrix plot of the original parameters for Type 2 (in blue) and simulated parameters using density estimation (in green).

Figure 3.7: Distribution of the original parameters for Type 2.

Figure 3.8: Distribution of the simulated parameters using copulas for Type 2.

Figure 3.9: Distribution of the simulated parameters using density estimation for Type 2.

Table 3.1: Parameter mean values comparison for Types 1 through 4 – copula method. The values of the generated data are below the actual ones.

		a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
Type 1		1	0.42	0.50													2.5	132
		1	0.42	0.49													2.6	137
Type 2		1	0.29	0.30	0.19	0.70	0.26										2.8	231
		1	0.29	0.30	0.19	0.71	0.27										2.8	232
Type 3	I	1	0.17	0.17	0.27	0.41	0.13	0.15	0.66	0.30							2.9	413
		1	0.18	0.17	0.28	0.41	0.12	0.15	0.66	0.29							2.9	411
	II	1	0.12	0.13	0.30	0.29	0.09	0.23	0.46	0.16	0.13	0.72	0.30				2.9	561
		1	0.12	0.13	0.31	0.29	0.09	0.23	0.46	0.16	0.13	0.72	0.30				2.9	561
	III	1	0.11	0.11	0.32	0.24	0.07	0.28	0.37	0.12	0.19	0.57	0.18	0.10	0.83	0.23	2.8	658
		1	0.11	0.11	0.31	0.25	0.07	0.28	0.38	0.12	0.20	0.58	0.18	0.10	0.83	0.25	2.8	644
Type 4	I	1	0.41	0.23	0.27	0.13	0.15	0.21	0.75	0.18							2.5	347
		1	0.42	0.23	0.31	0.12	0.14	0.19	0.76	0.18							2.6	362
	II	1	0.33	0.14	0.25	0.09	0.10	0.29	0.52	0.11	0.17	0.73	0.22				2.5	490
		1	0.32	0.15	0.26	0.09	0.10	0.29	0.53	0.11	0.16	0.73	0.22				2.5	479
	III	1	0.30	0.12	0.24	0.08	0.10	0.30	0.45	0.08	0.25	0.57	0.13	0.13	0.77	0.24	2.6	579
		1	0.30	0.13	0.23	0.08	0.09	0.31	0.45	0.08	0.26	0.58	0.13	0.11	0.79	0.25	2.6	563

Table 3.2: Parameter mean values comparison for Types 1 through 4 – density estimation method.
The values of the generated data are below the actual ones.

		a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
Type 1		1	0.42	0.50													2.5	132
		1	0.41	0.48													2.5	136
Type 2		1	0.29	0.30	0.19	0.70	0.26										2.8	231
		1	0.29	0.30	0.18	0.71	0.26										2.8	232
Type 3	I	1	0.17	0.17	0.27	0.41	0.13	0.15	0.66	0.30							2.9	413
		1	0.17	0.18	0.27	0.42	0.13	0.15	0.67	0.30							2.9	408
	II	1	0.12	0.13	0.30	0.29	0.09	0.23	0.46	0.16	0.13	0.72	0.30				2.9	561
		1	0.12	0.13	0.31	0.30	0.10	0.23	0.46	0.16	0.13	0.72	0.30				2.9	558
	III	1	0.11	0.11	0.32	0.24	0.07	0.28	0.37	0.12	0.19	0.57	0.18	0.10	0.83	0.23	2.8	658
		1	0.11	0.11	0.32	0.24	0.07	0.30	0.37	0.12	0.20	0.56	0.18	0.10	0.82	0.22	2.9	644
Type 4	I	1	0.41	0.23	0.27	0.13	0.15	0.21	0.75	0.18							2.5	347
		1	0.41	0.24	0.30	0.13	0.16	0.22	0.76	0.18							2.4	341
	II	1	0.33	0.14	0.25	0.09	0.10	0.29	0.52	0.11	0.17	0.73	0.22				2.5	490
		1	0.33	0.15	0.24	0.09	0.11	0.28	0.52	0.10	0.16	0.74	0.23				2.5	475
	III	1	0.30	0.12	0.24	0.08	0.10	0.30	0.45	0.08	0.25	0.57	0.13	0.13	0.77	0.24	2.6	579
		1	0.32	0.12	0.25	0.08	0.10	0.30	0.45	0.08	0.26	0.58	0.12	0.11	0.78	0.23	2.6	582

Table 3.3: Parameter mean values comparison for Type 5 profiles – copula method. The values of the generated data are below the actual ones.

Type	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	T(ms)
1-2	1	0.12	0.13	0.56	0.44	0.15	0.27	0.67	0.39							2.5	459
	1	0.11	0.11	0.59	0.42	0.19	0.17	0.73	0.52							2.6	501
2-1	1	0.13	0.14	0.15	0.31	0.10	0.42	0.66	0.20							2.4	460
	1	0.13	0.15	0.16	0.31	0.11	0.44	0.67	0.19							2.4	468
1-2 ^r																	
1-3.I	1	0.11	0.11	0.54	0.32	0.11	0.47	0.46	0.16	0.15	0.73	0.31				2.5	539
	1	0.12	0.13	0.47	0.35	0.11	0.41	0.48	0.17	0.13	0.77	0.25				2.6	510
1-4.I	1	0.11	0.12	0.22	0.33	0.11	0.47	0.50	0.13	0.25	0.70	0.27				2.6	521
	1	0.10	0.10	0.22	0.33	0.14	0.36	0.50	0.12	0.26	0.66	0.34				2.3	549
2-2	1	0.11	0.11	0.23	0.27	0.08	0.30	0.59	0.12	0.26	0.72	0.28				2.8	603
	1	0.11	0.11	0.23	0.28	0.08	0.33	0.58	0.12	0.24	0.73	0.28				2.9	600
3.I-1	1	0.09	0.09	0.26	0.22	0.08	0.08	0.45	0.23	0.39	0.84	0.13				3.5	666
	1	0.09	0.09	0.26	0.22	0.08	0.08	0.45	0.23	0.39	0.84	0.13				3.5	666
4.I-1																	
1-3.I ^r																	
2-2 ^r																	
1-3.II	1	0.10	0.11	0.58	0.40	0.14	0.21	0.56	0.08	0.21	0.71	0.14	0.07	0.87	0.23	3.0	585
	1	0.10	0.11	0.58	0.37	0.14	0.21	0.56	0.08	0.21	0.71	0.14	0.07	0.87	0.23	3.0	585
1-4.II	1	0.11	0.13	0.23	0.34	0.13	0.22	0.45	0.07	0.38	0.54	0.13	0.20	0.72	0.28	2.2	648
	1	0.11	0.13	0.17	0.29	0.12	0.31	0.38	0.07	0.45	0.48	0.15	0.23	0.71	0.32	2.1	664
2-3.I	1	0.09	0.10	0.26	0.24	0.07	0.39	0.46	0.09	0.35	0.60	0.15	0.14	0.82	0.20	2.4	620
	1	0.09	0.10	0.15	0.26	0.07	0.51	0.46	0.10	0.38	0.63	0.10	0.15	0.83	0.20	2.3	576
2-4.I	1	0.12	0.10	0.22	0.22	0.08	0.22	0.55	0.17	0.30	0.70	0.10	0.14	0.86	0.13	2.6	640
	1	0.10	0.10	0.20	0.22	0.08	0.30	0.57	0.24	0.29	0.73	0.11	0.11	0.86	0.11	2.7	715
3.I-2	1	0.10	0.11	0.27	0.26	0.08	0.18	0.40	0.14	0.31	0.73	0.16	0.21	0.86	0.18	2.5	670
	1	0.09	0.10	0.28	0.27	0.09	0.16	0.43	0.12	0.32	0.72	0.17	0.19	0.85	0.17	2.7	670
1-3.II ^r	1	0.35	0.11	0.68	0.09	0.10	0.30	0.50	0.09	0.25	0.63	0.11	0.14	0.79	0.19	2.4	562
	1	0.36	0.11	0.68	0.09	0.10	0.24	0.50	0.08	0.26	0.63	0.11	0.15	0.79	0.19	2.5	538
1-4.II ^r	1	0.66	0.11	0.91	0.10	0.10	0.42	0.39	0.09	0.04	0.76	0.06	0.19	0.88	0.14	2.4	458
	1	0.66	0.11	0.91	0.10	0.10	0.42	0.39	0.09	0.04	0.76	0.06	0.19	0.88	0.14	2.4	458

Table 3.4: Parameter mean values comparison for Type 5 profiles – density estimation method.
The values of the generated data are below the actual ones.

Type	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	T(ms)
1-2	1	0.12	0.13	0.56	0.44	0.15	0.27	0.67	0.39							2.5	459
	1	0.12	0.13	0.43	0.39	0.13	0.24	0.62	0.36							2.6	456
2-1	1	0.13	0.14	0.15	0.31	0.10	0.42	0.66	0.20							2.4	460
	1	0.12	0.13	0.18	0.31	0.10	0.45	0.63	0.22							2.4	436
1-2 ^r																	
1-3.I	1	0.11	0.11	0.54	0.32	0.11	0.47	0.46	0.16	0.15	0.73	0.31				2.5	539
	1	0.11	0.11	0.56	0.31	0.10	0.49	0.47	0.16	0.13	0.73	0.33				2.5	525
1-4.I	1	0.11	0.12	0.22	0.33	0.11	0.47	0.50	0.13	0.25	0.70	0.27				2.6	521
	1	0.10	0.10	0.22	0.33	0.14	0.36	0.50	0.12	0.26	0.66	0.34				2.3	549
2-2	1	0.11	0.11	0.23	0.27	0.08	0.30	0.59	0.12	0.26	0.72	0.28				2.8	603
	1	0.11	0.11	0.23	0.27	0.09	0.32	0.59	0.14	0.25	0.71	0.24				2.8	610
3.I-1	1	0.09	0.09	0.26	0.22	0.08	0.08	0.45	0.23	0.39	0.84	0.13				3.5	666
	1	0.09	0.09	0.26	0.22	0.08	0.08	0.45	0.23	0.39	0.84	0.13				3.5	666
4.I-1																	
1-3.I ^r																	
2-2 ^r																	
1-3.II	1	0.10	0.11	0.58	0.40	0.14	0.21	0.56	0.08	0.21	0.71	0.14	0.07	0.87	0.23	3.0	585
	1	0.10	0.11	0.58	0.37	0.14	0.21	0.56	0.08	0.21	0.71	0.14	0.07	0.87	0.23	3.0	585
1-4.II	1	0.11	0.13	0.23	0.34	0.13	0.22	0.45	0.07	0.38	0.54	0.13	0.20	0.72	0.28	2.2	648
	1	0.12	0.13	0.30	0.39	0.15	0.13	0.52	0.08	0.31	0.60	0.12	0.17	0.74	0.24	2.3	631
2-3.I	1	0.09	0.10	0.26	0.24	0.07	0.39	0.46	0.09	0.35	0.60	0.15	0.14	0.82	0.20	2.4	620
	1	0.09	0.09	0.18	0.24	0.06	0.40	0.47	0.10	0.34	0.63	0.13	0.13	0.84	0.22	2.4	609
2-4.I	1	0.12	0.10	0.22	0.22	0.08	0.22	0.55	0.17	0.30	0.70	0.10	0.14	0.86	0.13	2.6	640
	1	0.14	0.11	0.20	0.20	0.08	0.23	0.52	0.13	0.32	0.69	0.11	0.14	0.87	0.17	2.7	544
3.I-2	1	0.10	0.11	0.27	0.26	0.08	0.18	0.40	0.14	0.31	0.73	0.16	0.21	0.86	0.18	2.5	670
	1	0.10	0.11	0.26	0.25	0.08	0.12	0.40	0.14	0.29	0.72	0.15	0.19	0.85	0.13	2.7	700
1-3.II ^r	1	0.35	0.11	0.68	0.09	0.10	0.30	0.50	0.09	0.25	0.63	0.11	0.14	0.79	0.19	2.4	562
	1	0.36	0.11	0.76	0.09	0.10	0.27	0.52	0.09	0.21	0.66	0.11	0.12	0.84	0.17	2.4	526
1-4.II ^r	1	0.66	0.11	0.91	0.10	0.10	0.42	0.39	0.09	0.04	0.76	0.06	0.19	0.88	0.14	2.4	458
	1	0.66	0.11	0.91	0.10	0.10	0.42	0.39	0.09	0.04	0.76	0.06	0.19	0.88	0.14	2.4	458

Table 3.5: Prescribed vs detected profiles.

Type	Prescribed	Detected	
		Copula Method	Density Estimation Method
1	22	22	22
2	227	212	213
3.I	566	561	547
3.II	644	606	641
3.III	84	85	80
4.I	29	25	27
4.II	68	47	58
4.III	58	58	63
1-2	4	4	5
2-1	5	6	5
1-2 ^r	0	3	2
1-3.I	10	12	13
1-4.I	0	0	0
2-2	31	29	30
3.I-1	0	0	0
4.I-1	0	0	0
1-3.I ^r	0	2	0
2-2 ^r	0	1	0
1-3.II	2	2	4
1-4.II	3	3	3
2-3.I	8	12	9
2-4.I	6	6	8
3.I-2	10	9	5
1-3.II ^r	5	3	5
1-4.II ^r	2	2	2

3.4 Discussion

The different aspects of the synthetic pulses are discussed in the first part of this section. The inter-pulse characteristics are discussed in the second part.

3.4.1 On Synthetic Pulses

The produced synthetic pulses have the same dependence structure as shown in Figures 3.5 and 3.6. This shows that both the method using copulas and the density estimation method were appropriate. The advantage of using copulas is the fact that this method has the potential to be fully parametric. The dependence between parameters is contained in the Gaussian copula, which is parameterized by the correlation matrix of the data. If an appropriate model can be found for the individual parameter distributions, then the method using copulas would be fully parametric.

The individual parameter distributions have been captured as well by both methods, as shown in Figures 3.8 and 3.9. The parameter mean values of the simulated data are in good agreement with the mean values of the actual parameters, as shown in Tables 3.1 through 3.4.

The synthetic pulses were fed back to the curve fitting algorithm and a detection rate of 95% was found. In curve fitting the synthetic pulses, a lower threshold was set for the residual defining an adequate fit (average residual threshold of 0.001). This is because the synthetic pulses are expected to match closely the fitted curves. If a less strict threshold is used, the detection rate increases. However, a misinterpretation of the different types could occur (e.g., Type 3.I detected as Type 2 due to a high threshold). In Table 3.5, it can be seen that the number of prescribed pulses of each

type matches well the number of detected pulses of each type.

3.4.2 On Inter-Pulse Characteristics

The produced acceleration time history exhibits no significant drift in either velocity nor displacement as shown in Figure 3.4. The displacement is within the range of ± 1 metre for the most part, which is consistent with a wave height of 2 metres. Downward accelerations are lower than -1 g in a small number of cases. This occurs when a reasonable solution (i.e. large variation in displacement or significant drift) cannot be obtained with the cam profile y_{cam} , which was described in Section 3.2. The boat experiences mainly the pull of gravity. However, a larger downwards force is possible as a result of combined rotational and translational motion. This occurs when the forward portion of the boat pitches and heaves about a rotation point close to the centre of the boat, right before impact.

3.5 Conclusion

Data simulation procedures were employed to generate HSC acceleration time histories. It was shown that the characteristics of the simulated data are reasonably close to those of the actual data. Simulated time histories are useful for extending sea trial time histories by using their statistical characteristics and to determine the risk of injury of human occupants by computing measures such as vibration dose.

Chapter 4

Conclusion

The analysis of RHIB acceleration data yielded a five-type classification of impact acceleration pulses. It was found that the different profiles can be characterized by a combination of modified Gaussian curves. Equation parameters were obtained from curve fitting the data. The mean values of these parameters were provided and can be used to obtain standard profiles for testing RHIB seats. The developed profile detection algorithm is also intended as a general tool for the analysis of HSC acceleration data. In addition to pulse characterization, two methods for synthesizing acceleration time histories with similar characteristics to the original data were presented. Both methods were tested and shown to be reliable. Synthetic time histories can be used for numerical simulation of HSC seat response. Vibration and shock measures can then be computed such as vibration dose when sea trial data is not available or when it is required to extend short time histories. The main contributions of this work are:

1. Mathematical characterization of HSC acceleration pulses;

2. Classification of acceleration pulses into different profiles;
3. Development of an algorithm for the analysis of acceleration data; and
4. Development of an algorithm for the synthesis of acceleration time histories with similar pulse shape characteristics to actual data.

To the best of the author's knowledge, the contributions of this work are novel.

Using the work presented in this thesis, several follow-on projects can be conducted in the spirit of HSC shock mitigation:

1. An assessment of the severity of the different profiles can be made by subjecting HSC seats to these profiles. A test rig is, in fact, currently being built at Carleton University for this purpose.
2. Relationships between pulse shape characteristics and boat/sea parameters can be investigated. This can be achieved by applying the developed curve fitting algorithm to acceleration data of different types of HSC and in different sea conditions. Establishing such correlations could yield a useful tool for boat and seat design.

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Appendix A

Reduction of Additional Data Set

An additional data set was provided to the ADL by DRDC. The sensors used here were:

- 10 Silicon Designs 2460 triaxial accelerometers, using a sampling rate of 4000 Hz; and
- 1 Microstrain 3DM-GX3-25 Attitude Heading Reference System (RS232/USB #6223-4220) Gyroscope of unknown sampling rate. The accelerometer data was down sampled to 100 Hz and the gyroscope data was interpolated in order to be aligned with the accelerometer data. Sea trials were performed over a number of days. The results shown below are for the sea trial performed on October 24th, 2012 and for the acceleration at the forward deck of the boat.

The profile distribution seen in this data is not identical to the data set presented in the thesis. The main differences are: a smaller number of Type 3.II and 3.III profiles, a larger number of Type 1 and 4.I profiles. While the shape parameters of the pulses seen in both data sets are within the same range, significant differences can be observed between the peak accelerations and pulse durations of each set.

Figure A.1: Profile distribution – forward position.

Figure A.2: Distribution of Type 5 profiles – forward position.

Table A.1: Forward position – parameter mean values for Types 1 to 4 profiles.

		a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
Type 1		1	0.46	0.50													3.0	116
Type 2		1	0.31	0.28	0.20	0.68	0.27										3.2	203
Type 3	I	1	0.22	0.19	0.26	0.51	0.15	0.18	0.75	0.28							3.2	288
	II	1	0.20	0.17	0.24	0.40	0.11	0.31	0.56	0.15	0.14	0.82	0.25				3.0	334
	III	1	0.15	0.14	0.28	0.34	0.09	0.32	0.49	0.12	0.21	0.70	0.13	0.08	0.91	0.14	3.0	429
Type 4	I	1	0.40	0.22	0.19	0.14	0.14	0.20	0.76	0.20							2.9	254
	II	1	0.36	0.18	0.19	0.11	0.12	0.25	0.52	0.12	0.18	0.80	0.23				2.8	328
	III	1	0.36	0.15	0.21	0.10	0.13	0.20	0.53	0.09	0.23	0.67	0.12	0.13	0.86	0.17	3.1	368

Table A.2: Forward position – parameter mean values for Type 5 profiles.

Type	a_1	x_1	σ_1	a_2	x_2	σ_2	a_3	x_3	σ_3	a_4	x_4	σ_4	a_5	x_5	σ_5	$a(g's)$	$T(ms)$
1-2	1	0.16	0.14	0.61	0.50	0.16	0.22	0.78	0.24							3.0	330
2-1	1	0.17	0.18	0.17	0.40	0.12	0.45	0.81	0.19							2.5	320
1-2''	1	0.53	0.16	0.60	0.16	0.16	0.26	0.85	0.16							2.6	303
1-3.I	1	0.19	0.16	0.51	0.48	0.12	0.40	0.63	0.14	0.19	0.88	0.18				2.5	344
1-4.I	1	0.16	0.15	0.18	0.43	0.11	0.49	0.58	0.14	0.26	0.80	0.24				2.6	349
2-2	1	0.16	0.15	0.11	0.31	0.08	0.42	0.61	0.13	0.20	0.81	0.25				2.9	347
3.I-1	1	0.17	0.14	0.26	0.37	0.09	0.25	0.53	0.11	0.31	0.86	0.14				2.6	401
4.I-1	1	0.30	0.12	0.15	0.09	0.09	0.14	0.45	0.10	0.41	0.77	0.15				2.6	349
1-3.I''	1	0.48	0.14	0.74	0.14	0.15	0.20	0.66	0.09	0.19	0.81	0.17				2.5	349
2-2''	1	0.69	0.13	0.59	0.17	0.16	0.21	0.33	0.09	0.37	0.80	0.12				2.8	373
1-3.II																	
1-4.II	1	0.18	0.11	0.32	0.35	0.11	0.84	0.48	0.12	0.16	0.78	0.08	0.20	0.88	0.12	3.1	347
2-3.I	1	0.23	0.09	0.11	0.07	0.07	0.44	0.47	0.09	0.25	0.66	0.22	0.09	0.87	0.13	3.3	435
2-4.I	1	0.23	0.11	0.23	0.20	0.11	0.16	0.60	0.13	0.41	0.73	0.11	0.12	0.89	0.31	2.5	431
3.I-2	1	0.14	0.11	0.23	0.32	0.08	0.26	0.47	0.16	0.33	0.77	0.09	0.20	0.9	0.09	2.8	515
1-3.II''	1	0.43	0.13	0.70	0.13	0.12	0.15	0.58	0.07	0.29	0.67	0.11	0.22	0.83	0.17	2.4	362
1-4.II''	1	0.63	0.14	0.32	0.13	0.18	0.15	0.48	0.08	0.16	0.77	0.08	0.22	0.87	0.12	2.4	298

Appendix B

Data Pre-processing Code

```
function [removedgravityaz] = Removegravity(data)

%-----
% Data should be in the format [thetax thetay thetaz az];
%-----

Al = data;
n = length(data);
gl = zeros(3,n);

% get gravity vector components in the local frame at each data ...
% point
parfor i = 1:n

    gl(:,i) = [-sin(Al(i,1))*sin(Al(i,3))+cos(Al(i,3))*sin(Al(i,2)...
               )*cos(Al(i,1));
```

```

        -cos (Al (i,3)) *sin (Al (i,1)) -cos (Al (i,1)) *sin (Al (i,3) ...
        ) *sin (Al (i,2));
        -cos (Al (i,2)) *cos (Al (i,1))]];

end

% add components to acceleration
for i=1:n
    Al (i,4) = Al (i,4) +gl (3,i);

end

removedgravityaz = Al (:,4);

end

function [peak] = Extractpeaks (time,acceleration,treshold)

%-----
% Pulses are extracted from the data (time,acceleration) based
% on the specified treshold. The pulses are stored in the
% structure peak. peak contains 5 fields:
% peak.az(n).values(:) = acceleration values of pulse n
% peak.time(n).values(:) = time values of pulse n
% peak.aMax(n) = maximum acceleration of pulse n
% peak.duration(n) = duration of pulse n
% peak.durationtomax(n) = peak time of pulse n
%-----

az = acceleration;
n = length(acceleration)-1000;

```

```

t = time;
maximum = treshold;

aimpact= struct('values', []);
timeimpact = struct('values', []);

b = 1;
f = 1;
i = 1000;
j = 0;

while i <= n
    % look for values larger than the treshold
    if(az(i))>maximum
        j = j+1;
    % once a value is found go backwards and forward to get the 0
    % intercepts
    while az(i-b)>=0
        b = b+1;
    end
    while az(i+f)>=0
        f = f+1;
    end

    l = b+f+1;

    % store values in aimpact and timeimpact
    for k = 1:l
        aimpact(j).values(k) = az(i-b-1+k);
    end
end

```

```

        timeimpact(j).values(k) = t(i-b-1+k);
end

% interpolate so that the pulse passes by exactly (0,0) and ...
    (1,0)
timeimpact(j).values(1) = interp1([aimpact(j).values(1)...
    aimpact(j).values(2)], [timeimpact(j).values(1)...
    timeimpact(j).values(2)], 0);

aimpact(j).values(1)=0;

timeimpact(j).values(1) = interp1([aimpact(j).values(1-1)...
    aimpact(j).values(1)], [timeimpact(j).values(1-1)...
    timeimpact(j).values(1)], 0);

aimpact(j).values(1)=0;

% get the max, duration and peak time of pulse
[aMax(j), index] = max(aimpact(j).values(1:1));
duration(j) = timeimpact(j).values(1)-timeimpact(j).values...
    (1);
durationtomax(j) = timeimpact(j).values(index)-timeimpact(j)...
    .values(1);
i = i + f;
f = 1;
b = 1;
else
    i = i+1;

```

```
end

end

% store everything in the structure peak
peak = struct('az',aimpact,'time',timeimpact,'aMax',aMax,...
'duration',duration,'durationtomax',durationtomax);

end
```


Appendix C

Curve Fitting Algorithm

```
function [parameters] = Curvefitdata(peak,lim)
% matlabpool('open',4); use this if you have parallel
%computing
%-----
%Curve fits the pulses in structure peak based on the threshold
%averageresidual specified by lim => maximum residual =
%(lim*number of data points in peak). For the provided data
%lim = 0.0001 was used. For the generated data lim = 10^-6
%was used. The equation parameters are output in the structure
%parameters which is formatted as follows:
% equation 1 fit => parameters.eq1(n).values(:) = [sig1 x1...
%(residual) (pulse number in the peak strcuture)]
%equation 2 fit => parameters.eq2(n).values(:) = [a2 x2...
%sig1 sig2 x1 1-a1 (residual) (pulse number in the peak strcuture)]
%equation 3 fit => parameters.eq3(n).values(:) = [a2 x2 a3...
%x3 sig1 sig2 sig3 x1 1-a1 (residual) (pulse number in the peak
```

```

%strcuture)] and so on ...
% parameters.detected = % detected
% parameters.und(n) = pulse number in the peak structure of
%the nth undetected pulse
%-----

% setting up options for the curve fitting function
options = optimoptions('lsqcurvefit');
options.Display = 'off';
options.FinDiffType = 'central';

n = length(peak.az);
und = 0;
ku = 0;

k1 = 0; %y1 = zeros(2,2);
k2 = 0; %y2 = zeros(7,2);
k3 = 0; %y3 = zeros(11,2);
k4 = 0; %y4 = zeros(15,2);
k5 = 0; %y5 = zeros(19,2);

tmpeq1 = struct('values', []);
tmpeq2 = struct('values', []);
tmpeq3 = struct('values', []);
tmpeq4 = struct('values', []);
tmpeq5 = struct('values', []);

% go through each pulse, try equation 1 if the residual is too high ...
    try

```

```

% equation 2 and so on ...
% the fitting functions syntax is as follows:
% [parameters,residual] = Fiteqn(time,acceleration values, options, ...
    parameters starting points, parameters lower bound, parameters ...
    upper bound)
%for types 2 to 5 try two sets of starting guesses and bounds

parfor i = 1:n
    l = length(peak.time(i).values(:));
    xdata = (peak.time(i).values(1:l)-peak.time(i).values(1))./...
        peak.duration(i);
    adata = peak.az(i).values(1:l)/peak.aMax(i);
    m = peak.durationtomax(i)/peak.duration(i);
    L = lim*(length(peak.az(i).values)-2);
    [y1,Z1] = Fiteq1(xdata,adata,options,[0.1 m],[0 m-0.1; 1 m+0.1]);
    if Z1<=L
        tmpeq1(i).values(1:4) = [y1,Z1,i];
    else
        [y2,Z2] = Fiteq2(xdata,adata,options,[0.2 2*m 0.1 0.1 m 0],[0 0 0 0 ...
            m-0.1 0; 1 1 1 1 m+0.1 0.001]);
        if Z2<=L
            tmpeq2(i).values(1:9) = [y2,Z2,i];
        else
            [y2,Z2] = Fiteq2(xdata,adata,options,[0.2 2*m 0.1 0.1 m 0],[0 0 0 0 ...
                m-0.05 0; 1 1 1 1 m+0.05 0.001]);
            if Z2<=L
                tmpeq2(i).values(1:9) = [y2,Z2,i];
            else

```

```

[y3,Z3] = Fiteq3(xdata,adata,options,[0.25 2*m 0.25 0.6 0.1 0.1 0.1 ...
    m 0],[0 0 0 0 0 0 0 0 m-0.02 0; 1 1 1 1 1 1 1 m+0.02 0.001]);
if Z3<=L
tmpeq3(i).values(1:13) = [y3,Z3,i];
else
[y3,Z3] = Fiteq3(xdata,adata,options,[0.25 0.5*m 0.25 0.6 0.1 0.1 0...
    .1 m 0],[0 0 0 0 0 0 0 0 m-0.02 0; 1 1 1 1 1 1 1 m+0.02 0.001]);
if Z3<=L
tmpeq3(i).values(1:13) = [y3,Z3,i];
else
[y4,Z4] = Fiteq4(xdata,adata,options,[0.25 2*m 0.25 0.6 0.25 0.8 0.1...
    0.1 0.1 0.1 m 0],[0 0 0 0 0 0 0 0 0 0 m-0.02 0; 1 1 1 1,...
    1 1 1 1 1 1 m+0.02 0.001]);
if Z4<=L
tmpeq4(i).values(1:17) = [y4,Z4,i];
else
[y4,Z4] = Fiteq4(xdata,adata,options,[0.25 0.5*m 0.25 0.4 0.25 0.6 0...
    .1 0.1 0.1 0.1 m 0],[0 0 0 0 0 0 0 0 0 0 m-0.02 0 ; 1 1 1 1,...
    1 1 1 1 1 1 m+0.02 0.001]);
if Z4<=L
tmpeq4(i).values(1:17) = [y4,Z4,i];
else
[y5,Z5] = Fiteq5(xdata,adata,options,[0.25 2*m 0.25 0.6 0.25 0.8 0...
    .25 0.9 0.1 0.1 0.1 0.1 0.1 m 0],[0 0 0 0 0 0 0 0 0 0 0,...
    0 0 m-0.02 0; 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1...
    m+0.02 0.001]);
if Z5<=L
tmpeq5(i).values(1:21) = [y5,Z5,i];
else

```

```

[y5,Z5] = Fiteq5(xdata,adata,options,[0.25 0.5*m 0.25 0.6 0.25 0.8 0...
    .25 0.9 0.1 0.1 0.1 0.1 0.1 m 0],[0 0 0 0 0 0 0 0 0 0 0 0,...
    0 0 m-0.02 0; 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1...
    m+0.02 0.001]);

if Z5<=L
tmpeq5(i).values(1:21) = [y5,Z5,i];
end
end
end
end
end
end
end
end
end
end
end

eq1 = struct('values',[]);
eq2 = struct('values',[]);
eq3 = struct('values',[]);
eq4 = struct('values',[]);
eq5 = struct('values',[]);

for i = 1:length(tmpeq1)
    if isempty(tmpeq1(i).values(:))==0
        k1 = k1+1;
        eq1(k1).values = tmpeq1(i).values;
    end
end
end

```

```

for i = 1:length(tmpeq2)
    if isempty(tmpeq2(i).values(:))==0
        k2 = k2+1;
        eq2(k2).values = tmpeq2(i).values;
    end
end
for i = 1:length(tmpeq3)
    if isempty(tmpeq3(i).values(:))==0
        k3 = k3+1;
        eq3(k3).values = tmpeq3(i).values;
    end
end
for i = 1:length(tmpeq4)
    if isempty(tmpeq4(i).values(:))==0
        k4 = k4+1;
        eq4(k4).values = tmpeq4(i).values;
    end
end
for i = 1:length(tmpeq5)
    if isempty(tmpeq5(i).values(:))==0
        k5 = k5+1;
        eq5(k5).values = tmpeq5(i).values;
    end
end

detected = (k1+k2+k3+k4+k5)/n;

%store everything in the structure parameters

```

```

parameters = struct('eq1',eq1,'eq2',eq2,'eq3',eq3,'eq4',eq4,'eq5',...
    eq5,'detected',detected,'undetected',und);
end

```

```

function F = eq1(x,xdata)

```

```

    sig1 = x(1);

```

```

    M = x(2);

```

```

    m1 = -(2*M-1)/((2/(sig1^2))*M*(M-1))+M;

```

```

    k1 = 1/((M*(M-1))*exp(-(M-m1)^2/(sig1^2)));

```

```

    F = k1*(xdata).*(xdata-1).*exp(-(xdata-m1).^2/(sig1^2));

```

```

end

```

```

function [parameters,resnorm] = Fiteq1(xdata,adata,options,ini,cstr)

```

```

try

```

```

    [parameters,resnorm]= lsqcurvefit(@(x,data)eq1(x,xdata),ini,xdata,...
        adata,cstr(1,:)...
        ,cstr(2,:),options);

```

```

    if ismember(0,isreal(parameters)) == 1;

```

```

        parameters = zeros(2,1); resnorm = 1;

```

```

    end

```

```
catch
    resnorm = 1;
    parameters = zeros(2,1);
end

end
```


Appendix D

Classification of Fitted Pulses into the Different Profiles Code

```
function [types] = classify(parameters,peak)

%-----
%Classifies the fitted pulses into the different types.
%The parameters of each type are ouput in the structure
%types which is formated as follows:
%types.type1 = [sigma1 x1 residual...
%(pulse number in peak structure) (maximum acceleration)...
%(pulse duration)]
%types.type2 = [a2 x2 sigma1 sigma2 x1 1-a1 h2 residual...
%(pulse number in peak structure) (maximum acceleration)...
%(pulse duration)]
%types.type3sub1 = [a2 x2 a3 x3 sigma1 sigma2 sigma3 x1 1-a1 h2 ...
h3...
```

```

%residual(pulse number in peak structure) (maximum acceleration)...
%(pulse duration)]
%types.type5sub1 = [a2 x2 a3 x3 sigma1 sigma2 sigma3 x1 1-a1 h2 ...
    h3...
%residual (pulse number in peak structure) (maximum acceleration)...
%(pulse duration) xmin amin xmax2 amax2]
%types.freq(:) = [# of types 1, # of type 2,...]
%A bar plot of the profile distribution is output.
%-----

k1 = 0; type1 = struct('values',[]);
k2 = 0; type2 = struct('values',[]);
k31 = 0; type3sub1 = struct('values',[]);
k32 = 0; type3sub2 = struct('values',[]);
k33 = 0; type3sub3 = struct('values',[]);
k41 = 0; type4sub1 = struct('values',[]);
k42 = 0; type4sub2 = struct('values',[]);
k43 = 0; type4sub3 = struct('values',[]);
k51 = 0; type5sub1 = struct('values',[]);
k52 = 0; type5sub2 = struct('values',[]);
k53 = 0; type5sub3 = struct('values',[]);

if isempty(parameters.eq1(1).values) == 0
for i = 1:length(parameters.eq1(:))

k1 = k1+1;
    type1(k1).values(1:4) = parameters.eq1(i).values(1:4);
    X(k1,:) = parameters.eq1(i).values(1:4);

```

```

    type1(k1).values(5) = peak.aMax(parameters.eq1(i).values(4));
    type1(k1).values(6) = peak.duration(parameters.eq1(i).values(4))...
        ;
end
end
clear X

if isempty(parameters.eq2(1).values) == 0
for i = 1:length(parameters.eq2(:))

k2 = k2+1;
    type2(k2).values(1:9) = parameters.eq2(i).values(1:9);
    X(k2,:) = parameters.eq2(i).values(1:9);
    type2(k2).values(10) = peak.aMax(parameters.eq2(i).values(9)...
        );
    type2(k2).values(11) = peak.duration(parameters.eq2(i).values(9)...
        );
end
end

if isempty(parameters.eq3(1).values) == 0
for i = 1:length(parameters.eq3(:))

amax = peak.aMax(parameters.eq3(i).values(13));
m = peak.duratiomtomax(parameters.eq3(i).values(13))/peak.duration(...
    parameters.eq3(i).values(13));

%-----
% this section is to find xmin amin xmax2 amax2

```

```

x1 = linspace(parameters.eq3(i).values(8),parameters.eq3(i).values...
    (2),100);
y1 = eq3(parameters.eq3(i).values(1:9),x1);
[min1,ind1] = min(y1);
if parameters.eq3(i).values(8)<parameters.eq3(i).values(2)
xp1 = linspace(x1(ind1),1,100);
else
    xp1 = linspace(x1(ind1),0,100);
end
yp1 = eq3(parameters.eq3(i).values(1:9),xp1);
[max1,indm1] = max(yp1);
H(1,:) = [x1(ind1) min1 xp1(indm1) max1];
x2 = linspace(parameters.eq3(i).values(8),parameters.eq3(i).values...
    (4),100);
y2 = eq3(parameters.eq3(i).values(1:9),x2);
[min2,ind2] = min(y2);
if parameters.eq3(i).values(8)<parameters.eq3(i).values(4)
xp2 = linspace(x2(ind2),1,100);
else
    xp2 = linspace(x2(ind2),0,100);
end
yp2 = eq3(parameters.eq3(i).values(1:9),xp2);
[max2,indm2] = max(yp2);
H(2,:) = [x2(ind2) min2 xp2(indm2) max2];
%-----

if max([(max1-min1),(max2-min2)])>0.2
    k51 = k51+1;
    type5sub1(k51).values(1:13) = parameters.eq3(i).values(1:13);

```

```

X1(k51,:) = parameters.eq3(i).values(1:13);
[tmax,tind] = max([(max1-min1),(max2-min2)]);
type5sub1(k51).values(16:19) = H(tind,:);
type5sub1(k51).values(14) = peak.aMax(parameters.eq3(i).values...
    (13));
type5sub1(k51).values(15) = peak.duration(parameters.eq3(i)...
    .values(13));
elseif parameters.eq3(i).values(2)<m || parameters.eq3(i).values(4)<...
m
k41 = k41 +1;
type4sub1(k41).values(1:13) = parameters.eq3(i).values(1:13);
X2(k41,:) = parameters.eq3(i).values(1:13);
type4sub1(k41).values(14) = peak.aMax(parameters.eq3(i).values...
    (13));
type4sub1(k41).values(15) = peak.duration(parameters.eq3(i)...
    .values(13));
else
k31 = k31+1;
type3sub1(k31).values(1:13) = parameters.eq3(i).values(1:13);
X3(k31,:) = parameters.eq3(i).values(1:13);
type3sub1(k31).values(14) = peak.aMax(parameters.eq3(i).values...
    (13));
type3sub1(k31).values(15) = peak.duration(parameters.eq3(i)...
    .values(13));
end
end
end
clear x1; clear y1; clear x2; clear y2; clear xp1; clear xp2; clear ...
yp1; clear yp2; clear X1; clear X2; clear X3; clear H;

```

```

if isempty(parameters.eq4(1).values) == 0
for i = 1:length(parameters.eq4(:))

    amax = peak.aMax(parameters.eq4(i).values(17));
    m = peak.durationtomax(parameters.eq4(i).values(17))/peak.duration(...
        parameters.eq4(i).values(17));

%-----
% this section is to find xmin amin xmax2 amax2
x1 = linspace(parameters.eq4(i).values(11),parameters.eq4(i).values...
    (2),100);
y1 = eq4(parameters.eq4(i).values(1:12),x1);
[min1,ind1] = min(y1);
if parameters.eq4(i).values(11)<parameters.eq4(i).values(2)
xp1 = linspace(x1(ind1),1,100);
else
    xp1 = linspace(x1(ind1),0,100);
end
yp1 = eq4(parameters.eq4(i).values(1:12),xp1);
max1 = max(yp1);
[max1,indm1] = max(yp1);
H(1,:) = [x1(ind1) min1 xp1(indm1) max1];

x2 = linspace(parameters.eq4(i).values(11),parameters.eq4(i).values...
    (4),100);
y2 = eq4(parameters.eq4(i).values(1:12),x2);
[min2,ind2] = min(y2);

```

```

if parameters.eq4(i).values(11)<parameters.eq4(i).values(4)
xp2 = linspace(x2(ind2),1,100);
else
    xp2 = linspace(x2(ind2),0,100);
end
yp2 = eq4(parameters.eq4(i).values(1:12),xp2);
max2 = max(yp2);
[max2,indm2] = max(yp2);
H(2,:) = [x2(ind2) min2 xp2(indm2) max2];

x3 = linspace(parameters.eq4(i).values(11),parameters.eq4(i).values...
    (6),100);
y3 = eq4(parameters.eq4(i).values(1:12),x3);
[min3,ind3] = min(y3);
if parameters.eq4(i).values(11)<parameters.eq4(i).values(6);
xp3 = linspace(x3(ind3),1,100);
else
    xp3 = linspace(x3(ind3),0,100);
end
yp3 = eq4(parameters.eq4(i).values(1:12),xp3);
max3 = max(yp3);
[max3,indm3] = max(yp3);
H(3,:) = [x3(ind3) min3 xp3(indm3) max3];
%-----
    if max([(max1-min1),(max2-min2),(max3-min3)])>0.2

        k52 = k52+1;
        type5sub2(k52).values(1:17) = parameters.eq4(i).values(1:17);
        X1(k52,:) = parameters.eq4(i).values(1:17);

```

```

[tmax,tind] = max([(max1-min1),(max2-min2),(max3-min3)]);
type5sub2(k52).values(20:23) = H(tind,:);
type5sub2(k52).values(18) = peak.aMax(parameters.eq4(i).values...
    (17));
type5sub2(k52).values(19) = peak.duration(parameters.eq4(i)...
    .values(17));
elseif parameters.eq4(i).values(2)<m || parameters.eq4(i).values(4)<...
m || parameters.eq4(i).values(6)<m
k42 = k42 +1;
type4sub2(k42).values(1:17) = parameters.eq4(i).values(1:17);
X2(k42,:) = parameters.eq4(i).values(1:17);
type4sub2(k42).values(18) = peak.aMax(parameters.eq4(i).values...
    (17));
type4sub2(k42).values(19) = peak.duration(parameters.eq4(i)...
    .values(17));
else
k32 = k32+1;
type3sub2(k32).values(1:17) = parameters.eq4(i).values(1:17);
X3(k32,)=parameters.eq4(i).values(1:17);
type3sub2(k32).values(18) = peak.aMax(parameters.eq4(i).values...
    (17));
type3sub2(k32).values(19) = peak.duration(parameters.eq4(i)...
    .values(17));

end

end

end

```



```

clear x1; clear y1; clear x2; clear y2; clear x3; clear y3; clear ...
    xp1; clear xp2; clear xp3; clear yp1; clear yp2;
clear yp3; clear X1; clear X2 ;clear X3; clear H;

if isempty(parameters.eq5(1).values) == 0
for i = 1:length(parameters.eq5(:))

    amax = peak.aMax(parameters.eq5(i).values(21));
    m = peak.durationtomax(parameters.eq5(i).values(21))/peak.duration(...
        parameters.eq5(i).values(21));

%-----
% this section is to find xmin amin xmax2 amax2
x1 = linspace(parameters.eq5(i).values(14),parameters.eq5(i).values...
    (2),50);
y1 = eq5(parameters.eq5(i).values(1:15),x1);
[min1,ind1] = min(y1);
if parameters.eq5(i).values(14)<parameters.eq5(i).values(2)
xp1 = linspace(x1(ind1),1,50);
else
    xp1 = linspace(x1(ind1),0,100);
end
yp1 = eq5(parameters.eq5(i).values(1:15),xp1);
max1 = max(yp1);
[max1,indm1] = max(yp1);
H(1,:) = [x1(ind1) min1 xp1(indm1) max1];

x2 = linspace(parameters.eq5(i).values(14),parameters.eq5(i).values...
    (4),100);

```

```

y2 = eq5(parameters.eq5(i).values(1:15),x2);
[min2,ind2] = min(y2);
if parameters.eq5(i).values(14)<parameters.eq5(i).values(4)
xp2 = linspace(x2(ind2),1,100);
else
    xp2 = linspace(x2(ind2),0,100);
end
yp2 = eq5(parameters.eq5(i).values(1:15),xp2);
max2 = max(yp2);
[max2,indm2] = max(yp2);
H(2,:) = [x2(ind2) min2 xp2(indm2) max2];

x3 = linspace(parameters.eq5(i).values(14),parameters.eq5(i).values...
    (6),100);
y3 = eq5(parameters.eq5(i).values(1:15),x3);
[min3,ind3] = min(y3);
if parameters.eq5(i).values(14)<parameters.eq5(i).values(6)
xp3 = linspace(x3(ind3),1,100);
else
    xp3 = linspace(x3(ind3),0,100);
end
yp3 = eq5(parameters.eq5(i).values(1:15),xp3);
max3 = max(yp3);
[max3,indm3] = max(yp3);
H(3,:) = [x3(ind3) min3 xp3(indm3) max3];

x4 = linspace(parameters.eq5(i).values(14),parameters.eq5(i).values...
    (8),100);
y4 = eq5(parameters.eq5(i).values(1:15),x4);

```

```

[min4,ind4] = min(y4);
if parameters.eq5(i).values(14)<parameters.eq5(i).values(8)
xp4 = linspace(x4(ind4),1,100);
else
    xp4 = linspace(x4(ind4),0,100);
end
yp4 = eq5(parameters.eq5(i).values(1:15),xp4);
max4 = max(yp4);
[max4,indm4] = max(yp4);
H(4,:) = [x4(ind4) min4 xp4(indm4) max4];
%...
-----...

if max([(max1-min1),(max2-min2),(max3-min3),(max4-min4)])>0.2
k53 = k53+1;
type5sub3(k53).values(1:21) = parameters.eq5(i).values(1:21);
X1(k53,:) = parameters.eq5(i).values(1:21);
[tmax,tind] = max([(max1-min1),(max2-min2),(max3-min3),(max4-...
    min4)]);
type5sub3(k53).values(24:27) = H(tind,:);
type5sub3(k53).values(22) = peak.aMax(parameters.eq5(i).values...
    (21));
type5sub3(k53).values(23) = peak.duration(parameters.eq5(i)...
    .values(21));
elseif parameters.eq5(i).values(2)<m || parameters.eq5(i).values(4)<...
m || parameters.eq5(i).values(6)<m || parameters.eq5(i).values(8)...
<m
k43 = k43 +1;

```

```

type4sub3(k43).values(1:21) = parameters.eq5(i).values(1:21);
X2(k43,:) = parameters.eq5(i).values(1:21);
type4sub3(k43).values(22) = peak.aMax(parameters.eq5(i).values...
    (21));
type4sub3(k43).values(23) = peak.duration(parameters.eq5(i)...
    .values(21));
else
    k33 = k33+1;
    type3sub3(k33).values(1:21) = parameters.eq5(i).values(1:21);
    X3(k33,:) = parameters.eq5(i).values(1:21);
    type3sub3(k33).values(22) = peak.aMax(parameters.eq5(i).values...
        (21));
    type3sub3(k33).values(23) = peak.duration(parameters.eq5(i)...
        .values(21));
end
end
end

tot = k1+k2+k31+k32+k33+k41+k42+k43+k51+k52+k53;

figure
bar([1,2,3,4,5,6,7,8,9],[100*(k1/tot),100*(k2/tot),100*(k31/tot)...
    ,100*(k32/tot),100*(k33/tot),100*(k41/tot),100*(k42/tot),100*(k43...
    /tot),...
    100*((k51+k52+k53)/tot)]);set(gca,'XTick',1:9,'XTickLabel',{'1',...
    '2','3-v1','3-v2','3-v3','4-v1','4-v2','4-v3','5'});
ylabel('Relative Frequency %');
freq = [k1 k2 k31 k32 k33 k41 k42 k43 k51+k52+k53];
% store everything in types structure

```

```

types = struct('type1',type1,'type2',type2,'type3sub1',type3sub1,...
    'type3sub2',type3sub2,'type3sub3',type3sub3,'type4sub1',type4sub1,...
    'type4sub2',type4sub2,...
    'type4sub3',type4sub3,'type5sub1',type5sub1,'type5sub2',...
    type5sub2,'type5sub3',type5sub3,'freq',freq);
end

```

```

function [types] = classifytype5(types)

%classify type5sub1
[Y1,n1] = arrange(types.type5sub1,3);
for i = 1:n1
    xmin1(i) = types.type5sub1(i).values(16);
    xmax21(i) = types.type5sub1(i).values(18);
    for j = 10:18
        Y1(i,j) = types.type5sub1(i).values(j);
    end
end

k12 = 0; type5sub12 = struct('values',[]);
k21 = 0; type5sub21 = struct('values',[]);
k12r = 0; type5sub12r = struct('values',[]);
k21r = 0; type5sub21r = struct('values',[]);

for i = 1:n1
    if Y1(i,8)<xmin1(i)
        if Y1(i,2)>xmin1(i) && abs(Y1(i,2)-xmax21(i))<abs(Y1(i,4)-...
            xmax21(i))

```

```

        k12 = k12 + 1;
        type5sub12(k12).values(1:length(Y1(i,:))) = Y1(i,:);
    elseif Y1(i,2)<xmin1(i) && Y1(i,2)>Y1(i,8)
        k21 = k21 + 1;
        type5sub21(k21).values(1:length(Y1(i,:))) = Y1(i,:);
    end
else
    if Y1(i,8)>Y1(i,4)
        k21r = k21r + 1;
        type5sub21r(k21r).values(1:length(Y1(i,:))) = Y1(i,:);
    elseif Y1(i,8)>Y1(i,2)
        k12r = k12r + 1;
        type5sub12r(k12r).values(1:length(Y1(i,:))) = Y1(i,:);
    end
end
end

types.type5sub12 = type5sub12;
types.type5sub12r = type5sub12r;
types.type5sub21 = type5sub21;
types.type5sub21r = type5sub21r;

%classify type5sub2
[Y2,n2] = arrange(types.type5sub2,4);
for i =1:n2
    xmin2(i) = types.type5sub2(i).values(20);
    xmax22(i) = types.type5sub2(i).values(22);
    for j = 13:23
        Y2(i,j) = types.type5sub2(i).values(j);
    end
end

```

```

end

end

k131 = 0;   type5sub131 = struct('values', []);
k131r = 0;  type5sub131r = struct('values', []);
k141 = 0;   type5sub141 = struct('values', []);
k141r = 0;  type5sub141r = struct('values', []);
k22 = 0;    type5sub22 = struct('values', []);
k22r = 0;   type5sub22r = struct('values', []);
k311 = 0;   type5sub311 = struct('values', []);
k311r = 0;  type5sub311r = struct('values', []);
k411 = 0;   type5sub411 = struct('values', []);
k411r = 0;  type5sub411r = struct('values', []);

for i = 1:n2
    if Y2(i,11)<xmin2(i)
        if Y2(i,2)>xmin2(i)
            if abs(Y2(i,2)-xmax22(i))< abs(Y2(i,4)-xmax22(i))
                k131 = k131 + 1;
                type5sub131(k131).values(1:length(Y2(i,:))) = Y2(i,:);
            elseif abs(Y2(i,2)-xmax22(i))>abs(Y2(i,4)-xmax22(i)) &&...
                ...
                abs(Y2(i,6)-xmax22(i))>abs(Y2(i,4)-xmax22(i))
                k141 = k141 + 1;
                type5sub141(k141).values(1:length(Y2(i,:))) = Y2(i,:);
            end
        elseif Y2(i,2)<xmin2(i) && Y2(i,4)>xmin2(i) && Y2(i,2)>Y2(i...
            ,11)

```

```

        k22 = k22 + 1;
        type5sub22(k22).values(1:length(Y2(i,:))) = Y2(i,:);
elseif Y2(i,4)<xmin2(i) && Y2(i,6)>xmin2(i) && Y2(i,2)>Y2(i...
,11)
    k311 = k311 + 1;
    type5sub311(k311).values(1:length(Y2(i,:))) = Y2(i,:);
elseif Y2(i,4)<xmin2(i) && Y2(i,6)>xmin2(i) && Y2(i,2)<Y2(i...
,11) && Y2(i,4)>Y2(i,11)
    k411 = k411 + 1;
    type5sub411(k411).values(1:length(Y2(i,:))) = Y2(i,:);
end
else
    if Y2(i,2)<xmin2(i) && Y2(i,4)>xmin2(i) && Y2(i,4)>Y2(i,11)
        k131r = k131r + 1;
        type5sub131r(k131r).values(1:length(Y2(i,:))) = Y2(i,:);
    elseif Y2(i,2)<xmin2(i) && Y2(i,4)>xmin2(i) && Y2(i,4)<Y2(i...
,11)
        k141r = k141r + 1;
        type5sub141r(k141r).values(1:length(Y2(i,:))) = Y2(i,:);
    elseif Y2(i,4)<xmin2(i) && Y2(i,6)>xmin2(i) && Y2(i,1)>Y2(i...
,3)
        k22r = k22r + 1;
        type5sub22r(k22r).values(1:length(Y2(i,:))) = Y2(i,:);
    elseif Y2(i,6)<xmin2(i) && Y2(i,1)>Y2(i,3)
        k311r = k311r + 1;
        type5sub311r(k311r).values(1:length(Y2(i,:))) = Y2(i,:);
    elseif Y2(i,6)<xmin2(i) && Y2(i,1)<Y2(i,3)
        k411r = k411r + 1;
        type5sub411r(k411r).values(1:length(Y2(i,:))) = Y2(i,:);

```



```

        end
    end
end

types.type5sub131 = type5sub131;
types.type5sub141 = type5sub141;
types.type5sub22 = type5sub22;
types.type5sub311 = type5sub311;
types.type5sub411 = type5sub411;
types.type5sub131r = type5sub131r;
types.type5sub141r = type5sub141r;
types.type5sub22r = type5sub22r;
types.type5sub311r = type5sub311r;
types.type5sub411r = type5sub411r;

[Y3,n3] = arrange(types.type5sub3,5);
for i =1:n3
    xmin3(i) = types.type5sub3(i).values(24);
    xmax23(i) = types.type5sub3(i).values(26);
    for j = 16:27
        Y3(i,j) = types.type5sub3(i).values(j);
    end
end

k132 = 0; type5sub132 = struct('values',[]);
k132r = 0; type5sub132r = struct('values',[]);
k142 = 0; type5sub142 = struct('values',[]);
k142r = 0; type5sub142r = struct('values',[]);
k231 = 0; type5sub231 = struct('values',[]);
k231r = 0; type5sub231r = struct('values',[]);

```

```

k241 = 0; type5sub241 = struct('values', []);
k241r = 0; type5sub241r = struct('values', []);
k312 = 0; type5sub312 = struct('values', []);
k312r = 0; type5sub312r = struct('values', []);
k321 = 0; type5sub321 = struct('values', []);
k321r = 0; type5sub321r = struct('values', []);
k412 = 0; type5sub412 = struct('values', []);
k412r = 0; type5sub412r = struct('values', []);
k421 = 0; type5sub421 = struct('values', []);
k421r = 0; type5sub421r = struct('values', []);

for i = 1:n3
    if Y3(i,14)<xmin3(i)
        if Y3(i,2)>xmin3(i)
            if abs(Y3(i,2)-xmax23(i))<abs(Y3(i,4)-xmax23(i))
                k132 = k132 + 1;
                type5sub132(k132).values(1:length(Y3(i,:))) = Y3(i,:);
            elseif abs(Y3(i,2)-xmax23(i))>abs(Y3(i,4)-xmax23(i))
                k142 = k142 + 1;
                type5sub142(k142).values(1:length(Y3(i,:))) = Y3(i,:);
            end
        elseif Y3(i,2)<xmin3(i) &&...
            Y3(i,4)>xmin3(i)
            if abs(Y3(i,4)-xmax23(i))<abs(Y3(i,6)-xmax23(i))
                k231 = k231 + 1;
                type5sub231(k231).values(1:length(Y3(i,:))) = Y3(i,:);
            elseif abs(Y3(i,4)-xmax23(i))>abs(Y3(i,6)-xmax23(i))
                k241 = k241 + 1;

```

```

        type5sub241(k241).values(1:length(Y3(i,:))) = Y3(i,:);
    end
elseif Y3(i,4)<xmin3(i) && Y3(i,6)>xmin3(i) && Y3(i,2)>Y3(...
    i,14)
    k312 = k312 + 1;
    type5sub312(k312).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,4)<xmin3(i) && Y3(i,6)>xmin3(i) && Y3(i,2)<Y3(...
    i,14)
    k412 = k412 + 1;
    type5sub412(k412).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,6)<xmin3(i) && Y3(i,8)>xmin3(i) && Y3(i,2)<Y3(...
    i,14)
    k421 = k421 + 1;
    type5sub421(k421).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,6)<xmin3(i) && Y3(i,8)>xmin3(i) && Y3(i,2)>Y3...
    (i,14)
    k321 = k321 + 1;
    type5sub321(k321).values(1:length(Y3(i,:))) = Y3(i,:);

end
else
    if Y3(i,2)<xmin3(i) && Y3(i,4)>Y3(i,14)
        k132r = k132r + 1;
        type5sub132r(k132r).values(1:length(Y3(i,:))) = Y3(i,:);
    elseif Y3(i,2)<xmin3(i) && Y3(i,4)>xmin3(i) && Y3(i,4)<Y3(i...
        ,14)
        k142r = k142r + 1;
        type5sub142r(k142r).values(1:length(Y3(i,:))) = Y3(i,:);
    end
end

```

```

elseif Y3(i,4)<xmin3(i) && Y3(i,6)>xmin3(i) && Y3(i,14)>Y3(i...
    ,6)
    k231r = k231r + 1;
    type5sub231r(k231r).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,4)<xmin3(i) && Y3(i,6)>xmin3(i) && Y3(i,14)<Y3(i...
    ,6)
    k241r = k241r + 1;
    type5sub241r(k241r).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,6)<xmin3(i) && Y3(i,8)>xmin3(i) && Y3(i,1)>Y3(i...
    ,3)
    k312r = k312r + 1;
    type5sub312r(k312r).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,6)<xmin3(i) && Y3(i,8)>xmin3(i) && Y3(i,1)<Y3(i...
    ,3)
    k412r = k412r + 1;
    type5sub412r(k412r).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,8)<xmin3(i) && Y3(i,1)<Y3(i,3)
    k421r = k421r + 1;
    type5sub421r(k421r).values(1:length(Y3(i,:))) = Y3(i,:);
elseif Y3(i,8)<xmin3(i) && Y3(i,1)>Y3(i,3)
    k321r = k321r + 1;
    type5sub321r(k321r).values(1:length(Y3(i,:))) = Y3(i,:);
end
end
end

types.type5sub132 = type5sub132;
types.type5sub142 = type5sub142;
types.type5sub231 = type5sub231;
types.type5sub241 = type5sub241;

```

```

types.type5sub312 = type5sub312;
types.type5sub321 = type5sub321;
types.type5sub412 = type5sub412;
types.type5sub421 = type5sub421;

types.type5sub132r = type5sub132r;
types.type5sub142r = type5sub142r;
types.type5sub231r = type5sub231r;
types.type5sub241r = type5sub241r;
types.type5sub312r = type5sub312r;
types.type5sub321r = type5sub321r;
types.type5sub412r = type5sub412r;
types.type5sub421r = type5sub421r;

tot = k12+k21+k12r+ k131+k131r+k141+k22+k22r+k311+k411+ k132+k132r+...
      k142+k142r+k231+k241+k312;
freq5 = [k12 k21 k12r k131 k141 k22 k311 k411 k131r k22r k132 k142 ...
        k231 k241 k312 k132r k142r];
types.freq5 = freq5;
figure
bar([1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17],[100*(k12/tot),100*(...
      k21/tot),100*(k12r/tot),...
      100*(k131/tot),100*(k141/tot),100*(k22/tot),100*(k311/tot),100*(...
      k411/tot),100*(k131r/tot),100*(k22r/tot),...
      100*(k132/tot),100*(k142/tot),100*(k231/tot),100*(k241/tot)...
      ,100*(k312/tot),100*(k132r/tot),100*(k142r/tot)]);...
```

```

set(gca,'XTick',1:17,'XTickLabel',{'1-2','2-1','1-2r','1-3.I','...
    1-4.I','2-2','3.I-1','4.I-1','1-3.I','4.I-1','1-3.Ir','2-2r',...
    '1-3.II','1-4.II',...
    '2-3.I','2-4.I','3.I-2','1-3.IIr','1-4.IIr'});
ylabel('Relative Frequency %');

end

```

Appendix E

Synthetic Data Generation Code

```
function [Ysim] = Simwcopula(X,n-s,s)

%-----
%Simulates parameters using copulas. Inputs are the
%original data vector X, the number of observation to
%be simulated n-s and the seed number s
%-----

n = length(X(:,1));
var = length(X(1,:));
if any(X)~=0
    if n>1
        rho = corr(X);

    rho_s = rho;
    rng(s,'twister')
```

```

Z = mvnrnd(zeros(1,var),rho_s,n_s);

for i = 1:var
    invCDF(:,i) = sort(X(:,i));
end

U = normcdf(Z,0,1);

for i =1:var
    temp(:,1) = invCDF(:,i);
    Ysim(:,i) = temp(ceil(n*U(:,i))));
    clear temp
end
    elseif n==1
        Ysim = repmat(X,n_s,1);
    end
else
    Ysim = 0;
end

end

function [Ysim] = JointProbabilityG4(X,n_s,s)

%-----
%Simulates data using the histogram method.

```



```

%Inputs are the original data vector X with 4 variables
%the number of observation to be simulated n_s
%and the seed number s.
%-----

n = length(X(:,1));
var = length(X(1,:));
if any(X)~=0
    if n>1
X1(1,:) = X(:,1)';
X2(1,:) = X(:,2)';
X3(1,:) = X(:,3)';
X4(1,:) = X(:,4)';

lim = zeros(var,2);
delta = zeros(var,1);

n_c = repmat(15,1,var); %subdividing the range of each parameter ...
    into 15

for i = 1:var
lim(i,:) = [min(X(:,i)) max(X(:,i))];
delta(i,1) = (lim(i,2)-lim(i,1))/n_c(i);
end

lim1 = lim(1,1); delta1 = delta(1,1);
lim2 = lim(2,1); delta2 = delta(2,1);
lim3 = lim(3,1); delta3 = delta(3,1);
lim4 = lim(4,1); delta4 = delta(4,1);

```

```

n_c = n_c+1;
parfor i = 1:n_c(1)
    j1 = lim1+delta1*(i-1);
    t1(i).values = find(abs(X1(1,:)-j1)<delta1/2);
end
parfor i = 1:n_c(2)
    j2 = lim2+delta2*(i-1);
    t2(i).values = find(abs(X2(1,:)-j2)<delta2/2);
end
parfor i=1:n_c(3)
    j3 = lim3+delta3*(i-1);
    t3(i).values = find(abs(X3(1,:)-j3)<delta3/2);
end
parfor i = 1:n_c(4)
    j4 = lim4+delta4*(i-1);
    t4(i).values = find(abs(X4(1,:)-j4)<delta4/2);
end

counter = 0;

for i1 = 1:n_c(1)
    a1(1,:) = zeros(1,n);
    a1(1,t1(i1).values) = t1(i1).values;
    for i2 = 1:n_c(2)
        a2(1,:) = zeros(1,n);
        a2(1,t2(i2).values) = t2(i2).values;
    B1 = [a1;a2];
    B1(:,~any(B1,1)) = []; %cols
    D1 = length(find(~diff(B1)));

```



```

P(:,~any(P,1))=[];
P(~any(P,1),:)=[];
rng(s,'twister');

C = randsample(1:length(P),n_s,true,P/length(P));

Ysim = zeros(n_s,var);

for i = 1:n_s
    for u = 1:var
        Ysim(i,u) = L(C(i),u)+ delta(u,1)*rand(1)-delta(u,1)/2;
    end
end
else
    Ysim = repmat(X,n_s,1);
end
else
    Ysim =0;
end

end

function [v,d,j] = VDJeql(x,xdata,T)

%-----
%Outputs the velocity, displacement, and jerk for Type 1

```

```

%in the range xdata*T (so xdata ranges
%from 0 to 1 and T is the duration of the pulse). x contains
% the equation parameters.
%-----

sig1 = x(1);
M = x(2);

m1 = -(2*M-1)/((2/(sig1^2))*M*(M-1))+M;

k1 = 1/((M*(M-1))*exp(-(M-m1)^2/(sig1^2)));

vs = @(T,k1,m1,sig1,x) -1.0./T.^2.*(T.^2.*k1.*sig1.^2.*x.*exp(-m1....
    ^2.*1.0./sig1.^2-1.0./T.^2.*1.0./sig1.^2.*x.^2+...
    (m1.*1.0./sig1.^2.*x.*2.0)./T).*(1.0./2.0)+T.^3.*k1.*sig1.^2.*exp...
    (-m1.^2.*1.0./sig1.^2-1.0./T.^2.*1.0./sig1.^2.*x.^2+...
    (m1.*1.0./sig1.^2.*x.*2.0)./T).*(m1-1.0).*(1.0./2.0))+...
    sqrt(pi).*k1.*erfi(sqrt(-1.0./T.^2.*1.0./sig1.^2).*(x-T.*m1)).*1...
    .0./sqrt(-1.0./T.^2.*1.0./sig1.^2).*(m1.*-2.0+m1.^2.*2.0+sig1....
    ^2).*(1.0./4.0);

ds = @(T,k1,m1,sig1,x) sqrt(pi).*erfi(sqrt(-1.0./T.^2.*1.0./sig1.^2)...
    .*(x-T.*m1)).*1.0./sqrt(-1.0./T.^2.*1.0./sig1.^2).*...
    (T.*k1.*m1.^2.*2.0-T.*k1.*m1.^3.*2.0+T.*k1.*sig1.^2-T.*k1.*m1.*...
    sig1.^2.*3.0).*(1.0./4.0)+...
    T.^2.*k1.*sig1.^2.*exp(-m1.^2.*1.0./sig1.^2-1.0./T.^2.*1.0./...
    sig1.^2.*x.^2+...

```

```

(m1.*1.0./sig1.^2.*x.*2.0)./T).*(-m1+m1.^2+sig1.^2).*(1.0./2.0)+...
    sqrt(pi).*k1.*x.*erfi(sqrt(-1.0./T.^2.*1.0./sig1.^2).*(x-T.*...
    m1)).*1.0./...
sqrt(-1.0./T.^2.*1.0./sig1.^2).*(m1.*-2.0+m1.^2.*2.0+sig1.^2)....
    *(1.0./4.0);

js = @(T,k1,m1,sig1,x)1.0./T.^2.*k1.*x.*exp(-1.0./sig1.^2.*(m1-x./T)...
    .^2)+(k1.*exp(-1.0./sig1.^2.*(m1-x./T).^2).*(x./T-1.0))./T+...
    1.0./T.^2.*k1.*1.0./sig1.^2.*x.*exp(-1.0./sig1.^2.*(m1-x./T).^2)...
    .*(m1-x./T).*(x./T-1.0).*2.0;

v = vs(T,k1,m1,sig1,xdata*T);
d = ds(T,k1,m1,sig1,xdata*T);
j = js(T,k1,m1,sig1,xdata*T);

end

function[generatedpeak,gtypes,az,timevec,pres,vz,dz] = ...
    generatedatanewlast2(types,N,s)

```

```

%-----
%Generates time history with N-1 pulses using copulas
%based on the statistics of actual data which is in
%the structure types. The outputs are the structure
%generatedpeak which is similar to the structure peak
%(e.g. generatedpeak.az(n).values(:) =acceleration
%values of nth pulse),the time vector timevec, the

```

```

%acceleration az, velocity vz, displacement dz,
%and prescribed number of pulses of each type pres
%Data simulation for each type has a particular seed.
%The seeds range from 0 to 24. The input s is added
%to each seed: e.g. if s = 0, then the seeds will be
%from 1 to 25, if s = 10 then the seeds will be from
%11 to 35.
%-----

[Ys1,n(1)] = arrange(types.type1,1);
[Ysim1] = Simwcopula(Ys1,4*length(Ys1(:,1)),s+1);
[Ys2,n(2)] = arrange(types.type2,2);
[Ysim2] = Simwcopula(Ys2,4*length(Ys2(:,1)),s+2);
[Ys3,n(3)] = arrange(types.type3sub1,3);
[Ysim3] = Simwcopula(Ys3,4*length(Ys3(:,1)),s+3);
[Ys4,n(4)] = arrange(types.type3sub2,4);
[Ysim4] = Simwcopula(Ys4,4*length(Ys4(:,1)),s+4);
[Ys5,n(5)] = arrange(types.type3sub3,5);
[Ysim5] = Simwcopula(Ys5,4*length(Ys5(:,1)),s+5);
[Ys6,n(6)] = arrange(types.type4sub1,3);
[Ysim6] = Simwcopula(Ys6,4*length(Ys6(:,1)),s+6);
[Ys7,n(7)] = arrange(types.type4sub2,4);
[Ysim7] = Simwcopula(Ys7,4*length(Ys7(:,1)),s+7);
[Ys8,n(8)] = arrange(types.type4sub3,5);
[Ysim8] = Simwcopula(Ys8,4*length(Ys8(:,1)),s+8);
[Ys9,n(9)] = arrange(types.type5sub12,3);
[Ysim9] = Simwcopula(Ys9,4*length(Ys9(:,1)),s+9);
[Ys10,n(10)] = arrange(types.type5sub21,3);
[Ysim10] = Simwcopula(Ys10,4*length(Ys10(:,1)),s+10);

```

```

[Ys11,n(11)] = arrange(types.type5sub12r,3);
[Ysim11] = Simwcopula(Ys11,4*length(Ys11(:,1)),s+11);

[Ys12,n(12)] = arrange(types.type5sub131,4);
[Ysim12] = Simwcopula(Ys12,4*length(Ys12(:,1)),s+12);
[Ys13,n(13)] = arrange(types.type5sub141,4);
[Ysim13] = Simwcopula(Ys13,4*length(Ys13(:,1)),s+13);
[Ys14,n(14)] = arrange(types.type5sub22,4);
[Ysim14] = Simwcopula(Ys14,4*length(Ys14(:,1)),s+14);
[Ys15,n(15)] = arrange(types.type5sub311,4);
[Ysim15] = Simwcopula(Ys15,4*length(Ys15(:,1)),s+15);
[Ys16,n(16)] = arrange(types.type5sub411,4);
[Ysim16] = Simwcopula(Ys16,4*length(Ys16(:,1)),s+16);
[Ys17,n(17)] = arrange(types.type5sub131r,4);
[Ysim17] = Simwcopula(Ys17,4*length(Ys17(:,1)),s+17);
[Ys18,n(18)] = arrange(types.type5sub22r,4);
[Ysim18] = Simwcopula(Ys18,4*length(Ys18(:,1)),s+18);

[Ys19,n(19)] = arrange(types.type5sub132,5);
[Ysim19] = Simwcopula(Ys19,4*length(Ys19(:,1)),s+19);
[Ys20,n(20)] = arrange(types.type5sub142,5);
[Ysim20] = Simwcopula(Ys20,4*length(Ys20(:,1)),s+20);
[Ys21,n(21)] = arrange(types.type5sub231,5);
[Ysim21] = Simwcopula(Ys21,4*length(Ys21(:,1)),s+21);
[Ys22,n(22)] = arrange(types.type5sub241,5);
[Ysim22] = Simwcopula(Ys22,4*length(Ys22(:,1)),s+22);
[Ys23,n(23)] = arrange(types.type5sub312,5);
[Ysim23] = Simwcopula(Ys23,4*length(Ys23(:,1)),s+23);
[Ys24,n(24)] = arrange(types.type5sub132r,5);

```



```

[Ysim24] = Simwcopula(Ys24,4*length(Ys24(:,1)),s+24);
[Ys25,n(25)] = arrange(types.type5sub142r,5);
[Ysim25] = Simwcopula(Ys25,4*length(Ys25(:,1)),s+25);

%pick the quantity of each type based on the profile distribution of...
    the
%actual data
rng(27,'twister');
R = randsample(1:25,N,true,[types.freq(1:8) types.freq5(1:17)]/sum([...
    types.freq(1:8) types.freq5(1:17)]));
j1 = 0; j2 = 0; j3 = 0; j4 = 0; j5 = 0; j6 = 0; j7 = 0; j8 = 0; j9 = ...
    0; j10 = 0; j11 = 0;
j12 = 0; j13 = 0; j14 = 0; j15 = 0; j16 = 0; j17 = 0; j18 = 0; j19 = ...
    0; j20 = 0; j21 = 0; j22 = 0;
j23 = 0; j24 = 0; j25 = 0;
pres = [length(find(R==1)) length(find(R==2)) length(find(R==3)) ...
    length(find(R==4)) length(find(R==5)) length(find(R==6)) length(...
    find(R==7))...
    length(find(R==8)) length(find(R==9)) length(find(R==10)) length...
    (find(R==11)) length(find(R==12)) length(find(R==13)) length(...
    find(R==14))...
    length(find(R==15)) length(find(R==16)) length(find(R==17)) ...
    length(find(R==18)) length(find(R==19)) length(find(R==20)) ...
    length(find(R==21))...
    length(find(R==22)) length(find(R==23)) length(find(R==24)) ...
    length(find(R==25))];

aMax = zeros(1,N);
duration = zeros(1,N);

```

```

durationtomax = zeros(1,N);
a= struct('values',[]);
time = struct('values',[]);
timeindex(1) = 1;
timevec(1) = 0;
options = optimoptions('fsolve');
options.MaxFunEvals = 2000;
options.MaxIter = 2000;
options.Display = 'off';
options.Algorithm='Levenberg-Marquardt';
for i = 1:N

    if i<N
        h = 0;
        for k = 1:2
            if R(i+1)==R(i)
                h = 1;
            end
            if k==1
                r = R(i+1);
            elseif k==2
                r = R(i);
            end
        end
        if r == 1;
            if h==1
                if k ==1
                    j1 = j1+2;
                elseif k==2
                    j1 = j1+1;
                end
            end
        end
    end
end

```

```

        end

        else

            j1 = j1+1;

        end

x1 = Ysim1(j1,:);
gtypes.type1(j1).values = x1;
aMax(i) = x1(length(x1(1,:))-1);
duration(i) = x1(length(x1(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x1(2)*duration(i);
a(i).values = aMax(i)*eq1(x1(1:length(x1)-1),t);
[v,d,j] = VDJeql(x1(1:length(x1)-1),t,duration(i));
ty(i) = 1;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
    j1 = j1-2;
else
    j1 = j1-1;
end
end

end

if r == 2;

```

```

        if h==1
            if k ==1
                j2= j2+2;
            elseif k==2
                j2 = j2+1;
            end
        else
            j2 = j2+1;
        end

x2 = Ysim2(j2,:);
gtypes.type2(j2).values = x2;
aMax(i) = x2(length(x2(1,:)));
duration(i) = x2(length(x2(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x2(5)*duration(i);
a(i).values = aMax(i)*eq2(x2(1:length(x2)-1),t);
[v,d,j] = VDJe2(x2(1:length(x2)-1),t,duration(i));
ty(i) = 2;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
    j2 = j2-2;
else
    j2 = j2-1;

```

```

end
end
    end
    if r == 3;
        if h==1
            if k ==1
                j3 = j3+2;
            elseif k==2
                j3 = j3+1;
            end
        else
            j3 = j3+1;
        end
x3 = Ysim3(j3,:);
gtypes.type3sub1(j3).values = x3;
aMax(i) = x3(length(x3(1,:)));
duration(i) = x3(length(x3(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x3(8)*duration(i);
a(i).values = aMax(i)*eq3(x3(1:length(x3)-1),t);
[v,d,j] = VDJe3(x3(1:length(x3)-1),t,duration(i));
ty(i) = 31;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1

```

```

if h == 1
    j3 = j3-2;
else
    j3 = j3-1;
end
end
end
    if r == 4;
        if h==1
            if k ==1
                j4 = j4+2;
            elseif k==2
                j4 = j4+1;
            end
        else
            j4 = j4+1;
        end
    end
x4 = Ysim4(j4,:);
gtypes.type3sub2(j4).values = x4;
aMax(i) = x4(length(x4(1,:)));
duration(i) = x4(length(x4(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x4(11)*duration(i);
a(i).values = aMax(i)*eq4(x4(1:length(x4)-1),t);
[v,d,j] = VDJe4(x4(1:length(x4)-1),t,duration(i));
ty(i) = 32;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;

```

```

duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j4 = j4-2;
else
j4 = j4-1;
end
end
end
if r == 5;
if h==1
if k ==1
j5 = j5+2;
elseif k==2
j5 = j5+1;
end
else
j5 = j5+1;
end
x5 = Ysim5(j5,:);
gtypes.type3sub3(j5).values = x5;
aMax(i) = x5(length(x5(1,:))-1);
duration(i) = x5(length(x5(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x5(14)*duration(i);
a(i).values = aMax(i)*eq5(x5(1:length(x5)-1),t);
[v,d,j] = VDJeq5(x5(1:length(x5)-1),t,duration(i));

```

```

ty(i) = 33;

if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j5 = j5-2;
else
j5 = j5-1;
end
end
end
if r == 6;
if h==1
if k ==1
j6 = j6+2;
elseif k==2
j6 = j6+1;
end
else
j6 = j6+1;
end
x6 = Ysim6(j6,:);
gtypes.type4sub1(j6).values = x6;
aMax(i) = x6(length(x6(1,:)));
duration(i) = x6(length(x6(1,:))-1);

```



```

t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x6(8)*duration(i);
a(i).values = aMax(i)*eq3(x6(1:length(x6)-1),t);
[v,d,j] = VDJe3(x6(1:length(x6)-1),t,duration(i));
ty(i) = 41;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j6 = j6-2;
else
j6 = j6-1;
end
end
end
if r == 7;
if h==1
if k ==1
j7 = j7+2;
elseif k==2
j7 = j7+1;
end
else
j7 = j7+1;
end
end

```

```

x7 = Ysim7(j7,:);
gtypes.type4sub2(j7).values = x7;
aMax(i) = x7(length(x7(1,:)));
duration(i) = x7(length(x7(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x7(11)*duration(i);
a(i).values = aMax(i)*eq4(x7(1:length(x7)-1),t);
[v,d,j] = VDJe4(x7(1:length(x7)-1),t,duration(i));
ty(i) = 42;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j7 = j7-2;
else
j7 = j7-1;
end
end
end
if r == 8;
    if h==1
        if k ==1
            j8 = j8+2;
        elseif k==2
            j8 = j8+1;

```

```

        end

        else

            j8 = j8+1;

        end

x8 = Ysim8(j8,:);
gtypes.type4sub3(j8).values = x8;
aMax(i) = x8(length(x8(1,:)));
duration(i) = x8(length(x8(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x8(14)*duration(i);
a(i).values = aMax(i)*eq5(x8(1:length(x8)-1),t);
[v,d,j] = VDJeq5(x8(1:length(x8)-1),t,duration(i));
ty(i) = 43;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
    j8 = j8-2;
else
    j8 = j8-1;
end
end
end

    end

    if r == 9;
        if h==1

```

```

        if k ==1
            j9 = j9+2;
        elseif k==2
            j9 = j9+1;
        end
    else
        j9 = j9+1;
    end

x9 = Ysim9(j9,:);
gtypes.type5sub12(j9).values = x9;
aMax(i) = x9(length(x9(1,:)));
duration(i) = x9(length(x9(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x9(8)*duration(i);
a(i).values = aMax(i)*eq3(x9(1:length(x9)-2),t);
[v,d,j] = VDJe3(x9(1:length(x9)-1),t,duration(i));
ty(i) = 12;
if isempty(a(i).values(1))==1 || isnan(a(i).values(1))== 1
    a(i).values = ones(1,floor(duration(i)/p));
    aMax(i) = 0;
    duration(i) = 0;
    durationtomax(i);
end
if k==1
    if h == 1
        j9 = j9-2;
    else
        j9 = j9-1;
    end
end

```

```

end

    end

    if r == 10;

        if h==1

            if k ==1

                j10 = j10+2;

            elseif k==2

                j10 = j10+1;

            end

        else

            j10 = j10+1;

        end

x10 = Ysim10(j10,:);
gtypes.type5sub21(j10).values = x10;
aMax(i) = x10(length(x10(1,:)));
duration(i) = x10(length(x10(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x10(8)*duration(i);
a(i).values = aMax(i)*eq3(x10(1:length(x10)-2),t);
[v,d,j] = VDJe3(x10(1:length(x10)-1),t,duration(i));
ty(i) = 21;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end

if k==1

if h == 1

```

```

        j10 = j10-2;
else
    j10 = j10-1;
end
end
end

    if r == 11
        if h==1
            if k ==1
                j11 = j11+2;
            elseif k==2
                j11 = j11+1;
            end
        else
            j11 = j11+1;
        end
    end

x11 = Ysim11(j11,:);
gtypes.type5sub12r(j11).values = x11;
aMax(i) = x11(length(x11(1,:))-1);
duration(i) = x11(length(x11(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x11(8)*duration(i);
a(i).values = aMax(i)*eq3(x11(1:length(x11)-2),t);
[v,d,j] = VDJeq3(x11(1:length(x11)-1),t,duration(i));
ty(i) = 122;
if isempty(a(i).values(1))==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;

```

```

durationtomax(i);
end
if k==1
if h == 1
j11 = j11-2;
else
j11 = j11-1;
end
end
end
if r == 12;
if h==1
if k ==1
j12 = j12+2;
elseif k==2
j12 = j12+1;
end
else
j12 = j12+1;
end
x12 = Ysim12(j12,:);
gtypes.type5sub131(j12).values = x12;
aMax(i) = x12(length(x12(1,:)));
duration(i) = x12(length(x12(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x12(11)*duration(i);
a(i).values = aMax(i)*eq4(x12(1:length(x12)-2),t);
[v,d,j] = VDJe4(x12(1:length(x12)-1),t,duration(i));
ty(i) = 131;

```

```

if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j12 = j12-2;
else
j12 = j12-1;
end
end
end
    end
    if r == 13;
        if h==1
            if k ==1
                j13 = j13+2;
            elseif k==2
                j13 = j13+1;
            end
        else
            j13 = j13+1;
        end
    end
x13 = Ysim13(j13,:);
gtypes.type5sub141(j13).values = x13;
aMax(i) = x13(length(x13(1,:)));
duration(i) = x13(length(x13(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));

```



```

durationtomax(i) = x13(11)*duration(i);
a(i).values = aMax(i)*eq4(x13(1:length(x13)-2),t);
[v,d,j] = VDJe3(x13(1:length(x13)-1),t,duration(i));
ty(i) = 141;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j13 = j13-2;
else
j13 = j13-1;
end
end
end
if r == 14;
if h==1
if k ==1
j14 = j14+2;
elseif k==2
j14 = j14+1;
end
else
j14 = j14+1;
end
end
x14 = Ysim14(j14,:);

```

```

gtypes.type5sub22(j14).values = x14;
aMax(i) = x14(length(x14(1,:)));
duration(i) = x14(length(x14(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x14(11)*duration(i);
a(i).values = aMax(i)*eq4(x14(1:length(x14)-2),t);
[v,d,j] = VDJe4(x14(1:length(x14)-1),t,duration(i));
ty(i) = 22;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j14 = j14-2;
else
j14 = j14-1;
end
end
end
if r == 15;
if h==1
if k ==1
j15 = j15+2;
elseif k==2
j15 = j15+1;
end
end

```

```

        else
            j15 = j15+1;
        end

x15 = Ysim15(j15,:);
gtypes.type5sub311(j15).values = x15;
aMax(i) = x15(length(x15(1,:)));
duration(i) = x15(length(x15(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x15(11)*duration(i);
a(i).values = aMax(i)*eq4(x15(1:length(x15)-2),t);
[v,d,j] = VDJe4(x15(1:length(x15)-1),t,duration(i));
ty(i) = 311;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
    a(i).values = ones(1,floor(duration(i)/p));
    aMax(i) = 0;
    duration(i) = 0;
    durationtomax(i);
end
if k==1
    if h == 1
        j15 = j15-2;
    else
        j15 = j15-1;
    end
end
end

end

if r == 16;
    if h==1
        if k ==1

```

```

        j16 = j16+2;
    elseif k==2
        j16 = j16+1;
    end
else
    j16 = j16+1;
end

x16 = Ysim16(j16,:);
gtypes.type5sub411(j16).values = x16;
aMax(i) = x16(length(x16(1,:)));
duration(i) = x16(length(x16(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x16(11)*duration(i);
a(i).values = aMax(i)*eq4(x16(1:length(x16)-2),t);
[v,d,j] = VDJe4(x16(1:length(x16)-1),t,duration(i));
ty(i) = 411;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
    j16 = j16-2;
else
    j16 = j16-1;
end
end
end

```

```

end
if r == 17;
    if h==1
        if k ==1
            j17 = j17+2;
        elseif k==2
            j17 = j17+1;
        end
    else
        j17 = j17+1;
    end
end
x17 = Ysim17(j17,:);
gtypes.type131r(j17).values = x17;
aMax(i) = x17(length(x17(1,:)));
duration(i) = x17(length(x17(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x17(11)*duration(i);
a(i).values = aMax(i)*eq4(x17(1:length(x17)-2),t);
[v,d,j] = VDJe4(x17(1:length(x17)-1),t,duration(i));
ty(i) = 1312;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
    j17 = j17-2;

```

```

else
    j17 = j17-1;
end
end
end

if r == 18;
    if h==1
        if k ==1
            j18 = j18+2;
        elseif k==2
            j18 = j18+1;
        end
    else
        j18 = j18+1;
    end

x18 = Ysim18(j18,:);
gtypes.type5sub22r(j18).values = x18;
aMax(i) = x18(length(x18(1,:)));
duration(i) = x18(length(x18(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x18(11)*duration(i);
a(i).values = aMax(i)*eq4(x18(1:length(x18)-2),t);
[v,d,j] = VDJe4(x18(1:length(x18)-1),t,duration(i));
ty(i) = 222;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);

```

```

end
if k==1
if h == 1
    j18 = j18-2;
else
    j18 = j18-1;
end
end
end
    end
    if r == 19;
        if h==1
            if k ==1
                j19 = j19+2;
            elseif k==2
                j19 = j19+1;
            end
        else
            j19 = j19+1;
        end

        x19 = Ysim19(j19,:);
gtypes.type5sub132(j19).values = x19;
aMax(i) = x19(length(x19(1,:)));
duration(i) = x19(length(x19(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x19(14)*duration(i);
a(i).values = aMax(i)*eq5(x19(1:length(x19)-2),t);
[v,d,j] = VDJe5(x19(1:length(x19)-1),t,duration(i));
ty(i) = 132;

```

```

if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j19 = j19-2;
else
j19 = j19-1;
end
end
end
    if r == 20;
        if h==1
            if k ==1
                j20 = j20+2;
            elseif k==2
                j20 = j20+1;
            end
        else
            j20 = j20+1;
        end
    end
x20 = Ysim20(j20,:);
gtypes.type5sub142(j20).values = x20;
aMax(i) = x20(length(x20(1,:)));
duration(i) = x20(length(x20(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));

```



```

durationtomax(i) = x20(14)*duration(i);
a(i).values = aMax(i)*eq5(x20(1:length(x20)-2),t);
[v,d,j] = VDJe5(x20(1:length(x20)-1),t,duration(i));
ty(i) = 142;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j20 = j20-2;
else
j20 = j20-1;
end
end
end
end
if r == 21;
if h==1
if k ==1
j21 = j21+2;
elseif k==2
j21 = j21+1;
end
else
j21 = j21+1;
end
end
x21 = Ysim21(j21,:);

```

```

gtypes.type5sub231(j21).values = x21;
aMax(i) = x21(length(x21(1,:)));
duration(i) = x21(length(x21(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x21(14)*duration(i);
a(i).values = aMax(i)*eq5(x21(1:length(x21)-2),t);
[v,d,j] = VDJe5(x21(1:length(x21)-1),t,duration(i));
ty(i) = 231;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end
if k==1
if h == 1
j21 = j21-2;
else
j21 = j21-1;
end
end
end
if r == 22;
if h==1
if k ==1
j22 = j22+2;
elseif k==2
j22 = j22+1;
end
end

```

```

        else
            j22 = j22+1;
        end

x22 = Ysim22(j22,:);
gtypes.type5sub241(j22).values = x22;
aMax(i) = x22(length(x22(1,:)));
duration(i) = x22(length(x22(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x22(14)*duration(i);
a(i).values = aMax(i)*eq5(x22(1:length(x22)-2),t);
[v,d,j] = VDJe5(x22(1:length(x22)-1),t,duration(i));
ty(i) = 241;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
    a(i).values = ones(1,floor(duration(i)/p));
    aMax(i) = 0;
    duration(i) = 0;
    durationtomax(i);
end
if k==1
    if h == 1
        j22 = j22-2;
    else
        j22 = j22-1;
    end
end
end

    end

    if r == 23;
        if h==1

```

```

        if k ==1
            j23 = j23+2;
        elseif k==2
            j23 = j23+1;
        end
    else
        j23 = j23+1;
    end

x23 = Ysim23(j23,:);
aMax(i) = x23(length(x23(1,:)));
gtypes.type5sub312(j23).values = x23;
duration(i) = x23(length(x23(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x23(14)*duration(i);
a(i).values = aMax(i)*eq5(x23(1:length(x23)-2),t);
[v,d,j] = VDJe5(x23(1:length(x23)-1),t,duration(i));
ty(i) = 312;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
    a(i).values = ones(1,floor(duration(i)/p));
    aMax(i) = 0;
    duration(i) = 0;
    durationtomax(i);
end
if k==1
    if h == 1
        j23 = j23-2;
    else
        j23 = j23-1;
    end
end

```

```

end

    end

    if r == 24;

        if h==1

            if k ==1

                j24 = j24+2;

            elseif k==2

                j24 = j24+1;

            end

        else

            j24 = j24+1;

        end

x24 = Ysim24(j24,:);
gtypes.type5sub132r(j24).values = x24;
aMax(i) = x24(length(x24(1,:)));
duration(i) = x24(length(x24(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x24(14)*duration(i);
a(i).values = aMax(i)*eq5(x24(1:length(x24)-2),t);
[v,d,j] = VDJe5(x24(1:length(x24)-1),t,duration(i));
ty(i) = 1322;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;
durationtomax(i);
end

if k==1

if h == 1

```

```

        j24 = j24-2;
    else
        j24 = j24-1;
    end
end
end

    end

    if r == 25;

        if h==1

            if k ==1

                j25 = j25+2;

            elseif k==2

                j25 = j25+1;

            end

        else

            j25 = j25+1;

        end

x25 = Ysim25(j25,:);
gtypes.type5sub142r(j25).values = x25;
aMax(i) = x25(length(x25(1,:))-1);
duration(i) = x25(length(x25(1,:))-1);
t = linspace(0,1,floor(duration(i)/p));
durationtomax(i) = x25(14)*duration(i);
a(i).values = aMax(i)*eq5(x25(1:length(x25)-2),t);
[v,d,j] = VDJeq5(x25(1:length(x25)-1),t,duration(i));
ty(i) = 1422;
if isempty(a(i).values)==1 || isnan(a(i).values(1))== 1
a(i).values = ones(1,floor(duration(i)/p));
aMax(i) = 0;
duration(i) = 0;

```

```

durationtomax(i);
end
if k==1
if h == 1
j25 = j25-2;
else
j25 = j25-1;
end
end
end
end
if k == 1
s2 = (aMax(i)*j(1));
offsetnextcycle=(aMax(i)*(d(1)-d(length(d))))/duration(i);
vb = aMax(i)*v(1)+offsetnextcycle;
elseif k==2
s1 = aMax(i)*j(length(j));
offsetcurrentcycle=(aMax(i)*(d(1)-d(length(d))))/duration(i)...
;
end

end

if i==1
off = aMax(i)*v(1)+offsetcurrentcycle;
off2 = 0;
end

%The cam profiles and their integrals

```

```

f1 = @(k,t1,x) -k*sin((pi*x)/(2*t1));
f2 = @(k,t3,x) -k*cos((pi*x)/(2*t3));
intf1 = @(k,t1,x) k*((2*t1)/pi)*cos((pi*x)/(2*t1));
intf2 = @(k,t3,x) -k*((2*t3)/pi)*sin((pi*x)/(2*t3));

df1 = @(k,t1,x) k*((2*t1)/pi)^2*sin((pi*x)/(2*t1));
df2 = @(k,t3,x) k*((2*t3)/pi)^2*cos((pi*x)/(2*t3));

fs = @(k,t,x,o) 0.5*k*(sin((pi*(x+t/2))/t)-1)+o;
intfs = @(k,t,x,o) 0.5*k*(-(t/(pi))*cos((pi*(x+t/2))/t)-x)+o*x;
dfs = @(k,t,x,o) 0.5*k*(-(t/(pi))^2*sin((pi*(x+t/2))/t)-0.5*x.^2)+...
    o*x.^2;

fs2 = @(k,t,x,o) 0.5*k*(sin((pi*(x+t/2))/t)-1)+o;
intfs2 = @(k,t,x,o) 0.5*k*(-(t/(pi))*cos((pi*(x+t/2))/t)-x)+o*x;
dfs2 = @(k,t,x,o) 0.5*k*(-(t/(pi))^2*sin((pi*(x+t/2))/t)-0.5*x.^2)...
    +o*x.^2;

f1= @(k,x) -k;
intf1=@(k,x) -k*x;
df1=@(k,x) -k*x.^2;

if i==1
    lastd = 0;
    Ires=0;
end

```



```

g3 = @(x) [df1(abs(x(1)),abs(x(2)))+(intf1(abs(x(1)),-(abs(x(1))...
    *pi)/(2*s1),-(abs(x(1))*pi)/(2*s1))+aMax(i)*v(length(v))+off-...
    aMax(i)*v(1)-...
        intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),0)-intf1(abs(x...
            (1)),0))*abs(x(2))+...
    df1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),-(abs(x(1))*pi)...
        /(2*s1))+aMax(i)*v(length(v))+...
    off-aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*...
        s1),0))*-(abs(x(1))*pi)/(2*s1)+aMax(i)*d(length(d)...
        )+off2-...
    aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(abs(x...
        (1)),-(abs(x(1))*pi)/(2*s1),0)-...
    df1(abs(x(1)),0);...%(1) df1 = 0
    intf2(1,(1*pi)/(2*s2),(1*pi)/(2*s2))+intfs((1-abs(x...
        (1)),abs(x(3)),abs(x(3)),-abs(x(1)))+intf1(abs(x...
        (1)),abs(x(2)))+...
    intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),-(abs(x(1))*pi...
        )/(2*s1))+...
    aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(abs(x(1))...
        ,-(abs(x(1))*pi)/(2*s1),0)-intf1(abs(x(1)),0)-...
    intfs((1-abs(x(1))),abs(x(3)),0,-abs(x(1)))-intf2...
        (1,(1*pi)/(2*s2),0)-vb;...%(3) vtarget
    df2(1,(1*pi)/(2*s2),(1*pi)/(2*s2))+intfs((1-abs(x(1))...
        ),abs(x(3)),abs(x(3)),-abs(x(1)))+intf1(abs(x(1))...
        ,abs(x(2)))+...
    intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),-(abs(x(1))*pi...
        )/(2*s1))+aMax(i)*v(length(v))+...
    off-aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*...
        s1),0)-...

```

```

intf1(abs(x(1)),0)-intfs((1-abs(x(1))),abs(x(3)),0,-...
    abs(x(1))-intf2(1,(1*pi)/(2*s2),0))...
*((1*pi)/(2*s2))+dfs((1-abs(x(1))),abs(x(3)),abs(x(3)...
    ),-abs(x(1)))+...
(intf1(abs(x(1)),abs(x(2)))+intf1(abs(x(1)),-(abs(x...
    (1))*pi)/(2*s1),-(abs(x(1))*pi)/(2*s1))+aMax(i)*v(...
    length(v))+...
off-aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*...
    s1),0)-intf1(abs(x(1)),0)-...
intfs((1-abs(x(1))),abs(x(3)),0,-abs(x(1))))*abs(x(3)...
    )+...
df1(abs(x(1)),abs(x(2)))+...
df1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),-(abs(x(1))*pi)...
    /(2*s1))+...
(aMax(i)*v(length(v))+...
off-aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*...
    s1),0))*(-(abs(x(1))*pi)/(2*s1))+aMax(i)*d(length(...
    d))+off2-aMax(i)*d(1)+...
(off-aMax(i)*v(1))*duration(i)-...
df1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),0)-df1(abs(x(1))...
    ,0)+...
(intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),-(abs(x(1))*...
    pi)/(2*s1))+aMax(i)*v(length(v))+off-...
aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1)...
    ,0)-intf1(abs(x(1)),0))*abs(x(2))-dfs((1-abs(x(1))...
    ),abs(x(3)),0,-abs(x(1)))-...
df2(1,(1*pi)/(2*s2),0)]; %(4) displacement = 0

```

```

[sol3,fval3,exitflag3] = fsolve(g3,[0.2 0.1 duration(i)],...
    options);
if abs(fval3(1))<0.05 && fval3(2)>-0.05 && abs(fval3(3))<0.05 ...
    && -(abs(sol3(1))*pi)/(2*s1)+abs(sol3(2))+abs(sol3(3))+(pi)...
    /(2*s2)<5*duration(i)

k1 = abs(sol3(1));
k1 = abs(sol3(1));
t1 = -(k1*pi)/(2*s1);
o = -abs(sol3(1));
k2 = 1;
ks = k2-abs(sol3(1));
t1 = abs(sol3(2));
t2 = (k2*pi)/(2*s2);
ts = abs(sol3(3));

azz = [a(i).values, f1(k1,t1,(t1/10):(t1/10):t1), f1(k1)*ones(1,...
    length((t1/10):(t1/10):t1)),fs(ks,ts,(ts/10):(ts/10):ts,o),...
    f2(k2,t2,linspace(t2/10,t2,length(t2/10:t2/10:t2)))];

time1 = linspace(0,duration(i),floor(duration(i)/p));
time2 = ((t1/10):(t1/10):t1)+time1(length(time1));
time3 = ((t1/10):(t1/10):t1)+time2(length(time2));
time4 = ((ts/10):(ts/10):ts)+time3(length(time3));
time5 = (linspace(t2/10,t2,length(t2/10:t2/10:t2)))+time4(length(...
    time4));
T = [time1,time2,time3,time4,time5];
clear time5

```

```

d1 = off-aMax(i)*v(1);
d2 = aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,t1,0);
d3 = intf1(k1,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,...
    t1,0)-intf1(k1,0);
d4 = intf1(k1,t1)+intf1(k1,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v...
    (1)-intf1(k1,t1,0)-intf1(k1,0)-intfs(ks,ts,0,o);
d5 = intfs(ks,ts,ts,o)+intf1(k1,t1)+intf1(k1,t1,t1)+aMax(i)*v(length...
    (v))+off-aMax(i)*v(1)-intf1(k1,t1,0)-intf1(k1,0)-intfs(ks,ts,0,o)...
    -intf2(k2,t2,0);

vzz = [aMax(i)*v+d1, intf1(k1,t1,(t1/10):(t1/10):t1)+d2, intf1(k1,((...
    t1/10):(t1/10):t1))+d3, intfs(ks,ts,(ts/10):(ts/10):ts,o)+d4,...
    intf2(k2,t2,linspace(t2/10,t2,length(t2/10:t2/10:t2)))+d5];

d11 = off2-aMax(i)*d(1);
d22 = aMax(i)*d(length(d))+off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*...
    duration(i)-df1(k1,t1,0);
d33 = df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,...
    t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0);
d44 = df1(k1,t1)+df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v...
    (1)-intf1(k1,t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0)+(intf1(k1,t1,t1)+aMax(i)*v(length(v))+...
    off-aMax(i)*v(1)-intf1(k1,t1,0)-intf1(k1,0))*t1-dfs(ks,ts,0,o)...
    ;

```

```

d55 = dfs(ks,ts,ts,o)+d4*ts+ dfl(kl,tl)+dfl(kl,tl,tl)+(aMax(i)*v(...
length(v))+off-aMax(i)*v(1)-intf1(kl,tl,0))*tl+aMax(i)*d(length(d...
))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-dfl(kl,tl,0)-...
    dfl(kl,0)+(intf1(kl,tl,tl)+aMax(i)*v(length(v))+...
    off-aMax(i)*v(1)-intf1(kl,tl,0)-intf1(kl,0))*tl-dfs(ks,ts,0,o)...
    -df2(k2,t2,0);

dzz = [aMax(i)*d+d1*linspace(0,duration(i),length(d))+d11, dfl(kl,tl...
, (tl/10):(tl/10):tl)+d2*((tl/10):(tl/10):tl)+d22, dfl(kl, (tl/10)...
:(tl/10):tl))+...
    d3*((tl/10):(tl/10):tl))+d33,...
    dfs(ks,ts, (ts/10):(ts/10):ts,o)+d4*((ts/10):(ts/10):ts)+d44,...
    df2(k2,t2,linspace(t2/10,t2,length(t2/10:t2/10:t2)))+...
    d5*linspace(t2/10,t2,length(t2/10:t2/10:t2))+d55];

Ires = aMax(i)*d(length(d))+d1*duration(i)-aMax(i)*d(1);
off = vzz(length(vzz));
off2 = dzz(length(dzz));

else

g1 = @(x)      [intf2(abs(x(1)), (abs(x(1))*pi)/(2*s2), (abs(x(1))*pi)...
/(2*s2))+intf1(abs(x(1)),abs(x(2)))+intf1(abs(x(1)), -(abs(x(1))*...
pi)/(2*s1), -(abs(x(1))*pi)/(2*s1))+...
    aMax(i)*v(length(v))+off-aMax(i)*v(1)-...
    intf1(abs(x(1)), -(abs(x(1))*pi)/(2*s1), 0)-intf1(abs(x...
(1)), 0)-intf2(abs(x(1)), (abs(x(1))*pi)/(2*s2), 0)-vb;
    df2(abs(x(1)), (abs(x(1))*pi)/(2*s2), (abs(x(1))*pi)/(2*...
s2))+...

```

```

(intf1(abs(x(1)),abs(x(2)))+intf1(abs(x(1)),-(abs(x(1))...
    *pi)/(2*s1),-(abs(x(1))*pi)/(2*s1))+aMax(i)*v(length...
    (v))+...
off-aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1)...
    ,0)-intf1(abs(x(1)),0)-intf2(abs(x(1)),(abs(x(1))*pi...
    )/(2*s2),0))*((abs(x(1))*pi)/(2*s2))+...
df1(abs(x(1)),abs(x(2)))+df1(abs(x(1)),-(abs(x(1))*pi)...
    /(2*s1),-(abs(x(1))*pi)/(2*s1))+aMax(i)*v(length(v)...
    )+off-...
aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),0))...
    *(-(abs(x(1))*pi)/(2*s1))+aMax(i)*d(length(d))+...
off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(...
    abs(x(1)),-(abs(x(1))*pi)/(2*s1),0)-df1(abs(x(1)),0)...
    +...
(intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1),-(abs(x(1))*pi)...
    /(2*s1))+aMax(i)*v(length(v))+...
off-aMax(i)*v(1)-intf1(abs(x(1)),-(abs(x(1))*pi)/(2*s1)...
    ,0)-intf1(abs(x(1)),0))*abs(x(2))-df2(abs(x(1)),(abs...
    (x(1))*pi)/(2*s2),0)];

[sol1,fval1,exitflag1] = fsolve(g1,[0.5 duration(i)/4],options)...
;
A(:,i) = g1(sol1);

k1 = abs(sol1(1));
t1 = abs(sol1(2));

t1 = -(k1*pi)/(2*s1);
k2=k1;

```

```

t2 = (k2*pi)/(2*s2);
kl=kl;

azz = [a(i).values, f1(kl,t1,(t1/10):(t1/10):t1), f1(kl)*ones(1,...
    length((t1/10):(t1/10):t1)), f2(k2,t2,linspace(t2/10,t2,length(t2...
    /10:t2/10:t2)))]);
time1 = linspace(0,duration(i),floor(duration(i)/p));
time2 = ((t1/10):(t1/10):t1)+time1(length(time1));
time3 = ((t1/10):(t1/10):t1)+time2(length(time2));
time4 = (linspace(t2/10,t2,length(t2/10:t2/10:t2)))+time3(length(...
    time3));
T = [time1,time2,time3,time4];

d1 = off-aMax(i)*v(1);
d2 = aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(kl,t1,0);
d3 = intf1(kl,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(kl,...
    t1,0)-intf1(kl,0);
d4 = intf1(kl,t1)+intf1(kl,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v...
    (1)-intf1(kl,t1,0)-intf1(kl,0)-intf2(k2,t2,0);

vzz = [aMax(i)*v+d1, intf1(kl,t1,(t1/10):(t1/10):t1)+d2, intf1(kl,((...
    t1/10):(t1/10):t1))+d3, intf2(k2,t2,linspace(t2/10,t2,length(t2...
    /10:t2/10:t2)))+d4];

d11 = off2-aMax(i)*d(1);
d22 = aMax(i)*d(length(d))+off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*...
    duration(i)-dfl(kl,t1,0);

```

```

d33 = df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,...
t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0);
d44 = df1(k1,t1)+df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v...
(1)-intf1(k1,t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0)+(intf1(k1,t1,t1)+aMax(i)*v(length(v))+...
    off-aMax(i)*v(1)-intf1(k1,t1,0)-intf1(k1,0))*t1-df2(k2,t2,0);

dzz = [aMax(i)*d+d1*linspace(0,duration(i),length(d))+d11, df1(k1,t1...
, (t1/10):(t1/10):t1)+d2*((t1/10):(t1/10):t1)+d22, df1(k1, ((t1/10)...
:(t1/10):t1))+...
d3*((t1/10):(t1/10):t1))+d33,...
df2(k2,t2,linspace(t2/10,t2,length(t2/10:t2/10:t2)))+d4*linspace...
(t2/10,t2,length(t2/10:t2/10:t2))+d44];

if abs(fval1(1))<0.05 && fval1(2)>0.05

off = vzz(length(vzz));
off2 = dzz(length(dzz));
else

k1 =1;
g2 = @(x)    intf2(k1,(k1*pi)/(2*s2),(k1*pi)/(2*s2))+intf1(k1,...
abs(x))+intf1(k1,-(k1*pi)/(2*s1),-(k1*pi)/(2*s1))+...
    aMax(i)*v(length(v))+off-aMax(i)*v(1)-...
    intf1(k1,-(k1*pi)/(2*s1),0)-intf1(k1,0)-intf2(k1,(k1*pi...
)/(2*s2),0)-vb;

```



```

[sol2,fval2,exitflag2] = fsolve(g2,duration(i)/4,options);

t1 = abs(sol2);
t1 = -(k1*pi)/(2*s1);
k2=k1;
t2 = (k2*pi)/(2*s2);
k1=k1;

azz = [a(i).values, f1(k1,t1,(t1/10):(t1/10):t1), f1(k1)*ones(1,...
    length((t1/10):(t1/10):t1)), f2(k2,t2,linspace(t2/10,t2,length(t2...
    /10:t2/10:t2)))]];
time1 = linspace(0,duration(i),floor(duration(i)/p));
time2 = ((t1/10):(t1/10):t1)+time1(length(time1));
time3 = ((t1/10):(t1/10):t1)+time2(length(time2));
time4 = (linspace(t2/10,t2,length(t2/10:t2/10:t2)))+time3(length(...
    time3));
T = [time1,time2,time3,time4];

d1 = off-aMax(i)*v(1);
d2 = aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,t1,0);
d3 = intf1(k1,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,...
    t1,0)-intf1(k1,0);
d4 = intf1(k1,t1)+intf1(k1,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v...
    (1)-intf1(k1,t1,0)-intf1(k1,0)-intf2(k2,t2,0);

```

```

vzz = [aMax(i)*v+d1, intf1(k1,t1,(t1/10):(t1/10):t1)+d2, intf1(k1,((...
    t1/10):(t1/10):t1))+d3, intf2(k2,t2,linspace(t2/10,t2,length(t2...
    /10:t2/10:t2)))+d4];

d11 = off2-aMax(i)*d(1);
d22 = aMax(i)*d(length(d))+off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*...
    duration(i)-df1(k1,t1,0);
d33 = df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,...
    t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0);
d44 = df1(k1,t1)+df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v...
    (1)-intf1(k1,t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0)+(intf1(k1,t1,t1)+aMax(i)*v(length(v))+...
    off-aMax(i)*v(1)-intf1(k1,t1,0)-intf1(k1,0))*t1-df2(k2,t2,0);

dzz = [aMax(i)*d+d1*linspace(0,duration(i),length(d))+d11, df1(k1,t1...
    , (t1/10):(t1/10):t1)+d2*((t1/10):(t1/10):t1)+d22, df1(k1,((t1/10)...
    : (t1/10):t1))+...
    d3*((t1/10):(t1/10):t1))+d33, df2(k2,t2,linspace(t2/10,t2,...
    length(t2/10:t2/10:t2)))+d4*linspace(t2/10,t2,length(t2/10:t2...
    /10:t2))+d44];

if abs(df2(k2,t2,t2)+d4*t2+d44)<0.05 && fval2>0.05
off = vzz(length(vzz));
off2 = dzz(length(dzz));
elseif abs(fval1(1))<abs(df2(k2,t2,t2)+d4*t2+d44)
    k1 = abs(sol1(1));

```

```

t1 = abs(sol1(2));

t1 = -(k1*pi)/(2*s1);
k2=k1;
t2 = (k2*pi)/(2*s2);
k1=k1;

azz = [a(i).values, f1(k1,t1,(t1/10):(t1/10):t1), f1(k1)*ones(1,...
    length((t1/10):(t1/10):t1)), f2(k2,t2,linspace(t2/10,t2,length(t2...
    /10:t2/10:t2)))]);

time1 = linspace(0,duration(i),floor(duration(i)/p));
time2 = ((t1/10):(t1/10):t1)+time1(length(time1));
time3 = ((t1/10):(t1/10):t1)+time2(length(time2));
time4 = (linspace(t2/10,t2,length(t2/10:t2/10:t2)))+time3(length(...
    time3));
T = [time1,time2,time3,time4];

d1 = off-aMax(i)*v(1);
d2 = aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,t1,0);
d3 = intf1(k1,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,...
    t1,0)-intf1(k1,0);
d4 = intf1(k1,t1)+intf1(k1,t1,t1)+aMax(i)*v(length(v))+off-aMax(i)*v...
    (1)-intf1(k1,t1,0)-intf1(k1,0)-intf2(k2,t2,0);

vzz = [aMax(i)*v+d1, intf1(k1,t1,(t1/10):(t1/10):t1)+d2, intf1(k1,((...
    t1/10):(t1/10):t1))+d3, intf2(k2,t2,linspace(t2/10,t2,length(t2...
    /10:t2/10:t2)))+d4];

```

```

d11 = off2-aMax(i)*d(1);
d22 = aMax(i)*d(length(d))+off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*...
    duration(i)-df1(k1,t1,0);
d33 = df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v(1)-intf1(k1,...
    t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0);
d44 = df1(k1,t1)+df1(k1,t1,t1)+(aMax(i)*v(length(v))+off-aMax(i)*v...
    (1)-intf1(k1,t1,0))*t1+aMax(i)*d(length(d))+...
    off2-aMax(i)*d(1)+(off-aMax(i)*v(1))*duration(i)-df1(k1,t1,0)-...
    df1(k1,0)+(intf1(k1,t1,t1)+aMax(i)*v(length(v))+...
    off-aMax(i)*v(1)-intf1(k1,t1,0)-intf1(k1,0))*t1-df2(k2,t2,0);

dzz = [aMax(i)*d+d1*linspace(0,duration(i),length(d))+d11, df1(k1,t1...
    , (t1/10):(t1/10):t1)+d2*((t1/10):(t1/10):t1)+d22, df1(k1, ((t1/10)...
    :(t1/10):t1))+...
    d3*((t1/10):(t1/10):t1))+d33, df2(k2,t2,linspace(t2/10,t2,...
    length(t2/10:t2/10:t2)))+d4*linspace(t2/10,t2,length(t2/10:t2...
    /10:t2))+d44];
off = vzz(length(vzz));
off2 = dzz(length(dzz));
end

end

end

if i == 1

```

```

        timeindex(i+1) = length(azz);
        timevec(timeindex(i):timeindex(i+1)) = T;

time(i).values = time1;
az(timeindex(i):timeindex(i+1)) = azz;
vz(timeindex(i):timeindex(i+1)) = vzz;
dz(timeindex(i):timeindex(i+1)) = dzz;
    else
        timeindex(i+1) = timeindex(i)+length(azz);

        timevec(timeindex(i)+1:timeindex(i+1)) = T+timevec(timeindex(i));

time(i).values = time1;
az(timeindex(i)+1:timeindex(i+1)) = azz;
vz(timeindex(i)+1:timeindex(i+1)) = vzz;
dz(timeindex(i)+1:timeindex(i+1)) = dzz;
    end
    clear time1 time2 time3 time4 clear T

    end
end

figure
plot(timevec,az)
figure
plot(timevec,vz)
figure
plot(timevec,dz)

```

```
xlabel('Time (s)')
ylabel('Acceleration (g''s)')
generatedpeak = struct('az',a,'time',time,'aMax',aMax,'duration',...
    duration,'durationtomax',durationtomax,'ty',ty);

end
```

Appendix F

Code for Obtaining Parameter Mean Values

```
function [Yo,n] = organize(type,terms)

%-----
%Sorts the data for the type specified by the
%input type (e.g. types = types.type3sub2)
%like the function arrange but also organizes
%it in the following way:
%type 1 => [1-a1 x1 sig1]
%type 2 => [1-a1 x1 sig1 a2 x2 sig2]
%type 3 => [1-a1 x1 sig1 a2 x2 sig2 a3 x3 sig3]
%and so on ...
%-----

t = type;
```

```

n = length(type);
if terms == 1
    var = 2;
else
    var = 3*terms;
end

for i = 1:n
    s = isempty(t(i).values);
    if s == 0
        if terms == 1
            for j=1:var+4
                Y(i,j) = t(i).values(j);
            end
        else
            for j=1:var
                Y(i,j) = t(i).values(j);
            end

            Y(i,var+1) = t(i).values(var+terms-1+4);
            Y(i,var+2) = t(i).values(var+terms-1+3);

            % if var == 2
            %     Y(i,var+2) = t(i).values(var+1);
            %     Y(i,var+3) = t(i).values(var+2);
            % else
            %     Y(i,var+1) = t(i).values(var+1);
            %     Y(i,var+2) = t(i).values(var+2);
            % end

```



```

        end
    end
end

```

```

N = 0;
if terms==1
    Ysorted = Y;
else
    Ysorted = Y;
    for i = 1:n
        k = 0;
        for j = 1:terms-1
            k = k+2;
            l(j) = Y(i,k);
        end
        [sorted,ind] = sort(l);
        N = N+1;
        k = 0;
        for j = 1:2:2*(terms-1)
            k = k+1;
            Ysorted(N,j) = Y(i,2*ind(k)-1);
            Ysorted(N,j+1) = Y(i,2*ind(k));
        end
        k = 0;
        for j = 1:terms-1
            k = k+1;
            Ysorted(N,j+2*(terms-1)+1) = Y(i,2*(terms-1)+1+ind(k));
        end
    end
end

```

```

end
end

for w=1:n
    if terms ==1
        % Yo = Ysorted;
        Yo(:,1) = zeros(length(Ysorted(:,1)),1);
        Yo(:,2) = Ysorted(:,2);
        Yo(:,3) = Ysorted(:,1);
        Yo(:,4) = Ysorted(:,5);
        Yo(:,5) = Ysorted(:,6);
    else
        Yo = Ysorted;

        k = 0;
        Yo(:,1) = Ysorted(:,var);
        Yo(:,2) = Ysorted(:,var-1);
        Yo(:,3) = Ysorted(:,var-terms-1);
        k = 3;
        j = 0;
        for i = 1:terms-1
            Yo(:,k+1) = Ysorted(:,j+1);
            Yo(:,k+2) = Ysorted(:,j+2);
            Yo(:,k+3) = Ysorted(:,2*(terms-1)+1+i);
            k = k+3;
            j = j+2;
        end
    end
end

```

```
end
```

```
end
```

```
function [means] = getmeans(types)
```

```
%...
```

```
-----...
```

```
%Gets the mean values for Types 1 to 4 and outputs in means.
```

```
%The format of means is the same as the tables in the thesis:
```

```
% type 1 => means(1,1) = 1-a1, means(1,2) = x1, ...
```

```
% type 2 => means(2,1) = 1-a1, ...
```

```
%...
```

```
-----...
```

```
means = zeros(8,17);
```

```
[Y1] = organize(types.type1,1);
```

```
means(1,1:5) = mean(Y1);
```

```
[Y2] = organize(types.type2,2);
```

```
means(2,1:8) = mean(Y2);
```

```
[Y3] = organize(types.type3sub1,3);
```

```
means(3,1:11) = mean(Y3);
```

```
[Y4] = organize(types.type3sub2,4);
```

```
means(4,1:14) = mean(Y4);
```

```

[Y5] = organize(types.type3sub3,5);
means(5,1:17) = mean(Y5);
[Y6] = organize(types.type4sub1,3);
means(6,1:11) = mean(Y6);
[Y7] = organize(types.type4sub2,4);
means(7,1:14) = mean(Y7);
[Y8] = organize(types.type4sub3,5);
means(8,1:17) = mean(Y8);

```

```
end
```

```
function [means] = getmeanstype5(types)
```

```
%...
```

```
-----...
```

```
%Gets the mean values for Type 5 pulses and outputs in means.
```

```
% The format of means is the same as the tables in the thesis:
```

```
% type 1 => means(1,1) = 1-a1, means(1,2) = x1, ...
```

```
% type 2 => means(2,1) = 1-a1, ...
```

```
%...
```

```
-----...
```

```
means = zeros(17,17);
```

```
try
```

```

[Y12] = organize(types.type5sub12,3);
catch
Y12 = zeros(1,11);
end
if length(Y12(:,1))>1
means(1,1:11) = mean(Y12);

else
means(1,1:11) = Y12;

end
try
[Y21] = organize(types.type5sub21,3);
catch
Y21 =zeros(1,11);
end
if length(Y21(:,1))>1
means(2,1:11) = mean(Y21);

else
means(2,1:11) = Y21;
end

try
[Y12r] = organize(types.type5sub12r,3);
catch
Y12r = zeros(1,11);
end
if length(Y12r(:,1))>1

```

```

means(3,1:11) = mean(Y12r);

else
means(3,1:11) = Y12r;

end

%-----

try
[Y131] = organize(types.type5sub131,4);
catch
Y131 = zeros(1,14);
end
if length(Y131(:,1))>1
means(4,1:14) = mean(Y131);

else
means(4,1:14) = (Y131);

end
try
[Y141] = organize(types.type5sub141,4);
catch
Y141= zeros(1,14);
end
if length(Y141(:,1))>1
means(5,1:14) = mean(Y141);

```

```

else
means(5,1:14) = (Y141);

end
try
[Y22] = organize(types.type5sub22,4);
catch
Y22 = zeros(1,14);
end
if length(Y22(:,1))>1
means(6,1:14) = mean(Y22);

else
means(6,1:14) = Y22;

end
try
[Y311] = organize(types.type5sub311,4);
catch
Y311 = zeros(1,14);
end
if length(Y311(:,1))>1
means(7,1:14) = mean(Y311);

else
means(7,1:14) = Y311;

end
try

```

```

[Y411] = organize(types.type5sub411,4);
catch
Y411 = zeros(1,14);
end
if length(Y411(:,1))>1
means(8,1:14) = mean(Y411);

else
means(8,1:14) = (Y411);

end
try
[Y131r] = organize(types.type5sub131r,4);
catch
Y131r = zeros(1,14);
end
if length(Y131r(:,1))>1
means(9,1:14) = mean(Y131r);

else
means(9,1:14) = Y131r;

end
try
[Y22r] = organize(types.type5sub22r,4);
catch
Y22r = zeros(1,14);
end
if length(Y22r(:,1))>1

```



```

means(10,1:14) = mean(Y22r);

else
means(10,1:14) = Y22r;

end
%-----

try
[Y132] = organize(types.type5sub132,5);
catch
    Y132 = zeros(1,17);
end

if length(Y132(:,1))>1
means(11,1:17) = mean(Y132);

else
means(11,1:17) = Y132;

end
try
[Y142] = organize(types.type5sub142,5);
catch
Y142 = zeros(1,17);
end
if length(Y142(:,1))>1
means(12,1:17) = mean(Y142);

```

```

else
means(12,1:17) = Y142;

end
try
[Y231] = organize(types.type5sub231,5);
catch
Y231 = zeros(1,17);
end
if length(Y231(:,1))>1
means(13,1:17) = mean(Y231);

else
means(13,1:17) = Y231;

end
try
[Y241] = organize(types.type5sub241,5);
catch
Y241 = zeros(1,17);
end
if length(Y241(:,1))>1
means(14,1:17) = mean(Y241);

else
means(14,1:17) = Y241;

end
try

```

```

[Y312] = organize(types.type5sub312,5);
catch
Y312 = zeros(1,17);
end
if length(Y312(:,1))>1
means(15,1:17) = mean(Y312);

else
means(15,1:17) = Y312;

end
try
[Y132r] = organize(types.type5sub132r,5);
catch
Y132r = zeros(1,17);
end
if length(Y132r(:,1))>1
means(16,1:17) = mean(Y132r);

else
means(16,1:17) = Y132r;

end
try
[Y142r] = organize(types.type5sub142r,5);
catch
Y142r = zeros(1,17);
end
if length(Y142r(:,1))>1

```

```
means(17,1:17) = mean(Y142r);
```

```
else
```

```
means(17,1:17) = Y142r;
```

```
end
```

```
%-----
```

```
end
```