High-Energy Higgsstrahlung Cross-Section Suppression in the Minimal Composite Higgs Model

by

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Abstract

We compute the Higgsstrahlung cross section, $e^+e^- \rightarrow Z^0 H$, in an $SO(5) \times U(1)_{B-L}$ gauge-Higgs unification model, the Minimal Composite Higgs Model (MCHM) with the Medina-Shah-Wagner fermion embedding. We observe that the MCHM cross section is strongly suppressed at high energies in comparison with that of the Standard Model due to cancellations between successive Kaluza-Klein coupling contributions. We further investigate the role of particle mixing and decay widths in the cross-section couplings and behaviour. Finally, we determine that several characteristics of the model couplings which lead to this suppression – notably the magnitude and sign of the first KK mode coupling – are observable at an electron-positron collider such as the proposed International or Compact Linear Colliders.

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Statement of Originality

Chapters two and three of this thesis predominantly review material previously presented in the literature, and are referenced accordingly. The new research and analysis that make up the body of this thesis can be found in chapter four. This research is entirely new and original (to my knowledge), and has not yet been submitted to publication or as the final project of another degree. In the appendices we present further background and supporting information, notably the Feynman rules of the model, these have been derived independently and cross-checked against the literature where possible, as references indicate.

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Chapter 1

Introduction

In the Standard Model (SM) of particle physics, subatomic particles acquire mass when they interact with a single elementary particle called the Higgs boson. However, the SM Higgs mechanism has at least two problems. First, the flavour puzzle – it cannot explain why the masses of leptons and quarks fall into an ordered pattern. Second, the hierarchy problem – though loop diagrams contribute terms on the Planck scale ($M_p \approx 10^{19}$ GeV) to the calculation of the Higgs mass, the Higgs mass is required to be on the weak scale $M_w \approx 100$ GeV. Eliminating the Planck scale terms in the SM requires an undesirable and extreme fine-tuning of the model parameters.

Both problems can be solved by constructing a quantum field theory in a warped five-dimensional (5D) Anti-de-Sitter (AdS) spacetime bounded by two four-dimensional (4D) boundaries, an idea first proposed by Randall and Sundrum in 1999 [1]. The curvature of the fifth dimension causes the energy scale to decrease exponentially such that the first boundary has energies up to the Planck scale while the second has energies only up to the weak scale. The hierarchy problem can be solved by placing the Higgs boson near the weak-scale boundary, causing any large

mass contributions to be ‘warped down’ to the weak scale. The flavour puzzle can also be explained by the locations of the leptons and quarks in the fifth dimension, as particles close to the Higgs boson will have large masses while those farther away will have smaller masses.

According to the AdS/CFT conjecture [2], Randall-Sundrum (RS) models can be identified with a four dimensional strongly coupled conformal field theory (CFT₄); integrating the 5D theory over the fifth dimension provides an effective picture of this CFT. In this effective 4D perspective, the theory contains an infinite tower of 4D excitations – known as Kaluza-Klein (KK) modes – for each 5D particle; the zero modes of these towers correspond to Standard Model particles. Furthermore, as it is a 5D theory each four dimensional gauge boson must also have an associated scalar fifth component. As such, RS models have a much larger particle spectrum than the Standard Model. However, the masses of the higher order KK modes are constrained to be above a mass scale M_{kk} defined by the length of the fifth dimension, which must be larger than 1 TeV. Therefore the new Large Hadron Collider at CERN will be the first accelerator with the energy reach to observe the proposed extra particles.

A promising class of Randall-Sundrum models are those which employ Gauge-Higgs Unification (GHU), where the Higgs boson arises naturally as the scalar fifth component of one the gauge bosons. These models are dual to a 4D CFT that includes a composite¹ Higgs boson. The goal of this thesis is to gain a better theoretical understanding of how this composite Higgs interacts with other particles, how its fi-

¹It should be emphasized that for the purposes of this work the term ‘composite’ is meant to imply that a particle is not point-like, but has some finite size which influences its interactions with other particles. We do not propose to directly probe the constituents of the Higgs, or to comment upon the nature of its internal structure. Indeed, the effective Lagrangian of the 5D theory does not allow for that kind of analysis.

nite size would affect scattering processes, and whether such characteristics could be identifiable signatures of the model at experimental colliders. To this end, we present a calculation of the Higgsstrahlung, $e^+ e^- \rightarrow Z^0 H$, interaction in the Minimal Composite Higgs Model (MCHM) with the fermion representation proposed by Medina, Shah and Wagner (MSW) [3, 4]. This interaction is particularly suitable for our purposes as it is an s-channel interaction, ensuring that the initial interaction energy is transferred directly into the Higgs coupling. As the structure of this Higgs interaction is similar to that of the loop interactions that yield Planck scale mass corrections in the SM, we expect that the MCHM Higgsstrahlung cross section will demonstrate a similar suppression when compared with the SM cross section. The resulting rapid decay in the cross section with increasing collision energy would be characteristic of a composite particle form factor.

The structure of the thesis is as follows. In Chapter 2 we review the Standard Model, electroweak symmetry breaking (EWSB), and the problems that have led to investigation beyond the SM. In Chapter 3, we briefly introduce Randall-Sundrum models before providing a detailed description of the Minimal Composite Higgs model structure and the Medina-Shah-Wagner fermion representation. Chapter 4 provides the calculations, results and discussion of the MCHM Higgsstrahlung cross section and relevant coupling behaviour. We further provide a brief analysis of the feasibility of coupling extraction at both the International Linear Collider (ILC) and the Compact Linear Collider (CLIC). Although the most relevant formulas are provided in the text, detailed derivations of the model Feynman rules can be found in Appendix C.

Chapter 2

The Standard Model of Particle Physics

2.1 Introduction to the Standard Model

2.1.1 Forces and particles

The Standard Model (SM) of particle physics is the most complete theory to date that describes the fundamental interactions and behaviour of subatomic particles [5]. It contains three forces: the strong force, the weak force and the electromagnetic force.

The Standard Model does not include gravity. Gravity is an extremely weak force; even the Weak force itself is 10^{25} times stronger (see Table 2.1). As a result, gravity is negligible at the relevant scales for particle interactions, and may safely be neglected. Furthermore, gravity is described by General Relativity (GR), while the

Table 2.1 Forces and their relative strengths

Interaction	Strength	Range
Strong	1	10^{-15} m
Electromagnetism	10^{-2}	∞
Weak	10^{-5}	10^{-18} m
Gravity	10^{-39}	∞

Standard Model is a quantum field theory (QFT), and physicists have yet to successfully express GR in an effective QFT formalism. Until such a theory of quantum gravity is developed, unifying the two remains problematic.

Table 2.2 The particle spectrum of the Standard Model

Fermions				Bosons	
Generation	1	2	3	Gauge	Scalar
Leptons	ν_e neutrino	ν_μ neutrino	ν_τ neutrino	Z	Higgs, H
	Electron, e^-	Muon, μ	Tau, τ	$W^\pm$	
Quarks	Up, u	Charm, c	Top, t	Photon, γ	
	Down, d	Strange, s	Bottom, b	Gluon, g	

The particle spectrum of the Standard Model is outlined in Table 2.2. It contains twelve fundamental fermions – or spin 1/2 particles – as well as their anti-particles: six leptons (the electron, muon, tau and their associated neutrinos) and six

quarks (up, down, strange, charm, top and bottom). Unlike the leptons, the quarks are never found individually; they combine in pairs and triplets to form composite hadrons, such as the proton and neutron.

The SM also contains several vector (spin 1) gauge bosons which carry the three fundamental forces. The properties of these bosons are summarized in Table 2.3. The photon is a neutral, massless boson that carries the electromagnetic force, and therefore couples only to charged particles.

The gluons are also massless bosons, but they carry the strong force. There are eight different types of gluons, identified by combinations of the three strong charges, or colours. Gluons only couple to each other and to quarks, as they are the only other particles which carry colour charge.

The $W^{\pm 1}$ and Z are massive bosons that carry the weak force. The $W^{\pm}$ have charge ± 1 and have masses of 80.398 ± 0.023 GeV while the Z boson is neutral and has a mass of 91.1876 ± 0.0021 GeV [6]. They interact with each other, the fermions and the Higgs, and the charged $W^{\pm}$ additionally couple to the photon.

Finally, the Standard Model also includes a single scalar – spin 0 – boson, known as the Higgs. It is through their couplings to the Higgs that particles acquire their masses; therefore the Higgs couples only to massive particles at tree level.²

¹The W^+ and W^- have opposite charge but the same spin and mass; they are each other's anti-particles.

²“Tree level” indicates calculations up to zeroth order in perturbation theory; such calculations include only Feynman diagrams without loops (which sometimes look like trees).

Table 2.3: Properties of the Standard Model gauge bosons.

Interaction	Gauge Boson	Mass	Spin	Acts On
Strong	Gluon	0	1	Colour charge
Electromagnetism	Photon	0	1	Electric Charges
Weak	$W^\pm, Z^0$	80, 91 GeV	1	Weak Charge

2.1.2 Structure of the theory

The Standard Model is a non-abelian gauge theory as well as an effective quantum field theory. As a quantum field theory, the Standard Model describes the subatomic world in terms of fields. These fields are quantized, and their quanta correspond to the elementary particles. The theory is an *effective* field theory (EFT) in that it is accurate only up to some cut-off energy scale, and describes only the physics that is relevant over this energy range. Above the cut-off scale the SM is expected to break down due to the emergence of physics that is not included within the theory. It is possible that the SM is valid all the way up to the Planck scale ($M_p \approx 10^{19}$ GeV), where gravity becomes relevant. However, there are many compelling reasons to believe that the SM cutoff is in fact much lower (≈ 1 TeV), and that some form of new physics – as yet undetected – manifests at this scale; this concept will be discussed in further detail in Sec. 2.3.

As a gauge theory, the Standard Model Lagrangian is invariant under a certain symmetry group, $SU(3)_c \times SU(2)_L \times U(1)_Y$ [5]. For each generator of the group there is a corresponding gauge field, the quanta of which are the force-carrying gauge

bosons. It is non-abelian because the generators of the $SU(3)$ and $SU(2)$ sectors are non-commutative.

The $SU(3)$ symmetry describes the strong force sector, and its 8 generators correspond to the 8 types of gluons. This sector is described by an EFT within the SM known as Quantum Chromodynamics (QCD) [7].

The electromagnetic force is described by a $U(1)_{EM}$ EFT known as Quantum Electrodynamics (QED) [8]. The single generator of the abelian – or commutative – $U(1)$ symmetry group corresponds to the photon. However, in the Standard Model the electromagnetic sector is unified with the weak sector; the resulting electroweak sector is described by the $SU(2)_L \times U(1)_Y$ symmetry group which contains the $U(1)_{EM}$ group of QED. The three generators of $SU(2)_L$ along with the single generator of $U(1)_Y$ correspond to four gauge fields that mix to create the photon, $W_\mu^\pm$ and Z bosons.

2.2 Electroweak Symmetry Breaking

As seen in the preceding section, the gauge symmetry group of the Standard Model plays an important role in its structure. Such symmetries are very important in physics, as according to Noether’s theorem,

In any theory described by a Lagrangian, a symmetry of the Lagrangian corresponds to a conservation law [9].

As a gauge theory, the SM must obey gauge invariance, this requires that the Lagrangian remains invariant under local³ gauge transformations, which have the form

$$A_\mu \rightarrow U(x) A_\mu U^\dagger(x) + \frac{i}{g} U(x) \partial_\mu U^\dagger(x) \quad (2.1)$$

$$\phi \rightarrow U(x) \phi \quad (2.2)$$

for gauge fields A_μ (spin 1) and matter fields ϕ (spin 0 or $\pm\frac{1}{2}$) respectively [10]. Here $U(x)$ is an element of the symmetry group of the Lagrangian, and g is the gauge coupling.

However, a mass term for such a gauge boson does not obey this invariance. For example, consider the simplest case, a local $U(1)$ symmetry where $U(x) = \exp[i g \eta(x)]$. Then

$$\frac{1}{2} M^2 A_\mu A^\mu \rightarrow \frac{1}{2} M^2 (A_\mu A^\mu + 2 \partial_\mu \eta(x) A^\mu + \partial_\mu \eta(x) \partial^\mu \eta(x)) \quad (2.3)$$

Therefore for the gauge symmetry to be preserved the associated gauge bosons must be massless.

This presents a problem, for, as previously discussed, the weak bosons $W^\pm$ and Z are *not* massless, but have well-measured masses of about 80 and 91 GeV respectively [6]. As such, there must exist some mechanism by which the electroweak symmetry is spontaneously⁴ broken such that the weak gauge bosons are given masses but the photon remains massless. We therefore require the following symmetry-breaking scheme

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}, \quad (2.4)$$

³ If U is a constant, it is a *global* gauge transformation, a coordinate-dependent $U(x)$ results in a *local* gauge transformation.

⁴‘Spontaneous symmetry breaking’ means that the theory is defined so as to obey the symmetry, but the system it describes is always observed in a particular non-symmetric state.

where only the electromagnetic symmetry $U(1)_{EM}$ remains intact.

Such a mechanism must also introduce three new massless bosons – known as Goldstone bosons – to the theory according to Goldstone’s theorem,

“For every spontaneously broken continuous symmetry, the theory must contain a massless particle.” [11, 12]

However, one can always perform a gauge transformation to work in a particular gauge – known as unitary gauge – where the three extra degrees of freedom represented by the three Goldstone bosons become the longitudinal polarizations of the $W^\pm$ and Z bosons [10]. In essence, the process of electroweak symmetry breaking transforms a massless gauge boson with two polarizations into a massive boson with three polarizations.

2.2.1 The Higgs Mechanism

One method of breaking the electroweak symmetry is by use of the Higgs mechanism, which was first proposed by Peter Higgs in 1964 [13, 14]. The Higgs mechanism, in its original formulation, introduces a complex scalar doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i \phi_2 \\ \phi_3 + i \phi_4 \end{pmatrix} \quad (2.5)$$

into the SM Lagrangian, where ϕ^+ is charged, ϕ^0 is neutral, and $\phi_1, \phi_2, \phi_3, \phi_4$ denote real scalar degrees of freedom. The bosonic part of the electroweak Lagrangian then takes the form

$$L_B = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + |D_\mu \Phi|^2 - V(\Phi) , \quad (2.6)$$

where $V(\Phi)$ is a potential energy term that describes the self-interactions of Φ , we use the metric convention $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and the covariant derivative D_μ is given by

$$D_\mu = \partial_\mu - i \frac{g}{2} \sigma^a W_\mu^a - i \frac{g'}{2} Y B_\mu \quad (2.7)$$

In the above W_μ^a ($a = 1, 2, 3$) denotes the three bosons associated with the $\text{SU}(2)_L$ symmetry, B_μ is the single boson of the $\text{U}(1)_Y$ symmetry, σ^a are the Pauli matrices, g and g' are coupling constants and Y is the hypercharge

The most general renormalizable and $\text{SU}(2)_L$ invariant potential $V(\Phi)$ for the scalar doublet is given by

$$V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4, \quad (2.8)$$

where $\lambda > 0$ and $|\Phi|^2 = \Phi^\dagger \Phi$. If we require that $\mu^2 < 0$, the potential will have a minimum value at

$$\langle \Phi^\dagger \Phi \rangle = \frac{-\mu^2}{2\lambda} = \frac{v^2}{2} \quad (2.9)$$

As this defines only the expectation value of $\Phi^\dagger \Phi$, we may choose the expectation value of Φ to be

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (2.10)$$

In making this choice we have effectively chosen ϕ_3 to be a real scalar field with vacuum expectation value (vev) v , while ϕ_1 , ϕ_2 , and ϕ_4 are the massless Goldstone bosons which disappear from the particle spectrum to become the longitudinal polarizations of our massive bosons. We may therefore write the full scalar doublet in the unitary

gauge as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H(x) + v \end{pmatrix} \quad (2.11)$$

where $H(x)$ is a scalar field called the Higgs boson. The terms involving W_μ and B_μ in the covariant derivative result in the following interaction terms between the scalar and the vector bosons,

$$\begin{aligned} |D_\mu \Phi|^2 &= \left| \partial_\mu \Phi - i \frac{g}{2} \sigma^a W_\mu^a \Phi - i \frac{g'}{2} Y B_\mu \Phi \right|^2 \\ &= \left| \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} - \frac{i(v+H)}{2\sqrt{2}} \begin{pmatrix} g(W_\mu^1 - i W_\mu^2) \\ -g W_\mu^3 + g' B_\mu \end{pmatrix} \right|^2 \\ &= \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{1}{8} (v+H)^2 \left[g^2 |W_\mu^1 - i W_\mu^2|^2 + (-g W_\mu^3 + g' B_\mu)^2 \right] \end{aligned} \quad (2.12)$$

where we have used the fact that $Y = 1$ for the Higgs doublet Ψ , and $H = H(x)$. In Eq. (2.12) we can see that when $(v+H)^2$ is expanded the terms involving v^2 will produce mass terms for the gauge fields. However, the gauge fields we observe in nature are not in fact the gauge fields W_μ^1 , W_μ^2 , W_μ^3 and B_μ . We observe instead the W bosons, Z boson and the photon (V_μ),⁵ which are linear combinations of our original fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) \quad (2.13)$$

$$Z_\mu = \cos \theta_w W_\mu^3 - \sin \theta_w B_\mu \quad (2.14)$$

$$V_\mu = \sin \theta_w W_\mu^3 + \cos \theta_w B_\mu \quad (2.15)$$

⁵Here we represent the photon by V_μ to avoid confusion with A_μ^a , which will be used to generically represent gauge bosons in the next chapter

where

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \quad \text{and} \quad \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}. \quad (2.16)$$

The Weinberg angle θ_w has been experimentally determined to be $\theta_w = 28.75$ degrees [6]. Note that the coupling constants g and g' must be related to the electromagnetic constant e through $e = g \sin \theta_w = g' \cos \theta_w$ for the photon to have the proper coupling strength.

In terms of these superpositions, Eq. (2.12) becomes

$$\begin{aligned} |D_\mu \Phi|^2 = & \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{1}{8} (v^2 + 2vH + H^2) \\ & \times [2g^2 W_\mu^+ W^{\mu-} + (g^2 + g'^2) Z_\mu Z^\mu]. \end{aligned} \quad (2.17)$$

The masses of the bosons may be read off the Lagrangian as their squares are merely the mass term couplings; we therefore obtain

$$M_W^2 = \frac{g^2 v^2}{4}, \quad M_Z^2 = \frac{v^2}{4} (g^2 + g'^2) = \frac{g^2 v^2}{4 \cos^2 \theta_w}, \quad M_A^2 = 0. \quad (2.18)$$

Therefore the $W_\mu^\pm$ and Z_μ bosons gain mass terms due to the non-zero Higgs vev v . The photon, however, does not couple to the Higgs; therefore it remains massless.

We can also read off the couplings of W and Z bosons to the Higgs field H ,⁶

$$G_\mu G_\nu H H \rightarrow \frac{i g_G^2}{2} \eta_{\mu\nu}, \quad G_\mu G_\nu H \rightarrow \frac{i g_G^2 v}{2} \eta_{\mu\nu}, \quad (2.19)$$

where $G = Z, W^\pm$ and $g_W = g$, $g_Z = g \sec \theta_w$.

The Higgs itself also has a mass, which can be read off the first term of the Higgs potential

$$V(\Phi) = \frac{\mu^2}{2} (v + H)^2 + \frac{\lambda}{4} (v + H)^4 = -\mu^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4. \quad (2.20)$$

⁶Recall that when reading couplings from Lagrangian terms we must always multiply by i and a symmetry factor to account for all possible permutations of identical fields, for example, if we have a term $c Z^\mu Z_\mu$ the coupling is $2i c$ as the two Z_μ fields are interchangeable

along with two other couplings. Therefore we have

$$M_H^2 = -2\mu^2 = 2v^2\lambda, \quad H H H \rightarrow 6\lambda v, \quad H H H H \rightarrow 6\lambda. \quad (2.21)$$

Finally, through W boson interactions with muons the Higgs vacuum expectation value v may be related to the Fermi constant $G_F = 1.16639 \times 10^{-5} \text{GeV}^{-2}$ and evaluated,⁷

$$v = \left(\sqrt{2} G_F\right)^{-\frac{1}{2}} = \left(\frac{g^2}{4 M_W^2}\right)^{-\frac{1}{2}} = 246 \text{ GeV}. \quad (2.22)$$

Note that the Higgs mass cannot be determined directly from known experimental quantities as it depends on λ ; however, it can replace λ as a parameter of the theory (while μ can be replaced by known quantity v). The Higgs mass is constrained from experimental tests to be within the range $114 \text{ GeV} \leq M_H \lesssim 185 \text{ GeV}$. The lower bound of 114 GeV was obtained from direct experimental searches at the Large Electron-Positron Collider (LEP) [16], and is a general bound which applies to most models. However, the upper bound originates from electroweak precision fits to the SM; it is a model-dependent bound that does not necessarily apply to models beyond the SM [17]. Furthermore, results from the Tevatron collider forbid a SM Higgs of mass 160 – 170 GeV [18, 19].

The Higgs mechanism can also give fermions mass in a similar manner through Yukawa terms [11]. A Yukawa term describes the coupling of a Dirac fermion (Ψ) and anti-fermion to a scalar. By adding Yukawa terms involving the Higgs to the model Lagrangian and breaking the electroweak symmetry, we obtain

$$L_{\text{yuk}} = g_{\text{yuk}} \bar{\Psi} H \Psi \rightarrow g_{\text{yuk}} \bar{\Psi} (H + v) \Psi = g_{\text{yuk}} v \bar{\Psi} \Psi + g_{\text{yuk}} \bar{\Psi} H \Psi \quad (2.23)$$

⁷The Fermi Constant G_F is the coupling strength of the Fermi interaction, a four-fermion interaction proposed by Enrico Fermi in 1933 as part of his description of beta-decay [15].

where the first term on the right-hand side is a fermion mass term. We can see that with an appropriate choice of the Yukawa coupling constant g_{yuk} we can reproduce the appropriate fermion masses.

2.3 Problems with the Standard Model

Although the SM has prevailed through over 40 years of rigorous testing, it still suffers from several deficiencies. The most important of these is the Hierarchy Problem.

2.3.1 The Hierarchy Problem

The Standard Model is an incredibly successful theory. It has stood up to over four decades of rigorous experimental tests. However, there are still several questions and problems that it cannot successfully explain. One of the most prominent of these failures is known as the Hierarchy Problem. The Hierarchy Problem is essentially the question of why gravity is so weak; the gravitational force only becomes relevant at mass scales on the order of the Planck mass, $M_P \approx 10^{19} \text{ GeV}$, while the other forces operate at much lower scales. The Standard Model does not include gravitational effects, and it fails to provide an explanation for why there is such a prominent difference – or hierarchy – in the strength of the gravitational force when compared with the strong and electroweak forces.

In terms of Higgs physics, this problem is equivalent to asking why the Higgs mass parameter μ^2 is so small; this parameter sets the scale for the Higgs vev v , and by extension all of the particle masses, including that of the Higgs. Radiative corrections from loop diagrams such as those in Fig. 2.1 are expected to add quadratic

Planck scale corrections to μ^2 , causing it to become extremely large. But the particle masses are known to be much smaller – on the weak scale – by experiment, therefore any model that employs a Higgs mechanism must provide some way of protecting μ^2 from such large corrections. In the Standard Model there is no explicit mechanism or

Figure 2.1 Loop diagrams expected to contribute Planck scale contributions to the Higgs mass. Dashed lines represent Higgs, W or Z boson contributions. Solid lines represent fermion contributions, the largest is from the top quark.

symmetry to protect the Higgs mass. As such, there must be an extreme fine-tuning of the model parameters for the square of the Higgs mass to end up on the electroweak scale, roughly 34 orders of magnitude smaller than the model would naturally predict it to be. As most physicists consider this sort of fine-tuning to be undesirable and unnatural, it is believed to indicate that the Standard Model is not complete, there must be some further mechanism that manifests above the scale of the Higgs mass and which leads to the suppression or cancellation of the quadratic divergences. These new effects must appear around or below the TeV scale for $\mu^2 \simeq (100 \text{ GeV})^2$ to be considered natural.

There have been many proposals as to what that mechanism might be. The

most popular alternative is Supersymmetry [20, 21], a theory in which every Standard Model particle has an associated *superpartner*. The superpartners must be heavier than the SM particles – or we would have identified them experimentally by now – and the spin of each superpartner differs from that of the corresponding SM particle by $1/2$. In this scenario, the quadratic corrections to the Higgs mass from each SM particle are cancelled by those from their superpartner.

Another proposal is that of Little Higgs models [22], where the Higgs is a pseudo-Goldstone boson, it is the Goldstone boson of a spontaneously broken symmetry, but an additional explicit symmetry breaking gives it a small mass. In these cases, the Higgs mass is protected from large quadratic corrections by the symmetries of the model. There are many further possibilities as well, including models of composite Higgs bosons [23, 24, 25, 26], and models such as Technicolour, which achieve EWSB without a Higgs [27, 28, 29, 30, 31]. However, each of these models has their own draw-backs and many still retain hierarchy problems or fine-tuning, though to a lesser degree than the Standard Model.

2.3.2 Other problems with the Standard Model

Although the Hierarchy problem is the most common incentive and inspiration for proposals of Beyond the Standard Model (BSM) mechanisms, there are several other questions that the Standard Model fails to answer, and which BSM models try to account for.

One of these is the large range of different fermion masses, for example, the electron has a mass of $m_e = 0.510998910 \pm 0.000000013$ MeV, while the top quark's mass, $m_t = 172 \pm 2$ GeV [6], is larger by five orders of magnitude. Although the

SM can describe these masses with Yukawa terms, it cannot explain *why* the fermion masses are so different. This problem became even more pertinent with the relatively recent discovery that neutrinos also have very tiny non-zero masses [32]. This finding added several orders of magnitude to the range of fermion masses and created a whole new set of problems as the Standard Model does not account for non-zero neutrino masses at all.

A second significant problem is recent accumulation of experimental evidence for Dark Matter [33, 34]. Experimental measurements of the masses of galaxy-clusters have suggested that the universe contains a great deal more matter than can be accounted for by the Standard Model. It is estimated that Standard Model particles make up only 4.6 percent of the universe's mass and energy; roughly 23 percent is the extra Dark Matter, and the remaining percentage is Dark Energy. The primary characteristics of Dark Matter are that it must be massive, and electrically neutral as it does not emit radiation. There are several possibilities for what Dark Matter might be; however, the most common theory currently hypothesizes that it is some sort of Weakly Interacting Massive Particle (WIMP) [34]. Although the existence of Dark Matter has not yet been conclusively confirmed, the body of evidence is growing [35], and many BSM models attempt to include one or more particles which could be a WIMP or some other Dark Matter candidate.

Chapter 3

The Minimal Composite Higgs Model

3.1 An introduction to warped-geometry models

Five dimensional (5D) warped geometry models are a class of models based on the Randall-Sundrum (RS) model

3.1.1 Randall-Sundrum 1

The first Randall-Sundrum model (RS1) was proposed by Lisa Randall and Raman Sundrum in 1999 as a way of resolving the hierarchy problem [1]

This model is defined in a 5D Anti-de-Sitter (AdS_5) space. An AdS_n space is a maximally symmetric spacetime with a negative curvature. The fifth dimension of this spacetime – which we denote by y – is compactified on a circle, such that $y = y + 2\pi r_c$. Furthermore, the top and bottom halves of the circle are identified

with each other, $y = -y$, such that there are two boundaries, one at $y = 0$ and one at $y = L = \pi r_c$. This is formally known as the S^1/Z_2 orbifold, where S^1 refers to the circle and Z_2 to the transformation $y \rightarrow -y$.

The metric of this spacetime is obtained by solving Einstein's equations and is found to be

$$ds^2 = e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (3.1)$$

where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, k is on the order of the Planck scale, and $y = r_c\phi$ [1, 36, 37]

Energy scales on the boundary at $y = 0$ can be as high as the Planck scale, thus this boundary is known as the Planck – or ultraviolet (UV) – brane. However, due to the warped nature of the spacetime itself, the energy scale of the boundary at $y = L$ is much smaller, energies are warped down to the TeV scale, and so this is known as the TeV – or Weak, or infrared (IR) – brane. As such, through the warped spacetime the model can very naturally explain large hierarchies such as that of the Hierarchy problem.

To show this explicitly, consider the Einstein-Hilbert action [1, 36],

$$\begin{aligned} S &= 2M^3 \int d^4x \int_{-L}^L dy \sqrt{-\bar{g}_5} R_5 = 2M^3 \int d^4x \sqrt{-\bar{g}_4} R_4 \int_{-L}^L dy e^{-2k|y|} \\ &= \frac{M^3}{k} [1 - e^{-2kL}] \int d^4x \sqrt{-\bar{g}_4} R_4 \end{aligned} \quad (3.2)$$

where M is the 5D mass scale, R_5 is the 5D Ricci scalar, R_4 is the 4D Ricci scalar, $\bar{g}_5$ is the determinant of the 5D metric, and $\bar{g}_4$ is the determinant of $\eta_{\mu\nu}$. Comparing this to the usual 4D Einstein-Hilbert action, we find that

$$M_p^2 = \frac{M^3}{k} [1 - e^{-2kL}] \quad (3.3)$$

Given that k is on the order of the Planck mass, we can see that $M \approx M_p$; the mass scale relevant for gravitational interactions is not significantly altered by the warped spacetime.

However, consider the action relevant to a Higgs boson that is constrained to the TeV boundary

$$S = \int d^4x \sqrt{-\bar{g}_4} e^{-4kL} \left[\eta^{\mu\nu} e^{2kL} (D_\mu \Phi)^\dagger D_\nu \Phi - \lambda (\Phi^\dagger \Phi - v_0^2)^2 \right] \quad (3.4)$$

where v_0 is the Higgs vacuum expectation value. Rescaling the Higgs scalar field, $\Phi \rightarrow e^{kL} \Phi$, we find

$$S = \int d^4x \sqrt{-\bar{g}_4} \left[\eta^{\mu\nu} (D_\mu \Phi)^\dagger D_\nu \Phi - \lambda (\Phi^\dagger \Phi - v_0^2 e^{-2kL})^2 \right]. \quad (3.5)$$

This is the usual Higgs action, except the vev has become exponentially suppressed, $v = e^{-kL} v_0$. Therefore, we can see that the gravitational scale remains the same, but the Higgs vev – and by extension contributions to the Higgs mass – will be suppressed, creating a large hierarchy.

The warped 5D spacetime also has consequences with respect to the particle spectrum. Each particle corresponds to a five dimensional field. Due to the periodicity of the fifth dimension, these 5D fields can be expressed as a Kaluza-Klein decomposition [38, 39, 40]

$$A(x, z) = \sum_n^\infty f(m_n, z) A(m_n, x). \quad (3.6)$$

Here $A(x, z)$ denotes the full 5D field. $A(m_n, x)$ is a four dimensional component of $A(x, z)$ known as a Kaluza-Klein mode, which has mass m_n , and $f(m_n, z)$ is a profile that describes the KK mode's location in the fifth dimension. As a result, from a 4D effective perspective each 5D particle corresponds to an infinite tower of 4D KK modes. The zero modes of such towers correspond to Standard Model particles.

In the original RS1 model, all of the SM particles were confined to the TeV brane. However, a second formulation – RS2 – set $L \rightarrow \infty$ such that there was no TeV brane, and placed the particles on the Planck brane [41]. Subsequent models have placed the gauge bosons and fermions in the bulk of the fifth dimension [42, 43, 44], however, it is most common to keep the Higgs either on or very near to the TeV brane so as to maintain the mass scale hierarchy. Models further differ from each other by employing different gauge or global symmetry groups both in the bulk of the fifth dimension and on its boundaries, by including boundary-localized terms in the Lagrangian, and by considering a fifth dimension with a different or infinite length.

3.1.2 The AdS/CFT correspondence and composite Higgs bosons in RS Models

An interesting feature of Randall-Sundrum Models is that they are dual to 4D conformal field theories by the AdS/CFT correspondence [2]. Two theories are considered to be dual if their physical spectra and interactions are the same, even if their underlying dynamics are different. According to this correspondence, weakly coupled quantum field theories in an AdS_n spacetime can be identified with a strongly coupled conformal field theory on the $n - 1$ dimensional boundary. For example, certain Randall-Sundrum models can be shown to be dual to Technicolor [45], a class of models inspired by QCD which establish EWSB through strong gauge interactions, without the use of a Higgs boson [31].

The AdS/CFT duality is useful in many fields of physics – string theory, QCD, condensed matter – as many CFTs are extremely difficult to calculate because they are strongly coupled and perturbation theory cannot be used. The weakly-coupled 5D

perspective provides a perturbative way of approaching such theories. In the context of Randall-Sundrum models, only one side of the duality is known – the 5D side. As such, the details of the strong interactions and their elementary degrees of freedom are unknown, the 5D formalism provides only an effective Lagrangian in which the degrees of freedom are composite particles of the CFT. This is similar to the Chiral Effective Lagrangian description of QCD, in which the degrees of freedom are hadrons rather than quarks.

Due to this correspondence, Randall-Sundrum models have become a vehicle for studying the older idea that EWSB could arise from a strongly coupled gauge theory. Such theories generally divide into two classes: Technicolor theories [27, 28, 29, 30, 31] and composite Higgs theories [23, 24, 25, 26]. As mentioned above, RS models with bulk fermion field but a TeV brane constrained Higgs are dual to the former [45]. Similarly, an RS variation in which the 5D Higgs arises from the fifth component of one of the 5D gauge boson fields – formally known as Gauge-Higgs Unification (GHU) – is dual to a 4D CFT with a composite Higgs boson [46, 3, 47].

From the dual picture, a RS model with GHU is composed of two sectors, an elementary sector and a strongly coupled conformal sector [3]. However, the two sectors are not fully independent, but couple to each other. As a result, the elementary particle states mix with the composite states, yielding mass eigenstates that are superpositions of both elementary and composite states. The degree of compositeness in the 4D CFT is determined by the location of the particle in the 5D theory, light particles located near the Planck brane – such as SM particles – will be mostly elementary, while heavier particles near the TeV brane – such as KK modes – will be predominantly composite. The Higgs particle, however, is a fully composite bound state of the strong sector.

RS models with GHU provide a promising picture of EWSB, as RS models they have all the advantages discussed in the previous section, and they can be formulated so as to obey current electroweak precision tests and experimental constraints [48, 4, 49, 50], the violation of which is a common issue in Technicolor models [27]. A common version of GHU is that of the Minimal Composite Higgs Model (MCHM), which was first proposed by K. Agashe, R. Contino and A. Pomarol [3]. We will review the details of this model in the following section.

3.2 The Minimal Composite Higgs Model

For the purposes of this model, we will rewrite the metric, Eq. (3.1), in an equivalent form [3, 51],

$$ds^2 = \frac{1}{(kz)^2} [\eta_{\mu\nu} dx^\mu dx^\nu - dz^2] \equiv g_{MN} dx^M dx^N \quad (3.7)$$

where $M, N = 0, 3, 5$ are Lorentz indices and $x_5 = z = \frac{e^{ky}}{k}$ is the coordinate of the fifth dimension. The fifth dimension has range $L_0 \leq z \leq L_1$, where $L_0 = \frac{1}{k} \approx \frac{1}{M_p}$ and $L_1 = \frac{1}{M_{kk}} \approx \frac{1}{TeV}$. M_{kk} denotes the Kaluza-Klein mass scale.

The particles of the model are located in the bulk of the fifth dimension. The Higgs is located extremely close to the TeV brane, fermions are near the UV brane, and the gauge bosons are somewhere in between.

As discussed in Sec. 3.1.1, the primary strength of this model is that it provides a natural solution to the Hierarchy problem through the warped space-time of the fifth dimension [3]. Furthermore, placing the fermions in the bulk provides an explanation for the hierarchy of fermion masses, the mass of the fermion will be determined by its overlap with the Higgs in the fifth dimension. By placing light fermions very close to the UV brane, but keeping heavier ones such as the top quark nearer to the

Higgs, the proper spectrum of fermion masses – including those of neutrinos – can be produced [52]. There is also a possibility that the model could give rise to a dark matter candidate [52]. As such, this model has the potential to provide answers to many questions left unanswered by the Standard Model.

3.2.1 Symmetries of the model

In the bulk of the five dimensional space, the model has an $SU(3)_c \times SO(5) \times U(1)_{B-L}$ gauge symmetry. This symmetry is broken to $SU(3)_c \times SU(2)_L \times U(1)_Y$ on the Planck brane, $z = L_0$, and to $SU(3)_c \times SO(4) \times U(1)_{B-L}$ on the TeV brane, $z = L_1$ [3, 46]; here $SO(4) \approx SU(2)_L \times SU(2)_R$, $Y = Q_{B-L} + T^{3R}$ is the SM hypercharge and Q_{B-L} is the charge corresponding to $U(1)_{B-L}$.¹ A schematic representation of the model geometry is given in Fig. 3.1.

We will denote the bosons corresponding to the $U(1)_{B-L}$, $SO(5)$ symmetries by U_M , A_M^a respectively. The generator index a may take on the values $a = a_{L,R}, \hat{a}$ where $a_{L,R} = 1, 2, 3$ and $\hat{a} = 1..4$; there are ten in total. The $a_{L,R}$ generators correspond to the $SO(4) \approx SU(2)_L \times SU(2)_R$ symmetry that remains unbroken on the TeV brane, while the $\hat{a}$ are generators corresponding to the remaining broken $SO(5)/SO(4)$ symmetry.

The symmetry breaking is achieved by applying appropriate boundary conditions (BCs) to the 4-component gauge fields A_μ^a ; a Dirichlet BC ($f(z) = 0$) results in a broken gauge symmetry, while a Neumann BC ($\partial_z f(z) = 0$) preserves the gauge symmetry [3, 51]. Since we want an unbroken $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry

¹ $B - L$ refers to the baryon number ($B = \frac{1}{3}$ for a quark) minus the lepton number ($L = 1$ for a lepton).

Figure 3.1: A schematic representation of the MCHM spacetime. Note that the symmetry group $SU(3)_c$ is unbroken everywhere

on the TeV boundary, both the $A_\mu^{aL R}$ and U_μ bosons must have Neumann BCs at $z = L_1$, while the $A_\mu^{\hat{a}}$ bosons must have Dirichlet BCs. Similarly, at the Planck boundary the A_μ^{aL} must have Neumann BCs to preserve the $SU(2)_L$ symmetry. However, on this boundary we only want to preserve $U(1)_Y$, not $U(1)_{B-L}$. As such, there will be a linear combination of U_μ and A_μ^{3R} which will correspond to the SM hypercharge boson B_μ and will have Neumann BCs at both boundaries. The opposite linear combination, which we will call X_μ , will have a Dirichlet BC at the Planck boundary, but a Neumann BC at the TeV boundary. Finally, A_μ^{1R} , A_μ^{2R} and $A_\mu^{\hat{a}}$ bosons will all have Dirichlet BCs at the Planck boundary.

The scalar fifth components automatically must have opposite BCs to the corresponding gauge field [46, 3]. Therefore A_5^{aL} and B_5 have Dirichlet-Dirichlet BCs, A_5^{1R} , A_5^{2R} and X_5 have Neumann-Dirichlet BCs, and the $A_5^{\hat{a}}$ have Neumann-Neumann BCs. As Neumann-Neumann BCs are required for a zero-mode to arise, only the A_μ^{aL} ,

B_μ and A_5^a bosons will have zero-modes, these will ultimately become our SM gauge bosons and Higgs doublet

3.2.2 Model Lagrangians

The electroweak gauge sector of the model is described by the following Lagrangian [3, 38, 51]

$$L_{Gauge} = -\frac{\sqrt{\bar{g}}}{4} g^{MN} g^{RS} (F_{MR}^a F_{NS}^a + U_{MR} U_{NS}) + \sqrt{\bar{g}} L_{GF} \quad (3.8)$$

where L_{GF} is the gauge fixing term [46, 38],

$$\sqrt{\bar{g}} L_{GF} = -\frac{1}{2\epsilon k z} \left[\partial^\mu A_\mu^a - \epsilon z \partial_z \left(\frac{1}{z} A_5^a \right) \right]^2, \quad (3.9)$$

and ϵ is a gauge-fixing parameter.² The stress tensors are defined as

$$F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + g_5 A_{MN}^a, \quad (3.10)$$

$$U_{MN} = \partial_M U_N - \partial_N U_M, \quad (3.11)$$

where $A_{MN}^a T^a = -i [T^c, T^b]^a A_M^c A_N^b$. The generators of $SO(5)$, T^a , are given explicitly in Appendix B

The Lagrangian for a fermion is given by [3, 51]

$$L_f = \sqrt{\bar{g}} \left[\frac{i}{2} \bar{\Psi} e_A^M \Gamma^A D_M \Psi - \frac{i}{2} (D_M \Psi)^\dagger \Gamma^0 e_A^M \Gamma^A \Psi - M_\Psi \bar{\Psi} \Psi \right], \quad (3.12)$$

where $e_A^M = kz \delta_A^M$ is the vielbein,³ the 5-dimensional Dirac matrices are $\Gamma^M =$

²Recall that we defined $\bar{g}$ to be the determinant of the metric g_{MN} , it should not be confused with the weak coupling constant g . The trace of the metric is $(kz)^{-10}$, therefore $\sqrt{\bar{g}} = (kz)^{-5}$. Note also that we have omitted bulk mass terms in Eq. 3.8, we set them to zero.

³Note that the vielbein plays the role of the metric here, there is no extra metric factor implied in the contraction of the gamma matrix and the derivative.

$\{\gamma^\mu, -\gamma^5\}$, and the full covariant derivative is

$$D_M = \partial_M + \frac{1}{8}\omega_{MAB} [\Gamma^A, \Gamma^B] - i g_5 A_M - i g_5'' Q_{B-L} U_M, \quad (3.13)$$

where $A_M = T^a A_M^a$, U_M is the gauge boson corresponding to $U(1)_{B-L}$, and ω_{MAB} is the spin connection. However, it should be noted that in a diagonal metric such as Eq. (3.7) the spin-connection terms cancel out in Eq. (3.12).

3.2.3 The gauge boson sector

As previously discussed, from a four-dimensional perspective our 5D gauge bosons A_μ^a correspond to a tower of Kaluza-Klein (KK) modes. Standard Model particles correspond to the zero modes of these KK towers, which will only exist if the boson is subject to Neumann-Neumann boundary conditions. Before EWSB, all of the zero modes are massless, while the higher order gauge KK masses are located above the KK mass scale, $M_{kk} = \frac{1}{L_1}$, which is defined by the length of the fifth dimension. The first gauge KK modes (those of $A_\mu^{aL,R}$) appear at approximately $2.5 M_{kk}$, and subsequent KK modes of the same boson are separated by roughly $3.1 M_{kk}$. For a schematic example of the KK spectrum, see Fig. (3.2) near the end of the section.

In the case of the MCHM, the zero mode of the A_μ^{aL} KK tower corresponds to our usual SM $SU(2)_L$ bosons, we will therefore rename A_μ^{aL} as the more familiar W_μ^{aL} . The A_μ^{aR} bosons arise similarly from the extra $SU(2)_R$ symmetry of the MCHM, but have no SM analogue, let us call them W_μ^{aR} .

As in the Standard Model, the physical states may be obtained by performing rotations on our basis of states. The first rotation transforms $(W_\mu^{3R}, U_\mu) \rightarrow (B_\mu, X_\mu)$

through an angle θ_H ,

$$W_\mu^{3R} = \cos \theta_H B_\mu - \sin \theta_H X_\mu, \quad (3.14)$$

$$U_\mu = \sin \theta_H B_\mu + \cos \theta_H X_\mu, \quad (3.15)$$

where B_μ is the SM hypercharge gauge boson, and X_μ is a new gauge boson, their respective BCs have been previously discussed in Sec. 3.2.1. The mixing angle θ_H is related to the Weinberg angle through the relation $\cos \theta_H = \tan \theta_w$. This relation is determined by requiring that the coupling between the physical Higgs – which we will choose to be A_5^4 – and the photon V_μ be zero, as the photon is massless and the Higgs is neutral. As a result, both the Z and X couplings to the photon also disappear, indicating that both are neutral bosons. The zero mode of B_μ is the analogous particle of the SM, and so our familiar SM states can be found by performing the Standard Model rotation through the Weinberg angle – as described previously by Eqs. (2.14) and (2.15) – to transform $(W_\mu^{3L}, B_\mu) \rightarrow (Z_\mu, V_\mu)$. Similarly, the transformation for $W_\mu^{1,2} \rightarrow W_\mu^\pm$, Eq. (2.13), can be performed on both W_μ^{aL} and W_μ^{aR} . To obtain real coupling values it is also convenient to apply an analogous transformation to the A_μ^1 and A_μ^2 ,

$$A_\mu^\pm = \frac{1}{\sqrt{2}} \left(A_\mu^1 \mp i A_\mu^2 \right) \quad (3.16)$$

As a result, from the 4D perspective in the MCHM we have a tower of KK modes with a zero mode for each of the electroweak Standard Model bosons $W_{L\mu}^\pm$, Z_μ , and the photon V_μ . We also have seven extra gauge KK towers without zero modes due to the extended gauge symmetry $W_{R\mu}^\pm$, X_μ , $A_\mu^\pm$, A_μ^3 and A_μ^4 . In addition, each of these towers of vector bosons has an accompanying tower of scalar bosons from the corresponding A_5^a . These are Goldstone bosons, in the unitary gauge each $A_5^{a(n)}$ KK boson becomes the third polarization of its corresponding $A_\mu^{a(n)}$, where $n \neq 0$ denotes the n^{th} KK mode.

UV Boundary Condition	TeV Boundary Condition	Particles
Neumann	Neumann	$W_{L\mu}^{\pm(n)}, Z_{\mu}^{(n)}, V_{\mu}^{(n)}, H_5$
Dirichlet	Neumann	$W_{R\mu}^{\pm(n)}, X_{\mu}^{(n)}$
Dirichlet	Dirichlet	$A_{\mu}^{\pm(n)}, A_{\mu}^{\hat{3}(n)} \text{ and } A_{\mu}^{\hat{4}(n)}$

Table 3.1: The gauge boson spectrum of the MCHM.

However, the Z_5 , V_5 , $W_{L,R5}^{\pm}$ and X_5 towers have no zero-modes, for they have either Dirichlet-Dirichlet or Neumann-Dirichlet BCs as previously discussed. As only the $A_5^{\pm}$, $A_5^{\hat{3}}$ and $A_5^{\hat{4}}$ scalar towers obey Neumann-Neumann BCs, they are the only fifth component towers with zero modes. As such, these four zero-mode scalars are the Goldstone bosons that correspond to our SM zero modes; in other words, they are our Higgs doublet. As in the SM, three of them are eaten by the zero modes of the $W_{L\mu}^{\pm}$ and Z_{μ} towers – our SM $W_{\mu}^{\pm}$ and Z_{μ} – when the latter become massive due to EWSB, exactly as in the Standard Model.

In the end, only one scalar boson is left over; it becomes our Higgs boson. It does not matter which three $A_5^{\hat{a}(0)}$ bosons become polarizations and which becomes the Higgs; we will choose our Higgs to be the fourth boson, $H_5(x, z) = A_5^{\hat{4}}$.⁴ This mechanism where the Higgs arises naturally out of the gauge structure of the model is known as Gauge-Higgs Unification. Therefore, in the unitary gauge we are left with the spectrum of bosons outlined in Table 3.1.

⁴Here the 5 subscript on the H denotes that this is the 5D Higgs.

As discussed in the previous section, the 5D bosons are superpositions of these 4D KK modes along with their profile which describes their location in the fifth dimension. In the absence of EWSB, this profile $f_G(m, z)$ is obtained from the boson equation of motion which can be derived from the Lagrangian Eq (3 9),

$$\left[p^2 - \frac{1}{z} \partial_z + \partial_z^2 \right] f_G(p, z) = 0, \quad (3 17)$$

where $p = \sqrt{p^2}$ is equivalent to the KK mass [38, 51, 39]. It is convenient for future purposes to define even and odd solutions $C_A(m, z)$ and $S_A(m, z)$ with the boundary conditions $C_A(m_n, L_0) = 1$, $\partial_z C_A(m_n, L_0) = 0$, $S_A(m_n, L_0) = 0$ and $\partial_z S_A(m_n, L_0) = m_n$, [4, 53] such that

$$C_A(m_n, z) = \frac{\pi m_n}{2} z [J_1(m_n z) Y_0(m_n L_0) - J_0(m_n L_0) Y_1(m_n z)], \quad (3 18)$$

$$S_A(m_n, z) = \frac{\pi m_n}{2} z [J_1(m_n L_0) Y_1(m_n z) - J_1(m_n z) Y_1(m_n L_0)], \quad (3 19)$$

where $J_n(x)$ and $Y_n(x)$ are the n^{th} order Bessel functions of the first and second kind, details on these functions can be found in Appendix A. The profile of a boson with a Neumann (Dirichlet) boundary condition on the Planck brane is given by $C_A(m, z)$ ($S_A(m, z)$) normalized according to [51]

$$\int_{L_0}^{L_1} dz \frac{1}{kz} f_G^{(n)}(z) f_G^{(m)}(z) = \delta_{mn} \quad (3 20)$$

The zero mode profile can be determined by solving Eq (3 17) with $p = m_n = 0$, and imposing the normalization condition, yielding a profile independent of z

$$f_G^{(0)}(z) = \sqrt{\frac{k}{\ln(k L_1)}} \quad (3 21)$$

The mass spectrum m_n of the gauge bosons is determined by imposing the TeV boundary condition upon the appropriate profile. For example, the mass spectrum of a particle with a Neumann-Neumann boundary condition will be defined by the

mass condition $C'_A(m, L_1) = 0$ (where the prime indicates the derivative with respect to z)

Because the Higgs arises as the zero mode of the A_5^4 scalar, its profile does not have the same form as those of the gauge bosons. The requirement that zero-mode particles be massless before EWSB leads to the following Higgs profile [3],

$$f_H(z) = z \sqrt{\frac{2k}{L_1^2 - L_0^2}} \quad (3.22)$$

Note that although this profile is linear in the warped coordinate z , in the flat-space coordinate y the profile is extremely peaked toward the TeV brane.

3.2.4 The fermion sector

To properly incorporate fermions into a 5D model, they must be embedded in an appropriate representation of $SO(5) \times U(1)_{B-L}$. The choice of embedding is important, as it strongly affects the physics of the model [48, 54, 49]. We will make use of the Medina-Shah-Wagner (MSW) embedding [4], as it has been shown to satisfy electroweak precision constraints (EWPCs) [49, 4].⁵ In the MSW model, each generation of fermions is embedded in three multiplets, two **5**'s and one **10** of $SO(5) \times U(1)_{B-L}$. The left-handed quark doublet and the right-handed up-type quark singlet are each embedded in a different $\mathbf{5}_{2/3}$ representation, while the down-type quark singlet is embedded in a $\mathbf{10}_{2/3}$ representation (here $2/3$ is the $U(1)_{B-L}$ charge). The quark

⁵Other common fermion embeddings are the Hosotani Oda-Ohnuma-Sakamura (HOOS) embedding [55, 56], and the original MCHM4 spinorial embedding [3], however, in the former the ZZH coupling – which is key to our project – does not exist, and the latter is difficult to reconcile with EWPCs [48, 49].

embeddings have the form

$$\xi_{1L}^{q_i} = Q_{1L}^i \oplus \hat{t}_{1L}^i = \begin{pmatrix} \chi_{1L}^i(-, +)_{\frac{5}{3}} & t_{1L}^i(+, +)_{\frac{2}{3}} \\ \tilde{t}_{1L}^i(-, +)_{\frac{2}{3}} & b_{1L}^i(+, +)_{-\frac{1}{3}} \end{pmatrix} \oplus \hat{t}_{1L}^i(-, +)_{\frac{2}{3}} \quad (3.23)$$

$$\xi_{2R}^{q_i} = Q_{2R}^i \oplus \hat{t}_{2R}^i = \begin{pmatrix} \chi_{2R}^i(-, +)_{\frac{5}{3}} & t_{2R}^i(-, +)_{\frac{2}{3}} \\ \tilde{t}_{2R}^i(-, +)_{\frac{2}{3}} & b_{2R}^i(-, +)_{-\frac{1}{3}} \end{pmatrix} \oplus \hat{t}_{2R}^i(+, +)_{\frac{2}{3}} \quad (3.24)$$

$$\begin{aligned} \xi_{3R}^{q_i} &= T_{1R}^i \oplus T_{2R}^i \oplus Q_{3R}^i \\ &= \begin{pmatrix} \Xi_{3R}^i(-, +)_{\frac{5}{3}} \\ T_{3R}^i(-, +)_{\frac{2}{3}} \\ B_{3R}^i(-, +)_{-\frac{1}{3}} \end{pmatrix} \oplus \begin{pmatrix} \Xi'_{3R}^i(-, +)_{\frac{5}{3}} \\ T'_{3R}^i(-, +)_{\frac{2}{3}} \\ B'_{3R}^i(+, +)_{-\frac{1}{3}} \end{pmatrix} \oplus \begin{pmatrix} \chi_{3R}^i(-, +)_{\frac{5}{3}} & t_{3R}^i(-, +)_{\frac{2}{3}} \\ \tilde{t}_{3R}^i(-, +)_{\frac{2}{3}} & b_{3R}^i(-, +)_{-\frac{1}{3}} \end{pmatrix} \end{aligned} \quad (3.25)$$

where $\hat{t}$ denotes an $SU(2)_L \times SU(2)_R$ singlet, T_1 and T_2 transform under $(\mathbf{3}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{3})$ of $SU(2)_L \times SU(2)_R$, and Q 's denote bidoublets of $SU(2)_L \times SU(2)_R$ (where $SU(2)_L$ acts vertically and $SU(2)_R$ acts horizontally) [4, 50]. The final subscripts denote the electromagnetic charge, while the superscripts $i = 1, 2, 3$ denote the generation. The plus and minus signs represent even and odd boundary conditions respectively; the first entry corresponds to the Planck brane boundary condition, while the second corresponds to the TeV brane boundary condition. For fermions an odd boundary condition is the usual Dirichlet condition, but an even BC is a superposition of Dirichlet and Neumann BCs. However, fermions with even BCs $(++)$ on both boundaries still correspond to SM particles. In the case of a quark multiplet, t_{1L} and b_{1L} together correspond to the left-handed SM doublet of that generation, t_{1L} being the up-type quark and b_{1L} being the down-type quark. Similarly, $\hat{t}_{2R}$ and B'_{3R} correspond to the right-handed up and down-type quark singlets respectively.

It should be noted that in general 5D fermion fields are inherently non-chiral

Dirac spinors. Right- and left-handed Weyl spinors may be obtained from these Dirac spinors due to the Z_2 orbifold symmetry that is imposed on the fifth dimension, as discussed in Sec. 3.1.1 [40]. Fermions transform as $\Psi(-y) = \pm\gamma_5\Psi(y)$ under this symmetry, allowing the identification of right- and left-handed chiral states, $\Psi_{L,R} = \pm\gamma_5\Psi_{L,R}$. One of these chiral states will have an odd boundary condition, while the other will have an even boundary condition; as such, the boundary conditions of the multiplet of opposite chirality can be obtained simply by reversing the chiralities and boundary conditions in Eqs. (3.23-3.25).

The leptons are embedded in similar representations; the doublets and right-handed neutrinos are embedded in $\mathbf{5}_0$ representations, while the right-handed lepton singlets are embedded in $\mathbf{10}_0$ representations [52],

$$\xi_{1L}^{\ell_i} = Q_{1L}^{\ell_i} \oplus \hat{n}_{1L}^i = \left(\begin{array}{cc} \kappa_{1L}^i(-, +)_1 & n_{1L}^i(+, +)_0 \\ \tilde{n}_{1L}^i(-, +)_0 & \ell_{1L}^i(+, +)_{-1} \end{array} \right) \oplus \hat{n}_{1L}^i(-, +)_0 \quad (3.26)$$

$$\xi_{2R}^{\ell_i} = Q_{2R}^{\ell_i} \oplus \hat{n}_{2R}^i = \left(\begin{array}{cc} \kappa_{2R}^i(-, +)_1 & n_{2R}^i(-, +)_0 \\ \tilde{n}_{2R}^i(-, +)_0 & \ell_{2R}^i(-, +)_{-1} \end{array} \right) \oplus \hat{n}_{2R}^i(+, +)_0 \quad (3.27)$$

$$\begin{aligned} \xi_{3R}^{\ell_i} &= T_{1R}^{\ell_i} \oplus T_{2R}^{\ell_i} \oplus Q_{3R}^{\ell_i} \\ &= \left(\begin{array}{c} K_{3R}^i(-, +)_1 \\ N_{3R}^i(-, +)_0 \\ L_{3R}^i(-, +)_{-1} \end{array} \right) \oplus \left(\begin{array}{c} K'_{3R}^i(-, +)_1 \\ N'_{3R}^i(-, +)_0 \\ L'_{3R}^i(+, +)_{-1} \end{array} \right) \oplus \left(\begin{array}{cc} \kappa_{3R}^i(-, +)_1 & n_{3R}^i(-, +)_0 \\ \tilde{n}_{3R}^i(-, +)_0 & \ell_{3R}^i(-, +)_{-1} \end{array} \right) \end{aligned} \quad (3.28)$$

where $\hat{n}$ denotes an $SU(2)_L \times SU(2)_R$ singlet, and T_i , Q_i transform as in the quark multiplets. Similarly to the quark case, ℓ_{1L} , n_{1L} (L'_{3R} , $\hat{n}_{2R}$) is the left-handed (right-handed) SM lepton and its associated neutrino.

Similarly to the boson case, in the absence of EWSB, the fermion profiles of left and right chirality, $f_{L,R}(m, z)$ are solutions to

$$[\partial_z - z^{-1}(2 \pm c)] [\partial_z - z^{-1}(2 \mp c)] f_{L,R}^{(n)}(z) = -m_n^2 f_{L,R}^{(n)}. \quad (3.29)$$

which can be derived from the Lagrangian Eq. (3.12) [51]. The parameter c determines the location of the fermion along the fifth dimension, and is defined in terms of the fermion bulk mass M_Ψ by $M_\Psi = ck$. Note that there are three c values (c_1^i, c_2^i, c_3^i) for each generation of quarks and each generation of leptons, one for each multiplet $\xi_1^i, \xi_2^i, \xi_3^i$. We define the following solution to Eq. 3.29 [4, 50]:

$$S_c^\pm(m_n, z) = \frac{\pi m_n}{2k} (kz)^{\frac{5}{2}} \left[J_{\pm c + \frac{1}{2}}(m_n L_0) Y_{\pm c + \frac{1}{2}}(m_n z) - J_{\pm c + \frac{1}{2}}(m_n z) Y_{\pm c + \frac{1}{2}}(m_n L_0) \right]. \quad (3.30)$$

Up to a normalization constant, $S_c^+(m_n, z)$ ($S_c^-(m_n, z)$) is the profile of a left-handed (right-handed) fermion with a Dirichlet boundary condition on the Planck brane. The corresponding chiral partners have a Neumann BC – a mixed BC for fermions – on the Planck brane, and have profiles defined by [51, 4, 50]

$$\dot{S}_c^\pm(m_n, z) = \mp \frac{1}{m_n} \left[\partial_z - \frac{1}{z}(2 \mp c) \right] S_c^\pm(m_n, z). \quad (3.31)$$

Please see Table 3.2 for a summary of the functions associated with the profiles of different chirality. Again, the fermion mass spectrum m_n is defined by imposing the TeV boundary condition upon the profiles above.

The fermion profiles must satisfy the following normalization condition [51]

$$\int_{L_0}^{L_1} dz \frac{1}{(kz)^4} f_f^{(n)}(z) f_f^{(m)}(z) = \delta_{mn}. \quad (3.32)$$

By solving Eq. (3.29) with $m_n = 0$ and imposing the normalization condition, one obtains the zero-mode profile

$$f_L^{(0)}(z) = \sqrt{\frac{k(1-2c)}{(kL_1)^{1-2c} - 1}} (kz)^{2-c}. \quad (3.33)$$

Boundary Condition at the Planck Brane	Left-handed fermion	Right-handed fermion
Dirichlet	$S_c^+(m_n, z)$	$S_c^-(m_n, z)$
Neumann	$S_c^-(m_n, z)$	$S_c^+(m_n, z)$

Table 3.2 Profiles associated to fermion chiralities with different BCs

It is easier to see from Eq. (3.33) how the parameter c determines the location of the zero-mode fermion profile, as the power of the warped coordinate z is determined by c . If $c > 0.5$ ($c < 0.5$) the fermion's profile is angled toward the Planck (TeV) brane, while if $c = 0.5$ the profile is flat. As such, light fermions require $c > 0.5$, while the heavier top and bottom quarks are assigned $c < 0.5$. It should also be noted that when we speak of the profile being flat, it is with respect to the flat-space coordinate y , not the warped-space coordinate z .

Fermions are also subject to an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ invariant boundary mass Lagrangian on the TeV brane, which marries the $SU(2)_L \times SU(2)_R$ bidoublets and singlets via Dirac mass terms on the TeV brane and introduces mixing between fermion states of the same electromagnetic charge [3]. In the case of leptons, Majorana mass terms for the right-handed neutrino N_R may be added to either boundary. The lepton boundary mass Lagrangian takes the general form

$$\begin{aligned}
L_{\text{bound}} = & -2\delta(z - L_1) [\bar{N}'_L M_{\ell_1} N_R + \bar{L}_{1L} M_{\ell_2} L_{3R} + \text{h.c.}] \\
& - [M_{IR}\delta(z - L_1) - M_{UV}\delta(z)] N_R N_R
\end{aligned} \tag{3.34}$$

where $M_{\ell_{1,2}}$ are dimensionless matrices of the Dirac mass terms [52]. M_{IR} (M_{UV})

denotes the Majorana mass term on the TeV (Planck) brane, and is also dimensionless

The quark mass Lagrangian and boundary conditions are completely analogous if the Majorana mass terms are set to zero [4, 50],

$$L_{\text{bound}} = -2\delta(z - L_1) [\bar{u}'_L M_{q_1} u_R + \bar{Q}_{1L} M_{q_2} Q_{3R} + \text{h.c.}] \quad (3.35)$$

Similarly to the c parameter, there is a different set of $M_{\ell_1}^i$, $M_{\ell_2}^i$ (or $M_{q_1}^i$, $M_{q_2}^i$) for each generation of quarks or fermions

These boundary mass terms define the boundary conditions of the fermions. Consider the generic Lagrangian terms

$$L = -2\delta(z - L_1) \bar{\Psi}_L^1 M \Psi_R^2 - [M_{IR} \delta(z - L_1) - M_{UV} \delta(z - L_0)] \bar{\Psi}_R^2 \Psi_R^2, \quad (3.36)$$

where superscripts 1, 2 merely denote two different fermion fields and the different chiralities Ψ_L^1 , Ψ_R^1 , Ψ_R^2 and Ψ_L^2 have profiles $g_L(z)$, $g_R(z)$, $h_R(z)$ and $h_L(z)$ respectively. Considering Eq. (3.36) in the equations of motion leads to the following BCs on the fermion profiles [50, 52]

$$\lim_{\epsilon \rightarrow 0} g_R(L_1 - \epsilon) = -M h_R(L_1), \quad (3.37)$$

$$\lim_{\epsilon \rightarrow 0} h_L(L_1 - \epsilon) = M_{IR} h_R(L_1) + M g_L(L_1), \quad (3.38)$$

$$\lim_{\epsilon \rightarrow 0} h_L(L_0 - \epsilon) = M_{UV} h_R(L_0) \quad (3.39)$$

Although the MCHM evidently contains many more fermions than the Standard Model, only those identified with the SM – identified by $(+, +)$ BCs – will have zero modes. The rest exist only as heavy Kaluza-Klein modes, as in the gauge sector, and so do not significantly affect the physics at the SM particle mass scale. However, they do contribute to physics above the KK mass scale, particularly to the KK boson decay widths.

3.2.5 Electroweak Symmetry Breaking in the MCHM

In the SM, the Higgs boson only couples Z bosons to Z bosons and W^+ bosons to W^- bosons. However, in the MCHM the Higgs can couple to different bosons which are not each others' anti-particles. The possible Higgs coupling forms are summarized in Table 3.3.

Table 3.3: Coupling terms that lead to boson mixing after EWSB.

Sector	Bosons	Possible Mixing
Index	Involved	Coupling Forms
1	$W^{1L}, W^{1R}, A^{\hat{1}}$	$W^{1L}W^{1R}HH, W^{1L}A^{\hat{1}}H, W^{1R}A^{\hat{1}}H$
2	$W^{2L}, W^{2R}, A^{\hat{2}}$	$W^{2L}W^{2R}HH, W^{2L}A^{\hat{2}}H, W^{2R}A^{\hat{2}}H$
3	$Z, X, A^{\hat{3}}$	$ZXHH, ZA^{\hat{3}}H, XA^{\hat{3}}H$

For clarity, the first two groups are left in the original basis; in the physical basis, these two groups would be combined due to rotations of the form $(W^1, W^2) \rightarrow (W^+, W^-)$. The third group is given in the physical basis, as the corresponding original basis would also include the photon, which does not couple to neutral particles.

Because of these new couplings, the same EWSB mechanism that gives the zero-mode gauge bosons masses also causes certain particles to mix together. For example, consider the coupling between a Z an X and two Higgs, $c H H Z_\mu X^\mu$, where c is the coupling constant. If we let $H \rightarrow v$, we obtain $c v^2 Z_\mu X^\mu$. Physically, this term seems to describe the spontaneous transformation of a Z boson into an X boson,

which does not make sense. However, what this term actually indicates is that the Z boson after EWSB is not the same as the original Z , it is a superposition of the original Z and X bosons. In fact, due to the couplings in sector three of Table 3.3, the Z after EWSB is actually found to be a superposition of the original Z , X and A^3 bosons,⁶

$$Z \rightarrow Z_\mu(m, z, v) = a Z_\mu(m, z, 0) + b X_\mu(m, z, 0) + c A_\mu^3(m, z, 0), \quad (3.40)$$

where $a \gg b, c$. It is the coefficients a, b, c that introduce the dependence on the Higgs vev into the profile.

Of course, the dominant contribution to Z' is the original Z , one would expect the contributions of X and A^3 to be substantially smaller. Furthermore, the X and A^3 bosons after EWSB are also superpositions of the original Z , X and A^3 , with their own original states dominating over the other contributions. Similarly, (W^{1L}, W^{1R}, A^1) and (W^{2L}, W^{2R}, A^2) also form groups of bosons that become superpositions of each other after EWSB, due to the couplings outlined in the first two mixing sectors of Table 3.3.

This mixing of boson states alters the masses of some of the mixed bosons. For example, after EWSB the mass conditions of the Z and the A^3 bosons become related through the Higgs vev, this has the result of shifting the Z KK mode masses down slightly, while the A^3 KK mode masses are shifted slightly up. Furthermore, both bosons are described by the same profile evaluated at different mass values. On the other hand, the X mass condition remains unchanged and independent. Fig. (3.2) gives a rough picture of the KK mass spectrum of the third mixing sector both before and after EWSB.

⁶The origin of the A^3 mixing is somewhat clarified if one recalls that it is the zero mode fifth component $A_5^{3(0)}$ that acts as the neutral Goldstone which becomes the Z boson's third polarization.

Figure 3.2: A schematic of the KK mode mass distributions for the Z , X and A^3 bosons both before (solid grey lines) and after (black dotted lines) EWSB. Numeric values for a particular parameter set can be found in Table 3.8 in Sec. 3.2.7. Not to scale.

For the purposes of further discussion, we define the following terms

Mixed : Groups of bosons whose profiles after EWSB are linear superpositions of their original profiles.

Unmixed : Bosons whose profiles after EWSB are the same as their original profiles.

Combined : Mixed bosons which are described by the same mass condition and profile after EWSB.

Independent Mixed bosons with the same mass condition before and after EWSB

In the interests of space and clarity, we will simply summarize the procedure and effects of particle mixing here. A detailed overview of the calculations, transformations, and profiles discussed in this section is provided in Appendix C.3

Gauge Boson Mixing

The full mixing of gauge bosons through the Higgs vev v after EWSB would be difficult to achieve by considering KK profiles individually. Fortunately, the fully mixed vev-dependent particle profiles $f^a(m, z, v)$ can be related to the pre-EWSB profiles $f^a(m, z, 0)$ by means of the following gauge transformation [53, 4],

$$f^a(m, z, v) T^a = \Omega^{-1}(z, v) f^a(m, z, 0) T^a \Omega(z, v) \quad (3.41)$$

where $a = a_L, a_R, \hat{a}$, the transformation matrix is

$$\Omega(z, v) = \exp \left[-i g_5 v \int_{L_0}^z dz' f_h(z') \right] = \exp \left(-i \sqrt{2} \theta_G(z, v) T^4 \right), \quad (3.42)$$

and

$$\theta_G(z, v) = \frac{v g_5 (z^2 - L_0^2)}{2} \sqrt{\frac{k}{L_1^2 - L_0^2}} \quad (3.43)$$

The $f_\alpha(m, z, 0)$ are respectively defined by Eq. (C.21) in terms of the base functions $C_A(m_n, z)$ and $S_A(m_n, z)$ of Eqs. (3.18-3.19) and normalization coefficients C_α . The resulting EWSB profiles $f_\alpha(m, z, v)$ can be found in Eqs. (C.47-C.54).

This gauge transformation ‘turns on’ a non-zero vev v for the Higgs. This can be seen by applying the gauge transformation Eq. (3.42) to the Higgs $H_5 = H f_H$ in

analogy with Eq (2 1),

$$\begin{aligned}
H_5 T^4 &\rightarrow \Omega H_5 T^4 \Omega^\dagger + \frac{\imath}{g_5} \Omega \partial_z \Omega^\dagger \\
&\rightarrow \Omega H f_H T^4 \Omega^\dagger + \frac{\imath}{g_5} \Omega (-\imath g_5 v f_H) \Omega^\dagger = \Omega (H + v) f_H T^4 \Omega^\dagger, \quad (3 44)
\end{aligned}$$

where for simplicity we have dropped the coordinate dependence in the notation. It is also interesting to note that $\theta_G(L_0) = 0$ and therefore $\Omega(L_0, v) = 1$, as such, the mixing will not affect particles located near the Planck brane as strongly as those located near the TeV brane.

We may determine both the coefficients C_α – up to an overall normalization factor – and the masses m_n by applying the boundary conditions at the TeV brane. There are as many TeV boundary conditions as there are coefficients C_α , therefore the BCs provide us with a system of equations by which the coefficients may be determined. However, this system of equations will only have a solution if its determinant is zero, this extra condition results in the following mass conditions

$$\begin{aligned}
S_A(m_n, L_1) &= 0, \\
C'_A(m_n, L_1) &= 0, \\
S'^2_A(m_n, L_1) &= 0, \\
2 C'_A(m_n, L_1) S_A(m_n, L_1) + m_n k L_1 \sin \theta_G^2(L_1, v) &= 0, \\
2 C'_A(m_n, L_1) S_A(m_n, L_1) + m_n k L_1 \sec^2 \theta_w \sin \theta_G^2(L_1, v) &= 0, \quad (3 45)
\end{aligned}$$

where a prime represents the derivative with respect to the fifth coordinate z . Each of these conditions determines the mass parameters m_n for a given boson. The first two correspond to the A_μ^4 boson and photon V_μ masses, while the third corresponds to the $W_{R\mu}^\pm$ and X_μ masses, all of these particles retain the same mass spectrum as before EWSB. However, masses of both the Z_μ and A_μ^3 boson are described by the fourth

mass condition, while the fifth describes the $W_{L\mu}^\pm$ and $A_\mu^\pm$. The vev dependence of these combined mass conditions shifts the associated masses slightly.

Imposing each of these mass conditions on the system of equations yields a separate solution for the coefficients. Although all 10 equations – one for each boson – can be solved together, it is physically illuminating to note that the profile boundary conditions can be broken into four independent systems of equations containing a common set of coefficients C_α after EWSB. The particles within each group are listed in the first row of Table 3.4; the second row lists the coefficients to be determined by the corresponding system of equations. As one might expect, they reflect the mixing groups given by Table 3.3.

Table 3.4: Sectors of gauge bosons mixed by EWSB through the Higgs vev v . The third group of particles is given in the physical basis because it is in this basis that the boundary conditions are imposed.

Group	1	2	3	4
Particles	$W_\mu^{1L}, W_\mu^{1R}, A_\mu^{\hat{1}}$	$W_\mu^{2L}, W_\mu^{2R}, A_\mu^{\hat{2}}$	$Z_\mu, X_\mu, V_\mu, A_\mu^{\hat{3}}$	$A_\mu^{\hat{4}}$
Coefficients	$C_{1L}, C_{1R}, C_{\hat{1}}$	$C_{2L}, C_{2R}, C_{\hat{2}}$	$C_{3L}, C_{3R}, C_{\hat{3}}, C_B$	$C_{\hat{4}}$
Mixed Bosons	$W_\mu^{1L}, W_\mu^{1R}, A_\mu^{\hat{1}}$	$W_\mu^{2L}, W_\mu^{2R}, A_\mu^{\hat{2}}$	$Z_\mu, X_\mu, A_\mu^{\hat{3}}$	$A_\mu^{\hat{4}}$
Combined Bosons	$W_\mu^{1L}, A_\mu^{\hat{1}}$	$W_\mu^{2L}, A_\mu^{\hat{2}}$	$Z_\mu, A_\mu^{\hat{3}}$	–
Independent Bosons	W_μ^{1R}	W_μ^{2R}	X_μ	–
Unmixed Bosons	–	–	V_μ	$A_\mu^{\hat{4}}$

Although the photon V_μ is contained in the third mixing group, it turns out that it is in fact an unmixed particle. When the system of equations corresponding to group 3 is solved under the constraint of the photon mass condition, the resulting coefficients remove the vev dependence from the photon's EWSB profile $f^V(m, z; v)$. Furthermore, the photon coefficients also force $f^Z(m, z; v)$, $f^X(m, z; v)$, and $f^{\hat{3}}(m, z; v)$ to be zero, while the coefficients associated with the X , Z and $A^{\hat{3}}$ bosons force $f^V(m, z; v)$ to be zero. As such $f^V(m, z; v) = f^V(m, z; 0)$.

However, the masses and coefficients corresponding to the X boson do not force the Z and $A^{\hat{3}}$ profiles to be zero, or vice versa. This indicates that the mass eigenstate is not simply given by the mixed profile $f(z; v)$ with the appropriate coefficients. The profile of the true mass eigenstate is defined as a superposition of the mixed states weighted by their generators; the generic form is

$$F_G(m, z; v) T^{FG} = \sum_{\alpha} f_{\alpha}^G(m, z; v) T^{\alpha}, \quad (3.46)$$

where α runs over the particles contained in the mixing group and G indicates that the mixed profiles within the expression have been evaluated with the coefficients associated with that particle G 's mass condition. The normalization condition for such a profile is given by

$$\int_{L_0}^{L_1} \frac{dz}{kz} \sum_{\alpha} f_{\alpha}^G(m_n, z; v) f_{\alpha}^G(m_m, z; v) = \delta_{mn}. \quad (3.47)$$

As such, both Z and $A^{\hat{3}}$ are described by the same $F_G(m, z; v)$, as they are associated with the same mass condition and coefficient set. We can therefore see that what we identified as separate Z and $A^{\hat{3}}$ bosons before EWSB have mathematically become combined into a single 5D boson after EWSB (although the resulting KK modes can still be identified with the original Z and $A^{\hat{3}}$ KK modes based on their masses). The bosons of mixing groups 1 and 2 behave similarly, while $A^{\hat{4}}$ never mixes at all due to

the fact that its generator commutes with the mixing transformation.

As one might expect, the primary influence of this mixing manifests through the particle couplings. Due to the form of Eq. (3.46), the coupling of a given boson to other particles no longer involves solely its own generator, but also the generators of the bosons with which it is mixed. Consider the coupling of two gauge bosons, G_1 and G_2 , through a Higgs. Suppose that each is a superposition of two states; G_1 of states A and B , G_2 of states C and D . Their coupling through the Higgs will be the sum of all possible couplings between their components

$$\begin{aligned} G_1(m_1) G_2(m_2) H \rightarrow & C_{ACH} f_A^{G_1}(m_1) f_C^{G_2}(m_2) f_H + C_{ADH} f_A^{G_1}(m_1) f_D^{G_2}(m_2) f_H \\ & + C_{BCH} f_B^{G_1}(m_1) f_C^{G_2}(m_2) f_H + C_{BDH} f_B^{G_1}(m_1) f_D^{G_2}(m_2) f_H, \end{aligned} \quad (3.48)$$

where the C_{ABC} 's represent the usual coupling coefficient between particles A , B , C identified by generators T^A , T^B , T^C . The procedure is the same regardless of the number of mixed bosons involved in the coupling or the number of superimposed states per boson.

This complication of the couplings produces significant differences compared to the pre-EWSB couplings, as well as new couplings which were not previously allowed. For example, before EWSB a Z boson could not couple W_L^+ and W_R^- to each other; however, after EWSB W_L^+ (W_R^-) contains a small component of W_R^+ (W_L^-), which couples through the Z to W_R^- (W_L^+), allowing the coupling to be non-zero. Similarly, after EWSB the KK ZZH couplings contain a contribution from $ZA^{\hat{3}}H$ which significantly alters their value. However, it is worth noting that couplings involving zero modes – SM particles – are only slightly shifted by particle mixing; the dominant effects occur in couplings between KK modes. Furthermore, the profiles themselves are not substantially altered from those before EWSB. Although one cannot plot the

EWSB profile $F_G(m, z; v)$ due to the generator dependence of its definition, the profile normalization integrands before and after EWSB are indistinguishable when plotted together; in general the relative difference is of a few percent.

Fermion Mixing

The inter-mixing of fermion states through EWSB can also be accounted for via a similar gauge transformation on each of the fermion multiplets. The multiplets must each be written in vector form, with lepton multiplets $\xi_{\ell_{1L}}$, $\xi_{\ell_{2R}}$ and $\xi_{\ell_{3R}}$ corresponding to profile vectors $f_{\ell_{1L}}$, $f_{\ell_{2R}}$ and f_{ℓ_3} , and similarly for quarks. The vectors are defined by Eq. (C.81) in terms of the fermion profile base functions $S_c^\pm$ and $\dot{S}_c^\pm$ – Eqs. (3.30-3.31)) – and normalization coefficients C_f . However, these vectors are not defined in the same basis as our generators; to apply the gauge transformation, we must transform the fermion vector to the generator basis, and then transform back to the fermion basis so that boundary conditions can be applied. As such, our lepton profiles after EWSB are related to those before EWSB by

$$f_{\ell_{1L}}(m_n, z; v) = A\Omega(z, v)A^{-1}f_{\ell_{1L}}(m_n, z; 0), \quad (3.49)$$

$$f_{\ell_{2R}}(m_n, z; v) = A\Omega(z, v)A^{-1}f_{\ell_{2R}}(m_n, z; 0), \quad (3.50)$$

$$f_{\ell_{3R}}(m_n, z; v) = B\Omega(z, v)B^{-1}f_{\ell_{3R}}(m_n, z; 0) \quad (3.51)$$

where $\Omega(z, v)$ is defined above in Eq. (3.42) and the change-of-basis matrices A and B are given in Eq. (C.87) and Eq. (C.88) of Appendix C.3.

As in the boson case, we may determine the mass conditions of our mixed fermions by requiring that the determinant of the system of equations given by the fermion boundary conditions, Eqs. (3.37-3.39), be zero, and each mass condition will be associated with a particular solution set of coefficients C_f . As the boundary

conditions of the quarks may be obtained from those of the leptons by setting the Majorana mass parameters to zero, their coefficients and mass conditions may also be obtained from those of the leptons in this limit, and the mixing sectors are analogous

If no Majorana masses are incorporated, the system of boundary equations yields the following mass conditions,

$$\begin{aligned}
S_{c_3}^{-5} &= 0, \\
S_{c_2}^{-3} &= 0, \\
S_{c_1}^+ S_{c_3}^- + M_2^2 S_{c_1}^+ S_{c_3}^- &= 0, \\
2 S_{c_3}^+ \left(M_2^2 S_{c_3}^- S_{c_1}^- + S_{c_1}^- S_{c_3}^- \right) - (k z)^4 M_2^2 S_{c_1}^- \sin^2 \theta_G &= 0, \\
2 \left[M_1^2 S_{c_1}^+ S_{c_2}^- S_{c_2}^+ \left(M_2^2 S_{c_3}^- S_{c_1}^- + S_{c_1}^- S_{c_3}^- \right) + S_{c_2}^+ S_{c_2}^- \left(M_2^2 S_{c_1}^- S_{c_1}^+ S_{c_3}^- + S_{c_1}^- S_{c_1}^+ S_{c_3}^- \right) \right] \\
+ \left[M_2^2 S_{c_2}^+ S_{c_3}^- S_{c_2}^- + M_1^2 \left(2 M_2^2 S_{c_1}^+ S_{c_3}^- S_{c_1}^- + 2 S_{c_1}^+ S_{c_1}^- S_{c_3}^- - S_{c_2}^+ S_{c_2}^- S_{c_3}^- \right) \right] (k z)^4 \sin^2 \theta_G \\
- M_1^2 S_{c_3}^- (k z)^8 \sin^4 \theta_G &= 0
\end{aligned} \tag{3.52}$$

where c_1 , c_2 , and c_3 correspond to the parameter values of the respective multiplets and we have dropped the dependence on m_n , $z = L_1$ and v

The first three conditions correspond to non-SM fermions, while the fourth and fifth correspond to down-type and up-type fermions respectively. In total there are four mass conditions for the charge 1 (5/3) leptons (quarks), three for the charge -1 (-1/3) sector and five for charge 0 (2/3) sector. Both the upper and lower charged sectors have an unmixed state, resulting in five groups of equations to be solved for 12 mass conditions and coefficient sets. These five sectors are summarized in Tables 3.5 and 3.6

In a similar fashion to the gauge bosons, the full fermion profile F_ℓ^f associated with a given solution set f is the superposition of all of the $f_{\ell_i}(m_n, z, v)$, $i = 1, 2, 3$,

Table 3.5: Sectors of quarks mixed by EWSB through the Higgs vev v . Recall that t_{1_L}, b_{1_L} and $\hat{t}_{2_R}, B'_{3_R}$ are the left- and right-handed SM quarks.

Group	1	2	3	4	5
Mixed Quark	$\chi_1, \chi_3, \Xi_3, \Xi'_3$	χ_2	b_1, b_3, B_3, B'_3	b_2	$\tilde{t}_1, t_1, \hat{t}_1, \tilde{t}_2, t_2, \hat{t}_2, \tilde{t}_3, t_3, T_3, T'_3$
Combined Quark	χ_1, χ_3	—	b_1, b_3, B'_3	—	$(\tilde{t}_1, \tilde{t}_3), (t_1, \hat{t}_1, t_2, \hat{t}_2, t_3)$
Independent Quark	Ξ_3, Ξ'_3	—	$B_3,$	—	$\tilde{t}_2, T_3, T'_3$
Unmixed Quark	—	χ_2	—	b_2	—

evaluated with the specified coefficients.⁷ As such, the 4D coupling between a gauge boson G and two fermions f_1 and f_2 has the form

$$g_{G\bar{f}_1 f_2} = g_5 \int_{L_0}^{L_1} \frac{dz}{(kz)^4} \sum_{\alpha, i} f_{\ell_i}^{f_1 \dagger}(m_n, z; v) T^\alpha f_\alpha^G(m_n, z; v) f_{\ell_i}^{f_2}(m_n, z; v) \quad (3.53)$$

and the profile normalization condition is

$$\int_{L_0}^{L_1} \frac{dz}{(kz)^4} \sum_i f_{\ell_i}^{f \dagger}(m_n, z; v) f_{\ell_i}^f(m_n, z; v) = 1 \quad (3.54)$$

where i and f are defined as before. The generators for the **5** and **10** representations are given explicitly in Appendix B. It can be shown using the boundary conditions

⁷Any coefficients in the multiplet which were not a part of the solution set are taken to be zero

Table 3.6: Sectors of leptons mixed by EWSB through the Higgs vev v . Recall that n_{1_L}, ℓ_{1_L} and $\hat{n}_{2_R}, L'_{3_R}$ are the left- and right-handed SM leptons.

Group	1	2	3	4	5
Mixed Lepton	$\kappa_1, \kappa_3, K_3, K'_3$	κ_2	$\ell_1, \ell_3, E_3, E'_3$	ℓ_2	$\tilde{\eta}_1, \eta_1, \hat{\eta}_1, \tilde{\eta}_2, \eta_2, \hat{\eta}_2, \tilde{\eta}_3, \eta_3, N_3, N'_3$
Combined Lepton	κ_1, κ_3	—	ℓ_1, ℓ_3, E'_3	—	$(\tilde{\eta}_1, \tilde{\eta}_3), (\eta_1, \hat{\eta}_1, \eta_2, \hat{\eta}_2, \eta_3)$
Independent Lepton	K_3, K'_3	—	$E_3,$	—	$\tilde{\ell}_2, N_3, N'_3$
Unmixed Lepton	—	κ_2	—	ℓ_2	—

that the normalization coefficient for a given fermion will also normalize its chiral partner (when evaluated with the appropriate mass) [50].

3.2.6 The Coleman-Weinberg potential

As previously discussed, the Higgs boson in the MCHM arises as the zero mode of the A_5^4 scalar. Note that although this profile is linear in the warped coordinate z , in the flat-space coordinate y the profile is extremely peaked toward the TeV brane. Because the Higgs boson arises from the gauge sector in the MCHM, it does not have a potential at leading order as the SM Higgs does. Instead, the Higgs potential arises

primarily from loop contributions of the heavy gauge bosons and the top and bottom quarks.⁸ These one-loop interactions are the same effects that lead to the Planck scale corrections to the Higgs mass in the Standard Model.

The possibility that radiative corrections can induce spontaneous symmetry breaking – i.e., that the Higgs can gain a potential from loop corrections without having one at tree level – was first proposed by S. Coleman and E. Weinberg in 1973. To first order, the Higgs potential is given by their Coleman-Weinberg potential. In 4D theories, this has the following form [57]

$$V_{CW}^{(4D)}(\phi) = V_0(\phi) + i \sum_r \int_0^\infty \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^\infty \frac{1}{2n} \left(\frac{\frac{1}{2} g_r \phi^2}{k^2 - i\epsilon} \right)^n + \text{counterterms}, \quad (3.55)$$

where V_0 is the tree-level potential. The index r sums over all particles that obtain mass through the Higgs mechanism by coupling to the Higgs with strength g_r . The integral term is obtained by summing all one-loop diagrams whose external legs are the scalars ϕ . By using the series sum

$$-\ln(1 - z) = \sum_{k=1}^\infty \frac{z^k}{k}, \quad (3.56)$$

and Wick rotating ($k^2 \rightarrow -k^2$, $k_0 \rightarrow i k_0$), we obtain (taking $\epsilon \rightarrow 0$)

$$V_{CW}^{(4D)}(\phi) = V_0(\phi) + \frac{1}{2} \sum_r \int \frac{d^4 k}{(2\pi)^4} \ln \left(1 + \frac{g_r \phi^2}{2 k^2} \right) + \text{counterterms}. \quad (3.57)$$

Note that if the argument of the logarithm is set to zero and evaluated at the vev, $\phi = v$, it is equivalent to the mass condition for the propagating particle,

$$k^2 + \frac{1}{2} g_r v^2 = -m^2 + \frac{1}{2} g_r v^2 = 0 \rightarrow m^2 = \frac{1}{2} g_r v^2. \quad (3.58)$$

In 5D Kaluza-Klein theories, the Coleman-Weinberg potential – with ϕ re-

⁸We neglect the contributions of the light fermions in comparison with these.

placed by the vev v – is similarly given by [53, 4]

$$\begin{aligned}
V_{CW}(v) &= \sum_r \pm \frac{N_r}{(4\pi)^2} \int dp p^3 \ln(\rho_r(-p^2)), \\
&= \frac{3}{(4\pi)^2} \int_0^\infty dp p^3 [6 \ln(\rho_W(-p^2)) + 3 \ln(\rho_Z(-p^2)) \\
&\quad - 4 (\ln(\rho_t(-p^2)) + \ln(\rho_b(-p^2)))] , \tag{3.59}
\end{aligned}$$

where N_r is the number of degrees of freedom, and the plus applies to bosons while the minus applies to fermions. This form is analogous to that of Eq. (3.55); $V_0 = 0$ as there is no tree-level potential, counterterms are unneeded due to the convergence of the loop integrands, and the 4D mass condition is replaced by the analogous spectral functions, ρ . These are simply the particle mass conditions of the W boson, Z boson, bottom quark and top quark where the term independent of $\sin \theta_G$ has been divided out,

$$\rho_i(m^2) = 1 + F_i(m^2) \sin \theta_G(L_1, v)^2, \quad i = W, Z, b \tag{3.60}$$

$$\rho_t(m^2) = 1 + \frac{F_{t1}(m^2)}{2 F_d(m^2)}(m) \sin \theta_G(L_1, v)^2 + \frac{F_{t2}(m^2)}{2 F_d(m^2)}(m) \sin \theta_G(L_1, v)^4, \tag{3.61}$$

where the form-factors $F_i(m)$ are given by

$$\begin{aligned}
F_Z(m^2) &= \sec^2 \theta_w F_W(m) = \frac{m_n k L_1 \sec^2 \theta_w}{2 C'_A(m_n, L_1) S_A(m_n, L_1)} \\
F_b(m^2) &= -\frac{M_2^2 \dot{S}_{c_1}^-(k z)^4}{2 S_{c_3}^+ \left(M_2^2 S_{c_3}^- \dot{S}_{c_1}^- + S_{c_1}^- \dot{S}_{c_3}^- \right)} \\
F_d(m^2) &= M_1^2 S_{c_1}^+ \dot{S}_{c_2}^- \dot{S}_{c_2}^+ \left(M_2^2 S_{c_3}^- \dot{S}_{c_1}^- + S_{c_1}^- \dot{S}_{c_3}^- \right) + S_{c_2}^+ \dot{S}_{c_2}^- \left(M_2^2 \dot{S}_{c_1}^- \dot{S}_{c_1}^+ S_{c_3}^- + S_{c_1}^- \dot{S}_{c_1}^+ \dot{S}_{c_3}^- \right) \\
F_{t1}(m^2) &= M_2^2 S_{c_2}^+ S_{c_3}^- \dot{S}_{c_2}^- + M_1^2 \left(2 M_2^2 S_{c_1}^+ S_{c_3}^- \dot{S}_{c_1}^- + 2 S_{c_1}^+ S_{c_1}^- \dot{S}_{c_3}^- - \dot{S}_{c_2}^+ \dot{S}_{c_2}^- \dot{S}_{c_3}^- \right) \\
F_{t2}(m^2) &= -M_1^2 \dot{S}_{c_3}^-(k z)^8 \tag{3.62}
\end{aligned}$$

where the mass dependence in the $S_c(m, L_1; v)$ has been omitted.

Similarly to the SM case, the Higgs vacuum expectation value is determined by minimizing the Coleman-Weinberg potential, while its mass is determined by evaluating the second derivative at the vev [4]. As such, if the Coleman-Weinberg potential is divergent, so is the Higgs mass. This is the origin of the Hierarchy problem.

However, the 5D potential of Eq. (3.59) is finite and calculable. Fig. (3.3) plots the integrand of the CW potential for a particular parameter set; one can see that the integrand is exponentially suppressed as the momentum increases, creating an effective cut-off of the integration at an appropriate scale, eliminating the need for counterterms and avoiding the Hierarchy problem.

This exponential suppression can also be demonstrated analytically by using the asymptotic properties of Bessel functions (see Appendix A). Consider for example the Z form-factor $F_Z(m^2)$, which is inversely proportional to $C'_A(m_n, L_1) S_A(m_n, L_1)$. Assuming that $p L_1 \gg 1$, $p L_0 \ll 1$, and Eqs. (A.9-A.13), one can show that

$$\begin{aligned} C'_A(m_n, L_1) &= \frac{1}{2} \pi m^2 z (Y_0(m L_0) J_0(m L_1) - J_0(m L_0) Y_0(m L_1)) , \\ &\xrightarrow{\text{Wick rot}} \frac{1}{2} \pi i m^2 z \left(-\frac{2}{\pi} K_0(m L_0) I_0(m L_1) + I_0(m L_0) \frac{2}{\pi} K_0(m L_1) \right) \\ &\approx i m^2 z \left(\frac{\exp(m L_1)}{\sqrt{2 \pi m L_1}} \left(\ln \left(\frac{m L_0}{2} \right) + \gamma \right) + \sqrt{\frac{\pi}{2 m L_1}} \exp(-m L_1) \right) . \end{aligned} \quad (3.63)$$

Note that the first term is exponentially enhanced, while the second is exponentially suppressed. As such, the first term dominates and $C'_A(m_n, L_1)$ is approximately proportional to $\exp(m L_1)$. $S_A(m_n, L_1)$ can be shown to have a similar exponential proportionality, and thus the form factor $F_Z(m^2)$ is doubly exponentially suppressed. Furthermore, considering that $F_Z(m^2)$ is small, we may make use of the Taylor expansion

$$\ln(1 + x) = x - \frac{1}{2} x^2 + \frac{1}{3} x^3 \dots \quad (3.64)$$

to show that the Coleman-Weinberg integrand will be proportional to $F_Z(m^2)$, and

Suppression in the integrand of the CW potential

Figure 3.3: The integrand of the Coleman-Weinberg potential for $kL = 30$, $c_1 = 0.24$, $c_2 = -0.41$, $c_3 = 0.58$, $M_1 = 2.3$, and $M_2 = 0.4$; these values result in a Higgs vev corresponding to $\theta_G(L_1) = 0.318$.

therefore exponentially suppressed as well.

3.2.7 Parameters of the Minimal Composite Higgs Model

The Minimal Composite Higgs model depends on parameters g , θ_w , $M_z^{(0)}$, $kL = \ln(kL_1)$, and the various c_i and M_i that are necessary to specify the fermion multiplets. The first three of these are defined to be the usual SM parameters: $g = 0.649$, $\theta_w = 28.75^\circ$, $M_z^{(0)} = 91.18762$ GeV. The fourth parameter is usually taken to be

$kL \approx 30$ [4].

Once the fermion parameters of the third quark generation have been chosen, the Coleman-Weinberg potential may be minimized to determine the value of $\theta_G(L_1)$ [4].⁹ Using this value, k is determined by solving the Z boson mass condition such that $m = M_Z^{(0)}$. Having obtained k , L_0 and L_1 are given by their original definitions, $L_0 = 1/k$ and $L_1 = \exp(kL)/k$.

At this point the vev can be obtained by rearranging the equation for $\theta_G(L_1)$,

$$v = \frac{2\theta_G(L_1)}{g_5 \sqrt{k(L_1^2 - L_0^2)}}, \quad (3.65)$$

and, finally, the Higgs mass can be determined by evaluating the second derivative of the Coleman-Weinberg potential at the vev, $m_H = V''_{CW}(v)$, as previously discussed [4].

In an ideal world one would like to choose a set of parameters such that the resulting Higgs vev is that of the SM. However, in this parametrization scheme there is a close relationship between the Higgs vev v and the upper bound on the fifth dimension L_1 . This relationship is plotted in Fig. (3.4) for $kL = 30$; one can see that as the Higgs vev approaches the SM value of 246 GeV, the length of the fifth dimension approaches zero. As such, we cannot retain the SM Higgs vev value; we must strike a balance between keeping the vev value close to that of the SM and keeping L_1 near 1 TeV.

The choice of parameters has been further constrained by electroweak precision tests in the literature. In the MCHM with the MSW fermion embedding, if the light

⁹Note that the minimization condition can be written entirely in terms of the initial parameters and $\theta_G(L_1)$ – independently of k – by transforming the integration variable to pL_0 .

The relation between L_1 and the Higgs vev

Figure 3.4: A plot of the relationship between the Higgs vev v and L_1 for $kL = 30$. The dotted lines indicate the ideal values of $v = 246$ GeV and $L_1 = 1/\text{TeV}$.

fermions are placed close to the Planck brane – as is required to obtain realistic masses – then the KK mass scale $M_{kk} = \frac{1}{L_1} > 1.4$ TeV, implying that $L_1 < 0.714$ TeV^{-1} [49, 54]. The parameters of the third quark generation have further been constrained to the following regions of parameter space: $0 \leq |c_1| \leq 0.3$, $0.35 \leq |c_2| \leq 0.45$, $0.55 \leq |c_3| \leq 0.6$, $M_1 \geq 1$, and $M_2 < M_1$; ranges such that $c_1 > 0$ and $c_2 < -0.4$ are favoured by electroweak precision constraints (EWPC) [4, 50]. Furthermore, while light fermions generally require c values higher than 0.5 to obtain the right masses, EWPC require that $c < 0.75$ [52].

In all of the following work, we have used the initial parameters listed Table 3.7.

Table 3 7 Initial parameters used to solve the Coleman-Weinberg potential, as well as the resulting k , v , L_0 and L_1 . The parameters with superscript q_3 refer to those of the third quark generation, while c_{light} applies to all other fermions

kL	$c_1^{(q_3)}$	$c_2^{(q_3)}$	$c_3^{(q_3)}$	c_{light}	$M_1^{(q_3)}$	$M_1^{(q_3)}$	k [GeV]	v [GeV]	L_1 [GeV $^{-1}$]
30	0.24	-0.41	-0.58	0.70	2.3	0.5	$1.497 \cdot 10^{16}$	250.218	$7.140 \cdot 10^{-4}$

They are convenient as they abide by the constraints above, but result in a KK mass scale just above 1.4 TeV, so that the KK masses are as small as possible. Furthermore, parts of this parameter set was used for other purposes in the literature [50], which allowed for cross-checks of our work.

Table 3 8 Table of Z , X , and A^3 boson mass both before and after EWSB

KK order	Mass Before EWSB			Mass After EWSB		
	Z [TeV]	X [TeV]	A^3 [TeV]	Z [TeV]	X [TeV]	A^3 [TeV]
0	0		–	91.18	–	
1	3.442	3.368	5.367	3.437	3.368	5.372
2	7.809	7.732	9.826	7.804	7.732	9.831
3	12.199	12.121	14.249	12.194	12.121	14.254
4	16.595	16.515	18.661	16.590	16.515	18.667

As we will be neglecting zero-mode fermion masses, for simplicity we will not distinguish between the different light fermions, we will use the same parameters for

all leptons, neutrinos, and the first two quark generations. However, it should be noted that different choices of parameters will yield extremely different spectra of KK fermions.

For future reference, the parameters in Table 3.7 yield the spectrum of masses for the X , Z and A^3 bosons found in Table 3.8. As expected, the KK masses begin at roughly $2.5 M_{kk} = 2.5 \times 1.4 \text{ TeV} = 3.5 \text{ TeV}$. In accordance with Fig. 3.2, the X boson masses are left unchanged by the particle mixing. The $Z^{(0)}$ mass is shifted up to its SM value, while the Z KK masses are shifted down (but remain larger than the X masses). Similarly, the A^3 masses are shifted slightly upward.

Chapter 4

The Higgsstrahlung cross section

4.1 Higgsstrahlung as a probe of compositeness

In the Higgsstrahlung interaction an electron-positron collision produces a SM Z boson and a Higgs (see Fig. 4.1).

Figure 4.1: The Higgsstrahlung interaction, $e^+ e^- \rightarrow Z^0 H$.

We have chosen to study this interaction because the Higgs coupling has a similar structure to that of the $Z^{(n)}$ loop diagram that provides a correction to the Higgs mass. In the Standard Model this loop diagram is divergent, and contributes to the Hierarchy problem. However, it is not divergent in the MCHM, due to the exponential suppression in the integrand of the Coleman-Weinberg potential, as we saw in the previous chapter. In the effective 4D KK picture this is due to the warped geometry of the 5D model, in terms of the CFT picture it is due to the finite size of the composite Higgs boson.

Under either interpretation, one would expect the Higgsstrahlung cross section to display a similar suppression. The 5D structure of the Higgsstrahlung $Z^{(n)}Z^{(0)}H$ interaction is very similar to that of the $Z^{(n)}Z^{(n)}HH$ interaction, the only difference being the replacement of a n^{th} order Z boson profile with a zeroth order profile. Furthermore, one would expect the finite size of the composite Higgs to have the same effect on the Higgsstrahlung interaction as on the loop diagram.

Another attractive feature of Higgsstrahlung is that it is an s-channel interaction: therefore all of the initial energy from the fermions is transmitted directly through the propagator into the Higgs vertex. This allows for a clean high-energy interaction.

The unpolarized Standard Model cross section for this interaction is given by

$$\sigma_{\text{SM}} = \frac{G_f^2 M_z^{(0)}}{24\pi s^2} \lambda^{\frac{1}{2}}(s, M_z^{(0)2}, M_H^2) (C_V^2 + C_A^2) \frac{\left[12 M_z^{(0)2} s + \lambda(s, M_z^{(0)2}, M_H^2)\right]}{(s - M_z^{(0)2})^2 + \Gamma_z^{(n)2} M_z^{(0)2}}, \quad (4.1)$$

where G_f is the Fermi coupling constant, and

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \quad (4.2)$$

Note that when $s \gg M_z^{(0)2}$, $\lambda(s, M_z^{(0)2}, M_H^2) \approx s^2$ and therefore the cross section falls like s^{-1} . We will compare our MCHM Higgsstrahlung cross sections to Eq. (4.1), we

expect that the MCHM cross section will be strongly suppressed in comparison to the SM cross section above the KK mass scale

As the outgoing Higgs and $Z^{(0)}$ bosons are not charged, and the A_μ^a bosons do not couple to electron anti-electron pairs, the only bosons that can propagate through this interaction before mixing are the $Z_\mu^{(n)}$ and the $X_\mu^{(n)}$

4.2 Two equivalent perspectives

We have approached the calculation of the MCHM Higgsstrahlung cross section from two different but equivalent perspectives. Both involve integrating over the fifth dimension to obtain a four-dimensional effective theory

In the first case, we integrate the 5D Lagrangian before deriving Feynman rules. As a result, we obtain the usual 4D Feynman propagator for a vector boson. However, our 4D vertex rules now include an additional factor: the integral over z of the z -dependent 5D vertex components and the profiles of the relevant particles. In this case, we are treating our propagating bosons as KK modes. To obtain the full cross section, we would have to sum over all of the infinite number of KK modes, which is obviously not feasible. However, by truncating the sum we can obtain an approximate cross section. Despite the approximation, this approach has several advantages, notably that it is straightforward to include the zero-mode masses, gauge boson widths, and the effects of particle mixing in the calculation. We will refer to this approach as *Effective Vertex Approach* (EVA).

In the second case, we assemble the interaction amplitude using 5D Feynman rules, and then integrate over the fifth dimension. This method has the advantage of

including all of the propagating KK modes automatically, however, the inclusion of zero-mode masses, widths, and mixing effects becomes difficult. We will refer to this approach as *Effective Cross-section Approach* (ECA)

4.3 A first approximation: Higgsstrahlung without particle mixing

As a first approach to the analysis, we make an approximation in that we neglect particle mixing due to electroweak symmetry breaking. As discussed in Sec. 3.2.5, we expect the extra contributions to the Z from the X and A^3 bosons to be quite small, as the $Z^{(n)}$ boson after EWSB should still predominantly correspond to the original $Z^{(n)}$ boson, the same rationale applies to the mixing in the electron states. Therefore we expect this to be an acceptable approximation for our cross section.

Furthermore, we also neglect the effects of non-SM fermions, KK mode fermions channels are not included in any boson decay widths.¹

4.3.1 Feynman rules

A comprehensive summary of the 5D Feynman rules of the model – in the absence of mixed particle states – can be found in Appendix C. However, we will summarize those relevant to the cross section here.

In the EVA view each gauge KK mode is associated with the usual Feynman propagator for a heavy gauge boson. However, in the ECA view the 5D Z and X

¹The KK fermion channels in the gauge boson decay widths will be addressed in Sec. 3.2.5 when we compute the cross section with full particle mixing.

boson propagators from z to z' in unitary gauge are given by [38]

$$-i G(z, z', p) \left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) - i G(z, z', 0) \left(\frac{p^\mu p^\nu}{p^2} \right) \quad (4.3)$$

The function $G(z, z', p)$ is the Green's function of the equations of motion, Eq (3.17)

As such it is defined by

$$\left[p^2 - \frac{1}{z} \partial_z + \partial_z^2 \right] G(z, z', p) = k z \delta(z - z'), \quad (4.4)$$

where we have Fourier transformed the first four spatial coordinates x_μ to momentum space coordinates p_μ [51, 38]. The coordinate of the fifth dimension z has been left as a spatial coordinate. This differential equation has the general solution

$$G(u, v, p) = \frac{\pi}{2} \frac{k u v}{A D - B C} [A J_1(p u) + B Y_1(p u)] [C J_1(p v) + D Y_1(p v)], \quad (4.5)$$

where $u = \min(z, z')$, $v = \max(z, z')$, the coefficients A, B, C, D are determined by the appropriate BCs for the particle in question, and J and Y are the Bessel functions. The Z boson propagator will have $A = -Y_0(p L_0)$, $B = J_0(p L_0)$, $C = -Y_0(p L_1)$, and $D = J_0(p L_1)$, which correspond to Neumann-Neumann BCs. The X will have $A = -Y_1(p L_0)$, $B = J_1(p L_0)$, $C = -Y_1(p L_1)$, and $D = J_1(p L_1)$, which correspond to Dirichlet-Neumann BCs.

We also have the following vertices, which can be obtained by taking the functional derivative of the coupling terms in the model Lagrangian,

$$\begin{aligned} G_\mu^{(n)} Z_\mu^{(m)} H &\rightarrow \frac{i g_z g_G}{2 k z} \frac{g_5}{g} \eta^{\mu\nu} v f_H(z)^2 f_G^{(n)}(z) f_Z^{(n)}(z) \\ \bar{\Psi}_{L R} G_\mu^{(n)} \Psi_{L R} &\rightarrow \frac{g_G}{2(k z)^4} \frac{g_5}{g} C_{fG}^{L R} \gamma_\mu f_G^{(n)}(z) f_f(z)^2 \end{aligned} \quad (4.6)$$

where $G = Z, X$ are 4D gauge KK modes, H is the 4D Higgs boson, $f_f(z)$, $f_G(z)$ and $f_H(z)$ denote the appropriate fermion, gauge boson and Higgs profiles, given by

Eqs (3 18-3 21), Eqs (3 30-3 33) and Eq (3 22) in the non-mixing case We have defined the following constants,

$$C_{fZ}^{LR} = T^{3L} - Q \sin^2 \theta_w, \quad C_{fA}^{LR} = \frac{(Q - T^{3L}) \sin^2 \theta_w - T^{3R} \cos^2 \theta_w}{\cos 2\theta_w},$$

$$g_z = \frac{g}{\cos \theta_w}, \quad g_x = \frac{\sqrt{\cos 2\theta_w}}{\cos \theta_w}, \quad g_5 = g \sqrt{\frac{\ln(kL_1)}{k}} \quad (4.7)$$

Here $T^{3L,R}$ denotes the appropriate generator element for the right- or left-handed fermion in question Note that the first coupling of Eq (4 6) originates from a $G_\mu^{(n)} Z_\mu^{(m)} H H$ coupling where a single Higgs has been replaced with the vev, $H \rightarrow v$ This is why there are two factors of the Higgs profile, this coupling ultimately originates from the quartic gauge coupling in the gauge Lagrangian of Eq (3 8) The couplings of 5D particles would have the same form except that the profiles would be absent as they are included in the full 5D boson

It should also be noted that the left- and right handed fermion couplings have the same algebraic form due to the fact that the coupling terms in the fermion Lagrangian are the same for both chiralities However, due to the multiplet structure of the fermion sector, the values of $T^{3L,R}$ corresponding to our right- and left-handed SM fermions will be different, although the algebraic form is the same for all fermions the coupling values are not

4.3.2 Cross sections in the Effective Vertex Approach

Using the Effective Vertex Approach, the 4D effective amplitudes corresponding to the Higgsstrahlung cross section for generic fermions are

$$M_{G^{(n)}}^{LR} = -\frac{g_z g_G^2 v}{4} \epsilon^{\nu*}(p_4) Z_{ffG}^{(n)} Z_{GZH}^{(n,0)} C_{fG}^{LR} \frac{\bar{v}(p_1) \gamma_\nu (1 - \gamma_5) u(p_2)}{q^2 - M_G^{(n)2} + i\Gamma_G^{(n)} M_G^{(n)}}, \quad (4.8)$$

where p_1 , p_2 , and p_4 are the momenta of the anti-fermion, fermion, and external $Z^{(0)}$ boson respectively, $q = p_1 + p_2$ is the momentum exchange, and $\Gamma_G^{(n)}$ is the total decay width of the particle propagating in the s-channel. Here $G^{(n)} = Z^{(n)}, X^{(n)}$, L, R refer to the polarization of the fermions, the coupling factors $C_{fG}^{L,R}$ are given by Eq. (4.7), and the coefficients $Z_{Gff}^{(n)}(z)$ and $Z_{GZH}^{(n,0)}(z)$ are the integrals of the 5-dimensional profiles,

$$Z_{Gff}^{(n)} = \int_{L_0}^{L_1} dz \frac{1}{(kz)^4} \frac{g_5}{g} f_G^{(n)}(m_n, z) f_f^{(0)}(m_n, z)^2. \quad (4.9)$$

$$Z_{GZH}^{(n,0)} = \int_{L_0}^{L_1} dz \frac{1}{kz} \left(\frac{g_5}{g} \right)^2 f_G^{(n)}(m_n, z) f_Z^{(0)}(m_n, z) f_H(z)^2, \quad (4.10)$$

We have neglected the fermion masses and made use of the Dirac equation. Note that as we are integrating over the vertex by itself, all of the particles are external and on-shell; therefore the momentum dependence of the propagator profile is evaluated at the appropriate mass m_n .

In the center-of-mass frame the cross section corresponding to one of these terms is

$$\begin{aligned} \sigma_{G^{(n)}}^{L,R} &= \frac{g_z^2 g_G^4 v^2}{384\pi M_Z^{(0)2} s^2} \left[12 M_Z^{(0)2} s + \lambda(s, M_Z^{(0)2}, M_H^2) \right] \\ &\times \lambda^{\frac{1}{2}}(s, M_Z^{(0)2}, M_H^2) \frac{Z_{Gff}^{(n)2} Z_{GZH}^{(n,0)2} \left(C_{fG}^{L,R} \right)^2}{(s - M_G^{(n)2})^2 + \Gamma_G^{(n)2} M_G^{(n)2}}. \end{aligned} \quad (4.11)$$

The total cross section for a given polarization is given by

$$\begin{aligned} \sigma_{\text{tot}}^{L,R} &= \frac{g_z^2 v^2}{384\pi M_Z^{(0)2} s^2} \left[12 M_Z^{(0)2} s + \lambda(s, M_Z^{(0)2}, M_H^2) \right] \lambda^{\frac{1}{2}}(s, M_Z^{(0)2}, M_H^2) \\ &\times \left[\sum_{n=0}^{\infty} \frac{g_z^2 C_{fZ}^{L,R} Z_{zff}^{(n)} Z_{ZZH}^{(n,0)}}{(s - M_Z^{(n)2})^2 + \Gamma_Z^{(n)2} M_Z^{(n)2}} + \sum_{n=0}^{\infty} \frac{g_x^2 C_{fX}^{L,R} Z_{xzf}^{(n)} Z_{XZH}^{(n,0)}}{(s - M_X^{(n)2})^2 + \Gamma_X^{(n)2} M_X^{(n)2}} \right]^2. \end{aligned} \quad (4.12)$$

The unpolarized cross section σ corresponding to Eq. (4.1) can be obtained by adding the unpolarized cross sections and dividing by four,

$$\sigma_{\text{tot}} = \frac{1}{4} (\sigma_{\text{tot}}^L + \sigma_{\text{tot}}^R) . \quad (4.13)$$

4.3.3 Cross sections in the Effective Cross-section Approach

In the ECA approach, to obtain the effective 4D amplitude we integrate the five-dimensional components of the vertices together with the 5D propagator, $G(z, z'; q)$ of Eq. (4.5). The total effective 4D amplitude is therefore

$$M_G^{L,R} = -\frac{g_z v}{4} \epsilon^{*\alpha}(p_4) \bar{v}(p_1) \gamma_\alpha (1 - \gamma_5) u(p_2) g_G^2 Z_G(q, c) C_{fG}^{L,R} \quad (4.14)$$

where $G = Z, X$ and $G_Z(z, z'; q) = G^{(NN)}(z, z'; q)$ and $G_X(z, z'; q) = G^{(DN)}(z, z'; q)$; the superscripts indicate the appropriate boundary conditions for the boson in question. The parameter c refers to the electron profile parameter. The momenta p_1, p_2, p_3, p_4 , and q correspond to the anti-fermion, the fermion, the Higgs, the external Z boson, and the s-channel boson respectively. The constants $C_{fG}^{L,R}$ are the same as those defined in the previous section (Eq. (4.7)) and $Z_G(q, c)$ is the integral of all the 5-dimensional components over z and z' ,

$$\begin{aligned} Z_G(q, c) = & \int_{L_0}^{L_1} dz \int_{L_0}^z dz' \left(\frac{g_5}{g} \right)^3 \frac{1}{k z'} \frac{1}{(k z)^4} f_f^{(0)}(z)^2 f_Z^{(0)}(z') f_H(z')^2 G_G(z', z; q) \\ & + \int_{L_0}^{L_1} dz \int_z^{L_1} dz' \left(\frac{g_5}{g} \right)^3 \frac{1}{k z'} \frac{1}{(k z)^4} f_f^{(0)}(z)^2 f_Z^{(0)}(z') f_H(z')^2 G_G(z, z'; q) . \end{aligned} \quad (4.15)$$

The integration is performed so as to account for the fact that the propagator is in fact a piecewise function: $u = z$ and $v = z'$ if $z < z'$ while $u = z'$ and $v = z$ if $z > z'$. Note that in this case the propagator is not an external particle when we perform the integral, and therefore not on-shell; we therefore denote the propagator momentum

by q and not m . For analytic solutions to the integrals over the fifth dimension, please see Appendix E.

We therefore find that the total cross section is

$$\sigma_{L,R} = \frac{g_z^2 v^2}{384\pi M_z^{(0)2} s^2} \lambda^{\frac{1}{2}}(s, M_z^{(0)2}, M_H^2) [12 M_z^{(0)2} s + \lambda(s, M_z^{(0)2}, M_H^2)] \times \left[g_z^2 Z_Z(q, c) C_{fZ}^{L,R} + g_x^2 Z_X(q, c) C_{fX}^{L,R} \right]^2. \quad (4.16)$$

Let's consider the dimensions of $Z_G(q, c)$ for a moment. As each profile f has a dimension of $[\text{Energy}]^{\frac{1}{2}}$, $G(z, z'; q)$ has dimension $[\text{Energy}]^{-1}$ and g_5/g has dimension $[\text{Energy}]^{-\frac{1}{2}}$, we find that $Z_G(q, c)$ has dimension $[\text{Energy}]^{-2}$. Therefore $Z_G(q, c)$ has the same dimensions as the SM gauge boson propagator and the 4D amplitude $M_G^{L,R}$ is dimensionless, as it should be for a four body scattering process.

As noted previously, this approach to the calculation of the cross section is completely equivalent to the EVA approach if propagator widths and zero mode masses are neglected. This can be shown explicitly by means of the following relation which exists between the 4D gauge KK mode profiles and the 5D propagator [51],

$$G(z, z'; p) = \sum_{n=0}^{\infty} \frac{f_G^{(n)}(z) f_G^{(n)}(z')}{p^2 - M_G^{(n)2}}. \quad (4.17)$$

Using this identity, one finds that the integrals in Eq. (4.15) separate into the two integrals given by Eqs. (4.9) and (4.10), and therefore that

$$Z_G(c, q) = \sum_{n=0}^{\infty} \frac{Z_{Gff}^{(n)} Z_{GZH}^{(n,0)}}{q^2 - M_G^{(n)2}} \quad (4.18)$$

Therefore, the cross section obtained in Eq. (4.16) is completely equivalent to that in Eq. (4.12) in the limit that the gauge boson KK mode widths $\Gamma_G^{(n)}$ and zero-mode masses $M_Z^{(0)}$ are neglected (as Eq. (4.17) does not include EWSB, and therefore $M_Z^{(0)}$ is taken to be zero).

4.3.4 Comparison and analysis of cross sections

Although both the Z and X bosons may propagate in the $e^+e^- \rightarrow Z^{(0)}H$ cross section, the contribution of the X boson is effectively negligible for our choice of $c = 0.7$ for the electron, and remains so unless the electron c parameter becomes quite close to 0.5. As light fermions such as the electron ($c > 0.5$) have profiles peaked toward the Planck brane, their couplings to the X boson – whose profile is zero on the Planck brane due to the Dirichlet BC – are very weak. Therefore we may neglect the effects of the propagating X boson in the Higgsstrahlung cross section.

Fig. 4.2 compares the $e^+e^- \rightarrow Z^{(0)}H$ cross section of the SM, the MCHM from ECA, and the MCHM from EVA including the first 6 KK modes. We plot $q^2\sigma$ to cancel the q^{-2} falloff of the SM cross section at high q^2 . None of these cross sections include widths, which leads to the resonance spikes at $q = m_n$ for the Z KK modes.

The figure demonstrates the excellent correspondence between 4D and 5D approaches to calculating the cross sections. The four KK EVA MCHM cross section perfectly matches the full ECA MCHM cross section until the energy range approaches the mass of the fourth KK mode, where it begins to deviate slightly. After the fourth KK mode there is a sharp departure from the ECA curve – which encodes the full MCHM – exactly as one would expect. However, the six KK EVA cross section continues to match the ECA curve as we have not yet reached the limit of the KK modes that it includes.

This behaviour is supported by Fig. 4.3, where the cross section is plotted with an increasing number of KK modes included. As each curve reaches the final KK mode included it deviates sharply from the 5D curve, but flattens out at a smaller value. Smaller deviations from the 5D curve can also be seen when each EVA curve

Comparison of the MCHM and SM Higgsstrahlung Cross-sections

Figure 4.2: The $e^+e^- \rightarrow Z^{(0)} H$ from 4D (dotted line) and 5D (solid line) plotted with the SM cross section (dashed line). The units of σ and q are $[\text{GeV}]^{-2}$ and $[\text{GeV}]$ respectively. Note that the 4KK and 6KK curves are virtually indistinguishable from the full MCHM curve until the energy begins to approach their respective cut-off KK masses.

reaches the KK mode preceding the final mode included, indicating that the cross section becomes less accurate as it approaches the cut-off KK mode, as one would expect. This excellent correspondence between the truncated KK cross sections and the full cross section below the cut-off level indicates that the cross section is not substantially altered by KK modes that are much heavier than the center-of-mass (CoM) energy.

MCHM Cross-section with an increasing number of KK modes

Approximation. No particle mixing, no widths

Figure 4.3: A plot of the 4D $e^+e^- \rightarrow Z^{(0)} H$ cross section times q^2 including an increasing number of KK modes, up to 6 (dotted lines). Also plotted is the cross section obtained from 5D (solid line). The units of σ and q are $[\text{GeV}]^{-2}$ and $[\text{GeV}]$ respectively.

However, one might note a small discrepancy between the EVA and ECA cross sections at low energy in these two figures; the ECA cross section is substantially smaller than the others through the original SM peak from 200-1000 GeV. This feature is due to the fact that the ECA cross section does not include the zero-mode mass of the Z boson. If one plots the EVA cross sections with $M_Z^{(0)} = 0$ this discrepancy is resolved. Furthermore, all of the MCHM cross sections are slightly higher than that

of the SM; this is entirely due to the difference in vev value, v .

Figure 4.4: The influence of widths on the Higgsstrahlung cross section. The full MCHM does not include widths. Note that these widths do not include KK fermion states.

Fig. 4.4 plots the cross section with widths included in the KK mode propagators. Formulae for the relevant widths can be found in Appendix D. Gauge boson widths are obtained by calculating the boson's decay rate to lighter particles; their addition changes the cross section's divergence at the boson mass into a Breit-Wigner:

$$\frac{q^4}{\left(q^2 - M_G^{(n)2}\right)^2} \rightarrow \frac{q^4}{\left(q^2 - M_G^{(n)2}\right)^2 + M_G^{(n)2} \Gamma_G^{(n)2}}, \quad (4.19)$$

We have considered only two body decay states, these include decays to two gauge bosons, a gauge boson and a Higgs, or fermion anti-fermion pairs. Any combination of zero modes and KK modes may contribute, as long as their combined masses are light enough to be kinematically allowed.

One can see that the addition of the widths into the cross section does not substantially affect the curve near the first KK resonance, but merely shortens the resonance peak. However, as we progress to higher orders of KK modes the widths begin to affect the cross section between resonances, ultimately leading to an increased suppression at higher energies. Note that these widths do not include KK fermion decay modes, such modes will further widen the resonances and suppress the cross section, as we will see in section 4.4. Though the inclusion of widths significantly alters the cross section, it does not change the behaviour of the EVA KK cross section as it approaches the truncation point, we can see that the four KK EVA cross section becomes slightly offset after the third resonance, and deviates sharply from the six KK EVA cross section after the fourth just as it did before.

The most salient feature of Fig. 4.2, however, is that as the center-of-mass (CoM) energy increases the MCHM cross section in between the KK mode resonances becomes very suppressed in comparison with that of the SM, as expected. This is due to the influence of the MCHM's 5D structure. From the EVA perspective, it arises due to cancellations between KK contributions to the cross section.

To demonstrate this cancellation, let us consider each boson KK mode cross section to be at a point far from the mass resonance, i.e., in the limit that $M^2 \ll s^2$.

²In truth there is no such point, as there are an infinite number of KK modes so one cannot even theoretically reach an energy higher than all of them. However, the behaviour of the cross section at this hypothetical point will indicate its overall behaviour in the troughs between resonances.

At such a point, the individual propagators will become equivalent as s overpowers $M_z^{(n)2}$ in the denominator. Therefore the relative difference between the SM and MCHM cross sections will be due to the difference between the SM coupling and the squared sum of the contributing MCHM couplings.

$$\frac{\sigma_{L,R}^{\text{MCHM}}}{\sigma_{L,R}^{\text{SM}}} \approx \left(\sum_{i=1}^n \frac{C_{Zff_{L,R}}^{(i)} C_{ZZH_{L,R}}^{(i)} + C_{Xff_{L,R}}^{(i)} C_{XZH_{L,R}}^{(i)}}{C_{Zff_{L,R}}^{(\text{SM})} C_{ZZH_{L,R}}^{(\text{SM})}} \right)^2, \quad \text{if } s \gg M^{(n)2}. \quad (4.20)$$

Note that in the case of the Z couplings C_Z the constant coupling factors will be the same as those of the SM, except that the vevs are different. As such, if we neglect X contributions the ratio of MCHM to SM couplings is proportional to the ratio of the vev and the fifth dimension integrals defined in Eqs. (4.9-4.10),

$$\frac{\sigma_{L,R}^{\text{MCHM}}}{\sigma_{L,R}^{\text{SM}}} \approx \left(\sum_{i=1}^n \frac{v}{v_{\text{SM}}} Z_{ZZH}^{i,0} Z_{Zff}^i \right)^2, \quad \text{if } s \gg M^{(n)2}. \quad (4.21)$$

For the purpose of examining the overall mathematical structure and behaviour, we will consider sums of up to twenty KK modes. However, it should be noted that at some point above $4\pi M_{kk}$ our effective theory will begin to break down; the strong interactions of the CFT will come into play and perturbation theory will no longer provide an accurate picture of the physics. As such, the details of results above the tenth KK mode may not be a true representation of the physics involved.

As the number of included KK modes in this sum increases, the MCHM squared sum decreases sharply in comparison to the SM coupling, as seen in Figures 4.5 and 4.6 for the left- and right-handed sums respectively. The first reason for this behaviour is related to the form of the integrals $Z_{ZZH}^{i,0} Z_{Zff}^i$. Due to the Bessel functions in the gauge boson profiles, these integrals are oscillatory functions of the mass m centered about zero. When evaluated at the gauge masses, the odd order KK modes turn out to have negative couplings, while the even order KK modes have even couplings.

Figure 4.5: The ratio of the squared sum of the left-handed Higgsstrahlung interaction couplings with respect to the left-handed SM Higgsstrahlung coupling plotted versus the number of included KK modes.

The second reason for the coupling sum behaviour is that the first Z KK mode happens to have an overall coupling roughly ten percent larger than the zero mode in magnitude, but opposite in sign. As such, adding in the first KK mode's contribution completely cancels the zero mode; the coupling sum becomes negative, with about a tenth of the magnitude of the zero mode coupling. Adding the second KK mode – which has a positive coupling – cancels the remainder of the first two, and yields a positive sum of even smaller magnitude. This procedure continues, with the coupling sum oscillating about zero but becoming smaller in magnitude as more and more KK

modes are added in. When the sum is squared, the negative sum values are flipped up into the positive quadrant, where they fall into line with the positive sum values. This creates the relatively smooth downward curve seen in the figures.

However, one will note that on the right side of the figures the points begin to fall out of line and split into two tails. Mathematically this occurs because the alternating sequence is not perfectly centered about zero, the negative sum values become smaller in magnitude more quickly than the positive values, indicating that the sum sequence has a non-zero limit. As such, in Figs. 4.5 and 4.6 the curve of squared summation values divides into two lines, one representing the squares of negative values, while the other corresponds to positive values. Eventually a point will occur where new negative additions become too small to cancel the previous sum, this is the low point seen in 4.6. From this point on the sum will remain positive, and will approach a non-zero value. As long as the c parameter is relatively large – as it must be for the light electron – this summation limit will be much smaller than 1. Incoming additions to the sum will still add and subtract from it, causing it to oscillate about the summation limit as it approaches it.

It is at this point that the contributions of the X couplings become important.³ We can see in Fig. 4.5 that the combined left-handed sum of Z and X couplings does not split as rapidly as the sum of only the Z contributions. This is due to the fact that the overall left-handed coupling coefficients⁴ of the Higgsstrahlung interaction with an X boson propagating are of equal magnitude but opposite sign to those when

³Of theoretical importance that is. This behaviour occurs at energies too high to be relevant to our cross-section plots or further analysis.

⁴By coupling coefficients we mean the constant coefficients of the coupling that are not related to the integral over the fifth dimension.

Z propagates. Because $T^{3L} = T^{3R}$ for a left-handed electron,⁵ and $g_x^2 = g_z^2 \cos 2\theta_w$, the X coupling coefficients are

$$g_x^2 \frac{(Q - T^{3L}) \sin^2 \theta_w - T^{3R} \cos^2 \theta_w}{\cos 2\theta_w} = g_z^2 (Q \sin^2 \theta_w - T^{3L}), \quad (4.22)$$

which is the negative of the Z coupling coefficients

The only difference between the two couplings is the value of the integrals over the fifth-dimensional components, $Z_{ZZH}^{i0} Z_{Zff}^i$ and $Z_{XZH}^{i0} Z_{Xff}^i$. As it happens, the sums of these integrals approach similar values as more KK modes are included, as demonstrated in Fig. 4.5, allowing the X contribution to cancel that of the Z boson. The sum of both the X and Z left-handed contributions to the Higgsstrahlung interaction will have a limit much smaller than that of the sum of the Z contributions alone. The cancellation is not perfect, and the sum will still approach a smaller non-zero value eventually, but the effects of the non-zero limit – i.e. the splitting – will not become apparent until much higher energies are reached and many more KK modes are included.

However, including the X couplings in right-handed sums does not cause a reduction in the sum limit, in fact, one will note from Fig. 4.5 that their addition causes the deviation to increase. This is due to the fact that for right-handed electrons $T^{3L} = 0$ and $T^{3R} = -1 = Q$, so that the right-handed X coupling coefficients are

$$g_x^2 \frac{(Q - T^{3L}) \sin^2 \theta_w - T^{3R} \cos^2 \theta_w}{\cos 2\theta_w} = g_x^2 Q = g_z^2 Q \cos 2\theta_w \quad (4.23)$$

When added to the Z coupling of $-g_z^2 Q (\sin^2 \theta_w)$, the total becomes $-g_z^2 Q (\cos^2 \theta_w)$. Not only is this non-zero, but $\cos^2 \theta_w > \sin^2 \theta_w$, which is why the deviation in the

⁵If the Higgsstrahlung process were to occur with up-type fermions incoming, rather than down-type, the right-handed sum would approach zero but the left-handed would not.

Figure 4.6: The ratio of the squared sum of the right-handed Higgsstrahlung interaction couplings with respect to the right-handed SM Higgsstrahlung coupling plotted versus the number of included KK modes.

tails increases; the combined X, Z sum approaches a larger limit value than the sum with only Z contributions.

It is interesting to note that the left-handed sum is mostly independent of the value of c . When c is small (< 0.5), both the X and Z contributions sum to large values, as the fermion profiles are located farther away from the Planck brane and couple more strongly with the bosons. In this case, as discussed above, the X sum cancels that of the Z such that the resulting sum is small. When c is large, the X contributions are largely negligible as the fermions are very close to the Planck brane

where the X profile is zero. However, the Z contributions alone still sum to a limit on the same order of magnitude as that when c was larger. Furthermore, it seems that although the sum limit is small for all c , it is also non-zero for all c , if one explicitly constrains the fermions to the Planck brane, the alternating sequence of the sum still demonstrates the slight offset from the origin, suggesting that the ultimate limit is still non-zero. In fact, visually there is no difference between Fig. (4.5) and an analogous plot with the fermions constrained to the Planck brane.

The primary influence of c is on the right-handed sums, which may be quite large (greater than 1) for small values ($c < 0,5$), where both the X and Z sums are large and do not cancel. As the ratios of the Z MCHM couplings to SM couplings are the same for both right and left chiralities, as c become large – and the X contributions become negligible – the right-handed sum becomes equivalent to the left, as both obey Eq. (4.21).

As seen in Fig. 4.7, the suppression that results from this KK coupling cancellation is initially stronger than that of an exponential, but becomes less so as the mass of the largest included KK mode increases. The suppression with respect to mass can be parametrized by a function of form $y = x^a \exp[b + c(\ln x)^2]$, this fit was obtained by taking the logarithm of both variables, fitting a second order polynomial, and then reversing the transformation. The form of the fit suggests an initial exponential influence to the suppression of the cross section, however, the exponential factor is mitigated by both the power factor x^a and the logarithm in its argument as the energy increases. Indeed, the fit function can equivalently be expressed without an exponential as $y = d x^{a+c \ln z}$, where the constant $d = \exp b$.

Fig. 4.7 depicts the cross-section behaviour more accurately than a plot versus the number of KK modes, as the cross section spaces KK modes based on their mass,

Figure 4.7: The squared sum of Higgsstrahlung to SM coupling ratio (both left- and right-handed) plotted on a log-log scale versus the mass of the largest included KK mode. The parameters of the fit are $a = 3.7 \pm 0.2$, $b = -8.8 \pm 0.9$ and $c = -0.40 \pm 0.01$.

not their order. However, it is interesting to note that when plotted on a log-log scale versus $x = n + 1$ the squared sums are linear before the splitting behaviour begins, as seen in Fig. (4.8); this indicates a power-law relationship, $y = c x^b$, where $b < 0$ as the slope is negative. Therefore the relative suppression of the n KK mode squared sum with respect to the $n - 1$ KK mode squared sum is described by the following

Figure 4.8: The squared sum of Higgsstrahlung to SM coupling ratio plotted on a log-log scale versus the number of KK modes n plus one. The parameters of the fit are $a = -0.6 \pm 0.2$ and $b = -5.50 \pm 0.07$.

relationship

$$\frac{\left(\sum_{i=0}^n C_i\right)^2}{\left(\sum_{i=0}^{n-1} C_i\right)^2} \xrightarrow{i=j-1} \frac{\left(\sum_{j=1}^{n+1} C_j\right)^2}{\left(\sum_{j=1}^n C_j\right)^2} = \frac{a(n+1)^{-b}}{a n^{-b}} = \left(\frac{n}{n+1}\right)^b. \quad (4.24)$$

4.3.5 Coupling extraction

As with any model, it is important to determine how well the characteristic behaviour of the couplings can be identified at a collider. Let us consider the characterizability

of the MCHM of $e^+e^- \rightarrow Z^{(0)} H$ at the proposed International Linear Collider (ILC) and Compact Linear Collider (CLIC) Both of these accelerators will collide electrons with positrons

The ILC is the more developed of the two, it may be possible to commence construction as early as 2012 It is hoped that ILC will provide complementary data to that of the LHC The ILC proposes to scan center-of-mass (CoM) energies from 200 to 500 GeV – with the possibility of upgrading to 1 TeV – and is expected to attain a luminosity of 500 fb^{-1} over 4 years [58, 59] CLIC, on the other hand, will require a great deal more research and development before construction is possible, however, it is expected to reach much higher energies – scanning from 0.5 to 5 TeV with an optimum energy of 3 TeV – with twice the luminosity of the ILC [60]

In Table 4.1 we have tabulated the relevant statistics for the MCHM Higgsstrahlung cross section – without mixing or widths – measured at 500 GeV, 1 TeV, 3 TeV and 5 TeV with a luminosity of 500 fb^{-1} The expected number of events is found by multiplying the cross section by the luminosity

Note that these statistics include all possible decay modes of the $Z^{(0)} H$ final state Roughly twenty percent of the Z decays will be invisible, as they involve neutrinos which cannot be detected However, using Higgs decay data it is possible to reconstruct whether or not a given final state resulted from a $Z^{(0)}$ decay, and thus the Higgsstrahlung process This reconstruction would allow us to compensate for some of the lost percentage It could also be compensated for by increasing the luminosity by 20 percent, or a combination of both strategies

As a first scenario, let us assume that the masses and widths of both $Z^{(0)}$ ⁶

⁶The zero mode Z mass and widths have in fact been precisely measured at LEP

Table 4.1 Cross-section data used for coupling extraction

COM Energy, $\sqrt{s}$ [TeV]	$e^+e^- \rightarrow Z^{(0)} H$ cross section, σ [fb]	Number of Events, N	Statistical Error on N , $\sqrt{N}$	Statistical Error on σ , $\Delta\sigma$ [fb]
0.5	60.18	30089	173	$3.469 \cdot 10^{-1}$
1	15.81	7903	89	$1.778 \cdot 10^{-1}$
3	26.09	13046	114	$2.285 \cdot 10^{-1}$
5	$7.128 \cdot 10^{-1}$	356	19	$3.776 \cdot 10^{-2}$

the LHC ⁶ We would like to know whether the first MCHM Z KK mode could be identified at the ILC or CLIC through the Higgsstrahlung interaction, and what identifying information about the coupling could be determined

Fig. 4.9 plots a two-parameter $\Delta\chi^2$ test variable

$$\Delta\chi^2 = \sum_i \frac{(\sigma_{\text{th}}^i - \sigma^i)^2}{\Delta\sigma^i} \quad (4.25)$$

where σ and $\Delta\sigma$ are listed in the table above, $i = 1, 2, \dots, n$ denotes the n different energy values at which σ was measured and

$$\sigma_{\text{th}}(s, C_z^{(0)}, C_z^{(1)}) = \text{Coef}[s] \left| \frac{C_z^{(0)}}{s - M_z^{(0)2} + i\Gamma_Z^{(0)} M_Z^{(0)}} + \frac{C_z^{(1)}}{s - M_z^{(1)2} + i\Gamma_Z^{(1)} M_Z^{(1)}} \right|^2 \quad (4.26)$$

is a test function for the unpolarized $e^+e^- \rightarrow Z^{(0)} H$ cross section. Here $\text{Coef}[s]$ denotes the usual SM coefficients of the $e^+e^- \rightarrow Z^{(0)} H$ cross section and $C_z^{(0)}, C_z^{(1)}$

⁶The zero mode Z mass and widths have in fact been precisely measured at LEP

Figure 4.9: Extraction of $Z^{(0)}$ and $Z^{(1)}$ couplings from a χ^2 at 95 percent confidence using the following data points: 0.5 and 1 TeV (light grey, vertical); 1 and 3 TeV (medium grey, horizontal); 0.5 and 3 TeV (dark grey, centered); 0.5, 1, and 3 TeV (black, centered). The coupling ratios are expected to be $C_Z^{(0)} = 1.016$, $C_Z^{(1)} = -1.124$.

denote the magnitude of the $Z^{(0)}, Z^{(1)}$ couplings relative to the SM coupling. By ‘couplings’ here we mean the product of both the $Z^{(n)} \bar{f} f$ and the $Z^{(n)} Z^{(0)} H$ couplings, or the overall interaction coupling.

We can see from Fig. (4.9) that even at the ILC, which operates at energies much lower than the $Z^{(1)}$, the first Z KK mode coupling can clearly be differentiated from zero, and therefore identified through the Higgsstrahlung process. Furthermore, the two-point $\chi^2 = 5.99$ parameter space resulting from 0.5 and 1 TeV measurements clearly differentiates $C_Z^{(0)}$ from 1, this is due to the difference between the MCHM and SM vevs, and indicates that the MCHM $Z^{(0)}$ could be distinguished from the SM $Z^{(0)}$ at the ILC.

The 0.5 and 1 TeV parameter space relevant to the ILC also indicates that the $Z^{(1)}$ has the opposite sign of the SM coupling, however, it does not constrain its magnitude very precisely. CLIC would be able to show that $C_Z^{(1)}$ is both negative and roughly ten percent larger than the zero-mode coupling by providing data at the 3 TeV scale, recall that the mass of $Z^{(1)}$ is 3.44 TeV for our parameter choice. This is extremely important, as the sign and magnitude of the first Z KK coupling are key characteristics of the MCHM which lead to the suppression of the Higgsstrahlung cross section.

The best constraints for both parameters would be obtained from a data set that includes points at both the 0.5 and the 3 TeV range. If the former is omitted, $C_Z^{(0)}$ cannot be differentiated from C_{SM} , while if the latter is omitted the magnitude of $C_Z^{(1)}$ becomes vague. Attempts to include a 5 TeV data point yield a null parameter space, as the test function including only one KK mode deviates from the true cross section as energies move past the first KK resonance and begin to approach the second. This would provide evidence for the second KK mode.

Figure 4.10: Extraction of $Z^{(1)}$ and $Z^{(2)}$ couplings from a χ^2 at 95 percent confidence. The data points used, from the outside in, are: 0.5 and 3 TeV; 1 and 3 TeV; 0.5, 1, and 3 TeV; 0.5, 1, 3 and 5 TeV. The coupling ratios are expected to be $C_Z^{(1)} = -1.124$, $C_Z^{(2)} = 0.143$.

As a second scenario, let us assume that the Z zero mode coupling could also be determined at the LHC from Higgs production and decay measurements, along with the mass and width of the second Z KK mode at 7.8 TeV. Would it be possible to identify the coupling of this KK mode in such a case? Figure 4.10 plots the same χ^2 of Eq. (4.25) in a parameter space of the $Z^{(1)}$ and $Z^{(2)}$ couplings, $C_Z^{(1)}$ and $C_Z^{(2)}$, with the new test function

$$\sigma_{\text{th}}(s, C_Z^{(1)}, C_Z^{(2)}) = \text{Coef}[s] \left| \sum_{n=0}^2 \frac{C_Z^{(n)}}{s - M_Z^{(n)2} + i \Gamma_Z^{(n)} M_Z^{(n)}} \right|^2, \quad (4.27)$$

where $C_Z^{(0)}$ is now considered to be a known quantity. In this figure the χ^2 test is performed at the 95 percent confidence level, and is calculated from 2 data points (500 GeV and 3 TeV, 1 and 3 TeV), 3 data points (500 GeV, 3 TeV, and 5 TeV) and 4 data points (500 GeV, 1 TeV, 3 TeV, 5 TeV). Again, as long as a 3 TeV data point is included, the parameter spaces indicate that the first KK mode has the opposite sign and a slightly larger magnitude than that of the SM coupling. We can see that only the 4-point region - which includes the 5 TeV point - is clearly distinguishable from zero. Therefore the second KK mode will not be statistically identifiable unless CLIC is pushed to its maximum CoM energy. A full analysis of the interaction with backgrounds will be required to precisely determine CLIC's ability to properly identify the second Z KK mode. However, even if the coupling cannot be differentiated from zero, the 5 TeV point parameter space constrains the second Z KK coupling to be positive and smaller than the first KK mode coupling by a factor of four or more $|C_Z^{(2)}| \lesssim 0.25 |C_Z^{(1)}|$.

4.4 Higgsstrahlung with full particle mixing

4.4.1 Comparison and analysis of cross sections

Having completed the analysis in the original approximation, the next step is to investigate the effects induced by including full particle mixing and fermion KK decay modes. To incorporate the full particle mixing we must proceed by means of the EVA approach, making use of the mixed profiles and coupling definitions given in Section 3.2.5.

In the following, we continue to use the original parameters outlined in Table 3.7. As discussed in that section, we will use the same parameters for all of our light fermions; as such, we will choose $c_1 = c_2 = c_3 = 0.7$, $M_2 = 0.627$ and $M_1 = 1.5$ for all multiplets other than those of the third generation quarks. The choice of M_2 was made by requiring that the down-type fermion mass condition yield the electron mass, as this is the most important light fermion for our purposes.

The introduction of particle mixing does not substantially change the magnitude of couplings relevant to the Higgsstrahlung interaction. Fig. (4.11) compares the Higgsstrahlung cross section without widths both including and neglecting particle mixing; the cross sections are very similar.

The primary difference between the two is the extra resonances which occur between the primary Z KK peaks. These resonances correspond to our $A^{\hat{3}}$ bosons;⁷ due to their mixing with the Z boson KK modes, they are now able to propagate through the Higgsstrahlung interaction, though it was forbidden before EWSB.

The other difference is that the mixed cross section is slightly shifted down-

⁷Please refer to Table 3.8 for the $A^{\hat{3}}$ mass spectrum.

MCHM with mixing vs. without mixing (without widths)

Figure 4.11: The Higgsstrahlung cross section times q^2 with full mixing but no widths, compared with the approximate cross sections of Fig. (4.2) and the SM.

ward in comparison with the approximate cross section. This effect is entirely due to slight changes in the Z Higgsstrahlung couplings. Both the $Z\bar{f}f$ and ZZ^0H couplings individually become slightly smaller after particle mixing – by approximately 3 percent – which leads to an overall suppression of roughly 7 percent in the overall Higgsstrahlung interaction. This slight suppression in the couplings can also be observed in Fig. (4.12), which compares the right- and left-handed squared coupling sums before and after mixing; the centers of the mixed coupling icons (crosses and triangles) are slightly lower than those of the unmixed coupling icons (x's and diamonds).

In the case of the X couplings the fermion couplings are suppressed by approximately 2 percent after particle mixing, but the XZ^0H couplings become stronger by about the same amount. As a result the overall X Higgsstrahlung couplings alternate; odd mixed X KK modes have slightly stronger total couplings than the corresponding unmixed modes, but the even modes are slightly weaker. The overall change is less than one percent for all KK modes. As the change in all of the couplings is small, the X contribution to the cross section is still negligible in comparison to that of the Z .

Figure 4.12: A comparison of the square Higgsstrahlung coupling sum – divided by the corresponding SM coupling – including particle mixing to the squared sums depicted in figures 4.5 and 4.6 of the previous section.

As a result, there is virtually no change in the behaviour of the squared coupling sums. As depicted in Fig. 4.12, there is the same suppression as the number of KK modes increases, and the same deviation in the right-handed couplings indicating that the sum does not ultimately trend to zero. The initial ratio between the MCHM and SM cross sections displays the same suppression behaviour seen in Sec. 4.3.4, the initial suppression displays the same $y = x^a \exp[b + c(\ln x)^2]$ behaviour but eventually plateaus at some small value due to the slight offset in the alternating sum sequence. After this plateau the MCHM cross section would still be strongly suppressed compared to the SM, but the suppression would be constant, not exponential.

By far the primary influence of the particle mixing on the Higgsstrahlung cross section is through the boson decay widths. Although the couplings involving SM particles are not greatly affected by the mixing – as the mixing gauge transformation has no effect on the Planck brane – couplings between higher order KK gauge bosons are significantly altered. Furthermore, new decay modes become available due to couplings between lesser components of the mixed bosons, as previously discussed. These new effects greatly increase the decay widths of the propagating bosons, flattening the corresponding resonance peaks in the cross section.

In Fig. 4.13, a mixed cross section is plotted with the corresponding unmixed 6 KK mode cross section of Fig. 4.4. Both these cross sections include only the boson and SM fermion decay modes, we have not yet included KK fermion decay modes. Yet despite the fact that the widths are incomplete, the KK modes beyond $Z^{(1)}$ have become almost completely flat. The A^3 resonances have disappeared entirely, due to their small couplings before widths were included. Furthermore, the increase in the widths has further suppressed the cross section.

Figure 4.13: The Higgsstrahlung cross section times q^2 with 6 KK modes, widths and full particle mixing compared with the corresponding cross section without mixing, Fig. 4.4. Only boson and SM fermion decay channels are included in the widths.

However, note that the first KK mode resonance has only been shifted down very slightly, similar to the shift in the overall cross section. As the lightest boson KK mode, the $Z^{(1)}$ cannot decay into other KK bosons; therefore the width of the mixed $Z^{(1)}$ resonance in Fig. 4.13 includes only decays to zero mode particles. Since the couplings of these particles are only slightly shifted by mixing effects, the $Z^{(1)}$ resonance is not greatly altered by the mixing if KK fermion decay channels are omitted from the width.

Including the full KK fermion decay modes in the $Z^{(1)}$ width has the result of shortening the peak slightly, as can be seen in Fig. 4.14. The effect is not dramatic, as only the first top quark KK mode is light enough for the first Z to decay into it – i.e., $M_t < 0.5M_Z^{(1)}$ – when $M_1 = 1.5$ for the light fermions.

However, our original choice of this parameter M_1 was somewhat arbitrary. To gain a better idea of the influence of the fermions on the $Z^{(1)}$ width, we also include a full width for the case that $M_1 = 2.2$, in this case the first neutrino, up and charm KK modes are also light enough to be included. The resulting decrease in the resonance peak cross section is small, although numerically the size of the width jumps from 346 GeV to 442 GeV (roughly from ten to thirteen percent of the $Z^{(1)}$ mass). Had we chosen an entirely different parameter set the first fermion KK masses could become much smaller, on the order of several hundred GeV [4, 50]. Obviously this would significantly alter the boson widths, as such our results are significantly model dependent.

Even without lowering the first fermion KK masses, the widths of the KK modes beyond the first Z would become even more significantly suppressed if all of the KK fermion decay modes were included. Consider the second Z KK mode, at the very least, its higher mass allows decays to all of the first order fermion KK modes and the second KK modes of the top quark. Because of this, it will include decays to many exotic fermions which aren't available to $Z^{(1)}$. As there are 6 sets of fermion multiplets – each containing 20 fermions – as well as their corresponding chiral partners, one can see that the resulting contribution will be substantial.

Even if only the boson decay modes are included, the widths of the Z KK modes of order $n \geq 2$ are already between thirty and fifty percent of the boson's mass (for comparison, recall that the full width of the first Z KK mode is roughly

Fermion width effects in the first KK resonance

With full particle mixing

Figure 4.14: The first Z KK resonance peak in the Higgsstrahlung cross section times q^2 . The figure compares cross sections without widths, with only boson widths, and with both boson and fermion widths for $M_1 = 1.5, 2.2$.

ten percent of its mass). Once fermions are fully incorporated as well, it seems evident that the widths of higher order KK modes will soon begin to approach or exceed their masses. This indicates that the effective theory is breaking down; we are beginning to reach energies which probe the strongly coupled conformal field theory. At this range, the higher order KK modes become strongly coupled, and perturbation theory can no longer be used. As such, the width calculations are no longer accurate, as they contain terms proportional to couplings involving only heavy – and thus

predominantly composite – particles

However, it should be noted that the Higgsstrahlung cross section itself contains only couplings between KK modes and light SM particles. The same effect that keeps the masses of these particles light also requires that their couplings to the CFT sector be extremely weak. As such, the coupling behaviour of the cross section itself should not suffer from this deficiency. It is only the width calculations, which involve couplings between higher order KK modes, which become unreliable.

4.4.2 Coupling extraction

We will now apply the same coupling extraction procedure that was performed in Section 4.1.5 to the MCHM Higgsstrahlung cross section with full mixing and widths. The relevant statistics are tabulated in Table 4.2. For the record, we are still using the parameter $M_1 = 1.5$ for the light fermions.

Table 4.2 Cross-section data used for coupling extraction

COM Energy, $\sqrt{s}$ [TeV]	$e^+e^- \rightarrow Z^{(0)} H$ cross section, σ [fb]	Number of Events, N	Statistical Error on N , $\sqrt{N}$	Statistical Error on σ , $\Delta\sigma$ [fb]
0.5	52.91	26455	163	$3.253 \cdot 10^{-1}$
1	13.90	6947	83	$1.667 \cdot 10^{-1}$
3	21.51	10752	104	$2.074 \cdot 10^{-1}$
5	$5.819 \cdot 10^{-1}$	291	17	$3.412 \cdot 10^{-2}$

Figure 4.15: Extraction of $Z^{(0)}$ and $Z^{(1)}$ couplings from a χ^2 at 95 percent confidence level. The data points used, from the outside in, are: 1 and 3 TeV; 0.5 and 3 TeV; 0.5, 1, and 3 TeV; 0.5, 1, 3 and 5 TeV. The coupling ratios are expected to be $C_Z^{(0)} = 0.953$, $C_Z^{(1)} = -1.052$, with $M_1 = 1.5$.

Fig. 4.15 is analogous to Fig. 4.9, it depicts the different $Z^{(0)}$ and $Z^{(1)}$ coupling parameter spaces produced by a χ^2 evaluated at the four energy values. In some

respects the results are very similar to those of Fig 4 9, again, the zero-mode coupling can be differentiated from the SM for all data sets, and the first Z KK coupling is shown to be roughly ten percent larger in magnitude than $C_Z^{(0)}$ and of opposite sign. However, the new parameter spaces reflect the shift in the Higgsstrahlung couplings due to particle mixing by centering on the new coupling values. Both are smaller in magnitude by approximately 7-8 percent respectively, which is consistent with the results found in the previous section. Furthermore, both couplings are more strongly constrained.

The plot depicted in Fig 4 16 is almost identical – and extremely similar in appearance – to Fig 4 10. However, the small central parameter space (black) which includes the 5 TeV point is slightly shifted down in comparison with the plot in the previous section. As a result, even at the highest center-of-mass energy of CLIC the second Z KK mode coupling will not be distinguishable from zero. As in the previous section, Fig 4 16 implies the constraint that the magnitude of $C_z^{(2)}$ be approximately one fourth that of $C_Z^{(1)}$. However, recall that this cross section does not include the full fermion decay width for $Z^{(2)}$. To determine how changes in the width will change the constraint, we set the width of the second KK mode equal to its mass, $\Gamma_Z^{(2)} = M_Z^{(2)}$ (using $M_1 = 2.2$). This plot is depicted in Fig 4 17, one can see that $C_z^{(2)}$ is constrained to be smaller than 0.25 even when the boson's width approaches its mass.

Figure 4.16: Extraction of $Z^{(1)}$ and $Z^{(2)}$ couplings from a χ^2 at 95 percent confidence level. The data points used, from the outside in, are: 0.5 and 3 TeV; 1 and 3 TeV; 0.5, 1, and 3 TeV; 0.5, 1, 3 and 5 TeV. The coupling ratios are expected to be $C_Z^{(1)} = -1.052$, $C_Z^{(2)} = 0.134$, with $M_1 = 1.5$.

Figure 4.17: Extraction of $Z^{(1)}$ and $Z^{(2)}$ couplings from a χ^2 at 95 percent confidence level. The data points used, from the outside in, are: 0.5 and 3 TeV; 1 and 3 TeV; 0.5, 1, and 3 TeV; 0.5, 1, 3 and 5 TeV. Here we have set the width of $Z^{(2)}$ equal to its mass, with $M_1 = 2.2$.

Chapter 5

Conclusions

5.1 Discussion and Conclusions

In this thesis we studied the Higgsstrahlung cross section in the Minimal Composite Higgs Model at center-of-mass energies spanning the masses of the first several KK gauge excitations. We have calculated the MCHM Higgsstrahlung cross section both with and without including full particle mixing. The MCHM cross section exhibits a strong suppression in comparison with that of SM Higgsstrahlung. This suppression is primarily due to a systematic cancellation between the contributions of different Kaluza-Klein modes propagating through the interaction. In fact, the extent of the suppression is largely determined by both the magnitude and sign of the first Z KK mode coupling. As it is both slightly larger in magnitude and opposite in sign to the zero-mode coupling, it provides the dominant reduction of MCHM cross section; the cancellation between the two reduces the magnitude of the cross section by roughly 90 percent. The inclusion or omission of full particle mixing does not substantially alter this suppression; the relevant couplings become a few percent smaller across the

board, but the behaviour of the coupling cancellations remains the same

The suppression between resonances displays an energy dependence which can be fitted to the form $y = x^a \exp[b + c(\ln x)^2]$. Although this form suggests an exponential influence on the dependence, it is evidently not a simple exponential suppression, as one might anticipate from the suppression in the Coleman-Weinberg potential, the exponential behaviour is diminished both by a logarithmic function in its argument and by a multiplicative power law factor. Furthermore, the alternating coupling sum sequence is slightly offset from zero, suggesting that as the energy increases the suppression will deviate from the fit function behaviour entirely and approach a constant, rather than approach zero as a true exponential or power law suppression would. However, this offset is very small, and would still allow a suppression of at least seven or eight orders of magnitude when compared with the SM. In the end, although there are certain resemblances between the suppressions observed in the Higgsstrahlung cross section and the Coleman-Weinberg potential, the mechanisms behind each are obviously not completely analogous. The exact relationship between the two remains unclear.

A secondary suppression in the cross section is caused by the extremely large widths of the KK bosons. However, this effect is not easily quantifiable as width calculations for the higher KK modes become invalid when full particle mixing is incorporated, this is the primary effect of including the full mixing, as it significantly alters the couplings of heavy KK modes. Due to the influence of strong couplings between these modes, the widths become larger than the associated particle mass, indicating a breakdown of the perturbation theory.

We also performed a brief analysis of the feasibility of extracting the couplings of this model at electron-positron colliders such as the proposed ILC and CLIC. We

determined that the MCHM zero-mode Higgsstrahlung coupling should be clearly distinguishable from the SM coupling at both colliders. Furthermore, with the extended energy range of CLIC the magnitude and sign of the first KK mode coupling – the key characteristics which lead to the primary cross-section suppression – should also be identifiable. Furthermore, while the second KK coupling will not be differentiable from zero, CLIC would be able to constrain its magnitude, requiring that it be less than a fourth of the first KK mode’s magnitude, this would provide further evidence of the suppression, as the couplings must both become increasingly smaller in size and alternate in sign for the effect to occur.

5.2 Future Research

There are several avenues open for future research. The first is to develop a more rigorous relation between the suppression behaviour seen in both the Higgsstrahlung cross section and the Coleman-Weinberg potential. Although we have a strong mathematical understanding of the origin and exponential nature of the Coleman-Weinberg suppression, we have yet to achieve the same for the Higgsstrahlung interaction (our results are only numerical), or to systematically relate the two. This question bears further thought and investigation. Further work in this direction could involve delving deeper into the mathematics analytically or studying other Higgs interactions to determine whether or not similar suppressions arise.

A second option is to extend the current work by performing a detailed collider study of the Higgsstrahlung interaction at the ILC and CLIC, with the goal of both confirming and extending the introductory phenomenology presented in sections 4.3.5 and 4.4.2. Furthermore, a similar analysis – both theoretical and phenomenological –

to that presented in this thesis could be performed for Higgsstrahlung from a proton proton collision. Such an analysis would be more complicated due to the need to incorporate the parton distribution functions, but would yield results relevant to the Large Hadron Collider currently in operation at CERN in Geneva.

Finally, a third option is to attempt to develop methods of incorporating both widths and particle mixing – and by extension zero-mode masses – into the 5D propagator. If such methods were to be developed, the full mixing cross section could be calculated in both the EVA and ECA approaches. This might lend a different perspective to the mechanisms behind the coupling and cross-section behaviours of the Higgsstrahlung interaction.

Appendix A

Special Functions

A.1 Bessel Functions

Bessel functions of the first and second kind – $J_n(x)$ and $Y_n(x)$ respectively – are solutions to Bessel's differential equation [61, 62, 63],

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0 \quad (\text{A.1})$$

where n is a non-negative real number. The two solutions are independent only if n is not an integer; otherwise they are related through

$$Y_n(z) = \frac{J_n(z) \cos(n\pi) - J_{-n}(z)}{\sin(n\pi)} \quad (\text{A.2})$$

The Bessel functions are orthogonal according to

$$\int_0^a J_\nu \left(\alpha_n \frac{\rho}{a} \right) J_\nu \left(\alpha_m \frac{\rho}{a} \right) \rho d\rho = \frac{1}{2} a^2 [J_{\nu+1}(\alpha_m)]^2 \delta_{mn} \quad (\text{A.3})$$

where α_m is the m th zero of J_ν , and similarly for Y_ν . Their asymptotic approximations

are

$$J_n(x) \approx \begin{cases} \frac{1}{\Gamma(n+1)} \left(\frac{x}{2}\right)^n & \text{for } x \ll 1 \\ \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right) & \text{for } x \gg |n^2 - \frac{1}{4}| \end{cases} \quad (\text{A } 4)$$

$$Y_n(x) \approx \begin{cases} \frac{2}{\pi} [\ln(\frac{1}{2}x) + \gamma] & \text{for } m = 0, x \ll 1 \\ \frac{\Gamma(n)}{\pi} \left(\frac{2}{x}\right)^n & \text{for } m \neq 0, x \ll 1 \\ \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right) & \text{for } x \gg 1 \end{cases} \quad (\text{A } 5)$$

and they obey the recurrence relations

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x) \quad (\text{A } 6)$$

$$\frac{d}{dx} J_n(x) = \frac{1}{2} [J_{n-1}(x) - J_{n+1}(x)] = J_{n-1}(x) - \frac{n}{x} J_n(x) = \frac{n}{x} J_n(x) - J_{n+1}(x) \quad (\text{A } 7)$$

$$\frac{d}{dx} [x^m J_m(x)] = x^m J_{m-1}(x) \quad (\text{A } 8)$$

and similarly for $Y_n(x)$

The modified Bessel functions of the first and second kind – $I_n(x)$ and $K_n(x)$ – are solutions to Eq (A 1) under the transformation $x \rightarrow i x$ [61, 62, 63] The Bessel functions of the first kind are related through

$$I_n(x) = i^{-n} J_n(ix) \quad (\text{A } 9)$$

In the case where n is an integer, the Bessel functions of the second kind are related as follows

$$K_n(x) = \frac{\pi}{2} \frac{I_{-n}(x) - I_n(x)}{\sin(n\pi)} = \frac{\pi}{2} \frac{i^n J_{-n}(ix) - i^{-n} J_n(ix)}{\sin(n\pi)} \quad (\text{A } 10)$$

$$= -i^n \frac{\pi}{2} \frac{J_n(ix) \cos(n\pi) - J_{-n}(ix)}{\sin(n\pi)} = -i^n \frac{\pi}{2} Y_n(ix) \quad (\text{A } 11)$$

The modified Bessel functions obey the same recurrence relations as the orig-

nal Bessel function, and have the following asymptotic approximations,

$$I_n(x) \approx \begin{cases} \frac{1}{n!} \left(\frac{x}{2}\right)^n & \text{for } x \ll 1 \\ \frac{\exp(x)}{\sqrt{2\pi x}} & \text{for } x \gg 1 \end{cases} \quad (\text{A } 12)$$

$$K_n(x) \approx \begin{cases} \sqrt{\frac{\pi}{2x}} \exp(-x) & \text{for } x \gg 1 \\ -\ln\left(\frac{x}{2}\right) - \gamma & \text{for } x \ll 1, n = 0 \\ \frac{\Gamma(n)}{2} \left(\frac{2}{x}\right)^n & \text{for } x \ll 1, n \neq 0, \end{cases} \quad (\text{A } 13)$$

where $\gamma = 0.5772$ is the Euler-Mascheroni constant

A.2 Meijer G Functions

The Meijer G function $M_G(a_s, b_s, c_s, d_s, z)$ is defined as a complex countour integral over a number of Gamma functions [62, 63]. Its first four arguments are lists of numbers

$$\begin{aligned} a_s &= [a_1, a_2, \dots, a_k] \\ b_s &= [b_1, b_2, \dots, b_m] \\ c_s &= [c_1, c_2, \dots, c_n] \\ d_s &= [d_1, d_2, \dots, d_l] \end{aligned} \quad (\text{A } 14)$$

and it is defined as

$$M_G(a_s, b_s, c_s, d_s, z) = \frac{1}{2\pi i} \oint_{\gamma_L} dy z^y \frac{\left[\prod_{s=1}^k \Gamma(1 - a_s + y) \right] \left[\prod_{s=1}^n \Gamma(c_s - y) \right]}{\left[\prod_{s=1}^m \Gamma(b_s + y) \right] \left[\prod_{s=1}^l \Gamma(1 - d_s + y) \right]} \quad (\text{A } 15)$$

A.3 The Generalized Hypergeometric Function

The generalized hypergeometric function is defined as [62, 63]

$$F(a_s; b_s; z) = \sum_{n=0}^{\infty} \frac{z^n}{n!} \prod_{i=1}^k \prod_{j=1}^m \frac{\Gamma(a_i + n)}{\Gamma(a_i)} \frac{\Gamma(b_j)}{\Gamma(b_j + n)} \quad (\text{A.16})$$

The lists a_s and b_s are defined as in Eq. (A.14).

A.4 Lommel Functions

The first Lommel function $L_1(a, b, z)$ is defined in terms of the generalized hypergeometric function $F(a_s; b_s; z)$ of Eq. (A.16) as [62, 63]

$$L_1(a, b, z) = \frac{z^{a+1}}{(a-b+1)(a+b+1)} F\left(1; \frac{3}{2} - \frac{1}{2}(b-a), \frac{3}{2} + \frac{1}{2}(b+a); -\frac{z^2}{4}\right). \quad (\text{A.17})$$

A.5 Struve Functions

The Struve function $H_\nu(z)$ is defined in terms of Gamma functions as [62, 63]

$$H_\nu(z) = \left(\frac{1}{2z}\right)^{\nu+1} \sum_{k=0}^{\infty} \frac{(-1)^k (2z)^{-2k}}{\Gamma(k+3/2)\Gamma(k+\nu+3/2)}. \quad (\text{A.18})$$

A.6 The Confluent Hypergeometric Functions

The confluent hypergeometric functions of the first and second kinds, $K(\alpha, \gamma, z)$ and $W(\alpha, \gamma, z)$, are the two solutions to the confluent hypergeometric differential equation [62, 63]

$$z \frac{d^2}{dz^2} y(z) + (\gamma - z) \frac{d}{dz} y(z) - \alpha y(z) = 0. \quad (\text{A.19})$$

They are sometimes known as Kummel functions.

Appendix B

Generators

B.1 Definition

In the 5-representation, the generators of $\text{SO}(4) \approx \text{SU}(2)_L \times \text{SU}(2)_R$ are

$$T_{ij}^{a_{L,R}} = -\frac{i}{2} \left[\frac{1}{2} \epsilon^{abc} (\delta_i^b \delta_j^c - \delta_j^b \delta_i^c) \pm (\delta_i^a \delta_j^4 - \delta_j^a \delta_i^4) \right] , \quad (\text{B.1})$$

while the generators for $\text{SO}(5)/\text{SO}(4)$ are

$$T_{ij}^{\hat{a}} = -\frac{1}{\sqrt{2}} (\delta_i^{\hat{a}} \delta_j^5 - \delta_j^{\hat{a}} \delta_i^5) , \quad (\text{B.2})$$

where $a_{L,R} = 1, 2, 3$, $\hat{a} = 1 \dots 4$ and $i, j = 1 \dots 5$ [3]. Explicitely, the generators are given by

$$T^{1_{L,R}} = -\frac{i}{2} \begin{pmatrix} 0 & 0 & 0 & \pm 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ \mp 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad T^{2_{L,R}} = -\frac{i}{2} \begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \pm 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & \mp 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

B.2 Commutation relations

The generators obey the following commutation relations:

$$[T^{a_{L,R}}, T^{b_{L,R}}] = i \epsilon^{abc} T^{c_{L,R}}, \quad [T^{a_L}, T^{b_R}] = 0, \quad (\text{B.3})$$

$$[T^{\hat{a}}, T^{\hat{b}}] = \frac{1}{2} \left[\delta_i^{\hat{a}} \delta_j^{\hat{b}} - \delta_j^{\hat{a}} \delta_i^{\hat{b}} \right] = \frac{i}{2} \left[\epsilon^{abc} (T^{c_L} + T^{c_R}) + \delta_b^4 (T^{a_L} - T^{a_R}) \right], \quad (\text{B.4})$$

$$[T^{a_{L,R}}, T^{\hat{b}}] = \frac{i}{2} \left(\epsilon^{abc} T^{\hat{c}} \mp T^{\hat{a}} \delta_b^4 \pm T^{\hat{4}} \delta_b^a \right), \quad (\text{B.5})$$

where ϵ^{abc} is the totally antisymmetric tensor with $\epsilon^{123} = 1$.

B.3 Expansions of A_{MN}^a

Using these generators, our defined quantities $A_{MN}^a T^a = -i [T^c, T^b] A_M^c A_N^b$ become, where abc is an even permutation of 123,

$$A_{MN}^{a_L} = A_M^{b_L} A_N^{c_L} - A_M^{c_L} A_N^{b_L} + \frac{1}{2} \left(A_M^{\hat{b}} A_N^{\hat{c}} - A_M^{\hat{c}} A_N^{\hat{b}} + A_M^{\hat{a}} A_N^{\hat{4}} - A_M^{\hat{4}} A_N^{\hat{a}} \right), \quad (\text{B.6})$$

$$A_{MN}^{a_R} = A_M^{b_R} A_N^{c_R} - A_M^{c_R} A_N^{b_R} + \frac{1}{2} \left(A_M^{\hat{b}} A_N^{\hat{c}} - A_M^{\hat{c}} A_N^{\hat{b}} - A_M^{\hat{a}} A_N^{\hat{4}} + A_M^{\hat{4}} A_N^{\hat{a}} \right), \quad (\text{B.7})$$

$$\begin{aligned} A_{MN}^{\hat{a}} = \frac{1}{2} \left[\left(A_M^{b_L} + A_M^{b_R} \right) A_N^{\hat{c}} - A_M^{\hat{c}} \left(A_N^{b_L} + A_N^{b_R} \right) - \left(A_M^{c_L} + A_M^{c_R} \right) A_N^{\hat{b}} \right. \\ \left. + A_M^{\hat{b}} \left(A_N^{c_L} + A_N^{c_R} \right) - \left(A_M^{a_L} - A_M^{a_R} \right) A_N^{\hat{4}} + A_M^{\hat{4}} \left(A_N^{a_L} - A_N^{a_R} \right) \right]. \end{aligned} \quad (\text{B.8})$$

$A_{MN}^{\hat{4}}$ is given by

$$A_{MN}^{\hat{4}} = \frac{1}{2} \sum_{a=1}^3 \left[\left(A_M^{a_L} - A_M^{a_R} \right) A_N^{\hat{a}} - A_M^{\hat{a}} \left(A_N^{a_L} - A_N^{a_R} \right) \right], \quad (\text{B.9})$$

Appendix C

Derivation of Feynman Rules

C.1 Propagators

The relevant part of the gauge Lagrangian, Eq. (3.8), for the gauge boson propagator is

$$L_{prop} = \sqrt{\bar{g}} \left[-\frac{1}{4} g^{MN} g^{RS} (\partial_M A_R^a - \partial_R A_M^a) (\partial_N A_S^a - \partial_S A_N^a) + \frac{1}{2} m^2 k^2 g^{MN} A_M^a A_N^a + L_{GF} \right] ,$$

where $\bar{g}$ is the determinant of the metric. The gauge fixing term is chosen to be [46, 38]

$$\sqrt{\bar{g}} L_{GF} = -\frac{1}{2\epsilon k z} \left[\partial^\mu A_\mu^a - \epsilon z \partial_z \left(\frac{1}{z} A_5^a \right) \right]^2 . \quad (C.1)$$

In the following derivations of propagators, we assume that EWSB has not yet occurred, and omit any mass terms or mixing which might occur due to the Higgs vacuum expectation value.

C.1.1 Gauge Boson A_μ

We find that the propagator for a gauge boson A_μ^a , $\mu = 0 \dots 3$ with 4D momentum p^μ , moving between two points z and z' in the fifth dimension and with boundary mass m , is given by

$$-i G_m(z, z', p) \left(\eta^{\nu\mu} - \frac{p^\mu p^\nu}{p^2} \right) - i G_m(z, z', \frac{p}{\sqrt{\epsilon}}) \left(\frac{p^\mu p^\nu}{p^2} \right), \quad \text{in general,} \quad (\text{C } 2)$$

$$-i \eta_{\mu\nu} G_m(z, z', p) \quad \epsilon = 1, \quad (\text{C } 3)$$

$$-i G_m(z, z', p) \left(\eta^{\nu\mu} - \frac{p^\mu p^\nu}{p^2} \right) - i G_m(z, z', 0) \left(\frac{p^\mu p^\nu}{p^2} \right), \quad \epsilon = \infty, \quad (\text{C } 4)$$

$$-i G_m(z, z', p) \left(\eta_{\mu\nu} + \frac{p^\mu p^\nu}{p^2} \right) \quad \epsilon = 0, \quad (\text{C } 5)$$

in general, Feynman gauge, unitary gauge, and Lorentz gauge respectively [38]. The Green's function $G_m(z, z', p)$ is defined by

$$\left[p^2 - \frac{1}{z} \partial_z + \partial_z^2 - \frac{m^2}{z^2} \right] G_m(z, z', p) = k z \delta(z - z') \quad (\text{C } 6)$$

The solution to Eq. (C 6) is given by

$$G_m(u, v, p) = \frac{\pi}{2} \frac{k u v}{AD - BC} [A J_\alpha(p u) + B Y_\alpha(p u)] [C J_\alpha(p v) + D Y_\alpha(p v)] \quad (\text{C } 7)$$

where $\alpha = \sqrt{1 + m^2}$, $u = \min(z, z')$, $v = \max(z, z')$ and A, B, C, D will be determined by BCs¹. With Neumann-Neumann boundary conditions – which all SM particles will have, as they must be even under Z_2 symmetry – given by

$$\partial_u G(L_0, v, p) = \partial_v G(u, L_1, p) = 0, \quad (\text{C } 8)$$

we find that $A = -Y_{\alpha-1}(pL_0)$, $B = J_{\alpha-1}(pL_0)$, $C = -Y_{\alpha-1}(pL_1)$, and $D = J_{\alpha-1}(pL_1)$.

Similarly, with Dirichlet-Neumann boundary conditions, given by

$$G(L_0, v, p) = \partial_v G(u, L_1, p) = 0, \quad (\text{C } 9)$$

¹As we will be neglecting brane mass terms, for our purposes $m = 0$

we have $A = -Y_\alpha(pL_0)$, $B = J_\alpha(pL_0)$, $C = -Y_{\alpha-1}(pL_1)$, and $D = J_{\alpha-1}(pL_1)$

The third relevant set of boundary conditions for our purposes is the Dirichlet-Dirichlet condition,

$$G(L_0, v, p) = G(u, L_1, p) = 0 \quad (\text{C } 10)$$

which yields $A = -Y_\alpha(pL_0)$, $B = J_\alpha(pL_0)$, $C = -Y_\alpha(pL_1)$, and $D = J_\alpha(pL_1)$. However, as the A_μ^a do not couple to fermion anti-fermion pairs, this propagator is not relevant for our calculations

C.1.2 Fifth Component A_5^a

We find that the propagator for a fifth component A_5^a moving between two points z and z' is given by

$$\frac{i}{\epsilon} G_{\sqrt{m^2-1}}(z, z', \frac{p}{\sqrt{\epsilon}}), \quad \text{in general} \quad (\text{C } 11)$$

$$-i G_{\sqrt{m^2-1}}(z, z', p) \quad \text{in Feynman gauge } \epsilon = 1, \quad (\text{C } 12)$$

$$0, \quad \text{in unitary gauge } \epsilon = \infty, \quad (\text{C } 13)$$

where $G_m(z, z', p)$ is defined by Eq (C 6) and given explicitly by Eq (C 7) [38]. The unitary result is due to the fact that in the unitary gauge $A_5^a = 0$. However, note that for the A_5^a the gauge symmetry is broken by the BCs, and we cannot just choose the gauge of our choice, therefore $A_5^a \neq 0$, and this definition of the propagator is not compatible

C.2 Profiles without particle mixing

In the following derivations of profiles, we omit the effects of EWSB; profiles including these effects will be addressed in Sec. C.3.

C.2.1 Boson Profiles

We will need the 5-dimensional profiles for the Z and X KK gauge bosons, which correspond to the n^{th} 5D component $f_G^{(n)}(z)$ of the Kaluza-Klein (KK) decomposition,²

$$A_\mu(x, z) = \sum_n A_\mu^{(n)}(x) f_G^{(n)}(z). \quad (\text{C.14})$$

Using the Lagrangian of Eq. (3.8) and the principle of least action, it can be shown that $f_G^{(n)}(z)$ must satisfy [51]

$$\left[\partial_z^2 - \frac{1}{z} \partial_z + m_n^2 \right] f_G^{(n)}(z) = 0. \quad (\text{C.15})$$

This differential equation has the solution

$$f_G^{(n)}(z) = A_n k z [J_1(m_n z) + B_n Y_1(m_n z)], \quad (\text{C.16})$$

where the constants are defined by the BCs and the orthonormalization condition [51, 39]. The normalization condition can be derived by comparing the 5D theory to the 4D Standard Model (SM); if we integrate the kinetic terms of the 5D Lagrangian, we should obtain the appropriate kinetic terms of the 4D Lagrangian. Consider the

²Here A_μ denotes a generic boson; the A_μ^a and U_μ boson profiles have the same form and derivation.

kinetic term of two zero mode boson fields,

$$\begin{aligned} L_{4\text{eff}} &= \int_{L_0}^{L_1} dz L_{5\text{kin}} = \int_{L_0}^{L_1} dz \frac{1}{2kz} \partial_\mu A_\nu^{(0)}(x, z) \partial^\mu A^{(0)\nu}(x, z) \\ &= \frac{1}{2} \partial_\mu \tilde{A}_\nu^{(0)}(x) \partial^\mu \tilde{A}^{(0)\nu}(x) \int_{L_0}^{L_1} dz \frac{1}{kz} f_G^{(0)}(z) f_G^{(0)}(z), \end{aligned} \quad (\text{C } 17)$$

where $\tilde{A}_\nu^{(0)}(x, z)$ denotes the 4-dimensional SM field. We can see that for $L_{4\text{eff}}$ to correspond to the SM, the integral in the last line must evaluate to one. Accounting for the orthogonality of different modes, we obtain the following orthonormalization condition

$$\int_{L_0}^{L_1} dz \frac{1}{kz} f_G^{(m)}(z) f_G^{(n)}(z) = \delta_{mn} \quad (\text{C } 18)$$

It is convenient to define the following basis of solutions that are even (Neumann BC) or odd (Dirichlet BC) on the Planck brane,

$$C_A(m_n, z) = \frac{\pi m_n}{2} z [J_1(m_n z) Y_0(m_n L_0) - J_0(m_n L_0) Y_1(m_n z)] \quad (\text{C } 19)$$

$$S_A(m_n, z) = \frac{\pi m_n}{2} z [J_1(m_n L_0) Y_1(m_n z) - J_1(m_n z) Y_1(m_n L_0)] , \quad (\text{C } 20)$$

where $C_A(m_n, L_0) = 1$, $\partial_z C_A(m_n, L_0) = 0$, $S_A(m_n, L_0) = 0$ and $\partial_z S_A(m_n, L_0) = m_n$. Then the profiles of bosons with Neumann-Neumann (NN), Dirichlet-Neumann (DN) and Dirichlet-Dirichlet (DD) boundary conditions are given by

$$f_G^{(n)} = \begin{cases} C_G^{(n)} C_A(m_n, z) & \text{for NN BCs} \\ C_G^{(n)} S_A(m_n, z) & \text{for DN and DD BCs} \end{cases} \quad (\text{C } 21)$$

where the coefficients C_G are fixed using Eq. (C 18), and G denotes the gauge boson in question.³ By applying the appropriate boundary condition to these profiles on

³There are no gauge bosons in the MCHM with Neumann-Dirichlet BCs

the TeV brane, we obtain conditions that define the boson masses in the non-mixing limit,

$$Y_0(p L_0) J_0(p L_1) - J_0(p L_0) Y_0(p L_1) = 0 \quad \text{for NN BCs} \quad (\text{C } 22)$$

$$Y_1(p L_0) J_0(p L_1) - J_1(p L_0) Y_0(p L_1) = 0 \quad \text{for DN BCs} \quad (\text{C } 23)$$

$$Y_1(p L_0) J_1(p L_1) - J_1(p L_0) Y_1(p L_1) = 0 \quad \text{for DD BCs} \quad (\text{C } 24)$$

where we have used the relations in Eq (A 8) to evaluate the derivatives

The zero mode gauge boson profile can be found by solving Eq (C 15) with $m_0 = 0$. Taking Neumann BCs, $\partial_z f_G^{(0)}(z = L_0, L_1) = 0$ – where brane kinetic terms have been neglected – the solution has the form $f_G^{(0)}(z) = \text{constant}$. Using the orthonormality condition we find that,

$$f_G^{(0)}(z) = \sqrt{\frac{k}{\ln(k L_1)}} \quad (\text{C } 25)$$

C.2.2 Fermion Profiles

The Lagrangian for a fermion is given by Eq (3 12). As mentioned in the text, the spin connection ω_{MAB} cancels out, and the relevant part of the covariant derivative for the profile is simply the normal derivative ∂_M . We therefore have

$$L_f = \sqrt{g} \left[\frac{i}{2} \bar{\Psi}(kz) \Gamma^M \partial_M \Psi - \frac{i}{2} (\partial_M \Psi)^\dagger \Gamma^0(kz) \Gamma^M \Psi - M_\Psi \bar{\Psi} \Psi \right] \quad (\text{C } 26)$$

$$= \bar{\Psi} \left[i \sqrt{g}(kz) \Gamma^M \partial_M + \frac{i}{2} \Gamma^M \partial_M (\sqrt{g}(kz)) - \sqrt{g} M_\Psi \right] \Psi \quad (\text{C } 27)$$

$$= \bar{\Psi} \left[i \sqrt{g}(kz) \gamma^\mu \partial_\mu + \sqrt{g}(kz) \gamma^5 \partial_5 + \frac{1}{2} \gamma^5 \partial_5 (\sqrt{g}(kz)) - \sqrt{g} M_\Psi \right] \Psi \quad (\text{C } 28)$$

$$= \frac{1}{(kz)^4} \bar{\Psi} \left[\not{p} + \gamma^5 \partial_5 - \frac{1}{kz} (2k\gamma^5 + M_\Psi) \right] \Psi, \quad (\text{C } 29)$$

where we have applied integration by parts on the second term in the second line. The fermion fields can be broken into their chiral components $\Psi = \Psi_L + \Psi_R$ where

$\gamma_5 \Psi_{LR} = \mp \Psi_{LR}$ From Eq (C 29) it can then be shown that the left- and right-handed fermions are related through

$$[\partial_z - z^{-1}(2 - c)] \Psi_L^{(n)}(z) = -m_n \Psi_R^{(n)}, \quad (\text{C } 30)$$

$$[\partial_z - z^{-1}(2 + c)] \Psi_R^{(n)}(z) = m_n \Psi_L^{(n)} \quad (\text{C } 31)$$

where we have defined $M_\Psi = ck$ The corresponding KK decomposition is [51]

$$\Psi_{LR}(x, z) = \sum_n \Psi_{LR}^{(n)}(x) f_{LR}^{(n)}(z), \quad (\text{C } 32)$$

therefore $f_L^{(n)}(z)$ must satisfy

$$[\partial_z - z^{-1}(2 + c)] [\partial_z - z^{-1}(2 - c)] f_L^{(n)}(z) = -m_n^2 f_L^{(n)} \quad (\text{C } 33)$$

This differential equation has the solution

$$f_L^{(n)}(z) = A_n^f(kz)^{5/2} [J_{c+1/2}(m_n z) + B_n^f Y_{c+1/2}(m_n z)] , \quad (\text{C } 34)$$

where the constants are defined by the BCs and the orthonormality condition [51],

$$\int_{L_0}^{L_1} dz \frac{1}{(kz)^4} f^{(n)}(z) f^{(m)}(z) = \delta_{mn} \quad (\text{C } 35)$$

which can be derived by the same procedure as that of the gauge bosons Similarly to the gauge boson case, we define the basis function $S_c(m, z)$ as

$$S_c(m_n, z) = \frac{\pi m_n}{2k} (kz)^{5/2} \left[J_{c+\frac{1}{2}}(m_n L_0) Y_{c+\frac{1}{2}}(m_n z) - J_{c+\frac{1}{2}}(m_n z) Y_{c+\frac{1}{2}}(m_n L_0) \right] , \quad (\text{C } 36)$$

Up to a normalization constant, this basis function is the profile of a left-handed fermion with a Dirichlet boundary condition at the Planck brane, S_c^+ The profile of the corresponding right-handed chiral partner, S_c^+ , has a Neumann BC at the Planck brane and is defined by Eq (C 30) Similarly, the profile of a right-handed fermion

with a Dirichlet BC at the Planck brane, S_c^- , is given by reversing the sign of c in Eq. (3.30); its chiral partner, $\dot{S}_c^-$, is defined by Eq. (C.31). Therefore we have

$$S_c^\pm(m_n, z) = S_{\pm c}(m_n, z), \quad (\text{C.37})$$

$$\dot{S}_c^\pm(m_n, z) = \mp \frac{1}{m_n} \left[\partial_z - \frac{1}{z}(2 \mp c) \right] S_{\pm c}(m_n, z). \quad (\text{C.38})$$

The left-handed zero mode fermion profile can be found by solving Eq. (C.33) with $m_0 = 0$. Taking Neumann BCs where the brane kinetic terms have been neglected [51], the solution has the form $f^{(n)}(z) = A^f z^{2-c}$. Using the orthonormality condition we find that,

$$f_L^{(0)}(z) = \sqrt{\frac{k(1-2c)}{(kL_1)^{1-2c} - 1}} (kz)^{2-c}. \quad (\text{C.39})$$

A right-handed zero mode fermion profile can be obtained by substituting $c \rightarrow -c$ in Eq. (C.39).

C.2.3 Higgs Profile

Let the Higgs have the form $H_5(x, z) = H(x) f_H(z)$, and consider the square of the second gauge fixing term in Eq. (3.9). Substituting in our form for $H_5(x, z)$, it can be seen that when this term is integrated over z , the effective 4-dimensional term will be a mass term.

$$\left[\epsilon z \partial_z \left(\frac{1}{z} H_5(x, z) \right) \right]^2 = \left[\epsilon z \partial_z \left(\frac{1}{z} f_H(z) \right) \right]^2 H(x)^2. \quad (\text{C.40})$$

For the zero-mode the mass must be zero before EWSB. Therefore the factor preceding $H(x)^2$ must be equal to zero, yielding the relation

$$\partial_z \left(\frac{f_H(z)}{z} \right) = 0. \quad (\text{C.41})$$

Solving this differential equation, we find that the profile of $A_5^{a(0)}$ is $f_H(z) = c z$ where c is a constant. Using the orthonormalization condition of Eq. (C.18) we find that [3, 46]

$$f_H(z) = z \sqrt{\frac{2k}{L_1^2 - L_0^2}} \quad (\text{C.42})$$

C.3 Profiles with particle mixing

In this section we provide a detailed overview of the fully mixed particle profiles after EWSB. Explicit calculations and summaries can also be found in references [4, 50, 52, 64], however, it should be noted that these sources define the rotation angle θ_H in a different way. Furthermore, their profile coefficient solutions often take a different form from ours, due to differences in the solution method. However, the results are equivalent.

In the interest of notational clarity, we will omit the superscript n denoting the KK mode on all profiles $f^{(n)}$ and coefficients $C_G^{(n)}$ throughout this section, the order dependence will be expressed solely through the subscript on the mass parameters m_n .

C.3.1 Gauge Bosons

As indicated in the text, the fully mixed particle profiles with Higgs vev v , $f(m_n, z, v)$, can be obtained from the pre-EWSB basis profiles through the gauge transformation given in Eq. (3.41). The base profiles $f(m_n, z, 0)$ of the bosons are defined by

Eq (C 21) as

$$f_{a_L}(m_n, z, 0) = C_{a_L} C_A(m_n, z), \quad f_a(m_n, z, 0) = C_a S_A(m_n, z), \quad (\text{C } 43)$$

$$f_{1_R}(m_n, z, 0) = C_{1_R} S_A(m_n, z), \quad f_{2_R}(m_n, z, 0) = C_{2_R} S_A(m_n, z), \quad (\text{C } 44)$$

$$f_X(m_n, z, 0) = C_X S_A(m_n, z), \quad f_B(m_n, z, 0) = C_B C_A(m_n, z), \quad (\text{C } 45)$$

where $C_A(m_n, z)$ and $S_A(m_n, z)$ are the basis functions defined in Eqs (C 19-C 20), and the C_G are normalization coefficients [4]. Note that it is $f_X(m_n, z, 0)$ and $f_B(m_n, z, 0)$ that are defined in terms of the basis functions, rather than $f_{3_R}(m_n, z, 0)$ and $f_U(m_n, z, 0)$. This is because $C_A(m_n, z)$ and $S_A(m_n, z)$ are solutions with Neumann and Dirichlet BCs at the Planck brane respectively, and it is upon X and B that the boundary conditions are defined. We define $f_{3_R}(m_n, z, 0)$ and $f_U(m_n, z, 0)$ in terms of the X and B profiles in the work below.

In the 5-representation of the $\text{SO}(5)$ generators (please see Appendix B), the transformation matrix $\Omega(z, v)$ of Eq (3 42) can be rewritten as

$$\Omega(z, v) = 1 - i \sqrt{2} T^4 \sin \theta_G + \sum_{i=4}^5 \delta_{ii} (\cos \theta_G - 1) \quad (\text{C } 46)$$

where $\theta_G = \theta_G(z, v)$ is defined in Eq (3 43). In this representation the mixed profiles are found to be,

$$f_{1_L R}(v) = \frac{1}{2} (1 \pm \cos \theta_G) C_{1_L} C_A + \frac{1}{2} (1 \mp \cos \theta_G) C_{1_R} S_A \pm \frac{\sqrt{2}}{2} \sin \theta_G C_1 S_A, \quad (\text{C } 47)$$

$$f_{2_L R}(v) = \frac{1}{2} (1 \pm \cos \theta_G) C_{2_L} C_A + \frac{1}{2} (1 \mp \cos \theta_G) C_{2_R} S_A \pm \frac{\sqrt{2}}{2} \sin \theta_G C_2 S_A, \quad (\text{C } 48)$$

$$\begin{aligned} f_{3_L R}(v) &= \frac{1}{2} (1 \pm \cos \theta_G) C_{3_L} C_A \pm \frac{\sqrt{2}}{2} \sin \theta_G C_3 S_A \\ &\quad + \frac{1}{2} (1 \mp \cos \theta_G) [\cos \theta_H C_B C_A - \sin \theta_H C_X S_A], \end{aligned} \quad (\text{C } 49)$$

$$f_1(v) = \cos \theta_G C_1 S_A + \frac{1}{\sqrt{2}} \sin \theta_G (C_{1R} S_A - C_{1L} C_A) , \quad (\text{C.50})$$

$$f_2(v) = \cos \theta_G C_2 S_A + \frac{1}{\sqrt{2}} \sin \theta_G (C_{2R} S_A - C_{2L} C_A) , \quad (\text{C.51})$$

$$f_3(v) = \cos \theta_G C_3 S_A + \frac{1}{\sqrt{2}} \sin \theta_G (\cos \theta_H C_B C_A - \sin \theta_H C_X S_A - C_{3L} C_A) , \quad (\text{C.52})$$

$$f_4(v) = C_4 S_A , \quad (\text{C.53})$$

$$f_U(v) = \sin \theta_H C_B C_A + \cos \theta_H C_X S_A . \quad (\text{C.54})$$

where we have suppressed the z and m_n dependence; $f_G(v) = f_G(m_n, z, v)$, $C_A = C_A(m_n, z)$, and $S_A = S_A(m_n, z)$ [64]. Recall that $\cos \theta_H = \tan \theta_w$.

As discussed in section 4.3, the TeV boundary conditions provide a system of equations by which the coefficients C_G and mass eigenvalues may be determined. The boson masses are determined by setting the determinant of this system to zero, a requirement for the system to be solvable. The system of equations can be broken into four subsystems.

The W boson sector

The first two subsystems are mathematically identical, the only difference being that one involves the W^{1L} , W^{1R} and $A^{\hat{1}}$ bosons while the other involves the W^{2L} , W^{2R} and $A^{\hat{2}}$ bosons. We will represent both of these systems by W^{iL} , W^{iR} and $A^{\hat{i}}$ – where $i = 1, 2$ – with the corresponding system of TeV-brane boundary conditions

$$\partial_z f_{iL}^{(n)}(m_n, z; v) \Big|_{z=L_1} = 0 , \quad (\text{C.55})$$

$$\partial_z f_{iR}^{(n)}(m_n, z; v) \Big|_{z=L_1} = 0 , \quad (\text{C.56})$$

$$f_i^{(n)}(m_n, z; v) \Big|_{z=L_1} = 0 , \quad (\text{C.57})$$

which we use to solve for C_{iL} , C_{iR} and C_i [64]. Setting the determinant of this system to zero, we obtain the mass relations

$$S'_A(m_n, L_1) = 0, \quad (\text{C.58})$$

$$2 C'_A(m_n, L_1) S_A(m_n, L_1) + m_n k L_1 \sin \theta_G^2(L_1, v) = 0, \quad (\text{C.59})$$

where a prime represents the derivative with respect to the fifth coordinate z and we have used the Wronskian relation [4, 62],⁴

$$S'_A(m_n, z) C_A(m_n, z) - S_A(m_n, z) C'_A(m_n, z) = m_n k z. \quad (\text{C.60})$$

Eq. (C.58) corresponds to the W^{iR} state masses, which are the same as the original masses before mixing (Eq. (C.23)). Eq. (C.59) corresponds to the combined W^{iL} and $W^{\hat{i}}$ masses; note that as $v \rightarrow 0$ the left-hand side becomes the product of the left-hand sides of their original mass equations, $C'_A(m_n, L_1) S_A(m_n, L_1)$ (Eq. (C.22) and Eq. (C.24)).

Because there are two conditions that allow the determinant to be zero, there are two possible solutions for the coefficient set. The first, corresponding to the W^{iL} mass condition, Eq. (C.59), is

$$\begin{aligned} C_{iL}^{WL} &= \sqrt{2} C_i \frac{S_A(m_n, L_1) (1 + \cos \theta_G^2(L_1, v))}{C_A(m_n, L_1) \sin 2\theta_G(L_1, v)}, \\ C_{iR}^{WL} &= \frac{\sqrt{2}}{2} C_i \tan \theta_G(L_1, v), \quad C_i^{WL} = C_i. \end{aligned} \quad (\text{C.61})$$

where the remaining coefficient C_i will be determined by the normalization condition, and the superscript W_L identifies the coefficient set.

When these coefficients are substituted into the profiles $f_{iL}(m, z; v)$, $f_{iR}(m, z; v)$ and $f_i(m, z; v)$, all of these profiles are non-zero. This indicates that the

⁴Also known as Abel's identity.

full W^{iL} mass eigenstate profile is a superposition of these mixed profiles, weighted by their associated generators,

$$F_{iL}(m, z; v) T^{F_{iL}} = f_{iL}^{W_L}(m, z; v) T^{iL} + f_{iR}^{W_L}(m, z; v) T^{iR} + f_i^{W_L}(m, z; v) T^{\hat{i}}. \quad (\text{C.62})$$

where the superscript W_L on the profiles indicates that coefficients C_{iL} , C_{iR} , and $C_{\hat{i}}$ have been replaced by those in Eq. (C.61). Here $T^{F_{iL}}$ is meant to denote the generator that would be associated with the profile $F_{iL}(m, z; v)$; however, in practice the two are never separated. Depending upon the mass solution of Eq. (C.59) with which it is evaluated, Eq. (C.62) may represent a mass eigenstate that corresponds to either the pre-EWSB A_μ^{iL} , or the pre-EWSB $A_\mu^{\hat{i}}$.

The normalization condition for this profile is

$$\int_{L_0}^{L_1} \frac{dz}{kz} (f_{iL}^\alpha(m, z; v)^2 + f_{iR}^\alpha(m, z; v)^2 + f_i^\alpha(m, z; v)^2) = 1, \quad (\text{C.63})$$

The second solution set, corresponding to Eq. (C.58), is given by

$$C_{iL}^{W_R} = 0, \quad C_{iR}^{W_R} = -\sqrt{2} C_{\hat{i}} \tan \theta_G(L_1, v), \quad C_{\hat{i}}^{W_R} = C_{\hat{i}}. \quad (\text{C.64})$$

The corresponding full profile $F_{iL}(m, z; v) T^{F_{iR}}$ and its normalization condition are given by Eq. (C.62) and (C.63) with $W_L \rightarrow W_R$. We emphasize that the difference between the profiles of the three types of boson arises from the different coefficient sets obtained by solving Eqs. (C.55-C.57) and the corresponding mass eigenvalues.

The γ and Z sector

The third subsystem involves the Z , X , V and A^3 bosons where again we denote the photon by V_μ with the corresponding system of boundary conditions

$$\partial_z f_{3L}^{(n)}(m_n, z, v) \Big|_{z=L_1} = 0, \quad (\text{C } 65)$$

$$f_3^{(n)}(m_n, z, v) \Big|_{z=L_1} = 0, \quad (\text{C } 66)$$

$$\partial_z f_B^{(n)}(m_n, z, v) \Big|_{z=L_1} = \partial_z \left[\cos \theta_w f_{3R}^{(n)}(m_n, z, v) + \sin \theta_w f_U^{(n)}(m_n, z, v) \right] \Big|_{z=L_1} = 0, \quad (\text{C } 67)$$

$$\partial_z f_X^{(n)}(m_n, z, v) \Big|_{z=L_1} = \partial_z \left[\cos \theta_w f_U^{(n)}(m_n, z, v) - \sin \theta_w f_{3R}^{(n)}(m_n, z, v) \right] \Big|_{z=L_1} = 0, \quad (\text{C } 68)$$

which we use to solve for C_{3L} , C_X , C_B and C_3 [64] Setting the determinant of this system to zero, we obtain the mass relations

$$S'_A(m_n, L_1) = 0, \quad (\text{C } 69)$$

$$C'_A(m_n, L_1) = 0, \quad (\text{C } 70)$$

$$2 C'_A(m_n, L_1) S_A(m_n, L_1) + m_n k L_1 \sec^2 \theta_w \sin \theta_G^2(L_1, v) = 0 \quad (\text{C } 71)$$

The first two equations correspond to the masses of the X boson and photon state respectively, both are the same as the original masses before mixing (Eq (C 23) and Eq (C 22)) In particular, the zero-mode photon remains massless The third corresponds to the combined Z and A^3 masses, again, as $v \rightarrow 0$ the left-hand side of this equation becomes the product of the left-hand sides of the original Z and A^3 mass equations, $C'_A(m_n, L_1) S_A(m_n, L_1)$

In this case there are three possible solutions for the coefficient set The first, corresponding to the X boson mass condition, Eq (C 69), is

$$C_{3L}^X = C_B^X = 0, \quad C_X^\lambda = \sqrt{2} C_3 \tan \theta_G(L_1, v) \sin \theta_H, \quad C_3^X = C_3 \quad (\text{C } 72)$$

where the superscript is meant to identify the mass condition associated with the coefficient solution set.

The second solution set, corresponding to Eq. (C.71), is given by

$$\begin{aligned} C_B^Z &= \cos \theta_H C_{3_L}^Z = C_X \frac{S_A(m_n, L_1)}{C_A(m_n, L_1)} \frac{[1 - \cos^2 \theta_H + \cos^2 \theta_G(L_1, v) (1 + \cos^2 \theta_H)]}{\sin \theta_H (-1 + \cos^2 \theta_G(L_1, v)) (1 + \cos^2 \theta_H)} , \\ C_X^Z &= C_X , \quad C_3^Z = \frac{\sqrt{2} C_X}{\tan \theta_G(L_1, v) \sin \theta_H} . \end{aligned} \quad (C.73)$$

and the third, corresponding to the photon mass condition, Eq. (C.70), is given by

$$C_{3_L}^V = C_B \cos \theta_H , \quad C_X^V = C_3^V = 0 , \quad C_B^V = C_B . \quad (C.74)$$

When the coefficients corresponding to the X and Z mass conditions – Eq. (C.72) and Eq. (C.73) respectively – are substituted into the profiles $f_{3_L}(m, z; v)$, $f_X(m, z; v)$, $f_B(m, z; v)$ and $f_3(m, z; v)$, all of them are non-zero. However, while the profile of the Z boson

$$f_Z(m, z; v) = \cos \theta_w f_{3_L}(m, z; v) - \sin \theta_w f_B(m, z; v) , \quad (C.75)$$

is also non-zero, it turns out that the profile of the photon

$$f_V(m, z; v) = \cos \theta_w f_B(m, z; v) + \sin \theta_w f_{3_L}(m, z; v) , \quad (C.76)$$

disappears. The reverse is also true; when the coefficients corresponding to the photon mass condition, Eq. (C.70), are substituted into the profiles, only the photon profile is non-zero. As a result, the final profiles are the following,

$$F_X(m, z; v) T^{F_X} = f_Z^X(m, z; v) T^Z + f_X^X(m, z; v) T^X + f_3^X(m, z; v) T^{\hat{3}} , \quad (C.77)$$

$$F_Z(m, z; v) T^{F_Z} = f_Z^Z(m, z; v) T^Z + f_X^Z(m, z; v) T^X + f_3^Z(m, z; v) T^{\hat{3}} , \quad (C.78)$$

$$F_V(m, z; v) = \cos \theta_w f_B^V(m, z; v) + \sin \theta_w f_{3_L}^V(m, z; v) , \quad (C.79)$$

where, as in the previous section, the superscript indicates which set of coefficients has been applied. The normalization condition for each profile is analogous to Eq. (C.63). Notice that the photon profile is unaffected by the mixing.

The A_μ^4 sector.

The A_μ^4 boson does not mix with any other bosons, as such, its final profile is simply

$$F_4(m, z, v) = f_4(m, z, v) = f_4(m, z, 0) = C_4 S_A(m, z) \quad (\text{C.80})$$

C.3.2 Fermions

The procedure for the fermions is very similar to that of the gauge bosons, after performing the gauge transformation, we solve the coefficient matrices and obtain mass conditions from the matrix determinants.

The gauge transformation is slightly different for fermions. First, we write the fermion multiplets – in this case the lepton multiplets – in form

$$\begin{aligned}
 f_{\ell_{1L}}(m_n, z, 0) &= \begin{pmatrix} C_{\ell_1} S_{c_1}^+(m_n, z) \\ C_{\ell_2} S_{c_1}^+(m_n, z) \\ C_{\ell_3} S_{c_1}^-(m_n, z) \\ C_{\ell_4} S_{c_1}^-(m_n, z) \\ C_{\ell_5} S_{c_1}^+(m_n, z) \end{pmatrix} & f_{\ell_{3R}}(m_n, z, 0) &= \begin{pmatrix} C_{\ell_{11}} S_{c_3}^-(m_n, z) \\ C_{\ell_{12}} S_{c_3}^-(m_n, z) \\ C_{\ell_{13}} S_{c_3}^-(m_n, z) \\ C_{\ell_{14}} S_{c_3}^-(m_n, z) \\ C_{\ell_{15}} S_{c_3}^-(m_n, z) \\ C_{\ell_{16}} S_{c_3}^-(m_n, z) \\ C_{\ell_{17}} S_{c_3}^-(m_n, z) \\ C_{\ell_{18}} S_{c_3}^-(m_n, z) \\ C_{\ell_{19}} S_{c_3}^-(m_n, z) \\ C_{\ell_{20}} S_{c_3}^+(m_n, z) \end{pmatrix} \\
 f_{\ell_{2R}}(m_n, z, 0) &= \begin{pmatrix} C_{\ell_6} S_{c_2}^-(m_n, z) \\ C_{\ell_7} S_{c_2}^-(m_n, z) \\ C_{\ell_8} S_{c_2}^-(m_n, z) \\ C_{\ell_9} S_{c_2}^-(m_n, z) \\ f_{\ell_{2R}}^5(m_n, z, 0) \end{pmatrix} & &
 \end{aligned} \quad (\text{C.81})$$

where $S_c^\pm$ and $S_c^\pm$ are the fermion basis functions defined by Eqs (3.30-3.31) [4, 52, 64], and

$$f_{\ell_{2L}}^5(m_n, z, 0) = C_{\ell_{10}} \left(S_{c_2}^+ - M_{UV} S_{c_2}^- \right), \quad (\text{C } 82)$$

$$f_{\ell_{2R}}^5(m_n, z, 0) = C_{\ell_{10}} \left(-M_{UV} S_{c_2}^- + S_{c_2}^+ \right) \quad (\text{C } 83)$$

The vectors of opposite chirality can be read from these by replacing $S_c^\pm \leftrightarrow S_c^\pm$ and $f_{\ell_{2R}}^5 \rightarrow f_{\ell_{2L}}^5$. The quark profile vectors – $f_{\xi_{1L}}$, $f_{\xi_{2R}}$ and $f_{\xi_{3R}}$ – have an identical vector form, except that in their case $M_{UV} = 0$.

However, this form does not belong to the same basis as our $\text{SO}(5)$ generators in Appendix B. Therefore we must transform these vectors into our generator basis before the gauge transformation can be applied. As such, our gauge transformation takes the following form in the fermion case [4, 50],

$$f_{\ell_{1L}}(m_n, z, v) = A \Omega(z, v) A^{-1} f_{\ell_{1L}}(m_n, z, 0), \quad (\text{C } 84)$$

$$f_{\ell_{2R}}(m_n, z, v) = A \Omega(z, v) A^{-1} f_{\ell_{2R}}(m_n, z, 0), \quad (\text{C } 85)$$

$$f_{\ell_{3R}}(m_n, z, v) = B \Omega(z, v) B^{-1} f_{\ell_{3R}}(m_n, z, 0) \quad (\text{C } 86)$$

where the T^4 generator in $\Omega(z, v)$ is in 5-representation for the first two equations and 10-representation for the third. The change of-basis matrices A and B are given by [4, 50]

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & -1 & 0 & 0 & 0 \\ 0 & 0 & -i & 1 & 0 \\ 0 & 0 & i & 1 & 0 \\ -i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} \end{pmatrix} \quad (\text{C } 87)$$

and

$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & i & 0 \end{pmatrix} \quad (\text{C.88})$$

Performing this transformation, we obtain the following mixed profiles

$$f_{\ell_{1L}}(v) = \begin{pmatrix} C_{\ell_1} S_{c_1}^+ \\ \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_2} S_{c_1}^+ - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_3} \dot{S}_{c_1}^- - \frac{\sqrt{2}}{2} \sin\theta_G C_{\ell_5} S_{c_1}^+ \\ - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_2} S_{c_1}^+ + \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_3} \dot{S}_{c_1}^- - \frac{\sqrt{2}}{2} \sin\theta_G C_{\ell_5} S_{c_1}^+ \\ C_{\ell_4} \dot{S}_{c_1}^- \\ \frac{\sqrt{2}}{2} \sin\theta_G C_{\ell_2} S_{c_1}^+ + \frac{\sqrt{2}}{2} \sin\theta_G C_{\ell_3} \dot{S}_{c_1}^- + \cos\theta_G C_{\ell_5} S_{c_1}^+ \end{pmatrix} \quad (\text{C.89})$$

$$f_{\ell_{2R}}(v) = \begin{pmatrix} C_{\ell_6} S_{c_2}^- \\ \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_7} S_{c_2}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_8} S_{c_2}^- - \frac{\sqrt{2}}{2} \sin\theta_G f_{\ell_{2R}}^5(0) \\ - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_7} S_{c_2}^- + \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_8} S_{c_2}^- - \frac{\sqrt{2}}{2} \sin\theta_G f_{\ell_{2R}}^5(0) \\ C_{\ell_9} S_{c_2}^- \\ \frac{\sqrt{2}}{2} \sin\theta_G C_{\ell_7} S_{c_2}^- + \frac{\sqrt{2}}{2} \sin\theta_G C_{\ell_8} S_{c_2}^- + \cos\theta_G f_{\ell_{2R}}^5(0) \end{pmatrix} \quad (\text{C.90})$$

$$f_{\ell_{3R}}(v) = \begin{pmatrix} \cos\theta_G C_{\ell_{11}} S_{c_3}^- + \frac{i\sqrt{2}}{2} \sin\theta_G (C_{\ell_{18}} - C_{\ell_{15}}) S_{c_3}^- \\ \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{12}} S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{13}} S_{c_3}^- + \frac{i}{2} \sin\theta_G (C_{\ell_{19}} - C_{\ell_{16}}) S_{c_3}^- \\ \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{13}} S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{12}} S_{c_3}^- + \frac{i}{2} \sin\theta_G (C_{\ell_{19}} - C_{\ell_{16}}) S_{c_3}^- \\ \cos\theta_G C_{\ell_{14}} S_{c_3}^- - \frac{i\sqrt{2}}{2} \sin\theta_G C_{\ell_{17}} S_{c_3}^- + \frac{i\sqrt{2}}{2} \sin\theta_G C_{\ell_{20}} \dot{S}_{c_3}^+ \\ -\frac{i\sqrt{2}}{2} \sin\theta_G C_{\ell_{11}} S_{c_3}^- + \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{15}} S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{18}} S_{c_3}^- \\ \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{16}} S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{19}} S_{c_3}^- - \frac{i}{2} \sin\theta_G (C_{\ell_{12}} + C_{\ell_{13}}) S_{c_3}^- \\ -\frac{i\sqrt{2}}{2} \sin\theta_G C_{\ell_{14}} S_{c_3}^- + \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{17}} S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{20}} \dot{S}_{c_3}^+ \\ \frac{i\sqrt{2}}{2} \sin\theta_G C_{\ell_{11}} S_{c_3}^- + \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{18}} S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{15}} S_{c_3}^- \\ \frac{i}{2} \sin\theta_G (C_{\ell_{12}} + C_{\ell_{13}}) S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{16}} S_{c_3}^- + \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{19}} S_{c_3}^- \\ \frac{i\sqrt{2}}{2} \sin\theta_G C_{\ell_{14}} S_{c_3}^- - \sin^2\left(\frac{\theta_G}{2}\right) C_{\ell_{17}} S_{c_3}^- + \cos^2\left(\frac{\theta_G}{2}\right) C_{\ell_{20}} \dot{S}_{c_3}^+ \end{pmatrix} \quad (\text{C.91})$$

In terms of the vectorial form, the fermion boundary conditions of Eq. (3.37-3.39) can be written as

$$f_{\ell_{1L}}^{1, \cdot 4} + M_2 f_{\ell_{3R}}^{1, \cdot 4} = 0 \quad f_{\ell_{1L}}^5 + M_2 f_{\ell_{2R}}^5 = 0 \quad f_{\ell_{2L}}^{1, \cdot 4} = 0 \quad (\text{C.92})$$

$$f_{\ell_{3L}}^{1, \cdot 4} - M_2 f_{\ell_{1L}}^{1, \cdot 4} = 0 \quad f_{\ell_{2L}}^5 - M_1 f_{\ell_{1L}}^5 - M_{IR} f_{\ell_{2R}}^5 = 0 \quad f_{\ell_{3L}}^{5, \cdot 10} = 0 \quad (\text{C.93})$$

where the superscript denotes the vector component [4, 50, 52]. The quark profile boundary conditions are exactly the same, except that $M_{IR} = 0$.

As in the boson case, this system of equations can be divided into three independent subsystems, one for each fermion EM charge. In the lepton case we have charges 1, 0 and -1 , while in the quark case we have $5/3$, $2/3$ and $-1/3$.

In the following three subsections we will be dealing exclusively with the

fermion mass conditions and coefficient sets, which are all evaluated at the TeV brane. As such, we will omit any notational indication of functional dependence on the vev v , mass m_n , or the coordinate $z = L_1$ in $S_c^\pm$, $S_c^\pm$ and θ_G .

Exotic fermions ($Q = 5/3, 1$)

The first sector is entirely composed of exotic fermions with charge $5/3$ that do not appear in the Standard Model. There are five equations corresponding to these fermions, four can be solved for coefficients C_{ℓ_1} , $C_{\ell_{11}}$, $C_{\ell_{15}}$ and $C_{\ell_{18}}$, while the fifth contains only C_{ℓ_6} and is independent of the others.

The determinant of the corresponding matrix yields the following mass conditions,

$$S_{c_3}^2 = 0 \tag{C 94}$$

$$S_{c_1}^+ S_{c_3}^- + M_2^2 S_{c_1}^+ S_{c_3}^- = 0 \tag{C 95}$$

where all functions are evaluated at $z = L_1$.

The first mass condition corresponds to the following coefficients,

$$\begin{aligned} C_{\ell_1}^{\alpha\beta} &= 0, & C_{\ell_{11}}^{\alpha\beta} &= \frac{i\sqrt{2}}{2} \tan \theta_G (C_{\ell_{15}} - C_{\ell_{18}}), \\ C_{\ell_{15}}^{\alpha\beta} &= C_{\ell_{15}}, & C_{\ell_{18}}^{\alpha\beta} &= C_{\ell_{18}}, \end{aligned} \tag{C 96}$$

Because the condition appears twice the coefficient set is degenerate. Any two perpendicular vectors in the $C_{\ell_{15}}$ - $C_{\ell_{18}}$ space will provide two independent coefficient sets. We choose $(C_{\ell_{15}}, C_{\ell_{18}}) = (0, 1)$ and $(C_{\ell_{15}}, C_{\ell_{18}}) = (1, 0)$, we will label the former by superscript α and the second with superscript β .

The second mass condition gives rise to a third coefficient set,

$$C_{\ell_1}^\gamma = C_{\ell_{11}} \frac{\cos\theta_G}{M_2} \frac{\dot{S}_{c_3}^-}{\dot{S}_{c_1}^+}, \quad C_{\ell_{11}}^\gamma = C_{\ell_{11}}, \quad C_{\ell_{15}}^\gamma = -C_{\ell_{18}}^\gamma = \frac{i\sqrt{2}}{2} \tan\theta_G C_{\ell_{11}}. \quad (\text{C.97})$$

which we denote by γ . Note that is a combined fermion state that corresponds to the mass eigenstates of two different fermions (refer to Table 3.6 for the specifics of the combination).

The independent equation has the trivial mass condition $\dot{S}_{c_2}^- = 0$, and the sole coefficient C_{ℓ_6} is the normalization coefficient. We will denote this solution set by δ .

As such, in the end there are four exotic fermions per generation with profiles $F_{aL,R}^{\frac{5}{3}}$ defined by their action on an arbitrary matrix M as

$$F_{aL,R}^{\frac{5}{3}\dagger} M F_{aL,R}^{\frac{5}{3}} = \sum_{i=1}^3 f_{\ell_{iL,iR}}^{a\dagger} M f_{\ell_{iL,iR}}^a, \quad (\text{C.98})$$

$a = \alpha, \beta, \gamma, \delta$, and subject to the normalization condition

$$\int \frac{dz}{(kz)^4} \sum_{i=1}^3 f_{\ell_{iL,iR}}^{a\dagger} f_{\ell_{iL,iR}}^a = 1, \quad (\text{C.99})$$

where all profiles f_ℓ are evaluated at the same mass value. The notation $f_{\ell_i}^a$ indicates the vector f_{ℓ_i} with its coefficients evaluated according to coefficient set a ; any coefficients undefined by the solution are set to zero. Note that in the case of the independent fermion (δ solution set) Eq. (C.98) implies that

$$F_{\delta L,R}^{\frac{5}{3}} = C_{\ell_6} \dot{S}_{c_2}^- \quad (\text{C.100})$$

where C_{ℓ_6} is defined by the original fermion normalization condition, Eq. (C.35).

Down-type fermions ($Q = -1/3, -1$)

The second sector is composed of the appropriate Standard Model down-type fermion (depending on the fermion generation that we are considering) along with extra

fermions that do not appear in the Standard Model. As in the first sector, the one of the five equations involving these fermions – that involving C_{ℓ_9} – is independent of the others. The remaining four can be solved for coefficients C_{ℓ_4} , $C_{\ell_{14}}$, $C_{\ell_{17}}$ and $C_{\ell_{20}}$. The determinant of the corresponding 4x4 matrix yields the following mass conditions,

$$S_{c_3}^- = 0 \quad (\text{C } 101)$$

$$2 S_{c_3}^+ \left(M_2^2 S_{c_3}^- S_{c_1}^- + S_{c_1}^- S_{c_3}^- \right) - (k z)^4 M_2^2 S_{c_1}^- \sin^2 \theta_G = 0 \quad (\text{C } 102)$$

Here the second condition, Eq. (C 102), corresponds to the mixed masses of the SM down-type fermion with one of the extra fermions, we will call the resulting fermion our down-type fermion. The first condition, Eq. (C 101), simply corresponds to another extra fermion.

The first mass condition corresponds to the following coefficient set, which we denote by η ,

$$C_{\ell_4}^\eta = C_{\ell_{20}}^\eta = 0, \quad C_{\ell_{14}}^\eta = \frac{i\sqrt{2}}{2} \tan \theta_G C_{\ell_{17}}, \quad C_{\ell_{17}}^\eta = C_{\ell_{17}}, \quad (\text{C } 103)$$

The second mass condition, corresponding to our down-type fermion d , gives rise to a second coefficient set,

$$C_{\ell_4}^d = \frac{i\sqrt{2}}{M_2 \sin \theta_G} C_{\ell_{20}} \frac{S_{c_3}^+}{S_{c_1}^-}, \quad C_{\ell_{20}}^d = C_{\ell_{20}},$$

$$C_{\ell_{14}}^d = -\frac{i\sqrt{2}}{M_2 \tan \theta_G} C_{\ell_{17}}^d = \frac{i\sqrt{2}}{M_2 \tan \theta_G} C_{\ell_{20}} \frac{S_{c_3}^+}{S_{c_3}^-}, \quad (\text{C } 104)$$

which we denote by d . This mass condition and coefficient set correspond to yet another combined profile, that includes the mass eigenstates of three fermions (see Table 3.6)

The independent equation has the trivial mass condition $S_{c_2}^- = 0$, and the sole coefficient C_{ℓ_9} is the normalization coefficient. It corresponds to an extra fermion, we will denote it by ϵ .

In the end we have our down-type fermion with profile $F_{d_{L,R}}^{-\frac{1}{3}}$ and two extra fermions with profiles $F_{\epsilon_{L,R}}^{-\frac{1}{3}}$ and $F_{\eta_{L,R}}^{-\frac{1}{3}}$. The full profiles are defined by Eq. (C.98), where $a = d, \eta, \epsilon$.

Up-type fermions ($Q = 2/3, 0$)

The third sector is composed of the appropriate Standard Model up-type fermion along with extra fermions. In this case none of the equations are independent; all ten are solved together to define the remaining coefficients $C_{\ell_2}, C_{\ell_3}, C_{\ell_5}, C_{\ell_7}, C_{\ell_8}, C_{\ell_{10}}, C_{\ell_{12}}, C_{\ell_{13}}, C_{\ell_{16}}, C_{\ell_{19}}$. The resulting mass conditions are

$$\dot{S}_{c_3}^{-2} = 0 \quad (\text{C.105})$$

$$\dot{S}_{c_2}^{-} = 0 \quad (\text{C.106})$$

$$\dot{S}_{c_1}^{+} \dot{S}_{c_3}^{-} + M_2^2 S_{c_1}^{+} S_{c_3}^{-} = 0. \quad (\text{C.107})$$

$$\begin{aligned} & 2\dot{S}_{c_2}^{-} \left[S_{c_1}^{+} S_{c_2}^{-} M_{UV} \left(M_2^2 M_{IR} \dot{S}_{c_1}^{-} S_{c_3}^{-} - M_1^2 S_{c_1}^{-} \dot{S}_{c_3}^{-} \right) + \left(M_2^2 \dot{S}_{c_1}^{-} S_{c_3}^{-} + S_{c_1}^{-} \dot{S}_{c_3}^{-} \right) \right. \\ & \quad \times \left[M_1^2 S_{c_1}^{+} \dot{S}_{c_2}^{+} + \dot{S}_{c_1}^{+} \left(S_{c_2}^{+} - M_{UV} \dot{S}_{c_2}^{-} - M_{IR} \dot{S}_{c_2}^{+} + M_{IR} M_{UV} S_{c_2}^{-} \right) \right] \Big] \\ & - \left[2 \left(\dot{S}_{c_1}^{+} M_{IR} - S_{c_1}^{+} \right) \left(\dot{S}_{c_1}^{-} S_{c_3}^{-} M_2^2 + \dot{S}_{c_3}^{-} S_{c_1}^{-} \right) + M_2^2 \dot{S}_{c_2}^{-} S_{c_3}^{-} \left(M_{UV} \dot{S}_{c_2}^{-} - \dot{S}_{c_2}^{+} \right) \right. \\ & \quad \left. + \dot{S}_{c_2}^{-} \left(\dot{S}_{c_2}^{+} - M_{IR} S_{c_2}^{+} \right) \left(M_2^2 M_{IR} S_{c_3}^{-} + M_1^2 \dot{S}_{c_3}^{-} \right) \right] (kz)^4 \sin^2 \theta_G \\ & \quad - \left(M_1^2 M_{IR} S_{c_3}^{-} + M_1^2 \dot{S}_{c_3}^{-} \right) (kz)^8 \sin^4 \theta_G = 0. \end{aligned} \quad (\text{C.108})$$

Here the last condition corresponds to the mixed masses of the SM up-type fermion with one of the extra fermions; we will call the resulting fermion our up-type fermion. The other conditions simply correspond to other extra fermions.

The first mass condition yields two degenerate coefficient sets, which we denote

by ρ and α ,

$$\begin{aligned} C_{\ell_2}^{\rho,\sigma} &= C_{\ell_3}^{\rho,\sigma} = C_{\ell_5}^{\rho,\sigma} = C_{\ell_7}^{\rho,\sigma} = C_{\ell_8}^{\rho,\sigma} = C_{\ell_{10}}^{\rho,\sigma} = 0, \\ C_{\ell_{12}}^{\rho,\sigma} &= C_{\ell_{13}}^{\rho,\sigma} = C_{\ell_{13}}, \quad C_{\ell_{16}}^{\rho,\sigma} = -\frac{2i}{\tan\theta_G} C_{\ell_{13}} + C_{\ell_{19}}, \quad C_{\ell_{19}}^{\rho,\sigma} = C_{\ell_{19}}. \end{aligned} \quad (\text{C.109})$$

Here again we choose $(C_{\ell_{13}}, C_{\ell_{19}}) = (0, 1), (1, 0)$ to obtain two independent solutions that we will denote by ρ, σ .

The second condition results in the coefficient set

$$\begin{aligned} C_{\ell_2}^\tau &= C_{\ell_3}^\tau = C_{\ell_5}^\tau = C_{\ell_{10}}^\tau = C_{\ell_{12}}^\tau = C_{\ell_{13}}^\tau = C_{\ell_{16}}^\tau = C_{\ell_{19}}^\tau = 0, \\ C_{\ell_7}^\tau &= -C_{\ell_8}, \quad C_{\ell_8}^\tau = C_{\ell_8}, \end{aligned} \quad (\text{C.110})$$

which we denote by τ , and the third results in

$$\begin{aligned} C_{\ell_2}^\omega &= C_{\ell_2}, \quad C_{\ell_3}^\omega = C_{\ell_7}^\omega = C_{\ell_8}^\omega = C_{\ell_{10}}^\omega = C_{\ell_{13}}^\omega = 0, \quad C_{\ell_5}^\omega = \frac{\sqrt{2}}{2} \tan\theta_G C_{\ell_2}, \\ C_{\ell_{16}}^\omega &= -C_{\ell_{19}}^\omega = \frac{i}{2} \tan\theta_G C_{\ell_{12}}^\omega = \frac{i}{2} \tan\theta_G \frac{i}{2} M_2 \tan\theta_G C_{\ell_2} \frac{S_{c_1}^+}{\dot{S}_{c_3}^-} \end{aligned} \quad (\text{C.111})$$

which we will refer to by ω . This ω coefficient set corresponds to a profile representing three fermion eigenstates merged into a combined tower (see Table 3.6).

Finally, the mass condition associated with the up-type fermion yields the coefficient set u ,

$$\begin{aligned} C_{\ell_2}^u &= \frac{1}{\sqrt{2}} \tan\theta_G C_{\ell_5}^u = C_3 \frac{\dot{S}_{c_3}^- S_{c_1}^- + M_2^2 S_{c_1}^- \dot{S}_{c_1}^-}{\dot{S}_{c_1}^+ \dot{S}_{c_3}^- + M_2^2 S_{c_1}^+ S_{c_3}^-}, \quad C_{\ell_3}^u = C_3, \\ C_{\ell_{12}}^u &= \frac{\sin^2\theta_g}{1 + \cos^2\theta_g} C_{\ell_{13}}^u = i \tan\theta_G C_{\ell_{16}}^u = -i \tan\theta_G C_{\ell_{19}}^u = \frac{1}{2} \frac{M_2 C_3 (kz)^4 \sin^2\theta_G}{\dot{S}_{c_1}^+ \dot{S}_{c_3}^- + M_2^2 S_{c_1}^+ S_{c_3}^-}, \\ C_{\ell_7}^u &= C_{\ell_8}^u = \frac{1}{\sqrt{2}} \tan\theta_G \frac{S_{c_2}^+}{\dot{S}_{c_2}^-} C_{\ell_{10}}^u = -\frac{M_1}{M_2} \frac{\dot{S}_{c_3}^-}{\dot{S}_{c_2}^-} C_{\ell_{12}}^u + M_1 \frac{S_{c_1}^+}{\dot{S}_{c_2}^-} C_{\ell_2}^u, \end{aligned} \quad (\text{C.112})$$

which represents five combined fermions.

In the end we have our up-type fermion with profile $F_{u_{L,R}}^{\frac{2}{3}}$ and four extra fermions with profiles $F_{\eta_{L,R}}^{\frac{2}{3}}$, $F_{\rho_{L,R}}^{\frac{2}{3}}$, $F_{\tau_{L,R}}^{\frac{2}{3}}$ and $F_{\omega_{L,R}}^{\frac{2}{3}}$. The full profiles are defined by Eq. (C.98), where $a = u, \rho, \sigma, \tau, \omega$.

C.4 Vertices

In the following we provide the relevant vertices for our calculations. It should be noted that the final forms given are derived with respect to the 4D particle fields – and therefore will contain the profiles of the associated particles – but are not integrated over the fifth dimension. The effective 4D couplings can be found by integrating these forms over the fifth dimension, while 5D couplings can be obtained by dropping the interacting particles' profiles.

C.4.1 4-particle interaction terms: $W^{3_{L,R}} W^{3_{L,R}} H H$

For the interaction $f\bar{f} \rightarrow ZH$ we would like to determine all couplings of form (gauge boson) $\rightarrow ZH$. As the Z boson will arise from $A_\mu^{3_{L,R}} \rightarrow W_\mu^{3_{L,R}}$ as in the SM – and the Higgs will arise from the fifth component of one of the $SO(5)/SO(4)$ gauge bosons (A_5^a), we must consider all Lagrangian terms proportional to $A_\nu^{3_{L,R}} A_\mu^{3_{L,R}} A_5^a A_5^a$.⁵ Starting with the same Lagrangian as in Eq. (3.8), we take only the term of $F_{MR} F_{NS}$ involving a product of commutators, and let $M, N = \mu, \nu$ and $R, S=5$

$$L_{4v} = -\frac{\sqrt{g}}{2} g_5^2 g^{\mu\nu} g^{55} A_{\mu 5}^a A_{\nu 5}^a = \frac{1}{2kz} g_5^2 \eta^{\mu\nu} A_{\mu 5}^a A_{\nu 5}^a \quad (\text{C.113})$$

⁵Note that the metric factors prevent couplings of form $A_\mu A_\nu A_\alpha A_5$ or $A_\mu A_5 A_5 A_5$. Therefore an A_5 cannot be the propagator in an interaction such as $e^+ e^- \rightarrow ZH$.

From an inspection of Eq (B 6-B 9), we can see that we must consider only the $a = \hat{3}$ term, as it is the only one that contains terms with one A_5^4 field and one $A_\mu^{3L R}$ field. We find that

$$\begin{aligned}
L_{4v} &\supset \frac{1}{8kz} g_5^2 \eta^{\mu\nu} A_5^4 A_5^4 (W_\mu^{3L} W_\nu^{3L} - 2W_\mu^{3L} W_\nu^{3R} + W_\mu^{3R} W_\nu^{3R}) \\
&= \frac{1}{8kz} g_5^2 \eta^{\mu\nu} [H_5(x, z) + v f_H(z)]^2 (W_\mu^{3L} W_\nu^{3L} - 2W_\mu^{3L} W_\nu^{3R} + W_\mu^{3R} W_\nu^{3R}) \\
&= \frac{1}{8kz} g_5^2 \eta^{\mu\nu} (W_\mu^{3L} W_\nu^{3L} - 2W_\mu^{3L} W_\nu^{3R} + W_\mu^{3R} W_\nu^{3R}) \\
&\quad \times (H_5(x, z)^2 + 2 H_5(x, z) v f_H(z) + v^2 f_H(z)^2)
\end{aligned} \tag{C 114}$$

Therefore the relevant terms for $W_\mu^{3L R} W_\nu^{3L R} h_5$ interactions are

$$L_{4v} \supset \frac{1}{4kz} g_5^2 \eta^{\mu\nu} v f_H(z) H_5(x, z) (W_\mu^{3L} W_\nu^{3L} - 2W_\mu^{3L} W_\nu^{3R} + W_\mu^{3R} W_\nu^{3R}) \tag{C 115}$$

Substituting Eqs (2 14-2 15), Eqs (3 14 3 15), and Eqs (C 117) into Eq (C 115), we find the following vertex between two 5D Higgs bosons H_5 and two 5D photons,

$$H_5 H_5 V_\mu V^\mu \rightarrow \frac{1}{2kz} (\sin^2 \theta_w - 2 \sin \theta_w \cos \theta_H \cos \theta_w + \cos^2 \theta_w \cos^2 \theta_H) \tag{C 116}$$

As the Higgs is neutral, we require that this vertex be zero. This condition results in the following relationship between θ_H and the Weinberg angle θ_w ,

$$\cos \theta_H = \tan \theta_w \tag{C 117}$$

This fixes B_μ and X_μ in terms of W_μ^{3R} and U_μ . With this relationship, both the $H_5 H_5 A_\mu Z^\mu$ and $H_5 H_5 A_\mu X^\mu$ couplings also vanish. The former is fortunate, as the Z boson is known to be neutral, the latter tells us that the X boson is also neutral. Note that if we had chosen A_5^3 as our Higgs the relation between θ_H and θ_w would be the same, the choice of either A_5^1 or A_5^2 simply change the relation by a minus sign, $\cos \theta_H = -\tan \theta_w$.

Applying this relation, we finally find

$$L_{4v} \supset \frac{g_5^2}{4kz} \eta^{\mu\nu} v f_H(z) h_5(x, z) \left(\frac{1}{\cos^2 \theta_w} Z_\mu Z_\nu + \frac{2\sqrt{\cos 2\theta_w}}{\cos^2 \theta_w} X_\mu Z_\nu + \frac{\cos 2\theta_w}{\cos^2 \theta_w} X_\mu X_\nu \right) \quad (\text{C } 118)$$

Decomposing the 5D fields into 4D fields with profiles, taking the functional derivative of each term, and multiplying by $\imath$, we obtain the Feynman rules for the relevant 4D vertices,

$$Z_\mu^{(n)} Z_\mu^{(m)} H \rightarrow \frac{\imath g_z^2}{2kz} \frac{g_5}{g} \eta^{\mu\nu} v f_H(z)^2 f_Z^{(n)}(z) f_Z^{(m)}(z), \quad (\text{C } 119)$$

$$X_\mu^{(n)} Z_\mu^{(m)} H \rightarrow \frac{\imath g_z g_x}{2kz} \frac{g_5}{g} \eta^{\mu\nu} v f_H(z)^2 f_X^{(n)}(z) f_Z^{(m)}(z), \quad (\text{C } 120)$$

$$X_\mu^{(n)} X_\mu^{(m)} H \rightarrow \frac{\imath g_x^2}{2kz} \frac{g_5}{g} \eta^{\mu\nu} v f_H(z)^2 f_X^{(n)}(z) f_X^{(m)}(z), \quad (\text{C } 121)$$

where we have defined

$$g_z = \frac{g}{\cos \theta_w} \quad \text{and} \quad g_x = \frac{\sqrt{\cos 2\theta_w}}{\cos \theta_w} \quad (\text{C } 122)$$

Note that if we integrate the $Z_\mu^{(0)} Z_\mu^{(0)} H(x, z)$ vertex

$$\frac{g_z^2 v}{2} \frac{\ln k L_1}{k} \eta^{\mu\nu} \int_{L_0}^{L_1} \frac{dz}{kz} f_H(z)^2 f_G^{(0)}(z)^2 = \frac{g_z^2 v}{2} \eta^{\mu\nu} \int_{L_0}^{L_1} dz z \frac{2}{L_1^2 - L_0^2} = \frac{g_z^2 v}{2} \eta^{\mu\nu}, \quad (\text{C } 123)$$

we obtain an effective 4D term that is the same as the corresponding SM term

C.4.2 3-particle interaction: Three gauge bosons

The interaction of three gauge bosons arise from the terms in the gauge Lagrangian, Eq (3 8), proportional to both a derivative of the gauge field and a commutation of

gauge fields,

$$\begin{aligned}
L_{3v} &= -g_5 \frac{\sqrt{g}}{4} g^{MN} g^{RS} [(\partial_M A_R^a) A_{NS}^a - (\partial_R A_M^a) A_{NS}^a + A_{MR}^a (\partial_N A_S^a) - A_{MR}^a (\partial_S A_N^a)] \\
&= -\frac{g_5}{2kz} \eta^{\mu\nu} \eta^{\alpha\beta} [(\partial_\mu A_\alpha^a) A_{\nu\beta}^a - (\partial_\alpha A_\mu^a) A_{\nu\beta}^a] + \frac{g_5}{kz} \eta^{\mu\nu} [(\partial_\mu A_5^a) A_{\nu 5}^a - (\partial_5 A_\mu^a) A_{\nu 5}^a] \\
&= -\frac{g_5}{kz} \eta^{\mu\nu} [\eta^{\alpha\beta} (\partial_\mu A_\alpha^a) A_{\nu\beta}^a - (\partial_\mu A_5^a) A_{\nu 5}^a + (\partial_5 A_\mu^a) A_{\nu 5}^a] \\
&= -\frac{g_5}{kz} \eta^{\mu\nu} [\eta^{\alpha\beta} (\partial_\mu A_\alpha^a) A_{\nu\beta}^a + (\partial_5 A_\mu^a) A_{\nu 5}^a] , \tag{C 124}
\end{aligned}$$

where we have used the fact that $A_{\mu\nu}^a = -A_{\nu\mu}^a$. The last step is attained by neglecting all fifth components other than the Higgs ($a = 4$), as they are either zero, or are Goldstone bosons that will be eaten to become longitudinal polarizations of the vector bosons

Let us consider the first term of Eq (C 124), keeping only terms involving bosons with zero modes for the moment. This leaves only the interaction between W_μ^{1L} , W_μ^{2L} , and W_μ^{3L} ,

$$L_{w_{1L} w_{2L} w_{3L}} = -\frac{g_5}{kz} \eta^{\mu\nu} \eta^{\alpha\beta} (\partial_\mu A_\alpha^{aL}) A_{\nu\beta}^{aL} \tag{C 125}$$

$$= -\frac{g_5}{kz} \eta^{\mu\nu} \eta^{\alpha\beta} \epsilon^{aL bL cL} (\partial_\mu A_\alpha^{aL}) A_\nu^{cL} A_\beta^{bL} \tag{C 126}$$

Making replacements⁶ $A(x, z) = \int d^4p \exp(-ipx) A(p, z)$ in the corresponding action, decomposing the 5D fields into KK modes, taking the functional derivative [11], and multiplying by i we obtain our Feynman rule for the vertex $W_\mu^{1L}(k_1) W_\nu^{2L}(k_2) W_\lambda^{3L}(k_3)$,

$$\begin{aligned}
W_\mu^{1L(n)}(p_1) W_\nu^{2L(m)}(p_2) W_\lambda^{3L(l)}(p_3) &\rightarrow \frac{g_5}{kz} [\eta^{\mu\lambda} (p_3^\nu - p_1^\nu) + \eta^{\mu\nu} (p_1^\lambda - p_2^\lambda) \\
&\quad + \eta^{\nu\lambda} (p_2^\mu - p_3^\mu)] f_{1L}^{(n)}(z) f_{2L}^{(m)}(z) f_{3L}^{(l)}(z), \tag{C 127}
\end{aligned}$$

⁶Here we use the convention that the momenta p are incoming

where we use the convention that all particle momenta are incoming. This is the same vertex as found in the SM, except for the profiles, the factor of $(kz)^{-1}$ and the replacement $g \rightarrow g_5$ [11]. The rotation from this basis to the physical basis, $W_\mu^{1L} W_\nu^{2L} W_\alpha^{3L} \rightarrow W_\mu^+ W_\nu^- Z_\alpha$, works the same as way as in the SM, except that now the profiles are rotated with the 4D fields, as such, we find that in the physical basis

$$W_\mu^+(k_+) W_\nu^-(k_-) Z_\lambda(q) \rightarrow \frac{i g_z}{k z} \cos^2 \theta_w \frac{g_5}{g} [\eta^{\mu\nu} (k_+^\lambda - k_-^\lambda) + \eta^{\nu\lambda} (k_-^\mu - q^\mu) + \eta^{\lambda\mu} (q^\nu - k_+^\nu)] f_{W^+}^{(n)}(z) f_{W^-}^{(m)}(z) f_Z^{(l)}(z) \quad (\text{C } 128)$$

If we consider all vertices in the first term of Eq. (C 124) – not simply those corresponding to the SM – we find that the vertices will all have the same form as Eq. (C 127), with the following replacements of the indices

$$\begin{aligned} 2(a_{L,R}, \hat{b}, \hat{c}) &\rightarrow 2(1_R, 2_R, 3_R), (1_L, \hat{1}, \hat{4}), (1_L, \hat{2}, \hat{3}), (2_L, \hat{3}, \hat{1}), \\ &(2_L, \hat{2}, \hat{4}), (3_L, \hat{1}, \hat{2}), (3_L, \hat{3}, \hat{4}), -(1_R, \hat{1}, \hat{4}), (1_R, \hat{2}, \hat{3}), \\ &(2_R, \hat{3}, \hat{1}), -(2_R, \hat{2}, \hat{4}), (3_R, \hat{1}, \hat{2}), -(3_R, \hat{3}, \hat{4}), \end{aligned} \quad (\text{C } 129)$$

where outer factors $(-, 2)$ denote a difference of this overall factor in the vertex structure. In the physical basis, the coupling of three gauge bosons G_1 , G_2 and G_3 will be given by Eq. (C 127) with the coefficient replacement $g \rightarrow C_{G_1 G_2 G_3}$ – where $C_{G_1 G_2 G_3}$ is a function of θ_w proportional to g – and the profile replacement

$$f_{1L}^{(n)}(z) f_{2L}^{(m)}(z) f_{3L}^{(l)}(z) \rightarrow f_{G_1}^{(n)}(z) f_{G_2}^{(m)}(z) f_{G_3}^{(l)}(z) \quad (\text{C } 130)$$

The specific coefficients $C_{G_1 G_2 G_3}$ for the couplings relevant to our purposes are

$$\begin{aligned} C_{Z W_L^+ W_L^-} &= i g_z \cos^2 \theta_w, \quad C_{Z W_R W_R} = -i g_z \sin^2 \theta_w, \quad C_{Z A^\pm A^\mp} = \frac{i}{2} g_z \cos 2\theta_w, \\ C_{Z A^3 A^4} &= \frac{g_z}{2}, \quad C_{X W_R^+ W_R} = -i g_x, \quad C_{X A^\pm A^\mp} = -\frac{i}{2} g_x, \quad C_{X A^3 A^4} = \frac{g_x}{2}, \\ C_{W_L^\mp A^\pm A^3} &= C_{W_R^\mp A^\pm A^3} = \pm \frac{i}{2}, \quad C_{W_L^\mp A^\pm A^4} = -C_{W_R^\mp A^\pm A^3} = \pm \frac{1}{2} \end{aligned} \quad (\text{C } 131)$$

If we now consider the second term of Eq C 124), we find four further couplings involving the Higgs boson of form $A_\mu^{aI R} A_\nu^a H_5$

$$L_{A^{aL} R A^a H} = -\frac{g_5}{k z} \eta^{\mu\nu} (\partial_5 A_\mu^a) A_{\nu 5}^a = -\frac{g_5}{2 k z} \eta^{\mu\nu} \sum_{a=1}^3 \left[((\partial_z A_\mu^{aL}) A_\nu^a - (\partial_z A_\mu^a) A_\nu^{aL}) h_5 \right. \\ \left. - ((\partial_z A_\mu^{aR}) A_\nu^a - (\partial_z A_\mu^a) A_\nu^{aR}) h_5 \right] , \quad (C 132)$$

where we have kept only terms where $A_5^a = A_5^4$. As we are primarily interested in the couplings involving Z and X bosons, let's concentrate on the $a = 3$ term. Transforming to the physical basis, we find that the couplings of the 4D field components are

$$G_\mu^{(n)} A_\nu^{(m)} H \rightarrow \frac{1}{2 k z} C_{GA^a H} \frac{g_5}{g} f_H(z) \left[\left(\partial_z f_G^{(n)}(p, z) \right) f_A^{(m)}(p, z) \right. \\ \left. - \left(\partial_z f_A^{(m)}(p, z) \right) f_G^{(n)}(p, z) \right] , \quad (C 133)$$

where $G = Z, X, W^\pm$, $\hat{a} = +, -, \hat{3}$ and the particular coefficients are

$$C_{ZA^3 H} = g_z , \quad C_{XA^3 H} = g_x \quad C_{W_L^\pm A^\mp H} = -C_{W_R^\pm A^\mp H} = g \quad (C 134)$$

It is worth noting that in the absence of particle mixing all of the corresponding 4D effective couplings of this form will be zero if the relevant Z, X or W boson is a zero mode, as the profiles will integrate to zero

$$\frac{g_G}{2} \frac{g_5}{g} \int dz \frac{1}{k z} f_H(z) \left[\left(\partial_z f_G^{(0)}(m_G) \right) f_A^{(m)}(m_A, z) - \left(\partial_z f_A^{(m)}(m_A, z) \right) f_G^{(0)}(m_G) \right] \\ = -\frac{g_G}{2k} \frac{g_5}{g} f_G^{(0)}(m_G) \sqrt{\frac{2k}{L_1^2 - L_0^2}} \int dz \left(\partial_z f_A^{(m)}(m_A, z) \right) \\ = -\frac{g_G}{2k} \frac{g_5}{g} f_G^{(0)}(m_G) \sqrt{\frac{2k}{L_1^2 - L_0^2}} \left[f_A^{(m)}(m_A, L_1) - f_A^{(m)}(m_A, L_0) \right] \\ = 0, \quad (C 135)$$

by the Dirichlet boundary conditions on the coset bosons. Here we have used the fact that $f_G^{(0)}(z)$ is constant in z and required that the particles be on-shell. The fact that these couplings are zero provides a second reason – beyond the lack of fermion anti-fermion couplings – why the propagator in $e^+e^- \rightarrow Z^{(0)} H$ cannot be an $A^{\hat{a}}$ boson. However, note that these zero-mode 3-particle couplings reappear after mixing because the zero-mode gauge boson profiles are no longer flat.

C.4.3 3-particle interaction: Fermion-fermion-boson

By applying integration by parts to the derivative in the second term in Eq. (3.12), we find that the spin-connection contribution from Eq.(3.13) cancels out, while the derivative and boson terms double. Therefore the relevant terms for the coupling of two fermions to a Z or X gauge boson are

$$\begin{aligned} L_{\bar{f}fG} \supset & \frac{g_5}{(kz)^4} \eta^{\mu\nu} \left[\bar{\Psi}_L \gamma_\nu \left(\tilde{T}^{3L} W_\mu^{3L} + \tilde{T}^{3R} W_\mu^{3R} \right) \Psi_L + \bar{\Psi}_R \gamma_\nu \left(\tilde{T}^{3L} W_\mu^{3L} + \tilde{T}^{3R} W_\mu^{3R} \right) \Psi_R \right] \\ & + \frac{g_5''}{(kz)^4} \eta^{\mu\nu} \left[\bar{\Psi}_L \gamma_\nu Q_{B-L} \Psi_L U_\mu + \bar{\Psi}_R \gamma_\nu Q_{B-L} \Psi_R U_\mu \right], \end{aligned} \quad (\text{C.136})$$

where we have kept only the $3_{L,R}$ terms, and $g_5'' = g'' \frac{g_5}{g}$ is the 5D gauge coupling of $U(1)_{B-L}$, and defined

$$\tilde{T}_5^{3L,R} = A T_5^{3L,R} A^{-1}, \quad \tilde{T}_{10}^{3L,R} = B T_{10}^{3L,R} B^{-1}. \quad (\text{C.137})$$

The **5** and **10** representations of the relevant generators are defined in Appendix B; the basis transformation matrices A and B are defined in Appendix C.3.

We can relate Q_{B-L} to the hypercharge Y and the weak generator T^{3R} by requiring that the coupling of the hypercharge boson B_μ to fermions match that of the SM, $g' Y \bar{\Psi}_{L,R} \gamma^\mu B_\mu \Psi_{L,R}$, where $g' = g \tan \theta_w$. In our model B_μ is a mix of

U_μ and W_μ^{3R} , therefore the corresponding term arises from the second two terms of Eq. (C.136),

$$\frac{g_5}{(kz)^4} \eta^{\mu\nu} \bar{\Psi}_{L,R} \gamma_\nu \left[\tilde{T}^{3R} W_\mu^{3R} + g_5'' Q_{B-L} U_\mu \right] \Psi_{L,R} \quad (\text{C.138})$$

$$\supset \frac{\eta^{\mu\nu}}{(kz)^4} \bar{\Psi}_{L,R} \gamma_\nu \left[g_5 \tilde{T}^{3R} \cos \theta_H + g_5'' Q_{B-L} \sin \theta_H \right] B_\mu \Psi_{L,R} \quad (\text{C.139})$$

If we consider the zero-modes, the 5D components will integrate to 1 (as shown in Eq.(C.144)). We therefore require that

$$Y = \frac{g}{g'} \tilde{T}^{3R} \cos \theta_H + \frac{g''}{g'} Q_{B-L} \sin \theta_H = \tilde{T}^{3R} + Q_{B-L}. \quad (\text{C.140})$$

where we have made use of $g = \tan \theta_H g''$.

By making use of these relations as we substitute Eqs. (2.14-2.15) into Eq.(C.136) – and neglecting the photon fields – the vertex terms become

$$\begin{aligned} L_{\bar{f}fG}^{(L,R)} \supset & \frac{g_z}{(kz)^4} \frac{g_5}{g} \bar{\Psi}_{L,R} \left(\tilde{T}^{3L} - Q \sin^2 \theta_w \right) \gamma^\mu Z_\mu \Psi_{L,R} \\ & + \frac{g_x}{(kz)^4} \frac{g_5}{g} \bar{\Psi}_{L,R} \left(\frac{(Q - \tilde{T}^{3L}) \sin^2 \theta_w - \tilde{T}^{3R} \cos^2 \theta_w}{\cos 2\theta_w} \right) \gamma^\mu X_\mu \Psi_{L,R}, \end{aligned} \quad (\text{C.141})$$

where $Q = Y + \tilde{T}^{3L}$ – the electromagnetic charge in units of e , not to be confused with Q_{B-L} – is defined by requiring that the photon-fermion vertex be Qe . Note that if we take $\bar{\Psi}, \Psi$ to be specific fermions, rather than full fermion vectors, then $\tilde{T}^{3L,R}$ denotes the scalar element of the generator matrix that selects the appropriate fermions from the fermion spinors. Although the vertices of the right- and left-handed fermions have the same mathematical form, the element of $\tilde{T}^{3L,R}$ corresponding to each will not in general be the same; as such the right- and left-handed couplings will not have the same values, despite having the same form.

For right-handed SM particles, $T^{3L} = 0$. Therefore the fermion coupling to Z bosons in Eq. (C.141) matches that of the Standard Model in both the left- and

right-handed cases, as desired

$$\begin{aligned}
L_{ffG}^{(L\ R)} \supset & \frac{g_z}{(k\ z)^4} \frac{g_5}{g} \bar{\Psi}_L^{(SM)} \left(\tilde{T}^{3_L} - Q \sin^2 \theta_w \right) \gamma^\mu Z_\mu \Psi_L^{(SM)} \\
& + \frac{g_z}{(k\ z)^4} \frac{g_5}{g} \bar{\Psi}_R^{(SM)} \left(-Q \sin^2 \theta_w \right) \gamma^\mu Z_\mu \Psi_R^{(SM)}
\end{aligned} \tag{C 142}$$

Furthermore, note that for left-handed particles $T^{3_I} = T^{3_R}$, in this case the left-handed X coupling reduces to the same form as the Z coupling, but with opposite sign and an inverse factor of $\cos 2\theta_w$. This correspondence between the Z and X fermion couplings also holds in the case of right-handed up-type fermions. The correspondence is notable as the XZH vertex of the Higgsstrahlung interaction brings an extra factor of g_x^2 , which cancels the extra cosine factor and results in Z and X Higgsstrahlung coupling constants of equal but opposite magnitude.

Because $AT^{3_L R}A^{-1}$, $BT_{10}^{3_L R}B^{-1}$ and the matrix form of Q are all diagonal, Z and X can only couple a given fermion f to its own anti-fermion $\bar{f}$. However, the gauge boson A_μ^3 can couple different fermions of the same charge. This will become relevant once the electroweak symmetry is broken because both the X and Z become a superposition of states that includes the A_μ^3 , allowing KK decays to $f_1\bar{f}_2$. The Lagrangian term corresponding to the coupling of a fermion anti-fermion pair to A_μ^3 is simply given by

$$L_{ffG} \supset \frac{g_5}{(k\ z)^4} \eta^{\mu\nu} \bar{\Psi}_L \gamma_\nu T^3 A_\mu^3 \Psi_L \tag{C 143}$$

All of the possible couplings of Z , X or A_μ^3 bosons to fermions are summarized in Table C 1

Boson	Lepton pair	Quark pair	Q_ℓ, Q_q	$\tilde{T}^{3_L}$	$\tilde{T}^{3_R}$	$\tilde{T}^3$
Z, X	$\bar{\kappa}_1\kappa_1, \bar{\kappa}_2\kappa_2, \bar{\kappa}_3\kappa_3$	$\bar{\chi}_1\chi_1, \bar{\chi}_2\chi_2, \bar{\chi}_3\chi_3$	$1, \frac{5}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0
Z, X	$\bar{\eta}_1\hat{\eta}_1, \bar{\eta}_2\hat{\eta}_2, \bar{\eta}_3\hat{\eta}_3$	$\bar{t}_1\hat{t}_1, \bar{t}_2\hat{t}_2, \bar{t}_3\hat{t}_3$	$0, \frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
Z, X	$\bar{\eta}_1\eta_1, \bar{\eta}_2\eta_2, \bar{\eta}_3\eta_3$	$\bar{t}_1t_1, \bar{t}_2t_2, \bar{t}_3t_3$	$0, \frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
Z, X	$\bar{\ell}_1\ell_1, \bar{\ell}_2\ell_2, \bar{\ell}_3\ell_3$	$\bar{b}_1b_1, \bar{b}_2b_2, \bar{b}_3b_3$	$-1, -\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
Z, X	$\bar{\tilde{\eta}}_1\tilde{\eta}_1, \bar{\tilde{\eta}}_2\tilde{\eta}_2, \bar{N}_3N_3, \bar{N}'_3N'_3$	$\bar{t}_1\tilde{t}_1, \bar{t}_2\tilde{t}_2, \bar{T}_3T_3, \bar{T}'_3T'_3$	$0, \frac{2}{3}$	0	0	0
Z, X	$\bar{K}_3K_3$	$\bar{\Xi}_3\Xi_3$	$1, \frac{5}{3}$	1	0	0
Z, X	$\bar{L}_3L_3$	$\bar{B}_3B_3$	$-1, -\frac{1}{3}$	-1	0	0
Z, X	$\bar{K}'_3K'_3$	$\bar{\Xi}'_3\Xi'_3$	$1, \frac{5}{3}$	0	1	0
Z, X	$\bar{L}'_3L'_3$	$\bar{B}'_3B'_3$	$-1, -\frac{1}{3}$	0	-1	0
A_μ^3	$\bar{\tilde{\eta}}_3N'_3, \bar{N}_3\eta_3 + \text{c.c.}$	$\bar{t}_3T'_3, \bar{T}_3t_3 + \text{c.c.}$	$0, \frac{2}{3}$	0	0	$-\frac{\sqrt{2}i}{4}$
A_μ^3	$\bar{\tilde{\eta}}_3N_3, \bar{N}'_3\eta_3 + \text{c.c.}$	$\bar{t}_3T_3, \bar{T}'_3t_3 + \text{c.c.}$	$0, \frac{2}{3}$	0	0	$\frac{\sqrt{2}i}{4}$
A_μ^3	$\bar{\ell}_3L'_3, \bar{\ell}_3L_3 + \text{c.c.}$	$\bar{b}_3B'_3, \bar{b}_3B_3 + \text{c.c.}$	$-1, -\frac{1}{3}$	0	0	$-\frac{i}{2}$
A_μ^3	$\bar{\kappa}_3K'_3, \bar{\kappa}_3K_3 + \text{c.c.}$	$\bar{\chi}_3\Xi'_3, \bar{\chi}_3\Xi_3 + \text{c.c.}$	$1, \frac{5}{3}$	0	0	$\frac{i}{2}$
A_μ^3	$\bar{\eta}_1\hat{\eta}_1, \bar{\eta}_2\hat{\eta}_2 + \text{c.c.}$	$\bar{t}_1\hat{t}_1, \bar{t}_2\hat{t}_2 + \text{c.c.}$	$0, \frac{2}{3}$	0	0	$-\frac{1}{2}$
A_μ^3	$\bar{\eta}_1\hat{\eta}_1, \bar{\eta}_2\hat{\eta}_2 + \text{c.c.}$	$\bar{t}_1\hat{t}_1, \bar{t}_2\hat{t}_2 + \text{c.c.}$	$0, \frac{2}{3}$	0	0	$\frac{1}{2}$

Table C.1: Possible couplings of Z, X or A_μ^3 bosons to lepton anti-lepton pairs. The L, R subscripts have been suppressed. Refer to Eqs. (3.23-3.28) for the multiplet structure.

C.5 The Relation Between g and g_5

The relation between the 5D gauge coupling g_5 and the familiar SM gauge coupling g can be determined by comparing effective 4D MCHM couplings to those of the SM. It should be noted that particle mixing will alter the profiles used in the derivation below, and therefore cause a shift in the relationship between g and g_5 . However, as zero modes are not substantially altered by mixing this shift will be quite small.

C.5.1 From the four gauge boson vertex

To determine the relation between the 4D SM gauge coupling g and our 5D coupling constant g_5 , consider a vertex of four gauge bosons. Proceeding similarly to Eq (C 17), and neglecting all brane-restricted contributions to the Lagrangian, we must obtain g^2 when all z dependent contributions and the g_5^2 factor are integrated over z . Therefore we require that

$$g^2 = \int_{L_0}^{L_1} dz \frac{g_5^2}{k z} [f_G^{(0)}(z)]^4 = \frac{g_5^2}{k} \left[\frac{k}{\ln(k L_1)} \right]^2 \int_{L_0}^{L_1} dz \frac{1}{z} = \frac{k g_5^2}{\ln(k L_1)} \quad (\text{C 144})$$

If brane terms were included the relation would have form

$$\frac{1}{g^2} = \frac{\ln(k L_1)}{k g_5^2} + \frac{1}{g_{\text{UV}}} + \frac{1}{g_{\text{IR}}}, \quad (\text{C 145})$$

where g_{UV} and g_{IR} are possible contributions from the UV and IR brane terms [3], which are zero in our case.

C.5.2 From a $\bar{f}f A_\mu$ vertex

The coupling constant relation can also be derived by considering a vertex where a fermion and anti-fermion couple to a gauge boson. Specifically, let's consider $W_L^+ \rightarrow$

$e^- \bar{\nu}_L$ As before, the 4D constant g must be equal to the integral of all the 5D components Using Eq (C 39), Eq (C 25) and noting that both fermions have the same associated parameter c as they are part of the same $SU(2)_L$ doublet, we have

$$g = g_5 \int_{L_0}^{L_1} dz \sqrt{g} k z f_e^{(0)}(z) f_\nu^{(0)}(z) f_W^{(0)}(z) \quad (\text{C 146})$$

$$= g_5 \int_{L_0}^{L_1} dz \frac{1}{(k z)^4} \sqrt{\frac{k}{\ln(k L_1)}} \frac{k(1-2c)}{(k L_1)^{1-2c} - 1} (k z)^{4-2c} \quad (\text{C 147})$$

$$= g_5 \frac{k(1-2c)}{(k L_1)^{1-2c} - 1} k^{-2c} \sqrt{\frac{k}{\ln(k L_1)}} \int_{L_0}^{L_1} dz z^{-2c} \quad (\text{C 148})$$

$$= g_5 \frac{1-2c}{(k L_1)^{1-2c} - 1} k^{1-2c} \frac{1}{1-2c} (L_1^{1-2c} - L_0^{1-2c}) \sqrt{\frac{k}{\ln(k L_1)}} \quad (\text{C 149})$$

$$= g_5 \sqrt{\frac{k}{\ln(k L_1)}}, \quad (\text{C 150})$$

which is the same relation as found before

Appendix D

Gauge Boson Decay Widths

To account for the physical widths of the KK resonances in the cross section, we will need the total decay rate of each propagating boson. In the following formulae, all constant and 5D integral factors of the couplings will be simply expressed as overall couplings C , as when full particle mixing is implemented the couplings constants and 5D integrals can no longer be separated. The appropriate couplings can be found in Appendix C.4.

D.1 Boson decay to fermion final states

The decay rate in the center-of-mass frame for a gauge boson $G = Z, X$ of mass M_G to two fermions of mass m_1 and m_2 is given by

$$\Gamma_{Gff} = \frac{\lambda^{\frac{1}{2}}(M_G^2, m_1^2, m_2^2)}{48\pi M_G^5} \left[(C_{Gff}^{L\ 2} + C_{Gff}^{R\ 2}) \beta(M_G^2, m_1^2, m_2^2) \right. \\ \left. + 12 M_G^2 m_1 m_2 C_{Gff}^L C_{Gff}^R \right] \quad (\text{D.1})$$

where $M_z^{(n)}$ is the mass of the Z KK mode in question, C_{Gff}^{LR} are the appropriate overall left- and right-handed couplings – including 5D integral factors – and

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz, \quad (\text{D } 2)$$

$$\beta(x, y, z) = 2x^2 - y^2 - z^2 - xy - xz + 2yz \quad (\text{D } 3)$$

In the case where both fermions are zero modes and their masses may be neglected ($m_1 = m_2 = 0$), this reduces to

$$\Gamma_{Gff0} = \frac{M_G}{24\pi} [(C_{Gff}^{L2} + C_{Gff}^{R2})] \quad (\text{D } 4)$$

Note that for quarks these formulae will be multiplied by three to account for the different colours

D.2 Gauge boson decay to a Higgs final state

The decay-rate in the center-of-mass frame of a gauge boson KK mode $G_1 = Z, X$ of order n and mass M_1 to a second gauge boson KK mode $G_2 = Z, X$ of order m and mass M_2 and a Higgs boson is given by

$$\Gamma_{G_1 G_2 H}^{(n,m)} = \frac{C_{G_1 G_2 H}}{48 \pi M_1^{(n)3}} \lambda_2^1(M_1^{(n)2}, M_2^{(m)2}, M_H^2) \left[2 + \frac{(M_1^{(n)2} + M_2^{(m)2} - M_H^2)^2}{4 M_1^{(n)2} M_2^{(m)2}} \right], \quad (\text{D } 5)$$

where M_H is the mass of the Higgs boson, $\lambda(x, y, z)$ is defined in Eq (D 2), and $C_{G_1 G_2 H}$ is the appropriate coupling constant of the two gauge bosons to the Higgs. Here n cannot be smaller than m , this is kinematically forbidden

D.3 Gauge boson decay to two other gauge bosons

The decay rate in the center-of-mass frame of a gauge boson G_1 of order n and mass M_1 to two other gauge bosons G_2, G_3 of orders m, l and masses M_2, M_3 respectively, is given by

$$\Gamma_{G_1 G_2 G_3}^{(n,m,l)} = \frac{C_{G_1 G_2 G_3}}{48\pi M_1^{(n)3}} \lambda^{\frac{1}{2}} \left(M_1^{(n)2}, M_2^{(m)2}, M_3^{(l)2} \right) N(M_1^{(n)}, M_2^{(m)}, M_3^{(l)}), \quad (\text{D.6})$$

where $C_{G_1 G_2 G_3}$ is the appropriate coupling coefficient and

$$\begin{aligned} N(x, y, z) = & -8(x^2 + y^2 + z^2) + 2 \left(\frac{x^4}{y^2} + \frac{x^4}{z^2} + \frac{y^4}{x^2} + \frac{y^4}{z^2} + \frac{z^4}{x^2} + \frac{z^4}{y^2} \right) \\ & + \frac{1}{4} \left(\frac{x^6}{y^2 z^2} + \frac{y^6}{x^2 z^2} + \frac{z^6}{y^2 x^2} \right) - \frac{9}{2} \left(\frac{y^2 z^2}{x^2} + \frac{x^2 z^2}{y^2} + \frac{y^2 x^2}{z^2} \right). \end{aligned} \quad (\text{D.7})$$

It is useful to know that when $y = z$, $N(x, y, z)$ reduces to

$$N(x, y, y) = -12y - 17x + 4 \frac{x^4}{y^2} + \frac{x^6}{4y^4}. \quad (\text{D.8})$$

Appendix E

Analytic integrals for ECA cross-section calculations

E.1 Performing the integrations in the 5D coupling factor $Z_G(c, q)$

To perform the integrals, we must first substitute in Eqs. (C.144), (C.42), (C.39), (C.25) and (C.7), obtaining

$$\begin{aligned} Z_G(c, q) = & \text{Coef}(p) \int_{L_0}^{L_1} dz z^{1 \mp 2c} [C_G J_\alpha(p z) + D_G Y_\alpha(p z)] \\ & \times \int_{L_0}^z dz' z'^2 [A_G J_\alpha(p z') + B_G Y_\alpha(p z')] \\ & + \text{Coef}(p) \int_{L_0}^{L_1} dz z^{1 \mp 2c} [A_G J_\alpha(p z) + B_G Y_\alpha(p z)] \\ & \times \int_z^{L_1} dz' z'^2 [C_G J_\alpha(p z') + D_G Y_\alpha(p z')] , \end{aligned} \quad (\text{E.1})$$

$$\text{where} \quad \text{Coef}(p) = \pi \frac{k^{3\mp 2c}}{A_G D_G - B_G C_G} \frac{1 \mp 2c}{(k L_1)^{1\mp 2c} - 1} \frac{\ln(k L_1)}{(k L_1)^2 - 1} \quad (\text{E.2})$$

where the minus applies to the cases with left-handed fermions and the plus applies to the cases with right-handed fermions, as the former have positive c values while the latter have negative c values. Coefficients A , B , C , and D with Z subscripts are defined by Neumann-Neumann boundary conditions, while those with X subscripts are defined by Dirichlet-Neumann boundary conditions.

The integrals over z' can easily be solved using the basic properties of Bessel functions,

$$\int dz z^2 J_1(q z) = \frac{z^2}{q} J_2(q z), \quad \int dz z^2 Y_1(q z) = \frac{z^2}{q} Y_2(q z). \quad (\text{E.3})$$

The integration over z is complicated both by the variable c – which is not necessarily an integer or fraction – and the multiplication of Bessel functions in the integrand. Terms that involve only one Bessel function can be solved using the Meijer G function $M_G(a_s; b_s; c_s; d_s; z)$ and the first Lommel function $L_1(a, b, z)$. For certain values of c – those for which $1 \mp 2c$ is an integer – these functions simplify to yield results in terms of the Bessel functions and the Struve function $H_\nu(z)$. These special functions are defined in Appendix A and are incorporated as commands in programs such as Mathematica and Maple.

The single-Bessel integrals are given in general by

$$\begin{aligned} \int dz z^{1\mp 2c} J_1(zq) &= q^{-1\pm 2c} z [(1 \mp 2c) J_1(qz) L_1(\mp 2c, 0, qz) \\ &\quad - J_0(qz) L_1(1 \mp 2c, 1, qz)], \end{aligned} \quad (\text{E.4})$$

$$\int dz z^{1\mp 2c} Y_1(zq) = 2^{1\mp 2c} q^{-2\pm 2c} M_G(1; \pm c; 1/2 \mp c, 3/2 \mp c; 0, -c, 1/4 z^2 q^2). \quad (\text{E.5})$$

The integrals involving two Bessel functions cannot always be integrated analytically. Those involving combinations of $J_n(x) Y_m(x)$ and $Y_n(x) Y_m(x)$ must be performed numerically. However, the integral involving two Bessel functions of the first kind can be expressed in terms of the generalized hypergeometric function $F(a_s; b_s, z)$,

$$\int dz z^{1 \mp 2c} J_1(zq) J_2(zq) = \frac{q^3 z^{5 \mp 2c}}{16(5 \mp 2c)} F(5/2, 5/2 \mp c; 3, 4, 7/2 \mp c; -z^2 q^2). \quad (\text{E.6})$$

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