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Full Name of Author — Nom complet de l'auteur

MORTEZA NIKTASH

Date of Birth — Date de naissance

23, JAN, 1952

Country of Birth — Lieu de naissance

IRAN

Permanent Address — Résidence fixe

170 BLENHEIM DRIVE,
ROCKCLIFFE, OTTAWA, ONT., CANADA
K1L 5B5

Title of Thesis — Titre de la thèse

PERFORMANCE EVALUATION OF TWO-HOP SYSTEMS
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University — Université

CARLETON UNIVERSITY

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1985

Name of Supervisor — Nom du directeur de thèse

DEAN J.S. RORDON; PROFESSOR S. MAHMOUD

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Morteza Niktash

PERFORMANCE EVALUATION OF TWO-HOP SYSTEMS

IN LOCAL AREA NETWORKS

by

Morteza Niktash, B.Eng., M.Sc.(Applied)

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy

Department of Systems and Computer Engineering

Carleton University

Ottawa, Ontario

April 1985

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The undersigned recommend to the Faculty of Graduate
Studies and Research acceptance of the thesis:

**"PERFORMANCE EVALUATION OF TWO-HOP SYSTEMS
IN LOCAL AREA NETWORKS"**

submitted by Morteza Niktash in partial fulfilment of the
requirements for the degree of Doctor of Philosophy.

J. T. Jordan
.....
Thesis Co-Supervisor

A. M. Mahmoud
.....
Thesis Co-Supervisor

Jeremiah Hays
.....
External Examiner

B. Pagnier
.....
Chairman,
Department of Systems and
Computer Engineering

Carleton University

June 1985

ABSTRACT

Two-Hop Local Area Networks (LAN) extend data transmission services over larger geographic areas than those covered by single hop LANs. In such networks, the traffic generated by user terminals is relayed first to intermediate stations (controllers) each of which acts as a store-and-forward node and directs the collected traffic to the central station. Examples of such networks are two-hop centralized packet radio and two-hop CATV systems. Analysis of the two-hop LAN models is complicated, particularly when a sophisticated channel access protocol is used and the buffer capacity of each controller is greater than one message. In such cases, the number of system states increases rapidly as the system size grows.

This thesis presents analytical techniques for the evaluation of the performance of a number of CSMA_CD/CSMA_CD two-hop configurations. The approach presented is based on decoupling the outer loops from the inner loops and analyze them separately, taking into account their interdependence. Two major contributions are achieved. The first is the development of an analytical approach for examining the impact of the performance of the inner loop on that of the outer loops. As a second contribution, we have considered a number of two-hop centralized packet radio networks and a two-hop

CATV network, in which the outer-loops and the inner-loop employ the CSMA_CD protocol [74]. Numerical as well as simulation results are presented to illustrate the performance of these networks in terms of throughput and average delay. For controller buffer capacity greater than one message, results of the analytical model are shown to agree closely with the results of associated simulation runs.

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GLOSSARY OF NOTATION

a	the ratio of the propagation delay over the average packet transmission time for the inner-loop, i.e., τ/T
a_j	the ratio of the propagation delay over the average packet transmission time in the j th outer-loop, i.e., τ_j/b_1 .
A	a matrix with dimensions $(2k \times 2k)$.
$A_{i,j}$	a submatrix of the matrix A with dimensions $(k \times k)$.
A'	a matrix with dimensions $(2k \times 2k)$.
$A'_{i,j}$	a submatrix of the matrix A' .
A_1	$A_1 = A_{1,1} + A_{1,2}$.
A'_1	$A'_1 = A'_{1,1} + A'_{1,2}$.
b_1	the first moment of the distribution $\beta(y)$.
b_2	the second-moment of the distribution $\beta(y)$.
B_j	the blocking probability for the j th controller.
B	the blocking probability for the system.
C	the channel access control state transition matrix.
$C^*(s)$	the Laplace transform of the collision period in an outer-loop.
CR	the ratio of the speed of the inner-loop's channel to that of the outer-loop.
d_j	the mean packet delay for the j th outer-loop.
d	the mean packet delay for the inner-loop.
D	the mean packet delay for the network.
$E_a(k)$	number of $t_a^{(e+k)}$ epochs in a long period of time.

$E_l(k)$	number of $t_l(e+k)$ epochs in a long period of time.
$E_u(k)$	number of $t_u(e+k)$ epochs in a long period of time.
$G_j(z)$	the z transform of the distribution of the run time for a gambler with initial capital of x_j .
$h(\theta)$	probability distribution of the time period during which a controller's buffer is full.
h_1	first moment of $h(\theta)$.
h_2	second moment of $h(\theta)$.
$H^*(s)$	Laplace transform of $h(\theta)$.
$I(e+k)$	a random time period which starts at the end of a successful transmission time period and terminates when there are one or more user terminals of an outer-loop (or one or more controllers in the inner-loop) become ready.
l_j	in the protocol PC2, j th controller transmits l_j packets out of those resident in its buffer.
L_j	buffer capacity for the j th controller.
L_{MAX}	$L_{MAX} \geq L_j$ for $j=1,2,\dots,N$.
m_1	a macrostate.
m_2	a macrostate.
M	the cardinality of the state space.
M'	$M' = M/N$.
M_1	number of states in m_1 .
M_2	number of states in m_2 .
$n(t)$	total number of packets in the system at time t .
$n^*(x)$	total number of nonzero elements of state vector x .
N	number of controllers in the system.

N_u	total number of terminal users in the system.
P_0	probability that no arrival occurs during the time period α .
$P(k, j, \alpha)$	probability that k new packets arrived in the j th element of the state vector during time period α .
$P^{(j)}(n^*(x))$	the probability that a randomly chosen ready controller has only j packets in its buffer, when there are $n^*(x)$ ready controllers in the inner-loop.
$\text{Prob}(k, j, \alpha)$	transition probability associated with the j th element of state vectors x and z (i.e., $k = z_j - x_j$) for time period α .
P_1	the state transition probability matrix during a successful transmission time period.
P_2	the state transition probability matrix for the idle period followed by the Channel Assignment time period.
P_k	the one embedded step state transition probability matrix for the semi-Markov chain identified by the time points t_{e+k} $k=0, 1, 2, \dots$
P_u	the one embedded step state transition matrix for the semi-Markov chain identified by the time point $t_u^{(e+k)}$ $k=1, 2, \dots$
$[P^{(I)}]_0$	the vector state transition probability for the last time slot of an idle time period.
$q(z)$	the z transform of the number of the ready user terminals in an outer-loop.
q_i	the i th moment of $q(z)$.
$q_1(j)$	first moment of the queue length of the j th controller in the inner-loop.
$Q^{(\alpha)}$	state transition probability matrix between the start and the end of the time period α .
$[Q^{(\alpha)}]_{x,z}$	the (x, z) th element of the matrix $Q^{(\alpha)}$.
$[Q^{(\alpha)}]_x$	x th row of the matrix $Q^{(\alpha)}$.

$Q(c, r)$	the state transition matrix for the collision time period.
$[Q]^k$	the matrix resulted after multiplying the matrix Q with itself k times.
$r(e+k)$	time duration of a collision period given that a collision occurred.
r_1	the first moment of the random variable, $r(e+k)$, $k=0,1,2,\dots$
r_2	the second moment of the random variable $r(e+k)$, $k=0,1,2,\dots$
r_3	the third moment of the random variable $r(e+k)$, $k=0,1,2,\dots$
$R(z)$	the z transform of the distribution of the collision time period in the inner-loop.
$R^*(s)$	the Laplace transform of the distribution of the collision time period in the inner-loop.
S	the throughput of the inner-loop.
S_j	the throughput of the j th outer-loop.
$t_a(k)$	point of increase in the snapshot of the inner-loop channel; $k=0,1,\dots$
$t_l(e+k)$	the embedded points at departure instants $k=0,1,\dots$
$t_u(e+k)$	the embedded points at the beginning of a successful transmission time period.
t_{id}	$t_l(e+k+1) - t_l(e+k)$,
$\hat{t}_{id}$	$\hat{t}_{id} = r + T + \tau$.
T	transmission time of a packet for the inner-loop.
$U(e+1)$	number of arrivals during $\phi(e+1)$.
v_1	the first moment of Θ_1 when $\Theta_1 > T + \tau$.
v_2	the second moment of Θ_1 when $\Theta_1 > T + \tau$.
$y(e+1)$	number of arrivals during $\gamma(e+1) + \tau$.
$w(e+1)$	number of arrivals during $\Theta(e+1)$.

$x(e+k)$	the state variable for an outer-loop at time point $t_{\ell}(e+k)$, $k=0,1,2,\dots$
$\mathbf{x}$	the state vector for the inner-loop.
$\bar{X}$	the average number of packets in the system.
x_j	j th element of the state vector $\mathbf{x}$.
x_{N+1}	the index of the transmitting controller.
X	the state space.
$y(e+k)$	duration of a packet transmission time in the inner-loop.
$\gamma(e+k)$	duration of a successful transmission time period in an outer-loop.
$\beta(y)$	probability distribution of the packet transmission time for the outer-loops.
$\beta^*(s)$	s -transform of $\beta(y)$.
$\beta_1(y)$	probability distribution of $y+\tau_j$.
$\beta_1^*(s)$	s -transform of $\beta_1(y)$.
γ_j	mean of the random variable Θ normalized over τ_j .
δ_1	mean of Θ_1 when $\Theta_1 < T+\tau$.
δ_2	the second moment of Θ_1 when $\Theta_1 < T+\tau$.
$\Delta^*(s)$	the conditional Laplace transform of the distribution of the random variable Θ_1 when $\Theta_1 < T+\tau$.
ζ_1	mean of the random variable Θ_2 .
ζ_2	second moment of the random variable Θ_2 .
θ_1	mean of the random variable t_{1d} .
θ_2	the second moment of the random variable t_{1d} .
$\phi(e+k)$	duration of time that an outer-loop channel is set busy due to the fact that the buffer of its associated controller is full.
$\phi_1(e+k)$	$\phi_1(e+k) = t_{\ell}(e+k) - t_a(e+k)$, $k=0,1,2,\dots$
$\phi_2(e+k)$	$\phi_2(e+k) = t_{\ell}(e+k+j) - t_{\ell}(e+k)$, $j,k=0,1,2,\dots$

$\hat{\theta}(e+k)$	an approximation for $\theta(e+k)$.
$\hat{\theta}_1(e+k)$	an approximation for $\theta_1(e+k)$.
$\hat{\theta}_2(e+1)$	an approximation for $\theta_2(e+k)$.
λ_{jc}	the increased traffic load as a result of collision in the j th outer-loop.
λ_e	the total input traffic rate for the two-hop system.
λ	total input traffic rate to the controllers, in a finite buffer multiqueue single server system.
λ_{je}	total input traffic rate to an outer-loop.
λ_c	the increased traffic load as a result of collision in the inner-loop.
$\Lambda^{\#}(s)$	the conditional Laplace transform of the random variable θ_1 when $\theta_1 > T+\tau$.
$\xi_a(k)$	total number of packets in the system just before an arrival occurred.
$\xi(k)$	total number of packets in the system just after the departure of a packet.
$\xi_u(k)$	total number of packets in the system just before the beginning of a successful transmission period.
$\pi(X)$	the steady state probability distribution vector for the states of the state space X for embedded points $t_l^{(e+k)}$, $k=0,1,2,\dots$
$\pi(X)$	the steady state probability that the system is found to be in the state X for embedded points $t_l^{(e+k)}$, $k=0,1,2,\dots$
$\pi_u(X)$	the steady state probability distribution vector of the states of the state space X for embedded points $t_u^{(e+k)}$, $k=0,1,2,\dots$
$\pi_u(X)$	the steady state probability that the system is found to be in the state X for embedded points $t_u^{(e+k)}$, $k=0,1,2,\dots$
$\pi_a(X)$	the steady state probability distribution vector for the states of the state space X for time points just before an increase (i.e., arrival time of packets that are not lost).

$\pi(\mathbf{x})$	the steady state probability that the system is found to be in state $\mathbf{x}$.
$\bar{\pi}(\mathbf{x})$	approximation for $\pi(\mathbf{x})$.
$\hat{\pi}(\mathbf{x})$	approximation for $\pi(\mathbf{x})$.
σ	$(1-1/N)^{N-1}$. i.e., the system throughput during collision time period.
τ	propagation delay for the inner-loop.
τ_j	propagation delay for the j th outer-loop.
$\phi(e+k)$	time from the end of a successful transmission to the beginning of the next successful transmission.
$\psi(s)$	the Laplace transform of the distribution of the random variable $t_{1d} = r+T+\tau$.
$\Omega(s)$	the conditional Laplace transform of the distribution of the random variable Θ_2 .

CHAPTER 1

INTRODUCTION

1.1 Motivation

The significant increase in the number of independent computer systems used for business, scientific computing, process control, word processing and other office automation applications, coupled with the growing demand for local collection and dissemination of information, has motivated the need for Local Area Networks (LANs) to interconnect such systems [6,12,19,27,30,38,39,43,46-51,61,62,92,104,111,113,115,131].

Recently, automated office applications have generated the requirement for certain communication services. These services can be classified according to the traffic characteristics of the applications, e.g., data, voice and video communication services. Data services are further classified into two types: terminal-to-computer or vice versa and computer-to-computer data services.

Telephony and the associated twisted pair communication medium have been used where the main requirement is that of voice communications. To satisfy such requirements in a local environment, a class of LANs based on PABX devices has been designed for voice

communications [31,39,72].

For users with high volumes of data traffic and relatively low volumes of voice traffic, a class of LANs that are primarily designed for data communications has been developed with the potential of accommodating future voice communication requirements (e.g., ETHERNET [52]). Similarly, video and broadband transmission systems were designed to meet the requirement of video traffic [77,126].

LAN technology for support of the three communication services has been growing rapidly in the past decade. Cost effectiveness in meeting current and future office automation requirements dictate that these separately growing technologies should be merged into a single one, if for no other reason than to save wiring costs. The explosive growth of the local area communication services market was anticipated and several existing products were enhanced to meet new transmission requirements. For example, systems that were originally designed for voice communication services (e.g., PABX) are now being enhanced to satisfy data communication services (e.g., CBXs [105]). Likewise, systems that were designed exclusively for data communication services (e.g., ETHERNET [94]) are being extended and provided with the capability of transmitting digitized voice traffic [52]. As well, systems that were originally designed for video communication services, such as cable television networks, are enhanced to provide data and voice services (e.g., Broadband LANs) [31,39,72].

1.2 Definition of LANs

It is important to distinguish a Local-Area Network (LAN) from a Computer Bus and a Long Haul Network as it will reflect the differences among access schemes used for them [6,12,46-51]. The basic differences between a LAN and a Computer Bus are:

(1) Reliability issues surrounding a computer bus are somewhat different from those involved in a LAN. Except in fault tolerant systems, it is usually not critical that a computer bus continue to work if one of the devices attached to it has failed; it is usually acceptable to have the system halt momentarily until the failing device can be manually disconnected from the bus. In contrast, it is presumed that a LAN will continue to operate despite arbitrary failures of one or more nodes.

(2) Protocols that control communication on a LAN are often designed with the intention that messages can be exchanged between a LAN and a Long-Haul Network, an idea that is usually missing from the addressing and control structure of a computer bus.

(3) A computer bus often has a specialized interface, oriented towards the addressing, control structure and architecture of a computer. A LAN on the other hand, usually attempts to provide an interface equally suitable for a wide variety of nodes e.g., computers and other devices.

(4) A computer bus is often highly parallel with separate control, data and address lines; however, networks tend to

carry this information serially over a single set of lines. On the other hand, the idea of a computer bus which is completely serial is very attractive in the design of microprocessor systems, both to reduce component pinout and to eliminate problems of skew on parallel lines. Although parallel bus interfaces can accommodate very high transfer rates, they are not satisfactory for distances which exceed 200m or so because of differences in signal propagation rates and distances.

On the other hand the basic differences between LANs and long-haul networks are:

- (1) Topology: Long-haul networks often have unconstrained topology whereas LANs often involve ring or bus or star topologies.
- (2) Speed: LANs are high-data-rate networks in which the cost of transmission is very low compared to the costs associated with traditional data communication networks, providing some unique opportunities which conventional long-haul nets do not afford.
- (3) Distance: LANs typically span distances ranging from meters to tens of kilometers. Long-haul networks span distances ranging from meters to tens of thousands of kilometers (for intercontinental packet networks).
- (4) Protocols: The high bandwidth of LANs simplify some aspects of communication protocols: it simplifies schemes for allocation of network bandwidth, flow control and error detection and correction. Also long-haul networks

impose some constraints on certain modes of communications which can be accommodated easily in LANs, such as broadcast communications.

1.2.1 Single-Hop LANs

In single-hop LANs, the traffic generated by user terminals is relayed directly to the destination. No controller or central station is used.

The main characteristics that determine the operation of a LAN are: topology, transmission media, transmission mode and access protocols.

(1) Topology: Several LAN topologies have been developed and are commonly used: bus topology, e.g., ETHERNET [94]; ring topology, e.g., Pronet [114]; star topology, e.g., PABXs [2]; tree topology, e.g., CBXs [105], two-way buses implemented by CATV technology [150] etc.

(2) Transmission Media: The transmission media can be categorized into two classes: cable and free space. In the first category, user terminals are connected through a conductor. These conductors are of the following types: fiber optics, cable, twisted pair, telephone lines or power lines. Free space transmission media is used for radio broadcast and microwave communication systems [111].

(3) Transmission Mode: The transmission mode is categorized into two classes: baseband and broadband. In baseband, digital signals generated by each user is transmitted over a cable directly and modulation is not enforced. In broadband transmission mode, digital signals

generated by each user terminal are modulated and upconverted to higher frequencies and then transmitted over the cable [31,39,72].

(4) Access Protocols: A number of LAN systems which allow communications among computer systems and user terminals through a common channel are currently available commercially. Access to such common channels can be generally classified into two categories: Deterministic Access schemes (e.g., polling, TDMA, etc.) and Random Access schemes (e.g. ALOHA, CSMA, CSMA_CD, etc.).

A number of single-hop LAN's are shown in Figure 1.1(a) where all user terminals communicate with a central station which is the destination for all user terminals.

1.2.2 Two-Hop LANs

Two-hop LANs are employed to extend LAN services over larger geographic areas, where the traffic generated by user terminals is relayed first to intermediate stations (controllers) each of which acts as a store-and-forward node and directs the collected traffic to the destination [108,136,137].

Figure 1.1(a) and 1.1(b) depict examples of single-hop and two-hop systems respectively. Note that the single hop topologies (shown in Figure 1.1(a)) may be used for a two-hop system in which destinations are user terminals as well.

B

STAR

RING

Figure 1.1(a). Single Hop LAN Configurations
(Bus, Star and Ring topologies).

○ User Terminals

○ Controllers

Figure 1.1(b). Two Hop LAN Configurations

- User Terminals
- Controllers
- Central Station

Before we proceed further, we shall define the following terms:

(D1) Loop^{*}: A loop is composed of four major components:

i) A subset of user terminals or controllers that originate data packets called source-stations, each with finite buffer capacity.

ii) A single computer that receives packets originating from source-stations called a destination-station.

iii) A transmission medium connecting these stations.

iv) A channel access scheme.

(D2) Inner-Loop: A loop is called an inner-loop if the source stations are controllers and the destination station is a central station.

(D3) Outer-Loop^{**}: A loop is called an outer-loop if the source stations are user terminals and the destination station is a controller.

Note that every loop incorporates its own access scheme and transmission^{*} medium and these two distinguishing features of a loop are not necessarily the same for all loops in a two-hop system.

(D4) Stability: A loop is stable if arrivals that enter

* The terms LOOP and HOP are interchangeable throughout this thesis.

** The above definitions could easily be extended for multi-hop systems composed of a hierarchy of loops. In this case an intermediate-loop could be defined as an outer-loop in which the source stations as well as destination-station are controllers.

the loop depart from the loop after encountering a finite delay. A two-hop system is stable if and only if all loops in the system are stable.

(D5) Infinite buffer two-hop system: In such a system, each controller has infinite buffer capacity. Therefore the performance of the inner-loop does not have any impact on the performance of the outer-loops, and vice versa, under any input traffic load.

(D6) Finite buffer two-hop system: In such systems, controllers have finite buffer capacity. Consequently, the performance of the inner-loop is dependent on the performance of the outer-loops, and vice versa, i.e., the performance of the outer-loops is dependent on that of the inner-loop. Note that in such systems, the performance impact becomes insignificant as the average input traffic rate decreases and/or, as controller's buffer capacity increases. Arrivals, that enter a controller and find its buffer full are not lost. They attempt entry into the system at a future time, as a result of the invocation of some flow control procedure. Clearly in such systems, the inner-loop may never become unstable (as it forms a system with a finite buffer). Therefore as the average input traffic rate increases, the inner-loop remains stable whereas the outer-loops and subsequently the system become unstable.

(D7) Flow control mechanism: In finite buffer two-hop systems, a flow control mechanism is necessary as each

controller has a limited buffer size. In the two-hop system considered in this thesis, the flow control mechanism is implemented in a rather different manner. When a controller's buffer becomes nearly full, the controller sets the associated outer-loop channel busy until such time that it transmits a packet successfully and then it resets the channel idle. The invocation of such a mechanism results in the dependency between the inner-loop and the outer-loops.

1.3 Examples of Two-Hop Systems

1.3.1 Two-Hop Centralized Packet Radio Networks

A number of recent studies have focussed on the design and analysis of two-hop centralized packet radio networks.

A two-hop configuration involving a ring of repeaters around a central station was studied by Gitman [41]. In this network, traffic is generated by a set of terminals and relayed to a central station through a set of repeaters. The transmission range of terminals is short so that a terminal can reach only one repeater. The system has two channels, one for transmission from terminals to the central station, i.e., the uplink channel, and the other for the reverse traffic, i.e., the downlink channel. The access scheme for the uplink channel, studied by Gitman, is based on the slotted-ALOHA random access scheme where all devices (terminals as well as repeaters) compete for access to the uplink channel. Throughput results are

studied (as function of a number of repeaters) but packet delay was not considered.

A two-hop centralized packet radio network was studied by Tobagi [136]. In this network, traffic is generated by a set of terminals and is relayed to a central station through a set of store-and-forward repeaters; each repeater serves a cluster of terminals. Different cases were considered in which repeaters have finite capacity and the channel access schemes were the slotted-ALOHA [65] and the nonpersistent random access strategies [66,141-143]. In [136], the slotted-ALOHA access scheme was considered for two configurations: (1) star configuration as shown in Figure 1.2(a), in which each repeater is in line-of-sight and within the range of the central station only; (2) fully connected (FC) configuration as shown in Figure 1.2(b) in which all repeaters are within range and in line-of-sight of each other and of the station. Two protocols for repeaters are considered: (a) the Delayed-First-Transmission (DFT) Protocol in which a repeater transmits the head of its queue, in a time-slot with probability p ; (b) the Immediate-First-Transmission (IFT) Protocol in which a repeater transmits the head of its queue immediately (with probability one). A packet successfully transmitted by a given population of terminals can be "blocked" at the immediate destination repeater. Blocking is due to two factors: (1) the repeater (or any other repeater in FC

Figure 1.2(a). A Two Hop Star Configuration [136].

Figure 1.2(b). A Two Hop Fully Connected Configuration (136) ..

configuration) is in a transmit mode or (2) the repeater's receiver is on (and in the FC configuration, all other repeaters are quiet), but the repeater's buffer is full. The delay-throughput analysis for both protocols DFT and FC are studied in [136]. In [137], the nonpersistent CSMA access scheme was considered for the FC configuration. The transmission protocol is analogous to the IFT protocol considered in [136]. The delay-throughput analysis was performed for a single buffer repeater only.

A two-hop LAN for packet mobile radio data networks was studied by Riordon et al [108]. A number of access strategies was developed and evaluated for a set of channels over a two-hop path. The emphasis was on voice transmission. In that model, operating in the 800MHz band, users cannot rely on line-of-sight transmission from sender to receiver; all calls are therefore relayed by a base station. It is assumed that N_u uplink channels and N_d downlink channels are available. If N_u is large, these channels may be divided into groups of channels, with N_g channels per group. Users may either be active or inactive; an active user is one who has established communication (i.e., voice call) with another user. An inactive user achieves active status through a request to the call establishment protocol. Once a call is established, an uplink channel is assigned as required for the duration of each talkspurt. Talkspurt voice signals are digitized at 16 kbps and formed into packets. These are placed in buffers at the transmitter and then sent to

the base station. There the packets may be queued again if it is necessary to assign a downlink channel; if the latter is dedicated, of course, no queueing delay need be incurred. The probability of channel blocking has been calculated for different channel groups and various arrival traffic levels. Downlink blocking probabilities were also evaluated.

Recently, there have been a number of attempts made towards the development of analytical techniques for the evaluation of performance of packet radio networks with a multihop nature and general topology [10,11,109]. In [10,11], only throughput results are considered and throughput-delay tradeoffs were not provided. Ray et al [108] have described the nonpersistent carrier sense multiple access with busy-tone and collision detection (CSMA/BT_CD) protocol in a multihop environment. They have developed a continuous time Markov chain model of CSMA/BT_CD for a partially connected five node ring network, each node having a buffer capacity of unity. However, when the buffer capacity is greater than unity, their solution is intractable due to a large increase in the states of the network and therefore their model cannot provide results for such cases.

1.3.2 A Two-Hop Configuration for CATV Systems

A schematic diagram of a two-way CATV tree is shown in Figure 1.3(a). The tree consists of the Central Station (or Head-End), Main Trunk, Trunk Amplifiers, Distribution Branches, User Taps and User Terminals. These components are interconnected and form a 3-level cable hierarchy.

The inbound traffic originated from a user-tap and transmitted through the inbound channel goes to the central station. The central station then broadcasts the received traffic through the outbound channel to all user taps as shown in Figure 1.3(b).

In order to increase the reliability of a two-way CATV system, a degree of distributed control must be incorporated in the system. This can be realized by controlling the traffic from each distribution branch using control units residing at the trunk amplifiers. The idea is to divide the user terminals into subgroups which will essentially operate independently from each other. The terminals within each group communicate with their respective controllers. They time share the upstream channel to the controller and receive acknowledgement and information packets from it. The controllers in this configuration act as intermediate nodes. They collect packets from their respective terminals and empty them on the main trunk and vice versa. In order to achieve full control over the distribution branches and at the same time maximize the throughput, the controllers must be

Figure 1.3(a). Schematic Diagram of a CATV Tree.

HE = Head End (Central
Station)

MT = Main Trunk

TA = Trunk Amplifier

DB = Distribution Branch

FM = Feeder Multitap
(Power Splitter/
Combiner)

DA = Distribution Branch
Amplifier

UT = User Tap

O = User Terminal

Figure 1.3(b). Inbound and Outbound Channels in a CATV Tree.

HE = Head End
 (Central Station)
 MT = Main Trunk
 TA = Trunk Amplifier
 DB = Distribution Branch
 FM = Feeder Multitap
 (Power Splitter/
 Combiner)
 DA = Distribution Branch
 Amplifier
 UT = User Tap
 O = User Terminal

capable of processing store-and-forward functions and apply flow-control mechanisms. The controller will also be able to detect transmission errors midway between the terminals and the central station, reject damaged packets and detect a malfunctioning terminal (screamer).

By placing an active controller unit at each trunk amplifier, the system will in effect, be transformed into a two-hop configuration. The outer-loop operates between subgroups of terminals and their respective controllers and the inner-loop operates between the controllers and the central station. This configuration is depicted in Figure 1.3(c). This arrangement has two important advantages: (1) it provides a higher degree of protection against a failed terminal (screamer detection) and (2) it provides a shorter propagation delay within each subgroup, hence improving the performance of a CSMA type protocol employed as an access scheme for either the inner-loop or the outer-loops.

The two-hop configuration described above provides the system designer with greater flexibility and more options of access scheme combinations than that offered by an unmodified tree structure CATV system.

Figure 1.3(c). CATV Tree with Controllers.

HE = Head End
 (Central Station)
 MT = Main Trunk
 CN = Controller
 DB = Distribution Branch
 FM = Feeder Multitap
 (Power Splitter/
 Combiner)
 DA = Distribution Branch
 Amplifier
 UT = User Tap
 ○ = User Terminal

1.4 Scope and Objectives

Modelling and analysis of two-hop LANs become increasingly complex when the following factors are taken into account:

- (1) The finite buffer two-hop system is employed as opposed to the infinite buffer two-hop system.
- (2) The transmission protocol of the loops becomes more sophisticated, (e.g., CSMA, CSMA_CD).
- (3) The buffer capacity of each controller is greater than one. In this case state definition requires special care, because the number of states increases rapidly for certain state definitions and as the system grows in size.

The impact of the performance of the loops on each other, as a result of finite buffer capacity controllers, is the principal source of difficulty in the analysis of two-hop LANs. As far as the impact of the complexity of the protocols is concerned, it was mentioned in the previous section that the analysis for the slotted-ALOHA protocol was previously conducted for systems with small buffer size and small numbers of repeaters (i.e., buffers sizes of 1, 2, 3, 4, 5 and number of repeaters equal to 2). Due to the state definition employed, the approach does not provide tractable solutions for systems with a modest number of controllers (say 10 controllers) which is the case in practical CATV and packet radio systems. For the CSMA transmission protocol, the approach developed previously in [136] cannot be applied to analyze a finite buffer two-hop system in which the buffer capacity of each

controller is greater than unity.

In summary, the approaches for the analysis of two-hop systems introduced previously in the literature [10,11,109,136,137] are not useful in analysing practical systems since they can be used only in special cases such as when the controllers have a buffer size equal to one packet and the CSMA protocol is used.

The objective of this research is to develop analytic models for the evaluation of the performance of a number of two-hop system configurations with controller buffer capacity greater than unity. Two examples of systems which can benefit from the results of this research are:

- (1) Finite buffer two-hop centralized packet radio networks.
- (2) Finite buffer two-hop CATV systems.

1.5 Dissertation Structure

The remaining chapters of this dissertation are organized as follows:

In Chapter 2, we consider two-hop LANs with finite buffer controllers in which the outer-loops and the inner-loops employ the CSMA_CD protocol [74]. The chapter presents an approach for the analysis of two-hop LANs, which is based on decoupling the inner-loop from outer-loops and analyzing them separately, taking into account the interdependency of the loops.

Chapter 3 addresses the development of analytic

models for a number of channel access schemes with finite user buffers. The first segment of the chapter briefly surveys the shortcomings and limitations of the existing work on such problems. The following sections of this chapter presents a generalized model (G-model) for analysis of a number of deterministic as well as random access schemes. A number of random access schemes, based on the CSMA_CD [74], are presented. The G-model is employed in order to develop analytic models for the channel access schemes. New analytical as well as simulation results are presented for the channel access schemes.

In Chapter 4, an analytic model is presented for the analysis of an outer-loop based on the approach given in [74]. The model takes into account the effect of the performance of the inner-loop on that of the outer-loop.

In Chapter 5, the effect of the inner-loop on the outer-loop is addressed. As a result of the flow control mechanism explained earlier, the outer-loop channel is not available for transmission during the time period in which the associated controller's buffer is full (i.e., θ). Derivation of the law of this random time period (i.e., θ) is vital to the analysis of the outer-loop as it represents the effect of the inner-loop on the outer-loop. Using the analytic models presented in Chapter 3, Chapter 5 presents an approach to the specification of the closed form mathematical formula, under simplifying assumptions, for the law of the random time period (i.e., θ) during

which the buffer of a controller is full. This distribution is employed in the analytic model (developed for analysis of outer-loops and) given in Chapter 4.

In Chapter 6 we demonstrate the application of the analytic approaches, presented in this thesis, to the analysis of a number of two-hop LANs configurations. In this chapter, analytic as well as simulation results are presented for a two-hop centralized packet radio network and a two-hop CATV network. For controller's buffer capacity greater than one packet, results of the analytical model are shown to agree closely with the results of associated simulation runs.

Chapter 7, presents a summary of the main contribution of the dissertation along with recommendations for further research. Analytical formulations have been relegated to the appendices.

1.6 Contributions

The main contributions of this research are summarized in the following:

Chapter 2:

An approach to modelling and analysis of the two-hop LANs with finite buffer controllers is presented. This approach is based on decoupling the outer-loops from the inner-loop, which permits the analysis to be conducted for each loop separately. The model introduces a flow control mechanism in which the outer-loop channel is declared busy

when the controller's buffer is full. This flow control mechanism is essential in practice in order to prevent the system from falling into a congested (unstable) state.

Chapter 3:

(A) A generalized mathematical model is introduced for the analysis of the inner-loop. The model is based on a number of simplifying assumptions and alternative state definitions, and can be employed in the analysis of a number of random as well as deterministic access schemes with buffered users.

(B) A number of CSMA_CD protocols are presented for the inner-loop which are based on the CMSA_CD protocol given by Lam [74]. The distinguishing characteristics of these protocols are finite population of controllers and finite controller buffer capacity (greater than one packet). Furthermore, each controller, once it acquired the channel, follows a set of rules which allow the transmission of one packet or multiple number of packets.

(C) An analytic model (i.e., the small system size model) is presented for the analysis of the CSMA_CD protocols with a small number of network nodes (i.e., number of network nodes less than 5) and finite buffer capacity.

(D) Numerical as well as simulation results are presented for the analysis of the CSMA_CD protocols with small number of network nodes using the small size system model.

(E) An analytic model (i.e., the large system size model) is presented for the analysis of the CSMA_CD protocols with a large number of network nodes (i.e., number of network nodes is greater than or equal to 10) and finite buffer capacity.

(F) Numerical as well as simulation results are presented for the analysis of the CSMA_CD protocols with a large number of network nodes using the large size system model.

Chapter 4:

Following the approach presented by Lam [74], a mathematical model for the outer-loops is given in which user terminals employ the CSMA_CD protocol presented in [74]. This model takes into account the effect of the flow-control employed in the two-hop system.

Chapter 5:

An approach is presented for mathematical modelling of the impact of the performance of the inner-loop on that of the outer-loops. Results obtained from this approach are employed in the analytical model developed for the outer-loops.

Chapter 6:

(A) We have considered a number of CSMA_CD/CSMA_CD two-hop LANs in which the inner-loop employs the CSMA_CD protocols of Chapter 3 and the outer-loops employ the CSMA_CD protocol given by Lam [74]. Analytical as well as simulation results have been presented for the performance

behaviour of a CSMA_CD/CSMA_CD two-hop centralized packet radio network.

(B) Analytical as well as simulation results are presented for the performance behaviour of a CSMA_CD/CSMA_CD two-hop CATV network.

The approach presented in this thesis could be employed as a tool for design of the two-hop centralized networks. The designer of a two-hop centralized system is often faced with a number of design constraints that are required to be maintained by the values of various system parameters (e.g., number of controllers, maximum input traffic level, average delay, etc.). The analytic models presented in this thesis permits the designer to study the dynamic behaviour of the system for various values of system parameters (e.g., controller's buffer capacity, throughput, etc.) and select appropriate values of system parameters (e.g., controller's buffer capacity) in order to maintain the design constraints (e.g., an upperbound for average system delay when operating at a given input traffic level).

CHAPTER 2

PROBLEM DEFINITION AND APPROACH

2.1 Problem Definition

We consider a two-hop LAN with finite buffer controllers. Examples of such systems are: two-hop centralized packet radio networks and two-hop CATV networks. Each controller receives traffic from its outer-loop channel and transmits data over the inner-loop channel to the central station. A controller, in addition to performing the functions required to implement the inner-loop protocol, exercises data flow control over the incoming outer-loop connected to it. Data flow control is applied by sending a busy-tone signal on the outer-loop channel when the controller's buffer is full. Once the busy tone is detected the terminals refrain from transmitting any data packets to the controllers.

The central station (e.g., CATV head end) broadcasts the incoming traffic over a separate channel, called the outbound channel, to all user terminals. Our main interest in this thesis is to study the inbound traffic (from the terminals to the controllers and then from the controllers to the central station).

Access schemes to the inbound channel can be classified into two categories: deterministic access schemes and random access schemes.

In deterministic access schemes, each participating user terminal will transmit its packet either at the onset of a clock signal (as in TDMA) or upon receiving a polling signal from either a central controller (as in the case of centralized polling) or a neighbouring terminal which is releasing the channel itself (as in decentralized polling, e.g., token passing ring).

In random access schemes, the decision to transmit a packet rests with the generating terminal which executes a set of rules once it has a packet to transmit (which can happen randomly in time). These rules may involve sensing the status of the channel prior to transmitting the packet (as in Carrier Sense Multiple Access or CSMA), listening to the channel while transmitting to detect if the packet being transmitted is interfered with. Appropriate action such as backing off and halting further bit transmission will then be taken in the event that a packet error is detected. The packet error may be due to collisions with other packets or due to noise on the inbound channel. The latter access strategy is defined as CSMA with Collision Detection (CSMA_CD).

Figure 2.1 presents a model for the CSMA_CD/CSMA_CD two-hop system configurations. In this model, all the outer-loops and the inner-loops employ the CSMA_CD protocol [74]. This model will serve as the basis for the analysis conducted in this thesis. The main objective of the analysis is to determine the performance of the two-

hop configuration expressed in terms of the overall system stability and the delay-throughput characteristics. Our analysis will be restricted to the equilibrium behaviour of the system. Thus transient and overload situations are considered beyond the scope of this thesis.

As shown in Figure 2.1, the source of traffic to the broadcast channel of the j th outer-loop consists of:

- (i) An infinite population of users in each outer loop which collectively forms an independent Poisson process with an aggregate mean message generation rate of λ_j messages per second.
- (ii) Messages that have encountered collision and are rescheduled for transmission after a random time period. The mean message rearrival rate due to collision is denoted by λ_{jc} .
- (iii) Controllers do not perform assembly or disassembly of messages, i.e., the length of messages transmitted over outer-loops is equal to that transmitted over the inner-loop.

We define the random variable θ as the time period during which the controller's buffer remains full. As a result of the flow control mechanism explained earlier, the outer-loop channel is not available for transmission during the time period θ . The outer-loop becomes available again as soon as the controller succeeds in transmitting one or more packets on the inner-loop. Derivation of the probability distribution function of θ

Figure 2.1.1. A CSMA_CD/CSMA_CD Two Hop Configuration.

is vital for the analysis conducted to determine the interdependence between the inner-loop and the outer-loops.

2.2 Approach

The enforcement of a flow control mechanism in the presence of a finite controller buffer gives rise to the interdependence between the inner-loop and each of the outer-loops. When the inner-loop channel is congested and a controller buffer becomes full, the outer-loop channel will be declared busy and hence its associated terminals are blocked. Thus analysis of the outer loop will depend on knowledge of the pdf of the random variable θ for a given inner loop operating point. Similarly, analysis of the inner-loop will require knowledge of the departure process of the packets from the outer-loop. Analysis of the two-hop system will then require an iterative process in which we start by decoupling the loops, analysing them separately and then using the results of the analysis of one loop to feed into the analysis of the other loop. Analysis of the complete system will converge when the total aggregate input traffic into the outer-loops is equal to the output traffic of the inner-loop.

Consider the j th controller of the inner-loop depicted Figure 2.1. The arrival process into this controller is dependent on the contents of the controller's buffer. This is due to the fact that no arrival may enter during the time period in which the

buffer of this controller is full. Furthermore, the arrival process into this controller is also dependent on the contents of the other controller's buffer. This is because the time period in which the buffer of this controller is full increases when two or more controllers are waiting to transmit messages. Clearly this time period is shorter when there are no controllers waiting to transmit messages. Therefore the development of an exact model for the inner-loop is a cumbersome task. In this thesis, an approximate model is presented for the analysis of the inner-loop.

The performance behaviour of the inner-loop is approximated by the performance behaviour of the loss system shown in Figure 2.2. It is important to distinguish between packets that are lost in the model of Figure 2.2 and those packets that are blocked in an outer-loop, due to buffer overrun of a controller, waiting for the busy signal to stop. In the former case, a number of packets find the buffer of a controller full and are lost. These packets are not involved when approximating the model of the inner-loop by the model of Figure 2.2.

Consider the model given in Figure 2.2. The source of input traffic to the j th controller forms a Poisson process with mean arrival rate λ_j . Clearly, a number of arrivals find the buffer of the controller full and are lost. Let B_j represent the blocking probability for the j th controller. Arrivals that enter the j th controller

Figure 2.2.2. A Model for the Inner-loop.

form an input traffic process with mean arrival rate $\lambda_{je} = \lambda_j (1 - B_j)$. Note that λ_{je} is the mean message departure rate of the outer-loop. This arrival process to the j th controller is employed as an approximation to the arrival process of the j th controller of the inner-loop, shown in the two-hop system configuration of Figure 2.1.

A model for the outer-loop is demonstrated in Figure 2.3. Note that λ_{je} denotes the aggregate mean message generation rate and λ_{jc} denotes the mean regeneration rate of messages waiting for retransmission after encountering collision. The effect of a full controller buffer is taken into account by declaring the outer-loop channel busy during the time interval Θ . The pdf of Θ is obtained from the analysis of the inner-loop.

In summary, the approach presented is based on the following three steps:

- (1) The first step involves the development of a mathematical model for the analysis of a Multiqueue Finite Buffer Single Server System which represents the inner-loop. Using this model, the equilibrium results for the inner-loop (Figure 2.2) are approximated. Next, the distribution of the time period Θ for which a controller's buffer remain full is developed. This distribution is required in the analysis of each outer-loop (Figure 2.3).
- (2) In the second step, equilibrium results for an outer-loop are established through the analysis of the model based on Figure 2.3. The distribution of the random time period for which a controller's buffer remains full (which

Figure 2.3. A Model for the j th Outer-loop.

is established in step 1) is explicitly incorporated in the analysis of the outer-loop and, as mentioned earlier, it reflects the impact of the performance of the inner-loop on that of the outer-loop.

(3) Finally, in step 3 we establish an approximation for the equilibrium results for the two-hop system using the results of the analysis of the first two steps. The analytical results are verified through the use of a simulation model for the complete system.

The first step is addressed in Chapter 3. In this chapter, a generalized model (G-model) is presented for the analysis of a number of random as well as deterministic access schemes. The second step is addressed in Chapter 4. In this chapter, a model is given for an outer-loop which is an extension to that presented by Lam [74]. The model incorporates the impact of the inner-loop on that of the outer-loop. This is accomplished by incorporating in the outer-loop model the distribution of the time period θ for which a controller's buffer remains full. Chapter 5 presents this distribution under a number of simplifying assumptions. Finally, Chapter 6 addresses the third step, i.e., it presents numerical and simulation results for a number of two-hop system configurations.

2.3 Conclusions

In this chapter, we have considered a number of two-hop network configurations in centralized packet radio and in CATV networks. In these networks, the outer-loops employ a random access scheme (i.e., the CSMA_CD protocol given by Lam [74]) and the inner-loop may employ either either a random access scheme (i.e., the CSMA_CD with finite buffer controllers) or a deterministic access scheme (e.g., Polling [143], Reservation [33], etc.). The principle contribution of this chapter is an approach to modelling and analysis of the two-hop LANs with finite buffer controllers. This approach is based on decoupling the outer-loops from the inner-loop, which permits the analysis to be conducted for each loop separately. The model introduces a flow control mechanism in which the outer-loop channel is declared busy when the controller's buffer is full. This flow control mechanism is essential in practice in order to prevent the system from falling into a congested (unstable) state.

CHAPTER 3

INNER-LOOP ANALYSIS

The development of analytic models for buffered random access schemes is vital to the study of the behaviour of the inner-loop. The task of modelling and analysis of buffered random access schemes become increasingly complex when the following factors are taken into consideration:

(i) The transmission protocol becomes relatively more sophisticated (e.g., CSMA, CSMA_CD).

(ii) The buffer capacity of each controller is greater than one packet. In this case, state definition requires special care, because the number of states increases rapidly for certain state definitions as the system grows in size.

A number of attempts have been made to develop analytic models for buffered Random Access schemes [23,53,97,132].

These attempts tend to have the following limitations:

(i) The analytic model could only be applied to the ALOHA family of Random Access schemes [53,97,132].

(ii) The analytic model could only be applied to a very specific CSMA_CD Protocol with simplifying assumptions that do not correspond to practical system considerations [23].

In this chapter, a novel approach is presented for the

development of analytic models for a number of LAN protocols with finite buffer and finite number of stations. The model presented is applicable to analysis of a number of random as well as deterministic access schemes and can thus be considered a generalized model (G-Model).

Section 3.1 presents a survey of previously published techniques for analysis of buffered Random Access Schemes. A number of channel access schemes are described in Section 3.2. Section 3.3 presents a generalized model for analysis of the channel access schemes. New analytical as well as simulation results are presented in Section 3.4. Contributions made in this chapter are summarized in Section 3.5.

3.1 Survey

A number of attempts have been made in order to tackle the problem of buffered Random Access Schemes. In [53], Kamal and Mahmoud assumed a buffer with infinite capacity and the model presented was specifically designed for and limited to the Slotted ALOHA protocol.

In [97], Mittal and Venetsanopoulos introduced a model with two-dimensional vector $X=(x_1, x_2)$, where x_1 is the number of busy stations and x_2 is the total number of outstanding packets in the system. They assumed that the x_2 packets are equally distributed among the n busy stations. A general assumption made is that the time axis

is slotted into units equal to the packet transmission time. This limits the applicability of their analysis to ALOHA family of random access schemes such as the URN scheme.

In [132], Sykas et al has made an attempt to propose a new methodology for the queueing analysis of ALOHA Random Access schemes. Following the state definition approach chosen in [23], a two-dimensional vector $x=(x_1, x_2)$ was considered, where x_2 is the number of packets in the buffer of the selected station and x_1 is the number of busy stations among the remaining stations. Again the time axis was slotted into units equal to the packet transmission time which limits the capability of their model to the analysis of the ALOHA family of random access schemes.

In all of the above models, it was assumed that all stations have the same buffer capacity and similar input traffic process.

In [23], Coyle and Liu developed a continuous time Markov chain model for a very specific asynchronous CSMA-CD protocol. A two-dimensional vector $x=(x_1, x_2)$ is chosen where x_2 is the number of packets in the buffer of a station which is singled out arbitrarily and referred to as the isolated station and x_1 represents the number of busy stations among the remaining ones. Unlike the previous models, the isolated station has a buffer capacity of k packets whereas all other stations have buffer capacity of only one packet. Furthermore, it is

assumed that the remaining stations do not encounter buffer runaway and arrivals to those stations are not lost. This assumption is clearly quite unrealistic.

In [96], Mittal proposed a scheme called Efficient Contention Based Multiple Access Scheme. In this scheme, at the end of an ongoing transmission a subset of the total station population is given full access rights and the remaining stations are denied access completely. Upon collision, a collision resolving cycle begins. At each step in the Collision Resolving Algorithm (CRA), mutually exclusive subsets of the stations are given access rights to transmit. At each successive collision, each subset is divided into smaller subsets until the collision is resolved. A CRA is complete after every station involved in the original collision has transmitted its packet successfully. It is assumed that the unbuffered stations monitor the channel and hence can reach coordinated decisions. Analytical expressions are presented only for the maximum number of time slots wasted in resolving a collision, under heavy load conditions, simulation results are presented for the average delay versus the throughput for 8 and 16 stations. In summary, Mittal has proposed an access scheme based on tree search algorithms as well as simulation results for unbuffered stations.

In order to propose a mathematical model with applicability extending beyond the ALOHA family of random access schemes, the assumption that the time axis is

slotted into units equal to the packet transmission time must be relaxed. In the following, a new mathematical model is presented for a finite number of stations with limited buffer capacity. The model is applicable for the analysis of a number of random as well as deterministic access schemes and can thus be considered a generalized model (G_Model).

3.2 Channel Access Schemes for the Inner-Loop

The class of Random Access schemes studied in this research is based on the P_Persistent CSMA_CD Protocol studied by Lam [74] and are characterized in the following:

(i) Finite number of controllers, N , accessing the channel of the inner-loop.

(ii) The j th controller has a buffer of size L_j packets.

(iii) Arrival process to the j th controller forms a Poisson process with mean arrival rate of λ_j .

(iv) Arrivals to a full buffer are lost. Recall that the average loss rate for j th controller is $\lambda_j B_j$ where B_j is the blocking probability associated with the j th controller. Therefore the average departure rate from this controller is equal to $\lambda_{je} = (1 - B_j) \lambda_j$. It is important to note that this loss model (in which the average departure rate from the j th controller is equal to λ_{je}) is employed as an approximation to the model for the inner-loop in which the average arrival rate into the j th

controller is equal to λ_{je} .

(v) Every time that the j th controller gains access to the channel, it holds the channel for a duration of time according to a prespecified set of rules which applies to all controllers. One of the following options is usually employed:

PC1 - The controller transmits a single packet and then releases the channel; or

PC2 - The controller transmits up to a maximum of N_j packets already resident in its buffer at the time it gained channel access and then releases the channel; or

PC3 - Before releasing the channel, the controller transmits all the packets already resident in its buffer at the time it gained channel access; or

PC4 - The controller holds the channel until its buffer becomes empty, which means it transmits all the packets in its buffer plus the packets that arrive during the transmission time period of these packets.

(vi) Controllers are assumed to be synchronized in time so that following each successful transmission, the channel is slotted in time [74].

(vii) We assume here that collisions in the channel are detected and that users involved in a collision will back off, i.e., abort their transmission immediately upon detecting the collision [74].

(viii) Let τ be the time duration from the start of a transmission by one user to the instant when all users

sense the presence of this transmission. It is equal to the maximum propagation delay between two users in the network plus carrier detection time.

(ix) In order to implement the collision abort capability described above, the minimum duration of a time slot must be 2τ so that within a time slot, if a collision is detected and the collided transmissions are aborted immediately, the channel will be free of any transmission at the beginning of the next time slot [74]. It is important to distinguish between the time slot employed in the back off strategy (i.e., 2τ) and the one employed in our analysis (i.e., τ).

(x) Following a successful transmission, each ready controller transmits with probability 1 into the next time slot [74].

(xi) All controllers monitor the channel continuously and record its state. The channel can be in one of the following states:

- successful transmission state: a controller has successfully acquired the channel and transmission is in progress.
- idle state: a successful transmission has been terminated and the buffers of all controllers are empty.
- collision resolution state: the idle state has been terminated by a collision and the next successful transmission has not been started.

(xii) If a controller becomes ready and the channel is in the successful transmission state, it persists and transmits with probability one at the end of the successful transmission.

(xiii) If a controller becomes ready and the channel is in the collision resolution state; it selects a random retransmission time period using the adaptive algorithm [23,67].

(xiv) At the end of the collision resolution period, each ready controller in the system has equal chance of gaining access to the channel [23,67].

3.3 A Generalized Model for a Number of Buffered Channel Access Schemes

In this section a generalized model is presented for a number of deterministic as well as random access schemes. A mathematical model is then proposed which is based on a discrete time embedded Markov chain. The model is used in the analysis of different access schemes for cases where the input traffic from different controllers is uniform. The model can be also applied to buffered channel access schemes with non-symmetric input traffic.

3.3.1 The Generalized Model

The model presented in this Section is defined for a number of communication protocols that incorporate the following characteristics:

1. Finite number of N controllers; each controller is in one of two states: (i) ready state, i.e., it has at least one packet to transmit, and (ii) idle state, i.e., it has no packets in its buffer.
2. The j th controller has a limited buffer capacity of size L_j .
3. All users share a single communication channel in order to communicate with each other.
4. The input traffic to the j th controller forms a Poisson arrival process with arrival rate λ_j . Hence the total input traffic to the system forms a Poisson process with rate λ where:

$$\lambda = \sum_{j=1}^N \lambda_j \quad (3.1)$$

5. Arrivals that find a buffer full are lost.
6. Each protocol enforces a Channel Assignment Strategy (CAS). A CAS is the manner in which a ready controller is selected among all controllers and the channel is assigned to it. CAS could be deterministic (e.g., Reservation polling) or random (e.g., CSMA-CD). A random time period, called Channel Assignment Time, is required in order to implement CAS. The distribution of the channel assignment time may differ

from one CAS to another (e.g., sequences of reply times in polling; collision period in CSMA_CD).

7. Once a channel is assigned to a controller, it could transmit either a single packet or multiple number of packets depending on the specific protocol.

Figure 3.1 depicts a system configuration in which there are three controllers (i.e., $N=3$). Each controller has a maximum buffer capacity of three packets (i.e., $L_j=3$, $j=1,2,3$).

3.3.2 Analysis of the G-Model

We now proceed with the embedded-Markov Chain Model for the above access schemes.

Figure 3.2 depicts a snapshot of the broadcast channel for the multiple packet transmission protocols explained in the previous section. We observe on the time axis a sequence of subcycles. The time axis is slotted with time intervals of duration τ . Each subcycle is composed of a packet(s) transmission time period $y^{(e)}$, an idle period $I^{(e+1)}$ and a channel assignment time period $r^{(e+1)}$. Note that in this figure we have $\phi^{(e+1)} = I^{(e+1)} + r^{(e+1)}$. The packet transmission time period is followed by a time period of duration τ . This reflects the fact that after packet transmission is terminated by a controller, all users will sense the channel after a time period of the duration τ seconds. The state transition probability matrices between various time points are also shown. $Q^{(T)}$ represents the state transition probability

Figure 3.1 Inner Loop A System Configuration ($N=3, 1=3, 1, 2, 3$)

Figure 1. A map of the field of the field.

matrix between the start and the end of a packet transmission time period. P_1 is the state transition probability matrix between the start and the end of a successful transmission period. Finally P_2 represents the state transition probability matrix between the start of the idle time period and the end of the channel assignment time period. The random time period between t_{e+2} and t_{e+1} is represented by t_{10}^{e+1} . For the single packet transmission protocol, P_0 , t_{10}^{e+1} represents an interdeparture time period.

We consider two embedded Markov chains identified by their associated e-embedded points and u-embedded points. The e-embedded point is defined to be the start time of an idle period i.e., the time that a single or multiple departure have been successfully completed whereas an u-embedded point is identified by the start of a successful transmission period. In Figure 3.2, two successive e-embedded points are marked by t_{e+1} and t_{e+2} while two successive u-embedded points are marked by t_{u^e} and $t_{u^{e+1}}$.

The intervals of time between two consecutive embedded slots are defined as subcycles. Note that the time duration of the subcycles is a random variable.

The following definitions apply for both embedded Markov chains defined in Figure 3.2.

Consider the semi-Markov chain identified by the e-embedded or u-embedded points and let t_{e+k} and

$t_u(e+k)$, $k = 0,1,2,\dots$ denote the l -embedded and u -embedded points respectively. Let $\mathbf{x}$ denote the state vector of the system at these points. A number of state definitions are accommodated by the approach, as presented in the following. For simplicity, we consider first the following state definition: Let $\mathbf{x}(t)$, representing the state of the system at time t , be defined as follows:*

$$\mathbf{x}(t) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ x_{N+1} \end{bmatrix} \quad \begin{array}{l} 0 \leq x_j < L_j, \quad j = 1, \dots, N \\ 1 < x_{N+1} \leq N \end{array} \quad (3.2)$$

where x_j is the number of packets stored in the buffer of the j th controller, at time t , for $j=1,\dots,N$ and x_{N+1} is the index of the user who has acquired the channel access most recently. In Figure 3.1 the state of the system is shown at a time point say $t_i^{(e)}$. There are three controllers (i.e., $N=3$) and each controller has a maximum buffer capacity of three packets (i.e., $L_j=3$ for $j=1,2,3$). The first controller has a single packet in its buffer, the second controller has two packets in its buffer and the third controller has zero packets in its buffer. Assuming that the third controller was the one to which

* For simplicity and where there is no need to emphasize the parameter t , $\mathbf{x}(t)$ is also represented by $\mathbf{x}$.

the channel was assigned last, the state of the system at an embedded point, say $t=t_l^{(e)}$, is $(x_1=1, x_2=2, x_3=0, x_4=3)$:

$$x(t) = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$$

the state space is defined as:

$$X = \{x | 0 \leq x_j \leq L_j; 1 \leq j \leq N; 1 \leq x_{N+1} \leq N\} \quad (3.3)$$

the cardinality M of the state space is given by,

$$M = N \prod_{j=1}^N (L_j + 1) \quad (3.4)$$

Let P_i (or P_u) represent the state transition probability matrix between consecutive i -embedded (or u -embedded) points. P_i (or P_u) is the product of several state transition probability matrices which we define in the following.

Consider the consecutive i -embedded points (or u -embedded points) of Figure 3.2. Let $x(t)$ (defined by (3.2)) represent the state of the system at time t . The component x_{N+1} of the state vector $x(t)$ remains unchanged during the entire successful transmission time period $\gamma(e+k)$ (see Figure 3.2). As well x_{N+1} remains unchanged during the channel assignment time $r(e+k)$. This is a direct result from the definition of the component x_{N+1} of the state $x(t)$, as it represents the index of the

controller that acquired last channel access. Therefore x_{N+1} is changed at time points $t_u^{(e+k)}$ according to the channel assignment strategy applied. Let Q denote a state transition probability matrix which reflects the channel assignment strategy and is enforced at time points $t_u^{(e+k)}$. Let P_1 denote the state transition probability matrix for the successful transmission time period $\gamma^{(e+k)}$ and P_2 denote the state transition probability matrix for the idle period $I^{(e+k)}$ followed by the channel assignment time period $r^{(e+k)}$. Given Q , P_1 , and P_2 , the system state transition probability matrices P_s and P_u are easily calculated by:

$$P_s = P_2 Q P_1 \quad (3.5)$$

$$P_u = Q P_2 \quad (3.6)$$

The determination of the state transition probability matrices Q , P_1 and P_2 is accomplished based on appropriate probabilistic arguments associated with specific access strategy and input traffic characteristics. Appendices B and C presents the specification of these matrices in detail for different state definitions. Let y_i and z_j represent particular values of the state vector $x \in X$ associated with the i th row and the j th column of a state transition matrix (e.g., the matrix Q), respectively. The (i,j) th element of the matrices Q , P_1 , P_2 , P_s and P_u are

Note that in Equation (3.6), P_u represents the state transition probability matrix for time points just before the time instants $t_u^{(e+k)}$, $k=1,2,3,\dots$

denoted by $[Q]_{y,z}$, $[P1]_{y,z}$, $[P2]_{y,z}$, $[P3]_{y,z}$ and $[Pn]_{y,z}$ respectively. Note that, for simplicity, the subscripts i and j of y_i and z_j are eliminated. Similarly, the i th row of the above matrices are denoted by $[Q]_y$, $[P1]_y$, $[P2]_y$, $[P3]_y$ and $[Pn]_y$ respectively. In the following, the matrix Q is further clarified. Let us assume that the i th controller is the last one that had access to the channel and successfully transmitted a packet (or packets for multipacket transmission protocols). Let us further assume that the state of the system just before the successful transmission period is $y=(y_1, y_2, \dots, y_N, y_{N+1})$. Clearly, $y_{N+1}=1$. Assume that at the beginning of the successful transmission period the state of the system is $z=(z_1, z_2, \dots, z_N, z_{N+1})$. The state transition matrix Q takes effect just before the beginning of the successful transmission period. Therefore no packet may arrive during this sufficiently small time period and consequently $y_1, \dots, y_N$ do not change. In other words $[Q]_{y,z}=0$ if there exists j such that $y_j \neq z_j$. Note that Q is a deterministic state transition matrix for deterministic access schemes and is a probabilistic one for random access schemes. In Figure 3.3 we present the channel access state transition probability matrix for the CSMA-CD protocols described in Section 3.2. The system parameters used are: $N=2$ and $L_j=2$ for $j=1,2$.

In Figure 3.3, each row (or each column) of the matrix is associated with a unique state vector. The (i,j) th element of the matrix Q denoted by $[Q]_{y,z}$ is the

$$\lambda_3 = 1$$

$$\lambda_3 = 1$$

00 01 02 10 11 12 20 21 22 00 01 02 10 11 12 20 21 22

$$\lambda_3 = 1, \\ \lambda_1, \lambda_2 =$$

$$\lambda_3 = 2, \\ \lambda_1, \lambda_2 =$$

Figure 3.3 The Channel Access Control State Transition Probability Matrix For The CSMA_CD Protocols ($N=2, L=2$)

conditional probability of transition from the state vector y_1 , which is uniquely associated with the i th row, to the state vector z_j , which is uniquely associated with the j th column. The index values of each row (or column) is a two digit number. The first digit represents the first component of the state vector x (i.e., x_1) and the second digit represents the second component of the state vector x (i.e., x_2). Note that the value of the third component x_3 of the state vector x is indicated only once for all of the possible values of x_1 and x_2 . Clearly, the dimension of this matrix is $M \times M$ where M is the cardinality of the state space (given by (3.4)). In this example $M=18$.

As mentioned earlier, the state transition matrix Q takes place just before the beginning of a successful transmission period (i.e., $[Q]_{y,z}=0$ if there exists i such that $y_i \neq z_i$ for $1 \leq i \leq N$). Consider the state $y=(2,1,1)$ in Figure 3.3 (i.e., there are two packets in the buffer of the first controller; one packet in the buffer of the second one and the last successful transmission was accomplished by the first controller). Therefore there are two ready controllers (i.e., $y_1, y_2 \neq 0$) and each is in a position to complete a successful transmission. Recall that the channel access strategy for the CSMA_CD dictates that the ready controllers are selected with equal probability. For the state $y=(2,1,1)$, each controller is selected, with probability $1/2$, i.e., $[Q]_{y,z}=1/2$ for $y=(2,1,1)$ and $z=(2,1,1)$ or $z=(2,1,2)$ and $[Q]_{y,z}=0$ for

other values of the state vector $\mathbf{x}$. Also Figure 3.4 presents the matrix $\mathbf{Q}$ for deterministic polling protocols. In polling protocols, controllers are given channel access one after the other. For example consider the state $\mathbf{y}=(2,1,1)$ in Figure 3.4. Since the last successful transmission was accomplished by the first controller, the channel must clearly be assigned (with probability equal unity) to the second controller. Therefore $[Q]_{\mathbf{y},\mathbf{z}}=1$ for $\mathbf{y}=(2,1,1)$, $\mathbf{z}=(2,1,2)$ and $[Q]_{\mathbf{y},\mathbf{z}}=0$ for other values of the state vector $\mathbf{x}$.

The steady state results are presented in the following.

We define $\pi_i(\mathbf{X})$ to be the steady state probability distribution vector for all of the states of the state space $\mathbf{X}$ for the embedded points $t_i^{(e+k)}$:

$$\pi_i(\mathbf{X}) = \{\pi_i(\mathbf{x}) | \mathbf{x} \in \mathbf{X}\} \quad (3.7)$$

Clearly the Markov chain represented by the i -embedded points (or the u -embedded points) is an ergodic Markov chain since it is homogeneous (i.e., the transition probabilities are independent of the particular embedded point), positive recurrent (i.e., the average time to return to every state is finite or equivalently, it is possible to return back to every state after a finite number of transitions) and aperiodic (i.e., the number of transitions after which we return back to a state assume every positive value). The stationary distribution $\pi_i(\mathbf{x})$ is obtained by solving the following equation:

$x_3=1,$
 $x_1, x_2=$

$x_3=2,$
 $x_1, x_2=$

$x_3=1$									$x_3=2$								
00	01	02	10	11	12	20	21	22	00	01	02	10	11	12	20	21	22
00									1								
01										1							
02											1						
10												1					
11													1				
12														1			
20															1		
21																1	
22																	1
00	1																
01		1															
02			1														
10				1													
11					1												
12						1											
20							1										
21								1									
22									1								

Figure 3.4. State Transition Probability Matrix C For A Polling Protocol.

$$\underline{\pi}_\ell(X) = \underline{\pi}_\ell(X) P_\ell \quad (3.8)$$

For the semi-Markov chain represented by the ℓ -embedded points. Similarly, the stationary distribution

$$\underline{\pi}_u(X) = \{\pi_u(x) | x \in X\} \quad (3.9)$$

is established by solving the following equation:

$$\underline{\pi}_u(X) = \underline{\pi}_u(X) P_u \quad (3.10)$$

Consider the state transition probability matrix $[P_\ell]^k$ (or $[P_u]^k$) which is calculated by multiplying P_ℓ (or P_u) by by itself k times. Given that the stochastic process represented by P_ℓ (or P_u) is ergodic [64], and for large k (i.e., $k \rightarrow \infty$), all rows of the resulting state transition probability matrix $\lim_{k \rightarrow \infty} [P_\ell]^k$ (or $\lim_{k \rightarrow \infty} [P_u]^k$) are identical, each one comprising the steady state probability vector $\underline{\pi}_\ell(X)$ (or $\underline{\pi}_u(X)$) such that $\pi_\ell(x)$ (or $\pi_u(x)$) is the steady state probability of occupancy of state x .

Let $\underline{\pi}(X)$ be defined as

$$\underline{\pi}(X) = \{\pi(x) | x \in X\} \quad (3.11)$$

to be the steady state distribution of states $x \in X$ for all points in time. Let $\hat{\underline{\pi}}(X)$ be an approximation for $\underline{\pi}(X)$. It is important to note that $\underline{\pi}_\ell(X)$ (or $\underline{\pi}_u(X)$) represent the steady state distribution of state occupancy at the embedded points $t_\ell^{(e+k)}$ (or $t_u^{(e+k)}$), $k=0,1,2,\dots$. These distributions do not necessarily represent the steady state distribution of state occupancy at other time points

(i.e., in general $\pi(X) \neq \pi_q(X)$). However there are cases, indeed, where $\pi(X) = \pi_q(X)$ and the necessary conditions for this equality are shown in Appendix A. Appendices B and C present the specification of these matrices in detail for different state definitions employed for the CSMA_CD protocols given in Section 3.2. For further clarification of the above approach, a number of numerical examples are presented in Appendix D.

The marginal distribution of x_j is then given by

$$\text{Prob}(x_j=k) = \sum_{X'} \pi(X)$$

where

$$X' = \{X \in X | x_j=k; 1 \leq j, x_{N+1} \leq N\} \quad (3.12)$$

and the average queue length at the j th controller denoted by $q_1(j)$

$$q_1(j) = \sum_{k=0}^{L_j} k \cdot \text{Prob}(x_j=k) \quad (3.13)$$

In the G-model presented in Section 3.3.1, let B_j represent the blocking probability for the j th controller and B denote the blocking probability for the system. In order to establish the blocking probability B consider the following conservation law: the total number of messages entering the system is equal to the total number leaving the system for over a sufficiently long time period. Therefore, average arrival rate must be equal to the average departure rate. Let $E^{-1}(t_{1d})$ denote the average departure rate, the blocking probability B for the system

is therefore given by

$$(1-B) = E^{-1}(t_{id})$$

or

$$B = 1 - E^{-1}(t_{id}) \quad (3.14)$$

For the cases in which the specification of the average interdeparture time is not trivial (e.g., multiple packet transmission protocols PC2, PC3 and PC4), approaches are required to establish approximate values for the blocking probability B . One such approach is to approximate B_j , i.e., the blocking probability for the j th controller, by the limiting probability that the buffer of the j th controller is full, i.e., we have

$$B_j = \sum_{x \in X | x_j = L_j} p(x) \quad (3.15)$$

the blocking probability B is then established by solving the following equality: the total average arrival rate into the system equals the sum of the average arrival rates into the controllers, i.e.,

$$\lambda(1-B) = \sum_{j=1}^N \lambda_j (1-B_j)$$

or equivalently:

$$B = \sum_{j=1}^N \lambda_j B_j / \lambda \quad (3.16)$$

The throughput S is defined as the rate of successful packets received at the central station and is given by:

$$S = (1-B) \lambda T \quad (3.17)$$

In the following, the maximum throughput of the single packet transmission protocol PC1 is given. Let n_1 represent the first moment of the limiting distribution or the mean of the random variable $n^{(e+1)}$ (refer to Figure 3.2). Let the inner-loop utilization factor be defined as the ratio of the rate at which packets enter the inner-loop to the maximum rate at which these packets can be transmitted on the inner-loop channel [64]. The maximum average rate at which packets can be transmitted on the inner-loop channel is equal to one packet per $(T+1+r_1)$ seconds (refer to Figure 3.2). Therefore the inner-loop utilization factor is equal to $(1-B) \cdot (T+1+r_1)$. The requirement $(1-B) \cdot (T+1+r_1) < 1$ gives rise to the following upper bound on the channel throughput.

$$S < 1 - (1-B) \cdot (T+1+r_1) \quad (3.18)$$

The average number of packets in the system is given by

$$\bar{x} = \sum_{k=1}^N (x_k) \cdot p(x) \quad (3.19)$$

using Little's result [64], the average network delay is given by

$$d = \bar{x} / S \quad (3.20)$$

3.3.3 Extension to Larger Systems

Let $x(t^{(e+k)})$ denote the state of our system at the k th embedded point $t^{(e+k)}$. Then the stochastic process $\{x(t^{(e+k)}), k=0,1,2,\dots\}$ describes the dynamic behaviour of the system at time points $t^{(e+k)}, k=0,1,2,\dots$. At this stage, it is necessary to answer the following questions:

(1) The stochastic process defined at points $t^{(e+k)}, k=0,1,2,\dots$ describes the dynamic behaviour of the system at time points $t^{(e+k)}$. How do we specify the dynamic behaviour of the system at some other time points, specifically the time points immediately before an arrival enters the system?

(2) Under what conditions can a solution be obtained for the general stochastic process?

The first issue is studied in Appendix A. The second issue requires reduction of the stochastic process into a Markov process and by using simplifying assumptions, state transition probability matrices are developed and the steady state solutions are established. Determination of the state transition probability matrices is heavily dependent on the state definition. For state definitions that results in a large state space there are large amounts of information carried by each state vector and this simplifies the development of state transition matrices. Clearly less information is carried by a state vector that yields a small state space. This creates more difficulty in developing the state transition matrices.

In order to overcome this difficulty, simplifying assumptions are usually introduced.

Although the G-model is applicable to a number of deterministic and random access schemes, extension to larger systems requires new state definitions. In the following a number of state definitions are reviewed and an appropriate one is chosen for the G-model.

3.3.4 Alternative State Definitions

We consider again the state vector x given by:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \\ x_{N+1} \end{bmatrix} \quad 0 \leq x_j \leq L_j, 1 \leq j, x_{N+1} \leq N$$

where x_j is the number of packets stored in the buffer of the j th controller for $j=1, \dots, N$ and x_{N+1} is the index of the user who has acquired the channel most recently. In general, if there are N users and the j th user has buffer capacity L_j , an $N+1$ dimensional state description x defined as in equation (3.2) can be employed.

* Using the macro state approaches, it is possible to reduce the size of the state space by eliminating x_{N+1} element of the state vector x .

The advantages of this state definition is that it allows the development of the state transitions easily for variety of deterministic and random access protocols. This state definition is employed in Appendix E. In this appendix state transition probability matrices (associated with the protocols presented in Section 3.2) are given.

The main disadvantage is that the state space grows rapidly as system size increases. The size of the state space, in this case, is given by (3.4). However, Appendix E discusses a number of approaches for reducing the size of the state space.

2. We consider the state vector $\mathbf{x}$ given by:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_L \end{bmatrix} \quad 0 \leq x_k \leq N \quad (3.21)$$

Where L_{MAX} represents maximum buffer capacity among controllers and x_k is the number of controllers with k packets in their buffer, $k=1, \dots, L_{MAX}$. Clearly the number of controllers with zero packet in their buffer is equal to $N - x_1 - x_2 - \dots - x_{L_{MAX}}$.

The main resultant advantage of this technique is a smaller state space (as $L_{MAX} \ll N$ for large systems) and the size of the state space is of the order of $(N)^{(L_{MAX})}$.

However, this technique has the disadvantage of a state transition matrix which is more cumbersome than that of the state definition in (3.2). In addition, the definition does not allow the analysis of deterministic access schemes.

3. The state vector x can be defined as:

$$\begin{aligned}
 x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad & \begin{aligned} 0 \leq x_1 \leq N-1 \\ 0 \leq x_2 \leq \sum_{j=1}^N L_j \\ 1 \leq x_3 \leq N \end{aligned} \end{aligned} \quad (3.22)$$

Where x_1 is equal to the number of busy controllers, x_2 represents the total number of outstanding packets in the system and x_3 represents the index of the controller in the last successful transmission period.

The main advantage in using this definition is the smaller size of state space (i.e.,

$$M = N^2 \left(1 + \sum_{j=1}^N L_j \right).$$

The main disadvantage is that it does not allow analysis of random access schemes in a nonhomogeneous environment. A system is said to operate in a homogeneous environment when the controllers are identical, i.e., they have the same buffer capacity and the arrival process into the controllers are identical.

The distribution of x_2 packets among x_1 ready users is not usually known and hence a number of

assumptions are required (e.g., x_2 packets are distributed equally among all x_1 ready users).

4. The vector $\bar{x}$ is defined as follows:

$$\bar{x} = \begin{matrix} x_1 & 0 \leq x_1 \leq N-1 \\ x_2 & 0 \leq x_2 \leq L_{MAX} \\ x_3 & 1 \leq x_3 \leq 2 \end{matrix} \quad (3.23)$$

Where x_1 is the number of active controllers excluding a selected controller and x_2 is the number of packets in the buffer of the selected controller. A value $x_3=2$ indicates that the selected controller was the last transmitting controller while $x_3=1$ indicates that one of the $N-1$ controllers transmitted in the last successful transmission period.

The main advantages of this representation is a relatively smaller size state space, i.e., $M = 2N L_{MAX}$. Similar to the previous representation, it allows analysis of random access schemes in a homogeneous environment only, that is a system in which we have:

$$\lambda_j = \lambda_k; L_j = L; k = j = 1, \dots, N \quad (3.24)$$

The alternative state definitions are summarized in Table 3.1.

In the following sections, first we consider the state definition given by equation (3.2) for the G-model. This state definition is chosen because it permits the development of the state transition matrices P^l and P^u in a simple fashion. Furthermore, it permits a

Table 3.1

Alternative State Definitions for the G-Model

State Definition	Cardinality of the State Space
$x = (x_1, \dots, x_N, x_{N+1}),$ $0 \leq x_j \leq L_j$ $1 \leq j, x_{N+1} \leq N$	$M = N \prod_{j=1}^N (L_j + 1)$
$x = (x_1, \dots, x_{L_{MAX}})$ $1 \leq x_k \leq N$ $1 \leq k \leq L_{MAX}$	$M = (N)^{L_{MAX}}$
$x = (x_1, x_2, x_3),$ $0 \leq x_1 \leq N-1$ $0 \leq x_2 \leq \sum_{j=1}^N L_j$ $1 \leq x_3 \leq N$	$M = N^2 (1 + \sum_{j=1}^N L_j)$
$x = (x_1, x_2, x_3),$ $0 \leq x_1 \leq N-1$ $0 \leq x_2 \leq L_{MAX}$ $1 \leq x_3 \leq 2$	$M = 2 * N * (L_{MAX} + 1)$

simple illustration of application of the G-model. However, the cardinality of the state space (refer to Table 3.1) grows rapidly as the system increases in size. Thus the state definition (3.2) permits analysis of small size systems (i.e., $N \leq 5$, $L_j \leq 4$). Next, we consider the state definition given by equation (3.23) for the G-model. This state definition is chosen due to the fact that it permits the development of the state transition matrices P_u and P_v . Furthermore, for large size systems (i.e., $N \geq 10$, $L > 1$), the state space associated with this state definition has the smallest value of cardinality, among state definitions given in Table 3.1.

3.4 New Results

This section presents new results which are obtained from analytic models developed for the inner-loop.

Appendix B presents the details of the analytic model for small size systems. It employs the state definition given by (3.2) and specifies the state transition matrices P_1 , P_2 and Q .

Appendix C presents the details of the analytic model for large size systems. It employs the state definition given by (3.23) and presents the state transition matrices P_1 , P_2 and Q .

In Section 3.4.1, a number of numerical examples are discussed in order to clarify the analytical models presented in Appendices B and C. Section 3.4.2 presents numerical and simulation results for the protocol PC3

using the small size system model established in Appendix B. In 3.4.3, numerical results are presented for the protocol PC1 using the large size system model established in Appendix C. Finally, section 3.4.4 shows a number of examples for which a large size system model as well as a small size system model are developed for a system employing the PC1 protocol. For these examples, it is shown that results obtained from the simulation model, the small system size model and the large system size model have excellent agreement.

3.4.1 Clarification of the Analytical Models

This section reviews a number of numerical examples in order to clarify the analytic models presented in Appendices B and C. These examples are summarized in Appendix D. In this appendix, an example for small size system models followed by an example for large size system models are given.

Consider the example for small size system models given in Appendix D. In this example, the multiple packet transmission protocol PC3 is employed in a homogeneous environment in which there are two controllers (i.e., $N=2$) each of which has a finite buffer capacity of two packets (i.e., $L_j=2$, $j=1,2$), the ratio of the propagation delay over packet transmission time is equal to .1 (i.e., $a=0.1$) and mean arrival rate to each controller is $\lambda_1=\lambda_2=0.25$. The analytic model for this system is established in

Appendix B. First using the state definition 3.2, the states of the system are generated. Next the state transition matrices P_1 , P_2 and Q are established. Using the equation (3.6), the state transition matrix P_k is calculated. An initial state vector x_0 is arbitrarily chosen. Applying equation (3.10), the steady state distribution $\pi_0(x)$ is obtained after multiplication of the matrix P_k by itself 9 times. Using equation 3.5, the state transition matrix P is then calculated and after multiplying by itself 9 times, it converged to a matrix with same rows i.e., the steady state distribution π, X . Using the steady state distributions, the blocking probability B , throughput S and average delay d (given by equations (3.16), (3.17), and (3.20) respectively) are calculated for ~~v~~-embedded points and u-embedded points.

Consider the example for the large system size models given in Appendix D. The example is based on the analytic model presented in Appendix C. In the following, first the analytic model of Appendix C is reviewed. Next the example for the large system size model given in Appendix D is discussed.

In Appendix C the single packet transmission protocol PC1, given in Section 3.2, is employed in a homogeneous environment. The state definition given by (3.23) is employed. In this protocol, let $n^*(x(t))$ denote the number of ready controllers in the system at time t . Clearly, for $x(t) = (x_1, x_2, x_3)$, $n^*(x(t)) = x_1 + 1$ when $x_2 > 0$, otherwise for $x_2 = 0$, we have $n^*(x(t)) = x_1$. Recall,

that x_1 represents the number of ready controllers among the N controllers. Consider a ready controller among the x_1 ready controllers and assume that it has started the successful transmission of a packet at time t . Let $P(n^*(x, t))$ denote the probability that this controller had only one packet i.e., the packet which is transmitted successfully in its buffer at time t . The knowledge of $P(n^*(x, t))$ is vital to the determination of the state transition matrix P_1 . This is due to the fact that x_1 may decrease only when the transmitting controller has one packet stored in its buffer (Appendix C). The matrix P_1 and consequently the state transition matrices P^1 and P_u are all dependent on the values of $P(n^*(x(t)))$. The assumption that $P(n^*(x(t)))$ exists and has a constant value, reduces the general stochastic process $\{x(t+e+k), k=0,2,3,\dots\}$ to a Markov chain. Therefore the G-model could be applied in order to analyze the Markov chain. As it is shown in Appendix C, $P(n^*(x))$ is a function of the steady state distribution $\pi(x)$. Consequently, an iterative algorithm is employed which operates as follows: using an initial steady state distribution $\pi(x)$, new values are calculated for $P(n^*(x))$. Using these new values of $P(n^*(x))$, new values for $\pi(x)$ is calculated. This procedure is repeated until a convergence criterion (e.g., 0.00001 precision) is met for successive values obtained for $P(n^*(x))$. The overall numerical analysis of the large system size is summarized in Appendix C.

Consider the example for the large system size model given in Appendix D. The single packet transmission protocol PC1, given in Section 3.2, is employed in which there are 3 controllers (i.e., $N=3$) each of which has buffer capacity of 3 packets (i.e., $L_j=3$, $j=1,2,3$) and the ratio of the propagation delay over packet transmission time is equal to .01 (i.e., $a=0.01$). The convergence of the values of $P(n^*(x))$ for the two average input traffic rates $\lambda=0.4$ and $\lambda=0.6$ is presented.

3.4.2 Numerical Results for Small Size Systems

In this section, numerical results are obtained for various performance measures of the multiple packet transmission protocol PC3 in a homogeneous environment (i.e., homogeneous controller's input traffic and identical controller's buffer capacity). Appendix B presents the analysis of this protocol.

The results presented are for a typical small size system in which there are three controllers (i.e., $N=3$) each with a buffer capacity of three packets (i.e., $L_j=3$ for $j=1,2,3$). Without loss of generality, all time delays will be normalized with respect to the packet transmission time, T . The results are obtained for different values of the ratio of the propagation delay to packet transmission time: $a=0.1$ and $a=0.01$.

The simulation results presented in this section are based on a simulation model developed using the SLAM simulation package explained in [106]. The steady state

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behaviour of our system is studied through the following steps taken in a simulation run:

- (1) The simulation is started in an empty state.
- (2) The accumulated statistics of the system are cleared after 100 simulation time units (a simulation time unit is equal to the packet transmission time T).
- (3) The simulation model is executed for a total simulation time of 2000 simulation time units.

The validity of the results of the simulation model is enhanced by considering a number of (in some cases up to ten) simulation runs in which various random number generators are employed.

Figures 3.5 and 3.6 present the steady state average system delay at the embedded points $t_e^{(e)}$ and $t_u^{(e)}$, normalized to packet transmission time T , as a function of the total input traffic load to the system for $a=0.01$ and $a=0.1$. Simulation results show that the results at $t_e^{(e)}$ provide a good approximation (less than 5% error) for $\lambda < 0.7$. For heavier ($\lambda > 0.7$) input traffic load, the results at $t_u^{(e)}$ yield a good approximation. Clearly the system operates beyond the total input traffic load of $\lambda=1$. In such cases, more packets are lost and those in the system encounter higher delays due to collision. It is important to note that the adaptive algorithm employed (refer to Section 3.2) guarantees a minimum value for the throughput of the system for any average input traffic rate λ .

Figure 3.5. Average Delay Versus Total Input Traffic Rate At The Following Time Points:
 I - Start Of Successful Transmission Time Periods.
 II - Departure Time Points.

Figure 3.6. Average Delay Versus Total Input Traffic Rate At The Following Time Points:
 I - Start of Successful Transmission Time Periods.
 II - Departure Time Points.

Figures 3.7 and 3.8 present blocking probability as a function of total input traffic rate for the system with $a=0.01$ and $a=0.1$ respectively.

Figures 3.9 and 3.10 depict the average number of packets in the buffer of a controller as a function of total input traffic rate to the system for $a=0.01$ and $a=0.1$ respectively.

For the embedded points $t_l^{(e)}$ and $t_u^{(e)}$, the steady state distribution of queue size of a controller for two different input traffic rates $\lambda=0.3$ and $\lambda=0.9$ are given in Figure 3.11 and Figure 3.12 for $a=0.01$.

Finally Figure 3.13 presents the probability distribution of the queue size obtained at $t_l^{(e)}$ for a typical input traffic rate of $\lambda=0.4$ and propagation delay $a=0.1$.

This section has examined the performance behaviour of the multiple packet transmission protocol PC3 (refer to Section 3.2) for a typical small size system in which there are three controllers (i.e., $N=3$) each with a buffer capacity of three packets (i.e., $L_j=3$). The results are obtained for different values of $a=0.1$ and $a=0.01$. As expected, the steady state results obtained at time points $t_l^{(e+k)}$, $k=0,1,2,\dots$ differ from those obtained at all time points (by the simulation model). However, the steady state results obtained at time points $t_l^{(e+k)}$, $k=0,1,2,\dots$ yield good approximation when operating at moderate input traffic levels (i.e., $\lambda<0.7$). This implies

Figure 3.7. Blocking Probability Versus Total Average Arrival Rate.

Figure 3.8. Blocking Probability Versus Total Average Arrival Rate.

Figure 3.9. Average Queue Size Versus Total Average
 Arrival Rate At The Following Time Points:
 I - Start of Successful Transmission Time
 Periods.
 II - Departure Time Points.

Figure 3.10. Average Queue Size Versus Average Arrival Rate:
 I- At The Start Of Successful Transmission Time Periods.
 II-At Departure Time Points.

Figure 3.11. The PDF of The Queue Size Of A Controller.

QUEUE SIZE OF A CONTROLLER

Figure 3.12. The PDF of The Queue Size Of A Controller.

Figure 3.13. The PDF Of The Queue Size Of A Controller

that when operating at and up to such input traffic levels, the probability that a controller has two or more packets in its buffer is negligible and thus the protocol PC3 behaves similarly to the single packet transmission protocol PC1.

3.4.3 Numerical Results For Large Size Systems

In this section, numerical results are obtained for various performance measures of the single packet transmission protocol PC1 in a homogeneous environment (i.e., homogeneous controller's input traffic and identical controller's buffer capacity). Appendix C presents the details of the analysis for this protocol.

In the following, first the maximum throughput of the system (given by equation (3.18)) is examined for various values of N (i.e., the number of controllers in the system). Clearly, as $N \rightarrow \infty$, the snapshot of the broadcast channel (refer to Figure 3.2) for the protocol PC1 given in Section 3.2 reduces to that given by Lam [74]. We demonstrate that as $N \rightarrow \infty$, the maximum throughput in our model reduces to that given in Lam's model [74]. Next, a typical large size system is considered and the associated results are presented.

As we have indicated in Appendix B, consider the adaptive algorithm reviewed in Section 3.2. Let x represent the state of the system at the beginning of a time slot in the collision time period $r^{(e+1)}$ (refer to

Figure 3.2). At the beginning of each time slot associated with a collision time period, ready controllers transmit into the next time slot with equal probability (i.e., $1/n^*(x)$). Furthermore, let σ denote the probability that only one ready controller (among $n^*(x)$ ready controllers) transmit in this time slot.

Clearly we have

$\sigma = \text{Prob}(\text{one controller out of } n^*(x) \text{ active controllers transmit in a time slot})$

$$= n^*(x) \frac{1}{n^*(x)} \left(1 - \frac{1}{n^*(x)}\right)^{(n^*(x)-1)}$$

$$\text{or } \sigma = \left(1 - \frac{1}{n^*(x)}\right)^{(n^*(x)-1)} \quad (3.25)$$

when the system is operating at its maximum throughput capacity (i.e., all N controllers are ready), we have $n^*(x)=N$ and therefore

$$\sigma = \left(1 - \frac{1}{N}\right)^{(N-1)} \quad (3.26)$$

The conditional distribution of the random variable $r^{(e+1)}$ conditioned on the fact that there are two or more ready controllers in the system is equal to

$$\begin{aligned} \text{Prob}(r^{(e+1)} = k/n^*(x) \geq 2) \\ = \sigma(1-\sigma)^{k-1} \quad k=0,1,2,\dots \end{aligned} \quad (3.27)$$

with mean $r_1 = 2/\sigma$. It is important to note that as $N \rightarrow \infty$, the value of $\sigma \rightarrow 1/e = 0.367$ and the maximum throughput in our model, given by equation (3.18), reduces to that given

in Lam's model [74]. The maximum throughput of the system, given by equation (3.18) varies for different values of the mean r_1 of the random variable $r^{(e+1)}$ (refer to Figure 3.2) and the ratio of propagation delay to packet transmission time (i.e., $a = 1/T$). In Table 3.2, the maximum throughput of the system is tabulated for various values of N and a . Results indicate that when the system is operating at its maximum throughput capacity, as N increases, the maximum number of ready controllers in the system increases and as a result, r_1 increases which in turn decreases the maximum throughput. Clearly the values of a remain the dominant system parameter and as a decreases, the probability of collision decreases which in turn increases the maximum throughput.

The results presented are for a typical large size system in which there are ten controllers (i.e., $N=10$). Again all time delays are normalized with respect to the packet transmission time, T . The results are obtained for two values of $a=0.01$, and $a=0.1$ and for controller's buffer capacity $L_j = 1, 2$, and 3 .

Figure 3.14 presents the average system delay d , normalized with respect to the packet transmission time T , as a function of the total input traffic rate to the system for various values of $a=0.01$, 0.1 and $L_j=1, 2, 3$. Larger values of a contribute to larger probability of collision which in turn increases the average delay. For $a=0.1$ and larger values of λ (i.e., $\lambda > 0.6$), the probability of buffer overrun increases. This results in

Table 3.2

The Maximum Throughput as a Function of the Number
of Controllers for $a=0.1$; 0.01

N=Number of Controllers	Maximum Throughput for $a=0.1$	Maximum Throughput for $a=0.01$
2	.6666	.9524
5	.6296	.9420
10	.6187	.9420
20	.6135	.9403
50	.6104	.9400
∞	.6083	.9395

Figure 3.14. Average Delay Versus Average Arrival Rate.

the loss of larger numbers of packets, i.e., decreases the throughput which in turn yields a lower rate of increase for average delay. For small values of λ , the average delay remains the same as L changes. However, for larger values of λ , and as L grows, the total numbers of packets in the system increases, hence the average delay increases.

Figure 3.15 presents the blocking probability B as a function of the system throughput for $a=0.1$ and $L_j = 1, 2, 3$. For given values of S , as L_j increases, the blocking probability decreases. This is because larger values of L_j contribute to larger available space in the controller's buffer. This yields smaller probability of loss in the controller's buffer which in turn contributes to smaller probability of blocking. However, for larger values of S and as L_j increases, the probability of blocking decreases in a less significant manner. This is because for larger values of S , the system operates near its throughput capacity, at which point the buffers become nearly full regardless of their capacities. This makes the system less sensitive to the values of L_j .

Figure 3.16 depicts the system throughput S as a function of the total input traffic rate λ , for $a=0.1$. Note that for constant values of N , λ and a , as L_j increases, the throughput of the system increases. This is due to the fact that an increase in L_j yields an increase in system throughput. However, for larger values

Figure 3.15. Blocking Probability Versus Throughput

Figure 3.16. Throughput Versus Average Packet Arrival Rate.

of λ , the system is operating near its throughput capacity and for larger values of L_j , it yields insignificant decrease for the blocking probability. Therefore it contributes to an insignificant increase in the throughput of the system.

In Figure 3.17 the delay-throughput performance of the system is shown. Clearly for a given N , a and S , as L_j increases, the average delay increases. This is due to the fact that when L_j increases, fewer packets are lost; thus the average number of packets in the system increases, and therefore the average delay increases.

3.4.4 Analysis of the Numerical Results

In this section, results are obtained for the single packet transmission protocol, PC1 in a homogeneous environment using the three models: the small size system model, the large size system model and the simulation model.

Although it is interesting to show that the large size system model reduces to the small size model, it is not clear whether such convergence could be shown mathematically. Such an attempt requires the proof of the following conjecture:

Let F_1 be the set of all Markov chains that could be developed using the small size system model. Let F_2 represent the set of all Markov chains that could be developed using the large size system model. There exists

Figure 3.17. Average Delay Versus Throughput.

a mapping f with domain F_1 and codomain F_2 which maps any Markov chain f_1 selected from F_1 to a Markov chain f_2 in the set F_2 such that the steady state solution obtained from solving f_1 is equivalent to that obtained from solving f_2 .

The theory of lumpable Markov chains, reviewed in Appendix A, is not sufficient to prove the above conjecture: Using this theory, one may partition the state space into Macrostates by lumping 'equivalent' states (two or more states are called 'equivalent' if their steady state probability of state occupancies are equal) into a Macrostate. Clearly, in the process of reduction of a given Markov chain f_1 selected from F_1 to a chain f_2 in F_2 it is necessary to lump states that are not 'equivalent'.

In the absence of a mathematical proof we employ simulation and numerical results in order to show the reduction of the large size model to the small size model for a number of system configurations.

The results presented are for typical small size systems in which $N=3$. Each controller has a buffer capacity of three packets (i.e., $L_j=3$ for $j=1,2,3$). The packet transmission time T is considered to be equal to unity. The results are obtained for values of $\alpha=0.1$ and $\alpha=0.01$.

The simulation results presented in this section are based on an optimized simulation model developed using the SLAM simulation package [106]. The steady state

behaviour of our system is studied through the following steps taken in a simulation run:

- (1) The simulation is started in an empty state.
- (2) The accumulated statistics of the system are cleared after 100 simulation time units (a simulation time unit is equal to the packet transmission time T).
- (3) The simulation model is executed for a total simulation time of 2000 units.

The validity of the results of the simulation model is enhanced by considering a number of (in some cases up to ten) simulation runs in which various random number generators are employed.

Figure 3.18 presents the average system delay, normalized on packet transmission time T , as a function of the total input traffic rate to the system for $N=3$, $a=0.01, 0.1$, using the three modelling approaches. Figure 3.19 presents the blocking probability (using (3.16)) as a function of the total input traffic rate for $N=3$, $a=0.01, 0.1$. In Figures (3.18) to (3.19), the simulation results agree with the numerical results obtained from the small and the large size system models. Furthermore, results obtained from the large size system model show excellent agreement (i.e., results are the same up to the 3rd decimal digit) with those obtained from the small size system model.

Figure 3.18. Average Delay Versus Average Packet Arrival Rate.

Figure 3.19. Blocking Probability Versus Average Arrival Rate.

In this chapter, we have presented a number of analytic models for the analysis of a number of random as well as deterministic access schemes with buffered users. These models are employed to study the dynamic behaviour of the inner-loop. In the following chapter, i.e., Chapter 4, we study the dynamic behaviour of an outer-loop taking into account the impact of the inner-loop on outer-loops.

3.5 Conclusions

The principle contributions of this chapter can be summarized as follows:

- (1) A generalized mathematical model (G-model) is presented for the analysis of the inner-loop. The model is based on a number of simplifying assumptions and alternative state definitions and can be employed in the analysis of a number of random as well as deterministic access schemes with buffered users.
- (2) A number of CSMA_CD protocols (i.e., PC1, PC2, PC3 and PC4 protocols) are presented for the inner-loop which are based on the CSMA_CD protocol given by Lam [74]. The distinguishing characteristics of these protocols are finite population of controllers and finite controller buffer capacity. Furthermore, each controller, once it acquired the channel, follows a set of rules which allow the transmission of one packet (as in protocol PC1) or multiple numbers of

packets (as in protocols PC1, PC2, PC3 and PC4).

- (3) An analytic model (i.e., the small system size model) is presented for the analysis of the CSMA_CD protocols (i.e., protocols PC1, PC2, PC3 and PC4) with small numbers of network nodes (i.e., $N < 5$) and finite buffer capacity.
- (4) Numerical as well as simulation results are presented for the analysis of the CSMA_CD protocols PC1 and PC3, with small numbers of network nodes (i.e., $N=3$, $L=3$, $a=0.01, 0.1$) using the small size system model.
- (5) An analytic model (i.e., the large system size model) is presented for the analysis of the CSMA_CD protocol PC1 with large numbers of network nodes (i.e., $N \geq 10$) and finite buffer capacity.
- (6) Numerical as well as simulation results are presented for the analysis of the protocol PC1 with large numbers of network nodes (i.e., $N=10$, $L=1,2,3$, $a=0.1, 0.01$) using the large size system model.
- (7) Numerical as well as simulation results are presented for the analysis of the protocol PC1 using the three models: the small size system model, the large size system model and a simulation model for $N=3$, $L=3$ and $a=0.1, 0.01$. Results obtained from the three models show excellent agreement.

CHAPTER 4

OUTER-LOOP ANALYSIS

The flow control algorithm enforced by each controller affects the performance behaviour of its outer-loop. It is essential to take this impact into account when developing an analytic model for the outer-loop. The impact of the performance of the inner-loop on that of the outer-loop is represented by the random variable Q and the Laplace transform $H^*(s)$ of its PDF. Recall that the random variable Q is a time period during which the controller's buffer is full. Following the approach presented by Lam [74], a mathematical model is developed for the analysis of the outer-loop. The model incorporates the transform $H^*(s)$ of the random variable Q .

Section 4.1 presents the access schemes for outer-loops. Modelling and analysis of the outer-loops is given in Section 4.2. In Section 4.3 performance results are presented and discussed. Finally in Section 4.4 a summary of contributions made in this chapter is given.

4.1 Channel Access Schemes

We consider the CSMA_CD protocol presented by Lam [74]. All outer-loops employ this protocol. In the following we briefly review this protocol.

User terminals are synchronized so that following each successful transmission the channel is slotted in time. User terminals may start transmission only at the beginning of a time slot. A user terminal, once a packet is generated and is ready for transmission, senses the channel and operates as follows:

1. If the channel is sensed idle, the terminal transmits the packet with probability one.

2. If the channel is sensed busy, the terminal waits until the channel becomes idle (i.e., persisting on transmitting) and only then transmits the packet with probability one (hence, the name one-persistent).

Note that when the outer-loop controller's buffer become full, the controller will send a busy tone which will prevent the terminals from transmitting. This condition will be considered in modelling and analysis of the outer-loops.

3. Collisions on the channel are detected and the user terminals involved in a collision abort their transmission immediately upon detecting the collision.

4. Upon detection of a collision, each ready user terminal uses an adaptive algorithm for selecting its transmission probability (<1) in the next time slot.

The selection of the CSMA_CD protocol for the outer-loops is based on the fact that in practice each outer-loop incorporates a large number of bursty user terminals with high peak to average ratio of data traffic. This

protocol leads to high throughput as long as the length of the outer-loop and the resultant propagation delay are small in relation to the packet transmission time.

4.2 Mathematical Model of Outer-Loop

As mentioned earlier in Section 4.1, all outer-loops employ the CSMA_CD protocol similar to that introduced by Lam [74]. The protocol is similar to the p-persistent CSMA_CD in which users are assumed to be synchronized in time. Following each successful transmission, the channel is slotted in time as shown in Figure 4.1. Users may start transmission only at the beginning of a time slot. Assumptions and related approximations introduced in development of the analytic model of the outer-loops are listed below. The difference between the model given by Lam [74] and the model for outer-loop discussed here is that in the former model the channel is busy either during a successful transmission or when a collision takes place whereas in the latter model the channel is busy during a successful transmission, when a collision takes place or when the associated controller buffer becomes full.

Assumptions

1. All outer-loops are assumed to be of identical characteristics. Hence the analysis is made for a single outer-loop.
2. Errors due to random noise are insignificant relative to errors due to collisions and can be neglected.

Figure 4.1. Outer-loop: A Snapshot Of The Broadcast Channel.

3. In the 4th step of the protocol explained in the previous section, a suitable algorithm is used so that the probability of a successful transmission in the next time slot is equal to a constant σ which is equal to the throughput of Slotted-ALOHA (i.e., $\sigma = .367$).
4. The source of traffic to an outer-loop broadcast channel consists of an infinite population of users who collectively form an independent Poisson process with mean message generation rate λ messages per second. This approximates a large but finite population in which each user generates messages infrequently.
5. Each user terminal is allowed to store and attempt to transmit at most one packet at a time. Thus the generation of a new packet is equivalent to increasing the number of ready users by one. Therefore the effect of queueing messages at individual user terminals is neglected.
6. Each message may be transmitted in a time interval much less than the average time between successive messages generated by a given user.
7. Acknowledgements are assumed to be instantaneous.
8. The packet processing time at any device is assumed to be negligible, i.e., much shorter than the packet transmission time.

In this section, a message is defined to be a variable length packet with a given associated probability distribution.

9. The transmission time of each message is a random variable with the probability distribution function $p(y)$, which has a mean value b_1 , a second moment b_2 , and Laplace Transform $p^*(s)$.
10. Consider the j th outer-loop and let τ_j be the time from the start of transmission by one user to when all users sense the presence of this transmission. In order to implement the back-off strategy given in step 3 of the protocol, the minimum duration of a time slot for the back-off strategy is $2\tau_j$. Thus if a collision is detected and the collided transmissions are aborted immediately in a given time slot, the channel will be free of any transmissions at the beginning of the next time slot.

The users ready to transmit can be considered to form a distributed queue with the broadcast channel providing a random order of service. The equilibrium moment generating function of the distributed queue size is obtained using an embedded Markov chain analysis.

Under the assumption of Poisson arrivals and given that the messages depart the outer-loop channel one at a time, the moment generating function of the queue size obtained for the embedded points is valid for all points in time.

A snapshot of the outer-loop channel is illustrated in Figure 4.1. We define the following random variables:

$x(e)$ = Number of ready users at embedded point
 $t_l(e)$

- $\phi(e+1)$ = Time from the departure of the packet at time $t_f^{(e)}$ to beginning of the next successful transmission.
- $r(e+1)$ = The number of time slots in the collision resolution period. It includes the slot containing the initial collision.
- $I(e+1)$ = The number of time slots in the idle period.
- $y(e+1)$ = Duration of the packet transmission time in an outer-loop.
- $q(e+1)$ = Time duration in which the channel is set busy due to the fact that the buffer of the associated controller is full.
- $u(e+1)$ = Number of new (Poisson) arrivals during $\phi(e+1)$.
- $v(e+1)$ = Number of new (Poisson) arrivals during $y(e+1) + \tau_j$.
- $w(e+1)$ = Number of new (Poisson) arrivals during $q(e+1)$.

Clearly $I(e+1)$ is independent from $r(e+1)$ and we have:

$$\phi(e+1) = (I(e+1) + r(e+1)) \cdot (2\tau_j) \quad (4.1)$$

In assumption 9, we considered $\beta(y)$ to be the PDF for the random variable $y^{(e+1)}$. Let $\beta_1(y)$ be the PDF for the random variable $y^{(e+1)} + \tau_j$. The Laplace transform of $\beta_1(y)$ is thus

$$B_j^*(s) = B^*(s) e^{-s\tau_j} \quad (4.2)$$

The Laplace Transform of the Probability Distribution Function (PDF) of the random variable $Q^{(e+1)}$ is found through analysis of the inner-loop. $Q^{(e+1)}$ is the time elapsed from the moment a controller's buffer becomes full until the controller successfully completes the transmission of one or more packets, hence one or more buffer spaces become available. The time duration $Q^{(e+1)}$ depends on how fast the controller can accomplish the successful transmission of a single packet which in turn depends on the traffic conditions on the inner-loop. We assume that $Q^{(e+1)}$ is independent from $q^{(e+1)}$ and $y^{(e+1)}$. Furthermore, let h_1 , h_2 and $H^*(s)$ represent the mean, second moment and the Laplace Transform of the PDF of the random variable Q respectively.

The random variable $I^{(e+1)}$ represents the number of slots in an idle period. We note that $I^{(e+1)}$ is nonzero only if $x^{(e)} = 0$. Let $P(k, 2\tau_j)$ denote the probability of k new arrivals (i.e., ready users) in a time slot. Then we have:

$$P(k, 2\tau_j) = \frac{(2\tau_j \lambda_{je})^k e^{-2\tau_j \lambda_{je}}}{k!} \quad k = 0, 1, \dots \quad (4.3)$$

where λ_{je} is the mean arrival rate to the j th outer-loop. Given our earlier assumptions, we have:

$$\text{Prob}(I^{(e+1)} = k | x^{(e)} = 0) = (1 - P(0, 2\tau_j)) (P(0, 2\tau_j))^{k-1} \quad k = 1, 2, \dots \quad (4.4)$$

$r^{(e+1)}$ is the number of slots in the contention resolution period including the initial collision slot. Using assumption 3 we have:

$$r^{(e+1)} = 0 \quad ; \quad \text{if no collision}$$

$$\begin{aligned} \text{Prob}[r^{(e+1)} = k / \text{collision occurred}] \\ = \sigma(1-\sigma)^{k-1}, \quad k = 1, 2, \dots \end{aligned} \quad (4.5)$$

Consequently the Laplace Transform of the probability density function of a contention period given that a collision occurred is:

$$C^*(s) = \frac{\sigma e^{-s(2\tau_j)}}{1 - (1-\sigma)e^{-s(2\tau_j)}} \quad (4.6)$$

which has a mean equal to $2\tau_j/\sigma$ and second moment equal to $(2\tau_j)^2((2-\sigma)/\sigma^2)$. The second moment given in [74] for the above distribution is found to be an error although numerically the error is negligible; hence it did not affect Lam's delay-throughput results.

Let $x^{(e+k)}$ denote the number of packets in the system at time point $t_e^{(e+k)}$, $k=0,1,2,\dots$. The following relationship is evident from Figure 4.1:

$$x^{(e+1)} = x^{(e)} + U^{(e+1)} + V^{(e+1)} + W^{(e+1)} - 1 \quad (4.7)$$

where:

$$\begin{aligned} \text{(i)} \quad -V^{(e+1)} \quad \text{is an independent random variable} \\ \text{with Z transform } \beta_1^* (\lambda_{je} - \lambda_{je}z) \end{aligned} \quad (4.8)$$

$$\begin{aligned} \text{(ii)} \quad -W^{(e+1)} \quad \text{is an independent}^* \text{ random variable} \\ \text{with Z transform } H^* (\lambda_{je} - \lambda_{je}z) \end{aligned} \quad (4.9)$$

* Note that this is an assumption.

(iii) $U^{(e+1)}$ is dependent on $x^{(e)}$ in the following manner. Let $x^{(e)}$ represent the number of ready terminals at time point $t_k^{(e)}$ (refer to Figure 4.1). For $x^{(e)}=0$, i.e., there are no ready terminals in the outer-loop, the outer-loop channel remains idle for a random time period $I^{(e+1)}$. In the last time slot of the idle period, at least one terminal becomes ready. At the end of this time slot, if only one terminal is ready then there will be no collision and a successful transmission time period starts. Therefore the number of ready terminals at the end of the time period $\phi^{(e+1)}$ is equal to unity (i.e., $U^{(e+1)}=1$). However, if more than one terminal becomes ready during the last time slot of the idle time period then there will be a collision with probability equal to unity. Therefore the number of ready terminals at the end of the time period $\phi^{(e+1)}$ is equal to the sum of the number of packets arriving during the last time slot of the idle period plus the number of packets arriving during the collision time period $r^{(e+1)}$. For $x^{(e)}=1$, there is one ready terminal in the outer-loop. Therefore we have $\phi^{(e+1)}=0$ which in turn implies $U^{(e+1)}=0$. For $x^{(e)} \geq 2$, a collision time period starts with probability equal to unity. Thus the number

of ready controllers at the end of the time period $\phi^{(e+1)}$ is increased by the number of packets arrived during the collision time period $r^{(e+1)}$. The dependency of $U^{(e+1)}$ on $x^{(e)}$ is summarized in the following:

- 1) for $x^{(e)} = 0$ and $P(k, 2\tau_j)$ defined by Equation (4.3), we have

$$U^{(e+1)} =$$

$$1 \text{ with probability: } \frac{P(1, 2\tau_j)}{1 - P(0, 2\tau_j)}$$

$$k + \text{number of arrivals during a contention period with probability: } \frac{P(k, 2\tau_j)}{1 - P(0, 2\tau_j)}$$

- 2) for $x^{(e)} = 1$ we have

$$U^{(e+1)} = 0 \quad (4.10)$$

- 3) finally, for $x^{(e)} \geq 2$ we have

$$U^{(e+1)} = \text{number of arrivals during a contention period}$$

Given that a collision occurred, the number of new arrivals during a contention period is an independent random variable with Z transform $C^*(\lambda_{je} - \lambda_{je}z)$.

$$E(z^U / x \geq 2) = C^*(\lambda_{je} - \lambda_{je}z) \quad (4.11)$$

Let the Utilization Factor be defined as the ratio of the rate at which packets enter the outer-loop to the maximum rate at which these packets can be transmitted on the outer-loop channel [64]. Recall that b_1 denotes the mean packet transmission time in the outer-loop, h_1

represents the mean of the random variable Θ and r_1 denotes the average collision time period. The maximum average rate at which packets can be transmitted on the outer-loop channel is equal to one packet per $(b_1 + h_1 + \tau_j + r_1)$ seconds (refer to Figure 4.1). Therefore the utilization factor for the j th outer-loop is equal to $\lambda_{je}(b_1 + h_1 + \tau_j + r_1)$.

The equilibrium queue length probability q_k

$$q_k = \lim_{\alpha \rightarrow \infty} \text{Prob}[x^{(e+\alpha)} = k] \quad , \quad \alpha = 0, 1, 2, \dots \quad (4.12)$$

exist if the utilization factor remains less than one [64] i.e.:

$$\lambda_{je}(b_1 + h_1 + \tau_j + 2\tau_j/\sigma) < 1 \quad (4.13)$$

Define the z transform:

$$q(z) = \sum_{k=0}^{\infty} q_k z^k \quad (4.14)$$

by considering Equations (4.7-4.10) and considering the limiting random variables when $\alpha \rightarrow \infty$ for $t_\ell^{(e+\alpha)}$, we obtain after some algebraic manipulations, (See Appendix F) the following results:

$$\begin{aligned} q(z) = & \beta_1^{-1} (\lambda_{je} - \lambda_{je} z) H^{\#}(\lambda_{je} - \lambda_{je} z) \\ & \cdot \{q_1 z[1 - C^{\#}(\lambda_{je} - \lambda_{je} z)] + \\ & \frac{q_0}{1 - P(0, 2\tau_j)} [P(1, 2\tau_j) z(1 - C^{\#}(\lambda_{je} - \lambda_{je} z)) \\ & - C^{\#}(\lambda_{je} - \lambda_{je} z) (1 - e^{-2\tau_j \lambda_{je}(1-z)})]\} \\ & \cdot [z - \beta_1^{-1} (\lambda_{je} - \lambda_{je} z) H^{\#}(\lambda_{je} - \lambda_{je} z) \\ & C^{\#}(\lambda_{je} - \lambda_{je} z)]^{-1} \quad (4.15) \end{aligned}$$

where:

$$q_0 = \frac{1 - \lambda_j e^{-(b_1 + h_1 + \tau_j + 2\tau_j/\sigma)}}{2\tau_j e^{-(b_1 + h_1 + \tau_j + 2\tau_j/\sigma)} \left(\frac{1}{1-P(0,2\tau_j)} - \frac{1}{\beta_1^*(\lambda_{je}) H^*(\lambda_{je})\sigma} \right)} \quad (4.16)$$

and:

$$q_1 = \left(\frac{1}{\beta_1^*(\lambda_{je}) H^*(\lambda_{je})\sigma} - \frac{P(1,2\tau_j)}{1-P(0,2\tau_j)} \right) q_0 \quad (4.17)$$

Using Equations (4.15-4.17), the mean queue size is obtained. Application of Little's result [64] yields the mean message delay (time of arrival to time of departure) for the j th outer-loop, denoted by d_j to be:

$$d_j = b_1 + h_1 + 2\tau_j + 2\tau_j/\sigma + \frac{1 - P(0,2\tau_j)}{2[\beta_1^*(\lambda_{je}) H^*(\lambda_{je})\sigma - (1-P(0,2\tau_j))]} \left(\frac{2}{\lambda_{je}} - 4\tau_j \right) + \frac{\lambda_{je} [(b_2 + h_2 + 2b_1 h_1 + 2\tau_j (b_1 + h_1) + \tau_j^2) + 2(b_1 + h_1 + \tau_j) (2\tau_j/\sigma) + 4\tau_j^2 (2/\sigma)^2]}{2[1 - \lambda_{je} (b_1 + h_1 + \tau_j + 2\tau_j/\sigma)]} \quad (4.18)$$

4.3 Numerical Results and Observations

An important performance parameter is the ratio of the carrier sense time τ_j to the mean message transmission time b_1 . This ratio is represented by a_j :

$$a_j = \frac{\tau_j}{b_1} \quad (4.19)$$

The throughput of the j th outer-loop is represented by S_j and is defined to be the fraction of channel time utilized by data messages which under equilibrium conditions is:

$$S_j = \lambda_j e^{-b_1} \quad (4.20)$$

Let γ_j represent the mean of the random variable θ normalized on the propagation delay τ_j i.e.:

$$\gamma_j = \frac{h_1}{\tau_j} \quad (4.21)$$

The requirement of (4.13) gives rise to the following upper bound on the channel throughput:

$$S_j < \frac{\sigma}{2a_j + (1 + a_j \gamma_j + a_j) \sigma} \quad (4.22)$$

Under equilibrium conditions the total impact of the inner-loop on the upper bound of the channel throughput of the j th outer-loop is summarized in the parameter γ_j . In the case where the controllers are provided with infinite buffer, the parameter, γ_j , vanishes to zero and the inner-loop does not have any impact on the upper-bound of S_j .

Figures 4.2 to 4.4 demonstrates the impact of the inner-loop's performance on the outer-loop's performance. These numerical results are calculated for an exponential probability distribution function for the random variable $\theta(e+1)$ with mean $h_1 = 0, .5, 1, 4$.

Figure 4.2 shows the impact of the inner-loop on the throughput of an outer-loop. Clearly the throughput of

Figure 4.2. The Impact of The Performance of The Inner-loop on The Throughput of The Outer-loop.

the outer-loop decreases as h_1 increases. For constant values of h_1 and as τ_j decreases, the throughput of the outer-loop S_j increases. This increase becomes insignificant as large values of h_1 yield small throughput values.

Figures 4.3 to 4.4 shows the impact of the inner-loop on the average delay of an outer-loop for $a=0.01$ and $a=0.1$ respectively. In Figure 4.3 for throughput level of 0.6 and $h_1=0$, the average delay is 2 whereas for the same throughput level and $h_1=0.5$ the average delay is 13, i.e., a drastic impact on the average delay. However, the impact on the average delay is less pronounced for small throughput levels. These figures demonstrate that when an outer-loop is operating at small throughput levels (i.e., $S_j < 0.1$), only large values of h_1 (i.e., $h_1 \geq 1$) have significant impact on the average outer-loop delay.

In the following chapter, the difficult problem of the distribution of the time period θ for which a controller's buffer remains full, is addressed.

4.4 Conclusions

This chapter presented an extension to the analytic model given by Lam [74]. We considered the protocol and the analytic model given in [74]. The model is extended to incorporate the impact of the inner-loop on the outer-loop. Furthermore, we have presented (in Appendix F) mathematical analysis for the outer-loop.

Figure 4.3. The Impact Of The Inner-loop's Performance on The Throughput-Delay of An Outer-loop.

Figure 4.4. The Impact of The Inner-loop's Performance on The Throughput-Delay of An Outer-loop.

CHAPTER 5

INTER-LOOP DEPENDENCE

The enforcement of the flow control mechanism gives rise to the interdependence between the inner-loop and each of the outer-loops. When the inner-loop channel is congested and a controller buffer becomes full, the outer-loop channel will be declared busy and hence its associated terminals are blocked. The outer-loop channel will remain busy until such time that the controller successfully transmits one or more packets.

In Chapter 3, a model was presented for the analysis of a number of finite buffer LAN protocols, i.e., the G-model. This model is employed as an approximation to the model of the inner-loop. In order to develop an exact model for the inner-loop, it is necessary to develop a mathematical model for the finite buffer LAN protocols in which the arrival process into the controllers has an arbitrary interarrival time distribution. Such a model, however, tends to be intractable. Thus as an alternative, we have employed the G-model as an approximation to the model of the inner-loop. It is important to note that the first moment of the interdeparture time distribution of the outer-loops (i.e., $1/\lambda_{je}$) is known as an exact value while all other moments of this probability distribution are approximations.

As we have shown in Chapter 4, the analysis of the outer-loop depends on knowledge of the mean h_1 , second moment h_2 and the Laplace transform $H^*(s)$ of the distribution of the random variable Θ for a given inner-loop operating point.

Derivation of the exact mathematical formula for the Laplace transform $H^*(s)$ of the distribution of the random variable Θ is analytically intractable. In the G-model presented in Chapter 3, Θ denoted the random variable representing the time during which the buffer of a controller is full. Clearly, the probability distribution of the random variable Θ in the G-model is an approximation of the probability distribution of the random variable Θ . In this chapter, we consider the large system size model of Chapter 3 and using some simplifying assumptions, we present the Laplace transform of the distribution of the random variable Θ .

Section 5.1 employs the large system size model of Chapter 3 and presents the distribution of the random variable Θ , under heavy traffic assumption (i.e., when the system operates near its throughput capacity, there exists at least two ready controllers in the system at all times). Due to the complexity of this distribution, we present an estimate for the distribution of the random variable Θ , in Section 5.2. Section 5.3 presents numerical results obtained for the mean and second moment of the random variable Θ . Simulation results are obtained

for the mean and second moment of the random variable $\hat{\theta}$ for two network configurations, namely a two-hop centralized packet radio network and a two-hop CATV network. They are compared with the numerical results obtained for the mean and the second moment of the random variable θ . Conclusions are presented in Section 5.4.

The following two remarks should be noted in the analyses to follow:

(i) As mentioned earlier and throughout this thesis, we concern ourselves not with the time-dependent behaviour (which is inferred by the superscript $(e+k)$) but rather with the limiting distributions for the random variables which we denote by eliminating the superscript $(e+k)$.

(ii) We employ the following notation:

$$\begin{aligned}\hat{\pi}(*, x_2, x_3) &= \sum_{0 \leq x_1 \leq N-1} \hat{\pi}(x_1, x_2, x_3) \\ \hat{\pi}(*, x_2, *) &= \sum_{0 \leq x_1 \leq N-1} \sum_{x_3=1,2} \hat{\pi}(x_1, x_2, x_3) \quad (5.1) \\ \hat{\pi}(+, x_2, x_3) &= \sum_{1 \leq x_1 \leq N-1} \hat{\pi}(x_1, x_2, x_3)\end{aligned}$$

5.1 Impact of the Inner-Loop Congestion on the Outer-Loop

Recall that the random variable $\hat{\theta}^{(e+k)}$ represents the time duration in which the buffer of a controller is full. The start of this random time period is the departure time of a packet from the outer-loop (or equivalently, the arrival time of a packet into the outer-loop's controller) which finds $L-1$ packets waiting in the buffer of that

controller. This random time period terminates when the controller successfully accomplishes the transmission of a packet for the first time.

New packets that arrive into the associated outer-loop during the time period $\hat{Q}(e+k)$ (refer to Figure 5.1) are not lost. They act as if they have collided and hence they postpone their arrival into the outer-loop until the channel is reset idle, at which point they join the packets waiting to gain channel access. The impact of the flow control mechanism on the outer-loop is reflected in the random variable $\hat{Q}(e+k)$. As stated in Chapter 4 each successful transmission time period in the snapshot of the outer-loop's broadcast channel (e.g., $\gamma(e+k)$ in Figure 4.1) is followed by the associated random time period $\hat{Q}(e+k)$ which can assume a zero value. Here we specify cases in which $\hat{Q}$ is greater than zero.

Consider the large system size model presented in Chapter 3. Recall that the model employs the state definition given by (3.23). Figures 5.1 to 5.4 depict snapshots of the inner-loop's broadcast channel for the selected controller for various cases where $\hat{Q}(e+k)$ is greater than zero. The associated sequences of subcycles are also presented. Based on the heavy traffic assumption, each subcycle is composed of a collision time period followed by a successful transmission time period. As shown in these figures, the random variable $\hat{Q}(e+k)$ is equal to the sum of the two random variables $\hat{Q}_1$ and $\hat{Q}_2$ i.e., we have:

Figure 5.1. Arrival of a Packet into the Buffer of the Selected Controller during a Successful Transmission Time Period. The arrival find the system in one of the states $\underline{x} = (x_1, L-1, 1)$.

Figure 5.2. Arrival of a packet into the Buffer of the Selected Controller during a Successful Transmission Time Period. The system is found to be in one of the states: $\underline{x} = (x_1, L-1, 2)$.

Figure 5.3. Arrival of a Packet into the Buffer of the Selected Controller during a collision time period. The arrival finds the system in one of the states: $\underline{x} = (x_1, L-1, 1)$.

Figure 5.4. Arrival of a Packet into the Buffer of the Selected Controller during a Collision Time Period. The arrival finds the System in one of the states: $\underline{x} = (x_1, L-1, 2)$.

$$\hat{\theta}(e+k-1) = \hat{\theta}_1(e+k) + \hat{\theta}_2(e+k) \quad (5.2)$$

where $\hat{\theta}_1(e+k-1)$ is equal to the random time period starting from the arrival time of the new packet until the following departure epoch $t_k(e+k)$, and $\hat{\theta}_2(e+k)$ is equal to the random time period starting from the departure epoch $t_k(e+k)$ until the departure epoch $t_k(e+k+j)$ such that the state of the system, for the first time is found to be one of the states $x=(x_1, L-1, 2)$ where $0 \leq x_1 \leq N-1$ (refer to the state definition given by (3.23)). Note that for $j=0$ (refer to Figure 5.2) we have $\hat{\theta}_2(e+k)=0$.

In Figures 5.1 and 5.2, the packet arrives into the buffer of the selected controller during an ongoing successful transmission time period, whereas in Figures 5.3 and 5.4, it arrives into the buffer during an ongoing collision time period. In Figures 5.1 and 5.3, the arrival finds the system in one of the states $x=(x_1, L-1, 1)$, $0 \leq x_1 \leq N-1$ i.e., one of the $N-1$ controllers is transmitting (or transmitted last), whereas in Figures 5.2 and 5.4 it finds the system in one of the states $x=(x_1, L-1, 2)$, i.e., the selected controller is transmitting (or transmitted last). These cases are summarized as follows:

Case C1. The new packet arrived during a successful transmission period (i.e., $\hat{\theta}_1 < T+1$).

Case C1₁. The system is found in one of the states $x=(x_1, L-1, 1)$, $0 \leq x_1 \leq N-1$.

Case C12. The system is found in one of the states $x=(x_1, L-1, 2)$, $0 \leq x_1 \leq N-1$ and $L \geq 2$.

Case C2. The new packet arrived during a collision time period (i.e., $\hat{\theta}_1 > T + 1$),

Case C21. The system is found in one of the states $x=(x_1, L-1, 1)$ $0 \leq x_1 \leq N-1$.

Case C22. The system is found in one of the states $x=(x_1, L-1, 2)$, $0 \leq x_1 \leq N-1$.

Separate treatment is required for the conditional Laplace transform of the distribution of the random variable $\hat{\theta}$ associated with each of the above cases. In order to establish the unconditional Laplace transform of the distribution of the random variable $\hat{\theta}$, it is necessary to specify the steady state distribution of the state vector x as seen by arrivals conditioned on the fact that the arrivals enter the j th controller. Let this distribution be represented by $\pi_{cj}(X)$.

Section 5.1.1 presents an estimate $\hat{\pi}_{cj}(X)$ for the distribution $\pi_{cj}(X)$. Furthermore, it presents an estimate for the steady state probability that $\hat{\theta}$ is nonzero for the selected controller. The distribution of the random variable $\hat{\theta}_1$ is presented in Section 5.1.2. Using an approach similar to that presented by Tobagi [137], the conditional Laplace transform of the distribution of the random variable $\hat{\theta}$ for each of the above four cases are given in Section 5.1.3. The Laplace transform of the

distribution of the random variable θ is given in Section 5.1.4.

5.1.1 The Distribution $\pi_{cj}(X)$

As mentioned in Chapter 3, $\pi_l(X)$ represents the distribution of the state vector x as seen by an arrival, conditioned on the fact that the arrival is not blocked. $\pi_{cj}(X)$ denotes the distribution of the state vector x as seen by those arrivals that enter the j th controller. In general, $\pi_l(X)$ is not necessarily equal to $\pi_{cj}(X)$. Clearly, for an infinite buffer, multiqueue, multiserver system in a homogeneous environment, these limiting distributions are equal, i.e., $\pi_l(X) = \pi_{cj}(X)$, $1 \leq j \leq N$. However, for a single server, infinite buffer multiqueue system in a homogeneous environment, this equality is maintained if the server does not enforce any service priority discipline. In the single server, finite buffer, multiqueue system presented in Chapter 3, we have $\pi_l(X) \neq \pi_{cj}(X)$, $1 \leq j \leq N$. This is because $\pi_l(*, L, *) \neq 0$ (i.e., an arrival may enter the system and just before its entry, it observes L packets stored in the buffer of the j th controller) whereas $\pi_{cj}(*, L, *) = 0$ (i.e., an arrival may enter the j th controller) and just before its entry, it may not observe L packets stored in the buffer of j th controller). Consider the multiqueue system described in Chapter 3 and let $\pi_c(X)$ denote the distribution of the state vector x as seen by arrivals that enter the selected controller. We estimate $\pi_c(X)$ by $\hat{\pi}_c(X)$ defined as

follows:

$$\pi_c(\underline{x}) = \frac{\pi_k(\underline{x})}{1 - \pi_k(*, L, *)} \quad (5.3)$$

$$\underline{x} \in X, \underline{x} = (x_1, x_2, x_3)$$

$$0 \leq x_1 \leq N-1, 0 \leq x_2 \leq L-1,$$

$$x_3 = 1, 2$$

Note that $\pi_c(\underline{x})$ is approximated by the steady state distribution of the state vector $\underline{x}$ as seen by an arrival into the system, conditioned on the fact that the arrival found $L-1$ or less number of packets in the buffer of the selected controller.

The limiting probability, that a nonzero $\hat{\theta}$ is initiated by the selected controller, is estimated by $\hat{\pi}_c(*, L-1, *)$. Note that this estimate is biased for $L=1$ as we have $\hat{\pi}_c(*, L-1, *) = 1$. In this case, $\hat{\pi}_c(*, L-1, *)$ (i.e., $\hat{\pi}_c(*, 0, *)$) incorporates in itself the steady state probability that an arrival finds the system empty (and therefore $\hat{\theta} = 0$).

5.1.2 Distribution of the Random Variable $\hat{\theta}_1$

Recall that the random variable $\hat{\theta}_1$ represents a time period which starts from the arrival time of a new packet that enter the selected controller (with $L-1$ packet stored in its buffer). The time period $\hat{\theta}_1$ terminates at the following departure instant $t_d^{(e+k)}$ (refer to Figures 5.1 to 5.4). Under the heavy traffic assumption and large values of L (i.e., $L > 1$), the interdeparture times are represented by the random variable $t_{id}^{(e+k)}$, $k=0, 1, 2, \dots$

Clearly, we have $t_{id}^{(e+k)} = r^{(e+k)} + T + \tau$, $k=0,1,2,\dots$ and the interdeparture times $t_{id}^{(e+k)}$, $k=0,1,\dots$ form a renewal process [74] i.e., $t_{id}^{(e+k)}$, $k=0,1,\dots$ are independent and identically distributed random time periods. Define $F(t_{id})$ to represent the limiting distribution function of $t_{id}^{(e+k)}$, $k=0,1,2,\dots$. Also consider the distribution of the random variable r given by equations (C.9) of Appendix C. Since $t_{id} = r + T + \tau$, we have

$$\begin{aligned}
 F(t_{id} < y) &= \text{Prob}((r + T + \tau) < y) \\
 &= \text{Prob}(r < y - (T + \tau)) \\
 &= \begin{cases} \sum_{k=T+\tau}^{y-(T+\tau)} \sigma(1-\sigma)^{k-1} & \text{for } k=T+\tau, (T+\tau)+2\tau, \\ & (T+\tau)+4\tau, \dots \\ 0 & k \leq (T+\tau) \end{cases} \quad (5.4)
 \end{aligned}$$

Let $R^*(s)$ denote the Laplace transform of the distribution of the random variable r with mean r_1 and the second moment r_2 given by equations (C.12), (C.13) and (C.14) of Appendix C respectively. Also let $F^*(s)$ denote the Laplace transform of the distribution of the random variable t_{id} with mean θ_1 and the second moment θ_2 . Using (5.4) we have:

$$F^*(s) = R^*(s) e^{-s(T+\tau)} \quad (5.5)$$

Using this equation, the mean θ_1 of the random variable t_{id} is given by:

$$\theta_1 = r_1 + T + \tau \quad (5.6)$$

and the second moment σ_2 of the random variable t_{id} is given by:

$$\sigma_2 = r_2 - 2(r_1)(T + \tau) + (T + \tau)^2 \quad (5.7)$$

Using the heavy traffic assumption, the distribution of the random variable $\hat{\theta}_1$ is equal to the distribution of the residual life of a component with life time equal to the interdeparture time t_{id} . Let $f(\hat{\theta}_1)$ represent the density function of the random variable $\hat{\theta}_1$. $f(\hat{\theta}_1)$ is given by [64]:

$$f(\hat{\theta}_1) = (1 - \psi(\hat{\theta}_1)) / \sigma_1 \quad (5.8)$$

Let $\hat{F}^*(s)$ denote the Laplace transform of $f(\hat{\theta}_1)$. $\hat{F}^*(s)$ is given by:

$$\hat{F}^*(s) = (1 - R^*(s)e^{-s(T + \tau)}) / \sigma_1 s \quad (5.9)$$

Let $\hat{\theta}_1^{(n)}$ denote the nth moment of the random variable $\hat{\theta}_1$. Using (5.9), $\hat{\theta}_1^{(n)}$ is given by:

$$\hat{\theta}_1^{(n)} = \frac{E((t_{id})^n)}{(n+1) E(t_{id})} \quad (5.10)$$

Therefore

$$\hat{\theta}_1^{(1)} = \frac{E((t_{id})^2)}{2E(t_{id})} \quad (5.11)$$

and

$$\hat{\theta}_1^{(2)} = \frac{E(t_{id}^3)}{3E(t_{id})} \quad (5.12)$$

The probability that $\hat{\theta}_1$ is less than or equal to $T+\tau$ is of particular interest to us and it is calculated by substituting (5.4) into (5.8) and integrating over the range zero to $T+\tau$:

$$\begin{aligned} \text{Prob}(\hat{\theta}_1 < T+\tau) &= \int_0^{T+\tau} \frac{1 - \psi(\hat{\epsilon}_1)}{r_1 + \tau + T} d\hat{\epsilon}_1 = \int_0^{T+\tau} \frac{1}{r_1 + \tau + T} d\hat{\epsilon}_1 \\ &= (T+\tau)/(r_1 + \tau + T) \end{aligned} \quad (5.13)$$

Similarly, the probability that $\hat{\theta}_1$ is greater than $T+\tau$ is given by:

$$\begin{aligned} \text{Prob}(\hat{\theta}_1 \geq T+\tau) &= 1 - \text{Prob}(\hat{\theta}_1 < T+\tau) \\ &= (r_1)/(r_1 + T + \tau) \end{aligned} \quad (5.14)$$

5.1.3 Conditional Laplace Transforms

(a) The Conditional Laplace Transform $H_{x,11}^*(s)$

Recall that in this case, the conditions are (Figure 5.1):

- 1) the new packet arrive during a successful transmission period (i.e., $\hat{\theta}_1 < T + \tau$).
- 2) the new arrival that is going to enter the selected controller, finds the system in one of the states $\underline{x} = (x_1, L-1, 1)$, $0 \leq x_1 \leq N-1$

Theorem 5.1

The conditional Laplace transform of the distribution of the random variable $\hat{\theta}$ for this case is given by:

$$H_{\underline{x},1}^*(s) = \begin{cases} \int_0^{T+\tau} e^{-s\hat{t}_1} \left[\sum_{\underline{z} \in X} \text{Prob}_{\underline{x},\hat{t}_1}(\underline{z}|\underline{x}) H_{\underline{z}}^*(s) \right] d\hat{t}_1 \\ 0 \end{cases} \quad (5.15)$$

where $\int_0^{T+\tau} f(\hat{t}_1) d\hat{t}_1 / \text{Prob}(\hat{t} < T+\tau)$

- i) $\underline{x} = (x_1, L, 1)$, $\underline{z} = (z_1, L, 1)$ and $z_1 \geq x_1$
 ii) $\text{Prob}(\underline{z}|\underline{x}) = P(n^*(\underline{x})) \cdot \text{Prob}((z_1 - x_1 + 1), 1, \hat{\theta}_1) +$
 $\frac{x_1}{\hat{t}_1} (1 - P(n^*(\underline{x}))) \cdot \text{Prob}((z_1 - x_1 + 1), 1, \hat{\theta}_1)$

where $P(n^*(\underline{x}))$ is given by equation (C.1) in Appendix C. $\text{Prob}(k, 1, \hat{\theta}_1)$ is similar to equation (C.8) in Appendix C and is given by:

$$\text{Prob}(k, 1, \hat{\theta}_1) = \binom{N-x_1-1}{k} (1 - e^{-\hat{\theta}_1})^k (e^{-\hat{\theta}_1})^{N-x_1-k-1}$$

for $k=0, 1, \dots, N-x_1-1$

- (iii) $H_{\underline{z}}^*(s)$ is the $\underline{z}$ th element of the vector $H^*(s)$
 where:

$$H^*(s) = [I - P_4(s)]^{-1} \cdot P_3(s) \cdot P_5$$

I is the identity matrix and

$$[P_3(s)] = \sum_{\underline{z}, \underline{y}} \sum_{\underline{x}' \in X} [P_2]_{\underline{z}, \underline{x}'} (1/n^*(\underline{x}')) R^*(s) e^{-s(T+\tau)}$$

$$[P_1]_{\underline{x}', \underline{y}'}$$

$$[P_4(s)] = \sum_{\underline{z}, \underline{y}} \sum_{\underline{x}' \in X} [P_2]_{\underline{z}, \underline{x}'} (1 - 1/n^*(\underline{x}')) \cdot R^*(s) e^{-s(T+\tau)}$$

$$[P_1]_{\underline{x}', \underline{y}}$$

where

$$x' = (x_1', L, 1) \text{ \& } x_1' \geq z_1$$

$$y' = (y_1', L-1, 1) \text{ \& } y_1' \geq x_1'$$

$$\bar{y} = (y_1, L, 2) \text{ \& } y_1 \geq x_1' - 1$$

and P_5 is a column vector of ones.

(iv) $f(\theta_1)$ is given by equation (5.8).

(v) $\text{Prob}(\theta < T + \tau)$ is given by equation (5.13).

Proof:

Let $H_{x,11,\hat{\theta}_1}^*(s)$ denote the conditional Laplace transform

for the distribution of the random variable θ conditioned on the duration of θ_1 . Since $\theta_1 < T + \tau$, we have:

$$\begin{aligned} H_{x,11}^*(s) &= E(e^{-s(\hat{\theta}_1 + \hat{\theta}_2)} | C11, x) \\ &= \frac{\int_0^{T+\tau} E(e^{-s(\hat{\theta}_1 + \hat{\theta}_2)} | C11, x, \hat{\theta}_1 = \hat{\theta}_1) f(\hat{\theta}_1) d\hat{\theta}_1}{\text{Prob}(\hat{\theta}_1 \leq T + \tau)} \end{aligned} \quad (5.16)$$

now we are faced with the task of writing the expression for the expectation $E(e^{-s(\hat{\theta}_1 + \hat{\theta}_2)} | C11, x, \hat{\theta}_1 = \hat{\theta}_1)$.

Figure 5.1 depicts a snapshot of the inner-loop's channel. Consider the time point $t_k^{(e+k)}$, i.e., the first departure instant after the arrival of the packet. Also consider the arrival time of the packet i.e., time point $t_k^{(e+k)} - \theta_1$. The state transition probability row for the state vector x between the time points $t_k^{(e+k)} - \theta_1$ and $t_k^{(e+k)}$ is represented by $\text{Prob}_{x, \hat{\theta}_1}(z | x)$. Let $H_z^*(s)$ denote

the Laplace transform of the distribution of the random variable $\tilde{\theta}_2$ conditioned on the event that the state of the system is $\underline{z}$ at time point $t_l^{(e+k)}$. Considering that $\tilde{\theta}_1$ is a constant, we have

$$E(e^{-s(\tilde{\theta}_1 + \tilde{\theta}_2)} | C11, \underline{x}, \tilde{\theta}_1 = \tilde{\theta}_1) = e^{-s\tilde{\theta}_1} \cdot \sum_{\underline{z}} \text{Prob}(\underline{z} | \underline{x}, \tilde{\theta}_1) H_{\underline{z}}^{\#}(s) \quad (5.17)$$

where $\underline{x} = (x_1, L, 1)$ and $\underline{z} = (z_1, L, 1)$ and $z_1 \geq x_1$.

The state transition probability between the time points $t_l^{(e+k)} - \tilde{\theta}_1$ and $t_l^{(e+k)}$ depends on the number of packets found in the buffer of the transmitting controller. As we have shown in Appendix C, the probability that the transmitting controller has one packet in its buffer is denoted by $P(n^{\#}(\underline{x}))$, therefore we have

$$\text{Prob}(\underline{z} | \underline{x}) = P(n^{\#}(\underline{x})) \cdot \text{Prob}((z_1 - x_1 + 1), 1, \tilde{\theta}_1) + (1 - P(n^{\#}(\underline{x}))) \cdot \text{Prob}((z_1 - x_1), 1, \tilde{\theta}_1) \quad (5.18)$$

where

$$\text{Prob}(k, 1, \tilde{\theta}_1) = \binom{N-x_1-1}{k} (1 - e^{-\tilde{\theta}_1 \lambda_j})^k (e^{-\tilde{\theta}_1 \lambda_j})^{N-x_1-k-1} \quad \text{for } k=0, 1, \dots, N-x_1-1 \quad (5.19)$$

At this stage we are faced with the problem of writing an expression for $H_{\underline{z}}^{\#}(s)$. Given that the state of the system at time point $t_l^{(e+k)}$ is $\underline{z} = (z_1, L, 1)$, $0 \leq z_1 \leq N-1$, the state of the system at time point $t_u^{(e+k)}$ is $\underline{x}' = (x_1', L, 1)$

and $x_1' \geq z_1$ with the associated state transition probabilities $[P2]_{\underline{z}, \underline{x}'}$ (for the state transition probability matrix $P2$, refer to Figure 3.2). At this stage, either the selected controller is chosen with probability $1/n^*(x')$ or one of the controllers among the x_1' ready controllers is selected with probability $(1-1/n^*(x'))$. In the former case, the selected controller is chosen and after a successful transmission time period, the state of the system is found to be one of the states $y' = (y_1', L-1, 2)$ & $y_1' \geq x_1'$. The associated state transition probability matrix for this time period (refer to Figure 3.2) is equal to $[P1]_{\underline{x}', \underline{y}'}$. In the latter case, one of the controllers among the x_1' controllers is chosen to transmit successfully and after a successful transmission time period, the system is found to be in one of the states $y = (y_1, L-1, 2)$ and $y_1 \geq x_1'$. At this time point this procedure is repeated all over again until such time that the selected controller is chosen to transmit successfully. The conditional Laplace transform of the distribution of the random variable θ_2 for the above two cases are given by equations (5.20) and (5.30) respectively:

$$H_{\underline{z}, \underline{y}'}^*(s) = \sum_{\underline{x}'} [P2]_{\underline{z}, \underline{x}'} \cdot (1/n^*(x')) \cdot R^*(s) \cdot e^{-s(T+\tau)} \cdot [P1]_{\underline{x}', \underline{y}'} \quad (5.20)$$

$$H_{\underline{z}, \underline{y}}^*(s) = \sum_{\underline{x}'} [P2]_{\underline{z}, \underline{x}'} \cdot (1-1/n^*(x')) \cdot R^*(s) \cdot e^{-s(T+\tau)} \cdot [P1]_{\underline{x}', \underline{y}} \cdot H_{\underline{y}}^*(s) \quad (5.21)$$

where

$$x' = (x_1', L, 1)$$

$$z = (z_1, L, 1) \text{ and } z_1 \leq x_1'$$

$$y' = (y_1', L-1, 1) \text{ and } y_1' \geq x_1'$$

$$y = (y_1, L-1, 2) \text{ and } y_1 \geq x_1' - 1$$

Using the matrices $P3(s)$ and $P4(s)$ defined in part (iii) of the above theorem, we have:

$$H_z^*(s) = \left[\sum_{y' \in X} [P3(s)]_{z, y'} \right] + \left[\sum_{y \in X} [P4(s)]_{z, y} \cdot H_y^*(s) \right] \quad (5.22)$$

Define $H_z^*(s)$ to be a column vector with z th element equal to $H_z^*(s)$. Also, let $P5$ denote a column vector of one's. Therefore we have:

$$H_z^*(s) = P3(s) \cdot P5 + P4(s) \cdot H_z^*(s) \quad (5.23)$$

hence

$$H_z^*(s) = [I - P4(s)]^{-1} \cdot P3(s) \cdot P5 \quad (5.24)$$

Q.E.D.

(b) The Conditional Laplace Transform $H_{x,12}^*(s)$

In this case, the conditions are (Figure 5.2):

- 1 - the new packet arrives during a successful transmission period (i.e., $\theta_1 < T + \tau$)
- 2 - the new arrival, that is going to enter to the selected controller, finds the system is one of the states $x = (x_1, L-1, 2)$, $0 \leq x_1 \leq N-1$.

Note that, this case is relevant when $L > 1$. For $L=1$, no arrival may enter the selected controller during a successful transmission period associated with this controller.

Theorem 5.2

The conditional Laplace transform of the distribution of the random variable θ for this case is given by:

$$H_{\mathbf{x},12}^{\theta}(s) = \sum_{\mathbf{y}'} \left\{ \frac{\int_0^{T+\tau} [e^{-s\hat{e}_1} \text{Prob}_{\mathbf{x},\hat{e}_1}(y'_1 - x_{1,1}, \hat{e}_1)] \hat{f}(\hat{e}_1) d\hat{e}_1}{\text{Prob}(\hat{e}_1 < T + \tau)} \right\} \quad (5.25)$$

where

$$\mathbf{y}' = (y'_1, L-1, 2) \text{ \& } y'_1 \geq x_1$$

and $\hat{f}(\hat{e}_1)$, $\text{Prob}(\hat{e}_1 < T + \tau)$ and $\text{Prob}_{\mathbf{x},\hat{e}_1}(K, 1, \hat{e}_1)$ are given by equations (5.8), (5.13) and (5.15) respectively.

Proof:

In this case, the new packet arrives during a successful transmission time period associated with the selected controller. Therefore at $t_k^{(e+k)}$, the system will be in one of the states $\mathbf{y}'$ and $\hat{\theta}_2 = 0$. Hence

$$H_{\mathbf{x},12}^{\theta}(s) = E(e^{-s(\hat{\theta}_1 + \hat{\theta}_2)} | C12, \mathbf{x}) \quad (5.26)$$

$$= \frac{\int_0^{T+\tau} E(e^{-s\hat{e}_1} | C12, \mathbf{x}, \hat{e}_1) \hat{f}(\hat{e}_1) d\hat{e}_1}{\text{Prob}(\hat{e}_1 < T + \tau)}$$

where

$$E(e^{-s\hat{\theta}_1} | C12, \underline{x}, \hat{\theta}_1 = \bar{\theta}_1) = \sum_{\underline{y}'} e^{-s\hat{e}_1} \text{Prob}_{\underline{x}, \hat{e}_1} (y'_1 - x_{1,1}, \hat{e}_1) \quad (5.27)$$

Q.E.D.

(c) The Conditional Laplace Transform $H^*(s)_{\underline{x}, 21}$

In this case, the conditions are (Figure 5.3):

- 1 - the new packet that is going to enter the selected controller arrives during a collision time period (i.e., $\hat{\theta}_1 > T + \tau$).
- 2 - it finds the system in one of the states $\underline{x} = (x_1, L-1, 1)$, $0 \leq x_1 \leq N-1$

Theorem 5.3

The conditional Laplace transform of the random variable $\hat{\theta}$ for this case is given by:

$$H^*(s)_{\underline{x}, 21} = \int_{\hat{e}_1 = T + \tau}^{\infty} e^{-s\hat{e}_1} \left\{ \sum_{\underline{x}''} \text{Prob}_{\underline{x}, \hat{e}_1} ((x''_1 - x_1), 1, \hat{e}_1 - (T + \tau)) \cdot [1/n^*(\underline{x}'')] \sum_{\underline{y}'} [P1]_{\underline{x}'', \underline{y}'} + (1 - 1/n^*(\underline{x}'')) \sum_{\underline{z}} [P1]_{\underline{x}'', \underline{z}} \cdot H^*(s)_{\underline{z}} \right\} f(\hat{e}_1) d\hat{e}_1 / \text{Prob}(\hat{e}_1 \geq T + \tau) \quad (5.28)$$

where $f(\hat{\theta}_1)$, $H^*(s)_{\underline{z}}$ and $\text{Prob}(\hat{\theta} \geq T + \tau)$ are defined in equations (5.8), (5.15) and (5.13) respectively and

$\underline{x}'' = (x_1'', L, 1)$ and $x_1'' \geq x_1$.

$\text{Prob}(k, 1, \hat{\theta}_1 - (T + \tau))$ is similar to equation (C.8) in Appendix C and is given by:

$$\text{Prob}(k, 1, \hat{\theta}_1 - (T + \tau)) = \binom{N - x_1 - 1}{k} (1 - e^{-(\hat{\theta}_1 - (T + \tau))})^j e^{-k(\hat{\theta}_1 - (T + \tau))} e^{-(\hat{\theta}_1 - (T + \tau))} \binom{N - x_1 - k - 1}{j}$$

Proof:

Assume that the system is found in the state $\underline{x}$, at time point $t_a^{(e+k-1)}$ (refer to Figure 5.3) and the value of $\hat{\theta}_1$ is equal to $\hat{\theta}_1$. After a time duration $\hat{\theta}_1 - (T + \tau)$, i.e., at time point $t_u^{(e+k-1)}$, the system is found in one of the states $\underline{x}''$ where $\underline{x}'' = (x_1'', L, 1)$, $x_1'' \geq x_1$. At this time point, either the selected controller is chosen with probability $1/n^*(\underline{x}'')$ in which case we have $\hat{\theta}_2 = 0$, or one of the x_1'' ready controllers is chosen (to transmit successfully) with probability $(1 - 1/n^*(\underline{x}''))$ and after a time period of $T + \tau$, The system is found in one of the states $\underline{z} = (z_1, L, 2)$ and $z_1 \geq x_1''$ and consequently $\hat{\theta}_2 > 0$. Clearly, in the latter case, the conditional Laplace transform of the distribution of the random variable $\hat{\theta}_2$ is equal to $H_{\underline{z}}^*(s)$ given by equation (5.22) presented in Theorem 5.1. Hence we have:

$$\begin{aligned}
H_{x,21}^*(s) = & \left\{ \int_{\hat{\theta}_1 = T+\tau}^{\infty} e^{-s(\hat{\theta}_1 - (T+\tau))} \left[\sum_{x''} \text{Prob}_{x, \hat{\theta}_1}((x'' - x_1), 1, \hat{\theta}_1 - (T+\tau)) \right. \right. \\
& \left. \left[(1/n^*(x'')) \sum_{y'} [P1]_{x'', y'} e^{-s(T+\tau)} \right. \right. \\
& \left. \left. + (1 - 1/n^*(x'')) \sum_{z} [P1]_{x'', z} e^{-s(T+\tau)} H_{z,21}^*(s) \right] \right. \\
& \left. \left. \hat{f}(\hat{\theta}_1) d\hat{\theta}_1 \right\} / \text{Prob}(\hat{\theta}_1 \geq T + \tau) \quad (5.29)
\end{aligned}$$

$e^{-s(T+\tau)}$ could be taken out and therefore:

$$\begin{aligned}
H_{x,21}^*(s) = & \left\{ \int_{\hat{\theta}_1 = T+\tau}^{\infty} e^{-s\hat{\theta}_1} \sum_{x''} \text{Prob}_{x, \hat{\theta}_1}((x'' - x_1), 1, \hat{\theta}_1 - (T+\tau)) \right. \\
& \left[(1/n^*(x'')) \sum_{y'} [P1]_{x'', y'} \right. \\
& \left. + (1 - 1/n^*(x'')) \sum_{z} [P1]_{x'', z} e^{-s(T+\tau)} H_{z,21}^*(s) \right] \\
& \left. \left. \hat{f}(\hat{\theta}_1) d\hat{\theta}_1 \right\} / \text{Prob}(\hat{\theta}_1 \geq T + \tau) \quad (5.30)
\end{aligned}$$

Q.E.D.

(d) The Conditional Laplace Transform $H_{x,22}^*(s)$

In this case, the conditions are (Figure 5.4):

- 1 - the new packet that is going to enter the selected controller, arrives during a collision period (i.e., $\hat{\theta}_1 > T + \tau$)

Figure 5.16. Two Hop CATV Network:
 h_2 Versus System Throughput.

Note that for $L=1$, no arrival may enter the selected controller during a successful transmission time period associated with this controller. Thus $H^*(s)$ is given by:

$$H^*(s) = \text{Prob}(\Theta_1 < T + \tau)$$

$$\left[\sum_{\mathbf{x}} \frac{c(\mathbf{x}_1, 0, 1)}{\mathbf{x}, 11} H^*(s) + \sum_{\mathbf{x}} \frac{c(\mathbf{x}_1, 0, 2)}{\mathbf{x}, 12} H^*(s) \right]$$

$$+ \text{Prob}(\Theta_1 \geq T + \tau)$$

$$\left[\sum_{\mathbf{x}} \frac{c(\mathbf{x}_1, 0, 1)}{\mathbf{x}, 21} H^*(s) + \sum_{\mathbf{x}} \frac{c(\mathbf{x}_1, 0, 2)}{\mathbf{x}, 21} H^*(s) \right]$$

(5.32)

5.2 An Estimate For $H^*(s)$

The mathematical equation for $H^*(s)$ presented in the previous section is cumbersome and complicated. However, using this mathematical equation, an estimate is presented for $H^*(s)$.

5.2.1 Estimates for the Conditional Laplace Transforms

(I) An Estimate for the Conditional Laplace Transform

$$\frac{H^*(s)}{\mathbf{x}}$$

We assume that under heavy input traffic a constant number of n controllers are ready to transmit at time points $t_u(e+k)$. Hence the probability that the selected controller (given that its buffer is full) is chosen to

transmit next, is equal to a constant value and it is equal to $1/n$. Based on this assumption we have:

From The selected controller gains channel access after k subcycles of duration t_{1d}

$$= (1/n)^{k-1} \cdot 1/n \quad k=1, 2, \dots \quad (5.33)$$

$n > 1$

For constant value n , the random variable k is independent from the random variable n^{e+1} and Θ_2 is a random sum of random variables $t_{1d} \cdot n^{e+1}$ hence we have:

$$\Theta_2 = \sum_{i=1}^k t_{1d} n^{e+1} = \sum_{i=1}^k n^{e+1} T_i \quad (5.34)$$

k is a geometrically distributed random variable with z transform $K(z)$, hence we have

$$K(z) = \frac{1/n \cdot z}{1 - (1-1/n)z} \quad (5.35)$$

on the other hand consider the Laplace transform of the random variable $T_i + T_i \cdot n$ given by (5.5). Substitution of equation (5.12) into equation (5.5) yields:

$$f^*(s) = e^{-s(T_i + T_i \cdot n)} \quad R^*(s) = \frac{e^{-s(T_i + T_i \cdot n)}}{1 - e^{-s(T_i + T_i \cdot n)}} \quad (5.36)$$

Let $\tilde{f}^*(s)$ denote the conditional Laplace transform of the random variable Θ_2 given $\Theta_2 > 0$. $\tilde{f}^*(s)$ is equal to:

$$\tilde{f}^*(s) = K(z) \quad z = \tilde{f}^*(s)$$

$$H^*(s) = \frac{1-n}{1-(1-n)e^{-2Ts}} - (1-n) \quad (5.37)$$

Let μ_1 and μ_2 denote the mean and the second moment of the random variable Θ_2 . Using (5.37) the mean μ_1 is obtained and it is equal to:

$$\mu_1 = n(T+2T/T) \quad (5.38)$$

the second moment μ_2 is equal to:

$$\mu_2 = 2[n/2(T+T)]^2 - n[8^{-2}/T + 4T/T + (T+T)^2] \quad (5.39)$$

(II) An Estimate for the Conditional Laplace Transform

$$H_{X,1}^*(s)$$

Approximating $H_{X,1}^*(s)$ by $H^*(s)$ and substituting it in equation (5.15), an estimate $H_{X,1}^*(s)$ for $H_{X,1}^*(s)$ is given as follows:

$$\begin{aligned} H_{X,1}^*(s) &= \int_0^{T+} e^{-s\tau} \{1 - \text{Prob}_{X,1}(\tau)\} d\tau \\ &= \int_0^{T+} e^{-s\tau} \{1 - \text{Prob}_{X,1}(\tau)\} d\tau \\ &= L^*(s) L^*(s) \end{aligned} \quad (5.40)$$

where $L^*(s)$ is the conditional Laplace transform of the random variable Θ_1 , given $\Theta_1 < T+$, and is defined as follows:

$$\phi(s) = \int_0^{T+\tau} e^{-s t_1} f(t_1) dt_1 \text{Prob}(t_1 < T+\tau)$$

Substituting $f(\theta_1)$ using equation (5.8) in the above equation, we have:

$$\phi(s) = \int_0^{T+\tau} e^{-s t_1} \frac{1 - e^{-s(t_1 - T)}}{s} dt_1 \text{Prob}(t_1 < T+\tau)$$

but equation (5.4) suggests that

$$\phi(s) = 0 \text{ for } 0 \leq \theta_1 \leq T+\tau \quad (5.41)$$

hence

$$\phi(s) = \int_0^{T+\tau} e^{-s t_1} f(t_1) dt_1 \text{Prob}(t_1 < T+\tau)$$

Therefore we have:

$$\phi(s) = \frac{\int_0^{T+\tau} e^{-s t_1} f(t_1) dt_1}{\text{Prob}(t_1 < T+\tau)} = \frac{1 - e^{-s(T+\tau)}}{s \bar{s}_1 \text{Prob}(t_1 < T+\tau)} \quad (5.42)$$

Let $\bar{s}_1$ and $\bar{s}_2$ denote the mean and the second moment of the conditional Laplace transform $\phi(s)$. Using (5.42), $\bar{s}_1$ and $\bar{s}_2$ are given as follows:

$$\bar{s}_1 = \frac{(T+\tau)^2}{2(2\tau/\sigma + \tau + T) \text{Prob}(\theta_1 < T+\tau)} \quad (5.43)$$

and second moment,

$$\bar{s}_2 = \frac{(T+\tau)^3}{3(2\tau/\sigma + \tau + T) \text{Prob}(\theta_1 < T+\tau)} \quad (5.44)$$

(III) An Estimate for the Conditional Laplace Transform

$$H_{x,12}^*(s)$$

Let $H_{x,12}^*(s)$ denote an estimate for $H_{x,12}^*(s)$ and it is

defined as:

$$H_{x,12}^*(s) = \int_0^{T+\tau} e^{-s\hat{e}_1} f(\hat{e}_1) d\hat{e}_1 / \text{Prob}(\hat{e}_1 < T+\tau) \quad (5.45)$$

using equation (5.42) we have:

$$H_{x,12}^*(s) = \Delta^*(s) \quad (5.46)$$

(IV) An Estimate for the Conditional Laplace Transform

$$H_{x,21}^*(s)$$

Let $H_{x,21}^*(s)$ denote an estimate for $H_{x,21}^*(s)$. Substituting

$H_{x,21}^*(s)$ by $\Delta^*(s)$ in equation (5.28), we have:

$$\begin{aligned} H_{x,21}^*(s) &= \int_{\hat{e}_1=T+\tau}^{\infty} e^{-s\hat{e}_1} [1/n + (1-1/n)\Delta^*(s)] f(\hat{e}_1) d\hat{e}_1 / \text{Prob}(\hat{e}_1 > T+\tau) \\ &= [(1/n) + (1-1/n)\Delta^*(s)] \cdot \int_{\hat{e}_1=T+\tau}^{\infty} e^{-s\hat{e}_1} f(\hat{e}_1) d\hat{e}_1 / \text{Prob}(\hat{e}_1 > T+\tau) \end{aligned} \quad (5.47)$$

Let $\Delta^*(s)$ denote the conditional Laplace transform of the random variable $\hat{e}_1$ given $\hat{e}_1 > T+\tau$. Let v_1 and v_2 denote its first and its second moments respectively. $\Delta^*(s)$ is given as follows:

$$\Lambda^*(s) = \frac{\int_{T+\tau}^{\infty} e^{-s\tau} f(\tau) d\tau}{\text{Prob}(\tau > T+\tau)} = \frac{[F^*(s) - \int_0^{T+\tau} e^{-s\tau} f(\tau) d\tau]}{\text{Prob}(\tau > T+\tau)}$$

Using (5.9), we have:

$$\Lambda^*(s) = \frac{e^{-s(T+\tau)} (1-R^*(s))}{\int_0^{T+\tau} e^{-s\tau} f(\tau) d\tau} \quad (5.48)$$

Substituting (5.48) into (5.47) we have:

$$H^*(s) = \Lambda^*(s) [(1/n) + (1-1/n) \Lambda^*(s)] \quad (5.49)$$

the mean v_1 is equal to:

$$v_1 = \frac{2(T+\tau)(2\tau/\sigma) + (2\tau)^2((2-\sigma)/\sigma^2)}{2(2\tau/\sigma + T+\tau) \text{Prob}(\tau > T+\tau)} \quad (5.50)$$

and the second moment v_2 is equal to:

$$v_2 = \frac{3(2\tau/\sigma)(T+\tau)^2 + 3(T+\tau)(2\tau)^2((2-\sigma)/\sigma^2) + (2\tau)^3(6-6\sigma+\sigma^2)/\sigma^3}{3(2\tau/\sigma + T+\tau) \text{Prob}(\tau > T+\tau)} \quad (5.51)$$

Note that v_1 and v_2 are obtained from the following procedure:

- (1) For writing the expression for the mean v_1 , the following equality is employed:

$$\int_{T+\tau}^{\infty} \tau f(\tau) d\tau = \int_0^{\infty} \tau f(\tau) d\tau - \int_0^{T+\tau} \tau f(\tau) d\tau \quad (5.52)$$

- (2) For writing the expression for the second moment v_2 , the following equality is employed:

$$\int_{T+\tau}^{\infty} \hat{e}_1^2 \hat{f}(\hat{e}_1) d\hat{e}_1 = \int_0^{\infty} \hat{e}_1^2 \hat{f}(\hat{e}_1) d\hat{e}_1 - \int_0^{T+\tau} \hat{e}_1^2 \hat{f}(\hat{e}_1) d\hat{e}_1 \quad (5.53)$$

where (using the theory of residual life time [64]):

$$\int_0^{\infty} \hat{e}_1^k \hat{f}(\hat{e}_1) d\hat{e}_1 = (-1)^{k+1} \frac{d^{k+1}}{ds^k} (e^{-s(T+\tau)} R(s)) \Big|_{s=0} / ((k+1)\hat{\alpha}_1) \quad (5.54)$$

and

$$\int_0^{T+\tau} \hat{e}_1^k \hat{f}(\hat{e}_1) d\hat{e}_1 = (-1)^k \frac{d^k}{ds^k} \left(\frac{1 - e^{-s(T+\tau)}}{\hat{\alpha}_1 s} \right) \Big|_{s=0} \quad (5.55)$$

(V) An Estimate for the Conditional Laplace Transform

$H^*(s)$
X,22

As we have shown previously, $H^*(s)$ and $H^*(s)$ are
X,22 X,21

similar. Let $H^*(s)$ be an estimate for $H^*(s)$. $H^*(s)$ is
X,22 X,22 X,22

similar to $H^*(s)$ and it is given by equation (5.49).
X,21

5.2.2 The Estimate for $H^*(s)$

Substituting the estimates $H^*(s)$, $H^*(s)$, $H^*(s)$ and
X,11 X,12 X,21

$H^*(s)$ in equation (5.31) and using the notation similar to
X,22

that given by equation (5.1), i.e.,

$$\sum_X \pi_c(x_1, L-1, 1) = \pi_c^*(s, L-1, 1) \quad (5.56)$$

$$\tau_c(x_1, L-1, x_3) = \tau_c(s, L-1, s)$$

We have an estimate $H^s(s)$ for $H^s(s)$ when $L > 1$:

$$\begin{aligned} H^s(s) = & [\text{Prob}(\Theta_1 < T+\tau)] \{ [\tau_c(s, L-1, 1)] \cdot [L^s(s) \hat{L}^s(s)] \\ & + \tau_c(s, L-1, 2) \cdot \hat{L}^s(s) \} + \\ & [\text{Prob}(\Theta_1 \geq T+\tau)] [\tau_c(s, L-1, s)] \\ & \cdot [L^s(s) \{ (1/n) + (1-1/n) \hat{L}^s(s) \}] \\ & + 1 - \tau_c(s, L-1, s) \end{aligned} \quad (5.57)$$

The mean h_1 of the random variable Θ is given by:

$$\begin{aligned} h_1 = & [\text{Prob}(\Theta_1 < T+\tau)] \cdot \{ [\tau_c(s, L-1, 1)] \cdot [\hat{s}_1 + \hat{s}_1] \\ & + \tau_c(s, L-1, 2) \cdot \hat{s}_1 \} \\ & + [\text{Prob}(\Theta_1 \geq T+\tau)] \cdot [\tau_c(s, L-1, s)] \\ & \cdot [v_1 + (1-1/n) \hat{s}_1] \end{aligned} \quad (5.58)$$

The second moment of the random variable Θ is given by:

$$\begin{aligned} h_2 = & [\text{Prob}(\Theta_1 < T+\tau)] \cdot [\tau_c(s, L-1, 1)] \\ & \cdot [\delta_2 + 2\delta_1 \hat{s}_1 + \hat{s}_2] + \tau_c(s, L-1, 2) \delta_2 \\ & + [\text{Prob}(\Theta_1 \geq T+\tau)] \cdot [\tau_c(s, L-1, s)] \\ & \cdot [v_2 + 2(1-1/n) v_1 \hat{s}_1 + (1/n) \hat{s}_2] \end{aligned} \quad (5.59)$$

where $\text{Prob}(\Theta_1 < T+\tau)$, $\text{Prob}(\Theta_1 \geq T+\tau)$, $\hat{s}_1$, $\hat{s}_2$, δ_1 , δ_2 , v_1 and v_2 are given in equations (5.13), (5.14), (5.38), (5.39), (5.43), (5.44), (5.50) and (5.51) respectively.

5.3 Numerical Results

In this section, numerical results are obtained for the mean h_1 and the second moment h_2 of the random variable Θ . Also simulation results are presented for the mean h_1 and the second moment h_2 of the random variable Θ associated with two network configurations: a two-hop centralized packet radio network and a two-hop CATV network. As mentioned earlier, for a given input traffic level, the distribution of the random variable Θ is different for these network configurations and we approximate them with the distribution of the random variable Θ . In the two-hop configurations considered in this section, the outer-loops employ the CSMA_CD protocol given in Section 4.1., The inner-loop employs the single packet transmission protocol PC1 presented in Section 3.2.

The simulation results presented in this chapter are based on a simulation model developed for the two-hop systems as a whole (i.e., the outer-loops and the inner-loops are not decoupled) using a SLAM simulation package explained in [106]. The steady state behaviour of the two-hop system is studied through the following steps taken in a simulation run:

- (1) The simulation is started in an empty state.
- (2) The accumulated statistics of the system are cleared after 100 simulation time units (a simulation time unit is equal to the inner-loop packet transmission time T).

- (3) The simulation model is executed for a total simulation time of 7000 simulation time units.

Figures 5.5 to 5.7 depict the first moment of the random variable θ for a two-hop centralized packet radio network as a function of the system throughput for $L=3,2,1$ respectively. In Figure 5.5 where $L=3$, the estimated numerical results provide an excellent approximation for the average h_1 of the random variable θ . In Figure 5.6 where $L=2$, the estimated numerical results provide an excellent approximation for the average h_1 for $S > .5$. However, for smaller system throughput, due to the heavy traffic approximation established for the development of the Laplace transform $H^*(s)$, the estimated numerical results are noticeably larger than the average h_1 of the random variable θ . However, this discrepancy has insignificant impact on the results obtained (and presented in Chapter 6) for the outer-loop. In such configurations, outer-loops are operating at light throughput (i.e., $S_j < .1$) and consequently h_1 has to be increased significantly until a noticeable impact on the performance of the outer-loop is observed. In Figure 5.7 where $L=1$, the estimated numerical results do not provide good approximation for the average h_1 . As mentioned earlier, our estimation approach is based on the assumption that L is large (i.e., $L > 1$). Figures 5.8 to 5.10 depict the second moment of the random variable θ for a two-hop centralized packet radio network as a function

Figure 5.5. Two Hop Centralized Packet Radio: h_1 versus System Throughput.

Figure 5.6. Two Hop Centralized Packet Radio Network:
 h_1 Versus System Throughput.

Figure 5.7. Two Hop Centralized Packet Radio Network:
 h_1 Versus System Throughput.

Figure 5.8. Two Hop Centralized Packet Radio Network:
 h_2 Versus System Throughput.

Figure 5.9. Two Hop Centralized Packet Radio Network: h_2 Versus System Throughput.

Figure 5.10. Two Hop Centralized Packet Radio Network:
 h_2 Versus System Throughput.

of the system throughput for $L=3,2,1$. As in the previous case, the approximation provided by the estimated numerical results for the second moment h_2 of the random variable Θ deteriorates as L decreases.

Figures 5.11 to 5.13 depict the first moment of the random variable Θ for a two-hop CATV network as a function of the system throughput for $L=3,2,1$ respectively. The estimated numerical results provide a worst case approximation for the mean h_1 of the random variable Θ for such network configurations. This approximation deteriorates as L decreases. Figures 5.14 to 5.16 depict the second moment of the random variable Θ for a two-hop CATV network as a function of the system throughput for $L=3,2,1$. The discrepancies between the analytic and the simulation results are partly due to the approximated arrival process employed in the analytic model of the inner-loop. Recall that the arrival process to the j th controller of the inner-loop with mean arrival rate λ_{je} is approximated by the arrival process into the j th controller of the G-model (refer to Section 3.3) with mean arrival rate λ_j such that $\lambda_{je} = \lambda_j(1-B_j)$, where B_j is the blocking probability for the j th controller in a two-hop system and for a given operating traffic level (e.g., average arrival rate for the j th outer-loop is equal to λ_{je}), the arrival process to the j th controller of the inner-loop varies as a result of any variation of the channel speed ratio CR , i.e., the arrival process to the

Figure 5.11. Two Hop CATV Network:
 h_1 Versus System Throughput.

Figure 5.12. Two Hop CATV Network:
 h_1 Versus System Throughput.

Figure 5.13. Two Hop CATV Network:
 h_1 Versus System Throughput.

Figure 5.14. Two Hop CATV System:
 h_2 Versus System Throughput.

Figure 5.15. Two Hop CATV Network:
 h_2 Versus Throughput.

Figure 5.16. Two Hop CATV Network:
 h_2 Versus System Throughput.

jth controller is dependent on the values of CR. However, for the purpose of mathematical tractability, this dependency cannot be incorporated in the analytic models presented in Chapter 3.

Such discrepancies have insignificant impact on the results (presented in Chapter 6) for the outer-loop. This is mainly due to the fact that in the above system configurations, outer-loops operate at small throughput levels (i.e., $S_j < 1$) and, as we have shown in Chapter 4, at such throughput levels, variations of values of h_1 has insignificant impact on the average outer-loop delay.

The results presented in this chapter suggest that our approach provides good approximations for the first and the second moments of the random variable Θ for $L > 1$.

At this point, we have set the stage to study the dynamic behaviour of a number of two-hop systems. In Chapter 3, analytic models are presented for the analysis of the inner-loop. An analytic model is given for the analysis of an outer-loop, taking into account (the distribution of the random time period during which the buffer of the controller is full). The impact of the performance of the inner-loop on that of the outer-loop.

In Chapter 5, we presented an approach to the specification of the Laplace transform of the distribution of the random variable Θ (i.e., the time duration in which the buffer of a controller is full). In the following chapter, the analytic models developed in Chapters 3 to 5

are employed and analytic and simulation results are presented for the dynamic behaviour of a number of two-hop systems.

5.4 Conclusions

In this chapter, we have considered the large system size model presented in Chapter 3. The principle contributions can be summarized as follows:

- (1) The development of the Laplace transform $H^*(s)$ of the distribution of the random time period during which the buffer of a controller is full under heavy traffic assumption.
- (2) The development of an estimate $\hat{H}^*(s)$ for the Laplace transform $H^*(s)$.
- (3) Numerical and simulation results for the first and the second moments h_1 and h_2 of the random time period during which the buffer of a controller is full.

CHAPTER 6

CSMA_CD/CSMA_CD TWO-HOP LAN CONFIGURATIONS

In this chapter, numerical and simulation results are obtained for a number of CSMA_CD/CSMA_CD two-hop LANs in which the inner-loop employs the single packet transmission CSMA_CD protocol PC1 (given in Section 3.2) and the outer-loops employ the CSMA_CD protocol reviewed in Chapter 4.

Results presented in this chapter are associated with the CSMA_CD/CSMA_CD two-hop networks in which the inner-loop spans a larger geographic area than that covered by an outer-loop. Furthermore, the channel data transmission rate of the inner-loop channel is greater than that of an outer-loop channel. In addition, the length of packets transmitted over the outer-loops is equal to that transmitted over the inner-loop. Thus in such systems, the maximum throughput of the inner-loop is less than or equal to that of an outer-loop. In these systems, as the average arrival rate λ_e increases towards unity, the inner-loop approaches its throughput capacity, and due to the dependency of the outer-loops on the inner-loop, the outer-loops and subsequently the system become unstable. We consider two classes of CSMA_CD/CSMA_CD two-hop networks. In the first class of networks, i.e., the two-hop CATV networks, the maximum throughput of the inner-

loop is less than that of an outer-loop. The second class of networks is composed of the two-hop Centralized Packet Radio Networks in which the maximum throughput of the inner-loop is equal to that of an outer-loop [135,136].

Section 6.1 presents numerical and simulation results obtained for the CSMA_CD/CSMA_CD two-hop LAN configuration in CATV network. In Section 6.2, we present numerical and simulation results obtained for the CSMA_CD/CSMA_CD two-hop LAN configuration in centralized packet radio networks. Section 6.3 summarizes the contributions made in this chapter.

It is important to note that results presented in this chapter are for the worst situations in which the outer-loops as well as the inner-loop span large geographic areas (with diameters 10-20 km). Clearly for networks in which the outer-loops span smaller geographic areas (i.e., 2-3 km) the dependencies among the loops are less pronounced, thus results obtained from our analytic approaches will be better than those presented in this chapter.

6.1 Example: A Two-Hop CATV Configuration

In this section, numerical and simulation results are obtained for a number of CSMA_CD/CSMA_CD two-hop networks in which the ratio of the speed of the inner-loop channel to that of the outer-loop channel is equal to N . It is

Figure 6.2. Two Hop CATV network:
 I - Average Outer-loop Delay Versus System Throughput.
 II - Average Inner-loop Delay Versus System Throughput.

important to note that such two-hop networks represent a number of two-hop CATV as well as two-hop centralized packet radio networks.

Consider a representative CATV system with 10 equispaced two-way trunk amplifiers. Each of the amplifiers supports a maximum of 4 Distribution Branches. Intelligent controllers with finite buffer are colocated with the trunk amplifiers. These controllers regulate traffic flow and isolate the cluster of user terminals from the trunk of the CATV tree (refer to Figure 1.3(c) in Chapter 1). Consequently, the distribution lines will be isolated from the main trunk in the up-stream direction. Note that this does not alter or affect in any way the normal functions of the CATV system. These controllers will collect and buffer the traffic from their groups of terminals and then output this traffic into the main trunk. In addition to these functions, the controller imposes a flow control mechanism on the outer-loop by sending a busy tone signal when its buffer is full. The controllers together with the trunk form the inner-loop, whereas the cluster of user terminals associated with a controller together with the associated four distribution lines form an outer-loop. We assume that the four distribution lines associated with a controller form a single bus. Note that in practice we may consider each distribution line together with its associated cluster of user terminals to form an outer-loop. This reduces the contention among user terminals and consequently yields

better system performance. The length of the main trunk is assumed to be 20 km. The propagation delay is highly dependent upon the length and type of the cable. For a typical cable, the propagation delay is 5 μ sec. per 1 km. Thus the propagation delay for the inner-loop is 100 μ sec. We assume that the speed of the trunk is 1 Mbps and the ratio of the transmission rate over the inner-loop to the transmission rate on the outer-loop is equal to N. Therefore, for $N=10$, the speed of a distribution link is 100 Kbps. For a transmission speed of 1 Mbps and constant packet length of 1000 bits, the inner-loop packet transmission time T is one millisecond. Consequently, the ratio of the propagation delay to packet transmission time for the inner-loop is $a=.1$. In order to demonstrate results for the worst case situations, we consider $a_j = .01$ which corresponds to a distribution link with length equal to 20 km. Note that an outer-loop usually spans a geographic area with diameter equal to 2-3 km (i.e., the length of a distribution link is 2-3 km). Consider a CSMA_CD/CSMA_CD architecture for the system in which the inner-loop employs the single packet transmission protocol PC1 presented in Section 3.2 and the outer-loops employ the CSMA_CD protocol reviewed in Section 4.1.

The values of the system parameters are summarized in the following:

The inner-loop:

N = number of controllers = 10

τ_j = propagation delay = 100 μ sec. = 0.1 msec

T = packet transmission time = 1 msec

$a = \tau_j/T = 0.1$

The outer-loops:

τ_j = propagation delay = 100 μ sec. = 0.1 msec

b_j = packet transmission time = 10 msec

$a_j = \tau_j/b_j = 0.01$

We employ the approach presented in this thesis, i.e., decoupling the inner-loop from the outer-loops and analyze them separately, taking into account the interdependency of the loops. In the following the numerical and simulation results obtained for the above example is presented.

In the above two-hop system configuration, the maximum system throughput is equal to the maximum throughput of the inner-loop. The maximum throughput of the inner-loop is discussed in Section 3.4.3 (refer to Table 3.1 in Chapter 3). Recall that the value of the maximum throughput is dependent on the values of N and τ (e.g., for $N=10$ and $\tau=.1$, maximum throughput = .6187).

The simulation results presented in this chapter are based on a simulation model developed for the two-hop systems as a whole (i.e., the outer-loops and the inner-loops are not decoupled) using a SLAN simulation package explained in [106]. The steady state behaviour of the two-hop systems is studied through the following steps taken in a simulation run:

- (1) The simulation is started in an empty state.

- (2) The accumulated statistics of the system are cleared after 100 simulation time units (a simulation time unit is equal to the inner-loop packet transmission time T).
- (3) The simulation model is executed for a total simulation time of 7000 simulation time units.

The analytic results are obtained for the average inner-loop delay and are presented in Figures 6.1 to 6.3. These results are established using the analytic model (i.e., the large size system model) presented in Chapter 3. The average delay is calculated for various values of average arrival rate λ . For a given value of λ and using the analytic model (i.e., the large system size model), average delay and probability of blocking B are calculated. The average delay is employed as an approximation to the average delay associated with the inner-loop of a two-hop system with average departure rate λ_e where $\lambda_e = (1-B)\lambda$. Figures 6.1 to 6.3 also depict the analytic results for the average outer-loop delay. Using this analytic model (i.e., the large system size model) and the approach developed in Chapter 5, the mean h_1 , the second moment h_2 and the value of the Laplace transform of the random time period θ (i.e., time during which the buffer of a controller is full) at $s = \lambda_{je}$ (i.e., $\hat{H}^*(s)|_{s=\lambda_{je}}$) are calculated. These values (i.e., λ_{je} , h_1 , h_2 and $\hat{H}^*(s)|_{s=\lambda_{je}}$) are substituted in the formula for the average outer-loop delay given by the analytic model

Figure 6.1. I - Average Outer-loop Delay Versus System Throughput.

II - Average Inner-loop Delay Versus System Throughput.

Figure 6.2. Two Hop CATV network:
 I - Average Outer-loop Delay Versus System Throughput.
 II - Average Inner-loop Delay Versus System Throughput.

Figure 6.3. Two Hop CATV network:
 I - Average Outer-loop Delay Versus System Throughput.
 II - Average Inner-loop Delay Versus System throughput.

of Chapter 4. It is important to note that as λ_j increases from 0 to .618, λ_{j0} increases from 0 to .0618. (i.e., each outer-loop is operating under small average input traffic rate).

Figures 6.1 to 6.3 depict the average inner-loop delay and the average outer-loop delay versus system throughput for $L=1,2,3$ respectively. The average inner-loop delay is the smaller component of the average system delay. As system throughput increases, the average inner-loop delay increases and beyond $S=.5$, it is less than that predicted by the numerical results. This is due to the nature of the outer-loop's departure process. Recall that it was approximated by the arrival process into the finite buffer multiqueue system of Chapter 3. Clearly as b_1 increases and become large compared to the average inner-loop delay, the probability that there exists two or more packets in the buffer of a controller decreases. Furthermore, for large values of b_1 , the controller completes the transmission of the packet residing at the front of its queue before the arrival of the next packet. As a result, there is no significant difference between the inner-loop's average delay obtained for $L=2$ and for $L=3$. The average inner-loop delay for small values of system throughput (i.e., $S<.3$) is the same for $L=1,2,3$. For $S>.3$, the average inner-loop delay for $L=1$ is less than its $L=2$ counterpart. In this case, there is no difference between the average delay obtained for $L=2$ and its $L=3$ counterpart.

The average outer-loop delay is the predominant component of the total average system delay. This is due to the fact that b_1 is large compared to T (i.e., $b_1 \gg N \cdot T$). For $L=1$, the difference between the numerical results obtained for the average outer-loop delay and its corresponding simulation value is less pronounced. This is due to the fact that the difference between the numerical results obtained for the moments of the random variable θ , and the corresponding values obtained from the simulation become negligible when they are normalized to the outer-loop's packet transmission time b_1 . For $S < .3$, the outer-loop's average delay does not change for $L=1,2,3$. As S increases beyond .3, the difference between the average outer-loop delay for $L=1$, and $L=2$ increases. For a particular value of $S > .3$, the average outer-loop delay decreases as L increases. However, insignificant decrease is achieved by going from $L=2$ to $L=3$.

Figures 6.4 to 6.6 present the average system delay normalized to the inner-loop packet transmission time versus system throughput for $L=1,2,3$ respectively. The analytic results obtained for the average system delay are established by simply adding analytical results (given in Figures 6.1 to 6.3) associated with the inner-loop to that associated with the outer-loop. For a given value of λ_o , the average inner-loop delay calculated analytically at λ_o is added to the average outer-loop delay calculated analytically at λ_o . For $S < .3$, the average system delay

Figure 6.4. Two Hop CATV System: Average Delay Versus System Throughput.

Figure 6.5. Two Hop GATL System. Average System Delay Versus Throughput

Figure 6.6. Two Hop CATV System: Average System Delay Versus System Throughput.

remains the same for $L=1,2,3$. As L increases, the average inner-loop delay increases whereas the average outer-loop delay decreases. For $S>.3$, as L increases, the average system delay decreases. For example, for system throughput of $S=.41$ and $L=1$, the average system delay is equal to 17.87 whereas for a larger system throughput of $S=.47$ and $L=2$, the average delay is equal to 17.30. This is due to the fact that in such system configurations the average outer-loop delay is the dominant component of the average system delay and it clearly decreases as L increases. No significant decrease is achieved by going from $L=2$ to $L=3$.

6.2 Example: A Two-Hop Centralized Packet Radio Network

In this section, numerical and simulation results are obtained for a number of CSMA_CD/CSMA_CD two-hop networks in which the ratio of the speed of the inner-loop channel to that of the outer-loop channel is equal to unity. Such networks represent a number of two-hop centralized packet radio networks [135,136].

We consider a representative two-hop centralized packet radio network with 10 controllers where all controllers are within range and in line-of-sight of each other and of the central station. Traffic originates at terminals is destined to a central station and requires for its transport that packets be relayed by store-and-forward controllers (refer to Figure 1.2(b)). All devices

are provided with omnidirectional antennas. A population of terminals is associated with each controller. The terminals are in line-of-sight and within range of their controller station. Traffic originates at terminals and is destined to the central station. We consider inbound traffic only. Each controller is provided with a finite storage capacity which can accommodate L packets. The central station has an infinite storage capacity. The cluster of user terminals associated with a controller together with the associated channel form an outer-loop. The controllers together with the associated channel form the inner-loop. We assume that a controller may receive new packets, if there is available storage, while it is transmitting a packet, i.e., the controllers may receive and transmit at the same time. The controllers regulate traffic flow and impose a flow control mechanism on the outer-loop by sending a busy tone signal when its buffer is full.

In order to demonstrate results for the worst case situations, we consider $a_j = .1$ which corresponds to a geographic area, spanned by an outer-loop, with diameter 10 km and packet transmission time of .33 msec. Note that an outer-loop usually spans a geographic area with diameter 2-3 km.

We assume that the controllers are spread across a geographic area with diameter of 10 kilometers. Thus the maximum propagation delay for the inner-loop is 33 μ sec. We assume that the bit transmission rates of the inner-

loop and the outer-loops are equal to 150 kbps (transmitted over wideband at 200 KHz). For a transmission speed of 150 kbps and constant packet length of 50⁸ bits, the inner-loop packet transmission time T is equal to 0.33 msec. The values of the system parameters are summarized in the following:

The inner-loop :

N = number of controllers = 10
 τ = propagation delay = 33 μ sec
 T = packet transmission time = 0.33 msec
 $a = \tau/T = 0.1$

The outer-loops:

τ_j = propagation delay = 33 μ sec
 b_j = packet transmission time = 0.33 msec
 $a_j = \tau_j/b_j = 0.1$

We employ the approach presented in this thesis, i.e., decoupling the inner-loop from the outer-loop and analyze them separately, taking into account the interdependency of the loops. Using this approach and the analytical models presented, the numerical and simulation results are obtained for the above example and are presented in the following.

Figures 6.7 to 6.9 depict average inner-loop delay versus system throughput for $L=1,2,3$ respectively. The average inner-loop delay is the predominant component of

* Note that in practice, the length of a packet is 100-1000 bits and $\tau = 10 \mu$ sec. The values of system parameters presented are for worst case situations.

Figure 6.7. Two Hop Centralized Packet Radio Network: Average Inner-loop Delay Versus System Throughput.

Figure 6.8. Two Hop Centralized Packet Radio Network:
 • Average Inner-loop Delay Versus throughput.

Figure 6.9. Two Hop Centralized Packet Radio Network:
 / Average Inner-loop Delay Versus Throughput.

the average system delay. As system throughput increases, the average inner-loop delay increases. Figures 6.7 to 6.9 show excellent agreement between the numerical and simulation results obtained for the average inner-loop delay. The average inner-loop delay for small values of system throughput (i.e., $S < .3$) is the same for $L=1,2,3$. For $S > .3$, the average inner-loop delay for $L=1$ is less than its $L=2$ counterparts. In this case, there is no significant gain to be achieved by going from $L=2$ to $L=3$.

Figures 6.10 to 6.12 depict average outer-loop delay versus system throughput for $L=1,2,3$ respectively. For $L=1$, there is a significant departure between the numerical results obtained for the average outer-loop delay and the simulation counterparts. This is due to the approximation made for the Laplace transform of the distribution of the random variable Q . Recall that in equations (5.57) to (5.59) of Chapter 5, we have $L > 1$. In these equations, for $L=1$, $\pi_c(s, L-1, s)$ (i.e., the probability that $Q > 0$ or $\pi_c(s, 0, s)$) incorporates in itself the probability that the new arrival finds no ready controllers in the system. This yields larger values for $\pi_c(s, 0, s)$, and as a result larger values for h_1 , h_2 and $H^s(s)$ which increases outer-loop average delay (refer to Figure 6.10).

In Figures 6.10 to 6.12, as S increases beyond .3, the difference between the outer-loop delay for $L=1$ and $L=2$ increases. However, insignificant decrease is

Figure 6.10. Two Hop Centralized Packet Radio Network:
Average Outer-loop Delay Versus Throughput.

Figure 6.11. Two Hop Centralized Packet Radio Network:
Average Outer-loop Delay Versus Throughput.

Figure 6.12. Two Hop Centralized Packet Radio Network:
Average Outer-loop Delay Versus Throughput.

achieved by going from $L=2$ to $L=3$. Also note that the average outer-loop delay increases rapidly when operating beyond $S=.56$. This is due to the fact that the outer-loop is operating at small throughput levels $S_j < S/N = .0608$. Therefore going for $S=.3$ to $S=.5$ implies going from $S_j=.03$ to $S_j=.05$ i.e., (slight) increase to the throughput of an outer-loop. Beyond $S=.56$, n_1 increases and consequently it yields a sharp increase to the average outer-loop delay.

Figures 6.13 to 6.15 presents average system delay normalized on inner-loop's packet transmission time versus system throughput for $L=1,2,3$ respectively. In this case, the average inner-loop delay is the dominant component of the average system delay and it increases when L grows. Whereas the average outer-loop decreases as L increases. For system throughput of $S=.5$ and $L=1$, the average system delay is equal to 4.85 whereas for the same throughput of $S=.5$ and $L=2$, the average system delay is equal to 4.79, a 1.3% improvement. No significant improvement is achieved by going from $L=2$ to $L=3$.

6.3 Conclusions

the results presented in this chapter suggest that our approach provides good approximations for the delay-throughput performance of a number of two-hop centralized networks in which the controllers have finite buffer capacity greater than one packet. The important

Figure 6.13. Two Hop Centralized Packet Radio Network: Average System Delay Versus Throughput.

Figure 6.14. Two Hop Centralized Packet Radio Network: Average System Delay Versus Throughput.

Figure 6.15. Two Hop Centralized Packet Radio Network: Average Delay Versus System Throughput.

characteristics of these two-hop systems are:

- constant packet length (i.e., controllers do not perform assembly or disassembly of packets)
- the inner-loop bit transmission rate is greater than or equal to the outer-loop bit transmission rate
- the inner-loop span larger geographic areas than that covered by an outer-loop.

Examples of two-hop systems with the above characteristics are two-hop CATV networks and two-hop centralized packet radio networks. Results presented in this chapter are for the worst case situations in which the outer-loops as well as the inner-loop span large geographic areas (with diameters 10-20 km). Clearly, for networks in which the outer-loops span smaller geographic areas (i.e., 2-3 km) the dependencies among the loops are less pronounced, thus results obtained from our analytic approaches will be better than those presented in this chapter.

The contributions made in this chapter are summarized in the following:

(1) We have considered a number of CSMA_CD/CSMA_CD two-hop LANs in which the inner-loop employs the single packet transmission CSMA_CD protocol PC1 and the outer-loops employ the CSMA_CD protocol given by Lam [74]. Analytical as well as simulation results have been presented for the performance behaviour of a CSMA_CD/CSMA_CD two-hop centralized packet radio network. Results are obtained for worst case situations in which

inner-loops and outer-loops have equal bit transmission rates. Furthermore, inner-loops and outer-loops span a large geographic area (i.e., diameter equal to 10 km).

(2) Analytical as well as simulation results are presented for the performance behaviour of a CSMA_CD/CSMA_CD two-hop CATV network. Results are obtained for a worst case situation in which inner-loops as well as outer-loops span large geographic areas (i.e., 20 km).

CHAPTER 7

THESIS CONTRIBUTIONS AND SUGGESTIONS FOR FURTHER RESEARCH

7.1 Summary of Thesis Contributions

In this research, we have considered a number of two-hop network configurations in Centralized Packet Radio and in CATV networks. In these networks, the outer-loops employ a random access scheme (i.e., the CSMA_CD protocol given by Lam [74]) and the inner-loop may employ either a random access scheme (i.e., the CSMA_CD with finite buffer controllers) or a deterministic access schemes (e.g., Reservation Protocol [33]). The objective of this research was to develop analytical models for the evaluation of the system performance in terms of throughput-delay where the controller's buffer capacity is large (i.e., greater than one packet). The difficulty in the analysis has been primarily due to the dependencies that exist between the inner-loop and each of the outer-loops. In addition to this difficulty, the analytical models are required to represent systems with large number of controllers (i.e., $N \geq 10$). In this thesis, we have presented an approach in which the inner-loops and outer-loops are decoupled and studied separately while taking into account the interdependency between the loops. As an example for the application of the analytic models developed in this research, we have presented analytical

results for the CSMA_CD/CSMA_CD two-hop configurations. It is shown that for most practical cases, the analytical results obtained have excellent agreement with the associated simulation results.

The main contributions of this research are summarized in the following:

1. An approach to modelling and analysis of the two-hop LANs with finite buffer controllers is presented. This approach is based on decoupling the outer-loops from the inner-loop, which permits the analysis to be conducted for each loop separately. The model introduces a flow control mechanism in which the outer-loop channel is declared busy when the controller's buffer is full. This flow control mechanism is essential in practice in order to prevent the system from falling into a congested (unstable) state.

2. An approach is presented for mathematical modelling of the impact of the performance of the inner-loop on that of the outer-loops. Results obtained from this approach are employed in the analytical model developed for the outer-loops.

3. We have considered a number of CSMA_CD/CSMA_CD two-hop LANs in which the inner-loop employs the single packet transmission CSMA_CD protocol PC1 and the outer-loops employ the CSMA_CD protocol given by Lam [74]. Analytical as well as simulation results have been presented for the performance behaviour of a

CSMA_CD/CSMA_CD two-hop centralized packet radio network. Furthermore, analytical as well as simulation results are presented for the performance behaviour of a CSMA_CD/CSMA_CD two-hop CATV network.

A number of minor contributions and extensions of other work have been presented in this research and are summarized in the following:

1. A generalized mathematical model is introduced for the analysis of the inner-loop. The model is based on a number of simplifying assumptions and alternative state definitions, and can be employed in the analysis of a number of random as well as deterministic access schemes with buffered users.

2. A number of CSMA_CD protocols (i.e., PC1, PC2, PC3 and PC4 protocols) are presented for the inner-loop which are based on the CSMA_CD protocol given by Lam [74]. The distinguishing characteristics of these protocols are finite population of controllers and finite controller buffer capacity. Furthermore, each controller, once it acquires the channels follows a set of rules which allow the transmission of one packet (as in protocol PC1) or multiple packets (as in protocols PC2, PC3 and PC4). An analytical model (i.e., the small system size model) is presented for the analysis of the CSMA_CD protocols (i.e., protocols PC1, PC2, PC3 and PC4) with a small number of network nodes (i.e., $N < 5$) and finite buffer capacity.

3. Numerical as well as simulation results are presented for the analysis of the CSMA_CD protocols PC1

and PC3, with small number of network nodes (i.e., $N=3$, $L=3$, $a=0.01, 0.1$) using the small system size model.

4. An analytic model (i.e., the large system size model) is presented for the analysis of the CSMA_CD protocol PC1 with a large number of network nodes (i.e., $N \geq 10$) and finite buffer capacity. The model is solved using an approach similar to that presented by Sykas [132]. Numerical as well as simulation results are presented for the analysis of the protocol PC1 with a large number of network nodes (i.e., $N=10$, $L=1,2,3$, $a=0.01, 0.1$) using the large system size model.

5. Following the approach presented by Lam [74], a mathematical model for the outer-loops is given in which user terminals employ the CSMA_CD protocol presented in [74]. This model takes into account the effect of the flow-control employed in the two-hop system.

7.2 Suggestions for Further Research

The generalized model presented in this research could be applied for the analysis of a number of random as well as deterministic access schemes. In this work, we have demonstrated, as an example, the application of the model to the analysis of a number of random access schemes (i.e., the CSMA_CD protocols PC1, PC2, PC3 and PC4) when the controllers have finite buffer capacity. The generalized model could easily be applied to analysis of a deterministic access scheme (e.g., Polling and Reservation

Protocols). Using the decoupling approach, a number of finite buffer two-hop system configurations, in which outer-loops employ the CSMA_CD and the inner-loop employs a deterministic access scheme, could be analyzed. Results obtained could be compared to the associated results demonstrated for the finite buffer CSMA_CD/CSMA_CD configurations.

The approach presented in this research could be applied to the analysis of finite buffer two-hop configurations with other flow control mechanism. In the flow control mechanism considered in this thesis, a controller, with a full buffer, resets its associated outer-loop channel after it has accomplished a successful transmission of a packet. As an example for a different flow control mechanism, the controller with a full buffer could reset the outer-loop's channel as soon as it is able to determine that it is transmitting successfully (e.g., T seconds before it accomplishes a successful transmission).

In the two-hop configurations considered in this research, we have assumed that the controllers do not perform assembly or disassembly of packets, i.e., the length of packets transmitted over outer-loops is equal to that transmitted over the inner-loops. It would be useful from a practical point of view to relax this assumption and formulate analytical models for such two-hop system configurations.

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APPENDIX A

MULTIPLE PACKET TRANSMISSION PROTOCOLS

In this appendix, the interrelationship among the steady state solutions $\pi_q(X)$, $\pi_u(X)$, $\pi_a(X)$, $\pi(X)$, associated with the protocols presented in Section 3.2, are discussed.

For protocols in which only a single packet is transmitted when a controller gains channel access, the results obtained from the semi-Markov chain identified by the λ -embedded points are equal to those obtained at the time instants just before arrival epochs i.e., $\pi_q(X) = \pi_a(X)$. When the arrival process is Poisson and no arrival is lost, $\pi_q(X)$ is valid for all times [20], i.e., $\pi_q(X) = \pi_a(X) = \pi(X)$.

As for protocols in which multiple packets are transmitted when a controller gains channel access, the results obtained using the steady state distributions $\pi_q(X)$ and $\pi_u(X)$ are approximate solutions. This is due to the fact that although the state changes occur one at a time at arrival epochs (i.e., we do not treat batch arrivals), multiple state changes occur at the departure epochs. Consequently the results obtained from the semi-Markov chain identified by the λ -embedded points are not necessarily the same as those obtained just before arrival epochs.

Let $n(t)$ denote the total number of packets in the system at time t . Let, also, the points of increase, after some time $t=0$, be labelled consecutively $t_a^{(e+a)}$, the points of decrease, $t_i^{(e+a)}$ and the points at the beginning of a single/multiple decrease, $t_u^{(e+a)}$, $a=0, 1, 2, \dots$. Let $n(t_a^{(a)}-)$ be denoted by $\xi_a^{(a)}$, $n(t_i^{(e+a)}+)$, $\xi_i^{(a)}$ and $n(t_u^{(e+a)}-)$, $\xi_u^{(a)}$. Figure A.1 depicts an example of the evolution of the single packet protocol PC1. Under the assumption that the process $n(t)$ is ergodic, the steady state distribution exists and the time averages of the process is equal to the ensemble averages. Now, consider a long period of time and let $E_a^{(a)}$, $E_i^{(a)}$ and $E_u^{(a)}$ denote the number of $t_a^{(e+a)}$, $t_i^{(e+a)}$ and $t_u^{(e+a)}$ epochs respectively. Since all of the sample functions of the process $n(t)$ are, step functions with unit jumps, coupled with the ergodicity we have [20]

$$\lim_{a \rightarrow \infty} E_a^{(a)} = \lim_{a \rightarrow \infty} E_i^{(a)} = \lim_{a \rightarrow \infty} E_u^{(a)} \quad (A.1)$$

and

$$\begin{aligned} \lim_{a \rightarrow \infty} \Pr(\xi_a^{(a)} \leq k) &= \lim_{a \rightarrow \infty} \Pr(\xi_i^{(a)} \leq k) \\ &= \lim_{a \rightarrow \infty} \Pr(\xi_u^{(a)} \leq k+1) \end{aligned} \quad (A.2)$$

In other words,* since the state of the system increases by one and decreases by one, the statistical properties of

* We insist that (A.2) is valid only for single packet transmission protocols and its validity, as an assumption, for the multiple packet transmission protocols stands for some and not all of the parameters N and λ .

the points before an increase must be the same as those after a decrease if and only if, with probability one, $n(t)$ does not approach infinity for all times. Consequently, in systems with finite buffer capacity, $\pi_i(X)$ is the limiting distribution of the state of the system as seen by an arrival conditioned on the fact that the arrival enter the system. It is important to note that, in this thesis, $\pi_i(X)$ is employed as an approximation for the limiting distribution of the state vector x of the inner-loop.

Figure A.2 presents an example of the evolution of the system for a multipacket protocol (e.g., the protocol PC3). Due to the fact that $t_i^{(e)}$ is the time at which the service of multiple customers are completed, we have:

$$\lim_{a \rightarrow \infty} E_a(a) \geq \lim_{a \rightarrow \infty} E_i(a) = \lim_{a \rightarrow \infty} E_u(a) \quad (A.3)$$

This inequality is the result of the loss of the $t_i^{(e)}$ and $t_u^{(e)}$ associated with the first $i-1$ packets in a successful transmission time period of length i packets. Writing the equation (A.2) for the multiple packet transmission protocols is mathematically cumbersome. It becomes even more complicated for a system with finite buffer controllers. Therefore we have relied on simulation in order to establish a boundary for such values as N and i in which the assumption of the validity of the (A.2) is acceptable for our specific multiple packet transmission protocols.

Consider the multiple packet transmission protocols given in previous sections. For a system with small values of λ (e.g., $\lambda = .2$), there is a negligible probability that a controller's queue will have more than one packet stored in its buffer. This leads to a small probability that multiple packets are transmitted. Therefore in such systems, the single packet transmission model is an approximation for the multiple packet transmission model. Thus the statistics obtained at the time point after departure is an approximation to the statistics obtained at the time point before arrival. Simulation results for a number of system configurations employing the multiple packet transmission protocols has shown that the goodness of the above approximation for such systems extends to input traffic rate of up to $\lambda = .7$. This approximation becomes closer to the exact results as N increases.

In a system with small value of N (e.g., $N=3$) and higher input traffic rate (say $\lambda = .9$), simulation results have shown that for the multiple packet transmission protocol PC1, the statistics obtained from the time points $t_u(e+k)$, $k=0,1,2,\dots$ establishes an upperbound for the dynamic behaviour of the system (refer to Figures 3.5 to 3.10).

Figure A.1. Evolution Of the Total Number of Packets in the System For the Single Packet Transmission Protocol PC1.

Figure A.2. Evolution of the Total Number of Packets in the System For A Multiple Packet Transmission Protocol.

APPENDIX B

G-MODEL: SMALL SIZE SYSTEMS

This Appendix presents analysis of the Random Access Schemes given in Section 3.2. The state definition employed in the analysis is given by the equation (3.2). The analysis is performed for a general nonhomogeneous system of N controllers, i.e., the j th, $j=1,2,\dots,N$, controller has a buffer capacity of L_j packets and the mean arrival rate is λ_j . Note that the analysis is applicable to small system sizes (i.e., $N=1,\dots,5$, $L_j=1,\dots,4$). This is due to the fact that the state definition (3.2) is employed and the state space grows rapidly as system size increases.

B.1 Random Access Schemes: Analysis

Consider the state definition given by equation (3.2). Also consider a snapshot of the broadcast channel given in Figure 3.2.

Let $P(k,j,\tau)$ denote the probability that k new packets arrive in the buffer of the j th controller during a single propagation delay:

$$P(k,j,\tau) = \frac{(\tau\lambda_j)^k e^{-\tau\lambda_j}}{k!} \quad \begin{matrix} k = 0,1,2,\dots \\ j = 1,2,\dots,N \end{matrix} \quad (B.1)$$

Let also $P(k, j, T)$ denote the probability that k new packets arrive in the buffer of the j th controller during a packet transmission time T :

$$P(k, j, T) = \frac{(T\lambda_j)^k e^{-T\lambda_j}}{k!} \quad \begin{array}{l} k = 0, 1, 2, \dots \\ j = 1, 2, \dots, N \end{array} \quad (B.2)$$

For any state $\underline{x}$, we define $n^*(\underline{x})$ as follows

$$n^*(\underline{x}) = \sum_{j=1}^N U(x_j)$$

$$U(x_j) = \begin{cases} 1 & x_j \geq 1 \\ 0 & x_j = 0 \end{cases} \quad (B.3)$$

$n^*(\underline{x})$ is the number of nonzero elements of the state $\underline{x}$ (i.e., number of ready controllers). Considering the definition of the random variables $o^{(e+1)}$, $r^{(e+1)}$, $i^{(e+1)}$ and $y^{(e+1)}$ given in Section 3.3, we have:

$$\begin{aligned} \text{Prob} [I^{(e+1)} = k | n^*(\underline{x}) = 0] \\ = (1 - P_0) P_0^{k-1} \quad k = 1, 2, \dots \end{aligned} \quad (B.4)$$

where

$$P_0 = \prod_{j=1}^N P(0, j, 2\tau) \quad (B.5)$$

and clearly we have

$$P(k, j, 2\tau) = \frac{(2\tau\lambda_j)^k e^{-2\tau\lambda_j}}{k!} \quad \begin{array}{l} k = 0, 1, 2, \dots \\ j = 1, 2, \dots, N \end{array} \quad (B.6)$$

In the following, we present the conditional distribution of the random variable $r^{(e+1)}$ conditioned on

the fact that there are two or more ready controllers (i.e., $\text{Prob}[r^{(e+1)}=k | n^*(\underline{x}) \geq 2]$). Consider the adaptive algorithm reviewed in Section 3.2. At the beginning of each time slot associated with a collision time period, ready controllers transmit into the next time slot with equal probability (i.e., $1/n^*(\underline{x})$). Let $\underline{x}$ represent the state of the system at the beginning of a time slot in the collision time period $r^{(e+1)}$. Furthermore, let σ denote the probability that only one ready controller (among $n^*(\underline{x})$ ready controllers) transmit in this time slot. Clearly σ is a function of $n^*(\underline{x})$ and is given by:

$$\sigma = \text{Prob}(\text{one controller out of } n^*(\underline{x}) \text{ active controllers transmit in a time slot})$$

$$= n^*(\underline{x}) \frac{1}{n^*(\underline{x})} \left(1 - \frac{1}{n^*(\underline{x})}\right)^{n^*(\underline{x})-1}$$

or

$$\sigma = \left(1 - \frac{1}{n^*(\underline{x})}\right)^{n^*(\underline{x})-1} \quad (\text{B.7})$$

In this thesis, we propose the following worst case approximation for the value of σ i.e.,:

$$\sigma = \left(1 - \frac{1}{N}\right)^{N-1} \quad N = 2, 3, \dots \quad (\text{B.8})$$

The above approximate value for σ is known as an exact value when the system is operating at its maximum throughput capacity (i.e., all N controllers are ready and we have $n^*(\underline{x})=N$).

The conditional distribution of the random variable $r^{(e+1)}$ conditioned on the fact that there are two or more ready controllers is equal to:

$$\begin{aligned} \text{Prob}[r^{(e+1)}=k | n^*(x) \geq 2] \\ = \sigma(1-\sigma)^{k-1} \quad k = 1, 2, \dots \end{aligned} \quad (\text{B.9})$$

where σ is given by equation (B.8). The z transform of the above distribution is equal to:

$$R(z) = \frac{\sigma z}{1-(1-\sigma)z} \quad (\text{B.10})$$

The first, second and the third moments of the above distribution are denoted by r_1 , r_2 and r_3 respectively and are given in the following:

$$r_1 = 2\tau/\sigma \quad (\text{B.11})$$

$$r_2 = (2\tau)^2((2-\sigma)/\sigma^2) \quad (\text{B.12})$$

$$r_3 = (2\tau)^3(6-6\sigma+\sigma^2)/\sigma^3 \quad (\text{B.13})$$

So far, we have set the stage for writing expressions for the state transition matrices, Q , P_1 and P_2 for the protocols PC1, PC2, PC3 and PC4. We will proceed to do so in the following section.

(a) The State Transition Matrix Q

As mentioned earlier, the protocols PC1, PC2, PC3 and PC4 maintain the following property: at the beginning of each successful transmission, ready controllers have equal chance of acquiring the channel irrelevant of which controller had channel access during the last successful transmission time period. The (x, z) element of the matrix Q is defined as

$$[Q]_{x,k} = \begin{cases} 0 & \exists x_k/x_k \neq z_k, 1 \leq k \leq N \\ 1 & n^*(x) = 1 \text{ \& } j = z_{N+1} \text{ \& } z_j \neq 0 \\ 1/n^*(x) & n^*(x) > 1 \text{ \& } j = z_{N+1} \text{ \& } z_j \neq 0 \end{cases} \quad (B.14)$$

where $n^*(x)$ is given by the equation (B.3). In Figure 3.3, we have shown an example for Q matrix when $N=2$ and $L=2$. Note that in order to make Q a stochastic matrix, we have inserted ones in appropriate places.

(b) The State Transition Probability Matrix P_1

Figure 3.2 depicts a snapshot of the broadcast channel. A transmitting controller remains the same during the entire successful transmission time period. This implies that the transition rates, between the start and the end of a successful transmission time period, associated with states with different transmitting controller is equal to zero. As a result, the matrix P_1 is a sparse matrix and its nonzero elements form a set of $M/N \times M/N$ submatrices along its diagonal. Therefore, only these submatrices may be stored in an $M \times M'$ matrix where

$$M' = M/N \quad (B.15)$$

In the following, the state transition probability matrix P_1 is described for the protocols PC1, PC2, PC3 and PC4.

(c) The State Transition Probability Matrix P_1 for the Protocol PC1

For the single packet transmission protocol PC1, we have

$$P_1 = Q^{(T)} Q^{(\tau)} \quad (B.16)$$

where $[Q]_{x,z} = 0$ when one of the following conditions are met:

- (1) $x_j = 0$ when $j = x_{N+1}$ i.e., for a transmitting controller with zero packets.
- (2) $x_{N+1} \neq z_{N+1}$ i.e., there exists more than one transmitting controller during a successful transmission subcycle.

For $k = x_{N+1}$:

- (3) $x_k - z_k > 1$ i.e., transmitting controller shrinks by more than one packet; or
- (4) $x_k \neq 0$ and $\exists j \neq x_{N+1} / x_j - z_j > 0$ i.e., a nontransmitting controller shrinks by one or more packets.
- (5) $x_k = L_k$ and $z_k \neq L_k - 1$ i.e., a transmitting controller with L_k packets does not find itself in state $L_k - 1$ after a successful transmission.

When none of the above conditions are met, $[Q]_{x,z}$ is non-zero and is equal to

$$= \left(\prod_{j=1}^N \text{Prob}((z_j - x_j), j, T) \right) P((z_j - x_k + 1), k, T)$$

and $j \neq x_{N+1}$ when $k = x_{N+1}$ and $z_k < L_k - 1$

$$\begin{aligned}
&= \sum_{j=0}^N \text{Prob}(z_j - x_j, j, T) \text{ when } z_k = x_k = L_k^- \\
&= \sum_{\substack{j=0 \\ j \neq x_{N+1}}} \text{Prob}(z_j - x_j, j, T) - P(z_k - x_k = 0, k, T) \\
&\quad \text{when } k = x_{N+1} \quad \text{B.17} \\
&\quad \text{and } z_k = L_k^- \\
&\quad \text{and } x_k < L_k^-
\end{aligned}$$

where

$$\begin{aligned}
&\text{Prob}(z_j - x_j, j, T) \\
&= \begin{cases} 1 & z_j = x_j = L_j \\ 1 - P(z_j - x_j - 1, j, T) & x_j \neq L_j \text{ \& } z_j = L_j \\ P(z_j - x_j, j, T) & x_j \neq L_j \text{ \& } z_j \neq L_j \end{cases} \quad \text{B.18}
\end{aligned}$$

and $P(k, j, T)$ is given by the equation (B.2).

$Q^{(-)}$ is given by:

$$[Q^{(-)}]_{x,z} = \begin{cases} 0 & E_j / x_j - z_j > 0 \\ \sum_{j=1}^N \text{Prob}((z_j - x_j), j, -) & \text{otherwise} \end{cases} \quad \text{(B.19)}$$

where $\text{Prob}(k, j, -)$ is given by equation (B.1).

(ii) The State Transition Matrix P_1 for PC2

Let $[Q]^k$ represent the resulting matrix obtained from multiplying Q , k times by itself. Let also $[Q]_x^k$ denote the state transition row associated with the state x in the state transition matrix $[Q]^k$. Recall that in this protocol, a controller transmits up to ℓ_j packets out of

those it has at the time it acquired channel access.

Clearly we have:

$$P_{i,j}^1 = \begin{cases} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} & \text{for } \forall x \in X/0 = x_{N+1}, x_3 \neq 0 \\ & \text{and } k = \min\{x_3, x_2\} \\ & \text{otherwise} \end{cases}$$

a deterministic row with (x,x) th element equal to one and (x,z) th element equal to zero for $x \neq z$.

E.21

(iii) The State Transition Probability Matrix P_1^1 for PC3

In this protocol, a controller transmits all of the packets stored in its buffer at the time it acquired channel access. Clearly we have:

$$P_{i,j}^1 = \begin{cases} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} & \text{for } \forall x \in X/0 = x_{N+1}, \forall x \in X/x_3 \neq 0 \\ & \text{a deterministic row with } (x,x) \text{th element} \\ & \text{equal to one and } (x,z) \text{th element equal to} \\ & \text{zero for } x \neq z. \end{cases}$$

E.22

(iv) The State Transition Probability Matrix P_1 for PC4

Recall that in this protocol, a controller, say the j th one, who has successfully acquired channel access, holds the channel until its buffer becomes empty. Therefore the probability distribution of the successful transmission period is similar to that of ruin time of a gambler with initial capital of x_j where $\beta = x_{N+1}$. Let $G_j(z)$ denote the z transform of this probability distribution. The state transition row for the successful

transmission period for a given state $\underline{x}$ is equal to:

$$[P1]_{\underline{x}} = [G_j(\underline{Q}^{(T)})]_{\underline{x}} \underline{Q}^{(T)} \quad (B.23)$$

We are not interested in further analysis of this protocol as it is questionable whether it is worth the effort.

(C) The State Transition Probability Matrix $P2$

As mentioned earlier, the state transition matrix $P2$ is the same for the protocols PC1, PC2, P3 and PC4. The state transition matrix $P2$ is obtained by establishing the state transition row associated with each state. Immediately after a successful transmission period, the state space, associated with the state definition given by (3.2), is partitioned into three subsets:

- (1) The subset C-states = $\{\underline{x} \in X | n^*(\underline{x}) \geq 2\}$ i.e., collision will occur with probability equal unity
- (2) The subset I-states = $\{\underline{x} \in X | x_j = 0, j=1, \dots, N\}$ i.e., no ready controller is in the system
- (3) The subset S-states = $\{\underline{x} \in X | n^*(\underline{x}) = 1\}$ i.e., states in which there is a single ready controller.

In the following, the state transition rows associated with the states in each of the above partitions are presented.

(1) The "State Transition Probability Rows for the C-States

For each state of the C-States, collision occurs with probability one, hence we restrict $Q^{(2\tau)}$ into the C-States transition matrix $Q^{(C,2\tau)}$ as follows:

$$[Q]_{x,z}^{(C,2\tau)} = \begin{cases} 0 & n^*(x) \leq 1 \\ [Q^{(2\tau)}]_{x,z} & n^*(x) \geq 2 \end{cases} \quad (B.24)$$

using the z-transform of the distribution of the collision period given by (B.10) we have:

$$Q^{(C,r)} = R(Q^{(C,2\tau)}) \quad (B.25)$$

where $[Q^{(2\tau)}]_{x,z}$ is the (x,z) th element of the one step (of duration 2τ) state transition probability matrix $Q^{(2\tau)}$. The (x,z) th element of this matrix is defined as follows:

$$[Q]_{x,z}^{(2\tau)} = \begin{cases} 0 & n^*(x) \leq 1 \\ \prod_{j=1}^N \text{Prob}((z_j - x_j), j, 2\tau) & n^*(x) \geq 2 \end{cases} \quad (B.26)$$

where $\text{Prob}((z_j - x_j), j, 2\tau)$ is given by equation (B.18) when in this equation, T is replaced by 2τ , and $n^*(x)$ is defined by (B.3).

(ii) State Transition Probability Rows For The I-States

Define the one step state transition probability for a state say z in the last time slot of the idle period to be

$$\text{Prob}(z|Q) = \left[\sum_{j=1}^N \text{Prob}(z_j, j, 2-) \right] / \left(1 - \sum_{j=1}^N \text{Prob}(0, j, 2-) \right) \quad (\text{B.27})$$

Let $[P^{(I)}]_Q$ represent the vector state transition probability for the last time slot of an idle period:

$$[P^{(I)}]_{Q,z} = \begin{cases} 0 & n^*(z) \leq 1 \\ \text{Prob}(z|Q) & n^*(z) \geq 2 \end{cases} \quad (\text{B.28})$$

Those states for which $n^*(z) \leq 1$, do not go through a further collision, hence we have

$$[P_2]_{Q,z} = \text{Prob}(z|Q) \quad \forall z \in X | n^*(z) \leq 1 \quad (\text{B.29})$$

However states z for which $n^*(z) \geq 2$ go through a further collision period. Hence

$$[P_2]_{Q,z} = [P^{(I)}]_Q Q^{(c,r)} \quad \forall z \in X | n^*(z) \geq 2 \quad (\text{B.30})$$

where the elements $[P^{(I)}]_{Q,z}$ of the state transition probability vector $[P^{(I)}]_Q$ is given by (B.28). This completes the specification of the state transition matrix P_2 .

(iii) The State Transition Rows for the S-States

For $x \in S$ -States, there is only one ready user in the system hence we have:

$$[P2]_{x,z} = \begin{cases} 0 & \text{for all } z \in X \mid x \neq z \\ 1 & \text{for all } z \in X \mid x = z \end{cases} \quad (B.31)$$

Using equations (3.5) and (3.6), we now establish the state transition probability matrices $P_k(P_u)$ for protocols PC1, PC2, PC3 and PC4. The steady state distribution is evaluated by solving $\pi_k(X) = \pi_k(X) \cdot P_k$, $(\pi_u(X) = \pi_u(X) P_u)$. Using (3.15) to (3.20) the performance measures of the system are evaluated.

APPENDIX C

G-MODEL: LARGE SIZE SYSTEM MODEL

This Appendix presents an analytic model for the single packet transmission protocol PC1 given in Section 3.2. The analytic model employs the state definition given by the equation (3.23). Thus it can be applied to large size systems (e.g., $N=0, \dots, 20$ and $L_j=1, 2, \dots, 5$). Let $x(t^{(e+k)})$ denote the state of the system at the k th time point $t^{(e+k)}$ and is defined by the equation (3.23). The stochastic process $\{x(t^{(e+k)})\}; k=0, 1, 2, \dots\}$ describes the dynamic behaviour of the system at time points $t^{(e+k)}, k=0, 1, 2, \dots$. As mentioned in Section 3.3.4, the determination of the state transition matrix P is a difficult task. This is due to the fact that the state definition given by the equation (3.23) is employed. Recall that the first element of the state vector represents the number of ready controllers (i.e., controllers with one or more packets stored in their buffer) among the $N-1$ selected controllers. Note that the number of packets stored in the buffer of these controllers is not recorded in x . This is the primary source of the difficulty in writing the state transition matrix P . In the following, an example is given in order to clarify this difficulty. Let $x(t^{(e+k)}) = (x_1, x_2, x_3)$ represent the state of the system at time point $t^{(e+k)}$.

Assume that at time point $t_i^{(e+k)}$, we have $x_1 = N-1$ and $x_2 > 0$ (i.e., all controllers are ready). Furthermore, assume that a ready controller with two or more packets among the x_1 ready controllers, gains channel access and transmit a packet successfully. At the subsequent time point $t_i^{(e+k+1)}$, the element x_1 of the state vector $\mathbf{x}$ remains unchanged (i.e., $x_1(t_i^{(e+k)}) = x_1(t_i^{(e+k+1)}) = N-1$). However, if the ready controller had only one packet in its buffer at time $t_i^{(e+k)}$ and no new packet has arrived in its buffer, during the time period $t_i^{(e+k+1)} - t_i^{(e+k)}$, then the element x_1 of the state vector $\mathbf{x}(t_i^{(e+k+1)})$ is changed from $N-1$ to $N-2$ (i.e., $x_1(t_i^{(e+k)}) = N-1$ and $x_1(t_i^{(e+k+1)}) = N-2$).

The above example demonstrates that the knowledge of the probability that the controller, the one among the x_1 ready controllers, which transmitted successfully and it had only one packet in its buffer, is vital to the determination of the state transition probability matrix P . Let $n^*(\mathbf{x})$ denote the number of ready controllers when the system is in state $\mathbf{x}$. Recall that ready controllers are selected randomly at the beginning of each successful transmission period. At this time point, let $P^{(1)}(n^*(\mathbf{x}))$ denote the probability that a randomly chosen ready controller has only one packet in its buffer when there are $n^*(\mathbf{x})$ ready controllers in the system. Let's assume that $P^{(1)}(n^*(\mathbf{x}))$ exists and has a constant value. This assumption reduces the general stochastic process

$(x, t) \in e+k$, $k=0,1,\dots$) into a Markov chain. Clearly this Markov chain is ergodic since it is homogeneous i.e., the transition probabilities are independent of the particular embedded point, positive recurrent (i.e., it is possible to return back to every state after a finite number of transitions), and aperiodic (i.e., the number of transitions after which we return back to a state assume every positive value). Therefore the G-model could be applied in order to analyze the chain. We employ an approach similar to that given in [131] and specify the probability $p^{(1)}(n^*(x))$.

In the following, consider the single packet transmission protocol PC¹ given in Section 3.2 and the state of the system $x = (x_1, x_2, x_3)$ defined in equation (3.23). Consider a successful transmission of a packet by a controller randomly selected out of the x_1 active controllers. Clearly, the only time that the state element x_1 of the state x changes is when there is only one packet in the buffer of that controller.

Since a controller is selected among the $n^*(x)$ active controllers with equal probability, the probability $p^{(1)}(n^*(x))$ that a randomly chosen ready controller has only one packet in its buffer when there are $n^*(x)$ ready controllers is equal to the corresponding probability for the originally selected controller (i.e., the one its

* In $p^{(k)}(n^*(x))$, the superscript (k) denotes the number of packets in the buffer of the randomly selected controller. For simplicity, in cases where $k=1$, the superscript k may be deleted.

number of packets of which is represented by x_2):

$P^{(1)}(n^*(x)) = \text{Prob}(\text{The transmitting controller has one packet in its buffer when there are } n^*(x) \text{ ready controllers}).$
 $= \text{Prob}(\text{The selected controller has one packet in its buffer when there are } n^*(x) \text{ ready controllers given that the selected controller is ready; i.e., it has one or more packets})$

$$= \frac{\pi((n^*(x)-1), 1, *)}{\sum_{j=1}^L \pi((n^*(x)-1), j, *)} \quad \text{for } n^*(x)=1, 2, \dots, N \quad (C.1)$$

where

$$\pi(x_1, x_2, *) = \sum_{x_3=1}^2 \pi(x_1, x_2, x_3) \quad (C.2)$$

Consider the state definition given in the equation (3.23). Let $P(k, 2, \tau)$ denote the probability that k new packets arrive in the buffer of the selected controller during a single propagation delay, we have:

$$P(k, 2, \tau) = \frac{(\tau \lambda_j)^k e^{-\tau \lambda_j}}{k!} \quad k=0, 1, 2, \dots \quad (C.3)$$

Let also $P(k, 2, T)$ represent the probability that k new packets arrive in the buffer of the selected controller during a packet transmission time T :

$$P(k, 2, T) = \frac{T^k e^{-T}}{k!} \quad k=0, 1, 2, \dots \quad (C.4)$$

for any state $\underline{x}$, we define $n^*(\underline{x})$ as follows:

$$n^*(\underline{x}) = x_1 + U(x_2)$$

$$U(x_2) = \begin{cases} 1 & x_2 \geq 1 \\ 0 & x_2 = 0 \end{cases} \quad (C.5)$$

$n^*(\underline{x})$ is the number of ready controllers when the system is in state $\underline{x}$. Clearly we have:

$$\begin{aligned} \text{Prob}(I^{(e+1)} = k | n^*(\underline{x}) = 0) \\ = (1-P_0) P_0^{k-1} \quad k = 1, 2, \dots \end{aligned} \quad (C.6)$$

where

$$P_0 = (P(0, 1, 2\tau)) (P(0, 2, 2\tau)) \quad (C.7)$$

and for $k=0, 1, \dots, N-1$:

$$P_{\underline{x}}(k, 1, 2\tau) = \binom{N-x_1-1}{k} (1-e^{-2\tau\lambda_j})^k (e^{-2\tau\lambda_j})^{N-x_1-k-1} \quad (C.8)$$

As for the conditional distribution of the random variable $r^{(e+1)}$, it is equal to that presented in Appendix B (refer to the equations (B.7) to (B.13)), i.e., we have:

$$\begin{aligned} \text{Prob}(r^{(e+1)} = k | n^*(\underline{x}) \geq 2) \\ = \sigma(1-\sigma)^{k-1} \quad k = 1, 2, 3, \dots \end{aligned} \quad (C.9)$$

in which σ is given by (refer to equation (B.8) in Appendix B):

$$\sigma = (1 - 1/N)^{(N-1)} \quad N = 2, 3, \dots \quad (C.10)$$

The z transform of the above conditional distribution is similar to equation (B.10) and is given by:

$$R(z) = \frac{\sigma z}{(1 - (1 - \sigma)z)} \quad (C.11)$$

or equivalently, the Laplace transform of the above conditional distribution, denoted by $R^*(s)$, is given by:

$$R^*(s) = \frac{\sigma e^{-s(2\tau)}}{1 - (1 - \sigma)e^{-s(2\tau)}} \quad (C.12)$$

The first, second and the third moments of the above distribution are denoted by r_1 , r_2 and r_3 respectively (which are similar to those given by (B.11) to (B.13)) and are given as follows:

$$r_1 = 2\tau/\sigma \quad (C.13)$$

$$r_2 = (2\tau)^2 ((2 - \sigma)/\sigma^2) \quad (C.14)$$

$$r_3 = (2\tau)^3 (6 - 6\sigma + \sigma^2)/\sigma^3 \quad (C.15)$$

So far we have set the stage for writing expressions for the state transition matrices, Q , P_1 , and P_2 as we will proceed to do so in the following section.

(a) The State Transition Probability Matrix Q

All of the protocols maintain the following property: at the beginning of each successful transmission, all ready controllers have equal chance of acquiring the channel irrelevant of which controller had channel access in the last successful transmission. Using this property,

$$Q(c, r) = \sum_{k=1}^K \text{Prob}(r=k) \cdot Q(c, z^k)$$
[illegible][illegible]

Figure 0.1 The State Transition Diagram for the OSMA-1 system. For the OSMA-1 system, the state $x_5 = 1$ and $x_5 = 2$ are the two definitions of the equation 3.1.

(3) $k = x_3$,

$x_k - z_k > 1$: transmitting controller shrinks by more than one packet or more than one active controllers transmit their packet during a successful transmission period

4. $k = x_3$,

$x_k \neq 0$ & $z_k \neq x_k$ $x_k - z_k > 0$: a nontransmitting controller shrinks by one or more packets

5. $x_2 = 1$ and $z_2 \neq 1$ and $x_3 = 2$:

transmitting controller does not find itself in state 1- after a successful transmission

$$= P \cdot x \cdot [\text{Prob } z_1 = x_1 \cdot \dots \cdot \text{Prob } z_2 = x_2 \cdot \dots \cdot \text{Prob } z_3 = x_3]$$

$$= P \cdot x \cdot [\text{Prob } z_1 = x_1 \cdot \dots \cdot \text{Prob } z_2 = x_2 \cdot \dots \cdot \text{Prob } z_3 = x_3]$$

$$\text{for } x_3 = 2 \quad z_1 = x_1$$

$$= \text{Prob } z_1 = x_1 \cdot \dots \cdot \text{for } x_3 = 2 \text{ \& } z_2 = 1 \text{ \& } x_2 = 1$$

$$= \text{Prob } z_1 = x_1 \cdot \dots \cdot \text{Prob } z_2 = x_2 \cdot \dots \cdot \text{for } x_3 = 2$$

$$\text{\& } z_2 = 1$$

$$= \text{Prob } z_1 = x_1 \cdot \dots \cdot \text{Prob } z_2 = x_2 \cdot \dots \cdot \text{for } x_3 = 2 \text{ \& } z_2 = 1 \text{ \& } x_2 = 1$$

$$\text{for } x_3 = 2 \text{ \& } z_2 = 1 \text{ \& } x_2 = 1$$

$$= \text{Prob } z_1 = x_1 \cdot \dots \cdot \text{Prob } z_2 = x_2 \cdot \dots \cdot \text{for } x_3 = 2 \text{ \& } z_2 = 1$$

$$\text{for } x_3 = 2 \text{ \& } z_2 = 1$$

where

$\text{Prob } K, T$ = Probability that K controllers become busy given state X , during a time period T .

$$= \frac{1}{N!} \sum_{k=0}^{N-K} \frac{N!}{k! (N-K-k)!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{\lambda}{\mu} \right)^{N-K-k} \left(\frac{\lambda}{\mu} \right)^{N-K-k} \quad 0.18$$

$$K = 0, 1, \dots, N-1$$

and

$$\text{where } x_2 = x_2 = \dots$$

$$\text{Prob } K, 2, T = \frac{1}{N!} \sum_{k=0}^{N-K} \frac{N!}{k! (N-K-k)!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{\lambda}{\mu} \right)^{N-K-k} \left(\frac{\lambda}{\mu} \right)^{N-K-k} \quad \text{where } x_2 = 1 \text{ \& } x_2 = \dots$$

$$\text{Prob } K, 2, T = \frac{1}{N!} \sum_{k=0}^{N-K} \frac{N!}{k! (N-K-k)!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{\lambda}{\mu} \right)^{N-K-k} \left(\frac{\lambda}{\mu} \right)^{N-K-k} \quad x_2 = 1 \text{ \& } x_2 = \dots$$

$$0.19$$

and

$\text{Prob } X$ = Probability that the transmitting controller has only one packet in its buffer

Using equivalent statistical behaviour of all stations, this steady state probability is equivalent to the probability that there is one packet in the buffer of the selected controller and is given by $0.1 - 0.18$.

and

$$\text{such that } x_1 = 1, x_2 = 0, \dots, x_N = 0$$

$$\text{Prob } X, X = \dots$$

$$\text{Prob } X_1 = x_1, \dots, \text{Prob } X_2 = x_2, \dots$$

$$0.20$$

In Figure 0.2 we have shown an example for the matrix E for a system in which $N=3$ and $L=2$ non-zero elements

Figure 1. An Example for The State Transition Matrix $\lambda_5 = 2$. The State Definition is given by Equation 3.23.

are shown by an $\cdot$. Zero elements are denoted by an empty space.

(d) The State Transition Probability Matrix P_2

The state transition matrix P_2 is obtained by establishing the state transition row associated with each state.

Immediately after a successful transmission period, we partition the state space into three subsets:

(1) The subset C-States = $\{x \in X | n^*(x) \geq 2\}$

i.e., collision will occur with probability one

(C.21)

(2) The subset I-States = $\{x \in X | x_j = 0, j=1,2\}$

i.e., the idle time period starts with probability equal to unity.

(3) The subset S-States = $\{x \in X | n^*(x) = 1\}$

i.e., states in which there is a single ready controller

In the following, we present the state transition rows associated with the above subsets of state space separately. Note that the state transition matrix P_2 is the same for all of the protocols PC1, PC2, PC3 and PC4.

(1) The State Transition Probability Rows for the C-States

Let $q^{(c,r)}$ denote the state transition probability matrix for the C-states. For each state of the C-states, collision occurs with probability one, hence we restrict

$Q^{(2)}$ into the state transition probability matrix $Q^{(c,2)}$ as follows:

$$[Q^{(c,2)}]_{x,z} = \begin{cases} 0 & n^*(x) \leq 1 \\ [Q^{(2)}]_{x,z} & n^*(x) > 1 \end{cases} \quad (C.22)$$

where $[Q^{(2)}]_{x,z}$ is the (x, z) th element of the one step (of duration 2) state transition matrix $Q^{(2)}$ defined as:

$$[Q^{(2)}]_{x,z} = \begin{cases} 0 & \exists j | x_j - z_j > 0 \\ \text{Prob}((z_1 - x_1), 1, 2) \cdot \text{Prob}((z_2 - x_2), 2, 2) & \end{cases} \quad (C.23)$$

using the z transform of the distribution of the collision period given by equation (C.11), $Q^{(c,r)}$ is given as follows:

$$Q^{(c,r)} = R(Q^{(c,2)}) \quad (C.24)$$

(ii) The State Transition Probability Row Associated With the Idle States (i.e., I-states)

The one step state transition probability for a state say x in the last time slot of the idle period is given by:

$$\text{Prob}(x|Q) = [\text{Prob}(x_1, 1, 2) \cdot \text{Prob}(x_2, 2, 2)] / [1 - \text{Prob}(0, 1, 2) - \text{Prob}(0, 2, 2)] \quad (C.25)$$

where $\text{Prob}(k, 1, 2)$ and $\text{Prob}(k, 2, 2)$ are similar to the equations (C.14) and (C.15) respectively (change T to 2).

in these equations).

Let $[P^{(I)}]_0$ represent the state (those states for which $n^*(z) \geq 2$) transition probability vector over the last time slot of an idle period:

$$[P^{(I)}]_{0,z} = \begin{cases} 0 & n^*(z) \leq 1 \\ \text{Prob}(z|0) & n^*(z) \geq 2 \end{cases} \quad (C.26)$$

Those states z for which $n^*(z) \leq 1$ do not go through a further collision, hence we have

$$[P2]_{0,z} = \text{Prob}(z|0) \text{ for all } z | n^*(z) \leq 1 \quad (C.27)$$

However, states z for which $n^*(z) \geq 2$ go through a further collision period hence

$$[P2]_{0,z} = [P^{(I)}]_0 Q^{(c,r)} \text{ for all } z \in C\text{-States} \quad (C.28)$$

where the elements $[P^{(I)}]_{0,z}$ of the state transition vector $[P^{(I)}]_0$ is given by (C.21). This completes the state transition row for the idle state in the state transition matrix $P2$.

(iii) The State Transition Probability Rows for the S-States

For $x \in S$ -states, there is only one ready user in the system hence we have:

$$[P2]_{x,z} = \begin{cases} 0 & \text{for all } x \in X | x \neq z \\ 1 & \text{for all } x \in X | x = z \end{cases} \quad (C.29)$$

Using equation (3.5) we now establish the state transition probability matrix P for the protocol PC1. The steady state distribution is evaluated by solving equation (3.8). In this case the matrix P_1 is a function of $P^{(1)}(n^*(\underline{x}))$ and therefore it is a function of the steady state probabilities $\pi(\underline{x})$. We need to solve (3.8) and (C.1) numerically by assuming an initial distribution $\pi(x)$ and solving (C.1) to find new $P^{(1)}(n^*(\underline{x}))$. $P^{(1)}(n^*(\underline{x}))$ is employed to determine the matrix P^* . Using equation (3.8), new $\pi(x)$ is calculated. This process is repeated until a convergence criterion is met.

Note that the state transition probability matrix P_2 is calculated only once whereas P_1 is calculated for each new $P^{(1)}(n^*(\underline{x}))$. Numerical results have shown that for various system configurations, $P^{(1)}(n^*(\underline{x}))$ converged after repeating the above procedure nine times, i.e., a rapid convergence.

The marginal distribution of queue size for the selected controller is given by:

$$\text{Prob}(x_2=k) = \frac{\pi(\underline{x})}{\{\underline{x} \in X | x_2=k\}} \quad (\text{C.30})$$

and the average queue length at the buffer of the j th controller denoted by $q_1(j)$ is given by

$$q_1(j) = \sum_{k=0}^L k \cdot \text{Prob}(x_2=k) \quad (\text{C.31})$$

Appendix G gives the Laplace transform of the distribution of the interdeparture time. It also presents the average

interdeparture time $E(t_{id})$. The blocking probability for j th controller is given by (3.14).

The throughput S is then given by equation (3.17).

The average number of packets in the system is given by

$$\bar{x} = N * q_1(j) \quad (C.32)$$

using Little's result [64], the average network delay is given by (3.20).

APPENDIX D.

G-MODEL: EXAMPLES

This Appendix presents a number of numerical examples for the analytic models presented in Appendices C and D.

D.1 An Example for Small System Sizes

In this section, an example is presented for the small system size model presented in Appendix B. The multiple packet transmission protocol PC3, presented in Section 3.2, is employed in a homogeneous environment; i.e., there are two controllers (i.e., $N=2$) each of which has a buffer capacity of two packets (i.e., $L_1 = L_2 = 2$); the mean arrival rate to each controller is $\lambda_1 = \lambda_2 = .25$ (i.e., $\lambda = .5$) and $a = .1$.

(a) Generation of the States of the System

For $N=2$, $L=2$ we have

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad 0 \leq x_1, x_2 \leq L; 1 \leq x_3 \leq N$$

The states of the system are generated using the following algorithm:

Do for $I = 1, N$

- generate all N digit numbers on base $(L+1)$
- for all of the generated numbers, consider

$$x_{N+1} = I$$

Continue

The states generated for our example are:

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad x_9 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad x_{10} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \quad x_{11} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad x_{12} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad x_{13} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad x_{14} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$x_6 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad x_{15} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$x_7 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \quad x_{16} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$x_8 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \quad x_{17} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

(b) State Transition Probability P_1

As mentioned in earlier sections, a transmitting user remains the same during the entire time in a successful transmission subcycle, e.g.,

$$P_1(x = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} | z = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}) = 0$$

(iv) Using (B.22) and the T-step state transition probability matrix when the second controller is transmitting, the second matrix along the diagonal of P_1 is calculated:

(c) State Transition P2

A transmitting user remains the same during the entire time in a collision time period. We have represented the selection of the transmitting user by the matrix $\underline{Q}$ which takes place just before the beginning of a successful transmission. Using equations (B.24) - (B.31), the state transition probability matrix $\underline{P2}$ is calculated in the following steps:

(i) The C-states state transition probability matrix $\underline{Q}^{(c,r)}$ is established as follows: first, the state transition probability matrix $\underline{Q}^{(c,2^1)}$ is established for a time slot of the collision time period using the equation (B.24). Then using (B.25), the matrix $\underline{Q}^{(c,r)}$ is

$$Q(c, r) = \sum_{k=1}^{k'} \text{Prob}(r=k) \cdot Q(c, z)$$
[illegible][illegible]

(d) State Transition Probability Matrix $\underline{P}$

The state transition probability matrix $\underline{P}$ is developed using equation B.14 and stored in an $M \times M$ matrix. Note that the matrix $\underline{P}$ is composed of L identical submatrices of dimension $M' \times M'$.

(e) State Transition Matrix $\underline{P}_n$

Using equation 3.6, $\underline{P}_n$ is calculated as follows:

.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00

(f) Steady State Distribution $\underline{\pi}$

Using an initial state say $\underline{x}_0$, the matrix $\underline{P}_n$ is multiplied by itself and after 9 times, it converged to the steady state distribution:

.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00

(g) Calculation of the State Transition Probability

Matrix P

The elements of the matrix P by the matrix P is performed as follows. Consider the first row of each of the $M \times M$ submatrices of the $M \times M$ matrix P . These rows form an $M \times M$ matrix. If this matrix is the average of the homogeneous $M \times M$ submatrices, the elements of the matrix P are

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

11. Multiplying the above matrix P from left, matrix P is calculated:

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(h) Steady State Distribution π

After multiplication of P matrix by itself 9 times, it converged to a matrix with same rows, i.e., π :

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

D.2 An Example for Large System Sizes

In this section, an example is presented for the application of the G-model to analysis of large system sizes. The single packet transmission protocol [1] is employed in a homogeneous environment where there are three controllers, i.e., $N=3$, each of which has a buffer capacity of three packets, i.e., $L_i = 3$ for $i=1,2,3$. The mean arrival rate to each controller is $\lambda_i = 0.1, 0.2$ and $\lambda_3 = 0.1$, i.e., $\lambda_1 = 0.1, \lambda_2 = 0.2$ and $\lambda_3 = 0.1$.

The overall numerical analysis of large system sizes is summarized in the following algorithm.

- Generate system states
- $\pi(X)$ - initial state occupancy probabilities
- $P^{(0)}(n^*(X))$ - initial $P^{(0)}(n^*(X))$
- calculate matrix P_2
- Repeat 10 times:
 - calculate P_1 using $P^{(0)}(n^*(X))$
 - calculate P using equation (3.5)
 - compute steady state probabilities $\pi(X)$
 - calculate new $P^{(1)}(n^*(X))$ using equations (C.1) and (C.2)

until convergence criteria met

Please note that the state transition matrix P_2 is calculated only once whereas P_1 is calculated repeatedly using new values of $P^{(1)}(n^*(X))$.

(a) Generation of the States of the System

For $M=3$ and the state definition given by equation (3.1), we have

$$X = \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \quad x_1 = 0 \leq x_1 \leq M-1 = 2 \leq x_2 \leq 2, \quad x_3 = 0, 1, 2$$

The states of the system generated for our example are

$$\begin{array}{llll} X_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & X_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & X_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} & X_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ X_5 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} & X_6 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} & X_7 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} & X_8 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \\ X_9 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} & X_{10} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & X_{11} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} & X_{12} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \\ X_{13} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} & X_{14} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & X_{15} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} & X_{16} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \\ X_{17} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} & X_{18} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} & X_{19} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} & X_{20} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \\ X_{21} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} & X_{22} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} & X_{23} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} & X_{24} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \\ X_{25} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} & X_{26} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} & X_{27} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} & \end{array}$$

(b) Generation of the Matrix P_1

A transmitting user remains the same during the entire time in a successful transmission time period. Therefore the matrix P_1 is a sparse matrix with nonzero elements in the form of the two submatrices $(M/2 \times M/2)$ along its diagonal. Each of these submatrices are calculated separately so that an $(M \times M/2)$ storage is

required. The matrix P_1 is calculated for each convergence cycle through application of the equations 0.17 to 0.20. This procedure is almost similar to that applied in the small size system model. Each time that the state transition matrix probability P_1 is calculated, a new $P^{(1)}(n^*(x))$ is used.

The state transition probability matrix P_2 is calculated using equations 0.22 to 0.29. The procedure is similar to that given for small size system model.

Finally the state transition probability matrix P is calculated using (3.5).

The steady state distribution $\pi(X)$ is calculated using (3.8). Using values $\pi(X)$, new values for $P^{(1)}(n^*(x))$ is calculated and used for the determination of a new state transition probability matrix P_1 . This process is repeated until $P^{(1)}(n^*(x))$ converges.

$P^{(1)}(n^*(x))$ converged rapidly (i.e., after nine times) for variety of system configurations. This is evident in the convergence pattern associated with the following examples.

The convergence pattern of $P^{(1)}(n^*(x))$ for a system in which $N=3$, $L=3$, $a=0.01$ and $\lambda=0.4$ is as follows:

$n^*(x)=1$	$n^*(x)=2$	$n^*(x)=3$
.92941	.81614	.66445
.89924	.87754	.83800
.89733	.88366	.86541
.89722	.88431	.86898
.89722	.88438	.86943
.89722	.88439	.86948
.89722	.88439	.86949
.89722	.88439	.86949
.89722	.88439	.86949
.89722	.88439	.86949

each row represents the values associated with $P^{(1)}(n^*(x))$, $n^*(x)=1,2,3$. Note that $P^{(1)}(n^*(x))$ converges after 8 times. The convergence pattern of $P^{(1)}(n^*(x))$ for the same system configuration except $\alpha=.6$ is as follows:

$n^*(x)=1$	$n^*(x)=2$	$n^*(x)=3$
.89339	.74335	.53317
.81998	.77690	.70837
.81176	.78219	.74782
.81086	.78325	.75528
.81077	.78347	.75664
.81076	.78351	.75688
.81076	.78352	.75693
.81076	.78352	.75694
.81076	.78352	.75694

Note that $P^{(1)}(n^*(x))$ converges after 9 times.

APPENDIX E

STATE SPACE SIZE REDUCTION

This appendix reviews a number of analytical techniques for reduction of the size of the state space X associated with the state definition (3.2) and (3.23). It also presents a numerical technique which can be employed for reduction of the size of the state space X .

(1) Macro State Definition

It is evident that when the state definition given by (3.2) is employed, the number of states (given by (3.4)) increases rapidly when system size grows. The growth is due to the detailed information that must be recorded about the queues inside the system. There are a number of alternatives for reducing the size of the state space. One approach is to use the theory of "lumpable" Markov chains [88] to lump equivalent states into a Macro state and obtain a Markov chain of substantially smaller size. An extension of this approach suggests that we could merge states $\{x_j\}$, $j = 1, 2, \dots, M_1$ into Macrostate m_1 , states $\{x_k\}$, $k = 1, \dots, M_2$ into Macrostate m_2 and obtain an approximate Markov chain for the exact one. The Macrostate transition rates are obtained using the state transition probabilities in the exact Markov chain. Denote the state transition rate from state x_j to x_k in

the exact Markov chain by $P(z_k|x_j)$, then the state transition rate from Macrostate m_1 to Macrostate m_2 will be given by [88]:

$$P(m_2/m_1) = \frac{1}{M_1} \sum_{j=1}^{M_1} \sum_{k=1}^{M_2} P(z_k|x_j) \quad (E.19)$$

If the merged states in a Macrostate have equal steady state probability, no error is made in the approximation, otherwise, the resulting chain only approximates the exact one.

The above approaches easily extend the feasibility of the application of the generalized model to the larger system sizes. For example, the above averaging approach yields exact results for a system with homogeneous controller's input traffic in which $L_j=L$, $j = 1, \dots, N$ and the following Macrostate definition: for a given state x define the Macrostate $m(x)$ as:

$$m(x) = \{z \in X | x_j = z_k \text{ such that } x_j = z_k\} \quad (E.2)$$

i.e., $m(x)$ is a subset of the state space and it is the set of all states with equivalent nonzero elements.

(ii) A Numerical Approach

In a homogeneous environment, the size of the state space for the state definition given by (3.23), is equal to $M=2N(L+1)$. Using certain properties of the state transition matrices obtained from the analysis of the model for large size systems, a numerical approach is

presented for the reduction of storage requirement for the state transition matrices. In the following, first a theorem is presented for the matrices with certain properties. Next, it is shown that the application of this theorem reduces the size of the matrix P by 75 percent.

Theorem E.1

Let A be a $M \times M$ matrix and satisfies the following properties:

1 - M is an even integer

$$2 - A = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \text{ where } A_{1,1} = A_{2,1} \text{ and } A_{1,2} = A_{2,2}$$

3 - $A_{1,j}$ are submatrices of $M/2 \times M/2$.

Let A_1 be, an $M/2 \times M/2$ matrix, constructed by adding the two submatrices, $A_{1,1}$ and $A_{1,2}$ of the matrix A , i.e.,

$$A_1 = A_{1,1} + A_{1,2} \quad (E.3)$$

A^K is a matrix resulted from multiplication of the matrix A by itself K times. Let A^K be represented as follows:

$$A^K = \begin{bmatrix} A_{1,1}^{(K)} & A_{1,2}^{(K)} \\ A_{2,1}^{(K)} & A_{2,2}^{(K)} \end{bmatrix}$$

in the following, we prove that

$$(a) - A_{1,1}^{(K)} = A_{2,1}^{(K)} \quad \text{and} \quad A_{1,2}^{(K)} = A_{2,2}^{(K)} \quad (E.4)$$

$A^{(k)}$ is related to $A_{1,1}$ through the following relation:

$$A^{(k)} = A_{1,1} + A_{1,2} A^{(k-1)} = A_{1,1} A^{(k-1)} + A_{1,2} A^{(k-1)} \quad (E.5)$$

Proof of Part (a)

Multiplying A by itself, i.e., for $k=2$, we have:

$$A_{1,1}^{(2)} = A_{1,1} + A_{1,2} A_{1,1} \quad ; \quad A_{1,2}^{(2)} = (A_{1,1} + A_{1,2}) A_{1,2} \quad (E.6)$$

$$A_{2,1}^{(2)} = (A_{1,1} + A_{1,2}) A_{2,1} \quad ; \quad A_{2,2}^{(2)} = (A_{1,1} + A_{1,2}) A_{2,2}$$

Therefore $A_{1,1}^{(2)} = A_{2,1}^{(2)}$ and $A_{1,2}^{(2)} = A_{2,2}^{(2)}$. Now suppose we multiply A by itself $k-1$ times, and for $A^{(k-1)}$ we have:

$$A_{1,1}^{(k-1)} = A_{2,1}^{(k-1)} \quad \text{and} \quad A_{1,2}^{(k-1)} = A_{2,2}^{(k-1)}$$

multiplying $A^{(k-1)}$ by A from right we have:

$$A_{1,1}^{(k)} = (A_{1,1}^{(k-1)} + A_{1,2}^{(k-1)}) A_{1,1} \quad ; \quad A_{1,2}^{(k)} = (A_{1,1}^{(k-1)} + A_{1,2}^{(k-1)}) A_{1,2}$$

$$A_{2,1}^{(k)} = (A_{1,1}^{(k-1)} + A_{1,2}^{(k-1)}) A_{2,1} \quad ; \quad A_{2,2}^{(k)} = (A_{1,1}^{(k-1)} + A_{1,2}^{(k-1)}) A_{2,2} \quad (E.7)$$

Similarly, multiplying $A^{(k-1)}$ by A from left, we have:

$$A_{1,1}^{(k)} = (A_{1,1} + A_{1,2}) A_{1,1}^{(k-1)} \quad ; \quad A_{1,2}^{(k)} = (A_{1,1} + A_{1,2}) A_{1,2}^{(k-1)}$$

$$A_{2,1}^{(k)} = (A_{1,1} + A_{1,2}) A_{2,1}^{(k-1)} \quad ; \quad A_{2,2}^{(k)} = (A_{1,1} + A_{1,2}) A_{2,2}^{(k-1)}$$

Proof of Part (b)

For $k=2$, and using (E.6) we have

$$\begin{aligned} A_{1,1}'^{(2)} + A_{1,2}'^{(2)} &= (A_{1,1} + A_{1,2}) A_{1,1} + (A_{1,1} + A_{1,2}) A_{1,2} \\ &= (A_{1,1} + A_{1,2}) (A_{1,1} + A_{1,2}) = (A_{1,1} + A_{1,2})^2 = A_{1,1}^2 \end{aligned}$$

E.8

Similarly, suppose the following equality satisfies for A^{k-1} :

$$A_1^{(k-1)} = (A_{1,1} + A_{1,2})^{k-1} = A_{1,1}'^{(k-1)} + A_{1,2}'^{(k-1)} \quad \text{E.9}$$

multiplying A^{k-1} by A (from right) we have:

$$A_{1,1}'^{(k)} + A_{1,2}'^{(k)} = (A_{1,1}'^{(k-1)} + A_{1,2}'^{(k-1)}) A_{1,1} + (A_{1,1}'^{(k-1)} + A_{1,2}'^{(k-1)}) A_{1,2}$$

using (E.9) we have:

$$\begin{aligned} A_{1,1}'^{(k)} + A_{1,2}'^{(k)} &= (A_{1,1} + A_{1,2})^{k-1} A_{1,1} + (A_{1,1} + A_{1,2})^{k-1} A_{1,2} \\ &= (A_{1,1} + A_{1,2})^{k-1} (A_{1,1} + A_{1,2}) = (A_{1,1} + A_{1,2})^k \end{aligned} \quad \text{(E.10)}$$

using (E.3) we have:

$$A_{1,1}'^{(k)} + A_{1,2}'^{(k)} = (A_{1,1} + A_{1,2})^k = A_{1,1}'^{(k)} + A_{1,2}'^{(k)}$$

Note that (E.10) is obtained when multiplying A^{k-1} by A from left as well:

$$A_{1,1}'^{(k)} + A_{1,2}'^{(k)} = (A_{1,1} + A_{1,2}) A_{1,1}'^{(k-1)} + (A_{1,1} + A_{1,2}) A_{1,2}'^{(k-1)}$$

$$= (A_{1,1} + A_{1,2}) (A_{1,1}^{(k-1)} + A_{1,2}^{(k-1)})$$

Using E.9, we have:

$$\begin{aligned} A_{1,1}^{(k)} + A_{1,2}^{(k)} &= (A_{1,1} + A_{1,2}) (A_{1,1} + A_{1,2})^{k-1} \\ &= (A_{1,1} + A_{1,2})^k = A_1^k \end{aligned}$$

The above theorem is employed in order to reduce the size of the storage requirement for the state transition matrices P and P_u obtained from the analysis of the model for large size systems. Clearly, the state definition given by (3.23) yields a state space of size $M=2N(L+1)$ and M is an even integer. The structure of the matrices P_1 , P_2 and Q , as presented in Appendices B and C, for the random access schemes presented in Chapter 3, are of the following forms:

$$\begin{aligned} P_1 &= \begin{bmatrix} P_{1,1} & 0 \\ 0 & P_{1,2} \end{bmatrix} & Q &= \begin{bmatrix} Q_{1,1} & Q_{1,2} \\ Q_{1,1} & Q_{1,2} \end{bmatrix} \\ P_2 &= \begin{bmatrix} P_{2,1} & 0 \\ 0 & P_{2,1} \end{bmatrix} \end{aligned} \quad (E.11)$$

Note that $P_{1,j}$, $P_{2,j}$ and $Q_{1,j}$ are submatrices of $M/2 \times M/2$. Using (E.11), we have:

$$P^k = P_2 \otimes P_1 = \begin{bmatrix} P_{2,1,1} & \underline{C}_{1,1} & P_{1,1,1} & | & P_{2,1,1} \underline{C}_{1,2} & P_{1,2,2} \\ \hline & & & & & \\ P_{2,1,1} & \underline{C}_{1,1} & P_{1,1,1} & | & P_{2,1,1} \underline{C}_{1,2} & P_{1,2,2} \end{bmatrix} \quad (E.12)$$

$$P_u = \underline{C} P_1 P_2 = \begin{bmatrix} \underline{C}_{1,1} & P_{1,1,1} & P_{2,1,1} & | & \underline{C}_{1,2} & P_{1,2,2} & P_{2,1,1} \\ \hline & & & & & & \\ \underline{C}_{1,1} & P_{1,1,1} & P_{2,1,1} & | & \underline{C}_{1,2} & P_{1,2,2} & P_{2,1,1} \end{bmatrix} \quad (E.13)$$

Therefore, the matrices P_1 and P_u are of the following form:

$$P = \begin{bmatrix} P_{1,1} & | & P_{1,2} \\ \hline & & \\ P_{1,1} & | & P_{1,2} \end{bmatrix}$$

where $P_{1,1}$ and $P_{1,2}$ are submatrices of size $M/2 \times M/2$.

Application of the above theorem yields:

$$P^k = \begin{bmatrix} P'_{1,1}(k) & | & P'_{1,2}(k) \\ \hline & & \\ P'_{1,1}(k) & | & P'_{1,2}(k) \end{bmatrix}$$

$$P_1 = P_{1,2} + P_{1,2}$$

$$P'_{1,1}(k) + P'_{1,2}(k) = (P_{1,1} + P_{1,2})^k = P_1^k$$

which means instead of computing P^k , an $M \times M$ matrix, to find $\{\pi(x) | x \in X\}$, we compute P_1^k , an $M/2 \times M/2$ matrix, which

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provides $\{\pi(x_1, x_2, 1) + \pi(x_1, x_2, 2) | x \in I\}$. In other words, after the convergence of P_1^k , the $x=(x_1, x_2)$ th column of a row in P_1^k represents the steady state probability $\pi(x_1, x_2, 1) + \pi(x_1, x_2, 2)$. This approach requires less storage and less processing time compared to the approach which computes P^k . Furthermore, the errors due to the numerical manipulations are reduced. Note that $\pi(x_1, x_2, 1)$ and $\pi(x_1, x_2, 2)$ are not equal, in general. Therefore should the need arise to find $\pi(x_1, x_2, 1)$ and $\pi(x_1, x_2, 2)$ separately, it is necessary to compute P^k .

APPENDIX F

OUTER-LOOP: MATHEMATICAL ANALYSIS

Recall the following relationship which is evident from Figure 4.1:

$$x^{(n+1)} = x^{(n)} + U^{(n+1)} + V^{(n+1)} + W^{(n+1)} - 1 \quad (F.1)$$

Rewriting the above equation, we have:

$$z^{x^{(n+1)}} = z^{[(x^{(n)} + U^{(n)}) + V^{(n+1)} + W^{(n+1)} - 1]} \quad (F.2)$$

since $x^{(n)}$ and $U^{(n)}$ are dependent random variables (this dependency is expressed by (4.10)), $V^{(n+1)}$ an independent random variable and $W^{(n+1)}$ is an independent random variable (note (4.9)), we have:

$$z^{x^{(n+1)}} = z^{(x^{(n)} + U^{(n)})} \cdot z^{V^{(n+1)}} \cdot z^{W^{(n+1)}} \cdot z^{-1} \quad (F.3)$$

Assuming limiting random variables x , U , V and W exist, we apply the expectation function to both sides of the above equation:

$$E(z^x) = E(z^{x+U}) E(z^V) E(z^W) E(z^{-1}) \quad (F.4)$$

Now we write expressions for each of the terms in the above equation. Using (4.14) we have:

$$E(z^x) = \sum_{k=0}^{\infty} x^{(k)} z^{(k)} = q(z) \quad (F.5)$$

Using (4.8) we have:

$$-E(z^V) = g_1^*(\lambda_j - \lambda_j z) \quad (F.6)$$

Using (4.9) we have:

$$-E(z^W) = H^*(\lambda_j - \lambda_j z) e^{-(\lambda_j - \lambda_j z) \tau_j} \quad (F.7)$$

Now we are faced with the difficult task of writing the expression for $E(z^{X+U})$. Using the dependency defined for the random variables x and U in (4.10), we have:

$$\text{Let } y = x + U \text{ or } U = y - x \quad (F.8)$$

Hence:

$$E(z^{X+U}) = E(z^Y) = \sum_{k'=0}^{\infty} \text{Prob}(y=k') z^{k'} \quad (F.9)$$

we know that

$$\text{Prob}(y=k') = \sum_{k=0}^{k'} \text{Prob}(U=k'-k | x=k) q_k \quad (E.10)$$

where q_k is given by (4.12).

Substituting (F.10) in (F.9) we have:

$$E(z^{(X+U)}) = \sum_{k'=0}^{\infty} \sum_{k=0}^{k'} \text{Prob}(U=k'-k | x=k) q_k z^{k'} \quad (F.11)$$

Changing the order of the summations we have:

$$E(z^{(X+U)}) = \sum_{k=0}^{\infty} \sum_{k'=k}^{\infty} \text{Prob}(U=k'-k | x=k) q_k z^{k'} \quad (F.12)$$

Let I' be defined as follows:

$$I' = k' - k \quad (F.13)$$

Substituting (F.13) into (F.12) we have:

$$\begin{aligned} E(z^{k+U}) &= \sum_{k=0}^{\infty} \sum_{I'=0}^{\infty} \text{Prob}(U=I' | x=k) q_k z^{I'+k} \\ &= \sum_{k=0}^{\infty} \sum_{I'=0}^{\infty} (\text{Prob}(U=I' | x=k) z^{I'}) (q_k z^k) \end{aligned}$$

or

$$E(z^{x+U}) = \sum_{k=0}^{\infty} q_k z^k E(z^U | x=k) \quad (F.14)$$

where $E(z^U | x=k)$ is the z transform of the conditional distribution function of the random variable U given $x=k$.

Rewriting (F.14), we have:

$$\begin{aligned} E(z^{x+U}) &= q_0 E(z^U | x=0) + q_1 E(z^U | x=1) \\ &\quad + \sum_{k=2}^{\infty} q_k z^k E(z^U | x \geq 2) \end{aligned} \quad (F.15)$$

Using the equation (4.14), equation (F.15) is rewritten as follows:

$$\begin{aligned} E(z^{x+U}) &= q_0 E(z^U | x=0) + q_1 E(z^U | x=1) \\ &\quad + (q(z) - q_0 - q_1 z) E(z^U | x \geq 2) \end{aligned} \quad (F.16)$$

Now, it is necessary to derive expressions for $E(z^U | x=0)$, $E(z^U | x=1)$ and $E(z^U | x \geq 2)$. As indicated in the equation (4.10), let J be a random variable which represents the number of arrivals during the last time slot of an idle time period. Furthermore, let y , a random variable, be

defined as $y = U - J$, i.e., y denotes the number of packets arrived during the collision time period $r^{(e+1)}$ (refer to Figure 4.1). For $n \geq 2$, we have:

$$\begin{aligned}
 \text{Prob}(U=n|x=0) &= \text{Prob}(y+J=n|x=0) \\
 &= \sum_{k'=0}^n (\text{Prob}(J=n-k') \text{Prob}(y=k')) \\
 &\quad - \text{Prob}(J=0) \text{Prob}(y=n) \\
 &\quad - \text{Prob}(J=1) \text{Prob}(y=n-1)
 \end{aligned} \tag{F.17}$$

$$\begin{aligned}
 E(z^U|x=0) &= \sum_{n=0}^{\infty} \text{Prob}(U=n|x=0) z^n \\
 &= \text{Prob}(U=0|x=0) + z(\text{Prob}(U=1|x=0) \\
 &\quad + \sum_{n=2}^{\infty} \text{Prob}(U=n|x=0) z^n) \\
 &= \text{Prob}(J=0|x=0) + z \text{Prob}(J=1|x=0) \\
 &\quad + \sum_{n=2}^{\infty} \text{Prob}(U=n|x=0) z^n
 \end{aligned}$$

Since J may not take value zero, we have:

$$E(z^U|x=0) = z \text{Prob}(U=1|x=0) + \sum_{n=2}^{\infty} \text{Prob}(U=n|x=0) z^n \tag{F.18}$$

Using equation (F.17), we have:

$$\begin{aligned}
 \sum_{n=2}^{\infty} \text{Prob}(U=n|x=0) z^n &= \sum_{n=2}^{\infty} \left[\sum_{k'=0}^n (\text{Prob}(J=n-k') \right. \\
 &\quad \left. \text{Prob}(y=k')) - \text{Prob}(J=0) \text{Prob}(y=n) \right. \\
 &\quad \left. - \text{Prob}(J=1) \text{Prob}(y=n-1) \right] z^n
 \end{aligned} \tag{F.19}$$

Using (F.19), we have:

$$\sum_{n=2}^{\infty} \text{Prob}(U=n|x=0) z^n = \sum_{n=1}^{\infty} \text{Prob}(U=n|x=0) z^n \quad (\text{F.20})$$

or

$$\begin{aligned} & \sum_{n=2}^{\infty} \text{Prob}(U=n|x=0) \\ &= \sum_{n=1}^{\infty} \left[\left(\sum_{k'=0}^n \text{Prob}(J=n-k') \text{Prob}(y=k') \right) \right. \\ & \quad \left. - \text{Prob}(J=0) \text{Prob}(y=n) \right. \\ & \quad \left. - \text{Prob}(J=1) \text{Prob}(y=n-1) \right] z^n \end{aligned}$$

Since $\text{Prob}(y < 0) = 0$, we have:

$$\begin{aligned} &= \sum_{n=0}^{\infty} \left[\left(\sum_{k'=0}^n \text{Prob}(J=n-k') \text{Prob}(y=k') \right) \right. \\ & \quad \left. - \text{Prob}(J=0) \text{Prob}(y=n) \right. \\ & \quad \left. - \text{Prob}(J=1) \text{Prob}(y=n-1) \right] z^n \end{aligned}$$

Therefore we have

$$\begin{aligned} & \sum_{n=2}^{\infty} \text{Prob}(U=n|x=0) z^n \\ &= E(z^J)E(z^Y) - \text{Prob}(J=0) E(z^Y) - z \text{Prob}(J=1)E(z^Y) \end{aligned} \quad (\text{F.21})$$

Substituting (F.21) in (F.18) we have:

$$\begin{aligned} E(z^U|x=0) &= z \text{Prob}(U=1|x=0) + E(z^J)E(z^Y) - \\ & \quad - \text{Prob}(J=0)E(z^Y) - z \text{Prob}(J=1)E(z^Y) \end{aligned} \quad (\text{F.22})$$

where $E(z^J)$ is the z transform of the distribution of the number of arrivals during the last time slot of the idle

period (note that at least one packet arrives during this time slot). Using (4.3) we have:

$$E(z^J) = \frac{1}{1-P_0} (1 - e^{-2\tau_j \lambda_{je}(1-z)}) \quad (F.23)$$

$E(z^Y)$ is the z transform of the distribution of the number of arrivals during the collision time period $r^{(e+1)}$ (refer to Figure 4.1). Using (4.6), we have:

$$E(z^Y) = C^*(\lambda_{je} - \lambda_{je}Z) \quad (F.24)$$

$\text{Prob}(J=0)$ is denoted by P_0 and given by (4.3), i.e.,:

$$P_0 = \text{Prob}(J=0) = e^{-2\tau_j \lambda_{je}} \quad (F.25)$$

Similarly $\text{Prob}(J=1)$ is denoted by P_1 and given by (4.3), i.e.,:

$$P_1 = \text{Prob}(J=1) = \lambda_{je} \tau_j e^{-2\tau_j \lambda_{je}} \quad (F.26)$$

At this stage, we write the expression for $E(z^U | x=1)$.

Using (4.10), we have:

$$E(z^U | x=1) = \sum_{n=0}^{\infty} \text{Prob}(U=n | x=1) z^n \quad (F.27)$$

As indicated in (4.10), $x=1$ implies $U=0$, i.e.,:

$$\text{Prob}(U=0 | x=1) = \begin{cases} 1 & \text{for } U=0 \\ 0 & \text{elsewhere} \end{cases} \quad (F.28)$$

Substitution of (F.24) in (F.23) yields:

$$E(z^U | x=1) = 1 \quad (F.29)$$

As for $E(z^U | x \geq 2)$ collision occur with probability equal to unity (as $x \geq 2$). Therefore, U represents the number of arrivals during a collision time period

$$\begin{aligned} E(z^U | x \geq 2) &= \sum_{n=0}^{\infty} \text{Prob}(U=n | x \geq 2) z^n \\ &= C^u (\lambda_{je} - \lambda_{je} z) \end{aligned} \quad (F.30)$$

where $C^u(z)$ is given by the equation (4.6).

Substituting (F.22), (F.29) and (F.30) in (F.16), we have:

$$\begin{aligned} E(z^{X+U}) &= q_0 [E(z^J)E(z^Y) - \text{Prob}(J=0)E(z^Y) \\ &\quad - z \text{Prob}(J=1)E(z^Y) + z \text{Prob}(J=1)] \\ &\quad + q_1 z \\ &\quad + \sum_{k=2}^{\infty} q_k z^k E(z^Y) \end{aligned} \quad (F.31)$$

Substituting (F.31), (F.6) and (F.7) in (F.4) we have:

$$\begin{aligned} E(z^X) &= \{q_0 [E(z^J)E(z^Y) - \text{Prob}(J=0)E(z^Y) - z \text{Prob}(J=1)E(z^Y) \\ &\quad + z \text{Prob}(J=1)] \\ &\quad + q_1 z + E(z^Y) [E(z^X) - q_0 - q_1 z]\} E(z^V) E(z^W) z^{-1} \end{aligned} \quad (F.32)$$

Substituting (F.5) - (F.7), (F.23) - (F.26) into (F.32) we have:

$$\begin{aligned}
E(z^*) = q(z) &= \beta_1^* (\lambda_{je} - \lambda_{je} z) H^*(\lambda_{je} - \lambda_{je} z) \\
&\quad \{q_1 z [1 - C^*(\lambda_{je} - \lambda_{je} z)] \\
&\quad + \frac{q_0}{1 - \text{Prob}(J=0)} [\text{Prob}(J=1) \cdot z \cdot (1 - \\
&\quad C^*(\lambda_{je} - \lambda_{je} z)) - C^*(\lambda_{je} - \lambda_{je} z) \\
&\quad (1 - e^{-\lambda_{je}(2\tau)(1-z)})]\}
\end{aligned}$$

$$\begin{aligned}
&\times (z - C^*(\lambda_{je} - \lambda_{je} z) \beta_1^* (\lambda_{je} - \lambda_{je} z) H^*(\lambda_{je} - \lambda_{je} z))^{-1} \\
&\hspace{15em} (F.33)
\end{aligned}$$

In order to find q_0 and q_1 we solve the following simultaneous equations:

$$q_0 = q(z) \Big|_{z=0}$$

$$q_1 = \frac{d}{dz} q(z) \Big|_{z=0}$$

which yields:

$$\begin{aligned}
q_0 &= \frac{1 - \lambda_{je} [b_1 + h_1 + \tau_j + (2\tau_j)/\sigma]}{2\tau_j \lambda_{je} \left[\frac{1}{1 - \text{Prob}(J=0)} - \frac{1}{\beta_1^* (\lambda_{je}) H^*(\lambda_{je})^\sigma} \right]} \\
&\hspace{15em} (F.34)
\end{aligned}$$

$$\begin{aligned}
q_1 &= \left(\frac{1}{\beta_1^* (\lambda_{je}) H^*(\lambda_{je})} - \frac{\text{Prob}(J=1)}{1 - \text{Prob}(J=0)} \right) q_0 \\
&\hspace{15em} (F.35)
\end{aligned}$$

APPENDIX G

INNER-LOOP INTERDEPARTURE TIME PROCESS

In this Appendix, we consider the large system size model presented in Appendix C. First the Laplace transform of the distribution of the interdeparture time t_{id} (refer to Figure 3.2) is presented. Next the mean interdeparture time $E(t_{id})$ is given.

The random variables $t_{id}^{(e+k)}$ (refer to Figure 3.2) is dependent on the state of the system x found by a packet departing from the system at time point $t_k^{(e+k)}$. Consider the case in which a departing packet finds the system in one of the states $x = \{x \in X | n^0(x) = 0\}$, i.e., there exists no ready controller in the system. The interdeparture time $t_{id}^{(e+k)}$, in this case, is equal to the sum of an idle random time period plus either: (i) the sum of $T + \tau$; or: (ii) the sum of a random collision time period plus $T + \tau$. In the former situation, only one controller becomes ready during the last time slot of the idle period whereas in the latter situation, two or more controllers become ready during that time slot, hence a collision time period is encountered and is included in the interdeparture time period. Now, consider the case in which a departing packet finds the system in one of the states $x = \{x \in X | n^0(x) = 1\}$, i.e., there is only one ready controller in the system. The interdeparture time

$t_{id}^{(e+k)}$, in this case, is equal to the time duration of a successful transmission time period $T+\tau$. Finally, consider the case in which a departing packet finds the system in one of the states $\mathbf{x}=\{\mathbf{x} \in \mathbf{X} | n^*(\mathbf{x}) > 1\}$, i.e., there are two or more ready controllers in the system. The interdeparture time $t_{id}^{(e+k)}$, in this case is equal to the sum of a collision time period plus a successful transmission time period. Summarizing the above, we have:

$$\begin{aligned} ID^*(s) &= E(e^{-st_{id}}) = E(e^{-st_{id}} | n^*(\mathbf{x})=0) \cdot \text{Prob}(n^*(\mathbf{x})=0) \\ &\quad + E(e^{-st_{id}} | n^*(\mathbf{x})=1) \cdot \text{Prob}(n^*(\mathbf{x})=1) \\ &\quad + E(e^{-st_{id}} | n^*(\mathbf{x}) \geq 2) \cdot \text{Prob}(n^*(\mathbf{x}) \geq 2) \end{aligned} \quad (G.1)$$

where

$$\begin{aligned} (i) \quad E(e^{-st_{id}} | n^*(\mathbf{x})=0) &= \frac{P_1}{1-P_0} (I^*(s)e^{-s(T+\tau)}) + \\ &\quad (1 - \frac{P_1}{1-P_0}) (I^*(s) R^*(s) e^{-s(T+\tau)}) \end{aligned} \quad (G.2)$$

and P_k is the probability that k controllers become active during the last time slot of the idle period, i.e.,

$$P_1 = N(1-e^{-2\tau\lambda_j}) (e^{-2\tau\lambda_j(N-1)})$$

$$P_0 = e^{-2\tau\lambda_j N}$$

$$\begin{aligned} (ii) \quad E(e^{-st_{id}} | n^*(\mathbf{x})=1) &= e^{-s(T+\tau)} \end{aligned} \quad (G.4)$$

$$\begin{aligned}
 (iii) \quad E(e^{-st_{1d}} | n^*(x) \geq 2) \\
 = R^*(s) e^{-s(T+\tau)}
 \end{aligned} \tag{G.5}$$

$$(iv) \quad \text{Prob}(n^*(x)=0) = \pi_k(0,0,*)$$

$$\text{Prob}(n^*(x)=1) = \pi_k(1,0,*) + \pi_k(0,+,*) \tag{G.6}$$

$$\text{Prob}(n^*(x) \geq 2) = 1 - \pi_k(0,*,*) - \pi_k(1,0,*)$$

The mean of the interdeparture time t_{1d} is established through the following equation:

$$E(t_{1d}) = -\frac{d}{ds} E(e^{-st_{1d}})_{s=0} \tag{G.7}$$

Substituting (G.1) into (G.7) and solving (G.7) we have .

$$\begin{aligned}
 E(t_{1d}) = & \pi_k(0,0,*) \frac{2\tau}{1-e^{-2\tau\lambda_j N}} + 2\tau/\sigma + T + \tau \\
 & - \frac{N(1-e^{-2\tau\lambda_j}) (e^{-2(N-1)\tau\lambda_j})}{1-e^{-2\tau\lambda_j N}} \cdot 2\tau/\sigma \\
 & + [\pi_k(0,+,*) + \pi_k(1,0,*)](T+\tau) \\
 & + [1 - \pi_k(0,*,*) - \pi_k(1,0,*)] (2\tau/\sigma + T + \tau)
 \end{aligned} \tag{G.8}$$

7

END

1 0 0 4 8 6

FIN