INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI

A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor MI 48106-1346 USA
313/761-4700 800/521-0600

Multipath Mobile Wireless Channel: Models, Interpretations and Applications

by

Syed Aun Abbas, B.Sc, M.Eng

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy**

**Ottawa-Carleton Institute of Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University
Ottawa, Ontario, Canada**

© September, 1997

National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file *Votre référence*

Our file *Notre référence*

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-26842-X

The undersigned recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis

**"Multipath Mobile Wireless Channel: Models, Interpretations and
Applications"**

submitted by Syed Aun Abbas, B.Sc, M.Eng
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Chair, Department of Systems and Computer Engineering

Thesis Supervisor
Professor Asrar U. H. Sheikh

External Examiner
Professor Norman C. Beaulieu
Queens University
Kingston, Ontario, Canada

Carleton University
September 15, 1997

Abstract

In this thesis, the Multipath Mobile Wireless Channel is mathematically modeled in terms of a phasor sum. The model is evolved in suitable forms to help identify the propagation mechanism. The main focus is to provide appropriate physical interpretations for the propagation mechanism and link it to the channel fading parameter behavior. The issue of the segregation of so-called fast and slow fading, in particular the boundary between slow fading and fast fading, is investigated. Rice/lognormal and Nakagami/lognormal fading models have been studied and modifications suggested. The equivalency of Rice and Nakagami distributions is discussed and a simple approximation of the Nakagami m and Rice K parameter is found and verified. The issue of the physical model of the Nakagami fading is addressed. For narrowband channels, the physical model of the Rice/Rayleigh distribution is argued to be applicable to the Nakagami case. For wideband channels, a geometrical theory of Nakagami fading is suggested and physical interpretations are provided. A two-fold interpretation of the Nakagami m parameter, one each for narrowband and wideband cases, is provided. BER performance of the RAKE receiver in the case of a random number of signal paths is investigated. Radio link performance is investigated for the case of frequency-selective cochannel interference: A performance model is built based on frequency-selective desired and interfering Nakagami fading signals; and outage expressions are derived. Radio link performance is also investigated for the case of multipath self interference assuming a Poisson path arrival process and using a shot noise model for statistical characterization of multipath self interference.

Acknowledgments

The author would like to thank a number of individuals and organizations for their help and encouragement during the course of this thesis research. The author specifically thanks to his thesis supervisor, Professor Asrar Sheikh, for his patience, criticism, encouragement and understanding over the past six years of his association with him. Thanks are due to his fellow graduate students, Mohammed Al-Tarhuni, Bing Chu, Frank Van Heeswyk, Khalid Islam, Parsa Larijani, Reza Etemadi, Arif Obaid, Shahid Rasul Chaudry and Framarz Hindessi for useful discussions. Thanks are due to Ijlal, for being there to listen to me when needed. The help and understanding of my Colleagues at Nortel, Tom Taylor and Dia Helmy, respectively, is gratefully acknowledged. Julia Saunders helped with her expertise as a professional editor, as did Tom. The well placed articles are due to them, and those which are not well placed are entirely my fault. The author must also thank Professor David Coll for his last minute assistance as the Surrogate Supervisor in the absence of Professor Asrar Sheikh. The financial assistance of the Ministry of Science and Technology, Government of Pakistan, and the National Science and Engineering Research Council of Canada is gratefully acknowledged.

Thanks are due to my loving family. Thanks to my only brother for the encouragement and guidance. I thank to my parents for their unbounded patience, sufferings and encouragement. Thank you Abbo for your love, and understanding. I also thank to my wife's parents and her family for their encouragement. Thank you Shazia, my beloved wife, for your patience, support and encouragement. Without your support, this would not have been possible. Thank you Rida, the peace of my heart, my daughter, for helping me keep my chin up amidst the toughest moments. Above all, thanks to my Allah.

Table of Contents

Acceptancen Form ii

Abstract iii

Acknowledgements iv

Table of Contents v

List of Figures viii

List of Tables xii

List of Acronyms xiii

List of Symbols xiv

Chapter 1 Introduction 1

1.1 Background and Motivation 1

1.2 Literature Survey 4

1.3 Problem Identification 8

1.4 Contributions 10

1.5 Organization 11

Chapter 2 Multipath Mobile Wireless Channel 13

2.1 Introduction 13

2.1.1 Narrowband vs. Wideband Channel 14

2.1.2 Parameter Characteristics 16

2.2 Narrowband Channel: Theoretical Foundation 18

2.2.1 Variability in Received Signal Envelope and its Physical Interpretation 22

2.2.2 Propagation Loss 24

2.2.3 Fading 28

2.2.3.1 Statistical Distribution of Envelope Fading 31

2.2.4 Shadow Fading 32

2.2.5 Distribution of Phase 33

2.3 Wideband Channel: Theoretical Foundation 33

2.3.1 The Path Arrival Process 34

2.3.1.1 Turin's Model 34

2.3.1.2 Saleh's Model 37

2.3.1.3 Comparison and Comments on the Two Path Arrival Models	37
2.3.1.4 Delay Spread	38
2.4 Summary	39
Chapter 3 Physical Basis of Multipath Mobile Wireless Channels	41
3.1 Introduction	41
3.2 Narrowband Channel	41
3.2.1 Nature of Slow and Fast Fading	42
3.2.1.1 Distribution of Slow Fading	47
3.2.1.2 Segregation of Slow and Fast Fading	53
3.2.1.3 Distribution of Fading Envelope	61
3.2.1.4 Distribution of Phase	64
3.2.2 Radio Link Performance Methodology	65
3.2.3 Physical Basis of Nakagami Fading	67
3.2.3.1 Parameter Equivalency of the Rice and Nakagami Distributions	70
3.2.3.2 Complex Nakagami Fading Envelope	75
3.3 Wideband Channel	77
3.3.1 Geometric and Physical Interpretations	77
3.3.2 Parameter m and the Order of Multipath Diversity	83
3.4 Summary	88
Appendix 3A The parameter relationship of the Rice and Nakagami distributions	90
Chapter 4 BER Performance of Frequency Selective Signal Reception	92
4.1 Introduction	92
4.2 Performance of Wideband Receivers in Realistic Channels	92
4.2.1 Practical Considerations	102
4.3 Summary	105
Chapter 5 Radio Link Performance in Cochannel Interference	108
5.1 Introduction	108
5.2 System Model	110
5.3 Outage Analysis	112
5.3.1 Numerical Results	114
5.4 Comments on Frequency Selectivity/Non-Selectivity of Cochannel Interference	121
5.5 Comments on Performance Methodology	125

5.6	Summary	126
Appendix 5A	Derivation of Equation (5.13), (5.14) and (5.17)	127
Chapter 6	Radio Link Performance in Multipath Self Interference	131
6.1	Introduction	131
6.2	System Model	131
6.3	Statistical Analysis of Multipath Self-Interference	133
6.4	Outage Analysis	137
6.4.1	Rayleigh/Rayleigh-Poisson	139
6.4.2	Rayleigh/Nakagami-Poisson	143
6.4.3	Nakagami/Nakagami-Poisson	146
6.4.4	Rice/Nakagami-Poisson	149
6.5	Summary	150
Appendix 6A	Derivation of Equation (6.21) and Equation (6.24)	153
Appendix 6B	Derivation of Equation (6.37) and Equation (6.39)	156
Chapter 7	Summary of Results and Further Work	160
7.1	Introduction	160
7.2	Summary of Results	160
7.3	Further Work	166
References		167

List of Figures

Figure 2.1	Figure 5(b) of [10]. Measured and mean power in urban environments. The LOS propagation at 900 MHz. 23
Figure 2.2	Figure 8 of [10]. Fast variation of three different distance segments. 23
Figure 2.3	The Figure 3(b) of reference [10]. Measured and calculated (two-rays) mean power in a rural environments at 11GHz with monopole/monopole transmit/receive antennas combinations. 25
Figure 2.4	The Figure 6(b) of reference [10]. Comparison of measured power at 900 MHz on the Lexington Ave. and the calculated mean power based on the six-ray model over a 500 feet road segment. 25
Figure 2.5	The dB magnitudes versus the distance of the phasor summation of both the terms of (2.19). 26
Figure 2.6	The effect of the normalization on g_r by the g_L to yield g_f as in (2.26). The Figure 2.5 is normalized to g_L . 26
Figure 2.7	Taken from reference [39]; multipath propagation model. 30
Figure 2.8	Illustration of the modified Poisson Process. 36
Figure 3.1	The mechanism behind the median path loss. The dB magnitudes versus the distance of the phasor summation of LOS, GR and WR paths (solid). The plane earth model is also plotted (dashed). 43
Figure 3.2	The dB magnitudes versus the distance of the phasor summation of both terms of (2.16). 43
Figure 3.3	Normal distribution fit for the real part of the phasor sum of the diffracted paths. 45
Figure 3.4	Normal distribution fit for the imaginary part of the phasor sum of the diffracted paths. 45
Figure 3.5	Rayleigh distribution fit for the envelope or magnitude of the phasor sum of the diffracted paths. 46
Figure 3.6	The distribution of phase. The phase is nearly uniformly distributed over - 3.141 to 3.141 radians. When this condition is met this is enough evidence of that phasor sum to be a zero mean complex Gaussian process. 46
Figure 3.7	The Figure 11 of reference [8]. Measured slow fading (above) vs. distance to basestation, fast fading (centre) and corresponding Rician K factor (below). 48
Figure 3.8	Comparison of the Nakagami and lognormal distribution fit on the empirical distribution based on a 300 meter 3000 sample simulation run. 52
Figure 3.9	Path loss due to phasor sum of the LOS, GR and AWGN. 55
Figure 3.10	Comparison of the K parameter values with different averaging intervals. The zero values represent the unavailability of the K value for that interval. The solid curve is for the 2 meter averaging interval, the dashed is for the 10 meter and the dotted is for 20 meter. 55
Figure 3.11	Rice parameters, s (solid) and σ (dashed) as a function of distance for the 2 meter averaging interval case. The zero values represent unavailability of s

	and σ values for that interval. 57
Figure 3.12	Normalized path loss curve after the normalization of Figure 3.9 to distance dependent loss. 57
Figure 3.13	Normalization of the path loss curve in Figure 3.12 based on 2 meter averaging intervals. 58
Figure 3.14	Comparison of the K parameter values obtained for the normalized (solid) and non-normalized (dashed) cases. The zero values represent unavailability of K value for that interval. 60
Figure 3.15	s parameter (solid) and σ parameter (dashed) of the Rice distribution after curvefitting in the normalized envelope case. The zero values represent unavailability of s and σ values for that interval. 60
Figure 3.16	Variation of m with respect to K . 72
Figure 3.17	The rate of change of m with respect to K . 72
Figure 3.18	Validation of approximation in (3.29): The solid line corresponds to the m parameter values obtained after curve-fitting. The dashed lines are m values calculated from the K parameter values obtained after curve-fitting on the same data. The abscissa is the indexing of the simulation data set. 74
Figure 3.19	Illustration of the physical interpretation of the wideband channel in a microcellular environment showing guided propagation. 81
Figure 3.20	Figure 7 (a) of [5]: LOS measured impulse response profiles in a room. Squaring the magnitude will give the power delay profiles. 81
Figure 3.21	Average bit error probability as a function of average SNR per bit for $m=1$ (solid), $m=2$ (dashed), $m=4$ (dash-dot). BER performance is improved as the value of m is increased. 87
Figure 4.1	BER performance for $L=1$ (solid) $L=2$ (dashed) and $L=4$ (dash-dot). Performance improves as L increases. 96
Figure 4.2	BER performance for $\lambda=1$ (solid) $\lambda=2$ (dashed) and $\lambda=4$ (dash-dot). Performance improves as λ increases. 96
Figure 4.3	BER performance Comparison for $L=1$ (dashed) and $\lambda=1$ (solid). 97
Figure 4.4	BER performance Comparison for $L=2$ (dashed) and $\lambda=2$ (solid). 97
Figure 4.5	BER performance Comparison for $L=4$ (dashed) and $\lambda=4$ (solid). 98
Figure 4.6	BER performance Comparison for $L=1$ (solid) and $\lambda=1$ (solid-circle), $L=2$ (dashed) and $\lambda=2$ (dashed-circles), $L=4$ (dash-dot) and $\lambda=4$ (dash-dot-circles). 100
Figure 4.7	BER performance comparison: BER as a function of number of resolvable paths, L , (dashed) and average number of paths, λ , (solid) for an average SNR per bit of 10 dB. 100
Figure 4.8	BER performance comparison; BER as a function of L and λ for an average SNR per bit of 100 dB. 101
Figure 4.9	BER performance comparison; BER as a function of λ and L for an average SNR per bit of 10 dB and 100 dB together. 101
Figure 4.10	BER performance comparison when λ is random; dotted lines are $L=1$ and $L=4$ cases, solid lines are for $\lambda=1$ and $\lambda=4$, dashed lines when λ varies between 1 and 4 with equal probability and dash-dot line when $\lambda=1$ occurs with probability 0.2 and $\lambda=4$ occurs with probability 0.8. 103

Figure 4.11	Probability of at most four paths as a function of λ .. A probability of 0.9 corresponds to $\lambda = 2.4325$. 106
Figure 4.12	BER performance for a practical case: $L=4$ (solid), $L=1$ (dash-dot) and $\lambda = 2.4325$ (dashed). 106
Figure 5.1	Outage probability as a function of SIR in dB for the following parameters: $m_d=2$, $m_f=2$: Solid; $R_f=7$ dB, Dash-dash: $R_f=13$ dB, Dash-dot; $R_f=20$ dB. 116
Figure 5.2	Outage probability as a function of SIR in dB for the following parameters: $R_f=7$ dB, $I=2$: Solid; $N=2$, Dash-dash; $N=3$, Dash-dot; $N=4$. 116
Figure 5.3	Outage probability as a function of SIR in dB for the following parameters: $R_f=7$ dB, $N=4$: Solid; $I=4$, Dash-dash; $I=3$, Dash-dot; $I=2$. 117
Figure 5.4	The <i>pdf</i> of Signal-to-Interference Power Ratio as a function of N , I , and average SIR : Average $SIR=20$ dB, $N=4$; $I=4$ (solid), $I=3$ (dashed) $I=2$ (dash-dot). 119
Figure 5.5	The <i>pdf</i> of Signal-to-Interference Power Ratio as a function of N , I , and average SIR : Average $SIR=5$ dB, $N=4$; $I=4$ (solid), $I=3$ (dashed) $I=2$ (dash-dot). 119
Figure 5.6	Outage probability as a function of SIR in dB for the following parameters: $R_f=7$ dB, $N=4$: Solid; $\lambda_f=4$, Dash-dash; $\lambda_f=3$, Dash-dot; $\lambda_f=2$. 120
Figure 5.7	Outage probability as a function of I (solid) and λ_f (dashed) for $N=4$, $R_f=7$ dB: $SIR=5$ dB for upper two curves and $SIR=20$ dB for lower two curves. 122
Figure 5.8	Outage probability as a function of SIR in dB for the following parameters: $R_f=7$ dB, $\lambda_N=4$: Solid; $\lambda_f=4$, Dash-dash; $\lambda_f=3$, Dash-dot; $\lambda_f=2$. 122
Figure 5.9	Figure 5.6 and the Figure 5.8 superimposed. 123
Figure 5.10	Outage probability as a function of SIR in dB for the following parameters: $R_f=7$ dB, $I=2$: $\lambda_N=4$: Solid; $\lambda_f=4$, Dash-dash; $\lambda_f=3$, Dash-dot; $\lambda_f=2$. 123
Figure 5.11	Figure 5.2 and the Figure 5.10 superimposed. 124
Figure 6.1	Illustration of the (a) shot noise and (b) multipath arrival process. 134
Figure 6.2	Partial sums of the outage probability formula (6.27) for scenario 1 as a function of X , for the following parameter values: $\lambda T=3.55$, $X_a=1$. Solid: one term; dotted: two terms; dash-dot: three terms, dash-dash: four terms. 142
Figure 6.3	Scenario 1 outage probability as a function of X , for $R_f=5$, $X_a=1$. Solid: $\lambda T=10$; dotted: $\lambda T=3.55$; dash-dot: $\lambda T=0.8$; dash-dash: $\lambda T=0.6$. P_{out} approximated by the first four terms of Equation (6.27) on page 140. 144
Figure 6.4	Partial sums of scenario 1 outage probability as a function of X for the following parameter value: $R_f=5$, $\lambda T=0.6$, $X_a=1$. Solid: one term; dotted: two terms; dash-dot: three terms; dash-dash: four terms of Equation (6.27) on page 140. 144
Figure 6.5	Partial sums of scenario 1 outage probability as a function of X for the following parameter value: $R_f=5$, $\lambda T=10$, $X_a=1$. Solid: one term; dotted: two terms; dash-dot: three terms; dash-dash: four terms of Equation (6.27) on page 140. 145

- Figure 6.6 Scenario 2 outage probability as a function of x , for the following parameter values: $\lambda T=3.55$, $X_d=1$. Solid: $R_f=5$; dotted: $R_f=10$; dash-dot: $R_f=100$. 145
- Figure 6.7 Scenario 2 outage probability as a function of x , for the following parameter values: $R_f=5$, $\lambda T=3.55$, $\Omega=1$. Solid: $m=1$; dotted: $m=2$; dash-dot: $m=3$; dash-dash: $m=4$. 147
- Figure 6.8 Outage probability as a function of Ω_d , for the following parameters: $R_f=5$, $\lambda T=3.55$, $\Omega=1$, $m=1$: Solid: $m_d=1$. Dotted: $m_d=2$; Dash-dot: $m_d=3$ 151
- Figure 6.9 Outage probability as a function of Ω_d , for the following parameters: $R_f=5$, $\lambda T=3.55$, $\Omega=1$, $m_d=2$: Solid: $m=1$, Dotted: $m=2$; Dash-dot: $m=3$ 151

List of Tables

TABLE 3.1	Comparison of the lognormal and the Nakagami model for slow fading based on the minimum squared error sum (MSES) criterion. 52
TABLE 3.2	Comparison of the coverage outage predictions of the lognormal and Nakagami models with the true outages of the data. $P_{out,-5}$ and $P_{out,-10}$ denote the outage probability for -5dB and -10dB outage thresholds, respectively. 68
TABLE 6.1	Multiple fading scenarios 138

List of Acronymns

AWGN	Additive White Gaussian Noise
CCI	Co-Channel Interference
ED	Edge Diffracted
GR	Ground Reflected
ISI	Inter-Symbol Interference
LOS	Line-of-Sight
MSI	Multipath Self Interference
RAKE	An anti-multipath receiver based on a tapped delay line structure
SIR	Signal-to-Interference Ratio
SNR	Signal-to-Noise Ratio
WR	Wall Reflected
WSSUS	Wide Sense Stationary Uncorelated Scattering

List of Symbols

a_n	The coefficient of the power received from the n th path
A	A constant
C	A constant
D_l	Diffraction coefficients
$f(.)$	The probability density function of $(.)$
g_n	The random multiplicative disturbance of the n th unresolvable path
g_{sf}	The magnitude of the resultant of slow and fast varying phasors.
g_{ff}	The magnitude of the resultant of slow and fast varying phasors.
$g_l(t)$	The time-variant attenuation factor of the l th path
g_r	The received signal magnitude
G_t	The transmitter antenna gain
G_r	The receiver antenna gain
$h(\tau, t)$	The time-variant impulse response of the multipath channel.
$h(t)$	The time-invariant impulse response of the multipath channel
l	The number of multipath signals or the number of unresolvable interfering paths.
$Im(.)$	The imaginary part of $(.)$
I_N	The number of unresolvable multipath signals or the complex low pass noise
I_{LP}	The magnitude of complex low pass noise
I_c	The in-phase component of the complex low pass noise I_N
I_s	The quadrature component of the complex low pass noise I_N
K	Parameters of the $\Delta - K$ model or Rice K factor
L	The number of multipath signals
m	Parameter of the Nakagami distribution
N	The number of unresolvable multipaths

$n(t)$	The complex white Gaussian noise
P	The magnitude of the dominant multipath
P_s	The specular power component
P_r	The random power component
P_t	The transmitted power
P_i	The received power of the i th path
$p(t)$	The received power time response at a given position when an impulse is transmitted
r_i	The length of the i th path.
r	The length of the line of sight path
r_G	The length of the ground reflected paths
r_m	The length of the wall reflected paths
r_l	The length of the edge diffracted paths
R	The magnitude of the n dimensional multipath signal
R_G	Ground reflection coefficient
R_l	The signal-to-interference protection ratio
R_m	Wall reflection coefficient
$Re(.)$	The real part of $(.)$
S_1	State one of the $\Delta - K$ model
S_2	State two of the $\Delta - K$ model
t	The observation time
t_i	The associated arrival time at the receiver
t_0	The shortest resolvable multipath signal
T	The duration of the transmitted signal
v	The equivalent low pass waveform
$v_r(t)$	The received complex low pass signal
$v(t)$	The low pass transmitted signal

$v_m(t)$	The received signal due to multipath effect
$v_m(t)$	The n th resolvable multipath signal
w	The power of the narrowband or wideband desired signal
z	The power of the composite MSI or CCI signal
z_n	The Rayleigh distributed envelope of the complex additive white Gaussian noise
τ_m	The length of the impulse response
τ	The application time of the impulse
τ_n	The excess delay of n th path.
τ_m	The average excess delay
T_l	The arrival time of l th cluster in Saleh's model
τ_{kl}	The arrival time of the k th ray in the l th cluster in Saleh's model
τ_{rms}	The <i>rms</i> delay spread
$\tau_l(t)$	The time-variant propagation delay of the l th path
Λ	Mean arrival rate of Saleh's model
Ω	The average power of a random variable.
Δ	Parameters of the $\Delta - K$ model
λ	General mean path arrival rate or mean path arrival rate of Saleh's model or wavelength of the transmitted signal
λ_0	The signal wavelength
$\theta_{,}$	The phase of the resultant slow and fast varying phasors.
θ_{ff}	The phase of the resultant slow and fast varying phasors.
θ_n	The uniformly distributed phase of the complex additive white Gaussian noise
θ_n	The random phase of the n th unresolvable path
$\theta_l(t)$	The time-variant phase of the l th path.
φ	The phase of the LOS path
$\Delta\varphi_G$	The differential phase difference of the m th ground reflected path

$\Delta\varphi_m$	The differential phase difference of the m th wall reflected path
$\Delta\varphi_l$	The differential phase difference of the l th path diffracted path
ω_0	The frequency of the transmitted sinusoidal carrier in radians
Υ_n	The total time spread of the channel
s	Mean of a non-stationary, non-zero mean complex Gaussian process
σ^2	Variance of a non-stationary, non-zero mean complex Gaussian process
$f(g_f \sigma)$	Conditional probability density function of g_f given σ
$f_N(g_f (m, \Omega))$	Conditional probability density function of g_f given m and Ω
$f_N(g_f (K, \Omega))$	Conditional probability density function of g_f given K and Ω
$f(m)$	The probability density function of m
$f(K)$	The probability density function of K
$f(\Omega)$	The probability density function of Ω
$f(\theta_f)$	The probability density function of θ_f
μ	The mean of $\ln(s)$
α^2	The variance of $\ln(s)$
c	The velocity of light or equal to $2\sigma^2$

Chapter 1

Introduction

1.1 Background and Motivation

Telecommunication technology has come of age in recent years. However, it took about a century to migrate from the plain old telephone system to the modern personal communication era. In 1990, the United States Federal Communication Commission (FCC) issued a Notice of Inquiry into the establishment of new Personal Communication Services. Over hundred companies fielded comments [1]. The FCC identifies PCS as wire-less communication services “that free individuals from the constraints of the wireline public switched telephone network and enable them to communicate when they are away from their home or office telephone” [1]. Other attributes of PCS systems are the massive coverage to serve large user base using limited amount of space and spectrum and near-ubiquity meaning service to users almost wherever they are.

The commercial success of PCS systems will depend on the total number of subscribers which can be supported by the system for a given grade of service. The number of users may be increased through more frequent reuse of the available spectrum by decreasing the reuse distance between cells. The reduction of cell radius from 1 km to 100 m yields a hundred-fold reduction in cell area with corresponding increased system user capacity due to spectrum reuse [2]. This, however, does not come cheaply. Challenges

associated with this choice are numerous [2]. It requires better understanding of new propagation models, their characterization and methods for predicting radio link performance. Efficient resource management and handoff strategies are needed. Architectural issues such as integration of microcells with macrocells and sharing spectral resource among them must be answered. Finally, it requires a simple and economical base unit and infrastructure to connect it to backbone network. The motivation behind this thesis research is to look into one of the above issues; pertaining to better understanding of propagation mechanisms and development of new models, their characterization and methods for predicting radio link performance. Similar issues are perhaps also the driving force behind the recent urban channel measurement and characterization campaigns [4]-[20].

The importance of the communication channel can never be over-emphasized. It is the single most important part of any communication system and affects the design of all other system components. In general, measurements are carried out to obtain the statistical characterization of the parameters of the channel model. The amount of confidence one may express in a radio link design is directly related to the degree of accuracy with which it is possible to obtain the channel parameter values. A number of assumptions are made in channel modeling for the sake of simplicity and mathematical tractability, which though lend simplicity, they in most cases fail to relate the model with the physical environment. A system designed on an oversimplified channel model is likely to fail in practice and when successfully implemented will perform poorly.

When a signal is transmitted over a wireless channel, the channel modifies the transmitted signal parameters in a random manner. The physical properties of the channel, such as attenuation, fading, the amount and kind of interference etc. will determine the best signals which can deliver the information to the destination most reliably. When a suitably designed signal is passed through the channel it must be possible to detect it with suffi-

cient reliability, e.g., with sufficiently small probability of error or outage probability. This performance should, of course, be achieved at a reasonable cost and complexity. The final radio link design emerges through the trade-offs between three main ingredients of the system design, i.e., reliability, cost and complexity. For example, the probability of error of a BPSK signal transmitted over an AWGN channel is a function of received signal-to-noise ratio (SNR) per bit. It is known that to obtain a bit error rate of 10^{-3} or less, the SNR/bit must be 7 dB or more [71]. This may be assured by transmitting sufficient power.

When fading occurs, and it is described by the Rayleigh distribution, the average bit error rate of 10^{-3} requires an average SNR/bit¹ of 24 dB. The fade margin in this case is 17 dB. Increasing the power to overcome fading results in stronger interference to the cochannel users in a cellular system. It also results in an increase in the channel delay spread and system's operational cost. The time dispersive nature of a channel may introduce multipath induced intersymbol interference (ISI). It may also be used to advantage by exploiting the multipath diversity with the use of specialized receivers such as RAKE. In a RAKE receiver, the performance is strongly dependent on both the total number of paths and the inter arrival time of the multipath signals. If the number of resolvable paths is not equal to the number of fingers², the order of multipath diversity is accordingly decreased. Similarly, determination of the multipath arrival times requires a search algorithm whose window size depends on the channel impulse response length. The overall performance of the radio link may be limited either by the path loss or delay spread or both. In certain areas with low path loss and high delay spread, the performance will most probably be limited by the delay spread. In other areas, high propagation loss may limit the performance [29] [67].

1. Average SNR/bit = (SNR/bit) x (2nd moment of the fading signal strength).

2. In a RAKE receiver, received signal is delayed by a symbol duration and multiplied by a complex gain and then summed to provide the resultant signal. The number of fingers of the RAKE are equal to the number of delay elements.

1.2 Literature Survey

Wireless channels may be characterized by the impulse response of a linear time-variant filter. This characterization is quite useful in a number of ways. For example, the output signals corresponding to a variety of applied signals may be easily determined due to linearity assumptions about the channel impulse response. The fading dispersive channel's impulse response is mathematically modeled as a random phasor sum where each phasor corresponds to a constituent multipath. From a purely theoretical standpoint, this channel may be assumed to be characterized as a random linear time variant filter. Fading is due to the time variant nature of the channel while time dispersion is due to the significant inter-path delays. When relating dispersive channel bandwidth with the transmitted signal bandwidth, the channel may either be classified as narrowband or wideband. The issue of fading has been extensively studied in the past, however, there are a number of issues which remain to be clarified. The same is true for the time dispersive nature of multipath channels. This is discussed below.

The most popular fading model for dispersive fading channels is that of a Gaussian Wide Sense Stationary Uncorrelated Scattering (GWSSUS) model, initially proposed by Bello [22], and subsequently applied by Clarke [34] to mobile radio reception. It assumes that the fading is a complex, zero mean, Gaussian process whose envelope is Rayleigh faded while its phase is uniformly distributed. The corresponding physical model is the so-called scattering model which assumes that a large number of *nearly equal strength* multipath phasors combine at the receiver to yield the Rayleigh envelope. However, received signals appear to have two types of variations, namely, slow and fast fading. Clarke showed that if the slow fading is removed, the fast variations can be reliably modeled as Rayleigh distributed. This prompted the approach of moving mean normalization to segregate the slow and fast fading. To date, this is the most popular approach to separate the

two types of fading. Combined slow and fast fading had been observed in other channels, such as ionospheric propagation, and has been separated in the above manner as reported in [75]. The fact that the mean power changes from one small distance segment to other resulted in the proposition by Cox [26], and Parsons and Bajwa [37], to term it the Quasi Wide Sense Stationary Channel (QWSSC) meaning that it is stationary over small distances or areas but non-stationary over large distances or areas.

The notion of slow and fast fading may find usefulness in terms of understanding the environment in which a receiver operates. It is, however, not clear if this approach has any real benefits in radio link performance studies, such as coverage and multiuser interference analysis.

The Rayleigh distribution is not the only statistical characterization for fading mobile radio channels. Other statistical distributions have also provided a good fit to the fast fading data. The more important of them being the Rice and Nakagami distributions. The Rice fit can be easily explained in terms of the previously described scattering model. The physical interpretation may be extended to include the case when the strength of any path is noticeably greater than all other paths. In common wisdom the strongest path is the LOS path. Numerous other researchers describe this case as the direct path or the specular path.

There is no physical model that is associated with the Nakagami distribution. Braun and Dersh [38] attempted to provide an insight into the physical mechanism behind the channel parameters, but they do not specifically provide a clear physical interpretation of Nakagami parameters.

In the case of the wideband channel, the time-variant impulse response consists of a random number of paths each with fading strengths and different excess delays. The modeling efforts in this case are based on the work of Turin et al. [40], Suzuki [41] and Hashemi [42]. It is interesting to observe that in the modeling efforts apparently no com-

parison is made to the GWSSUS channel. The work of Turin et al. [40] is a mathematical representation of a parametric model of the channel where a set of four parameters (path strength, phase, excess delay and number of paths) completely describe the channel at a given location.

If a comparison is made of the Turin parametric model with Bello's GWSSUS channel some interesting observations may be made. The power delay profile for a given scattering function of a GWSSUS channel can be used to obtain the impulse response of the channel. Therefore, all the four parameters may be extracted from the power delay profiles¹. However, Cox [25]-[29], who derived the fading and delay spread characteristics of the mobile radio channel from measurements of the scattering function, reports on the three multipath parameters characteristics but omits the fourth parameter, number of path statistics. On the other hand, Suzuki [39] investigates at the path arrival process level and uses a statistical parametric model for the number-of-path statistics in addition to path strength, phase and excess delay. Regarding the path strength statistics of the multipath channel, Suzuki does not necessarily use the zero mean Gaussian assumption for the complex magnitude of the received multipath and also tries Rice, Nakagami and lognormal distributions for the fading statistics in addition to Rayleigh. His results show a good fit for Rice and Nakagami distributions while the Rayleigh fit is the poorest. The finding that a Rice distribution fits well is the evidence that the fading is a complex non-zero mean Gaussian random process for that particular environment. The Nakagami fit is slightly worse than the Rice fit. It is because the Nakagami distribution is an approximate solution to a constant plus random phasor sum while the Rice distribution is the exact solution. This should raise doubts about the uniformly distributed phase assumption in those channels which demonstrate Rician envelope behavior. Thus, it is contradictory to assume a channel with uniform

1. The power delay profile is the power impulse response which can be easily converted to the voltage impulse response using the appropriate relation.

phase and Rice envelope or, similarly, uniform phase and Nakagami distribution for each path. The uniformly distributed phase is frequently assumed and its validity is not even tested in measurement campaigns [40]. Whether zero or non-zero mean Gaussian processes are assumed for the path strength statistics, the statistics are not only non-stationary in t they are also non-stationary in τ . That is, the magnitude statistics at different delays (in other words, different resolvable paths) do not necessarily have the same mean values.

The properties of the multipath channel parameters, i.e., their values, variations and relation to other parameters may be explained through a physical model of the wideband channel. This approach was apparently first utilized by the Parsons and Bajwa [37] who used the received signal to determine the position of the scatterers. Braun and Dersch [38] used this approach to signify the importance of the measuring bandwidth and its relation to the details in the channel impulse response.

The understanding of the channel model, the statistical characterization of its parameters and their relation to the propagation environment is generally used in analysis and design of system parameters. One such case is the presence of interference in every mobile system. This interference can be of different types and due to different physical sources. Interference severely degrades the performance of the mobile radio systems. Cochannel interference arises from the reuse of the same RF channel. One criterion for cochannel performance evaluation is the received signal-to-cochannel interference power ratio. An associated measure is outage probability, which is the probability that this ratio will remain below a certain threshold known as the signal-to-interference power protection ratio.

Majority of earlier work on cochannel interference (multiuser interference) analysis has only considered the case of fading non-dispersive signals in the presence of fading non-dispersive interference [55]-[62]. To evaluate cochannel interference performance

information is needed about the statistical characterization of the desired and interfering signals. There may be one or more fading interfering signals. In the case of multiple interferers, the distribution of the interference sum is needed to find the distribution of the signal-to-interference power ratio. The outage probability expressions for the fading signal and cochannel interference scenario have very complicated analytical forms. This is because the analysis methodology is based on the segregation of slow and fast fading and in most cases yields non-closed-form solutions. For example, outage expression for Nakagami faded lognormal shadowed signal reception in the presence of Nakagami faded lognormal shadowed interference as given in [62]¹ is quite complex and apparently cannot be written in closed form. In some cases even numerical evaluation is not reasonable².

1.3 Problem Identification

Research on communication channels is perhaps one of the oldest areas in telecommunications research. There is a rich literature in this area³, however, ever evolving telecommunication systems and services keep invigorating researchers to either derive new channel models and their statistical characterization or to verify the applicability of previous models for these systems.

An editorial in the September 1963 issue of the *IEEE Transactions on Communication Systems* summarized a few approaches which may be used to characterize communication channels [3]. According to this editorial, these are *physiogeometric*, *system-function* and *performance-index* approaches. The physiogeometric approach focuses on the causal forces that shape the channel response, such as propagation mechanism, physical processes and geometries. The system-function approach is directed to the resultant channel

1. See equations (6) and (7) in [62].

2. See equation (25) in [62].

3. A complete list may perhaps never be compileable. This thesis cites a large body of research on the mobile radio channel for conventional cellular and microcellular channels. However, it cannot be claimed to be complete.

response, excluding additive disturbances (i.e., receiver noise), such as time-varying impulse response and corresponding system-function. In the performance-index approach, the end-to-end performance-index of greatest interest in a given application is picked as the vehicle for the channel characterization effects of the combined disturbances which are studied all together. An example of the performance index approach is the information theoretic model of the communication link wherein the channel is characterized by a matrix of transition probabilities between the input and output symbols.

More recent literature pertaining to channel studies [4]-[20] may be put in any one of the first two aforementioned approaches. Between the physiogeometric and system-function approaches, the system-function approach is rather quite mature in view of the very frequently quoted voluminous work of Bello [22], which is based on some earlier work of Zadeh [82]. The physiogeometric approach is dynamic in the sense that it pertains to the propagation mechanism, physical processes and geometries. As new communication systems are most likely to operate in new environments, there will always be a need to study the propagation mechanism, physical processes and associated geometries. Therefore, while the system-function approach will be applicable to model and characterize the communication channel of a new system, the statistical characterization and associated physical interpretation may not necessarily remain the same as in any other system. Examining the applicability of the previously understood concepts to a new scenario, which would seem to be a logical approach, has an associated danger of blocking the vision. Example of this are the notions of deceptive shadowing and the unsuitability of the typical procedure of segregation of slow and fast fading in Ricean/Nakagami LOS microcellular channels, as discussed later in this thesis.

This thesis research focuses on the physiogeometric approach of channel characterization. It is different from other channel studies in that it goes a step forward to explore the

nature of the parameters of the statistical distributions of the channel parameters, i.e., investigations of the physical basis, their variations in practical cases of interest, and their application in BER and link performance studies to help understand the impact of the parameter characteristics on performance methodologies as well as the performance results.

1.4 Contributions

Multipath channels are mathematically modeled in terms of a phasor sum which results in two different forms to help identify the propagation mechanism, the effect of different contributing paths, and model the variability in the received signal.

The physical basis of the slow and fast fading phenomena has been discussed. The concepts of *Effective Direct Path* and *Deceptive Shadowing* are introduced. The issue of separation of the so-called fast and slow fading, in particular, the boundary between slow fading and fast fading in microcellular channels, is investigated. It is shown that in Ricean channels, segregation of the two fadings is not possible and is redundant if the two fading types are segregated using running mean normalization. It is shown that the physical model associated with slow fading is not quite accurate and that the Nakagami distribution can be used to statistically characterize the slow fading.

The issue of the physical model of Nakagami fading is addressed for both narrowband and wideband channels. For narrowband channels, it is shown that the physical model of the Rice/Rayleigh distribution is applicable to the Nakagami case as well. The issue of the equivalency of Rice and Nakagami distributions is discussed. The relationship of the parameters of two distributions is derived and a simple approximation of the Nakagami m and Rice K parameters are found. The validity of the approximation is shown through simulations. For wideband channels, a geometrical theory of Nakagami fading is evolved and

a physical interpretation is provided. A twofold interpretation of the Nakagami m parameter, each for narrowband and wideband case, is provided. In the narrowband case, the interpretation of the m and K parameters are shown to be similar. For the wideband case, m is related to the order of multipath diversity obtainable from a time dispersed channel.

A realistic model of the wideband channel assumes random path arrival. Analysis of the RAKE receiver is performed for this case and its performance is quantified and compared with RAKE reception in the case of a known number of paths.

The effect of the presence of wideband frequency selective interference on radio link performance is investigated. Dispersive Nakagami fading is assumed for both desired and interfering signals. Outage expressions are derived and numerical results are presented.

Radio link performance is investigated for the case of multipath induced ISI, termed here as Multipath Self Interference (MSI). Assuming a Poisson path arrival process and using shot noise model for statistical characterization of MSI, outage probability expressions are derived for numerous fading scenarios.

1.5 Organization

This thesis is organized as following.

Chapter 2 begins with a review of the wideband channel and a general description of a mathematical wideband channel model is presented. It is shown that the narrowband channel is a special case of it. The characteristics of the parameters of the narrowband channel are presented and a theoretical formulation is made, based on which the associated physical mechanism is explained. The statistical nature of the multipath arrival process is presented and the time dispersive nature of the wideband channel is reviewed.

Chapter 3 visits in detail some of the issues reviewed in Chapter 2. Focus is on the physical basis of fading and time dispersive channels. The nature of fading in LOS chan-

nels, the relation between Rice and Nakagami fading and the physical basis of Nakagami fading is investigated. A twofold interpretation of the Nakagami m parameter, each for narrowband and wideband cases, is provided.

Performance of a wideband RAKE receiver is analyzed in Chapter 4 for the case of a realistic time dispersed channel, in which paths arrive randomly following a modified Poisson process. BER expressions are derived.

In Chapter 5, using the dispersive Nakagami fading channel model, the effect of the presence of time dispersed fading interference on radio link performance is investigated. Outage expressions are derived and numerical results are presented. The interpretation of the Nakagami m parameter in Chapter 3 and Chapter 4 is used to interpret the outage performance results.

Radio link performance is investigated in Chapter 6 for the case of multipath induced ISI or the so-called multipath self interference (MSI). A Poisson path arrival process is assumed and a shot noise model is used for statistical characterization of MSI. Outage probability expressions are found for multiple fading scenarios.

The results of the thesis are summarized in Chapter 7 and further work is suggested. Finally, a list of references is attached.

Chapter 2

Multipath Mobile Wireless Channel

2.1 Introduction

The conventional mobile radio channel experiences multipath propagation and is modeled as a linear time-variant tapped-delay line filter. The mobile radio environment is quite versatile and the propagation character may change significantly as the user moves from one type of environment to another. The tapped-delay line model remains valid in all of these environments as long as the parameters of the model are appropriately modified. If we assume that the mobile radio channel is characterized as a linear time-variant filter, then its time-variant impulse response $h(\tau, t)$ would completely characterize it; t is the observation time while τ is the application time of the impulse, [45]. For a multipath channel, the lowpass time-variant impulse response may be written as, [45],

$$h(\tau, t) = \sum_l g_l(t) \delta(\tau - \tau_l(t)) e^{j\theta_l(t)}, \quad (2.1)$$

where $g_l(t)$, $\tau_l(t)$ and $\theta_l(t)$ are the time-varying attenuation factor, propagation delay and phase of the l th path, respectively. The parameter l is the path number.

Let us suppose that $Re[v(t) \cdot e^{j\omega_0 t}]$ is sent and $Re[v_r(t) \cdot e^{j\omega_0 t}]$ is received after passing through the noisy multipath linear time-variant channel, then the received complex low pass signal $v_r(\tau, t)$ is of the form,

$$v_r(\tau, t) = \sum_l g_l(t) v(\tau - \tau_l(t)) e^{j\theta_l(t)} + n(t), \quad (2.2)$$

where $n(t)$ is complex white Gaussian noise, and the summation term on the right is obtained after the convolution of low pass transmitted signal $v(t)$ with the multipath channel impulse response $h(\tau, t)$. The received signal in (2.2) is either deterministic or random depending upon the nature of $h(t)$ through its parameters, $g_l(t)$, $\tau_l(t)$, $\theta_l(t)$, l and the noise $n(t)$. A complete characterization of the received signal requires the description of the above five parameters, the first four of which are just due to multipath propagation phenomena. The statistical characteristics of these parameters may be significantly different depending upon whether the channel is assumed to be narrowband or wideband; this is discussed next.

2.1.1 Narrowband vs. Wideband Channel

If the transmitted signal is finite over a duration T and zero elsewhere, the received signal may either be heavily or slightly dispersed in time. That is, due to multipath nature of channel a number of attenuated and delayed replicas of transmitted signal are received. Suppose τ_m is the duration over which the multipath signal is received¹. Then two possible cases arise. If $\tau_m \gg T$, then channel is termed wideband, while when $\tau_m \ll T$, it is known as a narrowband channel². A loose definition would be one where it is assumed that for the wideband case $\tau_m > T$ and $\tau_m \leq T$ for the narrowband case³. Let us suppose that the received signal due to multipath effect alone is $v_m(\tau)$ ⁴ and given by,

1. Ideally τ_m is infinite due, for example, to multiple reflections but those would only be significant if the receiver sensitivity is ideal, i.e. zero or $-\infty$ dB. Due to finite receiver sensitivity τ_m is always finite.

2. It must be noted that any given channel is narrowband or wideband depending on the choice of the transmitted signal whose duration would result in this classification. A wideband channel may be exploited for multipath diversity but this is obtained at the cost of increased signal bandwidth.

3. This is also dependent on the symbol duration. If a full width pulse of duration T is transmitted, where transmission rate is $1/T$, then this definition is loose. On the other hand if a half width symbol pulse is used then this condition is not loose rather it is quite exact.

4. For brevity, $v_r(\tau, t)$ will be denoted by $v_r(\tau)$ and the time dependence of channel parameters will not be shown in later sections.

$$v_m(\tau) = \sum_{l=1}^L g_l v(\tau - \tau_l) e^{j\theta_l}. \quad (2.3)$$

Let the L multipath terms in (2.3) can be separated in N summation terms of I_n terms each such that $L = \sum_{n=1}^N I_n$ as shown below.

$$v_m(\tau) = \sum_{i=1}^{I_1} g_{1,i} v(\tau - \tau_{1,i}) e^{j\theta_{1,i}} + \dots + \sum_{i=1}^{I_n} g_{n,i} v(\tau - \tau_{n,i}) e^{j\theta_{n,i}} + \dots + \sum_{i=1}^{I_N} g_{N,i} v(\tau - \tau_{N,i}) e^{j\theta_{N,i}}. \quad (2.4)$$

Let us make the following assumptions.

1. The first replica is received at $t=0$.
2. We impose the restrictions that $0 \leq \tau_{1,i} < T$, $T \leq \tau_{2,i} < 2T$ and $(N-1)T \leq \tau_{N,i} < NT$, that is, each individual summation term in (2.4) consists only of unresolvable multipath components.

The consequence of the above formulation and unresolvability assumption is such that each summation term on the right hand side of (2.4) may be treated as a phasor sum in the narrowband case. Equation (2.4) may now be written as

$$v_m(\tau) = \sum_{n=1}^N g_n v(\tau - \tau_n) e^{j\theta_n}, \quad (2.5)$$

where g_n , τ_n and θ_n are the multiplicative disturbance, delay and the phase associated with the n th path, respectively. Parameter N is the number of paths.

It is the sum of the form in (2.5) instead of (2.3) which is commonly referred to in literature whenever impulse response measurements of the multipath channel are performed. It is important to realize that both g_l and g_n are random variables. The numerous statistical characterizations reported in literature refer to g_n not g_l . However, g_l is also a random vari-

able whose random nature is due more to the receiver noise¹ than to the multipath interference phenomenon.

If we assume that $\tau_m \leq T$ then $N = 1$ in (2.5) and (2.4) is reduced to

$$v_m(\tau) = \sum_{i=1}^{L_1} g_{1,i} v(\tau - \tau_{1,i}) e^{j\theta_{1,i}}. \quad (2.6)$$

to give the narrowband case. Let us assume that $v(t) = A$, where A is a constant and represents the peak value of the transmitted carrier. Without loss of generality, we may assume $A = 1$ for all τ and drop one from the subscripts because no differentiation in terms of resolvability is needed. As a result, (2.6) reduces to,

$$v_m(\tau) = \sum_i g_i \cdot e^{j\theta_i}. \quad (2.7)$$

which is the desired phasor sum of the narrowband case.

2.1.2 Parameter Characteristics

The random linear time-variant tapped delay line filter channel is statistically characterized as Gaussian Wide Sense Stationary Uncorrelated Scattering (GWSSUS) channel, [22], [71], [23]. The $g_n(t)$ in (2.5) are assumed to form a sample function of a wide sense stationary Gaussian process and fading in each resolvable multipath strength of the impulse response is assumed to be uncorrelated thus the term GWSSUS. A *scattering function* or *delay Doppler spectrum*, which completely characterizes this channel², is related to the *spaced-time spaced-frequency correlation function* of the channel through a double Fourier transform. The two main effects, namely, delay spread and doppler spread,

1. If a pulse of sufficiently narrow width is transmitted such that each of the L paths are separately resolvable, then attenuation of each g_i in (2.3) is obtained mainly through free space loss where each path encounters the same receiver noise (AWGN). In this case the variability in each path strength is due to receiver noise only. If the above condition is not met, then variability is due to both multipath interference and receiver noise.

2. Any zero mean Gaussian process can be completely described in terms of its correlation function or its associated power spectrum. The mean is needed, in addition, for a non-zero mean Gaussian process.

are evident. One accounts for the time spread and the other accounts for the frequency spread; the two are inversely related to the so-called frequency selectivity and time selectivity of channel, respectively. The multipath channel parameters can also be derived from this perspective: $g_n(t)$ relate to the Doppler spread and time selectivity (coherence time) of the channel, whereas $\tau_n(t)$ relate to the delay spread¹ and frequency selectivity (coherence bandwidth) of the channel.

There can be two sources of time-variation in the impulse response of the mobile radio channel. The first is *temporal* (in time) *variation*. In this case, the spatial position of transmitter and receiver are fixed, however, two impulses applied at $t = t_1$ and $t = t_2$ would yield two different sets of channel parameter values, i.e. they are dependent on t . The main physical source of this variation is the movement of objects surrounding the transmitter and receiver. The second source of parameter variation is the *spatial variation* in the transmitter and receiver due to their relative motion. This may, however, be converted to temporal variations because the two are related through the velocity of motion. In this thesis, the time variance of the impulse response is assumed to be due to spatial variations only unless otherwise specified. However, although the time dependence of channel parameters in the above sense is assumed throughout this thesis, it is not always shown in notation for brevity.

The variations of the channel parameters are assumed to be random. Therefore, all of them are characterized statistically. The time dependent² envelope, $g_n(t)$, is reported to be distributed in accordance with Rice, Rayleigh, lognormal, or Nakagami distributions under different sets of conditions and environments [34]-[45]. The time dependent phase,

1. The commonly used term delay spread refers to rms delay spread and, of course, is not equal to $\tau_n(t)$. It is a measure of the time spread of the channel around its average value. It is defined in Section 2.3.1.4.

2. This time dependence is due to spatial variation and related to the velocity of motion through $r=st$, where r is the distance from transmitter, s is the speed and t is the time.

$\theta_n(t)$ is assumed to be a uniformly distributed random variable over $(0, 2\pi)$. The propagation delay $\tau_n(t)$ is strongly dependent upon the propagation environment and is related to the number of paths, N . A more meaningful parameter derived from the $\tau_n(t)$ is the inter-arrival time, defined as $\Upsilon_n = \{\tau_n - \tau_{n-1}\}$, and is related to the underlying path arrival process. In one theoretical formulation, [51], the parameter N is shown to have a Poisson distribution. Measurement results [39] [40], also support this characterization, albeit through some modifications in the Poisson distribution parameter (average arrival rate) which is found to vary randomly between two values and is modeled as a Markov process. In summary, the path arrival process may be assumed as a Poisson process, so that Υ_n will be an exponentially distributed continuous random variable and the N will be a Poisson distributed discrete random variable. It may be noted that the sum $\sum_{n=1}^N \Upsilon_n$ gives the total time spread of the channel.

All the other channel impairments may be incorporated into $n(t)$, which is statistically characterized as the complex Additive White Gaussian Noise (AWGN). The physical sources for this impairment may be diverse and are not of prime concern in this study.

2.2 Narrowband Channel: Theoretical Foundation

In this section we develop the theoretical basis of the narrowband LOS microcellular mobile radio propagation channel.

Let us suppose that $v(t) = \text{Re}\{g \cdot e^{j\omega_0 t}\}$ is sent and, after passing through a noisy multipath channel, $v_r(\tau)$ is received, where g is the amplitude and ω_0 is the frequency of the transmitted sinusoidal carrier in radians. The received multipath signal $v_r(\tau)$ is of the form

$$v_r(\tau) = \sum_i g_i \cdot e^{j\omega_0(\tau - \tau_i)} + n(\tau), \quad (2.8)$$

where i the number of paths and τ_i is the associated arrival time of the signal through that path. The noise $n(\tau)$ is a complex Gaussian noise which may be written as

$$n(\tau) = z_n \cdot e^{j\theta_n}, \quad (2.9)$$

where z_n is the Rayleigh distributed envelope of the complex additive white Gaussian noise and θ_n is assumed to be uniformly distributed over $\{-\pi, \pi\}$.

Also,

$$\theta_i = \omega_0(\tau - \tau_i) = 2\pi f_0 \cdot (\Delta r_i / c) = 2\pi \cdot \frac{\Delta r_i}{\lambda_0}, \quad (2.10)$$

where $\lambda_0 = c/f_0$, c is the velocity of light and Δr_i is the path length difference between the i th and reference signal paths; thus, we have

$$v_r(\tau) = \sum_i g_i \cdot e^{j\theta_i} + z_n \cdot e^{j\theta_n}. \quad (2.11)$$

Assuming free space propagation, in a multipath mobile radio propagation environment, the received signal power of a path, P_i , is given [23] by

$$P_i = P_t \cdot G_t \cdot G_r \left[\frac{\lambda_0}{4\pi r_i} \right]^2. \quad (2.12)$$

where P_t and P_i are the transmitted and received power of i th signal paths respectively. G_t and G_r are the transmitter and receiver antenna gains, λ_0 is the signal wavelength and r_i is the distance travelled over the i th signal path. As $g_i = \sqrt{P_i}$, that is, received signal voltage is equal to the square root of power when normalized to one Ohm load, (2.12) may be rewritten as

$$g_i = \frac{\sqrt{P_t G_t G_r} \cdot \lambda_0}{4\pi} \cdot \frac{1}{r_i} = C \cdot \frac{1}{r_i}, \quad (2.13)$$

where

$$C = \frac{\sqrt{P_t G_t G_r} \cdot \lambda_0}{4\pi} \quad (2.14)$$

The receiver is considered to perform the phasor summation over all the received signal paths. Each signal path is specified in terms of its strength and relative phase to a reference (usually the first) signal path. The path signal strength is obtained using (2.14) whereas phase is given by (2.10).

Because multipath in an urban environment is due to reflection and diffraction and their combinations, the received field in a multipath environment is given as a phasor sum of these component multipath signals as follows¹:

$$\bar{v}_r = g_r e^{j\theta_r} = \frac{C e^{j\varphi}}{r} + \frac{C R_G e^{j\Delta\varphi_G}}{r_G} + \sum_{m=1}^M \frac{C R_m e^{j\Delta\varphi_m}}{r_m} + \sum_{l=1}^L \frac{C D_l e^{j\Delta\varphi_l}}{r_l} + z_n \cdot e^{j\theta_n} \quad (2.15)$$

where C is the constant as defined in (2.15) and takes into account the effects of frequency, transmitted signal strength and antenna gains. R_G , R_m and D_l are ground reflection, wall reflection and diffraction coefficients, respectively. The path lengths of the line-of-sight (LOS), ground reflected (GR), wall reflected (WR) and edge diffracted (ED) paths are r , r_G , r_m , and r_l , respectively. The phase of the LOS path is denoted by φ which is the reference phase. The differential phase difference of the ground reflected, m th wall reflected and l th diffracted paths from the LOS path are denoted by $\Delta\varphi_G$, $\Delta\varphi_m$ and $\Delta\varphi_l$, respectively.

Equation (2.11) and (2.15) are equivalent; however, (2.15) explicitly accounts for the multipath propagation mechanism. Equation (2.11) may be rewritten from the perspective of slow and fast variation in the received signal strength, assuming that there are K paths

¹. Other terms may be added in (2.15) which account for the multiple reflections, diffractions and their combinations. But those may be neglected due to associated excessive reflection and distance dependent loss.

that constitute slow and $L+1$ paths that are responsible for fast variations in the received signal, such that

$$\bar{v}_r = \sum_{k=1}^K g_k \cdot e^{j\theta_k} + \sum_{l=1}^{L+1} g_l \cdot e^{j\theta_l}. \quad (2.16)$$

Let¹

$$g_{sf}e^{j\theta_s} = \sum_k g_k \cdot e^{j\theta_k} = Ce^{j\varphi}/r + \frac{CR_G e^{j\Delta\varphi_G}}{r_G} + \sum_{m=1}^M \frac{CR_m e^{j\Delta\varphi_m}}{r_m}, \quad (2.17)$$

and

$$g_{ff}e^{j\theta_f} = \sum_{l=1}^{L+1} g_l \cdot e^{j\theta_l} = \sum_{l=1}^L \frac{D_l e^{j\Delta\varphi_l}}{r_l} + z_n \cdot e^{j\theta_n}, \quad (2.18)$$

such that

$$\bar{v}_r = g_r e^{j\theta_r} = g_{sf} e^{j\theta_{sf}} + g_{ff} e^{j\theta_{ff}}, \quad (2.19)$$

where g_{sf} and g_{ff} are the magnitudes and θ_{sf} and θ_{ff} are the corresponding phases of the resultant slow and fast varying phasors. However, the total received signal strength is the resultant of the phasor sum in (2.19). Thus, in most general form,

$$\bar{v}_r = g_r e^{j\theta_r} = x_r + jy_r, \quad (2.20)$$

where

$$x_r = g_{sf} \cdot \cos\theta_{sf} + g_{ff} \cdot \cos\theta_{ff}, \quad (2.21)$$

and

$$y_r = g_{sf} \cdot \sin\theta_{sf} + g_{ff} \cdot \sin\theta_{ff}. \quad (2.22)$$

The magnitude and phase of the resultant vector are given by

1. This assumption is justifiable and has been verified through measurements. It will become clear as the discussion unfolds.

$$g_r = |\overline{v_r}| = \sqrt{x_r^2 + y_r^2}. \quad (2.23)$$

and

$$\theta_r = \arg(x_r + jy_r). \quad (2.24)$$

Parameter g_r in (2.23) is the received signal magnitude and is a function of receiver location. The (2.23) and (2.24) form the theoretical basis of the received signal strength and phase statistics extracted from the narrowband channel sounding. A narrowband channel simulator would also be based on (2.23) and (2.24).

2.2.1 Variability in Received Signal Envelope and its Physical Interpretation

Figure 2.1 and Figure 2.2 are taken from [10]. Figure 2.1 shows the mean power as a function of distance in a line of sight (LOS) microcellular environment at 900 MHz. Three important observations are made: there is a distance dependent path loss on which are superimposed some slow variations (noticeable at distances in the order of several hundred meters), residing on which are the fast variations (noticeable at distances in the order of a few meters). The fast variations for three different distance segments are shown in Figure 2.2. The segments which are at a larger distance from the transmitter show relatively larger variations around their local mean.

The main ingredient of g_r is the free space loss as in any propagation loss study. Because the ground is present, the next approximation to the microcellular environment is the plane earth propagation model. However there are reflectors and diffractors present so they will also contribute. As a result the total path loss will be contributed by all of these multipath components and is theoretically given by (2.15) or (2.19) both of which are equivalent. However, as it is not feasible to obtain path loss measurements at points very close to each other, i.e., distances λ/n where $n \gg 1$, therefore propagation loss measurements are reported in terms of median or mean path loss over a distance of a few λ and

Figure 2.1 Figure 5(b) of [10]. Measured and mean power in urban environments. The LOS propagation at 900 MHz.

Figure 2.2 Figure 8 of [10]. Fast variation of three different distance segments.

plotted against the distance. These plots smooth the actual path loss curve and rapid variations¹ in the path loss are averaged out. In other words, the shape of the median path loss curve is defined mainly by the strong contributors. A number of researchers have looked into this problem and it is the consensus that a simple four to six ray propagation model is quite adequate for median path loss computation or slow fading of the channel [6], [10]. In certain scenarios, just a two ray plane earth reflection model may be sufficient to model median propagation loss [10] (Figure 2.3). The Figure 2.4 is taken from [10] and shows the six ray approximation of the mean path loss. From the point of view of (2.19), only the first term on the right-hand side has been evaluated by the six ray model to give the curve in Figure 2.4.

To deal with the fading phenomenon separately, the distance dependent path loss in g_r should be separated from fading of the propagation loss. It may be split at first as follows:

$$g_r = g_L \cdot g_f, \quad (2.25)$$

where g_L is the distance dependent propagation loss and g_f is the total fading or variation in the propagation loss. The g_L provides information about the required transmission power. The statistical distribution of the g_f provides the information about the additional power requirement to allow for the fade margin. From (2.25), g_f would be obtained by normalizing the g_r by g_L . The effect of this normalization may be witnessed in Figure 2.6. In the next section the nature of g_L is investigated. The characteristics of the g_f are discussed in Section 2.2.3.

2.2.2 Propagation Loss

The ITU-T (formerly CCIR) used the results of the extensive propagation measurement program by Okumura [46] to specify the signal strength prediction method for con-

1. When fast fading is to be studied, $n \gg 1$ is a necessary condition. The confidence of the statistical study of rapid variations may perhaps be said to be proportional to n .

Figure 2.3 The Figure 3(b) of reference [10]. Measured and calculated (two-rays) mean power in a rural environments at 11GHz with monopole/monopole transmit/receive antennas combinations.

Figure 2.4 The Figure 6(b) of reference [10]. Comparison of measured power at 900 MHz on the Lexington Ave. and the calculated mean power based on the six-ray model over a 500 feet road segment.

Figure 2.5 The dB magnitudes versus the distance of the phasor summation of both the terms of (2.19).

Figure 2.6 The effect of the normalization on g_r by the g_L to yield g_r as in (2.26). The Figure 2.5 is normalized to g_L .

ventional cellular systems. However, Okumura's results are not intended for prediction at distances of less than one kilometer and antenna heights less than 30 meters. The parameters which are used in [46] are as follows:

- Frequency range: 150-2000 MHz
- Distance: 1-100 Kilometers
- Effective base station antenna height: 30-1000 meters

Therefore, there is no basis of comparison between measurements done using low antenna height LOS microcellular measurements reported in the literature [4]-[10] to the predictions of Okumura's or Hata's [47] empirical formulas, the latter being based on Okumura's measurements. In one instance, Harley [4] compared measurement results for a microcellular environment to Okumura's results after extrapolation to low antenna heights and distances using Hata's empirical formulas, and found them to give pessimistic results; they predict greater loss than indicated by his measurements. The point is that microcellular propagation has its own character as discussed below.

Unlike free space propagation, which exhibits a single slope linear dependence of log path loss in dB's on the distance, line-of-sight microcellular propagation data fall into two regions which are separated by a break point. These two regions produce two different slopes when log path loss in dB's is regressed linearly against distance. On close examination, the dominant character of the propagation is found to be similar to a two ray plane earth propagation model. Before the break-point, the radio signal oscillates severely, due to constructive and destructive addition of two rays. In contrast, after the break-point, loss increases more rapidly than in the first region. This two slope and break-point character is unanimously observed in the measurements reported in literature [4]-[10], and included in any mathematical models suggested therein. It is also observed for both the 900 MHz (cellular) and 1.9 GHz (PCS) bands.

A physical interpretation of this phenomenon in terms of Fresnel zone clearance is provided in [6]. The first Fresnel zone is described by an ellipsoid whose foci are the transmitting and receiving antennas. The break-point is then determined to be the distance between the antennas for which the ground just begins to obstruct the First Fresnel zone. The horizontal separation at which the first Fresnel zone just touches the ground is given by

$$d = \frac{1}{\lambda} \sqrt{(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 + \Delta^2) \left(\frac{\lambda}{2}\right)^2 + \left(\frac{\lambda}{2}\right)^4}, \quad (2.26)$$

where $\Sigma = h_b + h_m$ and $\Delta = h_b - h_m$. The h_b and h_m are the base and mobile antenna heights respectively. For higher frequencies, this expression is approximated as a simple function of wavelength and antenna heights as follows [6]:

$$d = \frac{4h_b h_m}{\lambda}. \quad (2.27)$$

Interestingly, the same approximate equation is also suggested in [7], though based on a different physical interpretation. The slope of the loss law before and after the break-point is strictly dependent on the propagation environment and may be 15-20dB/decade before and 25-40 dB/decade after the break-point [6] [7]. In system design, the best approach should be one which uses a channel simulator based on (2.23) and obtains the slope value by linear regression analysis.

2.2.3 Fading

The fading component g_f of g_r may be obtained by normalizing the g_f with g_L (Figure 2.6.) but how to segregate g_{sf} and g_{ff} is a question of important concern. Separation of shadowing and fast fading is a very grey area¹. The question in the case of finding the fast fading statistics is what criterion to use to divide the large distance into a number of small

1. A comprehensive discussion of these issues may be found in [23], pp 151, 214-215.

intervals to allow extraction of fast fading statistics over these small distances. It can be appreciated that using a relatively large interval will result in biasing due to inclusion of slow variation in the fast fading coefficient estimates, while too small an interval will tend to leave fast variation in the slow fading estimates. There are suggestions to use the averaging intervals such as 30λ [23], or 40λ [68]. The value of 90λ has also been used¹ [26].

Once the data are separated in an appropriate manner, the remaining issue is to find the statistical distribution of the fading components. The result of channel measurements and empirical curve fitting reveals that slow fading is lognormally distributed while fast fading is distributed according to Rayleigh, Rice, lognormal or Nakagami distributions.

The Nakagami distribution, compared with the Rice, Rayleigh and lognormal distributions, is perhaps the most general to fit the fading data of a multipath channel. However, the propagation mechanism behind the Nakagami distribution is neither very well understood nor reported elsewhere as far as the knowledge of this author is concerned. This is perhaps the reason why the Nakagami fading model is not as frequently used by mobile radio system designers as the Rice or Rayleigh fading models.

Clarke [34] is perhaps the first to propose the physical interpretation of the Rice and Rayleigh distribution in the context of mobile radio channel reception; his interpretation was further refined by Suzuki [39]. Figure 2.7 is taken from [39] and depicts the physical scenario to rationalize the conventional Rayleigh/lognormal fading model. It is explained in Suzuki's words as follows:

The main wave arrives at a cluster of buildings. Before undergoing a local "scattering" by the clusters of buildings, it has travelled a path which is subject to the influence of multipath reflections and/or diffractions by natural and man-made objects. Thus, the path strength, on arrival at the local cluster, will be a random variable. After arrival at the local cluster the path undergoes a breaking up into local subpaths as a result of 'scattering' from the local objects. The subpaths all

1. This corresponds to a distance of about 10-30 meters at a frequency of 900 MHz.

A model of urban radio multipath channels.

Figure 2.7 Taken from reference [39]; multipath propagation model.

arrive at the receiver with roughly the same delay but with different carrier phases. Therefore, the path strength seen by the receiver tends to be distributed....

As a special case, we can imagine the transmitter-to-cluster path to have a lognormal strength since it undergoes a multiplication effect, while the local-path distribution is Rayleigh because it is due to an additive scattering effect.

The Rice fading model is somewhat of an expansion of the above lognormal/Rayleigh fading model. In cases where there is a LOS or direct path between the transmitter and receiver then fast fading statistics become Rice distributed. Others have also advocated this model. For example, Parsons [23] writes

The assumption of similar-amplitude waves holds in a wide variety of scenarios because in general the mobile[terminal] has no line-of-sight path to the transmitter and there is no dominant incoming wave. However there are situations, e.g. in small cells of a cellular radio system, where there may be a line-of-sight path or at least a dominant specular component. We may then expect that the statistics will differ from those already described (Rayleigh). The problem is analogous to that of a sinusoidal wave plus random noise, which has been extensively treated by Rice...

The above physical models are associated with Rice/lognormal and Rayleigh/lognormal fading models in conventional urban radio propagation. A similar understanding is generally extended towards the urban microcellular radio propagation mechanism. In Chapter 3, the accuracy of these physical interpretations and their application to the Nakagami fading channels is investigated.

2.2.3.1 Statistical Distribution of Envelope Fading

The underlying random process to account for the random signal variability as suggested by Clarke [34], Cox [26], Suzuki [39], and Parsons and Bajwa [37], is a non-stationary zero-mean complex Gaussian process with variance σ^2 . Therefore, the received signal envelope g_r is considered to be a Rayleigh distributed random variable whose local mean value changes from one small area to another. Thus, the local statistics are provided by the conditional Rayleigh distribution

$$f(g_f|\sigma) = \frac{g_f}{\sigma^2} e^{-\frac{g_f^2}{2\sigma^2}} \quad g_f \geq 0 \quad (2.28)$$

From one small area to another σ is not constant but varies according to the lognormal distribution: thus,

$$f(\sigma) = \frac{1}{\sigma\alpha\sqrt{2\pi}} e^{-\frac{(\ln(\sigma) - \mu)^2}{2\alpha^2}} \quad -\infty \leq \sigma \leq \infty \quad (2.29)$$

where μ is the mean of $\ln(\sigma)$ and α is the standard deviation of $\ln(\sigma)$. From the total probability theorem,

$$f(g_f) = \int_{-\infty}^{\infty} f(g_f|\sigma) f(\sigma) d\sigma \quad 0 \leq g_f < \infty \quad (2.30)$$

Thus,

$$f(g_f) = \int_{-\infty}^{\infty} \frac{g_f}{\sigma^2} e^{-\frac{g_f^2}{2\sigma^2}} \cdot \frac{1}{\sigma\alpha\sqrt{2\pi}} e^{-\frac{(\ln(\sigma) - \mu)^2}{2\alpha^2}} \cdot d\sigma \quad 0 \leq g_f < \infty \quad (2.31)$$

The above distribution is the so called mixture distribution and is reported by Suzuki [39].

2.2.4 Shadow Fading

The physical mechanism of shadow fading, as quoted earlier, is commonly associated with the absence of a LOS path. The statistical characterization of shadow fading is usually done in terms of a lognormal distribution, the standard deviation of which gives the degree of shadow fading. The values of 3-7 dB are reported for conventional cellular propagation by Okumura [46], while for a LOS microcellular channel values of up to 8 dB are reported [9].

2.2.5 Distribution of Phase

The phase θ_f also varies randomly and is assumed to be a uniformly distributed random variable over $(0, 2\pi)$. A discussion on the justification of this assumption may be found in [40]. Unfortunately, it is hard to justify this assumption theoretically, except in the special case of Rayleigh fading. However, as it has been shown that this is generally not the case in the LOS microcellular environment, there is a need to find an appropriate distribution for θ_f . If there is a dominant path then the magnitude statistics are Rician distributed and it can be shown that the phase of the complex Rician process is given by [66]:

$$f(\theta_f) = \frac{1}{2\pi} \cdot e^{-K} \cdot \left[1 + \sqrt{\pi K} \cdot \cos \theta_f \cdot e^{K \cos \theta_f} \right] \cdot [1 + \operatorname{erf}(\sqrt{K} \cdot \cos \theta_f)]. \quad (2.32)$$

The plot of the distribution in (2.32) is a function of K and peaky in nature. It leads to an important implication that the phase of the resultant phasor sum is dominated by the phase of the dominant path in the phase distribution.

2.3 Wideband Channel: Theoretical Foundation

When a wide band pulse is used to probe the channel, both time and frequency selectivity parameters of the channel can be extracted. In this case those paths which arrive at the receiver over a period which is less than the transmitted pulse duration would be combined to form one pulse with faded magnitude and phase as discussed in Section 2.2. This gives the time selective behavior of the channel and the discussion and results of Section 2.2 are applicable. If the pulse width is less than the impulse response duration then receiver has more than one replica to deal with and the characteristics of the number of resolvable paths and time spread characteristics of the channel need to be accounted for. Both τ_n and N are random variables and are accordingly characterized in a statistical manner. For the time selective behavior of each pulse, the discussion of Section 2.2 is still

applicable. The path arrival process of the wideband channel addresses the statistical nature of both τ_n and N as described below.

2.3.1 The Path Arrival Process

The statistical characterization of the path arrival process was largely neglected in the past except for a few bodies of work [40]-[45]. In this section we look into the approaches which are available in the literature to statistically model the random multipath arrival process. There appear to be two major approaches to the statistical modeling of the path arrival process. One of them focuses on the outdoor mobile radio environment and is reported by Turin et al. [40], Suzuki [39], [41] and Hashemi [42], [43]. The other is due to the work of Saleh et. al., [44], and pertains to the indoor mobile radio environment. Hashemi [45] has more recently applied the outdoor model to the indoor environment as well.

2.3.1.1 Turin's Model

The work of Turin et al. hypothesized a Poisson arrival process. However, it was discovered that for a small Poisson parameter λ , the empirical distributions were more clustered around their mean than a Poisson distribution would dictate. This is interpreted to reflect the tendency of the near line of sight paths to arrive in groups. Moreover, as λ increases, the fit between the empirical and Poisson distributions improves, possibly suggesting that late arriving paths in the sequence are more accurately modeled by the Poisson arrival assumption.

Based on these observations, a modified Poisson process (or a so-called $\Delta - K$ model) was suggested [39]- [41] which takes into account the possibility that paths may arrive in groups which reflect the grouping property of buildings (or reflectors) in the environment.

This process was studied in detail by Suzuki [39] [41]. Some salient features are mentioned in the following.

The $\Delta - K$ model is depicted in Figure 2.8. There are two states S_1 and S_2 for the mean arrival rates, λ and $K\lambda$ respectively. The process starts in S_1 with mean arrival rate λ . When a path arrives at a certain time t , the state changes to S_2 with the mean arrival rate $K\lambda$ for a time interval of $(t, t + \Delta)$. If no other path arrives in this interval a transition is made back to the state S_1 . For $K=1$ or $\Delta = 0$, the $\Delta - K$ model reverts to a standard Poisson process.

The implementation of the $\Delta - K$ model revealed that the continuous time version of this model is too complex to handle analytically, so a discrete version was analyzed. It was also learnt that changing both the parameters K and Δ was not essential¹. Therefore, Δ was kept fixed for a suitable duration and K was changed to obtain an optimum value by curve fitting in the minimum mean square error sense. The details may be found in [41], [42] and [43].

Results of the work of the Turin et al. need to be applied carefully to the propagation environment for PCS systems. The available targets for system deployments necessitate a focus on the urban environment. The requirement for high user density has resulted in small cell (microcellular and picocellular) PCS systems, having low base station antenna heights and untraditional cell structures [49]. The model developed by Turin et. al., though general in nature, was tested on such an environment which does not much resemble those proposed for PCS, e.g., low antenna LOS microcellular. Consequently, the parameters derived by Turin et al. cannot be used right away. On the other hand, some of the general conclusions are very applicable, one of them being, as Suzuki [41] puts it "Urban areas are

1. This K is not to be confused with the Rice K factor. Later, when needed, the symbol K will be kept for the Rice K factor, and a new symbol will be assigned to the K parameter in the $\Delta - K$ model.

Figure 2.8 Illustration of the modified Poisson Process.

homogenous and more Poisson while suburban areas have clusters of buildings", thus being less Poisson or modified Poisson.

2.3.1.2 Saleh's Model

The statistical arrival model of the Saleh [44], though different from the Turin model in mathematical formulation, appears to be based on the same concept. We shall compare their mathematical forms for similarities and dissimilarities after Saleh's model is introduced.

This model is based on the physical reality that measured rays arrive in clusters. The arrivals of the first rays of successive clusters are modeled as a Poisson arrival process with some fixed mean arrival rate, Λ . The rays within each cluster of rays are also assumed to arrive with Poisson arrival process with another fixed mean arrival rate, λ . In general, $\lambda \gg \Lambda$, which means that each cluster consists of many rays.

The arrival time of l th cluster is denoted by T_l , where $l=0,1,2,\dots$, and the arrival time of the k th ray in the l th cluster is assumed to be τ_{kl} , where $k=0,1,2,\dots$. Also by definition, for the first cluster $T_0=0$ and $\tau_{0l} = 0$. Then according to this model, T_l and τ_{kl} are described by the following independent inter-arrival exponential probability density functions;

$$P(T_l | T_{l-1}) = \Lambda e^{\{-\Lambda(T_l - T_{l-1})\}}, \quad l > 0 \quad (2.33)$$

and

$$P(\tau_{kl} | \tau_{(k-1)l}) = \lambda e^{\{-\lambda(\tau_{kl} - \tau_{(k-1)l})\}}, \quad k > 0. \quad (2.34)$$

2.3.1.3 Comparison and Comments on the Two Path Arrival Models

On comparison, the gist of the two models appears to be the same. The Poisson process is modified to accommodate for the variability in mean arrival rate λ . In Saleh's case, λ and Λ can be computed after curve fitting for an environment, but in general, $\lambda \gg \Lambda$. In

Turin's case, λ and K could be either greater or less than one. The Turin model is more general in nature and takes into account clustering as well as homogeneity of the environment. Furthermore, Hashemi [45] successfully applied Turin's model to an extensive set of indoor measurements and fitted a path arrival process to the experimental data. The values of K which are reported are less than one and, therefore, contrary to the observations of Saleh. This suggests that the indoor environment can also be homogenous and Saleh's model will fail in this scenario, rendering Turin's model more suitable. In fact Saleh's model may be generalized by removing the condition that $\lambda \gg \Lambda$ and rendering it applicable to the homogenous environment as well. This, however, would not help differentiate the received paths in two distinct groups. In fact, the main difference between the two models is the way in which variability in the mean arrival rate is modeled. In the first case, it is modeled in a stochastic manner, whereas in the second case, a deterministic model is used.

2.3.1.4 Delay Spread

The path delays, τ_n , have been used to define a number of other useful measures of the channel time dispersion. In this section a summary of them is presented. One of the measures is the inter-arrival time whose sum provides the length of the impulse response or the power delay profile. Another is the so called excess delay defined as relative delays of the multipath with respect to the first path. The *rms* delay spread, τ_{rms} , represented by a number, provides useful information on the impulse response profile. It is defined as the square root of the second central moment of the power delay profile. It is a measure of the profile width and is used in communication system performance analysis. Its importance stems from the fact that the performance limitation due to time dispersion is a function of the width of the power delay profile.

For a multipath channel, if one defines the received power time response, $p(\tau)$, at a given position when an impulse is transmitted, then

$$p(\tau) = \sum_n a_n \cdot \delta_n(\tau - \tau_n). \quad (2.35)$$

where a_n is the coefficient of the power received from the n th path, while τ_n represents the excess delay of this path.

The average excess delay, τ_m , is given by

$$\tau_m = \frac{\int_0^{\infty} \tau p(\tau) d\tau}{\int_0^{\infty} p(\tau) d\tau} = \frac{\sum_n a_n \cdot \tau_n}{\sum_n a_n}, \quad (2.36)$$

and the *rms* delay spread is

$$\tau_{rms} = \left[\frac{\int_0^{\infty} \tau^2 p(\tau) d\tau}{\int_0^{\infty} p(\tau) d\tau} - \tau_m^2 \right]^{\frac{1}{2}} = \left[\frac{\sum_n a_n \cdot \tau_n^2}{\sum_n a_n} - \left(\frac{\sum_n a_n \cdot \tau_n}{\sum_n a_n} \right)^2 \right]^{\frac{1}{2}}. \quad (2.37)$$

It has been observed in [10] that the rms delay spread caused by the “dielectric canyon” varies inversely to the proximity to the microcell base antenna. The worst case rms delay spreads are found to be for omnidirectional monopole antennas; they could be reduced by increasing the directivity of the mobile antenna and thus suppressing the number of paths. A theoretical distribution density of delay spread for mobile radio environment has been obtained in [51].

2.4 Summary

This chapter provided an overview of the issues related to multipath mobile radio propagation channel with emphasis on LOS microcellular channels. In the beginning it

was shown that a narrowband channel may be considered to be a special case of the wideband channel under certain conditions. A theoretical approach in terms of a phasor sum was then derived to clearly demonstrate the propagation mechanism enumerating all the factors which affect the behavior of the received signal. The distance dependent loss as well as fading models were reviewed. The physical model behind the fading phenomenon was also introduced. The statistical characterization of the path arrival process in a wideband channel has been described; it is reported in the literature as a modified Poisson process. The concept of delay spread was reviewed and its salient features discussed.

In the next chapter, the mobile radio channel is revisited with emphasis on the physical basis of the statistical characterization of the channel parameters.

Chapter 3

Physical Basis of Multipath Mobile Wireless Channels

3.1 Introduction

In this chapter, the main emphasis is on the physical basis of the statistical characteristics of multipath wireless channel parameters, as well as modification to the statistical parametric characterization, where needed, to conform to the physical basis of the channel behavior. The focus is also on understanding the nature of slow and fast fading their segregation, as well as, the Nakagami fading model for the narrowband case. The extension of the Rayleigh/lognormal fading model to the Rice/lognormal or the Nakagami/lognormal fading models has also been investigated. Moreover, the Nakagami fading model for time dispersive channels is investigated.

3.2 Narrowband Channel

Narrowband channels are characterized in terms of two statistical parameters, the fading strength and the random phase. In this section, these parameters are studied in detail providing deeper insight into the physical mechanism which causes them. A number of new and interesting observations are made which provide deeper insight into the fading behavior of the signal envelope.

3.2.1 Nature of Slow and Fast Fading

Physical measurements do not allow the segregation of multipath components to permit the observation of their contribution to the fading envelope. On the other hand, the use of computer simulation of the channel provides the needed flexibility to permit this observation. A simulation¹ of the multipath propagation on Slater Street in downtown Ottawa, generated all the multipath components in (2.15) and also their summations, to help observe the nature and the physical basis of the slow and fast fading envelope. Figure 3.2 shows a typical simulated received signal envelope which is expected to be obtained by the physical measurements. Few observations can be made. First of all, both slow and fast envelope variations are evident. Secondly, using the popular approach, if the envelope is suitably divided into small intervals such that the mean value does not vary appreciably over the selected interval, it can be observed that, at distances very close and very far from the transmitter, the received signal has relatively larger variation around its mean than at the middle. The questions being investigated here are: What causes slow and fast variations? And how is the change in variability explained?

Starting with slow fading, the physical mechanism of slow or shadow fading, as mentioned before, is commonly associated with the absence of the LOS path. However, it would be erroneous to attribute the slow fading to the obstruction of the LOS path, as there are no significant obstructions in the LOS channels. It is suggested here that the source of slow fading is the random interference of the dominant multipath, such as the LOS, GR and single wall reflected paths. Figure 3.1 shows the phasor summation of the LOS, GR and once wall reflected paths. A comparison of Figure 3.1 and Figure 3.2 reveals

1. The following values are used for the channel simulation results presented in this thesis: Transmitter height = 8 m, $G_t = G_r = 0$ dB, Receiver height = 3.5 m, $R_G = 0.9$, $R_m = 0.6$, $D_l = 0.1$, Frequency = 910 MHz, Transmitter power = 1 W and the receiver dynamic range is 80 dB. The G_t and G_r are defined in (2.12) and R_G , R_m and D_l are defined in (2.15). The average separation between the row of buildings along the street is about 18 meters. The transmitter is placed approximately in the center of street. About 24 buildings are used on both sides of the street in the simulation of the path loss curves. A distance of 300 meters is traversed for illustration. The closest receiver position is about 10 meters away from the transmitter. For further details on the channel simulation methodology, the reader is referred to [54].

Figure 3.1 The mechanism behind the median path loss. The dB magnitudes versus the distance of the phasor summation of LOS, GR and WR paths (solid). The plane earth model is also plotted (dashed).

Figure 3.2 The dB magnitudes versus the distance of the phasor summation of both terms of (2.16).

that the composite signal amplitude is the result of superimposing fast variations in Figure 3.2 onto the solid curve of Figure 3.1. This confirms the hypothesis that the K paths in (2.16) do constitute the slow variations, while the $L+1$ paths in (2.16) constitute the rapid variation in the propagation loss which supports the above mentioned conclusion. The slow variation due to the K paths in (2.16) is termed the *deceptive shadowing*. The deceptive shadowing would be in addition to the blocked LOS path, if any. In the context of the LOS microcellular channel, a blocked direct¹ path reduces the value of K in (2.16) and, as a result, gives a modified slow fading interference pattern.

Another interesting observation is that the fast variations are mainly due to the edge diffracted paths and can be modeled as a complex Gaussian process as discussed below. The first term on the right hand side of (2.18) is the sum of diffracted component phasors:

$$g_d \cdot e^{j\theta_d} = \sum_{l=1}^L \frac{D_l e^{j\Delta\phi_l}}{r_l} = x_d + jy_d. \quad (3.1)$$

In (2.15), L is a large number and $L \gg M$. Curve-fitting on the simulated data for x_d and y_d showed that those are normally distributed with nearly zero mean and some variance, σ_d^2 , as shown in Figure 3.3 and Figure 3.4 respectively. The g_d is therefore Rayleigh distributed as shown in Figure 3.5, and the phase is approximately uniformly distributed as shown in Figure 3.6. The last term on the right hand side of (2.18) is the additive Gaussian noise term with zero mean and variance σ_n^2 whose envelope z_n is Rayleigh distributed and the phase θ_n is uniformly distributed. The sum of two Rayleigh phasors is known to yield another Rayleigh phasor [66]. In this case g_{ff} is Rayleigh distributed with $\sigma_{ff}^2 = 2(\sigma_d^2 + \sigma_n^2)$ and θ_{ff} is the uniformly distributed phase.

1. This may happen due to moving traffic on the street. Due to low antenna heights, a transit bus is fully capable of blocking the LOS path.

Figure 3.3 Normal distribution fit for the real part of the phasor sum of the diffracted paths.

Figure 3.4 Normal distribution fit for the imaginary part of the phasor sum of the diffracted paths.

Figure 3.5 Rayleigh distribution fit for the envelope or magnitude of the phasor sum of the diffracted paths.

Figure 3.6 The distribution of phase. The phase is nearly uniformly distributed over -3.141 to 3.141 radians. When this condition is met this is enough evidence of that phasor sum to be a zero mean complex Gaussian process.

At a given physical location, although the magnitude of the dominant path g_{sf} remains constant, if the location is changed then the g_{sf} varies with the inverse distance dependence of the free space propagation law as does the phasor summation of the comparably strong multipath. As a result, this becomes the *effective direct path* plus the noise case. Because the effective direct path changes with distance depending upon the first summation term on the right hand side of (2.19), it is expected that the K parameter value of the Rice distribution, defined as the ratio of the specular power to the scattered power, should also change from one small distance segment to another one. This varying K factor is termed the *effective K factor* of the Rice distribution. This distance dependent varying K factor behavior has been observed in the curve-fitting of measured fading data [8] as shown in Figure 3.7.

3.2.1.1 Distribution of Slow Fading

The statistical characterization of slow fading is commonly performed in terms of lognormal distribution. The standard deviation of slow fading is a measure of the variation in slow fading. The values of 3-7 dB for the standard deviation are reported for conventional cellular propagation by Okumura [46], while for the LOS microcellular channel, values of up to 8 dB are reported [9].

The fundamental question which needs to be asked is: What is the physical basis to characterize the slow fading by lognormal distribution besides the historical reasons? Suzuki proposed a physical justification of lognormal fading in terms of multiple reflected and/or diffracted paths in [41] as follows¹:

The main wave arrives at a cluster of buildings. Before undergoing a local “scattering” by the clusters of buildings, it has travelled a path which is subject to the influence of multipath reflections and/or diffractions by natural and man-made objects. Thus, the path strength, on arrival at the local cluster, will be a random variable...

1. Repeated in part from Chapter 2 for ready reference.

Figure 3.7 The Figure 11 of reference [8]. Measured slow fading (above) vs. distance to basestation, fast fading (centre) and corresponding Rician K factor (below).

As a special case, we can imagine the transmitter-to-cluster path to have a lognormal strength since it undergoes a multiplication effect, while the local-path distribution is Rayleigh because it is due to an additive scattering effect.(emphasis added)

Above is the justification of (2.16) in the light of Suzuki's interpretations based on a conventional cellular environment, while (2.16) suggests a different viewpoint.

This interpretation can *not* be justified in the light of equation (2.16). It is clear that each multiple reflected or diffracted path adds to the other paths at the receiver. The model is not multiplicative, but the sum of multiplied gains. The consequence of multiple reflection or diffraction is manifested in the value of the reflection or diffraction coefficient of the respective summation term. Furthermore, a multiple reflected path is weaker than the one-time reflected path, and each added reflection weakens the impact of that path on the envelope. For example, if the value of the reflection coefficient is assumed to be 0.5, then the one-time reflected path is at least 3 dB below the LOS path, the twice reflected path is 6dB, and each added reflection adds another 3 dB to the pure reflection loss of that path. Multiple reflections also result in the increased length of these paths, and they suffer more distance dependent loss in addition to the reflection loss. These weak multiple reflected paths ultimately go below the receiver noise threshold and their effect is not differentiable from the receiver noise.

A good source for both the history and a detailed description of lognormal distribution is [79]. Aitchison and Brown [79] have stated their belief that lognormal is as fundamental a distribution in statistics as is the normal, despite the stigma of the derivative nature of its name¹. Suzuki's proposed physical interpretation stems from his conjecture that lognormal distribution arises from a theory of elementary errors combined by a multiplicative process, just as the normal distribution arises from a theory of elementary errors combined by addition.

1. One interesting statement in [79] is: "Had man (human to be politically correct) been more adept at multiplication the 'exponential-lognormal', or normal, might have been the derivative distribution".

It appears to be a common belief that the success with which any particular frequency curve graduates empirical data is a sufficient criterion of its worth. There are, however, at least two reasons for seeking a more fundamental basis in the theory of probability for any *pdf* to which a more than transient importance is attached. Firstly, to provide a basis to obtain a clearer insight into underlying natural processes and, secondly, an understanding of the elementary assumptions from which the law of frequency distribution is derived. Although lognormal distribution is widely reported as a distribution of choice for slow fading, based on the previous discussion, it does not appear to provide a physical basis nor do its elementary assumptions coincide with the fundamental understanding of multipath propagation in terms of the phasor sum. More interestingly, while for fast fading, numerous distributions have been tested to curve-fit the fading data, there appears to be no comparative testing of slow fading data on distributions other than the lognormal distribution. A reasonable approach in light of the above is to test other distributions as well, in addition to the lognormal distribution. The use of the Nakagami distribution to curve-fit the slow fading data is based on the general mathematical model in terms of the multipath phasor sum which is applicable for both the fast and slow fading scenarios (i.e. small area and large area fading). Nakagami [36] has shown that the probability distribution of the resultant vector in (2.16) follows the *m*-distribution which is currently known as the Nakagami distribution and is given below:

$$f(g_{sf}) = \frac{2}{\Gamma(m)} \frac{m^m}{\Omega^m} \cdot g_{sf}^{2m-1} e^{-\left(\frac{m}{\Omega}\right)g_{sf}^2}, \quad (3.2)$$

where

$$\Omega = E[g_{sf}^2], \quad (3.3)$$

and

$$m = \frac{E[g_{sf}^2]^2}{E[(g_{sf}^2 - E[g_{sf}^2])^2]} = \frac{\Omega^2}{E[(g_{sf}^2 - \Omega)^2]}. \quad (3.4)$$

The Nakagami distribution is an approximate solution of the problem of finding the distribution of the magnitude of a sum of random vectors. Therefore, the Nakagami distribution should provide a reasonable model for the large area slow fading data. In the literature, it is also suggested that the Nakagami distribution also approximates the lognormal distribution under certain conditions [36] [41].

To this end, a comparative non-linear least squares curve-fitting for the lognormal and Nakagami distributions on the empirical distribution of the simulated slow fading data is performed. The Rayleigh and Rice distributions are not included in this comparison as these can be modeled by the Nakagami distribution quite well [80]. Other distributions are not considered due to the absence of any suitable physical basis. Goodness of the fit of the model distribution to the empirical distribution is based on the minimization of a merit function¹ (minimum squared error sum). The minimization process proceeds iteratively starting with a given trial value of the model distribution parameters. The procedure repeatedly modifies the value of the parameters until merit function stops decreasing. The best fit model distribution parameters are those which give the minimum value of the merit function.

The result of comparative curve-fitting is shown in Figure 3.8. The Nakagami distribution fits better than the lognormal based on the least squared error sum performance metric. An additional four runs on different simulated vehicle paths verify this observation (Table 3.1). However, in one case, the lognormal model fit was found to give a better fit than the Nakagami model. Based on the above observations, it can be concluded that the Nakagami distribution can be confidently used to model the large area slow fading.

1. For further details, the reader is referred to [84].

Figure 3.8 Comparison of the Nakagami and lognormal distribution fit on the empirical distribution based on a 300 meter 3000 sample simulation run.

TABLE 3.1 Comparison of the lognormal and the Nakagami model for slow fading based on the minimum squared error sum (MSES) criterion.

Case #	MSES-lognormal	MSES-Nakagami
1	0.0136	0.2101
2	0.0461	0.0139
3	0.0375	0.0057
4	0.1033	0.0536
5	0.0484	0.0245

3.2.1.2 Segregation of Slow and Fast Fading

In the following we test the hypothesis that slow and fast fading can be segregated using the multiplicative model of slow and fast fading using a moving mean normalization procedure in those channels which exhibit Ricean or Nakagami fading for $K>0$ and $m>1$ respectively¹.

The issue of the segregation of slow and fast fading using a multiplicative model is discussed in detail in several popular books [67] [68], and by Clarke [34]. The following must be asked: What is the need for and the benefit of segregating the slow and fast fading? Does it simplify or complicate the radio link performance analysis? Is it physically justifiable? The apparent reason for segregating the two types of fading is to associate physical interpretations to their occurrences. The segregation of slow and fast fading complicates the system performance methodologies without, perhaps, yielding any greater accuracies. The first issue is the criterion for segregation itself. As the notions of slow and fast are illusionary and vary from person to person, there remains unanswered a fundamental question on the criterion for the segregation of slow and fast fading, i.e., the choice of time/distance-segment/small area over which the mean may be assumed constant. The literature is rich from a variety of choices, such as 30λ [23], 40λ [68] and 90λ [26], all of which are deemed as suitable engineering judgements². The choice of the normalization interval plays a significant role in what the model distribution parameter values are and in some cases the type of distribution itself, as shown later, and should be given more than arbitrary consideration. More importantly, however, the consideration of the normalization interval is only significant if it is known that slow and fast fading can indeed be usefully segregated. If it is not the case then this issue is simply irrelevant. We show in the follow-

1. $K>0$ and $m>1$ corresponds to the non Rayleigh case.

2. This corresponds to a distance of about 10-30 meters at a frequency of 900 MHz.

ing that the slow and fast fading cannot be segregated in the Rice and the Nakagami fading cases for $K > 0$ and $m > 1$ respectively.

To illustrate the effect of the choice of the normalization interval for the segregation of slow and fast fading, a simulated path loss curve over a distance of 100 meters is shown in Figure 3.9. It is a summation of the LOS and the ground reflected paths, and the additive zero mean Gaussian noise with a noise power of - 80 dB. This curve is a good representation of the LOS microcellular propagation scenario with suitable antenna directivity. A similar experimental curve is reported by Rustako et al (Ref. [10] and Figure 3(b) therein). The same envelope is divided into 2, 10 and 20 meter intervals and curve-fitted for the Rice distribution. The associated K parameter values of the best fit Rice distribution in the minimum square error sum sense are shown in Figure 3.10. In some cases, where the Rice distribution did not fit at all, and thus parameter values were not available, the unavailability of parameters is assigned zero value for both K and Ω . The K and Ω equal to zero scenarios mostly arose for the 2 meter interval case. This confirms the earlier observation that the choice of interval affects the type of distribution which fits the data. It can also be observed that there is not a single representative K value for the entire interval which fluctuates over a large range from one to the other interval. As expected, the variability in K is less for larger intervals, i.e., the variability of K is larger for the 2 meter intervals than for the 10 meter intervals. One significant observation pertains to the 2 meter interval case, where $K=0$ quite exactly coincides with the notches in Figure 3.9. In fact, if even smaller distance segments are used, this accuracy increases. Another important observation pertains to relatively smaller K values for distances as close as 10 to 25 meters from the antenna.

Figure 3.9 Path loss due to phasor sum of the LOS, GR and AWGN.

Figure 3.10 Comparison of the K parameter values with different averaging intervals. The zero values represent the unavailability of the K value for that interval. The solid curve is for the 2 meter averaging interval, the dashed is for the 10 meter and the dotted is for 20 meter.

To explain these observations we have plotted the Rice parameter s and σ^2 in Figure 3.11. These are actual values obtained from the 2 meter interval curve-fitting procedure. As K parameter values are derived from s and σ through the relation $K = s^2/2\sigma^2$, Figure 3.11 will help illustrate the behavior of K near the transmitter². Note that variations in s are more or less proportional to the LOS plus the ground reflected paths or the plane earth propagation model. It also decreases with distance on the average. Also, note that Figure 3.9 was produced by assuming noise power equal to -80 dB (or $\sigma_n = 1 \times 10^{-4}$), and that for distances greater than 50 meters σ is nearly equal to σ_n (1×10^{-4}), however, close to the transmitter, the value of σ is higher but it decreases with distance to $\sigma_n = 1 \times 10^{-4}$ at the coverage boundary. Thus, it can be said that the main cause of variability and the shape of K is the changing mean power, s^2 , however, increased σ values close to the transmitter reduces the value of K for intervals close to the transmitter for large interval sizes. The source of increased variability close to the transmitter is deep notches produced as a result of interference of the LOS and ground reflected paths. These notches are too close to each other to be separated by the choice of interval size resulting in increased value of σ .

The discussion in the previous paragraph elucidated the mechanism of segregation of slow and fast fading, and its effects on distribution parameter values. Now the benefit of this approach for the Rician LOS microcellular channel is investigated. The benefit of the moving average normalization by Clarke [34] helped to improve the Rayleigh fit on the fast fading data. Is this also the case for the Rician fading LOS microcellular channel? Figure 3.12 shows the normalized path loss curve when normalized to the inverse distance squared free space loss to remove the downward distance dependency. Figure 3.13 shows the normalized path loss when normalized to the 2 meter interval mean. The resulting K

1. Note that σ^2 is the variance of a zero mean Gaussian process and is related to noise power when noise is modeled as Gaussian.

2. The Rice distribution is based on the underlying complex non zero mean Gaussian process. Its parameter, s and σ are related with the mean and variance of that process. For this relation, see [71].

Figure 3.11 Rice parameters, s (solid) and σ (dashed) as a function of distance for the 2 meter averaging interval case. The zero values represent unavailability of s and σ values for that interval.

Figure 3.12 Normalized path loss curve after the normalization of Figure 3.9 to distance dependent loss.

Figure 3.13 Normalization of the path loss curve in Figure 3.12 based on 2 meter averaging intervals.

parameter values are plotted in Figure 3.14 along with the corresponding 2 meter moving average curve from Figure 3.10 (solid curve in Figure 3.10 and Figure 3.14). It is interesting to note that the two curves are nearly the same and retain most of the same features. The corresponding s and σ values for the normalized envelope case are plotted in Figure 3.15¹. Comparison of s and σ values for the normalized and the un-normalized interval cases shows that s and σ values for the normalized case have been modified. The s and σ have, however, been modified in such a manner that $K = s^2/(2\sigma^2)$ is the same in both cases. Further explanation of this point is as follows. Knowing that each segment of the received envelope is obtained by a phasor summation of the type $s_i e^{j\theta_0} + n_i e^{j\theta_n}$ where s_i is the mean value of the i th interval with phase θ_0 and n_i is the magnitude of the zero mean complex Gaussian noise phasor with uniformly distributed phase θ_n and standard deviation σ_n . If θ_0 is assumed as the known reference phase of the specular path, then $s_i e^{j\theta_0} + n_i e^{j\theta_n}$ may be written as $s_i + n_i e^{j\theta_n}$. The $s_i + n_i e^{j\theta_n}$ forms a non-zero mean complex Gaussian process with mean s_i and standard deviation σ_n . When $s_i + n_i e^{j\theta_n}$ is normalized to the mean s_i then the resulting envelope is the magnitude of $1 + (1/s_i)n_i e^{j\theta_n}$. It can now be noted that the mean of this complex process is one, however, the standard deviation has been multiplied by a factor of $1/s_i$. This is what has approximately happened in Figure 3.15. The large variation in the s_i values has changed to approximately one. On the other hand, σ has been modified to reflect the multiplication by the ratio $1/s_i$. Clearly the ratios $K_i = s_i^2/(2\sigma_i^2)$ and $K_i = 1/(2(1/s_i^2)\sigma_i^2)$ are the same, and that is why the K parameter obtained for the normalized and un-normalized cases are the same. The main reason for this result is evidently the fact that the underlying process of the slow variation is an additive rather than a multiplicative process as is commonly assumed. Moreover, no

1. Again, s and σ equal to zero values in the curve shows the inability of the Rice distribution to represent the corresponding interval.

Figure 3.14 Comparison of the K parameter values obtained for the normalized (solid) and non-normalized (dashed) cases. The zero values represent unavailability of K value for that interval.

Figure 3.15 s parameter (solid) and σ parameter (dashed) of the Rice distribution after curvefitting in the normalized envelope case. The zero values represent unavailability of s and σ values for that interval.

benefit is obtained from normalization to improve the statistical fit as was observed in [34].

The conclusion to be drawn from the above discussion is that, *channels demonstrating fast Ricean fading with $K > 0$ do not permit the segregation of slow and fast fading using a moving mean normalization procedure which is redundant*. This conclusion can be extended to include the Nakagami fading channels with $m > 1$ due to the relation which exists between the m and K parameters. However, as the Rice parameters do vary from one arbitrarily selected distance segment to another, if it has been decided to adopt a segregation procedure for fading characterization, it is useful to statistically characterize them for use in the performance analyses to account for variability in the parameters.

In the next section, the distributions of the fading envelope are found when fading is optionally segregated in slow and fast fading for Rician channels.

3.2.1.3 Distribution of Fading Envelope

It was mentioned in Section 2.2.3.1 that the underlying random process to account for the random signal variability as suggested by Clarke [34], Cox [25], Suzuki [39], and Parsons and Bajwa [37], is a non-stationary zero-mean complex Gaussian process with variance σ^2 . The statistical distribution of the received signal envelope, g_f , under the Rayleigh fading and lognormally shadowing assumption is given as follows:

$$f(g_f) = \int_{-\infty}^{\infty} \frac{g_f}{\sigma^2} e^{\frac{-(g_f^2)}{2\sigma^2}} \cdot \frac{1}{\sigma\alpha\sqrt{2\pi}} e^{\frac{-(\ln(\sigma) - \mu)^2}{2\alpha^2}} \cdot d\sigma \quad 0 \leq g_f < \infty \quad (3.5)$$

Where μ is the mean of $\ln(\sigma)$ and α is the standard deviation of $\ln(\sigma)$. The above distribution was reported by Suzuki [39], and is the so-called mixture distribution.

For the LOS microcellular channel, the Rice/lognormal channel has been suggested [8]. This assumes a non-stationary, non-zero mean complex Gaussian process with parameters s and σ . To begin with one would tend to replace the $f(g_f|\sigma)$ in (2.30) with

$$f(g_f|\sigma) = \frac{g_f}{\sigma^2} e^{\frac{-(g_f^2 + s^2)}{2\sigma^2}} I_0\left(\frac{g_f s}{\sigma^2}\right), \quad g_f \geq 0 \quad (3.6)$$

However, this is not appropriately based on the physical understanding developed in Section 3.2.1.1 which assures that σ remains nearly constant but s significantly changes over reasonably small distance segments. In this case g_f should be conditioned on s instead of σ . The parameter s is also a random variable¹ and if it is assumed to be lognormally distributed, the statistical distribution of g_f shall then be found as follows:

$$f(g_f) = \int_{-\infty}^{\infty} \frac{g_f}{\sigma^2} e^{\frac{-(g_f^2 + s^2)}{2\sigma^2}} I_0\left(\frac{g_f s}{\sigma^2}\right) \cdot \frac{1}{s\alpha\sqrt{2\pi}} e^{\frac{-(\ln(s) - \mu)^2}{2\alpha^2}} \cdot ds \quad 0 \leq g_f < \infty. \quad (3.7)$$

where μ is the mean of $\ln(\sigma)$ and α is the standard deviation of $\ln(\sigma)$. The distribution in (3.7) is in a complicated form and apparently has no simpler closed form. Its approximation for the large and small values of g_f may be found to be as follows [66]:

$$g_f \approx \begin{cases} \frac{1}{g_f \alpha \sqrt{2\pi}} e^{\frac{-(\ln(g_f) - \mu)^2}{2\alpha^2}} & g_f \gg \sigma\sqrt{2} \\ \frac{g_f}{\sigma^2} e^{\frac{-g_f^2}{2\sigma^2}}, & g_f \ll \sigma\sqrt{2} \end{cases} \quad (3.8)$$

i.e., g_f is lognormal for values much larger than $\sigma\sqrt{2}$ and Rayleigh for values much smaller than $\sigma\sqrt{2}$.

1. The s here corresponds to g_{sf} .

The Rice distribution may also be expressed in an equivalent form in terms of the effective Rice K factor. It is desirable to rewrite (3.7) in terms of the effective K factor because K is a compact single indicator of variations in both s and σ and has been used in literature more frequently [52]. In this case, two parameters, K and Ω , are used to define the Rice distribution both of which depend on the s and σ . As σ may be assumed to be the same everywhere in the cell, therefore, both K and Ω will be random due to s only. Using the appropriate transformation relationship, the *pdf*'s of the K and Ω may be found to be as follows:

$$f(\Omega) = \frac{1}{(\Omega - c)\alpha\sqrt{2\pi}} e^{-\frac{\left(\frac{1}{2}\ln(\Omega - c) - \mu\right)^2}{2\alpha^2}}, \quad (3.9)$$

$$f(K) = \frac{1}{2Kc\alpha\sqrt{2\pi}} e^{-\frac{\left(\frac{1}{2}\ln(Kc) - \mu\right)^2}{2\alpha^2}}, \quad (3.10)$$

where c is equal to $2\sigma^2$.

In this case, the distribution in (3.6) is conditioned on the K and Ω . The *pdf* of g_r may be found from the three dimensional joint distribution of g_r , K and Ω as follows:

$$f(g_f) = \int_0^\infty \int_0^\infty f(g_f, K, \Omega) dK d\Omega = \int_0^\infty \int_0^\infty f(g_f|K, \Omega) f(K|\Omega) f(\Omega) dK d\Omega. \quad (3.11)$$

If

$$f_R(g_f|(K, \Omega)) = \frac{2g_f(1+K)}{\Omega} e^{-K - \frac{(1+K)g_f^2}{\Omega}} I_0(2g_f\sqrt{K(1+K)/\Omega}). \quad \begin{matrix} g_f \geq 0 \\ K \geq 0 \\ \Omega \geq 0 \end{matrix} \quad (3.12)$$

Then

$$\begin{aligned}
 f(g_f) = & \int_0^\infty \int_0^\infty \frac{2g_f(1+K)}{\Omega} e^{\left(-K - \frac{(1+K)g_f^2}{\Omega}\right)} I_0(2g_f\sqrt{K(1+K)/\Omega}) \cdot \\
 & \frac{1}{2Kc\alpha\sqrt{2\pi}} e^{\frac{-\left(\frac{1}{2}\ln(Kc) - \mu\right)^2}{2\alpha^2}} \cdot \\
 & \frac{1}{(\Omega - c)\alpha\sqrt{2\pi}} e^{\frac{-\left(\frac{1}{2}\ln(\Omega - c) - \mu\right)^2}{2\alpha^2}} \cdot dK d\Omega \cdot \quad 0 \leq g_f < \infty
 \end{aligned} \tag{3.13}$$

The (3.13) apparently does not have a closed form and will need to be computed numerically. If the slow fading is assumed to be the Nakagami as shown in Section 3.2.1.1, the distribution of the fading envelope would still be nearly as complicated as (3.13) and would depend on both m and Ω .

3.2.1.4 Distribution of Phase

It was mentioned in Section 2.2.5 that the phase of the complex Rician process is given by [66]

$$f(\theta_f) = \frac{1}{2\pi} \cdot e^{-K} \cdot \left[1 + \sqrt{\pi K} \cdot \cos\theta_f \cdot e^{K(\cos\theta_f)^2} \right] \cdot [1 + \operatorname{erf}(\sqrt{K} \cdot \cos\theta_f)]. \tag{3.14}$$

For the Rice/lognormal fading model, K is a random variable with the *pdf* given in (3.10), therefore, (3.14) is modified as follows:

$$\begin{aligned}
 f(\theta_f) = & \int_0^\infty \frac{1}{2\pi} e^{-K} \left[1 + \sqrt{\pi K} \cdot \cos\theta_f \cdot e^{K(\cos\theta_f)^2} \right] [1 + \operatorname{erf}(\sqrt{K} \cdot \cos\theta_f)] \\
 & \frac{1}{2Kc\alpha\sqrt{2\pi}} e^{\frac{-\left(\frac{1}{2}\ln(Kc) - \mu\right)^2}{2\alpha^2}} \cdot dK \cdot
 \end{aligned} \tag{3.15}$$

The link performance analysis disregards the phase information. The knowledge of the phase distribution should be more useful in the analysis and design of phase synchronization techniques.

3.2.2 Radio Link Performance Methodology

The understanding and characterization of received signal variability is typically used in coverage predictions and in the design of receivers. One measure of coverage prediction is the outage probability which is determined analytically by using the statistical characterization of fading. Two types of outages are commonly encountered: outage due to desired signal fading below a predetermined level, and outage due to signal-to-cochannel interference falling below a desired level. The physical understanding of outages is useful in system design. An understanding of the relation between fading and the propagation environment helps in understanding the environment in which a receiver operates.

A simpler measure of coverage prediction is outage due to signal fading. Lee [68] and Jakes [67] have used a lognormal distribution to model the variability in the mean or median signal strength. The outage is defined as the probability that the received signal strength is below a certain threshold value. Assuming r is the logarithmic signal strength, with the distance dependent mean signal strength μ and the standard deviation α , the outage expression is given by¹

1. If $g_f = e^r$ then (3.16) may be written using a linear scale as follows, where μ and α are the mean and the standard deviation in a linear scale:

$$P_{out} = Pr(g_f < g_0) = \int_0^{g_0} \frac{1}{g_f \alpha \sqrt{2\pi}} e^{-\frac{(\ln(g_f) - \mu)^2}{2\alpha^2}} dg_f.$$

$$P_{out} = Pr(r < r_0) = \int_{-\infty}^{r_0} \frac{e^{-\frac{(r-\mu)^2}{2\alpha^2}}}{\sqrt{2\pi}\alpha} dr. \quad (3.16)$$

where μ is the distance dependent mean signal strength. In conventional cellular scenarios, one of many empirical formulas are used to determine this parameter. If the received signal envelope is normalized, then μ is equal to zero dB. The value of α , which is environment dependent, is obtained through field trial. As mentioned before, values of 3-7 dB for α are reported for conventional cellular propagation by Okumura [46], while for the LOS microcellular channel, values of up to 8 dB are reported [9].

It was shown in this thesis that Nakagami distribution is also a good fit to the slow fading data, therefore, outage expression in this case becomes,

$$P_{out} = Pr(g_f < g_0) = \int_0^{g_0} \frac{2}{\Gamma(m)} \frac{m^m}{\Omega^m} \cdot g_f^{2m-1} e^{-\left(\frac{m}{\Omega}\right)g_f^2} dg_f, \quad (3.17)$$

where g_f is the received signal strength in a linear scale¹. An equivalent expression for the logarithmic signal strengths may be written using the appropriate transformation of the Nakagami distribution².

There is no clear preferred choice between the Nakagami and lognormal distributions based on analytical complexity for the case of outage due to signal fading, as the integrals in (3.16) and (3.17) can be easily computed. The preference should, however, be made based on the accuracy with which either of the two above mentioned model distributions predict the outage due to signal fading, when compared to the outage calculated from the empirical simulated data. The definition of outage due to signal fading (the probability of the lower tail) also suggests that the accuracy with which lower tails of the model distribution fits the empirical distribution will yield an outage probability closer to the true

1. $g_f = 10^{r/10}$ or $g_f = 10 \log(r_f)$.

2. See Nakagami [36].

outage. Figure 3.8 suggests that the Nakagami is better than the lognormal fit on the lower tail. In all the cases listed in Table 3.1, this has also been the case. This suggests that the Nakagami outage model should be close to the true outages when compared to the lognormal model. As explained below, this has been verified to be the case.

Equations (3.16) and (3.17) can be used to calculate and compare the coverage outage predicted by the use of the lognormal or Nakagami model distributions to the true outage. To do so, it is necessary that a reasonable common basis be devised for a fair comparison of the use of the two model distributions in the analytical model of the coverage outage prediction. To this end, a number of path loss curves were simulated and then normalized to the free space loss to remove the distance dependent loss. Curve-fitting was performed for both model distributions on the same data to obtain the distribution parameter values which are summarized in Table 3.2. Associated coverage outage predictions are also shown for -5 dB and -10 dB thresholds. In addition, true outage predictions are also recorded. In all cases the lognormal model provides an underestimation of the true outage probabilities. On the other hand, outage predictions based on the Nakagami model are very close to the true outages. It can also be observed that in most cases, the Nakagami model gives an overestimation of the true outage probability. From a system deployment perspective, the model which gives an overestimate would be more desirable than the one which underestimates, as the former model gives the worst case scenario.

3.2.3 Physical Basis of Nakagami Fading

In Section 2.2.3, the physical interpretation of the Rice/Rayleigh fading phenomenon in a mobile radio environment was described. In this section, a physical basis of the Nakagami fading phenomenon in multipath radio reception is provided. It is shown here that the physical model used for the Rice distribution can also be applied for the Nakagami distribution with an appropriate perspective.

TABLE 3.2

Comparison of the coverage outage predictions of the lognormal and Nakagami models with the true outages of the data. $P_{out,-5}$ and $P_{out,-10}$ denote the outage probability for -5dB and -10dB outage thresholds, respectively.

Case #	Lognormal Model				Nakagami Model				True Outage	
	μ	α	$P_{out,-5}$	$P_{out,-10}$	m	Ω	$P_{out,-5}$	$P_{out,-10}$	$P_{out,-5}$	$P_{out,-10}$
1	0.3356	0.5528	4.96%	0.36%	1.1203	2.4740	9.95%	2.89%	8%	2.07%
2	0.5206	0.5618	2.55%	0.15%	1.1755	3.5552	6.12%	1.64%	6.6%	1.5%
3	0.2576	0.562	6.91%	0.61%	1.1172	2.1216	11.7%	3.44%	11.6%	3.53%
4	0.3427	0.6184	6.88%	0.78%	1.0091	2.5805	11.3%	3.7%	12.2%	3.83%
5	0.3252	0.769	12.1%	2.74%	0.7932	2.8446	15.1%	6.21%	14.6%	5.57%

The theoretical model of the Rice distribution was developed by Rice [64]. The starting point is the equation (3.10-4) in Rice's voluminous work, *Mathematical Analysis of Random Noise* [64]¹. The goal is to determine the distribution of the envelope (or magnitude) of

$$I(t) = P \cos(pt) + I_N, \quad (3.18)$$

which is,

$$|I| = \sqrt{(P + I_c)^2 + I_s^2}. \quad (3.19)$$

Where I_c is the component of I_N in phase with $\cos pt$, and I_s is the component of I_N in phase with $\sin pt$. The (3.19) is the magnitude of the following phasor sum based on (3.18):

$$I = P + I_c + jI_s. \quad (3.20)$$

It was shown that the distribution of the magnitude of the complex sum of a sinusoidal wave plus additive Gaussian noise, as in (3.19), is the now well known Rician distribution. The (3.19) is also used by [50] and [66] to consider the distribution of a constant plus Rayleigh distributed phasor which is shown to be the Rice distribution.

To investigate the equivalency of the Rice and Nakagami distributions, the starting point is (2.19) which is reproduced below:

$$g_r e^{j\theta_r} = g_{sf} e^{j\theta_{sf}} + g_{ff} e^{j\theta_{ff}}. \quad (3.21)$$

If it can be assumed that $g_{sf} = P$, $\theta_{sf} = 0$, $I_c = g_{ff} \cos \theta_{ff}$, $I_s = g_{ff} \sin \theta_{ff}$ and both I_c and I_s are normally distributed with a zero mean and equal variance, then under this condition, the statistical distribution of g_r must be the Rice distribution. From the conditional equivalency of (3.20) and (3.21), it can be concluded that the physical model of

1. See page 100 of [64] for details.

the Rice distribution, i.e., a direct or specular path plus scattered paths, should also be applicable to the Nakagami distribution. Moreover, the parameters of the two distributions should be equatable under this special condition.

3.2.3.1 Parameter Equivalency of the Rice and Nakagami Distributions.

Let the average received power be defined as

$$\Omega = E\{g_f^2\}. \quad (3.22)$$

whereas the total received power can be identified to be comprised of the sum of two power components, namely, the effective direct or specular power component, P_s , and the scattered power component, P_r , whose ratio K , now termed as the *effective Rice K factor*, is defined as follows:

$$K = \frac{\text{power in specular component}}{\text{power in random component}} = \frac{P_s}{P_r}, \quad (3.23)$$

where, $P_s = g_{sf}^2$ and $P_r = E[(g_f^2 - g_{sf}^2)^2]$ while $g_{sf} = E[g_f]$.

The Rice distribution can now be written in terms of K parameters as follows. [Appendix 3A, (3A.6)],

$$f_R(g_f) = \frac{2g_f(1+K)}{\Omega} e^{\left(-K - \frac{(1+K)g_f^2}{\Omega}\right)} I_0(2g_f\sqrt{K(1+K)/\Omega}) \quad \begin{matrix} g_f \geq 0 \\ K \geq 0 \end{matrix}. \quad (3.24)$$

The Nakagami distribution can be written in terms of the K parameter as well if for convenience it is assumed that [36], [Appendix 3A, (3A.13)]

$$m = 1/\left[1 - \frac{K^2}{(1+K)^2}\right]. \quad (3.25)$$

giving,

$$f_N(g_f) = \frac{2}{\Gamma(m)} \cdot \frac{m^m}{\Omega^m} \cdot g_f^{2m-1} e^{-\left(\frac{m}{\Omega}\right)g_f^2}, \quad \begin{matrix} g_f \geq 0 \\ 0.5 \leq m < \infty \end{matrix} \quad (3.26)$$

If we substitute the value of K from (3.23) in terms of P_r and P_t in (3.25), it can be shown that [36]

$$m = \frac{1}{\text{var}_N(g_f^2)} \quad (3.27)$$

The parameter m is defined as the inverse of the normalized variance of the received signal power, $\text{var}_N(g_f^2)$, instead of being defined as the inverse of the normalized variance of the instantaneous received signal (a modified definition of the K parameter). It is therefore possible to say that m is a parameter which is similar in physical interpretation to that of K . That is the amount of specular power in the received signal with respect to random power. The relationship between m and K is not linear although they vary with respect to each other in a direct rather than an inverse manner. The actual relationship is given by (3.25).

To show that the physical interpretation of the Rice K parameter also applies to the Nakagami m parameter in the following, it is shown that a reasonably approximate linear relation exists between m and K . If m is plotted as a function of K through the relationship given in (3.25), the Figure 3.16 is obtained. This relationship can be broken down into two domains of K values. To identify these domains, the rate of change of m with respect to K versus the K has been plotted in Figure 3.17. It can be observed that the rate of change of m with respect to K varies over the K value interval (0,2) significantly, but in the interval (2, ∞) this rate of change is nearly constant. This observation is the basis for suggesting that in the interval (2, ∞), a linear approximation for the relation between m and K can be used instead of (3.25). This relation may be written as

$$m = s_m K + m_0 \quad (3.28)$$

Figure 3.16 Variation of m with respect to K .

Figure 3.17 The rate of change of m with respect to K .

where s_m is the slope and m_0 is the m -axis intercept. The s_m tend to the value of 0.5 as K increases (Figure 3.17). The value of $s_m = 0.4998528$ and $m_0 = 0.7622159$ was obtained after fitting the (3.28) on (3.25) for K in the interval $\{2-100\}$.

In the K value interval $\{0-2\}$, either (3.25) can be used, or its linear approximation may also be obtained in the above fashion. In one such curve-fitting we obtained $s_m = 0.422312$, and $m_0 = 0.9283$. In summary, the approximations may be written as

$$m = \begin{cases} 0.4221K + 0.9283 & 0 \leq K < 2 \\ 0.4998K + 0.7622 & 2 \leq K < \infty \end{cases} \quad (3.29)$$

To verify the approximation in (3.29), a set of ten simulated small area fading data is chosen to curve-fit both the Nakagami and the Rice distributions and to extract the m and K parameter values. In Figure 3.18, the solid lines correspond to the empirical m parameter value, and the approximation of the m parameter through the K parameter using (3.29) is shown dashed. It can be observed that the approximation is very good.

This relation between m and K as studied in Figure 3.18 also confirms that the measurement results obtained for the Rice channels can be confidently transformed to the equivalent Nakagami channels using (3.25). This transformation is quite accurate in the K value interval $(2, \infty)$. The use of linear approximation is not recommended in the transformation of the K parameter values to the equivalent m parameter values, as there is nothing to gain in terms of simplicity. The motivation behind the development of this approximation was to demonstrate the applicability of the use of physical interpretation of the K parameter to the m parameter. This is, apparently, the first time that the validity of the relation between m and K has been tested based on simulation data.

Figure 3.18 Validation of approximation in (3.29): The solid line corresponds to the m parameter values obtained after curve-fitting. The dashed lines are m values calculated from the K parameter values obtained after curve-fitting on the same data. The abscissa is the indexing of the simulation data set.

3.2.3.2 Complex Nakagami Fading Envelope

The Nakagami distribution is based on a random phasor sum. The resultant phasor which is the envelope is, of course, complex and thus has a magnitude and a phase. In the case of the complex Rician envelope, the magnitude of the envelope is Ricean distributed and there is an associated phase distribution. Along a similar line of thought, for a complex Nakagami envelope process, the magnitude of the envelope is approximately Nakagami distributed, as shown by Nakagami, but the associated phase distribution has not been discussed in the literature. It is of interest because the assumption of the Nakagami fading with uniform phase distribution is not valid except for $m=1$, which has nevertheless been assumed elsewhere [40]-[42] [53]. Fortunately, it is easy to find an approximate associated phase with the complex Nakagami fading process due to the m and K parameter relationship between the Nakagami and Rice distributions. The phase distribution associated with the Rice distribution shall also give the approximate phase of the Nakagami fading process if the Rice parameters are converted to the Nakagami parameters. The (3.14) can be used for this purpose. Using the relation in (3.25) or (3.28), the K in (3.14) can be written as a function of m . Thus (3.14) can be written as

$$f(\theta_f) = \frac{1}{2\pi} e^{-K(m)} \left[1 + \sqrt{\pi K(m)} \cdot \cos \theta_f \cdot e^{K(m)(\cos \theta_f)^2} \right] [1 + \operatorname{erf}(\sqrt{K(m)} \cdot \cos \theta_f)], \quad (3.30)$$

which now becomes the approximate phase of the complex Nakagami envelope.

In section 3.2.1.3, the distribution of the fading envelope was found for the Rice/lognormal model. Similarly, the Nakagami/lognormal model may be used. The *pdf* of g_f is obtained for the Nakagami/lognormal fading channel from (3.11) by using the relation between m and K as in (3.28) as follows. Assuming

$$f_N(g_f | (m, \Omega)) = \frac{2}{\Gamma(m)} \cdot \frac{m^m}{\Omega^m} \cdot g_f^{2m-1} e^{-\left(\frac{m}{\Omega}\right) g_f^2}, \quad \begin{matrix} g_f \geq 0 \\ 0.5 \leq m < \infty \end{matrix} \quad (3.31)$$

and,

$$f(m) = \frac{1}{(2m - 1.52)c\alpha\sqrt{2\pi}} e^{\frac{-\left(\frac{1}{2}\ln((2m - 1.52)c) - \mu\right)^2}{2\alpha^2}}, \quad m \geq 1.76 \quad (3.32)$$

while $f(\Omega)$ is given by (3.9), we have,

$$f(g_f) = \int_0^\infty \int_{1.76}^\infty \frac{2}{\Gamma(m)} \cdot \frac{m^m}{\Omega^m} \cdot g_f^{2m-1} e^{-\left(\frac{m}{\Omega}\right)g_f^2} \cdot \frac{1}{(2m - 1.52)c\alpha\sqrt{2\pi}} e^{\frac{-\left(\frac{1}{2}\ln((2m - 1.52)c) - \mu\right)^2}{2\alpha^2}} \cdot \frac{1}{(\Omega - c)\alpha\sqrt{2\pi}} e^{\frac{-\left(\frac{1}{2}\ln(\Omega - c) - \mu\right)^2}{2\alpha^2}} \cdot dmd\Omega, \quad 0 \leq g_f < \infty \quad (3.33)$$

The associated approximate phase for the complex Nakagami/lognormal fading model will now become

$$f(\theta_f) = \int_0^\infty \frac{1}{2\pi} e^{-K(m)} \left[1 + \sqrt{\pi K(m)} \cdot \cos \theta_f \cdot e^{K(m)(\cos \theta_f)^2} \right] \cdot \frac{1}{(2m - 1.52)c\alpha\sqrt{2\pi}} e^{\frac{-\left(\frac{1}{2}\ln((2m - 1.52)c) - \mu\right)^2}{2\alpha^2}} \cdot dK, \quad (3.34)$$

where $K(m)$ is given by (3.28).

An earlier outage performance analysis [62], assumed Ω being lognormally distributed in a Nakagami fading and lognormal shadowing model. The analysis based on Ω parameter is not completely accurate as the m parameter is a function of Ω , and therefore is also random. The variability in m should also be taken into account in this analysis. Equation (3.33) is based on the reality that both parameters of the Nakagami distribution are random.

3.3 Wideband Channel

It was shown in Section 2.1.1 that the output of a multipath channel to a wide band signal $v(\tau)$ may be written as

$$v_m(\tau) = \sum_{n=1}^N g_n(t) v(\tau - \tau_n), \quad (3.35)$$

i.e., each path has been multiplied by a complex Nakagami random process $g_n(t)$ with random excess delay, τ_n . The total number of paths is N .

Equation (3.35) can be subjected to a geometrical interpretation and a corresponding physical interpretation as discussed below.

3.3.1 Geometric and Physical Interpretations

For geometrical interpretation, it can be observed that the transmitted waveform $v(t)$ is of finite duration T . If the received signal is resolved into orthogonal time slots of the same duration T , then, $v(t-nT)$ for $n=1, 2, \dots, N$, forms a set of N orthogonal waveforms and as a consequence $v_m(t)$ may be interpreted as the weighted sum of the N signal vectors in an N -dimensional signal space and can be represented as a point in an N -dimensional signal space. This implies that g_n is a projection of $v_m(t)$ on the N -th signal axis. The magnitude of the random signal vector $v_m(t)$ denoted by R , can be written in terms of g_n 's as follows:

$$R = \sqrt{g_1^2 + g_2^2 + \dots + g_N^2}. \quad (3.36)$$

It is known that g_n are random and Nakagami distributed. Therefore the distribution of R may be obtained through the transformation relationship as given in (3.36). Thus, if

$$P(g_n) = \frac{2m_n^{m_n} g_n^{2m_n-1} e^{-\left(\frac{m_n}{\Omega_n}\right)g_n^2}}{\Gamma(m_n)\Omega_n^{m_n}}, \quad (3.37)$$

is the distribution of g_n , where Ω_n is the average power of each path and m_n is the associated m parameter of that path, then the distribution of R is then given below [36]:

$$P(R) = \frac{2m^m R^{2m-1} e^{-\left(\frac{m}{\Omega}\right)R^2}}{\Gamma(m)\Omega^m}, \quad (3.38)$$

subject to the condition that [36]

$$\frac{\Omega_n}{m_n} = C, \quad (3.39)$$

where C is a constant. The (3.38) is also a Nakagami distribution with parameters

$$m = \sum_n m_n \text{ and } \Omega = \sum_n \Omega_n.$$

In addition to [36], the (3.38) has been used by other authors [62]. No justification for the validity of the assumption and the associated physical interpretation has been provided earlier as its use is subject to the fulfillment of (3.39) and in this sense should be considered as limited in scope. This strict condition may perhaps never be met in reality, as is generally the case with other cases of theoretical modeling and their physical justifications¹. It appears, however, to be possible to meet this condition in an approximate sense as described below.

We know from (3A.12) that

$$m_n = \frac{[\overline{r_n^2}]^2}{[\overline{r_n^2 - r_n^2}]^2} = \frac{\Omega_n^2}{[\overline{r_n^2 - \Omega_n}]^2} = \frac{\Omega_n^2}{\text{Var}[r_n^2]}. \quad (3.40)$$

Therefore, using (3.40) in (3.39) gives

1. Consider the case of modeling by Turin et al [40]: the random path arrival process as the Poisson process based on finite observations.

$$\frac{\Omega_n}{m_n} = \frac{Var(r_n^2)}{\Omega_n} = C. \quad (3.41)$$

which may also be written as,

$$Var(r_n^2) = C \cdot \Omega_n. \quad (3.42)$$

The index n in (3.42) relates to the excess delay, and as a result, both Ω_n and $Var(r_n^2)$ are a function of the delay variable. The condition in (3.39) may now be interpreted as follows. The rate of change of the variance of the signal power with respect to the path delay should be a constant times the rate of change of the average signal power with respect to the multipath delay. This is a sufficient condition for the applicability of (3.38). This condition may be broken into two parts: firstly, both the average signal power and the variance of the signal power should change in the same direction, i.e., both should increase or decrease not vice versa and, secondly, the rates of change should be the same or their ratio should be a constant. The first part of the condition may be shown to be met frequently on the average basis, however, it is hard to verify the applicability of the second part of the condition. For an engineering approximation, it is not necessary that the second part be met exactly. As long as there is a suitable way of measuring the degree of approximation, the (3.38) may be used and the results may be put in proper perspective keeping in view the approximation involved. One way to quantify the approximation is to assume that there is a number ϵ such that

$$\frac{Var(r_n^2)}{\Omega_n} = D \quad \left| \quad C - \frac{\epsilon}{2} \leq D \leq C + \frac{\epsilon}{2} \right. \quad (3.43)$$

The ratio $\epsilon/2C$ in this case becomes the degree of approximation.

To examine the validity of the notion that both the average signal power and the variance of the signal power increase or decrease together, *on the average*, a physical interpretation of the geometric model is needed, and is discussed next.

The physical interpretation described herein is similar to that provided by [37] and [38] for conventional cellular radio propagation in an urban environment, and under the assumption that the multiple scattering can be neglected due to the associated excess losses, and thus is not differentiable from the receiver noise. Considering the transmitter and receiver positions as the foci, one can draw concentric ellipsoids such that the line joining the two vertices and passing through the foci is of lengths $d_n = c(t_0 + nT)$. Here, c is the velocity of light, $n = 1, 2, \dots, N$, t_0 is the propagation delay of the shortest path and T is the waveform duration. Each of the N terms on the right-hand side of the equation (3.35) corresponds to the summation of the multipath generated by the reflectors present within the space between the ellipsoids with the major axes $c(t_0 + (n-1)T)$ and $c(t_0 + nT)$. This is true because it is the property of an ellipse that the sum of any two lines joining the foci of an ellipse to any point on the ellipse is equal to the major axis of the ellipse. Therefore, only those reflectors contribute in the fading characteristics of a given fading signal path whose sum of the incident and reflected path lengths is greater than $c(t_0 + (n-1)T)$, but less than $c(t_0 + nT)$. This is illustrated in Figure 3.19, where two scenarios for microcellular environment are depicted. In Figure 3.19, there are two receiver positions, R_A and R_B . The transmitter is at point T_X . The ellipses 1, 2 and 3, represented by thick lines, correspond to the receiver position R_A and the ellipses a and b correspond to the receiver position R_B .

The Figure 3.20¹ is borrowed from [5] and may be explained in relation to the Figure 3.19. In LOS propagation, the first path shall always contain the LOS path, and most likely the ground reflected paths. Any other subpaths in the first time slot would traverse less dis-

1. This impulse response was measured in the LOS indoor environment.

Figure 3.19 Illustration of the physical interpretation of the wideband channel in a microcellular environment showing guided propagation.

Figure 3.20 Figure 7 (a) of [5]: LOS measured impulse response profiles in a room. Squaring the magnitude will give the power delay profiles.

tance than the subpaths in the next time slots. Therefore, the first path shall always have a higher specular power component, as well as the average power, when compared to the second path. The variance of the first path power shall also be quite high because of the interference of the LOS and Ground reflected paths. The same is true in relation with the second and third paths except that, instead of LOS and GR paths, single reflected and twice reflected paths may be assumed to be the dominant paths for the second and third paths respectively. This can be clearly observed from Figure 3.20. It is clear that the mean power, as well as the variance of the power decrease from τ_1 and τ_2 , thus confirms the approximate condition in (3.43).

Some other observations made during the channel impulse response measurements elsewhere [12], may be explained in terms of the above physical model. These are discussed next.

In general, the m and K parameter values of the first path should be higher than the m and K parameters of the second path because the first path shall always contain the LOS path and the ground reflected paths. Any other subpaths in the first time slot would traverse less distance than the subpaths in the next time slots. Therefore, the first path shall always have a higher specular power component when compared to the second path. As a result, the Rice K factor (or m parameter) of the first path would be greater than the remaining paths. Increasing the pulse width corresponds to larger distances between ellipses whose direct consequence is that the number of subpaths increase in the resolvable paths, and this would, on the average, result in the lower specular power and increased random power component in each path, thus giving lower K values. It was observed in [12] that the value of K for the first path was between 8-12 dB's and for the second path between 6-10 dB's for a transmitted pulse of 0.1 microsecond duration. When data was processed and analyzed for about one microsecond pulse duration, the K parameter value of all the paths is reduced to 2-4 dB's. This verifies the above conclusions. The

argument used for the K parameters of the first and second paths is also applicable to second and third paths, and so on, if these paths exist.

It can also be noted that Figure 3.19 clearly segregates the scatterers which contribute to the phasor sum of a given resolvable path. This ensures that the scatterers contributing to one path are not the same for the other path. This implies that the path loss and the fading of two different paths of the same impulse response, may not be highly correlated. On the other hand the same paths (e.g., the first path) of different impulse responses in close proximity should be expected to be highly correlated because scatterers contributing to that path remain nearly same.

3.3.2 Parameter m and the Order of Multipath Diversity

The Nakagami distribution in (3.38) is an important relation in that it takes into account the time dispersive nature of the received multipath signal. This is explained as follows.

If the multipath are unresolvable, then (3.35) becomes the narrowband case and may be written as

$$v_m(t) = g(t)v(t). \quad (3.44)$$

The $g(t)$ is then a complex Nakagami process, that is, the distribution of the magnitude of $g(t)$ is Nakagami distributed and given by (3.38), and, the phase is given by (3.30). However, as soon as the signal bandwidth is increased over and above the channel bandwidth, it is possible to resolve the received multipath signal. It can be realized that the geometrical interpretation of (3.35) is making use of the time orthogonality of the received signal. The received signal in (3.35) may be modeled as a tapped delay line which instantly suggests that no other than a RAKE type receiver is ideal for the reception of (3.35). In this case, (3.37) will give the distribution of the gain of each finger. The main objective of the RAKE is to exploit the time or the frequency diversity in the received multipath signal.

The maximum order of diversity which may be obtained is calculated either by dividing the total bandwidth over the coherence bandwidth, or by dividing the total time dispersion¹ to that of transmitted waveform duration. For a Rayleigh channel, i.e., $f(g_n)$ is Rayleigh distributed with all m_n 's=1, the value of m in (3.38) is nothing but the order of diversity which may be obtained from the channel. This is a novel formulation for the interpretation of m for the *time dispersive* Nakagami fading channel and has not been reported before. Charash [53], however, noted that increasing the m results in diversity-like improvement in bit error rate, but did not explain what the relation of m is with the order of diversity.

The above notion may further be extended if it is assumed that m_n 's are not necessarily equal to one and may have any value equal or greater than 0.5. In this case, one may have only two fingers of the delay line channel, but m_n might add up to some value greater than two. For example, if $m_1=1.8$ and $m_2=1.2$, the value of m in (3.39) is three. Thus a two finger RAKE type receiver in this dispersive Nakagami fading channel is expected to perform equivalent to a three finger RAKE receiver in a dispersive Rayleigh fading channel. As m is a real number equal or greater than 0.5, m_n might add up to some value which is not necessarily an integer. Therefore, it is possible to have a real order of diversity in this scenario. It is also possible to quantitatively substantiate the relation of m with the multipath diversity, using a simple analytical argument as discussed below. The basis of the argument is this: if it can be shown that the BER performance of the single Nakagami fading received signal and the space/time diversity reception of the Nakagami fading signals is *equal*, then the parameter of the two cases must be related as claimed above and the above interpretation should be valid. This is indeed the case as discussed below.

1. Length of impulse response or power delay profile.

The performance of certain modulation schemes over the Nakagami Fading channel have been analyzed in [53] and [78]. Assume that a BPSK modulated signal is received over a noisy and fading channel, and is passed through a matched filter receiver. Also, assume that noise is additive and Gaussian, and that fading is Nakagami distributed. For the AWGN only case, the probability of error, P_2 , for binary orthogonal signals is given by the following well known expression¹ [77]:

$$P_2(\gamma_b) = Q(\sqrt{\gamma_b}). \quad (3.45)$$

where $\gamma_b = (\alpha E_b)/N_0$. The E_b is the energy per bit and N_0 is the variance of the AWGN whereas α is the attenuation of the signal. $Q(\cdot)$ is the Q-function defined as follows:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt = 1 - \text{erf}(x). \quad (3.46)$$

The P_2 is the probability of error for the fixed α which can be viewed as the conditional error probability. For the random α , the P_2 may be averaged over the *pdf* of γ_b to incorporate the effect of fading. In this case [77],

$$P_2 = \int_0^{\infty} P_2(\gamma_b) p(\gamma_b) d\gamma_b. \quad (3.47)$$

The problem therefore becomes twofold: to determine the *pdf* of γ_b and the evaluation of the integral in (3.47). Assuming γ_b is Nakagami distributed, we have [77]

$$p(\gamma_b) = \frac{m^m}{\Gamma(m) \bar{\gamma}_b^m} \gamma_b^{m-1} e^{-(m\gamma_b)/\bar{\gamma}_b}. \quad (3.48)$$

where $\bar{\gamma}_b = E(\alpha^2)(E_b/N_0)$ is the average SNR per bit.

1. The notation of [77] is used to avoid any possible confusion.

The average probability of error for the Nakagami fading case is found elsewhere [78], and is given below:

$$P_e = \left(\frac{m}{2\bar{\gamma}_b + 4m} \right)^m \frac{\Gamma(2m)}{\Gamma(m)\Gamma(m+1)} \left(\frac{\bar{\gamma}_b}{\bar{\gamma}_b + 2m} \right)^{1/2} {}_2F_1 \left(1, m + \frac{1}{2}; m + 1; \frac{2m}{\bar{\gamma}_b + 2m} \right), \quad (3.49)$$

where,

$${}_2F_1(1, b; c; z) = 1 + \frac{b}{c}z + \frac{b(b+1)}{c(c+1)}z^2 + \frac{b(b+1)(b+2)}{c(c+1)(c+2)}z^3 + \dots, \quad (3.50)$$

is a hypergeometric function. A plot of the average probability of error is shown in Figure 3.21 for the Nakagami m parameter values of one (solid), two (dashed) and four (dash-dot). BER performance is improved as the value of m is increased. It is similar to the performance gain as in the diversity case. It is verified below, if it is the case indeed.

The case of a diversity receiver with maximal ratio combining as in [77] may be considered as a starting point. The approach is similar to the above in that it evaluates the integral in (3.47), however, the *pdf* of γ_b is the one which is now obtained after the summation of the power of each diversity path. Referring to [77], it has been found to be as follows:

$$p(\gamma_b) = \frac{1}{(L-1)! \bar{\gamma}_c^L} \gamma_b^{L-1} e^{-\gamma_b/\bar{\gamma}_c}, \quad (3.51)$$

where $\gamma_b = \frac{E}{N_0} \sum_{k=1}^L \alpha_k^2$. L is the number of diversity branches. $\bar{\gamma}_c = \frac{E}{N_0} E(\alpha_k^2)$ is the average SNR per branch and α_k is the random attenuation of the k th branch.

It can be shown that (3.51) is a special case of (3.48) as follows. Assuming m is an integer and is equal to L and $\bar{\gamma}_b = L\bar{\gamma}_c$, then replacing them in (3.48) for m and $\bar{\gamma}_b$ reduces (3.48) exactly to (3.51). Therefore, the average probability of a bit error given in (3.49) is also applicable to the case of the L branch diversity receiver.

Figure 3.21 Average bit error probability as a function of average SNR per bit for $m=1$ (solid), $m=2$ (dashed), $m=4$ (dash-dot). BER performance is improved as the value of m is increased.

As a summary, it has been shown that the m parameter of the Nakagami distribution may be interpreted in two ways. For a non-frequency selective signal or for any one of the resolvable paths of a frequency selective signal, increasing the value of m suggests the reduced amount of fading due to the increased effective direct path in the received signal. From the perspective of dispersive signals, m is the maximum possible diversity order available from the channel when compared to the bench-mark dispersive Rayleigh channel.

3.4 Summary

This chapter investigated the physical basis of the parameter of the statistical characterization of the multipath wireless channel. A number of new concepts have been introduced.

In the case of narrowband channels, the nature of slow fading their segregation and the statistical characterization of slow fading have been discussed. It is shown that slow fading may also be modeled by the Nakagami distribution based on comparative curve-fitting. This comparative fitting, it appears, has not been done before. It is shown that the segregation of slow and fast fading may not be very useful for radio link performance as it does not add any significant value. Moreover, it is a redundant exercise in the case of Ricean channels. The nature of Rice fading in the LOS microcellular channel is explained. The parameter relation between the Rice and Nakagami distributions is described and related in a simple manner. The Nakagami m parameter is interpreted in the same manner as the effective Rice K factor based on the Rice/Rayleigh physical model. The statistical distribution of the fading envelope is derived, which is applicable everywhere in the cell.

This chapter introduced a new approach to characterize the statistical distribution of the time dispersive Nakagami fading signal based on a geometrical interpretation of the received signal. The Nakagami m parameter has a particular physical basis for both fading

only, and dispersive and fading channels. The m parameter is related to the order of multipath diversity available from the time dispersive behavior of the channel.

In the following chapters, the BER and the radio link performance are investigated, based on the channel model and characterization as understood in this chapter.

Appendix 3A *The parameter relationship of the Rice and Nakagami distributions*

To derive the relationship between the m and K parameters, we first derive the Rice distribution in terms of the K and Ω parameters and then obtain the desired relationship.

Let Ω be the average received power, and suppose P_s and P_r are the specular and random power components in Ω , such that

$$\Omega = P_s + P_r. \quad (3A.1)$$

The Rician distribution in terms of P_s and P_r can be written as

$$f_R(g_f) = \frac{2g_f}{P_r} e^{-\frac{g_f^2 + P_s}{2P_r}} I_0\left(\frac{2g_f\sqrt{P_s}}{P_r}\right) \quad g_f \geq 0. \quad (3A.2)$$

here, $P_s = g_{sf}^2$ and $P_r = E\{g_f^2 - g_{sf}^2\}$ while $g_{sf} = E\{g_f\}$.

Let us define

$$K = \frac{\text{power in specular component}}{\text{power in random component}} = \frac{P_s}{P_r}. \quad (3A.3)$$

To write (3A.2) in terms of Ω and K , we proceed as follows. From (3A.1), after some algebraic manipulations, we have

$$P_r = \Omega / (1 + K). \quad (3A.4)$$

From (3A.3) we have

$$P_s = KP_r = (K\Omega) / (1 + K). \quad (3A.5)$$

Substituting (3A.4) and (3A.5) in (3A.2) we have the Rice distribution in terms of K and Ω as follows:

$$f_R(g_f) = \frac{2g_f(1+K)}{\Omega} e^{\left(-K - \frac{(1+K)g_f^2}{\Omega}\right)} I_0(2g_f\sqrt{K(1+K)/\Omega}) \cdot \begin{matrix} g_f \geq 0 \\ K \geq 0 \end{matrix} \quad (3A.6)$$

Moreover, the n th moment of the Rice distribution is

$$E\{g_f^n\} = P_r^{n/2} \Gamma\left(1 + \frac{n}{2}\right) F_1\left(-\frac{n}{2}, 1; -\frac{P_s}{P_r}\right). \quad (3A.7)$$

From (3A.7)

$$E\{g_f^2\} = P_s + P_r = \Omega. \quad (3A.8)$$

$$E\{g_f^4\} = P_s^2 + 4P_r P_s + 2P_r^2. \quad (3A.9)$$

Nakagami [36] has shown that the probability distribution of the resultant vector follows the m -distribution, which is currently known as the Nakagami distribution and is given below:

$$f(g_f) = \frac{2}{\Gamma(m)} \frac{m^m}{\Omega^m} \cdot g_f^{2m-1} e^{-\left(\frac{m}{\Omega}\right)g_f^2}, \quad (3A.10)$$

where

$$\Omega = E\{g_f^2\}. \quad (3A.11)$$

and

$$m = \frac{[\overline{g_f^2}]^2}{[\overline{g_f^2} - g_f^2]^2} = \frac{\Omega^2}{[g_f^2 - \Omega]^2}. \quad (3A.12)$$

The use of (3A.8) and (3A.9) in (3A.12) after some algebraic manipulations yields

$$m = 1 / \left[1 - \frac{K^2}{(1+K)^2} \right]. \quad (3A.13)$$

Chapter 4

BER Performance of Frequency Selective Signal Reception

4.1 Introduction

In the previous chapter, the characteristics of a dispersive Nakagami fading channel were highlighted. This chapter relates this understanding to the bit error rate performance. In particular, it addresses the BER performance of frequency selective (dispersive) signals received over fading channels with random path arrival character. This is related to the performance analysis of frequency selective RAKE receivers.

4.2 Performance of Wideband Receivers in Realistic Channels

The wideband frequency selective signals possess the potential to exploit the inherent multipath character of the mobile radio channel. The frequency selective reception helps obtain the time/frequency diversity from the channel. The RAKE receiver has been suggested for above mentioned purpose. The RAKE receiver usage is included in the specifications of the IS95 system [12]. One issue related to its usage for operation in a realistic channel is the random nature of the path arrival. Because paths randomly arrive at the receiver, efficient strategies are needed to maximize captured energy from the channel. In the classical RAKE receiver, a large number of fingers are available. Multipath combining

is, however, performed by enabling only those fingers corresponding to delays at which useful multipath components are observed [73].

In one approach, the position of the fingers on the delay line may be optimally selected by an adaptive algorithm [12]. In this implementation, fingers are not necessarily separated by the fixed duration as in the classical RAKE receiver [73], [77], but are determined by a search algorithm [12]. This search algorithm makes the finger assignment as per the following rule for the IS95 base station receiver: 1) assign one finger-pair at the global maximum of multipath intensity profiles, and 2) assign the second finger-pair at the local maximum of multipath intensity profile which was of greatest value, which was within 6dB of the global maximum, and for which two fingers were at least more than a chip duration apart. The effect of the random path arrivals on RAKE performance has apparently not been investigated to date. An analysis which takes into account the random path arrivals will be helpful in quantifying the performance degradation due to random path arrivals and may serve as the theoretical benchmark for comparison of multiple RAKE implementations designed to overcome this obstacle.

The performance of an ideal RAKE receiver, which exploits time/frequency diversity, is no different than the performance of an L branch space diversity receiver as shown in [77]. Therefore, the following expressions for average probability of error for binary orthogonal signals for space diversity systems¹ can be used as a starting point for RAKE [77],

$$P_e = \left[\frac{1}{2}(1 - \mu) \right]^L \sum_{k=0}^{L-1} \binom{L-1}{k} \left[\frac{1}{2}(1 + \mu) \right]^k, \quad (4.1)$$

where

1. with maximal ratio combining

$$\mu = \sqrt{\frac{\bar{\gamma}_c}{2 + \bar{\gamma}_c}}. \quad (4.2)$$

$\bar{\gamma}_c$ is the average SNR per branch and it is assumed that branch mean square values $E(\alpha_k^2)$ are equal. When these are unequal we have [77]

$$P_e = \frac{1}{2} \sum_{k=1}^L \pi_k \left[1 - \sqrt{\frac{\bar{\gamma}_k}{2 + \bar{\gamma}_k}} \right]. \quad (4.3)$$

Here,

$$\pi_k = \prod_{\substack{i=1 \\ i \neq k}}^L \frac{\bar{\gamma}_k}{\bar{\gamma}_k - \bar{\gamma}_i}. \quad (4.4)$$

In the case of RAKE, the average probabilities of bit error given in (4.1) and (4.3) are functions of the number of diversity branches or resolvable paths. This probability may now be considered as the average probability of bit error, conditioned on the number of resolvable paths, L . Average bit error probability which is now conditioned on L is given by

$$P_e = \sum_{N=1}^{\infty} P_e(L=N)P(N). \quad (4.5)$$

In this case $P(N)$ is given by the Poisson distribution [51], as follows;

$$P(N) = \frac{\lambda^{N-1}}{(N-1)!} e^{-\lambda}, \quad N \geq 1 \quad (4.6)$$

Here, N is the number of paths and λ is the average number of paths per total duration/length of RAKE receiver. Turin et al [40] suggested a doubly stochastic Poisson process for the statistical modeling of path arrivals where λ is also a random variable.

The average probability of error for BPSK signals is now given by the following expression.

$$P_e = \sum_{N=1}^{\infty} \frac{\lambda^{N-1}}{(N-1)!} e^{-\lambda} \left[\frac{1}{2}(1-\mu) \right]^{N-1} \sum_{k=0}^{N-1} \binom{N-1+k}{k} \left[\frac{1}{2}(1+\mu) \right]^k. \quad (4.7)$$

The average probability of error as a function of average signal to noise ratio per bit as given in (4.3) is plotted in Figure 4.1. The curves are drawn for the case of one two and four resolvable paths¹. The average probability of error, as given in (4.7), as a function of average signal to noise ratio per bit is plotted in Figure 4.2. These curves are drawn again for the three values of the average number of resolvable paths per given length of the RAKE, λ , i.e., one two and four. These integer λ values are deliberately chosen so that the average number of paths case may be compared to that of a known number of paths. Increasing the λ results in improved performance², however, the gain in performance is not as large as in the deterministic case. Thus, when performance is compared for the same value of number of paths and the parameter λ , a degradation is experienced for the random path arrival case. Figure 4.3 - Figure 4.6 are plotted to help understand the nature and quantify the performance degradation for these cases. The solid lines in these figures represent the random number of paths case whereas the dashed line represent the known number of paths case. Figure 4.3 compares the case of only one path and $\lambda=1$. It shows that $\lambda=1$ case is always better than $L=1$ for the range of average SNR per bit values chosen for that plot. Figure 4.4 and Figure 4.5 compare the cases of $\lambda=2$ and $\lambda=4$ respectively. In Figure 4.4, random number of paths case is slightly better up to an average SNR per bit of about 14 dB after which a very noticeable performance degradation is experienced compared to known numbers of paths case. In Figure 4.5, however, increasing the

1. The *resolvable paths* here are the same as space diversity branches. The term *paths* will be used for brevity in this chapter when referring to resolvable paths.

2. The use of performance improvement here refers to the improvement in performance of a time/frequency diversity case with non-diversity (which thus becomes narrowband), fading signal reception scenario.

Figure 4.1 BER performance for $L=1$ (solid) $L=2$ (dashed) and $L=4$ (dash-dot). Performance improves as L increases.

Figure 4.2 BER performance for $\lambda=1$ (solid) $\lambda=2$ (dashed) and $\lambda=4$ (dash-dot). Performance improves as λ increases.

Figure 4.3 BER performance Comparison for $L=1$ (dashed) and $\lambda=1$ (solid).

Figure 4.4 BER performance Comparison for $L=2$ (dashed) and $\lambda=2$ (solid).

Figure 4.5 BER performance Comparison for $L=4$ (dashed) and $\lambda=4$ (solid).

value of λ results in no advantage before the crossover point at about 10 dB after which a very noticeable performance degradation is experienced again when compared to the known number of paths case. Figure 4.3 through Figure 4.5 are plotted together in Figure 4.6 to make these differences in relative behavior more obvious.

Figure 4.7 and Figure 4.8 illustrate how performance is varied for L and λ for an average SNR per bit of 10 and 20 dB. From Figure 4.6, it may be observed that at high average SNR per bit the difference in BER performance is higher than at the lower values. This is shown in an enlarged fashion in Figure 4.7 and Figure 4.8. Note the crossover point at $\lambda=3$ and $\lambda=1.5$ in Figure 4.7 and Figure 4.8 respectively. Figure 4.7 and Figure 4.8 are plotted together in Figure 4.9 to help appreciate the relative magnitude of performance differences.

The P_e expression in (4.7) was obtained for the case of classical Poisson multipath arrival. If the modified Poisson process, i.e the $\Delta - K$ model, is assumed then we assume λ to be a random variable. Thus, the expression for P_e in (4.7) is to be regarded as a conditional one; conditioned on λ . Turin and Suzuki have suggested that random variations in λ may be modeled by a two state discrete Markov chain assuming $\Delta = 1$. This condition on P_e may be removed by averaging the P_e on the *pdf* of λ . The *pdf* of λ may be written as follows.

$$P(\lambda) = P_1 \delta(\lambda - \lambda_1) + P_2 \delta(\lambda - \lambda_2). \quad (4.8)$$

Here P_1 is the state probability that λ is in state $\lambda=\lambda_1$ and the P_2 is the state probability that λ is in state $\lambda=\lambda_2$. The state transition matrix may be obtained from the appropriate measurement data and thus consequently the state probabilities [69]. The expression for P_e may now be found as follows;

Figure 4.6 BER performance Comparison for $L=1$ (solid) and $\lambda=1$ (solid-circle), $L=2$ (dashed) and $\lambda=2$ (dashed-circles), $L=4$ (dash-dot) and $\lambda=4$ (dash-dot-circles).

Figure 4.7 BER performance comparison; BER as a function of number of resolvable paths, L , (dashed) and average number of paths, λ , (solid) for an average SNR per bit of 10 dB.

Figure 4.8 BER performance comparison; BER as a function of L and λ for an average SNR per bit of 100 dB.

Figure 4.9 BER performance comparison; BER as a function of L and λ for an average SNR per bit of 10 dB and 100 dB together.

$$P_e = \int_0^{\infty} P_e P(\lambda) d\lambda, \quad (4.9)$$

$$P_e = \int_0^{\infty} (P_e P_1 \delta(\lambda - \lambda_1) + P_e P_2 \delta(\lambda - \lambda_2)) d\lambda. \quad (4.10)$$

Using (4.7) for P_e in (4.10) and applying the sampling property of the impulse function, we obtain;

$$P_e = P_1 \cdot \sum_{N=1}^{\infty} \frac{\lambda_1^{N-1}}{(N-1)!} e^{-\lambda_1} \left[\frac{1}{2}(1-\mu) \right]^{N-1} \sum_{k=0}^{N-1} \binom{N-1+k}{k} \left[\frac{1}{2}(1+\mu) \right]^k + \quad (4.11)$$

$$P_2 \cdot \sum_{N=1}^{\infty} \frac{\lambda_2^{N-1}}{(N-1)!} e^{-\lambda_2} \left[\frac{1}{2}(1-\mu) \right]^{N-1} \sum_{k=0}^{N-1} \binom{N-1+k}{k} \left[\frac{1}{2}(1+\mu) \right]^k.$$

The effect of variability in the λ is illustrated in Figure 4.10. The solid lines correspond to the known λ case, i.e., when $\lambda=1$ and $\lambda=4$, respectively. These values are chosen to allow for sufficient spacing between the two curves to illustrate the effect of randomness in λ clearly. For reference, the top and bottom dotted lines are plotted for the $L=1$ and $L=4$ cases. Assuming that λ varies randomly between the two values of $\lambda=1$ and $\lambda=4$ with equal probabilities, i.e. $P_1=Pr(\lambda=1)=0.5$ and $P_2=Pr(\lambda=4)=0.5$, the performance is now given by the dashed line in Figure 4.10. This figure also shows the results for the case where the probability that λ assumes the value 1 or 4 is not equal; instead, it is given by $P_1=Pr(\lambda=1)=0.2$, and $P_2=Pr(\lambda=4)=0.8$. It can be noted that the two curves provide performance lying between the pure $\lambda=1$ and $\lambda=4$ cases.

4.2.1 Practical Considerations

Up to this point, the performance was compared on one-to-one basis for a known number of paths versus a given average number of paths where the actual number is a random

Figure 4.10 BER performance comparison when λ is random; dotted lines are $L=1$ and $L=4$ cases, solid lines are for $\lambda=1$ $\lambda=4$, dashed lines when λ varies between 1 and 4 with equal probability and dash-dot line when $\lambda=1$ occurs with probability 0.2 and $\lambda=4$ occurs with probability 0.8.

variable. It helped understanding of the general behavior of performance variation. A case of more practical interest is described below.

Given a maximum allowable transmitted power¹ in a channel and sensitivity of the receiver used, the length of the power delay profile can be measured or simulated. Furthermore, based on the targeted system capacity and other design factors, the optimal choice of transmitted pulse/symbol duration may be made. With the help of above information one can determine the maximum order of multipath diversity available from the channel. Suppose this order of diversity is four. It is justified to ask, if measurements are made, what range of λ values will be obtained. Intuitively, this value would not be greater than four because the constraints of the real equipment preclude the possibility of receiving more than four resolvable multipath symbols. As the arrival process over the duration of the average power delay profile is assumed to be approximately Poisson one can therefore use the Poisson distribution to find the probability of a given number of paths arriving over the duration of the average power delay profile. Thus, with a certain measure of confidence one can determine the upper bound on the value of λ . For example, assuming that it is known that the probability of receiving at most 4 paths is 90%, one can solve the following equation to find the value of λ which any λ measurement in that channel will not exceed. If $P(.)$ represents the probability of the event in braces, the probability of at most four paths per duration of the average power delay profile will be given as follows,

$$P(N \leq 4) = P(N = 0) + P(N = 1) + P(N = 2) + P(N = 3) + P(N = 4). \quad (4.12)$$

As $P(N)$ is given by Poisson distribution. Therefore (4.12) can be written as;

1. The maximum allowable transmitted power is typically specified in the Physical Layer standard specification. Its choice is primarily dependent on the regulatory EMI requirements in air as well as the emission regulations for equipment.

$$\begin{aligned}
 P(N \leq 4) &= e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2!} + \frac{\lambda^3 e^{-\lambda}}{3!} + \frac{\lambda^4 e^{-\lambda}}{4!}, \\
 &= e^{-\lambda} \left(1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \right).
 \end{aligned} \tag{4.13}$$

The probability of at most four paths, given the probability of the number of paths given by Poisson distribution, is plotted in Figure 4.11 as a function of λ . The value of λ corresponding to a probability of 0.9 (i.e. 90%) is 2.4325.

Thus, it is more realistic to compare the four finger RAKE performance with the $\lambda=2.4325$ case instead of $\lambda=4$. Three curves have been plotted in Figure 4.12. The solid curve corresponds to the fourth order diversity case; the dash-dot line represents Rayleigh or Nakagami ($m=1$) fading with no diversity. The $\lambda=2.4325$ case has been shown by a dashed line. To quantify the deterioration in performance for a BER of 10^{-3} , it may be noted that the theoretical 14 dB savings (approximate) due to the use of ideal four finger RAKE receiver compared with a simple matched filter receiver will be reduced to about 9.5 dB when the same RAKE receiver operates over a realistic channel with random path arrival at a rate of $\lambda=2.4325$. This is a performance loss of about 4.5 dB due to randomness.

The fact that λ may assume real values adds an interesting twist to the concept of order of diversity, which is usually given as an integer value. The notion of fractional diversity may be introduced for realistic channels, with the condition that fractional diversity can not be less than or equal to one.

4.3 Summary

This chapter investigated the BER performance of the RAKE type receiver over realistic multipath channels. This receiver makes use of the multipath diversity available from dispersive channels. The contribution of this chapter is to provide an extension to RAKE per-

Figure 4.11 Probability of at most four paths as a function of λ . A probability of 0.9 corresponds to $\lambda = 2.4325$.

Figure 4.12 BER performance for a practical case; $L=4$ (solid), $L=1$ (dash-dot) and $\lambda = 2.4325$ (dashed).

formance methodology so as to incorporate the effect of the random path arrival nature of real dispersive channels. The notion of fractional diversity is introduced.

In the next chapter, radio link performance is analyzed for signals which have passed through a frequency selective Nakagami fading channel in the presence of frequency selective Nakagami fading multi-user cochannel interference.

Chapter 5

Radio Link Performance in Cochannel Interference

5.1 Introduction

The allocation of a limited spectrum for mobile radio services necessitates the reuse of spectrum either in a different cell or in the same cell but in different sectors, depending on the type of access scheme, to bring the system user capacity to an economically viable level. The total system capacity is a function of the frequency reuse pattern. Frequency reuse results in a number of subscribers having their information bearing signal on the same carrier frequency but at some distance, known as reuse distance, apart. In this scenario, it is desirable to consider the effect of cochannel interference on the system performance. One criterion for the performance evaluation is the received signal to cochannel interference power ratio. Another useful measure is outage probability, which is the probability that this ratio will remain below a certain threshold by an amount known as the signal to interference power protection ratio.

The magnitude of the chosen transmission bandwidth relative to the channel coherence bandwidth determines whether the desired and interfering signals are termed frequency selective or non-selective. If the transmission bandwidth of the combination of the desired and interfering signals at the receiver is less than the channel coherence bandwidth, these signals are known as frequency non-selective. When the combined transmis-

sion bandwidth is greater than the channel coherence bandwidth, these signals are known as frequency selective. In the frequency selective case, the choice of receiver determines how the frequency selectivity is going to manifest itself on the receive side. If a RAKE receiver is used to derive benefit from multipath diversity, the multiuser interference will present itself in a time dispersed manner. Time dispersion may result in inter-symbol interference, but also presents the opportunity to design a suitable receiver to extract the desired signal. However, this approach impairs the transmission capacity. A trade-off may be made by allowing multipath induced inter-symbol interference in order to increase the transmission rate, generally at the cost of increased complexity of the receiver.

Based on above discussion, three interference scenarios may be identified of which one pertains to the frequency non-selective fading case whereas the other two belong to the frequency selective fading case. The majority of previous work on the cochannel interference analysis [55]-[62], comes under the frequency non-selective category.

In frequency selective scenarios both desired and interfering signals are fading and frequency selective. A first frequency selective scenario may be built around RAKE reception of the frequency selective desired signals. In a second scenario, frequency selectivity of the desired signal is manifested as the so called multipath self interference or MSI. There seems to be only one previous work [63], which investigates the frequency selective nature of the desired signal in interference analysis. In [63], Yao and Sheikh considered the problem of a frequency selective desired signal in the presence of multiple fading interferers resulting in intersymbol interference. The assumption in [63] is similar to the one made in the analysis of Chapter 6. However, the approach used in Chapter 6 is different from [63].

This chapter considers the outage performance of a desired signal which is passed through a frequency selective (dispersive) and fading channel in the presence of frequency selective and fading multiuser interference. In the following, outage performance is inves-

tigated for RAKE reception of the desired signal in the presence of time dispersed fading multiuser interference. Apparently, this problem has not been considered before. Analysis of this chapter is different from [63] in two respects: it assumes a RAKE reception and it assumes frequency selective interference.

5.2 System Model

Let us assume that the received signal is of the following form:

$$v_r(\tau) = d(\tau) + I(\tau) + n(\tau), \quad (5.1)$$

Where

$$d(\tau) = \sum_{n=1}^N g_n v(\tau - \tau_n) e^{j\theta_n} \quad (5.2)$$

is the desired multipath signal, and

$$I(\tau) = \sum_{i=1}^I \sum_{l=1}^{L_i} \alpha_{i,l} u_{i,l}(\tau - \tau_{i,l}) e^{j\beta_{i,l}} \quad (5.3)$$

is the multipath cochannel interfering signal. The $n(\tau)$ is the white Gaussian noise which is neglected in the following analysis.

Using the results of Section 3.2.3 the desired signal is Nakagami distributed with the *pdf* given as in (3.38) and is rewritten as follows:

$$f(R) = \frac{2m_d^{m_d} R^{2m_d-1}}{\Gamma(m_d)\Omega_d^{m_d}} e^{-\left(\frac{m_d}{\Omega_d}\right)R^2}, \quad R \geq 0, \quad (5.4)$$

where m_d is the Nakagami parameter of the desired signal's magnitude distribution and Ω_d is the mean power of the received desired signal. Of course, again,

$$m_d = \sum_n m_n, \quad (5.5)$$

and

$$\Omega_d = \sum_n \Omega_n. \quad (5.6)$$

As a special case, when $m_n = 1$ and $\Omega_n = \Omega$, $m_d = N$ and $\Omega_d = N\Omega$.

In (5.3), I interferers are assumed each of which reaches the receiver through L_i multipaths. Thus, cochannel interference at the receiver is given by expanding (5.3):

$$\begin{aligned} I(\tau) = & \sum_{l=1}^{L_1} \alpha_{1,l} u_1(\tau - \tau_{1,l}) e^{j\beta_{1,l}} + \dots + \sum_{l=1}^{L_i} \alpha_{i,l} u_i(\tau - \tau_{i,l}) e^{j\beta_{i,l}} + \dots \\ & + \sum_{l=1}^{L_I} \alpha_{I,l} u_I(\tau - \tau_{I,l}) e^{j\beta_{I,l}}. \end{aligned} \quad (5.7)$$

where the $u_i(\tau)$ are the low pass interfering signals, $\alpha_{i,l}$ and $\beta_{i,l}$ are, respectively, the strength and phase of the l th path of the i th interferer, and $\tau_{i,l}$ is the associated delay of the l th path of the n th interferer. At the receiver, the identity of the source of any interference component is not important because the knowledge of a particular co-channel interfering path does not provide the desired information. This permits rewriting the summation of (5.7) in the following form:

$$I(\tau) = \sum_{i=1}^I \alpha_i u_i(\tau - \tau_i) e^{j\beta_i}. \quad (5.8)$$

Because each resolvable interfering path is a sum of random interference phasors, the magnitude of each interfering path may be assumed to be Nakagami distributed. Thus

$$f(\alpha_i) = \frac{2m_i^{m_i} \alpha_i^{2m_i-1} e^{-\left(\frac{m_i}{\Omega_i}\right)\alpha_i^2}}{\Gamma(m_i)\Omega_i^{m_i}}, \quad \alpha_i \geq 0 \quad (5.9)$$

and

$$f(I) = \frac{2m_I^{m_I} I^{2m_I-1} e^{-\left(\frac{m_I}{\Omega_I}\right)I^2}}{\Gamma(m_I)\Omega_I^{m_I}}, \quad I \geq 0 \quad (5.10)$$

Again,

$$m_I = \sum_i m_i, \quad (5.11)$$

and

$$\Omega_I = \sum_i \Omega_i. \quad (5.12)$$

As the Nakagami distribution mimics the Rice and Rayleigh distributions, the consideration of Nakagami interferers is sufficient to account for other interference fading scenarios.

5.3 Outage Analysis

The *pdf* of the desired signal's power, $f(x)$, where x and R are related by $x=R^2$ is found to be [Appendix 5A, (5A.5)]

$$f(x) = \left[\frac{m_d}{\Omega_d} \right]^{m_d} \frac{x^{m_d-1}}{\Gamma(m_d)} e^{-\left(\frac{m_d}{\Omega_d}\right)x}, \quad x \geq 0. \quad (5.13)$$

The statistical distribution of the magnitude of multipath self-interference is given by equation (5.10). The *pdf* of the power of the cochannel interference, i.e., $z=R^2$ may be obtained using the same approach as in arriving at (5.13), giving,

$$f(z) = \left[\frac{m_I}{\Omega_I} \right]^{m_I} \frac{z^{m_I-1}}{\Gamma(m_I)} e^{-\left(\frac{m_I}{\Omega_I}\right)z}, \quad z \geq 0 \quad (5.14)$$

To obtain outage probability, in this case, we are interested in the signal to multipath interference power ratio, $r=x/z$. The *pdf* of signal to interference power ratio r can be found as follows [Appendix 5A, (5A.11)]:

$$f(r) = \left[\left(\frac{m_d}{\Omega_d} \right)^{m_d} \left(\frac{m_I}{\Omega_I} \right)^{m_I} \frac{\Gamma(m_d + m_I) r^{m_d-1}}{\Gamma(m_d) \Gamma(m_I)} \right] / \left[\frac{m_d r}{\Omega_d} + \frac{m_I}{\Omega_I} \right]^{m_d + m_I}. \quad (5.15)$$

Note that the *pdf* of the signal to interference power ratio depends on the parameters of distribution of x and z , i.e., m_d , m_I , Ω_d and Ω_I .

The outage probability may now be obtained by evaluating the following integral.

$$P_{out} = \int_0^{R_I} f(r) dr, \quad (5.16)$$

where $f(r)$ is given by equation (5.15). This leads to the following expression for the outage probability, [Appendix 5A, equation (5A.21)],

$$P_{out} = \frac{m_d^{m_d-1} R_I^{m_d} \Gamma(m_d + m_I)}{SIR^{m_d} m_I^{m_d} \Gamma(m_d) \Gamma(m_I)} \cdot {}_2F_1\left(m_d + m_I, m_d; 1 + m_d; \frac{-(m_d/m_I)}{SIR} R_I\right). \quad (5.17)$$

The above expression involves an ordinary hypergeometric series. SIR is the average signal to interference ratio, defined as $SIR = \Omega_d/\Omega_I$.

Parameter m_d in (5.17) is given by (5.5). If it is assumed that $m_n=1$ in (5.5) then $m_d=N$ is an integer and corresponds to the number of fingers of the RAKE. Then (5.17) may be written as

$$P_{out} = \frac{N^{N-1} R_I^N \Gamma(N + m_I)}{SIR^N m_I^N \Gamma(N) \Gamma(m_I)} \cdot {}_2F_1\left(N + m_I, N; 1 + N; \frac{-(N/m_I)}{SIR} R_I\right). \quad (5.18)$$

Similarly, if in (5.11), $m_I=1$, as a result, $m_I=I$, and (5.18) becomes,

$$P_{out} = \frac{N^{N-1} R_I^N \Gamma(N + I)}{SIR^N I^N \Gamma(N) \Gamma(I)} \cdot {}_2F_1\left(N + I, N; 1 + N; \frac{-(N/I)}{SIR} R_I\right). \quad (5.19)$$

If N is not deterministic, then (5.18) may be regarded as conditional probability of outage, conditioned on N , such that,

$$P_{out} = Pr\{r < R_I | N\}. \quad (5.20)$$

The condition may be removed as follows:

$$P_{out} = \sum_{N=1}^{\infty} Pr\{r < R_I | N\} \cdot P(N), \quad (5.21)$$

where $P(N)$, the probability of N paths, is assumed as in (4.6),

$$P(N) = \frac{\lambda_N^N}{(N-1)!} e^{-\lambda_N}. \quad (5.22)$$

Using (5.22) in (5.21),

$$P_{out} = \sum_{N=1}^{\infty} \frac{\lambda_N^N}{(N-1)!} e^{-\lambda_N} \cdot \frac{N^{N-1} R_I^N \Gamma(N+I)}{SIR^N I^N \Gamma(N) \Gamma(I)} \cdot {}_2F_1\left(N+I, N; 1+N; \frac{-(N/I)}{SIR} R_I\right). \quad (5.23)$$

Equation (5.23) takes into account the random number of paths of the desired signals. In another scenario, N may be assumed to be known while I may be assumed to be random with probability of occurrence as given below. Using (5.22) and replacing N with I and λ_N with λ_I ,

$$f(I) = \sum_{I=1}^{\infty} \frac{\lambda_I^I}{(I-1)!} e^{-\lambda_I}. \quad (5.24)$$

The outage probability using (5.19) and (5.24) is given by as follows:

$$P_{out} = \frac{N^{N-1} R_I^N}{SIR^N I^N \Gamma(N)} \cdot \sum_{I=1}^{\infty} \frac{\lambda_I^I}{(I-1)!} e^{-\lambda_I} \frac{\Gamma(N+I)}{\Gamma(I)} {}_2F_1\left(N+I, N; 1+N; \frac{-(N/I)}{SIR} R_I\right). \quad (5.25)$$

When both N and I are assumed to be random, the outage probability is given by

$$P_{out} = \sum_{N=1}^{\infty} \frac{\lambda_N^N}{(N-1)!} e^{-\lambda_N} \cdot \frac{N^{N-1} R_I^N}{SIR^N I^N \Gamma(N)} \cdot \sum_{I=1}^{\infty} \frac{\lambda_I^I}{(I-1)!} e^{-\lambda_I} \cdot \frac{\Gamma(N+I)}{\Gamma(I)} \cdot {}_2F_1\left(N+I, N; 1+N; \frac{-(N/I)}{SIR} R_I\right), \quad (5.26)$$

which is the most general form.

5.3.1 Numerical Results

The outage expressions, defined in the previous section, may be used in mobile radio system design as discussed below. Figure 5.1 illustrates the effect of choice of SIR protec-

tion threshold. Increasing this threshold requires more average *SIR* to obtain the same outage probability. For example, for an outage probability of one percent and *SIR* protection thresholds of 7 dB, 13 dB and 20 dB, the respective required average *SIRs* are about 19 dB, 25 dB and 32 dB.

The outage expression in (5.19) is a function of R_f , N , I and *SIR* through Ω_d and Ω_f , where R_f is the *SIR* protection threshold and N is related to the length of the RAKE receiver. If I is assumed to be less than or equal to N then it signifies the absence or presence of interference on the fingers of the RAKE receiver. For example if $I=2$ and $N=4$, this corresponds to a situation where a four finger RAKE is used for reception while only two fingers experience interference. The parameters N and I in (5.19) are obtained through (5.5) and (5.11) which are the summation of m parameters of each resolvable desired and interfering path. It was assumed for convenience that all m_n 's and m_i 's are equal to one. These assumptions imply that the received and the interfering signals on fingers have Rayleigh fading strengths. Since it is based on these assumptions, equation (5.19) may appear to have limited scope but, as discussed later in this section, this is not necessarily the case.

The effect of change in the length of RAKE receiver is shown in Figure 5.2. Assuming $I=2$ and $R_f=7$ dB and keeping them fixed, N is changed in Figure 5.2 from 2 to 3 and 4. This is equivalent to increasing the number of fingers of the RAKE receiver. For an acceptable outage requirement of one percent, a gain of about 3.5 dB in average *SIR* is obtained when multipath diversity increases from 2 to 4. This increase in gain is more significant when the outage requirement is lower.

In Figure 5.3, N is assumed to be 4 while I is increased from 2 to 4. A number of important observations may be made. With comparison to the Figure 5.2 it may be observed that similar variation in I has not affected the outage probability as significantly as the change in N does. In another observation, the value of *SIR* protection threshold serves as a cross over point below which decreasing the value of I results in lower outage

Figure 5.1 Outage probability as a function of SIR in dB for the following parameters: $m_d=2$, $m_l=2$: Solid; $R_f=7$ dB, Dash-dash; $R_f=13$ dB, Dash-dot; $R_f=20$ dB.

Figure 5.2 Outage probability as a function of SIR in dB for the following parameters: $R_f=7$ dB, $l=2$: Solid; $N=2$, Dash-dash; $N=3$, Dash-dot; $N=4$.

Figure 5.3 Outage probability as a function of SIR in dB for the following parameters: $R_F = 7$ dB, $N=4$: Solid; $l=4$, Dash-dot; $l=3$, Dash-dot; $l=2$.

probability. However, the decrease is insignificant both in terms of the magnitude of the difference in outage for a given average SIR and because this region is not where one wants to operate due to high outage probability. In the region above the threshold, decreasing the value of I results in slight deterioration in the outage performance. It is rather surprising because one would expect improvement in the performance. To further investigate this observation, the *pdf* of the signal to interference power ratio given by (5.15) is plotted for two different values of SIR (one above and other below the SIR protection threshold) while keeping other parameters fixed. It should be noted that N , I and SIR correspond to m_d , m_i and the ratio of Ω_d and Ω_i respectively. Noting the difference in the horizontal scale of the Figure 5.4 and Figure 5.5, it may be observed that reducing the value of I for fixed N and average SIR results in a *pdf* whose tails are heavier. The value of average SIR effects the spread of the *pdf*, which is not surprising as SIR is the ratio of two spread related parameters: the desired and interfering signal powers. The outage probability, in most cases, is the lower tail probability of the *pdf* for a given threshold value.

The computation of outage probability, as the average SIR ratio is increased, is in fact moving from the use of the *pdf* shown in Figure 5.5 to the use of the *pdf* shown in Figure 5.4. It can be noted that in the case of average SIR being lower than the protection threshold of 7 dB, outage probability is high because the *pdf* in Figure 5.5 is very peaky and concentrated towards the lower values of SIR power ratio. The outage probability in Figure 5.5 is not just the probability of lower tail. In the case of Figure 5.4, the *pdf* has spread out and outage probability is now the tail probability. Further increasing the average SIR will increase the spread but it will also ensure that a large value of I results in a lower outage compared to that obtained with lower values of I .

Figure 5.6 uses (5.25) with $N=4$ and λ_i equal to 2, 3 and 4. A comparison with Figure 5.3 indicates that considering I as random affects the performance negligibly. This is further confirmed by plotting the outage probability as a function of I and λ_i for $N=4$ and

Figure 5.4 The *pdf* of Signal-to-Interference Power Ratio as a function of N , l , and average SIR . Average $SIR=20$ dB, $N=4$; $l=4$ (solid), $l=3$ (dashed) $l=2$ (dash-dot).

Figure 5.5 The *pdf* of Signal-to-Interference Power Ratio as a function of N , l , and average SIR . Average $SIR=5$ dB, $N=4$; $l=4$ (solid), $l=3$ (dashed) $l=2$ (dash-dot).

Figure 5.6 Outage probability as a function of SIR in dB for the following parameters: $R_F = 7$ dB, $N=4$: Solid; $\lambda_I = 4$, Dash-dot; $\lambda_I = 3$, Dash-dot; $\lambda_I = 2$.

average SIR of 5 and 20 dB. For I and λ_I greater than two the two curves nearly overlap. It can be concluded that consideration of the random nature of interfering paths may be ignored to reduce computational complexity.

When the random nature of the desired paths is taken into account, the calculated performance degrades. Figure 5.8 is based on (5.26) and assumes that N is random with λ_N equal to four. Three curves are drawn for λ_I equal to 2, 3 and 4. The variation of λ_I does not affect the performance significantly. In Figure 5.9, Figure 5.6 and Figure 5.8 have been superimposed to compare the known and random N cases with other parameters remaining the same. Performance degrades severely after an average SIR of about 15 dB.

Figure 5.10 is drawn to compare the outage performance when the average number of desirable paths change. The cases of λ_N equal to 2, 3 and 4 have been plotted with $I=2$. Decreasing the λ_N results in the requirement for a higher average SIR to obtain the same outage probability. For one percent outage objective a channel with $\lambda_N=2$ requires about 3.5 dB more average SIR than a channel with $\lambda_N=4$.

Figure 5.11 is obtained by superimposing Figure 5.2 and Figure 5.10 to help compare the performance of known versus random numbers of desired paths. For an outage objective of one percent the difference in additional average SIR is no greater than 4 dB. However, this increases with the lower outage objectives.

5.4 Comments on Frequency Selectivity/Non-Selectivity of Cochannel Interference

The possibility of using the Nakagami distribution to statistically characterize the resolvable paths and their RAKE combinations for both desired and interfering signals simplifies the radio link performance analysis. But it requires careful and clear interpretation of the m parameters, which have gained much more significance in the analysis compared to what has been traditionally assigned to them. One interesting observations is that,

Figure 5.7 Outage probability as a function of I (solid) and λ_I (dashed) for $N=4$, $R_F=7$ dB: $SIR=5$ dB for upper two curves and $SIR=20$ dB for lower two curves.

Figure 5.8 Outage probability as a function of SIR in dB for the following parameters; $R_F=7$ dB, $\lambda_N=4$: Solid; $\lambda_I=4$, Dash-dash; $\lambda_I=3$, Dash-dot; $\lambda_I=2$.

Figure 5.9 Figure 5.6 and the Figure 5.8 superimposed.

Figure 5.10 Outage probability as a function of SIR in dB's for the following parameters; $R_I = 7$ dB, $l=2$: $\lambda_N=4$: Solid; $\lambda_I=4$, Dash-dot; $\lambda_I=3$, Dash-dot-dot; $\lambda_I=2$.

Figure 5.11 Figure 5.2 and the Figure 5.10 superimposed.

except through proper interpretation of the m parameters, the narrowband and the wideband cases cannot be distinguished. Analytically, the problem is in a simple form and outage analysis of a single fading/non-dispersive Nakagami distributed desired signal and a single fading/non-dispersive Nakagami distributed interfering signal is also applicable to the wideband case. An earlier work [83] has looked into the outage probability for this case and gives the outage expression. The results in this thesis show the applicability of the same result for time dispersed Nakagami fading channels. The results have been derived independently and analysis is extended to account for random path arrivals due to dispersive nature of the channel.

5.5 Comments on Performance Methodology

One extension of the analysis here would seem to be to use an approach which segregates slow and fast fading and incorporates the slow fading. However, this extension is not recommended in this thesis. The primary purpose of the modeling and statistical characterization of the variability in the received signal envelope is to be able to predict the probability that the received signal goes below a certain threshold value anywhere in a cell service area. It was pointed out in Chapter 3 that segregation of slow and fast fading is a redundant procedure for Rice/Nakagami channels and as such does not add anymore information or accuracy in the outage prediction. Therefore, using a single distribution based on fading data in an entire cell makes more sense and should be a preferred choice.

The parameter values (i.e., variance) of the empirical distribution gained from measurements will account for the increased variability due to the confounding of slow and fast fading. References [55], [56], and [57] provide precedents wherein the researchers have used the statistical distribution of the slow fading to approximate both slow and fast fading when standard deviation of the slow fading is large compared to fast fading .

An earlier result of this thesis showing that the Nakagami distribution may be used to statistically characterize the slow fading can also be used. However, it is clear that the analysis and the results presented in this chapter remain valid.

5.6 Summary

This chapter investigated the radio link performance when both desired and cochannel interference signals are frequency selective and Nakagami fading. A system model was developed and outage expressions were found. The results can be used for frequency selective Rice/Rayleigh fading scenarios due to their relation to the Nakagami distribution. The effect of change in the fading parameters of desired and interfering signals was investigated. The results show that the performance variation due to variation in statistics of the desired signal is quite significant compared with the effect of variations in interference statistics.

The case of radio link performance for frequency selective interference manifesting as intersymbol interference is analyzed in the next chapter.

Appendix 5A Derivation of Equation (5.13), (5.14) and (5.17)

We want to find the *pdf* of x when the *pdf* of R is known to be given by

$$f(R) = \frac{2m_d^{m_d} R^{2m_d-1} e^{-\left(\frac{m_d}{\Omega_d}\right)R^2}}{\Gamma(m_d)\Omega_d^{m_d}} \quad R \geq 0 \quad (5A.1)$$

The *pdf* of x may be obtained through the transformation relationship,

$$x = R^2. \quad (5A.2)$$

The problem is thus reduced to finding the *pdf* of x from the *pdf* of R as given by equation (5A.1) through the transformation relationship given by equation (5A.2).

We know that we must have

$$f(x)dx = f(R)dR, \quad (5A.3)$$

which gives

$$f(x) = f_R(\sqrt{x}) \left[\frac{dg_n}{dx} \right] = \left(\frac{1}{2\sqrt{x}} \right) f_R(\sqrt{x}). \quad (5A.4)$$

Thus we have

$$f(x) = \left[\frac{m_d}{\Omega_d} \right]^{m_d} \frac{x^{m_d-1} e^{-\left(\frac{m_d}{\Omega_d}\right)x}}{\Gamma(m_d)} \quad x \geq 0 \quad (5A.5)$$

Similarly it can be shown that the *pdf* of z , when the *pdf* of I is known, may be obtained through the transformation relationship $z=I^2$ and using the above procedure. It is given by

$$f(z) = \left[\frac{m_I}{\Omega_I} \right]^{m_I} \frac{z^{m_I-1} e^{-\left(\frac{m_I}{\Omega_I}\right)z}}{\Gamma(m_I)} \quad z \geq 0 \quad (5A.6)$$

To find the *pdf* of r , the signal to cochannel interference ratio, we use the known relationship

$$f(r) = \int_0^{\infty} z f_x(rz) f(z) dz \quad r \geq 0, \quad (5A.7)$$

where the *pdf* of x is given by (5A.5) and the *pdf* of z , by (5A.6). Substituting in (5A.7), we have,

$$f(r) = \int_0^{\infty} z \left[\frac{m_d}{\Omega_d} \right]^{m_d} \frac{(rz)^{m_d-1}}{\Gamma(m_d)} e^{-\left(\frac{m_d}{\Omega_d}\right)rz} \left[\frac{m_l}{\Omega_l} \right]^{m_l} \frac{z^{m_l-1}}{\Gamma(m_l)} e^{-\left(\frac{m_l}{\Omega_l}\right)z} dz. \quad (5A.8)$$

After some algebraic manipulations we may write,

$$f(r) = \left(\frac{m_d}{\Omega_d} \right)^{m_d} \left(\frac{m_l}{\Omega_l} \right)^{m_l} \frac{r^{m_d-1}}{\Gamma(m_d)\Gamma(m_l)} \int_0^{\infty} z^{m_d+m_l-1} e^{-\left[\frac{m_d r}{\Omega_d} + \frac{m_l}{\Omega_l}\right]z} dz. \quad (5A.9)$$

Using the result, [70], that

$$\int_0^{\infty} z^{v-1} e^{-uz} dz = \frac{1}{(u)^v} \cdot \Gamma(v) \quad \text{for} \quad (u > 0, v > 0), \quad (5A.10)$$

we have

$$f(r) = \left[\left(\frac{m_d}{\Omega_d} \right)^{m_d} \left(\frac{m_l}{\Omega_l} \right)^{m_l} \frac{\Gamma(m_d+m_l) r^{m_d-1}}{\Gamma(m_d)\Gamma(m_l)} \right] / \left[\frac{m_d r}{\Omega_d} + \frac{m_l}{\Omega_l} \right]^{m_d+m_l}. \quad (5A.11)$$

In order to find the outage probability, first rewrite the equation (5A.11) as follows,

$$f(r) = \left[B^{m_d} \frac{\Gamma(m_d+m_l) r^{m_d-1}}{\Gamma(m_d)\Gamma(m_l)} \right] / [1 + Br]^{m_d+m_l} \quad (5A.12)$$

where,

$$B = \frac{(m_d/\Omega_d)}{(\Omega_d/\Omega_l)}. \quad (5A.13)$$

As the outage probability is given by

$$P_{out} = \int_0^{R_I} f(r) dr. \quad (5A.14)$$

we get

$$P_{out} = \int_0^{R_I} \left[B^{m_d} \frac{\Gamma(m_d + m_I) r^{m_d - 1}}{\Gamma(m_d) \Gamma(m_I)} \right] / [1 + Br]^{m_d + m_I} dr. \quad (5A.15)$$

Noting that the above integration is with respect to r , we can remove the terms independent of r from the integrand, giving

$$P_{out} = B^{m_d} \frac{\Gamma(m_d + m_I)}{\Gamma(m_d) \Gamma(m_I)} \int_0^{R_I} \frac{r^{m_d - 1}}{(1 + Br)^{m_d + m_I}} dr. \quad (5A.16)$$

Now we know that [70]

$$\int_0^U \frac{x^{\mu - 1}}{(1 + \beta x)^{\nu}} dx = \frac{U^{\mu}}{\mu} {}_2F_1(\nu, \mu; 1 + \mu; -\beta U) \quad [|\arg(1 + \beta U)| < \pi, \operatorname{Re} \mu > 0]. \quad (5A.17)$$

Using above relation we have the P_{out} in terms of the ordinary hypergeometric series as follows:

$$P_{out} = \frac{B^{m_d} \Gamma(m_d + m_I) R_I^{m_d}}{\Gamma(m_d) \Gamma(m_I) m_d} \cdot {}_2F_1(m_d + m_I, m_d; 1 + m_d; -BR_I). \quad (5A.18)$$

The signal to cochannel interference ratio (SIR) is defined as the average-signal power to average-interference power ratio. Thus,

$$SIR = \frac{\Omega_d}{\Omega_I}, \quad (5A.19)$$

and (5A.13) may be written as

$$B = \frac{(m_d/m_I)}{SIR}. \quad (5A.20)$$

as a result, P_{out} may now be written as

$$P_{out} = \frac{m_d^{m_d-1} R_I^{m_d} \Gamma(m_d + m_I)}{SIR^{m_d} m_I^{m_d} \Gamma(m_d) \Gamma(m_I)} \cdot {}_2F_1\left(m_d + m_I, m_d; 1 + m_d; \frac{-(m_d/m_I)}{SIR} R_I\right). \quad (5A.21)$$

which is the desired equation and same as Equation (5.17).

Chapter 6

Radio Link Performance in Multipath Self Interference

6.1 Introduction

In the previous chapter it was pointed out that two interference scenarios for frequency selective signals may arise. The first case considered the RAKE reception of frequency selective signal and interference and was dealt with in last chapter. In the second case frequency selectivity results in multipath induced inter-symbol interference, hereafter referred to as Multipath Self Interference or MSI. This chapter contains an analysis of mobile radio link performance in the presence of MSI. The performance measure is the outage probability. The MSI analysis quantifies the effects of the frequency selectivity of the desired signal which is manifested as inter-symbol interference. This problem was considered earlier by Yao and Sheikh [63]. The analysis of this chapter is different from [63] in two respects: it takes into account the random path arrivals and it uses a different approach to statistically characterize the MSI.

6.2 System Model

In a multipath environment, suppose $Re[v(\tau) \cdot e^{j\omega_0\tau}]$ is sent and $Re[v_r(\tau) \cdot e^{j\omega_0\tau}]$ is received. The received complex low pass signal, $v_r(\tau)$, is of the form:

$$v_r(\tau) = \sum_{n=1}^N g_n \cdot v(\tau - \tau_n) e^{j\theta_n}. \quad (6.1)$$

where $v(\tau)$ is the input waveform, g_n is a random multiplicative disturbance, τ_n is a random excess delay, and θ_n is a random phase. If one can assume that objects which cause multipath propagation are randomly located and oriented in space, then, as a result, the arrival of resolvable signal paths will follow the Poisson distribution [41], [51]. That is, the total number of signal paths N is Poisson distributed.

The received signal $v_r(\tau)$ may be written as;

$$v_r(\tau) = g_s v(\tau - \tau_s) + \sum_{k=0}^K g_k v(\tau - \tau_k) = v_s(\tau) + I(\tau) \quad (6.2)$$

where, $v_s(\tau)$ is the desired signal with the shortest propagation delay and is given by

$$v_s(\tau) = g_s(t) v(\tau - \tau_s). \quad (6.3)$$

Here, g_s is a complex random process and τ_s is the propagation delay of the shortest path. The magnitude or envelope of the g_s may be Nakagami, Rayleigh or Rician distributed while the associated random phase of g_s may be assumed to be uniformly or nonuniformly distributed over $[-\pi, \pi)$.

The $I(\tau)$ in Equation (6.2) is the interference term due to multipath and is given as:

$$I(\tau) = \sum_{k=0}^K g_k v(\tau - \tau_k). \quad (6.4)$$

Where the complex random processes g_k on the right hand side may be assumed to have a Rice/Rayleigh or Nakagami-distributed envelope and uniformly or nonuniformly distributed associated phase. The number of self interferers K is derived from N : specifically, $K=N-2$. K may thus be assumed to be Poisson distributed.

The paths in (6.1) are dispersed in time and are resolvable. However, they do cause interference in the reception of those symbols which were transmitted afterwards. As a result, at the symbol sampling instant, both desired and multipath interfering components will be present. Although the multipath interfering components will be from different transmitted pulses, the collection of them is assumed to exhibit a Poisson arrival process. It is assumed here that the statistical characteristics of (6.4) are similar to those of (6.1). In the next section (6.4) is statistically characterized.

6.3 Statistical Analysis of Multipath Self-Interference

Let us start with the expression of the shot noise as used by Rice [64], as given below:

$$I(\tau) = \sum_{k=0}^K a_k v(\tau - \tau_k). \quad (6.5)$$

where $a_1, a_2, \dots, a_k, \dots$ are independent random variables, all having the same distribution. It is assumed that all of the moments, $\overline{a^n}$, exist and that the events occur at random according to a Poisson distribution. From the definition, $I(\tau)$ may be considered as the output of a linear system with an impulse response $v(\tau)$ for input Poisson impulses, $p(\tau)$, as shown in Figure 6.1. With the role of $v(\tau)$ and $p(\tau)$ reversed, which is permitted due to commutative property of convolution, one can easily see that the functional form of the impulse noise as given by Equation (6.5) and the functional form of the multipath self interference from the channel as in Equation (6.4) are similar. This observation suggests that one can use the impulse noise model in modeling the self interference due to multipath propagation. Of course in this case, the statistical characterization of the impulsive noise would also be valid for the multipath self-interference as long as the pulse shape $v(\tau)$, random variable¹ a_k and pulse arrival process are chosen appropriately. This equiva-

1. If a_k is the same as g_k in (6.4) then (6.5) is equivalent to (6.4)

Figure 6.1 Illustration of the (a) shot noise and (b) multipath arrival process.

lence is used later to determine the outage probability due to MSI. In the following, some of the results pertaining to the statistical characterization of shot noise are summarized.

The shot noise, $I(\tau)$, may be represented by a sum of the following type [64] [65]:

$$I(\tau) = \sum_{k=0}^K v(\tau - \tau_k), \quad (6.6)$$

where $v(\tau)$ is pulse of some arbitrary shape starting at $\tau = \tau_k$. Parameter K is random and is statistically characterized by a Poisson distribution with parameter λ . Equation (6.6) is different from (6.5) in that it assumes all the impulses to have equal amplitude. Campbell's theorem states that the average value of $I(\tau)$ is [64] [65]

$$\overline{I(\tau)} = \lambda \int_{-\infty}^{\infty} v(\tau) d\tau, \quad (6.7)$$

and the mean square value of its fluctuation about this average is

$$\overline{(I(\tau) - \overline{I(\tau)})^2} = \lambda \int_{-\infty}^{\infty} v^2(\tau) d\tau. \quad (6.8)$$

The density function of $I(\tau)$ has been derived by Rice [64] in a Fourier integral form by using the method of characteristic functions as follows:

$$f(I) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\frac{\lambda}{2\pi} \int_{-\infty}^{\infty} (e^{juv(\tau)} - 1) d\tau} e^{-jIu} du. \quad (6.9)$$

This result in its present form is not applicable to our problem because all the contributing impulses have equal amplitudes. However, Rice [64] has stated an extension of Campbell's theorem which considers the case of impulses having randomly varying amplitudes. The starting point is the expression in (6.5) which includes the random multipliers a_k . The

extension states that the n th semi-invariant of the probability density $f(I)$ of I , where I is given by (6.5), is

$$v_n = \lambda a^n \int_{-\infty}^{\infty} v^n(\tau) d\tau. \quad (6.10)$$

Here λ is the expected number of events per second and $a_k = a$. v_n is related to the moments of the distribution of I . Thus if $m_1, m_2, \dots$, denote the first, second, ..., moments about zero, we have [64]

$$\begin{aligned} \bar{I} &= m_1 = v_1 \\ \bar{I}^2 &= m_2 = v_2 + v_1 m_1 \\ \bar{I}^3 &= m_3 = v_3 + 2v_2 m_1 + v_1 m_2 \\ &\dots \end{aligned} \quad (6.11)$$

The probability density $f(I)$ of I , that is, the magnitude of $I(\tau)$ as defined by (6.5), is found to be [64]

$$f(I) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-juu} \lambda \int_{-\infty}^{\infty} q(a) da \int_{-\infty}^{\infty} (e^{juav(\tau)} - 1) d\tau e^{-jlu} du, \quad (6.12)$$

where $q(a)$ is the density function of a . Equivalently,

$$f(I) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{juu} \sum_{n=1}^{\infty} \frac{(ju)^n v_n}{n!} e^{-jlu} du, \quad (6.13)$$

where the first exponential in the integrand on right hand side is the series expansion of characteristic function and v_n is the n th semi-invariant given by (6.10). This form of the pdf is of course not very useful. However, a more useful form in terms of an infinite series may be obtained [64]. Let us define a normalized random variable w and set

$$v_2 = \sigma^2. \quad (6.14)$$

$$w = \frac{I - v_1}{\sigma} = \frac{I - \bar{I}}{\sigma}, \quad (6.15)$$

and then expand $\exp\left[\sum_{n=3}^{\infty} \frac{(ju)^n v_n}{n!}\right]$ as a power series in u , integrating term-wise:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} (ju\sigma)^n \exp\left[-ju\sigma w - \frac{u^2 \sigma^2}{2}\right] du = (-1)^n \sigma^{-1} \phi^{(n)}(w), \quad (6.16)$$

where

$$\phi^{(n)}(w) = \frac{1}{\sqrt{2\pi} d w^n} e^{-\frac{w^2}{2}}. \quad (6.17)$$

Finally, collecting the terms according to their order in the power of $\phi^{(n)}(w)$ gives [64]

$$\begin{aligned} f(w) \approx & \sigma^{-1} \phi^{(0)}(w) - \frac{v_3 \sigma^{-4}}{3!} \phi^{(3)}(w) + \frac{v_4 \sigma^{-5}}{4!} \phi^{(4)}(w) - \frac{v_5 \sigma^{-6}}{5!} \phi^{(5)}(w) + \\ & \frac{v_3^2 \sigma^{-7}}{72} \phi^{(6)}(w) + \dots \end{aligned} \quad (6.18)$$

This is the Edgeworth series [74]. The first term gives the normal distribution and the remaining terms which depend on λ through v_n show that the normal distribution is approached as $\lambda \rightarrow \infty$. Equation (6.18), as mentioned before, is the *pdf* of the MSI term in (6.2), and is used in the following outage analysis.

6.4 Outage Analysis

In this analysis, the outage probability due to multipath effect or so called multipath self-interference (MSI) is determined using the following approach. The received signal is written in the form of (6.2) such that the desired and undesired signals can be separated. The statistics of the magnitudes of these two components and their powers is then deter-

mined. Finally, the statistics of the desired to undesired signal power ratio is determined and outage is obtained from the statistical distribution of this ratio. The major difficulty in this approach lies in the compact statistical characterization of the received signal components and their ratios.

A number of scenarios for outage analysis can be built. Each scenario is based on the statistical characterization of channel parameters. The following scenarios are investigated and summarized in Table 1;

- The envelopes of the desired signal and of the interfering multipath components are Rayleigh distributed. The number of multipaths is Poisson distributed. This scenario is named as "Rayleigh Rayleigh-Poisson".
- The desired signal envelope is Rayleigh faded but the amplitudes of the interfering multipath components are Nakagami distributed. The number of multipath is Poisson distributed. (Rayleigh/Nakagami-Poisson)
- The desired signal envelope is Nakagami faded and amplitudes of the multipath components are Nakagami distributed. The number of multipath is Poisson distributed. (Nakagami/Nakagami-Poisson)
- The desired signal envelope is Rician faded and amplitudes of the multipath components are Nakagami distributed. The number of multipath is Poisson distributed. (Rice/Nakagami-Poisson)

TABLE 6.1 Multiple fading scenarios

Scenario	Desired signal envelope distribution	Interfering signal envelope distribution	Path arrival process
1	Rayleigh	Rayleigh	Poisson
2	Rayleigh	Nakagami	Poisson
3	Nakagami	Nakagami	Poisson
4	Rice	Nakagami	Poisson

6.4.1 Rayleigh/Rayleigh-Poisson

Our first objective is to find *pdf* of the ratio of the desired signal power x to multipath interference power z . Assuming that the envelope of the desired signal transmitted through the shortest path is Rayleigh distributed, and the signal power measured over an RF cycle is $g_s^2/2$, the *pdf* of random variable g_s , where $X = E\{g_s^2\}$, is given by

$$f(g_s) = \frac{g_s}{X} e^{-\frac{g_s^2}{2X}} \quad ; \quad g_s \geq 0, \quad (6.19)$$

and the *pdf* of $x = g_s^2/2$ is found to be [11]

$$f(x) = f_{g_s}(2\sqrt{x}) \frac{dg_s}{dx} = \frac{1}{X} e^{-\frac{x}{X}} \quad ; \quad x \geq 0. \quad (6.20)$$

The *pdf* of the normalized amplitude of the self interference due to multipath w as defined by (6.14) and (6.15) is given by (6.18) on the same page, and the *pdf* of its power $f(z)$ may be found to be [Appendix 6A, Equation (6A.7) on page 154]

$$f(z) = \frac{1}{2\sigma\sqrt{2\pi}} \cdot z^{-1/2} e^{-z/2} - \frac{V_3}{4\sigma^4\sqrt{2\pi}} \cdot e^{-z/2} + \frac{V_3}{12\sigma^4\sqrt{2\pi}} \cdot z e^{-z/2} + \dots \quad (6.21)$$

Defining the signal to multipath interference power ratio as $r = x/z$, we can derive the *pdf* of r as follows [64]:

$$f(r) = \int_0^{\infty} z f_{x,z}(rz, z) dz \quad r \geq 0. \quad (6.22)$$

As the random variables x and z are independent, we have

$$f(r) = \int_0^{\infty} z f_x(rz) f(z) dz \quad r \geq 0. \quad (6.23)$$

Using equations (6.20) and (6.21) in (6.23) we obtain [Appendix 6A, Equation (6A.13) on page 155]

$$f(r) = \frac{1}{2\sigma X\sqrt{2\pi}} \left[\frac{\Gamma(3/2)}{\left(\frac{1}{2} + \frac{r}{X}\right)^{3/2}} - \frac{v_3\Gamma(2)}{2\sigma^3\left(\frac{1}{2} + \frac{r}{X}\right)^2} + \frac{v_3\Gamma(5/2)}{6\sigma^3\left(\frac{1}{2} + \frac{r}{X}\right)^{5/2}} + \dots \right] \quad r \geq 0. \quad (6.24)$$

Assuming that the signal to multipath interference protection ratio is R_I , the outage probability due to multipath interference is given by

$$\begin{aligned} P_{out} &= \text{Prob}(r < R_I) \\ &= \int_0^{R_I} f(r) dr, \end{aligned} \quad (6.25)$$

where $f(r)$ is the *pdf* of the signal to multipath interference ratio given by (6.24). If we replace $f(r)$ in (6.25) then after using [70]

$$\int_0^{R_I} \Gamma(n) \left(\frac{1}{2} + \frac{r}{X}\right)^{-n} dr = \frac{X\Gamma(n)}{1-n} \left[\left(\frac{1}{2} + \frac{R_I}{X}\right)^{1-n} - \left(\frac{1}{2}\right)^{1-n} \right], \quad (6.26)$$

we obtain

$$\begin{aligned} P_{out} &= \frac{1}{2\sigma\sqrt{2\pi}} \left\{ -2\Gamma(3/2) \left[\left(\frac{1}{2} + \frac{R_I}{X}\right)^{-1/2} - \left(\frac{1}{2}\right)^{-1/2} \right] + \frac{v_3\Gamma(2)}{2\sigma^3} \left[\left(\frac{1}{2} + \frac{R_I}{X}\right)^{-1} - \left(\frac{1}{2}\right)^{-1} \right] \right. \\ &\quad \left. - \frac{v_3\Gamma(5/2)}{9\sigma^3} \left[\left(\frac{1}{2} + \frac{R_I}{X}\right)^{-3/2} - \left(\frac{1}{2}\right)^{-3/2} \right] + \dots \right\}. \end{aligned} \quad (6.27)$$

From (6.27) it can be seen that outage probability is a function of parameters X , the mean power of the desired signal, and v_n , the n th semi-invariant of MSI, obtained as the coefficients in the expansion of the logarithm of the characteristic function [64]:

$$\ln(E\{e^{ju}\}) = \sum_{n=1}^N \frac{v_n}{n!} (ju)^n + o(u^n), \quad (6.28)$$

and given by the following equation:

$$v_n = \overline{\lambda a^n} \int_{-\infty}^{\infty} v^n(t) dt. \quad (6.29)$$

Here λ is the expected number of paths and $v(t)$ is the lowpass transmitted waveform. If we assume the transmitted waveform to be a rectangular pulse of duration T and height one, then we may have,

$$v_n = T \overline{\lambda a^n}. \quad (6.30)$$

The n th moment of the Rayleigh random variable, a , is given by [71]

$$\overline{a^n} = E\{a^n\} = (2X_a)^{n/2} \Gamma(1 + n/2). \quad (6.31)$$

where X_a is the mean power of the interfering multipath. Equation (6.30) may now be rewritten as,

$$v_n = T \lambda (2X_a)^{n/2} \Gamma(1 + n/2). \quad (6.32)$$

If outage probability is plotted as a function of the received mean signal power X , Figure 6.2 is obtained. As expected, outage probability is a decreasing function of the increasing mean signal power. Because the outage probability is obtained from an infinite sum of terms, it is of interest to look into the rate of convergence of this sum. In Figure 6.2, partial sums are shown for one, two, three and four terms by dashed, dotted, dash-dot and dash-dash lines, respectively. We can observe that the effect of inclusion of two to four terms doesn't increase the accuracy much.

However, a close look at (6.18) will reveal that the convergence of the outage expression is directly related to the Poisson parameter λ : the greater the value of λ , the faster the convergence. In Figure 6.2, the value of λ is assumed to be 3.55 paths/microsecond (a number borrowed from measurement results by Turin et al. [40]), while $T=1$ microsecond.

Figure 6.2 Partial sums of the outage probability formula (6.27) for scenario 1 as a function of X , for the following parameter values: $\lambda T=3.55$, $X_d=1$. Solid: one term; dotted: two terms; dash-dot: three terms, dash-dash: four terms.

Representing the sum by the first four terms of (6.18), we have plotted P_{out} vs. X for different values of λ as shown in Figure 6.3. The effect of different λ values on convergence is now evident. For $\lambda = 10$ and 3.5 , the convergence is good, however, when λ is less than one, i.e., $\lambda = 0.8$ and 0.6 , additional terms are needed for a more accurate plot.

There is another way to look at the above mentioned convergence phenomenon. In the following, for a given value of λ we obtain the P_{out} vs. X plot for partial sums of one, two, three and four terms depicted by solid, dotted, dash-dot and dash-dash curves, respectively. The two chosen values for λ are 0.6 and 10 . The respective plots are shown in Figure 6.4 and Figure 6.5.

In Figure 6.4, the addition of each term changes the resultant outage probability significantly. On the other hand in Figure 6.5, after the addition of second term, adding the other terms doesn't affect the outage probability appreciably. In general one may conclude that for the case of practical interest, i.e., $\lambda = 3.55$, in P_{out} expression, the sum of the first four terms of equation (6.18) should be considered accurate enough. However, for a smaller number of paths (an environment devoid of large number of reflectors and diffractors) a larger number of terms is necessary.

The effect of changing the signal to interference ratio threshold, R_I has been illustrated in the Figure 6.6. One can observe that for higher threshold values higher signal power is required to obtain the same outage probability.

6.4.2 Rayleigh/Nakagami-Poisson

In this second case the outage probability is still given by (6.27), the only difference being that the n th moments in (6.32) for the n th semi-invariant of the *pdf* of self interference are obtained from a Nakagami distribution. The n th moment of a Nakagami distribution is given as [36]

Figure 6.3 Scenario 1 outage probability as a function of X , for $R_1=5$, $X_g=1$. Solid: $\lambda T=10$; dotted: $\lambda T=3.55$; dash-dot: $\lambda T=0.8$; dash-dash: $\lambda T=0.6$. P_{out} approximated by the first four terms of Equation (6.27) on page 140.

Figure 6.4 Partial sums of scenario 1 outage probability as a function of X for the following parameter value: $R_1=5$, $\lambda T=0.6$, $X_g=1$. Solid: one term; dotted: two terms; dash-dot: three terms; dash-dash: four terms of Equation (6.27) on page 140.

Figure 6.5 Partial sums of scenario 1 outage probability as a function of X for the following parameter value: $R_f=5$, $\lambda T=10$, $X_a=1$. Solid: one term; dotted: two terms; dash-dot: three terms; dash-dash: four terms of Equation (6.27) on page 140.

Figure 6.6 Scenario 2 outage probability as a function of X , for the following parameter values: $\lambda T=3.55$, $X_a=1$. Solid: $R_f=5$; dotted: $R_f=10$; dash-dot: $R_f=100$.

$$\overline{a^n} = E\{a^n\} = \frac{\Gamma(m+n/2)}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{n/2}. \quad (6.33)$$

where n is a positive number, and m and Ω are Nakagami parameters defined as $m = \Omega / (E\{(a^2 - \Omega)^2\})$ and $\Omega = E\{a^2\}$. For $m=1$ and $\Omega = 2X_a$, the Nakagami distribution degenerates to the Rayleigh distribution and (6.33) assumes the form in (6.31).

Equation (6.32) in this scenario becomes:

$$v_n = T\lambda \frac{\Gamma(m+n/2)}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{n/2}. \quad (6.34)$$

In Figure 6.7, outage probability is plotted for this scenario, for the parameter values specified in the figure caption. The main observation is the change in the outage probability as the value of Nakagami m parameter is changed: interestingly, increasing m results in poorer outage performance. This can be interpreted as follows. As m is the so called fading figure such that increasing the m means lesser amounts of fading in the interfering paths, therefore in this case a larger value of m corresponds to stronger multipath self-interference components, hence poorer performance. This scenario should be synonymous in effect to one where the desired path experiences severer fading relative to the interfering paths.

It is also evident that for $m=1$ the outage performance is same as that of the Rayleigh/Rayleigh-Poisson model because in this case the Nakagami distribution degenerates into Rayleigh distribution.

6.4.3 Nakagami/Nakagami-Poisson

In this scenario the desired signal is assumed to follow a Nakagami distribution whose amplitude *pdf* is given as

Figure 6.7 Scenario 2 outage probability as a function of X , for the following parameter values: $R_f=5$, λ , $T=3.55$, $\Omega=1$. Solid: $m=1$; dotted: $m=2$; dash-dot: $m=3$; dash-dash: $m=4$.

$$f(g_s) = \frac{2m_d^{m_d} g_s^{2m_d-1} e^{-\left(\frac{m_d}{\Omega_d}\right)g_s^2}}{\Gamma(m_d)\Omega_d^{m_d}}, \quad g_0 \geq 0 \quad (6.35)$$

where m_d and Ω_d are Nakagami parameters. The *pdf* of $f(x)$, where $x=g_s^2/2$, is given as follows after the transformation:

$$f(x) = \left[\frac{2m_d}{\Omega_d}\right]^{m_d} \frac{x^{m_d-1}}{\Gamma(m_d)} e^{-\frac{2m_d}{\Omega_d}x}, \quad x \geq 0 \quad (6.36)$$

The *pdf* of z is assumed to be given by (6.21). The *pdf* of the ratio $r=x/z$ can be found to be [Appendix 6B, Equation (6B.8) on page 157]:

$$f(r) = \frac{1}{2\sigma\sqrt{2\pi}} \left[\frac{2m_d}{\Omega_d}\right]^{m_d} \frac{(r)^{m_d-1}}{\Gamma(m_d)} \left[\frac{\Gamma(m_d + 1/2)}{\left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)^{m_d + 1/2}} - \frac{v_3 \Gamma(m_d + 1)}{2\sigma^3 \left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)^{m_d + 1}} + \frac{v_3 \Gamma(m_d + 2)}{6\sigma^3 \left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)^{m_d + 2}} + \dots \right] \quad (6.37)$$

The outage probability may now be calculated by numerically evaluating the integral

$$P_{out} = \int_0^{R_t} f(r) dr. \quad (6.38)$$

where $p(r)$ is given by Equation (6.37) on page 148. The following expression for P_{out} is derived in Appendix 6B [Appendix 6B, Equation (6B.14)]:

$$\begin{aligned}
P_{out} = & \left[A 2^{m_d+1/2} \Gamma(m_d+1/2) \frac{R_I^{m_d}}{m_d} {}_2F_1(m_d+1/2, m_d; 1+m_d; -BR_I) - \right. \\
& A \frac{\nu_3 2^{m_d+1} \Gamma(m_d+1) R_I^{m_d}}{2\sigma^3} \frac{R_I^{m_d}}{m_d} {}_2F_1(m_d+1, m_d; 1+m_d; -BR_I) + \\
& \left. A \frac{\nu_3 2^{m_d+2} \Gamma(m_d+2) R_I^{m_d}}{6\sigma^3} \frac{R_I^{m_d}}{m_d} {}_2F_1(m_d+2, m_d; 1+m_d; -BR_I) + \dots \right], \quad (6.39)
\end{aligned}$$

where, $A = \frac{1}{2\sigma\sqrt{2\pi}\Gamma(m_d)} \left[\frac{2m_d}{\Omega_d} \right]^{m_d}$ and $B = \frac{4m_d}{\Omega_d}$.

The outage probability is again plotted for this scenario. In Figure 6.8 the effect of change in the mean square power Ω_d of the desired signal component on outage probability is plotted for different values of m_d , the desired signal component fading figure, while keeping other parameters fixed. The results show that there is a threshold value of Ω_d . Above this threshold, increasing the value of m_d results in better outage; below it, increasing the value of m_d actually increases outage slightly.

Figure 6.9 shows that for Scenario 3, as for Scenario 2, the effect of an increase in the m parameter value of the distribution of the multipath self-interference components is to increase the probability of outage.

6.4.4 Rice/Nakagami-Poisson

If one assumes that the envelope of the desired signal is Rician distributed, and the signal power measured over a one RF cycle is $g_s^2/2$, the *pdf* of random variable g_s is given by:

$$f(g_s) = \frac{g_s}{X} \cdot e^{-\frac{(g_s^2 + s^2)}{2X}} I_0\left(\frac{g_s s}{X}\right) \quad ; \quad g_s \geq 0, \quad (6.40)$$

where $X = E\{g_s^2\}$. From this, the *pdf* of $x = g_s^2/2$ is found to be

$$f(x) = f_{g_i}(2\sqrt{x}) \frac{dg_s}{dx} = \frac{1}{X} e^{-\frac{2x+s^2}{2X}} I_0\left(\frac{s\sqrt{2x}}{X}\right), \quad x \geq 0 \quad (6.41)$$

which is a non-central chi square distribution with two degrees of freedom.

It has been shown earlier in this thesis (Chapter 3) that the Rician *pdf* can be approximated by the Nakagami distribution by relating their parameters through a simple relations. This is desirable because Rician *pdf* contains higher mathematical functions (such as Bessel functions). Therefore outage probability may be approximately given by (6.39), where $p(r)$ is given by (6.37) and parameter m_d is obtained from (3.25) in terms of the Rician K factor. As an illustration, the outage probability curves shown in Figure 6.8 for m_d equal to 1, 2 and 3, correspond to similar curves in Scenario 4 for K values of 0, 2.41, and 4.44 respectively.

One could use the linear approximation of (3.29) instead of (3.25). In this case, the corresponding estimated K values are 0.17, 2.47 and 4.47 respectively. The use of the exact expression (3.25) is advocated for the conversion between K and m_d as there is not much to gain from the use of the approximate expression. However it may be noted that the difference between values of K provided by the exact and approximate relations is relatively insignificant as the value of m is increased.

Similarly a Rice/Rice-Poisson model may be developed in which both m parameters of the Nakagami distributions for signal and the multipath self interference can be found for given values of K by equation (3.25).

6.5 Summary

In this chapter, the outage probability due to MSI with Poisson arrival process was investigated. It has been shown that the impulse noise models may be used to characterize the multipath self interference. A number of scenarios built around the different statistical

Figure 6.8 Outage probability as a function of Ω_d , for the following parameters; $R_f=5$, $\lambda T=3.55$, $\Omega=1$, $m=1$: Solid: $m_d=1$, Dotted: $m_d=2$; Dash-dot: $m_d=3$

Figure 6.9 Outage probability as a function of Ω_d , for the following parameters; $R_f=5$, $\lambda T=3.55$, $\Omega=1$, $m_d=2$: Solid: $m=1$, Dotted: $m=2$; Dash-dot: $m=3$

characterizations of the magnitudes of desired signal and interferers were analyzed. Outage probability expressions in the form of an infinite series were derived. It was shown that the rate of convergence of these series is dependent on the value of Poisson parameter, λ . For λ values of interest, which were obtained through measurements, it was found that the summation of first few terms gives sufficiently accurate results. Based on these results, a number of conclusions about performance were derived:

The outage probability was found to be a decreasing function of mean signal power (Figure 6.2-Figure 6.9). When the signal-to-interference protection ratio threshold is increased more signal power is needed to obtain the same outage probability (Figure 6.6). When interferers are assumed to have Nakagami distributed magnitudes, larger value of m (interfering path fading factor) results in poorer performance (Figure 6.7). The case of both the desired and the interfering signals having Nakagami distributed magnitudes was also considered. If the m parameter value (fading factor) of the desired signal is increased performance improves above a certain mean signal power and degrades below it. The rate of improvement above the mean signal power threshold is higher than the rate of degradation below it. Finally, it was argued that the performance of the Rice-Nakagami-Poisson model may be obtained through use of the Nakagami/Nakagami-Poisson model considering the fact that a Rice distribution may be approximated by a Nakagami distribution.

The next chapter summarizes the results of this thesis research. The chapter also contains proposals for future work to follow up.

Appendix 6A Derivation of Equation (6.21) and Equation (6.24)

The *pdf* of $z = w^2$ is to be found when the *pdf* of w is known to be given by

$$f(w) = \sigma^{-1} \varphi^{(0)}(w) - \frac{v_3 \sigma^{-4}}{3!} \varphi^{(3)}(w) + \frac{v_4 \sigma^{-5}}{4!} \varphi^{(4)}(w) - \frac{v_5 \sigma^{-6}}{5!} \varphi^{(5)}(w) + \frac{v_3^2 \sigma^{-7}}{72} \varphi^{(6)}(w) + \dots \quad (6A.1)$$

We know that we must have

$$f(z)dz = f(w)dw, \quad (6A.2)$$

which gives

$$f(z) = f_w(\sqrt{z}) \frac{dw}{dz} = \frac{1}{2\sqrt{z}} f(\sqrt{z}) = \frac{1}{2w} f(w). \quad (6A.3)$$

From Equation (6.17) on page 137

$$\varphi^{(n)}(w) = \frac{1}{\sqrt{2\pi}} \frac{d^n}{dw^n} e^{-\frac{w^2}{2}}. \quad (6A.4)$$

Equation (6A.1), therefore, may be rewritten as:

$$f(w) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{w^2}{2}} - \frac{v_3}{\sqrt{2\pi} \cdot \sigma^4 \cdot 3!} (3w - w^3) e^{-\frac{w^2}{2}} + \dots \quad (6A.5)$$

The derivation of Equation (6A.5) makes repeated use of the relation

$$\frac{d^n}{dw^n} \left(w^n \cdot e^{-\frac{w^2}{2}} \right) = (nw^{n-1} - w^{n+1}) e^{-\frac{w^2}{2}}. \quad (6A.6)$$

Using Equation (6A.5) in Equation (6A.3) and after some manipulation,

$$f(z) = \frac{1}{2\sigma\sqrt{2\pi}} \cdot z^{-1/2} e^{-z/2} - \frac{v_3}{4\sigma^4\sqrt{2\pi}} \cdot e^{-z/2} + \frac{v_3}{12\sigma^4\sqrt{2\pi}} \cdot ze^{-z/2} + \dots \quad (6A.7)$$

This is the same as Equation (6.21) on page 139.

Recall that the *pdf* of the signal to multipath interference power ratio r is given by Equation (6.23) on page 139:

$$f(r) = \int_0^{\infty} z f_x(rz) f(z) dz \cdot \quad r \geq 0 \quad (6A.8)$$

Also, the *pdf* of x , the power of the desired signal, is available from Equation (6.20) on page 139:

$$f(x) = \frac{1}{X} e^{-\frac{x}{X}} \quad ; \quad x \geq 0, \quad (6A.9)$$

Using (6A.7) and (6A.9) in (6A.8),

$$f(r) = \int_0^{\infty} z \frac{1}{X} e^{-\frac{rz}{X}} \left[\frac{1}{2\sigma\sqrt{2\pi}} \cdot z^{-1/2} e^{-z/2} - \frac{v_3}{4\sigma^4\sqrt{2\pi}} \cdot e^{-z/2} + \frac{v_3}{12\sigma^4\sqrt{2\pi}} \cdot ze^{-z/2} + \dots \right] dz \cdot \quad (6A.10)$$

After some algebraic manipulations,

$$f(r) = \left\{ \left[1/(2\sigma X\sqrt{2\pi}) \right] \int_0^{\infty} z^{1/2} e^{-\left[\frac{1}{2} + \frac{r}{X}\right]z} dz - \frac{v_3}{4\sigma^4 X\sqrt{2\pi}} \int_0^{\infty} ze^{-\left[\frac{1}{2} + \frac{r}{X}\right]z} dz + \frac{v_3}{12\sigma^4 X\sqrt{2\pi}} \int_0^{\infty} z^{3/2} e^{-\left[\frac{1}{2} + \frac{r}{X}\right]z} dz + \dots \right\} \cdot \quad (6A.11)$$

Using the result [70] that

$$\int_0^{\infty} z^{v-1} e^{-uz} dz = \frac{1}{(u)^v} \cdot \Gamma(v) \quad \text{for} \quad (u > 0, v > 0). \quad (6A.12)$$

the following relation is obtained:

$$f(r) = \frac{1}{2\sigma X \sqrt{2\pi}} \left[\frac{\Gamma(3/2)}{\left(\frac{1}{2} + \frac{r}{X}\right)^{3/2}} - \frac{v_3 \Gamma(2)}{2\sigma^3 \left(\frac{1}{2} + \frac{r}{X}\right)^2} + \frac{v_3 \Gamma(5/2)}{6\sigma^3 \left(\frac{1}{2} + \frac{r}{X}\right)^{5/2}} + \dots \right] \quad r \geq 0. \quad (6A.13)$$

which is the same as Equation (6.24) on page 140.

Appendix 6B Derivation of Equation (6.37) and Equation (6.39)

Recall that the *pdf* of the signal to multipath interference power ratio r is given by Equation (6.23) on page 139:

$$f(r) = \int_0^{\infty} z f_x(rz) f(z) dz \quad r \geq 0. \quad (6B.1)$$

Using now the *pdf* of x , the power of the desired signal, from Equation (6.36) on page 148

$$f(x) = \left[\frac{2m_d}{\Omega_d} \right]^{m_d} \frac{x^{m_d-1}}{\Gamma(m_d)} e^{-\left(\frac{2m_d}{\Omega_d}\right)x}, \quad x \geq 0, \quad (6B.2)$$

and the *pdf* of z from Equation (6.21) on page 139 or Equation (6A.7) on page 154,

$$f(z) = \frac{1}{2\sigma\sqrt{2\pi}} \cdot z^{-1/2} e^{-z/2} - \frac{v_3}{4\sigma^4\sqrt{2\pi}} \cdot e^{-z/2} + \frac{v_3}{12\sigma^4\sqrt{2\pi}} \cdot ze^{-z/2} + \dots \quad (6B.3)$$

and substituting (6B.2) and (6B.3) in (6B.1), we get

$$f(r) = \int_0^{\infty} z \left[\frac{2m_d}{\Omega_d} \right]^{m_d} \frac{(rz)^{m_d-1}}{\Gamma(m_d)} e^{-\left(\frac{2m_d}{\Omega_d}\right)rz} \left[\frac{1}{2\sigma\sqrt{2\pi}} \cdot z^{-1/2} e^{-z/2} - \frac{v_3}{4\sigma^4\sqrt{2\pi}} \cdot e^{-z/2} + \frac{v_3}{12\sigma^4\sqrt{2\pi}} \cdot ze^{-z/2} + \dots \right] dz. \quad (6B.4)$$

After some algebraic manipulations,

$$f(r) = C \left[\int_0^\infty z^{m_d-1/2} e^{-\left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)z} dz - \frac{v_3}{2\sigma^3} \int_0^\infty z^{m_d} e^{-\left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)z} dz + \right. \\ \left. \frac{v_3}{6\sigma^3} \int_0^\infty z^{m_d+1} e^{-\left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)z} dz + \dots \right], \quad (6B.5)$$

where C is given by;

$$C = \frac{1}{2\sigma\sqrt{2\pi}} \left[\frac{2m_d}{\Omega_d} \right]^{m_d} \frac{\Gamma(m_d)}{\Gamma(m_d)}. \quad (6B.6)$$

Using the result, [70], that

$$\int_0^\infty z^{v-1} e^{-uz} dz = \frac{1}{(u)^v} \cdot \Gamma(v) \quad \text{for} \quad (u > 0, v > 0). \quad (6B.7)$$

(6B.6) becomes

$$f(r) = \frac{1}{2\sigma\sqrt{2\pi}} \left[\frac{2m_d}{\Omega_d} \right]^{m_d} \frac{\Gamma(m_d)}{\Gamma(m_d)} \left[\frac{\Gamma(m_d+1/2)}{\left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)^{m_d+1/2}} - \frac{v_3 \Gamma(m_d+1)}{2\sigma^3 \left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)^{m_d+1}} + \right. \\ \left. \frac{v_3 \Gamma(m_d+2)}{6\sigma^3 \left(\frac{2m_d r}{\Omega_d} + \frac{1}{2}\right)^{m_d+2}} + \dots \right], \quad (6B.8)$$

which is same as Equation (6.37) on page 148.

The outage probability may be found by evaluating the following integral.

$$P_{out} = \int_0^{R_t} f(r) dr. \quad (6B.9)$$

Let us rewrite the (6B.8) as follows:

$$f(r) = A \left[\frac{r^{m_d-1} 2^{m_d+1/2} \Gamma(m_d+1/2)}{(1+Br)^{m_d+1/2}} - \frac{v_3 r^{m_d-1} 2^{m_d+1} \Gamma(m_d+1)}{2\sigma^3 (1+Br)^{m_d+1}} + \right. \\ \left. \frac{v_3 r^{m_d-1} 2^{m_d+2} \Gamma(m_d+2)}{6\sigma^3 (1+Br)^{m_d+2}} + \dots \right], \quad (6B.10)$$

where $A = \frac{1}{2\sigma\sqrt{2\pi}\Gamma(m_d)} \left[\frac{2m_d}{\Omega_d} \right]^{m_d}$ and $B = \frac{4m_d}{\Omega_d}$. Thus.

$$P_{out} = A \int_0^{R_i} \frac{r^{m_d-1} 2^{m_d+1/2} \Gamma(m_d+1/2)}{(1+Br)^{m_d+1/2}} dr - A \int_0^{R_i} \frac{v_3 r^{m_d-1} 2^{m_d+1} \Gamma(m_d+1)}{2\sigma^3 (1+Br)^{m_d+1}} dr + \\ A \int_0^{R_i} \frac{v_3 r^{m_d-1} 2^{m_d+2} \Gamma(m_d+2)}{6\sigma^3 (1+Br)^{m_d+2}} dr + \dots \quad (6B.11)$$

Rearranging the terms gives

$$P_{out} = A 2^{m_d+1/2} \Gamma(m_d+1/2) \int_0^{R_i} \frac{r^{m_d-1}}{(1+Br)^{m_d+1/2}} dr - \\ A \frac{v_3 2^{m_d+1} \Gamma(m_d+1)}{2\sigma^3} \int_0^{R_i} \frac{r^{m_d-1}}{(1+Br)^{m_d+1}} dr + \\ A \frac{v_3 2^{m_d+2} \Gamma(m_d+2)}{6\sigma^3} \int_0^{R_i} \frac{r^{m_d-1}}{(1+Br)^{m_d+2}} dr + \dots \quad (6B.12)$$

Now it is known that [70]

$$\int_0^U \frac{x^{\mu-1}}{(1+\beta x)^v} dx = \frac{U^\mu}{\mu} {}_2F_1(v, \mu; 1+\mu; -\beta U) \quad [|\arg(1+\beta U)| < \pi, \operatorname{Re} \mu > 0]. \quad (6B.13)$$

Using above relation, the P_{out} may be written in terms of the ordinary hypergeometric series as follows:

$$\begin{aligned}
P_{out} = & \left[A 2^{m_d + 1/2} \Gamma(m_d + 1/2) \frac{R_I^{m_d}}{m_d} {}_2F_1(m_d + 1/2, m_d; 1 + m_d; -BR_I) - \right. \\
& A \frac{v_3 2^{m_d + 1} \Gamma(m_d + 1) R_I^{m_d}}{2\sigma^3} \frac{R_I^{m_d}}{m_d} {}_2F_1(m_d + 1, m_d; 1 + m_d; -BR_I) + \\
& \left. A \frac{v_3 2^{m_d + 2} \Gamma(m_d + 2) R_I^{m_d}}{6\sigma^3} \frac{R_I^{m_d}}{m_d} {}_2F_1(m_d + 2, m_d; 1 + m_d; -BR_I) + \dots \right].
\end{aligned} \tag{6B.14}$$

which is Equation (6.39) on page 149.

Chapter 7

Summary of Results and Further Work

7.1 Introduction

This chapter summarizes the results of this thesis and suggests further work which could ensue from it. The summary of the results is presented in Section 7.2; it is intended to provide the reader with a coherent understanding of the impact of this thesis research. In Section 7.3, future work is proposed.

7.2 Summary of Results

Chapter 1 mentioned that there may be three approaches to characterize communication channels [3], namely, the *physiogeometric*, *system-function* and *performance-index* approaches. This thesis research focused on the physiogeometric approach to channel characterization, concerning itself with the propagation mechanism, physical processes and channel geometries. The physiogeometric approach is dynamic in the sense that new communication systems are most likely to operate in new environments and will require the study of new propagation mechanisms, physical processes and associated geometries. While the system-function approach will be applicable to model the communication channel of a new system, the statistical characterization and associated physical interpretation may not necessarily remain the same as in any previous system.

The main goal of this thesis research was to explore the nature of the parameters of the statistical distributions of the wireless multipath channel parameters, i.e., investigations of the physical bases, their variations in practical cases of interest, and their application in BER and link performance studies to help understand the impact of the parameter characteristics on performance methodologies as well as the performance results.

Based on the system function approach, the wireless multipath channel may be characterized by its time-variant impulse response as in Equation (2.1) on page 13 and a signal received through this channel may be written in most general form as in Equation (2.5) on page 15. There are four parameters in Equation (2.5): the random multiplicative disturbance g_n , the random delay τ_n , and the random phase θ_n associated with the n th path, and N the total number of multipath signals. The mathematical channel model of Equation (2.5) is applicable to all multipath channels, e.g., indoor or outdoor wireless channels. However, the difference may lie in the statistical characterization of the four parameters.

When the statistical characterization of the parameters of two different channels is the same, the difference between the channels may lie in the range of parameter values exhibited by each. Even if this is not differentiable, then there should be a difference in physical mechanism which gives rise to a specific parameter range of a given statistical distribution.

Aside from the knowledge of a particular statistical distribution which a channel parameter may assume, it is extremely important to understand the physical mechanism which is responsible for the parameter values obtained for that statistical distribution. This increases the capability of a system designer to understand the consequences of the design options and provides him/her with the necessary knowledge (of the flexibility available to him/her) to improve system performance. For example, the knowledge that availability of a direct path between transmitter and receiver will result in the migration of the statistical

characterization of multiplicative disturbance from Rayleigh distribution to the Rice distribution may be used to advantage in the microcellular environment.

With this in mind, this thesis research delved deeper into the investigation of the physical basis of the statistical characterization of the channel parameters and the parameters of these statistical distributions as discussed below. Among the four channel parameters, the main focus is on the multiplicative disturbance exhibited in the form of fading. The secondary focus is on the random nature of the total number of multipath signal components.

Initial contributions of this thesis pertain to the understanding of the nature of slow and fast fading with relation to the multipath signal components in line of sight channels. With the help of channel simulations, the material presented previously

- shows that the multipath component may be separated into two distinct groups, one responsible for slow fading and the other for fast fading. The group causing slow fading consists of stronger multipath components while weaker components are the source of fast fading.
- shows that the cause of fast fading is the diffracted components and they form a complex Gaussian process when summed alone.
- argues that the notion of shadow fading in line of sight channels is incorrect; instead the slow fading may be termed as deceptive shadowing. This is due to the physical mechanism of slow fading which is not obstruction of strong signal paths but the phasor summation of strong paths.
- shows that the slow fading may also be characterized by Nakagami distribution. The author disagrees with the physical basis of lognormally distributed slow fading as proposed by Suzuki [39], because the mathematical model is not multiplicative but is addition of multiplied gains. Suzuki's proposal results in no sound physical justification for a lognormal distribution. The author

argues that the use of the Nakagami distribution for slow fading characterization is as good as the lognormal distribution, if not better.

Further contributions of this thesis relate to the issue of segregation of slow and fast fading components from a fading envelope and the extraction of the types of statistical distribution and their parameters. With the help of simulation and measurement results, the thesis shows that

- the parameters of the Rician distributed fast fading are dependent on the choice of moving mean normalization interval. The K parameter is a function of effective direct path which is the phasor sum of strong contributing paths instead of one direct path and is termed the effective K parameter.
- the moving mean normalization procedure is redundant for the Rician channel as with or without the procedure the value of the Rice K parameter remains the same. This is because slow and fast fading components are additive not multiplicative. It may also be concluded that Rician fading channels do not permit the multiplicative segregation of slow and fast fading.

These results have an impact on the statistical characterization of the fading envelope everywhere in the coverage area. In this regard,

- the statistical distribution of the Rice/lognormal fading model is derived. It takes into account both K and Ω parameters of the Rice distribution, as K is a function of Ω and thus is also random. Expressions for a Nakagami/lognormal fading model are also derived.
- it is argued that phase associated with a complex Nakagami fading envelope may be approximated by the associated phase of Complex Rician fading envelope by using the parameter relation between the Rice and Nakagami distributions.

In addition to previously mentioned contributions, this thesis contributed towards better understanding of the dispersive/fading Nakagami channel hypothesis. There is ample evidence in literature for the empirical fit of Nakagami distribution on the fading envelope.

lope. However, the physical basis for Nakagami fading has not been very clear. Dwelling upon the fact that both Rice and Nakagami distributions are based on the random phasor sum modeling of the fading process it is argued that the physical basis of one should apply to the other. As the physical model behind Rice distribution is well known the same should apply to the Nakagami distribution. Nakagami, who called the Rice distribution the “ n distribution”, provided the parameter relation between the two distributions. The relation between Nakagami m and Rice K parameter was found to be non-linear.

It is shown here that a linear approximation can be made to the non-linear relation between the m and K parameters. On this basis the author proposes that the Rice physiogeometric model may be used for the Nakagami physiogeometric model and that the physical interpretation of m parameter is similar to that of the K parameter. The validity of the approximation has been tested based on simulation and has been found satisfactory.

For dispersive Nakagami fading channels the thesis shows that, based on time orthogonality of the dispersive multipath received signal, an interesting interpretation may be made:

- it is shown that a geometric interpretation may be attached to the dispersive multipath received signal. As a consequence of that interpretation it is possible to show that the resultant signal is Nakagami distributed. In doing so, a new interpretation of the m parameter in terms of order of diversity emerges.
- It is shown that BER performance for the non-dispersive and dispersive cases is the same. Therefore the m parameter of each contributing path, which is related to the m parameter of the resultant signal through summation, may also be interpreted as a measure of multipath diversity in comparison with the diversity gain in the Rayleigh fading case.

Due to the relationship between the K and m parameters, it is clear that dispersive Rician fading channels may also be similarly interpreted after appropriate transformation of the parameter values.

Armed with the above understanding of the channel, BER and radio link performance is investigated.

For a dispersive Nakagami fading channel, BER performance of the RAKE receiver is investigated with the realistic assumption that in a real channel the total number of paths will not be known and in this case

- there is a performance degradation which is quantified by extending the analysis of classical RAKE receiver to include the random nature of the number of multipath components. An existing random path arrival model is used which assumes a doubly stochastic Poisson process. An example of the use of the results is presented. The notion of fractional diversity is introduced.

Another major contribution of this thesis research is in terms of radio link performance for the case of cochannel interference. A comprehensive performance analysis model is built which is based on dispersive/frequency selective desired and interfering Nakagami fading signals. The case of non-dispersive/frequency non-selective fading desired and interfering signals becomes a special case of this model. The random path arrival character of the dispersive channels is also incorporated in the analysis. In this regard,

- outage expressions are derived and the numerical results are presented.
- it is argued that the analytical complexity involved in the segregation of slow and fast fading is perhaps unnecessary and the main goal of the analysis may be achieved in a simple manner. This however, requires that fading parameters be derived based on a non-segregated approach.

When dispersion/frequency selectivity manifests itself in the form of inter-symbol interference, it is referred to as multipath self interference (MSI) in this thesis. In this case,

- it is shown that a shot noise model may be used to statistically characterize the MSI.
- outage expressions are derived and numerical results are presented for a variety of fading scenarios.

7.3 Further Work

The main thrust of this thesis was to provide plausible understanding of the physical basis of the channel parameters. In doing so new physical understanding is provided for the parameters of the statistical characterization of the channels, in particular the fading parameter. As the main purpose of the channel measurements is to derive these statistical parameters, it would be quite useful to perform new measurements and derive these parameters based on the understanding developed in this thesis. For example,

- slow fading data should be tested for both Nakagami and lognormal distribution as discussed here to gather experimental evidence and to second-source this confirmation.
- envelope fading data may be gathered for line of sight Ricean fading channels to test the redundancy of the moving mean normalization procedure for segregation of slow and fast fading. The same data should be used to curve fit both Nakagami and Rice distributions and experimental evidence of the parameter relationship of the two distributions should be obtained.
- wideband channel measurements may be performed to extract the Poisson parameter, average number of paths per power delay profile length, which may be used for RAKE performance comparison.
- the experimental implementations of the IS95 RAKE receivers may be used to obtain the performance comparison.

References

- [1] Taylor, Jack T. and Omura, Jim K., "Spread Spectrum Technology: A Solution to the Personal Communication Services Frequency Allocation Dilemma", *IEEE Communications Magazine*, February 1991, pp. 48-51
- [2] Greenstein, Larry J. et. al., "Microcells in Personal Communications Systems" *IEEE Communications Magazine*, December 1992, pp. 76-88
- [3] Editorial, "The Characterization of Transmission Channels", *IEEE Transactions on Communications*, vol. CS-, no., September 1963, pp.
- [4] Harley, Phillips, "Short Distance Attenuation Measurements at 900 MHz and 1.8 GHz Using Low Antenna Heights for Microcells", *IEEE Transactions on Selected Areas in Communications*, vol. 7, no. 1, January 1989, pp. 5-11
- [5] Desch, Ulrich and Zollinger, Ernst, "propagation mechanisms in Microcell and Indoor Environment", *IEEE Transactions on Vehicular Technology*, vol. VT-43, No. 4, November 1994, pp. 1058-1066.
- [6] Xia, H. L., et. al., "Radio Propagation Characteristics for Line-of-Sight Microcellular and Personal Communications", *IEEE Transactions on Antennas and Propagation*, vol. 41, no. 10, October 1993, pp. 1439-1447
- [7] Schilling, Donald L., et. al., "Broadband CDMA for Personal Communications Systems", *IEEE communication Magazine*, November 1991, pp. 86-93
- [8] Berg J., et. al, "Path Loss and Fading Models for Microcells at 900 MHz", in *Proceedings 42nd IEEE Veh. Tech. Conference*, May 10-13, 1992, Denver, Colorado, pp. 666-671

- [9] Mogenson, P. E., et. al., "Urban Area Radio Propagation Measurements at 955 and 1845 MHz for Small and Micro Cells" in *Proceedings of GLOBCOM '91*, pp. 1297-1301
- [10] Rustoko, Jr., A. J., et.al., "Radio Propagation at Microwave Frequencies for Line-of-Sight Microcellular Mobile and Personal Communications", *IEEE Transactions on Vehicular Technology*, vol. VT-40, no. 1, February 1991, pp. 203-210
- [11] Arowojolu, A., et al., Time Dispersion Measurements in Urban Microcellular Environment", in *Proceedings 44th IEEE Veh. Tech. Conference*, 1995, pp. 150-154
- [12] Stewart, Kenneth, A., et al. "Wide band channel measurements at 900 MHz", in *Proceedings 45th IEEE Veh. Tech. Conference*, July 26-29, 1995, Chicago, Illinois, pp. 236-240
- [13] Whittaker, J.H. "Measurements of Path Loss at 910 MHz for Proposed Microcell Urban Mobile Systems", *IEEE Transactions on Vehicular Technology*, vol. VT-37, no. 3 August 1988, pp. 125-129
- [14] Green E, "Propagation Characteristics of a Narrowband Microcellular Radio Channel", Ph. D Dissertation, Department of Electronics and Computer Science, University of Southampton, 1990
- [15] Rappaport, T.S., et. al., "900-MHz Multipath Propagation Measurements for U.S. Digital Cellular Radiotelephones", *IEE Transactions on Vehicular technology*, vol. 39, no. 2, May 1990, pp. 132-139
- [16] Seidel, R.S. and Rappaport, T.S., "Path Loss and Multipath Diversity Statistics for Four European Cities for 900 MHz Cellular and Microcellular Communications", *Electron Letters*, 27th September 1990, vol. 26, no. 20, pp. 1713-1715

- [17] Blackard, K.L., et. al., "Path Loss and Delay Spread Models as a Function of Antenna Height for Microcellular System Design", in *Proceedings of the 42nd IEEE Vehicular Technology Conference*, June 1992, pp. 333-337
- [18] Feuerstein, Martin J. et. al., "Path Loss, Delay Spread, and Outage Models as a Function of Antenna Height for Microcellular System Design" *IEEE Transactions on Vehicular Technology*, vol. VT-43, no. 1, August 1994, pp. 487-498
- [19] Blackard, Kenneth L., et. al., "Path Loss and Delay Spread Models as Functions of Antenna Height for Microcellular System Design, in *Proceedings of the IEEE Vehicular Technology Conference*, 1991, VT-91, pp. 333-337
- [20] Bultitude, R. J. C., "Propagation Characteristics on Microcellular Urban Mobile Radio Channels at 910 MHz" *IEEE Journal in Selected areas in Communications*, vol. SAC-7, no. 1, January 1989, pp. 31-39
- [21] Bultitude, R. J. C., "Measured Characteristics of 800/900 MHz Fading Radio Channel with High Angle Propagation Through Moderately Dense Foliage". *IEEE Journal on Selected Areas in Communication*, vol. SAC-5, no. 2, February 1987, pp. 116-127
- [22] Bello, Phillip A., "Characterization of Randomly Time-Varaint Linear Channels". *IEEE Transactions on Communication Systems*, vol. CS-11, December 1963, pp. 360-393
- [23] Parsons, J.D. "Mobile Radio Propagation Channel" Pentech Press Limited, London, 1992, pp. 16-18
- [24] Young Jr., W. R., "Comparison of Mobile Radio Transmission at 150, 450, 900 and 3700 Mc", B.S.T.J., vol. 31, no. 6, November 1952, pp. 1068-1085

- [25] Cox, Donald, C., "Delay Doppler Characteristics of Multipath Propagation at 910 MHz in a Suburban Mobile Radio Environment", *IEEE Transactions on Antennas and Propagation*, vol. AP-20, No. 5, September 1972, pp. 625-635
- [26] Cox, Donald, C., "910-MHz Urban Mobile Radio Propagation: Multipath Characteristics in New York City", *IEEE Transactions on Communications*, vol. COM-21, No. 11, November 1973, pp. 1188-1194
- [27] Cox, Donald, C., "Distribution of Multipath Delay Spread and Average Excess Delay for 910-MHz Urban Mobile Radio Paths", *IEEE Transactions on Antennas and Propagation*, vol. AP-23, No. 2, March 1975, pp. 206-213
- [28] Cox, Donald, C., "A Measured Delay-doppler Scattering Function for Multipath Propagation at 910 MHz in an Urban Mobile Radio Environment" *Proceedings of the IEEE*, April 1973, pp. 479-480
- [29] Cox, Donald, C., "Multipath Delay Spread and Path Loss Correlation for 910-MHz Urban Mobile radio Propagation", *IEEE Transactions on Vehicular technology*, vol. VT-26, No. 4, November 1977, pp. 340-344
- [30] Jakes Jr., W. C., "Comparison of Mobile Radio Transmission at UHF and X Band", *IEEE Transactions on Vehicular Technology*, vol. VT-16, no. 1 October 1967, pp. 10-14.
- [31] Rustako Jr., A. J., "Evaluation of a Mobile Multiple Channel Diversity Receiver using Pre-Detection Combining", *IEEE Transactions on Vehicular Technology*, vol. VT-16, no. October 1967, pp. 46-57
- [32] Ossana Jr, J. F., "A Model for Mobile Radio Fading Due to Building Reflections: Theoretical and Experimental Fading Waveform Power Spectra", *B.S.T.J.*, vol. 43, no. 6, November 1964, pp. 2935-2971

- [33] Gilbert, E. N., "Energy Reception for Mobile Radio", *B.S.T.J.*, vol. 44, no. 8, October 1965, pp. 1779-1803
- [34] Clarke, R. H., "A Statistical Theory of Mobile Radio Reception", *B.S.T.J.*, vol. 47, no. 8, July-August 1968, pp. 957-1000
- [35] Lecours, M. et al., "Statistical Modelling of the Received Signal Envelope in a Mobile Radio Channel", *IEEE Transactions on Vehicular Technology*, vol. VT-37, no. 4, November 1988, pp. 204-212
- [36] Nakagami, M. "The m-Distribution - A General Formula of Intensity Distribution of Rapid Fading", in *Statistical Methods in Radio Wave Propagation*, pp 3-36, W. C. Hoffman Ed., Pergamon Press, Elmford NY, 1960
- [37] Parsons, J.D. and Bajwa, A.S., "Wideband Characterization of fading mobile radio channels", *IEE Proc.*, Vol. 129, Pt. F, No., 2, April 1982, pp. 95-105
- [38] Braun, W R., Dersch, U., "A Physical Mobile Radio Channel", *IEEE Transactions on Vehicular Technology*, vol. VT-40, no. 2, May 1991
- [39] Suzuki, H. "A Statistical Model for Urban Radio Propagation", *IEEE Transactions on Communications*, vol. COM-25, no. 7, July 1977
- [40] Turin G. L., et al. "A Statistical Model for Urban Multipath Propagation", *IEEE Transactions on Vehicular Technology*, vol. VT-21, no. 1, February 1972, pp. 1-9
- [41] Suzuki, H. "A Statistical Model for Urban Radio Propagation", Ph.D Dissertation, University of California, Berkeley, 1975
- [42] Hashemi, H., "Simulation of the Urban Radio Propagation Channel" *IEEE Transactions on Vehicular technology*, vol. VT-28, August 1979
- [43] Hashemi, H., "Simulation of the Urban Radio Propagation Channel", Ph.D Dissertation, University of California, Berkeley, 1977

- [44] Saleh. A. M., et al., "A Statistical Model for Indoor Multipath Propagation" *IEEE Journal on Selected areas in Communications*, vol. SAC-5, no. 2, February 1987, pp. 128-137
- [45] Hashemi, H., "The Indoor Radio Propagation Channel", *Proceedings of the IEEE*, vol. 81, no. 7, July 1993, pp. 941-968
- [46] Okumura, Y. et al., Field Strength and its Variability in VHF and UHF Land-Mobile Radio Service," *Review Electronic Communication Laboratory*, vol 16, pp. 825-873, 1968
- [47] Hata, M., "Empirical Formula for Propagation Loss in Land Mobile Radio Services", *IEEE Transactions on Vehicular Technology*, vol. VT-29, no. 3, August 1980, pp. 317-325
- [48] Joint Technical Committee Report, "Technical Report on RF Channel Characterization and System Deployment Modelling". September 23, 1994, JTC(AIR)/94.09.23-065R
- [49] Gudmundson, Mikael, "Cell Planning in Manhattan Environment", in *Proceedings 42nd IEEE Veh. Tech. Conference*, May 10-13, 1992, Denver, Colorado, pp. 435-438
- [50] Norton K. A., et al., "The Probability Distribution of the Amplitude of a Constant Vector Plus a Rayleigh Distributed Vector", *Proceeding IRE*, vol. 43, 1955, pp. 1354-1361
- [51] Yuanqing, Li, "A Theoretical Formulation for the Distribution Density of Multipath Delay Spread in a Land Mobile Radio Environment", *IEEE Transactions on Vehicular technology*, vol. VT-43, No. 2, May 1994, pp. 379-388, in particular Appendix B

- [52] Abu-Dayya, Adnan, A. and Beaulieu, N.C., "Microdiversity on Rician Fading Channels", *IEEE Transactions on Communications*, vol. COM-42, no. 6, June 1994, pp. 2258-2267
- [53] Charash, U., "A study of Multipath Reception with Unknown Delays", Ph.D Dissertation, University of California, Berkeley, 1974
- [54] Abbas, S. A., "Microcellular Mobile Radio Channel Simulation: A Geometrical Optics Approach", M.Eng Thesis, Department of Systems and Computer Engg. Carleton University, 1993, p. 69
- [55] French, R. C., "The effect of Fading and Shadowing on Channel Reuse in Mobile Radio", *IEEE Transactions on Vehicular technology*, vol. VT-28, No. 3, August 1974, pp. 171-181
- [56] Cox, D. C. "Co-channel Interference Considerations in Frequency Reuse Small Coverage Area Radio Systems" *IEEE Transactions on Communications*, vol. COM-30, no. 1, Jan 1982, pp. 135-142
- [57] Muammar, R. et al., "Co-channel Interference in High Capacity Mobile Radio Systems", *IEEE Transactions on Communications*, vol. COM-30, no. 8, Aug 1982, pp. 1973-1982
- [58] Yeh, Y. S., Schwartz, S.C., "Outage Probability in Mobile Telephony due to Multiple Lognormal Interferers", *IEEE Transactions on Communications*, vol. COM-32, no. 4, April 1984, pp. 380-387
- [59] Williamson, A. G. and Parsons, J.D., "Outage Probability in a Mobile Radio System Subject to Fading and Shadowing", *Electron Letters*, vol. 21, no. 14, July 1985, pp. 622-623

- [60] Sowerby, K. W. and Williamson, A. G., "Outage Probability Calculations for a Mobile Radio System having Multiple Rayleigh Interferers". *Electron Letters*, vol. 23, no. 11, May 1987, pp. 622-623
- [61] Yao, Y. D., and Sheikh, A. U. H., "Outage Probability Analysis for Microcellular Mobile Radio Systems with Co-channel Interferers in Rician/Rayleigh Fading Environment" *Electron Letters*, vol. 26, no. 13, June 1990, pp. 864-866
- [62] Abu-Dayya, Adnan, A. and Beaulieu, N.C., "Outage probabilities of Cellular Mobile Radio Systems with Multiple Nakagami Interferers". *IEEE Transactions on Vehicular Technology*, vol. 40, No. 4, November 1991, pp. 757-768
- [63] Yao, Y. D., and Sheikh, A. U. H., "Co-channel Interference Modelling and Performance Analysis of Microcell Systems for Wireless Personal Communications". *Canadian Journal of Electrical and Computer Engineering*, Vol. 19, No. 1, 1994, pp. 27-35
- [64] Rice, S. O., "A Mathematical Analysis of Random Noise". *The Bell System Technical Journal*, vol. 23, 1944 pp. 294-305
- [65] Papoulis, A., "*Probability, Random Variables, and Stochastic Processes*", McGraw Hill, 2nd ed., 1984, pp.379-385 & 136-137
- [66] Beckman, Petr, "*Probability in Communication Engineering*", Harcourt, Brace and World, Inc.
- [67] Jakes Jr, W. C., "*Microwave Mobile Communications*", New York Wiley, 1974
- [68] Lee, W. C. Y., "*Mobile Communication Engineering*", McGraw Hill, 1982
- [69] Allen, Arnold O., "Probability, Statistics and Queuing Theory" Academic Press New York, 2nd edition, 1990, pp.219-233
- [70] Ryzhik, I.M., and Gradshteyn, I. S., "*Table of Integrals Series and Products*" Academic Press New York, 1980, pp. 317 & 1039

- [71] Proakis, John G., "Digital Communications". McGraw Hill Book Co., 2nd edition, 1988, pp. 702-716 and 37-52
- [72] Turin G. L., "Communication Through Noisy Random-multipath Channel", MIT, Lincoln Lab. Technical Report no. 116. May 1956
- [73] Turin George L., "Introduction to Spread-Spectrum Antimultipath Techniques and their Application to Urban Digital Radio.", *Proceedings IEEE*, vol: 68, No. 3 March 1980, pp. 328-353
- [74] Edgeworth, F. Y, "The Law of Error", *Camb. Phil. Soc. Proc.* 20(1905), pp. 36-141
- [75] Baghdady Elie, J., "*Lectures on Communication System Theory*", McGraw Hill, 1961
- [76] Edgeworth, F. Y, "The Law of Error", *Camb. Phil. Soc. Proc.* vol. 20, 1905, pp. 36-141
- [77] Proakis, John G., "Digital Communications", McGraw Hill Book Co., 3rd edition, 1995, pp. 772-806
- [78] Panter, Philip F., "Communication Systems Design: Line-of-sight and Tropo-scatter Systems", McGraw Hill, New York, 1972, pp. 522-527
- [79] Aitchison and Brown, "The Lognormal Distribution". Cambridge University Press, London, 19XX
- [80] Abbas, Syed and Sheikh, A. U. H., "A Geometric Theory of Nakagami Fading Multipath Channels with Physical Interpretation" in *Proceedings 46th IEEE Vehicular Technology Conference*, April 27-30, 1996, Atlanta, Georgia, U.S.A. pp.
- [81] Abbas, Syed, "Multipath Wireless Channels: Models, Interpretations and Applications". Department of Systems and Computer Engineering, Ph.D. Thesis Proposal Carleton University, Ottawa, Canada, June, 1996

- [82] Zadeh, Lutfi. "Frequency Analysis of Variable Networks". Proceedings IRE. vol.38, March 1950, pp. 291-299
- [83] Wojnar, Andrzej H., "Unknown Bounds on Performance in Nakagami channels", *IEEE Transactions on Communications*, vol. COM-34, No. 1, January 1986, pp. 22-24
- [84] Press H. et al., "Numerical Recipes. The Art of Scientific Programming", Cambridge University Press, 1986. pp. 529-533

IMAGE EVALUATION TEST TARGET (QA-3)

APPLIED IMAGE, Inc.
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc., All Rights Reserved