1 OF/DE 2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

32181

National Library
of Canada

Bibliothèque nationale
du Canada

CANADIAN THESES
ON MICROFICHE

THÈSES CANADIENNES
SUR MICROFICHE

NAME OF AUTHOR / NOM DE L'AUTEUR Richard L. Stratton

TITLE OF THESIS / TITRE DE LA THÈSE Microstrip 3dB Branch-Line Couplers

UNIVERSITY / UNIVERSITÉ Carleton University

DEGREE FOR WHICH THESIS WAS PRESENTED /
GRADE POUR LEQUEL CETTE THÈSE FUT PRÉSENTÉE M. Eng.

YEAR THIS DEGREE CONFERRED / ANNÉE D'OBTENTION DE CE GRADE 1976

NAME OF SUPERVISOR / NOM DU DIRECTEUR DE THÈSE Professor J.P. Knight

Permission is hereby granted to the NATIONAL LIBRARY OF
CANADA to microfilm this thesis and to lend or sell copies
of the film.

L'autorisation est, par la présente, accordée à la BIBLIOTHÈ-
QUE NATIONALE DU CANADA de microfilmer cette thèse et
de prêter ou de vendre des exemplaires du film.

The author reserves other publication rights, and neither the
thesis nor extensive extracts from it may be printed or other-
wise reproduced without the author's written permission.

L'auteur se réserve les autres droits de publication; ni la
thèse ni de longs extraits de celle-ci ne doivent être imprimés
ou autrement reproduits sans l'autorisation écrite de l'auteur.

DATED / DATE June 25/77 SIGNED / SIGNÉ Richard L. Stratton

PERMANENT ADDRESS / RÉSIDENCE FIXE 2400 CARLING AVE.
APT #306
OTTAWA K2B 7H2

National Library of Canada

Cataloguing Branch
Canadian Theses Division

Ottawa, Canada
K1A 0N4

Bibliothèque nationale du Canada

Direction du catalogage
Division des thèses canadiennes

NOTICE

The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us a poor photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this film is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30. Please read the authorization forms which accompany this thesis.

**THIS DISSERTATION
HAS BEEN MICROFILMED
EXACTLY AS RECEIVED**

AVIS

La qualité de cette microfiche dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de mauvaise qualité.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, examens publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de ce microfilm est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30. Veuillez prendre connaissance des formules d'autorisation qui accompagnent cette thèse.

**LA THÈSE A ÉTÉ
MICROFILMÉE TELLE QUE
NOUS L'AVONS REÇUE**

MICROSTRIP 3dB BRANCH-LINE COUPLERS

by

RICHARD L. STRATTON, B.Eng.

A thesis submitted to the Faculty of Graduate Studies
and Research in partial fulfillment of the requirements
for the degree of

Master of Engineering.

1105 9006 01 XX

T.
M. Eng
1976
S97

Carleton University

Ottawa, Canada

September, 1976

© Richard L. Stratton 1976

The undersigned recommend to the Faculty of Graduate Studies
and Research acceptance of the thesis:

"Microstrip 3dB Branch-Line Couplers"

submitted by Richard L. Stratton in partial fulfillment of the requirements
for the degree of

Master of Engineering.

Thesis Supervisor

Chairman, Department of
Electronics Engineering

ACKNOWLEDGEMENTS

The author expresses deep appreciation to Professor J.P. Knight for his kind assistance and encouragement throughout the course of this work.

Also thanks are extended to Dr. D.R. Conn for supplying the thesis topic and making many helpful suggestions.

The funding of the project under NRC Research Grant A8453 is much appreciated. Many thanks go to Sandra Ryan who sacrificed some of her holidays to type this thesis.

ABSTRACT

Two designs for a 3 dB branch-line coupler are presented. One is for a conventional design employing T junctions. The other is for an easier-to-model design that uses transmission lines with the same impedance and symmetrical Y junctions. Detailed models are developed for the constituent elements of each coupler in order to account for their true non-ideal behaviour. Specifically, models are derived for the T and Y junctions as well as the required coax to microstrip connectors.

To experimentally model the T and Y junctions an indirect resonance technique is devised that circumvents the unknown reactances associated with the measurement system's connectors. T junction experimental results are compared with two existing theories and they are shown to support one theory more closely than the other. A new theoretical model is developed for the Y junction and experimental results are shown to be in reasonable agreement.

A completely general empirical model is developed for the coax to microstrip connector by adopting the well known sliding termination technique to microstrip. This model is applicable across X band and applies for any loading at the connector ports.

Using the individual models developed, the measured and calculated responses of the conventional coupler are compared and the connectors are shown to have a significant influence on the overall coupler performance. For the substrate material used all T junction models display a small and comparable effect on the coupler's characteristics.

Y junction effects are found to have a negligible influence on the new Y coupler's performance. Compared to the conventional coupler, this Y coupler is demonstrated to have a comparable bandwidth; to be easier to model and to be more readily tuneable.

TABLE OF CONTENTS

Acknowledgements

Abstract

Table of Contents

List of Symbols

Chapter 1 - Introduction	1
1.1 Introduction	1
1.2 Thesis Objectives and Organization	4
Chapter 2 - Directional Coupler Theory	6
2.1 Introduction	6
2.2 General Directional Coupler	6
2.2.1 Performance Specifications	8
2.3 Branch-Line Directional Coupler	9
2.3.1 Basic Configuration	9
2.3.2 Historical Development	11
2.4 Even-Odd Mode Analysis of n-Branch-Line Coupler	14
2.5 Even-Odd Mode Synthesis of Branch-Line Couplers	21
2.5.1 Introduction	21
2.5.2 Ideal Conventional 2-Branch-Line Coupler	22
2.5.3 Ideal Y Junction 2-Branch-Line Coupler	28
Chapter 3 - Microstrip Transmission Line	32
3.1 Introduction	32
3.2 Microstrip Transmission Line Configuration	32
3.3 Microstrip Characteristic Impedance and Guide Wavelength	34
3.4 Dispersive Behaviour in Microstrip	37
3.5 Losses in Microstrip Lines	38
3.5.1 Dielectric Loss	39
3.5.2 Conductor Loss	40
3.6 Microstrip Transmission Line Discontinuities	42

3.6.1	Introduction	42
3.6.2	Resonance Method for Measurement of Small Microstrip Reactances	44
3.6.3	Microstrip End Effect	46
3.6.4	Microstrip T Junction	47
3.6.4.1	Introduction	47
3.6.4.2	Theoretical Models	48
3.6.4.3	Experimental Model	58
3.6.5	Microstrip Y Junction	62
3.6.5.1	Introduction	62
3.6.5.2	Theoretical Model	65
3.6.5.3	Experimental Model	69
Chapter 4	Coax-Microstrip Connector	74
4.1	Introduction	74
4.2	Connector Mounting Structure	74
4.3	Mounted Connector Equivalent Circuit	77
4.4	Measurement of Z_{oc} and Z_{sc}	80
4.5	Measurement of Z_{50}	82
4.5.1	Direct Method	82
4.5.2	Sliding Termination Technique	84
4.5.2.1	Introduction	84
4.5.2.2	Theory of Sliding Termination Technique	84
4.5.2.3	Sample Problem	88
4.5.2.4	Measured Results	89
4.6	Experimental Verification of Connector Model	89
Chapter 5	Analysis of Coupler Performance	92
5.1	Introduction	92
5.2	Conventional Coupler	92
5.2.1	Effect of Branch-Line Length	92
5.2.2	Effect of T Junctions	95
5.2.3	Practical Coupler Configuration	98
5.3	New Y Junction Coupler	105

Chapter 6 - Conclusions and Recommendations	108
---	-----

6.1 Conclusions	108
-----------------	-----

6.2 Recommendations	110
---------------------	-----

Appendix A1	
-------------	--

A2	
----	--

A3	
----	--

References	
------------	--

LIST OF SYMBOLS

A	element a_{11} of 2x2 ABCD matrix
A_1	vector voltage wave exiting port 1
$A^i(t^2)$	element a_{11} of 2x2 ABCD matrix expressed as a polynomial of degree i in t^2
$A^i(\frac{1}{t^2})$	element a_{11} of 2x2 ABCD matrix expressed as a polynomial of degree i in $1/t^2$
A_E	element a_{11} of even mode 2x2 ABCD matrix
A_O	element a_{11} of odd mode 2x2 ABCD matrix
A'	element a_{11} of reverse ABCD matrix
a_i	admittance of i th main line in branch-line coupler and i th mode incident wave for 2-port S-parameter designation used in Chapter 4
α	attenuation in propagation constant $\gamma = \alpha + j\beta$
α_c	conductor loss in dB/cm
α_d	dielectric loss in dB/cm
B	element a_{12} of 2x2 ABCD matrix
$B^i(t^2)$	element a_{12} of 2x2 ABCD matrix expressed as a polynomial of degree i in t^2
$B^i(\frac{1}{t^2})$	element a_{12} of 2x2 ABCD matrix expressed as a polynomial of degree i in $1/t^2$

CONFIDENTIAL

B_E	element a_{12} of even mode 2x2 ABCD matrix
B_O	element a_{12} of odd mode 2x2 ABCD matrix
B_a, B_b	admittances in waveguide Y junction equivalent circuit
B_c, B_d	admittances in alternate representation of waveguide Y junction equivalent circuit
B'	element a_{12} of reverse ABCD matrix
b_i	admittance of i th branch line in branch-line coupler and i th mode incident wave for 2-port S-parameter designation used in Chapter 4
β	phase shift in propagation constant $\gamma = \alpha + j\beta$
C	element a_{21} of 2x2 ABCD matrix
$C^i(t^2)$	element a_{21} of 2x2 ABCD matrix expressed as a polynomial of degree i in $1/t^2$
$C^i(\frac{1}{t^2})$	element a_{21} of 2x2 ABCD matrix expressed as a polynomial of degree i in $1/t^2$
C_E	element a_{21} of even mode 2x2 ABCD matrix
C_O	element a_{21} of odd mode 2x2 ABCD matrix
C'	element a_{21} of reverse ABCD matrix
C_K	capacitor of known value
CF	coupling factor
c	constant in formula for microstrip end effect
D	element a_{22} of 2x2 ABCD matrix
$D^i(t^2)$	element a_{22} of 2x2 ABCD matrix expressed as a polynomial of degree i in t^2

$D^i(\frac{1}{t^2})$	element a_{22} of 2x2 ABCD matrix expressed as a polynomial of degree i in $1/t^2$
D_E	element a_{22} of even mode 2x2 ABCD matrix
D_O	element a_{22} of odd mode 2x2 ABCD matrix
D	element a_{22} of reverse ABCD matrix
D_{eff}	effective width of a microstrip or strip transmission line
D_i	effective parallel plate width of i th microstrip or strip transmission line or height of i th waveguide
D_m	effective width of a microstrip line
D_s	effective width of a stripline transmission line
dB	decibel
dc	direct current
d	main line reference plane location in T junction model
d'	branch line reference plane location in T junction model
ΔL	apparent increase in length of microstrip line due to end effect
E	subscript indicating coupler even mode also indicates electric field strength
ϵ_{eff}	effective dielectric constant for the propagating wave on microstrip
ϵ_0	free space dielectric constant = 8.85×10^{-12} farads/meter

ϵ_r	relative dielectric constant
f_D	frequency in gigahertz below which dispersion may be neglected
f_m	measured resonant frequency
f_0	centre design frequency
GHz	gigahertz
Γ	reflection coefficient
Γ_E	even mode coupler analysis circuit reflection coefficient
Γ_O	odd mode coupler analysis circuit reflection coefficient
Γ_L	load reflection coefficient
γ	complex propagation coefficient
H	magnetic field strength
H_m	thickness of dielectric substrate of microstrip line
H_s	stripline ground plane spacing
i	integer designation
I_i	complex current at port i
I_i'	$-I_i$
j	$\sqrt{-1}$

K	complex load impedance
L	microstrip physical line length
l	electrical length of a transmission line
l_i	electrical length of i th transmission line
L_K	inductor of known value
λ	wavelength
λ_a	free space wavelength
λ_g	guided wavelength
λ_0	wavelength at the centre design frequency f_0
λ_m	wavelength at the measured resonant frequency f_m
m	subscript to indicate microstrip
n	turns ratio in Vogel's T junction equivalent circuit
n'	turns ratio in Leighton and Milne's T junction equivalent circuit
O	subscript indicating coupler odd mode
P_i	power exiting port i
q	Wheeler's dielectric filling factor
R	radius of circle described by sliding termination technique

R_s conductor resistivity
 $S_{11}, S_{12}, S_{21}, S_{22}$ elements of S parameter matrix description of 2-port network
 s subscript to indicate stripline
 σ conductivity of a dielectric substrate
 T transmission coefficient
 T_i i th reference plane
 TEM transverse electromagnetic
 TE_{10} transverse electric waveguide mode
 TM_0 transverse magnetic waveguide mode
 T_E even mode coupler analysis circuit transmission coefficient
 T_O odd mode coupler analysis circuit transmission coefficient
 t abbreviation for $\tanh$
 t_m thickness of microstrip conductor
 t_s thickness of stripline conductor
 θ coupling factor in Crompton's Analysis
 θ_{22} angle of complex S-parameter - S_{22}
 μ_0 free space permeability
 V_i complex voltage at port i

VSWR voltage standing wave ratio

W_i physical width of i th microstrip or stripline transmission line

W_m physical width of a microstrip line

W_s physical width of a stripline transmission line

X reactance in model for waveguide E plane T junction

X-band band frequency band 8-12.4GHz

X_a, X_b reactances in parallel plate T junction model

$X_a(Z_1, Z_2), X_b(Z_1, Z_2)$ reactances in parallel plate T junction model between lines of main impedance Z_1 and branch impedance Z_2

X_c, X_d reactances in parallel plate Y junction model

X_u unknown reactance

Y admittance in Waveguide Handbook parallel plate T junction model

Y_0 characteristic admittance of a transmission line

Z_0 characteristic impedance of a transmission line

Z_i characteristic impedance of the i th transmission line

Z_{in} input impedance

Z_L load impedance

CARLTON

Z_{sc}

input impedance to a network with its output ports short circuited

Z_{oc}

input impedance to a network with its output ports open circuited

Z_{50}

input impedance to a network with its output ports terminated
in 50 ohms

CONFIDENTIAL

CHAPTER 1

INTRODUCTION

1.1 Introduction

With the increased development of microwave technology, interest has turned to the design of circuits that are less expensive, lighter and more compact than the commonly used waveguide configurations. Two such classes of circuits, which also have much larger bandwidths than waveguide, are stripline and microstrip. Stripline propagates only the transverse electromagnetic (TEM) mode from dc to the start of the first waveguide mode¹ while microstrip propagates a similar more complex Quasi-TEM mode³. Microstrip has the advantages that it is a simpler structure and accordingly it is less expensive and easier to use than stripline. It has the disadvantage that it is more often necessary to use cut and try design methods⁴ because the Quasi-TEM mode is not as well characterized as the TEM mode.

The 3db coupler is a microwave circuit element of particular importance⁵⁻¹² because of its wide usage in many areas such as balanced mixers; frequency discriminators, duplexers and phase shifters.

Microstrip 3db couplers generally fall into three main categories: coupled-line, rat race and branch-line. Fig. 1.1 shows the three types.

Coupled-line types have the widest bandwidths and a 90 degree phase difference between their output ports.

Fig. 1.1. Microstrip 3dB coupler types.

However, there is no dc path between the outputs and they are extremely sensitive to dimensional tolerances due to the tight coupling required between the lines. Also the output ports are not adjacent hence, crossover lines are sometimes required.

Rat race configurations are much less sensitive to dimensional tolerances and have a dc path between the ports. However, they have significantly narrower bandwidths; 180 degree phase shift between the output ports; and the outputs are not taken from adjacent arms. Cascading them to increase the total bandwidth is not easily done.

Branch-line couplers have slightly less bandwidth and are slightly more sensitive to dimensional tolerances than the rat race types. They have a 90 degree phase difference between the output ports and are much better suited to tight coupling than the coupled-line types. (Zero db couplers are possible). Their output is taken from adjacent arms and they have a dc path between the ports. They can also be easily cascaded to increase bandwidth. However, microstrip couplers with more than two branch lines are uncommon because line impedances become difficult to achieve; space is often at a premium; and the high losses of microstrip circuitry becomes a problem. One common application for 2-branch-line couplers is in balanced mixers which require 3db coupling, 90 degree phase difference between the outputs and adjacent outputs with a dc connecting path. Despite their widespread use, published theories for these structures in

microstrip do not agree with their measured performances. Current theories^{15,16} attribute most of the anomalous behaviour of these couplers to the T-shaped junctions between the microstrip lines (Fig. 1.1(c)). However, characterization of these junctions is very difficult and the majority of the work has been done on stripline^{49,50} not microstrip. Inclusion of the existing microstrip T models does not completely account for the measured performances of the couplers¹⁶.

It is well known^{49,54} that the coax-microstrip connectors between the external test equipment and the microstrip circuitry can cause significant discontinuity impedances with poor repeatability. Despite this, current published experimental microstrip models only emphasize the T junctions¹⁶. A complete experimental investigation of microstrip branch-line coupler performance should include the coax-microstrip connector as well as all relevant properties of the microstrip line.

The topic of this thesis is therefore the characterization of microstrip 3db branch-line couplers.

1.2 Thesis Objectives and Organization

The purpose of this thesis is to study the influence of various factors on the performance of microstrip 3db branch-line couplers in an attempt to develop a usable coupler model.

The second chapter contains a review of coupler theory in which the usual equations have been modified to account for the microstrip transmission line loss. A standard 2-branch-line coupler with T junctions is analyzed as well as an original design that employs transmission lines with a single impedance and symmetrical Y junctions.

Chapter 3 presents the properties of the microstrip line. An empirical T junction model is obtained for the 8-12 GHz band using a resonance technique. This model, which is used in Chapter 5 to analyze the coupler with T junctions, is shown to be in good agreement with an existing theoretical model¹⁵. Similarly an empirical Y junction model is obtained and shown to agree well with a new theoretical Y junction model which is developed. These Y junction models are discussed in Chapter 5 in conjunction with the Y coupler.

In Chapter 4, a completely general empirical model is presented for the coax-microstrip connector for the range 8-12 GHz.

Using a computer program, a comparison of the performances of various coupler models and measured coupler responses is made in Chapter 5.

Chapter 6 presents the conclusions and recommendations.

CHAPTER 2

DIRECTIONAL COUPLER THEORY

2.1 Introduction

As a preliminary to the analysis of practical microstrip couplers, this chapter introduces the relevant coupler theory. The performance criteria of a general coupler are defined; the branch-line configuration is illustrated; their historical development is outlined; current analysis and design techniques are presented and two microstrip designs are given. One is for a standard 2-branch-line coupler that employs T junctions. The other is for a new design that uses single impedance lines and symmetrical Y junctions.

2.2 General Directional Coupler

A general directional coupler can be represented by a 4-port network as shown in Fig. 2.1. A voltage wave $(1+j0)$ incident on port #1 causes voltage waves to exit from some or all of the ports of the coupler. These waves are vector quantities designated A_1 , A_2 , A_3 and A_4 for ports 1 to 4 respectively. The fraction of the incident power that leaves each port is similarly designated by the scalar quantities P_1 , P_2 , P_3 and P_4 .

A coupler is regarded as performing optimally at some frequency when it divides all of the power incident on port 1 in the required ratio between ports 2 (the output port) and 3 (the coupled port); no power is reflected at port 1 (the incident port); no power leaves port 4 (the isolated port); and no power is lost inside the coupler.

A_1 is the voltage vector of the wave leaving port 1.

P_1 is the fraction of the power incident on port 1 that leaves port 1.

Fig. 2.1. General 4-port coupler network.

2.2.1 Performance Specifications

8.

A directional coupler can be characterized by the coupling factor, voltage standing wave ratio, isolation, directivity and insertion loss.

These terms are defined as follows:

$$\text{Coupling Factor (db)} = 10 \log_{10} \left(\frac{1}{P_3} \right) \quad 2.1$$

P_3 is the fraction of the power incident on port #1 that exits the coupled output port #3.

$$\text{VSWR} = \frac{1 + |A_1|}{1 - |A_1|} \quad 2.2$$

$|A_1|$ is the amplitude of the voltage wave reflected at port #1.
VSWR is a dimensionless quantity.

$$\text{Isolation (db)} = 10 \log_{10} \left(\frac{1}{P_4} \right) \quad 2.3$$

P_4 is the fraction of the power incident on port #1 that exits from the isolated port #4.

$$\text{Directivity (db)} = 10 \log_{10} \left(\frac{P_3}{P_4} \right) \quad 2.4$$

Directivity expresses the ratio of the powers P_3 and P_4 exiting from ports #3 and #4 respectively. It is not an independent quantity i.e.

$$\text{Directivity (db)} = \text{Isolation (db)} - \text{Coupling Factor (db)}$$

$$\text{Insertion Loss (db)} = 10 \log_{10} \left(\frac{1}{P_2} \right) \quad 2.5$$

P_2 is the fraction of the power incident on port #1 that exits the output port #2.

Using the above performance definition, a 3db coupler exhibits optimum performance when:

- 1) Coupling Factor = 3db
- 2) VSWR = 1
- 3) Isolation $\rightarrow \infty$
- 4) Directivity $\rightarrow \infty$
- 5) Insertion Loss = 3db

2.3 Branch-Line Directional Coupler

2.3.1 Basic Configuration

A basic branch-line coupler is composed of a series of transmission lines of length:

$$l = \frac{\lambda_0}{4}$$

2.6

where: λ_0 is the guide wavelength at the design centre frequency f_0 of the coupler.

Figure 2.2(a) gives the equivalent circuit for an ideal n-branch-line coupler comprised of transmission lines. If the coupler is made of ideal transmission lines i.e. there is

- 1) no dispersive behaviour
- 2) constant characteristic impedance
- 3) no losses
- 4) no discontinuity impedances at the junctions

then it will be designated an ideal branch-line coupler. The normalized admittances

Input
Port 1

Port 2

10.

2

Figure 2.2. a) Equivalent Circuit of General n -Branch-Line Coupler.
b) Even Mode Analysis Circuit.
c) Odd Mode Analysis Circuit.

of the $n-1$ main lines are designated a_i where $1 \leq i \leq n-1$ while the normalized admittances of the n branch lines are designated b_i where $1 \leq i \leq n$. The lead-in lines have normalized admittances of unity.

From figure 2.2(a) it is seen that the branch-line coupler will always be symmetrical about axis 1-1. The coupler is termed a "symmetrical coupler" when it is also symmetrical about axis 2-2.

i.e.

$$a_i = a_{n-i} \quad 1 \leq i \leq n-1 \quad 2.7$$

$$b_i = b_{n-i+1} \quad 1 \leq i \leq n \quad 2.8$$

2.3.2 Historical Development

In 1945 Lipmann⁵ made one of the earliest contributions to branch-line coupler theory with his symmetry analysis technique which analyzed 4-port networks symmetrical about axis 1-1 (sec. 2.3.1) as a combination of two 2-port networks. (This method is outlined in section 2.4). Reid and Wheeler⁶ used this method in 1956 to design branch-line couplers with optimal performance at the centre frequency (section 2.2). In each of their couplers, the main lines had one impedance value; the centre branches another; and the end branches had yet another value. They found that the circuit bandwidth increased with the number of branch lines used. Also in 1956, Young⁷ presented design formulas for couplers with up to eight branches. He varied both the main and branch-line impedances symmetrically about the centre to obtain optimum responses at the design frequency.

The following year Crompton⁸ presented a first order method for calculating the overall coupling factor of a cascade of couplers.

He found that if a single coupler couples power

$$P_3 = \sin^2 \theta \quad 2.9$$

then 1 cascaded identical ones couple

$$P_3 = \sin^2 1 \theta \quad 2.10$$

He reasoned that the 1 couplers could be grouped so that their total reflection coefficient (ignoring second order reflections) formed a maximally flat (binomial) response. Fig. 2.3(a) illustrates a grouping of eight-21db couplers he used to attain a design for a 5 branch-line 3db device shown in Fig. 2.3(b). This design method worked best for weak coupling when the branch impedances were large and the reflected waves small. In the same year, Lomer and Crompton⁹ constructed this coupler and achieved performances reasonably close to those expected. In 1958 Reed¹⁰ presented an expansion of his design technique to waveguide couplers with any number of branches. Crompton's first order approximation of no multiple reflections was extended to Chebychev designs by Levy¹¹ in 1959. His designs varied the branch-line impedances symmetrically about the centre while all the normalized main-line impedances were fixed at unity. For the same number of elements, his Chebychev designs gave a wider bandwidth than Crompton's binomial designs. In 1962 Young¹² employed a quarter wave transformer prototype circuit to design couplers with binomial or Chebychev input reflection response that varied both the main and branch-line impedances symmetrically about the centre. Although this method was not exact he found that for the same bandwidth approximately half as

The 21 dB couplers are combined to form a 5 branch 3 dB coupler and the total branch admittances form a binominal series.

a) Grouping of 21dB Couplers

Relative Admittances as indicated

b) 5 branch-line 3db coupler

Figure 2.3 Illustration of Crompton's First Order Design Method

many branches were required as with the regular periodic structures of Reed and Wheeler⁶. He also presented empirical correction curves which enabled good estimates of the bandwidth but poorer estimates of the VSWR and directivity. Two years later, Kurzrok¹³ published an approximate formula which showed that loss in the transmission lines of 2-branch-line 3db couplers significantly affects their isolation. In 1968, branch-line coupler design tables based on a mathematical analysis and digital computation were developed by Levy and Lind¹⁴. These tables gave exact results for the reflection and isolation in the binomial case and nearly exact results for the Chebychev case. For the same number of branch lines, these designs gave better performances than all previous ones. T junction reactances and dimensional tolerance effects on microstrip 3db 2-branch-line couplers were theoretically considered by Leighton and Milnes¹⁵ in 1971. The next year Vogel¹⁶ used another T junction model to construct microstrip couplers that compensated for these effects. Although this compensation improved the agreement between his experimental and theoretical results, there was still considerable discrepancy between the two.

2.4 Even-Odd Mode Analysis of n-Branch-Line Coupler

The even-odd mode analysis technique^{5,6} determines the performance of branch-line couplers over any desired frequency range by treating these 4 port networks as a combination of 2-port networks. The method

does not require ideal transmission lines and can handle discontinuities of equal value symmetrically placed about the axis 1-1 (such as coax-microstrip connectors or reactances associated with various transmission line junctions). In conjunction with ABCD matrix theory^{A1} the technique readily lends itself to a computer program.

To illustrate the method, the circuit of Fig. 2.2(a) will be analyzed. Couplers with discontinuities would be analyzed identically with the exception that the ABCD matrices for the discontinuity elements would also be included.

For a voltage wave $(1+j0)$ incident on port #1, it is desired to determine the voltage waves A_1 , A_2 , A_3 and A_4 exiting ports 1 to 4 respectively. Invoking superposition, the incident wave at port #1 may be regarded as a summation of 2 waves:

$$1+j0 = (\frac{1}{2}+j0) + (\frac{1}{2}+j0) \quad 2.11$$

similarly a zero signal incident on port #4 may be regarded as

$$0+j0 = (\frac{1}{2}+j0) - (\frac{1}{2}+j0) \quad 2.12$$

Combining these signals gives the two cases of Figs. 2.2(b) and (c) which when added together give the original case of Fig. 2.2(a). The two in-phase signals cause a voltage maximum (open circuit) along the axis of symmetry. This is the even mode case. The odd mode case consists of the two out-of-phase signals causing a voltage minimum (short circuit) along the axis of symmetry. Since both cases are symmetrical about axis 1-1, they can be analyzed by considering only one half of the circuit with an appropriately placed open or short

circuit. These analysis circuits then consist of a cascade of $\frac{\lambda_0}{4}$ 16.
 transmission lines and $\frac{\lambda_0}{8}$ shunt stubs. The performances of these circuits
 can be determined using ABCD matrix methods and then related to the
 overall performance of the coupler. Fig. 2.4 gives the ABCD matrices for
 some typical cases.

In general the transmission lines are lossy therefore γ , the complex
 propagation constant is used.

where

$$\gamma = \alpha + j\beta$$

α is the attenuation factor in nepers/inch

β is the phase constant in radians/inch

For an ideal coupler, α is zero because there is no transmission
 line loss.

The ABCD matrix for each analysis circuit is obtained by
 multiplying the matrices of its constituent elements together.

For the even mode analysis circuit the resulting matrix is of
 the form¹⁴:

$$\begin{bmatrix} A_E & B_E \\ C_E & D_E \end{bmatrix} = \frac{1}{(1-t^2)^{n-1}} \times \begin{bmatrix} A^{(n-1)}(t^2) & tB^{(n-2)}(t^2) \\ tC^{(n-1)}(t^2) & D^{(n-1)}(t^2) \end{bmatrix} \quad 2.13$$

where

n = number of branches

$$t = \tanh \frac{\gamma \times \lambda_0}{8}$$

$A^{(n-1)}(t^2)$ indicates a polynomial in t^2 of degree $n-1$

Figure 2.4(a). Short Circuit Shunt Stub of Electrical Length ℓ and Admittance b_1 .

Figure 2.4(b). Open Circuit Shunt Stub of Electrical Length ℓ and Admittance b_1 .

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \frac{1}{\sqrt{1-t^2}} \begin{bmatrix} 1 & \tanh \gamma l \\ a_1 \tanh \gamma l & 1 \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Figure 2.4(c). Transmission Line of Length l and Admittance a_1 .

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \frac{1}{1 - \tanh^2 \gamma l} \begin{bmatrix} 1 + \tanh^2 \gamma l & \frac{2}{a_1} \tanh \gamma l \\ 2a_1 \tanh \gamma l & 1 + \tanh^2 \gamma l \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} \cosh 2\gamma l & \frac{\sinh 2\gamma l}{a_1} \\ \frac{a_1}{\sinh 2\gamma l} & \cosh 2\gamma l \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Figure 2.4(d). Transmission Line of Length $2l$ and Admittance a_1 .

For the odd mode analysis circuit the resulting matrix is of the form:¹⁴

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \frac{(-1)^{n+1} t^{2(n-1)}}{(1-t^2)^{n-1}} \times \begin{bmatrix} A^{(n-1)} \frac{1}{t^2} & \frac{1}{t} B^{(n-2)} \left(\frac{1}{t^2} \right) \\ \frac{1}{t} C^{(n-1)} \frac{1}{t^2} & D^{(n-1)} \frac{1}{t^2} \end{bmatrix}$$

2.14

From ABCD matrix theory, the reflection coefficient Γ_E for the even mode analysis circuit is:

$$\Gamma_E = \frac{A_E + B_E - C_E - D_E}{A_E + B_E + C_E + D_E} \quad 2.15$$

The transmission coefficient is:

$$T_E = \frac{2}{A_E + B_E + C_E + D_E}$$

Identical equations hold for the odd mode with the exception that the subscripts are 0 instead of E.

Fig 2.5 shows how these reflections and transmissions are related to the actual voltage waves A_1 to A_4 exiting the ports of the coupler.

Figure 2.5. 2-Port Even and Odd Mode Cases Combined to Give A_1 , A_2 , A_3 and A_4 .

$$A_1 = \frac{1}{2}\Gamma_E + \frac{1}{2}\Gamma_0$$

2.17 a)

$$A_2 = \frac{1}{2}T_E + \frac{1}{2}T_0$$

2.17 b)

$$A_3 = \frac{1}{2}T_E - \frac{1}{2}T_0$$

2.17 c)

$$A_4 = \frac{1}{2}\Gamma_E - \frac{1}{2}\Gamma_0$$

2.17 d)

As long as the coupler consists only of elements of equal value symmetrically located about axis 1-1 with known ABCD matrices, its performance can be determined using this method.

2.5 Even-Odd Mode Synthesis of Branch-Line Couplers

2.5.1 Introduction

Equations 2.13 - 2.17 relate the voltage waves A_1 through A_4 to the main and branch line normalized admittances. Therefore reversing the process and specifying the responses of A_1 to A_4 determines the necessary conditions that must be imposed on the admittances to achieve these responses.

In this study, two microstrip couplers will be designed assuming that they are ideal. Their measured performances will then be analyzed by taking into account the true non-ideal behaviour of both the microstrip and the coax-microstrip transitions used.

Figs. 2.6(a) and 2.8(a) show the two configurations of the 2-branch-line microstrip 3db coupler that will be studied. One is the conventional type that uses different main and branch line admittances as well as T junctions between the lines. The other is a unique concept with all admittances the same, a matching stub and symmetrical Y junctions. The dotted lines in both figures represent effective parallel plate widths. For design purposes, transmission line lengths are measured between these reference planes and the effects of the junctions are ignored. The criteria of optimum performance at the centre frequency (section 2.2) is used for synthesis. Other performance criteria are Butterworth or Chebychev¹⁴ however for these 2-branch structures, the resulting designs are all the same no matter what criteria is used³⁰.

2.5.2. Ideal Conventional 2-Branch-Line Coupler

Figs. 2.6(b,c, and d) give the equivalent circuit and even and odd mode analysis circuits for the conventional coupler of Fig. 2.6(a). The resulting even and odd mode matrices are determined from Figs. 2.4(a-d) as:

$$\begin{bmatrix} A_E & B_E \\ C_E & D_E \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b_1 \tanh \gamma l & 1 \end{bmatrix} \times \begin{bmatrix} \cosh 2\gamma l & \frac{\sinh 2\gamma l}{a_1} \\ a_1 & \frac{\sinh 2\gamma l}{\cosh 2\gamma l} \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ b_1 \tanh \gamma l & 1 \end{bmatrix}$$

(a) Microstrip Layout.

(b) Equivalent Circuit.

(c) Even Mode Analysis Circuit.

(d) Odd Mode Analysis Circuit.

Figure 2.6. Ideal Conventional 2-Branch-Line Coupler.

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{b_1}{\tanh \gamma l} & 1 \end{bmatrix} \times \begin{bmatrix} \cosh 2\gamma l & \frac{\sinh 2\gamma l}{a_1} \\ \frac{a_1}{\sinh 2\gamma l} & \cosh 2\gamma l \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ \frac{b_1}{\tanh \gamma l} & 1 \end{bmatrix}$$

2.19

At the centre frequency $l = \frac{\lambda_0}{8}$ and for a lossless coupler with $\alpha = 0$ the above equations reduce to:

$$\begin{bmatrix} A_E & B_E \\ C_E & D_E \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \pm j b_1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & \frac{1}{a_1} \\ j a_1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ \pm j b_1 & 1 \end{bmatrix}$$

Branch Main Branch

2.20

$$= \begin{bmatrix} \pm \frac{b_1}{a_1} & \frac{1}{a_1} \\ j \left(a_1 - \frac{b_1^2}{a_1} \right) & \pm \frac{b_1}{a_1} \end{bmatrix}$$

2.21

For optimum performance, the following is true:

a) $VSWR = 1$ or $A_1 = 0$

and from equation 2.17(a)

$$\frac{1}{2} \Gamma_E + \frac{1}{2} \Gamma_0 = 0$$

2.22

b) Isolation $\rightarrow \infty$ or $A_4 = 0$

and from equation 2.17(d)

$$\frac{1}{2} \Gamma_E - \frac{1}{2} \Gamma_0 = 0$$

2.23

Combining equations 2.22 and 2.23

$$\Gamma_E = \Gamma_0$$

2.24

That is:

$$\frac{A_E + B_E - C_E - D_E}{A_E + B_E + C_E + D_E} = \frac{A_0 + B_0 - C_0 - D_0}{A_0 + B_0 + C_0 + D_0} = 0 \quad 2.25$$

From equation 2.21 it is seen that $A_E = D_E$ 2.26

$\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$

Therefore,

$$B_E = C_E = B_0 = C_0 \quad 2.27$$

$$\frac{1}{a_1} = j \left(a_1 - \frac{b_1^2}{a_1} \right) \quad 2.28$$

$$1 + b_1^2 = a_1^2 \quad 2.29$$

The voltage wave A_3 exiting port #3 is:

$$A_3 = \frac{1}{2} (T_E - T_0) \quad 2.30$$

$$= \frac{1}{2} \left(\frac{2}{A_E + B_E + C_E + D_E} - \frac{2}{A_0 + B_0 + C_0 + D_0} \right) \quad 2.31$$

$$= \left(\frac{1}{-2b_1 + \frac{2j}{a_1}} \right) \left(\frac{1}{2b_1 + \frac{2j}{a_1}} \right) \quad 2.32$$

$$= -\frac{b_1}{a_1} \quad 2.33$$

The minus sign indicates that A_3 lags the incident voltage at port #1 by 180 degrees.

Also from section 2.2.1 the coupling factor (CF) is related to the voltage wave A_3 at port #3 by:

$$CF = 20 \log_{10} \left(\frac{1}{|A_3|} \right) \quad 2.34$$

$$|A_3| = \frac{b_1}{a_1} = 10^{-\frac{(CF)}{20}} \quad 2.35$$

Equations 2.29 and 2.35 uniquely specify a conventional 2-branch-line coupler.

From equation 2.35 for a 3db coupler

$$\frac{b_1}{a_1} = \frac{1}{\sqrt{2}} \quad 2.36$$

Substituting this into equation 2.29 gives the final coupler design that:

$b_1 = 1$ and $a_1 = \sqrt{2}$ for a 3db coupler. The microstrip configuration used is shown in Fig. 2.7(a) mounted on its test jig.

Fig. 2.7. Mounted coupler structures.
 a) Conventional coupler.
 b) New Y junction coupler.

2.5.3 Ideal Y Junction 2-Branch-Line Coupler

Fig. 2.8(a) gives the microstrip layout of a new design of branch-line coupler. All line admittances are made the same resulting in completely symmetrical Y junctions. These junctions require only 2 circuit elements to model their discontinuity reactances. (To be discussed in Chapter 3.) To have a 90 degree phase shift between ports 2 and 3 at the design frequency, the lengths of the branch lines are set at $\frac{\lambda_0}{4}$. Figs. 2.8 (b,c, and d) give the equivalent circuit and even and odd mode analysis circuits. The even and odd mode matrices are determined using the ABCD matrices of Figs. 2.4 (a-d) where $\alpha=0$ and γ is replaced by $j\beta$ because the coupler is assumed to be ideal. The even and odd mode matrices at frequency f_0 are:

$$\begin{bmatrix} A_E & B_E \\ 0 & 0 \\ C_E & D_E \\ 0 & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ \pm j & 1 \end{bmatrix}}_{\text{Branch}} \times \underbrace{\begin{bmatrix} \cos \beta l_1 & j \sin \beta l_1 \\ j \sin \beta l_1 & \cos \beta l_1 \end{bmatrix}}_{\text{Main}} \times \underbrace{\begin{bmatrix} 1 & 0 \\ j \tan \beta l_2 & 1 \end{bmatrix}}_{\text{Main}} \times \underbrace{\begin{bmatrix} \cos \beta l_1 & j \sin \beta l_1 \\ j \sin \beta l_1 & \cos \beta l_1 \end{bmatrix}}_{\text{Main}} \times \underbrace{\begin{bmatrix} 1 & 0 \\ \pm j & 1 \end{bmatrix}}_{\text{Branch}}$$

2/37

Comparing this expression with that of equation 2.20 and equating main-line and branch-line matrices, solutions for l_1 and l_2 can be found.

Figure 2.8. Ideal 2-Branch-Line Y Coupler.

Comparing branch line expressions, they will be equivalent if $b_1 = 1$.
 Therefore from 2.29, $a_1 = \sqrt{2}$ and the coupler is constrained to having
 3db coupling.

Equating the main-line matrices gives:

$$\begin{bmatrix} \cos\beta l_1 & j\sin\beta l_1 \\ j\sin\beta l_1 & \cos\beta l_1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ j\tan\beta l_2 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\beta l_1 & j\sin\beta l_1 \\ j\sin\beta l_1 & \cos\beta l_1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{1}{a_1} \\ ja_1 & 0 \end{bmatrix}$$

2.38

Writing the matrix expressions as separate equations gives:

$$\cos^2\beta l_1 - \sin^2\beta l_1 - \sin\beta l_1 \cos\beta l_1 \tan\beta l_2 = 0 \quad 2.39$$

$$2j\cos\beta l_1 \sin\beta l_1 + j\tan\beta l_2 \cos^2\beta l_1 = ja_1 \quad 2.40$$

$$2j\cos\beta l_1 \sin\beta l_1 - j\tan\beta l_2 \sin^2\beta l_1 = \frac{1}{a_1} \quad 2.41$$

Subtracting equation 2.41 from equation 2.40

$$\tan\beta l_2 = \frac{a_1^2 - 1}{a_1} \quad 2.42$$

$$\text{for } a_1 = \sqrt{2}$$

$$l_2 = .113\lambda_0 \quad 2.43$$

from equation 2.39

$$\cos^2 \beta \ell_1 - \sin^2 \beta \ell_1 - \sin \beta \ell_1 \cos \beta \ell_1 \tan \beta \ell_2 = 0$$

dividing by $\cos^2 \beta \ell_1$

$$\tan^2 \beta \ell_1 + \tan \beta \ell_2 \tan \beta \ell_1 - 1 = 0 \quad 2.44$$

substituting equation 2.42

$$\tan^2 \beta \ell_1 + \left(\frac{a_1^2 - 1}{a_1} \right) \tan \beta \ell_1 - 1 = 0 \quad 2.45$$

$$\tan \beta \ell_1 = \frac{(1 - a_1^2)}{2a_1} \pm \frac{(1 + a_1^2)}{2a_1} \quad 2.46$$

$$\tan \beta \ell_1 = \frac{1}{a_1} \text{ or } -a_1 \quad 2.47$$

Rejecting the negative solution as requiring excessive line lengths,

$$\text{for } a_1 = \sqrt{2},$$

$$\ell_1 = .113 \lambda_0 \quad 2.48$$

The final coupler design is indicated in figure 2.7(b) as it would appear mounted on the test jig.

CHAPTER 3

MICROSTRIP TRANSMISSION LINE

3.1 Introduction

To analyze the performance of the couplers of Figs 2.7(a) and (b), it is necessary to account for all of the factors that could significantly alter their electrical behaviour from the ideal. These factors include transmission line loss, lengths of lead-in lines, T and Y junction reactive effects and the connector electrical model.

Microstrip transmission line properties are considered in this chapter while Chapter 4 deals with the connector mounting structure and connector electrical model.

3.2 Microstrip Transmission Line Configuration

Figure 3.1 illustrates a typical microstrip transmission line configuration. A thin conducting strip of thickness t_m and width W_m is separated from a ground plane by a dielectric material of thickness H_m . Because there is an air-dielectric interface, the fields are not continuous. Components are present in the direction of propagation hence the mode of propagation is not purely TEM. If the frequency is not too high, this closely resembles a pure TEM mode and is often designated

t_m = strip conductor thickness
 H_m = substrate thickness
 W_m = strip conductor width

Fig. 3.1. Cross-section of microstrip transmission line.

Quasi-TEM³. The Quasi-TEM mode may be analyzed by using the electrostatic techniques³ which are applicable to pure TEM modes.

3.3 Microstrip Characteristic Impedance and Guide Wavelength

Employing the quasi-TEM assumption and a conformal mapping technique, Wheeler has obtained approximate solutions for the wave propagation on microstrip. Figure 3.2 shows that his results for characteristic impedance and guide wavelength are complex functions of the relative dielectric constant ϵ_r of the substrate and the ratio W_m/H_m . His values for characteristic impedance have been experimentally verified⁴² to within 5%. Wheeler also used the effective dielectric constant ϵ_{eff} such that:

$$\epsilon_{eff} = \left(\frac{\lambda_a}{\lambda_g} \right)^2$$

3.1

where:

λ_a is the freespace wavelength and

λ_g is the guide wavelength.

The copper clad dielectric substrate material from which all of the microstrip circuits used in this study were made is a commercially available teflon based product - RT/duroid 5870 whose properties are given in Appendix 3.

The guide wavelength on 50 ohm microstrip lines was found at 10GHz by measuring the input impedance of a long length of open-circuited line

Figure 3.2(a). Microstrip Characteristic Impedance^o Calculated from Work of Wheeler.

Figure 3.2(b). Ratio of Free Space Wavelength (λ_0) to Microstrip Wavelength (λ_g)
Calculated from Work of Wheeler.

and then cutting off a length of line until the same input impedance was obtained. The length of line removed was taken to be $\lambda_g/2$. The relative dielectric constant ϵ_r was determined to be 2.3 by measuring the capacitance of a large sheet. The measured value of λ_g at 10GHz was .842 inches which corresponded to $\epsilon_{eff}=1.968$. This was in excellent agreement with the value of 1.977 determined using Wheeler's expressions.

3.4 Dispersive Behaviour in Microstrip

At high frequencies, interaction of a TM_0 mode and the TEM mode causes the mode of propagation to be less like a pure TEM mode³⁷; the Quasi-TEM assumption becomes less valid and the actual wavelength is shorter than that predicted by Wheeler's curves of Fig. 3.2.

Jain³⁷ has found that these dispersive effects may be neglected at frequencies below

$$f_D = \frac{6.}{(\epsilon_r - 1)^{1/4}} \left[\frac{Z_0}{H_m} \right]^{1/2} \text{ GHz} \quad 3.2$$

where:

ϵ_r is the relative dielectric constant of the substrate material

Z_0 is the characteristic impedance of the microstrip line

H_m is the thickness of the substrate material in mils

f_D is the frequency in gigahertz below which dispersion may be neglected.

For the 50 ohm lines used in this study, $f_D = 12.6\text{GHz}$ and for 35 ohm lines $f_D = 10.5\text{GHz}$.

Syrett³⁸ found that for the substrate material used in this work, a more appropriate expression is:

$$f_D = \frac{.3}{(\epsilon_r - 1)^{1/4}} \left[\frac{Z_0}{H_m} \right]^{1/2} \text{ GHz} \quad 3.3$$

This would give $f_D = .6\text{GHz}$ for 50 ohm lines and $.5\text{GHz}$ for 35 ohm lines. He also found that the dispersive effect was so small across X-band (ϵ_{eff} changed by about .5%) that a single value - i.e. the value measured at 10GHz, was suitable for use across the band.

For this study, the measured value of $\epsilon_{\text{eff}} = 1.968$ was used across X-band for 50 ohm lines. This value was significantly different from the value of $\epsilon_{\text{eff}} = 2.09$ measured by Syrett. This discrepancy may possibly be attributable to variations in ϵ_r between sheets of copper clad dielectric. Measurements of the capacitance of many circular dots constructed on the same sheet indicated variations in ϵ_r as high as 10%.

3.5 Losses in Microstrip Lines

Loss in microstrip transmission lines is often much higher than in other types of guiding systems. Typically, at 10GHz, the loss is .1dB/metre in waveguide, .17dB/metre in air core coaxial line and as high as

.1dB/centimetre in microstrip. It is important to accurately determine the value of the microstrip loss because the isolation of branch-line couplers is particularly sensitive to loss¹³ and because long lead-in lines are required in the practical couplers.

Losses in microstrip with non-magnetic substrates may be attributable to two main sources⁴⁰⁻⁴²:

- 1) Dielectric loss (α_d dB/cm)
- 2) Conductor loss (α_c dB/cm)

3.5.1 Dielectric Loss

Welch and Pratt have derived a theoretical formula for dielectric loss in microstrip which is based on Wheeler's filling factor³. This expression was later used by Pucel et al⁴² in the following form:

$$\alpha_d = 4.34 \frac{q}{\sqrt{\epsilon_{eff}}} \sqrt{\frac{\mu_0}{\epsilon_0}} \sigma \text{ dB/cm} \quad 3.4$$

$$q = \frac{1}{\epsilon_{eff}(\epsilon_r - 1)} \quad 3.5$$

where:

q is Wheeler's filling factor³

ϵ_{eff} is the effective dielectric constant of the microstrip line

μ_0, ϵ_0 are the permeability and permittivity of free space

σ is the conductivity of the dielectric substrate.

Figure 3.3 shows a plot of α_d/λ_g for the frequency range 8-12GHz. The conductivity of the material is determined from the dissipation factor given in Appendix 3.

3.5.2 Conductor Loss

Caulton et al⁴¹ have derived an expression for the conductor losses in microstrip based on the assumptions that:

- 1) There is a uniform current across the width of the conductor.
- 2) The ground plane current is distributed uniformly under the conductor.

They found that:

$$\alpha_c = \frac{8.68 R_s}{Z_0 W_m} \text{ dB/cm} \quad 3.6$$

where:

R_s is the resistivity of the conductor and ground plane

Z_0 is the characteristic impedance

W_m is the width of the microstrip transmission line.

Strictly speaking, these assumptions are only valid for infinitely wide conductors. A more recent paper by Pucel et al⁴² uses Wheeler's incremental inductance rule⁴³ to estimate the conductor losses. Figure 3.3 is a plot of the attenuation factors predicted by the theories of both Pucel et al⁴² and Caulton et al⁴¹ for a 50 ohm line in the X-band range where the dielectric loss contribution is determined from the ex-

Figure 3.3. Theoretical Loss for 50 ohm Microstrip Lines Constructed on Duroid.

pression of Pucel et al⁴² as extrapolated by Syrett³⁸ to X-band. Also shown in this figure are the experimental values measured by Syrett³⁸ using a ring resonance technique.

In this study, the attenuation across X-band was taken to be the value determined by Syrett of $.12\text{dB}/\lambda$, which is in excellent agreement with the theory of Pucel et al.

3.6 Microstrip Transmission Line Discontinuities

3.6.1 Introduction

When the physical size of a microstrip transmission line configuration becomes a significant fraction of a wavelength, the discontinuities associated with various types of junctions can become an important consideration when determining the circuit's electrical behaviour. In general, these discontinuities are lossless and reciprocal.³⁵ They are lossless because the higher order modes which cause them are below cut-off. Hence, they can be modelled by purely reactive elements. However, most of the models are still approximate^{15,16} because an exact analysis of these modes is extremely difficult. The junctions are reciprocal because non-magnetic substrates are used. Figure 3.4 shows the configurations considered in this study.

The following sections present some existing theoretical models for the T junction and open-ended microstrip. Also, because there is no

(a) Open-Ended Line.

(b) T Junction.

(c) Y Junction.

Figure 3.4. Microstrip Junctions of Importance in This Study.

published theoretical model for the Y junction, one is developed. Experimental results are also presented for the T and Y models. The lack of data for the T has been pointed out⁴⁹ by other authors; no data exists for the Y; while the open-ended line has already been extensively investigated^{44,46,47}. The next section outlines the resonance technique used to measure these junction reactances.

3.6.2 A Resonance Method for Measurement of Small Microstrip Reactances

The poor repeatability of the electrical model for the coax-microstrip connector makes it extremely difficult to measure, by conventional reflection techniques, any small microstrip discontinuities located beyond this transition^{4,32,33,49,54}.

If the discontinuity is reactive, this problem can be circumvented by introducing it into a resonant circuit. The resulting resonant frequency which is extremely sensitive to even very small reactances can be easily measured and used to compute the discontinuity value. The circuit can be arranged so the coax-microstrip connectors have no effect on the resonant frequency.

A general description of the method is as follows. Consider Fig. 3.5. A transmission measurement is made between the coax-microstrip connectors. At some frequency f_m , the branch circuit will series resonate and cause a sharp high Q dip in the transmission. If the inductance L_K and the capacitance C_K are known, then the unknown reactance X_U can be found from f_m by

Figure 3.5. General Representation of the Resonance Technique..

solving the following:

$$2\pi f_m L_K - \frac{1}{2\pi f_m C_K} + X_U = 0 \quad 3.7$$

Since the resonating tuned circuit appears as an effective short circuit at the junction, the frequency determining elements are isolated from the measuring circuit - particularly the connectors.

3.6.3 Microstrip End Effect

Figure 3.4(a) shows an open-circuited microstrip line. Associated with the fields at the open end is a complex admittance. The real part (which may be neglected) for the 50 ohm lines used)⁴⁵ is due to radiated energy while the imaginary term is due to capacitive storage fields.

Many authors^{44,46,47} have modelled an open-ended microstrip line of length L as an open-circuited line of length $L + \Delta L$. Jain, Makios and Chudoblak⁴⁶ found that this effective length of line ΔL is given by:

$$\Delta L = \frac{1}{\beta} \operatorname{arccot} \frac{4c + 2W_m}{c + 2W_m} \cot(\beta c) \quad 3.8$$

where:

ΔL is the apparent increase in length of the center conductor due to fringing fields at the open end.

$$\beta = 2\pi/\lambda_g \quad 3.9$$

λ_g is the guide wavelength.

$$c = \frac{H_m \ln 4}{W_m} \quad 3.10$$

H_m is the height of the microstrip substrate.

W_m is the width of the microstrip line.

For $\beta c \leq .3$, the above expression may be approximated to within 3% by:

$$\Delta L = c \left[\frac{c + 2W_m}{4c + 2W_m} \right] \quad 3.11$$

The value of ΔL for both a 50 ohm and 35 ohm line is approximately 4 mils. Throughout this study, open-ended lines of physical length L will be referred to as open-ended lines of electrical length:

$$l = L + \Delta L \quad 3.12$$

3.6.4 Microstrip T-Junction

3.6.4.1 Introduction

Figure 3.4(b) shows a T-shaped junction between two microstrip or stripline transmission lines of widths W_1 and W_2 . This junction occurs not only in branch-line couplers but also in such structures as impedance matching networks and filters. Hence, an accurate representation of any associated discontinuity impedances would be of great value to the circuit designer.

In general, a 3-port network requires nine independent complex parameters to model its electrical behaviour at each frequency. However, because these junctions can be assumed to be lossless and reciprocal with one axis of symmetry, only four pure-imaginary elements are required.

3.6.4.2 Theoretical Models

Although both theoretical and experimental studies have been made of the discontinuity impedances of symmetrical stripline T junctions^{47,50}, the majority of the work on microstrip has been theoretical^{15,16}. In coupler analysis, the most often used stripline and microstrip theoretical T models are derived in the same way from existing waveguide E plane T equivalent circuits. The same waveguide model was used by Oliner and Altschuler⁴⁷ for stripline that was used by Leighton and Milnes¹⁵ for microstrip. Similarly, the Waveguide Handbook²⁹ model that was used by Franco and Oliner⁵⁰ for stripline was also used by Vogel¹⁶ for microstrip. Because this derivation is not strictly correct (TEM mode propagation is assumed), it is advisable to experimentally verify the results before using them.

Using the Waveguide Handbook equivalent circuit for illustrative purposes, the strip models are derived as follows.

a) The waveguide structure with known circuit model is converted to an equivalent parallel plate structure. Figure 3.6(a) shows the waveguide E plane T with the equivalent circuit of Fig. 3.6(b). Element values (labelled waveguide) are given in the graphs of Fig. 3.7. These results:⁵⁰

a) Waveguide E plane T structure.

b) Waveguide E plane T equivalent circuit.

c) Parallel plate structure.

Fig. 3.6. Transformation of the waveguide E plane T to the required parallel plate T junction.

d) New parallel plate structure derived from Babinet equivalence.

e) Equivalent circuit of new structure.

Fig. 3.6. Transformation of the waveguide E plane T to the required parallel plate T junction.

Fig. 3.7. Values for T junction model from Waveguide Handbook²⁹.

Fig. 3.7. Values for T junction model from Waveguide Handbook²⁹.

"apply equally well to a parallel plate E-plane tee if λ , the wavelength of the TEM mode in parallel plate line, is substituted for λ_g , the guide wavelength of the TE_{10} mode in rectangular waveguide." Also, ⁵⁰ "The parallel plate structure may be modified without disturbing the fields by truncating the width and inserting magnetic sidewalls."

Figure 3.6(c) illustrates the resulting parallel plate structure which has the same equivalent circuit as the waveguide configuration.

b) A Babinet equivalence is performed on the parallel plate structure to obtain a parallel plate configuration of the proper geometry. That is:

- 1) Magnetic and electric walls are interchanged.
- 2) Lines of E and H are replaced by lines of H and -E respectively.
- 3) Admittances in equivalent circuits become impedances and vice-versa.
- 4) Shunt and series elements are interchanged.

This gives the structure of Fig. 3.6(d) with equivalent circuit as shown in Fig. 3.6(e). Element values are again determined from Fig. 3.7 (labelled parallel plate).

c) The stripline or microstrip is converted to an equivalent parallel plate structure. For stripline, the transformation is: ⁵⁰

$$D_s = W_s + \frac{2H_s \ln 2}{\pi} + \frac{t_s}{\pi} \left[1 - \ln \frac{2t_s}{H_s} \right] \quad 3.13$$

where:

H_s is the spacing between ground planes

W_s is the original strip width

t_s is the strip thickness

D_s is the equivalent parallel plate width.

Figures 3.8(a) and (b) show the original stripline fields and the parallel plate representation. Since discontinuities in the center conductor are symmetrically placed with respect to the plates, only one-half of this structure is required to perform the analysis. Therefore, the two parallel plate representation of Fig. 3.8(c) can be used.

Microstrip lines of width W_m , height H_m and impedance Z_o are converted to equivalent parallel plate lines of width D_m using a transformation¹⁵ that preserves their height, impedance and wavelength.

$$D_m = \frac{H_m}{Z_o} \sqrt{\frac{\mu_o}{\epsilon_{eff} \epsilon_o}} \quad 3.14$$

$$\epsilon_{eff} = \left(\frac{\lambda_a}{\lambda_g} \right)^2 \quad 3.15$$

Figures 3.8(d) and (e) show the original microstrip fields and the parallel plate representation.

d) The parallel plate dimensions for the strip transmission lines are substituted into the derived equivalent circuits.

The other T model which was derived in a similar fashion and used by Leighton and Milnes for microstrip and Oliner and Altschuler for stripline is shown in Fig. 3.9. The two models may be related by the following:

a) Stripline field configuration.

b) Stripline equivalent parallel plate representation.

c) Simplified stripline parallel plate representation.

d) Microstrip field configuration.

e) Microstrip equivalent parallel plate representation.

Fig. 3.8. Transformation of stripline and microstrip to an equivalent parallel plate structure.

a) Layout.

b) Equivalent circuit.

$$X_a = \frac{Z_1 D_2}{\lambda_g} \left(\frac{n' \pi}{4} \right)^2$$

$$X_b = -\frac{X_a}{2} + \left[\frac{2D_1 Z_1}{(n')^2 \lambda_g} \right] \times \left[\ln \left(\frac{1.43 D_1}{D_2} \right) + 2 \left(\frac{D_1}{\lambda_g} \right)^2 \right]$$

$$n' = \frac{\sin \left(\frac{\pi D_2}{\lambda_g} \right)}{\left(\frac{\pi D_2}{\lambda_g} \right)}$$

Fig. 3.9. Leighton and Milnes' and Oliner and Altschuler's T junction model.

$$n^2 = (n')^2 \frac{\cos^2 \left(\frac{2\pi d'}{\lambda_g} \right)}{\cos^2 \left(\frac{2\pi d}{\lambda_g} \right)} \quad 3.16a)$$

$$\frac{Y}{Y_1} + n^2 \frac{Y_2}{Y_1} \tan \left(\frac{2\pi d'}{\lambda_g} \right) = 2 \tan \left(\frac{2\pi d}{\lambda_g} \right) \quad 3.16b)$$

$$\tan \left(\frac{2\pi d'}{\lambda_g} \right) = \frac{Z_1}{Z_2} (n')^2 \frac{X_b}{Z_1} \quad 3.16c)$$

$$\tan \left(\frac{2\pi d}{\lambda_g} \right) = - \frac{X_a}{Z_1} \quad 3.16d)$$

Leighton and Milnes theoretically analyzed the performance of microstrip 3dB directional couplers. However, no experimental verification was given. Vogel studied the same type of coupler and obtained experimental results that supported his theoretical T model. However, he still had disagreement and he never specifically measured the T junction reactances. Both authors indicated that the influence of the T junction was significant. Recently, Easter⁴⁹ obtained experimental results for T junctions on alumina substrates. Agreement with Leighton and Milnes and Vogel was poor. Thomson and Gopinath⁴⁸ developed a theoretical model which agreed well with one element in Easter's model. Unfortunately, they did not present results for lines constructed on substrates with low dielectric constant.

3.6.4.3 Experimental Model

Figures 3.10 and 3.11 show the two circuit configurations used to measure the T-junction reactances. For illustration, Leighton and Milnes' model is assumed. The transformer has been neglected because its theoretical turns ratio is very nearly unity. The reactances shown in the figures are represented at each frequency by: $X_a(Z_1, Z_2)$ and $X_b(Z_1, Z_2)$ where Z_1 is the main-line T impedance and Z_2 is the branch-line impedance. The reactances are measured for a number of different resonant frequencies across X-band.

To determine $X_b(50, 35)$, the circuit of Fig. 3.10 is used. The stub length ℓ_1 is cut to be approximately $\lambda_0/4$ where λ_0 corresponds to the anticipated resonant frequency. The frequency is then adjusted to find f_m , the true resonant frequency. Then, X_b is found from

$$X_b(50, 35) - j35 \cot\left(\frac{2\pi}{\lambda_m}\right)\ell_1 = 0 \quad 3.17$$

where:

λ_m is the wavelength at the measured resonant frequency.

To determine the main-line reactance, $X_a(50, 35)$, it is first necessary to determine $X_b(50, 50)$. This is done exactly as described above using the

(a) Microstrip Layout.

(b) Equivalent Circuit.

Figure 3.10 Resonant Structure Used to Measure $X_b(z_1, z_2)$ in Leighton and Milnes' T Model.

circuit of Fig. 3.10; only Z_2 is made 50 ohms instead of 35 ohms. After this first step, the circuit of Fig. 3.11 is used. The lines of length ℓ_0 and ℓ_1 have impedance of $Z_1=50$ ohms and the line of length ℓ_2 has impedance of $Z_2=35$ ohms.

At resonance, the stub of length ℓ_2 has input impedance.

$$Z_{in} = j X_b(50,50) - j Z_2 \cot \left(\frac{2\pi}{\lambda_m} \right) \ell_2 \quad 3.18$$

This value is extremely large since $\ell_2 = \frac{\lambda_0}{2}$ and may be ignored because it is in shunt with the rest of the circuitry. The input impedance of the total resonant structure is then given as:

$$Z_{in} = j X_b(50,50) + Z_1 \left[\frac{Z_L \cos \beta \ell_1 + j Z_1 \sin \beta \ell_1}{Z_1 \cos \beta \ell_1 + j Z_L \sin \beta \ell_1} \right] \quad 3.19$$

where:

$$Z_L = -j Z_1 \cot \beta \ell_2 + j 2 X_a(50,35) \quad 3.20$$

At resonance, $Z_{in}=0$.

Solving for Z_L in 3.19 gives:

$$Z_L = \frac{j Z_1 \sin \beta \ell_1 + j X_b(50,50) \cos \beta \ell_1}{\frac{X_b(50,50)}{Z_1} \sin \beta \ell_1 - \cos \beta \ell_1} \quad 3.21$$

$$= j Z_1 \sin \beta \ell_1 - j X_b(50,50) \quad 3.22$$

Substituting into 3.20

$$j X_a(50,35) = \frac{1}{2} \left[j Z_1 \sin \beta \ell_1 - j X_b(50,50) + j Z_1 \cot \beta \ell_3 \right] \quad 3.23$$

3.6.4.4 Results

The experimental results for the frequency range 8-12GHz are shown plotted in Fig. 3.12 along with the theories of Vogel and Leighton and Milnes. Both $X_b(50,50)$ and $X_b(50,35)$ are in close agreement with Leighton and Milnes' model rather than Vogel's. Results for $X_a(50,35)$ do not show too much scatter but do not support either model. Values are of the same order of magnitude as both theories.

3.6.5 Microstrip Y Junction

3.6.5.1 Introduction

Figure 3.4(c) shows a Y shaped junction between microstrip lines of the same width W_1 . The novel feature of this junction is that it is lossless, reciprocal and completely symmetrical (as opposed to the T which only has one axis of symmetry). Hence, to model it, the number of required

a) $X_b(50,50)$ in Leighton and Milnes' model.

b) $X_b(50,35)$ in Leighton and Milnes' model.

Fig. 3.12. Measured T junction results.

c) $X_a(50,35)$ in Leighton and Milnes' model.

Fig. 3.12. Measured T junction results.

independent reactive circuit elements is two compared with four for the T junction. This simpler junction which is used in the new coupler design given in Section 2.5.3 could be used to replace most constant impedance T junctions. A model for the microstrip Y junction has not appeared in the literature; accordingly in the following sections a theoretical model is developed and experimentally verified for 50 ohm lines.

3.6.5.2 Theoretical Model

Following the approach outlined in Section 3.6.4.2, this microstrip equivalent circuit is developed from the model for a waveguide E plane Y junction. Figures 3.13(a) and (b) give this waveguide junction and its corresponding equivalent circuit²⁹. A more useable representation is given in Fig. 3.13(c). Values for the susceptances $j \frac{B_c}{Y_0}$ and $j \frac{B_d}{Y_0}$ are determined as follows: Z_{sc} - the normalized input impedance at port 1 with ports 2 and 3 short circuited is computed for both the model of Fig. 3.13(b) and Fig. 3.13(c). Similarly, Z_{pc} is computed.

For the circuit of Fig. 3.13(b),

$$Z_{sc} = j \frac{3Y_0}{3B_a - B_b} \quad 3.24$$

$$Z_{pc} = j \frac{(3B_a - 2B_b)Y_0}{3(B_a B_b - B_a^2)} \quad 3.25$$

For the circuit of Fig. 3.13(c),

Figure 3.13 Development of Parallel Plate Y Junction Model.

$$jX_c = Z_0 \left\{ \frac{3Z_{oc} - Z_{sc} \pm \sqrt{(Z_{sc} - 3Z_{oc})^2 - 4Z_{sc}Z_{oc}}}{2Z_{sc}Z_{oc}} \right\}$$

$$jX_d = Z_0 \left[\frac{3 \left[1 - j \frac{X_c}{Z_0} \right] Z_{sc}}{2Z_{sc}} \right]$$

$$Z_{sc} = \frac{-31}{Z_0 (3B_a - B_b)}$$

$$Z_{oc} = \frac{j(3B_a - 2B_b)}{3Z_0 (B_a B_b - B_a^2)}$$

$$B_a = Y_0 \left(\frac{2D_1}{\lambda_g} \right) \times 0.6455$$

$$B_b = Y_0 \left(\frac{\lambda_g}{D_1} \right) \times \frac{2\sqrt{3}}{\pi}$$

Figure 3.13 Development of Parallel Plate Y Junction Model

$$Z_{sc} = \frac{-j3Y_o}{2B_d + 3B_c} \quad 3.26$$

$$Z_{oc} = \frac{-j(3B_c + B_d)Y_o}{3(B_c B_d + B_c^2)} \quad 3.27$$

Solving Equation 3.26 for B_d

$$B_d = \frac{-j3Y_o - 3B_c Z_{sc}}{2Z_{sc}} \quad 3.28$$

Substituting 3.28 into 3.27

$$Z_{oc} = \frac{-j(3B_c + \frac{(-j3Y_o - 3B_c Z_{sc})}{2Z_{sc}})Y_o}{3(B_c(-j3Y_o - 3B_c Z_{sc}) + B_c^2)} \quad 3.29$$

or

$$Z_{sc} Z_{oc} \left(\frac{jB_c}{Y_o} \right)^2 + (Z_{sc} - 3Z_{oc}) \left(j \frac{B_c}{Y_o} \right) + 1 = 0 \quad 3.30$$

The normalized susceptances jB_c and jB_d can now be calculated.

$$\frac{jB_c}{Y_o} = \frac{3Z_{oc} - Z_{sc} \pm \sqrt{(Z_{sc} - 3Z_{oc})^2 - 4Z_{sc} Z_{oc}}}{2Z_{sc} Z_{oc}} \quad 3.31$$

$$\frac{jB_d}{Y_o} = \frac{3(1 - \frac{jB_c}{Y_o} Z_{sc})}{2Z_{sc}} \quad 3.32$$

where Z_{sc} and Z_{oc} are determined from Equations 3.24 and 3.25 respectively.

The values of B_c and B_d are the desired discontinuity susceptances defined in terms of B_a and B_b which can be found from the formulas in the Waveguide Handbook.

As with the T junction analysis, this waveguide model applies equally well to a parallel plate structure. Performing the Babinet equivalence procedure of Section 3.6.4.2 (where admittances become impedances and shunt elements become series elements) gives the final parallel plate representation of Fig. 3.13(d) with equivalent circuit as shown in Fig. 3.13(e). This model applies to microstrip if Equation 3.14 is used to convert the microstrip to an equivalent parallel plate structure.

3.6.5.3 Experimental Model

Figures 3.14 and 3.15 show the circuits used to measure the reactances at the Y junction. For this study, the junction consists of 50 ohm lines.

The circuit of Fig. 3.14 is used to measure jX_c . The input impedance at T_6 for the arm of length $\ell_2 = \lambda_0/2$ is extremely large and since it is in shunt with the rest of the circuitry, it can be ignored. Since $\ell_3 = \lambda_0/4$ the input impedance of this arm at T_7 is close to a short circuit and the comparatively large shunting reactance jX_d can be neglected.

The input impedance of the total resonant structure is then given by:

Fig. 3.14. Equivalent circuit for measuring β_X in n - p junction model.

$$Z_{in} = jX_b(50,50) + Z_1 \left[\frac{Z_L \cos \beta \ell_1 + jZ_1 \sin \beta \ell_1}{Z_1 \cos \beta \ell_1 + jZ_L \sin \beta \ell_1} \right] \quad 3.33$$

where

$$Z_L = -j Z_1 \cot \beta \ell_3 + 2 jX_c \quad 3.34$$

Solving for jX_c gives

$$jX_c = \frac{1}{2} \left[jZ_1 \sin \beta \ell_1 - jX_b(50,50) + jZ_1 \cot \beta \ell_3 \right] \quad 3.35$$

Measured results are compared to the theory in Fig. 3.16(a) and the agreement is seen to be very good.

The circuit of Fig. 3.15 is used to determine jX_d . Both ℓ_2 and $\ell_3 = \lambda_0/2$ hence both input impedances are large and these arms can be ignored compared with jX_d . The resulting expression for jX_d is easily determined from Equation 3.35 with $jX_c + jX_d$ replacing $2jX_c$.

$$jX_d = -jX_c + j Z_1 \sin \beta \ell_1 - jX_b(50,50) + j Z_1 \cot \beta \ell_3 \quad 3.36$$

Experimental and theoretical results are compared in Fig. 3.16(b). The agreement is worse for jX_d than for jX_c . However, the experimental and theoretical results show approximately the same frequency dependence.

a) Values of jX_c . b) Values of $1/jX_d$.

Fig. 3.16. Y junction results.

CHAPTER 4.

COAX-MICROSTRIP CONNECTOR

4.1 Introduction

This chapter outlines the development of an empirical electrical model for the coax-microstrip connector mounted on each port of the microstrip couplers. The first part deals with the connector mounting configuration used to insure repeatability. The remainder of the chapter presents the development of a 2-port network representation for the mounted OSM-244-4A²⁷ connector used. Results are given in ABCD matrix form to facilitate calculation.

4.2 Connector Mounting Structure

The coax-microstrip connector is fastened to an end plate and base block as shown in Fig. 4.1. The centre conductor passes through a teflon-filled hole in the end plate with a diameter calculated²⁵ to maintain a 50 ohm impedance transmission line. The plate thickness is selected so that the wedge-shaped tip of the centre conductor just extends beyond the plate. Filling the space around the conductor with teflon enables the connector to be accurately held in position thus improving repeatability.

To achieve high repeatability in the contact point between the end plate and the base block, a ridge 4 mils in depth is cut in the base block 4 mils from the top of the block. Fig. 4.1 shows that

Figure 4.1. Connector mounted on 50 ohm Terminated Line.

d) S-parameter Matrices for circuit elements.

$$\frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}}{\frac{e^{-\gamma l}}{r_L} - S_{22}}$$

or for lossless lines and reciprocal connectors

$$\frac{b_1}{a_1} = S_{11} + \frac{(S_{12})^2}{\frac{e^{j\beta l}}{r_L} - S_{22}}$$

e) Signal flow graph.

Fig. 4.1. Connector Mounted on 50 Ohm Terminated Line

this ridge causes the plate to span the gap and contact the block within 4 mils of its top. Hence not only does the ridge improve repeatability of the contact point but it also minimizes discontinuities by insuring that the centre conductor and ground path are approximately equal in length.

4.3 Mounted Connector Equivalent Circuit

If the connector discontinuity impedances are associated with the wedge-shaped transition, the equivalent circuit of Fig. 4.1(c) can be used. At any given frequency a characterization of this 2-port network enables a complete specification of the connector's performance for any loading conditions at its ports.

Using the ABCD matrix (see Appendix A1) a 2 port network can be expressed as follows:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad 4.1$$

where V_1 , I_1 , V_2 , I_2 are complex voltages and currents at ports 1 and 2 when the impedances at these ports are normalized. A , B , C , and D are complex matrix elements which are functions of frequency.

It is seen that four complex independent equations are required to model a 2-port network at each frequency.

If the network is reciprocal,

$$AD - BC = 1$$

4.2

The remaining three required equations can be determined by making three measurements of the input impedance at port #1 with port #2 terminated in known output impedances. The input impedance is

$$Z_{in} = \frac{AV_2 + BI_2}{CV_2 + DI_2}$$

4.3

or when port #2 is terminated in a normalized impedance K

$$Z_{in} = \frac{AK + B}{CK + D}$$

4.4

The values of K selected are²⁹:

$$K = 0 \text{ ohms (short circuit)}$$

4.5

$$K = \infty \text{ ohms (open circuit)}$$

4.6

$$K = 1 \text{ (normalized termination)}$$

4.7

substituting these values into equation 4.4 gives:

2 2
OF/DE

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

$$Z_{sc} = (Z_{in} \text{ with } K = 0) = \frac{B}{D}$$

4.8

$$Z_{oc} = (Z_{in} \text{ with } k \rightarrow \infty) = \frac{A}{C}$$

4.9

$$Z_{50} = (Z_{in} \text{ with } K = 1) = \frac{A + B}{C + D}$$

4.10

Solving for A, B, C and D, from equation 4.8,

$$B = DZ_{sc}$$

4.11

from 4.9

$$A = CZ_{oc}$$

4.12

substituting 4.11 and 4.12 into 4.10

$$Z_{50} = \frac{CZ_{oc} + DZ_{sc}}{C + D}$$

4.13

$$C = \frac{D(Z_{sc} - Z_{50})}{(Z_{50} - Z_{oc})}$$

4.14

substituting 4.11 and 4.12 into 4.2

$$(CZ_{oc})D - (DZ_{sc})C = 1$$

4.15

substituting 4.14 into 4.15

$$\frac{Z_{oc} D^2 (Z_{sc} - Z_{50})}{(Z_{50} - Z_{oc})} - \frac{Z_{sc} D^2 (Z_{sc} - Z_{50})}{(Z_{50} - Z_{oc})} = 1$$

4.16

$$D^2 \left[\frac{(Z_{sc} - Z_{50})(Z_{oc} - Z_{sc})}{(Z_{50} - Z_{oc})} \right] = 1 \quad 4.17$$

$$D = \sqrt{\frac{(Z_{50} - Z_{oc})}{(Z_{sc} - Z_{50})(Z_{oc} - Z_{sc})}} \quad 4.18$$

and from 4.11, 4.14 and 4.12

$$B = DZ_{sc}$$

$$C = \frac{D(Z_{sc} - Z_{50})}{(Z_{50} - Z_{oc})}$$

$$A = \sqrt{Z_{oc}}$$

Hence measurement of Z_{sc} , Z_{oc} and Z_{50} will enable A, B, C and D to be determined.

4.4 Measurement of Z_{oc} and Z_{sc}

The structures used to measure Z_{oc} and Z_{sc} are the same as that in Fig. 4.1 with the exception that the termination is replaced by an open circuit. To measure Z_{oc} , the line length l is set to a half wavelength while to measure Z_{sc} , it is a quarter wavelength. The measured input impedances for these circuits for the frequency range 8-12GHz in .5GHz steps are given in Table 4.1.

Frequency	Z_{sc}	Z_{oc}	Z_{50}	A	B	C	D
8.0	1.33	-j4.7	1.02+j.13	.96+j.01	.00+j.32	.00+j.20	.97-j.01
8.5	1.33	-j4.5	.96+j.14	.93+j.00	.00+j.33	.00+j.21	1.00+j.00
9.0	1.39	-j3.3	.98+j.10	.92+j.00	.00+j.38	.00+j.28	.97+j.00
9.5	1.45	-j3.5	1.08+j.07	.98-j.02	-.01+j.41	.01+j.28	.91+j.02
10.0	1.49	-j3.5	1.12+j.12	.98-j.01	.00+j.44	.00+j.28	.9+j.01
10.5	1.54	-j4.0	1.13+j.27	.96+j.01	.01+j.49	.00+j.24	.92-j.01
11.0	1.51	-j5.5	.99+j.36	.92+j.01	.00+j.51	.00+j.17	.99-j.01
11.5	1.48	-j7.1	.9+j.42	.89+j.02	.01+j.51	.00+j.13	1.05-j.03
12.0	1.42	-j4.2	.77+j.3	.81+j.01	.00+j.47	.00+j.19	1.13-j.01

TABLE 4.1.

Measured and Calculated Connector Parameters

4.5 Measurement of Z_{50}

4.5.1 Direct Method

To measure Z_{50} directly, it is necessary to terminate port #2 of the connector in 50 ohms. Therefore, the input impedance, Z_{in} to the 50 ohm microstrip transmission line of length l , shown in Fig. 4.1, must be 50 ohms.

The input impedance, Z_{in} of a transmission line of characteristic impedance Z_0 terminated in load Z_L is given by:

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \quad 4.19$$

where: Z_0 is the characteristic impedance of the transmission line

Z_{in} is the input impedance

Z_L is the load impedance

γ is the complex propagation constant $= \alpha + j\beta$

α is the attenuation factor in nepers/in

β is the propagation phase shift in rad/in

l is the transmission line length in inches

If the transmission line is terminated in a load $Z_L = Z_0$ (Fig. 4.1) then for a 50 ohm line $Z_{in} = Z_0 = 50$ ohms. (Z_0 is assumed to be 50 ohms throughout this study because the comparatively small value of transmission line

loss does not introduce a significant reactive term). Proper termination of the line was not possible, however, since commercially available 50 ohm resistors were found to have a VSWR of approximately 1.2.

Another approach is to make a lossy transmission line of sufficient length such that $\tanh \gamma l \rightarrow 1$ causing Z_{in} to approach Z_0 independent of the value of Z_L . To test this method, a spiralled microstrip line of length $l = 84$ inches was constructed. This should have caused reflections from a load to be attenuated by 24 db at 10 GHz for a line of loss $.12\text{db}/\lambda_g$.

Measurement of Z_{in} across X-band displayed many large resonances which were absent when a short ($l < \frac{\lambda_0}{2}$) terminated line was used. This suggests that the long length of line rather than the connector discontinuities was the cause of these resonances since a line shorter than $\lambda_0/2$ cannot multiply resonate.

One possible explanation for the effect could be the variations in the permittivity of the dielectric material as discussed in Section 3.4. Coupling between adjacent lines of the spiral and radiation from the lines did not seem to be of significance. This method had too many unwanted reflections to be used.

4.5.2 Sliding Termination Technique

4.5.2.1 Introduction

This procedure does not require a perfectly terminated transmission line, however, it is more indirect than the previous methods. The circuit used is shown in Fig. 4.1 with l approximately one wavelength.

The experimental method is as follows:

- 1) Measure the input reflection coefficient of the circuit and plot it on a Smith Chart.
- 2) Remove the connector; cut a short length of line (approximately $\frac{1}{20}$ of a wavelength) off the connector end; and replace the connector on this shortened line.
- 3) Repeat steps 1 through 2 approximately 10 times.

The locus of points plotted on the Smith Chart will be circular (barring experimental errors). The impedance read off the chart at the "best-fit" centre of the locus may be taken as Z_{50} at that frequency.

4.5.2.2 Theory of the Sliding Termination Technique⁵²

The equivalent circuit of the configuration is given in Fig. 4.1(c) where the transmission line, because of its short length is assumed lossless. In the S-parameter designation the vector input signals a_1 and a_2 to ports #1 and #2 of the connector are related to the vector output signals b_1 and b_2 by:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

4.20

where:

$$a_1 = \frac{1}{2} \left(\frac{v_1}{\sqrt{Z_0}} + \sqrt{Z_0} I_1 \right)$$

$$b_1 = \frac{1}{2} \left(\frac{v_1}{\sqrt{Z_0}} - \sqrt{Z_0} I_1 \right)$$

$$a_2 = \frac{1}{2} \left(\frac{v_2}{\sqrt{Z_0}} + \sqrt{Z_0} I_2 \right)$$

$$b_2 = \frac{1}{2} \left(\frac{v_2}{\sqrt{Z_0}} - \sqrt{Z_0} I_2 \right)$$

The S-parameter matrix for each element in Fig. 4.1(c) is given below in Fig. 4.1(d). The connector is reciprocal therefore $S_{12} = S_{21}$ and the transmission line is assumed lossless because of its short length therefore $\gamma = j\beta$. The signal flow graph is given in Fig. 4.1(e) from which the input reflection coefficient can be derived using the non-touching loop rule⁵³.

$$\frac{b_1}{a_1} = S_{11} + \frac{(S_{12})^2}{1 - S_{22}}$$

4.21

If the microstrip line is perfectly terminated, $\Gamma_L = 0$ and the reflection coefficient is $\Gamma = \frac{b_1}{a_1} = S_{11}$. This is the desired value - the connector reflection coefficient when its output is properly terminated. The reflection coefficient is easily related to the connector input impedance by

$$Z_{in} = \frac{\Gamma + 1}{\Gamma - 1} \quad 4.22$$

and may be read directly from a Smith Chart.

When the microstrip line is not properly terminated, $\Gamma_L \neq 0$. However, as can be seen from Equation 4.21 as βl is varied through 2π radians, (as will happen as the line is shortened) the locus traces out a circle (this is one of the properties of such a linear fractional transformation⁵²). The centre of the circle is assumed to be S_{11} . Although this is not exactly correct, it seldom causes significant errors.

This can be seen by determining the true centre of the circle.

Rewriting 4.21

$$\frac{b_1}{a_1} = S_{11} + \frac{(S_{12})^2}{\frac{e^{j\beta l}}{\Gamma_L} - |S_{22}|e^{j\theta_{22}}} \quad 4.23$$

$$\text{where } S_{22} = |S_{22}| e^{j\theta_{22}}$$

Two points on opposite sides of the circle are given by:

$$S_{11} + \frac{(S_{12})^2}{\left[\frac{1}{|\Gamma_L|} - |S_{22}| \right] e^{j\theta_{22}}} \quad 4.24$$

and

$$S_{11} + \frac{(S_{12})^2}{\left[-\frac{1}{|\Gamma_L|} - |S_{22}| \right] e^{j\theta_{22}}} \quad 4.25$$

subtracting equation 4.25 from 4.24 gives the vector connecting these points whose magnitude is the diameter of the circle. This vector is:

$$\frac{2(S_{12})^2 |\Gamma_L| e^{j\theta_{22}}}{1 - |\Gamma_L|^2 |S_{22}|^2} \quad 4.26$$

Hence the radius R of the circle is given by:

$$R = \frac{|S_{12}|^2 |\Gamma_L|}{1 - |\Gamma_L|^2 |S_{22}|^2}$$

The centre of the circle is located half way along the vector of equation 4.26. Hence the coordinates of the centre are given by:

$$\frac{(S_{12})^2 |\Gamma_L| e^{j\theta_{22}}}{1 - |\Gamma_L|^2 |S_{22}|^2} - \frac{(S_{12})^2 e^{j\theta_{22}}}{\left[\frac{1}{|\Gamma_L|} + |S_{22}| \right]} + S_{11} \quad 4.28$$

$$= S_{11} + \frac{(S_{12})^2 |\Gamma_L|^2 e^{j\theta_{22}}}{1 - |\Gamma_L|^2 |S_{22}|^2} \quad 4.29$$

Therefore the magnitude of the distance between the true centre of the circle and S_{11} which is the error in estimating S_{11} is given by:

$$X = \frac{|S_{12}|^2 |r_L|^2 |S_{22}|}{1 - |r_L|^2 |S_{22}|^2} \quad 4.30$$

$$= R |r_L| |S_{22}| \quad 4.31$$

where X is the error in estimating S_{11}

R is the radius of the circle traced out by shortening the line

Typical values for the parameters are:

$$|r_L| = .02$$

$$|S_{22}| = .2$$

$$|S_{11}| = .2$$

$$R = .06$$

$$X = .001$$

Hence X - the error in estimating S_{11} is negligibly small. Another advantage to this method is that many measurements are required for each estimate of Z_{50} hence there is an inherent averaging of random measurement errors.

4.5.2.3 Sample Problem

Applying the sliding termination technique to a network with known impedance values illustrates that the error in estimating Z_{50} is very small.

The connector is assumed to be a series inductance of value $j1$ while the termination is assumed to have a VSWR of 2. As the line is shortened the impedance seen at port #2 of the connector follows locus A of Fig. 4.2 while the impedance at port #1 follows locus B. The centre of locus B is the estimate of Z_{50} . This estimated value is seen to be $1.05 + j.91$ which compares well with the actual value of $1+j1$ considering the extreme parameter values chosen.

4.5.2.4 Measured Results

The experimental loci obtained for the connector in the frequency range 8-12 GHz in .5 GHz steps are given in appendix A2. The "best-fit" centres of these loci are given in table 4.1. Although loss in the microstrip line causes the loci to spiral slightly outwards as the line is shortened, this effect is so small that it is not noticeable from the measured results in appendix A2.

4.6 Experimental Verification of Connector Model

To check on the correctness of the connector model, the input impedance to the circuit configuration of Fig. 4.3 (a) was both measured and computed for the 8-12 GHz range. Fig. 4.3 (b) shows the results. The deviation between the two becomes most pronounced in the range 11-12 GHz where the wavelengths are shorter and accurate measurements of Z_{oc} and Z_{sc} were accordingly more difficult.

The next chapter uses this connector model in the prediction of the performance of a microstrip coupler with a connector on each port.

Fig. 4.2. Example of the Sliding Termination Technique.

(a) Test configuration

(b) Measured and Computed Values for Z_{in}

Figure 4.3 Experimental Verification of Connector Model

CHAPTER 5

ANALYSIS OF COUPLER PERFORMANCE

5.1 Introduction

This chapter studies the applicability of various coupler models whose performances are determined using ABCD matrix theory and a computer program. Measured results of a conventional coupler are compared with some models and the most significant performance-determining factors are identified. Connectors are shown to be very significant while for the substrate material used, T junctions are comparatively unimportant.

A theoretical analysis of the Y junction coupler demonstrates its suitability to replace the conventional coupler in applications where fast, high precision modelling or tuneability are required.

5.2 Conventional Coupler

5.2.1 Effect of Branch Line Length

One of the simplest and most widely used models for the 2-branch-line coupler is shown in Fig. 5.1(a). If the main line lengths are $\lambda_0/4$, the performance curves of Figs. 5.1(b)-(d) have been computed for various branch-line lengths where f_0 is the resonant frequency (i.e., the frequency that corresponds to maximum isolation when the branch lines are also $\lambda_0/4$.)

It is seen that for main and branch lines of equal length, the coupling into both ports 2 and 3 is 3dB when the isolation is perfect. Port 3

a) Basic Equivalent Circuit

b) Isolation (Incident Power Leaving Port 4)

Figure 5.1 Effect of Branch-line Length on the performance of the Basic Coupler Equivalent Circuit

c) Insertion Loss (Incident Power Leaving Port 2)

d) Coupling Factor (Incident Power Leaving Port 3)

Figure 5.1 Effect of Branch-line Length on the Performance of the Basic Coupler Equivalent Circuit

coupling across the band is much flatter than that of port 2.

For the other cases, shortening the branch lines raises the resonant frequency while lengthening them lowers it. For the cases illustrated, the overall shape of the responses remains essentially independent of branch-line lengths while the resonant frequency is approximately mid-way between that predicted by just considering the branch-line lengths and that predicted by just considering the main-line lengths. However, increasing the branch-line to main-line length ratio increases the phase difference between the outputs - a property that may not always be desirable.

5.2.2 Effect of T Junctions

Figure 5.2(a) shows the basic coupler equivalent circuit of Fig. 5.1(a) with a T junction model added and all lines of length $\lambda_0/4$. All of the T junction models outlined previously can be put into this form. The capacitors in series with the branch lines shorten their electrical length while the inductors lengthen the main lines. As previously discussed, longer main lines would tend to lower the resonant frequency while shorter branch lines would tend to raise it. In all of the T models studied, the capacitive reactance is larger than the inductive; hence, the overall effect is a shortening of the branch lines. This can be determined from the computed responses for the various T models given in Figs. 5.2(b)-(d). Again, port 3 effects are small.

(a) Suggested Configuration and Equivalent Circuit

(b) Leighton and Milnes' Configuration and Equivalent Circuit

Figure 5.2 Effect of T Junctions on Basic Coupler Performance

(c) Insertion Loss

(d) Coupling Factor

Figure 5.2 Effect of T Junction on Basic Coupler Performance

It is seen that the T junction models have only a small influence (up to 0.5dB) on the coupler response and there is no appreciable difference between them. However, most authors indicate that T junction effects are significant^{15,16}. This is illustrated in Figs. 5.2(c)-(d) which also show the theoretical curves derived by Leighton and Milnes to demonstrate the influence of their T junction model on couplers with alumina substrates and a center frequency of 9GHz. The differences between these and present results can be attributed both to the larger discontinuities associated with alumina ($\epsilon_r \approx 10$) rather than Duroid^{A3} ($\epsilon_r \approx 2.3$) substrates and the dissimilar configurations used.

Figure 5.2(b) gives the layout and assumed equivalent circuit used by Leighton and Milnes. This configuration is poor because they have introduced a step junction and then neglected it in their equivalent circuit. They have assumed that inductance is in the main lines and capacitance is in the branch lines. This causes the theoretical coupling into port 2 to be low below the center frequency and high above it. If their branch lines and main lines had been interchanged as was done in the present study, the step junction would have been eliminated and no simplifications of the model would have been necessary.

5.2.3 Practical Coupler Configuration

In order to measure the coupler response, coax/microstrip connectors and lead-in lines must be added to the basic structure discussed in the

previous sections. Figure 5.3(a) is a reproduction of Fig. 2.7 which shows the necessary configuration along with a suggested equivalent circuit. Account has been made of all the anomalous effects discussed in the previous chapters. Measured and computed responses for this coupler are compared in Figs. 5.3(b)-(d).

Line loss was found to have a small influence on the computed outputs at ports 2 and 3 as was also noticed by Kurczok¹⁵. The only significant effect was to lower these outputs by approximately the loss attributable to the long lead-in lines (.3dB). However, the computed isolation¹³ was found to be significantly influenced by the loss. In practice, poor repeatability prohibited meaningful measurement of this effect.

Not only do main lines introduce loss but, more importantly, they change the phase of the connector model at the center coupling structure. The connector model and its position are overwhelmingly the most significant cause of deviations from an ideal coupler response. Inclusion of a T model also improves the agreement. However, as expected from the previous discussion in Section 5.2.2 the differences between the models are so small that it is not possible to determine conclusively which is the most appropriate model.

Comparing the computed and measured responses, it is seen that for the most detailed model (with lead-in lines, loss, T junctions and connectors),

Figure 5.3 Practical Coupler Configuration

b) Isolation (Incident Power Port 4)

Figure 5.3 Practical Coupler Configuration

c) Insertion Loss (Incident Power Leaving Port 2)

d) Coupling Factor (Incident Power Leaving Port 3)

Figure 5.3 Practical Coupler Configuration

the predicted couplings into ports 3 and 4 are in excellent agreement with the measured results. The discrepancies for port 2 are also less than for the other models but are still quite large. The best agreement is in the lower frequency range. This could be related to the connector model which showed poorer predictability at the higher frequencies. Also, the 50 ohm terminations used with the connectors were not ideal - hence, they could introduce errors.

The only known published results for this coupler configuration are given by Vogel¹⁶ with no mention being made of connector discontinuities or loss. His measured results for 3GHz couplers constructed on alumina are shown in Figs. 5.4(a)-(b). Also shown in this figure are the theoretical responses computed by the present author for this structure using the T junction models of Vogel and Leighton and Milnes. Vogel also designed couplers to compensate for his T junction model and claimed much improved agreement between measurement and theory. However, from Figs. 5.4(a)-(b), it is apparent that if one includes his T models in the response calculations, rather than shortening the line lengths to compensate for the T junctions, there is not a great improvement in the effectiveness of the model. Although he did not consider Leighton and Milnes' T model, it is seen to give agreement comparable to his own. Significantly, his results over most of the band also indicate less coupling into port 2 than predicted by the theory and good agreement for the other ports - the

(a) Insertion Loss. (Incident Power Leaving Port 2)

(b) Coupling Factor (Incident Power Leaving Port 3)

Figure 5.4 Comparison of Vogels' Measured Results With 2 Models

same phenomenon observed in couplers constructed for the present study.

Better agreement with theory is to be expected for Vogel's results compared with the present ones because of the higher quality substrates and lower frequency range used.

5.3 New Y Junction Coupler

Figure 5.5 (a) shows the equivalent circuit for the Y junction coupler whose theoretical performance curves are given in 5.5 (b). As can be seen, the coupling difference (i.e. the difference in power between the output ports) is zero dB at two frequencies rather than one as in the case for the conventional structure. Also the output coupling is within .5 dB over 2.5 GHz compared with 2.0 GHz for the conventional coupler and within .3 dB over 2.1 GHz compared to 1.6 GHz. The isolation curves in both types are observed to be almost identical. However, because the output response is assymetrical with respect to the center frequency, the Y coupler achieves wider coupling bandwidth at the expense of isolation. This coupler has many advantages over the conventional one. It is comprised entirely of Y junctions which are much easier to model than T junctions and because only a single impedance of line is used transmission line characterization (such as dispersive behaviour and loss) is also facilitated.

Figures 5.5 (c) and (d) show the characteristic of this coupler as the stub lengths are varied. It is seen that the resonant frequency and the point of zero dB coupling difference is tuneable over a slight range. This advantageous feature could also enable electrical tuning of the coupler by placing varactors at the ends of the stubs. Y junction effects are not shown because they were found to have a negligible effect on the circuit's performance.

a) Equivalent circuit

b) Performance with Stub Length = $.113\lambda_0$

Figure 5.5 Basic V coupler Performance Curves

c) Performance with Stub Length = $.113\lambda_0 \times .9$

d) Performance with Stub Length = $.113\lambda_0 \times 1.1$

CONCLUSIONS AND RECOMMENDATIONS6.1 Conclusions

This study has extensively investigated the characteristics of 3db 2-branch-line couplers. The factors that have a bearing on the coupler response have been modelled and by comparing computed and measured performances, the most significant influences have been identified.

In Chapter 2 branch line couplers were introduced and a new design was derived that consisted of easier-to-model Y junctions.

The next chapter presented the relevant microstrip theory. Although the T junction had been reported as having a major influence on the coupler response, neither of the commonly used microstrip models had been experimentally verified. A direct measurement of these very small T reactances was not possible because they were overshadowed by large unknown and unrepeatable connector impedances. To circumvent this problem, an indirect resonance technique was devised for measuring small reactances. This method was used to develop an experimental T model that was shown to support the theory of Leighton and Milnes more closely than that of Vogel's.

This method was also used to experimentally model the previously uncharacterized microstrip Y junction. A new theoretical model was derived for this Y from a waveguide E plane Y junction. Experimental and theoretical results were found to be in reasonable agreement considering the small values of the reactances being measured.

Chapter 4 studied the connector. The only published model for the particular connectors used was for high VSWR loading at only one terminal and one frequency. This was expanded to a completely general model across X-band for any port loadings. To circumvent the problem of unideal chip resistor terminations the commonly used sliding termination technique was adapted to microstrip with good success because the required multiple placement of the connector was shown to cause an averaging that minimized the troublesome repeatability errors. Experimental verification of the connector model showed that it was more accurate at the lower frequencies.

In Chapter 5 the 2-branch-line couplers were studied. Theoretical and practical coupling into port 2 was observed to be more frequency sensitive than that of port #3. Very large practical isolations were achievable at the centre frequency. Although the inclusion of the T model in the theoretical responses improved the agreement with the measured results, the differences between the T models were small. The T-effects were found to be less significant than suggested by Vogel while his configuration was found to be superior to the one used by Leighton and Milnes. The connector was found to be the most significant cause of the measured response deviating from the ideal. Lead-in-line loss was minor since it only caused a slight shift in the port coupling. However lead-in-line length was critical since it changed the phase of the connector model.

The new coupler using Y junctions displayed wider coupling bandwidths and poorer isolation than the conventional type. Y junction parasitics did not significantly alter the theoretical response.

An advantage to this Y coupler was shown to be its simplicity which should enable much more precise designs. Because it uses single impedance lines, it is completely balanced. Therefore any anomalous effects such as dispersion will effect all the lines in the same way. The symmetrical Y junctions used can be completely and easily experimentally modelled. Hence characterization of the coupler is facilitated. Because a tuning stub is a natural consequence of the structure, the device can easily be tuned and by placing varactors at the end of the stubs it can be tuned electrically.

6.2 Recommendations

The connector's influence should be minimized by lowering its associated discontinuities and improving repeatability. Also, it is recommended that more attention be given to the quality of the microstrip line with particular emphasis on the purity of the dielectric substrate as well as the precision of the etching process. Hence a more suitable substrate material for further study is high purity Alumina with deposited lines. The empirical T and Y junction models determined in this study should be adopted where applicable since the theoretical models were not rigorously derived. Symmetrical Y junctions should replace many of the existing T junction configurations

because the parasitics are much easier to model. More microstrip discontinuities modelled using high accuracy indirect resonance techniques would be of great practical significance. Further study into the coupler characteristics using improved connectors and circuit quality will probably significantly improve the theoretical and experimental agreement.

The Y junction coupler should replace the conventional coupler in some applications - particularly where fast, high precision modelling or stub tuning are required. It is recommended that an investigation be made into the possibility of electrically tuning this coupler using varactors.

APPENDIX A1

A1.1 ABCD Matrix for a General 2-Port Network

The complex ABCD matrix at a single frequency for general 2-port network shown in Fig. A1.1 relates the complex normalized voltages and currents V_1 and I_1 at port #1 to V_2 and I_2 at port #2.

Figure A1.1 2 Port Network

The relationship is written as:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

A1.1

where:

A, B, C and D are the matrix elements at a single frequency

V_1, I_1, V_2, I_2 are the complex normalized voltages and currents at the ports at a single frequency.

The main advantage of these matrices is that cascade connections can be calculated by multiplying individual matrices together.

A1.2 Reverse Matrix

The voltage and current at port #2 may be expressed in terms of the voltage and current at port #1.

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{(-1)} \times \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

A1.2

$$= \frac{1}{\det} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \times \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

A1.3

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{AD-BC} \times \begin{bmatrix} D & B \\ C & A \end{bmatrix} \times \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

A1.4

A.1.3

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

A1.5

where:

$$I_1' = -I_1$$

$$I_2' = -I_2$$

The currents I_1' and I_2' have been used to maintain a convention that current flows out of the independent port and into the dependent port. This convention facilitates cascading of the networks.

A1.3 Input Impedance of the Network

Referring to equation A1.1 the input impedance at port # 1 may be expressed as:

$$Z_{in} = \frac{V_1}{I_1} = \frac{AV_2 + BI_2}{CV_2 + DI_2} \quad A1.6$$

If port #2 is terminated in a complex normalized impedance K,

$$Z_{in} = \frac{AK + B}{CK + D} \quad A1.7$$

A1.4 Voltage Reflection Coefficient With a Matched Load

The complex input voltage reflection coefficient Γ of a network is related to the normalized input impedance Z_{in} by:

$$\Gamma = \frac{Z_{in} - 1}{Z_{in} + 1}$$

A1.8

Under matched conditions, the impedance at port #2 is $K = 1$ and substitution of eqn. A1.7 into eqn. A1.8 gives:

$$\Gamma = \frac{\frac{A+B}{C+D} - 1}{\frac{A+B}{C+D} + 1} = \frac{A+B-C-D}{A+B+C+D}$$

A1.9

A1.5 Voltage Transmission Coefficient With a Matched Load

Referring to Fig. A1.1, the complex voltage transmission coefficient T from port #1 to port #2 is:

$$T = \frac{V_2}{V_1} = \frac{V_2}{V_1} \times \frac{V_1}{V_1}$$

A1.10

where:

V_1 is the normalized voltage wave incident at port #1

V_2 is the normalized transmitted voltage wave - voltage at port #2.

The voltage at port #1 is given by:

$$V_1 = AV_2 + BI_2$$

Therefore for a properly terminated network where $V_2 = I_2$

$$\frac{V_2}{V_1} = \frac{1}{A + B}$$

A1.11

The voltage at port #1 is a summation of an incident voltage wave V_i and a reflected voltage wave V_r .

$$V_1 = V_i + V_r$$

$$\frac{V_i}{V_1} = \frac{(1 + \Gamma)}{A + B}$$

A1.13

Substituting the previously determined expression for Γ (equation A1.9).

$$T = \frac{1 + \frac{A + B - C - D}{A + B + C + D}}{A + B}$$

A1.14

$$= \frac{2}{A + B + C + D}$$

A1.15

A1.6 Reciprocity:

If the network is reciprocal, then the transmission coefficient from port #1 to port #2 is the same as the transmission coefficient from port #2 to port #1. ie.

$$\frac{2}{A + B + C + D} = \frac{2}{A' + B' + C' + D'}$$

$$= \frac{1}{AD - BC} \times \frac{2}{A + B + C + D}$$

A1.16

Therefore, for a reciprocal network

$$AD - BC = 1$$

A1.17

A1.7 Symmetry:

If the network is symmetrical the two ports #1 and #2 are indistinguishable. Therefore, the forward and the reverse matrices must be identical. ie.

$$\frac{1}{AD - BC} \times \begin{bmatrix} D & B \\ C & A \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

A1.18

Therefore, in a symmetrical network $A = D$ and $AD - BC = 1$ (the condition for reciprocity).

APPENDIX A2

A2.1 Sliding Termination Results

This appendix presents the experimental loci determined using the sliding termination technique of section 4.5.2 on the coax-microstrip connector.

Figure A.2.1. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 8GHz.

Figure A.2.2. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 8.5GHz.

Figure A.3. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 9GHz.

Figure A2.4. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 9.5GHz.

Figure A2.5. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 10GHz.

Figure A2.6. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 10.5GHz.

Figure A2.7. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 11GHz.

Figure A2.8. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 11.5GHz.

Figure A2.9. Smith Chart Plot of the Sliding Line Results for the Coax-Microstrip Connector at 12GHz.

APPENDIX A3

Properties of R/T Duroid 5870 Microstrip

Property	Value
Dielectric Strength, Short Time, volts/mil.	300
Dielectric Constant, 1 MHz	2.35
Dissipation Factor, 1 MHz	0.0005
Dielectric Constant, 10 GHz	2.35
Dissipation Factor, 10 GHz	.0012
Surface Resistivity, Ohms	3.0×10^{14}
H_m Dielectric Thickness, Mils	10
t_m Conductor Thickness, Mils	1
W_m Width of 50 Ohm Line, Mils	31
Width of 35 Ohm Line, Mils	50

REFERENCES

1. M.V. Schneider, "Microstrip Lines for Microwave Integrated Circuits", Bell System Technical Journal, May-June 1969, pp. 1421-1444.
2. R. Levy, "Directional Couplers, Advances in Microwaves", Vol. 1, L. Young, Ed., New York: Academic Press, 1966, pp. 155-157.
3. H.A. Wheeler, "Transmission Line Properties of Parallel Strips Separated by a Dielectric Sheet", IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-13, March 1965, pp. 172-185.
4. P. Troughton, "Design of Complex Microstrip Circuits by Measurement and Computer Modelling", Proc. IEE, Vol. 118, March-April 1971, pp. 469-474.
5. B.A. Lippmann, "Theory of Directional Couplers", M.I.T. Rad. Lab. Rep., No. 860; December 28, 1945.
6. J. Reed and G.J. Wheeler, "A Method of Analysis of Symmetrical Four-Port Networks", IRE Trans. on Microwave Theory and Techniques, Vol. MTT-4, October 1956, pp. 246-252.
7. L. Young, "Branch Guide Directional Couplers", Proc. Natl. Electronics Conf., Vol. 12, 1956; pp. 723-732.
8. J.W. Crompton, "A Contribution to the Design of Multi-Element Directional Couplers", Proc. IEE (London), Vol. 104, pt. C, pp. 398-402; 1957.
9. P.D. Lomer and J.W. Crompton, "A New Form of Hybrid Junction for Microwave Frequencies", Proc. IEE, Paper No. 2356R, May 1957, pp. 261-264.
10. J.R. Reed, "The Multiple Branch Waveguide Coupler", IRE Trans. Microwave Theory Tech., Vol. MTT-6, 1958, pp. 398-403.
11. R. Levy, "A Guide to the Practical Application of Chebyshev Functions to the Design of Microwave Components", Proc. Inst. Elec. Engrs. (London) C106, 1959, pp. 193-199.
12. L. Young, "Synchronous Branch Guide Directional Couplers for Low and High Power Application", IRE, Trans. Microwave Theory Tech., Vol. 10, 1962, pp. 459-475.
13. R.M. Kuzrok, "Isolation of Lossy Transmission Line Hybrid Circuits", IRE Trans. Microwave Theory Tech., February 1966, pp. 127-128.

14. R. Levy and L.F. Lind, "Synthesis of Symmetrical Branch-Guided Directional Couplers" IRE Trans Microwave Theory Tech., Vol. MTT-16, No. 2, February 1968 pp. 80-89.
15. W.H. Leighton JR. and A.G. Milnes "Junction Reactance and Dimensional Tolerance Effects on X-Band 3-db Directional Couplers", IEEE Trans. Microwave Theory Tech., Vol. MTT-19, No. 10, October 1971, pp. 818-824.
16. R.W. Vogel, "Effects of the T Junction Discontinuity on the Design of Microstrip Directional Couplers", IEEE Trans. on Microwave Theory and Tech., March 1973 pp. 145-146.
17. H.J. Riblet, "Comment on 'Synthesis of Symmetrical Branch-Guide Directional Couplers'", IEEE Trans. on Microwave Theory and Tech., Jan. 1970, pp.47-48.
18. R.B. Ekinge, "On the Design of Impedance - Transforming Directional Couplers" IEE Trans. Microwave Theory Tech., April 1971, pp. 415-416.
19. J.S. Wight, W.J. Chudobiak and V. Makios, "An All Harmonic Coupling-Band Hybrid", IEEE Trans. on Microwave Theory and Tech. Feb. 1976, pp. 122-124.
20. B. Schiek, "Hybrid Branchline Couplers- A Useful New Class of Directional Couplers", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-22, No. 10, Oct. 1974, pp.864-869.
21. K.G. Patterson, "A Method for Accurate Design of A Broad-Band Multibranch Waveguide Coupler", IRE Trans. Microwave Theory Tech., June 1959.
22. L. Young, "Tables for Cascaded Homogeneous Quarter-Wave Transformers", IRE, Trans. Microwave Theory Tech., Vol. 7, 1959 pp. 233-237; Vol. 8, 1960, pp. 243-244.
23. L. Young, "Correction to 'Tables for Cascaded Homogeneous Quarter-Wave Transformers'", IRE, Trans. Microwave Theory and Tech.,
24. L. Young, "The Quarter-Wave Transformer Prototype Circuit", IRE, Trans. Microwave Theory Tech., Vol. 8, 1960, pp. 483-489.
25. L. Young, "Stepped Impedance Transformers and Filter Prototypes", IRE, Trans. Microwave Theory Tech., Vol. 10, 1962, pp. 339-359.
26. L. Young, "Direct-Coupled Cavity Filters for Wide and Narrow Bandwidths", IRE, Trans. Microwave Theory Tech., Vol. 11, 1963, pp. 169-179.

27. OSM 244-4A Connector, Omni Spectra Corp.
28. "The Microwave Engineer's Handbook Buyer's Guide", Brookline, Mass.: Horizon House - Microwave Inc., 1969
29. N. Marcuvitz, "The Waveguide Handbook", Vol. 10, MIT Rad. Lab. Series, McGraw-Hill
30. R.E. Collin, "Field Theory of Guided Waves", McGraw-Hill, 1960.
31. G.L. Matthaei, L. Young and E.M.T. Jones, "Microwave Filters, Impedance-Matching Networks and Coupling Structures", New York, McGraw-Hill Book Co., 1964.
32. P. Troughton, "Measurement Techniques in Microstrip", Elec. Lett., Vol. 5, No. 2, Jan. 1969, pp. 25-26.
33. I.M. Stephenson and B. Easter "Resonant Techniques for Establishing the Equivalent Circuits of Small Discontinuities in Microstrip", Elec. Lett., Vol. 7, No. 19, Sept. 1971, pp. 582-584.
34. M. Caulton, J.J. Hughes, and H. Sobol "Measurements on the Properties of Microstrip Transmission Lines for Microwave Integrated Circuits", RCA Review, September 1966, pp. 377-391.
35. A.A. Oliner, "Equivalent Circuits for Discontinuities in the Balanced Strip Transmission Line", IRE Trans. on Microwave Theory and Tech., Vol. MTT-3, March 1955, pp. 134-143.
36. J.S. Wight, "Characterization of Microstrip Impedance Discontinuities and Application of a Novel Double Coupling Band Hybrid in the Design of a Coherent Image Recovery Mixer", Faculty of Engineering, Carleton University, Ottawa, Ontario, Canada, Technical Report, Nov. 1973.
37. O.P. Jain, "A Study of Dispersive Behaviour in Microstrip Transmission Line", Faculty of Engineering, Carleton University, Ottawa, Ontario, Canada, Technical Report, May 1971.
38. B.A. Syrett, "The Use of the Automatic Network Analyzer in the Development and Modelling of a Novel Broadband Bias Line for X-Band Microstrip Circuits", Faculty of Engineering, Carleton University, Ottawa, Ontario, Canada, Technical Report, January 1973.

39. J.D. Welch and H.J. Pratt, "Losses in Microstrip Transmission Systems for Integrated Microwave Circuits", NEREM Rec., Vol. 8, 1966, pp. 100-101.
40. F. Assadourian and E. Rimal, "Simplified Theory of Microstrip Transmission Systems", Proc. IRE, Vol. 40, December 1952, pp. 1651-1657.
41. M. Caulton, J.J. Hughes, and H. Sobol, "Measurements on the Properties of Microstrip Transmission Lines for Microwave Integrated Circuits", RCA Review, Sept. 1966, pp. 377-391.
42. R.A. Pucel, D.J. Masse, and C.P. Hartwig, "Losses in Microstrip", IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-16, June 1968, pp. 342-350.
43. H.A. Wheeler "Formulas for the Skin Effect", Proc. IRE, Vol. 30, Sept. 1942, pp. 412-424.
44. L.S. Napolì and J.J. Hughes, "Foreshortening of Microstrip Open Circuits on Alumina Substrates", IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-19, pp 559-561, June 1971.
45. H. Sobol, "Radiation Conductance of Open Circuits Microstrip", IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-19, Nov. 1971, pp. 885-887.
46. O.P. Jain, V. Makios and W.J. Chudobiak, "Open End and Edge Effect in Microstrip Transmission Lines", IEEE Trans. on Microwave Theory and Tech., Vol. MTT-20, Sept. 1972, pp. 626-628.
47. H.M. Oliner and A.A. Altschuler, "Discontinuities in the Centre Conductor of Symmetric Strip Transmission Lines", IRE Trans. on Microwave Theory and Tech., Vol. MTT-8, May, 1960, pp. 328-339.
48. A.F. Thomson and A. Gopinath, "Calculation of Microstrip Discontinuity Inductances", IEE Trans. on Microwave Theory and Tech., Aug 1975 pp. 648-655.
49. B. Easter, "The Equivalent Circuit of Some Microstrip Discontinuities", IEEE Trans on Microwave Theory and Tech., Aug. 1975, pp. 655-660.
50. A.G. Franco and A.A. Oliner "Symmetric Strip Transmission Line Tee Junction", IRE Trans. on Microwave Theory and Tech., March 1962, pp. 118-124.
51. J.S. Wight, O.P. Jain, W.J. Chudobiak and V. Makios, "Equivalent Circuits of Microstrip Impedance Discontinuities and Launchers", IEEE Trans. on Microwave Theory and Tech., Jan. 1974, pp. 48-52.
52. D.M. Kerns and R.W. Beatty, "Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis", Pergamon Press, 1967.

53. J.K. Hunton, "Analysis of Microwave Measurement Techniques by Means of Signal Flow Graphs", I.R.E. Trans. on Microwave Theory and Tech., March 1960, pp. 206-212.

54. E.H. England, "A Coaxial to Microstrip Transition", IEEE Trans. on Microwave Theory and Tech., Jan. 1976, pp.47-48.

END

FIN

ARC-375