Experimental Investigation of Visible Diffraction in Tilted Fibre Bragg Gratings

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Abstract

We present an analysis on the visible diffraction patterns of *Tilted Fiber Bragg Gratings* (TFBG), with applications to blue-laser fiber sensors. On the basis of our current understanding of a visible diffraction phenomenon called *sidetapping*, or *outtapping*, we compare theoretical predictions with experimental results.

In order to compare theory to experiment we obtain empirical observations of diffraction angles. A 1550nm-TFBG is connected to a Coherent Spectrum-70C laser source and measurements are taken for several wavelengths including red (647.1nm), yellow (568.2nm), green (514.5nm), blue (488.0nm) and violet (457.9nm) to show that our results are in definite agreement with theory. There are other variables that affect and govern this diffraction behaviour so a comprehensive parameter study is used to relate the input variables to output variables. Output variables are *power* and longitudinal *angles* of each radiating *diffraction order*. Input variables would include the *type of fiber* used, *grating pitch*, *grating tilt*, and *wavelength*. With information gathered from the input-output analysis, we continue by studying blue light diffraction. Different devices are compared to gain insight into the optimal conditions for a blue light TFBG sensor. The diffraction of blue light in particular is exploited to gain insight on specialized fiber nano-coatings used in biochemical sensors.

To start we review the existing methods of TFBG diffraction in the visible spectrum, including *Coupled Mode* analysis and a brief overview of alternate methods. Generally, depending on the wavelength and other factors there may or may not be coupling to radiation modes. The specific angles of these radiation modes with respect to the fiber axis, called longitudinal angles, are determined by theoretical phase-matching conditions related to the incoming beam wavelength.

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To Marina

“Into her sea,

in warmth and solace,

I am reborn.”

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List of Notations and Abbreviations

CMT	Coupled Mode Theory
DM	Degenerated Mode
DMG	Degenerated Mode Group
EMI	Electromagnetic Interference
FBG	Fiber Bragg Grating
IR	Infrared
OSA	Optical Spectrum Analyzer
RI	Refractive Index
SRI	Surrounding Refractive Index
TFBG	Tilted Fiber Bragg Grating
TIR	Total Internal Reflection
UV	Ultraviolet
VCM	Volume Current Method
λ_B	Bragg Wavelength
n_{eff}	Effective Refractive Index
Λ	Refractive Index Modulation Periodicity
θ	Tilt Angle

Chapter 1

Introduction

1.1 *Fiber Bragg Gratings* as Sensors

It is not surprising that the need for sensors is a natural one. We all depend on our senses to perceive and interconnect with our surroundings and sensors are technological extensions of our natural senses. But they also allow us to “datify” the physical world and interact with it in new ways. With the development of new areas of technology comes the continuous demand to monitor, or sense, physical quantities such as temperature, strain, pressure, load, and a multitude of other parameters in order to maintain and operate our gadgets, machines and appliances. That is why, with the growing importance that sensors have in the world we live in, research and development in sensing technology is a high priority for the scientific community.

In sensing research, *Fibre Bragg Gratings* (FBGs) present important characteristics that set them apart including high sensitivity and multiplexing capacity as well as low fabrication cost. Fiber Optic sensors in general provide advantages over electrical sensors in that they are electrically passive and immune to *electromagnetic interference* (EMI).

In FBG, the periodicity of the grating is controlled using Ultraviolet (UV) inscription technology. This periodic modulation of the refractive index (RI) determines the wavelength of the light meant to be reflected back; the Bragg wavelength, λ_B . Reflectivity of 100% is possible as a result of phase matching between the in-coupled light and grating periodicity, and the shifts in this reflected Bragg wavelength can be used to measure an array of parameters

including temperature and strain. The most appealing trait of these measurements is that FBG have a built-in referencing mechanism because the position of the Bragg wavelength is independent of the attenuation of signals in the system, therefore FBG sensors provide absolute quantities for the measured parameters. Notwithstanding the significant growth of FBG sensors in the last 35 years a new world of possibilities emerges when introducing a tilt to the grating planes of the FBG. This type of FBG is called a *Tilted Fiber Bragg Grating* (TFBG) and has very promising sensing capabilities.

For instance, in their planar form FBG cannot be used to measure *Surrounding-material Refractive Index* (SRI) since the reflected light is confined to the core mode. However, because the introduction of the tilt allows for backward-coupled cladding modes, these higher-order modes show up in transmission spectra as transmission loss peaks below the Bragg wavelength. In short, the TFBG is a device that senses its external environment while enjoying the same advantages as the untilted FBG.

1.2 Tilted Fiber Bragg Gratings as Diffraction Prisms

If a broadband visible light source is connected to a TFBG a phenomenon called diffraction would separate the incoming beam into different colours in a process that may be compared to the effect a prism on white light, although theoretically much different. Furthermore, when the tilt of the grating is large enough (above 30° with respect to the fiber axis) there is significant light coupled into the higher-order diffraction modes, so there would be several orders of each colour coming out of the side of the fiber, such as in **Figure 1.2.1** (see Appendix E for individual colour diagrams).

Figure 1.2.1 – A white-light source hitting a theoretical 1550nm FBG tilted by 37.5°.

In the scope of this thesis we investigate the sensitivity of such a TFBG sensor operating in the visible electromagnetic spectrum by studying the phenomenon of visible diffraction in TFBGs. The goal is to map output back to its input, namely the power and angular distribution of high-order diffraction modes (outputs) in relation to design characteristics such as the type of fiber used, the fabrication technique used to inscribe the TFBG into the fibre, and the pitch and tilt of the grating itself (inputs). The aim of such a study is to gain enough insight as to be able to reliably produce TFBGs with optimal visible light outcoupling. Though these results can be generalized to understand the behaviour of any colour in the visible spectrum, blue light will be emphasized for its particular relevance to ongoing research in molecular sensing^{1,2} as well as its reported enhancement of Ag nanocoated sensors^{3,4}. As far as its applications, the variation in the outcoupling behaviour due to external changes in SIR could allow for novel visible light sensors never before created. This research serves as the underpinning for such an endeavour.

1.3 Organization of the Report

In this project I investigate the interaction between the visible laser spectrum and a TFBG. I isolate for the effects of grating pitch, grating tilt, and cut-off wavelength on the azimuthal and longitudinal radiation profiles of outcoupled light. This comprehensive map is used to provide a broad understanding of the TFBG in the visible spectrum and potentially to help improve TFBG sensitivity in the visible range.

This report conveys the results obtained during the course of the Master's thesis at the Department of Electronics, Carleton University, Canada.

Chapter 2 contains background principles relating to FBG technology and serves as a summary of the fundamental methods and concepts behind TFBGs. The phase-matching criteria for sidetapping phenomena are reviewed along with the main methods used to understand grating diffraction. This chapter will serve as the elementary backdrop for understanding the latter results in the report, as well as the necessary explanation of the parameters to be tested.

In **Chapter 3**, I present an overview of the experiment, complete with a list of all the sources and components used in the apparatus, many of which are shown in diagrams. Reading this chapter is useful for anyone repeating the experiments described in this thesis.

Chapter 4 presents the azimuthal and longitudinal power distribution profiles versus wavelength for our TFBG. Results show that phase-matching conditions predict well the angular distribution, but not the power distribution of radiation modes for wavelengths from 457.9 nm to 647.1 nm. Results confirm that a change in the grating pitch is strongly correlated

to changes in the angular distribution of the modes as well as the out-coupled power. Since blue light is of particular importance, I also explore the effect of grating tilt on radiation distribution of blue light. I show that although the grating tilt does not significantly affect the angular distribution profiles of the radiated modes, it does have a strong impact on the power coupled into a specific mode.

In the course of this chapter, I use gratings inscribed in cladding suppressed fiber produced out-of-lab as references with which to compare several kinds of large-angle TFBGs made in our own laboratory. This chapter's main purpose is to summarize in graph and tabulated form all empirical results undertaken during the course of this project and to present the clear links between input and output.

Chapter 5 presents analysis and a discussion of the results from **Chapter 4**. To investigate the potential benefits of optimization, the parametric behaviour of several TFBG devices are considered. The conclusions drawn are used to predict a grating design range that may exhibit improved outcoupling in the blue light range. The main purpose of this chapter is to highlight the kind of TFBG fabrication setup needed to optimize the application of blue-light sensors.

Chapter 6 summarizes the thesis work and presents an outline for future work as well as considerations on innovative and versatile applications of our method in the engineering world; with a particular focus on blue light sensing in biochemical and DNA-detection technologies.

Chapter 2

Tilted Fibre Gratings in the Visible Spectrum

Herein the *Tilted Fiber Bragg Grating* (TFBG) will be introduced as a special form of the more conventional Fiber Bragg Grating (FBG). First, FBGs will be presented as the overarching family of short period modulated gratings and their unique properties will be examined through *coupled mode theory*. In the latter part of the chapter I will focus on the TFBG in particular. The properties that the tilt angle θ exhibits will be discussed including cladding-mode coupling and how this leads to further interesting characteristics such as coupling to cladding modes and for large tilts, high-order diffraction. To put the results of this thesis project into academic perspective, this chapter details the different methods used to quantify grating diffraction as well as the methods underlying much of TFBG sensing research in the visible spectrum. The fabrication process of TFBG is highlighted in order to introduce the key concepts of grating pitch and grating tilt. It is shown that a simple phase matching approach is theoretically sufficient to explain some of the empirically derived results of this research.

2.1 Fabrication

In 1978 *photosensitivity* was discovered by Hill et al⁵ at CRC Canada. The team ‘accidentally’ came across this photosensitivity in optical fibers when laser light at 488nm and 514.5nm was coupled into a Ge-doped silica fiber. The phenomenon causes permanent changes to the refractive index in the fiber, and can thus be used to photo-induce gratings into optical fiber.

This type of inscription results in a standing wave pattern in the fiber is known as a self-organizing grating. The disadvantage with this basic method is that the application of such gratings is limited since the reflected wavelengths are determined by the wavelength of the radiation used to create the pattern in the first place.

Later developments in photo-induction include a transverse holographic technique, known as dual-beam holography^{6,7}. By intersecting two beams of UV light one can control the period of the inscribed gratings. The advantage of this technique is that a wide range of grating periods can be obtained. This development in grating inscription technology facilitated their emergence and popularization into applications and markets including aerospace, medical and general sensing technology domains.

Modern commercial production of FBGs uses *phase-mask* techniques to inscribe gratings in optical fiber. A phase mask with the desired periodicity simply covers the optical fiber and UV light is applied to the fiber to write the grating (no need for a laser source with high spatial coherence). Phase mask techniques not only allow for mass production of identical gratings by applying an individual mask, but also allow for variability in the grating period itself; making chirped FBG fabrication possible. In this technique the quality of the inscribed gratings is fully dependent on the quality of the mask, so imperfections in the mask are reflected in the grating.

In this thesis project I compare the outcoupling behaviour for gratings made with different types of fiber. The difference in fabrication technique may have a very important impact on the visible radiation patterns. In this research, however I focus on the impact of the types of fiber used for inscription, which is explored in more detail in **Chapters 4 and 5**.

2.2 Total Internal Reflection

In order appreciate the more complex theoretical basis of TFBG properties, it is important to grasp the fundamental physical phenomenon upon which Fiber Optic cables work in general. Total Internal Reflection (TIR) serves as the backdrop for most inherent properties of FBG and TFBG alike.

When an EM wave crosses a boundary whereupon the refractive index changes, the light partially refracts and partially reflects as illustrated in **Figure 2.2.1a**. When crossing the interface from higher RI to lower RI, light refracts away from the boundary. For refractive indices $n_1 > n_2$, it can be seen that as the incoming angle, θ_i , increases the system will reach a point where the transmitted light travels along the boundary at a transmission angle, θ_t , of 90° . For any incident light at an angle greater than this particular angle (called the critical angle, θ_c) the light is no longer transmitted at all and it is totally reflected, as shown in **Figure 2.2.1b**.

Figure 2.2.1 – Vector diagram highlights the difference between refraction and TIR.

This phenomenon is known as Total Internal Reflection. Refraction at such a boundary is governed by Snell's Law:

$$n_i \sin(\theta_i) = n_t \sin(\theta_t) \quad (2.2.1)$$

When the incident beam reaches the critical angle the transmitted beam travels along the boundary perpendicular to the interface normal, as mentioned above. Substituting 90° for the transmitted angle, I obtain a formula for the critical angle in terms of the internal and external refractive indices. At incident angles above this angle, TIR occurs.

$$\theta_c = \sin^{-1}\left(\frac{n_t}{n_i}\right) \quad (2.2.2)$$

I will thus call upon the consequences of this phenomenon as they relate to TFBG sensing properties in subsequent chapters of the report.

2.3 Simple Fiber Grating

FBG is a periodic modulation of the refractive index in the core of an optical fiber where the planes of the inscribed gratings are perpendicular to the longitudinal axis of the fiber. The periodicity of this modulation is one half of the wavelength meant to be reflected; known as the Bragg wavelength.

$$\lambda_B = 2n_{eff}\Lambda \quad (2.3.1)$$

where λ_B is the Bragg wavelength, n_{eff} is the effective refractive index of the grating in the core and Λ is the grating period. This condition is known as the Bragg condition for the simplest case of a grating in single mode fiber. For this condition to be satisfied both energy and momentum

must be conserved. For energy, the frequency of the incoming light must match that of the reflected light.

$$h\nu_i = h\nu_r \quad (2.3.2)$$

where h is Planck's constant and ν represents frequency in units of Hz. For momentum conservation the wavevectors involved must respect the following condition (see **Figure 2.3.1**):

$$\mathbf{k}_i + \mathbf{K} = \mathbf{k}_r \quad (2.3.3)$$

where $\mathbf{k}_i$ and $\mathbf{k}_r$ are the incident and reflected wavevectors respectively and $\mathbf{K}$ is the grating wavevector. The grating wavevector is perpendicular to the grating planes and has magnitude:

$$K = 2\pi/\Lambda \quad (2.3.4)$$

For pure reflection the reflected wave is exactly equal and opposite the incident wave. Since we know the wavevector k_i will have magnitude $2\pi n_{eff}/\lambda_0$, we rearrange (2.3.3) and substitute for magnitudes to find the resulting first order condition.

Figure 2.3.1 – Long (top) and short (bottom) period gratings. Incident, radiated and grating wavevectors shown.

$$\left(2\pi n_{eff}/\lambda_0\right) - \left(-2\pi n_{eff}/\lambda_0\right) = -(-2\pi/\Lambda) \quad (2.3.5)$$

The free-space wavelength that satisfies this phase matching condition is labeled the Bragg wavelength and from (2.3.5) the first-order Bragg condition in (2.3.1) is easily deduced.

2.4 Fiber Bragg Grating Reflectivity

Typically, the refractive index profile of a uniform grating in the core of a single mode fiber can be presented by the following equation⁵:

$$n(z) = n_a + \Delta n \cos\left(\frac{2\pi z}{\Lambda}\right) \quad (2.4.1)$$

where n_a is the average refractive index of the core, Δn is refractive index perturbation amplitude (typically in the range of 10^{-4} to 10^{-3}), and z is the distance along the fiber longitudinal axis. To find reflectivity, one must begin with the Helmholtz equation:

$$\nabla^2 \tilde{E} + \tilde{n} k_0^2 \tilde{E} = 0 \quad (2.4.2)$$

where $\tilde{E}$ is the Fourier Transform of the electric field distribution with respect to time.

Since $n(z)$ is periodic in the z direction, reflectivity can be found by considering the Fourier Series expansion:

$$n(z) = \sum_{m=-\infty}^{+\infty} n_m \exp[2\pi i m(z/\Lambda)] \quad (2.4.3)$$

By applying appropriate boundary conditions, reflectivity can be expressed as following (**Figure 2.4.1**) – see Appendix A for more detail:

$$R(l, \lambda) = \frac{\kappa^2 \sinh^2(sl)}{\delta k^2 \sinh^2(sl) + s^2 \cosh^2(sl)} \quad (2.4.4)$$

where l is the grating length, κ is the coupling efficiency, $\delta k = k - \pi/\lambda$ represents wavevector detuning (while the propagation constant $k = 2\pi n_d/\lambda$) and $s^2 = \kappa^2 - \delta k^2$. For the center wavelength, there is no detuning; $\delta k = 0$, so the maximum reflectivity is given by:

$$R_{\max}(l, \lambda) = \tanh^2(\kappa l) \quad (2.4.5)$$

In practice, reflectivities of over 99% at the center wavelength are achievable and commonplace. The undesirable feature of FBG reflection spectrum is the presence of the sidelobes on either side of the stop band.

Figure 2.4.1: Sample reflection spectrum for FBG versus detuning from center wavelength.

2.5 Tilted Fiber Bragg Grating

TFBGs are simply a special type of short-period FBG at visible wavelengths in which the grating planes are slanted with respect to the longitudinal fiber axis. **Figure 2.5.1** shows a tilted grating with an effective grating pitch (Λ) that remains the same along the fibre axis as compared to a flat grating.

Figure 2.5.1 – The effective grating periodicity remains unchanged along the fiber axis.

As mentioned in **Chapter 1**, the tilt of the grating planes allows them to couple light from the forward propagating core mode to backward propagating core and cladding modes. This results in transmission loss peaks at discrete resonant wavelengths shorter than the Bragg wavelength as shown in **Figure 2.5.2**. The cladding modes travel near the cladding-external medium interface which renders them sensitive to SRI.

Figure 2.5.2 – A typical TFBG transmission spectrum showing higher order mode losses.

When the grating plane is tilted with respect to the fiber axis as shown in **Figure 2.5.3**, the phase matching condition predicts tilt and wavelength dependent diffraction. The three wavevectors of interest are the incoming, outgoing and grating wavevectors, such that mode coupling between incident and radiated waves is related to the grating wavevector.

By establishing a coordinate system to separate the axial and normal components we are able to explicitly derive the phase matching conditions. Along the z-axis, we add the z-components as follows:

$$k_I - k_G \cdot \hat{z} = k_R \cdot \hat{z} \quad (2.5.1)$$

$$\frac{2\pi n_{eff}}{\lambda} - \frac{m\pi}{\Lambda \cos \theta} \cdot \cos \theta = \frac{2\pi n_{core}}{\lambda} \cdot \cos \alpha \quad (2.5.2)$$

Tilt angle of the grating is eliminated from the equation because as the tilt increases so does the size of the grating wavevector, effectively canceling this term as shown in (2.5.3).

Figure 2.5.3 – Phase matching analysis of incoming k_i , grating k_G and radiation k_R wavevectors. This diagram applies only to short period gratings. Analysis for long period gratings differs.

Again, we see that the tilt of the grating does not actually affect the z-component of the phase-matching condition. Isolating for the angle, α , at which the diffracted wave exists the fiber as a function of incoming wavelength (λ) and diffraction order (m) we get:

$$\alpha_m(\lambda) = \cos^{-1} \left(\frac{\Lambda 2\pi n_{eff} - m\lambda}{\Lambda 2\pi n_{core}} \right) \quad (2.5.3)$$

This equation predicts the angle of the outcoupled radiation as a function of wavelength, mode number, and effective and core refractive indices. In **Chapter 4**, these predicted angles are compared with experimentally measured angles. Snell's law was taken into consideration as the outcoupled radiation crosses the fiber-air interface. We note importantly that only the transverse component of the phase-matching condition is affected by the tilt angle, alpha. This implies the longitudinal phase-matching condition is 'unaffected' by the grating tilt. This is supported since the gratings that were made used a fabrication configuration that maintains a constant Bragg wavelength⁸. In contrast with this coupled-wave approach, there is also the Volume Current Method approach adopted or explored by certain groups.⁹⁻¹³ The former coupled-wave approach is also adopted by other groups to explain optical fiber sidetap phenomena.¹³⁻²¹ In Kashyap's model¹⁷ (Section 2.7) the net filter response, or power, of the diffraction grating depends on the degree of both longitudinal and transverse phase matching. Also, in this very same model the conditions are shown for Bragg reflection, back reflection to cladding modes, forward coupling to cladding modes, coupling to radiation modes and the effect of higher order (m) grating wavevector. The effect of higher-order m number is equivalent to a longer period grating which results in forward coupling, hence explaining the multiple diffraction orders. A rigorous derivation of these fundamental cases is given in Tamir (1975)²⁵.

2.6 Visible Diffraction Efficiencies in Tilted Fibre Bragg Gratings

Many researchers have been involved with the progress to date of our understanding of visible light diffraction behaviour in fibre optics. Within the theoretical sphere, several useful methods called Volume Current Method⁹⁻¹³, Coupled Mode Analysis^{9,14-15}, and a Free Space Model¹⁶⁻¹⁷ are used interchangeably since either one has pros and cons. Volume Current Method (VCM) is an analytic approach to calculate the radiation pattern of tilted fiber gratings. Since the Coupled Mode Theory (CMT) inherently breaks down for radiation perpendicular to the fiber axis, and since this particular configuration is of great interest for grating outcoupling, the VCM as an approach is often better suited as a design tool for this purpose. However, this research is experimental in nature and not confined to perpendicular radiation so the theoretical derivations of the VCM are considered outside the scope of this research, though information on the VCM is readily accessible in the literature⁹⁻¹³.

A subset of antenna theory is used in Kashyap's¹⁷ model for power efficiency of visible diffraction in TFBGs and discuss the difficulties in implementing this theoretical framework.

Figure 2.6.1 presents a schematic of the problem under study. The scattered light creates a cone defined by the phase matched angle q (labeled ϕ in the remainder of this thesis). By considering a fibre with an infinite cladding (n_{co} , n_{cl}) with a UV written FBG with a period Λ tilted under the angle θ_g along the y-axis, the scattered power from the grating is given by integrating all radiating dipoles through the spherical surface with radius R :

$$P_{scat}(\lambda) = \int_{\phi=0}^{2\pi} \int_{q=0}^{\pi} R^2 S_{scat}(R, \phi, q, \lambda) \sin q dq d\phi \quad (2.6.1)$$

Figure 2.6.1 – Scattering from the fibre core with the FBG. Light scatters in cone shape.¹⁷

with q being the angle between the fibre axis (z) and the direction of the radiation, ϕ here being the azimuthal angle (x - y plane) and λ being the wavelength of the interacting light. The Poynting vector is defined through:

$$S_{scat}(R, \phi, q, \lambda) = \frac{n_{cl}}{2} \left(\frac{\epsilon_0}{\mu_0} \right)^{1/2} |E_{scat}(R, \phi, q, \lambda)|^2 \quad (2.6.2)$$

The scattered field can be written as follows:

$$E_{scat}(R, \phi, q, \lambda) = \frac{\Gamma}{R} \iint_{core} E_0(x, y) I_L(x, L) e^{-ikn_{cl}(x \cos \phi \sin q + y \sin \phi \sin q)} dx dy \quad (2.6.3)$$

with $\Gamma = k^2 n_{cl} \Delta n e^{ikn_{cl}R} / 2\pi$, $k = 2\pi/\lambda$, and Δn is the maximum refractive index change in the grating. Integration is over the fibre core area. Finally, the function depending on the total length of the grating, defined as

$$I_L(x, L) = \frac{e^{i\gamma x}}{2} \frac{e^{ivL} - 1}{iv} \quad (2.6.4)$$

will lead to longitudinal phase matching (see previous derivation in Section 2.5) where q_L is the angle at the longitudinal resonance. For the sake of simplicity, Kashyap considers $m = 1$.

Rewriting (2.6.3), the scattered field can be written as follows:

$$E_{scat}(R, \phi, q, \lambda) = \frac{\Gamma}{2R} \frac{e^{ivL} - 1}{iv} I_{core}(\gamma, q, \phi), \quad (2.6.5)$$

where the function I_{core} is the non-trivial integration expression:

$$I_{core}(\gamma, q, \phi) = \iint_{core} E_0(x, y) e^{\{i[x(\gamma - kn_{cl} \cos \phi \sin q) - ykn_{cl} \sin \phi \sin q]\}} dS. \quad (2.6.6)$$

When (2.6.6) is converted into polar form and substituted into (2.6.5), the scattered power:

$$P_{scat}(\lambda) = \frac{n_{cl}}{2} \left(\frac{\epsilon_0}{\mu_0} \right)^{1/2} \Gamma^* \Gamma \int_{\phi=0}^{2\pi} \int_{q=0}^{\pi} |I_{core}(\gamma, q, \phi)|^2 \frac{\sin^2(vL/2)}{v^2} \sin q dq d\phi \quad (2.6.7)$$

This model correctly predicts that the $m = 1$ mode is dominant, and this final form of scattered power is very promising for future theoretical considerations, however the I_{core} parameter is not trivial as it requires a complex analysis further explained in Kashyap's model. Furthermore, this derivation ignores the z-component of the electric field, which according to literature¹³ is not negligible for this phenomenon.

Finally, the experimental observations mentioned in this model did not clearly indicate whether these equations can be used to model all visible wavelengths as the reported validation is only at HeNe wavelength (632nm).

For these reasons, this model is considered a good starting point for future work on theoretical comparisons of experimentally observed efficiency measurements reported in Chapter 4 of this thesis, though it was beyond the scope of this experimental thesis to use this model.

From Kashyap¹⁷ the phase matching for FBG for non-zero tilt shows the phase matched angles contain the major part of the radiated power and are expressed as:

$$q_L = \cos^{-1} \left[\frac{1}{n_{cl}} \left(\frac{m\lambda}{\Lambda} \cos \theta_g - N \right) \right] \quad (2.6.8)$$

This equation is analogous and identical to the simple phase-matching derived equation (2.5.3).

2.7 Conclusion

The Fiber Bragg Grating as a basic component in optical sensors was introduced. The tilting of the grating planes presents interesting properties in terms of sensing and flexibility. The coupling of the core mode into backward coupled cladding modes renders these latter modes 'leaky' to external media. This process is rendered more significant by large-tilt FBGs (greater than 30°) due to the high coupling efficiencies into these higher order modes. In summary, TFBGs present extraordinary possibilities due to their versatility with regards to sensing and their ability to control crosstalk between the confined core mode and the more externally interactive cladding modes.

Chapter 3

Test Apparatus for Visible Diffraction in TFBG

This work began as part of the MOSAIC (Multimodal Optical Sensor Applications, Interfaces, and Controls) project, a federal-private-academic collaboration sponsored by NSERC. The focus area of this thesis in relation this wider project was the investigation of TFBG mode-coupling phenomena in the *visible light range*. Relative to common NIR bands used heavily in optical fibre communication, this particular band of light has not been deeply studied. Hence the need for an original and empirical probe into the effects of visible light radiation through, and out-tapped from, a Tilted Fibre Bragg Grating. In this chapter I look at the original apparatus design, followed by an explanation of the fabrication techniques used to generate various kinds of test fibres. Finally, the data analysis techniques are presented to establish the validation framework to compare empirical findings with theory, and the apparatus is shown to be an effective and versatile measurement tool for TFBG Visible Light Radiation.

3.1 Feasibility Study Using HeNe Laser

Before the main apparatus was finally developed, the initial work went into testing for visible light outcoupling, as shown in **Figure 3.1.1**. A camera was used to observe the images shown in **Figure 3.1.2.**, which proved that the setup design could work to observe diffraction maxima. The mounted camera was rotated by an angle θ around the x-axis on the y-z plane until maxima were observed. A rotator was used to rotate the fibre around the z-axis to account for the

polar (ψ) dependence of the radiation patterns, discussed in chapters 4 and 5. This served as the necessary validation to test the concept because this meant I could now replace the camera with a power meter. By sweeping the camera in the place of the power meter, we were able to reduce all output parameters into one single multivariable output. The output power as a function of input power (P_{in}), azimuthal angle (θ), polar angle (ψ) and input wavelength (λ), TFBG slant (ϕ) and TFBG pitch (Λ) can be measured as long as any apparatus design is able to adequately control, isolate and vary for each of these 6 parameters. This output is described by the function $P_{out}(P_{in}, \theta, \psi, \lambda, \phi, \Lambda)$.

Figure 3.1.1 – The idea was to test the feasibility of visible light diffraction in standard SMF-28 fibre.

Figure 3.1.2 – A sample of a radiation maximum ($m = 1$) observed at a particular angle θ_m .

3.2 List of Apparatus

The list of apparatus used in the main experimental procedure is as follows (a schematic is shown below in **Figure 3.2.1** indicating where the instruments are located relative to the fibre):

1. CVD Camera
 - For initial verification of radiation peaks (Section 3.1)
2. Goniometer
 - To measure diffracted angles
3. Circular Aperture Optical Power Meter – Attached to viewing end of Goniometer
 - To measure power of diffracted modes
4. Fibre Holder with Rotator
 - To hold fibre in place while allowing polar rotation
5. SMF-28 Fibre
 - Standard single mode fiber
6. UVS-INT Fibre
 - A type of cladding mode suppressed fibre
7. UVS-652 Fibre
 - A type of cladding mode suppressed fibre
8. Coherent® Innova 70C-Spectrum – Tunable laser (458 nm – 753 nm)
 - To control input power and input wavelength
9. Polarizer
 - To control input polarization
10. TFBG
 - Produced with varying tilt angle and Bragg wavelength

Figure 3.2.1 – Schematic of test apparatus with main test items identified.

3.3 Apparatus Design – Assembly and Procedure

As explained in section 3.1, an original setup was required to empirically probe the parameters involved with diffraction of visible light radiation in a Tilted Fibre Bragg Grating. A Coherent Spectrum 70C laser was used to control both the power and the wavelength of the input light. A Goniometer and a fibre holder were assembled into the apparatus shown below. This 180° Goniometer combined with a polar-rotating fibre stand allowed our setup the complete 3D freedom needed to measure the output radiated power in relation to all 6 input parameters; *wavelength* (λ), *power* (P_{in}), *azimuthal angle* (θ), *polar angle* (ψ), *grating tilt* (ϕ) and *pitch* (Λ).

In order to assemble the design shown in **Figure 3.3.1**, the TFBG was slotted through the fixed end of the Goniometer, at the centre of which a fibre holder was fixed in place used optical screws to ensure stable angle measurements. A small U-shaped metal rod was inserted into the fibre holder in order to rotate the fibre to measure polar, or azimuthal, dependence.

Figure 3.3.1 – TFBG is at the centre of the fibre holder (B). The left side of the Goniometer (C), replacing the CVD camera, was rotated around the TFBG. The rotator (D) was used to measure polar dependence.

Figure 3.3.2 – This picture illustrates clearly how the observing end of the Goniometer was used to rotate (red arrows) around the TFBG (A) at the centre of the fibre holder. A power meter was used to replace the observing lens in order to ‘scan’ the entire 180° range provided by the setup. The fibre holder was rotated using the rotator (B).

Once validated and tested, this apparatus proved to be the main data source for this thesis due to its versatility. Shown in **Figure 3.3.2** are the two angular ranges that allow the TFBG to be seen “in 3D” by rotating the viewing end of the Goniometer as well as the fibre rotator shown as “B” in the above figure. It is important to stress that the fibre holder must be attached to the Goniometer rigidly in order to have reproducible test results. The novel data collected from these tests were analyzed and compared favourably to theoretical expectations. Due to time constraints, the relationship between input and output power losses could not be adequately tested within the length of this research. These tests are encouraged and any future work may benefit from the continuation of the original approach used in this project.

3.4 Controlling Beam Wavelength and Power

Figure 3.4.1 shows the Coherent® Spectrum-70C Innova Laser used in this experiment. The freespace-to-fiber coupling setup is also shown in **Figure 3.4.2**. Finally, in **Figure 3.4.3**, the variation in input wavelength (λ_L) is highlighted by showing the laser tuned to blue, green and red. For the purposes of this comparative research five colours were used (indigo 458 nm, blue 488 nm, green 515nm, yellow 568 nm and red 647 nm). These were used to validate the observed diffraction behaviour versus that predicted by phase-matching conditions.

Figure 3.4.1 – The COHERENT SPECTRUM 70C laser, to the left, is adjusted using a digital interface above. Both power and wavelength can be finely tuned. The laser here is tuned to red – or 647 nm.

Figure 3.4.2 – The coupling apparatus is shown here. The input laser is safely directed into the fibre terminal. Upon careful adjustment, very efficient coupling is achieved, as seen in **Figure 3.4.3**.

Figure 3.4.3 – After the light is coupled into the fibre, the input wavelength can simply be altered by using the digital control shown in **Figure 3.4.1**. Here I show the fixed end of the Goniometer (see **A** in **Figure 3.3.1**). The “glow” is not related to diffraction but to the scattering from multimode interference. Visible light will usually operate in multimode when traveling through standard SMF-28 fibre.

3.5 TFBG Fabrication – Grating Slant and Grating Pitch

TFBGs are fabricated using the same tools and techniques as standard FBGs; from permanent refractive index changes induced in a fibre core by an interference pattern between two intense UV laser beams. There are several variants of this basic technique and they are described extensively in the literature^{5,23,24}.

3.5.1 Grating Slant

Since this research is aimed at understanding visible light diffraction, I only describe here the modifications required to make “large-angle” TFBGs, which are necessary for visible outcoupling to work. These large angle gratings make it possible for some of these modes to reach the cladding, or even leave the fibre altogether. The higher the tilt angle, the more orders are observed, and the more pronounced the outcoupling effects become. See **Figure 3.5.1** showing the setup used in TFBG fabrication.

Figure 3.5.1 – TFBG fabrication setup: tilt angles reported in this research are those marked in green.

3.5.2 Grating Pitch

Since the relationship between phase mask periodicity and TFBG grating pitch is discussed in Chapter 2, here I describe the relevance of testing grating pitch in the context of understanding visible light diffraction in TFBG. For a given phase mask, the fabrication setup shown in **Figure 3.5.2** will control the grating slant (or tilt angle) of the fibre. However, by using different phase masks, we are able to control the grating pitch itself. This allows an additional experiment where the tilt is kept constant to isolate and test for the effects of TFBGs with different values of Bragg wavelength. Though the results of these experiments are preliminary, results in Chapter 4 show that these relationships are important since these two parameters, namely grating tilt and grating pitch, affect the output in different ways, further discussed in Chapter 5. Although drastic changes in the tilt may affect the grating pitch, within the fabrication ranges considered in this research (tilt angles between 25° - 35°), the grating pitch is assumed to remain unaffected by grating tilt.

Figure 3.5.2 – In this configuration, the lens is fixed and the phase mask is rotated using the rotating stand. This setup can be used to produce gratings with tilts up to 30° .

3.5.3 Fiber Type

Since this research was undertaken with the goal of maximising visible light outcoupling, many parameters were considered. None turned out to be as important as the type of fiber used in the inscription process. Shown in **Figure 3.5.3** is a special type of cladding mode suppressed fiber (UVS-652) manufactured by a CorActive®, a specialty optical fiber manufacturer. As shown in **Chapter 4**, this fiber turned out to be a phenomenal test fiber for visible light outcoupling, showing drastic improvements in outcoupling efficiencies across the entire visible spectrum. These results were compared to standard SMF-28 fiber and UVS-INT fiber, another specialty type of fiber that exhibited major outcoupling, though not as pronounced as the UVS-652 shown here below.

Figure 3.5.3 – UVS-652 Specialty Optical Fiber Product. Manufactured by CorActive®.

3.6 Multimodal Theoretical Simulations Ignoring Dispersion Effects

Since most of the experimental work in this project was in multi-mode operation (visible light in SMF-28 fiber), a computational mode solver software called FIMMWAVE® was used. This program was used to model the shapes of individual modes in order to predict the effect of single- versus multi-mode coupling. The fundamental and first harmonic modes are shown in **Figure 3.6.1**. The remainder of the simulations are shown in Appendix B for the sake of concision. This set of data is the modal analysis for only one m-number, from which the spectral width of a radiation peak for this m-number is measured as the sum of the modes. If the gratings were fabricated with grating pitches nearer to visible light wavelengths (400-700nm) then I should expect much narrower spectral widths for individual radiation peaks, however this is a challenge yet unachieved in literature and far more work will required to fabricate single-mode visible light gratings that outcouples light at high efficiencies. It is also important to note that for the remainder of the thesis I will ignore the dispersion effects that cause higher losses in shorter wavelengths traveling in optical fibre. This is due to the very short distance (approximately 10-20 cm) traveled by incoming light before hitting the TFBG.

Figure 3.6.1 – Fundamental (left) and first-order (right) modes, for 632nm light traveling in SMF-28 fibre.

Figure 3.6.2 – A radiation profile showing spectral width of one diffracted mode. This spectral width was confirmed using FIMMWAVE® simulations with input wavelength 488.0nm.

This assumption should be valid, and it is also made in Kashyap’s model¹⁷. The input power is adjusted at the insertion point of the light right before hitting the TFBG and the output power is measured directly from the radiated light. By minimising the effects of dispersion losses, the output efficiency is calculated in the same way for all colours tested.

To explain the angular spread of outtapped modes shown in **Figure 3.6.2** one must consider the distribution characteristics of higher order modes in a multimode fiber¹⁹. A complete derivation of cladding modes power distribution is shown in Appendix A. This derivation represents the theoretical underpinning used to explain the relationship between parameters such as mode number and outcoupling behaviour. Computational simulators such as

FIMMWAVE® (see Appendix B) are also used to confirm that the angular spread shown in **Figure 3.6.2** is reasonable based on this relationship.

In summary, in order to validate the results later seen in this report, FIMMWAVE was used to understand the relationship between cladding modes and a broad radiation pattern observed of about 0.05° (or 0.4nm across) as shown in **Figure 3.6.2**. For ideal, single mode, operation there should theoretically be one sharp vertical line of radiation, so this image shows that this fiber supports a number of modes. Theoretically, due to the slight shifts in propagation constants of higher order modes, there would be slight shifts in the effective wavelengths of these modes as they make contact with a TFBG, which would change their exit angles slightly but measurably. The data and plots in Appendix B were extracted using FIMMWAVE, and the corresponding propagation constants were found and substituted into the phase matching equation in Section 2.5 (equation 2.5.2), predicting a spectral width of about 0.05° for a 30° TFBG, thus validating the span of output angles instead of just one sharp line. This effect, as mentioned, is related to the multimode operation of SMF-28 fibre in the visible region. Since higher-order cladding modes are bound to exist in such fibres at these wavelengths, the experimental observation is corroborated by the theoretical predictions. In conclusion, the observed spread makes sense according to our calculations.

3.6 Conclusion

Based on our findings, the phenomena of interest in visible light range do not occur until grating slant is at least 30° . The grating slant thus affects mainly the *power* but not the *angle* of any outcoupled light (this is explained in Section 2.5). Conversely, any variation in the grating

Pitch strongly affects the *angle* of outcoupled light. In the following sections I isolate for these two parameters by explicitly showing the outcoupled radiation patterns as a function of grating slant as well as grating pitch. First I show the validation of our setup by comparing our results with theoretical predictions for various wavelengths across the visible spectrum for a TFBG slanted at 37.5° and with a pitch of 512nm. The theoretical approach used was derived from the simple phase-matching condition shown in Chapter 2.

In this chapter I showed the original apparatus design and its validation. By implementing a validated setup for visible-light testing purposes a fundamental contribution was made toward the Advanced Photonic Components Group's broader goal of studying the interaction of both visible and IR light with TFBGs, with the ultimate purpose of improving our photonic sensors. Following the observation of near-perfect agreement between the empirical and predicted results, the validation hurdle was passed, paving the way for study of the effects of grating slant and pitch on the outcoupled radiation. The results of these effects are described in Chapter 4 and analyzed in Chapter 5.

Chapter 4

Results of Visible Diffraction Experiments: Angular and Power Distribution

In this chapter the main body of findings of our research is presented. This body is divided into several sections. In section 4.1 I show visual results from the initial validation experiment, which confirmed the feasibility of our novel apparatus. In section 4.2, our first attempts at producing visible out-tapping were compared to reference gratings (Table 4.1.1). These reference gratings showed relatively drastic outcoupling efficiencies, and an exponential dependence of outcoupled irradiation on grating tilt. Statistical agreement is shown between phase matching predictions and empirical angular distribution values (see Appendix D).

Figure 4.0.1 – Intense blue light outcoupling (white) observed for 30° TFBG, fiber shown horizontally in blue. The radiation efficiency measured for this grating surpassed that of the TFBG-R-37.5°.

In section 4.3, the results of reference gratings made with specialty optical fiber (see **Figure 3.4.3**) served as our first clue that our gratings may not have been written with optimal writing conditions. These gratings did not work efficiently due to incorrect laser orientation during fabrication, which caused incoherent TFBG fabrication. Upon rectifying these issues, an extreme improvement in outcoupled efficiencies was seen in our second generation gratings, surpassing those of the reference gratings. Finally, it is shown that in the linear optical regime the input power of light and its polarization have negligible effects on the distribution of outcoupled light. Parameter dependencies of outcoupled light on grating slant, grating pitch and input wavelength are summarized graphically. The important results of the chapter are summarized in the concluding section.

4.1 Apparatus Feasibility and Initial Results

When the concept of this research was devised, the first challenge was to design an apparatus capable of measuring the angular distribution of outcoupled diffraction orders as well as their respective power levels (or efficiencies). This was done by using the Goniometer described and shown in Sections 3.1-3.2. In addition, an optical power meter was used to measure input, transmitted and outcoupled power in order to calculate relative power efficiency values. The optical power meter was attached to the viewing end of the Goniometer and as it was rotated around the TFBG, the power readings were measured at regular angular intervals (see **Figure 3.2.2**). The results for this 10° 1610nm grating confirmed the feasibility of the apparatus and its readings, as shown in **Figure 4.1.1**. Azimuthal angles for HeNe were measured for validation.

Table 4.1.1 – Type R gratings are out-of-lab-fabricated gratings that serve as references for the in-lab gratings (Types A and B). *Type A gratings were produced before – Type B gratings were produced after – a change was made to the fabrication setup which resulted in different behaviour.

Grating Type	Fibre Type	Angles Tested	Bragg Wavelength
R	UVS-652	20°	1550nm
		25°	
		30°	
		35°	
		37.5°	
A*	SMF-28	6°	1610nm
		10°	
		30°	
B	SMF-28, UVS-INT, UVS-652	30°	1610nm
			976nm (SMF-28 only)
			663nm (SMF-28 only)

Table 4.1.1 above shows the distinction between gratings used as reference, type “R” gratings, and gratings used for testing purposes, type “A” and “B” gratings. The reference gratings These gratings were provided by Professor Tuan GUO, from Jinan University in Guangzhou, China. They were made in Fibercore cladding mode suppressed fiber similar to the CORACTIVE 652 fiber. The rest of the gratings were fabricated in-lab. The main distinction between type A and type B gratings lies in the fabrication setup. Due to misalignment in the excimer laser the lower spatial coherence leads to smaller fringe patterns, which in turn forms supposed “rough” gratings instead of holographic gratings. Type A gratings are included in this thesis because understanding their abnormal behaviour is of theoretical interest.

Figure 4.1.1 – Efficiency and azimuthal angles of scattered modes for sample 10° grating using HeNe laser (632nm input wavelength). $\theta_1 = 25.2^\circ$, $\theta_1 = 69.4^\circ$, $\theta_1 = 102.9^\circ$, $\theta_1 = 143.6^\circ$ (angles referenced to negative z axis direction). In this preliminary experiment, power readings were qualitative.

Three important observations were extracted from these results.

1) *the apparatus was accurate enough to an error of less than 0.2°*

This important result confirmed the feasibility of the apparatus in measuring angles, allowing further data to be acquired simply by repeating the experiment for different gratings and fibres. The power measurements were easily taken by placing a power meter at the “eye” of the Goniometer (**Figure 4.1.2**).

2) *the angles shown in **Figure 4.1.1** agreed with phase-matching predictions*

Even though the outcoupling efficiencies were later improved drastically, this showed an encouraging agreement between experiment and theory.

3) *the diffracted mode with the largest efficiency was the forward-propagating mode*

The outcoupling pattern is shown in **Figure 4.1.1**. Although angles were consistent with phase-matching, the theoretical expectation was that the backward-propagating mode should have the largest efficiency²⁵ and that the shape should be crescent-like¹⁷.

Figure 4.1.2 – Sample green (514.5nm) and blue (488.0nm) output radiation patterns are shown. An optical power meter was placed at in front of the slot shown, measuring the output power of each diffracted mode relative to input power.

4.2 Phase Matching – Reference Gratings

Since the gratings used in the preliminary tests in Section 4.1 did not agree well with theoretical predictions, more work was needed to understand whether the underlying issue was due to the gratings we had fabricated, from the apparatus itself, or from a previously unknown physical phenomenon. In order to isolate for these variables, holographic gratings were ordered and tested (TFBG-R). These gratings showed drastic improvements in two areas. First, the outcoupled power was extremely high, enough to see visible radiation several metres away on the wall in a dark room. Second, the shapes and angles of the diffracted modes agreed perfectly with theory. The crescent-like pattern reported in the literature¹⁷ was observed, while the agreement between predicted angles and observed angles are shown in **Figure 4.2.1** below. Note the strong overlap between predicted (filled) and experimental values (empty).

4.2.1 Phase-Matching Predictions versus Observations

Figure 4.2.1 – The agreement is clear between predicted and observed angular distribution for several colours including purple (457.9nm), blue (488.0nm), green (514.5nm), yellow (568.2nm), red (647.1nm). Predicted values are shown as filled circles, while observed values are shown as empty squares.

Figure 4.2.1 leaves no doubt as to the agreement between the phase-matching predictions of visible outcoupling in TFBG and observations made with our apparatus. **Chapter 5** contains more rigorous statistical comparisons showing a Chi-square value of 0.00444, meaning the results agree to a 95% confidence interval. Furthermore, the variance in the observed results is 0.3° , which is smaller than the reading error, meaning the observed values are strongly statistically correlated and that this experiment should be easily reproducible.

4.2.2 Relationship Between Output Efficiency and Grating Tilt

A total of five gratings were tested as reference, ranging from 20° to 37.5° . To track the relationship between output power and grating tilt (or slant), the backward-coupled mode was chosen and measured for each grating (20° , 25° , 30° , 35° , and 37.5°) as shown in **Figure 4.2.2**.

An extremely insightful result, **Figure 4.2.2** shows that for grating angles greater than 30°, the output power displays a marked jump. This relationship makes it clear that grating tilt plays a large role in outcoupling efficiencies. Further work is required to verify whether this trend continues onward to 45°, or if there is an “ideal” tilt at which the outcoupled radiation peaks, however previous literature has confirmed that 45° is expected to produce optimized radiation.¹⁷

Though not shown here for the sake of brevity, all five gratings showed exactly the same angular distributions (see **Figure 4.2.1**) within reading errors. Therefore, for the range tested, I observed an exponential or power law relationship between grating tilt and efficiency – and a negligible relationship between grating tilt and angular distribution – of the outcoupled radiation pattern for visible light diffraction in tilted fiber Bragg gratings.

Figure 4.2.2 – Graph showing relationship between outcoupled power of backward-coupled mode versus grating angle. These results are for blue light (488.0 nm) for reference.

4.3 Large-Angle Grating Fabrication: *Improved Design and Efficiency*

It was discovered that the laser orientation used in the fabrication process was sub-optimal, which would severely affect the quality of the gratings produced. Upon optimising this detail, the gratings showed behaviour consistent with Type-R gratings.

4.3.1 Clues of Sub-Optimal Fabrication Process: *"Low quality" TFBGs*

The corrections made to our gratings were a result of two separate sources of information. Initially, the first gratings tested (see Section 4.1) showed inconsistent behaviour with theoretical predictions (see **Figure 4.3.1**). Namely, the radiation pattern for these gratings was circular instead of crescent-like as seen for the gratings discussed in Section 4.2, and the initial gratings radiated primarily in the forward direction instead of the backward direction.

Figure 4.3.1 – A) The shape of the radiation profile was circular for the first generation of gratings, while **B)** the shape of the profile once corrections were made agreed with theory. Only the strongest radiation mode is shown, i.e. for **A)** the forward propagating mode – and for **B)** the backward-propagating mode.

Table 4.3.1 – The angles of radiation for four modes, and their respective efficiencies, are given for three gratings. Input power was fixed at 0.27W, efficiencies are included directly. All gratings have 30° slants. Table A (top) is for green light – 514.5nm, and Table B (bottom) is for blue light – 488.0nm.

Fibre Type	θ_1 (°)	ϵ_1 (%)	θ_2 (°)	ϵ_2 (%)	θ_3 (°)	ϵ_3 (%)	θ_4 (°)	ϵ_4 (%)
SMF-28	30.0	0.0043	68.0	0.0001	95.0	0.0001	123.0	0.0009
UVS-INT	30.7	0.0230	68.6	0.0003	95.4	0.0004	123.6	0.0027
UVS-652	30.0	0.0826	67.9	0.0012	94.7	0.0004	122.9	0.0015
Fibre Type	θ_1 (°)	ϵ_1 (%)	θ_2 (°)	ϵ_2 (%)	θ_3 (°)	ϵ_3 (%)	θ_4 (°)	ϵ_4 (%)
SMF-28	44.0	0.0011	74.0	0.0003	99.0	0.0004	127.0	0.0009
UVS-INT	44.6	0.0059	74.5	0.0006	99.7	0.0004	127.2	0.0025
UVS-652	43.5	0.0285	73.4	0.0020	98.6	0.0012	126.2	0.0035

This type of radiation is typical of surface relief gratings. Here they would result from poorly defined grating planes that would behave as omnidirectional scatterers instead of plane interfaces between regions of high and low refractive index. Those gratings were made with an incorrect orientation of the excimer laser beam.

4.3.2 Comparing TFBGs

As shown in **Figure 4.3.1**, the corrections made to the apparatus resulted in a marked change in the behaviour of visible light diffraction. Another important distinction was the drastic improvement in visible sidetapping efficiencies. In order to simultaneously account for the effect of fibre type on radiation intensity, three different fibres were used (see Table 4.1.1). The results are tabulated in Table 4.3.1 above, showing the four orders of diffraction modes for each grating and the corresponding power reading. Results for both green (514.5nm) and blue (488.0nm) input light are shown.

For red light and green light, the efficiencies of the TFBG-R-37.5° grating were very high for the backward-outcoupled mode, at 0.23% and 0.18%, respectively. However, given the nature of the rise in efficiency as a function of tilt, the fact that our 30° gratings displayed 0.083% efficiency for green light (nearly half the strength of TFBG-R-37.5°) is encouraging. As shown in **Figure 4.2.2**, the dependence of efficiency on grating slant is shows a marked rise as the grating angle increases above 30°. For this reason, it is remarkable that our 30° gratings showed even greater efficiencies for blue light outcoupling (shown in **bold** in **Table 4.3.2**) than even TFBG-R-37.5° (see **Figure 4.2.2**). The exact reason for this improvement in efficiencies between our Type B gratings and Type R gratings may be explained by the small difference in Bragg wavelength (1610nm for Type B, 1550nm for Type R), however more work is required to test this hypothesis. Pictures were taken of both green and blue outcoupled radiation to visually confirm these results, shown in **Figure 4.3.2** below.

4.4 Input Power, Polarization, Wavelength and Grating Pitch

In order to address the effects of input power, polarization, wavelength and grating pitch on output radiation, an experiment was devised to measure the efficiency and angular distribution of outcoupled radiation as a function of these parameters. It is shown that both input power and polarization have negligible impacts on angular distribution, while more work may be required to fully understand their effects on efficiency in the nonlinear regime. The pitch of the grating, which determines the Bragg wavelength of the TFBG, was used as a control parameter to address and further validate the phase-matching analysis in the context of fibre grating taps for visible light. Observations of the effect of input wavelength are also shown.

0.0285% Efficiency

0.0826% Efficiency

Figure 4.3.2 – Strong visible outcoupling of UVS-652 1cm 30° 1610nm TFBG. Shown on top are pictures taken from afar (intense outcoupling at TFBG in centre) and below through a spectroscope. It is important to note that these pictures represent the $m = 1$ diffraction mode order only. The multimodal nature of the guided radiation results in several lines showing up for each m number. The lines shown in these pictures do NOT represent different m numbers, only the spectral width of one m -number due to multimode operation of visible wavelengths in a 1610nm TFBG.

4.4.1 Independence of Angular Distribution on Input Power and Polarization

The polarization of the incoming light was adjusted using a standard polarization rotator (see **Figure 4.4.1**). Polarization was adjusted in increments of 5° for a complete 360° perspective. This experiment showed that the angular distribution of outcoupled light for all gratings tested (including type A, type B and type R gratings) was unaffected by polarization. This results establishes that, for the samples observed, the angular distribution of outcoupled radiation is independent of input polarization. Input power was varied in increments of 0.05mW from 0.05mW to 3.00mW and then in increments of 10mW from 10mW to 270mW to test its effect

on angular distribution. The result of this experiment showed no correlation, which indicates that angular distribution of outcoupled radiation is independent of input power. Indeed, neither the angular distribution nor the radiation profiles were affected by either input power or polarization. In the nonlinear regime, input power could affect mode distribution so these results confirm that the experiments conducted in this research were in the linear regime.

4.4.2 Effect of Input Power and Polarization on Sidetapping Efficiency

From section 4.4.1, it is clear that input polarization has no considerable effect on either output efficiency or radiation profiles. However, it is important to note that future work could include further control of the polarization in such a way that the directionality of the output

Figure 4.4.1 – Polarization rotator was incrementally adjusted to test effect on outcoupled radiation. Results showed that both output efficiency and angular distribution are independent of polarization.

Table 4.4.1 – The angles of radiation for four modes, and their respective efficiencies for sample grating (37.5°). Input power was not fixed, values are shown for each colour. Efficiencies are calculated directly.

Input Power	Input Wavelength	θ_1 (°)	ϵ_1 (%)	θ_2 (°)	ϵ_2 (%)	θ_3 (°)	ϵ_3 (%)	θ_4 (°)	ϵ_4 (%)
430mW	647.1nm	21.6	0.1047	69.6	0.0033	104.5	0.0052	149.4	0.0116
360mW	568.2nm	49.3	0.0015	82.6	0.0002	113.5	0.0004	159.5	0.0010
400mW	514.5nm	23.0	0.1200	62.9	0.0004	91.3	0.0002	120.0	0.0005
450mW	488.0nm	37.0	0.0129	69.4	0.0003	95.8	0.0003	123.7	0.0008
100mW	457.9nm	48.0	0.0005	76.1	0.0001	100.6	0.0002	127.8	0.0003

may be controlled. Using a sample grating (TFBG-R-35°) efficiencies for the backward-coupled mode were measured to be roughly the same (between 0.009-0.010%) for input power levels ranging from 0.1W to 0.43W for 647.1nm (red) input radiation. The range in these values is well within the error of the power readings, which was calculated to be $\pm 0.002\%$. Thus our results indicate sidetapping efficiency is negligibly affected by input power and polarization.

Figure 4.4.2 – The angles of outcoupled radiation are shown and collected into similar modes. That is, each mode is shown changing slightly as the wavelength is altered, exhibiting a linearly decreasing relationship to increasing wavelength. This can be verified by the phase-matching equation.

Table 4.4.2 – The angles of radiation, experimental and theoretical, for the backward-sidetapped mode for various wavelengths of a 976nm grating. Only theoretical values are shown for the 663nm grating.

Grating Pitch	Input Wavelength	Theoretical θ (°)	Experimental θ (°)
976nm	568.2nm	47.2	45.5
	514.5nm	40.7	38.4
	488.0nm	37.3	35.0
663nm	568.2nm	67.8	NA
	514.5nm	61.3	NA
	488.0nm	58.0	NA

Using a sample grating (TFBG-R-37.5°), all four orders of visible outcoupled light were measured for angles as well as power efficiency and the results are shown in **Table 4.4.1** above and **Figure 4.4.2** below. Angular distributions have a linear dependence on mode number.

4.4.3 Effect of Grating Pitch

To begin accounting for the effect of grating pitch on output radiation, three gratings with different Bragg wavelengths were chosen and tested. These gratings had Bragg wavelengths of 1610nm, 976nm and 663nm, respectively. Results agree weakly with phase-matching analysis.

The results for the 1610nm have been extensively reported in Section 4.3, so I discuss here only those for the 976nm and 663nm gratings. All gratings tested were in SMF-28 fiber. Both gratings showed significantly reduced outcoupling efficiencies and it was observed that the 663nm TFBG did not produce any outtapping at all. However, future work specialty optical fibres may yet produce different results and this is indeed part of the research path for this project. Therefore, only the experimental results for the 976nm grating are reported in **Table**

4.4.2 above, which shows the experimental and theoretical values of angular distribution for the main backward-sidetapped mode for 976nm, while also showing the theoretically expected angular values for the 663nm grating in the absence of experimental data.

4.5 Conclusion

To summarize the salient points of this chapter, our results indicate that simple phase-matching predictions are compatible with our experimental results, showing strong agreement. This indicates the success of our apparatus to empirically extract the angular distribution profiles of various TFBG configurations. The versatility of this apparatus is its most attractive feature, since it can easily isolate and thus compare the effects of many input variables including input power, polarization and wavelength as well as grating pitch while measuring output variables in a systematic way, including output power, angular distribution of diffracted radiation as well as azimuthal, radial and axial radiation profiles.

We are encouraged by the stark improvement in efficiencies and the progress made in the short span of this thesis work, and with our current understanding of the importance of photosensitive fibres it would be interesting to retest the effects of grating pitch to fabricate TFBGs that operate in single mode in the visible region. This should improve efficiencies even further. Finally, it would be necessary to produce 45° gratings at some point, as this angle is expected to maximize radiation efficiency.

Chapter 5

Analysis and Significance of Angular and Power Distribution Results

The main results are emphasized in this chapter and their significance in the context of future research is discussed. The results from the previous Chapter make it clear that the research of visible light sidetapping from Tilted Fibre Bragg Gratings has interesting research potential. In the short span of a 20-month thesis project, much progress was made in understanding the various components that affect and govern outcoupling radiation. The fabrication method of TFBG, the type of fibre used, the grating pitch, the grating tilt and the input wavelength will all affect the output radiation profile, which can be broken into relationships between input variables and output, namely:

- a) the angular distribution of diffracted modes (see **Figure 4.1.1** and **Figure 4.3.1**) and
- b) the power or efficiency of outcoupled radiation, as represented in **Figure 4.2.2** and **Tables 4.3.1** and **4.4.1**.

The main goal of this future research is in understanding and controlling blue light sidetapping for novel sensing solutions in DNA- and biomolecular-detection applications.

5.1 Fabrication Techniques

Although the first set of gratings made in Advanced Photonic Components Group (APCG) laboratory were “low quality” gratings (labeled TFBG-A in Table 4.1.1), this allowed us to study the effects of

varying the TFBG fabrication setup. As shown in **Figure 4.3.1**, these weak gratings completely changed the expected output characteristics of diffracted light. The salient points distinguishing these gratings from ‘good’ gratings are that:

a) the angular distribution remained unaffected, but the main diffraction mode was forward-coupled, not backward-coupled (as observed for TFBG-B gratings),

b) the outcoupled efficiency was so weak as to render the outcoupling nearly impossible to measure (even the forward-coupled mode), and

c) the radiation profile, as shown in **Figure 4.3.1**, was circular, instead of crescent-shaped.

Points **b)** and **c)** above are actually quite interesting and the physical phenomenon underpinning this behaviour could be useful to understand exactly what is happening in the fibre as a grating is fabricated. What happens to the grating inside the fibre that so drastically alters the output behaviour? Can this behaviour be controlled and optimized after all?

Regardless, once the corrections to the excimer laser orientation were made, further comparisons between different types of optical fiber were investigated as described in Sections 3.4 and 4.3. To control the grating tilt, the rotating setup shown below could be adjusted while the focusing lens was kept fixed (see green arrows in **Figure 5.1.1**) until a grating reached a desired tilt. As far as limitations, the largest angles that could be achieved using this fabrication setup were 30° gratings. Gratings shown in Section 4.3 were made using this technique. Preliminary work explored the option of rotating the lens in parallel with the rotating stage to attempt even larger angle gratings (see blue arrows in **Figure 5.1.1**), but so far these efforts have proven fruitless. In conclusion, the first technique worked best – shown in green in **Figure 5.1.1** – and more work is required to produce even larger angle gratings.

5.1.1 Power Optimisation – Effect of Grating Tilt

In order to optimize sidetapping efficiency, it is important to produce even larger-tilt gratings as discussed in the literature¹⁷ and corroborated by our own results in **Figure 4.2.2**. These results seem to show an exponential dependence of sidetapping efficiency on grating tilt angle; one of the more significant outcomes of this research. More work is required to investigate whether this exponential trend continues as grating tilt approaches 45° or if efficiency levels off. Either way it suffices to have shown that for gratings above 25°-30°, the trend definitely shows a drastic improvement in efficiency therefore, regardless of input wavelength, any attempt to optimize outcoupling should begin with the fabrication of 45° TFBGs.

Figure 5.1.1 – The setup holding the phase mask (right) was rotated to allow fabrication of up to 30° gratings, while the lens in the middle was tested in two configurations – 1) fixed (green), and 2) rotated in parallel with the rotating plate (blue). It was found that the first setup was more feasible.

5.1.2 Power Optimisation – Effect of Grating Pitch

I know from phase-matching predictions, which have been validated in the course of our research, that grating pitch strongly affects output power. In Section 4.4.4, I showed that for 30° gratings with varying grating pitch, that the outcoupled power efficiency was significantly reduced. However, much more work is required to comprehensively study the effect of grating pitch on visible sidetapping. This future work is important for two reasons. First, repeated experiments may yet show significant if not improved sidetapping when photosensitive fibres are used (see Section 4.3). Second, if successful, these gratings should yield perfect or near-perfect single mode operation in the visible spectrum. This would be a huge advantage in optimising the scattered radiation by controlling both efficiency and output angle. Furthermore, this would introduce new possibilities hitherto unexplored, such as systems that could exploit polarization-selectivity. Much potential lies in this type of sensor if significant control over output radiation is achievable.

5.1.3 Power Optimisation – Effect of Fibre Photosensitivity

Though well explored in Section 4.3.1, it is useful to reiterate the main result: fibre photosensitivity in the cladding plays a major role in optimising output power. Comparing SMF-28 fibres and UVS-652 fibres alone, **Table 4.3.1** showed that photosensitivity in the cladding results in an increase in efficiency by a factor of 30 for blue light, and by factor of 20 for green light. These are very significant results especially since the main goal of this research was maximising the efficiency of blue light sidetapping. It is highly encouraging that only three types of fibre were compared; SMF-28, UVS-INT and UVS-652.

Table 5.2.1 – Theoretical exit angles for blue light radiation for 1610nm, 976nm and 663nm TFBGs..

Grating Pitch (nm)	Input Wavelength (nm)	Theoretical θ (°)
663	488.0	58.0
976	488.0	37.3
1610	488.0	43.5

5.2 Controlling Angular Distribution

There are only two ways to control the scattering angles of visible light diffraction in TFBG: adjusting grating pitch, or adjusting input wavelength. In the context of this thesis work, we are primarily interested in blue light outcoupling, leaving only grating pitch as the technique of choice. If a certain exit angle is chosen, phase-matching predictions allow the calculation of an “ideal” grating pitch that should yield such radiation. For example, in **Table 5.2.1** I compare the theoretical output angles of blue light (488.0nm) for three gratings tested in this research. I note, however, that the relationship between grating pitch and scattered mode is not linear so any attempt to ‘control’ scattering angles should be very carefully studied. The reason this is not straight-forward is that more work is required to study the impact, if any, of scattering angle on blue-light sensing properties. For instance, although grating pitch clearly affects both scattering angle as well as multimode operation in the visible EM spectrum, it may be impossible to achieve a perfect balance and a trade-off may be necessary.

It is noted that although grating tilt does affect output efficiency, as summarized in Section 5.1.1, grating tilt does not affect the angular distribution profiles of outtapping. This result is clear from a phase-matching standpoint and simplifies any future work by limiting the focus to grating pitch alone.

5.3 Conclusion

The most encouraging aspects of this thesis research are the accurate prediction of coupling angles from phase matching analysis, and the strong outcoupling of visible light traveling through a TFBG especially in photosensitive optical fibre. The main significance of this work is in its simple approach, which proved to be flexible and versatile. The simplicity of the apparatus described in Chapter 3 gave way to the myriad results in Chapter 4, culminating with the first reported blue light outcoupling for a relatively low-tilt grating (i.e. without using a 45° TFBG).

How can we extend the knowledge gathered in this Master's thesis? What future work is required to address the limitations and restrictions encountered? Why is it important to pursue this research? Can this research be integrated into modern sensing systems to enhance current technologies, or branched out and used to produce completely novel ones?

The following chapter will attempt to summarize the main body of findings of this research work, and to answer some of these questions while discussing the context for future work on fibre grating taps for visible light.

Chapter 6

Discussion and Future Work

At the beginning of this undertaking, the literature had shown some interest in visible outcoupling but as of yet no publicly available findings has confirmed or reported successful blue light outcoupling. In this respect, this thesis work has gone above and beyond expectations by successfully achieving intense blue light radiation from large angle TFBGs produced in-lab at the Advanced Photonic Components research group at Carleton University.

Nonetheless, including time constraints, certain limitations meant that particular areas were left unexplored. For example, there may be many benefits to shifting from multimode to single mode operation such as potentially increased power efficiency, improved sensitivity as well as polarization selectivity. Also, it is recommended by this author that a priority is placed on future research to explore in more depth the effect of grating pitch on outcoupling properties, whether for blue light or other visible wavelengths. This would allow a build-up of empirical knowledge on the subject, which could eventually be summarized into a seminal understanding of visible light behaviour in optical fibres, SMF-28 or other, in general.

An increased understanding of the output power and angular distribution relationships to inputs such as grating tilt, grating pitch, input power, input wavelength, and azimuthal, axial and radial parameters could lead to completely novel solutions to sensing applications. However, it may be impossible or at least unfeasible to empirically test for all these parameters, which is why this author recommends an extensive effort to simulate the theoretical behaviour

of the outcoupling behaviour of visible light. Although the *angular distribution* has been quite carefully understood and validated using simple phase-matching analysis in the course of this research, it is especially important to further our understanding, empirically and theoretically, on the *output power* behaviour of visible light as a function of the aforementioned variables.

As has been mentioned repeatedly in this document, it is essential for optimisation of outcoupled power to produce 45° TFBGs, regardless of the pitch or other input parameters. This has been reported in the literature¹⁷ and any future expansion of this research will require this necessary step to improve outcoupling efficiency even further.

The drastic improvement in efficiency seen using photosensitive fibres as opposed to standard SMF-28 fibre is highly encouraging, especially since only two photosensitive fibres were tested (UVS-INT and UVS-652). If more research could be done on exploring a wider range of photosensitive fibres, even greater improvements in visible light efficiencies may yet be seen.

Finally, as a result of time limitations, no testing of these results directly into sensing properties was implemented. It is especially interesting that blue-light and the commonly used Ag nanocoatings in our laboratory could be combined to produce novel applications, as Ag is known to have a strong absorption peak at around 480nm. This is why it is paramount that the results of this and future research be applied directly to novel sensors, in order to validate their usefulness and exploit their potential in the future growth of research on the optimisation of visible light diffraction in tilted fibre Bragg gratings.

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Appendix A: Derivation of Cladding Modes Power Distribution

To find the coupling efficiency from the forward propagating core mode to backward coupled cladding modes in a TFBG, one must consider the power distribution characteristics that the blazing of the grating planes exhibits. The guided power in a multimode fiber can be expressed as follows²²:

$$P_{G,m} = \sum_{i=1}^{N_m} P_{G,m,i} \quad (\text{A.1})$$

Radiation power is similarly defined:

$$P_{R,m} = \sum_{i=1}^{N_m} P_{R,m,i} \quad (\text{A.2})$$

The relation between resonant (or degenerated) mode groups (DMG) and guided power can be represented by the following relation:

$$\sum_{i=1}^{N_m} P_{G,m,i} = \sum_{i=1}^{N_m} K_{m,i} P_{R,m,i} \quad (\text{A.3})$$

where $P_{G,m}$ and $P_{R,m}$ are the guided and radiation power of the m^{th} DMG, respectively, and $P_{G,m,i}$ and $P_{R,m,i}$ are the i^{th} degenerated mode (DM) in the m^{th} DMG, and $K_{m,i}$ is the reciprocal of the coupling coefficient for the i^{th} DM. The coupling coefficient for a specific mode is defined as:

$$K_m = \frac{\sum_{i=1}^{N_m} K_{m,i} P_{R,m,i}}{P_{R,m}} \quad (\text{A.4})$$

The total power of all the guided modes or essentially the total coupled power in the fiber of input light before reaching the TFBG is given by:

$$P_t = \sum_{i=1}^{N_m} P_{G,m} = \sum_{i=1}^{N_m} K_m P_{R,m} \quad (\text{A.5})$$

Appendix A: Derivation of Cladding Modes Power Distribution

We define the fraction of power coupled into a specific cladding mode m as the modal power reflection coefficient, or coupling strength, through:

$$R_{m,dB} = -10 \log \left(\frac{P_{R,m}}{P_t} \right) \quad (\text{A.6})$$

In order to find this coupling strength within a TFBG we consider that the grating tilt is equivalent to introducing a spatially dependent phase $\Phi(r, \varphi, \theta)$ to the wavefunction $\Psi(r, \varphi)$ in the grating with uniform refractive index modulation of $\widetilde{\Delta n}$. At small tilt angles, the RI modulation is:

$$\Delta n = \widetilde{\Delta n} \cos \left[\frac{2\pi z}{\Lambda} + \Phi(r, \varphi, \theta) \right] \quad (\text{A.7})$$

where

$$\Phi(r, \varphi, \theta) = \frac{2\pi r \cos \varphi \tan \theta}{\Lambda} \quad (\text{A.8})$$

Since the fundamental core mode has only radial dependence, $\Psi(r)$ will be used instead of $\Psi(r, \varphi)$ to represent this mode. We represent the modal field distributions with $\Psi_{lm}(r, \varphi)$. The coupling strength between the core mode and the modal distribution is described by first considering the effective refractive index modulation:

$$\widetilde{\Delta N} = \int_0^{2\pi} \int_0^\infty \widetilde{\Delta n} \Psi(r) \Psi(r, \varphi) e^{j\Phi} r dr d\varphi \quad (\text{A.9})$$

We define an overlap integral:

$$OL = \int_0^{2\pi} \int_0^\infty \Psi(r) \Psi(r, \varphi) e^{j\Phi} r dr d\varphi \quad (\text{A.10})$$

Appendix A: Derivation of Cladding Modes Power Distribution

So equation (A.7) becomes:

$$\widetilde{\Delta N} = \widetilde{\Delta n} \cdot OL \quad (\text{A.11})$$

The peak reflection amplitude at a wavelength λ is then given by:

$$R = \tanh^2(kl) \quad (\text{A.12})$$

where l is the grating length and

$$k = \frac{\pi \widetilde{\Delta N}}{\lambda} \quad (\text{A.13})$$

This effective index modulation can thus be used to measure the relative coupling strength between the core mode and resonant modes.

Appendix B: FIMMWAVE® Simulations Used For Spectral Width Validation

mode	neff	10	20	25	30	35	37.5
1	1.462654	78.39	78.04	77.78	77.46	77.09	76.89
2	1.462654	78.39	78.04	77.78	77.46	77.09	76.89
3	1.461392	78.38	78.03	77.77	77.45	77.08	76.87
4	1.461390	78.38	78.03	77.77	77.45	77.08	76.87
5	1.461390	78.38	78.03	77.77	77.45	77.08	76.87
6	1.461388	78.38	78.03	77.77	77.45	77.08	76.87
7	1.459772	78.37	78.02	77.75	77.43	77.06	76.86
8	1.459772	78.37	78.02	77.75	77.43	77.06	76.86
9	1.459769	78.37	78.02	77.75	77.43	77.06	76.86
10	1.459769	78.37	78.02	77.75	77.43	77.06	76.86
11	1.459264	78.37	78.01	77.75	77.43	77.06	76.85
12	1.459264	78.37	78.01	77.75	77.43	77.06	76.85
13	1.457870	78.36	78.00	77.74	77.42	77.05	76.84
14	1.457870	78.36	78.00	77.74	77.42	77.05	76.84
15	1.457143	78.35	77.99	77.73	77.41	77.04	76.83
16	1.457142	78.35	77.99	77.73	77.41	77.04	76.83
17	1.457141	78.35	77.99	77.73	77.41	77.04	76.83
18	1.457141	78.35	77.99	77.73	77.41	77.04	76.83

Figure B.1 – Table showing mode number (see below for visual representation – not to be confused with m number), effective index for each mode, and the outcoupling angle for each mode for gratings between 10° and 37.5°, where the tilt is shown at the top of the table. This table shows that for all value of tilt, the expected “spread”, or variation in angle, for one m – number (in this case m = 2 and wavelength of 632nm) is about 0.05°.

Appendix B: FIMMWAVE® Simulations Used For Spectral Width Validation

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5 1.46139
6 1.461388
7 1.459772
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10 1.459769
11 1.459264
12 1.459264
13 1.45787
14 1.45787
15 1.457143
16 1.457142
17 1.457141
18 1.457141

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Appendix B: FIMMWAVE® Simulations Used For Spectral Width Validation

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Appendix B: FIMMWAVE® Simulations Used For Spectral Width Validation

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Appendix B: FIMMWAVE® Simulations Used For Spectral Width Validation

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Appendix C: Derivation of Coupled Mode Analysis

We begin by assuming an input mode (E_1, H_1) entering a waveguide (E_2, H_2)

$$\vec{\nabla} \times \vec{E}_1 = -j\omega\mu\vec{H}_1 \quad \vec{\nabla} \times \vec{E}_2 = -j\omega\mu\vec{H}_2 \quad (\text{C.1ab})$$

$$\vec{\nabla} \times \vec{H}_1 = -j\omega\mu\vec{E}_1 + \vec{P}_1 \quad \vec{\nabla} \times \vec{H}_2 = -j\omega\mu\vec{E}_2 \quad (\text{C.2ab})$$

$(\vec{E}_2, \vec{H}_2)$ is a mode of the waveguide $\rightarrow$ i.e. one of the eigenmodes of the problem.

$(\vec{E}_1, \vec{H}_1)$ is a field generated by $\vec{P}_1$.

$$\nabla(\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1) = -j\omega\vec{P}_1 \cdot \vec{E}_2 \quad (\text{C.3})$$

$$\iint_{\infty} dx dy \frac{\partial}{\partial z} [\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1]_z = -j\omega \iint_{\infty} dx dy \vec{P}_1 \cdot \vec{E}_2 \quad (\text{C.4})$$

The eigenmodes are given by $(\vec{e}_n, n_n)$ pairs of transverse fields (neglecting $\hat{z}$ fields). The backward propagating eigenmode is identical to the forward one except that $\vec{h}_n = -\vec{h}_n$ in order to have $\vec{e}_n \times n_n = \pm \hat{z}$.

If we choose $(\vec{E}_2, \vec{H}_2)$ as a backward propagating mode n:

$$(\vec{E}_2)_t = \vec{e}_n \exp(+j\beta_n z) \quad (\vec{H}_2)_t = -\vec{h}_n \exp(+j\beta_n z) \quad (\text{C.5ab})$$

For field 1, the most general solution (of the Helmholtz equation) is:

$$\vec{E}_1 = \sum_m a_m \vec{e}_m \exp(-j\beta_m z) + b_m \vec{e}_m \exp(+j\beta_m z) \quad (\text{C.6})$$

$$\vec{H}_1 = \sum_m a_m \vec{h}_m \exp(-j\beta_m z) - b_m \vec{h}_m \exp(+j\beta_m z) \quad (\text{C.7})$$

$\vec{e}$ and $\vec{h}$ have the same amplitude factors because they are solved simultaneously.

If we replace into LHS of (4):

Appendix C: Derivation of Coupled Mode Analysis

$$\iint_{\infty} dx dy \frac{\partial}{\partial z} \left[\left(\sum_m a_m \vec{e}_m \exp(-j\rho_m z) + v_m \vec{e}_m \exp(+j\rho_m z) \right) \times (-\vec{h}_n) \exp(-j\beta_n z) \right. \\ \left. + \vec{e}_n \exp(-j\beta_n z) \times \left(\sum_m a_m \vec{h}_m \exp(-j\beta_m z) - b_m \vec{h}_m \exp(+j\beta_m z) \right) \right] \quad (C.8)$$

Note $\vec{h}_m = \vec{h}_m$ and $\vec{e}_m = \vec{e}_m$ and (C.8) = 0 if $n \neq m$.

$$\iint_{\infty} dx dy \frac{\partial}{\partial z} \left[\begin{aligned} &-a_m (\vec{e}_m \times n_n) e^{-j(\rho_n - \rho_m)z} - v_m (\vec{e}_m \times n_n) e^{-j(\rho_n + \rho_m)z} \\ &+ a_m (\vec{e}_n \times n_m) e^{-j(\beta_n - \beta_m)z} - v_m (\vec{e}_n \times n_m) e^{-j(\beta_n + \beta_m)z} \end{aligned} \right] \quad (C.9)$$

$$\iint_{\infty} dx dy \frac{\partial}{\partial z} \left[-2b_m (\vec{e}_m \times n_n) e^{-j(\rho_n - \rho_m)z} \right] \quad (C.10)$$

Using RHS, changing syntax for clarifying integrals and rearranging we obtain:

$$\frac{\partial}{\partial z} \left(-2b_m e^{-j(\beta_n - \beta_m)z} \right) \iint_{\infty} (\vec{e}_m \times n_n) ax dy = -j\omega \iint_{\infty} \vec{r}_1 \cdot \vec{e}_n e^{-j\beta_n z} ax dy \quad (C.11)$$

Using $m = n$ we further arrange (9) and obtain:

$$b'_m = \frac{j\omega}{2} \frac{\iint_{\infty} \vec{P}_1 \cdot \vec{e}_m e^{-j\beta_m z} ax dy}{\iint_{\infty} (\vec{e}_m \times n_m) ax dy} \quad (C.12)$$

Where $b'_m = \frac{\partial}{\partial z} b_m$ and similarly for a_m :

$$a'_m = -\frac{j\omega}{2} \frac{\iint_{\infty} \vec{P}_1 \cdot \vec{e}_m e^{-j\beta_m z} ax dy}{\iint_{\infty} (\vec{e}_m \times n_m) ax dy} \quad (C.13)$$

We note that for some derivations, the denominator (including the factor of 2) is arbitrarily set to 4. Note also that in some texts a_μ and b_μ are used instead of A_μ and B_μ .

Using the same eigenfunctions to write down $\vec{P}$:

Appendix C: Derivation of Coupled Mode Analysis

$$\vec{P}_t = \Delta\epsilon E_t = \Delta\epsilon \sum_v (a_v + b_v) E_{tv} \quad (C.14)$$

In the case of a single mode excitation (forward, with amplitude a_0):

$$\vec{P}_t = \Delta\epsilon \vec{e}_{0t} e^{-j\beta_0 z} = a_0 \epsilon_0 \vec{e}_{0t} \Delta n e^{-j\beta_0 z} = a_0 \epsilon_0 \Delta n \Delta n \vec{e}_{0t} e^{-j\beta_0 z} \quad (C.15)$$

Note this is valid for x or y polarized excitation.

Replace into (C.12) and (C.13):

$$b'_m = \frac{j\omega\epsilon_0 \Delta n}{\mathcal{Z}} \frac{\iint_{-\infty}^{\infty} a_0 \vec{e}_0 \cdot \vec{e}_m e^{-j(\beta_0 + \beta_m)z} \Delta n dx dy}{\iint_{-\infty}^{\infty} (\vec{e}_m \times n_m) y dx dy} \quad (C.16)$$

$$a'_m = \frac{-j\omega\epsilon_0 \Delta n}{\mathcal{Z}} \frac{\iint_{-\infty}^{\infty} a_0 \vec{e}_0 \cdot \vec{e}_m e^{-j(\beta_0 + \beta_m)z} \Delta n dx dy}{\iint_{-\infty}^{\infty} (\vec{e}_m \times n_m) y dx dy} \quad (C.17)$$

Cleaning up, we obtain for the change in amplitude of the backward going mode b'_m :

$$b'_m = ja_0 \left[\frac{\iint_{-\infty}^{\infty} \vec{e}_0 \cdot \vec{e}_m \Delta n ax dy}{\iint_{-\infty}^{\infty} (\vec{e}_m \times n_m) y dx dy} \right] e^{-j(\beta_0 + \beta_m)z} = ja_0 \vec{K}_{0m} e^{-j(\beta_0 + \beta_m)z} \quad (C.18)$$

Note β_0 is positive for backward mode and β_m is positive for forward mode. The change in amplitude of the a_0 mode is given by (due to $b_m e^{j\beta_m z} \vec{e}_m$):

$$a'_0 = -jb_m \left[\frac{\iint_{-\infty}^{\infty} \vec{e}_m \cdot \vec{e}_0 \Delta n ax dy}{\iint_{-\infty}^{\infty} (\vec{e}_0 \times n_0) y dx dy} \right] e^{j(\beta_0 + \beta_m)z} = -jb_m \vec{K}_{m0} e^{j(\beta_0 + \beta_m)z} \quad (C.19)$$

With positive β_0 and β_m , and where, for both (C.18) and (C.19):

Appendix C: Derivation of Coupled Mode Analysis

$$\vec{K}_{mn} = \omega \epsilon_0 n \frac{\iint \vec{e}_m \cdot \vec{e}_n \Delta n_0 dx dy}{\iint_{-\infty}^{\infty} (\vec{e}_n \times n_n) dx dy} \quad (C.20)$$

For sinusoidal perturbations:

$$\Delta n = \Delta n_0 \cos \frac{2\pi z}{\Lambda} = \Delta n_0 \cos Kz = \Delta n_0 \left(\frac{e^{iKz} + e^{-iKz}}{2} \right) \quad (C.21)$$

Substituting into (C.19) and (C.18), respectively, while noting that positive K term integrates to zero when used in (C.21) and vice-versa for (C.19), we obtain:

$$a'_0 = -jb_m \left[\frac{\iint \vec{e}_m \cdot \vec{e}_0 \Delta n_0 dx dy}{\iint_{-\infty}^{\infty} (\vec{e}_0 \times n_0) dx dy} \right] \frac{e^{j(\beta_0 + \beta_m - K)z}}{2} \quad (C.22)$$

$$b'_m = +ja_0 \left[\frac{\iint \vec{e}_0 \cdot \vec{e}_m \Delta n_0 dx dy}{\iint_{-\infty}^{\infty} (\vec{e}_m \times n_m) dx dy} \right] \frac{e^{-j(\beta_0 + \beta_m - K)z}}{2} \quad (C.23)$$

Here we have as the phase-matching condition $2\delta = \beta_0 + \beta_m - K \cong 0$, and with $mn = m0$ or $0m$:

$$K_{mn} = \frac{\omega \epsilon_0 n}{2} \frac{\iint \vec{e}_m \cdot \vec{e}_n \Delta n_0 dx dy}{\iint_{-\infty}^{\infty} (\vec{e}_n \times n_n) dx dy} \quad (C.24)$$

For the Bragg mode (C.24) reduces to:

$$K_{mm} = \frac{\omega \epsilon_0 n}{2} \Delta n_0 Z_0 \frac{\iint \vec{e}_m \cdot n_m dx dy}{\iint_{-\infty}^{\infty} (\vec{e}_m \times n_m) dx dy} \quad (C.25)$$

Appendix C: Derivation of Coupled Mode Analysis

Which further reduces to the well-known Bragg condition:

$$K_{mm} = \frac{\mathcal{Z} \pi f \varepsilon_0 \mathcal{N} \Delta n_0}{\mathcal{Z}} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\mathcal{N}} \eta = \pi f \sqrt{\varepsilon_0 \mu_0} \Delta n_0 \eta = \frac{\pi \Delta n_0}{\lambda_0} \eta \quad (\text{C.26})$$

So Bragg mode and backward going mode coupling is summarized through:

$$a'_0 = -j b_m K_{m0} e^{j2\delta z} \quad (\text{C.27})$$

$$b'_m = +j a_0 K_{0m} e^{-j2\delta z} \quad (\text{C.28})$$

Define:

$$A(z) = a_0(z) e^{-j\delta z} \quad (\text{C.29})$$

$$B(z) = b_m(z) e^{j\delta z} \quad (\text{C.30})$$

Relating this set of equations (C.27-30) we find:

$$\frac{dA}{dz} + j\delta A = -j K_{m0} B \quad (\text{C.31})$$

$$\frac{dB}{dz} - j\delta B = +j K_{0m} A \quad (\text{C.32})$$

Standard solution from above only works if $K_{0m} = K_{m0}$, which is true only if $\iint_{\infty} \vec{e}_m \times \vec{n}_m = \iint_{\infty} \vec{e}_0 \times \vec{n}_0$.

Appendix D: Statistical Comparison of Empirical/Theoretical Values

In order to confirm the statistical significance we use a Chi Squared Distribution

$$\chi^2 = \frac{1}{N-1} \sum_{m,\lambda} \frac{(\alpha_{m,\lambda-theory} - \alpha_{m,\lambda-experimental})^2}{\alpha_{m,\lambda-theory}}$$

where for the data in **Figure 4.2.1** we get a chi squared value of

$$\chi^2 = 0.00444$$

This value of chi squared implies a statistical significance at the 95th percentile.

Yet another validation is the sample variance calculated using

$$s^2 = \frac{1}{N-1} \sum_{m,\lambda} (\alpha_{m,\lambda-theory} - \alpha_{m,\lambda-experimental})^2$$

to get a variance of

$$s^2 = 0.1$$

The square root of this value is 0.3°. This value is called the “root mean square”, or RMS, and is in this case an average measure of variation between the experimental and expected values.

We note that the RMS value is larger than the reading error of the angle measurements. In summary, the observed results show a near-certain agreement with expected values.

Appendix E: Diagrams of Visible Diffraction Orders for 1550nm TFBG

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514.5 nm

488.0 nm

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