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**EXPERIMENTAL INVESTIGATION OF
TURBULENT SWIRLING FLOW IN AN ANNULUS**

by

Fathy Milad Yowakim, P. Eng.
B.Sc. (Honours) and M.Sc. (Mech. Eng.)

A thesis
submitted to the Faculty of Graduate Studies
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

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ABSTRACT

This thesis describes an experimental and theoretical investigation of a turbulent swirling flow in a stationary annulus. Isothermal air was used as the working fluid in an annulus of 0.625 hub/tip diameter ratio and with a single axial bulk Reynolds number of 3.1×10^5 . The swirling velocity component was imparted to the flow by inlet guide vanes which had radially constant outlet flow angle. The vanes were restaggered manually to provide nominal swirl angles of 15, 30 and 45 degrees and measurements on the the non-swirling flow were conducted with the vanes removed. In the former cases, a two-dimensional jet of air was ejected from the trailing edge of each of the vanes to minimize the velocity defect in the vane wakes so that conditions were as nearly axisymmetric as possible. Measurements were taken at five axial stations along the annular duct in all four flows.

For the swirling flows, the radial distributions of the three time-mean velocity components and of the six Reynolds stresses were determined from measurements taken with normal and slanted single-sensor hot wire probes. A three-hole cobra probe was used to measure radial profiles of total pressure and flow angle. It was also used as a Preston tube to measure the wall shear stress on the inner wall of the annulus. Clauser charts, modified for swirling flows, were used to determine the wall shear stresses on both walls. The radial distributions of the static pressure were computed by numerical integration of the radial momentum equation. In the data reduction for the non-swirling flow case, the radial and tangential mean velocity components and two components of Reynolds stress ($-\rho \overline{v'w'}$ and $-\rho \overline{u'w'}$)

were assumed negligible in order to obtain better results for the remaining parameters. The profiles of the eddy viscosity components (μ_{rx} and $\mu_{r\theta}$) and of the mixing lengths (L_{rx} and $L_{r\theta}$) were deduced from the measured data and are presented in the thesis. The experiments were performed semi-automatically using a HP-3054A automatic data acquisition and control system. Data reduction was also performed using the micro-computer of the system (HP-85A).

The degree of swirl and the wall curvature were found to induce significant effects on the flow parameters; in general, the effects were of opposite trends near the inner (convex) and the outer (concave) walls. Approximate relations are derived for the radial distributions of the axial and tangential Reynolds stresses and the relations between inner and outer wall-shear stresses were found to be in good agreement with the corresponding experimental results. New logarithmic laws of the wall were derived and found to predict the axial and tangential velocities near the walls with good accuracy. Moreover, the universal law of the wall was found to be valid for the resultant velocity only if the degree of swirl is less than 15 degrees. Furthermore, the present study indicates that, in the case of a swirling flow, the application of the Clauser chart technique to the axial or tangential velocity profiles will determine $U^* \sqrt{\tau_{x,w}/\tau_w}$ or $W^* \sqrt{\tau_{\theta,w}/\tau_w}$ and not the friction velocities U^* and W^* .

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NOMENCLATURE

A, B	law -of-the wall constants
C_μ	coefficient of axial eddy viscosity, Eq. (2.2)
C_u^*	$U^* \sqrt{\tau_{x,w}/\tau_w}$
C_w^*	$W^* \sqrt{\tau_{\theta,w}/\tau_w}$
D_h	hydraulic diameter of annulus, $D_h = 2(r_o - r_i)$
$\sqrt{e^2}$	linearized RMS value corresponding to $\sqrt{e^2_{br}}$, Eq. (E.6)
$\sqrt{e^2_{br}}$	RMS of the AC component of bridge output voltage of hot-wire anemometer
E	voltage, instantaneous
$\bar{E}$	linearized voltage corresponding to $\bar{E}_{br}$, time-mean value Eq. (E.5)
$\bar{E}_{br}$	bridge output voltage of hot-wire anemometer, time-mean value
h	pitch factor of hot-wire probe, Eq. (4.7), or annulus gap, $h = r_o - r_i$
k	kinetic energy of turbulence, $k = \frac{1}{2}(\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$, or yaw factor of hot-wire probe, Eq. (4.7)
K	von Karman's constant
l	mixing length
l_o	mixing length in the case of non-swirling flow
l_{rx}	axial mixing length, Eq. (3.10)
$l_{r\theta}$	tangential mixing length, Eq. (3.11)
l_s	dissipation length scale of energy-containing eddies, Eq. (2.2)
m	mass flow rate, Eq. (5.18)
P	time-mean static pressure
P_t	time-mean total pressure
P'	fluctuating pressure component

NOMENCLATURE (continued)

Q	resultant flow velocity
$\bar{Q}$	mean resultant flow velocity, Eq. (4.2)
Q^*	resultant frictional velocity
Q^+	law-of-the-wall variable, $Q^+ = Q/Q^*$
r	radius
r_i, r_o	inner and outer wall radii of annulus
r^*	non-dimensional radius, $r^* = (r - r_i)/(r_o - r_i)$
Re	Reynolds number
Ri	Richardson number, Eq. (3.16)
S	slope of linearized calibration curve of hot-wire probe, Eq. (4.11)
SN	swirl number, Eq. (4.3)
U, V, W	time-mean velocity components in x, r, θ directions
u, v, w	fluctuating velocity components in x, r, θ directions
U_{eff}	effective cooling velocity of hot-wire sensor, Eq. (4.7)
U_o^*	frictional velocity of non-swirling flow, $U^* = \sqrt{\tau_{o,w}/\rho}$
U^*, W^*	frictional velocities in x, θ directions, Eq. (5.29)
U^+, W^+	law-of-the-wall variables, $U^+ = U/C_u^*, W^+ = W/C_w^*$
$\bar{U}$	mean axial velocity, Eq. (5.22)
x, r, θ	axial, radial, tangential coordinate directions
x	axial distance from exit plane of inlet guide vanes
y	distance from wall
y^+, y_1^+, y_2^+	law-of-the-wall variables, $y^+ = yQ^*/\nu$, $y_1^+ = yC_u^*/\nu$, $y_2^+ = yC_w^*/\nu$
α	hot-wire probe angle (the complementary angle to the angle between the probe stem and the wire sensor, Fig. 4.10)
ϵ	rate of dissipation of turbulence energy

NOMENCLATURE (continued)

μ	viscosity
$\mu_{rx}, \mu_{r\theta}$	eddy viscosities in the axial, x, and tangential, θ , directions
ν	kinematic viscosity of air, $\nu = \mu_{lam}/\rho$
ρ	density of air
σ_k	Prandtl/Schmidt number for the kinetic energy of turbulence k
$\sigma_{r\theta}$	eddy viscosity ratio, $\sigma_{r\theta} = \mu_{rx}/\mu_{r\theta}$
τ	shear stress
$\tau_{rx}, \tau_{r\theta}$	turbulent shear stresses in x, θ directions, $\tau_{rx} = -\rho\overline{uv}$, $\tau_{r\theta} = -\rho\overline{uv}$
$\tau_{x,w}$	axial wall shear stress component
$\tau_{\theta,w}$	tangential wall shear stress component
τ_w	resultant wall shear stress, $\tau_w = \sqrt{\tau_{x,w}^2 + \tau_{\theta,w}^2}$
$\tau_{0,w}$	wall shear stress of non-swirling flow
ϕ	flow angle
ϕ'	relative flow angle, $\phi' = \{ \tan^{-1}(W/U) - \tan^{-1}(\tau_{r\theta}/\tau_{rx}) \}$
ζ, ξ, η	fixed coordinate system of hot-wire probe, Fig. 4.10
ψ	orientation angle of hot-wire sensor (i.e. angle through which the hot-wire probe is rotated about its stem), Fig. 4.10

NOMENCLATURE (continued)

subscripts

a	ambient
br	Wheatstone bridge of hot-wire anemometer
lam	laminar
lin	linearized
i	inner wall
o	outer wall or non-swirling flow conditions
rx	axial component
r θ	tangential component
t	total
tr	pressure transducer
w	wall
-	time-mean

CHAPTER I

INTRODUCTION

This thesis is concerned with swirling flow in annular passages. Swirling flow is an important and basic feature of turbomachinery. For example, the flow at the exit of an axial compressor rotor blade row has a strong swirl or circumferential velocity in addition to the axial or through-flow velocity component. This swirl velocity will decay along the duct, due mainly to the viscous forces between the fluid and the duct walls and between the fluid particles themselves. The axial velocity profile must also change, but cannot decay because it is constrained by the continuity equation. Depending on the axial spacing between the rotor and the subsequent stator, the change in flow angle due to the decay of swirl velocity and the change in the axial velocity profile will cause the incidence angle of the stator to be mismatched and may degrade the performance of this blade row. This situation could be serious in current turbomachinery design; for example, in the fan stage of a turbofan engine, a fairly large axial space between rotor and stator is required to reduce the fan noise generated by blade wake/stator interaction. The same problem can arise in an inter-turbine duct which has a large distance between the high pressure and the low pressure turbine stages.

The most widely used and comprehensive methods for predicting the internal flow of turbomachinery are the streamline curvature and matrix methods [1, 2]* and both of these methods assume essentially inviscid

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* Numbers in square brackets designate references at the end of the text.

flow between blade rows. These inviscid methods are capable of estimating air flow angles as a function of radial position at blade row exit, but they are unable to account for viscous effects. Therefore, empirical formulae have been developed and the annulus boundary layer is calculated to get a more realistic flow picture. Present capabilities for predicting turbulent swirling flows are however, rather limited. In order to predict the turbulent boundary layer flows, some knowledge of the turbulent (i.e. Reynolds) stresses is required. Predictions are sensitive to variations in the turbulence models used for this purpose and reliable models for swirling flows are not yet available. The objective of this thesis is to make a contribution in this area.

Experimental data are essential for the development of turbulence models. Previous work is reviewed in Chapter II but here it can be said that no suitable turbulence data for swirling flows have been published to date. In the experimental study by Scott and Rask [3], the axial and tangential eddy viscosities were estimated from the measured mean velocities and pressures with the aid of simplified momentum equations; no attempt was made to directly measure the turbulent shear stresses. In the theoretical study by Yamamoto and Millar [4], local isotropy of the axial and tangential eddy viscosities was assumed. In their computations no modifications were made to the turbulence model (a $k-\epsilon$ model) or to the universal law of the wall to account for the effects of swirl and streamline curvature. The results of their computations were in poor agreement with the experimental results of Scott and Rask. This indicates that swirl and streamline curvature have important effects on turbulence structure. The present work was initiated to help overcome the deficiencies in currently available methods for analyzing swirling

flows in annuli.

The present work comprised an experimental investigation of swirling flow in a stationary annulus. The objective was to gather data suitable for validation or further development of turbulence models for this type of flow.

The experimental apparatus consisted of an annular duct with a hub/tip diameter ratio of 0.625. Isothermal air near standard atmospheric conditions was used as the working fluid. The swirling flow component was imparted to the flow by a cascade of inlet guide vanes with constant outlet flow angle. The vanes could be restaggered manually to obtain various swirl angles. Swirl angles of 0, 15, 30 and 45 degrees were investigated. A jet of air was ejected at the trailing edges of the vanes to achieve axisymmetrical flow conditions with minimum blade wakes. All experiments were conducted with a mean axial velocity of 32.6 m/sec, corresponding to a Reynolds number of 3.11×10^5 based on the mean axial velocity and hydraulic diameter of the annulus. The apparatus and experimental techniques are described in detail in Chapter IV.

Radial distributions of the six Reynolds stress components and the three time-mean velocity components were measured at five axial locations in a swirling flow in an annulus, using single-sensor 45° slant and normal hot-wire probes. At each point in the flow six measurements of mean and RMS anemometer output voltage were taken, three using the slanted probe and the other three with the normal probe. The velocity and stress components were deduced from these measurements using the assumption of low turbulence intensity ($< 15\%$). Distributions of axial and tangential eddy viscosity and mixing length

were deduced and compared with selected turbulence models. The methods used for data reduction are described in detail in Chapter V.

A 3-hole cobra probe was used to measure profiles of total pressure and flow angle. Static pressure distributions and wall shear stresses were determined from appropriate data.

The experimental results are analyzed and discussed in Chapter VI. The effects of streamline curvature on the axial and tangential turbulent shear stresses and on the inner and outer wall shear stresses are illustrated with the guidance of some new relationships derived from simplified versions of the momentum equations. New versions for the logarithmic law of the wall are derived for the distributions of the axial and tangential velocities in the near wall regions. These new log-laws account for the effects of swirl angle and radius of curvature and are valid even if the turbulent stresses vary across the wall layers. The validity of the universal law of the wall for the resultant velocity of a swirling flow is examined. Also a new concept is introduced for applying the Clauser chart technique to determine the wall shear stresses in the case of swirling flows. The validity of some existing turbulence models is examined against the experimental results and recommendations are made for swirling annular flows.

CHAPTER II

LITERATURE REVIEW OF SWIRLING FLOWS

2.1 Preliminary Remarks

Non-swirling flows in annuli are not reviewed in detail here but an up-to-date review and some data on non-swirling turbulent flow through annuli with concentric core tubes are available [5-7]. Also, the effect of rotation of either the inner core tube or outer pipe of a long, concentric, annular duct with axial flow and its interesting illustrations of stabilization effects is discussed elsewhere [8, 9].

Internal swirling flows result from the application of a spiralling motion with an azimuthal, tangential or swirl velocity component being imparted to the flow via the use of inlet guide vanes (called swirlers in combustion literature) or by merging two streams, axial and tangential, through separate entries into a swirl generator. Moreover, the degree of swirl imparted to the flow has large effects on flow field parameters for both inert and reacting flows [10]. Some of the basic and interesting characteristics of swirling flows have been discussed by Scott and Rask [11]. Only a brief review of previous work related to the present study will be given here.

Streamline curvature produces large changes in the turbulence structure of shear layers. The stability of a turbulent flow influenced by centrifugal forces was first studied by Prandtl [12], who extended the analogy between buoyancy and curvature to turbulent flows. For flows over convex surfaces, the centrifugal force is largely balanced by a normal pressure gradient. If a fluid particle is displaced outward, across mean streamlines into regions of higher mean velocity, the

centrifugal force of the fluid particle will be less than the new normal pressure gradient, resulting in a restoring force. As a result convex wall curvature exerts a stabilizing influence on turbulent momentum exchange in boundary layers. The opposite is true for boundary-layer flows on concave walls and the de-stabilizing effects lead to increased turbulent momentum exchange between fluid particles [13]. Convex wall-curvature, in general, appears to have a stronger effect on the behaviour of the boundary layer than concave curvature of the same ratio of boundary layer thickness to wall radius. Experimental studies of boundary layers along curved surfaces [12-14] have indicated that over the convex wall the radial distribution of the shear stress quickly reaches a self-preserving form and the shear stress, turbulent intensities and length and velocity scales of turbulent motions are reduced, whereas concave curvature shows the opposite effect. It has also been observed that curvature has a relatively larger effect on the turbulent shear stress than on the turbulent kinetic energy. Turbulence may be nearly eliminated on highly-convex surfaces, while on highly-concave surfaces momentum transfer by quasi-steady longitudinal vortices dominates the ordinary turbulence processes [15].

2.2 Previous Experiments

The coordinate system and velocity component nomenclature used in the following discussion, and throughout this thesis, are shown in Fig.

3.1. Swirling flows represent a large category of flows with streamline curvature, including all flows whose mean streamlines are roughly helical. Classification and general discussion of various examples of swirling flows were reported by Bradshaw [15] in his review

on effects of streamline curvature on turbulent flows. A criterion for the instability of inviscid rotational flows was first investigated by Rayleigh [16, pp. 500]. He found that the flow becomes unstable if its peripheral velocity W decreases with the radius r , that is

$$W = \frac{\text{constant}}{r^n} \quad \text{with } n > 1 \quad (\text{unstable}) \quad (2.1)$$

A detailed discussion of the stability of viscous rotational flows can be found elsewhere [16, 17]. Mattendorf [18] attempted to isolate the effect of streamline curvature on turbulent flow by using a curved channel of constant curvature and cross section. He found that for fully developed, curved, internal flows, the eddy viscosity near the outer (concave) wall was up to four times the corresponding inner (convex) wall value. Eddy viscosity values were also greater than those for straight flow near the outer wall and less near the inner wall in accordance with Rayleigh's criterion.

Aerodynamics of a swirling flow in an annular channel of 0.65 inner to outer diameters ratio and for flows of up to 77 degrees of swirl was investigated by Sudarev [19]. The study was very briefly reported and no turbulence measurements were conducted. A three-hole probe was used to measure the axial and tangential velocity components. The results showed that the radial distribution of the two velocity components relative to their maximum values at each axial location did not change with variation of the axial velocity from 30 to 120 m/sec. Over an axial distance of 1.85 m and with the swirl angle increased from 50° to 77°, the maximum axial velocity was found to increase by 10 to 70 percent and the flow maximum angle to decrease by 15 to 30 percent of

the initial values at the vortex generator exit. No information was reported concerning the occurrence of reversed axial velocity zones at high swirl angles.

Scott and Rask [3] used a semi-analytic method to determine eddy viscosities from measurements of swirling flow in a stationary annulus with a single diameter ratio ($d_i/d_o = 0.4$) and at a single axial bulk Reynolds number of 1.3×10^5 . Isothermal air flow with initial swirl angle varying from 0 to 60 degrees was used as working fluid. Axial and tangential eddy viscosity profiles were computed from integral-differential forms of simplified momentum equations by using in the integrands the measured velocity profiles obtained by a 3-hole cylindrical probe. Turbulence intensity terms were not measured and were excluded from the momentum equations in their study. The inner and outer axial wall shear stresses $\tau_{x,w}$ were obtained by using a Clauser chart, while tangential wall shear stresses $\tau_{\theta,w}$ were obtained using the flow swirl angle ϕ at the wall, i.e. $\tau_{\theta,w} = \tau_{x,w} \tan \phi$. Also, a Preston tube was used to determine the resultant wall shear stress at the inner wall only. Results for 45° swirl showed that the outer wall tangential shear stress values were approximately twice the inner wall values. The axial and tangential eddy viscosities μ_{rx} and $\mu_{r\theta}$ were found to increase slightly near the outer wall as the swirl was increased from 10° to 60°. Near the inner wall $\mu_{r\theta}$ was found to decrease and μ_{rx} remained unchanged with increasing swirl. In general their results showed that $\mu_{rx}/\mu_{r\theta} > 1$, especially near the inner surface and at high degree of swirl. The method used in that study involves integration and differentiation of large numbers of experimental profiles - a procedure which virtually guarantees a large amount of

scatter. In order to test the validity of these simplified equations, measurements comprising a complete static pressure field, profiles of all six of the Reynolds stresses, and a complete mean velocity field for all three components are needed.

More recently, Scott and Bartlett [20] studied the decay of annular swirling flow with solid body rotation at inlet by using the experimental facilities of reference [3]. Also formulae for the radial distributions of the eddy viscosities were derived from simplified forms for the momentum equations and by assuming isotropic eddy viscosities. Their results showed a uniform axial decay of angular momentum and a profile shape independent of axial location. The shape of the tangential velocity profiles and the magnitude and distributions of the tangential eddy viscosity were found to be independent of the initial rate of swirl and axial location.

2.3 Turbulence Models

A considerable amount of experimental and theoretical work has been done in the search for suitable turbulence models. Studies in these areas have shed some light on the turbulence structure in swirling flows and have provided strong evidence of nonisotropy. Reviews of the general development are available [21, 22] and recent work on turbulent swirling flows is briefly reviewed below.

2.3.1 Theoretical Methods

Previous findings indicated that models of turbulent swirling flows need to include two important phenomena: the effect of swirl enhances the primary components of turbulent stress and eddy viscosity

(τ_{rx} and μ_{rx}), and the distribution of stress and eddy viscosity are nonisotropic. There is a strong link between these effects and the swirl angle and the Richardson number [10, 21 and 23].

Advanced work on calculating turbulent flows, see for example Launder and Spalding [24], has postulated that the axial eddy viscosity, μ_{rx} , may be described adequately by two quantities: the kinetic energy of turbulence k and a dissipation length scale L_s . The relation

$$\mu_{rx} = C_\mu \rho k^{1/2} L_s \quad (2.2)$$

is used. C_μ is an empirical coefficient and two extra differential equations are required, one for k itself and the other for a variable such as kL_s , k/L_s or the dissipation rate ϵ . Indications are that the ϵ -equation is preferable [21]. The two equation (k - ϵ) model was recently applied by Yamamoto and Millar [4] in an implicit numerical method for the computation of laminar and turbulent flow with and without swirl in a stationary annulus. For the turbulent flow case, local isotropy was assumed for the two components of the eddy viscosity, μ_{rx} and $\mu_{r\theta}$, and the radial velocity component was neglected. As mentioned earlier the computed results for swirling flows did not agree well with the experimental results of Scott and Rask [3]. This may be attributed to the assumed isotropy of the turbulence and application of the model without modifications to account for curvature effects. In general the model only successfully predicted turbulent flows without swirl. Many researchers [25-30] have reported that existing two-equation turbulence models successfully predict swirling flow only when one (or more) of the coefficients in the ϵ transport equation is

specifically "tuned" as a function of a suitable Richardson number to bring predictions into agreement with measurements. The Richardson number provides a measure of streamline curvature effects on turbulence structure, as suggested by Bradshaw [15]. In addition recent studies [30-36] have documented that the coefficient C_μ in the k - ϵ model is not a constant. Launder [35] showed that C_μ depends on the local ratio of turbulent kinetic energy production to dissipation as well as on the flow geometry. Predictions of developing two-dimensional, curved channel flow [35, 36] indicated that C_μ should be modified to include the direct influence of curvature effects on the length scale in near-wall regions of the flow.

Formulae for the variation of the turbulence velocity scales with Richardson number in a swirling flow have been derived by So [23] from the Reynolds stress equations by assuming local equilibrium, (production of turbulent energy balances dissipation). His results showed that the eddy viscosity is not isotropic and that the Richardson number is an appropriate parameter to use for the description of swirl effects on the turbulence. For weak swirling flows the Monin-Oboukhov formula, which involves a parameter closely related to the Richardson number, appears to satisfactorily represent the effects of streamline curvature on turbulence parameters as suggested by Bradshaw [15].

Prandtl's mixing length hypothesis (MLH) provides the simplest physical basis for turbulent flow computations. It has been proven to be reliable for predicting boundary layer flows on stationary walls [37] if the axial pressure gradients and curvature effects are small. Moreover, by virtue of its simplicity, its use in numerical solutions of the boundary layer equations is very economical of computer time.

Various extensions of the MLH to swirling flows have been proposed, the task being to link the tangential eddy viscosity, $\mu_{T\theta}$, with the axial viscosity, μ_{rx} , and to allow for the nonisotropy of the viscosity components [10, 21, 30]. The mixing length model produced satisfactory predictions for swirling flow in a pipe [21], and for axisymmetric and three-dimensional turbulent flows (e.g. infinite swept wing) [38]. Good results were also obtained by employing the MLH for calculating swirling flows near spinning cones, disks, cylinders [42] and turbulent annular flow with rotating core tube [39]. Galbraith et al. [40] in their study on mixing length in the wall region of turbulent boundary layers have modified the MLH to

$$l = K y \sqrt{\tau / \tau_w} \quad (2.3)$$

where K is von Karman constant and τ and τ_w are the shear stresses, at height y and at the wall, respectively. The above equation reduces to the conventional near-wall MLH if the shear stress τ is substantially constant through the wall region. Equation (2.3) is applicable even if the variation of the shear stress across the wall layer and/or the axial pressure gradients are significant.

It should be noted here that l_s in equation (2.2) refers to the dissipation length scale of the energy dissipating eddies of turbulence; it is usually expressed in terms of the kinetic energy of turbulence k and its dissipation rate ϵ i.e. $l_s = k^{3/2} / \epsilon$ [24, 33, 36]. Prandtl's concept for the mixing length l is analogous to the molecular mean free path (but much larger) in the kinetic theory of gases [16, 17]. The values of l and l_s in a certain flow case may be close [14] but they are

generally different as indicated by Bradshaw et al. [25].

Many solution procedures, using the MLH or $k - \epsilon$ turbulence models, are now available [10, 41-44] for parabolic boundary layer flows and more general elliptic recirculating flows, each of these being able to, or in principle adaptable to, predict the properties of weakly or strongly swirling flows.

2.3.2 Experimental Methods

Concerning experimental methods, the distributions of various Reynolds stress components and of time-mean velocities may be measured experimentally by hot-wire or laser anemometers. Such distributions allow values of components of the turbulent eddy viscosity to be determined at all points in the flow field. One can then check whether a certain turbulence model would be consistent with these eddy viscosity and stress distributions. As reported by Lilley in his review for swirling flows [21] this technique has been used in the study of swirling flow in a pipe, free swirling jets and the flow field near the recirculation zone at the exit of a swirl generator. In each case the turbulence is such that significant axial and radial variations of the components of turbulent eddy viscosity are found together with a considerable degree of nonisotropy.

The hot-wire probe technique is so far the most widely used approach to determine the flow field parameters and in particular the turbulent stresses. However, a hot-wire system is by no means as convenient as one might wish and the probe itself is exceedingly delicate and must be handled with great care. Regardless of the difficulties that may arise with the associated electronic equipment,

the calibration procedures are very lengthy and should be frequently repeated. In addition, the interpretation of the output signal does not follow any standard procedures. Good general discussions of the electronic aspects, probe calibration, factors influencing probe performance as well as signal interpretation of single or multiple wire-probes are documented in several references [45-49]. Single and/or multiple wire probes have been used to determine the mean velocity components and turbulent stresses for different flow geometries [49-56]. In general, determination of all turbulent stress components by a single hot-wire requires the use of both normal-wire and slanted-wire probes [12, 13, 57-60]. By choosing different wire positions and by rotating the probe around its own axis, a number of measurements can be made at any given point in the flow. Based on the analytical relationship between the hot-wire output signals (DC and RMS) and the flow parameters, a system of independent linear equations can be formulated and solved simultaneously to provide the required flow parameters.

In order to obtain accurate measurements by hot-wire probes, the effective cooling velocity, U_{eff} , experienced by the wire sensor must be carefully defined. The dependence of U_{eff} on the orientation of the hot-wire sensor is generally expressed in the literature [47, 48, 52, 61, 62] in terms of yaw and pitch coefficients, k and h , respectively. Yaw and pitch directions are illustrated in Fig. 4.8; the exact relationship between U_{eff} , k and h will be given in Chapter IV. Previous investigations of k and h are briefly reviewed below.

There have been several investigations of the factor k for a single yawed wire and there is somewhat conflicting evidence about the

correct value for the coefficient k . Hinze [62] suggested values between 0.1 and 0.3 (decreasing with Reynolds number). Webster [63] reported a value of 0.2 and suggested that it was independent of flow velocity. Champagne [57] concludes that the coefficient k is primarily a function of the length to diameter ratio of the wire and is equal to 0.2 for $l/d = 200$ and tends to zero for $l/d > 600$. Friehe and Schwarz [64] estimated, based on their experiments, values of $k = 0.4 - 0.2$ (45° slanted hotwire) for l/d values from 117 to 250. Also, Kjellstrom and Hedberg [65] and Jorgensen [61] showed that k was dependent on the yaw angle. Jorgenson, based on his measurements of directional characteristics, concluded that the yaw coefficient k depends on sensor geometry and is found to vary with the yaw angle α , (see Fig. 4.10), decreasing with increasing values of α . The value of k selected for $\alpha = 90^\circ$ may be used over the full range $0 \leq \alpha \leq 90^\circ$, resulting in errors from 4 to 10 percent, the error being largest for small prong spacing and wire length to diameter ratio. The pitch factor h depends on sensor geometry and prong length but is nearly independent of the pitch angle. If pitch corrections are neglected, errors between 5 and 12 percent will occur, the largest error occurring for small prong spacing, prong length, and wire length to diameter ratio. Using h selected for yaw angle $\alpha = 90^\circ$, errors reduce to between 1 and 2 percent. Several manufacturers are now producing straight probes ($\alpha = 90^\circ$) in which $h = 1$ and $k = 0.05$. This can be achieved [61, 66] by careful prong spacing and by gold plating the sensing wire near to the prongs. For more accurate results, h and k should be determined by calibration for each individual probe, as manufacturing variations affect these values. Methods of experimental determination of these

sensitivity coefficients are presented in several references [47, 48, 67-69].

2.4 Law of the wall and wall shear stresses

In the inner region of a turbulent boundary layer and outside the viscous sublayer (see Fig. 2.1), the velocity profile is generally described by the universal logarithmic law in the form ;

$$u^+ = A \ln y^+ + B \quad (2.4)$$

where u^+ and y^+ are "law of the wall" variables, defined in the list of symbols and A and B are universal constants having values of approximately 2.5 and 5.5 respectively [17]. The universal law applies to any wall flow over a smooth surface as long as the shear stress τ is effectively constant across the wall layer (small axial pressure gradient and/or small $d\tau_w/dx$). Some analytic formulae have been also proposed [38] and are applicable for the whole inner layer (viscous sublayer plus logarithmic law region). Dean [70] suggested a single formula that showed close agreement with experimental data for turbulent boundary layers in a wide range of pressure gradients. Van den Berg [75] derived an extended law of the wall valid for three-dimensional flows by taking into account effects of both the pressure gradient and the inertial forces. His wall law is valid only when the deviations from the universal law are not large. However, the important feature of his extended wall law is the prediction of the flow angle variations near the wall.

Fully developed turbulent non-swirling flow in smooth concentric

annuli has been studied in many experiments and numerous theoretical investigations [4-12, 72-74]. In all cases the velocity profile for the outer wall boundary layer was found to be in good agreement with the universal law of the wall. However, the velocity profile for the inner wall boundary layer was found to fit the logarithmic profile only if the inner to outer diameter ratio was greater than 0.375, and significant discrepancy occurred with decreasing diameter ratio. Modifications of the universal law to take into account the effect of the diameter ratio on the velocity profile for the inner wall cylinder have been suggested for the non-swirling flow case in several studies [42, 73, 75].

Yamamoto and Millar [4] in their study of swirling flows in annuli reported that the universal law of the wall was not as accurate as expected for predicting the axial velocity near the inner wall in annuli with small diameter ratio and for predicting the tangential velocity, especially near the outer wall. Scott and Rask [11] obtained good to relatively poor agreement between the experimental data and the universal law for the axial velocity profiles, especially at the inner wall for 0° to 60° swirl; higher values of swirl showed poorer agreement. No information was provided in their study concerning the validity of the universal law for the tangential velocity profiles.

Research on turbulent boundary layers with mild streamwise curvature by Shivaprasad and Ramaprian [14] indicated that while the velocity distribution across the convex and concave wall boundary layers exhibited a log-region close to the walls in both cases, the extent of the log-region was reduced by convex curvature and increased by concave curvature. So and Mellor [12, 13] in their experiments for turbulent boundary layers with high streamwise curvature concluded that the extent

of the log-region was reduced for both walls but for a convex wall it was relatively substantial in comparison with a very marginal extent for a concave wall. However, for turbulent swirling pipe flow [76], satisfactory logarithmic velocity profiles were obtained and curvature effects appeared to be negligible near the wall.

The Preston tube technique is known to be a simple and convenient method of determining the wall shear stress in two-dimensional turbulent boundary layers when the pressure gradients are not too severe [77, 78]. Previous researchers [79, 80] have shown that consistent values of skin friction can be obtained through the above technique even in three dimensional turbulent boundary layers if the Preston tube is aligned with the local flow direction. Prahlad [79] reported that only when both the pressure gradient and the diameter of the Preston tube are small, the direction given by a Preston tube approximates well the true wall shear-stress direction. He also concluded that both the pressure gradient component in the wall shear direction and that normal to it have to be small for the successful use of the Preston-tube technique in determining the wall shear stress vector in three dimensional flows. Ragaratnam and Muralidhar [80] suggested that both the direction and magnitude of the wall shear stress in a 3-dimensional boundary layer could be obtained by placing a conventional three-hole yaw probe on the wall and using it as a Preston tube. If the yaw probe is rotated until the pressure indicated by its side tubes are equal (i.e. nulling position) their method complies with the conventional Preston tube technique. Van den Berg [71] gives a prediction of the relative flow velocity angle with respect to the shear stress direction near the wall. The study also established a sound criterion for employing or

abandoning the use of the 3-hole yaw probe as a Preston tube for three dimensional flows in which the effects of wall curvature are small and deviations from the universal law of the wall are not large.

Previous studies [74, 81] for non-swirling fully developed flow through annuli of different diameter ratios indicated that the ratio of inner to outer wall shear stresses increased from 1.03 to 1.4 with decrease of the inner to outer diameter ratio from 0.8 to 0.2. Measurements [11] of flows with 45 degree swirl and in an annulus of 0.4 radius ratio, showed that tangential shear stress on the outer wall was approximately twice that on the inner wall and scatter in the axial wall shear stress values was such that no conclusion concerning it could be drawn. In a study of swirling flow in a cylindrical channel [82] the axial wall shear stress relative to its value at zero swirl was found to increase up to a value of 2 at 45 degree swirl and to decrease with any further increase in swirl angle.

2.5 Present Investigation

No turbulence measurements have previously been done in the swirling flow along a stationary annular duct. The lack of reliable experimental data for such a flow prevents existing turbulence models from being verified or developed to correspond to the real situation. Therefore further experimental study as well as theoretical approaches are warranted.

The present study, based on experimental measurements of the time-mean velocity components and the turbulent stresses, was initiated to help overcome the deficiency in the existing literature on swirling flow in an annulus. The task was focused on studying the effects of

swirl angle and streamline curvature on the distributions of the axial and tangential velocities, turbulent stresses, wall shear stresses, eddy viscosities and mixing lengths. The validity of some existing turbulence models for this flow type are examined against the new data and recommendations are made.

The main experimental facilities included an annular wind tunnel with hub/tip diameter ratio of 0.625, constant temperature hot-wire anemometer, miniature single-sensor hot-wire probes, 3-hole cobra probe, pressure transducers and an automatic data acquisition and control system. A cascade of inlet guide vanes was designed with constant outlet flow angle; it was installed at the tunnel entrance to provide swirl angles between 0 and 60 degrees.

CHAPTER III

MATHEMATICAL FORMULATION

3.1 The Governing Equations

Consider a cylindrical coordinate system (Fig.3.1) with x , r and θ as the axial, radial, and azimuthal coordinates. Let U , V and W be the time-mean velocities in the x , r and θ directions and u , v and w be the corresponding turbulent fluctuation velocities. Writing the continuity and Navier - Stokes equations in this notation, taking mean values with respect to time, requiring the mean motion to be steady, the mean density, ρ , to be constant, density fluctuations to be negligible, x derivatives of viscous and turbulence stresses to be negligible and symmetry with respect to θ , (i.e. $\partial/\partial \theta = 0$), results in the following equations [83]:

Continuity Equation

$$\frac{\partial}{\partial x}(\rho r U) + \frac{\partial}{\partial r}(\rho r V) = 0 \quad (3.1)$$

Momentum Equations

x - direction

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho r} \frac{\partial}{\partial r} (r \tau_{rx}) \quad (3.2)$$

r - direction

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial r} - \frac{W^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{1}{\rho} \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) - \frac{\tau_{\theta\theta}}{r} \quad (3.3)$$

θ - direction

$$U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial r} + \frac{VW}{r} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\theta}) \quad (3.4)$$

Stress terms in these equations consist of a laminar part, the laminar stresses and a turbulent part, the Reynolds stresses, i.e.

$$\tau_{ij} = \tau_{ij}^l + \tau_{ij}^t \quad (i, j = x, r, \theta)$$

where,

$$\left. \begin{aligned} \tau_{rx} &= \mu \frac{\partial U}{\partial r} - \rho \overline{uv} \\ \tau_{rr} &= -\rho \overline{v^2} \\ \tau_{r\theta} &= \mu \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right) - \rho \overline{vw} \\ \tau_{\theta\theta} &= -\rho \overline{w^2} \end{aligned} \right\} \quad (3.5)$$

Comments on Momentum Equations:

- a - The above equations are parabolized because the axial viscous and turbulent diffusion terms (i.e. the second order derivatives in the axial direction of viscous terms, such as $\mu \partial^2 (U, V, W) / \partial x^2$, are omitted from all momentum equations. Therefore, to solve this set of equations, a marching procedure can be employed. As long as there is no flow recirculation or separation (which is generally

characterized by elliptic equations), the obtained results are quite accurate [3, 4, 10].

- b - The terms $-\partial \bar{u}^2 / \partial x$, $-\partial \bar{u} \bar{v} / \partial x$ and $-\partial \bar{u} \bar{w} / \partial x$ in the right hand side of equations (3.2-4) are neglected due to their small value with respect to the other terms. This assumption that $\partial / \partial x \ll \partial / \partial r$ is confirmed by many previous workers [3, 4, 23].
- c - The terms $1/r \partial(r \tau_{rr}) / \partial r$ and $-\tau_{\theta\theta} / r$ in Equation (3.3) were also neglected by Refs. [3, 4, 23], and retained by Refs. [84, 85]. These terms mainly represent the effect of turbulent and laminar viscosity combined with the curvature effect on the radial pressure distribution. In the present case, the radial velocity component V is found to be negligible, as will be illustrated in Chapter VI, compared to the other velocity components U and W . Consequently, the error percentage in determining the gradient $\partial V / \partial r$ will be considerable. Therefore, the quantities $\mu \partial V / \partial r$ and $\mu V / r$ are ignored in evaluating τ_{rr} and $\tau_{\theta\theta}$ and only terms shown in equation (3.5) are considered in the computation of the radial pressure distribution (see Chapter V).

Obviously, in order to be able to solve equations (3.1-4) some knowledge of the Reynolds stresses is required. Accurate knowledge of Reynolds stresses does not exist due to the complexity of the turbulence, especially in shear flows. Thus, approximate relations, rather crude from the scientific point of view, are used; these however are very useful in solving engineering problems. The additional relations used in order to be able to solve equations (3.1-4) consist of the turbulence models.

Simultaneous solution of these equations by several numerical

techniques may be found in the literature [41-44]. The present study is focused on examining the validity of some turbulence models available in the literature for swirling flow in an annulus. The experimental findings can serve as the basis for recommendations for modifications for turbulence models.

3.2 Turbulent Shear Stresses

As was mentioned before, many models of turbulence have been put forward for calculating turbulent flows. Some are easy to use but are often less generally applicable; others are more complicated but possess wider range of applicability.

In engineering problems, the eddy viscosity approach is the most widely used concept to evaluate the turbulent shear stresses because it can give reasonable results with far less computation time than other approaches. Distributions of various stress components, time-mean velocities and velocity gradients, are deduced from the hot-wire measurements after smoothing the data*. Such distributions allow values of the eddy viscosity components to be determined at all points in the flow field. The eddy viscosity values thus obtained can be also used to check the consistency of the mixing length model and of the most popular 2-equation (k- ϵ) model, discussed before in section 2.3.1.

The axial and tangential components of the turbulent shear stresses, $-\rho \overline{uv}$ and $-\rho \overline{vw}$, in equation (3.5) will be expressed according to the eddy viscosity concept as follows:

$$-\rho \overline{uv} = \mu_{rx} \frac{\partial U}{\partial r} \quad (3.6)$$

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* The method of the least square curve fitting was adopted for this purpose, see Chapter V.

$$-\rho \overline{vW} = \mu_{r\theta} \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right] \quad (3.7)$$

where, μ_{rx} and $\mu_{r\theta}$ are called the eddy or turbulent viscosities in the axial and tangential directions. From the above two relationships and equation (3.5), the shear stresses τ_{rx} and $\tau_{r\theta}$ can be given by:

$$\tau_{rx} = \left(\mu + \mu_{rx} \right) \frac{\partial U}{\partial r} \quad (3.8)$$

$$\tau_{r\theta} = \left(\mu + \mu_{r\theta} \right) \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right] \quad (3.9)$$

The mixing lengths l_{rx} and $l_{r\theta}$ are computed in this study from results for μ_{rx} and $\mu_{r\theta}$ and from the velocity gradients by the following relationship [38, 39, 86]:

$$l_{rx}^2 = \frac{\mu_{rx} / \rho}{\left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{\frac{1}{2}}} \quad (3.10)$$

$$l_{r\theta}^2 = \frac{\mu_{r\theta} / \rho}{\left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{\frac{1}{2}}} \quad (3.11)$$

This type of model satisfies the requirement that μ_{rx} reduces to the usual Prandtl model for a nonswirling flow as W tends to zero. In Chapter VI, results obtained from equations (3.10) and (3.11) are compared with some of the previously proposed relationships for evaluating l_{rx} and $l_{r\theta}$.

More recent work on calculating turbulent flows [21, 24, 43], has postulated that the primary component, μ_{rx} , of eddy viscosity may be described by:

$$\mu_{rx} = C_\mu \rho k^2 / \epsilon \quad (3.12)$$

where k is the kinetic energy of the turbulence and ϵ is the turbulence energy dissipation rate per unit mass. The value of ϵ is computed from the experimental results using the transport equation for k (see Appendix A for more details) as follows:

$$\rho \epsilon = \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu + \frac{\mu_{rx}}{\sigma_k} \right) \frac{\partial k}{\partial r} \right] + G \quad (3.13)$$

G represents the turbulent energy production and is expressed in terms of velocity gradients and the turbulent viscosities as:

$$G = \mu_{rx} \left(\frac{\partial U}{\partial r} \right)^2 + \mu_{r\theta} \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \quad (3.14)$$

The recommended value of the constant σ_k is unity [24, 43].

Extensive research has taken place to adapt the above model to different flow geometries. The coefficient C_μ was considered to be of a constant value and equal to 0.09. In this study the data obtained for μ_{rx} , $\mu_{r\theta}$, k and the velocity gradients are substituted into equations (3.12-14) to determine the value of C_μ that actually prevails at different radii in the flow. In addition, profiles of C_μ at different swirl numbers are presented to exemplify the influence of swirl flow component on this coefficient.

The degree of nonisotropy for turbulent viscosities is designated by determining the viscosity ratio or number $\sigma_{r\theta}$ in the form

$$\sigma_{r\theta} = \frac{\mu_{rx}}{\mu_{r\theta}} = \left(\frac{r_{rx}}{r_{r\theta}} \right)^2 \quad (3.15)$$

Various formulae have been proposed and used in the past for calculating the viscosity ratio $\sigma_{r\theta}$ [15, 21, 23, 39], the task being to link this ratio to a Richardson number. A gradient Richardson number can be defined as [21, 23, 39]:

$$Ri = \frac{2 W \partial(rW)/\partial r}{r^2 \{ (\partial U/\partial r)^2 + [r \partial(W/r)/\partial r]^2 \}} \quad (3.16)$$

Using the experimental data for the eddy viscosities, the radial distributions of $\sigma_{r\theta}$ are calculated from equation (3.15) at selected axial stations. The obtained results are presented and discussed in Chapter VI.

3.3 Approximate Relationship for distribution of turbulent shear stresses

As in conventional boundary layers in mild pressure gradients, the convection terms in the axial momentum equation can be neglected near the walls. Under these conditions, equation (3.2) is simplified to:

$$\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rx}) = \frac{\partial P}{\partial x} \quad (3.17)$$

If $\partial P/\partial x$ is considered to be constant then the above equation will integrate, near inner and outer walls, to:

$$\tau_{rx} = \left\{ \tau_{x,wi} + \left(\frac{dP}{dx} \right)_{ri} \left[y \left(1 + \frac{y}{2r_i} \right) \right] \right\} \frac{1}{(1 + y/r_i)} \quad (3.18)$$

(inner wall)

$$\tau_{rx} = \left\{ \tau_{x,wo} - \left(\frac{dP}{dx} \right)_{ro} \left[y \left(1 - \frac{y}{2r_o} \right) \right] \right\} \frac{1}{(1 - y/r_o)} \quad (3.19)$$

(outer wall)

where y is the distance from the inner or outer wall. The above two equations illustrate the effect of axial pressure gradient as well as of the radius of curvature, convex at inner wall and concave at outer wall, on the radial distribution of axial shear stress. Values of τ_{rx} predicted by them are expected to be in reasonable agreement with the experimental results only in the near-wall regions. Equations (3.18-19) are generally valid for any swirling flow as long as the axial pressure gradient is effectively constant and $\partial U/\partial x$ or $\partial \tau_{rx}/\partial x$ is negligible.

Relationships for the radial distribution of $\tau_{r\theta}$ are deduced from the θ -momentum equation which can be written in another form by utilizing the continuity equation as follows:

$$\frac{1}{\rho r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\theta}) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 VW) + \frac{\partial}{\partial x} (UW) \quad (3.20)$$

Since approximate relations are to be deduced and the value of the velocity V is negligible, the term $(1/r^2) \partial(r^2 VW)/\partial r$ is neglected.

Integrating equation (3.20), near the inner and outer walls, results in:

$$\tau_{r\theta} = \left(\tau_{\theta,wi} + \frac{\eta_1}{r_i^2} \right) \frac{1}{(1 + y/r_i)^2} \quad (3.21)$$

(inner wall)

$$\tau_{r\theta} = \left(\tau_{\theta,wo} - \frac{\eta_0}{r_o^2} \right) \frac{1}{(1 - y/r_o)^2} \quad (3.22)$$

(outer wall)

where,

$$\eta_1 = \frac{\partial}{\partial x} \int_{r_i}^r \rho r^2 UW \, dr \quad (3.23a)$$

$$\eta_0 = \frac{\partial}{\partial x} \int_r^{r_o} \rho r^2 UW \, dr \quad (3.23b)$$

The functions η_0 and η_i are evaluated by numerical integration at each radius. Equations (3.21-23) illustrate the interaction between $\tau_{r\theta}$ and the decay of the velocity component W in the axial direction. Also, comparison of the $\tau_{r\theta}$ and τ_{rx} relationships indicates that the radius of curvature has more impact on $\tau_{r\theta}$ than on the τ_{rx} distribution. Equations (3.21-23) are valid for any axisymmetric swirling flow as long as the radial velocity V is negligible with respect to the main flow velocity components U and W .

3.4 Ratios of Outer to Inner Wall Shear Stresses

Multiplying the axial momentum equation by r and adding the continuity equation to its left hand side as well as executing some calculus procedures will give

$$\frac{\partial}{\partial x} (r U^2) + \frac{\partial}{\partial r} (r UV) = -\frac{r}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial r} (r \tau_{rx})$$

Integrating the above equation between the limits r_i and r_o results in:

$$\frac{r}{\rho} \tau_{rx} \Big|_{r_i}^{r_o} = \int_{r_i}^{r_o} \left[\frac{\partial}{\partial x} (r U^2 + \frac{rP}{\rho}) dr \right] + r VU \Big|_{r_i}^{r_o} \quad (3.24a)$$

The quantity $rVU \Big|_{r_i}^{r_o}$ is equal to zero at both radii r_i and r_o . The final relationship between outer and inner axial wall shear stresses

$\tau_{x,wi}$ and $\tau_{x,wo}$ is described by:

$$\frac{\tau_{x,wo}}{\tau_{x,wi}} = \frac{r_i}{r_o} \left[1 + \frac{\rho}{r_i \tau_{x,wi}} \frac{\partial}{\partial x} \int_{r_i}^{r_o} \left(U^2 + \frac{P}{\rho} \right) r dr \right] \quad (3.24b)$$

In case of fully developed flow ($\partial/\partial x \propto 0$) between two parallel plates

($r_o \rightarrow 0$ and $r_i \rightarrow \infty$) the two wall shear stresses will be equal. For non-swirling fully developed flow in an annulus, a decrease in the radius ratio r_i/r_o will decrease the wall shear stress ratio $\tau_{\theta,wo}/\tau_{\theta,wi}$ and vice versa [74, 81]. The influence of swirl number on wall shear stress ratio is presented in Chapter VI.

The ratio of outer to inner tangential wall shear stresses is obtained by integrating equation (3.20) between the limits r_i and r_o which gives the form:

$$r_o^2 \tau_{\theta,wo} - r_i^2 \tau_{\theta,wi} = \rho r^2 VW \Big|_{r_i}^{r_o} + \frac{\partial}{\partial x} \int_{r_i}^{r_o} \rho r^2 UW dr \quad (3.25a)$$

The quantity $r^2 VW \Big|_{r_i}^{r_o}$ is equal to zero at both radii r_i and r_o and the above equation reduces to:

$$\frac{\tau_{\theta,wo}}{\tau_{\theta,wi}} = \left(\frac{r_i}{r_o} \right)^2 \left[1 + \frac{\eta_{io}}{r_i^2 \tau_{\theta,wi}} \right] \quad (3.25b)$$

where,

$$\eta_{io} = \frac{\partial}{\partial x} \int_{r_i}^{r_o} \rho r^2 UW dr$$

The function η_{io} is determined by numerical integration at each axial station.

3.5 Logarithmic law of the wall for swirling flows

As mentioned in the literature review, the validity of the universal law of the wall defined by equation (2.4) for the velocity components U and W is questionable, because the effect of the swirl on

the eddy viscosity components cannot be ignored. Also, swirling flows are generally associated with curved surfaces and the type of curvature (convex or concave) has a considerable effect on the turbulence structure. The following analysis introduces two new versions for the logarithmic law of the wall for the distribution of the velocity components U and W in the log-law region for swirling flow cases. Also the validity of the universal law of the wall for the resultant velocity, Q , is discussed.

Outside the viscous sublayer, in the log-law region, the laminar viscosity is neglected and the mixing lengths (l_{rx} and $l_{r\theta}$) are assumed to be given by the following relationships proposed by Galbraith et al. [40]:

$$l_{rx} = Ky \sqrt{\tau_{rx} / \tau_{x,w}} \quad \text{and} \quad l_{r\theta} = Ky \sqrt{\tau_{r\theta} / \tau_{\theta,w}} \quad (3.26)$$

where K is the von-Karman constant and y is the distance from the wall surface. Substitution of equation (3.26) into equations (3.10-11) implies that the eddy viscosity components μ_{rx} and $\mu_{r\theta}$ are not isotropic and are expressed by

$$\mu_{rx} = K^2 y^2 \rho \left(\frac{\tau_{rx}}{\tau_{x,w}} \right) \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{1/2} \quad (3.27a)$$

$$\mu_{r\theta} = K^2 y^2 \rho \left(\frac{\tau_{r\theta}}{\tau_{\theta,w}} \right) \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{1/2} \quad (3.27b)$$

In general, for flows over a concave wall the gradients $\partial U / \partial r$ and $r \partial (W/r) / \partial r$ included in the above equation increase with increasing swirl and therefore, values of μ_{rx} and $\mu_{r\theta}$ are expected to be higher

than in non-swirling flows. However, for flows over a convex wall the reverse occurs, and a negligible increase or a decrease in μ_{rx} and μ_{re} would be expected with increasing of swirl. As will be illustrated in Chapter VI, calculated values for l_{rx} , l_{re} , μ_{rx} , and μ_{re} from equations (3.8-11) show that the two relationships (3.26-27) are valid in the log-region with good accuracy for the present flow case. By substituting the relationship (3.27) for μ_{rx} or μ_{re} into equations (3.8-9) and neglecting the laminar viscosity in the log-region, the wall shear stresses $\tau_{x,w}$ and $\tau_{\theta,w}$ are found to be given by:

$$\tau_{x,w} = K^2 y^2 \rho \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{\frac{1}{2}} \frac{\partial U}{\partial r} \quad (3.28)$$

$$\tau_{\theta,w} = K^2 y^2 \rho \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{\frac{1}{2}} \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right] \quad (3.29)$$

The resultant wall shear stress τ_w can be computed from

$$\tau_w = \sqrt{\tau_{x,w}^2 + \tau_{\theta,w}^2} \quad (3.30)$$

and can also be expressed in terms of the velocity gradients and the distance y by

$$\tau_w = K^2 y^2 \rho \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\} \quad (3.31)$$

substitution of equation (3.31) into equations (3.28-29) gives a simpler expression for $\tau_{x,w}$ and $\tau_{\theta,w}$ as follows:

$$\tau_{x,w} = K y \rho \left(\frac{\tau_w}{\rho} \right)^{\frac{1}{2}} \frac{\partial U}{\partial r} \quad (3.32)$$

$$\tau_{\theta,w} = K y \rho \left(\frac{\tau_w}{\rho} \right)^{\frac{1}{2}} \left[r \frac{\partial}{\partial r} \left(\frac{w}{r} \right) \right] \quad (3.33)$$

If the ratios $\tau_{rx} / \tau_{x,w}$ and $\tau_{r\theta} / \tau_{\theta,w}$ are considered unity then equation (3.26) will reduce to Prandtl hypothesis for the mixing length (i.e. $l_{rx} = l_{r\theta} = Ky$). Also the wall stresses $\tau_{x,w}$, $\tau_{\theta,w}$ and τ_w in equation (3.26) will reduce to the Prandtl's hypothesis for the mixing length (i.e. $l_{rx} = l_{r\theta} = Ky$). Also the wall stresses $\tau_{x,w}$, $\tau_{\theta,w}$ and τ_w in equations (3.23-33) will be replaced, after performing the previous procedures for the derivation, by their local values τ_{rx} , $\tau_{r\theta}$ and τ , respectively. Such an approach was applied by van der Berg [71] for developing a logarithmic law of the wall for three dimensional flow when the deviations from the universal law of the wall are small (i.e. τ_{rx} and $\tau_{r\theta}$ are effectively constant across the log-region). However, Galbraith et al. [40] concluded that unless the shear stress is substantially constant through the wall region, the mixing length must be defined by their proposed relation (i.e. equation 3.26) if a logarithmic distribution is to be obtained. In the present study, τ_{rx} and $\tau_{r\theta}$ were found to be non-constant in the near-wall regions; their behaviour is similar to that found in pipe flow, see Chapter VI. Therefore, the use of equation (3.26) to define the mixing lengths has eliminated the necessity for assuming a wall layer of constant shear stress distribution in order to derive a valid logarithmic wall-law. Since $\tau_{x,w}$, $\tau_{\theta,w}$ and τ_w are constants at each axial station, equations (3.32-33) can be rewritten in the form:

$$\tau_{\theta}^* = 2K y \left(\frac{\partial u}{\partial r} \right) \quad (3.34)$$

$$C_w^* = \pm K y \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right] \quad (3.35)$$

where,

$$C_u^* = U^* \sqrt{\frac{\tau_{x,w}}{\tau_w}} = \frac{U^{*2}}{Q^*} \quad \text{and} \quad C_w^* = W^* \sqrt{\frac{\tau_{\theta,w}}{\tau_w}} = \frac{W^{*2}}{Q^*} \quad (3.36)$$

and U^* , W^* and Q^* are the frictional velocities in the axial, tangential and resultant flow directions, respectively; their values are calculated from:

$$\left. \begin{aligned} U^* &= \left(\frac{\tau_{x,w}}{\rho} \right)^{1/2} \\ W^* &= \left(\frac{\tau_{\theta,w}}{\rho} \right)^{1/2} \\ Q^* &= \left(\frac{\tau_w}{\rho} \right)^{1/2} \end{aligned} \right\} \quad (3.37)$$

To derive a logarithmic law for the distribution of the velocity components U and W , in the log-region, the following non-dimensional terms U^+ , W^+ , y_1^+ , y_2^+ and r^+ are defined by:

$$\left. \begin{aligned} U^+ &= \frac{U}{C_u^*}, \quad W^+ = \frac{W}{C_w^*}, \quad y_1^+ = \frac{C_u^* y}{\nu}, \\ y_2^+ &= \frac{C_w^* y}{\nu} \quad \text{and} \quad r^+ = \frac{C_w^* r}{\nu} \end{aligned} \right\} \quad (3.38)$$

The above relationships can be used to rewrite equations (3.34-35) in the following non-dimensional forms:

$$\frac{dU^+}{dy_1^+} = \pm \frac{1}{K y_1^+} \quad (3.39)$$

$$\frac{d}{dy_2^+} \left(\frac{W^+}{r^+} \right) = \pm \frac{1}{K r^+ y_2^+} \quad (3.40)$$

3.5.1 Law of the wall for the axial velocity component

Integration of equation (3.39) gives the final relationship for the distribution of U in the log-region.

$$U^+ = A \ln y_1^+ + B \quad (3.41)$$

where $A = 1/K$ and B is a constant. Different values for K and B can be found in the literature. The most commonly used ones are $K=0.4$ and $B = 5.5$ [7, 17, 38, 72], whereas $K = 0.41$ and $B = 4.9$ were preferred by others [12-14]. In the present study, best fitting of equation (3.41) for the U component was obtained by $K = 0.4$ and $B = 5.5$ as will be illustrated in Chapter VI.

Although the above derived form of the log-law relationship for U appears the same as the well-known universal log-law, it differs from the latter in that the parameters U^+ and y_1^+ are defined by using U^2/Q^* or $U^* \sqrt{\tau_{x,w}/\tau_w}$ rather than U^* as in the conventional log-law. Therefore, the use of the Clauser chart for the U velocity component in the case of swirling flows leads to determination of the value of U^2/Q^* or $U^* \sqrt{\tau_{x,w}/\tau_w}$ and not of U^* .

Equation (3.41) indicates that the wall curvature does not induce any direct effect on the axial velocity distribution near the walls. The effect of swirl appears, however, in the newly introduced parameter $U^* \sqrt{\tau_{x,w}/\tau_w}$, which reduces to U^* for non-swirling flows. This equation

is generally applicable to the axial velocity component U of any swirling flow as long as the pressure gradient $\partial P / \partial x$ and/or $\partial \tau_{rx} / \partial x$ and $\partial \tau / \partial x$ are not too large.

3.5.2 Law of the wall for the tangential velocity

Another logarithmic law can be derived for the tangential velocity W by integrating equation (3.40) for the inner and outer wall layers, the formulae obtained are

$$W^+ = A \left(1 + \frac{y_2^+}{r_i^+} \right) \ln \left[y_2^+ / \left(1 + \frac{y_2^+}{r_i^+} \right) \right] + B \quad (3.42)$$

(inner wall)

$$W^+ = A \left(1 - \frac{y_2^+}{r_o^+} \right) \ln \left[y_2^+ / \left(1 - \frac{y_2^+}{r_o^+} \right) \right] + B \quad (3.43)$$

(outer wall)

where, $A = 1/K$, $r_i^+ = C_w^* r_i / \nu$ and $r_o^+ = C_w^* r_o / \nu$. If the radii of curvature r_i and r_o are large (case of three-dimensional flows over mildly curved surfaces or plane surfaces, $y_2^+ / r_i^+ \ll 1$ and $y_2^+ / r_o^+ \ll 1$) and then both equations (3.42-43) will reduce to

$$W^+ = A \ln y_2^+ + B \quad (3.44)$$

which in its turn reduces to the universal law of the wall in the case of a two-dimensional flow over plane surfaces if the constant B is taken to be equal to the standard value of 5.5. Values of $K = 0.4$ and $B = 5.5$ were found to provide the best fit for the experimental data for W as will be shown in Chapter VI. The derived logarithmic law is applicable to the velocity W in any swirling flow and to two dimensional flows over convex or concave surfaces as long as the pressure gradient $\partial P / \partial x$ and/or

$\partial \tau_{\theta\theta} / \partial r$ and $\partial \tau / \partial \theta$ are not too large.

The newly derived log-laws of equations (3.42-43) include the radius of wall curvature as a parameter and indicate that the distribution of the velocity W near the inner and outer walls will be different even if the shear stresses $\tau_{\theta, w}$ and τ_w are equal on both walls. Moreover, these two equations disclose the opposite effects of concave and convex curvature. The effect of swirl is implicitly included in these equations by the use of W^2/Q^* or $W^* \sqrt{\tau_{\theta, w}/\tau_w}$ rather than W^* or Q^* as the velocity scale in defining W^+ , y_2^+ , r_i^+ and r_o^+ . This latter concept implies that the application of the Clauser chart technique with the velocity W , in the case of swirling flows, yields the value of W^2/Q^* or $W^* \sqrt{\tau_{\theta, w}/\tau_w}$ and not W^* , see Chapter V.

The foregoing outlines the main differences between the newly derived log-laws and the universal law of the wall. In general, these differences increase with decreasing radius of curvature and increasing swirl.

3.5.3 Validity of the universal law of the wall for the resultant flow velocity

To examine the validity of the universal law of the wall for the resultant flow velocity, Q , recall equation (3.31) which relates the resultant wall shear stress τ_w to the gradient of Q .

$$\tau_w = K^2 y^2 \rho \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\} \quad (3.45)$$

To transform the above equation into a useful log-law relationship, the gradients of U and W should be re-expressed in terms of Q . This may be

achieved by using the following relations

$$U = Q \cos \phi \quad \text{and} \quad W = Q \sin \phi \quad (3.46)$$

where ϕ is the angle between the flow direction and the x-axis.

Differentiation of the above relations and substitution for the gradients of U and W in equation (3.45) gives the following result:

$$\tau_w = K^2 y^2 \rho \left[\left(\frac{\partial Q}{\partial r} \right)^2 - I \right] \quad (3.47)$$

where,

$$I = \frac{Q}{r} \sin^2 \phi \left(2 \frac{\partial Q}{\partial r} - \frac{Q}{r} \right) + Q^2 \frac{Q}{r} \left(\frac{\sin 2\phi}{r} - \frac{\partial \phi}{\partial r} \right) \quad (3.48)$$

If the quantity $[I / (\partial Q / \partial r)^2] \ll 1$, then the term I would be negligible with respect to $(\partial Q / \partial r)^2$ and in this case only, equation (3.47) can be integrated to give the universal law of the wall. This is true only for weakly swirling flows ($\phi < 15^\circ$) with small $\partial \phi / \partial r$, and as the degree of swirl increases, the difference between equation (3.47) and the universal law of the wall increases. This result is expected since the profiles of the velocity components U and W, especially W, do not follow the conventional form of the universal law of the wall.

Application of the equations derived in this chapter to the present experimental results is considered in Chapter VI. The effect of the swirl on the different turbulent stress components is also considered there. Deduction of the various flow field and turbulence parameters from the experimental data is described in Chapter V.

CHAPTER IV

APPARATUS AND EXPERIMENTAL METHODS

As the result of the literature review concerning swirling flow in an annulus, it was decided that the most useful contribution would be further experimental measurements of swirling flow. An annular wind tunnel described by Sjolander [87], was used for this work, modified as described below. The test rig consisted of the annular wind tunnel, with the air admitted to the test section through a row of inlet guide vanes (IGV's) which were adjustable to give different swirl angles. A 3-hole cobra-type pressure probe and slanted and normal hot-wire probes (constant temperature anemometer) were used to measure the three time-mean velocity components, the six turbulent stresses, wall shear stress and pressure distribution in the flow for various swirl angles. The test rig, instrumentation and experimental methods are described in more detail in the following subsections.

4.1 Test Rig

4.1.1 Annular Wind Tunnel

The annular wind tunnel was originally designed to study swirling flow in annuli [88]. It had been modified [87] to study the endwall boundary layer in an annular cascade of turbine nozzle guide vanes. Fig. 4.1 shows a cross-sectional drawing of the annular wind tunnel in the configuration used for the present experiments. The flow through the tunnel is produced by a Budworth two-stage axial flow compressor; this is driven by a variable speed DC-motor and can supply flow rates

up to about 4 Kg/sec with a maximum total pressure rise of about 5 KPa (50 cm H₂O). The tunnel is also instrumented with a thermocouple to measure the supplied air temperature. A more detailed description of the tunnel is provided in Appendix B.

4.1.2 Test Section

The test section is a constant area annular duct fabricated of mild steel. The cylindrical outer wall consists of five segments, each of 0.4 m in diameter and length. The segments were accurately aligned such that there were no significant discontinuities in the surface at the joints. The inner wall of the annulus was 0.254 m diameter, 1.8 m long and the overall length of the test section was 2 meters or 13 hydraulic diameters. A set of straightening vanes was installed at the outlet of the test section so that the flow was discharged into the room with negligible swirl. This arrangement allowed recovery of the dynamic pressure associated with the tangential velocity component and avoided mismatch of radial pressure gradient at outlet. The test section shown in Fig. 4.2 was attached to the settling chamber by a tongue and groove-type flange, and held in place by a quick-release clamp; it was supported on a wheeled dolly to facilitate removal and modification.

The outer cylinder wall is instrumented along a single axial ray with 18 static pressure taps of 0.5 mm diameter. The first pressure tap was located 5 cm downstream of the IGV's trailing edge while the remainder were uniformly distributed, being 10.1 cm apart. To check the axisymmetry of the wall static pressure, 8 static taps distributed uniformly around the circumference were installed at each of the first four axial static tap locations. Also, the outer wall was fitted with

18 instrumentation ports of 6.3 mm diameter along a single axial ray at the same axial stations as the static taps. These ports were used for radial traverses and could be sealed off when not in use. Only five of these ports were actually used in the present experiments and their locations are given in table 4.1. The first port used for traverses was 25.4 cm downstream of the trailing edge of the IGV's to assure that the IGV's wakes had dissipated before the first traverse plane. At each axial station seventeen radial points were surveyed with the majority concentrated close to outer and inner walls.

The inner cylinder wall was provided along a single axial ray with five static pressure taps of 0.9 mm diameter. They were in the same vertical plane as the five instrumentation ports used in the experiments.

A special attachment shown in Fig. 4.3 and made of plexiglass was used to assure tight sealing around the stem of the instrumentation probes. It also served as a protecting device for passing the hot-wire probes through the measuring ports in the outer wall of the test section.

4.1.3 Inlet Guide Vanes (IGV's)

As already mentioned, the swirl was introduced by a set of inlet guide vanes. The vanes were designed with constant outlet angle because untwisted blades are easier to manufacture. Since the resultant velocity increased across the vanes there was a favourable overall pressure gradient and they were relatively free of aerodynamic problems. The undisturbed flow upstream of the guide vanes had uniform axial velocity and zero tangential velocity. Since the annulus was

stationary and there was no energy exchange the total pressure of an incompressible flow would remain constant after the IGV's if losses due to fluid friction and wall boundaries are neglected [89]. However, there is the disadvantage that the magnitude of the resultant flow velocity varies from hub to tip with the highest velocity at the hub.

The cascade of IGV's consisted of 26 vanes distributed uniformly around the circumference. The vane thickness and the trailing edge thickness were kept as small as possible to minimize the vane wakes. Each vane was of 61.9 mm chord length, 5.55 mm maximum camber and 5.25 mm maximum thickness. Details of the cascade design are described in Appendix C. The guide vanes, Fig. 4.4, could be restaggered to obtain different swirl angles from 0 to 60 degrees. The vanes are hollow and have a slotted trailing edge (0.7 mm air gap) to permit ejection of a jet of air of uniform velocity over the vane height to compensate for the momentum deficit in the vane wakes. A Roots blower capable of delivering flow rates up to 0.1 Kg/Sec at total pressures up to 31 cm mercury gage was used to produce the air jets from the trailing edges of the vanes. The air from the blower was distributed to the vanes via a distributing chamber of 6.5 litre volume, see Fig. 4.5. The chamber pressure was calibrated to indicate directly the velocity of the air jet issuing from the trailing edges of the vanes.

In order to derive a simple relationship between the required air jet velocity V_j and the resultant flow velocity Q , for each swirl angle, the flat plate relations for zero pressure gradient were used to estimate the wake momentum deficit on both surfaces of the blade. Details of this derivation are illustrated in Appendix C and the final result is given by:

$$\bar{V}_j = \bar{Q} \sqrt{6.34 \text{Re}_l^{-1/5} + 1.36} \quad (4.1)$$

where Re_l is the Reynolds number at the blade trailing edge, $\bar{Q}/\nu$ and l is the vane chord length. For every swirl angle, $\bar{Q}$ was taken to be the mean of the resultant flow velocity over the annular height and at the first axial station, its value was calculated from

$$\bar{Q} = \frac{\int_{r_i}^{r_o} Q U r dr}{\int_{r_i}^{r_o} U r dr} \quad (4.2)$$

Values of Re_l varied from 1.24×10^5 to 1.75×10^5 depending on the value of $\bar{Q}$ and the temperature of the air supplied by the air compressor.

4.1.4 Wind Tunnel Operating Point,

The outer wall of the wind tunnel contraction was instrumented with two static pressure taps, one near the start and the other near the end of the contraction; the pressure difference between these two taps was directly related⁺ to the test section dynamic pressure and was therefore used to monitor the tunnel operating point. The pressure difference was measured with a pressure transducer and displayed continuously on a digital voltmeter. The operating point had to be chosen in accordance with the maximum achievable jet velocity of the air injected at the trailing edges of the IGV's. Also, the same Reynolds number, based on the axial flow velocity, had to be achieved for the flow through the annulus at different swirl angles. These factors were

⁺ To calibrate the tunnel, the vanes were removed (i.e. non-swirling flow case) and the three-hole cobra probe (see Fig. 4.7) was traversed at station # 0 (see Fig. 4.2) to measure the axial velocity profile. Various air mass flow rates ($\dot{m}$) were calculated using equation (5.20) and the results were related to the corresponding dynamic pressures of the test section.

considered when choosing the tunnel operating point. All experiments were conducted at the same nominal mass flow rate of 3.1 Kg/sec through the working section; this corresponds to a mean axial velocity of 32.6 m/sec and a Reynolds number equal to 3.11×10^5 based on the mean axial velocity and the hydraulic diameter of the annulus. These operating conditions represent the values under which the air compressor could provide the same mean axial velocity at all swirl angles without approaching surge at the highest swirl angle.

The major goal of the present study was to investigate the effect of the swirl on flow field parameters and therefore variation of the Reynolds number was not considered essential. Sudarev [19] in his aerodynamic study of a swirling flow in an annulus concluded that with a change in axial air velocity from 30 to 120 m/sec the radial distributions of the axial and tangential velocities relative to their maximum values are independent of the Reynolds number (i.e. non-dimensionalized experimental points were well grouped around a single curve).

To change the swirl velocity component imparted to the flow while maintaining the same mass flow rate, the inlet guide vanes were restaggered to provide nominal exit flow angles of 15, 30 and 45 degrees to the axial direction. For the measurements on the straight flow (zero swirl) case the inlet guide vanes were removed because the vane passages become of diffusive nature for outlet angles less than 8° .

Since the outlet flow angle at the cascade exit always deviates from its design value, especially in the vicinity of the wall surface, it is preferable to describe the degree of swirl of a certain flow by a dimensionless number. The most common dimensionless number used in the literature is the swirl number SN , defined by [10, 21, 50].

$$SN = \frac{\int_{r_i}^{r_o} U W r^2 dr}{r_o \int_{r_i}^{r_o} U^2 r dr} \quad (4.3)$$

where U and W are the time-mean velocity components of the flow in the axial and the tangential directions. Due to the decay of the swirl along the tunnel axis, SN will have higher values at the upstream sections than at the downstream portions of the flow. Therefore, calculated values of SN at the first measuring section were considered to be representative of the degree of the swirl of the flow. In the present experiments SN assumed values of 0, 0.23, 0.47 and 0.87 corresponding to nominal exit flow angles of 0, 15, 30 and 45 degrees respectively.

4.2 Traverse Gear, Pressure Probes and Transducers

As mentioned earlier, measuring probes were traversed radially at five axial stations along the test section. The following subsections describe the traverse gear and the different probes used to sense the local flow parameters.

4.2.1 Traverse Gear

The mechanism shown in Fig. 4.6 was used to traverse cobra and hot-wire probes and had a radial stroke length of 8 cm. The probes could be rotated a full 360 deg.; the rotation angle was determined to within 0.1 deg., and the radial backlash was 0.15 mm. Backlash effects, however, were negligible because the probes were always traversed in one direction, from the outer to the inner wall. At each axial station, measurements were taken at seventeen radial positions with the majority near the outer and inner walls; the distance between every two successive

points was 1.59 mm or a multiple of it. Radial location was determined within 0.03 mm and misalignment of the traverse gear axis with the radial direction of the test section was found to be less than 1° .

The datum position of the hot-wire probe for all radial traverses was established at the entrance section of the annulus, see Fig. 4.2. A shorting probe (DISA 55H30) and a set of thin metal strips (0.05 to 1.6 mm thickness) were used to set up the positioning attachment (two stopping pins were fixed to its outer wall) connected at the end of the probe holder (see Fig. 4.11a) such that when the shorting probe was replaced with the hot-wire probe, the centre of the latter would be at 3.2 mm from the inner wall. This procedure required measurements of the lengths of the hot-wire and shorting probes; these were performed using a microscope equipped with a vernier scale of 0.1 mm readability. Therefore the accuracy of the datum setting was within 0.15 mm. The foregoing procedures were repeated, without using the shorting probe, to establish the datum position of the 3-hole probe.

4.2.2 3-Hole Cobra Probe

The 3-hole cobra probe is shown in Fig. 4.7. This probe was mainly used to measure the radial distribution of the total pressure at each axial location. It was also used as a Preston tube to measure the wall shear stress at the inner wall surface. The probe stem is 3.2 mm in diameter and its tip consists of three tubes welded together, each tube is 0.73 mm OD and 0.44 mm ID and the two side tubes have 30° chamfer. This type of probe, having a short upstream projection (2.3 mm) was required to enable it to pass through the instrumentation ports in the outer wall of the annular test section. The proximity of the probe stem

to the tube openings (tube length/diameter ratio is 3) could cause interference effects in regions of large total pressure gradient by causing radial deflection of flow streamlines. Therefore, data from such regions must be interpreted with caution.

The probe sensitivity to yaw of 10° (see Fig. 4.8 for definition) was checked at three different air velocities of 22, 30 and 41 m/s; the corresponding error in the indicated total pressure was found to be 0.62, 1.08 and 1.5 per cent respectively. This variation is due to the velocity component perpendicular to the flow direction. In general if the probe is nulled, the centre tube would be insensitive to misalignments up to 5° between resultant velocity vector and the tube axis at its mouth (i.e. 5° yaw, Ref. 90). Also, similarity in the chamfer of the two side tubes of the probe was checked at the same three air velocities. The difference between the maximum pressure head indicated by the middle tube at the nulling position of the probe (when the probe stem was rotated 180° in a plane perpendicular to the flow direction such that each of its side tubes was in the place of the other) was found to be 1.2% at the most. These spot checks were conducted by using the experimental set-up described in section 4.4 which was mainly used for calibrating the hot-wire probes.

4.2.3 Pressure Transducers

Three capacitance type pressure transducers (Rosemount Inc., Minneapolis, Minnesota, Model #831A3) were used to transduce the pressures sensed by pressure probes or wall taps. Their working pressure range is 2.58 cm of Mercury above or below the reference pressure (atmospheric) and an external power source of 28 volts is

required for their operation. The output voltage for each of them was first calibrated against a manometer of adjustable inclination angle (between 2.3° and 90°) and containing fluid of specific gravity of 0.8. The sensitivity of these transducers was about 0.12 mm of water per 1 mv output voltage; the latter was measured with 1 μ v accuracy. Small drifts (less than 3%) were found when the transducers were recalibrated occasionally during the period of conducting the experiments.

4.3 Automatic Data Acquisition and Control System (ADACS)

The Hewlett Packard Model 3054A is a computer based system and combines precision measurements and control capability. The ADACS system provided a semi-automatic centralized data gathering and processing station for the present study. It also allowed monitoring and checking of real time data inputs received from the measuring probes. Visual indicators were installed in a special electronic circuit connected to the system (see sub-section 4.5.5) to inform the operator of any changes and to control the procedures of running the experiments. The main components of the ADACS system are briefly described below; for details see the operating manuals for the different components of the system [91].

4.3.1 System Computer

The HP-85A Programmable Desktop Computer is the System Computer. The computer is interfaced with the System instruments via the 82937A HP-IB I/O (input/output) Card. The HP-85A provides easy programmability with the use of an enhanced form of BASIC. Additional features of the HP-85A include a CRT, an internal printer, live keyboard, buffered I/O,

priority interrupt, high speed tape cartridge, auto-start and error trapping.

4.3.2 Digital Voltmeter

The 3456A Digital Voltmeter is a $3\frac{1}{2}$ to $6\frac{1}{2}$ digit integrating voltmeter. Its DC and RMS AC voltage maximum of 1000 volts and resistance measurement capability reduce the amount of signal conditioning necessary. The 3456A has selectable integration times from 0.01 to 100 power line cycles and offers guarding* to give higher accuracy in the presence of noise and common-mode voltages. Also, the 3456A offers two other ways of eliminating noise: an analog input filter and digital averaging. The other high speed digital voltmeter (3437A) was not used because the 3456A was more suitable for this experimental work.

4.3.3 Data Acquisition/Control Unit

The 3497A Data Acquisition/Control unit combines precision measurement capability with control functions. Standard equipment for the 3497A includes a built-in real time clock, and front panel display controls. Signals received from the measuring instruments were input to the system through three channels of a 20 channel input relay assembly (multiplexer card). Also only two channels of the 16 channel Mercury relay actuator/digital output (actuator card) were connected to the external control circuit that will be described in subsection 4.5.5.

The system package also included various application computer.

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* Coaxial cables were used to connect the different equipment with the ADACS system. The low-voltage terminal of each equipment was a common terminal for the guard and for the low-voltage terminals of a certain channel of the multiplexer input card.

programs and subprograms; few of them were used in the present study. The 3054A system components are inter-connected by the Hewlett-Packard Interface bus (HP-IB). An example of the communication linkage between the system components with HP-IB and the measuring instruments is illustrated later.

The ADACS system provided features such as on-line data deduction, precision low-level measurements, high speed data/program storage and access, sampling and time averaging, and above all substantial time savings. Without this system, the measurements with the hot-wire probe would have been excessively time consuming and thus impractical.

4.4 Hot-wire Anemometer

A constant temperature anemometer, Thermo-Systems Inc. Model TSI 1053A, was used to perform the hot-wire measurements of the present study. This type of anemometer operates by utilizing a feedback controlled Wheatstone bridge circuit to maintain the sensor at a constant temperature in a recommended operating range between 150° and 260 °C. The sensor wire constitutes one arm of the bridge and the other three arms are constant resistances. Any variation in the sensor resistance due to changes in the fluid velocity over the sensor is immediately corrected by automatic re-adjustment of the current through the sensor. The output of the anemometer is the voltage required to drive the necessary current through the sensor to maintain its temperature at a constant value.

Details of the anemometer operation and electronic circuitry are described in the operating and service manual [92]. The operational

procedures outlined in that manual should be followed with extreme care and full understanding to obtain results of good accuracy. A few of these operational procedures are discussed below.

Selection of sensor operating temperature is not only limited by the mechanical properties of the sensor material but also by nonlinear effects in the sensor response curve occurring at high operating temperature. However, high operating temperature improves the sensor response to fluctuations of the flow velocity. A value of 1.8 for the sensor overheat ratio* was used as recommended in TSI anemometer manual for tungsten wires; this corresponds to 230 C° sensor operating temperature. Also, the response of the anemometer circuit over a frequency range 0 to 20 KHz was optimized by using a square wave generator built into the anemometer unit, while the hot-wire was exposed to a jet velocity equivalent to the maximum operating air velocity in the annular test section. Coaxial cables were used to connect different instrumentation units to minimize the effects of noise.

The two types of hot-wire probes used in the present measurements were DISA miniature probes, the 55P11 normal probe and 55P12 45° slant probe. The slant angle of the latter when checked by a shadow-graph comparator was found to be very accurate (less than 0.2° difference). Therefore, the value of 45° as specified by the probe manufacturer was used in the equations for data reduction presented in Chapter V. The probe sensors are tungsten wires of 5 μ m diameter and 1.25 mm length; this provides an equivalent sensor length to diameter ratio of 250. The different equipment used for measurements with hot-wire probes are

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* This is the ratio between the sensor resistance at a selected sensor operating temperature and its resistance at ambient temperature of 20° C.

listed in table 4.2. Also Fig. 4.9 shows the arrangement of the hot-wire anemometer and the different components of the ADACS system. The instantaneous output voltage of the anemometer bridge is referred to as E_{br} , the time-mean or DC signal as $\bar{E}_{br}$ and the root-mean square of the signal as $\sqrt{e_{br}^2}$. The relationship between these three signals is given by [45, 62].

$$E_{br} = \bar{E}_{br} + e_{br} \quad (4.4a)$$

and

$$\sqrt{e_{br}^2} = \{ (E_{br} - \bar{E}_{br})^2 \}^{1/2} \quad (4.4b)$$

where e_{br} is the fluctuating component of the anemometer output voltage E_{br} . The $\sqrt{e_{br}^2}$ signal was measured by a true RMS voltmeter (Bruel & Kjaer 2425)* set at a 3 sec time constant. The outputs from this meter and the anemometer bridge were connected to two analog channels of the multiplexer assembly card of the ADACS system. Values of $\bar{E}_{br}$ and $\sqrt{e_{br}^2}$ recorded by the HP3456A DVM represent the average of 100 instantaneous readings scanned at a sampling rate of 6 readings/sec.

Fig. 4.10 shows the position of the hot-wire probe relative to the velocity components of the flow, U, V and W. Such a configuration was

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* The B & K voltmeter measures the RMS through measuring the amount of power dissipated from the AC component of the input voltage in linear electronic systems, see Ref. 93. Therefore, accuracy of the indicated RMS value would be less influenced by the frequency and complexity of the wave form of the input voltage than if the RMS is determined by mathematical operations which is the case with the HP 3456A voltmeter.

used in several previous studies [13, 14, 49, 52, 59, 94] of internal flows. In the present work the probe stem was normal to the nominal flow direction rather than aligned with it. This arrangement was, of course, necessitated by the need to introduce the probe through the small ports in the outer wall of the test section and to allow rotation of the probe to achieve the several sensor orientations at each measurement point. Interference effects from the prongs which support the hot-wire sensor are included in the full three-dimensional probe calibration which was necessary in any case for the present measurements. In portions of the flow where radial gradients are large these interference effects may differ somewhat from those in the uniform calibration flow; and less accuracy may also be expected in measurements performed by the 45° slant probe, since the sensor then spans a finite radial distance. However, the fact that the probe sensor is only 1.25 mm in length will minimize the effects of gradients along the sensor length. Also measurements were stopped at a distance of 3.2 mm from the inner and outer walls to avoid the steep gradients and the induced cooling effects that would have affected the hot-wire sensor in the vicinity of the wall. Analysis of errors due to steep velocity gradients near the wall is discussed in Appendix H. This analysis is based on experimental data published by Gessner and Moller [48].

4.5 Measurement Methods

As mentioned earlier, measurements were taken at seventeen radial locations at each of the five axial stations. First and last measurements were taken at distances of 3.2 mm from inner and outer wall surfaces. The instantaneous velocity components of the flow U' , V' and

W' in the axial, x , radial, r or y , and tangential, θ or z , directions, respectively, can be broken into two parts, a time-average or mean part and a fluctuating part; that is

$$U' = U + u, \quad V' = V + v \quad \text{and} \quad W' = W + w \quad (4.5)$$

where upper and lower case symbols denote time average and fluctuating quantities, respectively.

Table 4.3 summarizes the parameters of interest measured at the five traverse stations for swirl numbers 0, 0.23, 0.47 and 0.87. The pressure transducers described in sub-section 4.2.3 were used to transduce the pressure values sensed by probes or wall taps. The air mass flow rate through the annular test section was monitored using one of the transducers while running the experiments. Different computer programs compatible with the ADACS system were designed to calibrate the hot-wire probes and to monitor, record and process the experimental results. The following subsections outline the methods used for measuring the various flow parameters and the approaches used for determining related parameters.

4.5.1 Static Pressure

To accurately measure the static pressure in a swirling flow field requires great care because the streamline curvature induces a radial pressure gradient. Furthermore, in the boundary layer region, the turbulence intensity is significant and its effect on static pressure probe measurements will be appreciable.

The 3-hole cobra probe could be calibrated in a straight flow to

obtain the critical angles (α_{crit}) + needed to read the true static pressure [11]. However, the effect of swirl on the probe calibration curves is not known and no means of checking is available in the literature. Furthermore, the probe stem would produce substantial disturbances in the flow near the probe tip. The author of reference [13] also concluded that calibrations for static pressure obtained in straight flow could not be used in a swirling flow. Because of the uncertainty associated with this technique, the 3-hole probe was only used to measure the static pressure distribution at the first and last axial stations for each swirl number.

The radial static pressure distribution was computed in the present study by integrating the radial momentum equation. This approach depends upon using the time-mean and fluctuating velocity components measured by hot-wire probes; its details are provided in Chapter V. These results are compared with the corresponding data from the 3-hole probe.

Both outer and inner wall pressures were measured at each axial section by a pressure transducer connected to a wall static tap. Although the turbulence at the wall taps is not fully known, the turbulence velocity fluctuations fall to zero at the surface and the effect is believed to be small provided the wall is smooth and the hole diameter is quite small.

+ α_{crit} is the angular location of one of the probe's three holes at which the sensed pressure is just equal to the static pressure. Values of α_{crit} could be obtained by rotating the pressure probe until the indicated static pressure agreed with that measured using a static wall tap located in the outer wall at the same axial location.

4.5.2 Total Pressure and Flow Direction

The radial velocity is identically zero at both the inner and the outer walls. Only near the entrance to the annulus duct are the axial variations of the axial velocity significant and the mean axial velocity remains constant. Therefore, the radial component of the velocity is everywhere small (about 0.5 m/sec, see Chapter VI) and the direction of the flow can be described by a single angle ϕ between the resultant velocity and the axial direction.

Therefore, the 3-hole cobra probe described earlier was quite suitable for measuring the total pressure and flow angle at each point in the flow field. By nulling the cobra probe, the total pressure was indicated by the middle tube reading and the angle of probe rotation represented the flow angle. Two pressure transducers were used; one connected to the two side tubes of the probe for nulling the probe and the other connected to the centre tube to read the total pressure at each radius. A special computer program was designed, and used with the ADACS system to record the total pressure values. These recorded values represent the average of one hundred instantaneous readings taken at a sampling rate of 6 readings per second. Pressure values were recorded with an accuracy of 0.13 mm of water. The computer program used for this purpose is discussed later, along with other programs used for the hot-wire probe measurements. Flow angles were recorded manually to an accuracy of 0.1 degree using the protractor attached to the traverse mechanism.

The total pressure, P_t , distribution was also calculated using the time-mean and fluctuating velocity components measured by hot-wire probes and the computed values for the static pressure, P , distribution.

- After neglecting the radial velocity component, the total pressure in incompressible turbulent flow can be given by [90]:

$$P_t = P + \frac{1}{2} \rho (U^2 + W^2) + \frac{1}{2} \rho (\overline{u^2} + \overline{v^2} + \overline{w^2}) \quad (4.6)$$

The total pressure values calculated from the above equation are compared with those obtained using the 3-hole probe.

4.5.3 Wall Shear Stress

The methods used to measure indirectly the wall shear stress in the present study are outlined in this subsection. Previous experimental studies [79, 80] have indicated that a conventional 3-hole yaw probe can be used as a Preston tube to determine both the direction and magnitude of the wall shear stress in 3-dimensional turbulent flows providing that pressure gradients and the height (D in Fig. 4.7) of the probe are small. Analytical formulae for predicting the flow angle variations with respect to local shear stress direction near the wall were presented by Van der Berg [71]. His study provided more accurate criteria than other experimental work for examining the validity of using the 3-hole probe to measure wall shear stress in 3-dimensional flows. However, his results are valid only when the deviations from the simple law of the wall are not large and the effects of radius of curvature are negligible.

The 3-hole cobra probe was used as a Preston tube only at the inner wall. No measurements of this kind were taken at the outer wall because the flow is disturbed by the presence of the port opening required to admit the probe into the test section. The centre line of

the probe's mouth opening is 0.37 mm above the wall; this corresponds to a dimensionless distance ranging from $y^+ = 60$ to 80 depending on the value of wall shear stress. For y^+ less than 100, as discussed later in Chapter VI, the wall shear stress direction deviates by less than 0.5 degrees from the direction of the local resultant velocity of the flow. Consequently, if the cobra probe is placed on the wall surface and rotated until the pressures indicated by its two side tubes are equal (i.e. nulling position), then its direction indicates the direction of the wall shear stress to a very good approximation.

Obtained values for total pressure, P_{tot} , static pressure at the wall, P_w , and the dimensions of the cobra probe represent the required parameters for determining the resultant wall shear stress, τ_w . The universal non-dimensional relationship and the calibration data of Head and Vasanta Ram [78] for Preston tubes were applied as described in Chapter V to evaluate τ_w at the inner wall. Also an alternative method, the Clauser Chart, was employed to determine τ_w and its components, $\tau_{x,w}$ and $\tau_{\theta,w}$ at both outer and inner walls.

4.5.4 Mean Velocity and Turbulence Quantities

Mean velocity and turbulence quantities were measured using hot-wire anemometry. Several methods are available for measuring these quantities by means of a hot-wire anemometer. The methods can be categorized according to the number of hot-wire sensors used, the measuring positions of the wires and the type of wires [46-48].

Concerning the three-wire method, three fixed sensor wires make relatively quick measurements of all mean velocity and turbulence components possible. If the wires are placed along the axes of an

orthogonal system, the velocity components along these axes are easily derived. The drawback is that spatial resolution is poor and/or interference between wires cannot be eliminated. Furthermore, more measuring equipment is required as one channel of anemometry is required for each wire and each channel must be calibrated, making system calibration quite complex. Also, the hot-wire positions are not optimum with respect to every particular flow situation. Moreover, if the wire array remains fixed in space the wires will become non-uniformly dirty, thus altering the calibrations in a non-homogenous way. Wire breakage is also relatively difficult and costly to deal with.

Concerning single wire methods, the advantages here are that only a single calibration has to be performed, wire replacement is simple and less equipment is needed. The major disadvantage of these methods is that greater time is required to perform all the necessary measurements. The probe has to be set to a number of different orientations so that a more sophisticated probe holder may be necessary and the measurements are more time-consuming.

Based on the above discussion it was decided to use a single hot wire to perform the required experiments. A major factor in this decision was the geometry of the present apparatus, where any probes had to be introduced radially into the flow through small ports in the outer wall of the annular test section.

The method adopted for the present hot-wire measurements requires the presence of only one sensor in the flow at any given time. In order to deduce the three mean velocity components U , V and W and the six Reynolds stresses $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$, $\overline{uv}$, $\overline{vw}$ and $\overline{uw}$ at any given point in the flow, a total of six measurements must be made at that point. Each of

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+ In the analysis of the hot-wire equations, they refer to Reynolds stresses/fluid density(ρ).

the six measurements are for a different orientation of the hot-wire sensor. While some researchers [47] have suggested that all six measurements at each point could be taken by the same slanted probe, others [60] restricted the number to five measurements for the slanted probe with an extra one to be taken by a normal probe. Beer and Chigier [50] proposed a method that requires only a straight probe and four measurements to determine all the velocity components and Reynolds stresses. Their method also requires the knowledge of flow direction and assumes that the velocity fluctuations are of square wave form. In general, many researchers [12, 13, 49, 54, 56, 59, 60] have used more than one hot-wire probe (single or multiple wire) to obtain Reynolds stresses in 3-dimensional flows.

In the present work, preliminary experiments were conducted to test the validity of taking the required six measurements by one hot-wire probe (45° slanted); the obtained results showed unrealistic values for Reynolds stresses. Therefore, the author believes that a maximum of five measurements should be taken by a slanted probe in order to assure that the main determinant of the coefficients of the set of six equations, Chapter V, is different from zero.

The measurements were conducted by traversing the two DISA miniature probes mentioned earlier (45° slant probe and 90° normal probe). At each point in the flow three measurements were performed with the slant probe while the other three were taken with the normal probe. In every measurement both the time-mean or DC ($\bar{E}_{br}$) and RMS ($\sqrt{e^2_{br}}$) of the hot-wire anemometer output signal were recorded and processed later by the ADACS system. A computer program "MEAS" was designed to record any number between 1 and 6 of measurements for each

type of probe and at any point in the flow. The required number of measurements could be easily fixed by assigning certain numerical values to certain controlling variables in the program. A listing of this computer program and a brief description of its symbols are given in Appendix D. The angle of probe orientation ψ was adjusted manually with the help of the protractor attached to the traverse mechanism. The values of the angles ψ were chosen such that any interference between probe prongs and the main flow direction was minimized. This was especially important in the case of the normal probe. The positive direction for ψ is defined in Fig. 4.10. The specific values used for both probes with different swirl numbers are listed in table 4.4.

Details of signal analysis and data reduction are given in Chapter V. The necessary data comprise the DC and RMS ($\bar{E}_{br}$ and $\sqrt{e^2_{br}}$) of the anemometer output signal or the mean and mean-square of the computed linearized signals ($\bar{E}$ and $\bar{e^2}$), as explained later, for each probe orientation.

In general a hot-wire sensor has different sensitivities to the velocity components normal and tangential to the wire axis. In addition, support-prong interference effects give rise to slightly different sensitivities to the normal velocity components co-planar with and normal to the plane containing the support prongs, the "normal" and "binormal" components respectively. The sensor response can be considered in terms of two coordinate systems, shown in Fig. 4.10. The x, y, z , system represents the local axial, radial and tangential directions, respectively, in the annular flow passage. The ξ, η, ζ system is fixed to the probe and represents the normal, binormal and tangential directions, respectively, with respect to the hot-wire

sensor. The prongs of the probe are in the ξ , ζ plane and in the present work the probe stem is always in the y direction, except during calibration.

A hot-wire sensor responds to an effective cooling velocity, U_{eff} , which has to be defined carefully if accurate measurements are to be made in three-dimensional flows by using the hot-wire probe. Although several relationships for U_{eff} have been proposed [45, 47, 48, 61], the one suggested by Jorgensen [61] is used in the present work because of its suitability for the data processing technique. This relationship is given by:

$$U_{eff}^2 = U_n^2 + k^2 U_t^2 + h^2 U_b^2 \quad (4.7)$$

where U_n , U_t , and U_b are the normal (ξ direction), tangential (ζ direction) and binormal (η direction) velocity components, respectively [Fig.4.10]. k and h are known as the yaw and pitch coefficients respectively, or sensitivity factors in general. These factors should be included in hot-wire equations (see Chapter V) otherwise errors up to 15 percent can occur as reported by other investigators [66].

The calibration of a hot-wire probe consists of simultaneous velocity and voltage measurements from which the calibration constants in an assumed velocity-voltage relationship are determined. In the present study, the hot-wire probe was calibrated against a jet of air issuing from a nozzle of 3.8 cm diameter. The probe sensor was placed in the potential core of the jet along with a Pitot tube to determine the velocity of the air flow. Fig. 4.11 shows the experimental set-up used for calibrating hot-wire probes. To minimize the turbulence level,

the air flow was passed through a plenum containing multiple screens and of 0.046 m^3 in volume. The resulting jet had a very uniform velocity (less 1% variation) and a turbulence level of about 1% in its potential core. Its temperature was measured by a mercury thermometer and one of the pressure transducers mentioned earlier was used to transduce the output of the Pitot tube. Intervals between calibration data points should be as uniform as possible to optimize the accuracy of the hot-wire response curve in the range of most interest (0 to 55 m/sec in this study). At each calibration data point the time-mean of the anemometer bridge output voltage $\bar{E}_{br}$ was measured and the velocity of the air jet U_j was calculated by equation (E-1) to establish the general calibration curve; see Appendix E for details. The calibration data are mainly used to determine certain unknown parameters required to linearize the relationship between $\bar{E}_{br}$ and U_j as outlined below.

If variations in air flow temperature and density are negligible and the effects of probe geometry are incorporated through the sensitivity factors in the cooling law then the relationship between the instantaneous anemometer bridge output E_{br} and the effective cooling velocity can be generally expressed by

$$E_{br} = f (U_{eff}) \quad (4.8)$$

Due to the special relative configuration between the hot-wire sensor and the calibration air jet, equation (4.8) can be simplified (see Appendix E) to the following form

$$E_{br} = f (U_j) \quad (4.9)$$

The relation in equation (4.8) or (4.9) is highly non-linear. To minimize inaccuracies in highly turbulent flows and to simplify the data reduction equations, a linear relationship is often established between $\bar{E}_{br}$ and U_{eff} . Linearization can be accomplished by introducing an electronic linearizer module into the measurement system or by using an analytic expression to linearize during data reduction.

Various expressions have been derived and used [47, 48, 92] to linearize the hot-wire response curve. Following the early work of King [47], the anemometer output signal E_{br} and the effective cooling velocity are often related by

$$E_{br}^2 = A + B U_{eff}^n \quad (4.10)$$

where A and B are constants and can be determined by curve fitting the calibration data. The accuracy of the above equation in providing a straight line relationship depends on the exponent n which in its turn is a function of the velocity range of the flow [69].

In the present study, the linearization procedures described in the TSI manual for the electronic signal linearizer [92] were employed due to their generality and their ability to provide more accurate linearization than equation (4.10). Based on the procedures given in this manual, the ADACS system computer was used to perform the linearization procedures by approximating the calibration curve ($\bar{E}_{br}$ and U_j data) with a fourth order polynomial. Through these procedures the anemometer output voltage E_{br} can be converted to a corresponding linearized voltage E; the latter is directly proportional to the effective cooling velocity as follows:

$$E = S U_{eff} \quad (4.11)$$

where S is the slope of the linearized calibration curve. Calculating the value of U_{eff} that corresponds to a certain anemometer output signal E_{br} requires the determination of the slope S and certain coefficients (linearization coefficients) from the calibration data of $\bar{E}_{br}$ and U_j . These procedures were part of the computer program "CALIB" designed for calibrating the hot-wire probe and automatically executed as soon as the recording of calibration measurements was completed. The linearization coefficients and the slope S thus obtained were stored on a tape and executed later by another program "MEAS" to calculate the linearized signals $\bar{E}$ and $\sqrt{e^2}$ corresponding to $\bar{E}_{br}$ and $\sqrt{e^2}_{br}$ of the anemometer output for measurements taken in the test flow. Each calibration of the hot-wire probe was found to be valid for an operating time of about three hours after which the probe had to be recalibrated.

The yaw and the pitch factors (k and h) included in the cooling law of equation (4.7) were experimentally determined, rather than taken from previous studies, to account for any manufacturing defects and to obtain more accurate results. To simplify the procedures of determining the factors k and h , equation (4.10) with $n = 0.45$ was used for their deduction. The approach was based on a recent study published by Bruun and Tropea [69].

Appendix E provides a more detailed description of the hot-wire calibration and the linearization procedures in addition to the approach used for evaluating the sensitivity factors k and h . A listing of the computer program "CALIB", a summary of the symbols and a sample of its output are also given in Appendix E.

4.5.5 Control Circuit

Most of the experimental data were recorded and processed by using the ADACS system. The experiments were performed semi-automatically and a supplementary technique was required to control the recording of the measured data and to inform the user of any changes. This was achieved by using three external electric circuits connected in parallel to the ADACS system as shown in Fig. 4.12. The measured voltage at the terminals of the first circuit was used as a start/stop decision-making criterion, and a computer program was prepared to instruct the ADACS system to operate in sequential steps. The system digital voltmeter (DVM) was programmed to check the voltage at the terminals of this circuit before triggering to any signal received from the measuring instruments. The ON/OFF switch in Fig. 4.12 was activated by the experimenter. As long as the switch was in the "off" position the DVM measured zero voltage and continued repeating the checking procedure. The second circuit was also closed during this idling period and a green light was lit to indicate that probe movement or other changes were permissible. When the switch was placed in the "on" position, a yellow light was lit to indicate closing of the circuit and to warn the user that measurements would soon start. At that instant the DVM measured a voltage value greater than zero and the ADACS system, according to the program instructions, turned the green light off and closed the third circuit to light the red light, thus informing the user that recording of experimental data was in progress and no changes were permissible. The DVM kept triggering to the signal received from a certain instrument, while the red light was on, until it completed taking the specified number of samples, calculated the time average and

sent the final result to the system computer for storage. After this stage the red light was turned off and the green light on to inform the user that probe adjustments or other changes were allowable. At this point the experimenter turned the ON/OFF switch to the "off" position and made the necessary adjustments. These semi-automatic procedures were repeated until all radial locations at a given axial station were surveyed. The green light then started flashing while other lights were off. The experimenter then, could move to the next axial station or start processing and filing the data for immediate or later data reduction.

The computer program that executed the foregoing controlling procedures was a common 'module' in all programs designed to work with different measuring probes. Its listing is included in the programs listed in Appendices D and E; minor changes were introduced in each program to suit the specific task.

CHAPTER V

DATA REDUCTION

The experimental results obtained by different measuring instruments as described in the previous chapter were processed by using the micro-computer of the ADACS system. Due to the limited memory capacity of this micro-computer (32K RAM), the necessary data reduction was carried out using several computer programs. Whenever necessary, the results of any given program were saved on tape and recalled later by another program for further data processing. Two main computer programs "VELOC" and "CALCUL" were used to compute most of the required flow parameters. The program "VELOC" was designed to determine the time-mean velocity components and Reynolds stresses from the data measured by the hot-wire probes at each point in the flow field. Its output results were processed further by the second program "CALCUL" to compute the eddy viscosities, static pressure distribution and mixing lengths. Listings, sample outputs and symbol definitions for these two computer programs are provided in Appendix G. Computation of the other parameters outlined in Chapter III and presented in the next chapter was performed by separate computer programs not listed in this thesis due to space limitations and their relative simplicity. The following subsections outline the different techniques employed to determine some of the flow parameters of major interest in the present study.

5.1 Mean Velocity Components and Reynolds Stresses⁺

As outlined in section 4.5.4, at each point in the flow field three measurements were taken with a slant probe and three others with a

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⁺ In the analysis of hot-wire equations, they refer to Reynolds stresses divided by the fluid density, ρ .

normal probe. The six probe orientation angles ψ were chosen as listed in table 4.4 for every swirl number. In every measurement both $\bar{E}_{br}$ and $\sqrt{e^2}_{br}$ of the anemometer output signal were recorded and the corresponding linearized signals $\bar{E}$ and $\sqrt{e^2}$ were computed. The following analysis gives the relationship between the mean and mean-square, $\bar{E}$ and e^2 , signals and the three velocity components (U , V and W) as well as the six Reynolds stresses ($\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$, $\overline{uv}$, $\overline{vw}$ and $\overline{uw}$).

The instantaneous velocity components ($U + u$), ($V + v$) and ($W + w$) in the directions x , y and z of the annular tunnel can be resolved into another three components U_n , U_ξ and U_b in the directions ξ , ζ and η respectively, of the hot-wire probe coordinate system as shown in Fig. 4.10. The velocity components U_n and U_b are the normal (ξ direction) and bi-normal (η direction), respectively, and the component U_ξ (ζ direction) is tangential to the probe sensor. The angles α and ψ are defined as follows (see Fig. 4.10):

α is the complementary angle to the angle between the probe stem and the wire sensor.

ψ is the angle through which the probe is rotated about its stem (y axis). ψ is zero when the sensor wire is in the x - y plane and the positive direction is given by the right hand rule.

The transformation from the annular tunnel (x, y, z) axis system to probe-fixed coordinates (ξ, ζ, η) involves a rotation of the (x, y, z) axes about the y axis through the angle ψ and about the z -axis through the angle α . Since this is an orthogonal transformation, the velocity

components are transformed in the same way as the coordinates.

Therefore, the components U_n , U_t and U_b are given by:

$$\begin{aligned} U_n &= [(U + u) \cos \psi + (W + w) \sin \psi] \sin \alpha + (V + v) \cos \alpha \\ U_t &= [(U + u) \cos \psi + (W + w) \sin \psi] \cos \alpha - (V + v) \sin \alpha \\ U_b &= [(U + u) \sin \psi - (W + w) \cos \psi] \end{aligned} \quad (5.1)$$

As mentioned in section 4.5.4, the effective cooling velocity U_{eff} is related to the components U_n , U_t and U_b of equation (5.1) through the yaw and pitch factors, k and h , respectively. Also U_{eff} is related to the instantaneous linearized anemometer output E by the slope S of the linearized hot-wire calibration curve. Recall equations (4.7) and (4.11) for U_{eff} and E :

$$U_{eff}^2(\psi, \alpha) = U_n^2 + k^2 U_t^2 + h^2 U_b^2 \quad (5.2)$$

$$E(\psi, \alpha) = S U_{eff}(\psi, \alpha) \quad (5.3)$$

Substitution of equation (5.1) and (5.2) into (5.3) yields the definitive relationship between the instantaneous flow field parameters and the instantaneous linearized output voltage of the anemometer.

$$\begin{aligned} E(\psi, \alpha) &= S \left\{ [(U + u) \cos \psi + (W + w) \sin \psi] \sin \alpha + (V + v) \cos \alpha \right\}^2 \\ &\quad + k^2 \left\{ [(U + u) \cos \psi + (W + w) \sin \psi] \cos \alpha - (V + v) \sin \alpha \right\}^2 \\ &\quad + h^2 \left\{ (U + u) \sin \psi - (W + w) \cos \psi \right\}^2 \right\}^{1/2} \end{aligned} \quad (5.4)$$

The accurate interpretation of the above equation meets with

difficulty mainly due to the non-linear nature of the terms on the right hand side. The main differences between the several approaches found in the literature stem from the approximations made to simplify the voltage/velocity relation (Equation 5.4 or similar equations) to a form that can be manipulated to yield the required unknowns with acceptable accuracy. In conventional methods for reduction of hot-wire data, equation (5.4) is expanded in a binomial series and higher order terms in the fluctuating quantities are neglected. This approximation is justified if the turbulence intensity is relatively low, preferably less than 10% [48, 62]. Recently, Acrivlellis [60] claimed that by using an "improved" method of hot-wire signal analysis it is possible to accurately determine all of the three mean velocity components and the six Reynolds stresses in a flow of arbitrary turbulence intensity. His method was based on an incorrect equation and therefore led to erroneous results even in the case of low turbulence intensity, as reported by Bartenwerfer [95]. Acrivlellis' method appeared very attractive and the author, before being aware of the Bartenwerfer reference, spent a considerable period of time in applying Acrivlellis' method and in attempting to discover the reasons for the inconsistent results then obtained for the Reynolds stresses. Many methods have been published for making and reducing hot-wire anemometer measurements in three dimensional flows [47, 48, 49, 52, 57] and this experience illustrates the pitfalls that can be encountered. The error in Acrivlellis' method was finally discovered and another method for reducing the data was adopted. No suitable and correct methods are available which do not rely on the assumption of low turbulence intensity.

The approach adopted, in the present study, employs the

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conventional method for simplifying the exact hot-wire response equation (5.4). Like several other methods [12, 13, 59, 60], it is also based on using two types of hot-wire probes, 45° slant and normal, for performing the required measurements. After some algebraic manipulation, equation (5.4) can be simplified by expansion into a binomial series of the first degree. Since the turbulence intensity is about 12 percent for the present flow case, as seen in Chapter VI, neglecting higher order terms is justified. The following equations give the general relationship between the linearized hot-wire anemometer output signals $\bar{E}$, $\bar{e}^2$ and the required unknowns (for details of the derivation see Appendix F).

$$\frac{\bar{E}(\psi, \alpha)}{S} = (aV^2 + bU^2 + cW^2 + dUV + fVW + gUW)^{1/2} \quad (5.5)$$

$$\begin{aligned} \frac{\bar{e}^2(\psi, \alpha)}{S^2} = & \frac{1}{4\lambda^2} (a_1^2 \bar{v}^2 + b_1^2 \bar{u}^2 + c_1^2 \bar{w}^2 \\ & + 2a_1b_1\bar{uv} + 2b_1c_1\bar{uw} + 2a_1c_1\bar{vw}) \end{aligned} \quad (5.6)$$

where:

$$\begin{aligned} a &= \cos^2 \alpha + k^2 \sin^2 \alpha \\ b &= \sin^2 \alpha \cos^2 \psi + k^2 \cos^2 \alpha \cos^2 \psi + h^2 \sin^2 \psi \\ c &= \sin^2 \alpha \sin^2 \psi + k^2 \cos^2 \alpha \sin^2 \psi + h^2 \cos^2 \psi \\ d &= (1 - k^2) \sin 2\alpha \cos \psi \\ f &= (1 - k^2) \sin 2\alpha \sin \psi \\ g &= (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \sin 2\psi \\ a_1 &= dU + fW \\ b_1 &= 2bU + gW \end{aligned} \quad (5.7)$$

$$c_1 = 2c_W + gU$$

$$\lambda = (aV^2 + bU^2 + cW^2 + dUV + fVW + gUW)^{1/2}$$

S is the slope of the linearized hot-wire response curve
(volt/m/sec)

k, h are the yaw and pitch coefficients of the probe as defined earlier

Equations (5.5 and 5.6) are valid for both slanted and normal hot-wire probes providing that the turbulence intensity is less than 15 percent. As seen in equation (5.7), the coefficients a_1 , b_1 and c_1 of equation (5.6) are functions of the time-mean velocity components, angles α and ψ , and the coefficients k and h. Therefore, their values should be computed first at each local position in the flow field.

In the case of non-swirling flow where the tangential and radial components W and V are negligible, equation (5.5) is simplified to

$$\frac{\bar{E}(\psi, \alpha)}{S} = \sqrt{b} U \quad (5.8)$$

and the coefficients $\frac{a_1}{\lambda}$, $\frac{b_1}{\lambda}$ and $\frac{c_1}{\lambda}$ in equation (5.6) become independent of the flow velocity U. Also, if a normal probe is used ($\alpha = 0^\circ$) at a position of $\psi = 90^\circ$; equations (5.5) and (5.6) become

$$\frac{\bar{E}(90, 0)}{S} = h U \quad (5.9)$$

and

$$\frac{\bar{E}^2(90, 0)}{S} = h^2 U^2 \quad (5.10)$$

i.e., the signals $\bar{E}$ and $\bar{e}^2$ would be directly proportional to U and $\bar{u}^2$, respectively. These simple relationships can be found in the literature [45, 48, 62, 68] with the value of h taken as unity or with its true value.

For swirling flow cases, the velocity components U , V and W were computed by writing equation (5.5) for each of the three Ψ angles used with the slant probe as listed in table 4.4. Coefficients a , b , c , d , f and g , defined by equation (5.7) were calculated first for each Ψ value. The three linear equations thus obtained were simultaneously solved to determine the values of U , V and W . The resultant flow velocity Q is given by

$$Q = \sqrt{U^2 + V^2 + W^2} \quad (5.11)$$

For straight flow, V and W were neglected while the velocity U was computed on the basis of equation (5.8), the three measured values of $E(\Psi, 45)$ being combined in the following form

$$U = \frac{1}{S \sqrt{\frac{1+k^2}{2} + 2h^2}} \left\{ \bar{E}^2(90, 45) + \bar{E}^2(45, 45) + \bar{E}^2(-45, 45) \right\}^{1/2} \quad (5.12)$$

The values thus obtained for Q could be checked by utilizing the other measurements for $\bar{E}(\Psi, 0)$ performed by the normal probe ($\alpha = 0$) as shown in table 4.4. If the radial velocity V is small in comparison to U and W , Q can be computed by writing equation (5.5) for two values of Ψ (for $\alpha = 0$) and combining the two equations as follows:

$$Q = \left\{ \frac{\bar{E}^2(\Psi_1, 0) + \bar{E}^2(\Psi_2, 0)}{k^2 + h^2} \right\}^{1/2} \quad (5.13)$$

where ψ_1 and ψ_2 are the two orientation angles of the normal hot-wire probe (see table 4.4 for values).

Equations (5.12) and (5.13) were checked against a Pitot tube in the potential core of a one dimensional jet flow shown in Fig. 4.11 (used mainly to calibrate the hot-wire probes); good agreement was found with less than 0.8% differences in the values obtained for the jet velocity.

Finding all of the six components of the Reynolds stresses at a point in the flow requires a system of six independent linear equations; these are obtained by writing equation (5.6) for each of the six hot-wire sensor orientations (ψ, α) used in the measurements at each point (see table 4.4). Also, the coefficients a_1, b_1 and c_1 , defined by equation (5.7) were first computed for every (ψ, α) at each point. The six linear equations thus obtained were simultaneously solved to determine the six Reynolds stresses. For the non-swirling flow case, better results were obtained by neglecting⁺ the components $\overline{vw}$ and $\overline{uw}$. Therefore only four measurements were required and the mean value of the first two signals and the third signal for each of the hot-wire probes were used to determine $\overline{uv}$, $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$.

The data-reduction computations were carried out by the ADACS system computer. The program "VELOC" which was used to compute the three time-mean velocity components and the six different Reynolds stresses was written in a general form to facilitate its future use by other researchers. For every swirl number, the program uses the six

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⁺ In the absence of swirl ($W=0$), the production of turbulent kinetic energy due to the gradients $\partial W/\partial r$ and $\partial W/\partial x$ is negligible; this results in very small values (i.e. within the experimental error) for $\overline{vw}$ and $\overline{uw}$. Any attempt to determine their values by using the six values of $e^2(\psi, \alpha_j)$ will cause the determinant of the matrix of the coefficients of equation (5.6) to be equal to zero or obtaining data with very large scatter.

linearized values of each of $\bar{E}^2$ and $\bar{e}^2$ recorded by the computer program "MEAS", as explained in section 4.3.4, at the different measurement radii at each axial station. Equations (5.5) and (5.6) as well as the coefficients given by equation (5.7) are used in their general form in the program. Values of probe angle α , rotational angle ψ , pitch factor k and yaw factor h are provided as input data. Consequently, the necessary coefficients can be computed at each point in the flow field and for every sensor orientation. As already mentioned, the mean velocity components and the Reynolds stresses are obtained by simultaneous solution for the sets of equations obtained for each point in the flow. The subprogram used for this purpose was a part of the main program "VELOC" and can be found in reference [96]. The calculated results were saved on tape and were called by another program "CALCUL" to calculate the eddy viscosities, mixing lengths and static pressure distribution. A listing of the program "VELOC" and a sample of its output are provided in Appendix G.

5.2 Summary of Hot-wire Procedures

The following section summarizes the major steps, in sequential order, used to determine the time-mean velocity components and Reynolds stresses by means of hot-wire probes for every swirl number and at each of the five axial stations. These steps were described in more detail in the preceding section of this chapter.

- 1 - If any of the slanted or the normal hot-wire probes are used for the first time, determine the yaw factor k and the pitch factor h for this probe (see Appendix E).
- 2 - Calibrate the slanted hot-wire probe to determine the

linearization polynomial coefficients and the slope S of the linearized hot-wire response curve (use the computer program "CALIB", Appendix E).

- 3 - Traverse the slanted hot-wire probe ($\alpha = 45^\circ$) radially across the annulus and at each point measure both output signals $\bar{E}_{br}(\psi_i, 45)$ and $\sqrt{\bar{e}^2}_{br}(\psi_i, 45)$ where $i = 1, 2$ and 3 . Values of ψ_i depend on the swirl number and are listed in table 4.4. Computer program "MEAS" (Appendix D) is used for that purpose and also to calculate and record the corresponding linearized voltages $\bar{E}(\psi_i, 45)$ and $\sqrt{\bar{e}^2}(\psi_i, 45)$.
- 4 - Repeat steps 2 and 3 with the normal hot-wire probe ($\alpha = 0$) to obtain $\bar{E}(\psi_i, 0)$ and $\sqrt{\bar{e}^2}(\psi_i, 0)$ where $i = 4, 5$ and 6 . Values of ψ_i depend on the swirl number, see table 4.4.
- 5 - Use the computer program "VELOC" to calculate the required flow parameters from the data recorded in steps 3 and 4. Input required data for the program as follows:
  - Values of the angle α for slanted hot-wire probe (α for the normal probe was assumed zero).
  - The six values used for the angles ψ_i .
  - The yaw coefficient k and the pitch coefficient h for each of the two hot-wire probes.

The time-mean velocity components U , V and W are calculated from the three values of $\bar{E}(\psi_i, 45)$ for the slanted probe while Reynolds stresses are calculated from $\sqrt{\bar{e}^2}(\psi_i, 45)$ and $\sqrt{\bar{e}^2}(\psi_i, 0)$ obtained by measurements with both hot-wire probes. Repeat steps 2 to 5 for each of the five axial stations.

- 7 - Repeat steps 2 to 6 for each of the four swirl numbers (0, 0.23, 0.47 and 0.87).

5.3 Eddy Viscosities

The two main components of the eddy viscosity in the axial and tangential directions (μ_{rx} and $\mu_{r\theta}$) were computed from equations (3.6) and (3.7) which can be rewritten in the forms

$$\mu_{rx} = \frac{-\rho \overline{uv}}{\frac{\partial U}{\partial r}} \quad (5.14)$$

and

$$\mu_{r\theta} = \frac{-\rho \overline{vw}}{\left(\frac{\partial W}{\partial r} - \frac{W}{r} \right)} \quad (5.15)$$

The terms included in the right hand side of the above equations were directly substituted or computed from the results of the hot-wire measurements.

To determine the gradients $\partial U / \partial r$ and $\partial W / \partial r$ each of the profiles of U and W was divided into three segments (middle and two wall regions). The profile of each segment was approximated by a polynomial in the form

$$f_0 = \sum_{m=1}^M C_m r^{m-1} \quad (5.16)$$

where, f_0 stands for U or W. A third ($m = 1, 2, 3, 4$) or fourth ($m = 1, 2, 3, 4, 5$) degree polynomial was used for each segment of the velocity profile⁺. This was chosen such that the polynomials obtained for the three segments provided the best fit to the entire experimental profile of U or W. The polynomial coefficients, C_m , were determined by a least-squares curve fitting (LSCF) method for each segment. A computer

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⁺ Overlapping was employed between every two adjacent segments. The degree of the polynomial and/or the number of the overlapped points can be selected as required (see statements # 2070-2110, computer program "CALCUL", Appendix G).

subprogram for least-squares curve fitting found in reference [97] in FORTRAN was translated into BASIC and modified to compute the coefficients C_m . Differentiation of equation (5.16) gives the velocity gradient in the form

$$\frac{df_o}{dr} = \sum_{m=2}^n (m-1) C_m r^{m-2} \quad (5.17)$$

where df_o/dr stands for either $\partial U/\partial r$ or $\partial W/\partial r$. These procedures were part of the main data reduction computer program "CALCUL"; details are provided in Appendix G.

5.4 Mass Flow Rate and Mean Axial Velocity

The mass flow rate was calculated from the following basic formula.

$$\dot{m} = 2 \pi \bar{\rho} \int_{r_i}^{r_o} (U r) dr \quad (5.18)$$

The mean flow density $\bar{\rho}$ was calculated from the ideal gas law in the form

$$\bar{\rho} = [P_a + (P_{wo} + P_{wi})/2] / RT \quad (5.19)$$

where P_a is the atmospheric pressure and P_{wo} and P_{wi} are the gauge static pressures on the outer and inner walls, respectively. Equation (5.18) was numerically integrated by the trapezoidal method, see Fig. 5.1, as follows:

$$\dot{m} = 2 \pi \bar{\rho} \sum_{r_i}^{r_o} (U r)_j \Delta r \quad (5.20)$$

where,

$$(U r)_j = \frac{1}{2} [(U r)_{r_j} + (U r)_{r_{j+1}}]$$

$$\Delta r = r_{j+1} - r_j$$

The mean axial velocity $\bar{U}$ is defined by

$$\bar{U} = \frac{\dot{m}}{\bar{\rho} A} \quad (5.21)$$

where A is the annulus cross sectional area, $A = \pi (r_o^2 - r_i^2)$.

Substitution of equation (5.20) into equation (5.21) gives

$$\bar{U} = \frac{2}{(r_o^2 - r_i^2)} \sum_{r_i}^{r_o} (U r)_j \Delta r \quad (5.22)$$

The mean axial velocity varies slightly along the annular duct if the fluid density varies as a result of axial pressure gradients as will be illustrated in Chapter VI.

5.5 Static Pressure Distribution

Using the time-mean and fluctuating velocity components, the static pressure was computed at each radial location in the flow. In this approach the radial momentum equation (3.3) is directly integrated after neglecting the terms of small order of magnitude. The following equation gives the value of the gauge static pressure, P , at any radius r .

$$P = P_{wo} - \bar{\rho} \left[\overline{v^2} + \int_r^{r_o} \frac{\overline{w^2} + \overline{w'^2} - \overline{v^2}}{r} dr \right] \quad (5.23)$$

where P_{wo} is the gauge static pressure on the outer wall surface. In the above equation, terms that include the mean radial velocity component, V , are neglected because their values are relatively small (V is less than 0.5 m/sec as will be illustrated in Chapter VI) and because there is considerable scatter in the measured radial distribution of the V component. Equation (5.23) was normalized with respect to $\frac{1}{2} \bar{\rho} \bar{U}^2$ and numerically integrated by the trapezoidal method, see Fig. 5.2, as follows

$$\frac{P}{\frac{1}{2} \bar{\rho} \bar{U}^2} = \frac{P_{wo}}{\frac{1}{2} \bar{\rho} \bar{U}^2} - \frac{1}{\bar{\rho} \bar{U}^2 / 2} \left[\overline{v_m^2} + \sum_{r_j}^{r_o} f_m \Delta r \right] \quad (5.24)$$

where,

$$f_m = \frac{1}{2} \left[\left(\frac{\overline{w^2} + \overline{w'^2} - \overline{v^2}}{r} \right)_{r_j} + \left(\frac{\overline{w^2} + \overline{w'^2} - \overline{v^2}}{r} \right)_{r_{j+1}} \right]$$

$$\Delta r = r_{j+1} - r_j, \quad \overline{v_m^2} = \frac{1}{2} (\overline{v_j^2} + \overline{v_{j+1}^2})$$

$\bar{U}$ is the mean axial velocity calculated by equation (5.21) at each axial station. Equation (5.24) is expected to give more accurate values for the radial pressure distribution than different approaches used by previous workers [3, 11, 20] where the effects of swirl and/or turbulence were neglected. The aforementioned computational scheme was part of the computer program "CALCUL" prepared for the present data

reduction. The computed static pressure distributions were compared with those measured by the 3-hole probe at the first and last axial stations.

It should be noticed that at $r = r_i$ the pressure $P(r_i)$ will be equal to the inner wall pressure P_{wi} and $\bar{v}^2 = 0$; therefore

$$\frac{P_{wo} - P_{wi}}{\frac{1}{2} \bar{\rho} \bar{U}^2} = \frac{1}{\bar{U}^2/2} \left(\int_{r_i}^{r_o} f_m \Delta r \right) \quad (5.25)$$

The above equation indicates that the outside wall pressure P_{wo} is greater than the inner wall pressure which is of course to be expected. Using the measured outer wall pressures, the inner wall pressures were calculated from equation (5.25) and the results were compared with those measured using the pressure taps on the inner wall.

5.6 Wall Shear Stress

Experimental values for the resultant wall shear stress at the inner wall, τ_{wi} , were evaluated from the results obtained by the 3-hole probe when it was used as a Preston tube at the inner wall. The universal nondimensional relation for the difference between the total pressure P_t recorded by a Preston tube (placed on the wall) of external diameter D , aligned in the direction of the shear stress, and the static pressure at the wall P_w can be written as [78]:

$$\frac{(P_t' - P_w) D^2}{4 \bar{\rho} \bar{v}^2} = f \left(\frac{\tau_w D^2}{4 \bar{\rho} \bar{v}^2} \right) \quad (5.26)$$

where τ_w is the wall shear stress. The calibration data for Preston tubes as reported by Head and Vasanta Ram [78] were used to establish

the relationship of equation (5.26). τ_w can then be determined from the measured values for P_t and P_w . The axial and tangential wall shear stress components, $\tau_{x,w}$ and $\tau_{\theta,w}$, are related to the resultant, τ_w , by the relationship

$$\tau_w = \sqrt{\tau_{x,w}^2 + \tau_{\theta,w}^2} \quad (5.27)$$

and

$$\tau_{x,w} = \tau_w \cos \phi^*, \quad \tau_{\theta,w} = \tau_w \sin \phi^* \quad (5.28)$$

where ϕ^* is the angle of rotation of the nulled cobra probe with respect to the axial direction at the wall surface. The resultant friction velocity Q^* and its components U^* , W^* in the axial and tangential directions are given by

$$\left. \begin{aligned} Q^* &= \sqrt{\tau_w / \rho} \\ U^* &= \sqrt{\tau_{x,w} / \rho} \\ W^* &= \sqrt{\tau_{\theta,w} / \rho} \end{aligned} \right\} \quad (5.29)$$

An alternative method, the Clauser Chart (which is based on the newly derived laws of the wall) was also employed to determine Q^* , U^* and W^* at the outer and inner walls. If the parameters C_u^* and C_w^* are determined then the frictional velocities can be calculated; recall equation (3.36) for the definition of C_u^* and C_w^*

$$C_u^* = U^* \sqrt{\frac{\tau_{x,w}^*}{\tau_w^*}} = \frac{U^{*2}}{Q^*} \quad (5.30a)$$

$$C_w^* = W^* \sqrt{\frac{\tau_{\theta,w}^*}{\tau_w^*}} = \frac{W^{*2}}{Q^*} \quad (5.30b)$$

From the above relations, Q^* can be given by

$$Q^* = (C_u^{*2} + C_w^{*2})^{\frac{1}{2}} \quad (5.31)$$

and its components U^* and W^* were calculated from

$$U^* = (C_u^* Q^*)^{\frac{1}{2}} \quad (5.32a)$$

$$W^* = (C_w^* Q^*)^{\frac{1}{2}} \quad (5.32b)$$

To estimate C_u^* at each axial station, recall the logarithmic law of the wall given by equation (3.41).

$$U^+ = 2.5 \ln y_1^+ + 5.5 \quad (5.33)$$

The above equation can be rewritten in a more suitable form as follows:

$$U = 2.5 C_u^* \ln \frac{y}{\nu} + C \quad (5.34a)$$

where,

$$C = C_u^* (2.5 \ln C_u^* + 5.5) \quad (5.34b)$$

By assuming different values for C_u^* a family of straight lines can be constructed on a semi-logarithmic chart between the axial velocity U and $\frac{y}{\nu}$. If the experimental velocity values for U at a certain axial station are plotted on the same chart the specific value of C_u^* at this station can be estimated.

To estimate C_w^* , the new version of the wall law given by equations (3.42) and (3.43) was employed. This wall law can be expressed in a general form at the inner wall by

$$W = f_1 \left(C_w^*, \frac{y}{\nu} \right) \quad (5.35a)$$

and at the outer wall by

$$W = f_2 \left(C_w^*, \frac{y}{\nu} \right) \quad (5.35b)$$

The exact formulae for the functions f_1 and f_2 are defined by equations (3.42) and (3.43). By assuming different values of C_w^* , a family of curves can be constructed on a semi-logarithmic chart between W and $\frac{y}{\nu}$. If the experimental velocity profiles for W at a certain axial station are plotted on the same chart, then the value of C_w^* at this specific section can be estimated. The values of C_u^* , C_w^* and equations (5.31) and (5.32) were used to calculate the frictional velocities at each axial station. The aforementioned procedures are illustrated in Appendix H for the data of the swirl number 0.87 at the last axial station.

The estimated values of C_u^* and C_w^* were substituted in equations (5.31) and (5.32) to determine Q^* , U^* , and W^* at each axial station. The resultant wall shear stress τ_w and its components $\tau_{x,w}$ and $\tau_{\theta,w}$ were then calculated from equation (5.29).

CHAPTER VI

RESULTS AND DISCUSSION

The experimental and calculated results from the data obtained by the hot-wire and the three-hole probes for the three velocity components, flow angles, six Reynolds stresses, static and total pressure distributions, wall shear stresses, axial and tangential components of the eddy viscosity and the mixing lengths are presented and discussed in the following sections. Also, the applicability of the newly derived formulae for the law of the wall, the approximate relations for the axial and tangential turbulent stresses and the ratio of the outer to inner wall stresses is examined against the corresponding experimental results in the near wall regions. The different results are shown for the four tested swirl numbers 0, 0.23, 0.47 and 0.87 which correspond to their values as calculated by equation (4.3) at the first measuring station ($X/D_h = 1.67$). All results are plotted in a non-dimensional form and the parameter r^* defined by

$$r^* = \frac{r - r_i}{r_o - r_i} \quad (6.1)$$

is used as one of the axes in most cases. Estimated values for the possible experimental errors in the data of the velocity components, Reynolds stresses and static and total pressures are given in Appendix I.

6.1 Flow Angles, Velocity Profiles and the Law of the Wall

The obtained data for the axial, tangential, radial and resultant

flow velocities are presented in this section and the applicability of the new logarithmic laws of the wall to the distribution of U and W near the walls is illustrated. Also, the profiles of Q are compared with the universal law of the wall. Values of C_u^* ($U \sqrt{\tau_{x,w}/\tau_w}$), C_w^* ($W \sqrt{\tau_{\theta,w}/\tau_w}$) and Q^* substituted in any of these logarithmic formulae are those that obtained from Clauser charts as described in Chapter V.

The use of inlet guide vanes (IGV's) designed with a constant outlet-flow angle has imposed certain features on the velocity profiles. For example, the IGV outlet flow is generally characterised by higher velocities near the inner wall. Assuming uniform stagnation enthalpy, isentropic flow, negligible radial velocity and radial equilibrium, the axial and tangential velocity profiles at outlet from this type of IGV can be expressed by the following vortex flow relationships [101]

$$\left. \begin{aligned} W r^m &= C_1 & ; & & U r^m &= C_2 & ; \\ \phi &= \tan^{-1}(W/U) = \text{constant} \end{aligned} \right\} \quad (6.2)$$

where $m = \sin^2 \phi$ and C_1 and C_2 are constants.

In the near wall regions the existence of the boundary layer and the relatively high turbulence intensity render unrealistic the basis for the assumptions underlying equations (6.2). These relations can therefore only be expected to apply in the central portion of the flow.

6.1.1 Flow Angles

Figs. 6.1 to 6.3 show that the flow angles measured by the 3-hole probe are in good agreement (within 2-6%) with the corresponding angles calculated from the U and W velocity components determined from the hot-wire data. As can be seen, at all axial stations the flow angles are non-uniform even in the central portion of the flow; the variations are, however, only a few degrees and can be attributed to radial variation of the flow-deviation angle and, especially near the walls, to departures from the assumptions underlying equations (6.2).

6.1.2 Axial Velocity Profiles and the Law of the Wall

Figs. 6.4 to 6.7 show the results obtained by the slanted hot-wire probe for the radial distributions of the axial velocity. At the last station ($X/D_h = 10.33$) the axial velocity profiles exhibit a nearly fully-developed shape. At the first axial station ($X/D_h = 1.67$), in the range $r^* = 0.25$ to 0.75 , the velocity profiles are within 5% of the vortex flow equation (6.2). Important viscous effects clearly occur near the inner and outer walls.

The mean axial velocities $\bar{U}$, calculated by equation (5.22) for the four swirl numbers 0.87, 0.47, 0.23 and 0.0, are 32.7, 32.5, 32.3 and 31.7 m/sec, respectively. For each swirl number, differences between the values of $\bar{U}$ at the different axial stations were less than 1.2 percent, see table 6.1. Axial pressure gradients were small so the fluid density was virtually constant. Continuity therefore requires constancy of $\bar{U}$. Confidence in the profile data is inspired by the fact that values of $\bar{U}$ determined by integration of measured U profiles are so nearly constant for each swirl number.

The axial velocity profiles of Figs. 6.4 to 6.7 show no sign of reversed flow or flow separation at the inner or outer walls. Furthermore, as shown later, the wall static pressure drops continuously in the axial direction along both the outer and inner walls, and this emphasises the improbability of reversed flow. The favorable pressure gradient is generally stronger along the outer wall. Scott and Rask [11] reported that axial velocity is most sensitive to swirl at swirl angles greater than 45° . In their study the swirling flow approached reversed flow conditions at a swirl degree of approximately 60° .

The measured profiles for non-swirling flow, Fig. 6.4, are representative of a developing turbulent flow. Also, somewhat higher velocities and a thinner boundary layer are observed near the inner wall at all axial stations along the tunnel. If linear pressure drop and unchanging velocity profiles with increasing downstream distance are to be taken as the criteria for fully developed flow then the required development length for the present radius ratio ($r_i/r_o = 0.625$) is 15 times the hydraulic diameter [73]. Since the last surveyed axial station is only at 10.3 hydraulic diameters, fully developed flow in the non-swirling case has not been achieved in the present study. However, for fully developed annular flow with Reynolds number above 10^5 the radial location R_m of the maximum axial velocity can be described by the relationship [7, 73]

$$\frac{R_m - r_i}{r_o - R_m} = \left(\frac{r_i}{r_o} \right)^{0.343} \quad (6.3)$$

If this relationship is applied to the present case, the maximum axial velocity should occur at a radius equal to 16.2 cm or $r^* = 0.46$ which is

not too far from the position of the maximum velocity at the last axial station ($X/D_h = 10.3$) as seen in Fig. 6.4.

As the swirl number, SN, increases from 0.23 to 0.87 a peak in the axial velocity profiles at the first station becomes progressively more recognizable and appears closer to the inner wall as shown in Fig. 6.8. This is because the inlet guide vanes were designed with constant outlet flow angle which is generally characterized by higher axial velocities near the inner wall (see equation 6.2). However as the flow proceeds downstream the maximum-velocity position is shifted towards the outer wall, as shown by Fig. 6.9; this results from the greater drop in static pressure along the outer wall as compared to the inner wall in the case of the swirling flows.

The axial velocity data near the walls are compared with the modified law of the wall (equation 3.41 with $A = 2.5$ and $B = 5.5$) in Figures 6.10 to 6.13. While all profiles exhibit a log-region close to both walls, there are clear differences in their behavior in the wake region. In that region, when swirl is present, the profiles deviate upwards near the inner wall and downwards near the outer wall; this trend strengthens with increasing the swirl. This can also be ascribed to the favorable pressure gradient and axial increase in velocity near the outer wall compared to the approximate constancy of these parameters near the inner wall.

The results obtained for non-swirling flow are in agreement with those of previous workers [72-74, 102]. Scott and Rask [11] in their study of swirling flow in an annulus of 0.4 diameter ratio, found good to poor agreement near the inner wall between their axial velocity data and the universal law of the wall as the swirl was increased from 0 to

60°. The data for $\phi = 60^\circ$ showed the poorest agreement. They did not report any corresponding data or conclusions regarding either axial velocity profiles near the outer wall or the tangential velocity profiles. This may indicate that poor agreement was found in those cases.

6.1.3 Tangential Velocity and the Law of the Wall

The radial distributions of tangential velocity obtained by the slanted hot-wire probe are plotted in Figs. 6.14 to 6.16 for the five axial stations and for each swirl number. At the first axial station, the figures show that the vortex flow of equation (6.2) is in good agreement with the experimental velocity profiles. Also, all profiles exhibit a vortex type flow over a major radial portion of the annulus and this vortex flow character is still apparent at the last axial station. A boundary layer is evident near the inner wall; its thickness increases until at the fifth axial station it occupies about the entire inner half of the annulus.

The magnitude of the tangential velocity peak and its radial position change with distance from the inlet guide vanes. In the vortex flow the tangential velocity increases monotonically towards the inner wall. Of course near the walls viscous effects reduce the tangential velocity below that which would occur in the inviscid vortex flow. The radial position of the tangential velocity peak is thus determined by the thickness of the inner-wall boundary layer. Since this becomes thicker with increasing axial distance, the peak decreases in magnitude and moves radially outwards as seen by comparing Figs. 6.17 and 6.18.

Bearing in mind that the tangential velocity and the swirl number

SN are related by equation (4.3) and that the mean axial velocity, $\bar{U}$ remains constant, SN can be used as an index of the changes in the mean tangential velocity, $\bar{W}$, with respect to $\bar{U}$. Figs. 6.19 shows that the swirl number decays in the axial direction; the decay is greater for the higher swirl numbers. For example, for initial swirl numbers 0.87 and 0.23 at $X/D_h = 1.67$, by $X/D_h = 10.33$ the decrease in the higher swirl number was found to be four times the decrease in the lower one. This can be attributed to the significant increase in the tangential eddy viscosity, μ_{re} , and wall shear stresses, $\tau_{\theta,w}$, with increasing swirl number as discussed later. This underlines the importance of accurate estimation for the tangential eddy viscosity in predicting turbulent swirling flows as reported in previous studies [4].

In Figures 6.20 to 6.22 the experimental data for the tangential velocity distributions are compared with the new logarithmic laws of the wall expressed by equations (3.42) and (3.43) at the inner and outer walls, respectively, with the constants $K = 0.4$ and $B = 5.5$. Equation (3.44), which assumes that r_0 and r_i are very large, is also plotted in the figures for purposes of comparison. As can be seen, the experimental values of W^+ are in good agreement with the new logarithmic laws in the regions close to the inner and outer walls even for the highest swirl number (SN = 0.87). The agreement remains very good up to y^+ values of about 1000.

Near the outer concave wall, the new log-law coincides with equation (3.44) up to $y^+ = 250$ after which the latter starts to predict higher values than the experimental ones (3 to 5 percent at $y^+ = 1000$). On the other hand, near the inner convex wall, the new log-law coincides with equation (3.44) only up to $y^+ = 100$ after which the latter starts

to predict lower values than the experimental ones (7-10 percent at $y^+ = 1000$). Therefore, if equation (3.44) was used to predict the tangential velocity profile, the error in the calculated values near a convex wall would be approximately twice that near a concave wall.

Theoretical calculations for turbulent swirling flow in a concentric annulus of 0.4 diameter ratio [4] indicated that the tangential velocity profiles predicted by the universal law of the wall were in poor agreement with the corresponding experimental results of Reference [3] especially near the outer wall. The new logarithmic laws, equations (3.42) and (3.43), are plotted in Figs. 6.23 and 6.24 against some previous experimental results for flow over a highly curved convex surface of 29.3 cm radius of curvature [13] and for flows over mildly curved concave and convex surfaces of 249 and 259 cm radii of curvature respectively [14]. The constants A and B of equations (3.42) and (3.43) are given the same values as those used by the publishing authors for the universal law of the wall. As seen in Figs. 6.23 and 6.24, equations (3.42) and (3.43) are in good agreement with the general trends of the experimental data of Refs. [13, 14]. For the mildly curved surfaces, the curves of equations (3.42) and (3.43) are closer to the universal log-law than in the case of the highly curved convex surface. Fig. 6.23 shows that the wake region becomes negligible with respect to the new logarithmic law for the data of $x = 54.5$ and 59 inches.

In the present study, the discrepancy between the new logarithmic laws and equation (3.44) was quite considerable because of the small radii of curvature (20.3 and 12.7 cm at outer and inner wall, respectively). In general this discrepancy increases with decreasing radius of curvature for any flow over a convex or concave surface.

6.1.4 Radial Velocity Profiles

The hot-wire data for the radial velocity component, V , exhibited excessive scatter due to the small value of V in comparison with the main velocity components U and W ($V \approx 0.02 U$). Therefore, only the profiles for the highest swirl number ($SN = 0.87$) are shown in Fig. 6.25 as an example. Except to say that V was always small and could be positive or negative, (up to 118% error, see Appendix I), no firm conclusions could be drawn concerning the radial velocity, V .

The foregoing demonstrates the reason for neglecting the velocity V in the calculating of static pressure distribution by equation (5.24) and in the coefficients required in the hot-wire equations (5.7). The radial velocity component was also neglected by other researchers [3, 4] in their studies of swirling flows in an annulus.

6.1.5 Resultant Velocity Profiles and the Law of the Wall

The resultant velocity profiles calculated by equation (5.11) from the individual components U and W were in good agreement (about 2% difference) with those obtained directly by the normal hot-wire probe using equation (5.13). Since Q profiles were similar to those of the velocities U and W , they are not included in this thesis.

The results for Q obtained from the U and W data are plotted in Figs. 6.26 to 6.28 in the non-dimensional form Q^+ against the universal law of the wall, equation (2.4). The best fit was obtained with the constant $A = 2.44$ for all swirling flows and $B = 5$ for $SN = 0.87$ and 0.47 and $B = 5.2$ for $SN = 0.23$. The figures show that for $SN = 0.23$, the data for Q^+ near the inner and outer walls are in good agreement with the universal law of the wall. However as the swirl increases, the

universal log-law continues to hold for $y^+ < 600$ near the outer wall but a substantial decrease in the extent of the universal log-region and a wake region of higher strength became apparent near the inner wall, especially for $SN = 0.87$. As shown earlier, wall curvature was found to have negligible direct effect on the axial velocity U (for the present radius ratio $r_1/r_0 = 0.625$) but it did affect the tangential velocity W and the effect near the inner wall was more significant than near the outer wall. The resultant velocity, Q , will of course reflect these effects. Recall that the theoretical analysis of section 3.5.3 indicated that the universal law of the wall could be expected to apply to the resultant velocity Q only in weakly swirling flows. The present data supports this indication.

To summarize, the universal law of the wall was found to be valid for the resultant flow velocity but the range of validity was affected by wall curvature and was smaller than the range of validity of the modified laws of the wall derived for velocity components U and W . The universal law was valid for the resultant velocity, for $y^+ < 600$ and $y^+ < 400$ near the outer and inner walls, respectively. In general, annular turbulent flows with higher swirl numbers ($SN > 0.87$) and/or positive axial pressure gradient might involve reversed flow zones and extensive deviation from the universal law would then be expected.

6.2 Static Pressure Distribution

In all experiments the outer wall static pressure was found to be axisymmetrical with a maximum variation of ± 3 percent. Its value was checked through the pressure taps distributed uniformly (30° apart) around the circumference of the outer wall at the axial stations

$x/D_h = 0.33, 1, 1.66$ and 2.33 .

The static pressure distributions along the inner and outer walls are presented in Figs. 6.29 to 6.32. The figures show that the values for the wall pressure at the inner wall as computed from equation (5.25) at each axial station are in good agreement with the corresponding measured values. This agreement helps to justify the omission of the radial velocity component from equations (5.24) and (5.25) and also inspires confidence in the static pressure calculated inside the flow.

For the non-swirling flow case, the difference between the outer and inner wall pressures is negligible and axial pressure gradient is nearly uniform along both walls. As the swirl increases, the difference between the two wall pressures of course increases as the pressure difference must balance the centrifugal forces associated with swirl. Because the mean axial velocity U must remain constant, there must be a decrease in static pressure in the positive x -direction in order to balance the wall shear stresses $\tau_{x,w}$. As already pointed out, the swirl, as represented by SN , must also decrease due to the wall shear stresses $\tau_{\theta,w}$ and the static pressure difference between outer and inner walls must therefore decrease. Consequently the static pressure must decrease more rapidly along the outer wall, as compared to the inner wall. This behaviour is evident in Figs. 6.30 to 6.32. All the axial pressure distributions exhibit a favourable pressure gradient over the entire measurement region and reversed flow is therefore most unlikely.

The radial distributions of static pressure computed from equation (5.24) are shown in Figs. 6.33 to 6.35. The experimental data obtained by the 3-hole probe at the first and last stations are also presented in

the figures. For the non-swirling flow case, the static pressure was assumed to be uniform across the annulus height and equal to the outside wall pressure at each axial station. Figs. 6.33 to 6.35 show that the 3-hole probe data are not in good agreement with the calculated ones; large scatter and errors ranging from 10 to 25 percent are found. The highest error corresponds to $SN = 0.87$. These errors can be ascribed to the large disturbances induced locally in the flow by the probe tip at the measuring point and/or ignoring the effect of velocity gradients and swirl on the probe calibration. The present results are in agreement with the conclusions reported by Kuzay [8] where the calibration curves for both a cylinder and 3-hole cobra probe in straight annular flow were found to be invalid for a rotating flow case. In that study, the static pressure distribution was computed by integrating the radial momentum equation, after neglecting the turbulence terms.

The profiles of Figs. 6.33-6.35 also show that the radial static pressure gradient, dP/dr , is much higher near the inner wall than near the outer wall for the higher swirl numbers. This is because the radial pressure gradient is inversely proportional to the radius of curvature in the dominant term of equation (3.4) (i.e. $dP/dr \approx \rho W^2/r$) and because W increases with r near the inner wall while it decreases near the outer wall.

6.3 Total Pressure Distribution

The experimental total pressure profiles obtained by the 3-hole probe are shown in Figs. 6.36 to 6.39. For the first, third and fifth axial stations the figures also include the total pressure values calculated from equation (4.6) by substituting the static pressure

values computed from equation (5.24) and the hot-wire results for U , W and the normal Reynolds stresses. For each swirl number the profiles exhibit the characteristics of a developing flow: that is, boundary layers at the annulus walls with a core flow of uniform total pressure at upstream axial stations. The core gradually disappears as the flow proceeds down the annulus.

The swirl number has a significant effect on the total pressure distributions as can be seen by comparing the profiles of Figs. 6.36 to 6.39. As swirl increases the boundary layer on the inner wall tends to become thicker while that on the outer wall becomes thinner. Most of this effect appears to be due to increasing losses in the 'hub' region within the IGV blade row because the main changes with swirl number are already apparent in the profiles at the first axial station for each swirl number. To some extent the axial pressure gradient may be a contributing factor (less favorable along inner wall as compared to outer wall as SN increases). On the other hand the surface shear stress is generally greater on the outer wall when swirl is present and this would produce an opposite trend.

The directly-measured and calculated total pressure in Figs. 6.36 and 6.39 are generally in good agreement. The largest discrepancy is only 13 percent and occurs at the highest swirl number near the inner wall. The discrepancy was smallest in the central flow regions. The discrepancy can be ascribed to the steep radial gradients of velocity near the walls, since the 3-hole cobra probe has only a small upstream projection and the large disturbances may be produced by the probe stem near the tip in such flow conditions. Furthermore, the error in measuring the velocity components U and W by the slanted hot-wire (about

0.3 to 4 percent) affected the accuracy of the calculated values for the total pressure by 0.9 to 14.3 percent, see Appendix I. The generally good agreement between calculated and measured total-pressure distributions again emphasises the consistency of the experimental data and inspires confidence in it.

6.4 Wall Shear Stress

The axial distributions for the parameters C_u^* (i.e. $U \sqrt{\tau_{x,w}/\tau_w}$) and C_w^* (i.e. $W \sqrt{\tau_{\theta,w}/\tau_w}$) as determined from the velocity profiles of U and W , respectively, by using the Clauser chart technique at each axial station are plotted in Figs. 6.40 and 6.41 for the tested swirl numbers. The plotted values represent the ratios between C_u^* and C_w^* and the frictional velocity determined for the non-swirling flow at the inner wall $U_{0,i}^*$, or the outer wall $U_{0,o}^*$, at every axial station.

Fig. 6.40 shows that for every swirl number, the profiles of C_u^*/U_0^* exhibit no significant axial gradient along the outer wall while a weak negative gradient is observed along the inner wall. As the swirl increases from 0 to 0.87, a progressive increase of 3-8 percent is observed in C_u^*/U_0^* values at the outer wall (3 percent for $SN = 0.23$; 8 percent for $SN = 0.87$). However at the last four stations on the inner wall, an opposite trend is evident with a decrease of 2 percent ($SN = 0.23$) to 5 percent ($SN = 0.87$). These small variations can be attributed to two main interacting factors that influence the axial velocity profiles. First, these profiles, in general, are strongly governed by the continuity equation rather than by the swirl number and secondly, the development of U profiles (i.e. the growth in the boundary layer near the walls) in the axial flow direction was slightly enhanced

at the inner wall and retarded at the outer wall under the influence of changes in axial pressure gradient associated with increasing swirl. To generalize, Fig. 6.40 shows that the swirl number has no significant effect on the axial distributions of $\dot{C}_u/U_0^*$.

Fig. 6.41 shows that the obtained values for $\dot{C}_w/U_0^*$ at all axial stations exhibit an increase approximately proportional to the increase in the swirl number of the flow. This is an expected result and can be referred to the increase in the tangential velocity W , which is the major factor that influences the value of the tangential wall shear stress as the swirl increases. Furthermore, the figure also shows that the difference between the values of $\dot{C}_w/U_0^*$ at the outer and inner walls increases significantly with increasing swirl number. The latter can be mainly ascribed to the changes in the inner and outer wall boundary layers associated with increasing swirl. As noted earlier the boundary layers become thicker and thinner on the inner and outer walls, respectively, as swirl is increased and much of this appears to originate in the IGV blade row. The different axial pressure gradients along the inner and outer wall, discussed earlier, are probably also a contributing factor.

Regardless of the foregoing discussion concerning the effect of the swirl number on the parameters $\dot{C}_u/U_0^*$ and $\dot{C}_w/U_0^*$, the diameter ratio of the annulus imposes an additional influence on the wall shear stress $\tau_{x,w}$, $\tau_{\theta,w}$ and τ_w as will be discussed later. Investigation of this was not included in the scope of the present study, but available information in the literature will be related to the present results.

Based on the conventional concept of the Clauser Chart, $\dot{C}_u^*$ and $\dot{C}_w^*$ were incorrectly considered to stand for the frictional velocities U^* and W^* ,

respectively, in the previous studies concerning swirling flow in an annulus [3, 4, 11]. In the present study, this concept was found to be invalid for swirling flows and the frictional velocities U^* , W^* and Q^* were calculated from the values of C_u^* and C_w^* by equations (5.31 and (5.32). Fig. 6.42 shows that the axial distributions of Q^* for the different swirl numbers exhibit similar trends to those of C_u^* and C_w^* . The data for Q^* , shown in this figure, were used to calculate Q^+ and y^+ previously presented in Figs. 6.26 to 6.28.

The results for the axial and tangential wall shear stresses, $\tau_{x,w}$, and $\tau_{\theta,w}$, computed from equation (5.29) and the frictional velocities, are shown in Figs. 6.43 and 6.44. The figures show that the stresses obtained by the Preston tube near the inner wall are generally in good agreement with the corresponding stresses calculated from equation (5.29). This is to be expected because the τ_w and Q vectors were found to have the same direction (within 0.5 deg., as will be seen later) and the universal law of the wall was found to apply for Q at least up to $y^+ = 200$ on both walls and for all the tested swirl numbers (see Section 6.1.5). The non-dimensional height D of the Preston tube was always less than 100. The curves of Figs. 6.43 and 6.44 show, in general, similar trends to those of Figs. 6.40 and 6.41 for C_u^* and C_w^* . This is to be expected in view of the close relationship (equation (5.29) and (5.32)) between C_u^* , C_w^* and the wall shear stress components.

Fig. 6.43 shows that for the non-swirling flow case, the values of $\tau_{x,w}$ decline at the early axial stations where the flow is developing and then become approximately constant as the flow approaches the fully developed condition. At all axial stations the values at the inner wall are higher than at the outer wall. This reflects the fact that $\tau_{x,w}$

must decrease with increasing radius, at least where the convection terms in the momentum equation are small (see equations 3.18 and 3.19). The effect of r_1/r_0 on $\tau_{x,w}$ will be discussed later in more detail. With swirl the values of $\tau_{x,w}$ also decline near the duct entry and approach constant values as the flow becomes more fully developed. For $SN = 0.23$, values of $\tau_{x,w}$ and $\tau_{\theta,w}$ are nearly equal at inner and outer walls and further increases in the swirl number result in higher wall stresses at the outer wall especially at the last four axial stations. This illustrates the opposite effects induced by the swirl and wall curvature on the wall shear stresses. Near the inner (convex) wall, the positive radial gradients of angular momentum tend to stabilize the turbulence while near the outer (concave) wall the negative gradients are de-stabilizing.

The tangential wall shear stresses, $\tau_{\theta,w}$, plotted in Fig. 6.44 show the same trends, for the same reasons, as the axial stresses $\tau_{x,w}$ discussed in the preceding paragraph. The major difference is, of course, the increase in magnitude of $(\tau_{\theta,w} / \frac{1}{2} \bar{\rho} \bar{U}^2)$ as swirl increases the magnitude of $(W/\bar{U})$.

The effects of swirl number on the wall shear stresses $\tau_{x,w}$ and $\tau_{\theta,w}$ relative to the wall stress at zero swirl, $\tau_{0,w}$, are shown in Figs. 6.45 and 6.46. These figures show that $\tau_{x,w}$ and $\tau_{\theta,w}$ on both walls increase with increasing swirl number. The stresses on the outer wall are generally higher than on the inner wall, as is to be expected due to the de-stabilizing effects of the swirl. The increase of $\tau_{x,w}$ on the inner wall may be attributed not to the effects of the swirl but to the inlet guide vanes which produced higher axial velocities near the inner wall (see equation 6.2). Increasing the swirl number above 0.87

may be associated with positive axial pressure gradients along the inner wall and/or reversed flow zones which would tend to reduce wall shear stresses at both walls. Khalatov et al. [81], in their study for the friction law of swirled flow in a cylindrical channel, found that the axial wall shear stress reached its maximum value at $SN = 0.8$ after which $\tau_{x,wo}$ started to decline considerably.

Previous experiments on turbulent flows over curved walls [12-14] with flow velocities of about 20 m/sec have indicated that the tangential wall stress $\tau_{\theta,w}$ depends strongly on the radius of curvature and distance between the measuring station and flow entrance. Results for flows over mildly curved walls [14] showed that the stress on the outer (concave) wall becomes 1.6 percent higher than that on the inner (convex) wall after a distance of 1.7m (measured along the wall).

Experiments on highly curved walls [12, 13] showed that $\tau_{\theta,wo}$ (concave) is higher by 4 and 11 percent at distances of 0.6 and 1.6 m from flow entry respectively.

The data presented before in Figs. 6.43 and 6.44 were also used to calculate the ratio of outer to inner wall shear stresses $\tau_{x,wo}/\tau_{x,wi}$ and $\tau_{\theta,wo}/\tau_{\theta,wi}$ for each axial station and swirl number; the results are shown in Figs. 6.47 to 6.50. Other points shown in the same figures were computed from equations (3.24b) and (3.25b); general agreement between both sets of values is evident at all axial stations.

The non-swirling flow data of Fig. 6.47 show that $\tau_{\theta,wo}/\tau_{\theta,wi}$ is nearly uniform along the x-axis and its value is approximately 0.93. This would give a value of 1.08 for the inner to outer wall shear stresses which is in excellent agreement with data available in the literature as shown in Fig. 6.51. The latter is taken from a study

by Pletcher and Malik [74] on turbulent flow and heat transfer in annular geometries. The figure shows that $\tau_{o,wi}/\tau_{o,wo}$ decreases exponentially to a minimum value of unity with increasing annulus radius ratio r_i/r_o .

The swirling flow data of Figs. 6.48 to 6.50 shows that the ratios $\tau_{x,wo}/\tau_{x,wi}$ and $\tau_{\theta,wo}/\tau_{\theta,wi}$ are relatively higher than those obtained in the case of non-swirling flow. Moreover, all curves exhibit a positive axial gradient which increases with the swirl number and is slightly higher for the tangential stress ratio. This is due to the stabilizing/destabilizing effect of swirl on the flow near the inner/outer walls, respectively.

It should be noted here that the data of Figs. 6.48 to 6.50 were obtained for an annulus of diameter ratio 0.625 and varying this ratio while keeping the Reynolds and the swirl numbers the same would presumably alter the values of $\tau_{x,wo}/\tau_{x,wi}$ and $\tau_{\theta,wo}/\tau_{\theta,wi}$. The author is not aware of information in the literature on this point but physical understanding and data of Fig. 6.51 for the non-swirling case suggest that radius ratio would be significant.

Relations (3.24a) and (3.25a), obtained from the axial and tangential momentum equations, were integrated over the x-axis between the first and third axial stations, and between the first and last axial stations in order to check the consistency between wall shear stresses (Clauser chart values), velocity profiles and static pressure distributions. Table 6.2 presents the ratio of the left hand side to the right hand side of each relationship for the various swirl numbers used in the experimental investigation. In general, Table 6.2 shows good to reasonable consistency (about 5 to 20% difference from unity)

between the various measured data. The differences can be attributed to the experimental error in the values used in performing the integration, see Appendix I.

6.5 Turbulent Shear Stresses

The radial distributions for the axial and tangential turbulent shear stresses, τ_{rx} and $\tau_{r\theta}$, are shown in Figs. 6.52 to 6.58 for the tested swirl numbers at three axial stations; plotted values for wall shear stresses at $r^* = 0.0$ and 1.0 are those determined from Clauser charts. Also, the calculated values from equations (3.18 and 3.19) and (3.21 and 3.22) (using Clauser charts data) for τ_{rx} and $\tau_{r\theta}$, respectively, near the walls are presented in the same figures at the last axial station. The figures show that the experimental data at the last station are in good agreement with their corresponding calculated values in the near-wall regions; and the largest discrepancy can be seen in the central flow region especially with the τ_{rx} data corresponding to high swirl numbers. Equations (3.21) and (3.22) give more accurate values for $\tau_{r\theta}$ far from the walls than those predicted for τ_{rx} by equations (3.18) and (3.19) because the convection terms in the axial momentum were neglected in the derivation of the latter equations.

The regions of maximum axial or tangential velocities can be seen in Figs. 6.4 to 6.9 and 6.14 to 6.18 for all axial stations. Near the maxima of U or W the stresses τ_{rx} or $\tau_{r\theta}$, respectively, were found to be very small or approximately zero. This is an expected result since the total pressure is constant or nearly constant in these regions (see Fig. 6.36 to 6.39) and they are effectively regions of irrotational flow..

The non-swirling flow data of Fig. 6.52 show that the radial distributions of τ_{rx} near both walls are nearly symmetrical at all axial stations. As the swirl increases, steeper radial gradients can be seen in the τ_{rx} and $\tau_{r\theta}$ distributions near the inner wall while an opposite trend is observed near the outer wall, see Figs. 6.59 and 6.60. This can be attributed to the combined effects of the radius of curvature and swirl number. Near the inner wall, the swirl tends to enhance the stability of the turbulence structure while it destabilizes it near the outer wall. This diffusion is more effective near the outer wall and gradients there tend to be lower.

In general, Figs. 6.52 to 6.58 show that the radial distributions of τ_{rx} and $\tau_{r\theta}$ do not exhibit regions of constant shear stress although variations are relatively small very near the walls. The profiles are similar to those experienced in a turbulent pipe flow especially near the outer wall. The absence of constant stress regions near the walls did not offset the validity of the new logarithmic laws, as presented before, for the velocity profiles in the near wall regions. This is because the new log-laws are derived using a modified relationship for the mixing lengths (i.e. equation 3.26, suggested by Ref. 45) which includes the effect of stress variation near the walls. Moreover, in many simple internal flow cases such as pipe flow, the distribution of τ_{rx} follows the known straight-line relationship [9, 16, 17, 73]

$$\tau_{rx} = \tau_w \left(1 - \frac{y}{R} \right) \quad (6.4)$$

where R is the pipe radius, and the universal law of the wall is found

to be in excellent agreement with the experimental velocity profiles [9, 16, 17, 38, 73]. Also, the τ_{rx} data of Fig. 6.52 for the non-swirling flow case are similar to those obtained by other researchers for turbulent annular flows [72-74, 102] and in their studies the universal law of the wall was found to be in good agreement with their experimental data in the near wall regions if the diameter ratio of the annulus, r_i/r_o , was greater than 0.4. If the distribution of τ_{rx} for a pipe flow is considered as a criterion, then the logarithmic formulae would be valid for the velocity profiles in the near wall regions of an annulus if the numerical values of the radial gradients for τ_{rx} or $\tau_{r\theta}$ are less than or equal to $2\tau_{x,w}/h$ or $2\tau_{\theta,w}/h$, respectively (h is the annulus gap).

The resultant shear stress τ_w was found to align approximately with the flow direction in the near wall regions. Fig. 6.61 shows the distributions of the resultant shear stress angles ϕ' , relative to the flow direction at the first and last stations for $SN = 0.23, 0.47$ and 0.87 . The data for ϕ' represent the absolute difference between the angle of the stress τ_w ($\tan^{-1}(\tau_{r\theta}/\tau_{rx})$) and the flow angle ϕ ($\tan^{-1}(w/u)$). The figures show that for $y^+ < 200$, the stress τ_w can be assumed to be in the direction of the resultant flow velocity with good accuracy (about 0.5 degree difference). The latter justifies the use of the Preston tube (probe centre-line was located at $y^+ = 50 - 90$ from the wall surface) to measure the resultant wall stress τ_w as described in Chapter V. The angle ϕ' was found to increase with the distance from the wall and more scatter can be seen at the points closer to the central flow regions.

The experimental data for the turbulent shear stress $\tau_{x\theta}$ (i.e. -

$P \bar{u}w$) included large scatter due to its relatively small value in comparison with the other turbulent stress components (e.g. 2 to 4 percent of τ_{rx}). Therefore, only the profiles for the highest swirl number are shown in Fig. 6.62 as an example. The figure shows that $\tau_{x\theta}$ decreases with the distance from either wall until it reaches a minimum value at $0.35 < r^* < 0.52$. According to the eddy viscosity concept, this stress can be expressed by

$$\tau_{x\theta} = \mu_{x\theta} \frac{\partial W}{\partial x}$$

where the eddy viscosity $\mu_{x\theta}$ has approximately the same order of magnitude of μ_{rx} or $\mu_{r\theta}$ while the gradient $\partial W / \partial x$ is relatively small compared to the other velocity gradients. The axial distances between the measuring stations were too far to allow accurate estimation for $\partial W / \partial x$. Since this stress was, as expected, relatively small it was not of major interest in the present study and the $\tau_{x\theta}$ data were not processed further. The turbulent stress $\tau_{x\theta}$ was neglected in previous studies for swirling annular flows [3, 4, 11].

6.6 Eddy Viscosities

The radial distributions for the eddy viscosities μ_{rx} and $\mu_{r\theta}$ are shown in Figs. 6.63 to 6.70 for the four tested swirl numbers at the first, middle and last axial stations. The values of μ_{rx} and $\mu_{r\theta}$ were assumed to be zero at the wall surfaces. In the central flow regions that were characterized by approximately uniform velocity and very small turbulent shear stresses, unrealistic⁺ values for the eddy viscosities were always obtained at a few points in the core flow region

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⁺ i.e. values that do not appear to fall within the general trend of a fitted curve or exceeded the range of the selected scale.

and therefore their data were omitted from the figures.

All profiles show that the eddy viscosities increase with the distance from either wall up to a maximum value after which their values start to decrease until they reach a minimum value at a radial position r^* ranging from 0.4 to 0.52. Also for all swirl numbers, μ_{rx} and $\mu_{r\theta}$ have higher values near the outer wall than near the inner wall and the difference becomes more pronounced as the swirl number increases. For example, the non-swirling flow data show that the maximum value for μ_{rx} near the outer wall is higher by 15 percent than its corresponding value near the inner wall. For $SN = 0.87$, the maximum values of μ_{rx} and $\mu_{r\theta}$ near the outer wall are at least twice their values near the inner wall, see Figs. 6.71 and 6.72. Moreover, as the swirl increases, the eddy viscosities increase significantly near the outer wall while their values remain unchanged or increase only slightly near the inner wall. This is because swirl promotes the instability in turbulence structure near the outer wall (concave) while near the inner wall (convex) it tends to stabilize it.

The non-swirling flow data for μ_{rx} are generally in good agreement with those obtained by other researchers [6, 72, 74, 102] for straight flows in an annulus. Scott and Rask [3] in their study for swirling flows in an annulus concluded that the tangential eddy viscosities near the inner wall were considerably lower than the corresponding values near the outer wall and that the axial and the tangential eddy viscosities near the outer wall were slightly increased when the degree of swirl was increased from 0° to 60° . The latter, disagrees with the present results where the swirl was found to greatly increase the eddy viscosities μ_{rx} and $\mu_{r\theta}$ near the outer wall.

However, the accuracy of the results obtained by Scott and Rask is questionable because the axial and tangential viscosities were computed from their measured velocity and static pressure distributions with the aid of simplified momentum equations and no turbulence measurements were taken. Moreover, Scott and Bartelt [20] in a study of decaying annular swirl flow with solid body rotation at inlet (rotating outer wall only) concluded that the distribution and magnitude of the tangential eddy viscosity, near both walls, were independent of the axial location and of the initial rate of swirl. Again, the swirl should significantly increase the eddy viscosities near the outer wall as illustrated before and reported in other studies [21, 23, 86, 103, 104].

The distributions of the viscosity ratio $\sigma_{r\theta}$ ($\mu_{rx}/\mu_{r\theta}$) for $SN = 0.23, 0.47$ and 0.87 are shown in Figs. 6.72-75 at the first, middle and last axial stations. These figures show that $\sigma_{r\theta}$ can be considered unity in the near wall regions ($0.15 > r^* > 0.85$) with an error less than 5 percent while in the central flow region its value increases by 10 to 25 percent. Also increasing the swirl number from 0.23 to 0.87 has no significant effect on the magnitude and the distributions of $\sigma_{r\theta}$ as shown in Fig. 6.75.

The data for $\sigma_{r\theta}$ are also in good agreement with those reported by Lilley [21] in his review of turbulence nonisotropy in swirling flows. Complicated formulae for $\sigma_{r\theta}$ have been derived by So [23] from Reynolds-stress equations by assuming that production of turbulent energy balances viscous dissipation while Lilley [10, 21] proposed a simple empirical expression as a function of the swirl number only.

Bissonnette and Mellor [59] measured the mean velocities and the shear stresses for flow over a rotating cylinder at two different

rotational speeds. The mean value of $\sigma_{r\theta}$ was found to be 1.4 for ($y/\delta_\theta = 0.1$ to 0.9 where δ_θ is the circumferential displacement thickness) with a standard deviation of 20%. They also found that the scatter in the data was large and therefore no trend of $\sigma_{r\theta}$ with respect to x or r could be conclusively identified. Moreover, So [23] applied his formula to their results and found that for $y/\delta_\theta = 0.1$ to 0.6 the mean value of $\sigma_{r\theta}$ was 1.25 with no particular trend with respect to x or r . To generalize, the present data and previous studies show that $\sigma_{r\theta} > 1$, especially away from the walls, and no significant change was found in the values of $\sigma_{r\theta}$ with respect to axial location.

The computed values for the coefficient C_μ of equation (3.12), are plotted in Fig. 6.76 for the tested swirl numbers at the last axial station. As can be seen, C_μ near the walls is nearly constant and equal to the conventional constant 0.09 [24, 43], while in the central flow region larger scatter and higher values are observed as the swirl number increases from 0 to 0.87. The present results are in agreement with those published by Launder [34] where C_μ was found to be a function in the local ratio of the turbulent kinetic energy production to its dissipation and C_μ varied from 0.09 under the conditions of equilibrium to a value ranging between 0.12 to 0.56 near the axis of flow symmetry. The literature [31-36] contains different relations which take the effects of swirl and/or curvature radius into consideration for calculating the coefficient C_μ .

6.7 Mixing Length Distributions

The obtained values for the mixing lengths as calculated by equations (3.10) and (3.11) are shown in Figs 6.77 to 6.85 at the

first, middle and last axial stations. Data with a negative sign or unrealistic⁺ values, especially at the first station and/or in the central flow regions, are omitted. All the figures show that L_{rx} and $L_{r\theta}$ are approximately isotropic near the walls and that higher values are reached near the outer wall. Also as the swirl increases, the mixing lengths increase near the outer wall; very little effect is evident near the inner wall. This can be attributed to the effects of the positive velocity gradients and Richardson number near the inner wall which tends to stabilize the turbulence structure while their negative values near the outer wall induce an opposite effect. In the central flow region, despite the large scatter in the data, it was apparent that the mixing lengths increase with increasing swirl, see Figs. 6.84 and 6.85. The distributions of L_{rx} and $L_{r\theta}$ are generally similar for all swirl numbers and at all axial stations and no significant change is observed with respect to the axial position.

Mixing lengths as calculated by equation (3.26), i.e.

$$L_{rx} = K y \sqrt{\frac{\tau_{rx}}{\tau_{x,w}}} \quad \text{and} \quad L_{r\theta} = K y \sqrt{\frac{\tau_{r\theta}}{\tau_{\theta,w}}} \quad (6.5)$$

are also plotted for the last axial station in all figures. As can be seen, the above relations are in good agreement with the experimental data in the near wall regions but not in the central flow regions. This is not surprising as equations (6.5) and similar relations normally only represent conditions in the law-of-the wall region of shear flows. In the central flow regions shear stress and velocity gradients are small and eddy viscosities are therefore ill defined. Mixing lengths are therefore also ill defined, as seen from equations (3.10) and (3.11).

⁺ See footnote on page 108.

Unrealistic values or large scatter were consequently obtained in the central flow regions. Nevertheless, the experimental data indicate that the mixing lengths most likely reach their maximum or a certain non-zero value in the regions of low shear stress.

The mixing length distribution L_0 for the non-swirling flow case, Fig. 6.77, is in good agreement with the following empirical relation:

$$L_0 = 2 R' \left[0.07 - 0.04 \left(1 - \frac{Y}{R} \right)^2 - 0.03 \left(1 - \frac{Y}{R} \right)^4 \right] \quad (6.6)$$

where,

$$R' = r_0 - R_m \quad (\text{near outer wall}) \quad (6.7a)$$

$$\text{and } R' = R_m - r_i \quad (\text{near inner wall}) \quad (6.7b)$$

R_m is the radius at the point of maximum velocity in a fully developed flow with zero swirl in an annulus; its value can be calculated by equation (6.3). Equation (6.6) is also plotted in Fig. 6.77 for $R' = h/2$ where h is the annulus gap. In the latter case, equation (6.6) gives lower values for L_0 near the outer wall and higher values near the inner wall when compared with the experimental data. However, it gives a continuous profile in the central flow region which is in agreement with the experimental data while the values of R' as given by equations (6.7a) and (6.7b) result in a discontinuous profile if the radius is equal to R_m . Therefore, equations (6.7a) and (6.7b) are recommended for R' near the walls, with $R' = h/2$ in the central flow region; the transition point between relations should be chosen such

that the predicted mixing length L_0 is continuous. In the present study the experimental data for L_0 were used to calculate L_{rx} and $L_{r\theta}$ in the case of swirling flows as described below. The present data for the non-swirling flow case, are in agreement with those obtained elsewhere [72, 74, 102] where the mixing length was found to reach its maximum value at the location of maximum velocity or very close to the point of zero turbulent stress.

In the case of a pipe flow (i.e. $r_1 = R_m = R$) and $R' =$ pipe radius R), equation (6.6) reduces to the formula proposed by Nikuradse [16, 25] for the variation of the mixing length over the radius of a smooth or rough pipe; the mixing length was found to be independent of Reynolds number when values below 10^5 were excluded. Also, in the neighbourhood of the wall equation (6.6) can be simplified to

$$L_0 = y \left(0.4 - 0.44 \frac{y}{R} \right) \quad (6.8)$$

and it is apparent that Prandtl's hypothesis, $L_0 = Ky$, with $K = 0.4$, is confirmed for small distances from the wall. Equations (6.5) and (6.6) are plotted in Fig. 6.86 for fully developed flow in a pipe. The value required for τ_{rx}/τ_w in the first equation was calculated from equation (6.4). As can be seen, the difference between the two relations is small near the wall while at the pipe centreline equation (6.5) gives a zero value and equation (6.6) gives a maximum value for the mixing length. Nevertheless, Nikuradse's relation is often recommended in the literature [16, 25] for calculating the mixing length of a turbulent flow in a pipe. Accuracy of formulae for predicting eddy viscosities and mixing lengths is not critical in central regions of flows precisely

because velocity gradients and shear stress are small in these regions.

The mixing-length correction formula for a stratified atmosphere deduced by Monin and Oboukhov [15, 21, 23] is commonly used to account for effects of the external body force on the mixing length in the case of curved shear flows or swirling flows; this formula is given by:

$$L = L_0 (1 - \beta Ri) \quad (6.9)$$

where the mixing length L represents L_{rx} or $L_{r\theta}$, β is an adjustable parameter, Ri is the Richardson number defined by equation (3.16) and L_0 is the mixing length for non-swirling flow. Bradshaw [15] postulated that the Monin-Oboukhov formula can be generally applied to account for the effect of streamline curvature and centripetal accelerations on the mixing length in the boundary layer only if $W/r \ll (\partial W / \partial r)$ or the absolute value of Richardson number is small. Equation (6.9) was found to be only valid for weakly swirling flows in pipes [21, 23] with swirl number less than 0.4. In general, equation (6.9) is appropriate for flows with negative Richardson number (e.g. unstable flows on concave surfaces). For flows with positive Ri (e.g. stable flows on convex surfaces) equation (6.9) gives negative values for L if $\beta Ri > 1$. To avoid the latter, the following empirical formula is recommended [14].

$$L = L_0 / (1 + \beta Ri) \quad (6.10)$$

Using equations (6.9) and (6.10) and the experimental data for L_0 the distributions of the mixing lengths L_{rx} and $L_{r\theta}$ were calculated.

The values of β that provided best fitting for the experimental data were: $\beta = 2.5$ and 1.5 in the computed data near the outer and inner walls, respectively. Also, β was kept the same for both L_{rx} and $L_{r\theta}$ because the experimental data of Figs. 6.72 to 6.74 and 6.79 to 6.85 indicated that their values are approximately the same near both walls. However in the literature β ranged from 2 to 4.5 for $Ri < 0$ [14, 21, 23] and 4 to 7 for $Ri > 0$ [14], and in every case its value was selected to fit the experimental data. Differences can be attributed to flow geometry, the definition of Richardson number and whether the case is two or three dimensional flow. The author is unaware of data in the literature for comparison of mixing length distributions and β values for a swirling flow in an annulus.

Figs. 6.78 to 6.83 show that the values calculated from equations (6.5), (6.9) and (6.10) at the last axial station for L_{rx} and $L_{r\theta}$ are in good agreement with the experimental data in the wall layers for $SN = 0.23$ and that the discrepancy between predictions and data increases with increasing swirl number especially near the inner walls. Moreover, Figs. 6.80 to 6.83 for $SN = 0.47$ and 0.87 show that equation (6.5) predicts the mixing lengths more accurately than equations (6.9) and (6.10) especially near the inner wall. These results are in agreement with the general conclusions reported in many studies [14, 15, 21, 33] where the Monin-Oboukhov relation or equations (6.9) (6.10) were found to predict L_{rx} or $L_{r\theta}$ with acceptable accuracy in the wall layers only if the Richardson number is small or the swirl number is less than 0.4.

In other studies [21, 105] for swirling flows with combustion, the Monin-Oboukhov relation was used to calculate $\sigma_{r\theta}$ rather than the

mixing length L_{rx} . Lilley in his studies [10, 21] used equation (3.10) to calculate μ_{rx} for swirling flow cases and recommended a linear empirical formula for L_{rx} as a function of the swirl number, SN; see equation (4.3) for the definition of SN. The foregoing illustrates the difficulty involved in defining the mixing lengths for such complicated flow cases and points out the need for more theoretical or experimental work that deals with it.

6.8 Turbulence Intensities

The root mean square of the turbulent velocities $\sqrt{u^2}/\bar{Q}$, $\sqrt{w^2}/\bar{Q}$ and $\sqrt{v^2}/\bar{Q}$ ($\bar{Q}$ is defined by equation 4.2) are presented in Figs. 6.87 to 6.98 at the first, middle and last axial stations for the tested swirl numbers. The data show that the maximum measured value for any of the turbulent velocities is less than 12 percent and the turbulence level (i.e. $\sqrt{\frac{1}{3}(u^2 + v^2 + w^2)}/\bar{Q}$) is below 12 percent. The assumption of low turbulence intensity made in the data-reduction equations is therefore justified. All figures indicate that the turbulent velocities are generally nonisotropic (i.e. $\sqrt{u^2} > \sqrt{w^2} > \sqrt{v^2}$) and the degree of nonisotropy varies with the swirl number. For example, the non-swirling flow data show that $\sqrt{w^2}$ and $\sqrt{v^2}$ are nearly equal (within 25 percent difference) and their values are approximately 50 percent less than $\sqrt{u^2}$. The difference between the values of $\sqrt{u^2}$ and $\sqrt{w^2}$ decreases as the swirl increases until they become nearly equal and approximately twice to three times the value of $\sqrt{v^2}$ especially in the near wall regions for SN = 0.87, see Figs. 6.99 to 101. Also at the last station, the profiles of $\sqrt{u^2}$ have an approximately parabolic shape for all swirl numbers and the profiles of $\sqrt{w^2}$ are generally similar to those of $\sqrt{u^2}$.

for $SN = 0.47$ and 0.87 ; swirl has no significant effect on the $\sqrt{v^2}$ profiles. This is an expected result since in the presence of swirl both of the components $\sqrt{u^2}$ and $\sqrt{w^2}$ receive energy by production directly due to the velocity gradients of U and W . In all cases, the turbulent velocities exhibit higher values near the outer wall than near the inner wall. When swirl is present this can be attributed to the effect of the positive gradients of angular momentum near the inner wall which tends to stabilize the turbulence structure while the negative gradients near the outer wall induce an opposite effect. As the swirl number increased from 0 to 0.87, $\sqrt{w^2}/\bar{Q}$ increased by approximately 55 to 75 percent while no significant increase was found in the other components, $\sqrt{u^2}/\bar{Q}$ and $\sqrt{v^2}/\bar{Q}$, at all axial stations, see Figs. 6.99-101. This can be attributed to the significant increase in C_w^* relative to the small change in C_u^* with increasing the swirl number as explained before in section 6.4. Therefore, a small increase will be observed if $\sqrt{u^2}$, $\sqrt{w^2}$ and $\sqrt{v^2}$ are plotted relative to C_u^* , C_w^* and $\sqrt{C_u^* C_w^*}$, respectively. However, a noticeable increase would be observed in all turbulence velocities if they were plotted relative to the mean axial velocity $\bar{U}$ rather than $\bar{Q}$. Bearing in mind that the outer wall shear stress increases with increasing swirl, see Figs. 6.59 and 6.60, the present results can be considered consistent with those obtained by Brighten and Jones [72] for non-swirling annular flows where the turbulent velocities relative to the outer wall frictional velocity were found to be independent of the Reynolds number effects.

In all cases the minimum values of each of the three turbulent velocities occurred near the maxima of velocities U and W , where little or no local production takes place. For the non-swirling flow case,

$\sqrt{w^2}$ and $\sqrt{v^2}$ were nearly equal at the point of maximum velocity while $\sqrt{u^2}$ was about 40 percent higher. Also at this point and as the swirl increases, the difference between $\sqrt{u^2}$ and $\sqrt{w^2}$ decreases until for $SN = 0.87$ they become nearly equal and higher than $\sqrt{v^2}$ by 30 to 45 percent.

The present data at the last axial station for the non-swirling flow case are generally in good agreement with those obtained by other researchers for non-swirling turbulent flows in annuli [72, 73, 102]. The results of those studies confirmed that the turbulence velocities are not isotropic even if the flow field has reached the state of fully developed flow and the velocity gradient $\partial U / \partial r$ is zero. At this radial position, where U is maximum, the component $\sqrt{u^2}$ was found to be 15 to 35 percent higher than the other components $\sqrt{w^2}$ and $\sqrt{v^2}$. For non-swirling flow in a pipe Laufer [106] found that $\sqrt{u^2}$ was only slightly higher (about 5 percent) than $\sqrt{v^2}$ and $\sqrt{w^2}$ at the point of maximum velocity. However, Sandborn [107] reported that $\sqrt{u^2}$ was generally 20 to 30 percent higher than $\sqrt{v^2}$ and $\sqrt{w^2}$ on the axis of flow in a pipe. For fully developed flow between parallel walls, Laufer's [106] results showed that $\sqrt{u^2}$ was about 38 percent higher than $\sqrt{v^2}$ and $\sqrt{w^2}$ at the point of maximum velocity.

All figures of this section show that the turbulent velocities have higher values near the outer wall than near the inner wall. For example Fig. 6.89 for the non-swirling flow case shows that values of $\sqrt{u^2}$, $\sqrt{w^2}$ and $\sqrt{v^2}$ at $r^* = 0.96$ are higher than their values at $r^* = 0.04$ by 30, 32 and 15 percent, respectively. This may be attributed to the effect of the annulus radius ratio on the radial distributions of the turbulence velocities. The experiments by Rehme for straight flows

in annuli [6] indicated that the ratio of r_i/r_o has a significant effect on the values of the turbulent velocities near the inner wall and a slight effect near the outer wall. His results also showed that ratio of turbulent velocities near the outer wall to those near the inner wall decreased from about 2 to 1 when r_i/r_o was increased from .01 to 1. Other experiments for straight flows in annuli with $r_i/r_o = 0.56$ and 0.52 [72, 73] also confirmed that values of turbulent velocities at $r^* = 0.95$ are higher than their values at $r^* = .05$ by 20 to 30 percent for $\sqrt{u^2}$, 12 to 25 percent for $\sqrt{w^2}$ and 18 to 45 percent for $\sqrt{v^2}$.

Concerning the behavior of the turbulent intensities for the case of swirling flows in annuli, the author is not aware of any information available in the literature for comparison. The complexity associated with such flows is the main obstacle to conducting theoretical and/or experimental work that deals with swirling flows. As advanced equipment such as data acquisition and control systems become more available for different research institutes, more research may become available on swirling flows in annuli. The author hopes that the present study has shed light on some of the aspects of swirling flows in annuli and will lead to more accurate computational methods in future.

CHAPTER VII

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary

This thesis reports on an experimental and theoretical investigation of a developing turbulent swirling flow in a stationary annulus. The test section was a constant area annular duct of 0.4 and 0.25 m outer and inner wall diameters, respectively (0.625 diameter ratio). The effects of swirl number and transverse curvature on the radial distributions of the following flow parameters along the tunnel were studied:

- The resultant flow velocity and its axial and tangential components
- Static and total pressures
- Wall shear stresses
- Axial and tangential turbulent stresses
- Axial and tangential eddy viscosities
- Mixing lengths
- Turbulence intensities

Simplified versions of the momentum equations were used to derive approximate relations for the distributions of the axial and tangential turbulent stresses near the walls. Two new versions for the logarithmic law of the wall were derived for the distributions of axial and tangential velocities in the near wall regions. The latter introduced a modified Clauser chart technique for determining the wall shear stress in the case of swirling flows.

The three time-mean velocity components and the six Reynolds stresses were determined from measurements taken by single-sensor 45° slant and normal hot-wire probes. The total pressure and the flow angle were measured using a 3-hole cobra probe while the static pressure was computed by integrating the radial momentum equation. The cobra probe was used as a Preston tube to determine the inner wall shear stress while the Clauser chart method was employed to determine both inner and outer wall shear stresses. Mixing length distributions were computed from equations (3.10) and (3.11) and the experimental data for the velocity gradients and the eddy viscosities. These distributions were compared with the values predicted by several empirical relations.

Many consistency checks were applied to the data and in all cases close agreement was found between results arrived at by different methods and data. This gives considerable confidence in the validity of the data.

7.2 Conclusions

The following main conclusions may be drawn from the results obtained in the present work.

7.2.1 Velocity Profiles and the Law of the Wall

The velocity profiles for the non-swirling flow case exhibited the characteristics of a developing turbulent flow in an annulus. However, fully developed flow was not reached in the measurement length.

No indications of reversed or separated flow were seen in any of the results. As expected, the mean axial velocity was independent of swirl number and axial location. The tangential velocity was found to

decay continually in the axial flow direction and its rate of decay increased with increasing swirl number. The vortex equation (6.2) predicts the axial and tangential velocities within 5 percent in the central flow region at the first axial station for all swirl numbers. The radial velocity component was very small (less than 2% of the mean axial velocity) and its terms could be neglected in the data-reduction equations.

In the case of swirling flows, the universal law of the wall cannot be applied for predicting the axial or tangential velocities near the wall; it should be modified to account for the effects of swirl and streamlines curvature. However, the newly derived logarithmic laws, equations (3.41-43), are in good agreement with the experimental profiles for the axial and tangential velocities in the near wall regions. These new laws are applicable to any swirling flow or to flows with transverse curvature even if the turbulent stresses substantially vary across the wall layer. Both radius of curvature and swirl number induce large effects on the distribution of the tangential velocity near the walls; the stronger effects were found near the inner wall(convex). However, the radius of curvature did not affect the axial velocity profiles; these are only affected by the swirl number.

The universal law of the wall may be applied for predicting the resultant flow velocity near the wall. As the swirl number increases above about 0.2 there is a decrease in the radial extent of the region over which the universal log-law fits the data, especially near the inner(convex) walls. In these cases, it is advisable to calculate the resultant flow velocity from its axial and tangential components which can be calculated from the new log-laws over a wider radial extent.

7.2.2 Total and Static Pressure Distributions

A 3-hole cobra probe can be used to measure the total pressure in swirling flows if the radial velocity is negligible. However, its calibration curves for static pressure in a non-swirling flow were found to be invalid for swirling flows (i.e. errors up to 40 percent were found in the data for 0.87 swirl number). Therefore, the static pressure distribution was computed by integrating the radial momentum equation. The outer wall static pressure is higher than at the inner wall and the difference between them increases with increasing swirl number. Static pressures measured on the inner wall agreed closely with those obtained by integration of the radial momentum equation.

7.2.3 Wall Shear Stress

The application of the modified Clauser chart method to the axial and tangential velocities, in the case of swirling flows, determines C^*_U (i.e. $U^* \sqrt{\tau_{x,w}/\tau_w}$) or C^*_w (i.e. $w^* \sqrt{\tau_{\theta,w}/\tau_w}$), respectively. These values and equations (5.31) and (5.35) give the values of the frictional velocities U^* , w^* and Q^* . Another way of obtaining the wall shear stress and hence the frictional velocities is to use the 3-hole probe as a Preston tube. This could only be done on the inner wall in the present work. Frictional velocities obtained by the two methods were in good agreement. The swirl number was found to have significant effects on the parameter C^*_w and on its axial distribution but little or no effect on C^*_U .

For the non-swirling flow, the inner wall shear stress, $\tau_{\theta,wi}$ was 8 percent higher than the outer wall shear stress, $\tau_{\theta,wo}$. For swirl number 0.23 both wall stress components, $\tau_{x,w}$ and $\tau_{\theta,w}$, were

approximately equal and further increase in the swirl number resulted in higher wall stress on the outer wall than on the inner wall. It was also found that the ratios $\tau_{x,w}/\tau_{o,w}$ and $\tau_{\theta,w}/\tau_{o,w}$ on both walls increase with increasing the swirl number. This increase was expected near the outer wall due to the de-stabilizing effects of swirl on the turbulence structure. However, the apparent increase near the inner wall might be attributed not to the effects of swirl but to the use of inlet guide vanes with constant outlet flow angle which implies higher axial and tangential velocities near the inner walls.

7.2.4 Turbulent Shear Stresses

Equations (3.18, 19) and (3.21, 22) predict the axial and tangential turbulent shear stresses, τ_{rx} and $\tau_{r\theta}$, with good accuracy near the wall. At distances far from the wall, equations (3.21, 22) give more accurate values for $\tau_{r\theta}$ than those predicted for τ_{rx} by equations (3.18, 19). Near the points of maximum axial or tangential velocity, τ_{rx} or $\tau_{r\theta}$ were found to be very small or approximately zero.

For the non-swirling flow case, the distributions of τ_{rx} near both walls are nearly symmetrical at all axial stations. With increased swirl, steeper radial gradients were found in the τ_{rx} and $\tau_{r\theta}$ distributions near the inner wall while an opposite trend was observed near the outer wall. The profiles of τ_{rx} and $\tau_{r\theta}$ do not exhibit a constant stress region near the walls; the profiles are similar to those found in turbulent pipe flow, especially near the outer wall. The resultant shear stress was found to be aligned with the flow direction within 0.5° in the near wall regions.

7.2.5 Eddy Viscosities

The eddy viscosities μ_{rx} and $\mu_{r\theta}$ increase with the distance from either wall up to a maximum value after which they apparently start to decrease until they reach a minimum value at a radial position r^* ranging from 0.4 to 0.52. In all cases, μ_{rx} and $\mu_{r\theta}$ have higher maximum values near the outer wall than near the inner wall. Increasing the swirl number increases the maximum eddy viscosities significantly near the outer wall (e.g. doubled for $SN = 0.87$) while only a marginal increase was found near the inner wall. μ_{rx} and $\mu_{r\theta}$ are approximately isotropic (within 5% difference) near either wall while the viscosity ratio $\sigma_{r\theta}$ was always greater than unity in the central flow region (e.g. $\sigma_{r\theta} = 1.10 - 1.25$ near the axis of the annulus gap). No particular trend was found in the distribution of $\sigma_{r\theta}$ with respect to the axial flow direction.

7.2.6 Mixing Lengths

The mixing lengths L_{rx} and $L_{r\theta}$ are approximately isotropic near either of the walls and they reach higher maximum values near the outer wall than near the inner wall. Increasing the swirl number increases the maximum mixing lengths near the outer wall while no distinct trend was found near the inner wall. No significant change was found in the distributions of L_{rx} and $L_{r\theta}$ in the axial flow direction. Large scatter was found in the data of the central flow region; most likely the mixing lengths reach some maximum or a certain non-zero value at the point of zero shear stress.

Equations (6.5) predict the mixing lengths with good accuracy near both walls for all tested swirl numbers. The Monin-Oboukhov relations

(6.9) and (6.10) predict the mixing lengths accurately if the swirl number is less than 0.23; accuracy decreases with increasing swirl number, especially near the inner wall. For the non-swirling flow case, equation (6.6) also predicts the mixing length with good accuracy near both walls.

7.2.7 Turbulence Models

The aforementioned results indicate that for prediction purposes the eddy viscosities can be calculated in the case of swirling annular flows from equations (3.10) and (3.11), i.e.

$$\mu_{rx} = \rho L_{rx}^2 \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{1/2} \quad (7.1)$$

$$\mu_{r\theta} = \rho L_{r\theta}^2 \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{1/2} \quad (7.2)$$

with the mixing lengths L_{rx} and $L_{r\theta}$ to be computed from the relations recommended in the previous subsection.

The $k-\epsilon$ model given by equation (3.12) may also be used to compute the viscosity μ_{rx} in swirling flow cases if the coefficient C_μ is modified to be a function of radial location and swirl number. The viscosity $\mu_{r\theta}$ can be calculated using the viscosity ratio given by equation (3.15) and μ_{rx} . However, as mentioned in Chapter III, the $k-\epsilon$ model does not give good predictions of swirling flows unless some of the coefficients in the ϵ -equation are modified to be functions of the Richardson number. The latter was not investigated in this study, but in future studies relations for swirling flows in annuli can be compared with the present data to assess their validity.

7.2.8 Turbulence Intensities

The maximum measured value for any of the turbulent velocities was 12 percent and the turbulence level was below 12 percent. Turbulent velocities were generally nonisotropic and the degree of nonisotropy varied with the swirl number. For zero swirl flow, $\sqrt{v^2}$ and $\sqrt{w^2}$ were nearly equal while for the highest swirl number $\sqrt{u^2}$ and $\sqrt{w^2}$ were approximately equal. Increasing the swirl number from 0 to 0.87 increased $\sqrt{w^2}/\bar{Q}$ by 55 to 75 percent while no significant increase was found in the other components, $\sqrt{u^2}/\bar{Q}$ and $\sqrt{v^2}/\bar{Q}$, at all axial stations. In all cases the minimum values of each of the turbulent velocities occurred at the maxima of velocities U and W where no local production takes place. Also the highest values for each of the turbulent velocities were found to be near the outer wall in all experiments.

7.3 Recommendations for Future Research

The present study on turbulent swirling flow in an annulus of constant area has shown that there is further need to carry out both analytical and experimental work to investigate the effects of high swirl number ($SN > 0.9$) and annulus geometry on the various flow parameters. As a result of this study, the following recommendations are proposed for future research in the area of swirling flows in annuli:

- 1 - Numerical solution of equations (3.1-4) should be carried out using available computer programs for parabolic flows [48, 49], modified to apply the new logarithmic wall-laws and the turbulence model recommended in section 7.2-7. Predicted data for the mean

velocity profiles and major turbulent shear stresses, τ_{rx} and $\tau_{r\theta}$, could be compared with the experimental results of the present study and further progress may be achieved in the computation of swirling flows in general. Furthermore, it might be feasible to modify the general computer program of Spalding [48] to predict swirling flows in annuli of any geometry (i.e. different values for r_i/r_o ratio and for outlet to inlet area ratio).

- 2 - Further experimental investigations should be conducted to examine the effects of annulus geometry, in the case of swirling flows, on the various flow parameters.
- 3 - Efforts should be made to develop hot-wire techniques for measurement of 3-dimensional flows with turbulence intensity higher than 25 percent.
- 4 - Empirical correction factors should be developed for measurements with pressure probes (3-hole and 5-hole probes) to account for the effects of swirl and/or velocity gradients on calibrations obtained in uniform flows.

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Table 4.1

Location of the instrumentation ports along the outer wall

Station No.	Distance from Blade Trailing Edge in cm	Hydraulic Diameters
1	25.4	1.67
2	55.9	3.67
3	86.9	5.67
4	116.8	7.67
5	157.5	10.33

Table 4.2

Different equipment used for measurements with the hot-wire probes

Model	Equipment
(TSI) 1051-6	Monitor and power supply
(TSI) 1056	Variable decade probe resistance
(TSI) 1053 A	Constant temperature anemometer
HP-141A	Oscilloscope (used for setting the anemometer before use)
HP-3054A	Automatic Data Acquisition and Control System (ADACS) Its components are: a - HP85A desktop computer b - HP3456A digital voltmeter c - HP3497A data acquisition/control unit
Brueel & Kjaer 2425	True RMS voltmeter
DISA 55H20	Probe support
DISA 55P12	45 deg. slanted miniature hot-wire probe
DISA 55P11	Normal probe

Table 4.3

Parameters Measured At Each Traverse Station

Measured Quantity	Description	Probe Used	Reasons For Measuring
U, V and W	Time-mean velocity components (axial, radial and tangential directions respectively)	Single slant hot-wire (DISA P12)	Study the decay of the tangential velocity and the development of the axial velocity components along the annular duct
$\overline{uv}$, $\overline{vw}$, $\overline{uw}$ $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$	The six components of Reynolds stress tensor	Single slant and normal hot-wires (DISA P12 and DISA P11)	Study the effect of swirl on turbulent stresses and examine the validity of some turbulence models
P_t (r)	Total pressure at any radius	3-hole cobra probe (United Sensor CA-120)	Study the effect of swirl on total pressure distribution
τ_{wi}	Wall shear stress at inner wall	Same 3-hole probe is used as Preston tube	Study effect of swirl on wall shear stress and compare experimental data with Clauser chart results
P_{wo} and P_{wi}	Static pressure at the outer and inner walls	Wall static taps	Compute static pressure distribution and wall stress τ_{wi} . Also check for possibility of reversed flow.

Table 4.4

Values Of Probe Rotation Angles Ψ For Slanted And Normal
Hot Wires Recommended With Different Swirl Numbers.

Nominal swirl number SN	Ψ values for slanted probe ($\alpha = 45^\circ$)* (Degrees)	Ψ values for normal probe ($\alpha = 0$) (Degrees)
0.87	0 , 45 , 90	- 45 , 0 , 90
0.47	0 , 45 , 90	- 45 , 0 , 90
0.23	0 , 45 , 90	- 45 , 45 , 90
0	0 , 45 , 90	- 45 , 45 , 90

* α is the complementary angle to the angle between the probe stem
and the wire sensor, see Fig. 4.10.

Table 6.1

Values of axial mean velocity, U , at the different
axial stations for the tested swirl numbers

Swirl number	U (m/sec)				
	1	2	3	4	5
0	31.2	31.6	31.3	31.4	31.6
0.23	32.4	32.5	32.4	32.8	32.3
0.47	32.4	32.6	32.5	32.4	32.5
0.87	32.7	32.6	32.8	32.6	32.9

Table 6.2

Ratios between left and right hand sides of
x-integrated forms of equations (3.24a) and (3.25a)

Swirl Number	I ₁ /I ₂ Ratio*		I ₃ /I ₄ Ratio**	
	Bet. 1st and 3rd stations	Bet. 1st and last stations	Bet. 1st and 3rd stations	Bet. 1st and last stations
0	0.92	1.19	N/A	N/A
0.23	0.95	1.16	0.84	0.89
0.47	0.83	0.82	1.18	1.06
0.87	0.81	0.91	0.93	1.11

* From Eq. (3.24a) (axial momentum)

$$\underbrace{\frac{1}{\rho} \int_{x_a}^{x_b} [r_o \tau_{w,xo} - r_i \tau_{w,xi}] dx}_{I_1} = \underbrace{\int_{x_a}^{x_b} \frac{\partial}{\partial x} \left[\int_{r_i}^{r_o} (U^2 + \frac{p}{\rho}) r dr \right] dx}_{I_2}$$

** From Eq. (3.25a) (angular momentum)

$$\underbrace{\frac{1}{\rho} \int_{x_a}^{x_b} [r_o^2 \tau_{\theta,wo} - r_i^2 \tau_{\theta,wi}] dx}_{I_3} = \underbrace{\int_{x_a}^{x_b} \frac{\partial}{\partial x} \left[\int_{r_i}^{r_o} (U^2 + \frac{p}{\rho}) r dr \right] dx}_{I_4}$$

x_a = axial distance at first axial station

x_b = axial distance at third or last axial station

Fig. 2.1 Nomenclature of wall layer, Ref. [22]

Fig. 3.1 Definition sketch for coordinate system and velocity components

Fig. 4.1. Annular wind tunnel, Ref. [87]

Fig. 4.2 Test section

Fig. 4.3 Plexiglass attachment used to pass the hot-wire probes through the outer wall of the test section

Fig. 4.4a Inlet guide vanes

blade camber-line coordinates and the thickness distribution

x measured on chord-line mm	y camber-line mm	thickness mm	
0.00	0.00	---	
4.76	2.38	2.63	
11.11	3.97	3.43	
17.46	4.76	5.02	
23.81	5.55	5.25	max. thickness & position of axis of rotation
30.16	4.76	5.02	
36.51	3.97	3.82	
42.86	3.17	3.43	
49.21	2.38	2.63	
61.91	0.00	0.96	trailing edge thickness

Fig. 4.4b Dimensions of blade profile used for inlet guide vanes

Fig. A.5 Air distributing chamber used to distribute the air supplied to the inlet guide vanes

Fig. 4.6 Traverse gear

Fig. 4.8 Definition of pitch and yaw

Fig. 4.7 3-hole cobra probe

Fig. 4.9a Arrangement of the equipment used for the measurements with the hot-wire probes

Fig. 4.9b Equipment used for measurements with the hot-wire probes

Fig. 4.10

Schematic diagram showing the resolution of the resultant velocity vector Q in x, y, z and ξ, η, ζ coordinate systems

- 1 - Air jet nozzle
- 2 - Pitot tube
- 3 - Hot-wire probe holder
- 4 - Positioning attachment
- 5 - Vice
- 6 - Attachment to rotate probe about its own axis

Fig. 4.11a Experimental set-up
used for calibrating
hot-wire probes

Fig. 4.11b Schematic diagram of the experimental set-up used for calibrating hot-wire probes

Fig. 4.12 Schematic diagram of the control circuit

Fig. 5.2 Schematic diagram showing the elements for calculating the mass flow rate by equation (5.20)

$$f = \frac{W^2 + w^2 - v^2}{r}$$

Fig. 5.2 Schematic diagram showing the elements for calculating the static pressure at any radius r_j by equation (5.24)

Fig. 6.1 Radial distributions of flow angles at the different axial stations for $SN = 0.23$

Fig. 6.2 Radial distributions of flow angles at the different axial stations for SN = 0.47

Fig. 6.3 Radial distributions of flow angles at the different axial stations for $SN = 0.87$.

Fig. 6.4 Radial distributions of the axial velocity at the different axial stations for non-swirling flow.

Fig. 6.5 Radial distributions of the axial velocity at the different axial stations for $SN = 0.23$

Fig. 6.6

Radial distributions of the axial velocity at the different axial stations for $SN = 0.47$

3

Fig. 6.7 Radial distributions of the axial velocity at the different axial stations for SN = 0.87

Fig. 6.8 Radial distributions of the axial velocity at the first axial station for different swirl numbers

Fig. 6.9 Radial distributions of the axial velocity at the last axial station for different swirl numbers

Fig. 6.10 Semi-log plot of the axial velocity profiles for non-swirling flow

Fig. 6.11 Semi-log plot of the axial velocity profiles for $SN = 0.23$

Fig. 6.12 Semi-log plot of the axial velocity profiles for $SN = 0.47$

Fig. 6.13 Semi-log plot of the axial velocity profiles for $SN = 0.87$

Fig. 6.14 Radial distributions of the tangential velocity at the different axial stations for $SN = 0.23$

Fig. 6.15 Radial distributions of the tangential velocity at the different axial stations for $SN = 0.47$

Fig. 6.16 Radial distributions of the tangential velocity profiles for $SN = 0.87$

Fig. 6.17 Radial distributions of the tangential velocity at the first axial station for different swirl numbers

Fig. 6.18 Radial distributions of the tangential velocity at the last axial station for different swirl numbers

Fig. 6.19 Axial distributions of the swirl number

Fig. 6.20 Semi-log plot of the tangential velocity profiles for $SN = 0.23$

Fig. 6.21 Semi-log plot of the tangential velocity profiles for $SN = 0.47$

Fig. 6.22 Semi-log plot of the tangential velocity profiles for $SN = 0.87$

Semi-log plot of the velocity profiles to show the law of the wall region.
 $\circ, x = 24 \text{ in.}; \dots, x = 54.5 \text{ in.}; \square, x = 58 \text{ in.}; \Delta, x = 67 \text{ in.}; \bullet, x = 71 \text{ in.}$

Fig. 6.23 Comparison between equation (3.42) and the experimental results of So and Mellor [13] for turbulent boundary layer on convex wall

Fig. 6.24 Comparison between equations (3.42) and (3.43) and the experimental results of Shivaprasad and Ramaprian [14] for turbulent boundary layers along mildly curved surfaces

Fig. 6.25 Radial distributions of the radial velocity at different axial stations for $SN = 0.87$

Fig. 6.26 Semi-log plot of the resultant velocity profiles for $SN = 0.23$

Fig. 6.27 - Semi-log plot of the resultant velocity profiles for $SN = 0.47$

Fig. 6.28 Semi-log plot of the resultant velocity profiles for $SN = 0.87$

Fig. 6.29 Axial distributions of outer and inner wall pressures for non-swirling flow (P_a is the ambient static pressure)

Fig. 6.30 Axial distributions of outer and inner wall pressures for $SN = 0.23$

Fig. 6.31 Axial distributions of outer and inner wall pressures for $SN = 0.47$

Fig. 6.32 Axial distributions of outer and inner wall pressures for $SN = 0.87$

Fig. 6.33 Radial distributions of the static pressure at the different axial stations for $SN = 0.23$

Fig. 6.34 Radial distributions of the static pressure at the different axial stations for $SN = 0.47$

Fig. 6.35 Radial distributions of the static pressure at the different axial stations for $SN = 0.87$

Fig. 6.36 Radial distributions of the total pressure at the different axial stations for non-swirling flow

Fig. 6.37 Radial distributions of the total pressure at the different axial stations for $SN = 0.23$

Fig. 6.38 Radial distributions of the total pressure at the different axial stations for $SN = 0.47$

Fig. 6.39 Radial distributions of the total pressure at the different axial stations for $SN = 0.87$

Fig. 6.40 Axial distributions of the parameter C_u^* relative to the frictional velocity at zero swirl, U_o^* , for the tested swirl numbers (Clauser chart results)

Fig. 6.41 Axial distributions of the parameter C_w^* relative to the frictional velocity at zero swirl, $U_{0,0}^*$, for the tested swirl numbers (Clauser chart results)

Fig. 6.42 Axial distributions of the resultant frictional velocity for different swirl numbers

Fig. 6.43 Axial distributions of axial wall shear stress, $\tau_{x,w}$, for the tested swirl numbers.

Fig. 6.44 Axial distributions of the tangential wall shear stress, $\tau_{\theta,w}$, for the tested swirl numbers.

Fig. 6.45 Effect of swirl number on the axial wall shear stress, $\tau_{x,w}$, relative to its value at zero swirl, $\tau_{o,w}$

Fig. 6.46 Effect of swirl number on the tangential wall shear stress, $\tau_{\theta,w}$, relative to the axial wall shear stress at zero swirl, $\tau_{o,w}$.

Fig. 6.47 Axial distributions of outer to inner wall shear stresses for $SN = 0$

Fig. 6.48 Axial distributions of outer to inner wall shear stress components for $SN = 0.23$

Fig. 6.49 Axial distributions of outer to inner wall shear stress components for SN = 0.47

Fig. 6.50 Axial distributions of outer to inner wall shear stress components for SN = 0.87

Fig. 6.51 Ratio of inner to outer wall shear stress as presented by Pletcher and Malik [74] for non-swirling flow in an annulus of different radius ratios.

Fig. 6.52 Radial distributions of the axial turbulent shear stress at different axial stations for non-swirling flow. (Clauser charts data are plotted for τ_{rx} at $r^* = 0.0$ and 1.0)

Fig. 6.53 Radial distributions of the axial turbulent shear stress at different axial stations for SN = 0.23. (Clauser charts data are plotted for τ_{rx} at $r^* = 0.0$ and 1.0)

Fig. 6.54 Radial distributions of the tangential turbulent shear stress at different axial stations for $SN = 0.23$. (Clauser charts data are plotted for τ_{re} at $r^* = 0.0$ and 1.0)

Fig. 6.55 Radial distributions of the axial turbulent shear stress at different axial stations for $SN = 0.47$. (Clauser charts data are plotted for τ_{rx} at $r^* = 0.0$ and 1.0)

Fig. 6.56 Radial distributions of tangential turbulent shear stress at different axial stations for SN = 0.47.
(Clauser charts data are plotted for $\tau_{r\theta}$ at $r^* = 0.0$ and 1.0)

Fig. 6.57 Radial distributions of the axial turbulent shear stress at different axial stations for $SN = 0.87$. (Clauser charts data are plotted for τ_{rx} at $r^* = 0.0$ and 1.0)

Fig. 6.58 Radial distributions of turbulent tangential shear stress at different axial stations for $SN = 0.87$. (Clauser charts data are plotted for $\tau_{r\theta}$ at $r^* = 0.0$ and 1.0)

Fig. 6.59 Radial distributions of the axial turbulent shear stress at the last axial station for different swirl numbers. (Clauser charts data are plotted for τ_{rx} at $r^* = 0.0$ and 1.0)

Fig. 6.60 Radial distributions of the turbulent tangential shear stress at the last axial station for different swirl numbers.
(Clauser charts data are plotted for $\tau_{r\theta}$ at $r^* = 0.0$ and 1.0)

Fig. 6.61 Radial distributions of the Relative flow angle ϕ°

$$\phi^{\circ} = |\tan^{-1}(w/u) - \tan^{-1}(\tau_{r\theta} / \tau_{rx})|$$

Fig. 6.62 Radial distributions of the turbulent shear stress $\tau_{x\theta}$ at different axial stations for SN = 0.87

Fig. 6.63 Radial distributions of the axial eddy viscosity at different axial stations for non-swirling flow

Fig. 6.64 Radial distributions of the axial eddy viscosity at different axial stations for $SN = 0.23$

Fig. 6.65 Radial distributions of tangential eddy viscosity at different axial stations for $SN = 0.23$

Fig. 6.66 Radial distributions of the axial eddy viscosity at different axial stations for $SN = 0.47$

Fig. 6.67 Radial distributions of the tangential eddy viscosity at different axial stations for $SN = 0.47$

Fig. 6.68 Radial distributions of the axial eddy viscosity at different axial stations for SN = 0.87

Fig. 6.69 Radial distributions of the tangential eddy viscosity at different axial stations for $SN = 0.87$

Fig. 6.70 Radial distributions of the axial eddy viscosity at the last axial station for different swirl numbers

Fig. 6.71 Radial distributions of the tangential eddy viscosity at the last axial station for different swirl numbers

Fig. 6.72 Radial distributions of the eddy viscosity ratio, μ_{rx}/μ_{re} , at different axial stations for SN = 0.23

Fig. 6.73 Radial distributions of the eddy viscosity ratio, μ_{rx}/μ_{re} , at different axial stations for $SN = 0.47$

Fig. 6.74 Radial distributions of the eddy viscosity ratio, μ_{rx}/μ_{re} , at different axial stations for SN = 0.87

Fig. 6.75 Radial distributions of the eddy viscosity ratio, μ_{tx}/μ_{re} , at the last axial station for different swirl numbers

Fig. 6.76 Radial distributions of the coefficient C_μ at the last axial station for different swirl numbers

Fig. 6.77 Radial distributions of the mixing length L_0 for non-swirling flow.

Fig. 6.78 Radial distributions of the mixing length L_{rx} at different axial stations for $SN = 0.23$

Fig. 6.79 Radial distributions of the mixing length $L_{r\theta}$ at different axial stations for $SN = 0.23$

Fig. 6.80 Radial distributions of the mixing length L_{rx} at different axial stations for $SN = 0.47$

Fig. 6.81 Radial distributions of the mixing length $L_{r\theta}$ at different axial stations for $SN = 0.47$

Fig. 6.82 Radial distributions of the mixing length L_{rx} at different axial stations for $SN = 0.87$

Fig. 6.83 Radial distributions of the mixing length $L_{r\theta}$ at different axial stations for $SN = 0.87$

Fig. 6.84 Radial distributions of the mixing length L_{rx} at the last axial station for different swirl numbers

Fig. 6.85 Radial distributions of the mixing length $L_{r\theta}$ at the last axial station for different swirl numbers

Fig. 6.86 Radial distributions of the mixing length and the axial turbulent shear stress for fully developed turbulent flow in a smooth pipe

Fig. 6.87 Radial distributions of the turbulent velocities at the first axial station for non-swirling flow. ($\bar{Q} = 31.2$ m/sec)

Fig. 6.88 Radial distributions of the turbulent velocities at the middle axial station for non-swirling flow.
($\bar{Q} = 31.3$ m/sec)

Fig. 6.89 Radial distributions of the turbulent velocities at the last axial station for non-swirling flow. ($\bar{Q} = 31.6$ m/sec)

Fig. 6.90 Radial distributions of the turbulent velocities at the first axial station for $SN = 0.23$ ($\bar{U} = 33.6$ m/sec.)

Fig. 6.91 Radial distributions of the turbulent velocities at the middle axial station for $SN = 0.23$ ($\bar{Q} = 33.5$ m/sec)

Fig. 6.92 Radial distributions of the turbulent velocities at the last axial station for $SN = 0.23$ ($\bar{U} = 33.4$ m/sec)

Fig. 6.93 Radial distributions of the turbulent velocities at the first axial station for $SN = 0.47$
 $(Q = 37.4 \text{ m/sec})$

Fig. 6.94 Radial distributions of the turbulent velocities at the middle axial station for $SM = 0.47$ ($\bar{Q} = 37.0$ m/sec)

Fig. 6.95 Radial distributions of the turbulent velocities at the last axial station for $SN = 0.47$ ($\bar{Q} = 36.8$ m/sec)

Fig. 6.96 Radial distributions of the turbulent velocities at the first axial station for $SN = 0.87$ ($\bar{Q} = 45.5 \text{ m/sec}$)

Fig. 6.97 Radial distributions of the turbulent velocities at the middle axial station for $SN = 0.87$ ($\bar{Q} = 43.9$ m/sec)

Fig. 6.98 Radial distributions of the turbulent velocities at the last axial station for $SN = 0.87$ ($Q = 43.1$ m/sec)

Fig. 6.99 Radial distributions of $\sqrt{u^2}/\bar{u}$ at the last axial station for different swirl numbers

Fig. 6.100 Radial distributions of $\sqrt{w^2}/\bar{Q}$ at the last axial station for different swirl numbers

Fig. 6.101 - Radial distributions of $\sqrt{v^2}/\bar{Q}$ at the last axial station for different swirl numbers

APPENDIX A

TRANSPORT EQUATION FOR THE TURBULENT KINETIC ENERGY

The transport equation for the turbulent kinetic energy k , after neglecting the time dependent terms and using the boundary layer approximation [83] is given by:

$$\begin{array}{c}
 \text{convection} \quad \text{diffusion} \\
 \left| \frac{1}{r} \frac{\partial}{\partial r} (\rho r V k) + \frac{\partial}{\partial x} (\rho r U k) \right| = \left| \frac{\partial}{\partial r} \left[r \left(\mu \frac{\partial k}{\partial r} - \rho \overline{v k} - \overline{v p'} \right) \right] \right| \\
 \\
 \text{source} \\
 \left| + G - \rho \epsilon \right| \\
 \\
 \text{energy production} \quad \text{energy dissipation}
 \end{array} \tag{A.1}$$

where,

$$\begin{array}{c}
 \text{dominant terms} \\
 G = \rho \left(\overline{uv} \frac{\partial U}{\partial r} + \overline{vw} \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right] + \right. \\
 \left. \overline{v^2} \frac{\partial V}{\partial r} + \overline{u^2} \frac{\partial U}{\partial x} + \overline{uw} \frac{\partial W}{\partial x} \right)
 \end{array} \tag{A.2}$$

ϵ is the energy dissipation rate per unit mass. Modelling of the different terms in equation (A.1) is as follows:

Diffusion Term

The diffusion term is put in a typical diffusion form, in accordance with several authors [23, 24, 83]

$$\text{assuming} \quad \rho \overline{vk} + \overline{vP'} = - \frac{\mu_{rx}}{\sigma_k} \frac{\partial k}{\partial r}$$

$$\text{then :} \quad \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu \frac{\partial k}{\partial r} - \rho \overline{vk} - \overline{vP'} \right) \right] = \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu + \frac{\mu_{rx}}{\sigma_k} \right) \frac{\partial k}{\partial r} \right] \quad (\text{A.3})$$

Source Term

The turbulence production function G can be simplified by neglecting $\overline{u^2} \partial U / \partial x$ because the change in the axial direction of U is governed by the continuity equation and the effect of swirl on the mean axial velocity distribution is almost negligible, see Chapter VI, and $\partial U / \partial x$ is therefore small. The other non-dominant terms $\overline{v^2} (\partial V / \partial r) + \overline{uw} (\partial W / \partial x)$ are also neglected in comparison with the dominant terms. Consequently, the production term will be simplified as follows:

$$G = \rho \left[\overline{uv} \frac{\partial U}{\partial r} + \overline{vw} r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]$$

and modelled as

$$G = \mu_{rx} \left(\frac{\partial U}{\partial r} \right)^2 + \mu_{r\theta} \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right] \quad (\text{A.4})$$

With these steps the turbulent kinetic energy equation can be written in the following form:

$$\frac{1}{r} \frac{\partial}{\partial r} (\rho r V k) + \frac{\partial}{\partial x} (\rho r U k) = \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu + \frac{\mu_{rx}}{\sigma_k} \right) \frac{\partial k}{\partial r} \right] + G - \rho \epsilon \quad (\text{A.5})$$

and G is defined as above by equation (A-4). No evidence of reversed flow zones is observed and the radial velocity component is negligible, and variation of Uk with x is expected to be small; therefore convection terms in equation (A-5) are neglected and the final transport equation for the turbulent kinetic energy is approximated by:

$$\rho \epsilon = \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu + \frac{\mu_{rx}}{\sigma_k} \right) \frac{\partial k}{\partial r} \right] + G \quad (A.6)$$

This equation is used to determine the dissipation rate, ϵ , from the measurements.

APPENDIX B

AIR COMPRESSOR AND SETTLING CHAMBER

The wind tunnel shown in Fig. 4.1 is of the open circuit type and is located in the Hydraulics Laboratory of the Department of Mechanical and Aeronautical Engineering, Carleton University. The room is large enough that the compressor intake flow is not disturbed by the tunnel discharge. The compressor delivers air to a straight annular diffuser (Item 4). The diffuser has an area ratio of 2.72 and an equivalent cone angle of 14-deg. A hexagonal honeycomb (Item 5) of 0.95 cm cell width and 7.62 cm depth was added at the diffuser outlet to remove any residual swirl from the compressor.

To reduce the turbulence intensity of the flow supplied to the tunnel three sets of screens (Item 6) were added to the settling chamber; 8 mesh/cm, open area ratio 0.58, seamless stainless steel screening was used throughout. The first set is mounted 12.7 cm downstream of the diffuser outlet and uses three layers of screening at the centre, decreasing to one layer at the settling chamber wall. The low porosity at the centre is intended to spread the flow more uniformly over the settling chamber cross-section. The other two screens are single layer and are intended to further reduce the mean flow non-uniformities and turbulence. The three sets are spaced 14 cm or 112 mesh lengths apart.

At the downstream end of the settling chamber a ring of 36, 5.1 cm diam. holes (Item 7) is present to allow some of the tunnel flow to be bled off. The flow area of the bleed ring is 729 cm², compared with an axial discharge area of 790.3 cm² for the working section with 25.4 cm

centre - body, so that a large amount of bleed flow is possible. A sliding stainless steel band which can be adjusted manually, is used to block the bleed holes by any desired amount. By varying both the bleed opening and compressor speed it is possible to obtain a given combination of total and dynamic pressure at the test section inlet with a variety of compressor operating points; it is therefore usually possible to choose a compressor operating point which is well away from surge during start-up.

The exit of the settling chamber is fitted with an annular contraction. If the velocity is assumed uniform across the settling chamber a contraction ratio of 15 to 16 can be estimated. However, the velocity is probably highest near the centre-line and an effective contraction ratio of 10 to 12 seems more likely. The compressor and settling chamber are enclosed in a compartment having a louvered ceiling and lined with acoustic tiles because of the high noise level of the compressor.

A segmented straight annular duct was also available; this could be assembled to lengths varying from 40.6 to 203.2 cm. The outer wall diameter was fixed at 40.6 cm while different inner walls were available allowing annulus heights of 5.1, 7.62 and 10.16 cm to be used.

The above describes the basic facilities available for the present study. At the time the present work began the swirling flow in the annular duct had not been examined.

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APPENDIX C

INLET GUIDE VANES

The swirl velocity component was imparted to the flow in the present experiments, by using a set of turbine nozzle guide vanes. The vanes were designed and manufactured to provide an air jet at the trailing edge. The momentum of the ejected air is intended to eliminate the momentum deficit in the wake of each guide vane so that the swirling flow leaving the vanes is as close as possible to being truly axisymmetric.

It was decided to use a blade row with constant outlet angle (constant angle design) because untwisted blades are easier to manufacture. However, there is the disadvantage that the magnitude of the resultant main flow velocity varies from hub to tip with the highest velocity at the hub.

A paper by Horlock [98] has critically reviewed several different and widely used methods of designing turbine cascades. His review concluded that the optimum space-chord ratio of turbine cascades could be chosen on the basis of Zweifel's criterion [99] with an accuracy comparable to the best of the other methods. This criterion was accordingly used to choose the space-chord ratio for the present cascade. Since the flow approaches the blades axially the inlet flow angle is fixed at zero. The outlet flow angle however, varies from 0° to 60° and it is impossible to satisfy the optimum conditions for each angle using only one cascade set. An assessment indicated that the best results would be obtained by choosing 45° as the design outlet angle. The cascade was designed with 26 blades having a mean-radius spacing to

axial chord ratio of 0.787. The spacing at the mean radius is 3.99 cm and the blade chord length is 6.19 cm. This design gives approximately the same load coefficient at both 45° and 60° and a lower value at 15° outlet angle. Also the cascade was designed to have a positive incidence angle at 45° flow outlet angle and a negative incidence at 15° to minimize the profile losses as far as possible. Since the blade passages become of diffusive nature for outlet angles less than 8° , it was decided to remove the blades to study the straight flow case.

The blades are constructed by bending brass sheets of 0.13 mm thickness, with inserts to form the air passages, as in Fig. C-1. The blowing air is introduced into each blade at the outer wall of the annulus through a brass tube of 7 mm ID which is squeezed to 5 mm to match the maximum blade thickness. Small holes of different diameters, with the smallest near the hub, are drilled into the side of the brass tubes to allow the blowing air to enter the interior of the hollow blades. The air leaves the blades with uniform velocity along the blade height at the trailing edge through a 0.7 mm slot of 0.48 cm^2 area. The blades have maximum camber at 38 percent chord, thickness to chord ratio of .085, leading edge radius of 1 mm and a 0.96 mm in trailing edge thickness. Details of the blade camber line coordinates and the thickness distribution are given in Fig. 4.4b.

The velocity of the ejected air jets should be adjusted such that the momentum deficit of the blade wakes is just compensated by the momentum of the air jet. In order to get a simple relationship between the required air jet velocity V_j and the resultant velocity Q (see Fig. C.2) the flat plate relations were used to estimate the wake momentum deficit. The momentum thickness Θ on a flat plate for zero pressure

gradient can be estimated by using the expression [17]

$$\frac{\theta}{x} = 0.036 \text{Re}_x^{-1/5} \quad (\text{C.1})$$

for $\text{Re}_x = Q x / \nu < 10^7$. The dimension x is taken to be equal to the chord length ($l = 61.9 \text{ mm}$). From Fig. (C.2) a relationship between V_j and Q is expressed by

$$V_j = Q \sqrt{\frac{(\theta + \theta')}{b} + \frac{a}{b}} \quad (\text{C.2})$$

where,

a, b are the air gap and the blade trailing edge thickness.

θ, θ' are the momentum thickness at the blade trailing edge on the suction and pressure surfaces respectively.

Assuming that θ and θ' are equal and given by equation (C.1) and substituting the values of a and b , equation (C.2) becomes

$$V_j = Q \sqrt{\frac{2\theta}{b} + 1.3628}$$

or,

$$V_j = Q \sqrt{6.37 \text{Re}_l^{-1/5} + 1.36} \quad (\text{C.3})$$

Since Q varies along the vane height, a mean value defined by

$$\bar{Q} = \frac{\int_{r_i}^{r_o} Q U r \, dr}{\int_{r_i}^{r_o} U r \, dr} \quad (\text{C.4})$$

was used in equation (C.3). The final expression for V_j is given by

$$V_j = \bar{Q} \sqrt{6.37 \operatorname{Re}_\ell^{-1/5} + 1.36} \quad (\text{C.5})$$

Equation (C.5) gives the values of V_j in terms of $\bar{Q}$ and Re_ℓ ($\bar{Q} l / \nu$) for any swirl number used in the experiments. The values of V_j , Re_ℓ and the pressure of the distribution chamber used at each of the tested swirl numbers are given below.

Swirl Number	V_j (m/sec)	Re_ℓ	Pressure of Distribution Chamber cm mercury
0.0	Vanes were removed	N/A	N/A
0.23	47.1	1.30×10^5	12.3
0.47	52.2	1.45×10^5	14.8
0.87	63.2	1.76×10^5	20.9

Fig. C.1 Longitudinal cross section of blades used as inlet guide vanes

Fig. C.2 Sketch for boundary layers around the blades of inlet guide vanes

APPENDIX D COMPUTER PROGRAM "MEAS"

A brief description for the main parameters of the program and its statements are given below along with the flowchart and the program listing.

Parameter	Description
$C\phi(,)$	$\sqrt{e^2_{br}}$ or $\sqrt{e^2}$ or $\bar{e^2}$
$C1(,)$	E_{br} or E or E^2
$Y\phi(,)$	$\bar{e^2}$
$Y1(,)$	E^2
$Y( )$	Linearization coefficients used with equations (E.5) and (E.6)
B1	Voltage measured at the terminals of the first circuit, Fig. 4-12, when the "ON/OFF" switch is in "ON" position
A2	Variable used to keep track of the number of the angle ψ ($A2 = 1, 2, 3$)
A3	Refers to the number of the point in the flow being surveyed ($A3 = 1, 2, 3 \dots, 17$)
S	Slope of the linearized calibration curve, see equation (E.5)
$D( )$	Dimensionless radius r^* , $r^* = (r - r_0)/(r_0 - r_1)$
P	Output from Avdcv subprogram, P contains the coverage value of a 100 instantaneous readings of the measured voltage
P1	Input to "Avdcv" subprogram, it specifies the number of channel to be closed so that its terminals voltage can be measured by the 3456 DVM
$M\phi$	Parameter used to control the printing of the measured data. If $M\phi > 1$, values of E and $\sqrt{e^2}$ would not be printed and vice versa

Program Description

Line #	Description
10	Title of the program.
20	Specifies that all array elements dimensioned will be located at positions 1 through N rather than 0 through (N - 1).
30-50	Arrays are dimensioned to hold the collected data.
121-126	Reading the linearization coefficients, slope S of the linearized calibration curve and anemometer bridge output at zero velocity E_0 . See equation (E.5).
170-180	"Init" is the ADACS System Subprogram that performs the necessary initialization to assure proper communication between the computer and system instruments. It also pre-defines many of the variables used by other System Subprograms. P1 declares which instrument is to be the system voltmeter (either 3456A or 3497), see Ref. [91] for more details.
190-260	In this endless loop the system DVM will keep checking the voltage B1 present at the terminal of the first circuit, see Fig. 4.12, until the circuit is closed. In the latter case the value of B1 will be greater than 4 volts and the program will proceed to line 270. The experimenter can terminate the measuring procedures at any time by pushing "KEY #1" (line 220). The latter does not destroy any data taken before.
230	"Avdcv" is a subprogram which set up the 3456A Voltmeter to take a 100 instantaneous reading of the voltage signal present at the terminals of the specified closed channel with a sampling rate of 6 readings/sec. The subprogram then calculated the average voltage and returned that value to the calling program, see Ref. [91] for more details.
280	Turn red light on (close the third circuit shown in Fig. 4.12).
290-370	Measuring the voltages $\bar{E}_{br}$ and $\sqrt{e^2}_{br}$ taken with the three Ψ angles.
350	Turn red light off (open the third circuit shown in Fig. 4.12).
380	Wait 5 seconds.
410	Turn green light on (close the second circuit shown in Fig. 4.12).

Line #	Description
411-415	Print values of $\bar{E}_{br}$ and $\sqrt{e^2}_{br}$ taken with the three ψ angles.
430	Move to the next radial point in the flow.
450	Turn green light off (open the second circuit shown in Fig. 4.12.
490-550	Flash green light, if $A3 > 17$.
561-608	Linearization for the voltages $\sqrt{e^2}_{br}$ and $\bar{E}_{br}$.
840-1010	Print values of $\bar{E}$ and $\sqrt{e^2}$ if $M\phi \leq 1$.
1300-1380	Calculate $\bar{E}^2$ and $\bar{e}^2$
2730-3000	Save values of $\bar{E}^2$ and $\bar{e}^2$, values on tape and print related information.
3010-3360	Read values of $\bar{E}^2$ and $\bar{e}^2$ that were stored before and print them if $M\phi > 5$ (used only for checking purposes).

Fig. D.1 Flowchart of computer program "MEAS"

```

10 . PROGRAM ** MEAS ** TO MEAS
    URE AND RECORD HOT-WIRE READ
    INGS
20 OPTION BASE 1
30 DIM C0(19,3),C1(19,3)
40 DIM Y0(19,3),Y1(19,3),Y(5)
50 DIM D(21)
60 M0=5
70 N=0 @ B1=0 @ S=0 @ C3=0 @ N0
    =0 @ S0=0
80 FOR I=1 TO 19
90 FOR J=1 TO 3
100 C0(I,J)=0 @ C1(I,J)=0
110 NEXT J
120 NEXT I
121 ASSIGN# 1 TO "CAL"
122 FOR I=1 TO 5
123 READ# 1 , Y(I)
124 NEXT I
125 READ# 1 : S,E0
126 ASSIGN# 1 TO *
130 CLEAR @ A3=0
140 DISP "INPUT STARTING ROW NUM
    BER A3"
150 INPUT A3
160 A2=1
170 P1=3456
180 GOSUB 4010 : CALL Init
190 CLEAR 709
200 OUTPUT 709 , "D04,0"
210 P1=3
220 ON KEY# 1,"BDOWN" GOTO 480
230 GOSUB 4820 : CALL Avdcv
240 B1=P
250 IF B1>4 THEN GOTO 270
260 GOTO 210
270 CLEAR 709
280 OUTPUT 709 , "DC4,0"
290 P1=1
300 GOSUB 4820 : CALL Avdcv
310 C1(A3,A2)=P
320 P1=0
330 GOSUB 4820 : CALL Avdcv
340 C0(A3,A2)=P
350 OUTPUT 709 , "D04,0"
360 IF A2>=3 THEN GOTO 400
370 A2=A2+1
380 WAIT 5000
390 GOTO 210
400 CLEAR 709
410 OUTPUT 709 , "DC4,3"
411 PRINT USING 412 ; A3,C1(A3,1)
    ,C1(A3,2),C0(A3,1),C0(A3,2)
412 IMAGE 00,2X,2(MDZ,DDD),2(MZ,
    DDDD)
413 PRINT USING 415 ; C1(A3,3),C
    0(A3,3)
415 IMAGE 4X,2(MDZ,DDD)
420 IF A3>=17 THEN GOTO 480
430 A3=A3+1 @ A2=1

```

```

440 WAIT 5000
450 OUTPUT 709 ; "D04,3"
460 GOTO 210
470 !
480 IF C1(A3,1)>>0 THEN N=A3 ELSE
      N=A3-1
490 CLEAR 709
500 OUTPUT 709 ; "D04,3"
510 WAIT 800
520 OUTPUT 709 ; "D04,3"
530 ON KEY# 2, "ST CAL" GOTO 550
540 GOTO 490
550 OUTPUT 709 ; "D04,3"
551 !
552 CLEAR @ DISP "INPUT RMS-MERT
      ER ZERO READING E1"
553 DISP "INPUT E1" @ INPUT E1
560 CLEAR
561 FOR I=1 TO N
562 FOR J=1 TO 3 @ L1=Y(2) @ L2=
      0
563 FOR J1=3 TO 5
564 L1=L1+(J1)*(J1-1)*(C1(I,J)-
      E0)^(J1-2)
565 NEXT J1
566 L2=(C0(I,J)-E1)*L1 @ C0(I,J)
      =0 @ C0(I,J)=L2*S
567 NEXT J
568 NEXT I
580 !
600 FOR I=1 TO N
601 FOR J=1 TO 3
602 L1=Y(1)
603 FOR J1=2 TO 5
604 L1=L1+Y(J1)*(C1(I,J)-E0)^(J1
      -1)
605 NEXT J1
606 C1(I,J)=0 @ C1(I,J)=L1*S
607 NEXT J
608 NEXT I
610 DISP "
620 DISP " INPUT AIR TEMPERATURE
      "
630 DISP " " @ DISP " INPUT
      T2 "
640 INPUT T2
650 DISP "
660 DISP " INPUT SECTION-NUMBER
      N0 , SWIRL-ANGLE S0 AND TODA
      Y,S-DATE,D$"
670 DISP " "
680 DISP "INPUT N0,S0,D0$ " @ IN
      PUT N0,S0,D0$
690 PRINT "SECTION-NUMBER= ";N0
700 PRINT "SWIRL - ANGLE = ";S0
710 PRINT "TODAY,S - DATE= ";D0$
730 PRINT "CALIB - SLOPE = ";S
740 PRINT "
770 ! variable D is non-dimsionl
      ess radius , D=(R - Rinner)/
      (Router-Rinner)

```

```

750 D(1)=1 @ D(2)=46/48 @ D(3)=1
    5/16
790 D(4)=11/12 @ D(5)=7/8 @ D(6)
    =5/6 @ D(7)=3/4
800 D(8)=2/3 @ D(9)=7/12 @ D(10)
    =1/2 @ D(11)=5/12
810 D(12)=1/3 @ D(13)=1/4 @ D(14)
    =1/6 @ D(15)=1/8
820 D(16)=1/12 @ D(17)=1/16 @ D(
    18)=1/24
830 D(19)=0
840 IF M0>1 THEN GOTO 2000
850 FOR I=1 TO N
860 PRINT USING 870 ; I+1,D(I+1)
    ,C1(I,1),C1(I,2),C1(I,3)
870 IMAGE DD,2X,Z,DD,2(X,MODZ,DD
    D),9X,MODZ,DD
880 NEXT I
890 PRINT "
950 PRINT " *****
    *****" @ PRINT "
970 FOR I=1 TO N
990 PRINT USING 1000 ; I+1,D(I+1)
    ,C0(I,1),C0(I,2),C0(I,3)
1000 IMAGE DD,2X,Z,DD,2(MODZ,DD
    D),9X,MODZ,DD
1010 NEXT I
1300 FOR I=1 TO N
1310 FOR J=1 TO 3
1320 L1=0 @ L2=0
1330 L1=C1(I,J)^2
1340 L2=C0(I,J)^2
1350 C1(I,J)=L1
1360 C0(I,J)=L2
1370 NEXT J
1380 NEXT I
2000 !
2500 ON KEY# 4,"RECORD" GOSUB 27
    10
2700 GOTO 2700
2710 CLEAR
2720 DISP "
2730 DISP "INPUT DATA FILE NAME
    ,
    BUFFER NUMBER AND TAPE
    No. FOR STORING DATA"
2740 INPUT B$,M5,M4
2750 CREATE B$,N+1,48
2760 ASSIGN# M5 TO B$
2770 FOR I=1 TO N
2780 FOR J=1 TO 3
2790 PRINT# M5 ; C0(I,J),C1(I,J)
2800 NEXT J
2810 NEXT I @ PRINT# M5 ; S,TZ
2820 ASSIGN# M5 TO *
2830 DISP "
2840 DISP "**** DATA STORED ****
    "
2850 PRINT "
2860 PRINT "DATA STORED INFORMAT
    ION "

```



```

2870 PRINT "-----"
----" @ PRINT "
2880 PRINT "DATA FILE NAME = ";B
$
2890 PRINT "DATA BUFFER No = ";M
5
2900 PRINT "TAPE NUMBER = ";M
4
3000 PRINT "
"
3005 IF M0>1 THEN GOTO 4000
3010 ASSIGN# M5 TO B$
3020 FOR I=1 TO N
3030 FOR J=1 TO 3
3040 READ# M5 , Y0(I,J),Y1(I,J)
3050 NEXT J
3060 NEXT I @ READ# M5 , S8,T8
3070 ASSIGN# M5 TO *
3080 PRINT "DATA READ INFORMATIO
N"
3090 PRINT "-----"
-
3100 PRINT "DATA FILE NAME = ";B
$
3110 PRINT "BUFFER NUMBER = ";M
5
3120 PRINT "TAPE NUMBER = ";M
4
3130 PRINT "
"
3200 FOR I=1 TO N
3210 PRINT USING 3220 ; I+1,D(I+
1),Y1(I,1),Y1(I,2),Y1(I,3)
3220 IMAGE DD,2X,Z.DD,2(X,MODZ.D
DD),9X,MODZ.DDD
3230 NEXT I
3240 PRINT "
"
3250 PRINT " *****
*****" @ PRINT "
"
3300 FOR I=1 TO N
3310 PRINT USING 3320 ; I+1,D(I+
1),Y0(I,1),Y0(I,2),Y0(I,3)
3320 IMAGE DD,2X,Z.DD,2(MODZ.DDD
D),9X,MODZ.DDD
3330 NEXT I
3340 PRINT "
"
3350 PRINT "CALIB SLOPE = ";S8,"
volts/m/sec"
3360 PRINT "
"
3370 PRINT "AIR TEMPERATURE = ",
T8;" F"
3400 RETURN
4000 END
4010 ! SUBPROGRAM Init hp85
4020 ! QA 7/08/80 GAC
4030 ! P1=DvMeter
4040 DIM Q7$(23) ! DEVICE NAMES
4050 !
4060 ! INITIALIZE DEVICE ADDRES
SES
4070 !
4080 R0=709 ! 3497A SCANNER ADDR
ESS

```

```

4800 RETURN
4810 ! SUBEND
4820 ! SUBPROGRAM Avdcv hp85
4830 ! QA 7/11/80 GAC
4840 ! P1=Channel
4850 OUTPUT R1 USING "*" ! TEST
      FOR OVM
4860 RESUME R1 DIM 100 ! DROP AT
      N

```

```

5130 GOSUB 5170 ! CALL Warn
5140 P=9.E19
5150 RETURN
5160 ! SUBEND
5170 ! SUBPROGRAM Warn hp85
5180 ! QA 7/30/80 GAC
5190 ! R3=ERROR WORD
5200 ! R$=NAME OF CALLING ROUTIN
      E
5210 DIM Q8$(100),Q6$(15)
5220 O5=2 ! MAXIMUM NUMBER OF WA
      RNINGS ON EACH ERROR ( <=9
      )
5230 O2=0 ! ERROR FLAG

```

```

5820 DATA "ERROR 11: Datapoint<=
      0 on a log axis"
5830 DATA "ERROR 12: Level cross
      ing not found"
5840 DATA "ERROR 13: User define
      d error"
5850 DATA "ERROR 14: User define
      d error"
5860 RETURN
5870 ! SUBEND

```

APPENDIX E

HOT-WIRE LINEARIZATION AND CALIBRATION METHODS

The hot-wire probe was calibrated by using the experimental set-up shown in Fig. 4.11. For calibration purposes it is convenient to define the x , y , z axes such that x is in the flow (U_j) direction and the probe axis lies in the x - y plane, see Fig. E.1. α' is defined as the angle of inclination of the sensor wire to the x - z plane and ψ is the angle between the x - y plane and the plane of the prongs.

A computer program was used with the ADACS system to facilitate the calibration process. The principles on which the calibration method was based are outlined in the following paragraphs.

The probe mounting mechanism (see Fig. 4.11a) consists of a lathe vise and a fabricated attachment which fits its jaws as well as a probe holder (DISA 55H20). The vise allows the probe to be rotated in the horizontal plane while the extra attachment provides probe rotation about its own axis. The vise is also mounted on a base supported on three screws to allow leveling of the whole mechanism in the horizontal plane. Alignment of probes relative to the jet axis was carefully checked using a microscope to ensure that the reference orientation conditions of $\psi = 0$ and $\alpha' = 90^\circ$, see Fig. E.1, are accurately established.

Before taking any calibration measurements, the detailed operational procedures described in the ISI manual for the hot-wire anemometer [92] were performed carefully to obtain results of good accuracy. The Pitot tube was placed with its tip about 2 mm below the hot-wire sensor. The Pitot should be placed as close as possible to the

hot-wire sensor without introducing any interference effects. This was checked by observing the display of the DVM's for $\sqrt{e^2_{br}}$ and $\bar{E}_{br}$ as the Pitot tube was moved into position. Since the Pitot tube is insensitive to mis-alignments up to about 15° [90], it was adjusted visually to be parallel to the nozzle axis. As recommended by other researchers [100] both the hot-wire probe sensor and the Pitot tube tip were located at twice the nozzle exit diameter downstream of the nozzle exit. At this specific location a uniform velocity profile was obtained and the turbulence level was about 1%. A check showed a potential core of 2 cm in diameter with less than 1% variation in velocity at this position. Therefore, the hot-wire sensor and the Pitot tube were considered to read essentially the same velocity. The air jet velocity was calculated by:

$$U_j = \sqrt{\frac{2\Delta P}{\rho}} \quad (E.1)$$

The differential pressure Δp was measured with a pressure transducer; its output signal $\bar{E}_{tr}$ was related to Δp by a calibration process. A maximum of 21 data points were measured over a velocity range between 0 and 55 m/sec; at each point both $\bar{E}_{tr}$ and the anemometer output voltage $\bar{E}_{br}$ were recorded. These recorded values represent the average of 100 readings taken by the DVM (HP 3456A) at a sampling rate of 6 samples/sec with accuracy of 1 mV. The approach employed to establish the relationship between U_j and $\bar{E}_{br}$ or the corresponding linearized signal $\bar{E}$ is briefly illustrated below.

From the above discussion, it can be concluded that the potential core of the calibration jet is effectively one dimensional and

turbulence intensity is negligible. If the angle α is replaced by α' in equation (5.2), then the latter applies for this case in the form

$$\begin{aligned}\overline{U_{eff}^2}(\psi, \alpha') &= \overline{U_n^2} + k^2 \overline{U_t^2} + h^2 \overline{U_b^2} \\ &= (\sin^2 \alpha' \cos^2 \psi + k^2 \cos^2 \alpha' \cos^2 \psi + h^2 \sin^2 \psi) U_j^2 \quad (E.2)\end{aligned}$$

Moreover, if the hot-wire probe is positioned such that the sensor wire is perpendicular to the jet axis ($\alpha' = 90^\circ$) and the probe prongs are in the x-y plane ($\psi = 0$), see Fig. E.1, equation (E.2) is simplified to

$$\overline{U_{eff}}(0, 90) = U_j$$

Substituting the above relationship into equation (4.8) will give the general formula for calibrating the hot-wire probes.

$$\overline{E_{br}}(0, 90) = f(U_j) \quad (E.3)$$

The above relationship is to be expected because in that specific configuration the yaw and pitch effects on the wire response are eliminated.

An electronic signal linearizer ISI (Model 1052) was available and could have been used to furnish a linear relationship for the calibration curve expressed generally by equation (E.3). The linearizer unit would approximate equation (E.3) by a fourth degree polynomial, the anemometer output E_{br} would be passed through the linearizer to give a corresponding linearized output that would be directly proportional to the effective cooling velocity. However, for the present study the

ADACS system was also available and linearization was carried out by computation using the system computer, rather than by the electronic analogue method. This saved the considerable time that would otherwise have had to be devoted to establish the electronic linearizer set - points at each calibration session. Furthermore, accuracy was also improved and linearizer drift was avoided. Linearization was carried out by employing the same concept and the procedures detailed in TSI linearizer operating manual [92]. The computed linearized voltage E_{lin} is directly proportional to the effective cooling velocity as follows.

$$E_{lin} = S U_{eff} \quad (E.4)$$

The anemometer bridge response curve was approximated with a fourth degree polynomial and converted into linearized voltages by the relation

$$\bar{E}_{lin} = S \{ A_0 + A(\bar{E}_{br} - E_0) + B(\bar{E}_{br} - E_0)^2 + C(\bar{E}_{br} - E_0)^3 + D(\bar{E}_{br} - E_0)^4 \} \quad (E.5)$$

where E_0 and $\bar{E}_{br}$ are the bridge output voltages at zero and non-zero values, respectively, of the effective cooling velocity. The unknown coefficients in equation (E.5) were determined from the calibration data by using a computer program very similar to that listed in the TSI linearizer manual; this program was slightly modified and translated into BASIC to suit the HP-85 microcomputer of the ADACS system. This program was a part of the main data processing computer program "CALIB" used for the calibration of the hot-wire probes. E_0, S and the polynomial coefficients A, B, C and D were saved on a tape to be used in another computer program "MEAS" which monitored and processed the

hot-wire measurements taken during the experiments.

The method of computing the linearized RMS signal $\sqrt{e^2_{lin}}$ from the measured RMS of the bridge output signal e^2_{br} is outlined in the following.

Assume that for low turbulence cases

$$\sqrt{e^2_{lin}} = \Delta \bar{E}_{lin} \quad \text{and} \quad \sqrt{e^2_{br}} = \Delta \bar{E}_{br}$$

or

$$\sqrt{e^2_{lin}} = \frac{\Delta \bar{E}_{lin}}{\Delta \bar{E}_{br}} \sqrt{e^2_{br}}$$

taking the limits would give

$$\sqrt{e^2_{lin}} = \frac{\partial \bar{E}_{lin}}{\partial \bar{E}_{br}} \sqrt{e^2_{br}}$$

Differentiation of equation (E.5) determines $\partial \bar{E}_{lin} / \partial \bar{E}_{br}$ which if substituted in the above relationship will give the final expression for $\sqrt{e^2_{lin}}$.

$$\sqrt{e^2_{lin}} = S \{ A + 2B(\bar{E}_{br} - E_0) + 3C(\bar{E}_{br} - E_0)^2 + 4D(\bar{E}_{br} - E_0)^3 \} \sqrt{e^2_{br}} \quad (E.6)$$

As a check on the method, various voltage signals from the anemometer bridge unit were input to the TSI electronic signal linearizer and its outputs were compared with the values calculated from equations (E.5) and (E.6) and excellent agreement was found.

Since $\bar{E}_{br}$ and $\sqrt{e^2_{br}}$ will be only measured and processed to compute

the linearized values which in their turn were used in all the hot-wire formulae, the symbols $\bar{E}$ and $\bar{e}^2$ refer to $\bar{E}_{lin}$ and $\bar{e}^2_{lin}$ unless otherwise mentioned.

The purpose of calibrating the hot-wire probe, as outlined above, was thus to establish values for the slope S of the linearized calibration curve (equation E.4) and to determine the coefficients A, B, C and D in equations (E.5) and (E.6). A listing of the computer program "CALIB" used to perform the calibration procedures and a sample of the program printout are furnished at the end of this Appendix.

The approach used to determine the pitch and yaw factors k and h is based on a recent study published by Bruun and Tropea [69]. This approach is briefly described below.

Rewrite the relationship (4.10) to represent the hot-wire response curve in the form:

$$\bar{E}^2_{br}(\psi, \alpha') = A + B \bar{U}^n_{eff}(\psi, \alpha') \quad (E.7)$$

$\bar{U}^n_{eff}(\psi, \alpha')$ could be expressed as follows (equation E.2) for the calibration situation.

$$\bar{U}^n_{eff}(\psi, \alpha') = (\sin^2 \alpha' \cos^2 \psi + k^2 \cos^2 \alpha' \cos^2 \psi + h^2 \sin^2 \psi)^{n/2} U^n_j \quad (E.8)$$

Substituting equation (E.8) into (E.7) provides the general formula for determining k and h .

$$\bar{E}^2_{br}(\psi, \alpha') = A + B (\sin^2 \alpha' \cos^2 \psi + k^2 \cos^2 \alpha' \cos^2 \psi + h^2 \sin^2 \psi)^{n/2} U^n_j$$

The above equation can be rewritten in the form.

$$\bar{E}_{br}^2(\psi, \alpha') = A + B'(\psi, \alpha') U_j^n \quad (E.9)$$

where

$$B'(\psi, \alpha') = B(\sin^2 \alpha' \cos^2 \psi + k^2 \cos^2 \alpha' \cos^2 \psi + h^2 \sin^2 \psi)^{n/2} \quad (E.10)$$

The constant A is equal to the square of the anemometer bridge output at zero flow velocity, E_o^2 . Evaluation of each of the two sensitivity factors, k and h, requires two values of $B'(\psi, \alpha')$ determined by positioning the probe at different sets of ψ and α' as will be explained. The index n in equation (E.10) is taken to be 0.45 as suggested by previous investigators [47, 69]. To determine the coefficient $B'(\psi, \alpha')$, the probe is positioned at the chosen values of ψ and α' and then recalibrated as described before. The data for $\bar{E}_{br}^2(\psi, \alpha')$ versus U_j^n are fitted with a straight line relationship by applying the least square curve fitting method; the value of B' which best fits the data is thus found.

Evaluation of yaw factor K

Consider the position $\psi = 0$ but with two angles α'_1 and α'_2 then equation (E.10) becomes

$$B'(0, \alpha'_1) = B(\sin^2 \alpha'_1 + k^2 \cos^2 \alpha'_1)^{n/2} \quad (E.11a)$$

and

$$B'(0, \alpha'_2) = B(\sin^2 \alpha'_2 + k^2 \cos^2 \alpha'_2)^{n/2} \quad (E.11b)$$

division of the above equations gives the general formula used for computing the factor k

$$\left\{ \frac{B'(\alpha'_1)}{B'(\alpha'_2)} \right\}_{\psi=0}^{2/n} = \frac{\sin^2 \alpha'_1 + k^2 \cos^2 \alpha'_1}{\sin^2 \alpha'_2 + k^2 \cos^2 \alpha'_2} \quad (E.12)$$

The angles α'_1 and α'_2 are taken to be 45° and 60° for the 45° slanted probe and 30° and 45° for the normal hot-wire probe (see Fig. E.2).

These values were chosen to be as close as practicable to the setting of the probe in the test section. Although the latter setting corresponds to $\alpha' = 0$ for the normal probe, $\alpha' = 0$ is avoided in evaluating the factor k due to the interference of the probe prongs with the jet flow in that position. The values obtained for k^2 are 0.08 and 0.007 for slanted and normal probes respectively with slight changes between probes of the same kind. Also, these figures represent average values for k^2 over the entire velocity range. Since k^2 is a function of α' and of the flow velocity range, only values appropriate to the prevailing flow conditions should be used. This recommendation was followed in the present experiments. In the case of normal probes the value of k^2 was taken to be zero in some literature [45, 48].

Evaluation of pitch factor h

The pitch factor h is determined by setting and calibrating the probe at two positions first at $\psi = 0$, $\alpha' = 90^\circ$ and secondly at $\psi = 90^\circ$, $\alpha' = 90^\circ$ or 45° . The first is the calibration position used for determining the slope S ; therefore, the data obtained for that purpose

were reused. The α' values of 90° or 45° in the second position correspond to the cases of normal and slanted probes; results for h are essentially the same for both α' values as long as $\psi = 0$, see Fig. E.3. The coefficient $B'(\psi, \alpha')$ is determined as before and equation (E.10) is simplified for the two positions by:

$$B'(0, 90) = B \quad (E.13)$$

$$B'(90, 90) = Bh^n$$

From these two equations the factor h is evaluated from:

$$h = \left(\frac{B'(90, 90)}{B} \right)^{1/n} \quad (E.14)$$

Values of the factor h obtained from equation (E.14) are 1.17 and 1.09 for slanted and normal probes respectively with small variations between probes of the same kind. These values are considered to be average ones for the entire velocity range. Values appropriate to the operating conditions were always used. However, in some previous studies [12, 13, 49, 58, 59, 63, 64] the value of h was considered equal to unity.

Fig. E.1 Enlarged top view of hot-wire probes showing their orientation with respect to nozzle air jet during calibration.
(probe prongs // to x - y plane for $\psi = 0$)

Fig. E.2 Enlarged top view of hot-wire probe orientations used to determine the yaw factor k
 (probe prongs // to x-y plane for $\psi = 0$)

Fig. E.3 Enlarged top view of second hot-wire probe orientations used to determine the pitch factor h
 (probe prongs // to z-y plane for $\psi = 90^\circ$)

COMPUTER PROGRAM "CALIB"

A brief description for the main parameters of the programs and its statements are given below along with the flowchart, the program listing and samples of their print-out.

Parameter	Description
P1(21), P2( ) P1\$( ), P2\$( ), P3\$( )	Arrays and strings utilized in subprogram "PLOT", see Ref. [91].
A (,), F0( ), F1( ) E0( ), E1( ), M ( ) S ( ), W ( ), X5(,)	Arrays utilized in lines 1000-1930, see Ref. [92].
C0( )	Output voltage of the pressure transducer (Volts).
T ( )	Output voltage of pressure transducer in mv after subtracting its value at zero flow velocity.
C1( ), E ( )	Bridge output voltage of hot-wire anemometer.
B1	Voltage measured at the terminals of the first circuit (when the "ON/OFF" switch is in "ON" position, see Fig. 4.12..
F ( )	Flow velocity (m/sec).
N	Number of data points (its maximum value is 21).
B5	Output voltage of pressure transducer at zero flow velocity (mv).
T1	Air temperature in Celcius degrees.
T2	Air temperature in Kelvin.
L ( )	Coefficients used to set the electronics signal linearizer Model TSI 1052.
Y ( )	Linearization coefficients $A_0, A, B, C,$ and D used in equation (E.5).
S5	Slope S of linearized calibration curve.

Program Description.

Line #	Description
10-500	Same as explained before in computer program "MEAS", see Appendix D.
600-640	Calculation of jet velocity using equation (E.1) (i.e. $U_j = \sqrt{2\Delta P/\rho}$, $\Delta P = K1 \cdot E_{tr}$ where K1 is the pressure transducer constant in cm H ₂ O/mv and is experimentally determined by calibrating the transducer, E_{tr} is the transducer output voltage in mv.
1000-1930	This part can be found exactly in FORTRAN in TSI manual for the electronic linearizer [92] along with its detailed description. The function of these lines is to calculate the linearization coefficients and the slope S used with equations (E.5, 6) and those required for setting the electronic linearizer which was used for checking purposes.
2130-2140	Saving the coefficients of equation (E.5), slope S and the anemometer output voltage at zero flow velocity, E_0 , on a tape.
2310-2390	Printing the coefficients of the electronic linearizer.
2471-2474	Printing the coefficients of equation (E.5).
2500-2670	Plotting the calibration curves (i.e. E_{br} vs V_j and E vs V_j). For more details about the subprogram "PLOT", see Ref. [91].

Fig. E.4 Flowchart of computer program "CALIB"


```

10  SUBPROGRAM*CALIB**  CALI
    BRATION OF THE HOT-WIRE PROB
    ES
20  OPTION BASE 1
30  DIM C0(21),C1(21),P1(21),P1$
    [60],P2$[60],P3$[60]
50  DIM T(21),E(21),F(21),P2(21)
60  DIM A(8,8),F1(21),Y(5),E0(21)
    ,L(4),M(21),X(13),S(7),W(21)
    ,X5(3,8),F0(21),E1(21)
70  FOR I=1 TO 21
80  C0(I)=0 @ C1(I)=0 @ T(I)=0 @
    E(I)=0
90  NEXT I @ DISP "INPUT TODAY'S
    DATE"
100 A3=1 @ N=0 @ T1=0 @ S5=0 @ I
    NPUT D0$ @ PRINT "TODAY'S DAT
    E =" ,D0$ @ PRINT "
110 P1=3456
120 GOSUB 2740 ! CALL Ini
130 B1=0
140 CLEAR 709
150 OUTPUT 709 ; "D04,0"
200 P1=3
210 GOSUB 3550 ! CALL Rvdcm
220 B1=P @ ON KEY# 1, "N<20" GOTO
    420
230 IF B1>=4 THEN GOTO 250
240 GOTO 200
250 CLEAR 709
300 OUTPUT 709 ; "D04,0"
310 P1=0
320 GOSUB 3550 ! CALL Rvdcm
330 C0(A3)=P
340 P1=1
350 GOSUB 3550 ! CALL Rvdcm
360 C1(A3)=P @ PRINT USING 370 ,
    A3,C0(A3)*1000,C1(A3)
370 IMAGE DD,2X,MDDDDZ,DDD,3X,MDZ
    ,DDDDDD
380 OUTPUT 709 ; "D04,0"
390 IF A3 =21 THEN GOTO 420
400 A3=A3+1
410 WAIT 5000 @ GOTO 200
420 IF C0(A3)<C0(A3-1) THEN N=A3
    -1 ELSE N=A3
430 CLEAR 709
440 OUTPUT 709 ; "D04,3"
450 WAIT 800
460 OUTPUT 709 ; "D04,3"
470 ON KEY# 2, "ST CAL" GOTO 490
480 GOTO 430
490 OUTPUT 709 ; "D04,3" @ CLEAR
500 DISP "CALCULATIONS IN PROGRES
    S RESULTS WILL BE PRINTED S
    OON" @ B5=1000*C0(1)
510 FOR I=1 TO N
520 T(I)=1000*C0(I)-B5
530 E(I)=C1(I)

```

```

540 NEXT I
550 DISP "INPUT THE AIR TEMPERAT
    URE IN C"
570 INPUT T1
580 DISP "INPUT PRESSURE TRANSDU
    CER CONSTANT K1 "
590 INPUT K1
600 FOR I=1 TO N
610 T2=T1+273
620 S7=K1*T(I)*T2
630 F(I)=.74548*SQR(S7)
640 NEXT I
650 PRINT "      " @ PRINT " I ";
    VTRAN " " VELOC " " BOUTPUT
    " " TEMP"
660 FOR I=1 TO N
670 PRINT USING 680 ; I,T(I),F(I)
    ,E(I),T1
680 IMAGE DD,MDDOZ.D,X,MDOZ DD,2X
    ,MZ.DDD,2X,MDD.D
690 NEXT I
990 ! Calculations of the coeffic
    ients needed for linearizer
    in hot-wire anemometer
1000 D=4 ! D is the degree of th
    e polynomial
1010 Z0=E(1)
1020 Z1=F(1)
1030 FOR I=1 TO N
1040 F0(I)=F(I)-Z1
1050 E0(I)=E(I)-Z0
1060 NEXT I
1070 W(1)=100
1080 W(2)=80
1090 W(3)=60
1100 W(4)=40
1110 W(5)=20
1120 W(6)=10
1130 N1=N-1
1140 FOR I=7 TO N1
1150 W(I)=1
1160 NEXT I
1170 W(N)=100
1180 FOR I=1 TO 3
1190 X(I)=0
1200 NEXT I
1210 FOR I=1 TO 2
1220 S(I)=0
1230 NEXT I
1240 FOR I=1 TO N
1250 X(1)=X(1)+W(I)
1260 X(2)=X(2)+W(I)*E0(I)
1270 X(3)=X(3)+W(I)*E0(I)^2
1280 S(1)=S(1)+W(I)*F0(I)
1290 S(2)=S(2)+W(I)*E0(I)*F0(I)
1300 NEXT I
1310 N3=1
1320 L3=N3+1
1330 K3=L3+1
1340 FOR I=1 TO L3

```

```

1350 FOR J=1 TO L3
1360 I2=J-1+I
1370 A(I,J)=X(I2)
1380 A(I,K3)=S(I)
1390 NEXT J
1400 NEXT I
1410 FOR I=1 TO L3
1420 A(K3,I)=-1
1430 K2=I+1
1440 FOR J=K2 TO K3
1450 A(K3,J)=0
1460 NEXT J
1470 C=1/A(1,I)
1480 FOR I1=2 TO K3
1490 FOR J=K2 TO K3
1500 A(I1,J)=A(I1,J)-A(I1,I)*A(I
,J)*C
1510 NEXT J
1520 NEXT I1
1530 FOR I1=1 TO L3
1540 FOR J=K2 TO K3
1550 A(I1,J)=A(I1+1,J)
1560 NEXT J
1570 NEXT I1
1580 NEXT I
1590 FOR I=1 TO L3
1600 X5(I,K3-2)=A(I,K3)
1610 NEXT I
1620 IF N3=0 THEN GOTO 1740
1630 N3=N3+1
1640 J=2*N3
1650 X(J)=0
1660 X(J+1)=0
1670 S(N3+1)=0
1680 FOR I=1 TO N
1690 X(J)=X(J)+E0(I)^(J-1)*W(I)
1700 X(J+1)=X(J+1)+E0(I)^J*W(I)
1710 S(N3+1)=S(N3+1)+F0(I)*E0(I)
^N3*W(I)
1720 NEXT I
1730 GOTO 1320
1740 M1=D+1
1750 FOR I=1 TO M1
1760 Y(I)=A(I,K3)
1770 X5(I,4)=X5(I,4)*E0(N)^(I-1)
1780 NEXT I
1790 FOR J=2 TO N
1800 F2=Y(I)
1810 FOR I=2 TO 5
1820 F2=F2+Y(I)*E0(J)^(I-1)
1830 NEXT I
1840 F1(J)=F2*10/F0(N)
1850 E1(J)=(F2-F0(J))*100/F0(J)
1860 NEXT J
1870 S5=10/F0(N)
1880 FOR I=1 TO 4
1890 L(I)=X5(I+1,4)*S5
1900 NEXT I
1910 FOR I=2 TO N
1920 M(I)=F0(I)*S5

```

```

1930 NEXT I
1940 CLEAR
1950 DISP USING 1960
1960 IMAGE "**",2X,"Linearizer C
      oefficients",2X,"**"
1970 DISP "-----"
      "
1980 DISP "
1990 DISP USING 2030 ; L(1)
2000 DISP USING 2040 ; L(2)
2010 DISP USING 2050 ; L(3)
2020 DISP USING 2060 ; L(4)
2030 IMAGE 3X,"A =",MDZ.DDD
2040 IMAGE 3X,"B =",MDZ.DDD
2050 IMAGE 3X,"C =",MDZ.DDD
2060 IMAGE 3X,"D =",MDZ.DDD,2/
2070 DISP "*****"
      "*****"
2080 DISP "VELOCITY";" NFLOW ";
      " COUTPUT "; " PERROR"
2090 FOR I=2 TO N
2100 DISP USING 2110 ; F0(I),M(I)
      ,F1(I),E1(I)
2110 IMAGE MD0.DD,1X,MDZ.DDD,1X,
      MDZ.DDD,3X,MDZ.DD
2120 NEXT I
2130 CREATE "CAL",1,40
2135 ASSIGN# 1 TO "CAL"
2136 FOR I=1 TO M1
2137 PRINT# 1 ; Y(I)
2138 NEXT I
2139 PRINT# 1 ; S5,E(1)
2140 ASSIGN# 1 TO *
2145 !
2210 ON KEY# 4,"PRINT" GOTO 2270
2230 GOTO 2230
2260 !
2270 PRINT " " @ PRINT USING
      2280
2280 IMAGE "**",2X,"Linearizer C
      oefficients",2X,"**"
2290 PRINT "-----"
      "
2300 PRINT "
      "
2310 PRINT USING 2350 ; L(1)
2320 PRINT USING 2360 ; L(2)
2330 PRINT USING 2370 ; L(3)
2340 PRINT USING 2380 ; L(4)
2350 IMAGE 3X,"A= ",MDZ.DDD
2360 IMAGE 3X,"B= ",MDZ.DDD
2370 IMAGE 3X,"C= ",MDZ.DDD
2380 IMAGE 3X,"D= ",MDZ.DDD,2/
2390 PRINT "*****"
      "*****"
2400 PRINT "VELOCITY";" NFLOW ";
      " COUTPUT "; " PERROR"
2410 FOR I=2 TO N
2420 PRINT USING 2430 ; F0(I),M(
      I),F1(I),E1(I)

```

```

2430 IMAGE MD0.00,1X,MD2.000,1X,
      MD2.000,3X,MDZ.00
2440 NEXT I
2450 PRINT "
2460 PRINT " SLOPE =" ,S5."
      Volts/m/sec"
2470 PRINT "
2471 FOR I=1 TO M1
2472 PRINT USING 2473 : I,Y(I)
2473 IMAGE "COEFFICIENT ",D," =
      ",MD0Z.000000
2474 NEXT I
2475 PRINT "
2480 ON KEY# 5."Plot" GOTO 2500
2490 GOTO 2490
2500 P1=0 @ P2=60 @ P3=2 @ P4=5
      @ P5=7.9 @ P6=4.9 @ P7=0 @
      P8=0 @ P9=-N
2510 P1$="Air Velocity m/sec"
2520 P2$="Bridge Output volts"
2530 P3$="CALIB OF H-W "
2540 FOR I=1 TO N
2550 P1(I)=F(I)
2560 P2(I)=E(I)
2570 NEXT I
2580 GOSUB 5000
2590 COPY @ GCLEAR
2600 P1=0 @ P2=60 @ P3=0 @ P4=10
      @ P5=7.9 @ P6=11.9 @ P7=0
      @ P8=0 @ P9=-N
2610 P1$="Air Velocity m/sec " @
      P2$="Linearizer Output vol
      ts " @ P3$="LINEAR OUTPUT"
2620 FOR I=1 TO N
2630 P1(I)=F(I)
2640 P2(I)=F1(I)
2650 NEXT I
2660 GOSUB 5000
2670 COPY
2730 END
2740 ! SUBPROGRAM Init hp85
2750 ! QA 7/08/80 GAC
2760 ! P1=Dvmmeter
2770 DIM Q7$(23) ! DEVICE NAMES
2780 !
2790 ! INITIALIZE DEVICE ADDRES
      SES
2800 !

3520 !
3530 RETURN
3540 ! SUBEND
3550 ! SUBPROGRAM Rvdv hp85
3560 ! QA 7/11/80 GAC
3570 ! P1=Channel

```

```

3860 GOSUB 3900 ! CALL Warn
3870 P=9.E19
3880 RETURN
3890 ! SUBEND
3900 ! SUBPROGRAM Warn hp85
3910 ! QA 7/30/80 GAC
3920 ! R3=ERROR WORD
3930 ! R$=NAME OF CALLING ROUTIN
E
3940 DIM Q8$(100),Q6$(15)
3950 O5=2 ! MAXIMUM NUMBER OF WA
RINGS ON EACH ERROR ( <=9

```

```

4590 RETURN
4600 ! SUBEND
5000 ! SUBPROGRAM Plot hp85 rev
A 88/07/16
5010 ! P1=Xmin,P2=Xmax,P3=Ymin,P
4=Ymax,P5=Xtic,P6=Ytic,P7=X
log,P8=Ylog
5020 ! P1$=Xlabels,P2$=Ylabels,P
1()=X(*),P2()=Y(*),P9=N,P3$
=Title

```

```

6220 RETURN
6230 ! SUBEND

```

TODAY'S DATE = 16-9-82

1	-0	130	2	378570
2	1	100	2	721070
3	3	770	2	965880
4	6	520	3	104700
5	15	340	3	235530
6	28	210	3	363530
7	37	100	3	434690
8	51	040	3	506880
9	70	190	3	589990
10	88	370	3	657970
11	119	000	3	732640
12	179	040	3	850160
13	219	400	3	908610
14	287	240	3	992270
15	342	010	4	044800
16	423	100	4	106980
17	523	410	4	171960
18	680	210	4	249430
19	844	000	4	309350
20	1067	110	4	375160
21	1258	420	4	418130

	UTRAN	VELOC	BOUOUTPUT	TEMP
1	0	0	2	379
2	1	56	2	721
3	3	95	2	966
4	6	42	3	105
5	15	01	3	236
6	28	99	3	364
7	37	21	3	435
8	51	70	3	507
9	70	54	3	590
10	88	07	3	658
11	119	33	3	733
12	179	01	3	850
13	219	16	3	909
14	287	36	3	992
15	342	70	4	045
16	423	70	4	107
17	523	23	4	172
18	680	02	4	249
19	844	46	4	309
20	1067	88	4	375
21	1258	07	4	418

** Linearizer Coefficients **

A = 1.000000
 B = 1.000000
 C = 1.000000
 D = 1.000000

```

*****
VELOCITY NFLOW COUTPUT PERROR
1.66 0.312 0.287 -0.31
2.95 0.557 0.601 -0.35
4.42 0.834 0.837 -0.32
6.01 1.132 1.113 -0.17
7.99 1.505 1.450 -0.35
9.21 1.736 1.673 -0.36
10.70 2.016 1.930 -0.42
12.34 2.364 2.269 -0.45
14.07 2.652 2.586 -0.48
16.33 3.077 2.980 -0.34
20.01 3.771 3.712 -0.15
22.16 4.178 4.133 -0.05
25.36 4.778 4.810 0.03
27.70 5.220 5.284 0.12
30.72 5.800 5.898 0.16
34.23 6.450 6.606 0.15
39.02 7.352 7.545 0.26
43.46 8.120 8.349 0.29
48.88 9.211 9.315 0.11
53.07 10.000 9.996 -0.04

```

SLOPE =
 188437538137
 Volt m/sec

COEFFICIENT 1 = 0.020827
 COEFFICIENT 2 = 3.006450
 COEFFICIENT 3 = 4.507593
 COEFFICIENT 4 = -2.407382
 COEFFICIENT 5 = 2.806635

Bridge Output volts

APPENDIX F

HOT-WIRE EQUATIONS

General mathematical expressions relating the time-mean, $\bar{E}$, and the mean square, $\overline{e^2}$, of the linearized voltages of the hot-wire anemometer output to the velocity components U , V and W and the six Reynolds stresses $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$, $\overline{uv}$, $\overline{vw}$ and $\overline{uw}$ are derived below for slanted and/or normal ($\alpha = 0$) hot-wire probes. The present analysis is based on a common approach found in the literature [47, 48, 49, 53] for determining the parameters of a 3-dimensional flow by means of hot-wire technique.

Recall equation (5.4) which gives the relationship between the instantaneous flow field parameters and the instantaneous linearized output voltage of the hot-wire anemometer.

$$\begin{aligned} \frac{E(\psi, \alpha)}{S} = & \left\{ [(U + u) \cos \psi + (W + w) \sin \psi] \sin \alpha + (V + v) \cos \alpha \right\}^2 \\ & + k^2 \left\{ [(U + u) \cos \psi + (W + w) \sin \psi] \cos \alpha - (V + v) \sin \alpha \right\}^2 \\ & + h^2 \left\{ (U + u) \sin \psi - (W + w) \cos \psi \right\}^2 \quad (F.1) \end{aligned}$$

After some algebraic manipulation, it is possible to rewrite the above equation in the following form.

$$\begin{aligned} \frac{E(\psi, \alpha)}{S} = & \{ (V^2 + v^2) (\cos^2 \alpha + k^2 \sin^2 \alpha) \\ & + (U^2 + u^2) (\sin^2 \alpha \cos^2 \psi + k^2 \cos^2 \alpha \cos^2 \psi + h^2 \sin^2 \psi) \\ & + (W^2 + w^2) (\sin^2 \alpha \sin^2 \psi + k^2 \cos^2 \alpha \sin^2 \psi + h^2 \cos^2 \psi) \\ & + (VU + uv) (1 - k^2) \sin 2 \alpha \cos \psi \\ & + (VW + vw) (1 - k^2) \sin 2 \alpha \sin \psi \\ & + (UW + uw) (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \sin 2 \psi + D \}^{1/2} \quad (F.2) \end{aligned}$$

---

* In the analysis of the hot-wire equations, they refer to Reynolds stresses/fluid density (ρ)

Furthermore, equations (F.2) can be put into the following simpler form:

$$\frac{E(\psi, \alpha)}{S} = \lambda \left\{ 1 + \frac{av^2 + bu^2 + cw^2}{\lambda^2} + \frac{duv + fvw + quw}{\lambda^2} + \frac{D}{\lambda^2} \right\}^{1/2} \quad (F.3a)$$

and,

$$D = 2a vV + 2b uU + 2c wW + g(wU + uW) + d(vU + uV) + f(vW + wV) \quad (F.3b)$$

where:

$$\left. \begin{aligned} a &= \cos^2 \alpha + k^2 \sin^2 \alpha \\ b &= \sin^2 \alpha \cos^2 \psi + k^2 \cos^2 \alpha \cos^2 \psi + h^2 \sin^2 \psi \\ c &= \sin^2 \alpha \sin^2 \psi + k^2 \cos^2 \alpha \sin^2 \psi + h^2 \cos^2 \psi \\ d &= (1 - k^2) \sin 2\alpha \cos \psi \\ f &= (1 - k^2) \sin 2\alpha \sin \psi \\ g &= (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \sin 2\psi \end{aligned} \right\} \quad (F.3c)$$

$$\lambda = (a v^2 + b u^2 + c w^2 + d uv + f vW + g uW)^{1/2} \quad (F.3d)$$

Equations (F.3 a-d) give the exact and general relationship between the instantaneous velocity vector and the linearized output voltage signal of the hot-wire system. It also indicates that the output signal is influenced by a primary term λ which includes only time-mean velocity components plus extra terms which involve turbulent fluctuation components. However, in the case of flows in which the turbulence intensity $\sqrt{\frac{1}{3}(u^2 + v^2 + w^2)}/\bar{Q}$ is less than 15% [62], equation (F.3a) can

be simplified by expansion into a binomial series of the first degree i.e. $(1 + \Delta)^{1/2} = 1 + \Delta/2$ and written as follows:

$$\frac{E(\gamma, \alpha)}{S} = \lambda \left\{ 1 + \frac{a v^2 + b u^2 + c w^2}{2 \lambda^2} + \frac{d uv + f vw + g uw}{2 \lambda^2} + \frac{D}{2 \lambda^2} \right\} \quad (F.4)$$

Time-averaging of equation (F.4) will give:

$$\frac{\bar{E}(\gamma, \alpha)}{S} = \lambda \left\{ 1 + \frac{a \bar{v}^2 + b \bar{u}^2 + c \bar{w}^2}{2 \lambda^2} + \frac{d \bar{uv} + f \bar{vw} + g \bar{uw}}{2 \lambda^2} \right\} \quad (F.5)$$

This equation can be further simplified by neglecting the turbulence terms since they are relatively small (about 0.017 of $2 \lambda^2$; based on typical values for Reynolds stresses measured at $(r - r_i)/(r_0 - r_i) = 0.98$ and presented in Chapter VI for the highest tested swirl number, also see Appendix I) compared to the time-mean velocity components.

This simplified relationship is given by:

$$\frac{\bar{E}(\gamma, \alpha)}{S} = (a V^2 + b U^2 + c W^2 + d UV + f VW + g UW)^{1/2} \quad (F.6)$$

Equation (F.6) relates the time-mean output signal of the hot-wire, $\bar{E}$, directly to the three time-mean velocity components of the flow. It can be used for both slanted and normal probes. The coefficients a, b, c, d, f, g are functions of probe angle α , rotational angle γ and the yaw and pitch coefficients, k and h of the probe. k and h are considered to be sensitivity coefficients and their values depend upon probe geometry and the velocity range. The technique used to determine their values is

their values is outlined in Appendix E. Equation (F.6) is derived by making certain approximations and hence is restricted to low turbulence intensity level; Acrivellis [60] derived a similar formula using incorrect assumptions and claimed that it was valid for flows of any turbulence intensity level.

To determine the three mean velocity components, from equation (F.6), requires that a system of three independent linear equations be established by taking measurements for three suitable ψ values with the slanted hot-wire probe at each point in the flow.

If e is considered to be the fluctuating component of the linearized output signal from the anemometer system, it can be related to the linearized instantaneous and time-average signals, E and $\bar{E}$, by

$$\frac{e(\psi, \alpha)}{S} = \frac{E(\psi, \alpha) - \bar{E}(\psi, \alpha)}{S} \quad (F.7)$$

Substitution of equations (F.4) and (F.5) into equation (F.7) yields

$$\begin{aligned} \frac{e(\psi, \alpha)}{S} = \frac{1}{2\lambda} [& a(v^2 - \bar{v}^2) + b(u^2 - \bar{u}^2) + c(w^2 - \bar{w}^2) \\ & + d(uv - \bar{u}\bar{v}) + f(vw - \bar{v}\bar{w}) + g(uw - \bar{u}\bar{w}) + D] \end{aligned}$$

The above equation can be rewritten in the form

$$\frac{e(\psi, \alpha)}{S} = \frac{1}{2\lambda} (A + B + D) \quad (F.8)$$

where,

2

$$A = a(v^2 - \overline{v^2}) + b(u^2 - \overline{u^2}) + c(w^2 - \overline{w^2})$$

$$B = d(uv - \overline{uv}) + f(vw - \overline{vw}) + g(uw - \overline{uw})$$

D as defined by equation (F.3b)

Squaring equation (F.8) and time-averaging gives

$$\frac{\overline{e^2(\gamma, \alpha)}}{s^2} = \frac{1}{4\lambda^2} \{ \overline{(A + B)^2} + \overline{D^2} \}$$

The term $\overline{(A + B)^2}$ in the above equation is a function of $\overline{u^4}$, $\overline{u^2v^2}$, $\overline{uv^2}$, ... etc.) and for low turbulence intensity higher correlations can be neglected with respect to the other term, $\overline{D^2}$, which is a function of $\overline{v^2}$, $\overline{u^2}$, $\overline{w^2}$, $\overline{uv}$, $\overline{vw}$ and $\overline{uw}$. Consequently $\overline{e^2}$ is approximately given by:

$$\frac{\overline{e^2(\gamma, \alpha)}}{s^2} = \frac{1}{4\lambda^2} \overline{D^2}$$

Substitution of the expression for D from equation (F.3b) into the above equation and rearrangement gives:

$$\frac{\overline{e^2(\gamma, \alpha)}}{s^2} = \frac{1}{4\lambda^2} \overline{(a_1v + b_1u + c_1w)^2} \quad (F.9a)$$

where,

$$a_1 = 2a \overline{V^2} + d \overline{V} + f \overline{W}$$

$$b_1 = 2b \overline{U^2} + g \overline{W} + d \overline{V}$$

$$c_1 = 2c \overline{W^2} + g \overline{U} + f \overline{V} \quad (F.9b)$$

The final form to be used in the data reduction is obtained by expanding

the right hand side of equation (F.9a) to give

$$\frac{\overline{\sigma^2}(\psi, \alpha)}{s} = \frac{1}{4\lambda^2} (a^2 \overline{v^2} + b^2 \overline{u^2} + c^2 \overline{w^2} + 2a_1b_1 \overline{uv} + 2b_1c_1 \overline{uw} + 2a_1c_1 \overline{vw}) \quad (\text{F.10})$$

Due to the large scatter in the distribution of the velocity component V (error up to 118%, see Appendix I) and its small value relative to U and W ($V=0.02 \overline{U}$), it was preferred to neglect the terms that include V in equation (F.9b) as this minimized the scatter in the data of Reynolds stresses. Thus the coefficients a_1 , b_1 , and c_1 can be defined by

$$\begin{aligned} a_1 &= d U + f W \\ b_1 &= 2b U + g W \\ c_1 &= 2c W + g U \end{aligned} \quad (\text{F.11})$$

Equation (E.10) gives the general relation between the mean square of the fluctuating component of the linearized anemometer output signal and the Reynolds stresses. It is valid for both slanted and normal hot-wire probes provided that the turbulence intensity is less than 15%. The coefficients a_1 , b_1 and c_1 should be computed at each local position in the flow field and for every probe orientation angle ψ .

Finding all six Reynolds stress components requires that a system of six independent linear equations be established by selecting suitable α and ψ values for the hot-wire probe and making measurements for each of the selected values at each point in the flow field. For any chosen value of ψ , the probe prongs should not be in a position which will

place the wire sensor in the wake of one of the prongs. The possibility of such interference from probe prongs can be checked by rotating the probe about its axis and measuring the RMS of the output signal; this signal was found to increase dramatically when interference occurred and such values of ψ were avoided. The values used for ψ are given in table 4.4. Furthermore, whenever probe calibration is carried out, the probe stem should be maintained in the same orientation with respect to the main flow direction as in the test section of the swirl tunnel.

APPENDIX G

COMPUTER PROGRAMS "VELOC" AND "CALCUL"

A brief description for the main parameters of programs "VELOC" and "CALCUL" and their statements are given below along with the flowcharts (Figs. G.1 and G.2), the program listing and samples of their print-out.

COMPUTER PROGRAM "VELOC"

Parameter	Description
A7(,), A8(,), A9(,) B (,), C (,), D5(,) Y6( )	Arrays utilized in the subprogram of Matrix Inversion (lines # 1549-1589), see Ref. [96] for more details.
D ( )	Non-dimensional radius $r^*, r^* = (r - r_0)/(r_0 - r_i)$.
E (,), E1(,)	$\bar{E}^2$ and e^2 , respectively (six values were measured for each of voltages at every radial location; three were taken by each hot-wire probe).
U ϕ (,), V ϕ (,), W ϕ ( )	U, V and W, respectively.
U1(,), V1(,), W1( )	$\bar{u}_v$, $\bar{v}_w$ and $\bar{u}_w$, respectively.
U2( ), V2( ), W2( )	$\bar{u}^2$, $\bar{v}^2$, and $\bar{w}^2$ or their root-mean square values.
U3( )	Contains values of b,c,d,f,g given by equation (5.7).
A6(,)	Contains the coefficients of equation (5.6) which was written for each of the six ψ angles, see table 4.4.

Parameter	Description
W3( )	Values of ψ angles.
Z (,)	Values of λ as calculated from equation (5.7). Row number refers to the number of the point in the flow under consideration. Column number refers to the number of probe orientation angle ψ .
D4( )	Contains values of a_1 , b_1 and c_1 , see equation (5.7).
D ~~4~~ ( )	Radius in meters.
H1, K1	Pitch factor h and square of yaw factor k for straight hot-wire probe.
A1	Sensor angle, α , of slanted hot-wire probe.
H, K2	Pitch factor h and square of yaw factor k for slanted hot-wire probe.
T3	Air temperature in Kelvin.
X1	Air density ρ .
V3	Number of iterations.
M	Mass flow rate.
U9	Mean axial velocity U .

Program Description (see Fig. G.1)

Line #	Description
140-230	Reading values of $\bar{E}^2$ and $\bar{e}^2$ taken by slanted hot-wire probe.
240-330	Reading values of $\bar{E}^2$ and $\bar{e}^2$ taken by straight hot-wire probe.
1100-1400	Calculation of the velocity components U, V and W by the iteration method using the three values of $\bar{E}$ measured by slanted hot-wire probe. Lines # 1210, 1255 and 1295 represent equation (5.5) after substituting for $\Psi = 90^\circ, 45^\circ$ and 0° respectively to calculate U^2, V and W^2 .
1403-1430	Calculation of λ values using equation (5.7) which corresponding to every Ψ angle.
1432-1507	Calculation of the coefficients used in equation (5.6) corresponding to every Ψ angle.
1506-1592	Solving the set of the six linear equation by Gauss-Jordan elimination method with pivotal condensation to determine Reynolds stresses at each radial location, for more details see Ref. [96]. This method was preferred to the iteration method because the latter was faced with instability problem when the values of Reynolds stresses were very small which is the case with the results of the central flow region.
1640-1730	Calculation of mass flow rate and mean axial velocity $\bar{U}$ using equations (5.20) and (5.22).
2020-6200	Printing values of U, V, W, $-\overline{uv}/\frac{1}{2}\bar{U}^2$, $-\overline{vw}/\frac{1}{2}\bar{U}^2$, $\overline{uw}/\frac{1}{2}\bar{U}^2$, $\sqrt{u^2}$, $\sqrt{v^2}$ and $\sqrt{w^2}$. These results were also saved on a tape by statements 6300-6370.

Fig. 6.1 Flowchart of computer program "VELOC"

```

10  PROGRAM**VELOC** TO CALCULA
    TE MEAN VELOC COMPONENTS & P
    ENOLDS STRESSES
20  OPTION BASE 1
25  DIM A7(6,6),A9(6)
30  DIM Y6(19),A8(19),D(19)
40  DIM E(17,6),E1(17,6),U0(19),
    V0(19),W0(19),U1(19),V1(19),
    W1(19),U2(19),W2(19),V2(19)
50  DIM U3(19),A6(9,7),W3(19),D4
    (30),D5(19),D0(19)
55  DIM B(9,14),C(7,7),Z(19,9)
70  M0=0 @ N=0 @ A1=0 @ S1=0 @ H
    =0 @ I=0 @ S2=0 @ T1=0 @ T2=
    0 @ N0=0 @ S0=0
80  FOR I=1 TO 17
90  FOR J=1 TO 6
100 E(I,J)=0 @ E1(I,J)=0
110 NEXT J @ NEXT I @ CLEAR @ DI
    SP " "
120 DISP "INPUT NUMBER OF READIN
    GS N " @ INPUT N
130 M0=0
140 DISP "INPUT DATA FILE NAME ,
    BUFFER NUMBER AND TAPE NUMB
    ER FOR READING VEEL0C6"
150 DISP " " @ DISP " INPUT B1$,
    M1, M2"
160 INPUT B1$,M1,M2
170 ASSIGN# M1 TO B1$
180 FOR I=1 TO N
190 FOR J=1 TO 3
200 READ# M1 , E1(I,J),E(I,J)
210 NEXT J @ NEXT I
220 READ# M1 , S1,T1
230 ASSIGN# M1 TO *
240 DISP " " @ DISP "INPUT FILE
    NAME , BUFFER NUMBER AN
    D TAPE NUMBER FOR READING VE
    LOC2"
250 DISP " " @ DISP " INPUT B2$,
    M3,M4
260 INPUT B2$,M3,M4
270 ASSIGN# M3 TO B2$
280 FOR I=1 TO N
290 FOR J=4 TO 6
300 READ# M3 , E1(I,J),E(I,J)
310 NEXT J @ NEXT I
320 READ# M3 , S2,T2
330 ASSIGN# M3 TO *
560 CLEAR @ DISP
580 DISP "*" CALCULATION WILL *"
    @ DISP "*" BE STARTED SOON *
    "
601 CLEAR
602 DISP " INPUT PITCH FACTOR H1
    ,YAW FACTOR K1 FOR STRAIGHT
    PROBE"
603 DISP "INPUT H1 K1"
604 INPUT H1,K1

```

```

610 CLEAR @ DISP " "
620 DISP "INPUT PROBE-ANGLE  $\alpha$  ,
      PITCH-FACTOR H , YAW-FACTOR
      K2"
630 DISP " " @ DISP " INPUT
       $\alpha$  , H , K2"
640 INPUT A1,H,K2
650 DEG @ DISP " "
660 DISP " INPUT SECTION-NUMBER
      N0 , SWIRL-ANGLE S0, CONTROL
      LING GUARD M0"
670 DISP " " @ DISP "IF M0=1 e
      xp. results will be printed
      , PUT M0=3 if not required"
680 DISP " " @ DISP "INPUT N0,
      S0,M0 " @ INPUT N0,S0,M0
690 PRINT "SECTION-NUMBER= ",N0
700 PRINT "SWIRL - ANGLE = ",S0
710 !
740 PRINT " " @ PRINT " EXPER
      IMENTAL RESULTS"
750 PRINT " -----
      " @ PRINT " "
760 ! CALCULATION OF VELOCITY CO
      MPONENTS AND TURBULENT SHEAR
      STRESSES
770 ! variable D is non-dimsionl
      ess radius , D=(R - Rinner)/
      (Router-Rinner)
780 D(1)=1 @ D(2)=46/48 @ D(3)=1
      5/16
790 D(4)=11/12 @ D(5)=7/8 @ D(6)
      =5/6 @ D(7)=3/4
800 D(8)=2/3 @ D(9)=7/12 @ D(10)
      =1/2 @ D(11)=5/12
810 D(12)=1/3 @ D(13)=1/4 @ D(14)
      =1/6 @ D(15)=1/8
820 D(16)=1/12 @ D(17)=1/16 @ D(
      18)=1/24
830 D(19)=0
840 IF M0=2 THEN GOTO 890
850 FOR I=1 TO N
860 PRINT USING 870 ; I+1,D(I+1)
      ,E(I,1),E(I,2),E(I,3),E(I,4)
      ,E(I,5),E(I,6)
870 IMAGE DD,X,Z DD,4(MDZ,DD),8X
      ,2(MDZ DD),/
880 NEXT I
890 J6=.0254*.0254 ! J6 is a con
      version factor lin2=J6 m2
900 FOR I=1 TO N+2
910 D0(I)=.0254*(3*D(I)+5)
920 NEXT I
930 DIM A3(21),A5(21),A2(21),A4(
      21)
950 PRINT " *****
      *****" @ PRINT " "
960 IF M0=2 THEN GOTO 1011
970 FOR I=1 TO N
990 PRINT USING 1000 ; I+1,D(I+1)
      ,E1(I,1),E1(I,2),E1(I,3),E1
      (I,4),E1(I,5),E1(I,6)

```

```

1000 IMAGE DD,X,Z,DD,4(MZ,DDD),8
      X,2(MZ,DDD),/
1010 NEXT I
1011 T3=(5*(T1+T2)+460)*(5/9)
1012 X1=352.921/T3
1100 A=COS(A1)^2+K2*SIN(A1)^2
1105 B=SIN(A1)^2+K2*COS(A1)^2
1110 D=(1-K2)*SIN(2*A1) @ CLEAR
1125 DISP "input value of U3, U3
      =1 if residuals are require
      d to be printed, U3=3 if no
      t required"
1130 INPUT U3
1135 IF U3>1 THEN GOTO 1160
1140 PRINT " "
1145 PRINT "Values of residuals
      in calculating U,V,W " @ PR
      INT " "
1150 PRINT " I ", " ITER ", " Res
      ,U ", " Res,U ", " Res,W"
1155 PRINT
1160 V3=30
1165 FOR I=1 TO N
1170 U7=E(I,3)/S1^2
1175 V7=E(I,2)/S1^2
1180 W7=E(I,1)/S1^2
1185 V=.1 @ U0=20 @ U=U0
1190 IF S0>0 THEN W0=U0*TAN(S0)-
      4 ELSE W0=.1
1195 W=W0
1200 U4=1
1205 U5=U @ V5=V @ W5=W
1210 U2=1/H-2*(U7-(B*W^2+A*U^2+V
      *W*D))
1215 U=SQR(U2)
1225 L1=0/SQR(2) @ F1=L1*(U+W) @
      G1=1-F1
1240 L3=(U^2+W^2)*(B+H^2)/2
1245 F3=U*W*(B-H^2)
1255 V1=0 @ V1=G1*(V7-(A*U^2+L3+
      F3))
1260 V9=(V1+V)/2 @ V8=0
1265 IF U4<3 THEN V4=V9 ELSE V4=
      V1
1270 IF S0>0 THEN V8=V1 ELSE V8=
      V4
1280 V=V8
1285 F2=1/H-2
1295 W2=F2*(W7-(A*U^2+B*U^2+U*V*
      D))
1305 W4=SQR(ABS(W2*W^2)) @ W9=W4
      *(W2+W^2)/2
1310 IF U4>0 THEN W4=ABS(W9)/ABS
      (W2)
1315 IF S0>0 THEN W3=W2 ELSE W3=
      W4
1320 W=SQR(W3)
1325 U6=ABS(U-U5) @ V6=ABS(V-V5)
      @ W6=ABS(W-W5)
1330 IF U4>=V3 THEN GOTO 1360

```

```

1335 U4=U4+1
1340 IF U6<.06 THEN GOTO 1345 EL
SE GOTO 1355
1345 IF V6<.06 THEN GOTO 1350 EL
SE GOTO 1355
1350 IF W6<.1 THEN GOTO 1360
1355 GOTO 1205
1360 U0(I+1)=U @ V0(I+1)=V @ W0(
I+1)=W
1365 IF U3>1 THEN GOTO 1380
1370 PRINT USING 1375 ; I,U4,U6,
V6,W6
1375 IMAGE DD.3X,DD,X,3(MDZ.DDDD
)
1380 NEXT I
1400 !
1401 !
1402 PRINT
1403 FOR I5=1 TO N
1404 W3(1)=0 @ W3(2)=45 @ W3(3)=
90
1405 B=SIN(A1)^2+K2*COS(A1)^2
1406 L3=0 @ L4=W0(I5+1) @ L5=U0(
I5+1)
1407 A=COS(A1)^2+K2*SIN(A1)^2
1408 D=(1-K2)*SIN(2*A1)
1409 FOR J=1 TO 3
1410 F1=COS(W3(J)) @ F2=SIN(W3(J
)) @ F3=SIN(2*W3(J))
1411 G1=A @ G2=B*F1^2+(H*F2)^2
1412 G3=B*F2^2+(H*F1)^2
1413 G4=D*F1 @ G5=D*F2
1414 G6=(B-H^2)*F3
1415 Z(I5,J)=G1*L3^2+G2*L5^2+G3*
L4^2+G4*L5*L3+G5*L3*L4+G6*L
4*L5
1416 NEXT J
1417 !
1418 A2=0 @ A5=COS(A2)^2+K1*SIN(
A1)^2 @ B5=SIN(A2)^2+K1*COS
(A2)^2
1419 D5=(1-K1)*SIN(2*A2)
1420 W3(4)=-45 @ W3(5)=0 @ W3(6)
=90
1421 L3=0 @ L4=W0(I5+1) @ L5=U0(
I5+1)
1422 !
1423 FOR J=4 TO 6
1424 F1=COS(W3(J)) @ F2=SIN(W3(J
)) @ F3=SIN(2*W3(J))
1425 G1=A5 @ G2=B5*F1^2+(H1*F2)^
2 @ G3=B5*F2^2+(H1*F1)^2
1426 G4=D5*F1 @ G5=D5*F2
1427 G6=(B5-H1^2)*F3
1428 Z(I5,J)=G1*L3^2+G2*L5^2+G3*
L4^2+G4*L5*L3+G5*L3*L4+G6*L
4*L5
1429 NEXT J
1430 NEXT I5
1431 CLEAR

```



```

1432 DISP "CALCULATION FOR v,u,w
      ,uv,vw,uw IN PROGRESS"
1433 PRINT
1434 FOR I5=1 TO N
1435 W3(1)=0 @ W3(2)=45 @ W3(3)=
      90
1436 B=SIN(A1)^2+K2*COS(A1)^2
1437 N2=1 @ L3=0 @ L4=W0(I5+1) @
      L5=U0(I5+1)
1438 L1=(COS(A1)^2+K2*SIN(A1)^2)
      *2
1439 FOR J=1 TO 3
1440 U3(1)=(B*COS(W3(J))^2+H^2*S
      IN(W3(J))^2)*2
1441 U3(2)=(B*SIN(W3(J))^2+H^2*C
      OS(W3(J))^2)*2
1442 U3(3)=0*COS(W3(J))
1443 U3(4)=0*SIN(W3(J))
1444 U3(5)=(B-H^2)*SIN(2*W3(J))
1445 D4(N2)=L1*L3+U3(3)*L5+U3(4)
      *L4
1446 D4(N2+1)=U3(1)*L5+U3(5)*L4+
      U3(3)*L3
1447 D4(N2+2)=U3(2)*L4+U3(5)*L5+
      U3(4)*L3
1448 N2=3*J+1
1449 NEXT J
1450 FOR J=1 TO 3
1451 W3(J)=4*Z(I5,J)
1452 NEXT J
1453 N2=1
1454 FOR J=1 TO 3
1455 A6(J,1)=D4(N2)^2/W3(J)
1456 A6(J,2)=D4(N2+1)^2/W3(J)
1457 A6(J,3)=D4(N2+2)^2/W3(J)
1458 A6(J,4)=2*D4(N2)*D4(N2+1)/W
      3(J)
1459 A6(J,5)=2*D4(N2)*D4(N2+2)/W
      3(J)
1460 A6(J,6)=2*D4(N2+1)*D4(N2+2)
      /W3(J)
1461 N2=N2+3 @ NEXT J
1462 B(1,1)=E1(I5,1)/S1^2
1463 B(2,1)=E1(I5,2)/S1^2
1464 B(3,1)=E1(I5,3)/S1^2
1465 !
1466 B(4,1)=E1(I5,4)/S2^2
1469 B(5,1)=E1(I5,5)/S2^2
1470 B(6,1)=E1(I5,6)/S2^2
1477 !
1478 A2=0 @ A5=COS(A2)^2+K1*SIN(
      A1)^2 @ B5=SIN(A2)^2+K1*COS
      (A2)^2
1479 D5=(1-K1)*SIN(2*A2)
1480 N2=19 @ W3(4)=-45 @ W3(5)=0
      @ W3(6)=90
1481 L3=0 @ L4=W0(I5+1) @ L5=U0(
      I5+1)
1482 L1=(COS(A2)^2+K1*SIN(A2)^2)
      *2

```

```

1483 FOR J=4 TO 6
1484 U3(1)=(B0*COSCOS(W3(J))^2+H1^2
      *SIN(W3(J))^2)*2
1485 U3(2)=(B5*SIN(W3(J))^2+H1^2
      *COS(W3(J))^2)*2
1486 U3(3)=D5*COS(W3(J))
1487 U3(4)=D5*SIN(W3(J))
1488 U3(5)=(B5-H1^2)*SIN(2*W3(J)
      )
1489 D4(N2)=L1*L3+U3(3)*L5+U3(4)
      *L4
1490 D4(N2+1)=U3(1)*L5+U3(5)*L4+
      U3(3)*L3
1491 D4(N2+2)=U3(2)*L4+U3(5)*L5+
      U3(4)*L3
1492 N2=3*J+1
1493 NEXT J
1494 FOR J=4 TO 6
1495 W3(J)=4*Z(15,J)
1496 NEXT J
1497 N2=19
1498 FOR J=4 TO 6
1499 A6(J,1)=D4(N2)^2/W3(J)
1500 A6(J,2)=D4(N2+1)^2/W3(J)
1501 A6(J,3)=D4(N2+2)^2/W3(J)
1502 A6(J,4)=2*D4(N2)*D4(N2+1)/W
      3(J)
1503 A6(J,5)=2*D4(N2)*D4(N2+2)/W
      3(J)
1504 A6(J,6)=2*D4(N2+1)*D4(N2+2)
      /W3(J)
1505 N2=N2+3 @ NEXT J
1506 N0=6
1507 !
1508 FOR I=1 TO 6
1509 FOR J=1 TO 6
1510 A7(I,J)=A6(I,J)
1512 NEXT J @ NEXT I
1516 !
1518 FOR I=1 TO 6
1519 A9(I)=B(I,1)
1521 NEXT I
1522 FOR I=1 TO 6
1523 B(I,1)=A9(I) @ NEXT I
1524 N1=N0 @ N2=N0
1525 FOR I1=1 TO N1
1526 FOR I2=1 TO N2
1527 C(I1,I2+1)=A6(I1,I2)
1528 NEXT I2 @ NEXT I1
1529 FOR I1=1 TO N1
1530 C(I1,1)=B(I1,1)
1531 NEXT I1
1532 GOSUB 1549
1533 FOR I1=1 TO N1
1534 FOR I2=1 TO N2
1535 A6(I1,I2)=B(I1,I2)
1536 NEXT I2 @ NEXT I1
1537 FOR I1=1 TO N1
1538 B(I1,1)=C(I1,1)
1539 NEXT I1

```

```

1540 M1=N0 @ N1=N0 @ M2=N0 @ N2=
1      1
1541 FOR I=1 TO M1
1542 FOR J=1 TO N2
1543 C(I,J)=0
1544 FOR K=1 TO M1
1545 C(I,J)=C(I,J)+A6(I,K)*B(K,J)
1546 NEXT K @ NEXT J @ NEXT I
1547 M=N0
1548 GOTO 1590
1549 ! MATFIX INVERSION
1550 FOR I=1 TO N0
1551 FOR J=1 TO N0
1552 B(I,J+N0)=0 @ B(I,J)=A6(I,J)
1553 NEXT J
1554 B(I,I+N0)=1
1555 NEXT I
1556 FOR K=1 TO N0
1557 IF K=N0 THEN GOTO 1568
1558 M=K
1559 FOR I=K+1 TO N0
1560 IF ABS(B(I,K))>>ABS(B(M,K))
1561 THEN M=I
1562 NEXT I
1563 IF M=K THEN GOTO 1568
1564 FOR J=K TO 2*N0
1565 B(K,J)=B(M,J)
1566 B(M,J)=B
1567 NEXT J
1568 FOR J=K+1 TO 2*N0
1569 B(K,J)=B(K,J)/B(K,K)
1570 NEXT J
1571 IF K=1 THEN GOTO 1578
1572 FOR I=1 TO K-1
1573 FOR J=K+1 TO 2*N0
1574 B(I,J)=B(I,J)-B(I,K)*B(K,J)
1575 NEXT J
1576 NEXT I
1577 IF K=N0 THEN GOTO 1584
1578 FOR I=K+1 TO N0
1579 FOR J=K+1 TO 2*N0
1580 B(I,J)=B(I,J)-B(I,K)*B(K,J)
1581 NEXT J
1582 NEXT I
1583 NEXT K
1584 FOR I=1 TO N0
1585 FOR J=1 TO N0
1586 B(I,J)=B(I,J+N0)
1587 NEXT J
1588 NEXT I
1589 RETURN
1590 U2(I5+1)=C(2,1) @ V2(I5+1)=
1591 C(1,1) @ W2(I5+1)=C(3,1)
1592 U1(I5+1)=C(4,1) @ V1(I5+1)=
1593 C(5,1) @ W1(I5+1)=C(6,1)
1594 NEXT I5
1595 !

```

```

1600 U0(1)=0 @ V0(1)=0 @ W0(1)=0
1601 U2(1)=0 @ V2(1)=0 @ W2(1)=0
1602 U2(N+2)=0 @ V2(N+2)=0 @ W2(
N+2)=0
1610 U0(N+2)=0 @ V0(N+2)=0 @ W0(
N+2)=0
1620 U1(N+2)=0 @ V1(N+2)=0 @ W1(
N+2)=0
1630 U1(1)=0 @ V1(1)=0 @ W1(1)=0
1640 ! Mass flow rate & Average
velocity
1650 S3=0 @ S4=0 @ FOR I=2 TO N+
2
1660 L1=0 @ F1=0 @ G1=0
1670 L1=.5*(U0(I-1)+U0(I))
1680 F1=.5*(D0(I)+D0(I-1))
1690 G1=D0(I-1)-D0(I)
1700 S3=S3+L1*F1*G1 @ S4=S4+G1*F
1
1710 NEXT I
1720 M=2*PI*X1*S3
1730 U9=S3/S4 @ S5=2*PI*S4 ! S5
is the annular area
2000 PRINT "
2010 PRINT "*****"
*****" @ PRINT "
"
2020 PRINT " TIME-MEAN VELOCITY
COMPONENTS ,TURBULENT STRES
SES AND TURBULENT VELOCITIE
S"
2030 PRINT "
2040 !
2050 PRINT " I*"; " DIST "; " * U-R
XL "; " * V-RAD "; " * W-TAN"
2060 FOR I=1 TO N+2
2070 PRINT USING 2080 ; I,D(I),U
0(I),V0(I),W0(I)
2080 IMAGE 00,2X,Z.00,3(M00Z.000
)
2090 NEXT I
2100 PRINT "
2110 PRINT " *I*"; " DIST "; " ** uv
*"; " ** vu **"; " ** uu **"
2120 FOR I=1 TO N+2
2125 F1=0 @ F1=.5*U9^2 @ F2=1000
/F1
2130 L1=U1(I)*F2*(-1) @ L2=V1(I)
*F2*(-1) @ L3=W1(I)*F2
2140 U1(I)=L1 @ V1(I)=L2 @ W1(I)
=L3
2150 PRINT USING 2160 ; I,D(I),U
1(I),V1(I),W1(I)
2160 IMAGE 00,2X,Z.00,3(M00Z.000
)
2170 NEXT I
2180 PRINT "
2190 PRINT " *I*"; " DIST "; " ** u
*"; " ** v **"; " ** w **"
2200 FOR I=1 TO N+2

```

```

2210 L1=SQR(U2(I)) @ L2=SQR(V2(I)
    ) @ L3=SQR(W2(I))
2220 U2(I)=L1 @ V2(I)=L2 @ W2(I)
    =L3
2230 PRINT USING 2160 ; I,O(I),U
    2(I),V2(I),W2(I)
2240 NEXT I
2250 PRINT
2671 PRINT " "
2672 PRINT "*****"
    *****
2673 PRINT " "
2680 PRINT " "
2690 PRINT USING 2700 ; M
2700 IMAGE "Mass flow rate = ",M
    DDZ.DDD," Kg/sec"
2710 PRINT USING 2720 ; M*2 2046
2720 IMAGE " " or = ",M
    DDZ.DDD," lb/sec"
2730 PRINT USING 2740 ; U9
2740 IMAGE "Uaxial,velocity= ",M
    DDZ.DDD," m/sec"
2750 PRINT USING 2760 ; U9*3.280
    8
2760 IMAGE " " or = ",M
    DDZ.DDD," ft/sec"
2770 PRINT " " @ PRINT USING 27
    90 ; 55*10000 @ PRINT USING
    2800 ; 55/J6
2780 PRINT " " @ PRINT "*****
    *****" @
    PRINT " "
2790 IMAGE "Annular area " = ",M
    DDZ.DDD," cm2"
2800 IMAGE " " or = ",M
    DDZ.DDD," in2"
6200 CLEAR @ DISP " "
6210 DISP "ON-KEY OPERATIONS COU
    LD BE USED NOW "
6220 ON KEY# 3,"EXTCAL" GOTO 601
6230 ON KEY# 4,"RECORD" GOSUB 63
    00
6240 GOTO 6240
6300 DISP "INPUT DATA FILE NAME
    , BUFFER NUMBER AND TAPE NU
    MBER FOR STORING THE DATA"
6310 INPUT B$,M5,M4
6320 CREATE B$,N+1,144
6330 ASSIGN# M5 TO B$
6340 FOR I=2 TO N+1
6350 PRINT# M5 ; U0(I),V0(I),W0(
    I),U1(I),V1(I),W1(I),U2(I),
    V2(I),W2(I)
6360 NEXT I
6370 ASSIGN# M5 TO *
6380 DISP
6390 DISP "**** DATA STORED****"
6400 RETURN
6900 GOTO 6200
6910 !
7150 END

```

SECTION-NUMBER= 5
SWIRL - ANGLE = 45

EXPERIMENTAL RESULTS

*****

*****

TIME-MEAN VELOCITY COMPONENTS
TURBULENT STRESSES AND TURBULENT
VELOCITIES:

I*	DIST *	U-AXL *	V-RAD *	W-TAN *
1	1.00	0.000	0.000	0.000
2	0.96	28.382	-0.150	27.133
3	0.94	29.790	-0.310	28.322
4	0.92	30.779	-0.171	28.951
5	0.88	32.188	-0.351	30.000
6	0.83	33.167	-0.252	30.350
7	0.75	33.786	-0.321	31.319
8	0.67	34.076	0.331	31.648
9	0.58	34.775	-0.400	32.308
10	0.50	35.195	0.340	32.677
11	0.42	34.785	-0.050	32.987
12	0.33	34.475	0.052	32.468
13	0.25	33.067	0.150	31.459
14	0.17	31.299	0.210	30.080
15	0.13	29.680	-0.110	28.781
16	0.08	28.491	0.152	26.783
17	0.06	27.383	-0.120	25.494
18	0.04	26.014	0.160	24.180
19	0.00	0.000	0.000	0.000

I*	DIST	** UV ***	UV ***	UV **
1	1.00	0.000	0.000	0.000
2	0.96	-5.295	-5.095	0.210
3	0.94	-5.095	-4.895	0.170
4	0.92	-4.895	-4.695	0.112
5	0.88	-4.396	-4.296	0.160
6	0.83	-3.896	-3.696	0.190
7	0.75	-2.897	-2.797	0.091
8	0.67	-2.048	-1.898	-0.051
9	0.58	-1.049	-0.799	0.189
10	0.50	-0.120	-0.100	0.082
11	0.42	0.300	0.100	0.012
12	0.33	1.029	0.699	-0.041
13	0.25	1.718	1.399	0.031
14	0.17	2.498	2.198	0.051
15	0.13	3.097	2.897	0.352
16	0.08	3.796	3.447	0.110
17	0.06	4.096	3.796	0.160
18	0.04	4.396	4.019	0.250
19	0.00	0.000	0.000	0.000

I	DIST	** U ***	V ****	W **
1	1.00	0.000	0.000	0.000
2	0.96	4.526	1.939	3.664
3	0.94	4.353	1.896	3.620
4	0.92	3.922	1.983	3.189
5	0.88	3.577	1.638	2.931
6	0.83	2.801	1.681	2.414
7	0.75	2.198	1.078	1.853
8	0.67	1.422	0.985	1.336
9	0.58	1.078	0.647	0.905
10	0.50	0.776	0.431	0.647
11	0.42	0.819	0.474	0.690
12	0.33	1.634	0.517	0.776
13	0.25	1.465	0.496	1.336
14	0.17	2.629	1.121	1.767
15	0.13	2.888	1.336	2.284
16	0.08	3.319	1.250	2.629
17	0.06	3.620	1.379	3.060
18	0.04	3.577	1.509	3.103
19	0.00	0.000	0.000	0.000

*****

Mass flow rate = 3.054 Kg/sec
or = 6.732 lb/sec
Uaxial velocity = 32.879 m/sec
or = 107.869 ft/sec

Annular area = 790.464 cm²
or = 122.522 in²

*****

COMPUTER PROGRAM "CALCUL"

Parameter	Description
A3( ), A5( ), A8( ) U3( ), V3( ), W3( ) Yφ( ), Y6( )	Arrays utilized in the subprogram "LEAST" (lines # 7860-8700), see Ref. [97] for more details.
Uφ( ), Vφ( ), Wφ( )	U, V and W, respectively.
U1( ), V1( ), W1( )	$\overline{uv}$, $\overline{vw}$ and $\overline{uw}$, respectively.
U2( ), V2( ), W2( )	$\sqrt{u^2}$, or $\overline{u^2}$, $\sqrt{v^2}$ or $\overline{v^2}$ and $\sqrt{w^2}$ or $\overline{w^2}$, respectively.
M6	The degree of polynomial, input to subprogram "LEAST".
D4( ), D5( )	Contains the gradients $\partial U / \partial r$ and $\partial W / \partial r$.
Dφ( ), D( )	Same as described before in program "VELOC".
R1( ), R2( )	$L_{rx} / (r_0 - r_i)$ and $L_{r\theta} / (r_0 - r_i)$.
A7( )	Contains values of the static pressure in cm H ₂ O.
A2( ), A4( )	μ_{rx} / μ and $\mu_{r\theta} / \mu$
X1	Air density ρ .
X2	$\mu \times 1000 / \rho$, μ is the laminar viscosity.

Program Description (see Fig. G.2)

Line #	Description
1200- 1260	Reading values of $U, V, W, -\overline{uv}/\bar{u}^2, -\overline{vw}/\bar{u}^2, \overline{uw}/\bar{u}^2, \sqrt{u^2}, \sqrt{v^2}$ and $\sqrt{w^2}$.
2070- 2300	Calculation of $\partial U/\partial r$ and $\partial W/\partial r$ by equation (5.17). Every velocity profile was divided into three regions; a middle region and two wall regions. Lines 7170-7850 states the parameters of each region and calls for the least-square curve fitting subprogram (lines 7860-8700). The latter can be found in FORTRAN in reference [97] along with an explanation for its use.
2850- 3040	Calculation of μ_{rx}/μ and μ_{re}/μ using equations (5.14) and (5.15) and printing their values.
3100- 3380	Calculation of static pressure distribution using equation (5.24) and printing its values.
3400- 3533	Calculation of $L_{rx}/(r_0 - r_i)$ and $L_{re}/(r_0 - r_i)$ using equations (3.10) and (3.11) and printing their values.

Fig. 6.2 Flowchart of computer program "CALCUL"

```

10  PROGRAM**CALCUL** TO CALCUL
    RATE EDDY VISCOSITIES, PRESSUR
    E DISTRIBUTION AND MIXING LE
    NGTHE
20  OPTION BASE 1
30  DIM Y6(21),A6(21),Y0(19)
35
40  DIM U0(21),V0(21),W0(21),U1(
    21),V1(21),W1(21),U2(21),W2(
    21),V2(21)
50  DIM U3(21),V3(21),W3(21),D4(
    21),D5(21),D0(21)
60  !
70  M0=0 @ N=0 @ A1=0 @ S1=0 @ H
    =0 @ I=0 @ S2=0 @ T1=0 @ T2=
    0 @ N0=0 @ S0=0
80  !
100 DIM D(19),R1(19),R2(19)
120 N=17
560 CLEAR @ DISP
590 DISP "INPUT AIR TEMPERATURES
    ,T1 AND T2"
620 INPUT T1,T2
650 DEG
660 DISP " INPUT SECTION-NUMBER
    N0 , SWIRL-ANGLE S0 "
680 DISP " " @ DISP "INPUT N0,
    S0 " @ INPUT N0,S0
690 PRINT "SECTION-NUMBER= ",N0
700 PRINT "SWIRL - ANGLE = ",S0
760 !
780 D(1)=1 @ D(2)=46/48 @ D(3)=1
    5/16
790 D(4)=11/12 @ D(5)=7/8 @ D(6)=
    5/6 @ D(7)=3/4
800 D(8)=2/3 @ D(9)=7/12 @ D(10)=
    1/2 @ D(11)=5/12
810 D(12)=1/3 @ D(13)=1/4 @ D(14)=
    1/6 @ D(15)=1/8
820 D(16)=1/12 @ D(17)=1/16 @ D(
    18)=1/24
830 D(19)=0
880 !
890 J6=.0254*.0254 ! J6 is a con
    version factor lin2=J6 m2
900 FOR I=1 TO N+2
910 D0(I)=.0254*(3*D(I)+5)
920 NEXT I
930 DIM A3(21),A5(21),A7(21),A2(
    21),A4(21)
950 PRINT " *****
    *****" @ PRINT "
1010 !
1011 T3=(.5*(T1+T2)+460)*(5/9)
1012 X1=352.921/T3
1100 !
1110 !
1200 DISP "INPUT DATA FILE NAME
    BUFFER NUMBER"
1210 INPUT B$,M5

```

```

1220 ASSIGN# M5 TO B$
1230 FOR I=2 TO N+1
1240 READ# M5, U0(I),V0(I),W0(I),
    U1(I),V1(I),W1(I),U2(I),V
    2(I),W2(I)
1250 NEXT I
1260 ASSIGN# M5 TO *
1270 DISP "INPUT AXIAL MEAN VELO
    CITY U5"
1280 INPUT U5
1290 U6=.5*U5^2/1000
1300 FOR I=2 TO N+1
1310 U1(I)=U1(I)*U6
1320 V1(I)=V1(I)*U6
1330 W1(I)=W1(I)*U6
1370 NEXT I
1600 U0(1)=0 @ V0(1)=0 @ W0(1)=0
1601 U2(1)=0 @ V2(1)=0 @ W2(1)=0
1602 U2(N+2)=0 @ V2(N+2)=0 @ W2(
    N+2)=0
1610 U0(N+2)=0 @ V0(N+2)=0 @ W0(
    N+2)=0
1620 U1(N+2)=0 @ V1(N+2)=0 @ W1(
    N+2)=0
1630 U1(1)=0 @ V1(1)=0 @ W1(1)=0
1640 PRINT "
2070 ! I5,I6,I7 represents M5 va
    lues in the regions
2080 ! I1,I2,I4 are the number o
    f data points to be fitted b
    y one curve
2090 !
2091 ! X5, if X5=1 curve coeff w1
    ll be printed , if X5=3 coef
    f will not be printed
2092 X5=3
2093 !
2100 I1=9 @ I2=15 @ I4=7
2110 I5=3 @ I6=4 @ I7=3 @ I8=2 @
    I9=12 @ J1=4 @ J2=5 @ J3=1
    6 @ J4=17 @ J5=N+2
2120 FOR I=1 TO N+2
2130 A3(I)=U0(I)
2140 NEXT I @ Z1$="Uaxial (1-8)"
    @ Z2$="Uaxial (7-15)"
2150 Z3$="Uaxial (14-19)" @ GOSU
    B 7170
2160 FOR I=1 TO N+2
2170 U3(I)=A5(I)
2180 D4(I)=A7(I)
2190 NEXT I
2200 X5=3
2210 ! Tangential velocity
2220 X5=3 @ I6=4 @ I7=3 @ I8=2 @
    I9=12 @ J1=4 @ J2=5 @ J3=1
    6 @ J4=17 @ J5=N+2
2230 FOR I=1 TO N+2
2240 A3(I)=W0(I)
2250 NEXT I @ Z1$="Wtang (1-8)"
    @ Z2$="Wtang (7-15)"

```

```

2260 Z3$="wtans (14-19)" @ GOSU
      B 7170
2270 FOR I=1 TO N+2
2280 W3(I)=A5(I)
2290 D5(I)=A7(I)
2300 NEXT I
2310 !
2330 !
2850 ! Calculation of turbulent v
      iscosities . X2 is the lam
      kinematic viscosity, X2=10
      00(01-X1)
2870 X2= 013125494+.000103922*(T
      3-273 333333333)
2871 !
2872 X2=X2/1000
2880 FOR I=1 TO N+2
2890 A2(I)=X1*U1(I)/D4(I)
2900 L1=D5(I)-W3(I)/D0(I)
2910 A4(I)=V1(I)*X1/L1
2920 NEXT I
2970 PRINT " "
2980 PRINT "Turbulent viscositi
      es  $\mu_{rx}/\mu_1$  &  $\mu_{r\theta}/\mu_1$  "
2981 PRINT " "
2982 PRINT "I*") "  $\mu_{rx}/\mu_1$  *****
      "  $\mu_{r\theta}/\mu_1$  *****"
2990 PRINT " "
3000 FOR I=1 TO N+2
3010 PRINT USING 3020 ; I,A2(I)/
      (X1*X2),A4(I)/(X1*X2)
3020 IMAGE 00,X,(X,M000Z,0000),(
      5X,M000Z,0000)
3030 NEXT I
3040 PRINT " "
3050 PRINT "*****"
      "*****"
3060 PRINT " "
3090 !
3091 FOR I=1 TO N+2
3093 V0(I)=0
3095 NEXT I
3100 ! Calculation of static pre
      ssure distribution
3120 CLEAR @ DISP "INPUT OUTSIDE
      WALL PRESSURE IN cm H20"
3130 INPUT Y1
3140 FOR I=1 TO N+2
3150 L1=V0(I)^2+V2(I)^2
3160 L2=W0(I)^2+W2(I)^2
3170 A5(I)=(L2-L1)/D0(I)
3180 NEXT I
3190 L1=0 @ FOR I=1 TO N+1
3200 S=D0(I)-D0(I+1)
3210 L1=S* 5*(A5(I)+A5(I+1))
3220 A5(I)=L1 @ NEXT I
3230 Y2=1/(9.81*10)
3240 FOR I=2 TO N+2
3250 S=0
3260 L1=V0(I)^2+V2(I)^2

```

```

3270 FOR J=2 TO I
3280 S=S+A5(J-1)
3281 NEXT J
3290 A7(I)=Y1-Y2*X1*(S+L1)
3300 NEXT I @ A7(1)=Y1
3310 PRINT "Static pressure dist
ribution "
3311 A2(1)=1 @ A2(19)=1 @ A4(1)=
1 @ A4(19)=1
3320 PRINT "
3330 PRINT " I " " Pst cm H2O
"
3340 FOR I=1 TO N+2
3350 PRINT USING 3360 ; I,A7(I)
3360 IMAGE 00,2X,(2X,M000Z.0000)
3370 NEXT I
3380 PRINT "
3390 PRINT "*****
*****" @ PRINT "
3400 ! CALCULATION OF MIXING LENG
THS
3410 FOR I=1 TO N+2
3420 L1=D5(I)-W3(I)/D0(I)
3430 L2=D4(I)^2+L1^2
3440 L3=SQR(L2)
3450 L4=A2(I)/(L3*X1)
3460 L5=A4(I)/(L3*X1)
3461 IF L4<0 THEN F1=-1 ELSE F1=
1
3462 IF L5<0 THEN F2=-1 ELSE F2=
1
3470 L6=SQR(ABS(L4))
3480 L7=SQR(ABS(L5))
3490 L8=3*.0254
3500 R1(I)=F1*L6/L8
3510 R2(I)=F2*L7/L8
3520 NEXT I
3523 PRINT @ PRINT "*****
*****" @ PRI
NT
3524 PRINT "MIXING LENGTHS Lrx/(
Ro-R1) AND Lr0/(Ro-R1)" @ P
RINT
3530 FOR I=1 TO N+2
3531 PRINT USING 3532 ; I,R1(I),
R2(I)
3532 IMAGE 00,3X,2(M00Z.000)
3533 NEXT I
3534 PRINT
3535 PRINT "*****
*****" @ PRINT
6910 !
7150 END
7160 !
7170 ! PROGRAM ** LEAST ** FOR P
OLYNOMIAL CURVE-FITTING
7180 CLEAR @ DISP "SUB curve-fit
ting is PROCESSING "
7190 DIM C9(5)
7300 FOR I=1 TO 11

```

```

7310 Y6(I)=0 @ A8(I)=0 @ Y6(I)=A
      3(I) @ A8(I)=D0(I)
7320 NEXT I
7330 M6=I5 @ Z$=Z1$ @ N4=I1
7340 GOSUB 7860
7350 N1=1 @ N2=J1
7360 GOSUB 7700
7370 FOR I=N1 TO N2
7380 A5(I)=A2(I)
7390 !
7400 A7(I)=A4(I)
7410 NEXT I
7420 FOR I=1 TO I2
7430 Y6(I)=0 @ Y6(I)=A3(I+I8) @
      A8(I)=D0(I+I8)
7440 NEXT I
7450 Z$=Z2$ @ N4=I2 @ M6=I6
7460 GOSUB 7860
7470 N1=J2 @ N2=J3
7480 GOSUB 7700
7490 FOR I=N1 TO N2
7500 A5(I)=A2(I)
7510 !
7520 A7(I)=A4(I)
7530 NEXT I
7540 FOR I=1 TO I4
7550 Y6(I)=A3(I+I9) @ A8(I)=D0(I
      +I9)
7560 NEXT I @ Z$=Z3$ @ N4=I4
7570 M6=I7 @ GOSUB 7860
7580 N1=J4 @ N2=J5
7590 GOSUB 7700
7600 FOR I=N1 TO N2
7610 A5(I)=A2(I)
7620 A7(I)=A4(I)
7630 NEXT I
7640 !
7700 ! SUB PROGRAM
7710 FOR I=1 TO N+2
7720 A2(I)=0 @ A4(I)=0
7730 NEXT I
7740 FOR I=N1 TO N2
7750 S=C9(I)
7760 FOR J=2 TO N6
7770 S=S+C9(J)*D0(I)^(J-1)
7780 NEXT J
7790 A2(I)=S @ NEXT I
7800 FOR I=N1 TO N2
7810 S=C9(2) @ FOR J=3 TO N6
7820 S=S+(J-1)*C9(J)*D0(I)^(J-2)
7830 NEXT J @ A4(I)=S @ NEXT I
7840 RETURN
7850 RETURN
7860 DIM A6(5,5),B6(5)
7870 DIM G6(8)
7880 FOR I=1 TO 5
7890 FOR J=1 TO 5
7900 A6(I,J)=0 @ B6(I)=0 @ C9(I)
      =0
7910 NEXT J

```

```

7920 NEXT I
7930 IF X5=1 THEN GOTO 7960
7940 PRINT "=====
      =====" @ PRINT "
"
7950 PRINT "Curve-fitting coeffi
      cients for ";Z$
7960 N6=M6+1
7970 M7=M6*2
7980 FOR I=1 TO M7
7990 G6(I)=0
8000 FOR J=1 TO N4
8010 G6(I)=G6(I)+A8(J)^I
8020 NEXT J
8030 NEXT I
8040 !
8050 A6(1,1)=N4 @ FOR I=1 TO N6
8060 FOR J=1 TO N6
8070 K6=I+J-2
8080 IF K6<=0 THEN GOTO 8100
8090 A6(I,J)=G6(K6)
8100 REM
8110 NEXT J
8120 NEXT I
8130 B6(1)=0
8140 FOR J=1 TO N4
8150 B6(1)=B6(1)+Y6(J)
8160 NEXT J
8170 FOR I=2 TO N6
8180 B6(I)=0
8190 FOR J=1 TO N4
8200 B6(I)=B6(I)+Y6(J)*A8(J)^(I-
      1)
8210 NEXT J
8220 NEXT I
8230 N5=N6-1
8240 FOR K6=1 TO N5
8250 K5=K6+1
8260 L=K6
8270 FOR I=K5 TO N6
8280 IF ABS(A6(I,K6))-ABS(A6(L,K
      6))<=0 THEN GOTO 8300
8290 L=I
8300 REM
8310 NEXT I
8320 IF L-K6<=0 THEN GOTO 8410
8330 FOR J=K6 TO N6
8340 T6=A6(K6,J)
8350 A6(K6,J)=A6(L,J)
8360 A6(L,J)=T6
8370 NEXT J
8380 T6=B6(K6)
8390 B6(K6)=B6(L)
8400 B6(L)=T6
8410 REM
8420 FOR I=K5 TO N6
8430 F6=A6(I,K6)/A6(K6,K6)
8440 A6(I,K6)=0
8450 FOR J=K5 TO N6
8460 A6(I,J)=A6(I,J)-F6*A6(K6,J)

```



```
8470 NEXT J
8480 B6(I)=B6(I)-F6*B6(K6)
8490 NEXT I
8500 NEXT K6
8510 C9(N6)=B6(N6)/A6(N6,N6)
8520 I=N5
8530 I3=I+1
8540 S6=0
8550 FOR J=I3 TO N6
8560 S6=S6+A6(I,J)*C9(J)
8570 NEXT J
8580 C9(I)=(B6(I)-S6)/A6(I,I)
8590 I=I-1
8600 IF I>0 THEN GOTO 8530
8610 !
8620 !
8630 IF X5>1 THEN GOTO 8700
8640 PRINT "
8650 FOR I=1 TO N6
8660 PRINT USING 8670 ; I,C9(I)
8670 IMAGE 00,5X,MOD0Z.00000
8680 NEXT I @ PRINT "
8690 PRINT "=====
      ====="
8700 RETURN
```

SECTION-NUMBER= 5
 SWIRL - ANGLE = 45
 *****

Turbulent viscosities μ_{rx}/μ_l &
 $\mu_{r\theta}/\mu_l$

I μ_{rx}/μ_l ***** $\mu_{r\theta}/\mu_l$ *****

1	0.0000	0.0000
2	157.1427	154.8450
3	226.7730	222.7770
4	290.7090	284.7150
5	391.6080	390.6090
6	462.5370	448.5510
7	534.4650	508.4910
8	514.4850	460.5390
9	397.6820	334.6650
10	299.7000	239.7600
11	919.9800	715.9840
12	264.7350	239.7600
13	269.7300	259.7400
14	284.7150	274.7250
15	279.7200	269.7300
16	230.7610	235.7640
17	193.8060	195.8040
18	136.8630	138.8610
19	0.0000	0.0000

*****

Static pressure distribution

I	Pst cm H2O
1	0.3270
2	0.3205
3	0.2616
4	0.1962
5	0.1308
6	-0.1243
7	-0.3237
8	-0.7048
9	-1.3080
10	-1.7004
11	-2.2090
12	-2.8776
13	-3.4662
14	-4.1202
15	-4.4472
16	-4.8396
17	-5.0150
18	-5.2120
19	-5.4202

*****

*****

MIXING LENGTHS $L_{rx}/(R_o - R_i)$ AND L
 $r_o/(R_o - R_i)$

1	0.000	0.000
2	0.017	0.017
3	0.024	0.024
4	0.032	0.032
5	0.045	0.044
6	0.057	0.055
7	0.074	0.072
8	0.082	0.078
9	0.084	0.074
10	0.079	0.076
11	0.135	0.113
12	0.059	0.052
13	0.056	0.053
14	0.048	0.046
15	0.041	0.039
16	0.030	0.029
17	0.023	0.023
18	0.015	0.015
19	0.000	0.000

*****

APPENDIX H

DETERMINATION OF THE FRICTIONAL VELOCITIES BY CLAUSER CHART

The Clauser-chart method was adopted in the present study to determine the resultant frictional velocity Q^* and its components U^* and W^* as described in section 5.6. The steps of the method are briefly described below for the data of U and W near the outer and inner walls of the last axial station with swirl number of 0.87.

- 1 - By assuming different values for C_U^* and C_W^* and using equations (5.34a,b) and (5.35a,b) families of curves can be constructed as shown in Figs. H.1 and H.2.
- 2 - Plot the data of U and W near the outer and inner wall on Figs. H.1 and H.2 respectively.
- 3 - From Fig. H.1 read $C_U^* = 1.45$ for the outer wall and $C_U^* = 1.35$ for the inner wall.
- 4 - From Fig. H.2 read $C_W^* = 1.4$ for the outer wall and $C_W^* = 1.23$ for the inner wall.
- 5 - Use equation (5.31) to calculate Q^* and equations (5.32a,b) to calculate U^* and W^* .
- 6 - The obtained values for Q^* , U^* and W^* are 2.01, 1.71, 1.68 (m/sec) for the outer wall and 1.83, 1.57, 1.5 (m/sec) for the inner wall, respectively.
- 7 - Repeat the above steps for all axial stations and for the tested swirl numbers.

Fig. N.1 Clauser chart for determining the parameter C_{u^*} at the outer and inner walls of the last axial station for $SH = 0.87$

Fig. H.2 Clauser chart for determining the parameter C_w^* at the outer and inner walls of the left axial station for $SN = 0.87$

APPENDIX I

ERROR ANALYSIS

The calibration of hot-wire probes and the experiments were conducted in the same laboratory using isothermal air as a working fluid. Therefore, the influence of fluid temperature variations on hot-wire readings was neglected in the equation of its data reduction. Different approaches can be found in the literature for correcting hot-wire readings [109, 110] when the fluid temperature varies especially if the measured air velocity is below 10 m/sec.

However, the effects of the velocity gradients, neglect of turbulence terms in equation (F.6) and setting of the probe orientation angle ψ on the accuracy of the determined values for U and W in addition to other general sources of error are given below. The effects of these factors on the accuracy of Reynolds stresses and the static and total pressure distributions are also discussed. The analysis is focused on determining the highest value of the possible error in each individual case.

Influence of Velocity Gradients on the Response of Slanted Hot-wire Probe

Gessen and Moller [48] have investigated this problem and found that the heat-loss of a hot-wire in a velocity gradient of a certain flow depends on the magnitude of the gradient and the length/diameter ratio of the probe sensor. A shear parameter s was defined as

$$s = (\Delta U/U_c) (d/l) \quad (I.1)$$

The symbols in the above relationship are defined in Fig. 1.1. The correction curves for the readings of tungsten wire operating with constant temperature anemometer in a two-dimensional flow are shown in Fig. 1.2, the symbols being defined as follows:

- $\bar{E}_{s=0}$ Anemometer output voltage when the hot-wire sensor is exposed to a flow of uniform velocity.
- $\bar{E}_{s \neq 0}$ Anemometer output voltage when the hot-wire sensor is influenced by velocity gradients (e.g. measurements in boundary layers).
- l/d Length to diameter ratio of the hot-wire sensor.
- n Overheat ratio (i.e. ratio of sensor resistance at a selected operating temperature to its resistance at ambient temperature). The value of n was 1.8 in the present experiments.
- Re_∞ Reynolds number determined at the free-stream conditions, $U_\infty d / \nu$. The velocity U_∞ is taken to be the maximum velocity of U_m in this analysis and ν is the kinematic viscosity of air at 20° C.

Using the correction curves of Fig. 1.2, the possible error in the measured velocity U due to the influence of $\partial U / \partial r$ on hot-wire readings for the case of non-swirling flow is given below. The expected errors for swirling flow cases will be also discussed.

The highest velocity gradient, $\partial U / \partial r$, in the case of non-swirling flow was found to occur at $r^* = 0.96$ ($y = 3.15$ mm from the inner wall) of the first axial station. Therefore the highest possible error would be at that point. The gradient $\partial U / \partial r$ can be estimated from equation (3.34) i.e.

$$\frac{dU}{dr} = \frac{C_u^*}{K y} = \frac{1.45}{0.4 \times 0.00315} = 1050.8 \text{ (1/sec)}$$

or

$$\Delta U / \Delta r = 1050.8 \quad 1/\text{sec}$$

From Fig. 1.1, the value of Δr can be given by

$$\begin{aligned} \Delta r &= \text{sensor length} \times \sin(\text{probe angle } \alpha) \\ &= 1.25 \times 10^{-3} \times 1/\sqrt{2} \\ &= 0.884 \times 10^{-3} \quad \text{m} \end{aligned}$$

Using the above information and equation (1.1), the above shear parameter s can be given by

$$s = \frac{\Delta U}{\Delta r} \cdot \Delta r \cdot \frac{1}{U} \cdot \frac{d}{l}$$

$$\begin{aligned} s &= 1050.8 \times 0.884 \times 10^{-3} \cdot \frac{1}{28.4} \cdot \frac{5 \times 10^{-6}}{1.25 \times 10^{-3}} \\ &= 0.13 \times 10^{-3} \end{aligned}$$

$$\begin{aligned} Re_{\infty} &= \frac{U_{\max}}{\nu} \cdot d = \frac{33.1 \times 5 \times 10^{-6}}{0.016 \times 10^{-3}} \\ &= 10.3 \end{aligned}$$

From Fig. 1.2 for $s = 0.13 \times 10^3$, $Re_{\infty} = 10.3$ and $l/d = 250$ read

$$\left(\frac{E_{s=0}}{E_{s \neq 0}} \right)^2 = 1.002$$

$$\frac{\bar{E}_{s=0}}{\bar{E}_{s \neq 0}} = 1.001 \quad (1.2)$$

Recall equation (5.8) which relates the voltage $\bar{E}$ and the velocity U for the case of non-swirling flow

$$\frac{\bar{E}(\psi, \alpha)}{s} = \sqrt{b} U \quad (s \neq 0) \quad (1.3)$$

where

$$b = \sin^2 \alpha \cos^2 \psi + k^2 \cos^2 \alpha \cos^2 \psi + h^2 \sin^2 \psi$$

Differentiate equation (1.3) to get

$$\frac{\Delta \bar{E}(\psi, \alpha)}{s} = \sqrt{b} \Delta U \quad (s \neq 0) \quad (1.4)$$

The output voltage $\bar{E}_{s=0}$ and $\bar{E}_{s \neq 0}$ can be related by

$$\bar{E}_{s=0} = \bar{E}_{s \neq 0} + \Delta \bar{E}$$

Using equations (1.3) and (1.4) to substitute for the right hand side of the above equation; rearrangement and dividing by equation (1.3) gives

$$\frac{\bar{E}_{s=0}}{\bar{E}_{s \neq 0}} - 1 = \frac{\Delta U}{U} \quad (1.5)$$

From the above equation and equation (1.2), the percentage of error in the velocity U would be

$$\frac{\Delta U}{U} \times 100 = 0.1 \% \quad (1.6)$$

Equation (1.6) indicates that the influence of $\partial U / \partial r$ on the hot-wire readings is negligible. This can be assigned to two factors: first, the ratio l/d of the used hot-wire sensor was relatively small, and secondly, the measured shear parameter, s , was also small because no measurements were taken in the regions very close to inner or outer walls.

Fig. 1.2 shows that the voltage ratio depends mainly on l/d and the parameter s ; the strongest effect is due to l/d . Gessner and Moller [48] found that for a sensor of 5 μm diameter with $l/d = 400$, in a velocity gradient with a shear parameter $s = 2 \times 10^{-3}$, operating at an overheat ratio 1.8, the indicated velocity U at the centre of the wire using the $s = 0$ calibration curve is at most 4% too low, for $3.3 < Re_w < 33$. If l/d is 200 the error is about 1.3 %.

The gradient of the resultant velocity $\partial Q / \partial r$ may be used to estimate the maximum possible error in slanted hot-wire readings in the case of swirling flows. This gradient can be approximately given by:

$$\frac{\partial Q}{\partial r} = \left\{ \left(\frac{\partial U}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{W}{r} \right) \right]^2 \right\}^{1/2} \quad (1.7)$$

where the gradients of U and W were calculated from equations (3.34) and (3.35). The highest value of $\partial Q / \partial r$ was found with the data of the highest swirl number ($SN = 0.87$) at $r^* = 0.96$ ($y = 3.15$ mm from the inner wall) of the first axial station; its calculated value is

$$\frac{\partial Q}{\partial r} = 1683.5 \quad 1/\text{sec}$$

Following the procedure applied for non-swirling flow, the shear parameter, and Re_∞ would be

$$s = 0.14 \times 10^{-3}$$

$$Re_\infty = 16.8$$

Using the above information and Fig. 1.2, the voltage ratio is given by

$$\left(\frac{\bar{E}_{s=0}}{\bar{E}_{s \neq 0}} \right)^2 = 1.004$$

$$\frac{\bar{E}_{s=0}}{\bar{E}_{s \neq 0}} = 1.002$$

If the hot-wire probe is aligned in the resultant flow direction then equation (1.5) can be used to estimate the percentage of error in velocity Q

$$\frac{\Delta Q}{Q} \times 100 = 0.2 \% \quad (1.8)$$

Since the correction curves of Fig. 1.2 were produced from measurements in two-dimensional flow, exact estimates for the errors U and W in the case of swirling flows would not be possible. However, if the percentage of error in any of the velocities U , W and Q is assumed to have the same order of magnitude then

$$\frac{\Delta U}{U} \times 100 = \frac{\Delta W}{W} \times 100 = \frac{\Delta Q}{Q} \times 100 = 0.2 \% \quad (1.9)$$

In the following sections a fixed value of 0.2% is used to represent the

possible error either U or W at any point in the flow field for all tested swirl numbers.

Although no information is available on which to base estimates of error in the Reynolds stresses due to radial gradients, the small estimated errors for mean velocities gives confidence that errors in the Reynolds stresses will also be acceptably small.

Error Due to Neglecting Turbulence Terms in Equation (F.6)

The highest measured values for Reynolds stresses were found at $r^* = 0.96$ of the last axial station* for swirl number of 0.87.

Therefore, the highest possible error in the velocities U and W due to neglecting the turbulence terms in equation (F.6) would be expected to occur in the data of this specific point. To simplify the error analysis at this point, the following approximations are made:

- a - the radial velocity component is negligible
- b - the flow angle is 45° i.e. $U \approx W$
- c - the errors in U and W are equal; i.e. $\Delta U \approx \Delta W$

Under these conditions, equation (F.6) can be simplified to:

$$\frac{\bar{E}(\psi, 45)}{s} = \sqrt{b + c + g} \quad U \quad (I.10)$$

Differentiation of equation (I.10) and dividing by $\bar{E}(\psi, 45)$ gives the following relationship

$$\frac{\Delta \bar{E}(\psi, 45)}{\bar{E}(\psi, 45)} = \frac{\Delta U}{U} \quad (I.11)$$

Comparing equations (F.5) and (F.6), the value of $\Delta \bar{E}(\psi, 45)$ can be

estimated from the neglected turbulence terms as follows

$$\frac{\Delta E(\psi, 45)}{S} = \frac{1}{2\lambda} (a \overline{v^2} + b \overline{u^2} + c \overline{w^2} + d \overline{uv} + f \overline{vw} + g \overline{uw}) \quad (I.12)$$

Substitution for the probe angle $\alpha = 45^\circ$ (U , V and W were determined from the measurements taken by the slanted probe, see section 5.1) into equations (F.3c,d) simplifies the coefficients a , b , c , d , f , g , and to

$$\left. \begin{aligned} a &= \frac{1}{2}(1 + k^2) \\ b &= \frac{1}{2}(1 + k^2) \cos^2 \psi + h^2 \sin^2 \psi \\ c &= \frac{1}{2}(1 + k^2) \sin^2 \psi + h^2 \cos^2 \psi \\ d &= (1 - k^2) \cos \psi \\ f &= (1 - k^2) \sin \psi \\ g &= \frac{1}{2}(1 + k^2 - 2h^2) \sin 2\psi \\ \lambda &= \sqrt{b + c + g} \quad U \end{aligned} \right\} \quad (I.13)$$

From equations (I.10-12) the value of $\Delta U/U$ or $\Delta W/W$ can be estimated from Reynolds stresses as follows

$$\frac{\Delta W}{W} = \frac{\Delta U}{U} = \frac{1}{2(b + c + g) U^2} (a \overline{v^2} + b \overline{u^2} + c \overline{w^2} + d \overline{uv} + f \overline{vw} + g \overline{uw}) \quad (I.14)$$

From Figs. 6.57, 6.58, 6.62 and 6.98 for $SN = 0.87$ read the following values for Reynolds stresses at $r^* = 0.96$

$$\frac{\tau_{rx}}{\frac{1}{2} \rho U^2} = -5.3 \times 10^{-3}, \quad \frac{\tau_{r\theta}}{\frac{1}{2} \rho U^2} = -5.1 \times 10^{-3}, \quad \frac{\tau_{x\theta}}{\frac{1}{2} \rho U^2} = 0.21 \times 10^{-3}$$

$$\sqrt{u^2}/U = 0.105$$

$$\sqrt{w^2}/U = 0.085$$

$$\sqrt{v^2}/U = 0.45$$

where, $U = 28.4$, $\bar{U} = 32.9$, $\bar{Q} = 43.1$ (m/sec)

$k = 0.28$ and $h = 1.7$ for slanted probe.

When the above values were substituted in equation (I.14) for the three values of ψ (i.e. 0, 45 and 90, table 4.4), the highest value of $\Delta U/U$ or $\Delta W/W$ was found to correspond to $\psi = 45^\circ$; this value is given by

$$\frac{\Delta U}{U} \times 100 = \frac{\Delta W}{W} \times 100 = 1.7 \% \quad (I.15)$$

Error Due to Setting the Angle ψ

This type of error has a direct effect on the coefficients b , c and g of equation (I.10). For a certain measured voltage $E(\psi, 45)$ if these coefficients were calculated for angle $(\psi + \Delta\psi)$ instead of the correct value ψ then, the error in U can be estimated from equation (I.10) and given by

$$\frac{\Delta U}{U} = 1 - \frac{(b + c + g)^{\frac{1}{2}}_{\psi}}{(b + c + g)^{\frac{1}{2}}_{\psi + \Delta\psi}} \quad (I.16)$$

Since the angle ψ was set within an accuracy of 0.1 deg. then, $\Delta\psi$ is given by

$$\Delta\psi = 0.1 \text{ deg} \quad (I.17)$$

The maximum error was found when $\psi = 45^\circ$. Using equation (I.13) to calculate b , c and g for $\psi = 45^\circ$ and 44.9° and substituting the results into equation (I.16), the error $\Delta U/U$ is given by

$$\frac{\Delta U}{U} \times 100 = 0.523 \times 10^{-3} \% \quad (I.18)$$

This shows that the error in calculating U due to the inaccuracy in setting the angle ψ is negligible.

General Sources of Error

The method used in this work for determining the time-mean velocity components and Reynolds stresses (see Sec. 5.2) by means of hot-wire probes requires the performance of many experimental procedures. Accuracy of final results might be influenced, in general, by the following factors:

- Inaccuracy in the linearized calibration curve of hot-wire probes due to its deviation from the exact linear relationship given by equation (5.3) (about ± 0.04 to $\pm 3\%$ in the measured velocity range).
- The velocity components and Reynolds stresses at each radial location were reduced from six measurements for $\bar{E}_{br}$ and $\sqrt{e^2_{br}}$ which were not taken simultaneously but in a sequential order. The latter requires that velocity fluctuations be independent of time; this was approximated by averaging 100 instantaneous readings with a sampling rate of 6 readings/sec for each of $\bar{E}_{br}$ and $\sqrt{e^2_{br}}$.
- Inaccuracy in readjusting the operating conditions of the test section.

An overall value for the possible error due to the aforementioned factors was estimated by recalibrating the hot-wire probes and repeating the earlier measurements under the same nominal operating conditions at four radial locations ($r^* = 0.92, 0.75, 0.25$ and 0.08) of the last axial station with the highest swirl number ($SN = 0.87$). The obtained results are expressed in terms of the following parameter

$$\Delta \epsilon_i \% = \frac{F_{i,1} - F_{i,2}}{\frac{1}{2}(F_{i,1} + F_{i,2})} \times 100 \quad (I.19)$$

where i refers to the components U , V , W , $\overline{UV}$, $\overline{VW}$, $\overline{UW}$, $\sqrt{U^2}$, $\sqrt{V^2}$ or $\sqrt{W^2}$ and subscripts 1 and 2 refer to the two values determined for each of the components at the point under consideration. The highest and lowest obtained values for $\Delta \epsilon_i$ are given below

$$\begin{aligned} \Delta \epsilon_U \% &= 0.2 \text{ to } 3.5 \% \\ \Delta \epsilon_W \% &= 0.3 \text{ to } 3.6 \% \\ \Delta \epsilon_V \% &= 21 \text{ to } 118 \% \\ \Delta \epsilon_{\overline{UV}} \% &= 1.6 \text{ to } 10.1 \% \\ \Delta \epsilon_{\overline{VW}} \% &= 1.5 \text{ to } 13.6 \% \\ \Delta \epsilon_{\overline{UW}} \% &= 1.4 \text{ to } 16.4 \% \\ \Delta \epsilon_{\sqrt{U^2}} \% &= 0.8 \text{ to } 11.6 \% \\ \Delta \epsilon_{\sqrt{V^2}} \% &= 2.3 \text{ to } 19.1 \% \\ \Delta \epsilon_{\sqrt{W^2}} \% &= 0.9 \text{ to } 12.3 \% \end{aligned} \quad (I.20)$$

The above relations show that the error in V due to calibration errors, imperfect averaging for $\bar{E}$ and $\sqrt{v^2}$, and other factors affecting repeatability is extremely high, while the least error can be expected for U and W . This is consistent with the poor results of V and the low scatter in the profiles of U and W presented in section 6.1. The higher percentages of errors in Reynolds stresses were found to occur in values determined at $r^* = 0.75$ and 0.25 .

Total Error in the Velocities U and W

The highest value of the total error in U or W may be estimated

from

$$\frac{\Delta W}{W} = \frac{\Delta U}{U} = \left\{ \left(\frac{\Delta U}{U} \right)_1^2 + \left(\frac{\Delta U}{U} \right)_2^2 + \left(\frac{\Delta U}{U} \right)_3^2 + (\Delta \epsilon_U)^2 \right\}^{1/2} \quad (I.21)$$

The values $(\Delta U/U)_1$, $(\Delta U/U)_2$, $(\Delta U/U)_3$ and $(\Delta U/U)_4$ are given by equations (I.9), (I.15) and (I.18) and (I.20) respectively; substitution of these values into equation (I.21) gives the upper limit for

$$\frac{\Delta W}{W} = \frac{\Delta U}{U} = \frac{1}{100} \{ (0.2)^2 + (1.7)^2 + (0.523)^2 \times 10^{-6} + (3.5)^2 \}^{1/2}$$

$$\frac{\Delta W}{W} \times 100 = \frac{\Delta U}{U} \times 100 = 3.89 \% \quad (I.22)$$

i.e. the highest possible error in calculating U or W from the slanted hot-wire readings is 3.9 percent and can be mainly assigned to neglecting the turbulence terms in equation (F.6), calibration errors and imperfect averaging for $\bar{E}$ and $\sqrt{e^2}$. This exemplifies the importance of restricting the use of equation (F.6) for flows with turbulence level less than 15% and minimizing the errors in performing the experimental procedures.

Errors in Calculating Reynolds Stresses

Excluding the error due to the application of the binomial expansion in the derivation of equation (F.10) and the errors given by equation (I.20) the other major source of error would be due to the errors in calculating the coefficients a_1 , b_1 , c_1 which are functions in U and W as shown in equation (F.11). If the velocities U and W are rewritten in terms of Q and ϕ , i.e.

$$U = Q \cos \phi \quad \text{and} \quad W = Q \sin \phi$$

and substituted in equation (F.10) then, the latter becomes independent of Q and can be generally rewritten in terms of ϕ and Reynolds stresses as follows:

$$\overline{e^2} = f(\phi, \overline{v^2}, \overline{u^2}, \overline{w^2}, \overline{uv}, \overline{vw}, \overline{uw}) \quad (\text{I.23})$$

where,

$$\phi = \tan^{-1} \left(\frac{W}{U} \right) \quad (\text{I.24})$$

Moreover, under the assumption of $\Delta U = \Delta W$, equation (I.22) becomes independent of U and W and this in its turn makes equation (I.23) also independent of any errors included in the values of U and W . Exact determination for the errors in Reynolds stresses if $\Delta U \neq \Delta W$ is very complicated and the author hopes that it would not increase the upper limit of the error values given by equation (I.20) for the Reynolds stresses at any point in the flow.

Error in Static Pressure Distribution

The error in computing the distribution of the static pressure for each axial station is accumulated at the last point under consideration. Therefore, the highest error would be included in the value of the inner wall static pressure computed from equation (5.23). After neglecting the turbulence terms in this equation, it is simplified to

$$p_{w0} - p_{wi} = \rho \int_{r_i}^{r_0} \frac{W^2}{r} dr \quad (\text{I.25})$$

If an error ΔW is included in the velocity W then the above equation can be written as follows:

$$(P_{w0} - P_{w1})' = \rho \int_{r_1}^{r_0} \frac{(W + \Delta W)^2}{r} dr \quad (I.26)$$

Subtract equation (I.25) from (I.26) and divide the result by equation (I.25) to obtain

$$\frac{\Delta(P_{w0} - P_{w1})}{(P_{w0} - P_{w1})} = \frac{I'}{I} - 1 \quad (I.27)$$

where,

$$\Delta(P_{w0} - P_{w1}) = (P_{w0} - P_{w1})' - (P_{w0} - P_{w1})$$

$$I = \int_{r_1}^{r_0} \frac{W^2}{r} dr \quad ; \quad I' = \int_{r_1}^{r_0} \frac{(W + \Delta W)^2}{r} dr$$

$$\frac{\Delta W}{W} \% = 3.9 \% \text{ or } \frac{\Delta W}{W} \% = 0.3 \%$$

Application of equation (I.27) for the profile of the velocity W with, $SN = 0.87$ at the last axial station gives values of the maximum and minimum possible errors in calculating the static pressure; these values are given by

$$\begin{aligned} \frac{\Delta(P_{w0} - P_{w1})}{(P_{w0} - P_{w1})} \times 100 &= 7.95 \% \left(\frac{\Delta W}{W} = 3.9 \% \right) \\ &= 0.7 \% \left(\frac{\Delta W}{W} = 0.3 \% \right) \end{aligned} \quad (I.28)$$

The highest error in the computed static pressure distribution at any

axial station increases from zero at the outer wall to a maximum value equal to/or below 8 percent at the inner wall.

Error in Calculating the Total Pressure by Equation (4.6)

Differentiation of equation (4.6) and division by P_t gives the following relation:

$$\frac{\Delta P_t}{P_t} = \frac{\Delta P}{P_t} + \frac{\rho}{2P_t} [2(\Delta U U + \Delta W W) + \Delta(\overline{u^2} + \overline{v^2} + \overline{w^2})] \quad (1.29)$$

Highest errors in computed static pressure values would be in the data of the points near the inner wall at any axial station. To estimate

$\Delta P_t / P_t$, read the following data for the point with $r^* = 0.04$ at the last axial station for $SN = 0.87$ from Figs. 6.35, 6.39 and 6.98 .

$$\frac{P_t}{\frac{1}{2} \rho U^2} = 0.37 \quad ; \quad \frac{P}{\frac{1}{2} \rho U^2} = -0.8 \quad ; \quad \frac{\sqrt{u^2}}{\overline{Q}} = .083$$

$$\frac{\sqrt{v^2}}{\overline{Q}} = .035 \quad ; \quad \frac{\sqrt{w^2}}{\overline{Q}} = 0.72 \quad ; \quad \overline{U} = 32.9$$

$$U = 26.01 \quad ; \quad W = 24.2 \quad ; \quad \overline{Q} = 43.1$$

$$\frac{\Delta P}{P} = 0.7 \text{ to } 8\% \quad (m/sec)$$

Using equation (1.20) for error percentages of $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$ and substituting the above data into equation (1.29) give the value of

$\Delta P_t / P_t$, i.e.

$$\frac{\Delta P_t}{P_t} \times 100 = 0.9 \text{ to } 14.3 \% \quad (1.30)$$

Summary of Error Analysis

The estimated values for the possible experimental error ranges in the data for velocity components, Reynolds stresses and static and total pressures are summarized below.

parameter	possible error in percent	
U	0.2 to 3.9	} (1.31)
W	0.3 to 3.9	
V	21 to 118	
$\overline{uv}$	1.6 to 10.1	
$\overline{vw}$	1.5 to 13.6	
$\overline{uw}$	1.4 to 16.4	
$\sqrt{u^2}$	0.8 to 11.6	
$\sqrt{v^2}$	2.3 to 19.1	
$\sqrt{w^2}$	0.9 to 12.3	
P_{static}	0.7 to 8	
P_{total}	0.9 to 14.3	

Fig. 1.1 Influence of velocity gradient on slanted hot-wire probe

Fig. 1.2 Response correction curves for tungsten wires (after Gessner and Moller, Ref. [48])

END

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FIN