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degree of Master of Engineering

Thesis Supervisor

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ABSTRACT

Pressure die casting is a complex process whose performance is governed by a number of design variables. For optimum results a proper die design is essential. The development of a design procedure that is based on an accurate thermal analysis of the die is therefore of significant importance.

A full transient analysis of the heat flow in the die would obviously be the most accurate analytical technique.

In the present work a method of heat flow analysis is presented. The success of the method depends on finding a good estimate of the steady state temperature in the die and using this as the initial condition in the transient analysis.

The initial estimate of steady state condition is obtained by finding the total heat exchanged between the casting and a die block during one cycle of the casting process. The total heat exchange is divided by the cycle time to find a rate of heat flux through the cavity surface of the die. The flux is used as an input for a steady state finite element analysis of the die to obtain an initial state from which to start the transient analysis.

A 2D finite element transient thermal analysis procedure and related computer programs are developed for the analysis of a die consisting of two die blocks. A number of case studies are carried out to verify the validity of the analysis process.

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List of Symbols

A	Area of cross section of pipe(m^2)
c	Specific heat of the material $\left(\frac{J}{kgC}\right)$
c_c	Specific heat of the casting $\left(\frac{J}{kgC}\right)$
c_f	Specific heat of water (J/kgC)
f_cⁱ	Heat flux into the die No. <i>i</i>
f_{ct}	Flux from the casting (J/m^2)
f_oⁱ	Nodal flux of the casting element ($J/m.s$)
G	Conductance Matrix for the conducting body
Gⁱ	Conductance Matrix for the <i>ith</i> pressure die
h	Convection coefficient for exchange of heat from the pipe wall to the coolant $\left(\frac{J}{m^2sC}\right)$
h_c	Coating conductance on the cavity surface $\left(\frac{J}{m^2sC}\right)$
h_p	Coating conductance on the parting surface $\left(\frac{J}{m^2sC}\right)$
H	Capacitance Matrix for the conducting body
Hⁱ	Capacitance Matrix for the <i>ith</i> die
H_cⁱ	Matrix of size N^i by N^i with non zero elements corresponding only to the nodes common to the casting
H_cⁱ	Matrix of size N^i by p formed from those columns of H_cⁱ that correspond to the nodes common with the casting
H_oⁱ	Capacitance Matrix for the non zero elements associated with nodes on the outer boundary (ambient air)

H_p^{11}	N^1 by N^1 matrix with non zero elements corresponding to die 1 nodes on the parting surface
H_p^{12}	N^1 by N^2 matrix with non zero elements corresponding to common nodes of die 1 and die 2
k	Thermal conductivity of the material $\left(\frac{J}{msC} \right)$
k_f	Thermal conductivity of coolant fluid (J/msC)
K	Conductance matrix for the cooling line
L	Latent heat per unit mass $\left(\frac{J}{kgC} \right)$
L_e	Length of each pipe element (m)
M	Capacitance matrix for the cooling line
p	Length of the perimeter of pipe(m)
Q	Strength of heat source (J/m^3s)
$\bar{Q}$	Total heat flux across the pipe wall (J/m^3s)
R	Capacitance Matrix for the casting
S	Conductance Matrix for the casting
t	Thickness of the Casting (mm)
T	Temperature at the die wall of the cooling line ($^{\circ}C$)
T_f	Fluid temperature ($^{\circ}C$)
T_{f0}	Fluid temperature at the entry of the pipe ($^{\circ}C$)
T_{liq}	Liquidus temperature of the Casting ($^{\circ}C$)
T_{sol}	Solidus temperature of the Casting ($^{\circ}C$)
T_{∞}	Ambient temperature ($^{\circ}C$)
T_{fe}	Vector of fluid temperatures at the nodes of the element ($^{\circ}C$)

T_e	Vector of the temperatures at element nodes of the die($^{\circ}C$)
T_n	Vector of temperatures ($^{\circ}C$) at time $t = n\Delta t$
v	Velocity of flow, that is, the volumetric flow rate divided by the pipe cross- sectional area(m/s)
ρ	Mass density of the conducting material($\frac{kg}{m^3}$)
ρ_c	Mass density per unit area of the casting($\frac{kg}{m}$)
ρ_f	Mass density of water (kg/m^3)
ρ_s	Mass density (kg/m^3)
τ	Process Cycle time (s)
Δt	Time step used in the integration (s)
Δt_1	Time step between injection and liquidus (s)
Δt_2	Time step between liquidus upto ejection and during the open phase of the process cycle (s)

CHAPTER 1

INTRODUCTION

1.1 The Pressure die casting Process

The die casting process is used for the repeated manufacturing of extremely intricate machine components. In this process the molten metal from which the component is to be made is injected at extremely high pressures into a cavity in the die. The thermal energy from the molten metal is transferred to the cooler die; as a result the melt solidifies and takes the shape of the cavity. After solidification has taken place, the die is opened and the casting is ejected. Following ejection of the casting, the die is left open for a short period of time. The cycle is repeated beginning with the injection of fresh molten metal. The time for which the casting remains within the die is called the residence time. The duration for which the die remains open is termed as the open phase.

The feature that distinguishes die casting from other foundry processes is that in the former the same mould is used repeatedly. The casting materials are usually zinc, magnesium and aluminum based alloys. The die is made of hardened steel. The die cavity is created after the die steel has been hardened. A spark erosion process followed by machining is used to ensure that a high degree of accuracy is obtained. The die is usually comprised of two or four blocks which slide open to eject the component.

To keep the temperature in the die block within operating limits, cooling lines are incorporated in the die. A cooling fluid, usually water, is passed through the lines to carry away heat from the die.

1.2 Design Requirements for a Pressure Casting Die

To be successful, the die used in the process must meet certain important design requirements.

(a) The casting metal should cool rapidly so that solidification is obtained as soon as possible and the cycle time is kept at a minimum. However, the cooling should not be so rapid that solidification would take place during the cavity fill.

(b) The die temperatures should be fairly uniform. Large variations in the die temperatures may cause distortion of the die components and the casting cavity. Distortion of the die may ultimately result in flash, wall thickening and cracking.

(c) The casting temperatures should be relatively uniform at ejection to avoid distortion of the casting.

(d) The casting temperature should be well below the solidus at the point of ejection. If the casting is too hot at ejection, the casting may be unable to resist stresses occurring during the ejection process and may gall as the dies move away.

(e) The casting temperature should not be too low at the point of ejection, as the casting may stick to the dies. If a part of the casting gets too cold before ejection, shrinkage problems may cause distortion of the part.

Since a pressure casting die is very expensive, a proper design is necessary. Previous methods of die design were essentially empirical, based on trial and error. This could be very expensive if a trial configuration did not work. Evidently, good thermal analysis procedures that will help in design are desirable. They can be used to validate a trial design at a fraction of the cost involved in producing a prototype. In fact, any number of trial and error analytical studies can be carried out at a modest cost, in order to arrive at an optimum design.

1.3 Analysis Software for Die Casting

While other technologies in metalworking have benefited economically from computerization, die casting remains an exception. Computer aided machining and manufacturing has proven to be useful in the design of tooling but the casting process is not easy to analyze, owing to the complexities of mathematics and physics of the problem.

Development of accurate models for metal flow and solidification is an important task and a difficult one too. The difficulty results in inadequate design giving rise to problems of quality such as surface defects, porosity and out of tolerance parts. The availability of a good mathematical model that would be used in a computer analysis would provide numerous benefits in quality and productivity.

In the past the prediction of the results of die casting process was made experimentally. Experiments would certainly capture the behavioral aspects of the process but cannot help in understanding the physics involved or the mathematical aspects related to the problem. Ideally, experiments and computer analysis would complement each other to solve and understand the problem. A mathematical model for software analysis should be able to simulate the real life process with sufficient accuracy so as to provide useful results.

To summarize, good analysis software aided by experiments should be capable of addressing the various issues related to casting process such as :

(a) Material Science

Be able to test the alloys, mould steels and cooling material and model their rheological and thermal behavior as well as structural and mechanical performance.

(b) Process Technology

Be able to formulate a mathematical model to describe the material behavior during extrusion or injection.

(c) Product Engineering

Be able to carry out solid modeling and perform structural analysis and product design for manufacturing.

(d) System Integration

Be able to develop the product and process in parallel or simultaneously.

An integrated software system would have, in addition, interactive modeling capabilities and menu driven strategies.

1.4 Literature Survey

Various studies of the pressure die casting process have been helpful in identifying the different parameters that govern the performance of the casting process. Studies by Benett (1966), Nelson (1968), Thukkaram (1970) and Ku et. al (1985) have shown that the die temperature variation is one of the most important parameters affecting production rate and casting defects. The die surface temperature during filling of the cavity should be high enough to ensure that the cavity is completely filled with molten metal when the metal flow stops. The temperature should then be lowered to freeze the casting and cool it to the ejection temperature rapidly so as to achieve an acceptable production rate. The studies mentioned above have highlighted the great need for practical information which would enable the die caster to achieve a proper and efficient cooling of the die.

Caswell and Lorentzon (1968) and Dent (1972) found that the waterline arrangement affected not only the quality of casting but also the production rate.

Nelson (1968), Kaiser (1972) and Groenveld et al (1975) found that a larger coolant flow rate and a lower inlet coolant temperature gave better coolant heat removal and a better overall performance.

Prasad (1967) found that the die itself played as important a part as location and size of the waterline in determining the performance of die cooling system. In the process of solidification of the casting, the die acts like a heat exchanger.

Studies of the die casting process have revealed the complexity of the process as well as the operating and design parameters which govern its performance. This makes the practical design of dies very difficult. Trial and error methods while being expensive do not always converge to a successful design. In addition they often do not provide any guidance on how to rectify a design to solve an identified problem. These factors have provided the motivation for analytical studies of the heat flow in pressure die.

For the thermal analysis of sand casting, techniques exist that make it unnecessary to model the heat flow in the sand. Boundary conditions based on known analytical solutions are used to replace the mould. These techniques, although useful for sand casting problem, have limited application in the modeling of the pressure die casting process because heat flow analysis in the die is of primary importance.

Early analytical studies of the pressure die casting process (Benett, 1966; Thukkaram, 1970) were limited in their usefulness because of the restrictive assumptions dictated by limits on computer memory and speed. More recent analytical studies of the die casting process have tackled the problem in two different ways. In one approach the problem is treated as that of transient thermal flow in which the analysis is started with the die at room temperature and continued until the temperatures reach periodic steady state. A finite difference or finite element method of analysis is used and the discretized equations are solved by numerical time integration. A number of researchers have used this foregoing approach (Myers, 1978; Myers, 1980; Kearns, 1986). Myers (1978) determined the long time cyclic

steady state in the die by integrating the heat diffusion equations directly using finite difference method. The analysis required the use of very small time step to avoid numerical oscillation. This made the computations very expensive. To overcome the problem, Myers (1980) used a modal coordinate transformation in which the spatially discretized equations were transformed using eigenvectors of the system. The transformed and reduced equations were then numerically integrated. An obvious drawback of the procedure is that it cannot handle nonlinear boundary conditions. Studies by both Grant (1981) and Kearns (1986) found that the necessity of using a small time scale made transient analysis a very costly procedure.

The cycle time in a pressure die casting process is much smaller compared to the time scale of the heat conduction process in the die. As a result, the transient temperature distribution extends only a small depth from the cavity surface to the interior of the die. In this transient layer, the temperatures vary quite rapidly and over a large range during one cycle of casting. On the other hand, in the interior of the die the temperatures vary quite slowly, and when cyclic steady state is reached they become essentially independent of time. Hogge and Gerrekens (1983) have noted that the rapid temperature variations and the steep gradients within the transient layer not only make the analysis very time consuming, they also introduce numerical problems in the solution process. To overcome some of the numerical problems, Gear (1971) used a specialized numerical integration technique with two different time scales for the diffusion problem with a high temperature gradient near the boundary.

In another approach, several researchers have carried out a transient analysis of the heat flow in the pressure die over one cycle starting from an assumed steady state as the initial condition (Riegger, 1981; Ohtsuka, 1982; Granchi et al, 1983). The drawback of this approach is that the results are dependent on the starting condition which is usually determined quite arbitrarily.

More recently Caulk and his co-researchers have developed a new method for the thermal analysis of die casting problem which obviates the various difficulties outlined in the earlier paragraphs and is stated to lead to reasonably accurate results. The work has been reported in several publications (Caulk, 1990; Barone and Caulk, 1993; Caulk and Siefker, 1993; Barone, Caulk and Siefker, 1993).

Caulk's method of analysis is based on the assumption that the transient temperature distribution penetrates only a small depth from the cavity surface into the interior of the die; and that the temperature is essentially independent of time in the rest of the die.

The fact that the transient temperature distribution penetrates only a small depth has been recognized by other researchers. Barton (1964) showed that analytical solution for heat flow in a die can be considered as consisting of two separate parts, although not independent of each other. One part can be defined by Laplace's equation giving the average temperature distribution and the rates of heat transfer. The second component satisfies the usual transient heat equation and can be considered as a perturbation about the steady state temperature. The perturbation was shown to decay rapidly as one moved away from the cavity surface. Goodman (1964) used the transient penetration concept to develop a method of analysis for heat flow in a body. Goodman's method was, however, not effective in handling a periodic boundary condition of the type encountered in the cyclic die casting process.

Caulk and his coworkers have developed an analytic method based on constant penetration depth. This method has been implemented in a workstation based computer program for 3D boundary element analysis of die casting; dieCAS. The program has been applied at General Motors to die casting process and much experience has been gained on this application over the past four years. Since Caulk's

method has close relevance to the present work, the theoretical background for the method as well as the assumptions made are described briefly below.

Caulk's method is based on the following important assumptions (Barone and Caulk, 1993): (a) the nominal thickness of the casting is small compared with its overall size; (b) the thermal conductivity of the casting alloy is much greater than that of the die; (c) the casting cycle is short compared with the start-up transient in the die.

The phase change in the casting material, during which the casting releases its latent heat, makes the surface boundary conditions nonlinear. In order to simplify the problem, Barone and Caulk adjust the initial temperature of the casting to convert the latent heat into an equivalent amount of sensible heat. It is stated that this assumption does not significantly affect the time averaged die temperatures.

The assumptions made by Caulk imply that the transient layer must penetrate only a very short depth d and that the temperatures in the rest of the die must be independent of time. In the solution a fixed value is used for the depth d .

$$d = 1.5(\alpha t_c)^{1/2} \quad (1.1)$$

where α is the thermal diffusivity and t_c is the cycle time. The die cavity is considered to have negligible thickness compared to the die dimension; the casting is therefore represented as a surface with variable mass per unit area,

The transient layer of depth d in which the temperatures are periodic is called surface layer. It is assumed that both in the casting and in the surface layer the flow of heat is normal to the surface.

The temperature in the surface layer, θ , which is a function of space coordinates and time, is expressed as a polynomial of the form

$$\theta(x, z, t) = \theta_0(x) + \sum_{m=1}^N \theta_m(x, z, t) (1 - z/d)^m \quad (1.2)$$

where z is the depth measured normal to the cavity surface and θ_0 is the constant temperature at the interface of the surface layer and the die interior. The time dependent coefficients θ_m are determined by satisfying the heat conduction equation in the surface layer. This equation can be expressed as

$$\rho c \dot{\theta} + \partial q / \partial z = 0 \quad (1.3)$$

where $q = -k \partial \theta / \partial z$ is the normal flux. Equation 1.3 is satisfied in an average sense by using the standard Galerkin procedure giving

$$\int_0^d \left(\rho c \frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} \right) \left(1 - \frac{z}{d} \right)^n dz = 0 \quad n = 0, 1, \dots, N \quad (1.4)$$

Integration of Eq. 1.4 leads to the following system of equations in θ , the vector of coefficients θ_m .

$$\mathbf{G}^\beta \dot{\theta}^\beta + \mathbf{K}^\beta \theta^\beta = q_s^\beta \mathbf{1} \quad \beta = 1, 2 \quad (1.5)$$

where superscript β refers to the die number, $\mathbf{G}^\beta$ is an N by N matrix whose (n, m) entry is

$$\frac{\rho^\beta c^\beta d^\beta}{m + n + 1} \quad (1.6)$$

and $\mathbf{K}^\beta$ is an N by N matrix with its (n, m) entry given by

$$\frac{k^\beta}{d^\beta} \left[\frac{mn}{(m + n - 1)} \right] \quad (1.7)$$

and q_s^A is the surface flux.

The $2N$ equations given by Eq. 1.5 together with the boundary condition on the cavity surface are sufficient to provide q_s^A and the coefficients θ_m^A . The solution converges rapidly with increasing order of the polynomial used in Eq. 1.4. In their analysis Caulk et al. find $N = 4$ to be adequate.

The boundary condition at the surface in contact with the casting is obtained from the energy balance equation for the casting

$$\mu(x) c_c(\theta_c) \frac{\partial \theta_c(x,t)}{\partial t} = -(q_s^1(x,t) + q_s^2(x,t)) \quad (1.8)$$

where μ is the mass per unit area of the casting and c_c its specific heat. The casting temperature satisfies the initial condition

$$\theta_c(x,0) = \theta_c^* \quad (1.9)$$

In the solution θ_c^* is adjusted to account for the latent heat of casting. This adjusted value θ_c^* is given by

$$\theta_c^* = \theta_c^o + L/c_o \quad (1.10)$$

where L is the latent heat and c_o is the sensible heat capacity.

The relation given by Eq. 1.8 holds for the duration of residence time. On the parting surface, the following condition applies during the residence time,

$$q_s^1 = -q_s^2 = h_p(\theta_s^1 - \theta_s^2) \quad (1.11)$$

where h_p is the contact conductance across the parting surface.

In the open phase, the conditions on both the cavity surface and the contact surface are

$$q_s^\beta = h_o(\theta_a - \theta_s^\beta) \quad (1.12)$$

where θ_a is the ambient air temperature and h_o is the heat transfer coefficient on the outer surface.

In addition, the temperature θ must be continuous at $t = t_c$ and satisfy the periodicity condition

$$\theta^\beta(x, 0) = \theta^\beta(x, t_c) \quad (1.13)$$

The steady state temperatures in the interior of each die must satisfy Laplace's equation of heat flow along with the boundary condition on the outer surfaces and the cooling pipes. In addition, temperatures and the time averaged flux in the interior and the surface layer should satisfy the continuity condition at the interface.

The foregoing equations, boundary conditions and initial condition will lead to a unique solution for the problem. For details of the solution process reference may be made to Barone and Caulk (1993) and Caulk (1990).

1.5 Scope of Work

The present work represents a step in the development of analytical capabilities for solving the heat flow problem in the die casting process. In this work, analytical methods are developed for a full two dimensional transient analysis of a pressure die consisting of two blocks. The die blocks may have any specified geometry and cooling line locations. The casting, which is considered to be thin in comparison to the dimensions of the die, may have non-uniform thickness.

Detailed formulations are developed for the equations governing heat flow in the cooling lines and the die, taking into account the appropriate boundary conditions on the cavity surface, the parting surfaces and the outer surfaces. The assumptions made in the analysis are clearly outlined.

The analysis begins with an estimate for the temperature distribution within the die. A steady state solution in which the casting is represented by a source of uniform heat flux is used to obtain the initial conditions for the transient analysis. The transient analysis is continued until cyclic steady state has been achieved.

Computer algorithms are developed to carry out the analyses. The output from the computer program provides the entire history of temperature distributions in the die and the casting. The results from the program can be further processed to obtain isotherms as well as other graphical representations using available utility programs.

1.6 Layout of Thesis

In Chapter 1 the die casting process is introduced and the need for developing software for the die casting process is discussed. Also presented is a literature survey covering the related areas of application, parametric studies to understand the die casting variables, and the various methods of analysis developed.

Chapter 2 deals with the formulation of governing equations for heat flow in cooling lines. The chapter also presents the heat flow equation for the die casting process as well as the formulation for finite element thermal analysis of die casting.

Chapter 3 deals with the numerical techniques used in the solution of finite element equations developed in Chapter 2 and presents justification for some of the assumptions made in the development of these equations. Also presented are the results of several case studies carried out by using the formulation and the numerical techniques described. The results are displayed through plots of cavity surface temperatures and isotherms at various stages of a steady state cycle.

Chapter 4 presents a summary of the work done and the conclusions drawn from it. Recommendations for future work are also given.

CHAPTER 2

FORMULATION OF THE HEAT FLOW EQUATIONS

2.1 Introduction

The transient heat transfer problems in pressure die casting is illustrated in Fig 2.1 which shows two die sections enclosing a casting cavity.

Fig. 2.1 Die Casting Model

A series of cooling lines run along the length of each die block. The cooling water or oil running through these lines carries heat away from the die block.

At the beginning of a casting cycle, molten metal is injected into the cavity. The injection takes place over a period of 10 to 500 ms. Because this time is very small in comparison to the cycle time, injection is assumed to be instantaneous.

Heat conduction takes place in the casting and also in the die. Heat is transferred from the casting metal to the die across the cavity surface. The amount of

heat exchanged depends upon the difference between the casting temperature and the die temperature, and the thermal conductance through the lubricant applied on the cavity surface. Also, heat transfer takes place across the parting surfaces and is determined by the temperature difference between the two dies.

The heat transfer is complicated by the fact that the casting metal changes phase from a liquid to solid state, releasing latent heat of solidification in the process.

The die remains closed as the casting solidifies and gains sufficient strength to retain its shape. Soon afterwards the die opens and the casting is ejected. The open die blocks cool further as heat is carried away by the cooling lines and is lost by convection to air at the outer boundaries. A new casting cycle begins with closing of the dies and injection of fresh molten casting metal. During the closed phase, heat conduction in the die blocks is governed by the following boundary conditions

- (1) convective heat loss on the outer boundaries, usually to the ambient air.
- (2) heat loss to the cooling fluid (usually water) flowing in the cooling lines.
- (3) heat exchange between the die blocks across the parting surfaces in closed phase.
- (4) heat transfer from the casting to the dies.

In the open phase, the cavity surface as well as the parting surfaces are exposed to the air and convective heat loss to air takes place through the external surfaces. The heat loss also continues through the cooling lines.

A 2-dimensional finite element method is used for the thermal analysis of the casting process. The theoretical background for the process is presented in the following sections. Sections 2.2 through 2.5 present a one dimensional advection

diffusion analysis for cooling lines. The remaining sections in the chapter present 2D transient heat transfer analysis for the die.

2.2 Conduction Convection Problem in a Cooling Line

Figure 2.2b shows a small section of a cooling line of circular section carrying a coolant. Water is the coolant medium in the present study.

The water temperature is denoted by T_f and the temperature of the die wall by T . The conductivity of water is k_f , the area of cross section of pipe is A , p is the length of the perimeter of pipe, ρ_f is the mass density of water, c_f is the specific heat of water, v is the velocity of flow, that is, the volumetric flow rate divided by the pipe cross-sectional area, h is the convection coefficient for exchange of heat from the pipe wall to the coolant, L_c is the length of each pipe element.

Fig. 2.2 Control Volume of Cooling Line

The heat conducted into the section at the left cross-section is $-k_f A \frac{\partial T_f}{\partial z}$. The heat conducted out through the right cross-section is

$$-k_f \frac{\partial T_f}{\partial z} A - \frac{\partial}{\partial z} \left(A k_f \frac{\partial T_f}{\partial z} \right) dz \quad (2.1)$$

The net heat conducted into the pipe is

$$A k_f \frac{\partial^2 T_f}{\partial z^2} dz \quad (2.2)$$

The energy entering the pipe through the flowing water is given by $A \rho_f v c_f T_f$. The heat leaving the water flowing out at the right hand side is given by

$$A \rho_f v c_f T_f + \frac{\partial}{\partial z} (A \rho_f v c_f T_f) dz \quad (2.3)$$

The net energy entering the pipe section due to flow of water is given by

$$-A \rho_f v c_f \frac{\partial T_f}{\partial z} dz \quad (2.4)$$

The energy entering the pipe through convection around its circumference is given by

$$h p (T - T_f) dz \quad (2.5)$$

The energy content of the water in the pipe increases through a rise of temperature; the increase is given by

$$A \rho_f v c_f \frac{\partial T_f}{\partial t} dz \quad (2.6)$$

An energy balance equation is now written using equations 2.2, 2.4, 2.5 and 2.6

$$-k_f A \frac{\partial^2 T_f}{\partial z^2} + v A \rho_f c_f \frac{\partial T_f}{\partial z} - h p (T - T_f) + \rho_f c_f A \frac{\partial T_f}{\partial t} = 0 \quad (2.7)$$

For the heat flow in a pipe considered here, the conduction term $-k_f A \frac{\partial^2 T_f}{\partial z^2}$ is small in comparison to other terms and is neglected. The transient heat flow equation thus becomes

$$v A \rho_f c_f \frac{\partial T_f}{\partial z} - h p (T - T_f) + \rho_f c_f A \frac{\partial T_f}{\partial t} = 0 \quad (2.8)$$

2.3 Formulating FEM equations for coolant temperature

The governing equation of heat flow in a cooling line, Eq. 2.8 can be solved by finite element analysis. In such an analysis temperature T_f in an element is approximated by

$$T_{fe} = N_e T_{fe} \quad (2.9)$$

where T_{fe} is the vector of fluid temperatures at the nodes of the element and

N_e is the vector of linear shape function given as

$$N_e = [N_1 \quad N_2] \quad (2.10a)$$

and

$$\begin{aligned} N_1 &= 1 - \frac{z}{L}, \\ N_2 &= \frac{z}{L} \end{aligned} \quad 0 \leq z \leq L \quad (2.10b)$$

Representing $B_i = \frac{\partial N_i}{\partial z}$ and following a standard Galerkin procedure, the finite element equation can be written as

$$\begin{aligned} & \frac{1}{v} \int N_i^T N_i \dot{T}_e dz + \int N_i^T B_i T_e dz \\ & - \frac{hpL_e}{Av\rho_f c_f} \int N_i^T N_i T_e dz + \frac{hpL_e}{Av\rho_f c_f} \int N_i^T N_i T_e dz = 0 \end{aligned} \quad (2.11)$$

An overdot represents differentiation with respect to time and the linear shape functions N_1, N_2 are used to interpolate the die temperature along its length of the pipe

On integration over an element,

$$\begin{aligned} & \frac{L_e}{v} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} \dot{T}_{f1} \\ \dot{T}_{f2} \end{bmatrix} + \begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} T_{f1} \\ T_{f2} \end{bmatrix} \\ & + \frac{hpL_e}{Av\rho_f c_f} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} T_{f1} \\ T_{f2} \end{bmatrix} - \frac{hpL_e}{Av\rho_f c_f} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} T_{f1} \\ T_{f2} \end{bmatrix} = 0 \end{aligned} \quad (2.12)$$

A 2D thermal analysis for the die is based on the assumption that there is no heat flow in a direction perpendicular to the cross-section of the die. In other words, die temperature does not change in the direction of the length of die that is in the direction parallel to the pipe. Equation 2.12 therefore becomes

$$\begin{aligned} & \frac{L_e}{v} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} \dot{T}_{f1} \\ \dot{T}_{f2} \end{bmatrix} + \begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} T_{f1} \\ T_{f2} \end{bmatrix} + \\ & \bar{\beta} \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} T_{f1} \\ T_{f2} \end{bmatrix} = \bar{\beta} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} [T] \end{aligned} \quad (2.13)$$

where $\bar{\beta} = \frac{hpL_e}{Av\rho_f c_f}$

The heat equation for the pipe can be solved after assembly of Eqs. for the pipe elements and applying the boundary condition

$$T_f = T_{f0} \text{ at } z = 0, t \geq 0 \quad (2.14a)$$

and initial condition

$$T_f = T_{f0} \text{ at } 0 \leq z \leq L, t = 0 \quad (2.14b)$$

The assembled equation can be written as

$$\mathbf{M}T_f + \mathbf{K}T_f = \mathbf{P} \quad (2.15)$$

Equation 2.15 can be solved by an implicit integration technique, such as backward Euler difference. The method is unconditionally stable but a reasonably small value of time step must be used to ensure adequate accuracy. Thus if T_f^n represents the water temperature at time $n\Delta t$, Eq. 2.15 can be written as

$$\frac{1}{\Delta t}(\mathbf{M}T_f^{n+1} - \mathbf{M}T_f^n) + \mathbf{K}T_f^{n+1} = \mathbf{P}^{n+1} \quad (2.16)$$

or as

$$\left(\frac{1}{\Delta t}\mathbf{M} + \mathbf{K}\right)T_f^{n+1} = \mathbf{P}^{n+1} + \frac{1}{\Delta t}\mathbf{M}T_f^n \quad (2.17)$$

2.4 Upwind finite element scheme

As stated in the preceding section, the conduction term in the heat equation for a pipe is generally small and can be neglected. This assumption is valid whenever the Peclet number is greater than 100. When the conduction term is considered, difficulties arise in the numerical solution. These difficulties are related to the oscillations in the solution, whenever the mesh size exceeds a certain critical value.

In a finite difference solution of the convective-diffusion problem, the difficulties mentioned above have been overcome by using a central difference approximation for the conduction term and a backward difference (upwind) approximation for the convective term (Piva and Dicarolo, 1975). For the central difference expression to be stable the mesh size h should be such that the Peclet number γ satisfies the following relationship

$$\gamma = \frac{vh}{k} \leq 2 \quad (2.18)$$

In a finite element method that uses the standard Galerkin formulation, the differentials are approximated by expressions that are similar to those of central differences. Evidently, this is the cause of numerical oscillations in the solution. To overcome the observed difficulties, Zienkiewicz et al (1975) and Heinrich et al (1977) have suggested the use of weighting functions of non-symmetrical form different from the standard Galerkin functions. The selected functions which lead to upwinding are successful in avoiding the oscillation, although at the cost of some loss in the accuracy.

For the sake of completeness, the derivation of an upwind finite element scheme for the transient heat flow in a pipe involving both conduction and convection

is presented here. The derivation uses the weighting functions suggested by Heinrich et al (1977).

Using the weighted residual method, the finite element equation corresponding to Eq. 2.7 can be written as

$$\begin{aligned}
 & k_f A \int \left[\frac{\partial w}{\partial z} \right]^T B dz T_f + A \rho_f v c_f \int w^T B dz T_f \\
 & + A \rho_f v c_f \int w^T N dz \dot{T}_f + h_p \int w^T N dz T_f \\
 & = h_p \int w^T N dz T
 \end{aligned} \tag{2.19}$$

where w is row vector of weighting functions. In the standard Galerkin procedure

$w \equiv N$. In an upwind scheme

$$w = [w_1 \quad w_2]$$

$$w_1 = N_1 - \alpha F(z) \tag{2.20}$$

$$w_2 = N_2 + \alpha F(z)$$

where α is an appropriately selected parameter and function $F(z)$ is given by

$$F(z) = \frac{-3}{L_e^2} z(z - L_e) \tag{2.21}$$

Substitution of Equations 2.20 and Eq. 2.21 in Eq. 2.19 leads to following expression for an element.

$$\begin{aligned}
& \begin{bmatrix} -1/2 + \alpha/2 + \eta + \bar{\beta}/3 - \alpha\bar{\beta}/4 & 1/2 - \alpha/2 + \eta + \bar{\beta}/6 - \alpha\bar{\beta}/4 \\ -1/2 + \alpha/2 + \eta + \bar{\beta}/3 - \alpha\bar{\beta}/4 & 1/2 + \alpha/2 + \eta + \bar{\beta}/3 - \alpha\bar{\beta}/4 \end{bmatrix} \begin{bmatrix} T_{f1} \\ T_{f2} \end{bmatrix} \\
& + \frac{L_c}{v} \begin{bmatrix} 1/3 - \alpha/4 & 1/3 - \alpha/4 \\ 1/6 + \alpha/4 & 1/3 + \alpha/4 \end{bmatrix} \begin{bmatrix} T_{f1} \\ T_{f2} \end{bmatrix} = \bar{\beta} \begin{bmatrix} 1/2 - \alpha/2 \\ 1/2 + \alpha/2 \end{bmatrix} T
\end{aligned} \tag{2.22}$$

where

$$\eta = \frac{k_f}{c_f \rho_f v L_c} \tag{2.23}$$

When $\alpha = 0$, Eq. 2.22 reduces to a form equivalent to central difference, and when $\alpha = 1$, a stable 'upwind' (backward difference) scheme is obtained. For the one dimensional steady state problem, Christie et al(1976) have shown that the optimum value of α is given by

$$\alpha = \left(\coth \frac{\gamma}{2} \right) - \frac{\gamma}{2} \tag{2.24}$$

In the present work, it was found that the inclusion of the conduction term made little difference to the amount of heat carried away by the pipe. The conduction term was therefore ignored and a steady state convective solution was used as explained in the following section.

2.5 Steady state heat flow in a pipe

If sufficient time is allowed, the heat flow in the pipe represented by Eq. 2.8 will reach a steady state. The steady state condition can be evaluated by carrying out the numerical integration shown in Eq. 2.17 for a sufficient length of time. However, a closed form solution exists for the steady state and is much easier to use. The derivation of the transient finite element formulation follows standard procedure; however to the knowledge of the author, the closed form steady state expressions presented below have not been used in earlier studies.

The steady state heat flow equation is given by

$$\frac{\partial T_f}{\partial z} = \frac{hp}{vA\rho_f c_f} (T - T_f) \quad (2.25)$$

A solution of Eq. 2.25 with the boundary conditions $T_f = T_{f0}$ at $z = 0$ gives

$$T_f = T - (T - T_{f0}) e^{-\beta \frac{z}{L}} \quad (2.26)$$

where L is the length of the pipe, and $\beta = \frac{hpL}{Av\rho_f c_f}$

The heat flux from the pipe to the die can be represented as

$$d\bar{Q} = hp(T_f - T)dz \quad (2.27)$$

$$= hp(T_{f0} - T)e^{-\beta \frac{z}{L}} dz$$

The total heat flux along the entire length of the pipe is obtained from

$$\begin{aligned}\bar{Q} &= \int_0^L hp(T_{f_0} - T)e^{-\beta \frac{z}{L}} dz \\ &= \rho_f c_f A v (T_{f_0} - T)(1 - e^{-\beta})\end{aligned}\quad (2.28)$$

The average heat flux per unit length is given by

$$\frac{\rho_f c_f A v}{L} (T_{f_0} - T) (1 - e^{-\beta}) \quad (2.29)$$

Equation 2.29 is used in the 2-dimensional finite element thermal analysis of the die casting process presented in this work.

2.6 2D-Transient Heat Flow Analysis

The governing equations and the associated boundary conditions for a two dimensional transient heat flow analysis are presented in this section. The formulation is applicable to the thermal analysis of a die idealized by a 2D model. The equations must be solved by a numerical method of analysis. The finite element method provides an effective technique for such analysis. The FEM equations for the 2D transient heat flow are derived here.

Referring to Fig. 2.3 the governing equation of 2-D heat conduction is given by

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q = c\rho \frac{\partial T}{\partial t} \quad (2.30)$$

where k denotes thermal conductivity of the material $\left(\frac{J}{msK} \right)$, ρ is the mass density $\left(\frac{kg}{m^3} \right)$, c is the specific heat of the material $\left(\frac{J}{kgC} \right)$, and Q is strength of heat source (in Joules per cubic meter per second).

The boundary conditions associated with Eq. 2.30 are of the form

$$T = T_o \quad \text{on boundary } S_1 \quad (2.31a)$$

$$-k \frac{\partial T}{\partial n} = q_o \quad \text{on boundary } S_2 \quad (2.31b)$$

where n represents the outward normal to the boundary. An approximate solution will not satisfy the governing equation. Also while the solution may satisfy the essential boundary condition given by Eq. 2.31a, it will not, in general, satisfy the natural boundary condition given by Eq. 2.31b.

Fig. 2.3: Finite Element discretization of a 2-D domain

In a numerical method of analysis, such as the finite element method, we attempt to minimize the associated error by using a weighted residual technique. Denoting the weighting function by w , the weighted residual expression can be written as

$$\int_A w \left[k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q - c\rho \frac{\partial T}{\partial t} \right] dA + \int_{s_2} w \left(-k \frac{\partial T}{\partial n} - q_o \right) dS = 0 \quad (2.32)$$

Consider the integral

$$I = \int_A w \left[k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) dA \right] \quad (2.33)$$

Integrating by parts and using Green's theorem we have

$$I = - \int_A \left(k \frac{\partial w}{\partial x} \frac{\partial T}{\partial x} + k \frac{\partial w}{\partial y} \frac{\partial T}{\partial y} \right) dA + k \int_{s_2} w \frac{\partial T}{\partial n} dS \quad (2.34)$$

Substitution into Eq. 2.32 gives

$$\begin{aligned} & \int_A \left(k \frac{\partial w}{\partial x} \frac{\partial T}{\partial x} + k \frac{\partial w}{\partial y} \frac{\partial T}{\partial y} \right) dA + \int_A w c \rho \frac{\partial T}{\partial t} \\ &= \int_A w Q dA - \int_{s_2} w q_o dS \end{aligned} \quad (2.35)$$

If the domain is divided into finite elements as shown in Fig 2.4, Eq. 2.35 can be expressed as

$$\begin{aligned} & \sum_N \int_{A_e} \left(k \frac{\partial W}{\partial x} \frac{\partial T}{\partial x} + k \frac{\partial W}{\partial y} \frac{\partial T}{\partial y} \right) dA_e + \sum_N \int_{A_e} w c \rho \frac{\partial T}{\partial t} dA_e \\ &= \sum_N \int_{A_e} w Q dA_e - \sum_N \int_{s_{2e}} w q_0 dS \end{aligned} \quad (2.36)$$

where A_e is the domain of one element and summation extends to N elements in the region.

In the finite element procedure, we assume that within an element the temperature varies as

$$T = N_e T_e \quad (2.37)$$

where T_e is a vector of the temperatures at element nodes and N_e is a row vector of shape functions,

We have

$$\begin{aligned} \frac{\partial T}{\partial x} &= \left[\frac{\partial N_e}{\partial x} \right] T_e \\ &= B_1 T_e \end{aligned} \quad (2.38)$$

and

$$\begin{aligned} \frac{\partial T}{\partial y} &= \left[\frac{\partial N_e}{\partial y} \right] T_e \\ &= B_2 T_e \end{aligned} \quad (2.39)$$

$$\frac{\partial T}{\partial t} = N_e \dot{T}_e \quad (2.40)$$

In the Galerkin method the test functions are taken to be trial or shape functions.

Equation 2.36 thus reduces to

$$\begin{aligned} \sum_N \int_{A_e} \left(k \mathbf{B}_1^T \mathbf{B}_1 \dot{T}_e + k \mathbf{B}_2^T \mathbf{B}_2 \dot{T}_e \right) dA_e + \sum_N \int_{A_e} c \rho \mathbf{N}_e^T \mathbf{N} \dot{T}_e dA_e \\ = \int_{A_e} \mathbf{N}_e^T \mathbf{N} Q_e dA_e - \int_{S_e} \mathbf{N}_e^T \mathbf{N}_s q_o dS \end{aligned} \quad (2.41)$$

where $\mathbf{N} Q_e$ represents the spatial variation of heat source input as a function of its nodal values. In the special case when the heat source is located at a finite element node, say at i , the integral $\int_{A_e} \mathbf{N}^T \mathbf{N} Q_e dA_e$ reduces to a vector with a single element Q_i .

Equations 2.41 can be expressed as

$$\begin{aligned} \sum_N k \mathbf{G}_e \dot{T}_e + \sum_N c \rho \mathbf{H}_e \dot{T}_e \\ = \sum_N \mathbf{H}_e Q_e - \sum_N \mathbf{H}_{S_e} q_o \end{aligned} \quad (2.42)$$

where

$$\mathbf{G}_e = \int_{A_e} \mathbf{B}_1^T \mathbf{B}_1 dA + \int_{A_e} \mathbf{B}_2^T \mathbf{B}_2 dA \quad (2.43a)$$

$$\mathbf{H}_e = \int_{A_e} \mathbf{N}_e^T \mathbf{N}_e \, dA \quad (2.43b)$$

$$\mathbf{H}_{se} = \int_{A_e} \mathbf{N}_s^T \mathbf{N}_s \, dS \quad (2.43c)$$

Assembly of the equations gives

$$\mathbf{GT} + \mathbf{HT} = \mathbf{Q} - \mathbf{H}_s \mathbf{q}_s \quad (2.44)$$

where the summation sign indicates assembly

$$\sum_N k \mathbf{G}_e = \mathbf{G} \quad (2.45)$$

$$\sum_N c \rho \mathbf{H}_e = \mathbf{H}$$

$$\sum_N \mathbf{H}_{se} = \mathbf{H}_s$$

A complete derivation of the element matrices for three node triangular finite elements can be found in any finite element text book.

2.7 Analysis of Heat flow in a Die

This section presents a specialized form of the general FEM formulation described in the preceding section for application to the analysis of a die that has two blocks and a number of cooling lines. Appropriate boundary conditions for the analysis of the die casting process are presented. The assumptions made in the analysis are also outlined.

In the present work the problem of heat flow in a die as described in Section 2.1 is assumed to be two-dimensional in nature. This assumption implies that heat transfer takes place only in the plane of the cross-section shown in Fig. 2.1 and no heat flows along the length of the die. For this assumption to be valid, the casting should extend throughout the entire length of the die and the casting thickness should not vary along the length of the die.

Since heat flows in the cooling pipes along their length, the problem of heat flow in a die with cooling lines is strictly speaking three dimensional in nature. However, by averaging the heat flux across the pipe wall, as indicated by Eq. 2.29, the problem can be converted into a two-dimensional problem. The assumptions outlined above are justified in many practical situations. The following additional assumptions are made in the analysis procedure.

- 1) The thickness of the casting is very small in comparison to the die. This assumption implies that the cavity is taken to be of zero thickness and heat conduction within the casting is ignored. In other words, the transverse thermal gradient in the casting is ignored.
- 2) The injection of the molten casting into the die cavity is instantaneous.

3) The size of the cooling lines is small in comparison to the die size; as a result, the cooling lines can be represented by point sinks located at finite element nodes. The cooling fluid remains within the saturated region. It is also assumed that the fluid reaches steady state during each time increment used in the integration of heat flow equation.

4) No heat flow takes place between the dies during the open phase of the die casting cycle.

Figure 2.4 shows a finite element discretization of the die shown in Fig 2.1. For clarity the two die blocks and the casting have been shown separately. Die i has N^i nodes. The casting itself is divided into a number of elements. The nodes on the casting match the corresponding nodes in the two die halves. Similarly the die nodes on the parting surfaces match each other. The thickness of the casting may vary from element to element. Cooling pipes are located at finite element nodes.

The mesh near the cavity is more refined compared to the rest of the interior, in order to capture the sharp thermal gradients in the vicinity of the casting. In the analysis only one half of the dies are modeled considering symmetry about the central plane of the dies.

The finite element heat flow equation for the i^{th} die is

$$G^i T^i + H^i T^i = Q^i - H_j^i q_j^i \quad (2.46)$$

The cooling pipes can be viewed as heat sources with a negative heat input. A pipe located at node j gives

$$Q_j^i = \frac{\rho_{ff} c_{ff} A_j v_j}{L_j} (T_{f0j} - T_j^i) (1 - e^{-\beta_j}) \quad (2.47)$$

Fig . 2.4 Finite Element discretization of two die half blocks and a stepped casting
(symmetric half domain)

Thus

$$\mathbf{Q}' = -\overline{\mathbf{Q}}'\mathbf{T}' + \mathbf{p}' \quad (2.48)$$

where $\overline{\mathbf{Q}}'$ is a diagonal matrix whose j^{th} diagonal element is given by

$$\frac{\rho_{\beta} c_{\beta} A_j v_j}{L_j} (1 - e^{-\beta_j}) \quad (2.49)$$

and $\mathbf{p}'$ is a vector with the j^{th} element equal to

$$\frac{\rho_{\beta} c_{\beta} A_j v_j}{L_j} T_{f,j} (1 - e^{-\beta_j}) \quad (2.50)$$

The boundary condition on the outer surface of the die wall is

$$q_o = h(T - T_{\infty}) \quad (2.51)$$

The discretized version of this boundary condition is

$$\mathbf{H}_o \mathbf{q}_o = h_o \mathbf{H}'_o \mathbf{T}' - h_o \mathbf{H}'_o \mathbf{T}'_{\infty} \quad (2.52)$$

where the non-zero elements of $\mathbf{H}'_o$ are associated with the nodes on the outer surface of the die and $\mathbf{T}'_{\infty}$ is expanded to include all nodes of die i ; of course, only the elements corresponding to nodes on the outer boundary are relevant.

The boundary condition on the parting surface between the two dies is given by

$$q_0 = h_p(T' - T') \quad (2.53)$$

where h_p is the contact conductance for the parting surface. Referring to Die No. 1, the discretized version of the boundary condition becomes

$$\mathbf{H}_p \mathbf{q}_0 = h_c \mathbf{H}_p^{11} \mathbf{T}^1 - h_c \mathbf{H}_p^{12} \mathbf{T}^2 \quad (2.54)$$

where $\mathbf{H}_p^{11}$ is an N^1 by N^1 matrix with non zero elements corresponding to die 1 nodes on the parting surface and $\mathbf{H}_p^{12}$ is an N^1 by N^2 matrix with non zero elements corresponding to common nodes of die 1 and die 2.

The boundary condition along the cavity surface is given by

$$q_0 = h_c(T - T_c) \quad (2.55)$$

where h_c is the coating conductance on the cavity surface. The discretized version of this boundary condition is

$$\mathbf{H}_c \mathbf{q}_0 = h_c \mathbf{H}_c' \mathbf{T}' - h_c \bar{\mathbf{H}}_c' \mathbf{T}_c \quad (2.56)$$

When the number of nodes on the casting is p , $\mathbf{T}_c$ is a vector of size p . Matrix $\mathbf{H}_c'$ is of size N^i by N^i with non zero elements corresponding only to the nodes common to the casting and die No. i . $\bar{\mathbf{H}}_c'$ is of size N^i by p formed from those columns of $\mathbf{H}_c'$ that correspond to the nodes common with the casting.

On substituting Eqs. 2.48, 2.52, 2.54 and 2.56 in Eq. 2.46, the finite element equation for die No i becomes

$$\bar{G}^i T^i + H^i \dot{T}^i - h_p H_p^{ii} T^i - h_c \bar{H}_c^i T_c = p^i \quad (2.57)$$

where

$$\bar{G}^i = G^i + \bar{Q}^i + h_o H_o^i + h_p H_p^{ii} + h_c H_c^i \quad (2.58a)$$

$$\bar{p}^i = p^i + h_o H_o^i T_o \quad (2.58b)$$

In addition to the finite element heat flow equations for the two die blocks, an energy balance equation should also be satisfied for the casting. This equation is given by

$$c_c \rho_c \frac{\partial T_c}{\partial t} = h_c [T^1 - T_c] + h_c [T^2 - T_c] \quad (2.59)$$

where c_c is the specific heat of the casting and ρ_c is its mass density per unit area. For a casting of non-uniform thickness ρ_c will vary from element to element. Temperature T_c in an element on the casting is now represented by

$$T_c = N T_{ce} \quad (2.60)$$

where N is the shape function row vector and T_{ce} is vector of temperatures at the casting nodes.

Using Galerkin residual method as explained for heat flow in a two-dimensional region, and assembling the element equations, the discretized version of the casting energy balance equation is obtained as

$$\mathbf{S}\dot{\mathbf{T}}_c + \mathbf{R}\mathbf{T}_c - h_c(\overline{\mathbf{H}}^1)\mathbf{T}^1 - h_c(\overline{\mathbf{H}}^2)\mathbf{T}^2 = 0 \quad (2.61)$$

where $\mathbf{S}$ and $\mathbf{R}$ are p by p matrices obtained from

$$\mathbf{S} = \sum_N \mathbf{S}_e \quad (2.62)$$

$$\mathbf{R} = \sum_N \mathbf{R}_e$$

and

$$\mathbf{S}_e = c_c \rho_c \int_{L_e} \mathbf{N}^T \mathbf{N} dS \quad (2.63)$$

$$\mathbf{R}_e = 2h_c \int_{L_e} \mathbf{N}^T \mathbf{N} dS$$

The equations for the heat flow in the dies and the energy balance in the casting are coupled. The coupled equations can be expressed as

$$\begin{bmatrix} \mathbf{H}^1 & & \\ & \mathbf{S} & \\ & & \mathbf{H}^2 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{T}}_1 \\ \dot{\mathbf{T}}_c \\ \dot{\mathbf{T}}_2 \end{bmatrix} + \begin{bmatrix} \overline{\mathbf{G}}^1 & -h_c \mathbf{H}_c^1 & -h_p \mathbf{H}_p^{12} \\ -h_c (\mathbf{H}_c^1)^T & \mathbf{R} & -h_c (\mathbf{H}_c^2)^T \\ -h_p \mathbf{H}_p^{21} & -h_c \mathbf{H}_c^2 & \overline{\mathbf{G}}^2 \end{bmatrix} \begin{bmatrix} \mathbf{T}_1 \\ \mathbf{T}_c \\ \mathbf{T}_2 \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{P}}^1 \\ \mathbf{0} \\ \overline{\mathbf{P}}^2 \end{bmatrix} \quad (2.64a)$$

or

$$\mathbf{HT} + \mathbf{GT} = \mathbf{P} \quad (2.64b)$$

It should be noted $\mathbf{H}_p^{21} = (\mathbf{H}_p^{12})^T$, and that coefficient matrices in Eq. 2.58 are therefore symmetrical.

A simple model to simulate the modelling of the solidification of casting is considered. As the casting melt solidifies, it releases latent heat of solidification. This release begins at a temperature lower than the injection temperature known as liquidus temperature and terminates when the temperature drops to solidus temperature.

The release of latent heat can be accounted for by using an effective value for specific heat c_c expressed as a function of temperature. Thus if L denotes the latent heat per unit mass; T_{liq} denotes the liquidus temperature and T_{sol} the solidus temperature, the time dependent value of the specific heat is given by

$$\begin{aligned} c_c &= c_o \quad T_c > T_{liq} \\ &= c_o + L/(T_{liq} - T_{sol}) \quad T_{liq} > T_c > T_{sol} \\ &= c_o \quad T_{sol} > T_c \end{aligned} \quad (2.65)$$

The time dependence of c_c makes the coefficient matrices in Eq. 2.58 also time- dependent and the problem becomes non-linear.

Equations 2.64 are valid for the residence time, that is when the casting is in the cavity and the die is in the closed phase. With the opening of the dies and the ejection of the casting, the heat conduction equation for each die should be solved independently. These equations are given by

$$\left[\mathbf{G}^i + h\mathbf{H}_o^i + \bar{\mathbf{Q}}^i \right] \mathbf{T}^i + \mathbf{H}^i \mathbf{T}^i = \mathbf{p}^i + h\mathbf{H}_o^i \mathbf{T}_\infty^i \quad (2.66)$$

If during the open phase convective loss to outside air occurs on surfaces *P* and *C* as well, matrix $\mathbf{H}_o^i$ should be modified to include non-zero elements corresponding to the nodes on boundaries *P* and *C*.

2.8 Initial Conditions for the Transient Heat flow Analysis of the Die

The finite element equations governing heat flow in the pressure casting die, Eqs. 2.64 and 2.66, are solved by a numerical method of integration. The details of the method used will be presented in Chapter 3. The time integration must begin with an initial estimate of the die temperatures and continue until a cyclic steady state is reached.

When the objective is to evaluate the cyclic steady state, time marching need not begin at the room temperature. In fact, any initial guess that provides an estimate of the steady state may be used. A good estimate would lead to a substantial reduction in the computation time required to achieve convergence to the steady state.

As stated above, time marching should begin with initial values for the die temperatures. If the initial values provide a good estimate of the steady state condition, convergence is accelerated. In addition, the difficulties associated with steep temperature gradients near the casting are also overcome. As noted earlier, very large integration time required in a transient analysis and the oscillations induced by steep gradients are the two significant problems that make the transient analysis impractical or

prohibitively expensive. Starting the integration with a good estimate of the steady state temperatures overcomes these difficulties and makes transient analysis a practical tool.

Starting the time marching with an estimate of the steady state temperatures provides another significant advantage. In general, at the level of the cooling lines, the temperatures throughout the transient analysis remains close to the initial estimates. In fact, in the examples to be presented later, the temperature of the die wall around a cooling line varies only a little during a time step of numerical integration. This fully justifies the use of steady state expression Eq. 2.29 for the heat carried away by the pipe.

In order to obtain a good estimate for the steady state temperatures in the die, the casting is removed and replaced by a heat flux at the cavity surface. The total heat flowing from this heat source to the two die blocks, f_{ct} (in J/m^2), is obtained as

$$f_{ct} = \rho_c \{ (L + c_0(T_{inj} - T_{ejec})) \} \quad (2.67)$$

where ρ_c is the mass per unit area of casting, L is the latent heat of solidification, T_{inj} is the casting temperature at injection and T_{ejec} that at ejection, the point when the casting is removed from the die cavity.

If it is assumed that f_{ct} flows at a uniform rate during a cycle of the casting process, the heat flow rate is given by

$$\begin{aligned} f_c &= f_{ct} / \tau \\ &= \rho_c \{ (L + c_0(T_{inj} - T_{ejec})) \} / \tau \end{aligned} \quad (2.68)$$

where τ is the cycle time

In using Eq. 2.68 a value must be assigned to the ejection temperature T_{eject} . This temperature is, of course, not known at the beginning of the analysis and a reasonable assumption must therefore be made. In practice this is not a serious drawback because the objective is only to obtain a good estimate for the die temperatures at the start of the transient analysis. This estimate need not be precise, because irrespective of the accuracy of the initial estimate, the transient analysis will eventually converge to the same solution.

As a further simplification, the die blocks can be considered independent (no thermal interaction between the dies) in carrying out a steady state analysis. In order to do this, f_c must be apportioned between the two die blocks. Thus

$$f_c = f_c^1 + f_c^2 \quad (2.69)$$

where f_c^i represents the heat flux into the die No. i . Again, a reasonable assumption must be made in determining f_c^1 and f_c^2 . If the two die blocks are more or less similar in terms of geometry, cooling load and material properties, f_c may be divided equally between the two.

In order to use the flux f_c^i in the finite element heat flow equation for die No. i , it must first be converted into nodal values. Figure 2.5 shows two casting elements $j-1$ and j . Since the two elements may have different thicknesses, the heat flux across the two elements is also different.

Fig. 2.5 Casting Elements

The nodal value of the flux (in $J/m.s$) at node j is given by

$$f_{oj}^i = \frac{1}{2} f_{c,j-1}^i L_{j-1} + \frac{1}{2} f_{c,j}^i L_j \quad (2.70)$$

The vector of nodal flux values, $\mathbf{f}_o^i$ is obtained by assembling the elements f_{oj}^i .

The steady state heat flow equation for die No. i is now obtained from Eq. 2.66 by ignoring the time variation

$$\left[\mathbf{G}^i + h\mathbf{H}_o^i + \overline{\mathbf{Q}}^i \right] \mathbf{T}^i = \mathbf{p}^i + \mathbf{f}_o^i + h\mathbf{H}_o^i \mathbf{T}_\infty^i \quad (2.71)$$

Solution of Eq. 2.71 provides the initial value of the die temperatures for starting the transient analysis. Away from the casting, these estimates are expected to be close to the steady state values. In the vicinity of the casting, the temperatures of course vary during a cycle, from a high soon after injection to a low value at the end of the cycle. The temperature estimate for these locations obtained from Eq. 2.70 represent averages over a cycle.

2.9 Finite Element Modelling

In the present work, analysis is carried out of heat flow in a pressure die casting consisting of two blocks. The die region is discretized by triangular finite elements. A typical finite element mesh is shown in Fig. 2.6. The mesh is finer in the area near the cavity surface where the temperature gradients are steep. To minimize the bandwidth of the coefficient matrices, the numbering scheme shown in Fig. 2.6 is used. With this scheme and the ordering of equations as in Eq. 2.64, the bandwidth of the coefficient matrices is close to a minimum.

The cooling lines are modelled as concentrated heat sinks located at one or more finite element nodes in each of the die block. These sinks add terms to both the coefficient matrix and the load vector as given by Eqs. 2.49 and 2.50 respectively.

The convective loss to the air through the outer surface of the die is modelled by using one dimensional line elements. The temperature along the boundary with a standard convective boundary condition is given by

$$T = NT_e \quad (2.72)$$

where

$$T_e = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \quad (2.73)$$

$$N = [N_1 \quad N_2] \quad (2.74)$$

and $N_1 = 1 - S/L \quad (2.75)$

$$N_2 = S/L$$

Fig 2.6 Finite Element Mesh and Node Numbering Scheme
(symmetric half domain)

The ambient temperature T_∞ is assumed to vary in a similar fashion so that

$$T_\infty = NT_{\infty e}$$

The standard Galerkin formulation gives the element contribution to the right hand side vector as follows

$$\begin{aligned} &= - \int_0^{L_e} wh(T - T_\infty) \\ &= - \int_0^{L_e} hN^T(NT_e - NT_{\infty e})dS \\ &= - \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} T_{\infty 1} \\ T_{\infty 2} \end{bmatrix} \end{aligned} \quad (2.76)$$

The parting surfaces between the two dies are also modelled by one dimensional line elements. The detailed formulation is similar to that for convective boundary condition discussed above except that both the terms in the expression are unknowns and need to be determined as a part of the solution.

$$- \int_0^{L_e} h_p N^T(NT_e^1 - NT_e^2)dS \quad (2.77)$$

The contact surfaces between the casting and the dies are also represented by one dimensional line elements. Here again the linear shape function given by Eq. 2.75 are used to derive the related matrices.

In the open die phase the elements along the parting surfaces and along the surfaces that were in contact with the casting are replaced by standard convective line elements.

The casting is modelled by one dimensional line elements whose nodes coincide with the corresponding contact elements of the die. The casting thickness is assumed to be constant over one element but may vary from element to element. One of the data required for a casting element is the mass per unit area. This is obtained from the mass density of the casting and the thickness of the casting. Thus if ρ_s represents the mass density in kg/m^3 and t the thickness in mm, the mass density per unit area, ρ_c , is given by

$$\rho_c = \rho_s t / 1000 \quad (2.78)$$

CHAPTER 3

NUMERICAL SOLUTION OF THE HEAT FLOW EQUATION

3.1 Introduction

As described in Chapter 2, transient heat flow analysis of a pressure die reduces to the solution of Eq. 2.64 and Eq. 2.66. Obviously, a numerical method of integration must be used; but a number of difficulties present themselves in the solution.

In the vicinity of the casting the temperatures in the dies vary quite rapidly with time. This implies that a very short interval must be used as the time step in numerical integration. This is even more critical in the range of casting temperature between *injection* and *liquidus*. The heat flow in the portion of the die away from cavity surface is fairly slow; as a result a long time is required for the die to achieve a cyclic steady state. From a design point of view, the cyclic steady state is the most important, and the time integration must therefore be continued until such a state is achieved. The necessity of using a very small time step means that the time required to carry the computation to steady state is very large (Myers, 1978).

When the transient thermal analysis is started with the die at room temperature, very steep thermal gradients exist close to the casting. This introduces oscillations in the numerical solution, unless an extremely fine finite element mesh is used in the vicinity of the cavity surface. This can make the computation prohibitively

expensive (M. Hogge and P. Gerrekens, 1983). In fact the mesh refinement becomes unnecessary as time integration is continued, the temperatures become steady, and the temperature gradients are no longer very steep.

It should be noted that Eq. 2.64 is non-linear because of the time dependence of the effective casting specific heat c_c . The process of numeric computation used in the solution should take this into account.

The techniques used to overcome the difficulties outlined in this section as well as the numerical integration procedure employed are described in the subsequent sections.

3.2 Numerical Solution Procedure for the Incremental Equations

Equation 2.64 can be expressed in an incremental form as follows. Using a backward difference expression for $\dot{T}$, we have

$$\dot{T}_{n+1} = \frac{T_{n+1} - T_n}{\Delta t} \quad (3.1)$$

where T_n is the vector of temperatures at time $t = n\Delta t$ and Δt is the time step used in the integration. Substitution of Eq. 3.1 in Eq. 2.64b, specialized for $t = (n+1)\Delta t$, gives

$$GT_{n+1} + H \left[\frac{T_{n+1} - T_n}{\Delta t} \right] = P_{n+1} \quad (3.2)$$

where P denotes the load term in the finite element formulation.

We now write

$$\mathbf{T}_{n+1} = \mathbf{T}_n + \Delta \mathbf{T} , \quad (3.3)$$

Eq. 3.2 then gives

$$\left[\mathbf{G} + \frac{\mathbf{H}}{\Delta t} \right] \Delta \mathbf{T} = \mathbf{P}_{n+1} - \mathbf{G}\mathbf{T}_n \quad (3.4)$$

Eq. 3.4 represents a time marching scheme in which the solution provides $\Delta \mathbf{T}$ for a known value of $\mathbf{T}_n$. Having determined $\Delta \mathbf{T}$, $\mathbf{T}_{n+1}$ can be obtained. The marching should begin with a specified value of $\mathbf{T}_n$.

The numerical integration scheme expressed by Eq. 3.4 is known as implicit integration and is unconditionally stable. A small value of Δt must however be used to obtain reasonably accurate results.

When the objective is to evaluate the cyclic steady state, time marching need not begin at the room temperature. In fact, any initial guess that provides an estimate of the steady state may be used. A good estimate would lead to a substantial reduction in the computation time required to achieve convergence to the steady state. In the present work, the starting condition is derived by the method presented in Section 2.8.

3.3 Computer Algorithm

Two main computer programs were developed as a part of this work. The first is designed to provide a numerical solution of steady state equation 2.71 for each die block. The second program is designed to solve Eqs. 2.64 and 2.66 using the implicit numerical integration scheme given by Eq. 3.4.

Both programs require the development of the characteristic matrices and load

vectors for individual finite elements and the assembly of these matrices and vectors to obtain the global matrices and vectors. The steady state problem represented by Eq. 2.71 involves a one time solution of a set of simultaneous equations. In the transient analysis program such equations must be solved repeatedly.

The coefficient matrices in Eq. 2.71 and 3.4 are symmetric and very sparse. Also if the nodes are numbered appropriately, these matrices have a fairly small bandwidth. To conserve memory, the matrices are stored in upper triangular form as shown in Fig. 3.1.

		N								NBAND			
N	1	2						1	2	0	0		
	x	3	4	6	9			3	4	6	9		
		x	5	7	10			5	7	10	0		
		x	x	8	11	13		8	11	13	0		
		x	x	x	12	14	16	12	14	16	0		
				x	x	15	17	15	17	0	0		
					x	x	18	18	0	0	0		
		(a)								(b)			

Fig. 3.1: Upper triangular storage of a banded matrix; (a) full matrix (b) storage of the upper triangle

The simultaneous equations are solved using an active skyline solver routine that uses an LDL^T factorization of the coefficient matrix. In a transient analysis, if the coefficient matrix does not change over a time step, the factorization of the matrix is not repeated.

3.4 Time Step used in the Integration

The time step to be selected for integration should satisfy two criteria : (a) it should be small enough to minimize truncation errors in the numerical solution, and (b) it should be small enough so that the change of phase at liquidus and solidus is not missed. The latter condition must be satisfied because with change of phase, the effective specific heat must be modified according to Eq. 2.65. Thus as soon as a change is detected, the coefficient matrix must be modified and the LDL^T factorization repeated.

An optimum value of the time step can be selected by a process of trial and error in which repeated analysis is carried out over one cycle with different values of the time step. It was found that the need to detect the phase change governed the selection of time step. In fact, the temperature drop in the casting is very rapid between injection and liquidus and the time step required for timely detection of liquidus is an order of magnitude smaller than that required for timely detection of solidus. To maintain the efficiency of computation process, two different values of Δt were used in the integration process: Δt_1 between injection and liquidus and Δt_2 from liquidus upto ejection and during the open phase. The computer program was designed to accept a specified value of Δt_2 and an integer divisor M such that $\Delta t_1 = \Delta t_2 / M$.

The integration is continued until the solution converges to a cyclic steady state. Convergence is measured by comparing the temperatures at a selected node near the casting, at any pair of corresponding points of time in two successive cycles as illustrated in Fig. 3.2. When the difference between the two values is less than 0.05%, convergence is assumed to have been reached.

Fig. 3.2

3.5 Case Studies

Heat flow analyses were carried out for a number of pressure dies using the

formulation developed in this work and outlined in Chapter 2. The computer algorithms described in the earlier sections of the Chapter were used in the computation. The results obtained are presented in this section. In one case, a solution for the dies analysed is available in the literature. That solution is also presented for the purpose of comparison.

In all of the case studies presented here, the casting is assumed to be of aluminum alloy AIA380 (used for transmission casings) and the following data are used.

Properties of the Casting Material

Heat capacity, c_o	$= 1000 \frac{J}{kg \ K}$
Density, ρ_s	$= 2570 \frac{kg}{m^3}$
Conductivity, k_c	$= 108 \frac{J}{msK}$
Latent heat, L	$= 390,000 \frac{J}{kg \ K}$
Liquidus, T_{liq}	$= 593 \ ^\circ C$
Solidus, T_{sol}	$= 538 \ ^\circ C$

Properties of the Die Material

Heat Capacity, c	$= 584 \frac{J}{kg \ K}$
Density, ρ	$= 7760 \frac{kg}{m^3}$
Conductivity, k	$= 29 \frac{J}{msK}$

$$\text{Coating Conductance , } h_c = 10000 \frac{J}{m^2 sK}$$

The other parameters used in the solution are as listed below.

Process Parameters

$$\text{Injection temperature , } T_{inj} = 620 \text{ } ^\circ C$$

$$\text{Cycle time , } \tau = 80 \text{ sec}$$

$$\text{Residence time , } t_r = 30 \text{ sec}$$

Water temperature

$$\text{at inlet to cooling line, } T_o = 25 \text{ } ^\circ C$$

$$\text{Diameter of Cooling line, } d = 15 \text{ mm}$$

$$\text{Water heat transfer coefficient , } h = 10000 \frac{J}{m^2 sK}$$

Convective heat transfer

$$\text{coefficient to ambient air, } h_o = 10 \frac{J}{m^2 sK}$$

Heat transfer coefficient

$$\text{at parting surface, } h_p = 5000 \frac{J}{m^2 sK}$$

The die materials, heat transfer coefficients and water temperatures are the same in both the die halves. The specified values of the properties and other parameters are similar to those considered in earlier test cases reported in the literature, reflecting a typical die casting process for an aluminum transmission case.

3.5.1 Case Study No. 1

In this study the heat flow in a two die block for an Aluminum casting having a uniform thickness of 6 mm is considered. The geometry of the die and the locations

of cooling lines are shown in Fig. 3.3. The two die blocks are of identical geometry. Also the cooling lines are symmetrically located in the two die halves. There are two cooling lines in each die half, placed 10 cm. inside the die from the casting cavity and 25 cm. apart. The primary time step used in the integration of the heat flow equations was 1 second. This step used from injection to liquidus was 1/50th of the primary step. The convergence criterion specified in Section 3.4.2 was used with reference to the node shown in Fig. 3.5. The nodes on the dies and the casting are in contact .

Fig. 3.3 Cross - section of die showing casting and cooling lines

The initial value of die temperature at the level of cooling lines was 63.410°C . At the end of the transient analysis, when cyclic steady state was reached, die temperature at cooling line varied from a minimum of 63.381°C to a maximum of 63.528°C during one cycle. The variation from the initial to the cyclic steady state value is quite small, indicating that the temperatures at this depth were almost steady. During an integration time step, that is 1 sec, the maximum change in the die temperature at cooling lines was from 63.407°C to 63.382°C . Because the change is

Fig. 3.4 Temperature history at nodes 9, 13 & 87 during a steady state cycle (87 node of lower die, 13 node of upper die and 9 node of casting in contact are considered) - Symmetric Cooling load, 6 mm thick uniform AlA380 casting

Fig. 3.5 Temperature Distribution at ejection on the cavity surface. Distance is measured from point A ; casting solidification times. Symmetric cooling load and 6 mm thick uniform AIA380 casting.

Fig. 3.7 Steady state cycle Isotherms at Liquidus stage of casting, 6 mm uniform AlA380 casting, symmetric cooling load

Fig. 3.8 Steady state cycle Isotherms at Solidus stage of casting, 6 mm uniform AlA380 casting, symmetric cooling load

Fig. 3.9 Steady state cycle Isotherms at ejection, 6 mm uniform AlA380 casting and symmetric cooling load

Fig. 3.10 Temperature distribution at ejection during a steady state cycle, 6mm thick A1A380 uniform casting, symmetric cooling load

very small, it is justifiable to use the steady state Eq. 2.28 for the heat removed by the cooling fluid.

The variation of temperatures at finite element node Nos. 13 in the upper die, 87 in the lower die and 9 in the casting(see Fig. 3.3) during the last cycle of casting operation included in the analysis is shown in Fig. 3.4.

Figure 3.5 shows the distribution of temperature in the die along the cavity surface at the instant of ejection. The casting temperatures at that point in time are also shown. The dip in the curve corresponds to a point on the cavity surface closest to the location of cooling pipe. For the purpose of comparison, the results obtained by Caulk (1993) are also presented. The two results match quite closely. The plots of isotherms in the die region for various stages in the steady state cycle of the casting process are shown in Figs. 3.6 through 3.9. Figure 3.10 presents a coloured image of the temperature distribution in the dies at ejection.

3.5.2 Case Study No. 2

The die and casting for this case are the same as in Case No. 1, but the cooling lines in the two die halves are not located symmetrically. The upper die has two cooling lines located 10 cm. inside the die from the cavity surface and 25 cm. apart. The lower die has only one cooling line placed 10 cm. inside from the cavity surface along the center line of the die.

The distribution of the die temperature on the cavity surface at ejection as well as the casting temperatures at that instant are shown in Fig. 3.11. The isotherms for this case are shown in Figs. 3.12 through 3.15. Figure 3.16 presents a coloured image of the temperature distribution in the dies at ejection.

Fig. 3.11 Temperature distribution at ejection along the cavity surface from the center of die; casting solidification times. Asymmetric cooling load and 6mm thick AIA380 casting

Fig. 3.12 Steady state cycle Isotherms at start of cycle, 6 mm uniform AlA380 casting, asymmetric cooling load

Fig. 3.13 Steady state cycle Isotherms at Liquidus stage of casting, 6 mm uniform AlA380 casting, asymmetric cooling load

Fig. 3.14 Steady state cycle Isotherms at Solidus stage of casting, 6 mm uniform AlA380 casting, asymmetric cooling load

Fig. 3.15 Steady state cycle Isotherms at ejection, 6 mm uniform AlA380 casting, asymmetric cooling load

Fig. 3.16 Temperature distribution at ejection during a steady state cycle, 6mm thick uniform AlA380 casting, asymmetric cooling load

3.5.3 Case Study No. 3

This case is similar in die geometry and cooling line location to Case No. 1. However, a stepped casting varying in thickness from 4 mm to 6 mm is used in this case. The distribution of die temperatures along the cavity surface at ejection is shown in Fig. 3.17. The figure also shows the casting temperature at that instant. The temperature distributions are similar to those for a 6 mm thick uniform casting. The isotherms for the case are shown in Figs. 3.18 through 3.21. Figure 3.22 presents a coloured image of the temperature distribution in the dies at ejection.

3.5.4 Case Study No. 4

In this case again, the die geometry is the same as in Cases 1 and 2. The casting is however non uniform in thickness. The central 30 mm width of the casting is 16 mm thick, the rest of the casting is 4 mm in thickness. Each die block has two cooling lines located 10 cm. inside from the cavity surface and 25 cm. apart, one on either side of the center line. As indicated by the temperature distribution shown in Fig. 3.23; the central regions of the die become very hot in this case and the casting temperatures near that region are such that the casting does not solidify prior to ejection.

An alternative arrangement of cooling lines is considered, again with two cooling lines in each half, but with both lines placed along the center line of the dies at approximately 2.5 cm. and 7.5 cm. inside from the cavity surface as shown in Fig. 3.24. This arrangement has a strong sink effect and helps to reduce the difference between the casting and the die temperatures. The casting now attains solidification (at a point when casting releases 100% of its latent heat) prior to ejection.

Fig 3.17 Temperature distribution at ejection along the cavity surface from the center of die, and solidification times. Symmetric cooling load and 6 - 4 mm thick ALA380 casting

Fig. 3.18 Steady state cycle Isotherms at start of steady state cycle, 6-4 mm stepped AlA380 casting, symmetric cooling load

Fig. 3.19 Steady state cycle Isotherms at Liquidus stage of casting, 6-4 mm stepped AlA380 casting, symmetric cooling load

Fig. 3.20 Steady state cycle Isotherms at Solidus stage of casting. 6-4 mm stepped AlA380 casting, symmetric cooling load

Fig. 3.21 Steady state cycle Isotherms at ejection, 6-4 mm stepped AlA380 casting, asymmetric cooling load

Fig. 3.22 Temperature distribution at ejection during a steady state cycle, 6-4 mm thick stepped AlA380 casting, symmetric cooling load

Fig. 3.23 Temperature distribution on the cavity surface at the moment the casting is 100% solidified. Symmetric cooling load and 4-16mm thick A1A380 casting

Fig 3.24 Temperature distribution at ejection, along the cavity surface from the center of die; solidification times. Inline, symmetric cooling load and 4-16 mm stepped casting .

Fig. 3.25 Steady state cycle Isotherms at start of cycle, 4-16 mm stepped AlA380 casting and symmetric, inline cooling load

Fig. 3.26 Steady state cycle Isotherms at Liquidus stage of casting, 4-16 mm stepped A1A380 casting and symmetric, inline cooling load

Fig. 3.27 Steady state cycle Isotherms at Solidus stage of casting, 4-16 mm stepped AlA380 casting and symmetric, inline cooling load

Fig. 3.28 Steady state cycle Isotherms at ejection, 4-16 mm stepped AIAJ80 casing, symmetric, inline cooling load

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

Fig. 3.29 Temperature distribution at ejection during a steady state cycle; 4- 16 mm stepped AlA380 casting, symmetric, inline cooling load

The isotherms for the latter case are shown in Figs. 3.25 through 3.28. Figure 3.29 presents a coloured image of the temperature distribution in the dies at ejection.

3.6 Discussion of the Results of Case Studies

The case studies presented here show that a 2-dimensional transient thermal analysis is quite feasible when started with initial temperature estimate derived as explained in this work. Convergence to cyclic steady state is reached in a reasonable time and the difficulties in numerical solution caused by steep temperature gradients are overcome.

The temperature variation in the casting and on the die surface during a cycle are vividly illustrated by Fig. 3.4. At the beginning of the cycle, the casting loses heat rapidly to the dies and correspondingly the die temperatures rise. After liquidus, the cooling of the casting is slowed down because the casting releases its latent heat which must be absorbed by the dies. In general, the dies attain their maximum temperatures around solidus. After solidus, the cooling of the casting is slowed down because the casting releases its latent heat which must be absorbed by the dies. In general, the dies attain their maximum temperatures around solidus. Thereafter, the temperature drop in the casting again becomes rapid till the end of cycle.

The ability of a good thermal analysis tool to help in the design of a die is well illustrated by Case No. 4. In that case the first trial design indicated that the casting ejected prior to solidification and the design was not satisfactory. A second trial proved the effectiveness of an alternative cooling line layout. Similar configurations are published in the literature (Caulk, 1993).

CHAPTER 4

SUMMARY AND CONCLUSIONS

4.1 Summary

The pressure die casting is a complex process whose performance is governed by a number of different parameters. When process parameters are not properly controlled, the casting can have a number of defects and the die life can be adversely affected. Optimum conditions for casting can be assured by a proper design for the die. In the past, empirical procedures and trial and error methods were used to arrive at a satisfactory design. Because the cost of a prototype is very high, the trial and error method of design can be very expensive. Also, there is often no indication what improvements should be made to an unsatisfactory design to improve the performance.

In view of the considerations outlined above, there is a great need for a proper technique for the thermal analysis of die casting process. There again difficulties arise on account of the complexity of the process. To obtain an accurate solution, a full transient analysis is needed. However, when such an analysis started from the room temperature the steep temperature gradients in the vicinity of the casting cause numerical oscillations in the solution unless an extremely fine mesh is used. In addition, because the temperatures in the die surface near the casting vary quite rapidly over a cycle, a small time step must be used for numerical integration of the transient heat flow equations. Since it takes a long time for the temperatures to reach cyclic steady state, the computations become very costly.

Several methods have been used in the past by others researchers to overcome the problems identified in the preceding paragraph. Transformation to modal

coordinates followed by a transient analysis in the transformed domain has been attempted. However, this method cannot handle non linear boundary conditions. A recent method proposed by Caulk and his co-workers is based on the assumption that the transient temperature signal penetrates only a short depth in the die and that the temperature in the rest of the die is independent of time. The method is quite promising and has been successfully applied in practice. It is, however, based on several other simplifying assumptions. The transverse thermal flow in both the casting and the surface layer is neglected. The continuity of heat flux at the interface of the surface layer and the interior is satisfied only in an averaged sense. Also, the latent heat of solidification of casting is accounted for indirectly by adjusting the initial temperature of the casting at injection.

In the present work, an alternative form of transient heat flow analysis is proposed which overcomes the problems associated with steep temperature gradients and the short time step that must be used in the numerical integration. The success of the method depends on finding a good estimate of the steady state and using this as initial condition for starting the transient analysis. The steady state condition is obtained by finding the total heat exchanged between the casting and a die block during one cycle of the casting process. This computation requires a knowledge of the ejection temperature which is not known at the beginning of the analysis. However, all that is needed is to make a reasonable assumption for this value; the transient analysis converges to the true solution regardless of what value is used for the ejection temperature. The total heat exchanged is divided by the cycle time to obtain a rate of heat flux through the cavity surface. This flux is used as an input for a steady state finite element analysis of the die to obtain an initial state from which to start the transient analysis.

The feasibility of the method is demonstrated by a detailed development of a 2-D finite element transient thermal analysis procedure for a die consisting of two blocks. As in the Caulk's work, casting is assumed to be thin relative to the dimension of the dies and its conductivity high as compared to that of die. These assumptions are not however necessary and the procedure can be extended to remove them. Transverse heat flow in the surface layer is taken into account and the latent heat of solidification is accounted for in a direct manner.

It is assumed that the cooling lines can be treated as local heat sinks located at finite element nodes. Detailed expressions are derived for the heat carried away by the cooling lines and it is shown that the use of a closed form steady state expression is reasonable and justified. To convert the 3-D problem to a representative 2-D problem in which the heat flow perpendicular to the cross-section of the die is neglected, the total heat carried away by a cooling line is averaged over the length of the die.

Two computer programs are developed to implement the analysis procedure described. Details of the finite element discretization, node numbering, matrix storage and numerical integration technique used are presented. A number of case studies are carried out to demonstrate the feasibility of the analysis procedure. The results obtained are presented in the form of graphs, isotherms and colour fringes.

The results presented show that the transient analysis converges in a reasonable time and problems associated with steep gradients do not exist. It is straightforward to repeat the analysis with alternative cooling line locations and related parameters; remeshing is not required as long as cooling lines are located at finite element nodes. The effect of different casting widths and thicknesses and material properties is also easily explored.

4.2 Conclusions

The work presented here shows that a transient thermal analysis of the die casting process is feasible when the analysis is started from an initial condition representing an approximate steady state conditions. Such a steady state condition can be derived by assuming that the total heat exchanged between the casting and the die is uniformly distributed over a cycle.

It is also shown that the heat carried away by the pipes can be represented in the analysis by a closed form of steady state expressions.

The limited amount of case studies show the practical usefulness of the analysis procedure. They also demonstrate that cooling line configuration can have a significant bearing on cavity surface temperature which in turn has an important role in ensuring a good quality casting.

4.3 Recommendations for Future Work

The work presented here can be extended in a number of ways in order to develop a comprehensive and useful practical tool for the thermal analysis and design of pressure casting dies. The following are some areas in which additional work and extensions are desirable.

- (1) The two dimensional analysis method should be extended to three dimensions.

This will permit the modeling of castings for complicated 3-d geometry. It will also permit the modeling of cooling lines with more complex shapes.

- (2) Extension to 3-D analysis will permit a more accurate representation of heat carried away by the cooling lines because the die temperature can now be allowed to vary along the length and the cooling channel can be represented by one dimensional elements with different cross-sectional area if necessary.

- (3) The present analysis assumes a constant value for the heat transfer coefficient between the coolant and the die wall. The analytical procedure can be extended to include the effect of velocity, pressure, boiling and coolant viscosity.
- (4) The effect of cooling sprays should be included in the analysis procedure.
- (5) The effect of temperature gradient in the casting and the effect of heat flow in the casting should be taken into account
- (6) A useful extension would be to consider fluid flow in the casting cavity and the process of cavity fill.
- (7) In the present analysis, the contact conductance across the parting surface is assumed to be a constant. Studies by Johnson and Moses (1988) have shown experimentally that the conductance in case of short periodic cycles can change with time. This effect can be easily incorporated in the analysis procedure.
- (8) The effect of radiation may be included in the analysis.

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