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SYNCHRONIZATION AND DETECTION
OF TAMED FREQUENCY MODULATION
IN DIGITAL MOBILE RADIO SYSTEMS

by

Mohamed S. El-Tanany, M. Sc.

Thesis submitted to the faculty of Graduate Studies and
Research in partial fulfillment of the requirements for
the degree of;

Doctor of Philosophy

Department of Electronics
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ABSTRACT

The work reported in this thesis research represents a comprehensive study, at both the experimental and analytical levels, of a Tamed Frequency Modulation System intended for the transmission of digital speech over radio channels. Due to the multiplicity of the problems involved in this application, we have focused only on those which are relatively tractable and of primary importance. These are: the carrier synchronization, bit timing and the data detection.

The carrier synchronization problem has been investigated in great detail. Two synchronization schemes have been proposed and evaluated experimentally; the first operates on the output of a frequency doubler, while the second is based on a frequency quadrupler. These schemes do not suffer from problems such as signal drop-out which are common in practice.

A novel clock synchronizer is proposed, and an all digital implementation is presented. The loop is superior, in jitter performance, to the conventional data transition-tracking loop at lower signal-to-noise ratios. The loop's acquisition performance is also superior to the commonly used binary phase locked loops. The loop, as reported here, lends itself to today's VLSI technology.

A novel theory has been developed for the design of a simple quadrature coherent detector for Tamed FM signals. The theory yields the impulse response of a class of Weighted Average Matched Filters (WAMP). A performance analysis of this receiver revealed a near-optimum performance at lower signal-to-noise ratios, while the asymptotic degradation is approximately 0.6 dB compared to the optimum receiver.

Good agreement between theory and experiment is demonstrated in all cases. Although all experiments were conducted on a TFM system, the theories, analysis procedures, and the implementation formats presented in this thesis are directly applicable to a wide class of "Partial Response Continuous Phase Modulation" techniques with modulation index $1/2$ as long as they exhibit an open eye diagram.

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GLOSSARY OF NOTATIONS

B_n :	Noise Bandwidth
$D(\underline{a}, \underline{b})$:	Squared Euclidean Distance, in signal space, between the sequences $\underline{a}$ and $\underline{b}$
$D(nT)$:	Decision Variable of the Quad Demodulator at time nT
E_B :	Signal Energy per bit
$E\{x\}$:	Expectation of the random variable x
$E_k\{x\}$:	Expectation of x , over the parameter k
f_b :	Data rate in bits per second
f_c :	Carrier frequency.
$\{f_n\}$:	Coefficients of Partial Response System polynomial
$F(\omega)$:	Frequency response of the PR encoder
$G(\omega)$:	Transfer function of the phase-smoothing filter
$h_{amf}(t)$:	Impulse Response of Average Matched filter
$h_d(t)$:	Impulse Response of predetection filter
$h_{dl}(t)$:	Truncated Impulse Response of predetection filter
$I(t, \underline{a})$:	In-phase signal component
$n(t)$:	Random noise component
$n_c(t)$:	In-phase noise component
$n_s(t)$:	Quadrature noise component
N_0 :	Power spectral density of a white noise process
P_l :	Line energy
P_n :	Noise power
P_s :	Signal power
$P(\epsilon)$:	Probability of error
$P_I(\epsilon)$:	Probability of symbol error over the I-channel only

$q(t)$: Phase pulse
 $Q(\cdot)$: Q-function
 $r(t)$: Received signal plus noise
 $R(\cdot)$: Autocorrelation function
 $R_i(\cdot)$: Autocorrelation function of the In-phase signal component
 $R_n(\cdot)$: Autocorrelation function of noise
 $R_n^C(\cdot)$: Autocorrelation function of the In-phase noise component
 $R_q(\cdot)$: Autocorrelation function of the quadrature signal component
 $R_s(\cdot)$: Autocorrelation of signal
 $s(t, \underline{a})$: Modulated signal
SIR: Signal-to-interference ratio
SNR: Signal-to-noise ratio
 SNR_L : Signal-to-noise ratio in the loop bandwidth
 $\{t_n\}$: Sampling instants
 T : One bit interval
 $\underline{a}$: Data vector
 a^n : n^{th} component of $\underline{a}$
 $\underline{y}$: Difference data sequence
 y^n : n^{th} component of $\underline{y}$
 $\{\Delta\phi_n\}$: Set of possible phase shifts over one symbol interval
 Λ : Log Likelihood function
 $\phi(t, \underline{a})$: Phase function carrying the information vector $\underline{a}$
 $\phi_{nn}(f)$: Power spectral density of noise
 $\phi_{ss}(f)$: Power spectral density of signal
 $\omega(t, \underline{a})$: Instantaneous frequency of modulated carrier

ω_c : Carrier radian frequency
 ω_n : Loop natural frequency
 ω_s : Sampling radian frequency
 ξ : Damping coefficient

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CHAPTER I

INTRODUCTION

1.1 GENERAL

It has been widely predicted that the demand for data transmission over radio mobile channels will increase over the current decade. Examples of applications requiring such a service include; law enforcement agencies, taxi and other dispatch systems, automatic vehicle location systems, and control of channel assignment in cellular systems. In addition, it is anticipated that a large fraction of the 350,000 users, which constitute the current Canadian mobile radio community, will add to their systems a data transmission capability in the next ten years.

The increasing demand for voice and data communications over mobile radio channels, coupled with the scarce nature of the radio spectrum resource, have motivated the search for new techniques to achieve better utilization of that resource. One of these techniques relies on the integration of voice and data over the same channel. Essentially, this technique

makes use of the existence of gaps, between talk spurts, of average duration in the range 0.6 to 1.2 sec. The channel becomes idle during these gaps. The transmission of voice and data over the same channel can obviously be made possible, by transmitting data during the silence intervals of the channel, which is used primarily for voice communications.

Assuming data packet lengths in the range of 1000 to 2000 bits, as is the case in most applications, it is possible to fit these packets into the silent intervals of the channel, using appropriate transmission rates. This is possible at rates higher than 8 Kb/s. The realization of the above scheme is feasible only under the following conditions:

1. The availability of low-cost modem sets capable of transmitting data at rates of the order of 16 Kb/s, over the assigned channel bandwidth, with acceptable bit-error rate and fast carrier synchronization and timing recovery.
2. The ability of accurate, low cost speech detectors in the voice terminals, so that the carrier is suppressed when no input speech is detected.
3. The ability of the data terminal to sense the voice carrier on the channel, to determine if

the channel is busy (talk spurt) or free (gap). The data terminal will transmit its packet only when a gap is detected.

1.2 CONSTRAINTS ON MOBILE RADIO SIGNALS

The bandwidth allocated for each mobile channel in the VHF band, according to the Canadian department of communications regulations, is 30 kHz. The transmitted signal must satisfy the following requirements:

1. The transmitted signal must fit under the spectral mask of Fig. (1.1).
2. The spurious radiated power in the adjacent channel should be generally suppressed to at least 60 dB below that in the desired channel.
3. The transmitted signal must have a constant envelope to avoid any spectrum spreading due to the nonlinear operations associated with the R.F. section of the transmitter. This requirement is particularly important, since the use of power-efficient class C amplifiers is highly desirable in mobile terminals.
4. The system bit error rate, under relatively low signal-to-noise ratio conditions, must be acceptable.

Fig. 1.1 Canadian Department of Communications Spectral Mask.

Bandwidth between the 6 dB attenuation points 16 KHz.

Bandwidth between the 70 dB attenuation points 25 KHz.

Bandwidth between the 90 dB attenuation points 50 KHz.

5. The signal detection should, in principle, be independent of the received signal level. In general, the detection should be independent of any amplitude fluctuations in the received signal envelope. This is important due to the fading conditions, which are common in the land mobile communications environment.

Due to these constraints, angle modulation techniques are unavoidable. The most widely used are the QPSK and MSK modulations. Although QPSK is, in principle, a constant envelope modulation technique, its power spectrum roll-off is slow and does not meet the first two requirements. Attempting to limit the spectrum, by bandpass filtering in the IF stage, results in envelope nulls. These nulls ultimately lead to the regeneration of the signal sidebands due to the nonlinearities involved in the RF section [1], and again results in failure to meet the first and second constraints.

The spectral density of MSK [2] is more compact compared to QPSK. However, the requirement on the out-of-band radiation is not met. In a recent study by Prahbu [3], it is shown that the bit-error-rate performance of MSK systems deteriorates rapidly if the signal bandwidth is limited to less than 1.45 times the bit rate. For these reasons, we have concerned ourselves with a class of modulation techniques known as "Correlatively Encoded Continuous Phase Modulation". This

class is very efficient in its spectrum occupancy and power requirements, and yields constant envelope signals. These advantages are obtained at the cost of increased system complexity. This subject is discussed in more detail in Chapter II, with emphasis on "Tamed Frequency Modulation".

1.3 THESIS OBJECTIVES

Due to the multiplicity of the problems involved in the SCPC digital radio systems, this thesis research has been focused only on those problems which are relatively tractable, and of prime importance for the realization of these systems. A preliminary study of the existing, spectrally-efficient modulation techniques revealed that Tamed FM is particularly suitable for digital mobile radio systems. Therefore, we have concerned ourselves with TFM during the course of this work. Our main objectives are as follows:

a) Synchronization

To date very little has been reported about the carrier synchronization and bit-timing recovery in TFM systems. Therefore, it was felt that a quantitative study of the self synchronization properties of TFM is needed, and a comprehensive treatment of the synchronization is to be

carried out at both the experimental and analytical levels. The acquisition performance of these synchronizers is also important. For RF Carriers in the 800 MHz frequency band, and frequency stability of 10^{-6} , the carrier recovery loop must be capable of acquiring a worst-case offset of 1.5 kHz in a relatively short time.

b) Detection

The optimum receiver for the class of CPM modulation techniques is the well known Maximum Likelihood Sequence Estimator (MLSE). The MLSE consists of a bank of matched filters and true multipliers, which are used to generate the so called "branch metrics", [8]. The branch metrics are input to a viterbi processor, which decides in favour of the data sequence which maximizes the a-posteriori probability of the received signal. The number of filters needed for the MLSE, as well as the processing time, can be large. This makes the MLSE impractical for applications requiring relatively high speed data, especially if the system cost is a prime concern. Therefore, it was

felt that the search for a simpler suboptimum receiver is a worthwhile task to be undertaken, if the resulting degradation in the system signal-to-noise ratio can be kept small.

1.4 THESIS ORGANIZATION

This thesis is written in the following order;

Chapter II represents a quick review of the correlatively encoded CPM class of modulation techniques, with emphasis on Tamed FM. The spectral properties of TFM are discussed, and an upper bound on the performance of an optimum TFM receiver, in terms of the symbol error rate, is then given. Comparison to MSK is made whenever possible. The chapter is concluded by proposing a simple coherent receiver structure for TFM signals.

Chapter III is devoted to the modulator design. The first part represents a qualitative study of the possible approaches to the design problem. Several possibilities are investigated. A novel design format is proposed. This format makes an all-digital implementation possible. In part two, the most favoured modulator configuration is pointed out, and the necessary hardware design is discussed in detail. The chapter is concluded by presenting the experimental results obtained from a printed-circuit board version.

Chapter IV is started by a quantitative study of the self synchronization properties of TFM. The study revealed the presence of two discrete tones, centered at double the carrier frequency, in the squared signal spectrum. It also showed that a frequency-quadrupled signal contains a powerful tone at 4 times the carrier frequency. A second order loop, which operates on the squared signal, is then studied in great detail, in terms of the acquisition performance in the presence of large frequency offsets, and the steady state phase jitter is related to the input signal-to-noise ratio. The hardware design of this loop is investigated in detail, and the experimental results are presented. The chapter is concluded by an experimental investigation of a fourth order loop, which is based on a frequency-quadrupled signal.

Chapter V discusses the bit-timing recovery. The chapter is started by a brief review of the common clock synchronizers. A clock synchronizer format, which bears some similarities to the data-transition tracking loop is presented. This format is suitable for CPM systems which exhibit an open eye diagram. An expression for the variance of the loop timing error is derived, and it suggests that the loop is superior at relatively low signal-to-noise ratios. A discrete version of this loop is also discussed, and the necessary hardware design philosophy is explained in detail. The transient response

to a step in phase and a step in frequency is analyzed. The chapter is concluded by presenting the experimental results obtained from a printed circuit board.

Chapter VI is centered around the signal detection. First a comparison between CPM with modulation index $1/2$, and MSK is conducted. It is shown that under certain conditions, which are satisfied for TFM, de Buda's MSK demodulation strategy can be used for CPM systems. A quadrature coherent receiver is proposed, and optimized in the mean-square-error sense. The optimization yielded a class of suboptimum filters. This result is then applied to TFM, and the symbol error rate performance is analyzed and compared to that obtained using a 4th order Butterworth filter. The necessary hardware design is discussed. We conclude the chapter by the bit error rate measurements conducted on a PC board.

Chapter VII is dedicated for conclusions, and propositions for future work to be conducted in the research area of this thesis.

CHAPTER II

TAMED FREQUENCY MODULATION

2.1 INTRODUCTION

As radio communications developed, many researchers found that consumption of bandwidth, beyond that of the transmitted intelligence, could improve reliability. This trend was formalized by Shannon in 1948. Now, the transmission of analogue signals by digital techniques, techniques which consume far more bandwidth, is becoming the norm. Indeed most modern day communications engineers recognize bandwidth expansion as a sound investment which consistently yields returns in the form of message reliability.

As new communication services emerge, such as land mobile communications, satellite business systems, medical remote monitoring and computer networks, ever increasing demand will be placed on what is now recognized as a limited and, relatively speaking, diminishing resource - the radio spectrum. This leads to the question "How can bandwidth be reduced while maintaining a fixed grade of service without the need for increased signal power per

bit?". This lead many of the present day researchers in digital communications to investigate digital transmission methods which are efficient in both power and bandwidth. All approaches make use of heavy signal processing techniques. In fact, researchers are trading both power and bandwidth for a third commodity; processing complexity.

In a recent paper [4], Anderson et al. considered the theoretical limitations of narrow-band signalling and showed that constant envelope, phase modulated signals can, in principle, perform near the Shannon capacity of the additive White Gaussian noise channel.

To achieve such performance requires a significant amount of signal processing complexity, as well as very accurate carrier phase synchronization. T. Aulin, et al [5], [6], explored a wide variety of means to achieve narrow band digital transmission. The authors have determined the distance properties, relative performances, and spectral characteristics of Continuous Phase Modulated (CPM) signals as function of the modulation index, pulse shape, alphabet size and receiver complexity. They developed the principle that performance improvement can be achieved by allowing the channel pulses to extend over several data symbol intervals. Muilwijk [7] discussed the "Correlative Encoded Phase Shift Keying" approach to sophisticated phase modulation, and came to similar conclusions. Muilwijk's study is an outgrowth of the

Tamed FM (TFM) work [8] at Philip's Laboratories. Austin & Chang [9] have introduced the "Quadrature Overlapped Raised-Cosine" modulation. Gaussian MSK is introduced in [10], and Doubinary MSK in [11]. A comprehensive bibliography for similar techniques is given in [6].

TFM has been shown to be one of the most spectrally efficient techniques in the CPM family. In addition, it bears great similarities to MSK [2], a fact which makes it possible to reduce the system complexity without significant degradation in its performance, as will be shown later on.

In this chapter, we first discuss the definition of the correlative encoded CPM, and the TFM is presented as a member of this class.

A comparison between TFM and MSK will then be made. The optimum receiver for TFM is presented, and the TFM bit error rate performance is computed for the high Signal-to-Noise Ratio (SNR) case; and compared to that of MSK.

2.2 CORRELATIVE ENCODED CPM

In the modulation techniques reported in [4] and [6-11], heavy use is made of Partial Response Signalling (PRS), [12] at baseband. PRS, in principle, is equivalent to introducing controlled intersymbol interference (ISI) in order to reshape the spectral density of the baseband

signal, which in turn affects the spectral density of the modulated carrier. PRS is based on the premise that since the introduced ISI is known, it can be removed, or accounted for, at the receiver.

Consider a discrete input sequence $\{a^n\}$ with L levels, and a symbol rate $1/T$, which modulates the phase of a carrier such that we obtain a constant envelope signal;

$$s(t) = \sqrt{\frac{2EB}{T}} \cos(\omega_c t + \phi(t)) \quad (2.1)$$

The principle of Correlative CPM is defined by the following three points [7];

- a) The information from $\{a^n\}$ is encoded into the phase shifts $\{\Delta\phi_n\}$ so that the phase is continuous. The phase shift over one symbol interval is given by;

$$\Delta\phi_n = \phi((n+1)T) - \phi(nT) = C^n \cdot \frac{2\pi}{M} \quad (2.2)$$

Where M is a fixed positive integer, equal to the number of possible phase positions at $t=nT$. The integer C^n carries the information.

- b) Successive phase shifts are correlated, so that the phase shift over a symbol interval is a function of the input data level during that interval, and at least one preceding

interval. This is realized by deriving the sequences $\{C^n\}$ from the input sequence $\{a^n\}$ via a correlative encoding rule. The number of possible phase shifts must be larger than the number of input data levels, L .

- c) The phase path is smoothed, so that the derivative of the phase is a continuous function of time.

A basic modulator is shown in Fig. (2.1). It consists of a correlative encoder, a premodulation low pass filter, and a frequency modulator in cascade. The correlative encoder is described by a polynomial in the delay operator D , [13], with degree N ;

$$F(D) = f_0 + f_1 D + \dots + f_N D^N \quad (2.3)$$

The greatest common divisor of the coefficients $\{f_m\}$ is one. These coefficients are chosen in such a way to enforce the desired phase values at the end of each signalling interval. The encoder frequency response is periodic and is given by;

$$P(\omega) = F(D) \Big|_{D=e^{-j\omega T}} = \sum_{k=0}^N f_k e^{-j\omega k T} \quad (2.4)$$

The premodulation filter, $G(\omega)$, is designed to achieve the following objectives;

Fig(2.1): A Functional Diagram of a "Correlatively Encoded CPM" Modulator

- 1) Preserve the modulated signal phase at times $t=nT$, as set by the correlative encoder.
- 2) Smooth the phase path.
- 3) Band limit the phase function spectral density.

In order to satisfy the first requirement, $G(\omega)$ must satisfy the Nyquist's Third Criterion [14], which states that;

$$\int_{(2k-1)T/2}^{(2k+1)T/2} g(t) dt = \begin{cases} 1 & \text{for } k=0 \\ 0 & k \neq 0, k \text{ integer} \end{cases}$$

The second objective achieved by $G(\omega)$ is of prime importance due to the fact that if the phase function, $\phi(t)$, and its first k derivatives are continuous, the modulated signal spectrum rolls off according to $1/f^{2(k+1)}$, [44]. The overall transfer function, from the data input to the frequency modulator input constitutes a premodulation filter with characteristic function,

$$H_m(\omega) = P(\omega) \cdot G(\omega) \quad (2.5)$$

It is expected that the system spectral efficiency increases with the order of $P(D)$, at the expense of added system complexity. The most widely used schemes are of first and second order.

2.3 TAMED FREQUENCY MODULATION

TFM [8] is viewed as a modified version of MSK [2], which uses a PRS technique with a second order system polynomial i.e.

$$F(D) = \frac{\pi}{8T} (1 + 2D + D^2) \quad (2.6)$$

A block diagram of TFM modulator is depicted in Fig. (2.2). It is obvious that the signal level at the correlative encoder output, during the n th signalling interval, $(n-1)T < t < nT$, is given by;

$$y(t) = \frac{\pi}{8T} (\alpha^{n-1} + 2\alpha^n + \alpha^{n+1}) \quad (2.7)$$

where $\{\alpha^n\}$ are the input, bipolar, data symbols. The premodulation filter, $G(\omega)$, is designed to smooth the phase function, $\int y(t)dt$, while preserving its value at the end of each signalling period - i.e. at $t=nT$. The transfer function $G(\omega)$ can be put in the following form [8];

$$G(j\omega) = \frac{\omega T/2}{\sin(\omega T/2)} N(j\omega) \quad (2.8)$$

where $N(\omega)$ is any pulse satisfying the Nyquist first criterion. $N(\omega)$ is chosen to minimize the out-of-band radiated power of the modulated carrier. From Eqn. (2.6)

Fig(2.2): A Functional Diagram of a TFM Modulator.

The Modulation-Sensitivity of the VCO is Assumed to be Unity.

and (2.8), it can be shown that the overall premodulation filter transfer function, $H_m(\omega)$, is given by;

$$H_m(j\omega) = \frac{\pi}{2T} \cos^2 \frac{\omega T}{2} \frac{\sin(\omega T/2)}{\sin(\omega T/2)} N(j\omega) \quad (2.9)$$

where

$$N(j\omega) = \begin{cases} 1 & 0 < |\omega| < \pi(1-\eta)/T \\ \frac{(1 - \sin \frac{\omega T - \pi}{2\eta})/2}{2\eta} & \pi(1-\eta)/T < |\omega| < \pi(1+\eta)/T \\ 0 & \text{otherwise} \end{cases} \quad (2.10)$$

$N(\omega)$, as given by Eqn. (2.10) is the raised-cosine characteristic, and η is known as the roll-off factor.

It has been reported in [8] that values of η less than 1/2 give nearly minimal out-of-band radiation. It is worth noting that if the function $N(\omega)$ is set to $\frac{\sin(\omega T/2)}{(\omega T/2)}$, the resulting $H_m(\omega)$ would be;

$$H'(j\omega) = \frac{\pi}{2T} \cos^2 \frac{\omega T}{2} \quad (2.11)$$

A system employing $H_m(\omega)$ as given by Eqn. (2.11) is called Tamed Frequency Shift Keying (TFSK), [8].

The impulse response of the premodulation filters of TFM and TFSK are shown in Fig. (2.3).

Fig(2.3): The Computed Impulse Response of the TFM and TFSK Premodulation Filters, as Defined by eqn(2.9-2.11).
The Roll off Factor in eqn(2.10) is Set to Zero

The phase state diagrams of TFM and MSK are shown in Fig. (2.4). Notice that while MSK has only 4 phase positions, TFM has 8 positions. The power spectral density of TFM, as compared to MSK, is shown in Fig. (2.5). Note that the spectral density falls at least 60 dB below the unmodulated carrier at $|f-f_c| > f_b$. The phase function for a particular data sequence is shown in Fig. (2.6). From Fig. (2.4) and (2.6) it is clear that the TFM signal assumes the phase values $0, \pm \frac{\pi}{4}, \pm \frac{\pi}{2}$ and π at the data transition instants, $t=nT$. This is due to the fact that the phase change during the n th symbol interval is given by

$$\Delta\phi_n = \frac{\pi}{2} (a^{n-1} + a^n + a^{n+1}) \quad (2.12)$$

and therefore, $\Delta\phi_n$ takes on the values $0, \pm \frac{\pi}{4}$ and $\pm \frac{\pi}{2}$, depending on the data sequence.

2.4 TFM OPTIMAL RECEPTION AND BER

For the sake of simplicity, the TFM signal is assumed to be transmitted over an additive, White Gaussian noise channel, with a one-sided spectral density N_0 . Thus the signal available for observation is;

$$r(t) = s(t, \underline{a}) + n(t) \quad -\infty < t < \infty \quad (2.13)$$

Fig(2.4): The Signal Space Diagram for (a) TFSK and
TFSK, (b) MSK

Fig(2.5): Power Spectral Density of TFM, TFSK and MSK

Fig(2.6): The Phase Function of MSK, TFSK and TFM.
The corresponding Data Sequence is Indicated
on the Abcissa

where $\underline{a}$ is the transmitted data sequence, and $n(t)$ is a zero-mean Gaussian random process.

A receiver which minimizes the probability of erroneous decision must observe the received signal over the entire time axis and choose the infinitely long sequence which minimizes the probability of error. This is referred to as a Maximum Likelihood Sequence Estimator (MLSE), [15]. The implementation, as well as the analysis of this receiver could become unmanageable. A sub-optimum receiver, which approaches the optimum in performance for high SNR, [5] and [16], observes the received signal $r(t)$ for N symbol intervals to make a decision about a particular data bit.

The detector maximizes the log-likelihood function,

$$\Lambda_N\{r(t)\} = \int_0^{NT} \{r(t) - s(t, \underline{a})\}^2 dt \quad (2.14)$$

where there are 2^N different data sequences in the interval NT . However, since the detector is only interested in finding an estimate $\hat{a}^0$ of a^0 , the 2^N sequences may be formed into two groups;

$$\begin{aligned} & a^0, a^1, a^2, \dots, a^{N-1} \\ & b^0, b^1, b^2, \dots, b^{N-1} \end{aligned}$$

where $a^0 = +1$ and $b^0 = -1$, and the detector function is to decide if the first symbol is a^0 or b^0 . In other words, the detector must find which group of sequences jointly maximizes the log-likelihood function. Such a detector is depicted in Fig. (2.7). The probability that the detector makes an erroneous decision is upper bounded by [16];

$$P(\epsilon) < \frac{1}{2^{N-1}} \sum_k \sum_l Q \frac{D(\underline{a}_k, \underline{b}_l)}{2N_0} \quad (2.15)$$

The summation is taken over all pairs of the sequences $\underline{a}$ and $\underline{b}$. The subscripts k and l denote sequence permutations. $Q(x)$ is the so-called error function [15] and is given by;

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \quad (2.16)$$

and $D(\underline{a}_k, \underline{b}_l)$ is the Euclidean distance, in signal space, between the sequences $\underline{a}_k$ and $\underline{b}_l$, and is defined by;

$$\begin{aligned} D(\underline{a}_k, \underline{b}_l) &= \int_0^{NT} (s(t, \underline{a}_k) - s(t, \underline{b}_l))^2 dt \\ &= \frac{2E_B}{T} \int_0^{NT} (\cos\phi(t, \underline{a}_k) - \cos\phi(t, \underline{b}_l))^2 dt \end{aligned} \quad (2.17)$$

Fig(2.7): A Suboptimum Detector for CPFSK-type Signals

where the phase function, $\phi(t, \underline{a})$, is given by

$$\phi(t, \underline{a}_k) = \omega_c t + \int_{-\infty}^t \sum_{m=1}^N a_k^m h_m(t-iT) dt \quad (2.18)$$

and E_B is the signal energy per bit, T is the symbol interval, and ω_c is the carrier radian frequency. For $\omega_c T \gg 1$ Eqn. (2.17) is approximated by;

$$D(\underline{a}_k, \underline{b}_l) = 2E_B \left(N - \frac{1}{T} \int_0^{NT} \cos(t, \underline{a}_k - \underline{b}_l) dt \right) \quad (2.19)$$

Therefore, it is sufficient to consider only the difference data sequence;

$$\underline{y}_{kl} = \underline{a}_k - \underline{b}_l \quad (2.20)$$

and for high Signal-to-Noise (SNR), Eqn. (2.15) is approximated by [5];

$$P(e) \approx \Gamma_0 Q \left\{ \frac{D_{\min}}{2N_0} \right\} \quad (2.21)$$

where

$$D_{\min}^2 = 2E_B \cdot \min_{k,l} \left(N - \frac{1}{T} \int_0^{NT} \cos \phi(t, \underline{y}_{kl}) dt \right) \quad (2.22)$$

with the restriction that

$$y_{kl}^0 = +2\pi \text{ for all } k \text{ and } l \quad (2.23)$$

A useful tool for the analysis of continuous phase modulation techniques is the so-called "phase-tree". This is formed by drawing the ensemble of phase trajectories $\phi(t, \underline{a})$ having a common starting value of zero at $t=0$.

The ensemble is over the sequence $\underline{a}$. Fig. (2.8) shows a portion of the phase tree of TFSK. To calculate $D_{\min}$ for an observation interval NT , all pairs of phase trajectories over N symbol intervals must be considered. The phase trajectories must not coincide over the first symbol interval, however. The Euclidean distance is calculated for all these pairs, according to Eqn. (2.19), and the minimum value is the desired result. Since $D(\underline{a}_k, \underline{b}_l)$ is a monotonically increasing function of N (see Eqn. 2.17), a tight upper bound on $D_{\min}$ can be obtained [6], by considering only the pairs of sequences for which the phase trajectories merge as soon as possible. Two phase trajectories merge at a certain time if they coincide for ever after. The first time for which any phase difference trajectory can be made identically zero forever after is in general $(L+1)T$, where LT is the length of the premodulation filter impulse response, $h_m(t)$, on the time axis. From Figure 2.3

Fig(2.8): Phase Tree for TFSK Signals.

NOTE: The first merge, which yields a tight upper bound on $D_{\min}$, occurs at point C. $D_{\min}$ is computed as the distance between the trajectories labelled ABC and AB'C.

$L \mid_{\text{TPM}}=5$, and $L \mid_{\text{TFSK}}=3$. The first merge in the phase tree of TPM and TFSK yields the minimum Euclidean distance.

The phase difference trajectory giving the first merge is obtained by choosing the difference sequence $\underline{\gamma}$ [6] here;

$$\gamma_i = \begin{cases} 0 & i < 0, i > 1 \\ 2 & i = 0 \\ -2 & i = 1 \end{cases} \quad (2.24)$$

the corresponding phase difference trajectory is;

$$\phi(t, \underline{\gamma}) = \begin{cases} 0 & t < 0 \\ \gamma^0 q(t) & 0 < t < T \\ \gamma^0 [q((L+1)T) - q(LT)] & t > (L+1)T \\ \gamma^0 [q(t) - q(t-T)] & T < t < (L+1)T \end{cases} \quad (2.25)$$

where $q(t)$ is given by

$$q(t) = \int_{-\infty}^t h_m(x) dx \quad (2.26)$$

$q(t)$ is known as the phase pulse and is shown in Fig. (2.9) for TPM and TFSK. With the aid of Fig. (2.9) and assuming the difference sequence $\underline{\gamma}$ as given by Eqn. (2.24), the TFSK's phase difference trajectories are given by;

Fig(2.9): The Computed Phase Pulse $\{q(t)\}$ for TPM and TFSK Signals

$$\phi(t, \underline{y}) = \begin{cases} 0 & t < 0 \\ \frac{\pi t}{4T} & 0 < t < T \\ \frac{\pi}{4} + \frac{\pi(t-T)}{4T} & T < t < 2T \\ \frac{\pi}{2} - \frac{\pi(t-2T)}{4T} & 2T < t < 3T \\ \frac{\pi}{4} - \frac{\pi(t-3T)}{4T} & 3T < t < 4T \end{cases} \quad (2.27)$$

using Eqn. (2.27). In Eqn. (2.19), we obtain the minimum Euclidean distance for TFSK:

$$D_{\min}^2|_{\text{TFSK}} = 2E_B(1.453) \quad (2.28)$$

similarly;

$$D_{\min}^2|_{\text{TFM}} = 2E_B(1.575) \quad (2.29)$$

and

$$D_{\min}^2|_{\text{MSK}} = 2E_B(2) \quad (2.30)$$

From Eqn. (2.28)-(2.30), the degradation in the system SNR (with MSK as a reference) is 1.3 dB for TFSK and 1 dB for TFM. For high SNR, the bit error rates are approximated

by:

$$P(\epsilon) \Big|_{\text{TFSK}} = \frac{1}{2} \exp\left(-\frac{1.45E_B}{2N_0}\right)$$

$$P(\epsilon) \Big|_{\text{TFM}} = \frac{1}{2} \exp\left(-\frac{1.6E_B}{2N_0}\right)$$

$$P(\epsilon) \Big|_{\text{MSK}} = \frac{1}{2} \exp\left(-\frac{2E_B}{2N_0}\right)$$

(2.31)

The computed BER curves, according to Eqn. (2.31) are shown in Fig. (2.10).

2.5 ALTERNATE RECEIVER STRATEGY

In section 2.4 we made it clear that the optimum receiver for TFM is a MLSE. This receiver is not practical for mobile units where cost is of primary concern. In addition, the MLSE could represent a heavy burden on the designer for relatively high speed applications. An alternative strategy is to use the quadrature coherent receiver shown in Fig. (2.11). This structure is easier to implement, and yields near optimal performance if the predetection filters are optimized. The block diagram of Fig. (2.11) is the subject of chapters (4-6).

Fig(2.10):, Computed Probability of Error for TPSK, TFM and MSK, at High SNR

Fig(2.11): A Quadrature Coherent Receiver Structure for CPM with modulation Index 4. (The System must satisfy equ 6.11)

$r(t)$ is the output of an IF band pass filter, which is assumed to be wide enough to pass the signal without distortion, and narrow enough for the narrow-band approximation of noise to be valid.

CHAPTER III

TAMED FM MODULATOR DESIGN

This chapter is composed of two main sections. The first is devoted to a qualitative discussion of different approaches to the design problem with emphasis on some basic functions, and the limitations they may impose on the system. Towards the end of this section, the most favoured implementation strategy is singled out. Section two is concerned with the details of the hardware design of that format.

3.1 DIFFERENT APPROACHES TO THE MODULATOR DESIGN

A TFM modulator is viewed as a device which, upon receiving a data sequence $\underline{a}$, generates the waveform $s(t, \underline{a})$, where;

$$\underline{a} = \{a^0, a^1, \dots, a^n\} \quad (3.1)$$

and

$$s(t, \underline{a}) = \sqrt{\frac{2E_B}{T}} \cos(\omega_c t + \phi(t, \underline{a})) \quad (3.2)$$

$$\phi(t, \underline{a}) = \sum_{i=-\infty}^{+\infty} \alpha^i q(t - iT) \quad (3.3)$$

$$q(t) = \pi \int_{-\infty}^t h_m(y) dy \quad (3.4)$$

Where α^i is a binary non-return-to-zero (NRZ) data symbol, $h_m(t)$ is the system frequency pulse as defined by Eqn. (2.9), $q(t)$ is the system phase pulse and ω_c is the unmodulated carrier frequency.

There are two main approaches to the design problem, both of which are theoretically valid. The first approach is based on viewing TFM as a frequency modulated carrier, while the second approach views TFM as a quadrature modulation technique.

3.1.1 Frequency Modulator

The TFM signal instantaneous frequency is given, from Eqn. (3.2), by:

$$\omega(t, \underline{a}) = \omega_c + \pi \sum_{i=-\infty}^{\infty} a^i h_m(t-iT) \quad (3.5)$$

The frequency of a voltage controlled oscillator can be made to follow Eqn. (3.5) at any moment if the VCO free-running frequency is assumed constant and equal to ω_c , and the tuning voltage is made equal to

$$\frac{\pi}{K_V} \sum_{i=-\infty}^{\infty} a^i h_m(t-iT) \text{ where } K_V \text{ is the VCO modulation}$$

sensitivity. A block diagram illustrating this approach is given in Fig. (3.1). This approach has several advantages;

1. Since the input data is binary, the digital filter can be implemented as a binary transversal filter. Hence, no true multiplications are required. This fact results in the feasibility of handling high speed data easily.
2. The VCO output has a constant amplitude all the time, and therefore the output signal has a constant envelope at any moment.
3. Simplicity.

Fig(3.1): Frequency Modulator Approach to the Generation of TFM

In spite of these advantages, the frequency modulator has two major limitations:

1. The free-running frequency of almost any wide band VCO is quite unstable. This problem can cause the modulated signal to drift towards adjacent RF channels, thus resulting in interchannel interference, in addition to some degradation in the overall system performance.
2. The VCO modulation sensitivity is, strictly speaking, nonlinear, and shows some changes with temperature and time. This can lead to the deviation of the modulation index from $1/2$, and can exhibit itself in the form of carrier phase jitter.

These limitations can be dealt with in principle. In [8], an attempt has been made to stabilize the free-running frequency, and to compensate for the changes in the modulation sensitivity, using a two-dimensional loop. The authors have shown that, if acceptable performance is to be obtained, then the response time of the loop will have to be large. This may cause loss of the data trailing sequence. In addition, perfect compensation is not possible, and some phase jitter will still have to be tolerated. Due to these facts, we find this implementation quite inadequate for our application, where reliability is of primary importance.

3.1.2 I-Q Implementation

Viewing TFM as a quadrature modulation technique is based on the fact that the modulated signal can be written, using Eqn. (3.2), in the following form;

$$s(t, \underline{a}) = \sqrt{\frac{2E_B}{T}} (\cos \omega_c t \cos \phi(t, \underline{a}) - \sin \omega_c t \sin \phi(t, \underline{a})) \quad (3.6)$$

The I-Q implementation is directly based on Eqn. (3.6) above, and has been suggested in [6] and [8]. Fig. (3.2) shows a block diagram for the direct implementation of Eqn. (3.6). For this implementation a fixed frequency carrier is all that is needed, and can be provided by a crystal-stabilized frequency source. With the use of this implementation, the two limitations of the frequency modulator are alleviated. Due to the sensitivity of the modulated signal envelope to any mismatch between the I and Q channels, particular care should be taken in the design, if the constant envelope is to be approached. The block diagram of Fig. (3.2) has been adapted for our design, and therefore we shall discuss several possibilities for implementing the two basic building blocks involved. These are the digital waveform generation, and the multiplication by $\cos \omega_c t$.

Fig(3.2): A Functional Diagram of the I-Q Implementation
for the TPM Modulator

3.1.2.1 Digital Waveform Generators

Although the phase function is linearly related to the input data $\{a^n\}$, $\cos \phi(t, \underline{a})$ and $\sin \phi(t, \underline{a})$ are not. There does not exist any linear system that will accept $\underline{a}$ and output $\cos \phi(t, \underline{a})$.

There are two main approaches for the generation of $\cos \phi(t, \underline{a})$ and $\sin \phi(t, \underline{a})$ as shown in Fig. (3.3). The schematic in Fig. (3.3a) is simple and easy to implement. The input data $\{a^n\}$ is processed to produce $\phi(t, \underline{a})$ modulo 2π , using a binary transversal filter, which does not require any true multipliers. The phase function is quantized and used to address two look-up tables to generate $\cos \phi(t, \underline{a})$ and $\sin \phi(t, \underline{a})$. The memory size required for this implementation is approximately $2^m 2^m$ words where m is the number of bits used to approximate each phase sample. For example, for $m=8$, 512 words of memory are needed. In spite of its simplicity, this implementation is not desirable since the quantization noise adds directly to the information-carrying phase function, and hence causes more degradation in the system performance. This quantization noise can be reduced to any degree desired, but at the expense of larger memory sizes. For a 16 bit/phase sample, a 65K word of memory space is needed for each look-up table.

(a)

(b)

Fig(3.3): Alternative Approaches to the Realization of the Digital Waveform Generators

The implementation of Fig (3.3b) is favoured since the quantization noise would add to the signal amplitude, instead of phase. The theory of this schematic is explained as follows. The phase of the modulated signal, at the data switching instants, assumes one of eight possible phase states as illustrated in Fig. (3.4). The system phase states follow the recursive relationship;

$$\phi(nT) = \phi((n-1)T) + \frac{\pi}{8} (a^{n-1} + 2a^n + a^{n+1}) \quad (3.7)$$

Consequently, the phase function may be written as,

$$\phi(t, \underline{a}) = \phi(nT) + \sum_{i=n-1}^{n+l} a^i (q(t-iT) - q(iT)) \quad nT < t < (n+1)T \quad (3.8)$$

where the second term to the right represents the phase change instead of absolute phase, and the system frequency pulse duration is $(2l+1)T$. Therefore, the phase function, during any time slot, can be determined by the phase state at the beginning of this interval and a $(2l+1)$ bit data sequence. In Fig. (3.3b), the "phase state memory" is used in updating the system phase state, each T sec. interval, according to Eqn. (3.7). The memory is addressed by the old phase state and a 3-bit data sequence. The updated phase state, together with a $(2l+1)$ -bit data sequence are used to address two look-up tables where $\cos \phi(t, \underline{a})$ and $\sin \phi(t, \underline{a})$ are stored. The memory size needed for this implementation is given by;

Fig(3.4): Phase State Diagram for TFM

(2 X number of phase states X $2^{(2l+1)}$ X number of samples per T sec.) words. For example, for $l=2$ and 8 samples per T sec., the memory size is 4096 words. It is possible, in principle, to reduce the required memory size by a factor of 8 by storing only the sin and cos functions of the second term to the right of Eqn. (3.8). A schematic diagram illustrating this concept is shown in Fig. (3.5). This Schematic is attractive for VLSI purposes. However, the schematic of Fig. (3.3b) is more favourable for prototype work since it requires fewer EPROMS than that of Figure (3.5). Consequently, the approach of Figure (3.3b) has been adapted in our system.

3.1.2.2 Carrier Multiplication

Digital Approach

For the multiplication by a carrier with arbitrary frequency ω_c , one can sample the carrier, $\cos \omega_c t$, and its quadrature component at a rate identical to that of the I and Q baseband signals. The sampled carrier and the baseband signals are then applied to a digital multiplier to form a sampled version of $\cos \omega_c t \cos \phi(t, q)$ and $\sin \omega_c t \sin \phi(t, q)$. This technique has the following limitations;

Fig(3.5): A Realization for the Sine and COS Waveform Generators.

$\phi(t)$ as indicated here is modulo $\pi/4$. The memory size needed for this implementation is about $1/8$ of the memory space needed for the implementation of fig(3.3b).

1. The sampling frequency, f_s , must be an integral multiple of the data rate, f_b . For $f_s = nf_b$, then (nf_b/f_c) must be greater than 2. For $f_c = 455$ kHz and $f_b = 16$ Kb/s, the smallest value allowable for f_s is 1024 kHz. This implies that the total processing time for each sample must be less than 1 μ s. This constraint gets more difficult for higher bit rates.
2. Due to the requirements on f_s , the memory size needed for the digital waveform generators becomes uneconomical. For example, for a sampling frequency of 1024 kHz, the memories needed for the schematic of Fig. (3.3b) becomes 32K words.

Due to these limitations, we do not see this approach as practical.

An alternative approach to carrier multiplication is possible in the very special case where the carrier frequency is $1/4 \omega_s$. From Eqn. (3.2) we can form the output sequence;

$$s(t_n) = \sqrt{\frac{2E_B}{T}} \left\{ \cos(\omega_c t_n) \cos(t_n, \underline{a}) - \sin(\omega_c t_n) \sin(t_n, \underline{a}) \right\} \quad (3.9)$$

Where $t_n = n/f_s$. For $\omega_c = \omega_s/4$, we have $\omega_c t_n = \frac{n\pi}{2}$.
Therefore,

$$s(t_n) = \begin{cases} \sqrt{\frac{2E_B}{T}} (-1)^{(n-1)/2} \sin\phi(t_n, \underline{a}) & \text{for } n \text{ odd} \\ \sqrt{\frac{2E_B}{T}} (-1)^{n/2} \cos\phi(t_n, \underline{a}) & \text{for } n \text{ even} \end{cases} \quad (3.10)$$

By close examination of Eqn. (3.10), the carrier multiplication amounts to the following:

1. From $\cos\phi(t_n, \underline{a})$ and $\sin\phi(t_n, \underline{a})$, we form two new sequences;

$$\begin{aligned} &\text{and} \quad \left. \begin{aligned} x_c(m) &= \cos\phi(t_{2m}, \underline{a}) \\ x_s(n) &= \sin\phi(t_{2n+1}, \underline{a}) \end{aligned} \right\} \quad (3.11) \end{aligned}$$

2. Inverting every other sample of the newly formed sequences, $x_c(m)$ and $x_s(n)$, to yield $x'_c(m)$ and $x'_s(n)$.
3. Forming the output sequence, $s(n)$, by interleaving alternate samples of $x'_c(n)$ and $x'_s(m)$.

For implementation, $\cos\phi(t, \underline{a})$ and $\sin\phi(t, \underline{a})$ may be sampled in synchronization with alternate clock pulses, and by inverting every other sample the sequences $x'_c(n)$ and $x'_s(m)$ are directly obtained. This approach is quite attractive due to the possible savings in hardware,

in addition to not imposing any limitations on the system, except for the carrier frequency. At this point, an all digital implementation for the modulator becomes feasible, and it may follow the schematic diagram of Fig. (3.6).

This implementation strategy is due to the author.

Unfortunately, it has been discovered at a very late stage of this thesis research, and as a consequence it has not been implemented. The spectral density at the output of the DAC, appears periodic in the frequency domain, as

shown in Fig. (3.7). The signal component in the frequency interval $\left(\frac{kf_s}{2} + \frac{f_s}{4} < f < \frac{kf_s}{2} + \frac{3f_s}{4}\right)$ may be selected using a

simple analogue post-modulation filter. The requirements of this filter can be relaxed by either increasing f_s or decreasing k . The disadvantage of this implementation is that it constrains the carrier frequency to a multiple of the bit rate. In the course of this work, the final objective was interfacing the TFM system to existing mobile radio transmitters. Since the IF frequency in this equipment is set to a standard value of 455 KHz, all our effort was directed towards this value, which is incompatible with the constraints set on Fig. (3.6).

Analogue Multiplication

This approach is illustrated in Fig. (3.8). The baseband samples, from the digital waveform generators, are first converted into analogue signals, and then low pass

Fig(3.6): An All-Digital Realization of the TFM Modulator

Fig(3.7): Spectral Density Following the D/A Converter of fig(3.6).

Fig(3.8): Analogue Carrier Multiplication

filtered to reconstruct an analogue version. The analogue signals are multiplied by the carrier using analogue multipliers. This approach makes it possible to use arbitrary carrier frequency. In this approach, particular care should be taken to ensure that the I and Q channels are properly matched, and that the arm filter has linear phase response and flat amplitude response. These are important considerations in order for the output signal to have a constant envelope. The schematic of Fig. (3.8) has been followed closely in the implementation of our TFM Modulator.

3.2 MODULATOR HARDWARE DESIGN

In our hardware development, the I-Q implementation format, of Fig. (3.2), has been adapted, with the digital waveform generators of Fig. (3.3b), and the carrier multiplication according to Fig. (3.8). In this section we shall discuss the hardware design problem in some detail.

3.2.1 Phase State Generator

The modulated signal phase, at the data switching moments, is allowed to assume one of 8 possible values, which are referred to as the system phase states, as shown

$\alpha \backslash \theta_{n-1}$	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
$\{0,0,0\}$	θ_6	θ_7	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5
$\{0,0,1\}$	θ_7	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
$\{0,1,0\}$	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
$\{0,1,1\}$	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_0
$\{1,0,0\}$	θ_7	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
$\{1,0,1\}$	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
$\{1,1,0\}$	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_0
$\{1,1,1\}$	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_0	θ_1

Table(3.I): Phase State Transition Matrix
 $\underline{\alpha} = (\alpha^{n-1}, \alpha^n, \alpha^{n+1})$

in Fig. (3.4).

The system phase states follow the recursive relationship of Eqn. (3.7). Using Eqn. (3.7) the system's phase state transition matrix is easily generated, and is given in Table (3.1). In the system implementation, the binary representation of the subscripts of the phase states are used instead of the actual values. The phase state at any time is uniquely defined by the previous state and a 3-bit data sequence, as indicated in Table (3.1). Consequently, updating the system's phase state is accomplished by addressing the phase state memory [which contain the information of Table (3.1)] by the old phase state, and a 3-bit data sequence. The delay T , in Fig. (3.3b), is achieved with a 4076 CMOS latch, while the readily available, low cost 2716 EPROM is used for the look-up table.

3.2.2 SIN and COS Look-Up Tables

It has been pointed out that the TFM frequency pulse is basically infinite in duration. This fact implies that the I and Q channel signals at any moment depend on an infinitely long data sequence, and hence an infinite memory space is required. For any practical implementation, the frequency pulse must be truncated to a finite interval, LT . It has been mentioned that the

memory size for this case is in the order of 2^L . It is also known that the spectral efficiency of TFM is strongly dependent on L , [8]. From the results reported in [8], it is clear that for $L=5$, the sidelobe levels of the modulated signal are at least 50 dB below the unmodulated carrier. Considering $L=5$, the memory size needed for the SIN and COS Look-Up Tables is 2K bytes, if a sampling frequency 8 times the bit rate is considered. Therefore, truncating the system frequency pulse to $5T$ was found to be a good compromise between the system's cost and spectral efficiency. Under this condition, the system design equations can be written as follows;

$$\phi(t, \underline{a}) = \phi(nT) + \sum_{i=n-2}^{n+2} a^i q(t-iT) \quad nT < t < (n+1)T \quad (3.12)$$

$$q(t) = \int_{-\frac{5T}{2}}^t h_m(\tau) d\tau \quad (3.13)$$

$$h_m(\tau) = \begin{cases} K_0 \int_{-\infty}^{\infty} \cos \frac{2\omega\tau}{T} \cdot \frac{\frac{\omega T}{2}}{\sin \frac{\omega T}{2}} e^{j\omega\tau} d\omega & |\tau| < \frac{5T}{2} \\ 0 & \text{otherwise} \end{cases} \quad (3.14)$$

and K_0 is chosen such that

$$K_0 \int_{-\frac{5T}{2}}^{\frac{5T}{2}} h_m(\tau) d\tau = 4 \quad (3.15)$$

The last constraint must be met in order for the modulation index to be 1/2. The phase function can easily be computed, using Eqns. (3.12-3.15), for all possible combinations of phase states and data vector permutations. A computer program has been developed for this task. The program is implemented according to the following strategy:

1. First, the system phase state is assigned a value 0. The phase function is computed, and stored, for all possible permutations of a 5-bit data sequence.
2. Given the phase function, the SIN and COS functions are computed and quantized into 8-bit words.
3. The system phase state is incremented by $\frac{\pi}{4}$ and the same procedure is repeated until all possible phase states are accounted for.

Fig. (3.9) shows the phase trajectories generated for an initial phase state 0. It is noticed that although there are 32 trajectories, there are only 5 end points,

Fig(3.9): Phase Trajectories Corresponding to 32 Different Permutations of a 5-bit Long Data Sequence

corresponding to phase changes of 0 , $\pm\frac{\pi}{4}$ and $\pm\frac{\pi}{2}$ as expected. A flow chart of the program is shown in Fig. (3.10). The program output is stored in two data files, for SIN and COS functions, in Hexadecimal format, and the data structure in these files is illustrated in Fig. (3.11). Each of these files contains 2048 bytes of data. These files were loaded into two 2716 EPROM's (via a DATA I/O PROM programmer) which constitute the system SIN and COS Lock-Up Tables.

3.2.3 I And Q Low Pass Filters

The design goals of the I and Q channel filters are as follows:

1. Complete suppression of the higher frequency replicas of the baseband signals.
2. Flat magnitude response over the baseband signal bandwidth.
3. Linear phase response over the same band.
4. The I and Q channels must be identical in performance.

If these requirements are not met, the modulated signal envelope cannot be constant. No attempt has been made to analyze the effect of any filters' mismatch, on the modulated signal envelope. Instead, the basic filter

Fig(3.10): Flow Chart of Program to Generate the Contents of the Look-up Tables of $\sin\phi(t, \alpha)$ and $\cos\phi(t, \alpha)$

Fig(3.11): The Organisation of the SIN and COS files
 $t_j = j/f_s, j = 1 \dots f_s T$
 a_i is a data vector identical to the binary representation of i
 each word in the file is accessible by three entries; phase state, data vector a_i , and the sample count

parameters were made tunable, to allow for experimental trimming, if needed.

A sixth-order Butterworth filter, with a cut-off frequency of 64 KHz (4 times the bit rate) was found to approximate the linear phase requirement. The filter gives sufficient rejection (about 36 dB) to the higher frequency components, due to the fact that the nearest undesired component is one octave above the filter cut-off frequency. The filters were designed according to the theory presented in [17]. The LF356 operational amplifier was used for this purpose due to its low offset, and relatively high gain at 64 KHz. The measured magnitude and phase response is shown in Fig. (3.12).

3.2.4 Multipliers

The multiplication by $\cos(\omega_c t)$ and $\sin(\omega_c t)$, can easily be accomplished for the case of square wave carriers. This is achieved using the 4066 CMOS analogue switch and the LF356 operational amplifier, as shown in Fig. (3.13). Since the carrier is a square wave, the modulator output will contain replicas of the signal spectrum at the harmonics of f_c . These are easily suppressed using a 2nd order low pass filter.

Fig(3.12): Measured Magnitude and Phase Response of I and Q Arm Filters

Fig(3.13): A Circuit for the Multiplication of the
I or Q Baseband Signal by a Square Wave carrier

3.3 EXPERIMENTAL RESULTS

Following the strategy outlined in this chapter, a PC modulator board has been fabricated. The digital section consists of commercially available CMOS integrated circuits, in addition to 3 EPROMS. The analogue section consists basically of the LF356 op. amp. in addition to two AD561SD DACs. The board contains 42 ICs and about 35 passive elements (high tolerance). The data clock, and the IF carrier are on board, and can be bypassed if different frequencies are desired. The board is powered from a $\pm 5V$ dual supply for the analogue section, and a $+5V$ power supply for the digital section. The total power consumption (measured) is 2.27 Watts. Fig. (3.14) shows the I and Q channel baseband signals, before the arm filters, in addition to the random data sequence used for the generation of these waveforms. Close inspection of the waveforms reveals that they are a delayed version of what might be expected if the data sequence is operated on with the system design equations. This is due to the fact that the input data is in fact delayed by 2 clock cycles on the modulator board. Fig. (3.15) shows the I and Q channel eye diagrams, obtained before and after low pass filtering. A $(2^{10}-1)$ bit PRS sequence was used in the generation of these patterns. The flat portions at the 0, ± 1 and ± 0.707 levels are due to the fact that the two samples adjacent to the data transition instants are

Fig. 14. (a) Measured I and Q Baseband Signals
using a 7-Bit Long Data Sequence

(b) Measured Q Channel Baseband Signal,
and the Data Sequence Used

(b)

Fig(3.15): Measured I_{afn} Eye Diagrams
 (a) Before Low Pass Filters
 (b) After Low Pass Filters

further by using narrower low pass filters' bandwidths (which would lead to some jitter in the zero crossings), or by using a larger number of samples per symbol interval. In this case the required memory size will be doubled, and the 2716 EPROM can be replaced by the 2732 EPROM.

Fig. (3.16) shows the output of the I and Q multipliers, for an all '1' data sequence. Fig. (3.17) shows the measured spectral density of the I channel baseband signal following the low pass filter. It was found that the measured and computed spectral densities are in close agreement, for frequencies $< f_b$. The measured sidelobes are relatively higher than predicted; this is partially due to the quantization of the baseband signal. A note should be made of the discrete line at f_b . This discrete tone is believed to be due to the sampling of the I and Q baseband signals, and could be suppressed completely if narrower LPF bandwidth were used. The side-lobe levels is at least 45 dB below the carrier. However, additional filtering may be needed for the signal to fit under the mask of Fig. (1.1). This can be achieved by using a 2nd order BPF, and does not result in any noticeable envelope fluctuations. A photograph of the modulator board is shown in Fig. (3.18).

Fig. 1. The output of the 1 and 2 laser. Multi-layers, generated with 1000 repetitions.

Fig(3.17): Measured Spectral Density of the I Channel Baseband Signal

CHAPTER IV

CARRIER SYNCHRONIZATION

4.0 INTRODUCTION

Successful transmission of information through a phase coherent communication system requires, by definition, a receiver capable of determining or estimating the phase and frequency of the received signal carrier with as little error as possible. Quite often the data bearing signal is modulated on an RF carrier in such a way that a residual component at the RF frequency exists in the overall signal power spectrum. This component can be tracked with a narrow band phase locked loop and used to provide the desired reference signal. On the otherhand, the power contained in this component does not convey any information other than the frequency and phase of the carrier. Thus, it represents power not available for the transmission of data, and in practice, techniques which conserve power are always of interest. More specifically, the data is often angle modulated on a subcarrier which, in the absence of a d.c. component in the signal spectrum, requires some type of suppressed carrier-tracking loop for establishing the coherent

2

digital modulation techniques including TFM and MSK.

Since conventional PLLs requires a discrete spectral component at the frequency to be tracked, they cannot be used directly to provide a coherent reference. However, a conventional PLL can be used if the modulated signal is operated upon with a nonlinear device, which could generate a discrete component at some frequency related to the carrier in a known way.

4.1 SELF-SYNCHRONIZATION PROPERTIES OF TFM

The Class of Partial Response CPM with modulation index $1/2$, including TFM, has the property that the squared signal power spectrum contains two discrete lines at $(2f_c \pm f_b/2)$, where f_c is the carrier frequency and f_b is the data rate. The energy contained in these discrete components is strongly dependent on the parameters involved in the modulation process, such as the duration and shape of the premodulation filter impulse response. The energy in the discrete tones, relative to the height of the continuous component of the spectrum, at $(2f_c \pm f_b/2)$, falls rapidly with the duration of the system frequency pulse. Unfortunately, there is no easy way to find the power spectral density of the class of CPM signals which use Partial Response Signalling, and all the reported techniques involve heavy numerical computations.

The method adapted in our analysis is due to Rowe and Prabhu, [18], and is summarized in the following section. We then present the numerical results obtained for TFM, with the frequency pulse duration as a parameter, and for MSK. The results presented show the effect of two types of nonlinearities on the modulated signal power spectrum; that of a frequency doubler and that of a frequency quadrupler.

4.1.1 Power Spectra of CPM Signals

The results reported by Rowe and Prabhu [18], using their notations, is summarized as follows;

The CPM signal is given by;

$$x(t) = \cos \{2\pi f_c t + \phi(t)\}$$

$$\phi(t) = \int_0^t f_d(u) du$$

$$f_d(t) = \sum_k h_{s_k}(t-kT) \quad s_k = 1, 2, \dots, M \quad (4.1)$$

where f_c is the carrier frequency, $\phi(t)$ is the signal phase, and f_d is the frequency deviation of the carrier. The signalling alphabet consists of M waveforms (frequency pulses) $h_1, h_2, \dots, h_M$; one of these is transmitted for each signalling interval. The frequency pulses are time limited to the interval $L_k T < t < U_k T$. The CPM

signal can also be written as;

$$\begin{aligned} x(t) &= R_e (e^{j2\pi f_c t} v(t)) \\ v(t) &= e^{j\phi(t)} \end{aligned} \quad (4.2)$$

The authors have proved that the power spectrum is given by;

$$P_x(f) = \frac{1}{2} P_v(f-f_c) + \frac{1}{2} P_v(-f-f_c) \quad (4.2a)$$

The first term to the right is the spectrum of the complex baseband wave, $v(t)$, shifted to the carrier frequency $+f_c$, while the second term is the spectrum of $v(t)$ shifted to $-f_c$. Rowe and Prabhu defined $\underline{p}$ as a row vector containing the probabilities of different data symbols, and $\underline{w}_d$ as a diagonal matrix with it's diagonal containing the same probabilities. $r(t)$ was defined as,

$$r(t) = \begin{cases} \pi_x \begin{bmatrix} \frac{k-1}{2} \\ i=-\frac{k-1}{2} \end{bmatrix} q(t-iT) & ; k \text{ odd} \\ \pi_x \begin{bmatrix} k/2 \\ i=-\frac{k-2}{2} \end{bmatrix} q(t-iT) & ; k \text{ even} \end{cases} \quad (4.3)$$

KT is the length of each frequency pulse, Π_x denotes Kronecker matrix multiplication, and $\underline{q}(t) = q(t)$ where,

$$q(t) = \begin{cases} \left\{ e^{j\int_{L_k T}^t h_1(u) du} \dots e^{j\int_{L_k T}^t h_M(u) du} \right\} & ; L_k T \leq t < U_k T \\ 0 & ; \text{otherwise} \end{cases} \quad (4.4)$$

It was found in [18] that the spectral density, $P_x(f)$, contains discrete tones if and only if

$$| \underline{W}_k q(U_k) | = 1 \quad (4.5)$$

This condition has a simple physical interpretation. Every frequency pulse must introduce the same total phase shift (modulo 2π) in the carrier in order to have line components. If that phase change is equal to $2\pi f_1$, then the line components appear at frequencies $[f_c + (f_1 + n)/T]$, $n = -1, 0, 1, \dots$. Under these conditions, the spectral density contains a line component $P_L(f)$ and a continuous component $P_C(f)$ where;

$$P_{UL}(f) = \frac{1}{T^2} | \underline{W}^k \cdot R(f) |^2 \sum_{n=-\infty}^{\infty} \delta(f - \frac{n+f_1}{T}) \quad (4.6)$$

$$\begin{aligned}
 P_{UC}(f) = & \frac{1}{T} \underline{R}(f) \cdot \left\{ (W_d^K - W)^k \cdot W^k \right. \\
 & + \sum_{n=1}^{k-1} \left(W^n \times W_d^{k-n} \times W^n - W^k \cdot W^k \right) e^{-j2\pi n(fT-f)} \\
 & + \sum_{n=1}^{k-1} \left(W^n \times W_d^{k-n} \times W^n - W^k \cdot W^k \right) e^{-j2\pi n(fT-f)} \left. \right\} \\
 & \cdot R^*(f) \Big]
 \end{aligned}
 \tag{4.7}$$

Here $\times$ denotes the Kronecker product, the exponents in brackets indicate multiple Kronecker product and " $\cdot$ " denotes ordinary matrix multiplication.

It has been reported that [18], if $|\underline{W} \cdot q(u_k)| < 1$, then the power spectrum does not contain any discrete tones, and is given by

$$P_U(f) = \frac{1}{T} \underline{R}(f) \cdot (A + A^+) \cdot R^*(f) \tag{4.8}$$

where

$$\begin{aligned}
 A = & \frac{1}{T} W_d^K + \sum_{n=1}^{k-1} e^{-j2\pi n f T} \left\{ \underline{q}(u_k) \cdot W_d^n \right\} \times W_d^{k-n} \times W^n \\
 & + \frac{\left\{ e^{-j2\pi f T W} \cdot \underline{q}(u_k) \cdot W_d \right\}^k}{1 - e^{-j2\pi f T W} \cdot \underline{q}(u_k)}
 \end{aligned}
 \tag{4.9}$$

and " $+$ " denotes the Hermitian matrix transpose.

4.1.2 Spectra of Frequency-Doubled And Frequency-Quadrupled TFM Signal

We can see that for a CPM signal, with modulation index $1/2$, which uses equally likely binary data symbols;

$$|w \cdot q(u_k)| = \left| \frac{1}{2} \cdot \begin{bmatrix} e^{j\pi/2} \\ e^{-j\pi/2} \end{bmatrix} \right| = 0$$

Therefore, the constraint given by Eqn. (4.5) is not satisfied, and the power spectrum does not contain any discrete tones. By doubling the modulated signal frequency, the modulation index is also doubled. As a result, the change in the modulated carrier phase due to one frequency pulse will be π (modulo 2π), and f_1 becomes $1/2$. Consequently the power spectrum contains two discrete lines at $(2f_c \pm f_b/2)$. Similarly, by quadrupling the modulated signal frequency, the phase change, due to the two possible data symbols, becomes 0 (modulo 2π). Therefore, $f_1 = 0$ and the spectrum contains discrete lines at $4f_c$ and $(4f_c \pm f_b)$.

A computer program has been developed to carry out the computations of Eqn. (4.7-4.9). The program is designed to work with any CPM system, upon defining the frequency pulse and the system modulation index. The program starts by testing the condition given by Eqn. (4.5). If the condition is satisfied, the program carries out the computations of Eqn. (4.6) and Eqn. (4.7);

otherwise, the computations of Eqn. (4.8) and (4.9) are carried out. A noteworthy point in the program is the calculations of $R(f)$. If the frequency pulse width is KT , then the vector $R(f)$ involves the computation of the continuous Fourier transform of 2^k complex, time limited, baseband functions for each point in the spectral density. Therefore, the computation time of $R(f)$ is an exponential function of K , and becomes impractical for values of $K > 3$. The FFT algorithm is preferred for this purpose, and by sampling the baseband vector $r(t)$ at a rate of $64 f_b$ the program yields very reasonable accuracy. For example, the error at the carrier frequency was found to be less than 0.01 dB, and increases to 1.6 dB at a normalized frequency $(f-f_b)T=2$. By reducing the sampling frequency to $16f_b$ the error increased beyond 4 dB. This is due to the aliasing effect of the DFT.

The spectrum of a frequency doubled TFM signal, with the frequency pulse duration as a parameter, is shown in Fig. (4.1). The spectrum of a frequency doubled MSK signal is also shown for comparison. The energy in the line components at $\{2f_c + (n + \frac{1}{2})f_b\}$ are given in Table (4.1) for $n = +1, -1$. Table 4.1 also shows the quantity "SIR" which represent the ratio of the energy of the line components at $(2f_c \pm f_b/2)$ relative to the height of the continuous part of the spectrum at the same frequency. The parameter SIR plays an important role in the recovered carrier phase jitter when the input SNR is high. It is obvious that SIR decreases as the frequency pulse duration is increased.

L	1	3	5
SIR	1.07	0.507	0.495

Table(4.1): SIR for Frequency-Doubled Signal

Fig(4.1): Spectral Density of a Frequency Doubled TFM Signal

It is also evident that SIR for TFM is always smaller than SIR of MSK. We notice also that the strongest line components occur at $(2f_c \pm f_b/2)$, and can be utilized in the clock and the carrier recovery.

The same computations have been repeated for a frequency-quadrupled TFM and MSK signals, with the results shown in Fig. (4.2) and table (4.II). Note that the strongest spectral component, in the TFM case, occur at $4f_c$. The presence of this component is explained by the fact that the phase of a TFM signal stays almost constant for 25% of the time, at the levels $\pm\pi/4$ (modulo 2π). quadrupling the signal frequency, the $\pm\pi/4$ phases are transformed into $\pm\pi$ such that the resulting signal phase will always be equal to π for 25% of the time, while the signal frequency remains constant and equal to $4f_c$. This component can be utilized to recover the system's carrier phase and frequency.

4.2 DIFFERENT APPROACHES TO THE CARRIER RECOVERY

In this section we discuss three different approaches to the carrier synchronization problem. The first approach is identical to that suggested by de Buda [2], the second is aided by the data clock, while the third utilizes a fourth power loop.

L	1	3	5
SIR	—	0.980	0.962

Table(4.II): SIR for the Frequency
Quadrupled TFM Signal

Fig(4.2): Power Spectrum of a Frequency-Quadrupled Signal

4.2.1 Squaring Loop

In this scheme, the signal frequency is doubled, using a square law device. The squared signal spectrum contains two discrete lines at $(2f_c + f_b/2)$ and $(2f_c - f_b/2)$, which can be tracked using two narrow-band PLL's. The outputs of the PLL's are mixed to generate the sum and the difference of the two output frequencies $(2f_c + f_b/2)$. The sum yields $4f_c$, which can be separated, using a high pass filter, and divided by 4 to yield the carrier frequency. The difference yields f_b , which is easily low pass filtered, and yields the clock. This approach is illustrated in Fig. (4.3).

In spite of the hardware simplicity of this scheme, it has the following disadvantages [19];

1. Since the centre frequencies of the two PLL's are almost the same, both loops have the same bandwidth. As a consequence, the bandwidths of the carrier and clock phase estimates are exactly the same.
2. The second disadvantage is the difficulty of acquiring signals with relatively large frequency uncertainty. If the loops are to be swept, then particular care must be taken to assure that the two PLL's do not lock to the same tone.

3. There is a problem with signal drop-out with this implementation. This arises under certain data sequences and results in the transmission of only one tone. When this occurs, one or the other loop loses lock and returns to its free-running frequency, thus producing both carrier and clock phase errors.

These disadvantages are expected to be severe in the TFM case due to the relative weakness of the discrete tones at $(2f_c + f_b/2)$.

4.2.2 Clock-Aided Carrier Recovery

This technique is also based on the presence of the discrete tones at $(2f_c + f_b/2)$, and takes care of the disadvantages that the implementation of Fig. (4.3) suffers from. A functional diagram of this approach is shown in Fig. (4.4), and its operation is as follows: The received modulated signal is first squared, and then multiplied by the data clock at frequency $f_b/2$, thus generating a spectral line at $2f_c$, in addition to two tones at $(2f_c + f_b)$. The discrete tone at $2f_c$ contains about 12.5% of the signal energy.

The $2f_c$ tone is separated using a narrow PLL, and divided by 2 thus yielding the carrier frequency. This implementation has the following advantages;

Fig(4.3): de Buda's Carrier and Timing Recovery Scheme

Fig(4.4): Clock-Aided Carrier Recovery Scheme

1. It allows the clock recovery loop to use a centre frequency of f_b , and thus makes it possible to use a very narrow tracking bandwidth.
2. Introducing any frequency acquisition aid such as loop sweeping to help the loop acquire large frequency offsets is not endangered by false locking to unwanted sidebands. This is due to the fact that the nearest line components to $2f_c$ are at $(2f_c \pm f_b)$, in addition to their relative weakness.
3. The spectral line at $2f_c$ exists under the all 1 and all 0 sequences. This line disappears only under an alternating sequence (1010.....). Therefore, the probability of signal dropout is reduced by 50% as compared to the implementation of Fig. (4.3). This approach to the carrier recovery is analyzed in section (4.3), and evaluated experimentally in section (4.4).

4.2.3 Fourth Power Loop

In this approach, the signal frequency is first quadrupled using a 4th order nonlinearity. The output of the nonlinearity contains a discrete tone at $4f_c$, which contains about 25% of the signal energy. This line can be

Fig(4.5): I-Q Implementation of Clock-Aided Carrier Recovery Loop

Fig(4.6): An I-Q Implementation of the Fourth Power Loop

tracked using a narrowband PLL, and divided by 4 to yield the carrier frequency. An I-Q implementation of this loop may follow the block diagram shown in Fig. (4.6). This loop is investigated in detail in section (4.5).

4.3 ANALYSIS OF THE CLOCK-AIDED CARRIER TRACKING LOOP

In this section, we first derive an expression for the recovered carrier phase jitter, and then analyze the loop acquisition performance, in the presence of large frequency offsets.

4.3.1 Loop Signal-To-Noise Ratio

We assume that the clock recovery loop is in perfect lock. The input signal to the carrier recovery loop, Fig. (4.5), is assumed to be corrupted by additive white Gaussian noise (AWGN) of one-sided spectral density N_0 . The signal plus noise is first bandpass filtered with an ideal bandpass filter of bandwidth $2B$. The filter is assumed to introduce no distortion to the signal component. Therefore, the loop input can be written as;

$$r(t) = S(t, \underline{a}) + n(t) \quad (4.10)$$

where,

$$S(t, \underline{a}) = \sqrt{\frac{2E_B}{T}} \cos(\omega_c t + \phi(t, \underline{a})) \quad (4.11)$$

where ω_c is the carrier radian frequency, $1/T$ is the data rate, $\phi(t, \underline{a})$ is the information-carrying phase and E_B is the signal energy per bit. Also, the noise component can be written as;

$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t \quad (4.12)$$

where $n_c(t)$ and $n_s(t)$ are low-pass Gaussian noise processes which are statistically independent, each having a single-sided density N_0 , [23]. Therefore, the input signal plus noise can be rewritten as;

$$r(t) = \sqrt{\frac{2E_B}{T}} \{ \cos \phi(t, \underline{a}) + n_c(t) \} \cos \omega_c t - \sqrt{\frac{2E_B}{T}} \{ \sin \phi(t, \underline{a}) + n_s(t) \} \sin \omega_c t \quad (4.13a)$$

$$= \{ I(t) + n_c(t) \} \cos \omega_c t - \{ Q(t) + n_s(t) \} \sin \omega_c t$$

The outputs of the I and Q multipliers contain a component centered at double the carrier frequency, which is suppressed by the arm filters, in addition to the baseband components.

The inputs to multiplier M_1 (Fig. 4.5) are given by;

$$r_i(t) = \frac{1}{2} (I(t) + n_c(t))$$

$$r_q(t) = \frac{1}{2} (Q(t) + n_s(t))$$

(4.13b)

The output of multiplier M_1 is given by;

$$r_{m1}(t) = \frac{1}{4} (I(t)Q(t) + n_c(t)Q(t) + n_s(t)I(t) + n_c(t)n_s(t)) \quad (4.14)$$

Therefore, the input to the loop filter (apart from a scaling factor of 1/4) becomes;

$$e(t) =$$

$$(I(t)Q(t) + n_c(t)Q(t) + n_s(t)I(t) + n_c(t)n_s(t)) \cos \frac{\omega_b t}{2} \quad (4.15)$$

To find the signal-to-noise ratio within the loop bandwidth we first compute the spectral density of $e(t)$.

To do this, we compute the autocorrelation function of $e(t)$, and take its Fourier transform. By assuming that the loop bandwidth is much smaller than the data rate, the loop SNR is obtained by multiplying the loop noise bandwidth by the spectral density at d.c.

The autocorrelation function, $R_e(t, t+\tau)$, is given by;

$$R_e(t, t+\tau) = E(e(t)e(t+\tau)) =$$

$$\begin{aligned} & \left[E(I(t)I(t+\tau)Q(t)Q(t+\tau)) + E(I(t)Q(t)Q(t+\tau)) \cdot E(n_c(t+\tau)) \right. \\ & + E(I(t)Q(t)I(t+\tau)) \cdot E(n_s(t+\tau)) + E(I(t)Q(t)) \cdot E(n_c(t+\tau) \cdot n_s(t+\tau)) \\ & + E(I(t+\tau)Q(t+\tau)Q(t)) \cdot E(n_c(t)) + E(Q(t+\tau)Q(t)) \cdot E(n_c(t) \cdot n_c(t+\tau)) \\ & + E(I(t+\tau) \cdot Q(t)) \cdot E(n_c(t) \cdot n_s(t+\tau)) + E(Q(t)) \cdot E(n_c(t) \cdot n_s(t+\tau) \cdot n_c(t-\tau)) \\ & + E(I(t)I(t+\tau)Q(t+\tau)) \cdot E(n_s(t)) + E(I(t)Q(t+\tau)) \cdot E(n_s(t) \cdot n_c(t+\tau)) \\ & + E(I(t)I(t+\tau)) \cdot E(n_s(t) \cdot n_s(t+\tau)) + E(I(t)) \cdot E(n_s(t) \cdot n_s(t+\tau) \cdot n_c(t+\tau)) \\ & + E(I(t+\tau)Q(t+\tau)) \cdot E(n_c(t) \cdot n_s(t)) + E(Q(t+\tau)) \cdot E(n_c(t) \cdot n_c(t+\tau) \cdot n_s(t)) \\ & \left. + E(I(t+\tau)) \cdot E(n_c(t) \cdot n_s(t) \cdot n_s(t+\tau)) + E(n_c(t) \cdot n_c(t+\tau) \cdot n_s(t) \cdot n_s(t+\tau)) \right] \\ & \cdot \cos \frac{\omega_b \tau}{2} \end{aligned} \quad (4.16)$$

In Eqn. (4.16) we have assumed that the noise and signal are statistically independent. Since $n_c(t)$ and $n_s(t)$ are zero mean, and statistically independent, Eqn. (4.16) is simplified to,

$$\begin{aligned} R_e(t, t+\tau) = \cos \frac{\omega_b \tau}{2} & \left[E(I(t)Q(t)I(t+\tau)Q(t+\tau)) + \right. \\ & E(I(t)I(t+\tau)) \cdot E(n_s(t) \cdot n_s(t+\tau)) + E(Q(t)Q(t+\tau)) \cdot E(n_c(t) \cdot n_c(t+\tau)) \\ & \left. + E(n_c(t) \cdot n_c(t+\tau)) \cdot E(n_s(t) \cdot n_s(t+\tau)) \right] \end{aligned}$$

Therefore;

$$R_e(\tau) = \{ R_{ss}(\tau) + R_{ns}(\tau) + R_{nn}(\tau) \} \cos \frac{\omega_b \tau}{2} \quad (4.17b)$$

where $R_{ss}(\tau)$ is due to the multiplication of the I and Q baseband signal components, $R_{ns}(\tau)$ is due to the multiplication of signal by noise while $R_{nn}(\tau)$ is due to the multiplication of the low pass noise components $n_c(t)$ and $n_s(t)$.

Signal X Signal Term

Using (4.13a), $R_{ss}(\tau)$ can be written as;

$$\begin{aligned} R_{ss}(\tau) &= E\{I(t)Q(t) \cdot I(t+\tau)Q(t+\tau)\} \\ &= \left(\sqrt{\frac{2E_B}{T}}\right)^4 \left[E\{ \sin\phi(t, \underline{a}) \cos\phi(t, \underline{a}) \cdot \sin\phi(t+\tau, \underline{a}) \cdot \cos\phi(t+\tau, \underline{a}) \} \right] \\ &= \frac{E_B^2}{T^2} E\{ \sin 2\phi(t, \underline{a}) \sin 2\phi(t+\tau, \underline{a}) \} \quad (4.18) \end{aligned}$$

The right hand side of Eqn. (4.18) is recognized as the autocorrelation function of the frequency doubled signal, $s(t, \underline{a})$. Therefore, the spectral density $\phi_{ss}(f)$ contains two discrete lines at

$(2f_c + f_b/2) \cdot \phi_{ss}(f)$ and can be written as follows:

$$\phi_{ss}(f) = \frac{E_B^2}{T^2} \left\{ P_L \cdot \delta(f_b/2) + P_L \cdot \delta(-f_b/2) + \phi_{ss}^c(f) \right\} \quad (4.19)$$

Where P_L is the energy contained in each line component, and $\phi_{ss}^c(f)$ is the continuous part of the spectrum.

The contribution of this term to the spectral density of $e(t)$ (the loop filter input), at d.c. is given by;

$$\begin{aligned} \phi_e^{ss}(0) &= 4\phi_{ss}(f_b/2) + 4\phi_{ss}(-f_b/2) \\ &= \frac{E_B^2}{T^2} \left\{ P_L \cdot \delta(0) + \phi_{ss}^c(f_b/2) \right\} \end{aligned} \quad (4.20)$$

The first term to the right represents the useful signal component, while the second term is a "self-noise" component due to modulation. The computed $\phi_e^{ss}(f)$ is shown in Fig. (4.7).

Signal X Noise Term

Using Eqn. (4.13a), $R_{ns}(\tau)$ can be written as;

$$R_{ns}(\tau) = \frac{2E_B}{T} \{ R_n^c(\tau) \cdot R_L(\tau) + R_n^s(\tau) \cdot R_Q(\tau) \} \quad (4.21a)$$

Fig(4.7): Spectral Density at input of Loop Filter Due to the Signal X Signal Term

Where $R_n^C(\tau)$, $R_i(\tau)$, $R_n^S(\tau)$ and $R_q(\tau)$ are the autocorrelation functions of $n_c(t)$, $I(t)$, $n_s(t)$ and $Q(t)$ respectively. Therefore;

$$\begin{aligned}\phi_{ns}(f) &= \frac{2E_B}{T} \left\{ \phi_n^S(f) \cdot \phi_i(f) + \phi_n^C(f) \cdot \phi_q(f) \right\} \\ &= \frac{E_B N_0}{T} \cdot 2 \int_{-B+f}^{B+f} \phi_i(f) df\end{aligned}\quad (4.21b)$$

where $\phi_i(f)$ is the power spectral density of the I channel baseband signal with a unit amplitude. The contribution of this term to the spectral density of $e(t)$, at d.c, is given by;

$$\begin{aligned}\phi_e^{ns}(0) &= \frac{1}{2} \phi_{ns}(f_b/2) + \frac{1}{2} \phi_{ns}(-f_b/2) \\ &= 2 \frac{E_B N_0}{T} \cdot I_{ns}\end{aligned}\quad (4.22)$$

where

$$I_{ns} = \int_{-B+f_b/2}^{B+f_b/2} \phi_i(f) df \quad (4.23)$$

Noise X Noise Term

$$R_{nn}(\tau) = R_n^C(\tau) \cdot R_n^S(\tau) \quad (4.24a)$$

Therefore,

$$\phi_{nn}(f) = \phi_n^C(f) \cdot \phi_n^S(f) \quad (4.24b)$$

Since $n_c(t)$ and $n_s(t)$ are low pass noise processes with double-sided bandwidth $2B$, and density $N_0/2$, $\phi_{nn}(f)$ becomes as shown in Fig. (4.8). Therefore,

$$\begin{aligned}\phi_{nn}^{nn}(0) &= 4\phi_{nn}(f_b/2) + 4\phi_{nn}(-f_b/2) \\ &= \frac{N_0^2}{4} (2B - \frac{f_b}{2})\end{aligned}\quad (4.25)$$

From Eqn. (4.20), (4.22) and (4.25), the noise power, P_n , within the loop bandwidth (B_n) is given by;

$$P_n = \left[\frac{E_B^2}{T^2} \phi_{ss}^c(f) + 2 \cdot \frac{E_B N_0}{T} I_{ns} + \frac{N_0^2}{4} (2B - \frac{f_b}{2}) \right] B_n \quad (4.27a)$$

while the useful signal power is given by;

$$P_s = \frac{E_B^2}{T^2} P_i \quad (4.27b)$$

Therefore, the loop signal-to-noise ratio is given by;

$$\begin{aligned}SNR_l &= \frac{P_s}{P_n} = \frac{P_i \frac{E_B^2}{T^2}}{\left[\frac{E_B^2}{T^2} \phi_{ss}^c(f_b/2) + \frac{2}{T} E_B N_0 I_{ns} + \frac{N_0^2}{4} (2B - \frac{f_b}{2}) \right] B_n} \\ &= \frac{1}{\left[SIR^{-1} + 2 \frac{N_0}{E_B} \left(\frac{I_{ns}}{P_i} \right) + \frac{1}{8P_i} \frac{N_0^2}{E_B^2} (4BT - 1) \right] B_n T}\end{aligned}\quad (4.27c)$$

Fig(4.8): Spectral Density Due to X Noise Component

where

$$SIR = \frac{T \cdot P_t}{\phi_{ss}^C(f_b/2)}$$

SIR is given in Table (4.1) for different durations of the TFM frequency pulse. The factor SIR^{-1} in Eqn. (4.27c) is due to the self noise component, and sets an upper bound on the loop signal to noise ratio. Using $P_t=12.5\%$, and $SIR=0.495$ and assuming that $B=1/T$, Eqn. (4.27c) can be rewritten as,

$$SNR = \frac{1}{\left[2.02 + 15.2 \frac{N_0}{E_B} + 3 \frac{N_0^2}{E_B^2} \right] B_n T} \quad (4.28)$$

Fig. (4.9) shows the loop SNR as a function of E_B/N_0 , with the IF filter bandwidth as a parameter. It is noticed that the filter bandwidth has no significant effect on the loop SNR for values of E_B/N_0 higher than 6 dB. It is also noticed that as E_B/N_0 changes from - to 15 dB, the change in SNR_L is approximately 2 dB. Therefore, we can conclude that the loop SNR is practically determined by the self noise component for relatively high E_B/N_0 ratios.

The received signal phase, at the data transition instants, is an odd multiple of 45° with probability 50%. Any uncertainty in the carrier phase at these moments affects the sampling noise margin. For example, a carrier phase error of 5° results in 0.5 dB degradation in the

Fig(4.9): Loop SNR as a Function of $\frac{E_B}{N_0}$ with the IF Bandwidth as a Parameter
 $B_n T = 0.0085$

system SNR. To maintain the carrier phase error below 5° most of the time, a loop noise bandwidth of approximately $0.0085/T$ is required. At a bit rate of 16 Kb/s, the loop bandwidth is approximately equal to 136 Hz.

4.3.2 The Loop Acquisition Performance

The expected worst case frequency uncertainty is approximately 1.5 kHz which is much higher than the required loop bandwidth (136 Hz). Therefore, the carrier recovery loop of Fig. (4.5) cannot acquire this offset in a reasonable time, and it becomes necessary to provide the loop with a frequency acquisition aid, to enable the loop achieve lock quickly, without degrading the recovered carrier spectral purity after the loop has achieved lock.

There are methods to speed the loop acquisition process [21]. These are;

1. Loop sweeping: In this technique the VCO frequency is swept linearly in search for the received signal frequency, and once the two frequencies coincide the sweeping mechanism is disabled. For the successful implementation of this process the loop must be provided with a lock detector, the output of which is used to control the start and stop points of the sweeping mechanism. Otherwise, the sweep circuit could carry off the VCO frequency in

the event of a fast fade, and the entire sweep range would have to be searched before the signal could be acquired. Lock indicators are well known for sinusoidal signals [21].

However, if the loop input is a modulated signal, then the lock indicators could differ according to the statistical properties of the signal. Even if these detectors are determined, their output must be averaged over a relatively long period of time before a reliable indication is obtained. These factors make the loop sweeping undesirable for our application.

2. Wide-bandwidth methods: The speed of acquisition can be improved by widening the loop bandwidth. A loop can be built to have a wide bandwidth for rapid acquisition, and a much narrower bandwidth for good tracking in the presence of noise. The increase of the loop bandwidth can be successful only if the input signal-to-noise ratio is sufficiently large. If the change in bandwidth brings the loop close to threshold, acquisition becomes unlikely [21].
3. Discrimination-aided acquisition: A frequency discriminator can be used in a conventional AFC loop to bring the VCO frequency close to that of the input signal. Phase locking

occurs when the frequency error is brought within the loop lock limit. This technique is successful only for input SNR higher than 6 dB. After locking has occurred, the AFC path adds to the loop damping. It is possible to leave the AFC path active permanently, and merely weigh the relative contributions of the phase and frequency detectors so as to obtain the desired damping.

In this study, the third technique has been chosen to aid in the loop acquisition, due to the following reasons;

- a) For input SNR less than 6 dB, the system's BER increases rapidly. Hence, trying to achieve loop acquisition would be of no practical value. Therefore, a frequency discriminator can be useful, although its practical value is limited for $\text{SNR} > 6\text{dB}$.
- b) There is no significant degradation in the recovered carrier phase jitter, in the tracking mode, that may be caused by the frequency discriminator.
- c) The AFC loop implementation can be made mostly digital, and can be used as a "frequency memory" for the loop in order to further speed up the loop reacquisition in case of signal loss.

A block diagram for the carrier recovery loop, Fig. (4.5), combined with an automatic frequency control loop is shown in Fig. (4.10). This loop operates as follows; the VCO control voltage is the sum of two components, one generated by the AFC loop, and the other is generated in the loop phase detectors. When the loop frequency error is many times the loop natural frequency, the outputs of the phase detectors are essentially zero and so is the loop phase control signal. At this stage, the VCO frequency is controlled by the AFC loop. The frequency discriminator output is proportional to the difference between the VCO frequency and the centroid of the input signals spectral density. This component drives the VCO's frequency closer to the input carrier. Once the loop frequency error is within the loop bandwidth, the phase detectors take over completing the loop phase acquisition.

The loop filter is designed to have high gain at d.c. The transfer function, $F_1(s)$, has a proportional and integral terms such that;

$$F_1(s) = \frac{\tau_2}{\tau_1} + \frac{1}{\tau_1 s} \quad (4.29)$$

The frequency comparator utilized in the AFC loop is the so called "Rotational Frequency Detector", [22]. Its output is a linear function of the loop frequency error if it is less than 25% of the carrier frequency. The AFC

Fig(4.10): Carrier Recovery Loop, Combined with an APC Path to Aid Acquisition

loop filter is an integrator, with the transfer function $F_2(s)$, where;

$$F_2(s) = \frac{1}{\tau_f s} \quad , \quad (4.30)$$

The loop acquisition performance is strongly dependent on the signal modulation. In fact the loop phase acquisition time can be relatively long if the data stream is random in nature. However, the loop acquisition can be made easier if each data burst is preceded by an alternating sequence (1010...). Under this condition, the TFM signal has a constant phase and frequency, f_c , and may be treated like a sinusoidal signal. For this reason, our treatment of the loop frequency and phase acquisition will be focused only on the case of sinusoidal inputs in the absence of noise.

Let the IF input signal be expressed as;

$$v_i(t) = V_s \cos(\omega_0 t + \lambda_i(t)) \quad (4.31)$$

where,

$$\lambda_i(t) = \Delta\omega t + \theta_i$$

and $\Delta\omega$ is the initial loop frequency error, and θ_i is the initial phase. The VCO output signal is given by;

$$v_o(t) = V_o \cos(\omega_o t + \lambda_o(t)) \quad (4.32)$$

where $\lambda_o(t)$ is the VCO instantaneous phase.

The outputs of the I and Q phase detectors contain a component centered at $(\omega_i + \omega_o)$, which is suppressed by the arm filters, in addition to a low frequency component at $(\omega_i - \omega_o)$. Therefore, the phase control signal is given by;

$$U_1 = \frac{V_s V_o}{2} k_p \sin 2(\lambda_i(t) - \lambda_o(t)) \quad (4.33)$$

Where $\sqrt{k_p}$ is the sensitivity of the I and Q phase detectors. The output of the loop frequency comparator is given by;

$$U_2(t) = k_f \left[\frac{d\lambda_i(t)}{dt} - \frac{d\lambda_o(t)}{dt} \right] \quad (4.34)$$

Where k_f is the sensitivity of the frequency comparator. Using Eqn. (4.31 - 4.34), the loop may be modeled as shown in Fig. (4.11). It is obvious that the VCO control voltage is given by;

$$U_3(t) = \frac{\tau_2}{\tau_1} U_1(t) + \frac{1}{\tau_1} \int_0^t U_1(\tau) d\tau + \frac{1}{\tau_f} \int_0^t U_2(\tau) d\tau \quad (4.35)$$

and the VCO output phase is related to $U_3(t)$ by

$$\frac{d}{dt} \lambda_o(t) = k_v U_3(t) \quad (4.36)$$

Fig(4.11): Analytical Model of Carrier Recovery Loop Combined with AFC

where k_v is the modulation sensitivity of the VCO. By combining equations (4.31)-(4.36) we arrive at the following differential equation which control the loop in its acquisition mode;

$$\ddot{\lambda}(t) + \left[2k \frac{\tau_2}{\tau_1} \cos 2\lambda(t) + \frac{k_f}{\tau_f} \right] \dot{\lambda}(t) + \frac{K}{\tau_1} \sin 2\lambda(t) = 0$$

where

$$\lambda(t) = \lambda_i(t) - \lambda_o(t), \quad \ddot{\lambda}(t) = \frac{d^2}{dt^2} \lambda(t)$$

$$\dot{\lambda}(t) = \frac{d\lambda}{dt} \lambda(t), \quad K = k_p k_v \text{ and } K_f = k_f k_v$$

(4.37)

By writing

$$\ddot{\lambda}(t) = \frac{d\dot{\lambda}(t)}{dt} = \frac{d\dot{\lambda}(t)}{d\lambda(t)} \frac{d\lambda(t)}{dt} = \dot{\lambda}(t) \frac{d\dot{\lambda}(t)}{d\lambda(t)}$$

we obtain

$$\dot{\lambda}(t) \frac{d\dot{\lambda}(t)}{d\lambda(t)} + \left[2K \frac{\tau_2}{\tau_1} \cos 2\lambda(t) + \frac{K_f}{\tau_f} \right] \dot{\lambda}(t) + \frac{K}{\tau_1} \frac{\sin 2\lambda(t)}{\dot{\lambda}(t)} = 0 \quad (4.38)$$

For $\lambda(t) \ll \frac{\pi}{4}$, $\sin 2\lambda(t) \rightarrow 2\lambda(t)$, and the loop model of Fig. (4.11) can be used to derive the following transfer function;

$$\frac{\lambda_o(s)}{\lambda_i(s)} = \frac{\omega_n^2 + 2\xi\omega_n s}{\omega_n^2 + 2\xi\omega_n s + s^2} \quad (4.39)$$

where

$$\omega_n^2 = 2K\tau_2/\tau_1$$

$$2\xi\omega_n = 2\xi_0\omega_n + K_f/\tau_f$$

$$\text{and } \xi = \xi_0 + \frac{K_f/\tau_f}{2\omega_n} \quad (4.40)$$

where ξ and ξ_0 are the loop damping coefficients in the presence and the absence of AFC respectively. From Eqn. (4.40) it is clear that the AFC path adds to the loop damping, without changing the natural frequency, ω_n .

Therefore, by properly weighing the gain of the AFC path relative to that of the phase control path, the desired loop damping can in principle be obtained. Consequently, the AFC mechanism can be left active after the loop has achieved lock.

Substituting Eqn. (4.40) into Eqn. (4.38) yields

$$\frac{d}{d\lambda(t)} \left[\frac{\dot{\lambda}(t)}{2\xi_0\omega_n} \right] = - \left[\cos 2\lambda(t) + \frac{\xi}{\xi_0} - 1 \right] - \frac{1}{(2\xi_0)^2} \frac{\sin 2\lambda(t)}{\left[\dot{\lambda}(t)/2\xi_0\omega_n \right]} \quad (4.41)$$

Eqn. (4.41) is the nonlinear differential equation which governs the loop acquisition performance. There is no known analytical solution for this equation. However, it can be solved numerically using the so called "phase plane method" [23]. In this method a graph of Eqn. (4.41) is constructed in which $\lambda(t)$ is the abscissa, and $(\dot{\lambda}(t)/2\xi_0\omega_n)$ is the ordinate. This is achieved by computing, from Eqn. (4.41), the slope of the trajectories

in the $\dot{\lambda}(t) - \lambda(t)$ plane. Certain properties of such a graph are obvious from Eqn. (4.41);

1. For large $\dot{\lambda}(t)$, the second term to the right becomes practically zero and the trajectories are approximately sloping sinusoids.
2. For $\dot{\lambda}(t)=0$, the slope is always $-\pi/\xi_0$; i.e. always negative and increases in magnitude with the AFC loop gain. Also, the slope equals -1 when the AFC path is disabled.
3. For $\dot{\lambda}(t) = \pm\pi/2$, the slope is always $(2 - \pi/\xi_0)$ such that it is always positive and decreases linearly with the AFC loop gain.
4. The equation is periodic in $\lambda(t)$ with period π . Therefore, to describe the acquisition behaviour it is only necessary to plot the trajectories for the interval $-\pi/2 < \lambda(t) < \pi/2$.
5. At $\dot{\lambda}(t)=0$ and $\lambda(t) = n\pi/2$, the slope is indeterminate. These points are called singularities [23]; they are stable for n even, and are unstable for n odd.

The phase plane trajectories corresponding to Eqn. (4.41) have been generated by digital computations for different values of π/ξ_0 and ξ_0 and the results are the plots of Figs. (4.17 - 4.12). The following observations can be made from these plots:

1. For large positive $\dot{\lambda}(t)$, a relatively small decay is noted as the trajectory traverses the strip from $\lambda(t) = -\pi/2$ to $\lambda(t) = \pi/2$. The decay

Fig(4.12): Phase Trajectories of Carrier Recovery Loop

$$\xi_0 = 0.707,$$

$$E/E_0 = 1.0$$

Fig(4.13): Phase Trajectories of Carrier Recovery Loop

$$\xi_0 = 0.5,$$

$$\xi/\xi_0 = 1.0$$

Fig(4.14): Phase Trajectories of Carrier Recovery Loop

$$\xi_0 = 0.707,$$

$$\xi/\xi_0 = 1.01$$

Fig(4.15): Phase Trajectories of Carrier Recovery Loop

$$\xi_0 = 0.5, \quad \xi/\xi_0 = 1.01$$

Fig(4.16): Phase Plane Trajectories of Carrier Recovery Loop

$$\xi_0 = 0.707,$$

$$\xi/\xi_0 = 1.1$$

Fig(4.17): Phase Trajectories of Carrier Recovery Loop

$$\xi_0 = 0.5,$$

$$\xi/\xi_0 = 1.1$$

increases with the ratio (ξ/ξ_0) . The rate of decay increases as $\lambda(t)$ decreases, although this becomes more noticeable as (ξ/ξ_0) approaches unity. The decay continues until at $\lambda(t) = \frac{n\pi}{2}$ (n odd) the value of $(\dot{\lambda}(t)/2\xi_0\omega_n)$ falls below AA. At this point, the system stops skipping cycles, and it may be said to be in frequency lock.

2. It can be seen, by comparing the value of $\dot{\lambda}(t)/2\xi_0\omega_n$ for which the line AA crosses the point $\lambda(t) = -\frac{\pi}{2}$, that the region of frequency lock is extended as ξ/ξ_0 is increased.

These qualitative observations may be put on a somewhat more quantitative basis. From Eqn. (4.41), by setting the second term to the right to zero, we obtain;

$$d \left[\frac{\dot{\lambda}(t)}{2\xi_0\omega_n} \right] = \left[\cos 2\lambda(t) + \frac{\xi}{\xi_0} - 1 \right] d\lambda(t)$$

Therefore, for a given trajectory we have the following

$$\frac{\dot{\lambda}(t) - \dot{\lambda}(-\frac{\pi}{2})}{2\xi_0\omega_n} = - \left(\frac{\xi}{\xi_0} - 1 \right) \left[\frac{\pi}{2} + \lambda(t) \right] - \frac{1}{2} \sin 2\lambda(t) \quad (4.42)$$

Eqn. (4.42) shows that, for large $\dot{\lambda}(t)$, the trajectory is a sloping sinusoid, and that the decay in $(\dot{\lambda}(t)/2\xi_0\omega_n)$ is given by;

$$\frac{d}{d\lambda(t)} \left[\dot{\lambda}(t)/2\xi_0\omega_n \right] = - \cos 2\lambda(t) - \left(\frac{\xi}{\xi_0} - 1 \right) \quad (4.43)$$

$$\Delta \left\{ \frac{\dot{\lambda}(t)}{2\xi_0\omega_n} \right\} = \left\{ \frac{\xi}{\xi_0} - 1 \right\} \pi \quad ; \frac{\dot{\lambda}(-\frac{\pi}{2})}{2\xi_0\omega_n} > 3$$

Eqn. (4.43) suggests that the decay in $\lambda(t)$ is linearly increasing with the AFC loop gain. For values of $[\lambda(t)/2\xi_0\omega_n]$ smaller than 3, the actual decay is larger than predicted by Eqn. (4.43) due to the action of the phase comparators in the loop, and hence Eqn. (4.43) gives a lower bound on the frequency decay.

The time needed to traverse a single phase trajectory can also be upper-bounded as follows.

From Eqn. (4.42), we obtain the following:

$$\begin{aligned} \int_{t_0}^{t_1} d(2\xi_0\omega_n t) &= \int_{-\pi/2}^{\pi/2} \frac{d\lambda(t)}{\frac{\lambda(-\frac{\pi}{2})}{2\xi_0\omega_n} - \left(\frac{\xi}{\xi_0} - 1\right)\left\{\frac{\pi}{2} + \lambda(t)\right\} - \frac{1}{2} \sin 2\lambda(t)} \\ &= 2\xi_0\omega_n(t_1 - t_0) \end{aligned} \quad (4.44)$$

where $(t_1 - t_0)$ is the time needed to traverse a phase trajectory starting at $\lambda(-\frac{\pi}{2})$. The integrand in Eqn. (4.44) is sketched in Fig. (4.18). The two shaded areas A_1 and A_2 are almost equal and, therefore, their contribution to the integral cancel out. Consequently, the integral can be approximated by,

Fig(4.18): The integrand of eqn(4.44)

$$\begin{aligned}
 2\xi_0\omega_n(t_1-t_0) &= \int_{-\pi/2}^{\pi/2} \frac{d\lambda(t)}{\frac{\lambda'(-\pi/2)}{2\xi_0\omega_n} - \left(\frac{\xi}{\xi_0} - 1\right)\left(\frac{\pi}{2} + \lambda(t)\right)} \\
 &= \frac{1}{(\xi/\xi_0 - 1)} \ln \left[\frac{\left\{ \frac{\lambda'(-\pi/2)}{2\xi_0\omega_n} \right\}}{\left\{ \frac{\lambda'(-\pi/2)}{2\xi_0\omega_n} - \pi\left(\frac{\xi}{\xi_0} - 1\right) \right\}} \right]
 \end{aligned}
 \tag{4.45}$$

Eqn. (4.45) is plotted in Fig. (4.19), with $\frac{\xi}{\xi_0}$ as a parameter. It is evident that (t_1-t_0) increases as the trajectory gets closer to the frequency locking region. Fig. (4.19) can be used together with the loop phase plane trajectories to predict, approximately, the loop frequency acquisition time.

4.4 HARDWARE DESIGN AND MODELING OF THE CLOCK-AIDED LOOP

The design equations for this loop are given by Eqn. (4.40), and will be repeated here for convenience;

$$\omega_n^2 = 2K\tau_2/\tau_1 \tag{4.46}$$

$$2\xi\omega_n = 2\xi_0\omega_n + K_f/\tau_f \tag{4.47}$$

Fig(4.19): Time Taken to Traverse a Phase Trajectory Versus Frequency, λ ; at $\lambda(t) = -\pi/2$

$$\xi = \xi_0 + \frac{K_f}{2\omega_n \tau_f} \quad (4.48)$$

And the loop noise bandwidth is given by [23];

$$2B_n(\text{Hz}) = \int_{-\infty}^{\infty} |H_l(j2\pi f)|^2 df = \frac{\omega_n}{4\xi} (1+4\xi^2) \quad (4.49)$$

For a given ω_n , the loop noise bandwidth is minimized for $\xi=1/2$. However, values of ξ_0 in the range 0.707-1 are preferred in practice since they yield better performance in the acquisition mode, in addition to a good tracking performance [21]. To maintain the degradation in the system's SNR, due to carrier jitter, below 0.5 dB, it has been shown that $2B_n$ must be less than 268 Hz. For $\xi_0=0.707$, the loop natural frequency is $\omega_n=252$ rad/sec.

Numerical computations of the loop acquisition time (assuming a sinusoidal input) have indicated that a loop damping, ξ , of 0.95 and a natural frequency of 600 rad/s are required in order for the loop to acquire a worst case frequency offset of 1.5 KHz in less than 15 ms. Consequently, the following loop parameters are chosen for our design;

$$\xi = 0.707$$

$$\omega_n = 252$$

for tracking

and

$$\xi = 0.95$$

$$\omega_n = 600 \text{ rad/sec}$$

for acquisition

The actual hardware implementation follows closely the block diagram of Fig. (4.20).

4.4.1 Loop Components

Phase detectors: the loop I and Q phase detectors are implemented as X-OR operations using IC4030. The detector characteristic is linear, as shown in Fig. (4.21). However, the linearity exists only for sinusoidal inputs, under high SNR conditions. For modulated inputs, and especially under low SNRs, the detector characteristic ceases to be linear and takes on a sinusoidal shape [24].

Arm Filters: The I and Q channel arm filters are required to satisfy the following design goals;

- a) Best possible ripple rejection; i.e. the high frequency component at the output of the phase detectors, which is centered at twice the

Fig(4.20): Implementation of Clock-Aided Carrier Tracking Loop

Fig(4.21): Loop Phase Detector Characteristic

carrier frequency must be strongly suppressed in order not to modulate the VCO.

- b) Linear phase response is required to avoid the dispersion of the I and Q baseband signals.
- c) Allow 99.9% of the signal energy to go through in order to minimize the intersymbol interference in the I and Q baseband signals.

A sixth order Butterworth filter with cut off frequency of $1/T$ was found to be a good compromise between the hardware complexity and the design goals. The filters have been implemented using the LF356 operational amplifier, and regular high tolerance components. The measured phase and magnitude responses are shown in Fig.

(4.22), while the circuit diagram is shown in Fig. (4.23).

Multipliers: The loop multipliers should exhibit low d.c. offset since any offset will tend to increase the variance of the recovered carrier phase. In addition, the multipliers should have a minimal feed through. The multiplication of the I and Q baseband signals is satisfactorily accomplished using the AD536J analogue multiplier. The multiplication by the data clock is best accomplished using the 4066 CMOS analogue switch in the manner shown in Fig. (4.24). Initially, the AD536J analogue multiplier was tried for this operation, but the feed through at multiples of the clock rate was found relatively high for our purpose.

Loop Filter: The transfer function of the loop filter is given by

Fig(4.22) : Frequency Response of Carrier Recovery Loop Arm Filters

Fig(4.23): Circuit Diagram of the Arm Filters

Fig(4.24a): Circuit Used for Multiplication by the Clock

Fig(4.24b): Hardware Implementation of Loop Filter

$$F_1(s) = \frac{1 + \tau_2 s}{\tau_1 s} \quad (4.49)$$

Where τ_1 and τ_2 are related to the loop damping and natural frequency according to Eqn. (4.46) - (4.48). The natural frequency, ω_n , is independent of the APC path. Therefore, the switching of the loop bandwidth can be achieved by only switching the resistive elements of the loop filter, using the 4066 analogue switch. A circuit diagram of the loop filter is given in Fig (4.24). The resistance R_0 is much higher than R_1 , and is necessary to avoid the saturation of the operational amplifier. The transfer function of the filter, as shown in Fig. (4.24) is given by;

$$F_1(s) \begin{cases} \frac{1 + (R_1 C_1) s}{(R_1 C_1) s} & ; \text{for tracking} \\ \frac{1 + (R_1 C_1) s}{(R_1 + R_0) C_1 s} & ; \text{for acquisition} \end{cases} \quad (4.50)$$

Frequency Comparator: In the heart of the APC loop is the "Rotational Frequency Comparator", [22]. The different situations encountered by the comparator are illustrated in Fig. (4.25a), where f_2 is the frequency of the received sinusoidal signal, and f_1 is the VCO frequency. The phasor diagram of f_1 and f_2 at two instants $\frac{1}{f_2}$ seconds apart, is shown in Fig. (4.25b). It is

readily seen that the angle of rotation of f_2 is 2π , while the angle of rotation of f_1 is $2\pi(\frac{f_1}{f_2}-1)$ which is counter-clockwise if $f_1 > f_2$ and clockwise if $f_1 < f_2$. Therefore detecting the sign of the frequency offset is equivalent to determining the direction of rotation, while the magnitude is related to the angle of rotation.

A full cycle of the VCO output and its quadrature component is shown in Fig. (4.25c), and the cycle is divided into four quadrants A, B, C, and D. Each of these quadrants is defined uniquely by the logical levels of the two carrier components. When the loop is in lock, the low-to-high transition, of the incoming signal, takes place in the same quadrant all the time. In the presence of a frequency offset, the transition will change quarters with time in a direction which depends on the sign of the frequency offset. The frequency of this change is directly related to the frequency error, in absolute value. An implementation for the comparator is shown in Fig. (4.25d). In this implementation, the VCO output and its quadrature components are sampled with the rising edges of the received signal. Two consecutive pairs of samples are stored in the D Flip Flops $Q_1Q_2Q_3$ and Q_4 . Gate G_1 generates a narrow pulse whenever $Q_1Q_2Q_3Q_4=1101$ indicating that a transition from quadrant B to C has taken place. Similarly, G_2 generates a pulse whenever a transition from C to B

(a)

(b)

(c)

(d)

(e)

Fig(4.25):

- (a) Different Signals Encountered by F.C.*
- (b) Phase or Diagram
- (c) In-phase and Quad. Carrier
- (d) Implementation of F.C.
- (e) Error Characteristic

*F.C. = Frequency Comparator

occurs. Therefore, the output of G_1 carries a pulse train only when $f_1 > f_2$ while G_2 remains low. The frequency of occurrence of these pulses is directly proportional to the frequency offset. The transfer characteristic of this comparator is depicted in Fig. (4.25e). The mean and variance of the comparator output, when the incoming signal is modulated, are derived in Appendix A. It is shown that the mean is always proportional to the frequency difference between the carrier and the VCO output. However, the output deviates considerably from the mean in the presence of long strings of "Ones" or "Zeroes", and therefore cannot be used directly in the loop. To alleviate this problem, a variable length Random Walk filter was found to provide a reliable indication of the loop frequency error.

AFC Loop Filter: A functional diagram of the AFC filter is shown in Fig. (4.26). The random walk filter is a variable length Up/Down counter, which counts the negative and the positive offset pulses, and sends a negative or a positive pulse (according to the majority polarity), to the integrator whenever the counter underflows or overflows. The counter length is also programmable, thus providing a means for controlling the AFC loop gain. The filter length is set to 4 stages in the acquisition mode, therefore allowing for frequent frequency corrections. The filter length is set to 10 stages in the tracking mode, thus greatly reducing the effect on the carrier jitter due to noise and modulation. The integrator is implemented as an 8-stage Up/Down

Fig(4.26): APC Loop Filter.

counter followed by a digital to analogue converter, which operates on the counter reading. The 74LS193 programmable counter chip is utilized for all counters in the APC filter, while the AD561SD analogue to digital Converter is used in the integrator portion.

The integrator output at any moment is proportional to the number of all the positive pulses generated up to this moment less the number of the negative pulses generated in the same time interval. This output is incremented (decremented) by δ upon receiving a positive (negative) pulse. If the instantaneous frequency offset is Δf , then the positive frequency pulses going into the integrator will have a repetition rate $\alpha \Delta f$, where α is a proportionality constant. Therefore, the filter output is pumped by δ each $\frac{1}{\alpha \Delta f}$ sec. Assuming that the time constant of an equivalent analogue integrator is τ_f , then

$$\frac{k_f}{\tau_f} = \alpha \delta = \frac{V_{\max}}{2^8} \quad (4.51a)$$

where k_f is the gain of the frequency comparator, and $\delta = (\text{maximum swing at output of DAC})/2^8$. Therefore,

$$\delta = \frac{1}{\tau_f} \cdot \frac{k_f \Delta f}{\alpha \Delta f} = \frac{k_f}{\alpha \tau_f} \quad (4.51b)$$

4.4.2 Experimental Results

To justify the theoretical results of section 4.1 and 4.3, the carrier recovery loop in addition to the data detection circuitry has been fabricated on a printed circuit board. The board contains 40 commercial integrated circuits and about 70 passive, high tolerance elements. The loop is powered from a $\pm 15V$ and $\pm 5V$ dual power supplies, and dissipates about 2.5 watts.

Spectral Density Measurements

This experiment is mainly concerned with the discrete components from which the loop control signals are derived. The HP3582A spectrum analyzer is used throughout this experiment. The random data needed for the modulator board is derived from the HP3760A data generator. The experimental test set-up is shown in Fig. (4.28). With the carrier recovery loop open, the VCO frequency was detuned by $\Delta f = 3.5$ kHz from the carrier which is fixed at 460.947 kHz. The signals at the output of multiplier M_1 , (point A, Fig. 4.10), and at the loop filter input (point B, Fig. 4.10) were monitored on the spectrum analyzer. The spectral density, obtained at point A, is shown in Fig. (4.29a), while Fig. (4.29b) is for point B. The density in Fig. (4.29a) is that of a frequency doubled signal down converted to $2\Delta f$, and we

Fig(4.28): Measurement Set-up Used for Spectrum Measurements

.Uniform Window
.BW = 100KHz
.Scale: 2dB/div & 2.5KHz/div
.Δf = 3.5KHz

Fig(4.29a): Spectral Density Following the 1st Multiplier.
The Loop is out of Lock
Δf = 3.5 KHz

notice the presence of a discrete tone at $f=(2\Delta f+8)$ kHz. The strongest discrete tone in Fig. (4.29b), is at $2\Delta f$ and is denoted as "beat note" on the graph. This is the useful control signal that drives the loop into lock when it is closed. We also notice the presence of two tones at the clock frequency and its third harmonic, which result from the multiplication of the frequency-doubled signal by the data clock. To measure the power of the discrete tones of Fig (4.29a), we followed the following procedure;

1. Use an all "One" sequence to modulate the IF Carrier.
2. Under this condition the modulated signal is transformed into a single tone at (f_c+4) kHz. The spectral density, at A, corresponding to this signal is a discrete note at $(2\Delta f+8)$ kHz, and contain all the signal power. This is used to set the power reference of the 3582 spectrum analyzer.
3. A (2^{10-1}) bit long PRS sequence is used to modulate the carrier. While the spectral density at A is being monitored on the analyzer, the system marker is moved to coincide with the discrete tone at $(2\Delta f+8)$. This relative power reading is equal to the energy of this discrete tone relative to the total signal energy. This procedure is

.Uniform Window
 .BW = 100Hz
 .Scale: 2dB/div & 2.5KHz/div
 . $\Delta f = 1.5\text{KHz}$

Fig(4.29b): Spectral Density at Input of Loop Filter
 .The Loop is out of Lock

repeated for the "beat note" energy. The spectral densities obtained for this measurement are shown in Fig. (4.30).

Each of the two lines of Fig. (4.30) contains approximately 12.5% of the total signal energy, in good agreement with the numerical results of section (4.1). The spectral density at point B, after the loop has achieved lock, is shown in Fig. (4.31). We notice the presence of a strong d.c. control signal. The line component at 8 kHz is due to the data clock.

Noise Performance

The test set-up used for this experiment is shown in Fig. (4.32). The bandpass noise component is obtained by X-ORing a 104857 bit-long PRS sequence, at a rate of 300 kb/s, with a reference carrier at 460.947 KHz (available on the modulator board). The spectral density at the X-OR output is given by

$$\phi_n(f) = \left| \frac{\sin \frac{\pi(f-f_c)}{f_n}}{\frac{\pi(f-f_c)}{f_n}} \right|^2 + \left| \frac{\sin \frac{\pi(f+f_c)}{f_n}}{\frac{\pi(f+f_c)}{f_n}} \right|^2 \quad (4.52)$$

Where f_c is the carrier frequency, and the f_n is the rate of the noise sequence. Eqn. (4.52) is plotted in Fig. (4.33), in an IF bandwidth of 32 kHz, centered at

Fig(4.30): Power Spectrum of the Discrete Tones at $2f_c$ and $(2f_c + f_D/2)$.

Note: The Line $\theta 2f_c$ has been shifted down by 10dB for Distinction

.Uniform Window
.BW = 100 Hz
.Scale: 2dB/div, 2.5KHz/div

Fig(4.31): Spectral Density at Input of Loop Filter
The Loop is Phase Lock

Fig(4.32): Experimental Set-up Used Loop SNR Measurements

Fig(4.33): Spectral Density of the Noise Component near f_c

$f_c = 460.947$ kHz. It is obvious that $\phi_n(f)$ is practically constant within the IF bandwidth, and is used to approximate the band-limited white noise component. A true r.m.s. meter is used to measure the noise and signal powers in the IF filter bandwidth and, consequently, the ratio E_B/N_0 . Fig. (4.34) show the spectral density of the recovered carrier at a very high E_B/N_0 , and also at 6 dB. The spectral density of the reference carrier is also shown for comparison. Fig. (4.34) was obtained by beating the recovered carrier frequency down to 5 kHz, to take advantage of the capabilities of the 3582A spectrum analyzer. The measurement of the recovered carrier phase variance and, hence, the loop SNR was carried out as follows:

1. The recovered carrier is first mixed with the reference carrier using an X-OR gate.
2. The low frequency component at the X-OR output is low pass filtered in a bandwidth of 16 KHz. The X-OR acts like a linear phase detector. Therefore, the LPF output is given by;

$$v_n(t) = \frac{V_p}{\pi} \phi_n(t) \quad (4.53)$$

where V_p is the difference (Volts) between the X-OR logic levels, and

$\phi_n(t)$ is the phase jitter component, which is zero mean, and has a variance,

Fig(4.34): Spectral Density of,
a. Local Reference Carrier
b. Recovered Carrier at High SNR
c. Recovered Carrier at Low SNR

$$\sigma_{\theta}^2 = E(\theta_n^2(t)) \quad (4.54)$$

A true r.m.s. meter is used to measure the phase noise power. The spectral density of the jitter component, $\theta_n(t)$, is shown in Fig. (4.35) with E_B/N_0 as a parameter. The loop SNR is defined as [21],

$$SNR_L = \frac{1}{2\sigma_{\theta}^2} \quad (4.55)$$

and is shown in Fig. (4.36) as a function of the ratio E_B/N_0 . The computed SNR, according to Eqn. (4.28), is also shown for comparison. We note that the theory and the measurements agree within 1 dB for $E_B/N_0 > 6$ dB. The discrepancy at lower values of E_B/N_0 is due to the fact that a linear model was utilized in the derivation of Eqn. (4.28). This model holds only for high E_B/N_0 , but fails as E_B/N_0 approaches 6 dB.

4.5 FOURTH POWER LOOP

The fourth power loop, as a potential synchronization device for TFM signals, was first discovered by the author at a very late stage in this research program. As a consequence, no time has been devoted to the analysis of the loop performance in the

Fig(4.35): Power Spectrum of Phase Jitter Component

- (a) SMR = 40 dB
- (b) 30 dB
- (c) 20 dB
- (d) 10 dB
- (e) 6 dB

Fig(4.36): Measured Loop SNR as a Function of E_b/N_0

presence of noise. However, we have taken up the task of the necessary hardware design and development. Our approach to this device to date has been essentially experimental. In this section we introduce the design philosophy, and the experimental results obtained on a bread-boarded version of the loop.

4.5.1 Design Philosophy

There are two basic design formats for this loop, as illustrated in Fig. (4.6) and (4.37). The I-Q implementation seems to be attractive at first glance, due to the following reasons,

1. The hardware involved in the first and third topmost arms is also needed for the data detection process.
2. All the necessary multiplications are performed at baseband, and therefore, the use of the commercially available analogue multipliers is possible.

However, the implementation of Fig. (4.37) has been chosen for our experiment due to the availability of all the components, needed for the design, at very low cost, in addition to simplicity. The following discussion is all focused on Fig. (4.37).

Fig(4.37): Implementation of a 4th Power Loop

Fig(4.38): Implementation of a 4th Order Nonlinearity

Fourth-order nonlinearity: This device has been realized using the 74C221 monostable multivibrator, and the implementation follows closely the schematic diagram of Fig. (4.38). The incoming signal is first hard limited. Monostables I and III are triggered on the rising edges, while monostables II and IV are made to trigger on the falling edges. The width of the pulses generated by I and II is set to approximately $\frac{1}{4f_c}$, while that of III and IV is set to approximately $\frac{1}{8f_c}$. Therefore, for each cycle of the incoming signal, four pulses are generated at the output of the nonlinearity of Fig. (4.38). The output duty cycle can not be 50% all the time due to modulation, carrier drift and thermal drift of the passive elements. Therefore, the output of the nonlinearity contains all harmonics of the incoming signal, although the energy contained in the fourth harmonic can be maximized by trimming the individual pulses of all the monostables of Fig. (4.38). Fig. (4.39) shows the power spectrum obtained at the output of the nonlinearity. We notice that the component centered at $4f_c$ is approximately 20 dB higher than any undesired component.

The fourth harmonic is easily extracted using a fourth order Butterworth filter. Fig. (4.40) shows the power spectral density of the filtered 4th harmonic. Note the presence of three discrete tones at $4f_c$, $(4f_c + f_b)$ and $(4f_c - f_b)$. The energy in the $4f_c$

Fig(A.39): Power Spectrum at the Output of the 4th Order Nonlinearity

Fig(4.40): Spectral Density of the Frequency Quadrupled FPM Signal

tone is at least 2 times the energy in the $(4f_c + f_h)$ tones.

Loop Design: The following factors must be taken into account in the loop design;

1. Due to the frequency multiplication by 4, the uncertainty in the IF carrier frequency is also quadrupled. Therefore, the loop must be capable of acquiring a worst case frequency offset of 6 KHz. This problem can be dealt with in the same way as was used for the clock-aided second order loop, i.e. by adding an AFC path to the loop.
2. The loop signal to noise ratio is upper bounded, due to the self noise component. Consequently, for a given permissible amount of degradation in the system SNR, there is an upper limit for the loop noise bandwidth.
3. Loop stability: Using the numerical results of Fig. (4.2) and Table (4.II), the loop SNR, in the absence of noise, is given by,

$$SNR_L = \frac{0.25}{0.26B_n T} = 0.962/B_n T \quad (4.56)$$

where B_n is the loop noise bandwidth. Therefore, the recovered carrier at $4f_c$, has a phase variance,

$$\sigma_{\theta}^2 = \frac{1}{2\text{SNR}_L} = 0.52B_n T \quad (4.57)$$

The required carrier at f_c is obtained by dividing the VCO output frequency by 4 and, therefore, the phase variance is divided by 16 such that

$$\sigma_{\theta}^2 = \sigma_{\theta}^2/16 = 0.0325B_n T \quad (4.58)$$

For the degradation in the system SNR, due to phase jitter, to be less than 0.5 dB, σ_{θ} must be less than 0.0059. Using Eqn. (4.58) we find that $B_n T < 0.18$ is needed. This holds only for very high E_B/N_0 . For lower E_B/N_0 the loop bandwidth must be narrower to keep the degradation below 0.5 dB.

A loop bandwidth of $0.06/T$ has been chosen for the purpose of our experiment. A second order loop has been chosen due to its unconditional stability. The loop damping is 0.707, and the loop filter transfer function is of the form

$$F(s) = \frac{1 + \tau_2 s}{\tau_1 s}$$

The loop design equations are;

$$\left. \begin{aligned} \omega_n^2 &= \frac{K}{\tau_1} \\ 2\xi\omega_n &= K\tau_2/\tau_1 \\ 2B_n &= \frac{\omega_n^2}{2\xi}(1+4\xi^2) \end{aligned} \right\} \quad (4.59)$$

A ripple-rejection filter follows the loop phase detector, to suppress the high frequency component which is centered at $8f_c$, and is realized as a 4th order Butterworth filter implemented according to the theory in [17]. A complete schematic diagram for the 4th power loop is shown in Fig. (4.41).

4.5.2 Experimental Evaluation

* Spectral Measurements: The test set-up of Fig. (4.28) was used for the evaluation of a bread-boarded version of the 4th power loop. The relative energy in the discrete tones generated by the 4th order nonlinearity were measured according to the following procedure;

1. By applying an alternating sequence (1010...) to the TFM modulator, the modulated signal becomes sinusoidal, and so is the 4th harmonic generated by the nonlinearity. The 4th harmonic is monitored on the 3582A spectrum analyzer and used to establish the system power reference.
2. The IF carrier is then modulated using a $(2^{10}-1)$ bit-long PRS sequence, and the spectral density around $4f_c$ is measured, relative to the reference established in Step 1. By moving the system marker to

Fig(4.41): A Complete Schematic of the 4th Power Loop

coincide with the discrete tone at $4f_c$, a direct reading is obtained for the relative energy in this tone. The same procedure is repeated for the discrete tones at $(4f_c + 16)$ KHz. The result of this measurement is shown in Fig. (4.42), and the relative power was found approximately 23% and 12% at $4f_c$ and $(4f_c + 16)$ respectively. This is in good agreement with the predictions of section (4.3).

Noise Performance: The set-up of Fig. (4.32) is used for this experiment, and the procedure is the same as for the second order loop. Therefore, just the experimental results are presented here. Fig. (4.43) shows the spectral density of the recovered carrier at very high E_B/N_0 , and also for $E_B/N_0 = 15$ dB. The spectrum of the unmodulated carrier is also shown for comparison. Fig. (4.44) shows the loop SNR as a function of E_B/N_0 . Notice that the SNR_1 is bounded from above by approximately 25 dB due to the system self noise component. This bound is in good agreement with Eqn. (4.57). We also notice that as the input E_B/N_0 ratio drops from ∞ to the vicinity of 15 dB, the reduction in the loop SNR is less than 3 dB. Therefore, it is concluded that the loop phase jitter is practically dominated by the self-noise component for high E_B/N_0 . We also notice that the 4th power loop outperforms the

Fig(4.42): Spectral Density at Input to 4th Power Loop Filter (The Loop is Open).

.Line B is due to the line at $(4f_c - f_b)$

.Line C is due to the line at $(4f_c + f_b)$

Note that the energy in line A is about twice the energy in lines B or C.

Fig(4.43): Power Spectrum of Recovered Carrier and the Crystal-Controlled Reference Carrier.

Fig(4.44): Measured SNR of the 4th Power Carrier Recovery Loop as a function of E_b/N_0

recovered carrier phase jitter. This fact, in addition to the loop's independence from the data clock leads to the conclusion that a 4th power loop can be very suitable for any practical applications of TFM.

4.6 SUMMARY

We have shown that the squared TFM signal contains two discrete tones at $(2f_c + f_b/2)$. These two lines have been utilized in the unmodulated carrier recovery using a second order clock-aided loop. An analysis of the loop noise performance has indicated that the loop SNR is upper-bounded due to the modulated signal's self-noise component. The experimental verification of the theoretical results revealed that our analysis holds only for E_b/N_0 6 dB. This is expected due to the use of a linear model for the analysis.

It has also been shown that a frequency-quadrupled TFM signal contains a strong line component at $4f_c$. This component has been successfully used for carrier recovery in a 4th power loop. The experimental results presented show that a 4th power loop outperforms the 2nd order loop, in addition to its independence of the data clock. The only problem encountered with the 4th power loop is that it has to cope with frequency uncertainties which are double the uncertainties encountered in a 2nd

CHAPTER V

CLOCK SYNCHRONIZATION

Power efficient digital receivers require the existence of a digital clock, synchronized to the received data stream, to control the integrate-and-dump detection filters, as well as the timing of the output data. Self synchronization techniques, which extract the clock from a noisy NRZ data stream has met wide spread attention and use.

In this chapter, we first discuss the most common types of the synchronization techniques. We then present a clock synchronizer, which is specially suited for narrow band CPM systems, with modulation index $1/2$. The error detector characteristic is derived, and a small signal analysis is conducted. This analysis yields the loop normalized timing jitter variance, under high input signal to noise ratio conditions. A full digital implementation of our synchronizer is discussed, and an approximate model, for the digital loop, is given. The loop transient response is studied in some detail, using numerical methods. Finally the experimental measurements, conducted on a printed circuit board version of the loop is presented.

5.2 COMMON SELF-SYNCHRONIZATION TECHNIQUES

A brief description of the most common self-synchronization techniques, is presented in this section.

5.2.1 Nonlinear Bit Synchronizers

This technique works on the received noisy NRZ data stream as follows:

1. The data signal is first bandpass filtered to eliminate part of the input noise.
2. The discrete line is then extracted using a narrow bandpass filter, or a phase locked loop.

The widely used nonlinearities are the second order, fourth order, delay and multiply, absolute value and $\log[\cosh(x)]$. This technique has been discussed in several references [e.g. 25-28], and it has the advantage of simplicity, and fast acquisition. On the other hand, the timing jitter due to noise, may be unacceptable under low signal to noise ratio conditions. Another drawback of this technique is the systems tendency to fall out-of-lock in the presence of long strings of "Ones" or "Zeroes".

5.2.2 In-Phase Mid-Phase Bit Synchronizer

This is sometimes called the "data-transition tracking loop" (DTTL), because of its method of operation. It has been first suggested by Lindsay and Tausworth [29], and investigated in [30] and [31]. Both an In-Phase and Mid-Phase branch are utilized to provide a timing error discriminator [see Fig. (5.1a)]. The In-Phase branch determines the polarity of the data transitions if and when they occur, while the Mid-Phase branch determines the magnitude of the timing error. The Mid-Phase error signal z_k is multiplied by $I_k = \pm 1$ if a transition has been sensed, or by $I_k = 0$ if no transition has been detected. The filtered output of the multiplier is used to drive the VCO, which controls the integrate and dump operations. An advantage of this scheme is that during long periods between data transitions, caused by a long sequence of "ones" or "zeroes", the discriminator does not allow any noise to perturb the loop; it "holds" the last valid estimate. The error discriminator characteristic is shown in Fig. (5.1b). For large input SNRs [26], the loop control voltage $e(\epsilon)$ is proportional to ϵ , where ϵ is the loop timing error. The mean square timing error is given by [26],

$$\left(\frac{\sigma E^2}{T^2}\right)^2 = \frac{B_f T}{4R} \quad (5.1)$$

In-phase branch

Mid-phase branch

(a)

(b)

Fig(5.1): Mid-Phase In-Phase Bit Synchronizer

(a) Block Diagram

(b) Error Discriminator Characteristic

B T=normalized loop bandwidth, and $R=E_B/N_0$. It has been shown in [30] and [31] that the use of a Mid-Phase window of only T/2 yields a 3 dB improvement over the T window case.

5.2.3 Early-Late Gate Synchronizer

This is also referred to as the absolute-value Bit synchronizer (AVBS), Fig. (5.2). It uses an Early and a Late Integrate and dump channels, having an absolute value operation, which makes it bit-independent. It was found in [32] that this scheme provides a 3 dB improvement in noise jitter, over the DTTL case, and that it is also superior in terms of the mean time to first cycle slip. The acquisition time of the AVBS is longer compared to the DTTL [26]. This can be explained by comparing the respective discriminator characteristics of Fig. (5.1b) and Fig. (5.2b), where it is noticed that the DTTL has a stronger correction voltage for timing errors greater than T/4. The AVBS has the advantage of simple circuitry, when implemented by analog means, since any d.c. drift in the multipliers will hopefully cancel out when differencing the timing-error signals.

Fig(5.2): Early-Late Bit Synchronizer

(a) Block Diagram

(b) Error Characteristic

3

5.2.4 Maximum Likelihood Bit Synchronizer (MLBS)

If the input pulse waveform is strictly confined to one symbol interval, T , then there is a maximum likelihood estimate for the data clock [28]. Stiffler [33] and Mengalli [34] have devised tracking synchronizers which converge to MLBS and have the same error statistics. Mengalli's tracker is shown in Fig. (5.3). Apart from the tanh function, the loop implementation is simple. It was found in [34] that the jitter of the scheme of Fig. (5.3) is only 20% worse than optimum. Unfortunately, Mengalli's tracker does not work for the partial response signalling case, which is our main concern.

The synchronization techniques discussed in this section assume a noisy NRZ baseband data stream as input. In the rest of this chapter we will discuss a novel synchronizer, which is suitable for partial response CPM with modulation index $1/2$. The synchronizer is similar to the AVBS in its principle of operation.

5.3 BIT TIMING FOR TFM

In what follows we shall concern ourselves with Tamed FM, although most of the discussion can be generalized to several CPM systems with modulation index $1/2$. By reference to Fig. (2.1), we assume that the IF

Fig(5.3): Maximum Likelihood Bit Synchronizer

filter bandwidth is wide enough to pass the modulated signal without distortion and yet, narrow enough for the narrow band approximation of noise to be valid.

Therefore, the I and Q channel signals, prior to the predetection ~~arm~~ filters, are given by

$$\begin{aligned} I(t, \underline{a}) &= \sqrt{\frac{2E_B}{T}} \cos \phi(t, \underline{a}) \\ Q(t, \underline{a}) &= \sqrt{\frac{2E_B}{T}} \sin \phi(t, \underline{a}) \end{aligned} \quad (5.2)$$

If we assume that the 8-phase states of Fig. (2.4) are equally probable, then the phase function $\phi(t, \underline{a})$ will assume the values $\frac{n\pi}{2}$ at the data switching moments with probability 50%, and $n=0, \pm 1, 2$. Hence, we can conclude that the zero crossings of $I(t, \underline{a})$ and $Q(t, \underline{a})$, [Eqn. 5.2], coincide with the data switching instants, and consequently, can be used in deriving the data timing signal, in the manner shown in Fig. (5.4). A clock synchronizer which derives its control information from the polarity changes of $y(t)$, Fig. (5.4), is shown in Fig. (5.5). There we assume, without loss of generality, that the loop estimates for the data switching moments are $t=nT$, while the actual changes take place at time $t=nT+\epsilon$ as shown in Fig. (5.5b). The loop error discriminator is made of two integrate and dump filters, the early channel integrator window covers the interval $\frac{T}{4}+nT < t < nT+\epsilon$, while the late channel window is, $nT+\epsilon < t < nT+T/4$. By differencing the absolute values of the two integrals, a

Fig(5.4): Utilizing the I and Q channel zero crossings in the data timing recovery

$$y(t) = \text{Sgn } I(t, \underline{a}) - \text{Sgn } Q(t, \underline{a})$$

Fig(5.5a): Block Diagram of the Proposed Clock Synchronizer

Fig(5.5): Assumed timing reference (b), and error characteristic (c) of loop error discriminator

useful control voltage is obtained. The discriminator output is made use of only if a polarity change, of $y(t)$, is detected in the interval $(n-\frac{1}{2})T < t < (n+\frac{1}{2})T$.

In the following, we derive the error discriminator characteristic, using the assumed timing reference of Fig. (5.5b).

a. $|\epsilon| < T/4$

We assume that the input signal, $y(t)$, changes polarity in the time interval $(n-1/2)T$ to $(n+1/2)T$. The early and late integrals are given by

$$\begin{aligned} S_e(\epsilon) &= \int_{-T/4}^{\epsilon} \pm V_s dt = \pm V_s (\epsilon + T/4) \\ S_l(\epsilon) &= \int_{\epsilon}^{T/4} \pm V_s dt = \pm V_s (T/4 - \epsilon) \end{aligned} \quad (5.3)$$

The error voltage, $e(\epsilon)$, is obtained by differencing the absolute values of $S_e(\epsilon)$ and $S_l(\epsilon)$. Therefore,

$$e(\epsilon) = (|S_e(\epsilon)| - |S_l(\epsilon)|) / 2V_s = \epsilon, \quad |\epsilon| < \frac{T}{4} \quad (5.4)$$

b. $T/4 < \epsilon < T/2$

Here we distinguish two cases; for ϵ positive, the early window is $-T/4 < \epsilon < T/4$, while the late window

vanishes, and vice-versa for ϵ negative. Consequently,

$$e(\epsilon) = \frac{V_s \cdot \text{sgn}(\epsilon)}{2V_s} \frac{T}{2} = \frac{T}{4} \text{sgn}(\epsilon) \quad , \frac{T}{2} > |\epsilon| > \frac{T}{4} \quad (5.5)$$

The discriminator characteristic, as given by Eqn. (5.4) and (5.5) is shown in Fig. (5.5c). Notice the soft limiting effect due to the selection of window widths $T/2$.

5.4 LOOP SIGNAL-TO-NOISE RATIO

In what follows we assume that the input signal to the receiver is corrupted by AWG noise, with spectral density N_0 . We also assume that the IF filter bandwidth, B , is small enough, for the narrow band approximation of noise to be valid. The IF filter output can be expressed as [35];

$$\begin{aligned} r(t) &= \sqrt{\frac{2E_B}{T}} \cos(\omega_c t + \phi(t, \underline{a})) + n(t) \\ &= R(t) \cos(\omega_c t + \phi(t, \underline{a}) + \theta_n(t)) \end{aligned} \quad (5.6)$$

$\theta_n(t)$ is a phase jitter component caused by the input noise process, and $R(t)$ is a positive random envelope function. It is shown in [35] that, the probability density function of $\theta_n(t)$ is given by

$$f(\theta_n) = \frac{e^{-k^2/2}}{2\pi} + \frac{k \cos \theta_n(t)}{2\pi} \exp\left(-\frac{k^2 \sin^2 \theta_n(t)}{2}\right) \cdot F[k \cos \theta_n(t)] \quad (5.7)$$

where k^2 is twice the signal to noise ratio in the IF filter bandwidth, i.e.

$$k^2 = \frac{2}{BT} \left(\frac{E_B}{N_0} \right) \quad (5.8)$$

$F(x)$ is the cumulative density for the unit normal distribution and defined by;

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-y^2/2} dy \quad (5.9)$$

Using Eqn. (5.6), the I and Q baseband signals in Fig. (5.4) may be rewritten as

$$\begin{aligned} I(t, \underline{a}) &= \frac{R}{2} \cos(\phi(t, \underline{a}) + \theta_n(t)) \\ Q(t, \underline{a}) &= \frac{R}{2} \sin(\phi(t, \underline{a}) + \theta_n(t)) \end{aligned} \quad (5.10)$$

and the input signal to the clock synchronizer of Fig.

(5.5a) becomes

$$y(t) =$$

$$\text{sgn}[\cos(\phi(t, \underline{a}) + \theta_n(t))] \oplus \text{sgn}[\sin(\phi(t, \underline{a}) + \theta_n(t))] \quad (5.11)$$

where ' $\oplus$ ' indicates the module-2 addition. $y(t, \underline{a})$ will switch polarity whenever

$$\psi(t) = \phi(t_c, \underline{a}) \oplus \theta_n(t_c) = \frac{\pi}{2}, \quad l=0, \pm 1, 2 \quad (5.12)$$

A typical situation is illustrated in Fig. (5.6a). The Phase function $\phi(t, \underline{a})$ is assumed linear, and the timing error is ϵ and, therefore, Eqn. (5.12) is satisfied at $t_c = \epsilon$. However, in the presence of noise, the phase function may take on a different shape, and Eqn. (5.12) is satisfied at $t_c = \epsilon + \Delta\epsilon$, with $\Delta\epsilon$ strongly dependent on the statistics of $\theta_n(t)$. The numerical calculations of Eqn. (5.7), as well as the experimental observation of the I and Q channel baseband signals, in the presence of noise, have indicated that $\Delta\epsilon$ is practically limited to $T/2$, in absolute value, when the input E_B/N_0 is higher than 6 dB. Consequently, in order to solve Eqn. (5.12) for t_c , we shall concern ourselves with the interval $(n-1)T + \epsilon < t < (n+1)T + \epsilon$. The phase function, $\phi(t, \underline{a})$, in this interval is shown in Fig. (5.6b) using a piece-wise linear approximation. Note that there are 8 possible phase paths, with equal probability of $1/8$. The values of $\Delta\epsilon$, as a function of $\theta_n(t)$, for each possible phase path in Fig. (5.6b), are given in Table (5.I).

Using the error discriminator characteristic of Fig. (5.5c), together with Table (5.I), and assuming that ϵ is small, it can be shown that the mean value of the

Fig(5.6): Possible Phase Paths in the Time Interval

$$(n-1)T+\epsilon < t < (n+1)T+\epsilon$$

Phase Path	$\Delta\tau = a_1\theta_n$
a-e	$\frac{2T}{\pi} \theta_n(t)$
a-f	$\left\{ \begin{array}{ll} \frac{2T}{\pi} \theta_n(t) & , \theta_n(t) < 0 \\ \frac{4T}{\pi} \theta_n(t) & , \theta_n(t) > 0 \end{array} \right.$
b-e	$\left\{ \begin{array}{ll} \frac{4T}{\pi} \theta_n(t) & , \theta_n(t) < 0 \\ \frac{2T}{\pi} \theta_n(t) & , \theta_n(t) > 0 \end{array} \right.$
b-f	$\frac{4T}{\pi} \theta_n(t)$
c-g	$-\frac{4T}{\pi} \theta_n(t)$
c-h	$\left\{ \begin{array}{ll} -\frac{4T}{\pi} \theta_n(t) & , \theta_n(t) > 0 \\ -\frac{2T}{\pi} \theta_n(t) & , \theta_n(t) < 0 \end{array} \right.$
d-g	$\left\{ \begin{array}{ll} -\frac{4T}{\pi} \theta_n(t) & , \theta_n(t) < 0 \\ -\frac{2T}{\pi} \theta_n(t) & , \theta_n(t) > 0 \end{array} \right.$
d-h	$-\frac{2T}{\pi} \theta_n(t)$

Table (5.I)

loop error voltage is given by

$$E_{\theta_n} \{e(\epsilon, \theta_n)\} = \epsilon \quad (5.13)$$

The variance of $e(\epsilon, \theta_n)$ is, generally, dependent on ϵ .

However, we will restrict ourselves to the case $\epsilon=0$, which corresponds to the loop lock conditions. Therefore,

$$\sigma_{\epsilon}^2 = E_{i, \theta_n} \{e^2(a_i \theta_n)\} \quad (5.14)$$

where i is an enumeration on the phase paths of Table (5.1), and a_i are the coefficients of θ_n , and E denotes the expectation and,

$$e(a_i \theta_n) = \begin{cases} a_i \theta_n & |\theta_n| < \frac{T}{4|a_i|} \\ \frac{T}{4} \operatorname{sgn}(a_i \theta_n) & \frac{T}{4|a_i|} < |\theta_n| < \frac{T}{2|a_i|} \end{cases} \quad (5.15)$$

Therefore,

$$\sigma_{\epsilon}^2 = E\{(e - \bar{e})^2\} \quad (5.16)$$

since $\bar{e} = E\{e\} = 0$, we obtain

$$\sigma_{\epsilon}^2 = 1/8 \sum_{i=1}^8 E_{\theta_n} \{e^2(a_i \theta_n)\} \quad (5.17)$$

combining the equal terms together yields,

$$\begin{aligned} \frac{\sigma_e^2}{T^2} = & \frac{4}{\pi^2} \int_0^{\pi/8} \theta_n^2 f(\theta_n) d\theta_n + \frac{16}{\pi^2} \int_0^{\pi/16} \theta_n^2 f(\theta_n) d\theta_n \\ & + \frac{1}{16} \int_{\pi/16}^{\pi/8} f(\theta_n) d\theta_n + \frac{1}{16} \int_{\pi/8}^{\pi/16} f(\theta_n) d\theta_n \end{aligned} \quad (5.18)$$

Following the analysis in [24], we define the discriminator gain A as follows:

$$A = \left. \frac{\partial}{\partial \epsilon} e(\epsilon) \right|_{\epsilon=0} = 1 \quad (5.19)$$

We also define the spectral density of the error voltage, at zero frequency, $S(0)$, as

$$S(0)/T = \sigma_e^2 \quad (5.20)$$

In Eqn. (5.20), it has been assumed that the spectral density is uniform over the Nyquist band $1/T$.

Using Eqn. (5.20), the noise power within the loop noise bandwidth, B , will be,

$$P_n = S(0) \cdot B_l = \sigma_e^2 \cdot B_l T \quad (5.21)$$

and the loop signal to noise ratio becomes,

with $-\sigma_\theta^2$ as given by Eqn. (5.18).

Eqn. (5.22) may be written as

$$\text{SNR}_i = \frac{1}{T^2 \cdot I \cdot B_i T}$$

Therefore,

$$\left(\frac{\sigma_E}{T}\right)^2 = I \cdot (B_i T) \quad (5.23)$$

where I is the right hand side of Eqn. (5.18). The quantity I has been evaluated numerically, as a function of the signal to noise ratio in the IF filter bandwidth, and the results are shown in Fig. (5.7). The variance of the noise induced phase component $\theta_n(t)$ is also shown for comparison. The ratio σ_θ^2 / I , in dB's is shown also in Fig. (5.7). It is worth noting that by extending the loop integration windows to cover one symbol interval, the error characteristic becomes linear over the range $-\frac{T}{2} < \epsilon < \frac{T}{2}$; in this case the integral I will be approximately equal to σ_θ^2 and the loop becomes very similar to a DTTL. As such, we can draw the following observations, about the loop timing jitter;

1. The proposed loop outperforms the standard DTTL at low input SNR's, for the same loop bandwidth. The improvement is approximately 6 dB and 3 dB at input SNR's of 6 and 10 dB respectively.

Fig(5.7): The Integral I and the Phase Noise Variance as Functions of SNR

2. For very high SNR, the loop performance approaches that of a DTTL.

Therefore, for an IF bandwidth twice the data rate, the proposed loop definitely outperforms a DTTL for $E_B/N_0 < 15$ dB, which is the range of most interest in digital radio environments.

5.5 AN ALL DIGITAL IMPLEMENTATION

Due to the high reliability, and low cost of digital IC's, an all-digital realization, for the proposed loop, has been chosen. In this section we discuss the design philosophy, as well as the mathematical modelling of a digital version of the proposed synchronizer.

a). Voltage Controlled Oscillator

A "VCO" is essentially a frequency source, with a phase correction capability. This purpose can be achieved by a crystal stabilized frequency reference, and a programmable counting chain as depicted in Fig. (5.8).

The phase adjustments can be done by "adding" or "deleting" cycles during the frequency division process.

In Fig. (5.8), the reference frequency @ 9.6 MHz, is first divided by 5, resulting in a square wave at 1.92 Mhz. The prescaler output is then applied to a frequency counting chain, which consists of a programmable 4-bit synchronous counter, and a constant divide-by-12 counter. When no phase corrections are needed, the programmable counter repetitively divides by 10. Upon receiving a phase correction command, the programmable counter is preset once in a data-clock cycle, to a control word generated by the filtered output of the loop error discriminator.

Suppose that the control word is the binary equivalent of 5-the programmable counter will divide by 11 only once, and then keep on dividing by 10 for the remainder of the data-clock cycle. Consequently, the present clock period will be,

$$T'_C = 11T_0 + 11 \times 10T_0 = 121T_0 \quad (5.24)$$

If no correction command is received, division by 120 is provided; therefore

Fig(5.8): Digital "Voltage Controlled Oscillator"

Fig(5.9): Data Transition Detector

$$T_c = 120 T_0 \quad (5.25)$$

where $T_0 = 1/1.92 \times 10^{-6}$ sec.

By comparing Eqn. (5.24) to (5.25), the resulting change in the output clock phase is found to be,

$$\Delta = 2\pi/120 = 3^\circ \quad (5.26a)$$

This can be generalized in the form,

$$\Delta = 2\pi/N \quad (5.26b)$$

where N is the frequency division ratio following the prescaler. Similarly, if the control word is 7, the division chain following the prescaler in Fig. (5.8) will divide by 119, thus resulting in a phase change of -3° . Hence, we can conclude that the output clock phase can be advanced or regarded by integer multiples of Δ [Eqn. (5.26)]. One could define a parameter k_v , equivalent to the modulation sensitivity of analogue VCO's, as the phase change resulting from one count change in the control word i.e.

$$k_v \triangleq \Delta (\text{rad})/\text{count} = \frac{2\pi}{N} \text{ rad/count} \quad (5.27)$$

b). Data Transition Detector

The function of this block is to sense the occurrence of polarity changes in the input signal, $y(t)$ in Fig. (5.4), to the clock synchronizer. The input signal $y(t)$ is delayed using a D-type flip-flop clocked by the reference clock @ 1.92 MHz. Then $y(t)$ is X-ORed with the delayed version, resulting in short pulses at all data transition moments. These pulses are used to generate pulses extending from the data switching moment until the next rising edge of the in-phase clock is encountered. The stretched pulses are used to enable the preset function of the divide-by-N counter, Fig. (5.8). A schematic for the hardware realization is shown in Fig. (5.9).

c). Early and Late Integration Gating

The early integration window extends from the mid-phase clock rising edge, to the data transition moment, while the late channel integration window covers the time interval between the data transition moment to the next falling edge of the mid-phase clock. This is illustrated in Fig. (5.10a), while the hardware realization is shown in Fig. (5.10b)

d). Loop Error Discriminator

The loop error signal is generated by integrating the input signal across the windows depicted in Fig. (5.10a) and differencing the absolute values of the two integrals. Since the input is bipolar, the magnitude of the early and late integrals is proportional to the

Fig(5.10): (a) Early and Late Channels Gating Signals, and
(b) their Hardware Implementation

respective window width, which can be easily estimated digitally. This is accomplished by counting the number of cycles, of a high frequency reference, contained in each window. Therefore, a possible hardware realization for the error discriminator, may follow Fig. (5.11a). Further simplification is still possible by replacing the two binary counters and the adder with a Programmable Up/Down Counter, as shown in Fig. (5.11b). The error voltage will, therefore, be proportional to the counter reading at the end of any counting cycle.

In what follows we assume that the reference clock is in phase coherence with the data clock, and $2M$ times the frequency. The Up/Down counter maximum count is $2K-1$. The implementation of Fig. (5.11b) operates as follows:

1. The reference clock is first gated with the mid-phase data clock
2. The gated reference is then gated with the early and late gates individually. This results in the waveforms of Fig. (5.11c). There, L and L' are enumerations on the positive and negative half cycles of the reference clock, respectively. Let X_u and X_d be the number of pulses applied to the count Up/Down inputs, respectively. It is obvious

(a)

(b)

Fig(5.11): Different Realizations of the Loop Error Discriminator

Fig(5.11c): Waveforms Encountered by the Digital Error Discriminator

that,

$$x_d = L \quad (5.28)$$

for a data transition in time slot L or L'.

and

$$x_u = \begin{cases} M-(L-1) & , \text{ for data transition in L} \\ M-L & , \text{ for data transition in L'} \end{cases} \quad (5.29)$$

We assume that the U/D counter is preset to (K-1) in the beginning of each counting cycle. Then, using Eqn. (5.28a) and (5.29) results in the following value for the counter reading at the end of a counting cycle, during which a data transition occurs

$$R_c = \begin{cases} (K + M - 2L) \bmod 2K & , \text{ for a d.t. in L } \& |\epsilon| < \frac{T}{4} \\ (K + M - 2L - 1) \bmod 2K & , \text{ for a d.t. in L'} \end{cases} \quad (5.30)$$

and

$$R_c = \begin{cases} K + M - 1 & , \epsilon > \frac{T}{4} \\ K - M - 1 & , \epsilon < -\frac{T}{4} \end{cases} \quad (5.31)$$

To avoid counter overflow in any counting cycle, K and M should satisfy,

$$\left. \begin{aligned} K + M - 1 &< 2K - 1 \\ K - M - 1 &> 0 \end{aligned} \right\} \quad (5.32)$$

This condition is satisfied if

$$M < (K-1) \quad (5.33)$$

Setting $M=K-1$, Eqn. (5.30) and Eqn (5.31) may be rewritten as

$$R = \begin{cases} (2K - 2L - 1) & , \text{d.t. in } L' \text{ \& } |\epsilon| < \frac{T}{4} \\ (2K - 2L - 2) & , \text{d.t. in } L \text{ \& } |\epsilon| < \frac{T}{4} \\ 2K - 1 & , T/4 < \epsilon < T/2 \\ 0 & , -T/2 < \epsilon < -T/4 \end{cases} \quad (5.34)$$

When the loop is in lock, the counter reading R will remain equal to $(K-1)$. Therefore, the loop error signal is given by

$$\Delta R = R - (K-1) = \begin{cases} K-2L & , \text{d.t. in } L' \text{ \& } |\epsilon| < \frac{T}{4} \\ K-2L-1 & , \text{d.t. in } L \text{ \& } |\epsilon| < \frac{T}{4} \\ K & , T/4 < \epsilon < T/2 \\ 0 & , -T/2 < \epsilon < -T/4 \end{cases} \quad (5.35)$$

We note that a data transition occurs in time slot L if the loop phase error, ϕ , is such that,

$$\frac{\pi}{2M}(L - \frac{M+1}{2}) > \frac{\pi}{2M}(L + 1 - \frac{M+1}{2}) \quad , M \text{ odd} \quad (5.36)$$

This inequality, together with Eqn. (5.35), yield the digital discriminator characteristic, which is shown in Fig. (5.11d) for the case $K=8$. The error discriminator gain, A , in the analogue loop was defined as the slope of the error characteristic when the timing error is zero. An equivalent parameter, g , can be defined for the digital loop as follows:

$$g \triangleq (\text{Increment in counter reading}) / \text{Phase step size} \\ = \frac{1}{\pi/2M} = \frac{2M}{\pi} \quad (5.37)$$

The equivalent open loop gain in the absence of any filtering is given using Eqn. (5.27) and (5.37), by

$$G' = k_v \times g = \frac{2\pi}{N} \times \frac{2M}{\pi} = \frac{4M}{N} \quad (5.38)$$

Where N is the frequency divider following the prescaler of Fig. (5.8).

Loop Filters

The loop filter is needed to enhance the signal to noise ratio of the error discriminator output. This can be achieved using different types of filters specially suited for discrete phase locked loops [36-39]. For simplicity, we have adapted the filter reported in [39], where the filtering is accomplished by accumulating the

Fig(5.11d): Digital Discriminator Characteristic

error discriminator output over m data transitions, and then using the accumulated output for phase correction. The useful error signal is expected to add algebraically, while the noise components will add up on a power basis. Therefore, the SNR following the accumulator is enhanced by m . The open loop gain can easily be controlled by dividing the accumulator output by m' . For simplicity of implementation, m and m' are chosen to be powers of 2. Hence, the open loop gain is changed by m/m' , i.e.,

$$G = G' \frac{m}{m'} = \frac{4M}{N} \frac{m}{m'} \quad (5.39)$$

The hardware implementation followed closely the block diagram of Fig. (5.12)

5.6 TRANSIENT RESPONSE OF THE DIGITAL LOOP

Although the digital loop discussed in section (5.5) is very similar, in principle, to the analogue loop of section (5.3) and (5.4), the analysis for the two loops is different due to the discrete structure of the digital loop. A complete analysis for the latter is possible by following the procedures reported in [36] and [37], and that would lead eventually to the evaluation of the equivalent loop bandwidth [see section 5.4]. However, a complete analysis seems to be tedious, and time consuming. We shall satisfy ourselves here with the

Fig(5.12): Realization of the Discrete Loop Filter

numerical analysis of the loop response to a phase step, and frequency step in the absence of noise. The evaluation of the loop timing jitter will be carried out experimentally.

Let ϕ_1 , ϕ_0 and ϕ denote the received data clock phase, the loop estimate of the data clock, and the phase error respectively. Then, the loop dynamics are controlled by the difference equations;

$$\phi^k = \phi_1^k - \phi_0^k \quad (5.40)$$

$$\phi_0^{k+1} = \begin{cases} \phi_0^k + G \frac{\phi}{\Delta} \Delta & |\phi^k| < \pi/2 \\ \phi_0^k + G(M-1) \text{sgn}(\phi^k) & \pi/2 < |\phi^k| < \pi \end{cases} \quad (5.41)$$

where G and Δ are given by Eqn., (5.39) and (5.26) respectively.

Eqn. (5.40) and (5.41) have been programmed to obtain, numerically, the phase error as a function of the data transition count, normalized to (M/M') , for a set of the different loop parameters and initial conditions. Fig. (5.13a-d) show the results of these computations. From these figures we can draw the following observations:

1. The loop Phase acquisition time is basically determined by $(M/N)M/M'$, independently of the individual values of the parameters involved.
2. There is, generally, a steady state constant phase offset. This is essentially due to the

Fig(5.13a): Effect of the Number of Quantization Levels on the Loop Acquisition.

Transition Count is Normalized to $\frac{n}{M}$

Fig(5.13b): M and N are Varied such that M/N is Constant

Fig(5.13c): Effect of Quantization Step Size on Loop Acquisition

Fig(5.13d): Effect of Phase Step Size on the Loop Acquisition

dead-zones of the error detector characteristic (Fig. 5.11d). The steady state effect is such that $|\phi_{ss}| < \frac{\pi}{2M}$

In the presence of frequency offset between the data clock, and the loop estimate, the dynamics will still be controlled by Eqn. (5.40) and (5.41), in addition to

$$\phi_i^{k+1} = (\phi_i^k + \Omega_0 T') \bmod 2\pi \quad (5.42)$$

where Ω_0 is the radian frequency offset, and T' is the time elapsing between the k th and $(k+1)$ st phase corrections. T' is, in general, a function of the received data sequence. Therefore, we have restricted ourselves to an alternating sequence of the form (00110011...) such that T' will be given by,

$$T' = 2mT \quad (5.43)$$

By programming Eqn. (5.40-5.43), the loop phase error has been obtained as a function of the normalized data transition count, with $(\Omega_0 mT)$ as a parameter. The results are shown in Fig. (5.14). From this figure, we notice the following,

1. The loop will have a constant steady state phase error component which is dependent on the frequency step size.
2. In addition, there is a phase "ripple" component which depends on the actual data sequence in nature.

Fig(5.14): Loop Response to a step in frequency

3. For the periodic sequence (00110011...), the maximum tractable frequency offset such that the steady state phase ripple does not exceed Δ , is given by,

$$\Omega_0 = \frac{2\pi}{Nm'} \quad (5.44)$$

5.6.1 EXPERIMENTAL TESTING

Following the strategy outlined earlier in this chapter, a printed circuit board version of the digital clock synchronizer has been fabricated. The board is made mostly of TTL medium scale integrated circuits. It contains about 45 IC's and is powered from a +5 V supply at 1 amp. The loop design parameters are as follows:

- master clock frequency 5.000 MHz
- nominal data-clock frequency 16 kHz
- $N=6$, $m=120$, $m=8$ and $m'=64$

The board has been tested according to the following procedure.

5.6.1.A Loop Response To A Step In Phase And Frequency

In absence of frequency offset, the loop response to a phase step is affected only by the data transition count, independent of the actual nature of the data sequence.

Hence, an alternating sequence (1010,...) will suffice for this purpose. The measurement setup is shown in Fig. (5.15a). The output data from the hp data generator, is first delayed by half the clock period (which is equivalent to 180° change in the clock phase). The loop input is switched between the data and its delayed version periodically. In effect, this is equivalent to a 180° phase step applied to the loop input. To obtain a measure for the loop timing error, as a function of time, the recovered clock is X-ORed with the actual data clock (taken directly from the frequency synthesizer, after frequency division), and the X-OR output is monitored on a scope. Fig. (5.15b) shows the result of this measurement. The bottom trace is a train of pulses, with the pulse width equal to the loop timing error. We notice that, initially the loop timing error is equal to $T/2$, starts decaying gradually and then reaches a steady state value which is small, but not zero. The loop requires about 20 data transitions to acquire the 180° phase step. The top trace is a low pass filtered version of the bottom trace. It is worth noting that a binary-quantized loop, having the same open loop gain, needs at least 60 data transitions to acquire the same phase step.

The steady state phase error, resulting from a difference in frequency between the nominal and actual data clocks was also investigated. Fig. (5.15c-e) show the effect of different values of the frequency offset. We notice the increase in the average pulse width (lower trace) as the frequency offset increases. This

Fig(5.15a): Experimental Set-up for the Measurement of the Loop Response to a step in Phase.

Fig. 5.15b Loop Response to a 180° Phase Step.
Note that the loop requires 20 data transitions
to achieve lock.

Fig. 5-15c Steady State Loop Phase Error Due to a Frequency Step Equal to 0.195% of the Free Running Frequency.

Fig. 5.15d Steady State Phase Offset Due to a Frequency Step Equal to 0.2% of the Free Running Frequency.

Fig. 5.15e Steady State Phase Offset Due to a Frequency Step Equal to 0.55% of the Free Running Frequency.

observation agrees with the numerical results of section (5,5).

Timing Jitter

The experimental set-up is essentially the same as Fig. (4.32) with the addition of the block diagram of Fig. (5.15f). The output of the X-OR is directly fed to the clock synchronizer. To obtain a measure for the loop timing jitter, the recovered clock is first X-ORed with the reference clock (taken off the modulator board), and low pass filtered. A true r.m.s. meter is used to measure the variance of the filter output, which is proportional to the timing jitter variance. The proportionality constant is easily obtained as follows:-

1. The reference clock and its quadrature component are first X-ORed, and low pass filtered.
2. The corresponding r.m.s. reading is recorded. This reading corresponds to a constant timing error of $T/4$. Therefore, the proportionality constant is given by,

$$a = \frac{(T/4)^2}{(\text{r.m.s. reading } |\Delta c = T/4|)^2} \quad (5.45)$$

Fig. (5.15g) show the measured loop timing jitter variance as a function of (E_B/N_0) . Note that, as E_B/N_0 increases, the loop timing jitter approaches a constant value. This is explained by the dead zone in the characteristic of Fig. (5.11d).

Fig(5.15f):-Set-up for the Measurement of the Loop Timing Jitter
The Baseband Signals $I(t)$ & $Q(t)$ are taken off the Demodulator Board

Fig. (5.15g): Measured Timing Jitter of the Clock Recovery Loop.

CHAPTER VI

DEMODULATION OF TFM

6.1 INTRODUCTION

The optimum receiver for the family of CPM Systems, is the well known Maximum Likelihood Sequence Estimator (MLSE), [6, 13, 16]. It has been shown in [6], that the MLSE consists of a bank of matched filters and multipliers, which are used to generate the so called "branch metrics". The branch metrics are input to a Viterbi Processor, which decides in favour of the data sequence $\underline{a}_k$, which maximizes the a-posteriori probability $P(r|\underline{a}_k)$, where r is the received signal during the receiver observation interval. The number of filters needed for this receiver, as well as the number of multiplications, is proportional to 2^L , where L is the receiver observation interval. This number could be large, even for moderate values of L , and this makes the MLSE impractical for applications where the systems cost and data speed are primary concerns.

The class of CPM systems with modulation index 0.5, has been noticed to bear great similarities to MSK [2], under certain conditions. This fact has been noticed and taken advantage of in devising simple demodulation

schemes for that class of modulation techniques [8, 40, 41]. In [8, 41], the authors have approximated TFM by the so-called "equivalent baseband system", then used the linear theories which are applicable to that equivalent system, and derived a receiving filter which yielded a near optimal performance at low SNR's. This same receiver was analyzed in [40], and it was found to be about 0.2 dB worse than the MLSE at low SNR; however, its asymptotic degradation is about 2.2 dB.

In [40], a group of selected filters was analyzed in conjunction with different CPM modulation techniques with modulation index $1/2$. It was found in [40] that some of those filters performed better with some modulation techniques than others. Therefore, we felt that a somewhat general theory is needed. Although our main concern is Tamed FM, we have approached the problem in some generality. In our approach, we have assumed a fixed receiver structure, similar to de Buda's MSK demodulator [2], with the inclusion of two arm filters. By optimizing the arm filters, in the mean square error sense, we have arrived at a general formula for the impulse response of the arm filters. This formula is a function of the modulation index, and the premodulation filter impulse response. The theory has been applied to TFM and yielded a near-optimum performance for SNR's < 10 dB, while the resulting asymptotic degradations is < 0.6 dB.

In this chapter, we first present a brief comparison between MSK, and the class of CPM with modulation index $1/2$, and derive the conditions under which they could be demodulated using de Buda's technique [2]. The predetection (arm) filters are then optimized, and a simple expression is given for a class of sub-optimum filters. The general results are then specialized to the TFM case. The performance of the sub-optimum TFM receiver is then analyzed, and simulated numerically with emphasis on the I and Q channel eye diagrams, and the system Bit error rate performance. A comparison is made to fourth order Butterworth, and raised cosine filters. Finally, the receiver hardware design is discussed and evaluated experimentally.

6.2 PARTIAL RESPONSE CPM AND MSK

We first will derive the conditions under which a CPM system, with modulation index $1/2$, exhibits an open eye eye-diagram. The system frequency pulse, $h_m(t)$ is assumed to be time limited to the interval $-(\frac{M-1}{2})T < t < (\frac{M+1}{2})T$, and M is an odd integer. As such, the contribution of the j th data symbol to the system phase function, at time $t=nT$, can be written as,

$$\phi^j(nT) = a^j \int_{-\infty}^{nT} g(n-jT) d\eta = a^j \sum_{l=M-1}^{n-j} \lambda_l^j \quad (6.1)$$

where

$$A_l = \int_{lT}^{(l+1)T} g(\tau) d\tau \quad (6.2)$$

The absolute value of the phase function at time $t=nT$, can be split up into three components:

1. Contribution due to all frequency pulses which ended before time $t=nT$ - These are the pulses associated with $\{a^j\}$ where $j < (n - \frac{M-1}{2})$.

This component is given by,

$$\phi_1(nT) = \sum_{i=-\infty}^{n-\frac{M-1}{2}} a^i q(t-iT) = \frac{\pi}{2} \sum_{k=-\infty}^{\frac{M-1}{2}} a^{n+k} \quad (6.3)$$

2. Contribution due to $\{a^j\}$, $j = n - \frac{M+1}{2}, \dots, n$

Using equation 6.1, this component is given by

$$\begin{aligned} \phi_2(nT) &= \sum_{j=(n-\frac{M-1}{2}+1)}^n a^j \left(\sum_{l=-\frac{M-1}{2}}^{n-j} A_l \right) \\ &= \frac{\pi}{2} \cdot \sum_{k=-\frac{M-1}{2}+1}^0 a^{n+k} \cdot \sum_{k=1}^{\frac{M-1}{2}} a^{n-k+1} \sum_{l=k}^{\frac{M-1}{2}} A_l \end{aligned} \quad (6.4)$$

3. Contribution due to $\{a^j\}$ with $j = n+1, \dots, n + \frac{M-1}{2}$

Using Eqn. (6.1), this term is;

$$\phi_3(nT) = \sum_{k=1}^{\frac{M-1}{2}} a^{n+k} \sum_{l=k}^{\frac{M-1}{2}} A_l \quad (6.5)$$

The addition of Eqn's. (6.3-6.5) yields

$$\phi(nT) = \sum_{i=-\infty}^n \frac{\pi}{2} a^i + \sum_{k=1}^{\frac{M-1}{2}} \left[a^{n+k} \sum_{l=k}^{\frac{M-1}{2}} A_{-l} a^{n-k+1} \sum_{l=k}^{\frac{M-1}{2}} A_l \right] \quad (6.6)$$

The first term to the right is the phase of an MSK signal generated using the data sequence, a . The second term is due to the deliberately introduced intersymbol interference. Therefore, we write $\phi(nT)$ in the form

$$\phi(nT) = \phi_{msk}(nT) + \phi_i(nT) \quad (6.7)$$

where

$$\phi_i(nT) = \sum_{k=1}^{\frac{M-1}{2}} \left[a^{n+k} \sum_{l=k}^{\frac{M-1}{2}} A_{-l} a^{n-k+1} \sum_{l=k}^{\frac{M-1}{2}} A_l \right] \quad (6.8)$$

and;

$$\phi_{msk}(nT) = \begin{cases} 0, & \text{modulo } 2\pi, & n \text{ even} \\ -\frac{\pi}{2}, \frac{\pi}{2} & \text{modulo } 2\pi, & n \text{ odd} \end{cases} \quad (6.9)$$

using Eqn. (6.9) in Eqn. (6.7) yields:

$$\cos \phi(nT) = \cos \phi_{msk}(nT) \cdot \cos \phi_i(nT) \quad n \text{ even}$$

$$\sin \phi(nT) = \sin \phi_{msk}(nT) \cdot \cos \phi_i(nT) \quad n \text{ odd}$$

$$\left| \sum_{k=1}^{\frac{M-1}{2}} \left[a^{n+k} \sum_{l=k}^{\frac{M-1}{2}} A_{-l} a^{n-k+1} \sum_{l=k}^{\frac{M-1}{2}} A_l \right] \right| < \frac{\pi}{2} \quad (6.10)$$

The decisions of de Buda's demodulator [2], are based on the sign of $\cos \phi_{msk}(nT)$; therefore, Eqn. (6.10) suggests that the same demodulator can still be used for our purpose if the ISI term, $\phi_i(nT)$ is less than $\pi/2$ in absolute value. Using Eqn. (6.8), this constraint can be written as,

$$\left| \sum_{k=1}^{\frac{M-1}{2}} \left[\sum_{l=k}^{\frac{M-1}{2}} A_{-l} e^{j(n-k+l)\alpha} \right] \right| < \frac{\pi}{2} \max.$$

This is satisfied if,

$$\sum_{k=1}^{\frac{M-1}{2}} \left[\left| \sum_{l=k}^{\frac{M-1}{2}} A_{-l} \right| + \left| \sum_{l=k}^{\frac{M-1}{2}} A_l \right| \right] < \frac{\pi}{2} \quad (6.11)$$

Eqn. (6.11) is satisfied for several CPM Systems, including TFM [8], Duobinary MSK [6], Gaussian MSK [10], and Raised Cosine CPM [6]. Hence, the use of de Buda's demodulator for these systems is feasible, although some performance degradation must be expected due to the ISI factor in Eqn. (6.8). However, the de Buda's demodulator can be modified to yield better performance with any CPM system satisfying Eqn. (6.11), by including two arm filters, as shown in Fig. (6.1). By properly designing the arm filters, a near optimal performance can be achieved for moderate signal to noise ratios. The optimization, for the general case, is carried out in section (6.3) and specialized for the TFM case in section (6.5).

6.3 OPTIMIZATION OF THE PREDETECTION ARM FILTERS

In the following, the signal plus noise is assumed to be bandpass filtered within an IF bandwidth which introduces no signal distortion, and which is narrow enough for the narrow-band approximation of noise to be valid.

In the receiver of Fig. (6.1), perfect synchronization for the carrier and data clock is assumed. The channel is assumed additive white Gaussian, with spectral density N_0 . Therefore, the signal input to the receiver, after bandpass filtering, can be written as;

$$r(t) = s(t, \underline{a}) + n(t) \quad (6.12)$$

where

$$s(t, \underline{a}) = \sqrt{\frac{E_B}{2T}} \cos(\phi(t, \underline{a}) + \omega_c t) \\ = I(t, \underline{a}) \cos \omega_c t - Q(t, \underline{a}) \sin \omega_c t \quad (6.13)$$

$$\left. \begin{aligned} I(t, \underline{a}) &= \sqrt{\frac{E_B}{2T}} \cos \phi(t, \underline{a}) \\ Q(t, \underline{a}) &= \sqrt{\frac{E_B}{2T}} \sin \phi(t, \underline{a}) \end{aligned} \right\} \quad (6.13a)$$

Fig(6.1): A Quadrature Coherent Receiver Structure for CPM with modulation Index 4. (The System must satisfy equ 6.11)

$r(t)$ is the output of an If band pass filter, which is assumed to be wide enough to pass the signal without distortion, and narrow enough for the narrow-band approximation of noise to be valid.

$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t \quad (6.14)$$

E_B is the signal energy per bit, the data rate is $1/T$, $\phi(t, \underline{a})$ is the information carrying phase, and ω_c is the unmodulated carrier radian frequency and $n_c(t)$ and $n_s(t)$ are two statistically independent lowpass noise processes with density N_0 [23]. At this point, we concentrate our analysis on the I channel only; the analysis for the Q channel will parallel the I channel's, with $I(t)$ and $n_c(t)$ replaced by $Q(t)$ and $n_s(t)$ respectively.

Using Eqns. (6.12), (6.13) and (6.14), the input signal to the I-channel filter, after dropping the component centered at $2\omega_c$, is given by;

$$e_i(t) = \frac{1}{2} I(t, \underline{a}) + \frac{1}{2} n_c(t) \quad (6.15)$$

Consequently, the signal presented to the I-Channel sampler, at time $t = nT$ becomes

$$D(nT) = \frac{1}{2} \int_{-\infty}^{\infty} \{ I(nT - \tau, \underline{a}) + n_c(nT - \tau) \} h_d(\tau) d\tau \quad (6.16)$$

$h_d(t)$ is the impulse response of the I channel predetection filter. In absence of noise, and the ISI factor, $D(nT)$ is given by

$$D(nT) = D_m(nT) \sqrt{\frac{E_B}{2T}} \cos \phi_{msk}(nT) \quad (6.17)$$

with $\phi_{msk}(nT)$ as given by Eqn. (6.9). We define the error function, $e(nT)$, as follows:

$$e(nT) \triangleq D(nT) - D_m(nT)$$

and, hence, the mean-square-error function becomes,

$$E \{ e^2(nT) \} = E_k \left\{ \left[\frac{1}{2} \int_{-\infty}^{\infty} \{ I(nT-\tau, \underline{a}_k) + n_c(nT-\tau) \} h_d(\tau) d\tau - \sqrt{\frac{E_B}{2T}} \cos \phi_{msk}(nT) \right]^2 \right\} \quad (6.18)$$

Assuming that $I(t, \underline{a})$ and $n_c(t)$ are stationary, and statistically independent random processes, Eqn. (6.18) may be written as follows:

$$\begin{aligned} E \{ e^2(nT) \} &= \frac{1}{4} \iint_{-\infty}^{\infty} \left[R_I(\tau_2 - \tau_1) + R_n^C(\tau_2 - \tau_1) \right] h_d(\tau_2) \\ &\quad h_d(\tau_1) d\tau_2 d\tau_1 - \sqrt{\frac{E_B}{2T}} \int_{-\infty}^{\infty} \left[E_k \{ I(nT-\tau, \underline{a}_k) | D_m(nT) > 0 \} \right. \\ &\quad \left. - E_k \{ I(nT-\tau, \underline{a}_k) | D_m(nT) < 0 \} \right] h_d(\tau) d\tau + \frac{E_B}{2T} \end{aligned} \quad (6.19)$$

The second integrand to the right of Eqn. (6.19) is recognized as the impulse response of the I-channel average matched filter [40], and will be denoted by $h_{anf}(t)$. Therefore, we rewrite Eqn. (6.19) in the form:

$$E \{ e^2(nT) \} = k \iint_{-\infty}^{\infty} [R_1(\tau_2 - \tau_1) + R_n^C(\tau_2 - \tau_1)] h_d(\tau_2) h_d(\tau_1) d\tau_2 d\tau_1 - \int_{-\infty}^{\infty} h_{amf}(nT - \tau) h_d(\tau) d\tau + \frac{E_B}{2T} \quad (6.19a)$$

where

$$h_{amf}(nT - \tau) = \sqrt{\frac{E_B}{2T}} \left[E_k \{ I(nT - \tau, \underline{a}_k) | D_m(nT) > 0 \} - E_k \{ I(nT - \tau, \underline{a}_k) | D_m(nT) < 0 \} \right] \quad (6.19b)$$

$R_1(\tau)$ and $R_n^C(\tau)$ are the autocorrelation functions of $I(t, \underline{a})$ and $n_c(t)$, respectively and k is an enumeration of the data vector, $\underline{a}$, permutation.

The minimization of the mean square error, as given by Eqn. (6.19) can improve the system error performance, under the condition that the I (and Q) channel eye diagram, following the arm filter, is not closed for any permutation of the data vector, $\underline{a}$. Consequently, we shall constrain the minimum eye opening to a positive constant, C-i.e.,

$$\int_{-\infty}^{\infty} [E_k \{ I(nT - \tau, \underline{a}_k) | D_m(nT) > 0 \} - E_k \{ I(nT - \tau, \underline{a}_k) | D_m(nT) < 0 \}] h_d(\tau) d\tau = C \quad (6.20)$$

The quantity in square brackets in Eqn. (6.20) will be denoted by $h_{min}(t)$ i.e.

$$\int_{-\infty}^{\infty} h_{\min}(nT-\tau)h_d(\tau)d\tau = C \quad (6.20a)$$

where

$$h_{\min}(\tau) = E_k \{ I(nT-\tau, \underline{\beta}_k) | D_m(nT) > 0 \} - E_k \{ I(nT-\tau, \underline{\beta}_k) | D_m(nT) < 0 \} \quad (6.20b)$$

$\{\underline{\beta}_k\}$ is the set of data vectors which determine the inner boundary of the eye diagram, in the time interval $(n-1)T < t < (n+1)T$.

The minimization of Eqn. (6.19), under the constraint of Eqn. (6.20) is a standard problem in the calculus of variations, and its solution is outlined in [42]. Let us denote the m.s.e. expression by f_1 , and the left hand side of Eqn. (6.20) by f_2 , and form the functional f ;

$$f = f_1 + \lambda f_2 \quad (6.21)$$

Where λ is a Lagrange multiplier. We assume that the optimum impulse response, $h_d^+(t)$ has been found and therefore f must have a stationary point for $h_d(t) = h_d^+(t)$. We also perturb $h_d^+(t)$ by $\gamma \cdot \phi_d(t)$, where γ is a constant, and $\phi_d(t)$ is an arbitrary variation; i.e. we write

$$h_d(t) = h_d^+(t) + \gamma \delta h_d(t) \quad (6.22)$$

The substitution of Eqns. (6.22), (6.19) and (6.20) into Eqn. (6.21) yields

$$f(\gamma) = a_0 + a_1 \gamma + a_2 \gamma^2 \quad (6.23)$$

where

$$a_0 = \frac{1}{2} \iint_{-\infty}^{\infty} [R_i(\tau_2 - \tau_1) + R_n^C(\tau_2 - \tau_1)] h_d^+(\tau_2) h_d^+(\tau_1) d\tau_2 d\tau_1 \\ - \int_{-\infty}^{\infty} h_{amf}(nT - \tau) h_d^+(\tau) d\tau + \frac{E_B}{2T} - \lambda C \quad (6.24)$$

$$a_1 = \frac{1}{2} \iint_{-\infty}^{\infty} [R_i(\tau_2 - \tau_1) + R_n^C(\tau_2 - \tau_1)] h_d^+(\tau_2) \delta h_d(\tau_1) d\tau_2 d\tau_1 \\ - \int_{-\infty}^{\infty} h_{amf}(nT - \tau) \delta h_d(\tau) d\tau + \int_{-\infty}^{\infty} \lambda h_{min}(nT - \tau) \delta h_d(\tau) d\tau \quad (6.25)$$

$$a_2 = \frac{1}{2} \iint_{-\infty}^{\infty} [R_i(\tau_2 - \tau_1) + R_n^C(\tau_2 - \tau_1)] \delta h_d(\tau_2) \delta h_d(\tau_1) d\tau_2 d\tau_1 \quad (6.26)$$

If $h^+(t)$ is optimal, then (6.23) must have a stationary point at $\gamma=0$ - This is satisfied if

$$a_1 = \int_{-\infty}^{\infty} \left[\frac{1}{2} \int_{-\infty}^{\infty} \{ R_i(\tau_2 - \tau_1) + R_n^C(\tau_2 - \tau_1) \} \delta h_d^+(\tau_2) d\tau_2 \right. \\ \left. - \{ h_{amf}(nT - \tau_1) + h_{min}(nT - \tau_1) \} \right] \delta h_d(\tau_1) d\tau_1 = 0 \quad (6.27)$$

Since the variation of $\delta h_d(t)$ is arbitrary, we argue that the integrand in Eqn. (6.27) must vanish, independent of $\delta h(\tau_1)$, in order for a_1 to be zero. This leads to

$$\int_{-\infty}^{\infty} [R_i(\tau_2 - \tau_1) + R_n^C(\tau_2 - \tau_1)] h_d^*(\tau_2) d\tau_2 - [h_{amf}(nT - \tau_1) + \lambda h_{min}(nT - \tau_1)] = 0 \quad (6.27a)$$

Eqn. (6.27a) is the integral equation to be solved for the optimum filter response, $h^*(\tau)$. Taking the Fourier transform of Eqn. (6.27a) results in

$$H_d^*(j\omega) = \frac{H_{amf}(j\omega) + \lambda H_{min}(j\omega)}{\phi_i(j\omega) + \phi_n^C(j\omega)} e^{j\omega nT} \quad (6.28)$$

where $H_d(j\omega)$, $H_{amf}(j\omega)$, $H_{min}(j\omega)$, $\phi_i(j\omega)$ and $\phi_n^C(j\omega)$ are the Fourier transforms of $h_d(t)$, $h_{amf}(j\omega)$, $h_{min}(t)$, $R_i(\tau)$ and $R_n^C(\tau)$, respectively, and '*' denotes the complex conjugate. For the AWGN case, and under the low SNR conditions (6.28) is simplified to

$$H_d^*(j\omega) = [H_{amf}(j\omega) + \lambda H_{min}(j\omega)] e^{j\omega nT} \quad (6.29)$$

or equivalently,

$$h_d^*(\tau) = h_{amf}(\tau - nT) + \lambda h_{min}(\tau - nT) \quad (6.30)$$

The arm filter response, as given by Eqn. (6.29) and (6.30) is matched to a weighted average of the baseband signal - therefore, it will be called Weighted Average Matched Filter (WAMF). The WAMF as given by Eqn. (6.30) is, strictly speaking, nonrealizable. However, the implementation becomes feasible if $h_d(t)$ is restricted to a time interval, LT , and delayed by $LT/2$. We define $h_{dl}(t)$ as follows;

$$h_{dl}(t) = \begin{cases} h_d^+(t - \frac{LT}{2}) & 0 \leq t < LT \\ 0 & \text{elsewhere} \end{cases} \quad (6.31)$$

We will denote a filter with impulse response, $h_{dl}(t)$ by WAMFL. In the application of Eqn. (6.30) and (6.31), the Lagrange multiplier will be assigned the value λ_{opt} which minimizes the system symbol error rate, as discussed in section (6.4).

6.4 PERFORMANCE ANALYSIS

The signal presented to the I-channel sampler at time $t=nT$, with n even, is given by,

$$D_i(nT) = \int_0^{LT} \left[\sqrt{\frac{E_B}{2T}} \cos \phi(nT - \tau, \underline{a}_i) + \frac{1}{2} n_c(nT - \tau) \right] h_{dl}(\tau) d\tau$$

Where $h_{di}(\tau)$ is given by Eqn. (6.31), and i is an enumeration on the data vector permutation. For the AWG noise case, $D_i(nT)$ can be split up into a signal component,

$$D_s^i(nT) = \sqrt{\frac{E_B}{2T}} \int_0^{LT} \cos \phi(nT-\tau, \underline{a}_i) h_{di}(\tau) d\tau \quad (6.32)$$

and a noise component $D_n^i(nT)$, with zero mean, and variance σ^2 , where

$$\sigma^2 = \frac{N_0}{4} \int_0^{LT} h_{di}^2(\tau) d\tau \quad (6.33)$$

An erroneous decision will occur on the I-channel if $D_s^i(nT) < -D_n^i(nT)$.

Hence, the conditional probability of symbol error is given by,

$$P_I(\epsilon | \underline{a}_i) = P[D_s^i(nT) < -D_n^i(nT)] = Q(X)$$

where, using Eqn. (6.32) and (6.33)

$$X^2 = \frac{D_s^2(nT)}{\sigma^2} = d_i \frac{E_B}{N_0} \quad (6.34)$$

and

$$d_i \triangleq \frac{2}{T} \left[\int_0^{LT} \cos \phi(nt-\tau, \underline{a}_i) b_{di}(\tau) d\tau \right] / \left[\int_0^{LT} \cos^2 \phi(lt-\tau, \underline{a}_i) d\tau \right] \quad (6.35)$$

and the conditional probability of error becomes

$$P_I(\epsilon | \underline{a}_i) = Q\left(\sqrt{d_i} \frac{E_B}{N_0}\right)$$

and the average probability of error is

$$P_I(\epsilon) = \sum_{i=1}^2 P(\underline{a}_i) Q\left(\sqrt{d_i} \frac{E_B}{N_0}\right) \quad (6.36)$$

The total probability of bit error is, approximately double $P_I(\epsilon)$, since an error occurs every time an I or Q channel error occurs.

6.5 APPLICATION TO TAMED FM

The theoretical results of sections 6.3 and 6.4 have been applied to the TFM case. First, a computer subroutine, WAMP, had been developed to carry out the computations suggested by Eqns. (6.19) and (6.20) - i.e. $h_{anf}(t)$ and $h_{min}(t)$, for any specified observation interval, LT. The program is written in such a way that it could be used for any CPM scheme, by specifying the premodulation filter characteristic. Since the impulse response of the WAMPL is symmetric, only one half of $h(t)$ had to be computed. The program is implemented as follows:

1. A data sequence length, L, is specified
2. All possible permutations of the L-bit data vector are calculated.
3. For all permutations, $\cos \phi(t, q)$ is generated.
4. From that set of time domain waveforms, $h_{anf}(t)$ and $h_{min}(t)$ are computed according

to Eqn. (6.19) and (6.20).

A flow chart of this program is given as

Fig. (6.2). The program was used for the TFM modulation, by feeding in the filter characteristic of Eqn. (2.9). Several trial runs were done for different lengths, L , and it was noticed that increasing L beyond 8-bits did not yield any noticeable change in the computed $h_{\min}(t)$ and $h_{\text{anf}}(t)$. The program output, $h_{\text{anf}}(t)$ and $h_{\min}(t)$, is stored in a data file DATA, for subsequent use. Fig. (6.2a) show the functions $h_{\text{anf}}(t)$ and $h_{\min}(t)$ as a function of t . We notice that $h_{\text{anf}}(t)$ is practically limited to $4T$, while $h_{\min}(t)$ extends over $8T$ interval. Program BER2 was developed to carry out the symbol error rate computations, as given by Eqn. (6.36). In this program, we first specify the value of the Lagrange multiplier, λ , and the duration of the impulse response, $h_{d1}(t)$. BER2 then computes $h_{d1}(t)$, according to Eqn. (6.31). The program outputs a plot for the BER vs. (E_b/N_0) , and the minimum squared Euclidean distance. Fig. (6.3) shows a flow chart for program BER2. Several runs were done for the values of $L=4, 6, 8$ and 10 and for each L , a range of values for λ was considered. We have found that the values of $\lambda=0.225, 0.125, 0.115$, and 0.113 yielded almost the best BER performance when used in conjunction with $L=4, 6, 8$, and 10 respectively. Fig. (6.4) show the impulse responses generated for that set of L 's and λ 's. The impulse response of the AMF and that of a fourth order

Fig(6.2): Flow-Chart of Subroutine WAMP, used for the Computation of $h_{mf}(t)$ & $h_{min}(t)$ (according to equ 6.19 & 6.20)

- .L: Length of Data Vector
- f_s : Sampling Frequency Normalized to $1/T$
- N_{max} : Number of Data Permutations
- I: Enumeration on Data Vector Permutation
- $H \& H_s$: One-Dimensional Arrays Containing $h_{mf}(t)$ and $h_{min}(t)$

Fig(6.2a): Impulse Response of the Average Matched and Minimum Energy Filters for TFM

f_s : Sampling Frequency
 M_f : Filter Observation Interval
 λ : Lagrange Multiplier
 M_{max} : Number of Data Permutations
 I : Enumeration on Data Permutations
 D : Squared Euclidean Distance
 BER : Bit-Error-Rate

Fig(6.3): Subroutine BER to Compute Bit-Error-Rate and Minimum Squared Euclidean Distance

Fig(6.4): Impulse Responses of Different Predetection Filters

Butterworth filter are also shown in the same figure.

Increasing L beyond 8 did not cause any noticeable change in the filter responses, due to the fact that $h_{\min}(t)$

and $h_{\text{amf}}(t)$ are limited to an $8T$ time interval,

Fig. (6.2a). The frequency characteristics of the filters in Fig. (6.4), computed using the FFT algorithm, are shown in Fig. (6.5) (all filters are normalized to have unity gain at d.c.).

The BER curves, generated with the filter responses of Fig. (6.4) are shown in Fig. (6.6) for low SNR (<12 dB), and Fig. (6.7) for high SNR. The BER of the optimum QPSK [43] system is also shown as a reference. We notice in Fig. (6.6) that the performance of the TPM system, using a WAMP6 is only 1 dB worse than that of the optimum QPSK - i.e. the WAMP6 performance is almost identical to that of a TPM MLSE receiver for SNR <10 dB. We also note that at $\text{BER}=10^{-15}$, the WAMP6 yields approximately 1.6 dB degradation, as compared to QPSK - in other words, the degradation of the WAMP6 compared to WAMP6, is about 0.15 dB at $\text{BER}=10^{-15}$. Although the average matched filter (AMF) yields good performance at very low SNR, its performance is not acceptable in the practical range of SNR. The degradation in performance caused by the AMF at $\text{BER}=10^{-4}$ is about 2 dB, compared to WAMP6. The performance of the commonly used filters, Butterworth and Raised Cosine [43] was also investigated. For the Butterworth characteristic, a fourth order type was chosen. By changing the cutoff frequency, f_c , it

Fig(6.5): Frequency Response of Different Predetection Filters

Fig(6.6): BER Performance at Low SNR

Fig(6.7): Bit Error Rate Performance at High SNR

was found that the best BER performance is achieved for $f_c T = 0.32$, and is shown in Figs. (6.6) and (6.7) - we note that the degradation at $\text{BER} = 10^{-4}$ is about 1 dB compared to WAMP6, while the asymptotic degradation is 1.6 dB.

For the raised cosine characteristic, we have found that a roll-off factor [43] of 0.5, and a cutoff frequency of $0.35/T$ yielded best BER performance for this class of filters. The result is shown in Figs. (6.6) and (6.7). Notice that the degradation in performance is about 0.2 dB at $\text{BER} = 10^{-4}$, and 0.5 dB at $\text{BER} = 10^{-5}$, with the WAMP6 as a reference.

To complete the picture, we have investigated the effect of the I channel predetection filter on the eye diagrams. For this purpose, a simulation program has been developed. The program follows the following steps:

1. Generates a 256 bit-long Pseudo Random Sequence.
2. Generate the corresponding baseband signal, $\cos \phi(t, \underline{a})$.
3. Convolve $\cos \phi(t, \underline{a})$, with the filter response $h_{dl}(t)$. $h_{dl}(t)$ is normalized such that $\int h_{dl}^2(t) dt = 1$.
4. The filter output is divided into segments. Each corresponds to $2T$ time interval. These segments are output to a digital plotter, one at a time.

A flowchart of this program is shown in Fig. (6.6). The length of the PKS sequence was chosen as a compromise between the computation time, and the degree of randomness of data. The tap settings are made variable to make it possible to obtain different data sequences at will.

Fig. (6.9a) shows the unfiltered eye diagram of a TPM signal, with the premodulation filter impulse response truncated to five symbol intervals. We see that the eye diagram assume the values ± 1 and ± 0.707 at the sampling moments, as expected. Figs. (6.9b - 6.9e) show the effects of a 4th order Butterworth filter as the cutoff frequency is decreased from $1/T$ to $0.325/T$. We note that the eye opening is decreased from 0.707, Fig. (6.9b) to 0.5 (Fig. 6.9e). The horizontal opening also decreases from 1 to 0.85. The eye patterns generated with the WAMP's, using the characteristics of Fig. (6.4), are shown in Figs. (6.9f - 6.9i). The vertical eye opening is 0.41, 0.70, 0.70 and 0.69 for AMP, WAMP8, WAMP6, and WAMP4 respectively. We also note that the zero crossings in the WAMP4 case are less jittery, a fact which may be important in systems where the data clock is recovered from the baseband signal zero crossings. A useful parameter for the judgement on the system performance at high SNR's, is the minimum squared Euclidean distance [6], $d_{\min}$. The squared Euclidean distance was given by Eqn. (6.35). $d_{\min}$ was also computed for all filters considered so far, and the results are shown in Table (6.1):

Fig(6.8): Flow-Chart of Program to Generate Eye-Diagram Following I-Channel Filter

Fig(6.9): Eye Diagrams Following Predetection Filters

a) Butterworth: $f_c T = 4$

b) Butterworth: $f_c T = 1$

c) Butterworth; $f_T = .5$

d) Butterworth; $f_T = .375$

e) Butterworth: $f_c T = .325$

f) AMF

g) NAME 8

h) NAME 6

1) NAME 4

MLE	SPAM	MSKF	AMF	WAMF4	WAMF6	WAMF8
1.57	0.95	0.96	0.784	1.251	1.279	1.31

Table 6.1: Minimum Squared Euclidean Distance for
Different TFM Receivers

To conclude, we have seen that the class of WAMP filters introduced in the preceding sections performs better than any known filter characteristic for the TFM modulation. We have also shown that the raised cosine characteristics can yield good performance, by properly choosing the roll-off factor and cut-off frequency. The performance of a Butterworth filter was found to be 1 dB worse than the WAMP6 at $BER=10^{-4}$. The choice of the arm filters will depend solely on the application - i.e. in some situations, it can be desirable to trade off something in performance for hardware simplicity; in this case the use of a conventional Butterworth filter may be chosen.

6.6. HARDWARE DESIGN AND EXPERIMENTAL RESULTS

The demodulator design followed the block diagram of Fig. (6.6), with the carrier recovery loop as given in chapter 4 and the clock synchronizer of Chapter 5. The I and Q arm filters have been realized with fourth order Butterworth filters with a cutoff frequency of $0.32/T$, where $1/T$ is the data rate. The design of these filters was carried out, according to the theory in [17], using the 356 operational amplifier and regular high tolerance components.

The data detection circuitry have been designed and implemented according to the schematic diagram of

Fig. (6.10); IC 4013 is used to implement the sample and hold as well as the 2T delay. IC 4030 is used for the modulo-2 addition. The carrier recovery loop, arm filters and the data detection circuitry are contained in one printed circuit board. A photograph of the P.C. board is shown in Fig. (6.11). The board contains 40 commercial IC's, and dissipates about 2 watts.

The effect of additive white noise on the system I and Q channel eye diagrams, in addition to the system Bit Error Rate, has been investigated using the measurement setup of Fig. (6.12). The band-limited white noise component is generated as follows:

1. the HP3722 noise generator is used to obtain an infinite length binary random sequence, at a rate 300 Kb/s,
2. the random sequence is X-ORed with the unmodulated carrier at 460.749 KHz and bandpass filtered using the 3202 Krohn Hite tunable filter. The filter passband is set to $2/T$, where $1/T$ is the data rate. The noise component so generated approximates a band-limited white noise.

The modulated signal is also bandlimited, and summed on the noise component using a summing amplifier. The separate filtering of noise and signal was necessary to facilitate the measurement of the noise and signal powers individually. The signal plus noise is then passed through a 4049 CMOS inverter which is used as a hard

Fig(6.10): Schematic Diagram of Data Detection Circuitry

Fig(6.12): Set-up for the Measurements of BER and Eye Diagrams

limiter. The limiter output is then passed to the demodulator board, while the I and Q channel baseband signals are monitored on a 465B Tektronix oscilloscope. The output data, from the demodulator is input to the HP3761 Error Detector, which is clocked by the HP3706A Data Generator.

Fig. (6.13) shows the effect of SNR, within the IF bandwidth, on the system eye diagrams. The I and Q channel eye patterns are shown in Fig. (6.13a), for a SNR=39 dB. The vertical opening on both channels is essentially the same, and is approximately 50%. This compares favourably with Fig. (6.9e). The two channels are not identical; this is basically due to hardware mismatch on the I and Q channels in both the modulator and the demodulator boards. Due to the redundancy of information in Fig. (6.13a), we have chosen to show only one channel in Figs. (6.13b - 6.13f). The effect of decreased SNR on the system eye diagrams is apparent from this figure, and could be summarized as follows:

The vertical opening decreases from 50% at 39 dB to 28% at 3 dB.

The horizontal opening is decreased from 88% at 39 dB to 65% at 3 dB.

One important effect, which could not be photographed, is the effect of carrier recovery loop cycle slipping on the eye patterns. The rate of cycle slipping increases as the SNR is decreased. This fact

Fig. (6,13): Effect of SNR on the Receiver Eye Diagrams

(a): 39 dB

(b): 39 dB

(c): 25 dB

(d): 20 dB

(f): 10 dB

(h): 3 dB

(e): 15 dB

(g): 6 dB

manifests itself as "flicker" in the eye patterns. This effect becomes very serious as the SNR approaches 3 dB. At this point the loop has a strong tendency to loose lock, and that causes a complete closure of the eye patterns.

The range of E_b/N_0 = 6-14 dB (SNR = 3-11 dB) has been chosen for the measurement of the system BER. This choice is affected by the fact that, below 6 dB the loop cannot maintain lock long enough for a BER reading to be taken, and above 14 dB the BER is $\sim 10^{-7}$, and therefore each BER reading would require ~ 17 hours.

Fig. (6.14) shows the measured BER as a function of E_b/N_0 . The theoretical BER curves for a 4th order Butterworth filter and the WAMP6 is also shown for comparison. The degradation increases as the BER increases e.g. the degradation at error rate of 5×10^{-3} is 1.5 dB with the theoretical performance as a reference. The corresponding degradation at error rate 10^{-5} is only 0.57 dB. The increased degradation at high error rates (low SNR) is due basically to the recovered carrier phase jitter. The degradation at high SNR is believed to be due to the hard-limiter in front of the demodulator.

Fig(6.14): Measured System Bit Error Rate as a Function of E_b/N_0

CONCLUSIONS AND FUTURE WORK

7.1 CONCLUSIONS

This thesis contains a comprehensive treatment of several problems which are common to digital radio systems as well as SCPC satellite systems. The study has been carried out at both the theoretical and experimental levels, with good agreement. First, a thorough study of constant envelope, spectrally efficient modulation techniques has revealed that TFM is particularly suitable for the application of digital mobile communications.

A largely-digital TFM modulator has been developed, and excellent agreement between experiment and theory has been achieved.

The carrier synchronization problem has been dealt with in great detail. First, a clock-aided carrier tracking loop has been proposed. It was found possible to reduce the acquisition time for this loop, in the presence of large frequency offsets, without degrading the system performance, by the addition of an AFC path to the loop. The AFC path also makes the loop pull-in range larger. A compact formula for the carrier phase jitter variance has been presented in terms of the received signal energy-per-bit to noise ratio. The agreement between theory and experiment is excellent at high E_b/N_0 . The discrepancies at low E_b/N_0 is largely due to the fact

that the model used for the analysis is linear. A fourth power loop has also been investigated experimentally. This loop yields better jitter performance due to the relative strength of the discrete tone obtained at $4f_c$.

A novel clock recovery loop has been presented. The loop bears some similarities to the data transition tracking loop. However, our system outperforms the DTTL, at SNR's below 15 dB, in terms of the phase jitter variance. For example, the gain in performance is about 6 us and 3 dB at SNR of 6 dB and 10 dB respectively. At high SNR's the two systems yield almost identical performance. An all digital version of this loop has been implemented, and it showed superior acquisition performance, when compared with a binary quantized loop.

A novel theory for the design of a simple, quadrature coherent detector for TFM has been developed. The theory applies to all partial response CPM systems with modulation index $1/2$, which exhibit an open eye diagram. The theory yields a class of Weighted-Average-Matched filters, which are optimum in the mean square error sense. A sub-optimum version of these filters can yield a near-optimum performance at low SNR's. The performance of this receiver is analyzed in great detail for a TFM system, with a frequency pulse duration of 5 symbol intervals. It was found that the receiver performance, in terms of the probability of error, is nearly optimum at low SNR's, while the asymptotic degradation is nearly 0.6 dB. The performance

of a 4th order Butterworth filter, and that of a raised cosine filter have also been presented. A 4th order Butterworth filter is used in the demodulator implementation. The measured bit error rates were found close to the theory at SNR's higher than 8 dB. At lower SNR's the performance was about 1.5 dB worse than predicted. This is largely due to the excessive cycle slipping, and a tendency of the carrier recovery loop to fall out of lock.

Although all experiments were performed on a TFM system, all the analyses presented in this thesis are directly applicable to a wide class of CFM systems with modulation index $1/2$.

7.4 FUTURE WORK

Future work in the area of this thesis may include some, or all of the following points:

1. A comprehensive analysis of the noise performance of the 4th power carrier recovery loop is a worth-while task to be undertaken. We feel that this can be achieved, although it may involve lengthy numerical computations.

2. A detailed analysis of the discrete clock recovery loop, presented in Chapter V, is recommended. The analysis should be directed towards the transient and steady-state performance in the presence of noise.

3. A joint analysis of the dynamics of the clock-aided carrier recovery loop and the clock synchronizer may be attempted. The analysis should be performed in the absence of noise only. Phase plane plots can be a useful tool for this case.

4. The implementation of the class of WAMP filters for TFM can easily be accomplished, using the results reported in Chapter VI. The experimental evaluation of the modem performance with these filters is straight forward using the experimental set-ups of Chapter VI.

5. The application of the concepts developed in this thesis, to other CPM systems with modulation index $1/2$ can be undertaken. A comparative study of the self-synchronization properties is particularly interesting. The computation of the WAMP filters and the analysis of their performance in conjunction with different modulation techniques, may shed some light on the relative behaviour of these techniques. This is particularly important due to the proliferation of the newly reported techniques and the lack of any comparative study that is of practical value.

APPENDIX A

If the incoming signal is phase modulated, the angle of rotation becomes a function of the frequency offset as well as the phase modulation; i.e.

$$\phi_r = \frac{2\pi\Delta f}{f_2} (\theta_k - \theta_{k+1}) \quad \text{A.1}$$

the second term on the right represents the effect of modulation, where θ_k is the phase of the incoming signal at the k th transition of the VCO output. Assuming that $|\Delta f| < \frac{f_2}{4}$, the frequency comparator's output is readily given by:

$$u_{fd} = \Delta\omega + (\theta_k - \theta_{k+1})f_2 \quad \text{A.2}$$

and has a mean

$$u_{fd} = E(2\pi\Delta f + (\theta_k - \theta_{k+1}))2\pi\Delta f + E(\theta_k) - E(\theta_{k+1}) \quad \text{A.3}$$

Note that θ_k and θ_{k+1} are not independent. Using the probability density function of θ_{k+1} conditioned on θ_k , as depicted in Fig. (A.1), we can write

$$E(\theta_{k+1}|\theta_k) = \frac{1}{2}(\theta_k + (\theta_k - \frac{\Delta}{4})) + (\theta_k + \frac{\Delta}{4}) + 1/8((\theta_k - \frac{\Delta}{2}) - (\theta_k + \frac{\Delta}{2})) \quad \text{(A.4)}$$

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Where $\frac{\Delta}{2}$ is the maximum possible phase change, due to modulation during one cycle of the VCO output ($\frac{1}{f_2}$ sec.)

Therefore,

$$E(\theta_{k+1}) = \int P(\theta_k) E(\theta_{k+1} | \theta_k) d\theta_k = E(\theta_k) \quad (A.4a)$$

From Eqn. (A.1) and (A.4) we get

$$\mu_{fd} = 2\pi \frac{\Delta f}{f_2} + E(\theta_k) - E(\theta_k) = \frac{2\pi \Delta f}{f_2} \quad (A.5)$$

The variance of μ_{fd} is given by;

$$\begin{aligned} \sigma_u^2 &= E \left(\left(\frac{2\pi \Delta f}{f_2} + \theta_k - \theta_{k+1} \right) - \frac{2\pi \Delta f}{f_2} \right)^2 \\ &= E(\theta_k^2) + E(\theta_{k+1}^2) - 2E(\theta_k \theta_{k+1}) \end{aligned} \quad (A.6)$$

Using the pdf shown in Fig. (A.1);

$$\begin{aligned} E(\theta_{k+1}^2 | \theta_k) &= \frac{1}{4} \theta_k^2 + (\theta_k - \frac{\Delta}{4})^2 + (\theta_k + \frac{\Delta}{4})^2 \\ &\quad + \frac{1}{8} (\theta_k - \frac{\Delta}{2})^2 + (\theta_k + \frac{\Delta}{2})^2 = \theta_k^2 + \frac{3\Delta^2}{32} \end{aligned} \quad (A.7)$$

Therefore,

$$E(\theta_{k+1}^2) = \int (\theta_k^2 + \frac{3\Delta^2}{32}) P(\theta_k) d\theta_k = \frac{3\Delta^2}{32} + E(\theta_k^2) \quad (A.8)$$

similarly,

$$E(\theta_k \theta_{k+1} | \theta_k) = \theta_k E(\theta_{k+1} | \theta_k) = \theta_k^2 \quad (A.9)$$

Therefore

$$E(\theta_k \theta_{k+1}) = \int \theta_k^2 P(\theta_k) d\theta_k = E(\theta_k^2) \quad (A.10)$$

Using Eqn. (A.6), (A.8) and (A.10) yields,

$$\sigma_u^2 = E(\theta_k^2) + E(\theta_k^2) + \frac{3\Delta^2}{32} - 2E(\theta_k^2) = \frac{3\Delta^2}{32} \quad (A.11)$$

Note that no assumption has been made regarding the actual probability distribution of θ_k .

In Eqn. (A.11),

$$\frac{\Delta}{2} = \left(\frac{\pi}{2}\right) \left(\frac{f_b}{f_2}\right) = 0.055 \text{ rad} \quad (A.12)$$

Therefore,

$$\sigma_u^2 = (0.055)^2 \times \frac{3}{8} = 0.0013 \quad (A.13)$$

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