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Design and Simulation of a Momentum Controller for Odin:

**A Small Satellite with Versatile and
Stringent Pointing Requirements**

by

Nathalie David, B.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering

Department of Mechanical and Aerospace Engineering
Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario

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**Design and Simulation of a Momentum Controller for Odin:
A Small Satellite with Versatile and Stringent Pointing Requirements**

submitted by

Nathalie M. Y. David, B.Eng.

in partial fulfilment of the requirements for
the degree of Master of Engineering

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25 April 1995

ABSTRACT

A three-axis attitude controller for Odin is developed and tested using simulation. Odin is a small Swedish satellite, currently under development. It must be inertially stabilized to $\pm 15^\circ$ in one mode of operation and in the other, scan the Earth's limb with a real-time accuracy of $\pm 6'$ and a reconstructed accuracy of $\pm 2.4'$ (the goal is however $\pm 1.2'$). A review of attitude hardware is given, outlining reaction wheels and magnetorquers as the best set of actuators for this application. The attitude control law is derived, using a momentum and decoupling approach, and the simulation model is developed, using Matlab with Simulink. The controller is tested under flight, noise, delays and disturbance conditions representative of what will be experienced in orbit. The performance of the controller under those conditions and the resulting design considerations are discussed. The impact of using this type of controller on spacecraft operations is also addressed.

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NOTATION AND LIST OF SYMBOLS

For the most part, the notation in this document follows that described in Hughes (1986), with equations written in matrix form. For example, vectors are resolved in a particular frame and represented by a bolded letter. In a few cases the general notation of a vector (with the arrow) will be used instead. Frequently used symbols are listed below.

Selected Abbreviations

LOS	Line of Sight
NEA	Noise Equivalent Angle
SSC	Swedish Space Corporation

English Letters

a	Semi-major axis of the orbit
A_{cm}	3x4 transformation matrix defining the orientation of the spin axis of each wheel, relative to the control frame ($A_{cm} = F_c \cdot S_m^T$)

B	Geomagnetic field vector
e	Orbit eccentricity
E	Eccentric anomaly
F_c	Control frame (with basis vectors $\hat{c}_1$, $\hat{c}_2$ and $\hat{c}_3$ or $\hat{X}_c$, $\hat{Y}_c$ and $\hat{Z}_c$)
F_f	Flight frame (with basis vectors $\hat{f}_1$, $\hat{f}_2$ and $\hat{f}_3$ or $\hat{X}_f$, $\hat{Y}_f$ and $\hat{Z}_f$)
F_i	Inertial frame (with basis vectors $\hat{i}_1$, $\hat{i}_2$ and $\hat{i}_3$ or $\hat{X}_i$, $\hat{Y}_i$ and $\hat{Z}_i$)
F_p	Operating (or reference) frame
F_n	Coordinate frame along the principal axes of wheel n (with basis vectors $\hat{i}_{n1}$, $\hat{i}_{n2}$ and $\hat{i}_{n3}$)
F_c	Vectrix (Hughes, 1986, Ap.B) for the control frame
h_d	Momentum demand sent to the wheels (4-element matrix)
h_{sn}	Angular momentum of wheel n , in the direction of its spin axis, relative to the spacecraft
h_r	Angular momentum of the wheels, relative to the spacecraft, in control axes
h_s	4-element constructed column matrix, each element corresponding to the angular momentum of one wheel, in the direction of its spin axis, relative to the spacecraft ($h_s = [h_{s1} \ h_{s2} \ h_{s3} \ h_{s4}]^T$)
$H_{\oplus}$	Total spacecraft momentum in control coordinates, taken about the composite mass center
i	Orbit inclination
I_m	mxm identity matrix
J_c	Composite inertia matrix (of the spacecraft and wheels), resolved in the control frame ($J_c = F_c \cdot \bar{J}_c \cdot F_c^T$, where $\bar{J}_c$ is the composite second-moment-of-inertia dyadic taken about the composite mass center)

J_m	4x4 constructed diagonal matrix of wheel inertias about their spin axis ($J_m = \text{diag} \{ J_{s1}, J_{s2}, J_{s3}, J_{s4} \}$)
J_{sn}	Polar moment of inertia of wheel (taken about its mass center)
J_n	Moment of inertia of wheel n with respect to any axis transverse to its spin axis and passing through the mass center of the wheel
K_1	Position gain matrix in the attitude control law (3x3)
K_2	Rate gain matrix in the attitude control law (3x3)
K_p	Scalar gain in the wheel speed control loop
$\hat{T}_n$	Absolute angular momentum vector of wheel n (relative to inertial space)
l_{sn}	Absolute angular momentum of wheel n , in the direction of its spin axis
l_s	4-element constructed column matrix, each element corresponding to the absolute angular momentum of one wheel, in the direction of its spin axis ($l_s = [l_{s1} \ l_{s2} \ l_{s3} \ l_{s4}]^T$)
M	Mean anomaly
q_{ci}	Simulated attitude, representing the rotation from the inertial to the control frame
q_{ci}^M	Measured attitude, representing the rotation from the inertial to the control frame
q_{if}	Quaternion describing the rotation from flight to inertial axes
q_{pi}	Reference attitude, representing the rotation from the inertial to the operating frame
δq	Attitude error, in quaternion representation, from the reference (or operating) frame to the control frame
S_m	Constructed column matrix of four unit vectors, each along the spin axis of one reaction wheel ($S_m = [\hat{s}_1 \ \hat{s}_2 \ \hat{s}_3 \ \hat{s}_4]^T$)

$\hat{s}_n$	Unit vector in the direction of the spin axis of wheel n
T_c	External control torques in control coordinates
$(T_c)_{desat}$	External control torque for desaturation, in control coordinates
T_d	External disturbance torques in control coordinates
T_{ext}	Total external torques in control coordinates
T_{mn}	Motor torque applied about the spin axis of wheel n
T_m	Constructed column matrix of four motor torques, each applied about the spin axis of one wheel

Greek Symbols

$\delta\theta$	Column matrix of the three small Euler angles (about axis 1, 2 and 3) describing the attitude variation of the control frame F_c relative to the operating frame F_p
ν	True anomaly
ω	Argument of perigee (one of a set of parameters defining the position of the satellite at a certain time)
$\overline{\omega}$	Spacecraft angular velocity
ω	Spacecraft angular velocity in control coordinates
ω^M	Measured spacecraft angular velocity in control coordinates
$\delta\omega$	Simulated error in angular velocity from the operating condition
ω_m	Speed of wheel n
ω_{dn}	Speed demand to wheel n
ω_s	4-element constructed column matrix of wheel speeds relative to the spacecraft
ω_s^M	4-element constructed column matrix of measured wheel speeds

$\delta\omega,$	Simulated variation in wheel speeds from the biases
Ω	Right ascension of the ascending node of the orbit
Ω	Reference or commanded spacecraft angular velocity in F_p coordinates, ie. rotation of the operating frame F_p relative to inertial space
$\Omega,$	4-element constructed column matrix of wheel speed biases

To my family

CHAPTER 1. INTRODUCTION

The first part of this chapter outlines the objective of the thesis, the candidate spacecraft, and the step-by-step approach undertaken. The second part provides a brief background on Odin and describes the main aspects of its mission and the requirements they place on the attitude control system.

1.1 Thesis Outline

The recent trend in the space arena is to promote the use of smaller and lower cost satellites¹, with high performance payloads. The overall objective of this thesis is to develop, simulate and evaluate the attitude control system of a small satellite² in the mini-satellite class, with versatile yet very accurate pointing requirements.

¹Since the first satellite launch in 1957, satellites have generally become larger, more complex and longer-lived. This has led to larger and more powerful launcher demands, dramatic increases in costs, and fewer missions. The recent trend, because of low-cost launch opportunities, has been to promote the development of smaller and lower cost satellites (Ward, 1991). Small satellites could be of value in space science research, demonstration of new technologies, training and military applications (AIAA, 1989). The development of spacecraft components, suitable for small satellites, was primarily stimulated by defence interests in the late 1980s (Stuart and Gleave, 1990).

²Mass, volume and cost criteria are generally used to classify a satellite as small. In terms of mass alone, a micro-satellite commonly refers to a 10-50 kg platform, and a mini-satellite to a 50-500 kg platform (Ward, 1991).

The Swedish satellite Odin appears to be a perfect candidate for use as the satellite model. Design drivers for Odin include: low cost, low mass (< 250 kg) and low power budgets, a fast-paced development schedule, and requirements for very high pointing accuracies in various attitude orientations and flight modes.

The initial task in the design of an attitude control system consists in choosing a method of stabilization for the spacecraft and the required hardware. Factors which must be considered are the mission requirements, the vehicle configuration, the anticipated disturbance environment and the budgets. At the time of this study, the data available include a basic vehicle configuration, a baseline orbit, a set of required pointing profiles and a proposed set of attitude hardware (SSC, 1991). In Swedish Space Corporation (1993d) the proposed hardware is listed as:

- Two star trackers,
- Three coarse Sun sensor units,
- Three two-axis gyros,
- Four reaction wheels for control,
- Three magnetorquers for momentum dumping,
- Magnetometers for coarse attitude determination and momentum dumping support, and
- Control electronics and software

The first step of the study is then to review the configuration, orbit and pointing requirements (next section) and verify the suitability of the proposed hardware. To that effect, a review of attitude control methods, attitude hardware used on satellites, and the

selection process is given in Chapter 2. Based on the mission requirements, the satellite must be three-axis stabilized and the proposed sensor and actuator suite is judged appropriate.

The next step is to develop the attitude control law. The general approach is summarized at the end of Chapter 2. As reaction wheels are used, an option is to control the momentum rather than the torque applied to the satellite. The scope of the thesis is restricted to investigating the applicability of momentum control to this satellite, which can be implemented by applying a position plus rate control law to the rate of change of the wheel momentum, and by integrating this law to generate a momentum command. Using a speed control loop around each wheel, this momentum command can then be sent to the wheels in place of a torque command. Assuming a continuous system, the momentum command is therefore generated based on a proportional and an integral term. Such a system is expected to be less sensitive to noise than if derivative compensation is used, and is also expected to be less sensitive to wheel disturbances, because of the speed feedback control (Staley, 1994). Derivation of the basic attitude control law, using a decoupling approach, is presented in Chapter 4.

The following step is to develop the simulation model (see Chapter 3). Matlab combined with Simulink is used to implement the simulation. The principal building blocks include: a model of the plant (rotational dynamics and kinematics of the spacecraft), the actuators, and the environmental torque disturbances; a controller block; a sensor performance block; an orbit generator block; and a commanded attitude and angular velocity profile block.

The next objective is to test, using simulation, whether the controller developed in Chapter 4 will be able to meet the performance requirements. Test conditions representative of what would be experienced by Odin must therefore be defined (see Chapter 5). This involves specifying a set of pointing profiles, test orbits, noise, system delays, disturbances, and errors to be incorporated in the simulation model. The performance of the controller is then tested under those conditions. Controller flaws and limitations are identified, and corrective actions are suggested (see Chapter 6).

The last stage of this study is to review the performance of the attitude control system developed, identify which characteristics on Odin are major factors in the controller design (eg. in the selection of gains), and evaluate the impact of using this type of controller on spacecraft operations (eg. wheel and torque rod biases) and attitude estimation requirements for Odin.

1.2 Odin Overview and Attitude Control Requirements

The Odin mission, orbit, flight modes and configuration are briefly discussed below.

1.2.1 Background and Mission³

Odin is a small Swedish satellite project for astrophysical and atmospheric studies.

³Included in Appendix E is a two-page summary which was produced by the Swedish Space Corporation in the early phases of the project and which outlines the main aspects of Odin and its mission.

A feasibility study was conducted in 1990. A Phase A study was then ordered by the Swedish National Space Board (SNSB), administered by the Swedish Space Corporation (SSC), and completed in 1991.⁴ The Phase B studies were completed at the end of 1993 and the satellite is currently under development. Collaboration with Canada, Finland and France is planned, and the anticipated launch date is late 1997 (Wilson, 1994).

The satellite's payload supports two missions: one in astronomy and one in aeronomy. The scientific instruments include a radiometer, with receivers in four submillimeter and one millimeter wavelength bands, and an optical slit spectrograph.

Odin's research objective in astronomy is to study the interstellar medium by observing key molecules using the onboard radiometer. Targets of interest will include molecular clouds, comets, giant planets and shells around old stars. Spectral line surveys will also be conducted.

Odin's principal research objective in aeronomy is to study the extent of and mechanisms behind ozone depletion. The overall areas of study include ozone chemistry in the stratosphere and mesosphere, the summertime mesopause region and the coupling between atmospheric regions. The scientific instruments include the radiometer and optical spectrograph, which is used primarily in the study of aerosols.

⁴The Phase A study outlined the mission, technical trade-offs, critical areas of development, specifications, budget and development schedule (SSC, 1991). The SNSB is the Swedish government agency responsible for all publicly-funded national and international space programmes. The SSC is a government-owned company responsible for the executive management of space projects. They both operate under the Ministry of Industry and Commerce (Wilson, 1992).

1.2.2 Orbit⁵

The baseline orbit is Sun-synchronous⁶ and circular, and is shown in Figure 1-1. In early documentation (SSC, 1991), the target altitude was 600 km. In more recent documentation, mainly as a result of the increased launcher capability, the target altitude and inclination are given as 700 km and 98.19 degrees (SSC, 1993a). Some of the discussions which follow refer to the 600 km value, used in our initial analysis.⁷

With the ascending node crossing at 18:00 local time, the Sun's rays are nearly perpendicular to the orbital plane. No active orbit control is planned and accurate orbit injection is assumed. The need for a near-polar, circular and low Earth orbit is associated with the aeronomy part of the mission.

Attitude hardware is often shared between orbit and attitude control subsystems, and disturbance torques will arise during orbit maneuvers, which the controller must be able to handle. Since no active orbit control is planned, there are therefore no hardware requirements or orbit control related disturbances to consider. The type of orbit also largely influences the disturbance environment. With this type of orbit (low altitude and high inclination), aerodynamic and magnetic torques are often the dominant

⁵Key drivers for the orbit choice are given in Appendix A (section A.2.2.1).

⁶In a Sun-synchronous orbit, the orientation of the orbit plane stays nearly constant relative to the Sun as the Earth moves along its orbit, i.e. in the ecliptic plane (see Ap. A, section A.2.2.1).

⁷Later tests and assessments are based on the 700 km orbit. Changing the altitude to 700 km mainly resulted in a slightly different disturbance environment (see Ap. A).

environmental effects. In the case of earth-pointing operation, the precession rate⁸ of the orbit and the orbital rate⁹ result in a changing commanded attitude profile which must be tracked, even in the presence of delays. There will also be coupling between the commanded attitude rotation and the spacecraft internal momentum. However, the precession rate of a Sun-synchronous orbit is 0.9856 degrees per day, which is quite small relative to the orbital rate of 3.7 or 3.6 deg/min, depending whether the altitude of the circular orbit is 600 or 700 km.

1.2.3 Flight Modes

The satellite has two missions, with distinct pointing requirements. The principal flight profiles are shown in Figure 1-2, from the Sun's perspective. The orientation of the entire spacecraft is changed¹⁰ to meet the pointing requirements in either mode of

⁸The rotation rate of the line of apsides of an orbit relative to an inertial reference (e.g. the vernal equinox) is often referred to as the precession rate of that orbit. The line of apsides is the line connecting the ascending and descending nodes, which are the points where the orbit crosses the equator from South to North and North to South respectively (see Ap. A, Figures A-2 and A-3). In this context, the precession rate is then the rate of change of the right ascension of the ascending node (Ω), and is given by the equation below when only secular effects due to the Earth's oblateness are considered. As the equation indicates, the precession rate decreases with increased altitude and inclination (up to 90°) and with smaller eccentricity.

$$\frac{d}{dt}\Omega \approx -2.06474 \times 10^{-14} a^{-7/2} (1-e^2)^{-2} \cos i$$

(Equation 3-41d in Wertz, 1978, where a , e and i are the semi-major axis, eccentricity and inclination and a is in km)

⁹The orbital or orbit rate refers to the angular motion of the satellite relative to the Earth center. In the case of a circular orbit, the orbital rate is given by $360^\circ/P$, where P is the orbital period.

¹⁰The use of an articulated platform, instead of rotating the entire spacecraft, could have led to smaller control torques during scanning activity. Articulated platforms can provide high pointing accuracy and conduct more stable and accurate scanning operations (Larson and Wertz, 1992). However, such platforms usually add significant weight and cost, and occupy a larger space. Volume is constrained on Odin and equipment layout is being carefully planned to maximize the use of available space. The fields of view of equipment on articulated platforms are also vulnerable to shadowing by other components under

operation.

1.2.3.1 Astronomy Sessions

For astronomy sessions, the spacecraft is oriented so that the radiometer points at one celestial target (Figure 1-2a). In selected cases, continuous scanning of the target may be carried out instead, at a maximum angular velocity of 0.6 deg/min for the 119 GHz channel and 0.13 deg/min for the four other radiometer channels. Variants of this profile are discussed in Chapter 5. In Swedish Space Corporation (1991), the requirements for the astronomy mode include:

- (i) A pointing control and knowledge accuracy of ± 15 arcseconds (1σ).¹¹
- (ii) A full pointing accuracy maintained for up to sixty minutes,
- (iii) No seasonal constraints (i.e, with/without eclipse conditions),
- (iv) The ability to handle multiple celestial targets per orbit, and
- (v) A 25 degree angle limit¹² between the normal to the solar panels and the Sun vector (to shield components from the Sun), also applicable to the aeronomy mode.

While pointing accuracies of one degree or less are common for small satellites

various lighting conditions.

¹¹An arcsecond is an angular measurement unit, also represented by the symbol ". Sixty arcseconds are equivalent to one arcminute and sixty arcminutes are equivalent to one degree. The symbols for arcminute and degree are ' and °. An accuracy of $\pm 15''$ therefore corresponds to $\pm 0.0042^\circ$.

¹²In the Progress Design Review documentation (SSC, 1993d), the maximum Sun incidence angle allowed, relative to the normal of the solar array, had been increased to 30 degrees.

(Siahpush and Gleave, 1990), the 15" pointing accuracy requirement for the control system in this mode is quite stringent.

1.2.3.2 Aeronomy Sessions

During aeronomy sessions, the spacecraft pitches at a rate of one revolution per orbit to track the Earth's limb¹³ (Figure 1-2b). Superimposed on this limb tracking profile, the spacecraft performs a scanning motion in pitch (i.e., in the orbital plane), so that the line of sight (LOS) of the instrument scans the horizon between 15 and 120 km above the Earth's limb¹⁴ at a rate of 40 scans (20 upwards and 20 downwards scans) per orbit. For some measurements, the satellite will point the payload towards the polar regions during their respective winter periods, and will therefore perform a scanning motion out of the orbital plane. Variants of this flight profile are described in Chapter 5.

Pointing and reconstructed accuracy requirements are listed in Chapter 4, Table 4-1. During aeronomy events, the most stringent demand is in pitch, with a required pointing accuracy in real-time of $\pm 6'$ (360") and a required reconstructed pointing knowledge of $\pm 2.4'$ (1σ). The goal is however to achieve an even better pointing knowledge of $\pm 1.2'$. As later indicated in Table 4-1, pointing requirements for the aeronomy mission are also given in terms of the altitude variation in the

¹³This refers to the edge of the apparent disc of the Earth, as seen from the satellite.

¹⁴Assuming a 600 km circular orbit and a spherical Earth, and using the Earth's mean radius value of 6371 km, the satellite altitude scanning limits of 15 and 120 km correspond to 0.30 and 2.43 degrees relative to the limb. For a 700 km orbit, these altitude limits correspond to 0.28 and 2.24 degrees relative to the limb.

measurement plane.¹⁵ The inertial slew rates¹⁶ about the pitch axis will range from + 5 to - 1 deg/min, while slew rates about the other axes of the spacecraft will be of the order of +/- 1 deg/min.

Although the pointing accuracy requirement during aeronomy sessions is more relaxed than during astronomy measurements, the control system is subjected to a changing commanded attitude profile which it must be able to track, even in the presence of system delays. This is specially relevant to continuous scanning (at scan reversals) and stepwise scanning activity.

1.2.4 Vehicle Configuration¹⁷ and Internal Sources of Disturbances

Although alternate launchers are being considered, Odin has been configured for launch on the Pegasus vehicle with a target satellite mass smaller than 250 kg. An early configuration is illustrated in Appendix E. The satellite is a small platform, flying in a near-polar orbit, with an array of solar panels and Sun shields facing the Sun and protecting the equipment, and with the boresight of the radiometer antenna slightly tilted relative to the plane of the array. Proposed thermal control is mostly passive. The only

¹⁵For example, for a spherical Earth of 6371 km radius, variations in the altitude pointed at of +/- 1 km and +/- 2 km equate to angular variations in the instrument's boresight attitude of +/- 1.2' (or 0.02°) and 2.4' (or 0.04°) for a 600 km orbit, and +/- 1.1' and 2.2' for a 700 km orbit.

¹⁶The larger slew rates in pitch are associated with the limb tracking and scanning motion of the satellite. A circular orbit at 600 km altitude has a period of 96.69 minutes. For such an orbit, the vector in the orbital plane from the satellite's position to the Earth's limb has an inertial slew rate of approximately +3.7 deg/min in pitch. The superimposed scanning motion consists of approximately 40 scans per orbit between 15 and 120 km above the Earth's limb, with a slew rate relative to the limb of +/- 1 deg/min. The resulting inertial slew rate is +4.7 deg/min during downwards scans, when the limb tracking and scanning are in the same direction, and +2.7 deg/min during upwards scans. For a 700 km orbit, the period is slightly longer (98.77 minutes) and therefore the inertial slew rates are slightly smaller.

¹⁷Key drivers for the satellite bus configuration are listed in Appendix A (section A.2.1).

active cooling, associated with cooling of the radiometer mixers and amplifiers to 80K, will be provided by a Stirling cycle mechanical cooler.

The controller must be able to handle external and internal disturbances. Environmental disturbances are a function of the orbit and the orientation of the satellite, but are also largely dependent on the physical characteristics of the satellite. A model of the spacecraft and of the environmental disturbances is therefore required to quantify the disturbance environment (see Appendix A), and also to simulate the spacecraft dynamics. Our model, shown in Figure A-1 of Appendix A, is based on early drawings of Odin (SSC, 1991). Assumed mass center, inertia and surface characteristics are listed in Table A-2.

Internal sources of disturbance for Odin include the vibrations associated with the mechanical cooler, and the deformations of the body and of the solar panels as a result of temperature variations or flexibility.

1.2.5 Attitude Control System Design - Principal Considerations

Based on the mission overview given above, the principal considerations which affect attitude control system design for Odin include: the various pointing profiles of the astronomy and aeronomy modes; the stringent 15" pointing accuracy during astronomy sessions; and the tracking of the commanded attitude to the required accuracy, especially during aeronomy scanning operations. Also important are the disturbances, mass, volume and cost factors.

Figure 1-1. Baseline Orbit for Odin. (SSC, 1993a)

Figure 1-2. Principal Flight Profiles.
(Other flight profiles for each mode are outlined in Chapter 5.)

CHAPTER 2. ATTITUDE HARDWARE AND CONTROL LAWS

This chapter discusses the choice of attitude control hardware, describes how each selected component operates, and outlines the types of attitude control laws used on satellites. No orbit control is planned for Odin and therefore no maneuvering thrusters are required.

2.1 Selection Process

Various methods of attitude control are used in satellites, including gravity-gradient, magnetic, spin stabilization and three-axis control. Actuators include thrusters, gravity-gradient torquers, magnetic torquers and various types of flywheels. Attitude sensors include horizon sensors, Sun sensors, star sensors, magnetometers, gyroscopes, inertial measurement units and directional antennas.

The selection of the method of attitude control and of the sensors and actuators is an iterative process. Initial choices are based on the type of orientations that the spacecraft must sustain, the accuracy at which the attitude must be controlled or known, whether the spacecraft must be able to slew and at which rate, and whether the satellite must be capable of orbit maneuvers (Larson and Wertz, 1992). The final choices also

take into account the disturbances, mass, volume, power and cost constraints, and other criteria such as reliability and satellite lifetime.

2.1.1 Choice of Attitude Control Method

A suitable method of attitude control must first be determined. The options are two-axis or three-axis control, providing moderate or high pointing accuracy. Attitude control methods are discussed in Larson and Wertz (1992). Basic two-axis control methods are described below, but can be combined to achieve greater control accuracy or three-axis control:

- (i) Pure gravity-gradient control makes use of the natural tendency of a long object to align itself with the gravitational field, and provides passive control about the orbital pitch and yaw with limited accuracy;
- (ii) Magnetic control makes use of the natural tendency of the magnetic dipole in a spacecraft to align itself with the geomagnetic field, and provides passive control about two axes with limited accuracy.
- (iii) Pure spin control or dual spin control, where one section of the satellite is despun, use gyroscopic stiffness for passive control of the spinning portion about two axes with limited accuracy. Periodic active control is usually required to restore the attitude of the spacecraft to its nominal orientation. Three-axis control can be applied to the despun portion, when it is equipped with adequate sensors. The pointing capability and

limitations are then a function of the joint between the spun and despun sections.

Three-axis stabilization requires active control using thrusters or a combination of reaction, momentum or gimballed wheels and external actuators. Control can be achieved by keeping the internal momentum of the spacecraft nominally at zero, or by purposely keeping it at a nonzero level. Control accuracy about each axis depends on the type of actuators used. For example, a single momentum wheel, with some speed control but otherwise fixed relative to the body axes, will provide accurate control about the spin axis (which will tend to align itself with the orbit normal), but marginal control about the two orthogonal axes (via gyroscopic stiffness).¹⁸ Examples of control systems with potentially accurate control about all three axes are listed below:

- (i) A minimum set of six fixed thrusters can be used to control the satellite about all three axes and keep the angular momentum of the spacecraft at zero. Control accuracy is achieved at the expense of the time interval between firings and is ultimately limited to avoid continuous firing;

¹⁸Common momentum-based systems used on small satellites, for moderate to accurate three-axis control, are given in Siahpush and Gleave (1990). The list includes the following:

- Single momentum wheel/damper system (active control in one axis, gyroscopic stability in other two),
- Pitch momentum wheel/thruster system (offset roll-actuated control torque in roll and yaw, without direct yaw sensing),
- Single-gimbal momentum wheel/desaturation mechanism,
- Pitch momentum wheel/yaw reaction wheel system (with/without additional desaturation mechanism),
- Double-gimbal momentum wheel/thruster system,
- Three-reaction wheel/desaturation system, and
- Two offset horizon scanner/momentum wheel devices, with the spin axes in the yaw-pitch plane.

- (ii) A minimum set of three reaction wheels, orthogonally mounted and nominally operated at zero speed, can be used to provide accurate control about all three axes by storing angular momentum. They are periodically unloaded and brought back to zero speed, to restore the internal momentum of the spacecraft to zero. Control accuracy is affected by internal disturbances, such as friction and stiction at zero speeds. Wheels can also be nominally operated at nonzero speed (momentum wheel mode). In this case, control is achieved by building-up and maintaining a nonzero internal momentum and steering the angular momentum vector to counteract external torques;

- (iii) A minimum of one double-gimballed wheel, with limited speed control, nominally operated with a large angular momentum, can provide high control accuracy in all three axes. When no external torques are applied, the internal momentum vector remains fixed relative to inertial space. When external torques are present, a counteracting torque is applied by steering the momentum vector such that a torque sensed about one of the gimbals is absorbed by changing the angle (rotating) about the orthogonal gimbal. Control about the spin axis is achieved by changing the wheel speed. Unloading by means of external actuators is again required to restore the wheel speed to its nominal level of operation and the gimbal angles to zero. Control accuracy is limited by the friction experienced in the gimbals, and the friction effect opposing the change in wheel speed; and,

- (iv) A minimum of two double-gimballed wheels, operated at constant speed,¹⁹ can absorb and generate torques about all three axes by steering the combined momentum vector. Periodic momentum unloading is required to restore the gimbal angles to zero. Control accuracy is limited by the friction effects experienced in the gimbals.

Odin will be required to switch back and forth from an Earth-referenced mode of operation, during which it performs a scanning motion or holds a specified orientation relative to the horizon, to an inertial mode of operation, during which it is required to point at or scan various celestial targets. The various pointing requirements dictate that the satellite be three-axis stabilized. The required control accuracy then determines which type of actuators are needed.

2.1.2 Choice of Actuators

As previously discussed, potential actuators for accurate three-axis control include thrusters and flywheels, with or without gimbals or speed control, used in conjunction with external actuators for momentum unloading. For the astronomy part of the mission, the most demanding pointing accuracies occur during staring sessions, with the satellite in a fixed inertial orientation. In that mode, absolute and reconstructed pointing accuracies better than ± 15 arcsec are required about pitch and yaw. In the limb-referenced mode, the pointing accuracy about the pitch axis is the most demanding. The reconstructed pointing accuracy must be better than ± 2 arcmin or ± 2 km altitude

¹⁹A single or double-gimballed wheel spinning at a constant rate is often referred to as a control moment gyro or CMG (Wertz, 1978).

and the real-time control accuracy better than ± 6 arcmin or ± 5 km altitude (see Table 4-1).

Thrusters provide less accurate control than flywheels, typically between ± 0.1 to 5 degrees (Larson and Wertz, 1992), to avoid continuous firing of the thrusters. Other disadvantages listed in Wertz (1978) include the weight of the propellant, which is directly related to the satellite's lifetime, and the mass expulsion torques (associated with gas expulsion, leakage, misalignments, plume impingement).

Typical control accuracies for reaction and momentum wheels are of the order of ± 0.001 to 1 degree (Larson and Wertz, 1992). Gimballed wheels are usually heavier, more complex, and more expensive than reaction wheels, but can provide greater torque capability and therefore higher slew rates. As the torques required for slewing usually exceed those required for disturbance rejection (as shown in the computations in Appendix A), they define the minimum torque capability of the system, when on-orbit plane changes are not part of the mission.

It was first assumed that a 64 degree slew in ten minutes under maximum acceleration conditions (bang-bang control) was indicative of the largest maneuvering demand. This defines an acceleration of $\pm 1.24 \times 10^{-5} \text{ m/s}^2$, a maximum slew rate of about 0.21 deg/s, and a required torque capability of the order of $1.24 \times 10^{-5} (I_{\max}) \text{ N-m}$, where $I_{\max}$ is the estimated maximum moment of inertia of the satellite. If $I_{\max}$ for Odin is estimated to be below 100 kg-m^2 , the required torque capability is below $1.24 \times 10^{-3} \text{ N-m}$. Reaction wheels are then sufficient, since they typically generate maximum torques of 0.01 to 1 N-m, whereas control moment gyros are typically used

to generate torques in the 25 to 500 N-m range (Larson and Wertz, 1992). If the time for the maneuver is reduced to two minutes, the required torque capability increases to 0.03 N-m for the same I_{max} , still well within reaction wheel capabilities.

Flywheels must be used in conjunction with thrusters or magnetic torquers to restore the internal momentum of the spacecraft to its nominal level, when the wheel speeds have reached saturated levels. Thrusters can generate large torques at any point in the orbit and therefore complete momentum unloading²⁰ faster than magnetorquers can. Onboard propellant, however, limits the satellite's life and thruster operation can be detrimental to nearby spacecraft components. Magnetorquers use more power and are less efficient, as their output is a function of the magnitude and direction of the geomagnetic field at the satellite's position. However, they are generally lightweight, require no fuel, and are most efficient in a polar orbit as the geomagnetic field's magnitude is greatest at the magnetic poles, and its direction varies throughout the orbit. The mass and volume budgets for Odin will be more difficult to meet than the power budget. Magnetorquers are therefore better suited to the application.

An optimum set of actuators for Odin would therefore include three reaction wheels mounted orthogonally, plus a fourth for redundancy, and three magnetorquers, with a secondary coil in each for redundancy. The sizing of the reaction wheels and magnetorquers depends on the disturbance environment and required slewing capability. Sizing of the actuators is included in Appendix A.

²⁰Momentum unloading or desaturation of the flywheels consists of applying an external torque to the satellite and a corresponding counteracting internal torque to the wheels, to reduce the wheel speed to zero or to its nominal level of operation. $(T_{ext})_{desat}$ and the rate of change of the absolute momentum of the wheels cancel out, so that the net external desaturation torque on the satellite is zero.

2.1.3 Choice of Sensors

Two classes of onboard sensors are used to determine the attitude and position of a satellite. Reference sensors provide an absolute measurement relative to an Earth-referenced vector or to an inertial-referenced vector at the time of measurement. Inertial sensors keep track of the rotational and/or translational motion of the satellite relative to an earlier reference in inertial space. Inertial sensors cannot be used in isolation. Reference sensor measurements are needed at the beginning of the mission to compute absolute attitude, and absolute attitude updates are required thereafter as the output from inertial sensors drifts over time. One must first choose one or more reference sensors that will provide a sufficient number of measurements with the desired accuracy, supplement with inertial sensors if appropriate, add the sensors without which the chosen actuators cannot operate, and finalize the set with sensors which will provide redundancy and support other phases of the mission, such as initial attitude acquisition.

Usually a reference sensor outputs either an arc-length measurement, which corresponds to the angle between a spacecraft axis and an external vector, or a rotation angle about a spacecraft axis between two external vectors (Wertz, 1978).²¹ At least two measurements of either type are needed to define the attitude of a single spacecraft axis, yielding in some cases multiple solutions.²²

²¹The external vectors are, for example, the line of sight of a star, the Sun or nadir or the magnetic field vector B . The orientation of the spacecraft axis is known in body coordinates, and the orientation of the external vectors are known at the time of measurement with respect to inertial, Earth-referenced or orbital coordinates. Each measurement is provided with a certain degree of accuracy.

²²For example, the measurements required to determine the orientation in space of the spacecraft axis $\hat{A}$ could include an arc-length measurement between $\hat{A}$ and the Sun direction and an arc-length measurement between $\hat{A}$ and nadir. Single axis attitude determination is discussed in Wertz (1978, Chapter 10).

To define the three-axis attitude of the spacecraft, measurements involving a minimum of two nonparallel external vectors are needed, as in the single-axis attitude determination case. The direction of one of these vectors must however be fully known in spacecraft coordinates (e.g., the output from a two-axis Sun sensor). In other words, a minimum of three measurements are needed, two to locate the direction of a single spacecraft axis, and one more to determine the attitude about that axis. Usually, for three-axis attitude determination, measurements are made so that at least two external vectors are fully resolved in a spacecraft body coordinate frame (i.e., overdetermined problem). A rotation matrix approach can then be used to determine the attitude (Wertz, 1978, section 12.2.2).

For Odin, the object is to point the boresight of the two payloads (radiometer telescope and optical spectrometer) accurately and to keep the spectrometer slit parallel to the horizon (roll control).

2.1.3.1 Main Sensors

Star sensors provide the most accurate reference measurements, at the expense of computational time, mass and cost. Typical attitude accuracies are of the order of 1 arcsec to 1 arcmin (Larson and Wertz, 1992). However, very accurate star sensors also have small field-of-views, must be protected from direct Sun illumination and can be occulted from their targets by the Earth depending on the satellite altitude and orientation.

Sun sensor technology continues to improve, with state-of-the-art sensors now

providing better than 0.01° ($36''$) accuracy (Prospace, 1993). However, depending on the orbit inclination, altitude and eccentricity, the satellite experiences eclipse periods when no measurements are possible. In a 600 km circular Sun-synchronous orbit, the satellite will be eclipsed from the Sun on approximately 98 days (the duration of the eclipse ranging from less than a minute to about 21 minutes).

The most accurate Earth sensors would provide direct measurements in roll and pitch with sufficient accuracy during atmospheric measurements, but would be inadequate during astronomical observations.

Assuming no viewing limitations, two star sensors, each with at least two-target tracking capability, would provide the coordinates of two star vectors in a spacecraft frame, with redundancy for one star tracker failure.

Because of computational times and viewing limitations especially in the aeronomy mode, star or Sun sensors alone cannot provide the required control accuracy. Inertial sensors are required to determine attitude between star data updates. The maximum drift rate and the accuracy of the inertial sensors are a function of the largest anticipated time interval during which no star sensor measurements will be available, and the required control accuracy during that time interval.

Because of the proven accuracies of star sensors, and the desired redundancy for one reference sensor failure and one inertial sensor failure, the preferred option for reference measurements is then two star sensors with multiple target capability (two-target capability at least) and three two-axis gyros, with the star sensor boresights

oriented to minimize misalignment effects and to maximize observation times in the best compromise between the two modes of payload operation.

2.1.3.2 Secondary Sensors

Medium resolution Sun sensors are required to monitor the spacecraft orientation relative to the Sun, as the solar panel normal must stay within a 25-30 degrees half cone angle relative to the Sun vector. Low accuracy measurements in 4π steradians²³ are also required to assist or provide redundancy during initial attitude acquisition and recovery modes.

Magnetic field measurements about all three axes are required to condition commands to the magnetorquers during momentum unloading. A set of four single-axis, three double-axis, or two three-axis vector magnetometers would provide sufficient redundancy during desaturation. They can also assist or provide redundancy during initial attitude acquisition and recovery modes.

2.2 Operating Principles

2.2.1 Reaction Wheels

Reaction wheels are flywheels which are, in general, nominally operated with

²³One steradian is defined as the solid angle which cuts an area, on the surface of a sphere, equal to the square of the radius of the sphere. The surface of a sphere is therefore equal to 4π steradians or sr units.

zero angular momentum (i.e., at zero-bias). The axis of rotation of reaction wheels is fixed with respect to the body of the vehicle. A built-in torque motor accelerates or decelerates the wheel, changing the internal momentum of the spacecraft.

Over time, torques are either periodic, and result in cyclic variations in the momentum, or do not average to zero over the course of one orbit and therefore result in a secular (steady increase with time) momentum change. The wheel speed increases and decreases to counteract cyclic effects. The level about which the wheel speed cycles increases to compensate for secular effects, until the wheel speed waveform approaches saturation levels. Another actuator is then used to produce an external torque and restore the wheel speed to zero.

Sizing of the reaction wheels and sample characteristics are provided in Appendix A. Sizing is based on initial estimates of slewing rates and spacecraft mass and surface properties.

A reaction wheel is basically the rotor in a motor operating with no load. Torque balance equations for motors are given in several reference books including Tapley (1990) and Wertz (1978). The dynamics of the wheels, assuming that the power is provided by a DC motor, are developed in Chapter 3.

2.2.2 Magnetorquers

Magnetic coils, discussed in Wertz (1978), are also called magnetic torquers, magnetorquers, electromagnets, or torque rods. Most frequently, they consist of

magnetic coils, enclosing an air-core. The current flowing through the coils generates a magnetic moment along the coils longitudinal axis, which when scaled by the permeability of the core material, corresponds to the dipole moment produced by the torquer. The magnetic dipole moment interacts with the Earth's magnetic field²⁴ to generate an external control torque on the spacecraft. The control torque is given by:

$$\mathbf{N} = \mathbf{d} \times \mathbf{B} \quad (2-1)$$

where $\mathbf{d}$ is the magnetic dipole moment in A-m² and $\mathbf{B}$ is the geomagnetic flux density in Wb/m².

For redundancy, magnetorquers sometimes include two half-coils connected in series during normal operation, as was done for ROSAT (Baldwin, 1987). The halves can then be disconnected if one fails, and the same magnetic moment can be generated by applying twice as much current in a single half. Within a specified linear range and accuracy, the magnitude of the dipole moment is proportional to the input current.

²⁴The magnetic field of the Earth is mainly that of a single dipole, in a sphere of uniform magnetization, with the dipole's orientation slowly changing with time (Wertz, 1978). Gauss showed in 1839 that the geomagnetic field can be represented in large part as the gradient of a potential function expressed as a series of spherical harmonics (Langel et al., 1982). Work has been done to develop standard models of the Earth's main magnetic field (Peddie, 1982). Such work continues, and the International Geomagnetic Reference Field (IGRF) coefficients are periodically updated. Computational issues associated with field models are given in Wertz (1978, Ap.H).

If each torquer comprises two coils, then the value of the dipole moment generated is given by (Baldwin, 1987):

$$\begin{aligned} d &= m_0 n \text{ if bang-bang control is used, or} \\ d &= (A_1 s_1 + A_2 s_2) I p n = k_1 K I p n \text{ if proportional current control is used} \end{aligned} \quad (2-2)$$

where

$A_1 = \mu N A_1$ is the effective area enclosed in the first coil,

$A_2 = \mu N A_2$ is the effective area enclosed in the second coil,

μ is the permeability of the core material ($\mu = \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ for free space),

I is the current flowing in the coil,

N is the number of turns in the coil,

p is polarity i.e., direction of current flow (-1 or 1),

s_1 and s_2 are binary values indicating whether each coil is included in the circuit or not i.e., 0 for disconnect state and 1 for connected state,

n is a unit vector in the direction of the coil axis (normal to the plane of the loop),

k_1 is the number of coils connected,

K is the scale factor ($\text{A-m}^2/\text{A}$),

$$I = V (k/k_{\max}) / (r_1 + r_2),$$

k can vary between $+k_{\max}$ to $-k_{\max}$ in the linear region,

$k_{\max}$ is the maximum number of command levels associated with one of the rods,

V is the voltage of the power supply connected across the coils (varying within a given range),

r_1 and r_2 are the resistances across the coils (varying with temperature), and m_0 is the saturation moment.

A small residual dipole is present when power to the torque rod is off or when a torque rod has failed. Inaccuracies are associated with voltage and temperature variations.

Sizing of the torque rods for Odin and sample torque rod characteristics are provided in Appendix A. Based on initial estimates of spacecraft mass, properties, and secular torques, three torque rods of 40 to 50 A-m² moment each are required.

2.2.3 Magnetometers

There are two categories of magnetometers, again discussed in Wertz (1978). Quantum sensors use fundamental atomic properties. Induction magnetometers are based on Faraday's law.

Some magnetometers have been designed for use on small satellites where weight, size and power usage must be minimized (Ithaco, 1991 and 1992). In general, magnetometers are lightweight, reliable, operate over a wide range of temperatures, have no moving parts, but have limited accuracy. Typical accuracies are listed as 0.5 to 3° in Larson and Wertz (1992), when used for attitude determination. At higher altitudes, their use is limited as spacecraft residual dipoles start to dominate the external field measurements.

In Wertz (1978), the relationship between the actual field strength and the sensor

voltage measurement for a single axis fluxgate magnetometer is given by:

$$V = a(n^T H) + V_0 \quad (2-3)$$

where a is the scale factor (in Wb/V), n is the unit vector along the measurement axis, H is the local field vector and V_0 is the magnetometer bias voltage. The analog measurement is digitized using an A/D converter, and the value of the field along that axis can be computed by inverting Equation 2-3 (taking into account digitization effects).

The type of information provided by the manufacturer includes: the number of measurement axes, the degree of orthogonality and linearity, the field measurement range, the scale factor, the zero field bias, the RMS noise, the frequency response, the supply voltage, the power consumption, the temperature range, the dimensions and the mass (see Ithaco, 1991 and 1992).

2.2.4 Sun Sensors

There are three types of Sun sensors. Analog Sun sensors output a signal which is a continuous function of the Sun angle. Sun detectors indicate the presence of the Sun with a constant signal. Digital Sun sensors are the most accurate and output a discrete value which is a function of the Sun angle.

In the case of Odin, medium resolution analog Sun sensors will be used to monitor the orientation of the solar array relative to the Sun vector and to assist during initial attitude acquisition or recovery from a failure mode.

The basic detector of a common analog Sun sensor is a silicon photocell which

generates a current nearly proportional to the cosine of the Sun incidence angle. There are usually two detectors per axis. When at ± 45 degrees relative to the sensor boresight, the difference of the output of the two cells is nearly proportional to the Sun angle in the ± 45 degrees range. At larger angles, the difference indicates whether the Sun is facing the positive or the negative detector (i.e., the angle polarity about the measurement axis relative to the boresight). Such sensors are manufactured by Sodern (Prospace, 1993). Wide angle analog Sun sensors, in which up to three detectors are mounted on each face of a small pyramid, are manufactured by Lockheed (1990).

2.2.5 Gyroscopes

Gyroscopes are inertial sensors. Between calibrations, the gyroscope outputs are affected by a random drift. The output degradation between calibrations is minimized by using estimation techniques. A gyroscope is basically a spinning body - a disc, wheel or rotor - symmetric about the spin axis, with an angular momentum of fixed magnitude. In the absence of external torques acting directly on the gyro, and assuming no bearing friction and massless gimbals, the angular momentum vector of the gyro (parallel to the spin axis) will maintain a fixed direction and magnitude with respect to inertial space. This property is referred to as momentum stiffness (Kaplan, 1976).

Depending on the gyro configuration, the spin axis can change its orientation about one, two or sometimes three degrees of freedom relative to the gyroscope casing. The degrees of freedom are achieved by means of mechanical gimbals, which are now floated to remove bearing friction, by suspension in a fluid, an electrostatic or magnetic field, or by the use of flexural (or torsion) members. When mechanical gimbals are

used, a third degree of freedom ensures orthogonality of the other two and gimbal lock is avoided.

When the gyroscope casing is fixed to the rotating vehicle or component for which attitude is being monitored, the gyro is said to be operating in a strapdown mode. When one or more gimbals are present between the casing and the vehicle, the gyroscope is said to be operating in a gimballed mode.

On Odin, three two-axis dynamically-tuned gyros will be used. Dynamically tuned gyros or dry-tuned rotor gyros (DTGs) are two-degree-of-freedom gyros, where the rotor is attached to one gimbal by torsion bars, the gimbal is attached to the casing by another set of torsion bars, and the shaft rotating speed has been set to cancel the elastic restraint torques (tuning condition) (Mansour & Lacchini, 1993). The deflection about each input axis is nulled by applying current to magnetic torquers about the orthogonal axis. The rebalance current is proportional to the rate of the spacecraft about the gyro input axis. The time integral of the current is very nearly proportional to the angular displacement of the spacecraft about the input axis, for small angular displacements. The output of the gyro can then be given as a train of pulses, each pulse corresponding to a spacecraft angular displacement increment.

In Wertz (1978), the gyro rate measurements are related to the angular velocity as

$$\omega_g^M = KU\omega + b_g + n_g \quad (2-4)$$

where

ω_g^M is the column matrix of angular velocity measurements (i.e. about each gyro

axis),

ω is the spacecraft angular velocity,

U is the matrix of direction cosines of the gyro input axes with respect to the spacecraft frame of reference,

K is the matrix of linearity coefficients,

b_g are the bias drift rates about each gyro axis, and

n_g are white noise estimates.

2.2.6 Star Trackers

Star sensors, the most accurate of attitude sensors, are capable of accuracies in the arcsecond range. They are however usually heavy, bulky, expensive, and the associated software generally requires integration times of several seconds to compute the attitude measurements.

Star sensors can be classified based on the mechanism used to search or track the stars: scanners make use of the spacecraft rotation, gimbaled trackers use mechanical means, and fixed-head trackers conduct electronic scans of the field of view. However, the use of gimbals adds complexity, weight, and other sources of errors (Wertz, 1978).

Fixed-head trackers typically use image dissector tubes (IDT) or charged-coupled devices (CCD) as image detectors. In the first case, the sensor uses optics to focus the star image on the photocathode of an IDT, where electrons are then ejected. By controlling the current in deflection coils mounted next to the drift tube, electrons emitted

from a specific area of the photocathode reach the aperture of the multiplier section where they are counted electronically at the anode end to detect the presence of a star. The search or track pattern is determined by the sensor CPU which sends the current commands. A CCD star sensor uses optics to focus the star image onto an array of solid state detectors from which output is serially sent to a processor. Either the signal of each pixel is compared to a threshold value (search mode), or the centroid of the star position is computed more accurately using the output of adjacent pixels (track mode). CCD star sensors have definite advantages over the IDT sensors, including lower mass, lower volume, better stability to temperature variations and more resistance to the radiation environment (Prospace,1993).

Selection criteria for fixed-head star sensors include: field-of-view (FOV), maximum number of stars simultaneously tracked, accuracy, sensitivity (to star magnitude), sensor response (e.g., star acquisition time, update rate), mass, size, and power requirements.

2.3 Attitude Control Laws

The derivation of the attitude control law for a particular spacecraft starts with the formulation of the equations of motion²⁵. When a rigid-body model is assumed for the spacecraft, the Newton-Euler or vectorial formulation is most often used. The equation describing the rotational dynamics of the spacecraft, with all vectors resolved

²⁵Williams (1976) illustrates the use of the Lagrangian formulation and the Newton-Euler formulation to derive the equations of motion for nonrigid spacecraft, and discusses the merit of the various reference frames and attitude parameterizations in use.

in the same coordinate frame, is then given by:

$$\dot{H}_i = T_{ext} - \omega \times H_i \quad (2-5)$$

where $H_i = H_b + h_w = J_c \omega$, H_b is the momentum of the spacecraft body, h_w is the internal momentum of the system relative to the spacecraft body, J_c is the composite inertia matrix of the spacecraft and wheels, ω is the angular velocity of the rotating frame of reference relative to inertial space, and T_{ext} are the external torques to the system and include control and disturbance torques.

The next step is to choose the reference coordinate frame which we wish to control and with respect to which the vectors in Equation 2-5 will be resolved, and the variables which will describe the attitude history of that frame relative to inertial space. Choosing a frame parallel to the spacecraft principal axes significantly reduces the number of nonlinear and linear terms²⁶ in the expanded equations, by removing all terms containing products of inertia. If, in addition to being resolved along principal axes, there exists an axis of symmetry in the principal frame, the dynamics along that axis will be decoupled and linear.

The angular velocity cannot be directly integrated to provide the time history of the frame attitude. Euler symmetric parameters, direction cosines, and modified Euler angles can be used to describe the attitude of the chosen frame relative to inertial space²⁷. For each of these parameterizations, a relationship can be derived between the body rates and these attitude parameters and their time derivatives. When expressed in terms of

²⁶Linear terms in Equation 2-5 would arise from a constant internal momentum vector relative to the body axes.

²⁷Other parameterizations exist: e.g. Rodrigues parameters (Williams, 1976).

attitude parameters and their time derivatives, Equation 2-5 is then second-order, coupled and nonlinear.

For devising the control law, the modified Euler angles parameterization is standard practice, especially in the case of Earth-referenced motion²⁸. The methodology described below is discussed in Staley (1994). For inertial pointing and in the absence of internal momentum, Equation 2-5 can be linearized and decoupled by performing the required transformation to principal axes, assuming small angles and rates, and dropping second-order nonlinear terms. In the case of Earth-referenced motion, additional coupling terms in the yaw and roll axes will be present as a result of the orbit rate (and cannot be dropped as a result of the small angle approximation). During earth pointing activity, coupling terms will also be present as a result of the precession rate of the orbit. However, for a Sun-synchronous orbit, these coupling terms are expected to be much smaller than the coupling terms associated with the orbit rate. In the presence of internal momentum, Equation 2-5 is often linearized by assuming a fixed internal momentum state in addition to the small angle approximation. The equation is further simplified, if some terms are dropped because considered negligible relative to other terms. However, even when resolved along principal axes and linearized, when internal momentum is present and/or earth pointing is performed, Equation 2-5 will be coupled mainly as a result of the orbital rate and transverse momentum.

²⁸The use of quaternions and direction cosines to parameterize the attitude and the form of the resulting dynamics equations and potential control laws are reviewed in Bach and Paielli (1990). Derivation and simulation of an inertial quaternion regulator for spacecraft eigenaxis rotations is provided in Wie et al. (1989). Neto (1988) reviews the concepts of relative quaternions, which describe the attitude of the satellite relative to the orbital rather than the inertial frame, and linearizing techniques, to apply linear control design to the maneuvering of spacecraft.

T_{ext} includes the control (T_c) and disturbance torques (T_d) to the system. Three principal forms of control torque are used (see Wertz, 1978): position, position and rate, and bang-bang control. Bang-bang control is common when gas jets or magnetorquers are used as actuators. Various forms of bang-bang control exist, depending on the conditions selected to trigger the positive or negative constant effort (i.e., depending on the switching curve definition). Proportional and rate control is the form of control torque most often used, when the control signal is continuous. Coupling terms, especially nonlinear terms when they cannot be considered negligible, are often included in the control torque, in addition to the proportional and rate terms.²⁹

Equation 2-5 can be fully linearized, with coupling terms mostly or all cancelled out, when: T_{ext} is replaced with $T_c + T_d$, T_c is replaced with the control law chosen which includes some or all coupling terms, and some assumptions are made (see p.34). Classical or modern linear control theory can then be applied to solve for the gains in the attitude control loop.

²⁹Commands sent to the actuators are then function of the coupled and nonlinear terms. The single-axis control diagrams in Figures 2-1a and 2-1b show that proportional and rate (i.e., proportional and derivative or PD) compensation becomes lead compensation, if momentum rather than torque commands are sent to the wheels, proportional control effects the rate of change of wheel momentum, and adequate gains are chosen. Lead compensation is of the form $K(Ts + 1)(\alpha Ts + 1)^{-1}$, which combines PD and a first-order pole at frequencies higher than the numerator breakpoint (Franklin et al., 1991).

- (a) Torque commands to ideal actuators based on position and rate (or proportional and derivative) compensation

- (b) Momentum commands to wheels based on integrated position and rate errors, with proportional control of the wheel speed, reducing to lead compensation of the form $K(Ts+1)(\alpha Ts+1)^{-1}$ with α or in this case $|K_1(K_2K_3)^{-1}| < 1$.

Figure 2-1. Block Diagrams for Spacecraft Single-Axis Attitude Control
(The rotational (decoupled) dynamics for a single axis of the spacecraft are given by: $J\ddot{\theta} = T_c + T_d$. Three-axis spacecraft rotational dynamics are discussed in Chapter 4.)

CHAPTER 3. SIMULATION

This chapter describes the simulation tool and block diagrams, and outlines the advantages and disadvantages of the simulation method which is used.

3.1 Simulation Tool

Matlab version 4.1.1 and Simulink version 1.2d for UNIX are used to build the simulation model. Matlab is a software package for matrix computation and includes numerical integration, programming, graphics and signal processing capabilities. Simulink, an extension to Matlab, provides block diagram editing functions and built-in integration tools to simulate dynamic systems. Matlab and Simulink are produced by the company Mathworks.

A simulink model is first constructed to represent the differential equations of the system. Built-in simulation routines (Euler, Runge-Kutta, Adams, Gear or Linsim) are then used to integrate the system states. Outputs can be viewed as the simulation is running or can be stored in matrices in the workspace and later retrieved, processed and plotted using the capabilities of Matlab and Simulink.

Simulink is principally designed to create models as block diagrams in a window environment. Blocks are copied from built-in libraries of blocks. Lines which carry a single or a group of values connect the blocks. The graphical model is saved as an M-file. In subsequent Simulink sessions, invoking the name of the graphical file creates a S-function which is an internal representation of the model to Simulink. The built-in simulation algorithms and analysis tools act directly on this S-function. The Matlab/Simulink user has no access to the internal model associated with the graphical model.

The Simulink tool is described in the software documentation by Mathworks (1992). Each block has internal parameters, which can be specified by double clicking on the block and editing the required fields. For example the gain value is a parameter to the gain block. These internal parameters can be assigned a numerical value or a variable name. At the beginning of the simulation, the variables are initialized according to their value in the workspace. Built-in libraries of blocks include the:

- Connections library
- Discrete library (e.g. *discrete transfer function*, *discrete state-space* blocks)
- Linear library (e.g. *sum*, *integrator*, *state-space*, *gain* blocks)
- Nonlinear library (e.g. *product*, *fcn*, *saturation*, *transport delay*, *Matlab fcn* blocks)
- Sinks library (e.g. *scope*, *To workspace* blocks)
- Sources library (e.g. *step fcn*, *sine*, *clock* (system time) blocks)
- Extras library (e.g. additional more specialized blocks of the

various types listed above, demos)

Some blocks, available from the sources library, provide inputs to the system. For example, constants, step functions and data from the workspace or from specially formatted files can be inserted into the system. Examples of discrete blocks include the *zero-order-hold*, which samples and holds the input for one sample period, and the *unit delay*, which is the equivalent to a *zero-order-hold* which output is delayed further by one sampling period. Examples of blocks in the linear library include the *gain* block, which scales the input, the *integrator* block, which integrates the input, and the *sum* block which adds its inputs. Examples of nonlinear blocks include the *saturation* block, which limits the input to a specified range, the *fcn* block, which applies the specified mathematical expression to the inputs, the *memory* block, which retains the input value at the last integration step, and the *S-function* block, which enables a S-function to be inserted into a model as a block. Blocks in the sinks library record or monitor simulated values. For example, the *To Workspace* block enables data to be written to a matrix in the workspace. Connection blocks include the *inport*, *outport*, *multiplexer* and *demultiplexer* blocks. The latter two enable scalar or vector lines to be joined or separated.

Models or parts of models can be also coded into S-functions using Matlab commands, Fortran or C, and using the proper syntax, compilers, linkers and interfacing code. These nongraphical models are then accessed by means of the *S-function* block. Blocks can be grouped together to form a higher level block, and organize the block diagram into its various functions.

3.2 Simulation Approach : Three Stages

To increase the simulation speed and to compute the initial conditions required at later stages, the simulation is divided into three parts: generate the test orbit data, determine the initial speed biases required, and simulate the controlled dynamics of the spacecraft.

3.2.1 Orbit Data Generation

The following orbit data is required to model the environmental torques and compute the external control torques:

- q_{if} Quaternion describing the rotation from flight to inertial axes
- R Magnitude of the orbit radius (from the center of the Earth to the spacecraft center of mass)
- $V_{r,t}$ Magnitude and direction of the velocity of the center of mass (i.e., magnitude of the radial and tangential velocity components)
- S_i Direction of the Sun vector (resolved in a frame parallel to the inertial frame but with the origin located at the spacecraft center of mass)
- B_f Direction and magnitude of the geomagnetic field (resolved in flight coordinates), and
Sun visibility (i.e., whether the spacecraft is eclipsed from the Sun or not)

For each test orbit, this data needs to be computed only once and stored in a format which can be retrieved during subsequent simulations as required.

The orbit generator model is implemented in Simulink by feeding time into a *Matlab Fcn* block, which accesses the function *orbgen3*, and into a dummy integrator, to force the output of the function block to be computed at each integration time step. The graphical representation of the model is included in Appendix B (Figure B-1). The function *orbgen3* computes a number of orbital parameters, based on the simulation time and on variables passed from the workspace. The Matlab interpreter needs to be called at each integration step, and considerable time savings are achieved by computing the required data once for each test orbit. The data is stored as a matrix, with one row of data per time step. To limit the size of the matrix, the data is stored for each five seconds increment. The workspace matrix variable *orbdats* is then saved, and retrieved in later Matlab sessions.

The computations in the orbit generator model are summarized next.

First, the start time of the simulation (t_s) and the ephemeris for the test orbit are retrieved. The ephemeris, a collection of parameters defining the position of the satellite at a certain time and derived from tracking data, includes:

t_s	Epoch time
M_0	Mean anomaly at epoch time
Ω_0	Right ascension of the ascending node at epoch time
ω_0	Argument of perigee at epoch time
e_0	Eccentricity at epoch time
i_0	Inclination at epoch time
n_0	Mean motion at epoch time.

The orbit parameters $\{a, e, i, \omega, \Omega\}$ are illustrated in Figures A-2 and A-3 of Appendix A. The mean anomaly M in degrees is given by $360 (\Delta t/P)$, where Δt is the

time elapsed since the last perigee crossing and P is the orbital period. The mean motion of a Keplerian orbit is given by $n = (\mu/a^3)^{1/2}$, where μ is given by $G(m_{s/c} + M_E) \approx GM_E = 3.986005 \times 10^{14} \text{ m/s}^2$ and is the Earth's gravitational constant (Wertz, 1978). The mean motion is related to the period by $P = 2\pi/n$.

For the purposes of the orbit simulation, only the first-order secular perturbations are taken into account. The orbital parameters a , e and i are therefore assumed to be constant, while ω , Ω and n are updated to account for the oblateness of the Earth. At time t_s ,

$$\begin{aligned}
 e &= e_0 \\
 i &= i_0 \\
 a &= a_0 \\
 n &= n_0 \left[1 + \frac{\frac{3}{2}J_2(1-e^2)^{1/2}(1 - \frac{3}{2}\sin^2 i)}{a^2(1-e^2)^2} \right] \\
 M &= M_0 + n(t-t_0) \\
 \Omega &= \Omega_0 - \frac{3J_2 \cos i}{2a^2(1-e^2)^2} n(t-t_0) \\
 \omega &= \omega_0 + \left[\frac{3J_2}{2a^2(1-e^2)^2} (2 - \frac{5}{2}\sin^2 i) \right] n(t-t_0)
 \end{aligned} \tag{3-1}$$

(Escobal, 1976, Eqs 10.29 through 10.32)

Kepler's equation is given as $E - e \sin E = M$ and defines the position in the orbit plane in time in terms of the eccentric and mean anomalies. Derivation is included in Escobal (1976). The eccentric anomaly E is the angle, relative to the center of the orbit, tracking the location of the satellite on a circle which radius is equal to the semi-major axis, when the satellite's position is projected outwards in a direction perpendicular to the line of nodes (see Figure A-4). When M is known, Kepler's equation can be

solved explicitly for the eccentric anomaly in terms of a series of Bessel functions, or using a numerical approach with successive approximations. The test orbits are assumed to be perfectly circular ($e=0$), and the eccentric and mean anomaly are therefore equal. The values for the radius, velocity and true anomaly (v) are computed next, from:

$$\begin{aligned}
 R &= a (1 - e \cos E) \\
 \dot{R} &= \frac{(a\mu)^{1/2} e \sin E}{R} \\
 V_T &= R\dot{v} = \frac{[\mu a(1-e^2)]^{1/2}}{R} \\
 \sin v &= \frac{a(1-e^2)^{1/2} \sin E}{R} \\
 \cos v &= \frac{a(\cos E - e)}{R} \\
 v &= \tan^{-1} \frac{\sin v}{\cos v}
 \end{aligned} \tag{3-2}$$

(Staley, 1993, Eqs. of unperturbed two-body problem, and Escobal, 1976, Eqs. 3.267 and 3.268)

The rotation matrix from F_i to F_f is then given by:

$$C_{fi} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} (\omega + v)_3 (i)_1 (\Omega)_3 \tag{3-3}$$

where $(\theta)_i$ indicates a Euler rotation by angle θ about axis i . C_{if} corresponds to the reverse sequence of rotations and is stored in quaternion representation (q_{if}). F_i is the inertial frame, with its origin at the geocenter and the basis vectors $\hat{i}_1$ pointing in the direction of the Sun at vernal equinox, $\hat{i}_3$ pointing at the North pole, and $\hat{i}_2$ completing the right-hand orthonormal triad. F_f is an orbit-referenced frame, with its origin at the undeformed spacecraft center of mass, and with $\hat{f}_3$ along nadir (orbital yaw), $\hat{f}_2$ along the negative orbit normal, and $\hat{f}_1$ completing the dextral triad.

Equations provided in Werz (1978, Section 5.5) are used next to estimate the position of the Sun at the time of interest in inertial coordinates. The Sun vector is then resolved in a frame parallel to the inertial frame but centered at the spacecraft center of mass.

The next step is to compute the Greenwich hour angle, measured eastward between the mean vernal equinox and the Greenwich meridian (Escobal, Eq. 1.27), the declination and the longitude of the satellite at the time of interest. That information, along with the fifth generation set of IGRF coefficients (1990), is used to compute the direction and magnitude of the geomagnetic field (Wertz, 1978, Ap. H).

Spacecraft with altitudes lower than 1.385×10^6 km experience either total, partial or no eclipse at all. Equations to verify eclipse conditions, assuming a spherical Earth, are provided in Wertz (1978, section 3.5) and are also implemented in the orbit generator routine.

3.2.2 Open-loop Simulation: Spacecraft Dynamics and Build-up of Momentum as a Result of Environmental Disturbances

Under the influence of disturbance and control torques, the total angular momentum of the spacecraft will vary. The reaction wheel speeds will be controlled to absorb the momentum change as a result of external torques, except in the case of attitude maneuvers. When the wheel speeds are zero, additional friction effects are present and, as will be seen later, external torques have a larger effect on the spacecraft attitude. In order to avoid zero speed conditions, the wheels can be spun up to an initial

bias level, before payload operations are started. These speed bias levels are initial conditions to the closed-loop simulation.

The required speed bias levels to avoid zero-speed crossings are a function of the spacecraft flight profile and the associated disturbance environment (further discussed in Chapter 5). An open-loop simulation model is therefore developed to monitor the momentum build-up for the various test scenarios (described in Chapter 5) and determine the required initial bias conditions.

The open-loop block diagram for simulating inertial pointing activities is included in Appendix B. An instruction file is run in Matlab to initialize the required variables and matrices in the workspace. The diagram has three principal building blocks. In block G of Figure B-2a (blown up in Figure C-12a), the orbital data for the current test orbit is retrieved from the matrix *orbdatt*, at each integration step, using linear interpolation between the data stored in successive rows. This orbital data had been computed previously and saved in different files for each test orbit.

Blocks H.i define the reference or commanded attitude quaternion and angular velocity of the spacecraft. Five different flight scenarios will be tested using simulation, and are implemented in blocks H.1 through H.5 (included in Appendix C and further discussed in section 3.2.3). A different simulation model is created for each flight scenario, to reduce the number of blocks in a given model and increase the speed of simulation.

Aerodynamic, solar radiation, gravity-gradient and magnetic disturbance torques are modelled in block A.2. The sum of the environmental torques and the commanded reference angular velocity are input into block A.5, where the following spacecraft rotational dynamics are calculated:

$$\dot{H}_t = T_{ext} - \omega \times H_t \quad (3-4)$$

H_t is the total momentum of the spacecraft (including that of the wheels) and $\omega^\times$ is the matrix representation of the cross-product. The spacecraft rotational dynamics are discussed in more detail in Chapter 4. The momentum build-up to be absorbed by the wheels, in control coordinates, is given by:

$$h_r = H_t - J_c \omega \quad (3-5)$$

where J_c is the composite inertia matrix of the spacecraft and wheels. The momentum build-up in each wheel is a function of the orientation of the wheels in the control frame and the number of wheels active. h_r is stored in a matrix in the workspace, along with other parameters, and is later processed according to the equation below to generate the momentum build-up in each wheel:

$$h_s = A_{cm}^R h_r + \lambda v \quad (3-6)$$

where A_{cm}^R is the least-squares right inverse of A_{cm} , A_{cm} is the 3x4 transformation matrix defining the orientation of the spin axis of each wheel in the control frame, v is the vector space perpendicular to the row space of A_{cm} and λ is the scalar determining which combination of wheels is active. This momentum distribution algorithm is discussed in section 4.6.1. Results of the open-loop simulations are included in Chapter 5 and are used to determine appropriate bias levels for each test case.

3.2.3 Closed-Loop Simulation: Controlled Dynamics of the Spacecraft and Actuators

The closed-loop simulation diagrams are included in Appendix C. Again, an instruction file is run in Matlab to initialize the required variables and matrices in the workspace. The main diagram has eight principal building blocks.

Previously computed orbital data for the current test orbit is retrieved from the matrix *orbdats* in block G. External disturbances (about the control axes) and internal disturbances (about the spin axis of the three active wheels) are input into the system in block D using Simulink built-in step functions. In block F, band-limited white noise is selectively input in the system, by means of switches, to affect speed, rate and attitude measurements. The information which will be used to evaluate the performance of the system is saved in blocks I and J. In particular, the attitude and angular velocity errors between the simulated and the reference values and the variation in speed from the bias levels are computed in block J, where the attitude error is given by:

$$\delta q = q_{pi}^{-1} q_{ci} \text{ (using quaternion multiplication)}$$

where

$$q_{pi}^{-1} = \{ -q_{pi}(1) \quad -q_{pi}(2) \quad -q_{pi}(3) \quad q_{pi}(4) \}$$

$$q_{pi} = q_{ci,ref} = \text{Reference or commanded attitude quaternion}$$

(3-7)

and where the product of two quaternions, say $q_{23}q_{12}$, is equivalent to the transformation matrix describing the rotation from frame 3 to frame 1 (i.e. $C_{13} = C_{12}C_{23}$) and is given in matrix form as (Wertz, 1978, Ap. D):

$$q'' = qq'$$

$$\begin{bmatrix} q_1'' \\ q_2'' \\ q_3'' \\ q_4'' \end{bmatrix} = \begin{bmatrix} q_4' & q_3' & -q_2' & q_1' \\ -q_3' & q_4' & q_1' & q_2' \\ q_2' & -q_1' & q_4' & q_3' \\ -q_1' & -q_2' & -q_3' & q_4' \end{bmatrix} \begin{bmatrix} q_1' \\ q_2' \\ q_3' \\ q_4' \end{bmatrix} \quad (3-8)$$

The quaternion error is resolved into attitude errors about each axis using a 3-1-2 (or ψ - ϕ - θ) Euler angle sequence (Wertz, 1978, Ap.E). For small errors, $\delta q_1 \approx \phi/2$, $\delta q_2 \approx \theta/2$, $\delta q_3 \approx \psi/2$ and $\delta q_4 \approx 1$. The simulation keeps track, in block J.2, of the absolute value of the maximum error in each axis using memory blocks which, however, decrease the simulation speed.

Sensor performance is simulated in block B, using a very simple approach. Band-limited white noise is added to each measurement. Unit delays are applied to the attitude measurements, adding three discrete states to the system.

Blocks A, C and H generate the dynamics of the spacecraft and actuators, the control effort and the reference attitude profile. Several closed-loop simulation models are created with modified versions of blocks A, C and H. In some models, certain blocks are removed, to increase the speed of simulation and evaluate the effect of specific parameters. The principal versions are listed in Table 3-1, to help clarify the discussions below and in Chapter 6.

When desaturation is not enabled, only the dynamics of the spacecraft and the reaction wheels need to be computed. This is implemented in block A0. The initial

conditions associated with block A0 include the initial total spacecraft momentum, specified in the integrator of block A.5, the absolute momentum and the angular position of the rotors, specified in the integrators of block A.6, and the initial attitude of the control axes relative to inertial space, specified in the integrator of block A.4. The differential equations implemented in block A0 are summarized below. The attitude kinematics, implemented in block A.4, are given by (Wertz, 1978):

$$\dot{q}_\alpha = \frac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} q_\alpha \quad (3-9)$$

where q_α describes the rotation from inertial to control coordinates. Using a shorter notation, this is given by (Hughes, 1986):

$$\dot{q}_\alpha = \begin{bmatrix} \dot{\epsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \omega & -\omega^\times \\ 0 & -\omega^T \end{bmatrix} \begin{bmatrix} \epsilon \\ \eta \end{bmatrix} \quad (3-10)$$

with

$$\epsilon \equiv a \sin \frac{\phi}{2}$$

$$\eta \equiv \cos \frac{\phi}{2}$$

where ϕ is the Euler angle associated with the single rotation about the eigenaxis a to bring the inertial frame parallel to the control frame. The spacecraft rotational dynamics, implemented in block A.5, are given in Equation 3-4 above. The dynamics of the three active wheels, implemented in block A.6, are given by:

$$\dot{h}_a = T'_m - T_{d,int} - T_f \quad (3-11)$$

where $\dot{h}_a$ is the 4x1 matrix of absolute momentum of the four wheels, T'_m is the matrix of applied motor torques, $T_{d,int}$ is the matrix of internal disturbance torques about the spin axis of the wheels and T_f is the matrix of friction torques. The reaction wheel model, based on the ROSAT wheels model (Baldwin, 1987), assumes a brushless DC motor source, a constant maximum motor torque capability to reduce the speed of the wheel, and a constant maximum motor torque capability to increase the wheel speed, up to the saturation speed, above which the maximum motor torque available decreases linearly with speed (as a result of constant maximum voltage conditions). For wheel n at nonzero speeds, the components in Equation 3-11 are then given by:

$$\begin{aligned} T_{mn} &\leq T_{max} && \text{for } |\omega_{sn}| \leq \omega_x \\ T_{mn} &\leq T_{max} - (|\omega_{sn}| - \omega_x) \frac{k_e k_t}{R_a} && \text{for } |\omega_{sn}| > \omega_x \\ T'_{mn} &= \beta T_{mn} [1 + A_r \sin(f_r \theta_{sn})] \\ T_{fn} &= T_{vn} + T_{cn} + T_{sn} \\ T_{vn} &= B_n \omega_{sn} \\ T_{cn} &= \text{sgn}(\omega_{sn}) k_{cf} \\ T_{sn} &= \text{sgn}(\omega_{sn}) k_{sf} e^k \text{ with } k = -\frac{|\omega_{sn}|}{K_{dc}} \end{aligned} \quad (3-12)$$

where

- T_{mn} is the motor torque demand to wheel n
- T_{max} is the maximum motor torque available, assumed constant for speeds up to the saturation speed
- ω_{sn} is the speed of wheel n
- ω_x is the saturation speed, above which T_{max} linearly decreases with speed
- k_e is the back emf constant of the motor
- k_t is the motor torque constant

R_a	is the resistance in the armature
β	is the linearity coefficient
A_r	is the amplitude of the ripple torque
f_r	is the frequency of the ripple
θ_{sn}	is the angular position of the rotor of wheel n
T_n	is the friction torque for wheel n and includes viscous friction (T_{vn}), kinetic or Coulomb friction (T_{cn}) and static friction or stiction (T_{sn}) components
B_n	is the viscous friction coefficient for wheel n
$\text{sgn}(\omega_{sn})$	gives the sign or direction of the wheel speed
k_{cf}	is the Coulomb friction coefficient
k_{dc}	is the static friction (stiction) decay coefficient
k_{sf}	is the static friction (stiction) coefficient

The assumed reaction wheel model parameters are outlined in Table 3-2.

When the wheel is initially at standstill, the net torque responsible for accelerating the wheel relative to the platform must be greater than the breakaway torque for the wheel to move. The breakaway torque is the torque required to overcome the additional friction effects when the wheel is at standstill. In the simulation, for absolute wheel speed values smaller than 1×10^{-6} rad/sec, the net torque is computed to verify if it is indeed greater than the breakaway torque. If not, the wheel speed is set to zero. The net torque on the wheel at zero speed is derived in Appendix D.

Block A0 includes 13 continuous states: six for the momentum of the spacecraft (H_i) and the active wheels (l_i or h_a), three for the angular position of the rotors (θ_i) and four for the attitude quaternion (with one of the four states dependent on the others via the relation $1 = q_1^2 + q_2^2 + q_3^2 + q_4^2$).

When desaturation activity is included in the model, the dynamics of the spacecraft, the reaction wheels and the torque rods need to be computed. This is

implemented in block A. Magnetic control torques are computed in block A.3, according to:

$$\mathbf{T}_c = [\mathbf{A}_{cl'} (\mathbf{d} + \mathbf{d}_r)] \times [\mathbf{C}_{cl} \mathbf{B}] \quad (3-13)$$

where

$\mathbf{A}_{cl'}$ defines the orientation of the enabled torque coils in the control frame;

$\mathbf{d}$ are the commanded dipoles to the three enabled torque coils;

$\mathbf{d}_r$ are the remanent dipoles of the three enabled torque coils, set to zero when the coil is active, and to a fixed value of $\pm 0.2 \text{ Am}^2$ when inactive, where the sign is determined by the polarity of the coil when last active;

$\mathbf{C}_{cl}$ is the rotation matrix from flight to control coordinates;

$\mathbf{B}$ is the geomagnetic field in flight coordinates; and,

$\mathbf{A} \times \mathbf{B}$ represents the cross-product of vectors $\mathbf{A}$ and $\mathbf{B}$ in matrix form.

Except for block A.3, block A is identical to block A0. Block A contains the 13 continuous states of block A0, plus the three discrete states associated with the memory blocks in blocks A.3.1.1 through A.3.1.3. These memory blocks are used to keep track of the polarity of the three enabled torque coils when they were last active.

The controller is implemented in block C. The three principal variants of block C are included in Appendix C. Block C0 generates the control effort sent to the three enabled wheels, using a continuous integrator block to compute the momentum demand. In block C0d, block C.1B is added to generate the control effort to the magnetic coils and the compensating control effort to the wheels, to support desaturation activity. In block Cxd, the following is implemented:

- (i) Digitization using Tustin's or the bilinear approximation to compute the momentum demand,
- (ii) Quantization and delays due to data transport via telemetry, but no desaturation activity, and
- (iii) Feedback of the saturated momentum demand.

Autonomous desaturation commands are generated in block C.1B. Desaturation is triggered when the speed of any of the enabled wheels has exceeded a specified level. The desaturation flags are set to 1 when the triggering condition, verified in the Fortran coded block C.1.1.1B, is met. Desaturation is stopped for a given wheel, and the corresponding desaturation flag is set to zero, when its speed has been restored to the required bias level (condition verified in Fortran coded block C.1.1.2). Desaturation is stopped for all wheels when the time limit has been exceeded (block C.1.1.3B). Desaturation torque commands are computed in blocks C.1.2 and C.1.3, and are either set to a fixed value or according to Equation 4-43 in Chapter 4. If desaturation is disabled (desaturation flags are zero), these commands are set to zero. Block C.1.4 then computes the required magnetic dipoles and the corresponding wheel torque commands, dependent on the magnetic field direction. The associated equations are given in Chapter 4 (Equations 4-44 through 4-47).

Block C.2 computes the errors in attitude and in angular velocity and the deviation in speed from the bias level, similarly to Equation 3-7 but between measured and reference values.

The momentum demand to the wheels is calculated in block C.3, based on

Equations 4-31a (Fortran coded in the S-function block *dhc2f_3*) and 4-31(b) (Fortran coded in the S-function block *hc1f*) developed in Chapter 4. In block C.3x, the momentum demand is implemented based on Equation 4-29 instead. This approach is explained in Chapter 6, and is to support feedback of the saturated momentum demand and avoid integrator windup.

Speed control is achieved in block C.4. Quantization effects and delays are incorporated in blocks C.5, C.6, C.7, C.8 and C.10, using built-in quantizer and transport delay blocks.

Block C0 adds three continuous states to the system, associated with the integrator generating the momentum demand. Block C0d has three additional discrete states, associated with block C.1B, as a result of the unit delay in the desaturation time counter associated with each wheel. A single time counter should have been used, however, since the unloading time limit is set the same for all wheels. Block Cxd includes three continuous states, associated with the time derivative of the reference angular velocity, and nine discrete states, associated with the unit delays in the attitude measurements and the unit delays used to generate the discrete momentum demand.

3.3 Observations on the Simulation Tool and Approach Used

The three-axis digital control of the attitude of a spacecraft using reaction wheels and magnetorquers constitutes a nonlinear time-varying system with mixed continuous and discrete states. Matlab with Simulink is strictly used here to build the model and

obtain the time history of the states under various disturbances, noise conditions and delays. The Simulink tools to extract the linear dynamics around a given operating condition and for a given sampling time are not used.

The main advantages of using Simulink for this particular application include:

- the capability of changing the values of variables in the workspace and running the model again under different conditions,
- the capability of making simple changes in the model easily (redirecting the input of certain blocks to others),
- the capability of easily importing blocks from other models,
- the extensive graphics capabilities (e.g. for displaying results, for organizing model diagrams in their functional elements), and
- the capability to analyse additional parameters, for example uncertainties in plant parameters, by defining more variables in the workspace.

The principal disadvantages of using Simulink for this particular application are listed below:

- the version used has bugs and it is necessary to work around them. For example it is not possible to embed multiple S-function blocks of the same name, if the function name is that of a graphical model. The problem is avoided by using Fortran-coded Mex files, in the place of graphical

models, when a given S-function block is required more than once³⁰;

- the main disadvantage is the glacial simulation speed. It is necessary to avoid, as much as possible, memory blocks, scopes, Matlab function blocks, pulse train function blocks, and many routines have been re-coded in Fortran to avoid the Matlab bottleneck;
- when using the graphical model building approach, the internal representation of the model is not visible;
- PC memory limitations are a problem, which has been avoided by switching to fast, large memory unix work stations;
- unit delay blocks must be disconnected or bypassed by means of switches to remove their effect. Pure time delay and quantization effects, on the other hand, can simply be removed by setting the delay interval or the quantization interval in the next simulation run to zero;
- Simulink is used here strictly to generate the time histories of the states and other variables. The linear control analysis tools available with the software are not taken advantage of, as the operating conditions (wheel speeds) vary significantly as a result of the control algorithm used. The use of Simulink would be better suited to an application with a controller which can be linearized about an operating point, as built-in functions such as *dlinmod* (used to extract the linear dynamics of a nonlinear discrete dynamics model), *dbode* (used to plot the magnitude and phase of

³⁰For example, quaternion multiplication is implemented in a S-function block accessing the model name *qprod fo*. *qprod fo.F* is a modified version of the file *usersys.F*. *usersys.F* is a Fortran-coded skeleton file provided with Simulink, which when modified as required by the user and compiled using the command *fmex usersys.F simulink.F*, is used to create a task file which can be accessed via a S-Fcn block. This enables the user to insert Fortran-coded routines in Simulink models and also to significantly increase the speed of simulation as compared to models including the same algorithms implemented as Matlab Fcn blocks.

the linearized version of the discrete model), and others could assist in the design and analysis of the control system.

Table 3-1. Simulation Models with Varying Levels of Detail

Models/Versions	Description
Models 1.A to 1.E	Inertial pointing (i.e. block H.1 with each version listed below)
Models 2.A and 2.E	Inertial pointing with switching between two targets (i.e. block H.2 with versions A and E listed below)
Models 3.A and 3.E	Limb following (i.e. block H.3 with versions A and E listed below)
Models 4.A and 4.E	Lim following with continuous scanning (i.e. block H.4 with versions A and E listed below)
Models 5.A and 5.E	Limb following with stepwise scanning (i.e. block H.5 with versions A and E listed below)
Version A	Continuous model with no quantization, no noise, no delays, no anti-windup, no desaturation (i.e., with blocks A0, C0, D, H.i, I and J)
Version B	Continuous model with desaturation but no quantization, no noise, no delays, no anti-windup (i.e., with blocks A, C0d, D, H.i, I and J)
Version C	Same as version A, but modified to include digitization of the controller
Version D	Same as version C, but modified to include star tracker delays
Version E	Same as version D, but modified to include quantization, noise, delays, anti-windup but no desaturation activity (i.e., with blocks A0, B, Cxd, D, F, G, H.i, I, J)

Table 3-2. Reaction Wheel Model

Initial assumptions: **Baseline reaction wheel characteristics (SSC, 1993d):**

Brushless DC motor

Reaction torque of 0.02 N-m at 0 rpm, 0.018 N-m at 1000 rpm (105 rad/sec) and 0.014 N-m at 5100 rpm (534 rad/sec)

Ripple torque: 20 percent of demanded torque

Angular Momentum: 4 N-m-s (max) at 5100 rpm

Size/Weight/Power: 20.4 cm diameter, 3.46 kg, 9 W at 5100 rpm (at steady-state)

DC motor equations: **Assuming friction, no load torque and a stationary platform:**
(see Tapley, 1990)

$$\begin{aligned} (a) \quad k_t i_a &= J_m \dot{\omega}_m + B_n \omega_m + T_f \\ &= -T_{rx} + B_n \omega_m + T_f \end{aligned}$$

where T_{rx} is the reaction torque, and

$$(b) \quad v_a = R_a i_a + L_a \frac{d}{dt} i_a + k_e \omega_m$$

At steady-state and at fixed maximum torque conditions, (a) becomes:

$$T_{max} = -T_{rx} + B_n \omega_m + T_f$$

At steady-state and fixed voltage conditions, (a) and (b) are rewritten as:

$$\frac{k_t v_a}{R_a} = -T_{rx} + \left(B_n + \frac{k_e k_t}{R_a} \right) \omega_m + T_f$$

Table 3-2. (continued)

Wheel Model Parameters:

<u>Symbol</u>	<u>Description</u>	<u>Assumed value</u>
J_m	Inertia (kg-m^2)	4 N-m-s divided by 534 rad/sec (from the baseline characteristics)
B_m	Viscous damping coefficient (N-m/rad/sec)	Assumed as 0.048% of J_m i.e., $4.8 \times 10^{-4} J_m$
k_{cf}	Coulomb friction coefficient	$\approx 0.02 - (0.018 + 105 B_m)$ (assuming that the reaction torque specified at zero speed corresponds to the average torque available, i.e. is equal to the motor torque)
R_a	Resistance in the armature	Assumed equal to 10 ohms
k_e	Back emf constant (V/rad/sec)	From $k_e = (v_a - R_a i_a) / \omega_m$ at 5100 rpm (assuming $v_a = 24\text{V}$ and $i_a = 9\text{W}/24\text{V}$)
k_t	Motor torque constant (N-m/Amp)	From Eq. (a) above at 5100 rpm, i.e. $k_t = (.014 + k_{cf} + 534 B_m) / i_a$
ω_x	Saturation speed	Assumptions: (1) a constant maximum motor torque is available to increase the wheel speed to saturation speed, above which the maximum torque available is limited by the maximum voltage conditions; (2) at 100 rad/sec the wheel is in the nonsaturated regime and at 534 rad/sec the wheel speed is above the saturation level. The saturation speed is then at the intersection of the slopes in the two regimes and is given by: $\omega_x = \{ [R_a / (k_e k_t)] [0.004 - 429 * B_m - 534 k_{cf} / R_a] \}$

Table 3-2. (continued)**Wheel Model Parameters (continued):**

<u>Symbol</u>	<u>Description</u>	<u>Assumed value</u>
k_{sf}	Static friction (stiction) coefficient	Assumed equal to 0.00005
k_{dr}	Stiction decay constant (rad/sec)	Assumed equal to 1.07
T_{brk}	Breakaway torque	Assumed equal to $k_{sf} + k_{cf} + 0.00001$
T_{max}	Maximum torque in the nonsaturated regime	$T_{max} = 0.018 + k_{cf} + 105B_n$
A_r	Amplitude of torque ripple	Assumed equal to 0.2 (20 %)
f_r	Frequency of ripple f_r	Assumed equal to 2 (i.e. twice the angular position of the rotor)
β	Torque linearity coefficient	1

CHAPTER 4. BASIC ATTITUDE CONTROLLER

This chapter describes the basic controller implemented in block C of the simulation diagram (see Appendix C). Coordinates and controlled variables are first defined. Next, the angular velocity of the control frame is expressed in terms of the time derivative of the frame attitude error, so that the dynamics can later be written in terms of angular displacements and their derivatives. The Newton-Euler approach is then applied to derive the attitude motion of the spacecraft. Dynamic decoupling is used, reducing the system to a set of uncoupled second-order equations. Controller design is then direct with gains selected to meet the maximum dynamic error allowed in each axis. Wheel speed control is chosen over torque control, as the resulting system is expected to be less sensitive to noise and disturbances. In the autonomous mode, critical wheel speeds trigger desaturation. Magnetic torques and commands to slow the reaction wheels are applied simultaneously, such that the net torque on the spacecraft is zero.

4.1 Coordinates and Controlled Variables

Five dextral orthonormal triads are referred to in the present chapter, and are shown in Figure 4-1. Details of the triads and the context in which they are used are given below.

The control frame F_c is a body-fixed payload-referenced frame, and has its origin at the undeformed composite mass center location. The composite mass center refers to the mass center of the system composed of the spacecraft rigid body and the four reaction wheels. F_c has basis vectors $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$, also referred to in the document as $\hat{X}_c$, $\hat{Y}_c$ and $\hat{Z}_c$. $\hat{e}_1$ is along the line of sight of the operating instrument, $\hat{e}_2$ is along the nominal Sun direction, and $\hat{e}_3$ completes the right-hand orthogonal triad³¹. Control about $\hat{e}_2$ and $\hat{e}_3$ directly corresponds to the pointing control in pitch and yaw of the payload boresight and must meet the specified requirements. Ignoring thermal deformations, the orientation of the reaction wheels and of the magnetorquer coils is fixed in F_c .

The reference frame F_p corresponds to the desired orientation of the control frame under nominal operating conditions. F_c is slightly rotated relative to F_p and their origins coincide (except for the reaction wheels, a rigid spacecraft body is assumed).

F_f is an orbit-referenced frame, with its origin also at the undeformed composite mass center location. F_f has basis vectors $\hat{f}_1$, $\hat{f}_2$ and $\hat{f}_3$, also referred to in the document as $\hat{X}_f$, $\hat{Y}_f$ and $\hat{Z}_f$. $\hat{f}_3$ is along the nadir vector³² (orbital yaw), $\hat{f}_2$ is along the negative orbit normal (negative orbital pitch), and $\hat{f}_1$ completes the dextral triad (orbital roll which is the velocity vector in the case of a circular orbit). Using an orbit generator and a model of the geomagnetic field, the geomagnetic field vector is known in F_f .

F_i is the inertial frame, with its origin at the geocenter. The basis vector $\hat{i}_1$ (or

³¹The control frame was defined to be consistent with that specified in Swedish Space Corporation (1993a).

³²The nadir vector is the vector from the spacecraft to the center of the Earth.

$\hat{X}_1$) points along the vernal equinox direction³³, $\hat{i}_2$ (or $\hat{Z}_1$) points along the North pole, and $\hat{i}_2$ (or $\hat{Y}_1$) completes the right-hand orthonormal triad. The reference spacecraft attitude is known in F_1 .

F_m is a triad along the principal axes of wheel n , with its origin at the mass center of wheel n . Applied motor torques are resolved in F_m .

Three sets of measurements are available to the controller, as well as the three corresponding sets of reference signals:

- q_{pi} is the reference attitude, given as a column matrix of four Euler parameters describing the desired orientation of F_p relative to F_1
- Ω is the reference angular velocity in F_p coordinates
- Ω_s is the column matrix of four wheel speed biases, prescribed in certain control modes to avoid zero-speed crossings (4-1)
- q_{ci}^M is the measured attitude, given as a column matrix of four Euler parameters describing the measured orientation of F_c relative to F_1
- ω^M is the measured angular velocity in F_c coordinates
- ω_s^M is the column matrix of four measured wheel speeds

Measured errors in attitude, angular velocity and wheel speed are determined as follows:

³³The vernal equinox is to the direction in space from the center of the Earth to the point where the Sun crosses the equator from south to north (first day of spring). It is represented by the symbol γ , as that direction pointed at the constellation of Aries at one time (Escobal, 1976).

2

**PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT**

PRECISIONSM RESOLUTION TARGETS

$$\begin{aligned}
\delta q^M &= q_{pi}^{-1} q_\alpha^M \\
\delta \omega^M &= \omega^M - \Omega \\
\delta \omega_s^M &= \omega_s^M - \Omega_s \text{ with} \\
q_{pi}^{-1} &= q_{ip} = [-q_{pi}(1) \quad -q_{pi}(2) \quad -q_{pi}(3) \quad q_{pi}(4)]
\end{aligned}
\tag{4-2}$$

where δq^M and $\delta \omega^M$ are small variations and δq^M , the measured error in attitude, is computed using quaternion multiplication (see Wertz, 1978, sect. 18.3.1 & Ap. D) and converted to $\delta \theta^M$ using appropriate algebra (see Wertz, 1978, sect. 12.1). It will be assumed for the moment that there are no measurement errors, so that:

$$\begin{aligned}
q_\alpha^M &= q_\alpha \\
\omega^M &= \omega \\
\omega_s^M &= \omega_s
\end{aligned}
\tag{4-3a}$$

and therefore

$$\begin{aligned}
\delta q &= \delta q^M \text{ or } \delta \theta = \delta \theta^M \\
\delta \omega &= \delta \omega^M \\
\delta \omega_s &= \delta \omega_s^M
\end{aligned}
\tag{4-3b}$$

The control objective is to maintain the variation in wheel momenta ($\delta \omega_s$) and the dynamic errors ($\delta \theta$) within prescribed limits.

4.2 Angular Velocity of the Control Frame

A 3-1-2 Euler angle sequence can be used to express the rotation between F_p and F_c . The spacecraft angular velocity in control coordinates is then given by:

$$\omega = \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + (\theta)_2 \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + (\theta)_2(\phi)_1 \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + (\theta)_2(\phi)_1(\psi)_3 \Omega \quad (4-4)$$

where Ω is the angular velocity of the reference frame F_p in F_p coordinates, $(\psi)_3$ is a small rotation about axis 3, $(\phi)_1$ is a small rotation about axis 1 and $(\theta)_2$ is a small rotation about axis 2.

Expanding the Euler rotations yields:

$$\omega = \begin{bmatrix} C_\theta & 0 & -S_\theta C_\phi \\ 0 & 1 & S_\theta \\ S_\theta & 0 & C_\theta C_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} C_\theta C_\psi - S_\theta S_\phi S_\psi & C_\theta S_\psi + S_\theta S_\phi C_\psi & -S_\theta C_\psi \\ -C_\phi S_\psi & C_\phi C_\psi & S_\phi \\ S_\theta C_\psi + C_\theta S_\phi S_\psi & S_\theta S_\psi - C_\theta S_\phi C_\psi & C_\theta C_\psi \end{bmatrix} \Omega \quad (4-5)$$

Assuming small angles, Equation 4-5 reduces to

$$\omega = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + C_\varphi \Omega = \delta\dot{\theta} + \Omega - \delta\dot{\theta} \times \Omega = \Omega + \delta\omega \quad (4-6)$$

where

$$C_{\varphi} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} = [1 - \delta\theta^{\times}]$$

$\delta\theta = [\phi \ \theta \ \psi]^T$ is the variation from the reference attitude

$\delta\omega = \delta\dot{\theta} - \delta\theta^{\times}\Omega$ is the variation in angular velocity due to the variation in attitude rate between F_p and F_c and the coupling between the variation in attitude and the reference angular velocity (4-7)

$$\delta\theta^{\times} = \begin{bmatrix} 0 & -\delta\theta_3 & \delta\theta_2 \\ \delta\theta_3 & 0 & -\delta\theta_1 \\ -\delta\theta_2 & \delta\theta_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\psi & \theta \\ \psi & 0 & -\phi \\ -\theta & \phi & 0 \end{bmatrix}$$

is the skew-symmetric representation of the vector cross-product operation (see Hughes, 1986, Ap.B)

Equation 4-6 is later used to express the dynamics in terms of angular displacements and their derivatives.

4.3 Spacecraft Rotational Dynamics

Ignoring flexibility and other sources of internal momentum, the spacecraft can be treated as a rigid body with four symmetrical rotors. The attitude dynamics of a rigid body with one or more symmetrical rotors are derived and analysed in several textbooks (eg. Kane, Likins & Levinson, 1983, sect. 3.6 - fwd; Hughes, 1986, sect. 3.5 & chap. 6). Following the same approach as that described in the literature, by using the torque

and angular momentum relationship³⁴ and assuming that all torques due to internal forces cancel, the attitude motion of the spacecraft is given by:

$$\frac{d}{dt}\bar{H}_{\oplus} = \bar{T}_{ext} = \frac{\partial}{\partial t}\bar{H}_{\oplus} + \bar{\omega} \times \bar{H}_{\oplus} \quad (4-8)$$

where d/dt preceding a vector is the time derivative of that vector as seen in an inertial frame, $\partial/\partial t$ preceding a vector is the time derivative of that vector as seen in a frame rotating by $\bar{\omega}$ relative to inertial space, and $\bar{H}_{\oplus}$ is the total angular momentum of the spacecraft, taken about the composite mass center. In the rest of the document, most vector equations will be expressed in matrix form, with all vectors resolved in the same coordinate frame, as shown below.

When resolved into a rotating reference frame, the three scalar equations corresponding to Equation 4-8 are known as Euler's equations. Euler's equations, in rotating F_c coordinates and in matrix form, are then given by:

$$\dot{\bar{H}}_{\oplus} + \omega^{\times} \bar{H}_{\oplus} = \bar{T}_{ext} \quad (4-9)$$

where the dot is used here to indicate the time derivative of scalar quantities, and $\omega^{\times}$ is again the skew-symmetric representation of the vector cross-product operation (see Hughes, 1986, Ap. B), with $\omega^{\times}$ expressed in control coordinates as:

³⁴This leads to the Newton-Euler formulation, so called because it is based on the momentum equations developed by Newton and Euler (see Hughes, 1986, chap. 3):

$$\frac{d}{dt} \bar{p} = \bar{f} \text{ and } \frac{d}{dt} \bar{h} = \bar{T}$$

$$\omega^x = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad (4-10)$$

The total spacecraft angular momentum is written in matrix form as:

$$H_{\oplus} = J_c \omega + A_{cm} h_s$$

where

$$A_{cm} = F_c \cdot S_m^T \quad (4-11)$$

$$S_m = [\hat{s}_1 \ \hat{s}_2 \ \hat{s}_3 \ \hat{s}_4]^T$$

$$h_s = J_m \omega_s, \text{ and}$$

$$J_m = \text{diag}\{J_{s1}, J_{s2}, J_{s3}, J_{s4}\}$$

The first term in Equation 4-11 includes the body momentum and spin and transverse momenta of the wheels due to the body rotation. The second term consists of the spin momenta of the wheels due to the wheel rotation relative to the spacecraft. Other sources of internal momentum, such as flexibility and mechanisms, are ignored for the moment.

If speed or momentum control is to be applied, equations are formulated in terms of h_s and ω_s . If torque control is to be implemented, equations must be written in terms of motor torques. The absolute angular momentum of wheel n , resolved in F_m coordinates, is given by:

$$\bar{\mathbf{T}}_n = J_{sn}[\omega_{sn} + (\hat{\mathbf{i}}_{n1} \cdot \bar{\boldsymbol{\omega}})] \hat{\mathbf{i}}_{n1} + J_m(\hat{\mathbf{i}}_{n2} \cdot \bar{\boldsymbol{\omega}}) \hat{\mathbf{i}}_{n2} + J_m(\hat{\mathbf{i}}_{n3} \cdot \bar{\boldsymbol{\omega}}) \hat{\mathbf{i}}_{n3} \quad (4-12)$$

where $\hat{\mathbf{i}}_{n1}$, $\hat{\mathbf{i}}_{n2}$ and $\hat{\mathbf{i}}_{n3}$ are the unit vectors along the principal axes of wheel n , J_{sn} is the spin axis inertia of wheel n , and J_m is the transverse inertia of wheel n , assumed identical in any direction perpendicular to the spin axis.

As shown below, because of the wheel symmetry, the torque about the spin axis is only a function of the angular acceleration about that axis and is independent of the transverse momentum:

$$T_{mn} = (\bar{\mathbf{T}}_{net} + \bar{\mathbf{T}}_f) \cdot \hat{\mathbf{i}}_{n1} \quad (4-13)$$

where

$$\begin{aligned} T_{mn} &= \text{Motor torque about the spin axis of wheel } n \\ \bar{\mathbf{T}}_f &= \text{Friction torques (static, kinetic and viscous)} = T_f \hat{\mathbf{i}}_{n1} \\ \bar{\mathbf{T}}_{net} \cdot \hat{\mathbf{i}}_{n1} &= \left(\frac{\partial}{\partial t} \bar{\mathbf{T}}_n + \bar{\boldsymbol{\omega}} \times \bar{\mathbf{T}}_n \right) \cdot \hat{\mathbf{i}}_{n1} \\ &= \left(\frac{\partial}{\partial t} \bar{\mathbf{T}}_n \right) \cdot \hat{\mathbf{i}}_{n1} + (J_m - J_m)(\bar{\boldsymbol{\omega}} \cdot \hat{\mathbf{i}}_{n2})(\bar{\boldsymbol{\omega}} \cdot \hat{\mathbf{i}}_{n3}) (\hat{\mathbf{i}}_{n1} \cdot \hat{\mathbf{i}}_{n1}) \\ &= \left(\frac{\partial}{\partial t} \bar{\mathbf{T}}_n \right) \cdot \hat{\mathbf{i}}_{n1} \text{ (because of symmetry)} \end{aligned}$$

At this point, it is assumed that the friction torques are very small and that the net torque and the motor torque to the wheel are essentially the same. This assumption was not made when deriving the dynamics of the wheels in Chapter 3. It is made here to develop the equations of the controller, keeping in mind that the friction torques will

be disturbance torques to the system. Therefore, ignoring friction torques, the four axial motor torques are expressed in matrix form as:

$$T_m - T_f \approx T_m = \dot{h}_s = J_m \dot{\omega}_s + J_m A_{cm}^T \dot{\omega} \quad (4-14)$$

The rate of change of wheel spin momenta relative to the spacecraft is then:

$$\dot{h}_s = T_m - J_m A_{cm}^T \dot{\omega} \quad (4-15)$$

Substituting (4-11) into (4-9), the rotational dynamics are given by:

$$J_c \dot{\omega} = T_{ext} - A_{cm} \dot{h}_s - \omega \times (J_c \omega + A_{cm} h_s) \quad (4-16)$$

or, if written in terms of motor torques (using Equation 4-15):

$$(J_c - A_{cm} J_m A_{cm}^T) \dot{\omega} = T_{ext} - A_{cm} T_m - \omega \times (J_c \omega + A_{cm} h_s) \quad (4-17)$$

4.4 Decoupling Control

The control objective is twofold: maintain attitude and rate deviations within allowed limits and desaturate the wheels to bias levels when the speed limit has been exceeded. Each function can be treated separately. In the case of Odin, control should be effected to the largest extent by the reaction wheels, the most accurate actuators, with

torque rods active only when unloading is required³⁵. T_{ext} includes external control and disturbance torques. Although not implemented here, estimates of gravity gradient and magnetic disturbances (as the residual spacecraft dipole is measurable) could also be included to improve tracking.

In the discussion which follows, to develop the control law, some assumptions will be made and the equations of dynamics will be partially linearized. When the performance of the controller is tested using simulation (later chapters), the full nonlinear dynamics of the plant and actuators will be simulated. A first attempt at linearizing Equations 4-16 and 4-17 is made by letting:

$$\begin{aligned}
 \omega &= \Omega + \delta\omega \quad (\text{from Eq. 4-6}) \\
 \dot{\omega} &= \dot{\Omega} + \delta\dot{\omega} \\
 \omega_s &= \Omega_s + \delta\omega_s \\
 T_m &= (T_m)_{dev} + (T_m)_{desat} \quad (\text{commanded motor torques}) \\
 T_c &= (T_c)_{desat} \quad (\text{commanded magnetic torques}) \\
 T_d &= \text{Disturbance torques} \\
 T_{ext} &= T_c + T_d
 \end{aligned} \tag{4-18}$$

where Ω and $\dot{\Omega}$ are the angular velocity and acceleration under nominal operating conditions, Ω_s are the nominal wheel speed biases, and products of variations other than $\delta\omega_s$ are assumed to be negligible. Products of variations containing $\delta\omega_s$ will not be dropped (as indicated in Eq. 4-20), as $\delta\omega_s$ represents the variation away from the bias

³⁵Spacecraft with less stringent pointing requirements and equipped with wheels and magnetorquers can maintain magnetorquers enabled at all times to counteract the anticipated secular torques and to keep wheel speeds very close to bias levels while wheel motor torques can be applied based on deviations in attitude and angular velocity.

conditions, and it is assumed that the wheels will be allowed to vary from their nominal bias levels, with the magnetorquers operated only on a periodic basis.

To facilitate the selection of gains, axis decoupling control is developed, by rearranging Equation 4-17 for wheel torque control and Equation 4-16 for momentum control.

4.4.1 Torque Control

Substitution of Equations 4-18 into Equation 4-17 yields:

$$\begin{aligned} J'_c \delta \dot{\omega} = & -J'_c \dot{\Omega} + (T_c)_{desat} - A_{cm}(T_m)_{desat} - A_{cm}(T_m)_{dev} - \Omega \times H_{op} \\ & - [\Omega \times J_c - H_{op} \times] \delta \omega - [\Omega \times A_{cm} J_m] \delta \omega_s - [\delta \omega \times A_{cm} J_m] \delta \omega_s + T_d \end{aligned} \quad (4-19)$$

where

$$\begin{aligned} J'_c &= J_c - A_{cm} J_m A_{cm}^T \\ H_{op} &= J_c \Omega + A_{cm} J_m \Omega_s \end{aligned} \quad (4-20)$$

Most nonlinear terms vanish as products of variations other than $\delta \omega_s$ are assumed to be negligible, and $[\delta \omega \times A_{cm} J_m] \delta \omega_s$ is the only nonlinear term retained.

Setting the wheel torque demands such that:

$$\begin{aligned} A_{cm}(T_m)_{dev} = & K_1 \delta \theta + K_2 \delta \omega - J'_c \dot{\Omega} - \Omega \times H_{op} \\ & - [\Omega \times J_c - H_{op} \times] \delta \omega - [\Omega \times A_{cm} J_m] \delta \omega_s - [\delta \omega \times A_{cm} J_m] \delta \omega_s \end{aligned} \quad (4-21)$$

$$A_{cm}(T_m)_{desat} = (T_c)_{desat}, \text{ and}$$

$$T_m = (T_m)_{desat} + (T_m)_{dev} \text{ (as the total wheel command)}$$

where $(T_c)_{desat}$ is resolved in control coordinates and T_m along spin axes, reduces the dynamics in Equation 4-16 to :

$$J'_c \delta \dot{\omega} + K_2 \delta \omega + K_1 \delta \theta = T_d \quad (4-22)$$

Expanding $\delta \omega$ in terms of attitude angles and their derivatives (using Equation 4-6), Equation 4-22 becomes:

$$J'_c [\delta \ddot{\theta} + (\dot{\Omega}^x \delta \theta + \Omega^x \delta \dot{\theta})] + K_2 [\delta \dot{\theta} + \Omega^x \delta \theta] + K_1 \delta \theta = T_d \quad (4-23)$$

For inertial pointing (reference angular velocity and acceleration at zero), the dynamics in Equation 4-23 are those of a standard second-order system. For modes other than inertial pointing, to fully decouple the attitude angles and rates, motor torques are set such that:

$$A_{cm}(T_m)_{dev} = K_1 \delta \theta + K_2 \delta \omega - K_2 \Omega^x \delta \theta - J'_c (\dot{\Omega}^x \delta \theta + \Omega^x \delta \dot{\theta}) - J'_c \dot{\Omega} - \Omega^x H_{op} - [\Omega^x J_c - H_{op}^x] \delta \omega - [\Omega^x A_{cm} J_m] \delta \omega_s - [\delta \omega^x A_{cm} J_m] \delta \omega_s \quad (4-24)$$

while

$$A_{cm}(T_m)_{desat} = (T_c)_{desat}, \text{ and} \quad (4-25)$$

$$T_m = (T_m)_{desat} + (T_m)_{dev} \text{ (as the total wheel command)}$$

still apply, reducing the dynamics to:

$$\delta \ddot{\theta} + (J'_c)^{-1} K_2 \delta \dot{\theta} + (J'_c)^{-1} K_1 \delta \theta = (J'_c)^{-1} T_d \quad (4-26)$$

4.4.2 Momentum Control

Substitution of Equations 4-18 into Equation 4-16 yields:

$$J_c \delta \dot{\omega} = -J_c \dot{\Omega} + (T_c)_{desat} - A_{cm} \dot{h}_s - \Omega^\times H_{op} - [\Omega^\times J_c - H_{op}^\times] \delta \omega - [\Omega^\times A_{cm} J_m] \delta \omega_s - [\delta \omega^\times A_{cm} J_m] \delta \omega_s + T_d \quad (4-27)$$

It is first assumed that desaturation is disabled, such that $(T_c)_{desat}$ is zero. Based on discussions with Staley (1994), setting up an inner loop around the wheels with control of the form $T_m = K_p(\omega_d - \omega_s)$ proportional to the difference between speed demand and measured speed, and demanding wheel speeds such that:

$$\omega_d = J_m^{-1} A_{cm}^{-1} \delta h + \Omega_{s0} \quad (4-28)$$

where

$$\begin{aligned} \delta h &= A_{cm} \int_0^t \dot{h}_s dt = A_{cm} (h_d - J_m \Omega_{s0}), \text{ and} \\ \delta \dot{h} &= - \int_0^t [-K_1 \delta \theta - K_2 \delta \dot{\theta} + J_c (\dot{\Omega}^\times \delta \theta + \Omega^\times \delta \dot{\theta}) \\ &\quad + J_c \dot{\Omega} + \Omega^\times H_{op} + (\Omega^\times J_c - H_{op}^\times) \delta \omega \\ &\quad + (\Omega^\times A_{cm} J_m) \delta \omega_s + (\delta \omega^\times A_{cm} J_m) \delta \omega_s] dt \end{aligned} \quad (4-29)$$

and where Ω_{s0} are the initial conditions on wheel speeds, h_d is the column matrix of momentum demands along the spin axes and δh is along the control axes, reduces the dynamics to:

$$\delta\ddot{\theta} + (J_c)^{-1}K_2\delta\dot{\theta} + (J_c)^{-1}K_1\delta\theta = (J_c)^{-1}T_d \quad (4-30)$$

Rate terms can be taken out of the integral, so that:

$$\begin{aligned} a &= \int K_2\delta\dot{\theta}(t) dt = K_2[\delta\theta(t) - \delta\theta(0)] \\ b &= \int J_c[\dot{\Omega}(t)^\times\delta\theta(t) + \Omega(t)^\times\delta\dot{\theta}(t)] dt = J_c[\Omega(t)^\times\delta\theta(t) - \Omega(0)^\times\delta\theta(0)] \\ c &= \int J_c\dot{\Omega}(t) dt = J_c[\Omega(t) - \Omega(0)] \end{aligned} \quad (4-31a)$$

where $\Omega(0)$ and $\delta\theta(0)$ are initial values, and

$$\begin{aligned} \delta h &= a + b + c - \int_0^t [-K_1\delta\theta + \Omega^\times H_{op} + (\Omega^\times J_c - H_{op}^\times)\delta\omega \\ &\quad + (\Omega^\times A_{cm}J_m)\delta\omega_s + (\delta\omega^\times A_{cm}J_m)\delta\omega_s] dt \end{aligned} \quad (4-31b)$$

When desaturation is enabled, commands to magnetorquers and reaction wheels are set such that:

$$A_{cm}(T_m)_{desat} + (T_c)_{desat} = 0 \quad (4-32)$$

A first control approach attempted is to send $(K_p)^{-1}(T_m)_{desat}$ directly to the wheel speed control loops. Difficulties are encountered with this approach. The second approach is to include $(T_c)_{desat}$ in the integrand of Equation 4-29. This is found to be the appropriate approach. This matter is discussed further in Chapter 6.

The choice of attitude control and actuator gains K_1 , K_2 and K_p is discussed in the next sections.

By using momentum rather than torque control, the derivative of the attitude error is no longer needed if rate terms are taken out of the integral in Equation 4-31, thus probably improving performance under noise conditions. The system is also less sensitive to bearing irregularities and static friction (stiction) experienced in the wheels.³⁶

4.5 Choice of Attitude Control Gain Matrices K_1 and K_2 to Implement Momentum Control

Gain matrices K_1 and K_2 in Equation 4-30 are first computed, assuming that no noise or delays are present at any point in the system, based on the following criteria: maximum steady-state pointing error allowed under worst-case anticipated disturbance torques, while maintaining desired damping characteristics. The stability and performance of the system is then verified, taking into account potential noise and delays. The gains and consequently the system control bandwidth are modified if necessary.

³⁶For example, if it is assumed that the rotor equation follows that of a DC-motor, and that proportional control on the speed error is implemented such that:

$$T_m = K_p(\Omega_{sn} - \omega_{sn}) = J_{sn}\dot{\omega}_{sn} + B\omega_{sn} + T_f + T_l$$

Then, by using a large K_p , the effect of disturbance torques on speed are significantly reduced. At steady-state:

$$\omega_{sn} = \left[\frac{K_p}{B + K_p} \right] \Omega_{sn} - \left[\frac{T_f + T_l}{B + K_p} \right]$$

4.5.1 Maximum Dynamic Error in Each Mode

The dynamic error in each axis is defined here as the steady-state error caused by external disturbances. It must be a small fraction of the total error budget of the system. The required pointing accuracies for ODIN are given in Table 4-1.

When hardware is selected, all possible contributions to the total pointing error, known or estimated at the time, are tabulated to assist in choosing the best set of attitude sensors and actuators to meet the mission requirements. Such an error budget was compiled in Swedish Space Corporation (1993b). Errors in this case arise as a result of:

- Uncertainty in the attitude estimation associated with the operation of the star sensors and gyroscopes
- Uncertainty in the generated reference attitude profile
- Quantization and transmission delay of the sensor data to the controller in the telemetry network
- Uncertainty in the orientation of the control frame, sensor frames and actuator frames (misalignment during mounting or as a result of thermal deformations or structure flexibility)
- Quantization and transmission delay of actuator commands
- Uncertainty in the actuator performance function
- Error as a result of disturbances (including environmental disturbances, on-board mechanisms and actuator internal disturbance torques)

The root-sum-square values of the various sources of errors, other than those

associated with disturbances, are given in Table 4-2 for two sample cases. The largest contributions to the pointing error in the astronomy mode are associated with the random bias drift of the gyroscopes and the predicted satellite position. The values in Table 4-2 are based on the following assumptions, outlined in Swedish Space Corporation (1993b): the largest time interval during which the gyroscopes are anticipated to work without any star sensor update (referred to as bad sky areas) is five minutes, the gyroscopes have random drift characteristics of 0.5 deg/hr, and over a four-day period the estimated uncertainty in the satellite's position is four to five km for a 600 km baseline orbit. The gyroscopes would then accumulate 150" error over five minutes and the error in orbital position over a four-day prediction period would translate to 135", yielding a root-sum-square value of 202".

Comparing values in Tables 4-1 and 4-2, a dynamic error of 2" during astronomy sessions appears to be a reasonable goal. During astronomy sessions, the most demanding control is about the pitch axis. A dynamic error of 20", for disturbance rejection, appears to be an appropriate choice.

4.5.2 Bandwidth of the Decoupled Control Loop - Assuming no Noise and no Delays

4.5.2.1 Inertial Pointing

The following is assumed: a maximum steady state error of 2" in each axis

(although control accuracy is relaxed about the roll axis), a constant torque³⁷ of 10^{-4} N-m about one axis (for instance about the pitch axis or axis 2), and the following matrices³⁸:

$$J_c = \begin{bmatrix} 33.5300 & 0.0000 & 2.0670 \\ 0.0000 & 35.0281 & 0.0000 \\ 2.0670 & 0.0000 & 42.5399 \end{bmatrix} \quad (kg-m^2) \quad (4-33)$$

$$J_c^{-1} = \begin{bmatrix} 0.0299 & 0.0000 & -0.0015 \\ 0.0000 & 0.0285 & 0.0000 \\ -0.0015 & 0.0000 & 0.0236 \end{bmatrix} \quad (kg^{-1}-m^{-2})$$

Applying the final value theorem to the Laplace transformation of $\delta\theta$ in Equation 4-30, assuming a step disturbance torque input and assuming that $K_1' = (J_c)^{-1} K_1$ and $K_2' = (J_c)^{-1} K_2$ are diagonal matrices, then $K_1' = \text{diag}\{\omega_1^2, \omega_2^2, \omega_3^2\}$ and $K_2' = \text{diag}\{2\zeta\omega_1, 2\zeta\omega_2, 2\zeta\omega_3\}$, where ω_n is the pointing bandwidth of axis n , such that:

³⁷In both aeronomy and astronomy operating modes, worst-case torques are of the order of 10^{-4} N-m (see Appendix A).

³⁸The satellite model used is derived in Appendix A (see Figure A-1) and is based on an early configuration of Odin which is shown in Appendix E. The control frame axes are assumed to be parallel to the XsYsZs frame, with the origin at the undeformed composite center of mass location, Zs normal to the solar array and pointing towards the spacecraft structure, Xs in a plane parallel to that of the solar array and in the direction of the telescope boresight and Ys completing the dextral triad. The reference material indicates that different control frames slightly rotated from the XsYsZs frame and translated to the center of mass may be defined depending on the instrument used (SSC, 1993a). J_c and J_c^{-1} will therefore vary. This may result in different feedback gains being programmed into the computer when various control frames are used.

$$\omega_2^2 = \frac{(.0285 \text{ kg}^{-1}\text{m}^{-2}) (10^{-4} \text{ N-m})}{(2 \text{ arcsec}) \left(\frac{1^\circ}{3600 \text{ arcsec}}\right) \left(\frac{\pi}{180^\circ}\right)} = 0.2939 \text{ sec}^{-2}, \text{ and} \quad (4-34)$$

$$\omega_2 = 0.5422 \text{ rad/sec} \approx 0.09\text{Hz}$$

Assuming a damping factor³⁹ ζ of 0.707, the second diagonal element of K_2 is given by:

$$2\zeta\omega_2 = .7667 \text{ sec}^{-1} \quad (4-35)$$

Assuming that all decoupled axes have the same pointing bandwidth, and using the values computed in Equations 4-34 and 4-35, then the control gain matrices K_1 and K_2 are given by:

$$K_1 = J_c K'_1 = \begin{bmatrix} 9.8545 & 0.0000 & 0.6075 \\ 0.0000 & 10.2948 & 0.0000 \\ 0.6075 & 0.0000 & 12.5025 \end{bmatrix} \quad (4-36)$$

$$K_2 = J_c K'_2 = \begin{bmatrix} 25.7074 & 0.0000 & 1.5848 \\ 0.0000 & 26.8561 & 0.0000 \\ 1.5848 & 0.0000 & 32.6153 \end{bmatrix}$$

4.5.2.2 Aeronomy Mode

Assumptions now include a 20" steady-state error as a result of disturbances, and the same constant torque and inertia matrix as before. The pointing bandwidth is then

³⁹A damping ratio of 0.707 is often chosen with standard second-order systems, because of the desirable frequency response characteristics (Wertz, 1978, p.593).

approximately 0.03 Hz, the gains in matrix K_1 are a factor of ten smaller than in the inertial case and the gains in K_2 are slightly less than a third of the gains in the inertial case.

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.9854 & 0.0000 & 0.0607 \\ 0.0000 & 1.0295 & 0.0000 \\ 0.0607 & 0.0000 & 1.2502 \end{bmatrix} \\ K_2 &= \begin{bmatrix} 8.1292 & 0.0000 & 0.5011 \\ 0.0000 & 8.4924 & 0.0000 \\ 0.5011 & 0.0000 & 10.3136 \end{bmatrix} \end{aligned} \quad (4-37)$$

4.5.3 Effects of Sampling and Delays

Bode plots for the continuous decoupled system of Equation 4-30 are shown in Figure 4-2. In the staring mode, the phase margin is about 65° and the cross-over frequency about 0.82 rad/sec (~ 0.13 Hz). In the limb following mode, the system cross-over frequency is about 0.26 rad/sec (~ 0.04 Hz).

The main delays in the system will be associated with the operation of the star trackers, the transport delay of the sensor data to the controller, the computational time in the controller and the transport delay of commands from the controller to the actuators.

The effect of these delays on the decoupled loop are estimated below. First, the following assumptions are made:

- the attitude estimate is solely based on star tracker data and is updated at

10 Hz. This is simulated by a unit delay on the attitude error, which consists of a zero-order-hold (ZOH) with a sampling time of $T_s = 0.1$ second and an additional delay of one sampling period; and,

- sensor data is available to the controller within 1/16th of a second, commands are available to the actuators within 1/16th of a second, and the computational time delay in the controller is very small compared to the other delays; these three delay sources are approximated by a pure time delay of T_2 (ie 0.1 seconds)

The discrete open-loop transfer function is then⁴⁰:

$$G(z) = \frac{(z-1)}{z^{l+1}} Z \left\{ e^{mT_s} \frac{G(s)}{s} \right\} \quad \text{with} \quad (4-38)$$

$$\frac{G(s)}{s} = \frac{K_1 + K_2 s}{s^3}$$

where $l = 2$ and $m = 0$, as T_1 and T_2 combine into exactly two sampling periods.

Expressing $G(z)$ as a ratio of polynomials in z yields:

$$G(z) = \frac{(K_1 T^2 + 2K_2 T)z + (K_1 T^2 - 2K_2 T)}{2z^4 - 4z^3 + 2z^2} \quad (4-39)$$

⁴⁰The discrete transfer function of a continuous system with pure time delay is developed in [Franklin et al., 1990, sect. 2.4.2]. For simplicity, the time delay is assumed here to be a multiple of the sampling period. In this section, the sampling and delays are applied to both the rate and position error feedback paths (i.e. the diagram representation of Eq. 4-30), as a worst-case scenario. If gyros are used to provide rate estimates, the 10 Hz update rate will only affect the position error estimate.

The characteristic equation is then:

$$D_{G_i} + N_{G_i} = 2z^4 - 4z^3 + 2z^2 + (K_1T^2 + 2K_2T)z + (K_1T^2 - 2K_2T) \quad (4-40)$$

Bode plots for the discrete system are included in Figures 4-3 to 4-5. In Figure 4-3, because of the fast update rate of the star trackers, the small additional time delay, and the relatively low cross-over frequency of the continuous system, the phase margin in either mode is found to be little different from that of the continuous system ($\sim 53.5^\circ$ in the staring case and $\sim 62^\circ$ in the limb following case). However, a 10 Hz update rate is quite high for a star tracker with accuracies of a few arcseconds. Lower update rates significantly reduce the phase margin of the system, especially in the staring mode (see Figures 4-4 and 4-5).

4.5.4 Steady-state Error Associated with Changing Reference Inputs

By feeding forward coupled terms in the control law, the steady-state error associated with changing reference inputs disappears from the decoupled control loop. In other words, it is assumed that the controller can effect perfect tracking of the commanded states by computing in real-time the value of the coupled terms and sending compensating commands to the actuators. However this also assumes that:

- the inertia tensor J_c is perfectly known,
- there is no time delay in computing the commands,
- the actuators immediately and accurately effect the command, and
- there are no measurement or quantization errors.

The effect of delays, noise and quantization on the performance of the controller will be investigated using simulation. Results are included in Chapter 6.

4.6 Command Distribution and Actuator Gains to Implement Momentum Control

Because the satellite has a redundant reaction wheel and three redundant coils, algorithms must be defined to solve for ω_d in Equation 4-28 or h_d in Equation 4-29, and to determine the distribution of dipole commands. Actuator gains must then be selected to effect the commands. Commands and gains associated with normal operation are first discussed. Control during desaturation is addressed next.

4.6.1 Distribution of Wheel Speed Commands

In Equation 4-29, assuming that the four wheels are available for use, an infinite number of solutions exist for h_d . A steering law must determine how the momentum is distributed among the wheels. An algorithm based on the least squares right inverse⁴¹

⁴¹The inverse of nonsquare matrices is discussed in Wertz (1978, Ap. C.4). When it exists, the generalized right inverse of an $m \times n$ matrix A is given by:

$$\begin{aligned} A^R &= A^*(AA^*)^{-1} \\ &= A^T(AA^T)^{-1} \text{ if } A \text{ is a real matrix} \end{aligned}$$

where

$$AA^R = I_m \text{ (} m \times m \text{ identity matrix)}$$

of A_{cm} is written as ⁴²:

$$h_d = A_{cm}^R \delta h + \lambda v + J_m \Omega_{s0} \quad (4-41)$$

where v is the vector space which is perpendicular to the row space of A_{cm} ⁴³ and the value of λ is set to meet the constraints at time t of the simulation. For example, if all four wheels are active but the momentum demand to one wheel exceeds the allowed level, then λ is set to null the excess demand and redirect it to the other wheels. In the case of ODIN, only three wheels will likely be active at a time, with the fourth available in case of failure. If the third wheel is inactive or fails, λ is set to null the demand to that wheel and redirect all momentum demand to the other wheels. In the closed-loop simulation (see Appendix C), only the dynamics of the three active wheels are implemented to reduce the number of computations and the number of variables required on line, and consequently the simulation running time. As a result, the matrices for each wheel combination have to be stored. The advantage of using the distribution algorithm above is that only the matrix A_{cm}^R needs to be stored.

⁴²Such an algorithm for torque demand distribution among four wheels is presented in Wertz (1978, sect. 18.2.2).

⁴³Let

$$\begin{aligned} \delta h &= A_{cm}(\delta h_d)_1 = A_{cm}(\delta h_d)_2 \\ \text{where } (\delta h_d)_1 \text{ and } (\delta h_d)_2 &\text{ are two possible solutions for } \delta h_d \\ \text{and where } (\delta h_d)_1 &= A_{cm}^R \delta h \text{ is the solution with the minimum value for } |\delta h_d| \end{aligned}$$

Then

$A_{cm}[(\delta h_d)_2 - (\delta h_d)_1] = 0$ has two general solutions:

Solution 1: $(\delta h_d)_2 = (\delta h_d)_1$

Solution 2: $A_{cm}v = 0$ or $A_{cm}\lambda v = 0$, where $\lambda v = (\delta h_d)_2 - (\delta h_d)_1$, λ is a scalar and v is the vector space perpendicular to the row space of A_{cm} (Boas, 1983, sect.8)

4.6.2 Speed Control Loop around the Wheels

Simple proportional control can be implemented around each wheel to effect h_d . The motor torque equation for wheel n is then:

$$K_p(\omega_{dn} - \omega_{sn}) = T_{mn} = J_{sn}\dot{\omega}_{sn} + B_n\omega_{sn} + T_{fn} + T_{ln} \quad (4-42)$$

Ignoring viscous friction, the time constant of the loop is given by J_{sn}/K_p . K_p should be chosen such that the time response of the system is as small as possible. A lower bound on K_p can be set such that the closed-loop frequency is at least five times the bandwidth of the system. The effect of varying K_p will be investigated using simulation.

4.6.3 Desaturation Commands

Autonomous desaturation can be implemented such that desaturation of all wheels to bias levels will be triggered when the speed of any wheel has exceeded a prescribed limit. Dipole commands and unloading torque commands are computed at all times while one variable in the simulation keeps track of whether any of the enabled wheels has reached critical speed conditions. The value of that variable determines whether the desaturation commands are initiated.

The unloading torque commands can either be set to a specified value, for example 0.01 N-m (half the torque capability of the reaction wheel model) for a given time interval, or to the constant torque required to desaturate each wheel to its bias level

in the time t (ignoring friction):

$$T_n = \frac{B_m \left[\Omega_{in} - \omega_m e^{-\frac{B_m t}{J_n}} \right]}{\left(1 - e^{-\frac{B_m t}{J_n}} \right)} \quad (4-43)$$

The use of magnetorquers cannot produce torques parallel to the geomagnetic field vector. The net dipole and the corresponding torque which can be applied perpendicular to the field vector must then be calculated. Assuming that the field vector is computed in F_f coordinates using an orbit generator and a field model⁴⁴, T_n above is translated into the following dipole command, resolved in flight coordinates:⁴⁵

$$D_n = \frac{B^x C_{fr} T_n s_n}{B^T B} \quad (4-44)$$

where s_n is the spin axis of wheel n in control coordinates. The total dipole due to the

⁴⁴The direction and magnitude of the geomagnetic field in control coordinates could be derived from the onboard magnetometers.

⁴⁵The produced torque corresponding to the unloading torque command for wheel n is derived as follows. Let

$$\bar{T}_n = \bar{D}_n \times \bar{B}, \text{ where } \bar{D}_n \text{ must be computed}$$

Expanding the cross-product of the field vector by the unloading torque command yields:

$$\bar{B} \times \bar{T}_n = \bar{B} \times (\bar{D}_n \times \bar{B}) = (\bar{B} \cdot \bar{B})\bar{D}_n - (\bar{B} \cdot \bar{D}_n)\bar{B}$$

where the last term is zero since only the dipole component perpendicular to the field vector will generate the applied torque. To desaturate wheel n , the dipole command and the corresponding applied torque are then:

$$\bar{D}_n = \frac{\bar{B} \times \bar{T}_n}{\bar{B} \cdot \bar{B}} \text{ and } \bar{N}_n = -\bar{B} \times \left(\frac{\bar{B} \times \bar{T}_n}{\bar{B} \cdot \bar{B}} \right)$$

four wheel torque desaturation demands and the corresponding total external torque generated by this dipole are given by:

$$\begin{aligned} D &= \sum_{n=0}^4 \frac{B^x C_{fc} T_n s_n}{B^T B} \quad (\text{in flight coordinates}) \\ (T_c)_{desat} &= -C_{cf} B^x D \quad (\text{in control coordinates}) \end{aligned} \quad (4-45)$$

The dipole moment demand is directed to three of the six available coils. Which coils are active is assumed to be determined on the ground, and that information uplinked to the satellite. The dipole demand is given by:

$$d = (A_{cl'})^{-1} C_{cf} D \quad (4-46)$$

where

$$\begin{aligned} A_{cl'} &= F_c \cdot S_{l'}^T \\ S_l &= [\hat{l}_{1a} \ \hat{l}_{2a} \ \hat{l}_{3a} \ \hat{l}_{1b} \ \hat{l}_{2b} \ \hat{l}_{3b}]^T \\ \hat{l}_{ij} &\quad \text{Direction of rod } i \text{ coil } j, j=a,b \text{ for main and secondary coil} \\ S_{l'} &\quad \text{Reduced } S_l \text{ set, consisting of the basis vectors} \\ &\quad \text{associated with the three active rods} \end{aligned} \quad (4-47)$$

4.7 Digitization of the Controller

Up to this point, the controller has been developed assuming continuous control.

Tustin's or the bilinear approximation method⁴⁶, based on trapezoidal integration, is now used to digitize the controller. The integration for each axis in Equation 4-29 is modified as follows (implementation is shown in Appendix C, Block C.3.2):

$$\begin{aligned}\frac{U(s)}{X(s)} &= \frac{1}{s} \\ u(nT) &= u(nT-T) + \frac{T}{2} [x(nT-T) + x(nT)] \\ U(z) &= z^{-1}U(z) + \frac{T}{2} (1+z^{-1}) X(z)\end{aligned}\tag{4-48}$$

where u and x are the values along each axis of δh and of the integrand in Equation 4-29.

⁴⁶Digitization procedures and limits to their application are discussed in Franklin et al. (1991, sect. 8.3). They show that Tustin's method is more accurate than the matched pole-zero or modified matched pole-zero approaches, but requires more computations. They also state that sample rates of $20(\omega_n)$ or more will give reasonable results.

Table 4-1. Pointing and Reconstructed Accuracy Requirements of the Payload Boresight for Selected Activities (Information extracted from SSC, 1993a)

Activity (Sunlit and eclipse conditions)	Controlled Pointing Accuracy	Reconstructed Pointing Accuracy
Astronomy/Staring	Better than $\pm 15''$ (1 σ) in pitch and yaw	Better than $\pm 15''$ (1 σ) in pitch and yaw
Astronomy/Scanning		Better than $\pm 1.2'$ (1 σ) in pitch and yaw
Aeronomy/Limb pointing and scanning	Better than: pitch: $(\pm) 6'/5$ km altitude tracking error yaw: $(\pm) 50'/50$ km horizontal tracking error roll: $(\pm) 15' / 0.5$ km at slit ends with 80% confidence level	Better than: pitch: $(\pm) 2'/2$ km altitude tracking error yaw: $(\pm) 10'/10$ km horizontal tracking error roll: $(\pm) 15' / .5$ km at slit ends with 80% confidence level

Table 4-2 Root-Sum-Square Values of the Various Sources of Errors, Other than those Associated with Disturbance Rejection, for Two Sample Cases Compiled in Swedish Space Corporation (1993b).

Activity	Pointing error (arcseconds)	Reconstructed error (arcseconds)
Astronomy/Fine Pointing: Attitude estimation using two star sensors	pitch: $\sigma = 7.89+$ yaw: $\sigma = 8.74+$	pitch: $\sigma = 7.89+$ yaw: $\sigma = 8.74+$
Aeronomy/Limb Pointing: Combination Star tracker/Gyros	pitch: $\sigma = 202+$	pitch: $\sigma = 79+$

Figure 4-1. Selected Coordinate Frames. (see section 4.1)

The orientation of the spin axes of the four wheels was assumed to be as shown above. Reviewing the documentation, however, the orientation of the skewed wheel will be such that it is equally offset from the other three wheels (as indicated by *). This does not change the approach used in Chapters 4 to 6, but it will change the waveform of the wheel speeds or momentum for cases where the combinations including wheel 4 were used (eg., in Figures 5-1 to 5-24)

Figure 4-2. Bode Plots for the Continuous Decoupled System in Equation 4-30 for (a) Staring and (b) Limb Following Operating Modes.

(a)

(b)

Figure 4-3. Bode Plots for the Decoupled System in Equation 4-30, with Sampling and a Unit Delay of 0.1 Second and an Additional Pure Time Delay of 0.1 Second, for (a) Staring and (b) Limb Following Operating Modes.

Figure 4-4. Bode Plots for the Decoupled System in Equation 4-30, with Sampling and a Unit Delay of 0.5 Second and an Additional Pure Time Delay of 0.5 Second, for (a) Staring and (b) Limb Following Operating Modes.

Staring: $\omega_c \sim .84$ rad/sec, phase margin $\sim 5.5^\circ$

Limb following: $\omega_c \sim .26$ rad/sec, phase margin $\sim 46.5^\circ$

(a)

(b)

Figure 4-5. Bode Plots for the Decoupled System in Equation 4-30, with Sampling and a Unit Delay of 0.5 Second and an Additional Pure Time Delay of 0.1 Second, for (a) Staring and (b) Limb Following Operating Modes.

Staring: $\omega_c \sim .84$ rad/sec, phase margin $\sim 24.5^\circ$

Limb following: $\omega_c \sim .26$ rad/sec, phase margin $\sim 52.5^\circ$

CHAPTER 5. CONTROLLER PERFORMANCE : TEST CONDITIONS

This chapter describes the test conditions used to evaluate the performance of the controller developed in Chapter 4. Odin will be exposed to various flight conditions, which the controller must be able to handle. Sample flight conditions are defined in section 5.1. The effect of noise, disturbances and time delays in the system must also be investigated. Typical disturbances are defined in section 5.2. Representative noise and time delays associated with wheel speeds, attitude and angular velocity measurements are defined in section 5.3. As sensor data and actuator data to and from the controller will be sent using the onboard telemetry system, additional time delays and quantization effects will be present. These conditions are defined in section 5.4.

5.1 Flight Conditions

In this document, flight conditions refer to the satellite's orbit, attitude profile, angular velocity profile, momentum and disturbance environment.

The flight conditions are set by defining the reference signals into the controller and the spacecraft residual magnetic dipole. The reference signals include: reference

attitude quaternion, reference spacecraft angular velocity and wheel speed biases. The first two signals are generated in blocks H.i of the simulation diagrams (see Appendices B and C). The residual dipole defines the magnetic disturbance environment. It is often preferable to operate the wheels such that the speed direction is not reversed. This can be accomplished in certain cases by applying an initial bias to the wheel speed. The bias required to avoid zero-speed crossings is a function of the environmental torques anticipated, which in turn are a function of the spacecraft attitude profile and physical attributes.

The approach is then to specify a sample attitude and angular velocity profile, assume that the residual spacecraft dipole is measurable and at a given constant value over the next orbit, run the open-loop simulation to estimate solar radiation pressure, aerodynamic, gravity gradient and magnetic torque disturbances and the associated angular momentum buildup from zero, and then to estimate the wheel speed biases which are required to avoid zero-speed conditions.

The different types of attitude profiles which will be encountered during payload operation are described in section 5.1.1. Sample flight conditions other than speed biases, set up to reflect these various profiles, are detailed in section 5.1.2. The choice of initial bias levels for the reaction wheels is discussed in section 5.1.3.

5.1.1 Background Information on Commanded Attitude Profiles

The information in this section is extracted from a preliminary document (SSC, 1993c) describing the user requirements for the software of the onboard attitude control

and determination computer (ACDC).

According to the reference material, the ACDC will operate in five different modes: initial, safe, detumble, standby and normal. The controller developed in Chapter 4 will be tested under conditions representative of payload operation, when the ACDC is in its normal mode of operation and when the required accuracies are most stringent.

In its normal mode, Odin will follow one of four programs, summarized in Table 5-1. The type of information which will be available to the computer to generate the reference attitude profile during a given program, is included in Table 5-2.

During "Astronomy pointing and mapping sessions", the spacecraft will be required to point the boresight of the instrument at one or more celestial targets. A calibration target will be associated with each observation target and the spacecraft will be programmed to switch the commanded position between the two targets. Typically, calibration and observation targets will be offset by 10 to 30 arcminutes. The required control accuracy is 15 arcseconds and it is assumed that transients must settle as rapidly as possible.

During "Astronomy scanning" sessions, the spacecraft will perform a constant slow rotation from an initial to a final celestial target. In two of the axes, the steady-state tracking error during the rotation will have to be less than 1.2 arcminutes and the reconstructed pointing error less than 15 arcseconds. The control requirement about the third axis is more relaxed, and will be within two to five degrees (boresight axis).

During "Aeronomy limb pointing sessions", the line-of-sight (LOS) of the instrument will normally be pointing in the forward direction of the satellite's motion, at a constant altitude relative to the Earth's limb.⁴⁷ The orientation of the slit (rectangular field-of-view) of the instrument, parallel to the local horizontal plane, defines the third-axis orientation of the satellite. A few days prior to the event, the inertial components of the horizon-pointing and slit vectors will be simulated over the scheduled time of the event using orbit propagation routines. A set of Chebychev polynomials will be fitted to the simulated data. Shortly before the event, the computed sets of coefficients will be uplinked to the satellite with the other program data required. Using stored data, the computer will then be able to generate the target attitude quaternion at the time of interest⁴⁸. A first-order differentiation will provide the attitude quaternion rate which can then be converted to the required angular velocity.

"Aeronomy limb scanning" sessions will be performed in two stages. The spacecraft will first be placed in a "limb following" mode, during which the boresight of the instrument will point at a constant altitude above the Earth's limb. If required, the wheel speeds will be biased at this time. A scanning profile will then be superimposed on the limb following profile. Two types of scans will be implemented and, in most cases, scanning will be done in the orbital plane. The boresight of the instrument will either continuously scan between 10 and 120 km in altitude above the

⁴⁷In most "limb following" events, the instrument's LOS will remain in the orbital plane. When scanning is done out of the orbital plane, the angle between the scanning and the orbital plane will be restricted such that the angle of incidence of the Sun on the solar array does not exceed 25-30°.

⁴⁸If the components of the horizon and slit vectors are known in the body frame and can be calculated in the inertial frame using the stored Chebychev coefficients, the transformation matrix associated with the rotation from inertial to control axes can be computed using the algebraic method described in Wertz (1978, sect. 12.2.2). This rotation matrix can then be transformed into a quaternion representation as explained in Wertz (1978, Ap.D).

Earth's limb or will switch between fixed altitude levels. Using the Chebychev coefficients to generate the limb pointing attitude, and the programmed scan profile to generate the scan attitude, the computer will then generate the target attitude quaternion relative to inertial space and the quaternion rate.

5.1.2 Sample Flight Profiles and Spacecraft Magnetic Dipole

Based on the information presented in section 5.1.1, five general attitude profiles are set up in the simulation for testing the controller. They are implemented in blocks H.1 to H.5 of the open-loop (see Appendix B) and closed-loop (see Appendix C) simulation models and are described in Table 5-3. Three test orbits are defined, each with different Sun conditions, and are detailed in Table 5-4. Different sets of flight conditions are set up, with wheel speed biases not yet defined, and are included in Table 5-5.

5.1.3 Reaction Wheel Speed Biases and Triggering Levels for Desaturation Activity

Initial biases can be applied to reaction wheels, prior to initiating certain attitude profiles, to prevent zero-speed crossings. The initial biases required are case-specific. They depend on the commanded attitude profile, the orbit at the time of the event, the anticipated secular disturbances, how the momentum is to be distributed among the four wheels and the frequency at which momentum dumps are to be performed. An open-loop simulation is run for each set of flight conditions outlined in Table 5-5 to determine appropriate initial speed levels.

The open-loop simulation diagrams are included in Appendix B. For each case, the rotational dynamics of the spacecraft are integrated assuming perfect tracking of the reference inputs, and zero initial momentum in the wheels. Using the algorithm presented in Chapter 4, the momentum buildup is distributed among the reaction wheels, assuming that all four wheels are active or that any combination of three are active. When all four are active, the minimum norm is used as the criterion for choosing one out of the infinite possible solutions. For each of the cases in Table 5-5, except for case 5A where results are expected to be similar to those found for case 4A, the following is plotted (see Figures 5-1 to 5-24):

- (a) the momentum profile for individual wheels for each combination of active wheels,
- (b) the anticipated environmental disturbances and Sun incidence angle on the solar array, and
- (c) the spacecraft total momentum and wheel momentum along control coordinates, and the reference attitude and reference angular velocity.

The nominal bias speed required by each wheel, so that the speed does not cross zero when absorbing the effect of environmental disturbances, will depend on the wheel combination used. Although operating all four wheels minimizes the effort of each active wheel, only three wheels are likely to be used at any time with the fourth wheel available in case of failure. Further, the three orthogonal wheels are likely to be used on a regular basis, as the effort by the skewed wheel would have to be bigger because of its orientation. For some of the sets in Table 5-5, the biases required for the wheel combination leading to the worst-case cyclic variation in momentum are computed. For

the other sets, biases required for the orthogonal combination are calculated.

First let us consider the cases for which the reference frame F_p is nonrotating, as during inertial pointing activity. In such cases the three components of Ω are zero, where Ω is the commanded rotation of the reference frame F_p , resolved in F_p coordinates, relative to inertial space. To avoid zero-speed conditions, the momentum curve for each active wheel simply needs to be shifted in the direction of the secular momentum buildup by the maximum momentum change anticipated over the course of one orbit. It is further assumed that the wheels are operated between 10 and 95 percent of the range of speeds. For inertial pointing (with or without calibration slews), the minimum required bias level for wheel i , to avoid zero speed crossing, is then given by:

$$(\omega_{si})_{bias} = \text{sign}[(h_{sec,i})_0] \frac{[h_{min} + (\Delta h_{max,i})_0]}{J_{si}} \quad (5-1)$$

where

$h_{min} = 0.1 J_{si} (\omega_{si})_{max}$ (minimum wheel speed assumed to be at 10 percent of full range);

$(\omega_{si})_{max}$, the maximum speed, is set to 534 rad/sec (see the reaction wheel model described in Chapter 3);

J_{si} , the assumed polar moment of inertia for wheel i , is set to 0.0075 kg-m²;

$(\Delta h_{max,i})_0$ is the maximum momentum change for wheel i over one orbit, in the direction of $(h_{sec,i})_0$, assuming zero initial momentum; and,

$(h_{sec,i})_0$ is the secular momentum buildup for wheel i , one orbit following the previous ascending equatorial crossing, and assuming zero initial momentum.

The required biases to avoid zero-speed crossings for some of the inertial pointing

cases are included in Table 5-6.

When the reference frame F_p is rotating, as during limb tracking or scanning activity, the evolution of the system momentum is also a function of the initial momentum perpendicular to the reference frame rotation axis (i.e., we cannot simply shift the momentum curve as can be done for cases where F_p is nonrotating). We next attempt to estimate the required biases when Ω has nonzero magnitude, using the values of momentum buildup and variation computed over one orbit assuming zero initial wheel momentum.⁴⁹

If the external torques are assumed to be zero, the entire momentum of the system will remain constant in magnitude and direction. If we also assume that the reference inputs are perfectly tracked using the controller (i.e., the spacecraft control frame is aligned with the reference frame F_p and rotates with angular velocity Ω), that the

⁴⁹Let the system momentum initially be equal to H_o , and consist only of composite spacecraft body momentum. In other words, let the momentum of the wheels relative to the spacecraft body be zero at the start of the integration. Over time, with all vectors below resolved in the same coordinate frame and assuming perfect tracking of the reference inputs, the momentum would then be given by:

$$H_t = \int (T_{ext} - \Omega \times H_t) dt + H_o$$

Let a new variable H_t' be equal to $H_t + h_o$ and the system initial momentum be equal to $H_o + h_o$, where h_o is the momentum of the wheels relative to the body at the start of integration. This yields:

$$\begin{aligned} H_t' &= \int [T_{ext} - \Omega \times (H_t + h_o)] dt + H_o + h_o \\ &= H_t + \int -\Omega \times h_o dt + h_o \end{aligned}$$

which implies:

$$\int -\Omega \times h_o dt = 0$$

The solution of h_o will therefore consist of sinusoidal functions, and the relationship $H_t' = H_t + h_o$ will hold when the integration is done over one orbit (i.e., 360° or 2π radians) so that the integral of the coupling between the orbital rate and the bias momentum is zero.

commanded rotation of the spacecraft is fixed in magnitude and direction relative to inertial space, then the controller will issue commands to the wheels so that the bias momentum of the wheels is also constant in magnitude and direction relative to inertial space. As the spacecraft is rotating and the wheel spin axes are fixed relative to the body, the wheel speeds will cycle so that the combined momentum vector remains fixed in magnitude and direction.

Let us assume pure rotation of the reference frame F_p about the orbit normal. In other words, let us assume limb following activity with the spacecraft Z_p -axis aligned with the negative orbit normal, negligible precession rate of the orbit, and wheel 3 along the spacecraft Z_p -axis. If the three orthogonal wheels are used, the commanded rotation Ω of F_p is then $[0 \ 0 \ -\omega_o]$ when resolved in a coordinate frame formed by the three wheel spin axes, where ω_o is the orbital rate (i.e., in the earth-referenced mode, one of the orthogonal wheels is nominally aligned with the negative orbital pitch vector). Therefore, superimposed onto the secular and cyclic behavior required to absorb the external torques, a periodic variation in the speed of wheels one and two will also be present as a result of the initial bias momentum. Wheels one and two can therefore not avoid crossing the zero speed.

The cyclic variation in wheels one and two strictly due to the coupling between the orbit rate and the wheel initial bias momentum is given by (see footnote 49):

$$\int_0^{2\pi} -\Omega^x h_o \, dt = 0 \quad (5-2)$$

where $\Omega = [0 \ 0 \ -\omega_o]$, $h_1 = h \sin(-\omega_o t)$, $h_2 = h \cos(-\omega_o t)$, and $h_3 = 0$

When the wheel combination 1-2-4 is used, with wheels one and two nominally orthogonal to the orbital pitch vector and wheel four skewed, there is coupling between the orbital rate and all three wheels. A combination of wheel speeds can possibly be found so that the cyclic variation due to coupling of only one of the wheels will cross the zero speed. For example, the cyclic variations of the skewed wheel and one of the other wheels, if each shifted in opposite directions from zero by a sufficient bias, would result in a net variation crossing zero without each of their own variation crossing zero.

To completely avoid crossing the zero speed by any of the wheels, for cases when the reference frame F_p is rotating* (for example during limb following activity), an external torque could be applied, using the magnetorquers, opposing the coupling torque between the commanded Ω and the nominal wheel speed biases. Physically, this would correspond to rotating the bias momentum vector of the wheels in inertial space, so that the orientation of the bias momentum relative to the spacecraft axes remains fixed. This would be similar to desaturation activity, except that the net result would be that the wheels nominally remain at their bias level (rather than monotonically decreasing in speed as during desaturation), and only cycle and vary to absorb external disturbances. The minimum bias levels required could then be chosen as in the inertial pointing cases, as long as a continuous magnetic torque is applied to counteract the cyclic effect which would otherwise be introduced as a result of coupling between the orbit rate and wheel speed biases. In other words, the decoupling torque command could be sent to the torque rods rather than being included in the integrand of Equation 4-29.

The desaturation triggering conditions will be a function of the magnitude and direction of the field vector, the maximum dipole available from the torque rods and the maximum time allowed for unloading (see Appendix A). For a polar orbit, the magnitude of B at the equator can be approximated to M/R^3 and to twice that value at the poles, where M is the magnetic moment of the Earth, 7.96×10^{15} tesla-m³, and R is the radius from the center of the Earth to the spacecraft (Larson & Wertz, 1992). For a torque rod nominally perpendicular to B and capable of delivering 40 A-m², the torque generated will be of the order of 1 to 2×10^{-3} N-m. It is assumed that this torque is equally distributed between two wheels, i.e. 0.001/2 N-m, and that desaturation is limited to five minutes. Then, with no other torques present but aided by kinetic friction of 0.001 N-m, a resultant torque of 0.0015 N-m during five minutes would cause a change in speed of approximately 96 rad/sec if the wheel is initially at 300 rad/sec, 83 rad/sec if initially at 200 rad/sec, and 70 rad/sec if initially at 100 rad/sec (using our reaction wheel model). If desaturation to bias levels is to be effected in five minutes, then the average wheel speed over an orbit should be kept within 100 rad/sec from its bias level. An approximate triggering speed level could then be given by:

$$(\omega_{si})_{trigger} \approx (\omega_{si})_{bias} + 100 - \frac{\Delta h_i}{2J_{si}} \quad (5-3)$$

During "inertial pointing" activity, in the worst case scenario, celestial targets will be occulted from the spacecraft for up to 35 minutes (when the target is directly behind the Earth and a spherical Earth with 6371 km radius and a 700 km circular orbit are assumed). Desaturation should be triggered as much as possible while the target cannot be observed, by ground control, based on assessed secular buildup at the time.

The results in Table 5-6 and the wheel momentum distribution plots in Figures 5-1 to 5-24 have been obtained assuming the following orientation for the four reaction wheels (see Figure 4-1):

$$S_{ws} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -\sin(\theta) \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & \cos(\theta) \end{bmatrix}, \text{ where } \theta = \tan^{-1}(\sqrt{2}) \quad (5-4)$$

and where S_{ws} is the matrix of direction cosines of the spin axes relative to the satellite frame X,Y,Z_s . For the skewed wheel to be equally offset from the other three wheels, the sign of $S_{ws}(1,4)$ must be reversed.

5.2 Disturbances

Disturbances are generated in blocks A.2 and D of the closed-loop simulation. Environmental disturbances, including gravity gradient, solar radiation, aerodynamic and magnetic torques, are computed in block A.2. A spacecraft model, developed based on early Odin reference material, is described in Appendix A. The environmental disturbances can be "turned off" by disconnecting the output of block A.2 and feeding zero inputs into the summation block sum1 (of block A).

In block D, a step, a ramp or another type of input can be applied as an external torque about any of the three control axes. If control torques are excluded, the largest

torque anticipated during payload operation will be associated with the sum of all environmental torques experienced, of the order of 10^{-4} N-m. As only the solar radiation torque (of the order of 10^{-6} N-m, see Appendix A) steps from zero when exiting eclipse conditions, a step disturbance of 10^{-4} N-m would be an absolute worst-case scenario. The effect of such a step in each of the control axes will be investigated.

In block D again, an internal disturbance torque can be applied about any of the four wheel spin axes to investigate the effect of torque irregularities.⁵⁰ The effect of a 0.001 N-m internal step disturbance in one of the wheels will be investigated. This corresponds in our wheel model to five percent of the maximum motor torque available or to 100 percent of the Coulomb friction torque, but which is still smaller than the step experienced at zero-speed crossing.

The effect of zero-speed crossings will also be examined by setting the wheel speed biases at zero for some of the flight condition sets in Table 5-6.

A fictitious torque ripple has also been incorporated in the dynamics of the wheels (block A.6), and has been simulated as a simple sine wave with an amplitude corresponding to 20 percent of the nominal torque demand and a phase equal to twice the rotor angular position.

⁵⁰Bosgra and Prins (1982) discuss torque irregularities associated with ball-bearing DC-motor wheels. They report that the torque irregularities will be associated with temperature variations, with the motor commutation torque ripple, and with the movement of the retainer. They also describe the tests which can be conducted to evaluate torque irregularities, friction values and noise characteristics for a particular wheel (e.g. start-up tests, wheel speed reversal tests, wheel rundown tests and spectral tests).

5.3 Sensor Noise and Delays

Measurement of the spacecraft attitude, spacecraft angular velocity and wheel speeds is simulated in block B of the closed-loop simulation diagram, using a very simple approach.

The spacecraft total momentum and wheel spin momenta are computed over time in block A, by integrating the rotational dynamics of the system from a set of initial conditions. The spacecraft angular velocity and wheel speeds are then evaluated from the simulated momentum values. Integration of the kinematics of rotation yields the simulated spacecraft attitude.

Band-limited white noise⁵¹, and unit delays⁵² in the case of the attitude, are artificially applied to the simulated values, in an attempt to approximate the measurement error which would be present.

⁵¹When a variable step size integrator is used, if a new noise value is generated with every step, the program could iterate infinitely within a time interval without converging to a solution. Mathworks (1992) recommends using the built-in band-limited white noise block in Simulink, implemented by feeding white noise into a zero-order hold. The user then defines the noise variance per Hz, the seed for generating the pseudo-random sequence of errors (normally distributed and repeatable for a given seed), and the time for which to hold the random value which then determines the bandwidth. By fixing the step size during integration (to avoid problems with memory blocks - especially in terms of simulation speed), and by defining the sample time of the ZOH in the band-limited white noise block to be equal to the step size, numerical problems are avoided even for integrators which use intermediate steps to compute the value at the step size specified.

⁵²A unit delay is equivalent to a zero-order-hold, which samples and hold the data for a specified time interval T_s , with the output further delayed by a full sample period T_s .

5.3.1 Wheel Speed Measurements

Speed measurement is built-in with the reaction wheel unit, and the noise and time delay associated with the measurement is a function of the type of tachometer and pulse processing method used. For the purposes of this simulation, it is assumed that the tachometer consists of a wheel-based magnet and a body-fixed sensor, and that the pulse train is fed into a phase-locked loop system.⁵³ The time delay associated with counting the pulses and the quantization effect due to the fixed number of pulses per revolution are assumed to be negligible.

Band-limited white noise with 3σ equal to 0.1, 0.2 and 0.5 percent of the wheel speed will be investigated (i.e. an error of one, two or five pulses in a thousand). These error levels are however considered large.⁵⁴

5.3.2 Angular Velocity Measurements

Odin will be equipped with three two-axis dynamically tuned gyros (DTGs), one of which will be redundant. In addition to defining their power requirements, dimensions

⁵³Tachometers measure rotary speed. Different types exist and are described in Wertz (1978, sect. 6.6) and Tapley (1990, sect. 12.9.4). Electrical generators provide an analog output proportional to angular speed. Electromagnetic tachometers use magnet(s) fixed to the rotor and sensors fixed to the casing. Photoelectric tachometers can consist of a light beam generator, a disk with opaque and transparent patterns, and a light sensor. The latter two types generate a pulse train. The pulses are then counted over a given time interval, or the time associated with a fixed number of pulses is recorded. The pulse train can also be fed into a phase-locked loop, to reduce noise associated with the speed measurement.

⁵⁴Speed measurement error levels in commercial wheels are not readily available. The following is an example of the type of information provided. According to the manufacturer's data sheet, some of Teldix's wheels are equipped with a wheel drive electronics interface which receives the torque or speed commands and determines actual wheel torques, speeds and speed directions at an accuracy of up to 0.1 percent (Teldix, 1992).

and weight, Swedish Space Corporation (1993d) lists the following specifications for the baseline package:

- Dynamically tuned gyro
- Output in angular increment pulses
- Pulse scale factor 0.4"/pulse
- Bias drift stability of 0.1°/h
- Scale factor temperature sensitivity of 140 ppm/°C

It is assumed that the gyros will be used predominantly to provide rate measurements and that the computer will count the number of pulses and differentiate the resulting angular measurement at successive time intervals. The pulse train on Odin, however, is likely to be processed with the star sensor data such that filtered attitude and rate information is produced.

Each gyro has its own bias and noise characteristics. The flight package will therefore have to be thoroughly tested and characterized. The day-to-day drift or repeatability value is a measure of the variation in the output as a result of switching the gyro on and off. The random drift is a measure of the variation in the output over time. The gyro is usually judged based on its dynamic range, its repeatability and its long-term drift.

Gyro noise can be grossly approximated as containing white and colored components. White noise at a given time is independent from noise at any other time. Colored noise, on the other hand, is influenced by the noise value at some other time.

One method which is used to characterize long term noise effects is as follows (Cain, 1994). The gyro is mounted on a rate table with one axis parallel to the geographic north and the other axis along the local vertical. Over a period of a few days, with the table kept at standstill and at intervals of ΔT seconds, the root-mean-square value of the rate output (torque motor currents or train of angular increment pulses) over the last sampling interval is recorded. The power spectral density of the measurements, after the mean of the data has been removed (thus removing the bias), can be used to develop an approximate attitude and rate error model associated with noise.

In Cain (1994), the method outlined above is used to evaluate the performance of the Litton CSG-2 gyroscope in its two axes. The error model for the noise in the rate measurements is derived by fitting a first-order lag equation in the low frequency portion (mechanical noise), and by fitting a constant magnitude line in the high frequency region (electrical noise). The worst axis for the Litton gyro is found to have a drift rate of the order of $0.01^\circ/\text{hr}$ (ten times better than the baseline package for Odin). The model parameters for the x axis are given as:

$$\begin{aligned} Q_c &= .25 \text{ deg}/(\text{hr}(\text{Hz})^{1/2}) \\ \omega_c &= 2.744 \times 10^4 \text{ rad/sec} \\ Q_w &= .035 \text{ deg}/(\text{hr}(\text{Hz})^{1/2}) \\ \omega_c &= 1.678 \times 10^3 \text{ rad/sec} \end{aligned}$$

where ω_c is the corner frequency at which the first order lag is introduced and ω_w is the corner frequency after which white noise dominates. Since each gyro has its own noise characteristics, an arbitrary noise model on the rate is assumed for simulation purposes, based on the Litton gyro noise model, but with parameters scaled to reflect the difference

in drift rate characteristics between the Litton and the baseline gyro for Odin. The noise added to the gyro measurements is therefore assumed to be of the form:

$$W_t = \frac{W_c}{\tau s + 1} + W_w \quad (5-7)$$

where W_t is the total gyro noise, W_c is the colored noise component with variance $\sigma_c^2 = (10)^2 Q_c^2 (\omega_c / 2\pi)$, $\tau = \omega_c^{-1}$, W_w is the white noise component with variance $\sigma_w^2 = (10)^2 Q_w^2 (\Delta T)^{-1}$, $\Delta T = 2.64 \text{ min}$ is the time interval at which measurements are made in the Litton gyro experiment and the factor $(10)^2$ is included to account for the different drift rate characteristics.

Quantization effects and time delays associated with processing the pulse train and its time derivative are considered negligible, especially in view of the time delay associated with transporting the sensor data to the controller over the telemetry system.

5.3.3 Attitude Measurements

The attitude measurement accuracy will be a function of the star tracker performance characteristics, the estimation algorithm and the sensor frame alignment accuracy relative to the control frame.

The performance of star trackers is characterized by a stated accuracy in angular units about each measurement axis, a noise-equivalent-angle (NEA) value for the corresponding star magnitude range, the maximum tracking rate at which full accuracy

is maintained, the update rate in Hz, the field of view, and the number of stars which can be searched or tracked simultaneously (list nonexhaustive).⁵⁵

The accuracy is usually the root-sum-square⁵⁶ of errors from independent sources, each quantified in terms of a standard deviation⁵⁷. The stated accuracy may account for sensor biases arising from the fixed resolution of calibration procedures, the effect of neighboring stars and the precision of centroid computations. It may also account for noise and quantization of the output. The NEA⁵⁸ is a measure of the error associated with sensor noise⁵⁹.

Baseline specifications for the star tracker on Odin include (SSC, 1993d):

⁵⁵Examples of star tracker performance specifications are given in Prospace (1993) and Hughes Danbury Optical Systems (1990).

⁵⁶The root-sum-square (RSS) is the square root of the sum of squared elements.

⁵⁷The standard deviation σ is the root-mean-square of the deviations of a set of measurements from the mean. As deviations from the arithmetic mean are used, an unbiased value for σ is given by:

$$\sigma = (n-1)^{-1/2} \left[\sum_{i=1}^n (m_i - \bar{m})^2 \right]^{1/2}$$

where $\bar{m}$ is the arithmetic mean of the n measurements (Tapley, 1990).

⁵⁸It is assumed that when given in terms of arcsec (1σ), the noise-equivalent-angle or NEA represents the standard deviation or square root of the average power of the noise over the bandwidth of the sensor or a specified bandwidth. When given in terms of arcsec(Hz)^{-1/2}, it is assumed to represent the square root of the power density of the noise.

⁵⁹A measure of the sensor noise present is important. If the magnitude of the controller transfer function increases at higher frequencies, it will result in unwanted noise magnification and overheating of actuators. PD, lead, PI and lag compensation design and the importance of noise considerations is discussed in Franklin et al. (1991, pp. 296-312).

- 8° x 8° FOV
- 3.4" accuracy (including the effect of noise and output quantization)
- NEA of 1" to 5.5" for M=4 to M=6 sensitivity
- Tracking of up to five stars
- Tracking rate: up to 18°/min (full accuracy)
- Update rate: 10 Hz

When converting measured star angles into spacecraft attitude values, we are interested in knowing how the errors in the measurements translate into errors in the computed attitude. Covariance analysis is discussed in Wertz (1978, sect.12.3), and relates the variance and covariance of measurements to the variance and covariance of variables which are a function of those measurements. The covariance matrix P_c will depend mainly on the variances in the observation and reference star vectors and on the algorithm used to compute the attitude error. It will therefore also depend on the number of star measurements and sensors used, on the position and magnitude of the stars within the FOV of the instruments, and on the angular separation between the star vectors or sensors used. The relationship between measurement errors and control axes errors, for two sample sets of measurements, is included in Table 5-7.

Basing the attitude estimate on two star measurements alone is very conservative. Filtering of star position data (using more than two stars) would likely be implemented to reduce uncertainty in attitude estimates. Filtered gyro data (using more than three gyro measurements) would probably also be used to improve attitude estimates between star tracker updates.

Misalignment of the sensor frame relative to the control frame will also result in systematic errors in the attitude measurement.

It is decided to add attitude errors into the simulated system as follows (see block B.1). The simulated attitude from inertial to control axes (q_{ci}) is converted into the corresponding 3-1-2 Euler angle sequence, errors are added to each angle, and the resulting attitude is converted back into the quaternion representation. Since pointing control is more critical during inertial events, the effect of a step error of 2" and 4" in each Euler angle and the effect of band-limited white noise with a 3σ of 2" and 4" in all three angles will be investigated during such events. The effect of a 10 Hz, 5 Hz and 2 Hz update rate in star tracker measurements will also be examined.

5.4 Time Delays and Quantization Associated with the Onboard Telemetry System

On Odin, the sensor and actuator data will be received and sent by the controller using the telemetry system. The data transport delay will therefore be a function of the serial digital data rate and the amount of data telemetered (the traffic). The quantization level will be a function of the number of bits allotted to the particular measurement or command and its scaling.

The requirements document states that sensor data will be received by the controller after a maximum delay of one sixteenth of a second. Similarly, commands issued by the controller will reach the torque rods and the reaction wheels after a

maximum delay of one sixteenth of a second (SSC, 1993e). The effect of these delays will be investigated (implemented in blocks C.5, C.6, C.7 and C.8 in the simulation diagram).

Specifications for the telemetry system include 16-bit data words and 12-bit A/D resolution. Baseline specifications for other attitude equipment include: an 8°x8° FOV for the star trackers, a 534 rad/sec maximum wheel speed, a 18°/min maximum star tracking rate, and a wheel torque capability of 0.020 N-m or more. For comparison purposes, the following quantization effects will therefore be examined (also implemented in blocks C.5, C.6, C.7 and C.8).

Two quantization levels will be applied to the wheel speed measurements: (a) 0.0163 rad/sec and (b) 0.13 rad/sec. Level (a) corresponds to the quantization interval if a 16-bit signed data word length and a maximum value of 534 rad/sec are assumed. Level (b) corresponds to the quantization interval if a 12-bit unsigned data word length and a maximum value of 534 rad/sec are assumed.

Two quantization levels will be applied to the angular velocity measurements: (a) 1.598×10^{-7} rad/sec and (b) 4.439×10^{-8} rad/sec. Level (a) corresponds to the quantization interval if a 16-bit signed data word length and a maximum value of 18 deg/min (5.236×10^{-3} rad/sec) are assumed. Level (b) corresponds to the quantization interval if a 16-bit signed data word length and a maximum value of 5 deg/min (1.4544×10^{-3} rad/sec) are assumed.

The attitude measurements will be quantized to 0.88 arcsec in each axis. This

corresponds to the quantization interval if a 16-bit signed data word length and a maximum value of 8° are assumed. The measurement is quantized in block B.1, prior to being converted to quaternion representation.

Two levels of quantization will be applied to the torque commands: (a) $0.763 \mu\text{N-m}$ and (b) $6.105 \mu\text{N-m}$. Level (a) corresponds to the quantization interval if a 16-bit signed data word length and a maximum value of 25 mN-m are assumed. Level (b) corresponds to the quantization interval if a 12-bit unsigned data word length and a maximum value of 25 mN-m are assumed.

Table 5-1 General Attitude Profiles during Payload Operation
(Information extracted from SSC, 1993c)

Programs associated with the normal mode of operation of the ACDC ^a	Activities
1. Astronomy pointing/mapping	(a) Initial checks and preparations including program data validation (b) Large angle slew (c) Fine pointing to observation target (d) Slew to calibration target (e) Fine pointing to calibration target (f) Momentum management Repeat of activities (c) to (e) if mapping
2. Astronomy scanning	(a) Initial checks and preparations including program data validation (b) Large angle slew (c) Fine pointing to initial point of scanning (d) Astro scanning: Slow rotation about a given axis to final target (e) Momentum management
3/4. Limb pointing without/with scanning	(a) Initial checks and preparations including program data validation (b) Large angle slew (c) Fine pointing to initial target attitude (d) Limb pointing with wheel biasing (e) Scanning (continuous or step-wise) (f) Momentum management

^aAttitude Control and Determination Computer on Odin responsible for onboard attitude control computations including generating the required attitude, estimating the actual attitude, and issuing autonomous actuator commands.

Table 5-2 Example of the Type of Information (Nonexhaustive) Available to the Computer on Odin to Generate the Reference Attitude Profile
(Information extracted from SSC, 1993c)

Activity	Sample case-specific program data (loaded in memory prior to the event)
Fine pointing	Start time of event Attitude quaternion of observation target(s) Attitude quaternion of calibration target(s) Parameters specifying the switching profile between observation and calibration targets Control frame orientation in body axes Momentum management variables (eg desaturation triggering speed levels, active wheels, speed biases, active coils) Program-specific controller parameters (eg feedback gains)
Astro Scanning	Start time of event Attitude quaternion of initial pointing target Attitude quaternion of final target Constant velocity profile for a given time interval about a given axis Control frame orientation in body axes Momentum management variables Program-specific controller parameters
Limb following	Start time of event Attitude quaternion of initial target Chebychev polynomials coefficients used to compute the components of the slit and horizon vectors in the inertial frame loaded as: time interval, group index (1-n), group of six values (slit(3), horizon(3), where n is the

Table 5-2 (continued)

Activity	Sample case-specific program data (loaded in memory prior to the event)
	degree of expansion of the Chebychev polynomials) Control frame orientation in body axes Momentum management variables Program-specific controller parameters
Limb stepwise scanning phase	Start time of scanning phase Altitude steps sequence and timing information Momentum management variables Program-specific controller parameters
Limb continuous scanning phase	Start time of scanning phase Upscan constant velocity profile, downturn constant acceleration, downscan constant velocity profile, and upturn constant acceleration Momentum management variables Program-specific controller parameters

Table 5-3 General Types of Attitude Profile Implemented in the Simulation

Attitude profile type	Description	Object
1 (in block H.1)	Constant reference attitude quaternion and zero reference angular velocity	Simulate fine pointing
2 (in block H.2)	Commanded attitude switches between two reference quaternions while reference angular velocity is kept at zero	Simulate fine pointing with calibration slew
3 (in block H.3)	Reference angular velocity along the Zc axis is set to a constant orbital rate. Reference attitude is computed by integrating the kinematic equations from an initial reference attitude quaternion.	Simulate limb pointing (assuming a spherical earth and a circular orbit)
4 (in block H.4)	A continuous scanning profile between two altitude limits is superimposed onto scenario 3. The additional angular velocity command about Zc is the integrated output of a series of positive and negative pulses of constant angular acceleration which	Simulate continuous scanning between approx 10 km and 120 km altitude limits at a rate of approximately 20 cycles per orbit

Table 5-3 (continued)

Attitude profile type	Description	Object
	<p>results in constant velocity upscans and downscans between constant acceleration turns. The kinematic equations of the scan are integrated to generate the attitude quaternion relative to the limb pointing attitude. The scan attitude and the limb following attitude are combined using quaternion multiplication to generate the attitude quaternion relative to inertial space.</p>	
5 (in block H.5)	<p>Zero-order holds are applied to the attitude quaternion relative to the limb pointing attitude (generated in the continuous scan of scenario 4), to introduce fixed altitude steps, before multiplying it by the limb attitude quaternion to get the reference attitude quaternion relative to inertial space.</p>	Simulate stepwise scanning

Table 5-4 Test Orbits

Sample Orbit	Description
I	<p>Event time: 21-Jun-94 00:00-01:40Z</p> <p>Ephemeris epoch: 01-Jun-94 00:00Z</p> <p>Ideally sun-synchronous, circular, 700 km, 98.77 min period, mean anomaly and argument of perigee at epoch are 10 and 350 deg respectively (start of orbit at ascending equatorial crossing)</p> <p>Orbit experiencing eclipse conditions</p>
II	<p>Event time: 21-Mar-94 00:00-01:40Z</p> <p>Ephemeris epoch: 01-Mar-94 00:00Z</p> <p>Ideally sun-synchronous, circular, 700 km, 98.77 min period, mean anomaly and argument of perigee at epoch are 10 and 350 deg respectively (start of orbit at ascending equatorial crossing)</p> <p>Fully sunlit orbit</p>
III	<p>Event time: 13-Oct-94 12:00-13:40Z</p> <p>Ephemeris epoch: 13-Oct-94 12:00Z</p> <p>Ideally sun-synchronous, circular, 600 km, 96.69 min period, mean anomaly and argument of perigee at epoch are 10 and 350 deg respectively (start of orbit at ascending equatorial crossing)</p> <p>Orbit in which the sun vector and the orbit normal are aligned</p> <p>(To verify values of environmental torques computed in Appendix A)</p>

Table 5-5 Sample Flight Conditions

Set of conditions	Description
1A	<p>Inertial pointing (attitude profile type 1) , test orbit III</p> <p>Time of event: 13-Oct-94 12:00-13:40Z</p> <p>Control frame parallel to XbYbZb, with origin at undeformed mass center</p> <p>Target attitude: at ascending equatorial crossing XcYcZc axes are offset by -32 deg about X0 in X0Y0Z0, where Z0 is along the orbit positive normal, X0 is along the radius outwards positive and Y0 completes the dextral triad (to recreate test case in Ap.I)</p> <p>S/C Dipole = [0. 0. 2.] A*m*m (in control coordinates)</p> <p>Initial conditions (in control coordinates and N-m-s):</p> <p style="padding-left: 40px;">S/C total momentum: [0. 0. 0.]</p> <p style="padding-left: 40px;">Wheel relative momentum: [0. 0. 0.]</p>
1B	<p>Inertial pointing (attitude profile type 1), test orbit I</p> <p>Time of event: 21-Jun-94 00:00-01:40Z</p> <p>Control frame parallel to XsYsZs, with origin at undeformed mass center</p> <p>Target right ascension and declination: 25 and 10 deg</p> <p>Xs points at target and -Zs is as close as possible to the sun direction at the start time of the event</p> <p>S/C Dipole = [0. 2. 0.] A*m*m (in control axes)</p> <p>Initial conditions (in control coordinates and N-m-s):</p> <p style="padding-left: 40px;">S/C total momentum: [0. 0. 0.]</p> <p style="padding-left: 40px;">Wheel relative momentum: [0. 0. 0.]</p>
1C	<p>Inertial pointing (attitude profile type 1), test orbit II</p> <p>Time of event: 21-Mar-94 00:00-01:40Z</p> <p>Remaining conditions same as in 1B</p>

Table 5-5 (continued)

Set of conditions	Description
2A	<p>Inertial pointing with switching targets (attitude profile type 2), test orbit I</p> <p>Time of event: 21-Jun-94 00:00-01:40Z</p> <p>Control frame $X_cY_cZ_c$ parallel to $X_sY_sZ_s$, with its origin at the undeformed mass center</p> <p>Observation target right ascension and declination: 25 and 10 deg, with X_c pointing at target and $-Z_c$ as close as possible to the sun direction</p> <p>Calibration target is about 28 arcmin from the observation direction (3-1-2 Euler angle sequence of 20 arcmin, 60 arcmin and 20 arcmin from the observation target)</p> <p>Switching profile: 25 seconds on each target</p> <p>S/C Dipole = [0. 2. 0.] A*m*m (in control axes)</p> <p>Initial conditions (in control coordinates and N-m-s):</p> <p style="padding-left: 40px;">S/C total momentum: [0. 0. 0.]</p> <p style="padding-left: 40px;">Wheel relative momentum: [0. 0. 0.] Inertial pointing</p>
3A	<p>Limb following (attitude profile type 3), test orbit I</p> <p>Time of event: 21-Jun-94 00:00-01:40Z</p> <p>Control frame $X_cY_cZ_c$ parallel to $X_sY_sZ_s$, with its origin at the undeformed mass center</p> <p>X_s points at the limb in the forward motion direction with Y_sZ_s parallel to the orbital plane (6371 km radius spherical earth and 700 km circular orbit assumed)</p> <p>S/C Dipole = [2. 0. 0.] A*m*m (in control axes)</p> <p>Initial conditions (in control coordinates and N-m-s):</p> <p style="padding-left: 40px;">S/C total momentum: $J_c * C_{H'} * [0. -\omega_o \ 0.]'$, where J_c is the combined S/C and wheels inertia matrix, ω_o is the orbital rate and $C_{H'}$ is the rotation matrix from flight to commanded control axes</p> <p style="padding-left: 40px;">Wheel relative momentum: [0. 0. 0.]</p>

Table 5-5 (continued)

Set of conditions	Description
3B	<p>Limb following (attitude profile type 3)</p> <p>Conditions identical to those in 3A, with the exception of:</p> <p>S/C Dipole = [0. 0. 2.] A*m*m (in control axes)</p>
4A	<p>Limb continuous scanning (attitude profile type 4), test orbit 1</p> <p>Time of event: 21-Jun-94 00:00-01:40Z</p> <p>Control frame XcYcZc parallel to XsYsZs, with origin at the undeformed mass center</p> <p>Limb following profile identical to 3A</p> <p>Superimposed scan: continuous</p> <p>Scan cycle (upscan plus downscan): 290 seconds</p> <p>Reversal: Constant acceleration of $(2.05 \times 2)/(140 \times 5)$ deg/s*s during 5 seconds, alternating between positive and negative pulse every 145 seconds</p> <p>S/C dipole = [2. 0. 0.] A*m*m (in control coordinates)</p> <p>Initial conditions (in control coordinates and N-m-s):</p> <p style="padding-left: 40px;">S/C total momentum: $J_c \cdot C_{\theta} \cdot [0. -\omega_o \ 0.]'$, where J_c is the combined S/C and wheels inertia matrix, ω_o is the orbital rate and C_{θ} is the rotation matrix from flight to commanded control axes</p> <p style="padding-left: 40px;">Wheel relative momentum: [0. 0. 0.]</p>

Table 5-5 (continued)

Set of conditions	Description
4B	<p>Limb continuous scanning (attitude profile type 4)</p> <p>Conditions identical to those in 4A, with the exception of:</p> <p>S/C dipole = [0. 0. 2.] A*m*m (in control coordinates)</p>
5A	<p>Limb stepwise scanning (attitude profile type 5), test orbit I</p> <p>Time of event: 21-Jun-94 00:00-01:40Z</p> <p>Control frame XcYcZc parallel to XsYsZs, with origin at undeformed mass center</p> <p>Initial conditions and underlying limb following profile identical to 4A</p> <p>Zero-order-holds of 40 seconds on the scan profile to generate intervals of constant altitude</p> <p>S/C dipole = [2. 0. 0.] A*m*m (in control coordinates)</p>

Table 5-6 Wheel Speed Biases for Selected Combinations of Active Wheels for the Sets of Conditions Outlined in Table 5-5.

Set of flight conditions	Shown in Figures	Active wheels ^a	ω_{bias} Wheels 1,2,3,4 (rad/sec)	Secular speed variation in the enabled wheels (rad/sec)
1A	5-1 to 5-3	1,3,4 (largest Δh)	-80,0,73,-89	~ -8.4, 8.3, -15.3
1B	5-4 to 5-6	1,2,4 (largest Δh)	-79,-76,0,-90	~ -22.9,-16.9,-21.0
1B	5-4 to 5-6	1,2,3 (usual set)	-65,-59,-74,0	~ -10.8,-4.8,-12.1
1C	5-7 to 5-9	1,2,3 (usual set)	-69,-66,65,0	~ -13.9,-6.1,0.6
2A	5-10 to 5-12	1,2,3 (usual set)	-65,-59,-74,0	~ -10.7,-4.8,-12.1

^aThe sets of wheels include three orthogonal wheels (combination 1,2,3) and a fourth skewed wheel (orientation defined in section 5.1.3). Δh refers to the cyclic variation in momentum by each wheel, assuming zero initial wheel momentum.

Table 5-7 Relationship between Noise Equivalent Angle (NEA) Values and the Error Estimate in the Three Attitude Angles. (Values extracted from two sample cases in SSC, 1993b)

Sample Measurements	NEA	Roll	Yaw	Pitch
Sample set no 1: (algebraic method)	1"	~19"	~7"	~.7"
One star tracker used				
Two stars with 4° separation				
Zero error in guide star positions				
Sample set no 2:(algebraic method)	1"	~1.7"	~1.4"	~1"
Two star trackers used				
Two stars with 40° separation				
Zero error in guide star positions

Figure 5-1. Wheel Momentum Distribution during Inertial Pointing - Case 1A (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in third order Runge Kutta integration method, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-2. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Inertial Pointing - Case 1A (in Table 5-5).

Figure 5-3. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Inertial Pointing - Case 1A (in Table 5-5).
 (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in third-order Runge Kutta integration method, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-4. Wheel Momentum Distribution during Inertial Pointing - Case 1B (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in third-order Runge Kutta, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-5. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Inertial Pointing - Case 1B (in Table 5-5).

Figure 5-6. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Inertial Pointing - Case 1B (in Table 5-5).
 (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in third-order Runge Kutta integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-7. Wheel Momentum Distribution during Inertial Pointing - Case 1C (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in third-order Runge Kutta integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-8. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Inertial Pointing - Case 1C (in Table 5-5).

Figure 5-9. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Inertial Pointing - Case 1C (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in third-order Runge Kutta integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-10. Wheel Momentum Distribution during Inertial Pointing with Switching Targets - Case 2A (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-11. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Inertial Pointing with Switching Targets - Case 2A (in Table 5-5).

Figure 5-12. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Inertial Pointing with Switching Targets - Case 2A (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-13. Wheel Momentum Distribution during Limb Following - Case 3A (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-14. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Limb Following - Case 3A (in Table 5-5).

Figure 5-15. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Limb Following - Case 3A (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-16. Wheel Momentum Distribution during Limb Following - Case 3B (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-17. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Limb Following - Case 3B (in Table 5-5).

Figure 5-18. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Limb Following - Case 3B (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-19. Wheel Momentum Distribution during Limb Following with Continuous Scanning - Case 4A (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-20. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Limb Following with Continuous Scanning - Case 4A (in Table 5-5).

Figure 5-21. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Limb Following with Continuous Scanning - Case 4A (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Fig. 5-22. Wheel Momentum Distribution during Limb Following with Continuous Scanning - Case 4B (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, spin axes orientation relative to control axes as in Equation 5-6, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

Figure 5-23. Environmental Torques along Control Axes and Sun Incidence Angle on Solar Panels during Limb Following with Continuous Scanning - Case 4B (in Table 5-5).

Figure 5-24. Spacecraft and Wheel Momentum, and Reference Attitude and Angular Velocity during Limb Following with Continuous Scanning - Case 4B (in Table 5-5). (Assuming perfect tracking of reference inputs and zero initial wheel momentum, and using Simulink built-in *linsim* integration routine, 0.01 second fixed step size, 1e-6 tolerance)

CHAPTER 6. CONTROLLER PERFORMANCE - TEST RESULTS AND DISCUSSION

This chapter describes the performance of the controller when tested, using simulation, against various flight conditions, noise, disturbances and time delays. Test conditions, representative of those that would be experienced in flight, are defined in Chapter 5.

6.1 Overview

Simulations are carried out in the following order, in an attempt to isolate the effects of the different parameters in the system. All simulation runs are done assuming perfect knowledge of the plant parameters and, except for the last series of tests, with the spin axes of the three enabled wheels set parallel to the control axes. In the case of limb tracking or scanning events, the desired rotation of the spacecraft was assumed to be about the orbital pitch vector and of constant magnitude in the case of limb following events. The plane of the circular orbit was assumed to be fixed relative to inertial space, for the time intervals considered. Effects due to the precession rate of the orbit (nodal line rotation in equatorial plane of $0.9856^\circ/\text{day}$) were assumed to be negligible for the

time delays involved.

The first set of tests is done assuming inertial pointing requirements and no delays or noise in the system. The objective is to observe the effects of varying the attitude control gains K_1 and K_2 , and the wheel speed control loop gain K_p in the presence of disturbances and at different wheel speed conditions. The choice of K_p is shown to be largely dependent on the bias momentum of the system, as the disturbing effect of the torque ripple on the spacecraft is attenuated by the transverse momentum.

The objective of the second series of tests is to evaluate the performance of the controller for the five different flight scenarios, for set values of K_1 , K_2 and K_p , and assuming no delays and no noise. Certain flight conditions, as currently set up, are shown to lead to integrator windup. Saturation of the momentum demand resolves this problem.

Desaturation during inertial pointing is then simulated, with no noise or delays in the system, and assuming perfect knowledge of the field and actuator frame orientation. It is found that the desaturation torque applied must be fed forward to the integrator of the momentum demand for the system to settle quickly following start and stop of unloading.

In the fourth set of tests, computational delays in the controller, delays in the attitude measurements (as a result of computational delays associated with the star trackers) and transport delays in the telemetry stream are sequentially added and their effects on the system analyzed. These tests are carried out again for the inertial pointing

case. The stability margin is observed to be significantly reduced when deriving the control law from star tracker measurements alone, at the low update rate of 2 Hz.

The anticipated quantization levels are incorporated in the next set of simulations. Because of the proportional control implemented in the wheel speed loop, quantization in the speed measurements is shown to be a significant factor in the system's response.

The effect of step errors in the attitude measurements and noise in the attitude, angular velocity and wheel speeds is then examined.

In the final set of tests, baseline quantization, noise and delays are included. Five test cases (one for each flight scenario) are run with the wheel axes not parallel to the spacecraft control axes. During earth-referenced pointing, a large pitch error and a large variation in the pitch error into the controller are observed, introduced as a result of the delays in the system.

Each series of tests and the associated results are described in sections 6.2 through 6.8.

6.2 Variations in the Gains

6.2.1 Variations in K_1 and K_2

It is recalled that, in Chapter 4, an attitude control and decoupling law was

incorporated into the rotational dynamics of the spacecraft, resulting into the following second-order system of equations:

$$\delta\ddot{\theta} + (J_c)^{-1}K_2\delta\dot{\theta} + (J_c)^{-1}K_1\delta\theta = (J_c)^{-1}T_d \quad (6-1)$$

K_1 and K_2 were selected at that time based on the worst-case torque disturbance anticipated, the corresponding steady-state error allowed, and a damping ratio ζ of 0.707. The chosen gain matrices were different for the inertial pointing and limb following profiles, as the required pointing control accuracy is not as stringent in the latter case.

Figures 6-1 through 6-3 illustrate the effect of varying the attitude control gains. Increasing the gains results in tighter attitude and angular velocity control, but also in a larger system bandwidth (higher closed-loop natural frequency)⁶⁰. The smallest bandwidth possible which meets the pointing requirements is preferred, to avoid frequencies which may be associated with internal disturbances such as flexible modes or onboard mechanisms.

Also, scaling both K_1 and K_2 by a scales ζ and the undamped natural frequency of each axis by $(a)^{1/2}$. The poles of the transfer function for axis i are at $s = -\zeta_i\omega_{n,i} \pm j\omega_{n,i}(\zeta_i^2 - 1)^{1/2}$, where $K_1'(i,i) = \omega_{n,i}^2$, $0.5K_2'(i,i)(K_1'(i,i))^{-1/2} = \zeta_i$ and K_1' and K_2' are the diagonal matrices equal to $(J_c)^{-1}K_1$ and $(J_c)^{-1}K_2$. If $\zeta > 1$ as a result of scaling the gains, then the roots are real. A damping ratio ζ of 0.707 provides a small settling time and

⁶⁰Franklin et al. (1991, p.260) define the bandwidth as the maximum frequency at which the output tracks a sine input satisfactorily. They state that, by convention, the bandwidth of the system $R(s)^{-1}Y(s) = KG(s)/[1+KG(s)]^{-1}$ is the frequency at which the magnitude of the ratio $|y|/|r|$ has reduced to 0.707.

small overshoot response to the step disturbances. The increased oscillations in the attitude error transients, shown in Figures 6-2 and 6-3, are attributed to the interacting nonlinear dynamics of the spacecraft and actuators, under the combination of attitude and speed control gains used.

6.2.2 Variations in K_p

K_p is the scalar control gain in the speed control loop of each wheel. An initial value for K_p is chosen so that the bandwidth of the speed control loop is at least five times the bandwidth of the attitude control system. Assuming no speed measurement error, negligible electrical delay and proportional control of the form $T_{mn} = K_p (\Omega_{sn} - \omega_{sn})$, the dynamics of each wheel is given by:

$$K_p(\Omega_{sn} - \omega_{sn}) = J\dot{\omega}_{sn} + T_f + B\omega_{sn} + T_{st} \quad (6-2)$$

where T_{mn} is the rotor motor torque for wheel n , ω_{sn} is the actual speed of wheel n , Ω_{sn} is the desired speed, J is the rotor inertia and the remaining terms are kinetic, viscous and static friction. The characteristic equation is given by $Js + K_p + B = 0$. Ignoring viscous friction, the closed-loop natural frequency is then $\approx K_p/J$. In Chapter 4, the bandwidth of the attitude control system was found to be ≈ 0.54 rad/sec for inertial pointing and ≈ 0.17 rad/sec for limb following. Five times the bandwidth, for a wheel polar inertia of 0.0075 kg-m², results in a lower bound on K_p of ≈ 0.02 N-m/rad/sec in the inertial case and ≈ 0.0064 N-m/rad/sec in the limb following case.

Figures 6-1 and 6-4 through 6-8 show the effect of increasing the gain, in the

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**PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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presence of internal (torque ripple) and external disturbances and for different speed conditions. Increasing K_p results in tighter attitude and angular velocity control at start-up (i.e. when the torque demand is initially at zero). The final choice of K_p is, however, driven by the anticipated disturbances to reject in the speed control loop.

A major source of disturbance is a ripple of 20 percent in the motor torque (SSC, 1993d). For simulation purposes, the motor torque ripple for each wheel (T_m) has been modelled as a sine waveform with an amplitude variation equal to 20 percent of the motor torque command and a frequency equal to a multiple of the angular position of the rotor, such that $T_m = 0.2T_{mr}\sin(k\theta_{sn})$, where θ_{sn} is the angular position of wheel n relative to a reference position.

At near zero-speeds (Figures 6-6 to 6-8), increasing K_p leads to better attitude and angular velocity control without affecting the waveform of the torque demand⁶¹ (except during the start-up transient). This is because the change in speed demand is mainly regulated by the disturbance in the speed control loop. In the absence of external torques, the rotational dynamics of the spacecraft are given in matrix form by:

$$\begin{aligned} 0 &= \dot{H} + \omega^x H \\ 0 &= J_c \dot{\omega} + A_{cm} J_m \dot{\omega}_s + \omega^x (J_c \omega + A_{cm} J_m \omega_s) \end{aligned} \quad (6-3)$$

Assuming that the angular velocity and wheel speeds are very small (i.e. ω and $\omega_s \approx 0$), and ignoring the platform motion and friction torques, then for each wheel:

⁶¹For these test cases, the spin axes of the wheels were set parallel to the spacecraft control axes. As a result, the spacecraft dynamics are essentially decoupled near zero-speeds.

$$J\dot{\omega}_{sn} \approx \dot{l}_{sn} \approx T_{mn} = K_p(\Omega_{sn} - \omega_{sn}) \quad (6-4)$$

A disturbance in the motor torque (δT_{mn}) results in a variation in the wheel speed which is directly reflected in the spacecraft motion about that axis, and as a result in the momentum demand, such that $|\delta\omega_{sn}| \approx |\delta T_{mn} K_p^{-1}|$ and $\delta\omega \approx (J)^{-1} A_{cm} J_m \delta\omega_s$. If:

- the wheel spin axes are parallel to the spacecraft control axes;
- the average value of T_m is 2×10^{-3} N-m and the ripple is 20 percent of that value;
- K_p is 0.02 N-m/rad/sec;
- the wheel inertia is 0.0075 kg-m²;
- the diagonal terms of the inertia matrix (body and wheels) are of the order of 32 kg-m² (for two of the control axes); and,
- the effect of the off-diagonal terms in the inertia matrix is negligible.

then, as shown in Figure 6-6, the amplitude of the resulting variation in the spacecraft angular velocity would be roughly:

$$\frac{(\delta T_{mn})(0.0075)}{(0.02)(32)} \approx 4.7 \times 10^{-6} \frac{\text{rad}}{\text{sec}} \quad (6-5)$$

In Figures 6-1, 6-4 and 6-5, where the wheel speeds are nonzero with the spacecraft control axes still parallel to the spin axes, the variation in wheel speed caused by the internal disturbance is now reflected in all three spacecraft axes because of the

transverse momentum present (Equation 6-6).⁶² Consequently, the variation in the angular velocity of the corresponding spacecraft axis is smaller and significantly less reflected in the momentum demand and therefore in the torque demand. As K_p is increased, however, the ripple in the torque demand becomes greater as a result of the higher sensitivity to the momentum demand variation.

$$0 \approx J_c \delta \dot{\omega} + A_{cm} J_m \delta \dot{\omega}_s + \delta \omega^x [A_{cm} J_m (\omega_s + \delta \omega_s)] \quad (6-6)$$

K_p should therefore be set as a function of the bias momentum of the system, to at least 0.02 N-m/rad/sec for inertial pointing events, and to greater values when all the speeds are close to zero.

Design of the attitude control loop was done independently from the speed control loop. This is assumed to be valid for proportional control of the form $T_m = K_p(\omega_d - \omega_s)$ and a very small time constant in the speed loop. The interaction of K_p and the position and rate gains in a single-axis attitude control loop, and the complex nonlinear behavior of the system under certain gain combinations, are studied in Piper and Kwatny (1992).

⁶²If it is assumed that: J_c is diagonal, the wheel axes are parallel to the principal axes, the diagonal values of J_c are labelled J_d , the polar inertia of each wheel is J , there is bias momentum h only about principal axis 2, and the overall momentum is constant (no external torques), then a disturbance δT_m in the wheel along principal axis 1 will result in the following open-loop dynamics:

$$\begin{aligned} -\delta T_m &= J_d \delta \dot{\omega}_1 - h \delta \omega_3 \\ 0 &= J_d \delta \dot{\omega}_2 \\ 0 &= J_d \delta \dot{\omega}_3 + h \delta \omega_1 \end{aligned}$$

Let δT_m and h be positive. A negative change in $\delta \omega_3$, integrated, will result in a positive change in $\delta \omega_1$, which will reduce the effect of δT_m on the rate of change of $\delta \omega_1$.

6.3 Changes in the Reference Signals

For this series of tests, no delays and no noise are assumed and a continuous integrator block in Simulink is used to generate the momentum demand.

The basic controller developed in Chapter 4 is not designed to handle incompatible changes in the attitude and angular velocity commands. For example, in the case of inertial pointing with calibration slew, the spacecraft is commanded to alternate between two inertial pointing targets. The initial approach is to implement the switching as a sudden change in the commanded attitude quaternion with the reference angular velocity command kept constant at zero. The resulting unstable attitude of the spacecraft is shown in Figure 6-9.

The main problem with this approach is that the large errors (large attitude error initially and growing angular velocity error) cause the wheel output torques to saturate. Because momentum control is implemented, the errors keep being integrated, although the torque outputs from the wheels remain at their maximum value. A large error in the opposite direction must be experienced before the momentum demand can be restored to its proper value⁶³. Saturating the products $K_{\delta\theta}$ and $K_{\dot{\delta\theta}}$ in Equations 4-31a and 4-31b when the attitude error in any axis exceeds 30° reduces the integrator windup effect, although a large overshoot can still be observed (Figure 6-10). In the digital controller implementation, feeding back the momentum demand actually applied, when the actuator is at saturation levels, is a better approach (Figure 6-11).

⁶³The phenomenon of integrator windup is discussed in Franklin and al. (1991, pp.123-124).

Even when anti-windup measures are taken, the reorientation command is treated as a disturbance, and the decoupling and attitude control laws partially act against it because of the angular velocity error terms. An alternate approach to avoid integrator windup is to modify the commanded attitude and angular velocity profiles such that the transition between the two inertial targets is smooth.

Let ϕ_{exc} be the angle of rotation about the eigenaxis $\hat{e}$ yielding the minimum path maneuver between the two inertial targets. ϕ_{exc} and $\hat{e}$ are computed from the error quaternion for target one to target two as $q_1^{-1}q_2 = q_E = [\hat{e}\sin(\phi_{exc}/2), \cos(\phi_{exc}/2)]$. A commanded attitude profile can be generated by the onboard computer to approximate a minimum time maneuver for an assumed maximum effort.

In the case shown in Figure 6-12, incremental changes in attitude and new angular velocity commands are generated at intervals of 0.02 seconds, based on a maximum rate⁶⁴ of $(T\phi_{exc}/I)$ at half the maneuver, with T set to 0.015 N-m and I to 43 kg-m², assuming no bias momentum in the system (wheel speeds very close to zero) and no disturbances. The commanded maneuver is stopped when the second target is within ϕ_{exc} , equal to 5°, where ϕ_{exc} is the updated remaining part of the rotation.

As expected, overshoots in the attitude errors occur following the start, midpoint and stop of the maneuver, where steps in the angular acceleration are experienced. Although the path of the maneuver is optimized in the simulation run shown in

⁶⁴In a continuous system, a minimum time rotation about a given axis (disturbance or friction free) will display a linear increase in rate to a maximum of $(d\phi/dt)_{max} = (T\phi_{exc}/I)$ with respect to time. The rate $(d\phi/dt)$ will also be equal to $(d\phi/dt)_{max}(2\phi'/\phi_{exc})^{1/2}$ where ϕ' is the cumulative rotation to date in the first part of the maneuver and the remaining rotation angle in the second part of the maneuver.

Figure 6-12, there is only a slight improvement in terms of the time required to stabilize to the new attitude as maximum torque is initially applied in the case shown in Figure 6-11. A commanded profile could be generated to both maximize the use of the actuators and minimize the time required, given the initial momentum conditions of the system. However, in the case of small reorientations, the expected improvement does not warrant the computations required to generate the commanded attitude. In the case of large maneuvers, and especially if critical orientations should be avoided, generating a desired attitude profile would be required.

The control during limb following and limb scanning profiles, shown in Figures 6-13 and 6-14, is more relaxed as the gains K_1 and K_2 were chosen in Chapter 4 to allow a larger steady-state error.

Limb following activity with stepwise altitude scanning is illustrated in Figures 6-15 and 6-16. Control with anti-windup implemented as previously described is shown in Figure 6-15. The large instantaneous errors coincide with the steps in the commanded attitude (between 1000 and 4000 arcsec excursions in pitch in this example). Figure 6-16 shows the slight improvement in time response, when intermediate commanded attitude and angular velocity profiles are included. Simple anti-windup is however sufficient if the magnitude of the altitude steps is limited and the time interval between altitude changes is long enough.

6.4 Desaturation Activity

The effect of desaturation on inertial pointing activity is investigated next. Again, no delays and no noise are assumed and a continuous integrator block in Simulink is used to generate the momentum demand.

In the simulation run of Figure 6-17, the initial wheel speeds are set arbitrarily to 200 rad/sec and the desaturation triggering level to 200.04 rad/sec (so that desaturation will be initiated shortly after). A desaturation torque demand of 0.01 N-m to all wheels is triggered when one of the wheels reaches 200.04 rad/sec. These torque demands are translated into the required dipoles and the corresponding external torque $(T_e)_{desat}$ perpendicular to the field vector (Equations 4-45 and 4-46). $(T_e)_{desat}$ is included in the integrand of Equation 4-29 and therefore forwarded to the controller block which computes the momentum demand. However the initial torque demand for a given wheel should be of the order of 0.5 to 1×10^{-3} N-m, assuming that the field is normal to the demand, to avoid exceeding the capabilities of the magnetorquers (see section 4.6.3). For the case in Figure 6-18, the constant torque required to unload each wheel to its bias level in five minutes is computed instead (Equation 4-43). The dipole demand for two of the wheels still exceeds the 40 A-m^2 capability of the torque rods, as a result of the orientation of the field vector.

A torque demand as computed in Equation 4-43 or even a larger identical demand to all wheels can be used, as long as Equation 4-45 is modified to account for the maximum capability of the actuators. The response is shown in Figure 6-19, where the torque demand for desaturation to each wheel is set to 0.01 N-m, with the computed

dipole cut to the maximum value.

$$D = \sum_{n=0}^4 \frac{B^x C_{f_n} T_n s_n}{B^T B} \quad (\text{in flight coordinates}), \text{ with} \quad (6-7)$$

D' set to D and reduced as required so that $|D'| \leq \text{Maximum Dipole}$ and
 $(T_c)_{desat} = -C_{cf} B^x D'$ (in control coordinates)

In Figure 6-20, $(T_c)_{desat}$ is computed according to Equations 4-45, directly forwarded to the speed control loop, resolved along the spin axes, with each component scaled by a factor $(K_p)^{-1}$ and then added to the speed demand. Not integrating $(T_c)_{desat}$ introduces an external torque disturbance which causes attitude and angular velocity errors to build up, to which the system then responds. Although the system is stable, this approach causes unacceptable errors. Including $(T_c)_{desat}$ in the integrand of Equation 4-29 instead is therefore the favored approach.

In the simulation model implemented here, desaturation is stopped when the wheel has reached its bias speed level (Figure 6-17) or when a specified time has elapsed.

6.5 Delays

Computational delays in the momentum integrator associated with f equal to 50, 30 and 20 Hz update rates (implemented as unit delays of $1/f$ seconds) result in negligible variations in the maximum attitude error observed from start-up.

As expected, changing the star tracker measurements update rate has a significant effect on the system. The response is shown in Figure 6-1 for a simulation run with no

delays. For the case in Figure 6-21, update rates of 10Hz in the star tracker measurements and of 50 Hz in the controller are included and result in a ~ 10 percent increase in the maximum attitude error from start-up. Changing the star tracker update rate to 2 Hz results in a ~ 60 percent increase in the maximum error from start-up (Figure 6-22), relative to the no-delays run. In these simulation runs feedback of the saturated momentum to avoid integrator windup has not yet been implemented, and the rate terms have been removed from the integral of Equation 4-29. As a result, the term $\int K_2 d/dt(\delta\theta) dt$ of the control law, replaced by $K_2(\delta\theta - \delta\theta_0)$ is computed based on the delayed attitude measurements alone. The oscillations in the system response in Figure 6-22 indicate that the phase margin has decreased, as anticipated (see Figures 4-2 to 4-5).

To avoid windup, the rate terms have to remain in the integrand of Equation 4-29 to integrate the saturated momentum. Otherwise, $K_2(\delta\theta - \delta\theta_0)$ includes part of the excess momentum demand not applied. The term $\int K_2 d/dt(\delta\theta) dt$ is therefore computed as $\int K_2(\delta\omega - \Omega^x \delta\theta) dt$ (from Equation 4-6), and for inertial pointing is solely based on angular velocity measurements (as the commanded angular velocity Ω is zero). In the case of inertial pointing and when K_p is 0.02 N-m/rad/sec, the 2 Hz update rate in the attitude measurements has little effect on the response of the system to external step disturbances when the control law term $K_2 d/dt(\delta\theta)$ is based on angular velocity measurements (Figure 6-24) rather than on the star tracker measurements (Figure 6-23). This is not surprising as the model assumes that the sensor delays associated with the gyros are negligible. This holds even when the additional delays associated with the transport of sensor data to the controller and the transport of commands to the actuators are present (Figure 6-25). The effect of deriving the position and rate control law terms from star tracker measurements alone, in the presence of the transport delay in telemetry and for

an attitude update rate of 2 Hz, is shown in Figure 6-26.

The method by which attitude and rate errors are computed therefore significantly affects the response of the system at lower star tracker update rates. At a 10 Hz update rate, control is adequate even with errors solely derived from star tracker measurements (see Figure 6-21).

However, the pure time delay, combined with the star tracker and controller update rates, introduces an upper bound to the gain allowed in the speed control loop. Figures 6-27 and 6-28 show that increasing K_p to 0.2 N-m/rad/sec leads to instability in the speed control loop. This is also the case at near zero-speeds (Figure 6-29), where on the other hand it is preferable to have a larger gain to attenuate the effect of the ripple (Figure 6-8).

In a continuous system, a pure time delay increases the phase margin by $-\omega T$ (see Franklin et al., 1991, Chap.5). Without the time delay, the phase margin of the speed loop is approximately 90° . Assuming a total time delay of 0.125 seconds, the phase margin is reduced to zero at $\omega \approx 12.6$ rad/sec. This corresponds to the natural frequency of the speed loop when K_p is approximately 0.09 N-m/rad/sec, and the speed control loop is marginally stable.

6.6 Quantization Effects

The performance of the baseline system, including transport delay via telemetry,

5 Hz star tracker update rate, 50 Hz controller update rate, but no quantization of the measurements or control parameters, is shown in Figure 6-30. The effects of the quantization levels defined in Chapter 5 are shown in Figures 6-31 to 6-34 for K_p equal to 0.02 N-m/rad/sec. As shown in Figure 6-33, a quantization step of 0.13 rad/sec in the wheel speed measurements is too large for the current system (equivalent to a random 2.6×10^{-3} N-m internal disturbance), especially if K_p is increased. The quantization levels defined for the other measurements or commands passed via telemetry have no significant effect. Care should therefore be given to allocate a sufficient number of bits in telemetry to the measured wheel speeds. A quantization level of 0.13 rad/sec corresponds to a 1-bit change in a 12-bit unsigned data word, while a quantization level of 0.0163 corresponds to a 1-bit change in a 16-bit signed data word, both for a maximum value of 534 rad/sec.

6.7 Biases in Attitude Measurements and Noise

As mentioned in section 5.3.3, it is decided to include attitude errors into the simulated system in block B.1 by converting q_a to a 3-1-2 Euler sequence representation, adding errors to each angle, and then converting the resulting attitude back into the corresponding quaternion representation now labelled q_a^M (to simulate measured attitude). A fixed error of 4" (Figure 6-35) and of 2" (Figure 6-36) is added to each angle of the Euler angle sequence 3-1-2 at 30, 50 and 70 seconds into the simulation, along with external step disturbances of 10^{-4} N-m about each control axis at 20, 40 and 60 seconds and an internal step disturbance of 0.001 N-m in one of the wheels at 75 seconds.

The errors into the controller are computed by converting δq or $\delta q_{c,p}$ (i.e. the quaternion representing the rotation from the reference to measured control axes) to a 3-1-2 Euler sequence representation. For small errors, the Euler rotation axes can be considered orthogonal and the angular errors represent the independent rotations about axis 3, axis 1 and axis 2 to align F_p with F_c .

The Euler rotations axes when converting q_a and δq into a 3-1-2 sequence do not coincide, and therefore the error input into block B.1 and the simulated error output cannot be directly compared. The only exception is in the case of axis 2, when no errors are added about the two other axes. As expected, a 1:1 relationship is observed between the bias step at 50 seconds (about axis 2) introduced in block B.1, and the reaction in the controller error and the simulated attitude error. A better approach would be to introduce the attitude error in the controller block after the conversion of δq into the Euler angle sequence, so that the effect of the biases on all three angles can be directly assessed.

Noise effects are investigated next. Band-limited white noise is included independently in attitude, angular velocity and wheel speed measurements. In Simulink, band-limited white noise is implemented as white noise fed into a zero-order-hold (ZOH) (Mathworks, 1992). Three parameters are used to define the band-limited white noise generator block: variance, sampling time (T_s) of the ZOH, and seed for generating the pseudo-random normally distributed sequence (repeatable for a given seed). The variance is actually specified in terms of power spectral density units, for example in $(\text{rad/sec})^2(\text{Hz})^{-1}$ or in $(\text{rad})^2(\text{Hz})^{-1}$ depending on the measurement, and the noise bandwidth is $(T_s)^{-1}$.

The system's response under various noise conditions is provided in Figures 6-37 through 6-41. Because momentum control is implemented, the integrator providing the momentum demand acts as a filter on the measurement noise in attitude and angular velocity. Noise in the speed measurements also moderately affects the attitude error of the satellite. Noise amplification in the speed control loop is a larger problem, especially if K_p is increased. For the case in Figure 6-40, a noise of 3σ equal to 0.001 (75 rad/sec) is included and results in the torque demand variation due to the ripple being magnified several-fold over that shown in Figure 6-31. As for the biases, attitude measurement noise is added in block B.1 (and quantization effects as well) about axes not coincident to the control axes. A cause-effect relationship cannot be directly assessed (i.e., input axes in block B.1 are a function of the inertial attitude). Again, a better approach in the case of the attitude measurement noise would be to introduce it in the controller block after the conversion of δq into the Euler angle sequence, and assume that it also accounts for quantization effects.

6.8 Performance during the Various Flight Scenarios in the Presence of Quantization, Noise and Delays

The performance of the controller is now tested under flight scenarios 1B, 2A, 3A, 4A and 5A, in the presence of baseline quantization levels, noise and delays (5 Hz star tracker update rate, 50 Hz controller update rate and one sixteenth of a second pure time delay before and after the controller). Performance during the first 90 seconds (450 and 360 seconds for cases 4A and 5A) is shown in Figures 6-42 to 6-46. In addition to the modelled environmental torques, external disturbance torques are applied about each

control axes at 20, 40 and 60 seconds into the simulation, and a 0.001 N-m internal disturbance about wheel 2 is applied at 75 seconds from the start. A large variation in the pitch attitude error into the controller and a large error in the simulated pitch error is observed in all earth-referenced modes. This is initially believed to be due to the ramp input command in pitch attitude (constant angular velocity profile) combined with the attitude measurements updates at fixed 0.2 second intervals, the 0.0625 second delay between sensor updates and controller computations, and the 0.0625 second delay for the controller commands to reach the actuators.

The simulations are run again for a longer time frame (900 seconds) for cases 1B, 2A, 4A and 5A, under modelled environmental disturbance conditions and without the additional external and internal disturbances added in the previous runs. Results are plotted in Figures 6-47 to 6-50. The portion of the orbit for the 900 second simulation time frame is chosen to coincide with when large changes in the environmental torques are anticipated (see Figures 5-1 to 5-24 for the corresponding test cases).

The error in pitch observed in the earth-referenced modes, when all system delays are included, is larger than anticipated. Although smaller than the (real-time) control requirement in pitch of 360", it is a concern, since some sources of errors are not even included in the simulation, for example: the NEA values associated with star sensor measurement axes, which transform to larger boresight axis errors when only one star sensor is used, the errors in the generated reference profile based on information uplinked to the satellite prior to the event, and the uncertainties in plant parameters.

A single-axis model of the controller is developed, to investigate the sources of

the pitch error during earth-referenced operation. Performance is tested under a ramp input (at the desired pitch rate), a sine squared disturbance of maximum amplitude 10^{-4} N-m, and the same gains, viscous and friction torques as the three-axis control model. External disturbances are found to contribute $\sim 17''$ maximum error, and the 50 Hz controller update rate, 5 Hz star tracker update rate, 0.0625 second delay to and from the controller are found to contribute approximately $80''$. This holds even in the presence of the torque ripple, and the anti-windup scheme. At this point, the additional error observed is assumed to be caused by errors in the decoupling terms in the integrand of Equation 4-29, due to the difference between simulated and measured values as a result of noise, biases, quantization effects and delays.

The combined effect of delays, measurement errors and disturbances therefore have a major effect on the system, using this type of controller, during the earth-referenced modes. The model's performance would likely improve if the following options are included in the simulations:

- using a star tracker update rate of 10 Hz (the long simulations were done with an update rate of 5 Hz),
- replacing the unit delay associated with attitude measurements by a pure time delay block with the same T , (this would probably improve the system's performance significantly and would also be more representative of the actual situation (i.e. attitude estimates based on gyro measurements between star tracker updates),
- increasing the attitude control gains, and
- resetting the quaternions at certain simulation time intervals, to meet the

constraint $\sum q_i = 1$ (in the current simulation model, each component of q_d is integrated separately).

Drastic improvements would also be expected, if it was possible to time-tag the star tracker measurements, since this would effectively remove the delay associated with transport of data from the sensor to the controller via telemetry.

All simulation runs have been conducted assuming that there would be continuous onboard estimation of the gyro bias rates and that the biases would be subtracted from the measured values by the controller. In other words, only the gyro drift from the bias has been modelled (see section 5.3.2). If more simulation work was to be conducted, especially with respect to earth-referenced operations, the next steps would be: (a) to incorporate a more detailed model of the onboard attitude estimation block (i.e. with respect to the integration of the kinematics of rotation, integrator resets and gyro bias estimates), and (b) to run the simulation under conditions of no new star tracker updates for a specified time interval.

6.9 Other Sources of Errors

One important additional source of error is the uncertainty in plant parameters, especially since decoupled control is implemented. For example, an error in the composite inertia matrix estimation will result in an error in the generated momentum demand in the controller of the order of $\sim \delta J_c \Omega$ (terms with $\delta J_c \delta \theta$ and $\delta J_c \delta \omega$ products assumed negligible). Because of the proportional control in the wheel speed loops, this

will translate to an error in torque demand and therefore an additional disturbance torque in the system. This will affect pitch control when the spacecraft is in earth-referenced mode (eg. an additional torque disturbance of the order of 10^{-7} N-m in magnitude for a one percent error in the diagonal terms of J_c , a diagonal term ~ 42 kg-m², K_p equal to 0.02 N-m/rad/sec and the magnitude of Ω equal to $\sim 1.06 \times 10^{-3}$ rad/sec).

Further analysis could therefore address the magnitude of the disturbances introduced by the controller in the decoupled attitude control loop, as a result of:

- Uncertainty in the polar inertia of the wheels,
- Uncertainty in the orientation of the control frame and actuator frames relative to one another (for torque rods and wheels both), and
- Uncertainty in the magnetic field magnitude and direction

Roundoff errors associated with onboard computations will result in erroneous dipole and wheel speed commands. These errors are however assumed to be smaller than the quantization errors which were addressed in this study.

The entire study has been conducted assuming a perfectly rigid spacecraft, with the exception of the four reaction wheels. The vibrations of the Stirling cooler, the operation of onboard mechanisms and spacecraft deformations will however be present in the actual spacecraft. Associated frequencies should therefore be greater by a reasonable margin factor than the bandwidth of the controller. For the controller developed in this study, the bandwidths are ~ 0.1 Hz in the inertial pointing mode and ~ 0.03 Hz in the earth-referenced modes.

Figure 6-1. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_1 and K_2 Set According to Equation 4-36, K_3 Equal to 0.02 N-m/sec, and Nonzero Initial Wheel Speeds.

(K_1 and K_2 are discussed in sections 4.5.2.1 and 6.2.1, and are associated here with a damping ratio ξ of 0.707. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel. Hop X Dw is the coupling between nominal operating conditions and the angular velocity error, i.e. $(J_c \Omega + A_{cm} J_m \omega_c)^T \delta \omega$ with the magnitude of Ω equal to zero for inertial pointing, and hd is the momentum demand from the controller.)

Figure 6-2. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_1 and K_2 Equal to the Values in Equation 4-36 Scaled by 10, K_p Equal to 0.02 N-m/sec and Nonzero Initial Wheel Speeds.

(K_1 and K_2 are discussed in sections 4.5.2.1 and 6.2.1, and are associated here with a damping ratio $\xi a^{1/2}$ of $0.707(10)^{1/2}$. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-3. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_1 and K_2 Equal to the Values in Equation 4-36 Scaled by 100, K_3 Equal to 0.02 N-m/sec and Nonzero Initial Wheel Speeds.

(K_1 and K_2 are discussed in sections 4.5.2.1 and 6.2.1, and are associated here with a damping ratio $\xi a^{1/2}$ of $0.707(100)^{1/2}$ or 7.07. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-4. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_i and K_v Set According to Equation 4-36, K_v Equal to 0.2 N-m/sec and Nonzero Initial Wheel Speeds.

(K_v is discussed in section 6.2.2. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^4 N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel. Hop X Dw is the coupling between nominal operating conditions and the angular velocity error, i.e. $(J_c \Omega + A_{cm} \int_m \omega_c) \times \delta \omega$ with the magnitude of Ω equal to zero for inertial pointing, and hd is the momentum demand from the controller.)

Figure 6-5. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_p and K_v Set According to Equation 4-36, K_p Equal to 2 N-m/sec, and Nonzero Initial Wheel Speeds.

(K_p is discussed in section 6.2.2. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel. Hop X Dw is the coupling between nominal operating conditions and the angular velocity error, i.e. $(J_c \Omega + A_{\omega} J_{\omega} \omega)^X \delta \omega$ with the magnitude of Ω equal to zero for inertial pointing, and hd is the momentum demand from the controller.)

Figure 6-6. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_t and K_r Set According to Equation 4-36, K_p Equal to 0.02 N-m/sec, and Zero Initial Wheel Speeds.

(K_r is discussed in section 6.2.2. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel. Hop X Dw is the coupling between nominal operating conditions and the angular velocity error, i.e. $(J_c \Omega + A_{\omega} J_{\omega} \omega) \times \delta \omega$ with the magnitude of Ω equal to zero for inertial pointing, and hd is the momentum demand from the controller.)

Figure 6-7. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_p and K_v Set According to Equation 4-36, K_p Equal to 0.2 N-m/sec, and Zero Initial Wheel Speeds.

(K_p is discussed in section 6.2.2. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel. Hop X Dw is the coupling between nominal operating conditions and the angular velocity error, i.e. $(J_c\Omega + A_{cm}J_m\omega_c)^X\delta\omega$ with the magnitude of Ω equal to zero for inertial pointing, and hd is the momentum demand from the controller.)

Figure 6-8. System Response during Inertial Pointing Activity - Case 1B (in Table 5-5), with K_p and K_v Set According to Equation 4-36, K_p Equal to 2 N-m/sec, and Zero Initial Wheel Speeds.

(K_p is discussed in section 6.2.2. The rotational dynamics were simulated using Model 1.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: torque ripple included, and wheel spin axes and control axes parallel. Hop X Dw is the coupling between nominal operating conditions and the angular velocity error, i.e. $(J_c \Omega + A_{cm} J_m \omega_s)^X \delta \omega$ with the magnitude of Ω equal to zero for inertial pointing, and hd is the momentum demand from the controller.)

Figure 6-9. System Response during Inertial Pointing Activity, with Switching between Two Inertial Targets and No Anti-Windup Scheme - Case 2A (in Table 5-5).

(The rotational dynamics were simulated using Model 2.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_1 equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-10. System Response during Inertial Pointing Activity, with Switching between Two Inertial Targets, and with Terms $K_1\delta\theta$ and $K_2\delta\dot{\theta}$ Saturated to $\delta\theta = 30''$ - Case 2A (in Table 5-5).

(The rotational dynamics were simulated using Model 2.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_3 equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-11. System Response during Inertial Pointing Activity, with Switching between Two Inertial Targets, and with Anti-Windup Scheme - Case 2A (in Table 5-5).

(The rotational dynamics were simulated using Model 2.A (see Table 3-1) modified to include an anti-windup scheme discussed in section 6.3. The initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_3 equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-12. System Response during Inertial Pointing Activity, with Switching between Two Inertial Targets, and Commanded Maneuvering Profile between the Targets - Case 2A (in Table 5-5).

(The rotational dynamics were simulated using Model 2.A (see Table 3-1) modified to include an intermediate maneuvering profile between the two targets (modified block H.2), discussed in section 6.3. The initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36. K_3 equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-13. System Response during Limb Following Activity - Case 3A (in Table 5-5).

(The rotational dynamics were simulated using Model 3.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-37, K_p equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-14. System Response during Limb Pointing with Continuous Scanning Activity - Case 4A (in Table 5-5).

(The rotational dynamics were simulated using Model 4.A (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-37, K_3 equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-15. System Response during Limb Pointing with Stepwise Scanning Activity, and with Anti-Windup Scheme - Case 5A (in Table 5-5).

(The rotational dynamics were simulated using Model 5.A (see Table 3-1) modified to include an anti-windup scheme discussed in section 6.3. The initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-37, K_3 equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-16. System Response during Limb Pointing with Stepwise Scanning Activity, and Commanded Maneuvering Profile between Altitude Changes - Case 5A (in Table 5-5).

(The rotational dynamics were simulated using Model 5.A (see Table 3-1) modified to include an intermediate maneuvering profile between the two targets (modified block H.5), discussed in section 6.3. The initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-37, K_3 equal to 0.02 N-m/sec, nonzero initial wheel speeds, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-17. System Response during Inertial Pointing Activity and Desaturation, with (T_{desat}) Included in the Integrand of Equation 4-29 - Case 1B (in Table 5-5). (Desaturation torque demand set to 0.01 N-m, and then modified to account for the direction of B)

(The rotational dynamics were simulated using Model 1.B (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_1 equal to 0.02 N-m/sec, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-18. System Response during Inertial Pointing Activity and Desaturation, with $(T')_{desat}$ Included in the Integrand of Equation 4-29 - Case 1B (in Table 5-5). (Desaturation torque demand set to the torque required for 5-minute unloading, and then modified to account for the direction of B)

(The rotational dynamics were simulated using Model 1.B (see Table 3-1) from initial conditions of zero torque demand and zero attitude and angular velocity errors, with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_3 equal to 0.02 N-m/sec, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-19. System Response during Inertial Pointing Activity and Desaturation, with Saturated Dipole Demand (Equation 6-7) and with $(T_o)_{desat}$ Included in the Integrand of Equation 4-29 - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.B (see Table 3-1), modified to account for maximum magnetorquer capability of 40 A-m², from initial conditions of zero torque demand and zero attitude and angular velocity errors, and with a 0.001 second integration step size. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K , and K , set according to Equation 4-36, K_p equal to 0.02 N-m/sec, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-20. System Response during Inertial Pointing Activity and Desaturation, with (T_{desat}) Directly Forwarded to the Speed Control Loop - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.B (see Table 3-1), modified to forward the desaturation torque directly to the speed control loop rather than including it in the integrand of Equation 4-29. Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_3 equal to 0.02 N-m/sec, torque ripple included, and wheel and control axes parallel.)

Figure 6-21. System Response during Inertial Pointing Activity, with Controller Updates at 50 Hz, Star Tracker Measurement Updates at 10 Hz and $\int K_p d/dt(\delta\theta) dt$ Computed Based on Attitude Measurements - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.D (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_p and K_v set according to Equation 4-36, K_p equal to 0.02 N-m/sec, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-22. System Response during Inertial Pointing Activity, with Controller Updates at 50 Hz, Star Tracker Measurement Updates at 2 Hz and $\int K_2 \frac{d}{dt}(\delta\theta) dt$ Computed Based on Attitude Measurements - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.D (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_2 equal to 0.02 N-m/sec, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-23. System Response during Inertial Pointing Activity, with Controller Updates at 50 Hz, Star Tracker Measurement Updates at 2 Hz, $\int K, d/dt(\delta\theta) dt$ Computed Based on Attitude Measurements, and with Wheel Speeds Different than those in Figure 6-22 - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.D (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_3 equal to 0.02 N-m/sec, torque ripple included, and wheel spin axes and control axes parallel.)

Figure 6-24. System Response during Inertial Pointing Activity, with Controller Updates at 50 Hz, Star Tracker Measurement Updates at 2 Hz, and $\int K, d/dt(\delta\theta) dt$ Computed Based on Angular Velocity Measurements - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.D (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_3 equal to 0.02 N-m/sec, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one of the wheels, and wheel spin axes and control axes parallel.)

Figure 6-25. System Response during Inertial Pointing Activity, with Controller Updates at 50 Hz, Star Tracker Measurement Updates at 2 Hz, $\int K, d/dt(\delta\theta) dt$ Computed Based on Angular Velocity Measurements, and Delay Associated with Data Transport via Telemetry - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.E (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_3 equal to 0.02 N-m/sec, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one of the wheels, and wheel spin axes and control axes parallel.)

Figure 6-26. System Response during Inertial Pointing Activity, with Controller Updates at 50 Hz, Attitude Measurement Updates at 2 Hz, $\int K_2 d/dt(\delta\theta) dt$ Computed Based on Attitude Measurements, and Delay Associated with Data Transport via Telemetry - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.E (see Table 3-1), but with rate terms taken out of the integrand of Equation 4-29, and as a result $\int K_2 d/dt(\delta\theta) dt$ or $K_2 (\delta\theta - \delta\theta_0) dt$ computed based on attitude errors. Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_1 equal to 0.02 N-m/sec, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one of the wheels, and wheel spin axes and control axes parallel.)

Figure 6-27. System Response during Inertial Pointing Activity, with Controller and Star Tracker Updates at 50 and 10 Hz, $\int K, d/dt(\delta\theta) dt$ Computed Based on Angular Velocity Measurements, Delay Associated with Data Transport via Telemetry and K_p Equal to 0.02 N-m/rad/sec - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.E (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one of the wheels, and wheel spin axes and control axes parallel.)

Figure 6-28. System Response during Inertial Pointing Activity, with Controller and Star Tracker Updates at 50 and 10 Hz, $\int K_2 d/dt(\delta\theta) dt$ Computed Based on Angular Velocity Measurements, Delay Associated with Data Transport via Telemetry and K_p Equal to 0.2 N-m/rad/sec - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.E (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one of the wheels, and wheel spin axes and control axes parallel.)

Figure 6-29. System Response during Inertial Pointing Activity, with Controller and Star Tracker Updates at 50 and 10 Hz, $\int K_2 d/dt(\delta\theta) dt$ Computed Based on Angular Velocity Measurements, Delay Associated with Data Transport via Telemetry, K_2 Equal to 0.2 N-m/rad/sec, and Wheel Speeds Initially at Zero - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.E (see Table 3-1). Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. External disturbances include modelled environmental torques, and step disturbances of 10^{-4} N-m at 20, 40 and 60 seconds. Other simulation details are: K_1 and K_2 set according to Equation 4-36, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one of the wheels, and wheel spin axes and control axes parallel.)

Figure 6-30. Response of the Baseline System during Inertial Pointing Activity, with no Quantization Effects (Due to Transport via Telemetry) - Case 1B (in Table 5-5).

(The rotational dynamics were simulated using Model 1.E (see Table 3-1). The baseline system includes controller and attitude measurement updates at 50 and 5 Hz, a 0.0625 sec delay to and from the controller associated with sensor and actuator data transport via telemetry, and computes $\int K_2 \frac{d}{dt}(\delta\theta) dt$ based on angular velocity measurements. Initial conditions were zero torque demand and zero attitude and angular velocity errors, and the integration step size was 0.001 second. Other simulation details are: K_1 and K_2 set according to Equation 4-36, K_1 equal to 0.02 N-m/rad/sec, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one of the wheels, modelled environmental torques, external step disturbances of 10^{-4} N-m at 20, 40 and 60 sec, and wheel spin axes and control axes parallel.)

Figure 6-31. Response of the Baseline System during Inertial Pointing Activity, with Quantization Levels of $4.439\text{E-}8$ rad/sec for ω^M , 0.0163 rad/sec for ω_r^M , $0.763\text{E-}6$ for T_m and $0.88''$ for Attitude Measurements (Due to Transport via Telemetry) - Case 1B (in Table 5-5).

(The choice of quantization levels for the sensor and actuator data is discussed in section 5.4. Other simulation details are identical to those listed under Figure 6-30.)

Figure 6-32. Response of the Baseline System during Inertial Pointing Activity, with Quantization Levels of $1.598\text{E-}7$ rad/sec for ω^M , 0.0163 rad/sec for ω_s^M , $0.763\text{E-}6$ for T_m and 0.88° for Attitude Measurements (Due to Transport via Telemetry) - Case 1B (in Table 5-5).

(The choice of quantization levels for the sensor and actuator data is discussed in section 5.4. Other simulation details are identical to those listed under Figure 6-30.)

Figure 6-33. Response of the Baseline System during Inertial Pointing Activity, with Quantization Levels of $4.439\text{E-}8$ rad/sec for ω^M , 0.13 rad/sec for ω_s^M , $0.763\text{E-}6$ for T_m and 0.88° for Attitude Measurements (Due to Transport via Telemetry) - Case 1B (in Table 5-5).

(The choice of quantization levels for the sensor and actuator data is discussed in section 5.4. Other simulation details are identical to those listed under Figure 6-30.)

Figure 6-34. Response of the Baseline System during Inertial Pointing Activity, with Quantization Levels of $4.439\text{E-}8$ rad/sec for ω^M , 0.0163 rad/sec for $\omega_{,M}$, $6.105\text{E-}6$ for T_m and 0.88° for Attitude Measurements (Due to Transport via Telemetry) - Case 1B (in Table 5-5).

(The choice of quantization levels for the sensor and actuator data is discussed in section 5.4. Other simulation details are identical to those listed under Figure 6-30.)

Figure 6-35. Response of the Baseline System to a Step Error of 4° in Each of the Angles of the Euler Angle Sequence 3-1-2 Corresponding to q_d - Inertial Pointing Activity, Case 1B (in Table 5-5). (i.e., to a step of 4° in the angle about axis 1 at 30 sec, about axis 2 at 50 sec and about axis 3 at 70 sec)

(Simulation details include: Model 1.E, controller and attitude measurement updates at 50 and 5 Hz, 0.0625 sec delay to and from the controller, $\int K_2 \frac{d}{dt}(\delta\theta) dt$ computed based on $\delta\omega$ and $\delta\theta$, quantization levels of 4.439E-8 rad/sec for ω^M , 0.0163 rad/sec for ω^M , 0.763E-6 for T_m and 0.88° for attitude measurements, initial conditions of zero torque demand and zero attitude and angular velocity errors, integration step size of 0.001 sec, K_1 and K_2 set according to Eq. 4-36, K_p equal to 0.02 N-m/rad/sec, torque ripple included, internal disturbance of 0.001 N-m at 75 sec for one wheel, modelled environmental torques, external step disturbances of 10⁻⁴ N-m at 20, 40 and 60 sec, and wheel spin axes and control axes parallel.)

Figure 6-36. Response of the Baseline System to a Step Error of 2° in Each of the Angles of the Euler Angle Sequence 3-1-2 Corresponding to q_i - Inertial Pointing Activity, Case 1B (in Table 5-5). (i.e., to a step of 2° in the angle about axis 1 at 30 sec, about axis 2 at 50 sec and about axis 3 at 70 sec)

(Other simulation details are identical to those listed under Figure 6-35.)

Figure 6-37. Response of the Baseline System to Band-Limited White Noise of $3\sigma=4''$ in Each of the Angles of the Euler Angle Sequence 3-1-2 Corresponding to q_a - Inertial Pointing Activity, Case 1B (in Table 5-5).

(Other simulation details are identical to those listed under Figure 6-35.)

Figure 6-38. Response of the Baseline System to Band-Limited White Noise of $3\sigma=2^\circ$ in Each of the Angles of the Euler Angle Sequence 3-1-2 Corresponding to q_{ci} - Inertial Pointing Activity, Case 1B (in Table 5-5).

(Other simulation details are identical to those listed under Figure 6-35.)

Figure 6-39. Response of the Baseline System to Band-Limited White Noise in the Angular Velocity Measurements - Inertial Pointing Activity, Case 1B (in Table 5-5).

(The gyro noise is assumed to be of the form $W_t = (\tau s + I)^{-1} W_c + W_w$, with the variance specified in the band-limited white noise block associated with the colored noise component equal to σ_c^2/T_s , and the variance specified in the band-limited white noise block associated with the white noise component equal to σ_w^2/T_s . Parameter values associated with the gyro noise are listed below (see section 5.3.2 for explanation), and other simulation details are identical to those listed under Figure 6-35.)

$$\begin{aligned} Q_c &= .25 \text{ deg}/(\text{hr}(\text{Hz})^{1/2}) & \omega_c &= 2.744 \times 10^{-4} \text{ rad/sec} & Q_w &= .035 \text{ deg}/(\text{hr}(\text{Hz})^{1/2}) \\ \omega_c &= 1.678 \times 10^{-3} \text{ rad/sec} & \sigma_c^2 &= (10)^2 Q_c^2 (\omega_c/2\pi) & \tau &= \omega_c^{-1} \\ \sigma_w^2 &= (10)^2 Q_w^2 (\Delta T)^{-1} & \Delta T &= 3 \text{ sec} & T_s &= 0.001 \text{ (integr. step size)} \end{aligned}$$

(ΔT should have been set to 2.64 min to be consistent with the discussion in section 5.3.2. The noise situation tested here is therefore slightly worse than that defined in section 5.3.2.)

Figure 6-40. Response of the Baseline System to Band-Limited White Noise of $3\sigma = .001(75 \text{ rad/sec})$ in Each Wheel Speed Measurement - Inertial Pointing Activity, Case 1B (in Table 5-5).

(Other simulation details are identical to those listed under Figure 6-35.)

Figure 6-41. Response of the Baseline System to Band-Limited White Noise of $3\sigma = .002(75 \text{ rad/sec})$ in Each Wheel Speed Measurement - Inertial Pointing Activity, Case 1B (in Table 5-5).

(Other simulation details are identical to those listed under Figure 6-35.)

Figure 6-42. Response of the Baseline System with Delays, Quantization Effects, and Noise during 90 Seconds of Inertial Pointing Activity, Case 1B (in Table 5-5).

(Simulation details now include: Model 1.E, controller and attitude measurement updates at 50 and 5 Hz, 0.0625 sec delay to and from the controller, $\int K_2 \frac{d}{dt}(\delta\theta) dt$ computed based on $\delta\omega$ and $\delta\theta$, quantization levels of $4.439\text{E-}8$ rad/sec for ω^M , 0.0163 rad/sec for ω_r^M , $0.763\text{E-}6$ for T_m and 0.88° for attitude measurements, gyro noise as defined in Figure 6-39, tachometer noise with 3σ equal to 0.1% of wheel speed, $3\sigma=4^\circ$ error in each angle of the Euler angle sequence corresponding to q_{ci} , initial conditions of zero torque demand and zero attitude and angular velocity errors, integration step size of 0.001 sec, K_1 and K_2 set according to Eq. 4-36, K_2 equal to 0.02 N-m/rad/sec, torque ripple included, internal disturbance of 0.001 N-m at 75 sec into the simulation for one wheel, modelled environmental torques, external step disturbances of 10^{-4} N-m at 20, 40 and 60 sec into the simulation, and wheel spin axes and control axes parallel.)

Figure 6-43. Response of the Baseline System with Delays, Quantization Effects, and Noise during 90 Seconds of Inertial Pointing Activity Including Switching between Two Inertial Targets, Case 2A (in Table 5-5).

(The rotational dynamics were simulated using Model 2.E. Other simulation details are identical to those listed under Figure 6-42.)

Figure 6-44. Response of the Baseline System with Delays, Quantization Effects, and Noise during 90 Seconds of Limb Following Activity, Case 3A (in Table 5-5).

(The rotational dynamics were simulated using Model 3.E and K_1 and K_2 were set according to Eq. 4-37. Other simulation details are identical to those listed under Figure 6-42.)

Figure 6-45. Response of the Baseline System with Delays, Quantization Effects, and Noise during 450 Seconds of Limb Pointing with Continuous Scanning Activity, Case 4A (in Table 5-5).

(The rotational dynamics were simulated using Model 4.E and K_1 and K_2 were set according to Eq. 4-37. Other simulation details are identical to those listed under Figure 6-42.)

Figure 6-46. Response of the Baseline System with Delays, Quantization Effects, and Noise during 360 Seconds of Limb Pointing with Stepwise Scanning Activity, Case 5A (in Table 5-5).

(The rotational dynamics were simulated using Model 5.E and K_1 and K_2 were set according to Eq. 4-37. Other simulation details are identical to those listed under Figure 6-42.)

Figure 6-47. Response of the Baseline System with Delays, Quantization Effects, and Noise during 900 Seconds of Inertial Pointing Activity, Case 1B (in Table 5-5).

(Simulation details are identical to those listed under Figure 6.42.)

Figure 6-48. Response of the Baseline System with Delays, Quantization Effects, and Noise during 900 Seconds of Inertial Pointing Activity Including Switching between Two Inertial Targets, Case 2A (in Table 5-5).

(The rotational dynamics were simulated using Model 2.E. Other simulation details are identical to those listed under Figure 6-42.)

Figure 6-49. Response of the Baseline System with Delays, Quantization Effects, and Noise during 900 Seconds of Limb Pointing with Continuous Scanning Activity, Case 4A (in Table 5-5).

(The rotational dynamics were simulated using Model 4.E and K_1 and K_2 were set according to Eq. 4-37. Other simulation details are identical to those listed under Figure 6-42.)

Figure 6-50. Response of the Baseline System with Delays, Quantization Effects, and Noise during 900 Seconds of Limb Pointing with Stepwise Scanning Activity, Case 5A (in Table 5-5).

(The rotational dynamics were simulated using Model 5.E and K_1 and K_2 were set according to Eq. 4-37. Other simulation details are identical to those listed under Figure 6-42.)

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

The main conclusions and recommendations are listed below:

- Because of the various pointing profiles, the accuracies involved and the mass constraint, the best set of attitude control hardware for a satellite like Odin includes reaction wheels, magnetorquers, star sensors, gyroscopes, Sun sensors and magnetometers.
- Modelling of the satellite, the disturbance environment and the flight profiles led to the following specifications for the actuators: a torque and momentum capability of the order of 0.027 N-m and 3 N-m-s for each wheel and a dipole capability of the order of 40-50 A-m² for each rod. To derive those values, assumptions had to be made with respect to worst-case maneuvers, operating requirements and margin factors. The specifications should therefore not be treated as rigid.
- In both modes of operation, staring and limb pointing, the largest environmental effects are associated with the magnetic torques. The residual dipole of the spacecraft should therefore be kept as low as possible.
- The use of reaction wheels naturally leads to a momentum rather than to a torque

control approach. Including coupling terms in the control law and assuming a fast wheel response simplifies the design procedure considerably. Different attitude control gains are defined for the staring and limb following modes. In either mode, selection of K_1 and K_2 is based on the required pointing accuracy, the worst-case anticipated disturbance torque and a commonly used damping factor of 0.707. Simulation shows the choice of the attitude control gains to be adequate.

- The delays in the system (associated with the star tracker update rate, the controller computations and the relay of sensor data and commands via telemetry) are found to place an upper bound on the value of the proportional gain (K_p) in the speed control loop. At high gains, the torque demand to the wheel toggles between the positive and negative maximum torque levels which will lead to actuator wear. The upper bound on K_p is estimated to be approximately 0.09 N-m/rad/sec, for our wheel model and a pure time delay of 0.125 second.
- During inertial pointing activity and in the absence of transverse momentum, the motor torque ripple places a lower bound on the value of K_p . By extension, this also applies to other types of wheel disturbances. Undesirable oscillatory behavior is observed with $K_p=0.02$ N-m/rad/sec at near zero wheel speeds. This is partly due to the large ripple (20 percent of torque demand) and the large friction torque in our wheel model (eg. Coulomb friction of the order of $1\text{E-}3$ N-m compared to $1\text{E-}4$ N-m worst-case environmental torques). Simulation shows that transverse momentum or increasing K_p significantly reduces the effect of wheel disturbances on the attitude control loop. With this type of controller, and because of the upper bound on K_p , a bias should therefore be applied to at least one of the wheels during inertial pointing

activity to attenuate the effect of the ripple.

- Because of the ripple, the delays, and to avoid zero-speed crossings, the wheels should be biased in all modes of payload operation with this type of controller.
- For inertial pointing, the biases required to avoid zero-speed crossings are easily determined. The rotational dynamics are integrated using a model of the spacecraft and the disturbance environment, and assuming perfect tracking of the reference inputs. The biases are then a function of the flight profile, the orientation of the wheels, and the variation and direction of the secular buildup in the momentum of each wheel.
- When the control algorithm was developed initially, it was assumed that three wheels would be used at a time with the fourth kept for redundancy, and that nominally magnetorquers would be off unless unloading was effected. This was to reduce potential sources of disturbances. When the spacecraft is in earth-referenced mode, three wheels are active and the magnetorquers are not used, it is found that:
 - (1) coupling between the biases and the orbital rate cause large cyclic variations in the wheel momentum relative to the body axes; and
 - (2) zero-speed crossings cannot be avoided for two of the wheels if the orthogonal wheel combination is used, and for potentially only one of the wheels if wheel combination 1-2-4 is used.

During earth-referenced mode, to avoid zero-speed crossings by all three active

wheels, magnetorquers could be enabled at all times. The external torque by the rods would essentially rotate the bias momentum vector of the wheels in inertial space, so that the cycling effect in the speeds due to coupling between the biases and the orbital rate is removed, and the wheel speeds vary mainly to absorb the external disturbance torques.

- Simulation shows that the delays in the system adversely affect the attitude of the system during inertial pointing, when the momentum demand is generated solely based on star tracker measurements at low star tracker update rates (2 Hz). This is not the case at higher update rates (5 and 10 Hz). The gyros may therefore be required in the staring mode, to compute the attitude error rate terms in the control law, if the tracker specifications are not met.
- Anti-windup must be implemented with the integrator generating the momentum demand, especially when discrete changes occur in the commanded reference attitude and angular velocity.
- With this type of controller, the desaturation torque must be fed forward to the integrator generating the momentum demand.
- Because of the proportional control in the speed loop, the effect of noise and quantization in the speed measurements must be considered when choosing K_p and determining the accuracy at which the measurement should be passed in telemetry. The system is less sensitive to noise and quantization associated with other measurements, because of the integrator used to generate the momentum demand.

- During staring events, a static change in the attitude error is found to have an equivalent effect in the simulated attitude error. In the long inertial pointing simulation run, for which we had defined a set of baseline delays, quantization and noise conditions, the simulated error remained within $\pm 3''$. For the control requirement of $\pm 15''$ to be met with a margin factor of 1.5, the RMS pointing error in the axes normal to the sensor boresight (i.e., the pitch and roll control axes), associated with sources not accounted for or adequately represented in the simulation, should therefore be kept within $\sim 9.5''$ (eg., misalignments, thermal distortions).
- Because of star viewing constraints, the gyros must be used in the limb following modes. In the simulation tests, the attitude errors were computed based on the star tracker measurements, and the delay before and after the controller was set at the one sixteenth of a second worst-case scenario. During limb following events, however, the attitude errors would be computed using the gyros between star tracker updates and the delay would be random. The large variations in the pitch attitude error into the controller and the simulated pitch error, observed during the tests, are therefore expected to be reduced. Using this type of controller, the pitch error (found to be $< 170''$ during continuous scanning activity under baseline delays, quantization and noise conditions) is found to be larger than anticipated. This is attributed at the present time to errors in the decoupling terms, as these terms are based on noisy, sampled and delayed measurements, and most of them include the orbit rate as an operand. Therefore, although the pointing requirement is more relaxed in the aeronomy mode ($360''$ pitch pointing accuracy in real-time), the error budgeting must account for the decoupling errors, and for errors supplementary to the baseline conditions assumed in the simulation (eg., misalignments, thermal distortions, errors

in the attitude reference profiles, uncertainty in plant parameters).

- If this controller was to be flown, longer simulation tests would be recommended. In the case of earth-referenced modes, tests should investigate the effect of using magnetorquers at all times to remove the cyclic effect in the wheel speeds due to coupling between the orbit rate and the biases. It would also be recommended to model the detailed onboard attitude estimation approach, including mixing of the gyro and star tracker data and estimation of the gyro rate biases, and investigate the effect of the orbit precession rate, variations in plant parameters and uncertainty in the knowledge of B , and the transition between attitude control modes using simulation. The effects of internal torques (flexibility, thermal deformations, mechanisms) would also have to be addressed.
- It is felt that Simulink is probably not the best tool to test this particular controller in this particular application, at least not in the way the software has been used. Because of the changing operating conditions, no attempt has been made to linearize the system and take advantage of the built-in linear control design and analysis tools. Because Mex-files had to be used and because of the large demand on memory, our models are not easily exportable between different platforms and different versions of the software. The simulation speed has been the biggest limitation. The main advantages are the graphics capabilities, the ease at which sampling and delay blocks can be incorporated, and the built-in integration tools.

APPENDIX A

Sizing of the Attitude Control Actuators for Odin

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A.1 Control Objective and Sizing Criteria

The attitude control and determination subsystem (ACDS) of a spacecraft must be able to:

- (i) meet the pointing requirements dictated by the mission, while coping with environmental torques, cyclic and secular, and with internal torques such as those caused by equipment motion and structure flexibility;
- (ii) generate the torques required for attitude maneuvers, if slewing is part of the mission;

- (iii) stabilize the spacecraft following transient operations, such as separation from the launch vehicle, or orbital maneuvers if active control is part of the mission; and,
- (iv) assist in attitude recovery after specified hardware failures.

In the case of Odin, plans include various slewing activities but no active orbit control. Based on control accuracy and pointing requirements, preferred actuators are four reaction wheels to absorb or generate cyclical torques, and three orthogonally mounted magnetic torquers to dump momentum from the wheels. Selection reasoning is discussed in Chapter 2.

The methodology for sizing attitude control actuators is described in Larson and Wertz (1992, section 11.1). The size of the actuators is driven by the largest disturbance torques anticipated. However, these disturbances depend on:

- (i) the physical characteristics of the spacecraft, including geometry, mass distribution, surface properties and residual magnetism; and,
- (ii) the position, velocity and orientation of the spacecraft relative to the Earth, to the orbital frame and to inertial space.

Therefore, in order to quantify the disturbance environment and identify extreme conditions, an initial definition of the spacecraft attributes and the flight and attitude profiles is required. The actuators are then sized based on worst-case disturbance torques, the spacecraft attributes or configuration modified accordingly, and the sizing refined.

A.2 Quantifying the Disturbance Environment: Initial Assumptions and Modelling

A.2.1 Spacecraft Attributes

Spacecraft configuration design is described in Larson and Wertz (1992, section 10.2). Rules of thumb are given which assist in the early design stages and relate payload data (such as size, weight and power), overall spacecraft data, launcher fairing data and pointing requirements. The design process is iterative, and the starting points are usually the payload and mission requirements.

In a summary document following the Phase A studies (SSC, 1991), a baseline configuration for Odin is defined and the orbit is described as 600 km, circular and Sun-synchronous, with ascending node crossing at 1800 local time.

Key drivers for the satellite bus configuration described in the 1991 document include:

- (i) the weight, size and power of the astronomy and aeronomy instruments (especially the size of the reflector);
- (ii) the use of Pegasus as a baseline launch vehicle (constraining the diameter and length in the folded configuration);
- (iii) the requirement for solar panels (the body area being insufficient);
- (iv) the need to shield components from direct sunlight; and,
- (v) the required pointing control and knowledge accuracies (motivating the use

of fixed rather than articulated components to avoid additional inaccuracies associated with gimbals).

These factors led to fixed solar and Sun shield arrays attached at the bottom of the spacecraft, fixed payloads and attitude sensors optimally oriented, and rotation of the entire spacecraft to meet pointing requirements.

To characterize the disturbance environment, an approximate satellite model is derived from diagrams included in Swedish Space Corporation (1991). The model is illustrated in Figure A-1. The coordinates, geometry, surface characteristics and mass distribution which are assumed for the satellite model are outlined in Tables A-1 and A-2.

A.2.2 Flight and Attitude Profiles

To quantify worst-case disturbances, the satellite in Figure A-1 is flown for an entire orbit, at different times of the year, for two modes of operation. Orbit parameters are illustrated in Figure A-2.

A.2.2.1 Orbit Assumed

Torques are computed assuming the baseline orbit described in the 1991 summary document: 600 km, circular and Sun-synchronous (97.79°), with a period of 96.69 minutes and an ascending node crossing at 1800 local time.

The choice of the orbit described in the 1991 document is motivated by:

- (i) the need for atmospheric observations in the orbit plane (near the poles),
- (ii) power requirements (Sun-synchronicity and ascending node chosen to optimize power output from fixed solar panels),
- (iii) the need for constant viewing geometry, for similar lighting conditions throughout the orbit, and for sufficient height resolution (again during atmospheric measurements), and
- (iv) the Pegasus launch system capability.

Sun synchronicity occurs when the right ascension of the ascending node (Ω) changes at a rate of 0.9856 degrees per day or 360 degrees per sidereal year. Secular variations in Ω , ω and M_0 (mean anomaly at epoch t_0) are due to the nonspherical mass distribution of the Earth. When only Earth oblateness effects are considered, the rate of change of Ω can be expressed in terms of the semimajor axis, eccentricity and inclination (Wertz, 1978, Eq. 3-41d; Escobal, 1976, section 10.2). The proper combination of such parameters then results in a Sun-synchronous orbit.

When drag effects are ignored, the mean values of the semimajor axis, eccentricity and inclination are fixed. In Sun-synchronous orbits where drag effects are ignored, the angle between the Sun vector and the orbit normal measured in the equatorial plane is constant. The same angle measured in a plane perpendicular to the equator varies over the course of one year, as the ecliptic plane (plane of Sun motion in inertial space) is tilted by 23.45 degrees relative to the equatorial plane. For the assumed orbit (ignoring drag effects), the angle between the Sun vector and the orbit normal

measured in the equatorial plane is zero. The same angle measured in the plane perpendicular to the equator varies between -15.66 and 31.24 degrees over the course of one year, and the satellite experiences periods of eclipse near summer solstice, as illustrated in Figure A-6. Different times of year are then considered, to evaluate the effect on solar radiation torque levels.

In a design document issued at the end of 1993 (SSC, 1993a), as a result of the upgraded Pegasus launch system capability, the baseline orbit had been modified to 700 km, circular and Sun-synchronous (98.19°), with a period of 98.77 minutes and an ascending node crossing at 1800 local time. As a result of the increased altitude for the orbit, aerodynamic torques and drag are reduced. In the discussion below, results are extrapolated to account for the latest orbit choice.

A.2.2.2 Modes of Operation

To compute disturbance torques, two nominal modes of operation are considered:

- (i) **Aeronomy -- In-plane scanning for one entire orbit.** The boresight of the antenna scans the horizon between 15 and 120 km altitude, the scan in one direction lasting approximately two minutes. The vehicle scan axis is normal to the solar array and the solar array is parallel to the orbital plane; and,
- (ii) **Astronomy -- Fixed Inertial Orientation.** The satellite is in fixed inertial orientation with a solar incidence angle of up to 32° relative to the normal of the solar panels.

In the design review document issued at the end of 1993 (SSC, 1993a), seven modes of observation are listed. Observation modes in aeronomy include limb following, limb following with continuous altitude scanning and limb following with stepwise altitude scanning. Observation modes in astronomy include staring, staring with position switch, mapping of a sequence of n positions, and celestial target scanning. Although the two modes of operation assumed above do not represent all the required flight modes during payload operation, they illustrate the orders of magnitude and waveforms of torques to expect.

A.2.3 Margin Factors

When sizing the actuators, disturbances are scaled by a margin factor which should account for:

- (i) the satellite model approximation (e.g. uncertainties in the relative distances between the center of pressures of the various forces and center of mass, in the mass distribution, in moments and products of inertia, in surface properties), and
- (ii) orbit injection inaccuracies and drag effects (resulting in changes in orbit parameters)

A.3 Disturbance Torques

Anticipated disturbances for Odin include environmental torques, internal torques, torques associated with maneuvering, and momentum build-up during separation from the launch vehicle. Cyclic and secular effects are discussed below. Cyclic terms over an orbit average to zero (zero angular momentum build-up). Secular terms over an orbit are associated with torques components which do not average to zero over the course of one orbit, and which therefore result in a steady increase in angular momentum after each orbit.

A.3.1 Environmental torques - 600 km, Circular, Sun-synchronous Orbit

The four principal environmental torques acting on spacecraft are gravity-gradient, magnetic, solar radiation and aerodynamic torques.

A.3.1.1 Aerodynamic Torque

Aerodynamic drag is the resisting force acting on the satellite, generally antiparallel to the velocity vector, as a result of atmospheric molecules colliding with the spacecraft body. A torque is present when the line of action of the net drag force is offset from the center of mass of the vehicle.

An equation for the aerodynamic torque is given in Wertz (1978, section 17.2.3). In matrix form, the equation is:

$$\mathbf{T}_{aero} = \int_A \mathbf{r}_s^x df_{aero} = \int_A \mathbf{r}_s^x \left[-\frac{1}{2} C_D \rho (N^T \mathbf{V}) \mathbf{V} \right] dA \quad (\text{A-1})$$

where

df_{aero} is the aerodynamic drag on the surface element dA , antiparallel to the velocity vector of dA , assuming that the energy of the atmospheric particles is entirely absorbed during impact;

A is the overall satellite surface;

ρ is the atmospheric density;

N is a unit vector normal to the surface element dA ;

$\mathbf{r}_s$ is the vector from the center of mass to the surface element dA ;

$\mathbf{V} = \mathbf{V}_0 + \boldsymbol{\omega}^x \mathbf{r}_s$, and $\boldsymbol{\omega}^x$ is the skew-symmetric matrix representation of the cross-product;

$\mathbf{V}_0$ is the velocity of the center of mass relative to the velocity of the atmosphere,

$\boldsymbol{\omega}$ is the rotation of the satellite relative to the rotation of the atmosphere (which is approximately equal to the rotation of the Earth);

C_D is the drag coefficient, dimensionless and set to 2.0 when no measured value is available;

df_{aero} is set to zero when $N^T \mathbf{V}$ is less than zero; and,

df_{aero} , $\mathbf{r}_s$, $\mathbf{V}$, $\mathbf{V}_0$, N are all resolved in the same reference frame.

Atmospheric density varies with altitude, solar activity and within a shorter time frame, geomagnetic activity. At 600 km altitude, the maximum atmospheric density is $8.46 \times 10^{-13} \text{ kg/m}^3$ (extracted from the table at the end of Larson and Wertz (1992) and based on the MSIS atmospheric model (Hedin, 1986) with F10.7 index of 220). For a

600 km circular orbit, the velocity of the center of mass of the satellite is 7.558 km/sec. If the contribution of ω to the aerodynamic force is assumed to be small relative to the contribution of the velocity of the center of mass, terms in ω are neglected and the expression for the aerodynamic torque is reduced to:

$$T_{aero} = \frac{1}{2} C_D \rho \int_A (N^T V_0)(V_0^x r_s) dA \quad (A-2)$$

An estimate of the worst-case anticipated aerodynamic torque can be computed using (Larson and Wertz, 1992, section 10.4.2):

$$T_{aero} = \frac{1}{2} C_D \rho A V^2 (u_v^x l_{cp}) \quad (A-3)$$

where A is an estimate of the largest area perpendicular to V , V is the orbital velocity, u_v is the unit vector in the velocity direction, and l_{cp} is an estimate of the worst-case distance from the center of mass to the center of pressure, perpendicular to the direction of velocity.

As the satellite's orientation changes relative to the velocity vector, some surfaces can be hidden by others. For the satellite model in Figure A-1, a simple algorithm is created to approximate the effective area and the center of pressure of the surfaces partially hidden from the incoming flow. The algorithm is described in Table A-3.

During in-plane scanning, since the satellite's orientation remains almost fixed with respect to the orbital velocity vector, the aerodynamic torque over one orbit is nearly constant, except for a small cyclic effect as a result of the scanning motion of the

satellite. The constant nonzero aerodynamic torque arises as a result of the asymmetry of the satellite's surface relative to the orbital velocity vector.

When the satellite is in a fixed inertial orientation, the aerodynamic torque has large cyclic components along the body axes, as the velocity vector completes a 360 degree rotation relative to the spacecraft about an axis parallel to the orbit normal. The average torque over the orbit and the waveform of the torque components along X_b , Y_b and Z_b vary with the inertial orientation of the vehicle. A nonzero average torque arises as a result of the asymmetry of the spacecraft relative to the orbit normal and the non-symmetrical shading of components over the orbit. At large offset angles between the spacecraft Z -axis and the orbit normal, the waveforms of the torque components along X_b and Y_b show the effect of the shift in the center of pressure, which changes differently depending on whether the velocity vector is facing the front or back of the solar array.

Aerodynamic torque values are included in Table A-4 and the torque variation over an orbit for sample cases is shown in Figures A-8 through A-11. The magnitude of the torque is labelled T , and the torques labelled T_{xb} , T_{yb} and T_{zb} correspond to the torque components along the body axes defined in Table A-1.

Due to decreased orbital velocity and decreased atmospheric density, an increase in orbit altitude from 600 to 700 km results in the worst-case estimates being scaled by

the factor 0.103.

A.3.1.2 Solar Radiation Torque

Sources of electromagnetic radiation pressure include direct solar radiation, solar radiation reflected from the Earth, and Earth and atmospheric radiation. The only source usually considered is the direct solar radiation. The relative importance of the various sources of radiation as a function of orbit altitude are given in Wertz (1978, section 17.2.2).

An equation for estimating the solar radiation torque is given in Wertz (1978, section 17.2.2). In matrix form, the equation is:

$$\begin{aligned} \mathbf{T}_{srad} &= \int \mathbf{r}_s^x df_{srad} \\ &= -P \int \mathbf{r}_s^x \left\{ \left[(1 - C_s) S + 2 \left(C_s \cos\theta + \frac{C_d}{3} \right) N \right] \cos\theta \right\} dA \end{aligned} \quad (\text{A-4})$$

where

df_{srad} is the difference between incident and reflected radiation pressure on surface element dA ;

$P = 4.664 \times 10^{-6} \text{ kg/ms}^2$ is the mean momentum flux over the year (Larson and Wertz, 1992);

$\mathbf{r}_s$ is the vector from the center of mass to the surface element dA ;

C_s , C_d and C_a are the coefficients of specular reflection, diffuse reflection and absorption respectively, and sum to 1;

S is the unit vector from the surface element dA to the Sun;

N is the unit vector normal to the surface element dA ;

θ is the angle between S and N ;

$T_{\text{rad}} = 0$ for $\theta > 90$ deg, and

N , S and r_s are all resolved in the same reference frame.

In Larson and Wertz (1992, Table 11-9a), the following simplified equation is given for estimating the worst-case solar radiation torque:

$$T_{\text{rad}} = (F_s/c)A(1 + q)(\cos i)l_{cp} \quad (\text{A-5})$$

where $F_s = 1358 \text{ W/m}^2$ is the solar constant, $c = 3 \times 10^8 \text{ m/s}$ is the speed of light, A is the surface area, q is the reflectance factor and ranges from 0-1, i is the solar incidence angle relative to the surface, and l_{cp} is an estimate of the worst-case distance from the center of mass to the center of pressure.

For Earth orbits, the torque due to direct solar radiation depends entirely on the spacecraft physical properties, including surface optical properties and exposed area, and on the spacecraft orientation relative to the Sun. Sun distance variations have negligible effects. In the case of Odin, based on diagrams in Swedish Space Corporation (1991), the thermal blanket and the solar arrays shield the other components from the Sun for Sun incidence angles up to approximately 35 degrees relative to the normal of the solar array.

During scanning sessions, the solar radiation torque has cyclic components along the spacecraft axes. During in-plane scanning operations, the Sun angle relative to the

solar and thermal blanket arrays can be considered constant over one orbit, but varies over one year as indicated in Figure A-6. The largest solar radiation torque components for in-plane scanning sessions are observed during summer solstice, when the Sun angle is approximately 31.25 degrees (assuming ideal Sun-synchronous orbit injection and neglecting drag effects on the orbit parameters a , e and i).

During staring sessions, the direct solar radiation torque is constant over the course of one orbit. The largest solar torques during staring sessions are experienced when the Sun angle is at the maximum allowed value (32 degrees in our model), and when the orientation of the spacecraft is such that the distance between the center of pressure and the center of mass, normal to the Sun vector, is largest. Staring sessions are snap shots of in-plane scanning sessions under the same Sun angle conditions at a given point in the orbit.

Direct solar radiation torque values are included in Table A-4 and the torque variation over an orbit for sample cases is shown in Figures A-8 through A-11.

A.3.1.3 Gravity-gradient Torque

A gravity-gradient torque arises because the gravitational field varies with the squared distance from the Earth and is therefore not uniform over the mass of the satellite. Assuming a rigid satellite body and a spherical Earth, the gravity-gradient

torque about the center of mass is given in matrix form by:

$$\mathbf{T}_{grav} = \int \mathbf{r} \times \left(\frac{\mu}{R^3} \mathbf{R} \right) dm \quad (\text{A-6})$$

where

$d\mathbf{f}_{grav}$ is the gravitational force on element dm in the direction of $\mathbf{R}$;

$\mathbf{R} = \mathbf{R}_0 - \mathbf{r}$;

$\mathbf{R}$ is the vector from element mass dm to the center of the Earth;

$\mathbf{R}_0$ is the vector from the center of mass to the center of the Earth (along nadir);

$\mathbf{r}$ is the vector from the center of mass to the element mass dm ;

$\mu = 3.986005 \times 10^{14} \text{ m}^3/\text{s}^2$ is the gravitational constant of the Earth; and

$\mathbf{r}$, $\mathbf{R}$ and $\mathbf{R}_0$ are resolved in the same reference frame.

Setting $R = [\mathbf{R}^T \mathbf{R}]^{1/2}$ and $\mathbf{R} = \mathbf{R}_0 - \mathbf{r}$, expanding the term $[1 - 2(\mathbf{r}^T \mathbf{R}_0)R_0^{-2} + \mathbf{r}^2 R_0^{-2}]^{3/2}$ as a binomial series (assuming $r \ll R$) and dropping terms in r^2 or higher, the gravitational torque reduces to:

$$\mathbf{T}_{grav} = (3\mu/R_0^5) \int (\mathbf{r} \times \mathbf{R}_0)(\mathbf{r}^T \mathbf{R}_0) dm \quad (\text{A-7})$$

When the dot and cross products are expanded with both $\mathbf{r}$ and $\mathbf{R}_0$ resolved in body coordinates, the gravity-gradient torque can be expressed in terms of moments and products of inertia.

During scanning operations, the gravity-gradient torque is nearly constant, as the

orientation of the vehicle remains nearly fixed relative to nadir. Small cyclic effects are a result of the scanning motion.

During staring operations, the gravity-gradient torque has large cyclic components along the body axes, as nadir completes a 360 degree rotation relative to the spacecraft about the orbit normal. The average torque over the orbit again varies with the inertial orientation of the vehicle, with worst cases being experienced when the spacecraft Z-axis is offset from the Sun vector by the maximum allowed angle.

Gravity-gradient torque values are included in Table 1-4 and the torque variation over an orbit for sample cases is shown in Figures A-8 through A-11. The gravity-gradient torque estimates would be scaled by a factor of 0.96 as a result of increasing the orbit altitude from 600 to 700 km.

A.3.1.4 Magnetic Torque

Magnetic torques arise because the residual magnetism in spacecraft components and the magnetic field induced by current loops interact with the magnetic field of the Earth. The magnetic torque in matrix form is given by:

$$\mathbf{T}_{mag} = \mathbf{m} \times \mathbf{B} \quad (\text{A-8})$$

where $\mathbf{m}$ is the overall magnetic moment associated with the spacecraft, and $\mathbf{B}$ is the Earth's magnetic field, and both vectors are resolved in the same coordinate frame. The magnetic torque will have cyclic components along the vehicle axes, as the magnetic field changes direction throughout the orbit in the case of a polar, or in this case, a near-polar orbit. The direction of the magnetic torque however depends on the orientation of the

magnetic dipole of the spacecraft relative to the direction of the field. If the satellite dipole is fixed relative the body axes, potentially large secular effects will also occur during in-plane scanning.

In Larson and Wertz (1992, Table 11-9a), a simplified equation for estimating the largest magnetic torque in a polar orbit is given as:

$$T = D(2M/R^3) \quad (\text{A-9})$$

where

D is the estimated magnetic moment of the spacecraft (set to 1 A-m² for a small sized spacecraft)

M is the magnetic moment of the Earth (7.96×10^{15} tesla-m³)

R is the orbit radius in meters

A.3.2 Internal Torques

Excluding torques generated by the control subsystem, internal torques on Odin will be associated with the initial deployment of the solar and Sun shield arrays, with the reciprocal motion of the Stirling cooling engine during either or both modes of payload operation, and with the elastic deformations of the structure. These torques will affect the sensitivity of the control system and are not critical to the sizing of the actuators.

A.3.3 Attitude Maneuvers

Attitude maneuvers for Odin include:

- (i) initial attitude acquisition with the spacecraft still in launch configuration;
- (ii) required slewing for payload operation. Examples include:
 - a. scanning during aeronomy operations, with changes between $2.7^\circ/\text{min}$ and $4.7^\circ/\text{min}$ inertial rates,
 - b. scanning during astronomy sessions,
 - c. calibration slews during staring events, and
 - d. large angle slews prior to or between staring events (changing up to 180° in orientation); and,
- (iii) nominal attitude recovery (if the satellite becomes unstable)

Assuming bang-bang control, where a maneuver is conducted in minimum time under bounded acceleration and deceleration inputs, the required torque is given by:

$$T_{\text{man}} = \frac{4\theta I}{t^2} \quad (\text{N-m}) \quad (\text{A-10})$$

where θ is the angular displacement (radians), I is the moment of inertia about the rotation axis, and t is the duration of the maneuver (sec). A few estimates are given below:

Assumed Maneuver about a Single Axis	Required Torque
Large slew of 64° in 10 minutes, $I = 43\text{kg-m}^2$	$5.34 \times 10^{-4} \text{ N-m}$
Large slew of 64° in 2 minutes, $I = 43\text{kg-m}^2$	$1.33 \times 10^{-2} \text{ N-m}$
Calibration slew of 30 arcmin in 25 sec, $I = 43 \text{ kg-m}^2$	$2.4 \times 10^{-3} \text{ N-m}$
Changes in inertial rates during scanning, between 2.7 and $4.7^\circ/\text{min}$, with constant acceleration during 17 seconds, $I = 43\text{kg-m}^2$	$1.4 \times 10^{-3} \text{ N-m}$

A.3.4 Transient Operations -- Launch Separation

It is assumed that the maximum tip-off rate about each axis during separation from the launch vehicle is $1^\circ/\text{sec}$ and that the satellite in launch configuration is a 1.1 meter by 1.8 meter 225 kg cylinder of uniform density. The largest moment of inertia about the geometric center is then 78 kg-m^2 and the largest angular momentum imparted to the satellite about one axis would be $|\Delta H_{sep}| = I\omega \approx 1.36 \text{ N-m-s}$.

A.4 Worst-Case Disturbance Scenarios

A.4.1 Worst-Case Torques

At 600 km altitude, and for the satellite model discussed above, the largest environmental disturbances are associated with aerodynamic and magnetic torques. Combining worst-case maneuvering (assumed to be 64° in two minutes) and worst-case environmental torques yields $|T_w|_{\text{man,env}} \approx 13.4 \text{ mN-m}$.

A.4.2 Worst-Case Momentum Build-up

Angular momentum accumulation associated with the various disturbances will either occur once or a few times (as a result of transient operations), be cyclic (averaging to zero) over one orbit, or be secular (steadily increasing) with each orbit.

A.4.2.1 Launch Separation

The largest anticipated momentum accumulation as a result of a single disturbance will occur at separation from the launch vehicle, and was determined above to be of the order of 1.36 N-m-s.

A.4.2.2 Maneuvering

A rotation of 64° in two minutes about a single axis (with $I = 43\text{kg-m}^2$ and bang-bang control) is assumed to represent the largest maneuvering torque demand. Ignoring friction effects in the wheel, this introduces a variation of $|\Delta H_{\text{cyclic max man}}| \approx .8$ N-m-s in momentum in a single wheel, if the wheel axis is along the attitude maneuver axis.

A.4.2.3 Magnetic Torque Effect

Assuming that in either mode of operation the magnetic torque about one of the spacecraft axes is a sinusoidal disturbance of maximum amplitude A of 4.7×10^{-5} N-m at twice the orbital frequency, the maximum momentum accumulation then occurs during a quarter of the orbit. The average torque integrated between 0 and $\pi/2$ is $[\int d\theta]^{-1} [A \int \sin(2\theta) d\theta] = A \cdot .637$, and the corresponding momentum accumulation is $|\Delta H_{\text{cyclic max mag}}| \approx .25 * \langle \text{Orbital Period} \rangle * A \cdot .637 = .0434$ N-m-s.

A.4.2.4 In-plane Scanning Operations - Aerodynamic Effects

During in-plane scanning, the largest near constant environmental disturbance is the aerodynamic torque. The worst-case secular momentum increase during time interval t is then $|\langle T_a \rangle|t$, where $|\langle T_a \rangle|$ is $2.4 \times 10^{-5} \text{ N-m}$, the largest anticipated average aerodynamic torque during one orbit. For one orbit, the largest momentum change anticipated, solely due to aerodynamic effects, is $|\Delta H_{\text{secular scanning max aero}}| \approx .139 \text{ N-m-s}$.

A.4.2.5 Staring Operations - Combined Environmental Effects

Environmental torques are integrated over one orbit under worst-case solar incidence angle (32 degrees in our model), when the gravity-gradient torque is largest, and under zero solar incidence angle, when the aerodynamic torque is largest. Results are plotted in Figures A-12 and A-13. The largest secular effect is found to be $\approx .03 \text{ N-m-s}$.

A.4.3 Torque Changes - Critical Situations

Nearly instantaneous large torque changes will be experienced at the start and end of a maneuver, and especially at the mid-point, if bang-bang control is used. The rate of change of the aerodynamic torque along X_b and Y_b varies drastically during staring operations with large Sun angles, as a result of shadowing effects. Very rapid changes in direct solar radiation torques are experienced during eclipse season, when entering and exiting the Earth's shadow, although the torque changes are not as significant as during maneuvers.

A.5 Sizing of the Reaction Wheels

Reaction wheels are selected based on their angular momentum capacity, which is a function of both wheel inertia and angular velocity, on their maximum output torque, and on other factors such as operating wheel speed range, required power, size, weight, reliability, motor type, electronics and analog or digital interface.

The two main criteria in the selection of a reaction wheel are the minimum angular momentum capacity and the minimum torque capability that each reaction wheel must provide.

To maintain accurate pointing by the satellite, the minimum output torque must counteract the largest disturbance torque, estimated and then scaled by a margin factor, which could be experienced by the spacecraft. The minimum output torque must also be able to meet the slewing requirements for attitude maneuvers. Combining worst-case environmental and maneuvering torques, the worst torque anticipated is of the order of 27 mN-m (assuming a margin factor of 2). This is small and within the torque capability of the Honeywell reaction wheels listed in Table A-5.

The angular capacity of the reaction wheel must be larger than the largest integrated disturbance torque at any time between momentum dumping sessions, and is therefore also function of the frequency at which dumping sessions are conducted. Over one orbit, the worst-case momentum variation can be estimated by adding the worst-case cyclic variations due to maneuvering and magnetic effect. Over several orbits, the worst-case momentum variation can be estimated by adding the worst-case variation due to

magnetic effects, and the secular variation since the last momentum dump as a result of either in-plane scanning operations (secular effect estimated in section 4.2.4) or staring operations (secular effect estimated in section 4.2.5). Therefore, in the k th orbit of in-plane scanning operations since the last momentum dump, $|\Delta H_{max}| = .0434 + .139(k-1+frc)$ where frc is set to .25 as the maximum magnetic cyclic momentum build-up occurs in one-quarter orbit. Similarly, in the k th orbit of staring operations since the last momentum unloading session (assuming no maneuvers), $|\Delta H_{max}| = .0434 + .03(k-1+frc)$. The minimum angular momentum storage capacity of the wheel is then $|\Delta H_{max}|' = <Margin Factor> |\Delta H|_{max}$. A few estimates are given in the table below:

Margin Factor	Current Orbit Number Since Last Momentum Dump (k)	$ \Delta H_{max} '$ In-plane Scanning Operations	$ \Delta H_{max} '$ Staring Operations	$ \Delta H_{max} '$ Maneuvering and Magnetic Effects
1	0	.08 N-m-s	.05 N-m-s	.84 N-m-s
1	5	.63 N-m-s	.17 N-m-s	2.5 N-m-s
3	0	.23 N-m-s	.15 N-m-s	
3	5	1.9 N-m-s	.51 N-m-s	
1	0*	.12 N-m-s	.09 N-m-s	
1	5*	.67 N-m-s	.21 N-m-s	
3	0*	.34 N-m-s	.26 N-m-s	
3	5*	2.0 N-m-s	.62 N-m-s	

* If the maximum cyclic variation in momentum is set to .08 (see Figures A-12 and A-13) rather than .0434 N-m-s.

The HR01 model in Table A-5 is a possible candidate, if dumping sessions are frequent, the satellite is allowed to tumble following launch vehicle separation, and the

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magnetic torquers are used to initially acquire attitude. A wheel with larger capacity is however required, of the order of 4 N-m-s momentum storage capability, if the reaction wheels are to absorb all the momentum imparted during launch vehicle separation (assuming a margin factor of 3).

A.6 Sizing of the Magnetic Torquers

Magnetorquers are selected based on the moment that they can generate, the power (or current) they require, and their size, mass, reliability (or number of coils) and residual dipole. Sample catalog technical data for Ithaco torque rods are shown in Table A-6. Torque rods, when used in conjunction with reaction wheels, must provide a moment which is a function of the secular disturbance torques, the available geomagnetic field, and the desired frequency and duration of dumping sessions.

The peak geomagnetic field value in a polar orbit is approximated by $2M/R^3$, where $M = 7.96 \times 10^{15} \text{ tesla-m}^3$ (Larson and Wertz, 1992, Table 11-9a). In our model, this equates to approximately 4.7×10^{-5} tesla, only available twice per orbit. Assuming that momentum dumping lasts t_d and takes place over the poles, that k orbits have elapsed since the last dumping session, and that the intensity of the geomagnetic field during the session is at its peak throughout momentum unloading, the constant torque required to dump the accumulated momentum due to secular torques is approximated by $T = k |\Delta H_{\text{secular max}}| / t_d$, where $|\Delta H_{\text{secular max}}|$ was estimated earlier for both in-plane scanning and staring operations. The required dipole from the torque rod is then $D = (\text{Margin Factor})T/B$, where $B = 2M/R^3$, and the margin factor accounts for potentially

larger disturbance torques and variation in the magnitude of the total geomagnetic field.

A few estimates are given in the table below:

Margin Factor	Number of Complete Orbits Since the Last Momentum Dumping (k)	Duration of Dump (t_d) (minutes)	Minimum Dipole Required -- In-plane Scanning Operations	Minimum Dipole Required -- Staring Operations
1	1	15	$\approx 3.3 \text{ A-m}^2$	$\approx .7 \text{ A-m}^2$
1	5	15	$\approx 16.5 \text{ A-m}^2$	$\approx 3.5 \text{ A-m}^2$
3	1	15	$\approx 10 \text{ A-m}^2$	$\approx 2.1 \text{ A-m}^2$
3	5	15	$\approx 49 \text{ A-m}^2$	$\approx 10.5 \text{ A-m}^2$

Possible candidates are listed in Table A-6. Operational requirements will however constrain the duration of dumping, and may lead to the use of larger rods.

A.7 Conclusions

The main criteria in the selection of reaction wheels are minimum output torque and minimum angular capacity. The largest anticipated disturbance torque for Odin is associated with maneuvering, and the largest momentum build-up anticipated is following separation from the launch vehicle:

< Margin Factor > of 2

Order of Magnitude: $T_n \approx 27\text{mN-m}$

$$|\Delta H_{\max \text{ sep}}| \approx 2.6 \text{ N-m-s}$$

Secular components of environmental torques determine the size of the torque rods and the dumping strategy, and their magnitudes depend highly on the orbit and spacecraft attributes. Excluding magnetic effects, the largest secular components at 600 km are associated with aerodynamic effects during in-plane scanning. Again excluding magnetic effects, the largest secular components at 700 km are associated with gravity-gradient effects during staring events when the offset between the spacecraft Z-axis and the normal of the solar panels is at the maximum allowed angle.

At either altitude, the magnetic torque is the largest environmental torque experienced by the spacecraft, but can be measured and compensated for. Large secular effects are possible, especially during scanning operations (aeronomy), depending on the magnitude and orientation of the spacecraft magnetic dipole.

Critical torque conditions are anticipated at the start, end and mid-point of maneuvers. Although less critical at 700 km, shadowing effects result in large variations in the rate of change of external torques on the spacecraft. Finally, rapid changes in solar radiation torques occur during the eclipse season.

Notation	Origin and Orientation of Coordinate Frame
XYZ	<p>Origin is at the center of the active face of the solar array.</p> <p>Coordinate frame is right-handed orthogonal.</p> <p>Plane XY is parallel to the active face of the solar array.</p> <p>X is parallel to the boresight of the reflector projected onto the XY plane.</p> <p>Z is normal to the active face of the solar array.</p>
Xb'Yb'Zb'	<p>Origin is at the center of the active face of the solar array.</p> <p>The coordinate frame Xb'Yb'Zb', right-handed orthogonal, is rotated by -45° about the Z axis relative to the XYZ frame.</p>
XbYbZb	<p>The frame XbYbZb is parallel to Xb'Yb'Zb' but its origin is at the undeformed center of mass position.</p>
Xo'Yo'Zo'	<p>Origin is at the center of mass of the satellite model.</p> <p>Axes form a right-handed orthogonal system and do not rotate during the course of one orbit.</p> <p>Xo' points outwards along the orbit radius, towards the ascending node.</p> <p>Zo' is along the positive orbit normal.</p>

Table A-1. Systems of coordinates associated with the satellite model illustrated in Figure A-1.

Body	Cross-sectional or Body Shape	Identification / Mass / Area	Dimensions / Geometric Center (GC) in meters / Unit Normal ^a / Surface Properties (Ca, Cd, Cs) ^b
1		Four-panel solar array, each panel simulated as a rigid thin parallelepiped of uniform density Total Mass: 11.2 kg Total Area: 2.8 m ²	Dimensions of each panel: .64m x 1.095m x .01m GC: {0, 0, -.005} ¹ Surface density: 4kg/m ² Face exposed to Sun: Normal: {0, 0, 1} ¹ Ca = .805 and Cd = Cs = .0975
2		Four-panel Sun shield, each panel simulated as a rigid thin trapezium of uniform density Total mass: 7.1 kg Total area: 2.74 m ²	Dimensions of each trapezium: Parallel sides of .11m and 1.66 m distanced by .774 m, and with .005 m thickness GC: {0, 0, -.0025} ¹ Face exposed to Sun: Normal: {0, 0, 1} ¹ Ca = .2, Cd = 0, and Cs = .8
1-2		Bodies 1 and 2 combined to simplify moments and products of inertia computations Solar and Sun shield arrays simulated as a rigid thin disk, of uniform density, with a square hole Mass of thin disk without hole: 20.423 kg; Mass difference due to hole: 2.123 kg	Dimensions: Thin disk: 1.4 m radius, .01 thick Hollow rectangular parallelepiped: .8m x .8m x .01m

Table A-2. Characteristics of the 225 kg satellite model illustrated in Figure A-1 (a rigid six-body system approximation).

Notes (a) The location or orientation are expressed in XYZ coordinates which are defined in Table 1-1a. (b) Ca, Cd and Cs are the assumed coefficients of absorption, diffuse reflection and specular reflection, which are required for solar radiation torques computations.

Body	Cross-sectional or Body Shape	Identification/Mass/Area	Dimensions / Geometric Center ^a (GC) in meters / Unit Normal ^a / Surface Properties (Ca, Cd, Cs) ^b
3		Platform and top deck equipment, simulated as a rigid rectangular parallelepiped of uniform density Mass: 190.3 kg Volume .32 m ³	Dimensions: .8mx.8mx.5m GC: {0. 0. -.255} ^T Assumed surface properties of face exposed to Sun: Ca=.2, Cd=0. and Cs=.8
4		Reflector, approximated by a rigid thin disk of uniform density Mass: 6.3 kg	Dimensions: 1.1m dia, .03 m thick Orientation of boresight: {cos(28°) 0 sin(28°)} ^T GC: {0. 0. -1.} ^T
5		Rectangular support beam, assumed rigid and with uniform volumetric density of 8000 kg/m ³ Mass: 4.7 kg	Dimensions: .035mx.035mx.4775m GC: {-.3825 -.3825 -.74375} ^T

Table A-2. (continued)

Notes (a) The location or orientation are expressed in XYZ coordinates which are defined in Table 1-1a. (b) Ca, Cd and Cs are the assumed coefficients of absorption, diffuse reflection and specular reflection, which are required for solar radiation torques computations.

Body	Cross-sectional or Body Shape	Identification / Mass / Area	Dimensions / Geometric Center (GC) in meters / Unit Normal ^a
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6

Second rectangular beam to approximate the reflector support structure, assumed rigid with a uniform volumetric density of 8000 kg/m³
Mass: 5.4 kg

Dimensions: .035mx.035mx.56m
GC: {-.2 -.2 -.1}^T

Table A-2. (continued)

The satellite model is 225 kg, the center of mass^c is located at [-.0126 -.0126 -.2789] in Xb'Yb'Zb' coordinates and the inertia matrix, in XbYbZb coordinates, is:

$$I_b = \begin{bmatrix} 34.279 & - .7491 & - 1.4616 \\ - .7491 & 34.279 & - 1.4616 \\ - 1.4616 & - 1.4616 & 42.54 \end{bmatrix} \text{ (kg - m}^2 \text{)}$$

Notes (a) The location or orientation are expressed in XYZ coordinates which are defined in Table 1-1a. (b) Ca, Cd and Cs are the assumed coefficients of absorption, diffuse reflection and specular reflection, which are required for solar radiation torques computations. (c) Slightly different masses and geometric centers were used to compute the center of mass (a mass of 190.4 kg for body 3 with the GC at {0. 0. -.25}^T, and a mass of 4.6 kg for body 5, with the GC at {-.3825 -.3825 -.7325}^T).

Case	Description	Approximation
1	The velocity vector is parallel to the plane of the solar array.	<p>Surfaces exposed to drag at any time include one or two faces of the rectangular parallelepiped approximating the equipment box, and the back or front faces of the antenna.</p> <p>The drag associated with solar array thickness, thermal blanket thickness and antenna support beams is assumed to be negligible.</p>
2	The velocity vector is facing the front of the solar array.	<p>The solar and Sun shield arrays are assumed to be circular (Table A-2, body 1-2).</p> <p>Surfaces exposed to drag include the front face of the solar and thermal blanket arrays, the front face of the antenna, the front face of the rectangular box and one or two side faces partially shaded by the solar and Sun shield arrays.</p> <p>The area of each exposed side face of the equipment box is reduced by $l_1 \cdot l_2 \cdot \tan(\text{phy})$ where l_1 is the width of one of the four candidate surfaces (.8 meters) and $l_2 \cdot \tan(\text{phy})$ is the projected length of the array onto the box surface. l_2 is set to 1.0324 m, corresponding to the average length between cases when V is along Xb or Yb and when V is along X or Y.</p> <p>The center of pressure of the partially shaded box surfaces is shifted by $-.5 \cdot l_2 \cdot \tan(\text{phy})$ along Z from the geometric center of the surface.</p>

Table A-3. Effective Area and Center of Pressure of Surfaces Exposed to Drag.

Case	Description	Approximation
3	<p>The velocity vector is facing the back of the solar array.</p>	<p>The solar and Sun shield arrays are assumed to be circular (Table A-2, body 1-2).</p> <p>Surfaces exposed to drag include the back face of the solar and thermal blanket array (partially shaded), the back face of the antenna, the back and one or two side faces of the rectangular box.</p> <p>Up to $\phi_{vy} = 25$ deg, the equipment box shades about $.98 \text{ m}^2$ of the solar and blanket arrays, if the projected velocity vector is along X_b or Y_b. The center of pressure is then shifted by $.1263 \text{ m}$ in the direction of the projected velocity vector onto the XY plane. To simplify calculations, the above is assumed to hold for $\phi_{vy} \leq 32$ degrees and for any orientation of the projected velocity vector onto the XY plane.</p> <p>The back face of the equipment box (i.e., opposite to the solar array) is automatically shaded by the antenna. The net area exposed (A_d) is a function of the orientation of the velocity vector, and the center of pressure (C_p) is shifted in the direction of V.</p>

Table A-3. (continued)

Table A-4. Worst-case Estimates of Environmental Torques for the Satellite Model Illustrated in Figure A-1 and for a 600 km, Circular, and Sun-Synchronous Orbit (Ascending Node Crossing at 1800 Local Time).
The scaling required to resolve estimates for a 700 km, circular and Sun-synchronous orbit is included under column 1 in brackets.

Disturbance Torque	Model - In-plane Scanning Operations - Order of Magnitude of worst-case torque (T_w) and worst-case average (T_{av}) over one orbit	Model - Staring operations - Order of magnitude of worst-case torque (T_w) and worst-case average (T_{av}) over one orbit	Worst-case estimates based on simplified equations in Larson and Wertz (1992, Table 11-9a)
(1) Aerodynamic (Scale by .103 for a 700 km orbit)	Nearly constant $T_w \approx T_{av} \approx 2.4 \times 10^{-5}$ N-m	Mainly Cyclic $T_w \approx 2.4 \times 10^{-5}$ N-m (Sun angle of zero) $T_{av} \approx 1 \times 10^{-6}$ N-m (Sun incidence angle of 32°)	Scanning: 1.35×10^{-5} N-m ($A = 1.4 \text{ m}^2$, $l_{cp} = .2 \text{ m}$) Staring: 6.32×10^{-5} N-m ($A = 6.54 \text{ m}^2$, $l_{cp} = .2 \text{ m}$)
(2) Solar radiation	Mainly cyclic $T_w \approx 2.2 \times 10^{-6}$ N-m $T_{av} \approx 7 \times 10^{-7}$ N-m (summer solstice)	Constant $T_w = T_{av} \approx 2.2 \times 10^{-6}$ N-m (32° Sun incidence angle)	Staring: 9×10^{-6} N-m (25 deg solar incidence angle, $q = .6$, $l_{cp} = .2 \text{ m}$, $A = 6.17 \text{ m}^2$)
(3) Gravity-gradient (Scale by .96 for a 700 km orbit)	Nearly constant $T_w \approx T_{av} \approx 3 \times 10^{-6}$ N-m	Mainly cyclic $T_w \approx 1.1 \times 10^{-5}$ N-m and $T_{av} \approx 5 \times 10^{-6}$ N-m (e.g. 32° between XbYbZb and Xo'Yo'Zo' and Xo')	
(4) Magnetic (Scale by .96 for a 700 km orbit and by a factor of 2 for a dipole moment of 2 A-m^2)	Mainly cyclic $T_w \approx 8 \times 10^{-5}$ N-m $T_{av} \approx 2 \times 10^{-5}$ N-m	Mainly cyclic $T_w \approx 7 \times 10^{-5}$ N-m $T_{av} \approx 8 \times 10^{-6}$ N-m	Mainly cyclic Worst case: 4.7×10^{-3} N-m for $R = 6.971 \times 10^3 \text{ m}$, $D = 1 \text{ A-m}^2$ (spacecraft moment)
Sum of (1), (2) and (3)		$T_w \approx 7 \times 10^{-5}$ N-m and $T_{av} \approx 5 \times 10^{-6}$ N-m (e.g. 32° offset between XbYbZb and Xo'Yo'Zo' about Xo')	

Company	Model (Speed Range)	Output Torque	Angular Momentum	Mass	Other data
Ithaco	T-Reaction Wheel (0- +/-6000 rpm)	.02 N-m	1.3N-m-s (at 2000 rpm) 4N-m-s (at 6000 rpm)	3.6 kg (including electronics)	DC motor
Honeywell	HR02 (3500rpm)	.07 N-m	3.1 N-m-s	3.2 kg	DC motor 215.9 mm outer dia 96.5 mm high No integral electronics -24 to 61°C op. temp. range R4 bearing size
Honeywell	HR01 (2500rpm)	.046 N-m	1.9 N-m-s	2.3 kg	AC motor 235 mm outer dia 119 mm high No integral electronics -20 to 55°C op. temp. range R4 bearing size

Table A-5. Sample Characteristics of Reaction Wheels (extracted from Honeywell (1991) and Ithaco (1990) catalog technical data)

Characteristics	TR30CFR	TR60CFR
Linear Moment (min) (A-m ²)	35	60
Saturation Moment (min) (A-m ²)	40	70
Residual Moment (max) (A-m ²)	.2	.7
Linear Voltage (nominal) (V)	24.0	10.3
Saturation Voltage (nominal) (V)	28.0	12.6
Resistance of a single coil at 25°C (ohms)	132	40
Scale Factor of a single coil (A-m ² /mA)	.19	.25
Mass (including mounting blocks) (kg)	.95	1.7
Length (m)	.5	.64
Diameter (excluding mounting feet) (cm)	2.3	2.6
Number of Coils	2	2

(Ithaco, 1991)

Table A-6. Sample Torque Rod Characteristics (extracted from Itacho catalog data)

Figure A-1 Satellite model used to quantify disturbance torques.

(see Table A-1 and A-2 for details on coordinate frames and model characteristics)

Figure A-2. Parameters Defining the Shape of an Elliptical Orbit (reproduced from Wertz, 1978, Fig. 3-5). The shape of an orbit is usually defined by two parameters: the semimajor axis a , and the eccentricity e which is equal to the ratio c/a .

Figure A-3. Parameters Defining the Orientation of the Orbit Relative to Inertial Space (reproduced from Wertz, 1978, Fig. 3-7). The ascending node is the point in the orbit where the satellite crosses the equatorial plane from South to North. The right ascension of the ascending node (Ω) is measured eastward in the equatorial plane between the reference vector to Aries (direction of Sun at vernal equinox) and the vector from the center of the Earth to the ascending node. The inclination (i) is the rotation angle between the orbital and equatorial planes, about the ascending node vector. The angle of perigee (ω) is the angle measured in the orbital plane between the ascending node vector and the vector towards the perifocus, about the positive orbit normal.

Figure A-4. Location of the Satellite in the Orbit Plane (reproduced from Wertz, 1978, Fig. 3-8). E is the eccentric anomaly and v is the true anomaly. The classical set of parameters defining the position in inertial space of a satellite in an elliptical orbit at epoch time T_0 is $\{a, e, i, \Omega, \omega, p_6\}$, where the sixth parameter is either the time of perigee crossing, the eccentric anomaly or the true anomaly. For a circular orbit, the eccentricity is zero and ω is not defined. One alternate set of parameters is $\{R, i, \Omega, p_4\}$ defined at epoch T_0 , where R is the orbit radius and the fourth parameter is either the time of equatorial crossing at the ascending node or the anomaly measured from the ascending node.

Figure A-5. Orientation Relative to Inertial Space, over One Sidereal Year, of a 600 km Circular Sun-synchronous Orbit, with an Ascending Node at 1800h Local Time. Ω is the right ascension of the ascending node of the orbit, measured eastward in the equatorial plane between the vector to Aries and the vector to the ascending node (relative to the geocenter). The projection of Ω in the ecliptic plane is shown here, as well as the general location of the ascending node (which is the point in the orbit crossing the equatorial plane from South to North).

Figure A-6. Sun Angles, for a 600 Km Circular Sun-synchronous Orbit, with an Ascending Node at 1800h Local Time.

Figure A-7. Eclipse Season for a 600 Km Circular Sun-synchronous Orbit, with an Ascending Node at 1800h Local Time.

Figure A-8. Environmental torques with the spacecraft X-axis scanning between .3 and 2.3 degrees above the horizon. The scan axis is parallel to the vehicle Z-axis, and the XY vehicle plane is parallel to the orbit plane. T is the total torque, and T_{xb} , T_{yb} and T_{zb} are the components along the corresponding axes (defined in Table A-1). The Sun is at vernal equinox, and the Sun angle to the solar panels is 7.8 degrees. The spacecraft dipole was arbitrarily set to $[1. 0. 0.]$ A-m² in XbYbZb coordinates.

Figure A-9. Environmental torques with the spacecraft X-axis scanning between .3 and 2.3 degrees above the horizon. The scan axis is parallel to the vehicle Z-axis, and the XY vehicle plane is parallel to the orbit plane. T is the total torque, and T_{xb} , T_{yb} and T_{zb} are the components along the corresponding axes (defined in Table A-1). The Sun is at summer solstice, and the Sun angle to the solar panels is 31.25 degrees. The spacecraft dipole was arbitrarily set to $[1. 0. 0.]$ A-m² in XbYbZb coordinates.

Figure A-10. Environmental torques with the spacecraft axes $X_bY_bZ_b$ inertially fixed and offset from the non-rotating orbit axes $X_o'Y_o'Z_o'$ by -32 degrees about X_o' . No part of the orbit is eclipsed from the Sun and the spacecraft dipole is arbitrarily set to $[1. 0. 0.]$ A-m² in $X_bY_bZ_b$ coordinates.

Figure A-11. Environmental torques with the spacecraft axes $X_bY_bZ_b$ inertially fixed and aligned with the non-rotating orbit axes $X_o'Y_o'Z_o'$. Part of the orbit is eclipsed from the Sun and the spacecraft dipole is arbitrarily set to [1. 0. 0.] A-m² in $X_bY_bZ_b$ coordinates..

Figure A-12. Sum of the Environmental Torques Shown in Figure A-10 and the Resulting Momentum Accumulation. The spacecraft axes $X_bY_bZ_b$ are inertially fixed and offset by -32 degrees from the non-rotating orbit axes $X_o'Y_o'Z_o'$ about X_o' . H is assumed to be zero at the start of the integration.

Figure A-13. Sum of the Environmental Torques Shown in Figure A-11 and the Resulting Momentum Accumulation. The spacecraft axes $X_b Y_b Z_b$ are aligned with the non-rotating orbit axes $X_o' Y_o' Z_o'$. H is assumed to be zero at the start of the integration.

APPENDIX B

Simulink Block Diagrams for the Orbit Generation and Open-Loop Simulation Routines

Included here are the Simulink block diagrams of the orbit generation and open-loop simulation models.

The orbit model is used to generate the flight quaternions, Sun and magnetic field data associated with the test orbits described in Chapter 5. These parameters, stored at fixed time intervals in a workspace matrix variable, are saved in a data file and retrieved prior to open or closed-loop simulation runs. The associated equations are given in Chapter 3, section 3.2.1.

Open-loop simulations are run for each test orbit and flight profile type to determine the bias conditions required to avoid zero-speed crossings. Results are given in Chapter 5. Details and equations associated with the open-loop models are given in Chapter 3, section 3.2.2.

(a)

Block G0. The outputs are orbit parameters at time $t_s = t_{start} + t$ (t_{start} is defined in the workspace and t is the system time)
The orbit generation algorithm includes first order secular variations, and computes parameters at time t_s , based on ephemeris data at epoch t_0 .

(b)

Figure B-1. Orbit Generation Model. *Orbigen3* is a user-defined Matlab function, incorporated in the model using the Simulink built-in *Matlab fcn* block. Associated equations and details are given in Chapter 3, section 3.2.1.

(a)

(b)

Figure B-2. Open-Loop Simulation Model. This model simulates the spacecraft rotational dynamics, assuming perfect tracking of the reference inputs. Similar models were created for the other types of flight profiles. Blocks A.1, A.2 and A.5 are blown up in Appendix C.

APPENDIX C

Simulink Block Diagrams of the Closed-Loop Simulation Models

Included here are the Simulink block diagrams of the models used to simulate the controlled dynamics of the spacecraft and actuators. Details and equations associated with the closed-loop simulation models are included in Chapter 3, section 3.2.3 and Chapter 4. The principal models and versions which have been created are summarized in Chapter 3, Table 3-1.

(a)

H.2

(b)

H.3

(c)

H.4

(d)

H.5

(e)

Figure C-1. High-Level Diagrams for Closed-Loop Models 1-5, Version E. (Only the differing blocks are shown for models 2-5.)

Figure C-2. High-Level Diagram for Closed-Loop Model 1, Version A. (See Table 3-1 for details)

(a)

Figure C-3. Diagrams for Blocks A0 and A, which Include the Dynamics of the Spacecraft and Actuators. The dynamics of the torque rods are excluded from block A0 since it is used in the versions where desaturation activity is not modelled.

Block A.1 Embedded sub-system representing the module for
 Double click here for more info

- (c) (*qprod fo* is a fortran MEX file, created from the skeleton file *usersys.F* and implementing Equation 3-8 in Chapter 3)

Block A.2 Modelled environment / forces along control axes
 Double click here for more info

(d)

Figure C-3. (continued)

Block A.2.1 Estimate of gravity-gradient torque

(e) (*tgg_fo.F* is a modified version of *usersys.F* and implements Eq. A-7)

Block A.2.2 Estimate of torque due to solar radiation pressure

(f) (*tsrad_fo.F* is a modified version of *usersys.F* and implements Eq. A-4 discretized)

Block A.2.3 Estimate of aerodynamic torque

(g) (*taero_p.f* is a modified version of *usersys2.f* and implements Eq. A-2 discretized)

Block A.2.4 Estimate of magnetic torque

(h) (*tmag_fo.F* is a modified version of *usersys.F* and implements Eq. A-8 discretized)

Figure C-3. (continued)

(i)

(j)

Figure C-3. (continued)

Block A.3.1.1 Retained magnetization in enabled coil 1

(k)

Block A.4 Elimination of rotation
Double-click here for more info

- (l) ($qdot_fo.F$ is a modified version of $usersys.F$ and implements the inverted form of Eq.3-9)

Figure C-3. (continued)

- (m) (*cross_fo.F* is a modified version of *usersys.F* and implements the cross-product operation)

- (n) (*hadot3p.f* is a modified version of *usersys2.F* and implements Eqs. 3-11, 3-12 and D-6)

Figure C-3. (continued)

(a)

(b) (*euler312_fo.F* and *q312_fo.F* are modified versions of *usersys.F*, which implement the conversions between quaternion and 3-1-2 Euler angle sequence representations.)

Figure C-4. Diagrams for Block B, which Simulates Sensor Measurements

(c)

(d)

Figure C-4. (continued)

(a) (used in Version A - see Table 3-1)

Figure C-5. Principal Versions of the Controller Block. ((a) Version A, i.e. no desaturation activity, no delays, no quantization effects, continuous integration to generate the momentum demand (b) Version B, i.e. version A including desaturation activity (c) Version E, i.e. no desaturation but controller digitization, delays, quantization effects, noise included)

(b) (used in version B, see Table 3-1)

(c) (used in version L, see Table 3-1)

Figure C-5. (continued)

(a)

(b)

Figure C-6. Desaturation-Related Blocks

Block C.1.1.1B

(c) (*triggerf3.f* is a modified version of *usersys2.f*)

Block C.1.1.2 Stop condition no 1 for each wheel is when target speed has been met

(d) (*stopdes3.F* is a modified version of *usersys.F*)

Block C.1.1.3B Stop condition no 2 for each wheel is the time elapsed

(e)

Figure C-6.(continued)

Block C.1.1.3.1B Desaturation counter for enabled wheel no 1
(same for the two other enabled wheels, except for the initial conditions in the unit delay)

(f)

Block C.1.2 Computation of the constant torque required during 5 minutes to desaturate each wheel
($f(u)$ is a function of the model assumed for the reaction wheels)

(g) ($f(u)$ is given by Eq. 4-43)

Figure C-6.(continued)

Block C.1.3 Maximum applied torque of .01 N-m

(h)

Block C.1.4

- (i) (*autodd3x.f* is a modified version of *usersys2.f* and implements Eqs. 4-44 to 4-46 and 6-7)

Figure C-6. (continued)

Figure C-7. Diagram of Block C.2, which Computes the Errors into the Controller. (Errors computed according to Eq. 4-2. *qprod_{fo}.F* and *euler312.F* are modified versions of *usersys.F* and their compiled versions are accessed via S-function blocks to carry out quaternion multiplication and conversion from quaternion to Euler angle representation.)

(a)

(b)

Figure C-8. Diagrams for Blocks C.3, C.3x, C.3d and C.4, Associated with the Generation of the Wheel Momentum Demand and the Control of the Wheel Speed (or Momentum).

(c) (including the desaturation torque command into Eq. 4-31b)

Figure C-8. (continued)

Block C.3.1

(d) (*hc1f.F* is a modified version of *usersys.F* and implements Eq. 4-31a)

Block C.3.2 with continuous integration to generate the momentum demand

(e) (*dhc2f_3.f* is a modified version of *usersys2.f* and implements Eq. 4-31b)

Block C.3.2 x

(f) (*dhc.f*, modified version of *usersys2.f*, implements Eq. 4-29)

Figure C-8. (continued)

Block C.3.2d (with continuous integration to generate the momentum demand and with T_{desat} fed forward to be integrated as well)

(g) (Including the computed desaturation torque (T_{desat}) into the integrand of Eq. 4-29)

Figure C-8. (continued)

Block C.3.2: Wheel momentum distribution algorithm
 Double click here for more info

- (h) (If only three wheels are active, each matrix A_{cm} corresponding to a given wheel combination can be stored, and the momentum along each spin axis is given by $A_{cm}^{-1} h_c$, where h_c is in control coordinates)

Block C.3.3x: Wheel momentum distribution algorithm
 with three of four wheels active
 Double click here for more info

- (i) (If the dynamics of all four wheels are included, with all four wheels potentially active, then the momentum distribution algorithm could be implemented as Eq. 4-41 with λ set to the value of momentum demanded from the inactive wheel.)

Figure C-8. (continued)

Block C.4 Speed control loop around the wheels
Double click here for more info

(j)

Block C.4 Speed control loop around the wheels
(with monitoring of the saturated momentum demand to
feedback to the integrator in block C.3.2)
Double click here for more info

(k)

Figure C-8. (continued)

Block C.5 Delay and quantization due to transmission via the telemetry channels

(a)

Block C.6 Delay and quantization effects on measured wheel speeds as a result of transmission via the telemetry channels

(b)

Figure C-9. Blocks Included in the Controller to Account for Transport Delay and Quantization Effects on the Sensor and Actuator Data Sent through the Telemetry Steam

Block C.7 Delay and quantization of commanded dipoles due to transmission via the telemetry channel

(c)

Block C.8 Delay and quantization effects on torque commands as a result of transmission via the telemetry channels

(d)

Block C.10 Delay effect on measured attitude as a result of transmission via the telemetry channels (quantization effect handled in block B.1)

(e)

Figure C-9. (continued)

Figure C-10. Block D Diagram, Used to Input Disturbances into the System.

(a)

(b)

Figure C-11. Diagrams Associated with Block F, Used to Input Noise into the System.

(c)

(d)

Figure C-11. (continued)

(a)

(b)

Figure C-12. Diagrams Associated with Block G, Used to Retrieve Previously Computed Orbital Parameters for the Test Orbits and Profiles.

Block H.1: SAC reference attitude and angular velocity
Scenario 1: Initialization
 Double click here for more info

(a)

Block H.2: SAC reference attitude and angular velocity
Scenario 2: Initial attitude with calibration
Slow (continuous) switching
 Double click here for more info

(b)

Figure C-13. Diagrams Associated with Blocks H.1 to H.5, Used to Generate the Different Flight Profiles. (See Chapter 5, section 5.1 for details)

Block H.2: B/C-reference attitude and angular velocity
 Scenario 2: Inertial starting with calibration
 slows (non-instantaneous switching)
 Double click here for more info

(c)

Block H.2.1

(d)

Figure C-13. (continued)

Block H.2.1.1s

(c)

Block H.2.1.1.1

(f)

Figure C-13. (continued)

Block H.2.1.2

(g)

(h)

Figure C-13.(continued)

(i)

(j)

Figure C-13.(continued)

(k)

(l)

Figure C-13. (continued)

Figure C-14. Block I, Used to Store Selected Simulation Outputs

(a)

Figure C-15. Diagrams Associated with Block J, Used to Keep Track of the Simulated Attitude and Angular Velocity Errors and the Variations in Wheel Speed.

Block J.1

(b)

Block J.2

(c)

Figure C-15. (continued)

APPENDIX P

Net Torque on Reaction Wheels at Standstill

When the wheel is initially at standstill, the net torque responsible for the acceleration of the wheel relative to the platform must be greater than the breakaway torque for the wheel to move. In other words, the breakaway torque is the torque required to overcome all friction effects when the wheel is at standstill. In the simulation, for absolute wheel speed values smaller than 1×10^{-6} rad/sec, the net torque is computed to verify if it is indeed greater than the breakaway torque. If not, the wheel speed is set to zero. The net torque responsible for the acceleration of the wheel relative to the vehicle is derived below.

Let

A_{cm}	=	3x4 transformation matrix defining the orientation of the spin axis of each wheel, relative to the control frame
h_s	=	4-element constructed column matrix, each element corresponding to the angular momentum of one wheel, in the direction of its spin axis, relative to the spacecraft ($h_s = [h_{s1} \ h_{s2} \ h_{s3} \ h_{s4}]^T$)
h_{sn}	=	Angular momentum of wheel n, in the direction of its spin axis, relative to the spacecraft
H_t	=	Total spacecraft momentum in control coordinates
J_c	=	Composite inertia matrix (of the spacecraft and wheels), resolved in the control frame

J_m	=	Diagonal matrix of wheel inertias about their spin axis
l_s	=	4-element constructed column matrix, each element corresponding to the absolute angular momentum of one wheel, in the direction of its spin axis ($l_s = [l_{s1} \ l_{s2} \ l_{s3} \ l_{s4}]^T$)
l_{sn}	=	Absolute momentum of wheel n, along its spin axis
s_n	=	Direction of the spin axis of wheel n in control coordinates
T_{ext}	=	Total external torques in control coordinates
T_m'	=	Matrix of applied motor torques
T_{mn}'	=	Motor torque applied to wheel n
T_i	=	Matrix of internal disturbance torques about the spin axis of the wheels
T_{in}	=	Internal disturbance about the spin axis of wheel n
T_f	=	Matrix of friction torques.
T_{fn}	=	Friction torque for wheel n and includes viscous friction (T_m), kinetic or Coulomb friction (T_{cn}) and static friction or stiction (T_{sn}) components
ω	=	Spacecraft angular velocity in control coordinates
ω_s	=	4-element constructed column matrix of wheel speeds relative to the spacecraft
ω_{cn}	=	Speed of wheel n

The total momentum of the spacecraft and the absolute momentum of the wheels are given by:

$$\begin{aligned} H_t &= J_c \omega + A_{cm} J_m \omega_s \\ l_s &= J_m [\omega_s + A_{cm}^T \omega] = h_s + J_m A_{cm}^T \omega \end{aligned} \quad (D-1)$$

Euler's equation for the spacecraft, in matrix form, is:

$$T_{ext} = \dot{H}_t + \omega^x H_t = J_c \dot{\omega} + A_{cm} \dot{h}_s + \omega^x H_t \quad (D-2)$$

The dynamics of wheel n along the spin axis are given as:

$$\dot{l}_{sn} + (\omega^x l_n)^T s_n = \dot{l}_{sn} = T_{mn}' - T_{in} - T_{fn} \quad (D-3)$$

where the second term is zero as a result of wheel symmetry (see Equation 4-13 in

Chapter 4) and where, according to the reaction wheel model described in Chapter 3,

$$\begin{aligned}
 T_{mn} &\leq T_{\max} && \text{for } |\omega_{sn}| \leq \omega_x \\
 T_{mn} &\leq T_{\max} - (|\omega_{sn}| - \omega_x) \frac{k_e k_t}{R_a} && \text{for } |\omega_{sn}| > \omega_x \\
 T'_{mn} &= \beta T_{mn} [1 + A_r \sin(f_r \theta_{sn})] \\
 T_{fn} &= T_{vn} + T_{cn} + T_{sn} \\
 T_{vn} &= B_n \omega_{sn} \\
 T_{cn} &= \text{sgn}(\omega_{sn}) k_{cf} \\
 T_{sn} &= \text{sgn}(\omega_{sn}) k_{sf} e^k \text{ with } k = -\frac{|\omega_{sn}|}{K_{dc}}
 \end{aligned} \tag{D-4}$$

In matrix form, the dynamics of the four wheels are written as:

$$\dot{h}_s = T'_m - T_i - T_f = \dot{h}_s + J_m A_{cm}^T \dot{\omega} \tag{D-5}$$

Combining Equations D-2 and D-5 yields:

$$[1 - J_m A_{cm}^T J_c^{-1} A_{cm}]^{-1} [T'_m - T_i - T_f - J_m A_{cm}^T J_c^{-1} \dot{H}_i] = \dot{h}_s \tag{D-6}$$

This is a system of four linear equations with four unknowns, where the unknown variables are the friction torques of the wheels at standstill, with the corresponding $\dot{h}_{sn}$ set to zero, or the wheel accelerations relative to the platform for the wheels at nonzero speeds, with the corresponding friction torques set according to Equation D-4. The friction torques resolved for the wheels at standstill are limited to the breakaway torque value, and if larger, indicate that the net torque responsible for relative acceleration of the wheel is nonzero.

APPENDIX E

Odin Project

Included here is a two-page summary which was produced by the Swedish Space Corporation in the early phases of the project and which outlines the main aspects of Odin and its mission.

Main Scientific themes

Odin will open up sections of the electromagnetic spectrum, around wavelengths of 0.5 mm and 3 mm, normally closed to the scientist and will provide access to emission lines from important molecules such as water vapour, molecular oxygen, ozone and carbon monoxide. Complementary information for atmospheric studies will come from spectral lines at UV and optical wavelengths.

Astrophysics

The main objective is to perform detailed studies of the physics and the chemistry of the interstellar medium by observing key species. The following classes of objects will be main targets for observations:

- *Giant molecular clouds and nearby dark clouds:* Improve our understanding of the chemistry and the cooling process of the interstellar medium and thereby the conditions for star formation in particular by measuring lines from oxygen and water molecules. Detection of protostars.
- *Comets:* Studies of the physics of outgassing of water, the size of active regions and density estimates both in short and long period comets.
- *Planets:* Height distribution of trace elements in the atmospheres of Jupiter and Saturn. Detection of minor constituents brought up by convection would provide information not only on the dynamics but also on the physics and chemistry of the deep atmospheres.
- *Circumstellar envelopes:* Determination of mass loss rates.
- *Nearby galaxies:* Estimates of star formation activity from observations of CO and H₂O.

Aeronomy

This research will address scientific problem areas in the stratosphere and mesosphere by making measurements of various trace species. The scientific goals can be summarised as follows:

- *Stratospheric ozone science:* To elucidate the geographical extent of and mechanisms responsible for ozone depletion in the "ozone hole" region and to study dilution effects and possible heterogeneous chemistry even outside of the polar regions due to sulphate aerosols
- *Mesospheric ozone science:* To establish the relative role of odd hydrogen chemistry and the effects of ordered and turbulent transport and corpuscular radiation.
- *Summer mesospheric science:* To establish the variability of mesospheric water vapour including as assessment of the required fluxes for aerosol formation in the polar mesosphere.
- *Coupling of atmospheric regions:* To study some of the mechanisms that provide coupling between the upper and lower atmosphere, eg downward transport of aurorally enhanced NO with its effects on ozone photochemistry and the vertical exchange of minor species such as odd oxygen, CO and H₂O.

Spectral lines

The radiometer package covers transitions of aeronomical interest from the following molecules:

ClO, CO, NO₂, N₂O, H₂O₂, HO₂, H₂O, H₂¹⁸O, NO, N₂O, HNO₃, O₃ and O₂

and atomic and molecular transitions of astrophysical interest from:

Cl, H₂¹⁸O, H₂O, H₂S, NH₃, H₂CO, O₂, CS, ¹³CO, H₂CS, SO, SO₂

The optical spectrometer is aimed at detecting the following species in the Earth atmosphere:

Aerosols, ClO, O₃, O₂, O₄, NO and NO₂

Operating modes

The Odin spacecraft is designed to serve both astrophysics and aeronomy. However, the spacecraft has to be manoeuvred in different ways. For aeronomy the spacecraft follows the Earth limb - scanning the atmosphere up and down from 15 to 120 km at a rate of up to 40 scans per orbit. When observing astronomical sources Odin is stabilised pointing towards the object for up to 60 minutes after which the Earth momentarily blocks the view.

The Spacecraft

The drawing shows Odin with the Sun to the left illuminating four solar panels. In between these are shields to protect the upwards pointing antenna from the sun. Star trackers and payload elements are mounted on top of the upper platform. Subsystems are mounted on both platforms.

<i>Type</i>	3-axis stabilized with momentum wheels, star trackers and gyros
<i>Mass</i>	235 kg (160 kg bus and 75 kg payload)
<i>Size</i>	Width: 1.1 m, height: 1.8 m
<i>Power</i>	260 W from deployable fixed Si-arrays
<i>Pointing</i>	± 15 arcsec in staring mode ± 1.2 arcmin scanning (reconstructed)
<i>Datalink</i>	500 kbps to Esrange tracking station
<i>Storage</i>	80 Mbit in solid state memory
<i>Launch</i>	1996 on a Pegasus vehicle
<i>Orbit</i>	circular sunsynchronous, 600 km altitude $i = 97.8^\circ$ with ascending node at 18:00
<i>Lifetime</i>	2 years minimum

The Payload

Antenna

<i>Type</i>	Offset Cassegrain
<i>Diameter</i>	1.1 metre
<i>Surface</i>	10 μ m rms
<i>Material</i>	CFRP skins on honeycomb

<i>Sensitivity</i>	1 K in 1MHz with S/N=5 after 15 min
<i>Mixers</i>	cooled Schottky mixers
<i>LO</i>	local oscillators based on Gunn diodes followed by frequency multipliers
<i>LNA</i>	cooled HEMT low noise amplifiers
<i>Spectrometer</i>	1000 channel hybrid autocorrelator

Radiometer

The radiometer package has one receiver at a wavelength of 3 mm and four in the submm band ($\lambda = 0.5$ mm).

<i>Type</i>	Single sideband heterodyne receivers
<i>Frequencies</i>	119 GHz, 422 GHz, 488 GHz, 553 GHz and 575 GHz
<i>Coverage</i>	15-20 GHz in each submm band
<i>Bandwidth</i>	100 MHz to 1 GHz
<i>Resolution</i>	0.1 MHz to 1 MHz

Optical spectrometer

This instrument has four wavelength bands and views the limb through optics separate but aligned with the submm antenna.

<i>Type</i>	Grating spectrometers
<i>Wavelengths</i>	200 - 800 nm and 1270 nm
<i>Resolution</i>	optical: 1 nm, IR: 10 nm
<i>Aperture</i>	10 cm ²
<i>FOV</i>	0.02°x2°

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