**SUBTHRESHOLD CMOS LOGIC DESIGN USING PARALLEL TRANSISTOR
STACKS**

by

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ABSTRACT

The inverse-narrow-width effect in MOSFETs lowers the threshold voltage at narrower widths. The exponential dependence of the subthreshold current on the threshold voltage can result in higher than expected current at smaller widths. A design methodology that exploits this behavior is developed. A subthreshold optimum width that maximizes the current-to-capacitance ratio can be found. Using transistors of only this size either individually or in parallel stacks, as appropriate, leads to faster circuits in subthreshold. To design a CMOS digital logic circuit, a parallel transistor ratio is found that describes the ratio of PMOS-to-NMOS transistors to use. This is combined with the optimum subthreshold widths to determine an effective beta ratio, or effective PMOS-to-NMOS width ratio, to use with sizing strategies such as Logical Effort. Delay reductions of up to 64.5 % have been achieved in post-layout simulations with this design methodology compared to standard superthreshold designs.

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TABLE OF SYMBOLS

Symbol	Definition
α	Activity factor
b	Branching effort
β	Beta ratio, or PMOS-to-NMOS width ratio
β_{eff}	Effective beta ratio
β_{opt}	Optimum beta ratio
C	Capacitance
C_{bottom}	Diffusion bottom-plate capacitance
C_{DB}	Drain-to-body diffusion capacitance
C_{diff}	Diffusion capacitance
C_{eff}	Effective capacitance
C_G	Gate capacitance
C_{GC}	Gate-to-channel capacitance
C_{GDO}	Drain overlap capacitance
C_{GSO}	Source overlap capacitance
C_j	Junction capacitance per unit area
C_{jsw}	Junction capacitance per unit perimeter
C_L	Load capacitance
C_o	Overlap capacitance per unit width
C_{ox}	Gate oxide capacitance per unit area
C_{SB}	Source-to-body diffusion capacitance
C_{sw}	Diffusion side-wall capacitance
E	Energy
f	(i) Frequency; (ii) stage effort
f_{max}	Maximum frequency
g	Logical effort
h	Electrical effort, or fanout
GND	Ground
I	Current
I_{avg}	Average current
I_D	Drain current
I_{DS0}	Current at threshold
$I_{leakage}$	Leakage current
i_{peak}	Supply voltage maximum current
I_{static}	Static current
i_{supply}	Supply voltage instantaneous current
L	Transistor length
λ_d	Drain-induced barrier lowering coefficient
L_S	Junction side-wall length

m	Empirical parameter
N	Number of gates in ring oscillator
n	Process-dependent term describing depletion region characteristics
p	(i) instantaneous power; (ii) parasitic delay
P_{avg}	Average power
$P_{dynamic}$	Dynamic power
P_{peak}	Peak power
P_{static}	Static power
R	Resistance
R_{eq}	Equivalent resistance
R_{eqn}	NMOS equivalent resistance
R_{eqp}	PMOS equivalent resistance
T	(i) Period of oscillation; (ii) time interval
t	Time
τ	RC time constant
t_f	Fall time
t_p	Propagation delay
t_{pHL}	High-to-low propagation delay
t_{pLH}	Low-to-high propagation delay
t_r	Rise time
V_{DD}	Supply voltage
V_{DS}	Drain-to-source voltage
V_{GS}	Gate-to-source voltage
v_{out}	Output voltage
V_{supply}	Supply voltage
V_{swing}	Logic low to logic high voltage range
V_T	Threshold voltage
v_T	Thermal voltage
W	Transistor width
W_{eff}	Effective width
W_n	NMOS width
$W_{n,opt}$	Optimum NMOS width
W_p	PMOS width
$W_{p,opt}$	Optimum PMOS width
x_d	Lateral diffusion

TABLE OF ABBREVIATIONS

Abbreviations	Definition
AC	Alternating current
CMOS	Complementary metal-oxide-semiconductor
DIBL	Drain-induced barrier lowering
EDP	Energy-delay product
GP	General purpose transistor variant
INV	Logic gate that implements negation
INWE	Inverse-narrow-width effect
LOCOS	Local oxidation of silicon
LP	Low-power transistor variant
MOSFET	Metal-oxide-semiconductor field-effect transistor
NAND	Logic gate that implements negated AND
NAND2	2-input logic gate that implements negated AND
NAND3	3-input logic gate that implements negated AND
NMOS	n-channel MOSFET
NOR	Logic gate that implements negated OR
NOR2	2-input logic gate that implements negated OR
NOR3	3-input logic gate that implements negated OR
NWE	Narrow-width effect
PDP	Power-delay product
PG	Propagate and generate
PLS	Post-layout simulations
PMOS	p-channel MOSFET
PTR	Parallel transistor ratio
STI	Shallow trench isolation
VLSI	Very-large-scale integration

CHAPTER 1 – INTRODUCTION

Low-power design is an important concept for today's digital circuits. The driving motivator may be different but the goals of lower power dissipation and energy consumption are the same. A desktop computer, for example, is plugged into an AC outlet and so there is no worry about running out of power. However, much effort is placed into the cooling of components to keep them reliable in the heat generated through ever-increasing power dissipation. Laptop computers, smartphones and other portable devices add the need for increased battery life to avoid being tethered to one location for frequent charging. Increasing the battery life can provide a competitive advantage in the marketplace.

In this chapter, the motivations for pursuing a design methodology for subthreshold CMOS logic design are discussed. Also, the objectives of the thesis are presented alongside an outline of how the thesis is organized to meet these objectives.

1.1 Motivation

Certain applications, such as battery-operated wireless and medical devices, stress the importance of reduced power dissipation over high speed. Digital subthreshold circuit design has shown promise for low-power applications. In these circuits, the parasitic subthreshold leakage current of superthreshold circuits is used as the primary operation current [1]. It has been demonstrated that the minimum energy operation for CMOS circuits typically occurs in the subthreshold regime [2].

While researching the performance of asynchronous circuits [3, 4] in subthreshold, an interesting property of the transistor's subthreshold current was noted: the current was not a linear function of the transistor width. In fact, it appeared that the current would decrease with width as expected, but then increase once again at the narrowest widths. It was later observed that the specific current-width relationship would vary between PMOS and NMOS transistors and between technologies. Regardless, it was decided to investigate how to best take advantage of this anomaly.

The primary source of this anomaly is the inverse-narrow-width effect (INWE), the lowering of the threshold voltage as the width of the transistor narrows [5]. This translates into an increase in the subthreshold current due to its exponential dependence on the threshold voltage. Whether a transistor experiences INWE or its opposite, narrow-width effect (NWE), where the threshold voltage increases with decreasing widths, depends on the method of device isolation (e.g., local oxidation of silicon (LOCOS), shallow trench isolation (STI)) and the p- or n-type dopant implants [6-9]. The effects of different process steps such as additional implants, varied device structures and oxidation methods have also been investigated in regards to the narrow-width effects [10-12].

While analytical expressions for the threshold voltage that account for the narrow-width effects have been derived, they may prove unwieldy for subthreshold digital design. Also, access to specific process details for a given technology kit may be unavailable. Thus this thesis only demonstrates a design methodology based on the transistor performance that takes advantage of the INWE present in deep submicron technology kits.

1.2 Objectives

The objectives of this thesis are summarized below.

- Understand the transistor subthreshold behavior, in particular with regard to the currents and capacitances.
- Determine how to exploit the subthreshold behavior to maximize performance by reducing the propagation delay of CMOS digital logic circuits.
- Create a design methodology that maximizes subthreshold CMOS logic design performance.
- Demonstrate the effectiveness of the proposed design methodology through the use of application circuits.

1.3 Organization

The early chapters detail the relevant theory with which to formulate a design methodology. Chapter 2 focuses on the properties of the transistor such as current and how it is affected by the threshold voltage. A qualitative description of the cause of the narrow-width effects is provided. Leakage currents and the relevant capacitances are also examined. In Chapter 3, important VLSI concepts are discussed. These include propagation delay, power dissipation and energy consumption, as well as other quality metrics such as the power-delay product and the energy-delay product. A description of ring oscillator circuits follows, and the chapter concludes with a brief discussion of the use of Logical Effort as a gate sizing strategy to reduce delay.

For Chapter 4, the simulation results of actual technology kits are presented and a design methodology is developed. Starting from the transistor properties and their current-to-capacitance ratio, the simulations advance to ring oscillators which validate the optimum widths of transistors to use in subthreshold. The concept of the parallel transistor ratio is introduced. From the subthreshold optimum widths and the parallel transistor ratio, an effective beta ratio, or effective PMOS-to-NMOS width ratio, can be found for use in sizing strategies such as Logical Effort. Simulations of transistor stacks reveal the ineffectiveness of transistor scaling to replicate the current of a single transistor, reinforcing the usage of parallel stacks. Finally, all the steps of the design methodology are summarized.

In Chapter 5, the performance of a circuit designed with the custom methodology is compared to that of a standard design. Results from a minimum-sized circuit are also included. The circuits used for the comparison are a custom test circuit and a 32-bit carry-lookahead adder.

Chapter 6 summarizes the conclusions of this thesis and suggests possible future directions in which to expand upon this work.

CHAPTER 2 – MOS TRANSISTOR PROPERTIES

In this chapter, the relevant transistor properties such as current and capacitance are detailed, a qualitative explanation of the inverse-narrow-width effect is given, and sources of leakage are discussed.

2.1 Currents

Since operation in the subthreshold regime is the focus of this thesis, the reader is encouraged to read [13] and [15] for details on the transistor superthreshold drain current. While typically undesirable in superthreshold operation, subthreshold current is the name given to the drain current that occurs in subthreshold, and is discussed in the next section. The junction leakage and gate leakage currents are also discussed.

2.1.1 Subthreshold Current

When the gate-to-source voltage (V_{GS}) is less than the threshold voltage (V_T), current between the drain and source of a MOSFET is still possible due to weak-inversion conduction. An expression for this subthreshold current is given by

$$I_D = I_{DS0} e^{\frac{V_{GS}-V_T}{n v_T}} \left(1 - e^{\frac{-V_{DS}}{v_T}} \right) \quad (1)$$

where I_{DS0} is the current at the threshold that depends on process and device geometry, n is a process-dependent term describing the depletion region characteristics with a value typically in the range of 1.4 to 1.5, v_T is the thermal voltage and V_{DS} is the drain-to-source voltage [13].

Equation (1) shows the exponential dependence of the subthreshold current on the threshold voltage. Contrast this to the superthreshold current where the dependence on the threshold voltage is between linear and square depending on the severity of the carrier velocity saturation [14]. In superthreshold operation, the subthreshold current is called leakage and is undesirable. Here, it is the primary operation current [1, 13].

2.1.2 Junction Leakage

The p-n junctions between the source and drain diffusions and the substrate or well form diodes. Another diode is formed between a well and the substrate. The wells and substrate are biased to V_{DD} or GND to ensure a reverse-bias. Yet reverse-biased diodes still conduct a small amount of current referred to as junction leakage [13]. The low-power transistor variant in modern technology kits reduces this leakage.

2.1.3 Gate Leakage

Through a process called tunneling, charge carriers can pass through the gate oxide. Thinner oxides exponentially increase the opportunity for tunneling to occur [13]. As with the junction leakage, the low-power transistor variants reduce the gate leakage current.

2.1.4 Current Comparison

Tables 1 and 2 compare the drain, junction leakage and gate leakage currents that occur within a typical 65 nm process with general purpose (GP) and low-power (LP)

transistor variants at 1 V and 0.3 V. At both voltages, the junction and gate leakage currents are several orders of magnitude below the drain current thus supporting the use of the subthreshold drain current as the primary operation current. Also worth noting is that in superthreshold, the GP transistors conduct approximately twice the current of the LP variants, whereas in subthreshold, they conduct at least 30 times the current. This is an indication of how the low-power variant is effective at reducing the subthreshold current which in typical superthreshold applications is undesired.

Table 1: 65 nm, $V_{DD} = 1$ V; currents for 0.12 μm width transistors

Current	PMOS GP [μA]	NMOS GP [μA]	PMOS LP [μA]	NMOS LP [μA]
Drain	6.0×10^1	1.1×10^2	3.1×10^1	5.3×10^1
Junction leakage	4.0×10^{-7}	3.8×10^{-5}	6.0×10^{-9}	1.2×10^{-5}
Gate leakage	3.7×10^{-4}	4.6×10^{-4}	7.6×10^{-8}	2.7×10^{-7}

Table 2: 65 nm, $V_{DD} = 0.3$ V; currents for 0.12 μm width transistors

Current	PMOS GP [μA]	NMOS GP [μA]	PMOS LP [μA]	NMOS LP [μA]
Drain	2.0	1.2	1.9×10^{-2}	3.6×10^{-2}
Junction leakage	2.3×10^{-9}	1.2×10^{-7}	7.3×10^{-11}	1.9×10^{-11}
Gate leakage	1.8×10^{-5}	8.2×10^{-6}	3.9×10^{-9}	2.4×10^{-9}

2.2 Capacitances

The gate and diffusion capacitances, discussed in the next two sections, are the capacitances relevant to digital logic design.

2.2.1 Gate Capacitance

The gate capacitance (C_G) consists of three components: the gate-to-channel capacitance (C_{GC}) and the two overlap capacitances (C_{GSO} for the source and C_{GDO} for the drain). The gate-to-channel capacitance can be divided into three components that change in magnitude with the terminal voltages or operation region. These are the gate-to-source, gate-to-drain and gate-to-body capacitances. In superthreshold, the values of these component capacitances are often estimated for the operation region to ease analysis. If considering the gate capacitance as a load, these components may not matter since only the lumped total value will matter. The value of the gate capacitance does not vary much between the operation regions. If operating in subthreshold, this variation is further reduced.

The gate-to-channel capacitance can be approximated as

$$C_{GC} = C_{ox}WL \quad (2)$$

where C_{ox} is the gate oxide capacitance per unit area, W is the width and L is the length of the channel. The other components of the gate capacitance, the overlap capacitances, are only a function of the structure of the transistor. They are caused by the extension of the source and drain diffusions under the gate. The polysilicon gate and the source or drain form the plates of the capacitor while the thin oxide is the dielectric. They are expressed as

$$C_{GSO} = C_{GDO} = C_{ox}x_dW = C_oW \quad (3)$$

where x_d is the length of the extension of the source or drain under the gate oxide. Since x_d is process dependent, the overlap capacitance is often expressed in terms of the overlap capacitance per unit width C_o . Summing Equations (2) and (3), the gate capacitance can be estimated as [15]

$$C_G = C_{GC} + C_{GSO} + C_{GDO} = C_{ox}WL + 2C_oW \quad (4)$$

From Equation (4), it can be seen that the gate capacitance scales with transistor width.

2.2.2 Diffusion Capacitance

The diffusions for the source and drain within the substrate form reverse-biased p-n junctions which contribute the C_{SB} (source-to-body) and the C_{DB} (drain-to-body) parasitic capacitances, respectively. The depletion region width increases with the reverse-bias voltage, thus the capacitance is maximized when the source or drain and body are at the same voltage. Since the diffusion sits as a tub within the substrate, its capacitance is expressed as the sum of two junction capacitances: bottom-plate area and side-wall perimeter. The bottom-plate capacitance is given as

$$C_{bottom} = C_jWL_S \quad (5)$$

where C_j is the junction capacitance per unit area, W is the diffusion width (same as transistor width) and L_S is the side-wall length. The side-wall capacitance is

$$C_{sw} = C_{jsw}(W + 2L_S) \quad (6)$$

where C_{jsw} is the capacitance per unit perimeter. Note that for a first-order analysis, the diffusion-to-channel capacitance is ignored.

The total diffusion capacitance is then [15]

$$C_{diff} = C_{bottom} + C_{sw} = C_j W L_S + C_{jsw} (W + 2L_S) \quad (7)$$

When transistors are in series, it is common to share the diffusion areas. The drain of one transistor is merged with the source of another. Depending on whether a contact is required, the side-wall length may vary [13]. Similar to the gate capacitance, the diffusion capacitance scales almost linearly with the width.

2.3 Narrow-Width Effects

The inverse-narrow-width effect (INWE) is the reduction of the threshold voltage as the width of a transistor narrows. Conversely, the narrow-width effect (NWE) increases the threshold voltage as the transistor width narrows. As seen in Section 2.1.1, this could have a large effect on the subthreshold current because of its exponential dependence on the threshold voltage. Only a qualitative description of the narrow-width effects will be provided here since the subthreshold design methodology of this thesis does not depend on the exact mechanism or modeling of the INWE.

The predominant cause of the narrow-width effects is the type of device isolation used. Due to the limitations of the fabrication process, the polysilicon gate of a MOSFET extends beyond the channel width on both sides. This excess polysilicon overlaps the field oxide, creating a fringing capacitance. In the case of a transistor where shallow trench isolation (STI) is used, the fringing electric field lines terminate on charge located within the device channel, as shown in Figure 1(a). Thus the total gate-to-channel

capacitance consists of the typical parallel plate thin oxide capacitance plus the fringing capacitance. As the transistor width narrows, the fringing capacitance becomes more dominant in proportion to the parallel plate capacitance. Consequently, at narrower widths, the creation of an inversion layer is facilitated at a lower applied gate voltage, resulting in the INWE.

The fringing electric field lines of a transistor where local oxidation of silicon (LOCOS) is used terminate on charge located in the substrate, as seen in Figure 1(b). Also, the “bird’s beak” encroachment of the silicon dioxide (SiO_2) effectively reduces the designed width of the transistor, leading to a reduction in the gate-to-channel capacitance. At narrower widths, the proportion of charge accumulated in the channel is reduced compared to the charge accumulated in the substrate, thus necessitating a higher gate voltage to produce an inversion layer. This is termed the NWE [5, 8].

Figure 1: Cross section of a MOSFET fabricated with (a) shallow trench isolation and (b) local oxidation of silicon

CHAPTER 3 – VLSI CONCEPTS

Metrics and methods for characterizing and evaluating the performance of CMOS circuits such as propagation delay and power dissipation are discussed in this chapter. The relevant properties of ring oscillator circuits are detailed and Logical Effort, a delay estimation and circuit optimization technique, is also reviewed.

3.1 Propagation Delay

The propagation delay t_p defines how quickly a logic gate responds to a change at its input(s). It is measured as the time interval between the 50 % transition points of the input and output waveforms. Since a gate's response varies for rising and falling signals, two definitions of the propagation delay are required. t_{pHL} represents the propagation delay when the output transitions from high-to-low, while t_{pLH} covers the low-to-high transition. The propagation delay is then the average of the two:

$$t_p = \frac{t_{pLH} + t_{pHL}}{2} \quad (8)$$

In order to estimate the propagation delay, digital circuits are typically modeled as first-order RC networks. The transistor current is modeled by a constant linear resistance whose value is averaged over the interval of interest. The load capacitance is also modeled as a constant factor over this interval. While only diffusion and gate capacitances are typically considered for hand calculations, in reality, interconnect capacitances will also affect the performance.

If we assume a step input going from 0 to V is applied to a first-order RC network, the transient response is given by

$$v_{out}(t) = (1 - e^{-t/\tau})V \quad (9)$$

where $\tau = RC$, the time constant of the network.

From Equation (9), the time for the output to reach 50 % of its final value can be determined as

$$t = \ln(2)RC = 0.69RC \quad (10)$$

For digital circuits, a first-order estimate of the propagation delay is simply a matter of replacing R with the equivalent resistance R_{eq} of the transistor(s) and C with the load capacitance C_L in Equation (10). In most cases, the PMOS pull-up network and the NMOS pull-down network of a gate will have different equivalent resistances, represented by R_{eqp} and R_{eqn} , used to calculate t_{pLH} and t_{pHL} , respectively. Equation (8) can then be used to estimate the propagation delay.

Another equation for the propagation delay derived from the definition of current is

$$t_p = \frac{C_L(V_{swing}/2)}{I_{avg}} \quad (11)$$

where V_{swing} is the voltage range from logic low to logic high and I_{avg} is the average current over the period of interest [15].

3.2 Rise and Fall Times

Rise and fall times are properties of individual signals rather than gates. In the previous section, the estimate for propagation delay assumed a step input signal. The actual delay will be longer for inputs with lower slopes. Rise and fall times are used to quantify the transition rate of a signal. To avoid any uncertainty over where a transition begins and ends, the rise and fall times use the 10 % and 90 % points of a waveform. Thus the rise time, t_r , is the time for a signal to reach the 90 % point from the 10 % point in a rising transition, while the fall time, t_f , is the time for a 90 % to 10 % signal transition in a falling transition [15].

3.3 Power Dissipation

The peak power dissipation is important to consider when sizing supply lines. If the instantaneous power is $p(t)$, the peak power, P_{peak} , is defined as

$$P_{peak} = i_{peak}V_{supply} = \max[p(t)] \quad (12)$$

where V_{supply} is the supply voltage and i_{peak} is the maximum current drawn from it.

More important is the average power, P_{avg} , since this relates directly to battery and cooling requirements. It is defined as

$$P_{avg} = \frac{1}{T} \int_0^T p(t)dt = \frac{V_{supply}}{T} \int_0^T i_{supply}(t)dt \quad (13)$$

where i_{supply} is the instantaneous current from the supply voltage and T is the interval of interest.

Power dissipation can be further broken down into dynamic and static components. The dynamic power dissipation occurs during transients, i.e., when gates are switching. It is caused by the charging of capacitors and temporary current paths between the supply rails. The dynamic power is proportional to the number of switching events. The dynamic power dissipation is

$$P_{dynamic} = \alpha \times f \times C_{eff} \times V_{DD}^2 \quad (14)$$

where α is the activity factor, f is the switching frequency, C_{eff} is the effective capacitance being charged and discharged and V_{DD} is the supply voltage. The static power dissipation occurs at all times, even when the circuit is in steady-state. It is caused by the permanent conductive paths between the supply rails and by leakage currents such as gate leakage and junction leakage, and is described by

$$P_{static} = I_{leakage} \times V_{DD} \quad (15)$$

Operation in subthreshold will clearly have a significant impact in reducing both dynamic and static power dissipation by lowering the supply voltage seen in Equations (14) and (15). Moreover, many of the leakage currents have a very strong dependence on the terminal voltages. For example, gate leakage current, with its strong dependence on the gate voltage, becomes nearly negligible in subthreshold [1, 15].

3.4 Energy Consumption

For a first-order RC network with a step input going from 0 to V , the total energy E delivered is [15]

$$E = CV^2 \quad (16)$$

For digital circuits, C will be the load capacitance C_L and V is the supply voltage V_{DD} . From Equation (14), more energy will be consumed the greater the number of switching events.

3.5 Power-Delay Product

The power-delay product (PDP) is a quality measure for a switching device. The unit for power dissipation is the watt, equivalent to joules per second, and time is the measurement for propagation delay. Thus their product will be expressed in joules, the unit of energy. The PDP is therefore defined as the energy consumed by the gate per switching event [15].

3.6 Energy-Delay Product

Ideally, logic gates should be fast and consume little energy. The energy-delay product (EDP) can be considered an important quality metric since it combines both of these elements. The EDP can be found by multiplying the PDP by the propagation delay, and will thus have units of joule-seconds [15]. When discussing the efficiency of different circuit implementations, EDP is a reasonable parameter to consider.

3.7 Ring Oscillator Circuits

Ring oscillators typically consist of an odd number of inverter gates in a closed loop, where the output of one gate is fed directly as the input of the next. The circuit will

continue to oscillate due to the odd number of inversions. The period of oscillation can be found as

$$T = 2 \times t_p \times N \quad (17)$$

where N is the number of inverters. Note the factor of two arises since a complete cycle requires both high-to-low and low-to-high transitions [15].

Ring oscillators can also be built from other gate types. For example, if using NAND gates, both inputs can be tied together or one of the inputs can be tied to V_{DD} , depending upon the purpose of the circuit.

3.8 Logical Effort

Logical Effort is a method for estimating the delay of a logic circuit, and more importantly, optimizing it. This refers to the sizing of the gates as well as potentially adding buffers to minimize the delay. Logical Effort makes use of two factors that are normalized to an inverter: the parasitic delay, p , and the logical effort, g . The parasitic delay describes the delay inherent to the gate itself with no load attached. Logical effort, when referring to the factor g , describes the increase in input capacitance of a gate to produce the same current as an inverter, or equivalently, the reduction in output current for a gate having the same input capacitance as an inverter. Three other factors are the electrical effort, or fanout, h , the stage effort, f , and the branching effort, b . While these factors apply to individual gates, the same factors are defined separately for a path, or a series of gates.

A typical application of Logical Effort is sizing the gates of a logic path with a known load and a fixed-sized driving gate. By using equations that relate the path factors to the gate factors, the sizes for the gates that reduce the delay can be found. Also, the optimum number of stages can be determined, thus prompting the addition of buffers if desired [13, 15].

CHAPTER 4 – DESIGN METHODOLOGY DEVELOPMENT

The subthreshold performance of transistors is process dependent and is unique to each technology kit to some degree. Variations in performance between PMOS and NMOS transistors are also prevalent. In this chapter, data from actual technology kits will be presented. A subthreshold design methodology that takes advantage of the particulars of a given technology kit will be developed.

The main technology kit in use, in particular for the applications of Chapter 5, is a 65 nm 1.2 V low-power (LP) process. Data from other kits such as a 65 nm 1 V general purpose (GP) process and a 90 nm 1.2 V technology will also be presented to demonstrate that while the specifics may change between kits, the method being developed can still be used to maximize the subthreshold performance of a given technology. Comparisons to superthreshold operation will also feature extensively to highlight the differences in performance at the two supply voltage ranges. It should be noted that the minimum transistor width for these three kits is 0.12 μm .

4.1 Subthreshold Optimum Width from Current-to-Capacitance Ratio

The transistors are configured such that the gate and drain are tied together and connected to the appropriate power supply for maximum current drive as shown in Figure 2. The current, threshold voltage and gate and source diffusion capacitances versus width of both the PMOS and NMOS 65 nm LP transistors at 1.2 V are plotted in Figure 3. The currents and capacitances appear to be nearly linear with width. The threshold voltage drops as the width narrows, consistent with the inverse-narrow-width effect.

Figure 2: Transistor configuration for maximum current drive for (a) PMOS and (b) NMOS

Figure 3: 65 nm LP, $V_{DD} = 1.2$ V; current (I_D), threshold voltage (V_T) and gate (C_G) and source (C_{SB}) diffusion capacitances versus width for (a) PMOS and (b) NMOS

The same plots but for a power supply voltage of 0.3 V are shown in Figure 4. While the actual values of the threshold voltages and the capacitances vary, they follow the same trend with the width as compared to the superthreshold voltage. The big change is in the current. As expected, the current has dropped significantly, here by over three orders of magnitude. The real surprise is the way in which the current appears to be higher than what would be expected at the narrow widths. While this can be seen in the NMOS transistor, the extreme case is the PMOS transistor where the lowest current does not occur at the narrowest width.

Figure 4: 65 nm LP, $V_{DD} = 0.3$ V; current (I_D), threshold voltage (V_T) and gate (C_G) and source (C_{SB}) diffusion capacitances versus width for (a) PMOS and (b) NMOS

The superthreshold and subthreshold plots for the 65 nm GP technology are shown in Figures 5 and 6. The subthreshold currents are only between one and two orders of magnitude smaller than the superthreshold case. Again, it is the PMOS transistor that displays the surprising current plot. In fact, at the narrowest widths, the NMOS current is weaker than that of the PMOS.

Figure 5: 65 nm GP, $V_{DD} = 1.0$ V; current (I_D), threshold voltage (V_T) and gate (C_G) and source (C_{SB}) diffusion capacitances versus width for (a) PMOS and (b) NMOS

Figure 6: 65 nm GP, $V_{DD} = 0.3$ V; current (I_D), threshold voltage (V_T) and gate (C_G) and source (C_{SB}) diffusion capacitances versus width for (a) PMOS and (b) NMOS

Figures 7 and 8 are plots for the 90 nm technology kit. The big change here is the PMOS transistor exhibiting narrow-width effect where the threshold voltage increases at the narrowest widths. It is the NMOS transistor that displays the upturn in current at the narrowest widths in subthreshold. Finally, the PMOS subthreshold current is much weaker compared to that of the NMOS.

Figure 7: 90 nm, $V_{DD} = 1.2$ V; current (I_D), threshold voltage (V_T) and gate (C_G) and source (C_{SD}) diffusion capacitances versus width for (a) PMOS and (b) NMOS

Figure 8: 90 nm, $V_{DD} = 0.3$ V; current (I_D), threshold voltage (V_T) and gate (C_G) and source (C_{SD}) diffusion capacitances versus width for (a) PMOS and (b) NMOS

As seen in Equation (11), a reduction in propagation delay can be achieved by either increasing the current of the relevant transistors or by reducing the load capacitance. Consistent with what is shown in [16], both goals can be achieved by using transistor widths that maximize the current-to-capacitance ratio. In Figure 9, the current-to-capacitance ratio has been plotted for both the PMOS and NMOS 65 nm LP transistors in superthreshold. For the capacitance, the sum of the gate and source diffusion capacitances has been used. The gate capacitance is representative of a typical load and

the diffusion capacitance is representative of the self-loading that occurs in logic gates. The source diffusion capacitance has been used since it represents the worst case diffusion capacitance. As seen in the plot, the ratio continues to increase as the width approaches 1 μm .

Figure 9: 65 nm LP, $V_{DD} = 1.2$ V; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for (a) PMOS and (b) NMOS

Figure 10 plots the ratio for subthreshold. It is clear that the ratio is maximized at the minimum width of $0.12\ \mu\text{m}$ for both transistor types.

Figure 10: 65 nm LP, $V_{DD} = 0.3\ \text{V}$; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for (a) PMOS and (b) NMOS

Figures 11 and 12 plot the 65 nm GP superthreshold and subthreshold current-to-capacitance ratios. They are similar to the 65 nm LP case, except that the optimum width in subthreshold for the NMOS transistor is slightly larger than minimum at $0.15\text{ }\mu\text{m}$.

Figure 11: 65 nm GP, $V_{DD} = 1.0\text{ V}$; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for (a) PMOS and (b) NMOS

Figure 12: 65 nm GP, $V_{DD} = 0.3$ V; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for (a) PMOS and (b) NMOS

The 90 nm superthreshold current-to-capacitance ratios are shown in Figure 13. Again, they do not reach a maximum at less than 1 μm . Figure 14 shows how the 90 nm kit differs from both 65 nm kits as the PMOS transistor does not appear to have a subthreshold optimum width at less than 1 μm . The NMOS transistor is optimized at the minimum width of 0.12 μm .

In the next section, ring oscillator circuits will be used for confirming the subthreshold optimum widths found in a circuit context where both PMOS and NMOS transistors are employed together. Also, it will lead to finding a suitable width to use for the 90 nm PMOS transistor.

Figure 13: 90 nm, $V_{DD} = 1.2$ V; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for (a) PMOS and (b) NMOS

Figure 14: 90 nm, $V_{DD} = 0.3$ V; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for (a) PMOS and (b) NMOS

4.2 Subthreshold Optimum Width from Ring Oscillator Circuit

Ring oscillator circuits are the standard circuits used for comparing the performance of different technologies or circuit styles. They allow for a comparison that isn't affected by parameters such as fan-in and fan-out [15]. From Equation (11), the propagation delay can be seen to be a function of the transistor current and the load capacitance. Both the current and the load capacitance, composed of the gate and drain diffusion capacitances shown in Equations (4) and (7), scale almost linearly with the width. Thus, ideally, if the transistor widths of the inverters are scaled in a ring

oscillator, any increase in current will be balanced by an increase in capacitance, resulting in identical propagation delays.

The reality is that there are higher order effects that will lead to widths that maximize the oscillation frequency. In superthreshold, the variation in performance between different widths may not be very large. However, given the subthreshold current-to-capacitance ratio results from the preceding section, it would be expected that the highest frequencies are obtained when using the widths that maximized the ratio.

It is not necessary for the inverters in the ring oscillator to have the same width for the PMOS and NMOS transistors. In fact, for a symmetric response in an inverter's output, the PMOS transistor will typically be two to three times wider than the NMOS to compensate for the reduced mobility of holes compared to electrons. The ratio of width between the PMOS and NMOS transistors is called the beta ratio. But if symmetry is not required, a better performance may be achieved from a smaller beta ratio [15].

In order to find the transistor widths that optimize the performance, the oscillation frequency of a ring oscillator circuit consisting of nine inverters is simulated. In Figure 15, the oscillation frequency is plotted against the PMOS width for a fixed NMOS width of $0.12\text{ }\mu\text{m}$ for the 65 nm LP technology at 1.2 V. There are two local maximums, with the highest frequency of approximately 4.03 GHz occurring at a PMOS width of $0.15\text{ }\mu\text{m}$. Figure 16 shows the frequency as a function of the PMOS and NMOS widths. The frequency falls away as the difference in width between the two transistor types grows.

Figure 15: 65 nm LP, $V_{DD} = 1.2$ V; 9-inverter ring oscillator frequency versus PMOS width for 0.12 μm NMOS width

Figure 16: 65 nm LP, $V_{DD} = 1.2$ V; 9-inverter ring oscillator frequency as a function of PMOS and NMOS width

The maximum frequencies for each NMOS width simulated are summarized in Table 3. The corresponding PMOS widths and beta ratios are included as well. The widest NMOS transistor simulated was 1.08 μm , which is the width at which the highest frequency occurred for the data obtained. Had wider transistors been simulated, marginally higher frequencies may have been obtained. Regardless, working within the data presented, the drop-off in performance from the highest frequency obtained to the lowest is 13.9 %.

Table 3: 65 nm LP, $V_{DD} = 1.2$ V; 9-inverter ring oscillator maximum frequencies for varying NMOS widths

W_n [μm]	f_{max} [GHz]	$W_{p,\text{opt}}$ [μm]	β_{opt}
0.12	4.028	0.15	1.25
0.13	4.039	0.15	1.15
0.14	4.042	0.15	1.07
0.15	4.040	0.15	1.00
0.18	3.990	0.31	1.72
0.21	4.040	0.35	1.67
0.24	4.080	0.39	1.63
0.36	4.179	0.55	1.53
0.48	4.337	0.73	1.52
0.60	4.443	0.90	1.50
0.72	4.514	1.08	1.50
0.84	4.565	1.26	1.50
0.96	4.603	1.43	1.49
1.08	4.633	1.61	1.49

The subthreshold 65 nm LP oscillation frequency for a fixed 0.12 μm NMOS width is shown in Figure 17. The frequency monotonically decreases with increasing PMOS width. Also shown in Figure 18 is the plot of frequency for a fixed NMOS width

of $1.08\ \mu\text{m}$. It can be seen that even with such a wide NMOS transistor, the highest frequency occurs for a minimum-sized PMOS, although the presence of a second maximum around a width of $1.5\ \mu\text{m}$ is evident. In Figure 19, the frequency is plotted as a function of PMOS and NMOS width. The frequency is maximized when both transistors are sized at the minimum width of $0.12\ \mu\text{m}$, and falls away as the widths are increased.

Figure 17: 65 nm LP, $V_{DD} = 0.3\text{ V}$; 9-inverter ring oscillator frequency versus PMOS width for $0.12\ \mu\text{m}$ NMOS width

Figure 18: 65 nm LP, $V_{DD} = 0.3$ V; 9-inverter ring oscillator frequency versus PMOS width for 1.08 μm NMOS width

Figure 19: 65 nm LP, $V_{DD} = 0.3$ V; 9-inverter ring oscillator frequency as a function of PMOS and NMOS width

Table 4 summarizes the subthreshold results. The best performance occurs when both transistors are at the minimum width of 0.12 μm . This matches the subthreshold optimum widths found in the preceding section from the current-to-capacitance ratio when each transistor type was considered individually. Although the subthreshold optimum widths are in agreement here, should a discrepancy between the two methods result, priority should be given to the ring oscillator results since they are obtained in a method that is more in tune with a circuit design context.

Table 4: 65 nm LP, $V_{DD} = 0.3$ V; 9-inverter ring oscillator maximum frequencies for varying NMOS widths

W_n [μm]	f_{max} [MHz]	$W_{p,\text{opt}}$ [μm]	β_{opt}
0.12	4.012	0.12	1.00
0.13	3.917	0.12	0.92
0.14	3.827	0.12	0.86
0.15	3.740	0.12	0.80
0.18	3.343	0.12	0.67
0.21	3.139	0.12	0.57
0.24	2.965	0.12	0.50
0.36	2.458	0.12	0.33
0.48	2.201	0.12	0.25
0.60	1.994	0.12	0.20
0.72	1.821	0.12	0.17
0.84	1.667	0.12	0.14
0.96	1.555	0.12	0.13
1.08	1.450	0.12	0.11

The drop-off from the highest to lowest frequency within the dataset of Table 4 is 63.9 % compared to 13.9 % of the superthreshold case. Thus it is evident that in subthreshold, there may be a huge penalty in performance if the correct widths are not

used. Coupled with the knowledge that larger transistor sizes result in more energy consumption due to the increase in capacitance, it becomes clear that in the 65 nm LP technology kit, there is no justification for using larger transistors.

Post-layout simulations (PLS) were also performed to verify that the subthreshold optimum widths obtained from the schematic simulations are also valid when all the parasitics of a real circuit are considered. A sample 9-inverter ring oscillator layout is shown in Figure 20. Table 5 contains the post-layout simulation oscillation frequencies as well as those from schematic simulations. The post-layout simulation frequencies range from 38.0 to 49.2 % slower compared to those of the schematic simulations, and the results generally follow the same trends. Most importantly, the subthreshold optimum width of $0.12\ \mu\text{m}$ for both PMOS and NMOS transistors is confirmed.

Figure 20: 9-inverter ring oscillator layout where each inverter consists of one $0.15\ \mu\text{m}$ width PMOS and one $0.12\ \mu\text{m}$ width NMOS

Table 5: 65 nm LP, $V_{DD} = 0.3$ V; 9-inverter ring oscillator schematic and post-layout simulation frequencies

W_p [μm]	W_n [μm]	f – schematic [MHz]	f – PLS [MHz]
0.12	0.12	4.012	2.355
0.12	0.15	3.740	2.227
0.12	0.18	3.343	2.071
0.12	0.21	3.139	1.908
0.15	0.12	3.441	1.748
0.15	0.15	3.221	1.664
0.18	0.12	2.977	1.709
0.18	0.18	2.546	1.524
0.21	0.12	2.780	1.670
0.21	0.21	2.277	1.389

In Table 6, the 65 nm GP superthreshold maximum frequencies are summarized. For NMOS widths between 0.12 and 1.08 μm , the best width is 0.84 μm . There is a 9.0 % drop-off in performance from the highest to lowest frequency. Table 7 contains the subthreshold results. As with the current-to-capacitance ratios, the subthreshold optimum widths are 0.12 μm for PMOS and 0.15 μm for NMOS. The performance drop-off is 47.5 % from highest to lowest frequency.

Table 6: 65 nm GP, $V_{DD} = 1$ V; 9-inverter ring oscillator maximum frequencies for varying NMOS widths

W_n [μm]	f_{max} [GHz]	$W_{p,\text{opt}}$ [μm]	β_{opt}
0.12	7.445	0.15	1.25
0.13	7.513	0.15	1.15
0.14	7.561	0.15	1.07
0.15	7.593	0.15	1.00
0.18	7.500	0.30	1.67
0.21	7.624	0.34	1.62
0.24	7.715	0.38	1.58
0.36	7.913	0.54	1.50
0.48	8.006	0.70	1.46
0.60	8.122	0.86	1.43
0.72	8.176	0.95	1.32
0.84	8.180	1.10	1.31
0.96	8.172	1.25	1.30
1.08	8.152	1.39	1.29

Table 7: 65 nm GP, $V_{DD} = 0.3$ V; 9-inverter ring oscillator maximum frequencies for varying NMOS widths

W_n [μm]	f_{max} [MHz]	$W_{p,\text{opt}}$ [μm]	β_{opt}
0.12	315.8	0.12	1.00
0.13	322.1	0.12	0.92
0.14	326.5	0.12	0.86
0.15	329.6	0.12	0.80
0.16	315.9	0.12	0.75
0.18	315.6	0.12	0.67
0.21	311.6	0.12	0.57
0.24	305.2	0.12	0.50
0.36	274.4	0.12	0.33
0.48	247.5	0.12	0.25
0.60	229.0	0.12	0.20
0.72	212.1	0.12	0.17
0.84	197.3	0.12	0.14
0.96	184.4	0.12	0.13
1.08	173.0	0.12	0.11

For the 90 nm superthreshold results, Table 8 shows that the highest frequency for the widths simulated occurs at an NMOS width of 1.08 μm . The drop-off from highest to lowest frequency is 19.8 %. Table 9 contains the subthreshold frequencies. Recall from the preceding section that the subthreshold optimum width for the NMOS transistor was determined to be 0.12 μm , while no apparent optimum width for the PMOS was found. However, with these ring oscillator results, a PMOS width of 0.41 μm can be found that pairs with the 0.12 μm NMOS transistor to give the best performance. The drop-off from highest to lowest frequency is only 18.5 %, similar to the superthreshold scenario. It is clear that unlike the two 65 nm technologies, any gains obtained from exploiting the inverse-narrow-width effect will not be as substantial in the 90 nm technology since only the NMOS transistor has this property.

Table 8: 90 nm, $V_{DD} = 1.2$ V; 9-inverter ring oscillator maximum frequencies for varying NMOS widths

W_n [μm]	f_{max} [GHz]	$W_{p,\text{opt}}$ [μm]	β_{opt}
0.12	5.388	0.24	2.00
0.13	5.438	0.25	1.92
0.14	5.484	0.27	1.93
0.15	5.527	0.28	1.87
0.18	5.639	0.32	1.78
0.21	5.728	0.36	1.71
0.24	5.813	0.40	1.67
0.36	5.969	0.57	1.58
0.48	6.158	0.92	1.92
0.60	6.318	1.08	1.80
0.72	6.433	1.23	1.71
0.84	6.519	1.39	1.65
0.96	6.627	1.55	1.61
1.08	6.717	1.71	1.58

Table 9: 90 nm, $V_{DD} = 0.3$ V; 9-inverter ring oscillator maximum frequencies for varying NMOS widths

W_n [μm]	f_{max} [MHz]	$W_{p,\text{opt}}$ [μm]	β_{opt}
0.12	70.73	0.41	3.42
0.13	68.40	0.41	3.15
0.14	66.58	0.42	3.00
0.15	65.11	0.42	2.80
0.18	62.12	0.44	2.44
0.21	60.42	0.47	2.24
0.24	59.52	0.49	2.04
0.36	57.68	0.62	1.72
0.48	58.93	0.95	1.98
0.60	59.99	1.08	1.80
0.72	60.90	1.22	1.69
0.84	61.67	1.36	1.62
0.96	63.30	1.51	1.57
1.08	64.74	1.68	1.56

The optimum widths for all technologies in both superthreshold and subthreshold are summarized in Table 10.

Table 10: Ring oscillator optimum widths

Technology	Superthreshold		Subthreshold	
	PMOS [μm]	NMOS [μm]	PMOS [μm]	NMOS [μm]
65 nm LP	1.61	1.08	0.12	0.12
65 nm GP	1.10	0.84	0.12	0.15
90 nm	1.71	1.08	0.41	0.12

4.3 Parallel Transistor Ratio

The subthreshold optimum widths have been found in the previous two sections. For the ring oscillator simulations, the inverters have consisted of one PMOS and one NMOS transistor. While this may be satisfactory in most cases, there will be times when a larger gate will be required to drive a large load. It has been made clear that scaling the transistor widths is not a good solution in subthreshold due to the resulting dramatic decrease in current-to-capacitance ratio. Instead, we suggest that transistors at the subthreshold optimum width should be used in parallel.

From Table 10, the best inverter for the 65 nm LP technology in subthreshold consisted of both the PMOS and NMOS being sized at the minimum of 0.12 μm , thus having a beta ratio of 1. Now let us assume that an inverter of five times this size is required. Thus the inverter will have five PMOS transistors in parallel matched with five NMOS transistors in parallel. But is this the best solution?

A slight improvement can be made by finding a parallel transistor ratio, or the ratio of PMOS-to-NMOS transistors, to use. In Figure 21, ring oscillator frequencies for 65 nm LP 0.3 V operation are plotted again. This time, the width of the NMOS and PMOS transistors are fixed at 0.12 μm . Twenty NMOS transistors are placed in parallel (multiplicity of 20) while the PMOS multiplicity is swept. From the plot, it can be seen that 27 PMOS transistors gives the best frequency of 4.098 MHz. The resulting parallel transistor ratio is $27 / 20 = 1.35$. The increase in frequency from using a parallel transistor ratio of 1.35 instead of 1 is only 2.1 % at the cost of a $2.35 / 2 = 1.175$ times, or 17.5 %, increase in energy consumption.

Figure 21: 65 nm LP, $V_{DD} = 0.3$ V; 9-inverter ring oscillator frequency versus PMOS multiplicity for an NMOS multiplicity of 20

For the 65 nm GP kit, the optimum multiplicity of the PMOS transistor was 19 resulting in a frequency of 329.9 MHz. The parallel transistor ratio is thus 0.95 compared to the single transistor subthreshold optimum width ratio of 0.8. For the 90 nm kit, a frequency of 72.17 MHz was obtained with a PMOS multiplicity of 15, resulting in a parallel transistor ratio of 0.75 compared to the single transistor subthreshold optimum width ratio of 3.42.

4.4 Effective Beta Ratio

Many sizing strategies such as Logical Effort reference the total input width, or sum of the PMOS and NMOS widths for a given input, in their calculations. Thus to simplify calculations, the subthreshold optimum widths ($W_{p,opt}$ and $W_{n,opt}$) and the parallel transistor ratio (PTR) can be combined into an effective beta ratio (β_{eff}) as

$$\beta_{eff} = \frac{W_{p,opt}}{W_{n,opt}} \times PTR \quad (18)$$

The effective beta ratio can also be described as the ratio of the total PMOS width to the total NMOS width. Now when using the sizing strategies, by employing the effective beta ratio, there is no need to consider the subthreshold optimum widths or the number of parallel transistors. Simply follow the same procedures as superthreshold where a transistor in a logic gate can have any width, with the only constraint being the beta ratio between the PMOS and NMOS transistors.

Table 11 summarizes the subthreshold optimum widths, the parallel transistor ratio and the effective beta ratio for the three technologies.

Table 11: Subthreshold optimum widths, parallel transistor ratios and effective beta ratios

Technology	$W_{p,opt}$ [μm]	$W_{n,opt}$ [μm]	β	PTR	β_{eff}
65 nm LP	0.12	0.12	1	1.35	1.35
65 nm GP	0.12	0.15	0.8	0.95	0.76
90 nm	0.41	0.12	3.42	0.75	2.56

As an example, suppose that the total width of an inverter is calculated to be $3.5\ \mu\text{m}$ in the 90 nm technology. The PMOS portion of the width is $3.5\ \mu\text{m} \times (2.56 / 3.56) = 2.52\ \mu\text{m}$ while the NMOS portion is $3.5\ \mu\text{m} \times (1 / 3.56) = 0.98\ \mu\text{m}$. The number of PMOS transistors is calculated as $2.52\ \mu\text{m} / 0.41\ \mu\text{m} = 6$ while the number of NMOS is $0.98\ \mu\text{m} / 0.12\ \mu\text{m} = 8$. The 6 PMOS and 8 NMOS transistors satisfy the parallel transistor ratio of 0.75. This is illustrated in Figure 22.

Figure 22: Inverter split into parallel transistors

4.5 Transistor Stacks

Logic circuits require more than just inverters. Gates such as NAND and NOR require the series connection of transistors. Typically, the widths of two transistors in series, or in a double stack, are doubled, while in the case of a triple stack, the widths are tripled. This approximates the current drive of a single transistor, although the specific scaling factor required is a function of factors such as the body effect and velocity saturation [17, 18].

An equation developed in [17, 18] that describes the required scaled width when two transistors are in series is

$$W = (1 + \alpha)W_{eff} \quad (19)$$

where W is the width of each transistor in the stack and W_{eff} is the effective width if there was only a single transistor. The factor α is

$$\alpha = e^{\frac{-\lambda_d V_{DD}}{m v_T}} \quad (20)$$

where λ_d is the drain-induced barrier lowering (DIBL) coefficient (a negative number), m is an empirical parameter and v_T is the thermal voltage. From [17, 18], the theoretical and simulated scaling factors from predictive technology models for 65 nm 0.3 V are 3.28 and 3.20 for PMOS and 2.93 and 2.64 for NMOS. While Equation (19) may be useful for larger transistors, it does not reflect the reality of narrow transistors in actual technology kits. This will be illustrated in the simulation results that follow.

In Table 12, the data for 65 nm LP at 1.2 V is shown. The left side of the table displays the configuration (single, double stack or triple stack) of the transistors at the subthreshold optimum width with their corresponding current. The right side of the table displays the simulated scaled width required to achieve the current of the single transistor. Thus, for PMOS, the transistors have to be scaled by 2.42 and 3.92 times for double and triple stacks, while for NMOS, scaling factors of 2.08 and 3.25 are required. These values seem well within expectations.

Table 12: 65 nm LP, $V_{DD} = 1.2$ V; stack currents and scaled widths

Configuration	I [μ A]	Scaled W [μ m]	Scaled I [μ A]
1 x 0.12 μ m PMOS	48.34	N/A	N/A
2 x 0.12 μ m PMOS	25.04	0.29 (2.42x)	49.68
3 x 0.12 μ m PMOS	16.56	0.47 (3.92x)	49.17
1 x 0.12 μ m NMOS	84.11	N/A	N/A
2 x 0.12 μ m NMOS	47.65	0.25 (2.08x)	85.86
3 x 0.12 μ m NMOS	32.27	0.39 (3.25x)	85.27

Things are very different in subthreshold, as seen in Table 13. Here, a double stack of PMOS transistors must be scaled 16.83 times to achieve the current of a single transistor, while a triple stack requires a scaling factor of 30.75. For NMOS, the scaling factors are 11.17 and 21.50 for double and triple stacks, respectively. These values are impractical to implement, and thus it can be concluded that should additional current drive be required from a stack of transistors, they should be implemented as parallel stacks. The simulated double stack scaling factors of 16.83 and 11.17 for PMOS and NMOS, respectively, are much higher than the scaling factors calculated using Equations (19) and (20): 2.455 and 4.776.

Table 13: 65 nm LP, $V_{DD} = 0.3$ V; stack currents and scaled widths

Configuration	I [nA]	Scaled W [μ m]	Scaled I [nA]
1 x 0.12 μ m PMOS	18.74	N/A	N/A
2 x 0.12 μ m PMOS	6.299	2.02 (16.83x)	18.80
3 x 0.12 μ m PMOS	3.822	3.69 (30.75x)	18.77
1 x 0.12 μ m NMOS	36.32	N/A	N/A
2 x 0.12 μ m NMOS	12.24	1.34 (11.17x)	36.41
3 x 0.12 μ m NMOS	7.34	2.58 (21.50x)	36.40

The data for 65 nm GP is shown in Tables 14 and 15. While the scaling factors in subthreshold are less extreme than those of the 65 nm LP technology, they are still large enough to be impractical. The data for 90 nm is shown in Tables 16 and 17. The thing to note here is that the subthreshold scaling factors for the PMOS transistor are reasonable, right in line with those of superthreshold. This is not surprising since the subthreshold optimum width is relatively large and as we have already seen from Figure 8 that the PMOS transistor displays narrow-width effect and thus does not have a larger than expected current at the narrower widths.

Table 14: 65 nm GP, $V_{DD} = 1$ V; stack currents and scaled widths

Configuration	I [μ A]	Scaled W [μ m]	Scaled I [μ A]
1 x 0.12 μ m PMOS	60.42	N/A	N/A
2 x 0.12 μ m PMOS	31.75	0.28 (2.33x)	60.51
3 x 0.12 μ m PMOS	21.26	0.46 (3.83x)	61.06
1 x 0.15 μ m NMOS	130.8	N/A	N/A
2 x 0.15 μ m NMOS	72.97	0.29 (1.93x)	131.2
3 x 0.15 μ m NMOS	50.14	0.45 (3.00x)	133.1

Table 15: 65 nm GP, $V_{DD} = 0.3$ V; stack currents and scaled widths

Configuration	I [nA]	Scaled W [μ m]	Scaled I [nA]
1 x 0.12 μ m PMOS	2029	N/A	N/A
2 x 0.12 μ m PMOS	803.6	1.33 (11.08x)	2035
3 x 0.12 μ m PMOS	502.7	2.45 (20.42x)	2033
1 x 0.15 μ m NMOS	1409	N/A	N/A
2 x 0.15 μ m NMOS	542.6	0.65 (4.33x)	1425
3 x 0.15 μ m NMOS	341.0	1.20 (8.00x)	1415

Table 16: 90 nm, $V_{DD} = 1.2$ V; stack currents and scaled widths

Configuration	I [μ A]	Scaled W [μ m]	Scaled I [μ A]
1 x 0.41 μ m PMOS	157.3	N/A	N/A
2 x 0.41 μ m PMOS	75.86	0.85 (2.07x)	159.3
3 x 0.41 μ m PMOS	50.25	1.21 (2.95x)	157.7
1 x 0.12 μ m NMOS	111.8	N/A	N/A
2 x 0.12 μ m NMOS	68.01	0.22 (1.83x)	115.3
3 x 0.12 μ m NMOS	48.07	0.31 (2.58x)	112.3

Table 17: 90 nm, $V_{DD} = 0.3$ V; stack currents and scaled widths

Configuration	I [nA]	Scaled W [μ m]	Scaled I [nA]
1 x 0.41 μ m PMOS	527.0	N/A	N/A
2 x 0.41 μ m PMOS	222.6	0.90 (2.20x)	532.9
3 x 0.41 μ m PMOS	142.5	1.29 (3.15x)	527.8
1 x 0.12 μ m NMOS	811.0	N/A	N/A
2 x 0.12 μ m NMOS	294.2	0.97 (8.08x)	817.2
3 x 0.12 μ m NMOS	180.1	1.52 (12.67x)	812.1

The subthreshold optimum widths found in Sections 4.1 and 4.2 were for a single transistor or an inverter configuration. If two or more transistors are required in series, from the subthreshold current of Equation (1), it stands to reason that while V_{GS} and V_{DS} will drop, the other parameters, mainly V_T , will remain largely unaffected. Thus while the absolute value of the current may decrease, it will retain the same behavior. From the circuit configurations shown in Figure 23, Figure 24 plots the current-to-capacitance ratio for the 65 nm LP transistors in subthreshold. The subthreshold optimum widths are 0.12 μ m for both PMOS and NMOS, as was the case with a single transistor.

Figure 23: Double stack configurations for (a) PMOS and (b) NMOS

Figure 24: 65 nm LP, $V_{DD} = 0.3$ V; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for two series (a) PMOS and (b) NMOS

Likewise, Figure 25 confirms the 65 nm GP subthreshold optimum widths of 0.12 μm and 0.15 μm for PMOS and NMOS, respectively. The subthreshold behavior for the 90 nm technology also is the same, as seen in Figure 26. The NMOS transistor has a subthreshold optimum width of 0.12 μm , while the PMOS transistor's current-to-capacitance ratio keeps increasing with the width. These results indicate that the same subthreshold optimum widths obtained when simulating a single transistor apply when considering multiple transistors in a stack.

Figure 25: 65 nm GP, $V_{DD} = 0.3$ V; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for two series (a) PMOS and (b) NMOS

Figure 26: 90 nm, $V_{DD} = 0.3$ V; current-to-capacitance ratio ($I_D / (C_G + C_{SB})$) versus width for two series (a) PMOS and (b) NMOS

4.6 Ring Oscillator Tests for Parallel Transistor Stacks

As a test of the effectiveness of splitting a large transistor or a stack of large transistors into parallel transistor stacks, a 9-inverter ring oscillator will be simulated at 0.3 V in the 65 nm LP technology according to the two configurations shown in Figure 27. The standard inverter consists of large transistors with a beta ratio of 1.35, the subthreshold effective beta ratio for this technology. The parallel inverter configuration splits the widths for the PMOS and NMOS transistors into parallel transistors sized at the subthreshold optimum width of 0.12 μm . Similarly, the two 3-input NAND gate configurations shown in Figure 28 were simulated in a 9-gate ring oscillator. Input C was used for the loop path while inputs A and B were tied to V_{DD} .

Figure 27: (a) Standard and (b) parallel inverter configurations for 9-inverter ring oscillator

Figure 28: (a) Standard and (b) parallel 3-input NAND gate configurations for 9-gate ring oscillator

Table 18 contains the simulation results. The parallel configuration inverter is 2.63 times faster than its standard counterpart. The speed increase of the parallel 3-input NAND gate over its standard counterpart is 2.85 times. The reduction in energy-delay

product is 49.2 % and 56.2 % for the inverter and 3-input NAND gate, respectively. This is approximately a doubling of the efficiency.

Table 18: 65 nm LP, $V_{DD} = 0.3$ V; standard versus parallel configuration ring oscillator simulation results

Gate	f [MHz]	t_p [ns]	P_{avg} [pW]	PDP [aJ]	EDP [J·s]
INV - standard	1.555	35.73	384.3	13.73	4.906e-25
INV – parallel	4.092	13.58	1351	18.35	2.491e-25
NAND3 – standard	0.4824	115.2	388.3	44.72	5.150e-24
NAND3 - parallel	1.374	40.43	1545	62.47	2.256e-24

4.7 Subthreshold Logic Design Methodology

All the steps of the developed design methodology from this chapter will now be numerated. For a given technology:

1. Determine the subthreshold optimum width for each transistor type. This can be done either through the current-to-capacitance ratio or from the ring oscillator simulations.
2. Find the parallel transistor ratio through ring oscillator simulations. Note that all transistors should be sized at their subthreshold optimum width.
3. Calculate the effective beta ratio by multiplying the parallel transistor ratio with the ratio of the PMOS-to-NMOS subthreshold optimum widths.
4. Design the circuit with the sizing strategy of choice such as Logical Effort using the effective beta ratio.

5. Convert the transistor sizes into number of parallel transistors by dividing the designed size by the subthreshold optimum width and rounding to the nearest integer.
6. Create the circuit using parallel transistor stacks. The sizes of all transistors are fixed to the subthreshold optimum width including those of stacked transistors. The number of transistors in parallel was determined in step 5.

As an example using Logical Effort, suppose a NAND gate in the 90 nm technology kit is being designed. The effective beta ratio is 2.56, thus a normalized unit inverter will have PMOS and NMOS widths of 2.56 and 1, respectively. Assuming a doubling of the width for two series transistors, both of a unit NAND gate's PMOS transistors will have widths of 2.56, while the two NMOS transistors will have widths of 2. Now suppose that Logical Effort has resulted in a total width of 2 μm for a given input of the NAND gate. This will be divided as 1.12 μm for PMOS and 0.88 μm for the NMOS. The subthreshold optimum width for the PMOS transistor is 0.41 μm , thus 1.12 μm divided by 0.41 μm , rounded to the nearest integer, results in 3 parallel stacks of PMOS transistors. Similarly, given the subthreshold optimum width of 0.12 μm for the NMOS transistor, 0.88 μm divided by 0.12 μm results in 7 parallel stacks. The division of the gate into parallel transistors is shown in Figure 29.

Figure 29: NAND split into parallel transistors

CHAPTER 5 – APPLICATION CIRCUITS

The design methodology developed in the preceding chapter will be put to the test in two application circuits. Comparisons with a standard design methodology are provided. The first test circuit is a chain of five gates with two different loads. The second circuit is a 32-bit carry-lookahead adder where both schematic and post-layout simulations are performed.

5.1 Test Circuit

The propagation delays of a custom test circuit composed of five logic gates have been measured. The two propagation delays t_{pHL} (V_{in} rising, V_{out} falling) and t_{pLH} (V_{in} falling, V_{out} rising) are measured from V_{in} to V_{out} , as shown in Figure 30. The propagation delay t_p is the average of t_{pHL} and t_{pLH} . The circuit consists of the following gates: INV $\rightarrow$ NAND2 $\rightarrow$ NOR2 $\rightarrow$ NOR3 $\rightarrow$ NAND3. The chain has been constructed for worst case performance such that the input excited in the chain path is that furthest from the output of a given gate. In order to allow the circuit to function, the unused inputs of the NAND gates are tied to V_{DD} , while those of the NOR gates are tied to GND . The order of the gates was chosen in order to avoid having both either the pull-up or pull-down networks of the NAND or NOR gates active during one simulation. For example, should V_{in} be rising, the pull-up network of the NAND2 and the pull-down network of the NAND3 will be active. This is to help balance any performance discrepancy should any gate type be better at conducting in its pull-up or pull-down networks.

Figure 30: Test circuit schematic

V_{in} is generated through a driver circuit consisting of two inverters. The circuit has been designed to ensure that the rise and fall times of V_{in} are well within a 10 % difference of each other.

Two loading conditions have been tested: a single inverter composed of PMOS and NMOS widths of 0.24 and 0.12 μm , respectively, and 32 of these inverters in parallel. The application of Logical Effort for the gate sizing in the case of a single inverter load may have resulted in transistor widths below the minimum of 0.12 μm . In these cases, the minimum width was used.

Four beta ratio values have been chosen for the Logical Effort sizing. A value of 2 is being used since this is a typical value used in textbooks when teaching Logical Effort that approximates a symmetric inverter response. The second beta ratio chosen was the value that maximized the superthreshold ring oscillator frequency. From Tables 3, 6 and 8, it can be seen that the ratio varied depending upon the NMOS width. As such,

the ratio value of the highest frequency was used as a representative value for the technology. Third, a beta ratio calculated from the ratio of the PMOS-to-NMOS subthreshold optimum widths. These simulations will show the effect of neglecting the parallel transistor ratio. Finally, the effective beta ratios of Table 11 which were found for the subthreshold design methodology were simulated. For each beta ratio, two configurations were tested. The first is a standard configuration where the resulting widths from Logical Effort were simulated as a single transistor. The second configuration, parallel, splits this width into parallel transistors all sized at the subthreshold optimum width. Since only an integer number of transistors can be used, the total width of the transistors may be different from the calculated width. This is partially the reason that the sum of the widths of the transistors of the five gates in the chain are provided. Note that this width is for one input only. For example, a standard NAND3 gate with $0.24\text{ }\mu\text{m}$ PMOS and $0.36\text{ }\mu\text{m}$ NMOS transistors would contribute $0.6\text{ }\mu\text{m}$ to the total width, not the $1.8\text{ }\mu\text{m}$ for all the transistors of all three inputs.

One important note is how the logical effort values of the gates have been determined. As is typical, the logical effort of all gates is normalized to that of the unit inverter, thus the inverter, regardless of the sizing of the PMOS and NMOS transistors, has a logical effort of 1. If considering the NAND2 gate, the two series NMOS transistors would be scaled to provide the same current drive as the NMOS transistor of the inverter. However, since the beta ratios and thus inverter sizes have been determined from both superthreshold and subthreshold operation, the question becomes which operating voltage to use to determine the required scaling. In order to keep the focus on

comparing the beta ratios only and for simplicity, a convention of doubling the width of two and tripling the width of three series transistors will be used.

Also simulated are the propagation delays of a chain of gates composed of minimum sized transistors.

Before examining the simulation results, the design for the 65 nm GP parallel configuration with subthreshold effective beta ratio under the large load conditions will be detailed. This will demonstrate the proper application of Logical Effort with the subthreshold design methodology. The first step is to find the subthreshold optimum widths for both transistors. From the prior simulations and Table 11, we know that these widths are $0.12\ \mu\text{m}$ for PMOS and $0.15\ \mu\text{m}$ for NMOS. The parallel transistor ratio is 0.95. Thus the effective beta ratio is 0.76. Next, we apply Logical Effort to the test circuit shown in Figure 30.

The calculations are summarized in Table 19. A normalized unit width will be equivalent to $0.12\ \mu\text{m}$. Thus INV, which will be considered a unit inverter, will have PMOS and NMOS widths of 1 and 1.32, respectively. By definition, the logical effort of this first gate is 1. After finding the sizes of the other unit gates, their logical efforts can be calculated. The path logical effort is found by multiplying all the gate logical efforts together. The path input, i.e., the INV gate, has a total width of 2.32. The path output, or load, has a total width of 3 for one inverter multiplied by 32 inverters in parallel, thus 96. The path electrical effort can then be calculated as $96 / 2.32 = 41.4$. The path effort is then found by multiplying the path logical effort with the path electrical effort. Next, the best stage effort is calculated. With this, the total widths per input for the five gates are

found. This is then split between the PMOS and NMOS transistors, and then converted into microns. Now, the widths are divided by the subthreshold optimum widths to calculate the required number of transistors in parallel.

Table 19: 65 nm GP; parallel configuration with effective beta ratio under large load design example

	INV	NAND2	NOR2	NOR3	NAND3
Normalized unit gate PMOS / NMOS width	1 / 1.32	1 / 2.64	2 / 1.32	3 / 1.32	1 / 3.96
Logical effort	1	1.57	1.43	1.86	2.14
Path logical effort	8.94				
Path electrical effort	$(32 \times 3) / (1 + 1.32) = 41.4$				
Path effort	$8.94 \times 41.4 = 370$				
Best stage effort	$370^{1/5} = 3.26$				
Total normalized width per input	2.32	7.57	15.7	35.9	62.9
Normalized PMOS / NMOS width	1 / 1.32	2.08 / 5.49	9.46 / 6.24	24.9 / 11.0	12.7 / 50.2
PMOS / NMOS width [μm]	0.12 / 0.16	0.25 / 0.66	1.14 / 0.75	2.99 / 1.32	1.52 / 6.03
# of PMOS / NMOS transistors in parallel	1 / 1	2 / 4	10 / 5	25 / 9	13 / 40

Table 20 contains the simulation results for the 65 nm LP technology at 1.2 V. For the small load, a beta ratio of 2 using a standard configuration was best, although the difference to the standard configuration with a beta ratio of 1.49 (superthreshold beta ratio) was only 0.9 %. The parallel configurations were only slightly slower than their standard counterparts. The performance of the minimum sized gates was in line with those of the other configurations.

For the large load, the minimum sized gates were much slower. The 1.49 beta ratio standard configuration was best, although the difference to the other beta ratios was slight. The parallel configurations again lagged behind their standard counterparts.

Table 20: 65 nm LP, $V_{DD} = 1.2$ V; test circuit propagation delays

β	Configuration	Width [μm]	t_{pHL} [ps]	t_{pLH} [ps]	t_p [ps]
Small load – 1 inverter					
2	Standard	2.58	146.2	134.5	140.4
	Parallel	2.4	149.5	148.0	148.8
1.49	Standard	2.3	143.9	139.3	141.6
	Parallel	2.28	142.4	150.8	146.6
1	Standard	2.04	146.6	144.8	145.7
	Parallel	2.04	157.7	155.9	156.8
1.35	Standard	2.21	144.2	140.6	142.4
	Parallel	2.16	139.1	160.7	149.9
Min. size transistors (0.12 μm)		1.2	158.2	128.6	143.4
Large load – 32 inverters					
2	Standard	14.98	174.4	184.8	179.6
	Parallel	15.12	201.0	204.7	202.9
1.49	Standard	14.35	172.0	185.1	178.6
	Parallel	14.52	200.7	195.7	198.2
1	Standard	13.77	176.2	197.0	186.6
	Parallel	13.8	207.0	214.8	210.9
1.35	Standard	14.18	172.0	187.4	179.7
	Parallel	14.28	194.4	217.9	206.2
Min. size transistors (0.12 μm)		1.2	530.4	405.2	467.8

Table 21 contains the subthreshold results. Here, the parallel configurations completely dominate their standard counterparts. For the large load, the propagation delays for the standard configurations were more than double those of the parallel configurations. A beta ratio of 1.49 proved best for both loads, although the differences

to the other beta ratios were small. The performance of the minimum sized gates was quite good for the small load. This shouldn't be too surprising since in the 65 nm LP technology, the minimum width is the same as the subthreshold optimum.

For the 1.35 parallel configurations, designed according to the methodology of this thesis, the reduction in delay compared to the fastest standard configurations were 41.0 % and 61.4 % for the small and large loads, respectively.

Table 21: 65 nm LP, $V_{DD} = 0.3$ V; test circuit propagation delays

β	Configuration	Width [μm]	t_{pHL} [ns]	t_{pLH} [ns]	t_p [ns]
Small load – 1 inverter					
2	Standard	2.58	297.7	231.3	264.5
	Parallel	2.4	150.5	133.6	142.1
1.49	Standard	2.3	291.3	218.8	255.1
	Parallel	2.28	144.9	136.0	140.5
1	Standard	2.04	283.0	202.4	242.7
	Parallel	2.04	160.3	137.5	148.9
1.35	Standard	2.21	289.0	214.5	251.8
	Parallel	2.16	141.3	145.1	143.2
Min. size transistors (0.12 μm)		1.2	174.0	129.8	151.9
Large load – 32 inverters					
2	Standard	14.98	535.6	491.3	513.5
	Parallel	15.12	201.8	186.6	194.2
1.49	Standard	14.35	536.6	489.3	513.0
	Parallel	14.52	202.9	177.9	190.4
1	Standard	13.77	549.6	502.5	526.1
	Parallel	13.8	212.2	194.5	203.4
1.35	Standard	14.18	538.4	491.2	514.8
	Parallel	14.28	197.4	198.2	197.8
Min. size transistors (0.12 μm)		1.2	465.7	293.6	379.7

The 65 nm GP superthreshold results are shown in Table 22. Many of the same conclusions from the 65 nm LP superthreshold results still hold. A beta ratio of 2 proved best for the small load, while beta ratios of 2 and 1.31 proved almost equal for the large load.

Table 22: 65 nm GP, $V_{DD} = 1$ V; test circuit propagation delays

β	Configuration	Width [μm]	t_{pHL} [ps]	t_{pLH} [ps]	t_p [ps]
Small load – 1 inverter					
2	Standard	2.58	80.41	74.67	77.54
	Parallel	2.61	79.28	81.57	80.43
1.31	Standard	2.21	78.57	78.80	78.69
	Parallel	2.37	74.14	89.58	81.86
0.8	Standard	2.22	81.05	82.58	81.82
	Parallel	2.01	81.28	84.22	82.75
0.76	Standard	2.23	82.13	83.57	82.85
	Parallel	2.16	81.53	93.04	87.29
Min. size transistors (0.12 μm)		1.2	88.66	71.20	79.93
Large load – 32 inverters					
2	Standard	14.98	100.9	105.6	103.3
	Parallel	15.18	106.0	111.2	108.6
1.31	Standard	14.13	99.61	106.9	103.3
	Parallel	14.34	99.99	118.3	109.1
0.8	Standard	14.69	104.1	117.1	110.6
	Parallel	14.67	105.9	121.4	113.7
0.76	Standard	14.92	104.8	119.0	111.9
	Parallel	14.97	106.8	120.3	113.6
Min. size transistors (0.12 μm)		1.2	310.5	217.5	264.0

The subthreshold results for 65 nm GP are in Table 23. The gains from the parallel configurations compared to the standard configurations are not as great as in the 65 nm LP technology, however, they are still significant. A beta ratio of 0.8 was best for the small load, while a beta ratio of 0.76 proved best for the large load.

For the 0.76 parallel configurations, designed according to the methodology of this thesis, the reduction in delay compared to the fastest standard configurations were 19.8 % and 45.3 % for the small and large loads, respectively.

Table 23: 65 nm GP, $V_{DD} = 0.3$ V; test circuit propagation delays

β	Configuration	Width [μm]	t_{pHL} [ns]	t_{pLH} [ns]	t_p [ns]
Small load – 1 inverter					
2	Standard	2.58	3.022	2.381	2.702
	Parallel	2.61	2.143	1.898	2.021
1.31	Standard	2.21	2.774	2.193	2.484
	Parallel	2.37	1.984	1.973	1.979
0.8	Standard	2.22	2.696	2.058	2.377
	Parallel	2.01	1.977	1.752	1.865
0.76	Standard	2.23	2.680	2.063	2.372
	Parallel	2.16	1.947	1.857	1.902
Min. size transistors (0.12 μm)		1.2	2.239	1.640	1.940
Large load – 32 inverters					
2	Standard	14.98	5.108	4.601	4.855
	Parallel	15.18	3.081	2.809	2.945
1.31	Standard	14.13	4.803	4.306	4.555
	Parallel	14.34	2.602	2.636	2.619
0.8	Standard	14.69	4.713	4.293	4.503
	Parallel	14.67	2.512	2.449	2.481
0.76	Standard	14.92	4.733	4.323	4.528
	Parallel	14.97	2.531	2.394	2.463
Min. size transistors (0.12 μm)		1.2	12.64	2.729	7.685

Table 24 contains the 1.2 V simulations for the 90 nm technology. For the small load, three of the parallel configurations resulted in the same circuit considering the wide subthreshold optimum width of the PMOS transistor. Ultimately, these parallel configurations proved marginally better than the standard configurations.

For the large load, the standard configurations for beta ratios of 2 and 1.58 proved best, while all the standard configurations were reliably faster than the parallel configurations.

Table 24: 90 nm, $V_{DD} = 1.2$ V; test circuit propagation delays

β	Configuration	Width [μm]	t_{pHL} [ps]	t_{pLH} [ps]	t_p [ps]
Small load – 1 inverter					
2	Standard	2.58	107.0	107.5	107.3
	Parallel	2.89	114.8	95.13	105.0
1.58	Standard	2.36	106.4	113.2	109.8
	Parallel	2.89	114.8	95.13	105.0
3.42	Standard	3.24	113.7	102.5	108.1
	Parallel	3.59	132.4	104.4	118.4
2.56	Standard	2.85	110.1	103.8	107.0
	Parallel	2.89	114.8	95.13	105.0
Min. size transistors (0.12 μm)		1.2	131.9	112.7	122.3
Large load – 32 inverters					
2	Standard	14.98	119.6	138.0	128.8
	Parallel	15.07	150.3	145.5	147.9
1.58	Standard	14.45	118.3	139.3	128.8
	Parallel	14.56	141.8	151.4	146.6
3.42	Standard	16.79	129.4	151.6	140.5
	Parallel	16.74	146.0	163.7	154.9
2.56	Standard	15.71	122.8	141.6	132.2
	Parallel	16.11	140.6	153.4	147.0
Min. size transistors (0.12 μm)		1.2	332.5	416.1	374.3

In Table 25, the subthreshold 90 nm results are shown. In all cases, the parallel configurations proved faster than their standard counterparts, although not always by the greatest of margins. As mentioned previously, for the small load, three of the parallel configurations resulted in the same circuit. For the large load, a beta ratio of 2.56 proved best.

For the 2.56 parallel configurations, designed according to the methodology of this thesis, the reduction in delay compared to the fastest standard configurations were 7.4 % and 16.0 % for the small and large loads, respectively.

Table 25: 90 nm, $V_{DD} = 0.3$ V; test circuit propagation delays

β	Configuration	Width [μm]	t_{pHL} [ns]	t_{pLH} [ns]	t_p [ns]
Small load – 1 inverter					
2	Standard	2.58	9.847	9.511	9.679
	Parallel	2.89	9.530	7.221	8.376
1.58	Standard	2.36	10.40	10.47	10.44
	Parallel	2.89	9.530	7.221	8.376
3.42	Standard	3.24	9.312	8.788	9.050
	Parallel	3.59	9.951	7.313	8.632
2.56	Standard	2.85	9.597	8.970	9.284
	Parallel	2.89	9.530	7.221	8.376
Min. size transistors (0.12 μm)		1.2	18.71	14.43	16.57
Large load – 32 inverters					
2	Standard	14.98	11.87	13.39	12.63
	Parallel	15.07	12.03	10.13	11.08
1.58	Standard	14.45	11.77	13.51	12.64
	Parallel	14.56	11.78	10.87	11.33
3.42	Standard	16.79	12.89	14.79	13.84
	Parallel	16.74	10.97	10.63	10.80
2.56	Standard	15.71	12.20	13.80	13.00
	Parallel	16.11	10.79	10.42	10.61
Min. size transistors (0.12 μm)		1.2	30.40	48.52	39.46

For all three technologies, there was a benefit to using parallel stacks of transistors of the subthreshold optimum width. In the 65 nm LP and GP technologies, the increase in performance when using parallel stacks instead of the standard configuration was greater for the larger load. This can be attributed to the fact that the standard gates in the large load scenario have transistors whose widths are much larger than the subthreshold optimum width. That is one reason why the gains between the small and large loads in the 90 nm technology are comparable. Also, the 90 nm PMOS transistor exhibits narrow-width effect and thus does not benefit from being narrow in subthreshold.

The effective beta ratios for subthreshold design also proved best in two of the three technologies. Although it did not give the best results in 65 nm LP, the effective beta ratio of 1.35 performed reasonably when compared to the superthreshold optimum beta ratio of 1.49. These variations can be expected since there is a broad minimum for the delay with respect to the beta ratio.

5.2 Carry-Lookahead Adder

A 32-bit carry-lookahead adder produces a sum $S[32:1]$ and carry-out C_{out} from inputs $A[32:1]$, $B[32:1]$ and carry-in C_{in} . The adder being designed will be made from eight valency-4 blocks. One block is shown in Figure 31. Bitwise propagate and generate (PG) signals are evaluated for the four bits of A and B. Propagate and generate refer to the carry; the carry-in may be propagated to or a carry-out generated for the next part of the circuit. The bitwise PG signals are then combined with C_{in} to produce four

output bits of S . C_{in} refers to either the C_{in} input to the adder if this is the first block or is otherwise the signal C_{out} from a previous block. The bitwise PG signals are also used to generate group PG signals. The group PG signals are then combined with C_{in} in the carry logic to determine if there is a C_{out} . The C_{out} of the block then becomes the C_{in} of the next block, or is the C_{out} of the adder if considering the last block. By evaluating the carry-out of a 4-bit group in parallel with the sum bits and passing it on to the next block, the addition process is sped up compared to a carry-ripple adder where each sum bit must wait on receiving the carry from the previous full adder [13].

Figure 31: Valency-4 carry-lookahead adder block

Three versions of the carry-lookahead adder were designed, all with the 65 nm LP technology. The first was a standard design where the superthreshold optimal beta of

1.49 is used. This represents the typical superthreshold design. The second circuit was made from parallel transistor stacks composed of transistors of the subthreshold optimum width, which for this technology is $0.12\ \mu\text{m}$ for both transistor types. The associated effective beta ratio of 1.35 was used. This design represents the subthreshold design methodology of this thesis. The third circuit was a minimum design where all gates are made from minimum-sized transistors. This design would occupy the least area and also consume the least energy since the capacitance would be the smallest. Note that for the 65 nm LP technology, minimum-sized transistors are best for subthreshold, so it is expected that this design will perform favorably.

The carry-lookahead adder will be designed using Logical Effort. Each of the eight blocks will be identical. Within a block, each of the paths (bitwise PG, group PG, sum, carry) will be sized as a critical path with the appropriate accounting for load and branching.

The layouts will make use of a standard cell methodology, thus no optimization for area is made. Note that while the standard cell height of the kit ($2.44\ \mu\text{m}$ rail-to-rail) was appropriate for the minimum design, this height had to be extended to $2.655\ \mu\text{m}$ for the standard design and $4.15\ \mu\text{m}$ for the parallel stacks design. A sample NAND gate from the parallel stacks design with four PMOS transistors in parallel per input and six NMOS stacks in parallel is shown in Figure 32. It is clear that should a library of cells be designed for subthreshold logic gates, running the polysilicon gate horizontally must be considered in order to keep the heights of the cells within a limit.

Figure 32: NAND gate from parallel stacks design

With the exception of the height of the cells, the placement of gates and routing of interconnect was identical for the three designs to provide the fairest comparison of performance. The full layout of the carry-lookahead adder for the standard design is shown in Figure 33. The area is $1997 \mu\text{m}^2$. For the minimum design, the area was $1869 \mu\text{m}^2$. The parallel stacks design occupied $2887 \mu\text{m}^2$. These figures are provided for completeness and should only be used as a rough guide to the relative area requirements of the designs, keeping in mind the caveats from the preceding paragraph.

Figure 33: Standard design carry-lookahead adder layout

Schematic and post-layout simulations were done at both 1.2 and 0.3 V. The test cases are described below:

- All bits of A, B and C_{in} go from 0 to 1.
- All bits of A, B and C_{in} go from 1 to 0.
- A is fixed at 0; all bits of B are fixed at 1; C_{in} goes from 0 to 1.

- A is fixed at 0; all bits of B are fixed at 1; C_{in} goes from 1 to 0.
- Arbitrary case (a): A goes from 0 to 3642924507; B goes from 0 to 1232768243; C_{in} goes from 0 to 1.
- Arbitrary case (b): A goes from 0 to 2704529484; B goes from 0 to 3126743191; C_{in} is fixed at 0.

All inputs are driven through a driver circuit consisting of two inverters whose output rise and fall times are roughly equal. All outputs are loaded with an inverter with PMOS and NMOS widths of 0.24 and 0.12 μm , respectively.

Table 26 contains the superthreshold results with the post-layout simulation results presented in brackets. Other results besides the propagation delay are reported. The static current with all inputs at 0 and at 1 are shown. The average of these two values will be considered the static current. The energy of operation is found as follows: (i) calculate the total energy for the entire simulation by multiplying the average current by V_{DD} and the simulation time; (ii) find the total static energy by multiplying the static current by V_{DD} and the simulation time; (iii) find the dynamic energy by subtracting the total static energy from the total energy; and (iv) the energy of operation is then the sum of the dynamic energy and the static energy for the time of operation, found by multiplying the static current by V_{DD} and the propagation delay. The energy-delay product, a measure of efficiency, is found by multiplying the energy of operation by the propagation delay.

Table 26: 65 nm LP, $V_{DD} = 1.2$ V; carry-lookahead adder results (post-layout simulation results in brackets)

Standard 1.2 V (sim time = 10 ns)				
I_{static} with all inputs 0 = 43.74 (42.69) nA				
I_{static} with all inputs 1 = 43.32 (38.38) nA				
Test case	I_{avg} [μ A]	t_p [ps]	Energy [fJ]	EDP [J·s]
A, B, C 0→1	30.47 (91.05)	184.5 (374.4)	365 (1090)	6.74e-23 (4.09e-22)
A, B, C 1→0	45.92 (68.88)	174.0 (323.3)	551 (826)	9.58e-23 (2.67e-22)
A=0, B=1, C 0→1	20.33 (43.41)	889.8 (1943.1)	243 (521)	2.17e-22 (1.01e-21)
A=0, B=1, C 1→0	23.00 (40.96)	934.5 (1934.8)	276 (491)	2.57e-22 (9.50e-22)
Case (a)	23.99 (65.63)	262.4 (511.8)	287 (787)	7.54e-23 (4.03e-22)
Case (b)	26.44 (70.82)	338.9 (686.3)	317 (849)	1.07e-22 (5.83e-22)
Parallel stacks 1.2 V (sim time = 10 ns)				
I_{static} with all inputs 0 = 51.69 (56.05) nA				
I_{static} with all inputs 1 = 52.02 (57.86) nA				
Test case	I_{avg} [μ A]	t_p [ps]	Energy [fJ]	EDP [J·s]
A, B, C 0→1	35.29 (99.97)	189.6 (348.5)	423 (1200)	8.02e-23 (4.18e-22)
A, B, C 1→0	50.17 (76.02)	187.9 (318.9)	601 (912)	1.13e-22 (2.91e-22)
A=0, B=1, C 0→1	21.96 (47.51)	906.7 (1880.6)	263 (570)	2.38e-22 (1.07e-21)
A=0, B=1, C 1→0	24.27 (43.54)	1003.8 (1969.2)	291 (522)	2.92e-22 (1.03e-21)
Case (a)	27.48 (72.45)	262.3 (478.5)	329 (869)	8.63e-23 (4.16e-22)
Case (b)	30.07 (77.76)	350.6 (677.2)	360 (932)	1.26e-22 (6.31e-22)
Minimum 1.2 V (sim time = 10 ns)				
I_{static} with all inputs 0 = 30.45 (32.85) nA				
I_{static} with all inputs 1 = 25.68 (27.63) nA				
Test case	I_{avg} [μ A]	t_p [ps]	Energy [fJ]	EDP [J·s]
A, B, C 0→1	17.42 (79.93)	181.5 (660.8)	209 (959)	3.79e-23 (6.34e-22)
A, B, C 1→0	26.41 (52.84)	193.4 (519.1)	317 (634)	6.12e-23 (3.29e-22)
A=0, B=1, C 0→1	12.81 (37.85)	995.9 (3543.4)	153 (454)	1.53e-22 (1.61e-21)
A=0, B=1, C 1→0	15.46 (34.51)	1037.8 (3000.1)	185 (414)	1.92e-22 (1.24e-21)
Case (a)	13.36 (56.05)	268.9 (853.7)	160 (672)	4.30e-23 (5.74e-22)
Case (b)	14.60 (61.02)	352.8 (1099.2)	175 (732)	6.17e-23 (8.05e-22)

Considering the schematic simulation results, the standard design was fastest four times, while the parallel stacks and minimum designs were fastest once each. In terms of efficiency, the minimum design swept the results. The post-layout simulation results showed the parallel stacks design as fastest five times with the standard design coming out on top once. In terms of efficiency, the standard design had a clean sweep.

The subthreshold results are shown in Table 27. For the schematic simulations, the parallel stacks design was always fastest while the minimum design was always the most efficient. For the post-layout simulation results, the parallel stacks design was always the fastest and most efficient. Consider the second test case where all input bits fall from 1 to 0: the parallel stacks design resulted in a 64.5 % reduction in propagation delay compared to the standard design.

The reduction in propagation delay of the parallel stacks design compared to the minimum design is 41.6 %. However, for the schematic simulation, the reduction is only 5.8 %. It appears that when the parasitic resistances and capacitances are accounted for, the minimum design does not have the adequate current drive to compensate. A better design for all configurations may have been possible if wire capacitance estimates were included during the Logical Effort design, particularly since parasitics play a greater role in the deep submicron technologies. Logical Effort also tends to yield better designs when driving a larger load versus the small inverter considered here.

Table 27: 65 nm LP, $V_{DD} = 0.3$ V; carry-lookahead adder results (post-layout simulation results in brackets)

Standard 0.3 V (sim time = 10 μ s)				
I_{static} with all inputs 0 = 8.834 (8.853) nA				
I_{static} with all inputs 1 = 7.690 (6.920) nA				
Test case	I_{avg} [nA]	t_p [ns]	Energy [fJ]	EDP [J·s]
A, B, C 0→1	15.09 (28.68)	333.1 (1007.0)	21.3 (64.8)	7.10e-21 (6.52e-20)
A, B, C 1→0	19.61 (25.29)	395.6 (869.9)	35.0 (54.3)	1.39e-20 (4.72e-20)
A=0, B=1, C 0→1	13.65 (18.78)	1815.2 (4908.5)	20.7 (44.3)	3.75e-20 (2.17e-19)
A=0, B=1, C 1→0	14.57 (18.41)	1993.8 (4605.7)	23.9 (42.5)	4.76e-20 (1.96e-19)
Case (a)	13.82 (23.48)	487.9 (1322.7)	17.9 (49.9)	8.73e-21 (6.60e-20)
Case (b)	14.42 (24.98)	664.0 (1629.7)	20.1 (55.1)	1.34e-20 (8.99e-20)
Parallel stacks 0.3 V (sim time = 10 μ s)				
I_{static} with all inputs 0 = 13.82 (14.92) nA				
I_{static} with all inputs 1 = 13.50 (14.97) nA				
Test case	I_{avg} [nA]	t_p [ns]	Energy [fJ]	EDP [J·s]
A, B, C 0→1	22.08 (39.15)	178.2 (380.8)	26.0 (74.3)	4.63e-21 (2.83e-20)
A, B, C 1→0	25.63 (33.30)	195.2 (308.5)	36.7 (56.4)	7.17e-21 (1.74e-20)
A=0, B=1, C 0→1	20.45 (27.93)	924.9 (2078.8)	24.2 (48.3)	2.23e-20 (1.00e-19)
A=0, B=1, C 1→0	21.10 (26.71)	1004.4 (1807.6)	26.4 (43.4)	2.66e-20 (7.84e-20)
Case (a)	20.44 (32.79)	254.2 (521.3)	21.4 (55.9)	5.44e-21 (2.91e-20)
Case (b)	21.27 (34.30)	340.2 (703.6)	24.2 (61.2)	8.24e-21 (4.31e-20)
Minimum 0.3 V (sim time = 10 μ s)				
I_{static} with all inputs 0 = 8.000 (8.624) nA				
I_{static} with all inputs 1 = 6.670 (7.170) nA				
Test case	I_{avg} [nA]	t_p [ns]	Energy [fJ]	EDP [J·s]
A, B, C 0→1	10.99 (26.88)	181.2 (799.6)	11.4 (58.8)	2.06e-21 (4.71e-20)
A, B, C 1→0	14.24 (21.46)	207.3 (528.1)	21.2 (41.9)	4.39e-21 (2.21e-20)
A=0, B=1, C 0→1	10.68 (17.31)	1025.5 (4092.7)	12.3 (37.9)	1.26e-20 (1.55e-19)
A=0, B=1, C 1→0	11.69 (16.70)	1084.9 (2826.5)	15.5 (33.1)	1.68e-20 (9.36e-20)
Case (a)	10.48 (21.62)	267.4 (1021.3)	10.0 (43.6)	2.68e-21 (4.45e-20)
Case (b)	10.83 (22.83)	352.3 (1248.4)	11.3 (47.8)	3.97e-21 (5.96e-20)

CHAPTER 6 – CONCLUSIONS AND FUTURE WORK

The conclusions of this thesis will be described in this chapter. Also, possible future work to expand upon this research will be described.

6.1 Conclusions

In subthreshold, the current of a transistor may not increase linearly with its width. In fact, if the transistor exhibits the inverse-narrow-width effect, where the threshold voltage drops as the width narrows, the current's exponential dependence on the threshold voltage can result in surprising increases in current as the width is narrowed. Since a transistor's capacitances still continue to increase linearly with the width, it may be possible to find a width near the minimum size that clearly maximizes the current-to-capacitance ratio of a transistor, giving rise to what we call a subthreshold optimum width.

The subthreshold optimum width is the key component of the design methodology of this thesis to improve the subthreshold performance of logic circuits. Whether a gate requires transistors stacked in series or more current drive, different configurations of transistors locked to the subthreshold optimum width should be used. To complete a design for these cases, further refinements such as the parallel transistor ratio, describing the ideal ratio of PMOS-to-NMOS transistors sized at the subthreshold optimum width, have been developed. The subthreshold optimum widths and the parallel transistor ratio are then combined into an effective beta ratio to use with transistor sizing

strategies such as Logical Effort to design subthreshold circuits geared for maximum speed.

Technologies where both transistors exhibit the inverse-narrow-width effect such as 65 nm LP and GP both showed significant gains in subthreshold performance with the proposed design methodology. Even in the 90 nm technology, where the PMOS transistor exhibited the narrow-width effect, an increase in threshold voltage with narrowing widths, benefits could be seen from the proposed sizing for subthreshold operation. A 9-inverter ring oscillator, split into parallel transistors, had a 2.63 times increase in frequency and a 49.2 % reduction in energy-delay product, a near doubling of the efficiency, compared to its standard counterpart. Similarly, a ring oscillator composed of nine NAND3 gates with parallel transistor stacks was 2.85 times faster and reduced the energy-delay product by 56.2 %. A test circuit, composed of five gates of differing types, demonstrated clear performance improvements using the design methodology. In particular, large load conditions showed the most improvement, in one case resulting in a propagation delay reduction of 61.4 % compared to a standard design. In post-layout simulations of a 32-bit carry-lookahead adder done in the 65 nm LP kit, a reduction of 64.5 % in propagation delay when compared to a standard superthreshold design was demonstrated.

6.2 Future Work

The work performed in this thesis centered on a static CMOS logic style. The benefits of using the subthreshold optimum width with other logic styles such as cascode

voltage switch logic or pass-transistor could be explored. If work were to be pursued in this direction, consideration to the switching threshold and noise margins, and the overall robustness of the gates, would need to be given since these logic styles are not as robust as static CMOS.

The subthreshold supply voltage was fixed to 0.3 V due to previous work [17, 18]. However, optimizing this parameter for minimum energy operation could be a topic of further investigation.

The effects of process, temperature and voltage variations could be explored. Comparisons of the degradation in performance in the subthreshold case compared to superthreshold could be examined.

The design methodology developed in this thesis was geared towards performance. Should another measure be the ultimate goal, such as efficiency or least energy, it would be worthwhile to investigate what modifications to the methodology are necessary to meet those criteria. It has already been shown that the minimum energy operation can fall in the subthreshold region [2].

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