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Precision Tests of Perturbative QCD at the Z^0 Peak

by

Sukhpal Sanghera

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

Department of Physics
Carleton University
Ottawa, Canada, K1S 5B6
December 1991

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ISBN 0-315-75989-5

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The undersigned hereby recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis
"Precision Tests of Perturbative QCD at the Z^0 Peak"
submitted by
Sukhpal Sanghera, B.Sc., M.Sc.
in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

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Abstract

A QCD analysis of hadronic decays of the Z^0 in e^+e^- annihilation is presented. Jet production rates and other infrared safe observables (C planarity, oblateness and thrust) are studied in order to test the predictions of $o(\alpha_s^2)$ QCD calculations. To provide a complete accountability of theoretical uncertainties, different recombination schemes in the cluster jet algorithm have been used; thus the ambiguity caused by the degree of freedom provided by the theory to define jets is investigated. It is found that different schemes predict different jet rates, and the difference is reproduced by the experimental data. It is demonstrated that the scheme ambiguity can be totally removed by consistently applying the same scheme at theoretical and experimental levels and that the values of α_s , thus determined from different schemes agree with each other.

A special emphasis is given on estimating the theoretical uncertainties. The uncertainties due to the hadronization, parton virtuality, renormalization scale and the experimental systematics are estimated. The α_s values determined from different observables are found to be in agreement with each other and with the theoretical predictions based on the lower energy experiments. The theoretical uncertainties are found to be dominant over the experimental ones and hence the higher order calculations are required to further improve the results.

The theoretical predictions for the renormalization scale μ^2 optimized to $o(\alpha_s^2)$ are calculated using various QCD inspired procedures, and are compared to the optimized scale deduced from the experiment. An experimental procedure to eliminate the renormalization scale ambiguity is presented.

The overall conclusion is that QCD is apparently a valid theory of strong interactions, and higher order calculations are indispensable to further improve the precision of these tests.

Acknowledgements

The time of my Ph.D. has been a great learning experience. First of all, I thank P.J.S. Watson for allowing me a U turn from theory to experiment. I am grateful to R. K. Carnegie who opened the door for me to the wonderful world of Experimental High Energy Physics. I highly appreciate the support and encouragement provided by him in his own uniquely constructive way.

It is not possible in few words to express my gratitude toward my thesis supervisor Hans Mes for his regular help and encouragement throughout the course of this work. I am also thankful to him for his sincere efforts to break my initial phobia of hardware and for his constant struggle to convert my Punjabi-style English, that I usually use in my writeups, into a more standard one. In this age of complex theories full of problems, I have learned from him that there is always an underlying simplicity behind every apparent complexity and there is always a way around the problem.

Thanks are also due to R. J. Hemingway who got me started with the Monte Carlo aspect of experimental studies. Brief periods of working with him were full of learning and quest for learning. I did not learn only Physics from him but also the Sociology of Physics: how to physics and smile at the same time. I also thank Pat Kalyniak of Carleton and David Hanna of McGill for a careful and critical reading of the manuscript.

I am thankful to S. Bethke of Heidelberg who got me started with the jet analysis. It was a great pleasure to work with him, at CERN, on the OPAL jet papers. Most of the data presentation techniques used in this thesis were developed during that period. From him, I learned a great deal on the experimental aspects of QCD. Useful interactions with other members of the OPAL QCD group, namely George Azuelos, M. Dixit, J. W. Gary, Hannes Jeremie and David Ward, are also acknowledged.

This note would be incomplete without thanking the fellow graduate students who provided a very friendly environment at Carleton, and who know how to moo it when you can't do it. Mike Boyce, Victor Silalahi, André Turcotte and Sultan Sial are few to mention. Mention should also be made

of Sher Alam, the only graduate student with whom you can discuss any topic of physics at any time, but only if you don't mind hearing the word of God at the end of the discussion.

I would also like to thank Alan Barney and Bill Jack for settling the disputes between human and computer machine and putting the things back on the road again from time to time. Alan also helped in formatting the manuscript. I also appreciate the moral support and help provided by Marilyn McGregor of CRPP and Roselyn Tighe of Physics department during my stay at Carleton.

Surely, It was Carleton and not one of those acclaimed number one universities, but it offered me at least as much, if not more, work and training opportunities in particle physics as any of those number ones could possibly have. Working at CERN, with OPAL, at LEP was a dream came true; the most fantastic dream I ever had during my student life. This is the biggest achievement and reward of my life. The degree is just a bonus.

This is also the time to remember my folks back home: my mother who replaced me in the farm so that I could go to school, and my father who offered me an earthen lamp, a bamboo pen, and a dark corner in the house to begin the journey. I look back and see that I have come a long way and find myself able to say: Mom, Dad, I've made it! And along this way I appreciate the help of my uncle and aunt from Vancouver.

Finally, I thank my fiancé Reneé Kant for patiently waiting for the day that would change her last name, and for all her love that has been providing me a window from Physics to the outside world. Without her, I would have been lost somewhere between the experiment and theory. ya sure, you becha.

After all, we are human; I am also thankful to all those whom I have forgotten at this point in space-time.

Statement of Originality

In a big experiment like OPAL¹, which is run with the joint effort of a large number of people, it is very difficult to isolate the contribution of an individual. However, an attempt will be made to list the specific tasks performed by the author, which are related to this thesis.

Once the raw data collected by the detector is reconstructed, it is stored on the data summary tapes (DST). One can use the existing software, as well as write his own, to analyze the data on DST.

1. To the best of our knowledge, C planarity is studied on the experiment for the first time. Thus, the routine for this observable was developed. The routines for jet rates, oblateness and thrust were already existing.
2. Development of programs to calculate the experimental distributions of jet rates, C planarity, oblateness and thrust, using the standard routines mentioned above.
3. Development of fitting routines to fit the QCD predictions for the above observables to the experimental measurements.
4. Development of programs to calculate the predictions of the renormalization scale in the optimized QCD, using various theoretical models.
5. Analyzing the data, using these tools, to perform the study reported in this thesis.

In addition to this, the author also contributed to the experiment in the following areas:

a) Detector Related Work

The detector responsibilities of the author were with the Zed chambers, a sub detector designed to make a precise measurement of the z coordinate of

¹ A list of the members of the OPAL collaboration is presented in appendix B.

charged tracks in the OPAL central detector and hence to improve the polar angle and mass resolution. The following contributions were made:

1. Participation in testing the chambers before shipping them to CERN.
2. Development of a Monte Carlo program to study the resolution of Zed chambers.
3. Analysis of the test beam data to optimize the drift time determination technique.
4. A Monte Carlo study to determine the importance of Zed chambers in reconstructing K_s^0 tracks.
5. Offline analysis to monitor and to check the quality of Zed chambers data.
6. Various shifts on the Zed chambers and on the OPAL detector.

b) Data Reconstruction

The following contributions were made in the area of data reconstruction:

1. Participation in the software development of reconstructing the secondary vertices.
2. Offline and online shifts to run the data reconstruction programs on the OPAL raw data.

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Chapter 1

Introduction

Particle physics basically deals with the study of the fundamental particles and the interactions between them. In the past decade it has been occupied with studying interactions; a shift from previous times when it was overwhelmed with the search for particles. This shift occurred with the emergence of the Standard Model [1, 2, 3], a model to describe the fundamental interactions (except gravitation) of nature in a unified framework. Within this framework, all matter is composed of two types of fundamental fermions: quarks and leptons, and the interactions between them are described in terms of the exchange of fundamental bosons. The fundamental fermions are postulated to be grouped into generations:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \text{ quarks}^1$$

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \text{ leptons}$$

¹The t quark still remains to be observed.

Different quark types u , d , c , s , t and b are commonly called flavors. Corresponding to each particle listed above, there is an anti-particle. The Z^0 boson, along with W^+ and W^- bosons, mediates the weak force, the photon the electromagnetic force and gluons the strong force.

The mathematical basis of the standard model is the gauge group $SU(3) \otimes SU(2) \otimes U(1)$ representing a unified framework for electromagnetic, weak, and strong interactions; where $SU(2) \otimes U(1)$ and $SU(3)$ respectively represent the 'electroweak' and the 'strong' sector of the model. Quantum chromodynamics (QCD) is the $SU(3)$ gauge theory of strong interactions [4]. Clearly, the precision tests of QCD are as important as tests of the electroweak sector of the Standard Model.

In QCD, the perturbative approach is essentially the only viable method [5] to extract some testable quantitative predictions from the theory. e^+e^- annihilation into hadrons ² has been providing an adequate testing ground for QCD at PETRA, PEP and TRISTAN energies of 30-60 GeV [6, 7, 8, 9, 10]. There are a number of observables whose distributions can be calculated in perturbative QCD as a power expansion in α_s , often called the strong coupling constant. In analogy to electromagnetic theory where the probability for photon emission by a charged particle is given by the fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$, in QCD the probability for gluon emission by a colored quark is characterized by $\alpha_s = \frac{g_s^2}{4\pi\epsilon_0\hbar c}$; where g_s is the color charge. As the value of α_s apparently decreases with an increase in the center of mass energy E_{cm} , the prospects of testing perturbative QCD at present LEP energies (ie at $E_{cm} \approx M_{Z^0} \approx 91$ GeV) are richer than ever. In chapter 2, we present a review of some aspects of QCD relevant to this thesis. The details of QCD

²particles composed of quarks

theory and its steady development is reviewed, for example, in references [11, 12, 13, 14, 15, 16, 17, 18, 19].

In the framework of QCD, hadronic decays of the Z^0 are associated with the production of partons (that is quarks and gluons), which subsequently materialize into collimated showers of hadrons called jets. We study jet rates and other jet related observables such as C planarity, oblateness and thrust¹ to test the predictions of perturbative QCD. The analysis presented in this thesis is based on the data collected with the OPAL detector at the CERN e^+e^- collider LEP. The OPAL detector, the event selection criteria, and the cuts applied to the data in this analysis are described in chapter 3. For a detailed description of the detector one is referred to [20].

Perturbative QCD calculations for jet rates and other jet related observables are available only up to second order, and they are performed at the parton level while only the final hadrons are actually observed in the detector. The hadronization process (fragmentation of partons into hadrons) is not calculable in QCD. Thus, in order to make precision tests of QCD, one has to study the higher order effects and the hadronization effects. A complete description of the process $e^+e^- \rightarrow \text{hadrons}$ is commonly described by phenomenological models using Monte Carlo simulation programs. We study the higher order and the hadronization effects in chapter 4 using the Jetset (version 7.2) Monte Carlo (MC) [21].

In chapter 5, we compare the measured distributions of jet rates and C planarity to the Jetset MC. We find that the data is well described by the MC. Thus we conclude that this MC can reliably be used to make the

¹These observables are defined in chapter 2.

hadronization corrections to the data in order to compare it with $o(\alpha_s^2)$ QCD calculations.

Perturbative QCD contains the renormalization scale μ used to regularize the infinities arising in the calculations [22]. This is often called an unphysical parameter of which the physical quantities, calculated to all orders in the perturbative expansion, should be independent. But the perturbative expansion truncated at a finite order does depend upon the choice of the scale. The theory itself does not predict at which value of the scale the second order formulae should be evaluated. In this thesis, we vigorously deal with this issue.

In chapter 6, the experimental data corrected for the detector effects and for the hadronization effects is compared with $o(\alpha_s^2)$ QCD calculations. This is done in two ways: 1) by fixing the renormalization scale μ to E_{cm} , 2) by keeping the scale a free parameter in the fit and hence optimizing it. In order to give a complete accountability of the theoretical uncertainties we have used four different jet algorithms. For each observable, the QCD parameters are determined from the fit of only one bin and the comparison of the whole distribution, calculated using these values of the parameters, to the data is taken as a test of QCD.

In chapter 7, The strong coupling constant $\alpha_s(M_{Z^0})$ is determined from the differential distributions of 2- and 3-jet rates, and of C planarity. The uncertainties due to the higher order effects, the hadronization effects, the renormalization scale ambiguity, and due to the experimental systematics are estimated.

In chapter 8, we extend this analysis to two other observables: oblateness and thrust. The sensitivity of these observables to hadronization and higher order effects is studied. The differential distributions of these observables, calculated in second order QCD, are compared with the data and $\alpha_s(M_{Z^0})$ is thus determined.

Although QCD itself would not predict the renormalization scale at finite order, various QCD inspired theoretical procedures have been proposed to optimize the scale to finite order QCD. In chapter 9, we use these procedures to calculate the optimized renormalization scale for the observables under study. These theoretical predictions are compared with the optimized scale deduced from the experimental data. Also in chapter 9, we present an experimental technique to eliminate the scale ambiguity in the final results of $\alpha_s(M_{Z^0})$.

The main results and conclusions from this study are summarized in chapter 10.

Chapter 2

QCD at the Z^0 Peak

Only quarks and gluons, together called partons, experience and transmit strong forces. The description of these forces necessitated a new theoretical concept called *color*. Each quark flavor is available in three possible colors; red, blue and green. Antiquarks are anti-red, anti-blue and anti-green. Gluons bear two labels, one color and one anti-color, in such a way that the color is conserved at each quark-quark-gluon vertex. For example, a green quark can turn into a blue quark by emitting a green anti-blue gluon. Only colored particles can emit or absorb gluons. The corresponding theory of strong forces is called Quantum Chromodynamics (QCD).

In this chapter, we review some of the aspects of QCD, relevant to our study. In section 2.1, the effects of QCD in e^+e^- annihilation are discussed as a perturbation to electroweak theory ¹; and the definition of asymptotic freedom and color confinement is presented. The concept of higher order QCD calculations is introduced in section 2.2. In section 2.3, the strong

¹a unified theory of electromagnetic and weak interactions

coupling constant α_s is discussed. The main problems of testing QCD at the experiment are discussed in section 2.4. In section 2.5, the definitions of jets, C planarity, thrust and oblateness are presented.

2.1 e^+e^- Annihilation and QCD

The multihadron production in e^+e^- collisions at the Z^0 peak energy is described by the processes:

$$e^+e^- \longrightarrow Z^0 \longrightarrow q\bar{q}(g\dots) \longrightarrow \textit{hadrons} \quad (2.1)$$

and

$$e^+e^- \longrightarrow \gamma \longrightarrow q\bar{q}(g\dots) \longrightarrow \textit{hadrons} \quad (2.2)$$

A schematic picture of this process is shown in Fig. 2.1. An e^+e^- pair annihilates into a photon or a Z^0 boson, which decays into a primary $q\bar{q}$ pair. In the second step, quarks may radiate gluons, like electrons radiate photons. Unlike photons, gluons may further radiate. This may develop a shower of partons. It is this stage where QCD calculations are performed. In a third step, the partons fragment into a number of observable hadrons. This process, called hadronization, is not calculable in QCD and is described by phenomenological Monte Carlo (MC) models. In the final step, the unstable hadrons decay into other particles which are observed by the detector.

Fig. 2.1 A schematic picture of electron positron annihilation event.

In general, the primary process ($e^+e^- \rightarrow Z^0, \gamma \rightarrow q\bar{q}$) can be completely described by the electroweak interactions with the cross section given by [23]:

$$\sigma(s) = \frac{s}{(s - M_Z^2)^2 + (M_Z \Gamma_Z)^2} \left[\frac{12\pi \Gamma_e \Gamma_f}{M_Z^2} + \frac{I(s - M_Z^2)}{s} \right] + \frac{4\pi\alpha^2}{3s} Q_f^2 N_c^f \quad (2.3)$$

where M_Z is the mass of the Z^0 boson, s the square of the center-of-mass energy, Γ_Z the total decay width of the Z^0 , Γ_e and Γ_f the partial decay widths of the Z^0 to the electron and final state fermion respectively, Q_f and N_c^f the electric charge and the color number of the fermion f respectively, and α the QED ² fine structure constant. The first term in eq. 2.3 arises from the exchange of the Z^0 boson, the last from the photon exchange, and the second is the interference term.

From eq. 2.3, one can obtain the cross section σ_0 for producing the $q\bar{q}$ final state. One should notice that no strong interaction terms appear in eq. 2.3. Strong interactions emerge as perturbations when the primary quarks emit gluons, modifying σ_0 to

$$\sigma = \sigma_0 \sum_{i=0}^{\infty} c_i \left(\frac{\alpha_S}{\pi} \right)^i \quad (2.4)$$

where α_S is the strong coupling constant and the coefficients c_i depend upon the renormalization ³ scheme applied.

The strong interactions of colored quarks and gluons are believed to be described by QCD, a local non-abelian gauge field theory ⁴. The strong in-

²quantum electrodynamics, the theory of electromagnetic interactions

³The term *renormalization* is explained later in this chapter

⁴means the theory obeys the principle of local gauge invariance and allows the gluon-gluon coupling

teractions are assumed to be invariant under interchange of the color quantum number, that is to say that they are described by the symmetry group $SU(3)$. Since a quark can carry one of three possible colors, we can say that the quarks belong to the triplet representation of $SU(3)$. The local gauge invariance of QCD requires the introduction of eight massless gauge bosons, the gluons, which carry pairs of color labels. These are the mediators of the strong interaction and are postulated to belong to an octet representation of $SU(3)$.

The color in strong interactions is analogous to the electric charge in electromagnetic interactions, and both interactions are mediated by a massless vector boson (a gluon or a photon). Nevertheless, the strong forces transmitted by gluons differ significantly from the electromagnetic forces transmitted by photons. Electromagnetic interactions involve two types of charge and an uncharged mediating boson (photon), while strong interactions involve six types of charge (three colors and three anti colors) and eight charged (colored) mediating bosons (gluons). Gluons can couple directly to other gluons (reflecting the non-abelian nature of QCD) and this has no analogue in QED where photons cannot couple directly to other photons (that is to say QED is an abelian theory).

The most salient features of the QCD theory are asymptotic freedom and color confinement. Asymptotic freedom means that the effective coupling (α_s) decreases logarithmically at short distances, ie at high energy, justifying the perturbative approach at high energies. Color confinement is the property that the potential energy between two partons increases approximately linearly with the distance making the color ionization potential infinite. This means partons (quarks and gluons) cannot appear in isolation;

they can exist only within colorless composite hadrons, explaining why only colorless hadrons are observed.

All the observed hadrons may be divided into two classes: baryons and mesons. Baryons are fermions because their spins are half times an odd integer, while mesons are bosons due to their integral spins. Knowing that quarks are spin $\frac{1}{2}$ objects, one can construct a baryon out of three quarks and a meson out of a quark-antiquark pair. For example, the proton is composed of two u and one d quarks and the neutron is composed of two d and one u quarks. The pi meson consists of a (u, d) pair of quarks in three different states: $u\bar{d}$ composing π^+ , $d\bar{u}$ composing π^- , and a superposition of $u\bar{u}$ and $d\bar{d}$ composing π^0 . All other baryons and mesons are composed of other combinations of quark flavors. The colors of the constituent quarks are picked such that the resulting hadron is color neutral.

2.2 Higher Order QCD Calculations

The principle of asymptotic freedom promises a smaller strong coupling constant at high energies, and hence in this energy domain the precision tests of perturbative QCD can be performed. With increasing powers of α_s we obtain 2-partons($q\bar{q}$) , 3-partons($q\bar{q}g$), and 4-partons($q\bar{q}q\bar{q}, q\bar{q}gg$) final states corresponding to $\mathcal{O}(\alpha_s^0)$, $\mathcal{O}(\alpha_s^1)$, and $\mathcal{O}(\alpha_s^2)$ respectively. These parton processes and final states are illustrated in Fig. 2.2.

For simplicity, we will consider γ decay rather than Z decay because the cross section modified by the QCD corrections for the massless partons would be given by the same expression in both cases [24].

Fig. 2.2 Feynman diagrams for parton production: a) electroweak production of $q\bar{q}$, b) first order correction to $q\bar{q}$ production, c) first order $q\bar{q}g$ production, d) second order $q\bar{q}g\bar{q}$ production, and e) some diagrams contributing to second order corrections to $q\bar{q}g$ production.

The first order QCD cross section ($e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}g$) can be calculated from the Feynman diagrams shown in Fig. 2.2c, which turns out to be

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{dx_q dx_{\bar{q}}} = \frac{2\alpha_S}{3\pi} \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})} \quad (2.5)$$

where $x_q = 2E_q/E_{cm}$ and $x_{\bar{q}} = 2E_{\bar{q}}/E_{cm}$.

Considering the process $\gamma^* \rightarrow q\bar{q}g$ in the CM frame and assuming massless partons, it is trivial⁵ to show that

$$1 - x_q = \frac{2}{E_{cm}^2} E_q E_g (1 - \cos\theta_{qg}), \quad (2.6)$$

where θ_{qg} is the angle between the directions of the quark $\bar{q}$ and the gluon g . To calculate the first order QCD correction to the electroweak cross section given by 2.3, one is supposed to integrate equation 2.5 over x_q and $x_{\bar{q}}$ from 0 to 1. The integrand diverges as x_q or $x_{\bar{q}} \rightarrow 1$, an example of a problem commonly encountered in perturbative QCD calculations. From equation 2.6, one can notice that this situation would occur when the gluon is soft (ie $E_g \rightarrow 0$.) or when the gluon is collinear with one of the quarks (ie $\cos\theta_{qg}$ or $\cos\theta_{\bar{q}g} \rightarrow 1$). The former type of divergence is called an infrared divergence, and the latter one a collinear divergence or mass singularity.⁶

In order to proceed, one has to adopt a regularization procedure to handle infrared and collinear singularities. One way to do this is to assign a fictitious mass m_g to the gluon and to recalculate the cross section σ_m for Feynman

⁵ $1 - x_q = \frac{E_{cm}^2 - 2E_q E_{cm}}{E_{cm}^2} = \frac{1}{E_{cm}^2} (p_\gamma^2 - 2p_q \cdot p_\gamma) = \frac{1}{E_{cm}^2} (2p_q \cdot p_g) = \frac{2E_q E_g}{E_{cm}^2} (1 - \cos\theta_{qg})$

⁶ because if q or g had non zero mass, $\cos\theta_{qg} = 1$ would be kinematically impossible.

diagrams in Fig. 2.2c which previously led to divergences. A straightforward (though lengthy) calculation yields:

$$\frac{\sigma_m}{\sigma_0} = \frac{2\alpha_S}{3\pi} \left[\ln^2\left(\frac{m_g}{E_{cm}}\right) + 3\ln\left(\frac{m_g}{E_{cm}}\right) - \frac{\pi^2}{3} + 5 \right] \quad (2.7)$$

Notice that 2.7 is divergent as $m_g \rightarrow 0$, as expected. In order to complete the first order perturbation corrections, one has to include the Feynman diagrams in Fig. 2.2b, the so called virtual gluon diagrams. The contribution of these terms (σ_v) is calculated to be:

$$\frac{\sigma_v}{\sigma_0} = \frac{2\alpha_S}{3\pi} \left[-\ln^2\left(\frac{m_g}{E_{cm}}\right) - 3\ln\left(\frac{m_g}{E_{cm}}\right) + \frac{\pi^2}{3} - \frac{7}{2} \right] \quad (2.8)$$

The total first order QCD correction (σ_1) is the sum of 2.7 and 2.8 :

$$\frac{\sigma_1}{\sigma_0} = \frac{\sigma_m}{\sigma_0} + \frac{\sigma_v}{\sigma_0} = \frac{\alpha_S}{\pi} \quad (2.9)$$

which is finite and independent of m_g .

Thus the cross section corrected to first order becomes

$$\sigma = \sigma_0 \left[1 + \frac{\alpha_S}{\pi} \right] \quad (2.10)$$

Some of the Feynman diagrams for second order perturbation theory are shown in Fig. 2.2 . The second order calculations are of vital interest because QCD starts showing its full gauge structure (eg gluon self coupling, renormalization scale etc.) only from second order upward. Several independent

calculations of second order QCD are available [25, 26, 27, 28]. With the second order corrections, the QCD cross section is modified to

$$\sigma = \sigma_0 \left[1 + \frac{\alpha_S}{\pi} + c_2 \left(\frac{\alpha_S}{\pi} \right)^2 \right] \quad (2.11)$$

where the coefficient c_2 would depend upon the renormalization procedure.

2.3 The QCD Running Coupling Constant

In e^+e^- annihilation, a typical QCD cross section can be calculated in powers of α_S as given by equation 2.4. This is why various QCD tests at e^+e^- colliders involve the determination of α_S . The coefficients c_i are to be computed from the appropriate Feynman diagrams. In evaluating Feynman diagrams which contain loops, divergent integrals over loop momenta occur. The divergent expressions are made finite by some regularization procedure, the divergences are absorbed into the definitions of physical quantities, and the process is called renormalization. This is done by some specified (but arbitrary) prescription, which introduces a new dimensional scale μ . The renormalization scale dependence of the effective QCD coupling constant α_S is given by the equation [29, 30, 31]

$$\begin{aligned} \mu^2 \frac{d\alpha_S(\mu^2)}{d\mu^2} &= -b_0 \alpha_S^2 - b_1 \alpha_S^3 - b_2 \alpha_S^4 - \dots, \\ b_0 &= \frac{33 - 2n_f}{12\pi} \\ b_1 &= \frac{153 - 19n_f}{24\pi^2} \end{aligned}$$

$$b_2 = \frac{2857 - \frac{5033}{18}n_f + \frac{325}{54}n_f^2}{(4\pi)^3} \quad (2.12)$$

where n_f is the number of quark flavors involved⁷.

In solving this differential equation for α_s , a constant of integration Λ is introduced. This is one fundamental constant of QCD to be determined from experiment. The definition of Λ is arbitrary. One way is to parameterize the solution of equation 2.12 in terms of Λ as follows [32]

$$\begin{aligned} \alpha_s(\mu, \Lambda) = & \frac{1}{b_0 \ln(\mu^2/\Lambda^2)} \left[1 - \frac{b_1 \ln(\ln(\mu^2/\Lambda^2))}{b_0^2 \ln(\mu^2/\Lambda^2)} + \left(\frac{b_1 \ln(\ln(\mu^2/\Lambda^2))}{b_0^2 \ln(\mu^2/\Lambda^2)} \right)^2 \right. \\ & \left. - \frac{b_1^2 (\ln(\ln(\mu^2/\Lambda^2)) + 1) - b_0 b_2}{b_0^3 \ln^2(\mu^2/\Lambda^2)} \right] \\ & + o\left(\frac{\ln^2[\ln(\mu^2/\Lambda^2)]}{\ln^3(\mu^2/\Lambda^2)}\right). \end{aligned} \quad (2.13)$$

The coefficients b_0 and b_1 are independent of the choice of the renormalization scheme, while b_2 is scheme dependent and its value in 2.12 is given in the $\overline{MS}$ scheme⁸. However, physical quantities calculated to all orders of perturbation should be independent of the choice of the renormalization scheme.

2.4 Prospects and Problems of testing QCD

QCD stands as a building block of the standard model of the known interactions except gravitation, based on the gauge group $SU(3) \otimes SU(2) \otimes U(1)$.

⁷means the number of quark flavors kinematically accessible at a given energy. Quark flavors mean the quark types: u, d, c, s, t and b.

⁸a renormalization scheme known as modified minimal subtraction

Clearly, precise experimental tests of QCD are as important as tests of the electroweak sector of the standard model. However, testing QCD is more difficult than testing the electroweak theory. The coupling constant in QCD is not very small and the perturbative expansions are only relatively slowly converging because $\alpha_s \gg \alpha_{QED}$. In electroweak theory, leptons, photons, $W^\pm$, and Z^0 are fields in the theory as well as the particles observed in the detector; while QCD is a theory of confined quarks and gluons but only the hadrons are actually observed. Hence, non perturbative effects connected with soft parton⁹ shower and hadronization tend to obscure the underlying simplicity of parton dynamics. In addition, QCD calculations higher than second order are not available. Thus, the theoretical predictions of many physical quantities may be unable to meet the experimental accuracy. Hence, we must assess the associated theoretical uncertainties due to the unknown higher order corrections and the nonperturbative hadronization effects.

Although the physical quantities, calculated to all orders in perturbation theory, should not depend upon the renormalization scale, a truncated series may exhibit some dependence. One, therefore, has to address the question: what is the best choice for the renormalization scale μ ? The QCD theory itself does not provide the answer to this question. Even the higher order corrections would not "fix" the scale, but they might render the theoretical predictions less sensitive to its variation.

There are a number of quantities, known as infrared and collinear safe quantities, which can be predicted by the theory, and compared with the data. Infrared safe means that the quantity should not change abruptly if one soft particle is added to the final state, and collinear safe means that

⁹soft parton means parton with very small momentum

the quantity does not change abruptly if one particle in the final state is split into two, sharing its momentum. For many of these quantities the QCD calculations up to second order are available. However, for some of these, higher order corrections and hadronization effects could be important. Therefore, it has been proposed [33] that a good strategy of testing QCD at LEP would be to measure a number of such quantities, compare them to the theoretical predictions and look for the general agreement or disagreement. In this study we use jet rates, C planarity, oblateness and thrust as our tools to test QCD.

2.5 Jets, C Planarity, Thrust and Oblateness in QCD

In this thesis, we study the following observables:

1. Jets

It is a universal property of multihadron production that the final particles are not uniformly distributed over phase space. They are rather bundled into collimated clusters of particles called jets. In e^+e^- collisions, up to center of mass energy of about 3 GeV the dominant feature of the final state is resonance production(ie $e^+e^- \rightarrow \rho^0, \omega, \phi, \psi$ etc.). But at higher energy, parton dynamics suggests that final particles emerge in the form of jets traveling more or less in the directions of the original partons. The essential assumption leading to the expectation of jets is that the transverse momenta of parton fragmentation products come mainly from soft

processes and stay small, while their longitudinal momenta may increase with the original parton energy. Jets in e^+e^- annihilations began to be observed at the SPEAR energy range of $E_{cm}=6-8$ GeV, in 1975 [34]. At PEP and PETRA energies ($E_{cm}=30-40$ GeV), it was found to be the dominant feature of multihadron production [35, 36, 37, 38, 39].

Within the framework of QCD, multihadron production in e^+e^- collisions is associated with the production of partons which subsequently materialize into jets of hadrons. The relative multijet production rates are determined by the strong coupling constant α_s , which is expected to decrease with an increase in energy according to the concept of asymptotic freedom. Thus, the experimental studies of jet production rates are suitable to test the basic concepts of QCD and to determine the free parameters such as α_s . The jet axis angular distribution from the SPEAR data [34] provided evidence, for the first time, that quarks must have spin $\frac{1}{2}$. The study of 3-jet events collected by PETRA experiments confirmed the existence of gluons with zero electric charge and spin 1 [40, 41, 42], in accordance with the gluon bremsstrahlung process predicted by QCD. The Jade Collaboration [43] demonstrated, by studying the energy dependence of 3-jet event production rates, that the strong coupling constant α_s decreases with increase in energy as predicted by the principle of asymptotic freedom in QCD. First signs of the presence of the gluon self coupling were reported by AMY collaboration [44] from a study of 4-jet events around $E_{cm} = 56$ GeV.

The jet structure of the event can also be measured indirectly by introducing the infrared safe variables which do not require the explicit jet counting of the event. C planarity ¹⁰, oblateness and thrust, defined below, are few examples.

2. C Planarity

C planarity is extracted from the eigenvalues of the (infrared safe) momentum tensor [45, 46]

$$\theta_{ij} = \sum_a \frac{P_a^i P_a^j}{|P_a|} / \sum_a |P_a| \quad (2.14)$$

where the sum on a runs over all the final state hadrons and P_a^i is the i^{th} component of the three momentum of hadron a in the center of mass frame. If the tensor θ is normalized to have unit trace, then C planarity may be defined in terms of its eigenvalues λ_n as

$$C = 3(\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1) \quad (2.15)$$

From equations 2.14 and 2.15 one may grasp a physical insight of C planarity. For two back to back jets, there is only one degree of freedom and hence only one of the three eigenvalues λ_i has to be non zero. Thus from equation 2.15, the C value should ideally be zero. For planar events there are two degrees of freedom and therefore the C value can range from zero to 3/4. Thus, this observable is actually a measure of the planarity of events and hence the name C planarity. For events with larger number of jets, C planarity can extend its value up to 1.

¹⁰The name C parameter is used in the past, but the word "planarity" rather than "parameter" is a better representation of its meaning.

3. Thrust

The thrust T [47] is defined as

$$T = \max \left(\frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|} \right); \quad (2.16)$$

where i runs over all the final state particles, and the axis $\hat{n}$ is chosen to satisfy the equation. For a two parton final state (a perfect back to back jet) T would have a value of 1. For a three parton final state (a planar event) T is in the region $1 \geq T \geq 2/3$. For a larger number of final partons (≥ 4 -jet events) the T value can be as low as $1/2$.

4. Oblateness

The definition of oblateness [48] can be derived from equation 2.16. The axis $\hat{n}$ that satisfies equation 2.16 may be named as thrust axis $\hat{n}_{thrust}$. Another axis $\hat{n}_{major}$ is defined in a plane perpendicular to the thrust axis and satisfying the equation 2.16. The T value thus obtained is called T_{major} . Finally, an axis $\hat{n}_{minor}$ is chosen orthogonal to both $\hat{n}_{thrust}$ and $\hat{n}_{major}$, and the equation 2.16 evaluates what is called T_{minor} . The oblateness is simply defined by

$$O = T_{major} - T_{minor}. \quad (2.17)$$

The value of O will ideally be zero for collinear and spherical final states, and will extend from zero to $1/\sqrt{3}$ for 3-jet final state.

Chapter 3

The Apparatus and Data Selection

The data were recorded with the OPAL detector which is one of the four experiments (the other three are ALEPH, DELPHI and L3) situated at LEP, the e^+e^- collider at CERN. OPAL is a multipurpose apparatus with excellent acceptance for detecting the Z^0 decays over a solid angle of nearly 4π . The basic requirement guiding its design has been to detect all types of interactions occurring at e^+e^- collisions with efficient and accurate reconstruction and unambiguous classification of the events. The main features of the detector are:

- Tracking of charged particles in the central region of the solenoidal coil with measurements of their direction and momentum, particle identification by dE/dx and reconstruction of primary and secondary vertices at and near the interaction region.
- Identification of photons and electrons and measurement of their energy.

- Measurement of hadronic energy by total absorption using the magnet yoke instrumented as a calorimeter.
- Identification of muons by measurement of their position and direction within and behind the hadron absorber.
- Measurement of the absolute machine luminosity using Bhabha scattering¹ events in the very forward direction with respect to the beam line.

The general description of the OPAL detector is presented in section 3.1. In section 3.2, the trigger and the data acquisition system are discussed. The general trigger and event selection criteria are presented in section 3.3. The multihadron event selection and the cuts applied in this analysis are discussed in section 3.4. In section 3.5, some properties of multihadron events are discussed. The main properties of the OPAL detector and the main parameters of the LEP collider are listed in tables 3.1 and 3.2 respectively.

3.1 The Detector System

The general layout of the detector is shown in Fig. 3.1, indicating the location and relative size of the various components. Fig. 3.2 shows cross sections of the detector parallel and perpendicular to the beam axis. The detector consists of a system of central tracking chambers inside a solenoid which provides a uniform magnetic field of 0.435 T.

¹The elastic scattering process $e^+e^- \rightarrow e^+e^-$ is called Bhabha scattering.

Detector Element	Composition and Coverage	Main Properties
Beam pipe	0.1 mm aluminium pipe layers of carbon fiber $r_i = 78 \text{ mm}$	radiation length of carbon fiber = 23.5 cm
Vertex Chamber	36 sectors axial, 12wires/sector 36 sectors stereo (4°), 6wires/sector 1m long, $r_i=8.5\text{cm}$ $r_o=23.5\text{cm}$ $ \cos\theta \leq 0.95(0.91)$ for axial (stereo) layer :	$\sigma_{r\phi} = 55\mu\text{m}$ $\sigma_z = 700\mu\text{m}$
Jet Chamber	24 sectors, 159wires/sector 4 m long, $r_i = 24.5\text{cm}$, $r_o = 185\text{cm}$ $43^\circ \leq \theta \leq 137^\circ$ for 159 wires	$\sigma_{r\phi} = 135\mu\text{m}$ $\sigma_z = 6\text{cm}$
Zed Chambers	24 chambers, 8 cells/chamber, 6wires/cell 4 m long, $r_i=188\text{cm}$, $r_o = 196\text{cm}$ $44^\circ \leq \theta \leq 136^\circ$	$\sigma_{r\phi} = 1.5\text{cm}$ $\sigma_z = 100 - 350\mu\text{m}$
Magnet	6.3 m long, 2.18m in radius max. current 7000 A, max. power 5 MW overall magnet weight 2800 t	central field 0.435 T
T.O.F Barrel	160 scintillation counters 6.84m long, 2.36m mean radius $ \cos\theta \leq 0.82$	charged particle identification in the range of 0.6-2.5 GeV helps in rejecting cosmic rays
E.M. Calorimeter	16 presampler chambers $r = 2.39\text{m}$, length 6.62m 9440 lead glass blocks, $r=2.46\text{m}$ $ \cos\theta \leq 0.82$	detects e^- , e^+ , and γ in the range 10's MeV-100GeV provides π^0 γ and e^- - hadron discrimination
Hadron Calorimeter	9 layers of chambers radii 3.39 - 4.39 m	measures the energy of hadrons emerging from em calorimeter assists in μ identification
Muon Chamber	110 drift chambers, 2wires/chambers 10 m long, 5 m radius 93 % of 4π	identifies μ 's

Table 3.1: Properties of the OPAL Detector.

Parameter	Value
Beam energy (Mean)	45.5 GeV
Circumference	26.658 Km
Number of e^+ and e^- bunches	4
Bunch length	1.8 cm
Particles per bunch	4.16×10^{11}
Luminosity	$8 \times 10^{30} \text{cm}^{-2} \text{s}^{-1}$
Time separation between crossings	22.2 μs
$e^-(e^+)$ beam current	2.98 mA
Transverse dimensions of beam at intersections	$300 \times 20 \mu\text{m}$
Accelerating power	10 MW
Energy loss by synchrotron radiation	260 MeV/turn

Table 3.2: Main Parameters of LEP I.

OPAL 1991-1992 to 1993-1994
CERN

Fig. 3.1 A schematic diagram of OPAL detector.

Fig. 3.2 Cross section of a quadrant of the OPAL detector.

The solenoid coil is surrounded by a time-of-flight counter array, a lead glass electromagnetic calorimeter with a presampler, an instrumented magnet return yoke serving as a hadron calorimeter and four layers of outer muon chambers. A forward detector measures the luminosity.

OPAL's coordinate system is illustrated in Fig. 3.2 ; the x axis is horizontal and points approximately towards the center of LEP, the y axis is approximately vertical, and the z axis is in the e^+ beam direction. The polar angle, θ , is measured from the z axis, and the azimuthal angle, ϕ , from the x axis about the z axis.

The central tracking system is divided into a precision vertex chamber, a large volume jet chamber and z-chambers. The main tracking is performed with the jet chamber, a drift chamber of approximately four metre length and two metre radius with 159 layers of wires, providing both high redundancy and precision for the reconstruction of multihadronic events. The central tracking system operates at high pressure and is therefore contained inside a pressure vessel whose cylindrical structure provides mechanical support to the solenoidal coil mounted around it. The pressure vessel is closed at the two ends by bell shaped covers.

In the initial 1989 operation of the OPAL detector, the vertex drift chamber was operated at a drift field of 2.5 KV/cm and the average spatial resolution $\sigma_{r-\phi}$ was 55 μm . The jet chamber was operated at a drift field of 890 V/cm, and the average resolution $\sigma_{r\phi}$ was found to be 135 μm . The drift field for the Z chambers was 800 V/cm, and the average intrinsic z resolution (σ_z) was around 200 μm . In measuring the primary vertex in the central detector, the resolution of the impact parameter (measured with events $e^+e^- \rightarrow \mu^+\mu^-$)

was found to be $75 \mu\text{m}$ in the $r - \phi$ plane and 2mm in the r - z plane. The invariant mass resolution σ_{K^0} (for $K^0 \rightarrow \pi^+\pi^-$) is 8 MeV , consistent with the results from previous Monte Carlo studies [49].

The time-of-flight (TOF) system covers the barrel region $|\cos\theta| < 0.82$. It consists of 160 scintillation counters, 6.8 m long and 45 mm thick, located at a radius of 2.4 m . The TOF has performed with time resolution of 460 ps . (without using the external z information).

The main electromagnetic calorimeter consists of a cylindrical array of 9,440 lead glass blocks of 24.6 radiation lengths (X_0) thickness, covering $|\cos\theta| < 0.82$ in the barrel region, and 2,264 lead glass blocks of $20 X_0$ thickness in the endcaps, covering $0.81 < |\cos\theta| < 0.98$. Each block subtends a solid angle of approximately $40 \times 40 \text{ mrad}^2$ and projects towards the interaction region in the barrel region and along the beam direction in the endcaps. The two sections of the electromagnetic calorimeter together cover 98% of the solid angle.

Just inside the lead glass calorimeter and surrounding the pressure vessel, thin gas detectors (presamplers) provide measurements of the position and energy of electromagnetic showers which start in front of the lead glass. The overall energy resolution is improved by correcting for the energy lost in the material in front of the lead glass calorimeter. The measured energy resolution (σ_E/E) of the combined presampler and lead glass system is typically $(0.2 + \frac{6.3}{\sqrt{E}})\%$, where E is the electromagnetic energy in GeV .

The iron yoke of the magnet is made of welded plates, 10 cm thick, separated by air gaps. The yoke in the barrel and outer endcap region has air gaps of 2.5 cm width which are instrumented with streamer tubes and form

the main hadron calorimeter. The poles with air gaps of 1 cm are instrumented with thin gas multiwire chambers and complete the coverage of the entire solid angle for hadronic shower detectors. Hadronic showers are most of the time initiated in the lead glass calorimeter. For measuring the hadronic energy the signals of the hadron calorimeter are combined with those of the lead glass. The energy resolution of the combined hadron calorimeter varies from $\frac{100}{\sqrt{E}}\%$ for energies below 15 GeV to $\frac{140}{\sqrt{E}}\%$ at 50 GeV.

The entire iron structure is surrounded by several layers of chambers in order to identify muons by measuring the position and direction of all charged particles which have traversed the iron absorber.

The luminosity² of the colliding beams is measured by the observation of small angle Bhabha scattering with the forward detector. This device consists of two identical elements surrounding the beam pipe at either end of the central tracking system. Its acceptance covers angles from 40 to 150 mrad from the beam and 2π in azimuth. The two main elements of the forward detector are a lead-scintillator calorimeter of $24 X_0$ thickness, divided into 16 azimuthal segments, and three layers of position measuring tube chambers located behind the first $4 X_0$ of the calorimeter.

3.2 Trigger and Data Acquisition System

The trigger system [50] is designed to provide a high efficiency for the various physics reactions, and a good rejection of backgrounds arising from cosmic rays, from interactions of the beam particles with the gas inside the beam

²Luminosity is defined as number of events per unit time per unit area of cross section.

pipe or the wall of the beam pipe, and from noise. Most of the physics reactions are triggered by several independent conditions imposed on the subdetector signals. This redundancy leads to a high detection efficiency and greatly facilitates the measurement of this efficiency.

The 4π range in solid angle covered by the detector is divided into 144 overlapping bins, 6 bins in θ and 24 bins in ϕ . The subdetectors deliver trigger signals matched as closely as possible to this binning. This fine segmentation allows low thresholds for the calorimeters, since analogue sums are made over only a small region and thus noise is reduced. Besides the θ - ϕ signals, the subdetectors deliver "stand-alone" signals, derived from total energy sums or track counting.

The trigger signals from the various subdetectors are logically combined in the central trigger logic. θ - ϕ signals are used for hit counting, for the definition of back-to-back hits and to build detector coincidences correlated in space. Programmable conditions are imposed on the θ - ϕ matrix outputs and on the stand-alone signals to decide whether an event is accepted or rejected.

The DAQ system [51] is based on a multi-level tree structure with data being buffered at each level. For each event, data are collected from 16 acquisition subsystems (14 subdetectors, trigger and track-trigger), and from one of the stations of the slow control system. More than 150,000 analogue signals coming from the subdetectors produce after first data reduction 100 kbyte of data, on average, per multi-hadronic event. The data collection and processing is performed in a series of stages, briefly described below.

At OPAL, crossings of one of the four bunches of electrons and one of

the four bunches of positrons occur every $22.2 \mu\text{s}$. Based on fast information from the subdetectors preferably only those bunch crossings with a genuine e^+e^- interaction are selected by a programmable and flexible trigger system. The primary rate of 45 kHz is brought down to 1–5 Hz to be handled by the data acquisition system. The overall scheme of event triggering, readout, monitoring and data recording is shown in Fig. 3.3.

The read out system has a distributed tree structured architecture. Microprocessors of the 68020/68030 [52] type, running the OS9 [53] operating system, are used in a VME bus [54] based system for data compression and data moving as well as monitoring. The VME crates are interconnected using a fast parallel link for (sub-)event routing. The sub-events containing the digitized information of the subdetectors are buffered in memories controlled by the subdetector processors and then collected and merged into a single data structure by an “event builder” VME system. This system also acts as an event buffer for a microprocessor matrix, where up to ten 68030, in parallel, perform a first analysis of the complete events, and undesired background can be rejected at this stage. From here, events are transferred from the experimental area via an optical link to the surface, where they are buffered in another microprocessor controlled VME system. The selected events are sent from here to a set of fast processors for online event reconstruction. Finally, the events are transferred to the main experiment online computer, a VAX 8700, and stored on magnetic tape cartridges.

Fig. 3.3 Overall scheme of event triggering, readout, monitoring and data handling.

3.3 The Trigger Conditions and Event Selection

The analysis presented in this thesis is based on the data taken with the OPAL detector in 1990.

The data have been collected at center-of-mass (c.m.) energies between 88.28 and 95.04 GeV. For the present analysis, the difference between these energies is unimportant and data from the different c.m. energies are merged. The Monte Carlo event samples which are compared to these data were generated with a c.m. energy corresponding to the weighted mean of the energy of the events in the experimental sample.

The data acquisition system was triggered if at least one of the following conditions was fulfilled: (1) the energy deposited in the electromagnetic calorimeter exceeded 6 GeV; (2) there was a signal in at least three non-adjacent scintillator bars in the time-of-flight system; (3) there were at least two charged tracks detected by a trigger logic utilizing the jet chamber information, for which both tracks had a transverse momentum greater than 450 MeV/c with respect to the beam axis and pointed toward the vicinity of the collision point.

3.4 Multihadron Event Selection

An on-line event filter [55] using the lead glass and time-of-flight information selected hadronic Z^0 decay candidates. The combined trigger and selection efficiencies ($97 \pm 0.6\%$) and the residual background from τ pairs, two-photon

processes, beam-pipe interactions and cosmic radiation ($< 1\%$) have been determined by various methods which are described in [56]. The following four requirements defined a multihadron candidate: (i) at least 7 electromagnetic clusters, (ii) at least 5 tracks, (iii) a total energy deposited in the lead glass of at least 10% of the centre-of-mass energy

$$R_{vis} = \Sigma E_{clus} / \sqrt{s} > 0.1,$$

where E_{clus} is the energy of each cluster, and (iv) the energy imbalance along the beam direction satisfied

$$|R_{bal}| = |\Sigma(E_{clus} \cdot \cos \theta)| / \Sigma E_{clus} < 0.65,$$

where θ is the polar angle of the cluster. The cut on the number of clusters and the number of tracks efficiently eliminated Z^0 decays into charged lepton pairs. The R_{vis} cut discarded two-photon and beam-gas events. The cut in R_{bal} rejected beam-wall, beam-gas and beam-halo events, and cosmic rays in the end caps.

In addition to the event selection criteria, further requirements are applied to the data in order to prepare a set of well contained and completely reconstructed events. For each event, at least five charged tracks originating from the vertex region (with a closest distance to the vertex of at most 5 cm perpendicular to the beam axis and 25 cm in the beam direction) and a minimum particle momentum of 150 MeV transverse to the beam are required. Each charged track must be reconstructed by at least 40 good hits in the central jet chamber, and the angle between the particle direction and the beam line has to exceed 20 degrees. Electromagnetic energy clusters as reconstructed in the lead glass calorimeter are required to be detected by

at least two neighbouring lead glass blocks with a measured total energy of more than 200 MeV. No correction for energy deposited by associated charged particles is applied ³. At least eight particles thus reconstructed in the electromagnetic shower counters are required for each event, in the analysis of neutral particles only. For the reconstruction of particle energies, neutral particles are assumed to be photons, and charged particles to be pions. The angle between the principal event axis (the thrust axis) and the beam direction has to be larger than 26 degrees.

After these requirements, around 130,000 events with a mean centre of mass energy of 91 GeV remained in the analysis. The overall efficiency of the event selection was about 85 %.

3.5 Some Properties of Multihadron Events at LEP

A multihadron event at LEP contains, on average, 100 kbyte of data. The average number of charged particles [59] passing through the central tracking system is about 20. In a multihadron event, most of the particles have their momentum value [60] around 1 GeV or lower. The multiplicity and momentum/energy distributions of charged tracks in the central tracking system and clusters in the electromagnetic calorimeters are presented in Fig. 3.4.

³The double counting of energy from charged particles is largely reduced since charged tracks usually deposit energy only in one or two counters, while photon showers spread over many counters. The remaining effect of double counting does not affect the following analysis.

Some examples of the multihadron events, observed by the OPAL detector at LEP, are presented in Fig. 3.5. The charged tracks reconstructed in the central tracking system are drawn as helices. The energy deposited in electromagnetic and hadron calorimeters is represented by boxes. The arrow in Fig. 3.5 d represents a muon, possibly coming out of the semileptonic decay of a heavy (c or b) quark. One may say that events presented in figures 3.5 a, 3.5 b and 3.5 c are 2-, 3-, and 4-jet events respectively, but such a statement would be very adhoc. The number of jets that a jet algorithm would find in an event would depend upon some resolution parameter which would separate one jet from the other. Depending upon the value of the resolution parameter, the event shown in Fig. 3.5 d, for example, could be interpreted as a 2-, 3-, 4, or 5-jet event.

C planarity, oblateness, thrust and jets, studied in this thesis, are the properties related to the multihadron event shape. For other event shape properties, the reader is referred to [61].

a) Number of charged tracks per event.

b) Number of clusters per event.

c) Momentum distribution of charged tracks

d) Energy distribution of clusters

Fig. 3.4 The multiplicity and momentum distributions of charged tracks and electromagnetic clusters.

Fig. 3.5 Some examples of multihadron events at LEP, as observed by the OPAL detector.

Chapter 4

A Monte Carlo Study of Jet Rates and C Planarity

Comparison of the experimental data with finite order perturbative QCD faces two major difficulties which need to be considered:

1. The QCD calculations for final state observables are available at the most up to second order (with an exception of e^+e^- total hadronic cross section which is calculated up to second subleading order) while actually these observables are given by the infinite sum of Feynman diagrams. However, one may seek the remedy in the strong version of the KLN theorem¹ [57, 58] which states that for a suitably defined (infrared safe) physical quantity, the contributions of soft and collinear parton emission cancel. Thus such quantities can reliably be calculated in a finite order perturbative QCD.

2. The QCD calculations are performed at the parton level while our detector can see only the final hadrons. Here one has to rely on the assumption

¹The work of T. Kinoshita, M. Nauenberg and T. D. Lee

that the infrared safe quantities do not change abruptly from the parton level to the particle level.

Nevertheless, the situation remains that on one hand we have perturbative QCD truncated at second order, and on the other we cannot use QCD (as it stands now) to calculate the hadronization of partons into particles observed by detectors. Hence it is necessary to estimate the theoretical uncertainties introduced by the unknown higher order corrections and by the hadronization effects. Here one turns to the phenomenological models commonly used to provide a complete description of the $e^+e^- \rightarrow \text{hadrons}$ process. We use the Jetset shower Monte Carlo model(version 7.2) to study these effects for the jet cross sections and the C planarity as this model describes our data well in general [61].

In section 4.1, a brief introduction to the Jetset Monte Carlo model is presented. The production of Monte Carlo data is described in section 4.2. In section 4.3, we define the jet algorithm used to calculate the jet rates. The MC data analysis is presented in section 4.4 and the conclusions from this study are drawn in section 4.5

4.1 An Introduction to the Jetset Shower Monte Carlo

The Shower Monte Carlo is available as an option in Jetset (version 7.2). The generation of an $e^+e^- \rightarrow \text{hadrons}$ event can be divided into two major steps. In the first step, partons are produced from the primary process, and in the second step these partons are allowed to fragment into hadrons

(with unstable hadrons decaying further). Several options are available in the Jetset Monte Carlo program to perform these two steps. We choose the option of parton showering for the first step and the string fragmentation for the second.

The basic idea of the shower model is that the primary quark anti quark pair produced in the process $e^+e^- \rightarrow q\bar{q}$ may be very much off the mass-shell and may initiate a parton shower by successive branchings $q \rightarrow qg, g \rightarrow q\bar{q}, g \rightarrow gg$ etc. This evolves jetlike cascades of partons whose virtual masses² decrease with the development of the shower. The shower is halted at a cutoff mass, Q_g . The probability for each branching is given by the so called Altarelli-Parisi equations [16]. The total cross section for a shower is assumed to be proportional to the product of the individual probabilities at each tree level branching. That is to say no interference between tree level Feynman diagrams at various branches is taken into account, hence the name Leading Logarithmic Approximation (LLA). Energy and momentum are conserved exactly at each step of the shower.

A schematic picture of the parton shower evolution is presented in Fig. 4.1 . Partons at the end of the shower are fragmented into colorless hadrons using the Lund string model [63]. The simplest case is a back to back $q\bar{q}$ pair. The physical picture is that of a color flux tube being stretched between the partons. As q and $\bar{q}$ move apart, potential energy stored in the string increases, and it may break into two by producing a new $q\bar{q}$ pair, such that the initial $q\bar{q}$ color singlet is split into two color singlets $q\bar{q}$ and $q\bar{q}$. If the

²When the mass of a particle given by energy momentum conservation does not correspond to its real mass but is allowed by uncertainty principle for a very short time, the particle is called off the mass shell and its mass the virtual mass.

invariant mass of one of these two color singlet systems (or string pieces) is large enough, further breaks may occur.

Fig. 4.1 Schematic picture of parton shower evolution in e^+e^- events.

This string breakup process is assumed to proceed until only the on-mass-shell hadrons remain, each hadron corresponding to a small piece of string. Gluons come into the picture as momentum carrying kinks on the string spanned between a quark and an antiquark end. For example in a $q\bar{q}g$ event, the string is stretched from the q end via g to the $\bar{q}$ end.

The idea of quantum mechanical tunneling is invoked in order to generate $q\bar{q}$. In terms of the transverse mass m_T of q , the probability that $q\bar{q}$ will appear (ie the tunneling probability) is given by

$$\exp\left(-\frac{\pi m_T^2}{\kappa}\right) = \exp\left(-\frac{\pi m^2}{\kappa}\right) \exp\left(-\frac{\pi p_T^2}{\kappa}\right) \quad (4.1)$$

where $m_T^2 = E^2 - P_z^2 = m^2 + P_T^2$, where z is the longitudinal direction of the quark.

where κ is the string tension (mass per unit length), m is the mass of the q , and p_T is its transverse momentum. Hadron transverse momenta are obtained as a sum of transverse momenta of constituent quarks. With given mass and P_T of the hadron, only one degree of freedom remains to be used in four momentum phase space. For the original quark moving out in the z direction, one may select this degree of freedom, say the Z variable as the fraction of $E + P_z$ taken by the hadron out of the available $E + P_z$. This choice is longitudinal boost invariant¹. One may invoke another requirement namely that the fragmentation process as a whole should look the same irrespective of whether the iterative procedure to produce hadrons is started from the q end or $\bar{q}$ end. This requirement of left-right symmetry shapes the probability $f(Z)$ (that a given Z is picked) as

$$f(Z) = \frac{1}{Z}(1 - Z)^a \exp(-bm_T^2) \quad (4.2)$$

The parameters a and b are optimized to fit the experimental data.

4.2 Production of Monte Carlo Data

In order to study the hadronization and higher order effects for the jet cross sections and the C planarity, 50,000 e^+e^- multihadron events were generated using Jetset version 7.2 . Out of several options provided by the Monte Carlo, we chose the default option according to which the primary quark antiquark pair produced by e^+e^- annihilation initiates a parton shower which hadronizes to particles via the Lund string model.

¹invariant under Lorentz transformation in the longitudinal direction

Parameter	Monte Carlo name	Value used
Λ	PARJ(81)	0.29 GeV
σ_q	PARJ(21)	0.37 GeV
a	PARJ(41)	0.18
b	PARJ(42)	0.34 GeV^{-2}
M_Z	PARJ(123)	91.01 GeV
Γ_Z	PARJ(124)	2.467 GeV
E_{cm}	-	91.31 GeV
$\text{Sin}^2\theta_W$	PARU(102)	0.2325

Table 4.1: Values of the main Jetset72 parameters used.

We used the parameter values as optimized to describe our data [61]. Initial state and final state electromagnetic radiations were included. The five known flavors of quarks were allowed to generate according to the relevant probabilities as calculated by the program. The values of the main parameters used are presented in table 4.1 For a given value of the scale parameter Λ , parton showering is determined by the invariant mass cutoff Q_g of the parton below which it does not radiate gluons. The parameter Q_g represents our incapability of calculating all higher orders of QCD. One may study the sensitivity of observables to unknown higher order corrections by varying the value of Q_g . The Monte Carlo run was repeated over a wide range of Q_g values.

4.3 The Cluster Jet Algorithm and the Re-combination Schemes

The criteria for a good infrared safe jet definition is: 1) It is applicable at any order of perturbation theory; 2) It gives a finite cross section at any order of perturbation theory; 3) It is insensitive to hadronization; 4) It is simple to implement both experimentally and theoretically.

In e^+e^- annihilation, infrared safe jet algorithms are provided by the cluster algorithms based on an invariant mass cutoff. For all possible pairs of particles a and b of an event the scaled pair mass

$$y_{ab} = M_{ab}^2 / E_{vis}^2 \quad (4.3)$$

is calculated, where E_{vis} is the total visible energy of the event⁵. The two particles with the smallest value of y_{ab} denoted as y_i , are replaced by a pseudoparticle or "cluster" of four momentum $p_{i,j} = p_i + p_j$. This procedure is repeated until all the y_{ab} exceed a certain threshold value y_{cut} ⁶. The resulting number of clusters is called the jet multiplicity of the event. While this jet resolution parameter (y_{cut}) provides a means to compare experimental jet rates to the theoretical predictions, it also introduces an ambiguity in defining the jet algorithm. The results of the calculations depend on the detailed description for combining two unresolvable jets into a single jet. The $\mathcal{O}(\alpha_s^2)$ QCD calculations are performed for massless partons, whereas a jet formed by adding the two four-momenta of two previously unresolved

⁵charged particles are assumed to be pions and neutrals to be photons.

⁶the invariant mass cutoff

partons may not be massless. An algorithm must be chosen to deal with this mass. There is a certain amount of freedom in how we select this algorithm. The freedom in selecting this algorithm is what introduces the ambiguity.

Several schemes have been invented to combine two partons, with different treatments of the invariant mass of the resulting parton jet. The four most common schemes are "E0", "E", "p0" and "p" and are defined as follows :

a) E-scheme

The scaled invariant mass squared of a pair of partons i and j is calculated from the corresponding four-vectors, $\mathbf{p}_i$ and $\mathbf{p}_j$, according to

$$y_{ij} = \frac{(\mathbf{p}_i + \mathbf{p}_j)^2}{E_{cm}^2}. \quad (4.4)$$

If $y_{ij} < y_{cut}$, partons i and j are replaced by a parton jet k with four-momentum

$$\mathbf{p}_k = \mathbf{p}_i + \mathbf{p}_j. \quad (4.5)$$

This scheme is Lorentz invariant and energy and momentum are strictly conserved. In this sense, it represents a mathematically correct procedure. However the parton jet k has a non-zero mass value which cannot consistently be accounted for in the QCD calculations.

b) E0-scheme

The invariant pair mass is defined as in Eq. 4.4, while the four-momentum of the recombined parton jet k is calculated according to

$$E_k = E_i + E_j$$

$$\vec{p}_k = \frac{E_k}{|\vec{p}_i + \vec{p}_j|} \cdot (\vec{p}_i + \vec{p}_j). \quad (4.6)$$

The space-component $\vec{p}_k$ is rescaled so that the vector k has zero invariant mass. This scheme is not Lorentz invariant. It can be applied only in the laboratory frame and does not conserve the total momentum sum of an event.

c) p-scheme

As in the E0-scheme, the resulting four-vector p_k is constructed such that it has zero invariant mass, according to

$$\begin{aligned} \vec{p}_k &= \vec{p}_i + \vec{p}_j \\ E_k &= |\vec{p}_k|. \end{aligned} \quad (4.7)$$

While this scheme conserves the total momentum in an event, the total energy sum gradually decreases with each recombination of parton pairs.

d) p0-scheme

Jet recombination is treated as in the p-scheme; however with the modification that the effective scaled invariant jet pair mass is calculated according to

$$y_{ij} = \frac{M_{ij}^2}{E_{vis}^2}, \quad (4.8)$$

where E_{vis} is the actual, total energy sum of the event recalculated after each preceding recombination. This, in contrast to the p-scheme, keeps the effective cut-off mass below which two jets are to be recombined constant, despite the decrease in total energy after each recombination.

In our analysis, we used the modified version of JADE jet finding algorithm [43, 62], which includes all these four recombination schemes.

4.4 Data Analysis

Using the definitions and algorithms described earlier in this thesis, the MC data was analysed to study:

1. The relative jet rate production by all four recombination schemes.
2. The relative sensitivity of different recombination schemes to hadronization and parton showering.
3. The sensitivity of the C planarity to hadronization and parton showering.

To study the hadronization effects, the same observables were calculated both at the end of the parton shower and at the particle level. Fig. 4.2 presents the n-jet event rates as a function of y_{cut} at the parton level (discrete symbols) and at the particle level after hadronization (full line) for the E_0 -, F_0 -, P-, and E- schemes. The differential distribution for C planarity⁷ at the parton and at the particle level is presented in Fig. 4.6 . In these figures, the difference between the curves and the points indicate the changes in the distributions resulting from the fragmentation of partons into hadrons.

The hadronization effects are smallest for the E_0 -scheme ($\approx 5\%$), largest for the E-scheme ($\approx 25\%$), and moderate for the p_0 - and P-schemes. The hadronization effects for the differential C distribution are $\leq 15\%$ in the region of $C = 0.29-0.65$.

⁷The differential distribution of C planarity is the distribution of C planarity normalized as $C = N_T dN/dC$; where N_T is the total number of events and dN is the number of events in a bin of width dC .

To comprehend the difference in hadronization effects for different processes one should note that hadronization is a combined effect of fragmentation and the kinematics of the procedure involved. For example, in the E-scheme the particles have non zero mass compared to the zero mass in other schemes. As a result, a fraction of 2-jet events at parton level turns into 3-jet events at particle level because of the large jet masses at the particle level. This results in a relatively large hadronization effect.

All the above results were obtained using the soft parton shower (ie the parton cutoff mass $Q_g = 1 \text{ GeV}$)

The effect of parton showering on the observables was studied by varying the value of Q_g . Fig. 4.5 shows how the average number of partons at the end of the shower varies with the value of Q_g (in GeV). The typical jet rate dependence on Q_g for various recombination schemes is presented in Fig. 4.3 for jet rates at the parton level and in Fig. 4.4 for jet rates at the particle level, and corresponding to y_{cut} value of 0.02 for the E_{tr} , the P_{tr} , and the P-schemes and of 0.05 for the E-scheme. The dependence of the C planarity on Q_g (for $C = 0.15, 0.27$, and 0.63) is shown in Fig. 4.7 and 4.8. A similar dependence on Q_g was observed for other y_{cut} and C values.

One should notice from these diagrams that in the region of hard shower ($Q_g > 10$) the dependence of jet rates and C planarity on Q_g increases sharply. As expected from Fig. 4.5, the 2-jet rates start increasing and 3-jet rates start decreasing with increase in Q_g . Accordingly the value of C planarity starts decreasing with increase in Q_g which is consistent with its definition.

To quantify the sensitivity of the jet rates and the C planarity to parton showering over the entire physical range of y_{cut} and C, the fractional changes

Bin nr.	C Range	Q_y Range (GeV)	Maximum Change (%) in $C/N_T dN/dC$
1	0.00 - 0.06	1.00 - 10.00	81
2	0.06 - 0.13	1.00 - 10.00	42
3	0.13 - 0.17	1.00 - 10.00	15
4	0.17 - 0.21	1.00 - 10.00	19
5	0.21 - 0.25	1.00 - 10.00	18
6	0.25 - 0.29	1.00 - 10.00	16
7	0.29 - 0.33	1.00 - 10.00	18
8	0.33 - 0.37	1.00 - 10.00	12
9	0.37 - 0.41	1.00 - 10.00	12
10	0.41 - 0.45	1.00 - 10.00	11
11	0.45 - 0.53	1.00 - 10.00	9
12	0.53 - 0.65	1.00 - 10.00	8
13	0.65 - 0.77	1.00 - 10.00	7
14	0.77 - 1.00	1.00 - 10.00	25

Table 4.2: Effect of parton showering on the differential distribution of C planarity.

Y_{cut}	Q_g range	<i>maximum change in $R_2(\%)$</i>			
		E_0 scheme	P_0 scheme	P scheme	E scheme
0.01	0.65 - 8.00	8	6	7	12
0.02	0.65 - 8.00	2	4	4	8
0.03	0.65 - 8.00	1	2	3	7
0.04	0.65 - 8.00	2	2	3	5
0.05	0.65 - 8.00	1	2	2	5
0.06	0.65 - 8.00	1	2	2	3
0.08	0.65 - 8.00	1	2	1	2
0.10	0.65 - 8.00	1	1	1	2
0.12	0.65 - 8.00	1	1	1	1
0.14	0.65 - 8.00	1	1	1	1
0.17	0.65 - 8.00	1	1	1	1
0.20	0.65 - 8.00	1	1	1	1

Table 4.3: Effect of parton showering on 2-jet rate.

Y_{cut}	Q_g range	<i>maximum change in $R_3(\%)$</i>			
		E_0 scheme	P_0 scheme	P scheme	E scheme
0.01	0.65 - 8.00	5	4	4	5
0.02	0.65 - 8.00	1	2	2	4
0.03	0.65 - 8.00	1	2	2	5
0.04	0.65 - 8.00	1	1	2	4
0.05	0.65 - 8.00	1	2	2	4
0.06	0.65 - 8.00	1	2	1	4
0.08	0.65 - 8.00	1	1	1	3
0.10	0.65 - 8.00	1	1	1	2
0.12	0.65 - 8.00	1	1	1	1
0.14	0.65 - 8.00	1	1	1	1
0.17	0.65 - 8.00	1	1	1	1
0.20	0.65 - 8.00	1	1	1	1

Table 4.4: Effect of parton showering on 3-jet rate.

Fig. 4. n-jet rates as a function of y_{cut} determined from jetset shower MC.

Fig 4.3 Jet rate dependence on parton cutoff mass Q_q at parton level.

Fig 4.4 Jet rate dependence on parton cutoff mass Q_g at particle level.

Fig 4.5 Average number of partons as a function of cutoff mass Q_0 .

Fig 4.6 Jetset prediction of C planarity.

Fig 4.7 C planarity dependence on parton cutoff mass Q_0 at particle level.

Fig 4.8 C planarity dependence on parton cutoff mass Q_0 at parton level.

in these observables were calculated relative to $Q_g = 4$ GeV. The Q_g value of 4 GeV is chosen because it yields the parton multiplicity of about 4 (Fig. 4.5) to correspond to the second order QCD, and it is the second order QCD that we are going to compare with our data. In tables 4.2, 4.3, and 4.4, we record the maximum changes in the observables over the range of Q_g values shown in each table.

If the Monte Carlo Statistical errors cannot be seen in the figures it is because they are smaller than the symbols.

4.5 Comments on the Results

The main observations from this Monte Carlo study are:

1. The relative jet production rates are different for different schemes. The E-scheme produces relatively larger multijet rates.
2. The effects of the hadronization are also significantly different for different schemes (Fig. 4.2). They are rather large for the E-scheme, very small for the E_0 -scheme, and moderate for the P- and P_0 -schemes.
3. Choosing $Q_g = 4$ GeV as a reference point, the maximum change in the R_2^* value is 1 for the E_0 , ≤ 2 for the P_0 and the P, and ≤ 5 for the E-scheme corresponding to the $y_{cut} \geq 0.02$. The change for the R_3 value is ≤ 2 , ≤ 4 , ≤ 4 , ≤ 8 for the E_0 -, the P_0 -, the P-, and the E- schemes respectively. For lower y_{cut} values the sensitivity to Q_g increases in general.

*The two jet rates in percentage.

4. The hadronization effects for the differential distribution of C planarity are rather substantial. However, in most of the C range they are not larger than 15 %, and that is the range which we will use to extract physics information from the data.

5. The C planarity and jet rates are fairly stable over a wide range of Q_J values. The variations may be considered as a measure of the shifts in values predicted by a fixed order QCD.

Our conclusion is that the effects due to parton showering and hadronization are small enough that one may use the jet rates and the C planarity to extract the physics information from the data after applying the appropriate hadronization corrections. But these effects are big enough that one must estimate the resulting uncertainties introduced to any physical quantity such as α_s . The different magnitude of hadronization and the different jet rates predicted by different schemes suggest that the systematic uncertainty introduced by the degree of freedom to define the jet algorithm should not be ignored. A physics question that we are facing here is: can these systematics, caused by the freedom provided by the theory, be reproduced by the experimental data, and how would it influence the precision of the physics results obtained. That would be another check on QCD.

Chapter 5

Comparison of the Experimental Data to the Jetset Shower Monte Carlo

The evolution of partons into detectable hadrons (called hadronization or fragmentation) cannot be calculated in the framework of QCD from first principles. The hadronization process, at present, is only vaguely understood in terms of the QCD inspired Monte Carlo (MC) models. In order to compare the QCD calculations performed at the parton level to the experimental data at the hadron level, one has to apply the hadronization corrections to the data using an appropriate MC model.

In order to justify the hadronization corrections calculated by such a model one must first show how well the model describes the experimental data. We use the Jetset (version 7.2) shower QCD model to calculate the hadronization corrections and therefore we will compare the experimental data to this model.

The procedure for unfolding the OPAL data is explained in section 5.1. The comparison of the OPAL data with the Lund Shower MC is presented in section 5.2. The conclusions from this comparison are drawn in section 5.3.

5.1 Experimental data Unfolded for the Detector

To unfold the OPAL data for the effects of the limited detector resolution and acceptance, we employ bin-by-bin correction factors. To determine these factors, a distribution is generated in the form of a histogram for two MC samples: (I) with no detector simulation and (II) using the same MC but including detector simulation and initial-state radiation. The events of sample (II) are subjected to the same reconstruction algorithms and event selection criteria as are the real data. The MC sample (I) treats all particles with lifetimes greater than $3 \cdot 10^{-10}$ s as stable particles and includes all stable charged and neutral particles including neutrinos and neutrons. Let U_i^{MC} be the number of entries in bin i of a distribution and N_i^{MC} be the total number of events generated, for sample (I). Let D_i^{MC} and N_D^{MC} be the number of entries in histogram bin i and the number of events which survive after event reconstruction and selection, for sample (II). The correction factor C_i for bin i is then

$$C_i = \frac{(U_i^{MC}/N_i^{MC})}{(D_i^{MC}/N_D^{MC})} \quad (5.1)$$

This factor multiplies the number of entries D_i , measured experimentally, for bin i of the distribution, to give the unfolded experimental value U_i :

$$U_i = C_i \cdot D_i \quad (5.2)$$

It is important that the MC with detector simulation and initial-state radiation should provide a good description of the distribution at the detector level, for this technique to be applicable.

The OPAL detector simulation program [64] is based on the GEANT3 package [65] developed at CERN. It simulates the effects of energy loss, bremsstrahlung, Compton scattering, multiple scattering, delta-ray production, pair production, hadronic interactions, photoelectric interactions and positron annihilation on the MC particles which are tracked through a detailed model of the OPAL detector. As the end result we obtain MC events in the same format as the experimental events collected with our data acquisition system.

In tables 5.1, 5.2, 5.3, and 5.4 we present the experimentally observed multijet rates (in %) for the E_0 -, P_0 -, P-, and E- recombination schemes respectively. The data in these tables are corrected for detector acceptance and resolution and for initial state radiations. The jet rates are given for different values of y_{jet} ranging from 0.005 to 0.20, corresponding to the minimum jet pair masses of about 6.4 GeV to 40.7 GeV.¹ The experimental differential distribution of C planarity, corrected for detector acceptance and resolution and for initial state radiations, is listed in table 5.5. The correction factor for the first bin of C planarity is too large, and therefore we do not unfold

¹ $y_{jet} = \frac{m_{jet}^2}{T^2}$

y_{cut}	R_2	R_3	R_4	R_5
0.005	13.25 ± 0.17	43.10 ± 0.25	31.36 ± 0.24	12.29 ± 0.17
0.010	26.28 ± 0.21	51.76 ± 0.26	19.05 ± 0.21	2.96 ± 0.09
0.015	36.32 ± 0.24	50.44 ± 0.26	12.27 ± 0.17	1.04 ± 0.06
0.020	44.12 ± 0.24	47.14 ± 0.26	8.44 ± 0.15	0.44 ± 0.04
0.030	55.74 ± 0.24	40.20 ± 0.26	4.08 ± 0.11	0.08 ± 0.03
0.040	63.82 ± 0.23	34.14 ± 0.24	2.09 ± 0.08	0.01 ± 0.00
0.050	69.84 ± 0.22	29.11 ± 0.24	1.12 ± 0.06	—
0.060	74.65 ± 0.21	24.78 ± 0.23	0.63 ± 0.04	—
0.080	81.57 ± 0.19	18.31 ± 0.20	0.14 ± 0.02	—
0.100	86.44 ± 0.17	13.56 ± 0.18	0.02 ± 0.00	—
0.120	89.90 ± 0.15	10.11 ± 0.16	—	—
0.140	92.60 ± 0.12	7.40 ± 0.14	—	—
0.170	95.29 ± 0.10	4.71 ± 0.11	—	—
0.200	97.19 ± 0.08	2.81 ± 0.08	—	—

Table 5.1: Experimental jet rates for E_0 recombination scheme.

this bin. The errors of both the measurement and the correction factors contribute to the errors shown in the tables (in some cases, they are smaller than the symbol size in the figures). One may notice that at low y_{cut} values the E-scheme generates relatively large 4- and 5-jet rates.

Note that the experimental data for the jet rates and the C planarity presented in these tables are corrected for the detector acceptance and resolution, but not corrected for the hadronization effects. Thus these data, in principle, may be used for other QCD analyses without knowing the detailed properties of the detector.

y_{cut}	R_2	R_3	R_4	R_5
0.005	18.65 ± 0.19	44.38 ± 0.25	27.05 ± 0.23	9.44 ± 0.15
0.010	33.85 ± 0.24	48.27 ± 0.25	15.45 ± 0.18	2.18 ± 0.07
0.015	44.06 ± 0.24	45.30 ± 0.25	9.70 ± 0.15	0.82 ± 0.06
0.020	51.55 ± 0.25	41.63 ± 0.25	6.45 ± 0.13	0.32 ± 0.04
0.030	61.89 ± 0.24	34.82 ± 0.24	3.26 ± 0.09	0.04 ± 0.01
0.040	69.03 ± 0.23	29.35 ± 0.24	1.63 ± 0.07	0.01 ± 0.00
0.050	74.34 ± 0.21	24.83 ± 0.22	0.84 ± 0.05	—
0.060	78.58 ± 0.20	20.98 ± 0.20	0.45 ± 0.04	—
0.080	84.40 ± 0.18	15.48 ± 0.18	0.12 ± 0.02	—
0.100	88.42 ± 0.15	11.56 ± 0.16	0.02 ± 0.00	—
0.120	91.41 ± 0.14	8.59 ± 0.15	—	—
0.140	93.73 ± 0.12	6.27 ± 0.13	—	—
0.170	95.99 ± 0.09	4.01 ± 0.10	—	—
0.200	97.56 ± 0.08	2.44 ± 0.08	—	—

Table 5.2: Experimental jet rates for P_0 recombination scheme.

y_{cut}	R_2	R_3	R_4	R_5
0.005	19.56 ± 0.20	45.38 ± 0.25	26.23 ± 0.22	8.31 ± 0.14
0.010	35.32 ± 0.24	48.39 ± 0.26	14.28 ± 0.18	1.72 ± 0.07
0.015	45.77 ± 0.25	44.91 ± 0.25	8.62 ± 0.15	0.55 ± 0.04
0.020	53.42 ± 0.25	40.82 ± 0.25	5.51 ± 0.12	0.18 ± 0.02
0.030	64.05 ± 0.24	33.47 ± 0.24	2.47 ± 0.09	0.01 ± 0.00
0.040	71.43 ± 0.23	27.49 ± 0.23	1.10 ± 0.05	—
0.050	76.85 ± 0.21	22.63 ± 0.22	0.53 ± 0.04	—
0.060	80.94 ± 0.20	18.83 ± 0.20	0.24 ± 0.03	—
0.080	86.73 ± 0.17	13.25 ± 0.18	0.04 ± 0.02	—
0.100	90.71 ± 0.14	9.29 ± 0.15	0.01 ± 0.00	—
0.120	93.63 ± 0.12	6.36 ± 0.13	—	—
0.140	95.59 ± 0.11	4.41 ± 0.11	—	—
0.170	97.56 ± 0.08	2.43 ± 0.08	—	—
0.200	98.82 ± 0.05	1.18 ± 0.05	—	—

Table 5.3: Experimental jet rates for P recombination scheme.

y_{cut}	R_2	R_3	R_4	R_5
0.005	0.20 ± 0.02	8.17 ± 0.11	33.46 ± 0.21	58.01 ± 0.31
0.010	1.28 ± 0.04	32.84 ± 0.20	43.24 ± 0.27	22.81 ± 0.27
0.015	4.53 ± 0.07	50.18 ± 0.23	35.65 ± 0.27	9.93 ± 0.19
0.020	9.50 ± 0.11	58.97 ± 0.25	27.22 ± 0.26	4.79 ± 0.14
0.030	21.67 ± 0.17	61.90 ± 0.27	15.50 ± 0.22	1.34 ± 0.09
0.040	33.22 ± 0.20	57.72 ± 0.29	9.01 ± 0.18	0.38 ± 0.05
0.050	43.10 ± 0.21	51.81 ± 0.29	5.34 ± 0.14	0.06 ± 0.01
0.060	51.44 ± 0.22	45.64 ± 0.29	3.30 ± 0.11	
0.080	64.31 ± 0.21	34.94 ± 0.28	1.05 ± 0.05	
0.100	73.61 ± 0.19	26.32 ± 0.24	0.26 ± 0.02	
0.120	80.33 ± 0.18	19.71 ± 0.22	0.09 ± 0.04	
0.140	85.23 ± 0.17	14.85 ± 0.20	0.01 ± 0.00	
0.170	90.30 ± 0.14	9.81 ± 0.17		
0.200	93.90 ± 0.11	6.16 ± 0.14		

Table 5.4: Experimental jet rates for E recombination scheme.

Bin Nr.	C Range	$C/N_T \text{ dN}_T \text{ dC}$
1	0.000-0.060	---
2	0.060-0.130	0.324 ± 0.004
3	0.130-0.170	0.525 ± 0.018
4	0.170-0.210	0.508 ± 0.010
5	0.210-0.250	0.457 ± 0.012
6	0.250-0.290	0.460 ± 0.016
7	0.290-0.330	0.419 ± 0.012
8	0.330-0.370	0.403 ± 0.023
9	0.370-0.410	0.413 ± 0.022
10	0.410-0.450	0.349 ± 0.015
11	0.450-0.530	0.320 ± 0.014
12	0.530-0.650	0.302 ± 0.011
13	0.650-0.770	0.232 ± 0.006
14	0.770-1.000	0.077 ± 0.008

Table 5.5: The experimental differential distribution of C planarity.

5.2 Comparison with the Jetset Monte Carlo

The experimental data are compared with the predictions of the Jetset shower MC calculated using the optimized parameters described in the previous chapter. The comparison is shown in Fig. 5.1 for the jet rates and in Fig. 5.3 for the C planarity. The experimental data in these figures are unfolded for detector acceptance and the MC data is at the hadron level.

The detector correction factors for jet rates, corresponding to different recombination schemes, as a function of y_{cut} , are presented in Fig. 5.2. Over the entire y_{cut} region, the corrections are $\leq 10\%$ (except for the E-scheme where the deviations are up to 20%). From Fig. 5.2 one may get the impression that the detector corrections are consistently higher for 3-jet rates at high values of y_{cut} . However, this is not a problem. At higher values of y_{cut} R_2 is huge compared to R_3 , so that a small deviation of the same size in the values of R_2 and R_3 would be negligible for R_2 as compared to R_3 . For example in the E_0 algorithm, around a y_{cut} value of 0.10, R_2 is about 90% compared to a R_3 value of about 10%. A deviation of 1 in both (ie R_3 becomes 11 leaving R_2 89) would yield a correction factor of 1.1 for R_3 , but only 0.99 for R_2 .

The correction factors for the C planarity are presented in figure 5.4. The corrections are not larger than 5% for $c > 0.06$. The correction for the first bin ($c = 0.00-0.06$) is 37%. Thus we must not use this bin for a physics analysis. This rapid change in the value of the correction factor between the first and the second bin occurs because the effect of the detector is to migrate events towards the narrow 2-jet region, due to a finite granularity in the calorimeter so that the energy deposits in the core of a jet are merged.

As may be noticed, the MC describes the jet rate data well for each of the recombination schemes, in the entire y_{cut} range (ie 0.005 to 0.20). The MC also provides a good description of the data for C planarity. We have also verified that Jetset data after detector simulation provides a good description of the unfolded experimental data.

5.3 Concluding Remarks

From the comparison of the data with the Jetset shower MC one may draw the following main conclusions:

1. The experimental data are well described by the Jetset shower MC;
2. The difference in the production of relative jet rates by different recombination schemes (as predicted by the MC studies) is reproduced by the experimental data;
3. The general agreement of the data with the Monte Carlo justifies its use to calculate the hadronization corrections to be applied to the data in order to compare it with the second order QCD calculations.

Fig. 5.1 Measured n -jet rates as a function of y_{cut} compared to jetset shower MC.

Fig 5.2 The detector Correction factors for jet rates

Fig 3. C-planarity compared to Jetset shower Monte Carlo.

Fig 4. The detector correction factors for C-planarity

Chapter 6

Comparison of the Experimental Data with the Second order QCD calculations

We compare the data with the analytic $\mathcal{O}(\alpha_s^2)$ QCD calculations of Ellis, Ross, and Terrano [25] (commonly known as ERT) as parameterized by Z. Kunszt et al. [33]. The ERT calculations have been checked by the numerical analysis of Vermaseren et al. [66]. They evaluated the thrust distribution and found it in agreement with the thrust distribution calculated from the ERT formulae [67, 68]. Furthermore, the analytical formulae of ERT have been verified by Fabricius et al. [69]. Therefore, these may be considered as well established QCD calculations.

In section 6.1, we point out the differences of $\mathcal{O}(\alpha_s^2)$ QCD calculations from the parton shower QCD models. In section 6.2, we present the analytical $\mathcal{O}(\alpha_s^2)$ QCD expressions for the jet rates and the C planarity. The renormalization scale used in this analysis is described in section 6.3. The

experimental data for jet rates and C planarity are compared with the analytical $\mathcal{O}(\alpha_s^2)$ QCD calculations in section 6.4. The conclusions from this comparison are presented in section 6.5.

6.1 Finite Order QCD and Parton Shower Monte Carlo

In this analysis, we are correcting the data for hadronization using the parton shower MC, and comparing the corrected data to the analytic $\mathcal{O}(\alpha_s^2)$ QCD calculations. In order to appreciate this analysis, one must comprehend the difference between the parton shower MC and $\mathcal{O}(\alpha_s^2)$ QCD.

Finite order QCD, in which Feynman diagrams are calculated order by order, takes into account the exact kinematics and full interference and helicity¹ structure. This is why the calculations become increasingly difficult in higher orders and have only been carried out, in full, up to $\mathcal{O}(\alpha_s^2)$. In contrast to $\mathcal{O}(\alpha_s^2)$ QCD calculations, which allow the production of only up to 4 partons, QCD parton shower models provide the generation of a parton shower, with no explicit limit on the number of partons. Thus, in the parton shower approach, QCD is calculated to all orders but only the leading terms in all orders are taken into account. Thus the analytic $\mathcal{O}(\alpha_s^2)$ QCD calculations describe QCD, in full, up to second order; while the parton shower MC describes QCD, in leading logarithmic approximation, to all orders, and thus does not contain the full 3- or 4-parton matrix elements of second order

¹The helicity of a particle is defined by the relative direction of its velocity and spin; for example the helicity of a spin $\frac{1}{2}$ particle is +1 if its spin and velocity are parallel and it is -1 if they are antiparallel. The former is called right handed and the latter left handed.

theory. Thus, it has been suggested [70] that the parton shower MC cannot be used for the determination of α_s .

Due to these differences between $\mathcal{O}(\alpha_s^2)$ QCD and the parton shower MC, one could be more appropriate than the other for a given process. We calculate hadronization corrections from the parton shower MC. Thus the comparison of the $\mathcal{O}(\alpha_s^2)$ QCD to the corrected data would be the test of the theory and of the importance of the missing higher order terms. This approach provides a clean test of $\mathcal{O}(\alpha_s^2)$ QCD because of the clear separation of perturbative and hadronization components.

6.2 QCD Calculations for Jet Rates and C planarity

Having chosen a particular recombination scheme for the cluster jet algorithm, the relative jet production rates R_n of 2-, 3- and 4-jet events can then be calculated in QCD. In $\mathcal{O}(\alpha_s^2)$ QCD perturbation theory, the relative jet production rates R_n may be parameterized in powers of α_s as

$$R_3 \equiv \frac{\sigma_3}{\sigma_{tot}} = \frac{\alpha_s(\mu)}{2\pi} A_3(y_{cut}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 [A_3(y_{cut})2\pi b_0 \ln(\mu^2/E_{cm}^2) + B_{3,1}(y_{cut})] \quad (6.1)$$

$$R_4 \equiv \frac{\sigma_4}{\sigma_{tot}} = B_4(y_{cut}) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \quad (6.2)$$

where μ is the renormalization scale and

$$b_0 = \frac{33 - 2n_f}{12\pi} \quad (6.3)$$

where n_f is the number of flavors.

The values of the leading order coefficients $A_3(y_{cut})$ and $B_1(y_{cut})$ would be independent of the choice of the recombination scheme while the coefficient $B_{3,l}(y_{cut})$ for the next to leading order correction does depend upon the choice of the scheme. The values of A_3 , B_1 , and $B_{3,l}$ for E-, E_0 -, and P- schemes are given in ref. [33]. The values of $B_{3,l}$ for P_0 scheme were computed using the program supplied by P. Nason. Having calculated R_3 and R_1 from the above equations, R_2 is calculated from the unitarity requirement of $o(\alpha_s^2)$ (ie $R_2 + R_3 + R_1 = 1$)

The definition of the C planarity is already described in chapter 2. The cross section of multihadron events in terms of C planarity may be parameterized in powers of α_s as

$$\begin{aligned} \frac{C}{\sigma_{tot}} \frac{d\sigma}{dC} = & \frac{\alpha_s(\mu)}{2\pi} A(C) \\ & + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 [A(C)2\pi b_0 \ln(\mu^2/E_{cm}^2) + B(C)] \end{aligned} \quad (6.4)$$

The values for the coefficients A and B are listed in Reference [33].

6.3 Choice of the Renormalization Scale

The renormalization scale μ^2 , appearing in the above formulae, is often called an unphysical parameter because the QCD results, if calculated to all orders

in perturbation theory, are independent of it. However, the finite order calculations do depend upon the detailed choice of μ^2 , which has no direct physical interpretation but is an artifact of the incomplete perturbation series.

In order to compare the data with QCD one has to choose a value for the renormalization scale which is not determined by the theory. According to one school of thought the value of the scale should be of the order of the energy scale involved in the problem (that is to say $\mu^2 = E_{cm}^2$). This view seems to be promoted in ref. [33]. According to another school of thought [10, 71] the scale should be determined experimentally under the requirement of optimal description of the data by the theory. One may write $\mu^2 = f E_{cm}^2$ and treat f as a free parameter in fitting the theory with the data. Theoretically, this commonly means minimizing the unknown higher order corrections to a given perturbation order. However, minimizing higher order contributions to a given perturbation order does not obviously imply the best possible description of the data, and vice versa.

We attempt both of these approaches in the comparison of data to QCD. A more detailed study on the renormalization scale is presented in chapter 9.

6.4 Comparison of Data with QCD

In this section, the data are compared with the $o(\alpha_s^2)$ QCD calculations. The theoretical predictions were fitted to the experimental data. The fits were made for the fixed scale and for the optimized scale as explained below.

Because the same data sample is analyzed for different y_{cm} values, the

measured jet rates corresponding to different y_{cut} values are correlated. Thus we make a fit corresponding to only one y_{cut} value. Comparison of theoretical predictions, calculated using the QCD parameter values yielded by this fit, to the data for other y_{cut} values is then considered as a test of QCD. The choice of the y_{cut} used for the fit was made by three requirements: 1) Because the 5-jet rates are not calculated by the theory, the measured 5-jet rate corresponding to the chosen y_{cut} value should be zero, 2) As the 4-jet rates are calculated by the theory only to the leading order, the measured 4-jet rate corresponding to the chosen y_{cut} value should be less than 1 %, and 3) The chosen point should have good statistics. Meeting these three requirements, the y_{cut} value turned out to be 0.06 for the E_0 -, the P_0 - and the P- scheme and 0.08 for the E-scheme. Corresponding to these y_{cut} values the theory was fitted to the data for two different treatments of the scale factor f : 1) The QCD parameter $\Lambda_{\overline{MS}}$ was determined for fixed scale factor $f=1$ and 2) Both $\Lambda_{\overline{MS}}$ and f were treated as free parameters and were determined in a two parameter fit. These fits were made for 3-jet rates. Using the values of the QCD parameters determined from these fits, the 2-, 3-, and 4-jet rates were calculated in the region of $0.01 \leq y_{cut} \leq 0.20$ for all the four recombination schemes. Theoretical predictions of the jet rates thus determined are compared to the experimental data in Fig. 6.1 .

As is evident from Fig. 6.1, the data are well described by the theory with optimized scale in the region of $y_{cut} \geq 0.02$ for the E_0 -, P_0 - and P schemes, and in the region of $y_{cut} \geq 0.04$ for the E scheme. Note that the optimized theory fails to describe the data only in the region where the measured 5-jet rates are non zero, which is comprehensible as 5-jet rates are not calculated by the theory. It is interesting to note that the optimized theory provides a

Fig. 6.1 Measured n -jet rates as a function of y_{cut} , compared to $O(\alpha_s^2)$ QCD calculations.

Acknowledgements

The time of my Ph.D. has been a great learning experience. First of all, I thank P.J.S. Watson for allowing me a U turn from theory to experiment. I am grateful to R. K. Carnegie who opened the door for me to the wonderful world of Experimental High Energy Physics. I highly appreciate the support and encouragement provided by him in his own uniquely constructive way.

It is not possible in few words to express my gratitude toward my thesis supervisor Hans Mes for his regular help and encouragement throughout the course of this work. I am also thankful to him for his sincere efforts to break my initial phobia of hardware and for his constant struggle to convert my Punjabi-style English, that I usually use in my writeups, into a more standard one. In this age of complex theories full of problems, I have learned from him that there is always an underlying simplicity behind every apparent complexity and there is always a way around the problem.

Thanks are also due to R. J. Hemingway who got me started with the Monte Carlo aspect of experimental studies. Brief periods of working with him were full of learning and quest for learning. I did not learn only Physics from him but also the Sociology of Physics: how to physics and smile at the same time. I also thank Pat Kalyniak of Carleton and David Hanna of McGill for a careful and critical reading of the manuscript.

I am thankful to S. Bethke of Heidelberg who got me started with the jet analysis. It was a great pleasure to work with him, at CERN, on the OPAL jet papers. Most of the data presentation techniques used in this thesis were developed during that period. From him, I learned a great deal on the experimental aspects of QCD. Useful interactions with other members of the OPAL QCD group, namely George Azuelos, M. Dixit, J. W. Gary, Hannes Jeremie and David Ward, are also acknowledged.

This note would be incomplete without thanking the fellow graduate students who provided a very friendly environment at Carleton, and who know how to moo it when you can't do it. Mike Boyce, Victor Silalahi, André Turcotte and Sultan Sial are few to mention. Mention should also be made

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Fig 6.2 Measured C planarity compared to $O(\alpha_s^2)$ QCD calculations.

good description of the measured 4-jet rates. This is because in a two parameter fit the scale factor adjusts itself in order to account for the missing higher order corrections.

For the theory with fixed ($f=1$) scale, the data is well described only in the region of $y_{cut} \geq 0.04$ for the E_{0^-} , P_{0^-} and P^- schemes, and in the region of $y_{cut} \geq 0.08$ for the E scheme. This is because the fixed scale theory does not provide a good description of the measured 4-jet rates and thus, by the normalization condition $R_2 + R_3 + R_4 = 1$, must also fail to describe

the 2- and 3-jet rates in the regions where 4-jet rates are non zero. *The underproduction of 4-jet rates by the theory reflects the fact that the unknown next to leading order correction for 4-jet rates is substantial and is predicted to be positive.* This fact should be considered in any meaningful determination of a physical observable such as α_S .

A similar approach was taken in the case of C planarity as well. The theory was fitted with the data corresponding to one bin in the differential distribution. This bin was chosen to be $C=0.45-0.53$ to meet the requirements of minimum sensitivity to hadronization and higher order effects, and still have good statistics. The QCD parameter values determined from the fits of this bin, corresponding to fixed and optimized scale, were used to calculate the theoretical predictions of the other bins of the distribution. The distribution thus calculated is compared to the measured distribution in Fig. 6.2 . The optimized theory provides an overall better description of the data, a result consistent with the jet rate studies.

6.5 Main Observations

From the comparison of ERT calculations to the data, one may make the following observations:

1. The data is overall better described by the optimized theory than the fixed scale theory.
2. The failure of the fixed scale theory to describe the data at the low y_{int} values may be partially understood in terms of the unknown higher order corrections to the 4-jet rates.

3. The 4-jet rates are underevaluated by the theory and hence is an indication of an overall positive higher order contribution.

4. The theory optimized for 3-jet rates provides a good description of the measured 4-jet rates. This checks the consistency of the QCD calculations.

5. The fitted values of the scale factor f are significantly different for different recombination schemes.

6. We have fitted the theory to the data only at one point of the distributions; other points determined from that fit are in general agreement with the data. This provides a very convincing test of QCD.

Chapter 7

Determination of $\alpha_s(M_{Z^0})$ from Jet Rates and C Planarity

e^+e^- annihilation provides a rather simple experimental situation for precision tests of perturbative QCD. The initial state is completely known and for the final state the infrared safe quantities can be calculated as a function of a single parameter α_s . Therefore, various tests of QCD at e^+e^- experiments involve the determination of α_s . A general agreement (or disagreement) among the different measurements and among the values of α_s determined from different quantities would then reflect the consistency (or inconsistency) of the underlying theory.

In this study, we use two quantities (jet rates and C planarity) to carry out this investigation. As is claimed in ref. [33], the provision for different recombination schemes is not an ambiguity of the theory, but a freedom provided by the theory in defining an infrared safe jet algorithm. The algorithms based on all these schemes are equivalent in $o(\alpha_s)$ theory, but give different results in $o(\alpha_s^2)$ theory. As shown in the previous chapter, these differences

are reproduced by the experiment (which itself is at least a qualitative test of the theory). Assuming the validity of the claim that the freedom of different schemes is not an ambiguity of the theory, one would expect similar values for a physical quantity such as α_s determined by different schemes. If one does get similar values for α_s , then one should estimate the systematic uncertainties introduced in the determination of α_s by this freedom of algorithms. The α_s thus extracted from jet rates may be compared to that extracted from C planarity.

In this chapter, we describe the determination of α_s from the differential distributions of jet rates and C planarity. The errors on the determination of α_s from hadronization, parton virtuality, the renormalization scale ambiguity, as well as from the experimental systematics are estimated. The systematic error on the determination of $\alpha_s(M_{Z^0})$ due to the scheme ambiguity is also studied.

7.1 Determinations of $\alpha_s(M_{Z^0})$ from differential distributions of jet rates

As mentioned in the previous chapter, the relative jet rates as a function of y_{cut} are not statistically independent of each other, and hence neither are the errors on them. Therefore, we use instead the differential distributions of jet rates [72] defined as

$$D_i(y) = \frac{R_i(y) - R_i(y - \Delta y)}{\Delta y} \quad (7.1)$$

where $i=2$ and 3 for 2- and 3-jet rates respectively, and y is used for y_{cut} , for brevity. This can be viewed as a distribution of the y_{cut} value of events for which the jet multiplicity changes from 3 to 2 (for $i=2$ and also for $i=3$ in a 4- and 5-jet free region).

(One should note that each event will enter the differential distribution $D_i(y)$ only once, and hence the statistical errors in bins of $D_i(y)$ are independent of each other.

We use 2- and 3-jet differential distributions to fit the $\mathcal{O}(\alpha_s^2)$ QCD calculations [33] to the data, for two different choices of the renormalization scale:

1. The QCD parameter $\Lambda_{\overline{MS}}$ is determined by treating it as a free parameter in the fit for fixed renormalization scale $\mu^2 = E_{cm}^2$. The choice of the y region for the fit was determined by the two facts: 1) The R_i is calculated only to the leading order and, as observed in the previous chapter, does not describe the measured 4-jet rates [73, 74, 75] for the fixed scale; 2) the theoretical calculations are given for R_3 and R_4 only and we calculate R_2 from the unitarity condition $R_2 + R_3 + R_4 = 1$. As a result, the theory is bound to fail to describe the 2- and 3-jet production rates in y regions of 4-jet production. Thus for different schemes we used the y region in which the measured 4-jet rates are $\leq 1\%$. This turned out to be $y \geq 0.06$ in the E_0 and the P_0 - scheme, $y \geq 0.05$ in the P - scheme, and $y \geq 0.10$ in the E -scheme.

2. In the fit, both $\Lambda_{\overline{MS}}$ and the renormalization scale factor $f (= \mu^2 / E_{cm}^2)$ were treated as free parameters. The scale value thus obtained is called the optimized scale. The choice of the y region for the fit was motivated by the

two facts: 1) For very small values of y , the QCD calculations predict negative jet rates and hence this y region should be regarded as unphysical; 2) 5-jet rates are not calculated by the theory and hence the y region is chosen in which the measured R_5 is $\leq 1\%$. Having imposed these requirements, the y region turned out to be $y \geq 0.03$ in the E_0 - and the P_0 - scheme, $y \geq 0.02$ in the P- scheme and $y \geq 0.04$ in the E-scheme.

Having determined $\Lambda_{\overline{MS}}$ from these fits, $\alpha_s(M_{Z^0})$ is calculated by the second order expression [32]

$$\alpha_s(\mu) = \frac{1}{b_0 \ln(\frac{\mu^2}{\Lambda^2})} \left[1 - \frac{b_1}{b_0^2} \frac{\ln(\ln(\frac{\mu^2}{\Lambda^2}))}{\ln(\frac{\mu^2}{\Lambda^2})} \right] \quad (7.2)$$

by substituting M_{Z^0} for μ . The coefficients b_0 and b_1 depend upon the number of flavors n_f ($= 5$ at Z^0 peak) and are given by

$$b_0 = \frac{33 - 2n_f}{12\pi} \quad (7.3)$$

and

$$b_1 = \frac{153 - 19n_f}{24\pi^2} \quad (7.4)$$

The above mentioned fits were performed using the same scheme both at the experimental and the theoretical levels, and the results are recorded in tables 7.1 and 7.2 for 2-jet and 3-jet differential distributions respectively. The last column in each of these tables shows different number of degrees of freedom for optimized and fixed scale. This is because the number of parameters and the number of bins in the fit were different for each case.

Scheme	Λ	α_s	f	χ^2 of fit
E_0	145^{+15}_{-13}	$0.111^{+0.002}_{-0.001}$	$0.0028^{+0.0021}_{-0.0009}$	0.80/7
	357^{+39}_{-37}	0.127 ± 0.002	1.00	2.10/5
P_0	192^{+22}_{-18}	0.115 ± 0.002	$0.0349^{+0.0817}_{-0.0189}$	2.58/7
	315^{+40}_{-37}	0.124 ± 0.002	1.00	2.09/5
P	187^{+16}_{-15}	0.115 ± 0.001	$0.0447^{+0.0165}_{-0.0182}$	4.34/8
	281^{+32}_{-30}	0.122 ± 0.002	1.00	3.73/6
E	168 ± 15	0.113 ± 0.001	$0.000047^{+0.000002}_{-0.000002}$	4.22/6
	899^{+96}_{-91}	0.149 ± 0.003	1.00	2.48/3

Table 7.1: α_s from the differential distribution of the 2-jet rate, and for the same scheme at the experimental and theoretical level.

Scheme	Λ	α_s	f	χ^2 of fit
E_0	146^{+17}_{-16}	0.111 ± 0.002	$0.0027^{+0.0009}_{-0.0005}$	0.91/7
	332^{+43}_{-40}	$0.125^{+0.003}_{-0.002}$	1.00	4.17/5
P_0	183^{+25}_{-23}	0.114 ± 0.002	$0.0155^{+0.0090}_{-0.0046}$	2.67/7
	297^{+45}_{-41}	0.123 ± 0.003	1.00	3.06/5
P	186^{+26}_{-23}	0.115 ± 0.002	$0.0263^{+0.0284}_{-0.0105}$	3.47/7
	274^{+40}_{-37}	0.122 ± 0.003	1.00	2.62/6
E	125 ± 89	0.108 ± 0.001	0.000063 ± 0.000004	3.99/6
	872^{+98}_{-93}	0.148 ± 0.003	1.00	2.55/3

Table 7.2: α_s from the differential distribution of the 3-jet rate, and for the same scheme at the experimental and theoretical level.

The theoretical predictions for the best fit results are compared with the data in Fig. 7.1 for the 2-jet differential distributions for all the four recombination schemes. From these fits one may observe the following:

1. The theory provides a good description of the data in the ranges of y values chosen for the fit.
2. The optimized scales are significantly different for different recombination schemes: $f \approx 0.003$ (corresponding to $\mu^1 \approx 5\text{GeV}$) for the E_0 -scheme, $f \approx 0.03$ (corresponding to $\mu \approx 16\text{GeV}$) for the P_0 -scheme, $f \approx 0.09$ (corresponding to $\mu \approx 27\text{GeV}$) for the P-scheme and $f \approx 0.00005$ (corresponding to $\mu \approx 0.6\text{GeV}$) for the E-scheme.
3. The optimized scale factor f is the largest for the P-scheme, and the smallest for the E-scheme. Consequently, the difference between the fitted values of $\Lambda_{\overline{MS}}$ for optimized scale and for fixed scale is the largest for the E scheme and the smallest for the P-scheme.
4. The difference among the fitted $\Lambda_{\overline{MS}}$ values for various schemes is larger for the fixed scale as compared to the optimized scale.

Had we not used the same scheme at both theoretical and experimental levels, what would the contribution to the systematic error of α_s have been? We investigate this by fitting the distributions $D_i(y)$ computed from theoretical calculations for all the four schemes to the measured $D_i(y)$ for a given scheme. The results are presented in the tables 7.3 and 7.4 for $D_2(y)$ and in tables 7.5 and 7.6 for $D_3(y)$.

$$\mu^2 = f E_{cm}^2$$

Fig. 7.1 Differential 2-jet rates compared to analytic $O(\alpha_s^2)$ QCD calculations.

Scheme (OPAL) (ERT)	E_0	P_0	P	E
E_0	0.111 ± 0.002	0.104 ± 0.001	0.103 ± 0.001	0.132 ± 0.002
P_0	0.123 ± 0.002	0.115 ± 0.002	0.113 ± 0.001	0.151 ± 0.002
P	0.128 ± 0.002	0.118 ± 0.002	0.1160 ± 0.003	0.159 ± 0.003
E	0.097 ± 0.002	0.092 ± 0.002	0.0854 ± 0.001	0.113 ± 0.001
$\Delta\alpha_s(sch.)$	$+0.017$ -0.014	$+0.003$ -0.023	-0.031	$+0.046$

Table 7.3: α_s from the differential distribution of the 2-jet rate, and for the optimized scale.

Scheme (OPAL, (ERT)	E_0	P_0	P	E
E_0	0.127 ± 0.002	0.117 ± 0.002	0.113 ± 0.002	0.167 ± 0.004
P_0	0.136 ± 0.003	0.124 ± 0.002	0.113 ± 0.001	0.180 ± 0.004
P	0.138 ± 0.003	0.127 ± 0.003	0.122 ± 0.002	0.186 ± 0.004
E	0.114 ± 0.002	0.105 ± 0.002	0.102 ± 0.002	0.149 ± 0.003
$\Delta\alpha_s(sch.)$	$+0.011$ -0.013	$+0.003$ -0.019	-0.020	$+0.037$

Table 7.4: α_s from the differential distribution of the 2-jet rate, and for the fixed scale $\mu^2 = E_{cm}^2$.

Scheme (OPAL) (ERT)	E_0	P_0	P	E
E_0	0.111 ± 0.002	0.104 ± 0.002	0.101 ± 0.002	0.130 ± 0.002
P_0	0.123 ± 0.002	0.114 ± 0.002	0.112 ± 0.002	0.150 ± 0.002
P	0.128 ± 0.002	0.119 ± 0.003	0.115 ± 0.002	0.156 ± 0.002
E	0.094 ± 0.001	0.089 ± 0.002	0.086 ± 0.001	0.108 ± 0.001
$\Delta\alpha_s(Sch.)$	± 0.017	$+0.005$ -0.025	-0.029	$+0.018$

Table 7.5: α_s from the differential distribution of the 3-jet rate, and for the optimized scale.

Scheme (OPAL) (ERT)	E_0	P_0	P	E
E_0	0.125 ± 0.003	0.115 ± 0.003	0.112 ± 0.002	0.166 ± 0.004
P_0	0.134 ± 0.003	0.123 ± 0.003	0.120 ± 0.003	0.179 ± 0.004
P	0.137 ± 0.003	0.125 ± 0.003	0.122 ± 0.003	0.184 ± 0.004
E	0.112 ± 0.002	0.104 ± 0.002	0.101 ± 0.002	0.149 ± 0.003
$\Delta\alpha_s(sch.)$	$+0.012$ -0.013	$+0.002$ -0.019	-0.021	$+0.035$

Table 7.6: α_s from the differential distribution of the 3-jet rate, and for fixed scale $\mu^2 = E_{cm}^2$.

As may be noticed from these tables, by not applying the same scheme at both theoretical and experimental levels one may introduce a huge uncertainty to the measurements of α_s , and this systematic uncertainty can be avoided by consistently using the same scheme at both the theoretical and experimental levels.

Because there is not a unique and commonly accepted way to treat the renormalization scale in a fixed order QCD, we estimate the systematic error on α_s contributed by the scale ambiguity in a rather conservative way. For each scheme, we take the fitted values of $\Lambda_{\overline{MS}}$ for the optimized scale and for the fixed scale (ie $f=1$). From these two $\Lambda_{\overline{MS}}$ values, we then compute $\alpha_s(M_{Z^0})$ by using equation (7.2) for $\mu = M_{Z^0}$. The arithmetic mean of these two α_s values is quoted as the final value of α_s and the symmetric difference between the mean and the two extreme values is taken as the error on α_s due to the scale ambiguity, $\Delta\alpha_s(scale)$.

Because we applied hadronization corrections to the data, we must also estimate the systematic error on $\alpha_s(M_{Z^0})$ contributed by the hadronization corrections. A commonly accepted way to estimate this error is to apply corrections by using two different QCD and hadronization models, and to consider the variations in the results. In our analysis, we applied the hadronization and detector corrections using the Jetset shower MC. We repeated the analysis by using the Herwig QCD shower and cluster hadronization program [76] to correct the data for detector and hadronization effects. The resulting difference in the values of $\alpha_s(M_{Z^0})$ is taken as the error due to hadronization, $\Delta\alpha_s(had)$.

We concluded in chapter 4 that the uncertainty due to the parton virtuality² must be accounted for. To achieve our final results, we applied the hadronization corrections corresponding to the parton shower cutoff mass $Q_g=1$ GeV (ie the invariant mass of parton at which the QCD showering is stopped). The entire analysis was repeated for all the four schemes for values of Q_g ranging from 1 GeV to 10 GeV. The resulting values of α_s as a function of Q_g are plotted in Fig. 7.2 for the $D_2(y)$ fit and in Fig. 7.3 for the $D_3(y)$ fit. The error on α_s due to parton virtuality, $\Delta\alpha_s(Q_g)$, is estimated from these figures considering the range of Q_g values from 1 to 10 GeV. Because the final number of partons in the shower varies with the value of Q_g , the variations in α_s due to parton virtuality can also be interpreted as due to the variations of the influence of the higher order QCD contributions(inherent to the model).

The final results of $\alpha_s(M_{Z^0})$ including all these errors are listed in table 7.7. The systematic experimental error, $\Delta\alpha_s(syst)$, was calculated by comparing the results from two separate analyses, one which uses only the calorimeter information and the other which uses only the charged tracks.

7.2 Determinations of $\alpha_s(M_{Z^0})$ from differential distribution of C Planarity

The comparison of the QCD prediction of the differential distribution of C planarity with the data was already presented in Fig. 6.2. In the range of $C= 0.29$ to 0.65 , the differential distribution of C planarity was fitted

²ie the choice of the cutoff mass for parton where it stops showering in a Monte Carlo model.

Fig. 7.2 Values of α_s as a function of Q_g from the $D_2(y)$ fit.

Fig. 7.3 Values of α_s as a function of Q_g from the $D_3(y)$ fit.

Scheme	diff. dist.	α_s	$\Delta\alpha_s(stat.)$	$\Delta\alpha_s(syst.)$	$\Delta\alpha_s(had.)$	$\Delta\alpha_s(Q_g)$	$\Delta\alpha_s(scale)$
E_0	2-jet	0.119	± 0.002	-0.002	± 0.002	± 0.002	± 0.008
	3-jet	0.118	± 0.002	-0.001	± 0.002	± 0.001	± 0.007
P_0	2-jet	0.120	± 0.002	-0.001	± 0.002	± 0.004	± 0.005
	3-jet	0.119	± 0.002	-0.002	± 0.002	± 0.005	± 0.004
P	2-jet	0.118	± 0.001	-0.001	± 0.002	± 0.008	± 0.004
	3-jet	0.118	± 0.002	-0.001	± 0.002	± 0.008	± 0.003
E	2-jet	0.131	± 0.001	-0.002	± 0.002	± 0.002	± 0.018
	3-jet	0.128	± 0.001	-0.002	± 0.002	± 0.003	± 0.020

Table 7.7: Final results of α_s from the differential distributions of the 2- and 3-jet rates, and for the same scheme at the experimental and theoretical level.

Fig. 7.4 Values of α_s as a function of Q_g determined from C planarity.

with the data for the optimized scale and for the fixed scale. The errors due to hadronization, scale ambiguity, parton virtuality and experimental systematics were estimated in a similar fashion as described in the previous section. The dependence of α_s values on the parton shower cutoff mass Q_g is given in Fig. 7.4. The results on α_s from C planarity are listed in tables 7.8 and 7.9.

7.3 Main Observations

Some of the main observations from these results on α_s may be summarized as following:

1. One may notice from the listed values that one gets similar values of α_s for different algorithms in spite of getting different jet rate distributions. This demonstrates that the differences predicted by the theory are reproduced by the experiment in such a fashion as to provide similar values for the physical

C Range	Λ	α_s	f	χ^2 of fit
0.29-0.65	159^{+10}_{-7}	0.112 ± 0.001	$0.00030^{+0.00024}_{-0.00013}$	5.18/4
0.29-0.65	766^{+31}_{-30}	0.145 ± 0.001	1.00	18.41/5

Table 7.8: α_s from the differential distribution of C planarity for the optimized and fixed scale.

C Range	α_s	$\Delta\alpha_s(stat.)$	$\Delta\alpha_s(syst.)$	$\Delta\alpha_s(had.)$	$\Delta\alpha_s(Q_g)$	$\Delta\alpha_s(scale)$
0.29-0.65	0.128	± 0.001	± 0.001	± 0.001	± 0.002	± 0.016

Table 7.9: Final results of α_s from the differential distribution of C Planarity.

parameter α_s . This itself may be considered as a significant test of the validity of perturbative QCD.

2. The systematic uncertainty introduced in the determination of α_s by not consistently applying the same algorithm at both experimental and theoretical level may grow as high as 40 % (Tables 7.4 and 7.5). Thus we conclude that it is of vital importance to apply the same jet algorithm consistently at the theoretical and experimental levels.

3. The corrections and uncertainties are found to be different for different recombination schemes. The hadronization corrections are smallest for the E_{11} -scheme ($\approx 5\%$), largest for the E-scheme ($\approx 25\%$), and moderate for P- and P_{11} schemes. The E_0 -scheme is also the least sensitive to the parton virtuality (Table 7.7). Thus it suggests that the E_0 -scheme is the most suitable for jet studies in QCD.

4. Because the theoretical uncertainties dominate the overall error on the determination of α_s and because these errors are largely uncorrelated from one recombination scheme to another, we select the recombination scheme which yields the smallest overall error to obtain the final α_s value from jet rates

$$\alpha_s = 0.120 \pm 0.007 \quad (7.5)$$

and conclude that α_s from jet studies is determined to be around 0.120 with a total uncertainty of about 6 %.

5. The final value of α_s from C planarity is

$$\alpha_s = 0.128 \pm 0.016 \quad (7.6)$$

Thus we conclude that the α_s from C planarity is determined to be 0.128 with a total uncertainty of about 12 %.

6. The rather larger central value of α_s and the larger overall error on α_s in the E scheme are entirely due to the large renormalization scale uncertainty within this scheme. Such is the case for C planarity as well.

7. The α_s values determined from jet rates and from C planarity are consistent with each other and completely overlap within their errors.

Chapter 8

Determination of $\alpha_s(M_{Z^0})$ from Oblateness and Thrust

In the previous chapters, we have presented the QCD analysis of jet rates and C Planarity. In this chapter, we present the QCD analysis of two other infrared safe observables, oblateness and thrust. The differential distributions of these quantities, calculated in $\mathcal{O}(\alpha_s^2)$ perturbative QCD, are compared with the OPAL data and the strong coupling constant $\alpha_s(M_{Z^0})$ is thus determined. The uncertainties on the measurement of $\alpha_s(M_{Z^0})$ due to the hadronization, unknown higher order effects, experimental systematics, as well as due to the ambiguity of the renormalization scale are estimated.

The definitions of oblateness and thrust are already presented in chapter 2. In section 8.1, we present the second order QCD formulae for the differential distributions of oblateness and thrust. The sensitivity of these observables to hadronization and parton virtuality is studied in section 8.2. The measured distributions of these observables are compared with the Jetset shower MC in section 8.3. The determination of α_s from these observables

is presented in section 8.4. In section 8.5, we present an average of the individual results of $\alpha_s(M_{Z'})$ determined from different observables. Finally, in section 8.6 we present the conclusions from this study.

8.1 The Analytic $o(\alpha_s^2)$ QCD Calculations for Oblateness and Thrust

We use the analytic $o(\alpha_s^2)$ QCD calculations originally done by Ellis, Ross and Terrano [25], and parameterized by Kunszt et al. [33]. The differential distributions for oblateness and thrust may be parameterized in powers of α_s as

$$\begin{aligned} \frac{X}{\sigma_0} \frac{d\sigma}{dX} &= \frac{\alpha_s(\mu)}{2\pi} A(X) \\ &+ \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 [A(X)2\pi b_0 \ln(\mu^2/E_{cm}^2) + B(X)] \end{aligned} \quad (8.1)$$

where $X = O$ for oblateness and $1-T$ for thrust, σ_0 is the leading order cross section for e^+e^- annihilation into hadrons and

$$b_0 = \frac{33 - 2n_f}{12\pi} \quad (8.2)$$

where n_f is the number of quark flavors. The values for the coefficients A and B are listed in [33].

The physical cross section for O above 0.5 becomes negative and is meaningless to compare with the data. The second order correction to the oblateness

distribution is large and negative and has been looked upon as an interesting prediction of the theory [33]. It would be interesting to test this prediction with our data.

8.2 A Monte Carlo Study of Oblateness and Thrust

Before comparing the QCD calculations with the experimental data, a Monte Carlo (MC) analysis was performed in order to study the effects of hadronization and of parton virtuality on the observables under study. For this analysis 50,000 multihadron events were generated using the Jetset (version 7.2) Shower MC [21] corresponding to the virtual gluon cutoff mass Q_g (the mass of the gluon below which it does not shower) varying from 1 GeV to 10 GeV.

To study the hadronization effects, observables were calculated at the end of the parton shower and at the particle level. Fig. 8.1 presents the differential O distribution at the parton level (discrete symbols) and at the particle level after hadronization (full line). The detector correction factors and the hadronization correction factors for this distribution are presented in figures 8.2 and 8.3 respectively. The corresponding plots for the differential T distribution are presented in figures 8.4, 8.5 and 8.6. All these results were obtained using the soft parton shower (ie the parton cutoff mass $Q_g = 1$ GeV). The hadronization corrections in the region used for the fit are on average less than 10 % for the differential T distributions and are rather substantial (≈ 30 %) for the differential O distribution. The detector effects are considerably smaller in each case.

Fig. 8.1 Predictions of Jetset shower Monte Carlo for oblateness.

Fig. 8.2 The detector correction factors for oblateness.

Fig. 8.3 The hadronisation correction factors for oblateness.

Fig. 8.4 Predictions of Jetset shower MC for thrust.

Fig. 8.5 The detector correction factors for thrust.

Fig. 8.6 The hadronisation correction factors for thrust.

The effect of parton virtuality on the bin contents of the differential distributions was studied by repeating the MC runs for values of Q_g varying from 1 GeV to 10 GeV. To quantify the sensitivity of the observables to the parton virtuality, the fractional changes in bins of the differential distributions were calculated relative to $Q_g = 4$ GeV. The Q_g value of 4 GeV is chosen because it yields the parton multiplicity of about 4 to correspond to the second order QCD, and it is the second order QCD that we compare with our data. The maximum changes thus calculated in the bin contents of the differential distributions of O and T are tabulated in Table 8.1 and Table 8.2 respectively. The Q_g region of 1 to 10 GeV was used for T. The region of Q_g from 1 to 6 GeV was chosen for oblateness because the theory does not fit with the data corrected for hadronization using Q_g value larger than 6 GeV. The maximum changes in bin contents of the differential T distributions in its fit region (ie $T=0.715-0.935$) are about 10 % and they are large (≈ 20 %) for the fit region of O (ie $O = 0.06-0.35$).

Our conclusion is that the effects due to parton virtuality and hadronization at the Z^0 peak are moderate enough that one may use these infrared safe observables to compare the QCD calculations with data after applying the appropriate corrections, but these effects are large enough that one must estimate the resulting uncertainties introduced in the determination of α_s .

8.3 The Unfolding of Data and a Comparison with MC

In this analysis, we used the same data set as for jet rates and C planarity. The event selection criteria and other cuts are also the same as described

Bin nr.	Oblateness Range	Q_g Range (GeV)	Maximum Change (%) in O/N_t dN/dO
1	0.00 - 0.06	1.00 - 6.00	27
2	0.06 - 0.08	1.00 - 6.00	3
3	0.08 - 0.10	1.00 - 6.00	8
4	0.10 - 0.12	1.00 - 6.00	14
5	0.12 - 0.14	1.00 - 6.00	16
6	0.14 - 0.16	1.00 - 6.00	20
7	0.16 - 0.19	1.00 - 6.00	20
8	0.19 - 0.23	1.00 - 6.00	21
9	0.23 - 0.27	1.00 - 6.00	21
10	0.27 - 0.35	1.00 - 6.00	16
11	0.35 - 0.47	1.00 - 6.00	22
12	0.47 - 0.63	1.00 - 6.00	56

Table 8.1: Effect of parton virtuality on the differential distribution of oblateness.

Bin nr.	Thrust Range	Q_g Range (GeV)	Maximum Change (%) in $(1-T)/N_t$ dN/dT
1	0.500 - 0.615	1.00 - 10.00	200
2	0.615 - 0.715	1.00 - 10.00	19
3	0.715 - 0.755	1.00 - 10.00	5
4	0.755 - 0.795	1.00 - 10.00	11
5	0.795 - 0.815	1.00 - 10.00	5
6	0.815 - 0.835	1.00 - 10.00	6
7	0.835 - 0.855	1.00 - 10.00	7
8	0.855 - 0.875	1.00 - 10.00	11
9	0.875 - 0.895	1.00 - 10.00	10
10	0.895 - 0.915	1.00 - 10.00	10
11	0.915 - 0.935	1.00 - 10.00	15
12	0.935 - 0.955	1.00 - 10.00	14
13	0.955 - 1.000	1.00 - 10.00	9

Table 8.2: Effect of parton virtuality on the differential distribution of thrust.

in chapter 3. The charged and the neutral particles detected by the central tracking chambers and by the electromagnetic lead glass calorimeter respectively, were used in this analysis.

The differential distributions of oblateness and thrust are corrected for the limited detector acceptance and resolution and for the initial state radiation. The bin by bin correction factors were calculated from the two samples of MC events generated with the Jetset parton shower model, using the parameter values optimized to describe the OPAL data [61]. The first sample included the initial state photon radiation, a simulation of the OPAL detector and the same event reconstruction and event selection as applied to the real data. The second sample consisted of the MC events at the generator level including charged and neutral particles with lifetimes larger than 3×10^{-10} s, and no initial state radiation. The correction factors were determined by the ratio of the distribution from the second sample to the corresponding distribution obtained from the first sample. The differential distributions thus unfolded for the detector, are presented in Table 8.3 and Table 8.4.

The errors on the measurements shown in Tables 8.3 and 8.4, include the systematic experimental uncertainty which was calculated as explained below. The distributions were measured in three ways: using all charged tracks and clusters from electromagnetic calorimeters, using only charged tracks, and using only clusters. The largest difference between any pair of the three analyses is taken as the experimental uncertainty. The statistical and experimental uncertainties are then added in quadrature to yield the overall error presented in the Tables 8.3 and 8.4.

The measured differential distributions corrected for the detector are then

compared with the corresponding predictions of the Jetset Shower MC (calculated at the generator level) in figures 8.7 and 8.9 for oblateness and thrust respectively. As may be noticed from these figures, Jetset describes the data well for these distributions over their entire range. We have checked that the uncorrected distributions are also well described by the corresponding predictions of the Jetset data simulated through the OPAL detector. This combined with the result that the Jetset provides a good description of all the event shape distributions in a wide range of E_{cm} [61] suggests the use of this model to also correct the data for hadronization.

To compare with QCD calculations, the data are corrected for the detector effects and for the effects of hadronization. The bin by bin correction factors are obtained by the ratio of the corresponding differential distributions obtained from two samples of Jetset events. The first sample consists of the final partons in the QCD shower (halted at $Q_g=1$ GeV), the second sample includes initial state radiation, hadronization and a full simulation of the OPAL detector. The differential distributions thus corrected are compared with the predictions of $\mathcal{O}(\alpha_s^2)$ QCD calculations in figures 8.8 and 8.10.

8.4 Comparison with $\mathcal{O}(\alpha_s^2)$ QCD calculations and a Determination of $\alpha_s(M_{Z^0})$

The α_s values are obtained by fitting the theory to the data for the differential distributions of oblateness and thrust. From these fits, the QCD parameter $\Lambda_{\overline{MS}}$ is determined. In order to compare the data with QCD calculations one has to choose the value for the renormalization scale μ . As in the case of jet rates and C planarity, here also we make fits for two different choices of the

Bin nr.	O value	Oblateness Range	$O/N_i \text{ dN/dO}$
1	0.030	0.00 - 0.06	0.284 ± 0.005
2	0.070	0.06 - 0.08	0.326 ± 0.010
3	0.090	0.08 - 0.10	0.304 ± 0.011
4	0.110	0.10 - 0.12	0.294 ± 0.010
5	0.130	0.12 - 0.14	0.270 ± 0.011
6	0.150	0.14 - 0.16	0.248 ± 0.014
7	0.175	0.16 - 0.19	0.221 ± 0.014
8	0.210	0.19 - 0.23	0.205 ± 0.010
9	0.250	0.23 - 0.27	0.163 ± 0.006
10	0.310	0.27 - 0.35	0.107 ± 0.007
11	0.410	0.35 - 0.47	0.040 ± 0.003
12	0.550	0.47 - 0.63	—

Table 8.3: The measured Differential distribution of oblateness unfolded for the detector.

Bin nr.	T value	Thrust Range	$(1-T)/N_i \text{ dN/dT}$
1	0.5575	0.500 - 0.615	—
2	0.6650	0.615 - 0.715	0.037 ± 0.005
3	0.7350	0.715 - 0.755	0.092 ± 0.004
4	0.7750	0.755 - 0.795	0.126 ± 0.007
5	0.8050	0.795 - 0.815	0.163 ± 0.010
6	0.8250	0.815 - 0.835	0.185 ± 0.010
7	0.8450	0.835 - 0.855	0.212 ± 0.014
8	0.8650	0.855 - 0.875	0.225 ± 0.007
9	0.8850	0.875 - 0.895	0.269 ± 0.010
10	0.9050	0.895 - 0.915	0.321 ± 0.015
11	0.9250	0.915 - 0.935	0.363 ± 0.009
12	0.9450	0.935 - 0.955	0.417 ± 0.010
13	0.9775	0.955 - 1.000	0.246 ± 0.004

Table 8.4: The measured differential distribution of thrust unfolded for the detector.

Fig. 8.7 Measured oblateness distribution compared to Jetset shower MC.

Fig 8.8 Measured oblateness distribution compared to the analytic $O(\alpha_s^2)$ QCD calculations.

Fig. 8.9 Measured thrust distribution compared to Jetset shower MC.

Fig. 8.10 Measured thrust distribution compared to the analytic $O(\alpha_s^2)$ QCD calculations.

renormalization scale $\mu^2 = f E_{cm}^2$:

- The fits are made to determine $\Lambda_{\overline{MS}}$ while the renormalization scale factor f is fixed to 1 (ie $\mu^2 = E_{cm}^2$).
- Both the $\Lambda_{\overline{MS}}$ and the scale factor f are treated as free parameters and their values are determined from a two parameter fit.

The fits were made in the regions of $O = 0.06-0.35$ for oblateness and of $T = 0.715-0.935$ for thrust. The choice of these fit regions was motivated by the following requirements: 1) to be minimally sensitive to hadronization and perturbative effects, 2) to stay in the three jet region, 3) to stay away from singularities, and 4) to stay away from the unphysical regions where the theory predicts negative cross sections.

The fits of the theory with the data and the regions of fits are displayed in Fig. 8.8 for oblateness and in Fig. 8.10 for thrust. The corresponding values of $\Lambda_{\overline{MS}}$ and f along with their experimental errors from the fits are listed in Table 8.5. The main observations from the results may be summarized as follows:

- For thrust, the theoretical predictions with the optimized scale provide a better description of the data inside and outside the fit regions, as compared to the fixed scale ($f=1$) predictions.
- The optimized scale factor for thrust ($f=0.0017$) is closer to that from jet rates and C planarity. The optimized scale factor for oblateness ($f=0.60$) is closer to 1, significantly different from the other observables.

This may be explained by the negative second order corrections for oblateness in contrast to the positive second order corrections for other observables.

From the fit results of $\Lambda_{\overline{MS}}$, we determine $\alpha_s(M_{Z^0})$ using the formula 7.2 in chapter 7.

In the absence of a unique and commonly accepted theoretical prescription for the experimental choice of the scale and being consistent with the previous chapters, we determine $\alpha_s(M_{Z^0})$ from $\Lambda_{\overline{MS}}$ values quoted in Table 8.5 and take the mean of two extremes (ie $f=1$ and optimized f) as the final value of $\alpha_s(M_{Z^0})$ and the difference between the mean and the two extreme values is taken as the error on $\alpha_s(M_{Z^0})$ due to the scale ambiguity. The error due to the scale determined this way is artificially small ($=\pm 0.001$) for oblateness, because the optimized scale in this case is already closer to 1. Therefore, in the case of oblateness we estimated the scale error by taking the difference between the quoted α_s and the two extreme values calculated at $f=1$ and $f=0.25$. These final values of α_s are presented for oblateness and thrust in Table 8.6.

We estimate the error on our results due to the hadronization uncertainty by repeating our analysis in which the detector and hadronization corrections are now made using the Herwig QCD shower and cluster hadronization [76]. The resulting changes in the values of $\alpha_s(M_{Z^0})$ determine the errors due to the model dependence of the hadronization.

We also estimate the errors due to the parton virtuality to which the data are corrected. We repeat our entire analysis by correcting the data for

Fig. 8.11 Values of α_s as a function of Q_g from thrust.

Observable	Fit Interval	Scale Factor f	$\Lambda_{\overline{MS}}$ (MeV)	χ^2 of fit
Oblateness	0.06-0.35	$f(\text{fixed})=1.00$	269 ± 16	27.81/8
		$f(\text{optm.}) = 0.602^{+0.242}_{-0.144}$	300 ± 30	25.76/7
Thrust	0.715-0.935	$f(\text{fixed})=1.00$	776 ± 28	29.35/8
		$f(\text{optm.}) = 0.00172^{+0.00206}_{-0.00076}$	166 ± 18	13.22/7

Table 8.5: Fit results of $\Lambda_{\overline{MS}}$ for fixed and optimized scale factor f .

Observable	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(\text{exp.})$	$\Delta\alpha_s(\text{had.})$	$\Delta\alpha_s(Q_g)$	$\Delta\alpha_s(\text{scale})$
Oblateness	0.122	± 0.002	$+0.008$	$+0.046$	$+0.008$ -0.001
Thrust	0.129	± 0.003	± 0.001	-0.009	± 0.016

Table 8.6: Final results of $\alpha_s(M_{Z^0})$ from oblateness and thrust.

hadronization using the values of Q_g ranging from 1 GeV to 10 GeV for thrust, and from 1 GeV to 6 GeV for oblateness. In Fig. 8.11, the values of $\alpha_s(M_{Z^0})$ are plotted as a function of Q_g for thrust. The hadronization is sensitive to the value of Q_g and also the final number of partons in the QCD shower depends on it. Therefore, the uncertainty in the results calculated by varying Q_g may be interpreted as due to both the hadronization uncertainty and the uncertainty due to the higher order perturbative effects. The uncertainty due to Q_g on $\alpha_s(M_{Z^0})$ is taken from Fig. 8.11. The combined uncertainty due to hadronization and to Q_g is huge (+0.046) for oblateness and moderate (-0.009) for thrust.

The experimental errors listed in Table 8.6 include the statistical errors (of both the measurement and the correction factors) and the experimental systematic errors added in quadrature. The experimental systematic errors were estimated by repeating the analysis once only for the charged particles, then only for the neutral particles and measuring the shift in the values of the final results.

8.5 Average Value of $\alpha_s(M_{Z^0})$ Determined from Jet Rates, C Planarity and Thrust

So far, we have determined the values of $\alpha_s(M_{Z^0})$ from fits of the corresponding theoretical calculations to the experimental distributions; and this has been done separately for each observable. $\alpha_s(M_{Z^0})$ values thus determined from different observables are in general agreement with each other.

We shall now combine the results to obtain an average value of $\alpha_s(M_{Z^0})$.

We determine the average value of $\alpha_s(M_{Z^0})$ using the equation:

$$\overline{\alpha_s(M_{Z^0})} = \frac{\sum_i w_i \alpha_s^i(M_{Z^0})}{\sum_i w_i}, \quad (8.3)$$

where the weight w_i is the inverse of the square of overall error on $\alpha_s^i(M_{Z^0})$ determined from the observable i .

If the errors are asymmetric, ie of the form $^{+x_i}_{-y_i}$, w_i is given by $(\frac{x_i+y_i}{2})^{-2}$.

Because we have used the same procedures to estimate various errors on the determination of $\alpha_s(M_{Z^0})$ from different observables, the results may be correlated. Thus, it is not appropriate to take $(\sum_i w_i)^{-1/2}$ as the error on the weighted mean. The procedure that we adopted to calculate the error on the average result is explained here with an example. The average value of $\alpha_s(M_{Z^0})$ is computed using the sets of $\alpha_s^{(i)}$ values determined from different observables at fixed scale (ie at $f=1$), using the weights w_i as defined above. A second average value of $\alpha_s(M_{Z^0})$ is obtained using $\alpha_s^{(i)}$ determined at optimized scale (ie in a two parameter fit of f and $\Lambda_{\overline{MS}}$). The difference between these two values gives the contribution of the scale uncertainty to the overall error on $\overline{\alpha_s(M_{Z^0})}$. The same procedure is repeated for all the other sources of uncertainty, and the different errors thus calculated are added in quadrature to give the overall error on $\overline{\alpha_s(M_{Z^0})}$.

In case of jet rates, all the four recombination schemes yield an average value of $\alpha_s(M_{Z^0}) = 0.120^{+0.010}_{-0.006}$, which may be written in form of symmetric errors as $\alpha_s(M_{Z^0}) = 0.122 \pm 0.008$. All the jet rates combined with C planarity and thrust yield an average value of $\alpha_s(M_{Z^0}) = 0.121^{+0.010}_{-0.007}$, which may be written in form of symmetric error as $\alpha_s(M_{Z^0}) = 0.123 \pm 0.008$. To

Observable	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(\text{total})$
Jet rates:		
$D_2(y)$: E_0 -scheme	0.119	± 0.009
$D_2(y)$: P_0 -scheme	0.120	± 0.007
$D_2(y)$: P-scheme	0.118	± 0.009
$D_2(y)$: E-scheme	0.131	± 0.018
C planarity	0.128	± 0.016
Thrust	0.129	$+0.016$ -0.019
Oblateness	0.122	$+0.047$ -0.002
Average	0.123	± 0.008

Table 8.7: Final results of $\alpha_s(M_{Z^0})$ from jet rates, C planarity, thrust and oblateness. The average value is extracted from jet rates, C planarity and thrust. For details, see text.

obtain the average result, we did not include oblateness because it is highly sensitive to unknown higher order corrections and the renormalization scale and hence the $\alpha_s(M_{Z^0})$ value, determined from it, is not reliable.

The final values of $\alpha_s(M_{Z^0})$ from jet rates, C planarity, thrust and oblateness are listed in Table 8.7. All the errors from different sources are added in quadrature to obtain the total errors shown in the last column of the table.

8.6 Summary and Conclusions

We have determined $\alpha_s(M_{Z^0})$ from the differential distributions of oblateness and thrust in the hadronic Z^0 decays. We have estimated the experimental and theoretical uncertainties on the determination of $\alpha_s(M_{Z^0})$. The theoretical uncertainties are found to be dominant over the experimental ones and are different for each observable. The hadronization corrections are found to be moderate ($\approx 10\%$) for thrust and are rather large ($\approx 30\%$) for oblateness. The model dependence of hadronization was found to be large in oblateness where it corresponded to a variation in $\alpha_s(M_{Z^0})$ of 7%, while this is small in the case of thrust where it shifts the values of $\alpha_s(M_{Z^0})$ only by 1%. The combined effects of hadronization and parton virtuality are huge for oblateness and moderate for thrust.

The fit results of the scale factor f are different for each observable: $f=0.0017$ (corresponding to $\mu = 3.75$ GeV) for thrust, and $f=0.60$ (corresponding to $\mu = 70$ GeV) for oblateness. Because we are fitting the finite order theory with data which is not corrected for unknown higher order effects, the variations in the fit values of the scale factor f may be interpreted as accounting for the higher order contributions missing from the theory. Considering that the $\alpha_s(M_{Z^0})$ measurement from oblateness is far more sensitive to the parton virtuality, we expect that the higher order contributions for oblateness are substantially larger.

The final values of $\alpha_s(M_{Z^0})$ from oblateness and thrust are in agreement with each other and completely overlap within their uncertainties. They also agree with our measurements of $\alpha_s(M_{Z^0})$ from jet rates and C planarity.

The average value of $\alpha_s(M_{Z^0})$ obtained from jet rates, C planarity and thrust is:

$$\alpha_s(M_{Z^0}) = 0.123 \pm 0.008$$

Thus, we conclude that $\alpha_s(M_{Z^0})$ is determined to be around 0.123 with a total uncertainty of 7 %.

Chapter 9

The Optimization of the renormalization Scale

The renormalization scale μ^2 does not correspond to any physical quantity. It is often called an unphysical parameter of which the physical quantities should be independent. But this is true only if they are calculated to all orders in the perturbation expansion. Results obtained from perturbative QCD truncated at a finite order ($\mathcal{O}(\alpha_s^2)$ in our case) may depend upon the renormalization scale. As we have noticed in the previous chapters, they actually do. The structure of perturbative QCD does not predict at which value of μ we should evaluate the second order formulae. However, various procedures have been proposed to make a sensible choice of the scale at finite order. We attempted two experimental approaches in the previous chapter; to fix the scale μ to E_{cm} , and to treat it as a free parameter in the fit and hence to optimize it. In this chapter, we investigate the QCD inspired theoretical procedures to optimize the scale. For the sake of completeness, we modify (or extend) some of these procedures. The purpose of this study is twofold: 1) To

check the validity of the QCD ideas behind these procedures by comparing their predictions to the experimental results; 2) Because all procedures of optimizing the renormalization scale have some implied assumptions about the unknown higher order corrections¹, their predictions when compared to the experimental results may provide some clues about the unknown higher order corrections for a given observable.

In section 9.1, the definitions of these procedures are presented. In section 9.2, the mathematical form of these procedures is calculated for a general observable Q . The scale predictions for jet rates, C planarity, oblateness and thrust are calculated in section 9.3. These theoretical predictions are compared, in section 9.4, to the scale values deduced from the experimental data in chapters 7 and 8. In section 9.5, we present an experimental approach to eliminate the scale ambiguity in the final results. This study is pursued further in section 9.6. In section 9.7, we present the main conclusions from this study.

9.1 Theoretical Procedures for Optimizing the renormalization Scale

Various theoretical procedures to optimize the renormalization scale are discussed below.

a) The Principle of Minimal Sensitivity

For theoretical details of Stevenson's principle of minimal sensitivity (PMS) one is referred to [77]. The philosophy of the PMS is rooted in a simple ar-

¹without the unknown higher order corrections, there would be no scale problem.

gument: Since we know that the exact results should be independent of the scale μ^2 , the approximants should be least sensitive to small changes in the scale. Therefore, the calculated observable can only be a good approximation in the scale region where it is flat ie independent of μ^2 . Thus, mathematically speaking, in the PMS procedure the derivative of a given observable Q with respect to μ^2 should vanish ie

$$\frac{d}{d\mu^2}Q = 0. \quad (9.1)$$

This may also be viewed as partial equivalence of scale changes and variations of the (unknown) higher order corrections in such a way that they minimize the scale dependence of the (known) formula up to second order.

b) The MSD Procedure

Strictly speaking, Stevenson's approach represented by equation 9.1 is completely fulfilled only in infinite order perturbation theory. We modify the traditional representation of PMS by choosing the scale in such a way that the observable has a moderate dependence upon scale ie the first derivative of the observable with respect to scale is minimal. Thus the idea of Moderate Scale Dependence (MSD) would modify the equation 9.1 to

$$\frac{d^2}{d^2\mu^2}Q = 0. \quad (9.2)$$

To elaborate the difference between PMS and MSD, consider the observable $Q(x)$, where $x=\mu^2$, at a point around $x=x_0$. Now consider a small change in the scale x say Δx . One can write the Taylor expansion for Q as

$$Q(x_0 + \Delta x) = Q(x_0) + \Delta x Q'(x_0) + \frac{(\Delta x)^2}{2!} Q''(x_0) + \dots \quad (9.3)$$

Ignoring the higher order terms , the resulting change in the observable would be

$$Q(x_0 + \Delta x) - Q(x_0) = \Delta x Q'(x_0) + \frac{(\Delta x)^2}{2!} Q''(x_0) \quad (9.4)$$

PMS implies that this change is proportional to $(\Delta x)^2$, while MSD takes a more liberal approach by assuming that the change is a linear function of Δx .

c) The Fastest Apparent Convergence

The fastest apparent convergence (FAC) procedure, proposed by Grunberg [79], is based on absorbing the higher order terms into the strong coupling constant α_s . In second order theory, this means that the scale is chosen in such a way that the coefficient of the next to leading order correction vanishes.

d) The BLM Procedure

This procedure, proposed by Brodsky, Lepage and Mackenzie (BLM) [78], is based on absorbing terms which depend upon the number of active fermions n_f , into the strong coupling constant α_s . In second order theory this means that the coefficient of next to leading order correction $c_2(\mu^2, n_f)$ is independent of n_f that is

$$\frac{d}{dn_f} c_2(\mu^2, n_f) = 0 \quad (9.5)$$

e) The Modified BLM Procedure

Instead of demanding that the second order coefficient is independent of n_f , we modify the BLM scheme to demand that the n_f dependent part of the second order coefficient vanishes. This may be interpreted as making scale changes and variations in the (unknown) higher order terms in such a way that they cancel the n_f dependent part of the second order coefficient.

f) The Hard Scale Procedure

By hard scale we mean $\mu^2 = E_{cm}^2$ (ie $f=1$). It has been a tradition in the pre LEP era to fix the scale factor f to unity in comparing the theory to data. This is certainly one way of dealing with scale and it is useful to consider it in the framework of a procedure. It can be naturally achieved by imposing the requirement that the scale dependent term of the second order coefficient vanishes. This would lead to $\ln(f) = 0$, and hence $f=1$, for both the jet rates and other observables. This may be interpreted as varying the scale in the (unknown) higher order corrections in such a way as to cancel the scale dependent term in the second order coefficient as opposed to PMS where the scale dependence of the known terms (including α_s) is expected to be minimized.

9.2 Calculations of the Optimized Scale for Various Procedures

Consider a physical quantity Q that may be written in powers of α_S up to second order

$$Q = c_1 \alpha_S + c_2(\mu^2, n_f) \alpha_S^2 \quad (9.6)$$

Differentiation of this equation with respect to μ^2 yields

$$\frac{d}{d\mu^2} Q = c_1 \frac{d}{d\mu^2} \alpha_S + c_2(\mu^2, n_f) 2\alpha_S \frac{d}{d\mu^2} \alpha_S + \alpha_S^2 \frac{d}{d\mu^2} c_2(\mu^2, n_f). \quad (9.7)$$

Recall that the coefficient of the leading term, c_1 , does not depend upon the scale. The explicit form of the coefficient c_2 may be obtained by comparing the equation 9.6 with the equation 8.1.

Using the equation [33]

$$\frac{d}{d\mu^2} \alpha_S = -\frac{1}{\mu^2} (b_0 \alpha_S^2 + b_1 \alpha_S^3) \quad (9.8)$$

one can obtain from 9.7

$$\frac{d}{d\mu^2} Q = -\frac{1}{\mu^2} [(b_1 c_1 + 2b_0 c_2(\mu^2, n_f)) \alpha_S^3 + 2b_1 c_2(\mu^2, n_f) \alpha_S^4]. \quad (9.9)$$

Imposing the condition on 9.9 that the derivative of Q vanishes (PMS), and neglecting $Q(\alpha_S^4)$ term, we obtain

$$b_1 c_1 + 2b_0 c_2(\mu^2, n_f) = 0. \quad (9.10)$$

Equation 9.10 may be solved for a given observable Q to obtain the PMS prediction of optimized scale. Taking the second derivative of Q from equation 9.9, and letting it vanish we obtain

$$(b_1 - 2b_0^2)c_1 + 2b_0c_2(\mu^2, n_f) = 0. \quad (9.11)$$

Equation 9.11 can be solved to predict the MSD optimized scale for a given observable Q . The FAC prediction of the optimized scale can be calculated by equating c_2 to zero, and the BLM prediction can be calculated by solving equation 9.5. The predictions of the modified BLM (MBLM) procedure can be predicted by equating the n_f dependent part of $c_2(\mu^2, n_f)$ to zero.

9.3 Calculations of the Optimized Scale for Jet Rates and for other Infrared Safe Observables.

The general calculations in the previous section can be used to compute the scale prediction for a particular observable. We apply these calculations to the analytic formulae of 3-jet rates, C planarity, oblateness and thrust given in reference [33]. Replacing the general equation 9.6 with these formulae and following the PMS procedure², one obtains for the scale factor f

$$f = e^{-\frac{1}{2b_0} \left(\frac{b_1}{b_0} + \frac{B_{3,J}}{\pi A_{3,J}} \right)} \quad (9.12)$$

²In other words, solve equation 9.10 for f .

where the coefficients $B_{3,l}$ and $A_{3,l}$ depend upon the observable. In the case of jet rates, $B_{3,l}$ also depend upon the recombination scheme. We take the values of $A_{3,l}$ and $B_{3,l}$ from reference [33] as an input to equation 9.12 in order to compute the scale predictions of PMS. Similarly, we calculate the optimized scale using FAC, BLM and MSD procedures.

The f values calculated from the BLM, PMS, FAC and MSD procedures for the jet rates in the E_0 -scheme are listed in table 9.1. The f values calculated for the jet rates in the P_0 -, P-, and P schemes are listed in table 9.2. Corresponding to each y_{cut} value, there are three f values from the PMS, FAC, and MSD procedures respectively. The predictions of the optimized f for C planarity, oblateness and thrust are listed in tables 9.3, 9.4 and 9.5 respectively. The optimized scale factor f is plotted for these observables in figures 9.1 and 9.2.

9.4 Comparison of the Theoretical Predictions to the Experimental Results

For jet rates, the predicted scales are different for different recombination schemes, a result in agreement with the scale values deduced from the experimental data in chapter 7. One may notice that for the E_0 -scheme, the values of f predicted by FAC in the region of $y_{cut} \geq 0.08$ completely agree with the experimental value extracted from the D_3 fits. Such is also the case of MSD for the P_0 -scheme. For the P-scheme, the f values calculated from PMS and FAC completely agree with those deduced from the experimental data in the region of $0.04 \leq y_{cut} \leq 0.14$. In the E-scheme, predictions of PMS are closer to the experimental results.

Fig. 9.1 The theoretical predictions for the optimized scale.

Fig. 9.2 The theoretical predictions for the optimized scale.

y_{cut}	f			
	BLM	PMS	FAC	MSD
0.03	0.0018	0.011	0.015	0.030
0.04	0.0024	0.0058	0.0081	0.016
0.05	0.0031	0.0039	0.0054	0.011
0.06	0.0037	0.0030	0.0042	0.0082
0.08	0.0050	0.0022	0.0030	0.0060
0.10	0.0061	0.0019	0.0026	0.0052
0.12	0.0072	0.0018	0.0025	0.0048
0.14	0.0082	0.0018	0.0025	0.0049
0.17	0.0094	0.0019	0.0027	0.0052
0.20	0.0100	0.0020	0.0028	0.0054

Table 9.1: Predictions for the optimized scale factor calculated for 3-jet rates in E_{ij} -scheme using BLM, PMS, FAC and MSD procedures; f from experimental fits = $0.0027^{+0.0009}_{-0.0005}$

In the case of C planarity, the predictions of PMS for $C \geq 0.33$ completely overlap with the experimental value. For thrust, the f values predicted by PMS, FAC and MSD in the range of $0.81 \leq T \leq 0.93$ agree with the experimental value within two standard deviations.

For oblateness, the predicted values of f are closer to unity in contrast to the smaller values of f for other observables. This observation is consistent with the f values deduced from the experimental data.

A general observation from the theoretical predictions of the optimized scale factors f from PMS, FAC and MSD is that they all, except for oblateness, are considerably smaller than unity and much closer to the experimental values. The MBLM scheme predicts the scale factor f considerably greater than unity for all the observables and hence is not favored by the experimen-

y_{cut}	f		
	P_0	P	E
	(PMS,FAC,MSD)	(PMS,FAC,MSD)	(PMS,FAC,MSD)
0.03	0.051	0.070	0.00041
	0.071	0.097	0.00057
	0.14	0.19	0.0011
0.04	0.025	0.040	0.00026
	0.034	0.056	0.00036
	0.067	0.109	0.00070
0.05	0.017	0.029	0.00020
	0.024	0.041	0.00027
	0.046	0.080	0.00053
0.06	0.013	0.025	0.00016
	0.018	0.035	0.00022
	0.035	0.068	0.00043
0.08	0.0090	0.021	0.00012
	0.013	0.030	0.00017
	0.025	0.058	0.00033
0.10	0.0074	0.023	0.00010
	0.010	0.031	0.00014
	0.020	0.061	0.00028
0.12	0.0067	0.027	0.000090
	0.0093	0.038	0.00013
	0.018	0.074	0.00025
0.14	0.0061	0.037	0.000082
	0.0085	0.051	0.00011
	0.017	0.10	0.00022
0.17	0.0057	0.071	0.000066
	0.0079	0.099	0.000092
	0.015	0.19	0.00018
0.20	0.0055	0.18	0.000044
	0.0076	0.25	0.000062
	0.015	0.48	0.00012
f(exp.)	$0.016^{+0.009}_{-0.005}$	$0.026^{+0.028}_{-0.011}$	$0.000063^{+0.000004}_{-0.000004}$

Table 9.2: Predictions for the optimized scale factor, calculated for 3-jet rates in P_0 -, P - and E -scheme using PMS, FAC and MSD procedures.

C	f			
	PMS	FAC	BLM	MSD
0.09	0.011	0.016	0.00039	0.031
0.15	0.0023	0.0032	0.00089	0.0063
0.21	0.0015	0.0021	0.0016	0.0041
0.27	0.00075	0.0010	0.0024	0.0020
0.33	0.00049	0.00069	0.0035	0.0013
0.39	0.00041	0.00057	0.0046	0.0011
0.45	0.00042	0.00058	0.0059	0.0011
0.51	0.00033	0.00045	0.0072	0.00088
0.57	0.00029	0.00040	0.0086	0.00078
0.63	0.00032	0.00044	0.010	0.00086
0.69	0.00026	0.00036	0.012	0.00070

Table 9.3: Predictions for the optimized scale factor, calculated for C planarity using PMS, FAC, BLM and MSD procedures; f from experimental fits
 $= 0.00030^{+0.00024}_{-0.00009}$

O	<i>f</i>		
	PMS	FAC	MSD
0.07	19.96	27.74	54.26
0.09	5.63	7.82	15.31
0.11	3.73	5.18	10.14
0.13	2.35	3.27	6.40
0.15	1.71	2.38	4.06
0.17	1.49	2.08	4.34
0.19	1.59	2.22	3.58
0.21	1.32	1.83	5.06
0.25	1.86	2.59	9.10
0.31	3.35	4.65	34.72
0.35	12.77	17.75	0.012

Table 9.4: Predictions for the optimized scale factor, calculated for oblateness using PMS, FAC, and MSD procedures; f from experimental fits = $0.602^{+0.242}_{-0.141}$

T	<i>f</i>		
	PMS	FAC	MSD
0.735	0.000019	0.000026	0.000051
0.775	0.000095	0.00013	0.00026
0.805	0.00016	0.00022	0.00043
0.825	0.00019	0.00027	0.00053
0.845	0.00022	0.00031	0.00061
0.865	0.00030	0.00042	0.00083
0.885	0.00038	0.00053	0.0010
0.905	0.00037	0.00052	0.0010
0.925	0.00050	0.00070	0.0014

Table 9.5: Predictions for the optimized scale factor, calculated for thrust using PMS, FAC, and MSD procedures; f from experimental fits = $0.0017^{+0.0021}_{-0.0008}$

tal fits.

The experimentally optimized values of f agree with the predictions of the theoretical procedures based on the idea of minimal sensitivity which has its roots in the claim that the perturbative QCD solved to all orders has no scale dependence.

9.5 An Experimental Approach to Eliminate the Renormalization Scale Uncertainty

As we have noticed in chapter 7 and chapter 8, the theoretical errors on the determination of $\alpha_s(M_{Z^0})$ are dominant over the experimental errors and the major contribution to the theoretical errors comes from the renormalization scale ambiguity. The renormalization scale ambiguity arises due to the missing higher order contributions in the analytical calculations. The best solution to the problem is to calculate the higher order corrections which is of course a challenging task. In the absence of these calculations, two methods have been recently proposed [81, 82] to optimally use the existing calculations in order to reduce the scale ambiguity. We elaborate here on the method that we proposed in ref. [82].

$\Lambda_{\overline{MS}}$ is determined in a one parameter fit of the analytical calculations with the corresponding data for jet rates and other infrared safe observables. The fits are repeated for different values of f in its entire range in which the χ^2 of the fit is reasonable (ie ≤ 5 per degree of freedom). The $\Lambda_{\overline{MS}}$ thus determined is plotted as a function of f in Fig. 9.3. The line thickness of each $\Lambda_{\overline{MS}}$ curve in these plots corresponds to the percentage statistical fit error

which is about 10% for jet rates and 5% for C, O and T. The regions of y_{cut} , C, O and T in which the fits are performed are shown in the diagrams. These regions were chosen to exclude the 4-jet rates because they are calculated only to leading order, and to exclude the unphysical region where the predicted cross sections are negative and/or hadronization corrections are unacceptably high.

From plots in Fig. 9.3, the following conclusions may be drawn:

1. For jet rates, C planarity and thrust; the $\Lambda_{\overline{MS}}$ strongly depends on f around $f=1$, while it reaches a relatively stable region at smaller values of f which we shall call the optimized f region. Quantitatively, the values of f corresponding to the optimized f region can be defined by allowing $\Lambda_{\overline{MS}}$ to vary by the experimental uncertainty in $\Lambda_{\overline{MS}}$ in this region. The values of f obtained in this way are listed in Table 9.6.

2. The optimized f region for oblateness is not smaller than 1.

3. The optimized f region may, in general, be different for different observables.

The optimized region for each curve in plots of Fig. 9.3 was determined by the requirement that the variation of $\Lambda_{\overline{MS}}$ from its average value in this region should not exceed the statistical error.

The Delphi [83] collaboration has reported a similar scale dependence of $\alpha_s(M_{Z^0})$. At lower energy experiments a similar scale dependence of $\Lambda_{\overline{MS}}$ was observed from jet studies [84, 85].

We focus on three observations from Fig. 9.3 : 1) The optimized scale is,

Fig. 9.3 Λ as a function of scale factor f determined from the data.

in general, different for different observables; 2) The $\Lambda_{\overline{MS}}$ values determined from different observables in the entire scale region cover a wide range, 100 MeV to 1100 MeV; and 3) The values of $\Lambda_{\overline{MS}}$ determined from different observables are significantly closer to each other in their optimized scale regions and they overlap within an uncertainty of about ± 50 MeV which is the typical statistical error on $\Lambda_{\overline{MS}}$. This may be understood in terms of the difference in the size of the known second order and unknown higher order corrections for these observables. The difference in the size of the higher order corrections gives rise to the different optimized regions for the scale. Once the higher order corrections are correctly accounted for by the scale factor f , in these optimized regions, different observables yield similar values of $\Lambda_{\overline{MS}}$ with minimal scale dependence. Thus the final values of $\alpha_s(M_{Z^0})$ determined in the optimized region are more reliable than any other region. The average value of $\alpha_s(M_{Z^0})$ determined in the optimized f region for each observable is taken as the final value of $\alpha_s(M_{Z^0})$. The variation in this value in the optimized f region is smaller than the statistical errors and hence the scale uncertainty in this region is removed. The $\alpha_s(M_{Z^0})$ values thus determined are listed in table 9.6. All other errors are added in quadrature to give the total error on $\alpha_s(M_{Z^0})$ listed in the last column of the table.

The $\Lambda_{\overline{MS}}$ values determined in this fashion from jet rates, C planarity and thrust, fall in the range of 168 ± 52 MeV which translates to $\alpha_s(M_{Z^0}) = 0.113 \pm 0.005$; an excellent agreement with the QCD prediction of $\alpha_s(M_{Z^0}) = 0.11 \pm 0.01$ based on the extrapolation of lower energy³ experimental results [86]. Ellis, Nanopoulos and Ross [81] have proposed a theoretical procedure to account

³Once we know the value of $\Lambda_{\overline{MS}}$ at a given energy, it may be translated to an α_s value corresponding to any other energy scale using equation 2.13.

Observable	optimized f region	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(\text{total})$
Jet rates:			
E_0 -scheme	0.00075-0.025	0.112	± 0.004
P_0 -scheme	0.005-0.10	0.116	± 0.005
P-scheme	0.005-0.10	0.115	± 0.008
E-scheme	0.000075-0.0010	0.109	± 0.004
C Planarity	0.00020-0.0010	0.112	± 0.003
Thrust	0.00025-0.0010	0.111	$+0.003$ -0.010
Oblateness	0.75-10.00	0.121	$+0.017$ -0.002

Table 9.6: The final values of $\alpha_s(M_{Z^0})$ after the elimination of the scale ambiguity.

for the dominant unknown higher order terms. Applying their procedure to the published $\alpha_s(M_{Z^0})$ results from LEP data and combining the $\alpha_s(M_{Z^0})$ value thus obtained with the results from deep inelastic scattering, Υ decay etc.; they obtain the weighted mean of $\alpha_s(M_{Z^0}) = 0.113 \pm 0.004$; which is in excellent agreement with our result.

9.6 A Further Study of the Renormalization Scale Uncertainty

In the previous section, we extracted $\Lambda_{\overline{MS}}$, from the experimental data, as a function of f and demonstrated that there was a region of f in which the scale dependence of $\Lambda_{\overline{MS}}$ was minimal: the variations in the values of $\Lambda_{\overline{MS}}$ in that region were not greater than the statistical errors. Then we argued that the $\Lambda_{\overline{MS}}$ values in that region should be taken as the true values and hence there remained no scale uncertainty. This procedure was motivated by the

theoretical arguments concerning minimal scale dependence presented earlier in this chapter. This approach and these results are given further support from the data itself by examining the χ^2 properties of the fits of $\Lambda_{\overline{MS}}$ as a function of f presented in Fig. 9.3.

Fig. 9.3 shows the fitted values of $\Lambda_{\overline{MS}}$ as a function of f , for various observables. The corresponding $\chi^2/\text{d.o.f.}$ of these fits are plotted in Fig. 9.4. From these plots, we observe the following:

1. The $\chi^2/\text{d.o.f.}$ for all these observables is reasonable (ie. ≤ 5) in a broad range of f values.
2. Each χ^2 curve, in these plots, has a minimum, ie the data seems to prefer one f value against the other.
3. The minima on the χ^2 curves for C planarity, thrust and jet rates in the E_0 and E schemes are not around $f=1$. Thus, the data seems to suggest that $f=1$ is not necessarily a good choice.
4. The variations in the χ^2 for C planarity, oblateness, thrust and jet rates in the E_0 -scheme are substantial, while they are small for Jet rates in the P-, the P_0 -, and the E-scheme.

We choose to select a region of f around the point of minimum χ^2 ($\chi^2_{\min}$) by allowing the χ^2 to vary up to 100 % from its minimum value, ie $\chi^2_{\min} = 2\chi^2_{\min}$. We shall call this region the χ^2 preferred f region. The fits in the χ^2 preferred region yield a range of fitted values of $\Lambda_{\overline{MS}}$. The central value in this range is taken as the final value, and the symmetric difference between the central value and the two extreme values as the scale error. $\Lambda_{\overline{MS}}$ values thus obtained are translated to $\alpha_s(M_Z)$. The results are listed in Table 9.7.

Fig. 9.4 $\chi^2/\text{d.o.f.}$ of fits, as a function of scale factor f .

The last column in this Table shows the total error on $\alpha_s(M_{Z^0})$ modified by the reduction in the scale error.

In order to obtain an average value, the individual $\alpha_s(M_{Z^0})$ values from jet rates, C planarity and thrust, listed in Table 9.7, are combined using the equation:

$$\overline{\alpha_s(M_{Z^0})} = \frac{\sum_i w_i \alpha_s^i(M_{Z^0})}{\sum_i w_i} \quad (9.13)$$

where the weight w_i is the inverse of the square of the total error shown in the last column of Table 9.7. The error on this weighted mean is estimated as explained below. Using the weights w_i , we compute the average from the set of minimum values of $\alpha_s(M_{Z^0})$ from all observables in the χ^2 preferred f regions. Similarly, we obtain an average from the set of maximum values. The difference between the maximum and minimum average values is taken as the error on the weighted mean of $\alpha_s(M_{Z^0})$ due to scale. Similarly, the errors from other sources of uncertainty are calculated. These errors added in quadrature give the overall error on the weighted mean of $\alpha_s(M_{Z^0})$. The χ^2 method yields an average value of $\alpha_s(M_{Z^0}) = 0.114 \pm 0.005$, which is in excellent agreement with $\alpha_s(M_{Z^0}) = 0.113 \pm 0.005$, estimated in the previous section.

9.7 Summary of Conclusions

The main conclusions from this study may be summarized as following:

Observable	χ^2 preferred f region	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(\text{scale})$	$\Delta\alpha_s(\text{total})$
Jet rates:				
E_0 -scheme	0.001-0.050	0.113	± 0.002	± 0.004
P_0 -scheme	0.005-1.00	0.120	± 0.005	± 0.007
P-scheme	0.005-1.00	0.118	± 0.004	± 0.009
E-scheme	0.00005-1.00	0.127	± 0.020	± 0.020
C Planarity	0.00020-0.0025	0.113	± 0.001	± 0.003
Thrust	0.0005-0.25	0.123	± 0.012	$+0.012$ -0.015
Oblateness	0.5-25	0.124	± 0.004	$+0.047$ -0.001
Average	—	0.114	± 0.003	± 0.005

Table 9.7: The final values of $\alpha_s(M_{Z^0})$ extracted by using the χ^2 criteria. For details, see text.

1. The optimized values of the renormalization scale predicted by various QCD inspired models are in general agreement with the scale values deduced from the experimental data in both one parameter and two parameter fits.

2. The optimized scale is, in general, different for different observables. This difference may be understood in terms of the difference in the size of the known second order and unknown higher order corrections for these observables.

3. The optimized value of the scale factor **f** determined in a two parameter fit of **f** and $\Lambda_{\overline{MS}}$, given in chapter 7 and chapter 8, falls in the optimized region of **f** determined in a one parameter fit of $\Lambda_{\overline{MS}}$, as described in section 9.5.

4. The scale dependence of $\Lambda_{\overline{MS}}$ (and hence of α_s) is minimal in the optimized region of **f**.

5. The $\Lambda_{\overline{MS}}$ values determined from different observables are significantly closer to each other in the optimized f region, and vary over a wide range otherwise. Our interpretation is that these variations are due to the different higher order corrections which are correctly accounted for by the scale in the optimized f region.

6. The $\Lambda_{\overline{MS}}$ values determined from different observables in the optimized f region are in better agreement with each other and with the theoretical prediction based on the lower energy experiments.

7. The results obtained by considering the χ^2 of fits support the procedure presented in section 9.5 to eliminate the scale ambiguity using the idea of minimal scale dependence.

8. This study suggests that $f = 1$ is not necessarily a good choice for the QCD scale.

Chapter 10

Summary and Conclusions

We have studied the jet production rates, C planarity, oblateness and thrust in hadronic decays of the Z^0 . This study has been performed in order to test the predictions of the analytic $\mathcal{O}(\alpha_s^2)$ QCD calculations. Such a study not only checks the validity of perturbative QCD at the Z^0 peak in the e^+e^- annihilation cross section, but is also useful in predicting the outcome of future experiments at higher energy. Furthermore, precise and consistent knowledge of $\alpha_s(M_{Z^0})$ is a key input into the ongoing attempts [87, 88, 89] to use the precision LEP data in order to test proposed extensions of the Standard Model such as Grand Unified Theories.

In order to provide a complete accountability of the theoretical uncertainties, we have used four recombination schemes in the jet cluster algorithm, namely the E_0 -, the P_0 -, the P-, and the E-scheme. A Monte Carlo study of these observables using Jetset 7.2 is performed to study the effects of hadronization and of parton virtuality Q_g which defines a transition point between the QCD parton shower and hadronization process; varying Q_g may

be interpreted as varying the higher order QCD effects. Different recombination schemes are found to predict different jet rates from MC data. The hadronization corrections are small ($\sim 5\%$) for the E_0 -scheme, rather large ($\sim 25\%$) for the E-scheme, and moderate for the P_0 - and P-schemes. The hadronization corrections in the three jet dominant physical region are $\leq 15\%$ for C planarity, $\leq 10\%$ for thrust, and are rather large ($\approx 30\%$) for oblateness. Jet rates, C planarity and thrust show a moderate sensitivity to the parton virtuality, while oblateness is found to be highly sensitive to this.

The experimental data for jet rates (from all the four schemes), C planarity, oblateness and thrust, corrected for the detector effects, agree well with the MC data. This gives us the confidence to apply the hadronization corrections calculated by the MC to the experimental data in order to compare it with the $o(\alpha_s^2)$ QCD calculations.

The differences in the jet production rates among various recombination schemes predicted by the perturbative QCD are reproduced by the experimental data. This itself is a qualitative test of the theory.

The experimental data corrected for the detector effects and for the hadronization corrections are compared with the $o(\alpha_s^2)$ QCD calculations. This is done in two ways: 1) By fixing the scale factor f to unity; 2) By treating the scale factor as a free parameter in the fit and thus to optimize it. The data is overall better described by the theory by optimizing f than by fixing it to unity. The optimized values of f are different for different schemes in the jet algorithm, ranging from 0.04 (for the P- scheme) to 0.00005 (for the E-scheme). The optimized f is found to be 0.0003 for C planarity, 0.6 for oblateness, and 0.002 for thrust.

The strong coupling constant α_s is determined from the differential distributions of 2- and 3-jet rates in each recombination scheme, and from the differential distributions of C planarity, oblateness and thrust. The uncertainties due to the experimental systematics, hadronization corrections, parton virtuality and the scale ambiguity are estimated and included in the overall reported errors on α_s . It is found that using different schemes in the algorithm at the experimental and at the theoretical level adds a considerable ambiguity to the results and hence a consistent application of the same algorithm at both the theoretical and experimental level is recommended.

In the final results, errors due to the theoretical uncertainties dominate over the experimental errors. This indicates that the higher order terms for these observables are important, and their calculations are required in order to further improve the precision.

Although different recombination schemes produce different jet rates, they yield similar values of α_s which completely overlap within their errors. This is to say that with a consistent treatment of data and theoretical calculations, no algorithm uncertainty remains in the determination of the physical parameter α_s . This result is a significant test of the validity of perturbative QCD.

The final values of α_s are determined to be:

- 0.120 ± 0.007 from jet rates,
- 0.128 ± 0.016 from C planarity,
- $0.122^{+0.017}_{-0.002}$ from oblateness and

- $0.129^{+0.016}_{-0.019}$ from thrust.

Thus the values of $\alpha_s(M_{Z^0})$ determined from different observables are in good agreement with each other. This checks the consistency of the underlying theory. These values of $\alpha_s(M_{Z^0})$ are also in good agreement with the theoretical predictions based on the extrapolation of the experimental results at lower energy [86]

$$\alpha_s(M_{Z^0}) = 0.11 \pm 0.01 \quad (10.1)$$

By averaging the individual measurements from jet rates, C planarity and thrust, we obtain

$$\alpha_s(M_{Z^0}) = 0.123 \pm 0.008$$

The various procedures for optimizing the renormalization scale have been investigated for the observables under study. The theoretical predictions are compared with the experimental fit results. It is found that the experiment seems to favor the optimizing schemes based on the idea of minimal scale dependence which has its roots in the claim that the perturbative QCD solved to all orders is independent of the renormalization scale.

Based on the idea of minimal scale dependence, we have proposed a procedure to eliminate the scale ambiguity by determining $\alpha_s(M_{Z^0})$ in the optimized region of the scale. Using this procedure, the final value of $\alpha_s(M_{Z^0})$ from jet rates, C planarity and thrust is determined to be 0.113 ± 0.005 which is in excellent agreement with the theoretical prediction of $\alpha_s(M_{Z^0}) = 0.11 \pm 0.01$. This is also in excellent agreement with predictions from other lower energy

measurements; namely with $\alpha_s(M_{Z^0}) = 0.109^{+0.004}_{-0.005}$ predicted from the analysis of deep inelastic scattering (DIS) and prompt photon data [90] and with $\alpha_s(M_{Z^0}) = 0.112 \pm 0.003$ from an analysis of structure functions in DIS [91].

The overall general conclusion of this study is that QCD seems to be a valid theory of strong interactions. The unknown higher order radiative corrections are important. As a result, the theoretical uncertainties on the final results are dominant over the experimental ones. Thus in order to further improve the precision of the experimental tests of QCD, higher order calculations are indispensable.

Appendix A

List of Symbols

OPAL	omni purpose apparatus for LEP
q	quark
g	gluon
u,d,c,s,t,b	quark flavors
QCD	quantum chromodynamics; a theory to describe the interactions between quarks and gluons
E_{cm}	Energy in center of mass reference frame
LEP	Large Electron Positron collider at CERN
Z^0	a particle mediating weak interactions
MC	Monte Carlo
Q_q	parton cutoff mass, in GeV, at which the QCD shower is halted in MC
μ, μ^2	renormalization scale
f	scale factor defined by $\mu^2 = f E_{cm}^2$
α_s	strong coupling constant
$\Lambda_{\overline{MS}}$	QCD parameter in a renormalization scheme $\overline{MS}$
M_{Z^0}	mass of Z^0

y_{cut}	jet resolution parameter used in jet counting
n_f	number of quark flavors active in the experiment(= 5 at LEP)
C,O,T	C planarity, oblateness and thrust
$o(\alpha_s^2)$	second order in α_s
ERT	Ellis, Ross, Terrano
PMS	principle of minimal sensitivity
FAC	fast apparent convergence
PLM	Brodsky Lepage and Mackenzie
MSD	minimal scale dependence
LLA	leading logarithmic approximation
R_n	n-jet event production rate, in %, where n=2,3,4,5
D_n	differential n-jet rate, where n=2,3

Appendix B

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