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The Prediction of Low Cycle Fatigue For Die Casting With FEM

by

Bin Wang

B.E. Mechanical, Beijing Polytechnic University, China

A thesis submitted to

the Faculty of Graduate Studies and Research

in partial fulfillment of the requirements for the degree of

Master of Engineering

Department of Mechanical and Aerospace Engineering

Ottawa - Carleton Institute for Mechanical and Aerospace Engineering

Carleton University

Ottawa, Canada

December, 2000

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The Prediction of Low Cycle Fatigue For Die Casting With FEM

submitted by

Bin Wang

in partial fulfillment of the requirements for

the degree of

Master of Engineering

Thesis Supervisor

Chair, Department of Mechanical and Aerospace Engineering

Carleton University
Abstract

The objective of this thesis is using the finite element method to estimate the life of the die casting dies, i.e., the number of thermal cycles before the die surface between die and casting reaches the failure level. To get accurate results, 20-node brick element was used. Also refined meshing with 8 and 20 node brick elements were used in order to increase the accuracy of the solution.

Material model and material properties play an important role in the results of FEM. In this project temperature dependent material properties were developed in the material library for H13 steel and Aluminum 380.

A high resolution FEM thermo-visco-elastic stress analysis was performed for a 1-D simple die casting model and a more complex 3-D dumbbell die casting structure. The effective plastic strain increment was evaluated at die elements. The maximum effective plastic strain increment is always at the die surface between die and casting in this project. The point with the maximum effective plastic strain increment was assumed to be the most likely failure point in a die. The Coffin-Manson equation was used to estimated the life of the die.
To my Parents
Acknowledgments

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Contents

Acceptance Sheet ......................................................... ii
Abstract ................................................................. iii
Dedication ................................................................. iv
Acknowledgments ......................................................... v
Table of Contents ......................................................... vi
List of Figures ........................................................... xiv
List of Tables ............................................................. xv

1 Introduction ............................................................. 1

1.1 General view of die casting phenomena ......................... 1
   1.1.1 Advantages of die casting .................................. 3
   1.1.2 Quality issues during die casting process ............... 5
   1.1.3 Thermal failure .............................................. 7
1.2 FEM in reliable design of die casting dies .................... 8
1.3 Thesis organization .................................................. 10

2 Fatigue Failure of Die Casting Dies ................................. 12
  2.1 Mechanical behavior of die casting dies ......................... 14
    2.1.1 Stress strain relation-elastic ................................ 14
    2.1.2 Stress strain relation-plastic ................................. 16
    2.1.3 Stress strain relation-creep .................................. 22
    2.1.4 Stress strain relation-relaxation ............................. 23
    2.1.5 Thermal stress and low cycle fatigue ....................... 23
  2.2 Life prediction methods ........................................... 26
    2.2.1 Total life approach ........................................... 27
    2.2.2 Defect-tolerant approach .................................... 31

3 FEM Software .................................................................. 32
  3.1 General review of FEMO software ................................. 32
  3.2 The design environment for low cycle fatigue analysis ........ 37

4 FEM Mesh Refinement ................................................... 40
  4.1 The basic theory for mesh refinement ......................... 40
  4.2 Development of 20-node brick element by standard concept ... 44
    4.2.1 Face basis functions of 20-node brick element ............ 45
    4.2.2 Volume basis functions of 20-node brick element .......... 48
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.3</td>
<td>The geometry of 20-node brick element</td>
<td>50</td>
</tr>
<tr>
<td>4.2.4</td>
<td>The displacement field of 20-node brick element</td>
<td>52</td>
</tr>
<tr>
<td>4.2.5</td>
<td>The strain field of 20-node brick element</td>
<td>53</td>
</tr>
<tr>
<td>4.2.6</td>
<td>The B matrix of 20-node brick element</td>
<td>53</td>
</tr>
<tr>
<td>4.2.7</td>
<td>The stiffness matrix of 20-node brick element</td>
<td>54</td>
</tr>
<tr>
<td>4.2.8</td>
<td>The numerical integration of 20-node brick element</td>
<td>55</td>
</tr>
<tr>
<td>5</td>
<td>The Practical Problem</td>
<td>60</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>60</td>
</tr>
<tr>
<td>5.2</td>
<td>The 1-D analytical model</td>
<td>61</td>
</tr>
<tr>
<td>5.3</td>
<td>Design file</td>
<td>64</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Part information</td>
<td>65</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Parameter for thermal analysis</td>
<td>67</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Parameter for stress analysis</td>
<td>72</td>
</tr>
<tr>
<td>5.4</td>
<td>Material properties</td>
<td>76</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Aluminum</td>
<td>86</td>
</tr>
<tr>
<td>5.4.2</td>
<td>H13 Steel</td>
<td>87</td>
</tr>
<tr>
<td>5.5</td>
<td>The performance of thermal and stress analysis for 1-D model</td>
<td>89</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Temperature</td>
<td>90</td>
</tr>
<tr>
<td>5.5.2</td>
<td>Displacement</td>
<td>98</td>
</tr>
<tr>
<td>5.5.3</td>
<td>Stress</td>
<td>100</td>
</tr>
</tbody>
</table>
5.6 Predicted low cycle fatigue life ........................................ 103

5.7 The performance of thermal and stress analysis for 3-D dumbbell mesh 111

6 Conclusion and Further Work Suggested .................................. 117

Bibliography ........................................................................... 120
List of Figures

2.1 Stress-strain curves: a) small (elastic) loading, (b) loading beyond elastic limit. .......................................................... 16

2.2 Stress-strain curves with Bauschinger effect [7]. ......................... 17

2.3 Constant stress creep-time curve [8]. ........................................ 23

2.4 Stress-strain diagrams: (a) cycle dependent hardening material, (b) cycle dependent softening material [8]. ......................... 25

4.1 Node numbering for the 8- to 20-node linear, isoparametric hexahedron. 45

4.2 Node numbering for the 4- to 8-node linear, isoparametric quadrilateral. 45

4.3 4- to 8-node linear quadrilateral with one optional node: a. Subelements created by mid edge node; b. Basis function for node 5; c. Basis function for node 1, when node 5 is present. ......................... 47

4.4 The geometry of 20-node brick element. ..................................... 50

4.5 3x3 Gauss points in a square. ............................................... 56
5.1 The 1-D analytical model. ............................................ 62
5.2 The refined 8-node brick element mesh. ............................ 63
5.3 Density vs. Temperature for H13 steel [13]. ....................... 78
5.4 Young’s Modulus vs. Temperature for H13 steel [13]. .......... 79
5.5 Poisson’s Ratio vs. Temperature for H13 steel [13]. .......... 80
5.6 Coefficient of Thermal Expansion vs. Temperature for H13 steel [13]. 81
5.7 Specific Volume vs. Temperature for H13 steel [13]. .......... 82
5.8 Hardening Modulus vs. Temperature for H13 steel [13]. ....... 83
5.9 Volume Strain vs. Temperature for H13 steel [13]. .......... 84
5.10 Comparison of sinh model data from MMO software and from Ashbys Deformation Map. StressResults_0.4.dat, StressResults_0.6.dat, and StressResults_0.8.dat are the sinh model data from MMO software at 0.4, 0.6 and 0.8 melting temperature, dataFor0.4.dat, dataFor0.6.dat, dataFor0.8.dat are the sinh model data from Ashbys Deformation Map at 0.4, 0.6 and 0.8 melting temperature. ......................... 88
5.11 Die cavity temperature approaching steady state. .............. 91
5.12 Die cavity temperature [K] in the 6th time step of a load cycle. ... 92
5.13 Die cavity temperature [K] in the 9th time step of a load cycle. ... 93
5.14 Die cavity temperature [K] in the 11th time step of a load cycle. ... 93
5.15 The transient temperatures in casting and die for an 8-node refined mesh. ........................................ 95

5.16 The transient temperatures in casting and die for a 20-node refined mesh. ........................................ 95

5.17 Temperature distribution through the thickness of the layer at six different times during the cycles for an 8-node refined mesh. TemperatureVsX2 and TemperatureVsX3 are the second and third time step in the die open stage and TemperatureVsX7, TemperatureVsX8 are the second and third steps in the die closed stage. Finally the TemperatureVsX10 and TemperatureVsX12 are the 5th and 7th steps in die closed stage. ........................................ 96

5.18 Temperature distribution through the thickness of the layer at six different times during the cycles for a 20-node refined mesh. TemperatureVsX2 and TemperatureVsX3 are the second and third time step in the die open stage and TemperatureVsX7, TemperatureVsX8 are the second and third steps in the die closed stage. Finally the TemperatureVsX10 and TemperatureVsX12 are the 5th and 7th steps in die closed stage. ........................................ 97

5.19 Displacement in die cavity surface during the first 10 cycles for 8-node and 20-node refined mesh. ........................................ 98
5.20 The general view of the distribution of displacement [m] in the die. 99
5.21 A general view of the distribution of effective stress [Pa] in the die. 100
5.22 Effective stress in die cavity surface during the first 5 cycles for a coarse mesh. 101
5.23 Effective stress in die cavity surface during the first 5 cycles for a 8-node refined mesh. 102
5.24 Effective stress in die cavity surface during the first 5 cycles for a 20-node refined mesh. 102
5.25 A general view of the distribution of effective plastic strain in the die. 103
5.26 Effective plastic strain vs. time in die cavity surface during the first 10 cycles for a 8-node refined mesh. This data is for the Gauss point that has maximum effective plastic strain. 104
5.27 Effective plastic strain vs. time in die cavity surface during the first 10 cycles for a 20-node refined mesh. This data is for the Gauss point that has maximum effective plastic strain. 105
5.28 The geometry of 3-D dumbbell mesh. 112
5.29 The procedure of mesh refinement. 113
5.30 The temperature [K] distribution in a dumbbell mesh in the second time step of die closed stage. 114
5.31 The effective stress [Pa] distribution in a dumbbell mesh in the 7th time step of die closed stage. 115

5.32 The effective stress [Pa] distribution in a dumbbell mesh in die cavity surface in the 7th time step of die closed stage. 116
List of Tables

3.1 Sample object definition table. ........................................... 39

4.1 Table for two dimension square with 9 Gauss points. ............... 57
4.2 Table for 20-node brick element with 27 Gauss points. ............ 58
4.3 Table for 20-node brick element with 14 Gauss points. ............ 59

5.1 Properties of material used in this analysis [13]. ................... 77
5.2 Predicted life based on the 10th cycle for 8-node mesh. ............ 107
5.3 Predicted life based on the 10th cycle for 20-node mesh. .......... 107
5.4 Predicted life based on the 20th cycle for 8-node mesh. .......... 108
5.5 Predicted life based on the 20th cycle for 20-node mesh. .......... 108
5.6 Predicted life based on the 30th cycle for 8-node mesh. .......... 109
5.7 Predicted life based on the 30th cycle for 20-node mesh. .......... 109
5.8 Predicted life based on the 40th cycle for 8-node mesh. .......... 110
5.9 Predicted life based on the 40th cycle for 20-node mesh. .......... 110
Chapter 1

Introduction

1.1 General view of die casting phenomena

Die casting manufacturing is widely used for producing high-volume, mass-produced items in the metalworking industry. Die castings play an important role in our daily life from thousands of consumer, commercial to industrial products such as automobiles, household appliances, recreation, hobby and leisure-time products, farm and garden equipment, electrical equipment and ordnance, general hardware, power tools, computer and other business equipment, instruments, toys, novelties, and a great many others too numerous to mention. In fact, die castings have greater utility and are used in more applications than components produced by almost any other metal-forming process. Die casting component parts, decorative trim, and/or finished
products offer many features, advantages and benefits to those who specify this manufac-
turing process.

Die casting is a process involving the injection of molten metal at high pressures (as opposed to casting by gravity pressure). Die casting manufacturing process can produce accurately dimensioned, sharply defined, smooth or textured-surface metal parts. It is performed by forcing molten metal under high pressure into reusable metal dies. The process is often described as the shortest distance between raw material and finished product. The term "die casting" is also used to describe the finished part.

Depending on the complexity of the casting, die casting dies can be simple or complex, having movable slides, cores, or other sections. The complete cycle of the die casting process is by far the fastest known for producing precise non-ferrous metal parts. This is in marked contrast to sand casting which requires a new sand mold for each casting. While the permanent mold process uses iron or steel molds instead of sand, it is considerably slower, and not as precise as die casting.

In die casting manufacturing process two steel die halves called the cover and ejector die halves are generally used. Each of the die halves usually contain a portion
of the die cavity. The process sequences are: (a) Die closing. The die halves are closed and locked by the die casting machine. The required clamping force during the process may be hundreds of tons. (b) Cavity filling. The molten metal is injected into the die cavity under very high pressures and velocity for low cycle times. Typical filling times are measured in milliseconds with typical flow velocities of approximately 40 m/s (132 ft/s). (c) Casting solidification. The molten metal rapidly solidified, the die halves are opened, and the part is ejected. (e) Lubrication. The open die halves are sprayed with water-based lubricants and anti solder compounds.

1.1.1 Advantages of die casting

Generally the advantage of die casting can be summarized as [1][2]:

1. Die casting provides complex shapes within closer tolerances than many other mass-production processes.

2. Die castings are produced at high rates of production. Little or no machining is required.

3. Die castings can be produced with thinner walls than those obtainable by other casting methods and much stronger than plastic injection molding with the same dimensions.

4. Die casting provide parts which are durable, dimensionally stable, and have the
feel and appearance of quality.

5. Die casting dies can produce thousands of identical castings within specified tolerances before additional tooling may be required.

6. Zinc castings can be easily plated or finished with a minimum of surface preparation.

7. Die castings can be produced with surfaces simulating a wide variety of textures.

8. Die cast surfaces, as cast, are smoother than most other forms of casting.

9. Holes in die castings can be cored, and made to tap drill sizes.

10. External threads on parts can be readily die cast.

11. Die castings provide integral fastening elements, such as bosses and studs, which can result in assembly economies.

12. Inserts of other metals and some non-metals can be die cast in place.

13. Corrosion resistance of die casting alloys rates from good to high.

14. Die castings are monolithic. They combine many functions in one, complex shaped part. Because die castings do not consist of separate parts, welded or fastened together, the strength is that of the material, not that of threads or welds, etc.

15. Die casting is an efficient, economical process which, when used to its maximum potential, replaces assemblies of a variety of parts produced by various manufacturing processes at significant savings in cost and labor.
1.1.2 Quality issues during die casting process

Die casting is a high volume production process which produces geometrically complex parts of nonferrous metals with excellent surface finishes and low scrap rate. Production rates of 200 parts per hour and production batches of 300,000 parts are common.

The above considerable economic advantage of the die-casting process is contingent on the repeated utilization of a permanent casting cavity to produce high surface quality casting. Since the cast surface is a faithful production of the die surface, any degeneration of the die surface results in a deterioration of casting quality. Unfortunately, die wear and failure is a significant issue in the die casting process. The reason for die wear is that the die casting process inherently requires multiple reuse of the die (typically more than 100,000 castings are produced per die campaign with production rates of 2,500 shots per 24-h day). Low cycle times in die casting therefore dictate that molten metal be introduced into the die cavity at high flow velocities and that the molten metal rapidly solidify (large thermal gradients) to the part shape. Flow velocities of 40 m/s (132 ft/s) and die temperature gradients as high as 1000 °C/cm (4500 °F/in.) are common [3]. While these severe conditions are mandated to achieve these high production rates, they also limit the die materials that can be used and their respective production campaign. Wear phenomena are widely observed in
H-13 die steel, the most commonly used die material due to this severe mechanical and thermal loading. The major wear mechanisms leading to premature die failure are [3]:

1. Erosion or washout: This is a result of the high velocities with which the molten metal impinges parts of the die cavity causing steel to be washed away with the melt. Most die casting dies have complicated geometrical features, such as cores, pins, ribs, and corners, which are especially prone to erosive wear. This erosive wear reduces the ability of the die to maintain dimensional tolerances and often requires rebuilding of regions of the die that have suffered extensive washout.

2. Heat checking (thermal cracking): This is caused by the thermal fatigue due to the alternate heating and cooling of the die surface during die casting. The large thermal gradients created cause the die surface to be in compression during heating and tension during cooling of the die. This results in thermal cracking, which appears as cracks called heat checks on the die surface. degrades part surface finish, and ultimately leads to die failure.

3. Soldering and corrosion: This is caused by the chemical interaction of the casting alloy and the die material during filling and solidification. This results in parts of the molten metal sticking to the die surface (soldering), which obviously produces defective castings or corrodes part of the die surface.
Three causes contribute to die failure. The abrasive action of the metal injected into the die at high velocity causes erosion, and chemical attack of the die surface occurs as the "soldering", which takes place if part of the cast metal adheres to the die. However the most frequent cause of die failure arises from thermal fatigue.

1.1.3 Thermal failure

A metal subjected to a repetitive or fluctuating applied load will fail at a stress lower than that exerted by a static load. The fatigue strength of steels decreases with increasing temperature and failure can occur suddenly after a period of service. As a result of the gradient in temperature between the surface and interior of a die during casting, localized thermal stresses are developed as the expansion of the surface skin is restrained by the mass of the die. Continued thermal cycling causes cracking to develop.

The factors [1] which affect die life under conditions of thermal fatigue are summarized in the list below:

1. The composition of the steel must make it suitable for being brought to a sufficient hardness and strength to endure the conditions in the die.

2. The steel's physical properties determine the stresses set up under conditions of
rapid heating and cooling. Thus a low coefficient of expansion indicates a minimum dimensional change. High thermal diffusivity ensures the quick dissipation of heat.

3. Tensile, compressive and fatigue strengths at elevated temperatures affect the endurance of the steel under stress and the amount of dimensional change caused by that stress.

4. The steel quality determines whether the die components will have a long or a short life; any inclusion or crack is a stress raiser from which early fatigue failure will be initiated. A well made and carefully finished die is likely to have a longer life than one which has file marks, notches and inclusions.

5. Heat treatment can be the cause of long die life or early failure.

6. The thermal conductivity of the die cast metal has an effect on the transfer of heat. A die steel with greater conductivity than that of the metal being cast will cool more rapidly than a die into which a metal with higher conductivity is being cast.

1.2 FEM in reliable design of die casting dies

Fatigue in metals refers to a phenomena in which the strain is repeated and the ability to carry a load is gradually impaired and may finally be completely lost. It is a process which cannot be reversed, and which may ultimately lead to structural failure. The analysis of fatigue has a long history. During this long history different
fatigue life prediction approaches as one part of reliability engineering were developed. The reliability is generally defined as the probability that a system, equipment, or a component will perform its intended function without failure for a given period of time (e.g., product life cycle), in the environment for which it was designed. Modeling the reliability of a product is a combination of the statistical approach and the physics-based approach. This combination is the foundation of a design-for-reliability strategy which is the trend nowadays in the advanced industries in early design stages to enhance reliability and reduce product development cycle times.

The design-for-reliability process focuses on evaluating the functionality, cost, manufacturability, and reliability of possible design alternatives in the early design and product development stages prior to the construction of prototype hardware. To implement such a process requires a significant amount of numerical modeling and the understanding of the failure modes, thermal, and mechanical behavior, relevant materials and different approaches of life prediction.

Recently, FEM as one of the numerical modeling has been widely used to study the root cause of various failure mechanisms in die casting because it is a powerful tool used in understanding the deformation and stresses of structures.
1.3 Thesis organization

This thesis presents the fundamental principles for the prediction of low cycle fatigue life with FEM and its application in the real die casting practice. The simulation process can provide the tools to assist the die casting analysis in the design and production of the die casting structure. The objective of this thesis is to estimate the life of the die casting dies, i.e., the number of thermal cycles before the die reaches failure. Different meshing strategies and special solvers are used to solve the thermal stress problems by different models.

Chapter 1 will give an introduction of the die casting process characteristics.

Chapter 2 presents the theoretical aspects behind the phenomena as a result of applying the die casting process. The discussion of fatigue failure, the mechanical behavior of die casting dies such as elastic stress strain relation, plastic stress strain relation, and creep stress strain relation, and the different approaches of life prediction methods.

Chapter 3 describes the software used to predict low cycle fatigue. The software environment in MMO software which supported by Materials Manufacturing Ontario is also described.

Chapter 4 develops the different element types to suit the requirement of the accuracy of the analysis. The basic theory for high order \( C_0 \) family, the development of 20-node brick element such as basis function, B matrix, K matrix, Gauss integration,
and geometry presentation will be given a detail.

Chapter 5 presents the analysis process for a real die casting. The results of the thermal stress and thermal cycle life can be found in this chapter. A predictive framework is developed in an object oriented environment to support the large amount of data input for the analysis. The accuracy between 8-node brick element and 20-node brick element was compared.

Chapter 6 is a discussion of results and suggests further work.
Chapter 2

Fatigue Failure of Die Casting Dies

The word fatigue in engineering refers to the changes in properties resulting from the application of cyclic loads. Fatigue failure occur in many different forms, such as mechanical fatigue, creep fatigue, thermo-mechanical fatigue, corrosion fatigue, sliding contact fatigue, rolling contact fatigue and fretting fatigue [4].

In the long history of fatigue study, scores of scientists and engineers have made pioneering contributions to the understanding of fatigue in a wide variety of metallic and nonmetallic, brittle and ductile, monolithic and composite, and natural and synthetic materials. A. Woeher [5] conducted systematic investigations of fatigue failure during the period 1852-1869 in Berlin. His work led to the characterization of fatigue behavior in terms of stress amplitude-life (S-N) curves which many fatigue
life prediction in use today are based on. In another example Coffin (1954) [5] and Manson (1954) [5] established the notion that plastic strains are responsible for cyclic damage. They worked independently on problems associated with fatigue due to thermal and high stress amplitude. This so called Coffin-Manson relationship has remained the most widely used approach for the strain-based characterization of fatigue.

A fatigue failure is assumed to develop in three phases. In the initiation or nucleation phase micro-cracks are initiated. In the crack growth phase the cracks are growing at an increasing rate under repeated external forces. In the fracture phase the structure is spontaneously ruptured because the remaining cross section is too small to resist the external forces. The initiation phase was previously believed to cover the main part, say 50 to 75% [6], of the fatigue life of a structure. More careful microscopic studies have, however, revealed that micro-cracks are already generated after 1% [6] of the life-time. In addition, small surface defects acting as crack initiators may already be present on delivery from the manufacturer.

Die fatigue failure is a significant issue in die castings due to the high cost of dies. The reason for die fatigue failure is that the die casting process inherently requires multiple reuse of the die (typically more than 100,000 castings are produced per die campaign with production rates of 2,500 shots per 24-h day). Low cycle times in die
casting therefore dictate that molten metal be introduced into the die cavity at high flow velocities and that the molten metal rapidly solidify (large thermal gradients) to the part shape. Flow velocities of 40 m/s (132 ft/s) and die temperature gradients as high as 1000 °C/cm (4500 °F/in.) are common [3]. As a result of the gradient in temperature between the surface and interior of a die during casting, localized thermal stresses are developed as the expansion of the surface skin is restrained by the mass of the die. Continued thermal cycling causes cracking to develop.

While these severe conditions are mandated to achieve these high production rates, they also limit the die materials that can be used and their respective production campaign. Failure phenomena are widely observed in H-13 die steel, the most commonly used die material due to this severe mechanical and thermal loading.

2.1 Mechanical behavior of die casting dies

2.1.1 Stress strain relation-elastic

By definition, an elastic material may be deformed by a load or temperature so that the stress and strain go from O to A on the stress-strain curve in Figure 2.1a. Suppose further that when the load is removed, the material follows exactly the same curve back to the origin O. This property of a material by which it returns to its original
dimensions during release of the deforming loads or temperatures is called elasticity. The qualifying adjective of linear means that the load-deflection law may be represented by a linear relationship. The concept of linear elasticity was first announced by Robert Hooke in 1676 [6] and then generalized by Cauchy into the statement that the components of stress are linearly related to the components of strain. For linear isotropic elastic materials in Cartesian coordinates, the stress strain relations are [7]:

\[
\varepsilon_x = \frac{1}{E} \left[ \sigma_x - \nu (\sigma_y + \sigma_z) \right] + \alpha (T - T_0) \quad (2.1)
\]

\[
\varepsilon_y = \frac{1}{E} \left[ \sigma_y - \nu (\sigma_x + \sigma_z) \right] + \alpha (T - T_0) \quad (2.2)
\]

\[
\varepsilon_z = \frac{1}{E} \left[ \sigma_z - \nu (\sigma_x + \sigma_y) \right] + \alpha (T - T_0) \quad (2.3)
\]

\[
\gamma_{xy} = \frac{1}{G} \tau_{xy} \quad (2.4)
\]

\[
\gamma_{yz} = \frac{1}{G} \tau_{yz} \quad (2.5)
\]

\[
\gamma_{zx} = \frac{1}{G} \tau_{zx} \quad (2.6)
\]

where \( E \) is the Young’s modulus; \( \nu \) is the Poisson’s ratio; \( \alpha \) is the thermal coefficient of linear expansion; \( T_0 \) is the reference temperature; \( T \) is the instantaneous absolute temperature; \( \gamma \) is the shear strain; \( \tau \) is the shear stress; \( G \) is the shear modulus.
Figure 2.1: Stress-strain curves: a) small (elastic) loading, (b) loading beyond elastic limit.

2.1.2 Stress strain relation-plastic

Plasticity is the characteristic of a material by which it undergoes inelastic strains beyond those at the elastic limit (Point E in Figure 2.1). In the plastic region Hooke’s law no longer applies. Some other material laws such as plasticity and creep need to be considered. Unlike elastic deformation, plastic deformation is not a reversible process, and depends not only upon the initial and final states of loading but also upon the loading path by which the final state is achieved. Several aspects of real material behavior, such as the Bauschinger effect (Figure 2.2), cyclic hardening, plastic anisotropy, elastic hysteresis, and so forth, can be modeled by the theories of plas-
ticity. In this section, some well-established and most often used theories are briefly discussed.

Figure 2.2: Stress-strain curves with Bauschinger effect [7].

Yield surface

The yield surface is defined as the surface in stress space with stress components as coordinates. Within the yield surface, the stress vector may change without any plastic strain increment; stress increments begin from points in the surface, if directed toward the exterior, imply plastic strain increments.

Initial yield surface

For isotropic plasticity, the initial yield surface must be independent of the orientation of the reference axes. By choosing the axes of the principal stresses as the reference
axes, the initial yield surface may be expressed in terms of the principal stresses and represented by a surface in a stress space with $\sigma_1, \sigma_2, \sigma_3$ as coordinate axes. Thus the initial yield function may appear as

$$f(\sigma_1, \sigma_2, \sigma_3) = 0$$  \hspace{1cm} (2.7)

Furthermore, experiment indicates that the hydrostatic pressure has very little effect on the plastic deformation. Hence, the initial yield condition may be expressed in terms of the deviatoric stress invariant in the form

$$f(\bar{I}_2, \bar{I}_3) = 0$$  \hspace{1cm} (2.8)

where

$$\bar{I}_2 = \frac{1}{2} S_{ij} S_{ij}$$  \hspace{1cm} (2.9)$$

$$\bar{I}_3 = \frac{1}{3} S_{ij} S_{jk} S_{ki}$$  \hspace{1cm} (2.10)$$

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{\beta\beta} \delta_{ij}$$  \hspace{1cm} (2.11)$$

$\bar{I}_2$ and $\bar{I}_3$ are the deviatoric stress invariants and $S_{ij}$ is the deviatoric stress tensor. Two simple yield surface for the initial yield of isotropic materials with isotropic hardening that have provided highly useful descriptions of many real materials are briefly discussed in the following.

18
1. Von Mises yield condition (distortion energy theory) is based on the assumption that yielding occurs when the second deviatoric stress invariant attains a prescribed value $k$.

$$ f = I_2 - k^2 = \frac{1}{2} S_{ij} S_{ij} - k^2 = 0 \quad (2.12) $$

or

$$ f = \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)] - k^2 = 0 \quad (2.13) $$

where $k$ may be a function of plastic strain-hardening materials and the relation of $k$ to test data follows $k = \tau_y$ or $k = \frac{\bar{\sigma}_y}{\sqrt{3}}$ ($\bar{\sigma}_y$ is the yield stress in uniaxial tension).

2. Tresca yield condition (maximum shear theory) is based on the assumption that yield occurs when the maximum shear stress reaches a limiting value $k$.

$$ f = 4T_2^3 - 27T_3^2 - 36k^2T_2^2 + 96k^4T_2 - 64k^6 = 0 \quad (2.14) $$

or

$$ f = [(\sigma_1 - \sigma_2)^2 - 4k^2][(\sigma_2 - \sigma_3)^2 - 4k^2][(\sigma_3 - \sigma_1)^2 - 4k^2] = 0 \quad (2.15) $$

where $k = \tau_y$ or $k = \frac{\bar{\sigma}_y}{2}$. Experimental data appear to favor the use of the von Mises yield condition for most of the materials.
Constitutive yield surface

A general equation for determining the plastic stress-strain relation for any yield condition was proposed by Drucker [8]. Based on his definition of work-hardening materials \((d\sigma_{ij}d\epsilon_{ij} > 0\) upon loading; \(d\sigma_{ij}d\epsilon_{ij}^p \geq 0\) on completing a cycle), he stated that the plastic strain increment vector must be normal to the yield or loading surface at a smooth point on that surface, and must lie between adjacent normals at a corner point. That is,

\[
d\epsilon_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}}
\]  

(2.16)

The above equation is called the normality principle of plasticity and can be applied to plastic anisotropic hardening; \(d\lambda\) is a function that may depend on stress, strain, and strain history. For isotropic hardening, the von Mises yield condition is given by

\[
f = \frac{1}{2} S_{ij}S_{ij} - \frac{1}{3} \sigma_y^2
\]  

(2.17)

we can substitute this equation into Drucker’s equation, and we have

\[
d\epsilon_{ij}^p = d\lambda S_{ij}
\]  

(2.18)
or

\[ d\varepsilon_{ij}^p d_{ij}^p = (d\lambda)^2 S_{ij} S_{ij} \]  \hspace{1cm} (2.19)

Then the effective (equivalent) stress

\[ \bar{\sigma} = \sqrt{\frac{3}{2} S_{ij} S_{ij}} \]  \hspace{1cm} (2.20)

\[ \bar{\sigma} = \frac{\sqrt{2}}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \]  \hspace{1cm} (2.21)

\[ \bar{\sigma} = \frac{\sqrt{2}}{2} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} \]  \hspace{1cm} (2.22)

and effective (equivalent) plastic incremental strain

\[ d\bar{\varepsilon}_p = \sqrt{\frac{2}{3} d\varepsilon_{ij}^p d\varepsilon_{ij}^p} \]  \hspace{1cm} (2.23)

\[ d\bar{\varepsilon}_p = \left( \frac{\sqrt{2}}{3} \right) \sqrt{(d\varepsilon_1^p - d\varepsilon_2^p)^2 + (d\varepsilon_2^p - d\varepsilon_3^p)^2 + (d\varepsilon_3^p - d\varepsilon_1^p)^2} \]  \hspace{1cm} (2.24)

\[ d\bar{\varepsilon}_p = \left( \frac{\sqrt{2}}{3} \right) \sqrt{(d\varepsilon_x^p - d\varepsilon_y^p)^2 + (d\varepsilon_y^p - d\varepsilon_z^p)^2 + (d\varepsilon_z^p - d\varepsilon_x^p)^2 + d\gamma} \]  \hspace{1cm} (2.25)

where

\[ d\gamma = \frac{3}{2} [(d\gamma_{xy}^p)^2 + (d\gamma_{yz}^p)^2 + (d\gamma_{zx}^p)^2] \]  \hspace{1cm} (2.26)
2.1.3 Stress strain relation-creep

The stress-strain relations for the elastic and plastic materials previously described are obtained from tension tests involving only static loading of the material specimens and were time independent. However, the material develops additional strains over "long" periods of time and are said to creep. Creep is a mathematical model for rate-sensitive elastoplastic materials operating at elevated temperature. Creep strain may be broadly defined as viscous time-dependent deformation under constant load at "high" temperature.

Figure 2.3 shows a typical family of creep curves which can be obtained from "long time" uniaxial or shear tests at constant temperature and various stress conditions ($\alpha_4 > \alpha_3 > \alpha_2 > \alpha_1$). The slope of these curves ($d\varepsilon_i/dt = \tan \psi_i$) is referred to as the creep rate. At the first stage, the creep rate is undefined and the initial creep strain consists of either entirely elastic strain or partially elastic strain and partially plastic strain. During the second stage, the creep rate decreases with time because the effect of strain hardening is greater than that of annealing. These two effects are in equilibrium during the third stage, and the creep rate reaches essentially a steady state and changes very little with time. However, during the fourth stage, the creep rate increases rapidly with time until fracture occurs.
2.1.4 Stress strain relation-relaxation

Relaxation is the counterpart of creep. It is the decline in stress with time, in response to a constant applied strain, at constant temperature. One can imagine a series of tests in which the test specimens are given initial strains of $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, $\varepsilon_4$, ..., etc. In such tests, the stresses are typically computed from the load cell readings, taken at different times.

2.1.5 Thermal stress and low cycle fatigue

The majority of life-time relations applied in thermal fatigue situations have been developed within a low cycle fatigue (LCF) data base. The phenomenon of low cycle fatigue can be in principle described in terms of the theory of plasticity. In fact, low cycle fatigue is a cyclic elastoplastic deformation occurring until the expenditure of plasticity reserves. The material behavior at unloading and reversed loading, in par-
ticular the shape and size of hysteresis loops, is of essential significance in low cycle fatigue. The relationship between maximal stress and strains within a cycle generally differs from that in monotonic quasi-static loading. The cycle deformation relations depend on the type of loading process. They change whether this process is load- or displacement-controlled. Some materials reveal a tendency to cyclic hardening: under the displacement-controlled loading the maximal cycle stresses grow with the cycle number. Other materials reveal a tendency to cyclic softening. An intermediate place is occupied by the so-called plastically stabilizing materials. Depending on the microstructural state and temperature, the same material may behave in various ways. Typical diagrams of uniaxial tensile/compression deformation $\sigma(\varepsilon)$ are shown in Figure 2.4. They correspond to symmetrical cycle loading with the given strain amplitude $\varepsilon_a$. Figure 2.4a shows the behavior of a cycle dependent hardening material: Figure 2.4b shows the behavior of a cycle dependent softening material.

When the strain level is high, the number of cycle to fatigue failure is comparatively small, and significant one-sided residual deformations accumulate in the specimen. At a moderate strain level, it is convenient to represent test results with fatigue curves. Compared with high-cycle fatigue curves, low cycle fatigue curves are usually plotted on the plane of characteristic strain versus number of cycle to failure. Standard tests in tension or tension/compression are usually performed maintaining a
Figure 2.4: Stress-strain diagrams: (a) cycle dependent hardening material, (b) cycle dependent softening material [8].

constant range $\Delta \varepsilon$ of the nominal (averaged over all the working parts of a specimen) strain $\varepsilon$.

The simplest equation of low cycle fatigue was suggested by Coffin (1954):

$$\Delta \varepsilon_p N^\mu = C$$  \hspace{1cm} (2.27)

Here $\Delta \varepsilon_p$ is the range of plastic strain within a cycle, and $\mu$ and $C$ are empirical constants. For carbon steels $\mu = 0.4 \ldots 0.6$ [5]. This corresponds to a power exponent of $\Delta \varepsilon - N$ curves equal approximately to two. The constant $C$ is frequently connected with the ultimate strain $\varepsilon_U = \ln(1 - \Psi)^{-1}$. For symmetric cycles $\Delta \varepsilon_p = 2 \varepsilon_p$. This results in the Coffin-Manson equation in the form often used by engineers:

$$\varepsilon_p N^\mu = \frac{1}{2} \ln(1 - \Psi)^{-1}$$  \hspace{1cm} (2.28)

A number of analytical relationships have been suggested that combine low- and high-
cycle fatigue curves. Among them is a fatigue failure criterion connecting the range of complete (summed) strain $\Delta \varepsilon$ with the cycle number $N$:

$$\Delta \varepsilon = C N^{-\mu} + (\sigma_c / E) N^{-\nu}$$  \hspace{1cm} (2.29)

Here $E$ is Young's modulus and $\sigma_c$ is a material constant.

### 2.2 Life prediction methods

Different approaches to fatigue provide apparently different guidelines for the design of microstructural variables for optimum fatigue resistance. These differences are merely a consequence of the varying degrees to which the role of crack initiation and crack propagation are incorporated in the calculation of useful fatigue life. The concerning of microscopic mechanism by materials scientists and the resolution of the crack detection equipment (NDS) by engineers separate these approaches in two main area, total life approach and defect tolerant approach.

The development of reliable life prediction models which are capable of handling such complex service conditions is one of the toughest challenges in fatigue research. It is important to note here that a major obstacle to the development of life prediction models for fatigue lies in the choice of a definition for crack initiation.
2.2.1 Total life approach

In the total life approach, there are two distinguished sub approaches, the stress life approach and the strain life approach.

2.2.1.1 Stress life approach

The stress life approach, also called nominal stress (NS) approach, was widespread used in the fatigue analysis, mostly in applications where low-amplitude cyclic stresses induce primarily elastic deformation in a component which is designed for long life, i.e., in the so called high cycle fatigue applications. The NS approach was the first fatigue life prediction method for a long history. In this approach the effects of stress concentrations, mean stress, surface modifications, variable amplitude cyclic loads, and multi axial loads should be considered.

Basquin’s equation

Basquin’s equation estimates the number of cycles to failure in the high cycle fatigue for a fully reversed, constant amplitude fatigue test with number of load reversals to failure, $N_f$, and stress amplitude $\sigma_a = \frac{\Delta \sigma}{2}$

$$\frac{\Delta \sigma}{2} = \sigma_f (2N_f)^b.$$  \hspace{1cm} (2.30)

$\sigma_f$ is the fatigue strength coefficient and $b$ is known as the Basquin exponent which, for most metals, is in the range of -0.05 to -0.12 [5].
stress range $\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}$.

stress amplitude $\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}$.

mean stress $\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}$.

$N_f|_{\sigma_m=0}$ Number of cycles to failure for zero mean stress.

Soderberg Relation

Soderberg Relation can be written as $\sigma_a = \sigma_a|_{\sigma_m=0}(1 - \frac{\sigma_m}{\sigma_y})$.

This provides a conservative estimate of fatigue life for most engineering materials.

Modified Goodman Relation

Modified Goodman Relation is written as $\sigma_a = \sigma_a|_{\sigma_m=0}(1 - \frac{\sigma_m}{\sigma_{TS}})$.

This matches experimental observations for brittle materials built conservative for ductile materials. Because it is generally non-conservative for compressive mean stresses, one may assume that compressive mean stresses provide no beneficial effects on fatigue life.
2.2.1.2 Stain life approach

The strain life approach was used when considerable plastic deformation occurs during cyclic loading as, for example, a consequence of high stress amplitudes or stress concentrations, the fatigue life is markedly shortened. The design calls the so-called low cycle fatigue. In this approach the plastic strains play a important role in inducing permanent fatigue damage. Coffin (1954) and Manson (1954) proposed the following equation to estimate low cycle fatigue life based on the plastic strain amplitude, $\frac{\Delta e_p}{2}$, per load cycle

$$\frac{\Delta e_p}{2} = \epsilon_f'(2N_f)^c$$  \hspace{1cm} (2.31)

where $\epsilon_f'$ is the fatigue ductility coefficient and $c$ is the fatigue ductility exponent. Usually $\epsilon_f'$ is equal to the true fracture ductility and $c$ is in the range of -0.5 to -0.7 [5] for most metals. In a constant strain amplitude test, the total strain amplitude, $\frac{\Delta \epsilon}{2}$, can be written as the sum of the elastic strain amplitude $\frac{\Delta \epsilon_e}{2}$ and the plastic strain amplitude $\frac{\Delta \epsilon_p}{2}$.

$$\frac{\Delta \epsilon}{2} = \frac{\Delta \epsilon_e}{2} + \frac{\Delta \epsilon_p}{2}$$  \hspace{1cm} (2.32)

Noting that

$$\frac{\Delta \epsilon_e}{2} = \frac{\Delta \sigma}{2E} = \frac{\sigma_a}{E}$$  \hspace{1cm} (2.33)
Substituting for the stress amplitude in Basquin’s equation

\[ \frac{\Delta \varepsilon_e}{2} = \sigma'_f (2N_f)^b. \]  
(2.34)

The Coffin-Manson equation and Basquin’s equation can combined

\[ \frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \epsilon'_f (2N_f)^c. \]  
(2.35)

The transition from strain-based to stress-based failure can be estimated by equating the number

\[ (2N_f)_t = \left( \frac{\epsilon'_f}{\sigma'_f} \right)^{\frac{1}{b-c}}. \]  
(2.36)

Morrow assumed that a tensile mean stress reduces fatigue life such that

\[ \sigma_a = (\sigma'_f - \sigma_m)(2N_f)^b. \]  
(2.37)

and modified equation

\[ \frac{\Delta \varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E} (2N_f)^b + \epsilon'_f (2N_f)^c. \]  
(2.38)
2.2.2 Defect-tolerant approach

The prediction of crack propagation life using the defect tolerant approach involves empirical crack growth laws based on fracture mechanics. The basic premise here is that all engineering components are inherently flawed. The size of a pre-existing flaw is generally determined from nondestructive flaw detection techniques (such as visual, dye-penetrant or X-ray techniques or the ultrasonic, magnetic or acoustic emission methods). If no flaw is found in the component, proof tests are conducted whereby a structure is subjected to a simulation test a priori at a stress level slightly higher than the service stress. If no cracks are detected by the nondestructive test method and if catastrophic failure does not occurred during the proof test, the largest initial crack size is estimated from the resolution of the flaw detection technique. The useful fatigue life is then defined as the number of fatigue cycles or time to propagate the dominant crack from this initial size to some critical dimension.
Chapter 3

FEM Software

Recently finite Element Method (FEM) has become one of the most advanced numerical analysis with remarkable progress of computer technology. When it developed, it was used in the field of structural mechanics, but now it is applied more broadly to engineering and physics (for example, thermal conduction, fluid mechanics, acoustics and electromagnetic), and has become popular skill.

3.1 General review of MMO software

Like all FEA software, MMO software has three distinct parts:

The Pre-processor helps the engineer to build a file of input data. In this part, the engineer interacts with the computer very closely. The engineer builds up the shape of the product being analyzed, the materials, the sizes, loads, and
supports. Since meshing is done automatically from stereolithographics files that are generated easily from Pro-Engineer or other CAD systems, managing parts with complex geometry or assemblies with many parts with complex geometry does not increase the difficulty of doing an analysis.

The Solver proceeds almost entirely without user intervention. This part uses the file of input data and options prepared by the pre-processor. It solves the problem, and also generates a database or special data files of output results. Everything in this phase is strictly finite elements, and is computationally intensive. Recently many solvers have been developed in the MMO software such as the MrcoEnergySolver, MrcoStressSolver, and MrcoFillingSolver which analyze the thermal, stress behavior and simulated the real 3-D fluid flow virtual environment.

The Post-processor helps the user to view the output results graphically in a variety of ways. It also enables hard copying and the archiving of graphic outputs on to different kinds of media. Currently there are different software packages connected with the MMO software for post processing, such as Ensight7, Gnuplot, TecPlot, VTK.

The MMO software is fully object oriented, it is written in C++. With easy user interface the software was designed, developed and tested to be used routinely by designer in specific application domains such as die casting, electronic packaging and welding structure. The software is intended to do couplec fluid flow, thermal,
stress and microstructure analysis on real 3-D geometry.

The object-oriented paradigm provides four fundamental concepts: objects, classes, inheritance, and polymorphism. Software is organized into objects that store both its data and operators on the data. This permits developers to abstract out the essential properties of an object: those that will be used by other objects. This abstraction allows the details of the implementation of the object to be hidden, and thus easily modified. Objects are instances described by a class definition. Classes are related by inheritance. A subclass inherits behavior through the attributes and operators of the superclass. Polymorphism allows the same operation to behave differently in different classes and thus allows objects of one class to be used in place of those of another related class.

The application of object-oriented design to the finite element method has several advantages. The primary advantage is that it encourages the developer to abstract out the essential immutable qualities of the components of the finite element method. This abstraction forms the definition of objects that become the building blocks of the software. The class definitions encapsulate both the data and operations on the data, and provide an enforceable interface by which other components of the software may communicate with the object. Once specified, the object interfaces are frozen. It is possible to extend the interface later without affecting other code, but it should be noted that modifying the existing elements of an interface would require
changes throughout the rest of the program wherever the interface was used. The internal class details that are needed to provide the desired interface are invisible to the rest of the program and, therefore, these implementation details may be modified without affecting other code. Thus, the design forms a stable base that can be extended with minimum effort to suit a new task. Due to the encapsulation enforcement inherent in object-oriented languages, new code will work seamlessly with old code. In fact old code may call new code.

Currently, there are many classes were developed by the MMO software to suit the complex analysis of real world. The example classes can be written:

1. Material Class: This class is also called material library in MMO software, it includes different material and its properties such as enthalpy, Young’s modulus, yield stress, thermal expansion, bulk modulus, shear modulus, etc. which are important to the thermal stress analysis and low cycle fatigue life prediction.

2. Meshing Class: This class include any information of face, element, node, and Gauss point. Such as element and node type, position, Id number, element size, Gauss point number, etc. Currently there are different element types in the MMO software, but the most widely used element type are 8-node brick element and 20-node brick element with its different Gauss point schemes such as 8, 14, and 27.

3. Equation Solver Class: This class include different solvers for example Energy Solver, Stress Solver which support any type of element and Gauss point scheme
widely used for thermal, elastic and elastic-plastic analysis of die casting, welding structure and electronic packaging.

4. Design Files Class: This class is like the process planning in a real manufacturing process, it will include all necessary information which will be used in the analysis. For example the geometry object which will be analyzed, the boundary condition, and all design parameters which are readily accessible to the designer in text files. In this case, the designer should be able to query for the properties or results they want.

5. Time Manager Class: This class will give the time information during the solve process. It helps the user to check the CPU time and so on.

6. Data Dictionary Class: This class is a data structure that connects the data input and output during the analysis process.

Once all the relationships the object has to the rest of the system are established, the specific services that the object must implement can be defined. The interface definition table provides a formal description of the publicly available methods the object provides, including the argument and return types. A sample interface definition table is given in Table 3.1. The arguments and return types are given as a non-language specific description of the object type that is required. It is intended to be used directly to implement the design in a programming language.
3.2 The design environment for low cycle fatigue analysis

A decision on the feasibility of the design requires the integration of the individual modules. This generates the need for a design environment that provides tools for analyzing a given design.

Generally two design scenarios can be used for the real design, the static and evolving. The static scenario allows an evaluation of the given design, it checks the feasibility of the design based on the information supplied by the user but the information does not suffer any modification. It usually considers the parameters unchanged and only provides the information about the feasibility to produce product of good quality under the given conditions. The evolving scenario allows a progressive design based on the tools available to the designer to support the decision making process with accurate information. In order to reach such goals, the design approach has to be flexible enough to allow the user to perform the calculation or evaluation in a non-predefined and interactive form. The function of the design environment is to provide appropriate tools to help make proper decisions, but does not tie the user to a rigid procedure. It leaves the organization of the calculation procedure to the user.

In a highly specialized working environment such as die casting design or electronic packaging design practice, the user knows best how to make correct decision,
given the results. Therefore, the emphasis here is to provide accurate tools to make
the decisions process easier and more knowledgeable. It is not intended to substitute
for the enormous amount of experience and knowledge existing in the industry. Al-
though it could be also used for training purposes if desired. This is a fundamental
assumption that underlies the design environment. From the implementation point
of view, the design environment can be provided to the user by the integration or
embedding of C++ code with the Tcl language.

Tcl language is a command language used for developing interactive applica-
tions. Also it allows the extension of a basic set of commands with application-specific
commands. A fully programmable command language is obtained as a result of this
combination. Tcl language has a simple and minimal syntax and it provides basic
features such as variables, procedures, control structures, etc.
Table 3.1: Sample object definition table.

<table>
<thead>
<tr>
<th>Method</th>
<th>Arguments</th>
<th>Return and Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>GetPartId</td>
<td>element Id</td>
<td>part Id of part in the mesh</td>
</tr>
<tr>
<td>GetElementPosition</td>
<td>element Id</td>
<td>position of this element</td>
</tr>
<tr>
<td>GetNodeIds</td>
<td>element Id</td>
<td>node Id in this element</td>
</tr>
<tr>
<td>GetFaceIds</td>
<td>element Id</td>
<td>face Id in this element</td>
</tr>
<tr>
<td>GetFacePosition</td>
<td>face Id</td>
<td>position of this face</td>
</tr>
<tr>
<td>GetInBoundingBox</td>
<td>minX, minY, minZ, maxX, maxY, maxZ</td>
<td>all elements or nodes in this range</td>
</tr>
<tr>
<td>RefinePQR</td>
<td>element set to refined. the layer in direction of P, Q, R and child type</td>
<td>a refined subset</td>
</tr>
<tr>
<td>SetData</td>
<td>data name for example stress and its values</td>
<td>write a data in the local dictionary</td>
</tr>
<tr>
<td>GetData</td>
<td>data name for example stress and its values</td>
<td>get a data from the local dictionary</td>
</tr>
<tr>
<td>CurrentTime</td>
<td>N/A</td>
<td>time manager in the solve process</td>
</tr>
</tbody>
</table>
Chapter 4

FEM Mesh Refinement

4.1 The basic theory for mesh refinement

Many finite element analysis are complicated by the presence of localized phenomena. Using a uniform mesh of grading is inconvenient and results in element distortion which may reduce the accuracy of the solution. Based on this reason the software system should allow refinement of the mesh to improve the solution in an efficient manner as well as regulate the quality of the solution. Generally mesh generation is a time-consuming and error-prone process.

In the die casting processes the stress concentration appears in the surface which is a contact face between die and casting. In the simulation of such processes the starting mesh is well defined and can have the desired mesh density distribution. As
the simulation continues, the distortion of the die mesh is significant. Hence, it is necessary to generate a new mesh and interpolate the data from the old mesh to the new mesh in order to obtain accurate results.

Most of the present methods in numerical shape optimization are based on five approaches for mesh refinement:

1. H-refinement: This approach refers to a reduction in the size, $h$, of some of the elements in the mesh. The number of elements increase and the accuracy of the solution improves.

2. P-refinement: In this approach the degree, $p$, of the polynomial shape functions is increased for some elements in a mesh with a fixed number of elements. As we know the order of error in the approximation to the unknown function is $O(h^{p+1})$, where $h$ is the element size and $p$ is the complete polynomial present in the expansion. From this formulation it is clear that increasing the polynomial order will increase the order of error and convergence to the exact solution becomes more rapid. In this family of elements, basis functions corresponding to an element of degree $p$ are a subset of those corresponding to an element of degree $p + 1$. Thus, the stiffness matrix of the initial element (degree $p$) is a sub matrix of the stiffness matrix of the enriched element (degree $p + 1$). This means that when the order of the element is increase from $p$ to $p + 1$, only the new rows and columns of the stiffness matrix need to be determined [9].
3. P-h-refinement: This approach combines p-refinement and h-refinement.

4. R-refinement: one relocates the mesh points in order to get a better resolution of the solution with fixed number of unknowns and fixed mesh topology.

5. M-refinement: one switches to a different equation (= physical model) depending on the local behavior of the approximated solution. As an example one may use linearized equations only if the nonlinear terms of the physical model are negligible.

The selection of a suitable approach is dependent on the real problem with consideration of saving computer time and getting more accurate results for the solution. For example, h-refinement is widely used in refinement algorithms, but when large shape changes are involved, this approaches may be quickly lead to distorted elements which paralyze the smooth progress of shape optimization. And also this approach can create a finite element mesh with elements having large aspect ratios and irregular geometry. Analysis of these meshes using these elements especially for nonlinear analysis can lead us to wrong results. As an alternative, p-elements offers many advantages to overcome this problem.

With the firm establishment of the principles of finite element analysis, it is found that the development of element characteristics will follow a prescribed path once the shape function have been chosen. For instance in the analysis of plane stress or strain once the function describing the displacements within the element in
term of nodal values are known, standard expressions can be used and the element properties are uniquely defined. The possibilities of improvement of approximation are thus confined to devising alternative element configurations and developing new shape functions.

**Standard shape function**

This concept deals with the addition of a number of nodal points along the sides of such elements thus permitting a smaller number of variables to be used for solution of practical problems with a given degree of accuracy. This concept is the basis of most finite element programs. and the basis function can be defined as:

\[ u \approx \hat{u} = \sum_{i=1}^{n} N_i a_i = Na \quad (4.1) \]

where \( a_i \) are the unknown parameters to be determined, and \( u \) is a scalar variable.

This shape function should satisfy:

1. Shape function \( N \) chosen apart from the implication that \( N_i = 1 \) at node \( i \) and zero at all other nodes with a similar requirement for all other functions.

2. The continuity of the unknown only had to occur between elements (i.e., slope continuity is not required), or, in mathematical language, \( C_0 \) continuity was required.

3. The function has to allow any arbitrary linear form to be taken so that the con-
stant strain (constant first derivative) criterion could be observed.

4. The function should satisfy $\sum N_i = 1$.

Hierarchical shape function

This concept deals with the generating a polynomial of order $p$ along an element side with nodeless variable. It does not need to introduce nodes rather than use parameters without an obvious physical meaning.

4.2 Development of 20-node brick element by standard concept

In this project the 20-node brick element was developed and used in the thermal stress analysis of die casting process to increase the accuracy of the results. The 20-node linear hexahedron is formulated in the Figure 4.1. Nodes 1 to 8 are the corner nodes and node 9 to 20 are the optional mid edges nodes. Each face of this 20-node brick element is considered as a 8-node two dimensional quadratic element shown in the Figure 4.2. Nodes 1 to 4 are the corner nodes and node 5 to 8 are the optional mid edges nodes [10].
4.2.1 **Face basis functions of 20-node brick element**

As shown in the Figure 4.2 nodes 1 to 4 are the corner nodes and node 5 to 8 are the optional mid edges nodes. The basis function for 4- to 8-node quadrilateral can be formulated in a straightforward manner using the standard rules for isoparametric
elements:

\[ s_i(r_j, s_j) = \delta_{ij} \quad (4.2) \]

The mid edge nodes will create subelements within the original element, for example the mid edge node 5 create two smooth \((C^\infty)\) continuous subelements which was shown in the Figure 4.3a. Within each subelement the basis functions are linear. However, the basis functions for the mid edge node and its two adjacent corner nodes are creased at the junction of the subelements and are, therefore, \(C^0\) continuous. If node 5 for example were not present, element 1 would be a regular 4-node linear quadrilateral with linear basis functions; e.g., \(s_1^L, s_2^L\). However, \(s_5\) exists (Figure 4.3b) therefore \(s_1\) and \(s_2\) must be 0 at node 5. The value of the linear basis function \(s_1^L\) is 1/2 at node 5: the value of \(s_5\) at node 5 is 1. Therefore \(s_1\) (Figure 4.3c) and \(s_2\) become:

\[ s_1(r, s) = s_1^L(r, s) - \frac{1}{2} s_5(r, s) \quad (4.3) \]

\[ s_2(r, s) = s_2^L(r, s) - \frac{1}{2} s_5(r, s) \quad (4.4) \]

where:

\[ s_5(r, s) = \frac{1}{2} (1 - |r| (1 + s)) \quad (4.5) \]
Figure 4.3: 4- to 8-node linear quadrilateral with one optional node: a. Subelements created by mid edge node; b. Basis function for node 5; c. Basis function for node 1, when node 5 is present.
It is apparent that the presence of mid edge nodes affects the basis corner nodes on that edge. For this reason, the basis functions should be evaluated, in order, from $s_8$ to $s_1$. If a node is not present, it has no basis function. In effect, the value of the basis function and its derivatives are 0 everywhere. If a node is not present, its basis function and all subsequent references to the basis function should be removed.

The face basis functions for 20-node brick element are shown following:

\[ s_8 = \frac{1}{2}(1 + r)(1 - |s|) \]
\[ s_7 = \frac{1}{2}(1 - |r|)(1 - s) \]
\[ s_6 = \frac{1}{2}(1 - r)(1 - |s|) \]
\[ s_5 = \frac{1}{2}(1 - |r|)(1 + s) \]
\[ s_4 = \frac{1}{4}(1 + r)(1 - s) - \frac{1}{3}(s_7 + s_8) \]
\[ s_3 = \frac{1}{4}(1 - r)(1 - s) - \frac{1}{3}(s_6 + s_7) \]
\[ s_2 = \frac{1}{4}(1 - r)(1 + s) - \frac{1}{3}(s_5 + s_6) \]
\[ s_1 = \frac{1}{4}(1 + r)(1 + s) - \frac{1}{3}(s_5 + s_6) \]

### 4.2.2 Volume basis functions of 20-node brick element

As shown in the Figure 4.1 nodes 1 to 8 are the corner nodes and node 9 to 20 are the optional mid edges nodes. Just as in the two dimension situation, the mid edge nodes create smooth ($C^\infty$ continuous) subelements within the element. Within each
subelement the basis functions are linear. However, the basis functions for the mid edge nodes as well as their adjacent corner nodes are creased at the junction of the subelements and are, therefore, \( C^0 \) continuous.

The basis functions for 20-node brick element are shown following:

\[
\begin{align*}
s_{20} &= \frac{1}{4}(1 - t^2)(1 - s)(1 + r) \\
s_{19} &= \frac{1}{4}(1 - t^2)(1 - s)(1 - r) \\
s_{18} &= \frac{1}{4}(1 - t^2)(1 + s)(1 - r) \\
s_{17} &= \frac{1}{4}(1 - t^2)(1 + s)(1 + r) \\
s_{16} &= \frac{1}{4}(1 - s^2)(1 + r)(1 - t) \\
s_{15} &= \frac{1}{4}(1 - r^2)(1 - s)(1 - t) \\
s_{14} &= \frac{1}{4}(1 - s^2)(1 - r)(1 - t) \\
s_{13} &= \frac{1}{4}(1 - r^2)(1 + s)(1 - t) \\
s_{12} &= \frac{1}{4}(1 - s^2)(1 + r)(1 + t) \\
s_{11} &= \frac{1}{4}(1 - r^2)(1 - s)(1 + t) \\
s_{10} &= \frac{1}{4}(1 - s^2)(1 - r)(1 + t) \\
s_{9} &= \frac{1}{4}(1 - r^2)(1 + s)(1 + t) \\
s_{8} &= \frac{1}{8}(1 + r)(1 - s)(1 - t) - \frac{1}{2}(s_{15} + s_{16} + s_{20}) \\
s_{7} &= \frac{1}{8}(1 - r)(1 - s)(1 - t) - \frac{1}{2}(s_{14} + s_{15} + s_{19}) \\
s_{6} &= \frac{1}{8}(1 - r)(1 + s)(1 - t) - \frac{1}{2}(s_{14} + s_{18} + s_{13}) \\
s_{5} &= \frac{1}{8}(1 + r)(1 + s)(1 - t) - \frac{1}{2}(s_{13} + s_{17} + s_{16})
\end{align*}
\]
\[ s_4 = \frac{1}{8}(1 + r)(1 - s)(1 + t) - \frac{1}{2}(s_{11} + s_{12} + s_{20}) \]
\[ s_3 = \frac{1}{8}(1 - r)(1 - s)(1 + t) - \frac{1}{2}(s_{11} + s_{10} + s_{19}) \]
\[ s_2 = \frac{1}{8}(1 - r)(1 + s)(1 + t) - \frac{1}{2}(s_{10} + s_{9} + s_{18}) \]
\[ s_1 = \frac{1}{8}(1 + r)(1 + s)(1 + t) - \frac{1}{2}(s_{9} + s_{12} + s_{17}) \]

4.2.3 The geometry of 20-node brick element

Figure 4.4 shows the geometry of 20-node brick element. Let \((r, s, t)\) be the reference coordinate, then the Cartesian coordinates of an arbitrary point in the element are:

![Figure 4.4: The geometry of 20-node brick element.](image-url)
\[
\begin{pmatrix}
 x_1 \\
x_2 \\
\vdots \\
x_{20} \\
y_1 \\
y_2 \\
\vdots \\
y_{20} \\
z_1 \\
z_2 \\
\vdots \\
z_{20}
\end{pmatrix}
= 
\begin{pmatrix}
 s_1 & s_2 & \ldots & s_{20} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & s_1 & s_2 & \ldots & s_{20} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & s_1 & s_2 & \ldots & s_{20}
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\] (4.6)

Where: \( x, y \) and \( z \) are the Cartesian coordinates for any point of the 20-node brick element, \( s \) is the basis function.
4.2.4 The displacement field of 20-node brick element

The displacement field for 20-node brick element can be written:

\[
\begin{pmatrix}
    u \\
v \\
    w
\end{pmatrix}
= \begin{pmatrix}
    s_1 & 0 & 0 & s_2 & 0 & 0 & \ldots & s_{20} & 0 & 0 \\
    0 & s_1 & 0 & 0 & s_2 & 0 & \ldots & 0 & s_{20} & 0 \\
    0 & 0 & s_1 & 0 & 0 & s_2 & \ldots & 0 & 0 & s_{20}
\end{pmatrix}
\begin{pmatrix}
    u_1 \\
v_1 \\
w_1 \\
v_2 \\
w_2 \\
\ldots \\
v_{20} \\
w_{20}
\end{pmatrix}
\]

(4.7)

where \( u \), \( v \), and \( w \) are displacements of an arbitrary point in the element in the directions of the global \( x \), \( y \), and \( z \) axes. \( s \) is the basis function, the term inside the second right matrix are the displacement of the nodes in this element.
4.2.5 The strain field of 20-node brick element

The strain field can be written:

\[
(\varepsilon) = \begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx}
\end{pmatrix} = \begin{pmatrix}
u'_{,x} \\
v'_{,y} \\
w'_{,z} \\
u'_{,y} + v'_{,x} \\
v'_{,z} + u'_{,y} \\
w'_{,x} + u'_{,z}
\end{pmatrix}
\]  

(4.8)

where \(\varepsilon_x, \varepsilon_y, \varepsilon_z\) are normal strain, \(\gamma_{xy}, \gamma_{yz}, \gamma_{zx}\) are shear strain, the term inside the right matrix are displacement gradient.

4.2.6 The B matrix of 20-node brick element

The B matrix is a property of the element shape. It arises in the process of expressing the strain-displacement equations in matrix form \((\varepsilon) = [B](u)^e\). From equation 4.7 and 4.8 we can write the B matrix of 20-node brick element as:
\[ [B] = \begin{pmatrix}
  s_{1,x} & 0 & 0 & s_{2,x} & 0 & 0 & \ldots & s_{20,x} & 0 & 0 \\
  0 & s_{1,y} & 0 & 0 & s_{2,y} & 0 & \ldots & 0 & s_{20,y} & 0 \\
  0 & 0 & s_{1,z} & 0 & 0 & s_{2,z} & \ldots & 0 & 0 & s_{20,z} \\
  s_{1,y} & s_{1,x} & 0 & s_{2,y} & s_{2,x} & 0 & \ldots & s_{20,y} & s_{20,x} & 0 \\
  0 & s_{1,z} & s_{1,y} & 0 & s_{2,z} & s_{2,y} & \ldots & 0 & s_{20,z} & s_{20,y} \\
  s_{1,z} & 0 & s_{1,x} & s_{2,z} & 0 & s_{2,x} & \ldots & s_{20,z} & 0 & s_{20,x}
\end{pmatrix} \quad (4.9) \]

where \( \varepsilon_x, \varepsilon_y, \varepsilon_z \) are normal strain, \( \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \) are shear strain, \( s \) is the basis function.

### 4.2.7 The stiffness matrix of 20-node brick element

The stiffness matrix for an element is defined as

\[ [K]^e = \int_V [B]^T[D][B]dV \quad (4.10) \]

where \([B]\) is the B matrix from equation 4.9, \([B]^T\) is the transposed B matrix, and \([D]\) is the elasticity matrix which is a property of the material of the element.
4.2.8 The numerical integration of 20-node brick element

In the formulation of all the iso parametric elements, it is difficult to perform the integrations analytically. Hence, they are performed numerically. The natural coordinates for these elements vary from -1 to +1. This enables us to utilize the Gauss numerical integration scheme, an integral in the range of -1 to +1 can be obtained by a weighted sum of the values’ integrand at certain specific points, called the Gauss points.

For a two dimensional square it has the form:

\[
\int_{-1}^{+1} \int_{-1}^{+1} f(r, s) dr ds = \sum_{j=1}^{m} \sum_{i=1}^{n} w_i w_j f(r_i, s_j) \quad (4.11)
\]

where the \( w_i \) and \( w_j \) are called the weight factors. \( r_i \) and \( s_j \) are the Gauss points.

This form was used for the numerical integration of the face of 20-node brick element. 9 Gauss point was selected for this project which was shown in the Figure 4.5. The weight factors and corresponding Gauss points were shown in the Table 4.1. In this table \( a = 0.774596669241483 \), \( b = 0.0 \), \( w_1 = 0.5555555555555556 \), and \( w_2 = 0.888888888888889 \) [9].

For a three dimension square it has the form:

\[
I = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} f(r, s, t) dr ds dt = \sum_{k=1}^{p} \sum_{j=1}^{m} \sum_{i=1}^{n} w_i w_j w_k f(r_i, s_j, t_k) \quad (4.12)
\]
This form was used for the numerical integration of the volume of 20-node brick element. 27 and 14 Gauss points were selected for this project. The weight factors and corresponding Gauss points were shown in the Table 4.2 and Table 4.3. In Table 4.2 $a = 0.75878691099999995284$, $b = 0.7958224259999997168$, $c = 0.0$, $w_1 = 0.3351800549999997676$, $w_2 = 0.88642659300000004041$, and $w_3 = 2.0$ and in Table 4.3 $a = 0.774596669241483$, $b = 0.0$, $w_1 = 0.555555555555556$, and $w_2 = 0.888888888888889$ [9].
Table 4.1: Table for two dimension square with 9 Gauss points.

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<th>$w_i$</th>
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</tr>
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</table>
Table 4.2: Table for 20-node brick element with 27 Gauss points.

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Table 4.3: Table for 20-node brick element with 14 Gauss points.

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<td>9</td>
<td>-b</td>
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<td>c</td>
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<td>$w_3$</td>
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<td>b</td>
<td>c</td>
<td>$w_2$</td>
<td>$w_3$</td>
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<tr>
<td>11</td>
<td>c</td>
<td>-b</td>
<td>c</td>
<td>$w_3$</td>
<td>$w_2$</td>
<td>$w_3$</td>
</tr>
<tr>
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<td>c</td>
<td>b</td>
<td>c</td>
<td>$w_3$</td>
<td>$w_2$</td>
<td>$w_3$</td>
</tr>
<tr>
<td>13</td>
<td>c</td>
<td>c</td>
<td>-b</td>
<td>$w_3$</td>
<td>$w_3$</td>
<td>$w_2$</td>
</tr>
<tr>
<td>14</td>
<td>c</td>
<td>c</td>
<td>b</td>
<td>$w_3$</td>
<td>$w_3$</td>
<td>$w_2$</td>
</tr>
</tbody>
</table>
Chapter 5

The Practical Problem

5.1 Introduction

The objective of this chapter is to estimate the life of a die casting die, i.e., the number of thermal cycles before die reaches the failure level. A 1-D analytical model for thermally induced stresses in the mold surface during die casting was established. A coupled thermal stress analysis was done. The analysis uses 15 time step for die open and die closed stages for each load cycle. The point with maximum effective plastic strain increment was assumed to be the most likely failure point in that die casting. This thermal stress analysis of the die casting estimated that the maximum effective plastic strain increment is always at a point in a die cavity surface.
In this project the analysis used a full nonlinear analysis and a refined unstructured mesh in die casting that captures local deformation. Both 8- and 20-node brick element meshes were tested for this analysis. The comparison between these two element types was given in this chapter.

5.2 The 1-D analytical model

In this chapter a 1-D analytical model for solving the thermal and stress problems in the mold was used [11][12]. We consider the 1-D domain of Figure 5.1. In this geometry (Figure 5.1) element 1 and 2 are the die part, element 3 is the cooling line part and element 4 is considered as the casting part.

Before the meshing process, the STL files from solid model files on CAD system were first exported. STL files are composed of triangles and vertices, with different color to represent different parts. It is necessary to check STL files carefully before meshing to make sure different parts are connected properly, no gap or overlap exists between different STL parts. In this simple case the part for die with part Id 1, casting with part Id 3 and cooling line with part Id 2. When STL files are available, the meshing process can be performed for the analysis, i.e., create grid file. The used element type in this project are 8-nodes brick element with 8 Gauss point and

61
20-nodes brick element with 27 Gauss point. The element size is 1 cm.

In order to increase the accuracy of results, the above mesh has been refined. Because the stress concentration is always on the die cavity surface of die and casting, just the elements on this surface were refined in this project. And also because we are just interested in the normal direction of this surface, we just refined the element by the normal direction to the die cavity surface, that means we just refine pqr by the normal direction $X$ in this case. The refined 8-nodes brick element mesh in this project is shown in the Figure 5.2

For establishing the foundation of the 1-D model, some assumptions have to be made.
Figure 5.2: The refined 8-node brick element mesh.

First of all, because the melt is not able to produce any considerable mechanical resistance, the mold is almost free to expand in the x-direction. Second, because the model is symmetrical, the normal stresses in the y- and z-directions are equal. The next assumption is the mold wall does not bend because the casting is often very thin walled compared with the mold, and the mold wall is very well restrained by the rest of the mold. So all this leads to the following assumptions:

1. $\sigma_{xx} = 0$
2. $\sigma_{yy} = \sigma_{zz}$
3. $\sigma_{xy} = \sigma_{xz} = \sigma_{yz} = 0$
4. $\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0$
5. $\varepsilon_{yy} = \varepsilon_{zz}$ Varying in time

To follow the above assumptions, the outward normal dof was set to zero Dirichlet on all die and casting boundary nodes, but except the nodes between die and casting. This means that a face node would have only one dof Dirichlet. An edge node would have 2 dofs Dirichlet. A vertex or corner node would have 3 dof Dirichlet. In this case, all Dirichlet are zero Dirichlet. The face between die and cooling line is a convection boundary condition.

5.3 Design file

All necessary information such as geometry, part, material, initial conditions, environment condition, process parameters, and material model for analysis is input into the design file. A design file is composed of different sections. Each section starts with "@ (section title)" and ends with "@ENDOFSECTION". Usually, the following information is specified in a design file:
5.3.1 Part information

First of all the geometry, part, material and initial condition objects should be specified for this analysis. Generally all this information can be written in the "castingPartsInformation", "coolingLinePartsInformation" and "diePartsInformation". In each part-information section, we specify parts name, grid file name that deal with the geometry and the material type of each part. For example:

```plaintext
*DieCastingDesign
#
#
#
@castingPartsInformation
castPartName$ "casting"
`numberOfStlFiles% 0
nameOfStlFile$ " "
numberOfUSMFiles% 1
nameOfUSMFile$ "four_elements_umesh.cdf"
numberOfTheseUSMParts% 4
PartNumberInUSMFile% 7 8 9 10
initialTemperature! 910
```
material$ "Al_380"
materialStress$ "Al_380"
@endofsection
#
#
@
diePartsInformation
diePartName$ "die"
numberOfStlFiles% 0
nameOfStlFile$ " "

numberOfUSMFiles% 1
nameOfUSMFile$ "four_elements_umesh.cdf"

numberOfTheseU SMParts% 5
PartNumberInUSMFile% 1 2 3 4 5

initialTemperature! 300

initialTblTemperature! 300

estimatedDieCavityTemperature! 640

material$ "H13_Steel"

materialStress$ "H13_Steel"

@endofsection
5.3.2 Parameter for thermal analysis

All parameters for thermal analysis will be included in this section, such as the convection coefficient, number of cycles, time step for die open and die close, coolant temperature, and so on. The example for this section is shown as:

# Below is used for thermal problems

#
ambientTemperature! 300.0

initialTemperatureAirInTheCavity! 640

@ENDOFSECTION

#

#

#

@physicsParameters

convectionCoefCL! 30000.0

convectionCoefOL! 1000.0

cavityConductance! 10000.0

convectionCoefAir! 10.0

@ENDOFSECTION

#

#

#

@castingCycleInformation

coolingOnCycle% 0

nbOfCycles! 15

nbTimeStepsOpen% 4

totalDieOpenTime! 28.0
nbTimeStepsClosed% 9
totalDieClosedTime! 36.0
nbTimeStepsSlow% 1
totalDieSlowTime! 11000.0
nbTimeStepsEjected% 3
totalDieEjectedTime! 270.0

@ENDOFSECTION
#
#
#
#
@coolingLineGeneralInformation
modelSectionName$ "spotCoolingLineModel" "flowThroughCoolingLineModel"
coolantTemperature! 300

@ENDOFSECTION
#
#
#
#
@spotCoolingLineModel
nameOfSubModel! 1 2 3
coolingLineDiameter! 0.011 0.011 0.011
holeDiameterForSpotCooling! 0.022 0.022 0.022
tubeOuterDiameterForSpotCooling! 0.0222 0.022 0.022
tubeInnerDiameterForSpotCooling! 0.012 0.012 0.012
coolingLineTubeMaterialType$ "Brass" "Brass" "Brass"
coolingMedium$ "water" "water" "water"
flowRateOfCoolant! 1.2 1.2 1.2
initialCoolantTemperature! 300 300 300
outletCoolantPressure! 1.3 1.3 1.3
surfaceRoughness! 5.1e-7 5.1e-7 5.1e-7

@ENDOFSECTION
#
#
#
@flowThroughCoolingLineModel
nameOfSubModel! 4 5
coolingLineDiameter! 0.004138 0.127809
coolingLineTubeMaterialType$ "Brass" "Brass"
coolingMedium$ "water" "water"
flowRateOfCoolant! 1.2 1.2
initialCoolantTemperature! 298 298
outletCoolantPressure! 1.3 1.3

surfaceRoughness! 5.1e-7 5.1e-7

@ENDOFSECTION
#
#
#
#
@oilLineModel

heatingOilTemperature! 300.0

@ENDOFSECTION
#
#
#
#
@InitialCavityAirParameters

MassInitial! 2.1997e-5

nbMolesInitial! 7.59e-7

PressureInitial! 101000.0

TemperatureInitial! 293.0

@ENDOFSECTION
#
#
#
5.3.3 Parameter for stress analysis

All parameters for stress analysis will be written in this section, such as ICCG iterations, GS iterations, desired convergence ratio, number of linear iterations, relaxation factor, convergence criteria, number of time step, material model type, gravity vector and so on. The example of this section is shown as:

#Below is used for thermal problems
#
#
@stressSolverSection

nbOfLevels% 2
maxNbOfElements% 20000

nbOverlapLayers% 4

nbOfElements2Increment% 100

listDomainPartTypes$ "Die" "Casting"

listZeroDirichletPartTypes$

listNonZeroDirichletPartTypes$

listZeroNeumannPartTypes$

listNonZeroNeumannPartTypes$

@ENDOFSECTION

#

#

#

@stressSolverParameters
testMatrix! 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

ICCGIterations% 5000

GSIterations% 25

stopIfNoConvergence% 0

desiredConvergenceRatio! 1e-12

nbLinearIterations% 5000

relaxationFactor! 1.00
convergenceCriteria! 0.002

nbNLIterations% 5

case% 8

nbTimeSteps% 400

materialModel$ "SimoInfinitesimalStrain"

#materialModel$ "Elastic"

isGravity% 0

gravityVector! 0.0 0.0 -9.81

timeStepSize! 7.0

nbPoints% 41

coeffBottom! 21.5e-6

referenceCoord! 0.159025 0.119337

temperatures! 300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 
         300 373 373 300 \ 

300 373 373 300 \\n300 373 373 300 300 \\
times! 0 300 1200 2100 2700 3600 \\
4800 5700 6300 7200 \\
8400 9300 9900 10800 \\
12000 12900 13500 14400 \\
15600 16500 17100 18000 \\
19200 20100 20700 21600 \\
22800 23700 24300 25200 \\
26400 27300 27900 28800 \\
30000 30900 31500 32400 \\
33600 34500 35100 36000 37200 \\
@ENDOFSECTION
#
#
#
@contactConductanceParameters
gasThermalConductivity! 0.05
relativeEmissivity! 0.6
standardDeviation! 3.81E-03
5.4 Material properties

Many uncertainties arise in the design and analysis process. The material model is a common reason beyond to these uncertainties. The material model and material properties are very important for the analysis of FEM. Errors in the material model will lead to errors in the results.

Generally for thermal analysis, thermal conductivity, specific heat, specific enthalpy, density, and specific volume are important. The Young’s modulus, Poisson ratio, CTE, yield strength, viscosity and hardening coefficients are critical for stress analysis. In order to accurately predict the life of fatigue failure of die casting dies it is necessary to use reliable input data.

The material properties used were shown in the Table 5.1 and the temperature dependent properties were shown in the following Figures (5.3 to 5.9)
Table 5.1: Properties of material used in this analysis [13].

<table>
<thead>
<tr>
<th>MATERIAL PROPERTY</th>
<th>H13_STEEL</th>
<th>AL_380</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $kg/m^3$</td>
<td>Figure 5.3</td>
<td>2760.0</td>
</tr>
<tr>
<td>Enthalpy Liquid $kJ/kg$</td>
<td>8.15e9</td>
<td>3.297005e9</td>
</tr>
<tr>
<td>Enthalpy Solid $kJ/kg$</td>
<td>5.97e9</td>
<td>2.376855e9</td>
</tr>
<tr>
<td>Latent Heat $J/m^3$</td>
<td>2.18e9</td>
<td>9.2015e8</td>
</tr>
<tr>
<td>Specific Heat $J/kgK$</td>
<td>584</td>
<td>1170</td>
</tr>
<tr>
<td>Thermal Conductivity $W/mK$</td>
<td>24.6</td>
<td>109</td>
</tr>
<tr>
<td>Yield Strength $Pa$</td>
<td>1.65e9</td>
<td>1.59e8</td>
</tr>
<tr>
<td>Young’s Modulus $Pa$</td>
<td>Figure 5.4</td>
<td>6.9e10</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>Figure 5.5</td>
<td>0.33</td>
</tr>
<tr>
<td>Viscosity $Pa - s$</td>
<td>1.0e10</td>
<td>1e11</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion</td>
<td>Figure 5.6</td>
<td>21.5e-6</td>
</tr>
<tr>
<td>specific volume $m^3/kg$</td>
<td>Figure 5.7</td>
<td>4.103e-3 to 6.060e-9</td>
</tr>
<tr>
<td>hardening modulus $Pa$</td>
<td>Figure 5.8</td>
<td>2.0e9</td>
</tr>
<tr>
<td>bulk modulus $Pa$</td>
<td>140.0e9</td>
<td>40.0e9</td>
</tr>
<tr>
<td>shear modulus $Pa$</td>
<td>81.0e9</td>
<td>26.5e9</td>
</tr>
<tr>
<td>yield stress $Pa$</td>
<td>1.0e12</td>
<td>2.0e8</td>
</tr>
<tr>
<td>volume strain</td>
<td>Figure 5.9</td>
<td>—</td>
</tr>
</tbody>
</table>
Figure 5.3: Density vs. Temperature for H13 steel [13].
Figure 5.4: Young's Modulus vs. Temperature for H13 steel [13].
Figure 5.5: Poisson’s Ratio vs. Temperature for H13 steel [13].
Figure 5.6: Coefficient of Thermal Expansion vs. Temperature for H13 steel [13].
Figure 5.7: Specific Volume vs. Temperature for H13 steel [13].
Figure 5.8: Hardening Modulus vs. Temperature for H13 steel [13].
Figure 5.9: Volume Strain vs. Temperature for H13 steel [13].
It is well known that, the term visco-plastic stands for all the aspects of time
dependent response of stress to strain and vice versa. It is the characteristic that
contrasts the behavior of plastics most from that of metals. It manifests itself in
the form of many different time dependent phenomena. In the description of such
phenomena we may consider applied stress, loading types, loading rate, strain, strain
rate, time elapsed, and lastly the temperature, as independent variables. Temperature
is perhaps the strongest variable of all. Because of the large number of variables
involved, the description of visco-plastic has to be made necessarily through its as-
pects, namely creep, relaxation and recovery. Creep is the strain response to a stress
that is constant with time. Relaxation is the stress response to an applied constant
strain. Recovery is the strain response to a stress that has been removed. To describe
the time-dependent behavior of plastics in linear and moderately nonlinear regions
some material model can be used such as the Ashby’s sinh model [14].

The rate equation for this material model is:

$$\dot{\varepsilon} = A \left( \sinh \left( \frac{\alpha' \sigma_s}{G} \right) \right)^n \exp \left( \frac{-\Delta H}{RT} \right)$$

(5.1)

where: $\dot{\varepsilon}$ is the strain rate, $A$ is the Dorn constant, $\alpha'$ is the power low breakdown,
$\sigma_s$ is the shear stress, $G$ is the shear modulus, $n$ is the creep exponent, $\Delta H$ is the
activation energy, $R$ is the Boltzmann’s constant, and $T$ is the absolute temperature.
This equation has a time-independent and a time-dependent part, and involves different material constants. Ashby's constants are hard to find. The constants used for Aluminum and H13 Steel are presented in the following subsection. The most important use of Ashby's constants arises because the sinh(...) term is nonlinear and these constants should aim at a precision which corresponds with the general accuracy of experiments, which is about ±10% for the yield strength. Figure 5.10 shows the Comparison of sinh model data from MMO software and from Ashby's Deformation Map.

The above equation implies the following values for the viscosity as a function of temperature and stress.

\[
\mu = \frac{\sigma}{\dot{\varepsilon}} = \frac{\sigma}{A \left( \sinh \left( \frac{\sigma^{\prime}}{C} \right) \right)^n \exp \left( -\frac{\Delta H}{RT} \right)}
\]

\[\text{(5.2)}\]

5.4.1 Aluminum

For Aluminum the constants are evaluated at 42 °C and have values

\[A = 3.4e6 \text{ 1/s.}\]

breakdown \(\alpha' = 1000,\)

\[n = 4.1,\]

86
\[ \Delta H = 99700 \text{ J/mol}, \]

\[ R = 8.31 \text{ J/mol K}, \]

Normalised shear stress \( \frac{\sigma_s}{G} \).

Shear modulus \( G = 2.65 \times 10^8 \),

**5.4.2 H13 Steel**

For H13 Steel the constants are evaluated at 42 °C and have values

\[ A = 1.1 \times 10^4 \text{ 1/s}, \]

breakdown \( \alpha' = 3500 \),

\[ n = 6.0, \]

\[ \Delta H = 174000 \text{ J/mol}, \]

\[ R = 8.31 \text{ J/mol K}. \]

Normalised shear stress \( \frac{\sigma_s}{G} \).

Shear modulus \( G = 8.1 \times 10^{10}, \)
Figure 5.10: Comparison of sinh model data from MMO software and from Ashby's Deformation Map. StressResults_0.4.dat, StressResults_0.6.dat, and StressResults_0.8.dat are the sinh model data from MMO software at 0.4, 0.6 and 0.8 melting temperature, dataFor0.4.dat, dataFor0.6.dat, dataFor0.8.dat are the sinh model data from Ashby's Deformation Map at 0.4, 0.6 and 0.8 melting temperature.
5.5 The performance of thermal and stress analysis for 1-D model

Coupled thermal stress analysis was performed for the 1-D analytical model. The purpose of this analysis is to obtain the temperature, displacement, stress and effective plastic strain. Thermal problem was first analyzed. After getting the temperature field from thermal analysis the non-linear elastic and plastic stress analysis was done to get the displacement, stress and the effective plastic strain. After we have effective plastic strain, the maximum effective plastic strain increments in a die casting at the end of a specified load step can be evaluated. The maximum effective plastic strain increments will be used to estimate the number of cycles to failure of die casting dies.

The preferred domain on which the analysis is performed depends on the die casting stage. In the die closed stage the domain is the die, casting and cooling line. In the die open stage the domain is just the die part.

Also both 8-node brick elements and 20-node brick elements were used for this analysis. The results for them will be presented separately in the following sections. The comparison is also included in each section. The software for post processing is Ensight (version 7) and Gnuplot.
5.5.1 Temperature

In the thermal analysis of a die casting, the temperature field results from heat flow from the casting that has an initial temperature state. On every face on the boundary of the domain, boundary conditions have to be specified. This can be done in the design file.

In the die casting process the heating effect caused by the first casting remains in the die to affect solidification of the next casting, and so on. The die temperatures gradually increase as more castings are made, until the process reaches a cyclic steady state and the die temperatures become periodic in time. After the process reaches cyclic steady state, each casting is exposed to identical process conditions. The length of time required for a process to reach steady state depends on the square of the size of the die. Smaller dies approach steady state in just tens of cycles, and larger dies can take hundreds of cycles. Dies are usually designed for steady state conditions. Figure 5.11 show a cyclic steady state process for the die casting process. Figure 5.12 to Figure 5.14 show the temperature change process from die open to die closed in a load cycle.
Figure 5.11: Die cavity temperature approaching steady state.
Figure 5.12: Die cavity temperature [K] in the 6th time step of a load cycle.
Figure 5.13: Die cavity temperature [K] in the 9th time step of a load cycle.

Figure 5.14: Die cavity temperature [K] in the 11th time step of a load cycle.
The transient temperatures in casting and die were shown in Figure 5.15 and Figure 5.16. The casting temperature is shown only for the residence time, and the die cavity surface temperature is displayed for the whole cycle. Figure 5.15 and Figure 5.16 show that early in the cycle, the casting temperature drops abruptly as it loses sensible heat to the die. This is accompanied by a corresponding rise in the cavity surface temperatures. When the casting reaches liquidus temperature, it begins to cool more slowly since the die must now absorb the latent heat of solidification. After reaching solidus, the casting temperature drops rapidly again as it approaches equilibrium with the cavity surface.

Figure 5.17 and Figure 5.18 show the corresponding temperature distribution through the transient surface layer at six times during the cycle. In this case there are 15 time steps for die open and die closed stage. Figure 5.17 and Figure 5.18 show that 20-node brick element has much more smooth curves than 8-node brick elements.
Figure 5.15: The transient temperatures in casting and die for an 8-node refined mesh.

Figure 5.16: The transient temperatures in casting and die for a 20-node refined mesh.
Figure 5.17: Temperature distribution through the thickness of the layer at six different times during the cycles for an 8-node refined mesh. TemperatureVsX2 and TemperatureVsX3 are the second and third time step in the die open stage and TemperatureVsX7, TemperatureVsX8 are the second and third steps in the die closed stage. Finally the TemperatureVsX10 and TemperatureVsX12 are the 5th and 7th steps in die closed stage.
Figure 5.18: Temperature distribution through the thickness of the layer at six different times during the cycles for a 20-node refined mesh. TemperatureVsX2 and TemperatureVsX3 are the second and third time step in the die open stage and TemperatureVsX7, TemperatureVsX8 are the second and third steps in the die closed stage. Finally the TemperatureVsX10 and TemperatureVsX12 are the 5th and 7th steps in die closed stage.
5.5.2 Displacement

After we have the temperature field from thermal analysis, the stress analysis was performed and displacement was first analyzed. Figure 5.19 shows the cyclic steady state approaching of the displacement in die cavity surface during the 10 cycles. Figure 5.20 shows a general view of the distribution of displacement in the die with Ensight.

![Displacement graph](image)

Figure 5.19: Displacement in die cavity surface during the first 10 cycles for 8-node and 20-node refined mesh.
Figure 5.20: The general view of the distribution of displacement [m] in the die.
5.5.3 Stress

A general view of the distribution of effective stress in the die was shown in Figure 5.21. Figure 5.21 shows that the maximum effective stress always happens in the die cavity surface.

Figure 5.21: A general view of the distribution of effective stress [Pa] in the die.
Figure 5.22 to Figure 5.24 show the effective stress in die cavity surface during the 5 cycles. The effective stress in die cavity surface for a coarse mesh was first shown in Figure 5.22, from this figure the plastic behavior such as creep and relaxation were displayed very clearly. Figure 5.23 to Figure 5.24 show effective stress in die cavity surface for refined 8-node brick element and 20-node brick element.

![Graph showing effective stress vs. time](Image)

Figure 5.22: Effective stress in die cavity surface during the first 5 cycles for a coarse mesh.
Figure 5.23: Effective stress in die cavity surface during the first 5 cycles for a 8-node refined mesh.

Figure 5.24: Effective stress in die cavity surface during the first 5 cycles for a 20-node refined mesh.
5.6 Predicted low cycle fatigue life

When operating at high temperatures, die casting dies are expected to fail due to low cycle fatigue. The number of cycles to failure can be estimated with the Coffin-Manson equation. The increment in the effective plastic strain per cycle determined the number of cycles to failure. When we have the maximum effective plastic strain increment in a load cycle we can calculate the life based on it. The effective plastic strain was obtained from the above thermal stress analysis.

Figure 5.25 shows a general view of distribution of effective plastic strain in the die. Figure 5.26 to Figure 5.27 show the effective plastic strain vs. time at a point on the die cavity surface during 10 cycles.

Figure 5.25: A general view of the distribution of effective plastic strain in the die.
Figure 5.26: Effective plastic strain vs. time in die cavity surface during the first 10 cycles for a 8-node refined mesh. This data is for the Gauss point that has maximum effective plastic strain.
Figure 5.27: Effective plastic strain vs. time in die cavity surface during the first 10 cycles for a 20-node refined mesh. This data is for the Gauss point that has maximum effective plastic strain.
When we have the effective plastic strain as a function of time, the effective plastic strain increment of each element in the die will be obtained looping over each Gauss point of element. The maximum effective plastic strain increment was used to calculate the life for the die. The maximum effective plastic strain increment is always on a Gauss point closed to the die cavity surface between die and casting in this project.

Table 5.2 to Table 5.9 show the life of die based on the different number of cycles. In the refined die mesh there are 5 elements in die part. Element Id 0 contains the die cavity surface element and element Id 5 is the element on the cooling line side.

The suggests a life of die in the range of 50,000 to 60,000 cycles is given from Table 5.6 and Table 5.7 based on 30 cycles. From these two tables we can see that there is about a 10 percent difference between results of 8-node refined mesh and 20-node refined mesh.
Table 5.2: Predicted life based on the 10th cycle for 8-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.89e4</td>
<td>2.553e-3</td>
</tr>
<tr>
<td>1</td>
<td>1.94e5</td>
<td>1.215e-3</td>
</tr>
<tr>
<td>2</td>
<td>1.59e6</td>
<td>0.536e-3</td>
</tr>
<tr>
<td>3</td>
<td>83e9</td>
<td>7.78e-06</td>
</tr>
</tbody>
</table>

Table 5.3: Predicted life based on the 10th cycle for 20-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.65e4</td>
<td>2.64e-3</td>
</tr>
<tr>
<td>1</td>
<td>1.84e5</td>
<td>1.239e-3</td>
</tr>
<tr>
<td>2</td>
<td>1.37e6</td>
<td>0.567e-3</td>
</tr>
<tr>
<td>3</td>
<td>63e9</td>
<td>8.6e-06</td>
</tr>
</tbody>
</table>
Table 5.4: Predicted life based on the 20th cycle for 8-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.38e4</td>
<td>2.17e-3</td>
</tr>
<tr>
<td>1</td>
<td>4.3e5</td>
<td>0.891e-3</td>
</tr>
<tr>
<td>2</td>
<td>3.2e6</td>
<td>0.406e-3</td>
</tr>
<tr>
<td>3</td>
<td>271e14</td>
<td>5.51e-08</td>
</tr>
</tbody>
</table>

Table 5.5: Predicted life based on the 20th cycle for 20-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.03e4</td>
<td>2.24e-3</td>
</tr>
<tr>
<td>1</td>
<td>4.19e5</td>
<td>0.9e-3</td>
</tr>
<tr>
<td>2</td>
<td>2.8e6</td>
<td>0.43e-3</td>
</tr>
<tr>
<td>3</td>
<td>137e14</td>
<td>7.19e-08</td>
</tr>
</tbody>
</table>
Table 5.6: Predicted life based on the 30th cycle for 8-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.55e4</td>
<td>1.86e-3</td>
</tr>
<tr>
<td>1</td>
<td>8.8e5</td>
<td>0.67e-3</td>
</tr>
<tr>
<td>2</td>
<td>5.6e6</td>
<td>0.33e-3</td>
</tr>
</tbody>
</table>

Table 5.7: Predicted life based on the 30th cycle for 20-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.02e4</td>
<td>1.92e-3</td>
</tr>
<tr>
<td>1</td>
<td>8.8e5</td>
<td>0.674e-3</td>
</tr>
<tr>
<td>2</td>
<td>5.1e6</td>
<td>0.34e-3</td>
</tr>
</tbody>
</table>
Table 5.8: Predicted life based on the 40th cycle for 8-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.8e4</td>
<td>1.59e-3</td>
</tr>
<tr>
<td>1</td>
<td>1.74e6</td>
<td>0.516e-3</td>
</tr>
<tr>
<td>2</td>
<td>8.61e6</td>
<td>0.277e-3</td>
</tr>
</tbody>
</table>

Table 5.9: Predicted life based on the 40th cycle for 20-node mesh.

<table>
<thead>
<tr>
<th>Element Id</th>
<th>Max. Number of Cycles to Failure</th>
<th>Max. Plastic Strain Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.0e4</td>
<td>1.64e-3</td>
</tr>
<tr>
<td>1</td>
<td>1.83e6</td>
<td>0.507e-3</td>
</tr>
<tr>
<td>2</td>
<td>7.79e6</td>
<td>0.288e-3</td>
</tr>
</tbody>
</table>
5.7 The performance of thermal and stress analysis for 3-D dumbbell mesh

Although the 1-D model was useful for gaging the accuracy of the assumptions in the surface layer model, it is not general enough to illustrate the spatial coupling that can occur within a die component because of non uniform interior temperatures. These non uniformities stem from three basic sources: die geometry, cooling line positions, and casting thickness. In this section a 3-D nonlinear thermal stress analysis was also performed.

Figure 5.28 shows the geometry of 3-D dumbbell mesh, the blue part in this figure is the die and the red part in this figure is the casting. Similar to the 1-D model the dumbbell mesh was also refined in the normal direction to the die surface. Figure 5.29 shows the procedure of mesh refinement. Figure 5.30 to Figure 5.32 show the post-processed of temperature, and effective stress for the 20-node refined mesh. From Figure 5.32 we can see very clearly the effective stress distribution in die cavity surface.
Figure 5.28: The geometry of 3-D dumbbell mesh.
Figure 5.29: The procedure of mesh refinement.
Figure 5.30: The temperature [K] distribution in a dumbbell mesh in the second time step of die closed stage.
Figure 5.31: The effective stress [Pa] distribution in a dumbbell mesh in the 7th time step of die closed stage.
Figure 5.32: The effective stress [Pa] distribution in a dumbbell mesh in die cavity surface in the 7th time step of die closed stage.
Chapter 6

Conclusion and Further Work

Suggested

A coupled thermal stress for 1-D analytical model and 3-D dumbbell mesh was performed. The 1-D analytical model for thermally induced stresses in the mold surface during die casting was established. The low cycle fatigue life of die casting dies was predicted with this 1-D model. The suggests a life in the range of 50,000 to 60,000 load cycles based on an analysis with 30 load cycles.

A standard 20-node brick element with different Gauss point schemes was developed for this project. The 20-node brick element with 27 Gauss points and 8-node brick element with 8 Gauss points were used for the analysis. Special refined meshing
algorithms were created for increasing the accuracy of the solution. The results with 20-node brick element was about 10% more accuracy than the 8-node brick element on the finest mesh.

The interactions between thermal, mechanical processes presented in this project were included by coupling the thermal and stress analysis. The input data for the analysis include geometry, thermal properties, initial temperature, material model, material property, boundary conditions appropriate to the state of the die casting structure.

The accuracy of the computation was limited primarily by the knowledge of input data: nonlinear material properties, convection coefficient, high temperature viscosity of solids.

It is worth mentioning that the theoretical background of the high-pressure die casting process should consider fluid flow, heat transfer and stress/strain problems together. In this project the fluid flow problem was not considered. Mathematically, the temperature solution in the casting requires an initial value condition, and the temperature solution in the die satisfies a periodic or cyclic steady state condition. The combination of these two conditions makes the die casting problem very difficult.
to solve directly.

There are several opportunities for future work including the following.

1) Different casting mass distribution, die surface geometry, and cooling line locations can be achieved.

2) The transient temperature of die cavity surface can be deeply analyzed with consideration of flow problem. There is one method to compute the transient temperature of a very thin surface layer in MMO software. It deal with the fluid flow filling solver.

3) Since the transient time step must be small to accurately follow the rapidly fluctuating temperatures during the cycle and many cycles are typically needed to reach steady state, computation times can be enormous, especially in large dies. Also because the prediction of low cycle fatigue life should be based on cyclic steady state rather than the first few cycles, the Tcl script for coupled thermal stress analysis can be modified to suit this requirement.

4) Stress model SISII of MMO software can be used instead of stress model SISI of MMO software in this thesis. Stress model SISII can avoid the big spike of effective stress in Figures 5.22 to 5.24. This stress model need some changes in the material library. It needs hardningModulusIn, hardningModulusOut, yieldStressIn, yieldStressOut, viscosityIn and viscosityOut instead of single material property hardningModulus, yieldStress and viscosity in stress model SISI.
Bibliography


