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SER Prediction for Transmission of PSAM 16-QAM in Frequency Selective Fading Channels

by

Edward Pham, B. Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment
of the requirements for the degree of

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University

September, 1992

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submitted by **Edward Pham, B. Eng.**
in partial fulfillment of the requirements for
the degree of Master of Engineering

Thesis Co-Supervisor

Thesis Co-Supervisor

Chairman,
Department of Systems and Computer Engineering

Carleton University
September, 1992

Abstract

Pilot Symbol Assisted Quadrature Amplitude Modulation (PSAQAM) has been considered a viable modulation technique for use in frequency selective fading environment. It provides a more efficient use of channel bandwidth, at the same time, it offers some measure to combat channel fading distortions. The viability of using PSAQAM is studied by estimating the average Symbol Error Rate (SER) of the channel.

To this end, the SER is estimated using three different simulation methods. The first uses BOSS simulation program for a PSAQAM system in which synchronization between transmitter and receiver is obtained by having a common clock. This system uses fixed rate sampling. The second method also uses BOSS but with a clock recovery circuit in the receiver. The sampling in this system is dynamic, which tracks the peaks of the received signal envelope. In the above two systems, fading compensation is realized by estimating channel fading gains from three consecutive received pilot symbols. The third method is semi-analytical, involving brute-force Matlab calculation of the SER lower bounds. These simulations show a general trend of increasing SER for higher doppler frequency and larger delay spread.

The results obtained by these simulations can be used to establish measures for communication system design and performance evaluation.

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List of Abbreviations and Acronyms

BER -	Bit Error Rate
BOSS -	Block Oriented System Simulator
dB -	decibel
DFE -	Decision Feedback Equalizer
Equ. -	Equation
FBC -	Feedback Balance Coding
Fig. -	Figure
FSF -	Frequency Selective Fading
GHz -	Giga Hertz
Hz -	Hertz
ISI -	Inter-Symbol Interference
LOS -	Line Of Sight
MHz -	Mega Hertz
MSE -	Mean Square Error
MTFSK -	Multi Tone Frequency Shift Keying
PAM -	Pulse Amplitude Modulation
PSAM -	Pilot Symbol Assisted Modulation
PSAQAM -	Pilot Symbol Assisted Quadrature Amplitude Modulation
QAM -	Quadrature Amplitude Modulation
SER -	Symbol Error Rate
SNR -	Signal to Noise Ratio
TTIB -	Transparent Tone-In-Band

List of Symbols

α -	Magnitude of fading gain
B_d -	Doppler spread
c -	velocity of light
$c(t)$ -	Channel impulse response
$\delta(t)$ -	Unit impulse
E_b -	Energy per bit
$E\{\}$ -	Expected value
f_c -	Carrier frequency
f^d -	Maximum doppler frequency
$\text{Im}\{.\}$ -	Imaginary part of complex number
$I_0(.)$ -	Zeroth order modified Bessel function of first kind
k -	Integer index
λ -	Wavelength
μ -	Mean signal power
$n(t)$ -	Complex white noise
N -	Frame length
N_o -	Single-sided noise power spectral density
π -	3.14159
Q_1 -	Fading gain coefficient
$\text{Re}\{.\}$ -	Real part of complex number
θ -	Phase of fading gain
σ^2 -	Variance
τ -	Relative delay
t -	Continuous time variable

T -	Symbol period
T_c -	Coherence time
T_m -	Delay spread
t_s -	Optimal sampling instant
v_m	Mobile speed

Chapter 1

INTRODUCTION

The advent of mobile communications has introduced a new mode of transporting information. With cellular telephones, people can be reached even when they are on the move. Personal communicators, in the near future, will allow even easier access to many kinds of information [1]. All of the above communication methods make use of transmission of radio signals over a multipath channel which is markedly characterized by the severe envelope distortion and intersymbol interference [2-4].

The radio spectrum is also a limited resource and thus must be efficiently used. This calls for more bandwidth efficient modulation techniques such as M-ary QAM. Whatever modulation scheme is used, however, there associates a limitation in the average Symbol Error Rate (SER) performance. This limit must be known, or at least be approximated, so that system performance evaluation is made possible. It is this factor that makes SER prediction a useful step in communication systems design.

M-ary QAM signaling technique has been considered a desirable modulation format [5]. Its usage, however, comes with a cost of being highly susceptible to channel impairments. This means methods for combating or compensating these adverse effects must be developed to enhance the system performance. Some of the most common methods of reducing channel distortions are equalization, diversity, error correction coding, and pilot tones [6-11]. Another transmission method, Pilot Symbol Assisted quadrature amplitude Modulation (PSAM), has proved to be a simple but effective method in compensating for flat Rayleigh fading channels [12, 13]. In the current work, it is proposed to extend the use of PSAM in frequency selective fading channels and observe its error performance. Even though the performance of PSAM is not as good when compared to the more complex and elegant equalization, diversity, and error correction coding methods, the channel fading statistics has made it an attractive modulation format. The prediction of average SER in frequency selective fading

channels for systems using PSAM is thus a worthwhile problem to examine.

To this end, the SER is estimated using three different simulation techniques. The first two of these use Block Oriented System Simulator (BOSS) program to simulate the SER. These systems are similar; the only difference is in the technique of achieving symbol synchronization. Both systems cope with channel distortion by estimating, from three consecutive received pilot symbols, the channel gain subjected to each transmitted data symbol. These fading gain estimates are then used to remove the fading distortion from the received data.

The other simulation technique uses Matlab to calculate the absolute SER lower bounds of PSA-16QAM system. The calculation is based on an algorithm developed from PSA-16QAM system analysis.

In all of the above simulations, the frequency selective fading channel is modelled by a sum of two independent Rayleigh fading rays, each is a sum of nine sinusoids described in Jakes [14]

1.1 Thesis Objectives

Mobile radio channel over densely developed regions such as down town areas exhibits less severe frequency selective fading. That is, having elaborate distortion compensation methods is unnecessary. In this situation, a simpler compensation method is more desirable. This new method results in a degradation in SER performance; hence presenting a trade off factor in the utilization of such system. One of the methods for weighing this trade off factor is to examine the system's SER performance when a particular modulation method is used. Motivated by this, the objectives of this thesis are centered around the following topics:

- Study the error probability of Pilot Symbol Assisted 16-QAM by semi-analytical and simulation techniques.
- Study the limit in channel distortion above which elaborate detection schemes must be required in order to obtain satisfactory communication.

To narrow the thesis's scope, the objectives are pursued with a set of basic assumptions. These are:

- The channel is frequency selective slow fading. Such channel exists in densely built-up areas such as down town core or indoor wireless environment.
- The delay spread is not more than 25% of the symbol period. This constraint is put forward because there is no equalization utilized in the receiver.
- There are no line-of-sight paths.
- The long term fading is ignored. That is, shadowing is not considered in this work.
- There is no co-channel or adjacent channel interference.
- The additive noise is considered complex gaussian noise.
- There is no offset frequency at the receiver.
- There is no diversity scheme employed in the receiver.

These assumptions have a significant influence on the obtained error rates. Therefore, the results given in this thesis can only be referenced with an emphasis on the above stated assumptions.

1.2 Thesis Organization

The remainder of the thesis is organized into 4 chapters, each covering a relevant part of the thesis's scope.

Chapter 2 presents the background information and theories on frequency selective fading channels and methods of mitigating the fading effects on the transmitted signal. It also covers a survey of literature to ensure there is no repetition in the presented work.

Chapter 3 elaborates on the simulation of PSA-16QAM systems. Two types of receivers are considered. The first receiver obtains its sampling clock from the transmitter. The second derives the sampling clock from the peaks of the received signal envelope. This chapter also describes the functionalities of all system components as well as all the simulation parameters which directly control the resulting SER of the PSA-16QAM system.

Chapter 4 discusses the semi-analytical method of predicting the SER. An analysis of the SER, together with a set of practical assumptions give rise to an error rate calculation algorithm for an ideal receiver. The results obtained by this method can serve as the lower limit of the SER for PSA-16QAM systems. The chapter concludes with a comparison of SER performance of the simulated systems with reference to the

calculated results as well as other related published results.

Chapter 5 summarizes and concludes the findings of this research.

Chapter 2

BACKGROUND and LITERATURE SURVEY

In this chapter, theories and background information on the nature of the mobile channel are discussed. The characterization of this channel leads to a development of a channel model that is having a time-variant impulse response. The time varying characteristics of the channel is a direct result of the constantly changing transmission medium. Moreover, these changes, which are reflected in the atmospheric conditions, occur in a random manner. As a consequence, signals arriving at the receiver are randomly affected by the channel. These adverse effects, also known as channel distortions, put a constraint on the types and rates of data transmitted over the channel. It also presents others' approaches in dealing with the transmission of digital data in the form of M-ary QAM signals over such channel. Techniques of improving signal quality and methods of analyzing SER are examined.

The chapter begins with the characterization of the fading channels, from which, a statistical channel model is developed. Next, channel distortion compensation techniques are discussed. Particular emphasis on pilot symbol assisted modulation is treated because it is the technique used in the simulated system. Finally, a review of current research on the topics of M-ary QAM reception techniques and SER analysis are given. This literature survey serves as a guideline for the initial approach to the proceedings of this thesis.

2.1 Characterization of Fading Channels

One of the markedly observable characteristics of a fading channel is the large variations in the envelope of the received signal when a constant enveloped signal is transmitted. This is due to the multiple propagation path, or multipath, phenomenon. Under this phenomenon, the transmitted signal is dispersed into many rays through

the transmission medium. Each ray is then reflected by any sizable objects blocking its propagation path, or when it incidents on any interface with good signal reflective index. Signal receiving at any point in space is the vector sum of all signal rays converging at that point. These rays all originate from the same transmitter but now have different phases, amplitudes, and delays because of the different path they travelled.

The range of signal delays gives rise to the time spreading effect of the received signal. The larger the range, the more the spread. Depending on the phase and amplitude of each received ray, the resultant signal may add constructively to give good signal strength, or add destructively to nullify the transmitted signal. In the latter case, a signal fade is said to occur.

In addition, the transmission medium is constantly changing due to the random variations in the medium structure and in the distribution of signal reflectors. As a result, the phase, amplitude, and delay of each signal ray are also time varying. These random time variations suggest that the fading channel should be characterized statistically [15].

To mathematically characterize this, let the transmitted signal be

$$s(t) = \text{Re}\{u(t)e^{j2\pi f_c t}\} \quad (2.1)$$

where

$u(t)$ = equivalent lowpass signal

f_c = carrier frequency

then, assuming there are a discrete number of propagation paths, the received signal is given by

$$r(t) = \sum_n \alpha_n(t) s[t - \tau_n(t)] \quad (2.2)$$

where

$\alpha_n(t)$ = amplitude of the n^{th} ray

$\tau_n(t)$ = delay of the n^{th} ray

From this, the equivalent lowpass received signal would be

$$\begin{aligned} r(t) &= \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} u[t - \tau_n(t)] \\ &= \sum_n \alpha_n(t) e^{j\theta_n(t)} u([t - \tau_n(t)]) \end{aligned} \quad (2.3)$$

where

$\theta_n(t) = -2\pi f_c \tau_n(t)$ = phase of the n^{th} ray

By examining (2.3), the equivalent lowpass received signal is just a weighted sum of shifted versions of the transmitted signal. This means that the equivalent lowpass channel impulse response of a fading channel can be expressed as

$$c(t; \tau) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \delta[\tau - \tau_n(t)] \quad (2.4)$$

which is a time-variant impulse response.

When an unmodulated carrier at frequency f_c is transmitted, $u(t) = 1$ for all t , the received signal will be

$$\begin{aligned} r(t) &= \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \\ &= \sum_n \alpha_n(t) e^{j\theta(t)} \end{aligned} \quad (2.5)$$

From the above expression, it can be seen that $\theta_n(t)$ is changed by 2π rad whenever $\tau_n(t)$ changes by $1/f_c$. Since f_c is usually large, $\theta_n(t)$ can change drastically with relatively small variation in the medium.

Because the changes in the medium is random, it is expected that $\theta_n(t)$, hence $\tau_n(t)$ to be a random quantity. Therefore, it is reasonable to model the received signal as a random process.

When n is large, the central limit theorem dictates that $r(t)$ is a zero mean complex-valued gaussian random process. It can be expressed as a sum of inphase and quadrature components. That is,

$$r(t) = i(t) + jq(t) \quad (2.6)$$

where

$$\begin{aligned} i(t) &= \sum_n \alpha_n(t) \cos[2\pi f_c \tau_n(t)] = \text{inphase component} \\ q(t) &= \sum_n \alpha_n(t) \sin[2\pi f_c \tau_n(t)] = \text{quadrature component} \end{aligned}$$

Each $i(t)$ and $q(t)$ is a zero mean gaussian process. Their joint pdf is given by

$$p_{I,Q}(i, q) = \frac{1}{2\pi} e^{-\frac{i^2 + q^2}{2\sigma^2}} \quad (2.7)$$

where

$$\sigma^2 = \text{average received signal power}$$

The received signal can also be expressed in terms of amplitude and phase. That is,

$$r(t) = a(t)e^{j\phi(t)} \quad (2.8)$$

where

$$a(t) = \sqrt{i^2(t) + q^2(t)}$$

$$\phi(t) = \arctan \left[\frac{q(t)}{i(t)} \right]$$

Using Jacobian method, the joint pdf of A and Φ can be obtained as

$$p_{A\Phi}(a, \phi) = \frac{a}{2\pi\sigma^2} e^{-\frac{a^2}{2\sigma^2}}$$

$$= \frac{1}{2\pi} \frac{a}{\sigma^2} e^{-\frac{a^2}{2\sigma^2}} \quad (2.9)$$

from which

$$p_A(a) = \frac{a}{\sigma^2} e^{-\frac{a^2}{2\sigma^2}} \quad , a \geq 0 \quad (2.10)$$

and

$$p_\Phi(\phi) = \frac{1}{2\pi} \quad , -\pi \leq \phi \leq +\pi \quad (2.11)$$

The pdf of the envelope is a Rayleigh distribution and that of the phase is uniform. Any interval of 2π radians can be chosen as the domain of ϕ . The interval $[-\pi, +\pi]$ selected here is totally arbitrary. Such a channel is said to have Rayleigh fading characteristics.

When the mean of $r(t)$ is not zero, $P_A(a)$ will be a non-central chi-square pdf with two degrees of freedom, and is expressed as

$$p_A(a) = \frac{a}{\sigma^2} e^{-\left(\frac{a^2+s^2}{2\sigma^2}\right)} I_0\left(\frac{as}{\sigma^2}\right) \quad , a \geq 0 \quad (2.12)$$

where

$$s^2 = \sum_n \mu_n^2$$

μ_n = mean power of the n th path

$I_0(.)$ = zeroth order modified Bessel function of the first kind

This situation arises when there is a line-of-sight (LOS) path from the transmitter to the receiver. Since this is the Ricean distribution, the channel is said to have Ricean fading characteristics.

The characterization of fading channels can also be treated from the channel impulse response $c(\tau; t)$ point of view. Assume that $c(\tau; t)$ is wide-sense stationary, its autocorrelation function can be shown to be [15]

$$\phi_c(\tau_1; \tau_2; \Delta t) = \frac{1}{2} E[c^*(\tau_1; t) c(\tau_2; t + \Delta t)] \quad (2.13)$$

Further assume that the transmitted signal undergoes uncorrelated scattering. That is the phase, amplitude, and delay in each scattered ray are independent from the phase, amplitude, and delay of any other ray. Thus the autocorrelation function of the channel impulse response can also be expressed as a function of the difference in the delay. That is,

$$\begin{aligned} \phi_c(\tau_1, \tau_2; \Delta t) &= \phi_c(\tau_1 - \tau_2; \Delta t) \\ &= \phi_c(\tau_1; \Delta t) \delta(\tau_1 - \tau_2) \\ &\equiv \phi_c(\tau; \Delta t) \delta(\tau - \tau_0) \end{aligned} \quad (2.14)$$

where

$$\tau_0 = \tau_1 - \tau_2$$

The understanding of $\phi_c(\tau; \Delta t)$ and its Fourier transforms is essential in the characterization of any fading channel.

If $\Delta t = 0$ then $\phi_c(\tau; \Delta t) = \phi_c(\tau)$. This is the average power output of the channel as a function of the time delay τ . For this reason, it is also known as the **delay power spectrum** or **multipath intensity profile** of the channel.

As with any autocorrelation function, $\phi_c(\tau)$ has its maximum at $\tau = 0$. Also it is a monotonically decreasing function. The value of τ at which $\phi_c(\tau)$ is approximately zero is called the **multipath spread** or **delay spread** T_m of the channel.

Now if the Fourier transform of $\phi_c(\tau; \Delta t)$ in the variable τ is performed, $\phi_c(\Delta f; \Delta t)$ or the spaced-frequency, space-time correlation function is obtained.

The plot of $\phi_c(\Delta f; \Delta t)$ vs Δf when $\Delta t = 0$ gives a measure of frequency coherence of the channel, also known as the spaced-frequency correlation function of

the channel. The range of Δf over which $\phi_c(\Delta f)$ is essentially non zero is referred to as the **channel coherence bandwidth** $(\Delta f)_c$. It is approximately equal to the reciprocal of the channel delay spread T_m . That is,

$$(\Delta f)_c \approx \frac{1}{T_m} \quad (2.15)$$

When the bandwidth of the transmitted signal is smaller than $(\Delta f)_c$, the channel is said to be **frequency non-selective** or **flat fading**. Likewise, when it is larger than $(\Delta f)_c$, it is said to be **frequency selective**.

If $\Delta f = 0$, $\phi_c(\Delta f; \Delta t) = \phi_c(\Delta t)$. This is the spaced-time correlation function. The range of Δt over which $\phi_c(\Delta t)$ is non zero is called the **coherence time** T_c of the channel.

Taking a further step by Fourier transforming $\phi_c(\Delta f; \Delta t)$ in the Δt variable yields $S_c(\Delta f; \lambda)$. Setting $\Delta f = 0$ results in the doppler power spectrum. The range of λ over which $S_c(\Delta \lambda)$ is non zero is considered the doppler spread B_d of the channel. It is approximately equal to the reciprocal of the channel coherence time T_c . That is,

$$B_d \approx \frac{1}{T_c} \quad (2.16)$$

When the information symbol period T is less than the channel coherence time T_c , the channel undergoes **slow fading**. Otherwise, it is said to be **fast fading**.

2.2 Channel Model

In the following, a model of a non-LOS frequency selective, slow fading channel is developed. For this channel, the signal bandwidth is larger than the channel coherence bandwidth, and the symbol period is shorter than the channel coherence time.

The time-varying characteristics of the channel makes it hard to analyze. Since the channel is slow fading, it can be thought to be quasistatic. That is, the channel is time invariant over a few symbol periods. Thus, the channel impulse response can be expressed as

$$c(\tau; t) = \sum_n \alpha_n e^{j\theta_n} \delta[\tau - \tau_n] \quad , 0 \leq t \leq T \quad (2.17)$$

Figure 2.1 Channel model

$$\therefore r(t) = \sum_n \alpha_n e^{j\theta_n} u[t - \tau_n] \quad , 0 \leq t \leq T \quad (2.18)$$

Following Rummier and Jakes [16, 17], the channel can be represented by two independent Rayleigh fading rays. That is,

$$r(t) = \alpha_1 e^{j\theta_1} u(t) + \alpha_2 e^{j\theta_2} u(t + \tau) \quad , 0 \leq t \leq T \quad (2.19)$$

where

- α_i = independent identically Rayleigh distributed
- θ_i = independent identically uniformly distributed
- τ = relative delay between the two rays

Both α_i and θ_i are generated using Jakes' sum of nine sinusoids model. The resultant channel can be represented as outlined in Figure 2.1.

2.3 M-ary QAM Systems

An M-ary QAM signal can be represented as

$$s_m(t) = A_c u(t) \cos(2\pi f_c t) - A_s u(t) \sin(2\pi f_c t) \quad (2.20)$$

where

$m = 1, 2, \dots, M$

$c = 1, 2, \dots, C$

$s = 1, 2, \dots, S$

$M = C \times S$

A_c = coded amplitudes for inphase components

A_s = coded amplitudes for quadrature components

$u(t)$ = pulse shaping signal

In the following, modulation and demodulation method of M-ary QAM are discussed.

2.3.1 Modulation of M-ary QAM Signal

M-ary QAM is obtained by simultaneously transmitting pulse amplitude modulated (PAM) signal on two quadrature carriers. A block diagram showing QAM bandpass and baseband modulators is given in Figure 2.2. While these two modulators are equivalent, the baseband module is used in the simulation.

The binary data bits are first grouped into blocks of $n = \log_2(M)$ bits, where M is the modulation level. These n bits are further divided into two subgroups which are, in turn, coded into two sets of amplitude levels. The two sets of amplitude levels are then used to pulse amplitude modulate the two quadrature carriers. The resulting modulated signals are added together to produce QAM signal, which can be expressed as

When n is an even number, the subgroups contain all even or all odd numbered bits. However, when n is odd, the subgroup division is dependent on particular implementation. For example, when $M = 32$, $n = 5$, the subgroup bit sequence $b_0b_1b_2b_3b_4$ can be divided as

$$\begin{aligned} b_0b_2b_4 &\Rightarrow \{A_c\} \quad , j = 1, 2, \dots, 7 \\ b_1b_3 &\Rightarrow \{A_s\} \quad , k = 1, 2, \dots, 3 \end{aligned} \quad (2.21)$$

The mapping from binary digits to its corresponding amplitude level may be done linearly or according to some coding rule. For 16-QAM, $n = 4$ and the two set of modulation amplitudes are similar.

$$\{A_c\} = \{A_s\} = \{-3, -1, 1, 3\} \quad (2.22)$$

with the following mapping rule

Bits	$A_c; A_s$
00	-3
01	-1
10	1
11	3

Table 2.1 Mapping of binary bits to 16-QAM levels

Figure 2.2 a) QAM bandpass modulator b) QAM baseband modulator

For gray coding, the mapping is slightly different as can be seen in Table 2.2.

Bits	$A_c:A_s$
00	-3
01	-1
10	3
11	1

Table 2.2 Gray coded mapping of binary bits to 16-QAM levels

M-ary QAM can also be represented graphically as discrete points in a two dimensional signal space. The set of all signal points for a particular modulation level is known as its signal constellation. This is a better way of characterizing QAM signal. Using this representation, each signal point is a unique vector with certain amplitude and phase. The constellation of 16-QAM is depicted in Figure 2.3. This constellation is also useful in discussing the demodulation process.

2.3.2 Demodulation of M-ary QAM signal

Demodulation of M-ary QAM is achieved by first extracting the baseband symbol from the received signal. This received symbol is then compared to all symbols in the transmitting signal constellation. The constellation point nearest the received signal point is taken as the transmitted symbol. A block diagram of QAM demodulator is given in Figure 2.4

2.4 Symbol Synchronization

Symbol synchronization refers to the ability of the receiver to derive from the received signal a sampling clock that can sample the received signal at its largest eye opening instants. This is desired to minimize the effects of ISI on the received data. Since the thesis is not about synchronization, what follows is a description of a simple technique of achieving symbol synchronization for M-ary QAM modulation [15].

For QAM modulations, it is possible to sample the received signal at its envelope's peaks. The occurrence of these peaks depends directly on the transmitted data sequence. It is, therefore, possible that there are periods in which no peaks can be found in the

Figure 2.3 16-QAM constellation

[a]

[b]

Figure 2.4 a) Bandpass demodulator b) Baseband demodulator

envelope. During these periods, the sampling clock generator must insert appropriate number of clock pulses to maintain the synchronization. The basic operation to achieve symbol synchronization is depicted in Figure 2.5

2.5 Channel Distortion Compensation Methods

This section presents several techniques used to combat fading distortion. In particular, it elaborates on the fading compensation method based on pilot symbol assisted modulation for both flat and frequency selective fading channels.

2.5.1 Pilot Symbol Assisted Modulation

In Pilot Symbol Assisted Modulation (PSAM), digital data are arranged into fixed length frames. The first frame symbol is allocated to the transmission of a symbol known to the receiver. These symbols are called pilot symbols. PSAM works on the assumption that the channel is quasistatic. The pilot symbols are used to measure the channel distortions from which the receiver can estimate the channel distortion during each received symbol.

The pilot symbol used can be fixed or pseudo-random. Pseudo-random pilot is more desirable because it does not alter the source statistics. However, the use of fixed pilot renders a simpler overall system.

2.5.2 Compensation for flat Rayleigh fading channel

Consider a flat Rayleigh channel with slow fading, the received symbol can be expressed as

$$r(t) = c(t)u(t) + z(t) \quad .0 \leq t \leq T \quad (2.23)$$

where

$$c(t) = \alpha e^{j\theta}$$

α = Rayleigh distributed channel gain

θ = uniformly distributed channel phase

$z(t)$ = lowpass equivalent white noise

Figure 2.5 Symbol synchronization by envelope's peak detection

If the complex channel gain $c(t)$ can be estimated, its effects on the transmitted symbols may be removed. That is the compensated received signal can be expressed as

$$\hat{r}(t) \equiv \frac{r(t)}{c(t)} = \frac{c(t)}{c(t)} u(t) + \frac{z(t)}{c(t)} \quad .0 \leq t \leq T \quad (2.24)$$

$$\therefore r(t) \approx u(t) + \hat{z}(t) \quad (2.25)$$

The operation of PSAM is now discussed. Assume that perfect synchronization is achieved so that the receiver can differentiate between data and pilot symbols. The channel distortion imbedded in the pilot symbols are extracted to form estimates of channel distortion carried by each data symbol. The fading compensation is done by dividing the received symbol by the corresponding distortion estimate.

The channel distortion measurements can be extracted by first sampling the pilot symbols into a tap delay line. The length of this delay line is dictated by the estimation method being used. If more pilot symbols are used in the estimation, a better estimate can be obtained but the estimation delay will be longer.

The estimation is as follows. Consider the compensator's block diagram in Figure 2.6. Suppose there are N symbols in each frame, the sampled received signal is

$$r_k[mT] = c_k[mT]u_k[mT] + z_k[mT] \quad (2.26)$$

which is the m^{th} symbol in the k^{th} frame where

$$k = 1, 2, 3, \dots$$

$$m = 0, 1, 2, \dots, N-1$$

and T = symbol period

During the reception of pilot symbol ($m = 0$),

$$\begin{aligned} r_k[0] &= c_k[0]u_k[0] + z_k[0] \\ &\equiv c_k[0]p_k + z_k[0] \end{aligned} \quad (2.27)$$

from which

$$c_k[0] = \frac{r_k[0]}{p_k} - \frac{z_k[0]}{p_k} \quad (2.28)$$

Figure 2.6 Block diagram of fading compensator

Usually $|p_k| \gg z_k[\cdot]$ for moderately large SNR, thus

$$c_k[0] \approx \frac{r_k[0]}{p_k} \equiv \hat{c}_k[0] \quad (2.29)$$

That is, the channel complex gain during transmission of pilot symbol is estimated by Equ. (2.29). The assumption of slow fading channel enables interpolation of the channel gain during other data symbols. Any feasible interpolation method can be used. Cavers [13] suggested the use of Wiener filter where the number of taps depends on the extents of channel distortion. Sampei [12], on the other hand, employed a gaussian interpolator using three consecutive pilot symbols. He showed that this interpolator is suitable for estimating slow fading gains. Figure 2.7 gives the block diagram of such channel gain estimator.

In particular, let $\hat{c}_{k-1}[0]$, $\hat{c}_k[0]$, and $\hat{c}_{k+1}[0]$ be the three measured channel gains from three consecutive pilot symbols, the interpolated channel gain is given by [12]

$$\hat{c}_k\left[\frac{m}{N}\right] = Q_{-1}\left[\frac{m}{N}\right]\hat{c}_{k-1}\left[\frac{m}{N}\right] + Q_0\left[\frac{m}{N}\right]\hat{c}_k\left[\frac{m}{N}\right] + Q_{+1}\left[\frac{m}{N}\right]\hat{c}_{k+1}\left[\frac{m}{N}\right] \quad (2.30)$$

where

$$\begin{aligned} Q_{-1}\left[\frac{m}{N}\right] &= \frac{1}{2}\left[\left(\frac{m}{N}\right)^2 - \left(\frac{m}{N}\right)\right] \\ Q_0\left[\frac{m}{N}\right] &= 1 - \left(\frac{m}{N}\right)^2 \\ Q_{+1}\left[\frac{m}{N}\right] &= \frac{1}{2}\left[\left(\frac{m}{N}\right)^2 + \left(\frac{m}{N}\right)\right] \end{aligned}$$

The overall compensated signal will be

$$\hat{r}_k[mT] = \frac{c_k[mT]}{\hat{c}_k[mT]}u_k[mT] + \frac{z_k[mT]}{\hat{c}_k[mT]} \quad (2.31)$$

When the channel is not in deep fades, compensation is satisfied. During deep fades, $|c_k[mT]|$ is very small, thus enhancing the added noise. This results in higher error rate.

In summary, digital data transmitted through a flat Rayleigh, slow fading channel is scaled and rotated by the channel. With the use of pilot symbols, the compensator estimate the channel complex gain which is then used to normalize the overall channel distortion.

Figure 2.7 Block diagram of channel gain estimator

2.5.3 Compensation for frequency selective fading channel

When the channel is frequency selective, the same technique can be used provided the channel delay spread is not large. The overall performance of the compensator will degrade because of the presence of ISI in the received data stream.

Using the channel model given in Section 2.2, the received signal is

$$r(t) = \alpha_1 e^{j\theta_1} u(t) + \alpha_2 e^{j\theta_2} u(t - \tau) + z(t) \quad (2.32)$$

where

α_1 = gain of first path.

α_2 = gain of second path.

θ_1 = phase of first path.

θ_2 = phase of second path.

The compensation procedure is the same as in flat Rayleigh case, however, the estimated complex channel gain is now also a function of the delay spread τ . That is, the compensated output will be expressed as

$$\frac{r(t)}{\hat{c}(t)} = \frac{\alpha_1 e^{j\theta_1}}{\hat{c}(t)} u(t) + \frac{\alpha_2 e^{j\theta_2} u(t - \tau)}{\hat{c}(t)} + \frac{z(t)}{\hat{c}(t)}, 0 \leq t \leq T \quad (2.33)$$

or in sample domain as

$$\frac{r[mT]}{\hat{c}[mT]} = \frac{\alpha_1 e^{j\theta_1}}{\hat{c}[mT]} u[mT] + \frac{\alpha_2 e^{j\theta_2} u[mT - \tau]}{\hat{c}[mT]} + \frac{z[mT]}{\hat{c}[mT]} \quad (2.34)$$

where the estimated sampled channel complex gain $\hat{c}(t)$ is given in (2.30).

2.5.4 Other Methods

There are other methods to counteract the fading problems in mobile communication channel. These will now be briefly mentioned.

2.5.4.1 Transparent Tone-In-Band

Transparent Tone-in-Band or TTIB refers to a modulation technique whereby a pilot tone is transmitted along with data information to allow channel estimation and compensation at the receiver. The pilot tone is transmitted inside the bandwidth of the baseband signal.

In order to avoid corruption of data information, a notch is first created within the signal bandwidth. This is obtained by splitting the signal spectrum into two bands then inserting a gap (notch) in between. This notch is where the pilot tone is to be transmitted. The result of the pilot tone insertion is a slightly larger signal bandwidth. However, the frequency information of the original signal remains intact.

At the receiver, the pilot is removed and used for channel compensation. The notch in the received baseband spectrum is also removed to obtain the original baseband spectrum. The insertion and removal of pilot tone are completely transparent to the user of the channel. This technique is thus called Transparent Tone-in-Band [18].

There are many ways in implementing TTIB. In any case, the main problem in using such technique is the extraction of pilot tone because it must be done with a fairly narrow band filter. Moreover, the pilot signal is also changed by the doppler spread of the channel. This means that the pilot extraction filter must have band width large enough to accommodate the distorted pilot but small enough to avoid excessive noise. Fixed pilot filter bandwidth can be used but an adaptive filtering method can be employed to enhance the performance [19].

2.5.4.2 Equalization

A more effective method in dealing with multipath fading is the use of channel equalization. In this approach, the fading channel is modelled as transversal filter with adaptively adjusted coefficients (or tap weights.) There are two widely used criteria for optimizing the tap weights, namely minimization of peak distortion and mean square error (MSE)[15].

For minimal peak distortion, the equalizer must have a transfer function that is an inverse of the channel transversal filter. If this condition is met and there are an infinite number of taps then the fading induced intersymbol interference can be forced to zero. This is also called a zero-forcing filter, a member of linear equalizer family.

The discrepancy between transmitted and demodulated information is referred to as estimation error. In minimizing MSE, the tap weights are adjusted in a way such that the mean square estimation error is minimized. Such technique makes use of the steepest descent (or gradient) algorithm.

There are many forms of equalizer which can be categorized into T-spaced, frac-

tionally spaced, and decision feedback equalizers (DFE.) Using T-spaced zero forcing filters on M-ary QAM systems, it was observed [20, 9, 21] that equalized systems have better signatures and an associated decrease in SER compared to unequalized systems.

Decision feedback equalizers are also used on 16-, 64-, and 256-QAM systems [9, 22], from which it was concluded that DFE performs better than linear equalizers for low complexity levels. Also, the performance of DFE can be improved with more precise circuit implementation.

2.5.4.3 Diversity

Another effective method for counteracting multipath fading is to utilize the redundancy characteristics of multipath signal. The nature of multipath characteristics allows reception of the same signal at different locations or at the same location but different directions. The format results in space diversity, and the latter, angle diversity. Other methods of achieving diversity is to transmit the same information on different carriers, resulting in frequency diversity, or in different time slot to obtain time diversity.

Regardless of the diversity technique being used, the received signal in each diversity branch must be combined in some optimal way to achieve better communication. The combining methods can be grouped into four main categories which are now briefly described.

- **Selection Diversity:** In this technique, the branch with the highest received baseband SNR is selected. This is also known as the maximum SNR method.
- **Maximal Ratio Combining:** Also known as maximum output method. With this, individual received signal is cophased, weighted and summed together to produce the highest amplitude signal output.
- **Equal Gain Combining:** This is similar to maximal ratio combining except all weights are equal.
- **Feedback Diversity:** also known as switch diversity. It is similar to selection diversity but the selected branch is used until its SNR falls below some threshold. When this occurs, the branch with highest baseband SNR is selected.

These diversity techniques are better described by Jakes [17]

Historically, selection diversity is usually the approach taken in performance analysis. However, in practical implementation, maximal ratio combining is the preferred

technique [6]. Using such technique, Glance and Greenstein [23] pointed out that in frequency selective fading environment,

- Higher diversity order results in better performance.
- Larger delay spread raises the SER floor. For delay spread greater than 30% of the symbol time ($\tau \geq 0.3$), communication link become unsatisfactory.
- The shape of the delay spread spectrum has negligible impact on SER when delay spread is small.
- For system using raised cosine filter, the higher the rolloff factor, the better the SER.
- Combiner imperfections such as effects of time varying channel and errors in data estimation can cause the SER curve to be shifted up by a few dB.

Even though diversity techniques can enhance the SER performance of systems, each diversity system does have some costly disadvantages. For frequency diversity, it is the inefficient use of the radio spectrum. Each diversity branch also requires a separate transmitting antenna. Multiple receiving antennas are required for space diversity. When time diversity is used, the throughput of the channel becomes the critical design criterion.

2.5.4.4 Error Correction Coding

All of the above techniques emphasize on ways of affecting the overall channel characteristics. Coding offers yet another way of compensating multipath fading by building into the transmitted data enough redundancy to allow for error correction at the receiver. There are many forms of error control coding techniques which can be categorized into linear block codes and convolutional codes [24].

Coding is usually working together with a specific modulation scheme to provide optimal coding gain. For Rayleigh fading channel, an efficient coding method was developed for multi tone frequency shift keying (MTFSK) [10]. In contrast, very little efforts have been given to M-ary QAM systems. This stems from the fact that until recently, M-ary QAM was considered an unattractive modulation method in multipath fading channels [8].

Equalization, diversity, and error correction coding are important but not alternate choices for this research.

2.6 Review of Current Research

The thesis was initiated with a literature survey into the research of transmission of M-ary QAM, in particular 16-QAM, in digital mobile communication channels. The result of this survey can be grouped under different types of techniques used in combatting the effects of fading channel.

2.6.1 Pilot Symbol Assisted Systems

Sampei and Sunaga [12] proposed the use of pilot symbol assisted modulation for 16-QAM systems in digital land mobile radio channels. Based on the assumption that the channel is flat Rayleigh fading and that the receiver is perfectly synchronized to the transmitter, they used a second order gaussian interpolator to estimate the channel distortion from three consecutive received pilot symbols. This estimate was then used to remove the fading effects from the received signal. The final output was decoded by a matched filter.

By way of simulations, the effects of doppler spread, carrier frequency and phase jittering, frequency of pilot symbols, and the order of the estimator were studied. With no description of the simulated system, various SER curves were presented. An experiment using a 16-QAM modem was also conducted to verify the usefulness of the compensation technique for transmission of 16-QAM in digital land mobile radio channels.

J. Cavers [13] offers a thorough analysis of the SER of PSA-16QAM in flat Rayleigh fading channels. The method of fading compensation used in this analysis is very similar to that used by Sampei. The only difference between the two approaches is the distortion estimation filter. While the analysis provides the closed form expressions for the statistics of the decision variable which are required for the calculation of the SER, these expressions rely on the complete knowledge of the channel statistics which cannot be obtained in practice.

2.6.2 Pilot Tone Systems

The use of TTIB-based fading correction techniques was implemented by Martin, et al. [25] for 16-QAM mobile data system. An existing TTIB analog speech modem (GEC 2620) was converted to transmit 16-QAM data. The testing facility also included

a fading simulator. The channel distortion was estimated using the received pilot tone. It was found from tests at high signal strength that TTIB can provide very good synchronization. The SER of less than 10^{-4} was also observed. While the authors claimed to have obtained satisfactory results, no information was given on the type of channel being used in the system. There was no SER curve for the fading case, thus performance comparison with other systems cannot be carried out.

Yoshimasa, et al. [5] constructed yet another pilot tone system for M-ary QAM. In this system, a pilot carrier was injected into the data stream by appropriately biasing the inphase channel. Effectively, this method uses a feedback balance coding (FBC) encoder to create a notch in the signal spectrum where the pilot tone is to be injected. At the receiver, a FBC decoder is used to extract the pilot tone. Using such system, it was claimed that FBC is an effective method in extracting the pilot carrier, however, no elaboration on the SER performance was given.

2.6.3 Conclusion

The above literature survey indicates that the problem of SER estimation for transmission of 16QAM signals in frequency selective fading channels has not been studied. While M-ary QAM has been considered a desirable modulation method for digital land mobile communications, its SER analysis was limited to only the flat Rayleigh fading channels.

Chapter 3

SER SIMULATION of PSA-16QAM SYSTEMS

The error rate of a communication link depends on many factors. The most important factor is the nature of the channel itself. Not less important is the structure of the receiver. The system under study employs a method of compensating the fading distortion imposed on the transmitted signal by the channel. This method relies on the sampled received pilot symbols to estimate the fading gain and to carry out the compensation. Thus, the way in which the received signal is sampled can have a direct bearing on the resultant SER. In this chapter, two sampling methods, namely baud rate sampling and a peak—tracking sampling techniques are discussed.

The chapter begins with the system description where the functionality of each simulated hardware block is described. Related system parameters are treated in a separate section so that their references can easily be made. The simulation procedure is next discussed to emphasize the simulation objectives. Finally the simulation results are given in graphical formats.

3.1 System Description

The communication system under study is to be simulated using BOSS (Block Oriented System Simulator.) This simulator is selected because it is available and also there is a large collection of basic system modules that can be used to implement other communication systems. The built-in post processor provides a very good environment for system debugging and graphical results generation.

There are, however, some disadvantages in using BOSS. The execution of simulation programs requires external calls to library functions. The use of excessive external calls can cause long execution time, especially in a shared, multiprocess environment.

Figure 3.1 Baud rate sampling system configuration

Figure 3.2 Peak-tracking sampling system configuration

In spite of the above, extensive simulation was carried out to study the error performance of 16-QAM system in mobile communication channels. There are two systems used in the SER simulation. The first system uses baud rate sampling. The sampling clock is provided by the transmitter. The second system employs a peak detector to generate the sampling clock. These simulated communication systems are given in Figure 3.1 and 3.2.

The understanding of these systems can be best achieved by examining the functionality of each system module. Those modules specifically used in the fading compensation will be described in greater details than other modules.

3.1.1 Frame Synchronizer and Sampling Clocks Generator

This module provides synchronizing signal to control the pilot insertion and sampling clocks at the receiver. The sampling clocks include data symbol and pilot symbol clocks. The data clock is T-spaced whereas the pilot symbol clock is frame spaced. These clocks are affected by the channel delay and the selected sampling instance parameters. This module also provides the symbol number in each frame which is used by the Rayleigh Compensator to calculate the channel fading estimates.

3.1.2 16-QAM Source

The framing of data and QAM modulation is performed in this module. The pseudo-random binary sequence generator supplies a constant stream of binary data to the 16-QAM modulator. Section 2.3.1 discusses QAM modulation in more details. However, at the reception of the frame synchronizing pulse, the four bits corresponding to the pilot symbol are inserted to this binary sequence. Data framing is illustrated in Figure 3.3.

3.1.3 Raised Cosine Filter

This spectral shaping filter is used to achieve minimal intersymbol interference. It can be used as a square-root raised cosine with or without a cascade 1/sinc filter. In this particular system, the filter at the transmitter is square-root with 1/sinc cascaded to equalize the effect of the square nature of the transmitted pulse. At the receiver, just a square-root raised-cosine filter is used. The roll-off factor, which controls the excess

Figure 3.3 Pilot Symbol Assisted data framing

bandwidth, is one of the parameters that can be set at run time.

3.1.4 Fading Channel

The multipath fading channel given by Equ. (2.19) is emulated in this module. It consists of two independent Rayleigh fading branches, each is modelled after Jake's sum of nine sinusoids Rayleigh fader. Two rays are used because the relative delay between the rays results in additional ISI which characterizes the effects of frequency selective fading channels.

In one branch, the input signal is multiplied by the first fading signal. In the second branch, the input is delayed and attenuated before being multiplied by the second fading signal. The delay between these two paths is referred to as the *relative delay* and can be thought of as the *delay spread* of the channel. In addition, channel fading rate can also be controlled by specifying the maximum doppler frequency of the receiver. The higher the value chosen for the maximum doppler frequency, the higher the fading rate.

3.1.5 Rayleigh Compensator

This module extracts the pilot symbols from the received data stream to form an estimate of channel fading distortion. This distortion is used in the fading compensation of the sampled data. Both the extraction of pilot symbols and data symbols are achieved by selecting only one sample for each symbol received. As a result, the technique of sampling the received signal for fading compensation can largely influence the resulting SER. For the current simulation, the sampling instant t_s is set at

$$t_s = \frac{T + \tau}{2} \quad (3.1)$$

where

T = symbol length

τ = relative delay

This sampling instant was observed to produce the best average SER when the two rays have equal power. For more details regarding the implementation of the compensator, please refer to Figure 2.6.

3.1.6 Decision Maker

The compensator output is next coherently detected by the use of a QAM symbol splicer, referred to as a Decision Maker. It accepts a reference decision grid given by the following six thresholds

$$\begin{aligned}
 Re\{\} &= 0 \\
 Im\{\} &= 0 \\
 Re\{\} &= 2 \\
 Im\{\} &= 2 \\
 Re\{\} &= -2 \\
 Im\{\} &= -2
 \end{aligned}
 \tag{3.2}$$

to base its demodulation. The six thresholds divide the signal space into 16 decision cells as shown in Figure 3.4, with each cell corresponds to a source symbol. The demodulation is carried out by identifying in which decision cell the received, compensated symbol is located. Once the decision cell is identified, the corresponding source symbol is taken as the transmitted symbol.

3.1.7 QAM Error Counter

This module just counts the number of times the two appropriately delayed input symbol streams are different. The counting is enabled at the time the first detected symbol is output from the matched filter. This error count, together with the total number of transmitted symbols are used to generate the SER values. In particular, the error rate is given as

$$SER = \frac{\text{Number of symbol errors}}{\text{Number of transmitted symbols}}
 \tag{3.3}$$

Noting that this is the expression for the relative frequency of symbol error, however, to improve the accuracy of the obtained SER values, enough symbols are transmitted so that at least 100 errors are encountered for each case.

3.1.8 Sampling Clock Generator

In the second system, the receiver must derive its sampling clock from the received data. As explained in Section 2.4, the sampling clock can be generated from the

Figure 3.4 Example of decision cells. Dotted lines are decision thresholds

envelope's peaks. These peaks are determined by the transmitted sequence and that not every transmitted symbol results in a peak in the received envelope. Therefore, sampling clock pulses must be inserted during these peak missing periods. The number of clock pulses to be inserted as well as the insertion locations belong to a broader problem of symbol synchronization. Since this lies beyond the thesis objectives, a simpler method of sampling clock generation is considered.

Instead of always sampling at the peaks of the received envelope, the sampling clock generated is dependent on the location of the first legitimate peak. This peak is then taken to be approximately in the middle of the first received symbol.

Let the time of first peak occurrence be t_o , then the beginning of each received symbol is given by

$$t_n = t_o + \frac{(2n - 1)}{2} T \quad , \quad n = 1, 2, 3, \dots \quad (3.4)$$

where

T = symbol period

This symbol boundaries marking scheme does not provide the alignment of the actual received symbols. It does, however, offer a method of maintaining the integrity of received frame synchronization. In light of this, centering in each symbol period is an observation window of length less than T . If an envelope peak is found in this window then a sampling clock pulse is generated at the peak position. If there is no peak or more than one peak is found then a sampling clock pulse is generated at the center of the symbol. Please see Figure 3.5 for the generation of first clock pulse.

This module generates the sampling clock as well as the sampled received signal. Since only one sample is taken in each symbol period, the output stream is regarded as the received symbol stream. From Figure 3.5, the extreme distances between any two consecutive clock pulses are

Closest distance = $2t_d$

Farthest distance = $2(T - t_d)$

Figure 3.5 Reference for sampling clock generation

Since

$$0 < t_d < \frac{T}{2} \quad (3.5)$$

any value of t_d will result in irregular clock pulse separation. This calls for some method to regulate the output of these samples.

3.1.9 Rate Regulator

The above sampling technique results in a stream of unequal length symbols. That is, the output of the sampler appears to have a varying rate. These symbols must be regulated to have equal length so that the compensator can correctly extract data and pilot symbols.

A rate regulation algorithm was developed whereby the received symbol stream is further sampled at 4 times the baud rate. These samples are then stored in a shift register. Since the longest unregulated sample is less than $2T$ seconds, only samples taken within a $2T$ seconds time interval are kept in the shift register.

By counting the number of received signal sampling clock pulses and the number of expected received symbols, it is possible to select the correct sample from the shift register for each output symbol period. For more details about the design and implementation of the rate regulator, please consult Appendix A.

3.2 Parameter Description

In this section, all parameters which affect the outcomes of the simulation will be discussed. These are the factors controlling the SER of the channel and therefore must be kept in mind when reviewing the system performance.

3.2.1 Fading Rate

In mobile communications, fading rate plays an important role in affecting the SER of the channel. Most often, error occurs during deep fades where the effective SNR is relatively low. The more frequently the mobile unit encounters these deep fades, the higher will be the SER. As mentioned earlier, this fading rate is directly proportional to the mobile's speed and the carrier frequency. In particular, it depends on the doppler

frequency which is given by

$$f_d = \frac{v_m}{\lambda} = \frac{v_m f}{c} \quad (3.6)$$

where

v_m = mobile speed

λ = carrier wave length

f_c = carrier frequency

c = light velocity

For systems having carrier frequency below 1GHz, the doppler frequency of a mobile moving within the legal speed limits will be less than 90Hz.

Fading rate can also be specified by the doppler spread of the channel. This is the ratio of the maximum doppler frequency to the symbol rate of the transmitted data. This is a more commonly used parameter. For slow fading channel, this parameter is less than 1%. Figure 3.6 shows an example of a fading envelope.

3.2.2 Delay Spread

The amount of intersymbol interference in the received data of a frequency selective fading channel can be controlled by the channel delay spread. As with doppler spread, delay spread can be specified as percentage of the symbol period. For satisfactory communication under frequency selective fading, the delay spread cannot be more than 30% of the symbol period [23].

3.2.3 Rayleigh Variance

The fading signal generated by Jakes' sum of nine sinusoids has a Rayleigh distributed envelope and a uniform phase. It is required that the fading envelope must have unit median. The output from Jakes' model does not have unit median therefore it must be normalized before being used. The normalization factor for this was found to be $\frac{1}{\sqrt{1.5}}$ [17].

Figure 3.6 Example of a fading envelope for $f_d T = 0.005$

3.2.4 Noise Power

The power of added gaussian noise is also important in calibrating the E_b/N_o values. It was noticed that the noise variance of the complex white noise module, supplied by BOSS, does not correspond with $N_o/2$. In particular, the received noise power when the system uses raised cosine filters is found to be

$$\text{Noise Power} = \frac{2\sigma_n^2}{\text{Number of samples per symbol}} \quad (3.7)$$

where

$$\sigma_n^2 = \text{input noise variance}$$

3.2.5 Calibration of Average Received Signal to Noise Ratio

The received signal to noise ratio per bit (E_b/N_o) is important in presenting the SER results. Its calibration can be derived from Equ. 3.7 as follows.

$$\begin{aligned} E_b/N_o \equiv \gamma_b &= 10\log_{10}\left(\frac{P_{av}}{\text{noise power}}\right) \\ &= 10\log_{10}\left(\frac{N_s P_{av}}{8\sigma_n^2}\right) \end{aligned} \quad (3.8)$$

where

P_{av} = average received power per symbol

N_s = number of samples per received symbol

σ_n^2 = BOSS module's parameter equivalent to the variance of the added noise

$$\therefore \sigma_n^2 = \frac{N_s P_{av}}{8(10)^{\frac{\gamma_b}{10}}} \quad (3.9)$$

3.2.6 Roll-off Factor

This parameter control the bandwidth of both transmitter and receiver filters. Ideally this factor should not affect the SER at all because the use of raised cosine filter will ensure no ISI. However, unless sampling at the zero ISI instant is achieved, system

performance will depend, in some degree, on the value of this roll-off factor. In particular, there is a monotonic improvement in the SER as roll-off factor is monotonically increased [23].

Even though a value of 0.35 is usually chosen for mobile communication channel, it was set to 1.0 in all of the ensuing simulations. With this roll-off factor, the impulse response of the combined raised-cosine filter is depicted in Figure 3.7.

3.2.7 Frame Length

In PSAM, the transmission of pilot symbols helps the receiver estimate the channel fading parameters. A more frequent transmission of pilot symbols allows a more accurate estimation. However, pilot symbols do not carry any user information, hence reducing the throughput of the channel. It was suggested [12] that for 16Kbauds channel, 16 symbols per frame is the optimum value. This is more meaningful only when the fading rate is also specified.

It is reasonable to suggest that for slower fading channels, longer frames can be used. Within a particular channel, the range of optimal frame length must then be investigated. The work involved in this thesis does not include such objective, however, to show the effects of frame length on SER of the channel, transmission of 2- to 16-symbol frames were studied.

3.2.8 Sampling Instant

This parameter is important in minimizing the amount of ISI in the sampled received data. For single ray (flat fading) channel, it is possible to sample at the zero ISI instant. However, for selective fading channel, no such instant exists and therefore optimal sampling instant can be a major research topic.

This optimal sampling instant was determined by running short simulations for different sampling moments and then comparing the SER values. By doing this, the sampling moment is optimal in terms of producing the best average SER.

It was also noticed that in this system configuration, the optimal sampling instant fell between samples even if single ray channel is used and therefore ISI cannot be avoided. In order to minimize the ISI for the case of flat Rayleigh fading channel, a simple interpolation method is used. In this method, the pilot symbols are sampled at the

Figure 3.7 Impulse response of raised cosine filter

nearest possible instants on either side of the optimal point. The value of the received pilot is taken as the average of these two samples. The performance of this method was examined by transmitting pilot assisted 16-QAM data through a non fading channel and monitoring the received pilot symbols.

It can be concluded that from Figure 3.8 that the averaged sampled pilot signal is closer to its expected value of 3.0 than samples taken just before or just after the optimal sampling instant.

For frequency selective fading channel, this simple method can no longer be satisfactory because it will enhance the ISI due to delay spread. Instead, the sampling rate is increased to give more samples per symbol. This proved to be a more effective method of reducing the ISI compared to the averaging technique.

3.3 Simulation Procedure

It is desirable to examine how the SER is affected under various fading conditions. To this end, the simulations are organized into two parts. Part one deals with flat Rayleigh channel in which the effects of fading rate and frame length on SER are studied. Part two considers frequency selective fading with similar objectives in addition to monitoring the effects of delay spread on the SER performance of the system.

The number of symbols transmitted over the channel is large enough to ensure there are at least 100 errors have occurred. The parameters to be varied in the simulations are listed below.

$$\begin{aligned}\tau &= \{0, 2, 4, 8\} \text{ sample periods} \\ f_d &= \{10, 20, 40, 80\} \text{ Hz} \\ N &= \{4\} \text{ symbols} \\ \gamma_b &= \{20, 25, 30, 35, 40\} \text{ dB}\end{aligned}\tag{3.10}$$

where

τ = delay spread

f_d = maximum doppler frequency

N = frame length

γ_b = average received SNR per bit (E_b/N_0) in dB

[a]

[b]

[c]

Figure 3.8 Sampled Pilot in AWGN channels: a) Before optimal point, b) After optimal point, c) Averaged value

These parameters remain constant during the execution of each simulation. To obtain the worst fading condition in frequency selective channel, the gain on the second path is set equal to that of the first path (i.e. $\alpha_1 = \alpha_2$.)

The simulation proceedings can be summarized in the following steps:

- Measurement of the average received power per symbol. This is done for all doppler frequencies of interest.
- Calibration of Eb/No ratios. This involves the calculation of the input noise variance so that a specific Eb/No is attained at the receiver.
- Verification of system operation. This is to ensure the simulated system is operating as expected. The importance of this step cannot be overlooked because all delay values expected by the simulation program must be correctly set to avoid non system related error. The word 'system' here is used to denote the communication system.
- Obtainment of the optimal sampling moments. This is accomplished by running simulations for each delay spread at 30 dB SNR while the sampling instant is varied. The sampling instant that results in the best average SER is selected to be optimal.
- Simulation of steady state average SER for all doppler frequencies and delay spreads mentioned above.
- Verification of obtained results. The obtained results are compared to other related published results to verify the findings. If there are any reservations in the results, the simulated system must be modified then the simulations repeated.
- Preparation of results for final presentation.

3.4 Simulation Preparation

In this section, all the preparation works prior to the actual simulation of SER are given. To begin, the average power per received symbol for different doppler frequencies and different transmitted sequence lengths are measured and tabulated. These average power values are used in the calibration of the Eb/No ratios. It is noticed that for data sequence of 100,000 symbols, the average received power begins approaching its steady state value. Therefore, for longer sequences, only the steady state value of average received power is used.

From these average power values, together with the expression for the received noise power given in Equ. 3.9, provides a set of noise variances for each simulation

[a]

Maximum Doppler Frequency (Hz)	Average Received Symbol Power (Watts) for Selected Sequence Length (symbols)			
	10,000	20,000	50,000	100,000
10	11.49	9.26	8.21	7.48
20	9.23	8.57	7.47	7.34
40	8.53	7.36	7.31	7.31
80	7.37	7.62	7.38	7.31

[b]

Maximum Doppler Frequency (Hz)	Average Received Symbol Power (Watts) for Selected Sequence Length (symbols)			
	10,000	20,000	50,000	100,000
10	12.67	11.51	11.01	9.11
20	11.51	10.57	9.11	8.85
40	10.57	9.06	8.85	8.85
80	9.06	9.24	8.92	8.85

Table 3.1 Average received power per symbol for a) flat fading channel b) frequency selective fading channel

trial. These are given in Tables 3.2 and 3.3. Even though the average received power does not depend on the doppler spread, the measured values indicated that there is a small dependency. This is the result of unplanned long term fading produced by Jake's Rayleigh fader model.

The sequence lengths given in these tables are for flat Rayleigh fading channels. Shorter sequences can be used in the case of frequency selective fading channels. The length of each transmitted sequence is selected such that the resultant SER is observed to converge to some steady state value. This is only performed for the case of $f_d = 10\text{Hz}$, others can appropriately be inferred. All f_d values are given with respect to a data rate of 16Ksymbols/s.

In the SER simulations, however, enough symbols are transmitted so that at least 100 symbol errors are obtained before the average SER is accepted. As a verification measure, a confident interval is obtained for $f_d = 40\text{Hz}$ and $\tau = 12.5\%T$. Figure 3.9 shows that the one standard deviation is approximately 16% from the mean value. This is a relatively small interval for error rates of less than 10^{-2} .

Figure 3.9 SER spread

Maximum Doppler Frequency (Hz)	γ_b (dB)	Sequence Length (symbols)	Noise variance
10	20	10,000	4.5960e-1
	25	20,000	1.1713e-1
	30	50,000	3.2840e-2
	35	100,000	9.4615e-3
	40	500,000	2.9240e-3
20	20	10,000	3.9620e-1
	25	20,000	1.0840e-1
	30	50,000	2.9880e-2
	35	100,000	9.2844e-3
	40	500,000	2.9240e-3
40	20	10,000	3.4120e-1
	25	20,000	9.3097e-2
	30	50,000	2.9240e-2
	35	50,000	9.2465e-3
	40	300,000	2.9240e-3
80	20	10,000	2.9248e-1
	25	20,000	9.6386e-2
	30	50,000	2.9520e-2
	35	50,000	9.3350e-3
	40	200,000	2.9240e-3

Table 3.2 Noise variance for flat fading simulations

Maximum Doppler Frequency (Hz)	γ_b (dB)	Sequence Length (symbols)	Noise variance
10	20	10,000	5.0680e-1
	25	20,000	1.4559e-1
	30	50,000	4.4040e-2
	35	100,000	1.1523e-2
	40	500,000	3.5680e-3
20	20	10,000	4.6040e-1
	25	20,000	1.3370e-1
	30	50,000	3.6440e-2
	35	100,000	1.1194e-2
	40	500,000	3.5680e-3
40	20	10,000	4.2280e-1
	25	20,000	1.1460e-1
	30	50,000	3.5400e-2
	35	100,000	1.1194e-2
	40	300,000	3.5400e-3
80	20	10,000	3.6240e-1
	25	20,000	1.1688e-1
	30	50,000	3.5680e-2
	35	100,000	1.1283e-2
	40	200,000	3.5680e-3

Table 3.3 Noise variance for frequency selective fading simulations

3.5 Statistical Characteristics of Channel and Data

Before the results of simulated SER are given, it is instructive to present the statistical characteristics of the channel and its data. Following are a series of graphs showing the spectra, eye diagrams, and scatter diagrams of various signals along the communication link.

The distribution of transmitted data is offset from the expected equally likely source (Figure 3.10) because fixed pilot is used. This offset is ignored in this work.

The plots of received signal phase and envelopes pdf's (Figures 3.11 and 3.12) show that Jake's sum of nine sinusoid produces better estimates of Rayleigh fading signals for higher doppler frequency.

Figure 3.13 illustrates the spectra of the unfiltered transmitted 16_QAM signal and the filtered received faded signal.

Figure 3.14 shows that the eye of the received signal is completely closed even for very small doppler frequency. That is, the channel is useless without some form of fading compensation.

Figure 3.10 pdf of transmitted data

[a]

[b]

Figure 3.11 pdf of a) received envelope, b) received phase for $f_d = 10\text{Hz}$

[a]

[b]

Figure 3.12 pdf of a) received envelope, b) received phase for $f_0 = 80\text{Hz}$

[a]

[b]

Figure 3.13 Spectrum of a) 16-QAM signal, b) received signal

[a]

[b]

[c]

Figure 3.14 Eye diagram of a) transmitted signal, b) received signal, c) compensated signal¹ for $f_d = 10\text{Hz}$, $\tau = 0.25T$

¹ The compensated signal uses only one sample per symbol, resulting a flat eye diagram.

3.6 Simulation Results

The following are the set of SER vs E_b/N_0 curves obtained for both flat and frequency selective fading channels using two different sampling techniques. The performance of these two systems is first verified by comparing the SER results in flat Rayleigh fading channels with that obtained by Sampei [12] and Cavers [13]. Figure 3.15 shows matching performance among these systems for E_b/N_0 less than 35dB. At higher E_b/N_0 values, the simulated systems start to show a tendency to approach some irreducible error rates. This performance difference at high SNR indicates that the SER figures for high SNR given by Sampei and Cavers are only extrapolated values! Despite this discrepancy, the similarity in performance at lower SNR proves that the two simulated systems are reliable in predicting SER for fading channels.

For flat Rayleigh fading channels, there is not much difference in performance among different doppler spreads. This is because the channel are sampled frequently enough to allow good tracking of the channel fading gains at the compensator. Also the absence of ISI caused by frequency selective fading relieves the receiver from its constant effort to achieve symbol synchronization. In other words, once symbol synchronization is achieved, the receiver will remain in synchronism with the received data for the rest of the transmission duration. This, in turn, allows the compensator to retain its optimal performance throughout the reception period.

In frequency selective fading channels, a very noticeable characteristic of SER performance is the presence of irreducible error rates at any delay and doppler spread. The higher the delay or doppler spread, the higher the error floor. Delay spread, however, has a dominant effect on the irreducible error rates. This is because higher delay spread leads to more ISI in the received data stream. Increasing ISI causes increasing eye closure, thereby reducing the error margin at the receiver. While higher doppler spread can adversely affect the SER, its effects can be reduced by transmitting pilot symbols more often (i.e. smaller frames.) As an example, Figure 3.16 shows the SER of a very bad channel as a function of pilot symbol rate. Clearly, the more often pilot symbols are transmitted, the better the SER performance. This figure also indicates that for frame length of more than 6 symbols, there is essentially no changes in the SER performance. This is because with these frame lengths, the channel is not sampled often enough to

[a]

[b]

Figure 3.15 Simulated SER for flat Rayleigh fading channel. a) T-spaced sampling, b) peak-tracking sampling. Frame length = 4 symbols

allow the fading estimator to track the channel. As a result, the SER performance will approach that of an uncompensated system.

Figures 3.17 to 3.20 show the SER vs E_b/N_0 as a function of delay spread. In contrast, Figures 3.21 to 3.23 show the same but with doppler spread as the plotting parameter. These figures show that for $\tau \leq 12.5\% T$, and $f_d < 80\text{Hz}$, PSAQAM results in $\text{SER} < 2 \times 10^{-2}$ or equivalently, $\text{BER} < 5 \times 10^{-3}$ at $\gamma_b \geq 25\text{dB}$.

In an effort to observe the effect of the relative power in the two Rayleigh rays on the performance, the SER is simulated for different relative power ratios. The result of this, shows that (Fig. 3.24) the SER is very sensitive to the relative power unless the difference is less than 5dB.

Figure 3.16 Simulated SER of FSF channel with $f_d = 40$ Hz, $\tau = 12.5\%T$ as a function of frame length

[a]

[b]

Figure 3.17 Simulated SER of FSF channel with $f_d = 10\text{Hz}$.

a) T-spaced sampling, b) peak-tracking sampling. Frame length = 4 symbols.

[a]

[b]

Figure 3.18 Simulated SER of FSF channel with $f_d = 20$ Hz.

a) T-spaced sampling, b) peak-tracking sampling. Frame length = 4 symbols

[a]

[b]

Figure 3.19 Simulated SER of FSF channel with $f_d = 40\text{Hz}$.

a) T-spaced sampling, b) peak-tracking sampling. Frame length = 4 symbols.

[a]

[b]

Figure 3.20 Simulated SER of FSF channel with $f_d = 80\text{Hz}$.

a) T-spaced sampling, b) peak-tracking sampling. Frame length = 4 symbols.

[a]

[b]

Figure 3.21 Simulated SER of FSF channel with $\tau = 6.25\% T$.

a) T-spaced sampling, b) peak—tracking sampling. Frame length = 4 symbols.

[a]

[b]

Figure 3.22 Simulated SER of FSF channel with $\tau = 12.5\%$ T.

a) T-spaced sampling, b) peak-tracking sampling. Frame length = 4 symbols.

[a]

[b]

Figure 3.23 Simulated SER of FSF channel with $\tau = 25\% T$.

a) T-spaced sampling, b) peak-tracking sampling. Frame length = 4 symbols.

Figure 3.24 Simulated SER of FSF channel with $f_d = 80\text{Hz}$, $\tau = 25\%T$, $\text{SNR} = 40\text{dB}$ as a function of relative power ratio between the two branches.

3.6.1 Summary

In summary, the performance of the simulated system in flat Rayleigh fading channels is very comparable with those obtained by Sampei and Cavers. In frequency selective fading channels, the relative delay between the two paths has a dominant effect on the SER regardless of the doppler spread. With the relative delay of less than $12.5\%T$, the SER performance is satisfactory (error floor is in the 10^{-3} range.)

For frame length of less than 6 symbols, the SER performance shows monotonic improvement as the frame length is reduced. In a more realistic situation where the power level in each fading ray is different, the SER is better. The SER shows continued improvement as the power difference (in dB) between the two rays is increased. When this power difference exceeds 20dB, the SER performance approaches that of the flat fading case.

Chapter 4

SEMI-ANALYTICAL CALCULATION of LOWER SER BOUNDS

The SER obtained in the previous chapter depends directly on the simulation parameters. In this chapter, an analysis on the simulated system is carried out to gain a better understanding of the SER performance of such system. From this analysis, a new method of predicting SER is proposed. Under this new technique, the SER can be obtained much faster, thereby proven useful compared to the former simulation method.

The chapter begins with the analysis of the simulated system. It then proceeds with the SER analysis and the description of the new method of predicting the SER. Finally, the SER obtained under the new method is compared with results from system simulation.

4.1 System Analysis

The communication system used in the simulation is depicted in Figure 4.1. From this diagram, the received signal is given by

$$\begin{aligned} r(t) &= u(t) * h_t(t) * c(t; \tau) * h_r(t) + n(t) * h_r(t) \\ &= u(t) * h_t(t) * h_r(t) * c(t; \tau) + n(t) * h_r(t) \\ &= h(t) * c(t, \tau) + n_f(t) \end{aligned} \quad (4.1)$$

where

$$u(t) = \begin{cases} 1 & , 0 \leq t \leq T \\ 0 & , \text{otherwise} \end{cases}$$

T = symbol period

$h_t(t)$ = square-root raised-cosine, cascaded with $\frac{1}{\sin}$ filter

$h_r(t)$ = square-root raised-cosine filter

$c(t; \tau)$ = channel impulse response

$n(t)$ = complex white noise

Figure 4.1 Stochastic model of simulated system

where

$$\begin{aligned} h'(t) &= \frac{d}{dt}[h(t)] \\ h''(t) &= \frac{d^2}{dt^2}[h(t)] \end{aligned}$$

Rewriting $r(t)$ using Equ. 4.7 yields

$$\begin{aligned} r(t) &= [\alpha_1 e^{j\theta_1} + \alpha_2 e^{j\theta_2}] \sum_{k=n-L}^{n+L} I_k h(t-kT) \\ &+ \sum_{k=n-L}^{n+L} I_k \alpha_2 e^{j\theta_2} \left[-\tau h'(t-kT) + \frac{\tau^2}{2!} h''(t-kT) \right] + n_f(t) \\ &+ \alpha e^{j\theta} I_n h(t-nT) + \alpha e^{j\theta} \sum_{\substack{k=n-L \\ k \neq n}}^{n+L} I_k h(t-kT) \\ &+ \alpha_2 e^{j\theta_2} \sum_{k=n-L}^{n+L} I_k \left[-\tau h'(t-kT) + \frac{\tau^2}{2!} h''(t-kT) \right] + n_f(t) \end{aligned} \quad (4.8)$$

where

$$\alpha e^{j\theta} = \alpha_1 e^{j\theta_1} + \alpha_2 e^{j\theta_2}$$

Using the same approach in predicting the channel complex gains as described in Section 2.5.2 but further assume that perfect channel complex gain prediction can be achieved, the received, fading compensated signal has the following characteristic

$$\begin{aligned} r(t) &= I_n h(t-nT) + \sum_{\substack{k=n-L \\ k \neq n}}^{n+L} I_k h(t-kT) \\ &+ \frac{\alpha_2 e^{j\theta_2}}{\alpha e^{j\theta}} \sum_{k=n-L}^{n+L} I_k \left[-\tau h'(t-kT) + \frac{\tau^2}{2!} h''(t-kT) \right] \\ &+ \frac{n_f(t)}{\alpha e^{j\theta}} \\ &I_n h(t-nT) + X_I(t) + X_C(t) + \frac{n_f(t)}{\alpha e^{j\theta}} \quad , (n-1)T + \tau \leq t \leq nT \end{aligned} \quad (4.9)$$

where

$$X_I(t) = \sum_{k=n-L}^{n+L} I_k h(t - nT)$$

$$X_c(t) = \frac{\alpha e^{j\omega_c t}}{\alpha e^{j\omega_c t}} \sum_{k=n-L}^{n+L} I_k \left[-\tau h'(t - kT) + \frac{\tau^2}{2} h''(t - kT) \right]$$

In the above expression, the first term contains the desired symbol. The second term represents the ISI due to transceiver band limiting. The third term depicts the ISI due to channel dispersion. Finally, the last term shows the filtered noise after fading compensation.

If all the ISI factors and the compensated noise are lumped together into a single term, denoted as equivalent added noise $n_{eq}(t)$, that is,

$$\hat{r}(t) \equiv I_n h(t - nT) + n_{eq}(t) \quad (4.10)$$

the SER of the channel can be analyzed provided the statistics of $n_{eq}(t)$ are known. That is, the error rate can be analytically analyzed as in the case of additive gaussian noise, the only difference is that this added noise will have a different distribution. While this approach is novel, an analytical expression for the distribution of $n_{eq}(t)$ cannot be easily obtained.

One of the practical solutions in overcoming this difficulty is to fix the channel and data sequence. As a result, the compensated signal becomes a stochastic process that is directly related to the additive gaussian noise process. A decision variable U can be formed by sampling the compensated signal at some instant during each symbol period. The sampling instants are derived from the peak positions of the envelope of $\hat{r}(t)$. To this end, the pdf of U , conditioned on a fixed channel and data sequence, can be obtained.

Based on this pdf, the conditional SER is calculated. The predicted SER is obtained by averaging the conditional SER over all transmitted data sequences and across all channels being used.

Since the estimation error is not considered, this technique provides only a lower bound on the SER performance.

4.2 SER Analysis

The heart of this SER prediction is the pdf of the decision variable. Thus it is logical to first derive this pdf. To begin, recall that filtered noise is a narrow band noise, which can be represented as

$$n_f(t) = n_I(t) + j n_Q(t) \quad (4.11)$$

The statistics of this noise can be summarized as follows

$$\mu_{n_I} = \mu_{n_Q} = 0 \quad (4.12)$$

$$\sigma_{n_I}^2 = \sigma_{n_Q}^2 = \sigma_n^2 = \sigma^2 \quad (4.13)$$

and that the two quadrature components are uncorrelated.

For notational convenience, the subscript f is dropped from the filtered noise. References to additive complex gaussian noise will be made explicit.

In light of the above, for each data sequence and a particular realization of the channel, the compensated signal, sampled at some optimal instant, is a complex gaussian random variable which can be represented as

$$\hat{R} = \hat{R}_I + j \hat{R}_Q \quad (4.14)$$

where

$$\hat{R}_I = \text{Re} \{ \hat{r}(t) \}$$

$$\hat{R}_Q = \text{Im} \{ \hat{r}(t) \}$$

t denotes the optimal sampling instant.

This complex gaussian characteristic can be shown by examining the statistics of the inphase and quadrature components. These statistics are given as follows

Mean of inphase component is

$$\begin{aligned}
 \mu_I &= E[\hat{R}_I] \\
 &= E\left[Re\left\{I_n h(\hat{t} - nT) + X_t(t) + X_c(t) e^{-j\frac{n(t)}{\alpha}}\right\}\right] \\
 &= Re\{I_n h(\hat{t} - nT) + X_t(\hat{t}) + X_c(t)\} + E\left[Re\left\{\frac{n(t)}{\alpha} e^{-j\theta}\right\}\right] \\
 &= Re\{I_n h(\hat{t} - nT) + X_t(t) + X_c(t)\} \\
 &\quad + \frac{1}{\alpha} E[Re\{(n_I(t) + j n_Q(t))(\cos\theta - j \sin\theta)\}] \\
 &= Re\{I_n h(\hat{t} - nT) + X_t(\hat{t}) + X_c(t)\} + \frac{1}{\alpha} E[\cos\theta n_I(t) + \sin\theta n_Q(t)] \\
 &= Re\{I_n h(\hat{t} - nT) + X_t(\hat{t}) + X_c(t)\} + \frac{\cos\theta}{\alpha} \mu_{n_I} + \frac{\sin\theta}{\alpha} \mu_{n_Q} \\
 &= Re\{I_n h(\hat{t} - nT) + X_t(t) + X_c(\hat{t})\}
 \end{aligned} \tag{4.15}$$

since $\mu_{n_I} = \mu_{n_Q} = 0$ (4.12)

Similarly, mean of quadrature component is

$$\mu_Q = Im\{I_n h(\hat{t} - nT) + X_t(t) + X_c(t)\} \tag{4.16}$$

Variance of inphase component is

$$\begin{aligned}
 \sigma_I^2 &= E\left[\left(R_I - \mu_I\right)^2\right] \\
 &= E\left[\left(\frac{1}{\alpha}(\cos\theta n_I(\hat{t}) + \sin\theta n_Q(\hat{t}))\right)^2\right] \\
 &= \frac{1}{\alpha^2} \{ \cos^2\theta E[n_I^2(\hat{t})] + \sin^2\theta E[n_Q^2(\hat{t})] + \sin 2\theta E[n_I(\hat{t})n_Q(\hat{t})] \} \\
 &= \frac{1}{\alpha^2} \{ \sigma^2 \cos^2\theta + \sigma^2 \sin^2\theta \}, \\
 &= \left(\frac{\sigma}{\alpha}\right)^2
 \end{aligned} \tag{4.17}$$

since $\sigma_{n_I}^2 = \sigma_{n_Q}^2 = \sigma_n^2 = \sigma^2$ (4.13)

Similarly, variance of quadrature component is

$$\sigma_Q^2 = \left(\frac{\sigma}{\alpha}\right)^2 \tag{4.18}$$

where

σ^2 = variance of added complex gaussian noise

The covariance matrix of R_I and R_Q is given by

$$K = \begin{bmatrix} \sigma_I^2 & \rho_{IQ} \\ \rho_{QI} & \sigma_Q^2 \end{bmatrix} \quad (4.19)$$

where

$$\begin{aligned} \sigma_I^2 &= \sigma_Q^2 = \left(\frac{\sigma}{\alpha}\right)^2 = \sigma_r^2 \\ \rho_{IQ} &= E \left[(R_I - \mu_I)(R_Q - \mu_Q)^* \right] \\ &= E \left[\left(\frac{Re\{n(t)\}}{\alpha e^{j\theta}} \right) \left(\frac{Im\{n(t)\}}{\alpha e^{j\theta}} \right)^* \right] \\ &= \frac{1}{\alpha^2} E \{ Re\{n(t)\} Im\{n(t)\} \} \\ &= 0 \end{aligned} \quad (4.20)$$

Similarly,

$$\begin{aligned} \rho_{QI} &= \frac{1}{\alpha^2} E \{ Im\{n(t)\} Re\{n(t)\} \} \\ &= 0 \end{aligned} \quad (4.21)$$

Thus, R_I and R_Q are independent gaussian with their joint probability distribution, conditioned on a particular channel realization and a specific data sequence, given by

$$P_{R_I R_Q | \text{channel data}}(r_I, r_Q) = \frac{1}{\sqrt{2\pi\sigma_r^2}} e^{-j\frac{(r_I - \mu_I)^2}{2\sigma_r^2}} \frac{1}{\sqrt{2\pi\sigma_r^2}} e^{-j\frac{(r_Q - \mu_Q)^2}{2\sigma_r^2}} \quad (4.22)$$

With this probability distribution, the probability of making a correct decision, conditioned on a given channel and data sequence is given by

$$\begin{aligned} P_c | \text{channel, data} &= \int_{I_L}^{I_u} \frac{1}{\sqrt{2\pi\sigma_r^2}} e^{-j\frac{(r_I - \mu_I)^2}{2\sigma_r^2}} dr_I \int_{Q_L}^{Q_u} \frac{1}{\sqrt{2\pi\sigma_r^2}} e^{-j\frac{(r_Q - \mu_Q)^2}{2\sigma_r^2}} dr_Q \\ &= \frac{1}{\sqrt{\pi}} \int_{\frac{I_L - \mu_I}{\sqrt{2\sigma_r^2}}}^{\frac{I_u - \mu_I}{\sqrt{2\sigma_r^2}}} e^{-u^2} du \frac{1}{\sqrt{\pi}} \int_{\frac{Q_L - \mu_Q}{\sqrt{2\sigma_r^2}}}^{\frac{Q_u - \mu_Q}{\sqrt{2\sigma_r^2}}} e^{-v^2} dv \\ &= \frac{1}{4} \operatorname{erf} \left(\frac{I_L - \mu_I}{\sqrt{2\sigma_r^2}}, \frac{I_u - \mu_I}{\sqrt{2\sigma_r^2}} \right) \operatorname{erf} \left(\frac{Q_L - \mu_Q}{\sqrt{2\sigma_r^2}}, \frac{Q_u - \mu_Q}{\sqrt{2\sigma_r^2}} \right) \end{aligned} \quad (4.23)$$

where

I_l = lower edge of the inphase decision cell

I_u = upper edge of the inphase decision cell

Q_l = lower edge of the quadrature decision cell

Q_u = upper edge of the quadrature decision cell

$$\text{erf}(a, b) = \frac{2}{\sqrt{\pi}} \int_a^b e^{-t^2} dt \quad (4.24)$$

Thus, the conditional probability of making an error can be expressed as

$$\begin{aligned} P_e | \text{channel data} &= 1 - P_c | \text{channel data} \\ &= 1 - \frac{1}{4} \text{erf} \left(\frac{I_l - \mu_I}{\sqrt{2\sigma_I^2}}, \frac{I_u - \mu_I}{\sqrt{2\sigma_I^2}} \right) \text{erf} \left(\frac{Q_l - \mu_Q}{\sqrt{2\sigma_Q^2}}, \frac{Q_u - \mu_Q}{\sqrt{2\sigma_Q^2}} \right) \end{aligned} \quad (4.25)$$

Based on Equ. (4.25), the SER can be obtained by averaging over all possible data sequences and all channel being used. An algorithm for calculating SER must be developed and implemented. It is proposed that MATLAB is to be used in the implementation of such algorithm.

4.3 SER Calculation

4.3.1 Calculation Algorithm

The calculation of SER can be carried out in a straight forward manner. Since the ISI is assume to be contributed by the adjacent symbols, only 3-symbol sequences are used in the calculation. The average SER is obtained by transmitting all possible combinations of the 3-symbol sequences (48 in all) in each of the selected 1000 channels. It is clearer to present the algorithm in a flow chart format as depicted in Figure (4.2). For more details on the implementation of this algorithm, please consult Appendix C.

4.3.2 Channel selection method

In order to be compatible with BOSS simulations, Jake's model of a fading signal generator is employed. Unlike BOSS, a fading gain can be selected by specifying a

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Figure 4.2 SER calculation algorithm

particular time instant. That is, a particular fixed channel is selected by supplying to this fading signal generator a time instant. Therefore care must be taken so that the ensemble of the selected channels have a similar statistics as the continuous channel.

The employed technique of channel selection is to uniformly sample the fading signal within a fixed time interval [500,10500] seconds. As depicted in Figure 4.3, the resulting selected channels have a very similar ensemble statistics as the continuous channel.

[a]

[b]

**Figure 4.3 Histogram of a) BOSS fading channel,
b) MATLAB selected fading channels for $f_d = 80\text{Hz}$**

4.4 Calculation Results

In this section, the calculated SER results, in the form of SER vs E_b/N_0 curves, are presented. The performance of the algorithm is first verified by examining the SER for 16-QAM in AWGN channel. Figure 4.4 shows that the calculated SER is very similar to that given in Proakis [15]. This proves that the algorithm is accurate in predicting error rates.

For flat Rayleigh fading channels, the error rate curves at any doppler frequency has the same shape as AWGN error curve (Fig. 4.5.) This is because an ideal channel fading gain estimator is used. These results also show a general trend of having higher error rate for higher doppler spread. This is to be expected since higher doppler spread results in more frequent encounter of channel deep fades. Even though the gain estimator is ideal, during these deep fades, the effective received average SNR is drastically reduced. In an effort to compensate for the low signal level caused by the deep fade, the compensator applies a large gain to the received signal. As a result, the additive noise is also amplified by the same factor as the desired signal component. Since the effective SNR is already low, the compensated noise may effectively move the received signal point out of its corresponding decision cell. As a consequence, the receiver will more than likely make an erroneous decision as to which symbol was transmitted.

For frequency selective fading channels, there exhibit irreducible error rates at different values of E_b/N_0 , depending on the amount of channel delay spread as well as doppler spread of the receiver. The irreducible error rate is most visible when the delay spread is more than 10% of the symbol period and the maximum doppler frequency f_d is more than 10Hz (vehicular speed of more than 12Km/Hr for 900MHz carrier.) Moreover, the irreducible error rates tend to be in the 10^{-3} range. The error rate curves for FSF channels with delay spread as plotting parameter are given in Figures 4.6 to 4.9.

The presence of irreducible error rate at higher delay and doppler spread is caused by eye closure of the received signal and the higher probability of encountering a deep fade.

The calculated SER values, when compared to the simulated results, show that they provide a set of quite loose lower bounds.

Figure 4.4 SER of calculated AWGN channel

Figure 4.5 Calculated lower SER bound for flat Rayleigh fading channel

Figure 4.6 Calculated lower SER bound for FSF channel with $f_d = 10\text{Hz}$

Figure 4.7 Calculated lower SER bound for FSF channel with $f_d = 20\text{Hz}$

Figure 4.8 Calculated lower SER bound for FSF channel with $f_d = 40\text{Hz}$

Figure 4.9 Calculated lower SER bound for FSF channel with $f_d = 80\text{Hz}$

4.5.1 SER Comparison

In this section the simulated SER results are compared to the calculated ones as well as those obtained by others for similar communication systems. It is more instructive to itemize these comparisons in separate subsections.

4.5.1.1 SER Comparison for Flat Fading Channels

The performance of both systems under flat Rayleigh fading are very similar. Moreover, Figure 4.10 shows that they are in strong agreement with those obtained by Sampei [12] and Cavers [13]. Only towards high E_b/N_0 values ($> 40\text{dB}$), the simulated SER results begin to show a tendency to level off. This gives a strong indication that extrapolation was employed in these published works. However, for lower E_b/N_0 range, it shows that the SER values obtained by these simulations are highly reliable. The use of this system, therefore, can be extended to estimate the error rates of PSA-16QAM in frequency selective fading channels.

4.5.1.2 SER Comparison for Frequency Selective Fading Channels

The performance of T-spaced sampling system is comparable or better than peak-tracking sampling system for moderately large delay spread. It is observed that for delay spread of more than 10% of the symbol period, both systems encounter irreducible error rates at quite low an E_b/N_0 ratio. According to these findings, a baud rate clock, started at an appropriate time instant, can be used as a more effective sampling clock as that obtained by a more elaborate symbol timing loop. For this reason, only the results of T-spaced sampling system will be used in comparing with others.

While research in performance and performance improvement of 16-QAM systems were carried out by many [26, 27, 11, 28, 20], no explicit symbol error rate was given. For this reason, the calculated SER lower bounds must be used as benchmark. Comparisons of the simulated results with these lower bounds show that there is much room for improvement (Figure 4.11). Since the calculated SER was based on the assumption that perfect channel estimation is achieved, it is reasonable to conclude that SER performance can be improved with the use of better symbol synchronization and fading gain estimation techniques.

It is also more instructive to relate the SER performance of PSA-16QAM with the actual radio mobile channel which uses 48Kbits/s QPSK modulation. If the same bit rate

[a]

[b]

Figure 4.10 SER of simulated flat Rayleigh fading channel. a) T-spaced sampling, b) peak-tracking sampling

is to be used with PSA-16QAM, the effective symbol rate will be 12Ksymbols/s. Since all results presented are for 16Ksymbols/s, the equivalent performance can be obtained by multiplying f_d and τ by a scaling factor. This scaling factor is the ratio of

$$\frac{T}{T'} = \frac{16}{12} = 0.75 \quad 4.26$$

where

T = Symbol period in the simulation

T' = Symbol period for mobile radio

Thus, for example, the simulated performance at $f_d = 80\text{Hz}$ is equivalent to the mobile radio channel performance at $f_d = 60\text{Hz}$.

1.5.2 Summary

The absolute SER lower bounds for PSA-16QAM are obtained by averaging the SER results for all permutations of 3-symbol sequences are transmitted through 1000 different channels. These results show the same general trend (demonstrated in simulated results) of increasing SER for increasing relative delay and higher dopper spread. The differences between these bounds and the simulated results suggest that much improvement in the error performance can still be possible. These improvement can be realized by investigating the employment of a better symbol synchronization and fading estimate methods.

[a]

[b]

Figure 4.11 Comparison of simulated SER and the calculated lower bound . a) $f_d = 10\text{Hz}$, b) $f_d = 80\text{Hz}$

Chapter 5

CONCLUSION

Motivated by the desire to use more bandwidth efficient modulation techniques with the least order of system complexity, this research was set out to estimate the average Symbol Error Rate (SER) of a Pilot Symbol Assisted 16QAM (PSA-16QAM) system under the effects of frequency selective fading.

Under the operation of this system, the channel fading gains are periodically measured by transmitting known symbols (pilot symbols) at regular intervals. At the receiver, three consecutive fading measurements are used to predict (interpolate) the fading gains during the transmission of other data symbols.

The SER estimation was achieved by three different techniques. The first two techniques use BOSS (Block Oriented System Simulator). The last is semi-analytical which uses Matlab to calculate the lower error bounds. The results obtained by these methods are compared among each other as well as with other published findings to arrive at the following conclusions:

- PSAM works well for flat Rayleigh fading channels.
- In frequency selective fading channels, PSAM can offer satisfactory performance only if the fading is slow and the delay spread is not in excess of 12.5% of the symbol period.
- The SER is very sensitive to the amount of delay spread, regardless of the doppler spread. The larger the delay spread, the higher the error floor.
- For low delay spread ($\tau < 25\%T$), the SER is quite sensitive to the doppler spread. The higher the doppler spread, the higher the error floor.
- The shorter the frame length (faster pilot symbol rate or higher overhead), the better the SER performance. The improvement in performance is substantial only for frame lengths of less than 4 symbols.
- The larger the difference in the relative power of the two fading paths, the better

the SER performance. For relative power of less than 3dB, no further improvement can be achieved.

- Most of the error floors are contained within the 10^{-3} range for $\tau \leq 12.5\% T$ (Fig. 3.16.) This means that the Bit Error Rate is within the acceptant range (10^{-3} to 10^{-6}) [28].
- For $\tau > 12.5\% T$, the error floors are found within the 10^{-2} range.
- The calculated SER values are used to verify the validity of the simulated result (in the sense of absolute lower bounds.) This is because the fading compensator in these calculations is idealized. This is required since no equivalent SER performance was found in the available literatures.
- The performance of the peak-tracking sampling system is inferior compared to that of the T-spaced sampling system.

In light of the above, PSA 16QAM can be used in frequency selective fading environment only if the data rate is reduced such that the maximum delay spread will not exceed 12.5% of the symbol period. Better performance can still be obtained by increasing the amount of overhead.

These results also strongly emphasize the necessity for use of other more complicated techniques to combat fading effects should the bandwidth efficiency of 16-QAM be fully realized.

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Appendix A

Rate Regulator

In sampling the received signal according to its envelope's peaks, the resulting sampled stream contains unequal length symbols. It is desirable to equalize these symbols for further operation.

This rate regulator operates with the following specifications:

- No symbol is longer than twice the symbol period (T).
- Symbol slipping never exceeds $2T$. That is, the desired symbol never lies more than $2T$ from its expected position.

Conceptually, this regulator partitions the time axis into slots of length T , starting at the moment the first symbol is entered. This is indicated by the first pulse of the sampling clock. It keeps track of the number of actual symbols received to date as well as the number of expected received symbols. It also measures how long each symbol lasts. These three parameters can be used to decide where to sample the input symbol stream.

Figure A.1 Rate regulator design parameters specification

With reference to Figure A.1, consider the following definitions.

Received symbol count: Number of actual received symbols, obtained by counting the number of sampling clock pulses up to the current time instant.

Expected symbol count: Number of expected received symbols, obtained by counting the number of symbol periods has gone by since the first sampling clock pulse is detected.

t_{ni} : the time instant at the beginning of the expected n^{th} symbol.

t_{nf} : The time instant at the end of the expected n^{th} symbol.

Expected receive window: Time interval between t_{ni} and t_{nf} . This is the interval where the symbol indexed by the expected symbol count should occupy.

NRI: Received symbol count, sampled at t_{ni} .

NRF: Received symbol count, sampled at t_{nf} .

NE: Expected symbol count, sampled at t_{nf} .

SI: Time lapse between t_{ni} and the previous peak.

SF: Time lapse between t_{nf} and previous peak.

Having defined the above variables, let's consider the six general cases where the desired symbol is positioned with respect to the expected receive window. These cases are outlined in Figure A.2.

In all cases, the incoming symbol stream is sampled at 4 times the baud rate. That is, there is a sample at every $T/4$ seconds. Since the longest symbol is less than $2T$, the shift register needs be long enough to have 8 samples.

To make looking ahead in the received symbol stream possible, the input as well as all counter variables are delayed by $2T$ before feeding into the decision maker. At the end of each expected receive window, NRI, NRF, SI, and SF are latched into a decision maker to decide where the desired symbol lies and which of the stored samples is to be output. The decision for each case can be summarized as follows.

Case 1:

$$NE = NRI$$

$$NE < NRF$$

Output sample just before T-SF

Case 2:

Figure A.2 Different cases of received symbol position

$$NE = NRI$$

$$NE = NRF$$

Output sample at $T/2$

Case 3:

$$NE > NRI$$

$$NE = NRF$$

Output sample just after T-SF

Case 4:

$$NE > NRI$$

$$NE \leq NRF$$

Output sample just before T-SE

Case 5:

$$NE > NRI$$

$$NE < NRF$$

Output sample just after T-SF

Case 6:

$$NE > NRI$$

$$NE > NRF$$

Output sample at $T + T/2$

Figure A.3 shows the block diagram of the rate regulator. Details of other important blocks can be found in Appendix B. An example of the regulated output is given in Figure A.4 where the regulated stream is overlaid on the input symbol stream with appropriate time shift. It is also given an amplitude offset of 15 units to clearly show the effects of symbol length regulation on the incoming symbol stream.

Figure A.3 Block diagram of rate regulator

Figure A.4 Example of regulated output

Appendix B

Block Diagrams of Supporting System Modules

All block diagrams included in this appendix are BOSS system modules designed and implemented by the author. These are intended to show how the supplied BOSS primitive modules can be used to implement other systems.

Figure B.1 T-spaced sampling system

Figure B.2 Peak-tracking sampling system

Figure B.3 Pilot symbol assisted 16-QAM source

Figure B.5 Two ray channel

Figure B.6 Rayleigh fader

Figure B.7 Rayleigh compensator

Figure B.9 Peak detector

Figure B.10 Sampling clock generator

Figure B.11 Clock generator used in rate regulator

Figure B.12 Control signal generator used in rate regulator

Figure B.1.3 Sample select decision maker used in rate regulator

Figure B.14 1-of-8 demultiplexor

Appendix C

Matlab Source Listing

These are the program codes and supporting data for all the functions used in matlab to calculate the SER of the channel.

h.m

```

% function to evaluate h(t) = raised cosine pulse.
% Assume T = 1;

function out = h(BETA,t)

num = sin(pi*t) * cos(BETA*pi*t);
den = pi*t*(1 - (2*BETA*t)^2);

if t == 0,
    num = 1;
    den = 1;
elseif (den == 0) && (num ~= 1e-16),
    num = cos(pi*t)*cos(BETA*pi*t) - BETA*sin(pi*t)*sin(BETA*pi*t);
    den = 1 - 12*(BETA*t)^2;
end
out = num/den;

```

hl.m

```

% function to evaluate the first derivative of h.
function out = hl(BETA,t)

if (abs(t) <= 0.49) & (abs(t) <= 0.51),
    t = 0.4999* sign(t) ;
end

A = pi*t;
B = BETA*A;
C = BETA*t;

num = pi*A*(1-4*C^2)/(cos(A)*cos(B)-BETA*sin(A)*sin(B));
num = num - pi*(1-12*C^2)*sin(A)*cos(B);
den = A^2*(1-4*C^2)^2;
if t == 0,
    num = 0;
    den = 1;
end
out = num/den;

```

h2.m

```

% function to evaluate second derivative of h(t)
% Assume T = 1

function out = h2(BETA,t)

if abs(t) <= 0.91,
    t = 0.9999;
end

A = pi*t;
B = BETA*A;
C = BETA*t;

num = pi^2*t*(1 - 4*C^2)*(cos(A)*cos(B) - BETA*sin(A)*sin(B));
num = num - pi*(1-12*C^2)*sin(A)*cos(B);
den = A^2*(1 - 4*C^2)^2;

dnum = (1 + BETA^2)*sin(A)*cos(B) + 2*BETA*cos(A)*sin(B);
dnum = dnum*(pi^2*A*(4*C^2 - 1)) + 24*pi*BETA^2*t*sin(A)*cos(B);

dden = 2*pi^2*t*(1 - 4*C^2)*(1 - 12*C^2);
if (abs(t) <= 0.49) & (abs(t) <= 0.51),
    out = 0;
else
    out = (dnum*den - num*dden)/den^2;
end

```

fader.m

```

% program to generate Rayleigh fading signal
% Based os Jakes sum of 8 sinusoids model.

function [X,Y] = fader(id,fd,t)

for i = 1:8,
    beta(i) = pi*1.8;
    w(i)      = 2*pi*fd*cos(2*pi*i/37);
    offset(i) = pi*(1 + 2*id -2)/9;
end

I = 0.0;
Q = 0.0;

for i = 1:8,
    osc = cos(w(i)*t + offset(i));
    Q = Q + 2*sin(beta(i))*osc;
    I = I + 2*cos(beta(i))*osc;
end
X = 0.24*(I + cos(2*pi*fd*t));
Y = 0.24*(Q + cos(2*pi*fd*t));

```


get_mean.m

```

% Function to calculate the real and imag parts of the mean of
% the decision variable.
%
% Input parameters:
%
%   data      = transmitted data sequence
%   seq_len   = length of transmitted sequence
%   t         = time instant at which the mean is to be calculated
%   scale     = weighing scale for the ISI due to channel dispersion
%   tau       = channel delay spread
%   B         = rolloff factor
%
% Other used functions:
%
%   h(t) = raised-cosine pulse
%   h1(t) = first derivative of h(t)
%   h2(t) = second derivative of h(t)
%
function [mx,my] = get_mean(data,seq_len,t,scale,tau,B)

T = 1.0;
mu = 0.0; % initialize complex mean
for i = 1:seq_len,
    k = (seq_len-1)/2 + i - seq_len;
    mu = mu + data(i)*(h(B,t-k*T) + scale*(tau^2*h2(B,t-k*T) + 2
        - tau*h1(5,t-k*T)));
end

mx = real(mu);
my = imag(mu);
end

```

s.dat

```

1 + i1
3 + i1
1 + i3
3 + i3

```

These are the coordinates of the source alphabets. Only the alphabets in the first quadrant are used.

grids.dat

```

0.0000000e+00  2.0000000e+00  0.0000000e+00  2.0000000e+00
2.0000000e+00      Infinity  0.0000000e+00  2.0000000e+00
0.0000000e+00  2.0000000e+00  2.0000000e+00      Infinity
2.0000000e+00      Infinity  2.0000000e+00      Infinity

```

These are the decision cell boundaries in the order of

[X_{low} X_{high} Y_{low} Y_{high}]

ser.m

```

%*****
% Program to calculate the average SER of the channel
% Written by Edward Pham, June 11, 1992
%
%*****

load s.dat; % load transmitted alphabets %
load qm.dat % load integration thresholds

fprintf('Please enter the following parameters:\n')

fd = input('Maximum Doppler frequency (Hz) : ');
tau = input('Channel delay spread (samples) : ');
gamma = input('Eb/No (dB) : ');
w = input('Monitoring simulation (y/n)? : ','s');

fprintf('\n\nYou have selected the following simulation
        parameters:\n\n');
fprintf('Maximum Doppler frequency of the channel =
        %9.6f Hz',fd);
fprintf('Channel fixed delay spread =
        %9.6f samples',tau);
fprintf('Average received SNR per bit =
        %9.6f dB',gamma);

filename = ['ser_',num2str(fd),'_',num2str(tau),'_',num2str(gamma)];

if fd == 10,
    p_avg = 16.99;
elseif fd == 20,
    p_avg = 15.11;
elseif fd == 40,
    p_avg = 14.51;
else
    p_avg = 11.73;
end

fd = 0.25e-4*(fd/10); % convert to equivalent fd
tau = tau/32;

fprintf('\n\nCalculation of average SER in progress . . .\n');

%***** Main program %*****

```

```

avg_ser = 0.0;
ser      = 0.0;
len = 3; % sequence length
NUMPOINTS = 4; % number of points along each axis
NUMCHAN = 1000; % number of channels used
rand('uniform');
dt = 10; % channel sampling step size
t = 500; % initial sampling moment

if w ~= 'n',
    !~ watch &
else
    !~/matnice &
end

for chan = 1:NUMCHAN,

    t = t + dt; % select channel
    [x1,y1] = fader(1,fd,t); % get fading gain for first path
    [x2,y2] = fader(4,fd,t); % get fading gain for second path
    m2 = sqrt(x2^2 + y2^2); % mag of second fading gain
    p2 = atan(y2/x2); % phase of second fading gain
    m = sqrt((x1 + x2)^2 + (y1 + y2)^2); % mag of composite gain
    p = atan((y1 + y2)/(x1 + x2)); % phase of composite gain

% use all permutations for the selected sequence length.

    for a = 1:NUMPOINTS,
        for b = 1:NUMPOINTS,

            Lx = grids(b,1); % Decision grid
            Ly = grids(b,3);
            Ux = grids(b,2);
            Uy = grids(b,4);

            for c = 1:NUMPOINTS,

                data = [s(a),s(b),s(c)]; % data sequence
                scale = m2*exp(j*p2)/m*exp(j*p); % ISI weight
                t_opt = tau/2;
                [mx,my] = get_mean(data,len,t_opt,scale,tau,1);
                sigma = sqrt(p_avg/(8*10^(gamma/10))); % noise variance
                variance = (sigma/m)^2; % variance of decision variable
                std = sqrt(2*variance);

                x1 = (Lx - mx)/ std; % limits of integration
                x2 = (Ux - mx)/ std;

```

```

        y1 = (Ly - my) / std;
        y2 = (Uy - my) / std;

        error = 1 - erf(x1,x2)*erf(y1,y2)/4;
        ser = ser + error;
    end %c
end %b
end %a
ser = ser/NUMPOINTS*len;
avg_ser = avg_ser + ser;
end %chan

avg_ser = avg_ser / NUMCHAN;

s = ['save ',filename,'.dat avg_ser /ascii'];
eval(s);

```

END

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FIN