Postfix synchronization methods for OFDM and MIMO-OFDM systems

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Abstract

This thesis presents a novel synchronization method for OFDM and Multiple Input Multiple Output (MIMO)-OFDM systems which use preambles containing two identical training parts (Chu sequences) followed by a Flipped Postfix (FIP).

The proposed synchronization method is implemented in four steps: coarse time synchronization, estimation of the Carrier Frequency Offset (CFO) fraction and integer part, and fine time synchronization.

We extend the proposed and conventional Schmidl/Cox (SC) methods to MIMO systems. By using multiple antennas, the spatial diversity improves the accuracy of the symbol timing estimator and CFO estimator.

From the analysis and simulation, the proposed postfix synchronization method has a narrow and large timing metric peak value suitable to specify a threshold and is robust in continuous or burst communication scenarios under different channels with large CFO. It is also robust to the Doppler spread.

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List of Abbreviations

4G	Fourth Generation
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
CDMA	Code Division Multiple Access
CFO	Carrier Frequency Offset
CIR	Channel Impulse Response
CLT	Central Limit Theorem
CP	Cyclic Prefix
DAB	Digital Audio Broadcasting
DFT	Discrete Fourier Transform
DQPSK	Differential Quadrature Phase-Shift Keying
DVB-T	Digital Video Broadcasting
EGC	Equal Gain Combining
FCP	Flipped Cyclic Prefix
FDM	Frequency Division Multiplexing
FlP	Flipped Postfix
ICI	Intercarrier Interference
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Fast Fourier Transform
ISI	Intersymbol Interference

LAN	Local Area Network
MAN	Metropolitan Area Network
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MRC	Maximum Ratio Combining
OFDM	Orthogonal Frequency Division Multiplexing
PAPR	Peak to Average Power Ratio
PN	Pseudo-random Noise
QAM	Quadrature amplitude modulation
QPSK	Quadrature Phase-Shift Keying
RV	Random Variable
RVs	Random Variables
SC	Schmidl and Cox
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
SNR	Signal to Noise Ratio
STBC	Space Time Block Codes
STTC	Space Time Trellis Codes
SVD	Singular Value Decomposition
VCO	Voltage Controlled Oscillator
WSSUS	Wide-Sense-Stationary Uncorrelated Scattering
ZPP	Zero Padding Prefix

List of Symbols

<u>Symbol</u>	<u>Explanation</u>	<u>page where symbol is defined</u>
$\angle$	Argument of a complex number	38
$\mathbf{a}_{q,0}$	Identical part of the first training symbol from the q^{th} transmitter antenna	60
$\mathbf{a}_{q,k}$	Circular right shift k samples version of $\mathbf{a}_{q,0}$	60
$*$	Complex conjugation of a matrix or vector	37
$\mathbf{b}_{1,0}$	Second training symbol of the first transmitter antenna	62
$\mathbf{b}_{1,(q-1)G}$	Circular right shift $(q-1)G$ samples version of $\mathbf{b}_{1,0}$	62
B_d	Doppler spread of the channel	10
c	Speed of light	11
$\otimes$	Convolution operator	21
$\dagger$	Hermitian conjugation of a matrix or a vector	61
δ_f	Carrier frequency offset	28
γ	Carrier frequency offset normalized by the subcarrier spacing .	34
$(\Delta f)_H$	Coherence bandwidth of the channel	10
δ_{il}	Kronecker delta: $\forall i \neq l \delta_{il} = 0$ and $\delta_{ii} = 1$	15
$\delta(t)$	Dirac delta function	9

$(\Delta t)_H$	Coherence time of the channel	10
d_{opt}	Exact symbol start point excluding the prefix and postfix	55
$E\{\cdot\}$	Expectation of the argument	10
ϵ	Symbol time offset	27
f_c	Carrier frequency	11
f_m	Maximum Doppler frequency	11
γ_f	Fraction part of the carrier frequency offset	38
$\alpha_n(t)$	Complex amplitude of the n^{th} path delay	9
$H(f;t)$	Fourier transform of $h(\tau;t)$ respected to τ	10
h_m	Complex baseband channel impulse response at lag m	34
$h_n(t)$	n^{th} tap weight of the tapped delay line channel	14
$h_{ql,m}$	Complex baseband channel impulse response at lag m between the q^{th} transmitter antenna and the l^{th} receiver antenna	35
$h(\tau)$	Complex baseband time invariant channel impulse response ..	21
$h(\tau;t)$	Complex baseband time variant channel impulse response	9
$\Im\{\cdot\}$	Imaginary part of the argument	55
L	Number of the receiver antennas	34
L_p	$L_p = \frac{N}{2}$	49
M	Length of the tapped delay line channel model	14
M_d	$M_d = M_p(d_{opt}) - M_p(d_{opt+1})$	55
$M_l(d)$	Time metric of the proposed method for the l^{th} receiver antenna	63
$M_{MIMO}(d)$	Time metric of the proposed method for MIMO-OFDM systems	63
$M_{Minn}(d)$	Time metric of Minn's method	40
$M_p(d)$	Time metric of the proposed method for SISO-OFDM systems	49
$M_{SC}(d)$	Time metric of SC's method	37

N	Total number of subcarriers	15
G	Number of the cyclic prefix samples	19
n_ϵ	An integer representing the unknown arrival time of a symbol	34
n_k	k^{th} sample of the complex Gaussian noise with zero mean and variance σ_n^2	34
$n_{l,k}$	k^{th} sample of the complex Gaussian noise with zero mean and variance σ_n^2 at the l^{th} receiver antenna	35
$n(t)$	Additive White Gaussian Noise	21
P_d	$P_d = \ P_p(d_{opt})\ ^2 - \ P_p(d_{opt+1})\ ^2$	56
$\Phi_H(\Delta f)$	Fourier transform of $\phi_h(\tau)$	10
$\phi_h(\tau)$	Multipath intensity profile of the channel	10
Q	Number of the transmitter antennas	34
r_k^c	Carrier frequency corrected time domain sample	53
$r_{l,k}^c$	k^{th} carrier frequency corrected time domain sample at the l^{th} receiver antenna	66
R_d	$R_d = R_p^2(d_{opt+1}) - R_p^2(d_{opt})$	56
$\Re\{\cdot\}$	Real part of the argument	16
r_k	k^{th} received sample	34
$r_{l,k}$	k^{th} received sample at the l^{th} receiver antenna	35
$S_H(\lambda)$	Doppler power spectrum of the channel	10
$S_H(\tau; \lambda)$	Scattering function of the channel	11
$S_{q,k}$	Transmitted data on the k^{th} subcarrier from the q^{th} transmitter antenna	35
$s_{q,n}$	n^{th} time domain sample from the q^{th} transmitter antenna	35
τ_{max}	Delay spread or maximum channel delay	21
$\tau_n(t)$	n^{th} path delay	9
τ_{rms}	Root-mean-square delay spread	11

T_{cp}	Duration of the cyclic prefix	18
T_{FFT}	Duration of the OFDM symbol excluding the windowing and CP	18
θ	Carrier phase offset	28
θ_0	Initial phase	34
$\tilde{r}(t)$	Lowpass equivalent form of the received signal	21
$\tilde{s}(t)$	Equivalent lowpass signal	13
$\tilde{S}(f)$	Fourier transform of $\tilde{s}(t)$	13
T_m	Multipath spread of the channel	10
T	Transposition of a matrix or vector	34
T_s	OFDM symbol duration	2
T_{sa}	Sample duration	33
T_{win}	Duration of the windowing (one side)	20
v	Velocity of mobile	11
W	Bandwidth occupied by the real bandpass signal	13
W_s	Signal bandwidth excluding the bandwidth occupied by the windowing and CP	18
X_i	Complex modulated data on the i^{th} subcarrier	33
x_n	n^{th} sample of the time domain symbol	33

Chapter 1

Introduction

1.1 Thesis objectives

Orthogonal frequency division multiplexing (OFDM) is a digital modulation scheme for high data rate wireless transmission. The principle of OFDM is to split a high rate data stream (wide band signal) into a number of subcarriers, i.e. changing a high rate serial data stream into a slow rate parallel data stream. OFDM has been accepted in several wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), the IEEE 802.11a local area network (LAN) standard and the 802.16a metropolitan area network (MAN) standard. It is also a potential candidate for fourth generation (4G) mobile wireless systems.

- Multicarrier vs. Single carrier

A high rate data stream, transmitted serially on a single carrier has a narrow symbol duration, therefore single carrier systems are very sensitive to the delay spread that can cause significant intersymbol interference (ISI) in multipath fading channels. Using multicarrier systems can delimitate the ISI but increases the complexity of the system and sometimes introduces intercarrier interference.

- Orthogonal vs. Nonorthogonal [1]

In the classic way, a guard band is put between each subcarrier in the frequency domain, which makes the frequency use inefficient. To cope with the spectrum inefficiency, an idea proposed from the mid-1960s was to use parallel data and frequency division multiplexing (FDM) with overlapping subchannels. Each subcarrier is orthogonal with others in each symbol duration, i.e. the carrier space is a multiple of $1/T_s$ (excluding the cyclic prefix and windowing), where T_s is the OFDM symbol duration.

- OFDM is robust against narrow band interference [2]

Narrow band interference only affects a small fraction of the subcarriers. We can use powerful coding to correct those errors.

- OFDM can use discontinuous bandwidth for operation

OFDM is easy to deal with separate band resources and suitable for the wide band demand.

The main advantage of OFDM is that it has a low implementation cost when it transmits over a high frequency selective channel, but it suffers from two drawbacks. One is the peak to average power ratio (PAPR) problem due to the coherent summation of the sine waves, the other is sensitivity to synchronization errors [3]. Synchronization errors such as symbol time error and carrier frequency offset (CFO) cause intersymbol interference (ISI) and intercarrier interference (ICI). Therefore, designing a reliable synchronization scheme is very important for OFDM systems. There are several approaches for the OFDM synchronization: detecting the power of the arriving signal, using the cyclic prefix and using the training symbols.

In [4], the power of the arriving signal is used to find the beginning of the OFDM frame. It did not work well in fading channels and did not consider the CFO estima-

tion.

In [5], redundant information contained within the Cyclic Prefix (CP) is used to perform joint maximum likelihood symbol time and CFO estimation but reliability is low in multipath fading channels since the CP is short and corrupted by the previous symbol.

Using the training symbols can much more improve timing estimation but the overhead causes inefficiency. Schmidl and Cox (SC) use a training symbol with two identical halves in the time domain to find the start point of the symbol and the fraction part of the CFO. Then they use a second symbol containing differential modulated Pseudo-random Noise (PN) sequence information to estimate the integer part of the CFO [6]. However, due to the CP, there is an inherent plateau in the timing metric that leads to some uncertainty as to the start of the frame. To avoid this effect, the timing estimator is taken as the average of the left and right points at 90% peak value. SC's method is robust but the timing estimator has a large variance and its mean value is shift to the left by a significant amount. To avoid the occurrence of the plateau and hence some of the uncertainty, various training patterns have been subsequently proposed instead of the two identical halves (e.g. $[A \ A - A - A]$ proposed in [7] by Minn, $[+B + B - B + B]$ proposed in [8]). Methods such as Minn's method work well in the AWGN channel, but the mean value might exhibit shifts from the exact start point due to the multipath effect. Alternatively a constant envelop preamble weighted by a PN sequence that gives an impulse shaped timing metric in AWGN channel similar to that obtained in [9] has been proposed in [10]. But, [9] and [10] methods have three drawbacks. The major disadvantage is that the metric's peak value is quite small compared to SC's metric in ISI or fading channels since it only considers one path correlation and other paths are treated as noise. It is hard to choose a threshold for detection and the probability of false alarm

and missing detection is quite high. Another disadvantage is that in fading or ISI channels, the estimated start point will shift to the right and cause a large ISI when the instantaneous power of first path is not the biggest one among all paths. The third disadvantage is that since its metric cannot be calculated iteratively, the computation complexity is quite high when the number of the subcarriers is large.

Symbol timing estimation is often coupled with the CFO estimation. For example, [6] also provides a method to estimate the CFO, and [8] shows how Minn's training pattern (with repeated parts half the length of SC ones) [7] could be used to estimate the CFO. Since Minn's length of repeated parts of the training symbol is half of the SC's one, [8] shows Minn's training pattern yields little degradation compared to SC's training pattern.

A system which uses multiple antennas both on the transmitter and receiver is called a multiple-input multiple-output (MIMO) system. MIMO systems can be implemented in different ways to obtain either a spatial diversity gain to combat fading channel or a capacity gain in a multipath fading environment. When a MIMO system is designed for a frequency selective fading channel, to reduce the equalization complexity of the MIMO system, combining the MIMO system with OFDM modulation would be a good choice and the system becomes a MIMO-OFDM system [11].

MIMO-OFDM systems are also sensitive to synchronization errors. In [12], the author extended SC's synchronization method from OFDM systems to MIMO-OFDM systems. Due to the spatial diversity, the accuracy of the symbol timing estimator and CFO (fraction part) estimator is improved, but it did not give the method for estimating the integer part of the CFO and the estimated symbol timing shifted to the right. In [13] [14] and [15], the authors describe a synchronization method in the acquisition mode by four steps: coarse time synchronization and signal detection, frequency offset estimation in the time domain, residual frequency offset correction

and fine time synchronization. This method uses several repeated parts with CP to do the synchronization and channel estimation. The preamble is quite efficient, but its residual frequency offset estimator is inaccurate in a frequency selective channel and no theoretic support for setting the threshold in fine time synchronization is given.

This thesis investigates synchronization methods for OFDM and MIMO-OFDM systems. Since synchronization method using the training symbols is more reliable than other methods, we focus on synchronization methods that use training symbols in this thesis. In particular, a novel synchronization technique is proposed by removing the CP and adding a flipped postfix (FLP) in the first training symbol structure to make the timing metric more steep and the timing estimator more accurate. Proposed synchronization methods for OFDM and MIMO-OFDM systems are in four steps: coarse time synchronization, estimation of the fraction part of the CFO, estimation of the integer part of the CFO and fine time synchronization. Using analytical analysis, this thesis also explains how to set up the threshold for the fine time synchronization. Various FLP-based preambles are proposed and performance of the proposed synchronization technique is evaluated over AWGN, ISI and fading channels. For MIMO-OFDM systems, the performance of the proposed synchronization technique using different combining techniques and different number of antennas is evaluated under fading channels.

1.2 Thesis contributions

The main contribution of this thesis is the proposed postfix synchronization method for OFDM and MIMO-OFDM systems. By using FLP-based preambles in different channels, the symbol timing estimator becomes more accurate in term of the mean and variance of the estimator. A list of contributions is as follows:

- Introducing FLP-based preambles (section 4.1.1 and section 4.2.1) to solve the uncertainty of the timing estimators in different channels. When the accuracy of the symbol timing estimator increases, the CFO estimator one also increases.
- Giving an analysis of the FLP-based preambles effect in an AWGN channel (section 4.1.3 and appendix A).
- Describing the fine time synchronization method for the proposed method and SC's method (section 4.1.2). It is shown in chapter 5 that the performance of the timing estimator much improves at low SNR by using the fine time synchronization.
- Analyzing the false alarm and missing detection probability for fine time estimator and giving a theoretical support for setting the threshold (appendix B).
- Extending the proposed method and SC's method from OFDM systems to MIMO-OFDM systems (section 4.2). Due to the spatial diversity, the performance of the symbol timing estimator and CFO estimator is much improved.
- Evaluating, comparing and discussing the performance of the proposed, SC's and Minn's synchronization methods under different channels (chapter 5).

1.3 Thesis organization

- Chapter 2, first, introduces the multipath fading channel and gives a tapped delay line model. Then it presents background on OFDM and MIMO-OFDM systems. Finally, it gives the system models used in this thesis.
- Chapter 3 gives a review of the previous works on OFDM and MIMO-OFDM

systems synchronization and discusses the advantages and disadvantages of different synchronization methods.

- Chapter 4 describes the proposed methods for OFDM and MIMO-OFDM systems. Various FLP-based preambles are proposed for different channels and a theoretic analysis is given for AWGN channels.
- Chapter 5 gives the implementation details for the computer simulations. The proposed, Minn's and SC's methods are compared in terms of the mean and variance of the symbol timing estimator and CFO estimator in different channels by computer simulations. Different combining techniques and the effect of the antennas number are assessed for MIMO-OFDM systems.
- Chapter 6 summarizes the thesis and proposes future works.

Chapter 2

Background and system model

2.1 Multipath fading channel

In a wireless communication environment, radio signals generally propagate according to three mechanisms: reflection, diffraction and scattering [16]. Due to these three mechanisms, plane waves received on the receiver antenna arrive from many different directions with random amplitudes, frequencies and phases. This phenomena is called multipath. Since the wavelength is relatively short, small changes in the location of the transmitter, receiver and scattering object in the environment will cause large changes in the phases of the incident plane wave components. The constructive and destructive addition of plane waves combined with motion results in envelope fading. This phenomenon is known as channel fading.

Multipath-fading results in a doubly dispersive channel that exhibits dispersion in both the time and frequency domains. Time dispersion arises because the multipath components propagate over transmission paths having different lengths and, hence, they reach the receiver antenna with different time delays. Time dispersion causes intersymbol interference (ISI) that can be mitigated by using a time or frequency

domain equalizer in single carrier systems, a RAKE receiver in CDMA systems, or frequency domain equalization in OFDM systems. Channel time variations due to mobility are characterized by Doppler spreading in the frequency domain. Such time variant channels require an adaptive receiver to estimate and track the channel impulse response (CIR).

The complex baseband representation of a wireless channel impulse response can be described by [17]

$$h(\tau; t) = \sum_n \alpha_n(t) \delta(\tau - \tau_n(t)) \quad (2.1)$$

where $\tau_n(t)$ is the n^{th} path delay and $\alpha_n(t)$ is the corresponding complex amplitude. The complex baseband time variant channel impulse response $h(\tau; t)$ is a complex valued Gaussian random process in the t variable. When it is modeled as a zero mean complex valued Gaussian random process, which means none of the subpath is significantly stronger than others, the envelope $|h(\tau; t)|$ at any instant t is Rayleigh distributed and the channel is called a Rayleigh fading channel. When it is modeled as a nonzero mean complex valued Gaussian random process, which means there is one strong component along with the scatter components, the envelope $|h(\tau; t)|$ at any instant t is Ricean distributed and the channel is called a Ricean fading channel. There exist some other channel models, such as Nakagami, Weibull and Suzuki, derived mainly from experiments. This thesis focuses on the Rayleigh fading channel.

2.1.1 Wide-Sense-Stationary with Uncorrelated Scattering (WSSUS) channel

$h(\tau; t)$ is commonly assumed to be wide-sense-stationary with uncorrelated scattering for the path delays. This channel is then referred to as a WSSUS channel. We define

the autocorrelation function of $h(\tau; t)$ as:

$$E\{h^*(\tau_1; t)h(\tau_2; t + \Delta t)\} = \phi_h(\tau_1; \Delta t)\delta(\tau_1 - \tau_2) \quad (2.2)$$

Let $\Delta t = 0$, and we define

$$\phi_h(\tau) \triangleq \phi_h(\tau; 0) = E\{|h(\tau; t)|^2\} \quad (2.3)$$

$\phi_h(\tau)$ is called the multipath intensity profile or the delay power spectrum of the channel. $\phi_h(\tau)$ is the average power delay profile of the channel. The range of the value of τ over which $\phi_h(\tau)$ is essentially nonzero is called the multipath spread of the channel and is denoted as T_m . Let $\Phi_H(\Delta f)$ be the Fourier transform of $\phi_h(\tau)$. The range of the value of Δf over which $\Phi_H(\Delta f)$ is essentially nonzero is called the coherence bandwidth of the channel and is denoted as $(\Delta f)_H$. The relationship of the multipath spread and the coherence bandwidth is $(\Delta f)_H \approx \frac{1}{T_m}$. Let $H(f; t)$ be the Fourier transform of $h(\tau; t)$ with respect to τ . We define the autocorrelation function of $H(f; t)$ to be $\Phi_H(\Delta f; \Delta t)$, we have

$$\Phi_H(\Delta f; \Delta t) = E\{H(f + \Delta f; t + \Delta t)H^*(f; t)\} \quad (2.4)$$

Let $\Delta f = 0$, and we define $\Phi_H(\Delta t) \triangleq \Phi_H(0; \Delta t)$. The range of the value of Δt over which $\Phi_H(\Delta t)$ is essentially nonzero is called the coherence time of the channel and is denoted as $(\Delta t)_H$. Let $S_H(\lambda)$ be the Fourier transform of $\Phi_H(\Delta t)$. The function $S_H(\lambda)$ is the power spectrum that gives the signal intensity as a function of the Doppler frequency λ and it is called the Doppler power spectrum of the channel. The range of the value of λ over which $S_H(\lambda)$ is essentially nonzero is called the Doppler spread of the channel and is denoted as B_d . The relationship of the Doppler spread and the

coherence time is $(\Delta t)_H \approx \frac{1}{B_d}$.

The scattering function $S_H(\tau; \lambda)$ which provides a measure of the power dispersion imposed by the channel over delay τ and Doppler frequency λ is related to (2.4)

$$S_H(\tau; \lambda) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_H(\Delta f; \Delta t) e^{-j2\pi\lambda\Delta t} e^{j2\pi\tau\Delta f} d\Delta f d\Delta t \quad (2.5)$$

Commonly-used Doppler power spectrum: Jakes/Clarke's model

Jakes' model is a widely used model for the Doppler power spread spectrum of a mobile radio channel. In this model, $\Phi_H(\Delta t) = J_0(2\pi f_m \Delta t)$ where $J_0(x)$ is the zero-order Bessel function of the first kind and $f_m = f_c v/c$ is the maximum Doppler frequency, where f_c is the carrier frequency, v is the vehicle speed in meters per second (m/s) and c is the speed of light (3×10^8 m/s). The Doppler power spectrum is:

$$S_H(\lambda) = \int_{-\infty}^{\infty} \Phi_H(\Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t = \begin{cases} \frac{1}{\pi f_m \sqrt{1-(\lambda/f_m)^2}}, & (|\lambda| \leq f_m) \\ 0, & (|\lambda| > f_m) \end{cases} \quad (2.6)$$

Commonly-used multipath intensity profile/power delay profile: exponential model

Exponential model is a widely used model for the power delay profile of a mobile radio channel. In this model,

$$\phi_h(\tau) = \begin{cases} \frac{1}{\tau_{rms}} e^{-\frac{\tau}{\tau_{rms}}}, & \tau \geq 0 \\ 0, & \tau < 0 \end{cases} \quad (2.7)$$

where τ_{rms} is the standard deviation (or root-mean-square value) of the delay and is also equal to the mean of the delay in this model. The frequency correlation function

is

$$\Phi_H(\Delta f) = \frac{1}{2\pi\Delta f\tau_{rms}} \quad (2.8)$$

In practice, a truncated version of (2.7) to a finite range is used.

Multipath delay spread leads to time dispersion and frequency selective fading [18]. Doppler spread leads to frequency dispersion and time selective fading. They are independent with each other.

2.1.2 Fast fading vs. slow fading

The channel can be classified either as a fast fading or slow fading channel when we compare the rate of transmitted baseband signal change with the rate of the channel change [18]. If the channel impulse response changes rapidly within the symbol duration, the channel is called a fast fading channel. That is, the coherence time of the channel is smaller than the symbol duration or the Doppler spread is greater than the bandwidth of the baseband signal. On the contrary, if the channel impulse response changes at a rate much slower than the transmitted baseband signal, the channel is called a slow fading channel. That is, the coherence time of the channel is much greater than the symbol duration or the Doppler spread is much less than the bandwidth of the baseband signal.

2.1.3 Flat fading vs. frequency selective fading

Time dispersion due to multipath makes the transmitted signal under either flat or frequency selective fading scenarios [18]. When a mobile radio channel has a constant gain and linear phase response over a bandwidth which is greater than the transmitted signal bandwidth, the channel is called a flat fading channel. The coherence bandwidth is much greater than the signal bandwidth ($W \ll (\Delta f)_H$). On the contrary,

when a channel has a constant gain and linear phase response over a bandwidth which is smaller than the bandwidth of transmitted signal, the channel is called a frequency selective fading channel. The spectrum of the transmitted signal has a bandwidth which is greater than the coherence bandwidth of the channel ($W > (\Delta f)_H$). A frequency selective channel induces intersymbol interference (ISI). In this thesis, the bandwidth occupied by OFDM signals is greater than the coherence bandwidth of the channel. It undergoes frequency selective fading, but the bandwidth of each subcarrier is much smaller than the coherence bandwidth of the channel. Each subcarrier undergoes flat fading.

2.1.4 A Tapped delay line channel model [17]

A frequency selective slow fading channel can be represented by a tapped delay line channel model. Let W be the bandwidth occupied by the real bandpass signal. Then the bandwidth of the equivalent lowpass signal $\tilde{s}(t)$ is $|f| \leq \frac{1}{2}W$. From the sampling theorem, we have

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} \tilde{s}\left(\frac{n}{W}\right) \frac{\sin[\pi W(t - n/W)]}{\pi W(t - n/W)} \quad (2.9)$$

The Fourier transform of $\tilde{s}(t)$ is

$$\tilde{S}(f) = \begin{cases} \frac{1}{W} \sum_{n=-\infty}^{\infty} \tilde{s}\left(\frac{n}{W}\right) e^{-j2\pi f n/W}, & (|f| \leq \frac{1}{2}W) \\ 0, & (|f| > \frac{1}{2}W) \end{cases} \quad (2.10)$$

The noiseless received signal under a frequency selective channel is

$$\begin{aligned} \tilde{r}(t) &= \int_{-\infty}^{\infty} H(f; t) \tilde{S}(f) e^{j2\pi f t} df \\ \tilde{r}(t) &= \int_{-\infty}^{\infty} H(f; t) \tilde{S}(f) e^{j2\pi f t} df \\ &= \frac{1}{W} \sum_{n=-\infty}^{\infty} \tilde{s}\left(\frac{n}{W}\right) \int_{-\infty}^{\infty} H(f; t) e^{j2\pi f(t - \frac{n}{W})} df \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{W} \sum_{n=-\infty}^{\infty} \tilde{s}\left(\frac{n}{W}\right) h\left(t - \frac{n}{W}; t\right) \\
&= \frac{1}{W} \sum_{n=-\infty}^{\infty} \tilde{s}\left(t - \frac{n}{W}\right) h\left(\frac{n}{W}; t\right)
\end{aligned} \tag{2.11}$$

where $h(\tau; t)$ is the time variant channel impulse response (CIR) and $H(f; t)$ is the transfer function. We define

$$h_n(t) = \frac{1}{W} h\left(\frac{n}{W}; t\right) \tag{2.12}$$

Then we have

$$\tilde{r}(t) = \sum_{n=-\infty}^{\infty} h_n(t) \tilde{s}\left(t - \frac{n}{W}\right) \tag{2.13}$$

From (2.13), the time variant frequency selective channel can be modeled as a tapped delay line with tap space $\frac{1}{W}$ and tap weight coefficients $\{h_n(t)\}$. The lowpass channel impulse response is

$$h(\tau; t) = \sum_{n=-\infty}^{\infty} h_n(t) \delta\left(\tau - \frac{n}{W}\right) \tag{2.14}$$

It achieves a resolution of $\frac{1}{W}$ in the multipath delay profile. In practical scenarios, since the total multipath spread is T_m , we truncate the tapped delay line at $M = \lfloor T_m W \rfloor + 1$ taps and we have

$$\tilde{r}(t) = \sum_{n=1}^M h_n(t) \tilde{s}\left(t - \frac{n}{W}\right) \tag{2.15}$$

In a Rayleigh fading channel, the tap weights $\{h_n(t)\}$ are zero mean complex Gaussian random processes and they are statistically independent. The power delay profile of the channel, (2.3), determines each tap's mean power, or equivalently the variance of the $h_n(t)$. We assume all taps have the same Doppler power spectrum and

we can use the Jakes' model in section 2.1.1 to generate the tap weights.

Alternatively, for each tap $h_n(t)$, if the amplitude of $h_n(t)$ is fixed and the phase of $h_n(t)$ is fixed or uniformly distributed in $[0, 2\pi)$ (Phase does not change in the t variable), then (2.15) can be used to model ISI channels.

2.2 Representation of OFDM signal

OFDM is a multicarrier modulation technique which transmits data in parallel by modulating a set of orthogonal subcarriers. OFDM is attractive because it is achieving a high data rate under a frequency selective fading channel, is easy to equalize in the frequency domain (only a one-tap frequency domain equalizer is required) and allows the water pouring strategy (using adaptive modulation) to increase the overall bandwidth efficiency.

2.2.1 Transmitted OFDM signal (the simplest version)

OFDM is a modulation technique that spreads the data over a large number of carriers which are spaced apart at frequencies $f_n = nW/N$, where n is an integer, W is the total available bandwidth and N is the total number of subcarriers [3]. In the most simple case $W = N/T_s$, where T_s is the symbol duration. With this choice of subcarriers, the subcarriers are mutually orthogonal, since

$$\int_{kT_s}^{(k+1)T_s} e^{j2\pi f_i t} \cdot e^{-j2\pi f_l t} dt = T_s \delta_{il} \quad (2.16)$$

where δ_{il} is Kronecker delta: $\forall i \neq l \delta_{il} = 0$ and $\delta_{il} = 1$.

The modulation at this stage normally uses QPSK, DQPSK, QAM or any other modulation scheme. Let the complex transmit symbol at time instant k on the i^{th} subcarrier be $X_{i,k}$. Consider QPSK, $X_{i,k} = a_{i,k} + jb_{i,k}$, with $a_{i,k}, b_{i,k} = \pm 1$, with

$-N/2 \leq i \leq N/2$ and $i \neq 0$ (DC-subcarrier can be distorted by the electronics and therefore is usually not used). For simplicity of notation in this thesis, we use $0 \leq i \leq N-1$ instead of $-N/2 \leq i \leq N/2$ and $i \neq 0$.

The OFDM signal is

$$s(t) = \Re \{ \tilde{s}(t) e^{j2\pi f_c t} \} \quad (2.17)$$

where f_c is the center frequency of the occupied frequency spectrum and $\tilde{s}(t)$ is the complex envelope of the OFDM signal given by

$$\tilde{s}(t) = \sum_{k=-\infty}^{\infty} \tilde{s}_k(t) = \sum_{k=-\infty}^{\infty} \sum_{i=0}^{N-1} X_{i,k} g_i(t - kT_s) \quad (2.18)$$

where the basis pulse $g_i(t)$ is a frequency-shifted rectangular pulse

$$g_i(t) = \begin{cases} e^{j2\pi f_i t} = e^{j2\pi i t/T_s} & 0 \leq t \leq T_s \\ 0 & \text{otherwise} \end{cases} \quad (2.19)$$

2.2.2 OFDM digital implementation

We can use N oscillators to generate N subcarriers, then use N filters in the receiver to get the data. When N is large, the bandwidth of the subcarrier might be very narrow. It is hard and expensive to build N precise oscillators and filters. In 1971, Weinstein, S. and Ebert, P. published the paper [19] which introduces using the Discrete Fourier Transform (DFT) to implement the frequency division multiplexing (FDM). The DFT transforms a cyclic time domain signal into its equivalent frequency spectrum. This is done by finding the equivalent waveform generated by a sum of orthogonal sinusoidal components. The Inverse Discrete Fourier Transform (IDFT) performs the reverse process, transforming a spectrum signal (amplitude and phase of each component)

into a time domain signal. IDFT operation can be used to generate OFDM signals as shown in the following.

Consider only the first symbol interval and assume perfect synchronization in time and frequency domain. From (2.18), we have

$$\tilde{s}_0(t) = \sum_{i=0}^{N-1} X_{i,0} g_i(t) \quad (2.20)$$

If we sample $\tilde{s}_0(t)$ in (2.20) at $t = \frac{n \times T_s}{N}, n = 0, 1, \dots, N-1$, we have

$$\tilde{s}_0\left(\frac{n \times T_s}{N}\right) = \sum_{i=0}^{N-1} X_{i,0} e^{j \frac{2\pi i n}{N}}, n = 0, 1, \dots, N-1 \quad (2.21)$$

since $\frac{T_s}{N}$ is a constant, we can write $\tilde{s}_0(\frac{n \times T_s}{N})$ to be

$$\tilde{s}_0(n) = \sum_{i=0}^{N-1} X_{i,0} e^{j \frac{2\pi i n}{N}}, n = 0, 1, \dots, N-1 \quad (2.22)$$

The IDFT of the $X_{i,0}, i = 0, 1, \dots, N-1$ is

$$s(n) = \frac{1}{N} \sum_{i=0}^{N-1} X_{i,0} e^{j \frac{2\pi i n}{N}}, n = 0, 1, \dots, N-1 \quad (2.23)$$

Obviously, (2.22) and (2.23) are identical except for a constant scalar $\frac{1}{N}$. Hence it is seen that the OFDM signal can be generated at baseband by taking the IDFT of the complex modulated data symbols instead of generating the carriers separately. For the same reason, the receiver can be also implemented using the DFT.

The FFT or IFFT provides an efficient way to implement the DFT or IDFT [20]. It reduces the number of complex multiplications from N^2 to $\frac{N}{2} \log_2 N$ for N-point DFT or IDFT. Hence, with the help of the FFT and IFFT, the implementation of

OFDM is very simple.

2.2.3 Adding Cyclic Prefix

After OFDM modulation, a guard interval is inserted to suppress ISI which is caused by the multipath fading propagation. The guard interval is also called the cyclic prefix. As shown in Fig. 2.1, it is a copy of the last part of the present symbol. We

Fig. 2.1 The cyclic prefix is a copy of the last part of the present symbol

define a new base function

$$g'_i(t) = e^{j2\pi i \frac{W_s}{N} t} \quad -T_{cp} \leq t \leq T_{FFT} \quad (2.24)$$

where W_s is the signal bandwidth excluding the bandwidth occupied by the windowing and CP. $T_{FFT} = \frac{N}{W_s}$ is the duration of the OFDM symbol excluding the windowing and CP and T_{cp} is the duration of the cyclic prefix. Hence the symbol duration is $T_s = T_{FFT} + T_{cp}$ in this stage. It can be verified that

$$g'_i(t + N/W_s) = e^{j2\pi i \frac{W_s}{N} (t + N/W_s)} = e^{j2\pi i \frac{W_s}{N} t} = g'_i(t) \quad (2.25)$$

Considering the first symbol interval, if we use $g'_i(t)$ instead of $g_i(t)$ in (2.20), the complex envelope of the first OFDM signal is

$$\tilde{s}_0(t) = \sum_{i=0}^{N-1} X_{i,0} g'_i(t) \quad -T_{\text{cp}} \leq t \leq T_{\text{FFT}} \quad (2.26)$$

Therefore, for $0 \leq t \leq T_{\text{FFT}}$, $\tilde{s}_0(t)$ is equal to the "normal" OFDM signal (given for example by (2.20)). For $-T_{\text{cp}} \leq t < 0$, we obtain

$$\tilde{s}_0(t) = \sum_{i=0}^{N-1} X_{i,0} g'_i(t) = \sum_{i=0}^{N-1} X_{i,0} g'_i(t + N/W_s) = \tilde{s}_0(t + N/W_s) \quad (2.27)$$

Hence $\tilde{s}_0(t)$ is a copy of the last part of the present symbol. Equivalently, the samples of the complex envelope of the OFDM signal (with CP) are:

$$\tilde{s}_n = \tilde{s}_0(nT_{\text{FFT}}/N) = \begin{cases} \sum_{i=0}^{N-1} X_{i,0} e^{j2\pi i n/N} & n = 0, \dots, N-1 \\ \tilde{s}_0((n+N)\frac{N}{W_s}/N) = \tilde{s}_{N+n} & n = -G, \dots, -1 \end{cases} \quad (2.28)$$

where G is the number of the cyclic prefix samples. Similarly, if several data symbols are considered, then the complex envelope of the OFDM signal (with CP) is given by¹

$$\tilde{s}(t) = \sum_{k=-\infty}^{\infty} \sum_{i=0}^{N-1} X_{i,k} g'_i(t - kT_s) \quad (2.29)$$

where $g'_i(t)$ is given by (2.24) and $T_s = T_{\text{FFT}} + T_{\text{cp}} = N/W_s + G/W_s$.

When the duration of cyclic prefix is longer than the delay spread of the channel, the ISI caused by the multipath fading propagation would be diminished. All the delay echoes of the previous symbol will die away in the cyclic prefix. Otherwise the ISI will degrade the performance. The cyclic prefix makes OFDM robust against ISI

¹Note that in (2.29), a rectangular pulse shape is assumed.

and ICI.

2.2.4 Adding windowing

The windowing would be applied after adding the cyclic prefix to reduce out-of-band frequency leaking when the OFDM symbols have a discontinuous phase connection at the begin and end. For example, the windowing function $w(t)$ is the transmitter pulse prototype to smooth each OFDM symbol with a window around the edges of the OFDM frame to reduce out of band radiation resulting from Sinc function sidelobes. It is defined by:

$$w(t) = \begin{cases} \frac{1}{2}[1 + \cos \frac{\pi(t+T_{cp})}{T_{win}}] & -T_{win} - T_{cp} \leq t \leq -T_{cp} \\ 1 & -T_{cp} \leq t \leq T_{FFT} \\ \frac{1}{2}[1 + \cos \frac{\pi(t-T_{FFT})}{T_{win}}] & T_{FFT} \leq t \leq T_{FFT} + T_{win} \end{cases} \quad (2.30)$$

where T_{win} is the duration of the windowing (one side).

2.2.5 Transmitted OFDM signal (including CP and windowing)

The complex envelope of the OFDM signal (with CP and windowing) is given by

$$\tilde{s}(t) = \sum_{k=-\infty}^{\infty} \sum_{i=0}^{N-1} X_{i,k} e^{\frac{j2\pi i W_s(t-kT_s)}{N}} w(t - kT_s) \quad (2.31)$$

where $T_s = 2T_{win} + T_{cp} + T_{FFT}$.

2.2.6 Received signal (assuming perfect synchronization)

The complex envelope of the OFDM signal can be represented by (2.31). Assume that the transmitted OFDM signal goes through a time variant multipath fading channel which is expressed by its equivalent lowpass impulse response $h(\tau; t)$ plus additive

white Gaussian noise (AWGN) $n(t)$, then the lowpass equivalent form of the received signal can be written as

$$\tilde{r}(t) = h(\tau; t) \circledast \tilde{s}(t) + n(t) = \int_0^{\tau_{max}} h(\tau; t) \tilde{s}(t - \tau) d\tau + n(t) \quad (2.32)$$

where $\circledast$ is the convolution operator and τ_{max} is the channel maximum delay (or delay spread). $h(\tau; t)$ denotes the impulse response at time t (τ denotes the excess delay of the channel impulse response), which is zero outside the range of this convolution integral $\tau = [0, \tau_{max}]$ and $\tilde{s}(t)$ is the complex envelope of the OFDM signal given by (2.31).

Assuming the channel is time-invariant during the transmission of one OFDM symbol (slow fading channel), $\tau_{max} < T_{cp}$, and there is perfect time synchronization, the orthogonality of the tones is preserved and there is no interference from the previous OFDM symbol to affect the current one. In other words, ISI is suppressed. We assume that $h(\tau; t)$ is time invariant in one OFDM symbol interval and it can be represented as $h(\tau)$. In that case, considering the first symbol interval, the received signal after removing the windowing and CP can be represented by:

$$\tilde{r}_0(t) = h(\tau; t) \circledast \tilde{s}_0(t) + n(t) = \int_0^{\tau_{max}} h(\tau) \tilde{s}_0(t - \tau) d\tau + n(t) \quad (2.33)$$

$$\tilde{r}_0(t) = h(\tau; t) \circledast \tilde{s}_0(t) + n(t) = \int_0^{\tau_{max}} h(\tau) \tilde{s}_0(t - \tau) d\tau + n(t) \quad (2.33)$$

where

$$\tilde{s}_0(t) = \sum_{i=0}^{N-1} X_i e^{j2\pi i t / T_{FFT}} \quad (2.34)$$

and X_i is $X_{i,0}$. Sampling the received signal at nT_{FFT}/N yields

$$\tilde{r}_0(nT_{FFT}/N) = \int_0^{\tau_{max}} h(\tau) \tilde{s}_0(nT_{FFT}/N - \tau) d\tau + \tilde{n}_n \quad (2.35)$$

assuming a tapped delay line channel model (section 2.1.4)

$$h(t) = \sum_{m=0}^{M-1} h_m \delta(t - mT_{FFT}/N) \quad (2.36)$$

we have

$$\begin{aligned} \tilde{r}_0(nT_{FFT}/N) &= \int_0^{\tau_{max}} \sum_{m=0}^{M-1} h_m \delta(\tau - mT_{FFT}/N) \tilde{s}_0(nT_{FFT}/N - \tau) d\tau + \tilde{n}_n \\ &= \sum_{m=0}^{M-1} h_m \tilde{s}_0((n-m)T_{FFT}/N) + \tilde{n}_n \\ &= \sum_{m=0}^{M-1} h_m \tilde{s}_0(n-m) + \tilde{n}_n \end{aligned} \quad (2.37)$$

where $\tilde{s}_0(n-m)$ is a short notation for $\tilde{s}_0((n-m)T_{FFT}/N)$. An AWGN channel corresponds to $h_0 = 1$ and $h_1 = h_2 = \dots = h_{M-1} = 0$ (i.e. $h(\tau) = \delta(\tau)$).

Assuming an AWGN channel and no noise, the received samples are given by

$$\tilde{r}_0(nT_{FFT}/N) = \tilde{s}_0(n) = N \cdot IFFT\{X_{i,0}\} \Big|_{n^{th} \text{ sample}} \quad (2.38)$$

Therefore, it is seen that $\{X_{i,0}\}$ can be attained from $\{\tilde{r}_0(nT_{FFT}/N)\}$ by performing an FFT.

An FFT operation can also be used to attain the $X_{i,0}$'s for a tapped delay line channel model as shown as follows. In that case, the output of the FFT is

$$\begin{aligned} Y_k &= FFT\{\tilde{r}_0(iT_{FFT}/N)\} \Big|_{k^{th} \text{ sample}} = \sum_{i=0}^{N-1} \tilde{r}_0(iT_{FFT}/N) e^{-\frac{j2\pi ik}{N}} \\ &= \sum_{i=0}^{N-1} \left\{ \sum_{m=0}^{M-1} h_m \tilde{s}_0(i-m) + \tilde{n}_i \right\} e^{-\frac{j2\pi ik}{N}} \end{aligned}$$

$$\begin{aligned}
&= \sum_{m=0}^{M-1} h_m e^{-\frac{j2\pi mk}{N}} \left\{ \sum_{i=0}^{N-1} \tilde{s}_0(i-m) e^{-\frac{j2\pi(i-m)k}{N}} \right\} + \sum_{i=0}^{N-1} \tilde{n}_i e^{-\frac{j2\pi ik}{N}} \\
&= \sum_{m=0}^{M-1} h_m e^{-\frac{j2\pi mk}{N}} \left\{ \sum_{i=0}^{N-1} \left[\sum_{l=0}^{N-1} X_{l,0} e^{\frac{j2\pi(i-m)l}{N}} \right] e^{-\frac{j2\pi(i-m)k}{N}} \right\} + N_k \\
&= H_k \left\{ \sum_{l=0}^{N-1} X_{l,0} e^{\frac{j2\pi(k-l)m}{N}} \sum_{i=0}^{N-1} e^{\frac{j2\pi(l-k)i}{N}} \right\} + N_k \tag{2.39}
\end{aligned}$$

where $N_k = \sum_{i=0}^{N-1} \tilde{n}_i e^{-\frac{j2\pi ik}{N}}$ is the independent noise sample and $H_k = \sum_{m=0}^{M-1} h_m e^{-\frac{j2\pi mk}{N}}$. Since $\sum_{i=0}^{N-1} e^{\frac{j2\pi(l-k)i}{N}} = \delta_{kl}$, we have

$$\begin{aligned}
Y_k &= H_k \left\{ \sum_{l=0}^{N-1} X_{l,0} e^{\frac{j2\pi(k-l)m}{N}} \sum_{i=0}^{N-1} e^{\frac{j2\pi(l-k)i}{N}} \right\} + N_k \\
&= H_k \left\{ \sum_{l=0}^{N-1} X_{l,0} e^{\frac{j2\pi(k-l)m}{N}} \delta_{kl} \right\} + N_k \\
&= H_k X_{k,0} + N_k \tag{2.40}
\end{aligned}$$

therefore the complex data symbols can be attained from the Y_k 's using an one-tap frequency domain equalizer.

Remark, note that (2.40) can be viewed as the output of a matched filter, matched to the k^{th} subcarrier signal, assuming no ICI i.e.

$$Y_k = \frac{1}{T_{FFT}} \int_0^{T_{FFT}} \tilde{r}_0(t) \cdot e^{-j\frac{2\pi kt}{T_{FFT}}} dt \tag{2.41}$$

2.2.7 The OFDM system block diagram

The OFDM system block diagram is shown in Fig. 2.2 [3]. At the transmitter, the input binary data stream is encoded using any suitable modulation technique. Then this serial data stream is converted to the parallel data stream. This parallel data stream is then taken through an IFFT and this stage is called the OFDM modulation.

After OFDM modulation, a cyclic prefix is inserted to suppress ISI which is caused by the multipath fading propagation. The windowing would be applied after the cyclic prefix to reduce out-of-band frequency leaking. After that, the signal through a D/A converter becomes an analog baseband signal, upconverted to RF, and transmitted. After passing a time variant channel and adding a Gaussian noise, the signal arrives at the receiver antennas. For an OFDM receiver, the signal downconverted into an analog baseband signal, is passed through an A/D converter, then windowing and CP are removed, after the signal is demodulated using FFT (that converts the OFDM time domain samples into the frequency domain samples which correspond to the complex modulated data symbols). Then the obtained parallel data sequences are converted into a serial data sequence. After decoding, it becomes the output binary data stream.

Throughout this thesis, we employ the following assumptions:

- A cyclic prefix is used.
- The impulse response of the channel is shorter than the cyclic prefix.
- Channel noise is additive, white, and complex Gaussian.
- The fading is slow enough for the channel to be considered constant during one symbol interval. (For time variant fading channels, we model the CIR changes every sample.)

Fig. 2.2 The OFDM system block diagram

2.3 OFDM issues

2.3.1 Coding and Modulation

In the OFDM system the data is transmitted by N narrow subcarriers [1]. Each subcarrier has a different amplitude when it arrives to the receiver. Some of them may be in the deep fading in a frequency selective fading channel. That would cause high BER although most of subcarriers are detected without errors. To avoid performance dominated by the weakest subcarriers, some redundant codes are used to make the performance determined by the average received power. In the OFDM system block codes and/or convolution codes are normally used, sometimes two codes are combined or concatenated.

To deal with error bursts, we can use interleaving to randomize the occurrence of bit errors prior to decoding. After interleaving the adjacent bits are separated by several bits. Coding and interleaving can be combined to deal with the PAPR.

Because the rectangular constellations of QAM are easy to implement as they can be split into independent PAM components for both the in-phase and the quadrature part, we normally use them in combination with OFDM. The adaptive modulation uses different modulation schemes on different subcarriers according to the channel information of each subcarrier. The water pouring strategy increases the overall bandwidth efficiency.

2.3.2 Issues related to Cyclic Prefix

The cyclic prefix makes OFDM robust against the ISI and ICI. Some algorithms use it for the synchronization, but it takes 5% to 30% of the symbol duration. That makes it inefficient in terms of spectrum use. Another disadvantage is the SNR loss. The cyclic prefix constitutes “noise” when the signal is concerned and provides less power to the

signal frame by occupying signal space. By using a proper equalizer and new detection algorithm [21] we can eliminate the cyclic prefix. In [22], the author suggests to use a binary sequence that is multiplexed with the OFDM signal to replace the cyclic prefix. The sequence is used as a pilot for the time and frequency domain synchronization but we have to use a good equalizer reduce the ISI. The advantage is that we can put some information in that binary sequence, such as the user ID. In [22] the author says this method can also reduce the PAPR. It also increases the spectrum efficiency. We can also use those PN binary sequence to estimate the channel. The disadvantage is that we have to use a complex equalizer. In this thesis, we only consider a CP-based OFDM system.

2.3.3 The Peak to Average Power Ratio problem

Because of the coherent summation of the sine waves, we have a large peak to average power ratio which makes the problem for the linear bandwidth of the RF amplifier [23]. When N signals are added with the same phase, they produce a peak power that is N times the average power. To reduce the PAPR, we can use techniques such as amplitude clipping, clipping and filtering, coding, peak windowing tone reservation, tone injection, active constellation extension, partial transmit sequence, selected mapping and interleaving, but the price is the performance degradation and complexity increase.

2.3.4 Synchronization of OFDM systems [3]

Synchronization is an important issue of OFDM systems since it makes each subcarrier orthogonal with others. OFDM systems are highly sensitive to timing errors, carrier frequency offsets, phase noise and sampling clock offsets. The symbol synchronization means knowing when the symbol starts to make the symbol time offset ϵ as small as

possible. If it is larger than the cyclic prefix, the orthogonality is not maintained and ICI will occur. Carrier frequency offset is created by differences of the oscillators in transmitter and receiver, Doppler shifts or phase noise are introduced by nonlinear channels. Two destructive effects are caused by a carrier frequency offset in OFDM systems. CFO reduces the signal amplitude and introduces ICI from other subcarriers. Normally, the synchronization algorithms rely on the evaluation of the correlation function (training symbol or cyclic prefix).

Symbol time and carrier frequency offset

Symbol time and carrier frequency offset cause ICI. For single carrier systems the phase noise and frequency offset only cause the degradation of SNR rather than interference. OFDM is more sensitive to the frequency offset.

Assuming perfect synchronization, the received signal after the demodulation of OFDM (using FFT) becomes the received signal constellation point Y_k given by (2.40). Due to a frequency mismatch between the oscillators of the transmitter and receiver, a frequency offset δ_f and a carrier phase offset θ must be accounted for as a frequency shift in the lowpass equivalent received signal:

$$\tilde{r}'(t) = \tilde{r}_0(t) \cdot e^{j(2\pi\delta_f t + \theta)} \quad (2.42)$$

where $\tilde{r}_0(t)$ is given by (2.33). If furthermore a symbol time offset ϵ is present, the k^{th} sample of the received signal is

$$r_k = \tilde{r}'(kT_{FFT}/N + \epsilon) = \tilde{r}_0(kT_{FFT}/N + \epsilon) \cdot e^{j(2\pi\delta_f(kT_{FFT}/N + \epsilon) + \theta)} \quad (2.43)$$

Assume for simplicity, $\epsilon = -n_\epsilon \frac{T_{FFT}}{N}$, where n_ϵ is an integer representing the unknown

arrival time of a symbol. Let $\gamma = \delta_f T_{FFT}$ be the CFO normalized by the subcarrier spacing. From (2.43) and (2.37), we have

$$\begin{aligned} r_k &= \tilde{r}_0 \left((k - n_\epsilon) \frac{T_{FFT}}{N} \right) \cdot e^{j(2\pi(k \frac{\gamma}{N} - n_\epsilon \frac{\gamma}{N}) + \theta)} \\ &= \left\{ \sum_{m=0}^{M-1} h_m \tilde{s}_0(k - n_\epsilon - m) + \tilde{n}_{k-n_\epsilon} \right\} e^{j(2\pi \frac{k\gamma}{N} + \theta_0)} \\ &= \tilde{s}_{k-n_\epsilon} e^{j(2\pi \frac{k\gamma}{N} + \theta_0)} + n_k \end{aligned} \quad (2.44)$$

where $\theta_0 = \theta - 2\pi \frac{n_\epsilon \gamma}{N}$ is the initial phase. Note that demodulation can be done either using (2.40) or (2.41). In practice (2.40) is implemented. However to analyze qualitatively the effect of non-integer symbol time offset, it is more convenient to use (2.41). In the presence of a symbol time offset ϵ , the demodulation interval becomes $t \in (\epsilon, T_{FFT} + \epsilon)$. Hence we have

$$Y_k = \frac{1}{T_{FFT}} \int_{\epsilon}^{T_{FFT} + \epsilon} \tilde{r}'(t) \cdot e^{-j2\pi k \frac{(t-\epsilon)}{T_{FFT}}} dt \quad (2.45)$$

From [3], the final expression for the received signal without ICI term becomes:

$$Y_k = X_k \cdot H_k \cdot \text{sinc}(\delta_f \cdot T_{FFT}) \cdot e^{j\Psi_k} + N_k \quad (2.46)$$

and

$$\Psi_k = \theta + 2\pi\delta_f \left(\frac{T_{FFT}}{2} + \epsilon \right) + 2\pi\epsilon \left(\frac{k}{T_{FFT}} \right) \quad (2.47)$$

where $H_k = H(\frac{k}{T_{FFT}}) = \int_0^{\tau_{max}} h(\tau) \cdot e^{-j2\pi k (\frac{\tau}{T_{FFT}})} d\tau$ is the Fourier transform of $h(\tau)$ at the frequency $\frac{k}{T_{FFT}}$, N_k is the independent noise sample, and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. From (2.46), we can find the carrier frequency offset δ_f and carrier phase offset θ will cause attenuation of all subcarriers by the Sinc function and a common phase

rotation. H_k will introduce a phase rotation and an attenuation too. To minimize the effect of channel, we can estimate the channel when we assume the timing offset is not very large, or use differential modulation to cancel out the effect of the channel in a consecutive sequence. The carrier frequency offset also causes ICI of the adjacent subcarriers. The symbol time offset ϵ causes a progressive phase rotation (multiply by k) and the maximum phase rotation is found at the edges of the frequency band. When $\tau_{max} - T_{cp} \leq \epsilon \leq 0$, that means the channel impulse response dies away in the guard interval and no ISI, otherwise we have to deal with the ISI by the equalizer.

Sampling frequency synchronization

The received continuous time signal is sampled at instants determined by the receiver clock. There are two methods to deal with the mismatch in the sampling frequency. In the synchronized sampling system, a timing algorithm controls a VCO (voltage controlled oscillator) to lock the receiver clock with the transmitter clock. In the non-synchronized sampling system, the sampling rate is fixed and a postprocessing is required in the digital domain. The effect of the clock frequency offset not only rotates and attenuates the useful signal component but also introduces ICI. Non-synchronized sampling is more sensitive to the frequency offset, compared with a synchronized sampling system. In this thesis, we assume perfect sampling frequency synchronization and will focus on symbol time and carrier frequency offset estimation.

2.4 MIMO-OFDM systems background

2.4.1 MIMO systems

According to the number of antennas which are used at the transmitter and receiver, communication systems can be classified as four kinds [3][13]. Single-Input Single-

Output (SISO) is the basic wireless configuration, Single-Input Multiple-Output (SIMO) uses a single receiver antenna and multiple transmitter antennas, Multiple-Input Single-Output (MISO) has multiple transmitter antennas and single receiver antenna, Multiple-Input Multiple-Output (MIMO) uses multiple antennas both for the transmitter and receiver. MIMO systems can be implemented in different ways to obtain either a spatial diversity gain to combat fading channel or a capacity gain.

Spatial diversity is also called antenna diversity which can be divided into receive diversity and transmit diversity. Such techniques include delay diversity, space time block codes (STBC) and space time trellis codes (STTC). Types of diversity schemes include selection diversity, maximum ratio diversity and equal gain diversity. Transmit diversity is easy to be implemented at the base station but the transmitter must know well the channel information or use the space-time coding schemes without knowledge of the channel.

Spatial multiplexing gives systems a capacity gain. One popular example is Vertical Bell Laboratories Layered Space Time (V-BLAST) system. A MIMO system can linearly increase the channel capacity by using the same bandwidth spectrum and boosts the data rate multiple times, when a rich scattering environment provides independent transmission paths from each transmitter antenna to each receiver antenna. Each transmitter antenna transmits different data at the same time. Because of the multipath propagation, each receiver antenna gets the mixed data. Since we assume that the receiver has the complete knowledge about the channel, the data are processed by the signal processor to reconstruct the transmitted signal. It looks like using N different cables in same frequency band to transmit parallel data. It increases symbol rate N times and also reuses the spectrum N times.

The third way to use MIMO systems is to exploit the knowledge of the channel at the transmitter. It decomposes the channel coefficient matrix by singular value

decomposition (SVD), then uses these decomposed unitary matrices as pre-filters and post-filters at the transmitter and receiver to achieve near capacity.

In MIMO systems, there is a trade-off between the diversity gain and the capacity gain. When channel situation is bad, for decreasing the BER, we have to use the diversity and decrease the symbol rate (but we can use high level modulation to improve the bit rate). When channel is good, we can increase the symbol rate by transmitting different data on each transmitter antenna and give up a part of the diversity. We have to use the adaptive scheme to choose the best solution according to the channel situation and channel matrix.

2.4.2 MIMO-OFDM systems

Fig. 2.3 $Q \times L$ MIMO-OFDM system, where Q and L are the numbers of inputs and outputs respectively

When a MIMO system is designed for a frequency selective fading channel, to reduce the equalization complexity of the MIMO system, combining the MIMO system with OFDM modulation would be a good choice [11]. The system becomes the MIMO-OFDM system. In Fig. 2.3, we give a block diagram of a MIMO-OFDM system [13]. All MIMO-OFDM receivers must perform time synchronization, carrier and

sampling frequency offset estimation, and correction and parameter estimation. At the beginning of the frame, we use a preamble containing one or more training sequences. Once the acquisition phase is over, receiver goes into the tracking mode by using training pilots on some subcarriers.

2.5 System model

2.5.1 OFDM system model

Using the IDFT or IFFT to implement OFDM, from (2.23), the time domain samples of a complex value baseband OFDM symbol can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_i e^{j2\pi i n / N}, n = 0, 1, \dots, N-1 \quad (2.48)$$

where N is IFFT size, x_n is the n^{th} sample of the time domain symbol, X_i is the complex modulated data on the i^{th} subcarrier, and for convenience we use the normalized factor $\frac{1}{\sqrt{N}}$ instead of $\frac{1}{N}$. In a CP-based OFDM system, a CP which is the copy of the last part of the current symbol is added to each OFDM symbol. The signal then is passed through a window and a D/A converter, upconverted to RF, and transmitted. When the CP length is longer than the channel delay spread, the ISI caused by multipath fading is eliminated. At the receiver the signal is downconverted into a baseband signal $\tilde{r}(t)$, passed through a A/D converter, removed of its windowing and sampled at a rate $T_{sa} = T_{FFT}/N$, where T_{sa} is the sample duration. Taking into account the phase shift introduced by a CFO and a timing offset, the received sample can be written as (see (2.44) in section 2.3.4)

$$r_k = s_{k-n_e} e^{j(2\pi\gamma k/N + \theta_0)} + n_k \quad (2.49)$$

where n_ϵ is an integer representing the unknown arrival time of a symbol, γ is the CFO normalized by the subcarrier spacing, θ_0 is the initial phase, n_k is the k^{th} sample of the complex Gaussian noise with zero mean and variance σ_n^2 . For AWGN channels $s_k = x_k$. For ISI or fading channels, we use a tapped delay line model introduced in section 2.1.4 and s_k is given by

$$s_k = \sum_{m=0}^{M-1} h_m x_{k-m} \quad (2.50)$$

where M is channel length and h_m is the complex baseband channel impulse response at lag m .

2.5.2 MIMO-OFDM system model

Let us consider the $Q \times L$ space time system in Fig. 2.4 [14].

Fig. 2.4 Block diagram of a system with $Q \times L$ transmit-receive diversity

Such system has a diversity order of $Q \times L$, Let $\mathbf{S}_q = [S_{q,0}, \dots, S_{q,N-1}]^T$ be the transmitted OFDM symbol on the q^{th} transmitter antenna, $\mathbf{R}_l = [R_{l,0}, \dots, R_{l,N-1}]^T$

be the received OFDM symbol from the l^{th} receiver antenna. Similar as (2.48), the N point IDFT output sequence for the OFDM symbol $\mathbf{S}_q$ is given by

$$s_{q,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_{q,k} e^{j\frac{2\pi nk}{N}}, n = 0, 1, \dots, N-1 \quad (2.51)$$

where $S_{q,k}$ is the transmitted data on the k^{th} subcarrier from the q^{th} transmitter antenna and $s_{q,n}$ is the n^{th} time domain sample from the q^{th} transmitter antenna. For a fair comparison with SISO systems, the summation of all transmitter antennas power are normalized to be one and we have

$$\sum_{q=1}^Q \sum_{n=0}^{N-1} |s_{q,n}|^2 = \sum_{n=0}^{N-1} |x_n|^2 = 1 \quad (2.52)$$

At the receiver, for the l^{th} receiver antenna, after removing the windowing and CP, the received sample sequence is

$$r_{l,k} = \sum_{q=1}^Q \sum_{m=0}^{M-1} h_{ql,m} s_{q,k-n_\epsilon-m} e^{j(2\pi\gamma k/N + \theta_0)} + n_{l,k} \quad (2.53)$$

where $h_{ql,m}$ is the complex baseband channel impulse response at lag m between the q^{th} transmitter antenna and the l^{th} receiver antenna ($\mathbf{h}_{q,l} = [h_{ql,0}, \dots, h_{ql,M-1}]$) and M is the length of the channel. n_ϵ is an integer representing the unknown arrival time of a symbol, γ is the CFO normalized by the subcarrier spacing, θ_0 is the initial phase, $n_{l,k}$ is the k^{th} sample of the complex Gaussian noise with zero mean and variance σ_n^2 at the l^{th} receiver antenna.

Chapter 3

Existing synchronization methods

3.1 Synchronization methods using training symbols

3.1.1 Schmidl and Cox (SC)'s method [6]

The authors use two training symbols to estimate the symbol timing and carrier frequency offset. The first training symbol has two identical halves in the time domain. The training symbol is a PN sequence on the even subcarriers and zeroes on the odd subcarriers, so that the receiver can separate the training symbol and the data symbol. The first half is identical to the second half after passing the channel, except for a phase shift caused by the carrier frequency offset.

If the conjugate of a sample from the first half is multiplied by the corresponding sample from the second half ($\frac{T_{FFT}}{2}$ seconds later), the effect of the channel should be canceled out. Only a phase difference of $\phi = \pi\gamma$ will remain, we can use it to estimate the normalized carrier frequency offset γ . The phase difference is constant because the length between the samples L_{sc} is constant.

In order to increase the frequency capture range, the second training symbol is used. The second training symbol contains a PN sequence on the odd subcarriers

to measure these subchannels, and another PN sequence on even subcarriers to help determining the CFO.

At the start of the frame, the products of each of these pairs of samples will have almost the same phase, so the magnitude of the sum will be a large value. The sum of products can be expressed in the following equation:

$$P_{SC}(d) = \sum_{m=0}^{L_{sc}-1} (r_{d+m}^* \cdot r_{d+m+L_{sc}}) \quad (3.1)$$

where $L_{sc} = \frac{N}{2}$ samples. $P_{SC}(d)$ can be calculated iteratively,

$$P_{SC}(d+1) = P_{SC}(d) + r_{d+L_{sc}}^* r_{d+2L_{sc}} - r_d^* r_{d+L_{sc}} \quad (3.2)$$

The energy of the second half of the first training symbol to normalize the sum of the correlation value is used. The energy is calculated as:

$$R_{SC}(d) = \sum_{m=0}^{L_{sc}-1} |r_{d+m+L_{sc}}|^2 \quad (3.3)$$

A timing metric is defined as the normalized sum of products. It is:

$$M_{SC}(d) = \frac{|P_{SC}(d)|^2}{R_{SC}^2(d)} \quad (3.4)$$

The timing metric exhibits a plateau of length of the guard interval where the metric reaches a maximum when the samples are pairs with distances of half a symbol period. The start of the frame can be taken to be anywhere within this window without a loss in the received SNR. When the signal is propagating over a multipath fading channel, the length of the plateau will be shortened by the length of the channel delay time. This plateau leads to some uncertainty as to the start of the frame, as it will always

be a “rough” estimation of the symbol timing error. Authors propose an averaging method which find two points with 90% of the maximum value, one to the left and the other to the right of the maximum point. The timing estimate is taken as the average of the two 90% points.

We can use the main difference between the two halves of the first training symbol which is a phase difference to estimate the carrier frequency offset.

$$\phi = \pi\gamma \quad (3.5)$$

$$\hat{\phi} = \angle P_{SC}(\hat{d}_{SC}) \quad (3.6)$$

where $\angle$ denotes the argument of a complex number. $\hat{d}_{SC}$ is the estimated symbol start point using SC's method. Since (3.6) yields an estimate of ϕ only up to a multiple of 2π , only the fraction part of the CFO can be estimated from $\hat{\phi}$ as

$$\hat{\gamma}_f = \frac{\hat{\phi}}{\pi} \quad (3.7)$$

The actual frequency offset is

$$\gamma = \gamma_f + 2g \quad (3.8)$$

where g is an integer (possibly equal to 0), and γ_f is the fraction part of the carrier frequency offset. We have to use the second training symbol to estimate g . The second training symbol contains a PN sequence modulated differentially with respect to the first training symbol and it can be retrieved and compared with the reference sequence. After the fraction carrier frequency offset correction (by multiplying the sample r_n , given by (2.49), by $e^{-j2\pi n\hat{\gamma}_f/N}$), we obtain the corrected samples corresponding to the

two training symbols as:

$$\begin{aligned} r_n e^{-j2\pi n \hat{\gamma}_f / N} & \quad n = 0, \dots, N-1 \\ r_{n+N+g} e^{-j2\pi(n+N+g) \hat{\gamma}_f / N} & \quad n = 0, \dots, N-1 \end{aligned}$$

Let $x_{1,k}$ and $x_{2,k}$ be the FFT's of $r_n e^{-j2\pi n \hat{\gamma}_f / N}$ and $r_{n+N+g} e^{-j2\pi(n+N+g) \hat{\gamma}_f / N}$, and let v_k be the differentially modulated PN sequence on the even frequencies of the two training symbols.

$$v_k = X_{2,k} / X_{1,k} \quad k = 0, 2, \dots, N-2 \quad (3.9)$$

where $X_{1,k}$ and $X_{2,k}$ are frequency training sequences of first and second training symbol respectively and k is index of the subcarriers. The number of the even positions shifted can be calculated by finding $\hat{g}$ that maximizes

$$B(g) = \frac{|\sum_{k \in \mathbf{X}} x_{1,k+2g}^* v_k^* x_{2,k+2g}|^2}{2(\sum_{k \in \mathbf{X}} |x_{2,k}|^2)^2} \quad (3.10)$$

where $\mathbf{X}$ is the set of indices for the even frequency components given by $\mathbf{X} = \{0, 2, \dots, N-2\}$. The carrier frequency offset estimate is obtained as

$$\hat{\gamma} = \frac{\hat{\phi}}{\pi} + 2\hat{g} \quad (3.11)$$

We can also use the second training symbol to estimate the channel.

3.1.2 Minn's method [7]

First, Minn used

$$R_f(d) = \frac{1}{2} \sum_{m=0}^{N-1} |r_{d+m}|^2 \quad (3.12)$$

to replace the $R_{SC}(d)$ in (3.4), which decreases the variance of the symbol timing estimator. Then he proposed the new training pattern $[A \ A \ -A \ -A]$ where A represents samples of length $L_M = N/4$ generated by $N/4$ point IFFT of $N_u/4$ length modulated data of a PN sequence (N_u is the number of used subcarriers). The timing metric is given by

$$M_{Minn}(d) = \frac{|P_{Minn}(d)|^2}{R_{Minn}^2(d)} \quad (3.13)$$

where

$$P_{Minn}(d) = \sum_{k=0}^1 \sum_{m=0}^{L_M-1} r_{d+2kL_M+m}^* \cdot r_{d+2kL_M+m+L_M} \quad (3.14)$$

and

$$R_{Minn}(d) = \sum_{k=0}^1 \sum_{m=0}^{L_M-1} |r_{d+2kL_M+m+L_M}|^2 \quad (3.15)$$

The estimated start point of the symbol is

$$\hat{d}_{Minn} = \arg \max_d \{M_{Minn}(d)\} \quad (3.16)$$

When we use $R_f(d)$ in (3.12) to replace $R_{Minn}(d)$ in (3.13), we can also decrease the variance of the timing estimator obtained with Minn's training pattern.

Using Minn's method, the timing metric does not have the plateau any more, so the uncertainty of the timing estimation has been reduced. It works well in an AWGN channel, but in fading channels, due to the multipath effect, the mean value will not be in the exact start point and will shift to the right. It introduces large ISI since the FFT will include some samples which are from the next symbol.

For carrier frequency offset estimation, it uses the repeated parts length which is the half of the SC's method. Since the variance of the frequency estimator depends on the time error, SNR, and length of the training symbol, it has a little degradation

[8]. The estimated fraction part of CFO is

$$\hat{\gamma}_f = \frac{2}{\pi} \angle P_{Minn}(\hat{d}_{Minn}) \quad (3.17)$$

where $\hat{d}_{Minn}$ is the estimated symbol start point using Minn's method. Note that the '2' in (3.17) comes from the fact that the length of the repeated part of the training symbol in Minn's method is half SC's one. Minn did not propose any method for estimating the integer part of the CFO.

3.2 Synchronization methods for MIMO-OFDM systems

Similar to OFDM systems, MIMO-OFDM systems are also very sensitive to synchronization errors. Therefore, we have to design robust synchronization methods for them. Since multiple antennas are used, we can achieve better performance of the synchronization from the spatial diversity. In [13] [14] and [15], authors describe a synchronization method in the acquisition mode by four steps. The system model is same as the model described in section 2.5.2. The frame structure is given in Fig. 3.1.

Antenna 1

Antenna Q

Fig. 3.1 Frame structure for the $Q \times L$ MIMO-OFDM system

where G is the CP length, N_I is the length of the short training symbol, N is the length of the data symbol, Q is the number of transmitter antennas.

- Step 1) Coarse Time Synchronization and Signal Detection

Due to the CP (or guard interval), coarse time acquisition can be performed by autocorrelating the received, down converted discretized complex samples over a window of G samples that are at a distance of N_I from each other over a length G window:

$$n_{l,coarse} = \arg \max_n \{\phi_{l,n}\} \quad (3.18)$$

where

$$\phi_{l,n} = \sum_{k=0}^{G-1} (r_{l,n+k}^* \cdot r_{l,n+k+N_I}) \quad (3.19)$$

where $r_{l,k}$ is the k^{th} received sample at the l^{th} receiver antenna (given by (2.53)) and maximization of $\phi_{l,n}$ is performed only if $\phi_{l,n}$ exceeds a certain threshold which is 10% of the incoming signal energy contained in the correlation window.

- Step 2) Frequency Offset Estimation in the Time Domain

Let γ be the carrier frequency offset between the transmitter and receiver local oscillators normalized by the subcarrier spacing. The samples at a distance of N_I in the time domain have a phase shift $\phi = 2\pi\gamma N_I/N$. The frequency offset estimate of up to $\pm I/2$ subcarrier spacings is

$$\hat{\gamma}_{f,l} = \frac{I}{2\pi} \angle \{\phi_{n_{l,coarse}}\} \quad (3.20)$$

where $I = N/N_I$. By reducing the length of the training symbol by a factor of I , we increase the range of the frequency offset estimate by a factor of I . If we

multiply the received sample sequence with $e^{-j2\pi\hat{\gamma}k/N_I}$ during the preamble and $e^{-j2\pi\hat{\gamma}k/N}$ during the data portion, the fraction part of the CFO can be removed.

- Step 3) Residual Frequency Offset Correction

If the range of the CFO is over $\pm I/2$ subcarrier spacings, frequency domain processing can be used. Let $\{S_{q,k}\}_{k=0}^{N-1}$ be the frequency domain training sequence, obtained by taking N point FFT of the time domain training sequence $\{s_{q,n}\}_{n=0}^{N_I-1}$ transmitted from the q^{th} transmitter antenna. Suppose that the same frequency domain training sequence $\{S_{q,k}\}_{k=0}^{N-1}$ is transmitted from all the transmitter antennas. The residual frequency offset can be estimated by taking a cyclic cross-correlation of $\{S_{q,k}\}_{k=0}^{N-1}$ with the received, frequency corrected and demodulated symbol sequence $R_{l,n}^c = FFT_N\{r_{l,k}e^{-j2\pi\hat{\gamma}_{f,l}k/N_I}\}$ and we have¹

$$\chi_{l,k} = \sum_{n=0}^{N-1} S_{q,(k+n)_N}^* R_{l,n}^c \quad k = 0, 1, \dots, N-1 \quad (3.21)$$

where $(k+n)_N$ denotes $k+n \bmod N$. The integer frequency offset is estimated as

$$\hat{g}_l = \arg \max\{|\chi_{l,k}|\} \quad k = 0, 1, \dots, N-1 \quad (3.22)$$

The overall CFO estimate is then obtained as

$$\hat{\gamma}_l = \hat{\gamma}_{f,l} + \hat{g}_l \quad (3.23)$$

- Step 4) Fine Time Synchronization

When the frequency offset is removed, fine time synchronization can be performed by cross-correlating the received, frequency corrected samples with the

¹Note that $\chi_{l,k}$ does not depend on q since it is assumed the same frequency domain training sequence is used for all transmitter antennas.

transmitted sequences. The general fine time synchronization metric can be given by:

$$n_{l,fine} = \arg \max_n \{\psi_{l,n}\} \quad (3.24)$$

where

$$\psi_{l,n} = \sum_{q=1}^Q \sum_{k=0}^{N_I-1} (s_{q,k}^* r_{l,n+k}^c) \quad (3.25)$$

and

$$r_{l,k}^c = r_{l,k} e^{-j2\pi\hat{\gamma}_l k/N_I}$$

If the training sequences are orthogonal with each other, only one cross-correlation is needed per receiver antenna. The threshold is set at 10% of the energy contained in N_I received samples. Since it is a computationally consuming process, we should use a small window which is centered around the coarse time estimated point $n_{l,coarse}$. Finally, the estimated start point is $n_{opt} = \frac{1}{L} \sum_{l=1}^L n_{l,fine}$.

From the frame structure, we know the preamble is used not only for synchronization but also for channel estimation. The preamble is efficient, but this method also has some drawbacks. First, most of MIMO-OFDM channels are frequency selective fading channels, i.e. each subcarrier has a different frequency domain channel coefficient. In step 3, the cyclic cross-correlation would be affected by different channel coefficients and it makes the estimation of the residual frequency offset inaccurate. It also affects the performance of fine time synchronization. Secondly, in step 4, the threshold which is set at 10% of the energy contained in N_I received samples is chosen independently of SNR. And no theoretical support is provided for the choice of the threshold. Finally, [13, 14, 15] do not clearly explain how to use spatial diversity.

Unlike [13] [14] and [15], [12] and [24] focus on the spatial diversity for the synchronization in MIMO-OFDM systems. In [12], the author extends the SC's method from

OFDM systems to MIMO-OFDM systems. It uses a preamble containing two identical parts without CP. The length of the preamble is N . We assume Q transmitter antennas and L receiver antennas are used. The timing metric is given by:

$$P(d) = \sum_{i=0}^{L-1} w_i P_i(d) \quad (3.26)$$

where w_i is the combining weight. If $w_i = 1$, the system implements equal gain combining (EGC). If $w_i = \max(|P_i(d)|)$, the system is under a maximum ratio combining (MRC).

$$P_i(d) = \sum_{m=0}^{N/2-1} r_{i,d+m}^* r_{i,d+m+N/2} \quad (3.27)$$

where $r_{i,k}$ is the k^{th} received sample at the i^{th} receiver antenna.

The estimated start point of the symbol is

$$\hat{d}_{MVC} = \arg \max_d \{P(d)\} \quad (3.28)$$

The estimated fraction part of the CFO is given by:

$$\hat{\gamma}_f = \frac{\angle P(\hat{d}_{MVC})}{\pi} \quad (3.29)$$

From the simulation, the author concluded that MRC had better performance than EGC. There are two drawbacks of this method, one is that no method is given for estimating the integer part of the CFO, the other is that the estimated start point is not accurate in multipath channels and shifted to the right side of the exact start point.

Chapter 4

Proposed synchronization method for OFDM and MIMO-OFDM systems

4.1 Proposed synchronization method for OFDM systems

4.1.1 The training symbols structure of the proposed method

As seen from (3.5) and (3.6), the fraction CFO is estimated from the angle of P_{SC} at the estimated start point, so the CFO estimator's performance depends on the timing estimator's performance and an accurate timing estimator is essential.

As explained in section 3.1.1, CP causes timing metric uncertainty. Furthermore, since the first symbol contains two identical parts, to estimate the CFO integer part we can perform an $N/2$ point FFT on only the second half symbol in order to get the training data on even subcarriers. Therefore, having ISI in that symbol will not have an impact on the estimation of the CFO integer part. Since we are only using the time domain signals to do the estimation of the symbol timing and the fraction part

of the CFO, we can remove the CP from the first training symbol.

The proposed method uses two training symbols to do the time and carrier frequency synchronization. The first training symbol is used for estimating the coarse time and the CFO fraction part. The first symbol pattern of the proposed synchronization method for AWGN channels is illustrated in Fig. 4.1 b. For the proposed method, the first OFDM symbol contains two identical parts $[a \ a]$ which are Chu sequences with length $L_p = N/2$. Chu sequences are chosen because they are constant envelop sequences with a perfect autocorrelation function [25]. All circular autocorrelations are zero except for the circular shift D , when $\text{mod}(D/L_p) = 0$, where L_p is the length of sequences.

Cyclic Prefix G	Identical Part a $N/2$	Identical Part a $N/2$
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(a) SC's method first symbol

FCP $G/2$	Identical Part a $N/2$	Identical Part a $N/2$	FIP $G/2$
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(b) SISO proposed method first symbol in AWGN channels

$\frac{ZPP}{0}$ $G/2$	Identical Part a_{sh} $N/2 - G/2$	0 $G/2$	Identical Part a_{sh} $N/2 - G/2$	0 $G/2$	FIP $G/2$
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(c) SISO proposed method first symbol in ISI or fading channels

CP $G/2$	Identical Part $a_{q,0} = a_{1,(q-1)G}$ $N/2$	Identical Part $a_{q,0} = a_{1,(q-1)G}$ $N/2$	FIP $G/2$
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(d) MIMO proposed method first symbol in fading channels (q^{th} Tx antenna)

Fig. 4.1 First symbol structure of proposed method and SC's method

In addition to the two identical parts, in an AWGN channel, we add a Flipped cyclic prefix (FCP) and a Flipped Postfix (FIP) for the first training symbol (see

Fig. 4.1 b.). The FCP is the negative copy of the last $G/2$ samples of $\mathbf{a}$ and FIP is the negative copy of the first $G/2$ samples of $\mathbf{a}$, where G is the number of the CP samples. Let $\mathbf{a}$ be $[a_0, a_1, \dots, a_{N/2-1}]$. The first symbol $\mathbf{x}_{preamble}$ in an AWGN channel is

$$\left[-a_{\frac{N-G}{2}}, \dots, -a_{\frac{N}{2}-1}, a_0, a_1, \dots, a_{\frac{N}{2}-1}, a_0, a_1, \dots, a_{\frac{N}{2}-1}, -a_0, -a_1, \dots, -a_{\frac{G}{2}-1} \right]$$

In fading channels, multipath will cause the estimated start point to shift to the right side of the exact start point. Recalling the tapped delay line model in (2.49)-(2.50), the timing metric would be the summation of each tap timing metrics, hence the right shift. Usually, most of the channel power is located in first few taps (e.g. channel with exponential decaying power delay profile). To make the timing estimator robust in fading or ISI channels, we generate each half training sequences using Chu sequences with length $\frac{N}{2} - \frac{G}{2}$ ($\mathbf{a}_{sh}$, where 'sh' stands for short) and pad $\frac{G}{2}$ zeroes at each half end (see Fig. 4.1 c.). Such proposed preamble works well in burst communication scenarios, but in continuous communication scenarios, first several samples will be corrupted by the previous symbol which makes the estimated start point shift to the right. To make the timing estimator robust in continuous communication scenarios as well, the length of FIP was chosen to be $G/2$ samples instead of choosing G (SC method CP length) and we add a $G/2$ zero prefix at the beginning of the symbol (see Fig. 4.1 c.).

The second training symbol is used for estimating the CFO integer part and performing the fine time synchronization. It is a known Chu sequence with length N . Since the Chu sequences for the two training symbols are known and of different lengths, the CFO integer part can be estimated by using the differences of the two training symbols on the even frequencies.

4.1.2 Implementing the proposed synchronization method

The proposed method can be implemented in four steps:

- Step 1) Coarse Time Synchronization

The proposed method uses the timing metric similar as SC's method. Let r_k be the complex samples taken at the output of A/D (see Fig. 2.2), given by (2.49). Since the first symbol has two identical parts, after passing a channel, the first part is same as the second part except for a phase shift by the CFO. If the conjugate of a sample from the first half is multiplied by the corresponding sample from the second half, the effect of channel is canceled out. At the start of the frame, the products of each of these pairs of samples will have almost same phase, so the magnitude of the sum will be a large value. The sum of products can be expressed in the following equation:

$$P_p(d) = \sum_{m=0}^{L_p-1} (r_{d+m}^* \cdot r_{d+m+L_p}) \quad (4.1)$$

where $L_p = \frac{N}{2}$ samples. We will use the energy of the second half of the first training symbol to normalize the sum of the correlation value. The energy is calculated as:

$$R_p(d) = \sum_{m=0}^{L_p-1} |r_{d+m+L_p}|^2 \quad (4.2)$$

A timing metric is defined as the normalized sum of products. It is:

$$M_p(d) = \frac{|P_p(d)|^2}{R_p^2(d)} \quad (4.3)$$

If we adopt Minn's modified $R_f(d)$ given in (3.12) instead of $R_p(d)$ in (4.3), the variance of the timing estimator decreases at the expense of a slight increase

in complexity. The coarse estimation of the start point is taken at the peak of timing metric and it is given by

$$\hat{d}_{coarse} = \arg \max_d \{M_p(d)\} \quad (4.4)$$

- Step 2) Estimating the Fraction Part of the CFO

At the start of the frame, the products of each of these pairs of samples will have approximately same phase $\phi = \pi\gamma$, where γ is the CFO normalized by the subcarrier spacing. We can use the phase of those products summation to estimate the carrier frequency offset. Let

$$\hat{\phi} = \angle P_p(\hat{d}_{coarse}) \quad (4.5)$$

Since (4.5) yields an estimate of ϕ only up to a multiple of 2π , only the fraction part of the CFO can be estimated from $\hat{\phi}$ as

$$\hat{\gamma}_f = \frac{\hat{\phi}}{\pi} \quad (4.6)$$

The actual frequency offset is

$$\gamma = \gamma_f + 2g \quad (4.7)$$

where g is an integer (possibly null) and γ_f is the fraction part of the carrier frequency offset. The estimated fraction part of the CFO is

$$\hat{\gamma}_f = \frac{\hat{\phi}}{\pi} = \frac{\angle P_p(\hat{d}_{coarse})}{\pi} \quad (4.8)$$

In this step, the frequency offset estimation is up to ± 1 subcarrier spacing. To remove the fraction part of the CFO, we should multiply the received sample sequences r_k , given by (2.49) with $e^{-j2\pi\hat{\gamma}_f k/N}$.

- Step 3) Estimating the Integer Part of the CFO

To estimate the integer part of the CFO, we have to use the second training symbol. The second training symbol contains a known sequence on even subcarriers.

Since we did not use CP in the first training symbol, property of the circular convolution with the channel coefficients is not retained any more. But the first symbol contains two identical parts, only even subcarriers contain the information and all odd subcarriers are zeroes. The second part of the first training symbol retains property of the circular convolution with the channel coefficients. To estimate the CFO integer part we perform an $N/2$ point FFT on the second half of the first symbol in order to get the data on even subcarriers.

After the fraction CFO correction (by multiplying the samples r_k by $e^{-j2\pi\hat{\gamma}_f k/N}$), we obtain the corrected samples corresponding to the two training symbols as:

$$\begin{aligned} r_n e^{-j2\pi n \hat{\gamma}_f / N} & \quad n = 0, \dots, N-1 \\ r_{n+N+g} e^{-j2\pi(n+N+g) \hat{\gamma}_f / N} & \quad n = 0, \dots, N-1 \end{aligned}$$

Let $x_{1,k}$ and $x_{2,k}$ be the FFT's of $r_n e^{-j2\pi n \hat{\gamma}_f / N}$ and $r_{n+N+g} e^{-j2\pi(n+N+g) \hat{\gamma}_f / N}$. Let $\{c_k\}$ be the ratio sequence on the even frequencies of the two training symbols.

$$c_k = X_{1,k} / X_{2,k} \quad k = 0, 2, \dots, N-2 \quad (4.9)$$

where $X_{1,k}$ and $X_{2,k}$ are frequency training sequences of the first and second training symbol respectively and k is the index of the subcarrier. The number of the even positions shifted can be calculated by finding $\hat{g}$ that maximizes

$$G(g) = \frac{|\sum_{k \in \mathbf{X}} x_{2,k+2g}^* c_k^* x_{1,k+2g}|^2}{2(\sum_{k \in \mathbf{X}} |x_{1,k}|^2)^2} \quad (4.10)$$

where $\mathbf{X}$ is the set of indices for the even frequency components and $\mathbf{X} = \{0, 2, \dots, N-2\}$. The estimated integer part of the CFO is

$$\hat{g} = \arg \max_g \{G(g)\} \quad (4.11)$$

The estimated carrier frequency offset is obtained as

$$\hat{\gamma} = \hat{\gamma}_f + 2\hat{g} \quad (4.12)$$

where $\hat{\gamma}_f$ is given by (4.8). If we compare (4.9) and (4.10) with (3.9) and (3.10), we can find a slight difference of definitions of c_k and v_k . Since we add G zeroes in the first training symbol under ISI or fading channels, the frequency domain training sequence $X_{1,k}$ would not be a constant envelop sequence. The sample values on some subcarriers might be small. To reduce the effect of large value v_k to $B(g)$, we use the c_k and $G(g)$ instead of the v_k and $B(g)$.

If we multiply the received sample sequence r_k with $e^{-j2\pi\hat{\gamma}k/N}$, the CFO is removed.

- Step 4) Fine Time Synchronization

If the accuracy of the coarse time synchronization is not enough for the system demand, the fine time synchronization method can be used to achieve better

performance for the timing estimation. The price to pay is the computation complexity. First, we have to remove the CFO effect by multiplying the received sample sequence r_k with $e^{-j2\pi\hat{\gamma}k/N}$. The carrier frequency corrected time domain sample is

$$r_k^c = r_k e^{-j2\pi\hat{\gamma}k/N}. \quad (4.13)$$

Secondly, we calculate the cross-correlation of the carrier frequency corrected sequences $\{r_k^c\}$ and the transmitted preamble sequences. Since the first training symbol does not have a cyclic prefix, we use the cross-correlation function of the carrier frequency corrected sequences $\{r_k^c\}$ and the second time training sequences excluding CP $\{b_k\}$ obtained as follows:

$$\psi_n = \sum_{k=0}^{N-1} b_k^* r_{n+k}^c, \quad n \in [-G/2 + \hat{d}_{coarse} + N + 1.5G, \hat{d}_{coarse} + N + 1.5G + G/2] \quad (4.14)$$

From the central limit theorem (CLT), ψ_n is a Gaussian RV. We can also use the cross correlation of the carrier frequency corrected sequences $\{r_k^c\}$ and two training sequences excluding prefix and postfix by length $2N$ (In that later case, we have to remove G samples in the middle of $\{r_k^c\}$). The accuracy of timing estimation and complexity would be increased. In this thesis, since one symbol length cross correlation is enough for the system demand, we only use one symbol length cross-correlation to do the fine time estimation. All analysis and simulations consider this situation.

To get $\hat{d}_{fine}$, we search the first point $\hat{n}$ in $[-G/2 + \hat{d}_{coarse} + N + 1.5G, \hat{d}_{coarse} + N + 1.5G + G/2]$ (from smallest to greatest) whose $|\psi_{\hat{n}}|$ is greater than a threshold

η . Then we obtain fine time synchronization using

$$\hat{d}_{fine} = \hat{n} - N - 1.5G$$

Appendix B presents the methodology how to set the threshold for the fine time synchronization. From appendix B, we set

$$\begin{aligned} \eta_{AWGN} &= R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} \\ \eta_{ISI} &= \begin{cases} 0.2R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} > 5dB \\ 0.3R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} \leq 5dB \end{cases} \\ \eta_{Rayleigh} &= \begin{cases} 0.2R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} > 5dB \\ 0.3R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} \leq 5dB \end{cases} \end{aligned}$$

The complexity of the cross-correlation method depends partly on the search range for d_{fine} . Simulations (not presented in this thesis) have shown that increasing the range beyond $[-G/2 + \hat{d}_{coarse}, \hat{d}_{coarse} + G/2]$ did not improve significantly the accuracy of the timing estimator of the proposed method. Therefore to limit the computational complexity of the method, we propose to search d_{fine} in $[-G/2 + \hat{d}_{coarse}, \hat{d}_{coarse} + G/2]$. For SC's method, we can also implement the fine time synchronization, but the searching range should be $[-G + \hat{d}_{coarse}^{sc}, \hat{d}_{coarse}^{sc} + G]$ to get meaningful results. Comparing the range of the two methods, the proposed method has less complexity than SC's method.

4.1.3 Analysis of effect of the FIP in an AWGN channel

For simplicity, we consider an AWGN channel with unmodified $R_p(d)$. Analysis can be generalized to ISI or fading channels scenarios with $R_f(d)$ given by (3.12). We define the Random Variable (RV)

$$M_d = M_p(d_{opt}) - M_p(d_{opt+1}) \quad (4.15)$$

where d_{opt} is the exact symbol start point excluding the prefix and postfix, d_{opt+1} is the exact symbol start point shifted right by one sample and $M_p(d)$ is given in (4.3). Let each complex sample $r_m = s_m + n_m$ be made up of a signal and noise component. Let the variance of the real and imaginary components be $E\{\Re\{s_m\}^2\} = E\{\Im\{s_m\}^2\} = \sigma_s^2/2$, and $E\{\Re\{n_m\}^2\} = E\{\Im\{n_m\}^2\} = \sigma_n^2/2$, respectively. SNR is σ_s^2/σ_n^2 . From [6], we know the $|P_p(d)|$, $R_p(d)$, and $M_p(d)$ can be approximated by Gaussian RVs due to the central limit theorem. Since $|P_p(d)|$ and $R_p(d)$ have much larger means than their variances, $|P_p(d)|^2$ and $R_p^2(d)$ can also be approximated by Gaussian RVs with mean $L_p^2\sigma_s^4$, and $L_p^2(\sigma_s^2 + \sigma_n^2)^2$ respectively, also quite larger than their variances. Using $(1+b)/(1+c) \approx (1+b-c)$ (b and c are zero mean Gaussian RVs and their variances are quite smaller than one) we have

$$\begin{aligned} M_p(d_{opt}) &= \frac{|P_p(d_{opt})|^2}{R_p^2(d_{opt})} = \frac{L_p^2\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} \frac{1 + \delta P}{1 + \delta R} \approx \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2} (1 + \delta P - \delta R) \\ &= \frac{|P_p(d_{opt})|^2}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} - \frac{R_p^2(d_{opt})\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4} + \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2} \end{aligned} \quad (4.16)$$

where $1 + \delta P \triangleq |P_p(d_{opt})|^2/(L_p^2\sigma_s^4)$ and $1 + \delta R \triangleq R_p^2(d_{opt})/(L_p^2(\sigma_s^2 + \sigma_n^2)^2)$. Similarly

$$M_p(d_{opt+1}) \approx \frac{|P_p(d_{opt+1})|^2}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} - \frac{R_p^2(d_{opt+1})\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4} + \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2} \quad (4.17)$$

where we have used $L_p - 2 \approx L_p$ and $L_p - 1 \approx L_p$ when L_p is large. $M_p(d_{opt}), M_p(d_{opt+1})$ can be approximated by Gaussian RVs, so $M_d = M_p(d_{opt}) - M_p(d_{opt+1})$ is also a Gaussian RV given by

$$M_d \approx \frac{|P_p(d_{opt})|^2 - |P_p(d_{opt+1})|^2}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} + \frac{\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4} (R_p^2(d_{opt+1}) - R_p^2(d_{opt})) \quad (4.18)$$

To calculate the mean and variance of M_d , we have to calculate the means and variances of $P_d = |P_p(d_{opt})|^2 - |P_p(d_{opt+1})|^2$ and $R_d = R_p^2(d_{opt+1}) - R_p^2(d_{opt})$ and their covariance. Let $c_1 = \frac{1}{L_p^2(\sigma_s^2 + \sigma_n^2)^2}$ and $c_2 = \frac{\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4}$. The mean of M_d is $E\{M_d\} = c_1 \cdot E\{P_d\} + c_2 \cdot E\{R_d\}$, From appendix A, we have

$$E\{M_d\} = \begin{cases} \frac{4(L_p-1)\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} & \text{using the FIP preamble.} \\ \frac{2(L_p-1)\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} & \text{using SC's preamble.} \end{cases} \quad (4.19)$$

$$\text{and } \text{Var}(M_d) = c_1^2 \cdot \text{Var}(P_d) + c_2^2 \cdot \text{Var}(R_d) + 2 \cdot c_1 \cdot c_2 \cdot \text{Cov}(P_d, R_d)$$

$$= \begin{cases} \frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [4(L_p - 1)^2(3\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4) + 4(L_p - 1)(2\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) \cdot \\ (2\sigma_s^2\sigma_n^2 + \sigma_n^4) + 8(\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6)] + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 \\ + (32L_p - 28)\sigma_s^4\sigma_n^4] - 8\frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2) \\ + (L_p - 1)\sigma_s^4\sigma_n^2(\sigma_s^2 + 2\sigma_n^2) + (L_p + 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \text{ using the FIP preamble.} \\ \frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [(L_p - 1)^2\sigma_s^4(2\sigma_s^4 + 8\sigma_s^2\sigma_n^2 + 4\sigma_n^4) + 4(L_p - 1)[3\sigma_s^4(\sigma_s^2\sigma_n^2 + \sigma_n^4) + \\ (2\sigma_s^2\sigma_n^2 + \sigma_n^4)^2] + 8(\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6)] + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 + \\ (32L_p - 28)\sigma_s^4\sigma_n^4] - 8\frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2) + \\ (L_p - 1)\sigma_s^4\sigma_n^2(\sigma_s^2 + 2\sigma_n^2) + (L_p + 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \text{ using SC's preamble.} \end{cases} \quad (4.20)$$

When L_p is large, $Var(M_d)$ can be approximated as

$$Var(M_d) \approx \begin{cases} \frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [4(L_p - 1)^2 (3\sigma_s^6 \sigma_n^2 + \sigma_s^4 \sigma_n^4)] \\ + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2 \sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)^2] \\ - 8 \frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2 \sigma_s^4 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)] \\ \text{using the FLP preamble.} \\ \frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [(L_p - 1)^2 \sigma_s^4 (2\sigma_s^4 + 8\sigma_s^2 \sigma_n^2 + 4\sigma_n^4)] \\ + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2 \sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)^2] \\ - 8 \frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2 \sigma_s^4 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)] \\ \text{using SC's preamble.} \end{cases} \quad (4.21)$$

(4.19-4.21) show that the FLP doubles the mean of M_d (and decreases the variance of M_d when $SNR > 3dB$). Since M_d is approximated by a Gaussian RV, using the FLP decreases the probability of the start point shifting to the right side. Fig. 4.2 shows the comparison of the simulation and theoretic M_d 's mean and variance using SC and proposed preambles in AWGN channels, where $L_p = 512$, $N = 1024$ and $\sigma_s^2 = 1/N$. 100,000 simulation runs are done for each SNR.

Fig. 4.3 shows the timing metrics of different methods under a noise free and no distortion condition. The curve labeled "Proposed method" in Fig. 4.3 uses the AWGN preamble structure and the curve labeled "Without CP and postfix" uses as first training symbol only two identical parts. Park and Ren's methods are described in [9] and [10] respectively. From Fig. 4.3, we see the timing metric of the proposed method does not have the plateau of SC's method and the sub peaks of Minn's method. Its peak is quite sharp.

(a) Mean of M_d

(b) Variance of M_d

Fig. 4.2 Comparison of simulated and theoretical M_d statistic using SC and proposed preambles

Fig. 4.3 Timing metric in a noise free ideal channel.

4.2 Proposed synchronization method for MIMO-OFDM systems

From section 2.4, a MIMO-OFDM system uses spatial diversity, spatial multiplexing, or combining both to achieve better performance and/or capacity gain under frequency selective fading channels. It also needs reliable synchronization methods. In this section, we are going to extend our proposed method from OFDM systems to MIMO-OFDM systems. SC's method can be extended by generalization. In section 4.2.1, we analyze the requirements of MIMO-OFDM preambles and give a preamble structure. In section 4.2.2, we describe the proposed MIMO-OFDM synchronization method.

4.2.1 Preamble structures for MIMO-OFDM systems

If we use only one transmitter antenna, we can use the preamble discussed in section 4.1.1. If we use more than one transmitter antennas, the received signal would be a combination of the transmitted signals passed through the channel plus noise.

Now, let us analyze the optimum preamble in a multipath fading channel. Assume we use Q transmitter antennas and L receiver antennas. Let $h_{ql,m}$ be the channel impulse response at the lag m from the q^{th} transmitter antenna to the l^{th} receiver antenna. The first training symbol contains two identical parts (denoted as $[\mathbf{a}_{q,0}, \mathbf{a}_{q,0}]$ for the q^{th} transmitter). Let $\mathbf{a}_{q,k}$ denote the circular right shift k samples version of $\mathbf{a}_{q,0}$. From (2.53), the received signal of the l^{th} receiver antenna corresponding to the first symbol without noise is composed of two parts:

$$\begin{aligned} r_{l,I} &= \sum_{q=1}^Q \sum_{m=0}^{M-1} h_{ql,m} \mathbf{a}_{q,m} \\ r_{l,II} &= e^{j\phi} \left(\sum_{q=1}^Q \sum_{m=0}^{M-1} h_{ql,m} \mathbf{a}_{q,m} \right) \end{aligned} \quad (4.22)$$

where ϕ is the phase difference between the two identical parts. The correlation function of the two parts for the l^{th} receive antenna $P_l(d)$ defined as¹:

$$P_l(d) = \sum_{m=0}^{N/2-1} r_{l,d+m}^* r_{l,d+m+N/2} \quad (4.23)$$

where $r_{l,k}$ is given by (2.53). Let d_{opt} be the exact start point of the symbol excluding prefix and postfix. $P_l(d_{opt})$ can be expressed as the scalar product of the two vectors

¹MIMO version of (4.1).

$\mathbf{r}_{l,\text{I}}$ and $\mathbf{r}_{l,\text{II}}$:

$$\begin{aligned}
 P_l(d_{\text{opt}}) &= \mathbf{r}_{l,\text{I}}^\dagger \mathbf{r}_{l,\text{II}} \\
 &= e^{j\phi} \left(\sum_{q=1}^Q \sum_{m=0}^{M-1} |h_{ql,m}|^2 \mathbf{a}_{q,m}^\dagger \mathbf{a}_{q,m} + \sum_{q=1}^Q \sum_{p=1}^Q \sum_{m=0}^{M-1} \sum_{n=0, n \neq m}^{M-1} h_{ql,m}^* h_{pl,n} \mathbf{a}_{q,m}^\dagger \mathbf{a}_{p,n} \right)
 \end{aligned} \tag{4.24}$$

where $\dagger$ is Hermitian conjugation of a matrix or a vector.

In (4.24), the term $\sum_{q=1}^Q \sum_{m=0}^{M-1} |h_{ql,m}|^2 \mathbf{a}_{q,m}^\dagger \mathbf{a}_{q,m}$ always gives a positive contribution, but the second term $\sum_{q=1}^Q \sum_{p=1}^Q \sum_{m=0}^{M-1} \sum_{n=0, n \neq m}^{M-1} h_{ql,m}^* h_{pl,n} \mathbf{a}_{q,m}^\dagger \mathbf{a}_{p,n}$ might give a negative or positive contribution depending on the channel coefficients and training symbols. To eliminate the second term effect, we might set it to be zero, i.e., the training sequences of different antennas should be orthogonal and shift-orthogonal and each training sequence of different antennas should have a perfect autocorrelation function. Since we assume the channel tap length is no longer than the guard interval (CP length), we use Chu sequences and each circular shift G version of Chu sequences to be the half part of the first training symbol for each transmitter antenna. Let the first training symbol of the first transmitter antenna be $[\mathbf{a}_{1,0}, \mathbf{a}_{1,0}]$, where $\mathbf{a}_{1,0}$ is a Chu sequence. And let the first training symbol of the q^{th} transmitter antenna be $[\mathbf{a}_{1,(q-1)G}, \mathbf{a}_{1,(q-1)G}]$, where $\mathbf{a}_{1,(q-1)G}$ is circular right shift $(q-1)G$ samples version of $\mathbf{a}_{1,0}$. If the number of transmitter antennas Q is greater than $\lfloor \frac{N}{2G} \rfloor$, we cannot achieve the shift-orthogonality.

For the proposed method, to make the training sequences of different antennas orthogonal and shift-orthogonal with each other, we have to modify the first training symbol structure for ISI or fading channels. In addition to the two identical parts $[\mathbf{a}_{q,0}, \mathbf{a}_{q,0}] = [\mathbf{a}_{1,(q-1)G}, \mathbf{a}_{1,(q-1)G}]$ which are Chu sequences of length $L_p = N/2$, we

add a CP and a Flipped Postfix (FIP) of lengths $G/2$ to the first symbol of each transmitter antenna (see Fig. 4.1 d).

Let us denote the second training symbol (excluding CP) for the q^{th} transmitter antenna as $\mathbf{b}_{q,0}$. In OFDM systems, the differences of two training symbols on the even subcarriers can be used to estimate the integer part CFO. More specifically, using an approach similar to SC'method, the CFO integer part is obtained using the ratio sequence c_k defined as

$$c_k = X(q, 1, k)/X(q, 2, k) \quad k = 0, 2, \dots, N-2$$

where $\{X(q, 1, k)\}$ is the FFT of $[\mathbf{a}_{1,(q-1)G}, \mathbf{a}_{1,(q-1)G}]$ and $\{X(q, 2, k)\}$ is the FFT of $\mathbf{b}_{q,0}$, and k is the index of the subcarrier. For MIMO systems with multiple transmitter antennas, the $\{c_k\}$ must be identical for all transmitter antennas in order to get a better estimation for the CFO integer part. This can be achieved by letting $\mathbf{b}_{q,0} = \mathbf{b}_{1,(q-1)G}$, where $\mathbf{b}_{1,(q-1)G}$ is the circular shift $(q-1)G$ samples version of $\mathbf{b}_{1,0}$ and $\mathbf{b}_{1,0}$ is chosen as a Chu sequence. Since the second symbol shifts by the same number of samples as the first symbol for each transmitter antenna, by the circular time shifting property of DFT², the sequence $\{c_k\}$ is the same for all transmitter antennas.

4.2.2 Implementing the proposed synchronization method for MIMO-OFDM systems

Similar as the proposed synchronization method for OFDM systems, the proposed synchronization method for MIMO-OFDM systems has four steps:

²From [26], the circular time shifting property of the DFT is: Let $g[n]$ be a length N sequence and $G[k]$ be the N -point DFT of $g[n]$, then $e^{-j2\pi kn_0/N}G[k]$ is the N -point DFT of $g[(n-n_0)_N]$. where $(n-n_0)_N$ is $\text{mod}((n-n_0)/N)$.

- Step 1) Coarse Time Synchronization

First, we do the coarse time synchronization for each receiver antennas. Let the l^{th} receiver antenna timing metric be M_l :

$$M_l(d) = \frac{|P_l(d)|^2}{R_l^2(d)} \quad (4.25)$$

where $P_l(d)$ is given by (4.23) and

$$R_l(d) = \frac{1}{2} \sum_{m=0}^{N-1} |r_{l, d+m}|^2 \quad (4.26)$$

where $r_{l,k}$ is the k^{th} complex sample taken at the output of A/D of the l^{th} receiver antenna given by (2.53). Note that $R_l(d)$ corresponds to the so-called modified “R” following [7] (see (3.12)). The coarse time estimation of the l^{th} receive antenna is

$$\hat{d}_{l,coarse} = \arg \max_d \{M_l(d)\} \quad (4.27)$$

and the estimated SNR of the l^{th} receiver antenna is [6]

$$\widehat{SNR}_l = \frac{\sqrt{M_l(\hat{d}_{l,coarse})}}{1 - \sqrt{M_l(\hat{d}_{l,coarse})}} \quad (4.28)$$

The timing metric for MIMO-OFDM systems is defined:

$$M_{MIMO}(d) = \sum_{l=1}^L w_l M_l(d) \quad (4.29)$$

where w_l is the weight of each receiver antenna timing metric. If $w_l = 1$, the system implements equal gain combining. $w_l = \widehat{SNR}_l$ corresponds to maximum ratio combining. The coarse estimation of the start point is taken at the peak

of the timing metric $M_{MIMO}(d)$ and it is given by

$$\hat{d}_{coarse} = \arg \max_d \{M_{MIMO}(d)\} \quad (4.30)$$

- Step 2) Estimating the Fraction Part of the CFO

To estimate the fraction part of the CFO, we define

$$P_{MIMO}(\hat{d}_{coarse}) = \sum_{l=1}^L w_l P_l(\hat{d}_{coarse}) \quad (4.31)$$

where w_l is the weight of each receiver antenna defined as in step 1. Similar to the OFDM system, the estimated fraction part of the CFO is

$$\hat{\gamma}_f = \frac{\angle P_{MIMO}(\hat{d}_{coarse})}{\pi} \quad (4.32)$$

In this step, the frequency offset estimation is up to ± 1 subcarrier spacing. To remove the fraction part of the CFO, we should multiply the received sample sequences with $e^{-j2\pi\hat{\gamma}_f k/N}$.

- Step 3) Estimating the Integer Part of the CFO

The actual frequency offset is

$$\gamma = \gamma_f + 2g \quad (4.33)$$

where g is an integer. To estimate the integer part of the CFO g , we have to use the second training symbol. After the fraction CFO correction (by multiplying the samples by $e^{-j2\pi\hat{\gamma}_f k/N}$), let FFT of the l^{th} receiver antenna's first symbol be $x_{l,1,k}$ and the FFT of the l^{th} receiver antenna's second symbol be $x_{l,2,k}$. Let $\{c_k\}$

be the ratio sequence on the even frequencies of the two training symbols.

$$c_k = X(q, 1, k)/X(q, 2, k) \quad k = 0, 2, \dots, N-2 \quad (4.34)$$

where $X(q, 1, k)$ and $X(q, 2, k)$ are frequency training sequences of the first and second training symbol from the q^{th} transmitter antenna respectively and k is index of the subcarrier. As discussed in section 4.2.1, all transmitter antennas have same ratio sequences $\{c_k\}$. Let

$$G_l(g) = \frac{|\sum_{k \in \mathbf{X}} x_{l,2,k+2g}^* c_k^* x_{l,1,k+2g}|^2}{2(\sum_{k \in \mathbf{X}} |x_{l,1,k}|^2)^2} \quad (4.35)$$

where $\mathbf{X}$ is the set of indices for the even frequency components given by $\mathbf{X} = \{0, 2, \dots, N-2\}$. The number of the even positions shifted can be calculated by finding $\hat{g}$ that maximizes

$$G_{MIMO}(g) = \sum_{l=1}^L w_l G_l(g) \quad (4.36)$$

where w_l is the weight of each receiver antenna defined as in step 1. The estimated integer part of the CFO is

$$\hat{g} = \arg \max_g \{G_{MIMO}(g)\} \quad (4.37)$$

The estimated CFO is

$$\hat{\gamma} = \hat{\gamma}_f + 2\hat{g} \quad (4.38)$$

If we multiply the received sample sequences with $e^{-j2\pi\hat{\gamma}k/N}$, the CFO is removed.

- Step 4) Fine Time Synchronization

First, we have to remove the CFO effect by multiplying the received sample sequences of each receiver antenna with $e^{-j2\pi\hat{\gamma}k/N}$. The k^{th} carrier frequency corrected time domain sample at the l^{th} receiver antenna is

$$r_{l,k}^c = r_{l,k} e^{-j2\pi\hat{\gamma}k/N} \quad (4.39)$$

Secondly, we calculate the cross-correlation of the carrier frequency corrected time domain sequences $\{r_{l,k}^c\}$ and the transmitted preamble sequences of each transmitter antenna and average over all transmitter antennas to obtain:

$$\psi'_{l,n} = \sum_{q=1}^Q \left| \sum_{k=0}^{N-1} b_{q,k}^* r_{l,n+k}^c \right|$$

$$n \in [-G/2 + \hat{d}_{coarse} + N + 1.5G, \hat{d}_{coarse} + N + 1.5G + G/2] \quad (4.40)$$

where $\mathbf{b}_{q,0} = [b_{q,0}, \dots, b_{q,N-1}]$ is the second time training sequence of the q^{th} transmitter antenna excluding CP. Since the training sequences for different transmitter antennas are orthogonal, only one cross-correlation is needed per receiver antenna. Using the first transmitter antenna sequences $\{b_{1,k}\}$ to do the correlation, we have

$$\psi_{l,n} = \left| \sum_{k=0}^{N-1} b_{1,k}^* r_{l,n+k}^c \right|$$

$$n \in [-G/2 + \hat{d}_{coarse} + N + 1.5G, \hat{d}_{coarse} + N + 1.5G + G/2] \quad (4.41)$$

For each receiver antenna, we search the first point $\hat{n}$ in $[-G/2 + \hat{d}_{coarse} + N + 1.5G, \hat{d}_{coarse} + N + 1.5G + G/2]$ (from smallest to greatest) whose $\psi_{l,\hat{n}}$ is greater

than a threshold $\eta_{l, \text{Rayleigh}}$. Then we obtain fine time synchronization using $\hat{d}_{l, \text{fine}} = \hat{n} - N - 1.5G$ and

$$\hat{d}_{\text{fine}} = \left\lfloor \frac{1}{L} \sum_{l=1}^L \hat{d}_{l, \text{fine}} \right\rfloor \quad (4.42)$$

The choice of threshold for each receiver antenna is obtained similarly to the threshold for SISO OFDM systems. Assuming that the first tap power would be greater than -14dB (or equivalently $|h_{1l,0}| > 0.2$), it can be shown that a good threshold is $\eta_{l, \text{Rayleigh}} = 0.5E[\psi_{l, d_{\text{opt}}}] \approx 0.2R_l(\hat{d}_C)\sqrt{M_l(\hat{d}_C)}$. Since we can estimate SNR from the peak value of the timing metric, we can also set up a look up table depending on the estimated SNR for setting the threshold. For example, in the simulations of this thesis we set

$$\eta_{l, \text{Rayleigh}} = \begin{cases} 0.2R_l(\hat{d}_C)\sqrt{M_l(\hat{d}_C)} & \text{if } \widehat{\text{SNR}}_l > 5\text{dB} \\ 0.3R_l(\hat{d}_C)\sqrt{M_l(\hat{d}_C)} & \text{if } \widehat{\text{SNR}}_l \leq 5\text{dB} \end{cases}$$

Chapter 5

Simulation results

The performance of the proposed method is evaluated by computer simulations in terms of the mean and variance of the timing estimator and CFO estimator under AWGN, ISI or fading channels.

5.1 Simulation parameters

For all the simulations, the system uses 1024 subcarriers and 1024 point IFFT/FFT. The length of CP is $G = 102$ samples. The CFO is 12.4 subcarrier spacing. We use QPSK modulation and 10,000 simulations are run for each SNR. We did not consider the windowing in the simulations. All simulations consider a continuous communication scenario since its self interference is worse than the one in burst communication. For a fair comparison, the SNR is defined as the average signal power and the average noise power ratio on one OFDM symbol excluding the postfix and prefix.

Recall the OFDM system model in section 2.5.1 and MIMO-OFDM systems model in section 2.5.2. For the simplicity of computer simulations, we set $n_\epsilon = 0$ and $\theta_0 = 0$ in (2.49) and (2.53).

For OFDM systems, we compare the SC's, Minn's and proposed method in terms of the mean and variance of the timing estimator and CFO estimator under AWGN, ISI or Rayleigh fading channels. For the ISI channels, we use the ISI channel models described in [7] and [6]. We simulate two types of ISI channels. Both ISI channels have 16 paths with an equal path spacing of 4 samples. The delays τ_n are 0, 4, 8, $\dots$, 60 samples and path gains are given by

$$h_n = \frac{e^{-\tau_n/60}}{\sqrt{\sum_{k=1}^{16} e^{-\tau_k/30}}}, \quad n = 1, 2, \dots, 16. \quad (5.1)$$

The phases of all paths are zeroes in the first ISI channel and uniformly distributed in $[0, 2\pi)$ in the second ISI channel.

For Rayleigh fading channels, we use the tapped delay line channel model described in section 2.1.4. From section 2.1.1, the tap weights $\{h_n(t)\}$ are zero mean complex Gaussian random processes and statistically independent. The channel power delay profile is modeled as an exponential power delay profile, truncated to 16 paths (see section 2.1.1). All simulated Rayleigh fading channels have 16 paths with delays τ_n of 0, 4, 8, $\dots$, 60 samples and mean power of each path obtained from (2.3) and given by

$$E\{|h_n(t)|^2\} = \frac{e^{-\tau_n/30}}{\sqrt{\sum_{k=1}^{16} e^{-\tau_k/30}}}, \quad n = 1, 2, \dots, 16. \quad (5.2)$$

Both time-invariant Rayleigh fading channels ($\{h_n(t)\}$ does not change for all symbols and samples) and time-variant Rayleigh fading channels ($\{h_n(t)\}$ changes each sample period depending on the maximum Doppler frequency) are simulated. We assume all taps have the same (Jake/Clarke) Doppler power spectrum given by (2.6). To generate the Doppler power spectrum given by (2.6), we can use the simulation method given by [27], but it is difficult to create multiple uncorrelated fading waveforms. Therefore,

we use modified Jakes fading model provided by [28]. The carrier frequency of the system is 5GHz . The total bandwidth of the system is $W = 50\text{MHz}$ and the signal bandwidth is $W_s = 45.47\text{MHz}$.

In all OFDM simulation figures, the label “modified R” in the symbol timing estimator curves means that $R_f(d)$ given by (3.12) is used instead of $R_{SC}(d)$, $R_{Minn}(d)$ or $R_p(d)$ in each timing metric. Note that Minn’s [7] did not propose any method to estimate the integer part of the CFO hence Minn’s method is not included in the figures presenting performance of estimators of the entire CFO (e.g. fraction plus integer part of CFO).

For MIMO-OFDM systems, we only compare the SC method with the proposed method. We use the same parameters as for OFDM systems. The power of the transmitter antennas is given by (2.52) for a fair comparison. All the timing metrics use $R_l(d)$ (given by (4.26)) instead of $R_{SC}(d)$ or $R_p(d)$.

For the CFO estimator, for OFDM systems the label “fine time” means using $\frac{\angle P_p(\hat{d}_{fine})}{\pi}$ to estimate the fraction part of the CFO instead of $\frac{\angle P_p(\hat{d}_{coarse})}{\pi}$ (see (4.8)).

5.2 Simulation results in an AWGN channel

Figs. 5.1, 5.2 and 5.3 present the simulation results in terms of the means and variances of the symbol timing and CFO estimators for SC’s, Minn’s and proposed methods in an AWGN channel.

Fig. 5.1 shows the performance of the symbol timing estimators for different methods in an AWGN channel. Fig. 5.1(a) shows that the mean value of SC’s coarse timing estimator is quite far from the exact start point (“0 sample value”) and on the left side of it due to the CP. For example, Fig. 5.1(a) shows for SC’s method at $SNR = 2\text{dB}$, $\text{mean} = -50$. It means that the estimated start point has a mean which is 50 samples

before the exact start point. The mean values of the other two estimators and SC's fine timing estimator are almost at the exact start point. Comparing all coarse timing estimators, the proposed coarse timing estimator has the smallest variance (see Fig. 5.1(b)). The variance of the proposed fine timing estimator is less than 10^{-4} when $SNR \geq -2\text{dB}$ and the variance of the SC's fine timing estimator is less than 10^{-4} when $SNR \geq 2\text{dB}$. Fig. 5.1 also shows that all modified methods have smaller variance than the original methods, which is expected since the variance of $R_f(d)$ is half of the variance of $R_{SC}(d)$, $R_{Minn}(d)$ or $R_p(d)$ from the law of large number.

Fig. 5.2 and Fig. 5.3 show the performance of the CFO estimators for different methods in an AWGN channel. The variance of the proposed CFO estimator is slightly smaller than the one of SC's CFO estimator using the coarse time point. Using the fine time point or the coarse time point to estimate CFO yields almost same performance. For Minn's CFO estimator, since its repeated part length is half of the SC's one, its mean is not accurate at low SNR and its variance is quite large. It has the worst performance among all three CFO estimators. The integer part of the CFO is estimated correctly for 10,000 runs by SC's and proposed methods for each simulated SNR.

From the discussion above, we conclude that the proposed method has the best performance in an AWGN channel.

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.1 Performance of the timing estimator in an AWGN channel

(a) Mean of the fraction part of the CFO

(b) Variance of the fraction part of the CFO

Fig. 5.2 Performance of the CFO (fraction part) estimator in an AWGN channel

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.3 Performance of the CFO estimator in an AWGN channel

5.3 Simulation results in the first ISI channel

Figs. 5.4, 5.5 and 5.6 present the simulation results in terms of the means and variances of the symbol timing and CFO estimators for SC's, Minn's and proposed methods in the first ISI channel.

Fig. 5.4 shows the performance of the symbol timing estimators for different methods in the first ISI channel. Fig. 5.4(a) shows that the mean value of SC's coarse timing estimator shifts to the left side of the exact start point ('0' sample) due to the CP. The mean value of Minn's timing estimator shifts to the right side of the exact start point due to multipath effect. Note that Minn's results presented in Figs. 5.4, 5.5 and 5.6 were obtained by considering a training sequence (among all possible random sequences) that yielded performance similar to results presented in [7]. It was observed that this choice corresponded to almost the best training sequence in terms of performance of the timing estimator variance. If one uses Chu sequences instead then performance over the first ISI channel would be similar to the performance over the second ISI channel which will be presented in section 5.4. Fig. 5.4(a) shows that the mean values of the proposed coarse timing estimators (unmodified and modified "R") are quite close to the exact start point and always on the left side of it. As expected, mean values get closer to the exact start point as SNR increases. The mean values of the two fine timing estimators (SC and proposed) are almost at the exact start point. Since we pad $G/2$ zeroes at the beginning of the training symbol, the variance of the proposed coarse timing estimator is the smallest one at high SNR and slightly larger than Minn's estimator at low SNR. But since the mean value of the proposed estimator is on the left side of the exact start point, most of the estimated start points would be on the left of it and since we use CP for data transmission after the training, the resulting ISI would be small.

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.4 Performance of the timing estimator in the first ISI channel (*: uses training sequences similar to [7])

Fig. 5.4(b) shows that the proposed and SC's fine timing estimators have quite small variance. The variance of the proposed fine timing estimator is less than 10^{-4} when $SNR \geq 10\text{dB}$ and the variance of the SC's fine timing estimator is less than 10^{-4} when $SNR \geq 5\text{dB}$. Similar to AWGN results, Fig. 5.4(b) also shows that all modified methods have smaller variance than the original methods as expected.

Fig. 5.5 and Fig. 5.6 show the performance of the CFO estimators for different methods in the first ISI channel. The proposed CFO estimator has almost the same performance as SC's CFO estimator. Using the fine time point or the coarse time point to estimate CFO yields almost same performance. For Minn's CFO estimator, since its repeated part length is half of the SC's one, its mean is not accurate and its variance is quite large (see Fig. 5.5). It has the worst performance among all three CFO estimators in the first ISI channel, which was also observed in an AWGN channel. The integer part of the CFO is estimated correctly for 10,000 runs by SC's and proposed methods for each simulated SNR.

From the discussion above, we conclude that the SC's method (fine time) has the best performance in the first ISI channel.

(a) Mean of the fraction part of the CFO

(b) Variance of the fraction part of the CFO

Fig. 5.5 Performance of the CFO (fraction part) estimator in the first ISI channel (*: uses training sequences similar to [7])

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.6 Performance of the CFO estimator in the first ISI channel

5.4 Simulation results in the second ISI channel

Figs. 5.7, 5.8 and 5.9 present the simulation results in terms of the means and variances of the symbol timing and CFO estimators for SC's, Minn's and proposed methods in the second ISI channel.

Fig. 5.7 shows the performance of the symbol timing estimators for different methods in the second ISI channel. Fig. 5.7 shows that effects observed for the first ISI channel are also observed for the second ISI channel. For example, it is seen that the mean values of the proposed coarse timing estimators are quite close to the exact start point and always on the left side of it, whereas they are shifted to the left for SC's method (due to CP) and to the right for Minn's method. The mean values of the proposed and SC fine timing estimators are almost at the exact start point. Fig. 5.7(b) shows that the proposed and SC's fine timing estimators have quite small variance (less than 10^{-4} when $SNR \geq 5\text{dB}$ for the proposed estimator and less than 10^{-4} when $SNR \geq 0\text{dB}$ for SC's estimator). Fig. 5.7(b) also shows that all modified methods have smaller variance than the original methods in the second ISI channel as well.

Similarly, as seen in Fig. 5.8 and Fig. 5.9 performance of CFO estimators in the second ISI channel exhibits same trends as for the first ISI channels, except for Minn's CFO mean. The difference of behavior of CFO mean obtained by Minn's method between the first and second ISI channel is in fact due to the different choice of training sequence for the two channels (training sequence similar to [7] for the first ISI channel and Chu sequence for the second ISI channel). Preliminary simulations not shown in this thesis have showed that Minn's method performance is quite sensitive to the choice of training sequences. Fig. 5.9 shows that the proposed CFO estimator has almost the same performance as SC's CFO estimator. The integer part of the

CFO is estimated correctly for 10,000 runs by SC's and proposed methods for each simulated SNR. From the discussion above, we conclude that the SC's method (fine time) has the best performance in the second ISI channel. Note that it will be shown in section 5.5 that the proposed method (fine time) is better than SC's method (fine time) in Rayleigh fading channels. Therefore, it is to be expected that performance of the proposed method (fine time) might also outperform SC's method (fine time) in other ISI channels.

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.7 Performance of the timing estimator in the second ISI channel (*: The variance of SC's fine timing estimator is less than 10^{-4} when $SNR \geq 0\text{dB}$)

(a) Mean of the fraction part of the CFO

(b) Variance of the fraction part of the CFO

Fig. 5.8 Performance of the CFO (fraction part) estimator in the second ISI channel

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.9 Performance of the CFO estimator in the second ISI channel

5.5 Simulation results in the Rayleigh fading channel

5.5.1 Simulation results in the time-invariant Rayleigh fading channel

Fig. 5.10, 5.11 and 5.12 present the simulation results in terms of the means and variances of the symbol timing and CFO estimators for SC's, Minn's and proposed methods in the time-invariant Rayleigh fading channel.

Fig. 5.10 shows the performance of the symbol timing estimators for different methods in the time-invariant Rayleigh fading channel. Fig. 5.10(a) shows that the mean value of the proposed coarse timing estimator is quite closer to the exact start point than SC and Minn's methods. The mean values of the proposed and SC fine timing estimators slightly shift to the right of the exact start point because the first tap might be in deep fade and smaller than the threshold in some simulation runs. Fig. 5.10(b) shows that the proposed coarse timing estimator has quite small variance in the time-invariant Rayleigh fading channel. If we need a more accurate estimator, we can implement fine timing synchronization and the price is the computational complexity. The proposed fine timing estimator has better performance (in terms of variance) than SC's fine timing estimator at low SNR due to the FIP-based preamble. Similar to AWGN and ISI channels results, Fig. 5.10(b) also shows that all modified methods perform better than the original methods.

Fig. 5.11 and Fig. 5.12 show the performance of the CFO estimators for different methods in the time-invariant Rayleigh fading channel. The variance of the proposed CFO estimator is similar to SC's CFO estimator at low SNR and slightly bigger at high SNR. Using the fine time point or the coarse time point to estimate CFO yields almost same performance for both the proposed and SC's methods. Minn's CFO estimator has the worst performance among all three CFO (fraction part) estimators due to the short length of its repeated part. The integer part of the CFO is estimated

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.10 Performance of the timing estimator in the time-invariant Rayleigh fading channel

(a) Mean of the fraction part of the CFO

(b) Variance of the fraction part of the CFO

Fig. 5.11 Performance of the CFO (fraction part) estimator in the time-invariant Rayleigh fading channel

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.12 Performance of the CFO estimator in the time-invariant Rayleigh fading channel

correctly for 10,000 runs by SC's and proposed methods for each simulated SNR.

From the discussion above, we conclude that the proposed method (fine time) has the best performance in the time-invariant Rayleigh fading channel.

5.5.2 Simulation results in the time-variant Rayleigh fading channel

Figs. 5.13-5.21 present the simulation results in terms of the means and variances of the symbol timing and CFO estimators for SC's, Minn's and proposed methods in the time-variant Rayleigh fading channel with different maximum Doppler frequencies. Figs. 5.13-5.15, Figs. 5.16-5.18 and Figs. 5.19-5.21 presents results for a low vehicle speed ($v = 10\text{km/hr}$), a moderate vehicle speed ($v = 70\text{km/hr}$) and a high vehicle speed ($v = 250\text{km/hr}$) respectively. Figs. 5.13-5.15 results (mean and variance of the estimated start point, variance of the estimated CFO) are very close to the results obtained in Figs. 5.10-5.12 for the time invariant Rayleigh fading channel, which was to be expected for such a low vehicle speed ($v = 10\text{km/hr}$).

For each maximum Doppler frequency, the proposed fine timing estimator has the best performance. The variance of the proposed CFO estimator is similar to SC's CFO estimator one at low SNR and slightly bigger at high SNR. Performance when the fine time point or the coarse time point is used to estimate CFO is almost the same since $N = 1024$ is large. Minn's CFO estimator has the worse performance among all three CFO estimators due to the short length of its repeated part.

In Fig. 5.22 and Fig. 5.23, we compare the performance of the proposed timing and CFO estimators with different maximum Doppler frequencies. The timing estimators have almost the same performance when the maximum Doppler frequency changes. When the maximum Doppler frequency increases, the variance of CFO estimator also increases. The CFO estimator is more sensitive to the rate of the channel change. The integer part of the CFO is estimated correctly for 10,000 runs by SC's and proposed

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.13 Performance of the timing estimator in the time-variant Rayleigh fading channel, $v = 10\text{km/hr}$, $f_m = 46.3\text{Hz}$

(a) Mean of the fraction part of the CFO

(b) Variance of the fraction part of the CFO

Fig. 5.14 Performance of the CFO (fraction part) estimator in the time-variant Rayleigh fading channel, $v = 10\text{km/hr}$, $f_m = 46.3\text{Hz}$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.15 Performance of the CFO estimator in the time-variant Rayleigh fading channel, $v = 10 \text{ km/hr}$, $f_m = 46.3 \text{ Hz}$

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.16 Performance of the timing estimator in the time-variant Rayleigh fading channel, $v = 70\text{km/hr}$, $f_m = 324.1\text{Hz}$

(a) Mean of the fraction part of the CFO

(b) Variance of the fraction part of the CFO

Fig. 5.17 Performance of the CFO (fraction part) estimator in the time-variant Rayleigh fading channel, $v = 70\text{km/hr}$, $f_m = 324.1\text{Hz}$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.18 Performance of the CFO estimator in the time-variant Rayleigh fading channel, $v = 70 \text{ km/hr}$, $f_m = 324.1 \text{ Hz}$

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.19 Performance of the timing estimator in the time-variant Rayleigh fading channel, $v = 250\text{km/hr}$, $f_m = 1157.4\text{Hz}$

(a) Mean of the fraction part of the CFO

(b) Variance of the fraction part of the CFO

Fig. 5.20 Performance of the CFO (fraction part) estimator in the time-variant Rayleigh fading channel, $v = 250\text{km/hr}$, $f_m = 1157.4\text{Hz}$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.21 Performance of the CFO estimator in the time-variant Rayleigh fading channel, $v = 250 \text{ km/hr}$, $f_m = 1157.4 \text{ Hz}$

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.22 Performance of the timing estimator in the time-variant Rayleigh fading channel for various maximum Doppler frequencies

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.23 Comparing the performance of the CFO estimator in the time-variant Rayleigh fading channel for various maximum Doppler frequencies

methods for each simulated SNR.

Therefore, it is seen that the proposed method is quite robust to channel time variations.

5.6 Simulation results for MIMO-OFDM systems

5.6.1 Simulation results in the time-invariant Rayleigh fading channel for MIMO-OFDM systems

Fig. 5.24 and Fig. 5.25 present the simulation results in terms of the means and variances of the symbol timing and CFO estimators for SC's and proposed methods in the time-invariant Rayleigh fading channel using two transmitter antennas and two receiver antennas.

Fig. 5.24 shows the performance of the coarse and fine timing estimators for different methods using different combining methods in the time-invariant Rayleigh fading channel for a 2×2 antenna array. Fig. 5.24(a) shows that the mean value of the estimated start point is almost the same using EGC or MRC with a slight advantage for MRC. It also shows that the mean value of the proposed coarse timing estimator is quite close to the exact start point, whereas the SC's coarse timing estimator one shifts to the left of it. The mean values of the proposed and SC fine timing estimators slightly shift to the right of the exact start point because the first tap might be in deep fade and smaller than the threshold in some simulation runs. Fig. 5.24(b) shows that the proposed fine timing estimator has better performance than SC's fine timing estimator at low SNR due to the FLP-based preamble. The proposed coarse timing estimator has a smaller variance than the proposed fine timing estimator when $SNR \geq 15\text{dB}$. When $SNR \geq 10\text{dB}$, the proposed fine timing estimator has almost the same variance. This error floor is caused by the probability of false alarm and

missing detection of the fine timing estimator. It depends on the threshold setting and tap fading. Since the correlation function used for the proposed coarse timing estimator collects all tap signal powers, the accuracy of the proposed coarse timing estimator increases with SNR. The proposed coarse timing estimator variance using MRC is slightly lower than the one using EGC.

Fig. 5.25 shows the performance of the CFO estimators for different methods using different combining methods in the time-invariant Rayleigh fading channel for a 2×2 antenna array. It shows that the proposed CFO estimator has a similar variance as SC's CFO estimator one at low SNR and slightly bigger at high SNR. Performance using EGC or MRC is almost the same. The integer part of the CFO is estimated correctly for 10,000 runs by SC's and proposed methods for each simulated SNR.

In Fig. 5.26, we compare the performance of the proposed coarse timing estimator using different number of transmitter and receiver antennas with EGC. Due to the spatial diversity, the mean value of the proposed coarse timing estimator is closer to the exact start point and the variance is getting lower when the antenna number is increased. The receiver diversity gives more gain than transmitter diversity.

In Fig. 5.27, we compare the performance of the proposed fine timing estimator using different number of transmitter and receiver antennas with EGC. Since the total transmitted power is fixed, increasing the transmitter antennas means decreasing the transmitted power on each transmitter antenna. From (4.41), increasing the number of the transmitter antennas will decrease the mean value of $\psi_{l,n}$ and the probability of false alarm and missing detection of fine timing estimator will increase (appendix B). The performance of the proposed fine timing estimator depends on both the performance of the proposed coarse timing estimator and the probability of false alarm and missing detection of the fine timing estimator. At low SNR, the performance of the proposed fine timing estimator is dominated by the performance

of the proposed coarse timing estimator (in the searching range or not) and 2×3 has the lowest variance when $SNR = 0\text{dB}$. At high SNR, the performance of the proposed fine timing estimator is dominated by the probability of false alarm and missing detection of the fine timing estimator and 1×2 has the lowest variance when $SNR \geq 5\text{dB}$.

In Fig. 5.28, we compare the performance of the proposed CFO estimators using different number of transmitter and receiver antennas with EGC. Fig. 5.28 shows that increasing the number of transmitter antenna or receiver antenna improves the performance of the proposed CFO estimator. The receiver diversity gives more gain for the CFO estimation than the transmitter diversity.

Summarizing, it is seen that the performance when using EGC or MRC is almost the same with a slight improvement using MRC. For MIMO-OFDM systems, it is preferable to use the proposed fine timing estimator in low SNR scenario and the proposed coarse timing estimator in high SNR scenario to achieve the best performance. Overall, the proposed method works well in the time-invariant Rayleigh fading channel for MIMO-OFDM systems.

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.24 Performance of the timing estimator in the time-invariant Rayleigh fading channel, $Q = 2$, $L = 2$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.25 Performance of the CFO estimator in the time-invariant Rayleigh fading channel, $Q = 2$, $L = 2$

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.26 Performance of the proposed coarse timing estimator in the time-invariant Rayleigh fading channel using different number of antennas

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.27 Performance of the proposed fine timing estimator in the time-invariant Rayleigh fading channel using different number of antennas

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.28 Performance of the proposed CFO estimator in the time-invariant Rayleigh fading channel using different number of antennas

5.6.2 Simulation results in the time-variant Rayleigh fading channel for MIMO-OFDM systems

Figs. 5.29-5.34 present the simulation results in terms of the means and variances of the symbol timing and CFO estimators for SC's and proposed methods in the time-variant Rayleigh fading channel using two transmitter antennas and two receiver antennas with different maximum Doppler frequencies (corresponding to low, moderated and high vehicle speed).

For each maximum Doppler frequency, the proposed fine timing estimator has the best performance. The variance of the proposed CFO estimator is similar to SC's CFO estimator one at low SNR and slightly bigger at high SNR. Performance when the fine time point or the coarse time point is used to estimate CFO are almost the same.

In Fig. 5.35 and Fig. 5.36, we compare the performance of the proposed timing and CFO estimator with different maximum Doppler frequencies in a 2×2 MIMO-OFDM system. The timing estimators have almost the same performance when the maximum Doppler frequency changes. When the maximum Doppler frequency increases, the variance of the CFO estimator also increases. The CFO estimator is more sensitive to the rate of the channel change. Overall, it is seen that the proposed method is robust to channel time variations for MIMO-OFDM systems as well.

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.29 Performance of the timing estimator in the time-variant Rayleigh fading channel, $Q = 2$, $L = 2$, $v = 10\text{km/hr}$, $f_m = 46.3\text{Hz}$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.30 Performance of the CFO estimator in the time-variant Rayleigh fading channel, $Q = 2$, $L = 2$, $v = 10\text{km/hr}$, $f_m = 46.3\text{Hz}$

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.31 Performance of the timing estimator in the time-variant Rayleigh fading channel, $Q = 2$, $L = 2$, $v = 70\text{km/hr}$, $f_m = 324.1\text{Hz}$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.32 Performance of the CFO estimator in the time-variant Rayleigh fading channel, $Q = 2$, $L = 2$, $v = 70 \text{ km/hr}$, $f_m = 324.1 \text{ Hz}$

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.33 Performance of the timing estimator in the time-variant Rayleigh fading channel, $Q = 2$, $L = 2$, $v = 250\text{km/hr}$, $f_m = 1157.4\text{Hz}$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.34 Performance of the CFO estimator in the time-variant Rayleigh fading channel, $Q = 2$, $L = 2$, $v = 250\text{km/hr}$, $f_m = 1157.4\text{Hz}$

(a) Mean of the start point

(b) Variance of the start point

Fig. 5.35 Performance of the timing estimator using EGC in the time-variant Rayleigh fading channel for various maximum Doppler frequencies, $Q = 2$, $L = 2$

(a) Mean of the CFO

(b) Variance of the CFO

Fig. 5.36 Performance of the CFO estimator using EGC in the time-variant Rayleigh fading channel for various maximum Doppler frequencies, $Q = 2$, $L = 2$

Chapter 6

Conclusions and future works

6.1 Conclusions

This thesis presents a postfix synchronization method for OFDM and MIMO-OFDM systems that uses preambles containing two identical training parts (Chu sequences) followed by a Flipped Postfix (FIP). Specifically, the proposed preamble for AWGN channels consists of a Flipped Cyclic Prefix (FCP) and two identical training parts followed by a FIP. The FCP and FIP lengths are the Cyclic Prefix (CP) length of conventional preambles. The preamble for ISI or fading channels is similar except that the FCP is replaced by a Zero Padding Prefix (ZPP) of length $G/2$ and each identical part is a shorter Chu sequence of length $\frac{N}{2} - \frac{G}{2}$ followed by $\frac{G}{2}$ zeroes. For MIMO-OFDM systems, since the training sequences of different antennas should be orthogonal and shift-orthogonal and each training sequence of different antennas should have a perfect autocorrelation function in the fading channel, we use Chu sequences and each circular shift G version of Chu sequences to be the half part of the first training symbol for each transmitter antenna. The first symbol consists of a half length CP and two identical training parts followed by a FIP. We analyze the effect of the FIP in AWGN

channels. By using these FIP-based preambles, the accuracy of the timing estimator is highly improved compared to classical existing methods such as Schmidl and Cox's or Minn' methods without increasing complexity.

The proposed synchronization method is implemented in four steps. Step I is Coarse Time Synchronization. It uses the timing metric to find the coarse estimated symbol start point. Step II is Estimation of the Fraction Part of the CFO. It uses the phase difference between the two identical parts of the first training symbol. Since it uses the coarse estimated symbol start point as the symbol start point, its performance depends on the coarse time synchronization. Step III is Estimation of the Integer Part of the CFO. It uses the difference of the two training symbols on the even subcarriers. Specifically, after the fraction part of the CFO correction, the ratio sequence is retrieved and compared with the reference sequence in the frequency domain to get the estimated integer part of the CFO. Its performance depends on step I and step II. Step IV is Fine Time Synchronization. We calculate the cross-correlation of the CFO corrected sample sequences and the transmitted preamble sequences in a short range whose center is the coarse estimated symbol start point. We search the first point in that short range whose cross-correlation is greater than the threshold (from smallest to greatest). Using the fine time synchronization algorithm, the accuracies of the proposed method and SC's method are much improved in the low SNR scenario by setting a proper threshold. We give a theoretical support for setting the threshold. Since the cross correlation cannot be calculated iteratively, the computation complexity is high. Since the searching range of the proposed method is shorter than the one of SC's method, the proposed fine time estimation method has less complexity than SC's fine time estimation method. In fading channels, the fine time estimator has an error floor when SNR increases. This error floor is caused by the probability of false alarm and missing detection of the fine timing estimator and

depends on the threshold setting and tap fading. Since we can estimate SNR from the peak value of the timing metric, we can set up a look up table depending on the estimated SNR for setting the threshold. If we know the channel power delay profile, it is very helpful for setting the threshold. The performance is dominated by steps I, II and III in a low SNR scenario and by the probability of false alarm and missing detection of the fine timing estimator in a high SNR scenario.

MIMO-OFDM systems are also sensitive to synchronization errors. We extended the proposed method and SC's method to MIMO-OFDM systems. By using multiple antennas, the spatial diversity improves the accuracy of the symbol timing and carrier frequency offset estimator.

The performance of the proposed method for OFDM systems is evaluated by simulation over AWGN, ISI or fading channels in terms of the mean and variance of the symbol timing and carrier frequency offset estimators. Results show the proposed symbol timing estimator has a quite small variance and its mean value is quite close to the exact start point. Comparison to classical synchronization methods such as SC's and Minn's methods is also done. The performance of the proposed method for MIMO-OFDM systems is evaluated by simulation in fading channels. By comparing different combining techniques, such as EGC and MRC, we found the EGC had almost the same performance as MRC but is less complex. Looking from the point of view of synchronization, this thesis also shows that receiver diversity gives more gains than transmitter diversity.

In [29], the performance of the proposed method for OFDM systems is evaluated by simulation over AWGN, ISI and fading channels in terms of the mean and variance of the symbol timing and carrier frequency offset estimators using different simulation parameters. Obtained results are similar to the results presented in this thesis and can be seen as a complement of this thesis.

From the analysis and simulations, it is seen that the proposed synchronization method has a narrow and large timing metric peak value suitable to specify a threshold and is robust in continuous or burst communication scenarios under different channels with large CFO. It is also robust to the Doppler spread. In particular, simulations in various channels have shown the proposed symbol timing estimator's mean value is quite close to the exact start point with a small variance from the left side of it. The accuracy of the proposed CFO estimator is similar in low SNR scenario and slightly worse in high SNR scenario than SC's CFO estimator one. Taking into consideration both timing synchronization and CFO correction, this thesis showed that postfix synchronization methods are suitable for the practical synchronization of OFDM or MIMO-OFDM systems in various channels.

6.2 Future works

The proposed method is a synchronization method in the acquisition mode. After finding the start of the frame and estimated CFO, we have to estimate the channel. To make the preamble efficient, we need an algorithm to estimate the channel by using the information contained in the training symbols.

From appendix A, the FIP-based preamble increases the mean value of M_d . If we boost the power of the FIP, the mean value of M_d will be increased further. Future work could be to use power-boost FIP-based preambles to improve the accuracy of the symbol timing estimator under reasonable PAPR.

In this thesis, we assume a perfect sampling time. If there is a sampling clock offset, the performance of the proposed method will degrade. Using training symbols jointly to estimate the symbol timing, carrier frequency offset and sampling clock offset is another future work.

Generalized multicarrier (GMC) [30] transmission systems also need reliable synchronization methods. Future work could consist of adapting the proposed FLP-based preambles synchronization method to GMC systems and evaluating its performance.

Appendix A

The distribution of

$$M_d = M_p(d_{opt}) - M_p(d_{opt+1})$$

Using FIP-based preambles can decrease the probability that the peak of metric shifts to the right side of the exact start point. Let us recall the timing metric formula:

$$P_p(d) = \sum_{m=0}^{L_p-1} (r_{d+m}^* \cdot r_{d+m+L_p}) \quad (\text{A.1})$$

where $L_p = \frac{N}{2}$ samples and N is the IFFT size. We use the energy of the second half of the first training symbol to normalize the sum of the correlation value. The energy is calculated as:

$$R_p(d) = \sum_{m=0}^{L_p-1} |r_{d+m+L_p}|^2 \quad (\text{A.2})$$

A timing metric is defined as the normalized sum of products and can be given as:

$$M_p(d) = \frac{|P_p(d)|^2}{R_p^2(d)} \quad (\text{A.3})$$

Let us calculate the probability that the timing metric value of the exact start point

is smaller than the one of the right shift one point. To calculate that probability, we have to calculate the distribution of the random variable $M_d = M_p(d_{opt}) - M_p(d_{opt+1})$. From [6], we know the $|P_p(d)|$, $R_p(d)$, and $M_p(d)$ can be approximated by Gaussian Random Variables (RVs) from the central limit theorem (CLT). Since $|P_p(d)|$ and $R_p(d)$ have much larger means than variances, $|P_p(d)|^2$ and $R_p^2(d)$ can also be approximated by Gaussian RVs. Let each complex sample $r_m = s_m + n_m$ be made up of a signal and a noise component. Let the variance of the real and imaginary components be:

$$E[\Re\{s_m\}^2] = E[\Im\{s_m\}^2] = \frac{\sigma_s^2}{2} \quad (\text{A.4})$$

$$E[\Re\{n_m\}^2] = E[\Im\{n_m\}^2] = \frac{\sigma_n^2}{2} \quad (\text{A.5})$$

SNR is $\frac{\sigma_s^2}{\sigma_n^2}$ and we have:

$$\begin{aligned} P_p(d) &= \sum_{m=0}^{L_p-1} (r_{d+m}^* \cdot r_{d+m+L_p}) \\ &= \sum_{m=0}^{L_p-1} s_{d+m}^* s_{d+m+L_p} + s_{d+m}^* n_{d+m+L_p} + s_{d+m+L_p} n_{d+m}^* + n_{d+m}^* n_{d+m+L_p} \end{aligned} \quad (\text{A.6})$$

and

$$\begin{aligned} |P_p(d)| &\approx e^{-j\phi} \sum_{m=0}^{L_p-1} s_{d+m}^* s_{d+m+L_p} \\ &+ \sum_{m=0}^{L_p-1} \text{inPhase}_\phi(s_{d+m}^* n_{d+m+L_p} + s_{d+m+L_p} n_{d+m}^* + n_{d+m}^* n_{d+m+L_p}) \end{aligned} \quad (\text{A.7})$$

where from (3.5) $\phi = \pi\gamma$. We have

$$E\{s_{d_{opt}+m}^* s_{d_{opt}+m+L_p}\} = e^{j\phi} \sigma_s^2, \quad m = 0, 1, \dots, L_p - 1 \quad (\text{A.8})$$

$$\begin{aligned}
& E\{s_{d_{opt}+m+L_p}^* s_{d_{opt}+m+2L_p}\} \\
&= \begin{cases} -e^{j\phi} \sigma_s^2, m = 0, 1, \dots, G/2. & \text{using the FLP preamble.} \\ 0, m = 0, 1, \dots, G/2. & \text{using SC's preamble.} \end{cases} \quad (\text{A.9})
\end{aligned}$$

$|P_p(d_{opt})|$ is a Gaussian Random Variable (RV) with mean $L_p \sigma_s^2$ and its variance is quite small compared to its mean. When we use FLP, $E\{s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p}\} = -e^{-j\phi} \sigma_s^2$, $|P_p(d_{opt+1})|$ is also a Gaussian RV but its mean is $(L_p - 2) \sigma_s^2$. Its variance is also small compared to its mean when L_p is large. In the conventional (SC) structure, $s_{d_{opt}+2L_p}$ would be the first CP sample of the next symbol, so $E\{s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p}\} = 0$ and $|P_p(d_{opt+1})|$'s mean is $(L_p - 1) \sigma_s^2$.

For $R_p(d)$ we have

$$\begin{aligned}
R_p(d) &= \sum_{m=0}^{L_p-1} |r_{d+m+L_p}|^2 \\
&= \sum_{m=0}^{L_p-1} |s_{d+m+L_p}|^2 + s_{d+m+L_p}^* n_{d+m+L_p} + s_{d+m+L_p} n_{d+m+L_p}^* + |n_{d+m+L_p}|^2 \quad (\text{A.10})
\end{aligned}$$

$R_p(d)$ is also a Gaussian RV with mean $L_p(\sigma_s^2 + \sigma_n^2)$ and its variance is quite small compared to its mean.

Since $|P_p(d)|$ and $R_p(d)$ have much larger means than their variances, $|P_p(d)|^2$ and $R_p^2(d)$ can also be approximated by Gaussian RVs with means $L_p^2 \sigma_s^4, L_p^2(\sigma_s^2 + \sigma_n^2)^2$ respectively and their means are quite larger than their variances. Using the approximation $(1+b)/(1+c) \approx (1+b-c)$ (b and c are zero mean Gaussian RVs and their variances are quite smaller than one) we have

$$M_p(d_{opt}) = \frac{|P_p(d_{opt})|^2}{R_p^2(d_{opt})} \approx \frac{L_p^2 \sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} \frac{1 + \delta P_p}{1 + \delta R_p}$$

$$\begin{aligned}
&\approx \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2} (1 + \delta P_p - \delta R_p) = \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2} (1 + \delta P_p - \delta R_p - 1 + 1) \\
&= \frac{|P_p(d_{opt})|^2}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} - \frac{R_p^2(d_{opt})\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4} + \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2}
\end{aligned} \tag{A.11}$$

similarly we have

$$M_p(d_{opt+1}) \approx \frac{|P_p(d_{opt+1})|^2}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} - \frac{R_p^2(d_{opt+1})\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4} + \frac{\sigma_s^4}{(\sigma_s^2 + \sigma_n^2)^2} \tag{A.12}$$

$(|P_p(d_{opt+1})|^2)$'s mean is $(L_p - 2)^2\sigma_s^4$ or $(L_p - 1)^2\sigma_s^4$. Since L_p is large, we approximate $L_p - 2 \approx L_p$, $L_p - 1 \approx L_p$)

$M_p(d_{opt}), M_p(d_{opt+1})$ are approximated by Gaussian RVs, so $M_d = M_p(d_{opt}) - M_p(d_{opt+1})$ is also a Gaussian RV.

$$\begin{aligned}
M_d &= M_p(d_{opt}) - M_p(d_{opt+1}) \\
&\approx \frac{|P_p(d_{opt})|^2 - |P_p(d_{opt+1})|^2}{L_p^2(\sigma_s^2 + \sigma_n^2)^2} + \frac{\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4} (R_p^2(d_{opt+1}) - R_p^2(d_{opt}))
\end{aligned} \tag{A.13}$$

To calculate the mean and variance of the M_d , we have to calculate the means and variances of $P_d = |P_p(d_{opt})|^2 - |P_p(d_{opt+1})|^2$ and $R_d = R_p^2(d_{opt+1}) - R_p^2(d_{opt})$ and their covariance. Let $c_1 = \frac{1}{L_p^2(\sigma_s^2 + \sigma_n^2)^2}$ and $c_2 = \frac{\sigma_s^4}{L_p^2(\sigma_s^2 + \sigma_n^2)^4}$. We have

$$E\{M_d\} = c_1 \cdot E\{P_d\} + c_2 \cdot E\{R_d\} \tag{A.14}$$

$$Var(M_d) = c_1^2 \cdot Var(P_d) + c_2^2 \cdot Var(R_d) + 2 \cdot c_1 \cdot c_2 \cdot Cov(P_d, R_d) \tag{A.15}$$

First, let's calculate the mean and variance of P_d . Let us define

$$d_m = s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} + s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* + n_{d_{opt}+m}^* n_{d_{opt}+m+L_p} \tag{A.16}$$

Then

$$\begin{aligned}
|d_m|^2 &= d_m^* d_m \\
&= (s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} + s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* + n_{d_{opt}+m}^* n_{d_{opt}+m+L_p})^* \\
&\quad \cdot (s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} + s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* + n_{d_{opt}+m}^* n_{d_{opt}+m+L_p}) \\
&= (s_{d_{opt}+m} s_{d_{opt}+m+L_p}^* + s_{d_{opt}+m} n_{d_{opt}+m+L_p}^* + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m} + n_{d_{opt}+m} n_{d_{opt}+m+L_p}^*) \\
&\quad \cdot (s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} + s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* + n_{d_{opt}+m}^* n_{d_{opt}+m+L_p}) \\
&= |s_{d_{opt}+m}|^2 |s_{d_{opt}+m+L_p}|^2 + |s_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} + |s_{d_{opt}+m+L_p}|^2 s_{d_{opt}+m}^* \\
&\quad n_{d_{opt}+m} + n_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} + |s_{d_{opt}+m}|^2 s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} \\
&\quad + |s_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m} s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} \\
&\quad + n_{d_{opt}+m} s_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m} |s_{d_{opt}+m+L_p}|^2 n_{d_{opt}+m}^* \\
&\quad + s_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* + |s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m}|^2 \\
&\quad + |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} + s_{d_{opt}+m} s_{d_{opt}+m+L_p}^* n_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m} \\
&\quad n_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m+L_p}^* |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p} + |n_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2
\end{aligned} \tag{A.17}$$

Using that the first training sequence is not random and that constant envelopes (Chu sequences) sequences are used for the training sequences, we have

$$E\{|d_m|^2\} = \sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4 \tag{A.18}$$

$$\begin{aligned}
P_d &= |P_p(d_{opt})|^2 - |P_p(d_{opt+1})|^2 \\
&= \left(d_0 + \sum_{m=1}^{L_p-1} d_m\right)^* \left(d_0 + \sum_{m=1}^{L_p-1} d_m\right) - \left(d_{L_p} + \sum_{m=1}^{L_p-1} d_m\right)^* \left(d_{L_p} + \sum_{m=1}^{L_p-1} d_m\right)
\end{aligned}$$

$$\begin{aligned}
 &= |d_0|^2 + d_0^* \sum_{m=1}^{L_p-1} d_m + d_0 \sum_{m=1}^{L_p-1} d_m^* + \left| \sum_{m=1}^{L_p-1} d_m \right|^2 \\
 &\quad - \left(|d_{L_p}|^2 + d_{L_p}^* \sum_{m=1}^{L_p-1} d_m + d_{L_p} \sum_{m=1}^{L_p-1} d_m^* + \left| \sum_{m=1}^{L_p-1} d_m \right|^2 \right) \\
 &= |d_0|^2 - |d_{L_p}|^2 + (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m + (d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^* \\
 &= |d_0|^2 - |d_{L_p}|^2 + 2\Re \left\{ (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} \tag{A.19}
 \end{aligned}$$

From (A.8) and (A.9), we have

$$E\{d_m^*\} = \begin{cases} e^{-j\phi} \sigma_s^2, & m = 0, 1, \dots, L_p - 1. \\ -e^{-j\phi} \sigma_s^2, & m = L_p. \\ 0, & m = L_p. \end{cases} \begin{array}{l} \text{using the FLP preamble.} \\ \text{using SC's preamble.} \end{array} \tag{A.20}$$

Since d_0, d_{L_p} are independent of $d_m, m = 1, 2, \dots, L_p - 1$ (noises at each sample are independent and $s_{d_{opt}}, s_{d_{opt}+L_p}, s_{d_{opt}+2L_p}$ are independent of $s_{d_{opt}+m}, m = 1, 2, \dots, 2L_p - 1$), we have

$$\begin{aligned}
 &E \left\{ \Re \left\{ (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} \right\} = \Re \left\{ E \left\{ (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} \right\} \\
 &= \Re \left\{ E \{ (d_0^* - d_{L_p}^*) \} \sum_{m=1}^{L_p-1} E \{ d_m \} \right\} = \Re \left\{ (E \{ d_0^* \} - E \{ d_{L_p}^* \}) \sum_{m=1}^{L_p-1} E \{ d_m \} \right\} \\
 &= \begin{cases} 2(L_p - 1) \sigma_s^4 & \text{using the FLP preamble.} \\ (L_p - 1) \sigma_s^4 & \text{using SC's preamble.} \end{cases} \tag{A.21}
 \end{aligned}$$

From (A.18), (A.19) and (A.21), we have

$$\begin{aligned} E\{P_d\} &= E\{|d_0|^2\} - E\{|d_{L_p}|^2\} + 2E\left\{\Re\left\{(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m\right\}\right\} \\ &= \begin{cases} 4(L_p - 1)\sigma_s^4 & \text{using the FIP preamble.} \\ 2(L_p - 1)\sigma_s^4 & \text{using SC's preamble.} \end{cases} \end{aligned} \quad (\text{A.22})$$

To calculate $Var(P_d)$, first we calculate the second moment of P_d . We have

$$\begin{aligned} |P_d|^2 &= (|P_p(d_{opt})|^2 - |P_p(d_{opt+1})|^2)^* (|P_p(d_{opt})|^2 - |P_p(d_{opt+1})|^2) \\ &= \left[|d_0|^2 - |d_{L_p}|^2 + (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m + (d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^* \right]^* \\ &\quad \cdot \left[|d_0|^2 - |d_{L_p}|^2 + (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m + (d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^* \right] \\ &= (|d_0|^2 - |d_{L_p}|^2)^2 + 2(|d_0|^2 - |d_{L_p}|^2) \cdot 2\Re\left\{(d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^*\right\} \\ &\quad + \left[(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right]^2 + 2\left| (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right|^2 + \left[(d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^* \right]^2 \end{aligned} \quad (\text{A.23})$$

$$\begin{aligned} E\{|d_m|^4\} &= E\{|d_m|^2 \cdot |d_m|^2\} \\ &= (\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)^2 + \sigma_s^6\sigma_n^2 + \sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6 \\ &\quad + \sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6 + \sigma_s^2\sigma_n^6 + 2\sigma_s^6\sigma_n^2 + 4\sigma_s^4\sigma_n^4 + 2\sigma_s^2\sigma_n^6 \\ &= (\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)^2 + 6\sigma_s^6\sigma_n^2 + 8\sigma_s^4\sigma_n^4 + 6\sigma_s^2\sigma_n^6 \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned}
E\{|d_0|^2 |d_{L_p}|^2\} &= E\left\{(|s_{d_{opt}}|^2 |s_{d_{opt}+L_p}|^2 + |s_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} \right. \\
&\quad + |s_{d_{opt}+L_p}|^2 s_{d_{opt}}^* n_{d_{opt}} + n_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}}^* s_{d_{opt}+L_p} \\
&\quad + |s_{d_{opt}}|^2 s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + |s_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}+L_p}^* n_{d_{opt}} s_{d_{opt}}^* n_{d_{opt}+L_p} + n_{d_{opt}} s_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}} |s_{d_{opt}+L_p}|^2 n_{d_{opt}}^* + s_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} n_{d_{opt}}^* \\
&\quad + |s_{d_{opt}+L_p}|^2 |n_{d_{opt}}|^2 + |n_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} \\
&\quad + s_{d_{opt}} s_{d_{opt}+L_p}^* n_{d_{opt}}^* n_{d_{opt}+L_p} + s_{d_{opt}} n_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}+L_p}^* |n_{d_{opt}}|^2 n_{d_{opt}+L_p} + |n_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2) \\
&\quad \cdot (|s_{d_{opt}+L_p}|^2 |s_{d_{opt}+2L_p}|^2 + |s_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} \\
&\quad + |s_{d_{opt}+2L_p}|^2 s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + n_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p} \\
&\quad + |s_{d_{opt}+L_p}|^2 s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + |s_{d_{opt}+L_p}|^2 |n_{d_{opt}+2L_p}|^2 \\
&\quad + s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p} s_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} + n_{d_{opt}+L_p} s_{d_{opt}+L_p}^* |n_{d_{opt}+2L_p}|^2 \\
&\quad + s_{d_{opt}+L_p} |s_{d_{opt}+2L_p}|^2 n_{d_{opt}+L_p}^* + s_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} n_{d_{opt}+L_p}^* \\
&\quad + |s_{d_{opt}+2L_p}|^2 |n_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} \\
&\quad + s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^* |n_{d_{opt}+2L_p}|^2 \\
&\quad \left. + s_{d_{opt}+2L_p}^* |n_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p} + |n_{d_{opt}+L_p}|^2 |n_{d_{opt}+2L_p}|^2) \right\} \\
&= (\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4)^2 + 2\sigma_s^6 \sigma_n^2 + 4\sigma_s^4 \sigma_n^4 + 2\sigma_s^2 \sigma_n^6 \quad (A.25)
\end{aligned}$$

$$\begin{aligned}
E\{(|d_0|^2 - |d_{L_p}|^2)^2\} &= E\{|d_0|^4\} - 2E\{|d_0|^2 |d_{L_p}|^2\} + E\{|d_{L_p}|^4\} \\
&= 2[(\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4)^2 + 6\sigma_s^6 \sigma_n^2 + 8\sigma_s^4 \sigma_n^4 + 6\sigma_s^2 \sigma_n^6] \\
&\quad - 2[(\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4)^2 + 2\sigma_s^6 \sigma_n^2 + 4\sigma_s^4 \sigma_n^4 + 2\sigma_s^2 \sigma_n^6] \\
&= 8\sigma_s^6 \sigma_n^2 + 8\sigma_s^4 \sigma_n^4 + 8\sigma_s^2 \sigma_n^6 \quad (A.26)
\end{aligned}$$

$$\begin{aligned}
E\{|d_m|^2 d_m\} &= E\left\{(|s_{d_{opt}+m}|^2 |s_{d_{opt}+m+L_p}|^2 + |s_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} \right. \\
&\quad + |s_{d_{opt}+m+L_p}|^2 s_{d_{opt}+m}^* n_{d_{opt}+m} + n_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} \\
&\quad + |s_{d_{opt}+m}|^2 s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + |s_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2 \\
&\quad + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m} s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + n_{d_{opt}+m} s_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 \\
&\quad + s_{d_{opt}+m} |s_{d_{opt}+m+L_p}|^2 n_{d_{opt}+m}^* + s_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* \\
&\quad + |s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m}|^2 + |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} \\
&\quad + s_{d_{opt}+m} s_{d_{opt}+m+L_p}^* n_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m} n_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 \\
&\quad \left. + s_{d_{opt}+m+L_p}^* |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p} + |n_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2\right\} \\
&\quad \left(s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} + s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* + n_{d_{opt}+m}^* n_{d_{opt}+m+L_p}\right) \Big\} \\
&= (\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}+m}^* s_{d_{opt}+m+L_p}\} + (\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}+m}^* s_{d_{opt}+m+L_p}\} \\
&\quad + (\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}+m}^* s_{d_{opt}+m+L_p}\} + \sigma_n^4 E\{s_{d_{opt}+m}^* s_{d_{opt}+m+L_p}\} \\
&= (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + 4\sigma_n^4) E\{s_{d_{opt}+m}^* s_{d_{opt}+m+L_p}\} \\
&= \begin{cases} e^{j\phi} \sigma_s^2 (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + 4\sigma_n^4), & m = 0, 1, \dots, L_p - 1. \\ -e^{j\phi} \sigma_s^2 (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + 4\sigma_n^4), & m = L_p. \text{ using the FLP preamble.} \\ 0, & m = L_p. \text{ using SC's preamble.} \end{cases} \quad (\text{A.27})
\end{aligned}$$

$$\begin{aligned}
E\{|d_0|^2 d_{L_p}\} &= E \left\{ (|s_{d_{opt}}|^2 |s_{d_{opt}+L_p}|^2 + |s_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} + |s_{d_{opt}+L_p}|^2 s_{d_{opt}}^* n_{d_{opt}} \right. \\
&\quad + n_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}}^* s_{d_{opt}+L_p} + |s_{d_{opt}}|^2 s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + |s_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}+L_p}^* n_{d_{opt}} s_{d_{opt}}^* n_{d_{opt}+L_p} + n_{d_{opt}} s_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 + s_{d_{opt}} |s_{d_{opt}+L_p}|^2 n_{d_{opt}}^* \\
&\quad + s_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} n_{d_{opt}}^* + |s_{d_{opt}+L_p}|^2 |n_{d_{opt}}|^2 + |n_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} \\
&\quad + s_{d_{opt}} s_{d_{opt}+L_p}^* n_{d_{opt}}^* n_{d_{opt}+L_p} + s_{d_{opt}} n_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 \\
&\quad \left. + s_{d_{opt}+L_p}^* |n_{d_{opt}}|^2 n_{d_{opt}+L_p} + |n_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2 \right) \\
&\quad \cdot (s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p} + s_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+L_p}^* + n_{d_{opt}+L_p}^* n_{d_{opt}+2L_p}) \Big\} \\
&= (\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p}\} + (\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p}\} \\
&= (\sigma_s^4 + 3\sigma_s^2 \sigma_n^2 + 2\sigma_n^4) E\{s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p}\} \\
&= \begin{cases} -e^{j\phi} \sigma_s^2 (\sigma_s^4 + 3\sigma_s^2 \sigma_n^2 + 2\sigma_n^4), & \text{using the FIP preamble.} \\ 0, & \text{using SC's preamble.} \end{cases} \tag{A.28}
\end{aligned}$$

$$\begin{aligned}
E\{|d_{L_p}|^2 d_0\} &= E \left\{ (|s_{d_{opt}+L_p}|^2 |s_{d_{opt}+2L_p}|^2 + |s_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} \right. \\
&\quad + |s_{d_{opt}+2L_p}|^2 s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + n_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p} \\
&\quad + |s_{d_{opt}+L_p}|^2 s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + |s_{d_{opt}+L_p}|^2 |n_{d_{opt}+2L_p}|^2 \\
&\quad + s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p} s_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} + n_{d_{opt}+L_p} s_{d_{opt}+L_p}^* |n_{d_{opt}+2L_p}|^2 \\
&\quad + s_{d_{opt}+L_p} |s_{d_{opt}+2L_p}|^2 n_{d_{opt}+L_p}^* + s_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} n_{d_{opt}+L_p}^* \\
&\quad + |s_{d_{opt}+2L_p}|^2 |n_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} \\
&\quad + s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^* |n_{d_{opt}+2L_p}|^2 \\
&\quad \left. + s_{d_{opt}+2L_p}^* |n_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p} + |n_{d_{opt}+L_p}|^2 |n_{d_{opt}+2L_p}|^2 \right) \\
&\quad \cdot (s_{d_{opt}}^* s_{d_{opt}+L_p} + s_{d_{opt}}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}}^* + n_{d_{opt}}^* n_{d_{opt}+L_p}) \Big\} \\
&= (\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}}^* s_{d_{opt}+L_p}\} + (\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}}^* s_{d_{opt}+L_p}\} \\
&= (\sigma_s^4 + 3\sigma_s^2 \sigma_n^2 + 2\sigma_n^4) E\{s_{d_{opt}}^* s_{d_{opt}+L_p}\} = e^{j\phi} \sigma_s^2 (\sigma_s^4 + 3\sigma_s^2 \sigma_n^2 + 2\sigma_n^4) \tag{A.29}
\end{aligned}$$

$$E \left\{ \sum_{m=1}^{L_p-1} d_m^* \right\} = \sum_{m=1}^{L_p-1} E\{d_m^*\} = e^{-j\phi} \sigma_s^2 (L_p - 1) \quad (\text{A.30})$$

Since d_0, d_{L_p} are independent of $d_m, m = 1, 2, \dots, L_p - 1$, we have

$$\begin{aligned} & E \left\{ 2(|d_0|^2 - |d_{L_p}|^2) \cdot 2\Re \left\{ (d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^* \right\} \right\} \\ &= 4\Re \left\{ E \left\{ (|d_0|^2 - |d_{L_p}|^2)(d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^* \right\} \right\} \\ &= 4\Re \left\{ E \left\{ (|d_0|^2 - |d_{L_p}|^2)(d_0 - d_{L_p}) \right\} E \left\{ \sum_{m=1}^{L_p-1} d_m^* \right\} \right\} \\ &= 4\Re \left\{ (E\{|d_0|^2 d_0\} + E\{|d_{L_p}|^2 d_{L_p}\} - E\{|d_0|^2 d_{L_p}\} - E\{|d_{L_p}|^2 d_0\}) (e^{-j\phi} \sigma_s^2 (L_p - 1)) \right\} \\ &= \begin{cases} 0, & \text{using the FLP preamble.} \\ 4(L_p - 1)\sigma_s^4(\sigma_s^2 \sigma_n^2 + 2\sigma_n^4), & \text{using SC's preamble.} \end{cases} \quad (\text{A.31}) \end{aligned}$$

Since d_0, d_{L_p} are independent of $d_m, m = 1, 2, \dots, L_p - 1$, we have

$$E\{(d_0 - d_{L_p})^2 (\sum_{m=1}^{L_p-1} d_m^*)^2\} = E\{(d_0 - d_{L_p})^2\} E\{(\sum_{m=1}^{L_p-1} d_m^*)^2\}.$$

$$\begin{aligned} E\{(d_0 - d_{L_p})^2\} &= E\{(s_{d_{opt}}^* s_{d_{opt}+L_p} + s_{d_{opt}}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}}^* + n_{d_{opt}}^* n_{d_{opt}+L_p} \\ &\quad - s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p} - s_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} - s_{d_{opt}+2L_p} n_{d_{opt}+L_p}^* - n_{d_{opt}+L_p}^* n_{d_{opt}+2L_p})^2\} \\ &= E\{(s_{d_{opt}}^* s_{d_{opt}+L_p})^2\} - 2E\{(s_{d_{opt}}^* s_{d_{opt}+L_p} s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p})\} \\ &\quad + E\{(s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p})^2\} - 2E\{(s_{d_{opt}}^* n_{d_{opt}+L_p} s_{d_{opt}+2L_p} n_{d_{opt}+L_p}^*)\} \\ &= e^{j2\phi} \sigma_s^4 - 2(\sigma_s^2 + \sigma_n^2) E\{s_{d_{opt}}^* s_{d_{opt}+2L_p}\} + E\{(s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p})^2\} \\ &= \begin{cases} 2e^{j2\phi} (2\sigma_s^4 + \sigma_s^2 \sigma_n^2), & \text{using the FLP preamble.} \\ e^{j2\phi} \sigma_s^4, & \text{using SC's preamble.} \end{cases} \quad (\text{A.32}) \end{aligned}$$

As discussed above, $\sum_{m=1}^{L_p-1} d_m^*$ can be approximated by a Gaussian RV

$\mathcal{CN}(e^{-j\phi}\sigma_s^2(L_p - 1), \sigma_c^2)$. Its variance is quite smaller than its mean. We have

$$\begin{aligned} E \left\{ \left(\sum_{m=1}^{L_p-1} d_m^* \right)^2 \right\} &\approx E \{ [e^{-j\phi}\sigma_s^2(L_p - 1)]^2 + 2e^{-j\phi}\sigma_s^2(L_p - 1)\mathcal{CN}(0, \sigma_c^2) + \mathcal{CN}^2(0, \sigma_c^2) \} \\ &\approx e^{-j2\phi}\sigma_s^4(L_p - 1)^2 \end{aligned} \quad (\text{A.33})$$

$$\begin{aligned} E \left\{ (d_0 - d_{L_p})^2 \left(\sum_{m=1}^{L_p-1} d_m^* \right)^2 \right\} &= E \{ (d_0 - d_{L_p})^2 \} E \left\{ \left(\sum_{m=1}^{L_p-1} d_m^* \right)^2 \right\} \\ &= \begin{cases} 2(2\sigma_s^4 + \sigma_s^2\sigma_n^2)\sigma_s^4(L_p - 1)^2, & \text{using the FIP preamble.} \\ \sigma_s^8(L_p - 1)^2, & \text{using SC's preamble.} \end{cases} \end{aligned} \quad (\text{A.34})$$

$$\begin{aligned} E \left\{ (d_0^* - d_{L_p}^*)^2 \left(\sum_{m=1}^{L_p-1} d_m \right)^2 \right\} &= E \left\{ (d_0 - d_{L_p})^2 \left(\sum_{m=1}^{L_p-1} d_m^* \right)^2 \right\}^* \\ &= \begin{cases} 2(2\sigma_s^4 + \sigma_s^2\sigma_n^2)\sigma_s^4(L_p - 1)^2, & \text{using the FIP preamble.} \\ \sigma_s^8(L_p - 1)^2, & \text{using SC's preamble.} \end{cases} \end{aligned} \quad (\text{A.35})$$

Now let us calculate $E\{|(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m|^2\}$. We have

$$\begin{aligned} \left| (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right|^2 &= \left[(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right]^* \left[(d_0^* - d_{L_p}^*) \sum_{n=1}^{L_p-1} d_n \right] \\ &= (|d_0|^2 - d_0 d_{L_p}^* - d_0^* d_{L_p} + |d_{L_p}|^2) \left(\sum_{m=1}^{L_p-1} d_m^* \sum_{n=1}^{L_p-1} d_n \right) \end{aligned} \quad (\text{A.36})$$

$$\begin{aligned}
 E\{d_0 d_{L_p}^*\} &= E\{(s_{d_{opt}}^* s_{d_{opt}+L_p} + s_{d_{opt}}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}}^* + n_{d_{opt}}^* n_{d_{opt}+L_p}) \cdot \\
 &\quad (s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^* + s_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* + s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p} + n_{d_{opt}+L_p} n_{d_{opt}+2L_p}^*)\} \\
 &= \begin{cases} -\sigma_s^4, & \text{using the FIP preamble.} \\ 0, & \text{using SC's preamble.} \end{cases} \quad (A.37)
 \end{aligned}$$

$$E\{d_0^* d_{L_p}\} = (E\{d_0 d_{L_p}^*\})^* = \begin{cases} -\sigma_s^4, & \text{using the FIP preamble.} \\ 0, & \text{using SC's preamble.} \end{cases} \quad (A.38)$$

If $n \neq m, n = 1, 2, \dots, L_p - 1$ and $m = 1, 2, \dots, L_p - 1$, we assume that d_m and d_n are independent.

$$\begin{aligned}
 E\left\{\sum_{m=1}^{L_p-1} d_m^* \sum_{n=1}^{L_p-1} d_n\right\} &= E\left\{\sum_{m=1}^{L_p-1} |d_m|^2\right\} + E\left\{\sum_{m=1}^{L_p-1} d_m^* \sum_{n=1, n \neq m}^{L_p-1} d_n\right\} \\
 &= \sum_{m=1}^{L_p-1} E\{|d_m|^2\} + \sum_{m=1}^{L_p-1} E\{d_m^*\} \sum_{n=1, n \neq m}^{L_p-1} E\{d_n\} \\
 &= (L_p - 1)(\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4) + (L_p - 1)(L_p - 2)\sigma_s^4 \quad (A.39)
 \end{aligned}$$

Since d_0, d_{L_p} are independent of $d_m, m = 1, 2, \dots, L_p - 1$, we have

$$\begin{aligned}
 E\left\{\left|(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m\right|^2\right\} &= E\left\{(|d_0|^2 - d_0 d_{L_p}^* - d_0^* d_{L_p} + |d_{L_p}|^2) \left(\sum_{m=1}^{L_p-1} d_m^* \sum_{n=1}^{L_p-1} d_n\right)\right\} \\
 &= E\{|d_0|^2 - d_0 d_{L_p}^* - d_0^* d_{L_p} + |d_{L_p}|^2\} E\left\{\sum_{m=1}^{L_p-1} d_m^* \sum_{n=1}^{L_p-1} d_n\right\}
 \end{aligned}$$

$$= \begin{cases} 2(2\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)[(L_p - 1)(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) + (L_p - 1)(L_p - 2)\sigma_s^4] \\ \text{using the FIP preamble.} \\ 2(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)[(L_p - 1)(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) + (L_p - 1)(L_p - 2)\sigma_s^4] \\ \text{using SC's preamble.} \end{cases} \quad (\text{A.40})$$

From (A.23), (A.26), (A.31), (A.34), (A.35) and (A.40), we have

$$\begin{aligned} E\{|P_d|^2\} &= E\{(|d_0|^2 - |d_{L_p}|^2)^2\} + E\left\{2(|d_0|^2 - |d_{L_p}|^2) \cdot 2\Re\left\{(d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^*\right\}\right\} \\ &+ E\left\{\left[(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m\right]^2\right\} + E\left\{2\left|(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m\right|^2\right\} \\ &+ E\left\{\left[(d_0 - d_{L_p}) \sum_{m=1}^{L_p-1} d_m^*\right]^2\right\} \\ &= \begin{cases} 8\sigma_s^6\sigma_n^2 + 8\sigma_s^4\sigma_n^4 + 8\sigma_s^2\sigma_n^6 + 4(2\sigma_s^4 + \sigma_s^2\sigma_n^2)\sigma_s^4(L_p - 1)^2 \\ + 4(2\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)[(L_p - 1)(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) + (L_p - 1)(L_p - 2)\sigma_s^4] \\ \text{using the FIP preamble.} \\ 8\sigma_s^6\sigma_n^2 + 8\sigma_s^4\sigma_n^4 + 8\sigma_s^2\sigma_n^6 + 4(L_p - 1)\sigma_s^4(\sigma_s^2\sigma_n^2 + 2\sigma_n^4) + 2\sigma_s^8(L_p - 1)^2 \\ + 4(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)[(L_p - 1)(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) + (L_p - 1)(L_p - 2)\sigma_s^4] \\ \text{using SC's preamble.} \end{cases} \quad (\text{A.41}) \end{aligned}$$

From (A.22) and (A.41), we have

$$\begin{aligned}
 Var(P_d) &= E\{|P_d|^2\} - (E\{P_d\})^2 \\
 &= \left\{ \begin{aligned} &8\sigma_s^6\sigma_n^2 + 8\sigma_s^4\sigma_n^4 + 8\sigma_s^2\sigma_n^6 + 4(2\sigma_s^4 + \sigma_s^2\sigma_n^2)\sigma_s^4(L_p - 1)^2 \\ &+ 4(2\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)[(L_p - 1)(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) + (L_p - 1)(L_p - 2)\sigma_s^4] \\ &- [4(L_p - 1)\sigma_s^4]^2 \\ &\text{using the FLP preamble.} \end{aligned} \right. \\
 &= \left\{ \begin{aligned} &8\sigma_s^6\sigma_n^2 + 8\sigma_s^4\sigma_n^4 + 8\sigma_s^2\sigma_n^6 + 4(L_p - 1)\sigma_s^4(\sigma_s^2\sigma_n^2 + 2\sigma_n^4) + 2\sigma_s^8(L_p - 1)^2 \\ &+ 4(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)[(L_p - 1)(\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) + (L_p - 1)(L_p - 2)\sigma_s^4] \\ &- [2(L_p - 1)\sigma_s^4]^2 \\ &\text{using SC's preamble.} \end{aligned} \right. \\
 &= \left\{ \begin{aligned} &4(L_p - 1)^2(3\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4) + 4(L_p - 1)(2\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4) \\ &(\sigma_s^2\sigma_n^2 + \sigma_n^4) + 8(\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6) \\ &\text{using the FLP preamble.} \end{aligned} \right. \tag{A.42} \\
 &= \left\{ \begin{aligned} &(L_p - 1)^2\sigma_s^4(2\sigma_s^4 + 8\sigma_s^2\sigma_n^2 + 4\sigma_n^4) + 4(L_p - 1)[3\sigma_s^4(\sigma_s^2\sigma_n^2 + \sigma_n^4) \\ &+ (2\sigma_s^2\sigma_n^2 + \sigma_n^4)^2] + 8(\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6) \\ &\text{using SC's preamble.} \end{aligned} \right.
 \end{aligned}$$

Secondly, let us calculate the mean and variance of R_d . Let us define

$$q_m = |s_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^* + |n_{d_{opt}+m+L_p}|^2 \tag{A.43}$$

Recall $R_d = R_p^2(d_{opt+1}) - R_p^2(d_{opt})$ and from (A.10), we have

$$\begin{aligned}
 R_d &= R_p^2(d_{opt+1}) - R_p^2(d_{opt}) = \left(\sum_{m=1}^{L_p} q_m \right)^2 - \left(\sum_{m=0}^{L_p-1} q_m \right)^2 \\
 &= \left(q_{L_p} + \sum_{m=1}^{L_p-1} q_m \right)^2 - \left(q_0 + \sum_{m=1}^{L_p-1} q_m \right)^2 \\
 &= q_{L_p}^2 + 2q_{L_p} \sum_{m=1}^{L_p-1} q_m + \left(\sum_{m=1}^{L_p-1} q_m \right)^2 - q_0^2 - 2q_0 \sum_{m=1}^{L_p-1} q_m - \left(\sum_{m=1}^{L_p-1} q_m \right)^2 \\
 &= (q_{L_p}^2 - q_0^2) + 2(q_{L_p} - q_0) \sum_{m=1}^{L_p-1} q_m \tag{A.44}
 \end{aligned}$$

$$\begin{aligned}
 q_m^2 &= (|s_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^* + |n_{d_{opt}+m+L_p}|^2)^2 \\
 &= |s_{d_{opt}+m+L_p}|^4 + (s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p})^2 + (s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*)^2 + |n_{d_{opt}+m+L_p}|^4 \\
 &\quad + 2|s_{d_{opt}+m+L_p}|^2 s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + 2|s_{d_{opt}+m+L_p}|^2 s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^* \\
 &\quad + 2|s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m+L_p}|^2 + 2|s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m+L_p}|^2 \\
 &\quad + 2s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} |n_{d_{opt}+m+L_p}|^2 + 2s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^* |n_{d_{opt}+m+L_p}|^2 \tag{A.45}
 \end{aligned}$$

We have

$$E\{q_m^2\} = \sigma_s^4 + 4\sigma_s^2\sigma_n^2 + \sigma_n^4 \tag{A.46}$$

$$E\{q_m\} = \sigma_s^2 + \sigma_n^2 \tag{A.47}$$

Since q_m are independent with each other, we have

$$\begin{aligned}
 E\{R_d\} &= E\left\{ (q_{L_p}^2 - q_0^2) + 2(q_{L_p} - q_0) \sum_{m=1}^{L_p-1} q_m \right\} \\
 &= E\{q_{L_p}^2\} - E\{q_0^2\} + 2(E\{q_{L_p}\} - E\{q_0\}) E\left\{ \sum_{m=1}^{L_p-1} q_m \right\} = 0 \tag{A.48}
 \end{aligned}$$

$$\begin{aligned}
R_d^2 &= \left[(q_{L_p}^2 - q_0^2) + 2(q_{L_p} - q_0) \sum_{m=1}^{L_p-1} q_m \right]^2 \\
&= (q_{L_p}^2 - q_0^2)^2 + 4(q_{L_p}^2 - q_0^2)(q_{L_p} - q_0) \sum_{m=1}^{L_p-1} q_m + 4(q_{L_p} - q_0)^2 \left(\sum_{m=1}^{L_p-1} q_m \right)^2 \quad (A.49)
\end{aligned}$$

$$\begin{aligned}
E\{q_m^4\} &= E \left\{ [|s_{d_{opt}+m+L_p}|^4 + 4|s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^4 \right. \\
&\quad + 2(|s_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^2)(s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^* \\
&\quad \left. + (s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p})^2 + (s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*)^2] \right\} \\
&= (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4)^2 + 8\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)^2 + 2\sigma_s^4 \sigma_n^4 \quad (A.50)
\end{aligned}$$

$$\begin{aligned}
E\{q_0^2 q_{L_p}^2\} &= E \left\{ [|s_{d_{opt}+L_p}|^4 + 4|s_{d_{opt}+L_p}|^2 |n_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^4 \right. \\
&\quad + 2(|s_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^2)(s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^* \\
&\quad + (s_{d_{opt}+L_p}^* n_{d_{opt}+L_p})^2 + (s_{d_{opt}+L_p} n_{d_{opt}+L_p}^*)^2] \\
&\quad \cdot [|s_{d_{opt}+2L_p}|^4 + 4|s_{d_{opt}+2L_p}|^2 |n_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^4 \\
&\quad + 2(|s_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^2)(s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^* \\
&\quad \left. + (s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p})^2 + (s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^*)^2] \right\} \\
&= (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4)^2 \quad (A.51)
\end{aligned}$$

$$E\{(q_{L_p}^2 - q_0^2)^2\} = E\{q_{L_p}^4 - 2q_0^2 q_{L_p}^2 + q_0^4\} = 16\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)^2 + 4\sigma_s^4 \sigma_n^4 \quad (A.52)$$

$$\begin{aligned}
E\{q_m^3\} &= E\left\{\left(|s_{d_{opt}+m+L_p}|^4 + 4|s_{d_{opt}+m+L_p}|^2|n_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^4\right.\right. \\
&\quad + 2(|s_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^2)(s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*) \\
&\quad + (s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p})^2 + (s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*)^2) \\
&\quad \cdot \left(|s_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*\right)\left.\right\} \\
&= (\sigma_s^4 + 8\sigma_s^2\sigma_n^2 + \sigma_n^4)(\sigma_s^2 + \sigma_n^2) \tag{A.53}
\end{aligned}$$

$$\begin{aligned}
E\{q_0^2 q_{L_p}\} &= E\left\{\left(|s_{d_{opt}+L_p}|^4 + 4|s_{d_{opt}+L_p}|^2|n_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^4\right.\right. \\
&\quad + 2(|s_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^2)(s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^*) \\
&\quad + (s_{d_{opt}+L_p}^* n_{d_{opt}+L_p})^2 + (s_{d_{opt}+L_p} n_{d_{opt}+L_p}^*)^2) \\
&\quad \cdot \left(|s_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^2 + s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^*\right)\left.\right\} \\
&= (\sigma_s^4 + 4\sigma_s^2\sigma_n^2 + \sigma_n^4)(\sigma_s^2 + \sigma_n^2) \tag{A.54}
\end{aligned}$$

$$E\{q_{L_p}^2 q_0\} = (\sigma_s^4 + 4\sigma_s^2\sigma_n^2 + \sigma_n^4)(\sigma_s^2 + \sigma_n^2) \tag{A.55}$$

$$E\left\{\sum_{m=1}^{L_p-1} q_m\right\} = \sum_{m=1}^{L_p-1} E\{q_m\} = (L_p - 1)(\sigma_s^2 + \sigma_n^2) \tag{A.56}$$

Since q_m are independent, we have

$$\begin{aligned}
E\left\{(q_{L_p}^2 - q_0^2)(q_{L_p} - q_0) \sum_{m=1}^{L_p-1} q_m\right\} &= E\{(q_{L_p}^2 - q_0^2)(q_{L_p} - q_0)\} E\left\{\sum_{m=1}^{L_p-1} q_m\right\} \\
&= (L_p - 1)(\sigma_s^2 + \sigma_n^2) E\{q_{L_p}^3 + q_0^3 - q_{L_p}^2 q_0 - q_0^2 q_{L_p}\} \\
&= 8(L_p - 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 \tag{A.57}
\end{aligned}$$

$$\begin{aligned}
E\{q_0 q_{L_p}\} &= E\left\{\left(|s_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^2 + s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^*\right)\right. \\
&\quad \cdot \left(|s_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^2 + s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^*\right)\Big\} \\
&= (\sigma_s^2 + \sigma_n^2)^2
\end{aligned} \tag{A.58}$$

From (A.46) and (A.58)

$$E\{(q_{L_p} - q_0)^2\} = E\{q_{L_p}^2 - 2q_{L_p}q_0 + q_0^2\} = E\{q_{L_p}^2\} - 2E\{q_{L_p}q_0\} + E\{q_0^2\} = 4\sigma_s^2\sigma_n^2 \tag{A.59}$$

$$\begin{aligned}
E\left\{\left(\sum_{m=1}^{L_p-1} q_m\right)^2\right\} &= E\left\{\sum_{m=1}^{L_p-1} q_m^2 + \sum_{m=1}^{L_p-1} \sum_{n=1, n \neq m}^{L_p-1} q_m q_n\right\} \\
&= E\left\{\sum_{m=1}^{L_p-1} q_m^2\right\} + E\left\{\sum_{m=1}^{L_p-1} \sum_{n=1, n \neq m}^{L_p-1} q_m q_n\right\} \\
&= (L_p - 1)(\sigma_s^4 + 4\sigma_s^2\sigma_n^2 + \sigma_n^4) + \sum_{m=1}^{L_p-1} \sum_{n=1, n \neq m}^{L_p-1} E\{q_m\}E\{q_n\} \\
&= 2(L_p - 1)\sigma_s^2\sigma_n^2 + (L_p - 1)^2(\sigma_s^2 + \sigma_n^2)^2
\end{aligned} \tag{A.60}$$

$$\begin{aligned}
E\left\{(q_{L_p} - q_0)^2 \left(\sum_{m=1}^{L_p-1} q_m\right)^2\right\} &= E\{(q_{L_p} - q_0)^2\} E\left\{\left(\sum_{m=1}^{L_p-1} q_m\right)^2\right\} \\
&= 4\sigma_s^2\sigma_n^2 [2(L_p - 1)\sigma_s^2\sigma_n^2 + (L_p - 1)^2(\sigma_s^2 + \sigma_n^2)^2]
\end{aligned} \tag{A.61}$$

From (A.48), (A.52), (A.57) and (A.61)

$$\begin{aligned}
 Var(R_d) &= E\{R_d^2\} - (E\{R_d\})^2 \\
 &= E \left\{ (q_{L_p}^2 - q_0^2)^2 + 4(q_{L_p}^2 - q_0^2)(q_{L_p} - q_0) \sum_{m=1}^{L_p-1} q_m + 4(q_{L_p} - q_0)^2 \left(\sum_{m=1}^{L_p-1} q_m \right)^2 \right\} - 0 \\
 &= E \left\{ (q_{L_p}^2 - q_0^2)^2 \right\} + 4E\{(q_{L_p}^2 - q_0^2)(q_{L_p} - q_0) \sum_{m=1}^{L_p-1} q_m\} \\
 &\quad + 4E \left\{ (q_{L_p} - q_0)^2 \left(\sum_{m=1}^{L_p-1} q_m \right)^2 \right\} \\
 &= 16\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 + 4\sigma_s^4\sigma_n^4 + 32(L_p - 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 \\
 &\quad + 16\sigma_s^2\sigma_n^2[2(L_p - 1)\sigma_s^2\sigma_n^2 + (L_p - 1)^2(\sigma_s^2 + \sigma_n^2)^2] \\
 &= 16L_p^2\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 + (32L_p - 28)\sigma_s^4\sigma_n^4 \tag{A.62}
 \end{aligned}$$

Thirdly, let us calculate the covariance of P_d and R_d (both are real)

$$\begin{aligned}
 Cov(P_d, R_d) &= E\{(P_d - E\{P_d\})^*(R_d - E\{R_d\})\} \\
 &= E\{P_d R_d - E\{P_d\}R_d - 0(P_d - E\{P_d\})\} \\
 &= E\{P_d R_d\} - E\{P_d\}E\{R_d\} = E\{P_d R_d\} \\
 &= E \left\{ \left[|d_0|^2 - |d_{L_p}|^2 + 2\Re \left\{ (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} \right] \left[(q_{L_p}^2 - q_0^2) + 2(q_{L_p} - q_0) \sum_{n=1}^{L_p-1} q_n \right] \right\} \\
 &= E \left\{ (|d_0|^2 - |d_{L_p}|^2)(q_{L_p}^2 - q_0^2) \right\} + 4E \left\{ \Re \left\{ (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} (q_{L_p} - q_0) \sum_{n=1}^{L_p-1} q_n \right\} \\
 &\quad + 2E \left\{ (q_{L_p}^2 - q_0^2) \Re \left\{ (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} \right\} + 2E \left\{ (|d_0|^2 - |d_{L_p}|^2)(q_{L_p} - q_0) \sum_{n=1}^{L_p-1} q_n \right\} \tag{A.63}
 \end{aligned}$$

$$\begin{aligned}
E\{|d_m|^2 q_m^2\} &= E\left\{(|s_{d_{opt}+m}|^2 |s_{d_{opt}+m+L_p}|^2 + |s_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} \right. \\
&\quad + |s_{d_{opt}+m+L_p}|^2 s_{d_{opt}+m}^* n_{d_{opt}+m} + n_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} \\
&\quad + |s_{d_{opt}+m}|^2 s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + |s_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2 \\
&\quad + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m} s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + n_{d_{opt}+m} s_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 \\
&\quad + s_{d_{opt}+m} |s_{d_{opt}+m+L_p}|^2 n_{d_{opt}+m}^* + s_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* \\
&\quad + |s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m}|^2 + |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} \\
&\quad + s_{d_{opt}+m} s_{d_{opt}+m+L_p}^* n_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m} n_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 \\
&\quad + s_{d_{opt}+m+L_p}^* |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p} + |n_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2) \\
&\quad \cdot (|s_{d_{opt}+m+L_p}|^4 + 4|s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^4 \\
&\quad + 2(|s_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^2)(s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*) \\
&\quad \left. + (s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p})^2 + (s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*)^2\right\} \\
&= (\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4)(\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4) \\
&\quad + 2(\sigma_s^2 + \sigma_n^2)(\sigma_s^4 \sigma_n^2 + \sigma_s^4 \sigma_n^2 + \sigma_s^2 \sigma_n^4 + \sigma_s^2 \sigma_n^4) + 0 + 0 \\
&= (\sigma_s^2 + \sigma_n^2)^2 (\sigma_s^4 + 8\sigma_s^2 \sigma_n^2 + \sigma_n^4) \tag{A.64}
\end{aligned}$$

$$\begin{aligned}
E\{|d_0|^2 q_{L_p}^2\} &= E\left\{(|s_{d_{opt}}|^2 |s_{d_{opt}+L_p}|^2 + |s_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} \right. \\
&\quad + |s_{d_{opt}+L_p}|^2 s_{d_{opt}}^* n_{d_{opt}} + n_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}}^* s_{d_{opt}+L_p} \\
&\quad + |s_{d_{opt}}|^2 s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + |s_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}+L_p}^* n_{d_{opt}} s_{d_{opt}}^* n_{d_{opt}+L_p} + n_{d_{opt}} s_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}} |s_{d_{opt}+L_p}|^2 n_{d_{opt}}^* + s_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} n_{d_{opt}}^* \\
&\quad + |s_{d_{opt}+L_p}|^2 |n_{d_{opt}}|^2 + |n_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} \\
&\quad + s_{d_{opt}} s_{d_{opt}+L_p}^* n_{d_{opt}}^* n_{d_{opt}+L_p} + s_{d_{opt}} n_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}+L_p}^* |n_{d_{opt}}|^2 n_{d_{opt}+L_p} + |n_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2) \\
&\quad \cdot [|s_{d_{opt}+2L_p}|^4 + 4 |s_{d_{opt}+2L_p}|^2 |n_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^4 \\
&\quad + 2(|s_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^2)(s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^*) \\
&\quad \left. + (s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p})^2 + (s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^*)^2 \right\} \\
&= (\sigma_s^4 + 2\sigma_s^2 \sigma_n^2 + \sigma_n^4)(\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4) + 0 + 0 + 0 \\
&= (\sigma_s^2 + \sigma_n^2)^2 (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4)
\end{aligned} \tag{A.65}$$

Similarly, we have

$$E\{|d_{L_p}|^2 q_0^2\} = E\{|d_0|^2 q_{L_p}^2\} = (\sigma_s^2 + \sigma_n^2)^2 (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4) \tag{A.66}$$

$$\begin{aligned}
E\{(|d_0|^2 - |d_{L_p}|^2)(q_{L_p}^2 - q_0^2)\} &= E\{|d_0|^2 q_{L_p}^2 + |d_{L_p}|^2 q_0^2 - |d_{L_p}|^2 q_{L_p}^2 - |d_0|^2 q_0^2\} \\
&= E\{|d_0|^2 q_{L_p}^2\} + E\{|d_{L_p}|^2 q_0^2\} - E\{|d_{L_p}|^2 q_{L_p}^2\} - E\{|d_0|^2 q_0^2\} \\
&= -8\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)^2
\end{aligned} \tag{A.67}$$

$$\begin{aligned}
E\{d_0^* q_{L_p}\} &= E\left\{ (s_{d_{opt}} s_{d_{opt}+L_p}^* + s_{d_{opt}} n_{d_{opt}+L_p}^* + s_{d_{opt}+L_p}^* n_{d_{opt}} + n_{d_{opt}} n_{d_{opt}+L_p}^*) \right. \\
&\quad \cdot (|s_{d_{opt}+2L_p}|^2 + s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^* + |n_{d_{opt}+2L_p}|^2) \Big\} \\
&= (\sigma_s^2 + \sigma_n^2) E\{s_{d_{opt}} s_{d_{opt}+L_p}^*\} \\
&= e^{-j\phi} \sigma_s^2 (\sigma_s^2 + \sigma_n^2)
\end{aligned} \tag{A.68}$$

$$\begin{aligned}
E\{d_{L_p}^* q_0\} &= E\left\{ (s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^* + s_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* + s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p} + n_{d_{opt}+L_p} n_{d_{opt}+2L_p}^*) \right. \\
&\quad \cdot (|s_{d_{opt}+L_p}|^2 + s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^* + |n_{d_{opt}+L_p}|^2) \Big\} \\
&= (\sigma_s^2 + \sigma_n^2) E\{s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^*\} + \sigma_n^2 E\{s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^*\} \\
&= \begin{cases} -e^{-j\phi} \sigma_s^2 (\sigma_s^2 + 2\sigma_n^2) & \text{using the FLP preamble.} \\ 0 & \text{using SC's preamble.} \end{cases}
\end{aligned} \tag{A.69}$$

$$\begin{aligned}
E\{d_m^* q_m\} &= \\
&E\left\{ (s_{d_{opt}+m} s_{d_{opt}+m+L_p}^* + s_{d_{opt}+m} n_{d_{opt}+m+L_p}^* + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m} + n_{d_{opt}+m} n_{d_{opt}+m+L_p}^*) \right. \\
&\quad \cdot (|s_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^* + |n_{d_{opt}+m+L_p}|^2) \Big\} \\
&= (\sigma_s^2 + \sigma_n^2) E\{s_{d_{opt}+m} s_{d_{opt}+m+L_p}^*\} + \sigma_n^2 E\{s_{d_{opt}+m} s_{d_{opt}+m+L_p}^*\} \\
&= \begin{cases} e^{-j\phi} \sigma_s^2 (\sigma_s^2 + 2\sigma_n^2), & m = 0, 1, \dots, L_p - 1. \\ -e^{-j\phi} \sigma_s^2 (\sigma_s^2 + 2\sigma_n^2), & m = L_p. \\ 0, & m = L_p. \end{cases}
\end{aligned} \tag{A.70}$$

$$\begin{aligned}
E\{(d_0^* - d_{L_p}^*)(q_{L_p} - q_0)\} &= E\{d_0^* q_{L_p} - d_0^* q_0 - d_{L_p}^* q_{L_p} + d_{L_p}^* q_0\} \\
&= E\{d_0^* q_{L_p}\} - E\{d_0^* q_0\} - E\{d_{L_p}^* q_{L_p}\} + E\{d_{L_p}^* q_0\} \\
&= \begin{cases} -e^{-j\phi} \sigma_s^2 \sigma_n^2 & \text{using the FLP preamble.} \\ -e^{-j\phi} \sigma_s^2 \sigma_n^2 & \text{using SC's preamble.} \end{cases} \quad (A.71)
\end{aligned}$$

$$\begin{aligned}
E\left\{\sum_{m=1}^{L_p-1} d_m \sum_{n=1}^{L_p-1} q_n\right\} &= E\left\{\sum_{m=1}^{L_p-1} d_m q_m + \sum_{m=1}^{L_p-1} \sum_{n=1, n \neq m}^{L_p-1} d_m q_n\right\} \\
&= \sum_{m=1}^{L_p-1} E\{d_m q_m\} + \sum_{m=1}^{L_p-1} \sum_{n=1, n \neq m}^{L_p-1} E\{d_m q_n\} \\
&= \sum_{m=1}^{L_p-1} (E\{d_m^* q_m\})^* + \sum_{m=1}^{L_p-1} \sum_{n=1, n \neq m}^{L_p-1} \\
&E\left\{\left(s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} + s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* + n_{d_{opt}+m}^* n_{d_{opt}+m+L_p}\right) \cdot \left(|s_{d_{opt}+n+L_p}|^2 + s_{d_{opt}+n+L_p}^* n_{d_{opt}+n+L_p} + s_{d_{opt}+n+L_p} n_{d_{opt}+n+L_p}^* + |n_{d_{opt}+n+L_p}|^2\right)\right\} \\
&= (L_p - 1) e^{j\phi} \sigma_s^2 (\sigma_s^2 + 2\sigma_n^2) + (L_p - 1)(L_p - 2) e^{j\phi} \sigma_s^2 (\sigma_s^2 + \sigma_n^2) \\
&= e^{j\phi} [(L_p - 1)^2 \sigma_s^2 (\sigma_s^2 + \sigma_n^2) + (L_p - 1) \sigma_s^2 \sigma_n^2] \quad (A.72)
\end{aligned}$$

$$\begin{aligned}
&E\left\{\Re\left\{(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m\right\} (q_{L_p} - q_0) \sum_{n=1}^{L_p-1} q_n\right\} \\
&= \Re\left\{E\left\{(d_0^* - d_{L_p}^*) (q_{L_p} - q_0) \sum_{m=1}^{L_p-1} d_m \sum_{n=1}^{L_p-1} q_n\right\}\right\} \\
&= \Re\left\{E\left\{(d_0^* - d_{L_p}^*) (q_{L_p} - q_0)\right\} E\left\{\sum_{m=1}^{L_p-1} d_m \sum_{n=1}^{L_p-1} q_n\right\}\right\} \\
&= -\sigma_s^2 \sigma_n^2 [(L_p - 1)^2 \sigma_s^2 (\sigma_s^2 + \sigma_n^2) + (L_p - 1) \sigma_s^2 \sigma_n^2] \quad (A.73)
\end{aligned}$$

$$\begin{aligned}
E\{d_m^* q_m^2\} &= \\
E \left\{ (s_{d_{opt}+m} s_{d_{opt}+m+L_p}^* + s_{d_{opt}+m} n_{d_{opt}+m+L_p}^* + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m} + n_{d_{opt}+m} n_{d_{opt}+m+L_p}^*) \right. \\
&\quad \cdot [|s_{d_{opt}+m+L_p}|^4 + 4|s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^4 \\
&\quad + 2(|s_{d_{opt}+m+L_p}|^2 + |n_{d_{opt}+m+L_p}|^2)(s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*) \\
&\quad \left. + (s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p})^2 + (s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^*)^2 \right\} \\
&= (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}+m} s_{d_{opt}+m+L_p}^*\} + 2\sigma_n^2 (\sigma_s^2 + \sigma_n^2) E\{s_{d_{opt}+m} s_{d_{opt}+m+L_p}^*\} \\
&= \begin{cases} e^{-j\phi} \sigma_s^2 (\sigma_s^4 + 6\sigma_s^2 \sigma_n^2 + 3\sigma_n^4), & m = 0, 1, \dots, L_p - 1. \\ -e^{-j\phi} \sigma_s^2 (\sigma_s^4 + 6\sigma_s^2 \sigma_n^2 + 3\sigma_n^4), & m = L_p. \quad \text{using the FLP preamble.} \\ 0, & m = L_p. \quad \text{using SC's preamble.} \end{cases} \\
\end{aligned} \tag{A.74}$$

$$\begin{aligned}
E\{d_0^* q_{L_p}^2\} &= E \left\{ (s_{d_{opt}} s_{d_{opt}+L_p}^* + s_{d_{opt}} n_{d_{opt}+L_p}^* + s_{d_{opt}+L_p}^* n_{d_{opt}} + n_{d_{opt}} n_{d_{opt}+L_p}^*) \right. \\
&\quad \cdot [|s_{d_{opt}+2L_p}|^4 + 4|s_{d_{opt}+2L_p}|^2 |n_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^4 \\
&\quad + 2(|s_{d_{opt}+2L_p}|^2 + |n_{d_{opt}+2L_p}|^2)(s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^*) \\
&\quad \left. + (s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p})^2 + (s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^*)^2 \right\} \\
&= (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}} s_{d_{opt}+L_p}^*\} = e^{-j\phi} \sigma_s^2 (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4) \tag{A.75}
\end{aligned}$$

$$\begin{aligned}
E\{d_{L_p}^* q_0^2\} &= E \left\{ (s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^* + s_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* + s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p} \right. \\
&\quad + n_{d_{opt}+L_p} n_{d_{opt}+2L_p}^*) \cdot [|s_{d_{opt}+L_p}|^4 + 4|s_{d_{opt}+L_p}|^2 |n_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^4 \\
&\quad + 2(|s_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^2)(s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^*) \\
&\quad \left. + (s_{d_{opt}+L_p}^* n_{d_{opt}+L_p})^2 + (s_{d_{opt}+L_p} n_{d_{opt}+L_p}^*)^2 \right\} \\
&= (\sigma_s^4 + 4\sigma_s^2 \sigma_n^2 + \sigma_n^4) E\{s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^*\} + 2\sigma_n^2 (\sigma_s^2 + \sigma_n^2) E\{s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^*\}
\end{aligned}$$

$$= \begin{cases} -e^{-j\phi} \sigma_s^2 (\sigma_s^4 + 6\sigma_s^2 \sigma_n^2 + 3\sigma_n^4) & \text{using the FLP preamble.} \\ 0 & \text{using SC's preamble.} \end{cases} \quad (\text{A.76})$$

$$\begin{aligned} E\{(q_{L_p}^2 - q_0^2)(d_0^* - d_{L_p}^*)\} &= E\{q_{L_p}^2 d_0^* - q_0^2 d_0^* - q_{L_p}^2 d_{L_p}^* + q_0^2 d_{L_p}^*\} \\ &= E\{d_0^* q_{L_p}^2\} - E\{d_0^* q_0^2\} - E\{d_{L_p}^* q_{L_p}^2\} + E\{d_{L_p}^* q_0^2\} \\ &= -2e^{-j\phi} \sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2) \end{aligned} \quad (\text{A.77})$$

$$\begin{aligned} &E \left\{ (q_{L_p}^2 - q_0^2) \Re \left\{ (d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} \right\} \\ &= \Re \left\{ E \left\{ (q_{L_p}^2 - q_0^2)(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m \right\} \right\} \\ &= \Re \left\{ E \left\{ (q_{L_p}^2 - q_0^2)(d_0^* - d_{L_p}^*) \right\} E \left\{ \sum_{m=1}^{L_p-1} d_m \right\} \right\} \\ &= \Re \left\{ (-2e^{-j\phi} \sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)) (e^{j\phi} \sigma_s^2 (L_p - 1)) \right\} \\ &= -2(L_p - 1) \sigma_s^4 \sigma_n^2 (\sigma_s^2 + \sigma_n^2) \end{aligned} \quad (\text{A.78})$$

$$\begin{aligned}
E\{|d_m|^2 q_m\} &= E\left\{(|s_{d_{opt}+m}|^2 |s_{d_{opt}+m+L_p}|^2 + |s_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} \right. \\
&\quad + |s_{d_{opt}+m+L_p}|^2 s_{d_{opt}+m}^* n_{d_{opt}+m} + n_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m}^* s_{d_{opt}+m+L_p} \\
&\quad + |s_{d_{opt}+m}|^2 s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + |s_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2 \\
&\quad + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m} s_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + n_{d_{opt}+m} s_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 \\
&\quad + s_{d_{opt}+m} |s_{d_{opt}+m+L_p}|^2 n_{d_{opt}+m}^* + s_{d_{opt}+m} n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} n_{d_{opt}+m}^* \\
&\quad + |s_{d_{opt}+m+L_p}|^2 |n_{d_{opt}+m}|^2 + |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p}^* s_{d_{opt}+m+L_p} \\
&\quad + s_{d_{opt}+m} s_{d_{opt}+m+L_p}^* n_{d_{opt}+m}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m} n_{d_{opt}+m}^* |n_{d_{opt}+m+L_p}|^2 \\
&\quad \left. + s_{d_{opt}+m+L_p}^* |n_{d_{opt}+m}|^2 n_{d_{opt}+m+L_p} + |n_{d_{opt}+m}|^2 |n_{d_{opt}+m+L_p}|^2\right\} \\
&\quad \cdot (|s_{d_{opt}+m+L_p}|^2 + s_{d_{opt}+m+L_p}^* n_{d_{opt}+m+L_p} + s_{d_{opt}+m+L_p} n_{d_{opt}+m+L_p}^* + |n_{d_{opt}+m+L_p}|^2) \\
&= (\sigma_s^2 + \sigma_n^2)^3 + 2\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)
\end{aligned} \tag{A.79}$$

$$\begin{aligned}
E\{|d_0|^2 q_{L_p}\} &= E\left\{(|s_{d_{opt}}|^2 |s_{d_{opt}+L_p}|^2 + |s_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} \right. \\
&\quad + |s_{d_{opt}+L_p}|^2 s_{d_{opt}}^* n_{d_{opt}} + n_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}}^* s_{d_{opt}+L_p} \\
&\quad + |s_{d_{opt}}|^2 s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + |s_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}+L_p}^* n_{d_{opt}} s_{d_{opt}}^* n_{d_{opt}+L_p} + n_{d_{opt}} s_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 \\
&\quad + s_{d_{opt}} |s_{d_{opt}+L_p}|^2 n_{d_{opt}}^* + s_{d_{opt}} n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} n_{d_{opt}}^* \\
&\quad + |s_{d_{opt}+L_p}|^2 |n_{d_{opt}}|^2 + |n_{d_{opt}}|^2 n_{d_{opt}+L_p}^* s_{d_{opt}+L_p} \\
&\quad + s_{d_{opt}} s_{d_{opt}+L_p}^* n_{d_{opt}}^* n_{d_{opt}+L_p} + s_{d_{opt}} n_{d_{opt}}^* |n_{d_{opt}+L_p}|^2 \\
&\quad \left. + s_{d_{opt}+L_p}^* |n_{d_{opt}}|^2 n_{d_{opt}+L_p} + |n_{d_{opt}}|^2 |n_{d_{opt}+L_p}|^2\right\} \\
&\quad \cdot (|s_{d_{opt}+2L_p}|^2 + s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+2L_p} n_{d_{opt}+2L_p}^* + |n_{d_{opt}+2L_p}|^2) \\
&= (\sigma_s^2 + \sigma_n^2)^3
\end{aligned} \tag{A.80}$$

$$\begin{aligned}
E\{|d_{L_p}|^2 q_0\} &= E\left\{(|s_{d_{opt}+L_p}|^2 |s_{d_{opt}+2L_p}|^2 + |s_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} \right. \\
&\quad + |s_{d_{opt}+2L_p}|^2 s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + n_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* s_{d_{opt}+L_p}^* s_{d_{opt}+2L_p} \\
&\quad + |s_{d_{opt}+L_p}|^2 s_{d_{opt}+2L_p}^* n_{d_{opt}+2L_p} + |s_{d_{opt}+L_p}|^2 |n_{d_{opt}+2L_p}|^2 \\
&\quad + s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p} s_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} + n_{d_{opt}+L_p} s_{d_{opt}+L_p}^* |n_{d_{opt}+2L_p}|^2 \\
&\quad + s_{d_{opt}+L_p} |s_{d_{opt}+2L_p}|^2 n_{d_{opt}+L_p}^* + s_{d_{opt}+L_p} n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} n_{d_{opt}+L_p}^* \\
&\quad + |s_{d_{opt}+2L_p}|^2 |n_{d_{opt}+L_p}|^2 + |n_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p}^* s_{d_{opt}+2L_p} \\
&\quad + s_{d_{opt}+L_p} s_{d_{opt}+2L_p}^* n_{d_{opt}+L_p}^* n_{d_{opt}+2L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^* |n_{d_{opt}+2L_p}|^2 \\
&\quad + s_{d_{opt}+2L_p}^* |n_{d_{opt}+L_p}|^2 n_{d_{opt}+2L_p} + |n_{d_{opt}+L_p}|^2 |n_{d_{opt}+2L_p}|^2) \\
&\quad \cdot (|s_{d_{opt}+L_p}|^2 + s_{d_{opt}+L_p}^* n_{d_{opt}+L_p} + s_{d_{opt}+L_p} n_{d_{opt}+L_p}^* + |n_{d_{opt}+L_p}|^2)\Big\} \\
&= (\sigma_s^2 + \sigma_n^2)^3 + 2\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2) \tag{A.81}
\end{aligned}$$

$$\begin{aligned}
E\{(|d_0|^2 - |d_{L_p}|^2)(q_{L_p} - q_0)\} &= E\{|d_0|^2 q_{L_p} - |d_{L_p}|^2 q_{L_p} - |d_0|^2 q_0 + |d_{L_p}|^2 q_0\} \\
&= E\{|d_0|^2 q_{L_p}\} - E\{|d_{L_p}|^2 q_{L_p}\} - E\{|d_0|^2 q_0\} + E\{|d_{L_p}|^2 q_0\} \\
&= (\sigma_s^2 + \sigma_n^2)^3 - 2[(\sigma_s^2 + \sigma_n^2)^3 + 2\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)] + (\sigma_s^2 + \sigma_n^2)^3 + 2\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2) \\
&= -2\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2) \tag{A.82}
\end{aligned}$$

$$\begin{aligned}
E\{(|d_0|^2 - |d_{L_p}|^2)(q_{L_p} - q_0) \sum_{n=1}^{L_p-1} q_n\} &= E\{(|d_0|^2 - |d_{L_p}|^2)(q_{L_p} - q_0)\} E\{\sum_{n=1}^{L_p-1} q_n\} \\
&= (-2\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2))[(L_p - 1)(\sigma_s^2 + \sigma_n^2)] \\
&= -2(L_p - 1)\sigma_s^2 \sigma_n^2 (\sigma_s^2 + \sigma_n^2)^2 \tag{A.83}
\end{aligned}$$

From (A.63), (A.67), (A.73), (A.78) and (A.83), we have

$$\begin{aligned}
 & Cov(P_d, R_d) \\
 &= E\{(|d_0|^2 - |d_{L_p}|^2)(q_{L_p}^2 - q_0^2)\} + 4E\left\{\Re\left\{(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m\right\} (q_{L_p} - q_0) \sum_{n=1}^{L_p-1} q_n\right\} \\
 &+ 2E\left\{(q_{L_p}^2 - q_0^2) \Re\left\{(d_0^* - d_{L_p}^*) \sum_{m=1}^{L_p-1} d_m\right\}\right\} + 2E\left\{(|d_0|^2 - |d_{L_p}|^2)(q_{L_p} - q_0) \sum_{n=1}^{L_p-1} q_n\right\} \\
 &= [-8\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] + 4[-\sigma_s^2\sigma_n^2[(L_p - 1)^2\sigma_s^2(\sigma_s^2 + \sigma_n^2) + (L_p - 1)\sigma_s^2\sigma_n^2]] \\
 &\quad + 2[-2(L_p - 1)\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2)] + 2[-2(L_p - 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \\
 &= -4[(L_p - 1)^2\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2) + (L_p - 1)\sigma_s^4\sigma_n^2(\sigma_s^2 + 2\sigma_n^2) + (L_p + 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \\
 &\hspace{20em} (A.84)
 \end{aligned}$$

From (A.14), (A.22) and (A.48), we have

$$E\{M_d\} = c_1 \cdot E\{P_d\} + c_2 \cdot E\{R_d\} = \begin{cases} \frac{4(L_p-1)\sigma_s^4}{L_p^2(\sigma_s^2+\sigma_n^2)^2} & \text{using the FLP preamble.} \\ \frac{2(L_p-1)\sigma_s^4}{L_p^2(\sigma_s^2+\sigma_n^2)^2} & \text{using SC's preamble.} \end{cases} \quad (A.85)$$

From (A.15), (A.42), (A.62) and (A.84), we have

$$\begin{aligned}
 Var(M_d) &= c_1^2 \cdot Var(P_d) + c_2^2 \cdot Var(R_d) + 2 \cdot c_1 \cdot c_2 \cdot Cov(P_d, R_d) \\
 &= \left\{ \begin{aligned} &\frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [4(L_p - 1)^2(3\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4) + 4(L_p - 1)(2\sigma_s^4 + 2\sigma_s^2\sigma_n^2 + \sigma_n^4)(2\sigma_s^2\sigma_n^2 + \sigma_n^4) \\ &+ 8(\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6)] + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 + (32L_p - 28)\sigma_s^4\sigma_n^4] \\ &- 8\frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2) + (L_p - 1)\sigma_s^4\sigma_n^2(\sigma_s^2 + 2\sigma_n^2) \\ &+ (L_p + 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \quad \text{using the FIP preamble.} \end{aligned} \right. \\
 &\quad \left\{ \begin{aligned} &\frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [(L_p - 1)^2\sigma_s^4(2\sigma_s^4 + 8\sigma_s^2\sigma_n^2 + 4\sigma_n^4) + 4(L_p - 1)[3\sigma_s^4(\sigma_s^2\sigma_n^2 + \sigma_n^4) \\ &+ (2\sigma_s^2\sigma_n^2 + \sigma_n^4)^2] + 8(\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4 + \sigma_s^2\sigma_n^6)] + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2 \\ &+ (32L_p - 28)\sigma_s^4\sigma_n^4] - 8\frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2) + (L_p - 1)\sigma_s^4\sigma_n^2 \\ &(\sigma_s^2 + 2\sigma_n^2) + (L_p + 1)\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \quad \text{using SC's preamble.} \end{aligned} \right.
 \end{aligned} \tag{A.86}$$

When L_p is large, $Var(M_d)$ can be approximated by

$$\begin{aligned}
 Var(M_d) &= c_1^2 \cdot Var(P_d) + c_2^2 \cdot Var(R_d) + 2 \cdot c_1 \cdot c_2 \cdot Cov(P_d, R_d) \\
 &\approx \left\{ \begin{aligned} &\frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [4(L_p - 1)^2(3\sigma_s^6\sigma_n^2 + \sigma_s^4\sigma_n^4)] + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \\ &- 8\frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2)] \\ &\quad \text{using the FIP preamble.} \end{aligned} \right. \\
 &\quad \left\{ \begin{aligned} &\frac{1}{L_p^4(\sigma_s^2 + \sigma_n^2)^4} [(L_p - 1)^2\sigma_s^4(2\sigma_s^4 + 8\sigma_s^2\sigma_n^2 + 4\sigma_n^4)] + \frac{\sigma_s^8}{L_p^4(\sigma_s^2 + \sigma_n^2)^8} [16L_p^2\sigma_s^2\sigma_n^2(\sigma_s^2 + \sigma_n^2)^2] \\ &- 8\frac{\sigma_s^4}{L_p^4(\sigma_s^2 + \sigma_n^2)^6} [(L_p - 1)^2\sigma_s^4\sigma_n^2(\sigma_s^2 + \sigma_n^2)] \\ &\quad \text{using SC's preamble.} \end{aligned} \right.
 \end{aligned} \tag{A.87}$$

From (A.85), (A.86) and (A.87), we can see that using the FLP preamble will increase the mean of $M_d = M_p(d_{opt}) - M_p(d_{opt+1})$ and decrease the variance of $M_d = M_p(d_{opt}) - M_p(d_{opt+1})$ (for variance, when $SNR > 3dB$). Since $M_d = M_p(d_{opt}) - M_p(d_{opt+1})$ is approximated by a Gaussian RV, using the FLP preamble will decrease the probability of the start point shifting to the right side.

Appendix B

Setting the Threshold for Fine Time Synchronization

Fine time synchronization calculates the cross-correlation function of the frequency corrected sequences $\{r_k^c\}$ with the second transmitted training sequences $\{b_k\}$. Let us assume a perfect CFO estimation. After CFO correction, from (2.49), (2.50) and (4.13), we have

$$r_k^c = \left(\sum_{m=0}^{M-1} h_m x_{k-n_\epsilon-m} \right) e^{j\theta_0} + w_k \quad (\text{B.1})$$

where M is channel length, h_m is the channel impulse response, and w_k is the complex Gaussian noise with zero mean and variance σ_n^2 . θ_0 is the initial phase for each symbol. Since we assume θ_0 does not change in one symbol period and in this thesis the cross-correlation for fine time synchronization uses only the second training symbol, we can set $\theta_0 = 0$. For simplicity, we also assume $n_\epsilon = 0$. We have

$$r_k^c = \sum_{m=0}^{M-1} h_m x_{k-m} + w_k \quad (\text{B.2})$$

where x_k is the k^{th} sample of the complex transmitted sequences with mean power σ_x^2 . We have

$$E[\Re\{x_k\}^2] = E[\Im\{x_k\}^2] = \frac{\sigma_x^2}{2} \quad (\text{B.3})$$

$$E[\Re\{w_k\}^2] = E[\Im\{w_k\}^2] = \frac{\sigma_n^2}{2} \quad (\text{B.4})$$

We assume h_m is uncorrelated with x_k and the x_k 's are uncorrelated.

In [6], the author uses the value of $M(d_{opt})$ to estimate the SNR.

$$\widehat{SNR} = \frac{\sqrt{M(d_{opt})}}{1 - \sqrt{M(d_{opt})}} \quad (\text{B.5})$$

where $M(d)$ is the timing metric given by (3.4) and d_{opt} is the exact start point of first training symbol excluding prefix and postfix (Since we do not know the exact start point, we can use the estimated start point to estimate the SNR). Let d_{opt2} be the exact start point of the second training symbol excluding CP. From the preamble structure, we have $d_{opt2} = d_{opt} + G/2 + N + G$. Since the proposed method uses similar timing metric as SC's method, we use the estimated coarse time point instead of d_{opt} and the estimated SNR is

$$\widehat{SNR} = \frac{\sqrt{M_p(\hat{d}_{coarse})}}{1 - \sqrt{M_p(\hat{d}_{coarse})}} \quad (\text{B.6})$$

where M_p is given by (4.3).

Recall from appendix A (see A.10)) that $R(d)$ is a Gaussian RV with mean $N(\sigma_s^2 + \sigma_n^2)/2$, where σ_s^2 is defined in (A.4) and its variance is quite small compared to its mean. $R_f(d)$ in (3.12) also has the same mean given by $N(\sigma_s^2 + \sigma_n^2)/2$.

$$E\{R(d)\} = E\{R_f(d)\} = N(\sigma_s^2 + \sigma_n^2)/2 \quad (\text{B.7})$$

Recall the cross-correlation in (4.14), $\psi_n = \sum_{k=0}^{N-1} b_k^* r_{n+k}^c$. Recall from section 4.1.2, $\hat{d}_{fine} = \hat{n} - N - 1.5G$, where $\hat{n}$ is the first point in $[-G/2 + \hat{d}_{coarse} + N + 1.5G, \hat{d}_{coarse} + N + 1.5G + G/2]$ (from smallest to greatest) whose $|\psi_{\hat{n}}|$ is greater than a threshold η . In the following we will indicate how to set the threshold for various channels.

- Under AWGN channels

If $M = 1$ and $h_0 = 1$, we have $r_k^c = x_k + w_k$ and the channel is an AWGN channel. SNR is

$$SNR = \frac{\sigma_x^2}{\sigma_n^2} \quad (\text{B.8})$$

Since $x_{d_{opt2}+k} = b_k$, we have

$$\psi_{d_{opt2}} = \sum_{k=0}^{N-1} b_k^* r_{d_{opt2}+k}^c = \sum_{k=0}^{N-1} |b_k|^2 + \sum_{k=0}^{N-1} b_k^* w_{d_{opt2}+k} \quad (\text{B.9})$$

b_k is a Chu sequence, we assume the b_k 's are uncorrelated and b_k is uncorrelated with w_k .

$$E\{\psi_{d_{opt2}}\} = N\sigma_x^2 \quad (\text{B.10})$$

$$Var\{\psi_{d_{opt2}}\} = E\{\psi_{d_{opt2}}\psi_{d_{opt2}}^*\} - |E\{\psi_{d_{opt2}}\}|^2 = N\sigma_x^2\sigma_n^2 \quad (\text{B.11})$$

For $n \neq d_{opt2}$, since we search in a small range, we assume n is in the CP of the second symbol. The cross-correlation of $\{b_k\}$ and its circular shift version is zero.

$$\psi_{n, n \neq d_{opt2}} = \sum_{k=0}^{N-1} b_k^* w_{n+k, n \neq d_{opt2}} \quad (\text{B.12})$$

we have

$$E\{\psi_{n, n \neq d_{opt2}}\} = 0 \quad (\text{B.13})$$

$$\text{Var}\{\psi_{n,n \neq d_{opt2}}\} = N\sigma_x^2\sigma_n^2 \quad (\text{B.14})$$

Since $\sigma_s^2 = \sigma_x^2$, from (B.6) and (B.7) and using $R_f(\hat{d}_{coarse}) \approx N(\sigma_s^2 + \sigma_n^2)/2$, we have

$$\sigma_s^2 = \sigma_x^2 \approx \frac{2R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})}}{N} \quad (\text{B.15})$$

For AWGN channels, we propose to set the threshold

$$\eta_{AWGN} = 0.5E\{\psi_{d_{opt2}}\} = 0.5N\sigma_x^2 \approx R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})}$$

From the central limit theorem (CLT), ψ_n is a Gaussian RV. The probability of $|\psi_{d_{opt2}}| < \eta_{AWGN}$ and $|\psi_{n,n \neq d_{opt2}}| > \eta_{AWGN}$ for different SNR is given in Table B.1. Table B.1 shows that the probabilities of missing detection and false

Table B.1 The probability of $\psi_{d_{opt2}} < \eta_{AWGN}$ and $\psi_{n,n \neq d_{opt2}} > \eta_{AWGN}$ for different SNR under AWGN channels

SNR(dB)	-10	-5	0	5
$ \psi_{d_{opt2}} < \eta_{AWGN}$	2.100×10^{-7}	1.155×10^{-19}	6.389×10^{-58}	2.271×10^{-178}
$ \psi_{n,n \neq d_{opt2}} > \eta_{AWGN}$	2.100×10^{-7}	1.155×10^{-19}	6.389×10^{-58}	2.271×10^{-178}

alarm are quite small for the proposed choice of threshold.

- Under ISI channels

For ISI channels, we use the tapped delay line model given in section 2.1.4, but for each channel tap h_m , the amplitude of h_m is fixed and the phase of h_m is fixed or a uniform distribution in $[0, 2\pi)$. Recall the frequency corrected samples

r_k^c given in (B.2). We normalized the channel power

$$\sigma_h^2 = \sum_{m=0}^{M-1} |h_m|^2 = 1 \quad (\text{B.16})$$

The average SNR at the receiver is

$$SNR = \frac{E\{|\sum_{m=0}^{M-1} h_m x_{k-m}|^2\}}{E\{|w_k|^2\}} = \frac{(\sum_{m=0}^{M-1} |h_m|^2) \sigma_x^2}{\sigma_n^2} = \frac{\sigma_x^2}{\sigma_n^2} \quad (\text{B.17})$$

Since we use Chu sequences as the second training symbol, b_k has a perfect autocorrelation function.

$$\begin{aligned} \psi_{d_{opt2}} &= \sum_{k=0}^{N-1} b_k^* r_{d_{opt2}+k}^c = \sum_{k=0}^{N-1} b_k^* \sum_{m=0}^{l-1} h_m x_{d_{opt2}+k-m} + \sum_{k=0}^{N-1} b_k^* w_{d_{opt2}+k} \\ &= h_0 \sum_{k=0}^{N-1} |b_k|^2 + \sum_{k=0}^{N-1} b_k^* w_{d_{opt2}+k} \end{aligned} \quad (\text{B.18})$$

We have

$$E\{\psi_{d_{opt2}}\} = h_0 N \sigma_x^2 \quad (\text{B.19})$$

$$Var\{\psi_{d_{opt2}}\} = E\{\psi_{d_{opt2}} \psi_{d_{opt2}}^*\} - |E\{\psi_{d_{opt2}}\}|^2 = N \sigma_x^2 \sigma_n^2 \quad (\text{B.20})$$

For $n < d_{opt2}$, which means n is on the left side of the exact start point, similar to AWGN channels, we have

$$\psi_{n, n < d_{opt2}} = \sum_{k=0}^{N-1} b_k^* w_{n+k, n < d_{opt2}} \quad (\text{B.21})$$

$$E\{\psi_{n, n < d_{opt2}}\} = 0 \quad (\text{B.22})$$

$$Var\{\psi_{n, n < d_{opt2}}\} = N \sigma_x^2 \sigma_n^2 \quad (\text{B.23})$$

In most scenarios, the first tap power of ISI channels would be greater than -14dB , which means $|h_0| > 0.2$, so setting the threshold corresponding to half the mean of $\psi_{d_{opt2}}$ assuming that $h_0 = 0.2$ is reasonable i.e.

$$\eta_{ISI} = 0.5E\{\psi_{d_{opt2}}\} = 0.5h_0N\sigma_x^2 = 0.1N\sigma_x^2 \approx 0.2R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})}$$

From the central limit theorem (CLT), ψ_n is a Gaussian RV. The probability of $|\psi_{d_{opt2}}| < \eta_{ISI}$ and $|\psi_{n,n \neq d_{opt2}}| > \eta_{ISI}$ for different SNR is given in Table B.2. Two ISI channels are considered in this thesis with first tap gain $|h_0| = 0.37629$. Therefore, Table B.2 also provide probabilities for a threshold set to half the mean of $\psi_{d_{opt2}}$ considering the true $|h_0|$, e.g.

$$\eta_1 = 0.5E\{\psi_{d_{opt2}}\} = 0.5h_0N\sigma_x^2 \approx 0.19N\sigma_x^2$$

Table B.2 shows that if we choose the threshold to be equal to or little greater

Table B.2 The probability of $\psi_{d_{opt2}} < \eta_{ISI}$ and $\psi_{n,n < d_{opt2}} > \eta_{ISI}$ for different SNR under ISI channels

SNR(dB)	0	5	10	15
$ \psi_{d_{opt2}} < \eta_{ISI}$	4.731×10^{-19}	5.322×10^{-56}	2.592×10^{-172}	0
$ \psi_{n,n < d_{opt2}} > \eta_{ISI}$	0.00068714	6.334×10^{-9}	2.269×10^{-24}	1.068×10^{-72}
$ \psi_{d_{opt2}} < \eta_1$	1.251×10^{-9}	1.477×10^{-26}	1.436×10^{-79}	1.124×10^{-246}
$ \psi_{n,n < d_{opt2}} > \eta_1$	6.009×10^{-10}	1.511×10^{-27}	1.107×10^{-82}	1.682×10^{-256}

than the half of mean of $\psi_{d_{opt2}}$, the probability of missing detection and false

alarm will be at the same level, but we have to know the channel power of the first tap.

From the analysis, to get better performance, the threshold should depend on the SNR. Since we can estimate SNR from the peak value of the timing metric, we can set up a look up table depending on the estimated SNR for setting the threshold. For example, in the simulations presented in this thesis, the chosen threshold for ISI channels is

$$\eta_{ISI} = \begin{cases} 0.2R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} > 5dB \\ 0.3R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} \leq 5dB \end{cases}$$

- Under Rayleigh fading channels

For the Rayleigh fading channel, each tap amplitude is Rayleigh distributed, and we normalize the mean power of the channel to be one.

$$E\{\sigma_h^2\} = E\left\{\sum_{m=0}^{M-1} |h_m|^2\right\} = 1 \quad (\text{B.24})$$

The average SNR at the receiver is

$$SNR = \frac{E\{|\sum_{m=0}^{M-1} h_m x_{k-m}|^2\}}{E\{|w_k|^2\}} = \frac{E\{\sum_{m=0}^{M-1} |h_m|^2\}\sigma_x^2}{\sigma_n^2} = \frac{\sigma_x^2}{\sigma_n^2} \quad (\text{B.25})$$

Similar to ISI channels

$$\begin{aligned} \psi_{d_{opt2}} &= \sum_{k=0}^{N-1} b_k^* r_{d_{opt2}+k}^c = \sum_{k=0}^{N-1} b_k^* \sum_{m=0}^{M-1} h_m x_{d_{opt2}+k-m} + \sum_{k=0}^{N-1} b_k^* w_{d_{opt2}+k} \\ &= h_0 \sum_{k=0}^{N-1} |b_k|^2 + \sum_{k=0}^{N-1} b_k^* w_{d_{opt2}+k} \end{aligned} \quad (\text{B.26})$$

We have

$$E\{\psi_{d_{opt2}}\} = h_0 N \sigma_x^2 \quad (\text{B.27})$$

$$Var\{\psi_{d_{opt2}}\} = E\{\psi_{d_{opt2}} \psi_{d_{opt2}}^*\} - |E\{\psi_{d_{opt2}}\}|^2 = N \sigma_x^2 \sigma_n^2 \quad (\text{B.28})$$

For $n < d_{opt2}$, which means n is on the left side of exact start point, since we use Chu sequences, we have

$$\psi_{n, n < d_{opt2}} = \sum_{k=0}^{N-1} b_k^* w_{n+k, n < d_{opt2}} \quad (\text{B.29})$$

$$E\{\psi_{n, n < d_{opt2}}\} = 0 \quad (\text{B.30})$$

$$Var\{\psi_{n, n < d_{opt2}}\} = N \sigma_x^2 \sigma_n^2 \quad (\text{B.31})$$

Since $|h_0|$ is Rayleigh distributed, sometimes $|h_0|$ might be very small. If we set the threshold too small, the probability of the false alarm would be high and the fine time synchronization would be useless. On the other hand, if the first tap is too weak, we can ignore that tap and choose the first strong tap to be the first tap, which would not result in too much performance loss. Assuming that the first tap power is greater than -14dB (or equivalently $|h_0| > 0.2$), we can set the threshold for Rayleigh fading channels as

$$\eta_{Rayleigh} = 0.5 E\{\psi_{d_{opt2}}\} = 0.1 N \sigma_x^2 \approx 0.2 R_f(\hat{d}_{coarse}) \sqrt{M_p(\hat{d}_{coarse})}$$

$$\eta_{Rayleigh} = 0.5 E\{\psi_{d_{opt2}}\} = 0.1 N \sigma_x^2 \approx 0.2 R_f(\hat{d}_{coarse}) \sqrt{M_p(\hat{d}_{coarse})}$$

Similar results as ISI channels would be obtained.

Similar to ISI channels, better performance can be obtained by choosing the thresh-

old according to SNR. Since we can estimate SNR from the peak value of the timing metric, we can set up a look up table depending on the estimated SNR for setting the threshold. If we know the channel power delay profile, it is very helpful for setting the threshold. For example, in the simulations presented in this thesis, to obtain improved performance, while maintaining reasonable complexity for the look-up table, we propose to set the threshold for Rayleigh as

$$\eta_{Rayleigh} = \begin{cases} 0.2R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} > 5dB \\ 0.3R_f(\hat{d}_{coarse})\sqrt{M_p(\hat{d}_{coarse})} & \text{if } \widehat{SNR} \leq 5dB \end{cases}$$

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