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Special thanks to the members of the Mechanical Engineering workshop, particularly Mr. Bob Hill, for their sincere cooperation and manufacturing skills which made the experimental program possible.

This thesis is dedicated to my beloved parents, Khanom agha and Yadollah Zarabi.
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DESIGN AND TESTING OF INLET GUIDE VANES
FOR CENTRIFUGAL IMPELLERS

by

Houshmand Zarabi

A thesis submitted to the faculty of Graduate Studies in partial fulfilment of the requirements for the degree of Master of Engineering.

Faculty of Engineering
Carleton University
Ottawa, Ontario
July 1981
The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis "Design and Testing of Inlet Guide Vanes for Centrifugal Impellers" by Houshmand Zarabi, B.Sc., in partial fulfilment of the requirement for the degree of Master of Engineering.

[Signature]
Thesis Supervisor

[Signature]
Chairman, Department of Mechanical and Aeronautical Engineering
ABSTRACT

Theoretical and experimental investigations were carried out to determine the flow field downstream of inlet guide vanes. The theoretical study which was based on the Radial Equilibrium Equation indicated the superiority of the forced vortex type of distribution in reducing the incidence loss of typical inducers downstream of the vanes. The experimental program was involved in the design and testing of four sets of inlet guide vanes. Three of the sets were designed to produce constant-prewhirl angle with radius distribution while the fourth was designed to generate forced vortex distribution. The vanes of three sets had circular-arc section, while the fourth had flat plate section. Each set of vanes was tested in a 3.78 inches diameter circular duct in which axial flow velocity upstream of the vanes was kept constant at 136 ft/s. The results showed higher losses for the conventional flat plate vanes when compared with circular-arc vanes. In addition, it was found that a prewhirl angle of 30 degrees was the limit within which losses were tolerable. Also, the experimental values of prewhirl angles and axial velocity were in qualitative agreement with the theoretical predictions.
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<tr>
<td>(\sigma)</td>
<td>Solidity</td>
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<tr>
<td>(\frac{S}{C})</td>
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<td>(R_c)</td>
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$r_0$  Inlet duct radius
\[ R \]  Radius ratio
\[ b \]  Outlet channel width
\[ z \]  Number of vanes

Subscripts

\[ s \]  shroud
\[ h \]  hub
\[ o \]  stagnation condition
\[ CA \]  Circular arc
\[ FP \]  Flat plate
\[ d \]  design
INTRODUCTION

Centrifugal compressors are a type of turbomachinery, like fans, blowers and superchargers, in which air is sucked in and moved out to a higher pressure due to the dynamic action of the rotating vanes. These vanes increase the kinetic and pressure energies of the flowing air. A diffuser then converts part of the kinetic energy into pressure energy. The compression in centrifugal compressors is mainly due to the centrifugal action. This differs from the axial counterpart, where the compression is due to the diffusion of the flow. In high performance units, an inducer is provided to reduce losses as the flow is deflected from axial to radial direction.

At design flow operation of the compressor, the flow approaches the inducer axially, with no prewhirl component of velocity. The flow then enters the inducer at an angle corresponding to the inducer blade angle. At flow rates less than design, however, the flow is said to acquire prerotation and the flow enters the inducer with an incidence angle. At high incidence angles, flow separations occur in the inducer channel, which tends to deteriorate the performance of the machine. By providing inlet guide vanes at the inlet to the inducer, the direction of air can be altered in order to aid in maintaining the incidence
angles within a low-loss range. Thus, the performance of the machine can be improved. In the characteristic curves, this improvement becomes evident as an increase in the stable operating range which is often restricted due to surge.

In addition to the above, when the inlet guide vanes are rotated to an angle in the direction of the impeller rotation, the flow will approach the inducer with a pre-whirl. The head developed by the machine will then reduce, which is due to the subtractive term \( U_1 V_{g1} \) in the Euler's equation (Appendix A). In blowers, this aspect of inlet guide vanes is often used for the purpose to control the output head produced by the machine, which has the particular advantage of being able to keep the impeller speed constant.

The presence of the inlet guide vanes in the inlet duct, however, produces losses because of the vanes themselves. The losses are mainly due to the skin friction and the incidence losses. The effectiveness of the guide vanes to improve the performance, and particularly the efficiency of the machine is conditional on whether these losses are kept small or not.
The flexibility of using the stationary, but adjustable inlet guide vanes to impart a desired prewhirl to the flow has been found convenient by some investigators. Stepanoff [2], [12], Liveshitz [8], Shouman and Anderson [9], Rodgers [10] and Wallace [11] have studied the effect of prewhirl on the centrifugal impeller performance and have reported favourable results.

In view of the above, and the idea to further investigate the effects of inlet guide vanes on the centrifugal impeller performance, this thesis was commenced. Theoretical studies were carried out to investigate the effect of various distributions of prewhirl angles at the inlet to a hypothetical inducer. The radial variation of the incidence angles were then calculated, which allowed us to obtain the best type of prewhirl angle distribution based on the condition of a low-loss incidence range. The results were comparable to the work in Ref. [11]. Furthermore, to produce these idealized prewhirl angle distributions, two inlet guide vanes were designed to produce constant-prewhirl angle with radius distribution, in which one used a circular-arc section with constant-radius-of-curvature from hub to shroud, and the other a conventional flat plate section. In addition to a vane which was designed to generate forced vortex type of distribution with a circular-arc section. The vanes were of the simplest geometrical shapes possible.
The experimental program was aimed at examining the flow field downstream of the designed inlet guide vanes. Four sets of vanes were tested, in which three of the sets had constant-prewhirl angle with radius distribution, and one set had a forced vortex distribution. As will be shown, discrepancies exist between the theoretical and experimental values of prewhirl angles for the constant-prewhirl angle distribution which increases with higher angular positions of the guide vanes. These discrepancies are believed to indicate the general three-dimensional nature of the flow and the existence of secondary flows in the wall and central region of the duct. In case of the forced vortex type, the experimental values of prewhirl angles agreed well with the theoretical predictions for the design vane angle settings.

The fact that the inlet guide vanes introduce additional losses in the inlet duct makes them particularly critical in terms of the overall performance of the machine. Traditionally, flat plate guide vanes were used in practice due to its simplicity. The present investigation, however, has proved that a constant-radius-of-curvature vanes produce considerably less losses in comparison with the flat plate vane, for approximately the same prewhirl angle distribution. Further, there appears a limit for the
increase in prewhirl angles without the penalty in further losses even for the circular-arc vanes. The experimental results show this limit to be 30 degrees.
CHAPTER I

REVIEW OF LITERATURE

1.1 Introduction

Large amounts of literature exist in the field of centrifugal machinery, but very few have discussed the influence of inlet prewhirl on flow through the impeller channels. The following is a brief review of some of existing literature. In addition, a summary of the theory of centrifugal impellers is presented in the Appendix A.

1.2 Prerotation and Prewhirl

1.2.1 Prerotation

Prerotation is the phenomenon which arises due to the fact that the fluid, while entering the impeller, tries to do so with minimum possible disturbance; and in such a manner that the fluid flow angles correspond to the inducer blade angles. Theoretically, a uniform axial velocity with zero incidence angle to the inducer sections is possible only at the design conditions. For flow rates greater than the design, the fluid will acquire prerotation in the direction opposite to the impeller rotation; i.e., negative prerotation, and for flow rates lower than the design point, the prerotation will be in the direction of impeller rotation; i.e., positive prerotation.
Stepanoff [1] explained the phenomenon of prerotation using "the principle of least resistance"; namely, the resistance to flow is a minimum when the fluid enters the impeller at an angle approaching the inducer vane angle. Referring to figure (1.1a), at the design flows, the axial velocity is $V_z$ and the fluid enters the inducer with no incidence. At flows less than design, the axial velocity is $V'_z$ and the angle of incidence to the inducer is $i$. This is the case when the fluid approaches the inducer without prerotation. In actual practice, due to prerotation $a'$, the flow enters the inducer at an intermediate incidence angle $i'$, since sudden changes in direction cannot occur in fluids.

Pelikovsky [2] contradicted the above approach to explain the phenomenon of prerotation, and stated that the fluid in spite of its tendency to enter the impeller with minimum disturbance, cannot acquire prerotation; i.e., angular momentum, without a corresponding moment of external forces. It was pointed out that, at high incidence angles, vortex zones appear at the entrance edges of inducer, and their periodic penetration in the suction duct might impart to the incoming flow a certain prerotation.
Several investigators have observed that the backflow takes place at low flow rates. Murakami [3] in his experimental investigation observed the prerotation and backflow. He noticed backflows near the walls of the suction duct to a depth of 10%. Shepherd [4] has reported that another contributory factor for prerotation is the leakage flows from the high pressure side which disturbs the flow. Fisher and Thoma [5] have also considered the backflow from the impeller inlet to be the main cause of prerotation. Contrary to this, Stepanoff [1] argues that the backflow is the result of prerotation and not the cause. He states that prerotation develops a centrifugal head near the suction pipe wall, higher than the inlet head. Therefore, creating an adverse pressure gradient in that part of the pipe, causing the flow to reverse.

Acosta and Bowermann [6] stated that the backflow can occur over a considerable extent on the pressure side of the impeller vane at low flow rates. They further noticed that such backflows do not lead to high efficiency or stable operation. Johnson and Ginsburg [7] reported that at low flow rates, velocities on the front face at the impeller inlet decreases and sometimes they become negative. That is, an eddy forms in the passage. This tendency to eddy is greater with increased rotational speeds, lower flow rates and fewer vanes.
When the fluid approaches the impeller at an angle of incidence other than design, then losses occur, which tend to deteriorate the performance of the machine. These losses are termed "incidence losses" (or sometimes called "shock losses"), and are due to flow separations and turbulence within the impeller channel. Acosta and Bowerman [6] studied the influence of incidence angle. From their experimental results, they concluded that the inlet flow angles have a profound effect upon the characteristic curves. Experiments were performed on four impellers. The angle of incidence for best efficiency was observed to be in between $2^\circ$ and $-1.5^\circ$ depending on the impeller. Livshits [8] reported that the maximum efficiency occurs when the angle of incidence is zero. His experimental results indicated that the losses increase more quickly with positive incidence. It was argued that, at positive incidence angles, the vane inlet tends to act as a diffuser rather than as a nozzle. However, Johnson and Gilsburg [7] contradicted Livshits by their experiment. They observed that the negative incidence angles result in higher losses than the positive incidence angles. Further, they conducted their experiment at high subsonic mach numbers; whereas Livshits performed his experiment at low mach numbers, which can be the possible source of discrepancy between their results.
1.2.2 - Prewhirl

In view of the above, it becomes necessary to control the prerotation in such a manner that the flow will approach the impeller at a correct angle of incidence. To accomplish this, inlet guide vanes are provided in the suction duct, upstream of the impeller. By adjusting the position of the guide vanes, the desired prewhirl can be imparted to the flow to ensure smooth entry to the impeller. Figure (1.1b) shows the inlet velocity triangle with a positive prewhirl ($\alpha$).

When the flow approaches the impeller with a prewhirl in the direction of impeller rotation, the head developed by the machine reduces. This is evident from the Euler's equation (1.1) in Appendix A, where the second term $U_1V_{\theta 1}$ is not zero. If the prewhirl was given in the opposite direction to that of impeller rotation; i.e., negative prewhirl, then $U_1V_{\theta 1}$ changes its sign and as a result a higher head can be expected theoretically. Experimental results, however, indicate that there exists a limit for the head to increase. Stepanoff [1] reported that head cannot be increased further when a 20 degrees of negative prewhirl was given to the flow. Shouman and Anderson [9] reported this angle to be less than 15 degrees. They noticed high losses take place beyond this angle and, as a result, the efficiency and the head developed by the machine reduce considerably.
Rodgers [10] reported that the maximum efficiency of centrifugal compressor occurs with the guide vanes at zero prewhirl angle setting, with relatively large efficiency reduction at prewhirl angles greater than 40 degrees in the direction of rotation of the impeller. Wallace [11] observed that the introduction of 20 degrees of positive prewhirl leads to only a small reduction in the maximum efficiency. However, some investigators have observed that the efficiency increased with increase in guide vane prewhirl angle setting of up to 10 degrees, and dropped again at higher settings.

Stepanoff [12] investigated the effect inlet guide vanes on a centrifugal blower performance. He summarizes the influence of the guide vanes on the output head and efficiency as follows:

1) for small angles of flow deflection, head reduces such that:

\[ \eta = \text{Constant} \]

2) For "pure throttling" which is similar to when the flow is throttled at discharge, head reduces at the same rate as efficiency, or

\[ \frac{H}{H_a} = \frac{\eta}{\eta_a} \]
3) When the flow is throttled at the inlet, reduction of head as a result of gas density reduction across the guide vanes occur in such a manner that:

\[
\frac{H_a}{H} < \frac{\eta_a}{\eta}
\]

where subscript \( a \) is the variable with the guide vanes in active position. He concludes that, in practice, all of the above three factors appear simultaneously, and some power savings can be realized which can be utilized in multistage blowers.

Rodgers [10] indicated in his study that the effect of the use of variable inlet guide vanes was similar to changing the impeller rotational speeds; significant changes in the surge and choke flows are possible at low impeller rotational speeds and speeds greater than design. Inlet guide vanes are also being utilized in the turbocharger in diesel engines. According to Wallace [11], the stable range at operation of a turbocharger centrifugal compressors may be extended to lower flow rates by the introduction of positive prewhirl. In addition, the head produced by the compressor can be controlled at constant speed operation.

Shouman and Anderson [9] have suggested a method for controlling the power output of small gas turbine units with the help of guide vanes. By imparting prewhirl at
the inlet to the compressor, the pressure ratio could be varied. Therefore, the turbine temperature and pressure levels could be varied to adjust the output power. In other words, varying prewhirl at constant compressor speed is equivalent to varying compressor speed without prewhirl; as a result the compressor can operate over a wide range of pressure ratios.

The use of stationary guide vanes in high pressure ratio compressors is also a common practice. This is to improve the inlet flow conditions to the inducer by:

1. reducing the inducer tip relative Mach number and
2. reducing the inducer vane angle, which opens up the throat of the inducer passage.

According to Millar [13], the first eliminates shock losses at supersonic flows and the second increases the choking flow capacity of the machine at high speeds. Some of the disadvantages are:

1. Additional losses due to the guide vanes themselves;
2. increased diffuser losses;
3. higher noise level as wakes from the prewhirl vanes interact with the inducer vanes.
(4) steeper characteristic curves;
(5) a potential source of icing problems for operation in freezing conditions;
(6) additional cost.
CHAPTER II

INCIDENCE ANGLE STUDIES

2.1 Optimized Prewhirl Distribution

This theoretical study is conducted to compare four types of prewhirl distribution, and to determine the best type of distribution in order to keep the inducer incidence angles of a centrifugal impeller within a prescribed low-loss range. The study is based on a simplified analysis of the three dimensional swirling flow in a circular duct. From the results, the inlet guide vanes are designed according to a desirable type of distribution.

The inducer is considered as a number of radially stacked sections. At design flows, each section operates over a range of incidence angles for which the losses incurred in the flow within the section are relatively small. At off-design flows, however, each section is forced to operate at incidence angles outside the so-called low-loss range. Losses will rise rapidly due to flow separations in the inducer section, and the section tends to stall. At high incidence angle, the performance of the machine deteriorates considerably.

The objective of variable inlet guide vanes is to change the direction of air entering the inducer as the
flow is reduced in order to maintain the incidence angles at all sections within their individual low-loss range. This is shown in Figure 1.1. Throughout the study, the limits of operation are considered to be reached whenever any section reaches the selected limits of incidence range, either positive or negative from optimum. These limits are considered to define the usable incidence range for analysis purposes, whether or not stall or choke conditions are reached.

The data presented in the two-dimensional cascades of airfoils for single-stage axial flow rotors (18), show that the low-loss incidence ranges are \( \pm 2^\circ \) to \( \pm 6^\circ \) from the reference value of incidence in the shroud region, where the highest relative mach number and stagger angles occur. Over the rest of the blade height, the incidence angle ranges are \( \pm 6^\circ \) to \( \pm 10^\circ \). The ranges at values appear because the low-loss range of incidence angle applicable to a given blade section generally increases as the blade relative mach number and stagger angle decrease. In our analysis, due to similarities which exist between axial flow rotor and the inducer section of a centrifugal impeller, values of \( \delta_i \) chosen at \( \pm 6^\circ \) at the shroud and \( \pm 10^\circ \) at the hub region of the inducer.
The analysis is based on the familiar Radial Equilibrium approach, with the assumption of negligible "streamline curvature"; i.e., \( V_r = 0 \), no radial entropy and total temperature (or enthalpy) gradients. The types of prewhirl distributions investigated in detail are: free vortex, forced vortex, constant-prewhirl angle with radius and quadratic-prewhirl velocity with radius. All of these distributions follow the general equation \( V_\theta = AR^n \), where \( n \) is an integer except for the constant prewhirl angle with radius type, and \( A \) is a constant. Also, for the analysis, the incidence angle at the shroud of the inducer is kept constant at zero degrees, as a reference. This value was chosen because it is widely accepted [8] as the best efficiency point, where losses in the inducer are minimum. Various prewhirl angles are then assumed at the inducer shroud, and the effect on the incidence angles at other radii are calculated.

As one of the requirements of the prewhirl mechanism is that to be variable, the effect of rotating the vanes is also studied which initially generated the free vortex, forced vortex, etc. Rotation of the vanes through a fixed angle, has the effect of rotating the prewhirl angles at all radii through \( \Delta \theta \). The type of prewhirl generated thus differs from that obtained prior to the vane rotation; i.e., the free or forced vortex is destroyed. This however does not apply to the constant-prewhirl angle with radius type of distribution which is an advantage of using this type.
2.2. Analysis Procedure

The objective of this analysis is to find the most satisfactory type of distribution in order to control the incidence angles at the leading edge to an inducer. The radial variation of incidence angles is therefore computed for various types of distribution. Also, the effect of inlet guide vane rotation on the variation of incidence angles is studied.

2.2.1 Prewhirl Types

All four types of prewhirl velocity distributions which are assumed at the inlet to the inducer can be represented by the following equation:

\[ v_\theta = AR^n \]  \hspace{1cm} (2.1)

where

- \( R \) - non-dimensional radius ratio;
- \( A \) - a constant
- \( n \) - an integer

for free vortex type \( n = -1 \)
for forced vortex type (solid body rotation) \( n = 1 \)
for quadratic prewhirl velocity with radius type \( n = 2 \)
and for constant-prewhirl angle with radius type, the value of \( n \) is derived as shown in section 2.2.7, where \( n = - \sin^2 \alpha \).
2.2.2 Radial Equilibrium Equation

The radial component of the Navier-Stokes equation for an inviscid steady flow is known as Radial Equilibrium Equation. If we write this component of the equation for the space between the guide vanes where there are no radial forces exerted on the fluid by the guide vanes, we have,

\[-\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{\partial}{\partial r} \left( \frac{3V_r}{r} \right) + \frac{\partial}{\partial \theta} \left( \frac{\partial V_r}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{\partial V_r}{\partial z} \right) - \frac{V_r^2}{r}\]

if the flow is assumed to be axisymmetric, \(\frac{\partial}{\partial \theta} = 0\), then we have,

\[-\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{\partial}{\partial r} \left( \frac{3V_r}{r} \right) + \frac{\partial}{\partial z} \left( \frac{\partial V_r}{\partial z} \right) - \frac{V_r^2}{r} \]  \hspace{1cm} (2.2)

assuming the stream surfaces are cylindrical (streamline curvature is negligible), then \(V_r = 0\), and the flow is two-dimensional, we have,

\[-\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{V_r^2}{r}\]

if we non-dimensionalize the radius such that \(R = \frac{r}{r_o}\), where \(r_o\) is the inlet duct radius, we get,

\[-\frac{1}{\rho} \frac{\partial p}{\partial R} = \frac{V_r^2}{R} \] \hspace{1cm} (2.3)

The above equation (2.3) is the Simple Radial Equilibrium Equation which represents the radial pressure variation downstream of the guide vanes due to the centrifugal acceleration. This acceleration is caused by prewhirl. The equation stresses the fact that the static pressure will not be constant down-
stream from the guide vanes as is the case for pipe flow.

For a compressible flow, using the T ds relation, we have

\[ T \, ds = dh - \frac{dP}{\rho} \]

or

\[ \frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{\partial h}{\partial r} - \frac{T \, ds}{\partial r} \]  \hspace{1cm} (2.4)

from the relation for stagnation enthalpy,

\[ h_o = h + \frac{v^2}{2} \]

where

\[ v^2 = v_\theta^2 + v_z^2 \]

we have

\[ h_o = h + \frac{v_\theta^2}{2} + \frac{v_z^2}{2} \]

differentiating the above equation, we get

\[ \frac{\partial h}{\partial r} = \frac{\partial h_o}{\partial r} - v_\theta \frac{\partial v_\theta}{\partial r} - v_z \frac{\partial v_z}{\partial r} \]  \hspace{1cm} (2.5)

Combining equations 2.3, 2.4, and 2.5, we have

\[ \frac{\partial h_o}{\partial r} - \frac{T \, ds}{\partial r} = \frac{v_\theta^2}{r} + v_\theta \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_z}{\partial r} \]  \hspace{1cm} (2.6)

assuming radial gradients of enthalpy and entropy are negligible, i.e., no losses in the radial direction, then
equation 2.6 reduces to
\[ \frac{v_\theta^2}{r} + v_\theta \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_z}{\partial r} = 0 \]
or in non-dimensional form,
\[ \frac{v_\theta^2}{R} + v_\theta \frac{\partial v_\theta}{\partial R} + v_z \frac{\partial v_z}{\partial R} = 0 \]  
(2.7)

The above equation 2.7 is known as radial compatibility equation, where for any specified prewhirl distribution, the radial variation of axial velocity produced by the guide vanes can be calculated.

2.2.3 Prewhirl Angle Distribution

Differentiating equation 2.1, we have
\[ \frac{d v_\theta}{d R} = A_n R^{n-1} \]

Combining with equation 2.7, results in
\[ v_z \frac{d v_z}{d R} + R^{2n-1} \left[ A^2 (1 + n) \right] = 0 \]

integrating the above (Appendix B), we obtain
\[ v_z^2 = v_{zs}^2 + v_\theta^2 \left[ \frac{(1+n)}{n} (1-R^{2n}) \right] \]  
(2.8)
where subscript s refers to the condition at the shroud, now, applying the following condition to equation 2.4, at
\[ R = 1, \quad V_\theta = V_{\theta s} \]
where, \[ V_\theta = AR^n \]
we have, \[ A = V_{\theta s} \]
Therefore, \[ V_\theta = V_{\theta s} R^n \]
from the inlet velocity triangle, we have,
\[ \tan \alpha = \frac{V_\theta}{V_z} \]
or
\[ V_z = \frac{V_{\theta s} R^n}{\tan \alpha} \]
substituting the above equation in equation 2.8, we have,
\[ \alpha = \tan^{-1}\left\{ R^n \left( \frac{\tan^2 \alpha_s}{1 + \tan^2 \alpha_s \left[ \frac{(1+n)}{n} \left( 1 - R^{2n} \right) \right]} \right)^{1/2} \right\} \quad (2.9) \]
For a specified prewhirl angle at the shroud, equation 2.9 enables the prewhirl angles at all radii to be calculated. When the vanes are rotated through angle \( \Delta \alpha \), the original prewhirl distribution no longer exists (except for the constant prewhirl angle with radius distribution, and the prewhirl angles are calculated according to the following equation:}
$$\alpha = \tan^{-1} \left\{ \frac{R^{2n} \tan^2 \alpha_s}{1 + \tan^2 \alpha_s \left[ \frac{(1+n)}{n} \left( 1 - R^{2n} \right) \right]} \right\}^{1/2} + \Delta \alpha \quad (2.10)$$

2.2.3 Absolute and Axial Velocity Distribution

From inlet velocity triangle, we have,

$$V_z = V \cos \alpha \quad \text{and} \quad V_\theta = V \sin \alpha$$

differentiating with respect to R,

$$\frac{dV_z}{dR} = \frac{dV}{dR} \cos \alpha - V \frac{d\alpha}{dR} \sin \alpha$$

and

$$\frac{dV_\theta}{dR} = \frac{dV}{dR} \sin \alpha + V \frac{d\alpha}{dR} \cos \alpha$$

adding the above two equations will result in:

$$V \frac{dV_z}{dR} + V_\theta \frac{dV_\theta}{dR} = V \frac{dV}{dR} \quad (2.11)$$

combining equations 2.7 and 2.11, we have

$$V \frac{dV}{dR} = - \frac{V_\theta^2}{R}$$
dividing both sides of the equation by $V^2$,

$$\frac{dV}{V} = - \frac{dR}{R} \left( \frac{V_0}{V} \right)^2$$

or

$$\frac{dV}{V} = - \frac{\sin^2 \alpha}{R} dR$$

integrating the above, we get,

$$\ln \frac{V}{V_s} = - \int \frac{\sin^2 \alpha}{R} dR$$

now, if

$$X = - \int \frac{\sin^2 \alpha}{R} dR \quad \text{(2.12a)}$$

then

$$\frac{V}{V_s} = e^X \quad \text{(2.12)}$$

In order to perform the integration, the prewhirl angle distribution as a function of $R$ is given in equations 2.9 and 2.10. The integration is performed numerically (by trapezoidal rule) in the computer program. The absolute velocity parameter ($\frac{V}{V_s}$) is therefore obtained.

The axial velocity parameter distribution is given as:

$$\frac{V_z}{V_s} = \frac{V}{V} \cos \alpha \quad \text{(2.13a)}$$
and the prewhirl velocity parameter distribution as:

\[
\frac{V_0}{V_s} = \frac{V}{V_s} \sin \alpha
\]  

(2.13b)

2.2.5 Inducer Blade Angle Distribution

The inducer blade angle distribution is calculated according to the assumptions that the flow enters the impeller axially with no pre-rotation, and the axial velocity is constant with radius, figure 2.1. The above two assumptions are utilized in actual practice for the design of inducers for Centrifugal Compressors which is for a specified mass flow rate and rotational speed.

From figure 2.1, we have,

\[
\tan \beta_{0s} = \frac{U_s}{V_z}
\]

also

\[
\tan \beta_0 = \frac{U}{V_z}
\]

dividing the above equations, we have,

\[
\frac{\tan \beta_{0s}}{\tan \beta_0} = \frac{U_s}{U}
\]

but since,

\[
\frac{U}{U_s} = \frac{r}{r_s} = R
\]
Therefore,

\[ \tan \beta_0 = R \tan \beta_{os} \]  \hspace{1cm} (2.14)

where \( \beta_{os} \) is the inducer blade angle specified at the inducer shroud.

2.2.6 Incidence Angle Distribution

From the inlet velocity triangles,

\[ \tan \beta = \frac{U}{V \cos \alpha} - \tan \alpha \]  \hspace{1cm} (2.15)

at the shroud, we have,

\[ \tan \beta_s = \frac{U_s}{V_s \cos \alpha_s} - \tan \alpha_s \]

or

\[ U_s = V_s \sin \alpha_s + V_s \cos \alpha_s \tan \beta_s \]

also, since \( U = R U_s \), we have,

\[ U = R \left( V_s \sin \alpha_s + V_s \cos \alpha_s \tan \beta_s \right) \]  \hspace{1cm} (2.16)

Combining equations 2.15 and 2.16, we have,

\[ \tan \beta = \frac{R V_s \cos \alpha_s}{V \cos \alpha} \left( \tan \alpha_s + \tan \beta_s \right) - \tan \alpha \]  \hspace{1cm} (2.17)

From equation 2.17, the flow angle at each radial location is calculated. The incidence angles are therefore obtained from the following:

\[ i = \beta - \beta_0 \]  \hspace{1cm} (2.18)
For the analysis, the incidence angle at the shroud is taken as zero, so that,
\[ i_s = 0 \]

therefore,
\[ \beta_s = \beta_{os} \]

2.2.7 Special Cases

Two practical cases are of special interest:

a) the practical situation for the constant-prewhirl angle with radius \( j = a_s = \text{Constant} \), where the particular advantage is ease of construction of the required vanes. From equation 2.10, we have,

\[ \tan^2 \alpha = \frac{R^2 n \tan^2 a_s}{1 + \tan^2 a_s \frac{1+n}{n} \left(1-R^2 n\right)} \]

if \( a_s = \alpha = \text{Constant} \), then the above equation becomes,

\[ \tan^2 \alpha + n \tan^2 \alpha = -n \]

or

\[ n = -\sin^2 \alpha \]

The above value of \( n \) is derived in order to accommodate this particular case in the general equation 2.1.
Therefore,

\[ V_\theta = AR \sin^2 \alpha \]  

(2.19)

b) The practical situation when \( RV_\theta \) = Constant (free Vortex). This distribution is often used at the inlet to centrifugal and axial flow water turbines (known as Francis and Kaplan turbine). Also, for the design of rotor blades in axial flow compressors. The particular advantage is uniform axial velocity at the inlet to the bladings.

From equation 2.7, we have,

\[ V \frac{dV_z}{dR} + V_\theta \frac{dV_\theta}{dR} + \frac{V^2}{R} = 0 \]

Combining the last two terms,

\[ V \frac{dV_z}{dR} + \frac{V_\theta}{R} d(RV_\theta) = 0 \]

Since \( RV_\theta \) = Constant, therefore,

\[ \frac{d(RV_\theta)}{dR} = 0 \]

This leads to

\[ V \frac{dV_z}{dR} = 0 \]

or

\[ V_z = \text{constant} \]  

(2.20)
2.2.8. Computer Program

A computer program was prepared based on the above-mentioned analysis. The program language is Fortran IV.

The input data are:

1. inlet blade angle at the inducer shroud, ($\beta_{os}$).
2. prewhirl angle at the inducer shroud, ($\alpha_s$).
3. the radius ratio (R), from 0 to 1.0.
4. the radius ratio exponent ($n$), in the general prewhirl velocity distribution equation, ($V_\theta = AR^n$).
5. upper and lower limits of rotation, ($\alpha_u$, $\alpha_L$), respectively.
6. increment of rotation, ($\Delta\alpha$).

The outputs of the program are:

1. radial distribution of prewhirl angle, ($\alpha$);
2. radial distribution of absolute and axial velocity parameters, ($\frac{V}{V_s}$, $\frac{V_z}{V_s}$).
3. radial distribution of incidence angles, ($\imath$).

A list of the program is presented in the Appendix C, together with the symbols used in the program.
2.3 Results and Discussion

The results of the Computer program are shown in Figure 2.2 to Figure 2.7. In Figure 2.2, the radial variation of incidence angles is shown for the four types of prewhirl distribution, with a value of shroud prewhirl angle of $\alpha_s = 20^\circ$. The lower and upper dotted lines are a linear application of the incidence angle limitations selected earlier from stall considerations. The lower limit of radius ratio ($R$) was assumed at $R = 0.3$, which was a matter of convenience and did not effect our interpretations of the results. As shown in Figure 2.2, the free vortex distribution has the largest incidence change toward the hub; therefore, the largest incidence losses occur in this type. The forced vortex type shows the most favourable incidence variation (within $\pm 2.5^\circ$), although the other two types of prewhirl distribution are within incidence angle limitations. At other values of shroud prewhirl angle, the curves hold the same general form.

In Figure 2.3 and 2.4, the incidence angle variations are shown for the case when the vanes are rotated through an angle $\Delta\alpha$, at the shroud. Figure 2.3 for $\Delta\alpha = -20^\circ$, and Figure 2.4 for $\Delta\alpha = +20^\circ$. Again, the forced vortex type shows a distinctively good incidence distribution as compared with other three types of prewhirl distributions. The quadratic type is fairly good for positive rotation of
the vane; i.e., in the direction of rotation of impeller and the constant-prewhirl angle with radius type for negative rotations.

In figure 2.5, the inducer incidence angles are shown at the hub as a function of shroud prewhirl angles. The incidence angle limits selected previously are applied. The results show that the forced vortex type allows the maximum value of prewhirl angle at the shroud of \( \alpha_s = 45^\circ \), before the incidence at the hub exceeds its limit, whereas the free vortex type indicates a maximum prewhirl angle of \( 45^\circ \). Therefore, when the requirement for a specific inducer is to have a high prewhirl angle at the shroud, the forced vortex type is better adapted than other three types.

The radial variation of axial velocity is presented in Figure 2.6. The axial velocity is shown in the non-dimensionalized form as \( \frac{V_z}{V_s} \). The result for the free vortex type is a fairly constant axial velocity with radius, although it is expected to be a straight line according to equation 2.20 in Section 2.2.7b. The discrepancy is attributed to the numerical integration performed in order to solve equation 2.12. The results for other types of prewhirl distributions indicate higher axial velocities at the hub than the shroud region.
Figure 2.7 shows the approximate prewhirl angle required at the hub and shroud of the guide vanes for various prewhirl distributions. This figure is particularly applicable in the mechanical design of the vanes, because it represents the degree of the vane twist. When the difference between the hub and shroud prewhirl angles \( \alpha_s - \alpha_h \) is large, then the guide vanes require higher twist; therefore a greater mechanical complexity is resulted. As shown, the quadratic and free vortex type require the greatest guide vane twist. The constant-prewhirl angle with radius type is the most desirable if the simplicity in design is imperative.
CHAPTER III

INLET GUIDE VANE DESIGN

3.1 Effect of losses

The flow through inlet guide vanes can be idealized as a simple static pressure drop, which is accompanied by an appropriate increase in velocity relative to the vanes. In this case, the flow enters the guide vanes at design angle of attack with no frictional, pressure or secondary flow losses. However, when the effect of fluid viscosity is taken into account, these losses influence the overall compressor performance by reducing its efficiency.

The fact that the inlet guide vanes accelerate the flow from leading edge to trailing edge of the vane is an advantage, because many adverse effects of viscosity such as flow separations arising from unfavourable pressure gradients are avoided. Therefore, the losses are relatively lower than decelerating cascades. At high prewhirl angles, however, the flow is highly accelerated through the vanes and then suddenly decelerated, which incurs losses; and the advantage may not be realized.

If the inlet guide vanes are variable, then the losses increase when the vanes are rotated to other vane angle
settings other than design incidence angle (generally \( i = 0 \)). This design incidence angle corresponds to the best aerodynamic efficiency of the vanes. Unless a different set of guide vanes is designed for every desired prewhirl angle, or variable camber guide vanes are used, the losses due to high angles of attack are unavoidable. In axial flow compressors, the guide vanes are often fixed to the hub and shroud in the inlet; therefore, this problem is automatically eliminated.

3.2 Prewhirl angle distribution

The radial distribution of prewhirl angles produced by inlet guide vanes is an important aspect in the design of the vanes. In chapter II, from the results of the analysis for the four types of radial prewhirl velocity distribution, it was concluded that, apart from mechanical complexity, the forced vortex type of distribution is quite desirable for the design of the vanes. The constant-prewhirl angle with radius, however, is simpler to manufacture when flat plate sections are used. In this chapter, design procedures are given for the two constant-prewhirl angle with radius type: a) a conventional shape, which is often used in practice (centrifugal refrigeration machines), using a flat plate section
and b) an improved shape, using a circular-arc cambered plate section. In addition, a set of vanes is designed, based on a forced vortex type of distribution.

3.3 Inlet Guide Vane Sections

There are four geometrical sections used in the design of inlet guide vanes. These are as follows:

A - Constant thickness, flat plate section;
B - Constant thickness, circular-arc section;
C - a basic airfoil section;
D - a basic airfoil section fitted on a camber line.

The constant thickness, circular-arc sections have an advantage over flat plate sections. In the flat plates, due to the small leading edge radius, a laminar separation takes place at the leading edge, even at small angles of incidence. In the circular-arc sections, however, due to a more favourable incidence at the leading edge, laminar separation is delayed; therefore, extending low-loss incidence range of operation of the vanes. The circular-arc sections, therefore, are expected to be more effective (produce more turning and less losses) in comparison with the flat plates. This is when the vanes are rotated to other prewhirl angles other than design prewhirl angle.
Basic airfoil sections smoothly accelerate the flow up to their maximum thickness and then decelerate the flow toward their trailing edge, so that the flow is less likely to separate in the regions of adverse pressure gradient. The airfoil sections, therefore, have this advantage over flat plate or cambered plates; however the sections tend to be more complex to manufacture.

3.4 Preliminary Design Calculation

The original design calculation of the guide vanes was based on the measurements taken at the inlet to a centrifugal blower. The following is a summary of the calculation:

\[ \text{inlet duct diameter} = 3.78" \]
\[ \text{inducer hub diameter} = 1.2" \]

assuming:

(1) the vanes close tight in the inlet duct, with no gaps in between the vanes;

(2) there are six 6 vanes.

Then the position of the vanes in the duct will be as shown in figure 3.1a and 3.1b. From these figures,

\[ C_s = r_1 = 1.89" \]

to find the pitch (S) at the shroud, we must consider that the hexagon ABCD encloses another circle with radius \( r_2 \)
as shown in figure 3.1c, so that

\[ S_s = r_2 \]

and,

\[ r_2 = \frac{\sqrt{3}}{2} r_1 \]

therefore,

\[ \frac{S_s}{S_s} = 1.64" \]

also, since the two triangles ABO and A'B'O are similar, we have

\[ \frac{HO}{H'O} = \frac{HB}{H'B'} \]

or

\[ \frac{r_2}{r_3} = \frac{C_s}{C_h} \quad \text{where} \quad r_3 = 0.6" \]

therefore,

\[ C_h = 0.692" \]

also

\[ S_h = r_3 = 0.6" \]

The Pitch/Chord ratio at the hub and shroud are obtained as:

\[ (\frac{S}{C})_s = \frac{S}{C}_h = 0.866 \]

or

\[ \sigma_s = \sigma_h = 1.15 \]

3.4 The IGV Shape According to \( \alpha \)-Const. with Radius Distribution

The following are the design procedures used in order to obtain the shape of the guide vanes according to constant prewhirl angle with radius distribution. Two different sets of guide vanes are designed based on a) a circular-arc constant-thickness section, and b) a simple flat plate section.
3.5.1 - Circular-arc Constant-thickness Section

a) Camber angle determination:

To determine the camber angle (φ) for a circular-arc section, the following criteria are considered:

1. To be able to operate the vanes in a maximum possible low-loss incidence range;
2. To be able to deflect the flow from α = 0° to α = 30° in the direction of rotation of the impeller.

From the results of 2-Dimensional Cascade data for circular-arc sections, fig. 3.2* and 3.3*, the following are observed:

1. The largest variation in incidence angles for the low-loss incidence angle region occurs at Camber angles of up to 55°, for a solidity of 2.0, figure 3.2.

2. As the solidity is reduced, the low-loss incidence range decreases, for a given Camber angle, figure 3.2.

*These figures are adapted from Ref. 19 and 20 respectively. Figure 3.2 is basically a transformation of the same data used in figure 3.3.
3. As the Camber angle is reduced, the low-loss incidence angle variation increases, figure 3.2.

4. The negative variation of low-loss incidence angles is larger than positive variation in incidence angles, figure 3.3.

5. The deviation angle (δ) is approximately constant for negative variation in incidence angles, for a given solidity, figure 3.3.

Now, assuming the flow enters the vanes axially, tangent to the leading edge; i.e., i = 0, and leaves the vanes tangent to the trailing edge; i.e., δ = 0, figure 3.4a, the prewhirl angle produced by the vane is:

\[ \alpha = \phi \]  \hspace{1cm} (3.2)

When the flow leaves the vanes with a deviation, figure 3.4b, then

\[ \alpha = \phi - \delta \]  \hspace{1cm} (3.3)

Now, when we rotate the vanes to the limits of positive and negative incidence angle, we have, for negative limit of incidence angle, figure 3.4c:

\[ \alpha = \phi - \delta - i \]  \hspace{1cm} (3.4)

and for positive limits of incidence angle, figure 3.4d,
\[ \alpha = \phi - \delta + i \quad (3.5) \]

From the two-dimensional cascade data for circular-arc sections, we have,

\[ \delta = f(\sigma, i, \phi) \]

or \[ \alpha = f(\sigma, i, \phi) \quad (3.6) \]

Due to the number of variables involved, a trial and error procedure is used in order to find an appropriate camber angle which meets the criteria.

Assuming \( \phi = 40^\circ \), by interpolation of the data, figure 3.3, for \( \frac{S}{C} = 0.866 \), and above equations 3.3, 3.4 and 3.5, we have,

\[
\begin{array}{ccc}
\text{at} & i = 0^\circ & \delta = 10.5^\circ & \alpha = 29.5^\circ \\
\text{at} & i = -15^\circ & \delta = 9^\circ & \alpha = 16^\circ \\
\text{at} & i = 4^\circ & \delta = 13^\circ & \alpha = 31^\circ \\
\end{array}
\]

Therefore, the prewhirl angle limits for the low-loss incidence range is \( 16^\circ < \alpha < 31^\circ \) with \( \Delta \alpha = 15^\circ \); these values do not meet the criteria that we chose earlier. If, however, \( \phi = 25^\circ \) and \( \frac{S}{C} = 0.866 \), we have,

\[
\begin{array}{ccc}
\text{at} & i = 0^\circ & \delta = 4.5^\circ & \alpha = 20.5^\circ \\
\text{at} & i = -15^\circ & \delta = 4.7^\circ & \alpha = 5.3^\circ \\
\text{at} & i = 7.5^\circ & \delta = 7^\circ & \alpha = 25.5^\circ \\
\end{array}
\]

with \( 5.3^\circ < \alpha < 25.5^\circ \) and \( \Delta \alpha = 20^\circ \).
Therefore, \( \phi = 25^\circ \) is selected, which is particularly advantageous because it produces \( \phi = 20^\circ \) prewhirl angle when the flow enters the vanes at zero incidence angle (design point).

b) The radial profile distribution:
To determine the radial profile distribution of the circular arc vanes, we must specify the camber angle, chord length and stagger angle at every radial cross-section. However, in our particular design, for the condition that the prewhirl angle is constant with radius, the guide vanes are not twisted and therefore the stagger angle is omitted. Also, for simplicity and ease of manufacturing the vanes have a constant-radius-of-curvature, so that the camber angle can vary linearly with the chord length, as is demonstrated in Appendix D.

The flow at the inlet to the guide vanes can be assumed to be uniform and axial. As shown in figure 3.5, if a constant-radius of curvature vane is rotated so that the incidence angle at the shroud is zero, then there exists an incidence angle at the hub section. This incidence angle produces an increase in deviation angle \( \Delta \delta_h \) where, the
total deviation angle at the hub is given as:

$$\delta_h = \delta_{h0} + \Delta \delta_h$$

now to have a constant-prewhirl angle with radius at the outlet of the vanes

$$i_h = \delta_{os} - \delta_{oh} - \Delta \delta_h$$ \hspace{1cm} (3.7)

and from circular-arc sections geometry

$$i_h = \frac{\phi_s - \phi_h}{2}$$ \hspace{1cm} (3.8)

where $\delta_{oh}$ and $\delta_{oh}$ are the deviation angles at zero incidence angles for the shroud and hub sections of the vane respectively. In the following calculation, by trial and error procedure, the chord length at the hub section is adjusted so that the equations 3.7 and 3.8 match each other.

From the previous calculation, we have

$$C_s = 1.89^\prime \hspace{1cm} \left(\frac{S}{C}ight)_s = 0.866$$

$$C_h = 0.692^\prime \hspace{1cm} \left(\frac{S}{C}ight)_h = 0.866$$

also

$$\phi_s = 25^\circ$$
From Appendix D, we have

\[
\frac{\phi_s}{\phi_h} = \frac{C_s}{C_h}
\]

therefore,

\[
\phi_h = 9.2^\circ
\]

The deviation angles at zero incidence angle are found from accelerating cascade data, figure 3.6, adapted from Ref. 20, where the stagger angle for \( \phi = 25^\circ \) is within the limit of the specified stagger angle used in figure 3.6.

therefore,

\[
\delta_{os} = 3^\circ \quad \text{and} \quad \delta_{oh} = 0.9^\circ
\]

is obtained from figure 3.7, adapted from Ref. 21.

The figure is modified according to \( \sigma = 1.0 \), \( \frac{d\delta}{d\alpha} = 0.88 \), Ref. 19.

Now, from Appendix E, we have, \( \frac{d\delta}{d\iota} = 1 - \frac{d\delta}{d\iota} \), where \( d\iota = d\alpha' \), therefore,

\[
\frac{d\delta}{d\iota} = 0.76 \quad \text{or} \quad \frac{d\delta}{d\iota} = 0.24
\]

then,

\[
\frac{\Delta\delta}{\iota_h} = 0.24
\]
From equation 3.8, we have,

\[ i_h = 7.9^\circ \]

therefore,

\[ \Delta \delta_h = 1.89^\circ \]

From equation 3.7, we have

\[ i_h = 0.2^\circ \]

As shown, the value of \( i_h \) obtained from equations 3.7 and 3.8 are different. Therefore, \( C_h \) is increased in order to satisfy the condition. The results at the calculation are given in Table 1, Appendix F.

In Ref. 19, a set of circular-arc constant thickness guide vanes was designed, based on two-dimensional cascade data; and was tested at the inlet to an axial flow compressor. Their result indicates that near the outer wall, the large value of blade circulation in combination with the wall boundary layer tends to reduce the turning angle. Figure 3.8 shows the results adapted from Ref. 19 (note that the turning angle referred to in that reference is
analogous to our prewhirl angle). One way of compensating for the reduction in turning angle is to increase amount of vane interference upon each other by increasing the solidity (c).

Due to the similarities which exist between our design and the experimental investigation in Ref. 19, it was suggested to increase the chord length at the shroud by 20%.

The result of the trial and error calculation is given in Table 2, Appendix F. The final design parameters are as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>( \phi )</th>
<th>( C )</th>
<th>( \frac{S}{C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shroud</td>
<td>25°</td>
<td>2.27&quot;</td>
<td>0.72</td>
</tr>
<tr>
<td>Hub</td>
<td>22°</td>
<td>2.0&quot;</td>
<td>0.346</td>
</tr>
</tbody>
</table>

The vanes are drawn to scale as shown in Figure 3.9.

3.5.2 Flat Plate Section

The design of the flat plate section was relatively simple in comparison with the circular-arc section, which was due to the fact that there was no camber angle. The chord length was assumed to vary linearly with radius so that the pitch to chord ratio will be
kept constant with the radius. Experimental cascade data were not available for the flat plates; therefore, the prewhirl angles could not be estimated. However, the flow was assumed to follow the vane angle \( \alpha_0 \) at the outlet of the vanes. The flat plate vanes are drawn to scale as shown in Figure 3.10.

3.6 The Inlet Guide Vane's Shape According to the Forced Vortex Type of Distribution

The results of the theoretical study in Chapter II showed that a forced vortex type of distribution was very satisfactory when assumed at the inlet to an inducer. A set of inlet guide vanes based on this type of distribution was therefore designed as an improvement to the constant-prewhirl angle with radius type discussed in the previous section. An attempt was made to keep a number of design parameters constant between the two types of guide vanes, in order to obtain a more clear understanding of the effect of the guide vane design in the overall performance of a compressor. These controlled parameters are the camber angle and the chord length at the shroud section: \( \phi_s = 25^\circ \), \( C_s = 2.27" \). The leading edges of all radial sections were placed in the same plane so that the incidence angle will be kept constant with the radius, which results in a simplification in the design calculation. The use of two-dimensional cascade
data was not possible due to small values of camber angle at the hub, therefore an alternate method of design was adapted. The assumptions used in the design calculation are as follows:

1) There is a linear variation of camber angle with radius.

2) The flow through the guide vane is two-dimensional; i.e. $V_r = 0$.

From the empirical rule for deviation angle, Ref. 22 (also known as Carter's deviation rule), we have

$$\delta = m \phi \sqrt{\frac{S}{C}}$$

(3.9)

where,

$$m = 0.23 + 0.1 \frac{a}{50}$$

also from the circular-arc geometry, figure 3.4b, we have

$$\delta = \phi - \alpha$$

(3.10)

in the above two equations 3.9 and 3.10, there are four unknowns; therefore, when the values of two parameters are specified, then the other two are calculated.

For the shroud section, the parameters are the same as for the circular-arc constant prewhirl section. These are

\[ r_s = 1.84'' \quad \phi_s = 25^\circ \quad C_s = 2.27'' \]
applying the assumption 2, we have

\[ \phi = x r + b \]

with the boundary condition as

\[ r_o = 0, \quad \phi = 0 \]
\[ r_s = 1.64, \quad \phi = 25^\circ \]

we have

\[ \phi = \frac{25}{1.64} r \]

also from Appendix D and equation 3.11, we have

\[ \frac{C_h}{C_s} = \frac{r_h}{r_s} \]

Now, applying equations 3.9, 3.10, and 3.11, we have

at shroud \( r_s = 1.64 \) \( \alpha = 19.3^\circ \) \( \delta = 5.7^\circ \) \( \phi = 25^\circ \)

at hub \( r_h = 0.6 \) \( \alpha = 7.25^\circ \) \( \delta = 1.9^\circ \) \( \phi = 9.2^\circ \)

at mean \( r_m = 1.14 \) \( \alpha = 13.5^\circ \) \( \delta = 3.8^\circ \) \( \phi = 17.3^\circ \)

Table 3 in Appendix F gives the summary of the calculation for a set of inlet guide vanes based on a forced vortex type of distribution. \( R \) is the radius ratio defined as \( R = \frac{r}{r_0} \). These results are plotted in figure 4.12, together with the theoretical forced vortex prewhirl distribution. The discrepancy in the prewhirl angles are
within 10%; however, in the presence of many variables involved in the design of the vanes, it is considered satisfactory. The guide vanes are manufactured according to the drawing in figure 3.11.
4.1 Experimental Test Rig and Instrumentation

4.1.1 Experimental Test Rig

To investigate the flow conditions downstream of the inlet guide vanes, experiments were conducted in the inlet duct of a centrifugal blower. The blower had an impeller diameter and width of 26 inches and 1 inch, respectively; and was driven by a 15 HP induction motor. The inlet duct had an inside diameter of 3.78 inches, and a length of approximately 25 inches. It consisted of two sections so that the guide vanes could be properly fitted in between the two sections and together they could be bolted. The set-up is shown in Figure 4.1. In addition, the duct was provided with a bellmouth, as shown in Figure 4.2.

The inlet guide vanes were provided in the inlet duct in order to give the required prewhirl to the flow, which is shown in Figure 4.3. Four sets of guide vanes were tested, where each set consisted of either 6 or 9 identical vanes. The vanes were designed according to the procedure described in Chapter III; and were manufactured in the Mechanical Engineering machine shop as was
recommended in figures 3.9, 3.10 and 3.11. Each set of vanes is described in the following Table 4.

<table>
<thead>
<tr>
<th>Set #</th>
<th>type of section</th>
<th>type of distribution</th>
<th>number of vanes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Circular arc</td>
<td>$\alpha = \text{constant}$</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Flat plate</td>
<td>$\alpha = \text{constant}$</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Circular arc</td>
<td>$\alpha = \text{constant}$</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>Circular arc</td>
<td>forced vortex</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4

As shown, there were essentially two types of prewhirl distribution tested; namely, the constant-prewhirl angle with radius and forced vortex type. The three sets of guide vanes for the constant-prewhirl angle with radius type differed from one another by either the number of vanes or the type of section. The shape of each guide vane is shown in Figure 4.4.

To accommodate the vanes in the inlet duct, two rings were used. An aluminum ring for the 6-vane sets, and a steel ring for the 9-vane set. The rings had the same inside diameter as the inlet duct, a thickness of 1/4 inch and a width of 1 inch. These rings are shown in
Figure 4.5. Each vane was braze welded to a brass connector and together they could be mounted on to the rings by screws. As one of the requirements of the guide vanes was to be variable, allowances were made in the design of the rings, and the brass connector; so that the angular position of the vanes could be changed from full open to closed tight position. Figure 4.6 shows the 6-vane set assembly and Figure 4.7, the 9-vane set, positioned in their respective rings. A special fixture was designed and manufactured to adjust the vanes accurately, relative to the axial direction, which is referred to as the stagger angle. This stagger angle was set relative to the shroud of the vanes. The fixture is shown in Figure 4.8.

4.1.2 Instrumentation

To determine the effectiveness of the guide vanes, it was necessary to measure the prewhirl angles and the axial velocities downstream of the vanes, in addition to the stagnation pressure drop across the vanes. The average static pressure was recorded before the vanes by wall static pressure taps located at six equally spaced positions around the circumference of the inlet duct. Figure 4.1 shows the tubes used for this measurement. Because the flow in the inlet nozzle was essentially inviscid and incompressible, the average static pressure and the inlet
velocity were related according to Bernoulli's equation, and therefore evaluation of the mass flow rate through the system was possible.

To determine the flow pattern, a cobra probe was attached to a traversing mechanism which was fixed on the inlet duct. The mechanism consisted of a worm gear and collar arrangement, so that the probe was capable of moving with the gear in the radial direction and rotating about its axis. A dial was used on the traverse to measure the prewhirl angles, which had an accuracy of one degree. The traverse also had an arrangement to measure the distance in radial direction. The traverse is shown in Figure 4.9.

With the cobra probe traversed radially across the duct, it was possible to obtain the absolute velocity of the flow and its direction downstream of the vanes. The axial velocity ($V_2$) and prewhirl velocity ($V_0$) could therefore be calculated. The probe was also used to obtain the stagnation pressure by nullifying technique. The values gave an approximate idea of the amount of losses caused by the vanes. The tip of the probe was located approximately 1.35 inches for the circular-arc vanes and 1.5 inches for the flat plate vanes from the trailing edge at the vanes. The probe was calibrated with the help of the inlet average static pressure.
The stagnation pressure point and the nulling pressure points from the probe, together with the average static pressure point were all connected to a multitube manometer. The liquid used in the manometer was colored water, and the manometer had an accuracy of 1 mm. The ambient pressure and temperature were measured by a barometer and thermometer, respectively. Figure 4.10 shows the multi-tube manometer; together with the barometer and thermometer.

4.2 Experimental Procedure
4.2.1 Cobra probe calibration

To determine the static pressures downstream of the guide vanes, the cobra probe was used. However, since the probe did not sense the static pressure directly, the probe was calibrated with the help of the wall's average static pressure at the inlet. The static pressure holes were located approximately 9 inches from the plane of the transverse of the probe. This resulted in an error in our calculation due to the frictional losses associated with this distance; however, if these losses were neglected, which were relatively small, then the wall static pressure measurement would be related to the nulling pressures sensed by the probe according to the calibration.
The calibration was carried out by positioning the probe in the center of the duct with no guide vanes. The probe was then rotated until a nullified height was reached on the manometer, i.e. the probe was aligned with the axial flow stream lines. The flow rate was then varied by a throttling valve located at the outlet of the blower. The manometer fluid heights associated with the stagnation pressure \( h_T \), nullying pressure \( h_N \), and the wall static pressure \( h_S \) were then recorded for every new flow rate through the machine.

During the test, it was observed the stagnation pressure was essentially constant and atmospheric. This was expected because the flow in the central portion of the duct, outside the boundary layer is inviscid. The pressures and the fluid heights obtained from the test are related according to the following:

\[
P_T - P_S = \rho g h_m \quad (4.1)
\]

and

\[
P_T - P_N = \rho g h_n \quad (4.2)
\]

where

\[
h_m = h_S - h_T
\]

and

\[
h_n = h_N - h_T
\]

A plot of \( h_m \) vs. \( h_n \) was therefore obtained which is shown in Fig. 4.11. The result approximates a straight line, where the equation of the line was found as:

\[
h_m = 1.8 \, h_n \quad (4.3)
\]
4.2.2 Testing the Inlet Guide Vanes

The experiment was conducted systematically for the four sets of guide vanes which were described in Table 4. Each set was positioned for stagger angles from $0^\circ$ to $40^\circ$ in steps of $10^\circ$. The upper limit of $40^\circ$ was chosen, because for values greater, the manometer fluid showed large intermittency which tended to overflow the equipment. For the flat plate vanes (set #2), the stagger angles were coincided with the outlet vane angle, and the prewhirl angles were assumed to be equal to the stagger angles; i.e. $\alpha_{FP} = \gamma_{FP}$ whereas, for the circular-arc vanes due to the camber, the prewhirl angles were assumed to be greater than the stagger angles by $10^\circ$; i.e. $\alpha_{CA} = \gamma_{CA} + 10^\circ$

Therefore, theoretically, both the flat plate vanes and the circular-arc vanes would produce the same prewhirl angles when $\gamma_{FP} = \gamma_{CA} + 10^\circ$. This condition, however, does not take into account the effect of incidence angles and deviation angles as the angular position of the vanes were varied.

The speed of rotation of the impeller was constant at 3550 rpm. The velocity at the inlet to the guide vanes was essentially uniform and constant at 136 ft./s, corresponding to 100 mm of water from the measurement of
the wall static pressure. The mass flow rate was, therefore, constant at 0.785 lb/sec. This value of flow rate was chosen to ensure the safe operation of the motor and to prevent the possible overflow of the manometer fluid as the vanes were closed.

For every angular position of the vanes, the cobra probe was traversed from the wall to the center of the duct in steps of 1/4 inch. However, more data were taken at the two boundaries due to the complexity of the flow which existed there. After a traverse, the ring was rotated 15 degrees in case of 6-vane sets and 10 degrees for the 9-vane set; and the probe was traversed again. This was done up to a maximum of 60 degrees or 40 degrees depending on the number of vanes in order to sweep one vane thoroughly in the circumferential direction. One vane was sufficient since the flow through the vanes was assumed to be axisymmetric. At each measuring station, the probe was adjusted so that a nulling pressure was reached. The manometer reading corresponding to the nulling pressure and stagnation pressure was then recorded. The prewhirl angles were also read off the dial on the traverse. The ambient pressure was basically constant during the test, but varied from one day to another. The ambient temperature, however, varied during the test; but the changes were within ±3 degrees centigrade.
The data reduction equations are presented in the Appendix G.

4.3 Results and Discussion

The results of the experiment are shown in Figure 4.12 through Figure 4.28. Figure 4.12 through 4.25 are the plots of prewhirl angle (α), axial velocity ratio (\( \frac{V}{U} \)) and stagnation pressure loss (\( \frac{\Delta P_0}{1/2 \rho U^2} \)) as a function of radius ratio (R). Figure 4.26 and Figure 4.27 show the comparison between the theoretical and experimental values of axial velocity; and Figure 4.28 is a plot of the variation of stagnation pressure loss in the circumferential direction for the set #3 guide vanes. All the measured values represent an average from the calculation of five data points obtained along the circumference at each radial position. The results are not, therefore, exact; but rather give a qualitative presentation of the flow condition downstream of the guide vanes. For the circular-arc section vanes (Sets #1, 3 and 4), the stagger angles are shown from 0 to 30 degrees; and, for the flat plate section vanes (Set #2), from 0 to 40 degrees. All the figures are for a constant mass flow rate of 0.785 lb/s.
4.3.1 Radial Variation of Prewhirl Angles

The radial variation of prewhirl angles are presented in Figure 4.12 to Figure 4.16. Figures 4.12, 4.13, and 4.14 are for the constant-prewhirl angle with radius type of distribution (Set #1, 2 and 3) and Figure 4.16 is for the forced vortex type (Set #4). The theoretical values of prewhirl angles are also shown as the dashed lines for the constant-prewhirl angle with radius type. These theoretical values are comparable with the experimental values in such a manner that \( \alpha = \gamma_{FP} = \gamma_{CA} + 10^\circ \)

This condition was reached, based on the design calculation and the geometrical configuration of the vanes.

Figure 4.15 shows the trends for the Set #1, 2 and 3, which is the superposition of the plots in Figures 4.12, 4.13, and 4.14. The general results are that: 1) Near the shroud region of the vanes \((R = 0.868)\), the prewhirl angles are far better approximated by the theoretical values than any other radius; 2) All the prewhirl angles have a peak at radius ratio of approximately 0.2 to 0.45 near the hub region \((R = 0.132)\), which decrease with the increase in radius ratio; and show a fairly sharp decrease toward the center of the duct; and 3) For the prewhirl angles equal to 10 degrees, i.e., \(\gamma=0\) for circular-arc section vanes and \(\gamma=10\) degrees for the flat plate.
section vanes, the experimental values are within ±2 degrees of the theoretical values; however, as the prewhirl angles increase, the deviation of the peaks from the theoretical values increase in such an extent that for \( \alpha_0 = 40^\circ \), the discrepancy at the peak reaches +15° for the Set #1, +10° for the Set #2 and +20° for the Set #3.

The behavior of the guide vanes in the central region of the duct was somewhat expected, where due to the open area in that region, which has a 1/2 inch diameter, the flow tends to adjust itself in the direction parallel to the duct axis. However, since the flow cannot make a sudden change in direction, the peak moves further within the vane height. The general drop in prewhirl angles with radius ratio are believed to be due to the effects of three-dimensionality of the flow within the vane passage, which are caused by the small height of the vanes relative to its average chord length.

The comparison of the Set #1 and Set #3 in Figure 4.15 indicates a higher prewhirl angles for the Set #3, which is due to the increased number of vanes or solidity for the circular-arc vanes. This effect is more significant for the prewhirl angles greater than 20°. In the hub region, however, the effect of high solidity for the Set #1 or
Set #3 has produced prewhirl angles which are more extended toward the hub as compared with the Set #2 with a much smaller chord length at the hub.

The results of the prewhirl angles for the forced vortex are shown in Figure 4.17. Also, given in the figure, is the theoretical distribution based on the calculation in Chapter II and the design values in accordance with the procedure presented in Chapter III. The experimental values agree well with the theoretical values, except for the region close to the shroud. However, when the vanes were rotated to higher stagger angles, shown in Figure 4.16, the angles at the hub region tend to increase more rapidly than that in the shroud; so that, for \( \gamma = 20^\circ \) the plots more resembles a constant-prewhirl with radius type of distribution for \( R \geq 0.3 \). This does not mean that the latter type of distribution is permanent, but rather indicates the destruction of the forced vortex type of distribution when the vanes were rotated to other vane settings than design. In Chapter II, this was discussed; and, in fact, was noted as a disadvantage of the forced vortex type in comparison with the constant-prewhirl type of distribution.
4.3.2 Radial Variation of Axial Velocity

The results of the calculation for the non-dimensional axial velocity ratio \( \frac{v_z}{U} \) as a function of radius ratio \( R \) are shown in Figure 4.18 to Figure 4.21. For the vanes with constant-prewhirl angle with radius type of distribution, Set #1, 2 and 3, the results indicate an approximately constant variation of axial velocity with radius in the central portion of the vane height with \( 0.3 < R < 0.7 \).

Near the wall region of the duct, the axial velocities are lower and near the central region, the velocities are higher. For the low stagger angle position of the vanes, the plots indicate a possible violation of the continuity of mass law which is believed to be due to the errors produced in the averaging calculation of these values and the fact that the traversing probe might have missed the measurement of high velocity regions.

In the theoretical studies in Chapter II, it was convenient to non-dimensionalize the axial velocities based on the absolute velocity in the shroud. However, in order to be able to compare the theoretical and experimental values, the experimental values were converted in a manner to be comparable with the theoretical values. These results are presented in Figure 4.26 for the constant-prewhirl angle type and in Figure 4.27 for
the forced vortex type. The results are for the design prewhirl angle of 20° at the shroud. The plots in
Figure 4.26 indicate a more consistent values for the
Set #1 and Set #3 (circular-arc vanes) in compared with
the Set #2 (flat plate vanes). Also, from Figure 4.27,
it is apparent that a large discrepancy exists between the
theoretical and experimental values in case of the forced
vortex, although the trend is reasonably good.

4.3.3 Radial Variation of Stagnation Pressure Loss

The results of the calculation for the radial distribution
of stagnation pressure loss across the guide vanes are
presented in Figures 4.22 to 4.25. For the flat plate
vanes, Figure 4.23, the maximum stagnation pressure loss
occurs at the hub in such a manner that the curves show a
smooth increase in pressure loss values toward the hub, which
then decreases to the center of the duct. This characteristic
however was less evident for the circular-arc vanes, in
Figure 4.22 and Figure 4.24. In addition, higher stagnation
pressure losses were present for all the four sets in the
region close to the wall. These high stagnation pressure
loss regions at the hub and shroud indicate the general
three-dimensional nature of the flow, and the tendency of
the fluid to move from the high-pressure side to the low-
pressure side of the vane, which introduces a disturbing
flow of some form. Furthermore, due to a relatively small
diameter of the inlet duct, large wall frictions can be expected. In this case, the thick boundary layer upstream of the guide vanes interacts with the guide vanes which results in high separations and, consequently, large amount of losses downstream of the vanes.

In case of the forced vortex type, Figure 4.25, no losses were observed in the hub region which is an indication of no radial flows there. The cause of this behaviour may be attributed to the location of all the leading edges of the radial sections in the plane perpendicular to the inlet duct axis.

4.3.4 Circumferential Variation of Stagnation Pressure Loss

Figure 4.28 shows the variation of the stagnation pressure loss in the circumferential direction for Set #3 at R = 0.6. The R = 0.6 was chosen intentionally from Figure 4.24, since the losses were low at that radius. The results indicate clearly the presence of a wake downstream of the guide vanes. For γ = 10°, the losses are much lower which is due to the more favourable incidence angle at the leading edge for the design prewhirl angle. However, as the vanes are rotated to higher stagger angles, the width of the wake becomes wider in such a way that for γ = 30°, the losses increase substantially. Therefore,
with respect to the losses which already exist in other radii; i.e., Figure 4.24, it is evident that losses are present everywhere downstream of the vanes. That is, large amounts of separations prevail the flow field when the vanes are positioned to produce prewhirl angles greater than 30° or stagger angles greater than 20 degrees for the circular-arc vanes. With this observation and the fact that the flat plate vanes tend to produce higher losses than circular-arc vanes, it is expected that this limit be even lower for the flat plate vanes, Set #2.

4.3.5 Further Discussion

In Chapter III, the circular-arc guide vanes were designed with the assumption that the flow through the vanes were two-dimensional. This was assumed in order to enable us to use the available cascade data; Figure 3.2 and Figure 3.3. The two-dimensionality of the flow can be achieved if the vanes were infinitely long, so that there were no interference due to the bounding walls; or if the guide vanes were placed in an annulus with a large hub radius such that the hub to shroud radius ratio was approximately equal to one. In axial flow compressors, the requirement is to design the blades with high aspect ratios (3.0 or higher) to ensure two-dimensional flow over most of the blade height. The inlet guide vanes tested in
the present program however indicate an aspect ratio of approximately 0.8. Therefore, the flow through the vanes is expected to be three-dimensional.

The overall variation of the prewhirl angles for the constant-prewhirl angle with radius type of distribution guide vanes is shown in Figure 4.15. The fact that the figure is a superposition of Figure 4.12 to Figure 4.14 makes it particularly desirable for the purpose of interpretation, because it excludes the two variables, namely the number of vanes and the shape of the section. Apart from the behavior of the guide vanes in the region close to the hub and shroud, the results indicate high discrepancies in comparison with the theoretical estimations; particularly in the peak regions. However, it is of interest to note that the plots resemble a free vortex type of distribution, where Figure 4.18, 4.19 and 4.20 indicate a constant axial velocity across the duct which is the characteristic of this type of distribution. Also, the results of the theoretical studies in chapter II, which are shown in Figure 2.6 and Figure 2.7 agree approximately with the above observation. This is important, because it explains the significance of the radial compatibility equation and the assumptions made, in the prediction of the flow downstream of the guide vanes for the region unaffected by the wall or center of the duct.
In relation to the forced vortex type of distribution guide vanes, set #4, the results show an increasing prewhirl angle with radius, Figure 4.17 and a decreasing axial velocity with radius, Figure 4.27, which are in qualitative agreement with the theoretical results. This observation again indicates the validity of the theoretical predictions.

The calibration of the cobra probe was performed in the inlet duct, with the guide vanes removed; and to ensure a uniform velocity distribution with no radial component of velocity, the probe was positioned in the central portion of that duct. In general, however, in the regions of high velocity gradients such as wakes and boundary layers, due to the finite length between the two nulling points in the tip of the probe, the probe has to be adjusted to compensate for the change in magnitude of velocity which results in an error in the prewhirl angle measurement and the calculation of the absolute velocities from the nulling pressure measurements. In Figure 4.28, within the plane of the flow measurements, the wakes were shown to exist. Therefore, if the probe were traversed in the region within the wakes, then some of the discrepancies between the theoretical and measured values of prewhirl angles can be charged to this error. However, in relation
to the condition that these angles are an average of five data points, it is less likely that this source of error has a major effect on the results.

In comparison of the three sets of guide vanes in Figure 4.15, it must be noted that, although all the three sets produce small differences in the prewhirl angle, due to the type of section and number of vanes, the circular-arc vanes possess a major advantage over the flat plate vanes in the amount of losses they incur across the vanes. This is evident in Figure 4.22, and Figure 4.24, in comparison with Figure 4.23. The Figure 4.23 shows a considerably higher stagnation pressure losses; particularly for stagger angles greater than 20° for radius ratios less than 0.4. This fact can be utilized to improve the performance of the centrifugal machines.
CHAPTER V

CONCLUSIONS

1. The results of the theoretical study have shown that the performance of the centrifugal machines can be improved when a forced vortex type of distribution \( V_{\theta} = AR_\theta \) were assumed at the inlet to the inducer. This conclusion was drawn within the context of the assumptions made throughout the analysis.

2. The design of the inlet guide vane shape to produce a forced vortex type of distribution has proven to be very satisfactory. However, as the guide vanes were rotated to higher vane angle setting than design, the type of distribution was destroyed.

3. With respect to the constant-prewhirl angle with radius inlet guide vanes, the experimental values of prewhirl angles indicate discrepancies in comparison with the theoretical values, particularly in the region close to the hub. These discrepancies were charged to the effects of three-dimensional nature of the flow.
4. In case of the constant-prewhirl angle with radius type of distribution, the results indicate a considerably higher loss for the conventional flat plate section inlet guide vane, in comparison with the circular-arc section vanes; although they both produced an approximately similar prewhirl angle variation. This is of significance, because the effectiveness of the inlet guide vanes in the improvement of the performance; i.e., increased operating range and/or higher efficiency, can be achieved when the losses incurred by the vanes are kept to a minimum.

5. For the present experiment, it has been proven that losses exist everywhere downstream of the vane as the vanes were set to produce prewhirl angles greater than 30°.
RECOMMENDATIONS

Improvements in the theoretical and experimental investigations can be achieved as follows:

1. In the theoretical studies of Chapter II, values of the incidence angles were calculated in order to estimate the limits of operation of a compressor, for the case where inlet guide vanes were provided at the inlet to inducer section. The studies did not take into account the effects of inlet guide vane - inducer interaction and some means of correction is suggested.

2. In a particular case, when the requirement is to investigate the performance of a specific impeller, the inducer blade angles can be measured and input to the computer program for theoretical analysis. Furthermore, if the inlet guide vanes in the present experimental investigation are employed, then the theoretical analysis can further be improved by utilizing the measured values of prewhirl angles.
In future experimental investigations with the inlet guide vanes, an introduction of a half-body in the hub region of the inlet guide vane assembly is believed to smoothly accelerate the flow and thus reduce the losses.
Fig. 1.1 - Inlet velocity triangles with a) Positive prerotation  b) Positive prewhirl.
Fig. 1.2 - Centrifugal impeller

Fig. 1.3 - The velocity triangles: a) at the impeller inlet  
b) at the impeller outlet.
Fig. 1.4 - Ideal and actual characteristic curves.
Fig. 1.5 - Velocity profiles in the vane-to-vane plane.
Fig. 1.6 - Velocity profiles in the hub-to-shroud plane.

Fig. 1.7 - Cross flow in the vane channel.
Fig. 2.1 - Theoretical variation of the inducer blade angle with radius.
Fig. 2.2 - Radial variation of incidence angles at the inducer inlet for an assumed prewhirl angle at the shroud of 20 degrees.
Fig. 2.3 - Radial variation of incidence angles when the inlet guide vanes based on the original prewhirl distribution rotate through $\alpha = -20^\circ$, and the incidence angle at the shroud is kept constant at $i = 0^\circ$. 

- $RV_\theta$ constant (Free vortex)
- $V_\theta = AR$ (Forced vortex)
- $\alpha$ constant
- $V_\theta = AR^2$
Fig. 2.4 - Radial variation of incidence angles when the guide vanes, based on the original prewhirl distribution rotate through $\alpha = +20^\circ$, and the incidence angle at the shroud is kept constant at $i = 0^\circ$. 

\[ (\Delta \alpha = +20^\circ) \]

- $R \theta_\theta = \text{constant} \quad \text{(Free vortex)}$
- $\theta_\theta = AR \quad \text{(Forced vortex)}$
- $\alpha = \text{constant}$
- $\theta_\theta = AR^2$
Fig. 2.5 - Variation in the inducer hub incidence angle for various shroud angles assumed at the shroud, $R_h = 0.3$.

Shroud prewshroud angle ($\xi$), deg.

Incidence limits

$V_0$, AR = constant
$V_0$, AR = constant

Hub incidence angle ($\theta_h$), deg.
OF/DE
Fig. 2.6 - Radial variation of axial velocity downstream of inlet guide vanes for the four types of prewhirl distribution; theoretical.
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Fig. 3.2 - Variation of stalling incidence angle with camber angle for circular-arc airfoil sections set with tangent to mean profile arc at leading edge parallel to axis. The figure is adapted from reference [20].
Fig. 3.3 - Performance of cascades of circular-arc airfoil sections; the stalling points are indicated by asterisks; the values of camber angle are attached to the curves. The figure is adapted from reference [19].
Fig. 3.4 - Description of the inlet guide vanes with a) no incidence or deviation b) deviation c) negative incidence and d) positive incidence.

Fig. 3.5 - Geometries used for the design of constant-radius-of-curvature inlet guide vanes.
Fig. 3.6 - Performance of cascades of circular-arc section airfoils; at zero incidence and stagger angles from \(0^\circ\) to \(15^\circ\). The figure is adapted from reference [20].
Fig. A.3 - Position of the inlet guide vane assembly in the inlet duct, with the frontal section of the duct removed.
Fig. 3.8 - Radial variation of turning angle for typical circular-arc, sheet-metal inlet guide vanes for conventional axial flow compressor. Angle of incidence, 0°; annulus area ratio, 0.91; inlet mach number, 0.28; average solidity, 1.66. The figure is adapted from reference [19].
gage no. 16  
radius of curvature = 5.2°

Fig. 3.9 - Circular-arc inlet guide vane for the constant-prewhirl with radius type of distribution; drawn full scale.
Fig. 3.10—Conventional flat plate inlet guide vane for the constant-prewhirl with radius type of distribution; drawn full scale.
gage no. 16
radius of curvature = 5.2"

Fig. 3.11 - Circular-arc inlet guide vane for the forced vortex type of distribution; drawn full scale.
Fig. 3.12 - Circular-arc inlet guide vanes with the brass connector welded to the shroud.
Fig. 3.13 - Full scale drawing of the ring used to position the inlet guide vanes for various stagger angles.
Fig. 4.1 - Overall view of the centrifugal blower with the inlet duct.
Fig. 4.2 - Frontal view of the bellmouth.
Fig. 4.3 - Position of the inlet guide vane assembly in the inlet duct, with the frontal section of the duct removed.
Fig. 4.4 - Inlet guide vanes: a) Circular-arc section (set #1, 3) 
b) Flat plate section (set #2) 
c) Circular-arc section
Fig. 4.6 - 6-vane set assembly with the flat plate section in the guide vanes in position.
Fig 4.9 - Overall view of the traverse with the cobra probe fixed to its center.
Fig. 4.10 - Multi-tube accelerometer, barometer and thermometer.
$h_m = 1.8h_n$

Fig. 4.11 - Cobra probe calibration line.
Fig. 4.12 - Radial variation of the prewhirl angles downstream of the inlet guide vanes for various stagger angles. The theoretical estimations are shown by the dashed lines.
Fig. 4.13 - Radial variation of the prewhirl angle downstream of the inlet guide vanes for various stagger angles. The theoretical estimations are shown by the dashed lines.
Fig. 4.14 - Radial variation of the prewhirl angle downstream of the inlet guide vanes for various stagger angles. The theoretical estimations are shown by the dashed lines.
Fig. 4.15 - Overall variation of prewhirl angles for inlet guide vanes with constant-prewhirl with radius type of distribution.
Fig. 4.16 - Radial variation of prewhirl angles downstream of the inlet guide vanes for various stagger angles positions in the shroud.
The comparison between theoretical, design and experimental values of prewhirl angles for the forced vortex type of distribution (set # 4)

- Theoretical (eqn. 2.9)
- Design (eqn. 3.9)
- Experimental
Fig. 4.18 - Axial velocity variation downstream of the inlet guide vanes for various stagger angles.
Fig. 4.19 - Axial velocity variation downstream of the inlet guide vanes for various stagger angles.
Fig. 4.20 - Axial velocity variation downstream of the inlet guide vanes for various stagger angles.
Fig. 4.27 - Axial velocity variation downstream of the inlet guide vanes for various stagger angles; forced vortex distribution.
Fig. 4.22 - Radial variation of stagnation pressure loss across the inlet guide vanes for various stagger angles.
Fig. 4.23 - Radial variation of stagnation pressure loss across the inlet guide vanes for various stagger angles.
Fig. 4.24 - Radial variation of stagnation pressure loss across the inlet guide vanes for various stagger angles.
Fig. 4.25 - Radial variation of stagnation pressure loss across the inlet guide vanes for various stagger angles.
Fig. 4.26 - The comparison between the theoretical and experimental values of axial velocity for the constant-prewhirl with radius type of distribution at design prewhirl angle of 20 degrees.
Fig. 4.27 - The comparison between the theoretical and experimental values of axial velocity for the forced vortex type of distribution at design prewhirl angle in the shroud of 10 degrees.
Fig. 4.28 - Variation of stagnation pressure loss in the circumferential direction downstream of the inlet guide vanes for R=0.6.
APPENDIX A

THEORY OF CENTRIFUGAL IMPELLERS

A.1 Euler's Equation for Centrifugal Impeller

The Euler's equation for centrifugal impellers is derived by applying Newton's law for angular momentum to a surface surrounding an impeller, Figure 1.2. The torque required to produce the change of angular momentum is given as follows:

\[ T = \frac{\dot{\theta}}{g} (r_2 V_{\theta 2} - r_1 V_{\theta 1}) \]  \hspace{1cm} (1.1)

This torque is equal to the resultant moment about the axis of the impeller of the pressure and shear forces acting on the fluid in the impeller channels.

The power output to the impeller is given by

\[ P = \omega T \]

Also, since \( U = \omega r \), we have,

\[ P = \frac{\dot{\theta}}{g} (U_2 V_{\theta 2} - U_1 V_{\theta 1}) \]
and the specific work is derived as

$$w = \frac{1}{g} \left( U_2 V_{\theta 2} - U_1 V_{\theta 1} \right) \quad (1.2)$$

Equation 1.2 is Euler's pump and turbine equation or simply Euler's equation. If there is no inlet guide vanes to impart a prewhirl at the inlet to the impeller, then

$$w = \frac{1}{g} U_2 V_{\theta 2} \quad (1.3)$$

The Euler's equation represents the theoretical head produced by the machine. The following assumptions were made while deriving the equation:

1) There are infinite numbers of vanes, and the vanes are infinitely thin. Therefore, the flow is fully guided by the vanes in order to produce the ideal tangential velocities in equation 1.3; i.e., the flow is vane congruent.

2) The flow through the impeller channel is inviscid. Therefore, frictional losses due to effects of viscosity are neglected.

3) The flow through the impeller channels is one-dimensional; i.e., the specific angular momentum (rV_{\theta}) is constant across the inlet and outlet of the impeller.
From the velocity triangles, Figure 1.3, we find

\[ W^2 = V^2 + U^2 - 2UV \sin \alpha \]

and substituting the above equation 1.2, an alternative form of the Euler's equation is derived, which gives a physical significance to the equation. Therefore,

\[ w = \frac{V_2^2 - V_1^2}{2g} + \frac{U_2^2 - U_1^2}{2g} + \frac{W_1^2 - W_2^2}{2g} \quad (1.4) \]

The first term on the right hand side of equation 1.4 represents a gain in kinetic energy of flow from inlet to outlet. The second and third terms jointly represent an increase in static pressure. The second term can be considered to represent centrifugal energy due to the movement of fluid from one radius to the other, which is analogous to the centrifugal head developed in the case of forced vortex; and the third term represents the diffusion achieved in the impeller due to change in relative velocity from inlet to outlet. The ratio of static pressure terms to the total energy transfer is known as degree of reaction.
A.2 Theoretical and Actual Velocity Triangles

The velocity triangles are an important tool in the design and analysis of turbomachinery. The three sides of the triangle are represented by peripheral velocity \( U \), absolute velocity \( V \) and relative velocity \( W \). The two components of absolute velocity are \( V_\theta \) and \( V_z \) in the tangential and meridional directions respectively. For centrifugal machines, the velocity triangles are as shown in Figure 1.3.

Theoretical studies of impeller performance are based on the velocity triangles drawn on the vane angles, and the theoretical head is calculated according to Euler's equation. However, these velocity triangles give a considerably higher head than what is produced by the actual machines. In relation to the assumptions made in deriving the Euler's equation (section 1.1.), the velocity triangles at the outlet of the impeller is modified as shown in Figure 1.4. The figure is based on qualitative analysis, and shows the effects of finite number of vanes with finite thickness and fluid viscosity in reducing Euler's head. Other parameters such as impeller inlet diameter, vane shape, passage width and roughness also effect the actual flow angles at the outlet of the impeller. Ref. [14].
A.3 Theoretical and Actual Characteristic Curves

The equation 1.3 is the simplest form of the Euler's equation when there is no prewhirl imparted to the flow, it is given as:

\[ w = \frac{1}{g} U_2 v_{\theta 2} \quad (1.3) \]

from the velocity triangle, figure 1.2, we have

\[ v_{\theta 2} = v_{z2} \tan \beta_2 \quad \text{and} \quad v_{\theta 2} = U_2 - W_{\theta 2} \]

therefore,

\[ v_{\theta 2} = U_2 - v_{z2} \tan \beta_2 \quad (1.4) \]

substituting the above in equation 1.3, we have

\[ w = \frac{1}{g} \left( U_2^2 - v_{z2} U_2 \tan \beta_2 \right) \]

the mass flow rate at the outlet of the impeller is given as

\[ \dot{m} = \pi \rho D_2 b_2 v_{z2} \]

for an incompressible fluid, the flow rate is proportional to the meridional velocity, so that

\[ Q = \pi D_2 b_2 v_{z2} \]
Then, the equation 1.5 becomes

\[ \omega = \frac{1}{q} \left( \frac{U_2}{\pi D_2 b_2} \frac{Q}{\tan \beta_2} \right) \quad (1.6) \]

Equation 1.6 is the equation of a straight line for head-capacity characteristic. When \( \beta_2 = 0 \), the head is independent of \( Q \). Therefore, the characteristic curve is a line parallel to the \( Q \)-axis. When \( \beta_2 < 0 \), the second term in equation 1.6 is positive and the slope of the line is negative. When \( \beta_2 > 0 \), the second term is negative and the slope is positive. These results are shown in figure 1.4a.

To predict the general shape of the actual characteristic, we examine the way in which losses effect the ideal characteristic. The losses appear in the impeller due to the viscosity of the fluid, and are of two types, figure 1.4b. First, there are wall friction losses, which vary as the square of the flow rate. Second, there are the incidence losses due to separations produced on the curved part of the inducer when the flow enters the inducer at an incidence other than design. These losses vary parabolically at the design flow rate. There is also an effect called slip, caused by the finite number of the vanes. This slip effect
however is not a loss and represents the reduction in the input head at the same rate that it reduces the output head. A qualitative shape of the characteristic curve is shown in figure 1.4C. There are also other losses, such as windage losses due to frictional drag between the stationary housing and rotating impeller disk and leakage losses; but they do not occur in the impeller. In centrifugal compressors, the characteristic curve narrows down to a limited range of operation; such that, at low flow rates, surging occurs and, at high flow rates, the inducer chokes.

A.4 Flow Through the Impeller

The theoretical Euler's equation, derived in Section A.1, assumes that the air will follow the vane profile exactly. This can only be justified if the number of vanes are infinite, and the vanes are infinitely thin. It was also assumed that the fluid has no viscosity, and the flow is one-dimensional. In actual machines, however, these assumptions do not exist. Every vane must have a thickness so that infinite number of vanes is impossible and no fluid is perfect.

For a given vane angle and radius ratio, the effect of number of vanes in the impeller channel is similar to the
layer on the pressure side. Further, the fluid with less momentum is accumulated on the suction side of the vane, leading to large wakes of low energy fluid. The experimental results show that the cross-flow is larger for the double-shrouded impellers.

A.4.4 Pressure Distribution

In order to transmit power to the fluid, the pressure on the front or leading face of the vane must be higher than the pressure on the back or trailing face of the vanes. Therefore, at a given radius on the vane, the relative velocity on the back face is greater than the relative velocity on the front face. The velocity triangle in Figure 1.3 shows that, for a given vane angle, the head produced is lower with higher relative velocity or meridional velocity, since tangential velocity decreases. Therefore, higher relative velocity at the back of the vane will result in lower head and the total integrated head will be lower than that calculated for an average relative velocity flow within the channel. For large number of vanes, the average load on each vane is lower; therefore, the relative velocity on the back face of the vane is relatively lower, causing higher heads.
A.4.1 Relative Circulation

Relative circulation is one of the major secondary flows in a turbomachine. This is due to the inertia effect of the frictionless fluid particles explained in the second Helmholtz law. This law states that the vorticity of a frictionless fluid does not change with time; i.e., $\Gamma$ = Constant. Therefore, if the flow at the inlet to the impeller is irrational, then the absolute flow must remain irrational throughout the impeller. As the impeller has an angular velocity $\omega$, the fluid must have an angular velocity $-\omega$ relative to the impeller. Thus, if there is no flow through the impeller, then the flow will rotate with angular velocity opposite to the impeller rotation. Superimposing this relative circulation with the flow through the impeller in the vane to vane plane will result in an increase of the velocity in the suction side, and reduce the velocity in the pressure side of the vane as shown in Figure 1.5c. In addition, the streamlines will deviate from the direction of rotation of the impeller in such a way that the tangential velocity at the outlet ($V_{\theta 2}$) is reduced and the tangential velocity at the inlet ($V_{\theta 1}$) is increased, Ref. 1, as shown in Figure 1.5d. Also, due to the viscosity of the fluid, the fluid adheres to walls of the channel and the relative velocity profiles are further modified as shown in Figure 1.5e. The relative circulation is less with a greater number of vanes.
A.4.2 Velocity Profile in the hub-to-shroud Plane

In centrifugal machines, the flow has to make a 90° bend from the axial direction to the radial direction. This causes the meridional velocity in the impeller vane hub to increase, and the velocity in the impeller vane shroud to decrease. The result of this uneven velocity distribution is a reduction of the maximum head given by the Euler's equation. The velocity distribution at the outlet of the vanes is shown in Figure 1.6b. Assuming that the radial velocity varies linearly from $v_1^*$ in the vane shroud to $v_2^*$ in the vane hub, Stepanoff [1], has derived the following equation for the head:

$$ w = \frac{U_2^2}{g} - \frac{U_2 v_{z2}}{g \cot \beta_2} \left( 1 + \frac{v_2^* - v_1^*}{12 v_{z2}^2} \right) \quad (1.7) $$

also for sinusoidal velocity distribution Wislidenus [23] derived the following:

$$ w = \frac{U_2^2}{g} - \frac{U_2 v_{z2}}{g \cot \beta_2} \left( \frac{v^2}{8} \right) \quad (1.8) $$

where $v_{z2}$ is the average radial velocity. A comparison of equations 1.5 and 1.7 shows that the average theoretical head produced by an impeller with radial velocity varying
linearly is smaller than that for the average radial velocity $v_{z2}$. The same conclusion is observed between equations 1.5 and 1.8.

In deriving equation 1.7, viscosity was not taken into account. If, however, the viscosity was considered, the velocity of flow on the vane shroud and hub would have been equal to zero and the velocity profiles would be modified as shown in figure 1.6c. This effect of viscosity is to further reduce the Euler's head.

A.4.3 Cross-flow

Since the pressure on the front face of the vane is higher than the pressure on the back face of the vane, therefore, there is a pressure gradient within the channel perpendicular to the main flow. For a viscous flow, the fluid which is flowing in the shroud boundary layer has lesser momentum relative to the impeller, and this flow is subjected to the same pressure gradient as the main flow [16]. This leads to the further deflection of the flow toward the suction side of the blade passage than in the potential flow. A cross-flow is therefore resulted as shown in Figure 1.7. This cross-flow, which is partly caused by the viscosity of the fluid and partly by the pressure gradient, tends to produce a stable boundary
layer on the pressure side. Further, the fluid with less momentum is accumulated on the suction side of the vane, leading to large wakes of low energy fluid. The experimental results show that the cross-flow is larger for the double-shrouded impellers.

A.4.4 Pressure Distribution

In order to transmit power to the fluid, the pressure on the front or leading face of the vane must be higher than the pressure on the back as trailing face of the vanes. Therefore, at a given radius on the vane, the relative velocity on the back face is greater than the relative velocity on the front face. The velocity triangle in Figure 1.3 shows that, for a given vane angle, the head produced is lower with higher relative velocity or meridional velocity, since tangential velocity decreases. Therefore, higher relative velocity at the back of the vane will result in lower head and the total integrated head will be lower than that calculated for an average relative velocity flow within the channel. For large number of vanes, the average load on each vane is lower; therefore, the relative velocity on the back face of the vane is relatively lower, causing higher heads.
A.4.5 Non-active part of vane

In an actual or ideal centrifugal machine, the pressure difference on the two faces of the vane disappears at the vane tip, where the two streams from adjacent impeller channels join. This means that not all of the vane is active. The fluid leaves the vane tangentially at the high pressure or front face of the vane. The fluid on the back face, however, leaves the vane at an angle lower than the vane angle. The net result is that the fluid is discharged from the impeller at a mean angle which is less than the vane angle; i.e., there exists a fluid deviation similar to 2-dimensional cascades. This deviation or difference in tangential velocity is called the "slip" of the impeller.
APPENDIX B

Description of the integration used for derivation of equation

\[(2.8)\]

\[v_z \frac{dv_z}{dR} + R^{2n+1} \left[ A^2 (1 + n) \right] = 0\]

expanding the above equation, we have

\[v_z dv_z + A^2 R^{2n-1} dR + n A^2 R^{2n-1} dr = 0\]

integrating

\[v_z^2 + \frac{A^2}{n} R^{2n} + A^2 R^{2n} = B\]  \(\text{(B.1)}\)

where \(B\) is the Constant of integration.

Now, applying the following condition to equation B.1, and also to equation 2.1, we have,

\[\text{at } R = 1, \quad V_\theta = V_{\theta s} \quad \text{and} \quad V_z = V_{zs}\]

therefore \[B = \left( V_{zs}^2 + \frac{A^2}{n} + A^2 \right)\]

and \[A_n = V_{\theta s}\]

then, equation B.1 becomes:

\[v_z^2 = v_{zs}^2 + v_{\theta s}^2 \left[ \frac{1+n}{n} (1 - R^{2n}) \right]\]

which is similar to equation 2.8.
THIS PROGRAM CALCULATES INCIDENCE ANGLES AT THE INDUCER OF A CENTRIFUGAL MACHINE ACCORDING TO THE THEORETICAL BLADE ANGLES DISTRIBUTION.

DIMENSION AZAR(30), ROYA(30), AFSA(30)
DIMENSION WHLS(30), AXP(30), BLD(30), ADD(30)
DIMENSION ANT(30), VELR(30), FLAN(30), ENC(30)
DIMENSION KVAR(12), FARI(30), PLO(30)
COMMON/NEW/WHLS(30), SHIR(30), RR(10), YI(30), UPER, N, ZAR
INTEGER DUM
REAL LOWR, INCR, NORA

INPUT DATA

DATA AXP(1), AXP(2), AXP(3), AXP(4)/-1, 0, 1, 2/
DATA RR/0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0/
WHLS(1)=0.1746
BLDS=1.012
UPER=1.57
LOWR=-0.698
INCR=3.141
DO 55 L=1, 6
WRITE(*, 150)
150 FORMAT(70(1H*))
IF (L.EQ.1) GO TO 65
WHLS(L)=WHLS(L-1)+0.1745
DO 25 J=1, 4
KVAR(L)=WHLS(L)*57.3
WRITE(*, 90) AXP(J), KVAR(L)
25 CONTINUE
90 FORMAT(1HO, 10X, 16HRAD. EQU. EXP. =+12, 5X,
C20SHRHOUD WHIRL ANGLE =+F5.2///)

PREWHIRL ANGLE CALCULATION.

1H KO=(((RR(K)**(2.)*AXP(J)))*((TAN(WHLS(L)))
C**(2.)))
ROYA(K)=((((1.)+AXP(J))*(1.)-((RR(K)**(2.)*
CAXP(J)))/((2.)*AXP(J)))))
AFSA(K)=(ROYA(K)**(2.)*((TAN(WHLS(L)))**2))
WHLS(K)=ATAN((AZAR(K)/AFSA(K))**0.5
GO TO 50
WHLS(K)=WHLS(L)
YI(K)=$(SIN(WHEL(K))**2.5)/RR(K)
SHIR(K)=WHLS(K)*57.3
FORMAT(3X, 3HR, =F4.2, 5X, 13WHIRL ANGLE =+F9.5)
CONTINUE
32,100    ZAR=5.
32,200    M0=125 N=4,50
32,300    IF(ZAR.EQ.10.) GO TO 125
32,400    DUM=3
32,500    MAN=11
33,100 C    ******************************************************************************
33,100 C    ABSOLUTE VELOCITY AND AXIAL VELOCITY PARAMETER
33,100 C    CALCULATION.
33,300 C    ******************************************************************************
34,000    DO 40 I=2,10
34,100    IF(I.EQ.10) GO TO 95
34,300    GO TO 105
34,400 95    PARI(I)=COS(WHLS(L)),
35,500 105    K=MAN-1
35,600    ANT(K)=(YI(K)+YI(K+1))*0.05
37,500    ABD(I)=0
38,000    ADD(K)=ANT(K)+(ADD(K+1))
39,000    VELR(10)=1
30,000    VELR(K)=EXP(-ADD(K))
30,020    PARI(K)=COS(WHL(K)/VELR(K)
31,000    CONTINUE
32,000    VIDA=(COS(WHLS(L)))*((TAN(WHLS(L)))-(TAN(BLDS))
32,100 C    ******************************************************************************
32,200 C    INCIDENCE ANGLE CALCULATION
32,300 C    ******************************************************************************
33,000    DO 45 I=1,10
33,100    IF(WHL(I).LE.LOWR) GO TO 170
34,000    FLAN(I)=ATAN((TAN(WHL(I)))-(RR(I)
35,000    C*VELR(I)*VIDA)/(COS(WHL(I))))
36,000    BLDI(I)=ATAN(RR(I)*TAN(BLDS))
36,100    ENCI(I)=((BLDI(I)-FLAN(I)*360,))/(2.)*(3.14)
36,120 60    FORMAT(3X,3HR =F4.2,5X,12HVEL, RATID =F8.4,5X,3H,3X,12HAXIAL
36,130 C12HAXIAL VEL. =F8.4,)
36,200    WRITE(6,70)RR(I),SHI(R(I)),ENC(I)
36,300 70    FORMAT(3X,3HR =F4.2,5X,13HWHIRL ANGLE =F7.3,5X,
36,303 C11HINCIDENCE =F8.3)
36,304    GO TO 45
36,305 170    DUM=2
36,307 45    CONTINUE
36,309    IF(DUM.EQ.2) GO TO 156
36,310    FLOW=0
36,311 C    ******************************************************************************
36,312 C    VOLUME FLOW RATE CALCULATION
36,313 C    ******************************************************************************
36,317    DO 75 M=1,10
36,318    IF(M.EQ.10) GO TO 166
36,320    FLO(M)=2.*3.14*RRI(M)*0.1*PARI(M)
36,330    FLOW=FLOW+FLO(M)
36,334 166    IF(N.EQ.2) GO TO 75
36,335    WHLM=M=WHLM(M)-INCR
36,337 75    CONTINUE
36,340    WRITE(6,100)FLOW
36,350 100    FORMAT(1HO,5X,11HFLOW RATE =F10.5)
36,354    WRITE(6,155)
36,355 155    FORMAT(1HO,14X,35(1H*)
36,400 156    CALL ALAF
36.450 CONTINUE
25.500 CONTINUE
36.600 CONTINUE
37.000 STOP
38.000 END
38.100 C
38.200 C
38.300 C
39.000 SUBROUTINE ALAF
40.000 COMMON/NEW/WHL(30),SHIR(30),RR(10),YI(30),UPER,N,ZAP
41.000 DO 20 K=1,10
42.000 WHL(K)=WHL(K)+0.1745
43.000 SHIR(K)=WHL(K)*57.3
44.000 K(K)=((SIN(WHL(K)))**(2.))/RR(K)
45.100 IF(WHL(10).GE.UPER)GO TO 60
46.200 IF(N.GE.2)GO TO 50
47.210 IF(K.GE.2)GO TO 50
48.300 WRITE(*,40)
49.400 40 FORMAT(1H0,9X,42H10 DEGREES INCREMENT ROTATION OF THE VANES)
49.500 WRITE(*,70)
49.600 70 FORMAT(7X,51(1H*))
49.700 50 CONTINUE
49.800 30 FORMAT(8X,3HR =,F4.2,5X,13HWHIRL ANGLE =,F10.5)
49.900 GO TO 20
50.200 ZAR=10.
50.300 WRITE(*,160)ZAR
50.400 160 FORMAT(5X,5HZAR =,I3)
50.600 20 CONTINUE
51.000 RETURN
52.000 STOP
53.000 END
subroutine alaf

\[ K = 1 \]

\[ \alpha = \alpha + 10 \]

\[ \text{calc: } \frac{\sin \alpha}{R} \]

\[ \text{if } K : 10 \]

return
SYMBOLS USED IN THE COMPUTER PROGRAM

WHLS - Prewhirl angle at the inducer shroud (\( \alpha_s \)).
BLDS - Blade angle at the inducer shroud (\( \beta_{os} \)).
AXP - Radius ratio exponent (n).
RR - Radius ratio (R).
UPER - Upper limit of rotation (\( \alpha_U \)).
LOWR - Lower limit of rotation (\( \alpha_L \)).
INCR - Increment of rotation (\( \Delta \alpha \)).
WHL - Prewhirl angle (\( \alpha \)), in radians.
KHAR - Prewhirl angle in the inducer shroud, in degrees.
BLD - Inducer blade angle (\( \beta_o \)), in radians.
AZAR - The numerator term in equation 2.9.
ROYA - The term in bracket, in equation 2.9.
AFSA - The denominator term in equation 2.9.
YI - The term in equation 2.12a.
SHIR - Prewhirl angle in degrees.
PARI - Axial velocity parameter (\( \frac{V}{V_{so}} \)).
VELR - Absolute velocity parameter (\( \frac{V}{V_s} \)).
VIDA - The term \( \cos \alpha_s \) (\( \tan \alpha - \tan \beta_s \)) in equation 2.17.
PLAN - Inducer relative flow angle (\( \beta \)).
ENCI - Inducer incidence angle (i).
FLOW - Flow rate through the vanes (\( Q \)).
PLO - Flow through the radial loops (\( Q_R \)).
ALAF - Subroutine.
ZAR, DUM, MAN - Dummies.
APPENDIX D

Variation of camber angle with chord length for constant radius-of-curvature inlet guide vanes

\[ C = 2 R_c \sin \frac{\phi}{2} \]

By referring to the above figure, the chord length \( C \) can be found as:

Now, for two circular-arc sections which vary in the direction perpendicular to the page, we have

\[ \frac{C_1}{C_2} = \frac{\sin \frac{\phi_1}{2}}{\sin \frac{\phi_2}{2}} \]

assuming \( \sin \frac{\phi}{2} = \frac{\phi}{2} \) in radians

then, for \( \phi = 40^\circ \), the error by the approximation is within 2\% and for \( \phi = 25^\circ \), it's within 1\% which is reasonably small.
Therefore, for camber angles less than $40^\circ$, we can write,

\[
\frac{C_1}{C_2} = \frac{\phi_1}{\phi_2}
\]
APPENDIX E

The relationship between deviation angle - incidence angle slope and turning angle - incidence angle slope

By referring to figure 3.4d, we have,

$$\theta = \phi + i - \delta$$

differentiating with respect to i,

$$\frac{d\theta}{di} = \frac{d\phi}{di} + \frac{di}{di} - \frac{d\delta}{di}$$

where

$$\frac{d\phi}{di} = 0 \quad \text{and} \quad \frac{di}{di} = 1$$

therefore,

$$\frac{d\delta}{di} = 1 - \frac{d\theta}{di}$$
Results of the design calculation of the inlet guide vanes

\[ \phi_s = 25^\circ \quad C_s = 1.89" \quad (-\frac{S}{C})_s = 0.866 \]

<table>
<thead>
<tr>
<th>( C_h )</th>
<th>( \phi_h )</th>
<th>((-\frac{S}{C})_h)</th>
<th>( \frac{\phi_2 - \phi_1}{2} )</th>
<th>( \delta_2 - \delta_1 - \delta \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.69</td>
<td>9.2</td>
<td>0.866</td>
<td>7.90</td>
<td>0.20</td>
</tr>
<tr>
<td>1.30</td>
<td>17.2</td>
<td>0.533</td>
<td>3.90</td>
<td>1.60</td>
</tr>
<tr>
<td>1.50</td>
<td>19.3</td>
<td>0.462</td>
<td>2.58</td>
<td>1.77</td>
</tr>
<tr>
<td>1.60</td>
<td>21.2</td>
<td>0.433</td>
<td>1.90</td>
<td>1.93</td>
</tr>
</tbody>
</table>

Table 1 - Circular-arc design calculation (1st trial)

\[ \phi_s = 25^\circ \quad C_s = 2.27" \quad (-\frac{S}{C})_s = 0.72 \]

<table>
<thead>
<tr>
<th>( C_h )</th>
<th>( \phi_h )</th>
<th>((-\frac{S}{C})_h)</th>
<th>( \frac{\phi_2 - \phi_1}{2} )</th>
<th>( \delta_2 - \delta_1 - \delta \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.69</td>
<td>7.6</td>
<td>0.866</td>
<td>8.68</td>
<td>0.02</td>
</tr>
<tr>
<td>1.80</td>
<td>19.8</td>
<td>0.385</td>
<td>2.60</td>
<td>1.70</td>
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<tr>
<td>1.90</td>
<td>20.9</td>
<td>0.364</td>
<td>2.03</td>
<td>1.63</td>
</tr>
<tr>
<td>2.00</td>
<td>22.0</td>
<td>0.346</td>
<td>1.50</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Table 2 - Circular-arc design calculation (2nd trial)
<table>
<thead>
<tr>
<th></th>
<th>$R$</th>
<th>$\phi$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\frac{S}{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>hub</td>
<td>0.32</td>
<td>9.2</td>
<td>7.25</td>
<td>1.96</td>
<td>0.72</td>
</tr>
<tr>
<td>mean</td>
<td>0.60</td>
<td>17.3</td>
<td>13.50</td>
<td>3.80</td>
<td>0.72</td>
</tr>
<tr>
<td>shroud</td>
<td>0.80</td>
<td>25.0</td>
<td>19.30</td>
<td>5.70</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Table 3 - Forced vortex design calculation
APPENDIX G

DATA REDUCTION EQUATIONS

The following quantities were recorded during the experiment:

$h_T$ - the fluid height corresponding to the stagnation pressure hole of the probe, in mm.

$h_N$ - The fluid height corresponding to the nulling pressure holes of the probe in mm.

$h_s$ - The fluid height corresponding to the average static pressure at the duct inlet, in mm.

$a$ - Prewhirl angle in the nulling position of the probe, in degrees.

$P_{amb}$ - Ambient pressure in mm Hg.

$T_{amb}$ - Ambient temperature in degrees Centigrade.

Therefore, to convert these quantities into the flow parameters used in the plots of the experimental results, the following were derived:
1) **Stagnation pressure drop \( (\Delta P_o) \)**

The stagnation pressure drop is simply obtained as:

\[ \Delta P_o = \rho_w \cdot g \cdot h_T \]  \hspace{1cm} (4.4)

where \( \rho_w \) is the density of the manometer fluid (water), with the application of the proper conversion factor, we have,

\[ \Delta P_o = 1.42 \times 10^{-3} \cdot h_T \]  \hspace{1cm} (4.5)

where \( h_T \) is in mm, and \( \Delta P_o \) is in psi.

2) **Absolute velocity calculation \( (V) \)**

by combining equations 4.1 and 4.2 from section 4.2.1, we have,

\[ P_T - P_s = 1.8 \cdot \rho_w \cdot g \cdot h_n \]  \hspace{1cm} (4.6)

Now, writing the Bernoulli's equation for the flow just upstream of the probe in the nulling position, we obtain

\[ P_T - P_s = \frac{1}{2} \cdot \rho_o \cdot V^2 \]  \hspace{1cm} (4.7)
Therefore, from equations 4.6 and 4.7, we have,

\[ v = \left( \frac{3.6 \rho w g h_n}{\rho_o} \right)^{1/2} \]

\[ v = \left( 0.737 \frac{h_n}{\rho_o} \right)^{1/2} \]

where \( h_n \) is in mm, \( \rho_o \) is the air density in slug/ft\(^3\) and \( v \) is the absolute velocity, in ft/s. The air density is calculated from the perfect gas law equation using the ambient conditions.

3) Prewhirl angle (\( \alpha \))

The prewhirl angles were recorded directly from the readings on the traversing probe dial. The zero prewhirl angle was obtained when the flow was axial.

4) Axial velocity calculation (\( v_z \))

The axial velocities were calculated simply as:

\[ v_z = v \cos \alpha \]
5) **Inlet velocity calculation upstream of the vanes** \( (U) \)

The inlet velocity was approximately constant throughout the experiment at 136 ft./s, which corresponded to 100 mm H\(_2\)O. However, some changes were observed due to the variation in the ambient conditions from day to day. A more exact value of the inlet velocity was obtained as:

\[
P_S = P_a - 0.142 \quad (4.10)
\]

Where \(P_a\) and \(P_S\) are in psi. From these values, \(\frac{P_S}{P_a}\) was found and from the compressible flow tables, the value of Mach number was extracted. Therefore,

\[
U = 65.75 M \sqrt{T_a} \quad (4.11)
\]

where \(T_a\) is the ambient temperature in degrees kelvin, and \(U\) in ft./s.
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