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**LA THÈSE A ÉTÉ
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AN EVALUATION OF SAFETY
ASSOCIATED WITH TESTING TIMBER TRUSSES

by

Donald P.J. Duchesne, B. Eng.

A thesis submitted to the Faculty
of Graduate Studies and Research in
partial fulfillment of the require-
ments for the degree of Master of
Engineering.

Faculty of Engineering
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September 1980

The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis "An Evaluation of Safety Associated with Testing Timber Trusses" submitted by Donald P. J. Duchesne, B.Eng. in partial fulfillment of the requirements for the degree of Master of Engineering.

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Abstract

A technique is proposed to evaluate truss testing procedures by determining the level of safety that can be expected of a truss design given that it has passed a full scale test. The probability of survival of a single truss loaded to its design load is formulated as a function of its probability of surviving a test load. A solution is presented for materials not exhibiting creep rupture assuming a log normal distribution for resistance. A solution is also presented for materials exhibiting creep rupture using a damage accumulation model. Once the probability of survival of each component of a structure has been calculated, the probability of survival of the entire truss is formulated as a weakest link type system. The acceptance criteria is formulated as an Operational Characteristic equation relating the probability of survival of an individual truss specimen to the probability of the design passing the test.

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Table of Contents

	<u>Page</u>
1. Introduction.....	1
1.1 Background.....	1
1.2 Timber Truss Industry.....	1
1.3 Truss Testing.....	2
1.4 Truss Design Procedure.....	5
1.5 Factors Affecting Truss Tests.....	6
1.6 Factors Affecting Lumber Strength.....	7
1.6.1 Distributional Assumption.....	9
1.6.2 Duration of Load.....	9
1.6.3 Moisture Content.....	10
1.6.4 Grading Factor.....	11
1.6.5 Size Effect.....	11
1.7 Stochastic Evaluation of Truss Tests.....	12
1.8 Objectives.....	16
2. Structure Reliability.....	18
2.1 Concept of Structural Safety.....	18
2.2 Differential Reliability.....	24
2.3 System Reliability.....	26

	<u>Page</u>
3. Damage Effect.....	29
3.1 Duration of Load.....	29
3.2 Damage Model.....	30
3.3 Combined Loading.....	35
4. Evaluating Truss Testing.....	45
4.1 Acceptance Sampling.....	45
4.2 Risk Levels.....	46
4.3 Formulation of the Operational Characteristic Equation.....	46
4.4 Test Safety.....	50
4.5 Duration of Load.....	56
5. Applications.....	58
5.1 Example.....	58
5.2 Certification Test.....	59
5.3 Probability of Acceptance Based on Component Data.....	62
5.4 Probability of Surviving the Design Life.....	63
5.5 Sensitivity Analysis.....	66
5.5.1 Load Duration.....	66
5.5.2 Number of Tests.....	76
5.5.3 Load Ratio.....	81

	<u>Page</u>
5.6 Application To Decision Making.....	81
5.7 Code Calibration.....	88
6. Summary.....	90
6.1 Conclusions.....	90
6.2 Recommendations for Further Study.....	91
References.....	93

List of Figures

<u>Figure</u>	<u>Page</u>
1.1 Truss Analogues.....	4
1.2 Load Histories.....	7
1.3 Typical Beam Under Load.....	11
2.1 Load and Resistance Distributions.....	19
2.2 Definition of Safety Index.....	22
2.3 Probability of Failure for a Weakest Link Structure.....	28
3.1 Timber Element Under Combined Bending and Axial Load.....	36
3.2 Flow Chart to Determine Combined Interaction....	40
3.3 Concomitant Relationships Between Axial and Bending Strength.....	43
4.1 Operating Characteristic Curve.....	48
4.2 Relationship Between Safety and Variability.....	50
4.3 OC Curves Defined by Equation 4.....	53
4.4 Effect on Reliability Caused by Varying Load Ratio	54
4.5 Effect on Reliability Caused by Varying Number of Tests.....	55
5.1 Example Structure.....	58
5.2 OC Curves Defining Floor Truss Test.....	61
5.3 Reliability Associated with Trusses Tested Singly.....	64
5.4 Reliability Associated with Testing Trusses in Pairs.....	65

<u>Figure</u>	<u>Page</u>
5.5 Reliability Associated with 1 Hour Test.....	67
5.6 Reliability Associated with 12 Hour Test.....	68
5.7 Reliability Associated with 24 Hour Test.....	69
5.8 Reliability Associated with 48 Hour Test.....	70
5.9 Reliability Associated with Testing 1 Truss...	73
5.10 Reliability Associated with Testing 2 Trusses.	74
5.11 Reliability Associated with Testing 3 Trusses.	75
5.12 Reliability Associated with Testing 4 Trusses.	76
5.13 Test Reliability for Load Ratio of 1.5.....	77
5.14 Test Reliability for Load Ratio of 2.0.....	78
5.15 Test Reliability for Load Ratio of 2.5.....	79
5.16 Test Reliability for Load Ratio of 3.0.....	80
5.17 Definition of "Safety Factor", n	82
5.18 Probability of Survival Verses Factor of Safety for Floor Truss Test.....	85
5.19 Probability of Acceptance Verses Factor of Safety for Trusses Tested Singly.....	86
5.20 Probability of Acceptance Verses Factor of Safety for Trusses Tested in Pairs.....	87
5.21 Reliability Associated with Recalibrated Test.	89

Table of Nomenclature

a	- Material constant
b	- Material constant
c	- Test load ratio
f_D	- Duration of load factor
f_L	- Location factor
f_S	- Sampling factor
n	- Number of tests
y	- Number of successes
AQL	- Acceptance quality level
F_{ALL}	- Allowable stress
F_A	- Axial strength
F_B	- Bending strength
K_G	- Grade factor
K_H	- Height factor
K_M	- Moisture content factor
K_{SAFETY}	- Safety factor
K_T	- Time factor
K	- Rate of loading used for short term tests
L_W	- Design working load
P	- Probability of failure
Q	- Probability of survival

R	- Resistance
R_N	- Standard normal $N(0,1)$ variable
RQL	- Rejection Quality Level
S	- Standard deviation
T	- Time
U	- Load
V_N	- Standard normal $N(0,1)$ variable
α	- Probability associated with a type I error
α	- Damage
β	- Probability associated with a type II error
β	- Safety index, standard normal $N(0,1)$ variable
γ	- Coefficient of variation
n	- Over design safety factor referenced to median strength
λ	- Material constant
σ	- Stress ratio referenced to short term strength
τ_a	- Axial stress
τ_b	- Bending stress
ω	- Variability parameter

CHAPTER 1

INTRODUCTION

1.1 Background

It has been reported that, of the total amount of wood used in residential construction, one eighth is used for roof and floor trusses. In the United States this corresponds to approximately four billion board feet per year. With the growing concern for a more economical use of timber resources, it has become necessary to evaluate the efficiency and safety of timber structures. There is, therefore, a need for a methodology to study the level of safety associated with wood trusses and to investigate how certification testing can help in evaluating the reliability of these engineered wood structural systems.

These two aspects will be the main subject of this study. Other problem areas will be discussed and an approach for the solution will be suggested where appropriate. In some cases, these areas will be outside the scope of the study and they will be identified as topics for further research.

1.2 Timber Truss Industry

Metal plate connected trusses, first introduced in the early 1960's, have become popular alternatives in residential and light industrial construction. Trusses are being produced in increasingly complex and novel forms. Trusses are also being produced to compete

with standard floor joists and are available in a wide variety of shapes and forms.

Assumptions used in the analysis of common roof and floor trusses have been standardized throughout the industry with the specifications outlined by the "Truss Plate Institute of Canada (TPIC)" [1978]. Minimum requirements concerning plating, safety factors applied to plate properties, lumber quality and design loads, etc., are also specified by TPIC requirements.

1.3 Truss Testing

Truss testing procedures for the purpose of certification is one way of regulating product quality and reliability. The testing procedures considered in this study are those of the type usually specified by a building code or housing authority and designed to screen out poor designs. This is accomplished by rejecting truss designs which fail to pass a test. The test itself usually consists of loading representative truss specimens up to a specified load level and measuring deflections. If the specimen or specimens survive the test with acceptable strength and deflections, the design is accepted.

In Canada, as in many other countries, roof trusses may be certified if designs are based on sound engineering principles or if prototype testing is performed. Clause 9.23.13.16 of the National Building Code of Canada 1977 stipulates that trusses

"which are not designed in conformance with part 4 of this Bylaw shall be capable of supporting a load at least equal to the ceiling load plus $2 \frac{2}{3}$ the design roof load for 24 hours and shall not exceed the deflections shown in Table 9.23.13 B when loaded with the ceiling load plus $1 \frac{1}{3}$ the design roof snow load for one hour. Testing of lumber roof trusses shall be in conformance with CSA S307-1977 'LOAD TEST PROCEDURE FOR WOOD ROOF TRUSS FOR HOUSES AND SMALL BUILDINGS'".

CSA S307-1977 outlines the testing procedure for roof trusses.

Prototype testing is sometimes required by a regulatory body in order to account for

- (1) variability in lumber or joint strengths,
- (2) approximations used in modeling of unusual structural types or shapes.

Prototype testing can also be used by plate manufacturers who may not want to develop a model for a very low volume product. The option would only be economical for a product with very few variations since the cost of testing is quite often high.

Prototype testing is also used as an aid in the development of a model for use in designing a new product. A theoretical model is only as good as its ability to predict real structural behaviour. Wood is complex in nature, making it difficult to extrapolate theoretical results to real results in any exact manner.

FIGURE 1.1 TRUSS ANALOGUES

Granger [1969] points out that since a lack of knowledge exists on material behaviour, a conservative approach must be used for theoretical design. Prototype testing provides a "semi-empirical" approach that is of value in solving immediate problems, with tests often showing superior results to those from design calculations. A properly conducted prototype test therefore is one of the best ways to assess structural safety.

1.4 Truss Design Procedure

The design procedure adopted in Canada uses an analogue proposed by the "Truss Plate Institute of Canada" which standardizes design assumptions. Since conventional analysis techniques consider only one dimensional elements, the formulation of the mathematical analogue becomes difficult especially in the case of trusses with long heel joints. The question arises as to where the top and bottom chord analogue members should meet to adequately predict the behaviour of the truss. The TPIC analogue requires that axial forces in the members be calculated by assuming the members to be pin connected. The lines of action of members are defined as shown in Figure 1.1. Tests on full sized specimens have

shown that, for the case of roof trusses, this design technique is generally conservative.

1.5 Factors Affecting Truss Tests

It must be kept in mind that any one test tells little about a truss population because of large variability in lumber products. The level of confidence associated with a testing procedure increases with the number of trusses tested. There is, however, an economic and practical limit to the number of tests performed. A single test may cost anywhere from \$500.00 to \$2000.00 and, it may very quickly become uneconomical to continue testing in order to improve the estimate on truss performance. On the other hand, too few tests may result in an over-conservative design to obtain the same level of confidence associated with testing a large number of samples.

Since wood exhibits a duration of load effect, load history and lumber strength are not independent, thus causing strength properties to be a function of load level and time.

For a test to be practical, the length of time under load must be in the order of a few hours, whereas the length of time under service load is usually months or years. Furthermore, tests use constant load histories, ramp load histories or combinations of both, whereas service loads are a large number of random loads as shown in Figure 1.2.

FIGURE 1.2 LOAD HISTORIES

There are a number of other factors affecting truss testing as pointed out by Pearson [1965], including temperature and moisture content, but they will be assumed to have an equal effect on all testing procedures.

1.6 Factors Affecting Lumber Strength

At present (1980) the working stress design method is still used in Canada for wood. Allowable stresses for lumber are derived from short-term strength data of small, clear, straight grained specimens, in accordance with the American Society for Testing and Materials (ASTM) standard procedures. These strength values are then modified by a number of factors in order to reflect real lumber properties.

$$F_{ALL} = (K_T K_M K_H K_G \frac{1}{K_{SAFETY}}) (\bar{X} - 1.645 S) \quad (1.1)$$

where $\bar{X} - 1.645 S$ - calculates the 5th percentile value based on a normal distribution.

K_T - time factor where the 5th percentile value is modified by a value based on Wood's (1951) empirical duration of load curve.

K_M - modifies strength according to moisture content.

K_H - modifies strength depending on size.

K_G - modifies strength according to grade.

K_{SAFETY} - safety factor.

In the past few years this approach has been questioned as to its validity, and very complete discussions can be found in papers by Madsen [1975] and Aplin and Keenan [1977].

Of significance in this study is the distributional form assumed for strength and the method used to modify the allowable strength to account for duration of load since any study on reliability is highly dependant on the distributional assumptions.

1.6.1 Distributional Assumption

The assumption of a normal distribution for the strength of small clear specimens may be a valid one; however, its extrapolation to full sized lumber is dubious. As discussed by Warren [1974] and Madsen et al [1978], the strength properties of full sized lumber seem to fit reasonably well to a three-parameter Weibull distribution. Therefore, an estimate of the 5th percentile, based on the assumption of normality, is not considered to be representative of actual behaviour.

1.6.2 Duration of Load

It has been known for many years that wood exhibits higher strength when subjected to loads of short duration. This phenomenon was originally quantified by Wood [1951] by fitting an empirical curve through the mean times to fracture of small clear specimens loaded to different values. The design values for lumber are modified according to the cumulative time under load referred to a "normal" load duration of 10 years.

This practice of modifying the allowable stresses to account for duration of load has been fundamentally challenged by Madsen [1975]. He found that full sized, graded material is less sensitive to the duration of load factor at the 5th percentile than at the mean. Since the 5th percentile is used as the characteristic strength for design purposes, he argues that the method used to account for load duration is not correct.

Another serious drawback of the present method is that only the cumulative time under load is considered and not the actual load history. Barrett and Foschi [1978] argue that a rational discussion of safety in wood structures requires that duration of load effects be adequately considered because of the interdependence between load history and resistance. To this end, a damage model was formulated which relates the rate of damage accumulation to the load history. This damage model will be further discussed in Chapter 3.

1.6.3 Moisture Content

CSA 086- [1976] stipulates that allowable stresses be modified to reflect higher allowable strengths for dry material. Recent studies by Madsen [1975] on full sized specimens have shown that the effect of moisture content is almost negligible for material at the 5th percentile as compared to the mean. It was his recommendation that allowable stresses not be modified for the effect of moisture content.

1.6.4 Grading Factor

The present method requires that the strength of the small clear specimens be reduced by a grade factor to account for natural growth characteristics found in full size lumber. The grading factor is based on a set of standard grading rules. With the advent of ingrade testing, this grading factor will probably not be needed.

1.6.5 Size Effect

If a timber element is under load, as shown in Figure 1.3 the question may be asked: where will the beam fail?

FIGURE 1.3 TYPICAL BEAM UNDER LOAD

The beam need not fail at the point of highest shear, bending or bearing stress but rather it will fail at the point where the local stress resultant is higher than the local strength. In other words, the beam will fail when the weakest link fails.

This phenomenon was originally discussed by Weibull [1939] and again by Bolotin [1965] with the probability of failure formulated as a function of the stress distribution and the volume under stress. It was found that the larger the volume under stress, the lower the overall strength.

This type of behaviour was observed in timber structures where the volume of wood under stress was high as in the case of glue-laminated beams. Barrett [1976] has evaluated the characteristic strength of a timber element using the weakest link approach and had obtained results for shear and tension perpendicular to grain.

1.7 Stochastic Evaluation of Truss Tests

Leicester [1971] was the first to discuss prototype testing for timber trusses using statistical inference by formulating an approach to truss testing consistent with Limit States Design.

Leicester's acceptability criterion is that, R_{MIN} , the minimum strength of the sample structure in a load test, be no less than that given by equation (1.2)

$$R_{MIN} = f_L f_D f_S L_W, \quad (1.2)$$

f_L = location factor

f_D = duration of load factor

f_S = sampling load factor

L_W = Design working load

The statistical load factor, f_s , is formulated as a function of the number of tests and the coefficient of variation of strength. The method, although useful, does not directly consider type I and type II errors which are associated with imperfect testing.

Leicester realized the limitations of statistical analysis when used to evaluate test procedures. He states:

"It is apparent then, that subjective extrapolations of the available data must be made if a probabilistic analysis is used to derive specified test loads. In order to reduce the influence of erroneous assumptions about the statistical distributions, it is proposed that rather than specify absolute risk criteria, the test loads be evaluated on the criterion that the theoretical risk of failure for a structure accepted on the basis of a load test be related in some specified manner to the theoretical risk of failure evaluated for a timber structure of standard type. In addition it is proposed that the subjective assumptions made for the derivation of both these risks be similar."

There is a marked similarity between this statement concerning truss testing and that proposed by Suddath [1978] concerning differential reliability.

Sexsmith [1978] studied the problem of truss test evaluation by statistical hypothesis testing. The hypothesis to be tested, called the null hypothesis, is that the truss being tested comes from a poor distribution. The experimenter believes that his truss is indeed from a good distribution and wishes to reject the null hypothesis.

To test the null hypothesis in an objective manner, a decision maker, called a test statistic, is calculated based on information in the sample. In this case the test statistic is the number of trusses that survive the test.

The method studied by Sexsmith uses a sequential sampling plan where a test is performed and, based on the test statistic, a decision is made whether to accept or reject the null hypothesis or continue testing to acquire more information. For example, consider the case where three trusses are tested. It may be decided to reject the null hypothesis if two or three trusses survive. If, however, the test statistic is one, (i.e. one truss survives) then testing would continue to obtain more information.

Since any procedure which does not test the whole population is imperfect, there is some finite possibility of rejecting the null hypothesis when it is in fact true. This error, called a type I error for a statistical test, has a probability associated with it and is usually denoted by α . Alternatively, the null hypothesis may be accepted, given that it is in fact false. This error, called a type II error, has a probability associated with it and is usually denoted by β . For a fixed sample size, n , α and β are inversely related. Increasing the sample size, however, provides more information on which to base a decision which reduces both α and β .

For the case of truss testing, a type I error corresponds to accepting a truss design given that it is from a poor distribution. A type II error is associated with rejecting a truss design given that it comes from a good distribution.

If the null hypothesis is defined as: A_0 - the truss is from a poor distribution, and the alternate hypothesis is defined as A_1 - the truss is from a good distribution. The probability of a type I error, α , is defined as:

$$\alpha = P(A_0/\text{sufficient number of trusses survive to pass the test})$$

The probability of a type II error, β , is defined as:

$$\beta = P(A_1/\text{insufficient number of trusses survive to pass the test})$$

Using the binomial distribution, α and β , can be evaluated as:

$$\alpha = \sum_{i=1}^{n1} \frac{n!}{y!(n-y)!} P^y(A_0/B_i) Q(A_0/B_i)$$

$$\beta = \sum_{i=1}^{n2} \frac{n!}{y!(n-y)!} P^y(A_1/B_i) Q(A_1/B_i)$$

where

$n1$ = number of permutations of passing the test.

$n2$ = number of permutations of failing the test.

where y = number of successes

n = number of tests

$Q = 1-P$

Using Bayes' theorem $P(A_i/B_j)$ can be evaluated as:

$$P(A_i/B_j) = \frac{P(B_j/A_i)P'(A_i)}{\sum_{j=0}^n P(B_j/A_i)P'(A_i)}$$

where $P'(A_i)$ is a priori probability of A_i and B_j is the outcome of an experiment.

With this theory, Sexsmith evaluated the ability of a test to differentiate good designs from poor designs for a sequential test using different load levels. Following assumptions defining a poor distribution and a good distribution, Sexsmith concluded that more trusses tested at a lower load level gives more information than a few trusses tested at a high load level. This technique, however, requires that a priori probability $P'(A_i)$ be assumed and as such it is difficult to estimate safety based upon a test of a small number of trusses.

1.8 Objectives

In light of the previous work concerning reliability of truss tests, the objectives of this study were formulated as follows:

- (1) To evaluate the level of safety against collapse that can be expected from a truss test;
- (2) To study the effect of changing test parameters (load, number of samples and test duration) on safety;
- (3) To apply the methods developed in this theoretical study to the decision-making process of truss manufacturing with respect to design procedures;
- (4) To formulate a method to aid code writers in specifying testing procedures.

CHAPTER 2

STRUCTURE RELIABILITY

2.1 Concept of Structural Safety

With the advent of Limit States Design for timber structures, the amount of research on the reliability aspects of timber has escalated since 1976. The basic theory of safety as related to probability of failure has been discussed by Allen [1975], Pierce [1975], MacGreggor [1976], Zahn [1977] and Foschi [1979].

The safety of structures must be referenced to some measure that is independant of material or type of construction. In working stress design, safety is referenced to a "factor of safety" which is a factor applied to a characteristic ultimate resistance of a particular material to arrive at a "safe" working load.

The factor of safety used for steel is different from that used for wood or concrete as a result of the different coefficients of variation of each material. Safety factors are arrived at through experience and past performance. This approach has serious difficulties in that it is difficult to evaluate the safety of a new structural type of application.

Using a probabilistic design philosophy, the safety of a structure is related to a non-dimensional number called "the probability of failure". Using this technique, a target probability

of failure is chosen and designs referenced to this value. The underlying theory, although simple, is not straight forward in its implementation.

When a structure is designed, simplifying assumptions must be made in order for the design to be completed within a reasonable time. Although discrete values are assumed for both loads and resistances, they are in fact a continuum of values with different rates of occurrence. These values can be conveniently represented by statistical distributions and, as such, are capable of mathematical manipulations. Figure 2.1 represents the distributions of loads (U) and resistances (R) for a typical engineering material.

FIGURE 2.1 LOAD AND RESISTANCE DISTRIBUTIONS

The usual definition for the probability of failure is:

$$P_f = P(R < U) \quad (2.1)$$

and can be restated in a number of different formats including:

$$P_f = P(R - U < 0) \quad (2.2)$$

$$P_f = P(R/U < 1.0) \quad (2.3)$$

$$P_f = P(\ln(R/U) < 0) \quad (2.4)$$

Unless the absolute upper bound of the load distribution is less than the lower bound of the resistance distribution, there will always be a finite probability that failure will occur.

From basic statistical theory, the probability of occurrence of a load U between a value x and $x+dx$ is defined as the probability density function

$$p(x < U < x+dx) = f_U(x)dx \quad (2.5)$$

and similarly for resistance

$$p(x < R < x+dx) = f_R(x)dx \quad (2.6)$$

Then, given a load, the probability that the resistance is less than the given load, x , is

$$P(R < x) = \int_{-\infty}^x f_R(v)dv = F_R(x) \quad (2.7)$$

where $F_R(x)$ is defined as the cumulative density function. If the load and resistance functions are assumed independent, then the probability of them both occurring for each element dx is:

$$P(R < U) = \int_U f_U(x) dx F_R(x) \quad (2.8)$$

Integrating over all values of U :

$$P(R < U) = \int_{-\infty}^{\infty} f_U(x) F_R(x) dx \quad (2.9)$$

$$P(R < U) = \int_{-\infty}^{\infty} \int_{-\infty}^x f_U(x) f_R(v) dx dv \quad (2.10)$$

This general formulation can be applied to any distributional assumptions for R and U .

The effect on probability of failure caused by different distributional assumptions for R and U has been discussed by Pierce (1975). Comparisons were made for:

- (1) R and U both assumed normal
- (2) R and U both assumed log normal
- (3) U assumed normal and R assumed 2 parameter Weibull.

It was demonstrated that using different distributional assumptions had a marked effect on probability of failure.

The present limit states approach is distribution free

in that no explicit distribution is assumed for either load or resistance. The reliability is referenced to a "safety index" which is a standard normal $N(0,1)$ variable usually referred to as " β ". Once β is calculated, the probability of failure can be found using normal probability tables.

The factor " β " can be described as the number of standard deviations that the mean of $\ln(R/U)$ is from zero as shown in Figure 2.2.

FIGURE 2.2 DEFINITION OF SAFETY INDEX

The derivation of the safety index, β , has been shown by Allen [1975], MacGreggor [1976] and Foschi [1979]. The solution for R and U both assumed log-normal is:

$$\beta = \frac{\ln(\bar{R}/\bar{U}) - \frac{\ln(1+\gamma_R^2)/2 + \ln(1+\gamma_U^2)/2}{(\ln(1-\gamma_R^2) + \ln(1+\gamma_U^2))^{1/2}}}{1} \quad (2.11)$$

where γ_R is coefficient of variation of resistance

γ_U is coefficient of variation of load

and the first-order distribution-free approximation used for concrete and steel is

$$\beta^* = \frac{\ln(\bar{R}/\bar{U})}{(\gamma_R^2 + \gamma_U^2)^{1/2}} \quad (2.12)$$

The safety index, β , can be transformed into a safety checking equation of the form

$$\bar{U}\lambda = \phi \bar{R} \quad (2.12)$$

where $\phi = \exp(-0.75\beta^*\gamma_R)$

$\lambda = \exp(0.75\beta^*\gamma_U)$

as described by MacGreggor [1975].

During the formulation of the safety index, β , the assumption was made that loads and resistances are independent. Foschi [1979] states that this is not a good assumption in the case of wood because of the well known duration of load effect. He says:

"It is apparent, therefore, that the short-term strength distribution cannot be used to study the reliability of wood structures because of load-duration effects on strength. Furthermore, the distribution of strength τ_r required to compute a reliability measure β after N cycles, cannot be assumed to be equal to the distribution of short-term strength divided by a 'load-duration factor' independent of load history."

To account for duration of load in a rational manner, Foschi proposed that an accumulation of damage model relating damage to load history be used. This model is discussed in more detail in Chapter 3.

2.2 Differential Reliability

Although the statistical analysis of a structure may be straightforward as shown in section 2.1, the implementation of the theory to a real structure is more complicated. This results from inaccuracies made in the analysis techniques used to relate externally applied loads to internal stress resultants, and in the assumptions made in order to formulate limiting criteria. With these assumptions in mind, Suddarth et al (1978) presented a technique of interpreting failure probabilities called "differential reliability".

The method is described as follows:

"When studies of two or more cases are made, the contrast between the probabilities of failure for these cases allows strong analytical focus on the case differences. This strong analytical advantage occurs because all the assumptions involved in one case can also be carried through for the others in a completely formal way."

Where it is difficult to evaluate actual failure probabilities, it is possible to calculate the safety associated with a structure using simplifying assumptions, and compare this with the safety associated with a "standard" structure calculated using the same assumptions. What is of significance is the difference in safety levels and not the absolute values.

Using this technique, Suddarth [1978] investigated the effect of coefficient of variation and concomitance between bending and axial strength on the reliability of dimension lumber loaded in combined tension and bending. The interaction formula used was

$$\frac{f_b}{B} + \frac{f_t}{T} = 1 \quad (2.14)$$

where B - Bending strength of lumber (Random Variable)

T - Tensile strength of lumber (Random Variable)

f_b - Bending stress at extreme fibre

f_t - Average tensile stress.

The applied stresses, f_b and f_t , can also be random variables if either the load or geometric properties are taken as random variables. Failure was assumed to occur for $I > 1.0$.

Because of the difficulty involved in formulating an analytical solution for equation 2.14, a Monte-Carlo simulation technique was used. A deterministic load was applied to a typical simply supported beam and f_b and f_t were calculated using standard analysis techniques. The bending strength and axial strength were then simulated using a technique discussed by Suddarth (1978) where B and T were related to the modulus of elasticity.

It was found that, higher variability and higher degrees of concomitance between B and T resulted in larger probabilities of failure or lower reliability.

2.3 System Reliability

From a statistical point of view, systems can be divided into two main types: a parallel system and a series or weakest link system. A parallel system implies that failure of any one member will not necessarily cause failure of the whole system. A highly redundant structural system with many load paths can be viewed as a parallel system. A series system implies that failure of any component will automatically cause failure of the entire system. An example of a series or weakest link system is a statically determinate structure.

Although timber trusses are not strictly determinate structures, since top and bottom chords are usually continuous over joints, they are usually considered to be of the weakest link type. This is because most test criteria state that a truss must not show signs of distress in order to pass. In other words, if any member fails the truss is considered to have failed the test.

If the probability of survival is known for each one of the components of a weakest link structure, the probability of survival of the system can be calculated using equation 2.15, assuming all members are independent.

$$Q = \prod_{i=1}^n Q_{mi} \quad (2.15)$$

where Q = probability of survival of the system

Q_{mi} = probability of survival of component i

By way of example, consider a chain consisting of n links loaded to the median strength of the link population. The probability of survival of the chain is then 0.5^n . Figure 2.3 shows the relationship between the number of links and the probability of survival of the chain.

FIGURE 2.3 PROBABILITY OF FAILURE FOR A WEAKEST LINK STRUCTURE

Using the weakest link approach, the probability of survival of a truss can be calculated knowing the probability of survival of each component. This reliability measure can then be compared to the reliability of a standard structure and this difference in reliability can be evaluated.

CHAPTER 3

DAMAGE EFFECT3.1 Duration of Load

It has been known for many years that wood is stronger when subjected to short-term loads than when it is subjected to long-term loads. This phenomenon was originally quantified by Wood [1951] by fitting, to an empirical curve, the median times to fracture of small clear specimens loaded to various levels.

Allowable design stresses are referenced to a normal duration of load lasting 10 years for which the modification factor is 1.0. All other load durations are modified in relation to this 10-year loading. For example, CSA-086-1977 suggests that the duration of load factors, shown in Table 3.1, be used for design purposes.

TABLE 3.1

<u>Loading</u>	<u>Load-Duration Modification Factor</u>
Impact	2.0
1 Day (WIND)	1.33
7 Days (FORM WORK)	1.25
2 Months (SNOW)	1.15
Normal (FLOOR LOADS)	1.0
Continuous	0.90

This method has proven to be satisfactory; however, there are some problems associated with it.

Wood's curve was fitted to data obtained using small clear specimens. Recent work by Madsen [1975] has shown that properties of small, clear specimens may not be valid when extrapolated to full-sized lumber properties.

Another difficulty is the lack of a rational method for establishing the cumulative time under load. It may be difficult to prove that the actual cumulative time under snow load, for example, is in fact two months.

A final criticism is that the shape of the loading function cannot be directly used in order to calculate the time to fracture. The only type of loading function considered here is a constant load, making it impossible to evaluate the effect on strength caused by the actual load history.

3.2 Damage Model

Based on work originally proposed by Bolotin [1969], Barrett and Foschi [1978] developed two models for the creep rupture phenomenon observed in wood. These models are in the form of damage accumulation functions. Model I considered the damage rate to be a function of a stress dependant factor and a damage dependant factor as given in equation 3.1.

$$\sigma \geq \sigma_0 \quad \frac{d\alpha}{dt} = a(\sigma - \sigma_0)^b \alpha^c \quad (3.1)$$

$$\sigma < \sigma_0 \quad \frac{d\alpha}{dt} = 0$$

where σ - stress ratio referenced to a characteristic strength

σ_0 - threshold stress ratio

a, b, c , - material constants

α - damage

Damage is defined so that when $\alpha=0$ no damage has occurred and $\alpha=1.0$ corresponds to collapse.

Integrating equation 3.1, the time to fracture under a given stress history, $\alpha(t)$, is given by

$$a(1-c) \int_0^T (\sigma(\tau) - \sigma_0)^b dt = 1 \quad (3.2)$$

Model II considers a damage model where the rate of change of damage is a function of a stress dependent term and a damage dependent term as follows:

$$\sigma > \sigma_0 \quad \frac{d\alpha}{dt} = a(\sigma - \sigma_0)^b + \lambda \alpha \quad (3.3)$$

$$\sigma < \sigma_0 \quad \frac{d\alpha}{dt} = 0$$

The solution of the differential equation (3.3) for an arbitrary stress history, $\sigma(t)$, at time T is given by:

$$\sigma(T) = e^{\lambda T} \int_0^T a(\sigma(\tau) - \sigma_0) e^{-\lambda \tau} d\tau \quad (3.4)$$

Barrett and Foschi [1978 II] generalized Model II so that variability in time to fracture could be investigated. The damage rate was assumed to be a function of the applied stress and current damage and to vary randomly according to

$$\tau > \tau_0 \quad \frac{d\alpha}{dt} = a e^{\omega_1 R_N} \frac{(\tau - \tau_0)^{b + \lambda e^{\omega_2 V_N}}}{\tau_S} \alpha \quad (3.5)$$

$$\tau < \tau_0 \quad \frac{d\alpha}{dt} = 0$$

- where τ_S - short-term strength
 τ - applied stress
 τ_0 - threshold stress
 ω_1 and ω_2 - constants defining the variation in damage rate
 R_N and V_N - standard normal $N(0,1)$ variates
 a, b, λ - material constants

The variation constants, ω_1 and ω_2 , are related to the coefficients of variation through equation 3.6 when a log-normal distribution of times to fracture is assumed.

$$\omega_i = (\ln(\gamma^2 + 1))^{1/2} \quad (3.6)$$

where γ - coefficient of variation.

The variation in damage rate caused by the applied stress is associated with ω_1 and the variation in damage rate caused by the current damage level is controlled by ω_2 .

Equation 3.5 can be integrated to obtain the damage at any time, T , caused by a stress history $\tau(t)$ giving

$$\alpha(T) = a e^{\omega_1 R_N} \int_{t_0}^T (\tau(t) - \tau_0)^b \exp[\lambda e^{\omega_2 V_N} (T-t)] dt \quad (3.7)$$

Equation 3.7 can be evaluated for different loading histories and the time to fracture calculated by setting $\alpha(T)=1.0$. For the case of a constant load history, the time to fracture, T_c , is given by:

$$T_c = \frac{1}{\lambda e^{\omega_2 V_N}} \ln \left[1 + \frac{\lambda e^{\omega_2 V_N}}{a e^{\omega_1 R_N} (\sigma - \sigma_0)^b} \right] \quad (3.8)$$

where σ and σ_0 are stress ratios referenced to the short-term strengths.

For the case of a ramp load, the time to fracture, T_r , can be evaluated by solving equation 3.9.

$$ae^{\omega_1 R_{T_r}} \frac{(\tau_r - \tau_0)^{b+1}}{2\tau_r \tau_s^b} \int_{-1}^{+1} \frac{(1+\xi)^b}{2} \exp \lambda e^{\omega_2 V_{N_{T_r}} (\tau_r - \tau_0)} (1+\xi) d\xi = 1 \quad (3.9)$$

$$\text{where } \tau_r = K_r T_r$$

In the special case when the ramp loading rate, K_r , is chosen to be equal to the rate used to define short-term member strength, equation 3.9 can be re-written as

$$ae^{\omega_1 R_{T_s}} \frac{(1-\sigma_0)^{b+1}}{2} \int_{-1}^{+1} \frac{1+\xi}{2} \exp \lambda e^{\omega_2 V_{N_{T_s}} (1-\sigma_0)} (1+\xi) d\xi = 1 \quad (3.10)$$

from which the short-term time to fracture, T_s , can be found.

Barrett and Foschi simplified the model by assuming that λ and σ_0 were both equal to zero. The time to fracture under constant load was simplified to equation 3.11.

$$T_c = \frac{1}{ae^{\omega_1 R_{N_\sigma}}} \quad (3.11)$$

Using the simplifying assumptions, the short-term time to fracture, T_s , can be integrated in closed form to obtain

$$T_s = \frac{(b+1)}{ae^{\omega_1 R_N}} \quad (3.12)$$

For $\sigma_0 = 0.2$ in the general case, the material constants, a , b , and λ , were obtained by Barrett and Foschi [1978 I] by fitting the median times to fracture for small, clear Douglas-fir specimens in bending to obtain

$$a = 0.11 \times 10^7 \text{ (h}^{-1}\text{)}$$

$$b = 34.2$$

$$\lambda = 0.79 \times 10^{-3} \text{ (h}^{-1}\text{)}$$

For the simplified case, the material constants, a and b , were evaluated as

$$a = 444.4 \text{ (h}^{-1}\text{)}$$

$$b = 36.0$$

3.3 Combined Loading

The preceding section was based on the distribution of ultimate strengths in bending only. If the reliability of a beam, as shown in Figure 3.1, is to be evaluated, the problem is not so simple.

FIGURE 3.1 TIMBER ELEMENT UNDER COMBINED BENDING AND AXIAL LOAD

In order to determine the probability of failure for the combined loading case, some form of interaction equation must be assumed to relate bending stress and axial stress to strength. As discussed in section 2.2 the accepted method used for designing members under combined loading can be restated in the form

$$\frac{\tau_A}{F_A} + \frac{\tau_B}{F_B} \leq 1.0 \quad (3.13)$$

to be consistent with the notation adopted for this study.

where τ_A - applied axial stress

τ_B - applied bending stress

F_A - allowable axial strength,

F_B - allowable bending strength

Following the procedure suggested by Suddarth [1978],
equation 3.13 can be modified to reflect ultimate strength as

$$\frac{\tau_A}{F_A'} + \frac{\tau_B}{F_B'} \leq I \quad (3.14)$$

where F_A' - axial strength

F_B' - bending strength

The factor, I , is defined, following Freudenthal 1956,
to account for the fact that equation 3.14 is not an exact
representation of real behaviour. For the purposes of this
study it will be assumed that equation 3.14, with $I=1.0$,
reflects the behaviour of a lumber beam under combined tension/
bending and for the case of a short beam under combined compression/
bending. Therefore the probability of failure will be assumed
to be of the form

$$P_f = P \left[\frac{\tau_A}{F_A'} + \frac{\tau_B}{F_B'} \geq 1.0 \right] \quad (3.15)$$

where the axial and bending strength, F_A' and F_B' , must be consistent with loading history.

Following the general damage model presented by Barrett and Foschi 1978, the axial strength and bending strength under constant load can be evaluated. Rearranging equation 3.8 the axial and bending stress ratios can be given as

$$\sigma_A = \sigma_{OA} + \left[\frac{\lambda_A e^{\omega_{2A} V_{NA}}}{a_A e^{\omega_{1A} R_{NA}} (e^{T_c \lambda_A e^{\omega_{1A} V_{NA}}} - 1)} \right]^{1/b_A} \quad (3.16)$$

$$\sigma_B = \sigma_{OB} + \left[\frac{\lambda_B e^{\omega_{2B} V_{NB}}}{a_B e^{\omega_{1B} R_{NB}} (e^{T_c \lambda_B e^{\omega_{1B} V_{NB}}} - 1)} \right]^{1/b_B} \quad (3.17)$$

If the axial and bending stress ratios are defined as

$$\sigma_A = \frac{F_A'}{\tau_{SA}} \quad (3.18)$$

$$\sigma_B = \frac{F_B'}{\tau_{SB}} \quad (3.19)$$

where τ_{SA} - short-term axial strength

τ_{SB} - short-term bending strength

The axial and bending strengths, consistent with a constant load history, can be written as

$$F_A' = F_{SA}' \left[\sigma_{OA} + \frac{\lambda_A e^{\omega_{2A} V_{NA}}}{a_A e^{\omega_{1A} R_{NA}} (e^{T_c \lambda_A e^{\omega_{1A} V_{NA}}} - 1)} \right]^{1/b_A} \quad (3.20)$$

$$F_B' = F_{SB}' \left[\sigma_{OB} + \frac{\lambda_B e^{\omega_{2B} V_{NB}}}{a_B e^{\omega_{1B} R_{NB}} (e^{T_c \lambda_B e^{\omega_{1B} V_{NB}}} - 1)} \right]^{1/b_B} \quad (3.21)$$

The short-term strengths, τ_{SA}' and τ_{SB}' , can be evaluated by solving equation 3.10 for T_{SA} and T_{SB} using the appropriate material constants and substituting into equations 3.22 and 3.23.

$$\tau_{SA}' = K_A T_{SA} \quad (3.22)$$

$$\tau_{SB}' = K_B T_{SB} \quad (3.23)$$

where K_A is the rate of loading used for short-term axial strength test

K_B is the rate of loading used for short-term bending strength test.

Therefore, in order to evaluate the interaction at any time, T_c , under a constant load, the steps shown in Figure 3.2 must be followed:

FIGURE 3.2 FLOW CHART TO DETERMINE COMBINED INTERACTION

In the general case, the probability of survival for any member can be calculated using a Monte-Carlo simulation procedure choosing values for R_{NA} , R_{NB} , V_{NA} and V_{NB} for each trial. After a large number of iterations, the number of times that the result of equation 3.10 is less than 1.0, divided by the total number of trials, will approximate the probability of survival of the member.

Using the simplified model proposed by Barrett and Foschi [1978 II] and assuming that the material constants, a and b , are equal for the bending and axial case, a simple solution results. The times to fracture under constant load for axial and bending stresses are given by

$$T_{CA} = \frac{1}{ae^{\omega_1 A R_{NA}} \sigma_A^b} \quad (3.24)$$

$$T_{CB} = 1 \frac{1}{ae^{\omega_1 B R_{NB}} \sigma_B^b} \quad (3.25)$$

Using equations 3.17 and 3.18, the strength at time, T_c , under constant load can be written as

$$F_A = \tau_{AS} [ae^{\omega_1 A R_{NA} T_c}]^{-1/b} \quad (3.26)$$

$$F_B = \tau_{BS} [ae^{\omega_1 B R_{NB} T_c}]^{-1/b} \quad (3.27)$$

and the short-term strengths, τ_{AS} and τ_{BS} , can be written as

$$\tau_{AS} = \tilde{\tau}_{AS} e^{-R_{NA}\omega_{1A}} \quad (3.28)$$

$$\tau_{BS} = \tilde{\tau}_{BS} e^{-R_{NB}\omega_{1B}} \quad (3.29)$$

where $\tilde{\tau}_{AS}$ - median short-term axial strength

$\tilde{\tau}_{BS}$ - median short-term bending strength.

Equation 3.10 can now be written as

$$\frac{\tau_A}{\tilde{\tau}_{SA} e^{-R_{NA}\omega_{1A}} [ae^{\omega_{1A}R_{NA}T_c}]^{-1/b}} + \frac{\tau_B}{\tilde{\tau}_{SB} e^{-\omega_{1B}R_{NB}} [ae^{\omega_{1B}R_{NB}T_c}]^{-1/b}} = 1 \quad (3.30)$$

If it is further assumed that the variability associated with bending strength and axial strength are equal (i.e. $\omega_{1A} = \omega_{1B}$, and perfectly co-related (i.e. $R_{NA} = R_{NB} = R_N$), then R_N can be evaluated as

$$R_N = \frac{\ln\left(\frac{\tau_A}{\tilde{\tau}_{SA}}\right) + \frac{1}{b} \ln(aT_c)}{-\omega_1(1 + 1/b)} \quad (3.31)$$

Using R_N calculated from equation 3.31, the probability of survival, Q_m , can be found directly using normal probability tables.

3.4 Concomitance

The concomitance, or confunctioning, of F_a and F_b are not well understood as discussed by Suddarth [1978]. Concomitance is a measure of the level of co-relation of the residuals between F_A and F_B . Figure 3.3 shows two relationships between F_A and F_B .

FIGURE 3.3 CONCOMITANT RELATIONSHIPS BETWEEN AXIAL AND BENDING STRENGTH

If a linear fit is made through the data plotted in Figure 3.3(a), the co-relation coefficient, which is a measure of the dispersion of data about the fitted line, would be very near 1.0 indicating high co-relation. This means that, once the bending strength is known, then, the axial strength can be calculated using the linear fit of the two variables (i.e. $R_{NA} = f(R_{NB})$).

On the other hand, however, if the relationship between F_A and F_B is as shown in Figure 3.3(b) then there is no obvious functional relationship between the two variables. Bending strength and axial strength are said to be independant (i.e. $R_{NA} \neq f(R_{NB})$).

In this study it has been assumed that bending and axial strength are perfectly co-related (i.e. $R_{NA} = R_{NB}$).

Chapter 4

Evaluating Truss Testing

4.1 Acceptance Sampling

The question that must be addressed when evaluating any test is what effect has the passing of an arbitrary test on the structure's ability to perform safely during its service life. Stated in statistical format: What is the probability that a design will be accepted given its probability of failure during its service life?

When certifying roof trusses in lots of considerable size, the responsible agency would like all items to be satisfactory, but a more realistic situation would be to expect some defective units. One possible method of inspection would be to test every truss and classify it as defective or satisfactory. One hundred percent inspection methods are prohibitively expensive or impossible to carry out if destructive testing is involved. Furthermore, mechanical testing can introduce damage even though failure has not occurred and, therefore, a more realistic sampling plan must be used. Such a sampling plan must consider the question: What is the number of defective items in a sample of given size which will cause acceptance or rejection of the lot?

Because of the binary nature of the outcome (pass-fail), the binomial expansion forms the basis for the sampling plan. When binomial cumulative probabilities are plotted in a graph, they take on special significance in their relationship to sampling

plans. As discussed by Rickmers et al [1967] for a given size of sample, the vertical axis measures the probability of accepting a lot of quality described by the value found on the horizontal axis. This curve, known as the Operating Characteristic (OC) curve, is used to judge the worth of sampling plans.

4.2 Risk Levels

Consumer's Risk - The public should be protected from accepting a poor design characterized by a large number of substandard items. A Rejection Quality Level (RQL) must be established in order to define the lower limit of quality which will be accepted with a given probability.

Producer's Risk - On occasion some satisfactory designs will be rejected on the basis of a sample because of chance. Thus, the sampling plan must also recognize and maximize the chances of accepting high quality lots. This limit is known as the Acceptance Quality Level (AQL).

4.3 Formulation of the Operational Characteristic Equation

The result of a test is either survival (S) or failure (F) and therefore the outcome is a binomial experiment since:

- (1) The experiment consists of n identical trials.
- (2) Each trial results in one of two outcomes (S-F).
- (3) The probability of success on a single trial is equal to P and remains the same from trial to trial.

- (4) The trials are independant.
- (5) Interest is in y , the number of successes or failures observed in n trials.

The binomial probability density function is given by

$$P(y) = \frac{n!}{y!(n-y)!} p^y Q^{n-y} \quad (4.1)$$

y = number of successes

n = number of tests

P = probability of failure

$Q = 1-P$

Chapter 1 discussed the reasons and techniques used for truss testing. The acceptable performance criteria as outlined in CSA S307-1977 for example can be summarized as follows:

- (1) The design is accepted if two trusses meet the requirements for deflection and strength.
- (2) The design is not acceptable if both trusses fail to meet the requirements. If only one of the two trusses meet the requirements, a retest of two more trusses is permitted. If either or both of these fail to meet the requirements, the design is not acceptable.

These performance criteria can be restated in a probabilistic format using equation 4.2.

$$P_a = \frac{2!}{0!(2-0)!} P_f^0 Q^2 + \frac{2!}{1!(2-1)!} P_f Q \left(\frac{2!}{0!(2-0)!} P_f^0 Q^2 \right) \quad (4.2)$$

where P_a = probability of acceptance

Q = probability of survival of one truss

P_f = probability of failure of one truss.

Which simplifies to

$$P_a = Q^2 + 2P_f Q^3 \quad (4.3)$$

Equation 4.3 is plotted in figure 4.1 and is completely independent of the distributional forms of the load and resistance.

FIGURE 4.1 OPERATING CHARACTERISTIC CURVE

As the test stands, and assuming that the producer is willing to accept a Type II error of 10% and the consumer is willing to accept a Type I error of 10%, the acceptance quality level (AQL) is 15% and the rejection quality level (RQL) is 72%. This means that, through the implementation of this test, the consumer is willing to accept trusses having a probability of failure at the test load of 72% one out of ten times. Furthermore, the producer is willing to have 10% of his truss designs rejected, even though the probability of failure is only 15%.

As can be seen in Figure 4.1, the test cannot easily differentiate between good and poor designs. This is common for this type of test since the only way to improve its "strength" is to increase the number of trusses tested which is very expensive. If the load level is increased, the overall safety increases; however, the "strength" of the test is not affected.

Although useful, the "OC" curve, as is, tells us very little about the actual reliability of the test. To evaluate the safety of a test we must be able to determine the probability of acceptance given the probability of failure at the end of its design life.

4.4 Test Safety

FIGURE 4.2 RELATIONSHIP BETWEEN SAFETY AND VARIABILITY

Consider Figures 4.2(a) and (b) and assume the cross-hatched areas to the left of the test load are equal. This means that the probability of failure at the test load for case (a) and case (b) are equal which in turn, through equation 4.3, means that they have equal probability of acceptance. The safety of the design, however, is related to the area to the left of the design load and, as can be seen, case (a) is safer than case (b).

If both load and resistance are assumed to be log-normally distributed, the difference in safety can be quantified. The intent of this section is to describe the technique and, the assumptions made in this section have no bearing on the inferences made in succeeding chapters.

The standardized $N(0,1)$ variable for the log-normal distribution defined at the test load, β_t , and the design load, β_d , are given by

$$\beta_t = \frac{\ln U_t + \ln R}{S_{\ln R}} \quad (4.4)$$

$$\beta_d = \frac{\ln U_d + \ln R}{S_{\ln R}} \quad (4.5)$$

where U_t = test load

U_d = design load

R = median resistance

$S_{\ln R}$ = standard deviation of $\ln R = [\ln(\gamma^2 + 1)]^{1/2}$

Since the ratio of test load to design load is a constant, c , we can write

$$c = \frac{U_t}{U_D} = \frac{\exp(\beta_t S_{\ln R})}{\exp(\beta_d S_{\ln R})} \quad (4.6)$$

which can be rearranged to define a relationship between β_t and β_d

$$\beta_t = \beta_d - \frac{\ln(c)}{S_{\ln R}} \quad (4.7)$$

Using equation 4.7 for known values of c it is possible to calculate β_t as a function of β_d and the coefficient of variation of resistance. The probability of failure of a truss at test load P_f , can be obtained from standard normal tables using the value of β_t defined above. In turn, the probability of acceptance of the design can be calculated using equation (4.3). Figure 4.3 shows the OC curves for the Canadian roof truss test at different coefficients of variation, γ , of resistance.

There is an implicit assumption here in that the curves shown in Figures 4.3 are for materials that do not exhibit a duration of load effect; nevertheless, it shows some interesting results.

The most important result of Figure 4.3 is that safety significantly increases as the coefficient of variation decreases.

FIGURE 4.3 OC CURVES DEFINED BY EQUATION 4

$$P_a = Q^2 + 2P_f Q^3$$

FIGURE 4.4 EFFECT ON RELIABILITY CAUSED BY VARYING LOAD RATIO

$$P_a = Q^2 + 2PQ^3$$

FIGURE 4.5 EFFECT ON RELIABILITY CAUSED BY VARYING NUMBER OF TESTS

Figure 4.4 a, b, c and d shows the results of varying the load ratio for the test defined by equation (4.3):

As can be seen in Figures 4.4, changing the load ratio has the effect of shifting the curves along the x axis and has no effect on the slope. Physically, this means that the ability of the test to discriminate designs is not changed but the level of reliability of the test is increased or decreased.

Figure 4.5 shows a different result. By changing the number of trusses tested and keeping the load ratio constant, both the "strength" of the test and the level of reliability can be altered.

4.5 Duration of Load

Floor trusses, for example, are usually designed for "normal" loading with duration of 10 years. The test, on the other hand, is performed over a 24-hour period which would result in an apparent higher strength. Therefore, in order to obtain a measure of the reliability of a truss design, we must formulate its probability of failure to be compatible with design conditions.

Following the technique outlined in section 3.5 the probability of survival, or reliability of a structural system, can be calculated using the general approach and a Monte-Carlo simulation procedure.

Using the simplified procedure, the probability of survival for the constant load history case can be found using equation 3.31

if it is assumed that the damage incurred during testing is negligible. Once the probability of survival has been found, the probability of acceptance can be calculated using the characteristic equation for the test.

If the design life load history is of the constant load type, equation 3.31 can again be used to calculate the safety index, R_N , and a plot of reliability versus probability of acceptance can be made. Therefore, it is now possible to evaluate the probability of accepting a truss design given a measure of its reliability.

It should be noted here, that although this derivation was performed for a constant load history, it is possible to evaluate the safety of a truss test for any loading history by integrating equation 3.7 using the appropriate material constants.

CHAPTER 5

APPLICATIONS5.1 Example

In order to illustrate the test evaluation technique, consider the truss shown in Figure 5.1. The structure is a simply supported floor joist fabricated with metal or wood webs and dimension lumber on the top and bottom chords.

FIGURE 5.1 EXAMPLE STRUCTURE

The design load for the truss is subdivided into the ratios

- 4 - Top chord live load
- 1 - Top chord dead load
- 1 - Bottom chord dead load

The material properties for the truss components were assumed to be those shown in Table 5.1.

TABLE 5.1

Component	Median Tensile Strength (MPa)	Median Compressive Strength (MPa)	Median Bending Strength (MPa)	Area (mm) ²	Moment of inertia (mm) ⁴
Top Chord	18.4	20.0	32.0	3390.0	0.41×10^6
Bottom Chord	18.4	20.0	32.0	3390.0	0.41×10^6
Webs	8.5	7.1	-	1570.0	-

The values shown in Table 5.1 were derived assuming 38mm x 89 mm (2" x 4") SPF #1 lumber on the top and bottom chords and typical values for web strength. Webs were assumed to be pin connected.

5.2 Certification Test

For floor trusses, a test recently proposed for use in Canada, follows the guidelines set out by the "Central Mortgage and Housing Corporation" (1980). The strength portion of the test requires that trusses be tested to two times the design value for 24 hours, with the acceptance criteria outlined as follows:

For trusses tested singly:

- (a) Three trusses are tested and if all trusses survive, the design is accepted.
- (b) If one truss fails, one retest is permitted. If the retest survives, then the design is deemed acceptable.

For trusses tested in pairs:

- (a) Three pairs are tested and, if all pairs survive, the design is acceptable.
- (b) If one truss pair fails, one retest is permitted. If the retest survives, then the design is deemed acceptable.

Using the techniques outlined in Chapter 4, the characteristic equation can be formulated for the case where trusses are tested singly as:

$$P_a = Q^3 + 3 P_f Q^3 \quad (5.1)$$

where P_a = probability of acceptance

Q = probability of survival of a single truss

P_f = probability of failure of a single truss

5.2 OC CURVES DEFINING FLOOR TRUSS TEST

The characteristic equation for the case where trusses are tested in pairs is given by equation 5.2.

$$P_a = Q^6 + 6 P_f Q^7 + 3 P_f^2 Q^6 \quad (5.2)$$

Figure 5.2 shows the plots of equations 5.1 and 5.2. By examining Figure 5.2 it can be seen that, when trusses are tested singly, the test is not as "strong" as when they are tested in pairs. That is to say, it is more difficult for single tests to discriminate poor designs from good designs as compared to testing truss pairs. This is intuitively reasonable since more single trusses are evaluated when they are tested in pairs.

5.3 Probability of Acceptance Based on Component Data

In order to calculate the probability of acceptance, one must know the probability of survival of a single truss loaded to the test load for the duration of the test.

If it is assumed that:

- (1) any loadings applied previous to the strength test cause negligible damage as compared to the strength test,
- (2) the coefficients of variation of all components are equal,
- (3) the structure is of the weakest link type,
- (4) webs are pin connected,

it is possible to calculate the interaction as defined by equation [3.13] for each member using a standard structural analysis package such as "Purdue Plane Structures Analyser PPSA ". The characteristic strength used for equation 3.14 was the median short-term strength corresponding to the applied stress resultant. Example calculations are shown in Appendix A1.

Using the safety index, R_N , calculated using equation 3.31, the probability of survival of each element was calculated using normal probability tables and the probability of survival of the entire structure was calculated using equation 2.15. Once the probability of one truss surviving the test was known, the probability of acceptance was found using the appropriate characteristic equation.

5.4 Probability of Surviving the Design Life

The actual load history on a floor truss is very difficult to predict. It is, however, possible to predict approximate distributions of loads that a given truss design is expected to withstand. Using the distribution of loads, random load histories can be generated using a Monte-Carlo simulation. These random load histories can then be applied to randomly generated structures and, using the techniques discussed in Section 3.5, the probability of survival can be calculated.

For the purpose of comparing truss tests, the actual load distributions are not necessary. It is possible to compare tests even if the design load history is considered to be deterministic. To this end, it was assumed that the truss was to withstand the

FIGURE 5.3 RELIABILITY ASSOCIATED WITH TRUSSES TESTED SINGLY

FIGURE 5.4 RELIABILITY ASSOCIATED WITH TESTING TRUSSES IN PAIRS

full design load for a period of 10 years which is the load history assumed for design purposes. Figures 5.3 and 5.4 show the relationship between the probability of a truss surviving 10 years and the probability of the truss design being accepted for trusses tested singly and in pairs respectively.

As was expected, testing trusses in pairs is more reliable than testing single trusses if the applied load is the same. The difference in safety is affected to a large extent by the variability associated with the material. For a coefficient of variation in the order of 10% the reliability of the test is not altered much by testing pairs or single trusses, however, as the coefficient of variation increases, the effect on the relative reliability increases.

5.5 Sensitivity Analysis

Using the evaluation procedure, it is possible to change the parameters controlling the test in order to observe their effect on reliability. The parameters that are controllable by the testing agency are: (a) duration of test load, (b) number of tests performed and (c) the load ratio applied to the design load to arrive at a test load.

5.5.1 Load Duration

Figures 5.4, 5.5, 5.6 and 5.7 correspond to load durations of 1, 12, 24 and 48 hours respectively for four trusses tested singly. It was assumed that, in order for the design to be accepted, four trusses

FIGURE 5.5 RELIABILITY ASSOCIATED WITH 1 HOUR TEST

$$P_a = Q^4$$

FIGURE 5.6 RELIABILITY ASSOCIATED WITH 12 HOUR TEST

$$P_a = Q^4$$

FIGURE 5.7 RELIABILITY ASSOCIATED WITH 24 HOUR TEST

$$P_a = Q^4$$

FIGURE 5.8 RELIABILITY ASSOCIATED WITH 48 HOUR TEST

$$P_a = Q^4$$

must pass the test with no retest allowed. The characteristic equation for the test is given by equation 5.3.

$$P_a = Q^4 \quad (5.3)$$

Comparing Figures 5.5, 5.6, 5.7 and 5.8 it can be seen that a decrease in the test duration implies a decrease in test reliability.

It should be noted, however, that the duration of load effect is based on data obtained from small, clear specimens and, the results presented are not felt to be representative of state-of-the-art knowledge. Even though the results are not felt to accurately represent real behaviour, they are indicative of the trend associated with changing the duration of test load.

In order to obtain a better picture of the duration of load effect, data must be gathered for both the dimension lumber under axial and bending loads, and webs under axial compression and tension. The testing should include both short-term ramp tests as well as tests under constant loads of varying intensities. Using this data, the material parameters, a , b , λ , and σ_0 , of equation 3.7 can be evaluated for the webs and the dimension lumber following the method outlined by Barrett and Foschi [1978 I]. Once the constants defining the damage model for the webs and for the lumber have been calculated, a more accurate representation of the duration of load effect would be possible.

5.5.2 Number of Tests

The second parameter controlled by the test designer is the number of trusses required to be tested. It is, however, not enough to specify the number of trusses to be tested; the number required to pass the test must also be known. The effect on reliability caused by the number of trusses tested is governed by the characteristic equation.

There are a large number of variations possible for the characteristic equation but, in order to illustrate the sensitivity of changes in the characteristic equation, only four cases will be studied.

- (1) 1 truss tested - 1 required to survive, $P_a = Q$
- (2) 2 trusses tested - 2 required to survive, $P_a = Q^2$
- (3) 3 trusses tested - 3 required to survive, $P_a = Q^3$
- (4) 4 trusses tested - 4 required to survive, $P_a = Q^4$

The trusses were assumed to be loaded to two times the design load for 24 hours.

By comparing Figures 5.9, 5.10, 5.11 and 5.12, the effect on reliability caused by varying the characteristic equation can be evaluated. As expected, increasing the number of trusses tested increases the reliability associated with the test.

Allowing a retest also has a marked effect on the reliability associated with a test. By comparing Figure 5.11 to Figure 5.3 the effect of allowing a retest of one truss in the event of the failure of one of

FIGURE 5.9 RELIABILITY ASSOCIATED WITH TESTING 1 TRUSS

$$p_a = Q^4$$

FIGURE 5.10 RELIABILITY ASSOCIATED WITH TESTING 2 TRUSSES

$$P_a = Q^4$$

FIGURE 5.11 RELIABILITY ASSOCIATED WITH TESTING 3 TRUSSES

$$P_a = Q^4$$

FIGURE 5.12 RELIABILITY ASSOCIATED WITH TESTING 4 TRUSSES

$$P_a = Q^4$$

FIGURE 5.13 TEST RELIABILITY FOR LOAD RATIO OF 1.5

$$P_a = Q^4$$

FIGURE 5.14 TEST RELIABILITY FOR LOAD RATIO OF 2.0

$$P_a = Q^4$$

FIGURE 5.15 TEST RELIABILITY FOR LOAD RATIO OF 2.5

$$P_a = Q^4$$

FIGURE 5.16 TEST RELIABILITY FOR LOAD RATIO OF 3.0

$$P_a = Q^4$$

the original three tests can be examined. The comparison shows a marked effect on reliability with lower safety associated with a test allowing a retest.

5.5.3 Load Ratio

The effect on reliability caused by changing the load ratio, c , can be investigated by holding the load duration and characteristic equation associated with the test constant. This was done for a load duration of 24 hours and a characteristic equation given by equation 5.3. The sensitivity to load ratio was investigated for $c = 1.5, 2.0, 2.5$ and 3.0 with the resulting reliability curves plotted in Figures 5.13, 5.14, 5.15 and 5.16 respectively. It can be seen that there is a marked effect on reliability caused by varying the load ratios with higher safety associated with high load ratio.

5.6 Application To Decision Making

In order for a plate manufacturer to decide on what "safety factor" to apply to the web, plate grip values, the effect on the probability of acceptance of his designs must be known. The design grip value used for the webs was the median strength divided by a constant called a "safety factor", n , as shown in Figure 5.16. In this case the term "safety factor" reflects the more traditional concepts used before the Limit States Design formulation was adopted.

FIGURE 5.17 DEFINITION OF "SAFETY FACTOR", n

It was assumed that the median strengths were evaluated by tests of representative web specimens under short-term ramp loading. The truss test, however, is performed by ramp loading full trusses to c times its design load, and leaving the load on the trusses for a period of time.

At the present time, grip values for webs are not evaluated from tests on web specimens but are extrapolated from plate test data obtained from samples as recommended in CSA S347 - [1976]. This is not felt to be representative for two reasons. First, a single wood web is in fact a weakest link structure since a grip failure at either end of the web would cause failure of the web system. The median value for strength associated with this type of system would not be the median strength associated with single plate tests, but the median value associated with the distribution of minimums of sample size equals 2, taken from plate tests. This is to say that if strength values are predicted by single plate values, the median web strength will be over estimated.

The second point concerns joint eccentricity. When plate grip values are evaluated, the test specimens are usually loaded so that the resultant passes through the centroid of the tooth pattern. In many cases, however, plates on webs are loaded eccentrically, causing lower strength values than those predicted through plate tests.

Using equation 3.31 and assuming:

- (1) webs and lumber have the same variability,
- (2) damage occurs at the same rate as that for small, clear specimens,

it is possible to evaluate the probability of survival given the "safety factor" applied to the web grip value. Figure 5.18 shows

the effect of "safety factor" on the probability of survival for the example truss, assuming a weakest link structure. The load ratio, c , was assumed to be 2.0 with a duration of 24 hours. Figures 5.19 and 5.20 show the relationship between "safety factor" and probability of acceptance using characteristic equations 5.1 and 5.2 respectively.

If a producer wants to have 90% of his trusses accepted, he would have to apply a "factor of safety" of 3.0 for trusses tested singly with a coefficient of variation of 10%. For trusses tested in pairs, to obtain the same probability of acceptance, a "safety factor" of about 3.1 would be required. This seemingly paradoxical result arises because floor trusses are designed to pass the test rather than to survive the service load. Whether the test represents service load conditions is a different matter.

It is interesting to note that if a truss is designed so that the critical web (1st compression web in this case) is loaded to its median short-term strength ($n_{crit} = 2.0$) during the test, the probability of survival and hence the probability of acceptance of a truss design is nearly zero and not 0.5 as may be expected. This low survival rate can be explained by the fact that, in order for the truss to survive, all of the webs must survive. The probability of survival of the truss is then the product of a series of numbers smaller than one. To illustrate this further, consider the case where only the critical web can fail; all other members are grossly overdesigned. Since two webs can fail (one at each support), the

22
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FIGURE 5.18 PROBABILITY OF SURVIVAL VERSES FACTOR OF SAFETY FOR FLOOR TRUSS TEST

FIGURE 5.19 PROBABILITY OF ACCEPTANCE VERSES FACTOR OF SAFETY
FOR TRUSSES TESTED SINGLY

FIGURE 5.20 PROBABILITY OF ACCEPTANCE VERSES FACTOR OF SAFETY FOR TRUSSES TESTED IN PARIS

probability of survival of the truss in this case would be:

$$Q = (0.5)^2 = 0.25$$

This does not consider the duration of load effect nor the possibility of failure in the other members, yet the probability of survival is less than 0.50.

5.7 Code Calibration

If a code writing agency modifies a testing procedure, the new testing procedure must be at least as reliable as the existing procedure. Using the technique discussed in Chapter 4, it is possible to evaluate the reliability of a new test and compare it to the reliability of the existing test. The technique used to compare tests, called differential reliability, was discussed in Section 2.2.

Using this technique, it is possible to compare single as opposed to paired testing. The characteristic equation for the tests are given by equations 5.1 and 5.2, with the reliability associated with each test shown in Figures 5.3 and 5.4.

If it is required that trusses tested singly be at least as safe as the paired tests, either the load ratio, or the duration of test can be adjusted for calibration purposes. Figures 5.21 shows the results of adjusting the load ratio to achieve comparable reliability for both tests. It was found that a load ratio of 2.4 for the new test resulted in a test consistent with the existing one for coefficients of variation up to about 30%.

FIGURE 5.21 RELIABILITY ASSOCIATED WITH RECALIBRATED TEST

CHAPTER 6

SUMMARY6.1 Conclusions

This study has discussed a technique for the evaluation of the safety against collapse associated with testing timber trusses. The method allows the probability of survival of a truss to be calculated, from component data, for any load history. Once the probability of survival of a single structure subjected to a test load is known, then the probability of accepting the design can be calculated.

Parameters affecting the reliability of testing procedures were studied and the following conclusions made:

- (1) High variability in material strength properties corresponded to lower reliability.
- (2) Adjusting the ratio of test load to design load altered the level of reliability associated with a test, but did not change the ability of the test to discern good designs from poor designs.
- (3) Adjusting the number of single tests performed altered both the reliability and ability to discern good from poor designs.

- (5) Low test load ratio corresponded to lower reliabilities when compared to higher test, load ratios.
- (6) More single truss tests corresponded to higher reliability when compared to fewer tests at a given load ratio.
- (7) When more trusses are tested, the ability of the test to discern good from poor designs improves.
- (8) Using the technique it is possible for a manufacturer to choose a "safety factor" so that his truss designs will pass the strength portion of any test.
- (9) It is possible to calibrate one testing procedure against another so that the safety associated with each test is consistent.

6.2 Recommendations for Further Study

During the course of this study, the following areas were identified for further study.

- (1) The damage model should be formulated in such a manner as to allow variability in strength as a function of volume as well as coefficient of variation. This could be accomplished by assuming a three-parameter Weibull distribution rather than a log-normal distribution.

- (2) The effect of concomitance between bending and axial strength should be studied.
- (3) The duration of load data for metal plate connections should be acquired and fitted to the damage model to obtain the material constants for the joints.
- (4) Creep models should be adopted for the plates and for timber allowing the deflection aspects of the truss tests to be studied.

References

- Allen, D.E. 1975. Limit States Design - a probabilistic study. Canadian Journal of Civil Engineering, Volume 2, pp. 36-49.
- Aplin, E.N., Keenan, F.J. 1977. Limit States Design in wood: A Canadian perspective. Forest Products Journal, Volume 27, No. 7, pp. 14-18.
- Barret, J.D. 1976. Shear fracture of wood. Presented at the 1976 XVI Int. Union For. Res. Organ. World Congress, Oslo, Norway, Proc. Div. V, pp.160-166.
- Barrett, J.D., Foschi, R.O. 1978 I. Duration of load and probability of failure in wood. Part I. Modelling creep rupture. Canadian Journal of Civil Engineering, Volume 5, No. 4, pp. 505-514.
- Barrett, J.D., Foschi, R.O., 1978 II. Duration of load and probability of failure in wood. Part II. Constant, ramp and cyclic loadings. Canadian Journal of Civil Engineering, Volume 5, No. 4, pp. 515-532.
- Bolotin, V.V. 1969. Statistical methods in structural mechanics. Holden - Day Inc., San Francisco, CA. pp. 171-175.
- Canadian Standards Association 1976, Code for the engineering design of wood. Rexdale, Ont., CSA-086.
- Canadian Standards Association 1976, Method of test for evaluation of truss plates used in lumber joints, CSA-5347.
- Canadian Standards Association 1977, Load test procedure for wood roof trusses for houses and small buildings. Rexdale, Ont., CSA-5307.

- Foschi, R.O. 1979. A discussion on the application of the safety index concept to wood structures. Canadian Journal of Civil Engineering, Volume 6, No. 7, pp. 51-58.
- Freudenthal, A.M. 1956. Safety and the probability of structural failure. Transactions, ASCE, Volume 121, Proc. paper 2843, p. 1337-1375.
- Granger, G.D. 1969. Prototype testing of timber structures, Meeting of International Union of Forestry Research Organizations, Working Group on Structural Utilization, Vol. III London.
- Leicester, R.H. 1971. Test loads for proof and prototype testing of timber structures. Meeting of International Union of Forestry Research Organizations, Working Group on Structural Utilization, Madison WI, USA pp. 79-106.
- MacGreggor, J.G. 1976. Safety and Limit States Design for reinforced concrete. Canadian Journal of Civil Engineering, Volume 3, No. 4, pp. 484-513.
- Madsen, B. 1975. Strength values for wood and limit States Design. Canadian Journal of Civil Engineering, Volume 2, No. 3, pp. 270-279.
- Madsen, B. 1975. Moisture content - strength relationship for lumber subjected to bending. Canadian Journal of Civil Engineering, Volume 2, pp. 466-473.
- Madsen, B., Nielsen, P.C. 1978. In-grade testing accuracy and confidence computer simulation, Structural research series report No. 21, I. S.S.N. 0318-3378 University of British Columbia, Dept. of Civil Engineering, Vancouver B.C.
- National Building Code of Canada, 1977. National Research Council of Canada, NRCC No. 15555.

- Pearson, R.G. 1965. Assessment of adequacy of timber structures by test with special reference to timber roof trusses. The Australian Timber Journal, June 1965, pp. 91-97.
- Pierce, C.B. 1975. Classical probability of failure of a structure (timber) element under static uniform load. Meeting of International Union of Forestry Research Organizations. Wood Engineering Group. Proc. Vol. 2. Houtinstituut Inc. Delft Holland paper No. 21.
- Rickmers, A.D., Todd, H.N. 1967 Statistics an Introduction McGraw-Hill, Inc. New York.
- Sexsmith, R.G. 1977. Personal Communication with B. Madsen and N.E. Aplin re truss testing. August 26.
- Suddarth, S., Woeste, F.E. and Galligan, W.L. 1978. Differential Reliability: Probabilistic Engineering applied to wood members in bending/tension. United States Forest Product Laboratory. Research Paper FPL 302, 17 pp.
- Truss Plate Institute of Canada, 1978. Truss design procedures and specification for light metal plate connected wood trusses, Second edition - First draft.
- Warren, W.G., Madsen, B. 1977. Computer - assisted experimental design in forest products research: A case study based on testing the duration-of-load effect. Forest Products Journal, Vol. 27, No. 3, pp. 45-49.
- Weibull, W. 1939. Proc. Ing. Vetenskapsakad Akad., p. 151.
- Wood, L. 1951. Relation of strength of wood to duration of load. United States, Forest Products Laboratory, Madison, WI, Report No. 1916.
- Zahn, J.J. 1977. Reliability-based design procedures for wood structures. Forest Products Journal, Vol. 27, No. 3 pp. 21-28.

EXAMPLE CALCULATIONS

Using the properties shown in Table 5.1 the structure shown in Figure A1 was analysed using the computer program "Purdue Plane Structures Analyser (PPSA)."

Figure A1 Example structure

All webs were assumed to be pin connected. If the interaction is defined as

$$I = \frac{\tau_A}{\tau_{SA}} + \frac{\tau_B}{\tau_{SB}} \quad A1$$

where τ_A - applied axial stress

τ_B - applied bending stress

τ_{SA} - median short term axial strength

τ_{SB} - median short term bending strength

Then the maximum interaction, I for a member can be calculated directly by PPSA. If the length of time under load is assumed constant, equation A2 can be evaluated for each member.

$$R_N = \frac{\ln(I) + \frac{\ln(aT_c)}{b}}{-\omega(1 + \frac{1}{b})}$$

The constants, a and b were assumed to be

$$a = 444.4 \text{ (h}^{-1}\text{)}$$

$$b = 36$$

Table A1 shows the calculation of the probability of survival of each member based on the above structure.

MEMBER	INTERACTION I	TEST ($T_c = 24\text{h.}$)		DESIGN LIFE ($T_c = 10 \text{ YEARS}$)	
		R_N	Q_m	R_N	Q_m
1	.06	6.45	1.0000	5.88	1.0000
2	.11	4.92	1.0000	4.35	1.0000
3	.13	4.50	1.0000	3.93	1.0000
4	.14	4.31	1.0000	3.74	1.0000
5	.13	4.50	1.0000	3.93	1.0000
6	.11	4.92	1.0000	4.35	1.0000
7	.06	6.45	1.0000	5.88	1.0000
8	.06	6.45	1.0000	5.88	1.0000
9	.10	5.16	1.0000	4.59	1.0000
10	.11	4.92	1.0000	4.35	1.0000
11	.11	4.92	1.0000	4.35	1.0000
12	.10	5.16	1.0000	4.59	1.0000
13	.06	6.45	1.0000	5.88	1.0000
14	.17	3.82	.9999	3.25	.9994
15	.20	3.41	.9997	2.84	.9977
16	.12	4.70	1.0000	4.13	1.0000
17	.13	4.50	1.0000	3.93	1.0000
18	.06	6.45	1.0000	5.88	1.0000
19	.06	6.45	1.0000	5.88	1.0000
20	.00	10	1.0000	10	1.0000
21	.00	10	1.0000	10	1.0000

MEMBER	INTERACTION I	TEST ($T_c = 24h.$)		DESIGN LIFE ($T_c = 10$ YEARS)	
		R_N	Q_m	R_N	Q_m
22	.06	6.45	1.0000	5.88	1.0000
23	.06	6.45	1.0000	5.88	1.0000
24	.13	4.50	1.0000	3.93	1.0000
25	.12	4.70	1.0000	4.13	1.0000
26	.20	3.41	.9997	2.84	.9977
27	.17	3.82	.9999	3.25	.9994

$$Q_{TEST} = \prod_{i=1}^n Q_{mi_{TEST}} = .9992$$

$$Q_{DESIGN} = \prod_{i=1}^n Q_{mi_{DESIGN}} = .9942$$

The probability of acceptance can then be calculated using the appropriate operating characteristic equation. For example if

$$P_A = Q^4$$

$$\text{then } P_a = (.9992)^4$$

$$= .9968$$

Therefore a probability of surviving 10 years under full design load of .9942, the probability that that truss would be accepted would be .9968.

END

1 6 1 2 8 1 1

FIN