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SEISMIC RESPONSE OF MULTISTOREY REINFORCED
CONCRETE BUILDING STRUCTURES

by

• Pui-Hing Pang, B.Sc. (Eng.)

A thesis submitted to the Faculty of
Graduate Studies in partial fulfillment
of the requirements for the degree of
Master of Engineering

Department of Civil Engineering
Faculty of Engineering
Carleton University
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The undersigned recommend to the Faculty of Graduate Studies
and Research, acceptance of the thesis:

"SEISMIC RESPONSE OF MULTISTOREY REINFORCED CONCRETE
BUILDING STRUCTURES"

submitted by Pui-Hing Pang, B.Sc. (Eng.) in partial fulfillment
of the degree of

MASTER OF ENGINEERING

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ABSTRACT

The major objectives of this thesis are: the elastic response analysis of multistorey reinforced concrete shear wall-frame structures subjected to earthquake ground motions, and the inelastic analysis of reinforced concrete frames under earthquake motions with consideration of the effect of stiffness degradation on the dynamic response.

For the first part of this study, a computer program has been developed by the author to undertake the computational task. Modal analyses and dynamic analyses are carried out for selected shear wall-frames. Results are interpreted and examined. It is found that the triangular force distribution suggested by the National Building Code of Canada underestimates the shear in the upper storeys and overestimates the shear in the lower storeys of the wall-frames.

For the second part of this study, a program previously developed by Morris is used. Inelastic analyses are carried out for selected frames. Effect of stiffness degradation on the ductility requirements is examined. It is found that the stiffness degradation has a marked effect on short period frames and increases the ductility requirements considerably. For long period frames, responses of stiffness degrading and non-degrading frames do not differ much. The ratio of the

elastic member forces to those in the inelastic frame are also studied. It is found that for the beams which are designed to have a yield strength of one fourth of the maximum elastic moment, the force reduction ratios are of the order of 3. In the case of columns which are designed to remain elastic, this ratio is much smaller and is closer to 2. The beam ductility ratios are somewhat higher than the force reduction factor used in design.

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CHAPTER I

INTRODUCTION

1.1 BACKGROUND

A strong incentive for mankind to learn more about earthquakes is their catastrophic nature. Great loss of life and property has been caused by earthquakes over the past thousands of years, and the damage in terms of physical suffering is unaccountable. The most important cause of the loss of life and property during an earthquake is the collapse of buildings. This has stimulated engineers to improve their knowledge about the causes, nature and the effects of earthquakes, and their ability to design earthquake-resistant buildings with reasonable certainty of success at costs that are not prohibitive. The performance criteria implicit in most of the present codes of practice (1,2) require that a building structure be able to:

- (1) resist minor earthquakes without damage;
- (2) resist moderate earthquakes with minor structural and some nonstructural damage;
and
- (3) resist major earthquakes without collapse.

The basic aim is to prevent loss of life and to assure an acceptable level of public safety.

The major sources of difficulty in fulfilling the code requirements for earthquake-resistant building designs are:

- (1) Insufficient knowledge in the prediction of probability, intensity and detailed characteristics of an earthquake, to which a structure might be subjected during its life.
- (2) Complexities in analytical techniques.
- (3) Questions regarding the true response and resistance of structural elements.

The first point involves questions of whether, where, how much, what kind and when. The answers currently available are extracted from statistics of historic records. Since the number of actual measurements which have been made is very small in a statistical sense, the available past records may not adequately represent a future earthquake motion which could conceivably occur, even at the same site. As a guidance in earthquake-resistant building designs and to provide a standard for public safety, the codes of practice generally provide a seismic zoning map of the country and the associated maximum ground acceleration, which correspond to a given probability of exceedance. It is expected that with progressively increasing collected data and research, design earthquakes can be specified with increased assurance.

To improve the understanding of material and structural behaviour under a specified earthquake motion, a significant amount of research has been carried out during the past two decades. Observations of the behaviour of buildings subjected to actual earthquakes, analytical studies, as well as laboratory experiments by many investigators have all helped to shed light upon these points. The availability of large capacity electronic computers has made it possible to employ sophisticated analytical techniques in the computation of seismic response. The basic difficulty lies in the development of an analytical model to represent the material behaviour, including inelastic behaviour under seismic loading. Because of the increased popularity of reinforced concrete as a constructional material in modern buildings, it is worthwhile studying the seismic response of reinforced concrete structures.

This work is concerned with the study of seismic response of two types of concrete structures.

In the first part, reinforced concrete shear wall-frame structures are examined. Since the inelastic behaviour of shear walls is not yet very clearly understood, the analysis is confined to elastic range. It is believed that this study gives useful insight into the behaviour of such structure and into the distribution of shears along its height.

In the second part, the response of rigid frames of reinforced concrete is examined. The frames consist of horizontal beams and vertical columns. The analysis is extended into the inelastic range. The inelastic material behaviour is modelled in the form of a degrading hysteresis loop based on previous theoretical and experimental research carried out by other investigators.

1.2 REVIEW OF THE LITERATURE

Dynamic response of reinforced concrete building structures to intensive earthquakes is an area of research receiving much attention in recent years. The literature on this subject is voluminous and is increasing rapidly. What is cited here is only that which is directly relevant to this study.

The initial work on dynamic analysis of earthquake-resistant buildings was done in 1920 and early 1930 by researchers of Japan and the United States. Though the equations of motion were known, dynamic response could not be computed because computers were not available at that time. Instead, it was the usual practice to use a static analysis for earthquake-resistant building design. With the advent of computers, studies on dynamic response of single and multiple degrees of freedom systems became possible.

The earlier work on multistorey structures by Clough (1955)⁽³⁾, Blume (1958)⁽⁴⁾, Jennings and Newmark (1960)⁽⁵⁾, Berg (1961)⁽⁶⁾ and many others made notable contribution to the understanding of the dynamic behaviour of multistorey buildings. Experimental studies were also carried out by Hudson⁽⁷⁾ and Bouwkamp⁽⁸⁾ to examine the actual seismic behaviour of multistorey buildings subjected to simulated earthquake loadings, and to test the validity of mathematical models which were used for analyses.

Since the use of reinforced concrete shear walls became a very common means of resisting the lateral forces of wind and earthquakes in multistorey buildings, several studies were directed to this area. Many of these studies used the equivalent static loads suggested by codes for the analysis of shear wall-frame structures. Notable among them were the ones carried out by Seto⁽⁹⁾, MacLeod⁽¹⁰⁾, Khan and Sbarounis⁽¹¹⁾, and Rosenblueth and Holtz⁽¹²⁾. These studies also proposed some approximate methods for the analyses of shear wall-frame structures. Experimental work done by Coull and Irwin⁽¹³⁾, and Heidebrecht and Tso⁽¹⁴⁾ helped in obtaining a basic understanding of the actual behaviour of the shear walls and shear wall-frames under earthquake motions. Few studies have been carried out on the dynamic response of shear wall-frame structures.

— Based on the numerical integration method developed by Newmark⁽¹⁵⁾, and the capabilities of large scale digital computers, Clough and Wilson et al^(16,17) developed analytical techniques for calculating the dynamic response of multistorey buildings and presented an effective approach for the analysis of such structures. The development of the analytical techniques also enabled the studies of dynamic response of multi-storey building to be extended into the inelastic range.

It has been well recognized that even a moderate earthquake may be expected to produce inelastic deformation in multistorey buildings, and that the energy absorbed by the structure through inelastic deformations has a controlling influence on its deformation amplitudes. Due to the peculiarity of the inelastic behaviour of reinforced concrete, researchers have focused their work on the understanding of its actual behaviour under repeated load reversals into inelastic range, and on the development of a model that can simulate its true behaviour when loaded up to cracking of concrete under tension, yielding of tensile reinforcement, beyond yielding and beyond yielding but with load reversals after yielding.

The nonlinear nature of the inelastic behaviour of a reinforced concrete structure greatly complicates the inelastic dynamic analysis problem. To carry out the inelastic dynamic

analysis of multistorey reinforced concrete buildings, it is necessary to formulate two types of mathematical models:

- (1) A hysteretic model to represent the force-deformation relationship under load reversal.
- (2) An analytical model to represent the distribution of stiffness along a member.

For the development of a hysteretic model of reinforced concrete, experimental studies have been carried out by Takeda⁽¹⁸⁾, Van Kuren and Galambos⁽¹⁹⁾, Agrawal et al⁽²⁰⁾, Hanson and Connor⁽²¹⁾, Takeda, Sozen and Nielsen⁽²²⁾, and Bertero and Popov⁽²³⁾ on the behaviour of reinforced concrete members subjected to cyclic loadings. An elasto-plastic force-deformation relationship was used by Berg and DaDeppo⁽²⁴⁾, Heidebrecht et al⁽²⁵⁾, and Humar⁽²⁶⁾ for dynamic analysis because of its mathematical simplicity. Some researchers used a bilinear force-deformation model to simulate the strain hardening and Bauschinger effect in steel member. In some studies, reinforced concrete members were also idealized by a bilinear hysteresis model^(27,28). A stiffness degrading model was proposed by Clough⁽²⁹⁾ to simulate the behaviour of reinforced concrete members under load reversals. The

model was further refined by Takeda et al⁽²²⁾ who formulated a set of complicated hysteresis rules for this purpose.

For the analytical modelling of reinforced concrete members, a one-component model was proposed by Giberson⁽³⁰⁾. The member was assumed to have a linearly elastic prismatic element and two rigid-plastic rotational springs at its ends. All inelastic deformations were assumed to occur in these springs. This method of modelling the member has its weaknesses: first, the result is acceptable only when the inflexion point of the member stays close to the centre of the member during the oscillation; second, an assumed linear distribution of moment along the member is not true because of the existence of gravity loads and third, the inelastic deformation is not lumped at the ends of the member. An interesting model was proposed by Clough, Benuska and Wilson⁽²⁷⁾. In this model, a frame member was divided into two imaginary parallel elements: an elasto-plastic element to represent the yield phenomenon, and a fully elastic element to represent strain hardening behaviour. It was found that it is difficult for this model to simulate continuously varying stiffness. A connected two-cantilever model was proposed by Otani and Sozen⁽³¹⁾. The weakness of this model is that the member flexibility matrix is a function of the location of inflexion point, and when the inflexion point shifts, it causes a numerical problem.

When more accurate results are required, an alternative method is to divide a member into discrete elements, by using a finite element method. However, the computational effort required in using this model in the analysis of a building frame is prohibitive.

Inelastic dynamic response of multistorey reinforced concrete buildings was examined by Clough et al⁽²⁷⁾, Goel⁽³²⁾, and Otani⁽³³⁾ by using one of the above hysteretic and analytical models. From the results of analyses, it was found that the displacements of the elastic and the inelastic structure were of the same order⁽²⁷⁾. Ductility requirements of the members and effect of stiffness degradation of reinforced concrete members on the inelastic deformation and ductility demand were also examined in some of these studies, and in a study by Chopra and Kan⁽³⁴⁾.

1.3 SCOPE OF PRESENT STUDY

The purpose of this study is to examine the effect of factors, such as P-delta effect, shear deformation, finite joint size, and set-back of the shear wall, on the elastic response of reinforced concrete shear wall-frames. Shear distribution in a R.C. wall-frame is also investigated.

A second objective of this study is to examine the effect of stiffness degradation of reinforced concrete members on

the ductility demands in rigid frame members using Takeda's hysteresis model. Care has been taken in selecting the sample frames to avoid any inconsistency with the assumptions made for the analytical model. Another objective is to find a correlation between the elastic and the inelastic responses of concrete frames. To cover the objectives stated above, the present study is carried out in the following steps:

- (1) Survey of the related literature.
- (2) Development of a computer program for the elastic analysis of a reinforced concrete shear wall-frame.
- (3) Selection of appropriate hysteresis and analytical models to represent the behaviour of a concrete member and the selection of a computer program for the inelastic analysis of a reinforced concrete frame.
- (4) Selection of sample buildings for study.
- (5) Analysis of results.

Chapter 1 presents the background to this study, a literature review and the scope of the present study. Chapter 2 describes the development of a mathematical model of the structure, formulation of the equations of motion, method

of analysis, and the selection of important parameters and also presents a brief description of the computer programs.

Chapter 3 is devoted to the elastic analysis of reinforced concrete shear wall-frame, including descriptions of the sample wall-frame and the procedures of analysis. Results of analyses are presented by using tables and graphs. Effect of certain parameters on the elastic response are also examined in this chapter.

Chapter 4 is devoted to inelastic analysis of reinforced concrete frames. Hysteresis and analytical models used in the analysis are described. Results are presented in graphs, with focus on the member ductility requirements and correlation between elastic and stiffness degrading responses of the frames. Chapter 5 presents a summary, the conclusions drawn from the study, and recommendations for future studies.

CHAPTER 2

METHOD OF ANALYSIS

2.1 INTRODUCTION

Numerous theoretical studies have been made on the elastic and inelastic dynamic response of reinforced concrete multi-storey buildings subjected to intensive ground motion. The accuracy of analytical results depends on a clear understanding of the behaviour and the hysteretic properties of the material, on the selection of an appropriate mathematical model for the structure, and on a correct choice of the means for computation of its dynamic responses.

For the computation of response, a numerical method of integration of the equations of motion is adopted in this study. Also a number of simplifying assumptions are made in the mathematical modelling of the structure. Details of the idealization of structures, the formulation of mathematical model, the procedure adopted for numerical integration of the equations of motion, and the selection of important parameters, such as damping, time steps used in integration and the input ground motion are given in this chapter. A brief description of the computer programs developed or used for the present study is also presented.

2.2 MATHEMATICAL MODEL OF THE STRUCTURE

Because of the difficulty in modelling material behaviour, dynamic analysis of a multistorey reinforced concrete building is a very complex task. In obtaining a reasonably accurate analysis of such a structure, one must rely on engineering judgment. It is desirable to introduce as many simplifications in the computational process as possible so that reasonably accurate results could be secured without too high a cost of computing. A mathematical model of the structures is developed in this chapter for the above purpose. It is believed that with the assumptions made in modelling the structure, a complicated engineering problem is resolved into a comparatively simple mathematical problem, for which an effective computer solution technique can be used to arrive at a significant analytical result. At the same time, these assumptions clarify the limitations and applicability of the solution.

2.2.1 STRUCTURAL IDEALIZATION

The model developed in this study is idealized and simulates only the major sources of deformation of the structures, such as flexure and shearing. Although the method of analysis is general, it is based on certain assumptions and limitations, that are listed below:

- (1) A complete building considered here is composed of structural components which

can be separated into two sets of parallel rectangular plane frames, acting in perpendicular directions. Each frame is treated as an independent structure, with horizontal beams, vertical columns and fully rigid joints. The building is laid out symmetrically, hence no torsional effect is considered in frame analysis.

- (2) The floor systems are infinitely rigid in the horizontal direction. Thus, the effect of axial deformation of beams is neglected. Horizontal lateral loads are assumed to act at floor level and are transferred to the vertical elements through these rigid floor diaphragms. This means that only one sideways degree of freedom is associated with each storey. Hence the number of degrees of freedom is greatly reduced.
- (3) Vertical members such as columns and walls are fixed to an infinitely rigid foundation.

- (4) The base excitation is a horizontal motion in the plane of the frame. Rocking and vertical motion of base are neglected. The frame undergoes horizontal vibration only in its own plane.
- (5) In an elastic analysis, shear walls may be incorporated arbitrarily into the frame by treating them as columns of finite width.
- (6) Although the behaviour of members is primarily governed by flexure, axial deformations of vertical members are also considered in the analysis. Shear deformations may be considered as well.
- (7) Secondary moments introduced in columns due to axial loads interacting with deflection are small and can be neglected. However, the effect of axial column loads acting through sidesway deflection may be taken into account.
- (8) The mass of the structure is lumped at the floor levels and remains constant during the entire duration of motion.

- (9) Centroidal axis of beams in one floor is a continuous straight line and coincides with the centerline of the floor. Columns or walls in one vertical stack are assumed to have the same center line throughout the height of the frame. No offset is considered.
- (10) Sectional properties of both vertical and horizontal members are uniform throughout their lengths.
- (11) Joints are assumed to be entirely rigid. Deformations within joints are not accounted for.
- (12) Local or out of plane buckling of members is prevented.
- (13) The damping in the structure can be adequately represented as viscous type of damping. In other words, the damping force is assumed to be proportional to the velocity. For simplicity, the damping matrix is assumed to be the sum of a part proportional to the constant mass matrix and a part proportional to the stiffness matrix.

(14) In using a step-by-step numerical integration method, it is assumed that acceleration varies linearly over a small interval of time. Stiffness matrix of the structure remains constant during this small time interval. This will be discussed in more detail in the following sections.

(15) For inelastic analysis, location of inflexion point in a member is assumed to be at the center of the member.

2.2.2 MEMBER STIFFNESS MATRICES

A member is defined herein as a structural element which connects two adjacent joints of a frame. Under action of a load system applied at the joints, the behaviour of a member can be defined by the displacements of its joints. Consequently, when displacements of joints are defined, the overall behaviour of the frame is also defined.

Each independent displacement coordinate is defined as a degree of freedom. The number of degrees of freedom for different members is not unique. It is a function of the manner in which the real member has been idealized for analysis.

According to the assumptions made in Section 2.2.1, when members are confined to a plane, there are four degrees of freedom associated with an axially rigid member (beam), and six degrees of freedom associated with an axially deformable member (column). The convention used in numbering these degrees of freedom is shown in Fig. 2.1(a) and (b). The figures also show the positive directions of the degrees of freedom.

(A) ELASTIC MEMBER

A stiffness equation is actually a relationship between force and displacement. For those flexural members in which the deformations are entirely elastic, the stiffness equation relating the end moments and the end rotations can be expressed in matrix form as:

$$\{M\} = [K] \{\theta\} \quad (2.1)$$

Where $\{M\}$, $[K]$ and $\{\theta\}$ represent end moment, stiffness, and end rotation matrix respectively. When consideration is given to both the flexural and shear deformations of the member, the stiffness matrix $[K]_b$ for a beam can be expressed in the following form:

$$[K]_b = \begin{bmatrix} S_a & S_b \\ S_b & S_a \end{bmatrix} \quad (2.2)$$

where $S_a = \frac{2EI}{L} \frac{2 + \psi}{1 + 2\psi} \quad (2.2a)$

FIG. 2.1 MEMBER DEGREE OF FREEDOM

FIG. 2.2 FLEXURAL MEMBER DEFORMATION

$$S_b = \frac{2EI}{\ell} \frac{1 - \psi}{1 + 2\psi} \quad (2.2b)$$

$$\psi = \frac{6EI}{\ell^2 A_e G} \quad (2.2c)$$

In which A_e is the effective shear area, ψ is called the shear flexibility factor. E , I , ℓ and G denote Young's modulus, moment of inertia, member length and shear modulus respectively. In case shear deformations are not considered, ψ is equal to zero.

From Fig.2.2, Eq.2.1 can be expressed as:

$$\begin{Bmatrix} M_\ell \\ M_r \end{Bmatrix} = [K]_b \begin{Bmatrix} \theta_\ell \\ \theta_r \end{Bmatrix} \quad (2.3)$$

Where M and θ are the end moments and end rotations respectively. Subscripts ℓ and r refer to the left and the right end of the member. Subscript b indicates beam.

In the presence of shear walls, the significant effect of rigid finite joints has to be accounted for. Referring to Fig.2.3(a), it is seen that:

$$\theta_\ell = \theta'_L + a\theta'_L + b\theta'_R \quad (2.4a)$$

$$\theta_r = \theta'_R + a\theta'_L + b\theta'_R \quad (2.4b)$$

or in matrix form

$$\begin{Bmatrix} \theta_\ell \\ \theta_r \end{Bmatrix} = \begin{bmatrix} 1+a & b \\ a & 1+b \end{bmatrix} \begin{Bmatrix} \theta'_L \\ \theta'_R \end{Bmatrix} \quad (2.5)$$

(a) Deformation

(b) Moment Diagram

(c) Shear Diagram

FIG. 2.3 RIGID END EFFECT

Where a and b are the ratios of the left and right rigid zone lengths to the clear length of the member.

From Fig.2.3(b), the relationship between the moment applied at the center of the support and the moment at the face of the support can be expressed as:

$$\begin{Bmatrix} M_L' \\ M_R' \end{Bmatrix} = \begin{bmatrix} 1+a & a \\ b & 1+b \end{bmatrix} \begin{Bmatrix} M_\ell \\ M_r \end{Bmatrix} \quad (2.6)$$

By substituting Eq. 2.5 into Eq. 2.3, and Eq. 2.3 into Eq. 2.6, the following equation results:

$$\begin{Bmatrix} M_L' \\ M_R' \end{Bmatrix} = \begin{bmatrix} 1+a & a \\ b & 1+b \end{bmatrix} [K]_b \begin{bmatrix} 1+a & b \\ a & 1+b \end{bmatrix} \begin{Bmatrix} \theta_L' \\ \theta_R' \end{Bmatrix} \quad (2.7)$$

Eq. 2.7 shows the relationship between the end moments and end rotations of a beam in the local coordinate system. Considering that a beam is permitted to rotate and sway, the transformation between the member deformations and joint displacements is obtained from Fig.2.4. It is seen that:

$$\theta_L' = \theta_L - \frac{\Delta_L - \Delta_R}{L} \quad (2.8a)$$

$$\theta_R' = \theta_R - \frac{\Delta_L - \Delta_R}{L} \quad (2.8b)$$

or in matrix form

$$\begin{Bmatrix} \theta_L' \\ \theta_R' \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 1 & -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} \theta_L \\ \theta_R \\ \Delta_L \\ \Delta_R \end{Bmatrix} \quad (2.9)$$

FIG. 2.4 BEAM DEFORMATIONS AND JOINT DISPLACEMENTS

FIG. 2.5 COLUMN DEFORMATIONS

Where θ and Δ represent joint rotations and joint sways in the global coordinate system.

The relationship between joint moments and shears and the member end moments can also be expressed in a similar way:

$$\begin{Bmatrix} M_L \\ M_R \\ V_L \\ V_R \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{1}{L} & -\frac{1}{L} \\ \frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} M_L' \\ M_R' \end{Bmatrix} \quad (2.10)$$

In which V represents the joint shears. Substitutions of Eqs. 2.7 and 2.9 into Eq. 2.10 yields:

$$\begin{Bmatrix} M_L \\ M_R \\ V_L \\ V_R \end{Bmatrix} = [\bar{K}]_b \begin{Bmatrix} \theta_L \\ \theta_R \\ \Delta_L \\ \Delta_R \end{Bmatrix} \quad (2.11)$$

where

$$[\bar{K}]_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{1}{L} & -\frac{1}{L} \\ \frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} 1+a & a \\ b & 1+b \end{bmatrix} [K]_b \begin{bmatrix} 1+a & b \\ a & 1+b \end{bmatrix} \begin{bmatrix} 1 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 1 & -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \quad (2.12)$$

is the beam stiffness matrix in the global coordinate system, which accounts only for flexural or flexural and shear deformations. Letting:

$$[T]_b = \begin{bmatrix} 1+a & b \\ a & 1+b \end{bmatrix} \begin{bmatrix} 1 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 1 & -\frac{1}{L} & \frac{1}{L} \end{bmatrix} = \begin{bmatrix} 1+a & b & -\frac{1}{L} & \frac{1}{L} \\ a & 1+b & -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \quad (2.13)$$

Where ℓ and L are the clear length and the overall length of the member respectively as shown in Fig.2.4. Equation 2.12 can be expressed in a familiar form as:

$$[\bar{K}]_b = [T]_b^T [K]_b [T]_b \quad (2.14)$$

Matrix $[T]$ is called the transformation matrix. Again, subscript b denotes beam.

Usually the influence of axial deformation is significant in vertical members of multistorey buildings. Therefore, for members such as columns or shear walls, Eq. 2.2 should be revised as follows:

$$[K]_c = \begin{bmatrix} S_a & S_b & 0 \\ S_b & S_a & 0 \\ 0 & 0 & S_c \end{bmatrix} \quad (2.15)$$

Where S_a and S_b are similar to Eqs. 2.2a and 2.2b, and

$$S_c = \frac{AE}{l} \quad (2.15a)$$

In which E , A and l represent Young's modulus, cross sectional area and length of the member respectively.

With reference to Fig. 2.5, Eq. 2.3 becomes:

$$\begin{Bmatrix} M_l \\ M_u \\ P \end{Bmatrix} = [K]_c \begin{Bmatrix} \theta_l \\ \theta_u \\ \epsilon \end{Bmatrix} \quad (2.16)$$

Where M and θ indicate end moment and end rotation and P and ϵ represent the axial load and axial deformation respectively. Subscripts l and u refer to the lower and the upper end of the member, and c indicates column.

Referring to Fig. 2.6, transformation between column deformation and joint deformations is developed in a manner similar to that for the beams. The transformation matrix $[T]_c$ is given by

$$[T]_c = \begin{bmatrix} 1+a & b & \frac{1}{l} & -\frac{1}{l} & 0 & 0 \\ a & 1+b & \frac{1}{l} & -\frac{1}{l} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \quad (2.17)$$

FIG. 2.6 COLUMN DEFORMATIONS AND JOINT DISPLACEMENTS

Finally, Eq. 2.11 can be rewritten as:

$$\begin{Bmatrix} M_L \\ M_U \\ V_L \\ V_U \\ P_L \\ P_U \end{Bmatrix} = [\bar{K}]_c \begin{Bmatrix} \theta_L \\ \theta_U \\ \Delta_L \\ \Delta_U \\ \epsilon_L \\ \epsilon_U \end{Bmatrix} \quad (2.18)$$

where

$$[K]_c = [T]_c^T \cdot [K] \cdot [T]_c \quad (2.19)$$

is the column stiffness matrix expressed in global coordinate system. This stiffness matrix takes into consideration the axial deformation, which is induced by column load P .

The secondary effects arising from the column axial loads can be divided into two parts. First, the interaction of axial load with member deformation results in a reduction in the effective stiffness of the member. Previous studies^(35,36) have shown that the effect of the degradation of the member stiffness is insignificant, and can be neglected.

A second effect arises from the fact that when side-sway deflections are considerable, equilibrium should be formed on the deformed geometry of the structure, and the axial loads produce an unstabilizing moment which is proportional to the product of the loads and the sidesway deflections. This effect is commonly known as the P-delta ($P-\Delta$) effect.

When the P-delta effect is considered, then as shown in Fig.2.7 the additional shear required to be resisted by the member can be determined by using the following equation:

$$V = \frac{P}{\ell} \Delta \quad (2.20)$$

In matrix form

$$\begin{Bmatrix} V_L \\ V_U \end{Bmatrix} = -\frac{P}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta_L \\ \Delta_U \end{Bmatrix} = [K_G] \begin{Bmatrix} \Delta_L \\ \Delta_U \end{Bmatrix} \quad (2.21)$$

in which V , P , ℓ and Δ refer to the additional shear, axial column load, column length and sidesway deflection respectively. Matrix $[K_G]$ is commonly called the geometric stiffness matrix. It can be seen that the member stiffness tends to reduce when the P-delta effect is considered.

FIG. 2.7 P- Δ EFFECT ON COLUMN STIFFNESS

The modification of the member stiffness for the P-delta effect is carried out by adding the geometric stiffness matrix to the member stiffness matrix. It should be noted that P is unknown when the equations are formed and it would appear that an iterative process would be necessary if P-delta effect is to be taken into account. To obviate the iterative process, it is assumed that under static load, the sidesway is small. Neglecting the P-delta effect in the static analysis, the axial loads are obtained. These loads are then used to modify the column stiffness matrices before a dynamic analysis is carried out.

As the structure deforms due to ground motion, individual column loads will change, but the total of column loads in any storey still remains constant. Thus, the contribution of P-delta effect to the system stiffness matrix remains unaffected, and once accounted for will remain unchanged for the duration of the analysis.

(B) INELASTIC MEMBER

The member stiffness obtained as above is correct only if the member remains elastic during the entire period of motion. However, even moderate earthquakes may be expected to produce inelastic deformations in building structures, and inelastic deformations must therefore be accounted for in the calculation of response.

Observations have shown that structures designed for the lateral loads of codes^(2,37), which are generally quite low, have survived severe earthquakes with little or no damage. The ability of a structure to withstand much higher lateral loads as predicted by an elastic analysis for an intense earthquake is attributed mainly to the ability of ductile structures to dissipate energy through inelastic deformations. It is therefore the general practice to design building structures so that they will dissipate energy through inelastic deformation during a severe earthquake. The response values predicted by an elastic analysis are no longer valid and an inelastic analysis must be carried out for the estimates of ductility demand in a structure.

In order to obtain information on the ductility capacity of reinforced concrete members and on their force displacements characteristics, several experimental investigations have been carried out^(20,21,22). It was found that, because of the opening and closing of cracks in concrete, yielding and strain hardening of the steel reinforcement, gradual deterioration of bond between concrete and steel, and increase of shear deformation in plastic hinge zones under cyclic loading, the stiffness of a reinforced concrete member progressively decreases with increasing number of cycles of loading. This phenomenon is commonly referred to as stiffness degradation.

The difficulties in evaluating the stiffness matrix when the member is stressed beyond the elastic limit arise from the fact that the stiffness of the member section changes continuously with its stress history. In addition, the stiffness of the member section is not uniform along the length of the member but varies in accordance with the distribution of moment as shown in Fig.2.8(a).

In order to overcome the above difficulties, a reinforced concrete member is modelled as an elastic line element with an inelastic rotational spring at each end of the member. The distribution of curvature along the length of the member is shown in Fig.2.8(b). The hatched portions indicate the inelastic part of the deformation. The modelling of the member by an elastic line element with inelastic springs at either end implies that all of the inelastic rotation is concentrated at the ends. The incremental end rotation resulting from an incremental end moment is then obtained by the superposition of the elastic and inelastic components as expressed below:

$$\{\Delta\theta\} = \{\Delta\theta^E\} + \{\Delta\theta^I\} \quad (2.22)$$

in which $\Delta\theta$ is the incremental end rotation. Superscripts E and I denote contribution from elastic and inelastic components respectively.

(a) Moment Distribution

(b) Curvature Diagram

(c) Case I
Elementary Curvature
Corresponding to M_l (d) Case II
Elementary Curvature
Corresponding to M_r

FIG. 2.8 MOMENT - CURVATURE DISTRIBUTION IN AN INELASTIC MEMBER

The properties of the elastic line element can be easily defined as stated earlier. The characteristics of the inelastic rotational end spring is assumed to follow a set of hysteresis rules developed on the basis of experimental and theoretical investigations^(18,21). Hysteresis rules adopted in this study follow Takeda's hysteresis model⁽²²⁾ and simulate the characteristic behaviour of a reinforced member under load reversals into inelastic range so that the cracking of concrete, and yielding and strain hardening of reinforcement, can be taken into consideration.

The moment-rotation relationship of the inelastic spring is expressed as a function of its load history, indicating the possible paths the moment-rotation curve follows, while loading, unloading or reloading in both positive and negative bending directions. These hysteresis rules will be dealt with in detail in Chapter 4.

When using a step-by-step numerical method, incremental member end rotation and incremental member end moment are related through the following expression:

$$\{\Delta\theta\} = ([F^E] + [F^I]) \{\Delta M\} \quad (2.23)$$

in which $[F]$ is a flexibility matrix, with superscripts E and I to indicate the contribution from elastic line element and inelastic rotational end spring respectively. It is

evident that the flexibility matrix is made up of an elastic and an inelastic component.

It is assumed in this study that the inflexion point is located at the center of the member throughout the entire period of motion, and the moment-rotation relationship is idealized by a piece-wise linear primary curve, which is defined by two break points at cracking of tensile concrete, and yielding of longitudinal reinforcement. On the basis of the assumed hysteresis rules, the flexibility relation of the inelastic rotational end spring can be defined. Member stiffness is then obtained by inverting the flexibility matrix,

$$[K] = [[F^E] + [F^I]]^{-1} \quad (2.24)$$

Since a piece-wise linear relationship between end moment and end rotation has been assumed, the member stiffness remains unchanged over a short time increment, during which the member behaves in a linear manner. Therefore, the non-linear inelastic response is obtained as a sequence of linear responses of successively changing systems. Stiffness of members in the structure is evaluated at each time increment based on the state of deformation existing at the beginning of the time interval, and, of course, on the assumption made about the location of inflexion point in a member.

When consideration is given to the shear deformation, the elastic flexibility matrix in the right hand side of Eq. 2.23 is given by

$$[F^E] = \begin{bmatrix} 1+\psi & \psi \\ \psi & 1+\psi \end{bmatrix} \frac{l}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad (2.25)$$

in which ψ is the shear flexibility factor as expressed in Eq. 2.2c. It is evident that by inverting the flexibility matrix, the stiffness matrix of Eq. 2.2 will be obtained. It is also assumed in this study, that for any member, the ratio of the sum of deformations resulting from flexural and shear effect to the deformation due to flexure alone retains the same value throughout the entire load history even beyond the elastic limit. Therefore, the inelastic deformation due to the combined effect of flexure and shear, can be obtained by appropriately scaling the inelastic deformation due to flexure alone.

The finite joint effect and relationship between local coordinate system and global coordinate system are accounted for by transformations similar to those shown in Eqs. 2.13 and 2.17. Accordingly, member stiffness can be expressed in structural (global) coordinate system as:

$$[\bar{K}] = [T]^T [K] [T] \quad (2.26)$$

Where $[T]$, $[K]$ and $[\bar{K}]$ represent transformation matrix, stiffness matrix in the local coordinate system and stiffness matrix in the global coordinate system respectively.

Finally, the relationship between force and displacement can be expressed symbolically as:

$$\{\Delta M\} = [\bar{K}] \{\Delta \theta\} \quad (2.27)$$

which is the incremental form of Eq. 2.1 in the structural coordinate and can be used in the inelastic analysis.

Having determined the individual element stiffness matrices, the complete stiffness matrix of the frame is obtained by assembling the stiffness of each member through the direct correspondence between the member degrees of freedom and the structural coordinates.

Fig.2.9 shows the sequence used in numbering the structural coordinates. This sequence will give the minimum bandwidth for the structural stiffness matrix. The positive directions of forces and displacements are the same as the positive directions of degrees of freedom as shown in Fig.2.9.

2.3 EQUATIONS OF MOTION

The primary objective of structural dynamic analysis is the evaluation of the displacement time history of a given structure subjected to a given time-varying load. The mathematical expressions defining the relationship between forces and displacements, including dynamic effects, are called the equations of motion.

FIG. 2.9 STRUCTURAL COORDINATES

The equations of motion are comprised of a set of ordinary second order differential equations. These equations are formulated on the basis of the equilibrium condition of all forces and compatibility of deformations at each joint, and are expressed as follows:

$$[M] \{\ddot{Z}\} + [C] \{\dot{Z}\} + [K] \{Z\} = \{F\} \quad (2.28)$$

in which $\{\ddot{Z}\}$, $\{\dot{Z}\}$ and $\{Z\}$ represent the vector of accelerations, vector of velocities and vector of displacements respectively. $[M]$, $[C]$ and $[K]$ denote mass matrix, viscous damping matrix, and stiffness matrix respectively, and $\{F\}$ is the vector of applied forces. For base excitation, the applied forces are given by:

$$\{F\} = -[M] \{1\} \ddot{V}_g \quad (2.29)$$

where $\ddot{V}_g$ is the acceleration of base motion $[M]$ is the mass matrix, and $\{1\}$ is vector of unity. In this case Z is the vector of displacements relative to the base.

Solution of these equations of motion provides the required displacement history.

As will be discussed later, the equations of motion are

solved by a step-by-step numerical integration method in which the conditions at the end of a short increment of time are evaluated in terms of the conditions at the beginning of the interval. The equations of motion should therefore be written in an incremental form. Eqs. 2.28 become

$$[M] \{\Delta \ddot{Z}\} + [C] \{\Delta \dot{Z}\} + [K] \{\Delta Z\} = \{\Delta F\} \quad (2.30)$$

where

$$\{\Delta F\} = -[M] \{1\} \Delta \ddot{V}_g \quad (2.31)$$

As mentioned in Section 2.21, the building mass which remains constant during the entire period is assumed to be lumped at the floor levels. Therefore, $[M]$ is a diagonal matrix, composed of concentrated floor masses, and has non-zero terms only at the positions on the major diagonal, which correspond to the sidesway degrees of freedom. Stiffness matrix $[K]$ as described in Section 2.2.2 is assumed to remain constant over a short time increment and is evaluated at the beginning of each time increment.

2.3.1 DAMPING

In addition to the mass and the stiffness matrices, the damping matrix is also an important parameter in formulating the equations of motion. Damping characteristics of a structural system are however very complex and difficult to define.

Damping forces of various kinds may exist in a vibrating system. They may be proportional to velocity, or to some power of the velocity. They may even be proportional to the displacement or may be of a friction type⁽³⁸⁾. In this study, it is assumed that the damping is of viscous type and the damping matrix is proportional to a linear combination of the mass matrix and the stiffness matrix. Such a proportional damping matrix is named as Rayleigh damping matrix and can be written as:

$$[C] = \alpha [M] + \gamma [K] \quad (2.32)$$

where α and β are scalar multipliers discussed below.

For elastic analysis, the use of the orthogonality property of mode shapes enables one to uncouple the equations of motion and to obtain the response by using the method of mode superposition. The damping matrix given by Eq. 2.32 will also satisfy the orthogonality condition. The damping term in the i th uncoupled equation of motion is given by the following relationship:

$$2\xi_i \omega_i = \alpha + \gamma \omega_i^2 \quad (2.33)$$

Where ξ_i and ω_i indicate respectively the damping ratio and frequency of i th mode, and α and γ and the constants in Eq. 2.32.

By fixing modal damping ratios in the first two modes,

$$2\xi_1\omega_1 = \alpha + \gamma\omega_1^2 \quad (2.34a)$$

$$2\xi_2\omega_2 = \alpha + \gamma\omega_2^2 \quad (2.34b)$$

the damping constants can be expressed as follows:

$$\alpha = 2\omega_1\omega_2 \frac{\xi_2\omega_1 - \xi_1\omega_2}{\omega_1 - \omega_2} \quad (2.35)$$

$$\gamma = 2 \frac{\xi_1\omega_1 - \xi_2\omega_2}{\omega_1 - \omega_2} \quad (2.36)$$

Where ω_1 and ω_2 represent undamped frequencies, and ξ_1 and ξ_2 are associated damping ratios of the first two modes.

It is noted that the effective damping in all higher modes is established from the α and γ values as

$$\xi_j = \frac{\alpha + \gamma\omega_j^2}{2\omega_j} \quad (2.37)$$

Where ω_j and ξ_j are frequency and damping ratio of j th mode in which j is greater than 2.

Usually damping ratios must be evaluated directly by experimental methods. Alternatively, they can be assigned on the basis of the similarity of the analyzed structure to some previous structures. Full scale tests on buildings were carried out by Hudson, Ragget and Bouwkamp^(7,39,8) to obtain the modal damping ratio. Most of these tests were limited to low response amplitudes. As pointed out by Newmark and Hall⁽⁴⁰⁾, and Hart⁽⁴¹⁾, damping is a function of the stress levels introduced within the structure, and the use of damping values obtained from low amplitude test information can result in calculated responses exceeding those expected during earthquake excitations. Therefore, additional consideration must be given to large amplitude non-linear behaviour.

Refinements in predicting damping have lagged tremendously behind those used in estimating mass and stiffness. Experimental data are meager on this subject. Therefore, referring to some analytical results of full scale low amplitude tests⁽³⁹⁾, and those obtained from 12 southern California buildings during February 9, 1971, San Fernando earthquake⁽⁴¹⁾, a damping ratio of 5% of the critical damping has been used in the analyses throughout this study.

2.3.2 INTEGRATION OF THE EQUATIONS

In general, for elastic analysis, when the damping matrix satisfies the modal orthogonality condition, the set of equations of motion can be solved by using a mode superposition method. However, for inelastic analysis, due to variations in the stiffness matrix, and perhaps also in the damping matrix, the normal coordinate uncoupling of the equations of motion is not practical. A more general step-by-step numerical integration of the equations of motion is applicable to both elastic and inelastic systems, and is adopted throughout this study.

The main advantage of the method of numerical integration is its applicability to structures of any degree of complexity, such as arches, domes and framed structures, with any form of relationship between force and displacement ranging from linear elastic behaviour through any non-linear inelastic hysteretic behaviour up to failure, and to any type of dynamic loading, such as vibration, earthquake motion or blast.

(A) NUMERICAL METHOD OF INTEGRATION

The equations of motion are solved by a step-by-step integration procedure, in which it is assumed that the acceleration varies linearly during each time increment, while the properties of the system remain constant during

this interval. The linear variation of the acceleration and the corresponding quadratic and cubic variations of the velocity and displacement as shown in Fig.2.10 can be expressed as:

$$\{Z_{n+1}\} = \{Z_n\} + h \{\dot{Z}_n\} + \frac{h^2}{3} \{\ddot{Z}_n\} + \frac{h^2}{6} \{\ddot{Z}_{n+1}\} \quad (2.38)$$

$$\{\dot{Z}_{n+1}\} = \{\dot{Z}_n\} + \frac{h}{2} (\{\ddot{Z}_n\} + \{\ddot{Z}_{n+1}\}) \quad (2.39)$$

where subscripts n and $n+1$ represent values at time t_n and t_{n+1} respectively, and

$$h = t_{n+1} - t_n \quad (2.40)$$

is a small interval of time.

Since

$$\{\ddot{Z}_{n+1}\} = \{\ddot{Z}_n\} + \{\Delta\ddot{Z}\} \quad (2.41)$$

Eq. 2.38 gives

$$\{\Delta\ddot{Z}\} = \frac{6}{h^2} \{\Delta Z\} - \frac{6}{h} \{\dot{Z}_n\} - 3 \{\ddot{Z}_n\} \quad (2.42)$$

Substitution of Eq. 2.42 into Eq. 2.39 yields:

$$\{\Delta\dot{Z}\} = \frac{3}{h} \{\Delta Z\} - 3 \{\dot{Z}_n\} - \frac{h}{2} \{\ddot{Z}_n\} \quad (2.43)$$

Acceleration $\ddot{Z}$
(linear)

$$\ddot{Z} = \ddot{Z}_n + \frac{\ddot{Z}_{n+1} - \ddot{Z}_n}{h} t$$

Velocity $\dot{Z}$
(quadratic)

$$\dot{Z} = \dot{Z}_n + \int_{t_n}^t \ddot{Z} dt$$

Displacement Z
(cubic)

$$Z = Z_n + \int_{t_n}^t \dot{Z} dt$$

FIG. 2.10 LINEAR ACCELERATION METHOD

Substituting Eqs. 2.42, 2.43 and 2.32 into Eq. 2.30, the following expression is obtained:

$$[K^*] \{\Delta Z\} = \{\Delta F^*\} \quad (2.44)$$

where

$$[K^*] = \left(\frac{6}{h} + \frac{3}{h} \alpha\right) [M] + \left(\frac{3}{h} + \frac{3}{h} \beta\right) [K] \quad (2.45)$$

and

$$\{\Delta F^*\} = \{\Delta F\} + [M] \{A\} + (\alpha[M] + \beta[K]) \{B\} \quad (2.46)$$

in which

$$\{A\} = \frac{6}{h} \{\dot{Z}_n\} + 3 \{\ddot{Z}_n\} \quad (2.47)$$

$$\{B\} = 3 \{\dot{Z}_n\} + \frac{h}{2} \{\ddot{Z}_n\} \quad (2.48)$$

$[K^*]$ and $\{\Delta F^*\}$ may be interpreted as the effective stiffness matrix and the effective incremental force vector respectively.

The solution of the equations of motion expressed in Eq. 2.44 provides the incremental displacements at each structural joint. The incremental displacement at structural joints can be transformed to incremental member end

displacements. The incremental member forces are then obtained by multiplying the instantaneous member stiffness matrix by the incremental member displacements. The current values of the member forces are obtained by taking the sum of the previous and the incremental values of the member forces.

It is to be noted that the linear acceleration method adopted in this study is a special case of the more general Newmark's β method. In that method the displacement at time t_{n+1} is related to that at time t_n by the following equation:

$$Z_{n+1} = Z_n + h\dot{Z}_n + \left(\frac{1}{2} - \beta\right) \ddot{Z}_n h^2 + \beta \ddot{Z}_{n+1} h^2 \quad (2.49)$$

where β is used to indicate how much of the initial and final acceleration of the time interval enter into the relation for displacement at the end of the interval and can be assigned an appropriate value. It can be seen that a choice of $\beta = 1/6$ corresponds to a linear variation of acceleration in the time interval, and a choice of $\beta = 1/4$ corresponds to a constant acceleration, which is equal to the mean of the initial and final values of acceleration during the time interval.

For inelastic analysis, since a piece-wise force-displacement relationship idealization is adopted to represent

a non-linear member, the calculated member forces and displacements at the end of the time interval may not exactly comply with the specified force-displacement relationship, and suitable corrections must be made to avoid the propagation of errors.

To avoid the violation of equilibrium or compatibility conditions at a structural joint while correcting a member force or displacement to fit a given relationship between the two, two methods are used in this study. One is to retrace the response by using shorter time increments. Another is to use a correcting force to be added to the joint as an external force in the equation of motion in the next step of the response calculation.

Briefly, the analysis procedure involves the repeated application of the following steps for each time interval:

1. Evaluate stiffness of the structure. When an inelastic analysis is contemplated, it has to be based on the state of member deformation existing at the beginning of the interval.
2. Solve the equations of motion by a process of elimination and calculate the incremental displacement, assuming the accelerations to vary linearly during the interval.

3. Add the incremental displacements to the displacement state at the beginning of the interval to obtain the displacements at the end of the interval.
4. Calculate member forces from the displacements, taking into account the non-linear material property, when it is stressed beyond the elastic limit.
5. In an inelastic analysis, if the member forces and displacements at the end of the interval do not comply with the specified force-displacement relationship, repeat the above steps by using shorter time intervals.

(B) ERRORS IN THE METHOD

It can be seen that inevitable errors which result from round-off and truncation errors may be introduced into the numerical solution and may affect the accuracy of the final answer. The round-off errors are random in nature and can be reduced by using a higher precision in the computations. Thus they are not considered in this study.

Truncation errors are accumulated at each step. Their magnitude is the best indication of the accuracy of the numerical solution. A multipoint method, suggested by Humar, which makes use of the information at several rather than one previous point to assist in evaluating the dependent variable at each successive location, has been proved to have good performance with respect to stability, as well as in reducing the truncation errors when compared with the conventional single point formulae⁽²⁶⁾. However, owing to the change of stiffness and the consequent change of the differential equation from time step to time step, this multipoint method cannot be used in inelastic analysis.

Method of obtaining the truncation error for the numerical solution of differential equations is explained by Hamming⁽⁴²⁾. The errors tend to decrease as the time interval becomes shorter. However, too short a time interval will increase round-off errors and will also increase computation time.

(C) CHOICE OF INTERVAL

The choice of a time interval is the most important item in using the step-by-step numerical integration method. The determination of an appropriate time interval involves considerations of accuracy, stability, computational efficiency, and the convergence of an iterative type solution. Although

it is desirable to select as long a time interval as possible to augment the computational efficiency, the choice of time interval is limited by other considerations.

When the time interval exceeds a certain value, the errors have a tendency to grow and have an increasing effect on the computations at successive steps. At times, a spurious, unbounded oscillation may be introduced into the system; the solution then becomes meaningless and is said to be "unstable".

It has been pointed out by Newmark⁽¹⁵⁾ that when $\beta = 1/6$, a time interval h shorter than 0.55 times the highest mode period of the system, will ensure stability of the solution. This limitation has been kept in mind while selecting the integration time steps for the analyses reported in this study.

Since it is not necessary to use an iterative method for the type of differential equations involved in structural dynamics, convergence need not be a criterion for limiting the size of the time interval. Moreover, use of a direct solution of the simultaneous equations by a process of elimination, as adopted in this study, enables considerable economy in computing time to be achieved.

For systems with a large number of degrees of freedom, such as multistorey buildings, the stability criterion is always more restrictive compared to that for truncation error, because instability even in the highest mode, may ultimately render the solution unstable. In other words, for a system with a large number of degrees of freedom, when the time interval is so selected that the solution is stable, a fairly accurate result can always be obtained. Vice versa, for systems with only a few degrees of freedom, selection of time interval may be dominated by the consideration of accuracy rather than by the criterion of stability.

2.4 CHOICE OF GROUND MOTION

The most difficult problem encountered in seismic resistant design is the choice of a suitable ground motion. It involves the recognition of the probability of occurrence of an earthquake and its characteristics, which differ from region to region, and consideration of trade-off between safety and economy. Among these considerations, the characteristics of a strong ground motion is of particular interest to this study. The other factors are beyond the scope of this study and can be taken care of by fulfilling the requirements of the National Building Code of Canada 1977⁽¹⁾. Thus, they will not be discussed here.

The available data on the characteristics of strong ground motions is limited to the recordings of ground acceleration histories obtained during several earthquakes over the past 40 years, including those recently collected in Canada. Thus, one solution to the problem of selecting a suitable ground motion is to make use of a previously recorded ground motion which has been appropriately scaled to account for the characteristics of the site. However, particular bias may be inherent in any one record. Experience had shown that different records, even though of the same intensity, may give widely differing structural response. Therefore, the use of a single record may leave substantial uncertainty as to the significance of the analytical result.

In view of the difficulty in selecting a suitable ground motion; it is preferable to use an artificially generated ground motion, which is some form of average earthquake and is free of any bias. By a statistical treatment of the spectrum curves of earthquake normalized to give the same level of intensity, Newmark et al⁽⁴³⁾ have derived a set of average response spectra which represent a 90% probability of not being exceeded by the records. These design spectra are now used by the U.S. Atomic Energy Commission for the design of nuclear power plants⁽⁴⁴⁾ and also form the basis for the design spectra given in the National Building Code⁽¹⁾.

The above mentioned spectra can be used in an analysis, however, when a time series analysis is contemplated, it is necessary to have a time history of the ground motion rather than a spectrum curve. It would be appropriate to use ground motion which would produce a response spectrum similar to the design spectrum. A method of generating such spectrum compatible motions is proposed by Tsai⁽⁴⁵⁾. An earthquake-like motion can be generated by using a spectrum-suppressing or raising technique to modify a suitably selected ground motion record, so that its response spectrum closely matches and envelopes the design spectrum.

Two spectrum compatible motions were generated by Humar⁽⁴⁶⁾ using the above method. The north-south component of EL-Cento 1940 ground motion was chosen as the base ground motion. This ground motion was first scaled and then progressively modified till its spectrum was reasonably close to a target spectrum. The target spectra used were those specified by the U.S. Atomic Energy Regulatory Commission for two percent and five percent damping. The five percent of critical damping spectrum-compatible ground motion shown in Fig.2.11 was used as the input ground motion in the time series dynamic analysis throughout this study.

FIG. 2.11 DESIGN SPECTRUM AND ACCELERATION SPECTRUM OF COMPATIBLE MOTION

2.5 COMPUTER PROGRAMS

Two computer programs have been used in this study to carry out the elastic and inelastic analyses of multistorey buildings subjected to earthquake motion.

The elastic analysis program is based on a program which was developed by Humar⁽²⁶⁾. The program was revised by the author to include the effects of shear deformations and finite joint size. Thus it is suitable for use in the elastic analysis of multistorey buildings, in which shear walls are incorporated. The program can be used to carry out a combined static-dynamic analysis of a multistorey orthogonal rigid jointed plane frame, subjected to a specified ground motion, given as a set of discrete values at a regular interval of time. The output from this program comprises of: an echo print of the input data, including member properties and loading data; member forces under static loads; time history of elastic response including member forces at selected intervals of time; maximum displacements; shear envelopes and maximum element forces.

The major features of this program are:

1. Vertical uniform and concentrated gravity loads applied along the beam can be accounted for.

2. Effects of shear deformation and finite joint size, and P-delta effect can be considered at user's option.
3. Damping matrix is assumed to be proportional to mass matrix.
4. Frames with set-back can be analyzed.
5. Column axial deformations are taken into account.
6. Linear acceleration method is used in the solution of equation of motion.
7. Stiffness matrix of the frame is stored in a special matrix S with elements on diagonal occupying the first row of S matrix.
8. The stiffness matrix is symmetric and banded, and remains constant. In solving the simultaneous equations of motion, Cholesky method is used. Once the stiffness matrix is decomposed into a lower and an upper triangular matrix, the decomposed form is used throughout the analysis. Since the upper triangular matrix is simply the transform of the lower triangular matrix, only the lower triangular matrix is stored.

9. Double precision is used in calculations in this program to reduce the round-off errors.

The program used in this study for the inelastic analysis of frames was developed by Morris⁽⁴⁷⁾ using Takeda's hysteresis model. This program can be used in static analysis, in elastic seismic analysis or combined static-seismic analysis, and in modal analysis as well.

The major features of the program are:

1. Either stiffness degrading or non-degrading model can be analyzed.
2. Either linear or constant acceleration method may be used at user's option.
3. Calculations can be retraced by using shorter intervals when changes of linearity occur, and unbalanced joint loads can be added to the joint at the following step to fit a given force-displacement relation.
4. Effects of shear deformations, finite joint dimensions and P-delta effect may be considered.
5. Column axial deformations are included in the program.

6. Damping is expressed as a linear combination of mass matrix and the instantaneous stiffness matrix.
7. Frames with set-backs can be analyzed.
8. The output includes: an echo print of all input data; static analysis results for gravity load effects; natural frequencies and mode shapes; floor responses at an interval specified by the user; and member force and floor response envelopes.

CHAPTER 3
ELASTIC ANALYSIS OF REINFORCED
CONCRETE SHEAR WALL-FRAME

3.1 INTRODUCTION

Since the late 1940's, it has become a common practice in multistorey building designs to provide a combined system of reinforced concrete shear walls and frames to resist the lateral forces due to wind or earthquakes. It has been shown that stiffening effect of the walls provide effective control of the deflections caused by lateral loads and minimizes non-structural damage which could occur in rigid frames under severe earthquakes. Such a system has therefore been found to be economic and efficient for resisting the intensive lateral forces in high-rise buildings.

The analysis of shear wall-frame structures is both complicated and time consuming. A considerable amount of research has been directed to this subject and a number of papers have been published on the subject of interaction of shear walls with frames in multistorey buildings^(10,11,12,48,49). Under lateral loads, a frame would deflect as in Fig.3.1(a). A shear wall, when subjected to lateral loads, would deflect as a cantilever as shown in Fig.3.1(b). When the two basically different components are tied together to form a structure, each one will try to obstruct the other from bending in its

FIG. 3.1 TYPICAL DEFORMATION MODES

natural deflected shape. Thus, a redistribution of forces between the two components would be expected, and the whole structure would deflect in a shape as shown in Fig.3.1(c).

Often, shear walls are designed to resist all lateral forces while the frames are assumed to carry only vertical loads. The participation of the frames in resisting lateral loads is ignored. This may not always be conservative and it is therefore important that the interaction between walls and frames be considered in the design. A number of methods have been presented for evaluating the interaction of shear walls and frames under static lateral loads. In most of them, simplifying assumptions have been made and the results are therefore approximate. However, these methods^(10,11,49) are still suitable for preliminary design, for general understanding of the characteristics of the combined system under static lateral loads, and perhaps for the equivalent static load analysis in seismic design.

Khan⁽¹¹⁾ has presented a set of influence curves to assist the users in the analysis of shear wall-frame systems. He has also pointed out that the common design assumption of assigning all lateral loads to the shear walls, is not strictly correct over the entire height of the structure.

and further that the distribution of lateral shear between the frame and the shear wall depends not only on their relative stiffness, but also on the number of storeys. In Khan's method, the structure is reduced into an equivalent single frame and an equivalent single wall by addition of the properties of the separate vertical units. The shears on the frame, moments on the shear wall and the deflections of the structure are then found by using a set of charts. However, the applicability of the method is limited to certain types of loading cases and certain relative stiffnesses of beams, columns and walls. Furthermore, even though the computed frame shears agree quite well with the exact values over most part of the building, there is considerable discrepancy at the top of low buildings, where the frame shear is underestimated by about 50%.

A method known as the constant frame-shear method has been presented by MacLeod⁽¹⁰⁾. It is assumed that the frame takes constant shear and equations are presented for evaluating the interaction force at the top of the structure. This method has more flexibility than the Khan's method mentioned above, but is inaccurate for the case when the wall is more flexible than the frame. Additionally, this method is accurate only when all the walls in the structure exhibit the same

behaviour under lateral load. In other words, if the properties of the walls change with height or if they have different base conditions, the accuracy will be unpredictable.

It is important to know whether the results obtained from approximate methods are reasonably accurate for use in design. Variations of the relative stiffnesses of the two components, and variations of the properties of walls with height may affect the accuracy and make the results unreliable. For buildings with irregular shape, the degree of interaction between walls and frames can be much more varied, and the results obtained from the approximate methods can not be considered satisfactory for design. However, for static elastic analysis of regularly shaped shear wall-frame structures, these methods are still satisfactory particularly when the results are to be used for a preliminary design.

Experimental studies have also been carried out^(13,14,50,51) during the past decade, to investigate the elastic response behaviour of shear wall and shear wall-frame structures subjected to static lateral loads and simulated ground motions. These experiments have contributed much toward the present understanding of the behaviour of shear wall components.

For the earthquake-resistant design of multistorey buildings, an equivalent static method is suggested by most

building codes, including the National Building Code of Canada. This method, though rather empirical, is useful because of its simplicity. Reasonable accuracy can be obtained when the method is used for regular frame type structures. However, the method is not applicable to structures with an irregularity in the distribution of stiffness or mass or both. It is recognized that for wall-frame structures, whose behaviour is complicated by the interaction between walls and frames, shear allocation among the vertical components is no longer as simple as in a frame structure. Therefore, even purely elastic seismic response cannot be predicted precisely by an equivalent static analysis. A dynamic analysis is necessary for an understanding of the overall true behaviour of a wall-frame structure subjected to earthquake ground motions. The availability of large computers in the recent years has made such an analysis feasible.

It is, however, recognized that the structure may undergo substantial inelastic deformations when subjected to an intense ground motion. Strictly speaking therefore, this inelasticity should be taken into account in the dynamic analysis of a structure. The difficulties inherent in analyzing the inelastic behaviour of a reinforced concrete shear wall-frame are:

- (1) The difficulty in modelling the material behaviour when stressed beyond the elastic limit.
- (2) The difficulty in modelling the distribution of stiffness along a member, especially for members such as reinforced concrete shear walls.
- (3) The difficulty in evaluating the interaction between wall and frame. It becomes more complicated when the structural deformations extend into the inelastic range.

Very little research work has been carried out into the inelastic dynamic analysis of reinforced concrete shear wall-frame structure at the present stage. Meaningful results from inelastic dynamic analyses can be obtained only when more is known about the behaviour of concrete shear wall sections in the inelastic range. Fortunately, some understanding of the behaviour of a shear wall type structure can still be obtained through elastic dynamic analyses.

This chapter presents the results of elastic dynamic analysis of reinforced concrete shear wall-frames. The computer programs described earlier were used in these analyses.

Studies were carried out on the response of wall-frames, with heights of five storeys, ten storeys and fifteen storeys. The mass of these frames was adjusted to give different values of period. For each frame, the natural frequencies and mode shapes were calculated first through a modal analysis. A dynamic analysis was then carried out to obtain the response to a specified ground motion. The time history of response and the member forces were computed by the program and were printed out at desired intervals. The program also gave the maximum displacements, the maximum storey shears at each floor level, and the maximum member forces obtained during the entire history of response. The time history of displacements at certain levels were obtained and plotted. Effects of certain parameters, such as P-delta force, shear deformation and finite joint sizes, on the modal response were examined. The influence of set-backs along building heights on the response and shear distribution were also studied.

3.2 DESCRIPTION OF WALL-FRAMES

The five storey, ten storey and fifteen storey wall-frame shown in Figs. 3.2, 3.3, 3.4 and 3.5 were selected for study. Member properties of these wall-frames are also shown in the figures. Certain simplifications were made in selecting the wall-frames. The loading, frame dimensions and member properties were kept unchanged along the height to ensure that the analytical results were not obscured by irrelevant details.

I = Moment of Inertia in in^4
 A = Area in in^2
 A_e = Effective Shear Area in in^2

FIG. 3.2 FIVE STOREY SHEAR WALL FRAME

FIG. 3.3 TEN STOREY SHEAR WALL-FRAME

FIG. 3.4 FIFTEEN STOREY SHEAR WALL-FRAME

I = Moment of Inertia in in^4
 A = Area in in^2
 A_e = Effective Shear Area in in^2

FIG. 3.5 FIFTEEN STOREY SHEAR WALL FRAME WITH SET-BACK

The basic data for these wall-frames are:

1. Frame spacing = 24 ft.
2. Floor height = 10 ft.
3. Dead load = 100 p.s.f.
4. Live load = 50 p.s.f.
5. 25% snow load = 12 p.s.f.
6. Seismic zone = zone 2 of the NBC
7. Acceleration (A) = .048 g
8. Ductility coefficient (K) = 1.0.
9. Important factor (I) = 1.0
10. Foundation factor (F) = 1.0
11. Wind load not applicable

The material properties are:

1. Concrete
 - 28-day compressive strength (f'_c) = 4000 p.s.i.
 - Young's modulus (E) = $57 \sqrt{f'_c} = 3605$ k.s.i.
 - Poisson's ratio (ν) = .16
 - Shear modulus (G) = 1554 k.s.i.
2. Reinforcement
 - Yield strength (f_y) = 60 k.s.i.
 - Young's modulus (E_s) = 29,000 k.s.i.
 - Modular ratio (n) = 8.0

The codes referred to are:

1. The National Building Code of Canada 1977 (N.B.C.)⁽¹⁾
2. Code for the Design of Concrete Structures for Buildings (CSA Standard A23.3-1973)⁽⁵²⁾
3. Building Code Requirements for Reinforced Concrete (ACI 318-71)⁽⁵³⁾

The design method used in sizing the members are:

1. The positive and negative gravity moments in the beams are obtained by applying the formula of Clause 6.5.3 CSA A23.3-1973:

$$M = K w L^2 \quad (3.1)$$

2. Lateral loads due to earthquake motion are obtained by using the equivalent static load method suggested by NBC. The distribution of lateral loads between frame and shear wall is obtained by using the approximate methods suggested by MacLeod⁽¹⁰⁾, and Khan et al⁽¹¹⁾. Forces in members due to lateral loads are calculated by using the portal method.
3. Load combinations for dead load and live load, and dead load, live load and earthquake load are considered when sizing the members. The following equations are used:

$$M_u = 1.4 D + 1.7 L \quad (3.2a)$$

$$M_u = .75 (1.4 D + 1.7 L + 1.8 E) \quad (3.2b)$$

4. Beams are designed to have equal reinforcement at both the top and the bottom faces.
5. Square columns with reinforcement equally distributed on four faces are used for all wall-frames.
6. Members are assumed to be well confined by transverse reinforcement.
7. Shear walls are designed according to the requirements for walls in CSA Standard A23.3-1973.
8. ACI Design Handbook⁽⁵⁴⁾ is used in sizing the members.

3.3 RESPONSE ANALYSIS OF FIVE STOREY SHEAR WALL-FRAME

3.3.1 DETAILS OF ANALYSIS

For the five storey wall-frames shown in Fig.3.2, the storey mass was adjusted to give four different fundamental periods, ranging from 0.3 to 1.0 second, giving in effect, four different wall-frames. A modal analysis was performed for each frame to obtain the mode shapes and frequencies. A dynamic analysis was carried out next for the standard ground motion described in Chapter 2. For simplicity,

the damping coefficient γ was assumed to be equal to zero, and the damping coefficient α was obtained from the natural frequency of the fundamental mode, using Eq.2.34 and a 5% damping in the first mode.

Preliminary studies had shown that the strong phase of the ground motion used in the analysis was during the first ten seconds, and the response generally reached its maximum value within this period. The dynamic analyses were therefore restricted to a ten second duration of the adopted ground motion. The frequency of the highest mode, obtained from the modal analysis, was used as a reference to select the increment of time for the dynamic analyses according to the criterion outlined in Section 2.3.2. This ensured the stability of the analytical results. The shear wall-frame with a fundamental period equal to 0.3 sec. was selected to carry out a study on the effect of P-delta force, shear deformations and finite joint sizes on the modal frequencies and the mode shape vectors of the wall-frame. The results of these studies are presented below.

3.3.2 P-DELTA EFFECT

The modal frequencies, the fundamental period and the mode shapes of the selected wall-frame obtained from the modal analysis are listed in Tables 3.1 and 3.2. The vibration characteristics of the wall-frame are reflected in the

TABLE 3.1 MODAL FREQUENCIES OF FIVE STOREY WALL-FRAME

MODE	JOINT SIZE = 1 SHEAR = 1		P-Δ = 0		
	P-Δ = 1 (1)	P-Δ = 0 (2)	JOINT SIZE = 1 SHEAR = 0 (3)	JOINT SIZE = 0 SHEAR = 1 (4)	JOINT SIZE = 0 SHEAR = 0 (5)
1	20.922	20.943	22.767	18.049	19.413
2	91.164	91.193	131.212	82.443	118.567
3	190.479	190.519	375.115	172.555	333.603
4	276.533	276.581	762.818	248.181	643.552
5	334.358	334.411	1234.550	297.791	957.735
FUND. PERIOD (sec.)	0.300	0.300	0.276	0.348	0.324

TABLE 3.2 FUNDAMENTAL PERIOD MODE SHAPE VECTORS OF FIVE STOREY WALL-FRAME

STOREY LEVEL	JOINT SIZE = 1 SHEAR = 1			P-Δ = 0		
	P-Δ = 1 (1)	P-Δ = 0 (2)		JOINT SIZE = 1 SHEAR = 0 (3)	JOINT SIZE = 0 SHEAR = 1 (4)	JOINT SIZE = 0 SHEAR = 0 (5)
1	1.000	1.000	1.000	1.000	1.000	1.000
2	2.748	2.748	2.748	3.294	3.014	3.611
3	4.948	4.947	4.947	6.386	5.659	7.281
4	7.322	7.320	7.320	9.869	8.596	11.524
5	9.656	9.654	9.654	13.466	11.552	15.978

mode shape vectors, each of which represents the relative amplitudes of motion for each of the displacement components in a certain vibration mode. By comparing columns 1 and 2 in Table 3.2, it is seen that the fundamental mode shapes are almost unchanged by the inclusion of P-delta effect in the analysis. Since the dynamic response is generally dominated by the fundamental mode, it would appear that the effect of P-delta forces on responses will be small. The effect of the P-delta forces on the modal frequencies can be observed by comparing the results in columns 1 and 2 of Table 3.1. It is seen that modal frequencies decrease but only slightly by the inclusion of P-delta effect. The P-delta effect has therefore been ignored in the further investigations of the wall-frames.

3.3.3 EFFECT OF FINITE JOINT SIZE

By comparing columns 2 and 4, and columns 3 and 5 of Table 3.2, it is seen that the fundamental mode shape vectors change significantly when the effect of finite joint size is neglected. For facilitating the comparison, the fundamental mode shape vectors in columns 2, 3, 4 and 5 have been normalized so that the value of modal displacement at the top of the wall-frame is unity, and are plotted in Fig.3.6.

The effect of finite joint size on the modal frequencies can be seen from the results presented in Table 3.1. It is

FIG. 3.6 NORMALIZED FUNDAMENTAL MODE SHAPE VECTORS OF FIVE STOREY WALL-FRAME
 (with $T = 0.3$ sec., $P-\Delta = 0$)

2

A

seen, by comparing columns 2 and 4, that when the effect of finite joint size is neglected, the modal frequencies decrease by 10-14%. A comparison of columns 3 and 5 shows that the decrease is even more when shear deformation is ignored. Since the shear wall incorporated in the frame has a considerably larger width as compared to a column, the finite size of the beam shear wall joint adds significantly to the stiffness of the frame increasing its frequency of vibration. It would therefore be appropriate to include the effect of finite joint widths in the analysis of shear wall-frames.

3.3.4 EFFECT OF SHEAR DEFORMATION

The effect of shear deformation on the fundamental mode shape vectors can be seen from Fig.3.6. These fundamental mode shape vectors have been normalized so that deflections at the top of wall-frames are equal to unity. It is seen that even though the mode shape changes, the change is not drastic and the general shape remains the same. The effect of shear deformation on the modal frequencies is however very significant. Comparison of columns 2 and 3 in Table 3.1 indicates that exclusion of the effect of shear deformation results in a large increase in the modal frequencies of the wall-frame, the effect being more pronounced for the higher modes. It is obvious that this

large change in modal frequencies results from a change in the shear flexibility factor for the shear wall as seen from Eqs. 2.2b and 2.2c. The change in shear flexibility factor alters the stiffness matrix of the shear wall. For a normal size column, the shear flexibility factor is much smaller than unity, and the change in the stiffness matrix as shown in Eqs. 2.2, 2.2a and 2.2b is comparatively small. However, a shear wall has a great depth, its moment of inertia is therefore much larger than its area. Consequently, the shear flexibility factor would be of the same order as unity and may at times be greater than 1. This will result in a significant change in the values of the terms S_a and S_b in the member stiffness matrix. In fact, the sign of term S_b as shown in Eqs. 2.2a and 2.2b will become negative when the shear flexibility factor is greater than one. It is evident that shear deformation has to be taken into account in the analysis of reinforced concrete shear wall-frames.

3.3.5. RESPONSE OF FIVE STORY WALL-FRAME

Modal analyses of the wall-frames with different values of the storey mass were carried out to obtain their modal frequencies and mode shapes. As stated earlier, the P-delta effect was neglected, and finite joint widths and shear deformations were considered in analyses. The mode shapes are plotted in Fig. 3.7. They are common to the four wall-frames. Modal frequencies are listed in Table 3.3. It is

FIG. 3.7 MODE SHAPES OF FIVE STOREY WALL-FRAME

TABLE 3.3 MODAL FREQUENCIES OF FIVE STOREY WALL-FRAMES*

FRAME NO.	FUND. PERIOD (sec.)	FLOOR LOAD (lb./sq.ft.)	FLOOR MASS ($\frac{K - S^2}{\text{in.}}$)	FREQUENCY				
				1 st MODE	2 nd MODE	3 rd MODE	4 th MODE	5 th MODE
1	0.3	209.6	0.5858	20.943	91.193	190.519	276.581	334.411
2	0.5	582.1	1.6271	12.566	54.718	114.316	165.954	200.653
3	0.7	1140.9	3.1891	8.976	39.084	81.654	118.538	143.322
4	1.0	2328.4	6.5084	6.283	27.359	57.158	82.977	100.327

* Effect of finite joint width and shear deformation were included in analyses, whereas P-delta effect was ignored.

observed that the ratio of the second mode frequency to the first mode frequency is equal to 4.35, and the ratio of the third mode frequency to the second mode frequency is 2.09. The ratios of the frequency of a higher mode to that of the previous mode gradually decreases as the mode number increases. The decrease in these ratios is a pattern common to all wall-frames with different fundamental periods.

Elastic dynamic analyses were carried out for these wall-frames by using the computer program developed by the author. The damping was 5% in the fundamental mode and the ground motion was the standard motion described earlier. The maximum shear distribution along the height is plotted in Fig.3.8 for all wall-frames. In these plots, the maximum shear V_x at any level is expressed as a fraction of the total base shear V_b . The equivalent static load method of the code will give an inverse triangular type of force distribution for these frames⁽¹⁾. The resulting shear distribution is in the form of a parabola and is also shown on Fig.3.8. Comparing the results of the analysis with the shear distribution suggested by the code, it is seen that the code method underestimates the shear above 0.6 H, and overestimates it in the lower $\frac{6}{10}$ of the height of the wall-frame. Maximum displacements and relative displacements for these wall-frames are plotted in Fig.3.9. Both the maximum displacements and relative displacements increase with increasing fundamental period.

FIG. 3.8 MAXIMUM SHEAR DISTRIBUTION OF FIVE STOREY WALL-FRAMES

(a) MAXIMUM DISPLACEMENT (IN)

(b) MAXIMUM RELATIVE DISPLACEMENT (IN)

FIG. 3.9. RESPONSE OF FIVE STOREY WALL-FRAMES

Fig. 3.10 shows the ten second displacement time history at the top of the five storey wall-frame with a fundamental period of 0.3 second. It is seen that the wave-form of the displacement is strongly influenced by the free vibration period. The wall-frame oscillates at essentially the fundamental natural frequency. The maximum response appears at about 2.6 second after starting of the ground motion.

The shear diagram for the wall frame, the bending moments in vertical members and the axial forces in a column at the time of peak response are shown in Fig.3.11. A comparison of the bending moment diagram of the wall with that of the column as shown in Figs.3.11b and 3.11c, makes it clear that the behaviours of these two vertical members are essentially different. The shear wall is basically a cantilever column. Interaction with the frame, somewhat modifies its moment distribution. The column shows a pure frame action as can be seen from Figs.3.11c and 3.11d. The axial force in the wall is negligible in this case, and is not plotted.

3.4. ANALYSIS OF TEN STOREY WALL-FRAME

The method used in the analysis of the ten storey wall-frame is similar to that for the five storey wall-frame. The mass was adjusted to give four different wall-frames with fundamental periods of 0.5, 1.0, 1.5 and 2.0 sec. As illustrated earlier, the P-delta effect was considered insignificant and was ignored in the analyses; whereas, both the finite

FIG. 3.10 DISPLACEMENT TIME HISTORY OF FIVE STOREY WALL-FRAME
(with $T = 0.3$ sec.)

FIG. 3.11 RESPONSE OF FIVE STOREY WALL-FRAME AT $t = 2.6s$ (with $T = 0.3s$)

joint widths and the shear deformation effects were included. Modal analyses were first carried out to obtain the mode shapes and the modal frequencies. Fig.3.12 shows the first five mode shapes of the chosen wall-frame. The modal frequencies of the first five modes are listed in Table 3.4. As in the case of five storey wall-frame, the ratios of the higher mode frequency to the previous mode frequency decrease gradually when the mode number increases.

From the results of the modal analyses, the damping coefficients were found and dynamic analyses were then carried out for the standard ground motion with 5% damping in the first mode.

Fig.3.13 shows the maximum shear distribution along the height, expressed as the ratio of floor shear to the base shear. Again it is seen that the triangular force distribution suggested by the code results in an underestimation of the shear in the upper floor and in general an overestimation of the shear in the lower floors. For completeness, the maximum displacements and the relative displacements are also plotted in Fig.3.14. Again, with increasing fundamental period, both the maximum displacements and relative displacements increase.

Fig.3.15a shows the displacement time history of the ten storey wall-frame with a fundamental period of 1.0 sec.

FIG. 3.12 MODE SHAPES OF TEN STOREY WALL-FRAME

TABLE 3.4 MODAL FREQUENCIES OF TEN STOREY WALL-FRAMES

FRAME NO.	FUND. PERIOD (sec.)	FLOOR LOAD (lb./sq.ft.)	FLOOR MASS ($\frac{K - S^2}{in.}$)	FREQUENCY				
				1 st MODE	2 nd MODE	3 rd MODE	4 th MODE	5 th MODE
1	0.5	140.3	0.3923	12.566	55.013	123.887	206.321	291.430
2	1.0	561.4	1.5692	6.283	27.507	61.944	103.161	145.716
3	1.5	1263.2	3.5307	4.189	18.338	41.296	68.774	97.144
4	2.0	2245.7	6.2768	3.140	13.739	30.951	51.567	72.747

FIG. 3.13 MAXIMUM SHEAR DISTRIBUTION OF TEN STOREY WALL-FRAMES

FIG. 3.14 RESPONSE OF TEN STOREY WALL-FRAMES

The wall-frame vibrates basically in its fundamental vibration period. To study the effect of the frequency content of the base motion on the vibration frequency of the selected wall-frame, the time axis of the generated earthquake ground motion ~~was~~ compressed by a factor of 2.0, thus increasing the higher frequency content of the ground motion. From the displacement time history of the frame for the modified ground motion plotted in Fig.3.15b, it is observed that in spite of the different frequency content of the input ground motion, the vibration of the wall-frame is dominated by its fundamental period. It is also seen that the wall-frame does not develop its maximum response within the strong phase of the ground motion which is in the first four seconds but the maximum response is attained at $t=9.0$ sec. It may be recognized that the duration of the ground motion has a dominant influence on the magnitude of the responses.

The shear diagram, moment diagrams of the vertical members and column axial force at $t=9.0$ sec. for the above wall-frame are plotted in Fig.3.16. As shown in Fig.3.16b, the behaviour of the wall is essentially like a cantilever member, although interaction with the frame modifies its moment diagram slightly. Again, the column acts in a pure frame action.

FIG. 3.15 DISPLACEMENT-TIME HISTORY OF TEN STOREY WALL-FRAME (with $T = 1.0s$)

FIG. 3.16 RESPONSE OF TEN STOREY WALL-FRAME AT $t = 9.0 \text{ sec.}$

(with $T = 1.0 \text{ sec.}$)

3.5 ANALYSIS OF FIFTEEN STOREY WALL-FRAME AND EFFECT OF SET-BACK

Two fifteen storey shear wall-frames as shown in Figs.3.4 and 3.5 were selected for a study of the characteristics of their dynamic response. Fig.3.4 shows a fifteen storey wall-frame with the shear wall continued uniformly throughout the height. The wall-frame in Fig.3.5 has the shear wall reduced to half of its width above the 10th floor. The sudden change in stiffness along the height can be expected to have a significant effect on the dynamic response of such a frame.

Procedures used in the analysis of the selected wall-frames were similar to those used for the five and ten storey wall-frames. The masses of the two chosen wall-frames were adjusted to give four different wall-frames with fundamental periods of 1.0, 1.5, 2.0 and 2.5 sec. The modal frequencies of the first five modes and the last mode as obtained from a modal analysis are listed in Tables 3.5 and 3.6 for fifteen storey uniform wall-frame and set-back wall-frame respectively.

The variation in the modal frequencies of the four wall-frames as shown in Table 3.5 and 3.6 follows a pattern that is similar to the five and ten storey wall-frames. By comparing the modal frequencies in the set-back wall-frame to that of the related uniform wall-frame, it is of interest

TABLE 3.5 MODAL FREQUENCIES OF FIFTEEN STOREY UNIFORM WALL-FRAMES

FRAME NO.	FUND. PERIOD (sec.)	FLOOR LOAD (lb./sq.ft.)	FLOOR MASS ($\frac{K - S^2}{in.}$)	FREQUENCY					
				1 st MODE	2 nd MODE	3 rd MODE	4 th MODE	5 th MODE	15 th MODE
1	1.0	156.2	0.4365	6.283	27.603	63.688	109.335	161.455	535.891
2	1.5	351.4	0.9822	4.189	18.402	42.457	72.887	107.634	357.268
3	2.0	624.7	1.7461	3.142	13.801	31.843	54.666	80.725	267.952
4	2.5	976.2	2.7284	2.513	11.041	25.474	43.732	64.579	214.362

TABLE 3.6 MODAL FREQUENCIES OF FIFTEEN STOREY SET-BACK WALL FRAMES

FRAME NO.	FUND. PERIOD (sec.)	FLOOR LOAD* (lb./sq.ft.)	FLOOR MASS* $(\frac{K - S^2}{in.})$	FREQUENCY					
				1 st MODE	2 nd MODE	3 rd MODE	4 th MODE	5 th MODE	15 th MODE
1	1.0	150.6 (140.5)	0.4208 (0.3926)	6.283	22.271	47.823	84.001	122.759	541.630
2	1.5	338.7 (316.1)	0.9468 (0.8834)	4.189	14.847	31.881	56.000	81.838	361.085
3	2.0	602.2 (591.9)	1.6832 (1.5704)	3.142	11.135	23.911	42.000	61.379	270.818
4	2.5	941.0 (877.9)	2.63 (2.4538)	2.513	8.908	19.129	33.600	49.103	216.656

* Note: figures in bracket are for the set-back portion.

to find that the frequencies of the second mode and the third mode of the set-back wall-frame are smaller than those of the related uniform wall-frames; whereas the frequency of the highest mode is greater than that of the latter one. The ratio of the modal frequency of the set-back wall-frame to that of the same mode of the related uniform wall-frame decreases first and increases later on with increasing mode number.

The first five mode shapes of the two wall-frames have been normalized so that the value of modal displacement at the top of the wall-frame is unity, and have been plotted in Fig.3.17. It is observed that for the set-back wall-frame, although the shear wall is reduced by 50% of its width, so that the moment inertia is reduced to $1/8$ of the original value, the normalized mode shape vectors of the set-back wall-frame is not much different from that of the uniform wall-frame in the lower modes of vibration. The difference becomes more significant in the higher modes.

The shear distribution along the height of the uniform and set-back wall-frames are plotted in Figs.3.18 and 3.19. Again it can be seen that the triangular force distribution of the code will result in an underestimation of the shears in the upper storeys and an overestimation of the shears in

FIG. 3.17 MODE SHAPES OF FIFTEEN STOREY WALL-FRAMES

FIG. 3.18 SHEAR DISTRIBUTION OF FIFTEEN STOREY UNIFORM WALL-FRAMES

FIG. 3.39 SHEAR DISTRIBUTION OF FIFTEEN STOREY SET-BACK WALL-FRAMES

the lower storeys. The maximum displacements and relative displacements are plotted in Figs.3.20 and 3.21 for different fundamental periods of the two wall-frames.

The maximum displacements and relative displacements increase with increasing period. It is also seen that the displacements are dominated by the first mode shape. The accentuated response of the set-back portion is evident from the plot of relative displacements and Fig.3.22 which shows a marked increase above the 10th floor level. Displacement time history at top floor, 10th floor, and 5th floor are plotted in Fig.3.23. It is of interest to note that the displacement wave-forms of the two wall-frames with the same fundamental period of 1.5 sec. are almost the same in terms of their amplitudes and the time of peak displacement. Only a slight time lag can be found at the peak displacement. The response reaches the maximum at about 8.5 sec. for the uniform wall-frame, and only slightly later for the set-back wall-frame. For easy comparison, the shear diagrams, wall moment diagrams, column moment diagrams and column axial force diagrams are plotted at the same time of 8.5 sec. for the two wall-frames as shown in Figs.3.24 and 3.25. It is apparent that the column moment and axial force reduce considerably above 10th floor for the set-back wall-frame; whereas the change in wall moment diagram is not so significant, even

FIG. 3.20 RESPONSE OF FIFTEEN-STOREY UNIFORM WALL-FRAMES

FIG. 3.21 RESPONSE OF FIFTEEN STOREY SET-BACK WALL-FRAMES.

(a) MAX. DISPLACEMENT (IN)

(b) MAX. RELATIVE DISPLACEMENT (IN)

FIG. 3.22. DISPLACEMENT OF FIFTEEN STOREY WALL-FRAMES (with $T = 2.5$ sec.)

FIG. 3.23 DISPLACEMENT TIME HISTORY OF FIFTEEN STOREY WALL-FRAMES
(with $T = 1.5$ sec)

FIG. 3.24 RESPONSE OF FIFTEEN STOREY UNIFORM WALL-FRAME AT $t = 8.5 \text{ sec.}$

(with $T = 1.5 \text{ sec.}$)

FIG. 3.25 RESPONSE OF FIFTEEN STOREY SET-BACK WALL FRAME AT $t = 8.5$ sec.

(with $T = 1.5$ sec.)

though the wall stiffness is much lower in this portion. This is because the set-back portion undergoes increased displacement.

3.6 EFFECT OF BUILDING HEIGHT

Elastic analyses have been carried out for wall-frames of five storeys, ten storeys and fifteen storeys, as illustrated earlier. The results of earlier analyses have been summarized in this section, with a view to finding out the effect of the building height on the floor displacements and storey shears. Fig.3.26 shows the maximum floor displacements for wall-frames having 5, 10 and 15 storeys all with a fundamental period of vibration of 1.0 sec. The 15 storey set-back wall-frame is also included. It is observed that the floor displacement do not differ much. The floor displacements of the set-back wall-frame are in general smaller than those of the related uniform wall-frame, and the floor displacements of the 5 storeys frame are close to those of the 15 storey frame. As stated earlier, this is due to the fact that the displacement pattern is dominated by the fundamental mode; the change in higher modes has an insignificant effect on the floor displacements.

3.7 SHEAR COEFFICIENTS AND SHEAR DISTRIBUTION

A clearer understanding of the shear distribution along the height can be obtained by plotting the values of shear

FIG. 3.26 MAXIMUM FLOOR DISPLACEMENT OF WALL-FRAMES WITH
DIFFERENT HEIGHT (with $T = 1.0$ sec.)

coefficients. A seismic shear coefficient is defined as the ratio of the seismic shear at a particular level to the total weight of the wall-frame above that level. Thus

$$C_x = V_x / W_x \quad (3.3)$$

where C_x and V_x represent respectively the shear coefficient and storey shear at level x , and W_x represents the total weight of the wall-frame above that level. A normalized value of shear coefficient, given by C_x / C_b is used herein for easy comparison.

As stated earlier, the inverse triangular distribution of force proposed by the code underestimates the shears in the upper floors. This discrepancy becomes larger as the fundamental period of the building increases. Recognizing this, Humar and Wright⁽⁵⁵⁾ have proposed empirical equations for obtaining the distribution of shears along the height of the building. These equations relate the shear distribution to the height of the building and were obtained from the analyses of hybrid cantilever models of buildings. The normalized shear coefficient C_x / C_b is given by the following equation:

$$\frac{C_x}{C_b} = 1 + \frac{0.156 \frac{h_x}{H}}{0.6T - (T - 0.26) \frac{h_x}{H}} \quad (3.4a)$$

for $h_x \leq 0.6 H$, and

$$\frac{C_x}{C_b} = 1.6 + 3.85 T \left(\frac{h_x}{H} - 0.6 \right) \quad (3.4b)$$

for $h_x \geq 0.6 H$.

Where T stands for the period of the wall-frame, and C_b is given by:

$$C_b = \frac{V_b}{W} \quad (3.5)$$

where V_b is the base shear and W represents the total weight of the wall-frame.

Figs. 3.27 through 3.30 compare the exact values computed by the program, the triangular force distribution suggested by the National Building Code of Canada, and the empirical values suggested by Humar as stated above.

As stated earlier, it can be seen from the graphs that the triangular force distribution results in an underestimation of the shear in the upper storeys and an overestimation in the lower portion of the wall-frame. However, the empirical equations give a much better approximation to the shear distribution as shown.

To study the interaction between the wall and the frame the shears carried by each of these components at the time of peak displacement at the top floor have been plotted at Figs. 3.31 to 3.34 for the five storey, ten storey and the uniform and set-back fifteen storey wall-frames. For comparison, the total storey shears at the same instant as shown in Figs. 3.11a, 3.16a, 3.24a, and 3.25a have also been plotted after normalizing. It is seen that a major portion of the shear at levels below $0.6 H$ is sustained by the shear wall.

FIG. 3.27 SHEAR COEFFICIENTS FOR FIVE STOREY WALL-FRAMES

FIG. 3.28 SHEAR COEFFICIENTS FOR TEN STORY WALL-FRAMES

FIG. 3.29 SHEAR COEFFICIENTS FOR FIFTEEN STORY UNIFORM WALL-FRAMES

FIG. 3.30 SHEAR COEFFICIENTS FOR FIFTEEN STORY SET-BACK WALL-FRAMES

FIG. 3.3] SHEAR DISTRIBUTION IN FIVE STOREY WALL FRAME AT $t = 2.6$ sec.

(with $T = 0.3$ sec.)

FIG. 3.32 SHEAR DISTRIBUTION IN TEN STOREY WALL-FRAME AT $t = 9.0$ sec.
(with $T = 1.0$ sec.)

FIG. 3.33 SHEAR DISTRIBUTION IN FIFTEEN STOREY UNIFORM WALL FRAME AT $t = 8.5$ sec.

(with $T = 1.5$ sec.)

FIG. 3.34 SHEAR DISTRIBUTION IN FIFTEEN STOREY SET-BACK WALL-FRAME AT $t = 8.5$ sec.

(with $T = 1.5$ sec.)

The portion carried by the shear wall is of the order of 96% at the base to above 80% at $0.6 H$ of the wall-frame. The proportion of the shear sustained by the frame increases significantly towards the top. In the higher wall-frames, in fact, a negative force acts at the top of the wall to restrain the wall from bending in its natural deflected shape of a cantilever.

CHAPTER 4

INELASTIC ANALYSIS OF FRAMES

4.1 INTRODUCTION

The equivalent static lateral loading recommended by the codes^(1,2) for the design of structures against earthquakes are quite low as compared to those obtained through an elastic analysis of response to recorded ground motions. In fact, even a moderate earthquake causes significantly greater stresses and deformation than the code-forces do. In spite of this large difference the structure designed for the code forces have been known to have performed satisfactorily even during rather severe earthquakes. This satisfactory performance is attributed to the ability of ductile structures to withstand substantial inelastic deformation. A knowledge of the extent of inelastic deformation in building structures subjected to real ground motions is therefore necessary for the proper design of structural system and members.

The most difficult part in obtaining estimates of the inelastic deformation or ductility demand is the modelling of inelastic material behaviour. An elasto-plastic moment-curvature relationship as shown in Fig.4.1a has been used quite frequently^(24,25,26,56) because of its mathematical

FIG. 4.1 MOMENT-CURVATURE RELATIONSHIP

simplicity and good representation of the behaviour of structural steel under constantly increasing loads. A bilinear hysteretic model as shown in Fig.4.1b has also been used^(27,28) and has proved useful in simulating the effect of strain hardening in steel member. The Ramberg-Osgood model shown in Figs.4.1c and d has been used by Jennings⁽⁵⁷⁾ to represent the Bauschinger effect in structural steel.

On the basis of experimental and theoretical investigation, Hanson⁽²¹⁾ and Takeda⁽¹⁸⁾ found that the post yield moment-curvature relationship for reinforced concrete section was different from that for steel members. In concrete members, stiffness of the section does not return to the initial elastic stiffness as yielding ceases, rather, with increasing inelastic deformation, it degrades as a function of its stress history.

It is apparent that for a given amplitude of deformation, the energy absorbed per cycle in a system with degrading stiffness would be less than that in a non-degrading system. It is therefore necessary to investigate the effect of degrading stiffness on the inelastic deformation or ductility demand. An investigation into the effect of stiffness degradation on the ductility demand of single degree of freedom system was carried out by Clough⁽²⁹⁾ in 1968. He studied a simple single storey shear type building, and compared the ductility

requirements obtained from an elasto-plastic model with those obtained from a stiffness degrading model. It was found that stiffness degradation did not have a significant influence on ductility requirements of long period structures, whereas it had a detrimental effect on short period structures, and resulted in significant increase in the ductility demand.

Similar investigation was carried out by Chopra and Kan⁽³⁴⁾ on multi-degrees of freedom systems. They found that stiffness degradation had little influence on ductility requirements for flexible shear type buildings, but led to increased ductility demand for stiff buildings. Chopra and Kan's results were based on the study of shear type buildings in which the girders were assumed rigid and storey shear resistance was assumed to follow a degrading hysteretic cycle. In real buildings, girders are flexible and inelastic deformations are localized at certain sections of the members. The localized ductility requirements may be greater than those computed from the storey deflections of shear type buildings. Therefore, a more sophisticated model of inelastic member behaviour has to be used in order to obtain realistic estimates of ductility requirements.

Different analytical models for reinforced concrete members have been proposed by several researchers, and earthquake simulator tests have been carried out to examine the effectiveness of these models in simulating the observed response of inelastic model structures. Shake table tests were carried out by Takeda, Sozen and Nielsen⁽²²⁾, and Otani and Sozen⁽⁵⁸⁾ on concrete specimen and the experimental results were compared with those obtained from inelastic response analyses based on a hysteretic model proposed by Takeda⁽¹⁸⁾. It was found that the agreement between the analytical results and the experimental values measured through the tests was satisfactory.

This chapter is devoted to the study of the effect of stiffness degradation on the ductility demand of reinforced concrete frames. Takeda's hysteresis model with a simplified bilinear backbone curve has been adopted in this study to represent the stiffness degradation of a flexural reinforced concrete member. The correlation between the elastic and inelastic response is also investigated in order to evolve a rational design method in which the results of an elastic analysis can be used after suitable modification. A single storey frame, a five storey frame, and a ten storey frame, as shown in Figs.4.2, 4.3 and 4.4 respectively, are selected for this study. Inelastic dynamic responses are obtained

FIG. 4.2 SINGLE STOREY REINFORCED CONCRETE FRAME

FIG. 4.3 FIVE STOREY FRAME

FIG. 4.4 TEN STOREY FRAME

for both a degrading and non-degrading hysteresis relationship by using a computer program developed by Morris⁽⁴⁷⁾. The results obtained from the two sets of analyses are presented and compared with each other.

4.2 MOMENT-ROTATION RELATIONSHIP

The following assumptions are made in determining the shape of the moment-curvature curve for a flexural reinforced concrete section:

- (1) Plane sections before bending remain plane after bending.
- (2) The strains in reinforcement and the concrete at the same level are identical.
- (3) The stress strain curve for concrete subjected to compression as well as the limit of compressive concrete strain is known.

The first and second assumptions are nearly correct at all stages of loading up to flexural failure, provided good bond exists between the concrete and steel. The stress-strain curve for concrete subjected to compression is shown in Fig.4.5, and a value of .004 is adopted as the limit of compressive concrete strain in this study to define the ultimate strength point and hence the slope of the branch beyond yielding.

FIG. 4.5 STRESS-STRAIN RELATIONSHIP FOR CONCRETE

For a flexural reinforced concrete section, the moment-curvature relationship can be idealized by a trilinear relationship shown in Fig.4.6. The first segment represents the behaviour of the section up to cracking, the second to yield, and the third to the ultimate strength which is reached at the limit of strain in concrete. Based on the assumptions stated above and by using the principles of mechanics, the shape of the moment-curvature for a given section can be determined.

When a section is subjected to an axial load in addition to the moment, the moment curvature relationship is modified. In fact, an increase in axial load increases the ultimate moment, provided the maximum axial load is less than the balanced load. The moment-curvature relationship for sections carrying axial load in addition to moment can also be derived on the basis of assumptions similar to the above.

For a beam subjected to end moments of the same sign but of different magnitudes, the curvature diagram can be obtained from the known distribution of moments as shown in Figs.2.8a and b. The shaded portion in the diagram represents inelastic deformation. By integrating the curvature diagram, it is possible to obtain the end rotations. The moment-rotation relationship for an elastic member can be expressed as follows:

FIG. 4.6 MOMENT CURVATURE RELATIONSHIP FOR REINFORCED CONCRETE

$$\begin{Bmatrix} \theta_l \\ \theta_r \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} M_l \\ M_r \end{Bmatrix} \quad (4.1)$$

Where θ and M indicate respectively end rotation and end moment of the member, f is the flexibility influence coefficient and subscripts l and r refer to the left and right end of the member respectively.

The flexibility influence coefficients in the flexibility matrix on the right hand side of Eq. 4.1 depend only on the properties of the cross-section and are independent of the values of end-moments. When portions of the member become inelastic, the relationship between end moments and end rotations is not linear and a flexibility-relationship of the type of Eq. 4.1 can no longer be obtained. Since moment rotation relationships continuously change with the change in end-moments, an inelastic analysis becomes highly complex and time consuming.

As stated in Section 2.2.2(B), the member is modelled as consisting of an elastic line element with an inelastic hinge at each end of the member, which assumes that the inelastic rotations are concentrated at the ends of the member. The end rotation can then be obtained by the superposition of the elastic and inelastic components. Since the elastic flexibility influence coefficients do not depend on the end moments, they can easily be obtained. To derive the inelastic

flexibility influence coefficients, a reasonable assumption has to be made about the location of the inflexion point. It is assumed in this study that the inflexion point is located at the center of members. Disregarding the fact that inflexion point may shift from its assumed location, the inelastic moment-rotation relationship derived from the assumed location of the inflexion points is used throughout the analysis. The results obtained on the basis of this approximation are reasonable if the shift of the inflexion point is not extensive and if the spring characteristics are determined on the basis of an average location of the inflexion point.

In this study, since the frames selected for analysis are single bay and symmetrical, and gravity loads are not present, the inflexion point will always be located at the center of the beams. Although the inflexion point in some of the columns will not be exactly at the center, this will not introduce any significant errors because most columns will be found to remain elastic.

By using the moment-curvature relationship of Fig.4.6, and the moment distributions shown in Fig.4.7, the following expressions can be obtained for θ_c , θ_y and θ_u , which are the values of end rotation when the end moment reaches its value at cracking, yielding, and ultimate respectively.

FIG. 4.7 DERIVATION OF CRACKING, YIELDING AND ULTIMATE ROTATIONS

$$\theta_c = \frac{M_c L}{6EI} \quad (4.2)$$

$$\theta_y = \frac{L}{6} [(1-\beta^3)\phi_y + \beta^2\phi_c] \quad (4.3)$$

and

$$\begin{aligned} \theta_u = \frac{L}{12} [& \{(2+\lambda_2)(1-\lambda_2)(1+\lambda_2\lambda_3 - \lambda_2)/\lambda_3 \\ & + \lambda_2(1+\lambda_2) - 2\lambda_1^3\} \phi_y/\lambda_2 + 2\lambda_1^2\phi_c] \end{aligned} \quad (4.4)$$

where M_c = Moment at cracking of tensile concrete,

M_y = Moment at yielding of tensile reinforcement,

M_u = Moment at ultimate (crushing of compressive concrete),

ϕ_c = Curvature at cracking of tensile concrete,

ϕ_y = Curvature at yielding of tensile reinforcement,

ϕ_u = Curvature at ultimate (crushing of compressive concrete),

$$\beta = \frac{M_c}{M_y},$$

$$\lambda_1 = \frac{M_c}{M_u},$$

$$\lambda_2 = \frac{M_y}{M_u},$$

$$\lambda_3 = \frac{M_u - M_y}{\phi_u - \phi_y} \cdot \frac{\phi_y}{M_y}$$

In deriving the above expressions, it is assumed that the effective moment of inertia of the member is constant throughout the length.

Since M_c , M_y and M_u are known, the three values of θ_c , θ_y and θ_u can be obtained by using Eqs. 4.2, 4.3 and 4.4. The complete trilinear moment-rotation curve with break points

(θ_c, θ_c) , (θ_y, θ_y) and (θ_u, M_u) is plotted in Fig. 4.8.

The trilinear moment-rotation relationship can be further simplified if the discontinuity at cracking moment is disregarded. As shown by a dashed line in Fig. 4.8. The relationship is then represented by two straight line segments OB and BC. This simplified moment rotation relationship is used throughout this study.

As a part of this study, the stiffness degrading response of an example frame was obtained with the two alternative hysteretic models, one with a trilinear primary curve, and the other with an idealized bilinear curve. It was found that there was no significant difference between the results obtained from the two sets of analyses.

4.3 HYSTERESIS MODELS

As stated earlier, experimental observation of reinforced concrete members under static load reversals has revealed that the bilinear hysteresis relationship is not representative nor realistic. Rather, the stiffness of members degrades with increasing cyclic inelastic deformation. The behaviour can be explained by considering the cantilever beam shown in Fig. 4.9. Initially, the force-displacement relationship is straight line AB, and can be easily evaluated from elementary mechanics. After initial cracking, there is an immediate redistribution

FIG. 4.8 PRIMARY MOMENT-ROTATION CURVE

FIG. 4.9 BEHAVIOUR OF A CYCLICALLY LOADED CANTILEVER

of the internal forces and the member loads thereafter at a reduced slope to the point C where the tensile reinforcement starts to yield. The post yield slope k_{CD} of line segment CD depends primarily on the strain hardening properties of the reinforcement. If the member is now gradually unloaded, the slope k_{DE} of line segment DE is influenced by the lack of fit of the cracked surfaces, its value is reduced by an exponential function of previous maximum deformation⁽¹⁸⁾ as expressed by the following equation:

$$k_{DE} = k_{AC} (R)^{\alpha} \quad (4.5)$$

where k_{DE} and k_{AC} are the slopes of DE and AC respectively, R is the ratio of the maximum displacement attained at loading to the yield displacement, and α is taken as 0.5.

On load reversal and loading in the uncracked direction, the slope of the curve is influenced by cracking on the tension side and closing of cracks on the compression side. Prior to tensile cracking, the slope is relatively large, but thereafter it exhibits considerable reduction. Unloading along FG is more or less similar to that along DE. The reloading slope for the second and later cycles can be represented by an almost straight line joining the unloading point and the maximum point attained earlier.

Based on the general principles stated above, a set of stiffness degrading hysteresis rules has been proposed by

Takeda et al⁽²²⁾ to simulate the flexural behaviour of reinforced concrete members under repeated cycles of loading with light to medium amount of axial loads. these rules are illustrated in Fig.4.10 and described below:

- Rule 1: Loading on the primary curve before cracking.
- Rule 2: Loading on the primary curve before yielding.
- Rule 3: Loading on the primary curve after yielding.
- Rule 4: Unloading from the maximum point attained during Rule 2.
- Rule 5: Unloading from the maximum point attained during Rule 3.
- Rule 6: Reloading after Rule 4 or 5 towards the maximum point on the primary curve.
- Rule 7: Unloading without re-attaining the maximum point after Rule 6.
- Rule 8: Reloading towards the maximum point on the primary curve.
- Rule 9: Unloading without re-attaining the maximum point after Rule 8.
- Rule 10: Reloading towards the maximum point attained during Rule 6.
- Rule 11: Unloading from the maximum point attained during Rule 10.
- Rule 12: Loading towards the maximum point attained during Rule 8.
- Rule 13: Unloading from the maximum point attained during Rule 12.
- Rule 14: Loading in the uncracked direction after cracking in the other direction.

- 1 Rule 1
- 2 Rule 2
- 3 Rule 4
- 4 Rule 14
- 5 Rule 2
- 6 Rule 3
- 7 Rule 5
- 8 Rule 6

(a)

- 1 Rule 1
- 2 Rule 2
- 3 Rule 3
- 4 Rule 5
- 5 Rule 15
- 6 Rule 16
- 7 Rule 3
- 8 Rule 5
- 9 Rule 6

FIG. 4.10 TAKEDA'S HYSTERESIS RULES

FIG. 4.10 TAKEDA'S HYSTERESIS RULES (Cont'd)

Rule 15: Loading in the uncracked direction after yielding in the other direction.

Rule 16: Loading towards the yield point after Rule 15.

A simplified stiffness degrading hysteresis model based on a bilinear primary curve is adopted for this study. The set of simplified Takeda's hysteresis rules are similar to the above rules except that the break point at cracking is ignored.

The non-degrading behaviour is modelled by neglecting the stiffness deterioration under load reversals. Thus, both the unloading and reloading slopes are assumed to be the same as the loading slope on the primary curve as shown in Fig.4.1b.

4.4 DUCTILITY DEMAND

It is generally recognized that significant amounts of energy is absorbed in inelastic deformations by a structural system subjected to intensive earthquake ground motions. Therefore, the overriding criterion in the seismic design process is to ensure that the structure is capable of deforming in a ductile manner well into the inelastic range to dissipate the energy input into the system during a severe earthquake without collapse. Because of the importance of ductility in seismic performance of structures, the expected inelastic deformations or the ductility demand should be estimated in some manner and compared to an admissible value. The best

measure of ductility is the maximum strain attained. In practice, however, ductility can be measured in terms of either a displacement, a curvature, a spring rotation or the dissipated strain energy, as shown in Fig.4.11a,b,c and d.

In this study, rotational ductility demand is used as a measure of inelastic deformation. Referring to Fig.4.11c, rotational ductility is defined as the ratio of the maximum absolute rotation to the yield rotation and is given by:

$$\mu = \frac{\theta_{\max}}{\theta_y} \quad (4.6)$$

4.5 RESPONSE OF SINGLE DEGREE OF FREEDOM SYSTEM

When a building structure is subjected to an earthquake ground motion, it is expected that the structure will undergo substantial inelastic deformations. As indicated earlier, an inelastic analysis is highly complex and time consuming. Because of this and the difficulty in modelling the material behaviour in the inelastic range and the uncertainty inherent in specifying a design ground motion, a sophisticated inelastic analysis is seldom used in design. Instead, it is customary to use the forces obtained from an elastic analysis, adjusted by a reduction factor to account for the effect of ductility and inelastic deformation. In the National Building Code of Canada 1977⁽¹⁾, the reduction factor used in scaling down the elastic force is taken as equal to μ , the ductility factor,

(a) Ductility Based on Displacement

(b) Ductility Based on Curvature

(c) Ductility Based on Rotation

(d) Ductility Based on Energy

FIG. 4.11 MEASURES OF DUCTILITY

except when the fundamental period of the structure is less than 0.5 second. This is based on the reasoning that the total elastic displacement of the system is nearly equal to the total displacement of an elasto-plastic system with the same initial stiffness. The limitations of this approach are:

1. The assumption that the total elasto-plastic displacement is the same as the displacement of the corresponding elastic system is only an approximation even for a single degree of freedom system. In fact, the relationship is very much dependent on the ground motion, and large differences may exist between the two displacements.
2. The force-displacement relationship for a concrete frame is not elasto-plastic, and stiffness degradation introduces additional complexity in the behaviour.
3. The localized member ductility requirements are not the same as the force reduction factor used in design; and may, in fact be larger.

To investigate the correlation between elastic and inelastic response and the effect of stiffness degradation

on ductility demand, the single storey concrete frame shown in Fig.4.2 was analysed for 25 seconds of the standard ground motion scaled to represent an earthquake with a maximum ground acceleration of 50% of gravity. The mass of the frame was adjusted to give eight different values of the period ranging from 0.3 to 2.7 second, giving in effect eight different frames. The damping ratio was taken as 5%. For each frame, the following analysis procedure was used:

1. Carry out an elastic dynamic analysis.
2. Set the yield strengths of beams and columns at one fourth of the maximum earthquake moment obtained from the elastic analysis.
3. Carry out inelastic analyses, first with a non-degrading hysteresis model and then with a degrading hysteresis model.

Results of the analyses are presented in the following sections.

4.5.1 DISPLACEMENT TIME HISTORY

A sample plot of the time history for moment rotation relationship of a degrading hysteresis frame with fundamental period equals to 0.3 sec. is shown in Fig.4.12 to indicate how the hysteresis rules are functioning.

FIG. 4.12 HISTORY OF BEAM MOMENT-ROTATION RELATIONSHIP FOR DEGRADING HYSTERESIS

MODEL IN SINGLE STOREY FRAME (at $t=0-3.33s$, with $T=0.3s$.)

Figs. 4.13 through 4.17 show the displacement time histories for non-degrading and degrading systems with initial free vibration periods of 0.3s, 0.6s and 1.8s. The displacement histories are plotted for a duration of 25 seconds of the earthquake. The beginning and the end of yielding are identified by arrows. The time displacement curves show that the response is strongly influenced by the initial free vibration period. In case of the non-degrading response, although yielding occurs in many of the vibration cycles, the motion continues to take place at essentially the free vibration frequency. For the degrading system, the loss of stiffness resulting from the large yield deformation increases the effective period of vibration and smoothes the displacement curves considerably for the subsequent motion.

4.5.2 DUCTILITY REQUIREMENTS

Fig. 4.18 compares the deflection ductility requirements for the degrading and non-degrading systems. In non-degrading systems, the deflection ductility requirements are in general somewhat higher than the force reduction factor of 4, for periods lower than 1.5s. For frames with longer periods, the ductility requirements are smaller than the force reduction factor. The above information can also be presented in the form of a plot of inelastic to elastic relative displacement ratio as shown in Fig. 4.19. Since the member strengths have

(a) Non-Degrading Hysteresis Model

(b) Degrading Hysteresis Model

FIG. 4.13 DISPLACEMENT TIME HISTORY OF SINGLE STOREY FRAME (with $T=0.3s$.)

FIG. 4.14 DISPLACEMENT HISTORY OF SINGLE STOREY STIFFNESS NON-DEGRADING FRAME (with $T=0.6s$.)

FIG. 4.15 DISPLACEMENT HISTORY OF SINGLE STOREY STIFFNESS DEGRADING FRAME (with $T=0.6s$.)

FIG. 4.16 DISPLACEMENT HISTORY OF SINGLE STOREY STIFFNESS NON-DEGRADING FRAME (with $T=1.8s.$)

FIG. 4.17 DISPLACEMENT HISTORY OF SINGLE STOREY STIFFNESS DEGRADING FRAME (with $T=1.8s$.)

FIG. 4.18 DEFLECTION DUCTILITY IN A SINGLE STOREY FRAME

FIG. 4.19 RELATIVE DISPLACEMENT DUCTILITY IN A SINGLE STOREY FRAME

been adjusted so that yielding begins at one fourth the maximum elastic force, the deflection ductility is simply four times the displacement ratio mentioned above. The localized rotation ductility requirements as shown in Fig.4.20 follow the same pattern as the deflection ductility, although in all cases, they are higher than the corresponding deflection ductility requirements.

Stiffness degradation seems to have a marked effect on short periods structures, and increases the ductility requirements considerably. For longer periods though, the degrading and non-degrading response do not seem to differ much and the difference if any is not systematic.

4.6 RESPONSE OF FIVE STOREY FRAMES

To investigate the effect of inelasticity and stiffness degradation on the response of multistorey reinforced concrete frames, the five storey frame shown in Fig.4.3 was analyzed for the first ten seconds of the standard ground motion scaled to 50% g. As in the case of a single storey frame, the mass of the frame was adjusted to give four different frames with fundamental periods equal to 0.5s, 0.965s, 1.5s and 2.0s respectively. An elastic analysis was first carried out with a damping value of 5%. The yield strengths of the girders were then set at one fourth the maximum moment

FIG. 4-20 GIRDER DUCTILITY IN A SINGLE STOREY FRAME

obtained from the elastic analysis. The column strengths were kept higher than the maximum value of moment obtained during an elastic analysis to eliminate the possibility of yield in column sections.

Figs. 4.21a, 4.22a, 4.23a and 4.24a compare the maximum storey displacements for the elastic, stiffness degrading and non-degrading models. In the non-degrading inelastic frame, the maximum displacements are always less than the displacements in the elastic frame. In the stiffness degrading frames with periods of 0.5s and 0.965s, the displacements are more or less similar to the elastic displacements in the lower storeys but are higher in the upper storeys. For the stiffness degrading frame with a period of 1.5s, the upper floor displacements are still higher than those in the elastic floor but the lower floor displacements are slightly less than the elastic displacements. The maximum displacements of the elastic frame are considerably higher than those of the degrading stiffness frame in the frame with a fundamental period of 2.0s period.

The relative storey displacements are shown in Figs. 4.21b, 4.22b, 4.23b and 4.24b.

The degrading frame shows an accentuated response towards the top of the frame except for the frame with a period of 2.0s. The girder ductilities are shown in Fig. 4.25. The girder ductility requirements for the non-degrading frames are close to the

FIG. 4.21 RESPONSE OF FIVE STOREY FRAME (with $T=0.5s$.)

— Elastic
 --- Non-Degrading
 - - - Degrading

FIG. 4.22 RESPONSE OF FIVE STOREY FRAME (with $T = .965s$.)

— Elastic
 - - - Non-Degrading
 — Degrading

FIG. 4.23 RESPONSE OF FIVE STOREY FRAME (with $T=1.5s.$)

FIG. 4.24 RESPONSE OF FIVE STOREY FRAME (with $T=2.0s.$)

reduction factor of 4 for long period frames, but become higher as the period reduces. In the frame with a period of 0.5s, they are considerably higher than 4. For the degrading frames, the girder ductilities are substantially higher and even much higher for the frame with a period of 0.5s. The deflection ductilities are shown in Fig.4.26. They follow a pattern similar to that of girder ductilities for both the degrading and non-degrading frames.

Another response parameter of interest is the effective reduction factor for the beam and column moments. An effective reduction factor is defined as the ratio of the elastic response to the inelastic response. If the moment-rotation relationship for the beam were to be perfectly elasto-plastic, the reduction factors for the beam moments should be uniformly equal to 4. However, because the moment rotation relationship is bilinear, the slope of the second branch being about 5% of the slope of initial branch, the reduction factors are lower than 4 and are close to 3. The interesting part is that the reduction factors for columns which remain elastic are considerably lower and are slightly less than 2 for short period frames and close to 2 for long period frames, as shown in Fig.4.27.

4.7 RESPONSE OF TEN STOREY FRAME

The frame shown in Fig.4.4 had a fundamental period of 1.89s. The response of single degree of freedom system

FIG. 4.25 GIRDER DUCTILITIES IN FIVE STOREY FRAMES

FIG. 4.26 DEFLECTION DUCTILITIES IN FIVE STOREY FRAMES

FIG. 4.27 EFFECTIVE REDUCTION FACTORS IN FIVE STOREY FRAMES

shows that for a period in this range both the degrading and non-degrading displacement response is smaller than the elastic response. Fig.4.28 shows that this is indeed the case for the ten storey frame. Towards the upper storeys the degrading response starts increasing presumably, because of the effect of higher modes of vibration. The higher modes which have smaller periods are expected to show an accentuated degrading response as evident from the results of analysis for single degree of freedom systems. Fig.4.29 shows the relative storey displacements for the three models of ten storey frames. The non-degrading response is smaller than the elastic response. The degrading response, although smaller than the elastic response in the lower floors, exceeds the elastic response in the upper floors. This is obviously the influence of the higher modes of vibration which have a shorter period.

This influence of higher modes is again evident in the girder ductility ratios shown in Fig.4.30. In the lower five storeys, the degrading and the non-degrading models show similar response but in the higher storeys the girder ductilities for the degrading model are substantially more.

The column and beam moment reduction factors are shown in Fig.4.31. As in the case of five storey frame, the beam

FIG. 4.28 MAXIMUM DISPLACEMENT OF TEN STOREY FRAME

FIG. 4.29 RELATIVE DISPLACEMENT OF TEN STOREY FRAME

FIG. 4.30 GIRDER DUCTILITIES IN TEN STOREY FRAME

FIG. 4.31 REDUCTION FACTORS IN TEN STOREY FRAME

reduction factors are less than 4 because of a bilinear moment rotation relationship. The reduction factors for columns that remain elastic are almost half of those for the beams.

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CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 SUMMARY

The elastic response of reinforced concrete shear wall-frames and inelastic response of reinforced concrete frames to earthquake ground motion have been examined in this study. The work was approached in the following stages:

- (1) Review of literature to establish the present state of knowledge in the related field.
- (2) Development and selection of computer programs for the elastic and inelastic analysis of reinforced concrete structures.
- (3) Development of appropriate mathematical models for members and structures.
- (4) Selection of important parameters.
- (5) Carrying out modal analyses and dynamic analyses of selected structures for a range of parametric values.
- (6) Interpretation of the results of analyses.

Based on the results presented in this study, several important conclusions have been drawn and are given in this chapter. The author recognizes that the conclusions drawn are based on the limited number of studies presented herein and that too broad a generalization is not possible. One also has to bear in mind the limitations of the mathematical models and the simplifying assumptions made in the analysis. Accordingly, the conclusions drawn herein are valid only when:

- (1) out of plane movements are not present,
- (2) torsion is absent,
- (3) the effect of gravity loads on inelastic response is ignored.
- (4) all members are reinforced concrete members, and
- (5) the shear wall is not strained into the inelastic range.

5.2 CONCLUSIONS DRAWN FROM THE ELASTIC RESPONSE ANALYSES OF REINFORCED CONCRETE SHEAR WALL-FRAMES

The following conclusions can be drawn from the modal analysis and elastic dynamic analysis of reinforced concrete shear wall-frames:

- (1) P-delta effect has an insignificant effect on the modal response and can thus be ignored.
- (2) Effect of shear deformation on the modal response is quite significant. However, in the case of mode shapes, the change is not drastic and the general shape remains the same. The effect of shear deformation on the modal frequencies is more significant. Exclusion of this effect results in a large increase in the modal frequencies of the wall-frame, and the effect is even more pronounced for the higher modes. Therefore, shear deformation must be taken into account in the analysis of wall-frames.
- (3) For the wall-frames studied when the effect of finite joint width is neglected, the modal frequencies are found to decrease by 10-14% in comparison with those computed with the finite joint sizes accounted for. The decrease is even more when shear deformation is also ignored. It would therefore be appropriate to include the effect of finite joint width in the analysis of shear wall-frames.
- (4) In spite of the different frequency content in the input ground motions, a wall-frame vibrates basically in its fundamental vibration period.

- (5) Maximum displacements and relative displacements increase with increasing fundamental period of the wall-frame.
- (6) The triangular force distribution suggested by the code underestimates the shear in the upper storeys and overestimates the shear in the lower storeys. Empirical values, suggested Humar⁽⁵⁵⁾, may give a better estimation of shear distribution along the height of the wall-frame.
- (7) Behaviour of wall is essentially like a cantilever, although interaction with the frame changes its moment diagram slightly; the column acts in a predominantly frame type action.
- (8) For the wall-frames studied, more than 80% of the storey shear at levels below $0.6H$ is sustained by the shear wall. The portion of the storey shear sustained by the frame increases significantly towards the top of the wall-frame. In taller wall-frames, interaction between wall and frame may result in a negative force acting at the top of the wall to prevent it from bending in its natural shape of a cantilever.

- (9) For the set-back wall-frame as selected in this study, although the shear wall is reduced by 50% of its width in the set-back portion, the mode shapes, frequencies, and the displacement wave-form do not differ significantly from the related uniform wall-frame. The set-back portion however shows an accentuated response as evident in the plot of relative displacements. The moments induced in the set-back wall are of magnitudes comparable to those in the uniform wall even though the set-back wall has a much smaller stiffness.

5.3 CONCLUSIONS DRAWN FROM THE INELASTIC RESPONSE OF REINFORCED CONCRETE FRAMES.

Conclusions drawn from dynamic analyses of inelastic, stiffness degrading, and non-degrading reinforced concrete frames are:

- (1) Response of a stiffness non-degrading frame is strongly influenced by its initial free vibration period. The frame vibrates at essentially its free vibration period. Stiffness degradation increases the effective period of vibration of the frame and the response of the frame is dominated by such increased period.

- (2) Stiffness degradation seems to have a marked effect on short period frames and increases the ductility requirements considerably. For long period frames, responses of stiffness degrading and non-degrading frames do not differ much.
- (3) The localized rotation ductility requirements are in general slightly higher than the deflection ductility requirements.
- (4) The girder rotation ductility requirements for the non-degrading frames are close to the reduction factor for long period frames, but become higher as the period reduces. Degrading stiffness significantly increases the ductility requirements in short period frames and also in the upper storeys of tall frames where the response is affected by the higher modes.
- (5) The effective reduction factors for the beam moments obtained by comparing the elastic and inelastic response of a frame are lower than 4 (the value adopted in fixing the beam strength) and are close to 3 because of the strain hardening effect. Reduction factors for moments and axial loads in columns which remain elastic are considerably lower and are close to 2.

5.4 RECOMMENDATIONS FOR FUTURE STUDIES

During the course of this study, a few topics have been identified which need additional study and research. These are mentioned below:

- (1) It is well recognized that inelastic deformation may inevitably be produced during earthquakes. Many researchers are currently investigating the inelastic behaviour of reinforced concrete. There is need for extensive studies on:
 - (a) development of mathematical models for inelastic reinforced concrete shear wall-frames,
 - (b) effect of inelasticity on the wall-frame interaction with different relative stiffnesses of the wall and the frame, and
 - (c) application of inelastic analytical results in practical design.
- (2) Improvement is also needed in the modelling of reinforced concrete members subject to cyclic loading in the inelastic range. These improvements should make it possible to give consideration to the gravity loads and to obviate the need for an assumption about the location of the inflexion point along a member which may be unrealistic for certain structures.

- (3) Further studies are required to find the correlation between the elastic and inelastic response of a reinforced concrete frame. This will enable one to find the design forces in the members of a stiffness degrading structure by applying appropriate adjustment to the forces obtained in an elastic analysis.

NOMENCLATURE

Symbols are defined where they first appear in the text. In addition, they are defined below in alphabetical order. Whenever possible the same definition of a symbol is retained throughout; when a symbol has been used in more than one sense, all the alternative definitions are given below.

A	= cross-sectional area; ground acceleration
A_e	= effective shear area
a	= ratio of the left or lower end rigid zone length to the clear length of the member
b	= ratio of the right or upper end rigid zone length to the clear length of the member; subscript indicating beam
$[C]$	= damping matrix
C_b	= base shear coefficient
C_x	= shear coefficient at level x
c	= subscript indicating column
D	= dead load
E	= Young's modulus; earthquake load
F	= seismic foundation factor; applied force
$[F]$	= flexibility matrix
f	= flexibility influence coefficient
f'_c	= concrete 28 day compressive strength
f_y	= yield strength of reinforcement
G	= shear modulus
h	= storey height; time increment
h_x	= height of level x

I	= moment of inertia of the cross-section; seismic important factor
$[K], [R]$	= stiffness matrix
K	= a constant; seismic ductility coefficient
k	= slope of a line segment
L	= live load; over-all length of a member; subscript indicating left or lower end of a member
l	= clear length of a member; subscript indicating left or lower end of a member
M	= moment
$[M]$	= mass matrix
M_c, M_y, M_u	= cracking, yielding and ultimate moments
n	= modulus ratio; subscript indicating the location of interval of time at which the subscripted variable is evaluated
P	= axial load
T	= period of vibration
$[T]$	= transformation matrix
t	= elapsed time
U, u	= subscript indicating the upper end of a member
V	= storey shear
V_x	= shear at level x
V_b	= base shear
$\ddot{V}_g$	= acceleration of base motion
W	= effective weight of the building
W_x	= weight assigned to level x
$Z, \dot{Z}, \ddot{Z}$	= value of displacement, velocity and acceleration relative to the base

- α = damping coefficient
 γ = damping coefficient
 $\phi, \phi_c, \phi_y, \phi_u$ = curvature and curvature at cracking, yielding and ultimate
 $\theta, \theta_c, \theta_y, \theta_u$ = rotation and rotation at cracking, yielding and ultimate
 Δ = displacement
 $\Delta\theta$ = incremental rotation
 ΔF = incremental force
 $\Delta Z, \Delta \dot{Z}, \Delta \ddot{Z}$ = incremental displacement, incremental velocity and incremental acceleration
 $\Delta \ddot{V}_g$ = incremental acceleration of base motion
 ϵ = axial deformation
 ξ_i = damping ratio in the i th mode
 ω_j = natural frequency in the i th mode

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