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**The time course of activation and retrieval
of multiplication facts in adults**

**by
Maria-Antonietta Francini**

**A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Arts
Department of Psychology
Carleton University**

**Ottawa, Ontario
August, 1993
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Chair, Department of

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Abstract

The present research examined the representation and retrieval of multiplication facts in adults. By "representation" it is meant the organization in long term memory of the 36 basic "products". A "product" is defined as the outcome of the multiplication of two simple digits. "Retrieval" refers to the process by which skilled adults access the stored knowledge. Multiplications were presented in the form $A \times B = C$. A delay (SOA) of 0, 60, 100, 200, 300 or 400 ms., was interposed between the first multiplicand (" $A \times \dots$ "), and the rest of the operation (" $\dots B = C$ "). The presented answers were "false" or "true". Two types of false answers were presented: 50% were "IN" false answers (e.g., multiples of the first operand), and the remaining 50% were false "OUT" answers (e.g., multiples of the second operand). Latencies were slower to "false-IN" than to "false-OUT" answers for problems having a product less than 28. The opposite finding was obtained for problems having a product equal or greater than 28. The results were interpreted within a table-like mental multiplication format. Further considerations regard the long latencies obtained at SOA=60, 300 and 400, as compared with the short latencies observed at SOA=200. These findings suggested that at longer SOAs the stimulus was re-encoded and that at SOA=200 the system reached asymptote.

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Table of Contents

Table and Figure list	1
Introduction	2
Arithmetic Models	Counting Models 4
	Network Retrieval Model 8
	Distribution of Association Model 10
	Network Interference Model 15
	Summary 17
Present Research	Introduction 19
	Assumptions 22
	Experimental Design 23
Method	Subjects and Materials 28
	Procedure 30
Results	False Answers Analysis 31
	Time Course Analysis 36
	False Answers Error rate 39
	True Answers and Error rate 41
Discussion	"Small" problems 43
	"Large" problems 51
	True problems 54
Conclusions	55
References	58

Table and Figure list

Table 1 Ashcraft's table	66
2 Multiplication table	67
3 Simplified Table	68
4 Example of false-IN Answers	69
5 Example of false-OUT Answers	70
6 Example of IN and OUT	71
7 Means latencies and Error %	72

Figure 1 Problem-size by SOA	73
2 SOA by False Answers Location	74
3 Family by False Answers Location	75
4 Family by Problem Size	76
5 Three-way interaction	77
6 Time course IN vs OUT	78
7 Small problems: IN vs OUT	79
8 Large problems: IN vs OUT	80
9 Percent E or by SOA	81
10 True Answers	82

Appendix A: Problem Set	83
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Introduction

Various version of Associative Interference theory have dominated the psychological study of memory since the beginning of this century. Sharp criticisms of this approach were made by Bartlett (1932), who proposed that organized mental structures play a key role in remembering. Not until the 1960's did organizational theory prevail over the behaviouristic "stimulus-response" oriented approach exemplified by interference theory. Research on organizational processes in long term memory appeared (e.g., Mandler, 1968; Neisser, 1967; Tulving, 1962) that emphasized the importance of active strategic encoding for the recall of various kind of words lists. Attention began to be paid to the critical importance of retrieval cues in remembering (Tulving & Pearlstone, 1966; Tulving & Osler, 1968) and studies of knowledge retrieval and semantic memory were initiated (Atkinson & Shiffrin, 1968; Briggs & Blaha, 1969; Shiffrin & Shneider, 1977; Waugh & Norman, 1965).

The main stream view that currently prevails in the vast field of Cognitive Psychology is the Information Processing approach. A key concept in this approach is that knowledge is represented in an organized fashion. For example, in categorization research, memory has been conceptualized as the grouping or clustering of elements based on meaning. The models of semantic organization that have received most attention are: the Clustering model (Bousfield, 1953; Bower,

1970); the Set-Theoretical model (Meyer, 1970; Meyer & Schvaneveldt, 1971); the Semantic-Feature Comparison model (Smith, Shoben & Rips, 1974; Rosh, 1975); and the Spreading-Activation theory (Quillian, 1962, 1967; Collins & Quillian, 1969, 1972; Collins & Loftus, 1975; Anderson, 1981, 1983; Anderson & Bower, 1973).

Investigations in the word recognition area have also been focused on the representation and activation of words in the mental lexicon (Collins & Loftus, 1975; Meyer & Schvaneveldt, 1971; Neely, 1976, 1977). As in the categorization literature, research on word recognition suggests that concepts (words) are highly organized in the mental lexicon.

The goal of the present research was to examine the organization of mathematic facts within the context of multiplication. In doing so it was hoped that a better understanding of mental representation, and thus a more general theory of cognitive processing, could be achieved.

Arithmetic Models

Counting Models. Historically, the educational literature contains a number of studies (Knight & Beherens, 1928) which show that response time can be roughly ordered as an increasing function of the sum of two numbers to be added. Thomas (1963) attempted to give a more precise account of this ordering; he assumed that the processes underlying simple calculation behave as an information

processing system for which the time needed to perform a given arithmetic operation is proportional to the amount of information that must be handled by the system. Given this premises, Thomas derived the prediction that reaction time to simple addition problems should be a linear function of:

$$\log(p+q)$$

where p and q are the operand in the addition problem.

Other early studies were those of Dansereau (1969) and Dansereau and Gregg (1966). They constructed a computer simulation model of multidigit multiplication performance, and assumed that multiplication consists of a table-look-up procedure. They did not attempt any further specification of the processes involved.

In an investigation of the counting algorithm used by children, Groen (1967; Suppes & Groen, 1967; Suppes, Hyman, & Jerman, 1967; Restle, 1970), proposed a variety of simple counting models, all of which assume a counter with two operations: setting to a value and incrementing by one. The time required to set the counter is independent of the value to which it is set. If the incrementing time is constant, then the reaction time is a linear function of the number of times the counter is incremented. Groen (1967) described RT to individual problems using the formula:

$$RT = a + bx$$

where x is a structural variable and a and b are constant.

Groen found that the only model to yield a satisfactory fit was the model with $\min(p,q)$ (p and q are the operand in the operation, as stated above) as the structural variable x . It assumed that the counter was set to $\max(p,q)$ and that the number of increments was equal to $\min(p,q)$. This model gave a good account of the overall data, with the exception of the ties, which had uniformly lower latencies than most non ties problems with comparable sums. (Ties problems refers to one number added to itself: $2+2, 3+3$, and so on).

In further studies, Groen (Parkman & Groen, 1971; Groen & Parkman, 1972) used a verification task to study RT of simple addition in adults. In the verification task subjects are required to indicate whether an answer is "True" or "False". Parkman (1972) extended the verification method to the study of simple mental multiplication, and its underlying processes. Problems were presented in the form $p \times q = r$, and subjects were asked to decide whether or not the displayed answer (r) was correct.

The verification procedure can be viewed as having four successive stages: 1) encoding of the stimulus, 2) computation or retrieval of the product (multiplication stage), 3) comparison of the computed or retrieved product to the third number (comparison stage), and 4) a response-execution stage. The total reaction time is assumed to be the sum of the duration of the component processes, one of which is the multiplication process. The stimulus processing

and the response-execution stages are assumed to be constant (except for practice effects) and to be independent from the multiplication and comparison stages.

An important issue in the Groen and Parkman verification task is whether the effects of the comparison operation can be separated from the multiplication effects. Parkman (1972) notes that responses are typically longer to false than to true answers (difference of about 50 ms). However, the slope estimate from linear regression for various structural variables are very similar both for the true and false data. In other words, the data for true and false instances show a consistent intercept difference, but similar slopes. This suggests that the variables determining the slopes are affecting a different stage than are the variables determining the difference between intercepts, and that the processing durations of these respective stages are additive (Sternberg, 1969a, b). Because the slopes presumably reflect structural variables in multiplication, there is a compelling reason for associating these slopes with the multiplication stage. The variable determining the intercept difference is the relation True/False and is associated with the comparison and response stages (Parkman, 1971, 1972).

The topic of memory retrieval is fundamental to mental multiplication. The fundamental question is whether the subject forms the product of two single-digit numbers by use of some memory retrieval procedure or whether such a product

is generated by a computational algorithm. Parkman (1972) suggests that adults may use calculation of indexes for memory retrieval. Such indexes or cues, either provide a memory location (address) of a desired number fact or are used in a content-addressable fashion to locate the desired fact directly.

The counting model of addition proposed by Parkman and Groen (1971) can easily be modified to encompass a memory retrieval operation if it is assumed that a retrieval index is generated rather than a sum, and that the RT for memory retrieval is independent of the actual index used. Another possibility, examined by Parkman (1972), is that arithmetic "tables" are stored as hierarchical networks in long term memory and the time to retrieve a given solution is a function of that solution's location in the network, due, perhaps, to linear, terminating searches through the networks.

Counting models assume that verification consists of prediction plus comparison: The product is retrieved and then compared with the presented answer. Existing counting theories apply directly to addition and not to multiplication. But in principle there could be a counting model for multiplication. Subjects could count in units of the multiplier, with the number of counts specified by the multiplicand. For example, one could solve 3 times 4 by counting 3 steps of units of 4. The notion that counting may

be used is supported in research comparing the performance of children versus adults. Groen and Parkman (1972) found that the slope of the regression line for children was 400 ms whereas the corresponding slope for adults was 20 ms. They point out that 400 ms is compatible with the hypothesis of "counting to themselves", but 20 ms is not. They compare adults slope to the memory search rate obtained by Sternberg (1969a,1969b) (38 ms) and suggest that the counting process defined by their model can be viewed as a memory search process in which the "setting" operation corresponds to generating a list of numbers and the "incrementing" process corresponds to finding the next item on a list. But they conclude that the correspondence between the memory search that might be used in adult addition and the exhaustive search postulated by Sternberg, is not exact.

Network Retrieval Model. During the 1970's, Ashcraft studied issues concerning category typicality, semantic distance, and the processes of retrieval (1976). When he became interested in mental arithmetic, he hypothesized that simple arithmetic facts must serve an analogous function to the conceptual entries or nodes in semantic memory, and that semantic distance or relatedness must be the basic metric of semantic memory for simple arithmetic facts. On this view, he assumed that the fundamental mechanism by which multiplication facts are accessed is a process of spreading activation. At the heart of Ashcraft's model is the notion

of problem difficulty and the conviction that adult performance is not explainable in terms of simple counting models (Ashcraft, 1982, 1987; Ashcraft & Stazyk, 1981).

Ashcraft and Battaglia (1978), studying addition in adults, found an increase in RT as a problem increased in size (problem size effect). They suggested that RT performance on these problems reflected retrieval of stored information, with retrieval time increasing as a function of the magnitude of the problem: The numerical magnitude of the problem is proportional to the response time, the larger the problem the longer the response time.

The description of Ashcraft's (1987) model of mental arithmetic development is based on three critical and interrelated assumptions: First, the basic facts of addition and multiplication are assumed to be stored in long term memory as a network of interconnected nodes. The digits from 0 to 9 are "parents"; the set of nodes linked to a parent are referred to as "family members". Each directed pathway from a parent to a family member has a particular strength value associated with it (S_{ij}), where i is the index for the first addend and j for the second. RT for any node is a function of the strength value for the corresponding pathway. A corresponding printed table implies an equidistant node structure, whereas the ideal structure would be distorted from true square, by the strength or distance values S_{ij} . (My interpretation of this statement is

that the ideal table should be elongated like a rhomboid toward the corner of the problems with greater distance value). This model was suggested by Ashcraft (1987), but was not explained in more details. A computer simulation was presented as demonstration of the theory (Ashcraft, 1987).

A second assumption is that the strength for any problem node is a function of experience or practice with that problem. That is, the more the experience, the shorter the RT for retrieving the answer to a problem.

A third assumption is that retrieval is accomplished by a spreading activation process. Under this assumption, activation spreads not only to the target information in memory, but also to related facts and relationships.

Distribution of Association Model. Siegler and Shrager (1984) developed the distribution of associations model to account for strategy choices in pre-schoolers' addition and subtraction. The basic organization of the model, the strategy choice mechanism, and the factors influencing acquisition, are hypothesized to apply to many tasks, among them older children's multiplication (Siegler, 1988).

The basic organization of the model is an associative network in which each problem (e.g., 3×8) is associated with the correct answer (24) and with some incorrect answers. Associations across arithmetic operations (e.g., addition and multiplication) are also hypothesized to be present in the network.

When the associative strength between an individual problem and the correct answer is high, relative to other answers, that problem is supposed to have a peaked distribution of associations. Consequently the child will retrieve the correct answer quickly. On the contrary, if the associative strength between an individual problem and the correct answer is low, relative to other answers, that problem has a flat distribution of associations: The consequence being that the child will use some strategy different from retrieval, and the answer will be generated slowly, as compared with an answer for a problem with a peaked distribution.

In sum, the organization of the Siegler and Shrager's (1984) model is based on the associative strength among several answers. The peakedness (or not) of a problem's distribution is hypothesized to be a key determinant of error percentage, RT, and strategies used. Siegler, as opposed to Ashcraft's view, does not propose a table-like format, nor a spreading activation mechanism. For Siegler the functional unit is the problem itself, composed of first and second operand related by the sign of the operation to be performed. This problem-unit has links with several possible answers, some links are stronger and some are weaker.

The strategy-choice mechanism in the model depends on a "confidence criterion" and on "search length". The

confidence criterion is a value that must be exceeded by the associative strength of a retrieved answer for the child to state that answer. The search length indicates the maximum number of retrieval efforts that the child will make.

Retrieval efforts will continue as long as the associative strength of each retrieved answer is below the confidence criterion and the number of searches does not exceed the search length.

When retrieval efforts fail, an alternate form of retrieval may be used: the sophisticated guessing approach. In such a case the confidence criterion drops to 0, and the child states whichever answer comes to mind. If this approach is not used, the child may write the problem and be able to retrieve the correct answer because of a visual stimulation that triggers the correct associations. Finally the child may use repeated additions by writing one multiplicand the number of times indicated by the other, then perform an addition, and state the sum as the answer.

Direct instruction from teachers and textbooks, and the frequency of problem presentation, are considered to be the main factors influencing the acquisition of arithmetic knowledge. Accordingly, Siegler's predictions about development suggest that third and fourth graders' percentage correct on retrieval trials will predict older children's and even adults' RT, on that problem, better than any structural variable (product). Individual children

appear to use multiple strategies rather than a single one and there are parallels in relative problem difficulty between children and adults. The error patterns of children have been found to parallel the RT patterns of adults of much greater skill. The developmental changes are often changes in the distribution of strategy use over time (more or less frequent use), rather than a substitution of a strategy for another. The relative peakedness of the distribution of associations (called also "the associative strength variable", as the operational definition of the percentage of associative strength concentrated in the correct answer on the problem), is preserved over age, "despite the influence of 10 years of experience with multiplication" (Siegler, 1987, 1988). The implication of this statement seems to be that Siegler probably disagrees with the theory of the importance of problem size effects as determinant of the response time in adults. On the contrary, the frequency of problem presentation is considered to be the main factor influencing performance, as explained above.

An important point of Siegler and Shrager's (1984) model is that it does not postulate any table-like organization of the arithmetic knowledge representation. Instead, it only states that problem-answer links differ in associative strength or accessibility. Siegler and Shrager suggest that a tabular representation would make it difficult to explain: First, why people retrieve incorrect answers that are not

part of any single-digit arithmetic table (non table errors); Second, why factors other than problem size (e.g., 5 as a multiplicand) should influence RT and errors rate. The model suggests that the problem size effect and the multiplicand-related errors are attributable to the answers generated by backup strategies.

The distribution of associations model is a self-governing cognitive system because: First, it chooses the best strategy to obtain the most accurate performance (retrieval or back-up). Second, increased associative knowledge is assumed to be accompanied by increasing use of retrieval. Third, it is assumed that the system has the ability to recover from error. These self-governing characteristics are produced by ".... "mindless" cognitive activities..., decisions concerning what to do may be produced by surprisingly low level processes." (Siegler, 1988).

In conclusion, the Siegler and Shrager (1984) model states the following: First, the RT to access a correct answer might vary as a function of the number of different candidates (peaked or flat distribution). Second, the relative strength between a problem and the correct answer increases with practice. Third, back-up strategies are used at all ages when the retrieved answer does not match the confidence criterion and the search is longer than the "search length". Fourth, the cognitive system is self

governing.

The Network Interference Model. Campbell and Graham (1985) studied RT and errors for simple multiplication problems in children and adults using a production method. The production method consist in presenting a problem but not its answer. The subject is instructed to produce the answer and the response time is recorded at voice onset. The production method does not include a verification and comparison step and in this sense is different from the verification method. A further difference is that the production method is voice activated and the verification method is key activated.

Campbell and Graham (1985) proposed a network-interference theory to explain the slow course of acquisition and differential problem difficulty in terms of interference by false associations. The network-interference model is a theory of the processes underlying retrieval of simple addition and multiplication facts. Problems are represented by magnitude codes and by physical codes which, for each individual problem, are combined in a "node" (Campbell,1992).

Campbell has developed his model for about 10 years and in several papers (Campbell 1987a, 1987b, 1991). His main point is that when a problem is encoded, several answers compete for retrieval, and such competition is expressed by a mutual inhibition. An answer is produced when one of the

nodes in the network, storing an individual problem together with its magnitude code and its physical code (Campbell, 1992), reaches a critical threshold level of activation (Campbell 1992).

Campbell and Graham (1985) studied "table" errors. For example, in the problem 9×7 , the correct answer is 63, but they found that 13% of adults give the false answer: 72, which is the correct response for the problem 9×8 . Note that 9×8 shares the first operand (9) with the stimulus problem and differs by 1 on the non-shared operand (8 vs. 7). That is, in table errors, the erroneous response is an answer to a single digit problem that shares an operand with the stimulus problem, and differs by plus or minus 1 on the non shared operand. Consider, for example: $6 \times 8 = 48$, and $6 \times 7 = 42$. The so called "operand distance effect" would produce an error of the type $6 \times 8 = 42$, or $6 \times 7 = 48$ (e.g., 7 vs 8). This finding refers to the fact that the erroneous responses are usually correct for a problem that not only shares an operand with the stimulus problem, but is also close in magnitude with respect to the other operand.

Campbell and Graham (1985) also studied the problem size effect. They suggested that it is related to proactive inhibition and frequency of presentation, and that the differences in difficulty are determined by the numerical magnitude of the problems. Finally, they suggested that strength of associations between problems and answers is a

function of both frequency and order of learning.

Summary of the Current Arithmetic Models. Opinions with respect to the representation and the process of multiplication facts are quite diverse. In early studies great emphasis was given to the study of the developmental aspect of addition. Multiplication was assimilated to addition models, and never studied as an independent category.

The importance of the problem size effect is considered relevant in the research of Ashcraft (1987), who suggests that the best predictor of RT is the sum or the product. This notion is dismissed by Siegler and Shrager (1984). They focused on addition, and suggest that the frequency of problem presentation is important and not the size of the sum or product.

With respect to the representational assumptions, the opinions are also diverse. Ashcraft (1987) postulates a table like format for the ideal organization and retrieval of addition facts and assumes that the same organization is valid also for multiplication. On the contrary, Siegler and Shrager (1984) very clearly specify that they do not postulate a table-like format for the organization of addition facts, but consider that each problem is independent and related by associative strength to a number of possible outcomes.

There is also a diversity of opinion concerning how

arithmetic facts are related to each other. Ashcraft (1987) talks about spread of activation between nodes of a network, in which each "parent" node (single digit) activates its "family" at the moment of encoding. Siegler and Shrager (1984) refer to the peakedness of the distribution of associations between a complete problem composed of two single digit (related to each other by the addition symbol), and its correspondent and sum. Campbell (1992) refers to a network of interference in which each node stores an individual problem, a magnitude code and a physical code. Such a network applies equally to addition and to multiplication.

There are some points of general agreement, for example, that adults mostly use retrieval to access arithmetic facts. Nevertheless, according to Siegler, adults may, at times also use counting to solve addition problems. All researchers agree that response time decreases with practice, and that ties (e.g., $2+2$, 4×4 , $6+6$ and so on) are answered faster both in addition and in multiplication. Nevertheless, Parker (1972) found that in multiplication the RT for ties increases as a function of problem size.

The Present Research.

Introduction

The essential goal of Cognitive Science is to understand how knowledge is stored in the brain and how it is retrieved. The information processing approach assumes that the brain is a symbol processor and that the mental processes occur in real time.

The methodology that studies the time course of information processing in the human nervous system is called mental chronometry. In the present experiment mental chronometry was used with the intention of elaborating a model representing the mental architecture of stored arithmetic facts.

The alphabet of simple arithmetic digits is quite small, it is composed only of ten units: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, that are related to an abstract concept of magnitude. In order to perform arithmetic operations, numbers need to be associated with some category. The category is represented by the symbol that relates two or more operand. In arithmetic there are four categories: division, subtraction, addition and multiplication.

The representational model of multiplication that is used in the present research is similar to Ashcraft's Network Retrieval Model (1987) for addition. Accordingly, the terminology that Ashcraft employs will be used in the present research. The basic terms are defined as follows: "parent" digits are the nine single digits, each of them is the source of a "family". In the addition table (see Table

1), the family is obtained by adding one unit to each successive sum, in the multiplication table (see Table 2), the family increases by adding as many units as the magnitude of the parent digit. (e.g., $4+3=4+1+1+1=7$; on the contrary $4 \times 3=4+4+4=12$).

 Insert Table 1 and 2 about here

A multiplication table-like format is shown in Table 2. Note, however, that Table 2 contains many redundancies. Indeed the upper right half of Table 2 is identical to the lower left. For example, the products 18, 16 and 27 form the upper right and also the lower left corners. A more cognitively economical representation is shown in Table 3.

 Insert Table 3 about here

Table 3 shows a triangular format in which all the products from Table 2 are represented but with a substantial reduction in the number of nodes. In Table 3 only five products are repeated twice. The format shown in Table 3 is adopted for the present research. It is easy to deduce, by looking at the Table 3, that all the possible multiplications can be performed by using the nine parent digits and the 36 true products.

The present research adopts the assumptions concerning the spread of activation between nodes in a network as proposed by Collins and Quillian (1972), Collins and Loftus (1975), and by Anderson (1983a, 1983b). The theory is based on Quillian's theory of semantic memory search and semantic priming (1967). The theory made a number of assumptions about structure and processing in human semantic memory. Concepts can be represented as nodes in a network, and search in memory between concepts involves tracing out in parallel along the links between the nodes. Activation can only start out at one node at a time (serial), but it continues in parallel from other nodes that are encountered as it spreads out from the node of origin. Activation decreases over time or intervening activity. Activation from different sources summate and when the summation at the point of intersection reaches threshold, the path in the network producing the intersection will be evaluated. The semantic network is organized along the lines of semantic similarity: the more properties two concepts have in common, the more closely they will be related. The RT to decide that semantically unrelated concepts are different will be faster than to decide that related concepts are different. This aspect of Collins and Quillian's theory is applied in the present research with respect to multiples belonging to the same family of products.

Assumptions. The present research is concerned with the

operations related to multiplication of the digits in the range from 3×4 to 8×7 . There are only 6 possible single digits and 15 possible correct products. Two classes of assumptions will be considered: representation and processing assumptions.

Representation assumptions: a) In the ideal multiplication table, as presented in Table 3, there are 36 nodes representing the 36 products. In skilled adults, these are assumed to be organized and stored in long term memory in a network of interconnected nodes. b) The nodes are assumed to represent one product each. c) The products can be envisioned as ideally related along columns and rows, and interacting with each other in a systematic fashion. As shown in Table 3, the products are envisioned as clustered in a table-like format.

Processing assumptions: a) The activation vector spreads along the path of the operand. Consider the example: $3 \times 4 = 12$. The activation is assumed to spread horizontally from the first operand, 3, on the y-axis. This activation travels horizontally through the nodes 6 and 9 and then vertically down through 12, 15, 18, 21, 24, and 27. The operand 4 would result in activation of 8, 12, and 16, along the horizontal, followed by activation of 20, 24, 28, 32, and 36, along the vertical.

b) The node activated by both multiplicands will receive double activation and the skilled subject will perceive that

node as being the correct answer.

c) Errors are viewed as weak associations of the parents with some members of the family, as weak connections among answer nodes, or as interferences and inhibitions from other nodes.

Experimental Design. A priming paradigm was used to investigate the organization of mathematic facts and the connections between digits and numbers that are relevant to multiplication. As described above, in the Network Retrieval Model approach to mental multiplication (Stazyk, Ashcraft & Hamann, 1982; Ashcraft, 1987) it is assumed that skilled adults use retrieval to answer simple multiplication problems, and that multiplicands activate answers via spread of activation.

The facilitating and inhibiting effects of "priming" are consistent with both the network-interference (Campbell, 1987a; Campbell & Graham, 1985) and the distribution of associations (Siegler, 1986; Siegler & Shrager, 1984) models of memory for arithmetic facts. According to these accounts, a problem activates a number of response candidates to various levels of activation, and retrieval performance is determined by the activation level of the correct response relative to the activation level of its competitors (Anderson, 1983a).

The priming of multiplication problems was studied by Stazyk, Ashcraft & Hamman (1982) and by Campbell (1987b,

1991). In Campbell's experiments, multiplication problems were preceded by a numerical prime that remained on the screen for 300 ms (in the 1987b experiment) or for 200 ms (in the 1991 experiment), and the problem appeared immediately after. He used two kind of primes: "in-table" and "unrelated". "In-table" primes included only true products (a true product is the outcome of the multiplication of a single digit by another single digit). "Unrelated" primes were any number that is not a true product in the multiplication table. Campbell has shown that a false "in-table" prime reduced performance more than a false "unrelated" prime. No difference in RT between unrelated and neutral false primes was found. This suggests that the priming of an associated competitor interferes with correct retrieval. This effect can be explained by assuming that priming activates specific false associations to varying degrees and that interference during retrieval varies with the activation level of false candidates. Similarly, Koshmider and Ashcraft (1991), using the verification method, found that "false related" answers affected RT and error rate more than "false unrelated" answers.

Evidence for automatic processing in a task involving arithmetic knowledge was presented by LeFevre, Bisanz and Mrkonjic, (1988). They used a matching task and subjects indicated whether the target digit matched either of two

digits in the prime. When the target digit was the sum of the two digits in the prime, the RT was longer as compared to a neutral probe condition. This interference effect was only obtained at short SOA duration (120 ms). This suggested that activation of the sum of two digits was automatic and obligatory, even though the task did not require addition.

In the present research a simple multiplication was presented to the subject on a computer screen in the form $Ax \dots B=C$ (the dots represent a variable delay). In accord with previous findings, it was assumed that the multiples of the digit "A" become activated during the delay period. When the second multiplicand and a false answer were encoded and processed a few ms later ($B=C$), a number of possible true answers would have been pre-activated. On this view it was predicted that there would be competition between the "true" and "false" answers and thus a delay in the RT and an increase in error rate.

The results are meant to show that the multiples of a family of products are organized in a systematic fashion, that the "parent" digits activate their families by mean of spreading activation and that such activation has a specific time course.

It is assumed that the mental multiplication table has an ideal table-like format in which the columns and the rows are formed by products that belong to the same family. Therefore, each row or column will be activated by the

encoding of the parent digit. The spreading activation will be more evident and strong the closer the product is to its parent. Therefore, the left side of the table ("small problems") will be more activated by the first multiplicand than the right side ("large problems"). Moreover, the right side of the table ("large problems") will show the effect of the size and of the less frequent use of the large problems. As a consequence of the above, in the "small" side of the table, the response time for the "false" answers that are multiples of the first multiplicand will be slower than the RT to "false" answers that are multiples of the second multiplicand. The reverse will be true in the "large" side of the table.

The stimuli are represented by the "parent" digits, related by the symbol for multiplication, and by "true" and "false" answers. Half of the answers presented are true and half false. The false answers are of two kinds: 50% of the false answers are related to the first multiplicand ("IN" false answers), and 50% are related to the second multiplicand ("OUT" false answers). The false "IN", answers are multiples of the first multiplicand. Consider for example, the problem $3 \times \dots 7 = X$. The false "IN" answers are 18 and 24. These false answers are always a multiple of the first operand (e.g., $18 = 3 \times 6$, $24 = 3 \times 8$) and are located in the multiplication table either one preceding, or one following position with respect to the correct answer (21).

 Insert Table 4 about here

The other 50% false answers are "OUT", answers. They are not multiples of the first multiplicand, but instead multiples of the second multiplicand. With the example $3 \times \dots 7 = X$, the "OUT" false answers are 14 and 28. These false answers are always a multiple of the second operand (e.g., $14 = 2 \times 7$, $28 = 4 \times 7$), and are located in the multiplication table one position before, or one position after, with respect to the correct answer (21).

 Insert Table 5 about here

The pilot tests preliminary to the present study showed an activation of the nodes associated with the first operand that occurred at short SOAs. For this reason a control SOA=0 ms, and experimental SOA=60, 100, and 200 ms were planned. Moreover, in order to further study the time course of activation of multiplication facts, SOAs=300 and 400 ms were also included in the study.

Method

Subjects.

A total of 120 Carleton University's introductory psychology students participated to partially fulfil a course requirement. Median age of the subjects was 23:4 (years:months).

Materials.

Each problem consisted of a multiplication of the type $axb=c$, ranging from 3×4 to 8×7 . Excluded were ties, all the products of 2 and all the products of 9. The complete set of stimuli is presented in Appendix A.

The 30 problems (from 3×4 to 8×7) that were selected for the experiment are represented in the multiplication table by the products: 12, 15, 18, 21, 24, 20, 24, 28, 32, 30, 35, 40, 42, 48, 56. For the purpose of a more selective study of the multiplication table, and of the problem size effect, the group of problems with products less than 28 were classified as group "small", and the group of problems with products equal or larger than 28 were classified as group "large".

Each of the 30 problems selected was presented for 4 times with the correct answer, and 4 times with a false answer. The logic for selecting the false answers was as follow:

A) False answers were either multiples of the first multiplicand ("IN") or of the second multiplicand ("OUT"),

as specified above.

B) Each "IN" false answer was either "low" (one preceding position with respect to the correct product, e.g., $3 \times 5 = 12$), or was "high" (one following position with respect to the correct product, e.g., $3 \times 5 = 18$). Similarly, each "OUT" false answer was either "low" (one preceding position with respect to the correct answer, e.g., $3 \times 4 = 10$), or was "high" (one following position with respect to the correct product, e.g., $3 \times 4 = 20$).

According to this classification, there were four block of 60 problems and a total of 240 problems.

In summary, the criteria for the selection of the 4 false answers for each problem was related to "IN-OUT" and to "low-high" as explained above. Moreover, according to the location of the problem, in the left or right side of the table (having a product less than 28, or equal or more than 28), its location was classified as "small problem" or "large problem".

Each subject was presented with 240 problems, 120 of which were "true", and the remaining were either false "IN", or false "OUT" answers. The false "IN" could be "low" or "high" and equally the false "OUT" could be "low" or "high" as explained above. The "IN-OUT" and the "low-high" were randomized. (The term "IN" refers to being multiples of the first multiplicand and the term "OUT" refers to being multiple of the second multiplicand, as indicated above).

Insert Table 10 about here

The stimulus onset asynchrony (SOA) between the first multiplicand and the rest of the operation were six: 0, 60, 100, 200, 300, 400 ms. The 240 problems were presented once at each level of SOA, to different groups of 20 subjects each. For half of the problems the correct response was "yes". Each of the four blocks of 60 problems was ordered randomly with the constraint that no more than 3 "yes" or "no" items, no more than 2 of a problem type appeared consecutively. Two different list of the same problems were created, one list being the reverse of the other. Twenty subjects were randomly assigned to each SOA.

Procedure.

Items were presented on a video display controlled by a microcomputer. The subjects responded by pressing an appropriate key on a response panel with three keys arranged horizontally. The subjects were instructed to place the middle finger of the preferred hand below the middle key and to return the finger to that position after each keypress. Each trial was initiated by pressing the middle key. After a 450 ms delay, the first multiplicand followed by the multiplication sign was presented (e.g., 4x). Following a variable SOA (0, 60, 100, 200, 300, or 400 ms) the rest of the operation appeared (e.g., 6=24). The problem remained on

the screen until the subject responded "yes" or "no" by pressing the appropriate key on the response panel. The position of the "yes" key was counter balanced across subjects.

Latency and accuracy were recorded, and subjects received auditory feedback concerning the correctness of each response. An ascending series of four tones followed the correct response, and a short buzzer followed an incorrect response. Following a response, the problem was removed from the screen and replaced by the prompt "**", which remained until the middle key was pressed to initiate the next trial.

Before the experimental trial, the subjects were given 10 practice trials that included auditory feedback. Practice trials included items that were similar but not identical to the experimental problem types. The subjects were instructed to respond accurately but as fast as possible.

Results

Latencies and percent errors on "False" and "True" answers were analyzed separately.

Problems were characterized as "small" (problems, having a product less than 28), or as "large" (problems, having a product equal or larger then 28). The mean response latencies to false answers and percent errors are shown in Table 7.

Insert Table 7 about here

False Answers Analysis.

Overall Manova analysis

Latencies to "False Answers" were analyzed with a 2(Family: "IN" vs "OUT")x 2 (Location: "low" vs "high")x 2 (Problem-Size: "small" vs "large")x 6 (SOAs 0, 60, 100, 200, 300, 400 ms) ANOVA with repeated measures on the first three factors.

A significant main effect of "Problem-Size" showed that "small" problems (1227 ms) were responded to faster than "large" problems, $F(1,114) = 105.78$ MSe = 32996, $p < .001$. This reflects the ubiquitous problem size effect reported through the literature. The interaction of "Problem-Size" by SOA was significant, $F(5,114) = 2.29$, MSe = 32996.40, $p < .050$. Tests of simple effects were used to assess effect of Problem-size at each SOA. The Problem-size effect was significant at SOA=0, $F(1,114) = 5.36$, MSe = 32996.40, $p < .02$; at SOA=60, $F(1,114) = 7.95$, $p < .001$; and at SOA=400, $F(1,114) = 9.28$, $p < .001$. The problem-size effect was approaching significance at SOA=100, $F(1,114) = 3.27$, $p < .10$; and was not significant at SOA=200 and SOA=300.

Insert Figure 1 about here

SOA also interacted with Location ("low" vs "high"), $F(5,114) = 3.13$, $MSe = 11572$, $p < .011$. The difference between low and high False answers was significant only at $SOA=60$, $F(1,114) = 4.73$, $MSe = 11572.65$, $p < .05$, and at $SOA=400$, $F(1,114) = 17.67$, $p < .001$ (See Figure 2).

 Insert Figure 2 about here

In the overall analysis there was a Family ("IN" vs "OUT") by Location ("low" vs "high") interaction, $F(1,114) = 10.32$, $MSe = 8859$, $p < .002$, that showed longer latencies for the "IN" than for the "OUT" Family in the "Low" false answers location (1286 vs 1260 ms), $F(1,114) = 6.15$, $p < .05$. On the contrary, the "High" false answers location showed longer latencies for the "OUT" than for the "IN" Family (1295 vs 1308 ms), $F(1,114) = 7.7$, $p < .05$. The longer latencies for the "IN"- "low" false answers show the confusion effect produced by the activation of the multiples of the first multiplicand ("IN") in the "low" false answers. On the contrary, the longer latencies for the "OUT"- "high" answers show the confusion effect produced by the activation of the multiples of the second multiplicand ("OUT").

 Insert Figure 3 about here

Similarly, there was a Family by Problem-size

interaction, $F(1,114) = 5.38$, $MSe = 13940$, $p < .020$, that showed longer latencies for the "IN" Family, as opposed to the "OUT" Family, for the "small" problems. This difference was significant, $F(1,114) = 7.7$, $p < .05$. The type "large" problems showed longer latencies for "OUT" than for "IN" false answers, $F(1,114) = 7.2$, $p < .05$. This suggests that the activation of the multiples of the first multiplicand produced a confusion effect in the "small" problems and the activation of the multiples of the second multiplicand produced a confusion effect in the "large" problems.

 Insert Figure 4 about here

There was a significant three-way interaction of Location ("low" vs "high") by Problem-Size ("small" vs "large") by Family ("IN" vs "OUT"), $F(1,114) = 22.05$, $MSe = 11660$, $p < .001$ (See Figure 5).

The analysis of the interaction for the "small" problems showed longer latencies for the "IN" Family than for the "OUT" Family in the "low" false answers Location, ($F(1,114) = 7.08$, $p < .05$). On the contrary, in the "high" Location the latencies were longer for the "OUT" than for the "IN" Family ($F(1,114) = 6.3$, $p < .05$). For the "large" problems, the "low" false answers showed longer latencies in the "OUT" Family, ($F(1,114) = 6.42$, $p < .05$), and there was no significant difference between "IN" and "OUT" in the "high" Location.

These findings suggest that the first multiplicand ("IN") activated its multiples and this activation made it difficult to reject the false answers in Location "low". Therefore, the notion that the spread of activation starts at the "parent digit" and spreads along the column of its multiples, is supported. On this view, the "IN-low" items are more intensely activated and thus, more difficult to reject than the "IN-high" items. As can be observed in Figure 5, the "small"-high and the "large"-low Locations are parallel. This suggests that in the middle part of the multiplication table the activation of the multiples of the second multiplicand prevails and the false answers that belong to the Family of the second multiplicand are more difficult to reject. The lack of difference between "IN" and "OUT" in the "large"-high Location suggests that on the right side, at the end of the table, for the larger problems, the different activation of the Families has faded.

 Insert Figure 5 about here

Overall IN-OUT Effects.

Tests of simple effects were significant for "small" problems at SOA=60 ($F(1,114) = 4.88, p < .050$) in which "IN" was greater than "OUT", and approached significance at SOA=300 ($F(1,114) = 3.34, p < .10$). The same $F(1,114)$ ratio

approached significance for "large" problems at SOA=60 ($F(1,114) = 3.03, p < .10$), but in opposite direction: "OUT" was greater than "IN". At all other SOAs the F was not significant. These results showed a significant difference in response time between "IN" and "OUT" for "small" problems at SOA=60, due to the confusion effect produced by the activation of the multiples of the first multiplicand.

 Insert Figure 6 about here

Overall "Time Course" Analysis.

The overall "Time Course" for all False answers was analyzed comparing the overall response time at SOA=0 (1186 ms) with the overall response time at SOA=60 (1401 ms), the difference was significant, $F(1,39) = 5.35, p < .05$. The RT at SOA=0 was also compared with the overall RT at SOA=300 (1287 ms), the difference was significant, $F(1,39) = 5.26, p < .05$. Similarly, the difference was significant for the comparison between the overall SOA=0 and the SOA=400 (1375 ms), $F(1,39) = 5.32, p < .05$. On the contrary the overall difference between SOA=0 and SOA=200 was not significant.

These results show that the size of the SOA is relevant in certain types of priming experiments, like the present one, in which one cognitive unit (3×7) is artificially divided in two chunks. The first chunk (first multiplicand followed by the multiplication sign: $3 \times$) is meaningful by it

self, is encoded as a meaningful unit and is processed accordingly. The second chunk is presented with a delay, is not meaningful by itself (second multiplicand equal a number: 7=nn), is processed up to the first levels of features analysis, but cannot be processed further because its relation with the first chunk is not obvious, and it has no meaning to be further developed. At this point the whole input has to be re-evaluated, because the task demand requires that an answers has to be given. The re-evaluation of the input produces a new entity (e.g., the entire operation: $3 \times 7 = 14$) that is further processed as a verification problem.

Time Course of "small" problems.

The time course of activation of the "small" false problems was examined by analysing latencies at each SOA. "IN" false answers were generally rejected more slowly than "OUT" false answers.

 Insert Figure 7 about here

A Tukey HSD test showed that the differences between "IN" vs "OUT" for small problems were significant between $SOA=0$, vs $SOA=300$ and $SOA=60$. Similarly, the differences were significant between $SOA=100$ vs $SOA=300$ and $SOA=60$. Finally the differences between $SOA=200$ was significant vs $SOA=300$ and $SOA=60$ ($q.05 = 4.03$, $MSe = 13940.15$, $HSD = 40$).

Tests of the simple, simple effects showed that the differences between "IN" vs "OUT" false answers were significant at SOA=60 (58 ms difference, $F(1,19) = 3.5$, $p < .01$). At SOA=300 (48 ms difference, $F(1,19) = 3.5$, $p < .01$). Similarly, at SOA=400 the 29 ms difference between "IN" vs "OUT" was significant, $F(1,19) = 2.9$, $p < .01$. At SOA=0, 100 and 200 the differences between "IN" vs "OUT" were not significant.

This finding is consistent with the hypothesis that the first multiplicand activates its multiples and produces a confusion effect. At SOA=60 the initial encoding of the first multiplicand produces a fast, obligatory activation of the "IN" family. At SOA's of 300 and 400 ms, the same phenomena occurs because of a re-encoding, or re-processing of the first multiplicand.

Time Course of "large" problems.

The time course of activation of the "large" false problems were examined by analysing the response latencies at each SOA.

 Insert Figure 8 about here

A Tukey HSD test showed that the differences between "IN" vs "OUT" for large problems were significant between SOA=60 vs SOAs 0, 100, 200 and 400 ($q_{.05} = 4.03$, $MSe = 13940.15$, $HSD = 40$).

Test of the simple simple effects showed that "IN" false answers were rejected more slowly only at SOA=300 (15 ms difference, $F(1,19) = 3.7$, $p < .01$). On the contrary, at SOAs=60, 100, and 400, the "OUT" False answers were rejected more slowly. The differences between "OUT" and "IN" were significant at SOA=60 (46 ms difference, $F(1,19) = 3.5$, $p < .01$); at SOA=100 (10 ms difference, $F(1,19) = 3.7$, $p < .01$); and at SOA=400 (29 ms difference, $F(1,19) = 3.7$, $p < .01$). These results suggest that the activation of the Family of the second multiplicand (OUT) produced a confusion effect that made the false answers more difficult to reject for the "large" problems, at all SOAs, but not at SOA=300, where the "IN" answers were more difficult to reject.

False Answers Error Rate.

The error rate for "False" answers was analyzed with a 2 (Family: "IN" vs "OUT") $\times$ 2 (Location: "low" vs "high") $\times$ 2 (Problem-Size: "small" vs "large") $\times$ 6 (SOA 0, 60, 100, 200, 300, 400) ANOVA, with repeated measures on the first three variables. The overall error rate analysis showed a significant effect of Problem-size, $F(1,114) = 87.7$, $MSe = 88.82$, $p < .001$. Large false answers were more difficult to reject than small false answers, but more important is the interaction. There was a significant SOA by Problem-size interaction, $F(1,114) = 3.51$, $MSe = 88.82$, $p < .005$. The simple effects at each SOA showed that the differences between "small" versus "large" problems were significant at

all SOAs, but not at SOA=200. At SOA=60: 6.6% vs 12.5%, $F(1,114) = 5.44$, $MSe = 88.82$, $p < .05$. At SOA=100: 6.1% vs 11%, $F(1,114) = 4.22$, $p < .05$. At SOA=300: 7.8% vs 15.5%, $F(1,114) = 11.05$, $p < .001$. At SOA=400: 5.8% vs 11.5%, $F(1,114) = 5.95$, $p < .05$.

There was a significant effect of Location ("low" vs "high"), $F(1,114) = 20.25$, $MSe = 55.73$, $p < .001$, where fewer errors were made for "low" (8.1%) as compared to "high" (10.9%) false answers.

There was a significant effect of Family, $F(1,114) = 12.64$, $MSe = 52.17$, $p < .001$. The percent error for "OUT" (10.6%) was larger than for "IN" (8.3%).

 Insert Figure 9 about here

Location interacted with problem-size, $F(1,114) = 10.28$, $MSe = 65.36$, $p < .002$. Subjects made more errors on large than on small problems at both locations. On small problems, subjects made more errors on items at high than at low location (8.5 vs 4.7%), whereas on large problems, errors did not vary as much with location (13% vs 11.4%). Thus, the effect of location was greater for small than for large problems. The difference between "large" and "small" problems was significant for False answers of location "low" ($F(1,114) = 9.20$, $p < .001$), and also for location "high" ($F(1,114) = 8.9$ $p < .001$).

The average error rate for "small" problems was 6.58% and for "large" problems was 12.36%.

"True" Answers Analysis and Error Rate.

Latencies to "True" answers were analyzed with a 2 (Problem-Size: "small" vs "large") x 6 (SOA 0, 60, 100, 200, 300, 400) ANOVA. The problem Size effect was observed in the "True small" (996 ms) versus "True large" (1150 ms) answers, $F(1,114) = 4.08$ $p < .05$. The pattern of response time at different SOAs parallels the one found for the false answers.

 Insert Figure 10 about here

The overall mean error rate for "True" answers was 5.6%. The highest error rate was at SOA 0 (6.4%) and the lowest error rate was at SOA 200 (3.9%). No significant effects were found.

Discussion

The goal of the present research was to examine the organization and activation of mathematic facts within the context of multiplication. The architecture of Ashcraft's (1987) table-like model for addition was adapted to a table-like format for multiplication following the same criteria with respect to the function and location of the "parent digits" and of their associated "family" of products. It was

assumed that access to a single parent digit pre-activates its family of products by means of spreading activation.

The effect of the spreading activation from a parent digit to its family of products was examined using a priming paradigm. The logic was as follow: If the first multiplicand and the symbol for multiplication "x" were followed by a pause or lag of a few milliseconds (SOAs of 0, 60, 100, 200, 300, 400 ms) before the appearance of the rest of the operation (e.g., second multiplicand and answer), the family of products of the first multiplicand would be activated during the delay period. When the second multiplicand and a false answer were encoded and processed a few ms later, a number of possible answers would have been pre-activated. On this view it was predicted that there would be confusion between the "true" and "false" answers and thus a delay in the RT and an increase in error rate.

Problem size effects were examined by dividing the problems in two groups: problems with products less than 28 ("small" problems), and problems with products equal or larger than 28 ("large" problems). The division of the stimuli in a group of "small" versus a group of "large" problems permitted the analysis of the effects of the spreading activation in relation to the problem size effect. It was assumed that the activation of the family of the first multiplicand would be intense in the range of the "small" problems, but would suffer a delay in reaching the

"large" problems, possibly due to a distance effect (or to a low-frequency effect).

The results of the present research support the hypothesis that activation spreads from the first multiplicand to its "family" of products. The findings are consistent with the notion that the "parent-digits" are related to the products of the multiplication table in a systematic fashion. They further suggest that there is a relationship of neighbourhood between products and that the spread of activation travels along paths that can be referred to as columns and rows of a mental multiplication table.

The assumption that the activation originating from the first multiplicand spreads to its multiples is more precisely observed in the analysis of the "small" part of the table that includes problems smaller than 28. The effect of the spreading activation was separately analyzed in the two groups of "small" and "large" problems and at different SOAs of 0, 60, 100, 200, 300, 400 ms.

The "small" Problems Discussion.

Two aspects of the results must be discussed. The first aspect refers to the difference in RT between the two families of products (the "IN" and "OUT" difference). The second aspect refers to the significant longer latencies of both families at SOA=60, SOA=300 and SOA=400, as compared to the latencies at SOA=0 and at SOA=200, as reported above in

the results section.

Analysing the first aspect, the difference in RT between the two Families at SOA=60 suggests that the encoding of the first multiplicand produced an early, obligatory, activation of its Family of multiples and a confusion effect that was manifested by the longer latencies for the "IN" false answers. Latencies for "false" answers of the family of the first multiplicand were 58 ms slower than those for the "false" answers belonging to the family of the second multiplicand, and this difference was significant. This can be explained as follows: The early activation of "false" answers that belong to the same family (and are first degree neighbours) of the correct answer (that is activated at the presentation of the problem) produced a confusion effect (at the decision stage of the verification task) that made the "false" answers of the family of the first multiplicand difficult to reject. On the contrary, the "false" answers that belong to the second multiplicand received a reduced amount of activation and did not produce confusion. Therefore, they were easier to reject.

A second result is that significantly longer latencies were observed for both families at SOA=60, at SOA=300 and SOA=400, as compared to SOA=0. This effect can be explained by borrowing some concepts from the neurophysiology of the central nervous system. Consider that the lag of 60, 300 or 400 ms, after the presentation of the first multiplicand and

the "x" symbol, produced an unusual pause in the flow of information, followed by a confusion effect, as discussed above. This condition brings to mind some comment by Edelman (1990) who states: "In Neural Darwinism I pointed out that pairs of channels responsive to different neurotransmitters or voltage levels could cooperate or not, depending on the "time" at which they were in a particular state. Outside of a particular "time-window" they could no longer act together. An alteration in the interaction between several neurotransmitters could lead to a disorder in communication between re-entrant maps". This explanation fits quite well the present experiment and can be assumed to be the possible reason for the prolonged RT at some SOAs. If the "time-window" was not appropriate for the transmission of further information, this lead to a disorder in communication manifested by the longer RT. If this is so, it suggests something new about the early stages of encoding, the flow, and the transmission of partial information. It is possible to assume a neuro-chemical "refractory period" concomitant to a false alarm, or a stressing event. The chronometric evidence for this "refractory period" was robust for both families and in all conditions. Nevertheless, the condition of stress, or disorder in communication, did not affect the spread of activation from the first multiplicand to its family. This is probably due to the fact that the multiples were activated at earlier stages, and the process was not

inhibited by the disorder in communication that happened a few milliseconds later.

Continuing the analysis of the group of "small" problems, it is possible to note that at SOA=100 and SOA=200, the difference in RT, between the "false" answers related to the family of the first multiplicand and the "false" answers related to the family of the second multiplicand, is not present. On the contrary, the RT of both merges, as can be seen in the graph. Moreover, there is a reduction in total RT: At SOA=200 the RT is the same as at SOA=0. This effect suggests that at SOA=200, the multiplication network system reached asymptote: The correct answer was easy to retrieve, the false answers were easy to reject, and there was a higher confidence level due to the absence of the confusion effect.

The following SOA=300, shows that the latencies are longer, as compared to SOA=200, and are similar to SOA=60. The RT for "false" answers that belong to the family of the first multiplicand are 48 ms slower with respect to the RT for "false" answers that belong to the family of the second multiplicand. This pattern is similar to the one observed at SOA=60 and suggests that the confusion effect due to the fast, obligatory, activation of the family of the first multiplicand (as pointed out above), is present again. This suggests that the first multiplicand has been re-encoded during the prolonged delay and has re-activated its family

of products.

The re-encoding phenomena is assumed to be due to the fact that the stimulus is still present on the screen and the subject is focusing the attention on it, therefore the system automatically re-starts the encoding and processing activity. This point will be further discussed below, with more detailed explanations and assumptions.

The assumption of a re-encoding phenomena is relevant to the study of the time course of encoding and processing of information. The findings described above suggest that the encoding and processing of the information relative to a simple multiplication problem is performed in a period of time of approximately 200 ms, when the system reaches asymptote. This result is in agreement with recent studies using Event-Related-Potential (ERP). Posner (1992) says: "While we do not yet understand precisely the brain mechanism that makes possible the recognition of visual word forms, we know that the left ventral occipital lobe ...is involved in this learned skill.... By recording from scalp electrodes, we have been able to show that this computation takes place about 200 ms after the word appears." Similarly, Compton, Grossenbacher, Posner, and Tucker (1991), exploring the time course of detection of letter features, letter strings and words, using scalp electrodes (ERP), found that "...the feature related analysis begins within the first 100 ms and the visual word form discriminates words from strings

by about 200 ms."

The above consideration is relevant with references to studies of priming. For example, what comes to mind is the study by Balota and Chumbley (1985) on frequency effect in a delayed naming task. Subjects viewed words for about 1 s and then named the word when a cue was presented. Balota and Chumbley assumed that, since the lexical access of the word should have been completed after about 1 s, any frequency effect could not be an access effect, but a pronunciation output effect. They concluded that word frequency modulates output and not lexical access. The point that should be considered is that it is possible that the subject re-encoded the word. It can be observed in the present experiment that, when the activation starts to fade, the stimulus is re-encoded. Consequently, Balota and Chumbley's experiment does not guaranty that the frequency effects where not due to re-encoding and therefore to lexical access.

With reference to the same topic, it is notable that in most priming studies the possibility that primes were re-encoded is not taken into account. Therefore it would be interesting to pursue further investigations about the time course of the encoding and re-encoding of primes in the study of word recognition and lexical access. Equally, the time course of the optimal performance with relation to the type of stimulus has never been fully investigated.

Consequently, some effect may have been wrongly attributed to experimental conditions and were instead produced by the spread of activation or by the asymptote of the system under study. Therefore the time course of the asymptote and of the period of instability in systems currently under study should be more precisely investigated.

Related to the studies of priming are some findings that suggest post-target processing deficit resulting from attention factors. Such hypothesis is relevant to the present research. Reeves and Sperling (1986), Weichselgartner and Sperling (1987), and Sperling and Weichselgartner (1990), suggested an "attention gating" in short term visual memory. More recently, the temporary suppression of visual processing was equally observed by Raymond, Shapiro and Arnell (1992) in a Rapid Serial Visual Presentation (RSVP) task. Raymond et al. (1992) support the view of an opening and closing of an attentional gate to regulate the flow of perceptual visual information to recognition centres of the brain (Reeves & Sparling, 1986). Raymond et al. (1992) further suggest that: "When confusion is present, .. the attentional gate is both shut and locked, making the initiation of the next attentional episode a more time consuming process than if a locking operation had not been conducted".

The hypothesis that the attention gates may open and close in case of a confusion effect, can be related to the

neuro-chemical refractory period discussed above. Moreover, a detailed analysis of the data obtained in the present experiment fits nicely the hypothesis and explains the delayed RT at different SOAs.

For example: At SOA=60 the difference of the RT with respect to SOA=0 is of 200 ms, equally for the "small" and "large" problems (1119 vs 1319 ms for the small problems, and 1252 vs 1482 for the large ones). During the experiment, the subject focused the attention on the first multiplicand for 60 ms (e.g., 3x3x3x..). All the multiples of the first multiplicand were fully activated, as reported above, and it can be assumed that, when the second part of the multiplication was presented (e.g., 7=x), this new encoding produced a confusion effect that prevented further processing at higher levels. It can be assumed that the whole problem was encoded and processed about 200 ms later when the confusion effect was less intense and the gates of attention re-opened.

Equally, at SOAs=100 and SOA=200 the difference in RT, as compared to the SOA=0, is 150 and 50 ms respectively for the small problems, and 100 and 0 for the large problems. Here it is possible to assume that the prolonged attention to the first multiplicand produced a condition of neural fatigue or of habituation (e.g., "3x.." for 100 or for 200 ms), therefore the activation declined. The confusion effect produced by the presentation of the second half of the

operation was not so intense and the closing of the gates was shorter. This assumption also fits the data.

The RT rises again at $SOA=300$. This can be explained by assuming that, after 200 ms of fixing the attention to the first multiplicand, the corresponding activation has faded. Consequently, approximately 250 ms after the beginning of the trial, the first stimulus is re-encoded and about 50 ms later the complete operation is encoded and processed. This condition is an exact repetition of $SOA=60$ and the response time is equivalent. Similarly, at $SOA=400$ there are possibly one or two re-encoding of the first multiplicand (e.g., at 200 and 350 ms) from the beginning of the trial. It can be assumed that the activation and the confusion are intense at the moment of the presentation of the second half of the problem, and there is a closing of the attentional gates. The refractory period seems to last about 200 ms after which the whole problem is encoded and processed. The RT shows this 200 ms delay for both families.

The "large" Problems Discussion.

The discussion of the analysis of the "large" problems, that have a product larger than 28, involves several aspects. At $SOA=60$ the RT of the family of the second multiplicand is 46 ms longer than the RT of the family of the first multiplicand. This is the opposite to what happens in the small problems. To explain this effect, not only the spread of activation between the "Parent-digit" and the

Family has to be taken into account, but also the strength of the pathways inside the family itself. It is currently assumed that the more frequent the association between two items, the greater the strength of the pathway between the items. In the case of the families of products, the problem size effect suggests that the strength of the pathways between nodes of large problems are weaker than those of the small problems that are more frequently used in daily life.

This notion deserves a more detailed explanation. Anderson (1983a,b) states that: "A pattern of activation in long-term memory is set up in response to the input of activation from the focused units. These elements of focused working memory units are referred to as 'sources of activation'. The amount of activation they emit is a function of their strength" (Anderson, 1983a, 1983b). In the present study, the "sources of activation" are the "parent" digits and the nodes in this activation pattern are the products of the "families". Further, Anderson (1983a) suggests that there is a limit to the total amount of activation that will be pumped into the network from a source. This amount of activation is distributed among the nodes in the network "...with nodes getting more activation to the degree that they are closely and strongly connected to sources of activation" (Anderson, 1983a). More activation will accumulate in those part of the network that have a higher frequency of exposure. In the case of the

multiplication table, the part of the network that has a higher frequency of exposure, and that is also closely and strongly connected to sources of activation, is the small part, with products smaller than 28.

It is possible to assume that the activation reaching the products larger than 28 is relatively weak and will fade faster when it progressively spreads to more remote and less used family members. If it is assumed that the activation due to the first multiplicand is reduced and fades faster for the large problems, this explains the observation that the RT that corresponds to the activation of the family of the second multiplicand is 46 ms longer than the activation of the family of the first multiplicand. Indeed, it appears that the activation produced by the second multiplicand is responsible for the confusion effect observed at SOA 60. This can be explained by the assumption that the activation of the family of the first multiplicand is fading by the time when the second multiplicand is encoded. Therefore, the activation of the family of the second multiplicand, relative to the one of the first multiplicand, will be stronger. This explains the confusion effect due to the activation of the family of the second multiplicand for the large problems.

At SOA=200 an asymptote is reached and the RT is reduced to a minimum: The same occurrence as observed in the small problems. At SOA=300 and SOA=400 the RT becomes longer

again, suggesting a re-encoding of the stimulus and the starting of a new process, as assumed above.

The assumptions related to the refractory period and to the closing and opening of the attentional gates, as discussed above, can also apply to the "large" problems.

The True Problems.

The general behaviour of the "True" answers parallels that of the "False" ones for both small and large problems. The same reduction in RT at SOA 200 can be observed also in the "True" answers. This confirms that the system reaches asymptote at 200 ms.

For the large problems at SOA=200 the RT is significantly faster than at SOA=0 (87 ms). This suggests that there is a facilitation effect at SOA=200 with respect to the SOA=0. The facilitation effect can be explained by assuming that the spread of activation that occurred immediately after encoding is partially fading after about 200 ms. Nevertheless, some degree of activation probably still remains in the "family" of the first multiplicand. Such residual activation is not sufficient to create a confusion effect. On the contrary, the correct answer sums its residual activation to the activation it receives at the moment of the encoding of the entire problem. The double activation of the true answer, and its capacity to inhibit the weaker activation of its neighbours, makes it faster to affirm in the true trials.

Conclusions.

In the present research I focused on the fundamental questions:

- 1- Are multiplication facts associated in cluster-like format?
- 2- Do single digits activate their multiples (Ashcraft) or is it necessary to encode the whole operation to activate the products (Siegler)?
- 3- Is the first operand interacting with the second or do they independently activate a product?
- 4- What contribution can I bring to the understanding of the ubiquitous problem size effect?

The results of the present research showed that:

- 1) The problem size effect was significant for products larger than 28. This replicates the problem-size effect found throughout the literature and therefore validates the method used in the present research.
- 2) The hypothesis that a single multiplicand activates its multiples was supported. Such activation is implicit and obligatory.
- 3) The effect of neighbourhood and interaction between the products of the first and second multiplicand were significant, suggesting a clustering of multiplication facts.

These findings suggest that the 36 products used in daily life for simple mental multiplications, are stored

quite close to one another in a dynamic cluster that can be referred to as having a table-like format, probably bi-dimensional, in which one vector is always represented by the first multiplicand and the other vector by the second multiplicand.

In conclusion, with respect to the architecture, it is possible to affirm that the stored multiplication facts are functionally clustered. The depiction of this cluster as having a table-like format, is supported. With respect to the process, the access into the system is a specific pattern of spreading activation. This activation comes into effect when an operand is encoded and spreads along the path that represents the family of that operand.

Moreover, the present study has implications for different aspects of Cognition. It is a contribution to the understanding of the representation and process related to multiplication and to the study of the time course of the priming paradigm. It is equally relevant to the understanding of the response time of high versus low frequency items and helps interpret the findings related to neighbours and associated items. Fundamental aspects of storage and retrieval of multiplication facts, representational and processing assumptions, the problem size effect, the spread of activation, and the operand distance effect, have been considered. Moreover, suggestions for further studies have been proposed, relative to the

investigation of the time course of priming.

The possibility that the brain and its chemical neurotransmitters may cooperate or not in the processing of information, depending on a particular "time-window", is a very challenging assumption that deserves further consideration.

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TABLE 1.

ADDITION TABLE as proposed by Ashcraft

1	2	3	4	5	6	7	8	9
2	4	5	6	7	8	9	10	11
3	5	6	7	8	9	10	11	12
4	6	7	8	9	10	11	12	13
5	7	8	9	10	11	12	13	14
6	8	9	10	11	12	13	14	15
7	9	10	11	12	13	14	15	16
8	10	11	12	13	14	15	16	17
9	11	12	13	14	15	16	17	18

TABLE 2

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	15	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

TABLE 3.

SIMPLIFIED MULTIPLICATION TABLE

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
	2	3	4	5	6	7	8	9	

TABLE 4

MULTIPLICATION TABLE: EXAMPLES OF
"FALSE-IN" ANSWERS.

1								
2	4							
3	6	9						
4	8	12	16					
5	10	15	20	25				
6	12	<u>18</u>	24	30	36			
7	14	21	28	35	42	49		
8	16	<u>24</u>	32	40	48	56	64	
9	18	27	36	45	54	63	72	81
	2	3	4	5	6	7	8	9

EXAMPLE : $3x \dots 7 = x$

$3x \dots 7 = 18$

$3x \dots 7 = 24$

TABLE 5

MULTIPLICATION TABLE : EXAMPLE OF
"FALSE-OUT" ANSWERS.

1								
2	4							
3	6	9						
4	8	12	16					
5	10	15	20	25				
6	12	18	24	30	36			
7	<u>14</u>	21	<u>28</u>	35	42	49		
8	16	24	32	40	48	56	64	
9	18	27	36	45	54	63	72	81
	2	3	4	5	6	7	8	9

Example: $3x \dots 7 = x$

$3x \dots 7 = 14$

$3x \dots 7 = 28$

TABLE 6

Example of "IN" and "OUT" Family's relationships.

a)- The problem 3×5 may have two kind of "False" "IN" answers:

$$3 \times 5 = 12 \quad \text{and} \quad 3 \times 5 = 18$$

(note that both 12 and 18 are multiples of the first multiplicand: 3).

b)- The same problem may also have two kind of "False" "OUT" answers:

$$3 \times 5 = 10 \quad \text{and} \quad 3 \times 5 = 20$$

(note that both 10 and 20 are multiples of the second multiplicand: 5).

	/	IN-low	/	
	/	12	/	
-----/-----/-----				
OUT-low	/	"TRUE"	/	OUT-high
10	/	15	/	20
-----/-----/-----				
	/	IN-high	/	
	/	18	/	

TABLE 7.

SOA												
0		60		100		200		300		400		
RT	E½	RT	E½	RT	E½	RT	E½	RT	E½	RT	E½	
Small Problems												
IN												
low	1140	6.4	1364	3.9	1273	4.6	1179	4.3	1279	4.6	1286	2.8
high	1107	7.1	1334	8.2	1199	5.3	1131	6.0	1264	9.3	1317	7.1
OUT												
low	1084	5.7	1241	4.6	1220	5.0	1115	3.5	1183	5.3	1220	5.0
high	1145	9.6	1341	9.6	1256	9.6	1192	8.9	1263	12	1325	8.5
Large Problems												
IN												
low	1233	11	1417	13	1321	9.0	1214	7.8	1324	15	1308	11
high	1269	7.1	1500	8.2	1352	5.3	1239	6.0	1344	9.3	1488	7.1
OUT												
low	1246	10	1469	13	1365	11	1219	11	1324	18	1441	13
high	1261	8.8	1540	16	1329	19	1224	11	1314	20	1512	15

Figure 1. Overall analysis of False Answers: Problem-size by SOA interaction.

Figure 2. Overall analysis: SOA by False Answers Location interaction.

Figure 3. Overall analysis: Family by False Answers Location interaction.

Figure 4. Overall analysis: Family by Problem-size interaction.

Figure 5. Overall Analysis: Three-ways interaction. Problem-size by Family by False Answers Location interaction.

Figure 6. Time Course. Problem-size and "IN" vs "OUT".

Figure 7. "Small" problems: Time Course of "IN" vs "OUT"
Families.

Figure 8. "Large" problems: Time course of "IN" vs "OUT"
Families.

Figure 9. Error analysis: Percent error by SOA interaction.

Figure 10. "Small" and "Large" problems: True Answers analysis.

Appendix A: Problem Set

Problem	True	False Answers			
		"IN"Family		"OUT"Family	
		Low	High	Low	High
3x4	12	9	15	8	16
4x3	12	8	16	9	15
3x5	15	12	18	10	20
5x3	15	10	20	12	18
3x6	18	15	21	12	24
6x3	18	12	24	15	21
3x7	21	18	24	14	28
7x3	21	14	28	18	24
3x8	24	21	27	16	32
8x3	24	16	32	21	27
4x5	20	16	24	15	25
5x4	20	15	25	16	24
4x6	24	20	28	18	30
6x4	24	18	30	20	28
4x7	28	24	32	21	35
7x4	28	21	35	24	32
4x8	32	28	36	24	40
8x4	32	24	40	28	36
5x6	30	25	35	24	36
6x5	30	24	36	25	36
5x7	35	30	40	28	42
7x5	35	28	42	30	40
5x8	40	35	45	32	48
8x5	40	32	48	35	45
6x7	42	36	48	35	49
7x6	42	35	49	36	48
6x8	48	42	54	40	56
8x6	48	40	56	42	54
7x8	56	49	63	48	64
8x7	56	48	64	49	63

END

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FIN