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Thermal Stresses in Lowrise Loadbearing Concrete Masonry Buildings: A Finite Element Study

By

Khaled S. Ibrahim

B.Sc. (Civil)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

MASTER OF ENGINEERING

**DEPARTMENT OF CIVIL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, CANADA
APRIL, 1989.**

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Concrete Masonry Buildings:
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submitted by

Khaled S. Ibrahim

in partial fulfilment of the requirements for the degree of

Master of Engineering

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April, 1989

Abstract

In a typical lowrise concrete masonry wall building, the main structural elements are: concrete masonry wall, concrete roof slab (composite with steel joists), and concrete foundation. Since these elements have different stiffnesses and are made of different building materials which in turn have different properties, the potential exists for relative movements at the interfaces between the structural elements when they are exposed to temperature differentials. Since the structural elements restrain each other at their respective interfaces, significant forces may be generated and may result in cracking. Such cracking may expose the structural elements to environmental distressing factors such as freeze-thaw action and corrosion which in turn may influence the safety, serviceability and maintenance costs of the structure. This study investigates these thermal stresses and the effect of the key parameters that may govern these stresses.

Since thermal loads are likely to be somewhat relieved by the slip and separation between the different structural elements at their respective interfaces, a nonlinear finite element analysis was performed to determine the thermal stresses in a more realistic manner. The analysis employed a special joint element to model such interfaces and utilized an iterative technique for solving the equation of equilibrium.

The extreme temperatures to which the structural elements of a typical lowrise masonry wall building are subjected under typical Canadian climatic conditions were determined. All climatological factors including solar radiation were considered.

Also, a linear finite element analysis was carried out in order to evaluate the feasibility of using this method for predicting the thermal stresses generated in a lowrise loadbearing masonry wall building subjected to temperature differentials. Stresses generated from this analysis were compared against those obtained using a nonlinear finite element analysis.

The stress analyses indicated that the masonry wall may undergo extensive cracking under wintertime thermal loads. The concrete roof slab may undergo cracking under summertime and wintertime thermal loads. The linear stress analysis overestimates the thermal stresses, therefore this method is not the recommended method for this class of problems.

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Notation

A	area perpendicular to heat flow
$A_{1,2,3}, B_{1,2}$	functions of direction cosines and joint stiffnesses
B	strain/displacement transformation matrix
C	matrix of tangent moduli in stress-strain relation
C_{nn}, C_{ss}, C_{ns}	joint stiffnesses, nn is normal to joint, ss is tangential to joint, ns is coupling between normal displacement and shear force and vice versa
E_m	modulus of elasticity of masonry in compression
E	modulus of elasticity of concrete
E_c, E_f	joint stiffnesses normal to joint planes at different stress levels
f	moisture correction factor
f'_m	masonry compressive strength
f'_c	compressive strength of concrete
F_d	vector of internal resisting forces for joint elements
F	effective load vector
G_c, G_f	shear stiffnesses of joint element at different stress levels
H	masonry wall height
h_c	convective heat transfer coefficient
h_r	radiative heat transfer coefficient
h	interpolation function
H/L	aspect ratio
k	coefficient of thermal conductivity of a material
k_{br}, k_{bl}	thermal conductivity of brick and block, respectively

k_{bdr}, k_{bdl}	same as above, but in oven-dry condition
K_c, K_d	global stiffness corresponding to linearly elastic continuum and to joints, respectively
$[K]$	joint element stiffness matrix
L	masonry wall length
l	length of the joint element
N	total number of joint elements
n	number of iterations
P_m	ratio, average tensile stress/maximum tensile stress at the middle of the wall
P	vector of the applied load
Q	heat flow
q	rate of heat flow
R_d	degree of restraint, decimal fraction of unity
R	thermal resistance of a material
T_i	inside temperature
T_o	outside temperature
$[T]$	transformation matrix from natural coordinates to global coordinates
t	thickness of joint element
U	individual nodal displacements at corners of joint element
u	overall coefficient of heat transmission
ϵ_u	ultimate tensile strain
ϵ_d	drying shrinkage
ϵ_t	temperature contraction
w	moisture content in percent by volume

ν	fraction solid of block masonry
ρ	density of the material
α	fraction of block cores grouted
ψ	ratio of net block to gross block area
ξ	nondimensional coordinates in interpolation function
ϕ	angle of friction
$\epsilon_{n,s}$	normal and shear strain components, respectively, in joint element
$\sigma_{n,s}$	normal and shear stress components, respectively, in joint element

Superscripts:

T, B	top and bottom sides of joint element
------	---------------------------------------

Subscripts:

n, s	natural coordinates of joint element perpendicular and tangential to plane of joint
x, y	global coordinates, plane geometry

1.1 General

Movements in masonry buildings are a major concern because they can result in overstressing and cracking. Such cracking may expose the structural elements to environmental distressing factors such as freeze-thaw action and corrosion which in turn may influence the safety, serviceability and maintenance costs of the structure.

Movements occur as the result of temperature changes, shrinkage due to moisture content changes, chemical processes, and creep deformations in the component materials of a building. Among these factors, this thesis will concentrate on the effects of temperature changes on the performance of masonry buildings.

When building materials expand and contract due to temperature changes, considerable forces may be produced in restrained structural elements. If these forces are not properly considered, the resultant stresses can lead to serious failures (usually cracking) in materials and structural elements. These stresses can be relieved or reduced to tolerable limits by constructing control joints. Such joints are particularly important to allow movements to take place along certain preselected lines. Overall, failures may occur in the case of restrained elements when they are not strong enough to withstand the stresses induced, when clearances of control joints are insufficient or connectors do not allow movements or deformations.

One example of the importance of considering the temperature variations is in the case of highrise loadbearing masonry wall buildings with exterior masonry walls subjected to seasonal temperature variations. Here, external walls change their length relative to the interior intersecting walls, which remain unchanged in

a controlled environment. Another typical example are lowrise masonry buildings which have very long masonry walls in which significant thermal stresses may be generated due to temperature variations.

Two types of masonry walls are normally employed in masonry buildings: single wythe walls and cavity walls with exterior cladding. Among these types, this thesis models lowrise loadbearing single wythe concrete masonry wall buildings under the effect of temperature variations. Since the different structural elements of such buildings are exposed to the natural environment, they are subjected to seasonal and daily variations in the ambient air temperature as well as to solar radiation in the daytime. Consequently, the temperature fluctuations between daily as well as annual extremes need to be determined.

In a typical lowrise concrete masonry wall building, the main structural elements are: concrete masonry wall, concrete roof slab (composite with steel joists), and concrete foundation. Since these elements exhibit different stiffnesses and are made of different building materials which in turn have different properties such as coefficient of thermal expansion, Poisson's ratio and modulus of elasticity, the potential exists for relative movements at the interfaces between the structural elements when they are exposed to different temperature differentials. Since the structural elements restrain each other at their respective interfaces, significant forces may be generated and may cause considerable distress in the elements. Such relative movements at the different interfaces, which may be accompanied by debonding, sliding and recontacting of the two sides of each interface, may have a great influence on the thermal stresses. Therefore, modeling the interfaces may result in a realistic prediction of the thermal stresses.

For an interface separating two continua, slip, separation, and recontact may all occur during a given movement-time history. Different laboratory tests have been conducted to examine the physical behaviour of these joints. Such test results indicated the nonlinearity in the behaviour of the joints. Since classical analytical solutions are possible only for a limited class of idealized nonlinear interface prob-

lems, there is a growing interest in numerical solutions for such complex problems.

In simulating the gross behaviour of certain physical interface problems, some investigators, with the objective of representing the joint behaviour in rocks using the finite element method, have developed so called "joint elements" and have achieved success in such modeling. Therefore, in an inspiration of their work and using the finite element method, the problem of relative movements at the different interfaces between the elements constituting a lowrise loadbearing masonry building has been modeled. This work employed a special joint element to model the nonlinear behaviour of such interfaces while the domains above and below such interfaces were kept linearly elastic.

1.2 Scope and Objectives

The overall purpose of this study is to predict the movements and stresses generated in lowrise loadbearing concrete masonry buildings due to temperature fluctuations. This study will utilize the finite element method (FEM) and will consider the interfaces between different structural elements.

In modeling the lowrise building using FEM, three interfaces have been considered:

- a. Soil-foundation interface.
- b. Foundation-masonry wall interface.
- c. Masonry wall-roof interface.

In summary, the work presented in this thesis involves the following:

- Development of a computer program capable of solving problems where interfaces and discontinuities are embedded in an elastic medium.
- Examination of the behaviour of lowrise loadbearing concrete masonry buildings under thermal differentials where interfaces between different structural elements are modeled.

- Study of key parameters that affect thermal stresses in lowrise buildings.
- Development of a rational approach for the determination of control joint spacings.

1.3 Thesis Overview

The work in this thesis is divided as follows:

- In Chapter 2, a literature review of relevant background information is presented. This review includes a brief summary on interface elements, an overview of experimental results from laboratory tests conducted by several investigators to obtain interface material properties, and a literature review on movements, cracking and crack control in concrete masonry walls.
- In Chapter 3, the stress analysis preliminaries are discussed. The physical stress analysis problem is explained, followed by the material properties employed in the study. Finally, a thermal analysis has been conducted to determine the temperature differentials in each structural element of the building under study.
- Chapter 4 is concerned with the development of the finite element program which is capable of solving plane stress problems in which interfaces are embedded in an elastic medium. The derivation of the stiffness matrix of the joint element, the solution of nonlinear equations of equilibrium, the iterative technique employed, the relative displacement coding, and the computer program scheme are explained.
- Chapter 5 is devoted to a case study analysis of a typical lowrise masonry building under different temperature differentials. Evaluation of the linear stress analysis as a method for predicting the thermal stresses in this class of buildings is also presented. Also, the convergency of the iterative scheme is examined.
- Chapter 6 is focused on an examination of the key parameters that govern

the thermal stresses in lowrise concrete masonry buildings.

- Finally, the summary, conclusions, and recommendations for future work are presented in Chapter 7.

2.1 General

In the previous chapter, the necessity for modeling the interfaces between different structural elements of lowrise masonry buildings was discussed. In this chapter, a literature review on the aforementioned special joint elements used to model the interfaces is presented. Also, included is a literature review of the laboratory tests which obtained suitable stress-displacement relationships for the nonlinear behaviour of the interface model. Then, the nature of movements of concrete masonry walls and the resultant different types of cracks are explained. Finally, the present code criteria for handling such movement problems are presented.

2.2 Summary of Literature on Interface Elements

Much work has been carried out to model joints and material interfaces using finite element analysis. In this section, a summary of various finite interface elements is presented.

The pioneers Ngo and Scordelis [59], in their attempt to model the bond between steel and concrete, developed a linkage element, as shown in Fig. 2.1. This element consists of two linear springs parallel to a set of orthogonal axes. This linkage element has no physical dimensions at all, and only its mechanical properties are of importance. Two spring stiffnesses are used to define the mechanical behaviour of such an element. For simplicity, they assumed a linear relationship between bond-slip and bond-stress which means that the bond-slip at any section of the beam is directly proportional to the bond-stress.

Zienkiewicz, Valliappan, and King [84], in modeling a mass rock containing cracks and fissures which can not sustain tension, developed a solution based on elastic compression behaviour which permitted no tension. Such a solution presents a valuable "bound" in many real situations from which stability can be assessed. They have demonstrated how such solutions can be obtained by using a finite element formulation through an iterative "stress transfer" method. Two variants were suggested in this method. In the first, the elastic solution was obtained using original elastic properties by holding the stiffness matrix constant during iteration. In the second the elastic constants were modified by reducing the modulus in the tensile direction. The second variant converged more rapidly but the first presented many computational advantages. In both cases the same final solution was always achieved.

Goodman, Taylor, and Brekke [38] developed a two-dimensional interface element to predict the behaviour of the compound structure consisting of rocks and discontinuities. This element, as shown in Fig. 2.2, is a four-nodal point element with a finite length L and zero width as the nodal point pairs (1,4) and (2,3) initially have identical coordinates. They assumed that both normal and shear displacements vary linearly along the length of the element. The derivation of the interface stiffness matrix is based on the unit joint stiffness in the normal and tangential directions. The final stiffness matrix has 32 non-zero terms, yet depends only on two quantities, the unit joint stiffness in the normal and tangential directions.

Goodman et al. [38] presented the modes of behaviour of the interface element such that if the joint normal stress is tensile in any interface element, the unit joint stiffnesses in the normal and tangential directions are set equal to zero for this element. The problem then is repeated. Also, the joint cohesion, joint friction, and residual tangential stiffness are read as data and the shear strength is calculated for the indicated normal pressure on each joint. If the joint shear stress exceeds the shear strength, then the joint tangential stiffness is set equal to the residual tangential stiffness and the problem is repeated.

Zienkiewicz, Best, Dullage, and Stagg [83] developed a simple joint element. As shown in Fig. 2.3a, they consider the pairs of nodes 1-4 and 2-3 geometrically so close that it is permissible to specify them by the same coordinates in the general problem description providing the thickness t is used when computing the joint element properties. Thus, the geometric coordinates of the element rectangle are fully specified by the two points A and B and the thickness t as shown in Fig. 2.3b. The properties of the element can now be directly established using linear interpolation shape functions. Further, Zienkiewicz et al. [83] developed a curved, variable thickness, cubic isoparametric joint element in which the geometric coordinates are also specified by the mid-coordinates and the middle surface of the joint is defined by using a suitable shape function.

Mahtab and Goodman [52] developed plane joint elements of triangular and quadrilateral shape (Figs. 2.4a and 2.4b) by extending the line joint element concept presented by Goodman et al. [38]. The strains in a joint element are given by the relative displacement between the top and bottom faces, while the stresses are given by stress-strain relationships derived from direct shear tests.

Ghaboussi, Wilson and Isenberg [36] presented a method for the finite element analysis of discontinuities embedded in continuous systems. The discontinuities are intended to represent rock joints, faults, and interfaces. They developed a new joint element using relative displacements as independent degrees-of-freedom. Their new element is a four-nodal point element. The relative normal and tangential displacements are assumed to vary linearly along the element. The stresses and the strains are related through the following material property matrix, C :

$$\left. \begin{aligned} \begin{Bmatrix} \sigma_n \\ \sigma_s \end{Bmatrix} &= \begin{bmatrix} C_{nn} & C_{ns} \\ C_{sn} & C_{ss} \end{bmatrix} \begin{Bmatrix} \epsilon_n \\ \epsilon_s \end{Bmatrix} \\ \sigma &= C \epsilon \end{aligned} \right\} \quad (2.1)$$

where σ_n , σ_s and ϵ_n , ϵ_s are the normal and shear stresses and strains, respectively.

When the joint element is used to simulate the interfaces between different structures and where the contact is perfect, the material constant, C_{nn} , can have two values: zero (debonding) or infinity (contact). The element permits slip to occur under these circumstances. In case of debonding, the joint element is physically non-existent and disappears from the assembly of the global stiffness matrix. In case of contact, the relative displacements in the direction normal to the joint plane are zero.

Ghaboussi, Wilson, and Isenberg [36] also developed an axisymmetric joint element and derived the stiffness matrix for it. Their new element is able to handle the non-dilatant joints and the dilatant joints where a volume change occurs due to shearing strains.

Schäfer [74] proposed a special bond element which makes use of the energy dissipation process in the contact area. He derived the contact element stiffness matrix with the aid of the principle of virtual work assuming that a relation between the shear stress and the frictional deformation is known. For several one- and two-dimensional bond elements, the element stiffness matrices were given, allowing a more accurate description of contact problems than with conventional spring elements.

Wilson [81] extended the interface element developed by Ghaboussi et al. [36] using the relative displacement as the independent degrees-of-freedom to develop a three-dimensional interface element. He illustrated the numerical problem (ill-conditioning) by solving a simple one-dimensional problem in which the original displacements are used as independent degrees-of-freedom. Also, he emphasized that the important advantage of the formulation of the interface elements in terms of relative deformations is that this method can be easily used for two-dimensional as well as three-dimensional interface elements. A schematic diagram of the eight-noded solid elements and a four-noded interface element is shown in Fig. 2.5.

Goodman and St. John [37], using the interface finite element developed in 1968

by Goodman et al. [37], introduced an iterative solution to simulate real properties of joints. The linear equations developed with the displacements of nodal points provide a first solution, but this solution will usually presume tension in some joints or excessive shear in others. Through an iterative process, they computed deformations and stresses consistent with physical joint behaviour.

Goodman et al. [37] suggested various iterative processes which consider a direct shear specimen with an initial shear load F_o being forced by an initially compressed spring as illustrated in Fig. 2.6. First, they chose the initial stiffness K_1 representative of the elastic portion of the load-deformation curve for the joint. Owing to the initial load in the spring, the spring and joint come to equilibrium at point R_1 , which is unacceptable. So, they repeated the problem by aiming the solution towards a point on the constitutive curve near R_1 , for example, point R_1^C .

For the second iteration, they suggested two methods to approach the final correct solution. The first method is the *variable-stiffness method* which redefines the stiffness matrix to K_2 so that a solution will pass through R_1^C . For this method, the new solution will be $R_{2,VS}$, which is still unacceptable but closer to the correct answer R_∞ . The second method is the *load transfer method*, developed by Zienkiewicz et al. [83], in which the second iteration is started with the same stiffness matrix ($K_1 = K_2$) but a new initial load $F_{o,1}$, which is simply $F_o + \Delta N_1$ (where $\Delta N_1 = R_1^C - R_1$), and yields a solution point $R_{2,VL}$.

The *load transfer method* generally requires more iterations than the *variable-stiffness method* to reach convergence but requires fewer calculations per iteration [37]. Goodman et al. [37] stated that other modification paths are possible and may be preferable in certain instances. They suggested that one can choose a path like F_{CE} with a slope such that the area under $F_{CE} R_1^C$ is the same as the area under the actual load-deformation curve up to R_1^C . Such an approach will not converge in all cases. All these methods can be viewed as modifications of the Newton-Raphson scheme for nonlinear functions in which the stiffness is updated to the value of

the slope of the constitutive surface in the neighborhood of the current solution [37]. Finally, they introduced a summary of steps in the finite element analysis of jointed rock masses by the *load-transfer method* and presented some examples of computations with joints including a comparison of different ways to reach the convergence.

Desai [25] used a three-dimensional quadrilateral interface element to model the interface between a pile and the surrounding soil. The element relations can be formulated for the quadrilateral element with eight nodes and zero or non-zero thickness. For convenience in using the element for different planes, Desai obtained the stiffness relations for the quadrilateral element as the sum of the stiffness of the two component triangles as illustrated in Fig. 2.7. Incremental, incremental-iterative, and residual stress and strain methods have been used in his problems.

Dhlrendra, Duragohain, and Shah [28] developed curved isoparametric line and axisymmetric interface elements for situations involving straight or curved contact surfaces.

Dhlrendra et al. [28] also developed a quadratic isoparametric surface element of arbitrary curvature to handle other situations where the contact surfaces are arbitrarily curved or the boundaries of the contact surface are irregular, as in the case of hyperbolic paraboloid shell foundations and layered plates and shells with interlayer slip. In such situations, the problems cannot be tackled using straight or curved line elements or flat surface elements of regular shapes. Their element is an eight-noded isoparametric quadratic surface element of zero thickness as shown in Fig. 2.8.

Pande and Sharma [61] developed the eight-noded parabolic isoparametric interface element shown in Figs. 2.9a and 2.9b based on relative displacement as independent degrees-of-freedom. This work is an extension of the element developed by Ghaboussi et al. [36]. Pande and Sharma [61], by solving a simple example on accurate modern computers, stated that the differences in results obtained from (a)

conventional isoparametric parabolic elements and (b) an isoparametric parabolic element based on relative displacement are insignificant up to an extremely large length/thickness ratio of the interface element.

Herrmann [45] suggested three approaches in modeling contact surfaces. The first involves a "bond link element" that is inserted between the contacting materials; the properties of the bond element are selected to approximate the interface behaviour. A second procedure treats the two contacting surfaces as distinct and mates them mathematically by means of Lagrange multipliers. The third procedure considered is viewed as a combination of these two approaches. In modeling bond behaviour of reinforced concrete, he placed a pair of nodes at the same initial geometric location, one attached to the steel and the other to concrete, and linked them with a pair of nonlinear springs normal and tangential to the steel. He has made an attempt to select the nonlinear stiffness characteristics of the tangential bond springs so as to model the force slip relationship observed in "pull-out" tests. He states that the principal advantages of this procedure are the simplicity of the bond stress calculation and the straightforward manner in which slippage is introduced. Three behavioural regimes were defined, the "nonslip", the "slip", and the "separation" modes. A compatibility model, a combination compatibility-equilibrium model and an equilibrium model have been used, respectively, for the behaviour modes illustrated in Figs. 2.10 and 2.11.

Xiurun and Hsu-Jun [82] suggested new models for the analysis and description of non-linear properties of joints and interfaces in rock masses. Their models involved a shear displacement model, normal deformation model, shear strength model, and a relationship between unit shear stiffness and normal stress. Their non-linear analysis was based on an incremental procedure. They mentioned that their proposed method is suitable for different types of shear-displacement curves and can also be adapted to the complicated process of loading-unloading-reloading.

Katona [49] introduced a simple friction-contact interface element which simu-

lates frictional slippage, separation, and re-bonding of two bodies initially mating at a common interface and subsequently deforming with an arbitrary static loading schedule. Constraint equations between initially mating node pairs and the general principle of virtual work were used to formulate the interface element for a finite element solution procedure. Katona [49] outlined some advantages of his element which include easy implementation into standard finite element programs, direct determination of interface forces without round-off problems, and the generality afforded by the virtual work formulation to include other non-conservative models in the system.

Heuze and Barbour [46] recently presented a new model for axisymmetric interfaces. Their axisymmetric element was formulated in two independent ways which gave the same results: (1) a direct joint formulation, starting from strain-displacement relations and (2) a formulation starting from a transversely isotropic rectangle in a material with zero Poisson's ratio, and a vanishing element thickness.

2.3 Interface Material Properties

Page [60] presented an analytical method that accounts for the nonlinear behaviour in masonry. His investigation has been limited to the in-plane behaviour of masonry panels and does not take into account failure by buckling instability. Using the finite element technique developed by Goodman et al. [38] to analyze rock mechanics problems, Page proposed an analogy with the behaviour of jointed rock to model masonry behaviour. Uniaxial tests on masonry that subject the mortar joints to various combinations of normal and shear stresses have been carried out to provide sufficient information to define the joint element characteristics in Page's analytical model. The mortar joint deformation characteristics have been determined indirectly from the test performed on masonry prisms by subtracting the masonry unit deformation, which can be precisely determined, from the overall masonry measured deformation. The stress-strain curves for masonry, brick, and mortar in normal and shear directions are shown in Figs. 2.12 and 2.13.

Ali, Page, and Kleeman [1] described a finite element model capable of reproducing the behaviour of solid concrete brick masonry subjected to concentrated loads. In their study, the deformation relations and failure criteria for the bricks and the mortar have been determined from tests on individual bricks and samples of brickwork. Solid concrete bricks were used in their investigation to minimize variability in brick properties.

Ali et al. [1] derived the mortar stress-strain curves indirectly from prism tests (for the normal and parallel strains), and from brickwork couplets with sloping bed joints (for shear strains). The tests and the derived curves are shown in Figs. 2.14 and 2.15. In each case, the mortar stress-strain curve was derived by subtracting the contribution of the brick deformations from the overall measured deformations. The stress-plastic strain curves for the mortar are thus obtained. This procedure is similar to that described for tests on brickwork.

Hamid, Drysdale, and Heidebrecht [42] carried out an experimental study in which they investigated the effects of mortar type, grout strength, and bed-joint reinforcement on the shear strength characteristics of concrete masonry bed joints. They also considered different levels of precompression which was applied normal to the bed joints. Their results indicated that the mortar type was not as important as the effects of grouting and precompression. Fig. 2.16 indicates the effect of precompression on shear strength of masonry joints. Also, they concluded that it is likely that the shear bond strength is not only a function of the mortar properties, but also a function of the physical properties of the block such as its surface roughness and initial rate of absorption.

Qinglin and Wenzong [64] studied the failure mechanism of an unreinforced masonry wall under combined normal and shear stresses. They studied the response of the shear strength and the failure patterns with respect to the anisotropic behaviour of a masonry wall. Also, they presented formulas for calculating the shear strength envelope of a masonry wall according to different failure patterns due to

different stress combinations.

Pook, Stylianou, and Dawe [62] presented an experimental investigation on the influence of precompression on the shear strength of masonry joints. They tested the mortar joint shear strength using hollow block masonry prisms. They found that this shear strength increases linearly as the transverse compression increases. Also, they tested the effect of repetitive cycling of the shear force from zero to maximum at a fixed compression level on the masonry joint shear strength. The relationship between the initial ultimate shear strength and compressive stress can be seen in Fig. 2.17.

Atkinson et al. [10] examined the horizontal bed joint shear failure mode and the shear load-displacement behaviour for unreinforced brick masonry during static and cyclic loading. Atkinson et al. [10] carried out two types of shear tests: type I, consisting of monotonic loading to failure with several cycles of shear force reversal and type II, consisting of an initial application of small amplitude displacement cycles to a level not exceeding the displacement necessary to mobilize the peak shear load followed by monotonic loading to failure with several cycles of shear load reversal.

The masonry bed joint shear strength was described using the Mohr-Coulomb criterion:

$$\tau = C + \sigma \tan \phi \quad (2.2)$$

where C is the cohesive strength and $\sigma \tan \phi$ is the frictional strength with $\tan \phi$ being the friction coefficient. Table 2.1 gives the peak and residual values of C and $\tan \phi$ for type I and II tests and Fig. 2.18 shows the peak and residual shear strength envelopes for both tests. Also, Figs. 2.19 and 2.20 give the shear load-relative shear displacement response curves for type I and II tests.

Table 2.1 Results of the linear regression analysis for type I and II tests

Test type	Peak values		Residual values	
	$\tan \phi_p$	$C_p(MN/m^2)$	$\tan \phi_p$	$C_p(MN/m^2)$
Type I	0.708	0.077	0.695	$-0.056(\simeq 0)$
Type II	0.669	0.291	0.687	$-0.013(\simeq 0)$

2.4 Movements in Concrete Masonry Walls

2.4.1 General

The characteristics of movements of concrete block masonry walls are greatly affected by the properties of the materials used in the manufacture of masonry units and by the methods of manufacturing employed. Also, they are influenced by the type and the properties of the mortars used in joining the masonry units. In this section, the above influences will be described followed by a discussion of the effects of different construction methods on the movements of masonry walls.

2.4.2 Influence of Materials and Manufacture

2.4.2.1 Concrete Masonry Units

Concrete units made of a particular aggregate are likely to display increased shrinkage as cement and/or sand content is increased [72]. Therefore to control shrinkage movements, it is wise to avoid using unnecessarily strong units which may be rich in cement or sand.

Concrete bricks and blocks made with lightweight aggregates generally display greater shrinkage than those made of crushed rock aggregates which in turn tend to display a shrinkage greater than those units made of gravel aggregates [72].

The adopted curing regime may have a significant influence on the total shrinkage after casting. However, Kalousek et al. [48] found that curing at atmospheric pressure, whether in moist air, low temperature steam or high temperature steam curing, resulted in nearly the same final shrinkage in units. On the other hand, high pressure steam curing or so called autoclaving reduced shrinkage to about half of that obtained by other curing methods.

When concrete masonry units, which had been previously dried to equilibrium with low humidity, were subsequently sprayed on one face for 2 hours to simulate rain or painted with water cement paint, they re-expanded approximately by one third of their original shrinkage from a saturated condition [71]. ASTM specifications provide for a maximum moisture content of concrete masonry units ranging from 25% to 45% depending on concrete masonry units' shrinkage and average annual relative humidity at the construction site [5].

The range of thermal expansion coefficient given in BS 5628: Part 3: 1985 [16] is between 7 and 14×10^{-6} per $^{\circ}$ C for concrete masonry units, while the current draft of Eurocode 6 - Masonry [20] gives a design value of 10×10^{-6} per $^{\circ}$ C for the same units.

Since a close estimate of thermal movement in concrete masonry depends upon knowledge of the materials used, BS 5628 suggests that manufacturers may be in a position to supply thermal coefficients for their particular products.

2.4.2.2 Mortar

The main outcome of studies concerning the effect of mortar on masonry movements is that the choice among normal mortars will not likely lead to significant differences in movement and that knowledge of the shrinkage characteristics of particular mortars is not really useful in this context.

Grimm [40] reported that the unrestrained, 28-day shrinkage of mortar specimens cast in metal molds had a mean shrinkage value of about 0.065% with a standard deviation of 0.015%. He also estimated the ultimate (23 years) unre-

strained mortar shrinkage to have a mean of 0.18% with a standard deviation of 0.042% [41].

Ritchie [65] found that the horizontal shrinkage of mortar in bed joints is restrained by shear with the masonry units. This in turn reduces the mortar shrinkage. From the three aforementioned studies [40,41,65], it is estimated that the mortar shrinkage can be reduced by about 30% with standard deviation of 22% as a mean effect of restraint. Anderegg [8] found that the mortar left in contact with a brick had a shrinkage about 50% less than that of mortar cast in metal molds.

Hedstrom et al. [44], reporting on tests using eight different mortars, concluded that concrete block walls built with the weakest mortar accommodated about twice as much shrinkage before cracking as did walls using the strongest mortar.

As mortar sand fineness increases, water demand to provide workability increases and in turn shrinkage increases [43]. Thus, Anderegg [8] and Conner [21] found that mortar joint cracking increased with sand fineness. Also, mortar shrinkage increases with water-cement ratio, which increases with the lime content [24,33,56,68,79]. Voss [78] found that some mortars made with dolomitic hydrate shrink more than others made with high calcium putty.

Some mortar materials have been shown to give rise to greater movement. Ritchie [69] noted that the use of dolomitic lime has such an effect and Beard et al. [11] found a similar effect for mortars containing high alumina cement and crushed limestone in place of Portland cement and lime.

Concerning the movements due to temperature changes, the range of the coefficient of thermal expansion for mortars given in BS 5628: Part 3: 1985 [16] is between 11 and 13×10^{-6} per°C.

2.4.2.3 Concrete Masonry

Jessop [65] reported a wide range for the coefficient of thermal expansion, 6×10^{-6} per°C to 13×10^{-6} per°C for concrete block masonry. Also, according to PCA [63], the coefficients are 7.7×10^{-6} per°C for light weight and 9.4×10^{-6} per°C

for normal weight concrete masonry.

BIA [15] gives a series of values for the coefficient of thermal expansion of concrete block masonry according to the type of aggregate used manufacturing. For light weight aggregates such as expanded slag, the coefficient varies between 5.6×10^{-6} per $^{\circ}$ C and 8.3×10^{-6} per $^{\circ}$ C; for dense aggregates such as gravel or crushed stone it is 9.4×10^{-6} per $^{\circ}$ C.

Table 2.2 shows *movement coefficients* for concrete masonry which include the coefficient of thermal expansion, moisture movements percentages, and the modulus of elasticity given in BRE Digest 228 [14]. In this table, a wide range of values are given. Table 2.3 shows the variations of the movement coefficients for dense aggregate concrete made with different aggregates. These values may assist in fine tuning the values shown in Table 2.2 when the type of the aggregate used in the masonry is known. The irreversible portion of the moisture shrinkage of concrete blocks can be virtually removed by high pressure steam curing or by precarbonation during low pressure steam curing.

Table 2.2 Concrete masonry properties

Materials	Coefficient of linear thermal expansion α $\times 10^{-6}$ per $^{\circ}$ C	Reversible moisture movement %	Irreversible moisture movement (+) expansion (-) shrinkage %	Modulus of elasticity E kN/mm ²
Concrete brickwork and blockwork:				
Dense aggregate	6-12	0.02-0.04	0.02-0.06(-)	10-12
Lightweight aggregate (autoclaved)	8-12	0.03-0.06	0.02-0.06(-)	4-16
Aerated (autoclaved)	8	0.02-0.03	0.05-0.09(-)	3-8

It should be noted that the value of the coefficient of thermal expansion for concrete block masonry published by different sources varies over a wide range. One of the reasons is that no common conditions exist for all the parameters among the different research reports.

Table 2.3 Dense aggregate concrete properties

Materials	Coefficient of linear thermal expansion α $\times 10^{-6}$ per $^{\circ}\text{C}$	Reversible moisture movement %	Irreversible moisture movement (+) expansion (-) shrinkage %	Modulus of elasticity E kN/mm^2
Cement-based composites				
Gravel aggregate	12-14	0.02-0.06	0.03-0.08(-)	15-36
Crushed rock (except limestone)	10-13	0.03-0.10	0.03-0.08(-)	15-36
Limestone	7-8	0.02-0.03	0.03-0.04(-)	20-36

2.4.3 Construction Influences

Complete knowledge of the construction procedures, including purchase, handling, laying and curing of the masonry units, can assist in a close determination of the expected movements which will take place during the life time of the structure.

North American practice provides an option to construction: units can be provided in a "moisture controlled" or "non-moisture controlled" condition and the acceptable moisture content for the units is defined. Table 2.4 shows the moisture content and the shrinkage expectation according to the CSA Standard (A165.1-M1977) on concrete masonry units [17].

High pressure steam curing, precarbonation or protected storage for an adequate period before use are the best ways to reduce the shrinkage potential. Allan [2] studied the effect of "age-at-use" on shrinkage of concrete masonry panels and

Fig. 2.21 shows typical results. From this figure, it can be seen that a significant amount of shrinkage could be avoided by using 28-day or older masonry units.

Table 2.4 Moisture controlled unit according to CSA Standard (A165.1-M1977)

Linear shrinkage %	Moisture Content, max., percent of total absorption (average of 5 units)	
	Humidity conditions at job site or point of use	
	R.H. over 75 % †	R.H. under 75 % †
0.03 or less	45	40
0.03 to 0.045	40	35
over 0.045	35	30

† Average annual climatic relative humidity (R.H.) percent at point of manufacture

In addition, protection from water during storage and handling is essential. Also, finished masonry work should be protected until final weatherproofing is completed.

2.5 Cracking

2.5.1 General

Cracking is probably the most important cause of masonry performance failure [73] and has been an engineering concern for at least 150 years. Cracks are usually caused by high stresses developed as a result of restraining free movements in structural members. It is necessary to fully understand the nature of these stresses in order to produce a design with improved masonry wall performance. Proper performance depends on many factors including the materials specified for construction (masonry units and mortar types), the design of the masonry structure, and the method of construction itself.

2.5.2 Crack Classification

Masonry cracks can be classified by: structure type, masonry type, location, pattern, and causes. Overstressing in masonry may be generated by the imposition of loads or by restraint of volume changes in the masonry materials. Loads may be imposed by movements of foundations, structural frames, shelf angles, roof slabs, or wood expansion.

Cracks can also be classified by their width. Crack widths smaller than 0.1 mm are considered insignificant to water permeance since it is the least width through which wind-driven rain will enter [43]. The maximum width of a crack that will not affect appearance nor cause alarm is said to be 0.25 to 0.38 mm [62]. Bidwell [63] classified cracks as fine (up to 1.5 mm), medium (1.5 to 10 mm), and wide (above 10 mm). Rainer [63] classified crack widths as very slight (less than 1 mm), slight (1 to 5 mm), moderate (5 to 15 mm), and severe (more than 15 mm). Kaminetzky [63] classified crack widths as negligible (less than 0.1 mm), very light (0.4 mm), light (0.8 to 3.2 mm), moderate (3.2 to 12.7 mm), and extensive (12.7 to 25.4), or very extensive (more than 25.4 mm).

2.5.3 Crack Types and Causes

An extensive survey on cracking of apartment houses with external walls of masonry was carried out by Johnson Engineering Firm in cooperation with masonry unit manufacturers and builders in Sweden [31]. The investigation consisted of mapping cracks and studying the causes of cracking in 150 buildings. The masonry units were of the Swedish light weight cellular concrete type. Some of them were laid in lime mortar, some in lime-cement mortar, and some were laid without any mortar. The concrete slabs rested directly on the walls. From the survey, it was concluded that the method of freeing slab and wall from each other was superior in reducing the incidence of cracking.

The results of the aforementioned survey are condensed in Fig. 2.22. Also, the frequencies of the different types of cracks are shown in the same figure. ("Frequency"

is expressed as the percent of the observations of the specific crack type related to all crack observations: one crack observation is noted for one particular type of crack on one building, but additional observations of the same type of crack on the same building are not counted). Some of the different types of observed cracks and their major causes are outlined below.

Type 1a: Horizontal cracks at the topmost slab

This type of crack is due to differential movements in the concrete slabs and the external walls. Part of this differential movement comes from the difference in the coefficients of thermal expansion between the masonry wall and concrete slab. Another part of the differential movement is due to the rotation generated at the slab-wall interface. This movement can be reduced by altering the design to allow for a reduced rotation at the interface.

Type 1b: Horizontal cracks at intermediate floor slabs

The cause of this type of crack is the same as for crack Type 1a. This type of crack occurs mainly where the vertical force in the wall is low, for example under large windows. These cracks can be prevented by avoiding large openings, especially close to building corners.

Type 2: Cracks in foundation wall

This type of crack occurs mainly due to a poor foundation, section changes, and stress concentrations, in combination with temperature differences. This distress can be avoided by proper design of the foundation.

Type 3: Vertical cracks at the corner of the building

Lantz [51] has explained the mechanism for the cause of this type of crack. Due to the difference in the movement coefficients between concrete slab and the supporting masonry walls, when the concrete slab shrinks, the external walls are relatively free to follow the slab between the corners, but at the corners the movement

is prevented by in-plane forces in the external walls. The result is a deflection of the external walls with high bending moments occurring at the corners of the building. Also, the same type of crack can be produced by in-plane tensile stresses generated in the external walls due to different temperature gradients in the insulated roof slab and supporting masonry walls.

Type 4: Horizontal cracks around the window in the top story of the building

The moment caused by the deflection of a concrete slab due to loading and uneven shrinkage is given as a probable cause for this type of crack. Also, the effect may be magnified by a low level of normal loads with high load eccentricity. By choosing a short slab span, this class of crack can be avoided.

Type 5a: Cracks around staircases

This type of crack appears around the entrances at the lower part of a building. These cracks may be due to the section changes in the wall and the foundation.

Type 5b: Cracks around balconies

This type of crack has the same causes as Type 5a. In addition, the yearly and daily thermal movements of the balcony concrete slabs, which are unshielded from temperature fluctuation, often result in cracks.

Type 6: Vertical cracks below window openings

One of the possible causes for this type of crack is movement in the foundation. Also, the difference between the very high load in the wall on either side of windows and the absence of load immediately below the window could cause excessive stresses in the wall.

Type 7: Horizontal cracks at the building corners

The cause of this type of cracking is a vertical uplift of the slab corners which

is well-known from the theory of plates. If the slab is freed from the wall, then the slab can lift without cracking the masonry wall.

Type 8: Vertical cracks at the mid-length of the long walls

For walls with an excessive length to height ratio, this condition can result in a vertical crack near the center of the wall. For a wall which is restrained at the base by bond and friction and may or may not be restrained at the top, cracks typically are wider at the top. Copeland [23] provided an empirical method for recommending a maximum ratio of wall length to height.

Also, in trying to obtain a better understanding of the nature of movements in long masonry buildings, CIRIA [22] has carried out a survey of the cracking problems actually experienced in some 85 buildings incorporating both clay bricks and concrete blocks.

Results from the CIRIA survey indicated that:

- There are certain configurations where cracking from in-plane movements may be expected. These are shown in tabular form in Table 2.5 and diagrammatically in Figs. 2.23 and 2.24. Also, some specific building details at which cracking due to movements may occur are illustrated in Fig. 2.25.
- Clearly, as illustrated in Fig. 2.26, cracking or sliding is much less likely in walls designed within the BS5628 limits. Walls of intermediate length (two to three of these BS5628 limiting unbroken lengths) appear to be more vulnerable to cracking.
- Very long walls appear to be less affected; this applies to both brick- and block-work. No explanation for the reduced cracking in very long walls could be given.

- Some signs of movements, however slight, were seen in more than half of the buildings examined, though masonry repairs would have to be considered in less than 10% of the cases.
- The main factor which prevents movement and causes cracking is uniform in-plane restraint; restraint is present to some degree in all walls.
- Control joints can help accommodate movements, thus reducing the likelihood of cracking.

Table 2.5 Influence of structural form and details

Vulnerable to cracking	Less vulnerable to cracking
Short return in long straight wall	Long straight walls
Spandrel walls	Stepped or corrugated facades
Link bridges	Long returns (greater than 900 mm)
Long parapets	Simple unbroken shapes
Stronger mortars	Weaker mortars
Discontinuous movements joints	Movement joints in walls
Discontinuous damp proof course	Restrained walls
Brick slips	Walls under uniform vertical load
Incompressible joint fillers	Bed reinforcement in walls
Abrupt curtailment of bed reinforcement	
Changes of vertical loads	
Slender panels between large walls	
Changes in shapes, thickness, and height of wall	
Eccentrically confined walls	
Bonding to dissimilar materials (e.g. concrete)	

Other investigators also examined the problem of crack types and causes in masonry walls. Since the two surveys presented give an overview of the typical cracks in masonry wall buildings and their causes, the work of these investigators will not be reiterated.

2.6 Crack Control in Concrete Masonry Walls

As discussed in Section 2.5, various effects such as shrinkage due to moisture or temperature may cause movements and in turn cracking in restrained masonry. In concrete masonry walls, the dominant movements due to shrinkage and thermal deformations will tend to cause serious stresses and cracking. To accommodate these masonry movements without introducing serviceability or safety problems, two techniques may be employed for controlling such movements:

- Limitation on horizontal distance between vertical control joints.
- Installation of bed joint reinforcement or bond beams.

2.6.1 Control Joints

CSA A37-M1980 presents a definition for control joint as a continuous joint in the structure to regulate the location and amount of cracking and separation resulting from movement. Fig. 2.27 illustrates the cracking of a long masonry wall restrained at the bottom edge and the control joint function.

To date, the design of control joints is based more on art than science, i.e. experience plays a major concern in the spacing and widths of joints. This is so because movements are highly variable and depend on different parameters such as climatic factors during and after construction, the raw materials used in manufacture of masonry units, the manufacturing process, the degree of restraint imposed upon the masonry, the workmanship, and maintenance.

At the present time, no exact rules can be stated for the location of control joints. Each case must be studied individually to determine where joints can be placed without endangering structural integrity.

The following recommendations for control joint spacings [54] apply to walls built according to ASTM Specifications for Hollow Load-Bearing Concrete Masonry Units, Type I (moisture controlled) concrete masonry units. Bed reinforcement is to consist of two No. 9 cold drawn steel wires, one in each faceshell bed. With no

bed reinforcement, the maximum control joint spacing is 18 ft (5.5 m) in exterior walls in climates where the average annual relative humidity is between 50 and 70%. The control joint spacing is increased 33% to 24 ft (7.3 m) for a 16-in. (400 mm) spacing of bed joint reinforcement and increased 67% to 30 ft (9.1 m) for an 8-in. (200 mm) spacing of bed joint reinforcement. These spacings are increased 25% for interior walls. All control joint spacings are increased 6 ft (1.8 m) in dry climates, where the average annual relative humidity is less than 50% and reduced by 6 ft (1.8 m) in humid climates where the average annual relative humidity exceeds 75%. At critical points of concentrated stresses, for example at changes in wall height or thickness, a control joint should be provided.

According to PCA's "Building Movements and Joints" [61], control joint spacings should not be more than 20 ft (6.1 m) apart in exterior walls with frequent openings. In walls without openings the joint spacing may be a little greater but should never be more than 25 ft (7.6 m) to be most effective, and it is desirable to have a joint within 10 or 15 ft (3 or 4 m) of a corner, if possible.

2.6.2 Horizontal Reinforcement

Horizontal reinforcement is normally used to control crack widths in masonry walls by distributing the tensile stresses due to thermal and moisture deformations so that there will be many fine cracks instead of a few wider ones. The reinforcement may be provided in bed joints, reinforced collar joints or bond beams. ACI recommendations [4] for spacing of control joints and its relationship with the vertical spacing of joint reinforcement in non-reinforced masonry are given in Table 2.6.

In large wall expanses, bond beams may be used in lieu of bed joint reinforcement. Trough or U-shape concrete masonry units may be used in a continuous horizontal course; reinforcing steel is placed in these units and the horizontal void is filled with grout.

**Table 2.6 Maximum spacing of control joints in
non-reinforced masonry**

	Vertical spacing of joint reinforcement			
	None	24 in. on center (60 cm)	16 in. on center (40 cm)	8 in. on center (20 cm)
Maximum L/H	2	2.5	3	4
Maximum L	40 ft (12 m)	45 ft (13.5 m)	50 ft (15 m)	60 ft (18 m)

Fig. 2.1 Linkage element

Fig. 2.2 Joint element (width = 0) and local coordinate system

(a) A simple joint element of rectangular shape. Note that points 1-4 and 2-3 are assumed to have the same co-ordinates

(b) A curved, variable thickness, cubic isoparametric joint element

Fig. 2.3 Different interface elements for modeling straight and curved interfaces

(a) **Triangular joint element**

(b) **Quadrilateral joint element**

Fig. 2.4 Plane joint elements for modeling joints in three-dimensional finite element analysis

Fig 2.5 Three-dimensional interface element

Fig 2.6 Alternative iterative schemes to constrain the solution to obey the real material behavior

Fig 2.7 Three-dimensional interface element

Fig 2.8 Isoparametric interface surface element

(a) Interface element with two adjacent continuum elements

(b) Relative displacements at interface

Fig 2.9 Isoparametric parabolic interface element

Fig. 2.10 Relative displacement at interface

Fig. 2.11 Idealized model of bond between contacting surfaces

Fig. 2.12 Compressive stress-strain curves for masonry, brick, and mortar

Fig. 2.13 Stress-strain curves in shear for masonry, brick, and mortar

Fig. 2.14 Compressive stress-strain curves for masonry, brick, and mortar

Fig. 2.15 Stress-strain curves in shear for masonry, brick, and mortar

Fig. 2.16 Effect of precompression on shear strength of concrete masonry joints

Fig. 2.17 Different relationships between precompression and shear strength of concrete block masonry

(a) Type I tests.

(b) Type II tests.

Fig. 2.18 Peak and residual shear strength envelopes

(a) Global cyclic loading

(b) Prepeak loading response

Fig. 2.19 Shear load-relative shear displacement response curve for Type I test: Constant normal load equal to 44 kN.

(a) Global cyclic loading

(b) Prepeak loading response

Fig. 2.20 Shear load-relative shear displacement response curve for Type II test: Constant normal load equal to 94 kN.

Fig. 2.21 Influence of age-at-use on panel shrinkage

Description	Type of crack	Number of cracks of type
		All crack observations (%)
Horizontal cracks at the topmost slab		12.1
Horizontal cracks at intermediate floor slabs		9.9
Cracks in the foundation		13.3
Vertical cracks at the corner of the building		10.2
Horizontal cracks around the windows of the uppermost story of building		9.5
Cracks between roof and wall		8.8
Cracks around staircases		8.3
Cracks around balconies		5
Cracks in the mortar joints in the masonry		2.9
Vertical cracks below window openings		9.9
Horizontal cracks in the portion of the wall below the window		3.9
Horizontal cracks at the building corners		2.0
Vertical cracks at slab levels		4.2

Fig. 2.22 Observed types of cracks from a survey on 150 buildings

Corrugated facades

Simple unbroken shapes

Elevation

Bed reinforcements

Movement joints in walls

Fig. 2.23 Plans and elevations less vulnerable to masonry cracking

Short returns in long walls

Parapets

Rapid changes in load

Elevation

Spandrel walls

Narrow panels between walls

Fig. 2.24 Plans and elevations more vulnerable to masonry cracking

Fig. 2.25 Certain building configurations for which cracking due to movements are expected

Fig. 2.26 Incidence of cracking in blockwork

Fig. 2.27 Cracking of long masonry wall by contraction and the function of a control joint

3.1 General

The nature of the movement problem in loadbearing concrete block masonry walls has been discussed in the previous chapter. In this chapter, the physical formulation of the problem is treated, the properties of materials related to the analysis of the problem are discussed, and the analysis methods are described. Finally, the thermal analysis of the problem is presented followed by the temperature distributions along the suggested model.

3.2 The Stress Analysis Problem

A typical lowrise building with concrete block loadbearing masonry walls has been used for the stress analysis in this work. The dimensions of the building have been assumed as shown in Fig. 3.1. The roof loads are transmitted to the loadbearing longitudinal masonry walls as illustrated in Fig. 3.1. The transverse wall, because of its low out-of-plane stiffness, was assumed to offer no restraint to the ends of the longitudinal masonry wall.

Fig. 3.2 shows a typical cross section of the masonry wall. As illustrated in this figure, the model for the stress analysis study consists of a combination of concrete foundation, single wythe concrete block masonry wall with a lintel at its top, and concrete slab-joist roof system. Therefore, for the analysis in this work, three interfaces were considered and needed to be modeled:

1. Soil-foundation interface.
2. Foundation-masonry wall interface.
3. Masonry wall-lintel interface.

The model of the connection between the roof concrete slab and the lintel is explained later in detail under Section 3.4.

Two major loads were applied to the above mentioned model. The first one is the gravity load of the masonry wall, roof, and foundation. The second one derives from differential temperatures in the masonry wall, roof, and foundation.

Since the temperatures of the masonry wall, roof, and foundation are affected by the surrounding environment and the thermal properties of materials, the temperature profile in each of them will be different. The overall thermal analysis and the temperature profiles will be discussed under Section 3.5.

3.3 Properties of Materials

Although there are different materials that are normally used in typical building construction, concrete block masonry, concrete, and reinforcing steel are the structural materials which are relevant to this study. In the following subsections, the properties selected or assumed for the aforementioned materials are discussed.

3.3.1 Concrete Block Masonry

The following properties of concrete block masonry are of particular interest to this study:

1. Thermal properties (coefficient of thermal expansion).
2. Mechanical properties which include:
 - a. Compressive strength.
 - b. Tensile strength.
 - c. Modulus of elasticity.

Coefficient of Thermal Expansion

From the literature review in Section 2.4, the value of the coefficient of thermal expansion for concrete block masonry published by different sources varies over a wide range. A key reason for this variability is that there exists no common set of conditions for all the parameters among different research reports.

As long as there is no specific research information on the parameters influencing the thermal expansion coefficient, a value for that coefficient must be selected from the available sources. A value of 9.5×10^{-6} per°C for the coefficient of thermal expansion for normal weight concrete blocks made of dense aggregate seems to be reasonable and was assumed in the present study.

Studying the stability of the coefficient of thermal expansion over the range of temperature relevant to this study, Weigler and Fischer [80] have reported that the influence of temperature on the coefficient of thermal expansion is negligible at temperature below 100°C. Also, Berwanger [12] conducted some tests on concrete slabs which indicated that in the temperature range from -35°C to 25°C there is no significant change in the coefficient of thermal expansion of concrete. A constant value of 9.5×10^{-6} per°C therefore was assumed throughout.

Compressive Strength

Concrete block masonry is an orthotropic material. Hamid and Drysdale [42] have reported experimental results showing almost the same compressive strength in the principal directions for both grouted and ungrouted block masonry. However, loaded at skew angles, both types indicated a lower strength and the ungrouted specimens showed a larger drop in strength than the grouted specimens. It should also be mentioned that the compressive strength is customarily expressed on the basis of the block's net area including the grouting area, if any. Concrete block masonry orthotropy has not been considered in the present study.

Compressive strength of concrete block masonry is a function of the compressive

strength of the constituents such as concrete block, mortar, and grout. Workmanship and thickness of mortar joints are also major factors affecting compressive strength. In North American practice, strength of 10 to 12 MPa are typically used. For the analysis work presented in this thesis, 10 MPa was assumed as the compressive strength of concrete masonry; this can be achieved by using blocks of 18 MPa compressive strength and type S mortar.

In studying the effect of temperature on the compressive strength of concrete, Lankard et al. [50] reviewed a number of related studies and reported that the trend of results indicated about a 6% decrease in the compressive strength of concrete at a temperature of 50°C as compared to the strength at 20°C when free moisture is presented during heating.

According to Schellenbach and Schimmelpfennig [75], the compressive strength of concrete generally decreases with an increase in temperature but because of the multiplicity of the factors affecting the compressive strength, a great scattering is shown in the available results. Since similar test results for masonry are not available, a constant compressive strength of 10 MPa was used for the entire range of temperature in this study.

Tensile Strength

The tensile strengths of ungrouted block masonry differ greatly along the two principal axes [29]. The tensile strength parallel to bed joints is approximately twice as much as that perpendicular to bed joints. Grouted masonry, on the other hand, exhibits almost the same tensile strength in both directions.

Tensile strength of grout and mortar and block bond strength are the major factors affecting the tensile strength of masonry in the direction perpendicular to bed joints while the block tensile strength has almost no effect on the masonry tensile strength in that direction. On the other hand, the tensile strength in the direction parallel to bed joints is greatly influenced by the concrete block tensile strength especially when the failure occurs across the blocks; grouting has almost

no effect on that tensile strength.

Drysdale et al. [29] have reported test results which show that depending on the percentage of grouting and the strength of the grout and mortar, the tensile strength of concrete block masonry may vary from 0.25 to 1 MPa in the direction perpendicular to bed joints. The tensile strength in the direction parallel to bed joints was found to vary slightly from 0.7 to 0.8 MPa.

For the purpose of the nonlinear analysis in this work in which debonding was expected perpendicular to bed joints, a uniform tensile strength of the concrete block masonry of 0.5 MPa seems to be reasonable for a partially grouted masonry wall with medium strength grout. Such a strength will be assumed in this study.

Modulus of Elasticity

The modulus of elasticity of masonry, like concrete, is related to its compressive strength. The most widely used relation is the one presented by the CSA code (CAN3-S304-M84) [19] in which the masonry modulus of elasticity can be calculated as follows:

$$E_m = 1000f'_m \quad (3.1)$$

where

E_m = modulus of elasticity of masonry in compression, MPa

f'_m = masonry compressive strength, MPa

Sahlin [73] has reported test results which indicate that the CSA code equation gives a conservative but realistic estimate for the modulus of elasticity.

In studying the effect of temperature on the modulus of elasticity, Berwanger [12] has reported that by decreasing the temperature below the freezing point of water, an increase in the modulus of elasticity of concrete occurs. He attributes this

rise to an increase in the cohesive force between the particles in concrete caused by the expansive force produced by the freezing of free moisture on which the increase of modulus depends. Since no specific relationship between the moisture content and the increase in modulus of elasticity for concrete block masonry exists, this phenomenon was not taken into account in the present study. For this study, the CSA code equation was used. Therefore, for a masonry of compressive strength equal to 10 MPa, the modulus of elasticity was taken equal to 10 000 MPa.

3.3.2 Concrete and Steel

For normal density concrete, the CSA code (CAN3-A23.3-M77) [18] for the design of concrete structures provides the following formula for the modulus of elasticity for concrete:

$$E = 5000 f'_c {}^{(0.5)} \quad (3.2)$$

where

E = modulus of elasticity of concrete, MPa

f'_c = compressive strength of concrete, MPa

For a 20 MPa concrete typically used in lowrise construction, the modulus of elasticity and the coefficient of thermal expansion were taken equal to 22 360 MPa and 10×10^{-6} per°C, respectively.

Steel with a yield strength equal to 400 MPa is widely used for reinforcing in building construction. The modulus of elasticity of steel is almost constant and equal to 200 000 MPa. The coefficient of thermal expansion of steel is about 11.5×10^{-6} per°C. The mechanical and thermal properties of the materials used in this study are summarized in Table 3.1.

Table 3.1 Mechanical properties of materials

Material	Compressive strength, MPa	Tensile strength, MPa	Modulus of elasticity, MPa	Thermal coefficient of expansion per °C
Concrete	10.0	0.5 †	12 000	0.0000095
block masonry		0.7 ‡		
Concrete	20.0	2.0	22 360	0.00001
Steel	400.0	400.0	200 000	0.0000115

† perpendicular to bed joints

‡ parallel to bed joints

3.4 Method of Analysis

Copeland [23] presented two methods to estimate the control joint spacings in blank (without openings) walls. These theoretical methods were not offered as exact methods or rigorous mathematical treatments of the problem of movements in lowrise masonry buildings.

In the first method, by considering a blank wall restrained at the top and bottom edges, Copeland derived an equation for the wall length at which the tensile stress will become critical. Fig. 3.3 is a simple illustration of the horizontal forces and stresses acting on a wall which is free at the end but is restrained along the top and bottom edges. The increasing and decreasing curves of the shear diagram are not linear since for this method, it is not essential to know the exact form. The tensile resistance of the wall is equal to the tensile unit strength times the cross-sectional area that is effective in resisting tension; this assumes no transfer of stress occurs across vertical mortar joints and that the opposing force, or shear, will be equal to the shearing unit strength times its effective area [23]. By equating these forces Copeland came up with the following equation which gives the wall length at which tensile stress will become critical:

$$L = 2\left(\frac{f_t}{f_v}\right)\left(\frac{A_t}{A_v}\right)H \quad (3.3)$$

where

f_t = average tensile unit strength of wall psi

f_v = maximum shearing unit strength of mortar joint, psi

A_t = wall area effective in resisting tensile stress, per foot
of wall height, sq.in.

A_v = joint area effective in resisting shear, per foot of wall
height, sq.in.

While the above approach provides a rough rule for control joint spacing, it does not properly take into account the effect of variations in the shrinkage, thermal contraction, and extensibility of masonry.

The second method is based largely on unpublished experimental data developed by Carlson and Reading in studies conducted for the Portland Cement Association on the distribution of shrinkage stresses in walls [23]. The Carlson-Reading studies on the effect of the length/height ratio pertained to walls restrained only at the base. Copeland, by making many assumptions, derived an equation based on the aforementioned experimental data. This equation, which must be satisfied to avoid cracking, is as follows:

$$P_m \leq \frac{\epsilon_u}{R_d(\epsilon_d + \epsilon_t)} \quad (3.4)$$

where

P_m = ratio, average tensile stress/maximum tensile stress
at the mid-length of the wall.

ϵ_u = ultimate tensile strain, in. per in.

ϵ_d = drying shrinkage, in. per in.

ϵ_t = temperature contraction, in. per in.

R_d = degree of restraint, decimal fraction of unity

Therefore, using a relation derived from experimental data [23] between the average stress and the maximum principal stress at a restrained edge for walls of various length/height ratios as shown in Fig. 3.4, the proper masonry wall length can be found.

The degree of restraint can be computed by approximate methods such as suggested by Saemann, Warren, and Washa [77]. Note that the relative movements at the different interfaces, the temperature profiles, and the effect of openings were not considered in Copeland's study.

After spending an initial period looking at the possibility of using classical methods for the stress analysis of the work presented in this thesis, it was concluded that the problem was too complex to be analyzed using classical methods. Therefore, a decision was made to use a finite element method for the stress analysis work.

Rahman [65] has presented an analytical study to calculate the thermal stresses in intersecting masonry walls in highrise buildings. He used both linear and nonlinear finite element methods in his study. One of the conclusions that he obtained is that a linear elastic analysis can adequately predict the thermal cracking and overstressing effects for the problem he presented. Owing to the above conclusion and recognizing that the slippage and debonding at different interfaces of the model used for lowrise buildings are expected to be of great importance, the approach for this study was to solve the problem for local nonlinear behaviour at the interfaces while keeping the other domains linear, elastic and isotropic.

First, a linear elastic stress analysis was conducted to estimate the magnitude and the general distribution of stresses. Based on the linear analysis, the location of high stresses, at which slippage and debonding were expected to occur, were identified. A nonlinear stress analysis was necessary and therefore was carried out.

Local (weak) [36] nonlinear finite element analysis has been used in the stress analysis under study. For this method, the complete problem domain was handled as a linear elastic domain except for highly stressed areas; there, special interface

finite elements were used to represent the nonlinear behaviour. This nonlinear behaviour includes the slippage and debonding between the two sides of each interface.

For linear finite element analysis, available software (SP23B) developed by Bowes [13] was used after some modification. But for the nonlinear finite element analysis, none of the available software was able to handle interface problems. Because of time limitations and financial resources required for purchasing, installation, and adaptation of a new software in the campus computer, the option of obtaining the required software from outside sources did not appear to be feasible. Therefore, a finite element stress analysis computer program, suitable for the problem of this study, was developed. This program is able to handle the problem of relative movements at different interfaces including slippage and debonding.

Using the technique described by Bowes and Russel [13] to reduce a T-section to a co-planer system of plates joined by short bar elements, the 3-D roof, masonry wall, and foundation system was reduced to a 2-D co-planer system in which the roof and wall were connected by short bar elements as shown in Fig. 3.5. It should be noted that the roof slab will be shown rotated 90° in all figures presented in Chapters 5 and 6. Also, it should be mentioned that the advantage of symmetry was taken into consideration and only half of the building was analyzed.

3.5 Thermal Analysis

3.5.1 General

Several factors control the temperature distribution along a typical cross section of a lowrise masonry building. Owing to the fluctuations in the environmental conditions, that temperature distribution is transient and changing every moment. On the other hand, since the temperature inside a building in North America is typically controlled within a narrow range suitable for human habitation, the temperature variation in the interior of a building with respect to time is insignificant. Therefore, the temperature of the interior air will be assumed constant with time for the purpose of this study and will be taken equal to 20°C.

The objective of the thermal study in this section is to determined the temperature distributions in the different structural elements (masonry wall, roof and foundation) of a typical lowrise building under the maximum differentials of temperature. Two maximum temperature differentials, one in summer and the other in winter, are considered in the thermal analysis.

3.5.2 The Thermal Analysis Problem

As previously mentioned, the aim of this study is to determine the temperature distribution in a lowrise masonry building model which, as shown in Fig. 3.2, includes three elements, namely a concrete block masonry wall, roof slab, and concrete foundation.

To illustrate the nature of the heat flow problem, consider that the temperature outside a building is higher than that inside the building, as during a typical summer day; heat therefore will flow from outside to inside the building across the wall and roof cross sections. In this case, heat flow is a three-dimensional problem in which the heat flows along and across the wall and the roof in both vertical and horizontal directions. Uneven shading against solar radiation and air movements may be the reason for heat flow in the plane of the wall or the roof.

Since those factors can not be well identified in the thermal analysis, for the purpose of this study the building is assumed unshaded at all times. Also, air movements are assumed to have a negligible effect on the heat flow. Therefore, using these assumptions, the problem can be treated as a one-dimensional problem in which the heat flow only goes across the wall and the roof cross sections.

3.5.3 Extreme Temperature Distributions Along the Masonry Wall

Rahman and Suter [66] have presented results from transient and steady state heat flow analyses in masonry walls. They determined the extreme temperatures in the loadbearing wythe of typical exterior wall sections under common summer and winter conditions in Canada. Their analyses were carried out for two types of walls: loadbearing masonry walls covered on the outside by a clay brick masonry cladding, and single wythe loadbearing walls with no cladding. Fig. 3.6 shows typical cross sections for both types of walls.

In the following subsections, the parameters of heat flow across the above wall sections are explained, followed by the data they used and by the analytical results obtained for both transient and steady state heat flow.

3.5.3.1 Parameters of Heat Flow Across Wall Sections

Climatological factors such as the amount of total incident thermal radiation, ambient temperature, and wind, as well as the thermal properties of the materials used in the wall, including the absorptivity and emissivity, conductivity and thermal mass, govern the temperature at any point in a single or multiple wythe wall.

Among the climatological factors, *solar radiation incident* on the wall surface is the most important, especially in summer. Several factors including time of the day and year, latitude of the location, orientation of the surface and atmospheric conditions such as cloud cover and water vapor content of the atmosphere, control the total solar radiation reaching a surface on the earth.

Rahman et al. [66] used a transient heat flow analysis computer program,

developed by Fenton [35], which required the data describing the location in terms of latitude, longitude, standard time meridian, atmospheric conditions, and wall orientation. Location data for Ottawa were used in their study. After a series of analyses regarding the wall orientation as a variable, walls oriented west-southwest, i.e. with an azimuth angle 245° with the north, reached the highest temperature.

Absorptivity is the capacity of a body to gain a certain amount of heat by absorbing radiation. Inversely, the maximum amount of heat that a body loses during radiation is called *emissivity*. The available data on solar absorptivity of common building materials including masonry is rather sparse. Several sources [65] indicated solar absorptivity of 0.7 to 0.9 for dark colored bricks and 0.55 to 0.75 for concrete masonry surfaces.

The ambient air temperature is another important parameter when determining the exterior wall surface temperature; 37.8°C and -36.1°C are recorded as the highest and lowest temperature in Ottawa over a period of 43 years as reported by Environment Canada [31].

Wind induced convection at a wall surface can significantly increase the heat loss, depending upon the wind characteristics. Hutcheon and Handegord [65] have developed a theoretical relationship between wind velocity and convective heat transfer. The same equation is given by ASHRAE [6] but with different units. Environment Canada reported that the average wind speed in Ottawa is 3.25 m/sec in July and 4.45 m/sec in January [30]. These wind speeds were measured in the airport location. Since the wind speed in protected areas in the city are likely to be lower, Rahman et al. [66] assumed a nominal wind speed of 1 m/sec for the maximum temperature analysis and 2 m/sec for the minimum temperature analysis.

Based on a literature survey conducted by Fenton [35] to determine the thermal characteristics of the materials typically used in masonry wall construction, Rahman et al. [66] used the following formulas for thermal conductivities for clay brick and concrete block masonry in a thermal analysis to determine the maximum and minimum temperature distributions:

$$k_{br} = f k_{dbr} \quad (3.5)$$

$$k_{bl} = [\nu(f - 1.0) + 1.0] k_{dbl} \quad (3.6)$$

where

$$k_{dbr} = (\rho/2505.46)^{6.642} + 0.25917 \times 10^{-3} \rho - 0.0504 \quad (3.7)$$

$$k_{dbl} = 0.6078 \times 10^{-6} \rho^2 - 1.31 \times 10^{-3} \rho + 0.7702 \quad (3.8)$$

$$f = 1 + 0.3157w^{0.6} - 0.01567w \quad (3.9)$$

$$\nu = 1.0 - (1.0 - \alpha)(1.0 - \psi) \quad (3.10)$$

where, in the above Eqs.,

k_{br}, k_{bl} = thermal conductivity of brick and block, respectively, W/m°C

k_{dbr}, k_{dbl} = same as above, but in oven-dry condition, W/m°C

ρ = density of the material, kg/m³ (500 < ρ < 2600)

f = moisture correction factor

w = moisture content in percent by volume

ν = fraction solid of block masonry

α = fraction of block cores grouted

ψ = ratio of net block to gross block area

The formula for the thermal conductivity of 50 mm thick styrofoam insulation is given below:

$$k = 0.1686 \times 10^{-4}(T) - 0.0066 \quad (3.11)$$

where

T = temperature in °K

3.5.3.2 Steady State Analysis of Heat Flow Across Wall Sections

The fundamental analytical expression for the conduction of heat through solids is given by Fourier [47] as:

$$\frac{dQ}{dt} = k \cdot A \cdot \frac{dT}{dx} \quad (3.12)$$

where

dQ/dt = rate of heat flow

k = coefficient of thermal conductivity of a material

A = cross sectional area perpendicular to heat flow

dT/dx = rate of change of temperature, t , with respect to the thickness, x ,
of the element through which heat flows

When the flow of heat in a body has reached a steady state, i.e., the flow is constant, the temperature at each point in the body under consideration remains constant. Hence, the temperature is a function of position, but not of time.

Under such conditions

$$\frac{dQ}{dt} = q \quad (3.13)$$

$$\frac{dT}{dx} = \frac{(T_i - T_o)}{x} \quad (3.14)$$

Therefore Eq. 3.12 can be rewritten as:

$$\frac{q}{A} = \left(\frac{1}{R}\right)(T_i - T_o) \quad (3.15)$$

or

$$\frac{q}{A} = u(T_i - T_o) \quad (3.16)$$

where

$R = x/k$, thermal resistance of a material of thickness x

u = overall coefficient of heat transmission

T_i = inside temperature

T_o = outside temperature

q = rate of heat flow

A = area perpendicular to heat flow

For heat flow through different layers:

$$R_{tot} = 1/u = R_1 + R_2 + R_3 + \cdots + R_n$$

where

$R_1, \dots, R_n$ = individual resistance of the elements

The effect of radiation and convection has been included in the thermal resistance of air films at the inside and outside surfaces. This is done by deriving the thermal resistance of air films and air spaces from the following equation:

$$R_{air} = \frac{1}{f} \quad (3.17)$$

where

$$f = h_c + h_r$$

h_c = convective heat transfer coefficient

h_r = radiative heat transfer coefficient

Table 3.2 [65] gives the conductances of the inside and outside air films and the air spaces between the cladding and the loadbearing masonry wall.

Table 3.2 Conductances of air films and air spaces

Element	Conductance, $W/m^2 K$	
	Summer	Winter
Outside air film:		
Brick wall	18.14	20.12
Block wall	17.82	20.12
Inside air film	8.22	7.96
Air space	10.16	6.14

Using the sol-air temperature concept, the effect of solar radiation can be reflected in the outside temperature. Sol-air temperature is a fictitious air temperature which will cause the same heat flow by surface conductance alone as that caused by radiation, convection, and conduction at the surface of the wall. Rahman et al. [66] computed the maximum sol-air temperature in summer as 76.7°C and set the wintertime minimum temperature in the wall equal to the minimum ambient temperature in winter, -36.1°C . Also, they assumed the inside air temperature equal to 20°C which is the standard temperature given for reasonable comfort in both winter and summer.

The results of the steady state heat flow analysis conducted by Rahman et al. [66] are shown in Tables 3.3 through 3.6. The tables give the steady state minimum and maximum temperatures in cavity and single wythe walls.

3.5.3.3 Transient Analysis of Heat Flow Across Wall Sections

BIA [15] conducted research which indicated that the actual heat transfer through masonry walls may be up to 20% less than the values calculated by a steady state method. Also, NBS [58] reported that heat flows calculated on the basis of steady state conditions were 29% to 69% greater than those measured under dynamic conditions for masonry walls.

Also, Rahman et al. [66] conducted a transient heat flow analysis to determine more realistic maximum and minimum temperatures. The maximum and minimum temperatures by the transient analysis for both cavity and single wythe walls are shown in Table 3.7.

In summer, the calculated differentials are 21.3°C and 28.5°C for cavity and single wythe walls, respectively; in winter, the differentials are 38.7°C and 45.9°C , respectively.

**Table 3.3 Steady state maximum temperatures
in cavity wall**

Element	Conductance $W/m^2\text{ }^\circ C$	Resistance $m^2\text{ }^\circ C/W$	Temperature drop $^\circ C$	Temperature $^\circ C$
				76.7
Outside air film	18.14	0.055	1.7	75.0
Brick cladding	14.67	0.068	2.1	72.9
Air space	10.16	0.098	3.1	69.8
Block	7.60	0.132	4.2	65.6
Insulation	0.80	1.250	39.3	26.3
Wall finish	12.50	0.080	2.5	23.8
Inside film	8.22	0.122	3.8	20.0

Total resistance = 1.805

**Table 3.4 Steady state maximum temperatures
in single wythe wall**

Element	Conductance $W/m^2\text{ }^\circ C$	Resistance $m^2\text{ }^\circ C/W$	Temperature drop $^\circ C$	Temperature $^\circ C$
				76.7
Outside air film	17.82	0.056	1.9	74.8
Block	7.60	0.132	4.6	70.2
Insulation	0.80	1.250	43.2	27.0
Wall finish	12.50	0.080	2.8	24.2
Inside film	8.22	0.122	4.3	20.0

Total resistance = 1.640

**Table 3.5 Steady state minimum temperatures
in cavity wall**

Element	Conductance $W/m^2{}^{\circ}C$	Resistance $m^2{}^{\circ}C/W$	Temperature drop ${}^{\circ}C$	Temperature ${}^{\circ}C$
				-36.0
Outside air film	20.12	0.050	1.5	-33.5
Brick cladding	14.67	0.068	2.0	-31.5
Air space	6.14	0.163	4.8	-26.7
Block	7.60	0.132	3.9	-22.8
Insulation	0.80	1.250	36.9	14.1
Wall finish	12.50	0.080	2.4	16.5
Inside film	8.22	0.126	3.6	20.0

Total resistance = 1.869

**Table 3.6 Steady state minimum temperatures
in single wythe wall**

Element	Conductance $W/m^2{}^{\circ}C$	Resistance $m^2{}^{\circ}C/W$	Temperature drop ${}^{\circ}C$	Temperature ${}^{\circ}C$
				-36.0
Outside air film	20.12	0.050	1.7	-33.3
Block	7.60	0.132	4.4	-28.9
Insulation	0.80	1.250	42.0	13.1
Wall finish	12.50	0.080	2.7	15.8
Inside film	8.22	0.126	4.1	19.9

Total resistance = 1.638

**Table 3.7 Maximum and minimum temperatures by
transient analysis**

Type of wall	Temperature, °C, and hour of occurrence			
	Maximum		Minimum	
	External surface	Block mean	External surface	Block mean
Cavity wall	64.6	41.3	-30.3	-18.7
	17:00	22:00	5:00	10:00
Single wythe wall	57.6	48.5	-28.8	-25.9
	17:00	19:00	5:00	7:00

In the stress analysis work presented in Chapter 5, the transient analysis temperature differentials for the single wythe wall were applied as thermal loads on the modeled masonry wall.

3.5.4 Extreme Temperature Distributions Across the Roof Slab

In roof construction today, the insulation of the roof is always applied on the top of the structural slab. Because of that, the temperature variations in the structural slab are very limited.

The roof is exposed to solar radiation and ambient temperature and due to its orientation, the maximum summer temperature is expected to be higher than that of the masonry wall. But in fact, different factors affecting the roof surface temperature should also be considered, such as differences in the color and thermal properties of the roofing materials.

In determining the maximum and the minimum temperature distributions across the different layers of a typical cross section of a roof, -36.1°C and 76.7°C were assumed to be the minimum and maximum outside air temperatures in winter and summer while the inside air temperature was assumed to be equal to 20°C . Although -36.1°C is a dependable value as a minimum winter temperature, 76.7°C

may contain some approximation due to the factors mentioned above concerning the ambient air and solar radiation. But, in using the type of roofing in which the insulation is applied on the top of the concrete slab, the variations in the slab temperature are known to be very limited.

A steady state thermal analysis on a typical roof indicated an average minimum temperature of 17.5°C and an average maximum temperature of 23.4°C in the concrete slab. Tables 3.8 and 3.9 give the thermal properties and the steady state minimum and maximum temperatures in the different layers of the roof. The thermal properties of the roof materials used in this study were chosen from the values reported in Ref. [53]. Also, in Fig. 3.7, the maximum and minimum temperature distribution profiles across typical roof layers are illustrated.

3.5.5 Extreme Temperature Distributions Across the Foundation

The problem of heat flow across the foundation is two-dimensional in nature since the heat flow goes through three media: masonry wall, concrete foundation, and surrounding soil. The objective is to determine the temperature distributions along the foundation when the mean temperature in the loadbearing masonry wall is at a maximum and a minimum.

Fig. 3.8 shows an analytical model of the two-dimensional heat flow in a foundation. In this model, it is assumed that the temperature distribution is uniform along the thickness of the foundation. Since soil temperature, t_s , is maintained on both sides of the foundation wall along its depth, it can be assumed that at a distance L from O , the point on the wall-foundation interface, the temperature of the foundation wall equals t_s .

Since under steady state conditions no heat is stored, the following relationship can be written for the heat flow into and out of a prism with length Δx and unit thickness:

**Table 3.8 Steady state minimum temperatures
in roof materials**

Element	Resistance $m^2^{\circ}C/W$	Temperature drop $^{\circ}C$	Temperature $^{\circ}C$
			- 36.0
Outside air film	0.030	0.9	-35.1
Crushed stone (20 mm)	0.012	0.4	-34.7
Insulation	1.735	50.8	16.1
Concrete	0.029	0.9	17.0
Inside air film	0.105	3.1	20.0

Total resistance =1.911

**Table 3.9 Steady state maximum temperatures
in roof materials**

Element	Resistance $m^2^{\circ}C/W$	Temperature drop $^{\circ}C$	Temperature $^{\circ}C$
			76.7
Outside air film	0.030	0.9	75.8
Crushed stone (20 mm)	0.012	0.4	75.4
Insulation	1.735	50.8	23.9
Concrete	0.029	0.9	23.0
Inside air film	0.105	3.1	20.0

Total resistance =1.911

$$q_1 = q_2 + 2 q_3 \quad (3.18)$$

where

$$q_1 = -k \times b_f \times \left(\frac{dT}{dx} \right)_z \quad (3.19)$$

$$q_2 = -k \times b_f \times \left(\frac{dT}{dx} \right)_{z+\Delta x} \quad (3.20)$$

$$q_3 = \frac{1}{R_{sh}} \times (T - T_s) \times \Delta x \quad (3.21)$$

and

k = thermal conductivity of concrete

R_{sh} = thermal resistance of sheathing paper

By neglecting the higher order of the Taylor's expansion,

$$\left(\frac{dT}{dx} \right)_{z+\Delta x} = \left(\frac{dT}{dx} \right)_z + \left(\frac{d^2T}{dx^2} \right) \Delta x \quad (3.22)$$

Substituting Eqs. 3.19 through 3.22 in Eq. 3.18,

$$\frac{d^2T}{dx^2} - \frac{2}{k b_f} \frac{1}{R_{sh}} (T - T_s) = 0 \quad (3.23)$$

Let

$$T' = T - T_s \quad (3.24)$$

and

$$\lambda = \sqrt{\frac{2}{k b_f} \frac{1}{R_{sh}}} \quad (3.25)$$

Therefore Eq. 3.23 can be rewritten as,

$$\frac{d^2T'}{dx^2} - \lambda^2 T' = 0 \quad (3.26)$$

The general solution of the above equation is

$$T' = Ae^{-\lambda x} + Be^{\lambda x} \quad (3.27)$$

But

$$\text{at } x = 0 \quad T = T_i \quad \text{i.e.} \quad T' = T_i - T_s$$

$$\text{at } x = L \quad T = T_s \quad \text{i.e.} \quad T' = 0$$

Using the above boundary conditions in Eq. 3.27, the coefficients A and B can be found as

$$A = -\frac{e^{2\lambda L}}{1 - e^{2\lambda L}}(T_i - T_s) \quad (3.28)$$

and

$$B = \frac{(T_i - T_s)}{1 - e^{2\lambda L}} \quad (3.29)$$

Substituting Eqs. 3.24, 3.28 and 3.29 in Eq. 3.27 and rearranging, the following equation can be obtained:

$$T = \frac{T_i - T_s}{1 - e^{2\lambda L}}(e^{\lambda x} - e^{2\lambda L}e^{-\lambda x}) + T_s \quad (3.30)$$

Using a value of $0.011 \text{ m}^2\text{C/W}$ for R_{sh} , 0.2 m for b_f and $2.22 \text{ m}^2\text{C/W}$ for k , a value of 20.24 was computed for λ .

Letting the distance L , where T becomes equal to T_s , be 0.5 m , an assumption which is reasonable since temperature differentials became insignificant from a practical point of view at greater depths, Eq. 3.30 can be used to plot the foundation temperature profiles along its depth using T_s equal to 1.4°C for the winter case and 19.4°C for the summer case, according to data provided by Environment Canada [32]. The temperature profiles along the foundation depth for the winter and summer cases are shown in Fig. 3.9. Also, the soil temperature profiles for the aforementioned cases are illustrated in the same figure.

In summary, the temperature differentials in each structural element of the chosen model for a lowrise concrete block masonry wall building are calculated assuming that there are no thermal stresses when the mean temperature equals to 20°C in the masonry wall and roof slab, and 11°C in the concrete foundation. Fig. 3.10 shows the temperature differentials for the complete model for both the winter and summer cases; these differentials will be used in the thermal stress analysis presented in Chapters 5 and 6.

Fig. 3.1 View of a typical lowrise concrete block loadbearing masonry building

Fig. 3.2 Typical cross section for a masonry wall building

Fig. 3.3 Simplified illustration of horizontal tension and shear due to shrinkage in wall restrained at top and bottom according to Copeland

Fig. 3.4 Relation between average and maximum tensile stress at restrained edges for walls of various L/H ratios

Fig. 3.5 Details of the modeled connection between the masonry wall and the concrete roof slab

Fig. 3.6 Typical cross sections in different types of concrete block masonry walls

(a) Wintertime case

(b) Summertime case

Fig. 3.7 Temperature profiles across the roof assembly
(Steady state analysis)

Fig 3.10 Temperature differentials along the different structural elements for wintertime and summertime cases

2

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4.1 General

The finite element method has become widely accepted as a valuable technique for solving complex problems. Originating in the field of structural engineering, the method is now recognized as having a general basis with wide applicability to many additional areas of engineering and mathematical physics including steady-state and time-dependent heat transfer, fluid flow, and electric potential problems.

In this chapter, a special joint finite element which is used to model the relative movements at the different interfaces of a masonry wall building is explained. Also, the special technique that was used to solve for the nonlinear behaviour of such interfaces is described and the problems associated with the computer coding are discussed.

4.2 Assumptions Associated with Joint Behaviour

The numerical modeling technique for solving problems associated with interfaces are based on certain assumptions and it is important to outline these assumptions before presenting a detailed development of the technique. These assumptions are described as follows:

- I. The physical behaviour of joints and interfaces is nonlinear. Laboratory tests have been carried out to examine the physical behaviour of joints (Goodman [37], Desai [26] and Page [1]). These test results indicated nonlinearity of the

joints' behaviour. In the present study, the nonlinear joint behaviour characteristics are as follows:

1. For the masonry unit-mortar interface, joint material stress-strain curves under normal and shear loading are assumed to have bilinear relationships as shown in Fig. 4.1.
2. For the soil-foundation interface, joint material is assumed to be elastic-perfectly plastic as indicated in Fig. 4.2.
3. For the above mentioned interfaces, dilatency and strain softening effects have been neglected.
4. Soil-foundation interface joints are incapable of withstanding any tension in the normal direction. Masonry unit-mortar interface joints are capable of tolerating a tensile stress in the normal direction of the interface up to 0.5 MPa which is assumed as the ultimate tensile strength of the masonry bed joints in the normal direction.
5. For both types of interfaces, the peak shear strength of the joint is assumed to be a function of the normal stress developed in the joint and also a function of the cohesion and roughness angle using a Mohr-Coulomb yield criterion:

$$\tau = C + \sigma \tan \phi \quad (4.1)$$

where:

τ = maximum shear strength

σ = developed normal stress in the joint

C, ϕ = cohesion and angle of friction, respectively

6. If the joint shear stress exceeds the shear strength, then the joint tangential stiffness is set equal to the residual tangential stiffness and the problem is repeated for the next iteration.
7. In case of recontact of the two side of the interface, the joint residual shear

strength is assumed to be a function of the normal compressive stress and the residual friction angle.

- II. The material properties of the continuum on either sides of the interfaces are assumed to be homogeneous, isotropic and linearly elastic.

4.3 Interface Finite Element for Joints and Material Interfaces

In the last few years, there has been considerable interest in the application of the finite element method to discontinuous systems, especially in the field of rock mechanics where rock joints, faults, and various geological interfaces are of great importance. Other problems in which interface discontinuities play an important role in the physical behaviour of a system include soil-structure interaction in some underground structures, analysis of fuel element assemblies in nuclear reactors, and machine part interfaces, such as gears and rollers.

Masonry is still a field of discontinuity problems in which the inelastic nonlinear behaviour needs to be more fully investigated. In masonry, the discontinuities are block or brick-mortar interfaces. Such interfaces constitute a "plane of weakness" and the ability to describe their behaviour with a reasonable degree of accuracy is essential and depends on a knowledge of the geometric and material properties of the basic microcomponents: block or brick units and mortar.

For an interface separating two continua, slip, separation, and recontact may all occur during a given stress-time history. Tangential slip may be initiated when the tangential component of the interface-stress vector exceeds the shear strength of the discontinuity. Considering the sliding friction law, subsequent relative slip or separation or both may also occur if the normal component of the interface-stress vector exceeds the tensile strength of the discontinuity.

Since, from a mathematical standpoint, analytical solutions are possible only for a limited class of idealized nonlinear interface problems, there is a growing interest in numerical solutions to such complex problems.

Several investigators ([36],[37],[39],[83]), with an objective of representing the joint behaviour in rocks by finite element methods, have developed so-called "joint elements" and have achieved success in simulating the gross behaviour of some physical problems. Also, a few investigators, with an objective of representing masonry block or brick-mortar interfaces, achieved success in modeling the inelastic, nonlinear behaviour of masonry.

In the next subsections, the choice of interface finite element will be explained and some of its features will be described. The derivation of the stiffness matrix and the nonlinear behaviour of the joint element will also be explained.

4.3.1 Choice of Interface Element

In the analysis work presented in this thesis, the interface element developed by Ghaboussi et al. [36] has been used to model the joints and material interfaces. This interface element is a four noded element with nonzero thickness. A few features of the interface element are summarized as follows:

1. The joint element uses relative displacements as independent degrees-of-freedom. The displacements of one side of the slip surface are transformed into the relative displacements between the two sides of the slip surface.

Wilson ([36],[81]) has advocated the use of relative nodal displacements as independent degrees-of freedom for avoiding the numerical difficulties which may arise from ill-conditioning of the stiffness matrix due to very large off-diagonal terms or very small diagonal terms which are generated by the elements which use the nodal displacements as the independent degrees-of-freedom in some cases.

2. Since the relative displacements are used as independent degrees-of-freedom, the thickness of the interface element can be chosen with any small value without causing numerical difficulties.
3. Since the thickness of the interface element is small compared to its length, the strain along the thickness of the element is assumed to be constant.

Some investigators [61] assumed a linear variation of strain along the thickness of the interface element and developed an eight-noded element with mid-side nodes.

4.3.2 Derivation of Joint Element Stiffness Matrix

The joint element developed by Ghaboussi et al. [36] uses relative displacements as the independent degrees-of-freedom. The displacement degrees-of freedom of one side of the interface are transformed into the relative displacements between the two sides of the slip surface. The transformation relations are as follows:

$$\left. \begin{aligned} U_{xi}^T &= U_{xi}^B + \Delta U_{xi} ; & U_{yi}^T &= U_{yi}^B + \Delta U_{yi} \\ U_{xj}^T &= U_{xj}^B + \Delta U_{xj} ; & U_{yj}^T &= U_{yj}^B + \Delta U_{yj} \end{aligned} \right\} \quad (4.2)$$

The superscripts, T and B , refer to the top and the bottom elements with respect to the slip surface. As illustrated in Fig. 4.3, the degrees-of-freedom of the upper element, which are on the slip surface, are transformed while the degrees-of-freedom of the lower element are the original displacement quantities.

Because of the assumptions of having the relative displacements as independent degrees-of-freedom, in a two-dimensional problem the joint element will have only four degrees-of-freedom. The relative normal and tangential displacements, ΔU_n and ΔU_s , are assumed to vary linearly along the element length as follows:

$$\left. \begin{aligned} \Delta U_n &= h_i \Delta U_{ni} + h_j \Delta U_{nj} \\ \Delta U_s &= h_i \Delta U_{si} + h_j \Delta U_{sj} \end{aligned} \right\} \quad (4.3)$$

in which h_i and h_j = the linear interpolation functions and,

$$\left. \begin{aligned} h_i &= \frac{1}{2}(1 - \xi) \\ h_j &= \frac{1}{2}(1 + \xi) \end{aligned} \right\} \quad (4.4)$$

where ξ is a nondimensional coordinate (Fig. 4.4). The quantities, ΔU_{ni} , ΔU_{si} , ΔU_{nj} , and ΔU_{sj} , are the nodal point values of the relative displacements at points i and j , respectively.

Two strain components are assumed for the joint element: ϵ_n = normal strain, and ϵ_s = shear strain. These two strains are related to the relative displacements through the following relations:

$$\left. \begin{aligned} \epsilon_n &= \frac{1}{t} \Delta U_n \\ \epsilon_s &= \frac{1}{t} \Delta U_s \end{aligned} \right\} \quad (4.5)$$

The substitution of Eqs. 4.3 and 4.4 into Eq. 4.5 results in the strain-displacement relation for the element:

$$\left. \begin{aligned} \left\{ \begin{array}{c} \epsilon_n \\ \epsilon_s \end{array} \right\} &= \frac{1}{2t} \begin{bmatrix} (1-\xi) & 0 & (1+\xi) & 0 \\ 0 & (1-\xi) & 0 & (1+\xi) \end{bmatrix} \left\{ \begin{array}{c} \Delta U_{ni} \\ \Delta U_{si} \\ \Delta U_{nj} \\ \Delta U_{sj} \end{array} \right\} \\ \epsilon &= B \Delta U \end{aligned} \right\} \quad (4.6)$$

The stresses and the strains are related through the following material property matrix C:

$$\left. \begin{aligned} \left\{ \begin{array}{c} \sigma_n \\ \sigma_s \end{array} \right\} &= \begin{bmatrix} C_{nn} & C_{ns} \\ C_{sn} & C_{ss} \end{bmatrix} \left\{ \begin{array}{c} \epsilon_n \\ \epsilon_s \end{array} \right\} \\ \sigma &= C \epsilon \end{aligned} \right\} \quad (4.7)$$

The stiffness matrix for the joint element is formed in the n-s coordinate system as follows:

$$K_{ns} = \int_{vol} B^T C B dv \quad (4.8)$$

By transforming to the x-y coordinate system, it is found to be

$$K = T^T K_{ns} T \quad (4.9)$$

where

T = the transformation matrix containing the direction cosines

$$[K] = \frac{l}{6t} \begin{bmatrix} 2(A_1 - 2B_1) & 2(A_3 + B_2) & (A_1 - 2B_1) & (A_3 + B_2) \\ & 2(A_2 + 2B_1) & (A_3 + B_2) & (A_2 + 2B_1) \\ \text{Symmetric} & & 2(A_1 - 2B_1) & 2(A_3 + B_2) \\ & & & 2(A_2 + 2B_2) \end{bmatrix} \quad (4.10)$$

in which

$$A_1 = C_{ss} a^2 + C_{nn} b^2$$

$$B_1 = C_{ns}$$

$$A_2 = C_{ss} b^2 + C_{nn} a^2$$

$$B_2 = C_{ns} (a^2 + b^2)$$

$$A_3 = (C_{nn} - C_{ss}) ab$$

$$a = (1/l)(x_j - x_i) ; b = (1/l)(y_j - y_i)$$

For the purpose of the present work, the joints are assumed to be nondilatant which means that shearing produces no joint expansion or contraction. Therefore, the shear and normal components of deformation are uncoupled and the stress-strain relations are as follows:

$$\begin{Bmatrix} \sigma_n \\ \sigma_s \end{Bmatrix} = \begin{bmatrix} C_{nn} & 0 \\ 0 & C_{ss} \end{bmatrix} \begin{Bmatrix} \epsilon_n \\ \epsilon_s \end{Bmatrix} \quad (4.11)$$

Note that C_{nn} and C_{ss} are nonlinear functions.

The material property functions C_{nn} and C_{ss} for different joints will be explained in detail in the following subsection.

4.3.3 Nonlinearity of the Interface Elements

To model the nonlinear behaviour of the joints and interfaces, different laboratory tests have been carried out by several investigators to obtain stress-strain relationships for interface materials. Some of these laboratory results have been

utilized to model the physical joint behaviour in the numerical method used in the present work. Since we have two types of interfaces, two sets of data are required: block-mortar interface data and soil-foundation interface data which are described as follows:

4.3.3.1 Block-Mortar Interface Data

A limited number of laboratory test results were available to determine the deformation relations and failure criteria for the block-mortar interface. Ali, Page, and Kleeman [1] carried out a series of tests to come up with those relations. The mortar stress-strain curves were derived indirectly from prism tests (for the normal and parallel strains), and from brickwork couplets with sloping bed joints (for shear strains). The test results and derived curves are shown in Figs. 2.14 and 2.15. For each test result, the mortar stress-strain curve was obtained by subtracting the contribution of the brick deformations from the overall measured deformations. The stress-plastic strain curves for the mortar were thus obtained.

There are two reasons for using the above test results for the analysis work in this thesis. First, the technique used to determine the mortar shear-strain relationship enables one to obtain the shear stress-strain curves for variable normal stresses which are not available from other methods using a direct shear test. Second, the authors used solid concrete bricks in their investigation which provides a compatibility with the joining mortar very similar to the one that exists between concrete blocks and mortar in the masonry wall used in the present analysis work. In other words, the materials used in their experimental tests, in comparison with other test results in which for example clay brick units were used, seem to be representative of the masonry wall under study. The importance of the compatibility between the masonry unit and the mortar and its effect on the mechanical behaviour of the mortar joint, especially under shear stress, can be realized from the experimental study conducted by Hamid et al. [42]; Hamid et al. concluded that the shear bond strength is not only a function of mortar properties, but also a function of the

physical properties of the masonry units such as its surface roughness and initial rate of absorption.

The irregular stress-strain curves obtained from laboratory tests are difficult to use in typical finite element calculations. Therefore, the joint material behaviour is idealized, as shown in Fig. 4.1, to a bilinear relationship. Owing to the shortage of experimental data describing the plastic behaviour of mortar joints, Fig. 4.1 appears to be the closest idealization of the experimental data and can be supported by the different experimental test results reviewed in Chapter 2 under Section 2.3.

As mentioned before under Section 4.2, the peak shear strength of the joint is assumed to be a function of the normal stress developed in the joint and of the cohesion and roughness angle. Hamid, Drysdale and Heidebrecht [42] have published such relationships that were derived from a series of direct shear tests. Fig. 2.16 shows the relationships between the shear strength and the normal compression stress applied to the joint for ungrouted and grouted masonry. Two types of grouting were involved in the study. Since the grouting has no effect on the shear strength of the interface joint elements at the wall-foundation and wall-lintel interfaces, the relationship between the shear strength and applied compressive stress for ungrouted masonry has been used.

4.3.3.2 Soil-Foundation Interface Data

To obtain the nonlinear joint properties at the soil-foundation interface, different laboratory tests have been carried out by a number of researchers in the form of direct shear tests or triaxial tests on intact joints specimens.

Among these tests, Desai et al. ([26],[27]) have described a new device for testing large size interfaces between structural and geologic materials. They reported test results for a sand-concrete interface in which they expressed the shear stress as a function of normal stress, relative displacement, number of loading cycles and initial density. The soil used was a standard Ottawa sand passing the No. 20 sieve

and retained on the No. 40 sieve. This sand is highly uniform and has subrounded grains.

Fig. 4.5 shows a static direct shear test setup between sand and concrete. Fig. 4.6 illustrates observations for two typical tests where σ equals 4.77 and 9.55 kg/cm². Typical results from finite element calculations, using a thin-layer element to simulate the interface behaviour, were compared with the observations in Fig. 4.6; this led to the following criterion for the choice of thickness t of the interface:

$$0.01 < \frac{t}{l} < 0.1 \quad (4.12)$$

where

l = average length of the adjoining soil and structural element

In the analytical study presented in this thesis, the t/l ratio was taken equal to a value close to 0.01 for both types of interfaces.

In the present analytical work, the soil was assumed to be rigid in the normal direction of the soil-foundation interface and a very high numerical value was assigned to the normal stiffness coefficient E_c when the normal stress in the joint is compression; when the normal stress is tension, E_c was assumed to be zero as indicated in Fig. 4.2. The nonlinear behaviour of the soil-foundation interface in shear was idealized as an elastic-perfectly plastic material as shown Fig. 4.2. The laboratory test results by Desai et al. [26] were used to obtain the shear modulus value G_c .

Also, the published results of the experimental tests conducted by Desai et al. [27] to obtain the relationships between the peak shear strength and corresponding normal stress of the sand-concrete interface were used to simulate the soil-foundation interface properties. Fig. 4.7 shows the envelopes of peak interface shear stress versus normal stress obtained from static tests (0.5 in./min) with relative sand densities $D_r=15\%$, 65% and 80% . In the study presented in this thesis, the peak shear-normal stress relationship corresponding to $D_r=80\%$ was used in

the numerical analysis because it represented the only test data available in the literature.

4.4 Other Finite Elements Involved in the Analysis

Other finite elements are involved in the present study's computer program as follows:

- The *constant strain triangular* (CST) element was used to represent the continuum on the two sides of the interfaces as a linear elastic domain. This element is a simple element with three corner nodes and with two degrees-of-freedom at each node. The two degrees-of-freedom are the two displacements perpendicular to each other. To obtain more accurate results, an increasing number of small CST elements was used. It must be noted that the use of the CST element does not require a large band width in the stiffness matrix which is advantageous for reducing the storage space in the computer memory.
- A *bar element* is used to model the steel reinforcement and to model any increase in the stiffness, as for example due to lintels or connectors between wall lintel and roof slab. The bar element has two degrees of freedom at each node which are the two displacements perpendicular to each other.
- Also, the *link-bond element* developed by Ngo and Scordelis [59] (Fig. 2.1) is introduced in the finite element program used in the analysis. This element is mainly used in a coarse linear finite element analysis to investigate the need for nonlinear finite element analysis.

The above elements are so widely used that there is no need here to derive the stiffness matrices for them. However, their stiffness matrices can be found in any text on finite element methods.

4.5 Finite Element Program to Solve Joint and Interfaces Problems

4.5.1 General

A computer implementation was carried out to model the joints and interfaces in the analysis of the two-dimensional problems. The program is able to handle different types of interfaces. The nonlinear behaviour is simulated by using an iterative technique suggested by Ghaboussi et al. [35]. In the next subsections, the iterative technique used in the finite element computer program to solve the nonlinear equations of equilibrium and the relative displacement coding are explained.

4.5.2 Solution of Weakly Nonlinear Equations of Equilibrium

Ghaboussi et al. [35] have used the term "weak nonlinearity" to describe systems in which nonlinearity is local and constitutes a small portion of a linear system. In mathematical terms it can be stated that in weakly nonlinear problems the variation of the norm of the stiffness matrix of the system, $\|K\|$, is small compared to the considerable variation of such a norm in strongly nonlinear problems.

In this section the analysis will be confined to a simple solution technique which is applicable to such weakly nonlinear systems where the continuum material is linearly elastic but the joints are inherently nonlinear.

The stiffness matrix of such systems can be symbolically divided into K_c , representing the constant stiffness of the linearly elastic continuum, and K_d , representing the tangent stiffness of the joints. It should be noted that the size of the stiffness matrix, K_c , is usually large compared to K_d .

The equation of equilibrium for this system under small displacements can be written as follows:

$$K_c U_{n+1} + F_{dn} + K_{dn}(U_{n+1} - U_n) = P_{n+1} \quad (4.13)$$

in which

U = the vector of the displacements

F_d = the vector of the internal resisting forces for the joint elements

P = the vector of the applied loads

n = a subscript denoting the number of the iteration

The preceding equation of equilibrium can be written in a form more suitable for finding a solution to determine U_{n+1} when U_n is known:

$$K_n U_{n+1} = F_{n+1} \quad (4.14)$$

in which

$$K_n = K_c + K_{d_n}$$

$$F_{n+1} = P_{n+1} + K_{d_n} U_n - F_{d_n}$$

Since the number of joint elements in most problems is much less than the continuum elements, the stiffness matrix, K_{d_n} , will be very sparse with few nonzero elements. Therefore, the matrix multiplication $K_{d_n} U_n$, can be carried out at element level:

$$\sum_{m=1}^N K_{d_n}^m U_n \quad (4.15)$$

in which

K_d^m for the nonlinear element = the stiffness of the m^{th} element

N = the total number of joint elements

The internal resisting force vector, F_{d_n} , is formed directly from the stresses in the joint elements:

$$F_{d_n} = \sum_{m=1}^N B^T \sigma_n \quad (4.16)$$

This form of equation of equilibrium can be used for an incremental solution,

iterative solution, or incremental solution with some iterations [36].

For the purpose of the present work, the *iterative solution* was used to solve the equation of equilibrium. In this scheme, the total loads are applied from the beginning and the solution is approached iteratively. The convergence is measured by a norm of the difference of the displacement vectors of two consecutive iterations: $U_{n+1} - U_n$. The effect of the stiffness reformulation in this scheme is to increase the rate of convergence.

4.5.3 Relative Displacement Coding

Ghaboussi, Wilson, and Isenberg [36] have introduced a coding technique where only one of the sides of a joint element is recognized explicitly as a line of nodal points in the mesh; the opposite side of the joint is considered to belong to one of the contiguous blocks as an internal feature. Its effect is incorporated in the block stiffness through an expansion of the block's degrees-of-freedom.

Using the transformation matrix $[T]$, the 8×8 stiffness matrix $[K]$ of the quadrilateral block $IJKL$ can be expanded to a 12×12 matrix $[K']$ as follows:

$$[K'] = [T]^T [K] [T] \quad (4.17)$$

where

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.18)$$

If the side IJ of the block contacts side AB of a contiguous block across joint element $ABJI$ as shown in Fig. 4.8, the stiffness terms of the joint stiffness matrix

(Eq. 4.10) must be added to the rows and columns 9 through 12 of the expanded stiffness matrix $[K']$.

Since,

$$\left. \begin{aligned} U_I &= U_A + \Delta U_A; & V_I &= V_A + \Delta V_A \\ U_J &= U_B + \Delta U_B; & U_J &= U_B + \Delta U_B \end{aligned} \right\} \quad (4.19)$$

the vector of nodal-point displacements for the block $IJKL$ can be written as follows:

$$(U') = (U_A \quad V_A \quad U_B \quad V_B \quad U_K \quad V_K \quad U_L \quad V_L \quad \Delta U_A \quad \Delta V_A \quad \Delta U_B \quad \Delta V_B)^T \quad (4.20)$$

Also, the force vector computation is now changed to $[F'] = [T]^T [F]$.

Solution of the structural equation will then directly yield the displacements of the nodal points A , B , K , and L and the relative displacements ΔU_A , ΔV_A , ΔU_B , ΔV_B due to the joint. The last terms of the relative displacements will be equal to zero for a very stiff joint.

This technique allows one to compute successfully with high joint stiffness [37]. In general, a very high stiffness term on the main diagonal tends to fix a nodal point. In the relative displacement scheme above, a high joint stiffness will zero only the relative displacement of the joint; the results should be identical whether a mesh without a joint or one with a very stiff joint is used [37].

For examining the assembly process of the global stiffness matrix, several problems were solved using the computer program including different cases of loading, geometry and using a range of values for the interface element shear and normal stiffness constants up to very high numerical values. The problems solved resulted in identical results as compared to solutions obtained for the same problems without using a joint interface. No ill-conditioning problems arose even when very high joint stiffness constants were used.

Fig. 4.8 shows a simple problem in which the upper block $IJKL$ is connected to the lower block $ABCD$ across the joint element $ABJI$; it also illustrates how the stiffness matrices of the different elements are assembled in the global stiffness matrix. In this figure, it can be seen how the band width is enlarged by the transformation of the stiffness matrix of the upper continuum element.

4.5.4 Computer Programs

Program FE-PRIDE

Using the iterative technique suggested by Ghaboussi et al. [36] for solving problems which included joints and interfaces as explained under Section 4.5.2, a computer program was written to solve two-dimensional problems utilizing the finite element method and considering the relative displacements between the interfaces as independent degrees-of-freedom. The finite element program FE-PRIDE is capable of solving problems in which interfaces are embedded in an elastic medium. The program is written in FORTRAN. Two versions of the program compatible with the CP-6 and APOLLO domains are available. Fig. 4.9 illustrates a flow diagram showing the sequence of the key functions performed by the finite element program.

Program DATAGEN

To minimize human error, the data to be input manually were reduced by almost fully automating the data generation process. A data generation program DATAGEN was developed to generate the mesh data which contained the minimum possible band width. The mesh data were developed according to the numbering scheme explained previously under Section 4.5.3 in which a special numbering system for the interface elements was considered.

The basic data that is to be input manually to the program FE-PRIDE include the following:

1. Material properties which include Young's modulus, Poisson's ratio, density and coefficient of thermal expansion.

2. Initial temperature (if other than 20°C).
3. Interface material properties C_{nn} and C_{ss} for different types of interfaces.

Plotting Program

The program PLOT was developed to draw the nodal displacements of each structural element. The applied boundary restraints are indicated on these plots. Also, the program plots the principal stresses and shearing stress at each node, as well as the shear stress distribution at different selected levels and interfaces. The program uses some of the routines from the graphic package CALPLOT of the CP-6 library.

Fig. 4.1 Idealised stress-strain curves for masonry unit-mortar interface

Fig. 4.2 Idealized stress-strain curves for soil-foundation interface

Fig. 4.3 Geometry of the joint element

Fig. 4.4 Coordinate systems for the joint element

Fig. 4.5 Analysis of direct shear test for choice of interface thickness

Fig. 4.6 Typical direct shear test results ($D_r=80\%$) [Desai, 1976; Desai, et al., 1984]

Fig. 4.7 Static Mohr-Coulomb strength envelopes

Fig. 4.8 Coding scheme and stiffness matrices assembly for a simple problem associated with interface element

Fig. 4.9 Scheme of execution of the computer program

5.1 General

The main goal of this chapter is to determine the stresses that may develop in the selected model for a lowrise concrete block loadbearing masonry wall building under differential thermal and gravity loads. A nonlinear analysis was carried out using the finite element program discussed in Chapter 4, in which nonlinear behaviour is maintained in the system by modeling the different interfaces using a special joint element. Also, for a comparison between linear and nonlinear stress analyses, a linear finite element analysis was carried out for the same model and under the same thermal and gravity loads.

In the previous chapters, the principal assumptions upon which the stress analysis is based were explained in detail. Some of those assumptions are highlighted below:

1. Three interfaces have been modeled: soil-foundation interface, foundation-wall interface, and wall-roof slab interface.
2. The materials constituting the domains above and below each interface are assumed to be homogeneous, isotropic, and linearly elastic.
3. The wall and roof slab constituting the structure are assumed to be thin. Therefore, the effect of slab rotation at the top of the masonry wall can be ignored and the loadings are such that they are resisted by the structural elements through membrane action. Also, possible reduction of the contribution of the slab stiffness to the selected model due to bending precracking was neglected.

The advantage of the symmetry of loading and geometry was considered and only half of the wall was analyzed. Also, as mentioned before, in all graphics the concrete slab was rotated 90° and plotted in the same plane of the wall and foundation.

5.2 A Typical Analysis

5.2.1 General Description and Data

Fig. 3.1 shows the loading system and the nominal dimensions of the masonry lowrise building which is modeled in the present study. A typical cross section of the different structural elements and dimensions is given in Fig. 3.2. A description and data concerning the different structural materials are as follows:

Masonry Wall:

The typical nominal thickness of the concrete masonry blocks used to construct the masonry wall is 190 mm. However, the effective thickness of the wall, which is defined as the transformed area of all the solid parts such as block material, grout and reinforcement in the cross section of the wall per unit length, is smaller than the nominal thickness and may vary from wall to wall. For the purpose of this study, the CSA code (CAN3-S304-M84) [19] requirements have been taken into consideration. Therefore, it is assumed that the wall is grouted at a nominal spacing of 1200 mm on centers. Also, the total reinforcement has been assumed to be 0.2 percent of the gross cross sectional area of the wall which is the minimum reinforcement required by the code for reinforced masonry loadbearing walls. This reinforcement was assumed to be equally distributed in the horizontal and vertical directions.

Although the grouting increases the effective thickness of a masonry wall in the vertical direction which will develop a certain degree of orthotropy in the wall, this orthotropy was ignored in the stress analysis work.

Concrete Slab:

The roof slab of the building under study is assumed to be of one-way composite construction and to have a nominal thickness of 64 mm. This thickness was increased to 67 mm to account for the steel reinforcement that exists in the slab. The contribution of the steel joists to the in-plane stiffness of the slab was ignored in the present analysis.

Concrete Footing:

Two types of footing are applicable to most of Canada and the northern United States; the conventional footing and the insulated footing presented by Robinsky and Bessflug [70]. For the purpose of this study, the conventional type of footing is considered. The thickness and height of the concrete footing wall are assumed to be 200 mm and 250 mm, respectively. Also, 250 mm and 600 mm are assumed for the thickness and width of the footing, respectively. The density of the backfill is taken equal to 1700 kg/m^3 .

According to the thermal analysis presented in Chapter 3, two loading cases were assumed to account for winter and summer temperature variations. Equivalent gravity loads were used in both cases except for the weight of the snow which was added to the gravity loads for the winter case thermal stress calculations. Also, it should be mentioned that the live and snow load values were taken from the NBC for the Ottawa region.

The data used in the analysis can be summarized as follows:

Type of Building	Industrial
Length of Longitudinal Wall (L)	12.5 m
Length of Transverse Wall (L_s)	14.0 m (Fig. 3.1)
Wall Height (H)	5.0 m
Foundation Depth	1.5 m

Wall Thickness:	nominal	= 190 mm
	effective	= 84 mm

Slab Thickness:	nominal	= 64 mm
	effective	= 67 mm

Foundation Wall Thickness:	nominal	= 190 mm
	effective	= 200 mm

Gravity Loads:	Dead Load of Slab	= 2.0 kPa
	Live Load	= 1.0 kPa
	Snow Load	= 2.3 kPa
	Masonry Unit Load	= 2.4 kPa

Temperature: (see Fig. 3.10)

Material Properties:

Modulus of Elasticity:

block masonry	= 10 000 MPa
reinforced concrete	= 22 360 MPa

Poisson's Ratio:

block masonry	= 0.15
reinforced concrete	= 0.15

Coefficient of Thermal Expansion:

block masonry	= $9.5 \times 10^{-6} \text{ per } ^\circ\text{C}$
reinforced concrete	= $10 \times 10^{-6} \text{ per } ^\circ\text{C}$

5.2.2 Results

The deformations of the foundation-wall-slab model for the wintertime and summertime cases can be seen in Fig. 5.1. These deformations are plotted to the scale indicated at the bottom of each plot.

The principal stresses in the footing, masonry wall, lintel, and concrete slab for the wintertime and summertime cases are plotted in Figs. 5.2 and 5.3, respectively. An enlarged symbol of the proper type is used to identify the maximum tensile and compressive stresses and these are written on the top of each plot. The magnitudes and directions of the principal stresses are represented by the length and direction of the lines plotted at each node.

Fig. 5.4 illustrates the shear stress distributions along different selected levels of the masonry wall for both the wintertime and summertime cases. Horizontal stress distributions along different vertical cross sections of the masonry wall for wintertime case are shown in Fig. 5.5. Also, horizontal stress distributions along different cross sections of the masonry wall for wintertime case are illustrated in Fig. 5.6.

In examining the applicability of using the linear stress analysis to determine the generated thermal stresses and deformations, a linear finite element analysis was carried out on the same model and under the same temperature profiles for the wintertime and summertime cases. The deformations of the different structural elements of the masonry wall building for both cases using the linear finite element analysis are shown in Fig. 5.7. Using the linear stress analysis, the principal stresses in the footing, masonry wall, lintel, and concrete slab are plotted in Figs. 5.8 and 5.9. Also, Fig. 5.10 illustrates the shear stress distributions along different horizontal levels of the masonry wall generated from the analysis.

In comparing stresses based on the linear and the nonlinear analyses, Fig. 5.11 shows the horizontal stress distributions along the mid-span vertical cross section of the wall based on both methods. Also, a comparison between the horizontal stress distributions along selected horizontal cross sections is shown in Fig. 5.12.

5.3 Discussion

As illustrated in Fig. 5.1, the deformed shapes of the different elements of the selected model for both wintertime and summertime cases are beneficial in explaining how the whole building behaves, where the restraining points are, and how the different structural elements affect each other.

Concerning the stresses plotted in Figs. 5.2 through 5.6, the following observations can be made:

Wintertime Thermal Stresses

1. The masonry wall is subjected to a thermal stress field which is predominantly tensile as illustrated in Fig. 5.2. This tensile stress pattern starts at the free end of the wall with an inclination angle that is close to 45° with the wall-foundation interface and 135° with the wall-lintel interface and reduces gradually until it reaches the horizontal at the mid-length of the masonry wall at which the tensile stresses are more significant.
2. The lintel is subjected to very high thermal tensile stresses with a maximum magnitude of 2.91 MPa as can be seen in Fig. 5.2.
3. The concrete roof slab is subjected to a thermal stress field which is predominantly compression as illustrated in Fig. 5.2. The compression field pattern starts at the free end of the roof slab with an inclination angle close to 45° with the horizontal axis and reduces gradually until it almost reaches the horizontal at the building mid-length. The maximum principal compression stress is equal to 3.96 MPa as indicated in Fig. 5.2.
4. Considering the other principal stress distribution referred to as principal stress 2 in Fig. 5.2, the concrete roof slab is subjected to very high thermal tensile stresses close to the free end of the roof. These stresses are more significant at the corner of the roof at the masonry wall and have a maximum magnitude of 2.1 MPa.

5. The concrete roof and footing restrain the masonry wall. Therefore, the maximum horizontal shear stress distributions, parallel to bed joints, are found to be at the top and bottom of the masonry wall at its interfaces with the lintel and foundation. These shear stress distributions, shown in Fig. 5.4, indicated that the maximum horizontal shear stress at the bottom and top of the masonry wall are equal to 0.41 MPa and 0.55 MPa, respectively, for the wintertime case.
6. As shown in Fig. 5.5, the masonry wall at its mid-span is subjected to very significant horizontal tensile stresses under the wintertime case. These tensile stresses display higher values at the top and bottom of the wall and reduce gradually towards the wall mid-height. The maximum horizontal tensile stress of 1.1 MPa is found to be at the top of the wall at mid-span.
7. Also, Fig. 5.6 shows different profiles of horizontal stress distributions along selected horizontal cross sections. It can be seen that among the different horizontal cross sections, the top of the masonry wall displays the highest tensile stresses and the lowest rate of stress variation along the masonry wall length.

Summertime Thermal Stresses

1. The masonry wall is subjected to a thermal stress field which is predominantly compressive as illustrated in Fig. 5.3.
2. As shown in Fig. 5.3, the lintel is subjected to high compressive thermal stresses with a maximum magnitude of 1.76 MPa.
3. The roof slab is subjected to a thermal stress field which is predominantly tensile with a maximum value of 2.17 MPa. The direction and location of this stress is shown in Fig. 5.3.
4. From Fig. 5.4, for the summertime case, it can be seen that the maximum horizontal shear stresses at the bottom and top of the masonry wall are equal to 0.30 MPa and 0.24 MPa, respectively.

Overstressing and Cracking Under Thermal Stresses

Wintertime Case

1. From Fig. 5.2 and referring to the plot for principal stress 2, it can be seen that the roof slab is subjected to a maximum thermal tensile stress of 2.1 MPa at the exterior corner of the slab at an angle of approximately 135° with the positive x-axis. By comparing this tensile stress with a typical value of 2.0 MPa for the tensile strength of concrete, a few cracks may be expected in the concrete slab in the vicinity of the exterior corner at an angle of about 45° with respect to the positive x-axis.
2. Figs. 5.2 and 5.5 illustrate that the masonry wall is subjected to very high horizontal tensile stresses at its mid-length ranging between 0.74 to 1.1 MPa. Using 0.7 MPa for the tensile strength of the concrete block masonry wall in the direction parallel to bed joints, a vertical crack may occur at the wall mid-length. This crack, due to the thermal tensile stress distribution shown in Fig. 5.5, may be wider at the top of the wall and may taper down towards the wall bottom. This class of cracking and its relationship to masonry wall geometry will be discussed in Chapter 6.
3. Fig. 5.4 illustrates that the maximum shear stresses parallel to bed joints at the bottom and top of the masonry wall are 0.41 MPa and 0.55 MPa, respectively. Using an allowable shear stress of $0.015f'_m$ given by the CSA code [19], which results in a value of 0.15 MPa, the masonry wall is overstressed in shear.

Also, assuming an average compressive stress acting on the masonry wall of 0.42 MPa and employing the relationship between the shear strength of concrete masonry and the applied compressive stress published by Hamid et al. [42] (Fig. 2.16), the shear strength of the masonry wall is calculated as 0.83 MPa. This is greater than the maximum shear stresses at the top and bottom of the masonry wall. Therefore, the masonry wall is not expected to crack under thermal shear stresses.

Summertime Case

1. From Fig. 5.3 and referring to principal stress 1, it can be seen that the roof slab is subjected to a maximum thermal tensile stress of 2.17 MPa at the exterior corner of the slab at an angle of almost 45° with the positive x-axis. By comparing this tensile stress with a typical value of 2.0 MPa for the tensile strength of concrete, a few cracks may occur in the slab in the vicinity of the exterior corner at an angle of about 135° with the positive x-axis.
2. Fig. 5.4 illustrates that the maximum shear stress in the direction parallel to the bed joints at the bottom and top of the masonry wall are 0.30 MPa and 0.24 MPa, respectively. Using an allowable shear stress equal to $0.015 f'_m$ ($=0.15$ MPa) according to the CSA code [19], the masonry wall is overstressed in shear. Also, assuming an average compressive stress acting on the masonry wall equal to 0.26 MPa and employing the relationship between the shear strength of concrete masonry and the applied compressive stress published by Hamid et al. [42] (Fig. 2.16), the shear strength of the masonry wall is taken as 0.71 MPa which is greater than the maximum shear stresses at the top and bottom of the masonry wall. Therefore, the masonry wall is not expected to crack under thermal shear stresses.

5.4 Evaluation of Linear Stress Analysis

To evaluate linear elastic stress analysis as a method for predicting the thermal stresses generated in the lowrise loadbearing masonry wall building subjected to temperature differentials, a linear finite element analysis was carried out to compare resultant stresses with stresses previously determined using a “Weak” nonlinear finite element analysis.

Figs. 5.1 and 5.7 provide a comparison between the deformed shapes of the different elements of the selected model for wintertime and summertime cases using nonlinear and linear stress analyses. While Fig. 5.1 illustrates the slippages

occurred at the different interfaces between the structural elements, Fig. 5.7 illustrates the high degree of restraint that the structural elements apply on each other; such restraint caused an overestimation of stresses in the selected model.

From Figs. 5.8 through 5.10 it can be seen that the thermal stress fields in the roof slab, masonry wall and foundation derived from a linear stress analysis display much the same patterns as those predicted using nonlinear stress analysis. But even a cursory examination of the stresses plotted in those figures indicates significant differences in the magnitudes of stresses determined using the linear and the nonlinear methods. The linearly derived stresses typically are much higher than those previously discussed for Figs. 5.2 to 5.6.

For example, Fig. 5.11 shows a comparison between the horizontal stress distributions along the mid-span vertical cross section of the wall based on the linear and nonlinear stress analyses. Not only is the stress distribution profile based on the linear stress analysis different than that of the nonlinear stress analysis, but the stress values are also overestimated: The horizontal stress at the bottom of the wall based on the linear stress analysis is almost 4 times the stress that would be obtained using the nonlinear stress analysis; at the top of the wall, the same horizontal stress based on the linear stress analysis is 2.2 times that based on the nonlinear stress analysis.

Fig. 5.12 shows a comparison between the horizontal stress distributions along different horizontal cross sections of the masonry wall based on the linear and nonlinear stress analyses. Again, this figure shows the considerable difference that exists between the stresses generated from the two methods.

While the locations where cracking and sliding are expected to occur could be predicted using the linear elastic stress analysis, the extent of cracking, mortar joint opening and a more realistic value for stresses might not be determined by such an analysis. Therefore, the linear finite element analysis was only used to predict the locations of high stresses at which point special joint elements were used to model the nonlinear behaviour.

5.5 Convergence of the Nonlinear Finite Element Solution

As mentioned in Chapter 4, the convergence of the nonlinear finite element analysis is measured by the norm of the difference of the displacement vectors of two consecutive iterations. This convergence is depicted in Fig. 5.13 where the convergence is plotted versus the number of iterations for the wintertime case. As shown in this figure, the convergence is achieved just after 6 iterations.

**Fig. 5.1 Deformation of lowrise masonry wall building:
Nonlinear stress analysis**

Fig. 5.2 Wintertime principal stresses for nonlinear stress analysis

Fig. 5.3 Summertime principal stresses for nonlinear stress analysis

Fig. 5.4 Shear stress distributions along different horizontal planes:
Nonlinear stress analysis

Fig. 5.5 Horizontal stress distributions along different vertical cross sections of the masonry wall:
Nonlinear stress analysis

Fig. 5.6 Horizontal stress distributions along different horizontal cross sections of the masonry wall:
Nonlinear stress analysis

Fig. 5.7 Deformation of lowrise masonry wall building:
Linear stress analysis

Fig. 5.8 Wintertime principal stresses for linear stress analysis

Fig. 5.9 Summertime principal stresses for linear stress analysis

Fig. 5.10 Shear stress distribution along different horizontal planes:
Linear stress analysis

Fig. 5.11 Comparison between the horizontal stress distributions along wall mid-span vertical cross section based on linear and nonlinear stress analyses

Fig. 5.12 Comparison between horizontal stress distributions along different selected horizontal sections of the masonry wall based on linear and nonlinear stress analyses

Fig. 5.13 Convergence of the finite element solution
for the wintertime case

6.1 General

In the previous chapter, it was established that lowrise loadbearing masonry wall buildings may be greatly affected by thermal stresses and that cracking of walls may ensue. In order to take these thermal stresses into design consideration, it is necessary to know both the governing factors and the extent to which they control the thermal stresses. In this chapter, several of these factors are identified and examined in detail to determine their relationship with thermal stresses.

Since the major distress of the masonry wall is cracking due to tensile stresses developed from thermal contraction and since a major concern of this study is to determine the control joint spacing, only the wintertime case will be examined in the parametric study presented in this chapter. Also, it should be noted that for the majority of the parameters examined, the horizontal tensile stress distribution at the mid-length of the masonry wall is used to visualize the effect of the key parameters.

6.2 Effect of Wall Height/Length Ratio

Since one of the main objectives of the present study is to determine the spacing of control joints, a parametric study was carried out for different masonry wall height/length ratios (aspect ratios). Fig. 6.1 illustrates horizontal stress distributions for different aspect ratios; it can be seen that the magnitude of thermal tensile stress increases with an increase in length of the wall. Also, Fig. 6.2 shows the horizontal stress distributions along the masonry wall length at different wall

interfaces with the foundation and lintel.

As mentioned in Chapter 3, based on an experimental study, the masonry tensile strength in the direction parallel to a bed joint was taken equal to 0.7 MPa. Therefore from Fig. 6.3, which indicates the relationship between the maximum horizontal stresses at the masonry wall mid-length and the aspect ratio, it can be seen that for an aspect ratio (wall height/length ratio) less than 0.5 the horizontal stresses exceed the assumed tensile strength of the masonry in the direction parallel to the bed joints. In other words, for such walls a vertical crack at the wall mid-length is expected to occur. For aspect ratios equal to or greater than 0.5, stresses are below the tensile strength and hence walls with such aspect ratios are not expected to crack.

Also, from Fig. 6.3, it can be concluded that the horizontal stresses at the top of the wall tend to be greater than those at the bottom over a wide range of aspect ratios. Therefore, if a crack developed at the mid-length of the masonry wall due to thermal tensile stresses, this crack will be tapered with a greater width at the top and then narrower towards the bottom of the wall.

Fig. 6.4 illustrates the relationship between the maximum shear stresses adjacent to the wall-foundation and wall-lintel interfaces and the different aspect ratios. Assuming an average compressive stress acting on the masonry wall of 0.42 MPa and employing the relationship between the shear strength of concrete masonry and the applied compressive stress published by Hamid et al. [42] (Fig. 2.16), the shear strength of the masonry wall is equal to 0.83 MPa. By comparing the maximum shear stresses shown in Fig. 6.4 with the aforementioned experimental test results, successive slippage might occur in the masonry courses in the vicinity of the wall-roof and foundation-wall interfaces for walls with aspect ratios less than 0.2. Also note that the shear strength of the masonry wall is a function of the vertical compressive stress, which in turn depends on the geometry of the building; the shear strength therefore may be less than 0.83 MPa if the wall compressive stress is below 0.42 MPa.

Fig. 6.5 shows the horizontal thermal stresses developed in the lintel along its length for different masonry wall aspect ratios. From this figure, it can be seen that for different aspect ratios the lintel is subjected to large tensile stresses close to 3.0 MPa. These significant restraint stresses should be taken into consideration in the design of the lintel.

From the aforementioned discussion it can be concluded that, for a typical concrete masonry building of walls with aspect ratios less than 0.5, a control joint should be provided at mid-length where maximum thermal tensile stresses are typically found. Also, for masonry walls with aspect ratios less than 0.2, successive cracking due to shear failure in the masonry wall courses are expected to occur in the vicinity of the wall-roof and wall foundation interfaces close to the wall free ends.

Therefore, it can be concluded that control joints should be provided for a masonry wall such that the distance between them or the distance between a control joint and the wall free end is less than twice the height of the wall.

6.3 Effect of Temperature Profile Across the Foundation

In Chapter 3, the two-dimensional thermal study of the heat flow from the masonry wall across the concrete foundation resulted in the temperature profiles plotted in Fig. 3.9. In studying how a temperature profile along the foundation depth can affect the overall stresses, different temperature profiles were assumed and the resulting thermal stresses compared.

Three wintertime temperature profiles along the concrete foundation depth were considered in the parametric study:

1. The first is the profile that was proposed in Chapter 3 (Fig. 3.9) which is based on a 2-D heat flow analysis; this is assumed to be the most realistic one.

2. For the second temperature profile, it is assumed that there is no heat transfer between the masonry wall and the concrete foundation. Also, it is assumed that the temperature profile along the foundation is similar to that along the surrounding soil.
3. The third temperature profile is assumed to vary linearly along the depth of the concrete foundation. This linear temperature distribution begins with a temperature at the top of the foundation equal to that of the masonry wall and varies linearly until it reaches the soil temperature at the base of the foundation.

Fig. 6.6 shows the effects these temperature profiles have on the horizontal stress distribution along the masonry wall height at its mid-length.

By comparing the horizontal stresses generated at the wall mid-length it can be shown that:

- a. At the bottom of the wall, the horizontal tensile stress developed using profile (1) is almost equal to that developed using profile (2) and is 1.44 times the one developed using profile (3).
- b. At the top of the wall, the horizontal stress developed using profile (1) is 0.98 times the one developed using profile (2) and 1.17 times the one developed using profile (3).

In studying the effect a temperature profile along the foundation depth has on the horizontal tensile thermal stresses generated at the wall mid-length, it can be seen that there is practically no change in the results between profiles 1 and 2. This is a result of the very steep temperature profile in the vicinity of the wall-foundation interface; a very fine finite element mesh may be required to represent the temperature profile more accurately.

Also, from Fig. 6.6, it can be seen that modeling the temperature profile along the foundation depth by a linear relationship beginning with the wall temperature at the foundation top and proceeding linearly to the soil temperature at the foundation bottom is a coarse approximation which leads to an underestimation of stresses.

6.4 Effect of Mortar Bedding Area

Depending on the method of construction, the mortar can be applied to different bedding areas on the concrete block unit. Fig. 6.7 shows two types of mortar bedding areas; in the first case, the mortar is applied to the concrete block faceshells and in the second, the mortar is applied to the whole block net area. From this figure, it can be seen that by accounting for the 22 percent increase in the interface area of the concrete block unit, the horizontal tensile stresses at the wall mid-length increase approximately by 35 percent. This is due to the increase in the interface shear area at the wall-foundation and wall-lintel interfaces, which in turn increases the degree of restraint of the wall at the top and bottom edges.

6.5 Comparison Between Finite Element Method and Copeland Method

As discussed in Chapter 3, Copeland adapted the data generated from unpublished field measurements conducted by Carlson and Reading [23] to include walls restrained at the top and bottom.

Copeland also developed a relationship between the average stress at the centerline of the wall and the maximum principal stress at the restrained edges for walls of different length/height ratios. This relationship is the basis of his method to determine the control joint spacings and can be seen in Fig. 3.4.

Fig. 6.8 is a comparison of the finite element results with the aforementioned relationship developed by Copeland. This figure indicates that there is general agreement between the Copeland method, which is based on field measurements, and the finite element method.

Using Eq. 3.4 to calculate the ratio of average tensile stress/maximum tensile stress at the wall mid-length and utilizing Fig. 6.8 to obtain the corresponding aspect ratio, it can be noticed that, for a certain stress ratio, the method developed by Copeland is conservative compared to the finite element method. The largest

difference between Copeland's method and the finite element method occurs for small length/height ratios less than 1.5. For such short walls, a control joint typically will not be required and hence the difference between the methods is of little practical significance.

Fig. 6.1 Wintertime horizontal stress distributions at the masonry wall mid-length along its height for different H/L ratios

**Fig. A.16 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.5, wintertime stresses**

Fig. 6.3 Wintertime horizontal stresses at the masonry wall mid-length vs. H/L for selected horizontal cross sections

Fig. 6.4 Wintertime maximum shear stresses vs. H/L for selected horizontal cross sections

Fig. 6.5 Horizontal stress distributions in the lintel along its length
for different H/L ratios

Fig. 6.6 Effect of different temperature profiles across the footing on the horizontal stress distribution along the masonry wall height at mid-length

Fig. 6.7 Effect of mortar bedding area on the horizontal stress distribution at the masonry wall mid-length along its vertical cross section

Fig. 6.8 Comparison between finite element method and Copeland method for the relationship between average stress and L/H ratio

Summary, Conclusions & Recommendations

7

7.1 General

Movements in masonry buildings occur as the result of temperature changes, shrinkage due to moisture content changes, chemical processes, and creep deformations in the component materials. Such movements can result in overstressing and cracking which may expose the structural elements to environmental distressing factors such as freeze-thaw action and corrosion which in turn may influence the safety, serviceability and maintenance costs of the structure. The focus of this thesis is the effect of temperature changes on the performance of loadbearing lowrise concrete masonry buildings.

When building materials expand and contract due to temperature changes, considerable forces may be produced in restrained structural elements. If these forces are not properly considered, the resultant stresses can lead to serious failures (usually cracking) in materials and structural elements. These stresses can be relieved or reduced to tolerable limits by constructing control joints. Such joints are particularly important to allow movements to take place along certain preselected lines.

Even today, the design of the control joints is based more on art than science, i.e. experience plays a major role in the spacing and widths of joints. This is so because movements are highly variable and depend on many parameters such as climatic factors during and after construction, the raw materials used in manufacture of masonry units, the manufacturing process, the degree of restraint imposed upon

the masonry, workmanship, and maintenance. No exact rules can be stated for the location of control joints. Each case must be studied individually to determine where joints can be placed without endangering structural integrity.

In the introductory chapter of this thesis, the potential for relative deformations at the interfaces between different structural elements of lowrise loadbearing concrete masonry wall buildings under different temperature differentials was described.

A review of the available literature showed that little work had been carried out to investigate the above potential and its influence on the generated thermal stresses in the aforementioned class of building. Therefore, the need to investigate this potential was established. This work was undertaken with the objectives to estimate the stresses in lowrise loadbearing concrete masonry wall buildings generated particularly by differential temperature, to identify and study the key parameters influencing these stresses on the structural and serviceability behaviour of the different structural elements, and to contribute to a rational method for calculating the control joint spacings.

A summary of the important phases of this research work and the important conclusions thereby drawn are given in the following sections.

7.2 Summary and Conclusions

The finite element method appeared to be the best available analytical tool to determine the thermal stresses. However, no suitable computer program was readily available to solve the problem of movement in buildings where interfaces and discontinuities are involved. A nonlinear finite element stress analysis program was therefore developed employing an iterative technique and using a special joint element to model the interfaces while the upper and lower continua were kept linearly elastic.

Using the above program, stress analyses of a lowrise loadbearing masonry building were carried out for differential thermal loads. Both summertime and

wintertime thermal loads were considered and combined with the gravity loads.

Also, in trying to evaluate the linear stress analysis as a method for predicting the thermal stresses in this class of building, a linear finite element analysis was carried out and the results were compared with those obtained using the developed finite element program.

The developed program was again used to examine the key parameters that may govern the generated thermal stresses in lowrise masonry buildings. The parameters considered were, masonry wall height/length ratio, temperature profile across the foundation, and mortar bedding area. Also, a comparison between the nonlinear finite element method and the Copeland method was carried out.

Based on the linear and nonlinear analyses of a lowrise loadbearing concrete masonry building, the following conclusions can be stated:

Case Study:

1. A rational method to determine control joint spacings and to realistically predict the thermal stresses that may develop in a class of lowrise masonry buildings has been established. This method takes into account the relative deformation that may occur at different interfaces of the modeled building.
2. In winter, the masonry wall is likely to be subjected to a tensile stress domain distributed along its height with maximum stress values at the wall mid-length where a vertical crack is likely to occur.
3. In summer, the concrete roof slab is likely to be subjected to a dominant tensile stress field which may lead to a local cracking distress at the free end corner in the vicinity of the wall-roof interface.
4. A linear elastic stress analysis using plane stress elements considerably overestimates the thermal stresses. Therefore, a linear stress analysis is not the appropriate method to handle this problem where sliding, debonding and re-contacting are expected to occur between different structural elements at their interfaces.

Parametric Study:

5. From the study of the effect of wall height/length ratio under modeled thermal conditions, it was found that a masonry wall with an aspect ratio less than 0.5 is likely to have a vertical crack at its mid-length due to horizontal tensile stresses. A masonry wall with an aspect ratio less than 0.2 is likely to have horizontal cracks in the vicinity of the interfaces with the roof slab and foundation. At the outset, these cracks are likely to be very narrow and close to the masonry wall free ends.

From the aforementioned results, control joints should be provided for a masonry wall at spacings such that the length of continuous wall does not exceed twice its height.

6. When studying the effect of a temperature profile across the foundation depth on the overall thermal stresses, care should be taken in assuming a profile because of the errors that may result in the values of the thermal stresses due to a coarse assumption for the profile.
7. From the study of the effect of the mortar bedding area on thermal stresses, it was found that the stresses increased considerably by increasing the shearing area (mortar bedding area) at interfaces. Therefore, applying the mortar solely to the faceshell bedding area seems to be a superior construction method for reducing the degree of restraint at the wall-foundation and wall-roof slab interfaces.

7.3 Recommendations for Future Work

The study has indicated the need for future research in the following areas:

1. Further experimental studies are needed to obtain a better theoretical model for the mortar joint stress-strain relationships under compressive and shear loads, especially at the plastic stage.
2. Also, experimental studies are needed to obtain the stress-strain relationships

under normal and shear loads for typically damp-proof-course (dpc) materials that may be applied to different interfaces. Using certain types of low-friction materials may provide more freedom for the structural elements to move relative to each other. This in turn will cause a significant reduction in the thermal stresses.

3. In this work, the thermal loads were applied at one time for a short duration. In other words, cycling of the thermal loads and the time-dependent properties of masonry, such as creep, were not considered. However, creep is not likely to cause any significant reduction in the thermal stresses, since it does not take more than a few weeks for the temperature differential to reach its extreme from the neutral level. Nevertheless, creep and cycling of the thermal load should be included in future research at least until it is established whether they have a significant effect on potential cracking.
4. Shrinkage and moisture-induced deformations may be coupled with the thermal loads for a more comprehensive study.
5. The orthotropic behaviour of concrete block masonry walls needs to be analytically modeled.
6. While the foundation of the masonry building is assumed to be of a rigid type, more analyses of masonry buildings on flexible foundations are needed for a more comprehensive study.

Appendix A

Thermal Stresses in Lowrise Masonry Wall Buildings with Different Aspect Ratios

Fig. A.1 Deformation of lowrise masonry wall building with wall aspect ratio = 0.2: Wintertime case, nonlinear stress analysis

Fig. A.2 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.2, principal stress 1

Fig. A.3 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.2, principal stress 2

**Fig. A.4 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.2, wintertime stresses**

Fig. A.5 Deformation of lowrise masonry wall building with wall aspect ratio = 0.3: Wintertime case, nonlinear stress analysis

Fig. A.6 Wintertime principal stresses for nonlinear stress analysis:

H/L ratio = 0.3, principal stress 1

Fig. A.7 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.3, principal stress 2

Fig. A.8 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.3, wintertime stresses

Fig. A.9 Deformation of lowrise masonry wall building with wall aspect ratio = 0.4: Wintertime case, nonlinear stress analysis

**Fig. A.10 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.4, principal stress 1**

Fig. A.11 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.4, principal stress 2

**Fig. A.12 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.4, wintertime stresses**

Fig. A.13 Deformation of lowrise masonry wall building with wall aspect ratio = 0.5: Wintertime case, nonlinear stress analysis

Fig. A.14 Wintertime principal stresses for nonlinear stress analysis:

H/L ratio = 0.5, principal stress 1

Fig. A.15 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.5, principal stress 2

**Fig. A.16 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.5, wintertime stresses**

Fig. A.17 Deformation of lowrise masonry wall building with wall aspect ratio = 0.6: Wintertime case, nonlinear stress analysis

Fig. A.18 Wintertime principal stresses for nonlinear stress analysis:

H/L ratio = 0.6, principal stress 1

Fig. A.19 Wintertime principal stresses for nonlinear stress analysis:

H/L ratio = 0.6, principal stress 2

Fig. A.20 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.6, wintertime stresses

Fig. A.21 Deformation of lowrise masonry wall building with wall aspect ratio = 0.7: Wintertime case, nonlinear stress analysis

Fig. A.22 Wintertime principal stresses for nonlinear stress analysis:

H/L ratio = 0.7, principal stress 1

Fig. A.23 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.7, principal stress 2

**Fig. A.24 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.7, wintertime stresses**

Deflection scale: 2,000 mm

Aspect ratio = 0.8

Fig. A.25 Deformation of lowrise masonry wall building with wall aspect ratio = 0.8: Wintertime case, nonlinear stress analysis

Fig. A.26 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.8, principal stress 1

Fig. A.27 Wintertime principal stresses for nonlinear stress analysis:
H/L ratio = 0.8, principal stress 2

**Fig. A.28 Shear stress distributions along different horizontal planes:
Aspect ratio = 0.8, wintertime stresses**

Appendix B

Finite Element Mesh For the Selected Model

3 of/de 3

Microl

Fig. B.1 Finite element mesh for the selected model

REFERENCES

- [1] **Ali, S., Page A. W., and Kleeman P. W.**, "Non-Linear Finite Element Model For Concrete Masonry with Particular Reference to Concentrated Loads", Proceedings of the 4th Canadian Masonry Symposium, Vol. 1, New Brunswick, 1986, pp. 137-149
- [2] **Allan, W. D. M.**, "Shrinkage Measurements of Concrete Masonry", Journal of the American Concrete Institute, Detroit, MI, Vol. 26, No. 6, 1930, pp. 669-713.
- [3] **American Concrete Institute**, "Control by Cracking in Concrete Structures", ACI 224 R-80, Detroit, MI, July 1985, p. 13.
- [4] **ACI Committee 531**, "Concrete Masonry Structures-Design and Construction", Journal of the American Concrete Institute, Proceedings Vol. 67, No. 5, May 1970, pp. 380-403, and No. 6, June 1970, pp. 442-460.
- [5] **American Society for Testing and Materials**, 1985 Annual Book of Standards, Philadelphia, Vol. 4.05, 1985.
- [6] **American Society of Heating, Refrigerating and Airconditioning Engineers**, "ASHRAE Handbook and Product Directory", 1977 Fundamentals, 1978, New York.
- [7] **Anderegg, F. O. and Anderegg, J. A.**, "Some Volume Changes in Mortar and Concrete", ASTM Bulletin, American Society for Testing and Materials, Philadelphia, December 1955, pp. 60-63.
- [8] **Anderegg, F. O.**, "Some Properties of Mortars in Masonry", Proceedings, American Society for Testing and Materials, Philadelphia, Vol. 40, 1940, p. 1134.
- [9] **Arya, S. K. and Hegemler, G. A.**, "Finite Element Method for Interface Problems", Journal of the Structural Division, ASCE, Proceedings of the American Society of Civil Engineers, Philadelphia, Vol. 108, 1982, pp. 327-342.
- [10] **Atkinson, R. H., Amadei, B., Sture, S., and Saeb, S.**, "Deformation and Failure Characteristics of Masonry Bed Joints Under Shear Loading", Proceedings of Fourth North American Masonry Conference, August, 1987, Paper No. 61.
- [11] **Beard, R., Dinnie, A., and Richards, R.**, "Movement of Brickwork: Causes and Effects", British Ceramic Society, Transactions, Vol. 68, 1969, pp. 74-90.
- [12] **Berwanger, C.**, "The Modulus of Concrete and the Coefficient of Expansion of Concrete and Reinforced Concrete at Below Normal Temperatures", Temperature and Concrete, ACI Publication SP-25, pp. 191-233.

- [13] **Bowes, W. H. and Russell, T.**, "Stress Analysis by The Finite Element Method for Practicing Engineers", Lexington Books, D. C. Heath and Company, 1975, pp. 88-90.
- [14] **BRE Digests 227, 228, and 229**, "Estimation of Thermal and Moisture Movements and Stresses", 1979.
- [15] **Brick Institute of America**, "Heat Transmission Coefficients of Brick Masonry Walls", Technical Note on Brick Construction, No. 4 Revised, Aug/Sept, 1974.
- [16] **British Standards Institution**. BS5628 : Part 3, "Materials and Components, Design and Workmanship", 1985.
- [17] **Canadian Standards Association**, "CSA Standards on Concrete and Masonry", A165.1-M1977, March, 1977, pp. 15-20.
- [18] **Canadian Standards Association**, "Code for the Design of Concrete Structures for Buildings", CAN3-A23.3-M77, 1977.
- [19] **Canadian Standards Association**, "Masonry Design for Buildings", CAN3-S304-M84, 1984.
- [20] **Commission of the European Communities**, "Draft Eurocode No. 6-Masonry Structures", 1984.
- [21] **Conner, C. C.**, "Some Effects of the Grading of Sand on Masonry Mortar", Proceedings of the American Society for Testing and Materials, ASTM, Philadelphia, Vol. 53, 1953, pp. 933-945.
- [22] **Construction Industry Research and Information Association**, "Movement and Cracking in Long Masonry Walls", CIRIA practice note, Special Publication 44, Westminster, London, 1986.
- [23] **Copeland, R. E.**, "Shrinkage and Temperature Stresses in Masonry", Journal of American Concrete Institute, ACI, Detroit, MI, Vol. 53, 1957, pp.769-780.
- [24] **Davis, R. E. and Troxell, G. E.**, "Volumetric Changes in Portland Cement Mortars and Concrete", Proceedings of the American Society of Civil Engineers, Philadelphia, Vol. 25, 1929, pp. 210-260.
- [25] **Desai, C. S.**, "Deep Foundations", Numerical Methods in Geotechnical Engineering, Ed. Desai, C. S., and Christian, S. J., 1977, pp. 235-271.
- [26] **Desai, C. S.**, "Static and Cyclic Response of Interfaces for Analysis and Design of Soil-Structure Interaction Problems", Geotechnical Modeling and Applications, Sayed M. S., Editor, Gulf Publishing Company, 1987, pp. 147-180.

- [27] **Desai, C. S., Drumm, E. C., and Zaman, M. M.**, "Cyclic Testing and Modeling of Interfaces", *Journal of the Geotechnical Engineering*. Vol. 111, No. 6, June, 1985, pp. 793-815.
- [28] **Dhlrendra, Buragohain, N., and Shah, V. L.**, "Curved Isoparametric Interface Surface Element", *Journal of Structural Division, Proceedings of the American Society of Civil Engineers*, Vol. 104, No. ST1, January, 1978, pp. 205-209.
- [29] **Drysdale, R. G, Hamid, A. A., and Heidebrecht, A. C.**, "Tensile Strength of Concrete Masonry", *Journal of the Structural Division, Proceedings of the American Society of Civil Engineers*, Vol. 105, No. ST7, July, 1979, pp. 1261-1276.
- [30] **Environment Canada, Atmospheric Environment Services**, Annual Meteorological Summary of Ottawa International Airport, 1975-1984.
- [31] **Environment Canada, Atmospheric Environment Services**, Principal Station Data, 1982.
- [32] **Environment Canada**, "Daily Soil Temperature Data", 1981.
- [33] **Evans, D. N.**, "Properties of Some Masonry Cements", *Journal of the National Bureau of Standards, Research Paper 2427*, Washington, D.C., Vol.51, No.1, July 1953, pp. 11-16.
- [34] **Faruque M. O.**, "The Role of Interface Elements in Finite Element Analysis of Geotechnical Engineering Problems", M.Eng. Thesis, Carleton University, Ottawa, Canada, 1980.
- [35] **Fenton, G. A.**, "Differential Movements and Stresses arising in Masonry Veneers of High-rise Structures", M. Eng. Thesis, Carleton University, Ottawa, Canada, 1984.
- [36] **Ghaboussi, J., Wilson, E. L., and Isenberg, J.**, "Finite Element for Rock Joints and Interface", *Journal of Soil Mechanics and Foundations Division, ASCE*, Vol. 99, No. SM10, Proc. Paper 10095, October, 1973, pp. 833-848.
- [37] **Goodman, R. E., and St. John, C.**, "Finite Element analysis for Discontinuous Rocks", *Numerical Methods in Geotechnical Engineering*, Ed. Desai, C. S., and Christian, S. J., 1977, pp. 148-175.
- [38] **Goodman, R. E., Taylor, R., and Brekke, T. L.**, "A Model for Mechanics Jointed Rock", *Journal of Soil Mechanics and Foundations Division, ASCE*, Vol. 94, No. SM3, Proc. Paper 5937, May, 1968, pp. 637-659.
- [39] **Goodman, R. E.**, "Methods of Geological Engineering in Discontinuous Rocks, West Publishing Company, 1976, pp. 301-368.

- [40] **Grimm, C. T.**, "Probabilistic Design of Expansion Joints in Brick Masonry", Proceedings of the 4th Canadian Masonry Symposium, University of New Brunswick, Fredericton, NB, June, 1986.
- [41] **Grimm, C. T.**, "Durability of Brick Masonry- A Review of Literature", Masonry: Research, Application, and Problems, ASTM STP 871, American Society for Testing and Materials, Philadelphia, PA, 1985, pp. 202-234.
- [42] **Hamid A. A. and Drysdale, R. G., and Heidebrecht, A. C.**, "Shear Strength of Concrete Masonry Joints", Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 105, No. ST7, July, 1979, pp. 1227-1240.
- [43] **Harris, H. A.**, Editor, "Masonry: Materials, Design, Construction, and Maintenance", ASTM, STP 992, 1988, pp. 257-276.
- [44] **Hedstrom, R. O., Litvin, A., and Hanson, J. A.**, "Influence of Mortar and Block Properties on Shrinkage Cracking of Masonry Walls", 1968, pp. 34-51.
- [45] **Herrmann, L. R.**, "Finite Element Analysis of Contact Problems", Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 104, No. EM5, October, 1978, pp. 1043-1057.
- [46] **Heuze, F. E., and Barbour, T. G.**, "New Models for Rock Joints and Interfaces", Journal of Geotechnical Engineering Division, Proceedings of the American Society of Civil Engineers ASCE, Vol. 108, No. GT5, May, 1982, pp. 757-776.
- [47] **Jumikis, A. R.**, "Thermal Geotechnics", Rutgers University Press, 1977, pp. 125-145.
- [48] **Kalousek, G. L., O'Heir, R. J., Ziems, K. L., and Saxer, E. L.**, "Relation of Shrinkage to Moisture Content in Concrete Blocks", ACI Journal, November, 1953, pp. 225-240.
- [49] **Katona, M. G.**, "A Simple Contact-Friction Interface Element with Application to Buried Culverts", International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 7, 1983, pp. 371-384.
- [50] **Lankard, D. R., Birkimer, D. L., Fondriest, F. and Snyder, M. J.**, "Effects of Moisture Content on the Structural Properties of the Portland Cement Concrete Exposed to Temperatures up to 500°F", Temperature and Concrete, ACI Publication SP-25, pp. 59-102.
- [51] **Lantz, Hakan**, "How to Avoid Cracking in External Walls of Light Weight Cellular Concrete", Byggnadisindustrin No. 14.67, Stockholm, 1967.
- [52] **Mahtab, M. A., and Goodman, R. E.**, "Three Dimensional Finite Element Analysis of Jointed Rock Slopes", Proceedings of the Second Cong. ISRM, 1970, Belgrade, Vol. 3, pp. 353-360.

- [53] **Masonry Council of Canada**, "Guide to Energy Efficiency in Masonry and Concrete Buildings", April 1982, pp. 114-117.
- [54] **Masonry Structural Design for Building**, TM 5-809-3/NAVFCA DM-2.9/AFM 88-3, Chap. 3, Department of the Army, Navy, and the Air Force, Washington, D.C., January, 1985.
- [55] **McBurney, J. W.**, "Cracking in Masonry caused by Expansion of Mortar", Proceedings of the American Society for Testing and Materials, ASTM, Philadelphia, Vol. 52, 1952, pp. 1-20.
- [56] **Mears, A. R. and Hobbs, D. W.**, "The Effect of Mix Proportions Upon the Ultimate Air-Drying Shrinkage of Mortars", Magazine Of Concrete Research, Slough, England, Vol. 24, No. 79, June 1972, pp. 77-84.
- [57] **Mohamed, M. A.**, "Analysis of Lowrise Masonry Walls Subjected to Thermal and Moisture Deformations", M.Eng. Thesis, Carleton University, Ottawa, Canada, 1984.
- [58] **National Bureau of Standards**, Building Science Series No. 45, Washington, D.C., July, 1973.
- [59] **Ngo, D., and Scordelis, A. C.**, "Finite Element Analysis of Reinforced Concrete Beams", Journal of American Concrete Institute, Vol. 64, No. 3, March, 1967, pp. 152-163.
- [60] **Page, A. W.**, "Finite Element Model For Masonry", Journal of Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 104, No. ST8, August, 1978, pp. 1267-1285.
- [61] **Pande, G. N., and Sharma K. G.**, "On Joint/Interface Elements and Associated Problems of Numerical Ill-Conditioning", International Journal of Numerical and Analytical Methods in Geomechanics, 1978, pp. 293-300.
- [62] **Pook, L. L., Stylianou, M. A., and Dawe, J. L.**, "Experimental Investigation of the Influence of Compression on the Shear Strength of Masonry Joints", Proceedings of the 4th Canadian Masonry Symposium, Vol. 2, New Brunswick, 1986, pp. 1056-1065.
- [63] **Portland Cement Association**, "Building Movement Joints", Skokie, IL, 1982, p. 30.
- [64] **Qinglin, W. and Wenzong, Y.**, "The Shear Strength of Masonry Wall Under Combined Stresses", Proceedings of the 4th Canadian Masonry Symposium, Vol. 2, New Brunswick, 1986, pp. 599-611.
- [65] **Rahman, A. H.**, "Thermal Stresses in Intersecting Masonry Walls - A Finite Element Study", Ph. D. Thesis, Carleton University, Ottawa, Canada, 1986, pp. 36-44.

- [66] **Rahman, A. H. and Suter, G. T.**, "Extreme Temperatures and Associated Stresses in Intersecting Masonry Walls, Proceedings of the 4th Canadian Masonry Symposium, Vol. 1, New Brunswick, 1986, pp. 137-149
- [67] **Ritchie, T.**, "Dimensional Changes of Masonry in Relation to Soundness of Bricks and Mortars", National Research Council of Canada, Ottawa, 1963.
- [68] **Ritchie, T.**, "Effect of Restraint on the Shrinkage of Masonry Mortars", Materials Research and Standards, American Society for Testing and Materials, Philadelphia, Vol. 6, No. 1, January, 1966, pp. 13-16.
- [69] **Ritchie, T.**, "Influence of Lime in Mortar on the Expansion of Brick Masonry", Reaction parameters of Lime, ASTM, STP 472, American Society for Testing and Materials, Philadelphia, 1969, pp. 67-81.
- [70] **Robinsky, E. I. and Bessflug, K. E.**, "Design of Insulated Foundations", Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers. Vol. 99, No. SM9, September, 1973, pp. 649-667.
- [71] **Russell, W. A.**, "Shrinkage Characteristics of Concrete Masonry Walls", Housing and Home Finance Agency, Housing Research Paper 34, Superintendent of Documents, U.S. Government Printing Office, April 1954.
- [72] **Ryan, N. M.**, "Movements in Masonry Walls - A State of Knowledge Review and Proposal for a Method of Prediction", The National Institute for Physical Planning and Construction Research, Publication No. Ct. 342, Ireland, 1988, pp. 11-23.
- [73] **Sahlin, S.**, "Structural Masonry", Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1971, pp. 212-217.
- [74] **Schäfer, H.**, "A Contribution to the Solution of Contact Problems with the Aid of Bond Elements", Computer Methods in Applied Mechanics and Engineering, Vol. 6, No. 3, Nov., 1975, pp. 335-354.
- [75] **Schellenbach, G. and Schimmelpfennig, K.**, "High Temperature Strains in Reinforced Concrete Structures", Technische Vereinigung der Grosskraftwerksbetreiber, VGB Kraftwerkstechnik, Vol. 53, No. 10, October, 1979, pp. 729-741.
- [76] **Selvadurai, A. P. S., and Voyiadjis, G. Z.**, Editors, "Mechanics of Material Interfaces", Proceedings of the Technical Sessions on Mechanics of Material Interfaces held in the ASCE/ASME Mechanics Conference, Albuquerque, New Mexico, 1985.
- [77] **Saemann, J. C., Warren, C. and Washa, G. W.**, "Effect of Curing on the Properties Affecting Shrinkage Cracking of Concrete Block", ACI Journal, May 1955, Proc. Vol. 51, pp. 833-852.
- [78] **Voss, W. C.**, "Lime Characteristics and Their Effect on Construction", Symposium on Lime, American Society for Testing and Materials, Philadelphia, 1939, pp. 14.

- [79] **Watstein, D., and Seese, N. A.**, "Properties of Masonry Mortars of Several Compositions", ASTM Bulletin, No. 147, American Society for Testing and Materials, Philadelphia, August, 1974, pp. 77-81.
- [80] **Weigler, H. and Fischer, R.**, "Influence of High Temperatures on Strength and Deformations of Concrete", Concrete for Nuclear Reactors, Vol. 1, ACI Special Publication SP-34, pp. 481-494.
- [81] **Wilson E. L.**, "Finite Elements for Foundations, Joints, and Fluids", In Finite Elements in Geomechanics, Ed. Gudehus, G., Chapt. 10, Wiley, Chichester, 1977, pp. 319-350.
- [82] **Xiurun, G.**, "Non-linear Analysis of a Joint Element and its Application in Rock Engineering", International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 5, 1981, pp. 229-245.
- [83] **Zienkiewicz, O. C., Best, B., Bullage, C., and Stagg, K. G.**, "Analysis of Nonlinear Problems in Rock Mechanics with Particular Reference to Jointed Rock System", Proceedings of the Second Congress, International Society for Rock Mechanics ISRM, Vol. 3, 1970, pp. 501-509.
- [84] **Zienkiewicz, O. C., Valliappan, S., and, King I. P.**, "Stress Analysis of Rock as a 'No Tension' Material", Geotechnique, Vol. 18, 1968, pp. 56-66.

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