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**SIMULATION OF
A NC-FSK MAXIMUM LIKELIHOOD RECEIVER
FOR FAST RAYLEIGH FADING**

by

Robin D. Addison, B. Eng.

**A thesis submitted to the Faculty
of Graduate Studies and
Research in partial fulfillment
of the requirements for the degree of**

Master of Engineering.

**Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University**

21 December 1989

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21 December 1989

ABSTRACT

Implementational tradeoffs of the Linear Predictive Receiver for noncoherent FSK are examined in light of computer simulation results. This receiver exploits fast fading environments for which it obtains better performance than is possible in slow fading environments, by using the inherent time diversity of the flat fast fading channel. The Linear Predictive Receiver can be used in low data rate mobile communications and can be implemented using an existing DSP chip. Simulation results for binary and M-ary NC-FSK are presented. Estimated channel parameter values are required by the Linear Predictive Receiver and its sensitivity to errors in these values is examined. Reduction of the computational complexity can be achieved by decreasing the length of the prediction filter. Performance is evaluated for a variety of predictor lengths and fading rates.

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SYMBOLS AND ABBREVIATIONS

Abbreviations

AWGN	Additive White Gaussian Noise
dB	Decibel
dBW	Decibels with reference to 1 Watt
DFT	Discrete Fourier Transform
DSP	Digital Signal Processor
FBW	Fading-bandwidth (in Hz)
FBWSR	Fading-bandwidth-to-symbol-rate ratio (unitless)
FSK	Frequency Shift Keying
GCR	General Gaussian Receiver
IMSL	Commercial FORTRAN routines for mathematical applications
LPEF	Linear Predictive Error Filter
LPR	Linear Predictive Receiver
LPR(P)	Linear Predictive Receiver of order P
M-ary	Multiple symbols (up to M symbols)
MAP	Maximum a posteriori
μ VAX	DEC microcomputer (executable code is compatible to VAX)
NC-FSK	Noncoherent Frequency Shift Keying
VAX	DEC computer
VMS	Operating system on VAX computers
WGN	White Gaussian Noise

X

Operators

$\mathcal{F}\{X\}$	Fourier Transform of X
$\mathcal{F}^{-1}\{X\}$	Inverse Fourier Transform of X
X^*	Conjugate of X
X^{-1}	Inverse of X
X^h	Conjugate transpose of X (Hermitian)
X^{-h}	Inverse of the conjugate transpose of X
X^T	Transpose of X

Greek Symbols

β_i	Residue of the i^{th} pole located strictly on the right half of the s -plane
Δf	Frequency step size for DFT (in Hz)
$\epsilon_{m,i}$	Error of the i^{th} order predictor for $\underline{Y}_m$
ϕ	Phase angle (in radians)
$\phi_0(s)$	Moment-generating function of the log-likelihood ratio
ϕ_m	Random starting phase of the m^{th} tone (in radians)
λ_i	i^{th} eigenvalue of $\mathbf{v}_1 \mathbf{v}_2^{-1}$
Λ'_m	Intermediate likelihood of m^{th} symbol being received
Λ_m	Likelihood of m^{th} symbol being received
σ	Standard deviation of noise (Variance = σ^2)
σ_1^2	Variance of the noise for the in-phase channel

σ_Q^2	Variance of the noise for the quadrature channel
σ_T^2	Total noise variance
τ	Time difference for continuous autocorrelation (in s)
ω_o	Transfer function scaling frequency (in radians/s)
ω_m	Frequency of the m^{th} tone (in radians/s)

Normal and Script Symbols

$a_j^{(i)}$	i^{th} prediction coefficient for the j^{th} order predictor
A	Forward predictor matrix ($N \times N$, lower triangular)
$\mathcal{A}$	Tone amplitude (in V)
A_k	Residue of the k^{th} pole of the transfer function
B	Filter bandwidth (in Hz)
d_i	The i^{th} root of the log-likelihood function
D	Error weighting matrix ($N \times N$, diagonal)
$\frac{E_b}{N_o}$	Signal-to-noise ratio per bit
E_i	Error in the i^{th} order predictor
$\underline{E}_m$	Error in the linear prediction of $\underline{Y}_m$
f	Frequency (in Hz)
f_0	Transfer function scaling frequency (in Hz)
f_m	Frequency of the m^{th} tone (in Hz)
$h(n)$	Discrete impulse response
$h_a(t)$	Analog impulse response
$H(z)$	Transfer function in the z -domain

$H_a(s)$	Analog transfer function
$H_{aF}(s)$	Analog colouring transfer function of the multiplicative noise (fading-process)
$H_{aFN}(s)$	Analog colouring transfer function for multiplicative plus additive noise (fading-process-plus-noise)
$H'_{aF}(s)$	Normalized analog colouring transfer function of the multiplicative noise (fading-process)
$H_F(z)$	Colouring transfer function of the multiplicative noise (fading-process) in the z domain
$H_{FN}(z)$	Colouring transfer function for multiplicative plus additive noise (fading-process-plus-noise) in the z domain
i	Integer index
I	In-phase channel
Im	Imaginary component of a complex number
k	Integer index
κ	Time difference for autocorrelation (in number of samples)
k_i	The i^{th} reflection coefficient
m	Symbol number (range 1 to M)
$\hat{m}$	Predicted symbol number (the decision made by the receiver)
M	Number of possible symbols (as in M -ary)
n	Sample number (range 1 to N)
n_I	Noise sample for the in-phase channel
N	Number of samples per symbol
N	Number of time samples for a DFT
N_0	Noise power spectral density of the AWGN (in dBW/Hz)

$p(Y X)$	Conditional probability of Y given X
P	Order of linear prediction (range 0 to N-1)
P_b	Probability of bit error
$q_{m,i}$	i th element of $\underline{Q}_m$
Q	Quadrature channel
$\underline{Q}_m$	Vector of N samples of m th inverse-transformed $\underline{Y}$
r_κ	Autocorrelation element (for time difference of κ samples)
$R(\kappa)$	Discrete autocorrelation function (for time difference of κ samples)
$R_s(\tau)$	Continuous autocorrelation function (for time difference of τ)
$R_{sm}(\tau)$	Continuous autocorrelation function of the m th tone (for time difference τ)
$\Re$	Real component of a complex number
$R_m(\kappa)$	Discrete autocorrelation function of the m th tone (for time difference of κ samples)
R_s	Symbol rate
s	Complex frequency
s_k	k th pole of the transfer function
t	Time (in s)
T	Sample period (in s)
$\mathcal{T}_m$	Linear transformation matrix to go from $\mathcal{V}$ to $\mathcal{V}_m$
T_s	Symbol period (in s)
$u(n)$	Discrete step function (steps at n samples)
$u_s(t)$	Continuous unit step (steps at time t)
v_{ik}	Element (row i, column k) of the covariance matrix

$v_{m,ik}$	Element (row i , column k) of the m^{th} covariance matrix
$\mathcal{V}$	Covariance matrix of the receive signal
V_i	Normalized expected squared error for the i^{th} order predictor
$\mathcal{V}_m$	Covariance matrix of the receive signal for the m^{th} symbol
$\mathcal{V}_m'$	Covariance matrix for the m^{th} symbol faded but without AWGN
$X_m(t)$	Transmitted signal for the m^{th} symbol
$\underline{X}_m$	Complex vector of N samples of the m^{th} symbol
y_i	i^{th} sample of the receive signal
$y_{m,i}$	i^{th} element of $\underline{Y}_m$
$Y(t)$	Received signal
$\underline{Y}$	Complex vector of N samples of the receive signal
$\underline{Y}_m$	Complex vector of N samples of the signal with the m^{th} symbol's frequency removed

CHAPTER 1

1.0 INTRODUCTION

1.1 The Problem

The terrestrial mobile radio channel is often modelled by a Rayleigh flat-fading channel[1]. Communication over a Rayleigh fading channel is difficult when the fading rate is high, especially when the fading rate exceeds the symbol rate. Such high fading rates cause problems because in addition to possible signal attenuation, the phase is altered in a rapidly changing fashion. Coherent demodulation schemes are not possible in a fast fading environment because carrier phase recovery is difficult if not impossible due to the rapid phase changes. Noncoherent methods, such as the noncoherent detection of frequency shift keying (FSK), can be used but do not provide good performance. We seek detection methods that provide improved performance in these conditions.

1.2 The Rayleigh Fading Channel

Two examples of environments that can be characterized by a Rayleigh fading channel are the mobile radio environment and the atmosphere following nuclear explosion. In mobile radio, the received signal is typically a linear superposition of many copies of the transmitted signal, where each copy is reflected from a different object. This phenomena is known as multipath propagation. Depending on the reflection and propagation time, the

different signals have differing amplitudes and phases. Differing rates of change of path length result in different Doppler frequency shifts on the various component signals. If the difference in propagation times, for any selected pair of paths is very small relative to the inverse of the signal bandwidth, the fading process is flat (i.e., not frequency selective). When the difference in propagation times is significant, more than a quarter of the symbol period, some frequencies may add constructively whereas other frequencies may add destructively for the same difference in propagation times.

Since many independently attenuated copies are summed, the Central Limit theorem [2] indicates that their distribution of the real (in-phase or I) component is a zero-mean Gaussian distribution. Similarly, the imaginary (quadrature or Q) component is an independent zero-mean Gaussian distribution. Given that there are many reflections, when the signals are added together the resultant signal is characterized by a Rayleigh distributed amplitude and a uniformly distributed phase (0 to 2π).

For Rayleigh fading, two terms, fading rate and fading bandwidth are used interchangeably to quantify the rate of change of the fading process' variables (i.e., the gain and phase shift induced by the channel). Fast fading rate means a large fading bandwidth and rapid changes of the fading process' variables. Slow fading rate means a small fading bandwidth and slow changes of the fading process' variables. The reciprocal of fading bandwidth is a measure of coherence time[3]. The symbol interval must be much less than the coherence time to enable the symbol to be received without loss of coherence. The coherence bandwidth is defined as the largest bandwidth for

which all frequencies experience approximately the same gain and phase shift at any given time. The reciprocal of coherence bandwidth is a measure of multipath spread of the channel[3]. When the bandwidth of the communication system is less than the coherence bandwidth of the fading, then the fading is said to be frequency non-selective or flat.

The mobile band at 900 MHz, using normal car speeds, is characterized by flat fading bandwidths of 10-150 Hz. For military satellite communications it is important to operate continuously, even after a nuclear explosion (it is assumed that both the satellite and ground station survive). The scintillation after such an event causes Rayleigh fading with fading bandwidths of 15 Hz - 50 kHz at a centre frequency of 20 GHz.

1.3 Thesis Objectives

The objective of the thesis is to investigate, via simulation, the Linear Predictive Receiver in fast flat Rayleigh fading. This includes:

- a. definition of an implementation, based upon the theory of linear prediction, for the detection of FSK signals transmitted over Rayleigh flat-fading channels. Both binary and M-ary signalling will be addressed;
- b. determination of the computational complexity of this implementation relative to a direct approach;
- c. characterization of the performance of the technique by determining the range of useful fading rates;
- d. examination of the sensitivity of the technique to errors in the estimated channel parameters; and

- e. determination of strategies to reduce the computational complexity of the technique - perform a study of complexity versus performance.

1.4 Thesis Outline

Chapter 2 introduces the theory for the detection of NC-FSK transmitted over a Rayleigh fading channel. Both conventional noncoherent detection and the maximum likelihood detection scheme are described. As an example of the latter class, a Gaussian detector as presented by Van Trees in [4] is introduced. This is developed into the General Gaussian Receiver and then into the Linear Predictive Receiver (LPR). These maximum likelihood detectors are compared for complexity versus performance followed by a summary of the analysis done by Mason in [5]. The chapter ends with an examination of the receiver in the two extremes of very slow and very fast fading.

The details of the simulation implementation are covered in Chapter 3. This coverage is subdivided into the transmitter, channel and receiver portions. The last section of Chapter 3 examines the error rate measurement method used for the results.

Chapter 4 provides the simulation results that characterize the conventional NC-FSK and the Linear Predictive Receiver. First, binary FSK results are presented for various signal-to-noise ratios and fading bandwidths. These results are compared to the analytical results and to those of conventional NC-FSK. M-ary FSK results are then presented. The sensitivity of the receiver to channel parameter estimation errors was investigated by simulation and the results are shown for errors in fading bandwidth and

signal-to-noise estimates. Simulation results for a reduced-complexity version of the LPR are also presented. Finally, two issues are examined: effects of different channel models on simulation results, and a comparison of the LPR performance in fast fading to performance of conventional diversity schemes in slow fading conditions.

In Chapter 5, summary and conclusions are provided along with suggestions for further work. Tables of all the values used for the graphs are given in Appendix A. Also included are the standard deviations for the simulated points and references for the analytical curves. An illustrative example of an LPR analytical probability of bit error calculation is given in Appendix B.

CHAPTER 2

2.0 THE NC-FSK RECEIVER IN FLAT RAYLEIGH FADING

2.1 The Model for the Modulated Signal and the Channel

In M-ary FSK, one of M equally likely tones is sent over a flat Rayleigh fading channel which is characterized by multiplicative coloured Gaussian noise (to model fading) followed by additive white Gaussian noise. This is shown in Fig. 2.1 where the following definitions apply:

$X_m(t)$ is the m^{th} symbol,

$Y(t)$ is the received signal,

$H_{af}(s)$ is the colouring transfer function of the multiplicative noise,

N_0 is the noise spectral density of the AWGN.

Modelling in discrete time using complex baseband, the m^{th} symbol is

$$\mathbf{X}_m = \left\{ \mathcal{A} e^{j(2\pi f_m(n-1)T + \phi_m)} ; 1 \leq n \leq N \right\}, \quad (2.1)$$

where

$\mathbf{X}_m$ is the complex vector of N samples of the m^{th} symbol,

$\mathcal{A}$ is the signal amplitude,

f_m is the frequency of the m^{th} tone,

n is the sample number,

T is the sample interval,

ϕ_m is the starting phase for the m^{th} tone and is a uniform random variable $[0, 2\pi)$.

Fig. 2.1 Flat Rayleigh fading channel model.

This tone is then passed through the channel. Fig. 2.2 shows the steps to demonstrate that a tone subjected to fading with a spectrum $|H_{af}(s)|^2$, and AWGN, is equivalent to a frequency translation of the fading-plus-noise spectrum $|H_{aFN}(s)|^2$. From Fig. 2.2a to Fig. 2.2b, the tone and multiplicative fading source positions are exchanged. Since AWGN is white, it is unaffected by a frequency translation. This allows the tone to be moved past the AWGN to produce Fig. 2.2c. In Fig. 2.2d, the AWGN of spectral density N_0 , which is equivalent to WGN of variance 1 followed by a gain of $\sqrt{N_0}$, is included by summing the spectra to produce the new colouring transfer function $H_{aFN}(s)$, where $|H_{aFN}(s)|^2 = |H_{af}(s)|^2 + N_0$. In fact, $H_{aFN}(s)$ is never derived because the spectrum of this new transfer function is all that is necessary for the receiver's computations.

The noise source, in Fig. 2.2d, produces independent Gaussian random variables. As they are independent and Gaussian, they are jointly Gaussian. They are then passed through a linear filter that is the new colouring transfer function. Since the filter is linear, the noise samples are still jointly Gaussian [2], though modified. The last operation is a frequency shift which can be accomplished equivalently by multiplying the samples by an invertible matrix that has exponentials on the main diagonal. This is clearly a linear operation, so the resultant signal samples (the receive signal samples) are jointly Gaussian. This fact is important for the Gaussian detector, GGR and LPR.

2.2 Conventional NC-FSK Detection

A conventional NC-FSK demodulator is derived from [3] and shown in Fig. 2.3a. The complex signal is multiplied by the conjugate of the each of the

Fig. 2.2 Equivalent model for FSK over a fading channel.

- FSK tone with multiplicative fade and additive white noise
- Coloured noise multiplied by FSK tone with AWGN
- Coloured noise with AWGN multiplied by FSK tone
- New coloured noise (includes AWGN) multiplied by FSK tone

a.

b.

Fig. 2.3 Conventional NC-FSK demodulator block diagram.

a. Analog form

b. Discrete time form

possible transmit symbols, followed by integrate-and-dump filters and a squarer. In discrete form, shown in Fig. 2.3b, the complex signal is a vector of samples of a sinusoid, the integrate-and-dump filter is replaced by a summer (of the samples of a symbol) and the squarer is unchanged. This demodulator is used as a basis of performance for comparison with the maximum likelihood detection schemes.

2.3 The Direct Approach to Maximum Likelihood Detection of FSK

A vector $\underline{Y}$ is a Gaussian random vector when its components $y_1, y_2, y_3, \dots, y_N$ are jointly Gaussian random variables. In this case, the components are the time samples of the received signal. If the samples passed through the channel are jointly Gaussian, then the received samples can be modelled by a multivariate Gaussian random process and the number of variables of the process is exactly the number of samples per symbol (N). As shown previously, the samples passed through a Rayleigh fading channel can be modelled by a complex multivariate Gaussian random process.

The conditional probability of receiving a vector $\underline{Y}$ of N complex samples from this multivariate Gaussian random process, given that the m^{th} symbol was sent, is

$$p(\underline{Y}|\underline{X}_m) = \frac{1}{(2\pi)^N |\nu_m|} e^{-\frac{1}{2} \underline{Y}^h \nu_m^{-1} \underline{Y}}, \quad (2.2)$$

where

$\underline{Y}^h$ is the complex conjugate transpose of $\underline{Y}$.

ν_m is the covariance matrix for samples of the m^{th} symbol passed through the channel.

A maximum likelihood receiver that calculates these probabilities can be built, but it will require computation of M different covariance matrices and their inverse. This type of receiver, designated as the Gaussian Detector, is shown in Fig. 2.4.

2.4 The General Gaussian Receiver (GGR)

Each of the received signal spectra associated with the covariance matrices $\mathcal{V}_m$ has the same shape. The only difference is that it is centered at a frequency determined by the transmitted symbol. If this offset were removed first, computations would only involve one covariance matrix. This is the basis of the GGR.

2.4.1 Conditional Probability Density Calculation for the GGR

Fig. 2.5 shows the GGR that first removes the assumed symbol's frequency offset by multiplying by the conjugate of the symbol, without the random phase term. (For symbols other than $\mathbf{X}_m$, the frequency offset is changed but is still non-zero.) This signal in the m^{th} branch can be expressed as

$$\mathbf{Y}_m = \mathbf{X}_m^* \mathbf{Y}. \quad (2.3a)$$

$$\mathbf{Y}_m = \left\{ e^{-j2\pi f_m(n-1)T} ; 1 \leq n \leq N \right\} \mathbf{Y}. \quad (2.3b)$$

Then the probability is calculated with

$$p(\mathbf{Y}_m | \mathbf{X}_m) = \frac{1}{(2\pi)^{N/2} |\mathcal{V}|} e^{-\frac{1}{2} \mathbf{Y}_m^h \mathcal{V}^{-1} \mathbf{Y}_m}, \quad (2.4)$$

Fig. 2.4 Gaussian Detector block diagram.

Fig. 2.5 General Gaussian Receiver block diagram.

where

$\mathbf{Y}_m$ is the received signal vector with the frequency of the m^{th} symbol removed,

$\mathcal{V}$ is the covariance matrix of the received signal with no modulation, in fact the covariance matrix of the channel.

It will be shown that $p(\mathbf{Y}_m|\mathbf{X}_m)$ is the same as $p(\mathbf{Y}|\mathbf{X}_m)$, and therefore the GGR is equivalent to the Gaussian Detector.

2.4.2 Conditional Probability Density Function of the GGR Compared to that of the Gaussian Detector

This section makes use of the channel model shown in Fig. 2.2d. Let $R_a(\tau)$ denote the autocorrelation function corresponding to the fading plus AWGN spectrum (the spectrum of H_{aFN}). If this spectrum is translated in the frequency domain by a frequency f , then, using the Fourier transform frequency shifting property [2], the resultant autocorrelation function would be $R_a(\tau)e^{-j2\pi f\tau}$.

If the m^{th} tone was sent, the autocorrelation function of the faded tone would be

$$R_{am}(\tau) = R_a(\tau) e^{-j2\pi f_m \tau}. \quad (2.5)$$

In the discrete time domain the autocorrelation function is

$$R_m(\kappa) = R(\kappa) e^{-j2\pi f_m \kappa T}, \quad (2.6)$$

where T is the sample period.

For N samples per symbol, the covariance matrix $\mathcal{V}$ associated with no tone ($f = 0$) is

$$\mathcal{V} = \begin{bmatrix} r_0 & r_1 & r_2 & \dots & r_{N-1} \\ r_1^* & r_0 & r_1 & \dots & r_{N-2} \\ r_2^* & r_1^* & r_0 & \dots & r_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{N-1}^* & r_{N-2}^* & r_{N-3}^* & \dots & r_0 \end{bmatrix}, \quad (2.7a)$$

$$\text{or } v_{ik} = \begin{cases} r_{i-k} & ; i \geq k \\ r_{k-i}^* & ; i < k \end{cases}. \quad (2.7b)$$

For the m^{th} tone, the covariance matrix elements are

$$v_{m,ik} = \begin{cases} r_{i-k} e^{-j2\pi f_m(i-k)T} & ; i \geq k \\ r_{k-i}^* e^{+j2\pi f_m(k-i)T} & ; i < k \end{cases}. \quad (2.8)$$

A linear transformation $\mathcal{T}_m$ is chosen such that

$$\mathcal{T}_m = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & e^{-j2\pi f_m T} & 0 & \dots & 0 \\ 0 & 0 & e^{-j2\pi f_m (2)T} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & e^{-j2\pi f_m (N-1)T} \end{bmatrix}, \quad (2.9)$$

then $\mathcal{V}'_m$ can be expressed as

$$\mathcal{V}'_m = \mathcal{T}_m^h \mathcal{V} \mathcal{T}_m, \quad (2.10)$$

and

$$\mathcal{V}'_m{}^{-1} = \mathcal{T}_m^{-1} \mathcal{V}^{-1} \mathcal{T}_m^{-h}. \quad (2.11)$$

Due to the symmetry, $\mathcal{T}_m^{-1} = \mathcal{T}_m^h$. Substituting for this gives

$$\nu_m^{-1} = \tau_m^h \nu^{-1} \tau_m. \quad (2.12)$$

Substitution of (2.12) in (2.2) yields

$$p(\underline{Y}|\underline{X}_m) = \frac{1}{(2\pi)^N |\tau_m^h \nu \tau_m|} e^{-\frac{1}{2} \underline{Y}^h \tau_m^h \nu^{-1} \tau_m \underline{Y}}. \quad (2.13a)$$

$$= \frac{1}{(2\pi)^N |\tau_m^h| |\nu| |\tau_m|} e^{-\frac{1}{2} \underline{Y}^h \tau_m^h \nu^{-1} \tau_m \underline{Y}}. \quad (2.13b)$$

Since $|\tau_m^{-1}| = \frac{1}{|\tau_m|}$ therefore

$$p(\underline{Y}|\underline{X}_m) = \frac{1}{(2\pi)^N |\nu|} e^{-\frac{1}{2} \underline{Y}^h \tau_m^h \nu^{-1} \tau_m \underline{Y}}. \quad (2.14)$$

If $\underline{Q}_m$ is defined as

$$\underline{Q}_m = \tau_m \underline{Y}, \quad (2.15)$$

then the probability becomes

$$p(\underline{Y}|\underline{X}_m) = \frac{1}{(2\pi)^N |\nu|} e^{-\frac{1}{2} \underline{Q}_m^h \nu^{-1} \underline{Q}_m}. \quad (2.16)$$

Since τ_m is a diagonal matrix, then τ_m^{-1} is also a diagonal matrix with elements which are the inverse of the τ_m elements. From (2.9) and (2.15) each element of $\underline{Q}_m$ can be written as

$$q_{m,i} = y_i e^{-j2\pi f_m(i-1)T}. \quad (2.17)$$

This is equivalent to multiplying the receive signal by the conjugate of the complex tone f_m . (2.17) is the same as the definition of $\underline{Y}_m$ given in (2.3b), thus $\underline{Q}_m$ and $\underline{Y}_m$ are identical. Substituting $\underline{Y}_m$ into (2.16) gives

$$p(\mathbf{Y}|\mathbf{X}_m) = \frac{1}{(2\pi)^{N/2} |\mathcal{V}|} e^{-\frac{1}{2} \mathbf{Y}_m^h \mathcal{V}^{-1} \mathbf{Y}_m}. \quad (2.18)$$

The right hand side of (2.18) is the same as the right hand side of (2.4),

so

$$p(\mathbf{Y}|\mathbf{X}_m) = p(\mathbf{Y}_m|\mathbf{X}_m). \quad (2.19)$$

Equation (2.19) shows that the GGR and the Gaussian Detector have the same probabilities. The GGR is easier to implement (in an adaptive form) as it only requires inverting one covariance matrix ($\mathcal{V}$) rather than the M required by the Gaussian Detector ($\mathcal{V}_1$ to $\mathcal{V}_M$) when the channel estimates are changed. Also, it will be shown that there are additional computational benefits inherent in the GGR.

2.4.3 Log Likelihood Functions of the GGR

A maximum a posteriori (MAP) probability receiver, or an optimum receiver [6], is one that chooses the estimated transmit symbol that has the highest probability of generating the received signal. The channel characteristics and the probabilities of the possible transmitter inputs are required to determine the MAP probabilities.

When the probabilities of possible transmitter inputs are not known, as is generally the case in a communication system, the MAP receiver cannot be built. Instead, the different transmit symbols are assumed to be equiprobable and the maximum likelihood receiver is used. This receiver chooses the estimated transmit symbol that maximizes the probability, or equivalently the likelihood, of generating the received signal when the transmit symbols are

equally likely. The maximum likelihood receiver is the optimum receiver when the transmit symbols are equiprobable.

Determination of the likelihoods for this receiver starts with the conditional probability given in (2.18). The factor preceding the exponential function is constant and positive for all values of m , so it does not affect the choice of a maximum probability. The likelihood can be expressed as

$$\Lambda'_m = e^{-\frac{1}{2} \mathbf{Y}_m^h \boldsymbol{\psi}^{-1} \mathbf{Y}_m}. \quad (2.20)$$

The exponential function increases monotonically as its exponent increases so can be omitted. Taking the natural logarithm and multiplying by 2 yields

$$\Lambda_m = -\mathbf{Y}_m^h \boldsymbol{\psi}^{-1} \mathbf{Y}_m. \quad (2.21)$$

The GGR then chooses the symbol m associated with the largest likelihood Λ_m . It is assumed that $\boldsymbol{\psi}$ is available from a channel estimator.

2.5 The Linear Predictive Receiver (LPR)

With a large number of samples/symbol (e.g., $N = 64$) the GGR has to compute the inverse of a large covariance matrix (64×64). Though the calculations are simplified because the matrix is Hermitian, this is time consuming for very large numbers of samples/symbol.

The LPR uses an alternate method for calculating the likelihood using linear prediction that saves on computation. Linear prediction and its application to the likelihood calculations are covered followed by an analysis of the computational benefits of the LPR.

2.5.1 Linear Prediction

Linear prediction is a method of predicting a signal based on a model of the signal where the output can be computed as a linear function of past inputs and outputs, and the present input. This is called the pole-zero model [7] because the coefficients of the inputs map to zeros of the transfer function in the z domain and the coefficients of the outputs cause poles. If the inputs of the signal are unknown, a linear function involving only the past outputs provides the all-pole model. In predicting the fading-plus-noise process, the inputs are unknown, so the all-pole model is appropriate. In this application, the linear prediction error filter will be used (the output of this filter is the error between the actual received signal and the predicted signal) and the associated error output, appropriately weighted, will be minimized to find the greatest likelihood.

The coefficients of the linear prediction error filter must be calculated. Given that the autocorrelation coefficients are known then the Levinson-Durbin recursive method of computing the prediction parameters (the linear coefficients) is computationally efficient. This algorithm, taken from [7], is shown below

$$E_0 = r_0, \quad (2.22)$$

$$V_0 = 1,$$

For $1 \leq i \leq p$,

$$k_i = - \frac{\left(r_i + \sum_{j=1}^{i-1} a_j^{(i-1)} r_{i-j} \right)}{E_{i-1}},$$

$$a_i^{(i)} = k_i,$$

$$a_j^{(i)} = a_j^{(i-1)} + k_i a_{i-j}^{(i-1)} \quad ; 1 \leq j \leq i-1,$$

$$E_i = (1 - k_i^2) E_{i-1},$$

$$V_i = \frac{E_i}{r_0},$$

where

E_i is the minimum mean square error for the predictor of order i ,

r_i is the autocorrelation of a stationary process with a time difference of i samples (for the LPR this is the autocorrelation of the fading process-plus-noise),

k_i is the i^{th} reflection coefficient (only used as an intermediate value in this application),

$a_j^{(i)}$ is the i^{th} coefficient of the j^{th} order linear predictor,

V_i is the normalized error for the i^{th} order linear predictor.

To obtain the discrete time autocorrelation coefficients r_i needed, the autocorrelation function of the fading plus noise process is first computed by taking the inverse Fourier Transform of the fading plus noise spectrum. This is expressed as

$$R_s(\tau) = \mathcal{F}^{-1} \{ |H_{sFN}(s)|^2 \}. \quad (2.23)$$

The discrete time autocorrelation coefficients are appropriately spaced samples of the continuous autocorrelation function.

2.5.2 Using Prediction to Calculate the Inverse Covariance Matrix

Using the autocorrelation coefficients, the prediction coefficients $a_j^{(i)}$ and the normalized error coefficients V_i can be used to calculate the inverse covariance matrix. From [8], the inverse of the covariance matrix is

$$\psi^{-1} = \frac{1}{r_0} A^h D A, \quad (2.24)$$

where

- A is the forward predictor matrix, defined below,
- D is the error weighting matrix, defined below,
- r_0 is the autocorrelation coefficient for a time difference of 0.

The forward predictor matrix can be expressed as

$$A = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ a_1^{(1)} & 1 & 0 & \dots & 0 \\ a_2^{(2)} & a_2^{(1)} & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{N-1}^{(N-1)} & a_{N-1}^{(N-2)} & a_{N-1}^{(N-3)} & \dots & 1 \end{bmatrix}. \quad (2.25)$$

The error weighting matrix is

$$D = \begin{bmatrix} \frac{1}{V_0} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{V_1} & 0 & \dots & 0 \\ 0 & 0 & \frac{1}{V_2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{V_{N-1}} \end{bmatrix}. \quad (2.26)$$

2.5.3 Likelihood Index Computation Using Weighted Prediction Errors

The formula (2.24) can be substituted into the likelihood equation (2.21), omitting the constant term, to get

$$\Lambda_m = -Y_m^h A^h D A Y_m. \quad (2.27)$$

Numerically, equation (2.27) is equivalent to using the forward linear prediction error filter to obtain the prediction errors and then squaring the errors, taking the magnitudes, and finally weighting these magnitudes by the inverse normalized error terms ($\frac{1}{V_i}$). The likelihood is the negative sum of all the weighted squared errors. The forward linear prediction error vector $\underline{e}_m$ is

$$\underline{e}_m = A Y_m. \quad (2.28)$$

By substituting this into (2.27), the likelihood is obtained

$$\Lambda_m = - \mathbf{\hat{\epsilon}}_m^h D \mathbf{\hat{\epsilon}}_m. \quad (2.29)$$

Though (2.29) is mathematically equivalent to (2.27), the calculations involved in (2.29) can be simplified by expanding the matrix operations and omitting products involving 0's. For a full length predictor the prediction errors are given by

$$\epsilon_{m,i} = \sum_{j=1}^i y_{m,j} a_{i-j}^{(i-1)} \quad ; \quad 1 \leq i \leq N, \quad (2.30)$$

where

$$a_0^{(i)} = 1,$$

$i-1$ is the predictor order.

Then the likelihood is

$$\Lambda_m = - \sum_{i=1}^N \frac{|\epsilon_{m,i}|^2}{V_{i-1}}, \quad (2.31)$$

2.5.4 Computational Advantage of the Weighted Prediction Errors Over Inverse Covariance Matrix

The likelihood calculation for the GGR is given in equation (2.21). This involves multiplying a vector (length N) by an $N \times N$ matrix followed by a multiplication by another vector (length N). By taking advantage of the conjugate symmetry and by skipping the diagonal terms (which are the same in all branches), the computations can be completed in $N^2 - N$ complex

multiplications and $\frac{1}{2}N^2 - \frac{1}{2}N - 1$ complex additions. For $N = 64$, this results in 4032 multiplications and 2015 additions per symbol.

For the LPR, the likelihood calculation is based on equations (2.30) and (2.31). To calculate each prediction error term $\epsilon_{m,i}$, taking into account that $a_0^{(i)} = 1$, requires $i - 1$ complex multiplications and $i - 1$ complex additions. Since there are N prediction error terms, this totals $\frac{1}{2}N^2 - \frac{1}{2}N$ complex multiplications and additions. The likelihood portion in equation (2.31) involves N magnitudes (equivalent to $\frac{1}{2}N$ complex multiplications), $N - 1$ complex multiplications (given $V_0 = 1$ and assuming the division by V_i is replaced by the product with its reciprocal) and $N-1$ additions. The total likelihood calculation for a $N = 64$ LPR amounts to 2111 complex multiplications and 2079 complex additions. This is comprised of 2016 complex multiplications and 2016 complex additions for the $\epsilon_{m,i}$ calculations plus 95 complex multiplications and 63 complex additions for the likelihood using the $\epsilon_{m,i}$.

For 64 samples/symbol, the LPR requires approximately one half of the multiplications and the same number of additions as required by the GGR. This remains true for larger N . At 16 samples/symbol, the LPR requires two-thirds of the multiplications, but 10% more additions than the GGR.

2.6 Suboptimal Order Linear Predictive Receiver (LPR(P))

Further computational savings can be achieved at the expense of optimality. During fast fading, the coherence time of the samples is short. A full order predictor is of little benefit since most of the higher order predictors are not significantly better than the lower order predictors. If the predictor order was constrained to some value P less than the optimum $(N-1)$

the calculations would be reduced without significantly affecting performance. In this case, the prediction errors are

$$\epsilon_{m,i} = \begin{cases} \sum_{j=1}^i y_{m,j} a_{i-j}^{(i-1)} & ; 1 \leq i \leq P \\ \sum_{j=1}^{P+1} y_{m,i-P-1+j} a_{P+1-j}^{(P)} & ; P+1 \leq i \leq N \end{cases} , \quad (2.32)$$

and the likelihood is

$$\Lambda_m = - \sum_{i=1}^P \frac{|\epsilon_{m,i}|^2}{V_{i-1}} - \frac{1}{V_P} \sum_{i=P+1}^N |\epsilon_{m,i}|^2 . \quad (2.33)$$

For the shortened predictor, the $\epsilon_{m,i}$ calculations need $\frac{1}{2}P(P-1) + P(N-P)$ complex multiplications and additions if the multiply by $a_0^{(i)} = 1$ is omitted. The likelihood equation requires $\frac{1}{2}N + P$ complex multiplications and $N-1$ complex additions. For the complete calculations, a predictor with order $P=7$ will require 459 complex multiplications and 483 complex additions. This is one tenth of the complex multiplications required by the GGR and one quarter the number of complex additions.

It can be seen that a lower order predictor can provide significant computational savings given that the signal fading is fast. It will be shown in Chapter 4 that a lower order predictor can provide almost equally good performance as well.

2.7 Analysis of Gaussian Detector for Binary NC-FSK

To confirm the simulations, analysis provided in [5] is available for the Gaussian Detector which has the same performance as the GGR and LPR. The analysis starts with the covariance matrices of the faded tones. If $\mathcal{V}_1$ and $\mathcal{V}_2$ are the covariance matrices of $\underline{X}_1$ and $\underline{X}_2$ passed through the channel, then it is shown in [5] that the moment-generating function of the log-likelihood ratio is

$$\phi_0(s) = \prod_{i=1}^N \frac{1}{(1-s(1-\lambda_i))} , \quad (2.34)$$

where

N is the number of samples/bit,

s is the complex frequency

λ_i is the i^{th} eigenvalue of $\mathcal{V}_1 \mathcal{V}_2^{-1}$.

It is also shown that the probability of error can be calculated from

$$P_b = - \sum_{i=1}^k \beta_i, \quad (2.35)$$

where

β_i is the residue of the i^{th} pole of $\frac{\phi_0(s)}{s}$
located strictly on the right hand half
of the s -plane,

k is the number of eligible poles.

Appendix B contains an example of the calculations for the model chosen in the simulations. The example illustrates the use of the above formulas.

2.8 The Likelihood Function at Fading Rate Extremes

This section will examine the likelihood values at very slow and very fast fading rates. The GGR likelihood function is mathematically equivalent to the LPR likelihood function, and will be used because the GGR likelihood calculations are easier to conceptualize. The likelihood equation, given by (2.21), depends on the covariance matrix and hence the autocorrelation of the fading process.

In very slow fading conditions, the normalized autocorrelation of the fading process approaches unity. If a small amount of white Gaussian noise is added to the fading process, the autocorrelation of N samples of a symbol passed through the channel are

$$r_k = \begin{cases} 1+\sigma^2 & ; k = 0 \\ 1 & ; 1 \leq k \leq N-1 \end{cases} \quad (2.36)$$

When these elements are put in the covariance matrix, the resultant matrix is almost constant. This matrix, when inverted, results in

$$q'^{-1} = \frac{1}{\sigma^2(N+\sigma^2)} \begin{bmatrix} N+\sigma^2-1 & -1 & -1 & \dots & -1 \\ -1 & N+\sigma^2-1 & -1 & \dots & -1 \\ -1 & -1 & N+\sigma^2-1 & \dots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \dots & N+\sigma^2-1 \end{bmatrix} \quad (2.37)$$

If the above value is put into the likelihood function (2.21) and the multiplications are carried out, the following equation results

$$\Lambda'_m = \frac{-N^2}{\sigma^2(N+\sigma^2)} \left[\frac{N+\sigma^2}{N^2} \left(\sum_{i=1}^N |y_{m,i}|^2 \right) - \left(\frac{1}{N} \sum_{i=1}^N y_{m,i} \right) \left(\frac{1}{N} \sum_{i=1}^N y_{m,i}^* \right) \right] \quad (2.38a)$$

If N is large compared to σ^2 , as is the case in the simulations, then the following likelihood results

$$\Lambda_m = \frac{-N^2}{\sigma^2(N+\sigma^2)} \left[\left(\frac{1}{N} \sum_{i=1}^N |y_{m,i}|^2 \right) - \left(\frac{1}{N} \sum_{i=1}^N y_{m,i} \right) \left(\frac{1}{N} \sum_{i=1}^N y_{m,i}^* \right) \right] \quad (2.38b)$$

The constants in front do not affect the likelihood and can be removed. The term in the square brackets is recognizable as the variance of the elements of $\underline{Y}_m$. The likelihood is, therefore, proportional to negative of the variance of the received samples (once they have the assumed modulation removed). The correct symbol will be a (noisy) constant and therefore have low variance. The incorrect symbol will still have a frequency offset (in addition to noise) resulting in a high variance. It is significant that the likelihood in very slow fading rate conditions increases with the square of the sum of the samples - this is what occurs in coherent combining. In NC-FSK, the samples of one symbol are combined coherently.

At the other extreme of fade rates is the very fast fading case. This case, when the fade changes a large number of times over one symbol period, does not usually occur in practical communications systems because of the high data rates used, but will be examined for comparative purposes. The very fast

case will be so fast that the samples are uncorrelated, resulting in an autocorrelation function of

$$r_k = \begin{cases} 1+\sigma^2 & ; k = 0 \\ 0 & ; 1 \leq k \leq N-1 \end{cases} \quad (2.39)$$

The associated covariance is proportional to an identity matrix, which when inverted will give another identity matrix. If this is put into equation (2.21) the following is obtained

$$\Lambda_m = \frac{1}{1+\sigma^2} \sum_{i=1}^N |y_{m,i}|^2 \quad (2.40)$$

This indicates that the likelihood is proportional to the sum of the powers in the individual samples. This is noncoherent combining or integration of the samples of one symbol. Since all symbols have equal power, they will all result in the same likelihood - they are indistinguishable in very fast fading conditions.

It has been shown that in very slow fading rate conditions, the GGR (and the LPR) approaches coherent combining of the samples to form the likelihood. But in very fast fading rate conditions, the GGR approaches noncoherent combining and in the limiting case, the tones are indistinguishable.

CHAPTER 3

3.0 SIMULATION IMPLEMENTATION

3.1 Description of the Simulation Implementation

In this section an overview of the simulation approach is presented followed by the parameters used in the simulation. The simulation approach covers the simulation method, simulation model and finally the computer hardware and software used.

3.1.1 Overview of the Simulation Approach

The simulation method used is the Monte Carlo method. This involves performing the simulation many times, using appropriately distributed random variables, and measuring the results. The results are considered to be samples of the performance to be expected from an actual system.

To enable all elements of the system to be implemented on a digital computer, a discrete-time representation was chosen. Thus, the signals are a sequence of samples of the equivalent analog signal equally spaced in time. Floating point numbers are used to allow a full range of values without scaling.

The transmitted signals for an actual communication system are modulated on a high frequency carrier. In the simulation, a complex baseband representation (i.e., the signal deviation from a 0 Hz carrier) is used.

This represents accurately the high frequency carrier provided that the sampling rate is sufficiently high [3].

The simulation completes all processing for one symbol before starting to simulate the next. A symbol is generated in the transmitter, subjected to the fading and additive noise processes by the channel simulation, and detected by the receiver with any bit or symbol errors tabulated before the next symbol is simulated.

The simulation is implemented primarily on a VAX 750 running VMS 4.7 using VAX FORTRAN-77 V4.8 language. Supporting subroutines for random number generation and numerical methods are taken from built-in FORTRAN language functions and the IMSL MATH, STAT and SFUN V1.0 combined libraries. The simulations were also run without modification on two VAX 780s, a VAX 8200 and a μ VAX II.

3.1.2 Parameters Used by the Simulation

For the simulations, several parameters had to be chosen. Two channel parameters are selected at run-time: noise spectral density and fading bandwidth. The communication parameters, such as the FSK tone spacing, are chosen to characterize the system. Other parameters, signal power and sample period, are chosen for simplicity. For example, to obtain different signal-to-noise ratios, the transmitted signal power was chosen to be 0 dBW and the noise level was varied. The signal-to-noise ratio is a good example of the last type of parameter, the derived parameter, which is derived from settings of other parameters (signal level and noise spectral density).

3.1.2.1 Input Channel Parameters

Noise Spectral Density(N_o) - The noise power spectral density in dBW/Hz is input at the beginning of each simulation run. This density is used to determine the variance of the Gaussian noise samples.

Fading Bandwidth(FBW) - The other channel parameter is the fading bandwidth, or fading rate, in Hz. Fading bandwidth corresponds to the bandwidth of the fading spectral shaping filter and is used to determine the fading-bandwidth-to-symbol-rate ratio (FBWSR).

3.1.2.2 Variable Simulation Parameters

Samples/Symbol(N) - The number of samples per symbol was chosen to ensure that the all the faded signal bandwidth can be properly represented. This meant that N must satisfy the Nyquist criterion in that one must sample at greater than twice the highest frequency component. For the simulation runs used here, this value was either 16 or 64. The simulation ran much faster with a smaller number of samples (16) but the simulation of high fading-bandwidth-to-symbol-rate ratios and M-ary FSK required selecting the higher number of samples/symbol (64) because of the larger bandwidths involved.

Maximum Errors per Run - This parameter was normally set to 100. This meant that 100 symbol errors occurred before the run was finished. Typically, runs with 100 error events provide results with a reasonable degree of statistical accuracy.

Maximum Symbols per Run - If insufficient error events occurred because the error rate was too low, this value stopped the simulation from running indefinitely. Normally, this parameter was set to 100000.

Number of Runs per Point - Typically, there were eight runs per point for each simulated result. This enabled a variance to be calculated and these are presented in Appendix A.

3.1.2.3 Communication Parameters

FSK Frequencies ($\omega_m = 2\pi f_m$) - The frequencies for the FSK symbols are chosen such that they are orthogonal over the symbol period and have the widest possible spacing for a chosen number of samples per symbol. Fading effectively smears the spectrum of the FSK tones, resulting in a loss of complete orthogonality as well as an increase in bandwidth. As the fading-rate-to-symbol-rate ratio approaches one half of the tone separation, the tone spectra start to overlap significantly causing errors. For the 64 samples/symbol at 1 symbol/s, the binary frequencies used are $f_m = \pm 16$ Hz. This gives 32 Hz separation allowing reasonable results for fading-rate-to-symbol-rate ratios of up to almost 16. For 16 samples/symbol the frequencies are $f_m = \pm 4$ Hz. M-ary cases were run at 64 samples/symbol and consisted of: 4-ary $f_m = \pm 4, \pm 12$ Hz and 8-ary $f_m = \pm 4, \pm 12, \pm 20, \pm 28$ Hz. This provides the largest symmetrical spacing available. (Note that the 4-ary frequencies of widest spacing are $\pm 8, \pm 24$ Hz, but for simplicity in M-ary simulations were chosen to be the same as the first 4 of the 8-ary case).

Receiver Bandwidth (B) - The receiver input bandwidth determines the amount of noise that reaches the detector. The bandwidth should be large enough to pass all of the signal. The highest frequency component will not exceed half the sampling rate, so the minimum single-sided bandwidth is half the sampling rate. This was the bandwidth used in the simulations. For 16

samples/symbol at 1 symbol/s, the receiver bandwidth is 8 Hz and for 64 samples/symbol the bandwidth is 32 Hz.

3.1.2.4 Fixed Parameters

Signal Amplitude (a) - The absolute signal amplitude turns out to be unimportant because the signal-to-noise ratio is determined by the relative levels of signal and noise. Yet a value is required to generate the FSK tones. For numerical simplicity the signal power level is chosen to be 0 dBW, giving a signal (sinusoidal) amplitude of 1 V into 1 Ω .

Symbol Period (T_s) - The symbol period selected is 1 s. All parameters (except for fading-bandwidth-to-symbol-rate ratio) are based on 1 symbol and thus do not depend on the period.

3.1.2.5 Derived Values

Average Signal-to-Noise Ratio ($\frac{E_b}{N_o}$) - The average signal-to-noise ratio is a derived parameter based on the signal power level (fixed at 0 dBW) and the noise spectral density (which is input at runtime). This is an average ratio because fading causes the gain of signal amplitude to vary. The signal-to-noise ratio is calculated as follows

$$\frac{E_b}{N_o} = 0 \text{ dBW} - N_o \text{ (dBW)}. \quad (3.1)$$

Fading-Bandwidth-to-Symbol-Rate Ratio (FBWSR) - Symbol duration (fixed at 1 symbol/s) and fading rate are used to calculate this parameter. It provides a measure of the fading rate with respect to the symbol rate. A high number indicates a fast fading environment and the associated high degree of

time diversity that can be exploited by the receiver. Fading-bandwidth-to-symbol-rate ratio is given by

$$\text{FBWSR} = \frac{\text{FBW}}{R_s}, \quad (3.2)$$

where: $R_s = \frac{1}{T_s}$ is the symbol rate.

3.2 The Transmitter Structure

The transmitter is shown in Fig. 3.1. A data source and modulator are the two components of the simulated transmitter. This does not include any error correction coding or any upconversions. The coding can be added between the data source and the modulation to improve the data error rate. Various codes have been well studied elsewhere and code performance for random errors at a given channel error rate is well known. That information can be applied to the results in this thesis to extrapolate coded data performance.

In order to simplify the simulation calculations, upconversion to the transmitted frequency, and associated downconversion in the receiver, are omitted. Because a complex baseband representation is used, this does not affect the results. All modulation is done using a centre frequency 0 Hz.

3.2.1 Random Data Source

FORTRAN's RND function is used to generate a uniformly distributed random number between 0 and 1. This is multiplied by M (of M-ary FSK) and then truncated to arrive at one of M symbols which is both forwarded to the modulator and stored for later comparison.

Fig. 3.1 Transmitter block diagram.

3.2.2 Modulation

One of the M possible FSK tones is generated based on the symbol produced by the data source. Since the complex baseband representation is used, both positive and negative frequencies are used.

The transmitted symbol vector is comprised of samples of the complex sinusoid

$$x = A e^{j(2\pi ft + \phi)}, \quad (3.3)$$

where

A is the signal amplitude ($A = 1$),

f is the tone frequency,

ϕ is the random starting phase.

It is assumed that the start phase of each symbol is uniformly distributed over the range 0 to 2π radians. The random starting phase of the tone ensures that the downconverted signal starts at different (and non-zero) phases.

3.3 Implementation of the Channel Model

The channel portion of the simulation first induces fading on the signal by multiplying a complex Gaussian signal with a predetermined spectral shape. After the signal has been subjected to fading, additive white Gaussian noise is introduced.

3.3.1 The Rayleigh Fading Process

A block diagram of the Rayleigh fading section is shown in Fig. 3.2. To simulate Rayleigh fading, white noise is passed through an appropriate spectral shaping filter to provide the fading process. This is done

Fig. 3.2 Channel simulation model.

independently for the I and Q components. The complex fade gain is then multiplied by the complex modulated signal to give the faded signal. Each symbol involves multiple samples that each require fading gains. A vector of samples of white noise is generated and then filtered to provide a vector (dimension N) of complex fade gains.

To produce the vector of white noise samples, the RNNOF function from IMSL is used. This function produces Gaussian distributed random samples of unit variance and zero-mean which are then passed through the spectral shaping filter.

This filter is based on the fading spectrum presented in [9]. The lowpass filter transfer function is

$$H_{af}(s) = \frac{1}{(0.897s^2 + 0.31s + 1)(1.543s^2 + 0.841s + 1)(1.944s + 1)} \cdot (3.4)$$

The corresponding spectrum is shown in Fig. 3.3. For the simulations a discrete time filter is needed and to convert the filter to a discrete time form, the impulse invariant method given in [10] was used. The impulse invariant method will be described and then applied to the filter.

The impulse invariant method forces the digital filter to have a discrete impulse response that consists of samples of the impulse response of the original analog filter. If the analog filter transfer function is represented by

$$H_s(s) = \sum_{k=1}^N \frac{A_k}{s-s_k} \cdot (3.5)$$

Fig. 3.3 Fade colouring filter power spectrum.

Using the inverse Laplace transform, the analog impulse response is

$$h_a(t) = \sum_{k=1}^N A_k e^{s_k t} u_a(t), \quad (3.6)$$

where $u_a(t)$ is the unit step at time t .

Sampling at intervals of T gives

$$h(n) = h_a(nT) = \sum_{k=1}^N A_k e^{s_k nT} u(n), \quad (3.7)$$

where $u(n)$ is the discrete unit step at sample n .

Which may be rearranged to yield

$$h(n) = \sum_{k=1}^N A_k (e^{s_k T})^n u(n). \quad (3.8)$$

The resulting digital filter is expressed by

$$H(z) = \sum_{k=1}^N \frac{A_k}{1 - e^{s_k T} z^{-1}}. \quad (3.9)$$

This digital filter has the sampled impulse response of the analog filter but has its gain proportional to $\frac{1}{T}$ [10]. This may produce extremely high gains at high sample rates. To eliminate this problem, the transfer function is scaled by T giving

$$H(z) = \sum_{k=1}^N \frac{A_k T}{1 - e^{s_k T} z^{-1}} \quad (3.10)$$

The impulse invariant method will now be used on the analog filter. The analog filter taken from [9], normalized for a fading-bandwidth of 1 Hz has been given in (3.4). Partial fraction expansion yields

$$H'_{aF}(s) = \frac{0.23+0.20j}{s+0.17-1.04j} + \frac{0.23-0.30j}{s+0.17+1.04j} - \frac{0.47+0.32j}{s+0.27+0.76j} - \frac{0.47-0.32j}{s+0.27-0.76j} + \frac{0.49}{s+0.51} \quad (3.11)$$

Transforming into the digital domain using (3.5) and (3.10) results in the transfer function

$$H_F(z) = \frac{0.0014+0.0012j}{1-(0.99-0.0064j)z^{-1}} + \frac{0.0014-0.0012j}{1-(0.99+0.0064j)z^{-1}} - \frac{0.0030+0.0020j}{1-(0.99-0.0047j)z^{-1}} - \frac{0.0030-0.0020j}{1-(0.99+0.0047j)z^{-1}} + \frac{0.0031}{1-0.97z^{-1}} \quad (3.12)$$

By regrouping the conjugate poles and scaling to normalize the output power, the final filter becomes

$$H_F(z) = \frac{0.146-0.152z^{-1}}{1-1.974z^{-1}+0.979z^{-2}} - \frac{0.303-0.306z^{-1}}{1-1.964z^{-1}+0.967z^{-2}} + \frac{0.157}{1-0.968z^{-1}} \quad (3.13)$$

The simulations of different fading bandwidths is accomplished by pre-scaling the analog filter by the fading bandwidth. The fading bandwidth, FBW, results in a scale factor

$$f_0 = \text{FBW}. \quad (3.14)$$

The scaled transfer function is

$$H_{aF}(s) = 2\pi f_0 H'_{aF}\left(\frac{s}{2\pi f_0}\right). \quad (3.15)$$

The resulting analog filter is then transformed using the impulse invariant method described above.

Once two vectors of Gaussian distributed samples are generated, the vectors are passed through identical independent discrete lowpass filters. The two output real vectors are then combined to make a complex vector of fading process samples. The faded signal sample vector is the product of the complex signal vector and the complex fading sample vector.

3.3.2 Additive White Gaussian Noise

In addition to the fading process, additive white Gaussian noise is simulated in the channel. Rather than simulate white noise with a power spectral density of N_0 and a receiver input filter, the two are combined by producing additive Gaussian noise samples of the variance observed on the output of a receive filter.

The total power in complex noise, with a power spectral density of N_0 , passed through a low-pass filter of single-sided bandwidth B is

$$\sigma_T^2 = N_0(2B). \quad (3.16)$$

The receive filter used is the anti-aliasing filter with a bandwidth of half of the sampling rate that allows all the information to pass through the

filter. It is important that the receive filter be wide enough to allow all the faded signal energy to get through. Putting this bandwidth into (3.16) yields

$$\sigma_T^2 = \frac{N_0}{T} . \quad (3.17)$$

The total noise variance is made up of components from both the I and Q channel which independently are

$$\sigma_I^2 = \sigma_Q^2 = \frac{1}{2} \sigma_T^2 . \quad (3.18)$$

Therefore, a noise sample for any one channel (I or Q) can be generated from a normally distributed Gaussian random variable of zero mean and unit variance, $N(0,1)$, by

$$n_I = \sigma_I N(0,1). \quad (3.19)$$

The standard deviation of the noise is

$$\sigma_I = \sqrt{\frac{N_0}{2T}} . \quad (3.20)$$

Here the Gaussian source used is the RNNOF function of IMSL. Two independent vectors of Gaussian samples are produced, one each for the I and Q channels. These are then added to the complex faded signal vector with the sum being the received signal vector.

3.4 The Receiver Structure

The receiver is shown in Fig. 3.4. The receiver is made up of M branches, one for each possible symbol. Each branch computes a likelihood based on the output of linear prediction error filters (LPEF) used in the receiver to compute prediction errors. The error for one symbol is obtained by squaring, weighting and summing each of the sample errors. The decision

Fig. 3.4 Linear Predictive Receiver block diagram.

is based on the path with the maximum likelihood associated with the minimum error.

3.4.1 Demodulation

Each of M separate branches (for M -ary FSK) corresponds to one of the M possible symbols. In each path, the received complex faded noisy symbol vector is multiplied by the complex conjugate vector of the associated symbol. This will produce a fading process plus noise spectrum centered either at 0 Hz (for the correct symbol) or at a frequency offset by the frequency difference (for incorrect symbols) which is then fed into the linear prediction error filter.

3.4.2 Linear Prediction Error Filter

The linear prediction error filter follows demodulation to obtain the errors for a sequence of linear predictions of the fading plus noise spectrum, using a given composite spectrum. For the correct symbol the actual composite spectrum of the process after demodulation should be similar to the given composite spectrum. Under ideal conditions, the given and actual spectra will be identical. The corresponding covariance matrix is

$$\mathcal{V} = \begin{bmatrix} r_0 + \sigma^2 & r_1 & r_2 & \dots & r_{N-1} \\ r_1 & r_0 + \sigma^2 & r_1 & \dots & r_{N-2} \\ r_2 & r_1 & r_0 + \sigma^2 & \dots & r_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{N-1} & r_{N-2} & r_{N-3} & \dots & r_0 + \sigma^2 \end{bmatrix} \quad (3.21)$$

The autocorrelation coefficients of the fading-plus-noise spectrum are required for the computation of the linear predictor's coefficients. The additive Gaussian noise only affects the diagonal terms with addition of noise variance to the fading process autocorrelation.

Two methods of obtaining discrete autocorrelation values from a continuous filter spectrum are possible. The first method, as given in (2.23), is to apply an inverse Fourier Transform to the filter spectrum equation and obtain the continuous autocorrelation function which would then be sampled to produce the autocorrelation coefficients. The second method is done entirely on computer and consists of applying an inverse discrete Fourier Transform (DFT) to samples of the spectrum. The results are the autocorrelation coefficients. The latter method was chosen because it is easier to implement on a computer.

The desired autocorrelation coefficients, since they are the correlation of the samples, must be separated by one sample interval T . For a DFT, the relationship is

$$T = \frac{1}{N\Delta f} \quad , \quad (3.22)$$

where

N is the number of points on the DFT and

Δf is the frequency separation of samples.

For the simulation, the important value to select is N such that the following requirements are met:

- i) $\mathcal{N}$ is a power of two.
- ii) $8\Delta f$ is smaller than the filter bandwidth (fading bandwidth).
- iii) $\mathcal{N}\Delta f$ is at least four times wider than the filter bandwidth.
- iv) $\mathcal{N}$ is minimized within the above constraints.

Requirements two and three ensured that sufficient information on the shape of the filter spectrum was retained after sampling. The second requirement forced at least 8 frequencies within the passband whereas 8 frequencies are needed in the stopband by the third requirement. The second and third requirements provided sufficient resolution but may have demanded more points than were necessary. This selection procedure is repeated each time the fading bandwidth is changed.

Once the autocorrelation coefficients are available, the Levinson-Durbin algorithm (2.22) is used to produce the linear prediction coefficients $a_j^{(i)}$. Also computed are the expected errors V_i for each length of predictor. The full order forward prediction error filter output is calculated using (2.28) and (2.25). The demodulated receive signal vector is premultiplied by the forward predictor matrix to generate the prediction errors $\epsilon_{m,i}$ for the predictor orders 0 to N-1 where N is the number of samples/symbol.

3.4.3 Computation of the Likelihood Index from the Prediction Errors

Equation (2.31) is now used to compute the likelihood index. The prediction errors $\epsilon_{m,i}$ from the LPEF are squared to obtain the energy in the errors. These are then weighted by the inverse of their expected error V_i . For a full order predictor, the sum of these weighted errors yields the likelihood

index. The path with the maximum likelihood index is chosen and the corresponding symbol is the maximum likelihood one.

3.5 Error Statistics Collection

Once the maximum likelihood symbol is obtained, it is compared to the original symbol generated by the transmitter. If they are different, a symbol error is logged. The individual bits of the original symbol and the maximum likelihood symbol are then compared to obtain the count of bit errors. This completes the simulation cycle for one symbol.

Many simulation cycles are required to obtain statistically valid data. Simulation for very low error rates, such as 10^{-6} , are excessively time consuming. Fewer error events, and thus results with higher variance, were collected for the points with very low error rates that occurred in the simulations of high FBWSR.

3.5.1 Data Recording for Error Rates Greater Than 10^{-4}

For error rates of 10^{-4} or higher, the simulation of one point consists of 8 runs of 100 symbol error events. One hundred is considered to be a reasonable number of events to obtain statistically significant data. Each run is done eight times (in sequence so that distinct random number seeds were used) to get eight readings. These are then averaged to obtain a mean error rate and a standard deviation is computed (the standard deviations are given in Appendix A). The standard deviation enables one to judge the validity of the data. This is done by providing a measure of the variation of error rates from measurement to measurement.

3.5.2 Data Recording for Error Rates Less Than 10^{-4}

Simulation of one symbol at 64 samples/symbol takes approximately 2 s on a VAX 750. This computer requires 5 years to collect 100 symbol errors for 8 runs at an error rate of 10^{-5} . This is impractical.

Rather than ignoring these low error rates completely, the simulations were run for 1,000,000 symbols which took one or two weeks each. Since there was only one run, no standard deviation can be computed. These simulations lead to results that have less statistical significance, but are still informative.

CHAPTER 4

4.0 SIMULATION RESULTS

4.1 Introduction to Results

Most of the simulations involved binary FSK. For this type of modulation, there are analytical solutions for some of the points allowing confirmation of simulation technique and results. The simulations, though slow, worked on all points and enabled examination of effects that are mathematically intractable. Simulations were run for both the conventional NC-FSK receiver and the LPR at various fading-bandwidth-to-symbol-rate ratios as well as various signal-to-noise ratios.

Binary FSK passes less data than M-ary FSK for a fixed fading-bandwidth-to-symbol-rate ratio, or equivalently the FBWSR for the M-ary case is greater than the FBWSR of the binary case for a given data rate. Two types of M-ary FSK systems, 4-ary and 8-ary, were simulated.

Once M-ary FSK is covered, the sensitivity of the LPR to various effects is investigated. To examine the decrease in performance of the receiver when it has poor channel estimates, binary simulations were done at one setting of FBWSR and $\frac{E_b}{N_o}$ while varying the channel estimates used by the receiver. The receiver estimates of interest are the signal-to-noise ratio and the fading-bandwidth-to-symbol-rate ratio for the channel model used. These estimates are used by the receiver to calculate the coefficients for the linear prediction error filter.

It is expected that the performance loss suffered by the LPR, due to the use of shorter length of predictors, will decrease as the fading rate increases. This is because the coherence time of the fading process shortens as the fading rate increases. Results were obtained by fixing FBWSR and $\frac{E_b}{N_0}$ and shortening the length of the prediction filter from the optimum. If the maximum length of the prediction filter can be significantly reduced, without dramatically degrading the receiver performance, the computational complexity of the receiver could be greatly reduced.

Issues that became noteworthy during simulation development will also be examined. These include the effects of alternative channel fading process models and comparison of the LPR to conventional diversity.

In the following sections, several graphs showing analytical and simulation results will be introduced. The values for these graphs and the standard deviations for the simulations results have been tabulated in Appendix A. An example of the analytical calculation is given in Appendix B.

4.2 Conventional Binary NC-FSK Performance Under Fading Conditions

The simulation results for the conventional binary NC-FSK receiver are shown in Fig. 4.1. These simulations were run with a symbol duration of 1 s, at 64 samples/symbol and with FSK frequencies of ± 16 Hz. As expected, the conventional receiver's performance is, at best, as good as the optimum receiver in slow fading conditions, and the performance degrades with increasing FBWSR. At high FBWSR, as shown in Fig. 4.2, the conventional binary NC-FSK receiver provides very poor performance. These results are the opposite of those for the LPR which will be shown to provide better

Fig. 4.1 Conventional binary NC-FSK simulation results
for various $\frac{E_b}{N_0}$.

Fig. 4.2 Conventional binary NC-FSK simulation results for various FBWSR.

performance, at high FBWSR, than the optimum receiver in slow fading conditions.

4.3 Binary NC-FSK Performance Under Fading Conditions for LPR

Binary NC-FSK is the only modulation scheme for which analytical results for any fading rate were made available recently [5] though the analysis was started in [4]. To have confidence in the simulation, it was necessary to establish a basis of comparison. Early in the simulation work, binary FSK results were obtained for the slow fading case and the results reported in [3] were confirmed.

4.3.1 LPR Binary NC-FSK at various $\frac{E_b}{N_0}$

Fig. 4.3 shows the simulation results for binary FSK for various $\frac{E_b}{N_0}$. Curves are shown for fading-bandwidth-to-symbol-rate ratios of $\frac{1}{8}$ to 2. Also shown on this graph is the theoretical curve for the optimum NC-FSK receiver in slow fading conditions. It can be seen that the LPR has better performance during fast fading conditions than the optimum NC-FSK receiver in a slow fading environment! In addition, as the FBWSR decreases, the performance approaches that of the optimum NC-FSK receiver in slow fading. Thus the LPR behaves as an optimum receiver in slow fading environments but its performance is better for a range of faster fading rates.

These LPR simulations were run with a symbol duration of 1 s, 16 samples/symbol and FSK frequencies of ± 4 Hz. The FSK frequencies are 8 Hz apart, so there is no spectral overlap as long as the FBWSR is less than 2.

Fig. 4.3 LPR binary NC-FSK simulation results
 for various $\frac{E_b}{N_0}$.

This frequency separation enabled the simulation to be run much faster with only 16 samples/symbol than for 64 samples/symbols.

The results of the simulation shown previously can be compared to the analytical results, based on [5], which are shown in Fig. 4.4. (For illustrative purposes, a probability of bit error calculation example is given in Appendix B.) The analytical results match the simulation curves and confirm the simulation algorithms.

4.3.2 LPR Binary NC-FSK at Various FBWSR

The simulation results as a function of FBWSR are shown in Fig. 4.5. Due to the length of time required to generate a point with a probability of bit error less than 10^{-4} , these simulations were stopped early and thus have higher variance. The curves show the dip at which performance improves at higher ratios because of diversity gain until the non-coherent integration loss exceeds the diversity gain beyond which performance degrades. One can see that at an average $\frac{E_b}{N_0} = 14$ dB comparatively good performance is obtained over a decade range of FBWSR. As the $\frac{E_b}{N_0}$ increases, the optimum FBWSR increases. The increase in performance at the optimum FBWSR, compared to slow fading, is modest at $\frac{E_b}{N_0} = 10$ dB and becomes quite significant at higher $\frac{E_b}{N_0}$. For $\frac{E_b}{N_0} = 18$ dB, the FBWSR can range from 2.5 to 15 and the error rate is still less than 10^{-4} . It is important in practical applications to have a wide range of FBWSR with good performance to allow a good range of vehicle speeds. Unlike some current communications systems for Rayleigh faded conditions that require an $\frac{E_b}{N_0}$ of 28-30dB, the LPR provides good performance at a relatively low $\frac{E_b}{N_0}$ of 18 dB.

Fig. 4.4 LPR binary NC-FSK analytical results
 for various $\frac{E_b}{N_0}$.

Fig. 4.5 LPR binary NC-FSK simulation results
for various FBWSR.

These results can be compared with the analytical ones shown in Fig. 4.6, and close agreement can be observed. Unfortunately the algorithm in [5] becomes ill-conditioned (in that the probability of bit error, calculated on a finite precision computer, was based on the difference of two almost identical numbers) at high FBWSR and the right hand sides of the curves in Fig. 4.6 are incomplete.

These simulations were run with a symbol duration of 1 s, 64 samples/symbol and FSK frequencies of ± 16 Hz. Note that the number of samples/symbol and the frequencies are greater than for the previous section. This permitted fading-bandwidth-to-symbol-rate ratios below 16 without degradation due to overlapping of the faded tone spectra.

4.3.3 Summary of Binary NC-FSK

Using the optimum receiver in slow fading conditions as a basis of comparison, the conventional FSK receiver performed slightly worse in slow fading and even poorer in fast fading conditions. In contrast, the LPR's performance in slow fading conditions matched the optimum receiver's and the LPR's performance improved significantly in fast fading conditions. The LPR provides the optimum tradeoff between noncoherent integration loss and diversity gain. This tradeoff will be investigated further in section 4.9.

An example of this improvement is given by bit error rate performance at an average $\frac{E_b}{N_0} = 18$ dB. For the LPR, the range of good performance, with probability of bit error less than 10^{-4} , is from FBWSR of 2.5 to 15. In comparison, the conventional NC-FSK receiver has a bit error rate poorer than

**Fig. 4.6 LPR binary NC-FSK analytical results
for various FBWSR.**

10^{-1} over the same range and the optimum receiver in a slow fading environment has a bit error rate of 1.5×10^{-2} .

4.4 M-ary NC-FSK Under Fading Conditions for LPR

For the binary NC-FSK results, it was found that there exists a range of FBWSR over which good performance is obtained. The fading rates encountered in a mobile radio environment systems are dependant on the carrier frequency and vehicle speed. The vehicle speed is determined by the traffic regulations and the centre frequency is based on the frequency allocation over which the user has little control. Thus a communication system has a characteristic range of fade rates. In current mobile radio systems, these fade rates are quite low (e.g., 10-150 Hz at 900 MHz) and if this is matched to the optimum FBWSR of the LPR for binary FSK the resultant data rate is very low (e.g., 10 b/s). To increase the data rate, a modulation scheme with many symbols (as in M-ary FSK) is needed.

For M-ary FSK simulations, the energy per bit is held constant regardless of M. This means that the energy per symbol increases with higher M. The graph in Fig. 4.7 shows the results of the 4-ary and 8-ary compared with binary FSK for various FBWSR with $\frac{E_b}{N_0} = 14$ dB. It can be seen that as M increases, the bit error performance also improves. Note that this improved performance comes at the expense of a greater signal-bandwidth-to-bit-rate ratio. The curves have the same general shape (over the common domain) as those of the binary case. The M-ary FSK curves do not extend past FBWSR = 2 because the large bandwidth required for 8-ary FSK would require an impractical number of samples/symbol for the simulations.

Fig. 4.7 LPR M-ary NC-FSK simulation results
for various FBWSR at $\frac{E_b}{N_0} = 14$ dB.

There are no analytical results available for the M-ary NC-FSK LPR in fast fading conditions. For slow fading conditions, analysis is done in [3] but only for the conventional FSK receiver. These slow fading results for various signal-to-noise ratios are plotted in Fig. 4.8. The curves exhibit the same slope regardless of M. Higher values of M perform slightly better in the slow fading environment. By comparing the slow fade area of Fig. 4.7 (FBWSR tending to 0) to Fig. 4.8 at the appropriate $\frac{E_b}{N_0}$, one can see that the M-ary LPR performs the same as the optimum receiver in slow fade and improves with higher FBWSR in the same fashion as did the LPR for binary FSK.

The M-ary FSK simulations were run with a symbol duration of 1 s, 64 samples/symbol and frequencies of ± 16 Hz for binary, ± 4 , ± 12 Hz for 4-ary and ± 4 , ± 12 , ± 20 , ± 28 Hz for 8-ary. The $\frac{E_b}{N_0}$ was fixed at 14 dB and the FBWSR ranged from $\frac{1}{8}$ to 2.

4.5 LPR Sensitivity to Receiver Parameter Estimate Errors

The LPR relies on knowledge of the fading plus noise spectrum, in particular the autocorrelation coefficients of this spectrum are required to calculate the linear prediction error filter coefficients. Given the channel model presented in 3.3, the autocorrelation coefficients can be calculated if the model is described by two parameters: average $\frac{E_b}{N_0}$ and FBWSR. Both of these must be measured/estimated for the LPR. In a practical system, there can be errors associated with these estimates during communication as well as transient errors at startup. A useful communication system must not fail catastrophically if there are minor errors and must be able to start up in a reasonable fashion.

Fig. 4.8 Conventional M-ary NC-FSK
in slow fading conditions.

Sensitivity test simulations were run with the LPR given erroneous parameters which were then used to calculate the linear predictor error filter coefficients. This modelled the situation where the LPR received erroneous estimates from some channel estimator "black box." The block diagram of Fig. 4.9 shows the sensitivity simulation setup. For these simulations, the channel parameters were fixed at $\frac{E_b}{N_0} = 14$ dB and FBWSR of 1. An FBWSR of 1 was chosen to allow maximum deviation on either side of the nominal allowing the erroneous estimates to range from 0.1 to 10. For the $\frac{E_b}{N_0}$ sensitivity simulations the symbol duration was 1 s, with 16 samples/symbol and the FSK tones were ± 4 Hz. Because of the large fading bandwidths involved in the FBWSR sensitivity simulations, they were run with 64 samples/symbol with FSK frequencies of ± 16 Hz.

4.5.1 LPR Sensitivity to Noise Estimate Errors

The dashed line in Fig. 4.10 shows the effects of receiver estimate errors in average $\frac{E_b}{N_0}$. From -10 dB to +10 dB noise errors, the performance is hardly affected. If the actual $\frac{E_b}{N_0}$ was 14 dB and the receiver assumed that it was 4 dB, the bit error rate would go from 8×10^{-3} to 1.1×10^{-2} . The LPR is insensitive to errors in the noise estimate over at least two decades.

4.5.2 LPR Sensitivity to Fading Bandwidth Estimate Errors

The solid line in Fig. 4.10 shows the effects of errors in the estimate of fading-bandwidth-to-symbol-rate ratio. To allow these results to be plotted with the $\frac{E_b}{N_0}$ sensitivity, the error of the FBWSR is expressed in dB from

Fig. 4.9 Block diagram of simulation to test LPR sensitivity to estimate errors.

Fig. 4.10 LPR binary NC-FSK simulation results with estimate errors for $\frac{E_b}{N_0} = 14$ dB and FBWSR = 1.

the nominal value of 1. So a FBWSR estimate error of 3 dB corresponds to the receiver estimate of $\text{FBWSR} = 2$ while the channel remains at $\text{FBWSR} = 1$. Similarly, a FBWSR estimate error of -10 dB corresponds to a receiver estimate of $\text{FBWSR} = 0.1$. It can be seen that the LPR is more sensitive to these errors than those of average $\frac{E_b}{N_0}$. The sensitivity curve is unimodal, with the performance slowly decreasing as the error increases. The performance decreased from an error rate of 9×10^{-3} with no estimate error to an error rate of 5×10^{-2} at an FBWSR estimate error of a factor of 10. This is a small degradation for such a large estimate error and indicates that the LPR is only mildly sensitive to FBWSR estimate errors.

4.5.3 Estimate Errors in Ongoing Communication for LPR

Since the fading-bandwidth-to-symbol-rate estimate error curve is unimodal, a simple optimizing estimator will operate well. The $\frac{E_b}{N_0}$ curve is so flat that an optimizing estimator may have trouble converging to the peak quickly without overshoot. Since the LPR is not sensitive to these errors, almost any simple estimation scheme will work.

4.5.4 Estimate Errors in Startup of LPR

In startup, the $\frac{E_b}{N_0}$ estimate is not critical because of the insensitivity of the LPR to this estimate. The FBWSR estimate, however, is more important and performance may be poor if it is in error by more than a factor of 10. As shown previously, the LPR typically provides good performance over a decade of FBWSR. Given that a system would be designed to operate over this decade of FBWSR, one would start the receiver with an estimate of FBWSR in the middle

of this decade and be no further than a factor of 3.16 (equivalently 5 dB) from the actual, well within the factor of 10. An alternate solution is to use a preamble to communications to circumvent any startup errors. Because the fade is fast, the preamble necessary to characterize the fading bandwidth will be very short

4.6 Sensitivity of the LPR to Predictor Length

During communications, the LPR spends most of its time calculating the linear predictor error filter outputs. These filtering operations consist of matrix and vector calculations where the dimensions are the number of samples per symbol. With 64 samples/symbol the LPR deals with operations of order 64^2 . If the predictor length was shortened to half or less, there would be substantial savings in processor time. One must be first satisfied that a shortened predictor will still provide good performance.

In a fast fading environment, the coherence time is less than the duration of a symbol. There is no requirement to use a full covariance matrix when many of the off diagonal elements are close to zero and it may be possible to approximate these terms with 0. From this observation, it was expected that if the prediction filter length were constrained to a suboptimal order the performance would degrade only slightly. Binary simulations were run at 64 samples/symbol with maximum predictor lengths of 63 (optimum), 31, 15, 7 and 3. The symbol duration was 1 s with $\frac{E_b}{N_0} = 14$ dB and FSK frequencies are ± 16 Hz.

Fig. 4.11. shows the simulation results as a function of fading-bandwidth-to-symbol-rate ratio. At fading-bandwidth-to-symbol-rate ratios

Fig. 4.11 LPR binary NC-FSK simulation results for shortened predictors at $\frac{E_b}{N_o} = 14$ dB.

less than 1, the optimum provided better performance than the shortened. At ratios greater than 1 there is no significant improvement of the optimum predictor over the shortened ones.

Because the reasoning behind a suboptimal order predictor is based on the fast fading case, the shortened predictor from 63 to 7 provided acceptable but poorer than optimal performance in slow fading conditions. In fast fading conditions, though, the shortened predictor provided almost optimal performance. These shortened predictors are virtually indistinguishable (within the variance of the results) except at very slow fade. The predictor length of 3 was marginally poorer in all FBWSR. The computation savings associated with the 7th order predictor (compared to the 63rd) are very attractive (the computational savings are detailed in section 2.6).

4.8 Sensitivity of the LPR to Differences in Channel Fading Process Model

All the results, analytical and simulation, are based on this channel model, so it is important to have confidence that the receiver's performance does not change dramatically if the model is changed or if there is an error in the assumed spectrum. The channel fading model chosen was the one presented in [9]. It is demonstrated in [9] that the model chosen is a reasonable model for a Rayleigh fading channel. It will be shown that the receiver performance is not significantly affected by using some other channel models.

In [5], Mason calculates the performance of a binary NC-FSK system, but uses a model different than in [9]. The Rayleigh fading model used in [5] is the Gaussian autocorrelation function for the fading process taken from [11]. This

differs from the 10-pole spectral model used in the simulations. Yet the analytical performance for the two is similar. (The simulations for the 10-pole spectral model also matched the analytical performance.) When one examines the power spectra of the two models at a given bandwidth, the shapes are distinct; the 10-pole model first increases and then drops off with increasing frequency whereas the Gaussian model has a Gaussian shape (decreasing monotonically with frequency). Both models are accepted models of a Rayleigh fading channel. The task of testing all possible models is beyond the scope of this work, but in the two examined here, the LPR was found to be insensitive to model shape.

In analyzing the sensitivity of the LPR to channel parameter estimate errors (section 4.5.2), further support to the insensitivity to the LPR to different channel models is obtained. Given an error in fading-bandwidth of a factor of 2 (3 dB), the performance of the receiver was not significantly affected. This means that if the receiver assumed that the fading-bandwidth was twice as large as the fading bandwidth of the actual channel, the LPR still performed well. This corresponds to a major alteration of the shape of the model. Even with the model bandwidth twice the size of the LPR's assumed fading bandwidth, the LPR's performance was not significantly affected.

From the above one can conclude that the LPR is insensitive to the channel model (within reason) and that the channel model chosen is a reasonable representation of a Rayleigh fading channel.

4.9 Comparison of LPR with NC-FSK Receiver Using Conventional Diversity

Proakis, in [3], presents an analysis of FSK in a slow fading conditions with diversity using a square-law combiner, which is referred to here as conventional diversity. The performance of binary NC-FSK as a function of the order of diversity is shown in Fig. 4.12. It shows that performance improves as diversity increases up to a certain point beyond which it begins to fall off. This is due to noncoherent combining loss that increases with the order of diversity.

Diversity, because it depends on independent fades for each bit, causes the energy to be combined noncoherently, thus the loss. Since the energy per data bit is fixed, at low orders of diversity this energy is split into a small number of diverse bits which are transmitted over independently fading channels. At low orders of diversity, there is only a small number of noncoherent (i.e., square law) combining of different diverse bits and most of the energy is combined coherently (over bit duration within each diverse bit). For example, at diversity 1, all the bit energy is combined coherently over time and there is no noncoherent combining loss; when this bit undergoes severe fade it is completely lost. Techniques with high orders of diversity use many lower energy diverse bits combined in a noncoherent fashion to obtain the result. This means that the probability of all diverse bits undergoing severe fade is much lower but because of the large number of diverse bits there is a significant noncoherent combining loss. Following from the previous example, the energy of the diversity 1 bit is split into many bits each of which is subject to independent fading. The chances of the majority of diverse bits being lost is much less, but the energy cannot be combined coherently. In the

Fig. 4.12 Conventional binary NC-FSK as a function of order of diversity.

extreme case, the energy per diverse bit is so low that it is indistinguishable from the noise giving poor performance. Therefore, there is an optimum diversity for each signal-to-noise ratio (as is shown in [3]).

The simulation result in Fig. 4.5 and the analytical results shown in Fig. 4.6 demonstrate that there is an optimum fading bandwidth for each signal-to-noise ratio. Fig. 4.12 shows the performance of NC-FSK in slow fading conditions using square law combining as a function of order of diversity. The curves would be very similar to those shown in Fig. 4.5 and Fig. 4.6 if the order of diversity was replaced with FBWSR. A rough rule of thumb is that the performance due to FBWSR is equivalent to twice the order of diversity (e.g., FBWSR = 5 is equivalent to $L = 10$). This is not exact, but is close as demonstrated by examining the optimum performance points of the two graphs. For $\frac{E_b}{N_0} = 10$ dB the optimum point is diversity $L = 4$ or FBWSR = 1, for 14 dB $L = 9$ or FBWSR = 4 and for 18 dB $L = 21$ or FBWSR = 8. This supports the rule of thumb $L = 2 \times \text{FBWSR}$.

Further support for the rule of thumb can be found in examining the degrees of freedom involved in a sampled fading process and in diversity. For simplicity, let the symbol rate be 1 symbol/s so the FBW = FBWSR. With a system with diversity of order L , there are L different fadings per symbol, each with independent fading on I and Q totaling $2 \times L$ degrees of freedom. Consider the ideal case where the fading process filter is replaced with a brick wall filter of the same fading bandwidth. If fading process is sampled at $2 \times \text{FBW}$ then the resultant spectrum is white. This implies that the samples are uncorrelated and since there are $2 \times \text{FBW}$ complex samples per symbol, there are $4 \times \text{FBW}$ degrees of freedom. In a poorly designed system where the

sample rate is decreased below this, one obtains fewer samples and necessarily fewer degrees of freedom. If the sample rate is increased beyond $2 \times \text{FBW}$, despite acquiring more samples, there is no further information in the samples (Nyquist sampling theorem shows that sampling at twice the bandwidth retains all the information) and thus can be no more degrees of freedom. In a properly designed system where the sample rate is at least $2 \times \text{FBW}$, after sampling the fading process, the LPR obtains $4 \times \text{FBW}$ degrees of freedom whereas a diversity system obtains $2 \times L$ degrees of freedom. If the both the diversity system and the LPR use all the information, then the two degrees of freedom must be equal which occurs only when the rule of thumb $L = 2 \times \text{FBWSR}$ is satisfied.

CHAPTER 5

5.0 SUMMARY AND CONCLUSIONS

5.1 Summary

The Linear Predictive Receiver for detection of NC-FSK signals transmitted over a Rayleigh flat-fading channel was presented and shown to be mathematically equivalent to the General Gaussian Receiver derived from [4]. The LPR uses linear prediction to reduce the complexity of the likelihood calculations for the received symbol decision. At 64 samples per symbol, the likelihood computations for the LPR require approximately one half of the operations needed for the GGR. For 16 samples/symbol, the LPR needs less than two-thirds the operations of the GGR.

For the binary case, equations were presented for the theoretical analysis given by [5]. The analysis shows that the LPR approaches coherent combining of the samples under very slow fading while in a fast fading environment approaches noncoherent combining. In the limiting case of the fast fading, infinitely fast, the tones are indistinguishable.

The binary NC-FSK simulation results demonstrate that the LPR, in fast flat Rayleigh fading conditions, provides much better performance than the optimum receiver in slow fading conditions. The conventional NC-FSK receiver has very poor performance in fast fading conditions. Both receivers converge to the optimum receiver at slow fading conditions, but the LPR is better at higher fading rates. The LPR results showed improved performance

with increasing fading-bandwidth-to-symbol-rate ratio but beyond a certain FBWSR, the performance degrades. This feature results in optimum performance at a certain FBWSR which is greater at higher average $\frac{E_b}{N_0}$. Given a threshold for acceptable performance, one can determine a range of FBWSR where the LPR can operate. At $\frac{E_b}{N_0} = 18$ dB, the error rate was below 10^{-4} for FBWSR between 2.5 and 15.

The LPR results for 4-ary and 8-ary FSK show improved performance compared with binary NC-FSK. The improvement is similar to the one seen when conventional binary FSK is compared to M-ary FSK during slow fading conditions. M-ary FSK allows higher information rates to be sent over a channel with given fade rate ranges.

Sensitivity of the LPR to estimation errors in channel parameter were examined. Given the 10-pole fading process spectrum model, the channel can be characterized by fading-bandwidth and average $\frac{E_b}{N_0}$. By simulations, at one nominal setting of these two parameters in the channel and different values assumed in the receiver, the LPR was found to be relatively insensitive to errors in the estimate of the noise level. If the noise estimates were off by an order of magnitude, the error rate changes from the nominal of 8×10^{-3} to 1.1×10^{-2} . The LPR was found to be only mildly sensitive to errors in the estimate of FBWSR. Here, an error of a factor of 6 produced an error probability of 3×10^{-2} compared to the nominal 9×10^{-3} .

An LPR with shortened maximum order predictors was simulated. For 64 samples/symbol at FBWSR = 1 and $\frac{E_b}{N_0} = 14$ dB, the 63rd (optimal), 31st, 15th and 7th order predictors had almost identical performance during fast fading conditions and at very slow fading conditions the 63rd order predictor is better

with an error rate of 3.5×10^{-2} while the 7th had an error rate of 6.5×10^{-2} . The 3rd order predictor performed poorer than the others in all cases. The 7th order predictor requires one quarter of the computations needed for a 63rd order predictor, whereas the 3rd order predictor needs only one eighth demonstrating that the savings involved for shortened predictors are substantial.

The LPR was found to be insensitive to the selection of fade models by obtaining the same analytical results using two different Rayleigh fading channel models and remains so even when the FBWSR is changed by a factor of 4.

The LPR exploits the inherent time diversity in a signal subjected to fast fading. The simulation results showed that the performance of the LPR at a given FBWSR was similar to the performance of a conventional FSK receiver in a slow fading environment with an order of diversity equal to twice the FBWSR of the LPR. The effect of the fast fading bandwidth is to produce an order of time diversity approximately equal to twice the FBWSR.

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5.2 Conclusions

A binary NC-FSK system is practical using the LPR because the range of good performance of the LPR can be matched to the range of vehicular speeds. The LPR, at $\frac{E_b}{N_0} = 18$ dB, performed well with fading-bandwidth-to-symbol-rate ratios of 2.5 to 15. The 900MHz mobile band, at normal car speeds, has fading bandwidths from 10-150Hz. This suggests a bit rate of 10b/s which is slow and useful only for slowly changing data such as telemetry or vehicular dispatch systems. At higher centre frequencies such as with the new allocated bands in the 4 GHz area, the LPR becomes more useful. In this eventuality, the fading bandwidths increase and the bit rate can go to 50b/s. The bandwidth required for binary FSK will be four times the maximum FBW to ensure the faded tones do not overlap. In this case the bandwidth required is 3 kHz. The LPR becomes more useful with higher fading bandwidths, which occur with higher centre frequencies or faster vehicle speeds.

An M-ary NC-FSK system with an LPR will have yet a higher data rate. The useful bit rate for 16-ary FSK LPR in the 900MHz band is 40b/s. In the 4 GHz band the data rate is 200b/s. Larger values of M allow higher data rates.

The scintillation effects following a nuclear explosion are characterized by fast Rayleigh fading. Military satellite communications are possible during scintillation using the LPR. Here, because of the high centre frequencies and the fast fading, the data rate will be significantly higher - in the kbits/s range. This provides a survivable communication link which does not require large signal power margins for the scintillation fading.

The LPR is insensitive to noise estimate errors which simplifies the design and implementation of the noise estimator. It also means that the LPR

is robust in changing noise level conditions. The LPR, though, is slightly more sensitive to fading-bandwidth estimate errors. In either parameter, the dropoff due to a poor estimate is gradual and non-catastrophic allowing easy acquisition and tracking of the channel parameters. A short preamble can be used to speed up the initial channel estimate.

The reduced order predictor did not noticeably degrade performance but allows significant computational savings. For 64 samples/symbol, the predictor of length 7 will probably be the order of choice unless performance in slow fading conditions was critical.

The combination of an M-ary NC-FSK system using a shortened LPR provides an effective method for passing low rate data over a Rayleigh flat fast-fading channel. By exploiting the inherent time diversity of the channel, the receiver performs better in fast fading conditions than in slow fading conditions. This is a distinct improvement over non-diversity receiver systems which typically fare worse at fast fading rates than in slow. The LPR, which operates well at a $\frac{E_b}{N_0} = 18$ dB does not require the large signal power margins needed in some existing Rayleigh fading communications systems which need an $\frac{E_b}{N_0}$ of around 28 dB. Furthermore, the LPR implementation is simpler and cheaper than is the case for receivers using spatial or frequency diversity. A straight forward, low cost, implementation on a single DSP chip is possible.

5.3 Areas for Further Work

The simulation algorithms were developed with the DSP implementation in mind. They can be transferred to a floating-point chip such as a TMS320C30 without major modification. Once the algorithms are modified for fixed point operation, they also can be easily implemented on a fixed-point chip such as the TMS32010. The actual system can then be tested at much lower error rates than were simulated (due to the time required for simulation at low error rates).

The other important requirement for an actual LPR is the channel parameter estimator. This estimator will estimate the signal-to-noise ratio and the fading-bandwidth-to-symbol-rate ratio (which are then used to derive autocorrelation coefficients) or directly estimate the autocorrelation coefficients of the channel. As discussed previously, this design of the estimator is simplified by the insensitivity of the LPR to noise estimate errors and the mild sensitivity to fading-bandwidth estimate errors.

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APPENDIX A - TABLES OF VALUES

This appendix lists the values used for the graphs presented in the Chapters 3 and 4. The tables are given in order of the corresponding figure number. The key settings for the simulation or analysis (such as FSK frequencies or sample per symbol) precede each table and where the graphs are based on analysis, a reference to the appropriate section in this paper or open literature is given. For the simulation data, the standard deviation is listed along with the measured value.

With the exception of the shortened predictor data, all simulations involving the LPR use the optimum length predictor. For 64 samples/symbol this is a 63rd order predictor and a 15th order predictor is used for 16 samples/symbol.

3.3 Fade colouring filter power spectrum (partial listing)

Settings: $f_0 = 1$ Hz

Reference: (3.4) and (3.15)

Frequency [Hz]	Power [dB]
0.000	0.00
0.105	0.01
0.210	0.12
0.300	0.35
0.405	0.90
0.510	1.81
0.600	2.78
0.705	3.82
0.810	4.25
0.900	4.04
1.005	2.81
1.110	-0.90
1.200	-5.58
1.305	-11.02
1.410	-15.90
1.500	-19.62
1.605	-23.52
1.710	-27.04
1.800	-29.80
1.905	-32.78
2.010	-35.53
2.100	-37.75
2.205	-40.17
2.310	-42.46
2.400	-44.31
2.505	-46.38
2.610	-48.33
2.700	-49.94
2.805	-51.73
2.910	-53.45

4.1 Conventional Binary NC-FSK simulation results for various $\frac{E_b}{N_0}$

Settings: $N = 64$, $f = \pm 16$ Hz, $T_s = 1$ s

$\frac{E_b}{N_0}$	FBWSR = 0.125		FBWSR = 0.25	
	P_b	σ	P_b	σ
10.0	8.57×10^{-2}	5.69×10^{-3}	9.00×10^{-2}	8.76×10^{-3}
12.0	5.98×10^{-2}	5.91×10^{-3}	6.27×10^{-2}	5.44×10^{-3}
14.0	3.82×10^{-2}	2.34×10^{-3}	4.09×10^{-2}	3.22×10^{-3}
16.0	2.57×10^{-2}	1.44×10^{-3}	2.70×10^{-2}	2.83×10^{-3}
18.0	1.62×10^{-2}	1.61×10^{-3}	1.72×10^{-2}	9.58×10^{-4}
20.0	1.08×10^{-2}	1.43×10^{-3}	1.16×10^{-2}	1.36×10^{-3}
22.0	6.40×10^{-3}	3.96×10^{-4}	7.16×10^{-3}	8.24×10^{-4}

$\frac{E_b}{N_0}$	FBWSR = 0.5		FBWSR = 1.0	
	P_b	σ	P_b	σ
10.0	1.13×10^{-1}	8.82×10^{-3}	2.07×10^{-1}	2.51×10^{-2}
12.0	8.02×10^{-2}	5.90×10^{-3}	1.57×10^{-1}	1.00×10^{-2}
14.0	5.36×10^{-2}	3.47×10^{-3}	1.07×10^{-1}	8.13×10^{-3}
16.0	3.59×10^{-2}	3.49×10^{-3}	7.59×10^{-2}	5.83×10^{-3}
18.0	2.21×10^{-2}	1.42×10^{-3}	5.22×10^{-2}	6.97×10^{-3}
20.0	1.48×10^{-2}	7.17×10^{-4}	3.35×10^{-2}	4.05×10^{-3}
22.0	1.03×10^{-2}	8.99×10^{-4}	2.21×10^{-2}	1.53×10^{-3}

$\frac{E_b}{N_0}$	FBWSR = 2.0	
	P_b	σ
10.0	3.09×10^{-1}	5.18×10^{-2}
12.0	2.50×10^{-1}	2.32×10^{-2}
14.0	1.98×10^{-1}	1.66×10^{-2}
16.0	1.46×10^{-1}	1.72×10^{-2}
18.0	9.39×10^{-2}	9.15×10^{-3}
20.0	6.46×10^{-2}	8.63×10^{-3}
22.0	4.22×10^{-2}	5.25×10^{-3}

4.2 Conventional Binary NC-FSK simulation results for various FBWSR

Settings: $N = 64$, $f = \pm 16$ Hz, $T_s = 1$ s

FBWSR	$\frac{E_b}{N_0} = 10$ dB		$\frac{E_b}{N_0} = 14$ dB	
	P_b	σ	P_b	σ
0.125	8.57×10^{-2}	5.69×10^{-3}	3.82×10^{-2}	2.34×10^{-3}
0.25	9.00×10^{-2}	8.76×10^{-3}	4.09×10^{-2}	3.22×10^{-3}
0.5	1.13×10^{-1}	8.82×10^{-3}	5.36×10^{-2}	3.47×10^{-3}
1.0	2.07×10^{-1}	2.51×10^{-2}	1.07×10^{-1}	8.13×10^{-3}
2.0	3.09×10^{-1}	5.18×10^{-2}	1.98×10^{-1}	1.66×10^{-2}
3.0	3.40×10^{-1}	1.38×10^{-2}	2.33×10^{-1}	1.64×10^{-2}
4.0	3.88×10^{-1}	3.09×10^{-2}	2.90×10^{-1}	2.86×10^{-2}
5.0	4.08×10^{-1}	3.28×10^{-2}	2.93×10^{-1}	3.10×10^{-2}
6.0	4.05×10^{-1}	2.27×10^{-2}	3.33×10^{-1}	2.10×10^{-2}
7.0	4.54×10^{-1}	2.87×10^{-2}	3.38×10^{-1}	2.16×10^{-2}
8.0	4.25×10^{-1}	3.38×10^{-2}	3.66×10^{-1}	3.20×10^{-2}
10.0	4.11×10^{-1}	4.22×10^{-2}	3.69×10^{-1}	3.06×10^{-2}
12.0	4.67×10^{-1}	2.13×10^{-2}	4.02×10^{-1}	4.16×10^{-2}
14.0	4.68×10^{-1}	3.35×10^{-2}	4.01×10^{-1}	3.02×10^{-2}
15.0	4.74×10^{-1}	3.09×10^{-2}	4.20×10^{-1}	3.93×10^{-2}

FBWSR	$\frac{E_b}{N_0} = 18$ dB	
	P_b	σ
0.125	1.62×10^{-2}	1.61×10^{-3}
0.25	1.72×10^{-2}	9.58×10^{-4}
0.5	2.21×10^{-2}	1.42×10^{-3}
1.0	5.22×10^{-2}	6.97×10^{-3}
2.0	9.39×10^{-2}	9.15×10^{-3}
3.0	1.34×10^{-1}	9.62×10^{-3}
4.0	1.59×10^{-1}	1.14×10^{-2}
5.0	1.85×10^{-1}	1.83×10^{-2}
6.0	2.31×10^{-1}	1.40×10^{-2}
7.0	2.40×10^{-1}	2.47×10^{-2}
8.0	2.52×10^{-1}	1.64×10^{-2}
10.0	2.80×10^{-1}	1.18×10^{-2}
12.0	2.92×10^{-1}	1.81×10^{-2}
14.0	3.07×10^{-1}	1.40×10^{-2}
15.0	3.22×10^{-1}	1.99×10^{-2}

4.3 LPR Binary NC-FSK simulation results for various $\frac{E_b}{N_0}$

Settings: $N = 16$, $f = \pm 4$ Hz, $T_s = 1$ s

$\frac{E_b}{N_0}$	FBWSR = 0.125		FBWSR = 0.25	
	P_b	σ	P_b	σ
10.0	8.61×10^{-2}	1.45×10^{-2}	8.01×10^{-2}	1.29×10^{-2}
12.0	5.37×10^{-2}	1.09×10^{-2}	4.39×10^{-2}	5.35×10^{-3}
14.0	3.55×10^{-2}	2.57×10^{-3}	2.56×10^{-2}	1.60×10^{-3}
16.0	2.04×10^{-2}	2.70×10^{-3}	1.25×10^{-2}	1.42×10^{-3}
18.0	1.21×10^{-2}	1.55×10^{-3}	5.89×10^{-3}	6.17×10^{-4}
20.0	6.21×10^{-3}	6.17×10^{-4}	2.71×10^{-3}	2.67×10^{-4}
22.0	3.05×10^{-3}	2.84×10^{-4}	1.10×10^{-3}	9.99×10^{-5}

$\frac{E_b}{N_0}$	FBWSR = 0.5		FBWSR = 1.0	
	P_b	σ	P_b	σ
10.0	5.72×10^{-2}	5.54×10^{-3}	5.26×10^{-2}	5.11×10^{-3}
12.0	3.24×10^{-2}	4.36×10^{-3}	2.39×10^{-2}	1.88×10^{-3}
14.0	1.50×10^{-2}	4.37×10^{-4}	8.18×10^{-3}	7.91×10^{-4}
16.0	7.11×10^{-3}	1.41×10^{-3}	3.14×10^{-3}	1.50×10^{-4}
18.0	2.82×10^{-3}	3.03×10^{-4}	8.14×10^{-4}	6.74×10^{-5}
20.0	1.01×10^{-3}	1.25×10^{-4}	2.14×10^{-4}	1.51×10^{-5}
22.0	3.55×10^{-4}	2.59×10^{-5}	4.65×10^{-5}	5.53×10^{-6}

$\frac{E_b}{N_0}$	FBWSR = 2.0	
	P_b	σ
10.0	5.41×10^{-2}	3.95×10^{-3}
12.0	2.08×10^{-2}	8.84×10^{-4}
14.0	5.71×10^{-3}	8.77×10^{-4}
16.0	1.21×10^{-3}	6.29×10^{-5}
18.0	1.87×10^{-4}	1.31×10^{-5}
20.0	2.23×10^{-5}	4.56×10^{-6}
22.0	1.87×10^{-6}	1.55×10^{-6}

4.4 LPR Binary NC-FSK analytical results for various $\frac{E_b}{N_0}$

Settings: $N = 16$, $f = \pm 4$ Hz, $T_s = 1$ s

Reference: section 2.7 (Appendix B contains an example)

$\frac{E_b}{N_0}$	FBWSR				
	0.125	0.25	0.5	1.0	2.0
10.0	8.319×10^{-2}	7.578×10^{-2}	6.097×10^{-2}	5.349×10^{-2}	5.459×10^{-2}
11.0	6.773×10^{-2}	5.904×10^{-2}	4.467×10^{-2}	3.625×10^{-2}	3.423×10^{-2}
12.0	5.454×10^{-2}	4.510×10^{-2}	3.194×10^{-2}	2.359×10^{-2}	2.001×10^{-2}
13.0	4.343×10^{-2}	3.376×10^{-2}	2.232×10^{-2}	1.474×10^{-2}	1.089×10^{-2}
14.0	3.417×10^{-2}	2.478×10^{-2}	1.525×10^{-2}	8.866×10^{-3}	5.514×10^{-3}
15.0	2.655×10^{-2}	1.785×10^{-2}	1.020×10^{-2}	5.135×10^{-3}	2.602×10^{-3}
16.0	2.034×10^{-2}	1.263×10^{-2}	6.691×10^{-3}	2.870×10^{-3}	1.147×10^{-3}
17.0	1.535×10^{-2}	8.789×10^{-3}	4.297×10^{-3}	1.550×10^{-3}	4.744×10^{-4}
18.0	1.140×10^{-2}	6.023×10^{-3}	2.703×10^{-3}	8.106×10^{-4}	1.847×10^{-4}
19.0	8.338×10^{-3}	4.072×10^{-3}	1.666×10^{-3}	4.109×10^{-4}	6.806×10^{-5}
20.0	5.996×10^{-3}	2.720×10^{-3}	1.006×10^{-3}	2.022×10^{-4}	2.383×10^{-5}
21.0	4.242×10^{-3}	1.797×10^{-3}	5.953×10^{-4}	9.683×10^{-5}	7.972×10^{-6}
22.0	2.956×10^{-3}	1.176×10^{-3}	3.453×10^{-4}	4.517×10^{-5}	2.556×10^{-6}

4.5 LPR Binary NC-FSK simulation results for various FBWSR

Settings: $N = 64$, $f = \pm 16$ Hz, $T_s = 1$ s

FBWSR	$\frac{E_b}{N_0} = 10$ dB		$\frac{E_b}{N_0} = 14$ dB	
	P_b	σ	P_b	σ
0.125	8.74×10^{-2}	7.04×10^{-3}	3.35×10^{-2}	2.87×10^{-3}
0.25	7.46×10^{-2}	7.43×10^{-3}	2.52×10^{-2}	2.68×10^{-3}
0.5	6.25×10^{-2}	8.49×10^{-3}	1.53×10^{-2}	1.76×10^{-3}
1.0	5.44×10^{-2}	5.13×10^{-3}	8.67×10^{-3}	7.10×10^{-4}
2.0	5.21×10^{-2}	4.65×10^{-3}	5.44×10^{-3}	7.86×10^{-4}
3.0	5.58×10^{-2}	3.92×10^{-3}	4.26×10^{-3}	1.95×10^{-4}
4.0	6.79×10^{-2}	8.11×10^{-3}	4.05×10^{-3}	3.47×10^{-4}
5.0	7.05×10^{-2}	5.68×10^{-3}	4.44×10^{-3}	2.52×10^{-4}
6.0	8.24×10^{-2}	4.63×10^{-3}	4.96×10^{-3}	6.67×10^{-4}
7.0	9.79×10^{-2}	9.18×10^{-3}	6.05×10^{-3}	6.85×10^{-4}
8.0	9.82×10^{-2}	9.38×10^{-3}	6.59×10^{-3}	8.38×10^{-4}
9.0	1.15×10^{-1}	1.03×10^{-2}	6.96×10^{-3}	6.67×10^{-4}
10.0	1.15×10^{-1}	6.95×10^{-3}	7.32×10^{-3}	6.79×10^{-4}
11.0	1.20×10^{-1}	9.12×10^{-3}	8.43×10^{-3}	4.88×10^{-4}
12.0	1.28×10^{-1}	9.36×10^{-3}	9.88×10^{-3}	6.75×10^{-4}
13.0	1.35×10^{-1}	1.23×10^{-2}	1.12×10^{-2}	1.04×10^{-3}
14.0	1.47×10^{-1}	1.87×10^{-2}	1.54×10^{-2}	1.49×10^{-3}
15.0	1.61×10^{-1}	1.46×10^{-2}	2.23×10^{-2}	1.82×10^{-3}

FBWSR	$\frac{E_b}{N_o} = 18 \text{ dB}$		
	P_b	σ	Errors in 10^6
0.125	1.10×10^{-2}	1.10×10^{-3}	
0.25	6.03×10^{-3}	5.99×10^{-4}	
0.5	2.79×10^{-3}	2.47×10^{-4}	
1.0	7.84×10^{-4}	7.30×10^{-5}	
2.0	1.52×10^{-4}	1.90×10^{-5}	
3.0	6.20×10^{-5}		62
4.0	2.40×10^{-5}		24
5.0	2.10×10^{-5}		21
6.0	2.20×10^{-5}		22
7.0	1.00×10^{-5}		10
8.0	1.10×10^{-5}		11
9.0	1.10×10^{-5}		11
10.0	1.70×10^{-5}		17
11.0	7.00×10^{-5}		7
12.0	9.00×10^{-6}		9
13.0	1.50×10^{-5}		15
14.0	3.60×10^{-5}		36
15.0	1.18×10^{-4}		118

When the error rate was too low, the simulations were only run for 10^6 points. Since there is only one run, no standard deviation could be computed.

4.6 LPR Binary NC-FSK analytical results for various FBWSR

Settings: $N = 64$, $f = \pm 16$ Hz, $T_s = 1$ s

Reference: section 2.7 (Appendix B contains an example)

FBWSR	$\frac{E_b}{N_0}$		
	10 dB	14 dB	18 dB
0.125	8.29×10^{-2}	3.39×10^{-2}	1.12×10^{-2}
0.25	7.53×10^{-2}	2.45×10^{-2}	5.91×10^{-3}
0.5	6.04×10^{-2}	1.49×10^{-2}	2.59×10^{-3}
1.0	5.26×10^{-2}	8.49×10^{-3}	7.38×10^{-4}
2.0	5.29×10^{-2}	5.02×10^{-3}	1.45×10^{-4}
3.0	5.88×10^{-2}	4.26×10^{-3}	5.53×10^{-5}
4.0	6.62×10^{-2}	4.19×10^{-3}	2.98×10^{-5}
5.0	7.42×10^{-2}	4.44×10^{-3}	2.02×10^{-5}
6.0	8.23×10^{-2}	4.87×10^{-3}	1.59×10^{-5}
7.0	9.02×10^{-2}	5.43×10^{-3}	1.38×10^{-5}
8.0	9.80×10^{-2}	6.09×10^{-3}	1.25×10^{-5}
9.0	1.05×10^{-1}	6.85×10^{-3}	1.32×10^{-5}
10.0	1.12×10^{-1}	7.70×10^{-3}	
11.0	1.19×10^{-1}	8.64×10^{-3}	
12.0	1.26×10^{-1}	9.51×10^{-3}	
13.0	1.34×10^{-1}		
14.0	1.46×10^{-1}		
15.0	1.67×10^{-1}		

Some high FBWSR results are not available because the algorithm became numerically ill-conditioned.

4.7 LPR M-ary NC-FSK simulation results for various FBWSR

Settings: $N = 64$, $T_s = 1$ s, $\frac{E_b}{N_0} = 14$ dB

FBWSR	Binary ($f = \pm 16$ Hz)		4-ary ($f = \pm 4, 12$ Hz)	
	P_b	σ	P_b	σ
0.125	3.55×10^{-2}	2.57×10^{-3}	2.01×10^{-2}	1.50×10^{-3}
0.25	2.56×10^{-2}	1.60×10^{-3}	1.07×10^{-2}	1.07×10^{-3}
0.5	1.50×10^{-2}	4.37×10^{-4}	5.99×10^{-3}	7.02×10^{-4}
1.0	8.18×10^{-3}	7.91×10^{-4}	2.24×10^{-3}	2.13×10^{-4}
2.0	5.71×10^{-3}	8.77×10^{-4}	7.38×10^{-4}	6.43×10^{-5}

FBWSR	8-ary ($f = \pm 4, 12, 20, 28$ Hz)	
	P_b	σ
0.125	1.41×10^{-2}	1.06×10^{-3}
0.25	8.82×10^{-3}	1.31×10^{-3}
0.5	3.37×10^{-3}	1.97×10^{-4}
1.0	1.02×10^{-3}	1.64×10^{-4}
2.0	2.05×10^{-4}	1.66×10^{-5}

4.8 Conventional M-ary NC-FSK in slow fading conditions

Settings: orthogonal frequencies

Reference: equation (7.7.7) of [3] pp. 494

$\frac{E_b}{N_0}$	Probability of Bit Error		
	Binary	4-ary	8-ary
10.0	8.333×10^{-2}	5.481×10^{-2}	4.544×10^{-2}
12.0	5.602×10^{-2}	3.595×10^{-2}	2.954×10^{-2}
14.0	3.687×10^{-2}	2.326×10^{-2}	1.900×10^{-2}
16.0	2.391×10^{-2}	1.492×10^{-2}	1.214×10^{-2}
18.0	1.536×10^{-2}	9.512×10^{-3}	7.721×10^{-3}
20.0	9.803×10^{-3}	6.041×10^{-3}	4.896×10^{-3}
22.0	6.230×10^{-3}	3.828×10^{-3}	3.099×10^{-3}
24.0	3.949×10^{-3}	2.421×10^{-3}	1.959×10^{-3}
26.0	2.499×10^{-3}	1.530×10^{-3}	1.237×10^{-3}
28.0	1.579×10^{-3}	9.667×10^{-4}	7.816×10^{-4}
30.0	9.980×10^{-4}	6.104×10^{-4}	4.934×10^{-4}
32.0	6.301×10^{-4}	3.853×10^{-4}	3.114×10^{-4}
34.0	3.977×10^{-4}	2.431×10^{-4}	1.965×10^{-4}

4.10 LPR Binary NC-FSK simulation results with estimate errors

Settings: $N = 64$, $f = \pm 16$ Hz, $T_s = 1$ s, $\frac{E_b}{N_0} = 14$ dB, FBWSR = 1

FBWSR Scale	FBWSR Scale [dB]	P_b	σ
0.1	-10.00	3.66×10^{-2}	3.87×10^{-3}
0.125	-9.03	3.24×10^{-2}	2.28×10^{-3}
0.17	-7.70	2.62×10^{-2}	1.62×10^{-3}
0.25	-6.02	2.28×10^{-2}	2.18×10^{-3}
0.33	-4.81	1.94×10^{-2}	1.57×10^{-3}
0.5	-3.01	1.21×10^{-2}	1.10×10^{-3}
1.0	0.00	8.67×10^{-3}	7.16×10^{-4}
2.0	3.01	1.23×10^{-2}	1.03×10^{-3}
3.0	4.77	1.80×10^{-2}	1.33×10^{-3}
4.0	6.02	2.31×10^{-2}	1.97×10^{-3}
6.0	7.78	3.26×10^{-2}	4.13×10^{-3}
8.0	9.03	4.21×10^{-2}	3.06×10^{-3}
10.0	10.00	5.11×10^{-2}	4.76×10^{-3}

Settings: $N = 16$, $f = \pm 4$ Hz, $T_s = 1$ s, $\frac{E_b}{N_0} = 14$ dB, FBWSR = 1

$\frac{E_b}{N_0}$ Scale	$\frac{E_b}{N_0}$ Scale [dB]	P_b	σ
0.1	-10.00	9.68×10^{-3}	9.66×10^{-4}
0.125	-9.03	9.46×10^{-3}	5.75×10^{-4}
0.17	-7.70	9.72×10^{-3}	8.97×10^{-4}
0.25	-6.02	1.01×10^{-2}	1.02×10^{-3}
0.33	-4.81	9.00×10^{-3}	8.30×10^{-4}
0.5	-3.01	8.96×10^{-3}	1.20×10^{-3}
1.0	0.00	8.18×10^{-3}	7.91×10^{-4}
2.0	3.01	9.33×10^{-3}	3.36×10^{-4}
3.0	4.77	9.67×10^{-3}	8.14×10^{-4}
4.0	6.02	9.36×10^{-3}	7.39×10^{-4}
6.0	7.78	1.01×10^{-2}	1.50×10^{-3}
8.0	9.03	9.82×10^{-3}	1.01×10^{-3}
10.0	10.00	1.04×10^{-2}	1.11×10^{-3}

4.11 LPR Binary NC-FSK simulation results for shortened predictor

Settings: $N = 64$, $f = \pm 16$ Hz, $T_s = 1$ s, $\frac{E_b}{N_0} = 14$ dB

FBWSR	Order = 63		Order = 31	
	P_b	σ	P_b	σ
0.125	3.35×10^{-2}	2.87×10^{-3}	3.45×10^{-2}	2.37×10^{-3}
0.25	2.52×10^{-2}	2.66×10^{-3}	2.55×10^{-2}	2.29×10^{-3}
0.5	1.53×10^{-2}	1.76×10^{-3}	1.62×10^{-2}	4.64×10^{-4}
1.0	8.67×10^{-3}	7.10×10^{-4}	9.15×10^{-3}	1.11×10^{-3}
2.0	5.44×10^{-3}	7.88×10^{-4}	5.32×10^{-3}	4.21×10^{-4}
4.0	4.05×10^{-3}	3.47×10^{-4}	4.42×10^{-3}	5.05×10^{-4}
6.0	4.96×10^{-3}	6.67×10^{-4}	5.01×10^{-3}	5.43×10^{-4}
8.0	6.59×10^{-3}	8.38×10^{-4}	6.06×10^{-3}	2.88×10^{-4}
10.0	7.32×10^{-3}	6.79×10^{-4}	8.22×10^{-3}	4.59×10^{-4}
12.0	9.88×10^{-3}	6.75×10^{-4}	1.01×10^{-2}	9.46×10^{-4}
14.0	1.54×10^{-2}	1.49×10^{-3}	1.46×10^{-2}	7.04×10^{-4}
16.0	4.09×10^{-2}	5.01×10^{-3}	3.89×10^{-2}	1.41×10^{-3}

FBWSR	Order = 15		Order = 7	
	P_b	σ	P_b	σ
0.125	4.07×10^{-2}	3.54×10^{-3}	5.31×10^{-2}	4.85×10^{-3}
0.25	3.02×10^{-2}	2.75×10^{-3}	3.96×10^{-2}	4.11×10^{-3}
0.5	1.57×10^{-2}	1.06×10^{-3}	2.32×10^{-2}	1.39×10^{-3}
1.0	1.02×10^{-2}	1.05×10^{-3}	1.18×10^{-2}	1.40×10^{-3}
2.0	5.65×10^{-3}	5.02×10^{-4}	5.90×10^{-3}	6.11×10^{-4}
4.0	4.37×10^{-3}	4.04×10^{-4}	5.14×10^{-3}	4.70×10^{-4}
6.0	4.93×10^{-3}	6.29×10^{-4}	5.43×10^{-3}	6.78×10^{-4}
8.0	6.57×10^{-3}	6.34×10^{-4}	7.00×10^{-3}	7.49×10^{-4}
10.0	7.94×10^{-3}	9.11×10^{-4}	7.90×10^{-3}	7.59×10^{-4}
12.0	9.68×10^{-3}	1.39×10^{-3}	1.04×10^{-2}	1.18×10^{-3}
14.0	1.35×10^{-2}	9.16×10^{-4}	1.46×10^{-2}	1.13×10^{-3}
16.0	4.02×10^{-2}	3.05×10^{-3}	4.11×10^{-2}	3.82×10^{-3}

FBWSR	Order = 3	
	P_b	σ
0.125	7.25×10^{-2}	8.39×10^{-3}
0.25	5.26×10^{-2}	6.77×10^{-3}
0.5	3.17×10^{-2}	1.67×10^{-3}
1.0	1.68×10^{-2}	1.96×10^{-3}
2.0	8.97×10^{-3}	1.08×10^{-3}
4.0	5.61×10^{-3}	5.87×10^{-4}
6.0	7.29×10^{-3}	5.33×10^{-4}
8.0	9.38×10^{-3}	1.57×10^{-3}
10.0	9.44×10^{-3}	1.01×10^{-3}
12.0	1.05×10^{-2}	1.12×10^{-3}
14.0	1.70×10^{-2}	2.80×10^{-4}
16.0	4.38×10^{-2}	4.04×10^{-3}

4.12 Conventional Binary NC-FSK as a function of order of diversity

Settings: orthogonal frequencies

Reference: equations (7.4.15), (7.4.30), (7.4.36) of [3] pp. 474-479

L	$\frac{E_b}{N_0}$		
	10 dB	14 dB	18 dB
1	8.33×10^{-2}	3.68×10^{-2}	1.53×10^{-2}
2	5.53×10^{-2}	1.35×10^{-2}	2.61×10^{-3}
3	4.87×10^{-2}	7.71×10^{-3}	7.66×10^{-4}
4	4.77×10^{-2}	5.49×10^{-3}	3.05×10^{-4}
5	4.89×10^{-2}	4.46×10^{-3}	1.49×10^{-4}
6	5.12×10^{-2}	3.93×10^{-3}	8.43×10^{-5}
7	5.41×10^{-2}	3.66×10^{-3}	5.32×10^{-5}
8	5.75×10^{-2}	3.53×10^{-3}	3.65×10^{-5}
9	6.10×10^{-2}	3.51×10^{-3}	2.68×10^{-5}
10	6.47×10^{-2}	3.56×10^{-3}	2.08×10^{-5}
11	6.85×10^{-2}	3.65×10^{-3}	1.69×10^{-5}
12	7.23×10^{-2}	3.79×10^{-3}	1.42×10^{-5}
13	7.61×10^{-2}	3.95×10^{-3}	1.24×10^{-5}
14	8.00×10^{-2}	4.15×10^{-3}	1.11×10^{-5}
15	8.37×10^{-2}	4.38×10^{-3}	1.01×10^{-5}
16	8.75×10^{-2}	4.62×10^{-3}	9.46×10^{-6}
17	9.12×10^{-2}	4.89×10^{-3}	8.97×10^{-6}
18	9.49×10^{-2}	5.18×10^{-3}	8.63×10^{-6}
19	9.85×10^{-2}	5.49×10^{-3}	8.41×10^{-6}
20	1.02×10^{-1}	5.81×10^{-3}	8.28×10^{-6}
21	1.05×10^{-1}	6.16×10^{-3}	8.23×10^{-6}
22	1.08×10^{-1}	6.52×10^{-3}	8.25×10^{-6}
23	1.12×10^{-1}	6.89×10^{-3}	8.33×10^{-6}
24	1.15×10^{-1}	7.29×10^{-3}	8.47×10^{-6}
25	1.18×10^{-1}	7.69×10^{-3}	8.65×10^{-6}
26	1.21×10^{-1}	8.12×10^{-3}	8.88×10^{-6}
27	1.25×10^{-1}	8.55×10^{-3}	9.16×10^{-6}
28	1.28×10^{-1}	9.00×10^{-3}	9.48×10^{-6}
29	1.31×10^{-1}	9.46×10^{-3}	9.85×10^{-6}
30	1.34×10^{-1}	9.94×10^{-3}	1.02×10^{-5}
31	1.36×10^{-1}	1.04×10^{-2}	1.07×10^{-5}
32	1.39×10^{-1}	1.09×10^{-2}	1.12×10^{-5}

APPENDIX B - EXAMPLE OF ANALYSIS

The analysis done by Mason in [5] can be directly applied to the LPR for binary NC-FSK. The equations are presented in section 2.7 are sufficient to repeat the analysis, but it is difficult to work them through. This appendix contains an illustrative example of a reasonable setting of FBWSR and $\frac{E_b}{N_0}$ with 4 samples per symbol. The small number of samples per symbol was chosen to allow the various matrices to be suitably shown within page boundaries. To keep the numbers tractable, only 3 significant digits will be shown despite the fact that all the calculations were done in double precision to around 17 digits of precision.

Once the starting parameters are stated, the spectrum of the fading process will be calculated. This will then be inverse Fourier transformed into the autocorrelation coefficients. The covariance matrices for each of the faded tones are calculated and then modified to include the effect of AWGN. These resultant covariance matrices are used to determine the eigenvalues. The eigenvalues are then used to express the moment-generating function of the log-likelihood ratio as a product of single roots. By partial fraction expansion, the intermediate function (moment-generating function of the log-likelihood + s) is expressed as a sum of poles. The residues of these poles are then used to calculate the probability of bit error.

The selected point has a fading-bandwidth (f_o) of 0.5 Hz and $\frac{E_b}{N_o} = 18$ dB. Since there is one samples per second ($T_s = 1$ s) and 4 samples per symbol ($N = 4$) the resultant sample period is 0.25 s ($T = 0.25$ s) or a sampling frequency of 4 Hz. The corresponding FBWSR given the FBW and symbol rate is 0.5. A tone frequency spacing of 2 Hz is chosen because it is the maximum that can be handled with a sample rate of 4 Hz. The tone amplitude, for simplicity, will be 1V or 0 dBW. Therefore, the noise spectral density of -18 dBW/Hz or 1.58×10^{-2} W/Hz

At this point it is necessary to determine the spectrum of the fading process filter model in the analog domain. The normalized filter transfer function, given by (3.4) is scaled using (3.15). The amplitude and frequency scaling factor, from the above paragraph is $\omega_o = 2\pi f_o = \pi$. The spectrum of the fading process will be $|H_{af}(j2\pi f)|^2$.

In determining the autocorrelation coefficients, it is necessary to meet the requirements of 3.4.2 by sampling the fading process filter model spectrum in sufficient detail before doing the inverse Fourier transform. A 128 point transform ($N = 128$) meets these requirements and using (3.22) gives a frequency separation of the spectrum samples of $\Delta f = 3.125 \times 10^{-2}$ Hz. For the inverse transform, the first 65 points consist of the spectrum samples and the other 63 are mirrored about the centre.

The spectrum is the sampled at intervals of 0.03125 Hz giving the input to the inverse FFT. A partial listing of the spectrum samples is below.

Sample Number	Frequency [Hz]	Spectrum Power
1	0.60	1.00
2	0.03125	1.00
3	0.0625	1.00
4	0.09375	1.02
5	0.125	1.05
63	1.9375	2.20×10^{-7}
64	1.96875	1.86×10^{-7}
65	2.00	1.58×10^{-7}

After mirroring the samples to get 128 points, the DFFTCB routine of IMSL is used to get the inverse FFT resulting in samples of the autocorrelation function of the fading process spaced at the sample period. Only the first N (4 in the case) are needed and they are given here normalized for a signal amplitude of 1.

Index (k)	Time Difference (τ) [s]	Autocorrelation (r_k)
0	0.00	1.00
1	0.25	0.847
2	0.50	0.459
3	0.75	1.18×10^{-2}

At this point the covariance matrices for the faded tones (note that the additive noise has not yet been addressed) are arranged from the autocorrelation coefficients. For the first tone, assume a 0 Hz centre frequency and the other tone will be at 2 Hz which requires (2.6) to modify the autocorrelation coefficients. The covariance matrices follow.

0 Hz: Ψ_1'

$$\begin{bmatrix} 1.00 & 0.847 & 0.460 & 1.19 \times 10^{-2} \\ 0.847 & 1.00 & 0.847 & 0.460 \\ 0.460 & 0.847 & 1.00 & 0.847 \\ 1.19 \times 10^{-2} & 0.459 & 0.847 & 1.00 \end{bmatrix}$$

2 Hz: Ψ_2'

$$\begin{bmatrix} 1.00 & -0.847-j7.40 \times 10^{-8} & 0.459+j8.03 \times 10^{-8} & -1.19 \times 10^{-2}-j3.11 \times 10^{-9} \\ -0.847+j7.40 \times 10^{-8} & 1.00 & -0.847-j7.40 \times 10^{-8} & 0.460+j8.03 \times 10^{-8} \\ 0.460-j8.03 \times 10^{-8} & -0.847+j7.40 \times 10^{-8} & 1.00 & -0.847-j7.40 \times 10^{-8} \\ -1.19 \times 10^{-2}+j3.11 \times 10^{-9} & 0.460-j8.03 \times 10^{-8} & -0.847+j7.40 \times 10^{-8} & 1.00 \end{bmatrix}$$

Additive white noise of -18 dBW/Hz or 1.58×10^{-2} W/Hz must now be considered. The noise variance of the samples is obtained from the noise power spectral density by using (3.20). The resultant variance, 6.34×10^{-2} , is added to the diagonals to produce the final covariance matrices.

 Ψ_1

$$\begin{bmatrix} 1.06 & 0.847 & 0.459 & 1.186 \times 10^{-2} \\ 0.847 & 1.06 & 0.847 & 0.459 \\ 0.459 & 0.847 & 1.06 & 0.847 \\ 1.19 \times 10^{-2} & 0.459 & 0.847 & 1.06 \end{bmatrix}$$

 Ψ_2

$$\begin{bmatrix} 1.06 & -0.847-j7.40 \times 10^{-8} & 0.459+j8.03 \times 10^{-8} & -1.19 \times 10^{-2}-j3.11 \times 10^{-9} \\ -0.847+j7.40 \times 10^{-8} & 1.06 & -0.847-j7.40 \times 10^{-8} & 0.459+j8.03 \times 10^{-8} \\ 0.459-j8.03 \times 10^{-8} & -0.847+j7.40 \times 10^{-8} & 1.06 & -0.847-j7.40 \times 10^{-8} \\ -1.18 \times 10^{-2}+j3.10 \times 10^{-9} & 0.459-j8.03 \times 10^{-8} & -0.846+j7.40 \times 10^{-8} & 1.06 \end{bmatrix}$$

The eigenvalues of $\Psi_1 \Psi_2^{-1}$ are needed, so first the IMSL routine DLINCG is used to get the inverse of Ψ_2 .

$$\nu_2^{-1}$$

$$\begin{bmatrix} 4.15-j3.01 \times 10^{-23} & 4.38+j3.83 \times 10^{-7} & 0.627+j1.10 \times 10^{-7} & -1.35-j3.54 \times 10^{-7} \\ 4.38-j3.83 \times 10^{-7} & 8.34-j3.85 \times 10^{-23} & 5.25+j4.59 \times 10^{-7} & 0.627+j1.10 \times 10^{-7} \\ 0.627-j1.10 \times 10^{-7} & 5.25-j4.59 \times 10^{-7} & 8.34+j6.62 \times 10^{-23} & 4.38+j3.83 \times 10^{-7} \\ -1.35+j3.54 \times 10^{-7} & 0.627-j1.10 \times 10^{-7} & 4.38-j3.83 \times 10^{-7} & 4.15+j1.76 \times 10^{-24} \end{bmatrix}$$

Now the complex matrix multiplication operation is invoked with DMCRCR of the IMSL package.

$$\nu_1 \nu_2^{-1}$$

$$\begin{bmatrix} 8.40-j3.71 \times 10^{-7} & 14.2+j1.95 \times 10^{-7} & 9.00+j5.01 \times 10^{-7} & 1.16-j1.07 \times 10^{-7} \\ 8.09-j3.38 \times 10^{-7} & 17.3-j1.15 \times 10^{-7} & 15.2+j4.05 \times 10^{-7} & 5.14+j1.42 \times 10^{-7} \\ 5.14-j1.42 \times 10^{-7} & 15.2-j4.05 \times 10^{-7} & 17.3+j1.14 \times 10^{-7} & 8.09+j3.38 \times 10^{-7} \\ 1.16+j1.07 \times 10^{-7} & 9.00-j5.01 \times 10^{-7} & 14.2-j1.95 \times 10^{-7} & 8.40+j3.71 \times 10^{-7} \end{bmatrix}$$

The IMSL routine DEVLG determines all the eigenvalues (λ_i) of the resultant matrix. Each of the product terms in the moment-generating function of the log-likelihood ratio of (2.34) can be rearranged to a simple root form. In this case the roots, designated d_i , are equal to $\frac{1}{1-\lambda_i}$ giving product terms of $\frac{-d_i}{s-d_i}$.

i	λ_i	d_i
1	2.38×10^{-2}	1.02
2	0.107	1.12
3	9.34	-0.120
4	42.0	-2.44×10^{-2}

Now $\frac{\phi_0(s)}{s}$, the intermediate function, can be expanded by partial fractions to yield

$$\sum_{i=1}^4 \frac{\beta_i}{(s-d_i)} + \frac{\beta_0}{s}$$

(2.35) calls for the residues of all the poles located strictly on the right hand half of the s -plane which corresponds to the d_i greater than 0. This occurs only with poles $i = 1$ and 2. The needed β_i were computed and are tabulated below.

i	β_i
1	-2.860×10^{-2}
2	2.213×10^{-2}

Using (2.35) with $k = 2$, since there are only two eligible poles, the probability of bit error is 6.47×10^{-3} .

END

1 9/0 2/9 1

FIN