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**The RADARSAT Antarctic Yaw Maneuver:
An Alternative Based On
Controlled Momentum Partitioning**

by

MATHIAS WUHR, B.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Department of
Mechanical and Aerospace Engineering
Carleton University
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An Alternative Based On Controlled Momentum Partitioning**
submitted by
Mathias Wuhr, B.Eng.
in partial fulfilment of the requirements for
the degree of Master of Engineering

A handwritten signature in black ink, reading "Douglas A. Staley", written over a horizontal line.

Dr. D.A.Staley
Thesis Supervisor

A handwritten signature in black ink, reading "Robert Bell", written over a horizontal line.

Dr. R.Bell
Chairman, Department of Mechanical and Aerospace Engineering

Carleton University

Jan. 11, 1995

Abstract

The RADARSAT earth observation satellite will be required, at some point, to perform a 180° yaw reorientation. This thesis examines an alternative means of performing not only this maneuver, but maneuvers in general. Known as the 'momentum exchange' or 'momentum transfer' method, this approach relies solely on the conservation of angular momentum.

By using controlled momentum partitioning, the momentum exchange approach is seen to offer a desirable (i.e. fast, safe, and simple) means of reorienting even the most asymmetric of spacecraft. The strategies contained herein do not rely upon nutational energy damping for their viability.

Extensive use is made of a visualization technique that displays certain key dynamic parameters in the form of a 'momentum sphere' and an 'energy ellipsoid'.

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Initially, I wanted to “go against the flow” and thank, by name, all my friends and fellow graduate students here in the department. I didn’t want to just say “thanks to my colleagues”, because I felt that too impersonal. Now that I’m actually listing those who I’d like to acknowledge, though, I find that list beginning to grow “exponentially”. I guess it’s true that there’s never enough space.

To all the friends that I’ve made during my 7 years here at Carleton, all I can say is “Thanks guys”. Together we made it, and I couldn’t have made it alone! There’s something about sharing a common, stressful academic program that unites people in a special way.

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Dedication

to Konrad

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Chapter 1

Introduction

1.1 RADARSAT and the Antarctic Maneuver.

The RADARSAT earth observation satellite (see Fig. 1.1) is the object of, motivation behind, and the hopeful beneficiary of this thesis investigation. RADARSAT provides an ideal opportunity to examine satellite attitude acquisition maneuvers. In this thesis, several previously known ideas are combined, expanded upon, and applied to a practical end.

At some point in its operational orbit, RADARSAT will be required to reorient 180° about its yaw axis. This reorientation will allow for special observation of the Antarctic region. Designing a maneuver to effect this change in attitude is not a straightforward process. Large inertial asymmetries are present and an external gravity-induced torque must be accounted for.

The RADARSAT project office has already approved a strategy for fulfilling this yaw maneuver requirement. The maneuver will be accomplished in an intricate fashion: the spacecraft will perform a pitch reorientation, shed all of its stabilizing angular momentum, slowly torque itself 180° , reacquire its stabilizing angular momentum, and capture its desired final attitude by performing a second pitch reorientation. A successful maneuver would require approximately 200 minutes. A

Figure 1.1: The RADARSAT earth observation satellite.

failure during the “no momentum” phase would require the initiation of a complex recovery strategy.

1.2 Momentum Exchange as the Basis for an Alternative/Backup Plan.

The principle of conservation of angular momentum serves as the basis for one particular method of performing satellite attitude acquisition maneuvers, referred to as either the ‘momentum exchange’ or the ‘momentum transfer’ method¹. This method is found to be suitable for many arbitrary situations. It relies solely on the inherent dynamics of a system of connected bodies. Each body, assumed rigid, is capable of storing angular momentum by virtue of its rotational state.

A maneuver thus implemented is accomplished by partitioning angular momentum in a predetermined way amongst the various system components. In the case of a satellite, the momentum is ‘exchanged’, or ‘transferred’, amongst the various internal wheels and the spacecraft body itself. The momentum distribution is controlled by controlling the wheel speeds. Momentum is either added to or taken away

¹In the area of satellite dynamics, the term ‘momentum’ is widely used as a short form substitute for ‘angular momentum’.

from the wheels, at the expense of the momentum stored in the spacecraft body. In the absence of external torques, the combined vector sum of all component momenta must remain constant.

This thesis proposes such a method as an alternative or backup plan for performing the RADARSAT Antarctic yaw maneuver. In a more general sense, it also shows the power and capability of such an approach for arbitrary reorientations of arbitrary spacecraft. Advantages of using a momentum exchange approach, to be further expanded upon later, include:

- significant reduction in maneuver time;
- inherent safety, due to the maintenance of system angular momentum;
- simplicity of implementation, since no additional hardware or fuel is required.

In controlling the wheel speeds, one must be wary of the effects of internal wheel torques. Often, the satellite body gains an excess of rotational kinetic energy which must somehow be dissipated. Any excess energy that remains at the end of the maneuver manifests itself as a residual nutation about the desired attitude.

Momentum exchange maneuvers have been successfully implemented on previous satellites. As further discussed in chapter 2, they have been used in conjunction both with body-stabilized satellites in acquiring 3-axis stabilized orientations and with dual-spin satellites in recovering from a certain class of failure mode. To date, however, the control possibilities and versatility that these maneuvers are truly capable of have not yet been fully realized in practice.

1.3 Implementing a Momentum Exchange for the Antarctic Maneuver.

Taking as an overall goal the completion of the desired Antarctic yaw maneuver with minimum residual nutation, this thesis proposes two strategies for the implementation of a controlled momentum exchange. Neither strategy requires the use of nutational energy damping.

RADARSAT, in its operational mode, is 3-axis stabilized with approximately 50 Nms of angular momentum stored internally in its momentum and reaction wheels. Despinning these wheels would transfer all momentum to the spacecraft body alone.

Part of the system momentum could then be transferred to the yaw reaction wheel simply by spinning it up; i.e. by controlling wheel speeds, a momentum transfer could be effected. It is proposed to transfer either +20 Nms or -20 Nms to the yaw wheel.

Thus partitioned, the system momentum would cause a large, yet prescribed, nutation of the spacecraft body. This spacecraft reorientation would occur, and occur in a known fashion, because of the dynamic interplay inherent in dividing the system angular momentum amongst two components: an internal wheel and the spacecraft body itself.

After a sufficient period of time has passed and an appropriate change in body reorientation has occurred, the yaw reaction wheel would be despun. All system momentum would once again reside in the spacecraft body alone.

A return to a 3-axis stabilized state could then be enacted by appropriately spinning up the momentum and reaction wheels so as to match the desired final attitude condition.

Various control strategies, based on the above framework, are possible. For the large reorienting nutation, one has control over the amount of momentum transferred to the reaction wheel and over the amount of time that this transfer is left in effect before the wheel is despun. In acquiring the desired final attitude, one has control over the way in which all the wheels are spun up so as to achieve a minimum residual nutation.

1.4 Overview of Analysis and Results.

Representative data of the fully deployed RADARSAT were used, including a modeling of the gravity gradient environment. All required dynamic parameters and time derivatives were appropriately expressed. A simulation study was then performed.

Various maneuver control strategies were conceived and planned through a visualization technique advanced by Barba & Aubrun [3]. Introducing what's known as a 'momentum sphere' and an 'energy ellipsoid', this technique provided a physical "feel" to the numerical results. The momentum sphere (MS) and energy ellipsoid (EE) together gave a three-dimensional geometric representation of certain key dynamic quantities.

Though it's felt that all required data for this study would be known to a relatively high degree of accuracy, an error modeling technique was adopted and adapted from the work of Vigneron & Staley [19]. Preset timing sequences were used under "off-design" conditions to model the combined effects of various uncertainties.

Two effective control strategies were singled out as possible candidates for the RADARSAT Antarctic yaw maneuver. In the face of no parameter uncertainties, one strategy would perform the maneuver in 40 minutes and leave a residual nutation of less than 3° . The second would require 100 minutes and leave a residual nutation less than 7° . Both candidate proposals would return the satellite to a 3-axis stabilized orientation. Should it be desired to forgo their preprogrammed finishes, the attitude control system could be activated 20 minutes into either strategy and capture the desired final attitude itself.

Due to the inertia distribution of RADARSAT, any residual nutation will be highly elliptical in nature, as opposed to being circularly conical. Thus, minimum and maximum limit values were quoted when describing an expected nutational error. Even with what's deemed to be maximum parameter uncertainty, the predicted elliptical nutation would only be 7° – 17° .

The maximum linear acceleration expected of a point midway along a solar array (approximately 5 m from the center of mass) was found to be on the order of 10 micro-G (i.e. 10^{-5} times the acceleration due to gravity at the earth's surface). Any concerns of a momentum exchange approach being too harsh were thus alleviated.

Since the maneuver relies solely on the use of the momentum wheels, an investigation was made into the effects of possible failure of these wheels part way through the proposed maneuver strategies. As the wheels would naturally despin by friction, all the momentum would accumulate in the spacecraft body itself. Depending on

the timing of the failure, a possibly large nutation of the spacecraft might ensue. At best, the spacecraft would enter into a zero nutation spin about its maximum principal moment of inertia. At worst, the nutation would be so large that the spacecraft would appear to continually roll over upon itself. Contrary to what it might appear, however, this would not be a random tumbling; the nutation would always occur in a prescribed fashion due to the retention of the system's angular momentum.

After such a failure, the solar arrays would be periodically in and out of the sun's view. Concerns that a failure might result in an orientation that would continually drain the batteries were thus alleviated.

A comparison was made to maneuvers planned without consideration of the gravity gradient environment. It was found that gravity indeed needs to be considered for an accurate completion of the required maneuver. A 4° – 9° nutation would occur if a "no-gravity" solution were used in a realistic environment, even in the face of no parameter uncertainty. However, the "no gravity" case retains its usefulness for theorizing and for probing the effects of various changes in spacecraft configuration and control strategy.

Chapter 2

Background

2.1 Attitude Acquisition Maneuvers Based On Momentum Exchanges.

2.1.1 General.

When reorienting a satellite, it's common to make use of various external torquing devices, such as thrusters or magnetic torque rods. Maneuvers thus performed are simple to understand from a physical point of view: apply an external torque and the satellite is forcefully reoriented.

In contrast, maneuvers performed via a momentum exchange are more difficult to visualize and require some additional analysis; but, from the point of view of hardware, they are simpler to implement: adjust the speeds of various internal wheels and the satellite reorients itself.

A spacecraft can be thought of as a system of constrained rigid bodies: various internal wheels and the satellite body itself. If the spacecraft, as a whole, possesses angular momentum, then a 'momentum exchange' maneuver can be implemented. Such a maneuver is based on the conservation of total system angular momentum.

In reality, a spacecraft's total angular momentum vector will remain only *ap-proximately* constant in inertial space (both in magnitude and direction) since the satellite experiences only a *near* torque-free environment. Nevertheless, assume for the moment that the satellite is an isolated system; i.e. that no external torque

acts on the spacecraft. By partitioning the total momentum amongst the various components (wheels and body) a nutation can be set in motion. No matter how the momentum is partitioned, the vector sum of all components must remain constant.

As the momentum is partitioned and the spacecraft nutation begins, the spacecraft body reorients itself with respect to the constant, inertially fixed total momentum vector. If viewed from within the rotating spacecraft body, it would seem as if it were the total momentum vector which was reorienting itself; i.e. the total momentum vector, which in reality is inertially fixed, would appear to be rotating within the body-fixed frame of reference while its magnitude stayed constant.

By controlling the amount of momentum partitioning and its duration, various reorientations of the spacecraft body are possible. A return to a stationary body state, known as the '3-axis stabilized state', requires knowing how the momentum must be partitioned amongst the wheels for the body to remain stationary in its final desired orientation.

In actuality, a spacecraft body might only be stationary with respect to an orbiting frame of reference. In keeping a certain "face" pointed toward the earth as it orbits, the spacecraft body would in reality rotate relative to an inertial observer. Such an orbital motion of the satellite would imply that there would always be some angular momentum remaining in the body itself. To achieve the desired final attitude, an accurate momentum balance between the wheels and the body would be needed.

2.1.2 Origins.

In the early days of satellite technology, satellites were constructed with an axis of rotational symmetry. In orbit, they were set spinning about this axis to provide gyroscopic stability. This generation of satellites is known as the 'spinners'.

The desire to place various earth-monitoring and communication devices on board required a new design of satellite: one that would remain 'earth-pointing' (i.e. with one particular side always facing the earth). The stability benefits from maintaining large spacecraft angular momentum, as with the spinners, were too

great to be ignored. Earth-pointing spacecraft, that only rotated as they orbited, just did not have high enough spin rates to provide this stability. This was especially true of the emerging generation of satellites that orbited at 'geosynchronous' altitude; i.e. those that had an orbital period that just matched the earth's rotation, so that the earth-pointing face was always pointed toward the exact same spot on the earth. Two design innovations arose.

One was to place spinning wheels inside the satellite body. Thus, angular momentum was, in effect, attached to the relatively stationary satellite body. The wheels became known as 'momentum wheels', and the satellites as '3-axis stabilized' or 'body stabilized' satellites. Additional wheels were later included for finer control over a satellite's attitude; these became known as 'reaction wheels'.

The second innovation that emerged was to actually divide the spacecraft into two parts: a spinning section, known as the 'rotor', and an earth-pointing section, known as the 'platform'. This class of spacecraft, successfully flown by the Hughes Aircraft Co., garnished the title of 'dual-spin' satellites, or simply 'dual-spinners'.

Essentially, both the 3-axis stabilized satellites and the dual-spinners were similar in nature, though different in design. They both could be modeled as connected, two-body systems. Each had an earth-pointing portion and each had a rotating component that provided the bulk of the stabilizing angular momentum.

Due to their similar natures, the concept of using momentum exchange maneuvers developed its own parallel analogy for both the 3-axis and the dual-spin spacecraft. It became virtually universal for a geosynchronous, 3-axis stabilized spacecraft to perform a certain attitude acquisition maneuver, while at the same time it became imperative for a dual-spin satellite to be capable of recovering from a certain possible failure mode.

2.1.3 On-orbit Attitude Acquisition of a Geosynchronous 3-Axis Stabilized Satellite.

While injecting a satellite through a transfer orbit to geosynchronous altitude, it is customary to set it spinning about the axis of its largest moment of inertia.

Textbooks, such as [5] and [6], often highlight that a rigid body spinning about such an axis is in a stable configuration.

Two benefits arise from such a spin through transfer orbit. The first is a gyroscopic stability in attitude due to the angular momentum. The second is a thrust averaging effect due to the rotational motion. (Any major thruster required for the final geosynchronous orbit injection is aligned, as nearly as possible, with this axis of spin. In practice, though, it's impossible to achieve a perfect alignment. Unwanted torques about the satellite's center of mass will always be present. Even the smallest such torques, in the virtually torque free environment of outer space, can create havoc. By spinning the satellite, any non-axial thrust component averages out when the thruster is finally ignited.)

On achieving the desired final orbit location, see Fig. 2.1, the spacecraft must shed the angular momentum in the body (used for the transfer orbit phase) and replace it with momentum in the wheels (for the earth-pointing phase). In addition, a 90° reorientation of the spacecraft is usually required. This points the unusable face, the face with the major thruster, away from the earth.

Various strategies were conceived to perform this post-transfer orbit attitude acquisition. Some used an external torquing approach; others used a momentum exchange approach. One of the earliest momentum exchange analyses ever performed was in support of the Canadian CTS satellite (also called Hermes), [2]; however, the approach was rejected for this project. Such an approach was subsequently incorporated by the Astrospace Division of RCA (later G.E., later Martin Marietta), [14] and [10]. Patented as their 'stabilite turn', this attitude acquisition technique was used on several series of their 3-axis stabilized spacecraft, such as the Anik E series.

Certain advantages presented themselves in the use of a momentum exchange in lieu of an external torquing scheme. Momentum exchanges were simple to implement (a wheel, 90° to the transfer orbit orientation, was spun up to transfer the momentum from the spinning satellite body into the wheel, while at the same time reorienting the body). They were safe (if at any time the wheel had failed, the momentum would have returned to the spacecraft body). And, they were faster (the complex torquing maneuver, which required precision control and sensor tracking, often could only be

Figure 2.1: Attitude acquisition of a 3-axis stabilized spacecraft.

done after certain critically timed conditions had been met).

2.1.4 Dual-Spin Satellite Recovery From Flat-Spin Failure.

Should a failure result in the loss of relative rotation on a dual-spinner, the spacecraft would behave as a single rigid body, rather than two. All angular momentum would be conserved in the spacecraft, and the satellite would eventually seek its minimum energy state. This minimum energy state would be a stable spin about the spacecraft's axis of maximum moment of inertia, often a transverse axis 90° to its operational attitude as in Fig. 2.2. Such a spin became known as a 'flat-spin'.

When an attempt towards recovery would be ready, the rotor section would be spun up. Angular momentum would transfer back into the rotor section, which would imply a return to the spacecraft's operational attitude due to the conservation of angular momentum.

The transfer of momentum from the composite two-body system (with no relative rotation between bodies) to just one of the two bodies in particular (accompanied by a 90° reorientation of the satellite) is analogous to the 3-axis stabilized spacecraft maneuver outlined above.

It's noteworthy to mention that this use of a momentum exchange was for recovery from *a failure*. Thus, should all momentum wheels fail in a 3-axis stabilized spacecraft like RADARSAT, and all the momentum be transferred to the body itself, a recovery is possible by appropriately spinning up the secondary set of wheels.

2.1.5 Implementations, To Date, of Momentum Exchange Maneuvers.

Though much talk is made over the attractiveness of the momentum exchange principle as a basis for spacecraft attitude reorientations, only very simple applications have thus far been made. To date, and to this author's knowledge, only the 3-axis attitude acquisitions and the dual-spin recoveries outlined above have been the beneficiaries of this concept. Furthermore, these two applications have only used momentum exchange techniques in their simplest sense.

Figure 2.2: Recovery from a flat-spin by a dual-spinner.

In transferring angular momentum from the spacecraft body to a particular wheel (or rotor), the usual method has thus far been to make a total and immediate transfer. A constant torque would be applied to the momentum wheel, resulting in a linear increase in its speed over time. The wheel would thereby be spun up, in one single stage, to a value which would allow it to possess all of the spacecraft's angular momentum. Such a single stage transfer, however, would often result in excessive residual nutation about the desired attitude. The larger the wheel torque, the faster the spin-up, the more residual nutation that would result. The paper by Barba & Aubrun [3], does a fine job in explaining this phenomenon.

Any excess kinetic rotational energy in the spacecraft body, the cause of any residual nutation, would have to be dissipated. The dissipation would be either natural and gradual ('passive' damping), or be forceful and rapid ('active' damping). Much work has therefore been focused on trying to adequately implement nutation damping so as to take advantage of the momentum exchange basis.

One alternative and highly innovative scheme, for fulfilling the initial attitude acquisition requirement via a momentum exchange, was presented by Vigneron & Staley in [19], building upon earlier work by Staley in [16] and [17]. They approached the momentum transfer as a three stage process, rather than as a single stage wheel spin-up. They proposed spinning the momentum wheel up by a small initial amount, allowing inherent dynamic interplay to reorient the satellite by its required 90° , and then (*after* this reorienting) continuing the spin-up of the momentum wheel to the required final value. By first allowing the inherent dynamics to reorient the spacecraft before fully spinning up the momentum wheel, their concept avoided the unwanted nutation of previous proposals and they were able to propose using maximum wheel torque in each of the wheel spin-up stages. This was truly a minimum time maneuver, more so than anything that had thus far (or since) been published. Their work, however, went largely unnoticed.

2.2 Some Key Equations.

2.2.1 Euler's Equation; Motion Expressed in a Rotating Frame.

For the analysis to be pursued here, vector quantities will be expressed in a coordinate frame that is fixed in the spacecraft body and attached to its center of mass. (For a preview of the 4 possible choices of body-fixed coordinate frame convenient to RADARSAT, see Figs. 3.2, 3.3, and 3.4 on pages 30 and 32.) Since this coordinate frame rotates with the body, all inertia properties remain constant when expressed relative to it.

The key equation to be used is Euler's equation:

$$\tau_{gg} = \left(\frac{\partial \mathbf{H}}{\partial t} \right)_{\text{rel}} + \boldsymbol{\omega} \times \mathbf{H},$$

where

$\mathbf{H}$ represents the total angular momentum of the entire spacecraft;

τ_{gg} represents the external torque acting about the center of mass due to the gravity gradient;

$\boldsymbol{\omega}$ represents the angular velocity of the rotating coordinate frame, in this case the angular velocity of the spacecraft body itself; and

$()_{\text{rel}}$ represents the vector's derivative with respect to the rotating coordinate frame.

2.2.2 Coordinate Transformation Notation.

In referring to a coordinate transformation due to a rotation about some axis, say the x -axis, the following notation will be used: $(\theta)_x$, where for this case

$$(\theta)_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}.$$

2.2.3 Gravity Gradient Modeling.

Because an orbiting satellite is essentially a freely rotating body, even the smallest of disturbance torques can create significant changes in attitude when considered over the long term. In the case of RADARSAT, the largest of these disturbance torques is due to the gradient in the earth's gravitational field. The gravitational field is not uniform, but varies according to an inverse square law. This results in the center of gravity not coinciding with the center of mass, which in turn creates a disturbance torque about the center of mass.

Let $[C]$ be a coordinate transformation matrix used to transform a vector's coordinates as expressed in an earth-pointing orbital frame to coordinates as expressed in a body-fixed satellite frame (again, it may be helpful to see Fig. 3.2 on page 30):

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{body}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ \vdots & \ddots & \vdots \\ C_{31} & \cdots & C_{33} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{orbit}}.$$

For a spacecraft in a circular orbit, a first-order expression for the gravity gradient torque as expressed in a body-fixed coordinate frame is, from [9, 15]:

$$\tau_{gg} = 3\omega_0^2 \begin{bmatrix} (I_{zz} - I_{yy})C_{23}C_{33} - I_{xy}C_{13}C_{33} + I_{xz}C_{13}C_{23} - I_{yz}(C_{33}^2 - C_{23}^2) \\ (I_{xx} - I_{zz})C_{33}C_{13} - I_{yz}C_{23}C_{13} + I_{xy}C_{23}C_{33} - I_{xz}(C_{13}^2 - C_{33}^2) \\ (I_{yy} - I_{xx})C_{13}C_{23} - I_{xz}C_{33}C_{23} + I_{yz}C_{33}C_{13} - I_{xy}(C_{23}^2 - C_{13}^2) \end{bmatrix},$$

where ω_0 is the orbital frequency in (rad/s), and terms like I_{xy} are calculated as $I_{xy} = -\int xy \, dm$.

2.3 The Momentum Sphere and the Energy Ellipsoid.

2.3.1 Free Rotation of a Single Rigid Body.

There exists a geometric means of gaining qualitative information about the free rotation of a *single* rigid body (for example, see any of [1, 5, 6, 8, 11, 12, 13, 18]).

For the moment, a torque-free rotation will be assumed; this implies the constancy of two quantities: the body's angular momentum and its kinetic energy of rotation. The body's angular momentum vector will remain constant both in magnitude and in direction when viewed from an inertial frame of reference; however, only the magnitude will appear constant when this momentum vector is viewed from a frame that is fixed within the body and rotating with it.

When properly expressed relative to a body-fixed frame that's aligned with the *principal inertia axes*, the relations for the constancy of angular momentum and rotational energy take the form of three-dimensional closed surfaces: spheres and/or ellipsoids.

The angular momentum of a rigid body, $\mathbf{H}$, may be expressed either in terms of its components, H_x, H_y, H_z ; or in terms of the body's angular velocity components, $\omega_x, \omega_y, \omega_z$:

$$\begin{aligned} \mathbf{H} &= H_x \mathbf{i} + H_y \mathbf{j} + H_z \mathbf{k} \\ \left\langle \text{or } \mathbf{H} &= [\mathbf{I}] \boldsymbol{\omega} \right. \\ &= I_x \omega_x \mathbf{i} + I_y \omega_y \mathbf{j} + I_z \omega_z \mathbf{k}. \end{aligned}$$

Consider the constant magnitude of the momentum vector, H , as follows:

$$\begin{aligned} H^2 &= \mathbf{H} \cdot \mathbf{H} \\ &= H_x^2 + H_y^2 + H_z^2 \end{aligned} \tag{2.1}$$

$$\left\langle \text{or } H^2 = I_x^2 \omega_x^2 + I_y^2 \omega_y^2 + I_z^2 \omega_z^2 \right\rangle. \tag{2.2}$$

The value of H would be known from initial conditions. If H_x, H_y, H_z are thought of as unknown rectangular coordinates, then equation 2.1 represents the equation of a sphere with radius H (cf. $r^2 = x^2 + y^2 + z^2$). Equation 2.2 is of the form of an ellipsoid, if one thinks of $\omega_x, \omega_y, \omega_z$ as the unknown rectangular coordinates (cf. $k^2 = a^2 x^2 + b^2 y^2 + c^2 z^2$).

The rotational kinetic energy of the rigid body can be expressed as:

$$\begin{aligned}
 E &= \frac{1}{2} \mathbf{H} \cdot [\mathbf{I}]^{-1} \mathbf{H} \\
 &= \frac{1}{2} \left(\frac{H_x^2}{I_x} + \frac{H_y^2}{I_y} + \frac{H_z^2}{I_z} \right)
 \end{aligned} \tag{2.3}$$

$$\left\langle \begin{aligned} \text{or } E &= \frac{1}{2} \boldsymbol{\omega} \cdot [\mathbf{I}] \boldsymbol{\omega} \\ &= \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2) \end{aligned} \right\rangle \tag{2.4}$$

When thought of geometrically, as above, equations 2.3 and 2.4 both have the form of ellipsoids.

Though this author is unsure of who first coined the phrases ‘momentum sphere’ and ‘energy ellipsoid’, these terms have come to be used in describing the constructs of equations 2.1 and 2.3, respectively. It seems fairly safe, though, to credit Barba & Aubrun with providing a wider acceptance of these terms within the satellite community as a result of their paper [3]. (The geometrical constructs of equations 2.2 and 2.4 are often referred to as “Poincot’s construction”. “MacCullagh’s ellipsoid” is yet another name that is often associated with these styles of presentation.)

From here on, only the momentum component expressions 2.1 and 2.3 will be considered; i.e. a momentum sphere (MS) and energy ellipsoid (EE) approach will be taken.

In examining the free rotation of a rigid body, the goal of an analytical study would be to find $\mathbf{H}$ as a function of time; i.e. to find expressions for each component H_x , H_y , H_z as functions of time. The MS & EE relations donate two “integrals of the motion” which may be used in solving for these three unknowns. Incorporating Euler’s equation along with the use of Jacobi elliptic functions, the analysis can be completed [15].

2.3.2 The MS & EE as a Geometric Visualization Technique.

From a qualitative point of view, the MS & EE relations can provide a pictorial aid to understanding the dynamics inherent in Euler’s equation.

Figure 2.3: MS & EE for a rigid body in torque free rotation.

Using initial conditions for H and E , a plot of the MS & EE relations 2.1 and 2.3 in three-dimensional space (using H_x , H_y , H_z as rectangular coordinates) would result in the depiction of an intersecting sphere and ellipsoid, see Fig. 2.3. A particular point (H_x, H_y, H_z) representing the current momentum state can be located somewhere along the intersection of the MS & EE, since the H_i components must simultaneously satisfy both geometric relations. One can think of this point as describing the tip of the momentum vector $\mathbf{H}$ (more specifically this vector's orientation) as viewed from within a body-fixed principal-axes coordinate frame.

Recalling that $\mathbf{H}$ is actually fixed in inertial space, an idea is obtained as to how the body might actually be oriented. The orientation of $\mathbf{H}$ serves as a connection between the rotating body-fixed frame and the stationary inertial frame, since the axis of $\mathbf{H}$ behaves like an inertial reference datum. This is one advantage of using the H_i -component expressions 2.1 and 2.3 over the ω_i expressions 2.2 and 2.4; ω does not remain inertially fixed and provides no such mental connection between the two frames. Other advantages of the MS & EE approach will be outlined later in §2.3.4.

There must *always* exist a point, on the line of intersection of the MS & EE, which defines the current momentum state $\mathbf{H}$. As the body rotates relative to inertial space, the $\mathbf{H}$ vector appears to trace a path along this line of intersection (Fig. 2.3). The apparent nutation of $\mathbf{H}$ along this curve is in accord with Euler's equation; i.e. the tip of the $\mathbf{H}$ vector traces along the line of intersection in a direction and with a speed determined by Euler's equation. This relation between the MS & EE presentation and Euler's equation can best be understood by recalling that Euler's equation is itself expressed in body coordinates and, from this perspective, describes the change in $\mathbf{H}$:

$$\left(\frac{\partial \mathbf{H}}{\partial t} \right)_{rel} = \boldsymbol{\tau}_{gg} - \boldsymbol{\omega} \times \mathbf{H}.$$

where $\boldsymbol{\tau}_{gg}$ has been assumed zero in these discussions.

2.3.3 Two-body Systems; MS & EE as an Aid in Visualizing Momentum Exchange Maneuvers.

Textbooks often mention, in their discussions on the free rotation of a rigid body, the aforementioned geometrical view of nutation. They do not, however, present the usefulness of this visualization technique for explaining the rotational motion of connected two-body systems. Barba & Aubrun, in their paper [3], seem to have been the first to apply the MS & EE concept to a multi-body problem.

Their insight into the application of such an approach to the developing field of satellite technology was truly ground-breaking. Satellites are the epitome of bodies in torque-free rotation. Momentum exchange-based maneuvers had been discussed prior to their paper, see [2] for example, but their melding of the MS & EE with these maneuvers provided a new clarity and an assured understanding of stability. Prior to their work, the strictly mathematical investigations must have seemed somewhat vague to the more physically-minded reader, who might have been investigating the practical implications of such an approach for an actual satellite.

Consider, now, the case of a satellite with internal wheels. In spinning up these wheels, the satellite's angular momentum is partitioned into a body part, $\mathbf{H}_B$, and

a wheel part, $\mathbf{h}$. The total momentum would be expressed as:

$$\begin{aligned}\mathbf{H} &= \mathbf{H}_B + \mathbf{h} \\ &= [\mathbf{I}]\boldsymbol{\omega} + \mathbf{h}\end{aligned}\tag{2.5}$$

which in terms of components along the *principal inertia axes* would be:

$$\begin{aligned}H_x &= H_{B_x} + h_x = I_x\omega_x + h_x \\ H_y &= H_{B_y} + h_y = I_y\omega_y + h_y \\ H_z &= H_{B_z} + h_z = I_z\omega_z + h_z\end{aligned}\tag{2.6}$$

Note that h_x, h_y, h_z represent the principal axis components of the amount of momentum stored in the momentum wheels; h_x, h_y, h_z do not individually equate to specific wheels unless these wheels are themselves aligned with the spacecraft's principal axes.

The equation of the momentum sphere remains unchanged, since it expresses the constancy of total system angular momentum:

$$H^2 = H_x^2 + H_y^2 + H_z^2.\tag{2.7}$$

The equation of the ellipsoid is modified, however, as it deals with the constancy of rotational kinetic energy stored in the satellite body, exclusive of the wheels (constancy, that is, for a given momentum partition):

$$E = \frac{1}{2}\mathbf{H}_B \cdot [\mathbf{I}]^{-1}\mathbf{H}_B,\tag{2.8}$$

which in principal axis components is

$$E = \frac{1}{2}\left(\frac{H_{B_x}^2}{I_x} + \frac{H_{B_y}^2}{I_y} + \frac{H_{B_z}^2}{I_z}\right)$$

Relating $\mathbf{H}_B$ to $\mathbf{H}$ and $\mathbf{h}$ with the aid of 2.6 gives

$$E = \frac{1}{2}\left(\frac{(H_x - h_x)^2}{I_x} + \frac{(H_y - h_y)^2}{I_y} + \frac{(H_z - h_z)^2}{I_z}\right).\tag{2.9}$$

Continuing to use the total system components H_x, H_y, H_z as rectangular coordinates, a three-dimensional plot of 2.9 appears as an ellipsoid shifted from the

Figure 2.4: MS & EE for a composite body in torque free rotation.

coordinate origin by a vector displacement of $\mathbf{h}$ (cf. the general equation of an ellipsoid $k^2 = \frac{(x-c_1)^2}{a^2} + \frac{(y-c_2)^2}{b^2} + \frac{(z-c_3)^2}{c^2}$); i.e. the ellipsoid appears to be translated from the origin, with its center located at (h_x, h_y, h_z) . The intersection curve between a shifted ellipsoid and a sphere, see Fig. 2.4, would bear a resemblance to the line of stitching on a baseball.

2.3.4 A Few Points.

A few points should be gathered together and highlighted to assist in proper understanding of the multi-body case:

The multi-body MS & EE relations that will henceforth be dealt with, 2.7 and 2.9, are still plotted in the rectangular coordinates H_x, H_y, H_z . A vector drawn between any two points will be a momentum vector, since the axes are in units of momentum. The coordinates are still those of a rotating frame of reference that is fixed in the spacecraft body and aligned parallel to its principal inertia axes. Thus, momentum vectors that are portrayed in this MS & EE plot still have orientations

that would be seen by an observer fixed in and rotating with the satellite body itself.

The vector that points from the origin of this plot, at the center of the MS, to any point (H_x, H_y, H_z) on the intersection of the MS & EE represents $\mathbf{H}$, the total system angular momentum, and not $\mathbf{H}_B$, the body angular momentum. In the absence of external torques, $\mathbf{H}$ (not $\mathbf{H}_B$) remains constant both in magnitude and direction with respect to an inertial observer. Thus, the discussion of §2.3.2 still applies to this multi-body case: the orientation of $\mathbf{H}$ serves as an inertial reference datum, and therefore as a mental connection between the rotating body frame and the stationary inertial frame.

The MS & EE must always intersect in at least one point. The conservation law implies that $\mathbf{H}$ always exists, so a geometric solution in the form of an intersection between the MS & EE must always be present. If this intersection is simply a point, then the body does not nutate about the $\mathbf{H}$ vector. If the intersection is a closed curve, then the body does nutate about $\mathbf{H}$; from the perspective of body coordinates, such a nutation is manifested by the $\mathbf{H}$ vector tracing along the curve.

The geometry of the energy ellipsoid is significant to both the body and the momentum wheels. The center of the ellipsoid signifies the wheel momentum state; the distance that this center (h_x, h_y, h_z) is from the center of the MS represents the amount of momentum stored in the wheels. Thus, the momentum partition between body and wheels can be immediately discerned by observing the position of the EE relative to the MS. A set of wheel speeds that fix a given value of momentum storage causes $\mathbf{h}$ to remain fixed with respect to body coordinates; i.e. the center of the EE remains fixed relative to the MS. Nutation of the body is reflected by the motion of the $\mathbf{H}$ vector, not the motion of the $\mathbf{h}$ vector. If it's desired to have all momentum reside in the wheels and none in the body, then it is necessary for the center of the EE to reside on the surface of the MS.

The size of the energy ellipsoid signifies the body's rotational energy state. A large angle nutation is reflected in a large path of intersection between the MS & EE, and results from an excess of rotational energy in the spacecraft body; i.e. from the energy ellipsoid being too large. If it's desired to have all momentum in the wheels and none in the body, then it is necessary and sufficient for the EE to shrink until

it vanishes and becomes a mere point somewhere on the surface of the MS.

Finally, the shape of the EE gives a sense of the spacecraft inertial distribution. Each of the ellipsoidal axes, see equation 2.9, is parallel to one of the principal inertia axes. The largest ellipsoid axis corresponds to the axis of the spacecraft's largest principal inertia, and the smallest axis to the smallest inertia.

Now, Barba & Aubrun note advantages of using an MS & EE presentation over the use of the Poincot (or $\omega_x, \omega_y, \omega_z$) formulation. For one, it's easier to plot and visualize intersection curves on a momentum sphere, rather than on an ellipsoid. Furthermore, as outlined above, the fundamental parameter of stored wheel momentum is readily recognized from the position of the EE. Lastly, the effects of energy dissipation can be easily visualized if one imagines how the intersection curves might change as the EE shrinks about its fixed center. Though it's felt that they probably consider them obvious, explicit mention can also be made of a couple more points. First, the mapping of solutions onto a momentum *sphere* grants one a symmetric "world view", similar to visualizing direction within a celestial sphere. Second, as noted in §2.3.2, **H** provides an inertial reference datum that ω cannot because momentum is subject to a conservation law.

By no means is any of the above insight to be considered original, but it's hoped that yet another explanation of this material serves to promote the use of and understanding of these concepts.

2.3.5 The MS & EE as an Aid in Conceptualizing and Planning Momentum Exchange Maneuvers.

The MS & EE conveniently lend themselves to the conceptual planning of momentum exchange maneuvers.

By adding momentum to or subtracting it from a spacecraft's wheels, a shift in the center of and a change in the size of the EE are observed. Depending on the size and location of the EE, some form of intersection curve becomes apparent between the MS & EE. The current location of **H** is located at some point on the intersection.

In holding this momentum partition between body and wheels, $\mathbf{H}$ is allowed to trace along the curve of the intersection; i.e. the spacecraft body reorients as it nutates about the inertially fixed $\mathbf{H}$.

As $\mathbf{H}$ approaches some location of interest on the curve, the momentum wheels can be despun, or spun up further, to effect a new change in the EE size and location. The $\mathbf{H}$ vector is now constrained to travel along a new curve of intersection.

Through such momentum transferring, it's possible to eventually "catch" or "trap" a desired momentum state, with little if any residual nutation. In effect, what's required is to create an intersection between the MS & EE that is reduced, as near as possible, to a point. This will be further explained by way of example in chapter 4 as a momentum exchange analysis is made for the proposed RADARSAT yaw maneuver.

2.3.6 Changes in the Sizes of the MS & EE.

As has been assumed throughout these discussions, both the system angular momentum and the spacecraft body's rotational energy have been conserved. Strictly speaking, the angular momentum only stays constant when there are no torques external to the system, and the body energy only stays constant when both the external torque is zero and when the wheels are held to constant values of momentum storage.

In reality, small external disturbance torques are always present, such as that due to the earth's gravitational gradient. Such disturbance torques gradually affect the system momentum vector $\mathbf{H}$. Over the short term, the vector might drift a little in orientation and fluctuate by a few percent in magnitude. For the time frames involved in momentum exchange maneuvers, the following thesis investigation shows such deviations to be minor irritants that affect accuracy rather than major annoyances that affect feasibility.

For example, the MS might expand or shrink in radius as the momentum magnitude varies. The size of the EE will also vary; even though it's formulated from $\mathbf{H}_B$ (see equation 2.8) the EE is related both to changes in $\mathbf{H}$ and in h (see equation 2.5).

Mathematically, the changes in size of both the MS & EE can be related to external and internal torques. For the MS (cf. equations 2.1 and 2.7):

$$\begin{aligned}\frac{d}{dt}(H^2) &= \frac{d}{dt}(\mathbf{H} \cdot \mathbf{H}) \\ &= 2\mathbf{H} \cdot \frac{d\mathbf{H}}{dt} \\ &= 2\mathbf{H} \cdot \boldsymbol{\tau}_{gg}\end{aligned}$$

And for the EE (cf. equations 2.4, 2.5 and 2.8):

$$\begin{aligned}\frac{d}{dt}(E) &= \frac{d}{dt}\left(\frac{1}{2}\mathbf{H}_B \cdot [\mathbf{I}]^{-1}\mathbf{H}_B\right) \\ &= [\mathbf{I}]^{-1}\mathbf{H}_B \cdot \frac{d\mathbf{H}_B}{dt} \\ &= \boldsymbol{\omega} \cdot \frac{d\mathbf{H}_B}{dt} \\ &= \boldsymbol{\omega} \cdot \left(\frac{d\mathbf{H}}{dt} - \frac{d\mathbf{h}}{dt}\right) \\ &= \boldsymbol{\omega} \cdot (\boldsymbol{\tau}_{gg} - \boldsymbol{\tau}_{wh})\end{aligned}\tag{2.10}$$

where $\boldsymbol{\tau}_{wh}$ is the net torque experienced by the wheels, applied and frictional.

2.4 Solution Methods to Momentum Exchange Maneuvers.

In solving for the time histories of momentum exchange maneuvers, certain mathematical formulations are possible:

- Perturbation methods, that model solutions to a first-order approximation, have been used by Gebman & Mingori [7] and Staley [16].
- Jacobi elliptic functions were used by Vigneron & Staley [19] and Cochran, Shu & Rew [4] in obtaining closed form solutions.
- Numerical integration of Euler's equation, and of other parameters of interest, is always a ready option when vector quantities are appropriately expressed in some body-fixed coordinate frame.

The first two analytic techniques can be applied to simple situations and simple geometries, but further study is needed to expand their applicability. Even in such "simple" cases, the differential equations are nonlinear and coupled. The complexity of the RADARSAT problem warranted a simulation study.

2.5 Error Modeling.

There will always be some uncertainty as to the value of various parameters, be they inertial quantities, wheel speeds, or orbital conditions.

In modeling the errors introduced by such uncertainties, an interesting procedure was used by Vigneron & Staley [19]. For the momentum exchange maneuver that they studied, they planned to hold a certain amount of momentum in one of the wheels for a specific duration; in terms of the MS & EE, the EE would be "shifted" by a certain amount for a specified amount of time. To model the effects of uncertainties, they maintained the same timing strategy, but used perturbed values for the amount of momentum transferred to the wheel; i.e. the timing was set and optimized for a certain "design" value of EE shift, but the EE was shifted by "off-design" values.

Chapter 3

The RADARSAT Antarctic Yaw

3.1 The Requirement.

At some point in its operational orbit, RADARSAT will be required to perform a 180° yaw reorientation, see Fig. 3.1. This attitude reorientation will allow the satellite to perform a special examination of the Antarctic region.

Control failures that occur during the yaw maneuver must lead to recoverable conditions. After any such failure, the spacecraft must not allow its batteries to overly deplete by remaining in an orientation where the solar arrays would not receive sufficient illumination.

3.2 The Situation.

Though the maneuver requirement is straightforward, many complexities exist:

The RADARSAT geometry is highly asymmetrical, especially in its operational orientation. A body-fixed coordinate frame can be set in alignment with this nominal operating attitude, see Fig. 3.2. Known as the 'control' frame, this body-fixed frame would be kept parallel to an orbit-oriented 'flight' frame by the attitude control system during normal operation. Table 3.1 expresses spacecraft inertial properties relative to the control frame. *Note:* terms like I_{xy} include the "minus" sign; i.e. $I_{xy} = - \int xy \, dm$.

Often, the location of the principal inertia axes is of major interest. Fig. 3.2b

(a)

(b)

Figure 3.1: RADARSAT before and after the yaw maneuver.

Figure 3.2: Flight frame, control frame, and principal axes.

RADARSAT is in its nominal orientation when the body-fixed "control" frame is perfectly aligned with the orbit-oriented "flight" frame.

	$kg \cdot m^2$		$kg \cdot m^2$
I_{xx}	4532	I_{xy}	-1830
I_{yy}	16372	I_{yz}	-842
I_{zz}	15453	I_{zx}	169

Table 3.1: RADARSAT inertias as expressed in the “control” frame.

	$kg \cdot m^2$
I_{xx}	4256
I_{yy}	17097
I_{zz}	15006

Table 3.2: RADARSAT inertias as expressed in the “principal axes” frame.

depicts these axes, and Table 3.2 lists the inertial properties relative to them.

To an observer, it would seem that the satellite is flying in a “tilted” orientation. A coordinate frame aligned with the “natural” geometry of the spacecraft is known as the ‘mechanical build’ frame, see Fig. 3.3. This frame is parallel with the box-shaped body and the synthetic aperture radar (SAR) antenna. To transfer coordinates from the control frame to the mechanical build frame, the following coordinate transformation must be used (according to the convention of §2.2.2):

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{mechanical build}} = (-4.95^\circ)_y (-29.8^\circ)_x \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{control frame}}$$

Furthermore, the momentum and reaction wheel axes are offset from these mechanical build axes, see Fig. 3.4, to more or less align the wheels with the control frame. Such an alignment is not perfect, however, and the coordinate transformation from the control frame to the ‘wheel axis’ frame is:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{wheel axis}} = (+30^\circ)_x (-4.95^\circ)_y (-29.8^\circ)_x \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{control frame}}$$

Figure 3.3: Mechanical build frame and principal axes frame.

Figure 3.4: Wheel axis frame and mechanical build frame.

The x -wheel axis corresponds with the x -mechanical build axis.

Wheel	Axial Inertia $kg \cdot m^2$	Max. Axial Torque $N \cdot m$
roll	0.067	0.21
pitch	0.091	0.12
yaw	0.067	0.21

Table 3.3: Momentum & reaction wheel properties.

Orbital Angular Freq.	rad/s	0.00104
Orbital Period	min	101

Table 3.4. Orbital properties.

Wheels were treated as bodies having only axial inertia; due to their symmetrical nature, all transverse inertia components were assumed included in the spacecraft body. Inertia properties and maximum axial torque limits for the wheels are presented in Table 3.3.

[*Note to those familiar with RADARSAT:* The axis direction for the roll control wheel has been assumed positive in the positive direction of the roll axis. Certain RADARSAT documents seem to imply that this particular wheel axis has been assumed positive in the negative roll direction.]

A gravity gradient environment also had to be considered in order to approximate the actual conditions. Other minor disturbance torques were neglected in the analysis. Neglected torques would have included those due to a magnetic dipole, aerodynamic forces, and the solar pressure. As outlined in §2.2.3, modeling the gravity gradient torque requires knowing the orbital frequency, which is tabulated in Table 3.4.

RADARSAT's mission calls for what's known as a 'sun-synchronous' orbit. Such an orbit plane nominally has its normal pointed toward the sun throughout the year. As the seasons pass, however, the sun will not always remain exactly aligned with this normal. Nevertheless, as an average location, such a perfect alignment was assumed for purposes of checking sun angle incidence during maneuver failures.

This orbit "normal" is aligned with the pitch axis of the flight coordinate frame.

The solar array normals were assumed to point parallel to the y -axis of the satellite's control frame; thus, during normal unperturbed flight, the arrays were assumed to be perfectly perpendicular to the solar illumination. In actuality, the arrays will be slightly angled relative to this control frame axis; but, since the position of the sun was averaged, it was felt acceptable to use a simplification for the arrays, too. As currently planned for, the arrays will be oriented 180° to one another just prior to commencing an Antarctic yaw maneuver; i.e. one array will be facing directly into the sun, and one will be facing directly away from it. Such an arrangement was selected for RADARSAT to ensure illumination of at least one array throughout as much of the maneuver as possible, including a possible failure mode. Thus, in checking the "sun angle" to the array normal (more on this in §4.5.4), an incidence of either 0° or 180° corresponded to direct illumination of one of the arrays.

3.3 The Current Solution.

Currently, the RADARSAT yaw maneuver will be performed in a sequence of controlled steps, requiring a total of approximately 200 minutes:

- RADARSAT will pitch downward by about 90° . With its longest axis thus aligned parallel to the orbit radius, the satellite will be in a gravity gradient stabilized attitude. *As a note in passing*, this pitch axis adjustment can also be considered as a rotation about the spacecraft's angular momentum vector.
- Through simultaneous external torquing and wheel despinning, all of the angular momentum will be shed while remaining in this "pitched" attitude; i.e. *the spacecraft will now be in a zero-momentum state*.
- A 180° controlled yaw reorientation will slowly proceed about the (then current) flight frame's yaw axis. This, too, will be accomplished by means of external torquing.

- The 50 Nms of stabilizing wheel angular momentum will be reacquired.
- Another 90° pitch adjustment will be performed, undoing the previous “pitch” and restoring RADARSAT to an operational attitude.

If successfully completed, the spacecraft would now be effectively “yawed” 180° from its pre-maneuver condition.

Should an attitude control failure occur during the zero-momentum phase, a complex recovery scheme would be initiated. Circumstances during a possible failure are of major concern in mission planning. The possibility of excessive battery drain, due to a prolonged interruption of direct illumination of the solar arrays, limits the seasons during which performance of the maneuver can be risked.

3.4 The Proposal.

This thesis proposes an alternative/back-up strategy for consideration. More specifically, it is proposed that the Antarctic yaw requirement could be fulfilled by strictly using momentum exchange principles.

The satellite could retain all of its angular momentum along the flight frame’s pitch axis while it reorients itself into the required attitude. From the point of view of a body-fixed control frame, it would appear as if this total angular momentum vector $\mathbf{H}$ was being rotated, or “transferred”, through 180° from the negative y -control axis to the positive y -control axis.

A sequence of momentum sphere/energy ellipsoid diagrams could be used as a visualization tool for explaining the maneuver as it unfolds. Furthermore, the sphere/ellipsoid presentation could even serve as an aid to conceptualizing various ways of implementing the momentum transfer.

3.5 Thesis Justification.

This thesis merits consideration for several reasons:

1. *Practicality with versatility.* A real situation is studied, but the approach taken remains general in application and in scope. Satellite maneuvers will continue to be performed and be of increasing importance as the frontiers of space are explored. The material presented here shows a usefulness for the most complex of situations, and not just for interesting studies of "special cases."
2. *Novelty of approach.* A momentum exchange is successfully implemented by performing it in stages. Though Vigneron & Staley [19, 16, 17] have previously thought along these lines, this approach has not yet been implemented in practice. They applied a "staged" momentum exchange to a simple geometry in the absence of gravity; the RADARSAT problem is complex both in geometry and in its inclusion of gravity gradient torques. They studied an attitude acquisition technique from a spinning body state; this situation expands the scope of such an approach by considering a generic on-orbit maneuver: the transfer of a satellite from a 3-axis stabilized state, through a spinning body state, and back to a 3-axis stabilized state.
3. *A tool uncovered.* The momentum sphere and energy ellipsoid (MS & EE) presentation can be actively used as a planning "tool". Though brilliantly introduced to satellite dynamics by Barba & Aubrun [3], the MS & EE remain largely unappreciated. They remain a novelty in explaining simple dynamics "after the fact", rather than being widely used as a powerful tool in the advance planning of complex sequences of dynamic interchanges. The MS & EE have the potential to be to spacecraft maneuver planning what the map is to the traveler.
4. *Control demonstrated.* Momentum exchange maneuvers are shown to be controllable by nature, and not just the product of random tumbling. Perhaps the main reason for momentum exchange maneuvers having not been more widely explored in the past is that no company wishes to risk the well-being of a multi-million dollar satellite on what's perceived to be an uncontrollable venture. Such fears are shown to be unfounded. Ironically, a momentum ex-

change approach could well eliminate the costly safeguards required by the risks inherent in more "conventional" approaches.

5. *Phenomenal benefits.* The approach taken herein presents a *fast, safe and simple* approach to performing satellite attitude reorientations. The term "fast" is in comparison with the time scales of maneuvers thus far implemented by the technical community; "safe" refers to the inherent basis for recovery even in times of equipment failure (as evidenced by the use of momentum exchanges in recovering dual-spinners from flat-spin *failures*); "simple" refers to that aesthetic quality of using simple dynamics to achieve a complex goal. "Simple" also applies to the ease of implementation: any satellite possessing a momentum storage device is capable of performing a momentum exchange maneuver (*Note: this means that many satellites currently in orbit and approaching the end of their useful lives could be used for "proof of concept" verification*).

Chapter 4

A Controlled Momentum Exchange Solution

4.1 An Initial Maneuver Concept.

For an initial feasibility study, an *ideal* situation was assumed: no gravity gradient torques; no orbital motion; wheel axes perfectly aligned with principal axes; and angular momentum vector $\mathbf{H}$ initially aligned with the axis of maximum moment of inertia. (*In reality:* RADARSAT will experience a gravity gradient torque and an orbital motion; the pitch and yaw wheel axes are approximately 30° from the y - and z -principal axes, respectively; and the $\mathbf{H}$ vector will initially be aligned with the y -control frame axis, almost 30° from the maximum moment of inertia (or y -principal) axis.)

A sequence of MS & EE's, see Fig. 4.1, were used in developing an initial maneuver concept, which will be explained in the following paragraphs. *Note: the location and size of the energy ellipsoid, as given by equations 2.9 and 2.10, will be important throughout this chapter; furthermore, the concepts of §2.3.4 will also be used throughout.*

RADARSAT, during normal operation, will nominally have 50 Nms of angular momentum stored in its momentum wheels. So, in developing the preliminary concept, 50 Nms of momentum was assumed initially stored solely in the y -axis pitch wheel, which was itself assumed aligned with the axis of maximum moment of iner-

Figure 4.1: Initial concept.

A sequential presentation of the maneuver is shown via MS & EE's. Diagrams are representative, but not to scale.

tia. And in keeping with the assumption of no orbital motion, the spacecraft body was taken to be initially at rest. Thus, what with all the momentum in the pitch wheel, the energy ellipsoid would have collapsed to a mere point on the surface of the momentum sphere, Fig. 4.1a: i.e. an ellipsoid with center at $(0, -50, 0)$ and zero size, resulting in $(0, -50, 0)$ as the only point of intersection between the MS & EE.

Despinning the pitch wheel would transfer all 50 Nms of momentum from the wheel to the satellite body. During this "spin-down", the ellipsoid center would migrate from $(0, -50, 0)$ toward $(0, 0, 0)$, Fig. 4.1b. Furthermore, the ellipsoid would grow in size as the body began rotating. The only point of contact between the MS & EE would remain at $(0, -50, 0)$, indicating that $\mathbf{H}$ would remain fixed in body coordinates and that momentum would be directly transferred from the wheel to the body. Thus, the body would commence spinning stably about its maximum moment of inertia.

Eventually, no momentum would remain in the wheel; all would reside in the body. The ellipsoid center would coincide with that of the MS at $(0, 0, 0)$, Fig. 4.1c. The EE would be just large enough to contact the sphere only at $(0, -50, 0)$ and at $(0, 50, 0)$. The satellite would behave as a single rigid body, spinning with no nutation.

An impulsive spin-up of the yaw reaction wheel, assumed aligned with the z -principal axis, would set up a momentum partition in the yaw direction: +20 Nms of momentum to the yaw wheel, -20 Nms to the body. The ellipsoid's center would shift a corresponding amount along the z -principal axis to $(0, 0, 20)$, Fig. 4.1d, and its size would increase as the internal wheel torque added energy to the body. Because the shift would be impulsive, the location of the $\mathbf{H}$ vector could serve as a boundary value: both before and after the impulse, $\mathbf{H}$ would point toward $(0, -50, 0)$.

This momentum partition would be *held* for a certain time duration, allowing the $\mathbf{H}$ vector to trace along the intersection curve in a direction determined by Euler's equations, Fig. 4.1e.

As soon as $\mathbf{H}$ reached the body coordinate point $(0, 50, 0)$, Fig. 4.1f, the yaw reaction wheel would be impulsively despun, undoing the momentum partition and transferring all momentum back to the body alone, Fig. 4.1g.

Finally, the y -axis pitch wheel would be appropriately spun up so as to transfer all 50 Nms of momentum away from the body and back into the wheel, Fig. 4.1h. Reversing the process that began the maneuver and collapsing the EE back to a mere point, this would once again return the satellite body to a 3-axis stabilized state.

A simulation program was created to confirm this concept. In performing the simulations, use was made of realistic wheel torque limits, rather than the infinite torques necessary to truly effect impulsive changes. It was observed from these studies that the more momentum transferred to the yaw reaction wheel in Fig. 4.1d, the faster the nutation of Fig. 4.1e.

The EE "shifts" of Fig. 4.1d could have been performed in any number of directions. In the hopes of finding the best results, these shifts were done in symmetrical fashion. Ellipsoid shifts were only considered in the z -principal axis directions and in the x -principal axis directions. The x -direction shifts were found to be more susceptible to modeling errors and were not pursued further in any of the subsequent studies—even in the more realistic analyses that later followed. (A brief explanation of why the x -shifts were neglected is given in §4.5.2, the section on parameter uncertainty error modeling.)

As a note in passing, it was found useful to divide the MS & EE equations (2.7 and 2.9) through by H^2 so as to "normalize" the momentum coordinates H_x , H_y , H_z . The MS was thus considered as a unit sphere, and the value E/H^2 became an interesting parameter for comparison.

4.2 Simulation.

A computer program was built around a Runge-Kutta-Fehlberg integration routine that incorporated error control. The program integrated total system angular momentum $\mathbf{H}$, wheel momentum $\mathbf{h}$, and an inertially-referenced orientation quaternion $\mathbf{q}$. The vector equations used in the general investigation are presented in Appendix A.

A maneuver was broken down into discreet intervals of integration according to

the on/off switching of the axial wheel torques. A trial and error approach was taken in optimizing the durations of certain key intervals (more on these intervals in §4.4).

Maneuver success was quantified by observing the residual nutation from a body-fixed coordinate frame. The nutation was quantified by observing the size and shape of the intersection of the MS & EE. And the intersection, in turn, was quantified by noting the motion of the $\mathbf{H}$ vector about its *desired* location.

4.3 Early Attempts with the Real Situation.

Attempts were made at applying the straightforward maneuver strategy of §4.1 to the *actual* RADARSAT situation: i.e. it was attempted to spin down all momentum wheels, "shift" the EE, hold this shift for a certain time, undo the EE shift, and spin up all wheels so as to match the desired final attitude. This approach was found to be ineffective; the residual nutations were always too great.

[These early attempts remain instructive, nonetheless, and will be briefly outlined in the following paragraphs. Though undesirable, the numerical results of this section are included for the sake of completeness. Strategically speaking, though, two key points emerged as a result of these efforts. The first was that the EE shift did not *need* to be in a principal axis direction, but could conveniently be in one of the wheel axis directions, see Fig. 4.2. The second was that one is neither required nor compelled to follow the undoing of the EE shift with an *immediate* attempt at recovering 3-axis stabilization. Recognition of this last point lead to the discovery of the true potential contained within a "multi-staged" approach, to be further discussed in the next section, §4.4.]

The major difference between the real situation discussed here and the ideal situation of §4.1 was the introduction of the 30° offset between the control frame and the principal axis frame (see Fig. 3.2). The resulting "skewed" orientation of the initial and desired momentum states (Figs. 4.3a and 4.3b) replaced the "convenient" states used in §4.1 (Figs. 4.1a and 4.1b).

Simulations revealed that the initial spin-down of the momentum wheels, when

Figure 4.2: Alternative EE shifts.

Figure 4.3: Real situation—initial and desired momentum states.

done at maximum torque, did not leave the satellite body spinning perfectly about its maximum moment of inertia (as in Fig. 4.1c). Instead, due to the asymmetry of the initial state (Fig. 4.4a), the EE protruded considerably after the spin-down (Fig. 4.4b). This was not deemed a failure, though, since this was not the final desired state, but merely an intermediate one.

The EE shift (Fig. 4.4c, cf. Fig. 4.1d) along the z -principal axis was then accomplished by appropriately spinning up the various momentum wheels in combination. The term "in combination", just used, emphasizes how no one particular wheel was aligned with this principal axis direction. The first of the two key points was soon realized: shifting the ellipsoid *only* along a principal axis was an artificial restriction. Potential difficulties might even exist in actually trying to achieve such an effect. (In synchronizing the various wheels, it was thought best to model a change in momentum "state" by having only one wheel experience maximum torque; the others would receive appropriately lower torques so that all might achieve their desired final speeds in equal time frames.) A much "easier" shift of the EE was later modeled through the spinning up of a *single* reaction wheel. For example, though the yaw reaction wheel did not coincide with a principal axis, its axis was almost perfectly within the principal- y -principal- z plane. Spinning up this wheel shifted the ellipsoid in a "skewed" fashion, Fig. 4.2b.

Whether it was the EE shift of Fig. 4.2a (equivalently Fig. 4.4c) or Fig. 4.2b, a momentum partition was simulated and held for a given period of time. As the $\mathbf{H}$ vector approached its desired location, by nutating along the curve of intersection, the EE shift was undone by despinning the reaction wheel(s). After undoing this shift, the wheels were spun up in an attempt to immediately "capture" a 3-axis stabilized orientation. The final wheel speeds corresponded to those necessary for body-stabilization in the desired final attitude. At the time, it was felt that such an *immediate* attempt at 3-axis stabilization was the natural response to undoing an EE shift; but, the results consistently showed excessive residual nutation.

Variations were attempted by performing the EE shift in both the positive and negative z -principal axis directions and the positive and negative z -wheel axis directions, Fig. 4.2. Attempting the EE shift along the negative z -principal axis,

Figure 4.4: Real situation—early attempts.

A sequential presentation of the initial phases of a maneuver are shown via MS & EE's. Depicted are the momentum wheels' spin down, (a) to (b), and a principal axis EE shift, (b) to (c). Diagrams are representative, but not to scale.

Fig. 4.2c, actually shrunk the EE by such an extent that the $\mathbf{H}$ vector didn't even come near to its desired location. Results of these attempts are tabulated in Tables 4.1 and 4.2. *(Due to the inertia distribution of RADARSAT, residual nutation will be highly elliptical in nature. Throughout this chapter, minimum and maximum limits will be quoted for any such nutation.)*

The tabulated results that are marked with an asterisk bear a little clarification. The observation of §4.1 still applied: nutation of the $\mathbf{H}$ vector proceeded quicker with larger values of EE shift. Yet, the hold times for these higher-valued shifts seem to exhibit a sudden increase rather than a continuing decrease when compared with the non-asterisk values. This oddity arises because of the sheer ineffectiveness of the strategy here employed. So much residual nutation remains, even with the best of these attempts, that strange effects become apparent because of the three-dimensional nature of the motion. For the lower (non-asterisk) values of momentum transfer, the EE shifts were undone (as one might expect) as the $\mathbf{H}$ vector approached its desired location. For the higher (asterisk) values of momentum transfer, the best results were actually obtained by allowing the $\mathbf{H}$ vector to "overshoot" its target destination, hence the requirement for longer hold times.

4.4 The Multi-Staging Approach.

The failure of the above-mentioned "early attempts" exposed an unnecessary constraint. *Only one* segment of the maneuver had thus far been designated as an actual "hold" stage. At every other instant, wheels were either being spun-up or despun. *Only* after the momentum had been partitioned and the corresponding shift in the EE had occurred were wheel speeds "held", allowing the dynamics to interact unhindered.

Now, there is no particular reason why wheel speed "holds" should be restricted to such singular occurrences. A "hold" stage could conceivably be introduced after *any* change in the wheel speed profiles.

Considering a maneuver as a sequence of "stages" lead to new possibilities. Rather than "forcing" changes in the momentum state one immediately after an-

EE Shift Nms	Hold Time min.	Residual Nutation
+5	18.0	15–33°
+10	12.0	20–47°
* +15	23.0	25–61°
* +20	20.0	28–65°
* +25	17.0	31–65°
Nearest Approach		
-5	154°(EE did not fully emerge from MS)	
-10	64°	
-15	66°	
-20	68°	
-25	70°	

Table 4.1: Early attempts—Shifting along the z-principal axis.

EE Shift Nms	Hold Time min.	Residual Nutation
+5	22.0	9–20°
+10	14.0	13–32°
* +15	19.0	17–34°
* +20	16.0	14–31°
* +25	13.0	12–29°
-5	38.0	4–10°
-10	20.0	7–15°
-15	15.0	6–16°
-20	12.0	7–18°
-25	10.0	9–20°

Table 4.2: Early attempts—Shifting along the z-wheel axis.

other, the introduction of “hold stages” added an element of relaxation. Motions were allowed to unfold naturally during “holds”. Since nutation is periodic by nature, one became able to effectively “wait” for the most opportune time to initiate a change in the momentum state.

If one considers the vectors of equation 2.10, the large residual nutations observed in §4.3 can now be accounted for. After undoing the EE shift, an immediate attempt at regaining 3-axis stabilization didn’t shrink the ellipsoid (i.e. didn’t eliminate the nutation) as hoped for because the vector geometry just wasn’t receptive to such an attempt. If the satellite had been allowed to remain in an all-body momentum state for a certain period of time (i.e. if a *second* hold had been allowed), a more “natural” geometry could have been obtained.

In allowing multiple hold phases and a “multi-staging” approach to momentum exchange maneuvers, adaptability was discovered. Arbitrarily complex reorientations now seemed possible. Responding to this design freedom, two new and effective strategies emerged as potential candidates for the RADARSAT yaw maneuver.

4.4.1 Proposal #1.

The first proposal is very similar to the attempted strategy of §4.3, except for the introduction of a second holding phase, see Fig. 4.5.

- From the initial state, Fig. 4.5a, all momentum wheels would be despun, transferring all momentum to the spacecraft body, Fig. 4.5b.
- An immediate spin-up of the yaw reaction wheel, in the *negative* sense, would shift the EE along the negative *z-wheel* axis, Fig. 4.5c (also, cf. Fig. 4.2d).
- This momentum partition would be held—the *first* hold of the maneuver—allowing for the main reorientation of the satellite body, Fig. 4.5d.
- After a time, the yaw wheel would be despun, undoing the EE shift, Figs. 4.5e and 4.5f. The timing of this spin-down would be chosen so as to match the energy level of this second all-body momentum state with that of the first (cf. Figs. 4.5b and 4.5f).

Figure 4.5: Proposal #1.

A sequential presentation of the maneuver is shown via MS & EE's. Diagrams are representative, but not to scale.

EE Shift Nms	1 st Hold sec.	2 nd Hold sec.	Total Time Reqd. min.	Residual Nutation
-5	1990	720	59.8	< 2°
-10	1030	730	44.8	< 2°
-15	770	770	41.9	< 2°
-20	620	720	39.4	< 2.5°
-25	510	680	37.7	< 2.5°

Table 4.3: Proposal #1 results.

- The body would remain nutating in this state for a certain duration—the *second* hold of the maneuver—Fig 4.5g.
- Finally, all momentum would be transferred back to the wheels in reestablishing 3-axis stabilization, Fig. 4.5h.

Behind this first proposal was the idea of removing all residual nutation through “mirroring” the MS & EE’s of the final wheel spin-up (Figs. 4.5g and 4.5h) with those of the initial wheel spin-down (Figs. 4.5a and 4.5b).

Simulations have verified that this particular strategy does successfully place RADARSAT in the proper momentum state, Fig. 4.5h; however, a post-maneuver pitch readjustment would be necessary to establish proper orientation about the axis of the momentum vector **H**. Such a requirement should be no cause for concern, though, since similar readjustments are also required in the currently approved maneuver strategy, see §3.3.

Required hold times, total maneuver time, and expected residual nutation are tabulated in Table 4.3. The various values of momentum transfer listed represent different amounts of EE shift in Fig. 4.5c. All values are negative, since positive EE shifts were found not to lend themselves to this strategy (they did, however, lend themselves to the strategy of the next proposal). Positive shifts caused the EE to grow by such an extent that the “mirror” goal, mentioned above, was impossible to achieve.

4.4.2 Proposal #2.

The second proposal demonstrates some of the variety that can be used with momentum exchange maneuvers. The preliminary stages are similar to those listed above.

- From the initial state, Fig. 4.6a, all momentum wheels would be despun, transferring all momentum to the spacecraft body, Fig. 4.6b.
- An immediate spin-up of the yaw reaction wheel, in the *positive* sense, would shift the EE along the positive *z-wheel* axis, Fig. 4.6c (also, c.f. Fig. 4.2b).
- This momentum partition would be held—the *first* hold of the maneuver—allowing for the main reorientation of the satellite body, Fig. 4.6d.
- After a time, the yaw wheel would be despun, undoing the EE shift, Fig. 4.6e and 4.6f. The timing of this spin-down would be chosen so as to make this second all-body momentum state as nearly as possible a pure spin about the maximum moment of inertia (c.f. Fig 4.6f and 4.1g).
- The spacecraft would remain in such a spin—the *second* hold of the maneuver—just long enough so that, at the end of the entire maneuver, it would not only be 3-axis stabilized but have the proper attitude as well; i.e. this holding phase would serve as a “fine tuning adjustment” for the maneuver as a whole, ensuring proper angular orientation at the end.
- The wheels would be spun up so as to capture a (temporary) 3-axis stabilized state in alignment with the principal axes; i.e. not the final desired state, but a state where 50 Nms of angular momentum would nonetheless be in the wheels. Capturing this intermediate state would allow the EE to virtually collapse down to a point on the surface of the MS, Fig. 4.6g (cf. Fig. 4.1h).
- With the 50 Nms of momentum residing in the wheels, a slow readjustment of wheel speeds would gradually reorient the **H** vector from the maximum moment of inertia location to its final desired location, Fig. 4.6h. The body would

Figure 4.6: Proposal #2.

A sequential presentation of the maneuver is shown via MS & EE's. Diagrams are representative, but not to scale.

EE Shift Nms	1 st Hold sec.	2 nd Hold sec.	Total Time Reqd. min.	Residual Nutation
+5	2050	460	92.6	3-8°
+10	1390	1180	94.4	5-11°
+15	1090	1520	95.9	3-8°
+20	890	1700	96.3	3-7°
+25	750	1800	96.5	1-5°

Table 4.4: Proposal #2 results.

still be 3-axis stabilized, but this time with the proper momentum distribution. The body would also be properly oriented about the **H** vector because of the “fine tuning” step referred to above. The wheel speed readjustment would be done slowly, over 35–40 minutes, to keep the wheel torque term of equation 2.10 reasonably small; this, in turn, would prevent the EE from growing unchecked, thereby minimizing residual nutation.

Motivating this second proposal was the allure of attaining not only the desired momentum state, but the desired orientation about the **H** vector (a quality lacking in the first proposal).

In comparison to Proposal #1, simulations showed that this strategy would require a little more time and would result in a little more residual nutation; however, this strategy does offer a complete maneuver in itself—no final pitch readjustment would be necessary.

Tabulated values, summarizing the maneuver, are presented in Table 4.4. Results are quoted for varying degrees of the EE shift shown in Fig. 4.6c. Just as positive *z*-shifts did not lend themselves to Proposal #1, negative *z*-shifts did not lend themselves to Proposal #2.

4.4.3 One Additional Alternative.

The above two proposals were designed as 'open loop' control strategies; i.e. pre-program the wheels to readjust themselves at certain times, and in certain ways, *et voilà!*

Alternatively, one might wish to forego the "preprogrammed finishes" that these strategies offer and opt for an actively controlled (or 'closed loop') termination. Twenty minutes, or so, into either flight proposal, the spacecraft would be near enough to its final desired orientation to activate the attitude control system's "capture circuit". This control system could then *actively* bring the satellite to its final resting state.

4.5 Further Evaluation Of The Proposals.

4.5.1 Two Candidates Chosen.

Two candidates, one from each of the above 'Proposals', were singled out for further study. Selected were the trials that placed 20 Nms of momentum in the yaw reaction wheel. Why the ones involving 20 Nms? In general, these were chosen for speed and for stability.

As discussed in §§4.1 and 4.3, the greater the EE shift, the faster the nutation of the main reorientation phase (e.g. Fig. 4.1e). So, since the RADARSAT roll and yaw reaction wheels were each designed to accommodate up to 20 Nms of angular momentum (cf. the pitch wheel which can hold about 50 Nms), the maximum possible shift values were opted for.

Furthermore, in accordance with the results of the next section, the uncertainty modeling favored the higher valued shifts (the results of the 5 and 10 Nms shifts, in particular, were not as desirable).

4.5.2 Parameter Uncertainty Error Modeling.

Since the two proposed candidates are both 'open loop' control strategies, one would expect input parameter uncertainty to have some effect on the final nutational state. Successful implementations require both accurate wheel speed control and knowledge of the various spacecraft and wheel inertias. In practice, both of these requirements can be fulfilled to a high degree of accuracy; nevertheless, to show forth the stability of momentum exchange maneuvers, the following error analysis was made. The technique was adopted from a similar approach taken by Vigneron & Staley [19].

In designing a maneuver sequence, residual nutation was reduced by optimizing certain hold times. As mentioned above, the amount of momentum passed to the yaw reaction wheel (the size of the EE shift) determined the speed of the main reorientation (the first hold). *In modeling the combined effects of the various uncertainties, hold times were first decided upon for one particular value of this EE shift (the "design" value), then used with a different value of EE shift (an "off-design" value).* By varying the degree of deviation from the design value, various levels of uncertainty were modeled.

For example, in the case of the two chosen candidates, the maneuvers were designed for 20 Nms of EE shift and the timing sequences were optimized appropriately. By using these predetermined hold times with 21 or 19 Nms valued momentum shifts, 5% overall uncertainty errors were modeled.

This approach to error modeling seemed reasonable, since the discrepancy introduced in this one early stage would propagate and magnify through the following stages. The range of residual nutations thus observed were felt to be fair indicators of overall maneuver success.

Results of introducing off-design values of EE shift with the two chosen candidates are tabulated in Table 4.5.

As noted in §4.1, no consideration was given to shifting the EE along the x -axis direction. Both the x -wheel axis, corresponding to the roll reaction wheel, and the x -principal axis are closely aligned on the RADARSAT. Thus, if momentum was transferred to the roll wheel, the EE would very nearly shift along the axis of

EE Shift Nms	Residual Nutation Proposal #1	EE Shift Nms	Residual Nutation Proposal #2
-19.0	7-17°	+19.0	5-14°
-19.5	2-8°	+19.5	5-10°
-20	<2.5°	+20	3-7°
-20.5	2-7°	+20.5	< 5°
-21.0	6-13°	+21.0	1-6°

Table 4.5: Parameter uncertainty results.

minimum moment of inertia.

These x -shifts were not studied because of the effects of perturbing the EE location (an “off-design” EE shift is essentially a perturbation of the EE from its “design” location). Consider a perturbation, δ , of the EE from its design location, Fig. 4.7. Due to the highly eccentric shape of RADARSAT’s energy ellipsoid, δ -shifts in the x -direction (Fig. 4.7a) would affect the MS & EE intersection curve to a greater degree than would the same shifts in the y - z -plane (Fig. 4.7b). If further studies conclude that parameter uncertainty is not a major factor, then x -shifts could be further considered.

4.5.3 Maximum Acceleration.

In performing a satellite maneuver, a key advantage of using momentum exchange principles is the “speed” gained over conventional approaches. A valid concern arises as to whether such maneuvers might in fact be “too fast” for the sensitive and flexible appendages of modern day spacecraft.

In response to this concern, linear accelerations were computed for a point 5 m from the satellite’s center of mass. This point was chosen along the axis of the solar array, so as to approximate conditions at the array’s mid-point. Should some distance other than 5 m be desired, the results can be scaled appropriately. As shown in Table 4.6, the maximum acceleration experienced at such a location is on the order of 10 micro-G (i.e. 10^{-5} times the acceleration due to gravity at the earth’s

perturbation of EE in x-direction

perturbation of EE in y-z plane

Figure 4.7: Perturbations in EE position.

Proposal #	Max. Acceleration m/s^2
1	1.2×10^{-4}
2	1.6×10^{-4}

Table 4.6: Maximum acceleration of a point 5 m from c.o.m.

surface). Large “whiplash” effects, therefore, should not be a problem.

The timing of this maximum acceleration was found to occur when the spacecraft was mostly in an all-body momentum state, transferring momentum to or from the yaw reaction wheel. As a note in passing, this author presumes that such an acceleration level has already been designed for, since RADARSAT must be capable of recovering from all-body momentum spins.

Maneuver quaternions for these two proposed strategies (at 30 second intervals) are listed in Appendix B.

4.5.4 Solar Illumination During Failure.

Should a failure occur in wheel speed control, friction would gradually slow the momentum wheels to a halt. All angular momentum would then reside in the spacecraft body alone. Depending on the timing of such a failure, the body could exhibit any of several possible nutational states.

If failure should occur just prior to commencing the maneuver (or, for that matter, while in the nominal operational mode), the EE would gradually grow and translate. Momentum would slowly be transferred to the body, and the spacecraft would eventually find itself in a stable spin about its maximum moment of inertia, Fig. 4.8. With regards to failure situations, this would be ideal. The satellite would rotate (with zero nutation) about a known, fixed axis. Furthermore, at least one solar array would always be sun-oriented and receive sufficient solar illumination (the other *might* have its back to the sun, see §3.2).

Should the failure occur at some point during the main yaw reorientation, the EE would already be fairly large. The ellipsoid would tend to remain large as any

prior to failure

wheels partly spun down

all momentum in body

Figure 4.8: Failure cases—the ideal situation.

A sequential presentation of the failure is shown via MS & EE's. Diagrams are representative, but not to scale.

wheel momentum was transferred to the body. Its final size would depend on the location of the $\mathbf{H}$ vector at the time of failure. The body would likely possess a large residual nutation; possible MS & EE configurations are shown in Fig. 4.9a,b,c. With such large nutations, the solar arrays would at times be illuminated, and at times be shadowed. Whether or not the durations of insufficient illumination would excessively drain the batteries is of major concern.

Depending on the amount of residual body energy (i.e. depending on the size of the EE), the nutational period could vary anywhere from 30–75 minutes. To a casual observer, a scenario like that of Fig. 4.9c would, at first glance, appear to be totally unacceptable. The body would *seemingly* tumble “head over heels”. In reality, though, the body would be nutating in a prescribed and periodic fashion, thanks to the conservation of angular momentum. The solar arrays would alternately be sun-oriented, then deep-space oriented.

To describe the solar illumination environment, “sun-angle” calculations were made according to the assumptions of §3.2. Wheel failures were simulated at various times during the yaw wheel spin-up, during the hold phase, and during the yaw wheel spin-down. Two levels of illumination were chosen, 60° and 45° , and the percentages of nutational period within each of these limits were found. Specifically, the questions asked were: what percentage of the time was there an array experiencing an illumination of 60° incidence or less? of 45° incidence or less? (Zero incidence corresponded to direct perpendicular illumination.) The results were most satisfying, and are graphically shown in Figs. 4.10 and 4.11. The vertical dashes on these graphs delineate the zones where the failure was initiated: yaw wheel spin-up, hold phase, and yaw wheel spin-down.

4.5.5 Planning a Maneuver While Neglecting Gravity.

Gravity gradient torques exist in the “real world”; but, are they necessary to consider? If not, then one can assume perfect conservation of angular momentum, and any analytic solution that might eventually be found would indeed be useful.

In setting forth to explore this issue, hold times were optimized for the “no-

Figure 4.9: Failure cases—that occur during the yaw reorientation. Diagrams are representative, but not to scale.

Figure 4.10: Failure—Proposal #1 solar illumination percentages.

The horizontal axis signifies different initiation times for the wheel failure. The zero value corresponds to the beginning of the main yaw reorientation phase (the *first* hold); i.e. once the yaw wheel had fully achieved its 20 Nms value. Regions (a) and (c) signify, respectively, failures that initiate during the yaw wheel spin-up prior to and the spin-down after completion of “the hold”. The vertical axis depicts the solar illumination scenario once all momentum resides in the body alone.

Figure 4.11: Failure—Proposal #2 solar illumination percentages.

The horizontal axis signifies different initiation times for the wheel failure. The zero value corresponds to the beginning of the main yaw reorientation phase (the *first* hold); i.e. once the yaw wheel had fully achieved its 20 Nms value. Regions (a) and (c) signify, respectively, failures that initiate during the yaw wheel spin-up prior to and the spin-down after completion of “the hold”. The vertical axis depicts the solar illumination scenario once all momentum resides in the body alone.

Proposal #	Optimized For	Simulated In	1 st Hold sec.	2 nd Hold sec.	Total Time Reqd. min.	Residual Nutation
1	no-grav	no-grav	610	900	42.2	< 0.4°
"	no-grav	grav	"	"	"	4-9°
"	grav	grav	620	720	39.4	< 2.5°
2	no-grav	no-grav	940	1740	97.8	2-5°
"	no-grav	grav	"	"	"	3-9°
"	grav	grav	890	1700	96.3	3-7°

Table 4.7: A "no-gravity" vs. "gravity" comparison.

[Maneuver strategies were optimized, as indicated, for either a "no-gravity" or "gravity" environment. The "no-gravity" solutions were then used in a gravity environment, for purposes of comparison, resulting in the nutations shown.]

gravity" situation. These timing sequences were then used in a second set of simulations that included "gravity". As one might expect, residual nutations were observed. Nevertheless, though optimized for an ideal no-gravity environment, these solutions worked fairly well in the more realistic gravity environment, as shown by Table 4.7.

One concluding remark can be drawn. "No-gravity" analyses lend themselves nicely to theoretical studies, but gravity should be considered for as accurate a maneuver completion as possible.

Chapter 5

Concluding Remarks

5.1 Conclusions.

1. Two strategies have been proposed and verified as viable alternative/backup means of performing the RADARSAT Antarctic yaw maneuver.
2. Both strategies, in their reliance upon a "multi-staged" momentum exchange, offer highly favorable results:
 - The first will perform the maneuver in 40 minutes, the second in 100.
 - The first will leave less than 3° of residual nutation, the second less than 7° , if inertial and wheel control properties are known to a high degree of accuracy.
 - Failure of the momentum wheels will *not* result in a spacecraft orientation that will continuously deplete the batteries. Large nutations may result, *but* will always occur in a prescribed fashion.
3. Momentum exchange maneuvers are fast, safe, and simple. A multi-staged approach offers versatile control.
4. Use of a momentum sphere and an energy ellipsoid (MS & EE) offers geometric assurance of the control and stability inherent in a momentum exchange approach.

5. The MS & EE are a formidable tool in planning various multi-staged strategies.
6. Gravity gradient torques should be considered so that residual nutations can be reduced as much as possible.

5.2 Future Research.

1. An exact solution for the no-gravity situation might well be possible, using Jacobi elliptic functions.
2. Higher values of momentum partition (energy ellipsoid shift) result in faster nutations. There could be a fairly straightforward explanation as to why.
3. Further study should be made into the level of uncertainty that one might actually expect in inertial properties and in wheel speed control.
4. A modeling of body energy dissipation, such as that due to fuel slosh, could be added.
5. The proposals made in this thesis were "open loop" control strategies. A modeling of the "attitude capture" control system would allow for "closed loop" strategies to be considered.
6. Many satellites currently in orbit and approaching the end of their useful lives could be used for "proof of concept" verification of these momentum exchange techniques.

5.3 Summary of Original Contributions.

1. Two practical alternatives have been proposed and investigated for the RADARSAT Antarctic maneuver.
2. A known method of presentation (the MS & EE) was combined with a known method of approach (multi-staged momentum exchanges) to give a general format for performing arbitrary spacecraft reorientations.

3. Momentum exchange maneuvers were explained, it's hoped, in an original way.
4. (cf. §3.5—Thesis Justification.)

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Appendix A

Equations Used.

This section details the equations that formed the basis of the simulation study.

- **Angular momentum of the spacecraft body.**

$$\mathbf{H}_B = \mathbf{H} - \mathbf{h}$$

where

$\mathbf{H}$ is the total spacecraft angular momentum; and

$\mathbf{h}$ is the total wheel angular momentum.

- **Angular velocity of the spacecraft body.**

$$\boldsymbol{\omega} = [\mathbf{I}]^{-1} \mathbf{H}_B$$

where

$[\mathbf{I}]$ is the spacecraft inertia tensor.

- **Rotational kinetic energy of the body.**

$$E = \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{H}_B.$$

• **A Quaternion and its derivative.**

A quaternion can be expressed in a “vector” form as:

$$\mathbf{q} = q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} + q_4$$

where q_4 is the scalar part of the quaternion.

Its derivative can be expressed in “matrix” form as:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

where ω_i are the components of the body’s angular velocity expressed in a body-fixed coordinate frame.

• **Coordinate transformation matrix from a quaternion.**

A 3×3 coordinate transformation matrix $[C]$, equivalent in effect to a quaternion $\mathbf{q}$, would be found as follows:

$$[C] = \begin{bmatrix} q_4^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & q_4^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & q_4^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}.$$

• **Angular momentum derivatives.**

A body-coordinate expression for the derivative of the spacecraft’s total angular momentum is (cf. §2.2.1 and §2.3.2):

$$\dot{\mathbf{H}} = \left(\frac{\partial \mathbf{H}}{\partial t} \right)_{rel} = \boldsymbol{\tau}_{gg} - \boldsymbol{\omega} \times \mathbf{H}.$$

Momentum wheels were assumed to only have axial inertias (transverse inertias were assumed incorporated into the spacecraft). Therefore, the i^{th} wheel momentum derivative is simply equal to the axial torque on the i^{th} wheel:

$$\dot{h}_i = \tau_{ax_i}.$$

• **Relative wheel speed.**

Wheel speeds are most often quoted relative to the spacecraft. Thus, in calculating the absolute momentum of any wheel, account must also be made of the spacecraft's angular velocity, ω :

$$h_i = J_i(s_i + \omega \cdot \hat{e}_i)$$

where

J_i is the axial inertia of the i^{th} wheel;

s_i is speed of the i^{th} wheel relative to the spacecraft;

$\hat{e}_i$ is a unit vector aligned with the axis of the i^{th} wheel.

So rearranging for the relative wheel speed:

$$s_i = \frac{h_i}{J_i} - \omega \cdot \hat{e}_i.$$

• **Wheel torque.**

The total torque experienced by a wheel has both an axial component and a bearing component:

$$\begin{aligned} \tau_{wh} &= \tau_{ax} + \tau_{bearing} \\ &= \tau_{ax_i} \hat{e}_i + \omega \times h_i \hat{e}_i. \\ \left(\text{cf. } \tau_i &= \dot{h}_i + \omega \times h_i. \right) \end{aligned}$$

• **Sun angle.**

$\angle$ between y -flight axis & y -control axis.

• **Residual nutation.**

$\angle$ between H 's location & H 's desired location (seen from a body-fixed frame).

• **Linear acceleration of an arbitrary point.**

First, the angular acceleration of the spacecraft must be found:

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = [\mathbf{I}]^{-1} \dot{\mathbf{H}}_B = [\mathbf{I}]^{-1} (\dot{\mathbf{H}} - \dot{\mathbf{h}}).$$

Then the linear acceleration, relative to the center of mass (c.o.m.), of a point located with a position vector $\mathbf{r}$, also relative to the c.o.m., would be:

$$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}).$$

Appendix B

Maneuver Quaternions.

Following are listings of maneuver quaternions for the 20 Nms versions of the two proposals. They are presented so as to allow others "to see" how the satellite body reorients. These quaternions are expressed relative to an orbit-oriented flight frame, see Fig. 3.2a; i.e. in compiling these quaternions, the orbital motion has already been included. The scalar and vector parts of these quaternions are as defined in Appendix A.

Proposal #1

Time (min.)	q1	q2	q3	q4
0.000	0.0000E+00	0.0000E+00	0.0000E+00	1.0000E+00
0.500	-9.0639E-04	-1.7583E-03	-2.3861E-04	1.0000E+00
1.000	-3.5061E-03	-7.0236E-03	-1.0699E-03	9.9997E-01
1.500	-7.5767E-03	-1.5780E-02	-2.6516E-03	9.9984E-01
2.000	-1.2855E-02	-2.8011E-02	-5.1160E-03	9.9951E-01
2.500	-1.9054E-02	-4.3697E-02	-8.5666E-03	9.9883E-01
3.000	-2.5886E-02	-6.2821E-02	-1.3076E-02	9.9760E-01
3.500	-3.3068E-02	-8.5363E-02	-1.8687E-02	9.9563E-01
4.000	-4.0343E-02	-1.1130E-01	-2.5416E-02	9.9264E-01
4.500	-4.7480E-02	-1.4060E-01	-3.3254E-02	9.8837E-01
5.000	-5.4289E-02	-1.7323E-01	-4.2174E-02	9.8248E-01
5.500	-6.0619E-02	-2.0912E-01	-5.2135E-02	9.7462E-01
6.000	-6.6363E-02	-2.4818E-01	-6.3091E-02	9.6438E-01
6.500	-7.1460E-02	-2.9027E-01	-7.4995E-02	9.5132E-01
7.000	-7.5915E-02	-3.3517E-01	-8.7744E-02	9.3499E-01
7.500	-8.1498E-02	-3.8027E-01	-9.7898E-02	9.1606E-01
8.000	-9.1189E-02	-4.2406E-01	-1.0441E-01	8.9496E-01
8.500	-1.0738E-01	-4.6631E-01	-1.0892E-01	8.7130E-01
9.000	-1.2971E-01	-5.0702E-01	-1.1529E-01	8.4428E-01
9.500	-1.5564E-01	-5.4617E-01	-1.2671E-01	8.1328E-01
10.000	-1.8437E-01	-5.8312E-01	-1.4361E-01	7.7804E-01
10.500	-2.1503E-01	-6.1714E-01	-1.6629E-01	7.3841E-01
11.000	-2.4668E-01	-6.4739E-01	-1.9486E-01	6.9430E-01

11.500	-2.7834E-01	-6.7299E-01	-2.2925E-01	6.4580E-01
12.000	-3.0901E-01	-6.9298E-01	-2.6918E-01	5.9315E-01
12.500	-3.3772E-01	-7.0646E-01	-3.1418E-01	5.3680E-01
13.000	-3.6353E-01	-7.1257E-01	-3.6356E-01	4.7740E-01
13.500	-3.8560E-01	-7.1060E-01	-4.1644E-01	4.1585E-01
14.000	-4.0320E-01	-7.0006E-01	-4.7179E-01	3.5321E-01
14.500	-4.1571E-01	-6.8073E-01	-5.2845E-01	2.9075E-01
15.000	-4.2272E-01	-6.5274E-01	-5.8517E-01	2.2982E-01
15.500	-4.2397E-01	-6.1660E-01	-6.4072E-01	1.7184E-01
16.000	-4.1941E-01	-5.7324E-01	-6.9392E-01	1.1819E-01
16.500	-4.0919E-01	-5.2397E-01	-7.4371E-01	7.0087E-02
17.000	-3.9361E-01	-4.7043E-01	-7.8927E-01	2.8555E-02
17.500	-3.7314E-01	-4.1450E-01	-8.3002E-01	-5.6995E-03
18.000	-3.4833E-01	-3.5816E-01	-8.6565E-01	-3.2311E-02
18.500	-3.1981E-01	-3.0338E-01	-8.9614E-01	-5.1269E-02
19.000	-2.8809E-01	-2.5215E-01	-9.2165E-01	-6.3139E-02
19.500	-2.5295E-01	-2.0768E-01	-9.4214E-01	-7.2549E-02
20.000	-2.1503E-01	-1.6965E-01	-9.5823E-01	-8.2381E-02
20.500	-1.7517E-01	-1.3624E-01	-9.7053E-01	-9.4000E-02
21.000	-1.3428E-01	-1.0423E-01	-9.7986E-01	-1.0482E-01
21.500	-9.2748E-02	-7.2709E-02	-9.8660E-01	-1.1287E-01
22.000	-5.0647E-02	-4.2077E-02	-9.9080E-01	-1.1825E-01
22.500	-8.0552E-03	-1.2700E-02	-9.9252E-01	-1.2112E-01
23.000	3.4962E-02	1.5093E-02	-9.9185E-01	-1.2162E-01
23.500	7.8346E-02	4.1019E-02	-9.8883E-01	-1.1994E-01
24.000	1.2204E-01	6.4834E-02	-9.8355E-01	-1.1629E-01
24.500	1.6600E-01	8.6335E-02	-9.7606E-01	-1.1087E-01
25.000	2.1016E-01	1.0536E-01	-9.6640E-01	-1.0390E-01
25.500	2.5447E-01	1.2178E-01	-9.5460E-01	-9.5632E-02
26.000	2.9884E-01	1.3551E-01	-9.4069E-01	-8.6287E-02
26.500	3.4318E-01	1.4649E-01	-9.2465E-01	-7.6119E-02
27.000	3.8739E-01	1.5470E-01	-9.0649E-01	-6.5381E-02
27.500	4.3133E-01	1.6015E-01	-8.8620E-01	-5.4335E-02
28.000	4.7486E-01	1.6289E-01	-8.6377E-01	-4.3250E-02
28.500	5.1779E-01	1.6301E-01	-8.3921E-01	-3.2399E-02
29.000	5.5995E-01	1.6062E-01	-8.1251E-01	-2.2056E-02
29.500	6.0113E-01	1.5587E-01	-7.8371E-01	-1.2497E-02
30.000	6.4111E-01	1.4896E-01	-7.5284E-01	-3.9889E-03
30.500	6.7970E-01	1.4011E-01	-7.1998E-01	3.2099E-03
31.000	7.1668E-01	1.2957E-01	-6.8520E-01	8.8584E-03
31.500	7.5186E-01	1.1762E-01	-6.4862E-01	1.2737E-02
32.000	7.8506E-01	1.0457E-01	-6.1035E-01	1.4653E-02
32.500	8.1610E-01	9.0725E-02	-5.7056E-01	1.4461E-02
33.000	8.4369E-01	7.7097E-02	-5.3111E-01	1.3085E-02
33.500	8.6733E-01	6.4210E-02	-4.9344E-01	1.1100E-02
34.000	8.8740E-01	5.2120E-02	-4.5797E-01	8.4724E-03
34.500	9.0426E-01	4.0895E-02	-4.2499E-01	5.2181E-03
35.000	9.1826E-01	3.0610E-02	-3.9480E-01	1.4026E-03
35.500	9.2973E-01	2.1337E-02	-3.6761E-01	-2.8599E-03
36.000	9.3900E-01	1.3143E-02	-3.4359E-01	-7.4083E-03
36.500	9.4633E-01	6.0763E-03	-3.2292E-01	-1.2040E-02
37.000	9.5199E-01	1.7081E-04	-3.0570E-01	-1.6518E-02
37.500	9.5617E-01	-4.5643E-03	-2.9204E-01	-2.0580E-02
38.000	9.5907E-01	-8.1448E-03	-2.8204E-01	-2.3952E-02
38.500	9.6081E-01	-1.0614E-02	-2.7575E-01	-2.6362E-02
39.000	9.6148E-01	-1.2042E-02	-2.7322E-01	-2.7555E-02

Proposal #2

Time (min.)	q1	q2	q3	q4
0.000	0.0000E+00	0.0000E+00	0.0000E+00	1.0000E+00
0.500	-9.0639E-04	-1.7583E-03	-2.3861E-04	1.0000E+00
1.000	-3.5061E-03	-7.0236E-03	-1.0699E-03	9.9997E-01
1.500	-7.5767E-03	-1.5780E-02	-2.6516E-03	9.9984E-01
2.000	-1.2855E-02	-2.8011E-02	-5.1160E-03	9.9951E-01
2.500	-1.9054E-02	-4.3697E-02	-8.5666E-03	9.9883E-01
3.000	-2.5886E-02	-6.2821E-02	-1.3076E-02	9.9760E-01
3.500	-3.3068E-02	-8.5363E-02	-1.8687E-02	9.9563E-01
4.000	-4.0343E-02	-1.1130E-01	-2.5416E-02	9.9264E-01
4.500	-4.7480E-02	-1.4060E-01	-3.3254E-02	9.8837E-01
5.000	-5.4289E-02	-1.7323E-01	-4.2174E-02	9.8248E-01
5.500	-6.0619E-02	-2.0912E-01	-5.2135E-02	9.7462E-01
6.000	-6.6363E-02	-2.4818E-01	-6.3091E-02	9.6438E-01
6.500	-7.1460E-02	-2.9027E-01	-7.4995E-02	9.5132E-01
7.000	-7.5836E-02	-3.3519E-01	-8.7852E-02	9.3498E-01
7.500	-7.5371E-02	-3.8113E-01	-1.0398E-01	9.1556E-01
8.000	-6.5727E-02	-4.2701E-01	-1.2330E-01	8.9339E-01
8.500	-4.4564E-02	-4.7259E-01	-1.4343E-01	8.6839E-01
9.000	-1.2334E-02	-5.1732E-01	-1.5960E-01	8.4069E-01
9.500	2.8210E-02	-5.6061E-01	-1.6765E-01	8.1044E-01
10.000	7.5912E-02	-6.0206E-01	-1.6612E-01	7.7728E-01
10.500	1.2932E-01	-6.4100E-01	-1.5368E-01	7.4080E-01
11.000	1.8667E-01	-6.7649E-01	-1.2921E-01	7.0058E-01
11.500	2.4585E-01	-7.0735E-01	-9.2001E-02	6.5631E-01
12.000	3.0448E-01	-7.3217E-01	-4.1854E-02	6.0783E-01
12.500	3.5998E-01	-7.4945E-01	2.0752E-02	5.5526E-01
13.000	4.0975E-01	-7.5772E-01	9.4577E-02	4.9902E-01
13.500	4.5142E-01	-7.5572E-01	1.7762E-01	4.3995E-01
14.000	4.8302E-01	-7.4262E-01	2.6723E-01	3.7921E-01
14.500	5.0323E-01	-7.1807E-01	3.6034E-01	3.1825E-01
15.000	5.1147E-01	-6.8233E-01	4.5376E-01	2.5869E-01
15.500	5.0790E-01	-6.3618E-01	5.4448E-01	2.0213E-01
16.000	4.9330E-01	-5.8087E-01	6.2987E-01	1.5006E-01
16.500	4.6891E-01	-5.1794E-01	7.0788E-01	1.0374E-01
17.000	4.3627E-01	-4.4913E-01	7.7707E-01	6.4136E-02
17.500	3.9700E-01	-3.7626E-01	8.3654E-01	3.1913E-02
18.000	3.5268E-01	-3.0115E-01	8.8593E-01	7.4349E-03
18.500	3.0478E-01	-2.2556E-01	9.2528E-01	-9.2044E-03
19.000	2.5460E-01	-1.5119E-01	9.5498E-01	-1.8154E-02
19.500	2.0323E-01	-7.9593E-02	9.7569E-01	-1.9775E-02
20.000	1.5152E-01	-1.2194E-02	9.8827E-01	-1.4602E-02
20.500	1.0014E-01	4.9774E-02	9.9372E-01	-3.3169E-03
21.000	4.9529E-02	1.0527E-01	9.9312E-01	1.3293E-02
21.500	-5.3514E-05	1.5348E-01	9.8755E-01	3.4362E-02
22.000	-4.8520E-02	1.9378E-01	9.7807E-01	5.8974E-02
22.500	-9.5918E-02	2.2575E-01	9.6561E-01	8.6187E-02
23.000	-1.4240E-01	2.4911E-01	9.5102E-01	1.1504E-01
23.500	-1.8812E-01	2.6374E-01	9.3501E-01	1.4428E-01
24.000	-2.3245E-01	2.7022E-01	9.1896E-01	1.6872E-01
24.500	-2.7528E-01	2.7101E-01	9.0333E-01	1.8649E-01
25.000	-3.1674E-01	2.6882E-01	8.8730E-01	1.9759E-01
25.500	-3.5753E-01	2.6544E-01	8.7138E-01	2.0593E-01
26.000	-3.9804E-01	2.6099E-01	8.5308E-01	2.1379E-01
26.500	-4.3817E-01	2.5546E-01	8.3298E-01	2.2111E-01
27.000	-4.7779E-01	2.4884E-01	8.1110E-01	2.2783E-01

27.500	-5.1678E-01	2.4114E-01	7.8745E-01	2.3391E-01
28.000	-5.5499E-01	2.3239E-01	7.6205E-01	2.3928E-01
28.500	-5.9226E-01	2.2262E-01	7.3497E-01	2.4390E-01
29.000	-6.2844E-01	2.1188E-01	7.0626E-01	2.4772E-01
29.500	-6.6336E-01	2.0024E-01	6.7602E-01	2.5070E-01
30.000	-6.9689E-01	1.8777E-01	6.4434E-01	2.5282E-01
30.500	-7.2886E-01	1.7456E-01	6.1134E-01	2.5407E-01
31.000	-7.5917E-01	1.6069E-01	5.7715E-01	2.5443E-01
31.500	-7.8771E-01	1.4628E-01	5.4189E-01	2.5392E-01
32.000	-8.1437E-01	1.3142E-01	5.0571E-01	2.5256E-01
32.500	-8.3910E-01	1.1620E-01	4.6875E-01	2.5037E-01
33.000	-8.6184E-01	1.0074E-01	4.3112E-01	2.4741E-01
33.500	-8.8258E-01	8.5128E-02	3.9296E-01	2.4371E-01
34.000	-9.0129E-01	6.9444E-02	3.5437E-01	2.3934E-01
34.500	-9.1797E-01	5.3773E-02	3.1545E-01	2.3436E-01
35.000	-9.3266E-01	3.8185E-02	2.7627E-01	2.2884E-01
35.500	-9.4536E-01	2.2742E-02	2.3690E-01	2.2285E-01
36.000	-9.5610E-01	7.4976E-03	1.9738E-01	2.1645E-01
36.500	-9.6492E-01	-7.5034E-03	1.5776E-01	2.0972E-01
37.000	-9.7184E-01	-2.2224E-02	1.1804E-01	2.0272E-01
37.500	-9.7689E-01	-3.6636E-02	7.8248E-02	1.9552E-01
38.000	-9.8007E-01	-5.0713E-02	3.8376E-02	1.8819E-01
38.500	-9.8141E-01	-6.4439E-02	-1.5774E-03	1.8077E-01
39.000	-9.8090E-01	-7.7799E-02	-4.1618E-02	1.7334E-01
39.500	-9.7854E-01	-9.0780E-02	-8.1751E-02	1.6592E-01
40.000	-9.7431E-01	-1.0337E-01	-1.2198E-01	1.5858E-01
40.500	-9.6819E-01	-1.1557E-01	-1.6229E-01	1.5136E-01
41.000	-9.6015E-01	-1.2737E-01	-2.0267E-01	1.4429E-01
41.500	-9.5015E-01	-1.3875E-01	-2.4309E-01	1.3740E-01
42.000	-9.3815E-01	-1.4972E-01	-2.8351E-01	1.3072E-01
42.500	-9.2412E-01	-1.6025E-01	-3.2386E-01	1.2428E-01
43.000	-9.0802E-01	-1.7035E-01	-3.6406E-01	1.1808E-01
43.500	-8.8982E-01	-1.8001E-01	-4.0403E-01	1.1215E-01
44.000	-8.6951E-01	-1.8921E-01	-4.4363E-01	1.0648E-01
44.500	-8.4709E-01	-1.9794E-01	-4.8274E-01	1.0108E-01
45.000	-8.2257E-01	-2.0620E-01	-5.2121E-01	9.5947E-02
45.500	-7.9598E-01	-2.1397E-01	-5.5888E-01	9.1059E-02
46.000	-7.6738E-01	-2.2124E-01	-5.9558E-01	8.6403E-02
46.500	-7.3685E-01	-2.2802E-01	-6.3114E-01	8.1956E-02
47.000	-7.0450E-01	-2.3430E-01	-6.6540E-01	7.7691E-02
47.500	-6.7043E-01	-2.4007E-01	-6.9819E-01	7.3576E-02
48.000	-6.3480E-01	-2.4534E-01	-7.2938E-01	6.9573E-02
48.500	-5.9776E-01	-2.5011E-01	-7.5883E-01	6.5643E-02
49.000	-5.5948E-01	-2.5439E-01	-7.8642E-01	6.1744E-02
49.500	-5.2012E-01	-2.5818E-01	-8.1208E-01	5.7832E-02
50.000	-4.7986E-01	-2.6150E-01	-8.3574E-01	5.3860E-02
50.500	-4.3886E-01	-2.6434E-01	-8.5735E-01	4.9785E-02
51.000	-3.9729E-01	-2.6672E-01	-8.7690E-01	4.5560E-02
51.500	-3.5528E-01	-2.6864E-01	-8.9438E-01	4.1142E-02
52.000	-3.1297E-01	-2.7011E-01	-9.0981E-01	3.6486E-02
52.500	-2.7048E-01	-2.7112E-01	-9.2322E-01	3.1553E-02
53.000	-2.2789E-01	-2.7168E-01	-9.3465E-01	2.6304E-02
53.500	-1.8566E-01	-2.7179E-01	-9.4405E-01	2.0788E-02
54.000	-1.4641E-01	-2.7167E-01	-9.5106E-01	1.5580E-02
54.500	-1.1074E-01	-2.7150E-01	-9.5598E-01	1.0767E-02
55.000	-7.8702E-02	-2.7141E-01	-9.5922E-01	6.3139E-03
55.500	-5.0336E-02	-2.7146E-01	-9.6113E-01	2.1825E-03
56.000	-2.5655E-02	-2.7170E-01	-9.6204E-01	-1.6606E-03
56.500	-4.6600E-03	-2.7211E-01	-9.6224E-01	-5.2403E-03
57.000	1.2651E-02	-2.7264E-01	-9.6200E-01	-8.5679E-03

57.500	2.6285E-02	-2.7321E-01	-9.6152E-01	-1.1637E-02
58.000	3.6250E-02	-2.7373E-01	-9.6102E-01	-1.4420E-02
58.500	4.2553E-02	-2.7404E-01	-9.6063E-01	-1.6868E-02
59.000	4.5203E-02	-2.7403E-01	-9.6047E-01	-1.8907E-02
59.500	4.4303E-02	-2.7353E-01	-9.6062E-01	-2.0473E-02
60.000	4.2203E-02	-2.7179E-01	-9.6117E-01	-2.2148E-02
60.500	3.9932E-02	-2.6845E-01	-9.6216E-01	-2.4082E-02
61.000	3.7521E-02	-2.6344E-01	-9.6359E-01	-2.6109E-02
61.500	3.5012E-02	-2.5681E-01	-9.6542E-01	-2.8067E-02
62.000	3.2453E-02	-2.4863E-01	-9.6760E-01	-2.9796E-02
62.500	2.9897E-02	-2.3909E-01	-9.7004E-01	-3.1153E-02
63.000	2.7395E-02	-2.2841E-01	-9.7265E-01	-3.2016E-02
63.500	2.5001E-02	-2.1686E-01	-9.7535E-01	-3.2286E-02
64.000	2.2761E-02	-2.0478E-01	-9.7802E-01	-3.1897E-02
64.500	2.0719E-02	-1.9249E-01	-9.8060E-01	-3.0817E-02
65.000	1.8910E-02	-1.8037E-01	-9.8299E-01	-2.9047E-02
65.500	1.7363E-02	-1.6874E-01	-9.8515E-01	-2.6625E-02
66.000	1.6098E-02	-1.5795E-01	-9.8703E-01	-2.3623E-02
66.500	1.5128E-02	-1.4826E-01	-9.8863E-01	-2.0141E-02
67.000	1.4453E-02	-1.3991E-01	-9.8992E-01	-1.6306E-02
67.500	1.4068E-02	-1.3307E-01	-9.9093E-01	-1.2264E-02
68.000	1.3956E-02	-1.2784E-01	-9.9166E-01	-8.1714E-03
68.500	1.4089E-02	-1.2425E-01	-9.9214E-01	-4.1915E-03
69.000	1.4431E-02	-1.2224E-01	-9.9240E-01	-4.8462E-04
69.500	1.4936E-02	-1.2170E-01	-9.9245E-01	2.7985E-03
70.000	1.5548E-02	-1.2244E-01	-9.9234E-01	5.5236E-03
70.500	1.6209E-02	-1.2423E-01	-9.9209E-01	7.5786E-03
71.000	1.6855E-02	-1.2677E-01	-9.9175E-01	8.8788E-03
71.500	1.7424E-02	-1.2976E-01	-9.9135E-01	9.3707E-03
72.000	1.7858E-02	-1.3285E-01	-9.9093E-01	9.0338E-03
72.500	1.8104E-02	-1.3570E-01	-9.9055E-01	7.8818E-03
73.000	1.8124E-02	-1.3800E-01	-9.9025E-01	5.9617E-03
73.500	1.7890E-02	-1.3944E-01	-9.9006E-01	3.3513E-03
74.000	1.7391E-02	-1.3975E-01	-9.9003E-01	1.5635E-04
74.500	1.6632E-02	-1.3873E-01	-9.9018E-01	-3.4949E-03
75.000	1.5631E-02	-1.3622E-01	-9.9053E-01	-7.4568E-03
75.500	1.4422E-02	-1.3215E-01	-9.9106E-01	-1.1573E-02
76.000	1.3047E-02	-1.2650E-01	-9.9176E-01	-1.5681E-02
76.500	1.1559E-02	-1.1933E-01	-9.9259E-01	-1.9624E-02
77.000	1.0012E-02	-1.1076E-01	-9.9352E-01	-2.3251E-02
77.500	8.4616E-03	-1.0099E-01	-9.9450E-01	-2.6429E-02
78.000	6.9627E-03	-9.0262E-02	-9.9547E-01	-2.9045E-02
78.500	5.5640E-03	-7.8872E-02	-9.9639E-01	-3.1014E-02
79.000	4.3078E-03	-6.7142E-02	-9.9721E-01	-3.2281E-02
79.500	3.2283E-03	-5.5411E-02	-9.9792E-01	-3.2822E-02
80.000	2.3515E-03	-4.4020E-02	-9.9849E-01	-3.2650E-02
80.500	1.6946E-03	-3.3298E-02	-9.9894E-01	-3.1809E-02
81.000	1.2666E-03	-2.3541E-02	-9.9926E-01	-3.0373E-02
81.500	1.0682E-03	-1.5007E-02	-9.9948E-01	-2.8445E-02
82.000	1.0929E-03	-7.9005E-03	-9.9963E-01	-2.6150E-02
82.500	1.3265E-03	-2.3653E-03	-9.9972E-01	-2.3628E-02
83.000	1.7483E-03	1.5201E-03	-9.9978E-01	-2.1031E-02
83.500	2.3307E-03	3.7459E-03	-9.9982E-01	-1.8513E-02
84.000	3.0401E-03	4.3703E-03	-9.9985E-01	-1.6225E-02
84.500	3.8374E-03	3.5168E-03	-9.9988E-01	-1.4310E-02
85.000	4.6791E-03	1.3693E-03	-9.9991E-01	-1.2892E-02
85.500	5.5192E-03	-1.8365E-03	-9.9991E-01	-1.2077E-02
86.000	6.3106E-03	-5.8220E-03	-9.9989E-01	-1.1942E-02
86.500	7.0080E-03	-1.0278E-02	-9.9984E-01	-1.2540E-02
87.000	7.5701E-03	-1.4876E-02	-9.9976E-01	-1.3887E-02

87.500	7.9625E-03	-1.9284E-02	-9.9965E-01	-1.5971E-02
88.000	8.1598E-03	-2.3182E-02	-9.9952E-01	-1.8746E-02
88.500	8.1471E-03	-2.6269E-02	-9.9938E-01	-2.2139E-02
89.000	7.9211E-03	-2.8287E-02	-9.9923E-01	-2.6048E-02
89.500	7.4901E-03	-2.9021E-02	-9.9909E-01	-3.0347E-02
90.000	6.8729E-03	-2.8317E-02	-9.9897E-01	-3.4836E-02
90.500	6.0971E-03	-2.6085E-02	-9.9886E-01	-3.9540E-02
91.000	5.1962E-03	-2.2306E-02	-9.9876E-01	-4.4120E-02
91.500	4.2074E-03	-1.7029E-02	-9.9867E-01	-4.8479E-02
92.000	3.1682E-03	-1.0373E-02	-9.9856E-01	-5.2466E-02
92.500	2.1139E-03	-2.5213E-03	-9.9843E-01	-5.5949E-02
93.000	1.0752E-03	6.2887E-03	-9.9825E-01	-5.8812E-02
93.500	7.6836E-05	1.5774E-02	-9.9802E-01	-6.0968E-02
94.000	-8.6317E-04	2.5621E-02	-9.9772E-01	-6.2360E-02
94.500	-1.7335E-03	3.5497E-02	-9.9738E-01	-6.2964E-02
95.000	-2.5290E-03	4.5069E-02	-9.9701E-01	-6.2788E-02
95.500	-3.2494E-03	5.4014E-02	-9.9662E-01	-6.1874E-02
96.000	-3.8986E-03	6.2038E-02	-9.9624E-01	-6.0296E-02

Appendix C

Additional Clarifications.

A few notes might be added for additional clarification (for symbolic notation, cf. Appendix A).

First, one must distinguish between a wheel's "relative" axial momentum and its "absolute" axial momentum. Relative momentum would be calculated using the wheel's rotational speed *relative to* the satellite body; i.e.

$$h_{\text{rel},i} = J_i s_i.$$

Absolute momentum (the kind of momentum used throughout this thesis) would be calculated using an inertially referenced angular velocity; i.e. one must account not only for the relative wheel speed, but also for the satellite's absolute inertial angular velocity

$$h_i = J_i(s_i + \omega \cdot \hat{e}_i).$$

With reference to the RADARSAT application pursued herein, it can be noted that since h_i is of the order of 10, J_i the order of 0.1, and ω the order of 10^{-3} ,

$$h_i \approx h_{\text{rel},i}.$$

However, no such approximation was used in this thesis investigation.

Second, to properly separate the wheel momentum from the body momentum, a proper definition of the satellite body's inertia matrix should be made. In the spacecraft industry, one often measures (or calculates) a total satellite inertia matrix, $[I_T]$, that includes both body and wheel inertias. The body inertia will be

defined as this total inertia less the axial inertias of the wheels

$$[\mathbf{I}] = [\mathbf{I}_T] - [\mathbf{J}]$$

where

$[\mathbf{J}]$ is an appropriately constructed 3×3 inertia matrix that expresses all of the—and only the—wheel axial inertias.

For example, in the case of 3 wheels that have mutually perpendicular axes, the wheel matrix could be expressed as

$$[\mathbf{J}] = [\mathbf{C}] \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} [\mathbf{C}]^T$$

where

$[\mathbf{C}]$ is a coordinate transformation matrix that relates the 'wheel axis' frame to the body frame in which $[\mathbf{I}_T]$ is measured.

In the case of RADARSAT (Tables 3.1 and 3.3), it can be said that

$$[\mathbf{I}] \approx [\mathbf{I}_T]$$

to approximately 5 significant figures.

Third, the relation between the various momentum and inertia concepts just discussed can be expressed as

$$\mathbf{H} = \underbrace{[\mathbf{I}]\boldsymbol{\omega}}_{\mathbf{H}_B} + \mathbf{h} = [\mathbf{I}_T]\boldsymbol{\omega} + \mathbf{h}_{\text{rel}}.$$

Lastly, and to further clarify the multi-staging process, the following pages depict tables and time history plots of certain parameters. Plotted for each of the two proposed strategies and for each of the three wheels are net axial wheel torque, absolute axial wheel momentum, and relative wheel speed. [In keeping with the assumption of §3.2, the roll wheel speeds were assumed positive in the sense of the positive roll direction; i.e. opposite the sense assumed in some RADARSAT

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documents.] By initiating the given torquing sequences, the wheel momenta were controlled. Equally effective would have been a tracking of the given wheel speed profiles.

Plots of the body's rotational kinetic energy (actually, of a semi-normalized energy— E/H^2) are also presented for each of the two strategies. The gravity gradient torque prevented the energy level from remaining constant during the periods of constant wheel momentum (i.e. during the holds).

Stage #	Timing Limits on Stage		Wheel		
			Roll	Pitch	Yaw
	from $t =$ min	until $t =$ min	τ_{ax} Nm	τ_{ax} Nm	τ_{ax} Nm
1	0	6.93	0.0103	0.12	0.0074
2	6.93	8.52	0	0	-0.21
3 (the 1 st hold)	8.52	18.85	0	0	0
4	18.85	20.44	0	0	0.21
5 (the 2 nd hold)	20.44	32.44	0	0	0
6	32.44	39.36	0.0099	0.12	0.0011

Table C.1: Proposal #1—stage definition and net axial wheel torques.

Time Into Maneuver	Wheel					
	Roll		Pitch		Yaw	
	h Nms	s rad/s	h Nms	s rad/s	h Nms	s rad/s
0	-4.3	-64	-49.9	-548	-3.1	-46
6.93	0	0	0	0	0	0
8.52	0	0	0	0	-20	-299
18.85	0	0	0	0	-20	-299
20.44	0	0	0	0	0	0
32.44	0	0	0	0	0	0
39.36	4.1	61	49.9	548	0.5	7

Table C.2: Proposal #1—axial momenta and wheel speeds at stage junctures.

Figure C.1: Proposal #1—wheel torque profiles.

Displayed are plots of “net axial wheel torque” vs “time into maneuver”. The vertical dashed lines define the various stages of the maneuver.

Figure C.2: Proposal #1—wheel angular momentum profiles. Displayed are plots of “absolute axial wheel momentum” vs “time into maneuver”. The vertical dashed lines define the various stages of the maneuver.

Figure C.3: Proposal #1—wheel speed profiles.

Displayed are plots of “relative wheel speed” vs “time into maneuver”. The vertical dashed lines define the various stages of the maneuver.

Stage #	Timing Limits on Stage		Wheel		
			Roll	Pitch	Yaw
	from $t =$ min	until $t =$ min	τ_{ax} Nm	τ_{ax} Nm	τ_{ax} Nm
1	0	6.93	0.0103	0.12	0.0074
2	6.93	8.52	0	0	0.21
3 (the 1 st hold)	8.52	23.35	0	0	0
4	23.35	24.94	0	0	-0.21
5 (the 2 nd hold)	24.94	53.27	0	0	0
6	53.27	59.38	-0.0178	0.12	-0.0621
7	59.38	96.32	0.0048	0.0026	0.0105

Table C.3: Proposal #2—stage definition and net axial wheel torques.

Time Into Maneuver	Wheel					
	Roll		Pitch		Yaw	
	h min	s Nms	h Nms	s rad/s	h Nms	s rad/s
0	-4.3	-64	-49.9	-548	-3.1	-46
6.93	0	0	0	0	0	0
8.52	0	0	0	0	20	299
23.35	0	0	0	0	20	299
24.94	0	0	0	0	0	0
53.27	0	0	0	0	0	0
59.38	-6.5	-97	44.0	484	-22.8	-340
96.32	4.1	61	49.9	548	0.5	7

Table C.4: Proposal #2—axial momenta and wheel speeds at stage junctures.

Figure C.4: Proposal #2--wheel torque profiles.

Displayed are plots of "net axial wheel torque" vs "time into maneuver". The vertical dashed lines define the various stages of the maneuver.

Figure C.5: Proposal #2—wheel angular momentum profiles. Displayed are plots of “absolute axial wheel momentum” vs “time into maneuver”. The vertical dashed lines define the various stages of the maneuver.

Figure C.6: Proposal #2—wheel speed profiles.

Displayed are plots of “relative wheel speed” vs “time into maneuver”. The vertical dashed lines define the various stages of the maneuver.

Figure C.7: Semi-normalized energy profiles.

Displayed are plots of the body's semi-normalized rotational kinetic energy (E/H^2) vs "time into maneuver". The vertical dashed lines define the various stages of the maneuver.

END

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