Modeling of the Surface Marks Formation in an Immovable Mold during Continuous Casting of Steel

by

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Abstract

Continuous casting is the main process used today for manufacturing of billets. During continuous casting the surface marks (SM) are forming on the surface of continuous casting billets. The surface marks can cause cracking and can decrease the yield of the casting process because the billet surface containing marks has to be grinded away to avoid crack growth. Using a mathematical model, it is possible to understand how to decrease the negative impact of the SM formation process.

Most of previously developed models of surface mark formation had a phenomenological, qualitative character and the major factor determining process of mark formation in these models was the mold oscillation. The basic mathematical model of SM formation in which the surface of the liquid metal in the mold is free from additional factors such as reciprocating mold movement does not exist. The objective of this work is to create the mathematical model of the SM formation during continuous casting in an immovable mold.

In order to numerically describe the complex process of SM formation the two stage approach for the description of the cycle of SM formation was developed. For the thermal analysis the equation of transient heat transfer with the source function has been derived. An initial condition and boundary conditions were formulated based upon a physical conception of SM formation process. The temperature distribution on meniscus during first stage was calculated. Adequacy testing of the mechanical part of model (using mathematical statistics) demonstrates excellent correspondence between the model and experimental data.

The developed model can be considered as a basic model for the simulation of the initial solidification process under different conditions during continuous casting.

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List of symbols

V_m	instantaneous mold velocity, mm/min;
$V_{average}$	average mold velocity, mm/min;
V_C	the casting velocity, mm/min;
s	the stroke length, mm;
f	the oscillation frequency, 1/min;
t	the cycle time, s;
t_c	total cycle time, s;
t_N	negative strip time interval in each cycle of mold, s;
a	the Laplace capillary constant;
S, R_S	forces acting on meniscus, N;
R	radius of curvature, mm;
φ	angle of inclination at any point on the meniscus, rad;
dl	length of the infinitely small section of meniscus, mm;
T	transverse force, N;
M	bending moment, N*m;
$d\varphi, dX, dY$	increments of the angle and coordinate;
dS, dT, dM	increments of the appropriate values;

Q	the pressure of liquid metal (distributed load) - from the side of liquid metal, N/m^2 ;
q	the pressure of liquid slag (distributed load) - from the side of liquid slag, N/m^2 ;
R_S^{beg}	longitudinal reaction at the beginning of a meniscus section, N;
R_S^{end}	longitudinal reaction at the end of a meniscus section, N;
Δb	width of the considered section of meniscus in the direction, perpendicular to the plane of drawing, mm;
σ	the coefficient of interphase tension between liquid metal and liquid slag, N/m;
R_Q	the resultants of distributed load Q , N;
R_q	the resultants of distributed load q , N;
g	gravity acceleration, $\text{N/m} \cdot \text{s}^2$;
ρ_m	the density of liquid metal, Kg/m^3 ;
ρ_{sl}	the density of fluid slag, Kg/m^3 ;
θ_0	initial angle, rad;
δx	the height of the not solidifying part of the meniscus (initial section of meniscus), mm;
Δh_{sl}	slag rim thickness, mm;
Δy	solid slag thickness on the mold wall (below slag rim), mm;

Δh_{sl-f}	thickness of the liquid layer between the meniscus and the solid slag and rim, mm;
l_0	initial base's length of RE, mm;
δ_T	size of prism in the direction, perpendicular to the tangent to the meniscus (thickness of the layer of thermal perturbation), mm;
Δl	the RE (control section of meniscus) extension during meniscus stretching, mm;
ΔQ_V	the summary heat content of the RE; J
ΔQ_F	the RE heat content due to the substance, which left from the core during the meniscus stretching, J;
ΔQ_Δ	heat transfer from the RE to the mold wall, J;
c	the coefficient of the liquid metal heat capacity, kJ/(kg °C.);
λ	the coefficient of the thermal conductivity, W/(m °C);
T_M	the metal temperature in the core of ingot, °C;
T	temperature of metal on the boundary RE - liquid slag °C;
T_{sl-f}	liquid slag temperature °C;
T_{cr-sl}	temperature on the boundary liquid slag - solid slag, the temperature of the liquid slag crystallization °C;
T_{sl}	temperature of solid slag on the boundary solid slag - the mold wall °C;

T_{w-s}	temperature of the mold wall from the solid slag side °C;
T_{w-w}	temperature of the mold wall from the cooling water side °C;
T_{wat}	temperature of the cooling water °C;
α	thermal diffusivity coefficient, m ² /sec;
R_{Σ}	total heat resistance
u_{∞}	average velocity rate out of the boundary layer limits
Pr	Prandtl number;
Re	the Reynolds number;
ω	the speed of the liquid metal flow;
μ	the kinematics viscosity coefficient of liquid metal, poise;
r	the displacement of the starting point, mm;
RSS	sum of squares of residuals;
$N - p - 1$	The number of freedom;
N	the number of experimental data;
p	the number of the model parameters;
F^{α}	criterion of Fisher;
Δh_{gap}	gap thickness mm.

1. Literature Review

1.1. Introduction. A brief history of the continuous casting process

There are two basic casting methods used today for billets manufacturing: ingot casting and continuous casting. The benefits of the latter, such as:

- Higher yield
- Smaller number of necessary manufacturing steps
- More mechanized casting process
- More even composition
- Better surface finish.

make it the main process for billets manufacturing. The first continuous strip casting process was proposed by Bessemer in 1856 (Figure 1.1).

Figure 1.1 The twin-roll caster suggested by Bessemer [1]

At that time the major problem was “breakouts”(Figure 1.2), which occur when the solidifying steel shell sticks to the mold, tears, and allows molten steel to pour out from the mold [2] . This problem was overcome by Junghans in 1934 by introducing a vertically oscillating mold, utilizing the concept of “negative strip” wherein the mold travels downward faster than the steel shell during some portion of the oscillation cycle to dislodge any sticking. A similarly important development was the introduction of

Figure 1.2 Process following rupture of billet skin in the mold [2]

sinusoidal oscillation, first used on two Russian slab casters in 1959 [3]. Now the sinusoidal mode is the standard regime of oscillation worldwide. It is relatively simple to design, provides easy implementation and features a smooth character of movement. Many other developments and innovations have transformed the continuous casting process into the sophisticated process currently used to produce over 90% of steel in the world, including plain carbon, alloy and stainless steel grades [4].

The scheme of the modern continuous casting machines is shown in Figure 1.3 [4]. Melt flows through ceramic tubes (nozzles) from a ladle to a tundish and further to a mold. The nozzles are submerged in the melt. In the mold, the mold slag (flux) covers the melt surface. The mold slag protects the metal from contact with air and may act as a lubricant. Once in the mold, the molten steel freezes in the water-cooled copper oscillating mold and forms a solid shell. During solidification the defects, such as oscillation marks (OM) are produced on the surface of the billets.

Driving rolls continuously withdraw the shell from the mold at a rate or “casting speed” that matches the flow of incoming metal, so the process ideally runs in steady state. Beneath the mold are a secondary cooling zone and a straightening zone. Water or air

Figure 1.3 A vertical and radial continuous casting machine [4]

mist sprays cool the surface of the strand between the support rolls. The spray flow rates are adjusted to control the strand surface temperature until the molten core becomes solid. After the center of the strand is completely solid (at the “metallurgical length” of the caster, which is 10 – 40m) the strand is cut with oxyacetylene torches into slabs or billets of any desired length.

1.2. Factors influencing the initial process of the shell

solidification: mold slag and mold oscillation parameters.

The most critical part of the continuous casting process is the initial solidification at the meniscus (Figure 1.4). The meniscus is the free surface of a liquid which is near the walls of vessel and which is curved because of surface tension [5]. This is the location where the surface of the final product is created, and where defects such as oscillation marks and surface cracks can form.

Figure 1.4 Schematic of the meniscus region

Mold flux powder, normally consisting of silica (SiO_2), lime (CaO), sodium oxide (Na_2O), spar (CaF_2), MgO , Al_2O_3 , covers the surface of liquid metal in the mold. Graphite is also used as a flux material to control the flux melt rate. The casting flux serves [6 - 12] to:

- Prevent oxidation
- Provide thermal insulation, prevent meniscus solidification
- Absorb inclusions
- Provide lubrication
- Determine the meniscus shape

The following major flux properties must be controlled to give optimum flux performance: row flux insulation, rate of inclusion absorption, flux melting rate, flux viscosity and crystallization temperature.

The oscillation of the mold is one of the most important factors influencing the OM formation process [3, 6, 13, 14]. There are a number of parameters that describe the oscillation, the most important are:

Instantaneous mold velocity for sinusoidal cycle [3]:

$$V_m = 2 \cdot \pi \cdot s \cdot f \cdot \cos(2\pi \cdot f \cdot t / 60), \quad (\text{mm/min}) \quad (1.1)$$

where s - is the stroke length, mm, f - is the oscillation frequency, 1/min

and t - is the cycle time, s.

Average mold velocity [3]:

$$V_{average} = 2 \cdot s \cdot f, \quad (\text{mm/min}) \quad (1.2)$$

Total cycle time [3]:

$$t_c = 60 / f, \quad (\text{s}) \quad (1.3)$$

Negative strip time is the interval in each cycle of mold oscillation when downward velocity of the mold exceeds the withdrawal rate of the strand. Positive strip time is the interval in each cycle of mold oscillation when withdrawal rate of the strand exceeds the downward velocity of the mold.

Negative strip time interval in each cycle of mold [13]:

$$t_N = (60 / \pi \cdot f) \cdot \arccos (V_C / \pi \cdot f \cdot s), \quad (\text{s}) \quad (1.4)$$

where V_C - is the casting velocity.

The depth of oscillation marks decrease with increasing oscillation frequency [15 - 18] and increases with increasing oscillation stroke length [16, 18, 19] and negative-strip time [20, 21].

1.3. Formation of Ripples & Oscillation marks

1.3.1. Models of ripple formation

A particular group of defects characterized by ripples and laps in the ingots (ingot casting) and OM in strands (continuous casting) have been studied previously and a number of possible mechanisms were introduced to explain their formation. Waters [23] and Thornton [24] described a qualitative model which involves meniscus solidification during casting. Figure 1.5 illustrates the mechanism proposed by Thornton. As there is no

mold/liquid steel contact, the meniscus tends to build up until, when the internal pressure becomes too great, the meniscus bends to the mold wall at the limit of the frozen surface layer, as shown in Figure 1.5.

Figure 1.5 Ripple formation mechanism in ingots [24]

The steel surface is shown by :

$a b c d$ at time T ;

$a g c' d'$ at time $T + (1/4)\text{sec}$;

$a g' I c'' d''$ at time $T + (1/2)\text{sec}$.

The position of the boundary between solid and liquid steel is shown as:

$e f b$ at time T

$e' f' g$ at time $T + (1/4)\text{sec}$

$e'' f'' j k$ at time $T + (1/2)\text{sec}$

The precise position of the boundary cannot be determined. The dashed lines $f'hg$ and $f''h'g$ show an estimate of its position allowing for the reduced rate of chilling at the hollow (Figure 1.5 at time $T+(1/2)s$)) of the ripple where there is a gap between steel and mold.

Saucedo I, Beech J and Davis G.J. [29] studied ripples formation in the ingots during ingot casting . Their conclusions were:

1. The cooling factor (rate of heat extraction) of the mold is the major factor in the formation of the laps and ripples in ingot.
2. An increase of superheat and casting speed reduce these defects.
3. All of the observations made are consistent with solidification over the meniscus being the main step in the formation of surface rippling.

1.3.2. Models of oscillation mark formation

With the development of the continuous casting of steel, the mold reciprocation needed to be taken into account in the OM formation models. It has been proposed that mold reciprocation represented an additional mechanical effect. Consequently the model proposed by Savage [25] (see Figure 1.6) and Sato [26] incorporated mechanical and thermal mold – metal interactions.

Figure 1.6a illustrates a shell of the solidifying ingot in the mold at the time the mold is just beginning its upstroke. It is postulated that when the stress rate and maximum stress are sufficiently high, the thin and weak shell is broken near the meniscus level at position P_1 .

Figure 1.6 Reciprocation mark formation mechanism in steel billet [25]

The top element of the shell is then carried upwards by the mold, and it is assumed that during the upstroke some slipping may occur between the surface of the mold and the element E_1 . Consequently, E_1 is carried upwards by a smaller distance than the mold stroke, and arrives at the position shown in Figure 1.6b. In this time, element C has continued its downward movement at the casting speed. Consequently, a substantial gap

between E_1 and C is formed, which begins to freeze as the element B_1 , Figure 1.6b. The mold downstroke then begins, and continuous to the point shown in Figure 1.6d. In this period the stress applied by the mold to the ingot shell is either zero or slightly compressive, depending on whether the mold moves at the same or at a slightly greater speed than the ingot. Consequently, only a very weak mechanical link such as B_1 is required to permit the element C to move the element E_1 . Now when the upper edge E_1 penetrates the liquid steel meniscus, it is suggested that a fine lap L_1 is produced, as indicated in Figure 1.6c. During the reminder of downstroke, the lap L_1 passes beyond the meniscus level to reach the position shown in Figure 1.6d. When the mold is again reversed on the subsequent cycle, it is proposed that tearing occurs again as shown at P_2 , and the cycle of events is repeated.

The use of flux in the slab casting of steel introduced a further complexity. In this case it has been proposed that a mechanical interaction exists between flux and mold that changes the conditions under which the solid shell was formed. The next mechanism proposed by Davies and Sharp [28] for slabs considers that the negative strip in combination with the encapsulating effect of the solid flux can give rise to a pressure in the molten slag between strand and mold. The pressure acts so as to bend the solid shell inwards, producing an overflow of the liquid metal so forming the reciprocation mark.

In 1979 Saucedo I, Beech J and Davis G.J. suggested that the simple consideration of the casting speed can be used to understand how and when reciprocation marks are formed. [29]. The strongest rippling occurs at the point where mold and strand speed are equal. Lesser rippling would also be expected before and after this stage.

Meniscus solidification can account for the formation of reciprocation marks and the presence of smaller ripples in the region between them. After analyzing the model and experiments they suggested [29]:

1. Metallographic evidence shows that the same basic meniscus solidification mechanism is responsible for the formation of the reciprocation marks in continuous casting and ripples in ingots. In the former case this mechanism is modified by the mechanical effects produced by the mold reciprocation.
2. The final shape and size of the surface feature depends upon both the casting conditions and the rate of the heat transfer. For example the solidifying meniscus shell may be bent back towards the mold walls or it may be ruptured and bleeding can occur.

In 1980 Saucedo et al. [29] explained that the oscillation marks, or ripples, form because the meniscus solidifies. No marks will form if the rate of heat extraction is lowered. In 1991 Saucedo [30] presents the theory in more detail. The oscillation marks form when oscillation forces the liquid metal to regain contact with the mold wall. This can happen in two ways, either an overflow occurs, or the shell is bent towards the mold by the metallostatic pressure, or that these two processes combine. Figure 1.7 shows the different ways the liquid metal can come into contact with the mold wall above the frozen meniscus. The left mark is usually termed the folding mark (1), and the mark in the middle is called an overflow mark (2).

Figure 1.7 The mark formation starts with solidification of the meniscus [30]

1. The rising liquid pushes the solid shell towards the wall
2. The liquid overflows the shell
3. The first two are combined

The following conclusions were given [30]:

1. Samples indicate that initial solidification and oscillation mark formation are strongly influenced by meniscus effects.
2. The surface topography of oscillation marks has dendritic features, indicating that they were formed by meniscus solidification, without contacting the mold.
3. Oscillation marks produced in turbulent conditions can "split" in two, indicating that they are not produced by mechanical deformation effects.
4. Oscillation marks can have multiple depressions, indicating that there are not solely the products of mechanical effects during the mold downstroke.
5. Longitudinal streak lines on the surface of as-cast strands are produced by

meniscus solidification effects.

In 1980 Tomono et al [31] performed experiments with organic substances. They observed the formation of surface marks and concluded that the two mark types, i.e. folding marks and overflow marks are formed differently. They suggested that oscillation marks were formed when the meniscus was subjected to compressive force by particles sticking to the wall, and folding marks were formed independently of the oscillation. They used the Bikerman equation [31] for calculating the meniscus shape and connected the oscillation mark properties to the discrepancy between observed and calculated meniscus shape [3, 31].

In 1984 Takeuchi and Brimacombe [22] described a mechanism in which the meniscus is covered with a rigid solid skin (or without, depending on superheat and local convection) and the pressure generated in the liquid slag channel varies and draws the meniscus back towards the mold wall during the negative strip. The pressure is positive when the mold is moving downward faster than the strand and is negative when the strand is moving faster. The difference between marks with and without hooks is assumed to be caused by the difference in strength of the meniscus skin. If the skin is strong, an overflow will occur, and a hook forms (Figure 1.8). If the skin is weaker, the shell is pressed against the wall and no overflow is needed, and no hook forms (Figure 1.9). Takeuchi and Brimacombe described how the meniscus follows the oscillation and suggested the classification of oscillation marks [22, 32, 33].

Figure 1.8 Takeuchi and Brimacombe model of OM formation with subsurface hook [22]

Figure 1.9 Takeuchi and Brimacombe model of OM formation without subsurface hook [22]

Figure 1.10 Schematic drawing for ripple mark formation [34]

- a) Balance of forces at the tip of solidified shell
- b) Growth of shell and movement of melt surface

Suzuki et al [34] suggested that the meniscus in surface tension balance with the shell moves upwards as the solid shell grows inwards (see figure 1.10). Further J. Elfsberg [3] used this idea for her calculation.

In 1989 Delhalle et al. [35] described three different mechanisms for the formation of oscillation marks, as shown in Figure 1.11. The three mechanisms are based upon meniscus solidification. The size and shape of the oscillation marks is said to depend on the heat extraction, the oscillation parameters and the interfacial properties [35].

Based on the results of three experiments Lanez and Busturia [36] suggested that solidification does not start at the meniscus, but further down in the mold, at the lower part of the solid slag rim (Figure 1.12). The oscillation mark formation was connected to

this region and it has been noted that marks formed under “positive strip” (where the mold downward speed is at maximum). They termed their model E.L.V. (“Extra Liquid Volume”) solidification model, Figure 1.12.

Figure 1.11 Delhalle mark formation mechanism [35]

A. Mold oscillation and product withdrawal cause liquid steel to overflow the solid hook. The overflowing liquid freezes against the mold wall and a new shell tip forms.

B. Same as A, but here partial or total remelting of the solid shell tip is assumed.

C. The metallostatic pressure bends the solid shell back against the mold [35]

In 2003 J. Elfsberg [3] proposed that there can be two types of oscillation marks: overflow marks and folding marks. It was suggested that they both were caused by overflows. The difference in their appearance depends on that they formed at different times of the oscillation cycle. The overflow marks are formed during the down strokes, and the folding marks during the upstrokes. The oscillation marks may form patterns. One deeper mark is followed by some shallow marks.

Figure 1.12 E.L.V. solidification model
("Extra Liquid Volume") [36]

Then there comes a deep one and so on. She suggested that the deeper marks form when the interfacial tension and the oscillation co-operates (acting in the same direction). Also J. Elfsberg suggested that it is possible to avoid the oscillation mark formation by avoiding the overflows. If the oscillation frequency is chosen so that the maximum meniscus height is never reached, the overflows will not occur. In other words, if the mold is switching to upward movement just as the maximum meniscus height is reached, the overflow will be suppressed. The optimal oscillation frequency was suggested to be

[3]:
$$f = V_C / (\sqrt{2} \cdot a) \quad (1.5)$$

Sato [26, 27] suggests that the marks are formed in two steps, first the solid meniscus shell is lifted by the upward moving mold. This lift causes the formation of two convex surfaces, ab and bc in Figure 1.13. Then, as the mold turns, the two convex surfaces are forged together and the mark is formed. Figure 1.13 shows the conditions for casting with the use of mold slag. The presence of the slag will cause a different pressure [27].

Figure 1.13 The formation of oscillation marks in continuous casting with the use of mold slag. The mark forms during the upstroke. The solid meniscus lifted by the mold movement [27]

Edward S. Szekeres [13] suggested “the secondary meniscus mechanism”. According to [13] the oscillation mark forms at a secondary meniscus, not at the primary meniscus. In Figure 1.14 section A demonstrates the state just after the start of the cycle and an element of solid steel is assuming to be adhered to the mold wall and moving

upward with the mold. The surface of liquid metal between the meniscus shell and downward moving strand shell is a secondary meniscus. There is no tearing at this junction because the meniscus shell is not yet connected to the strand shell. Section B represents the start of negative strip. It is possible that the secondary meniscus freezes over as early as the start of negative strip which establishes a solid bridge between the meniscus shell and the strand shell.

Figure 1.14 Meniscus shell attachment mechanism [13]

As the negative strip condition intensifies, the meniscus shell breaks free from the mold wall. Also the author pointed out that the Savage model included a tearing healing action at the junction between upward - moving solid and the downward - moving strand. Although the tear and heal mechanism was referred to for many years, it has been abandoned because “Metallographic examinations of as-cast surfaces have revealed no evidence of tearing” [13].

Information about SM formation in an immovable mold was presented in 1990 [38]. The mechanism of slag film renewal during continuous casting in an immovable mold was suggested. The new research was published in 2002. After experiments with electro magnetic casting (EMC) of steel (Figure 1.15) Joonpyo Park et al. [39] concluded: in ordinary casting, the mold is oscillated to prevent sticking between the mold and the billet. In EMC, the magnetic pressure increased a curvature form in the meniscus, thus

Figure 1.15 The concept of electromagnetic casting of steel [39]

the inflow of the mold flux was improved. Therefore, it was suggested that EMC could be performed without mold oscillation. If the casting can be performed without mold oscillation, the casting plant could be dramatically simplified [39].

1.3.3. Classification of the oscillation marks

In 1979 Saucedo I, Beech J and Davis G.J. [29] described the different types of the OM (presented in Figures 1.16 – 1.18) and their formation. It is possible to observe that the lap can be covered by liquid overflow (Figure 1.16), also it is possible to observe an outward bending of an originally inward-growing shell (Figure 1.17), solid formation outside reciprocation marks, remelting of the reciprocation mark valley and bleeding through (Figure 1.18).

Figure 1.16 Bending of the originally lapped interface, x15 [29]

Figure 1.17 a) Reciprocation mark showing strong outward bending of solid shell, x24;
b) Solidification outside the reciprocation mark, x60 [29]

Figure 1.18 Remelting of the reciprocation mark valley and bleeding through, x60 [29]

Later, in 1985 Brimacombe J.K. et al. [32] suggested the following classification of OM (Figure 1.19):

Types of Oscillation Mark	Types of Positive Segregation		
	Type 1	Type 2	Type 3
Hooks in subsurface structure	 large segregation area	 weak segregation line	 weak segregation line
Hooks absent in subsurface structure	Type 4 segregation layer		

Figure 1.19 Types of oscillation marks [32].

The OM with subsurface hooks were studied by B.G Thomas et al.[37] in details. Hook formation is initiated by meniscus solidification. The instantaneous shape of the meniscus dictates the curvature of the line of hook origin. It has been established that hooks formation usually occur at the beginning of the negative strip period [37] as schematically illustrated in Figure1.20.

Figure 1.20 Schematics illustrating formation of curved hook in an ultralow -carbon steel slab by meniscus solidification and subsequent liquid steel overflow [37]

- a) meniscus freezing;
- b) meniscus overflow;
- c) line of hook origin;
- d) shell growth.

Oscillation marks (OM) are formed by steel shell growth after overflow. The meniscus subsequently overflows the line of hook origin, as shown in Figure 1.20(b). The final shape of the hook is formed as the hook tip fractures and is carried away, as shown in Figure 1.20(c). The hook protruding from the solidifying shell captures inclusions and bubbles in the liquid steel until the shell finally solidifies past the hook. These events are illustrated in Figure 1.20(d).

1.4. Conclusions. Objectives of the thesis

As has been demonstrated in the review, the most of the presented OM models have a qualitative character [13, 23, 24, 25, 27, 29, 30], i.e. there is no numerical simulation of the meniscus behavior. Therefore it is difficult to utilize them in specific applications. Usually, such models are based on a particular experimental data (templates with the sections of the billets surface, on which the meniscus was fixed). All these models have lack of completeness. There is no connection of the observed phenomenon to the previous history of process. Most researchers proposed models in which mold oscillation is the major factor determining the OM process [3, 33]. A large number of different models exist because there is no basic model of initial solidification wherein the meniscus is free from additional forces. The objective of this work is to create a mathematical model describing the mechanical behavior of the meniscus and the thermal processes acting upon the meniscus during continuous casting in an immovable mold.

2. Qualitative description of the proposed model

The developed in this thesis model gives a picture of the process of initial shell formation through a description of the mark formation process. The model contains a description of meniscus movement and deformations that are used in calculations of the temperature distribution on meniscus. Behavior of the meniscus was considered during the stationary process of ingot formation. The stationary process means that at the end of a mark formation cycle the next cycle started from the same point that the previous cycle began. In the developed model the mark formation process was divided into two stages, which followed each other successively, thus making a cycle.

2.1. The first stage of the mark formation

The scheme of the 1st stage of process is shown in Figure 2.1. This stage begins when the previous cycle comes to an end and solidified shell starts to move downwards relatively to the slag rim. At this moment solidified shell loses contact with the rim. The point on the boundary between the solidified portion of the meniscus and liquid (the top of solid mark) becomes the support for the meniscus of the liquid metal (Figure 2.1). Surface tension force keeps the liquid metal above the solid mark and prevents contact with solid slag on the mold wall. The meniscus stretches and moves in accordance with the balance of surface tension forces and gravity. The meniscus growth arises from two processes: the meniscus moving as a whole and the meniscus stretching. The initial condition of the 1st stage is shown in Figure 2.1(a). The top of the solid mark started moving downwards with the billet (Figure 2.1 (b)). The curved liquid meniscus (CLM)

expanded between two points: the first point is the top of the solid mark (T) and the second point is point of junction (J) between the curved meniscus and the flat portion of surface of liquid metal in the mold. The CLM started stretching and moving. The gap between the meniscus and rim increases and liquid slag flows into the gap (Figure 2.1 (b)). When the top of the solid mark moved, the meniscus height increased. Under the action of the ferrostatic pressure, growing meniscus is pressed to join the slag rim again. In addition, liquid slag is forced to move out from the gap (Figure 2.1 (c)). Thus, at the first stage, the billet grows due to the meniscus height increasing and overflow of the liquid metal over the solid mark. The first stage is divided into two phases.

Figure 2.1 The scheme of the 1st stage mark formation process

1- solid mark; 2 – meniscus; 3 – liquid slag; 4 – slag rim

The first one – when the meniscus moves from the slag rim, as shown in Figure 2.1(a-b) and second one when meniscus moves to the slag rim as show in Figure 2.1(c-d). When the meniscus contacted the slag rim the upper part of the meniscus stops. The contact of liquid metal and solid slag (Figure 2.1 (d)) leads to splitting of the meniscus into upper

and lower parts. We called this moment the sticking of meniscus and this is the end of stage 1. The lower meniscus still continues to follow the movement of the solid mark.

At the first stage we assume that up to the moment when meniscus stops at the mold wall it has not been solidified: less heat escapes the meniscus than accrues to it. This happens because new portions of superheated liquid metal are being added to the external surface of meniscus.

2.2. The second stage of mark formation

The second stage begins when the meniscus touched the mold wall and two menisci formed: upper and lower. The lower meniscus continues to follow the movement of the solid mark. After the meniscus had made contact with solid slag, part of the liquid slag becomes closed (under the point E, see Figure 2.1 (d). From this moment liquid slag cannot move upward on the liquid metal surface in the mold from the gap.

Figure 2.2 “Lower” meniscus transformation

1 – solid mark; 2 – meniscus of the liquid metal; 3 – liquid slag;
4 – slag rim; 5 – solid part of meniscus; 6 – closed slag.

The slag closing inside liquid metal leads to a changing of the forces which had provided a balanced interaction on the lower meniscus at the second stage. Thus, at the moment of slag closing the meniscus changes its shape very rapidly i.e. there is a transition from the form shown in Figure 2.1 (d), to that shown in Figure 2.2 (a). The lower meniscus becomes convex on the side of liquid metal. The top of the solid mark continues to move, therefore the ferrostatic pressure acting on the lower meniscus increases. The lower meniscus moves and deforms as a result of the increasing force. The process of the lower meniscus transformation is shown in Figure 2.2 (a, b). The bottom support point of the lower meniscus moves along the solid mark. The upper support point of lower meniscus moves down and its position is defined by the solidification and deformation processes. As the solid mark moved downwards the lower meniscus leaned on it, and following the solid mark became extended. As a result, fresh portions of liquid metal contact the external surface of the lower meniscus thereby adding superheat. If during the movement of the lower meniscus the heat extraction rate does not exceed the rate of the heat supply via fresh portions of liquid metal, the casting process became unstable. The liquid core is poured out under the mold through the non-solidified lower meniscus. On the contrary, if the level of heat withdrawal from the lower meniscus exceeds the level of incoming additional superheat then the gap between the sticking upper meniscus and solid mark solidifies. In the thesis we considered only the first stage of surface mark formation. To

describe the evolution of upper meniscus shape and its heat state we need to develop the mechanical and thermal models of the meniscus state. Based on the model of meniscus mechanical behavior (Chapter 3), the model of its thermal condition will be discussed in Chapter 4.

3. Mechanical model of the first stage

The mechanical model is formulated based on the equilibrium existing at the surface of meniscus between surface tension forces and the pressure of the liquid metal and slag (Figure 3.1).

3.1. Basic assumptions. Forces acting on meniscus

Meniscus is considered to be an interphase film. The coefficient of interphase tension assumed to be constant along meniscus length. To evaluate the mechanical state of meniscus we consider the unit width element of the meniscus along the ingot axis.

Figure 3.1 Forces acting on the meniscus

S , R_s , Q , q - forces acting on meniscus.

The forces, which act on this element from the remaining part of the meniscus, perpendicular to the plane of the drawing direction are neglected and only the forces, which act in the plane of drawing (Figure 3.1), are discussed, hence a 2-dimensional problem is considered. We assume, that the top of the solidified mark (point T) moves along an axis X (Figure 3.1), without changing coordinate Y .

Such an assumption is equivalent to the assumption that within the limits of the mark formation cycle neither solidification, nor a melting at the top of the solid mark occur.

To derive a mathematical model describing the mechanical behavior of the liquid metal meniscus, we examine the equilibrium of the forces acting on an infinitely small section of meniscus (Figure 3.2).

Figure 3.2 Diagram of the forces acting on an infinitesimally small section of the meniscus

Δb - unit width of the considered section of meniscus in the direction,

perpendicular to the plane of drawing;

φ - angle between tangent to the meniscus and axis X;

dl - length of the infinitesimally small section of meniscus;

S - longitudinal force - is always directed tangential to the surface of the considered

section; at any point of meniscus always has one and the same value (the surface tension force);

$d\varphi$, dx , dy – increments of the appropriate values.

Q - the pressure of liquid metal (distributed load), acts from the side of liquid metal;

q - the pressure of liquid slag (distributed load), acts from the side of liquid slag.

Within the limits of an infinitesimally small section of meniscus with the length dl we neglect the changes of Q and q .

3.2. Differential equation of equilibrium and its solution in parametrical form

Distributed loads Q and q can be presented by the resultant forces

$$R_Q = Q \cdot dl \cdot \Delta b \quad \text{and} \quad R_q = q \cdot dl \cdot \Delta b. \quad R_Q \text{ and } R_q \text{ are applied in the middle of the}$$

infinitesimally section of meniscus. The forces of interphase tension are equal to

$$S = \sigma \cdot \Delta b,$$

where σ - the coefficient of interface tension between liquid metal and liquid slag.

Equation of equilibrium forces (sum of the forces projections on the direction of the resultants of distributed forces):

$$2S \cdot \sin \frac{d\varphi}{2} - (Q - q) \cdot \Delta b \cdot dl = 0$$

To simplify this expression we use

$$S = \sigma \cdot \Delta b \text{ and } \Delta b = 1.$$

For small $d\varphi$, $\sin d\varphi \approx d\varphi$, we have:

$$(Q - q) - \sigma \cdot \frac{d\varphi}{dl} = 0.$$

Here $(Q - q)$ is a function of meniscus height x : $Q(x) = \rho_m \cdot g \cdot x$; $q(x) = \rho_{sl} \cdot g \cdot x$.

Finally differential equation of the force balance looks like:

$$(\rho_m - \rho_{sl}) \cdot g \cdot x - \sigma \cdot \frac{d\varphi}{dl} = 0, \quad (3.1)$$

where g - the gravity acceleration ;

ρ_m - the density of liquid metal;

ρ_{sl} - the density of fluid slag.

At a certain point on the meniscus with the coordinates (x, y) the projection of elementary section dl on axis X is equal to $dx = dl \cdot \cos \varphi$, where φ - angle of tangent inclination to the meniscus at point on the meniscus with coordinates (x, y) .

The curvature of the considered section is determined by the known expression:

$$\frac{1}{R} = \frac{d\varphi}{dl} = \frac{\frac{d\varphi}{dx}}{\frac{dl}{dx}}$$

R – radius of curvature;

from which after the transformation follows:

$$\frac{d\varphi}{dl} = \cos \varphi \cdot \frac{d\varphi}{dx} \quad (3.2)$$

then equation (3.1) becomes:

$$(\rho_m - \rho_{sl}) \cdot g \cdot x \cdot dx - \sigma \cdot \cos \varphi \cdot d\varphi = 0 \quad (3.3)$$

To simplify we introduce the variable: $B = \frac{(\rho_m - \rho_{sl}) \cdot g}{2 \cdot \sigma}$.

Integrating (3.3) over the meniscus height x_h

$$\int_0^{x_h} 2B \cdot x \cdot dx - \int_{\theta_0}^{\varphi_h} \cos \varphi \cdot d\varphi = 0;$$

we get:
$$B \cdot x_h^2 = (\sin \varphi_h - \sin \theta_0), \quad (3.4)$$

where θ_0 - initial angle at the point J (paragraph 2.1) where $x = 0$ (see Figure 3.3).

3.3. Meniscus line equation and it solution in Cartesian coordinate system

Notations for the description of the meniscus line are shown in Figure 3.3. An axis Y is directed along the flat portion of molten metal surface. An increase in the y values occurred to the right side from the beginning of coordinates.

Figure 3.3 Notations used in mechanical model

The axis X is parallel to the strand's axis and crosses the top of the solidified mark (point C_0 - point where the new cycle of the mark formation begins), directed downward.

C_{H_G} - the point corresponds to an extremum on the meniscus line;

C_h - a certain point on the meniscus at a distance h from the beginning of coordinates;

C_H - the top of solidified mark in the end of the first stage (height of the meniscus = max);

h - the current height of meniscus $\delta x \leq h \leq H_{\max}$, which corresponds to the displacement of the mark top C_0 on the distance h_c ;

H_δ - displacement of the top of solid mark;

H_h - limiting height of meniscus for concrete conditions;

h_l - the height of the not solidifying part of the meniscus (initial section of meniscus);

H_G - distance from the beginning of coordinates (along the axis X) to

the point of extremum on the meniscus C_{H_G} ;

x - coordinate of the current point on meniscus;

$y(x, h)$ - coordinate of the current point on meniscus;

y_{H_G} - coordinate of point H_G ;

Δy - the thickness of solid slag on the mold wall (out of the slag rim zone);

θ_0 - initial angle; theoretically this angle should be constant and equal to $-\pi/2$, but in reality the surface of the liquid metal in the mold is not absolutely plane surface (liquid metal continuously moving) therefore we assume θ_0 is one of the fitting parameters close to $-\pi/2$;

J – junction point (Paragraph 2.1).

The solution of the equation (3.4) we present as function $y = f(x)$.

From Figure 3.2 $\frac{dy}{dx} = \tan \varphi = \frac{\sin \varphi}{\cos \varphi}$;

substituting from equation (3.4):

$$\frac{dy}{dx} = \frac{\sin \varphi}{\cos \varphi} = \frac{B \cdot x^2 + \sin \theta_0}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \quad (3.7)$$

then

$$dy = \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx \quad (3.8)$$

or in integral form:

$$y(x) = y(x_0) + \int_{x_0}^x \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx \quad (3.9)$$

3.4. Basic parameters of the meniscus

Maximum height of the meniscus

The maximum height of meniscus can be found from equation (3.4):

$$x = \sqrt{\frac{\sin \varphi - \sin \theta_0}{B}}.$$

The height of meniscus is maximal if the residual $(\sin \varphi - \sin \theta_0)$ is maximal. In this case $(\sin \varphi - \sin \theta_0)_{\max} = 2$ when $\varphi = \pi/2$, $\theta_0 = -\pi/2$

Substituting these values and expression for B in equation (3.4):

$$\frac{(\rho_m - \rho_{sh}) \cdot g \cdot H_{\max}^2}{2} = 2.$$

From last equation we get:

$$H_{\max} = \sqrt{\frac{4 \cdot \sigma}{(\rho_m - \rho_{sl}) \cdot g}} = \sqrt{\frac{2}{B}}.$$

Substituting the numerical values of the parameters:

$$\rho_m = 7200 \text{ kg/m}^3; \rho_{sl} = 2667,3 \text{ kg/m}^3; g = 9,81 \text{ m/s}^2; \sigma = 1,2058 \text{ H/m},$$

We obtain the following:

$$H_{\max} = \sqrt{\frac{4 \cdot 1,2058}{(7200 - 2667,3) \cdot 9,81}} = 10.42 \text{ mm}$$

We assumed that the height of the studied meniscus is less than the maximum possible (in this case the random fluctuations will not lead to the destruction of the meniscus). In the further descriptions we accept the height of the meniscus $H_h = 0.985 \cdot H_{\max}$.

Points of the extremum on the line of meniscus

The position of the extremum of function $y(x)$ can be found from condition:

$$\frac{dy}{dx} = \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} = 0,$$

therefore $dy = (B \cdot x^2 + \sin \theta_0) = 0$,

the position of the extremum point is:

$$x = H_G = \sqrt{\frac{-\sin \theta_0}{B}}. \quad (3.10)$$

After substitution $\theta_0 = -\pi/2$, and finally we obtain $H_G = \sqrt{\frac{1}{B}}$.

The J - point coordinates at the maximum height of the meniscus

The coordinate $y(x_0)$ of the junction point J, (Figure 3.3) can be found from equation (3.9):

$$y(x) = y(x_0) + \int_{x_0}^x \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx.$$

Substituting $x_0 = 0$, $x = H_{\max}$ and the point on meniscus with coordinate $x = H_{\max}$

has the coordinate $y(H_{\max}) = 0$:

$$y(H_{\max}) = y(0) + \int_0^{H_{\max}} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx = 0;$$

then

$$y(0) = - \int_0^{H_{\max}} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx. \quad (3.11)$$

The height of the initial section of the meniscus

The initial section of meniscus is a part of the meniscus (higher than point C_0), which does not solidify during the period of mark formation. At the beginning of the new cycle and at the end of the stage 1, the $y(h_l)$ is equal to 0.

$$\text{From (3.9)} \quad y(h_l) = y(x_0) + \int_{x_0}^{h_l} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx = 0;$$

$$\text{From (3.11)} \quad y(x_0) = y(0) = - \int_0^{H_{\max}} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx, \text{ then}$$

$$\int_{x_0}^{h_l} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx = \int_{x_0}^{H_{\max}} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx \quad (3.12)$$

This equation 3.12 is implicit relatively to h , and solved by the numerical methods, where $y(0)$ can be found from (3.11).

Parametric equation for the meniscus line

It is important to note that dependence (3.9) has two parameters, i.e. in addition to the determination of the coordinate of the observation point x , the value of the meniscus height h at the given moment of time is required (equation 3.11, Figure 3.4).

Figure 3. 4 Parameters of the equation of meniscus line

For the considered point C the dependence (3.9) can be presented as two-parametrical:

$$y(x, h) = y(C) = -Y(h) + Y(x), \quad (3.13)$$

where $Y(h)$ - depends only on the height of the meniscus at considered moment in time

as:

$$Y(h) = \int_0^h \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx; \quad (3.14)$$

Function $Y(x)$ depends only on coordinate x of the point of observation:

$$Y(x) = \int_0^x \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx \quad (3.15)$$

The depth of the mark (3.13):

$$\begin{aligned} \Delta Y &= y(h_l, H_h) - y(H_G, H_h) = -Y(H_h) + Y(h_l) - [-Y(H_h) + Y(H_G)] = \\ &= Y(h_l) - Y(H_G) \end{aligned}$$

where $Y(h_l)$ and $Y(H_G)$ we determine from formula (3.15).

The equation of the slag line

Solid slag on the mold wall has two regions:

1. The slag rim (in the interval $H_G > x > 0$), i.e. coordinates of points on this section can be described by functional dependence on coordinate x ;
2. The solid slag with a constant thickness located under the slag rim (in the interval $x > H_G$).

Curve of the slag rim in the rim limits ($H_G > x > 0$) can be described by (3.9) when $h = H_h$

In parametric form the equations of the slag rim:

$$y_{sl}(x, H_h) = -Y(H_h) + Y(x) ; \quad (H_G > x > 0)$$

for $x > H_G$, $y_{sl}(x, H_h) = \text{const}$.

The generalized description of the solid slag line $y_{sl}(x)$, which includes rim and solid slag with a constant thickness, can be done by symbols of Boolean algebra:

$$y_{sl}(x, H_h) = y_{sl}(x, H_h) \cap (H_G > x > 0) + y(H_G, H_h) \cap (x > H_G),$$

$$\text{or} \quad y_{sl}(x, H_h) = [-Y(H_h) + Y(x)] \cap (H_G > x > 0) + [-Y(H_h) + Y(H_G)] \cap (x \geq H_G) \quad (3.17)$$

where $Y(H_h)$, $Y(H_G)$, $Y(x)$ can be found from formula (3.14) and (3.15) and the sign $\cap$ indicates the union of sets - domain of existence of the functions values that simultaneously stand to the left and to the right of sign.

The solid slag thickness

Outside of the slag rim we assume the solid slag thickness is constant and equals to Δy . In the slag rim zone the thickness of the rim is added to the thickness of the solid slag. The slag rim thickness depends on the coordinate x and h as:

$$\begin{aligned} \Delta h_{sl}(x, h) &= \Delta y + y_{sl}(H_G, H_h) - y_{sl}(x, H_h) = \\ &= \Delta y + [-Y(H_h) + Y(H_G)] - [-Y(H_h) + Y(x)] = \\ &= \Delta y + Y(H_G) - Y(x) \end{aligned} \quad (3.18)$$

where $H_G > x > 0$;

$\Delta h_{sl}(x, h)$ - slag rim thickness;

Δy - solid slag thickness on the mold wall (below slag rim) $\Delta y = \text{const}$;

$Y(H_G)$, $Y(x)$ - is defined by equation (3.14) and (3.15).

Thickness of the liquid slag layer

Thickness of the liquid layer between the meniscus and solid slag on the mold wall is the distance along an axis Y between points on the solid slag and on the meniscus at constant x . To find this we use the equation (3.17) and (3.13). Thickness of the liquid layer between the meniscus and the solid slag and rim Δh_{sl-f} can be expressed in the parametrical form as:

$$\begin{aligned}\Delta h_{sl-f} &= y(x, h) - y_{sl}(x, H_h) = Y(x) - Y(h) - \\ &- [(Y(x) - Y(H_h)) \cap (H_G > x > 0) + (Y(H_G) - Y(H_h)) \cap (x \geq H_G)] = \\ &= [Y(H_h) - Y(h)] \cap (H_G > x > 0) + \\ &+ [Y(x) - Y(h) + Y(H_h) - Y(H_G)] \cap (x \geq H_G)\end{aligned}\quad (3.19)$$

where h - current height of meniscus;

H_h - limiting height of meniscus for given conditions;

It can be noted that in the section of meniscus $(H_G > x > 0)$, the Δh_{sl-f} does not depend on x , but it depends only on the height of the meniscus h .

The length of the meniscus

The length of the meniscus is required for a description of the meniscus' thermal condition during moving and lengthening. The length of the line from a reference point x_0 up to some point C_x is determined by the formula:

$$L = \int_0^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx,$$

where x_0 and X - the beginning and the end of the studying part of curve $y = f(x)$ accordingly.

After the substitution the value of derivative $\frac{dy}{dx}$ defined by (3.8) for $x_0 = 0$; $x = h$ we obtain:

$$L_C = \int_0^x \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx$$

Or for the meniscus with height h :

$$L_C = \int_0^h \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx \quad (3.20)$$

3.5. Numerical simulation of the mechanical model at the first stage

The solution of differential equation (3.9) was done with the help of the mathematical package "MathCAD". For the best illustration of the meniscus displacements, the result of calculations has been represented as graphs. These graphs determined the form and position of solid slag, and also the end and intermediate positions of meniscus.

The physical values for calculation

The basic physical characteristics of the considered materials:

Density of the liquid metal	7500 kg / m ³
Density of the liquid slag	2500 kg / m ³
Coefficient of the surface tension	0.2 kg / m ²

We assumed that the metal remains liquid and neglected dependence of their basic physical properties on temperature. Furthermore, we assumed that the height of the studied meniscus is less than the maximum possible (in this case the random fluctuations will not lead to the destruction of the meniscus). In the further descriptions we accept the height of the meniscus $H_h = 0.985 \cdot H_{\max}$.

Influence of the basic parameters on the form and dimensions of the mark.

The results of the calculations are shown in figures 3.4 (a – h), where solid slag is marked by green line. Dotted blue line is the x coordinate of the point of extremum. The meniscus and solidified mark are blue. The top of solidified mark is located on an axis X ($y = 0$).

a)

b)

b)

d)

e)

f)

Figure 3.5 Calculated intermediate positions of the meniscus

To the figure 3.4:

- a - $H_{\delta} = 0$ - the initial position;
- b - $H_{\delta} = 0.25 \cdot (H_h - h_l)$ - 25% of the solidified mark movement;
- c - $H_{\delta} = 0.50 \cdot (H_h - h_l)$ - 50% of the solidified mark movement;
- d - $H_{\delta} = 1.00 \cdot (H_G - h_l)$ - exactly on the line of solid slag lower boundary;
- e - $H_{\delta} = 0.75 \cdot (H_h - h_l)$ - 75% of the solidified mark movement;
- f - $H_{\delta} = 0.85 \cdot (H_h - h_l)$ - 85% of the solidified mark movement;
- h - $H_{\delta} = 0.95 \cdot (H_h - h_l)$ - 95% of the solidified mark movement;
- g - $H_{\delta} = 1.00 \cdot (H_h - h_l)$ - 100% of the solidified mark movement.

Two phases of the first stage

From the previous numerical analysis of the meniscus displacement during the first stage of the mark formation, it is possible to separate two phases of meniscus movement, such as:

1. From the beginning of the mark formation cycle (point C_0 - see Figure 3.3) until the $x = H_G$ (interval $h_l > x > H_G$). During the first phase the line of the meniscus moves along the axis Y from the slag rim (Figure 3.4 a-d). The maximum distance $y(H_G)$ achieved at $x = H_G$ (Figure 3.4 d).
2. The second phase starts when the top of solidified mark passed through the point $x = H_G$ (Figure 3.4 d) and continues until the meniscus contacts the slag rim, i.e. till the ending of the first stage mark formation (Figure 3.4g).

During the second phase the line of the meniscus, moving along an axis Y , comes nearer to the slag rim. Here the distance between meniscus and slag rim in the section $H_G > x > 0$ varies from $y(H_G)$ up to zero.

4. The model of the thermal processes on the meniscus at the first stage of mark formation process

In the previous chapter the model of mechanical behavior of meniscus was described. In present chapter we consider how the temperature of each selected point of meniscus changes during the first stage of mark formation process.

4.1. Basic assumptions. The representative element

We can assume that it is possible to neglect the heat removal from the meniscus at the first phase of the first stage of mark formation. At the first phase when the meniscus moved away from the solid slag, the gap between meniscus and solid slag increased. The fresh portions of fluid slag with the high temperature moved into the gap. The temperature equalization occurred across the entire surface of meniscus, and became equal to temperature of the liquid metal in its core T_M . Reduction of the meniscus temperature starts only at the beginning of second phase after meniscus height became equal H_G . In the second phase the gap between the meniscus and the solid slag decreases. Fresh liquid slag stops coming into the gap and liquid slag starts to escape from the gap. It leads to appearance of a temperature gradient in the liquid slag providing intensive heat removal from the meniscus. For description of the meniscus thermal state during first stage, we consider the thermal state of n - points of observation on the meniscus starting from the end of the first phase.

First, we choose some point B on the meniscus with coordinate $x = x_0$ and

$y = y(x_0, H_G)$ (see Figure 3.3). At the point $B(x_0)$ we place the representative element (RE) with infinitesimally width (see figure 4.1) and from this point we start to consider changes of the heat state of RE. This RE is represented as orthogonal prism which is created by two planes parallel and two planes perpendicular to the plane of drawing. The centerline of the RE is perpendicular to tangent to the meniscus. One base of RE is located on the meniscus and the second base in the liquid metal (figure 4.1). The length of the RE is equal to the thickness of thermal boundary layer δ_T (see Chapter 4.6).

Figure 4.1 The RE geometry changes as a result of meniscus growth

a) initial position (the beginning of the second phase);

b) current position.

$B(x_0)$ - initial position of point of observation;

$B(x_h)$ - current position;

l_0 - initial width of RE base;

$l = f(h)$ - current width;

δ_T - length of RE (thickness of the thermal boundary layer);

h - current height of meniscus;

δ_0 - initial meniscus height;

x_0 - X- coordinate of point $B(x_0)$;

x_h - X- coordinate of point $B(x_h)$;

J – junction point;

θ_0 - initial angle.

Considering the process of the meniscus stretching the following special features of the RE stretching were taken into consideration:

- the RE stretched with the meniscus, the stretching is accompanied with liquid metal coming into RE from the core of the billet;
- the new portions of metal are evenly distributed throughout entire RE;

We study only 1 D transient heat transfer, i.e. we consider heat transfer only in the direction of the RE axis. We neglect heat exchange between the RE and the adjacent sections of meniscus. The heat transfer occurs only through the bases of RE.

4.2. Derivation of differential equation

The basic principles of the heat balance equation

The infinitesimally element du was selected inside RE (see Figure 4.2). Thermal conditions for du determined by:

- heat transfer through the bases of du ;
- change of the du heat content;
- heat comes with fresh portion of metal from the core of billet

Heat balance for the du

The change of the du heat content during the time $d\tau$ (see Figure 4.2):

$$dQ_V = dQ_\Delta + dQ_F, \quad (4.1)$$

where dQ_V - the total change of the heat content in the du during $d\tau$;

$dQ_\Delta = (-dQ_1 + dQ_2)$ - change of the heat transferred through the bases of the du during $d\tau$;

dQ_F - change of the heat content of the RE which comes with the fresh portion of metal from the core of billet.

Figure 4. 2 The RE boundary conditions

The total change of the heat content in the du during $d\tau$

A change of the heat content in the du is determined by the equation:

$$dQ_V(\tau) = c \cdot \rho \cdot b \cdot [l(\tau_2) \cdot T(u, \tau_2) - l(\tau_1) \cdot T(u, \tau_1)] \cdot du, \quad (4.2)$$

- c - coefficient of heat capacity of liquid metal;
 ρ - density of liquid metal;
 b - width of infinitesimally section of meniscus;
 u - the coordinate along the prism axes ($u_1 < u < u_2$).

Change of the heat transfer through the bases of the du

The notation of the coordinate of the RE bases: $u = 0$ on the meniscus, and $u = \delta_T$ inside liquid metal area. For the du element respectively: at $u = u_2$ the heat enters in the du , and at $u = u_1$ the heat exits from the du . According to the Fourier law the heat transfer through the ends of the du for the time from τ_1 to τ_2 :

$$dQ_\Delta = b \cdot \left[\lambda \frac{\partial T}{\partial u}(u_2, \tau) - \lambda \frac{\partial T}{\partial u}(u_1, \tau) \right] \cdot l(\tau) \cdot d\tau, \quad (4.3)$$

where $l(\tau) = l_0 + \Delta l(\tau)$ - is the du extension along the meniscus (tangentially to it);

$\Delta l(\tau)$ - lengthening of the RE along the meniscus as the function of time $\tau_1 \leq \tau \leq \tau_2$.

Change of the heat content of du due to the metal flows from the core

The meniscus stretching is accompanied by metal influx from the core of the strand. The temperature of this metal is equal to the temperature of the liquid metal, which was not cooled. In this case the heat, which entered into du is proportional to the extension of the du along the meniscus $\Delta l(\tau)$. The contribution to the du thermal equilibrium from the metal, can be determined as a difference in the values of the heat content of outgoing metal at τ_2 and at τ_1 :

$$dQ_F(\tau) = c \cdot \rho \cdot b \cdot T_M \cdot [l(\tau_2) - l(\tau_1)] \cdot du \quad (4.4)$$

After substitution of dQ_Δ , dQ_V and dQ_F into equation of heat balance (4.1) we get:

$$\begin{aligned}
& c \cdot \rho \cdot b \cdot [l(\tau) \cdot T(u, \tau_2) - l(\tau_1) \cdot T(u, \tau_1)] \cdot du = \\
& b \cdot \left[\lambda \frac{\partial T}{\partial u}(u_2, \tau) - \lambda \frac{\partial T}{\partial u}(u_1, \tau) \right] \cdot l(\tau_2) \cdot d\tau + c \cdot \rho \cdot b \cdot T_M \cdot [l(\tau_2) - l(\tau_1)] \cdot du \quad (4.5)
\end{aligned}$$

Equation (4.2) can be transformed:

$$\begin{aligned}
dQ_V &= c \cdot \rho \cdot b \cdot \frac{\partial [l(\tau) \cdot T(u, \tau)]}{\partial \tau} \cdot du \cdot d\tau = \\
&= c \cdot \rho \cdot b \cdot \left\{ [l(\tau)] \cdot \frac{\partial T(u, \tau)}{\partial \tau} + T(u, \tau) \cdot \frac{\partial [l(\tau)]}{\partial \tau} \right\} \cdot du \cdot d\tau
\end{aligned}$$

After substitution dQ_V into equation of heat balance (4.5) and transformation:

$$\begin{aligned}
c \cdot \rho \cdot b \cdot \left\{ [l(\tau)] \cdot \frac{\partial T(u, \tau)}{\partial \tau} \right\} \cdot du \cdot d\tau &= b \cdot \left\{ [l(\tau)] \cdot \frac{\partial \left[\lambda \cdot \frac{\partial T}{\partial u}(u, \tau) \right]}{\partial u} \right\} \cdot du \cdot d\tau + \\
&+ c \cdot \rho \cdot b \cdot \left\{ T_M \cdot \frac{\partial [l(\tau)]}{\partial \tau} - T(u, \tau) \cdot \frac{\partial [l(\tau)]}{\partial \tau} \right\} \cdot du \cdot d\tau
\end{aligned}$$

After reducing on $du \cdot d\tau$ the differential equation of heat balance takes the form

(taking into account, that the du is the uniform prism):

$$\begin{aligned}
\lambda \cdot [l(\tau)] \cdot \frac{d^2 T}{du^2} + c \cdot \rho \cdot [T_M - T(u, \tau)] \cdot \frac{\partial [l(\tau)]}{\partial \tau} &= [l(\tau)] \cdot c \cdot \rho \cdot \frac{dT}{d\tau}; \text{ or} \\
\frac{dT}{d\tau} &= \frac{\lambda}{c \cdot \rho} \cdot \frac{d^2 T}{du^2} + \frac{T_M - T(u, \tau)}{l(\tau)} \cdot \frac{\partial [l(\tau)]}{\partial \tau}.
\end{aligned}$$

This differential equation corresponds to the classical form of the parabolic differential equation with the source function:

$$\frac{dT}{d\tau} = \frac{\lambda}{c \cdot \rho} \cdot \frac{d^2 T}{du^2} + F(\tau), \quad (4.6)$$

$$\text{where } F(\tau) = \frac{T_M - T(u, \tau)}{l(\tau)} \cdot \frac{\partial[l(\tau)]}{\partial \tau} - \text{source function}; \quad (4.7)$$

Initial conditions

At τ' , when the first phase completes, the RE has the temperature in the whole volume:

$$T(u, \tau) \Big|_{\tau=\tau'} = T_M = \text{const} \quad (4.8).$$

Boundary conditions

Boundary conditions are formulated for two RE bases: BC₂ is located at the distance δ_T from the surface of meniscus:

$$T(u, \tau) \Big|_{u_2=\delta_T} = T_M = \text{const} \quad (4.9)$$

At the surface of the meniscus, boundary condition (BC₁) is:

$$\lambda \frac{\partial T}{\partial u}(u, \tau) \Big|_{u=0} = q_{RE}(\tau, x_0), \quad (4.10)$$

where $q_{RE}(\tau, x_0)$ - heat-flux density extracted from the base of the RE (it depends on the time of observation and coordinate of initial observation point). In the process of the meniscus stretching, RE is moving and, therefore heat-flux density is the function of the RE position (initial point is the point x_0).

4.3. Equations which describe the dynamics of RE

Functional description of the RE position is determined by meniscus displacement and stretching. The change of the position of the RE is the result of two simultaneous motions addition:

- displacement together with the meniscus resulting from meniscus height

increasing;

- displacement resulting from meniscus stretching.

The RE heat state at a given moment of time is defined by the heat transfer from RE depending on the RE displacement from the wall, and additional superheat from the core of the strand depending on the RE stretching. We assume that the start of the change in the RE heat state coincided with the start of the second phase. For the solution of the differential equation of heat conductivity it is necessary to know the functional dependence of RE location and stretching on the meniscus height.

RE position as a function of the meniscus height

In the end of the first phase the length of meniscus L_0 is equal (3.20):

$$L_0 = \int_0^{H_G} \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx, \quad (4.11)$$

The length of the part of meniscus L_{B_0} between point J and point $B(x_0)$ is equal:

$$L_{B_0} = \int_0^{x_0} \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx, \quad (4.12)$$

If the height of meniscus will increase to h ($H_h \geq h \geq H_G$), the meniscus length L_h becomes:

$$L_h = \int_0^h \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx, \quad (4.13)$$

Aspect ratio of meniscus : $\frac{L_h - L_0}{L_0}$.

Point $B(x)$ comes from point $B(x_0)$ into point $B(x_h)$ and length of the part of meniscus between point J and point $B(x_h)$ will be:

$$L_{B_h} = L_{B_0} \cdot \frac{L_h}{L_0}, \quad (4.14)$$

The coordinate x_h of the point $B(x_h)$ can be found from implicit equation:

$$L_{B_0} \cdot \frac{L_h}{L_0} = \int_0^{x_h} \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx, \quad (4.15)$$

where L_h - can be found from equation (4.13);

L_0 - can be found from equation (4.11);

L_{B_0} - can be found from equation (4.12);

x_0 - is known a priory;

x_h - unknown.

The equation (4.15) shows that $x_h = f(h, x_0)$

Coordinate $y(x_h, h)$ can be found from equation (3.13):

$$y(x_h, h) = Y(h) - Y(x_h),$$

Where $Y(h)$ and $Y(x_h)$ found from equation (3.14) and (3.15).

RE stretching as a function of h

Let us assume that RE width equal to l_0 when meniscus height equal to H_G .

Then if the meniscus height equal to $H_G < h < H_h$ then RE will stretched to l_h .

$$l_h = l_0 \cdot \frac{L_h}{L_0}, \quad (4.16)$$

where $L_h = f(h)$ (4.13);

L_0 - can be found from equation (4.11).

4.4. Transformation of the differential equation

For further calculations we need transformation from variable τ to variable h . This single-valued conversion of variables leads to a significant simplification in the calculations (4.6-4.10). Let assume that during $\Delta\tau = \tau - \tau_1$ the top of solidified mark moved along axis X and the height of the meniscus h :

$$h = H_G + V_C \cdot \Delta\tau = H_G + V_C \cdot (\tau - \tau_1),$$

we start to consider the cooling process from the second phase when meniscus height already equal to H_G (see Figure 3.3);

$V_C = \text{const}$ - the velocity of the displacement of the top of solidified mark.

At the time equal τ the length of meniscus equal to L_h and the width of RE equal to l_h .

For the source function the derivative $\frac{\partial[l(\tau)]}{\partial\tau}$ can be determined from (4.11):

$$\frac{\partial[l(\tau)]}{\partial\tau} = \frac{\partial[l_h]}{\partial\left(\frac{h}{V_C}\right)} = \frac{l_0}{L_0} \cdot V_C \cdot \frac{\partial L_h}{\partial h}$$

In this case from the expression (4.7) it follows, that the sources function:

$$\begin{aligned}
 F(\tau) &= \frac{T_M - T(u, \tau)}{l(\tau)} \cdot \frac{\partial[l(\tau)]}{\partial \tau} = \frac{T_M - T(u, \tau)}{l_0 \cdot \frac{L_h}{L_0}} \cdot \frac{l_0}{L_0} \cdot V_C \cdot \frac{\partial L_h}{\partial h} = \\
 &= \frac{T_M - T(u, \tau)}{L_\tau(h)} \cdot V_C \cdot \frac{\partial L_h}{\partial h} = F(h)
 \end{aligned}$$

$\frac{\partial L_h}{\partial h}$ can be determined from equation (4.12) by differentiation of integral:

$$\begin{aligned}
 \frac{\partial L_h}{\partial h} &= \frac{\partial \left\{ \int_0^h \sqrt{1 + \left[\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right]^2} dx \right\}}{\partial h}, \text{ or:} \\
 \frac{\partial L_h}{\partial h} &= \sqrt{1 + \left[\frac{(B \cdot h^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot h^2 + \sin \theta_0)^2}} \right]^2},
 \end{aligned}$$

Thus, source function became:

$$F(h) = \frac{T_M - T(u, h)}{L_h} \cdot V_C \cdot \sqrt{1 + \left[\frac{(B \cdot h^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot h^2 + \sin \theta_0)^2}} \right]^2}, \quad (4.17)$$

$$\text{where } L_h = \int_0^h \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx;$$

$$\text{Known, that } \frac{dT}{d\tau} = V_C \cdot \frac{dT}{dh}$$

After substitution of this expression in (4.6), we obtain:

$$\frac{dT}{dh} = \frac{\lambda}{V_C \cdot c \cdot \rho} \cdot \frac{d^2T}{du^2} + \frac{T_M - T(u, h)}{L_h} \cdot \sqrt{1 + \left[\frac{(B \cdot h^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot h^2 + \sin \theta_0)^2}} \right]^2}, \quad (4.18)$$

$$f(h) = \frac{T_M - T(u, h)}{L_h} \cdot \sqrt{1 + \left[\frac{(B \cdot h^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot h^2 + \sin \theta_0)^2}} \right]^2}.$$

4.5. BC₁ calculation

To determine boundary condition the estimation of the heat-flux density from the meniscus to the water through the slag and the mold wall is required. We examine heat transfer from the RE at $\tau > \tau'$.

Basic assumptions

Heat-flux density is determined according to the heat transfer through the multilayered wall. The coefficient of the liquid slag specific heat is more than 5 times lower than the coefficient of the specific heat of the liquid metal. Therefore for determination of the heat flux density function, we neglect the quantity of heat, introduced by liquid slag into the gap between the meniscus metal and the solid slag on the mold wall. We also neglect the heat, absorbed by liquid slag from the liquid metal, (low heat capacity of slag and its small quantity). Since boundary liquid slag – solid slag - is the boundary of the phase transition between liquid and solid slag, then the temperature of this boundary is constant and equal to the temperature of the slag crystallization (we neglect the interval of crystallization of slag). We assume the steady configuration of the slag rim observed during periodic displacements of meniscus. We consider only 1D heat transfer from RE to the examined the non-uniform prism, which is

located at a distance x from the beginning of coordinates. This prism is notes as the heat-transfer prism. The centerline of heat-transfer prism is parallel to axis Y . The heat transfer prism consists consecutively from liquid slag, solid slag and water-cooled mold wall. One base of heat-transfer prism is the base of the RE (located on meniscus of liquid metal). Second base of the prism is part of the surface of the water-cooled mold wall. RE and the heat-transfer prism are inclined toward each other at the angle φ .

For evaluation of the heat exchange from the RE to the water, the law of stationary heat exchange was used. Since we examined the stationary heat exchange (from the RE to the cooling water), an equal quantity of heat passed through each part of the considered prism, i.e., heat-flux density in all parts (thermal resistances) is equal

$$q = \lambda_i \cdot \frac{(T_{i+} - T_{i-})}{h_i} = \alpha_j \cdot (T_{j+} - T_{j-}), \quad (4.19)$$

where:

$$q = \lambda_i \cdot \frac{(T_{i+} - T_{i-})}{h_i} - \text{heat-flux density according to Fourier's law of thermal}$$

conductivity;

λ_i - the coefficient of the thermal conductivity of i -thermal resistance;

$(T_{i+} - T_{i-})$ - the temperature differential in the i -thermal resistance ;

h_i - the thickness of i -thermal resistance;

$q = \alpha_j \cdot (T_{j+} - T_{j-})$ - heat-flux density according to the Newton's law of heat emission;

α_j - the thermal transfer coefficient of j -thermal resistance;

$(T_{j+} - T_{j-})$ - the temperature differential on the j - boundary of two surfaces.

Considering structure of thermal resistance on the way from the surface of the meniscus to the cooling water, density of the thermal flux determined from the equation (without taking into account that the RE base is not perpendicular to the prism axis):

$$q = (T_m - T_{wat}) \cdot R_{\Delta T}, \quad (4.20)$$

where $R_{\Delta T}$ - thermal resistance on the way from the liquid metal meniscus to the cooling water.

Heat-flux density in the gap

Taking into account that the thermal resistance of the gap between liquid metal and mold wall, filled with solid and liquid slag has up to 80-90% of all thermal resistance on the way from the meniscus to the cooling water [42], we have:

$$q \approx \frac{b \cdot (T_m - T_{wat})}{\left(\frac{\Delta h_{sl-f}}{\lambda_{sl-f}} + \frac{\Delta h_{sl}}{\lambda_{sl}} \right)}; \quad (4.21)$$

where b varied from 0.8 to 0.9 depending on slag thickness, slag conductivity, mold wall material;

$\frac{\Delta h_{sl-f}}{\lambda_{sl-f}}$ - heat conductivity of fluid slag;

$\frac{\Delta h_{sl}}{\lambda_{sl}}$ - heat conductivity of solid slag.

I.e. the change of heat flux was determined by the change of the slag thermal resistance which resulted from a change in the thickness of the layers of solid slag and liquid slag (it is determined by definition of RE position with coordinate x):

$$\frac{1}{R_{sl}} = \frac{\Delta h_{sl-f}}{\lambda_{sl-f}} + \frac{\Delta h_{sl}}{\lambda_{sl}}.$$

Hence, heat-flux density (4.21) is a function of the thickness of the layers of solid and liquid slag.

During the estimation of the heat-flux density from the RE we consider not only RE position (coordinate x), but also angle between the RE and the axis X (projection of the RE exothermal base on the axis X). Finally, heat-flux density from the RE with the correction on the angle between the ax of the heat-transfer prism and RE:

$$q_{RE}(h, x_0) = \frac{b \cdot (T_m - T_{wat}) \cdot \cos \varphi(x_h)}{\left[\frac{\Delta h_{sl-f}(x_h)}{\lambda_{sl-f}} + \frac{\Delta h_{sl}(x_h)}{\lambda_{sl}} \right]}, \quad (4.22)$$

For x_h definition we use equation 4.15, for $\cos \varphi(x_h)$ definition we use equation:

$$\cos \varphi(x_h) = \frac{1}{\sqrt{1 + tg^2 \varphi(x_h)}} = \frac{1}{\sqrt{1 + \left(\frac{dy}{dx} \right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{B \cdot x_h^2 + \sin \theta_0}{\sqrt{1 - (B \cdot x_h^2 + \sin \theta_0)^2}} \right)^2}}.$$

4.6. Thermal boundary layer

The meniscus and meniscus area as parts of the shell of the billet are subject of the action of the liquid metal flow. The typical distribution of flow near the liquid metal surface in the mold is shown on Figure 4.3 (standard technology of continuous casting of steel).

Figure 4. 3 Distribution of the liquid metal flows in the mold [40]

The layer of liquid metal near the meniscus consists of the thermal boundary layer (TBL) and the velocity boundary layer (VBL). Thickness of the TBL is functionally connected to the thickness of the VBL and for liquid metals thickness of the TBL exceeds the thickness of the VBL [41].

Estimation of the TBL

Thickness of the TBL can be defined as the function of VBL thickness δ on the basis of criterial equation (Prandtl number). For liquid metals it is possible [42] to use dependence:

$$\delta \approx \delta_T \cdot \text{Pr}^{\frac{1}{3}}, \quad (4.23)$$

where δ - the VBL thickness;

δ_T - the thickness of the TBL;

Pr - Prandtl number.

$$\text{Pr} = \frac{\mu \cdot c_p}{\lambda},$$

where μ - coefficient of kinematics viscosity of liquid metal;

λ - coefficient of thermal conductivity of liquid metal;

c_p - the coefficient of heat capacity of liquid metal.

Estimation of the thickness of VBL is done on the basis of the criterial equation of the flow (Reynolds number):

$$\delta \approx \frac{z}{\sqrt{\text{Re}}}, \quad (4.24)$$

where z - distance from the end of plate, where the flow began;

$\text{Re} = \frac{\omega \cdot p}{\mu}$ - the Reynolds number;

p - the characteristic size, determined from the dimensions of the liquid metal flow;

ω - the speed of the liquid metal flow.

Let us consider that the velocity of the liquid metal flow ω is directly proportional to the speed of casting (extraction speed of ingot from the mold is V_C), $\omega \approx k \cdot V_C$ where k - coefficient of proportionality. After substituting Re , p , ω , k , δ in (14):

$$\delta_T \approx \frac{\delta}{\text{Pr}^{\frac{1}{3}}} = \frac{z}{\text{Pr}^{\frac{1}{3}} \cdot \sqrt{\text{Re}}} = \frac{z}{\left(\frac{\mu \cdot c_p}{\lambda}\right)^{\frac{1}{3}} \cdot \sqrt{\frac{\omega \cdot p}{\mu}}} = \frac{z \cdot \mu^{\frac{1}{6}}}{\left(\frac{c_p}{\lambda}\right)^{\frac{1}{3}} \cdot \sqrt{V_C} \cdot \sqrt{k \cdot p}} = \frac{C_\delta}{\sqrt{V_C}}$$

where

$$C_\delta = \frac{z \cdot \mu^{\frac{1}{6}}}{\left(\frac{c_p}{\lambda}\right)^{\frac{1}{3}} \cdot \sqrt{k \cdot p}}, \quad (4.25)$$

In the limits of the mark formation cycle it is possible to accept the physical characteristics of the liquid metal as constants: $\mu = \text{const}$, $c_p = \text{const}$, $\lambda = \text{const}$. Taking into account that the meniscus length is small in comparison with the flow extension above the solidified shell, it is possible to accept that the flow parameters above the meniscus are the same as above solidified shell. I.e. for any point of meniscus $z = \text{const}$, $p = \text{const}$. Thus, it is possible to assume $C_\delta = \text{const}$ and

$$\delta_T \approx \frac{\text{const}}{\sqrt{V_C}} = \frac{C_\delta}{\sqrt{V_C}}, \quad (4.26)$$

Some thermophysical parameters in expression (4.26) are unknown a priori. The speed of the flow of liquid metal near meniscus for the specific case is also unknown; we do not know exact value of the distance from the end of plate z , where the flow began (equation 4.24). Hence, the direct determination of δ_T from equation (4.26) is not possible and δ_T was chosen as fitting parameter. The following procedure was developed for estimation of the thickness of the TBL. We assume that at the end of the 1st stage meniscus is still liquid, i.e. its minimal temperature (T) is not less than temperature of liquidus (T_L). For some assigned value of δ_T at one fitting step we determine temperature distribution on meniscus (using the thermophysical model of meniscus). The iteration process is organized as follows. The minimum value T of the temperature distribution is compared with T_L . If $|T - T_L| \geq 0.2$ then we change value δ_T and repeat determination of the temperature distribution on meniscus and comparison T and T_L , i.e. do the next fitting step. The process ends when $|T - T_L| \leq 0.2$. It has been found that value $\delta_T = 0.3$ mm.

4.7. Summary of equations

Basic differential equation

$$\frac{dT}{dh} = \frac{\lambda}{V_C \cdot c \cdot \rho} \cdot \frac{d^2T}{du^2} + f(h)$$

Source function

$$f(h) = \frac{T_M - T(u, h)}{L_h} \cdot \sqrt{1 + \left[\frac{(B \cdot h^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot h^2 + \sin \theta_0)^2}} \right]^2},$$

where L_h - can be found from equation (4.9):

$$L_h = \int_0^h \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx,$$

x_h - the coordinate determining RE position and can be found from equation:

$$L_{B_0} \cdot \frac{L_h}{L_0} = \int_0^{x_h} \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx,$$

$$L_{B_0} = \int_0^{x_0} \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx,$$

$$L_0 = \int_0^{H_G} \sqrt{1 + \left(\frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \right)^2} \cdot dx,$$

$$H_G = \sqrt{\frac{-\sin \theta_0}{B}}, \quad (\text{Chapter 3.4})$$

Initial condition

$$T(u, h)|_{h=H_G} = T_M = \text{const}$$

Boundary conditions

$$T(u, h)|_{u=\delta_T} = T_M = \text{const},$$

$$\text{where } \delta_T \approx \frac{\text{const}}{\sqrt{V_C}} = \frac{C_\delta}{\sqrt{V_C}}.$$

$$\lambda \frac{\partial T}{\partial u}(u, h) \Big|_{u=u_1} = q_{RE}(h, x_0)$$

$$\text{where } q_{RE}(h, x_0) = \frac{(0,8...0,9) \cdot (T_m - T_{wat}) \cdot \cos \varphi(x_h)}{\left[\frac{\Delta h_{sl-f}(x_h)}{\lambda_{sl-f}} + \frac{\Delta h_{sl}(x_h)}{\lambda_{sl}} \right]},$$

$$\Delta h_{sl}(x_h) = \Delta y + Y(H_G) - Y(x_h), \quad (\text{Chapter 3.4})$$

$$\begin{aligned} \Delta h_{sh-f}(x_h) = & [Y(h) - Y(H_h)] \cap (H_G > x_h > 0) + \\ & + [Y(h) - Y(x_h) - Y(H_h) - Y(H_G)] \cap (x_h > H_G), \end{aligned} \quad (\text{Chapter 3.4})$$

$$\cos \varphi(x_h) = \frac{1}{\sqrt{1 + \left(\frac{(B \cdot x_h^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x_h^2 + \sin \theta_0)^2}} \right)^2}}$$

4.8. Analysis of the model of thermal processes on meniscus

A special program was developed for the temperature field of meniscus calculation (using MathCAD). This program was a continuation of the program which was developed for the model of the mechanical state of meniscus. Calculation was done for the second phase of the first stage of mark formation. Twenty points of observation were selected on the meniscus. For each point the differential equation of thermal

conductivity was solved. In addition, the displacement of the observation point was taken into account. Thus, the values of temperatures of meniscuses were determined.

Initial data for evaluating of the meniscus thermal condition

Thermophysical characteristics published for some slag and steels are shown in tables 4.1 and 4.2.

Table 4.1 Thermophysical characteristics of the mold slag

Thermophysical characteristics of the mold slag					
Name of parameters	Notation	Unit	Value of parameters		
			For steel 0.05%C	For low carbon steel	For medium-carbon steels steel
Temperature of slag on the surface in the mold	T_{sl}	°C			
Temperature of crystallization of slag		°C	950	1043	1163
Coefficient of heat conductivity of solid slag	λ_{sl}	W/(m °C)	1.5	1.73	1.83
Thickness of the solid slag on the mold wall	d_{sl}	mm		0.3...1.0	0.4...0.9
CaO/SiO ₂				0.96	1.41

Table 4.2 Thermophysical characteristics of steel

Thermophysical characteristics of steel						
Name of parameters	Notation	Unit	Value of parameters			
			Steel ¹ 0.1- .2%C	Steel ² 0.05%C	Steel ³ 0.5%C	Steel ⁴ 0.2%C
Temperature Liquidus	T_L	°C	1510	1529	1495	1520
Temperature solidus	T_S	°C	1475	1509	1445	1480
Density of liquid metal	ρ_M	kg/m ³	7400	7400	7500	7500
Coefficient of thermal conductivity of liquid metal	λ_M	W/(m °C)	23.26		23.2	29.1
Coefficient of heat capacity of liquid metal	C_M	kJ/(kg °C)	0.691		0.84	0.71
thermal diffusivity coefficient of liquid metal	$a \cdot 10^{-6}$	m ² /sec	4.546		3.682	5.465
Temperature in the core (with superheat) of strand	T_M	°C	1540	1550	1515	
Specific heat of crystallization	L	kJ/kg			268	268
Extraction velocity	V_C	m/sec	0.2			

Thermophysical characteristics of the mold slag mixtures used in experimental castings are shown in table 4.1. It is the medium carbon slag with CaO/SiO₂ equal to 1.41.

The results of the temperatures determination

The calculations were done for the 0,5% -carbon low-alloy steel with the thermophysical characteristics, shown in the tables 4.1. The appropriate solutions of the equation for the mechanical equilibrium of meniscus are used as the initial data for the solution of the differential equation of thermal conductivity. The changes of the temperature at the meniscus points were considered from time, when meniscus has the height $h = H_G$ and its temperature was constant throughout all length and equal to temperature of metal in the ingot's core $T = T_M$. While the position of the RE point $B(x_0)$ moved and the RE temperature field was calculated in the process of RE displacement. The examples of these calculations are shown in Figures 4.5 – 4.9.

Figure 4.4 Variation of RE temperature resulting from the changes of the meniscus height and determined for $h = 0.1 \cdot H_G$, 3D view

For the case shown in Figure 4.4 the casting temperature was 1515 °C and the point $B(x_0)$ is located at the distance $0.1 \cdot H_G$ from the surface of liquid metal. The axis U in Figure 4.4 is the distance along the RE centerline, the axis H is the parameter of the meniscus height ($H = (h - H_G) / H_h$). In Figure 4.4 and its 2-D projection, Figure 4.5 we can see only small changes of metal temperature because of close location of $B(x_0)$ to the point J.

Figure 4.5 2D views of variation of RE temperature resulting from the changes of the meniscus height and determined for $h = 0.1 \cdot H_G$

For the case shown in Figure 4.6 the casting temperature is equal to 1515 °C and the point $B(x_0)$ located at the distance $0.9 \cdot H_G$ from the surface of liquid metal. Here we have significant changes of temperature because of close location $B(x_0)$ to the mold wall.

Figure 4.6 Temperature of RE changes resulting from the changes of the meniscus height and determined for $h = 0.9 \cdot H_G$, 3D view

Below 2-D images of the 3-D graph for better illustration of the dependences T vs. U and T vs. h are shown.

Figure 4.7 2D views of temperature of RE changes resulting from the changes of the meniscus height and determined for $h=0.9 \cdot H_G$

Representation and analysis of the meniscus temperatures

Figure 4.8 Temperature field of meniscus at the end of first stage ($h = H_h$), 3D view

Resulting temperature field of the meniscus at the end of the first stage is shown on Figure 4.8. The set of the temperature values for this graph inferred from the sets of

the twenty computed values of temperatures for different position of RE. Below are 2-D image of the 3-D graph (with the liquidus temperature) is shown.

Figure 4.9 Temperatures of the meniscus at the end of first stage

As is evident, to the end of first stage of surface mark formation, curve for the temperature of meniscus is located above the temperature of liquidus.

Figure 4.10 Calculated shape of meniscus (blue) and calculated meniscus temperature field (red)

In the Figure 4.10 the changing of the temperature of the meniscus along the meniscus height is shown. The blue dashed line is the calculated shape of meniscus, red line is the temperature distribution along the meniscus height.

After obtained meniscus temperature field at the end of first stage we can verify that 1D heat transfer equation can be used in proposed thermal model. Let the q_t is the heat flux density in the direction tangent to meniscus and q_n is the heat flux density in horizontal direction. The heat-flux density according to Fourier's law of thermal conductivity:

$$q = \lambda \cdot \Delta T / h ;$$

From Figures 4.9 and 4.10 and table 4.2 we have: $\Delta T = 20^0 C$; $h = 10mm$;

$$\lambda = 23.2 W / m \cdot ^0 C .$$

$$q_t = 46,400 W / m^2 ;$$

For q_n we have: $\Delta T = 1475^0 C$; $h = 4mm$; $\lambda = 1.9 W / m \cdot ^0 C .$

$$q_n = 700,600 W / m^2 ;$$

hence $q_t / q_n = 7\%$. It confirms validity of our assumption about 1D heat transfer from the meniscus .

4.9. Conclusions.

- A mathematical model of the thermal state of meniscus was developed. Meniscus displacement and stretching were taken into consideration. In the estimation of the meniscus thermal state, the source function was introduced.
- Developed mechanical and thermal model of the meniscus can also be used for the mark formation study in the case of mold oscillation.
- The developed model can be further supplemented by the meniscus solidification calculation. It will permit to use the model for description of the meniscus thermal conditions at the second stage of mark formation.

5. Analysis of the adequacy of the developed model

Approbation of the developed mechanical model was done for the purpose of testing the model by experimental data and possible correction model parameters. The level of correspondence between model and data was evaluated using some methods of mathematical statistics.

5.1. Analysis of the adequacy of the mechanical model of the first stage of mark formation

Experimental data

The data were obtained from the surface marks on real billets were used for adequacy test of the model. These data were represented by measurements of fixed

SCALE 1 : 0.5 mm

Figure 5.1 Photo of template with position of meniscus marked as yellow dots

position of the meniscus, and were obtained by measurements of the templates images of surface marks. Figure 5.1 gives the sample of measurements for the medium-carbon low-alloyed steel. The experimental data values (Appendix B) represent a two-dimensional numerical file x_i, y_i . Quantity of pair's numbers we denote as N .

Optimization of the model fitting parameters

The developed model required a set of parameters that do not have an exact a priori determination. These fittings parameters are:

H_h - height of the meniscus at the end of the first stage of mark formation, thus

$$H_G \leq H_h \leq H_{\max};$$

θ_0 - initial angle of the meniscus (Chapter 3.3).

Since the parameters H_h and θ_0 cannot be exactly assigned a priori, it is necessary to find them first.

The coordinate system of the theoretical model does not obviously coincide with coordinate system of measurements. The parameter, which corrects the value x_i was introduced:

$$z_i = x_i + r ,$$

where r - correction of initial point, which corrects the parameter x ; r does not depend on the number of observation i .

Now equation 3.13 can be re-written as:

$$y(x_i, H_h, \theta_0, r) = -Y(H_h, \theta_0) + Y(x_i, \theta_0, r), \quad (5.1)$$

$$\text{where } Y(H_h, \theta_0) = \int_0^{H_h} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx;$$

$$Y(x_i, \theta_0, r) = \int_0^{x_i+r} \frac{(B \cdot x^2 + \sin \theta_0)}{\sqrt{1 - (B \cdot x^2 + \sin \theta_0)^2}} \cdot dx.$$

Thus, during adequacy testing the following parameters were corrected: H_h, θ_0, r .

The level of correspondence was evaluated using the residual sum of squares (RSS). RSS of the calculated and measured values:

$$RSS = \sum_{i=1}^N [y_i - y(x_i, H_h, \theta_0, r)]^2, \quad (5.2)$$

where x_i, y_i - pairs of numbers, which determine the position of the point on the meniscus, these pairs were found using experimental measurements;

$y(x_i, H_h, \theta_0, r)$ - predicted by the model.

The best approximation is obtained if we will get the minimum value of expression (5.2).

From this condition fitting parameters were found (Table 5.1). Minimization was done using mathematical software written for "MathCAD" (Appendix B, "Adequacy test").

Test of adequacy between experimental data and model

The conclusion that the model is adequate to the experimental data was done on the basis of Fisher test. Fisher's test is just the determination of the ratio of the estimated variances of two distributions. If the following inequality (5.3) is satisfied, then the model is adequate to the experimental data.

$$\frac{s_{ad}^2}{s_{rep}^2} \leq F_{1, N-p-1}^\alpha, \quad (5.3)$$

where $s_{ad}^2 = \frac{RSS}{N-p-1}$ - the variance of adequacy;

$N-p-1$ - the number of degree of freedom for variance of adequacy;

N - the number of experimental data;

$p = 3$ - the number of the parameters, determined by selection of their values for the best correspondence between experimental data and model;

s_{rep}^2 - variance of reproducibility;

1 - the number of the denominator degrees of freedom (for the variance of reproducibility);

$F_{1,N-p-1}^\alpha$ - Fisher ratio, the value of Fisher ratio we can find from the table of F-

distributions [43]. In this case the level of significance accepted as

$\alpha = 0,05$ (probability that the null hypothesis would be reject even if null

hypothesis is true [44]).

The dispersion of reproducibility was evaluated as follows:

Within the present research it was found that the maximum error of positioning of the point on the mark surface is $\pm 0,25mm$ (systematic error, i.e. half of scale, see Figure 5.1). Hence, the length of interval is $\Delta l = 0,50mm$. It is known that for the normal distribution 95% confidence interval corresponds to ± 2 standard deviation. From here we can write: $\Delta l = 4 \cdot s_{rep}$. Then the dispersion of reproducibility can be determined:

$$s_{rep}^2 = \left(\frac{\Delta l}{4} \right)^2.$$

By the fact that the image of template was increased by 20 times, the value of the dispersion of reproducibility needs to be corrected:

$$s_{rep}^2 = \left(\frac{\Delta l}{4 \cdot 20} \right)^2 = \left(\frac{0,5}{4 \cdot 20} \right)^2 = 0.000039$$

5.2. Results of the model adequacy testing.

Adequacy testing of the model was done for four samples, obtained from the surface of the different ingots. An example of the parameter H_h and θ_0 optimization results is given in Figure 5.2:

Figure 5.2 Results of optimization for the parameters H_h and θ_0 ,
number of points $N = 15$; $\theta_0 = 0,05^\circ$

green wide line – solid slag;

black line – meniscus;

blue crosses – experimental points

The results of calculations for four samples are given in the Table 5.1.

Table 5.1 The results of calculations

Sa mp les	Num ber of point s	Calculated parameters of the model		RSS	$N-p-1$	$\frac{S_{ad}^2}{S_{rep}^2}$	$F_{1,N-p-1}^\alpha$	Corres ponden ce
		H_h/H_{max}	$\theta_0 + 90$					
1	15	0.963	0.05	0.02033	12	43.37	243.91	yes
2	9	0.798	0.5	0.008617	6	36.76	233.99	yes
3	5	0.877	0.05	0.005893	2	75.43	199.50	yes
4	5	0.937	0.05	0.0095	2	121.60	199.50	yes

Table 5.1 data shows, that the model corresponds well to the experimental data, obtained in varied conditions. The Proposed model has quantitative and qualitative correspondence. Qualitative correspondence can be seen in Figure 5.2. This figure demonstrates that the points, obtained on the solidified part of the meniscus, do not exceed the top of the solidified mark, what corresponds to the prediction of the model. The points correspond to the meniscus extremum (model presentation) also coincides well with the experimental data.

6. Conclusions. Advantages of the developed model

The SM formation process occurs at the initial stage of solidification of the billet's shell and defines stability of continuous casting and quality of the billets surface. In order to describe the complex process of SM formation a two stage approach to the full cycle of SM formation was suggested. The first stage describes the changes of the meniscus position before touching the wall of the mold, after which the second stage begins. A mathematical model describing the first stage of the SM formation process was developed in this thesis. Oscillation factor which considerably complicates the modeling process was eliminated. Basic results of the modeling were tested during the continuous casting process in an immovable mold.

Novel features of the mechanical model

The developed mechanical model describes meniscus displacement and stretching during the first stage of SM formation process.

- The numerical analysis of the meniscus displacement and stretching during the first stage shows the existence of the two separate phases of meniscus movement. During the first phase meniscus moves away from the mold wall and during second phase meniscus comes to the mold wall.
- The top of the solidified mark is located at the same distance from the mold wall during the whole cycle of the mark formation;
- The upper meniscus after the first stage of mark formation is immovable.

- The developed numerical model has been tested against the experimental data. The level of correspondence between simulated and measured data has been evaluated and shows that the model corresponds well to the experimental data.
- The model was used to completely describe the process of displacement and deformation of meniscus. Results generated by the mechanical model were further used in the simulation of the thermal processes on meniscus.

Distinctive novel features of thermal model

- For purpose of thermal analysis the equation of 1 D transient heat transfer has been derived and the source function was obtained. Initial condition and boundary conditions were formulated on the basis of physical conception of mechanical model of SM formation process.
- The source function was obtained on the basis of detailed description of the meniscus stretching and considers the superheat which comes with portions of liquid metal from the core of billet to the meniscus.
- The numerical model of the temperature distribution on meniscus during its evolution was developed.
- The thermal model was used to predict of the thickness of the thermal boundary layer (δ_T) accepting that at the end of the first phase the minimal temperature of the meniscus equals to the temperature of liquidus.

Bibliography

1. Irving, W.R. "Continuous casting of steel", The Institute of Materials, London 1993
2. J. Savage, W.H. Pritchard, "The problem of Rupture of the Billet in the Continuous Casting Of Steel", the Journal of the Iron and Steel Institute, Vol. 178, November 1954, pp.269-277.
3. J. Elfsberg, "Oscillation Mark Formation in Continuous Casting Process", Casting of Metals, Royal Sweden Institute of Technology, 2003.
4. B.G. Thomas, "Continuous Casting: Modeling," *The Encyclopedia of Advanced Materials*, (J.Dantzig, A. Greenwell, J. Michalczyk, eds.) Pergamon Elsevier Science Ltd., Oxford, UK, Vol.2, 2001, 8p., (Revision 3, Oct. 12, 1999).
5. McGraw-Hill Dictionary of Physics, Sybil P. Parker (Editor in Chief), New-York, 1985.
6. Riboud P. V., Larrecq M., Steelmaking Conference Proceedings, Vol. 74, Washington D.C., USA, 1991, pp. 78-82.
7. Mills K. C., Steel Technology International, 1994.
8. Abratis H., Höfer F., Jünemann M., Sardemann J., Stoffel H., Stahl und Eisen 116 (1996), Nr. 4, pp. 85-91.
9. Pinheiro C. A., Samarasekara I.V., Brimacombe J.K., Iron and Steelmaker, October 1994, pp.55-56
10. Branion R.V., Mold Powders for Continuous Casting and Bottom Pour Teeming, Iron and Steel Society, AIME, pp. 3-14.
11. Mc Cauley W. L., Apelian D., Iron and Steelmaker, August 1983.

12. Gray R., Marston H., *Steelmaking Conference Proceedings*, Vol. 74, Washington D.C., USA, 14-17 Apr 1991, pp. 93-102.
13. Szekeres E. S., "Overview of mold oscillation in continuous casting", *Iron and steel Engineer*, July 1996, pp. 29-37.
14. Wolf M. M., *Mold Powders for Continuous Casting and Bottom Pour Teeming*, Iron and Steel Society, AIME, pp. 33-44.
15. N.A. McPherson and R.E. Mercer. *Ironmaking and Steelmaking*. 1980, vol. 67, pp.167-79.
16. T. Okazaki, H. Tomono, K. Ozaki, and Y. Akabane: *Tetsu-to-Hagane*, 1982, vol. 68, p.S929.
17. H. Oka, Y. Eda, T. Koshikawa, H. Nakato, T. Nozaki, and Y. Habu: *Tetsu-to-Hagane*, 1983, vol. 69, p. S1032.
18. T. Kuwano, N. Shigematsu, F. Hoshi, and H. Ogiwara: *Ironmaking and Steelmaking*, 1983, vol.10, pp.75-81.
19. T. Emi, H. Nakato, Y. Iida, K. Emoto, R. Tachibana, T. Imai, and H. Bada: *Proc. 61 st NOH-BOSC*, 1978, pp.350-61.
20. Kawakami, T. Kitagawa, H. Mizukami, H. Uchibori, S.Miyahara, M. Suzuki, and Y. Shiratani: *Tetsu-to- Hagane*, 1981, vol.67, pp.1190-99.
21. N.A. McPherson, A.W. Hardie, and G. Patric: *ISS Transactions*, 1983, vol.3, pp.21-36.

22. Takeuchi E., Brimacombe J.K., "The Formation of Oscillation Marks in the Continuous Casting of Steel Slabs", Metallurgical Transactions B, Vol. 15 B, September 1984, pp. 493-509.
23. Waters, B.H.C., "Continuous casting of Non-Ferrous Metals, Parts V and VI", Metal Treatment & Drop Forging, Vol.20, 1953, p.79 and 103.
24. Thornton, D. R., " An Investigation on the Function of Ingot Mold Dressings", Journal of the Iron and Steel Institute, vol. 7, July 1956, p.300-315.
25. Savage J., "A New Reciprocation Mold Cycle to Improve Surface Quality of Continuously Cast Steel", Iron and Coal Trades Review, Vol. 182, 4, April 14, 1961, p.787-795.
26. Sato R., " Powder Fluxes for Ingot Making or Continuous Casting", Bull. Jap. Institute of Metals, Vol. 12, 1973 (No. 6), p. 391.
27. Sato R., "Powder Fluxes for Ingot Making and Continuous Casting", 62nd National Open Hearth and Basic Oxygen Steel Conference proceedings, Vol. 62, Detroit, Michigan, 25-28 Mar, American Institute of Mining, Metallurgical, and Petroleum Engineers, Vol.62, 1979, pp. 48-67.
28. Davies I.G., and Sharp R.M., "The Formation of Reciprocation Marks During Continuous Casting of Slabs", British Steel Corporation Technical Note No. PDN/433/77, 1977.
29. Saucedo I.G., Beech J., Davies G. J., "The Development of Surface in Steel Ingots and Strands", Proceedings of 6th International Vacuum Metallurgy Conference on Special Melting: San Diego, California, April 23-27, 1979, pp. 885-904.

30. Saucedo I. G., "Early Solidification During the Continuous Casting of Steel", 74th Steelmaking Conference Proceedings, Vol. 74, Washington D.C., USA, 14-17 Apr 1991, pp. 79-89.
31. Tomono H., Ackermann P., Kurz W., Heinemann W., "Surface mark formation in continuous casting of steel", Proceedings of a Symposium: Continuous Casting of Small Cross Sections, Pittsburgh, Pa., 8 Oct 1980, Metallurgical Society of AIME, pp. 55-73.
32. Takeuchi E., Brimacombe J.K., "Effect of Oscillation-Mark Formation on the Surface Quality of Continuously Cast Steel Slabs", Metallurgical Transactions B, Vol. 16B, September 1985, pp. 605-625
33. Samarasekera I.V., Brimacombe J.K., Bommaraju R., "Mold Behaviour and Solidification in the Continuous Casting of Steel Billets", ISS Transactions, Vol. 5, 1984, pp.79-94.
34. Suzuki T., Miyata Y., Kunieda T., J. Japan Inst. Metals, Vol. 50, No.2 (1986), pp.208-214
35. Delhalle A., Larrecq M., Petegnief J., Radot J.P., La Revue de Métallurgie –CIT, June 1989, pp.483-489.
36. Lainez E., Busturia J. C., "The E.L.V. Solidification Model in Continuous Casting Billet Molds Using Casting Powder", 1st European Conference on Continuous Casting, Florence, Italy September 23-25, 1991, pp. 1.621-1.631.
37. J. Sengupta, H.J. Shin, B.G Thomas, S.H. Kim, "Micrograph evidence of meniscus solidification and sub-surface microstructure evolution in continuous cast ultralow-carbon steels", Acta Materialia, 54(2006), pp.1165-1173.

38. Mikloukhine S, Tsukerman V., “The mechanism of the slag renewing during continuous casting with low extraction speed”, proceeding of a 9th Conference: Problems of ingots casting, Volgograd, USSR, 1990.
39. Joonpyo Park, Heetae Jeong, Hoyoung Kim and Jongkeun Kim, “Laboratory Scale Continuous Casting of Steel Billet with High Frequency Magnetic Field”, ISIJ International, Vol. 42 (2002), No. 4, pp. 385–391.
40. Rajil Saraswat, A. B. Fox, K. C. Mills, P. D. Lee and B. Deo, “The factors affecting powder consumption of mold fluxes”, Scandinavian Journal of Metallurgy Vol. 33, 2004, pp. 85-91
41. Hermann Schlichting, Boundary layer theory. New York, McGraw-Hill Book Company, 1968, p.271.
42. Samoylovich Y.A., Krulevitskiy S.A., Goriainov V.A., Kabakov Z.K. Heat processes in continuous casting of steel. Moscow, Metallurgy, 1982, p.152.
43. D. Hudson, Statistic for physics. Moscow, World, 1970.
44. W. T. Eadie, D. Dryard, F.E. James, M. Roos, B. Sadoulet, “Statistical methods in experimental physics”. Nort-Holland Publishing Company - Amsterdam-London, 1971

Appendix A

Thermophysical model of meniscus for 1st stage formation

Parametric repres. of solution

Parameter of limiting value of meniscus dbh

Parameter of top of solidified mark displacement dd

Liquid steel density, kg/m³

$$\text{RoM} := 7200$$

Liquid slag density, kg/m³

$$\text{RoS} := 2667.3$$

Gravity acceleration, m/s²

$$g1 := 9.807$$

Coefficient of interphase tension, N/m

$$\text{SmS} := 1.2058$$

$$B := \left[(\text{RoM} - \text{RoS}) \cdot \frac{g1}{2\text{SmS}(10)^6} \right] \quad B = 0.018$$

Upper meniscus   Overflow process

Initial angle, degree

$$\text{Om} := 5$$

Calculated initial angle, radian

$$\text{Om l}(\text{Om}) := \frac{-\pi}{2} + \frac{\text{Om}\pi}{180}$$

sin of initial angle

$$\text{pp}(\text{Om}) := \sin(\text{Om l}(\text{Om}))$$

$$\text{pp}(\text{Om}) = -0.996$$

Coordinate $x=H_G$ along meniscus height, (point of extremum on meniscus) $y=y_{\max}$

$$H1(\text{Om}) := \sqrt{\frac{1 - \text{pp}(\text{Om})}{B}}$$

$$H1(\text{Om}) = 10.407$$

$H_{\max}$, initial angle $\text{Omega} > -90^\circ$

$$h1(\text{Om}) := \sqrt{\frac{-\text{pp}(\text{Om})}{B}}$$

$$h1(\text{Om}) = 7.352$$

First derivative
 $f1(x, \text{Om})$

$$f1(x, \text{Om}) := \frac{(\text{pp}(\text{Om}) + B \cdot x^2)}{\sqrt{1 - (\text{pp}(\text{Om}) + B \cdot x^2)^2}}$$

Assign limiting height of meniscus $H_h = bb$

Parameter of limiting height of
meniscus $0 < dbh < 1 \quad x H_G < H_h < H_{ma}$

$$dbh := 0.9$$

$$bb(dbh, Om) = 10.101$$

$$bb(dbh, Om) := h1(Om) + dbh(H1(Om) - h1(Om))$$

$$H1(Om) - bb(dbh, Om) = 0.306$$

Junction point(meniscus and liquid surface of metal coord.

$$Bh0(dbh, Om) := - \left[\int_0^{bb(dbh, Om)} fl[x, (Om)] dx \right]$$

Profile of rim

$$Bh0(dbh, Om) = 14.314$$

Vertical coordinate axis located on the top of solidified mark

$$Fb2(x, dbh, Om) := \left[Bh0(dbh, Om) + \int_0^x fl[x, (Om)] dx \right] \cdot (h1(Om) \geq x \geq 0)$$

Rim line equation.

$$Fb(x, dbh, Om) := Fb2(x, dbh, Om) + Fb2(h1(Om), dbh, Om) \cdot (x > h1(Om))$$

Solid slag line equation

Initial location of the top of solidified mark δx (along $Fb2(20, dbh, Om) = 0$
axis X) $zz1 \quad Fb2(h1(Om), dbh, Om) = -1.55 \quad Fb(h1(Om), dbh, Om) = -1.55$

Given

$$aa := .0001$$

$$z := 0 + aa$$

Liquid meniscus length

In the end of 1 phase

$$Fb(z, dbh, Om) = 0 \quad zz1 := \text{Find}(z) \quad zz1 = 3.979$$

Parameter of displacement of solidified mark dd

$$dd(Om) = 14.946$$

Value of displacement $db(bb)$

$$LL\alpha(Om) := \int_0^{h1(Om)} \sqrt{1 + fl[x, (Om)]^2} dx$$

$$dd := .80$$

Coordinate of the top of solidified mark X after
its displacement ($bb1$) = current height of
meniscus h

$$db(dd, dbh, Om) = 4.898$$

$$db(dd, dbh, Om) := dd \cdot (bb(dbh, Om) - zz1)$$

Coordinate of the extremum point along
X, ($y = y_{max}$) on solidified mark

$$bb1(dd, dbh, Om) := db(dd, dbh, Om) + zz1$$

$$bb1(dd, dbh, Om) = 8.877$$

Coordinate of junction point if h

$$hh3(dd, Om) := h1(Om) + db(dd, dbh, Om)$$

$$hh3(dd, Om) = 12.249$$

$$Bh(dd, dbh, Om) := \left[\int_0^{bb1(dd, dbh, Om)} fl[x, (Om)] dx \right]$$

$$Bh(dd, dbh, Om) = 15.508$$

Meniscus line equation, meniscus leaned on solidif. mark

$$Fb3(x, dd, dbh, Om) := \left[Bh(dd, dbh, Om) + \int_0^x fl[x, (Om)] dx \right] \cdot (x \leq bb1(dd, dbh, Om))$$

$$x := 5 \quad Fb3(x, dd, dbh, Om) = 0.373 \quad Fb3(zz1, 1, dbh, Om) = -2.555 \times 10^{-6}$$

Meniscus line equation, meniscus leaned on solidif. mark +
section of solid. mark + solid slag.

$$Fb4(x, dd, dbh, Om) := Fb2(x - db(dd, dbh, Om), dbh, Om) \cdot (x - db(dd, dbh, Om) > zz1) + Fb2(h1(Om), dbh, Om) \cdot (x - db(dd, dbh, Om) \geq h1(Om)) + Fb$$

$$Fb4(h1(Om), dd, dbh, Om) = -0.356$$

$$Fb4(hh3(dd, Om), dd, dbh, Om) = -3.101$$

$$DFb(x, dd, dbh, Om) := (Fb4(x, dd, dbh, Om) - Fb(x, dbh, Om))$$

Gap between solid slag
And meniscus

$$DFb(0, dd, dbh, Om) = 1.194$$

$$dbh = 0.9$$

$$DFb(h1(Om), dd, dbh, Om) = 1.194$$

$$x := 0, .1 \dots 2 \cdot H1(Om)$$

$$dd = 0.8$$

$$Om = 5$$

$$dd := .95$$

Meniscus height parameter dd if
 The volume = max
 ($0 < dd < 1$)

$$dd0 := \frac{h1 - zz1}{db(0.9)}$$

$$dd0 = 0.66$$

Length of meniscus when the
 volume = max – base length

$$LLBQ(dd0, dbh, Om) := \int_0^{bb1(dd0, dbh, Om)} \sqrt{1 + f1[x, (Om)]^2} dx_*$$

$$LLBQ(dd0, dbh, Om) = 19.62$$

Initial position of observation point on the bases part of meniscus for
 Temperature changes

Assign $B(x_0)$ with coordinate $x = Bx$ ($0 < Bx < bb1(dd0, dbh)$)

Parameter of position xx ($0 < xx < 1$)
 Find coordinate $x B(x_0)$ along X ($0 < Bx < hh(dd0, dbh)$)

$$BxQ(xx) := xx \cdot bb1(dd0, dbh, Om)$$

$$YxQ(xx) := Fb3(BxQ(xx), dd0, dbh, Om)$$

$$xx := .5$$

$$BxQ(xx) = 4.011$$

$$YxQ(xx) = 1.456$$

Parameter of displacement
of solid mark from the base position 1>dd01>dd0

Meniscus length up to the point
 $B_0(x)$

$$LLBX(xx) := \int_0^{BxQ(xx)} \sqrt{1 + fl[x, (Om)]^2} dx$$

$$LLBX(xx) = 15.183$$

Liquid meniscus length after
top displ.to dd01

$$dd01 := .8$$

$$bb1(dd01, dbh, Om) = 8.877$$

Meniscus lengthening after top
displ. from base position

$$LLBQ(dd01, dbh, Om) = 20.527$$

Meniscus length. from
point $B(x_0)$ to $B(x)$

$$DLI(dd01) = 1.046$$

$$DLI(dd01) := \frac{LLBQ(dd01, dbh, Om)}{LLBQ(dd0, dbh, Om)}$$

Coordinate x for $B(x)$: $x=Bx1$

$$LLx(xx, dd01) := DLI(dd01) \cdot LLBX(xx)$$

$$LLx(xx, dd01) = 15.885$$

$$LLx(xx, dd01) = 15.885$$

$$LI(Bbx) := \int_0^{Bbx} \sqrt{1 + fl[x, (Om)]^2} dx$$

$$DL(xx, dd01, Bbx) := LLx(xx, dd01) - LI(Bbx)$$

$$dd01 := 1 \quad xx := 1$$

$$Bbx := .1 \text{ Given} \quad DL(xx, dd01, Bbx) = 0$$

$$Bbx1(xx, dd01) = 10.101$$

$$COS(xx, dd01) := \sqrt{\frac{1}{1 + (fl(Bbx1(xx, dd01), Om))^2}}$$

Tangent in point $B(x)$: cos

$$COS(xx, dd01) = 0.467$$

Thickness of solid slag in point $B(x)$:
 $DY0$

$$x := h1(Om)$$

$$DYQ(x, dbh, Om) = 0$$

$$dd := 0.8$$

$$DYQ(x, dbh, Om) = 0$$

Finding the point B(c), which cam to the top

Gap between meniscus and solid

slag in point B(x): DFb

$$DFb(x, dd, dbh, Om) = 1.194$$

Meniscus length after top displ.

$$dd := 1$$

$$LLBQ(dd, dbh, Om) = 19.62$$

Meniscus length to the final position B(c) $bb1(dd0, dbh, Om) = 8.022$

Length of the base part of meniscus

$$LLBQ(0, dbh, Om) = 15.138$$

$$LLBQ(dd0, dbh, Om) = 19.62$$

Meniscus length to B(c)

$$ULI(dbh, Om) := \frac{LLBQ(dd, dbh, Om)}{LLBQ(dd0, dbh, Om)} \quad ULI(dbh, Om) = 1.135$$

Liquid meniscus lengthening in respect to base

$$LLQ(dbh, Om) := \frac{LLBQ(0, dbh, Om)}{ULI(dbh, Om)}$$

$$LLQ(dbh, Om) = 13.34$$

$$LI(Bx) := \int_0^{Bx} \sqrt{1 + f1(x, Om)^2} dx$$

Coordinate x finding for point B(c): $x=Bcx$

$$DLQ(dbh, Om, Bx) = 0$$

$$DLQ(dbh, Om, Bx) := LLQ(dbh, Om) - LI(Bx)$$

$$Bx := .1 \quad \text{Given}$$

$$Bcx(dbh, Om) = 2.864$$

Coordinate y finding for B(c): $x=Yc01$

$$By(dbh, Om) = 2.893$$

$$By(dbh, Om) := Fb3(Bcx(dbh, Om), dd0, dbh, Om)$$

Parameter of coordinate x for point B0(c): $xc=Bcx$

$$xc := \frac{Bcx(dbh, Om)}{bb1(dd0, dbh, Om)} \quad xc = 0.357$$

Coordinate B_h(c) after meniscus stopped

$$LL(h1(Om)) = 18.946$$

$$LI(Bbx2) := \int_0^{Bbx2} \sqrt{1 + f1[x, (Om)]^2} dx$$

The liquid meniscus length if the liq. slag vol.=max

$$LLBQ(dd0, dbh, Om) = 19.62$$

Initial position of the point of observation (for temperature calc.)

Meniscus length till the point $B_0(c)$ $xx := xc$ $dd01 := 1$

$$LLQ(dbh, Om) = 13.34$$

Meniscus length after top displacement
to 1

$$LLBQ(dd01, dbh, Om) = 22.265$$

Meniscus lengthening after the top displacement $DLI(dd01) := \frac{LLBQ(dd01, dbh, Om)}{LLBQ(dd0, dbh, Om)}$

$$DLI(dd01) = 1.135$$

Liquid meniscus length till point
 $B(c)$ after top displacement from the
base position till the end of first stage

$$LLx(xx, dd01) := DLI(dd01) \cdot LLBx(xx)$$

$$LLx(xx, dd01) = 15.138$$

Coordinate x for point $B(c)$ finding:

$$x = Bx1$$

$$LLx(xx, dd01) = 15.138$$

$$LL(Bbx) := \int_0^{Bbx} \sqrt{1 + f1[x, (Om)]^2} dx$$

$$DL(xx, dd01, Bbx) := LLx(xx, dd01) - LL(Bbx)$$

$$Bbx := .1 \quad \text{Given} \quad DL(xx, dd01, Bbx) = 0 \quad xx := xc$$

$$Bbx1(xx, dd01) = 3.979 \quad Bby3 := Fb3(Bbx1(xx, dd01), dd01, dbh, Om)$$

$$x := 0, .2 \dots 2 \cdot H1(Om) \quad dbh = 0.9 \quad Om = 5 \quad dd := dd0$$

Heat constants

Coef. of solid slag heat conductivity $K_{sl} \equiv 1.83$

Coef. of liquid slag heat conductivity $K_{sl\_f} := 2$

Solid slag thickness outside rim, MM $D_{sl} \equiv 1.5$

Extraction speed, MM/sec $VV \equiv 3.33$

Thermal diffusivity RE MM^2/sec $A_m \equiv 3.68$

Coef. of liquid metal heat conductivity $K_{tm} \equiv 23.2$

Thickness of disturbance layer (RE length) $L_m := .3$

Water temperature $T_w \equiv 15$

Liquid metal temperature (with superheat) $T_m \equiv 1515$

Temperature solidus $T_{sol} \equiv 1445$

Temperature liquidus $T_{lik} \equiv 1495$

Differential equation of unsteady state heat transfer

$$dd0 = 0.66 \quad xx := xc \quad dd := 1 \quad bb1(dd, dbh, Om) = 10.101$$

$$T1 := 1 - dd0 - .000001 \quad Dc = 6.766 \quad bb1(dd0, dbh, Om) = 8.022$$

$$Dc := \frac{A_m \cdot (bb(dbh, Om) - zzl)}{VV} \quad LLBQ(dd01, dbh, Om) = 22.265$$

$$dd01 := 1 \quad fl(Bbxl(xx, dd01), Om) = -0.992 \quad COS(xx, dd01) = 0.71$$

$$DYQ(xx, dd01, Om) = 12.119 \quad dd0 = 0.66 \quad dbh = 0.9 \quad Om = 5 \quad Bbxl(xx, dd0) = 2.864$$

$$DFb(Bbxl(xx, dd01), dd01, dbh, Om) = 0 \quad (fl(Bbxl(xx, dd01), Om))^2 = 0.985$$

Given

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(T_m - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = T_m \quad spacepts \equiv 5 \quad timepts \equiv 10$$

$$u_x(0, t) = \frac{[(T_m - T_w) \cdot 0.8 \cdot COS(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{K_{sl}} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{K_{sl\_f}} \right] \cdot K_{tm}}$$

$u(Lm, t) = Tm$
 $u := \text{Pdesolve}\left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts}\right]$
 $HHh(t) := bb1(dd0, dbh, Om) + (bb1(1, dbh, Om) - bb1(0, dbh, Om)) \cdot t$
 $A := \text{CreateMesh}(u, 0, Lm, 0, T1)$

$u(0, T1) = 1.50856 \times 10^3$
 $xx = 0.357$
 $n := 20$
 $i := 0..n$
 $j := 0..n$
 $xL_i := \frac{Lm}{n} \cdot i$
 $xT_i := \frac{T1 \cdot i}{n}$
 $HT_i := bb1(dd0, dbh, Om) + (bb1(1, dbh, Om) - bb1(0, dbh, Om)) \cdot (xT_i)$
 $UU_{0,i} := u(xL_i, T1)$
 $UU_{0,0} = 1.509 \times 10^3$
 $UT_{i,0} := u(0, xT_i)$
 $TTU_{i,j} := u(xL_i, xT_j)$
 $UU_{0,0} = 1.509 \times 10^3$

$$xx := 0$$

Given

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{0,i} := u(xL_i, T1)$$

$$UU_{0,0} = 1.515 \times 10^3$$

$$UH_{0,i} := u(0, xT_i)$$

$$xx := 0.05 \quad \text{Given}$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(L_m, t) = T_m$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ L_m \end{pmatrix}, t, \begin{pmatrix} 0 \\ T_1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{1,i} := u(xL_1, T_1) \quad UU_{1,0} = 1.515 \times 10^3 \quad UH_{1,i} := u(0, xT_i)$$

$$xx := 0.1 \quad \text{Given}$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(T_m - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = T_m$$

$$u_x(0, t) = \frac{[(T_m - T_w) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl_f} \right] \cdot Ktm}$$

$$u(L_m, t) = T_m \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ L_m \end{pmatrix}, t, \begin{pmatrix} 0 \\ T_1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{2,i} := u(xL_1, T_1)$$

$$UH_{2,0} = 1.515 \times 10^3$$

$$UU_{2,i} := u(xL_i, T1) \quad UU_{2,0i} = 1.514 \times 10^3$$

$$xx := 0.15 \quad \text{Given}$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = Tm$$

$$u_x(0,t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx \cdot t + dd0)]}{\left[\left(\frac{DYQ(Bbx l(xx \cdot t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbx l(xx \cdot t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{3,i} := u(xL_i, T1) \quad UU_{3,0} = 1.514 \times 10^3 \quad UH_{3,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.2$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbx l(xx \cdot t + dd0), Om)^2}$$

$$u(x, 0) = Tm$$

$$u_x(0,t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx \cdot t + dd0)]}{\left[\left(\frac{DYQ(Bbx l(xx \cdot t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbx l(xx \cdot t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{4,i} := u(xL_i, T1) \quad UU_{4,0} = 1.513 \times 10^3 \quad UH_{4,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.25$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbx l(xx \cdot t + dd0), Om)^2}$$

$$u(x, 0) = Tm$$

$$u_x(0,t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx \cdot t + dd0)]}{\left[\left(\frac{DYQ(Bbx l(xx \cdot t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbx l(xx \cdot t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{5,i} := u(xL_i, T1) \quad UU_{5,0} = 1.512 \times 10^3 \quad UH_{5,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.3$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{6,i} := u(xL_i, T1) \quad UU_{6,0} = 1.511 \times 10^3 \quad UH_{6,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.35$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{7,i} := u(xL_i, T1)$$

$$UU_{7,0} = 1.509 \times 10^3$$

$$UH_{7,i} := u(0, xT_i)$$

$$xx := 0.4 \quad \text{Given}$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{8,i} := u(xL_i, T1)$$

$$UU_{8,0} = 1.507 \times 10^3$$

$$UH_{8,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.45$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{9,i} := u(xL_i, T1)$$

$$UU_{9,0} = 1.504 \times 10^3$$

$$UH_{9,i} := u(0, xT_i)$$

$$xx := 0.5$$

Given

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{10,i} := u(xL_i, T1) \quad UU_{10,0} = 1.501 \times 10^3 \quad UH_{10,i} := u(0, xT_i)$$

Given $xx := 0.55$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{11,i} := u(xL_i, T1) \quad UU_{11,0} = 1.499 \times 10^3 \quad UH_{11,i} := u(0, xT_i)$$

Given $xx := 0.6$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{12,i} := u(xL_i, T1)$$

$$UU_{12,0} = 1.497 \times 10^3$$

$$UH_{12,i} := u(0, xT_i)$$

$$xx := 0.65 \quad \text{Given}$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{13,i} := u(xL_i, T1) \quad UU_{13,0} = 1.497 \times 10^3 \quad UH_{13,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.7$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{14,i} := u(xL_i, T1)$$

$$\text{Given } xx := 0.75$$

$$UU_{14,0} = 1.497 \times 10^3 \quad UH_{14,i} := u(0, xT_i)$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm \quad u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{15,i} := u(xL_i, T1) \quad UU_{15,0} = 1.499 \times 10^3 \quad UH_{15,i} := u(0, xT_i)$$

$$\text{Given } xx := 0.8$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(L_m, t) = T_m$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ L_m \end{pmatrix}, t, \begin{pmatrix} 0 \\ T_1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{16,i} := u(xL_1, T_1)$$

$$UU_{16,0} = 1.501 \times 10^3 \quad UH_{16,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.85$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(T_m - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = T_m$$

$$u_x(0, t) = \frac{[(T_m - T_w) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(L_m, t) = T_m$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ L_m \end{pmatrix}, t, \begin{pmatrix} 0 \\ T_1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{17,i} := u(xL_1, T_1) \quad UU_{17,0} = 1.503 \times 10^3 \quad UH_{17,i} := u(0, xT_i)$$

$$\text{Given} \quad xx := 0.9$$

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(T_m - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)^2}$$

$$u(x, 0) = T_m$$

$$u_x(0, t) = \frac{[(T_m - T_w) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(L_m, t) = T_m$$

$$u := \text{Pdesolve}\left[u, x, \begin{pmatrix} 0 \\ L_m \end{pmatrix}, t, \begin{pmatrix} 0 \\ T_1 \end{pmatrix}, \text{spacepts}, \text{timepts}\right]$$

$$A := \text{CreateMesh}(u, 0, L_m, 0, T_1)$$

$$xx := 0.95$$

Given

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$UU_{19,i} := u(xL_i, T1) \quad UU_{19,0} = 1.508 \times 10^3 \quad UH_{19,i} := u(0, xT_i)$$

$$xx := 1$$

Given

$$u_t(x, t) = Dc \cdot u_{xx}(x, t) + \frac{(Tm - u(x, t)) \cdot (bb(dbh, Om) - zzl)}{LLBQ(t + dd0, dbh, Om)} \cdot \sqrt{1 + fl(Bbxl(xx, t + dd0), Om)}^2$$

$$u(x, 0) = Tm$$

$$u_x(0, t) = \frac{[(Tm - Tw) \cdot 0.8 \cdot \cos(xx, t + dd0)]}{\left[\left(\frac{DYQ(Bbxl(xx, t + dd0), dbh, Om) + Dsl}{Ksl} \right) + \frac{DFb(Bbxl(xx, t + dd0), t + dd0, dbh, Om)}{Ksl\_f} \right] \cdot Ktm}$$

$$u(Lm, t) = Tm$$

$$u := \text{Pdesolve} \left[u, x, \begin{pmatrix} 0 \\ Lm \end{pmatrix}, t, \begin{pmatrix} 0 \\ T1 \end{pmatrix}, \text{spacepts}, \text{timepts} \right]$$

$$xx = 1 \quad dd0 = 0.66 \quad t := T1t + dd0 = 1 \quad Bbx1(xx, t + dd0) = 10.101$$

$$\text{COS}(xx, t + dd0) = 0.467$$

$$fl(Bbx1(xx, t + dd0), Om)^2 = 3.595 \quad LLB(t + dd0, dbh, Om) = 22.265$$

$$UU_{20,i} := u(xL_i, T1) \quad UU_{20,0} = 1.51 \times 10^3 \quad UH_{20,i} := u(0, xT_i)$$

$$j := 0..n \quad UT_{j,i} := 0.9975 Tlik \quad LiK_{j,i} := Tlik \quad xH_j := \frac{bb(dbh, Om) \cdot j}{n}$$

UU

```

x := 0, .2 .. 2 · H1(Om)

dbh = 0.9

Om = 5

dd := .9995

kk := 5

kk := FRAME■      dd := dd0 + (1 - dd0) ·  $\frac{kk}{n}$ 

xHHLj :=  $\frac{bb1(dd, dbh, Om) \cdot j}{n}$ 

```

Appendix B

Adequacy test

Physical characteristics of materials:

Density of liquid slag, kg/m^3

RoM := 7200

Density of liquid steel, kg/m^3

RoS := 2667.3

Gravity acceleration, m/s^2

g1 := 9.807

Coefficient of interphase tension, H/M

SmS := 1.2058

Experimental data

$$\text{MAS} := \begin{pmatrix} 0 & 0 \\ .95 & .11 \\ 1.14 & .15 \\ 1.34 & .21 \\ 1.56 & .28 \\ 1.76 & .37 \\ 1.93 & .47 \\ 2.13 & .58 \\ 2.33 & .71 \\ 2.51 & .84 \\ 2.67 & .97 \\ 2.84 & 1.11 \\ 3.03 & 1.29 \\ 3.19 & 1.47 \\ 3.37 & 1.68 \\ 3.57 & 1.93 \end{pmatrix}$$

$$B := \left[(RoM - RoS) \cdot \frac{g1}{2 \cdot SmS 10^6} \right] \quad B = 0.018$$

Liquid metal overflow

$$\Omega_{gal} := .07$$

$$\Omega := \frac{-\pi}{2} + \frac{\Omega_{gal} \cdot \pi}{180}$$

$$\Omega = -1.57$$

tan of initial angle

$$tt := \tan(\Omega)$$

$$tt = -818.511$$

sin of initial angle

$$pp := \frac{|tt|}{\sqrt{1 + (tt)^2}}$$

$$pp = 1$$

Max meniscus height, if initial angle equal 90^0

$$H := \sqrt{\frac{2}{B}}$$

$$H = 10.416$$

Max meniscus height, if initial angle
equal $j < 90^0$ ($pp = \tan j$)

$$pp = 1$$

$$H1 := \sqrt{\frac{1 + pp}{B}}$$

$$H1 = 10.416$$

Coordinate (x) along meniscus height in
which $y = y_{\max}$

$$h := \sqrt{\frac{1}{B}}$$

$$h = 7.366$$

$$h1 := \sqrt{\frac{pp}{B}}$$

$$h1 = 7.366$$

First derivative as a function $f(x)$

$$f1(x) := \frac{-(pp - B \cdot x^2)}{\sqrt{1 - (pp - B \cdot x^2)^2}}$$

Meniscus line $y=F(x)$

$$F1(x) := \int_0^x f1(x) dx$$

$$F(x) := \int_{0.1}^x f(x) dx_*$$

Define meniscus height using parameters dbh

$$dbh := 0.9$$

$$ms := 2$$

$$bb(dbh) := h1 + dbh(H1 - h1)$$

If we place coordinate origin into point of junction of meniscus and surface of liquid metal (ext. system), that coordinate of current location of the top of solidified mark $Bh(dbh)$ – displacement of the coordinate system along X

$$Bh(dbh) := \int_0^{bb(dbh)} f1(x) dx$$

$$Bh(dbh) = -36.57$$

Profile of rim.

Equation of line of rim.

$$Fb2(x, dbh) := - \left(Bh(dbh) - \int_0^x fl(x) dx \right) \cdot (bb(dbh) \geq x \geq 0)$$

Equation of solid slag line

$$Fb(x, dbh) := Fb2(x, dbh) \cdot (x < h1) + Fb2(h1, dbh) \cdot (x \geq h1)$$

$$Fb(6, dbh) = -1.308$$

Initial position of the top of solid mark on the rim (before displacement) zz1

$$aa := .0001$$

$$z := 0 + aa$$

Given

$$Fb(z, dbh) = 0$$

$$zz1(z, dbh) := \text{Find}(z)$$

$$zz1(z, dbh) = 3.996$$

$$rO := \text{rows}(\text{MAS}) - 1$$

$$rO = 15$$

Base for counting X

$$M^{\langle \emptyset \rangle} := \overrightarrow{\left(zz1(z, dbh) + ms + \text{MAS}_{rO,0} - \text{MAS}^{\langle \emptyset \rangle} \right)}$$

$$i := 0.. \text{rows}(M) - 1$$

$$M_{i,0} := \text{ztl}(z, \text{dbh}) + \text{ms} + \text{MAS}_{\text{rO},0} - \text{MAS}_{i,0}$$

Base for counting Y

$$M^{(1)} := \overrightarrow{\text{MAS}^{(1)}} - \text{MAS}_{\text{rO},1}$$

$$\text{ztl}(z, \text{dbh}) = 3.996$$

$$M^{(0)} =$$

	0
0	9.566
1	8.616
2	8.426
3	8.226
4	8.006
5	7.806
6	7.636
7	7.436
8	7.236
9	7.056
10	6.896
11	6.726
12	6.536
13	6.376
14	6.196
15	5.996

$$M^{(1)} =$$

	0
0	-1.93
1	-1.82
2	-1.78
3	-1.72
4	-1.65
5	-1.56
6	-1.46
7	-1.35
8	-1.22
9	-1.09
10	-0.96
11	-0.82
12	-0.64
13	-0.46
14	-0.25
15	0

$$\text{mean}(M^{(1)}) = -1.169$$

$$\text{SMM}(\text{dbh}, \text{ms}) := \sum_{i=0}^{\text{rows}(M)-1} \left[\left(\text{Fb}(\text{ztl}(z, \text{dbh}) + \text{ms} + \text{MAS}_{\text{rO},0} - \text{MAS}_{i,0}, \text{dbh}) - M_{i,1} \right)^2 \right]$$

$$\text{dbh} := 0.8 \quad \text{Given} \quad -1 < \text{ms} < 1$$

$$P := \text{Minimize}(\text{SMM}, \text{dbh}, \text{ms}) \quad \text{dbh1} := P_0 \quad \text{dbh1} = 0.963$$

$$\text{ms} := -2, -1.6..2 \quad \text{dbh} := 0.7, 0.75..1 \quad \text{dbh} := P_0 \quad \text{ms} := P_1$$

$$\begin{aligned}
 i &:= 0.. \text{rows}(M) - 1 & M_{i,0} &:= \text{zzl}(z, P_0) + P_1 + \text{MAS}_{rO,0} - \text{MAS}_{i,0} \\
 x &:= 0, .1.. 2H1 & b &:= 10 \text{round} \left(2 \frac{H1}{10} \right) \\
 P &= \begin{pmatrix} 0.963 \\ 0.059 \end{pmatrix} & \text{SMM}(P_0, P_1) &= 0.02260957 & \text{Fb}(h1, dbh) &= -2.017 & \text{Omegal} &= 0.07 \\
 & & \text{zzl}(z, P_0) &= 3.57 \\
 rO &= 15
 \end{aligned}$$

	0
0	-1.93
1	-1.82
2	-1.78
3	-1.72
4	-1.65
5	-1.56
6	-1.46
7	-1.35
8	-1.22
9	-1.09
10	-0.96
11	-0.82
12	-0.64
13	-0.46
14	-0.25
15	0

X =

$$FM_i := Fb(\text{zzl}(z, dbh) + ms + \text{MAS}_{rO,0} - \text{MAS}_{i,0}, dbh)$$

$$i := 0.. \text{rows}(M) - 1$$

$$X := M^{(1)}$$

$$Y := FM$$

Y =

	0
0	-2.013
1	-1.852
2	-1.792
3	-1.718
4	-1.598
5	-1.528
6	-1.437
7	-1.32
8	-1.19
9	-1.062
10	-0.939
11	-0.799
12	-0.63
13	-0.476
14	-0.291
15	-0.069