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A Comparison of Two Interference Cancellation Techniques for Turbo Equalization

by

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A thesis submitted to the

Faculty of Graduate Studies and Research

In partial fulfillment of the requirements for the degree of

Master of Applied Science

Ottawa-Carleton Institute for Electrical and Computer Engineering

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Abstract

In this thesis, an improved interference canceller (IC) for intersymbol interference is proposed. The IC is derived under the assumption that the *a priori* probabilities of the transmitted symbols are not necessarily equal. Some comparisons, in terms of the bit error rate, are made between this improved IC and the classic IC, for different channel models, transmitted symbol block sizes, error control coding schemes, modulation schemes, and bit/symbol interleaving mappings. Simulation results show that the improved IC system is mostly beneficial for severe ISI channels. When larger block sizes or stronger error control codes are used, the relative difference between these two systems decreases. Using a dynamic variance (one that changes on a symbol-by-symbol basis) to calculate the branch metrics can also lead to an improved performance when the ISI is severe. When the residual ISI at the output of the equalizer is large, there is a great advantage to use bit interleaving in terms of performance. However, when the bits in a symbol are not independent, such as when 16-QAM is used, there is a slight advantage to using symbol interleaving.

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List of Symbols

$\underline{a}$	Source symbols
n_a	Number of bits in one source symbol
N_a	Number of source symbols
$\underline{c}$	Coded symbols
n_c	Number of bits in one coded symbol
N_c	Number of coded symbols
N_s	Number of states for convolutional encoder
L_c	Constraint length of the convolutional code
$\underline{d}$	Interleaved symbols
n_d	Number of bits in one interleaved symbol
N_d	Number of interleaved symbols
$\underline{v}$	Transmitted samples
N_v	Number of transmitted samples
σ_v^2	Average variance of the transmitted samples
$\underline{r}$	Received samples
N_r	Number of received samples
L	Channel delay spread
$\underline{h}$	Channel coefficients
$\underline{w}$	AWGN noise
N_0	Single-sided noise power spectral density
$\underline{z}$	Equalized symbols
N_z	Number of equalized symbols
$\sigma_{z,n}^2$	Variance of the equalized symbols
$\bar{\sigma}_z^2$	Average variance of equalized symbols
ε_n	The difference between the achieved equalizer output and the desired output
J_n	Mean square value of ε_n
$\underline{\gamma}$	Metric values
$\underline{\mu}$	De-interleaved metric values
$\hat{\underline{a}}$	Estimation of the source symbols
$\underline{\psi}$	Soft feedback information
$\underline{\phi}$	Interleaved soft feedback information
$\bar{\underline{v}}$	Mean value of the feedback symbols
$\bar{\sigma}_{v,m}^2$	Variance of the feedback symbols
$\bar{\sigma}_v^2$	Average variance of the feedback symbols

$\underline{p}$	p filter coefficients
$\underline{q}$	q filter coefficients
K_1	Non-causal part of the p filter
K_2	Causal part of the p filter
Γ	$(K_1 + K_2 + 1) \times (K_1 + K_2 + 1)$ matrix for LE
$\underline{\xi}$	$(K_1 + K_2 + 1)$ vector
Ψ	$(K_1 + K_2 + 1) \times (K_1 + K_2 + 1)$ matrix for IC

List of Acronyms

APP	<i>a posteriori</i> probability
AWGN	Additive white Gaussian noise
BER	Bit error rate
BICM	Bit-interleaved coded modulation
CE	Convolutional encoder
dB	Decibel
DFE	Decision feedback equalizer
IC	Interference canceller
ISI	Intersymbol interference
LC	Low complexity
LE	Linear equalizer
MAP	Maximum <i>a posteriori</i> probability
MLSE	Maximum likelihood sequence estimation
MMSE	Minimum mean square error
QAM	Quadrature amplitude modulation
SNR	Signal-to-noise ratio
TCM	Trellis coded modulation
ZF	Zero forcing

Chapter 1 Introduction

In many digital communication systems, especially wireless communication systems, data transmitted over a channel is mainly corrupted by inter-symbol interference (ISI), fading and additive white Gaussian noise (AWGN). Environmental factors, such as thermal noise, cause the AWGN. In wireless systems the ISI is usually caused by what is called multi-path interference. When a signal is transmitted over the air, more than one different copy of this signal may arrive at the receiver at different times because of media reflection and so on. This is called the multi-path interference. In the receiver, the received data is not just the transmitted data; it is composed of three parts: the transmitted data itself, the sum of the interference from neighboring symbols and the AWGN. When one designs a receiver, ISI must be considered. When error control coding is used to protect the transmitted data, it is the receiver's responsibility to recover the original message from the encoded signal corrupted by the ISI and AWGN. An optimal receiver will consider the error control code and the ISI channel jointly. That is, it will treat the error control code and the ISI channel as a single code. Unfortunately, such an optimal receiver is difficult to implement because of the computational complexity, particularly when interleaving is used. People try to find some sub-optimal ways to do this with the trade off between the performance and complexity.

To reduce the complexity, a sub-optimal receiver design is typically considered. This approach includes performing two separate tasks in the receiver:

1. Using an equalizer to combat the ISI, and

2. Using a decoder for the error control code to combat the residual ISI and additive noise.

Data received from the channel is processed with an equalizer to eliminate as much of the ISI as possible and produce an estimate of the transmitted data, which is passed to the decoder to get an estimate of the original message. The focus of this thesis is on the implementation of the equalizer.

1.1 Traditional Equalization Techniques

Many different equalization schemes have been proposed over the years. Probably the simplest one is the linear equalizer (LE), proposed by Lucky [1][2][3], which just consists of a linear filter to process the received data. The current and past values of the received signal are linearly weighted by equalizer coefficients and summed to produce the output. The coefficients can be chosen using the zero forcing (ZF) criterion or the minimum mean square error (MMSE) criterion. The ZF criterion tries to minimize the worst-case residual ISI at the output of the equalizer. When the length of the filter is infinite, the ISI can be eliminated fully. The main disadvantage of this criterion is that when the equalizer eliminates the ISI, it enhances the additive noise too. If the channel contains a spectral null in its frequency response, the zero-forcing LE will give an infinite gain at that frequency to compensate for the null. This will bring infinite additive noise. The MMSE criterion is more robust. Its coefficients are chosen to minimize the mean-square error between the actual and desired equalizer output, thereby reducing excessive noise enhancement.

The decision feedback equalizer (DFE) is another method with similar complexity to the LE, which is proposed by Austin [3][4]. A DFE is implemented using two linear filters: a feed-forward filter operating on samples of the received signal and a feedback filter operating on symbol decisions made on earlier outputs of the equalizer. The equalizer output is the output of the feed-forward filter minus the output of the feedback filter. The idea is that, if the past decisions are correct then the ISI contributed by these symbols can be completely cancelled by subtracting the past symbol values, with appropriate weighting, from the equalizer output. When a feedback decision is not correct, it will affect future decisions. This is called error propagation and it is a key drawback of the DFE.

The LE and DFE are simple but when the ISI is severe, their performance is poor. Maximum likelihood sequence estimation (MLSE) equalization and maximum *a posteriori* probability (MAP) equalization are two more powerful equalization schemes at the price of greater computational complexity. MLSE is an equalization method that minimizes the probability of a sequence error, which was proposed by Forney [3][5]. Over channels with ISI that spans L symbols, the MLSE method is equivalent to solving the problem of estimating the state of a discrete-time finite state machine. The finite state machine is the equivalent discrete time channel and its states are given by the L most recent channel inputs (the transmitted symbols). If the transmitted symbols are M -ary, the channel filter has M^L states. Then an M^L -state trellis can be constructed and the Viterbi algorithm may be applied to determine the most probable path through the trellis.

MAP equalization, proposed by Chang and Hancock [6], is an equalization method that minimizes the probability of a symbol error. Like MLSE, it is based on an

M^L -state trellis, but it processes the received data twice, once in a forward direction then again in reverse, so it is roughly twice as complex as MLSE. Its primary advantage over MLSE is that it can provide soft output, the *a posteriori* probability (APP) distribution of the transmitted data, which can be used as soft input to the decoder for the error-correcting code.

All the equalization techniques described above treat the equalization and decoding as two independent tasks. The equalizer does not make use of any of the redundancy introduced by the error control code. More effective equalization techniques make use of this redundancy.

1.2 Turbo Equalization

In 1993, Berrou, Galvieux and Thitimajshima first introduced the “turbo concept” as an iterative decoding algorithm for parallel concatenated convolutional codes (turbo codes) [7]. This method uses two disjoint decoders, implemented using the soft-output MAP algorithm, repeatedly exchanging soft information in an iterative manner. This method is effective although sub-optimal. The turbo concept has since been applied to various communication problems, such as trellis-coded modulation (TCM), code division multiple access (CDMA) and turbo equalization.

In 1995, Douillard et al. proposed a receiver called the turbo-detector [8] (MAP-based turbo equalizer), which combined a MAP equalizer with a MAP decoder through an iterative process. The performance has proved to be near-optimum over many channels. But both the MAP equalizer and MAP decoder have high computational complexity. In particular, when the transmitted symbol alphabet (M) or the ISI spread (L)

In 2002, Tuchler, Singer and Koetter proposed a new alternative to the MAP-based turbo equalizer [10]. This method is called minimum mean square error equalization using *a priori* information. They use an LE to replace the MAP equalizer and use the MMSE criterion to determine the LE's coefficients. Unlike a traditional LE, where the coefficients are calculated under the assumption that the transmitted symbols are independent and identically distributed (iid), they allow each symbol to have an arbitrary *a priori* distribution (although they still maintain the independence assumption). The *a priori* distribution is provided as output from the decoder (on the first iteration, the iid assumption is used). As a result, the equalizer coefficients change with each iteration. In fact, the optimal coefficients differ on a symbol-by-symbol basis, which leads to a prohibitive computational complexity. The authors also provide a sub-optimal mechanism for calculating the coefficients that is called the low complexity (LC) algorithm. The LC algorithm uses the time average of the feedback *a priori* information to determine the coefficients, which just changes on a per-iteration basis, not a per-symbol basis.

1.3 Motivation and Contributions of this Thesis

The classic IC system proposed by Glavieux, Laot and Labat [9] has the advantage of lower complexity, with fewer filter taps compared with Tuchler's system. Furthermore, the filter coefficients do not need to be recalculated with each iteration. However, because the filter coefficients are calculated based on the assumption that the feedback information is a perfect estimate of the transmitted data, its performance may not be good, particularly in channels with severe ISI.

The initial motivation of this research was to try to improve the classic IC by applying some of the assumptions and techniques used in the derivation of Tuchler's algorithm, while still maintaining low complexity. Also we want to provide a comparison of the performance of the classic IC system and the improved IC system.

The main contributions of this thesis are:

1. Derivating the improved IC based on the classic IC, including the coefficients of the filters, and the mean and variance of the equalized symbols. We also show that this improved IC is identical to Tuchler's low complexity (LC) solution.
2. Showing the relationship between the improved IC, the classic IC and the LE in terms of a single parameter: the "confidence" index about feedback information. If the confidence index is 1 then the improved IC reduces to the classic IC. If the index is 0 then the improved IC reduces to the LE.
3. Providing a performance comparison between the classic IC and the improved IC. The comparison is done for different channel models, block sizes, modulation schemes and error control codes. The effects of using bit vs. symbol interleaving on the performance of the classic and improved IC's based on 16-QAM is also investigated. Since the results presented here show that the improved IC system only has a performance advantage over the classic IC system for the severe ISI channel, the various issues that contribute to the performance difference over the severe ISI channel are discussed.

1.4 Organization of the Thesis

This thesis is composed of four chapters, including this introduction. Chapter 2 contains a description of the discrete-time communication system model. The improved IC algorithm is also derived in this chapter. Chapter 3 presents simulation results of the performance comparison between the classic IC and the improved IC. The conclusions and future work are summarized in Chapter 4. A detail description of the improved IC can be found in the appendix.

2.2 The Transmitter

Figure 2.2 shows the structure of a transmitter. It includes three parts: the error control coding, the interleaver and the signal mapper.

Figure 2. 2 Structure of the transmitter.

2.2.1 Error Control Coding

Error control coding is useful for combating noise and improving system performance. Although there are many kinds of error control coding schemes, in this thesis we concentrate on convolutional coding since it is widely used and convenient to implement. A convolutional encoder (CE) is a finite state machine with $N_s = 2^m$ states, where m is the number of bit registers in the encoder. We can use (n_c, n_a, L_c) to denote a CE that has input message symbols with n_a bits per symbol, output code symbols with n_c bits per code symbol and L_c is the constraint length of the code.

The input to the CE is the source message, $\underline{a}$ (with N_a symbols), and the output is $\underline{c} = [c_0 \ c_1 \ \dots \ c_{N_c-1}]$, which consists of $N_c = N_a + L_c - 1$ code symbols. Each code symbol consists of n_c bits and can be expressed as $c_n = [c_n^{(n_c-1)} \ c_n^{(n_c-2)} \ \dots \ c_n^{(0)}]$.

As an example, for the (2,1,3) convolutional encoder with generators $(5,7)_8$ is shown in Figure 2.3.

Figure 2. 3 (2,1,2) Convolutional encoder with generators $(5,7)_8$.

2.2.2 Interleaver

Convolutional codes are designed to combat statistically independent errors caused by memoryless channels such as the AWGN channel. For ISI channels, which have memory, convolutional codes are not effective when used directly. However an interleaver can separate the code symbols such that the ISI channel looks like a memoryless channel. Then a convolutional code can be used.

An interleaver works on either a symbol-by-symbol basis or a bit-by-bit basis. With a symbol interleaver the code sequence is changed based on the symbol level. The order of the bits within each symbol remains intact. A bit interleaver will change the code sequence at the bit level and each bit in each symbol is involved in this process.

2.2.2.1 Symbol Interleaver

A symbol interleaver changes the order of the input code symbols, but keeps each code symbol intact. The interleaved code symbols are denoted by

$$\underline{d} = [d_0 \dots d_1 d_{N_d-1}], N_d \text{ symbols,}$$

$$d_n = [d_n^{(n_d-1)} d_n^{(n_d-2)} \dots d_n^{(0)}], n_d \text{ bits/symbol,}$$

where $N_d = N_c$, $n_d = n_c$. The interleaving algorithm is $d_n = c_{IL[n]}$, where

$\underline{c} = [c_0 c_1 \dots c_{N_c-1}]$ are the input and $IL[\bullet]$ is the one-to-one interleaving mapping, which is defined over $n \in \{0, 1, \dots, N_d-1\}$ and $IL[n] \in \{0, 1, \dots, N_c-1\}$. There are several kinds of mapping schemes such as the rectangular mapping scheme and the random mapping scheme. The interleaver plays an important role in system performance, so the mapping should be chosen carefully. However choosing an optimal interleaving method is not a research topic in this thesis, and only random interleaving is considered for it is easy to implement. The input code symbols are moved to new positions using a predefined, randomly selected permutation of the starting positions.

2.2.2.2 Bit Interleaver

Bit interleaving involves three processes: a symbol-to-bit converter, an interleaver and a bit-to-symbol converter. Figure 2.4 shows the structure.

Since the encoder's output is symbol-based, the symbol-to-bit converter breaks each symbol into bits. The input is the code symbol sequence $\underline{c}$ and the output is $\underline{c}' = [c'_0 c'_1 \dots c'_{N_c-1}]$, which has N_c bits, where $N_c = N_c n_c$ and $c'_m \in \{0, 1\} \forall m \in \{0, 1, \dots, N_c - 1\}$. The converter output is related to the input by $c'_{m_c+i} = c_n^{(i)}$.

Figure 2. 4 Structure of a bit interleaver.

The interleaver reorders the code bits. The method is the same as for symbol interleaving except that it operates on bits. The output is expressed as $\underline{d}' = [d'_0 \ d'_1 \ \dots \ d'_{N_d-1}]$ with $d'_m = c'_{IL[m]}$, where $N_d = N_c$ is the number of interleaved bits and $m \in \{0, 1, \dots, N_d - 1\}$, $IL[m] \in \{0, 1, \dots, N_c - 1\}$.

The interleaver is followed by a bit-to-symbol converter, which reorganizes the interleaved bits into symbols. The output is expressed as

$$\underline{d} = [d_0 \ d_1 \ \dots \ d_{N_d-1}] \ N_d \text{ symbols,}$$

$$d_n = [d_n^{(n_d-1)} \ d_n^{(n_d-2)} \ \dots \ d_n^{(0)}] \ n_d \text{ bits/symbol,}$$

$$N_d n_d = N_d' = N_c' = N_c n_c.$$

The converter output is related to the input by $d_n^{(i)} = d'_{nn_d+i}$.

2.2.3 Symbol Mapper

The interleaved symbols are mapped into the signal constellation, which is defined by the mapping scheme. We use $\underline{v} = [v_0 \ v_1 \ \dots \ v_{N_v-1}]$ to denote the mapped symbols, where $N_v = N_d$ is the number of symbols. That is:

$$v_n = \text{SM}[d_n], \quad (2.2)$$

where $\text{SM}[\bullet]$ reflects the symbol modulation scheme. There are many kinds of widely used modulation schemes, such as M -ary phase shift keying (M -PSK), M -ary frequency shift keying (M -FSK) and M -ary amplitude shift keying (M -ASK). Gray mapping is often used, which means that there is one bit difference between the information bits assigned to one symbol and its adjacent symbols.

2.3 Discrete-Time ISI Channel

We use $\underline{r} = [r_0 \ r_1 \ \cdots \ r_{N_r-1}]$ to denote the received samples after the ISI channel, where $N_r = N_v + L - 1$ is the number of received samples and L is the channel delay spread. The ISI channel is modeled by an equivalent discrete-time linear filter with a finite length impulse response (FIR), $\underline{h} = [h_0 \ h_1 \ \cdots \ h_{L-1}]$.

We assume that the channel impulse is perfectly known at the receiver¹, and without loss of generality, that the channel is normalized (i.e., $\sum_{l=0}^{L-1} |h_l|^2 = 1$). The received symbols are given by

$$r_n = \sum_{l=0}^{L-1} h_l v_{n-l} + w_n, \quad (2.3)$$

where $\underline{w} = [w_0 \ w_1 \ \cdots \ w_{N_r}]$ is the thermal noise. Each w_n , $n \in \{0, 1, \dots, N_r - 1\}$, is complex additive white Gaussian noise with the properties of

¹ When the channel is unknown, channel estimation techniques, like the least mean square (LMS) algorithm, could be used to estimate the channel. This requires a training sequence.

$$E[w_n] = 0, E[w_{n-m}^* w_n] = N_0 \delta_m, \quad (2.4)$$

where $\delta_m = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$ is the Kronecker delta function and N_0 is the single-sided noise

power spectral density.

We should be careful about the samples at the edges of the received block. Note that

$$\begin{aligned} r_0 &= h_0 v_0 + h_1 v_{-1} + \dots + h_{L-1} v_{-(L-1)} + w_0 \\ r_1 &= h_0 v_1 + h_1 v_0 + \dots + h_{L-1} v_{1-(L-1)} + w_1, \\ &\vdots \\ r_{L-2} &= h_0 v_{L-2} + h_1 v_{L-3} + \dots + h_{L-1} v_{-1} + w_{L-2} \end{aligned} \quad (2.5)$$

so these symbols depend on $\{v_{-(L-1)}, \dots, v_{-2}, v_{-1}\}$, and that

$$\begin{aligned} r_{N_v} &= h_0 v_{N_v} + h_1 v_{N_v-1} + \dots + h_{L-1} v_{(N_v-L-1)} + w_{N_v} \\ r_{N_v+1} &= h_0 v_{N_v+1} + h_1 v_{N_v} + \dots + h_{L-1} v_{N_v-(L-2)} + w_{N_v+1}, \\ &\vdots \\ r_{N_v+L-1} &= h_0 v_{N_v+L-1} + h_1 v_{N_v+L-2} + \dots + h_{L-1} v_{N_v} + w_{N_v+L-1} \end{aligned} \quad (2.6)$$

so these symbols depend on $\{v_{N_v}, v_{N_v+1}, \dots, v_{N_v+L-1}\}$. Since

$\{v_{-1}, v_{-2}, \dots, v_{-(L-1)}, v_{N_v}, v_{N_v+1}, \dots, v_{N_v+L-1}\}$ are not related to the transmitted symbols (they are not defined), we have several options to treat them.

Option 1. Use $v_n = 0 \ \forall n \notin \{0, 1, \dots, N_v - 1\}$. That means that the channel is quiet outside of the transmitted block.

Option 2. Use $v_n = SM[0] \ \forall n \notin \{0, 1, \dots, N_v - 1\}$. This will allow the trellis to start and end in the zero state.

Option 3. Assuming that a training sequence is transmitted before and after the data. This will allow the trellis to terminate at some known states.

In this thesis, option 1 is used, and the symbols outside the block are not used in the BER analysis.

2.4 Receiver Based on a Linear Equalizer

The one-pass linear equalizer (LE) based receiver is one of the simple receivers that treat the equalization and decoding independently. Figure 2.5 shows the structure of this system. The receiver is composed of a linear equalizer, a metric calculation unit (MCU), a de-interleaver and a MAP decoder.

Figure 2. 5 Linear equalizer based receiver.

2.4.1 Linear Equalizer

A linear equalizer is a sub-optimal equalizer that uses a transversal filter as shown in Figure 2.6. The equalizer output corresponding to the n^{th} transmitted symbol is

$$z_n = \sum_{k=-K_1}^{K_2} p_k r_{n-k} \quad n \in \{0, 1, \dots, N_z\}, \quad (2.7)$$

where $N_z = N_v$, $\{p_k\}$ are the $(K_1 + K_2 + 1)$ complex-valued tap coefficients of the filter, and $\{r_n\}$ are the received samples. The equalizer outputs, $\{z_n\}$, should be estimates of

the transmitted symbols, $\{v_n\}$. The minimum mean-square-error criterion is the most widely used method to choose the equalizer tap weight coefficients.

Figure 2. 6 Block diagram of a linear equalizer.

The difference between the achieved equalizer output and the desired output is

$$\varepsilon_n = z_n - v_n. \quad (2.8)$$

The MMSE criterion tries to optimize the coefficients to minimize the mean square value of ε_n

$$J_n = E[|\varepsilon_n|^2] = E[|z_n - v_n|^2]. \quad (2.9)$$

The coefficients $\{p_k\}$ that minimize J_n are the solution to the Wiener-Hopf equations:

$$\Gamma \underline{p} = \underline{\xi}, \quad (2.10)$$

where $\Gamma = E[\underline{r}_n \underline{r}_n^H]$ is a $(K_1 + K_2 + 1) \times (K_1 + K_2 + 1)$ Hermitian covariance matrix with

$\underline{r}_n = [r_{n+K_1} r_{n+K_1-1} \cdots r_{n-K_2}]^T$. The elements of Γ are

$$\Gamma_{l,k} = \sum_{a=0}^{L-1} h_a h_{a+k-l}^* + N_0 \delta_{l-k}, \quad \forall l, k \in \{-K_1, -K_1+1, \dots, K_2\}, \quad (2.11)$$

where the channel tap coefficients, h_i , are taken to be zero for $i < 0$ and $i \geq L$.

In Equation (2.11), $\underline{p} = [p_{-K_1} p_{-K_1+1} \dots p_{K_2}]^T$ are the filter coefficients and $\underline{\xi}$ is a $(K_1 + K_2 + 1)$ column vector with elements

$$\xi_l = \begin{cases} h_{-l}^* & \text{for } -(L-1) \leq l \leq 0 \\ 0 & \text{otherwise} \end{cases}. \quad (2.12)$$

The optimal equalizer tap weight coefficients are

$$\underline{p} = \Gamma^{-1} \underline{\xi}. \quad (2.13)$$

The optimal coefficients will result in the minimum mean square error as

$$J_{\min} = 1 - \underline{\xi}^H \Gamma^{-1} \underline{\xi} = 1 - \sum_{k=-K_1}^{K_2} p_k h_{-k}^*. \quad (2.14)$$

2.4.2 Metric Calculation Unit

We use $\underline{\gamma}$ to denote the metric. Assuming after the equalizer output, z_n , has a Gaussian distribution.

$$\gamma_n(d) = f(z_n | d_n = d) = f(z_n | v_n = SM[d]). \quad (2.15)$$

That is

$$\gamma_n(d) = \exp \left\{ -\frac{1}{\sigma_z^2} |z_n - E[z_n | v_n = SM[d]]|^2 \right\}, \quad (2.16)$$

for $n \in \{0, 1, \dots, N_z - 1\}$, $d \in \{0, 1, \dots, M - 1\}$, where $E[z_n | v_n = v] = v \sum_{k=-K_1}^{K_2} p_k h_{-k}$ and

$$\sigma_z^2 = E[|z_n - E[z_n | v_n = v]|^2 | v_n = v] = N_0 \sum_{k=-K_1}^{K_2} |p_k|^2.$$

2.4.3 De-interleaver

The de-interleaver process is the reverse process of the interleaver. It gives back the original code sequence. Paired with the transmitter side, there are two kinds of de-interleavers: a symbol de-interleaver and a bit de-interleaver.

2.4.3.1 Symbol De-interleaver

Use $\underline{\mu}$ to denote the de-interleaved metrics. For $n \in \{0, 1, \dots, N_v - 1\}$,

$d \in \{0, 1, \dots, M - 1\}$,

$$\mu_{IL[n]}(d) = \gamma_n(d), \quad (2.17)$$

where $IL[\bullet]$ is the interleaver mapping scheme, as described in section 2.2.2.

2.4.3.2 Bit De-interleaver

This is the twin process of the bit interleaver. It includes the symbol-to-bit metric converter, de-interleaver and bit-to-symbol converter. Figure 2.7 shows the structure.

Figure 2. 7 Structure of a bit de-interleaver.

The symbol-to-bit metric converter breaks the metric from symbol-based to bit-based. Assume bits are independent each other. Then the bit metric can be expressed as

$$\begin{aligned}
\gamma_n^{(i)}(k) &= f(z_n | d_n^{(i)} = k) \\
&= \frac{f(z_n, d_n^{(i)} = k)}{\Pr\{d_n^{(i)} = k\}} \\
&= \sum_d \frac{f(z_n, d_n = d, d_n^{(i)} = k)}{\Pr\{d_n^{(i)} = k\}} \\
&= \sum_d f(z_n | d_n = d, d_n^{(i)} = k) \Pr\{d_n = d | d_n^{(i)} = k\} \\
&= \sum_d f(z_n | d_n = d) \Pr\{d_n = d | d_n^{(i)} = k\} \\
&= \sum_d f(z_n | d_n = d) \prod_{j=0}^{n_d-1} \Pr\{d_n^{(j)} = d^{(j)} | d_n^{(i)} = k\} \\
&= \sum_{d | d_n^{(i)} = k} \gamma_n(d) \prod_{\substack{j=0 \\ j \neq i}}^{n_d-1} \Pr\{d_n^{(j)} = d^{(j)}\}.
\end{aligned} \tag{2.18}$$

Take the 4-QAM ($n_d = 2$) as example,

$$\begin{aligned}
\gamma_n^{(0)}(0) &= \gamma_n(00) \Pr\{d_n^{(1)} = 0\} + \gamma_n(01) \Pr\{d_n^{(1)} = 1\} \\
\gamma_n^{(0)}(1) &= \gamma_n(10) \Pr\{d_n^{(1)} = 0\} + \gamma_n(11) \Pr\{d_n^{(1)} = 1\} \\
\gamma_n^{(1)}(0) &= \gamma_n(00) \Pr\{d_n^{(0)} = 0\} + \gamma_n(10) \Pr\{d_n^{(0)} = 1\} \\
\gamma_n^{(1)}(1) &= \gamma_n(10) \Pr\{d_n^{(0)} = 0\} + \gamma_n(11) \Pr\{d_n^{(0)} = 1\}
\end{aligned} \tag{2.19}$$

The converter output is given by $\gamma'_{nm_d+i}(k) = \gamma_n^{(i)}(k)$.

The de-interleaver reorders the bit metrics into the original sequence of the transmitted code bits, using

$$\mu'_{IL[m]}(k) = \gamma'_m(k), \tag{2.20}$$

for $m \in \{0, 1, \dots, n_d N_d - 1\}$, $IL[m] \in \{0, 1, \dots, n_c N_c - 1\}$, $k \in \{0, 1\}$, where γ'_m is the input and μ'_m is the output.

The de-interleaver is followed by a bit-to-symbol metric converter, which converts the metric from bit-based to symbol-based. Let

$$\mu_n^{(i)}(k) = \mu_{nn_c+i}^{(i)}(k) = f(z_n | c_n^{(i)} = c^{(i)}), \quad (2.21)$$

then the symbol metric can be expressed as:

$$\mu_n(c) = \prod_{i=0}^{n_c-1} \mu_n^{(i)}(c^{(i)}) \quad n \in \{0, 1, \dots, N_c - 1\}, c \in \{0, 1, \dots, 2^{n_c-1}\} \quad (2.22)$$

2.4.4 MAP Decoder

The MAP decoding algorithm uses the symbol metrics, $\mu_n(c)$, to decode the convolutional code. It uses forward and backward recursions to calculate the *a posteriori* probabilities (APP's) of the message symbols. The APP's, which is fed into the decision device, can be expressed as $\Pr\{a_n = a | \underline{z}\}$. The decision device just picks the value of a that gives the largest APP as the decoding result.

2.5 Receiver Based on a Hard-Decision IC

As mentioned in the introduction of this thesis, linear equalizers, although simple, may suffer from excessive noise enhancement. Decision feedback equalizers, which use hard decisions based on previous equalizer outputs to suppress the ISI, have less noise enhancement. An even better approach is to use a hard-decision interference canceller, which is based on the assumption that estimates of future as well as previous transmitted symbols are available.

Figure 2. 8 Receiver based on a hard-decision IC.

2.5.1 Hard-decision Interference Canceller

From Equation (2.3) the n^{th} received symbol is expressed as

$$r_n = \sum_{l=0}^{L-1} h_l v_{n-l} + w_n. \quad (2.23)$$

Observe that information regarding the n^{th} transmitted symbol, v_n , occurs in the samples $\{r_n, r_{n+1}, \dots, r_{n+L-1}\}$. If all these samples are weighted by the conjugate of the channel coefficients and summed up, the combined sample is

$$x_n = \sum_{k=0}^{L-1} h_k^* r_{n+k} = v_n \sum_{k=0}^{L-1} |h_k|^2 + \sum_{\substack{b=-(L-1) \\ b \neq 0}}^{L-1} q_b v_{n-b} + w'_n, \quad (2.24)$$

where $q_b = \sum_{k=-(L-1)}^{L-1} h_k^* h_{b+k}$ and $w'_n = \sum_{k=0}^{L-1} h_k^* w_{n+k}$. The first term represents the desired symbol, v_n , the second term represents the ISI from other symbols, and the last term represents the noise.

If the interfering symbols are somehow known at the receiver, their effect on x_n could be removed, leading to perfect interference suppression. Furthermore, under the assumption that the channel is normalized (i.e., $\sum_{l=0}^{L-1} |h_l|^2 = 1$), we have

$$z_n = x_n - \sum_{\substack{b=-(L-1) \\ b \neq 0}}^{L-1} q_b v_{n-b} = v_n + w_n, \quad (2.25)$$

where w_n has a complex Gaussian distribution, with zero mean and a variance of N_0 . By using a hard IC, an ISI channel is therefore equivalent to transmitting symbols over an AWGN channel.

2.5.2 Turbo Equalization with Hard-decision IC

The above discussion assumes perfect knowledge of the other transmitted symbols, which, of course, is entirely unreasonable. However, by exploiting the “turbo” concept, it is possible to generate reasonably reliable estimates of these symbols.

Consider the receiver structure shown in Figure 2.8. The received samples are passed through a linear equalizer and processed as described in section 2.4. Once a block of hard decisions have been made by the MAP decoder, these hard decisions are re-encoded, and mapped to channel symbols using the same operation as used in the transmitter (as described in section 2.2). The resulting estimates, $\hat{\underline{v}} = [\hat{v}_0, \hat{v}_1, \dots, \hat{v}_{N_v-1}]$, can be used by a hard IC to better suppress the ISI found in the received samples. The IC output is

$$z_n = \sum_{k=-(L-1)}^0 h_{-k}^* r_{n-k} - \sum_{\substack{b=-(L-1) \\ b \neq 0}}^{(L-1)} q_b \hat{v}_{n-b}. \quad (2.26)$$

The IC output is then deinterleaved and passed through the MAP decoder again. Since the IC output is a more accurate estimate of v_n than the LE output, a lower probability of error should result. Furthermore, by re-encoding the new MAP decoder output, the accuracy of $\hat{v}$ should also improve, so the IC could be reapplied using the new values of $\hat{v}$. In fact, this process can be repeated several times, having performance improving with each iteration. Note that when a new block of samples is received, a linear equalizer is used on the first iteration and the IC is used on the subsequent iterations.

2.6 Receiver Based on the Classic IC

The receiver described in the previous section is simple and surprisingly effective, but suffers from the drawback that when hard decisions are made, valuable information is needlessly discarded. Only the hard decisions, $\hat{v}$, are available to the IC, with no measure of how confident the receiver is in these decisions. The receiver based on the classic IC [9], as shown in Figure 2.9, overcomes this deficiency.

Figure 2. 9 Receiver based on the classic IC.

In this system, the MAP decoder provides the soft information

$\psi_n(c) = \Pr\{c_n = c \mid \underline{z}\}$ that will be used as *a priori* information in the next iteration. The feedback interleaver and soft symbol generator are described in sections 2.6.1 and 2.6.2, and the classic IC is described in section 2.6.3.

2.6.1 Feedback Interleaver

The feedback interleaver reorders the soft feedback information with the exact same algorithm that is used in the transmitter, except it works on probabilities instead of bits. Either symbol or bit interleaving is used, corresponding to what was used in the transmitter.

2.6.1.1 Symbol Feedback Interleaver

The symbol interleaver is expressed as

$$\phi_n(c) = \psi_{IL[n]}(c), \quad (2.27)$$

for $n \in \{0, 1, \dots, N_d - 1\}$, $IL[n] \in \{0, 1, \dots, N_c - 1\}$, and $c \in \{0, 1, \dots, M - 1\}$,

where $\phi_n(\bullet)$ is the output and $\psi_n(\bullet)$ is the input.

2.6.1.2 Bit Feedback Interleaver

The bit feedback interleaver includes a symbol-to-bit metric converter, an interleaver and a bit-to-symbol metric converter. Figure 2.10 shows the structure.

Figure 2. 10 Structure of a feedback bit interleaver.

The symbol-to-bit converter converts the soft feedback information from symbol-based to bit-based. From $\psi_n(c) = \Pr\{c_n = c \mid \underline{z}\}$ we calculate

$$\Pr\{c_n^{(i)} = k \mid \underline{z}\} = \sum_{c|c^{(i)}=k} \psi_n(c). \quad (2.28)$$

Let $\psi'_{nn_c+i}(k) = \Pr\{c_n^{(i)} = k \mid \underline{z}\}$, which is the converter output.

The interleaver reorders the soft feedback information with the exact same mapping that is used in the transmitter. That is

$$\phi'_n(k) = \psi'_{IL[n]}(k), \quad (2.29)$$

for $n \in \{0, 1, \dots, n_c N_c - 1\}$, $IL[n] \in \{0, 1, \dots, n_d N_d - 1\}$, and $k \in \{0, 1\}$.

The bit-to-symbol converter converts the interleaved soft feedback information from bit-based back to symbol-based, which is required by the soft symbol generator. If we define $\phi_n^{(i)}(k) = \phi'_{nn_c+i}(k)$, then the feedback symbol information is

$$\phi_n(d) = \prod_{i=0}^{n_d-1} \phi_n^{(i)}(d^{(i)}) \quad n \in \{0, 1, \dots, N_d - 1\}. \quad (2.30)$$

2.6.2 Soft-symbol Generator

The soft-symbol generator finalizes the processing of the feedback information. It generates the mean value of the transmitted symbol using

$$\bar{v}_n = \sum_{d=0}^{M-1} \text{SM}[d] \Pr\{d_n = d\} = \sum_{m=0}^{M-1} \text{SM}[d] \phi_n(d). \quad (2.31)$$

Note that if the receiver is confident about the feedback information (i.e., $\phi_n(d)$ is close to 1 for one value of d and close to zero for the others), then $\bar{v}_n$ will be close to one of the points in the signal constellation. If the receiver does not have much confidence, then $\bar{v}_n$ will be close to zero.

2.6.3 Classic Interference Canceller

The structure of the classic IC is shown in Figure 2.11. It is composed of two filters: p and q . The input to the p filter is the channel output r_n and the input to the q

filter is the soft decision feedback of previous iteration, $\bar{v}_n$. For the first iteration there is no feedback so the IC cannot be used and an LE is used instead. The output of the IC is

$$z_n = \sum_{k=-(L-1)}^0 p_k r_{n-k} - \sum_{\substack{b=-(L-1) \\ b \neq 0}}^{L-1} q_b \bar{v}_{n-b}, \quad (2.32)$$

Using the MMSE criterion we can get the tap weight coefficients of the p and q filters.

Under the assumption that $\bar{v}_n = v_n$, which means that perfectly correct decision feedback is available, the coefficients of the filters are [9]

$$p_k = \frac{\sigma_v^2}{\sigma_v^2 + N_0} h_{-k}^* \quad k \in \{-(L-1), \dots, -1, 0\}, \quad (2.33)$$

and

$$q_b = \frac{\sigma_v^2}{\sigma_v^2 + N_0} \sum_{k=-(L-1)}^{L-1} h_{-k}^* h_{b-k} \quad b \in \{-(L-1), \dots, -1, 1, \dots, (L-1)\}, \quad (2.34)$$

where $\sigma_v^2 = E[|v_n|^2]$ is the average transmitted energy per symbol.

Figure 2. 11 Classic interference canceller.

The p filter is a matched filter, which improves the signal to noise ratio but increases the ISI spread. The q filter is to estimate the ISI in output of the p filter using the feedback $\bar{v}_n$. After the IC, we assume that the equalizer output z_n is just corrupted by

AWGN. That is $z_n \sim N\left(\frac{\sigma_v^2}{\sigma_v^2 + N_0} v_n, \left(\frac{\sigma_v^2}{\sigma_v^2 + N_0}\right)^2 N_0\right)$.

2.7 Receiver Based on MMSE Equalization Using *a priori* Information

By eliminating the use of hard decisions, the classic IC is able to deliver better performance than the hard IC, while effectively exploiting the soft feedback information. The equalizer proposed by Tuchler et al. provides a different way for using the soft feedback information. With Tuchler's scheme, a linear equalizer is used to combat the ISI in an iterative receiver structure as shown in Figure 2.12. What distinguishes this approach from the previous ones is the linear equalizer, called the MMSE equalizer with *a priori* information. Unlike the linear equalizer described in section 2.4.1, where the filter coefficients are derived under the assumption that the transmitted symbols have an equal *a priori* probabilities of

$$\Pr\{v_n = v\} = \frac{1}{M}, \quad (2.35)$$

with this equalizer, the transmitted symbols can have an arbitrary *a priori* probability distribution.

The soft feedback information from the decoder is used as the *a priori* probabilities. For the first iteration, before the decoder has had a chance to decode the code, equal *a priori* probabilities are used. Since the filter coefficients depend on the *a*

priori probabilities, and the *a priori* probabilities change with each symbol in each iteration, this decoder is prohibitively complex to implement. A low complexity (LC) solution can be obtained if the time average of the soft feedback information for each iteration is used. The equalizer output changes with each iteration. Therefore, iterative decoding can be used to improve the performance.

Figure 2. 12 Receiver based on MMSE equalization using *a priori* information.

2.8 Receiver Based on the Improved IC

One shortcoming of the classic IC is that, although soft decision feedback information is used to drive the q filter, the actual filter coefficients are derived under the assumption that the feedback information is a perfectly estimate of the transmitted symbols. To overcome the deficiency, we propose to determine the p and q filter coefficients of the classic IC under the assumption that the *a priori* probability of the transmitted symbols, $\{v_n\}$, are given by the feedback information. This is very similar to

the method proposed by Tuchler et al., except that while Tuchler et al. optimized the coefficients of a linear equalizer under this assumption, we optimize the coefficients of an IC. The improved IC will be introduced briefly below, and a detail derivation can be found in the appendix. The whole receiver is shown in Figure 2.13.

Figure 2. 13 Receiver based on the improved IC.

2.8.1 Finding the p, q Filter Coefficients

We generalize the IC somewhat by allowing K_1 non-causal coefficients and $K_2 + 1$ causal coefficients in the feedforward filter and place no restriction on the number of taps in the feedback filter. The IC output is then

$$z_n = \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \quad \forall n \in \{0, 1, \dots, N_v - 1\}. \quad (2.36)$$

It is shown in the appendix A.2 that the index of the second summation ranges over $\{-K_1, \dots, -1, 1, \dots, K_2 + L - 1\}$. To minimize the mean square error,

$J_n = E [|z_n - v_n|^2]$, we need to find $\{p_l\}$ and $\{q_l\}$ so that $\frac{\partial J_n}{\partial p_l} = 0$, $\frac{\partial J_n}{\partial q_l} = 0$. This

implies that we need

$$E[(z_n - v_n)r_{n-l}^*] = 0 \quad \forall l \in \{-K_1, \dots, K_2\}, \quad (2.37)$$

and

$$E[(z_n - v_n)\bar{v}_{n-l}^*] = 0 \quad \forall l \in \{-\infty, -1, 1, \dots, \infty\}. \quad (2.38)$$

When calculating the expected values in Equations (2.37) and (2.38), we assume that the *a priori* probabilities of the transmitted symbols are specified by the feedback information. That is

$$\Pr\{d_m = d\} = \phi_m(d), \quad (2.39)$$

where $\phi_m(d)$ is given by Equation (2.27) if symbol interleaving is used, and Equation (2.30) if bit interleaving is used. However, when calculating the coefficients for z_n (at time n), it is important that the feedback information at time n not be used to prevent excessive error propagation. Therefore we define

$$\bar{v}_m = E[v_m] = \begin{cases} \sum_{d=0}^{M-1} SM[d] \Pr\{d_n = d\} & m \neq n \\ 0 & m = n \end{cases}. \quad (2.40)$$

We also define

$$\sigma_{v,m}^2 = E[|v_m|^2] - |\bar{v}_m|^2 = \begin{cases} \sum_{d=0}^{M-1} |SM[d]|^2 \Pr\{d_n = d\} - |\bar{v}_m|^2 & m \neq n \\ \sigma_v^2 & m = n \end{cases}, \quad (2.41)$$

where $\sigma_v^2 = \frac{1}{M} \sum_{d=0}^{M-1} |SM[d]|^2$ is the average transmitted signal energy under the assumption that all signals are equally likely to be transmitted. Solving Equation (2.38) we can get the solution for q filter coefficients from appendix A.2,

$$q_b = \sum_{k=-K_1}^{K_2} p_k h_{b-k} \quad \forall b \neq 0. \quad (2.42)$$

Therefore z_n can be expressed as

$$z_n = \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} q_b \bar{v}_{n-b}. \quad (2.43)$$

From appendix A.3, Equation (2.37) can be expressed as

$$\sum_{k=-K_1}^{K_2} p_k [\sigma_v^2 h_{-k} h_{-l}^* + \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* \sigma_{v,n-b}^2 + N_0 \delta_{l-k}] = \sigma_v^2 h_{-l}^*. \quad (2.44)$$

The optimal filter coefficients, $\underline{p}$, can be found by solving this set of equations. Unfortunately, the solution depends on n , which means the filter coefficients vary from symbol to symbol. Since solutions for the coefficients requires a matrix inversion, using the optimal coefficients is computationally prohibitive. To simplify the problem we use the time average of the signal variance. Defining

$$\bar{\sigma}_v^2 = \frac{1}{N_v} \sum_{m=0}^{N_v-1} \sigma_{v,m}^2, \quad (2.45)$$

and using it in place of $\sigma_{v,m}^2$ in Equation (2.44) gives

$$\sum_{k=-K_1}^{K_2} p_k [\sigma_v^2 h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k}] = \sigma_v^2 h_{-l}^*. \quad (2.46)$$

It is convenient to rewrite Equation (2.46) using matrix form as

$$\Psi \underline{p} = \underline{\xi}, \quad (2.47)$$

where $\underline{\xi} = \sigma_v^2 [h_{-K_2} \ h_{-K_2+1} \ \cdots \ h_{K_1}]^H$, $\underline{p} = [p_{-K_1} \ p_{-K_1+1} \ \cdots \ p_{K_2}]^T$, and Ψ is a $(K_1 + K_2 + 1) \times (K_1 + K_2 + 1)$ matrix with elements (for $l, k \in \{-K_1, -K_1 + 1, \dots, K_2\}$)

$$\begin{aligned} \Psi_{l,k} &= \sigma_v^2 h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \\ &= (\sigma_v^2 - \bar{\sigma}_v^2) h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{b=-\infty}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \\ &= (\sigma_v^2 - \bar{\sigma}_v^2) h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{a=0}^{L-1} h_a h_{a+k-l}^* + N_0 \delta_{l-k}. \end{aligned} \quad (2.48)$$

Then p filter coefficients can be found as

$$\underline{p} = \Psi^{-1} \underline{\xi}. \quad (2.49)$$

Although these do not need to be calculated on a symbol-by-symbol basis, they do need to be calculated for each iteration.

2.8.2 The Mean and Variance of z_n

It is well shown that the distribution of the equalizer output, z_n , is well approximated by a Gaussian distribution [10][19]. From appendix A.5, the mean of z_n under the hypothesis that $v_n = v$ can be expressed as

$$E[z_n | v_n = v] = v q_0, \quad (2.50)$$

and the variance of z_n can be expressed as

$$\sigma_{z,n}^2 = \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} \sigma_{v,n-b}^2 |q_b|^2 + N_0 \sum_{k=-K_1}^{K_2} |p_k|^2. \quad (2.51)$$

From Equation (2.51), it is clear that $\sigma_{z,n}^2$ varies from symbol to symbol, and we call this the dynamic variance. A simpler alternative for the dynamic variance is its time average. We call this the static variance, which is expressed as

$$\bar{\sigma}_z^2 = \frac{1}{N_v} \sum_{n=0}^{N_v-1} \sigma_{z,n}^2 = \bar{\sigma}_v^2 \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} |q_b|^2 + N_0 \sum_{k=-K_1}^{K_2} |p_k|^2. \quad (2.52)$$

2.8.3 The Confidence Index

The quantity $\bar{\sigma}_v^2$ as given by Equation (2.45) represents how confident the MAP decoder is in its decisions. At one extreme, $\bar{\sigma}_v^2 = 0$ corresponds to the case that the decoder has complete confidence in the feedback (i.e., perfect, error-free feedback). At the other extreme, $\bar{\sigma}_v^2 = \sigma_v^2$ corresponds to the case that the decoder has absolutely no confidence in the feedback (i.e., no feedback). We examine these two cases separately.

2.8.3.1 Perfect Feedback

When $\bar{\sigma}_v^2 = 0$, Equation (2.46) reduces to

$$\sum_{k=-K_1}^{K_2} p_k [\sigma_v^2 h_{-k} h_{-l}^* + N_0 \delta_{l-k}] = \sigma_v^2 h_{-l}^*, \quad (2.53)$$

or

$$\sigma_v^2 h_{-l}^* q_0 + N_0 p_l = \sigma_v^2 h_{-l}^*, \quad (2.54)$$

where

$$q_0 = \sum_{k=-K_1}^{K_2} p_k h_{-k}. \quad (2.55)$$

Therefore

$$p_l = \frac{\sigma_v^2}{N_0} h_{-l}^* (1 - q_0). \quad (2.56)$$

Also

$$q_0 = \sum_{k=-K_1}^{K_2} p_k h_{-k} = \sum_{k=-K_1}^{K_2} \frac{\sigma_v^2}{N_0} h_{-k}^* (1 - q_0) h_{-k} = \frac{\sigma_v^2}{N_0} (1 - q_0) \sum_{k=-K_1}^{K_2} |h_k|^2, \quad (2.57)$$

therefore

$$q_0 = \frac{\sigma_v^2}{\sigma_v^2 + N_0}, \quad (2.58)$$

and

$$p_l = \frac{\sigma_v^2}{\sigma_v^2 + N_0} h_{-l}^*. \quad (2.59)$$

These are identical to the coefficients of the classic IC, as given by Equation (2.33).

When perfect feedback is available the improved IC becomes identical to the classic IC.

2.8.3.2 No Feedback

In the absence of feedback, $\bar{\sigma}_v^2 = \sigma_v^2$, and Equation (2.46) becomes

$$\Psi_{l,k} = \sigma_v^2 \sum_{a=0}^{L-1} h_a h_{a+k-l}^* + N_0 \delta_{l-k}. \quad (2.60)$$

Using these values to solve Equation (2.47) gives solutions identical to the filter coefficients for the linear equalizer, as described in Section (2.4.1). When no feedback is available the improved IC becomes a linear equalizer.

2.8.4 A Comparison with Tuchler's Equalizer

The filter coefficients given by Equations (2.42) and (2.49) have been derived for an interference canceller under the assumption that the *a priori* probabilities are not

all equal, similar to the approach taken by Tuchler et al. in finding the coefficients of their linear equalizer (low complexity solution). However, a closer comparison between the two systems reveals that these two equalization schemes are in fact identical, although the presentations are very different.

In Tuchler's LC algorithm [10], they use K received symbols, $\underline{r}_n = [r_{n-K_2} r_{n-K_2+1} \dots r_{n+K_1}]^T$, to estimate the transmitted symbol v_n , where $K = K_1 + K_2 + 1$, K_1 is the length of non-causal part of estimator filter and K_2 is the length of causal part. The estimate is denoted by z_n ,

$$z_n = \underline{a}_n^H r_n + b_n, \quad (2.61)$$

where $\underline{a}_n = [a_{n,K_2}^* a_{n,K_2-1}^* \dots a_{n,-K_1}^*]^T$ and b_n are the coefficients of the estimator. Using the MMSE criterion to find the optimized coefficients gives

$$\underline{a}_n = \text{Cov}(\underline{r}_n, \underline{r}_n)^{-1} \text{Cov}(\underline{r}_n, v_n), \quad (2.62)$$

and

$$b_n = E[v_n] - \underline{a}_n^H E[r_n]. \quad (2.63)$$

Then z_n can be expressed as

$$z_n = E[v_n] + \text{Cov}(v_n, \underline{r}_n) \text{Cov}(\underline{r}_n, \underline{r}_n)^{-1} (\underline{r}_n - E[\underline{r}_n]). \quad (2.64)$$

Similar to Equations (2.40) and (2.41), define

$$E[v_m] = \bar{v}_m = \begin{cases} \sum_{SM[i] \in S} SM[i] \Pr\{v_n = SM[i]\} & m \neq n \\ 0 & m = n \end{cases}, \quad (2.65)$$

and

$$\sigma_{v,n}^2 = \text{Cov}(v_n, v_n) = \begin{cases} \sum_{SM[i] \in S} |SM[i]|^2 \Pr\{v_n = SM[i]\} - |\bar{v}_n|^2 & m \neq n \\ \sigma_v^2 & m = n \end{cases}. \quad (2.66)$$

For the very first loop (no feedback), $E[v_n] = \bar{v}_n = 0$, $\sigma_{v,n}^2 = \sigma_v^2$. After that, the feedback *a priori* information is used to get the mean and covariance. Use these to substitute where the $\sigma_{v,n}^2$ applies, then the equalized symbol, z_n , can be expressed as

$$\begin{aligned} z_n &= \underline{p}^T (\underline{r}_n - \bar{\underline{r}}_n + \bar{v}_n \underline{s}) \\ &= \underline{p}^T \underline{r}_n - \underline{p}^T (\bar{\underline{r}}_n - \bar{v}_n \underline{s}), \end{aligned} \quad (2.67)$$

$$\text{where } \underline{p} = (N_0 \mathbf{I}_K + \sigma_{n,v}^2 \mathbf{H}^* \mathbf{H}^T)^{-1} \underline{s}^*, \quad (2.68)$$

$$\underline{s} = H [\underline{0}_{1 \times (K_2+L-1)} \quad 1 \quad \underline{0}_{1 \times K_1}]^T, \quad (2.69)$$

$$\bar{r}_n = E[r_n] = \sum_{k=0}^{L-1} h_k \bar{v}_{n-k}, \quad (2.70)$$

$$\bar{\underline{r}}_n = E[\underline{r}_n] = [\bar{r}_{n-K_2} \bar{r}_{n-K_2+1} \cdots \bar{r}_{n+K_1}]^T, \quad (2.71)$$

and

$$\mathbf{H} = \begin{bmatrix} h_{L-1} & h_{L-2} & \cdots & h_0 & h_{-1} & \cdots & h_{-K} \\ h_L & h_{L-1} & h_{L-2} & \cdots & h_0 & h_{-1} & \cdots & h_{-(K-1)} \\ & & & \ddots & & & & \\ h_{K+L-1} & \cdots & h_L & h_{L-1} & h_{L-2} & \cdots & h_0 \end{bmatrix}. \quad (2.72)$$

$\mathbf{H}$ is a $K \times (K+L-1)$ channel convolution matrix and $\mathbf{I}_K$ is a $K \times K$ identity matrix.

Below we will show that Tuchler's LC algorithm is identical to the improved IC.

From Equation (2.69), $\underline{s}$ can be expressed as

$$\underline{s}^* = H^* [\underline{0}_{1 \times (K_2+L-1)} \quad 1 \quad \underline{0}_{1 \times K_1}]^T = [h_{-K_2}^* \quad h_{-K_2+1}^* \quad \cdots \quad h_{K_1}^*]^T. \quad (2.73)$$

This is the $\underline{\xi}$ in Equation (2.47).

Also in Equation (2.68), let $\Psi = N_0 I_K + \sigma_{n,v}^2 H^* H^T$, then

$$\Psi_{l,k} = \sigma_v^2 h_{-k} h_{-l}^* + \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* \sigma_{v,n-b}^2 + N_0 \delta_{l-k}. \quad (2.74)$$

With the LC solution, a time average value of the covariance is used, that is

$$\bar{\sigma}_v^2 = \frac{1}{N} \sum_{k=0}^{N_v-1} \sigma_{k,v}^2, \quad (2.75)$$

then Equation (2.74) can be expressed as

$$\begin{aligned} \Psi_{l,k} &= \sigma_v^2 h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \\ &= (\sigma_v^2 - \bar{\sigma}_v^2) h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{b=-\infty}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \\ &= (\sigma_v^2 - \bar{\sigma}_v^2) h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{a=0}^{L-1} h_a h_{a+k-l}^* + N_0 \delta_{l-k}. \end{aligned} \quad (2.76)$$

Equation (2.76) is the same as Equation (2.48). Then the expression of p filter is the same as the p filter in the improved IC algorithm. Equation (2.67) can be expressed as

$$\begin{aligned} z_n &= \underline{p}^T \underline{r}_n - \underline{p}^T (\bar{\underline{r}}_n - \bar{\underline{v}}_n \underline{s}) \\ &= \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{b=-K_1}^{K_2} p_b \left(\sum_{a=0}^{L-1} h_a \bar{v}_{n-b-a} - \bar{v}_n h_{-b} \right) \quad \forall n \in \{ 0, 1, \dots, N_v-1 \}. \end{aligned} \quad (2.77)$$

Define $c = a+b$, then z_n can be expressed as

$$\begin{aligned} z_n &= \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{b=-K_1}^{K_2} p_b \left(\sum_{c=-K_1}^{K_2+L-1} h_{c-b} \bar{v}_{n-c} - \bar{v}_n h_{-b} \right) \\ &= \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{b=-K_1}^{K_2} p_b \sum_{\substack{k=-K_1 \\ k \neq 0}}^{K_2+L-1} h_{k-b} \bar{v}_{n-k} \end{aligned}$$

$$\begin{aligned}
&= \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{\substack{k=-K_1 \\ k \neq 0}}^{K_2+L-1} \sum_{b=-K_1}^{K_2} p_b h_{k-b} \bar{v}_{n-k} \\
&= \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} \sum_{k=-K_1}^{K_2} p_k h_{b-k} \bar{v}_{n-b} \\
&= \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} q_b \bar{v}_{n-b} \quad \forall n \in \{ 0, 1, \dots, N_v - 1 \}, \tag{2.78}
\end{aligned}$$

$$\text{where, } q_b = \sum_{k=-K_1}^{K_2} p_k h_{b-k} \quad \forall b \neq 0. \tag{2.79}$$

It is clear that Equations (2.79) and (2.78) are equal to Equations (2.42) and (2.43) respect. Then we can conclude that the improved IC algorithm is identical to Tuchler's LC algorithm.

Chapter 3 Simulation Results

In this chapter, we will present some simulation results. Two IC-based systems will be considered. The first system, called the classic IC system, uses the classic IC with symbol interleaving. The second system, called the improved IC system, uses the improved IC with bit interleaving. In general, we expect that the improved IC system should have better performance than the classic IC system at the price of higher computational complexity. The goal of this research is to quantify the performance gains of the improved IC system over the classic IC system.

Section 3.1 investigates the impact of different channels. Then Section 3.2 discusses different block sizes and convolutional coding schemes. Section 3.3 discusses different mapping schemes. Section 3.4 provides a more detailed comparison of the systems over the Proakis C channel.

3.1 System Comparison with Different Channel Models

The channel is one of the most important factors for comparing these two solutions, since it is the cause of the ISI. In the following, five different discrete time ISI channel models are considered.

1. The Proakis C channel [3], with channel taps of $(\frac{1}{\sqrt{19}}, \frac{2}{\sqrt{19}}, \frac{3}{\sqrt{19}}, \frac{2}{\sqrt{19}}, \frac{1}{\sqrt{19}})$,
2. The Proakis B channel [3], with channel taps of $(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$,
3. The Porat and Friedlander channel [12], with channel taps of $(\frac{2-0.4j}{\sqrt{16.98}}, \frac{1.5+1.8j}{\sqrt{16.98}}, \frac{1}{\sqrt{16.98}}, \frac{1.2-1.3j}{\sqrt{16.98}}, \frac{0.8+1.6j}{\sqrt{16.98}})$,

4. The 2-tap channel [3], with channel taps of $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, and
5. The Proakis B fading channel. Rayleigh fading is considered here. The variances for the taps are $(\frac{1}{6}, \frac{4}{6}, \frac{1}{6})$. For each transmitted block the taps are randomly chosen according to a complex Gaussian distribution, then normalized so that $\sum_{l=0}^{L-1} |h_l|^2 = 1$.

Figures 3.1 to 3.4 show the impulse and frequency responses of the first four channel models, where T is the symbol interval. We can see that Channel 1, with a deep spectral null in the middle of the frequency band, causes severe ISI; channel 2 causes moderate ISI; channels 3 and 4 have mild ISI.

Figure 3. 1 Impulse response (a) and frequency response (b) of the Proakis C channel.

(a)

(b)

Figure 3. 2 Impulse response (a) and frequency response (b) of the Proakis B channel.

(a)

(b)

Figure 3. 3 Impulse response (a) and frequency response (b) of the Porat and Friedlander channel.

Figure 3. 4 Impulse response (a) and frequency response (b) of the 2-tap channel.

In this section, the performance of the two IC systems under these different channel models is discussed. For all simulations, the number of message symbols for each block is 4048 symbols ($N_a = 4048$) and each message symbol consists of one bit ($n_a = 1$). The number of simulated blocks is 2500. The rate $\frac{1}{2}$ convolutional encoder has generator $(5, 7)_8$ with a constraint length of three ($L_c = 3$). The modulation scheme is 4-QAM. If a symbol interleaver is used, the size of the interrelaver is $(N_a + L_c - 1)$. If a bit interleaver is used, the size is $2 \times (N_a + L_c - 1)$. All the interleavers are random interleavers. For the first iteration of turbo equalizer a linear equalizer with 15 taps ($K_1 = 5, K_2 = 9$) is used. For the classic IC, the p filter (feedforward filter) has L taps ($K_1 = L - 1, K_2 = 0$) and the q filter (feedback filter) has $(2 \times L - 1)$ taps, where L is the length of the channel impulse response. Also when calculating branch metrics, the equalizer output z_n after the first iteration is treated as a Gaussian random variable with a

mean of $\frac{\sigma_v^2}{\sigma_v^2 + N_0} v_n$ and variance of $\left(\frac{\sigma_v^2}{\sigma_v^2 + N_0} \right)^2 N_0$ (that is: $z_n \sim N\left(\frac{\sigma_v^2}{\sigma_v^2 + N_0} v_n, \left(\frac{\sigma_v^2}{\sigma_v^2 + N_0}\right)^2 N_0\right)$). For the improved IC,

the p filter has 15 taps ($K_1 = 5$, $K_2 = 9$) and q filter has $(15+L-1)$ taps. Also when calculating branch metrics, the equalizer output after the first iteration is $z_n \sim N(v_n q_0, \sigma_{z,n}^2)$, as given by Equations (2.50) and (2.51).

Figure 3.5 shows the performance of the classic IC and improved IC systems over the Proakis C channel after the first, second, third and sixth iterations. Note that the first iteration for the classic IC system is an LE with symbol interleaving and the first iteration for the improved IC system is an LE with bit interleaving.

Figure 3. 5 Comparison of classic IC system (solid line) and improved IC system (dashed line) over the Proakis C channel.

From this figure we can see that there is an obvious gain by using the improved IC system instead of the classic IC system when the channel has severe ISI. When using the improved IC system, the performance after two iterations is almost the same as the performance of the classic IC system with six iterations. Comparing the results after six iterations, we will find that there is around 7 dB gain at a BER of 10^{-4} by using the improved IC system. We will investigate the contributing factors to this difference in performance in Section 3.4. Another point is that by using the improved IC system, after six iterations, there is around 3 dB degradation at a BER of 10^{-4} comparing with the system over AWGN channel without ISI. This means that even the improved IC system does not approach optimal results when the ISI is severe.

Figure 3.6 provides a similar system performance comparison for the Proakis B channel, which yields moderate ISI. We see that the performance gain of using the improved IC system is not as significant as it is in the Proakis C channel. After six iterations, the performance of the improved IC system is around 0.5 dB better than the classic IC system at a BER of 10^{-4} . Also, the improved IC system with three iterations has almost the same performance of the classic system with six iterations. With the improved IC system, the performance after six iterations is almost the same as the system over the Gaussian channel without ISI.

Figure 3. 6 Comparison of classic IC system (solid line) and improved IC system (dashed line) over the Proakis B channel.

Figures 3.7 to Figure 3.9 provide a system performance comparison for the mild ISI channels. We see that for channels with mild ISI, the advantage for using the improved IC system is so little that it can be neglected. The reason might be that in mild ISI channels the LE can eliminate most of the ISI, so the MAP decoder can reliably decode the convolutional code. As a result, the feedback information is very reliable, satisfying the correct-feedback assumption of the classic IC. Also using the IC scheme, after six iterations, the performance is very similar to the Gaussian channel, with has no ISI.

Figure 3. 7 Comparison of classic IC system (solid line) and improved IC system (dashed line) over the Porat and Friedlander channel.

Figure 3. 8 Comparison of classic IC system (solid line) and improved IC system (dashed line) over the 2-tap channel.

Figure 3. 9 Comparison of classic IC system (solid line) and improved IC system (dashed line) over the Proakis B fading channel.

From the discussion in Chapter 2, after a sufficiently large number of iterations the filter coefficients of the improved IC system should approach those of the classic IC system. Because the q filter coefficients depend on the p filter, we only focus on the p filter. All the following filter coefficients are based on SNR = 10 dB. From Figure 3.10, we can see that for the severe ISI Proakis C channel, the filter coefficients of the classic and improved systems are quite different during the second iteration. During the third iteration there is still some difference, but during the sixth iteration they are essentially identical. As the feedback becomes more reliable the filter coefficients approach the coefficients of the classic IC system in a general manner.

Figure 3. 10 Proakis C channel: Filter coefficients updates.

From Figure 3.11, we see that for the moderate ISI channel, Proakis B channel, during the third iteration, the filter coefficients of the classic and improved systems are quite similar. For the improved IC system, the coefficients have no difference during the third iteration or sixth iteration.

Coefficients of optimized IC
during 2nd iteration

Coefficients of optimized IC
during 3rd iteration

Coefficients of optimized IC
during 6th iteration

Classic IC coefficients

Figure 3. 11 Proakis B channel: Filter coefficients updates.

From Figure 3.12 and Figure 3.13 we see that for mild ISI channels like the Porat and Friedlander channel and 2-tap channel, during the second iteration the p filter's coefficients of the improved IC system and classic IC system are the same.

Coefficients of optimized IC
during 2nd iteration

Coefficients of optimized IC
during 3th iteration

Coefficients of optimized IC
during 6th iteration

Classic IC coefficients

Figure 3. 12 Porat and Friedlander channel: Filter coefficients updates.

Coefficients of optimized IC
during 2nd iteration

Coefficients of optimized IC
during 3rd iteration

Coefficients of optimized IC
during 6th iteration

Classic IC coefficients

Figure 3. 13 2-tap channel: Filter coefficients updates.

From the figures above we can see that the coefficients of the improved IC system gradually approach the coefficients of the classic IC system. The improved IC treats the feedback information as an estimate of the transmitted symbol, which is different from the transmitted symbol itself. In the first several iterations, especially for the Proakis C channel, the feedback information is very unreliable, so the estimate is quite different from the actual value. Using the estimated value to get the coefficients of the IC is more effective against the feedback errors. As the number of iterations increases, the feedback

estimation is getting closer and closer to the real value so more and more errors are eliminated. Then the coefficients of the improved IC are closer and closer to the coefficients of the classic IC, which treats the estimation symbol and actual symbol as the same. For mild ISI channels, the LE and MAP decoder are so effective at eliminating errors that the improved IC system only needs a few iterations to get what we call “confident” results of transmitted symbols. This can explain why after six iterations there is almost no difference between the improved IC system and the classic IC system.

3.2 Effect of Block Size and Code Complexity

In general, longer block size brings better performance, but at the expense of longer decoding delay. In this section we will try a shorter block size to investigate the impact on the improved and classic IC systems. All the simulation conditions are the same as in Section 3.1 except the block size is 500 and the number of simulated blocks is 20000. Figure 3.14 shows the performance using this shorter block size over the five channels. All the results are based on 6 iterations.

Figure 3. 14 Performance of the classic IC system (solid line) and the improved IC system (dashed line) over different channels, with block size of 500 symbols.

As could be expected, decreasing the block size leads to a corresponding degradation in system performance. This degradation occurs for both systems and for all the five channel models. More interesting is how the relative performance of the two systems changes when the block size is decreased. Whereas for the Proakis C channel the classic IC system performs 7 dB worse than the improved IC system for the long block size at a BER of 10^{-4} , the difference is 8 dB when the shorter block size is used. Similarly for the Proakis B channel the difference between the two systems increases from 0.5 dB to 1 dB. For mild ISI channels, however, the difference remains negligible

(less than 0.1 dB). These results suggest that increasing the block size can help overcome the performance difference between the two systems, particularly when the ISI is severe.

The coding scheme is another important factor for the system performance. The $(5, 7)_8$ convolutional code used in Section 3.1 is a relatively weak code. In this section, we will try the $(23, 35)_8$ code. It gives better performance at the price of more computational complexity. Figure 3.15 shows the generator structure, which has a constraint length of 5 ($L_c = 5$). All the other simulation conditions are the same as in Section 3.1. Figure 3.16 compares the performance of both systems using $(23, 35)_8$ code over the five channels. All the results are based on 6 iterations.

Compared with the weaker code, there is some performance improvement when the stronger code is used for both IC systems over all the five channels. For the mild ISI and moderate ISI channels the performance difference between the improved IC system and the classic IC system remains less than 0.1 dB. The more powerful code helps the IC

Figure 3. 15 Encoder for $(23, 35)_8$ convolutional code.

to better eliminate the ISI so even for moderate ISI channels, like the Proakis B channel, during the sixth iteration, the difference between the improved IC system and the classic IC system is also less than 0.1 dB. Only for the Proakis C channel is there an obvious advantage to use the improved IC system. There is about 3.3 dB gain at BER of 10^{-4} by using the improved IC system instead of the classic IC system, compared with a 7 dB gain when the weaker code is used. As with increasing the block length, increasing the strength of the convolutional code helps overcome the deficiencies of the classic IC system

Figure 3. 16 Comparison of $(23,35)_8$ coding of the classic IC system (solid line) and the improved IC system (dashed line) over different channels.

3.3 Effect of the Modulation Scheme

The results in the previous section are all based on 4-QAM, which maps 2 bits into one symbol. When greater spectral efficiency is needed 16-QAM can be considered, which maps 4 bits into one symbol. The drawback is the probability of error will increase as a result. In this section we will investigate the system performance when 16-QAM is used. Figure 3.17 shows the constellation of 16-QAM, where $A = \frac{1}{\sqrt{10}}$ to ensure the average symbol energy is 1.

All the simulation conditions are the same as in Section 3.1 except the mapping is 16-QAM instead of 4-QAM. Figure 3.18 compares the performance using 16-QAM over the five channels. All the results are based on 6 iterations. General speaking, the performance of both IC systems is degraded when 16-QAM is used instead of 4-QAM.

Figure 3. 17 16-QAM Constellation.

Figure 3. 18 Comparison of the classic IC system (solid line) and the improved IC system (dashed line) over different channels when 16-QAM is used.

For the Proakis C channel, the classic IC system performs very poorly. Even when the SNR is 24 dB, the bit error rate is still around 0.5. Using the improved IC system, the bit error rate can drop below 10^{-5} after 23 dB. For the Proakis B channel, there is about 7 dB gain at a BER of 10^{-4} by using the improved IC system instead of the classic IC system. The difference is much larger than the 4-QAM system, which is about 0.5 dB. But for the other mild ISI channels, such as the Porat and Friedlander, and 2-tap channels,

the performance of the improved IC system is even worse than that of the classic IC system.

This result is quite surprising since one would expect that the superior design methodology (and higher complexity) of the improved system should lead to better results. Two obvious differences between the two IC systems are bit vs. symbol interleaving and improved vs. classic IC. Let's build two sub-optimal systems

1. Improved IC with symbol interleaving, and
2. Classic IC with bit interleaving.

Then we can isolate which of either the bit interleaving or the improved IC causes this degradation in performance. In this section we use one more channel, the 2-tap Rayleigh fading channel. The variances for the taps are $(\frac{1}{2}, \frac{1}{2})$, and the taps are normalized in all simulations. The 2-tap channel, 2-tap fading channel, Porat and Friedlander channel, Proakis B channel and Proakis B fading channel will be considered.

Figure 3.19 shows results for the 2-tap channel after six iterations. The system with the improved IC and symbol interleaving gives the best performance, followed by the classic IC with symbol interleaving system, the improved IC with bit interleaving system, and finally the classic IC with bit interleaving system. Clearly, it is the use of bit interleaving that is causing the performance degradation. This observation is consistent with the results for the 2-tap fading channel, as shown in Figure 3.20, and the Porat and Friedlander channel, as shown in Figure 3.21.

Figure 3. 19 Comparison of 2-tap channel, 6 iterations.

Figure 3. 20 Comparison of 2-tap fading channel, 6 iterations.

Figure 3. 21 Comparison of Porat and Friedlander channel, 6 iterations.

The algorithm used to implement the metric conversions for bit interleaving is based on the assumption that the bits in each equalizer output sample, z_n , are independent. This assumption holds if BPSK is used, and since 4-QAM is essentially just two BPSK signals transmitted in quadrature, this assumption is also reasonably held for 4-QAM. For 16-QAM, however, this independence assumption is clearly violated, leading to a reduction in performance when bit interleaving is used instead of symbol interleaving.

There is, however, another factor affecting system performance. If the residual ISI affecting one equalizer output sample is large, all the bits in that symbol may be unreliable. It is desirable to disperse these bits into different code symbols, by using bit interleaving, to make the decoder more effective. Therefore, if the residual ISI is large, it

is preferable to use bit interleaving instead of symbol interleaving. This is confirmed by the results in Figure 3.22 for the Proakis B channel, which causes moderate ISI. The advantage of using bit interleaving to disperse the residual ISI is sufficiently large to overcome the disadvantage due to violating the independence assumption.

This situation is also confirmed in the results for the Proakis B fading channel, shown in Figure 3.23. When the improved IC is used, the residual ISI is not a problem, so symbol interleaving is preferred, but when the classic IC is used, the residual ISI becomes more dominant, so bit interleaving is preferred.

Figure 3. 22 Comparison of Proakis B channel, 6 iterations.

Figure 3. 23 Comparison of Proakis B fading channel, 6 iterations.

3.4 Further Discussion on Applications Based on the Proakis C Channel

The results in Section 3.1 show that improved IC system performs about 7 dB better than the classic IC system for the Proakis C channel. To investigate the cause of this, we note the differences between the two systems:

1. The improved IC system uses bit interleaving and the classic IC system uses symbol interleaving.
2. The improved IC system uses improved filter coefficients, and dynamic symbol mean and variance.
3. The improved IC system uses a longer filter.

Simulations below will show how much these three factors affect the difference in performance. All the results in this section are based on 6 iterations using 4-QAM.

Figure 3.24 shows the impact of the bit vs. symbol interleaving. From this figure, we can see that for Proakis C channel, bit interleaving performs much better than symbol interleaving. The system with the improved IC and bit interleaving has a 2 dB gain over the system with the improved IC and symbol interleaving at a BER of 10^{-4} , and the system with the classic IC and bit interleaving has a 4 dB gain over the system with the classic IC and symbol interleaving. When the bit interleaving is used, in the receiving side, the de-interleaver and feedback interleaver need to break the symbol metric into bit metric as Equations (2.18) and (2.28), also they need to convert the bit metric to symbol metric as Equations (2.22) and (2.30). The symbol interleaving does not need these steps. Although bit vs. symbol interleaving accounts for the most of the difference between the classic and improved IC systems, there still remains a difference of about 3 dB.

Figure 3. 24 Comparison the performance: bit interleaving vs. symbol interleaving and improved IC vs. classic IC.

The second difference between the classic and improved IC systems involves the mean and variance of the equalizer output, as used when calculating the branch metrics in Equation (2.16). From Chapter 2 we know that for the improved IC system the equalizer output samples are assumed to be Gaussian random variables. That is $z_n \sim N(v_n q_0, \sigma_{n,z}^2)$, defined in Equations (2.50) and (2.51), whose variance varies from symbol to symbol. This is called improved IC with dynamic variance. Also we can take the time average of the variance, $\bar{\sigma}_z^2$, as defined by Equation (2.52), and this is called improved IC with static variance. These dynamic and static solutions can be applied to the classic IC, which is called classic IC with dynamic variance and static variance. The dynamic variance is updated every symbol, the static variance is updated every iteration and the fixed variance has no updating.

Figure 3.25 shows the different variances impact on system performance when bit interleaving is used. Using the dynamically calculated variance can bring about 0.5 dB gain at a BER of 10^{-4} comparing with using the static variance for the improved IC. For the classic IC, using the dynamic variance has around 0.8 dB gain over using a fixed variance at a BER of 10^{-4} . Figure 3.26 shows the different variances impact on system performance when symbol interleaving is used.

Figure 3. 25 Variance impact on the system performance with bit interleaving.

Figure 3. 26 Variance impact on the system performance with symbol interleaving.

From the figure above, we can see that using the dynamically calculated variance can bring about 0.5 dB gain at a BER of 10^{-4} comparing with using the static variance for the improved IC. For the classic IC, using the dynamic variance has around 3 dB gain over using a fixed variance at a BER of 10^{-4} . We can conclude that using the dynamic variance can help the classic IC to improve its performance.

Figures 3.27 and 3.28 show the effect of the filter length when bit and symbol interleaving are used. The improved IC's p filter length is $(K_1 + K_2 + 1)$ and q filter has

length $(K_1 + K_2 + L)$ where $K_1 = 9$, $K_2 = 5$, $L = 5$. So the p filter's length is 15 and q filter's length is 19. The classic IC's p filter has length $L = 5$ and its q filter has length $(2L - 1 = 9)$. In general, longer filter should give better performance. The improved IC's filter length can reduce to the length of the classic IC's, so its p filter has a length of 5 and q filter has a length of 9. We call this an improved IC with shorter filter.

Figure 3.27 shows the results based on the bit interleaving. The length of the filter has a great impact on the performance. There is about 2.2 dB performance degradation if the shorter filter is used instead of the normal length filter. The improved IC with shorter filter almost has the same performance with the classic IC with dynamic variance and bit interleaving.

Figure 3. 27 Different filter length on bit interleaving.

Figure 3.36 shows the filter length's compact when symbol interleaving is used. The improved IC with shorter filter and symbol interleaving is even worse than the classic IC with dynamic variance.

In summary, of the 7 dB difference in performance between the classic IC system and the improved IC system at a BER of 10^{-4} , about 4 dB can be attributed to using bit interleaving instead of symbol interleaving, 0.8 dB is due to using dynamic instead of fixed variance in the MCU, and 2.2 dB is due to the longer filter.

Figure 3.28 Different filter length on symbol interleaving.

Chapter 4 Conclusions and Future Work

4.1 Conclusions

In this thesis, we attempted to derive an improved interference canceller using techniques similar to those employed by Tuchler et al. The resulting IC turned out to be identical to Tuchler's linear equalizer although these two methods start from different points: Tuchler's method tries to optimize an LE using the feedback *a priori* information and the improved IC method tries to improve the classic IC using the feedback *a priori* information. We also introduced the concept of the confidence index, and showed that when the receiver has full confidence (perfect feedback), the improved IC is identical to the classic IC, and when the receiver has no confidence (no feedback), the improved IC is identical to an LE.

We compared the performance of the improved IC system with the classic IC system proposed by Glavieux et al. The comparison was done for different channel models, block sizes, error control codes and modulation schemes. The effects of using bit vs. symbol interleaving on the performance was also investigated. The impact on the performance of using different methods for determining the variance of the equalizer output (which must be used in calculating the branch metrics) was also studied. In particular, the following results have been observed:

- The improved IC system is mostly beneficial for severe ISI channels. For the mild and moderate ISI channels there is not much gain from using the improved IC system instead of the classic IC system.
- When larger block sizes or stronger error control codes are used, the relative difference between these two systems decreases.

- Symbol interleaving is less complex than bit interleaving, but when the residual ISI at the output of the equalizer is large, there is a great advantage to use bit interleaving in terms of performance. However, when the bits in a symbol are not independent, such as when 16-QAM is used, there is a slight advantage to use symbol interleaving.
- Using a dynamic variance (one that changes on a symbol-by-symbol basis) to calculate the branch metrics can lead to improved performance when the ISI is severe.

4.2 Future Work

The results presented in this thesis raise some interesting topics for future research:

- In this thesis, only time-invariant channels are used to compare the performance of these two systems. It will be interesting to apply some time-variant frequency selective channels to these two systems to see if this affects the conclusions presented here. In particular, the effect of using imperfect channel estimation must be studied thoroughly.
- For simplicity, the fading channels used in this thesis have been normalized on a block-by-block basis. A more realistic model would not have this normalization, but doing this would greatly increase the time required for the simulation.
- In this thesis, only M -ary QAM is used. Different modulation schemes, such as trellis coded modulation (TCM) and bit-interleaved coded modulation (BICM) should be explored in future work.

- In this thesis, we focus on time domain processing. Frequency domain interference cancellation will be an interesting topic to explore.
- Finally, we have only considered random interleaving. Searching for an optimal interleaving mapping would be beneficial, and may alter the relative performance of the two systems.

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Appendix Optimization of the Interference Canceller

A.1 Introduction

Since the soft decision feedback information is used as one of the inputs to the classic IC instead of hard decisions, the classic IC can give better performance. However, performance is potentially limited because the actual filter coefficients are derived under the assumption that the feedback information is perfectly correct. To overcome this deficiency, we propose to optimally determine the p and q filter coefficients of the classic IC under the assumption that the *a priori* probability of the transmitted symbols, $\{v_n\}$, are given by the feedback information. This is very similar to the method proposed by Tuchler et al, except that Tuchler et al. optimized the coefficients of a linear equalizer under this assumption, and we optimize the coefficients of an IC.

We also generalize the IC somewhat by allowing K_1 non-causal coefficients and $K_2 + 1$ causal coefficients in the feedforward filter and place no restriction on the number of taps in the feedback filter. Then the IC output is

$$z_n = \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \quad \forall n \in \{0, 1, \dots, N_v - 1\}. \quad (\text{A.1})$$

We will later show that the index of the second summation ranges only over $\{-K_1, \dots, -1, 1, \dots, K_2 + L - 1\}$. To minimize the mean square error,

$J_n = E[|z_n - v_n|^2]$, we need to find $\{p_l\}$ and $\{q_l\}$ so that $\frac{\partial J_n}{\partial p_l} = 0$, $\frac{\partial J_n}{\partial q_l} = 0$. This

implies that we need

$$E[(z_n - v_n)r_{n-l}^*] = 0 \quad \forall l \in \{-K_1, \dots, K_2\}. \quad (\text{A.2})$$

and

$$E[(z_n - v_n) \bar{v}_{n-l}^*] = 0 \quad \forall l \in \{-\infty, -1, 1, \dots, \infty\}. \quad (\text{A.3})$$

When calculating the expected values in Equation (A.2) and (A.3), we assume that *a priori* probabilities of the transmitted symbols are specified by the feedback information.

That is

$$\Pr\{d_m = d\} = \phi_m(d), \quad (\text{A.4})$$

where $\phi_m(d)$ is given by Equation (2.27) if symbol interleaving is used, and Equation (2.30) if bit interleaving is used. However, when calculates the coefficients for z_n (at time n), it is important that the feedback information at time n not be used to prevent excessive error propagation. Therefore we define

$$\bar{v}_m = E[v_m] = \begin{cases} \sum_{d=0}^{M-1} SM[d] \Pr\{d_n = d\} & m \neq n \\ 0 & m = n \end{cases}. \quad (\text{A.5})$$

We also define

$$\sigma_{v,m}^2 = E[|v_m|^2] - |\bar{v}_m|^2 = \begin{cases} \sum_{d=0}^{M-1} |SM[d]|^2 \Pr\{d_n = d\} - |\bar{v}_m|^2 & m \neq n \\ \sigma_v^2 & m = n \end{cases}. \quad (\text{A.6})$$

A.2 Finding the q Filter Coefficients

To find $\{q_l\}$ we need to solve the set of equations given by Equation (A.3),

which can be expressed as

$$E[z_n] \bar{v}_{n-l}^* = E[v_n] \bar{v}_{n-l}^* \quad \forall l \in \{-\infty, \dots, -1, 1, \dots, \infty\}. \quad (\text{A.7})$$

We know that

$$E[v_n] = \bar{v}_n = 0, \quad (\text{A.8})$$

since $\bar{v}_n = 0$ from Equation (A.5), and from Equation (A.1)

$$E[z_n] = \sum_{k=-K_1}^{K_2} p_k E[r_{n-k}] - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b}. \quad (\text{A.9})$$

Also from Equation (2.3)

$$E[r_{n-k}] = \sum_{a=0}^{L-1} h_a \bar{v}_{n-k-a} = \sum_{a=-\infty}^{\infty} h_a \bar{v}_{n-k-a}, \quad (\text{A.10})$$

where we define $h_a = 0 \quad \forall a \notin \{0, 1, \dots, L-1\}$. If we substitute $b=a+k$, then

$$E[r_{n-k}] = \sum_{b=-\infty}^{\infty} h_{b-k} \bar{v}_{n-b}. \quad (\text{A.11})$$

Substituting Equation (A.11) into (A.9) gives

$$\begin{aligned} E[z_n] &= \sum_{k=-K_1}^{K_2} p_k \sum_{b=-\infty}^{\infty} h_{b-k} \bar{v}_{n-b} - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \\ &= \sum_{b=-\infty}^{\infty} \bar{v}_{n-b} \sum_{k=-K_1}^{K_2} p_k h_{b-k} - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \\ &= \bar{v}_n \sum_{k=-K_1}^{K_2} p_k h_{-k} + \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} \left(\sum_{k=-K_1}^{K_2} p_k h_{b-k} - q_b \right) \bar{v}_{n-b} \\ &= \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} \left(\sum_{k=-K_1}^{K_2} p_k h_{b-k} - q_b \right) \bar{v}_{n-b}. \end{aligned} \quad (\text{A.12})$$

Substituting Equation (A.12) and Equation (A.8) into Equation (A.7) gives

$$\sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} \sum_{k=-K_1}^{K_2} (p_k h_{b-k} - q_b) \bar{v}_{n-b} = 0 \quad \forall l \neq 0. \quad (\text{A.13})$$

Although this is an undetermined set of equations (one equation but an infinite number of unknowns), one possible solution is

$$q_b = \sum_{k=-K_1}^{K_2} p_k h_{b-k} \quad \forall b \neq 0. \quad (\text{A.14})$$

Note when $(b-k) < 0$, $h_{b-k} = 0$, so $q_b = 0$ for all $b < -K_1$. Also when $(b-k) > L-1$, $h_{b-k} = 0$, so $q_b = 0$ for all $b > K_2 + L-1$. Therefore from Equation (A.1) z_n can be expressed as

$$z_n = \sum_{k=-K_1}^{K_2} p_k r_{n-k} - \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} q_b \bar{v}_{n-b}. \quad (\text{A.15})$$

A.3 Finding the Coefficients of p Filter

To find $\{p_k\}$ we must solve the set of equations given by Equation (A.2). That is, solve

$$E[z_n r_{n-l}^*] = E[v_n r_{n-l}^*] \quad \forall l \in \{-K_1, \dots, K_2\}. \quad (\text{A.16})$$

Note that

$$\begin{aligned} E[v_n r_{n-l}^*] &= E \left[v_n \left(\sum_{a=0}^{L-1} h_a^* v_{n-l-a}^* + w_n^* \right) \right] \\ &= \sum_{a=0}^{L-1} h_a^* E[v_n v_{n-l-a}^*] \\ &= \sum_{a=0}^{L-1} h_a^* [E[|v_n|^2] \delta_{l+a} + \bar{v}_n \bar{v}_{n-l-a}^* (1 - \delta_{l+a})] \end{aligned}$$

$$\begin{aligned}
&= \sigma_v^2 h_{-l}^* + \sum_{a=0}^{L-1} h_a^* \bar{v}_n \bar{v}_{n-l-a}^* - h_{-l}^* |\bar{v}_n|^2 \\
&= \sigma_v^2 h_{-l}^*,
\end{aligned} \tag{A.17}$$

since $\bar{v}_n = 0$. Also,

$$E[z_n r_{n-l}^*] = \sum_{a=0}^{L-1} h_a E[z_n v_{n-l-a}^*] + E[z_n w_{n-l}^*], \tag{A.18}$$

with

$$\begin{aligned}
E[z_n w_{n-l}^*] &= \sum_{k=-K_1}^{K_2} p_k E[r_{n-k} w_{n-l}^*] - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} E[w_{n-l}^*] \\
&= \sum_{k=-K_1}^{K_2} p_k \left(\sum_{a=0}^{L-1} h_a E[v_{n-k-a}] E[w_{n-l}^*] + E[w_{n-k} w_{n-l}^*] \right) \\
&= \sum_{k=-K_1}^{K_2} p_k N_0 \delta_{l-k} = N_0 p_l,
\end{aligned} \tag{A.19}$$

and

$$E[z_n v_{n-l-a}^*] = \sum_{k=-K_1}^{K_2} p_k E[r_{n-k} v_{n-l-a}^*] - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \bar{v}_{n-l-a}^*. \tag{A.20}$$

Equation (A.20) can be simplified using

$$\begin{aligned}
E[r_{n-k} v_{n-l-a}^*] &= \sum_{m=0}^{L-1} h_m E[v_{n-k-m} v_{n-l-a}^*] \\
&= \sum_{m=0}^{L-1} h_m [\bar{v}_{n-k-m} \bar{v}_{n-l-a}^* (1 - \delta_{l+a-k-m}) + E[|v_{n-l-a}|^2] \delta_{l+a-k-m}] \\
&= \sum_{m=0}^{L-1} h_m [\bar{v}_{n-k-m} \bar{v}_{n-l-a}^* + (E[|v_{n-l-a}|^2] - |\bar{v}_{n-l-a}|^2) \delta_{l+a-k-m}] \\
&= \sum_{m=0}^{L-1} h_m [\bar{v}_{n-k-m} \bar{v}_{n-l-a}^* + \sigma_{v,n-l-a}^2 \delta_{l+a-k-m}]
\end{aligned}$$

$$= \bar{v}_{n-l-a}^* \sum_{m=-\infty}^{\infty} h_m \bar{v}_{n-k-m} + \sigma_{v,n-l-a}^2 h_{l+a-k} . \quad (\text{A.21})$$

Substituting $b = k+m$ gives

$$E[r_{n-k} v_{n-l-a}^*] = \bar{v}_{n-l-a}^* \sum_{b=-\infty}^{\infty} h_{b-k} \bar{v}_{n-b} + \sigma_{v,n-l-a}^2 h_{l+a-k} . \quad (\text{A.22})$$

Substituting Equation (A.22) into Equation (A.20) gives

$$\begin{aligned} E[z_n v_{n-l-a}^*] &= \sum_{k=-K_1}^{K_2} p_k \bar{v}_{n-l-a}^* \sum_{m=-\infty}^{\infty} h_{b-k} \bar{v}_{n-b} + \sum_{k=-K_1}^{K_2} p_k \sigma_{v,n-l-a}^2 h_{l+a-k} \\ &\quad - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \bar{v}_{n-l-a}^* \\ &= \bar{v}_{n-l-a}^* \left[\sum_{b=-\infty}^{\infty} \bar{v}_{n-b} q_b - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} \bar{v}_{n-b} q_b \right] + \sigma_{v,n-l-a}^2 q_{l+a} \\ &= \bar{v}_{n-l-a}^* [\bar{v}_n q_0 + \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} \bar{v}_{n-b} (q_b - q_b)] + \sigma_{v,n-l-a}^2 q_{l+a} \\ &= \bar{v}_n \bar{v}_{n-l-a}^* q_0 + \sigma_{v,n-l-a}^2 q_{l+a} \\ &= \sigma_{v,n-l-a}^2 q_{l+a} . \end{aligned} \quad (\text{A.23})$$

Substituting Equation (A.23) and Equation (A.19) into Equation (A.18) gives

$$\begin{aligned} E[z_n r_{n-l}^*] &= \sum_{a=0}^{L-1} h_a^* E[z_n v_{n-l-a}^*] + E[z_n w_{n-l}^*] \\ &= \sum_{a=0}^{L-1} h_a^* \sigma_{v,n-l-a}^2 q_{l+a} + N_0 p_l \\ &= \sum_{a=-\infty}^{\infty} h_a^* \sigma_{v,n-l-a}^2 q_{l+a} + N_0 p_l . \end{aligned} \quad (\text{A.24})$$

Define $b = l + a$ and use Equation (A.14) gives

$$\begin{aligned}
E[z_n r_{n-l}^*] &= \sum_{b=-\infty}^{\infty} h_{b-l}^* \sigma_{v,n-b}^2 q_b + N_0 \sum_{b=-K_1}^{K_2} p_b \delta_{l-b} \\
&= \sum_{b=-\infty}^{\infty} \sum_{k=-K_1}^{K_2} p_k h_{b-k} h_{b-l}^* \sigma_{v,n-b}^2 + N_0 \sum_{k=-K_1}^{K_2} p_k \delta_{l-k} \\
&= \sum_{k=-K_1}^{K_2} p_k \left[\sigma_{v,n}^2 h_{-k} h_{-l}^* + \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} \sigma_{v,n-b}^2 h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \right]. \tag{A.25}
\end{aligned}$$

Using Equation (A.17) and Equation (A.25), the set of linear equations given by Equations (A.16) can be expressed as

$$\sum_{k=-K_1}^{K_2} p_k \left[\sigma_v^2 h_{-k} h_{-l}^* + \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* \sigma_{v,n-b}^2 + N_0 \delta_{l-k} \right] = \sigma_v^2 h_{-l}^*. \tag{A.26}$$

The optimal filter coefficients, $\underline{p}$, can be found by solving this set of equations. Unfortunately, the solution depends on n , which means the filter coefficients vary from symbol to symbol. Since solutions for the coefficients requires a matrix inversion, using the optimal coefficients is computationally prohibitive.

A.4 Suboptimal Filter Coefficients

To simplify the problem we use the time average of the signal variance. Define

$$\bar{\sigma}_v^2 = \frac{1}{N_v} \sum_{m=0}^{N_v-1} \sigma_{v,m}^2, \tag{A.27}$$

and use it in place of $\sigma_{v,n}^2$ in Equation (A.26) giving

$$\sum_{k=-K_1}^{K_2} p_k \left[\sigma_v^2 h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \right] = \sigma_v^2 h_{-l}^*. \tag{A.28}$$

Since these equations do not depend on n , the filter coefficients only need to be recalculated once every iteration, not for each symbol. It is convenient to rewrite Equation (A.28) using matrix form as

$$\underline{\Psi} \underline{p} = \underline{\xi}, \quad (\text{A.29})$$

where $\underline{\xi} = \sigma_v^2 [h_{-K_2} \ h_{-K_2+1} \ \cdots \ h_{K_1}]^H$, $\underline{p} = [p_{-K_1} \ p_{-K_1+1} \ \cdots \ p_{K_2}]^T$, and

Ψ is a $(K_1 + K_2 + 1) \times (K_1 + K_2 + 1)$ matrix with elements (for $l, k \in \{-K_1, -K_1 + 1, \dots, K_2\}$).

$$\begin{aligned} \Psi_{l,k} &= \sigma_v^2 h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \\ &= (\sigma_v^2 - \bar{\sigma}_v^2) h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{b=-\infty}^{\infty} h_{b-k} h_{b-l}^* + N_0 \delta_{l-k} \\ &= (\sigma_v^2 - \bar{\sigma}_v^2) h_{-k} h_{-l}^* + \bar{\sigma}_v^2 \sum_{a=0}^{L-1} h_a h_{a+k-l}^* + N_0 \delta_{l-k}. \end{aligned} \quad (\text{A.30})$$

Then the filter coefficients can be found as

$$\underline{p} = \underline{\Psi}^{-1} \underline{\xi}. \quad (\text{A.31})$$

A.5 The Mean and Variance of z_n

When calculating the branch metrics

$$\gamma_n(d) = f(z_n | v_n = SM[d]), \quad (\text{A.32})$$

in Equation (2.15) we assume that z_n has a Gaussian distribution. To evaluate the likelihood function we need to know the mean and variance of z_n under the hypothesis that $v_n = v$ for some signal point v .

The mean of z_n is

$$\begin{aligned}
E[z_n | v_n = v] &= \sum_{k=-K_1}^{K_2} p_k E[r_{n-k} | v_n = v] - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \\
&= \sum_{k=-K_1}^{K_2} p_k \sum_{a=0}^{L-1} h_a \bar{v}_{n-k-a} - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \\
&= \sum_{k=-K_1}^{K_2} p_k \sum_{b=-\infty}^{\infty} h_{b-k} \bar{v}_{n-b} - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \\
&= \sum_{b=-\infty}^{\infty} q_b \bar{v}_{n-b} - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b \bar{v}_{n-b} \\
&= \bar{v}_n q_0 \\
&= v q_0
\end{aligned} \tag{A.33}$$

The variance of z_n is

$$\begin{aligned}
\sigma_{z,n}^2 &= E[|z_n - v q_0|^2 | v_n = v] \\
&= E[|z_n|^2 | v_n = v] - |v|^2 |q_0|^2 \\
&= \sum_{l=-K_1}^{K_2} p_l^* E[z_n r_{n-l}^* | v_n = v] - \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b^* \bar{v}_{n-b}^* E[z_n | v_n = v] - |v|^2 |q_0|^2 \\
&= \sum_{l=-K_1}^{K_2} p_l^* E[z_n r_{n-l}^* | v_n = v] - v q_0 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b^* \bar{v}_{n-b}^* - |v|^2 |q_0|^2.
\end{aligned} \tag{A.34}$$

In the first term of Equation (A.34), $E[z_n r_{n-l}^* | v_n = v]$ can be expressed as

$$E[z_n r_{n-l}^* | v_n = v] = v q_0 \sum_{b=-\infty}^{\infty} h_{b-l}^* \bar{v}_{n-b} + \sum_{b=-\infty}^{\infty} \sigma_{v,n-b}^2 q_b h_{b-l}^* + N_0 p_l. \tag{A.35}$$

Therefore

$$\begin{aligned}
\sigma_{z,n}^2 &= \sum_{l=-K_1}^{K_2} p_l^* \left[v q_0 \sum_{b=-\infty}^{\infty} h_{b-l}^* \bar{v}_{n-b} + \sum_{b=-\infty}^{\infty} \sigma_{v,n-b}^2 q_b h_{b-l}^* + N_0 p_l \right] \\
&\quad - v q_0 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b^* \bar{v}_{n-b}^* - |v|^2 |q_0|^2 \\
&= v q_0 \sum_{b=-\infty}^{\infty} q_b^* \bar{v}_{n-b}^* + \sum_{b=-\infty}^{\infty} \sigma_{v,n-b}^2 q_b q_b^* + N_0 \sum_{l=-K_1}^{K_2} |p_l|^2 - v q_0 \sum_{\substack{b=-\infty \\ b \neq 0}}^{\infty} q_b^* \bar{v}_{n-b}^* - |v|^2 |q_0|^2 \\
&= v q_0 q_0^* \bar{v}_n^* + \sum_{b=-\infty}^{\infty} \sigma_{v,n-b}^2 |q_b|^2 + N_0 \sum_{l=-K_1}^{K_2} |p_l|^2 - |v|^2 |q_0|^2 \\
&= \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} \sigma_{v,n-b}^2 |q_b|^2 + N_0 \sum_{l=-K_1}^{K_2} |p_l|^2, \tag{A.36}
\end{aligned}$$

since $\bar{v}_n = E[v_n | v_n = v] = v$, and $\sigma_{v,n}^2 = E[|v_n - \bar{v}_n|^2 | v_n = v] = 0$, we have

$$\sigma_{z,n}^2 = \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} \sigma_{v,n-b}^2 |q_b|^2 + N_0 \sum_{l=-K_1}^{K_2} |p_l|^2. \tag{A.37}$$

Note that $\sigma_{z,n}^2$ depends on n , so it must be calculated for each symbol. We call this dynamic variance. A simpler alternative for the dynamic variance is its time average. We call this the static variance, which is expressed as

$$\bar{\sigma}_z^2 = \frac{1}{N_v} \sum_{n=0}^{N_v-1} \sigma_{z,n}^2 = \bar{\sigma}_v^2 \sum_{\substack{b=-K_1 \\ b \neq 0}}^{K_2+L-1} |q_b|^2 + N_0 \sum_{k=-K_1}^{K_2} |p_k|^2. \tag{A.38}$$