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Analysis of Linear Combining
and Its Application to Frequency-Hopped Signals

by

Rolands Eriks Ezers, B.A.Sc., M.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Ontario, Canada
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submitted by

Rolands Eriks Ezers, B.A.Sc., M.Eng.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Electronics

Thesis Supervisor

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Carleton University
May 3, 1993

Abstract

In this thesis, fast-frequency-hopped (FFH) noncoherent frequency-shift-keyed (NCFSK) communication systems with time diversity and diversity combining are discussed. A novel analytical method has been developed for obtaining performance for systems employing linear diversity combining for demodulation as well as satellite uplink time synchronization. The novel method combines circularly symmetric function theory, Fourier-Bessel series, and Fourier series. With this novel method, performance measures are obtained for the demodulation of FFH NCFSK systems with time diversity and linear diversity combining, and for the time synchronization of a FFH NCFSK satellite-communication system with time diversity and linear diversity combining in broad-band noise and multiple-tone jamming. Where possible, the results obtained with this method were compared to other reported results, and the two methods' results agreed well. In addition, a novel analytical result was derived for computing the bit-error probability for the demodulation of FFH NCFSK systems with time diversity and 4-2- or 2-1-moment-method diversity combining in independent Rayleigh fading with a level of diversity of two. Also, a complete literature review of diversity-combining techniques is provided.

Analysis using the above techniques leads to the following observations. For demodulation, linear diversity combining is an excellent method in broad-band noise. In severe multiple-tone jamming, linear diversity combining is good. In weak multiple-tone jamming, linear diversity combining is not a good method because it is susceptible to Houston-sense multiple-tone jamming with a small jamming fraction. In independent Rayleigh fading, square-law combining is the best method. For uplink coarse-time synchronization, time diversity and linear diversity combining could be used to improve performance. For uplink fine-time synchronization, the estimator for the delay-advance method is biased in broad-band noise jamming.

A binary NCFSK modulator and demodulator with time diversity and linear diversity combining was implemented. This implementation shows the viability and practicality of linear diversity combining.

Acknowledgements

I would like to thank my supervisor Professor J.S. Wight for his guidance throughout this work.

I would like to thank Dr. E.B. Felstead for his supervision of my work at CRC.

I would like to thank Dr. T.A. Gulliver for his help.

I would like to thank Mr. J.S. Butterworth, Dr. L.J. Mason, Mr. C.J. Bélisle, Mr. D.J. Barlow, Mr. J. Oldham, Mr. W. Robertson, Mr. D.G. McLachlan, Miss V. Maier, Mr. R. Keightley, Mr. J.D. Lambert, Mr. W. Seed., Mr. R. Addison, Mr. A. McEwen, and Mr. R. Burrill for their help with the theoretical, practical, and computer work at CRC.

I would like to thank Mr. R.J. Griffith, Mr. W.W. Fergusson, Mr. M. Gagnon, and Mr. A. Castonguay for their help solving my computer problems and thus allowing me to produce this thesis.

I would like to thank Mr. G. Asselin, Mrs. B. Lynn, and Mr. D. Piamonte for their help throughout my stay at Carleton University.

I would like to thank the Department of Communication and the Department of National Defence of the Government of Canada for providing the facilities to conduct the work and their financial support.

I would like to thank NSERC for financially supporting the work through a Postgraduate Scholarship.

I would like to thank Telesat Canada and the Association of Old Crows for their financial support.

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List of Symbols

α - fraction of channels jammed
 b_a - adaptive-threshold coefficient
 b_f - fixed-threshold coefficient
 Δf - frequency separation
 ΔT - timing error
 E - number of empty bins
 E_b - energy per bit
 E_h - energy per hop
 ϵ_m - normalized decision statistic
 ϵ_t - normalized timing error
 $f()$ - probability density function
 $F()$ - cumulative distribution function
 ${}_1F_1(, ,)$ - confluent hypergeometric function
 γ - fraction of bins jammed
 $I_0()$ - zeroth-order modified Bessel function of the first kind
 J_o - equivalent jammer-power-spectral density
 $J_m()$ - m th-order Bessel function of the first kind
 $K_m()$ - m th-order modified Bessel function of the second kind
 L - level of diversity
 Λ_m - decision statistic
 M - M -ary signal
 $N_0/2$ - two-sided noise-power-spectral density
 N_J - jammer-power-spectral density
 p - uncoded channel-error rate
 P_b - probability of bit error
 P_d - probability of detection
 P_{fa} - probability of false alarm
 P_s - symbol-error probability
 $\Phi()$ - two-dimensional characteristic function
 R - maximum argument for Fourier-Bessel function

ρ - jamming fraction

$S()$ - square wave

SIR - signal-to-interference ratio

SJR - signal-to-jammer ratio

SNR - signal-to-noise ratio

T_s - symbol period

T_h - hop period

v_m - decision statistic

V - period of square wave

W_{ss} - spread-spectrum bandwidth

W_J - jammer bandwidth

$\hat{y}_{mi}$ - complex matched-filter-with-envelope-detector output

z_{mi} - matched-filter-with-envelope-detector output

Chapter 1

Introduction

1.1 Motivation

The demand for reliable, secure, and robust communications is ever increasing. Modern communication systems are required to be able to send data, voice, video, and facsimile to numerous locations - both fixed and mobile. Integral parts of such systems are digital-radio communication links. Such links can be susceptible to interference - such as noise or tones - and to signal fading. Both may prohibit communication.

There are two principal types of users for communications systems: military and civilian. Military users need communication systems to meet the command, control, communication, computer, and intelligence (C⁴I) needs in their militaries. Civilian users need communication systems to undertake business and social activities. Military communication systems can be subjected to hostile interference and environmental effects. Civilian communication systems can be subjected to unintentional interference and environmental effects. Both civilian and military users wish to communicate in these situations. Hence, both require robust communications.

Communication can be prohibited in two ways. First, synchronization between the transmitter and receiver can be lost. Second, the communication signal can be disrupted such that the information is demodulated incorrectly.

Frequency-hopping spread-spectrum techniques can be used in digital-radio communication systems for combatting interference effects in the demodulation process.

However, under certain circumstances, frequency-hopping techniques alone cannot reduce the probability of bit error to acceptable target levels. Error-correction coding can lower moderate bit-error probabilities to acceptable target levels. However, error-correction-coding techniques are ineffective at high channel bit-error probabilities. With high channel bit-error probabilities, time diversity can be used to lower the bit-error probabilities to acceptable target levels. Time diversity at the transmitter requires diversity combining at the receiver. Time diversity and diversity combining can also provide some anti-jam protection for the synchronization process.

1.2 Objectives

Fast-frequency-hopped noncoherent frequency-shift-keyed (FFH NCFSK) communication systems have inherent time diversity which can be used to improve anti-jam capabilities for demodulation. Time diversity is introduced at the transmitter by fast hopping and is removed at the receiver by an operation called diversity combining. There are many methods of diversity combining. Considerable work has been done in obtaining measures of performance for these methods in various interference and fading environments. However, because of the complexity of the problem, most work has involved special cases, simplifications, or computer simulation, and many aspects of the problem have not been considered. Fast frequency hopping is used in both terrestrial- and satellite-communication systems.

Time diversity and diversity combining can also be used to improve the anti-jam performance for uplink coarse-time synchronization and uplink fine-time synchronization of a FFH NCFSK satellite-communication system. The diversity-combining methods are similar to those used for demodulation. Little work has been done in obtaining measures of performance for coarse-time or fine-time synchronization for any of the diversity-combining methods.

Analytical techniques would provide insight into some aspects of the demodulation and synchronization problems; however, such techniques are quite difficult to derive. In this thesis, the objective is to develop analytical techniques for calculating performance measures for FFH NCFSK systems. The analytical techniques can then

be used to gain insight into the demodulation performance of FFH NCFSK systems, into the uplink-coarse-time-synchronization performance of a FFH NCFSK satellite-communication system, and into the uplink-fine-time-synchronization performance of a FFH NCFSK satellite-communication system.

1.3 Scope

In this thesis, a novel analytical technique is developed which can provide the general solution for one type of diversity-combining technique - linear diversity combining. Measures of performance are obtained in broad-band noise, multiple-tone jamming, and Rayleigh fading for demodulation in FFH NCFSK systems. Measures of performance are also obtained in broad-band noise, and multiple-tone jamming for uplink coarse-time synchronization in a FFH NCFSK satellite-communication system and in broad-band noise for uplink fine-time synchronization in a FFH NCFSK satellite-communication system.

Where possible, results are compared to other diversity-combining methods. To do this, a novel analytical technique was developed for obtaining performance of a special case of 4-2- or 2-1-moment-method diversity combining in Rayleigh fading for demodulation of FFH NCFSK systems.

1.4 Outline

In Chapter 2, the analysis method for linear combining is presented. In Chapter 3, background information for modulation and demodulation in FFH NCFSK systems with time diversity and diversity combining is presented. The modulation and demodulation techniques can apply to both terrestrial and satellite systems. Performance results are presented for linear combining and these results are compared to those for some of the other diversity-combining techniques. In Chapter 4, background information for uplink time synchronization in a FFH NCFSK satellite-communication system is presented. Both coarse-time and fine-time synchronization are considered. For coarse-time synchronization, a method which employs linear combining is described, and performance results are presented. For fine-time

synchronization, the delay-advance method which employs linear combining is described, and performance results are presented. In Chapter 5, an experimental system for obtaining performance results for demodulation in NCFSK systems with time diversity and linear diversity combining is described. Performance results are presented and they are compared to analytical results. In Chapter 6, conclusions are presented.

Chapter 2

Analysis Method for Linear Combining

2.1 Introduction

In this chapter, a novel analytical technique for calculating performance measures for systems which use linear combining is presented. The novel analytical technique consists of the unique combination of two procedures. The novel technique is applicable for solutions to a wide range of problems as is demonstrated in Chapters 3 and 4. In this chapter, the method itself is described.

2.2 Outline of the Method

A receiver often used in communications and radar is shown in Fig. 2.1. The input, $x(t)$, can be modelled as one or more tones plus additive white Gaussian noise (AWGN). The tones, if present, can be a desired signal or interference. The receiver is assumed not to have knowledge of the phase of the desired signal. The desired signal has a basic period of duration T . This input is applied to a filter noncoherently matched to the desired-signal frequency and then sampled at intervals of T to generate a complex sample, $\tilde{y}_i$, in the i th interval. Envelope detection is then performed to generate the envelope sample, z_i . Finally, L samples over the period LT are added together in an operation sometimes called "linear diversity

combining” or just “linear combining”, to obtain the decision statistic, v . The term “linear” refers to the sample combining, and not to the envelope-detection process which is actually nonlinear. This distinction is important since it is common practice to replace the envelope detector with a square-law detector. This second type of combining is often called “square-law combining” even though the actual combining is linear.

Figure 2.1: Simplified block diagram of process to be analyzed.

In order to analyze the performance of such a system, it is required to derive the probability density function (pdf) and cumulative distribution function (cdf) of the L -sample sum. There has been considerable previous work in deriving the pdf and cdf for the process where a square-law detector is used in place of the envelope detector. Much less has been done for envelope detection. In this chapter, a method of analysis is described that provides the pdf and cdf for the output, v , of the envelope detector and linear combiner of Fig. 2.1.

In previous work on envelope detection with linear combining, the pdf and cdf of the sum have been derived for some special cases of the stochastic process, $x(t)$. Brennan [1] computed the cdf of the sum of independent Rayleigh random variables, which occur if the input, $x(t)$, consists of white Gaussian noise, by doing repeated convolutions using numerical integration. Marcum [2] derived a method of evaluating the cdf of the sum of independent Rayleigh random variables, and of the sum of independent Rician random variables, which occur if the input, $x(t)$, consists of a sine wave plus white Gaussian noise, using Gram-Charlier series. Beaulieu [3] derived an efficient way of evaluating the pdf and cdf of the sum of independent Rayleigh random variables using Fourier series. Helstrom [4] derived a method of evaluating the pdf and cdf of the sum of independent Rayleigh and the sum of independent Rician random variables using Laplace transforms and saddlepoint in-

tegration. These methods either are not general enough to cover all cases of interest or are very difficult and labourious to implement computationally.

In this chapter, the technique developed by Bird [5] to analyze the pdf for a single sample using circularly symmetric function theory and Fourier-Bessel series is combined for the first time with a method [3],[6] which uses Fourier series to analyze the pdf and cdf of the sum of independent random variables in order to provide a method of analysis that is capable of calculating the pdf and cdf of the decision statistic, v . This novel method is not only computationally efficient and accurate, but is applicable to more general problems.

Once the pdf and cdf have been found, a variety of performance values can be determined. In particular, it can be used for computing the probability of error in demodulation problems, the probability of detection and false alarm in detection problems, and the mean and variance of estimates in estimation problems.

In later chapters, this method will be demonstrated through the analysis of a fast frequency-hopped (FFH) M -ary noncoherent frequency-shift-keyed (NCFSK) system. The method is applicable to obtaining performance measures for both demodulation and time synchronization of such systems. For demodulation of the frequency-hopped signal [7],[8],[9], there are M branches with M outputs, v_m , for $m = 1, \dots, M$, as shown in Fig. 2.2. The largest value is selected as corresponding to the symbol received. The demodulation problem is described in more detail in Chapter 3. The method can also be applied to the synchronization problem when the synchronization is performed by the transmitter. This type of synchronization requires that the transmitter send synchronization probes which are similar to the FFH NCFSK communication signal. For coarse-time-synchronization algorithms that require the detection of synchronization probes, there is at least one branch like that in Fig. 2.1 to detect the probes, and possibly others to sample the interference background [9]-[14]. For fine-time-synchronization algorithms that employ synchronization probes, two or more branches are used, and the outputs, v_m , are used to calculate an estimate of the timing error [9],[15],[16]. The synchronization problem is described in more detail in Chapter 4. In this thesis, the analysis method is applied to diversity combining for the demodulation of FFH NCFSK systems and for uplink coarse-time and fine-time synchronization of a FFH NCFSK satellite-communication system.

2.3 Mathematical Approach

Consider the general block diagram shown in Fig. 2.2. Let the input consist of a signal tone, plus one or more interfering tones, plus white Gaussian noise. In the i th transmission interval of duration, T , a sample is taken in every branch. Then, L of these are accumulated over the total symbol period LT . The output of the m th branch is

$$v_m = \sum_{i=1}^L z_{mi} \quad (2.1)$$

where z_{mi} is the envelope sample for the m th branch during the i th interval. To find the pdf and cdf of v_m , the pdf of z_{mi} is derived first.

Figure 2.2: A block diagram of the basic process with M branches.

Let the signal tone at the m th branch be

$$\sqrt{2}s_{mi} \cos(2\pi f_m t + \phi_{mi}) \quad (2.2)$$

where s_{mi} is the rms amplitude, f_m is the frequency, and ϕ_{mi} is a random phase

angle. The interference tone is

$$\sqrt{2}a_{mi} \cos(2\pi f_m t + \alpha_{mi}) \quad (2.3)$$

where a_{mi} is the rms amplitude, f_m is the frequency, and α_{mi} is a random phase angle. The noise is a white Gaussian random process with zero mean and two-sided noise power-spectral density, $N_0/2$. The sum of these three signals passes through the matched filter and envelope detector of Fig. 2.3.

Figure 2.3: A matched filter with envelope detector.

The output sample of the matched filter with envelope detector, z_{mi} , is the magnitude of a two-dimensional vector. This vector can be written as a complex number, $\hat{y}_{mi}$, where the real part, y_{mic} , is the sampled output of the in-phase integrator, and the imaginary part, y_{mis} , is the sampled output of the quadrature integrator. Then, $\hat{y}_{mi}$ is

$$\hat{y}_{mi} = y_{mic} + jy_{mis} = \tilde{s}_{mi} + \tilde{a}_{mi} + \tilde{n}_{mi} \quad (2.4)$$

so that

$$\tilde{s}_{mi} = s_{mi} \cos(\phi_{mi}) + js_{mi} \sin(\phi_{mi}), \quad (2.5)$$

$$\tilde{a}_{mi} = a_{mi} \cos(\alpha_{mi}) + ja_{mi} \sin(\alpha_{mi}), \quad (2.6)$$

$$\tilde{n}_{mi} = n_{mic} + jn_{mis}. \quad (2.7)$$

It can be shown that the noise components, $n_{m,c}$ and $n_{m,s}$, are Gaussian random variables with zero mean and variance,

$$\sigma^2 = \frac{N_0}{2T}. \quad (2.8)$$

Note that $\tilde{s}_{m,i}$, $\tilde{a}_{m,i}$, and $\tilde{n}_{m,i}$ are circularly symmetric random variables. Hence, $\tilde{y}_{m,i}$ is also circularly symmetric. Because of the circular symmetry, the pdf of $z_{m,i} = |\tilde{y}_{m,i}|$ is [5]

$$f_{z_{m,i}}(z_{m,i}) = \int_0^\infty z_{m,i} \rho J_0(\rho z_{m,i}) \Phi_{\tilde{y}_{m,i}}(\rho) d\rho \quad (2.9)$$

where $\Phi_{\tilde{y}_{m,i}}(\rho)$ is the two-dimensional characteristic function of $\tilde{y}_{m,i}$ given by [5]

$$\Phi_{\tilde{y}_{m,i}}(\rho) = \int_0^\infty J_0(\rho z_{m,i}) f_{z_{m,i}}(z_{m,i}) dz_{m,i}, \quad (2.10)$$

and $J_0()$ is the zeroth-order Bessel function of the first kind. Since the random variable, $\tilde{y}_{m,i}$, is the sum of independent components, the two-dimensional characteristic function of $\tilde{y}_{m,i}$ is the product of the characteristic functions of its corresponding components. Therefore, the two-dimensional characteristic function of each component is required so that (2.9) can be used to find the pdf of $z_{m,i}$.

For a constant signal tone of duration, T , the characteristic function is found from (2.10) as [5]

$$\Phi_{\tilde{s}_{m,i}}(\rho) = J_0(\rho s_{m,i}). \quad (2.11)$$

For a constant interference tone of duration, T , the characteristic function is [5]

$$\Phi_{\tilde{a}_{m,i}}(\rho) = J_0(\rho a_{m,i}). \quad (2.12)$$

The characteristic function for the noise is [5]

$$\Phi_{\tilde{n}_{m,i}}(\rho) = \exp\left(-\frac{\sigma^2 \rho^2}{2}\right). \quad (2.13)$$

The characteristic function of $\tilde{y}_{m,i}$ is

$$\Phi_{\tilde{y}_{m,i}}(\rho) = \Phi_{\tilde{s}_{m,i}}(\rho) \Phi_{\tilde{a}_{m,i}}(\rho) \Phi_{\tilde{n}_{m,i}}(\rho). \quad (2.14)$$

The pdf of $z_{m,i}$ can be found by substituting (2.14) into (2.9). Unfortunately, the integral in (2.9) cannot usually be solved analytically for most signal combinations of interest. Previous work in evaluating the pdf and cdf of $z_{m,i}$ has followed three

different approaches. The first approach leads to series solutions in terms of Laguerre polynomials [17]-[19]. The second involves numerical integration [20],[21]. These two approaches are complicated and specialized to each individual problem. The third uses Fourier-Bessel series which leads to an efficient numerical solution. It can be shown that [5]

$$f_{z_{m1}}(z_{m1}) = 2z_{m1} \sum_{q=1}^{\infty} \frac{\Phi_{\dot{y}_{m1}}(\lambda_q/R)}{[J_1(\lambda_q)]^2 R^2} J_0\left(\frac{\lambda_q z_{m1}}{R}\right) \quad (2.15)$$

where λ_q are the zeroes of $J_0()$ for $q = 1, 2, \dots, \infty$ in ascending order, and R is a value chosen such that for $z_{m1} > R$, $f_{z_{m1}}(z_{m1})$ is sufficiently close to zero. The cdf of z_{m1} can be obtained by integrating (2.15) with respect to z_{m1} term by term leading to [5]

$$F_{z_{m1}}(z_{m1}) = 2z_{m1} \sum_{q=1}^{\infty} \frac{\Phi_{\dot{y}_{m1}}(\lambda_q/R)}{[J_1(\lambda_q)]^2 R \lambda_q} J_1\left(\frac{\lambda_q z_{m1}}{R}\right). \quad (2.16)$$

Thus, both the cdf and pdf of z_{m1} can be efficiently evaluated using Fourier-Bessel series. The series in (2.15) and (2.16) are shown to converge in [5]. Thus, it is sufficient to compute the pdf and cdf with increasing numbers of terms until the final result does not change by more than the required accuracy [22].

There are two types of truncation error involved with evaluating (2.15) or (2.16). It is necessary to truncate the series after Q terms and to choose a value of R . For a signal with tone interference and Gaussian noise, it is suggested in [5] that R be chosen to be

$$R = R_S + R_I + 10\sigma \quad (2.17)$$

where R_S is the rms amplitude of the signal tones, and R_I is the sum of the rms amplitudes of the interfering tones. This selection makes the truncation error due to the choice of R negligible [5]. Then, Q is chosen so that the truncation error due to the finite series is sufficiently small. In order to guarantee a sufficiently small truncation error, calculations of the pdf and cdf are repeated with increasing numbers of terms in the series of (2.15) and (2.16) until the final results do not change by more than the required accuracy.

The pdf of z_{m1} is now used to determine the pdf and cdf of v_m given by (2.1). Note that the z_{mi} for $i = 1, \dots, L$ are independent random variables. Beaulieu [3] and Bird [6] have provided similar means of determining the pdf and cdf of a sum of

independent random variables. The technique used below more closely follows the approach of Beaulieu [3] with suitable modifications for the problem at hand.

In the derivation of (2.15), it is assumed that $f_{z_{m_i}}(z_{m_i}) = 0$ for $z_{m_i} > R$ and also for $z_{m_i} < 0$. Since v_m is the sum of the z_{m_i} , it follows that the pdf of v_m , $f_{v_m}(v_m)$, is zero for $v_m > LR$ and also for $v_m < 0$. A typical $f_{v_m}(v_m)$ is shown in Fig. 2.4. Now from [3], the artifice is introduced of multiplying $f_{v_m}(v_m)$ by a square wave, $S_V(v_m - L\epsilon_m)$, also shown in Fig. 2.4. Here, $\epsilon_m L$ is a constant offset. The square wave is defined by

$$S_V(v) = \sum_{u=-\infty}^{u=\infty} \text{rect}\left(\frac{v - \frac{V}{2} - uV}{V}\right) \quad (2.18)$$

where

$$\text{rect}(v) = \begin{cases} 1, & |v| \leq 0.5, \\ 0, & |v| > 0.5. \end{cases} \quad (2.19)$$

The period of the square wave is chosen to be $V = 2LR$. The cdf of v_m is

$$F_{v_m}(\epsilon_m L) = \int_0^{\epsilon_m L} f_{v_m}(v) dv, \quad (2.20)$$

$$= 1 - \int_{\epsilon_m L}^{LR} f_{v_m}(v) dv. \quad (2.21)$$

Since $S_V(v - \epsilon_m L) = 1$ for $\epsilon_m L < v < LR$, and $S_V(v - \epsilon_m L)f_{v_m}(v) = 0$ for $v < \epsilon_m L$ and $v > LR$, (2.21) can be written as

$$F_{v_m}(\epsilon_m L) = 1 - \int_{\epsilon_m L}^{LR} S_V(v - \epsilon_m L) f_{v_m}(v) dv. \quad (2.22)$$

$$= 1 - \int_{-\infty}^{\infty} S_V(v - \epsilon_m L) f_{v_m}(v) dv. \quad (2.23)$$

$$= 1 - E[S_V(v_m - \epsilon_m L)] \quad (2.24)$$

where $E[\]$ denotes expectation over the random variable, v_m . Thus, the cdf of v_m can be expressed in terms of the expected value of the periodic square wave. Note that (2.24) is true only if $f_{v_m}(v_m)$ does not have an impulse at a discontinuity of $S_V(v_m - \epsilon_m L)$ [3].

The Fourier series representation of $S_V(v)$ is [3]

$$S_V(v) = \frac{1}{2} + \sum_{k=-\infty, k \text{ odd}}^{k=\infty} \frac{e^{jk\omega v}}{\pi k j} \quad (2.25)$$

where $\omega = 2\pi/V$. Combining (2.24) and (2.25) gives [3]

$$F_{v_m}(\epsilon_m L) = \frac{1}{2} - \sum_{k=-\infty, k \text{ odd}}^{k=\infty} \frac{E[e^{jk\omega(v_m - \epsilon_m L)}]}{\pi k j}. \quad (2.26)$$

Figure 2.4: The pdf of v_m .

After considerable algebraic manipulation, the pdf and cdf of v_m can be found from the pdf of z_{mi} by the Fourier series expansion and are given by [3]

$$F_{v_m}(v_m) = \frac{1}{2} - \frac{2}{\pi} \sum_{k=1, k \text{ odd}}^{\infty} \frac{A_{mk} \sin \theta_{mk}}{k}, \quad (2.27)$$

$$f_{v_m}(v_m) = \frac{4}{V} \sum_{k=1, k \text{ odd}}^{\infty} A_{mk} \cos \theta_{mk} \quad (2.28)$$

where

$$A_{mik} = \sqrt{(E[\cos(k\omega z_{mi})])^2 + (E[\sin(k\omega z_{mi})])^2}, \quad (2.29)$$

$$\theta_{mik} = \tan^{-1} \left(\frac{E[\sin(k\omega(z_{mi} - \epsilon_m))]}{E[\cos(k\omega(z_{mi} - \epsilon_m))]} \right), \quad (2.30)$$

$$A_{mk} = \prod_{i=1}^L A_{mik}, \quad (2.31)$$

$$\theta_{mk} = \sum_{i=1}^L \theta_{mik}, \quad (2.32)$$

$$\epsilon_m = \frac{v_m}{L}. \quad (2.33)$$

Beaulieu [3] has shown that the series in (2.27) and (2.28) converge. The expected values, $E[\cos(k\omega z_{mi})]$ and $E[\sin(k\omega z_{mi})]$, are calculated by

$$E[\cos(k\omega z_{mi})] = \int_0^R \cos(k\omega z_{mi}) f_{z_{mi}}(z_{mi}) dz_{mi} \quad (2.34)$$

and

$$E[\sin(k\omega z_{mi})] = \int_0^R \sin(k\omega z_{mi}) f_{z_{mi}}(z_{mi}) dz_{mi} \quad (2.35)$$

using numerical integration where $f_{z_{mi}}(z_{mi})$ is given by (2.15). The coefficients, A_{mk} and θ_{mk} , can be found from (2.29)-(2.35). The choice of $V = 2LR$ simplifies the computations since the numerical integrations of (2.34) and (2.35) need only be evaluated once for each value of k for any given pdf. The pdf and the cdf of v_m can then be found using (2.27) and (2.28). The series in (2.27) and (2.28) are truncated after K terms. Note that $(K-1)/2$ terms are zero because the summation is only over odd terms. Thus, to calculate the pdf and cdf of v_m , only $K+1$ single integrations are needed. In order to guarantee a sufficiently small truncation error, calculations of the pdf and cdf are repeated with increasing numbers of terms in the series of (2.27) and (2.28) until the final results do not change by more than the required accuracy.

2.4 Comparison with Other Results

In this section, a comparison is made between the numerical results generated using the above method and previously published results.

2.4.1 Sum of Five Rayleigh Random Variables

Consider the problem of finding the cdf of the sum of five Rayleigh random variables, z_{mi} , for $i = 1, \dots, 5$, where each random variable has a pdf of

$$f_{z_{mi}}(z_{mi}) = z_{mi} \exp\left(-\frac{z_{mi}^2}{2}\right), z_{mi} \geq 0. \quad (2.36)$$

Beaulieu [3] has shown that for the Rayleigh random variable of (2.36), the integrals of (2.34) and (2.35) are

$$E[\cos(k\omega z_{mi})] = {}_1F_1\left(1, \frac{1}{2}, -\frac{k^2\omega^2}{2}\right), \quad (2.37)$$

$$E[\sin(k\omega z_{mi})] = \sqrt{\frac{\pi}{2}} k\omega \exp\left(-\frac{k^2\omega^2}{2}\right) \quad (2.38)$$

where ${}_1F_1(\cdot, \cdot)$ is the confluent hypergeometric function. The cdf of the sum can then be found from (2.27). Beaulieu [3] shows an efficient way of evaluating (2.37).

The series of (2.27) must be truncated after a certain number of terms. The cdf was computed with various numbers of terms, and the results are shown in Table 2.1 along with the previously published results of Helstrom [4]. Note that there is agreement to 6 decimal places between the results with 30 terms and 40 terms; therefore, 30 terms are sufficient for an accuracy of 6 decimal places. In addition, the results of [4] and those calculated in this chapter agree.

v_m	20 terms	30 terms	40 terms	Helstrom [4]
1.0	-0.000046864425	0.000000087051	0.000000242536	0.000000246072
3.0	0.006013209643	0.006040403871	0.006040461398	0.00604046
5.0	0.198147788091	0.198135172457	0.198135073842	0.198135
6.0	0.446517392095	0.446545531179	0.446545628911	0.446546
7.0	0.702846895639	0.702811444082	0.702811389564	0.702811
9.0	0.961571441004	0.961548568638	0.961548621161	0.9615486
11.0	0.998229594502	0.998237077575	0.998237134497	0.99823713
13.0	0.999946637776	0.999969488835	0.999969472545	0.9999694733

Table 2.1: The CDF of the Sum of 5 Rayleigh Random Variables

2.4.2 Sum of Sixteen Rician Random Variables

Consider the problem of finding the cdf of the sum, v_m , of sixteen Rician random variables, z_{mi} , for $i = 1, \dots, 16$, where each random variable has the pdf,

$$f_{z_{mi}}(z_{mi}) = z_{mi} \exp\left(\frac{-z_{mi}^2 + s_{mi}}{2}\right) I_0(z_{mi}s_{mi}), z_{mi} \geq 0. \quad (2.39)$$

The integrals of (2.34) and (2.35) were solved numerically using the adaptive-Romberg algorithm [23]. The integration was stopped when the error was less than a fraction, $\delta = 10^{-6}$, of the value of the integral. These integrals can also be solved analytically as

$$E[\cos(k\omega z_{mi})] = \exp\left(\frac{-s_{mi}^2}{2}\right) \sum_{j=0}^{\infty} \left(\frac{s_{mi}^2}{2}\right)^j \left(\frac{1}{j!}\right) {}_1F_1(j+1, \frac{1}{2}, -\frac{k^2\omega^2}{2}) \quad (2.40)$$

and

$$E[\sin(k\omega z_{mi})] = \exp\left(\frac{-s_{mi}^2}{2}\right) \sqrt{2}k\omega \exp\left(\frac{-k^2\omega^2}{2}\right) \times$$

$$\sum_{j=0}^{\infty} \left(\frac{s_{mi}^2}{2}\right)^j \frac{\Gamma(j+1.5)}{(j!)^2} {}_1F_1\left(-j, \frac{3}{2}, \frac{k^2\omega^2}{2}\right) \quad (2.41)$$

where ${}_1F_1(.,.)$ is the confluent hypergeometric function, and $\Gamma()$ is the gamma function. Both (2.40) and (2.41) agree with the corresponding expressions in [24]. The numerical results generated from (2.34) and (2.35) using (2.39) and analytical results generated from (2.40) and (2.41) agree with each other.

The cdf, $F_{v_m}(v_m)$, was computed for $v_m = 32.28729$ and several values of s_{mi} with various numbers of terms, and the results are shown in Table 2.2 along with the previously published results of Helstrom [4]. Note that there is agreement to 8 decimal places between the results with 50 terms and 70 terms; therefore, 50 terms are sufficient for an accuracy of 8 decimal places. In addition, the results of [4] and those calculated in this chapter agree.

s_{mi}	30 terms	50 terms	70 terms	Helstrom [4]
1.2604192	0.932001282965	0.932011262474	0.932011262683	0.9320114
1.5867737	0.644568109834	0.644600482984	0.644600482906	0.644601
1.9976298	0.134545925833	0.134562469337	0.134562469396	0.134563
2.5148669	0.001217999774	0.001246864269	0.001246864363	0.00124687

Table 2.2: The CDF of the Sum of 16 Rician Random Variables

2.5 Summary

A novel method for obtaining the pdf and the cdf of a sum of independent random variables has been presented. The method is independent of the underlying pdf. An efficient way of evaluating the pdf and the cdf of the sum has been provided. The cdf computed using this method has been compared to the cdf computed using a previously known method, and the results agree well. This novel analytical technique can be used to obtain performance results for systems employing linear combining. The application is demonstrated in the next two chapters.

Chapter 3

Demodulation of FFH NCFSK Signals

3.1 Introduction

In this chapter, a background in the modulation and demodulation of fast-frequency-hopped (FFH) noncoherent frequency-shift-keyed (NCFSK) communications systems with time diversity and diversity combining is presented. Several NCFSK systems are described, and their performance under jamming is presented. A need for anti-jam protection is shown, and hence, the need for fast-frequency hopping, time diversity and diversity combining are justified. The jamming assumptions that are made are as follows: 1) jammers are power limited; 2) jammers have complete knowledge of the communication system except for the hopping pattern; 3) jammers are able to use Gaussian-noise or multiple-tone jamming; 4) jammers are able to jam part or all of the communication bandwidth. Diversity-combining methods are discussed, and a complete literature review is presented. Performance results are presented for linear diversity combining in broad-band noise, multiple-tone jamming plus broad-band noise, and independent Rayleigh fading plus broad-band noise using techniques of Chapter 2.

3.2 NCFSK System

A noncoherent frequency-shift-keyed (NCFSK) system is a basic communication system. A NCFSK transmitter is shown in Fig. 3.1. Binary data is converted to M -ary symbols with period, T_s . The M -ary symbols are applied to a frequency synthesizer where the output signal is a tone at frequency, f_m , having one of M possible frequencies. The transmitted waveform consists of a series of tones at frequency, f_m , of duration, T_s . As shown in Fig. 3.2, the received signal is passed through a bank of M matched filters with envelope detectors. Each filter is matched to one of the M tones, f_m , and uses noncoherent detection. A matched filter with envelope detector is shown in Fig. 3.3. A decision is then made as to which symbol was sent based on the M output samples from the matched filter with envelope detector. The M -ary symbols are then converted to binary.

Consider a binary NCFSK system operating in broad-band noise with two-sided power-spectral density, $N_0/2$. The probability of bit error for such a system is [25]

$$P_b = \frac{1}{2} \exp\left(\frac{-E_b}{2N_0}\right) \quad (3.1)$$

where E_b is the energy per bit. This is the baseline performance and is plotted as a function of signal-to-noise ratio in Fig. 3.4 where signal-to-noise ratio is given by

$$SNR = \frac{E_b}{N_0}. \quad (3.2)$$

A jammer trying to prevent communications can easily jam a NCFSK system. The jammer can flood the relatively small bandwidth, $2/T_s$, with noise of two-sided power-spectral density, $N_J/2$. This would reduce the bit-error probability to

$$P_b = \frac{1}{2} \exp\left(\frac{-E_b}{2(N_0 + N_J)}\right). \quad (3.3)$$

Under usual jamming conditions, $N_J \gg N_0$. If E_b is fixed, then, N_J would determine the bit-error-probability performance according to (3.3). If P_b is too high due to a large N_J , the communication will be unreliable. For example, a signal-to-jammer ratio ($SJR = E_b/N_J$) of 5 dB would cause a bit-error probability of $P_b > 0.1$. Some form of anti-jam protection is required.

Figure 3.1: A NCFSK system transmitter.

Figure 3.2: A NCFSK system receiver.

Figure 3.3: A matched filter with envelope detector.

Figure 3.4: P_b for an NCFSK system with $M = 2$ in broad-band noise.

3.3 FH NCFSK System

Frequency hopping (FH) can offer a degree of anti-jam protection. A FH NCFSK transmitter is shown in Fig. 3.5. Binary data is converted to M -ary symbols with period, T_s . The M -ary symbols are applied to a frequency synthesizer where the output signal is a tone at frequency, f_m , having one of M possible frequencies. For the i th symbol period, the symbols are mixed with a frequency tone, f_{hi} , generated by a frequency synthesizer driven by a pseudonoise-code generator. The transmitted waveform consists of a series of tones at frequency, $f_{hi} + f_m$, of duration, T_s . As shown in Fig. 3.6, the received signal is mixed with the output of a local frequency synthesizer driven by the same pseudorandom code used at the transmitter. The output of the mixer is then a tone, f_m , of duration, T_h . This tone is passed through a bank of M matched filters with envelope detectors. The outputs are the samples, v_m , for $m = 1, \dots, M$. A decision is then made as to which symbol was sent based on these M samples. The M -ary symbols are then converted to binary.

The improvement in performance due to frequency hopping is caused by forcing the jammer to spread its power over a larger bandwidth. The communicator hops over a bandwidth, W_{ss} , called the "spread-spectrum bandwidth" which is much larger than the NCFSK bandwidth, M/T_s . If the jammer chooses to jam the entire spread-spectrum bandwidth with Gaussian noise, the jamming level must be lower by a factor equal to the ratio of the spread-spectrum bandwidth to the NCFSK bandwidth. This means that the signal-to-jammer ratio, E_b/N_J , will increase. Thus, the anti-jam protection can be achieved by increasing the spread-spectrum bandwidth until E_b/N_J is large enough to achieve the required bit-error probability.

The jammer at this point would realize that broad-band jamming is ineffective and would search for another strategy. Partial-band-noise or multiple-tone jamming turn out to be effective strategies. For partial-band-noise jamming, a part, W_J , of the spread-spectrum bandwidth is jammed with Gaussian noise. The fraction of the spread-spectrum bandwidth which is jammed is $\rho = W_J/W_{ss}$. The jammer can optimize ρ to give the maximum bit-error probability. The worst-case probability of bit error for a binary FH NCFSK system in partial-band noise jamming is

Figure 3.5: A FH NCFSK system transmitter.

Figure 3.6: A FH NCFSK system receiver.

approximately [9]

$$P_b = \begin{cases} \frac{\exp(-1)}{E_b/N_J}, & E_b/N_J > 2, \\ \frac{1}{2} \exp(\frac{-E_b}{2N_J}), & E_b/N_J \leq 2. \end{cases} \quad (3.4)$$

For multiple-tone jamming, tones are placed in the data-tone slots (bins) throughout the spread-spectrum bandwidth. For random-multiple-tone jamming, the jamming tones are located randomly in a fraction, γ , of the total number of bins. For Houston-multiple-tone jamming [26], a fraction of the total number of binary channels are jammed by a single jammer tone. There is never more than one tone per channel. The fraction of bins jammed is still γ . Generally, the worst-case jamming involves using tones of the same power. Again, the jammer can optimize γ to give the maximum bit-error probability. The worst-case multiple-tone jamming for a FH NCFSK system is Houston-multiple-tone jamming. Example tone distributions for both types of multiple-tone jamming are shown in Fig. 3.7. The worst-case probability of bit error for a binary FH NCFSK system in multiple-tone jamming is approximately [9]

$$P_b = \begin{cases} \frac{1}{E_b/N_J}, & E_b/N_J > 2, \\ \frac{1}{2}, & E_b/N_J \leq 2 \end{cases} \quad (3.5)$$

where N_J is the equivalent jammer power-spectral density. The performance of a FH binary NCFSK system in broad-band noise, worst-case partial-band noise and multiple-tone jamming is shown in Fig. 3.8 as a function of signal-to-jammer ratio, $SJR = E_b/N_J$. The performance in worst-case multiple-tone jamming is 4.3 dB worse than the performance in worst-case partial-band-noise jamming. Both worst-case partial-band-noise jamming and worst-case multiple-tone jamming cause considerably worse performance than broad-band noise at high signal-to-noise ratios. At low signal-to-noise ratios, the performances are similar. Clearly, a counter measure is required for partial-band jamming.

3.4 FH NCFSK System with FEC Coding

Forward-error-correction (FEC) coding can be used to suppress the detrimental effects of partial-band jamming. Thus, FEC coding can be used for anti-jam protection. A FH NCFSK system transmitter with forward-error-correction coding is

Figure 3.7: Two types of multiple-tone jamming.

Figure 3.8: P_b for a FH NCFSK system with $M = 2$.

shown in Fig. 3.9. Binary data is passed through an error-correction encoder. Both block and convolutional coding can be used. The coded bits are then converted to M -ary symbols with period, T_s . The M -ary symbols are applied to a frequency synthesizer where the output signal is a tone at frequency, f_m , having one of M possible frequencies. For the i th hop period, the symbols are mixed with a frequency tone, f_{hi} , generated by a frequency synthesizer driven by a pseudonoise-code generator. The tone, f_{hi} , has a hop duration, T_s . The transmitted waveform consists of a series of tones at frequency, $f_{hi} + f_m$, of duration, T_s . As shown in Fig. 3.10, the received signal is mixed with the output of a local frequency synthesizer driven by the same pseudorandom code used at the transmitter. The output of the mixer is then a tone, f_m , of duration, T_s . This tone is passed through a bank of M matched filters with envelope detectors. The outputs are the samples, v_m , for $m = 1, \dots, M$. A decision is then made as to which symbol was sent based on these M statistics. The M -ary symbols are then converted to binary and passed to the error-correction decoder. Finally, the binary data is received from the decoder.

Both convolutional and block codes are used in FH NCFSK systems. If a Golay (23,12) code with hard decisions is used, the bit-error probability for a FH binary NCFSK system is approximately [27]

$$P_b = \sum_{n=4}^{23} \frac{n}{23} \binom{23}{n} p^n (1-p)^{23-n} \quad (3.6)$$

where p is the uncoded-channel-symbol error rate. The rate, p , is obtained from (3.3), (3.4), and (3.5) with E_b replaced by rE_b where r is the code rate. Here, $r = 12/23$. The results for broad-band noise jamming, partial-band noise jamming, and multiple-tone jamming are shown in Fig. 3.11. There is an improvement in performance for high SJR , but for low SJR , the performance is worse. Thus, error-correction codes are ineffective at severe jamming levels [27],[28]. Repetition codes are used when a system must be robust at severe jamming levels. In FH systems, repetition coding is called "time diversity", and the decoding algorithm is called "diversity combining".

Figure 3.9: A FH NCFSK system transmitter with FEC coding.

Figure 3.10: A FH NCFSK system receiver with FEC coding.

Figure 3.11: P_b for a FH NCFSK system with Golay(23,12) decoding and $M = 2$.

3.5 FFH NCFSK System with Time Diversity and Diversity Combining

A FFH NCFSK transmitter with time diversity and diversity combining is shown in Fig. 3.12. Binary data is passed through an error-correction encoder. The coded bits are then converted to M -ary symbols with period, T_s . The M -ary symbols are applied to a frequency synthesizer where the output signal is a tone at frequency, f_m , having one of M possible frequencies. For the i th hop period, the symbols are mixed with a frequency tone, f_{hi} , generated by a frequency synthesizer driven by a pseudonoise-code generator. The tone, f_{hi} , has a hop duration, $T_h = T_s/L$. The hop rate for the system is fixed. The integer, L , is the number of hops per symbol and is termed the diversity level. If $L \geq 1$, the system is called fast frequency hopped; in this thesis, only fast-frequency-hopped systems are considered. The transmitted waveform consists of a series of tones at frequency, $f_{hi} + f_m$, of duration, T_h .

As shown in Fig. 3.13, the received signal is mixed with the output of a local frequency synthesizer driven by the same pseudorandom code used at the transmitter. The output of the mixer is then a tone, f_m , of duration, T_h . This tone is passed through a bank of M matched filters with envelope detectors which is shown in Fig. 3.14. Each filter is matched to one of the M tones, f_m , and uses noncoherent detection. The outputs are the samples, z_{mi} , for $m = 1, \dots, M$, and $i = 1, \dots, L$. The subscript, m , denotes the frequency bin, and the subscript, i , denotes the hop period. The diversity combiner forms the decision statistics, v_m , based upon the $L \times M$ samples, z_{mi} , using one of many diversity-combining techniques. A decision is then made as to which symbol was sent based on these M statistics. The M -ary symbols are then converted to binary and passed to the error-correction decoder. Finally, the binary data is received from the decoder. In the following sections, error-correction coding is not considered. Only time diversity and diversity combining are employed.

Figure 3.12: A FFH NCFSK system transmitter with time diversity.

Figure 3.13: A FFH NCFSK system receiver with diversity combining.

Figure 3.14: A bank of matched filters with envelope detectors.

3.6 Optimum Diversity Combiner

The optimum receiver for a FFH NCFSK system can be found using Karhunen-Loève expansions and Bayes' criterion [29]. For additive white Gaussian noise (AWGN), this results in a receiver consisting of a bank of matched filters with envelope detectors followed by a diversity combiner. The optimum method of diversity combining involves calculating the *a posteriori* probability for each hypothesis [29]. Therefore, the decision statistic is

$$\Lambda_m = \text{Probability}(\text{tone } m \text{ sent} \mid z_{mi}, i = 1, \dots, L, m = 1, \dots, M). \quad (3.7)$$

The largest Λ_m indicates the most probable tone that was sent.

For M -ary communication in AWGN, the optimum statistic can be derived from (3.7) and is given by

$$v_m = \sum_{i=1}^L \ln(I_0(\frac{z_{mi}s_i}{\sigma^2})) \quad (3.8)$$

where s_i is the rms signal-tone amplitude on the i th hop period, $\sigma^2 = N_0/2T_h$, $N_0/2$ is the two-sided noise power-spectral density, and $I_0()$ is the zeroth-order modified-Bessel function of the first kind. The largest v_m indicates the transmitted tone. This result was first derived for the binary case in [30]. The combining method of

(3.8) is often called “optimum combining” [30],[31], but it requires knowledge of s_i and σ . Since these are not known *a priori*, they must somehow be estimated, and therefore, it is not practical to use this method in an actual system. It is desirable to have a decision statistic which does not require knowledge of s_i or σ . Below, many other diversity-combining methods are presented.

3.7 Diversity-Combining Methods

3.7.1 Selection Diversity Combining

This combining technique, sometimes called switched diversity combining, picks the largest z_{mi} over the L hop periods as the decision statistic. Thus, [32]

$$v_m = \max_i(z_{mi}). \quad (3.9)$$

The largest v_m indicates the transmitted tone.

3.7.2 Linear Diversity Combining

If $z_{mi}s_i/\sigma^2 \gg 1$, the statistic of (3.8) can be approximated using $I_0(x) \approx \exp(x)/\sqrt{2\pi x}$ [33]. Taking the logarithm of both sides leads to $\ln(I_0(x)) \approx x - (0.5 \ln(2\pi x))$. For $x \gg 1$, $x \gg 0.5 \ln(2\pi x)$. Now, the statistic is [30],[31],[34]-[37]

$$\sum_{i=1}^L \ln(I_0(\frac{z_{mi}s_i}{\sigma^2})) \approx \sum_{i=1}^L \frac{z_{mi}s_i}{\sigma^2}. \quad (3.10)$$

Assuming that s_i and σ do not change from hop to hop, a decision statistic is

$$v_m = \sum_{i=1}^L z_{mi}. \quad (3.11)$$

The largest v_m indicates the transmitted tone. This combining method is called “linear combining” and should also have near-optimal performance in AWGN.

3.7.3 Square-Law Diversity Combining

If $z_{mi}s_i/\sigma^2 \ll 1$, the statistic of (3.8) can be approximated using $I_0(x) \approx 1 + (x^2/4)$ and $\ln(1+x) \approx x$ [33]. This results in [30],[31],[32],[38]-[43]

$$\sum_{i=1}^L \ln(I_0(\frac{z_{mi}s_i}{\sigma^2})) \approx \sum_{i=1}^L \frac{z_{mi}^2 s_i^2}{4\sigma^4}. \quad (3.12)$$

Assuming that s_i and σ do not change from hop to hop, the decision statistic can be chosen as

$$v_m = \sum_{i=1}^L z_{mi}^2. \quad (3.13)$$

The largest v_m indicates the transmitted tone. This combining method is called “square-law combining” and should have near-optimal performance in AWGN.

3.7.4 Hard-Decision-Majority-Vote Diversity Combining

Hard-decision-majority-vote (HDMV) diversity combining performs a hard decision on each hop. This means that [7],[37],[44]

$$f_{hdmv}(z_{mi}) = \begin{cases} 1, & z_{mi} \geq z_{ki} \text{ for all } k \\ 0, & \text{otherwise} \end{cases}. \quad (3.14)$$

Then, the decision statistic is

$$v_m = \sum_{i=1}^L f_{hdmv}(z_{mi}). \quad (3.15)$$

The largest v_m indicates the transmitted tone.

3.7.5 Ratio-Statistic-Envelope Diversity Combining

Ratio-statistic-envelope diversity combining normalizes samples on each hop by the largest of the samples on that hop and then sums the normalized samples. Thus, the decision statistic is [31],[41],[45],[46]

$$v_m = \sum_{i=1}^L \frac{z_{mi}}{\max_m(z_{mi})}. \quad (3.16)$$

The largest v_m indicates the transmitted tone.

3.7.6 Ratio-Statistic-Square-Law Diversity Combining

Ratio-statistic-square-law diversity combining normalizes the squares of the samples on each hop by the square of the largest sample on that hop and then sums the normalized samples. Thus, the decision statistic is [41],[46]

$$v_m = \sum_{i=1}^L \frac{z_{mi}^2}{\max_m(z_{mi}^2)}. \quad (3.17)$$

The largest v_m indicates the transmitted tone.

3.7.7 Self-Normalized-Envelope Diversity Combining

Self-normalized-envelope (SNE) diversity combining, which is sometimes called normalized-envelope detection (NED), normalizes samples on each hop by the sum of the samples on that hop and then sums the normalized samples. Thus, the decision statistic is [8],[28],[37],[41],[43],[44],[47],[48]

$$v_m = \sum_{i=1}^L \frac{z_{mi}}{\sum_{m=1}^M z_{mi}}. \quad (3.18)$$

The largest v_m indicates the transmitted tone.

3.7.8 Self-Normalized-Square-Law Diversity Combining

Self-normalized-square-law diversity combining normalizes the squares of the samples on each hop by the sum of the squares of the samples on that hop and then sums the normalized samples. Thus, the decision statistic is [41],[49]

$$v_m = \sum_{i=1}^L \frac{z_{mi}^2}{\sum_{m=1}^M z_{mi}^2}. \quad (3.19)$$

The largest v_m indicates the transmitted tone.

3.7.9 Clipped-Linear Diversity Combining

Clipped-linear diversity combining clips the samples at a prescribed level above the desired-signal level and then sums the clipped samples. This means that [35]

$$f_{cl}(z_{mi}) = \begin{cases} C, & \text{for } z_{mi} \geq C \\ z_{mi}, & \text{otherwise} \end{cases}. \quad (3.20)$$

Then,

$$v_m = \sum_{i=1}^L f_{cl}(z_{mi}). \quad (3.21)$$

The largest v_m indicates the transmitted tone. It requires side information to set the threshold, C .

3.7.10 Clipped-Square-Law Diversity Combining

Clipped-square-law diversity combining clips the squares of the samples at a prescribed level above the desired-signal level and then sums the clipped samples. This means that [41],[42]

$$f_{cal}(z_{mi}) = \begin{cases} C, & \text{for } z_{mi}^2 \geq C \\ z_{mi}^2, & \text{otherwise} \end{cases}. \quad (3.22)$$

Then,

$$v_m = \sum_{i=1}^L f_{cal}(z_{mi}). \quad (3.23)$$

The largest v_m indicates the transmitted tone. It requires side information to set the threshold, C .

3.7.11 Ratio-Threshold-Square-Law Diversity Combining

The ratio-threshold technique orders the input data from smallest to largest. It then takes the ratio of the two largest samples on each hop and assigns on each hop a quality factor, q_i , of 1 (good) if the ratio is greater than a threshold and assigns a factor of 0 (bad) if the ratio is less than the threshold. The threshold is chosen to be greater than 1. The decision statistic is formed by [50]

$$v_m = \sum_{i=1}^L q_i z_{mi}^2. \quad (3.24)$$

The largest v_m indicates the transmitted tone. If all $q_i = 0$ for $i = 1, \dots, L$, then a decision is based on the z_{mi} with the largest sample ratio.

3.7.12 Ratio-Threshold-Majority-Logic Diversity Combining

The ratio-threshold-majority-logic method assigns the same quality factor to each hop period as the ratio-threshold-square-law method. Then, a hard decision is performed on each hop. This means that [50],[51]

$$f_{rml}(z_{mi}) = \begin{cases} 1, & z_{mi} \geq z_{ki} \text{ for all } k \\ 0, & \text{otherwise} \end{cases}. \quad (3.25)$$

The decision statistic is formed by

$$v_m = \sum_{i=1}^L q_i f_{r_{iml}}(z_{mi}). \quad (3.26)$$

The largest v_m indicates the transmitted tone. If all $q_i = 0$ for $i = 1, \dots, L$, then a decision is based on the z_{mi} with the largest sample ratio.

3.7.13 4-2-Moment-Method Diversity Combining

The 4-2-moment-method diversity combining forms a decision statistic, [8],[37]

$$v_m = 2\left(\frac{1}{L} \sum_{i=1}^L z_{mi}^2\right)^2 - \left(\frac{1}{L} \sum_{i=1}^L z_{mi}^4\right). \quad (3.27)$$

The largest v_m indicates the transmitted tone. This method attempts to subtract the interference power from the samples.

3.7.14 2-1-Moment-Method Diversity Combining

In the 2-1-moment-method diversity combining, a decision statistic is formed [8],[28],[37]

$$v_m = 2\left(\frac{1}{L} \sum_{i=1}^L z_{mi}\right)^2 - \left(\frac{1}{L} \sum_{i=1}^L z_{mi}^2\right). \quad (3.28)$$

The largest v_m indicates the transmitted tone. This method also attempts to subtract the interference power from the samples.

3.7.15 1-1/2-Moment-Method Diversity Combining

The 1-1/2-moment-method diversity combining forms a decision statistic, [37]

$$v_m = 2\left(\frac{1}{L} \sum_{i=1}^L \sqrt{z_{mi}}\right)^2 - \left(\frac{1}{L} \sum_{i=1}^L z_{mi}\right). \quad (3.29)$$

The largest v_m indicates the transmitted tone. This method also attempts to subtract the interference power from the samples.

3.7.16 Order-Statistic Diversity Combining

The use of order statistics (OS) for diversity combining requires the sorting of the L values in each bin so that

$$z_{mi_1} \leq z_{mi_2} \leq \cdots \leq z_{mi_{L-1}} \leq z_{mi_L}, \quad m = 1, 2, \dots, M \quad (3.30)$$

where z_{mi_1} is the smallest OS of the m th bin, and z_{mi_L} is the largest OS of the m th bin. For a previously specified value of l , the decision statistic is chosen as [37],[52]-[54]

$$v_m = z_{mi_l}. \quad (3.31)$$

The largest v_m indicates the transmitted tone.

3.7.17 Weighted-Order-Statistic Diversity Combining

Order-statistic processing is done first by (3.30) in order to rank z_{mi} . Then, the ranked values are summed by [36],[54]

$$v_m = \sum_{l=1}^L w_{mi_l} z_{mi_l} \quad (3.32)$$

where w_{mi_l} are weights. The largest v_m indicates the transmitted tone.

3.7.18 OSSNE Diversity Combining

The order-statistic and self-normalized-envelope methods can be combined to form OSSNE combining. Order-statistic processing is done first by (3.30) in order to rank the z_{mi} . Then, the ranked values are normalized to

$$\widehat{z_{mi_l}} = \frac{z_{mi_l}}{\sum_{m=1}^M z_{mi_l}}. \quad (3.33)$$

The decision statistic is formed by [53],[54]

$$v_m = \sum_{l=1}^L \widehat{z_{mi_l}}. \quad (3.34)$$

The largest v_m indicates the transmitted tone.

3.7.19 OSHDMV Diversity Combining

The order-statistic and hard-decision-majority-vote methods can be combined to form OSHDMV combining. Order-statistic processing is done first by (3.30) in order to rank z_{mi} . Then, on each hop period, a hard decision is made such that

$$f_{oshdmv}(z_{mi}) = \begin{cases} 1, & z_{mi} \geq z_{ki} \text{ for all } k \\ 0, & \text{otherwise} \end{cases}. \quad (3.35)$$

Then, the decision statistic is

$$v_m = \sum_{i=1}^L f_{oshdmv}(z_{mi}). \quad (3.36)$$

The largest v_m indicates the transmitted tone.

3.7.20 WOSSNE Diversity Combining

In WOSSNE combining, a weighted sum of the modified values in the OSSNE method is formed. The decision statistic is [54]

$$v_m = \sum_{i=1}^L w_{mi} \widehat{z_{mi}} \quad (3.37)$$

where w_{mi} are the weights. The largest v_m is chosen as the transmitted symbol.

3.7.21 Recursive-Excision Diversity Combining

The average of the L values in each bin is computed as [54]

$$v_m = \frac{1}{L} \sum_{i=1}^L z_{mi}. \quad (3.38)$$

The ratios,

$$q_{mi} = z_{mi}/v_m, \quad (3.39)$$

are calculated on each iteration. If any of the q_{mi} exceed the threshold, C , the corresponding z_{mi} is deleted from the L -bin values, and L is decremented for that bin only. The calculations (3.38) and (3.39) are iterated until $q_{mi} \leq C$ for all remaining bin values. The decision statistics are then the final averages from (3.38). The largest v_m is chosen as the transmitted symbol.

3.8 Literature Review

This section provides a literature review of previous work in obtaining performance measures for the demodulation of FFH NCFSK systems with time diversity and diversity combining. Previous work, which is summarized in Table 3.1 and Table 3.2, presents results from theoretical analysis, computer simulation, or experimental analysis of various systems in different interference and fading environments. Prior to reviewing the literature, some terminology pertaining to the types of interference and fading must be introduced. For broad-band-noise interference, the system is subjected to AWGN across its entire bandwidth. For partial-band-noise interference, the system is subjected to AWGN across part of its bandwidth. For tone-jamming interference, the system is subjected to multiple-tone interference. For independent-Rayleigh fading, the fluctuating signal amplitude has a Rayleigh probability distribution. For independent-Rician fading, the fluctuating signal amplitude has a Rician probability distribution. For correlated fading, fluctuations in signal amplitude are correlated across hop periods.

Optimum diversity combining is discussed in [30] and [31]. In [30], the probability of error for a FFH binary NCFSK system in broad-band noise is presented. Results are generated by computer simulation for levels of diversity ranging from $L = 1$ to $L = 10$. In [31], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: partial-band noise with broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 4$.

Selection diversity combining is discussed in [32]. The probability of error for a binary NCFSK system in broad-band noise with independent Rayleigh fading is derived. Exact analysis is used for an arbitrary level of diversity. Results are shown for $L = 2, \dots, 10$.

Linear diversity combining is discussed in [30],[31], and [34]-[37]. In [30], the probability of error for a FFH binary NCFSK system in broad-band noise is presented. Results are generated by computer simulation for $L = 1, \dots, 10$. In [34], the probability of error for a FFH M -ary NCFSK system is presented for three cases: broad-band noise, partial-band noise, and tone jamming. Each case is treated separately. The analysis makes simplifying assumptions and uses bounds. Side informa-

Combining Method	Ref.	Analysis	Interference
optimum	[30]	simulation	broad-band noise
	[31]	simulation	tone jamming, partial-band noise
selection	[32]	exact	Rayleigh fading, broad-band noise
linear	[30]	simulation	broad-band noise
	[34]	bounds	partial-band noise, tone jamming
	[35]	exact	partial-band noise
	[36]	simulation	partial-band noise
	[31]	simulation	tone jamming, Rayleigh fading, partial-band noise
	[37]	simulation	broad-band noise
square-law	[32]	exact	Rayleigh fading, broad-band noise
	[38]	exact	Rayleigh fading, broad-band noise
	[39]	exact	Rician fading, broad-band noise
	[40]	exact	Rician fading, broad-band noise
	[30]	simulation	broad-band noise
	[41]	assumptions	partial-band noise
	[42]	bounds	partial-band noise
	[31]	simulation	tone jamming, Rayleigh fading, broad-band noise
	[43]	exact	Rician fading, partial-band noise
hard-decision-majority-vote	[7]	exact	partial-band noise, tone jamming
	[44]	experiment	partial-band noise, tone jamming
	[37]	simulation	broad-band noise, tone jamming
ratio-statistic-envelope	[41]	simulation	partial-band noise
	[45]	exact	partial-band noise
	[31]	simulation	partial-band noise, tone jamming, Rayleigh fading
	[46]	exact	Rician fading, partial-band noise
ratio-statistic-square-law	[41]	simulation	partial-band noise
	[46]	exact	Rician fading, partial-band noise

Table 3.1: Literature Review of Diversity-Combining Techniques

Combining Method	Ref.	Analysis	Interference
self-normalized-envelope	[41]	simulation	partial-band noise, tone jamming
	[47]	assumptions	tone jamming
	[44]	experiment	partial-band noise, tone jamming
	[37]	simulation	broad-band noise, tone jamming
	[8]	simulation	broad-band noise, tone jamming
	[28]	simulation	broad-band noise, tone jamming
	[48]	exact	Rician fading, partial-band noise
self-normalized-square-law	[41]	simulation	partial-band noise, tone jamming
	[49]	exact	partial-band noise
clipped-linear	[35]	exact	partial-band noise, Rician fading
clipped-square-law	[41]	simulation	partial-band noise
	[42]	exact	partial-band noise
ratio-threshold-sq.-law	[50]	exact	Rician fading, partial-band noise
ratio-threshold-majority-logic	[50]	exact	Rician fading, partial-band noise
	[51]	exact	tone jamming
4-2-moment-method	[37]	simulation	partial-band noise, tone jamming
2-1-moment-method	[37]	simulation	partial-band noise, tone jamming
	[8]	simulation	partial-band noise, tone jamming
	[28]	simulation	partial-band noise, tone jamming
1-1/2-moment-method	[37]	simulation	partial-band noise, tone jamming
order-statistic	[52]	exact	partial-band noise
	[37]	simulation	broad-band noise, tone jamming
	[53]	simulation	broad-band noise, tone jamming
	[54]	simulation	broad-band noise, tone jamming
weighted-order-statistic	[36]	simulation	partial-band noise
	[54]	simulation	broad-band noise, tone jamming
OSSNE	[53]	simulation	broad-band noise, tone jamming
	[54]	simulation	broad-band noise, tone jamming
WOSSNE	[54]	simulation	broad-band noise, tone jamming
recursive-excision	[54]	simulation	broad-band noise, tone jamming

Table 3.2: Literature Review of Diversity-Combining Techniques continued

tion concerning the hops that are jammed is assumed to be available. Results are presented for $M = 16$ and the optimum level of diversity. In [35], the probability of bit error is calculated for a FFH M -ary NCFSK system in partial-band noise jamming using an analytical method based on repeated convolutions which is only practical for small levels of diversity. Results are shown for $L = 3$. In [36], the probability of error for a FFH M -ary NCFSK system is presented for partial-band noise with broad-band noise. Results are generated by computer simulation for $L = 1, 3$ and $M = 4, 8, 16$. In [31], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: tone jamming with broad-band noise, and broad-band noise with independent Rayleigh fading. Results are generated by computer simulation for tone jamming with $L = 2$ and for Rayleigh fading with $L = 8$. In [37], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

Square-law diversity combining is discussed in [30],[31],[32], and [38]-[43]. In [32], the probability of error for a binary NCFSK system is presented for three cases: independent Rayleigh fading with broad-band noise, correlated Rayleigh fading with broad-band noise and independent Rayleigh fading with correlated noise. Exact analysis is used. Results are presented for the above three cases with $L = 1, \dots, 20$, $L = 2$, and $L = 2, \dots, 10$, respectively. In [38], the probability of error for a M -ary NCFSK system is presented for independent Rayleigh fading with broad-band noise. Exact analysis is used. Results are presented for $M = 2$ with $L = 1, \dots, 20$, for $M = 4$ with $L = 1, \dots, 10$, and for $M = 8$ with $L = 1, \dots, 5$. In [39], the probability of error for a M -ary NCFSK system is presented for independent Rayleigh and Rician fading with broad-band noise. Exact analysis is used for arbitrary M and L . Results are presented for $M = 2$ with $L = 1, \dots, 19$. In [40], the probability of error for a binary NCFSK system is presented for correlated Rayleigh and Rician fading with broad-band noise. Exact analysis is used. Results are presented for $M = 2$ with $L = 1, \dots, 15$. In [30], the probability of error for a FFH binary NCFSK system in broad-band noise is presented. Results are generated by computer simulation for $L = 1, \dots, 10$. In [41], the probability of error for a FFH M -ary NCFSK system is presented for two cases: broad-band noise, and partial-band noise. For broad-band

noise, exact analysis is used for arbitrary M and L . Results are presented for $M = 8$ with $L = 1, \dots, 64$. For partial-band noise, the analysis assumes no broad-band noise. Results are presented for $M = 8$ with $L = 1, \dots, 8$. In [42], the probability of error for a FFH binary NCFSK system is presented for partial-band noise with broad-band noise. Exact analysis is performed for arbitrary L . Results are presented for $L = 1, \dots, 6$. Results from exact analysis are compared to results calculated by using bounds. In [31], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: tone jamming with broad-band noise, and broad-band noise with Rayleigh fading. Results are generated by computer simulation for $L = 2, 4$. In [43], the probability of error for a FFH M -ary NCFSK system is presented for partial-band noise with broad-band noise and independent Rician fading. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 4$.

Hard-decision-majority-vote diversity combining is discussed in [7],[37], and [44]. In [7], the probability of error for a FFH M -ary NCFSK system is presented for two cases: tone jamming with broad-band noise, and partial-band noise with broad-band noise. Exact analysis is presented for arbitrary M and L . Results are presented for $M = 2, 8$ and $L = 1, \dots, 9$. In [44], the probability of error for a FFH 8-ary NCFSK system is measured for two cases: tone jamming with broad-band noise, and partial-band noise with broad-band noise. Experimental data is presented for $L = 1, \dots, 32$. In [37], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: tone jamming with broad-band noise, and broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

Ratio-statistic-envelope diversity combining is discussed in [31],[41],[45], and [46]. In [41], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and partial-band noise. Results are generated by computer simulation for $L = 1, \dots, 6$. In [45], the probability of error for a FFH binary NCFSK system is presented for partial-band noise with broad-band noise. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 5$. In [31], the probability of error for a FFH 8-ary NCFSK system is presented for three cases: partial-band noise with broad-band noise, tone jamming with broad-band noise, and broad-band noise with Rayleigh fading. Results are generated by computer simulation for $L = 4$. In [46], the probability of error for a FFH binary NCFSK

system is presented for partial-band noise with broad-band noise and independent Rician fading. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 4$.

Ratio-statistic-square-law diversity combining is discussed in [41] and [46]. In [41], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and partial-band noise. Results are generated by computer simulation for $L = 1, \dots, 6$. In [46], the probability of error for a FFH binary NCFSK system is presented for partial-band noise with broad-band noise and independent Rician fading. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 4$.

Self-normalized-envelope diversity combining is discussed in [8],[28],[37],[41],[43],[44],[47], and [48]. In [41], the probability of error for a FFH 8-ary NCFSK system is presented for three cases: broad-band noise, partial-band noise, and tone jamming. Results are generated by computer simulation for $L = 1, \dots, 6$. In [47], the probability of error for a FFH binary NCFSK system is presented for tone jamming. Approximate results are derived for arbitrary L since broad-band noise is not considered. In [44], the probability of error for a FFH 8-ary NCFSK system is measured for two cases: tone jamming with broad-band noise, and partial-band noise with broad-band noise. Experimental data is presented for $L = 1, \dots, 32$. In [8], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: tone jamming with broad-band noise, and broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$. In [28], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: tone jamming with broad-band noise, and broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 28$. In [37], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: tone jamming with broad-band noise, and broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 28$. In [48], the probability of error for a FFH binary NCFSK system is presented for partial-band noise with broad-band noise and independent Rician fading. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 4$. In [43], the probability of error for a FFH M -ary NCFSK system is presented for partial-band noise with broad-band noise and independent Rician fading. Exact analysis is presented for

arbitrary L . Results are presented for $L = 1, \dots, 4$.

Self-normalized-square-law diversity combining is discussed in [41] and [49]. In [41], the probability of error for a FFH 8-ary NCFSK system is presented for three cases: broad-band noise, partial-band noise, and tone jamming. Results are generated by computer simulation for $L = 1, \dots, 6$. In [49], the probability of error for a FFH binary NCFSK system is presented for partial-band noise with broad-band noise. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 4$.

Clipped-linear diversity combining is discussed in [35]. The probability of error for a FFH M -ary NCFSK system is presented for partial-band noise with broad-band noise. Exact analysis is presented for arbitrary L and M . Results are presented for $M = 32$ and $L = 3, 5$.

Clipped-square-law diversity combining is discussed in [41] and [42]. In [41], the probability of error for a FFH 8-ary NCFSK system is presented for three cases: broad-band noise, partial-band noise, and tone jamming. Results are generated by computer simulation for $L = 1, \dots, 6$. In [42], the probability of error for a FFH binary NCFSK system is presented for partial-band noise with broad-band noise. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, 2$.

Ratio-threshold-square-law diversity combining is discussed in [50]. The probability of error for a FFH binary NCFSK system is presented for three cases: partial-band noise with broad-band noise, independent Rayleigh fading with broad-band noise, and independent Rician fading with broad-band noise. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 5$.

Ratio-threshold-majority-logic diversity combining is discussed in [50] and [51]. In [50], the probability of error for a FFH binary NCFSK system is presented for three cases: partial-band noise with broad-band noise, independent Rayleigh fading with broad-band noise, and independent Rician fading with broad-band noise. Exact analysis is presented for arbitrary L . Results are presented for $L = 1, \dots, 5$. In [51], the probability of error for a FFH binary NCFSK system is derived for partial-band noise with broad-band noise, and and tone jamming with broad-band noise. Results are presented for $L = 4$ and $M = 2, 8$.

The 4-2-moment-method diversity combining is discussed in [8] and [37]. In

[8], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: partial-band noise with broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$. In [37], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: partial-band noise with broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

The 2-1-moment-method diversity combining is discussed in [8],[28], and [37]. In [8], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: partial-band noise with broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$. In [37], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: partial-band noise with broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$. In [28], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: partial-band noise with broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 28$.

The 1-1/2-moment-method diversity combining is discussed in [37]. The probability of error for a FFH 8-ary NCFSK system is presented for two cases: partial-band noise with broad-band noise, and tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

Order-statistic diversity combining is discussed in [37], and [52]-[54]. In [52], the probability of error for a FFH binary NCFSK system is derived for partial-band noise with broad-band noise. Results are presented for $L = 1, \dots, 8$. In [37], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and multiple-tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$. In [53], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and multiple-tone jamming. Results are generated by computer simulation for $L = 1, \dots, 32$. In [54], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and multiple-tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

Weighted-order-statistic combining is discussed in [36] and [54]. In [36], the

probability of error for a FFH M -ary NCFSK system is presented for partial-band noise with broad-band noise. Results are generated by computer simulation for $M = 2, 4, 8, 16$ and $L = 1, 3$. In [54], the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and multiple-tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

OSSNE combining is discussed in [53] and in [54]. In both papers, the probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and multiple-tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

WOSSNE combining is discussed in [54]. The probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and multiple-tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

Recursive-excision combining is discussed in [54]. The probability of error for a FFH 8-ary NCFSK system is presented for two cases: broad-band noise, and multiple-tone jamming with broad-band noise. Results are generated by computer simulation for $L = 1, \dots, 32$.

Some observations pertaining to the use of the various diversity-combining methods in the demodulation of FFH NCFSK systems are presented. Optimum diversity combining is not a practical method because it involves estimation of signal and noise powers. Selection diversity combining is not a practical method because it is susceptible to partial-band interference. Clipped-linear, clipped-square-law, ratio-threshold-square-law, and ratio-threshold-majority-logic diversity combining are not practical methods because side information is required to set a clipping level or a threshold. Hard-decision-majority-vote diversity combining is not a good method because it does not utilize all the information from the matched-filter output samples. 4-2-moment-method, 2-1-moment-method, and 1-1/2-moment-method diversity combining perform poorly in fading. Linear diversity combining is an excellent method in broad-band noise. Square-law diversity combining is an excellent method in fading. Ratio-statistic-envelope, self-normalized-envelope, ratio-statistic-square-law, self-normalized-square-law, order-statistic, OSSNE, OS-

HDMV, WOSSNE, weighted-order-statistic, and recursive-excision diversity combining are excellent methods in multiple-tone jamming. The methods based on order statistics are effective, but some methods are computationally intensive. Linear-square-law, ratio-statistic-envelope, self-normalized-envelope, ratio-statistic-square-law, self-normalized-square-law, order-statistic, weighted-order-statistic, OSSNE, OSHDMV, WOSSNE, and recursive-excision diversity combining are all possible methods of choice.

3.9 Performance Results

Previous error-performance analyses for FFH M -ary NCFSK receivers with linear diversity combining have primarily used computer simulation [30],[31],[36],[37] or approximations [34]. However, Keller and Pursley [35] calculate the probability of bit error, P_b , in partial-band noise using an analytical method based on repeated convolutions which is only practical for small levels of diversity and cannot be used for multiple-tone interference. Also, previous analyses have considered only small values of L , typically $L \leq 5$, whereas larger values of L are of practical interest. An analytical method is needed to compute P_b for an arbitrary level of diversity for the following situations: 1) partial-band-noise jamming plus system noise, 2) multiple-tone jamming plus system noise, and 3) fading plus system noise. The following sections of this chapter present the application of the analytical method presented in the last chapter to the demodulation problem in FFH M -ary NCFSK systems with linear diversity combining. In addition, previous error-performance analyses for FFH M -ary NCFSK receivers with 2-1- and 4-2-moment-method diversity combining have primarily used computer simulation [8],[28],[37]. A later section of this chapter presents an analytical method for the FFH binary NCFSK demodulation problem with 2-1- and 4-2-moment-method diversity combining and $L = 2$ in independent Rayleigh fading.

3.10 Broad-Band Noise

The probability of bit error was calculated for various levels of diversity in broad-band AWGN for linear diversity combining. The noise can originate from both system noise and jamming. The signal-to-noise ratio after dehopping is given by

$$SNR = \frac{E_h}{N_0} = \frac{s_{mi}^2}{2\sigma^2} \quad (3.40)$$

where E_h is the energy per hop, $N_0/2$ is the two-sided noise power-spectral density, and $\sigma^2 = N_0/2T_h$. This SNR is appropriate for a fixed-hop-rate system. Perfect time synchronization and frequency synchronization are assumed. The probability of symbol error is given by

$$P_s = 1 - \int_0^\infty [F_n(v)]^{M-1} f_{sn}(v) dv \quad (3.41)$$

where $F_n(v)$ is the cdf of the decision statistic with only broad-band noise present, and $f_{sn}(v)$ is the pdf of the decision statistic with broad-band noise and a signal tone present. The probability of bit error is then

$$P_b = \frac{M}{2(M-1)} P_s. \quad (3.42)$$

Therefore, the calculation of P_b requires only a single integration. The adaptive-Romberg method was used for the numerical integration [23] with $\delta = 10^{-8}$. In the adaptive-Romberg method, δ is the required accuracy. To calculate the Fourier-Bessel series of (2.15) and (2.16), 150 terms were used. To calculate the Fourier series of (2.27) and (2.28), 40 nonzero terms were used. With the above choices, the results are accurate to 8 decimal places.

The probability of bit error was calculated for various levels of diversity in broad-band AWGN for linear combining and $M = 8$. This noise could originate from either system noise or jamming. The parameters are the same as in [8] where results are generated by computer simulation; therefore, $M = 8$ and $SNR = -0.2$ dB. From (3.41) and (3.42), the probability of bit error was calculated for various levels of diversity and is plotted in Fig. 3.15. The results generated in this chapter agree very closely with the results presented in [8]. As expected, the probability of bit error decreases as the level of diversity is increased. The probability of bit error

was also calculated for various levels of diversity in broad-band AWGN for linear combining and $M = 2$. The results are shown in Fig. 3.16.

Since linear and square-law combining are near-optimal combining schemes in broad-band AWGN, a comparison between the two would be interesting. The probability of bit error with $M = 2$ for square-law combining is given by [25]

$$P_b = \frac{1}{2^{2L-1}} \exp\left(-\frac{LE_h}{2N_0}\right) \sum_{n=0}^{L-1} c_n \left(\frac{LE_h}{2N_0}\right)^n \quad (3.43)$$

where

$$c_n = \frac{1}{n!} \sum_{k=0}^{L-1-n} \binom{2L-1}{k}. \quad (3.44)$$

Table 3.3 shows the performance of the two diversity-combining methods with $SNR = -5$ dB and $SNR = 0$ dB. The P_b is almost the same for both methods. Thus, linear and square-law combining have similar performance. Recall that linear combining has near-optimal performance for large SNR in broad-band AWGN and that square-law combining has near-optimal performance for small SNR in broad-band AWGN. Simulation results from [30] show that both combining methods have similar performance and that the performance is near-optimal independent of the signal-to-noise ratio.

3.11 Multiple-Tone Jamming

The probability of bit error was calculated for binary NCFSK with $L = 8$ in multiple-tone jamming plus broad-band AWGN for two types of jamming strategies. First, results are presented for randomly distributed multiple-tone jamming. Then, these results are compared to results for Houston-sense multiple-tone jamming.

3.11.1 Random-Multiple-Tone Jamming

It is assumed that the jammer has a fixed-total-power constraint and this jamming consists of tones randomly located in a fraction, γ , of the total number of bins. All jammer tones are of the same power, $a_{m_i}^2$, which is proportional to $1/\gamma$. Each tone is located at the centre of a frequency bin. The signal-to-noise ratio is given

Figure 3.15: P_b with $M = 8$ and $SNR = -0.20$ dB in broad-band noise.

Figure 3.16: P_b with $M = 2$ in broad-band noise.

SNR in dB	L	P_b for square law	P_b for linear
0	8	0.069145	0.068921
0	12	0.034761	0.034492
0	16	0.018060	0.017842
0	20	0.009571	0.009411
0	24	0.005141	0.005035
0	28	0.002788	0.002719
0	32	0.001523	0.001479
-5	8	0.289804	0.293143
-5	12	0.248320	0.252187
-5	16	0.216027	0.220198
-5	20	0.189728	0.194068
-5	24	0.167742	0.172158
-5	28	0.149040	0.153467
-5	32	0.132937	0.137328

Table 3.3: P_b with $M = 2$ in Broad-band Noise

by (3.40). The ratio of signal to tone jammer after dehoppping is given by

$$SJR = \frac{E_h}{J_o} = \frac{s_{mi}^2}{\gamma a_{mi}^2} \quad (3.45)$$

where E_h is the energy per hop, and J_o is the equivalent jammer power spectral density. For L hops, the bin with the desired signal will contain a signal tone plus noise every time, but could contain a jammer tone l_s times where l_s is a random integer with $0 \leq l_s \leq L$. Similarly, for L hops, the empty bin could contain noise every time, but would contain a jammer tone l_e times where l_e is a random integer with $0 \leq l_e \leq L$. The probability of each combination of tone jamming can be calculated from:

$$P_{comb}(l_s, l_e) = \binom{L}{l_s} \binom{L}{l_e} \gamma^{l_s+l_e} (1-\gamma)^{2L-l_s-l_e}. \quad (3.46)$$

The probability, $P_{correct}(l_e, l_s)$, of a correct decision for each combination of tone jamming is calculated using

$$P_{correct}(l_e, l_s) = \int_0^\infty F_e(v) f_s(v) dv \quad (3.47)$$

where $f_s()$ is the pdf of the signal-bin decision statistic, and $F_e()$ is the cdf of the empty-bin decision statistic. The overall probability of a correct decision is then

$$P_{correct} = \sum_{l_s=0}^L \sum_{l_e=0}^L P_{comb}(l_e, l_s) P_{correct}(l_e, l_s), \quad (3.48)$$

and P_b is

$$P_b = 1 - P_{correct}. \quad (3.49)$$

Calculation of P_b for $M > 2$ follows in a similar manner except that the combinatorial relation (3.46) is replaced by a somewhat more complex form. Therefore, the calculation of P_b becomes much more labourious.

From (3.46)-(3.49), P_b was calculated for $SNR = 10$ dB and for various SJR , and is plotted as a functions of γ in Fig. 3.17. The adaptive-Romberg method was used for the numerical integration [23] with $\delta = 10^{-8}$. To calculate the Fourier-Bessel series of (2.15) and (2.16), 150 terms were used. To calculate the Fourier series of (2.27) and (2.28), 200 nonzero terms were used. With the above choices, the results are accurate to 8 decimal places.

For $SNR = 10$ dB and $SJR = -5$ dB, P_b is at a maximum of 0.2910 for a jamming fraction of $\gamma = 1.0$. As γ approaches zero, P_b drops considerably. Thus, jamming all the bins is the most detrimental jamming strategy at this level of interference.

For $SNR = 10$ dB and $SJR = 0$ dB, P_b is at a maximum of 0.1127 for a jamming fraction of $\gamma = 0.019$. As γ approaches zero, P_b drops considerably. Note that $P_b = 0.1123$ for $\gamma = 1.0$. Thus, jamming all the bins is almost as detrimental as jamming the worst-case fraction of bins at this level of interference.

For $SNR = 10$ dB and $SJR = 5$ dB, P_b is at a maximum of 0.0408 for a jamming fraction of $\gamma = 0.006$. As γ approaches zero, P_b drops considerably. For $\gamma = 1.0$, $P_b = 0.0016$. Thus, jamming with $\gamma = 0.006$ is much worse than jamming with $\gamma = 1.0$.

For $SJR = -5$ dB, the highest P_b occurs at full-band jamming. For $SJR = 0$ dB and 5 dB, the highest P_b occurs at a relatively small jamming fraction ($\gamma = 0.019$ and $\gamma = 0.006$ respectively). For small jamming fractions ($\gamma < 0.005$), P_b is almost the same for all three values of SJR . Full-band jamming is very effective at $SJR = 0$ dB and -5 dB, but is not effective for $SJR = 5$ dB.

Figure 3.17: P_b for randomly distributed multiple-tone jamming, $L = 8$, $M = 2$, $SNR = 10$ dB.

The undulations in the curves are related to break points in the value of P_b for the tone jamming, in the absence of noise, as a function of γ . The simplest type of break point occurs at values of γ for which all l_e of the jamming tones fall in the nonsignal bin to give a total amplitude $l_e a_{m1}$. In the absence of noise, if $l_e a_{m1} < L s_{m1}$, then, no error occurs, whereas if $l_e a_{m1} > L s_{m1}$, then, an error occurs. Since from (3.45) $a_{m1}^2 = s_{m1}^2 / \gamma SJR$, a break point occurs at $\gamma = l_e^2 / L^2 SJR$. More complex forms of break point occur for various numbers of jamming tones also falling in the signal bin in L hops. To illustrate the effect of break points, the example of Fig. 3.17 with $SJR = 0$ dB and $SNR = 10$ dB was recalculated with $SJR = -0.41$ dB and without noise using a new approximation for the signal-bin decision statistic,

$$v_s = (L - l_s) s_{m1} + l_s \sqrt{s_{m1}^2 + a_{m1}^2}. \quad (3.50)$$

For the approximation, the decision statistic is assigned its median value. Both results are shown in Fig. 3.18. The break points are clearly seen. The first break point as γ increases from zero occurs at $\gamma = 1/L^2 SJR$.

3.11.2 Houston-Multiple-Tone Jamming

For Houston jamming [26], a fraction, α , of the total number of M -bin channels are jammed by a single jammer tone. The fraction of bins jammed is $\gamma = \alpha/M$. It is the worst-case jamming when diversity is not employed. All jammer tones are of the same amplitude, a_{m1} . Each tone's centre is at the centre of the frequency bin. The signal-to-noise ratio is given by (3.40). The ratio of signal to tone jammer after dehopping is given by (3.45).

Consider a FFH binary ($M = 2$) NCFSK system. For L hops, the bin with the desired signal will contain a signal tone plus noise every time, but could contain a jammer tone l_s times where l_s is a random integer with $0 \leq l_s \leq L$. Similarly, for L hops, the empty bin could contain noise every time, but would contain a jammer tone l_e times where l_e is a random integer with $0 \leq l_e \leq L - l_s$. The probability of each combination of tone jamming can be calculated from

$$P_{comb}(l_s, l_e) = \binom{l_s + l_e}{l_s} \binom{L}{l_s + l_e} \left(\frac{1}{2}\right)^{l_s + l_e} \alpha^{l_s + l_e} (1 - \alpha)^{L - l_s - l_e}. \quad (3.51)$$

Figure 3.18: P_b for randomly distributed multiple-tone jamming, without broadband noise, $L = 8$, $M = 2$.

The probability, $P_{correct}(l_e, l_s)$, of a correct decision for each combination of tone jamming is calculated by (3.47) using (3.51). The overall probability of a correct decision is then calculated using (3.48), and P_b is given by (3.49). From (3.47)-(3.51), P_b was calculated for $SNR = 10$ dB and for $SJR = -5$ dB, 0 dB, and 5 dB, and is plotted as a function of γ in Fig. 3.19. The adaptive-Romberg method was used for the numerical integration [23] with $\delta = 10^{-8}$. To calculate the Fourier-Bessel series of (2.15) and (2.16), 150 terms were used. To calculate the Fourier series of (2.27) and (2.28), 200 nonzero terms were used. With the above choices, the results are accurate to 8 decimal places.

For $SNR = 10$ dB and $SJR = -5$ dB, P_b is at a maximum of 0.2834 for a jamming fraction of $\gamma = 0.5$. Thus, jamming all the channels is the most detrimental jamming strategy at this level of interference.

For $SNR = 10$ dB and $SJR = 0$ dB, P_b is at a maximum of 0.1138 for a jamming fraction of $\gamma = 0.019$. Note that $P_b = 0.0996$ for $\gamma = 0.5$. Thus, jamming with $\gamma = 0.019$ is worse than jamming with $\gamma = 0.5$.

For $SNR = 10$ dB and $SJR = 5$ dB, P_b is at a maximum of 0.0411 for a jamming fraction of $\gamma = 0.006$. Here, jamming with $\gamma = 0.006$ is much worse than jamming with $\gamma = 0.5$.

For $SJR = -5$ dB, the highest P_b occurs at full-band jamming. For $SJR = 0$ dB and 5 dB, the highest P_b occurs at a relatively small jamming fraction ($\gamma = 0.019$ and $\gamma = 0.006$). Houston-sense jamming is worse than randomly distributed jamming at $SJR = 0$ dB and 5 dB; however, randomly distributed jamming is worse than Houston-sense jamming at $SJR = -5$ dB. This second point is very important. It is often stated in the literature [37],[53],[54] that Houston-sense jamming is the worst case. However, for $L = 8$, $M = 2$, and $SJR = -5$ dB, this is not true. Thus, Houston-sense jamming is not necessarily the worst-case jamming.

3.12 Comparison of Performances in Jamming

Table 3.4 summarizes the worst-case P_b performance for the four different types of jamming presented in this section – broad-band noise jamming, random multiple-tone jamming with broad-band noise, Houston multiple-tone jamming with broad-

Figure 3.19: P_b for Houston-sense multiple-tone jamming, $L = 8$, $M = 2$, $SNR = 10$ dB.

band noise, and random multiple-tone jamming without broad-band noise – at the three different signal-to-interference ratios (SIR) with $L = 8$ and $M = 2$. The signal-to-interference ratio is given by

$$SIR = \frac{E_h}{N_0 + J_o}. \quad (3.52)$$

At low SIR , broad-band noise causes the greatest performance degradation. However, at high SIR , multiple-tone jamming causes the greatest performance degradation.

SIR(dB)	BBN Jamming	Random MTJ with BBN	Houston MTJ with BBN	Random MTJ without BBN
-5.14	0.2984	0.2910	0.2834	0.2396
-0.41	0.0851	0.1127	0.1145	0.1127
3.81	0.0017	0.0408	0.0411	0.0448

Table 3.4: Worst-Case P_b for Linear Combining, $L = 8$ and $M = 2$

3.13 Rayleigh Fading

In this section, the performances of linear, square-law, 4-2-moment method, and 2-1-moment-method diversity combining are considered in Rayleigh fading. Results are derived for $M = 2$ and $L = 2$.

3.13.1 The PDF for z_{mi} in Independent Rayleigh Fading

The output sample of the matched filter with envelope detector, z_{mi} , is the magnitude of a complex number,

$$\tilde{y}_{mi} = \tilde{s}_{mi} + \tilde{n}_{mi}, \quad (3.53)$$

where

$$\hat{s}_{m1} = s_{m1} \cos(\phi_{m1}) + j s_{m1} \sin(\phi_{m1}), \quad (3.54)$$

$$\hat{n}_{m1} = n_{m1c} + j n_{m1s}. \quad (3.55)$$

The noise components, n_{m1c} and n_{m1s} , are Gaussian random variables with zero mean and variance,

$$\sigma_n^2 = \frac{N_0}{2T_h} \quad (3.56)$$

where $N_0/2$ is the double-sided power spectral density of the noise, and T_h is the hop period. The signal amplitude, s_{m1} , is a random variable with pdf,

$$f_{s_{m1}}(s_{m1}) = \frac{s_{m1}}{s_0^2} \exp\left(-\frac{s_{m1}^2}{2s_0^2}\right), s_{m1} \geq 0. \quad (3.57)$$

For $s_{m1} < 0$, $f_{s_{m1}}(s_{m1}) = 0$. The mean power of the signal is $2s_0^2$. From (2.10), the two-dimensional characteristic function of the matched filter output sample is

$$\Phi_{\hat{y}_{m1}}(\rho) = \exp\left(-\frac{\rho^2(\sigma_n^2 + s_0^2)}{2}\right). \quad (3.58)$$

The pdf of the output sample with signal and noise present is

$$f_{zn}(z_{m1}) = \frac{z_{m1}}{\sigma_T^2} \exp\left(-\frac{z_{m1}^2}{2\sigma_T^2}\right), z_{m1} \geq 0, \quad (3.59)$$

where $\sigma_T^2 = \sigma_n^2 + s_0^2$. For $z_{m1} < 0$, $f_{zn}(z_{m1}) = 0$. The pdf of the output sample with only noise present is

$$f_n(z_{m1}) = \frac{z_{m1}}{\sigma_n^2} \exp\left(-\frac{z_{m1}^2}{2\sigma_n^2}\right), z_{m1} \geq 0. \quad (3.60)$$

For $z_{m1} < 0$, $f_n(z_{m1}) = 0$.

3.13.2 The 2-1- and 4-2-Moment Method

In the 2-1-moment method, a decision statistic, v_m , is formed by (3.28). For $L = 2$, the decision statistic is after simplification

$$v_m = z_{m1} z_{m2}. \quad (3.61)$$

Define a variable $q = z_{m2}$. The joint pdf of v_m and q is found from the joint pdf of z_{m1} and z_{m2} . Because of the independence of z_{m1} and z_{m2} , the joint pdf of z_{m1} and

z_{m2} is the product of the individual probability density functions. Thus, the joint pdf is

$$f_{z_{m1}, z_{m2}}(z_{m1}, z_{m2}) = \frac{z_{m1} z_{m2}}{\sigma^4} \exp\left(\frac{-z_{m1}^2 - z_{m2}^2}{2\sigma^2}\right), z_{m1} \geq 0, z_{m2} \geq 0. \quad (3.62)$$

The magnitude of the jacobian of the transformation from (z_{m1}, z_{m2}) to (v_m, q) is

$$|J(z_{m1}, z_{m2})| = q. \quad (3.63)$$

The joint pdf of v_m and q is [55]

$$f_{v_m, q}(v_m, q) = \frac{f_{z_{m1}, z_{m2}}(z_{m1}, z_{m2})}{|J(z_{m1}, z_{m2})|} \quad (3.64)$$

where $z_{m1} = v_m/q$ and $z_{m2} = q$. Thus, substituting (3.62) and (3.63) into (3.64),

$$f_{v_m, q}(v_m, q) = \frac{v_m}{q\sigma^4} \exp\left(\frac{-(\frac{v_m}{q})^2 - q^2}{2\sigma^2}\right), v_m \geq 0, q \geq 0. \quad (3.65)$$

The pdf of v_m can be found by integrating (3.65) with respect to q . Thus,

$$f_{v_m}(v_m) = \int_0^\infty \frac{v_m}{q\sigma^4} \exp\left(\frac{-\frac{v_m^2}{q} - q^2}{2\sigma^2}\right) dq, v_m \geq 0. \quad (3.66)$$

Simplifying [56],

$$f_{v_m}(v_m) = \frac{v_m}{\sigma^4} K_0\left(\frac{v_m}{\sigma^2}\right), v_m \geq 0, \quad (3.67)$$

where $K_0()$ is the zeroth-order modified-Bessel function of the second kind. By integrating (3.67), the cdf of v_m can be found as

$$F_{v_m}(v_m) = \int_0^{v_m} f_{v_m}(x) dx, v_m \geq 0. \quad (3.68)$$

Simplifying [33],

$$F_{v_m}(v_m) = 1 - \frac{v_m}{\sigma^2} K_1\left(\frac{v_m}{\sigma^2}\right), v_m \geq 0, \quad (3.69)$$

where $K_1()$ is the first-order modified-Bessel function of the second kind.

The bit-error probability, P_b , for binary NCFSK can be found by

$$P_b = 1 - \int_0^\infty F_n(v) f_{sn}(v) dv \quad (3.70)$$

$$= \int_0^\infty \frac{v^2}{\sigma_n^4 \sigma_T^2} K_0\left(\frac{v}{\sigma_T^2}\right) K_1\left(\frac{v}{\sigma_n^2}\right) dv \quad (3.71)$$

$$= \left(\frac{\sigma_n}{\sigma_T}\right)^4 {}_2F_1(2, 1; 3; 1 - \left(\frac{\sigma_n}{\sigma_T}\right)^4) \quad (3.72)$$

$$= \left(\frac{\sigma_n}{\sigma_T}\right)^4 \sum_{k=0}^{\infty} \frac{1}{k+2} \left(1 - \left(\frac{\sigma_n}{\sigma_T}\right)^4\right)^k \quad (3.73)$$

$$= \frac{\left(\frac{\sigma_n}{\sigma_T}\right)^4}{\left(1 - \left(\frac{\sigma_n}{\sigma_T}\right)^4\right)^2} \left(4 \ln\left(\frac{\sigma_T}{\sigma_n}\right) + \left(\frac{\sigma_n}{\sigma_T}\right)^4 - 1\right) \quad (3.74)$$

where $F_n()$ is the cdf of v_m with only noise present, $f_m()$ is the pdf of v_m with signal plus noise present, and ${}_2F_1(,;:)$ is the Gauss hypergeometric function. The expression for P_b in (3.74) is a new result. The performance of the 4-2-moment method is identical to that of the 2-1-moment method for $L = 2$ and $M = 2$ in independent Rayleigh fading.

3.13.3 Linear Diversity Combining

For linear diversity combining, a decision statistic, v_m , is formed by (3.15). For $L = 2$, the cdf of v_m can be computed as

$$F_{v_m}(v_m) = \int_0^{v_m} \int_0^{v_m - z_{m1}} \frac{z_{m1} z_{m2}}{\sigma^4} \exp\left(\frac{-z_{m1}^2 - z_{m2}^2}{2\sigma^2}\right) dz_{m1} dz_{m2}. \quad (3.75)$$

Simplifying,

$$F_{v_m}(v_m) = 1 - \exp\left(\frac{-v_m^2}{2\sigma^2}\right) - \frac{v_m \sqrt{2}}{2\sigma} \exp\left(\frac{-v_m^2}{4\sigma^2}\right) \operatorname{erf}\left(\frac{v_m}{2\sigma}\right), v_m \geq 0, \quad (3.76)$$

where

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) dt, v_m \geq 0. \quad (3.77)$$

The pdf of v_m can be found by differentiating (3.76). Thus,

$$f_{v_m}(v_m) = \frac{v_m}{2\sigma} \exp\left(\frac{-v_m^2}{2\sigma^2}\right) - \frac{\sqrt{\pi}}{4\sigma^3} (v_m^2 + 2\sigma^2) \exp\left(\frac{-v_m^2}{4\sigma^2}\right) \operatorname{erf}\left(\frac{v_m}{2\sigma}\right), v_m \geq 0. \quad (3.78)$$

The probability of bit error for binary NCFSK must be found numerically by integrating

$$P_b = 1 - \int_0^\infty F_n(v) f_m(v) dv. \quad (3.79)$$

3.13.4 Square-Law Diversity Combining

For square-law diversity combining, a decision statistic, v_m , is formed as (3.18). For $L = 2$, the pdf and cdf are given by [25]

$$f_{v_m}(v_m) = \frac{v_m}{4\sigma^4} \exp\left(-\frac{v_m}{2\sigma^2}\right), v_m \geq 0, \quad (3.80)$$

and

$$F_{v_m}(v_m) = 1 - \left(1 + \frac{v_m}{2\sigma^2}\right) \exp\left(-\frac{v_m}{2\sigma^2}\right), v_m \geq 0, \quad (3.81)$$

After considerable algebraic manipulation, the probability of bit error is given by

$$P_b = \frac{4 + 3\beta}{8 + 12\beta + 6\beta^2 + \beta^3} = \frac{4 + 3\beta}{(2 + \beta)^3} \quad (3.82)$$

where $\beta = SNR = s_0^2/\sigma_n^2$. This result agrees with [32].

3.13.5 Results

The bit-error probability was computed for $L = 2$ and $M = 2$ in independent Rayleigh fading using (3.79), (2.27), and (2.28) for linear combining, using (3.74) for the 4-2- and 2-1-moment-method combining, and using (3.82) for square-law combining. The results are shown in Fig. 3.20. Square-law combining has the best performance. The 4-2- and 2-1-moment-method combining have the second best performance. Linear combining has the worst performance. However, linear combining is only 1.9 dB worse than square-law combining. Square-law combining can be shown to be the optimum combining method in independent Rayleigh fading.

3.14 Numerical Accuracy

The numerical accuracy of the results is related to δ , the fractional error of the numerical integrations that are used to calculate $P_{correct}$. For $\delta = 10^{-n}$, $P_{correct}$ will be accurate to n significant figures. Since $P_b = 1 - P_{correct}$, P_b will be accurate to n decimal places. However, to achieve accuracy for small P_b , the number n must be somewhat larger than m , where P_b has the form $a.b \times 10^{-m}$, where a and b are integers.

Alternatively, the numerical accuracy, at small P_b , can be improved by finding P_b directly. For example, (3.47) could be written

$$P_{error}(l_e, l_s) = \int_0^\infty (1 - F_e(v)) f_s(v) dv. \quad (3.83)$$

Then, (3.48) becomes

$$P_b = \sum_{l_s=0}^L \sum_{l_e=0}^L P_{comb}(l_e, l_s) P_{error}(l_e, l_s). \quad (3.84)$$

Figure 3.20: P_b with $M = 2$ and $L = 2$ in independent Rayleigh fading.

3.15 Summary

In this chapter, performance results for the demodulation of FFH NCFSK systems with time diversity and linear diversity combining were presented. These results were obtained using a novel analytical technique described in the previous chapter. Specific observations uncovered with the use of this technique include the following. In broad-band noise, for linear combining, the probability of bit error decreases as the level of diversity is increased. Also in broad-band noise, linear and square-law combining have similar performance. In multiple-tone jamming, Houston-sense jamming is not necessarily the worst-case jamming. In Rayleigh fading with $M = 2$ and $L = 2$, square-law combining performs better than the 4-2-moment-method, 2-1-moment-method and linear combining.

Chapter 4

Application to Uplink Time Synchronization for a FFH NCFSK Satellite-Communication System

4.1 Introduction

In this chapter, the application of the novel analytical technique to the uplink time synchronization of a FFH NCFSK satellite-communication system is presented. For this system, time synchronization is performed differently than for many other communication systems. The system uses a single dehopper on the satellite and has many users per satellite transponder. These users are usually separated by frequency-division multiple accessing. Also, the use of a processing transponder makes it necessary to demodulate signals on board the satellite. For these reasons, the satellite maintains the master clock. Thus, the satellite's clock must not change. Hence, the ground terminals must change their timing to accommodate the satellite. This means that for the uplink, the transmitter is required to synchronize. In many communication systems, time synchronization is done by the receiver. Transmitter synchronization is usually performed by the use of synchronization probes. These probes are discussed later in this chapter. Time synchronization is performed in

two stages. First, coarse synchronization is performed to align the two signals to within a half of a hop period. Second, fine synchronization is performed to align the two signals to within the required tolerance. Both uplink coarse-time and fine-time synchronization can make use of time diversity and diversity combining to improve performance. In the following sections, both coarse-time and fine-time synchronization systems are described which use linear diversity combining. Also, performance results are presented. Coarse-time synchronization is described first followed by fine-time synchronization.

4.2 Coarse-Time Synchronization

4.2.1 Introduction

The overall uplink coarse-time synchronization system for a particular FFH NCFSK satellite-communication systems is shown in Fig. 4.1. In this application, the transmitter of Fig. 3.12 is used to send a series of synchronization-probe bursts in a serial-search process. On board the satellite, the payload processor makes a decision as to whether a probe burst has been received. Only the probe bursts falling within the on-board hop-pattern window can be detected. An acknowledgement is sent back to the originating terminal for use in adjusting the hop-pattern timing. Synchronization-search strategies are considered elsewhere [9],[12]. Once coarse-time synchronization has been achieved, fine-time synchronization aligns the two hop periods to within the required tolerance [9],[15],[16].

The synchronization probe, shown in Fig. 4.2, consists of a tone at a frequency, $f_{h_i} + f_p$, for the i th hop period. The probe frequency, f_p , is usually chosen as one of the M frequencies of the M -ary NCFSK channel so that no additional hardware is required. In Fig. 4.2, the probe arrives at the receiver with a timing error of ΔT relative to the on-board hop pattern. The dehopping process gates and downconverts the probe so that only the section of the probe falling within the on-board dehop period is seen at the output of the dehopper. If $|\Delta T|$ is greater than T_h , there is no probe energy available.

Following the dehopper is a bank of $E + 1$ matched filters and envelope detectors

Figure 4.1: Simplified coarse-time synchronization system.

of the form shown in Fig. 3.14. One filter is matched to the probe frequency, f_p , and E are matched to frequencies other than the probe frequency. The outputs are the sampled values, z_{pi} and z_{ei} , for $i = 1, \dots, L$, and $e = 1, \dots, E$. The empty bins are used to obtain noise and interference statistics. The diversity combiner forms decision statistics, v_p and v_e for $e = 1, \dots, E$, using one of the diversity-combining methods described previously. The decision statistics are then used to make the decision as to whether a detection has been made.

The analysis method of Chapter 2 is applied to uplink coarse-time synchronization of a FFH NCFSK satellite-communication system with time diversity and linear diversity combining. The decision statistics, v_p and v_e for $e = 1, \dots, E$, are formed from the matched-filter output samples, z_{pi} and z_{ei} for $i = 1, \dots, L$, and $e = 1, \dots, E$, by linear combining. The probe-bin statistic, v_p , is compared to a threshold, v_t . There are numerous methods of obtaining a value for the threshold, v_t . For illustration, the “adaptive-threshold method” is used. For comparison, the ideal value

Figure 4.2: The gating of the dehopper.

of v_t based upon perfect knowledge of σ is used.

In the adaptive-threshold method, a new value of v_t is obtained for every new ν_p and is based upon the sum, v_e , obtained in a single empty bin ($E = 1$). The means of obtaining the value of v_t is based on the following observations and assumptions. Suppose that the interference in the empty bin were only AWGN of two-sided power-spectral density, $N_0/2$. Then, [57]

$$E[v_e] = L\sqrt{\frac{\pi}{2}}\sigma \quad (4.1)$$

where $\sigma^2 = N_0/2T_h$. Thus, an estimate of σ is

$$\hat{\sigma} = \frac{v_e}{L}\sqrt{\frac{2}{\pi}}. \quad (4.2)$$

It can be shown that for a particular value of the probability of false alarm, the threshold is $v_t = c_a\sigma$ where c_a is a constant that is independent of σ for $L = 1$. It has been observed numerically that for a particular value of the probability of false alarm, c_a is also independent of σ and L . The constant, c_a , is computed separately once. Then, each new threshold can be calculated from the estimate, $\hat{\sigma}$, given via (4.2) by

$$v_t = \frac{c_a}{L}\sqrt{\frac{2}{\pi}}v_e = b_a v_e \quad (4.3)$$

where $b_a = (c_a/L)\sqrt{2/\pi}$ is a constant. A similar method for setting the threshold is

described in [58]. Clearly, if the interference is not Gaussian, the threshold will be set improperly, and the probability of false alarm will differ from its expected value.

For the ideal value of v_t , the actual (not estimated) value of σ for WGN is used to obtain $E[v_e]$ according to (4.1). The threshold is set to $v_t = b_f E[v_e]$ where b_f is a constant chosen to give the desired false-alarm probability.

The probability of synchronization-probe-burst detection was computed for two different types of jamming: (1) broad-band noise, and (2) multiple-tone jamming plus broad-band noise. It is assumed in the calculations that the synchronization-probe burst is either perfectly aligned, i.e. $\Delta T = 0$, or else $|\Delta T| > T_h$. This means that there is no spill over from the probe bin to the empty bin. The analysis can be easily modified for $|\Delta T| \neq 0$, or $|\Delta T| < T_h$.

4.2.2 Broad-Band Noise

For the adaptive-threshold method, the probability of detection in broad-band noise is given by

$$P_d = \int_0^{LR} (1 - F_{v_p}(b_a v_e)) f_{v_e}(v_e) dv_e, \quad (4.4)$$

and the probability of detection with an ideal threshold in broad-band noise is given by

$$P_d = 1 - F_{v_p}(b_f E[v_e]) \quad (4.5)$$

where the required cdf and pdf are calculated from (2.15),(2.16),(2.27), and (2.28) with the subscript, m , replaced by p or e . The signal is a constant tone with rms amplitude, s_{pi} , and the noise is AWGN. The signal-to-noise ratio after dehoppping is given by (3.40).

The constants are set so that $P_{fa} = 0.01$. The probability of false alarm can be computed using (4.4) and (4.5) with $F_{v_e}()$ substituted for $F_{v_p}()$ and P_{fa} substituted for P_d . Various constants are tried until $P_{fa} = 0.01$. The constants are set as follows: $b_f = 2.4215$ for $L = 1$ and an ideal threshold, $b_a = 1.8744$ for $L = 8$, $E = 1$ and an adaptive threshold, and $b_f = 1.4584$ for $L = 8$ and an ideal threshold. The adaptive-Romberg method was used for the numerical integration [23] with $\delta = 10^{-8}$. To calculate the Fourier-Bessel series of (2.15) and (2.16), 150 terms were

used. To calculate the Fourier series of (2.27) and (2.28), 40 nonzero terms were used. With the above choices, the results are accurate to 8 decimal places.

Fig. 4.3 shows the performance in broad-band noise. Three curves are shown: $L = 1$ and an ideal threshold, $L = 8$ and an ideal threshold, and $L = 8$, $E = 1$, and an adaptive threshold. The performance with $L = 8$, $E = 1$ and an adaptive threshold is about 2.7 dB better than the performance without diversity and an ideal threshold. However, the method with $L = 8$ requires eight times as many probes as the method without any diversity. The performance with $L = 8$ and an ideal threshold is a further 3.2 dB better than the performance with $L = 8$, $E = 1$ and an adaptive threshold. The ideal-threshold method represents the performance with a perfect threshold-setting algorithm. It is unattainable in practice, but provides a bound on achievable performance. If more empty bins are used to set the threshold, this bound can be approached. For example, the performance of the adaptive-threshold method with $E = 8$ is shown in Fig. 4.4 and found to be within 0.5 dB of that with an ideal threshold.

4.2.3 Random-Multiple-Tone Jamming

The probability of detection was computed for random-multiple-tone jamming plus broad-band noise for the adaptive-threshold method with $E = 1$. Jammer tones are randomly located in a fraction, γ , of the total number of bins and are assumed to have the same amplitude, $a_{pi} = a_{ei}$. Each tone's center is at the center of the corresponding frequency bin. Out of L hops, the probe bin would contain a signal tone and noise every time, but would contain a jammer tone l_p times where l_p is a random integer with $0 \leq l_p \leq L$. Similarly, out of L hops, the empty bin would contain noise every time, but would contain a jammer tone l_e times where l_e is a random integer with $0 \leq l_e \leq L$. The probability of each combination of tone jamming is calculated from (3.46). The probability of detection for each combination of tone jamming, $P_d(l_e, l_p)$, is calculated using (4.4) with $P_d(l_e, l_p)$ substituted for P_d where the required cdf and pdf are calculated by (2.15) and (2.16) with the appropriate characteristic functions used. The probability of false alarm for each combination of tone jamming, $P_{fa}(l_e, l_p)$, is calculated using (4.4) with $P_{fa}(l_e, l_p)$

Figure 4.3: The probability of detection in broad-band noise ($P_{fa} \approx 0.01$).

Figure 4.4: The probability of detection in broad-band noise ($P_{fa} = 0.01$).

substituted for P_d and $F_{i_e}()$ substituted for $F_{i_p}()$ where the required cdf and pdf are calculated by (2.15) and (2.16) with the appropriate characteristic functions used. The probability of probe-burst detection is given by

$$P_d = \sum_{l_e=0}^L \sum_{l_p=0}^L P_{comb}(l_e, l_p) P_d(l_e, l_p). \quad (4.6)$$

The probability of probe-burst false alarm is given by

$$P_{fa} = \sum_{l_e=0}^L \sum_{l_p=0}^L P_{comb}(l_e, l_p) P_{fa}(l_e, l_p). \quad (4.7)$$

The signal-to-noise ratio is given by (3.40). The ratio of signal to tone jammer is given by (3.45). The constant was set equal to $b_a = 1.8744$ so that the P_{fa} would be 0.01 in broad-band noise. The adaptive-Romberg method was used for the numerical integration [23] with $\delta = 10^{-6}$. To calculate the Fourier-Bessel series of (2.15) and (2.16), 150 terms were used. To calculate the Fourier series of (2.27) and (2.28), 200 nonzero terms were used. With the above choices, the results are accurate to 6 decimal places. Fig. 4.5 and Fig. 4.6 show the performance with $L = 8$, $E = 1$, and an adaptive threshold in random-multiple-tone jamming plus broad-band noise for $SNR = 10$ dB and $SJR = 0$ dB and 5 dB. The probability of detection is at a maximum for a jamming fraction of $\gamma = 0.0$ and at a minimum for broad-band jamming ($\gamma = 1.0$) for both levels of jamming.

For $SNR = 10$ dB and $SJR = 0$ dB, the probability of false alarm has a maximum at $\gamma \approx 0.08$ and a minimum at $\gamma = 0.0$ and $\gamma = 1.0$. In separate calculations, for broad-band noise jamming of the same total power ($SNR = -0.41$ dB), it was found that the probability of detection is $P_d \approx 0.16$, and the probability of false alarm is $P_{fa} = 0.01$.

For $SNR = 10$ dB and $SJR = 5$ dB, the probability of false alarm has a maximum at $\gamma \approx 0.04$ and a minimum at $\gamma = 0.0$ and $\gamma = 1.0$. In separate calculations, for broad-band noise jamming of the same total power ($SNR = 3.8$ dB), it was found that the probability of detection is $P_d \approx 0.57$, and the probability of false alarm is $P_{fa} = 0.01$. The probability-of-detection curve for $SNR = 10$ dB and $SJR = 5$ dB is not monotonically decreasing as is the one for $SNR = 10$ dB and $SJR = 0$ dB. The reason for this is the same as the one for the undulations in the probability-of-bit-error curves - break points caused by multiple-tone jamming.

Figure 4.5: Noise plus tone-jamming performance for adaptive-threshold method, $L = 8$, $SNR = 10$ dB, $SJR = 0$ dB.

Figure 4.6: Noise plus tone-jamming performance for adaptive-threshold method, $L = 8$, $SNR = 10$ dB, $SJR = 5$ dB.

Note that the lowest probability of detection is achieved with broad-band tone jamming ($\gamma = 1.0$). Thus, jamming only a fraction of the total number of bins is not effective in reducing the probability of detection. However, the probability of false alarm has a maximum at some intermediate value of jamming fraction ($0.0 < \gamma < 1.0$). Hence, jamming only a fraction of the total number of bins can be used to increase the false-alarm probability. Also, the probability of detection and probability of false alarm are both lower for broad-band tone jamming with broad-band noise than for only broad-band noise jamming.

4.3 Fine-Time Synchronization

4.3.1 Introduction

In this section, uplink fine-time synchronization for a fast-frequency-hopped satellite-communication system is presented. The overall uplink fine-time synchronization system for a particular FFH NCFSK satellite-communication system is shown in Fig. 4.7. In this application, the transmitter of Fig. 3.12 is used to send a series of synchronization-probe bursts. On board the satellite, the payload processor processes the probes and makes an estimate of the timing error. This estimate must include the magnitude of the timing error as well as whether the timing error is early or late. Algorithms to form the estimate are discussed in [9],[15],[16]. The timing-error estimate is returned to the ground terminal, and the ground terminal alters its timing accordingly.

4.3.2 Delay-Advance Method

This method was first discussed in [59] where this method was used for fine-time synchronization of a frequency-shift-keyed communication system which did not employ frequency hopping. The method was modified in [9] for use with frequency-hopped systems. The method has further been modified [16] for improved performance. The approach of [9] is discussed below, and analytical results are presented.

Fig. 4.8 shows the relative timing of the various signals involved. The satellite's payload processor has an on-board dehop period during which it expects to see

Figure 4.7: Simplified fine-time synchronization system.

the ground terminal's communication signal. Assume that the ground terminal's communication signal arrives with a timing error, ΔT , as shown in Fig. 4.8. The communications signal consists of a tone at a frequency, $f_{hi} + f_m$, for the i th hop period where the data frequency, f_m , is one of the M frequencies of the M -ary NCFSK channel. The goal of uplink fine-time synchronization is to align the ground terminal's communication signal with the on-board dehop period and thus, to reduce ΔT to zero. In order to obtain an estimate for the timing error, it is necessary to make measurements on the system under two different conditions. In [9], it is suggested that a series of synchronization probes be sent which are intentionally delayed by half of a hop period relative to the current synchronization position and then a series be sent which are intentionally advanced by half of a hop period. This allows the determination of the sign and magnitude of the timing error, ΔT . The advanced and delayed synchronization probes are shown in Fig. 4.8. The synchronization

probe consists of a tone at a frequency, $f_{hi} + f_p$, for the i th hop period. The probe frequency, f_p , is usually chosen as one of the M frequencies of the M -ary NCFSK channel so that no additional hardware is required. The dehopping process gates and downconverts the probe so that only the section of the probe falling within the on-board dehopping period is seen at the output of the dehopper. If there is not any overlap between the probe and the on-board dehopping period, there is no probe energy present in this period.

The output of the dehopper is applied to a matched filter with envelope detector which is matched to the probe frequency, f_p . The output of the matched filter with envelope detector are the samples, z_{ai} and z_{di} , for the advanced and delayed probes. The diversity combiner combines a burst of probes into advanced and delayed estimator statistics, v_a and v_d . If there are L probes in each burst and if linear diversity combining is used, then,

$$v_a = \sum_{i=1}^L z_{ai}, \quad (4.8)$$

and

$$v_d = \sum_{i=1}^L z_{di}. \quad (4.9)$$

The estimator statistics are used by the estimator to form an estimate, $\widehat{\Delta T}$, for ΔT . This estimate is returned to the ground terminal. The ground terminal then corrects its timing.

In the absence of noise, the samples are

$$z_{ai} = s_{mi} \left(\frac{1}{2} - \epsilon_t \right), \quad (4.10)$$

and

$$z_{di} = s_{mi} \left(\frac{1}{2} + \epsilon_t \right), \quad (4.11)$$

where s_{mi} is the rms amplitude of the dehopped probe, and ϵ_t is the relative timing error,

$$\epsilon_t = \frac{\Delta T}{T_h}. \quad (4.12)$$

The estimator statistics are

$$v_a = L z_{ai} = L s_{mi} \left(\frac{1}{2} - \epsilon_t \right), \quad (4.13)$$

Figure 4.8: Synchronization probes.

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

and

$$v_d = Lz_{di} = Ls_{mi}(\frac{1}{2} + \epsilon_i). \quad (4.14)$$

An estimate, $\hat{\epsilon}_i$, for the timing-error estimate, ϵ_i , is

$$\hat{\epsilon}_i = \frac{1}{2}(\frac{v_d - v_a}{v_a + v_d}). \quad (4.15)$$

The performance of this estimator in noise is now of interest.

4.3.3 Results

The analysis method of Chapter 2 is applied to the uplink fine-time synchronization problem in order to determine the performance of the estimator in noise. A measure of performance is the mean of the estimator under different signal-to-noise ratios. The mean of $\hat{\epsilon}_i$ is given by

$$E[\hat{\epsilon}_i] = \int_0^\infty \int_0^\infty \frac{1}{2}(\frac{v_d - v_a}{v_a + v_d})f_{v_a}(v_a)f_{v_d}(v_d)dv_adv_a \quad (4.16)$$

where $f_{v_a}(v_a)$ is the pdf of the advanced estimator statistic, and $f_{v_d}(v_d)$ is the pdf of the delayed estimator statistic. The mean of $\hat{\epsilon}_i$ can be computed using the method of Chapter 2 by

$$E[\hat{\epsilon}_i] = \int_0^{ul_a} \int_0^{ul_d} \frac{1}{2}(\frac{v_d - v_a}{v_a + v_d})f_{v_a}(v_a)f_{v_d}(v_d)dv_adv_a \quad (4.17)$$

where

$$ul_a = s_{mi}(\frac{1}{2} - \epsilon_i) + 10\sigma, \quad (4.18)$$

and

$$ul_d = s_{mi}(\frac{1}{2} + \epsilon_i) + 10\sigma. \quad (4.19)$$

The pdf, $f_{v_a}(v_a)$ and $f_{v_d}(v_d)$, can be computed using (4.17). The integral of (2.28) was solved numerically using the adaptive-Romberg algorithm for double integrals [23]. The signal-to-noise ratio after dehoppping is given by

$$SNR = \frac{E_h}{N_0} = \frac{s_{mi}^2}{2\sigma^2}. \quad (4.20)$$

The mean of $\hat{\epsilon}_i$ was computed for various ϵ_i , for various SNR , and $L = 8$, and the results are plotted in Fig. 4.9.

From Fig. 4.9, note that $E[\hat{\epsilon}_i] \neq \epsilon_i$. Hence, the estimator has a bias [60]. This is not a desirable characteristic for an estimator. Other estimators which do not have a bias are considered in [16] and [61].

Figure 4.9: $E[\hat{\epsilon}_t]$ for various SNR and for $L = 8$.

4.4 Summary

The time synchronization of a FFH NCFSK satellite-communication system is discussed in this chapter. Both the coarse-time and fine-time synchronization systems are discussed.

The performance of an uplink-coarse-time-synchronization algorithm for a FFH NCFSK satellite-communication system with time diversity and linear diversity combining in broad-band noise jamming and in multiple-tone jamming plus broad-band noise is presented. Results show that time diversity and linear diversity combining can be used to improve performance in broad-band noise. Results also show that the adaptive-threshold method does not have a constant false-alarm probability for multiple-tone jamming. With multiple-tone jamming and $\Delta T = 0$, the minimum probability of detection is achieved by jamming the entire band; however, the maximum probability of false alarm is achieved by jamming only part of the entire band.

The performance of the delay-advance method for uplink fine-time synchronization of a FFH NCFSK satellite-communication system with time diversity and linear diversity combining in broad-band noise jamming is presented. Results show that for the delay-advance method in broad-band noise jamming, the estimator is biased. As the signal-to-noise ratio is decreased, the bias in the estimator increases.

Chapter 5

Experimental System and Results

5.1 Introduction

It is desirable to show that linear diversity combining is a practical and viable method of diversity combining. Thus, a “proof-of-concept” experiment must be performed. The goal of the experiment is not to compare the performance of linear diversity combining to other diversity-combining techniques, but rather to determine whether it is possible to implement linear diversity combining in an actual system. In this chapter, a hardware-based experiment is described to test the bit-error-rate performance of a binary NCFSK modulator and demodulator with time diversity and linear diversity combining. This hardware-based experiment is designed to reflect the bit-error-rate performance of an actual fast-frequency-hopped binary NCFSK satellite-communication system. Since a frequency-hopped satellite-communication channel is unavailable, a simplified system that could be implemented on a laboratory bench has been built. This experiment could not possibly incorporate all the elements of an actual satellite system. However, all the essential elements of the satellite system pertinent to the modulation and demodulation processes have been included. One important element in an actual system is that phase is not continuous from hop to hop. Hence, phase continuity is not used to aid in demodulation.

5.2 NCFSK Modulator and Demodulator

Fig. 5.1 shows a block diagram for a binary NCFSK system. This system reflects the performance of an actual fast-frequency-hopped binary NCFSK satellite-communication system such as shown in Fig. 3.12 and Fig. 3.13. Binary data is passed to a frequency synthesizer which chooses one of the two data tones based on the data. Noise is then added to the data tone, and the sum is applied to matched filters with envelope detectors. The diversity combiner combines the output samples of the matched filters with envelope detectors to form decision statistics using linear diversity combining. Then, a decision is made, and hence, the data is received.

Figure 5.1: NCFSK modulator and demodulator.

5.3 Experimental System

5.3.1 Introduction

The experimental system is shown in Fig. 5.2. The modulator uses a data-error analyzer to generate pseudorandom binary data. This binary data selects one of two data tones on the frequency synthesizer. Noise is added to the data tone, and the sum is applied to the matched filters with envelope detectors. The demodulator is based on a discrete-Fourier transform (DFT). The matched filter with envelope detector is partially implemented in hardware and partially in software. The digital-signal processor performs the software portion of the matched filter with envelope

detector and provides the control signals for the experiment. Data is returned to the data-error analyzer so that bit-error rate measurements can be performed. In the following sections, each part of the experiment is described.

5.3.2 Data-Error Analyzer and Pulse Generator

In this system, a Hewlett Packard 1645A Data-Error Analyzer is used. The analyzer generates pseudorandom binary data with a pulse width of the external transmitter clock synchronized such that a pulse-generator negative edge causes a data transition. The received data, which has a pulse width of the external receiver clock, is compared with the transmitted data. Differences between transmitted and received bits are called bit errors. Based on a nominal amount of transmitted data, a bit-error rate can be measured by dividing the number of bit errors by the number of transmitted bits. Fig. 5.3 shows the relevant signals for the data-error analyzer. The external transmitter and receiver clocks are tied to the pulse generator which is controlled by the digital-signal processor. The data-out signal is used as the binary-data source for the system. The data-in signal is the received data from the demodulator. The pulse generator provides a clock which has a period of 5.0 ms. Thus, the data has a pulse width of 5.0 ms, and hence, the data rate is 200 bits per second. The clock generated by the pulse generator is controlled by the TMS320C25 digital-signal processor. Interfacing circuitry produces the correct clock.

5.3.3 Frequency Control and Frequency Synthesizer

The frequency synthesizer used in this experiment is a Wavetek Model 5155A Fast Switching Frequency Synthesizer. The programmable synthesizer operates in the 100 kHz to 1 GHz frequency range with 0.1 Hz frequency resolution. The synthesizer has a binary-coded-decimal (BCD) remote-programming-control capability (accessible through a 50-pin Delta connector and a 14-pin Delta connector) which can be used to remotely control both the frequency and amplitude of the synthesizer. The pins of 50-pin and 14-pin connectors are to be connected to the data lines, the latch strobes, and the execute strobes. The connector pin numbers are given in [62]. Negative-true TTL logic is used. The remote-enable line must be low

Figure 5.2: Experimental system.

Figure 5.3: Data-error analyzer.

for the front-panel control to be disabled, and the remote control to be established. All data lines are double buffered. A rising clock edge on the latch-strobe lines will store the data in the first rank of buffers. A second rising clock edge on the execute-strobe line will move the data into the second rank of buffers and cause a change in frequency and amplitude of the synthesizer. Fig. 5.4 shows the relative timing of the pulse generator, the inverted pulse generator, and the binary data. The pulse generator can be used for the latch strobes, and the inverted pulse generator can be used as the execute strobe.

Figure 5.4: Latch and execute strobe timing.

Interfacing is required from the data-error analyzer and pulse generator to the frequency synthesizer. This interfacing is shown in Fig. 5.5. The analyzer data output is applied to the data lines either unaltered or inverted as required to denote the required frequency on the data lines. A data-error-analyzer output bit of "1" corresponds to the frequency of 21.4008 MHz, and a data-error-analyzer output bit of "0" corresponds to a frequency of 21.3992 MHz. Several inverters (74LS14) are used for the output bits to avoid any fan-out problems. For the pulse-generator, the unaltered output is applied to the latch strobes, and the inverted output is applied to the execute strobe. Inverters are again used for the pulse-generator output to avoid any fan-out problems. The frequencies have been chosen for the minimum orthogonal signalling. This means that the frequency separation, Δf , is related to the hop period by

$$\Delta f = \frac{1}{T_h}. \quad (5.1)$$

For the experimental system, the data rate is 200 bits per second, and the level of diversity is $L = 8$. Thus, the equivalent hop period is $T_h = 625\mu s$. Thus, a frequency separation of $\Delta f = 1.6$ kHz is required, and the hop rate is $R_h = 1.6$ khops/s.

5.3.4 Noise Generator

The noise generator consists of a noise source, amplifiers, and filters. The noise source is a Digital Communications Corp. IF Noise Source Model 3002 which generates broad-band noise. Since the power-spectral density is not high enough directly from the output of the noise source, amplification is required. A chain of filters and amplifiers are used to do this as shown in Fig. 5.6. All filters are centered about 21.4 MHz. The chain consists of the following: K & L Microwave Inc. 500 kHz filter, Avantek UTO-521 50-dB 12-dBm amplifier, Piezo Technology Inc. 63.6 kHz filter, Avantek UTO-523 25-dB 13-dBm amplifier, Avantek UTO-523 25-dB 13-dBm amplifier, Piezo Technology Inc. 54.68 kHz filter, and Avantek UTO-1005 13-dB 22-dBm amplifier.

Figure 5.5: Frequency control and frequency synthesizer.

5.3.5 Matched Filter with Envelope Detector

The matched filters with envelope detectors were implemented using the narrow-band bandpass-signal representation [25]. This representation allows a discrete-time implementation. Two possible signals can be received. For a data bit of “1”, the received signal without noise is

$$s_1(t) = \sqrt{2}a_1 \cos(2\pi f_1 t + \phi_{1i}) \quad (5.2)$$

where a_1 is the rms signal amplitude, $f_1 = 21.4008$ MHz, and ϕ_{1i} is a random phase. For a data bit of “0”, the received signal without noise is

$$s_0(t) = \sqrt{2}a_c \cos(2\pi f_0 t + \phi_{0i}) \quad (5.3)$$

Figure 5.6: Noise generator.

where a_0 is the rms signal amplitude, $f_0 = 21.3992$ MHz, and ϕ_{0i} is a random phase. Both data signals can be rewritten as

$$s_1(t) = \sqrt{2}a_1 \cos(2\pi f_c t) \cos(2\pi(\Delta f/2)t + \phi_{1i}) - \sin(2\pi f_c t) \sin(2\pi(\Delta f/2)t + \phi_{1i}), \quad (5.4)$$

and

$$s_0(t) = \sqrt{2}a_0 \cos(2\pi f_c t) \cos(2\pi(\Delta f/2)t + \phi_{0i}) + \sin(2\pi f_c t) \sin(2\pi(\Delta f/2)t + \phi_{0i}) \quad (5.5)$$

where $\Delta f = 1.6$ kHz and $f_c = 21.4$ MHz. A complex low-pass signal can be defined by writing $s_1(t)$ and $s_0(t)$ as

$$s_1(t) = \text{Re}[u_1(t) \exp(j2\pi f_c t)] \quad (5.6)$$

and

$$s_0(t) = \text{Re}[u_0(t) \exp(j2\pi f_c t)] \quad (5.7)$$

where

$$\begin{aligned} u_1(t) &= \sqrt{2}a_1 \exp(2\pi(\Delta f/2)t + \phi_{1i}) \\ &= \sqrt{2}a_1 (\cos(2\pi(\Delta f/2)t + \phi_{1i}) + j \sin(2\pi(\Delta f/2)t + \phi_{1i})) \end{aligned} \quad (5.8)$$

and

$$\begin{aligned} u_0(t) &= \sqrt{2}a_0 \exp(-2\pi(\Delta f/2)t + \phi_{0i}) \\ &= \sqrt{2}a_0 (\cos(2\pi(\Delta f/2)t + \phi_{0i}) - j \sin(2\pi(\Delta f/2)t + \phi_{0i})). \end{aligned} \quad (5.9)$$

The received signal, $v(t)$, consists of a data tone corrupted by additive noise. Thus,

$$v(t) = s_i(t) + n(t) \quad (5.10)$$

for $i = 0, 1$ where $n(t)$ is a real noise signal. Let $r(t)$ be the complex low-pass equivalent received signal such that

$$v(t) = \text{Re}[r(t) \exp(j2\pi f_c t)]. \quad (5.11)$$

Thus,

$$r(t) = r_c(t) + jr_s(t) = u_i(t) + n_c(t) \quad (5.12)$$

for $i = 0, 1$ where $r_c(t)$ is the real in-phase component, $r_s(t)$ is the real quadrature component, and $n_c(t)$ is a complex noise signal. The matched filters with envelope detectors produce outputs,

$$z_{1i} = \left| \int_{(i-1)T_h}^{iT_h} r(t) u_1^*(t) dt \right| \quad (5.13)$$

and

$$z_{0i} = \left| \int_{(i-1)T_h}^{iT_h} r(t) u_0^*(t) dt \right| \quad (5.14)$$

where $*$ indicates a complex conjugate. The random phases can be assumed to be zero for $u_1^*(t)$ and $u_0^*(t)$ in (5.13) and (5.14). The diversity combiner forms decision statistics,

$$v_1 = \sum_{i=1}^L z_{1i}, \quad (5.15)$$

and

$$v_0 = \sum_{i=1}^L z_{0i}. \quad (5.16)$$

The largest decision statistic is chosen as the data bit.

The matched filter with envelope detector can be implemented as shown in Fig. 5.7. A complex low-pass equivalent representation of the received signal can be generated by multiplying the received signal by $\cos(2\pi f_c t)$ for the in-phase component and by $-\sin(2\pi f_c t)$ for the quadrature component and low-pass filtering. The received low-pass signal is multiplied by $u_1^*(t)$ and $u_0^*(t)$. The product is integrated for a hop period and the magnitude of the output is taken. The outputs are passed to the diversity combiner to form decision statistics. The largest decision statistic is chosen as the transmitted bit. In an actual implementation, the amplitudes of $u_1^*(t)$ and $u_0^*(t)$ would be set to some nominal value. The actual amplitudes, a_1 and a_0 , would not be estimated.

The second set of multiplications and the integrations can be implemented more easily in software. To do this, the outputs of the low-pass filters are sampled at a sampling rate of f_{sample} . Thus, the time between samples is $t_{\text{sample}} = 1/f_{\text{sample}}$. Since only one A/D converter is used, an analog multiplexer is used to switch the input to the A/D converter back and forth between the in-phase channel and the quadrature channel. Thus, both the in-phase and quadrature samples are not generated at the

Figure 5.7: Matched filter with envelope detector implementation.

same time. The quadrature samples, $r_s(n)$, are generated as

$$r_s(n) = r_s(t_s) = r_s((i-1)T_h + nt_{sample} + \varepsilon), \quad (5.17)$$

and the in-phase samples, $r_c(n)$, are generated as

$$r_c(n) = r_c(t_c) = r_c((i-1)T_h + nt_{sample} + 2\varepsilon), \quad (5.18)$$

where $t_{sample} = 25\mu s$, and $\varepsilon = 2\mu s$. A discrete multiplication can then be done in software, and the integrals of (5.13) and (5.14) can be replaced by summations. Thus, the matched filter output samples are computed by

$$z_{1i} = \left| \sum_{n=0}^{N-1} r_c(n) \cos(\pi \Delta f t_c) + r_s(n) \sin(\pi \Delta f t_s) + j[r_s(n) \cos(\pi \Delta f t_s) - r_c(n) \sin(\pi \Delta f t_c)] \right|, \quad (5.19)$$

and

$$z_{0i} = \left| \sum_{n=0}^{N-1} r_c(n) \cos(\pi \Delta f t_c) - r_s(n) \sin(\pi \Delta f t_s) + j[r_s(n) \cos(\pi \Delta f t_s) + r_c(n) \sin(\pi \Delta f t_c)] \right| \quad (5.20)$$

where N is the number of samples per hop. In this case, $N = 25$. Both (5.19) and (5.20) are the equivalent of doing a discrete-Fourier transform. The magnitude, diversity combining, and decision can also be done in software.

The hardware configuration for the matched filters with envelope detectors is shown in Fig. 5.8. The received signal is passed through a Piezo Crystal Co. Model 9690123 band-pass filter with a 21.4 MHz center frequency and a 10 kHz bandwidth. The output is applied to the input port of a Olektron HJ-302 3-dB two-way 0° power divider. Each output is applied to the RF port of a Watkins-Johnson WJ-M1 mixer. The LO ports of the mixers are each connected to the output ports of a Mini-Circuits ZMSCQ-2-21.4 two-way 90° power divider. The input port of the two-way 90° power divider is driven by a Wavetek Model 5155A fast-switching frequency synthesizer set at 21.4 MHz and 7 dBm. Thus, the outputs of the two-way 90° power divider are signals at 21.4 MHz with a relative phase shift of 90°. The IF ports of the mixers are connected to K & L Microwave Inc. 4L50-5.0-0 low-pass filters. Signal conditioning is done to the outputs of the filters in order to ensure that

both the in-phase and quadrature components have the same amplitude. Fig. 5.9 shows one leg of the signal-conditioning circuitry. The feedback resistor is $1k\Omega$. The variable resistor is implemented using a $10k\Omega$ potentiometer. The operational amplifiers are Texas Instrument TL081's. The potentiometers can be adjusted to give the required gain. The output of the signal-conditioning circuitry is applied to a Motorola MC14053 analog multiplexer. The choice of channel is controlled by the digital-signal processor. Fig. 5.10 shows the circuit diagram for the signal conditioning and the analog multiplexer. To null the dc offset of the operational amplifiers, $10k\Omega$ potentiometers are used with the center tap connected to the -5 V supply, and the two ends connected to pins 1 and 5. The control signal from the digital-signal processor is applied to pin A of the MC14053. The two signal paths are applied to pins X0 and X1. The output is available on pin X.

5.3.6 Analog-to-Digital Converter

The output of the analog multiplexer is sampled by an Analog-Devices MOD-1205 12-bit video analog-to-digital (A/D) converter. It is a twelve-bit analog-to-digital converter having a resolution of 1 mV . It can digitize input signals with swings from -2.047 to 2.047 Volts . It produces a 12 parallel bit output in NRZ form compatible with TTL. The output is in 2's complement notation. The analog-to-digital converter requires an encode signal to tell it when to sample. The encode signal is a 100 ns wide positive-going TTL pulse. The data resulting from an encode command is available two conversion cycles plus 275 ns after the encode command. This implies that three encode commands are required to get the data to the output. The data remains on the output lines until the next set of data is ready. The analog-to-digital converter can encode every 200 ns periodically or randomly. The encode commands are supplied by the digital-signal processor. The samples are then read into the digital-signal processor.

5.3.7 Digital-Signal Processor

The digital-signal-processing system used in this experiment is the TMS320C25 system, although an actual TMS320C25 digital-signal processor with associated

Figure 5.8: Matched filter with envelope detector – hardware.

Figure 5.9: Signal conditioning.

memory is not used. Instead, a special board has been installed in a Honson 286 personal computer allowing the Honson to emulate the operation of the TMS320C25. The board and its associated software are called the TMS320C25 Software Development System (SWDS). The SWDS interfaces to the target system through a 68 pin socket. The target system perceives the SWDS to be the TMS320C25 digital-signal processor. The SWDS performs all digital-signal-processor operations plus reads and writes to the memory which is located inside the Honson 286 computer. The reason for using the development system is that it allows easy software manipulation and also, simplifies the hardware design by not having to implement the memory and other components. The digital-signal processor's hardware and software are described below beginning with the hardware.

The TMS320C25 digital-signal processor is shown in Fig. 5.11. VSS and VCC are the power supply connections; VSS is tied to 0 Volts and VCC is tied to +5 Volts. The READY line is tied to +5 Volts to indicate that external devices are prepared for bus transfer of information. In the experimental system, all devices are fast enough that a wait cycle is not required. The HOLD input can be used to put data,

Figure 5.10: Signal conditioning and analog multiplexer.

address, and control lines in a high impedance state although this is not needed for the experiment. Hence, the HOLD line is also tied to +5 Volts. The SYNC input allows clock synchronization of two or more TMS320C25 microprocessors. Since only one processor is used here, the SYNC line is tied to +5 Volts. If the BIO line is low, the TMS320C25 executes a branch. This is not needed in the experiment; hence, it is tied to +5 Volts. INT0, INT1, and INT2 are external user interrupt inputs. Since external interrupts are not needed, these lines are also tied to +5 Volts. D0 through D15 are parallel data bus lines used for data input and data output to and from the microprocessor. D0 is the least significant bit, and D15 is the most significant bit. A0 through A3 are the four least significant parallel address lines. A0 is the least significant line. These address lines are needed to address the I/O space. IS is the I/O space select signal. It is always high except when it is asserted low for communicating to the I/O space. The D0-D15, A0-A3, and IS lines are used to send control signals to the rest of the system and to receive data from the rest of the system. The TMS320C25 has a 100-ns instruction cycle. The interfacing between these lines and the various components in the system are discussed in the following.

Figure 5.11: TMS320C25 digital-signal processor.

Fig. 5.12 shows the interfacing for the TMS320C25 digital-signal processor. The TMS320C25 interfaces to the I/O space through two 74AS138 integrated circuits. These 74AS138's are used for the interface to the pulse generator, data-error an-

alyzer, analog multiplexer, and A/D converter. The 74AS138 is a 3-line to 8-line decoder. The three input lines - A, B, and C - are used to select one of eight output lines - Y0 through Y7. There are three enable lines - G1, G2A, and G2B. When G1 is high, and G2A and G2B are low, the three input lines select one output line to go low. The remaining output lines are high. Other combinations of enable lines cause all the output lines to be high. The microprocessor when given the software command "OUT d,n" causes an address to be put on the address lines A0 through A3 corresponding to the number n and causes data to be put on the data lines which are located in memory location d . In general,

$$0 \leq n \leq 15. \quad (5.21)$$

Thus, a given n causes one of the output lines of the 74AS138 to go low. The Y0 and Y6 outputs are used to control the pulse generator. The Y1 and Y2 outputs are used to send return data to the data-error analyzer. The Y3 output of the first 74AS138 and the Y0 output of the second 74AS138 are used to control the analog multiplexer. The Y4 output is used to input data from the A/D converter to the digital-signal processor. The Y5 output provides the encode command for the A/D converter.

The 74AS109 integrated circuit is a dual J-K positive-edge-triggered flip-flop with clear and preset. The CLK line is tied to ground. This means that the R and S lines control the output line, Q, of the flip-flop. The setting of J and K do not matter. When R and S are high, the output line holds its value. When S is low, and R is high, Q is high. When R is low, and S is high, then, Q is low. When both are low, the output is unknown. Note that there are two flip-flops on each integrated circuit. The 74AS109's are used to provide the pulse generator signal, and the analog-multiplexer control signal, and to return the data to the data-error analyzer.

The most-significant eight bits of the sampled 12-bit output of the A/D converter are transferred to the digital-signal processor by using two 74AS245's. The 74AS245 is an octal bus transceiver with 3-state outputs. The 74AS245 transfers signals from B1 through B8 to A1 through A8 if the DIR line is tied low and the G line goes low. If the DIR line is high, the transfer goes the opposite way when the G line

goes low. When the G line is high, the A1-A8 lines and B1-B8 lines are isolated. A 16-bit word is created from the most-significant eight bits of the converter by making the least-significant eight bits of the 16-bit word zero. Thus, the 8 most significant converter output bits are tied to the B1-B8 lines of one 74AS245 and the B1-B8 lines of the other 74AS245 are grounded. The DIR line is tied low for both 74AS245 chips. The A1-A8 lines of one 74AS245 are tied to the D0-D7 data lines of the TMS320C25. The A1-A8 lines of the other 74AS245 are tied to the D8-D15 data lines of the TMS320C25. This is done so that the 8-bit A/D converter output is transferred to the upper 8-bits of the digital-signal processor's 16-bit data words. The Y4 output of one of the 74AS138 is used as the enable line and is wired to the G lines of both 74AS245's. Thus, the digital-signal processor selects the Y4 line when it wants to read the output of the analog-to-digital converter.

The TMS320C25 digital-signal processor must provide signals for the pulse generator, data-error analyzer, analog multiplexer, and A/D converter. It must also do the computation for the linear-diversity-combining algorithm. Software was written in the TMS320C25 assembly language to provide the control signals as well as do the computation. Fig. 5.13 shows the simplified computer program.

The first step is to initialize the processor. Program and data locations are allocated. Various modes, registers, and pointers are initialized. Then, constants are loaded into the required memory locations. A pulse is sent to the pulse generator. This marks the beginning of the symbol period. Since $L = 8$, there are 8 hop periods within a symbol period. Within each hop period, 25 samples are required. However, because of the time required to compute the envelopes in (5.19) and (5.20), only 24 samples can be processed. Hence, only 24 are taken. After each set of 24 is taken, the envelopes of (5.19) and (5.20) are computed. Once 4 envelopes have been computed, another pulse is sent to the pulse generator. This denotes the midpoint of the symbol period. Then, samples are acquired, and envelopes are computed until four more envelopes have been computed. Then, two decision statistics are formed. Using these two decision statistics, a decision is made. The received bit is applied to the data-error analyzer. Then, another pulse is sent to the pulse generator. The whole process repeats itself.

Figure 5.13: Simplified control signals computer program.

5.4 Experimental Results

The experimental system described above was tested. Binary NCFSK modulation was used, and the diversity level was $L = 8$. A bit-error-rate curve was generated for broad-band noise as a function of signal-to-noise ratio, $SNR = E_b/N_0$. The power of the signal, P_s , was measured using the Hewlett-Packard 8568B spectrum analyzer. Then, the energy per hop can be computed by $E_h = P_s T_h$. The noise power-spectral density, N_0 , was also measured with the spectrum analyzer using the method outlined in [64]. The measurement is conducted as follows [64]: 1) measure the noise power, P_n , in dBm with the video bandwidth much smaller than the resolution bandwidth; 2) add 2.5 dB; 3) normalize to the proper noise-power bandwidth. The correction term of 2.5 dB arises from the operation of the spectrum analyzer. The spectrum analyzer performs a measurement by envelope detection, logging, and averaging. The combination of envelope detection and logging underestimates the true rms value of the noise voltage. It is shown in [64] that for this Hewlett-Packard spectrum analyzer, a total correction of 2.5 dB is required. The proper noise-power bandwidth has been measured to be 1.2 times the resolution bandwidth, BW_r . Then, the noise power-spectral density is

$$N_0 = \frac{P_{nc}}{1.2BW_r} \quad (5.22)$$

where P_{nc} is the corrected noise power which includes the correction factor. Measurements of bit-error rate were obtained, and the results are shown in Fig. 5.14. The signal-to-noise ratio was varied by attenuating the signal or the noise by putting attenuators in the signal or noise path as appropriate. It is interesting to compare the experimental results with the analytical results. Therefore, analytical results were generated using the method of Chapter 2. Both results are shown in Fig. 5.14. The performance of the experimental results is 1.2 to 3.1 dB worse than the analytical results. This is due to several factors.

First, only 24 samples were used. The use of 25 samples was not possible because the square root in the envelope computation takes a great deal of time. Thus, one sample was dropped to allow for the square-root computation time. The square root is a major disadvantage with linear diversity combining which is not obvious

Figure 5.14: The bit-error rate in broad-band noise ($L = 8$ and $M = 2$).

at first glance. A square root is not necessary, for example, for square-law diversity combining. The use of 24 samples causes two problems: signal degradation and loss of orthogonality. Both cause performance degradation. The degradation could be avoided by the use of a second digital-signal processor as was done in [63]. The use of two processors allows one digital-signal processor to control the experiment and the other to do the computation.

Since perfect filters are assumed and actual filters are never perfect, there is a degradation caused here by aliasing of noise and signal distortion.

Truncation and round-off errors also exist in the digital-signal processor. There are contradictory requirements for dynamic range. Signals should be kept small to avoid an overflow condition, but signals should be large to reduce quantization noise.

Since frequency tracking is not performed, there may be frequency errors caused by drifting of the frequency synthesizers.

The noise level is difficult to measure. It often fluctuates and makes readings difficult.

Mismatches in devices may also cause performance degradation. One key part is the quadrature hybrid. For the system to achieve the least degradation, the two outputs of the quadrature hybrid must be as close to 90° out of phase as possible. Any variation will cause performance degradation. Also, each of the electronic components adds noise to the system. This causes additional degradation.

Despite the implementation impairments, the experiment proves the viability of linear diversity combining for fast-frequency-hopped NCFSK communication systems.

5.5 Summary

In this chapter, the implementation of a binary NCFSK modulator and demodulator with time diversity and linear diversity combining is described. The implementation shows that this type of diversity combining is a viable and practical approach for fast-frequency-hopped satellite-communication systems. The performance of the experimental results is 1.2 to 3.1 dB worse than the analytical results.

This is due to several implementation issues.

Chapter 6

Conclusions

6.1 Summary

A novel analytical method has been developed for one type of diversity combining – linear diversity combining. The method uses the unique combination of circularly symmetric function theory, Fourier-Bessel series, and Fourier series. With this method, a wide range of problems can be solved. These include obtaining performance measures for the demodulation and time synchronization of fast-frequency-hopped communication systems which employ time diversity. Where possible, results generated with the novel method have been compared to previously published results, and the results have agreed well. Also, numerous new results have been generated.

A background in the modulation and demodulation of fast-frequency-hopped noncoherent frequency-shift-keyed communications systems with time diversity and diversity combining is presented. A literature review of diversity-combining techniques and analysis methods as applied to those techniques is presented. Such a review has not been previously presented. Performance results are presented for linear combining in broad-band noise jamming, and multiple-tone jamming plus broad-band noise using the novel analytical technique. Linear combining was shown to be a near-optimal scheme in broad-band noise. Its performance is similar to that of square-law combining in broad-band noise. In multiple-tone jamming, the worst-case jamming depends on signal-to-noise ratio for linear combining. For high signal-

to-noise ratios, the worst-case is Houston-sense jamming with a relatively small jamming fraction. Randomly distributed jamming with a relatively small jamming fraction produces almost the same performance. However, for low signal-to-noise ratios, the worst-case is jamming every bin. In addition, a novel analytical result was derived for the 4-2- and 2-1-moment methods in independent Rayleigh fading. An expression is obtained for the probability of bit error with $L = 2$ and $M = 2$. For this situation, linear combining is a little bit worse than square-law combining, 2-1-moment-method combining, and 4-2-moment-method combining. However, other situations should be examined before conclusions can be drawn.

The time-synchronization problem in a fast-frequency-hopped system has been discussed. For coarse-time synchronization, performance results are presented for a system using linear diversity combining. Results show that an improvement can be achieved with the use of time diversity and diversity combining. For fine-time synchronization, performance results are presented for a system using linear diversity combining. Results show that for the delay-advance method, the estimator is biased for broad-band noise jamming.

A binary NCFSK modulator and demodulator with time diversity and linear diversity combining have been implemented in a hardware-based experiment. The implementation demonstrates that linear diversity combining is a viable and practical method for fast-frequency-hopped communication systems. The experimental performance results show an implementation loss of 1.2 to 3.1 dB.

6.2 Future Work

Future work should include: other diversity-combining techniques, extension of fading work, acquisition-time computation for coarse-time synchronization, and analysis of other methods of fine-time synchronization.

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