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ILLUSTRATION OF REGIONAL DISPARITIES IN
NORTH AMERICA / BY THE USE OF THE GRAVITY MODEL

BY

DAVID H. DOUGLAS

A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
Master of Arts

Carleton University
Ottawa, Ontario
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The undersigned hereby recommend to the Faculty of
Graduate Studies acceptance of this thesis, submitted by
David H. Douglas, in partial fulfilment of the requirements
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A B S T R A C T

This Treatise proposes and examines a general expression for distance interaction models with a view to practical application. Two methods of manipulating distance are suggested and one of these, resulting in a model named the proximity model, is tested with three variables for North America.

The potential model as developed by William Warntz is tested with data from Canada, United States, Mexico, and North America as a whole for the purpose of: comparing it with the suggested proximity model; examining the effects of each country on the potential surface of the others; and comparing the effect of a different spatial delineation of data zones.

The use of disparities at point locations between values obtained by similar interaction models using different variables, as a method of illustrating broad regions of intensity is also proposed and tested.

In summary, three interaction models, using population, retail sales, and manufacturing employment, and the disparities between identical models using different variables, are computed and mapped for North America. All maps were produced by computer using the Symap V mapping program. By experimentation with the program's special electives a method of reproducing maps in a format of quality suitable for publication purposes was developed.

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I N T R O D U C T I O N

The gravity concept of human interaction issues from the presumption that there is a measureable, generally applicable, form of interaction between individuals. Because individual behavior remains unpredictable, measurement must be made in the form of probability frequencies. An analogy to the physical situation is that the inability to measure the behavior of individual molecules does not hinder the ability to measure the behavior of masses. Boyle's and Charles' laws are classic illustrations. Random behavior of multitudes of molecules in gaseous form results in mathematically predictable relationships between pressure, temperature, and volume. Had the activity of the individual molecules, instead of the statistically distributed mass of molecules, been observable, relationships applying to the mass would not have been as obvious.

To produce interaction between persons, some channel of communication must be open between them. The probable frequency of interaction of an individual is a function of the number of individuals with whom he can interact and of the ease by which the channels of communication to him are traversed. Conversely, the probable frequency of interaction is inversely proportional to the difficulty all other individuals

have in traversing those channels. This difficulty is considered to be proportional to the intervening physical distance between the interacting individuals.

Relating masses of individuals by their distances from other masses is the concern of this treatise. "Individuals", as used in the concept are replaced by aggregate measures of a particular observation. Problems associated with the replacement of individuals by masses of individuals, the distance function and practical applications are discussed. Models are applied to Canada, the United States, and Mexico and maps of proximity and potential distributions are produced for the continent as a whole.

CHAPTER I

THE BASIC MODEL

The probability frequency of the interaction of an individual with all others may be expressed:

$$I_i = \sum_{j=1}^n f(d_{ij}) \quad (1)$$

where: I_i = probability frequency of interaction of individual i with all others;

n = the number of all other individuals; and,

$f(d_{ij})$ = some function of the physical distance between individual i and individual j .

The contribution of all individuals is considered to be equal in the aggregation of cumulative probability frequency of interaction felt by individual (i).

The availability of aggregated data makes calculation possible. Given equation (1) adjusted for a subset of the world set:

$$I_i = \sum_{j=k}^m f(d_{ij}) \quad (2)$$

where: $(m-k)$ = the population of a region.

Equation (2) is essentially the same as:

$$I_i = (m-k) f(d_{ij}) \quad (3)$$

where: d_{ij} = the average distance of the region from i .

The probability frequency of interaction felt by individual (i) equals the population of region (r), that is, $(m - k)$ divided by some function of the distance of region (r) from (i), providing that this distance is large. Aggregation of population to a point allowed the first calculations of this kind to be applied to populations.

The first explicit formulation of the gravity concept was made in such aggregate form by H. C. Carey, an American publisher. He reasoned that social phenomena was based on the same fundamental laws as physical mechanics and suggested that the populations of cities and their distance apart were factors of great importance. He suggested that people in

cities exert an influence on people in other cities, and that this influence varies directly with the populations of the cities and inversely with the distances. Carey's original statement of this concept was generally ignored for about a century.

The earliest practical use of the gravity concept of human interaction was made by direct application of the Newtonian gravitation formulae to population masses.¹ That is, the force of attraction (or interaction) between two masses is directly proportional to the product of their masses and indirectly proportional to the square of the distance between them.

$$F = - G \frac{M_i M_j}{d_{ij}^2}$$

where

F = the force of attraction;

G = the universal gravitational constant;

d_{ij} = the distance between i and j ;

M_i = the mass of i ;

M_j = the mass of j ; and,

d = unit vector along the line which defines d_{ij} since F is a vector quantity.

This represents force which is a vector quantity. In the case of an aggregate accumulation, the resultant forces on one particular mass would equal zero. The gravitational energy formula, therefore, is more easily adaptable to illustrate

1. John Q. Stewart, "An Inverse Distance Variation for Certain Social Influences", Science, 93: 2404 (1941), 89-90.

locations of aggregate centrality.

$$E_i = G \frac{M_j}{d_{ij}} \quad (4)$$

where E_i = the energy at i generated by mass j .

Applied to social phenomena, masses are simply replaced by the population of an area, or another socio-economic aggregate. Equation (4) is the same as Equation (3) but $f(d) = (1.0/d)$ and $(m-k) = M$. Because individual observations (k) to (m) are in the same region, all are a similar distance from (i) . These distances can, therefore, be considered equal. However equation (4) is of little use in calculating the potential a distribution in a region generates to locations within the region. The distances of observations (m) to (k) cannot be averaged to a simple (d) , and if any one distance approaches zero, E_i approaches infinity. A simple solution is to limit the minimum distance of a region to itself to 1.0 unit. Such a method, however, gives regions of similar magnitude similar "self interaction" values and does not take the density of observations into account. As an approximation, the point of maximum potential can be derived by integration if there is an expression for the distribution of individual observations over the area.

If an area is circular and there is a uniform distribution of observations over the area, an expression for the highest value may be derived.

Figure 1. Circular Area for Self Interaction Calculation.

Referring to Figure (1) the interaction at the center of a circle (I_o) from a band of width Δr is the number of observations in the band ($P_{\Delta r}$) multiplied by the selected function of the average distance of the band from the centre (which is one half the sum of r_1 and r_2).

$$\text{Therefore:} \quad I_o = P \Delta r f(r) \quad (5)$$

The number of observations in the band Δr equals the area of Δr multiplied by the density of observations.

$$P\Delta r = \pi(r_1 - r_2)^2 \frac{P}{\pi R^2}$$

$$I_o = \frac{P}{R^2} (r_1 - r_2) (r_1 + r_2) f(r)$$

For $\Delta r \rightarrow 0$

$$I_o = \frac{P}{R^2} f(r) 2r\Delta r$$

$$I_o = \frac{2P}{R} \int r f(r) dr \quad (6)$$

In this case the assumption was that the number of observations were evenly distributed over this circular area. Other expressions have been derived for other symmetrical areas such as ellipses and for other distributions of observations in the area.¹ If the density of observations could also be given as a function of the distance from the center, it could be seen that the solution was a simple matter of substitution. Density as an exponential function of the dis-

1. John Q. Stewart and William Warntz, "Physics of Population Distribution", Journal of Regional Science, 1 (1958), p. 120.

tance from the center promises the easiest solution to integrals.

The total interaction probability frequency for location (i) and can now be calculated.

$$I_i = G \left[I_o + P_1 f(d_{i1}) + P_2 f(d_{i2}) + \dots + P_n f(d_{in}) \right] \quad (7)$$

$$I_i = GI_o + G \sum_{\substack{j=1 \\ j \neq i}}^n P_j f(d_{ij}) \quad (8)$$

where: G = a constant selected to match units of measurement.
usually taken to be 1.0

As (I_i) can be calculated for every place, isarithms can be drawn joining all places of equal interaction to discern relative levels of interaction probability frequency.

CHAPTER II

THE FUNCTION OF DISTANCE

Most discussions of the distance variable have centered on how particular distances should be calculated.¹ It has been suggested that transportation costs in different portions of the region of study should be taken into account in the recording of distances. Should distances crossing a river or circumnavigating a large body of water such as a strait or bay be taken into account?

These questions are concerned with distances between individual observations, but it must be borne in mind that the number of individual distances to be recorded increases as the square of the number of observations.

Ten observations require 100 distances to be measured. Five hundred observations require 250,000 measurements. This kind of individual attention to the distances between particular observations is clearly impossible. For practical purposes, distances can be computed from co-ordinates of the locations only by an electronic computer.

1. Gunnar Olsson, Distance and Human Interaction (Philadelphia, Pennsylvania) Regional Science Research Institute, 1965) pages 57-65.

If a region over which a model is calculated is small, then the Pythagorean theorem using rectilinear Cartesian co-ordinates, will suffice for the calculation. If the longest area is less than one radian, latitude and longitude can be converted easily to a Cartesian system in 3 dimensions. Within that distance (4000 miles) the greatest error would be a 4% underestimation. For world scale models calculation by spherical trigonometry is required. Each of these three methods of calculating distances takes progressively longer time on the computer; this justifies the use of the shorter methods.

A question of major concern with respect to distances is the function of distance itself. Distance decay may be arithmetic, exponential, or complex.

Arithmetic forms can be used in location analysis where transport costs, for instance, vary as a direct arithmetic function of distance. The general form is:

$$f(d) = 1.0 - \frac{Md}{P} \quad (9)$$

where: M = the rate of decay;
P = weight at the location; and,
d = distance from the location.

Substitute in Equation 8

Equation (8) reduces to:

$$I_i = I_o + \sum_{\substack{j=1 \\ j \neq i}}^n P_j - M(d_{ij}) \quad (10)$$

if $M(d_{ij}) > P_j$, $M(d_{ij}) = P_j$

Arithmetic decays have been in general use in location analyses ever since the concept was introduced by Alfred Weber, but aggregate models resulting in maps have not. Aggregate models defined by Equation (10) would very likely be useful in assisting in the selection of regions for types of industry which are both market and transport oriented.

Much attention has been given to the exponent by which the distance variable should be raised.¹ Stewart and Warntz have used distance raised to the power of 1.0 depending on

1. Gunnar Olsson, Distance and Human Interaction (Philadelphia, Pennsylvania) Regional Science Research Institute, 1965) pages 57 - 65.

analogies to Newtonian mechanics. Most other researchers have argued that the correct function is distance raised to some other power.

$$\text{that is: } I_i = I_o + \sum P_j / d_{ij}^\alpha \quad (12)$$

Walter Isard devoted a great deal to the evaluation of exponents by deriving the gravity model from the probability of interaction point of view.¹ If interaction data, such as telephone calls, or interlocation transport registers, are plotted on double logarithm paper, the slope of the straight regression line is taken to be the exponent.

Equipotential maps of Canada calculated by D. M. Ray using exponents of 1.0 and 1.4 appeared to have the same peaks of potential.² Linear correlation analysis was performed using the point values from the two models as variables. There was a strong positive correlation between the two models, as was expected, but correlation co-efficients of 0.9839 -

1. Walter Isard, Methods of Regional Analysis; An Introduction to Regional Science, (Cambridge, Massachusetts.. 1967), p.p. 483 - 516.

2. These were maps computed as a part of the research resulting in an unpublished report which was submitted to the Watkins Task Force on Foreign Ownership of Manufacturing in Canada. See D. Michael Ray, Regional Aspects of Foreign Ownership of Manufacturing in Canada.

for retail sales potential and 0.9825 for population potential seemed to indicate that the greater exponent did not affect the results significantly. Changing the levels of the isarithms allowed construction of almost identical maps.¹

The reason the resultant surface does not change is that potential generated by an observation is not fixed at the observation. As (d) approaches zero, potential approaches infinity. Raising the exponent does not change the distance decay significantly except for very low values of (d). Therefore, the only thing that happens is that the peaks of potential are merely more pronounced. Similarly, low values of the exponent by which (d) is raised tend only to subdue the peaks.

This is clearly illustrated in figure (2) where a gravity model, using equation (12), was constructed for a distribution of points along a line. The exponent (α) was 1.0, 1.5 and 2.0 for figure (2) band A, B and C respectively. Dotted lines indicate actual gravity curves plotted from the values at the locations of each individual observation. Solid lines represent the curve drawn through values obtained at

1. There is some contradiction with Walter Isard on this point, however he examines the theoretical limits of the exponent, Walter Isard, op. cit., footnote on page 516.

Figure 2. Linear Gravity Models.

regularly spaced intervals, that is, at every centimeter.

The resulting curves having different exponents, are similar in appearance and the linear correlation co-efficients obtained by regression of the values were higher than 0.9646.

Returning to the Newtonian gravitational energy formula:

$$E = G \frac{M}{d} \quad (13)$$

where: G = the universal gravitational constant.

describes an energy generated to all other points as a function of the distance from the center of that mass. From this equation it could be deducted that the energy generated by the mass is infinity at its center, but this is not the case. Figure (3) illustrates graphically the performance of the energy curve represented by equation (13). This results in the curve $\overline{AB}$. Note that as (d) approaches zero, the value of (e) approaches infinity. Curve $\overline{OCB}$ represents the actual result of energy as a function of distance in the physical situation where (r) represents the radius of a spherical mass. For a circle in a plane, curve $\overline{OC}$ would be a straight line. If the distance of a socio-economic mass to itself is considered to be one distance unit, the resultant curve is $\overline{e_2GB}$, where (e_2) equals an energy value of the constant (G) (usually taken to be one), multiplied by (M) .

Inside the body the energy value does not approach infinity as dictated by equation (5), but approaches zero.¹

The mass of a spherical body as a function of the radius is:

$$M = \frac{4}{3} \pi r^3 \sigma$$

where: σ = mass per unit volume.

Let $r = d$

Then, by substitution in (13)

$$E = \frac{4}{3} \frac{\sigma \pi r^3}{r}$$

Reducing constants;

$$E = K r^2 \tag{14}$$

1. See Appendix III for a detailed proof regarding the gravitational potential inside a sphere.

This situation is not applicable for calculation of a model from socio-economic data. What happens inside an individual unit observation is not of concern.

Figure 3. Interaction Curves Generated by a Mass.

In the potential model where $f(d) = 1/d$, equation (6) integrates so that self-potential equals $2P/R$. Graphically this integration results in a potential surface defined by the line $\overline{e_1FB}$. A constant added to the distance variable moves the ordinate to the right, where O' , the new origin,

equals that constant. This is the basis of a new model called the "proximity model."¹

The proximity model with a complex distance decay, is represented by the equation:

(15)

$$P_x = \frac{M}{(1 + \alpha d)}$$

This equation represents proximity as a function of distance from the generating object in exactly the same way as does the potential model, except the maximum value is fixed at the object. It does not tend to infinity as (d) approaches zero. Because of this, the decay factor can be powerfully adjusted by merely changing the value of the constant. This constant may be selected with a view to halving the proximity at a certain distance ((αd) must equal 1.0 at that distance).

$$f(d) = \frac{1}{2} \quad (16)$$

1. Theodore Anderson first added a constant to the distance variable in order to overcome the problem of dividing by zero, however, this was applied to aggregates and not to individuals. See Theodore R. Anderson "Potential Models and the Spatial Distribution of Population", Papers and Proceedings of the Regional Science Association, II 1956, p. 53.

The general form of the gravity model equation becomes:

$$Px_i = G Px_o + G \sum_{\substack{j=1 \\ j \neq i}}^n \frac{1}{1+\alpha d} \quad (17)$$

P_{x_o} is calculated from the general expression for self interaction for a circular area of even distribution. See Equation (6).

$$\begin{aligned} Px_o &= \frac{2P}{R^2} \int_{r=0}^R \frac{rdr}{(1+\alpha r)} \\ &= \frac{2P}{R^2} \left[\frac{1}{\alpha^2} \{1 + \alpha r - \log(1 + \alpha r)\} \right]_{r=0}^R \\ &= \frac{2P}{R^2 \alpha^2} \{ \alpha R - \log(1 + \alpha R) \} \quad (18) \end{aligned}$$

Figure (2) bands D, E and F illustrate the performance of the proximity curve in a linear situation when α is changed so that proximity is halved at 2 millimeters, 2 centimeters and 4 centimeters respectively. The resulting curves clearly show the increases given to central locations as a result of

lower distance decays. This is in direct contrast to the potential models illustrated in bands B, C, and D. This model was specifically developed to allow the introduction of low distance decays in the study of proximity to population, market and manufacturing in North America.

CHAPTER III

POPULATION POTENTIALS: CANADA, UNITED STATES AND MEXICO

Figure (4) illustrates population potential for Canada, the counterineous United States, and Mexico using the 230 Canadian counties and census divisions, including one point at Whitehorse for the combined regions of the Yukon and Northwest Territories, 306 United States aggregated Congressional Districts, and the 32 states and territories of Mexico for calculation. Potentials for the three countries were calculated independently and isarithms were interpolated from the area control points. The heavy two-headed arrows appearing at the borders may serve as a reminder that the potential surfaces were calculated and interpolated separately for the three countries, and that in fact there are three separate maps. Figure (5) on the other hand, shows population potential for North America and uses these same 568 data points for Canada, the United States and Mexico. The only exception is that data points for Alaska and Hawaii were included for computation as well as interpolation control points (even though they lay beyond the borders of the illustrated map).

Figure 4. Population Potential: Canada, United States and Mexico, 1961.

For all four population potential models computed, the highest value for any single point was indexed at 1000. For North America and United States this point was New York at 262.38×10^4 persons per mile and 256.38×10^4 persons per mile respectively. For Canada it was Montreal at 47.37×10^4 persons per mile and for Mexico it was Mexico City at 94.30×10^4 persons per mile. The isarithm interval used between indices 1000 and 400 was 200, between 400 and 100: twenty, and below 100: ten.

These intervals were purposely selected to allow direct comparison with the map of United States produced for the American Geographical Society by William Warntz.¹

The population potential surface for Canada presents a fragmented and multi-insular pattern. There are at least eight identifiable peaks separated by valleys or divides.

The 18% isarithm (18% of the maximum potential) includes sub-peaks occurring at Ottawa and Quebec City within the Montreal massif. Within the massif centered at Toronto are all the cities of Southern Ontario from London to Oshawa. The 18% isarithm joins the two massifs into a Central Canada Axis. Except at Winnipeg, potentials above 16% occur nowhere

1. William Warntz, "A New Map of the Surface of Population Potentials for the United States, 1960", Geographical Review, LIV, 2 (1964), 171 - 184

else in Canada, although lesser peaks occur at Calgary, Edmonton and while not isolated by isarithms drawn at these intervals, at Regina, Saskatoon, Halifax and St. John's. Major divides occur at Kingston and Fort Francis, while minor ones separate the Vancouver sub-peak from the Calgary-Edmonton ridge, the Calgary-Edmonton ridge from Central Saskatchewan, Central Saskatchewan from Winnipeg, and Western from Eastern Newfoundland.

Predominance of Montreal over Toronto, and of Winnipeg over Vancouver illustrates weaknesses inherent in the self-potential calculation which assumes an even distribution over the area. Potential models computed with the self potential, taken to be population divided by 1.0 and not by the definite integral, give Toronto predominance over Montreal, and Vancouver over Winnipeg, which would seem to be more in accord with the expected situation. The area of the County of Montreal and Jesus Islands is much smaller than the area of York County. This gives Montreal a smaller demonimator in the self-potential computation and accounts for its predominance. Also, data is given for Census Division 20 in Manitoba which is defined by the boundary of Winnipeg. Vancouver on the other hand is part of Census Division 4 of British Columbia which is large enough to include vast areas of wilderness. Resolution of these anomalies awaits a more sophisticated, but universally applicable method of computing

potentials on the micro level, some of which were mentioned previously.

There were only 306 data points used in the computation of population potential for the United States in this case, as opposed to the 3,105 used by William Warntz in the construction of the map for the American Geographical Society. In spite of this difference the two maps are remarkably similar in both their appearance and the values resulting for specific points. On the Warntz map the highest value indicated is 250×10^4 persons per mile compared to 256×10^4 for this one.

A complex "Y" shaped demographic basin divides the United States into three demographic massifs. The Axis formed by Los Angeles and San Francisco defines the California massif. Another, more minor massif has its peak at Portland, Oregon, however, the major feature of United States potential is the third and major massif centered at New York City. The New York and California massifs are joined by a slight undulation in the basin which passes through Salt Lake City which could be called a col. Peaks at Boston, New York, Philadelphia, Baltimore and Washington form a ridge line along the mid-Atlantic coast. These are outlined by the 20% isarithm which is a fair representation of the boundary of the Eastern Seaboard Megalopolis. A region including Detroit, Cleveland,

Pittsburgh, and Cincinnati, (Lake Erie Megalopolis) form a plateau also defined by the 20% isarithm. This isarithm isolates a minor plateau joining Chicago and Milwaukee, as well as the cities of St. Louis and San Francisco. That region, generally referred to as the American Manufacturing Belt, is isolated roughly by the 18% isarithm which forms an axis from New York to Chicago. It is interesting to notice that the 18% isarithm in Canada joins the Montreal massif to the Toronto massif, although they are not equivalent. As a matter of fact, the potential at Montreal, using Canadian observations only, would not be great enough to include it above the 20% isarithm using United States values.

Mexico presents an entirely different picture than does United States or Canada. Both of the latter countries, but more especially Canada, are broken up into several massifs with demographic peaks, cols, passes, divides and basins presenting very complex patterns over the continent. The potential surface of Mexico resembles a uniform cone or a rotated hyperbola with its apex at Mexico City. The potential declines in all directions from Mexico City with but four locations presenting minor exceptions. These exceptions are subtle sub-peaks at Toluca in the Entidade of Mexico, Puebla in Puebla Entidade and Villahermosa in Tabasco Entidade. A pit of low potential occurs at Pachuca in the Antidade of Hidalgo.

One interesting plateau in the conical surface occurs at the 38% isarithm and joins Mexico City to the sea at Tampico. Main transportation routes however, join Mexico City to the sea at Vera Cruz, which is approximately 200 miles south of Tampico. A sloping ridge-line, with its axis in easterly direction from Mexico City, more closely approximates Mexico City's connection to the Gulf.

CHAPTER IV

POPULATION POTENTIAL: NORTH AMERICA

The area over which a gravity model is constructed must be meaningfully delineated. The great temptation that exists is to select a national, or subnational territory. One reason, of course, is the availability of a standard definition of data representative of one point in time for all observations. Another is nationalistic interest which tends to ignore that which is outside a nation's boundary. But a problem of far greater concern is the effect of the boundaries themselves. Most certainly boundaries affect the distance decay factor for specific types of interaction.

Gravity model study for a specific purpose, such as the determination of market for a particular product, may require the observation-weights, or the distance decay factor to be adjusted if the line joining pairs of observations crosses a customs frontier. Adjustments of this type may be affected by the inclusion of a simple test in the computation. Proximity and potential models of population, however, are concerned with all types of interaction and an adjustment caused by a boundary must be very complex indeed.¹

-
1. For a discussion of the boundary effect on Telephone calls from Montreal see: J. Ross MacKay, "The Interaction Hypothesis and Boundaries in Canada: A Preliminary Study", Canadian Geographer, 11 (1958) 1 - 8.

Every individual observation in the world contributes to the aggregated values of every other. Elimination of the rest of the world-set from the area subset must be carefully justified. A population potential model computed from eastern Ontario townships would hardly have meaning owing to the overwhelming effect of outside influences. This argument can be extended eliminating all efforts in this direction, save a model constructed on a world scale. But justification can be found if there is reason to believe that the aggregate influence of all locations outside this area would be so small, or uniform over the entire area that they can be neglected or at least accounted for in one's intellectual approach to the results. The selection of the United States as a region of study may perhaps be justified on these grounds. The effect of Canada on United States has been expected to be small, and other population centers are far away. Mexico for the same reasons may be a reasonable selection.

Canada, on the other hand, cannot be considered a meaningful isolated area for potential studies. Tenuously stretched along the northern frontier of the American monolith, with southern Ontario forming a conspicuous arrowhead jutting into its heart, a gravity model constructed, with Canadian data only, must almost be meaningless.

Figure (5) represents population potential for North America, computed in exactly the same way as it was for the three countries separately. The same data points were used except that points representing Hawaii and Alaska were included. Population figures were taken from the respective censuses and converted through time to 1 July 1961, using United Nations Tables. The effect of the boundaries was ignored to allow comparison of the extremes of absolute, as illustrated in Figure (4), and negligible barriers, as in Figure (5).

Cursory comparison of Figures (4) and (5) reveals almost no change in the patterns for the United States. The same "Y" shaped basin of low values separates the California, Portland and New York massifs. The Eastern Seaboard Megalopolis and the combined cities of Detroit, Cleveland, Pittsburgh, and Cincinnati are on isolated plateaus. Slight changes of the levels of the isarithms account for some of the apparent changes such as the exclusion of Rockford, Illinois from the New York-Chicago Axis and the formation of an island at Salt Lake City instead of a ridge in the "Y" shaped basin. It appears therefore that population potential of the United States does not suffer a very meaningful disruption owing to the inclusion of data from Canada and Mexico.

Figure 5. Population Potential, North America, 1961.

Closer examination reveals the actual effects of the populations of Canada and Mexico. New York's potential is raised by 2.2% (although it is still indexed at 1000) of which 1.7% is due to Canada. The Megalopolis plateaus are isolated by the 22% instead of the 20%, isarithm and the New York-Chicago Axis is defined by the 20% instead of the 18% isarithm. The absolute values of potential of all locations are raised somewhat. The value at Chicago is raised by 5.6% and at Los Angeles by 7.9%. Values at locations closer to borders are raised more than the rest. For example, the value at Syracuse, New York is raised by 14.5%; Fargo, North Dakota by 18.2%; Buffalo, New York by 21.6%; Montpelier, Vermont by 22%; and Laredo, Texas, by an amount of 73.1%. The inclusion of Canada tends to draw the potential surface of the North Eastern United States northward. The 18% isarithm is conspicuously further to the north passing through Kingston, Ontario instead of Rochester, New York. Canada and Mexico together tend to raise the levels of all locations of lower values and especially those in the western "Y" shaped basin.

Visual examination of the patterns in Mexico indicates that Mexico is not affected very much by the rest of North America. Mexico City still commands a position of dominance with values falling away the further one moves from it. In fact, the absolute value of potential at Mexico City is raised

by 16.5%, while those of the locations around are almost double. Oaxaca de Juarez in the Entidade of Oaxaco, south-east of Mexico City is raised by 93%. Monterrey in Nuevo Leon is raised by 98% but values in north west Mexico are raised by the greatest amount. Hermosillo in the Entidade of Sonora is raised by a factor of 3.5 while Mexicali in Baja California is raised by a factor of five.

Although all values in United States and Mexico are raised to a greater or lesser degree, there are no major interruptions in either country. The centers remain the same, while massifs and divides all remain in the same locations.

Canada is another case altogether. The most prominent change that occurs is the shift of the location of highest value from Montreal to Windsor which is now included in the Lake Erie Megalopolis defined by the 22% isarithm. The absolute value of potential at Windsor is raised by a factor of eleven, and Toronto by a factor of 2.5, while Montreal is raised by only 71%. The divide between the Southern Ontario Massif and the Montreal Massif remains, but is not as prominent. The value for Kingston in Frontenac County is raised by a factor of 6.7. The isarithm that isolates central Canada from the rest of Canada penetrates far into the south of United States and includes the cities of St. Louis, Missouri, and Nashville, Tennessee. Except for the Frontenac Axis, there are no other prominent divides. Instead, the surface

suggests a plane which declines gently to the periphery from its high point centered just west of New York.

In general the Potential map of North America highlights New York as the dominant location, which is a value almost twice that of any other place. It is the demographic centre of more than two thirds of the continent, and successive isarithms which surround it outline several hierarchies of city systems. In turn cities of the Eastern Seaboard Megalopolis, the New York-Chicago Axis and eastern North America are defined as systems by isarithms which hypothetically surround New York City. It is interesting to note that in the regions outside the isarithm which surrounds the New York-Chicago Axis there are only four locations with potentials above 20 per cent. These are San Francisco, Los Angeles, Mexico City, and Montreal. The latter two of these represent centres for different language cultures.

CHAPTER V

POTENTIAL AND PROXIMITY IN NORTH AMERICA

Population density in North America is illustrated by isopleth in Figure 6. This map was interpolated from central data points with values obtained by dividing the population in the data area by its area in square miles. Although the map does give a general picture of where the most densely populated areas are it also clearly illustrates the weakness of plotting values of this type by the isoline technique. Nowhere is this weakness more evident than around Denver, Colorado. The Congressional District which is represented by Denver is defined by the city limits of Denver itself. The density value, therefore, for Denver is an urban one which rises so high above the plane of low values around it so that a vast volcano shaped cone of density is interpolated. Surrounding areas with low values, more representative of the Great Plains' region, merely appear as pits in this cone and are only vaguely discernible.

The areal units selected for this study of United States were Congressional Districts which all have approximately the same population. The variations in areas, of course, is what accounts for the different densities. If the areas were all the same, the populations would have to be different. A

Figure 6. Population Density: North America, 1961. The isopleth interval is such that the same number of observations appear in each of seven levels. The isopleth values are: 335; 112; 66; 44; 25 and 11 persons per square mile.

slight change in a boundary can vastly affect the density. Compare Winnipeg and Vancouver for example. As mentioned previously, the boundaries of the Winnipeg census division are coincident with the city limits, and density there rises in a cone. The census division of which Vancouver is the control point is large in comparison, and parts of it are unpopulated. While both cities likely have a similar urban density, Winnipeg appears to have a far greater density.

One of the tools most often used to illustrate regional patterns is the simple ratio measure of two variables, or, in its more sophisticated form: the concentration quotient. Figures 7 and 8 represent such measures of manufacturing employees and population, and retail sales and population. Both figures give a broad measure of intensity, but both are misleading in the illustration of regions of manufacturing and retail sales intensity. Northern British Columbia is certainly not in contention with Michigan as a manufacturing region although this impression is implied in Figure 16. Similarly a person looking for marketing regions may be misled if he assumed Nevada provided more opportunities than Ontario.

The gravity model is useful in smoothing out such a fragmented distribution and gives a much better picture of aggregations of population.

7. Manufacturing Employees per Capita: North America, 1961. The isopleth values are: 21; 18; 15; 12; 9; 6 and 3 manufacturing employees per 100 persons.

Figure 8. Retail Sales per Capita: North America, 1961. The isopleth values are: 1500; 1200; 900; 700; 200 and 100 United States Dollars per capita.

Figures 9 to 17 are gravity models (the potential model and two proximity models) for North America using population, retail sales, and manufacturing employment as weights. If these maps are removed and arranged in a three by three matrix for viewing, comparisons may be easily made. Figures 9, 10 and 11, have population as a weight; figures 12, 13, and 14 have retail sales; and Figures 15, 16 and 17 have manufacturing employment as a weight.

Figures 9, 12 and 15, are potential models computed by equation (12) with $\alpha = 1.0$.

The other figures of this series are proximity models computed by Equation (17). The proximity is considered to be halved at 5 miles for Figures 10, 13 and 16, and halved at 100 miles for Figures 11, 14 and 17. All of these maps have the highest value indexed at 1000, and all of the isarithms are at the same levels. These levels were selected to allow the best illustration of patterns for the whole series.

The primary concern of this chapter is the comparison between models using the same variable as a weight, therefore, discussion of comparisons between the different variables using the same model will be postponed until the following chapter.

Figure 9 is the same map as Figure 5, except that the isarithm levels are different. With the isarithms at these

Figure 9 Population Potential.

Figure 10. Proximity to Population, Proximity Considered to be Halved at 5 Miles.

Figure 11. Proximity to Population, Proximity Considered to be Halved at 100 Miles.

Figure 12 . Retail Sales Potential.

Figure 13. Proximity to Retail Sales, Proximity Considered to be Halved at 5 Miles.

Figure 14. Proximity to Retail Sales, Proximity Considered to be Halved at 100 Miles.

Figure 15. Manufacturing Employment Potential.

Figure 16. Proximity to Manufacturing Employment, Proximity Considered to be Halved at 5 Miles.

Figure 17. Proximity to Manufacturing Employment, Proximity Considered to be Halved at 100 Miles.

levels, the extreme gradient of potential from a high point at New York is obvious. New York appears as a spire rising almost more than twice as high as the second highest point. The Eastern Seaboard Megalopolis is not outlined until the fifth level, the highest value of which is 36% of New York. Only a few cities outside the New York cone have potentials above the lower limit of level 5. These are Toronto, Boston, Pittsburgh, Cleveland, Cincinnati, Chicago, Milwaukee and Mexico City. The model results in an extremely variable surface which highlights peaks of local potential.

The histogram shows that most locations are in a region of lower levels.¹ Proximity to population considered halved at 5 miles is also a very high decay factor, but only 1/5 as high as the Potential Model's.² The map of Figure 10 is remarkably similar to the potential map. Figure 11 represents a different picture. Most of the values in Figure 10 are in the higher levels and the area in level 1 has greatly increased. This emphasizes that most of the area in the

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1. Because the observations are of different sizes the histogram has no statistical meaning. It was included, however, so that comparisons between maps can be made.
 2. The distance decays of course cannot be compared this easily because of the different functions used. One fifth is only an approximation.

region of the New York-Chicago Axis has a value of proximity above 81% of the highest value. The whole surface is greatly smoothed out and the whole continent (with the exception of Mexico) resembles one huge massif with its high point at New York, and its center the New York-Chicago Axis.

The greater emphasis given to overall centrality, using the proximity model with low distance decays (see Figure 2), has resulted in the decline of even San Francisco and Los Angeles as local centers.

The models computed for retail sales and manufacturing react to the different distance decays in exactly the same fashion. Low decays highlight overall centrality and high decays accentuate local concentrations.

CHAPTER VI

GRAVITY MODEL DISPARITIES

If disparities between raw variables can be used to indicate concentration, and gravity models, measures of interaction probability, then the disparity between two gravity models may be thought of as a probability measure of concentration.¹

$$Dis_i = \frac{M_{1i} - M_{2i}}{M_{1i} + M_{2i}}$$

where: Dis_i = Disparity at location i ;
 M_{1i} = Gravity model value with variable 1 at i ;
 M_{2i} = Gravity model value with variable 2 at i .

Remembering that high distance decays highlight local concentrations, and low ones overall centrality, the concentration quotient of two models may define more accurately than before, broad regions of disparity. Figures 18 to 26 represent the test and examination of this hypothesis. These maps also can be removed and arranged in a three by

1. "Regional Aspects of Foreign Ownership of Manufacturing in Canada", A Study prepared for the Task Force on the Structure of Canadian Industry, Sept. 1967).

Figure 18. Disparities between the Potentials of Population and Retail Sales.

Figure 19. Disparities between the Proximities to Population and Retail Sales, Proximities Considered to be Halved at 5 Miles.

Figure 20. Disparities between the Proximities to Population and Retail Sales, Proximities Considered to be Halved at 100 Miles.

Figure 21. Disparities between the Potentials of Manufacturing Employment and Population.

Figure 22. Disparities between the Proximities to Manufacturing Employment and Population, Proximities Considered to be Halved at 5 Miles.

Figure 23. Disparities between the Proximities to Manufacturing Employment and Population, Proximities Considered to be Halved at 100 Miles.

Figure 24. Disparities between the Potentials of Manufacturing Employment and Retail Sales.

Figure 25. Disparities between the Proximities to Manufacturing Employment and Retail Sales, Proximities Considered to be Halved at 5 Miles.

Figure 27. Disparities between the Proximities to Population and Retail Sales, North America, Proximities Considered Halved at 100 Miles.^a

^a This map cannot be considered valid owing to the non-compatibility of Mexican Retail Sales Data.

three matrix for comparison. Figures 18, 19, and 20 are disparities using the potential model while figures 21, 22 and 23 are disparities using the proximity models where the proximities are considered to be halved at 5 miles. Figures 24, 25 and 26 are disparities, again using the proximity model, where proximities are considered to be halved at 100 miles. All figures have the value range divided into equal parts for the isarithms. Since the levels indicate a comparison only, the values at which the isarithms are located are not significant. The intervals, therefore, represent one seventh the range.

Mexico has been excluded from all model and disparity computations for retail sales and population owing to the non-compatibility of retail sales data.¹

Regions with a greater probability of interaction with population than with retail sales are at an economic disadvantage with the rest.

The disadvantaged regions that stand out in the two models with high decay are the south east of United States,

1. The inclusion of Mexico Retail Sales data in the computation of Potential and Proximity models for North America, (see Figures 10, 11 and 12), did not affect the location of the isarithms significantly in the other countries. It greatly affected the disparities between retail sales and population, (Figure 28) but not manufacturing employment and retail sales. See Appendix V for Proximity, Potential and Disparity Models for Mexico.

Quebec and the Atlantic Provinces, and the northern Prairie Provinces. Advantaged regions are centered in the mid-Atlantic States, Denver, Colorado, and the California coast.

When a low distance decay is used there are significant changes although the southeastern United States and the Atlantic Provinces remain the most disadvantaged regions. The northern Prairie Provinces lose significance and proximity to New York and the mid-Atlantic States shifts the location of disadvantaged regions from Montreal and southern Quebec to southern Ontario. Denver, Minneapolis, Portland and Dallas lose significance as advantaged regions. Instead, there are east and west coast centers of advantage separated by an hourglass shaped corridor of disadvantage which passes through Ohio and southern Ontario.

Regions with a greater probability of interaction with manufacturing employment than with population may be called manufacturing regions. Figures 21, 22 and 23 illustrate the patterns. The New York-Chicago Axis stands out for lower decays with minor regions appearing near Los Angeles and the Pacific North West. Darkest regions join the Niagara Peninsula to the Atlantic coast between Baltimore and Portland, Maine. Other areas of highest intensity, appear at Chicago, Cleveland and Pittsburg.

Figure 28. Disparities between the Potentials of Manufacturing Employment and Population: Canada, United States and Mexico. These disparities were computed from Potential Models which were calculated independently for the three countries. Isarithms were not interpolated across boundaries.

For low distance decays the region of highest intensity is a block defined by vertices at Buffalo, Montreal, Portland and Norfolk, Virginia.

The disparities between models of manufacturing employment and retail sales are similar to those with manufacturing employment and population, except that in the latter, some regions with relatively low retail sales emerge as manufacturing regions. The most prominent areas to appear thus, are the southern Appalachian region and central Mexico.¹ The Golden Horseshoe, Montreal and region, also stand out as important manufacturing regions, and an axis between New York and Montreal becomes apparent.

As a further illustration Figure 28 shows the disparity between the potentials of manufacturing employment and population for the three countries calculated and plotted separately. Also, all of the possible models presented in Chapters V and VI, were calculated and mapped for each of the three North American countries, however the present scope of this study does not allow a thorough discussion of each.

1. As Mexico retail sales data is not comparable the regions in Mexico cannot be compared to regions of the same levels elsewhere. The inclusion of Mexico did not, however, significantly alter the levels in the other two countries.

C O N C L U S I O N

Gravity models have been explored and two new models were suggested to allow the introduction of different distance decays. One of the two new models was tested with two different distance decays to show the extremes.

The potential model was applied to North American data, and in some detail the effects of each of the three countries on each other were examined and found to be significant in specific cases. A new concept, the disparity between similar models with different variables, was introduced to illustrate manufacturing regions and regions of retail sales disadvantage. The success of this approach was not statistically tested, but there is a strong suggestion of its usefulness by visual impression.

While not forming a specific part of the body of this thesis, one important portion of the research was involved with computer mapping techniques and photographic reproduction. The figures in this text bespeak the results.

AREAL UNITS AND CENTRAL DATA POINTS
FOR NORTH AMERICA GRAVITY MODELS

<u>Areal Unit</u>		<u>Central Data Point</u>
<u>CANADA</u>		
Newfoundland	Census Div. 1	St. John's
	" 2	Grand Bank
	" 3	Port Aux Basques
	" 4	Stephenville
	" 5	Corner Brook
	" 6	Grand Falls
	" 7	Clareville
	" 8	Lewisport
	" 9	St. Anthony
	" 10	Goose Bay
Prince Edward Island	Kings	Souris
	Prince	Summerside
	Queens	Charlottetown
Nova Scotia	Annapolis	Middleton
	Antigonish	Antigonish
	Cape Breton	Sidney
	Colchester	Truro
	Cumberland	Amherst
	Digby	Digby
	Guysborough	Canso
	Halifax	Halifax
	Hants	Windsor
	Inverness	Inverness

<u>Areal Unit</u>	<u>Central Data Point</u>
CANADA - continued	
Nova Scotia - continued	
Kings	Kentville
Lunenburg	Bridgewater
Pictou	New Glasgow
Queens	Liverpool
Richmond	Richmond
Shelburne	Shelburne
Victoria	Dingwell
Yarmouth	Yarmouth
New Brunswick	
Albert	Albert
Carleton	Woodstock
Charlotte	St. Stephen
Gloucester	Bathurst
Kent	Richibucto
Kings	Sussex
Madawaska	Edmundston
Northumberland	Chatham
Queens	Chipman
Restigouche	Campbellton
St. John	St. John
Sunbury	Oromocto
Victoria	Grand Falls
Westmorland	Moncton
York	Fredricton

APPENDIX I -- continued

<u>Areal Unit</u>	<u>Central Data Point</u>
Canada (continued)	
Quebec	Val D'or
Abitibi	Lachute
Argenteuil	Victoriaville
Arthabaska	Actonvale
Bagot	St. Georges
Beauce	Valleyfield
Beauharnois	St. Raphael
Bellechasse	Berthierville
Berthier	Bonaventure
Bonaventure	Sutton
Brome	Jacques Cartier
Chambly	Cape Madelaine
Champlain	Baie St. Paul
Charlevoix	Chateauguay
Chateauguay	Chicoutimi
Chicoutimi	East Angus
Compton	St. Eustache
Deux-Montagnes	Lac Etchemin
Dorchester	Drummondville
Drummond	Lac Megantic
Frontenac	Chandler
Gaspé	Hull
Hull	Huntingdon
Huntingdon	

<u>Areal Unit</u>	<u>Central Data Point</u>
Canada (Continued)	
Quebec - continued	
Iberville	Iberville
Joliette	Joliette
Kamouraska	St. Anne
Labelle	Mont Laurier
Lac-St. Jean	Alma
Laprairie	La Prairie
L'Assomption	Repentigny
Levis	Levis
L'Islet	L'Isletville
Lotbiniere	St. Croix
Maskinonge	Louiseville
Matane	Matane
Megantic	Thetford Mines
Missisquoi	Cowansville
Montcalm	Rawdon
Montmagny	Montmagny
Montmorency	Beaupre
Ille-de-Montreal	Montreal
Napierville	St. Remi
Nicolet	Nicolet
Papineau	Buckingham
Pontiac	Shawville
Portneuf	St. Raymond

APPENDIX I -- continued

<u>Areal Unit</u>	<u>Central Data Point</u>
Canada (continued)	
Quebec - continued	
Quebec	Quebec
Richelieu	Sorel
Richmond	Asbestos
Rimouski	Rimouski
Rouville	Marieville
Saguenay	Sept Iles
Shefford	Granby
Sherbrooke	Sherbrooke
Soulanges	La Station
Stanstead	Megog
St. Hyacinthe	St Hyacinth
St. Jean	St Jean
St. Maurice	Trois Rivieres
Temiscamingue	Rouyn
Temiscouata	Riviere du Loup
Terrehonne	St. Jerome
Vaudreuil	Dorion
Vercheres	Beloeil
Wolfe	Disraeli
Yamaska	Pierreville
Ontario	
Algoma	S.S. Marie
Brant	Brantford
Bruce	Walkerton
Carleton	Ottawa

APPENDIX I -- continued

<u>Areal Unit</u>	<u>Central Data Point</u>
Canada (continued)	
Ontario - continued	
Cochrane	Timmins
Dufferin	Orangeville
Dundas	Morrisburg
Durham	Port Hope
Elgin	St. Thomas
Essex	Windsor
Frontenac	Kingston
Glengarry	Alexandria
Grenville	Prescott
Grey	Owensound
Haldimand	Dunnville
Haliburton	Haliburton
Halton	Burlington
Hastings	Bellville
Huron	Goderich
Kenora	Kenora
Kent	Chatham
Lambton	Sarnia
Lanark	Smiths Falls
Leeds	Brockville
Lennox & Addington	Napanee
Lincoln	St. Catherines
Manitoulin	Little Current

APPENDIX I -- continued

<u>Areal Unit</u>		<u>Central Data Point</u>
Canada (continued)		
Ontario - continued		
6	Middlesex	London
	Muskoka	Huntsville
	Nipissing	North Bay
	Norfolk	Simcoe
	Nurthumberland	Coburg
	Ontario	Oshawa
	Oxford	Woodstock
	Parry Sound	Parry Sound
	Peel	Brampton
	Perth	Stratford
	Peterborough	Peterborough
	Prescott	Hawsbury
	Prince Edward	Picton
	Rainy River	Fort Frances
	Renfrew	Pembroke
	Russell	Rockland
	Simcoe	Barry
	Stormont	Cornwall
	Sudbury	Sudbury
	Thunder Bay	Port Arthur
	Timiskaming	New Liskard
	Victoria	Lindsay
	Waterloo	Waterloo

APPENDIX I -- continued

<u>Areal Unit</u>	<u>Central Data Point</u>
Canada (continued)	
Ontario - continued	
Welland	Welland
Welland	Guelph
Wellington	Hamilton
Wentworth	Toronto
Manitoba	
Division 1	Steinbach
" 2	Morden
" 3	Killarney
" 4	Boissevain
" 5	Selkirk
" 6	Portage LaPrairie
" 7	Brandon
" 8	Virden
" 9	Stonewall
" 10	Neepawa
" 11	Shoal Lake
" 12	Gimli
" 13	Russell
" 14	Roblin
" 15	Swan River
" 16	Flin Flon
" 17	Dauphin
" 18	Winnipegosis
" 19	Powerview
" 20	Winnipeg

APPENDIX I -- continued

<u>Areal Unit</u>		<u>Central Data Point</u>
Canada (continued)		
Sask.	Division 1	Estevan
	" 2	Weyburn
	" 3	Assiniboia
	" 4	Maple Creek
	" 5	Melville
	" 6	Regina
	" 7	Moose Jaw
	" 8	Swift Current
	" 9	Yorkton
	" 10	Wynyard
	" 11	Saskatoon
	" 12	Biggar
	" 13	Kindersley
	" 14	Melfort
	" 15	Prince Albert
	" 16	North Battleford
	" 17	Lloydminster
	" 18	Creighton
Alberta	" 1	Medicine Hat
	" 2	Lethbridge
	" 3	Pincher Creek
	" 4	Hanna
	" 5	Drumheller

APPENDIX I -- continued

<u>Areal Unit</u>		<u>Central Data Point</u>
Canada (continued)		
Alberta - continued		
	Division 6	Calgary
	" 7	Stettler
	" 8	Red Deer
	" 9	Banff
	" 10	Camrose
	" 11	Edmonton
	" 12	St. Paul
	" 13	Barrhead
	" 14	Minton
	" 15	Grand Prairie
British Columbia		
	Division 1	Kimberly
	" 2	Trail
	" 3	Penticton
	" 4	Vancouver
	" 5	Victoria
	" 6	Kamloops
	" 7	Powell River
	" 8	Prince George
	" 9	Prince Rupert
	" 10	Dawson Creek
Yukon and Northwest Territories		
	Division 1	Whitehorse

APPENDIX I -- continued

<u>Areal Unit</u>				<u>Central Data Point</u>
United States				
Alabama	Congressional Dist. 1-8			Birmingham
Alaska	"	"	1	Anchorage
Arizona	"	"	1	Phoenix
	"	"	2	Tucson
	"	"	3	Flagstaff
Arkansas	"	"	1	Jonesboro
	"	"	2	Little Rock
	"	"	3	Fort Smith
	"	"	4	Hot Springs
California	"	"	1	Eureka
	"	"	2	Chico
	"	"	3	Sacramento
	"	"	4	Vallejo
	"	"	5-6	San Francisco
	"	"	7-10	Oakland
	"	"	11	San Mateo
	"	"	12	Salinas
	"	"	13	Santa Barbara
	"	"	14	Richmond
	"	"	15	Stockton
	"	"	16	Fresno
	"	"	17	Los Angeles
	"	"	18	Bakersfield
	"	"	33	Ontario
	"	"	34	San Diego

APPENDIX I -- continued

	<u>Areal Unit</u>			<u>Central Data Point</u>
United States - continued				
California - continued				
	Congressional Dist. 38			Riverside
Colorado	"	"	1	Denver
	"	"	2	Aurora
	"	"	3	Colorado Springs
	"	"	4	Grand Junction
Connecticut	"	"	1	Hartford
	"	"	2	Norwich
	"	"	3	New Haven
	"	"	4	Bridgeport
Delaware	"	"	1	Dover
Florida	"	"	1	Pensacola
	"	"	2	Jacksonville
	"	"	3 -4	Miami
	"	"	5	Daytona Beach
	"	"	6	Fort Lauderdale
	"	"	7	Lakeland
	"	"	8	Gainesville
	"	"	9	Tallahassee
	"	"	10	Tampa
	"	"	11	Orlando
	"	"	12	Petersburg
Georgia	"	"	1	Savannah

APPENDIX I -- continued

	<u>Areal Unit</u>		<u>Central Data Point</u>
United States - continued			
Georgia - continued			
	Congressional Dist. 2		Albany
	"	" 3	Columbus
	"	" 4	La Grange
	"	" 5	Atlanta
	"	" 6	Macon
	"	" 7	Rome
	"	" 8	Valdosta
	"	" 9	Gainsville
	"	" 10	Augusta
Hawaii	"	" 1	Honolulu
Idaho	"	" 1	Nampa
	"	" 2	Boise City
Illinois	"	" 1-11	Chicago
	"	" 12- 14	Maukegan
	"	" 15	Aurora
	"	" 16	Rockford
	"	" 17	Bloomington
	"	" 18	Peoria
	"	" 19	Rock Island
	"	" 20	Springfield
	"	" 21	Mount Vernon
	"	" 22	Decatur ,
	"	" 23	Mattoon
	"	" 24	Alton

APPENDIX I -- Continued

	<u>Areal Unit</u>			<u>Central Data Point</u>
United States - continued				
Indiana	Congressional Dist. 1			Gary
	"	"	2	Lafayette
	"	"	3	South Bend
	"	"	4	Fort Wayne
	"	"	5	Anderson
	"	"	6	Terre Haute
	"	"	7	Bloomington
	"	"	8	Evansville
	"	"	9	Columbus
	"	"	10	Muncie
	"	"	11	Indianapolis
Iowa	"	"	1	Davenport
	"	"	2	Cedar Rapids
	"	"	3	Waterloo
	"	"	4	Ottumwa
	"	"	5	Des Moines
	"	"	6	Sioux City
	"	"	7	Council Bluffs
Kansas	"	"	1	Salina
	"	"	2-3	Kansas City
	"	"	4-5	Wichita
	"	"	6	Pittsburg
Kentucky	"	"	1	Paducah
	"	"	2	Owensboro

APPENDIX I -- continued

	<u>Areal Unit</u>	<u>Central Data Point</u>
United States - continued		
Kentucky - continued		
	Congressional Dist. 3	Louisville
	" " 4	Covington
	" " 5	Middlesborough
	" " 6	Lexington
	" " 7	Ashland
Louisiana	" " 1-2	New Orleans
	" " 3	Lafayette
	" " 4	Shreveport
	" " 5	Nonro
	" " 6	Baton Rouge
	" " 7	Lake Charles
	" " 8	Alexandria
Maine	" " 1	Portland
	" " 2	Bangor
Maryland	" " 1	Salisbury
	" " 2	Havre De Grace
	" " 3-4	Balitmre
	" " 5	Annapolis
	" " 6	Cumberland
Massachusetts	" " 1-14	Boston
Michigan	" " 1	Detroit
	" " 2	Ann Arbor
	" " 3	Kalamazoo

APPENDIX I -- continued

<u>Areal Unit</u>			<u>Central Data Point</u>
United States - continued			
Michigan	Congressional Dist. 4		Benton Harbour
	"	" 5	Grand Rapids
	"	" 6	Flint
	"	" 7	Warren
	"	" 8	Saginaw
	"	" 9	Muskegon
	"	" 10	Bay City
	"	" 11	Sault Ste. Marie
	"	" 12-17	Marquette
	"	" 18	Pontiac
Minnesota	"	" 1	Rochester
	"	" 2	Mankato
	"	" 3	Minneapolis
	"	" 4-5	St. Paul
	"	" 6	St. Cloud
	"	" 7	Moorhead
	"	" 8	Duluth
Mississippi	"	" 1	Columbus
	"	" 2	Greenville
	"	" 3	Jackson
	"	" 4	Merridian
	"	" 5	Biloz

APPENDIX I -- continued

	<u>Areal Unit</u>		<u>Central Data Point</u>
United States - continued			
Missouri	Congressional Dist. 1-5		St. Louis
	" "	6	St. Joseph
	" "	7	Springfield
	" "	8	Columbia
	" "	9	St. Charles
	" "	10	Cape Girardeau
Montanna	" "	1	Butte
	" "	2	Great Falls
Nebraska	" "	1	Lincoln
	" "	2	Ohaha
Nevada	" "	1	Reno
New Hampshire	" "	1	Concord
New Jersey	" "	1	Atlantic City
	" "	2	Long Branch
	" "	3	Trenton
	" "	4	Morristown
	" "	5	Elizabeth
	" "	6 -14	Jersy City
	" "	15	New Brunswick
New Mexico	" "	1	Sante Fe
New York	" "	1 -6	Hempstead
	" "	6 -24	New York City
	" "	25-26	White Plains

APPENDIX I -- continued

<u>Areal Unit</u>		<u>Central Data Point</u>
United States - continued		
New York - continued		
Congressional Dist.27		Newburgh
"	" 28	Kingston
"	" 29-30	Albany
"	" 31	Watertown
"	" 32	Utica
"	" 33	Binghamton
"	" 34	Syracuse
"	" 35	Auburn
"	" 36-37	Rochester
"	" 38	Jamestown
"	" 39	Buffalo
North Carolina	" 1	Greenville
"	" 2	Wilson
"	" 3	Goldsboro
"	" 4	Raleigh
"	" 5	Winston Salem
"	" 6	Greensboro
"	" 7	Fayetteville
"	" 8	Charlotte
"	" 9	Salisbury
"	" 10	Gastonia
"	" 11	Asheville

APPENDIX I -- continued

		<u>Areal Unit</u>	<u>Central Data Point</u>
United States - continued			
North Dakota	Congressional Dist.	1	Fargo
	"	2	Bismarck
Ohio	"	1	Cincinnati
	"	2	Dayton
	"	3	Lima
	"	5	Defiance
	"	6	Portsmouth
	"	7	Springfield
	"	8	Marion
	"	9	Toledo
	"	10	Lancaster
	"	11	Warren
	"	12	Columbus
	"	13	Loraine
	"	14	Akron
	"	15	Zanesville
	"	16	Canton
	"	17	Mansfield
	"	18-19	Steubenville
	"	20	Cleveland
Oklahoma	"	1	Enid
	"	2	Muskogee
	"	3	Ardmore
	"	4	Shawnee

APPENDIX I -- continued

	<u>Areal Unit</u>		<u>Central Data Point</u>
United States - continued			
Oklahoma - continued			
	Congressional Dist.	5	Midwest City
	"	6	Lawton
Oregon	"	1	Salem
	"	2	Klamath Falls
	"	3	Portland
	"	4	Eugene
Pennsylvania	"	1-5	Philedelphia
	"	6	Reading
	"	7	Chester
	"	8	Allentown
	"	9	Lancaster
	"	10	Scranton
	"	11	Milkes-Barre
	"	12	Altoona
	"	13	Morristown
	"	14	Pittsburgh
	"	15	Easton
	"	16	Harrisburg
	"	17-18	Williamsport
	"	19-20	York
	"	21	New Kensington
	"	22	Johnstown
	"	23	Oil City

APPENDIX I -- continued

<u>Areal Unit</u>			<u>Central Data Point</u>
United States - continued			
Pennsylvania - continued			
	Congressional Dist. 24		Erie
	" " 25		New Castle
	" " 26		Washington
Rhode Island	" " 1		Pawtucket
South Carolina	" " 1		Charlestown
	" " 2		Columbia
	" " 3		Anderson
	" " 4		Greenville
	" " 5		Rock Hill
	" " 6		Florence
South Dakota	" " 1		Sioux Falls
	" " 2		Rapid City
Tennessee	" " 1		Johnson City
	" " 2		Knoxville
	" " 3		Chattanooga
	" " 4		Murfreesboro
	" " 5		Nashville
	" " 6		Clarksville
	" " 7		Jackson
	" " 8		Dyersburg
	" " 9		Mephis
Texas	" " 1		Texarkana
	" " 2		Beaumont

APPENDIX I -- continued

	<u>Areal Unit</u>		<u>Central Data Point</u>
United States - continued			
Texas - continued			
	Congressional Dist. 3		Tyler
	" "	4	Sherman
	" "	5	Dallas
	" "	6	Bryan
	" "	7	Lufkin
	" "	8	Houston
	" "	9	Galveston
	" "	10	Austin
	" "	11	Waco
	" "	12	Fort Worth
	" "	13	Wichita Falls
	" "	14	Corpus Christi
	" "	15	Laredo
	" "	16	El Paso
	" "	17	Abilene
	" "	18	Amarillo
	" "	19	Lubbock
	" "	20	San Antonio
	" "	21	San Angelo
Utah	" "	1	Ogden
	" "	2	Salt Lake City
Vermont	" "	1	Montpelier
Virginia	" "	1	Newport News

APPENDIX I -- continued

	<u>Areal Unit</u>		<u>Central Data Point</u>
United States - continued			
Virginia - continued			
	Congressional Dist. 2		Norfolk
	" "	3	Richmond
	" "	4	Petersburg
	" "	5	Danville
	" "	6	Roanoke
	" "	7	Staunton
	" "	8	Charlottesville
	" "	9	Bristol
	" "	10	Arlington County
Washington	" "	1-2	Seattle
	" "	3	Vancouver
	" "	4	Yakima
	" "	5	Spokane
West Virginia	" "	1	Wheeling
	" "	2	Morgantown
	" "	3	Charlestown
	" "	4	Huntington
	" "	5	Bluefield
Wisconsin	" "	1	Racine
	" "	2	Madison
	" "	3	La Crosse
	" "	4-5	Milwaukee
	" "	6	Sheboygan

APPENDIX I -- continued

	<u>Areal Unit</u>	<u>Central Data Point</u>
United States - continued		
Wisconsin - continued		
	Congressional Dist. 7	Kausau
	" " 8	Green Bay
	" " 9	Bau Claire
	" " 10	Superior
Wyoming	" " 1	Cheyenne
District of Columbia	" " 1	Washington

APPENDIX I -- continued

	<u>Areal Unit</u>	<u>Central Data Point</u>
<u>MEXICO</u>	Aguascalientes	Aguascalientes
	Daja California	Mexicali
	Baja California Terr.	La Paz
	Campeche	Campeche
	Coahuila	Torreon
	Colima	Colima
	Chiapas	Tuxtla Gutierrez
	Chihuahua	Chihuahua
	District Federal	Mexico City
	Durango	Durango
	Guanajuato	Leon
	Guerrero	Acapulco de Juarez
	Hidalgo	Pachuca de Soto
	Jalisco	Guadalajara
	Mexico	Toluca
	Michdacan	Morelia
	Morelos	Cuernauaca
	Nayarit	Tepic
	Nueuo Leon	Monterrey
	Oaxaco	Oaxaco de Juarez
	Puebla	Puebla de Zaragoza
	Queretaro	Queretaro

APPENDIX I -- continued

	<u>Areal Unit</u>	<u>Central Data Point</u>
<u>MEXICO</u>	Quintana Roo	Chetumal
	San Luis Potosi	San Luis Potosi
	Sinaloa	Culiacan
	Sonora	Hermosillo
	Tabasco	Villahermosa
	Tamaulipas	Cuidad Victoria
	Tlaxcala	Arizaco
	Vera Cruz	Vera Cruz Llave
	Yucatan	Merida
	Zacatecas	Fresnillo

APPENDIX II

CARTOGRAPHY AND MODEL COMPUTATION

A major part of the research for this thesis was involved in the mapping of data. The system used throughout was the Symap program Version 5, which was developed by the Laboratory for Computer Graphics and Special Analysis, Harvard Center for Environmental Design Studies, Harvard University. Maps were produced on the University of Ottawa IBM 360/65 computer.

Symap, Version 5.

Symap 5 is used for mapping spacially disposed quantitative and qualitative data illustrating special distributions in one or more of three basic types of map. The three types are the isoline, or contour map, the conformant map and the proximal map.

The conformant map is used to illustrate the distribution of data for which the areal limits are significant. These may be spaces, lines or points, such as countries, roads or stations. When used with spaces the resultant map is a choropleth map.

The proximal map is based on proximity to a data point. It is similar in appearance to a choropleth map, however the

boundaries of the data areas are defined not by the user, but by the program on the criterion that each point over the entire area is assigned a value according to the data point it is nearest.

The isoline map (contour map in the case of topographic heights) is used to represent continuous or related data in an elastic medium such as temperature or concentration. It is based on the conception of a three dimensional surface defined by known points on it. Isolines are closed curves representing all points on the surface a given distance from a fixed (usually horizontal) plane. The position of the isolines is interpolated from the "X", "Y" and "Z" co-ordinates of the known points of which as many as 1,000 may be used. This requires the computation of the "Z" value for every "X" and "Y" print location on the map surface.

The Symap program requires the preparation of an input deck of cards containing sub-decks called "packages". The first package defines the outline of the map from the "X" and "Y" co-ordinates of its vertices. The second package locates the data points, the third any legends, such as titles, bodies of water, or roads. The fourth or "values package" assigns a numerical value, or "Z" value to the "X" and "Y" of the data points. This instructs the computer to make a map pursuant to certain specified electives suited to the users requirements.

The computer output device used to print the map is the ordinary printer with Hollerith characters with the technique of overprinting. The maps are constructed by a distribution of textures and shades not unlike a newspaper photograph. For the map of North America the latitude and longitude of each of approximately 1500 vertices was recorded in degrees and minutes. This allowed experimentation with various projections. (See Appendix: Figures A and B for stereographic projections, that is: the view from a point in space, 3000 miles and 600 miles above St. John's, Newfoundland). It was decided that a sinusoidal projection about the 100th meridian would be best because of its true depiction of area and fairly close approximation of distance. Conical and stereographic projections were ruled out because it was found necessary to include fine-lines representing latitude and longitude to give the observer a true impression of North and South. If they were not included, eastern and western extremities appeared far to the north. Because of the nature of the Symap output extra fine-lines quickly clutter the finished map. The sinusoidal projection allowed the parallels of latitude to be straight lines across the page.

Figure A.

Figure B.

Stereographic Projections.

C PROGRAM TO CONVERT LAT. AND LONG. TO SINUSOIDAL
C COORDINATES IN A PLANE

```

1      DIMENSION X(1000), LABEL(2, 1000), ID(1000)
2      READ 2,  XZ, YZ, XM, YM, XMZ, YMZ, XMM, YMM
3      PRINT 2, XZ, YZ, XM, YM, XMZ, XMM, YMM
4      2 FORMAT (8F10.0)
5      XS=(XMM-XMZ)/(XZ-XM)
6      YS=(YMM-YMZ)/(YZ-YM)
7      DO 14 I=1, 1000
8      READ 3, ID(I), (LABEL(K, I), K=1, 2), XD, XM, YD, YM
9      3 FORMAT (I2, 3X, I2, I3, 61X, F2.0, F2.0, F3.0, F2.0)
10     LABEL(1, I)=LABEL(1, I)*1000+LABEL(2, I)
11     X(I)=XD+.0156667*XM
12     Y(I)=YD+.0156667*YM
13     Y(I)=((Y(I)-100.0)*(COS(X(I)*(3.14159/180.0))))+100.0
14     IF (X(I)) 4, 14, 14
15     4 NC=I-1
16     GO TO 20
17     14 CONTINUE
18     DO 21 I=1, NC
19     X(I)=(XZ-X(I))*XS
20     Y(I)=(YZ-Y(I))*YS
21     DO 30 I=1, NC
22     IX=X(I)+.5
23     IY=Y(I)+.5
24     IF (I-NC) 22, 28, 28
25     22 IP=I+1
26     DO 25 J=IP, NC
27     JX=X(I)+.5
28     IF (IX-JX) 25, 23, 25
29     23 JY=Y(J)+.5
30     IF (IY-JY) 25, 24, 25
31     24 X(J)=-9000000.-X(J)
32     25 CONTINUE
33     28 IF (X(I)+9000000.) 26, 26, 29
34     29 PUNCH 27, X(I), Y(I), ID(I), LABEL(1, I)
35     26 PRINT 27, X(I), Y(I), ID(I), LABEL(1, I)
36     27 FORMAT (10X, 2F10.0, 43X, I2, I5)
37     30 CONTINUE
38     STOP
39     END

```

The standard print patterns of the Symap output were found unsuitable for photographic reproduction especially for the large reduction required. (50" to 7") Other patterns were tested and photographed and a group of patterns were selected which suitably illustrate seven levels. The patterns were:

Level: 1 2 3 4 5 6 7

Print: A I : : : : .
 V I : : : :
 N X X / -
 Z Z X /

Overprinting represented by columns

Each computer printer has unique characteristics depending on the make, age, maintenance, etc., therefore a short program was found to be useful for testing purposes.

```
C  PROGRAM TO TEST OVERPRINT
C  PATTERNS
1      DIMENSION C(4),CHAR(4,124)
2      READ 8,ALPHA
3      8 FORMAT(A1)
4      7 READ5,(C(I),I=1,4)
5      5 FORMAT (4A1)
6      DO 6 I=1,4
7      DO 6 K=1,124
8      6 CHAR(I,K)=C(I)
9      4 DO 3 I=1,26
10     PRINT 1, (CHAR(1,K),K=1,124)
11     1 FORMAT (1X,124A1)
12     3 PRINT 2, (ALPHA,(CHAR(I,K),K=1,124),I=2,4)
13     2 FORMAT (125A1)
14     GO TO 7
15     END
```

For photographic reproduction it was found that Kodalith Ortho Film (4" X 5") processed as a "line" negative was extremely satisfactory. Maps were printed on Koda Bromide A3 paper.

All data processing and model computation was carried out on the Carleton University G.E. 415 Computer.

A basic proximity program was modified as necessary and used for all models.

C PROXIMITIES TO POPULATION, RETAIL SALES, AND
 C MANUFACTURING EMPLOYMENT, AND THE DISPARITIES
 C BETWEEN THESE PROXIMITIES.

```

1      DIMENSION ID (600),X(600),Y(600),Z(600),R(600),
2      1PROX(3,600),DIS(3,600),PM(3),W(3,600)
3      DO 8 I=1,600
4      DR=.0174533
5      DM=.01666667
6      READ 5, ID(I),AREA, (W(K,I),K=1,3),XD,XM,YD,YM
7      5 FORMAT (5X,I5,4F10.0,I7,1X,I2,I7,1X,I2)
8      X(I)=(XD+DM*XM)*DR
9      Y(I)=(YD+DM*YM)*DR
10     CT=COS(X(I))
11     X(I)=COS(Y(I))*CT
12     Y(I)=SIN(Y(I))*CT
13     Z(I)=SIN(X(I))
14     R(I)=SQRT(AREA*100./3.1415927)
15     DO 6K=1,3
16     6 PROX(K,I)=(0.01*R(I)-ALOG(1.0+.01*R(I)))*2.0*W(K,I)/
17     1(.0001*R(I)**2)
18     4 IF (ID(I))7,8,8
19     7 NR=I-1
20     GO TO 39
21     8 CONTINUE
22     39 DO 11 K=1,3
23     11 PM (K)=0
24     DO 13 I=1,NR
25     DO 9 J=1,NR
26     IF (J-I)9,9,10
27     10 D=DIST(X(I),Y(I),Z(I),X(J),Y(J),Z(J))
28     38 IF (D.LT.1.0)DT=1.0
29     IF (D.GE.1.0)DT=D
30     DO 9- K=1,3
31     PROX(K,I)=PROX(K,I)+W(K,J)/(1.0 + 0.01*D)
32     PROX(K,J)=PROX(K,J)+W(K,I)/(1.0 +0.01*D)
33     9 CONTINUE
34     DO 13 K=1,3
35     IF(PROX(K,I)-PM(K))13,13,12
36     12 PM(K)=PROX(K,I)
37     13 CONTINUE
38     PRINT18, (PM(K),K=1,3
39     18 FORMAT(20X,3E10.5)
40     DO 17 I=1,NR

```



```

40      15  PROX(K,I)=PROX(K,I)*1000/PM(K)
41          DIS(1,I)=(PROX(1,I)-PROX(2,I))/(PROX(1,I)+PROX(2,I))
42          DIS(2,I)=(PROX(3,I)-PROX(1,I))/(PROX(1,I)+PROX(3,I))
43      17  DIS(3,I)=(PROX(3,I)-PROX(2,I))/(PROX(2,I)+PROX(3,I))
44          PRINT 14,((PROX(K,I),K=1,3),(DIS(M,I),M=1,3),ID(I),I=1,NR)
45          PUNCH 14,((PROX(K,I),K=1,3),(DIS(M,I),M=1,3),ID(I),I=1,NR)
46      14  FORMAT(5X,3F100,5X,3F103,I10)
47          STOP
48          END

1      FUNCTION DIST(AX,AY,AZ,BX,BY,BZ)
2      DX=AX-BX
3      DY=AY-BY
4      DZ=AZ-BZ
5      D2=DX*DX+DY*DY+DZ*DZ
6      D=SQRT(D2)
7      DIST=(0.47*D2*D+D)*3955
8      RETURN
9      END

```

This program computes a model for each of three variables as well as the disparities between them. It calculates from the latitude and longitude a number of distances equal to one half the number of observations squared, and one distance calculation suffices for the computation of all the variables. For 570 observations this is 162,450 distances. The distance function calculates the straight line distance from point to point. For the largest distance in North America this represented approximately 5% underestimation. Three distance functions were tested including one using spherical trigonometry. However the program chosen, using Cartesian co-ordinates in three dimension, was found to be sufficiently accurate and represented a significant saving of time.

POTENTIAL INSIDE A SPHERE

The law of gravitation assumes the gravitational constant to be universal.

$$\vec{F} = -G \frac{M_1 M_2}{r^2} \hat{r} \quad (1)$$

where: $\vec{F}$ = the gravitational force due to the attraction of the mass M_1 and mass M_2 , their centers being separated by a distance r ; and,
 $\hat{r}$ = a unit vector along the line which defines r since $\vec{F}$ is a vector quantity.

The gravitational field therefore can be defined:

$$\vec{f} = \frac{\vec{F}}{M_2}$$

This is the force exerted by M_1 per unit mass. The gravitational field at a distance r from a mass M is:

$$\vec{f} = - \frac{GM}{r^2} \hat{r}$$

A gravitational field is a conservative one, therefore potential may be defined in terms of which the field is the gradient of this potential.

$$\vec{f} = -\vec{\nabla}\psi$$

The vector quantity is now a scalar quantity.

$$\nabla\psi = - \frac{\partial}{\partial r} \left(- \frac{GM}{r} \right) \hat{r}$$

Therefore $\psi = \frac{-GM}{r}$

for simple geometry of point masses.

For a density distribution for which we know the potential change as a function of r , that is;

$$\psi = \text{function of } r$$

it is possible to integrate:

$$\psi = \int d\psi$$

(the potential energy of a mass M_0 in a field of potential ψ is $VM_0\psi$. The negative sign in equation (1) indicates attraction.)

Consider a hollow sphere with thin walls and constant density.

Let r_0 be the radius of the sphere and Δt its thickness.

The potential d at a point P a distance R from the center, which is taken to be the origin, caused by a ring element (perpendicular to R), is:

$$d\psi = \frac{-G (2\pi r_0^2) \Delta t \sigma \sin \theta d\theta}{r} \quad (2)$$

where: $2\pi r_0 \sin \theta$ = circumference of the ring;

$r_0 d\theta$ = area of the surface of the ring;

Δt = thickness of the shell; and,

σ = density of the shell.

$$dM = 2\pi r_0^2 \sin \theta d\theta \Delta t \sigma$$

This element is integrated from A to B to cover the whole sphere from $r_0 - R$ at A to $r_0 + R$ at B. Thus:

$$\psi = - \int 2\pi G \sigma \Delta t r_0^2 \frac{\sin \theta d\theta}{r} \quad (3)$$

$$\text{Now } r^2 = r_0^2 + R^2 - 2 r_0 \cos \theta \quad (\text{law of cosines})$$

$$2 r dr = 2 r_0 \sin \theta d\theta$$

$$\sin \theta d\theta = \frac{r dr}{r_0 R}$$

Replacing this in (3)

$$\psi = -2\pi G \sigma \Delta t r_0^2 \int_{(r_0 - R)}^{(r_0 + R)} \frac{1}{r} \frac{r dr}{r_0 R}$$

$$\begin{aligned}
 \psi &= \frac{-2\pi G \sigma \Delta t r_o}{R} \int_{(r_o - R)}^{(r_o + R)} dr \\
 &= \frac{-2\pi G \sigma \Delta t r_o}{R} \left| (r_o + R) - (r_o - R) \right| \\
 &= -4\pi G \sigma \Delta t r_o
 \end{aligned}$$

But the total mass $M = 4\pi r_o \Delta t \sigma$

$$= \frac{GM}{r_o} = \text{constant}$$

$$-\vec{\nabla}\psi = \vec{f} = 0$$

Conclusion: the field inside a shell of finite thickness is zero.

In fact inside a shell of finite thickness T , f is zero since it would be the sum of their finite shells t , each of which produces zero field.

The potential inside a solid sphere at a point P a distance R from the center of the sphere will have two parts; one from a spherical shell of inside radius equal to R , and one from the sphere with outside radius equal to R . All spheres have the same center. The potential from the outside sphere has been shown to equal zero.

Thus: $\vec{f} = \frac{-M'G}{R^2} \hat{R}$

$$M' = \frac{4}{3} \pi R^3 \sigma$$

$$\begin{aligned}\text{Then } \vec{f} &= - \frac{4}{3}\pi \frac{R^3}{R^2} \sigma \hat{R} \\ &= - \frac{4}{3}\pi G \sigma R \hat{R}\end{aligned}$$

Finally the potential giving rise to this field is:

$$\psi = \frac{2}{3} \pi \sigma G R^2 \quad \text{or,}$$

$$-\vec{\nabla}\psi = - \frac{\partial}{\partial R} \hat{R} \psi$$

Therefore ψ increases as R^2 until r_0 and then falls off as $\frac{1}{r}$.

APPENDIX IV.

THE DATA.

Population was recorded in hundreds of persons for each areal unit: counties in Canada, Congressional Districts in United States and Entidades in Mexico. In Canada, population refers to 'de jure' population on July 1, 1961, in the United States 'de jure' April 1, 1960 and in Mexico 'de facto' population June 8, 1960. The difference between 'de facto' and 'de jure' was ignored, but this series was projected through time to June 1, 1961 using time series tables in the United Nations Monthly Bulletin of Statistics.

Manufacturing statistics were also relatively compatible. The number of employees as reported in the various manufacturing censuses was recorded for each areal unit. The census of manufacturing for each country reported yearly averages based on monthly submissions by employers of salaried and wage earning employees. There are no United Nations tables based on reports of manufacturers, rather the tables used to estimate employment increase were those based on census of labour reports.

Retail sales data were comparable on a broad basis between Canada and the United States, however they were not

comparable with Mexico. Mexico's sales data covered only specific products and did not represent a blanket coverage of all retail outlets.

The time periods were: Canada, January 1 to December 31, 1961, United States, January 1 to December 31, and Mexico, January 1 to December 31, 1960. United Nations time series tables based on sample surveys carried out by the various census bureaus were used to project these figures to July 1, 1961. Mexican pesos and Canadian dollars were converted as of June 31, 1961 using the Reports of the International Monetary Fund.

APPENDIX V

GRAVITY MODELS AND DISPARITIES: MEXICO

Figure C. Gravity Models: Mexico

Population
Potential

Retail Sales
Potential

Manufacturing Em-
ployment Potential

Proximity to
Population. 5 miles

Proximity to
Retail Sales
5 miles

Proximity to Manu-
facturing Employment
5 miles

Proximity to Popula-
tion. 100 miles

Proximity to
Retail Sales.
100 miles

Proximity to Manu-
facturing Employment
100 miles

Figure D. Disparity Models: Mexico

Potentials of Popula-
tion and Retail Sales

Potentials of Manu-
facturing Employment
and Population

Potentials of
Manufacturing
Employment and
Retail Sales

Proximities to Popula-
tion and Retail Sales
5 miles

Proximities to Manu-
facturing Employment
and Population

Proximities to
Manufacturing
Employment and
Retail Sales

Proximities to Popula-
tion and Retail Sales.
100 miles

Proximities to Manu-
facturing Employment
and Population

Proximities to
Manufacturing
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