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HEAT TRANSFER BETWEEN CONTACTING NON-CONFORMING SURFACES —
EXPERIMENTAL AND NUMERICAL STUDIES WITH SPECIAL EMPHASIS ON
PROBLEMS INVOLVING CONTACT PATCHES OF HIGH ELLIPTICITY

University — Université

CARLETON UNIVERSITY, OTTAWA

Degree for which thesis was presented — Grade pour lequel cette thèse fut présentée

DOCTOR OF PHILOSOPHY IN MECHANICAL ENGINEERING

Year this degree conferred — Année d'obtention de ce grade

1984

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HEAT TRANSFER BETWEEN CONTACTING NON-CONFORMING
SURFACES - EXPERIMENTAL AND NUMERICAL STUDIES WITH
SPECIAL EMPHASIS ON PROBLEMS INVOLVING CONTACT
PATCHES OF HIGH ELLIPTICITY

by

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A thesis

submitted to the Faculty of Graduate Studies and Research

in partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

in

Mechanical Engineering

Faculty of Engineering

Department of Mechanical and Aeronautical Engineering

Carleton University

May 1984

Thesis
Ph.D.
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The undersigned recommend to the Faculty of Graduate Studies and Research, acceptance of the thesis,

"Heat Transfer Between Contacting Non-Conforming Surfaces
- Experimental and Numerical Studies With Special Emphasis
on Problems Involving Contact Patches of High Ellipticity"

as submitted by Thomas Charles Currie, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering.

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ABSTRACT

This thesis describes experimental and numerical studies of heat transfer between surfaces in contact over a highly elliptical area in the presence of a gas or in vacuum. Such heat transfer could result from contact between pressure and calandria tubes in a CANDU reactor, either under normal operating conditions or following a loss-of-coolant accident (LOCA) with delayed emergency-core-coolant (ECC) flow, for instance.

The steady-state thermal resistance of a contact formed between two cylindrical steel surfaces was measured in air and in vacuum at different contact loads in the experiments. The radius of curvature of one contact surface was much smaller than that of the other (76 cm vs 0.95 cm) and the planes of curvature of the surfaces were orthogonal. The resulting contact patches were elliptical in shape, with a major/minor diameter ratio of 16 according to Hertz's theory of elastic contact. Tests were performed with smooth and rough contact surfaces. Although roughening the large radius contact surface to a roughness of $\sim 1\mu\text{m}$ by bead blasting caused a 5-20% reduction in the resistance of the contact in vacuum, presumably due to spreading of the contact patch, the resistance of the contact in air was essentially unaffected by the roughness. The resistance of the contact in air was only 25-50% of its resistance in vacuum.

Analytical models for predicting the resistance of the contact with smooth or rough surfaces in the presence of a gas or in vacuum are described

and used to obtain theoretical results for comparing with the experimental data. Theoretical and measured contact resistances are shown to differ by less than 12.5% in all cases. The theoretical results are obtained with a finite-difference model formulated in the ellipsoidal coordinate system.

The contact problem is also solved with the synthesis and multigrid methods. The former method, which synthesizes 3-D solutions by linearly combining known 2-D finite-difference solutions obtained at different axial locations in the problem of interest, is also used to perform transient and non-linear analyses of heat transfer between contacting pressure and calandria tubes in a CANDU reactor.

ACKNOWLEDGEMENTS

The author would like to thank Dr. J.T. Rogers for his supervision of the research described in this thesis. Also thanked are Drs. H.I. H. Saravanamuttoo and J. Goldak for moral support and encouragement extended during their respective periods as departmental chairmen and Professor M.M. Yovanovich of the University of Waterloo for having generously provided a large amount of valuable reference material. The author is also grateful to Dr. Laubitz of the Physics Division of the National Research Council of Canada for his donation of a bar of Armco-iron. My sincere thanks are extended as well to the management of Davis Engineering Ltd. for permitting me to work part-time at their firm while I completed this thesis.

Special thanks are extended to Diane Dodds for her patience in typing this manuscript.

I also wish to acknowledge financial support received from the National Research Council of Canada by way of a postgraduate scholarship. The work described in this thesis was undertaken for and partly supported by the Atomic Energy Control Board of Canada. Additional support was provided by the Natural Sciences and Engineering Research Council under Research Grant A6342.

PREFACE

This thesis consists of two documents, of which the first contains all of the thesis outside of documentation for a computer program used in the research and the second contains only that documentation. The documentation referred to is found in Part IV of the thesis (Parts I - III are found in the first document) and is provided with its own Table of Contents since it is intended to be a stand-alone user's manual.

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NOMENCLATURE

This nomenclature applies to Parts I-III of the thesis. All variables are defined when they are first used in the text - only variables which are used repeatedly are defined below.

- A - apparent contact area
- A_c - contour area (area of contact patch)
- A_{cs} - contact surface area
- A_i - cross-sectional area of Armco-iron heat flow metering bar
- $A_s(i,j,k)$ - surface area of boundary node i,j,k
- A_{st} - cross-sectional area of 0-1 steel thermal conductivity specimen
- $A_{SZ}(i,j,k)$ - surface area of boundary node i,j,k ($k=1$ or NZ) located on either one of the ends of a cylindrical body
- B_j - geometric factor used to calculate the heat conducted across the gas gap between a node on the pressure tube outer surface and the opposing node on the calandria tube inner surface (Equation III.3.13)
- C - nodal capacitance matrix
- C_a, C_b - constants in Dyson's formulae for contact dimensions a and b (Equations I.3.17 and I.3.18)
- C_1, C_2, C_3, C_4 - geometric parameters used in Hertz theory (see Equations I.3.1 - I.3.4)
- D - left-hand side coefficient matrix for the system of equations ($DZ=R$) defining the blending vector (Z) in a steady-state synthesis analysis ($D=W^T KW$)
- D' - left-hand side coefficient matrix for the system of equations ($D'Z^{t+1}=R'$) defining the blending vector (Z^{t+1}) at time $t+\Delta t$ in a transient synthesis analysis ($D'=W^T(C/\Delta t+K)W$)

- $\bar{D}, \bar{D}'$ - partitioned forms of D and D' respectively
- D_e - matrix multiplying Z^t in the transient synthesis method with forward-differenced time derivatives ($D_e = (I - \Delta t(W^T C W))^{-1} W^T K W$)
- E - Young's modulus of elasticity, or the error (residual) vector in the synthesis method (use should be obvious from context)
- $E(k)$ - complete elliptic integral of the second kind with modulus k
- $E(\beta, k)$ - incomplete elliptic integral of the second kind with argument β and elliptic modulus k
- F - matrix with the two-dimensional trial vectors used in a continuous synthesis analysis as its columns, or the right-hand side function for the differential equation $\mathcal{L}U = F$ (Equation II.4.1a) (use should be obvious from context)
- F_k - matrix with the two-dimensional trial vectors used at axial node K in the discontinuous synthesis method as its columns
- $F(\beta, k)$ - incomplete elliptic integral of the first kind with argument β and elliptic modulus k
- F^n - vector containing values of F evaluated at grid points on the n th grid in the multigrid algorithm
- $\bar{F}^n$ - right-hand side vector for the equations solved on the n th grid in the FAS multigrid algorithm
- G_a, G_b - coefficients in Hertz's formulae for contact dimensions a and b respectively (See Equations I.3.9 and I.3.11)
- G'_a, G'_b - coefficients in Dyson's approximate formulae for contact dimensions a and b respectively (See Equations I.3.17 and I.3.18)
- G - n th discretization level (grid) in a multigrid analysis
- H - matrix used in transient synthesis analysis (see Equation III.2.10) or hardness of softer contact surface (use should be obvious from context)
- $\bar{H}$ - partitioned form of matrix H
- I_n^{n-1} - averaging (restriction) operator used for the fine-to-coarse grid residual transfers in the multigrid algorithm.

- I_{n-1}^n - interpolation (prolongation) operator for the coarse-to-fine grid correction interpolations in the multigrid algorithm
- K - conductance (i.e., left-hand side coefficient) matrix for a finite-difference or finite-element discretization
- K' - equivalent "conductance" matrix in a transient analysis ($K' = C/\Delta t + K$)
- $\bar{K}, \bar{K}'$ - partitioned forms of K and K' respectively
- K_g - overall gas conductance (area integrated value) between opposing nodes on the two contact surfaces
- $K(k)$ - complete elliptic integral of the first kind with elliptic modulus k
- $K(i,j,k)_n$ - internodal conductance between node n and node i,j,k
- Kn - Knudsen number (L_m/δ)
- L - number of nodes per $r-\theta$ plane ($=NR \times NT$) in a finite-difference or finite-element $r-\theta-z$ discretization, or the length of the substitute contact specimen (use should be obvious from context)
- $\mathcal{L}$ - differential operator
- L_h - parameter in Hertz's theory (See Equation I.3.10)
- L_m - mean free path of gas molecules
- L^n - finite-difference or finite-element coefficient matrix for the n th discretization level (grid) in the multigrid algorithm
- L_p - surface profile length
- L_s - length of lower contact specimen (2.92 cm)
- L_{sp} - distance between two most central garter springs in a CANDU fuel channel
- M - finest discretization level in a multigrid analysis
- N - number of trial functions used in a continuous synthesis approximation
- N_e - total number of nodes in a finite-difference or finite-element mesh (same as total number of simultaneous nodal equations)

- N_i - number of x-axis nodes
- N_j - number of y-axis nodes
- N_k - number of trial functions used at axial node k in the discontinuous synthesis method
- N_R - number of radial (r) mesh points in a cylindrical grid
- N_T - number of circumferential (θ) mesh points in a cylindrical grid
- N_Z - number of axial (z) mesh points in a cylindrical grid
- P - contact load
- Pr - Prandtl number
- Q - right-hand side (heat flow) vector for a system of finite-difference or finite-element equations.
- Q' - heat flow vector for a transient analysis ($=Q+CT^t/\Delta t$)
- $\bar{Q}, \bar{Q}'$ - partitioned forms of Q and Q' respectively
- Q_c - total heat flow rate between contact surfaces
- Q_{ci}, Q_{hi} - heat flow rates in the lower (cold) and upper (hot) Armco-iron heat flow metering bars respectively
- Q_g - heat flow rate through the gas between the contact surfaces
- Q_H - heat flow rate through the Hertzian contact area
- Q_p - heat flow rate through the contact patch
- R - right-hand side vector for the system of equations ($DZ=R$) defining the blending vector (Z) in a steady-state synthesis analysis ($R=W^TQ$) or flux tube radius (use should be obvious from context)
- R' - right-hand side vector for the system of equations ($D'Z^{t+1}=R'$) defining the blending vector (Z^{t+1}) at time $t+\Delta t$ in a transient synthesis analysis ($R'=W^TQ'$)

R, R'	- partitioned forms of R and R' respectively
R_b	- resistance of substitute contact specimen with both ends isothermal (calculated using ellipsoidal finite-difference model)
R_c	- measured contact resistance
R_{ct}	- total contact resistance (i.e., not based on ΔT_c)
R'_c	- theoretical contact resistance
R_{ca}, R'_{ca}	- measured and predicted resistances of the contact in air respectively
R_{cv}, R'_{cv}	- measured and predicted resistances of the contact in vacuum respectively
R_g	- thermal resistance of the gas layer between the contact surfaces
R_{mac}, R_{mic}	- macroscopic and microscopic constriction resistances of a contact respectively
R_{qc}	- resistance of contact in air predicted with the quasi-coupled model
R_{tc}	- transient constriction resistance
R_1, R_2	- small contact radii of bodies 1 and 2 respectively
R'_1, R'_2	- large contact radii of bodies 1 and 2 respectively
S^M	- set of nodes on grid M in the multigrid algorithm
S_b	- boundary of body
S_y	- uniaxial yield strength
$S_1, S_2, \dots$	- boundary segments
T	- vector of nodal temperatures
T'	- synthesis approximation to T
$\dot{T}$	- time derivative of nodal temperatures (i.e., dT/dt)
$\dot{T}'$	- synthesis approximation to $\dot{T}$
T_c	- local temperature within contact area

- T_c - average temperature of contact area
- T_n - nth trial function used in the synthesis method
- T_0, T_1 - sink and source temperatures respectively
- T_∞ - environment or sink temperature
- ΔT_c - extrapolated temperature drop at the interface between two contacting surfaces

- U - exact solution to the differential equation $\mathcal{L}U = F$
- U^n - exact solution to the system of equations $L^n U^n = F^n$
- $U(i,j,k)_n$ - shape factor for conduction between node n and node i,j,k

- V - total number of 3-D trial functions used in a synthesis analysis ($= \sum_{k=1}^{NZ} N_k$)
- V^n - vector of corrections for the nth grid in a multigrid analysis ($= U^n - u^n$)
- $V(i,j,k), V(l)$ - volume of finite-difference cell for node i,j,k or node l

- W - "trial vector" matrix in the synthesis method - contains the trial vectors as its columns

- Z - mixing (blending) vector defined by the relation $DZ=R$ in a synthesis analysis (also, $T'=WZ$) or mean plane separation (use should be obvious from context)
- $\bar{Z}$ - partitioned form of vector Z

- a - semi-major diameter of elliptical contact or radius of circular contact
- a_r - one-half of length of contour area formed between rough surfaces
- b - semi-minor diameter of elliptical contact

- c_m - material specific heat
- d - number of dimensions (1, 2 or 3)
- d_{ii} - spacing between the i th fringe on one side of the contact center and the i th fringe on the other side in an interferometric surface radius measurement
- f - fraction of surface element area located with the contact patch
- f^n - right-hand side "residual" vector for the equations solved on the n th grid in the CS multigrid algorithm
- g_i - metric coefficient for the i th coordinate direction
- h_c - contact conductance
- $h_{cu}, h_{cu}, h'_{cu}, h_n$ - contact (approximation) conductances defined by Equations II.2.51, II.2.59, II.2.64 and II.2.58 (or II.2.63) respectively.
- h_{cv} - rough surface contact conductance in vacuum (see Equation I.3.42)
- h_f - film coefficient
- h_g - gas conductance (k_g/δ)
- i, j, k - nodal indices
- k - elliptic modulus ($= \sqrt{1-b^2/a^2}$)
- k' - complementary elliptic modulus ($= b/a$)
- k_g - thermal conductivity of gas
- k_i - thermal conductivity of Armco-iron
- k_{og} - thermal conductivity of gas under continuum conditions
- k_s - thermal conductivity of solid
- k_{st} - measured thermal conductivity of 0-1 steel
- $\bar{m}$ - average absolute slope of asperities

$\bar{m}_e$	- $\sqrt{\bar{m}_1^2 + \bar{m}_2^2}$, where $\bar{m}_1$ and $\bar{m}_2$ are the values of $\bar{m}$ for contact surfaces 1 and 2 respectively
m, n	- exponents in Dyson's formulae for a and b respectively (Equations I.3.15 - I.3.18)
p_a	- apparent contact pressure
p_h	- mean Hertzian contact pressure over the contact area
q_g, q_r	- rate of heat transfer by gas conduction and radiation respectively between opposing nodes on the pressure tube outer and calandria tube inner surfaces
q_v	- volumetric heat generation rate
q''	- surface heat flux
r	- radius
s	- arc length
t	- time, <u>or</u> thickness (use should be obvious from context)
u^n	- approximation to U^n in the multigrid algorithm
v^n	- approximation to V^n in the multigrid algorithm
w	- width
x, y, z	- Cartesian coordinates and the contact-centered coordinates system
$\underline{x}, \underline{y}, \underline{z}$	- transformed coordinates for the ellipsoidal finite-difference model
x_i	- generalized coordinates ($i=1, \dots, d$)

α	- thermal diffusivity ($k_s/\rho_m c_m$)
α_1, α_2	- accommodation coefficients for surfaces 1 and 2 respectively
β	- mesh size ratio (Δ_n/Δ_{n-1}), <u>or</u> elliptic argument <u>or</u> $H\Omega/\bar{m}_e$ (use should be obvious from context)
γ	- angle between planes of curvature containing R_1 and R_2
$\underline{\gamma}$	- ratio of specific heats for the gas in the gas gap
δ	- gap between contact surfaces
δ^*	- gap between rough contact surfaces within the Hertzian contact area
δ_{ij}	- Kronecker delta (=1 when $i=j$; =0 otherwise)
ϵ	- ratio of circular contact radius to sphere or flux tube radius ($=a/R$) <u>or</u> contact area fraction (use should be obvious from context)
$\epsilon(i,j,k)$	- residual (error) in the finite-difference or finite-element equation for node i,j,k
ϵ_H	- contact area fraction within the Hertzian area
n	- Z/σ (i.e., ratio of mean plane separation to rms surface roughness)
θ	- azimuthal coordinate in the cylindrical coordinate system, <u>or</u> the weighting factor in the generalized difference equation for transient heat conduction
$\kappa(A)$	- condition number of the matrix A ($= A \ A^{-1} $) (A is used in a generic sense here)
λ	- coordinate (ellipsoid) in the ellipsoidal coordinate system
$\lambda(A)_i$	- i th eigenvalue of the matrix A (A is used in a generic sense here)
λ_{Na}	- wavelength of yellow sodium light (5893A)
μ	- coordinate (one-sheeted hyperboloid) in the ellipsoidal coordinate system
ν	- coordinate (two-sheeted hyperboloid) in the ellipsoidal coordinate system
ν_1, ν_2	- Poisson's ratio for material 1 or material 2

ξ_T	- r- θ temperature variation (of trial function) defined by Equation II.3.1
$\rho(A)$	- spectral radius (maximum modulus eigenvalue) of the matrix A (A is used in a generic sense here)
ρ_m	- material density
σ	- standard deviation <u>or</u> rms surface roughness
σ_1, σ_2	- surface roughnesses of contact surfaces 1 and 2
σ_e	- equivalent root-mean square roughness ($=\sqrt{\sigma_1^2 + \sigma_2^2}$)
$\sigma_x, \sigma_y, \sigma_z$	- principal stresses in the x, y and z directions respectively
τ^n	- truncation error ($=L^n U^n - F^n$) for the nth grid in the multigrid algorithm
τ_G	- maximum octahedral shear stress
ϕ	- Kirchoff's transformation ($= \int_{T_r}^T k_s(T') dT'$, where T_r is a reference temperature)
ϕ_1	- $1/[1+2.5\beta^{1.2}]$
ϕ_2	- $1/[1+1.05\beta]$
ψ	- factor related to the shape of a contact (see Table I.2.1)
Ω	- $(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2$
Δ	- mesh spacing
$\Delta x, \Delta y, \Delta z$	- mesh spacings in the Cartesian coordinate system
$\Delta r, \Delta \theta, \Delta z$	- mesh spacings in the cylindrical coordinate system

PART I

A FUNDAMENTAL STUDY OF HEAT TRANSFER BETWEEN
CONTACTING, NON-CONFORMING SURFACES

CHAPTER I.1

INTRODUCTION

I.1.1 Heat Transfer Between Contacting Solids

Thermal problems involving contacting solid bodies can be subdivided into two main categories, specifically those involving "conforming", nominally flat surfaces and those involving non-conforming surfaces. Such a classification, while simple in theory, is difficult in practice since real surfaces are never totally flat, but rather have characteristic roughness and waviness* and hence never conform completely. In general, the actual contact area between the contiguous surfaces is only a small fraction of the apparent** contact area. When heat flows through such a contact, the high local heat fluxes at the points of contact of the surfaces produce a temperature drop (ΔT_c) at the interface, permitting definition of a thermal contact conductance, in analogy with the convective film coefficient, as $h_c = Q_c / A \Delta T_c$ where Q_c is the total heat

* Note the distinction between surface roughness and waviness. Roughness refers to the relatively finely spaced irregularities on a surface, while waviness refers to irregularities of longer wavelength.

** The apparent contact area in this context is the area defined by the outer boundaries of the contact region.

flow rate through the contact and A is the apparent contact area. The temperature drop ΔT_c is obtained by extrapolating temperature gradients measured in regions of one-dimensional heat flow on either side of (but remote from) the interface to the contact plane, as will be discussed in greater detail in Section 1.1.3.2. The contact conductance h_c is smallest for contacts in vacuum and increases in value when an interstitial fluid through which heat can be conducted is present around the points of contact of the surfaces and/or when the contact surface temperatures are high enough for radiative heat transfer to be important.

Contact between "nominally flat" but nevertheless non-conforming surfaces is not considered in any depth in this investigation. This type of contact, and the thermal resistance associated with it, has been the subject of extensive investigation and the heat transfer literature is replete with examples of such studies [e.g., 1-4]. For the purposes of this investigation, non-conforming contacts are defined as contacts involving surfaces having identifiable radii of curvature, such as spheres and cylinders. The ability to predict heat transfer rates between non-conforming surfaces is required in a large number of problems, some of which are described in the next section.

1.1.2 Examples of Heat Transfer Problems Involving Non-Conforming Contacts

A particular class of non-conforming contact problems has been studied in depth by previous researchers, specifically those problems which involve heat transfer between contacting spheres or spheres in contact with flat surfaces, either in a stagnant gas or in vacuo. Problems of this type arise in the study of:

- (1) Heat transfer in packed beds. Fixed-bed catalytic reactors in which large amounts of heat are absorbed or released in gas-phase

chemical reactions, packed tube heat exchangers and fixed-bed regenerators are examples of applications in which heat transfer between particles which are frequently spherical or cylindrical [5] in shape is of interest. The interstitial gas is not usually stagnant in these applications but knowledge of the zero-flow effective thermal conductivity of the bed is required to predict the effective thermal conductivities of the bed in directions parallel to and normal to the flow direction when flow is present [6], particularly at low gas flow rates. For this reason, the stagnant fluid case has been extensively investigated [e.g., 7-10]. This information is also needed to analyze heat transfer in the German AVR helium gas cooled pebble-bed nuclear reactors [11].

(2) Heat transfer in nuclear fuel pins containing spheres, rather than cylindrical pellets, of fuel [12]. Sphere-Pac fuel, which is being investigated for use in fast reactors [13], is a specific example of this type of fuel. Sphere-Pac fuel pins contain contacting spheres of carbide fuel and use stagnant helium gas as the interstitial fluid.

(3) Heat transfer through ball-bearings in vacuo - this is important in spacecraft instrumentation applications [14-16]. Spacecraft instruments are often mounted on gimbaled platforms, so heat generated by the instruments must generally be conducted across rolling element bearings before it can be transmitted to the spacecraft heat sink. Heat transfer analysts must know the thermal resistances of the bearings in order to predict component temperatures.

(4) Insulators and stand-offs used in cryogenic equipment [17].

Other types of non-conforming contact heat transfer problems (i.e., problems involving other than sphere-sphere contacts) have been reported in the literature. Yovanovich has investigated, analytically and experimentally, the thermal resistance of a roller bearing in vacuum in connection with spacecraft instrument heat rejection [18]. Several researchers have investigated the thermal resistance of a wire mesh sandwiched between two flat surfaces [e.g., 19, 20]. Each wire/flat contact is an example of a non-conforming contact in this arrangement. It was originally thought that it might be possible to use a screen to reduce the interfacial resistance between flat surfaces [19], but experiments indicated that the screens actually functioned quite well as rigid (with respect to normal loads) insulators.

Hwang and Wang [21] performed an analysis to determine the heat flow through the spacer rods separating the inner and outer hexcans of an LMFBR (liquid-metal fast breeder reactor) fuel channel during a LOPI (loss-of-piping integrity) accident. The fuel channel, shown in Figure 1.1.1, consists of a 19 pin fuel bundle surrounded by inner and outer hexcans which are separated by spacer rods. The gap between the inner and outer hexcans is filled with inert gas to minimize heat losses from the inner hexcan, which contains sodium coolant. A bypass flow of sodium is provided outside the outer hexcan. It is predicted that the sodium inside the inner hexcan will boil during a LOPI transient, in which case heat generated in the fuel must be dissipated by heat transfer from the inner to the outer hexcan, and subsequently to the bypass sodium flow. The inner hexcan heats up and expands during the transient, thereby loading the points of contact between the inner and outer hexcans and the spacer rods. According to Hwang and Wang,

the amount of heat conducted through the spacer rods is greater than that transmitted by radiation between the hexcans at all times in the transient, although details of how this result was obtained are not given in reference [21].

Figure I.1.1 LMFBR Fuel Channel

Heat transfer between parallel contacting tubes carrying hot and cold fluids was investigated by Simpson in connection with a proposed method for detecting coolant channel blockages in CANDU reactors [22]. The proposed method involved monitoring the temperatures of DN (delayed neutron) lines (tubes) carrying small bleed flows of the coolant from each fuel channel to a central monitoring station. The coolant-carrying tubes would have been bundled together for a considerable distance and excessive heat exchange between the tubes would have jeopardized the feasibility of the proposed method for detecting channel blockages.

The present study was motivated by two examples, arising from CANDU nuclear reactor safety analyses, of heat transfer problems involving contacting, non-conforming surfaces. The first example relates to contact between a bowed fuel element and either a pressure tube or another fuel element in a CANDU reactor. In the event of a loss-of-coolant accident (LOCA), fuel element bowing is extremely likely since cooling of the fuel element will be highly asymmetrical. Veeder and Schankula [23] demonstrated that thermal distortion of a fuel element, arising from asymmetrical cooling of the element, is the most likely cause of bowing. If a LOCA is accompanied by an impairment or loss of emergency-core-coolant (ECC) flow, conditions may exist where some pressure tubes contain only stagnant steam or a stratified layer of liquid with superheated steam flowing above it. In the former case in particular, removal of the decay heat generated in the fuel bundles will occur by predominantly radial heat transfer within the fuel bundle, pressure tube and calandria tube assembly, unaided by two-phase forced convection or boiling. In order to predict fuel temperatures and hence evaluate the likelihood or extent of fuel and sheath melting, sheath ballooning (due to gaseous fission product pressure), hydrogen production, etc., under these conditions, accurate information relating to contacting fuel element-fuel element and fuel element-pressure tube heat transfer rates is required. It should be recognized that fuel element-fuel element and fuel element-pressure tube contact will probably increase greatly in the later stages of a LOCA with impaired or non-existent ECC flow, as fuel elements overheat and sag under the influence of gravity.

The second example of a CANDU-related non-conforming contact heat transfer problem also arises from a LOCA-related scenario. If flow stagnation* occurs in a highly-rated fuel channel early in the blowdown phase of a LOCA, when the coolant pressure is still high, the pressure tube in the affected channel may overheat to the point where it balloons (due to internal coolant pressure) into contact with the calandria tube which encloses it. The resulting contact will be nominally uniform circumferentially and hence not the type of contact to which this research is directly addressed. If the ECC flow is delayed or impaired and the flow stagnates in a highly rated fuel channel late in the blowdown when the coolant pressure has dropped to a low value, however, the pressure tube in the affected channel will overheat and eventually sag onto the calandria tube which encloses it. The resulting contact will be non-conforming. For such a contact to form, the pressure tube in a modern CANDU reactor (e.g., Bruce A) will have to sag approximately 0.87 cm. over a 1-2m length.**

The rate of heat transfer between contacting pressure and calandria tubes during a LOCA with late stagnation and impaired or non-existent ECC flow must be predicted along with local heat fluxes and temperatures in order to assess the likelihood of fuel melting and fuel sheath, pressure tube, and calandria tube damage during the accident. This

* When and if flow stagnation occurs in any fuel channel during a LOCA depends on a number of factors, such as the size and location of the break, the primary coolant circulating pump characteristics, etc.

** Garter springs are inserted between the pressure and calandria tubes in a CANDU reactor at 1-2 m. intervals to prevent the tubes from contacting (due to creep of the pressure tube under normal operating conditions) over the design life of the reactor.

problem is discussed in greater detail by Rogers and Currie in references [24, 25, 26 and 27].

I.1.3 Scope and Objectives of Research

The objectives of the research being reported here will be outlined following a brief description of the distinguishing features of a heat transfer analysis involving contacting, non-conforming surfaces. Topics excluded from the study will be identified as they are mentioned in the preliminary discussion.

I.1.3.1 Mechanical Models

Although it is not possible to evaluate the rate of heat transfer between contacting non-conforming surfaces without determining the mechanical (physical) characteristics of the contact beforehand, the mechanical and thermal analyses can be performed independently if the contact area is not influenced (significantly) by thermal distortions.* The parameters that are needed to characterize the contact for the purposes of the thermal analysis vary depending on the radii of the contact surfaces, their surface finish, the contact load, etc., but the size and shape of the contact patch (often referred to as the contour area) are generally very important products of the mechanical analysis, particularly for contacts in vacuum. These parameters are important because they determine the contact's "macroscopic constriction resistance", defined by Clausing and Chao [28] (for contacts in vacuum)

* Some of the criteria which must be satisfied in order for this to be the case are discussed in Section I.2.1.2.

as the resistance produced when the heat flow through the contact is constricted from the cross-sectional area of the original heat flow channel to the area of the contact patch. It will be seen later that this resistance is primarily a function of the dimensions of the contact if those dimensions are small relative to the dimensions of the contacting bodies, essentially because most of the resistance to heat flow through the contact is encountered in its immediate vicinity (where the heat flow is most severely constricted).

Hertz's theory [30] can be used to predict the size and shape of a contact patch formed between surfaces with known radii of curvature provided:

- (1) the contact is formed elastically
- (2) the contact surfaces are smooth
- (3) the contact dimensions are small relative to the dimensions of the contacting bodies, and
- (4) the contact materials are isotropic and homogeneous.

Also, the contact geometry of interest must be describable in terms of the general geometry shown in Figure I.1.2. Bodies 1 and 2 in that figure both have two principal radii of curvature, where the small and large radii on body 1 are denoted R_1 and R_1' respectively and the corresponding radii on body 2 are designated R_2 and R_2' . The principal planes of curvature (i.e., the planes containing the principal radii of curvature) in each body are assumed to be orthogonal while the planes of curvature containing R_1 and R_2 (or equivalently R_1' and R_2') intersect at an angle γ . The axes labelled p_1 and p_2 are the axes of curvature for the principal radii R_1' and R_2' respectively while x and y are the major and minor axes respectively of the cross-hatched elliptical contact area. The

Figure I.1.2 Bodies in Contact.

illustrated force P is co-linear with the z -axis.

It is possible to describe the geometry of either of the CANDU reactor-related contacts mentioned previously in terms of the general geometry shown in Figure I.1.2, as illustrated in Figures I.1.3 and I.1.4, for a fuel element contacting a pressure tube and a pressure tube contacting a calandria tube respectively. The value of γ in each of the contact configurations shown in those figures is 0° since the principal radii R_1 and R_2 (or R_1' and R_2') lie in the same plane in each case. The bowing/sagging-related radius of curvature of the fuel element or pressure tube (i.e., R_1') would be its radius of curvature at the point of contact, of course. Also, the radius R_2 is shown as being negative for both contacts because the centers of curvature for the surfaces with radii R_1 and R_2 are found on the same side of the contact in each case whereas those centers are shown as being on opposite sides of the contact in the general problem illustrated in Figure I.1.2. This convention is used in Hertz's theory.

The conditions which must be satisfied in order for Hertz's theory to be applicable (i.e., conditions (1) - (4) mentioned previously) preclude its being used to obtain anything other than a first approximation to the contact size in the pressure/calandria tube or fuel element/pressure tube contact problem. In the case of a fuel element contacting a pressure tube, localized buckling and plastic deformation of the thin fuel element sheath (typically 0.375 mm. wall thickness in CANDU reactors) may occur in the vicinity of the contact while in the case of a sagging pressure tube contacting a calandria tube, localized distortion of the hot pressure tube due to yielding and thermal warping may occur. Also, the load (P) on the contact cannot be easily determined in either case

Figure I.1.3 Fuel Element Contacting a Pressure Tube

Figure I.1.4 Pressure Tube Contacting the Calandria Tube Which Encloses It

and contact dimensions will probably not be small relative to the dimensions (e.g., tube or sheath wall thicknesses) of the contacting bodies.

Although it is difficult to obtain an accurate estimate of the size and shape of either of the contacts shown in Figures I.1.3 and I.1.4 for the reasons just mentioned, there is no doubt that both of those contacts would have large aspect (length/width) ratios simply because of geometrical considerations. If it is assumed for the purpose of obtaining a rough estimate of the aspect ratio of the pressure/calandria tube contact that (1) Hertz's theory is approximately applicable and, (2) that the deflected shape of the pressure tube approximates that of a uniformly loaded (elastic) beam with fixed ends (at the garter springs), for instance, an aspect ratio of 8.8^{*} is obtained for the garter spring spacing of 1.02 m that is used in Bruce B reactor units 1-4^{**} (the approximate and idealized procedure that was used to obtain the aforementioned aspect ratio is described in more detail in Appendices B and C of Part IV). An aspect ratio of 15.5 is obtained using the same procedure for the garter spring spacing of 1.83 m. used in Bruce A reactor units 1 and 2. While these aspect ratios are very approximate, they should be of the correct order of magnitude for the contact patch that would form when the pressure and calandria tubes first contacted and the contact was lightly loaded.

While contact dimensions are easily evaluated for large, smooth, solid bodies in elastic contact by using Hertz's theory [30], the

* This result is independent of the contact load.

** And Bruce A reactor units 3 and 4.

presence of roughness on the surfaces increases the size of the contact above that predicted by Hertz's theory and makes contact dimensions difficult to estimate [29]. Furthermore, the surface asperities also reduce the ratio of actual contact area to contour area and by so doing introduce an additional thermal resistance, termed the "microscopic constriction resistance" (R_{mic}) by Clausing and Chao [28] for contacts in vacuum. This resistance is produced when heat flow through the contact is constricted from the contour area of the contact to the actual metal-to-metal contact points, termed microcontacts or a-spots, within the contact patch. The resulting interfacial conductance within the contact patch, $h_c (=1/R_{mic} A_c$, where A_c is the contour area), is analogous to the contact conductance defined previously for nominally flat surfaces in contact. Any thermal resistance introduced by the presence of oxide layers is usually considered to act in series with R_{mac} and R_{mic} .

A mechanical model for rough surfaces in contact must generally provide both the size and shape of the contour area (contact patch) and the interfacial conductance (h_c) within it. The interfacial conductance and its local variations within the contour area are intimately related to the metrology of the contiguous surfaces, of course, since their prediction requires estimates of the density, distribution, and radii of the microcontacts as a function of the load applied normal to the contact. Deciding which parameters best characterize the surface texture for the purpose of evaluating the quantities mentioned above is a major problem. Parameters frequently used are average

roughness height (rms* or CLA**), average asperity tip radius, mean slope of sides of asperities and others. Assumptions must be made as to the mode of deformation of the asperities (i.e., elastic and/or plastic), bearing in mind that the deformation of the asperities must be coupled to the deformation of the mean lines of the contacting surfaces. These matters are discussed further in Section I.3.5, where a simple model of rough non-conforming (curved) surfaces in contact is developed.

I.1.3.2 Thermal Models

The information obtained from the mechanical model must be incorporated into a thermal model of the problem of interest. Although almost all realistic problems defy exact analysis using, for instance, separation of variables or conformal mapping techniques, exact solutions to simpler problems can often be used to obtain thermal models which are sufficiently accurate for most engineering purposes. Examples of such models are given in Chapter I.2 along with a compendium of the exact solutions from which they are derived. More complex problems must be analyzed with a numerical technique such as the finite-difference, finite-element, or boundary-element method of course. The two CANDU reactor-related problems mentioned previously are examples of

* Root mean square.

** Center-line average.

problems which must be solved numerically because of their complexity. Although the thermal analysis of such problems should, strictly speaking, be coupled with mechanical (i.e., structural) analyses since component temperatures and deformations in such analyses may be interrelated to some extent (as mentioned previously), thermal/mechanical interactions were not considered in this investigation.

While numerical methods must be used to solve heat transfer problems as complex as the reactor-related examples discussed previously, it has never been conclusively demonstrated that approximate techniques can provide satisfactory solutions to problems in which small contacts are present, particularly in the immediate neighbourhood of the contact patch where the abrupt change in boundary conditions causes the solution to have a singular nature. This characteristic, combined with the fact that contact sizes are typically very small relative to the dimensions of the contacting bodies, means that the use of a very fine mesh near the contact is absolutely mandatory if a purely numerical solution procedure is adopted. As will be discussed in greater detail in Part II, the use of a highly graded mesh, that is a mesh in which the spatial discretization is much finer in some regions than in others, can lead to ill-conditioning of the algebraic equations defining the discrete solution and, in addition, cause certain iterative procedures for solving these equations, such as successive overrelaxation, to converge prohibitively slowly. Highly graded grids are also somewhat difficult to construct in three-dimensional analyses.

Although there are undoubtedly problems in which the accurate resolution of temperatures and temperature gradients near the contact is not particularly important, there are cases of practical significance in which these parameters should be determined as accurately as possible. The heat flow in the neighbourhood of the contact is particularly important in the case of the pressure tube-calandria tube contact, for instance, since it governs the maximum heat flux on the calandria tube outer surface. If that heat flux exceeds the critical heat flux corresponding to the prevailing moderator subcooling, the calandria tube wall may be damaged by dryout.

This report describes both a fundamental study of heat transfer through contacts with large aspect ratios, such as might result from a sagged pressure tube contacting the calandria tube which encloses it or a bowed fuel element contacting a pressure tube, as well as the application of numerical techniques to problems involving such contacts. The primary purpose of the fundamental investigation, which consisted of experimental and analytical studies of a simple contact problem involving a contact patch of large aspect ratio, was to demonstrate that the heat transfer rate through such a contact can be predicted fairly easily from first principles without recourse to contact-specific empirical data when the contact surfaces are sufficiently smooth. This capability had already been demonstrated for circular contacts, such as are produced by contacting spheres, as will be discussed in greater detail in Section I.2. The essential features of the contact problem that was investigated experimentally and analytically in the preliminary

part of this study, termed the "model contact problem", are described in the next section.

I.1.3.3 The Model Contact Problem

The essential geometric features of the contacts shown in Figures I.1.3 and I.1.4 were retained in the model contact problem, as seen in Figure I.1.5. The contact surface on specimen 1 had a small, 0.95 cm. (3/8"), radius of curvature (R_1) to simulate the curvature of a pressure tube (or fuel element) surface in a plane normal to the tube's longitudinal axis. The contact surface on specimen 2 had a larger, 76 cm. (29.94") radius of curvature (R_2) to simulate the bowed radius of curvature of a pressure tube or fuel element. The radii of curvature on the remaining principal planes of the contact specimens were infinite. The planes of curvature of the cylindrical contact surfaces were orthogonal, so application of a load normal to the interface produced an elliptical contact patch with an aspect ratio of 16, as predicted by Hertz's theory.

In the heat transfer experiments performed with the contact shown in Figure I.1.5, a heat source connected to specimen 2 and a heat sink connected to specimen 1 caused a steady one-dimensional heat flow to be established in the laterally-insulated source and sink specimens in regions remote from the contact. Extrapolation of the undisturbed (i.e., one-dimensional) temperature gradients in the source and sink specimens to the contact plane produced the extrapolated interfacial temperature drop, ΔT_c , shown in Figure I.1.5. The thermal resistance of the

Figure I.1.5 The Model Contact

contact (R_c) was then defined as;

$$R_c = \Delta T_c / Q_c \quad (I.1.1)$$

and used to characterize the contact for the purpose of comparing experimental and analytical results, as is common practice in experiments of this type [28].

The gap between a pressure tube and the calandria tube which encloses it contains nitrogen gas or CO_2 in modern CANDU reactors so contact between these tubes would occur in the presence of a conducting fluid (eg., nitrogen). Contact between a fuel element and either another fuel element or a pressure tube during a LOCA with delayed or impaired ECC flow would also occur in the presence of a conducting fluid, which would probably be largely superheated steam as mentioned previously. In order to assess and model the effect of a conducting fluid (gas or vapor) around a non-conforming contact such as either of those just mentioned, the resistance of the model contact (R_c) was determined experimentally and analytically with the contact in air and in vacuum. Accurate assessment of both the analytical gas and solid conduction models was then possible. In addition, contacts with both rough and smooth surfaces were investigated experimentally and modelled analytically although the study focussed, primarily, on contacts with smooth surfaces.

It should be stressed that the experiments performed in this study were not intended to apply directly to the two, very complex, reactor-related heat transfer problems described previously. The experiments were undertaken to assist in the verification of a thermal model which, once verified, could be applied to more complex problems. It will be apparent after reading the literature review given in Section 1.2 that the experiments performed in this study and, in some respects, even the chosen analytical approach, represent logical extensions of fundamental studies performed previously by other contact conductance researchers.

1.1.3.4 Analytical Procedures

An analytical model of the contact problem described in the preceding section is formulated in Chapters 1.3 and 1.4 and compared to experimental results in Chapter 1.5. The analytical model described in Chapter 1.4 is formulated in the ellipsoidal coordinate system, which has ellipsoids and one and two-sheeted hyperboloids as its coordinate surfaces. This coordinate system is almost optimal for the analysis of the model contact problem because it permits the elliptical contact to be represented exactly in the numerical model and naturally provides finer discretization in regions near to the contact than in regions remote from it. It will be seen in Chapter 1.5 that this model yields accurate results with very few nodes.

Although the analytical model formulated in the ellipsoidal coordinate system is probably optimal when applied to the model contact problem, it is of limited applicability in general because the coordinate surfaces of the ellipsoidal coordinate system do not match commonly encountered

boundary shapes (e.g., fuel rod surfaces, etc.) very well at all. For this reason, other more widely applicable numerical models are applied to the model contact problem in Part II of the thesis and compared, in accuracy, to the analytical model formulated in the ellipsoidal coordinate system. Solutions obtained with the latter model are used as benchmarks and assessments of the other models are made based not only on their ability to predict the resistance of the contact accurately but also on their ability to predict heat fluxes through and around the contact with fidelity. The analyses are, of necessity, three-dimensional so the results are of general applicability. While a two-dimensional model might yield results of acceptable accuracy in many problems involving large aspect ratio contacts (a two-dimensional line contact is effectively a contact with an infinite aspect ratio) analysts should actually be cautious when assuming that a two-dimensional analysis can be used to determine heat transfer rates between, for instance, two cylinders in line contact because continuous line contacts are probably rarely achieved in practice. The fraction of the apparent contact length over which contact actually occurs in such cases is governed by the waviness of the surfaces in the axial direction and the contact load (for given material properties), but it is probable that many contacts envisaged as being continuous line contacts are actually very intermittent.* This observation will be discussed further, with

* The intermittency being referred to here is the large-scale (macroscopic) intermittency that would result from waviness, not the microscopic non-conformity that would result from surface roughness.

reference to two specific examples, in Chapter I.2. It might not be necessary to perform a full three-dimensional heat transfer analysis to calculate the heat transfer rate between two surfaces in intermittent line contact since the heat transfer within each zone of actual contact would probably be largely 2-D in most cases, but it would certainly be necessary to use the actual contact length (if that parameter could be estimated) instead of the apparent contact length to evaluate the heat transferred by conduction through the contact patch.

One of the numerical models developed in Part II is applied in Part III to simplified non-linear and transient analyses of the pressure/calandria tube contact problem mentioned previously to demonstrate its applicability to such problems. The formulation of the method for such problems is also described in Part III.

CHAPTER I.2

LITERATURE SURVEY

I.2.1 Introduction

Although a fairly large number of studies relating to heat transfer between nominally flat contacting surfaces have been reported in the literature, there are relatively few papers dealing with heat transfer between contacting non-conforming surfaces (as defined in Section I.1.1). The studies of the latter type that have been reported can be divided into two categories, specifically those dealing with contacts in a vacuum environment and those dealing with contacts in gases. Only the thermal models differ between the two categories - the mechanical models are, of course, the same unless thermal strains are considered.

I.2.2 Non-Conforming Contacts In Vacuum

I.2.2.1 Thermal Models

Non-conforming contacts in vacuum are obviously more easily modelled analytically than those in gases since, in the absence of radiation, the boundary conditions of the associated thermal problem are greatly simplified. The relatively simple boundary conditions encountered in such analyses permit the derivation of thermal models using classical mathematical techniques, at least for problems similar to that of the model contact.

For such problems, particularly those involving smooth surfaces where it is assumed that the microscopic constriction resistance is negligible,* an expression for the macroscopic constriction resistance of the contact geometry in question constitutes the thermal model.

For reasons that will be discussed in Chapter I.3, it is generally assumed in analyses of the heat flow through contact configurations of the type shown in Figure I.1.5 that the contact patch itself is approximately isothermal if R_{mic} is negligible. With that assumption, the interface condition becomes, in effect, a boundary condition on each specimen and the total resistance of the contact can be obtained by summing the constriction resistances of the isothermal contacts on the two specimens. If h_c is uniform over the contour area (A_c) and the resulting value of R_{mic} ($\approx 1/h_c A_c$ in this case) is much greater than R_{mac} , however, the heat flux over the contact will be approximately uniform [28]. The constriction resistances of the contacts on the two contact specimens can still be evaluated separately in that case, only for a uniform contact heat flux, and combined in series with R_{mic} to obtain the total resistance of the contact. Certain distributions of h_c could produce an approximately uniform heat flux distribution over the contour area even if R_{mic} was not large relative to R_{mac} and h_c was not uniform [2], so the uniform contact temperature and uniform contact heat flux boundary conditions are two possibilities that have been studied fairly extensively.

* The validity of this assumption has been proven experimentally, as will be discussed later.

Table I.2.1 lists various constriction resistance expressions for several uniform temperature and uniform flux* contacts. This is by no means a comprehensive listing of such expressions, but rather is a collection of the cases of greatest relevance to this study.

Several expressions are cited for the resistances of contacts 3 and 4. These expressions are based on different mathematical models which yield approximately the same resistance values for small contacts ($\epsilon \leq 0.1$, where $\epsilon = \text{contact radius } (a)/\text{flux tube radius } (R)$) but differ markedly for large contacts ($\epsilon \geq 0.4$). It should be noted that the expressions defining ψ^T for contact 3 all predict that $\psi^T \rightarrow 1$ as $\epsilon \rightarrow 0$, so the constriction resistance of an isothermal contact spot on a flux tube of infinite radius is $R_C^T = 1.0/4k_s a$. This result is in agreement with the corresponding equation for contact 1a. The constriction resistance of a contact on an insulated half-space (e.g., contacts 1 and 2) is therefore an asymptotic value which can be used to approximate the resistance of a contact when the characteristic dimensions of the associated heat flow channel are large compared to the contact dimensions.

* The constriction resistance of a contact subject to a uniform heat flux is defined, in general, using the area-average contact temperature.

Table I-2.1 Constriction Resistance Expressions for Different Types of Contacts

Resistances, $(R_c \& R_{ct})^1$ and geometric factors (ψ) for isothermal and uniform flux contacts are written with the superscripts T and q respectively.				
Contact Number	Contact Description	Contact Diagram	Macroscopic Constriction Resistance	Researcher (Reference)
1	Circular contact spot on infinite half-space ^{1,2}		$R_{ct}^T = \frac{1.0}{4k_s a}$ (Isothermal contact) $R_{ct}^q = \frac{1.09}{4k_s a}$ (Uniform heat flux over contact)	Holm, R. [31] Yovanovich, M.M., Cordier, H., and Coutanceau, J. [32]
2	Elliptical contact spot on infinite half-space		$R_{ct}^T = \frac{\psi^T(k)}{4k_s a}$ where $\psi^T(k) = \frac{2}{\pi} \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2 \sin^2 \theta}} = \frac{2}{\pi} K(k)$ (1) $k = \sqrt{1 - \frac{b^2}{a^2}}$ The integral in Equation (1) (i.e., $K(k)$) is the complete Jacobian elliptic integral of the first kind of modulus k . As $k \rightarrow 1$, $\psi \rightarrow \frac{2}{\pi} \ln \left(\frac{4a}{b} \right)$. As $k \rightarrow 0$, $\psi \rightarrow 1$.	Yovanovich, M.M., [15]

Table 1.2.1 continued

Contact Number	Contact Description	Contact Diagram	Macroscopic Constriction Resistance	Researcher (Reference)
3	Circular contact spot on cylindrical heat flux tube		$R_C^T = \frac{T(\epsilon)}{4k_s a}$ <p>where $\epsilon = a/R$</p> <p>Several expressions for ψ^T (derived by different investigators) follow.</p> $(a) \psi^T = \frac{8}{\pi \epsilon} \sum_{n=1}^{\infty} \frac{\sin(\alpha_n R \epsilon)}{(\alpha_n R \epsilon)^3} J_1(\alpha_n R \epsilon)$ <p>with $J_1(\alpha_n R) = 0$</p> <p>For $0 < \epsilon < 0.6$, (a) can be approximated by the following expression:</p> $\psi^T = 1 - \frac{4\epsilon}{\pi}$ <p>(b) $\psi^T = 1 - \epsilon$</p> <p>(c) $\psi^T = \frac{2}{\pi} \arctan\left(\frac{1}{\epsilon}\right) - \frac{4\epsilon}{\pi}$</p> <p>(d) $\psi^T = \frac{2}{\pi} \arctan\left(\frac{1}{\epsilon} - 1\right)$</p> <p>(e) $\psi^T = 1 - 1.40925\epsilon + 0.29591\epsilon^3 + \dots$</p> <p>(f) $\psi^T = 1 - \frac{4\epsilon}{\pi} - \epsilon^2$</p> $R_C^T = \frac{T(\epsilon)}{4k_s a}$ <p>where $\psi^T = \frac{16}{\pi \epsilon} \sum_{n=1}^{\infty} \frac{J_1^2(\alpha_n R \epsilon)}{(\alpha_n R \epsilon)^3 J_0^2(\alpha_n R)}$</p> <p>with $J_1(\alpha_n R) = 0$</p>	<p>Mikic, B.B. [33]</p> <p>Bowden, F.P. and Tabor, D. [34]</p> <p>Holm, R. [35]</p> <p>Veziroglu, T.N. and Fishenden, M. [36]</p> <p>Roess, C.L. [37]</p> <p>Yovanovich, M.M. [38]</p> <p>Mikic, B.B. [33]</p>

Table I.2.1 continued

Contact Number	Contact Description	Contact Diagram	Macroscopic Constriction Resistance	Researcher (Reference)
4	Strip contact centrally located on end of rectangular heat flux tube (planar problem)		$R_C^T = \frac{\psi^T(c)}{\pi k_s}$ <p>where $c = b/t$</p> <p>Several expressions for ψ^T (derived by different investigators) follow:</p> <p>(a) $\psi^T = \frac{1}{\pi c} \sum_{n=1}^{\infty} \frac{1}{2} J_0(n\pi c) \sin(n\pi c)$</p> <p>(b) $\psi^T = \ln \left(\frac{1}{\sin \left(\frac{\pi c}{2} \right)} \right)$</p> <p>(c) $\psi^T = \left(\ln \left[\frac{1}{c} + \sqrt{\frac{1}{c^2} - 1} \right] - c \sinh \left[\frac{1}{c} + \sqrt{\frac{1}{c^2} - 1} \right] \right)$</p> <p>For $c \leq 1$, c can be approximated by:</p> $\psi^T = \left[\ln \left(\frac{2}{c} \right) - 1 \right]$ <p>$R_C^Q = \frac{\psi^Q(c)}{\pi k_s}$ (per unit length)</p> <p>where $\psi^Q = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^3} \sin^2(n\pi c)$</p>	<p>Mikic, B.B. [33]</p> <p>Veziroglu, T.N. and Chandra, S. [39]</p> <p>Yovanovich, M.M. [40]</p> <p>Mikic, B. B. [33]</p>

Table I.2.1 continued

Contact Number	Contact Description	Contact Diagram	Macroscopic Constriction Resistance	Researcher (Reference)
5	Strip contact eccentrically located on end of rectangular heat flux tube (planar problem)		$R_{ct}^T = \frac{T}{q} \quad (\text{per unit length of contact})$ $\text{where } T = \ln \left(\frac{1}{\sin \left(\frac{\pi b}{2t} \right) \cos \left(\frac{\pi e}{2t} \right)} \right)$ $R_{ct}^Q = \frac{q}{\pi k_s}$ $\text{where } q = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \left(\frac{n\pi b}{2t} \right) \cos^2 \left(\frac{n\pi e}{2t} \right) \frac{1}{\left(\frac{\pi b}{2t} \right)^2}$	Sexl, R.U. and Burkhard, D.G. [41] Veziroglu, T.N. and Chandra, S. [39]
6	Strip contact on semi-cylinder with isothermal diametral plane		$R_{ct}^T = \frac{x(k')}{2k_s} K(k) \quad (\text{per unit length of contact})$ $\text{where } k = \sin \alpha; k' = \sqrt{1 - k^2} \text{ and } K \text{ is the complete elliptic integral of the first kind (see contact #2)}$ $\text{For small } \alpha,$ $R_{ct}^T = \ln [2(1 + \cos \alpha)] / \sin \alpha / \pi k_s$	Schneider, G.E. [42] Yovanovich, M.M. and Coutanceau, J. [42a]

1. For all contacts except those on flux tubes, the constriction resistance is written as R_{ct} instead of R_c to indicate that the expression refers to the total resistance instead of the resistance based on ΔT_c (as in Equation (1.1.1)). To obtain the total resistance of a flux tube of cross-sectional area A and length L (the diagram for model 3 shows L) with a contact on one end, the resistance of the flux tube for uniform axial heat flow (i.e., $L/k_s A$) must be added to R_{ct} . The resulting total resistance will only be accurate when the heat flow at a distance L from the contact is essentially undisturbed by its presence (i.e., effectively one-dimensional), however.

2. The resistance of a contact on an infinite half-space can be regarded as the resistance between the contact patch and an isothermal, hemispherical surface of infinite radius. The contact patch is located at the center of the otherwise-insulated equatorial plane of the hemisphere.

In addition to the constriction resistance expressions shown in Table 1.2.1, other constriction resistance expressions for contacts on insulated half-spaces and flux tubes have been derived for a wide variety of contact shapes and contact boundary conditions. Yovanovich has formulated an integral technique by which the constriction resistance of an arbitrarily shaped planar contact area on an insulated half-space can be determined for arbitrary heat flux distributions over the contact area [43]. Yovanovich's boundary integral technique is based on the superposition of solutions to Laplace's equation for point heat sources distributed over the contact region. Yovanovich has used the method to predict the constriction resistances of several contact shapes (e.g., rectangular, annular, ring, triangular, elliptical, semi-circular) for easily specified flux distributions (e.g., uniform flux) [43-44].

The resistance (R_{ct}) of a general contact, which may or may not be isothermal, is usually defined as the difference between the average temperature of the contact area ($\bar{T}_c$) and the temperature far from the contact (T_∞), divided by the total heat flow rate through the contact area (Q_c). Mathematically, this definition can be written as:

$$R_{ct} = (\bar{T}_c - T_\infty) / Q_c, \quad \bar{T}_c > T_\infty$$

where

$$\bar{T}_c = \frac{1}{A_c} \iint_{A_c} T_c \, dA_c$$

T_c is the temperature of the differential surface element dA_c and A_c is the total contact area. Yovanovich et al. [44] showed that

non-dimensionalizing the constriction resistance of an arbitrarily shaped singly-connected contact area by writing it in the form $R_{ct} k_s \sqrt{A_c}$ produced a dimensionless resistance which was remarkably constant for a wide range of contact shapes (circular, semi-circular, triangular, etc.). The dimensionless resistance became even less a function of contact shape when the resistance R_{ct} was defined using the temperature at the centroid of the contact area instead of the area-average contact temperature. While this result was obtained for contacts subjected to a uniform heat flux, Schneider demonstrated that it was also true for hollow, rectangular, isothermal contacts of different aspect ratios [45]. These conclusions indicate that the resistance of a contact is a much stronger function of its area than its shape. It can be shown using the resistance equation given in Table 1.2.1 for contact 2 that the constriction resistance of an isothermal elliptical contact with an aspect ratio of four on an insulated half-space is only 11% less than the constriction resistance of a circular contact of the same area, for instance.

While Yovanovich's integral technique can be used to determine constriction resistances for arbitrary contacts with Neumann* boundary conditions,** explicit solutions for contacts with prescribed Dirichlet***

* Prescribed surface heat flux.

** Yovanovich's integral technique only yields an explicit expression for the resistance of a Neumann contact if the boundary integral in that technique can be integrated directly. In general, direct integration of the boundary integral is not possible and numerical quadrature must be used.

*** Prescribed surface temperature.

conditions are only available for special cases, most of which are listed in Table 1.2.1. Schneider applied a numerical technique, the boundary element method, to the problem of determining the constriction resistance of an arbitrary Dirichlet contact on an insulated half-space [45]. The constriction resistances of several hollow rectangular contacts with different aspect ratios were computed by Schneider, as mentioned earlier, to demonstrate the applicability of the boundary element method to problems of this type.

Strong, Schneider and Yovanovich used a finite-difference model formulated in the oblate spheroidal coordinate system to determine the constriction resistance of a circular contact on an insulated half-space with arbitrary heat flux distributions prescribed over the contact area [46]*. The study by Strong et al. was performed to test the validity of an assumption made by some contact conductance researchers, viz, that the constriction resistance of a circular contact subjected to any flux distribution is bounded by the uniform temperature and uniform heat flux resistances given for contact 1 in Table 1.2.1. There does not appear to be any reason for suspecting that this would be the case, and Strong et al. succeeded in demonstrating that this assumption is invalid.

Strong, Schneider and Yovanovich formulated their finite-difference solution in the oblate spheroidal coordinate system because it was expected that heat flow through the contact would produce a flow net, comprised of isotherms and flux lines, that coincided (approximately)

* Yovanovich's boundary integral technique could have been used for the study described in reference [46], but the introduction of that technique followed the study described in reference [46] chronologically.

with the coordinate surfaces. The heat flow between an isothermal circular contact on the otherwise insulated equatorial plane of one-half of an isothermal oblate spheroid and the surface of the oblate semi-spheroid itself is one dimensional with respect to the oblate spheroid coordinate surfaces. When the bounding isothermal oblate semi-spheroid is large enough, it approximates a "hemispherical" semi-infinite domain. While the heat flow from the circular disc (contact) at the center of the equatorial plane of the "hemisphere" is not one-dimensional with respect to the oblate spheroidal coordinate surfaces except when the disc is isothermal, Strong et al. expected that the natural grading of this coordinate system would offer computational advantages, in the form of reduced truncation errors, even when the disc was non-isothermal. The same finite-difference model was later used by Schneider, Strong and Yovanovich to study transient heat flow through a circular contact on an insulated half-space [47]. The half-space was assumed to be at zero temperature initially and, for time $(t) \geq 0$, heat was supplied to the half-space via the contact. The results were presented in terms of the transient constriction resistance (R_{tc}), which was defined as the average temperature of the contact area divided by the instantaneous heat flow rate through the contact. Graphs showing the dimensionless constriction resistance ($R_{tc} k_s a$) of a circular contact as a function of the Fourier modulus $\alpha t/a^2$ (where α is the thermal diffusivity of the half-space) are presented in [47] for various contact boundary conditions (i.e., uniform flux, uniform temperature, etc.).

Schneider, Strong and Yovanovich also used the oblate spheroidal finite-difference model to study transient heat flow between two semi-infinite bodies communicating through a small circular contact area [48].

The two bodies, which were allowed to have different thermal conductivities and diffusivities, were assumed to be at uniform but different temperatures before being brought into contact. The constriction resistance between the two bodies, defined as the ratio of the initial temperature difference between the bodies to the instantaneous rate of heat flow through the contact, was correlated as a function of a modified Fourier modulus for the range of physical parameters considered. The constriction resistance was non-dimensionalized by dividing it by the steady-state constriction resistance of the contact and the Fourier modulus was defined using the harmonic mean of the thermal diffusivities of the two contacting bodies.

It should be noted that with the exception of the last model discussed, all of the thermal models considered so far have treated contact areas as boundary areas over which certain boundary conditions are prescribed. A difficulty arises in applying these models to general problems in that contact areas are interfaces and, as such, they are subject to "boundary conditions" which are usually unknown. When the contacting bodies and imposed boundary conditions are mirror images about the contact plane and $R_{\text{mac}} \gg R_{\text{mic}}$, the contact can be assumed isothermal as mentioned previously. Similarly, if $R_{\text{mac}} \gg R_{\text{mic}}$ and h_c is uniform over the contact area, the contact heat flux can be assumed uniform. With the exception of these two cases, however, neither the heat flux nor the temperature distribution over a contact can be obtained without solving for the temperature distributions in the two contacting bodies simultaneously. While Strong et al. [46] have demonstrated, for a circular contact, that the constriction resistances for a uniform contact temperature and

a uniform contact heat flux do not bracket the resistances for all other heat flux/temperature distributions, all of the contact boundary conditions investigated by those researchers produced constriction resistances which differed by less than 11% from the mean of the uniform temperature and uniform flux resistances.* Thus the assumed flux or temperature distribution over a contact is probably not too important in cases where the resistance of the contact is the result of main interest.

I.2.2.2 Experimental Investigations

The first definitive paper related to contact between non-conforming surfaces in vacuum was that of Clausing and Chao [28], in which contact of specimens with spherical caps (contact surfaces) of large radii (3.35m. - 211.9m.) was modelled analytically and experimentally. It was intended that the spherical caps simulate surface waviness on nominally flat contact surfaces so the radii of the spherical caps were deliberately made very large. The experimental method used was similar to that outlined in section I.1.3.3. The tests were performed using several different metals (although identical metals were used in a particular contact) and the resistance of the contact (as defined in Section I.1.3) served as the basis for comparing experimental and analytical results.

* It can be seen in Table I.2.1 that the uniform temperature and uniform flux resistances of a circular contact on a half-space (contact 1) differ by 9% themselves.

The contact surfaces were highly polished to a surface roughness of approximately 0.1 microns rms and the radius of the contact was predicted using Hertzian theory. The total thermal contact resistance was assumed to be equal to the sum of three resistances, viz: (1) the macroscopic resistance, predicted using model (3e) in Table I.2.1, (2) the microscopic resistance, predicted using a simple expression outlined in [28], and; (3) the film resistance, arising from oxide films. The film resistance was actually not modelled since most of the tests were performed using clean contact surfaces. The experimental results were in excellent agreement with the proposed contact model and showed that the microscopic constriction resistance was negligible compared to the macroscopic constriction resistance.

Clausing later published a paper [49] wherein contact between dissimilar metals was investigated using the spherical cap model employed previously. Other researchers (e.g., [50-51]) had observed that the thermal resistance of a contact formed between dissimilar metallic specimens was a function of the direction and magnitude of the heat flow through the contact. Clausing's study verified this observation and a qualitative explanation, based on thermal strains in the vicinity of the contact, was offered to explain this phenomenon. The spherical contact surfaces on the specimens used by Clausing in his study had large radii of curvature, as in his previous investigation, so it was argued that the thermally induced curvatures (strains) of the contact surfaces would have been comparable in magnitude to the initial (uniform temperature) curvatures. It followed that the thermally induced curvatures of the contact surfaces, which would not have been complementary because the contacting materials

were different, would have affected the macroscopic contact area.

Powell et al. [52] investigated the directional effect in connection with the operation of a thermal comparator* and found no evidence of the effect when heat flowed through the small circular contact area formed between the small steel ball (6.3 mm. radius) of the comparator and a flat aluminum test surface. The same result was obtained when the steel comparator ball was brought into contact with germanium and ceramic surfaces (i.e., poor conductors). Powell et al. attributed their results to the fact that the initial curvature of one of their test surfaces (the steel ball) was much greater than any curvature changes that might have resulted from thermal effects. Consequently, it was argued that thermal effects would not have significantly altered the contact area, or the contact resistance, in this case.

Other researchers have attempted to model the "thermal rectification" (i.e., directional) effect, either analytically or empirically, and an excellent discussion and comparison of these models is given by Somers et al. in reference [53].

* The thermal comparator is a simple device for estimating thermal conductivities. With this device, the thermal conductivity of a material is estimated by bringing a small metal sphere, which is initially at a different temperature, into contact with the material and measuring the rate of change of the sphere's temperature. The latter parameter can be related to the test material's thermal conductivity because the time constant of the sphere's thermal response is governed by the large constriction resistance at the sphere/flat contact, which in turn is a function of the contact load (controlled) and the harmonic mean of the thermal conductivities of the test and sphere materials. Other factors can affect the results, however, such as gas conduction and test material surface roughness.

Yovanovich [15] developed an analytical model, which he later verified experimentally, for predicting the total resistance of a dry, unlubricated ball bearing assembly in vacuo. This resistance is required in spacecraft heat flow analyses since heat generated by spacecraft instruments, which are often mounted on gimballed platforms, must, in this case, be conducted across rolling element bearings before it can be transmitted to the spacecraft heat sink. Heat transfer analysts must know the thermal resistances of the bearings in order to predict component temperatures. Yovanovich's model considered the thermal resistance of a ball bearing with N balls to consist of N resistances connected in parallel. Each of the parallel resistances was assumed to be the sum of the macroscopic constriction resistances at the inner race/ball contact and the outer race/ball contact. These constriction resistances were predicted using the resistance equation (R_{ct}^T) for contact 2 (with $a=b$) in Table I.2.1, and contact sizes were predicted using Hertzian theory. Only symmetrically-loaded (i.e., axially loaded) ball bearings, in which all of the balls are equally loaded,* were considered. Yovanovich obtained very good agreement between experimental results (for a stationary bearing assembly) and the predictions of this model.

* The load supported by each ball is not directly proportional to the mechanical load applied to the bearing assembly since thermal expansion of the balls and races, if greater than the initial bearing clearance, introduces an additional load which is a function of the temperature gradient across the bearing.

The thermal resistance of a symmetrically-loaded ball-bearing in vacuo was also investigated by Hallmann [54, 55], again in connection with spacecraft instrument heat rejection. Hallmann did not use the resistance equation (R_{ct}^T) for contact in Table I.2.1, or equivalently that for contact 2 with $a=b$, to predict constriction resistances in his analytical model but used instead an expression developed by Yovanovich in earlier work for the thermal resistance of diametrically-opposed isothermal contacts on a solid sphere [56]. Hallmann's approach to the problem essentially paralleled that of Yovanovich in other respects and, as in the case of Yovanovich, he obtained good agreement between experimental and theoretical results.

The resistance between smooth spheres and smooth flat surfaces in contact was investigated by Kitscha and Yovanovich [16; 57]. The total thermal resistance of the contact was assumed to equal the macroscopic constriction resistance of the circular contact patch (i.e., $2 \times R_{ct}^T$ for contact 1 in Table I.2.1 or $2 \times R_{ct}^T$ for contact 2 with $\psi^T = 1$). The radius of the contact was calculated using Hertzian theory. Again, the agreement between the analytical and experimental results was excellent in that the difference between these results was typically less than 6% over the load range studied. The sensitivity of the sphere-flat contact resistance (in vacuum) to increased roughness of the contiguous surfaces was investigated by Kitscha and Yovanovich in [57] by bead-blasting the flat surface to produce a surface roughness of 0.75 microns rms (30 μ in). A decrease in the resistance of the contact at light loads was observed and it was postulated that this effect was caused by spreading of the contact patch beyond the Hertzian size.

Burde [58, 59] investigated the thermal resistance of a smooth sphere/rough flat contact in vacuum both experimentally and analytically.

His analytical resistance model consisted of:

(a) a mechanical model for predicting the most probable number, size and distribution of microcontacts within the contour area, along with the extent of the contour area itself, as a function of known material properties, surface characteristics and contact load. Burde allowed both the microcontact density and the average microcontact radius (microcontacts were assumed circular) to decrease with increasing distance from the center of the contact area, in contrast to earlier models developed by Yip [60] and others (e.g., [34, 61, 62]) in which both of these parameters were assumed constant within the contour area.

(b) a thermal model for predicting the thermal resistance of the contact from the results of the mechanical analysis. Different combinations of contact (contour area) and microcontact boundary conditions were considered, specifically:

1. both contact region and microcontacts isothermal
2. contact region isothermal but microcontacts under uniform flux
3. both contact region and microcontacts under uniform flux
4. contact region under uniform flux but microcontacts isothermal.

Constriction resistances in each case were obtained using Yovanovich's boundary integral technique, described previously. The heat flux distribution over an isothermal contact or a single, isolated, microcontact was assumed to be that determined by Roess [37], i.e.,

$$q''(r) = \frac{q''_0}{\sqrt{1 - \left(\frac{r}{a}\right)^2}}$$

where q''_0 is the heat flux at the center of the contact, r is the radial distance (in the contact plane) from the center of the contact, and a is the contact radius. Note that the heat flux at the edge (i.e., at $r=a$) of an isothermal circular contact on a large (relative to the contact size) flux tube is infinite according to Roess's formula, a factor which makes it difficult to obtain high accuracy in that region with a numerical solution.

Burde obtained excellent agreement between theoretical and experimental results by assuming that both the contour area and the microcontacts were isothermal.

The thermal resistances in vacuum of contacts other than those involving spheres have also been investigated, usually to obtain information required in spacecraft heat flow analyses. Yovanovich [18] has investigated, analytically and experimentally, the thermal resistance of a roller bearing in vacuum in connection with spacecraft instrument heat rejection. The rollers in the bearing were assumed to be cylindrical in shape, equally loaded, and thermally connected in parallel. The thermal resistance associated with each element (roller) was assumed to be the sum of the constriction resistances at the roller/outer race and roller/inner race contacts. The constriction resistances in the races were predicted using Veziroglu's expression (model 4b in Table I.2.1) and the resistances of the two contacts on each cylindrical roller were predicted using the Yovanovich-Coutanceau relationship (model 6 in Table I.2.1). Contacts were assumed isothermal and contact sizes were predicted using Hertzian theory. Non-uniformly spaced rollers were also

considered and model 5 in Table I.2.1 was used to predict the constriction resistances in the races in that case. Good agreement between experimental and theoretical results was reported, although the resistance of several 0.8 mm. diameter x 2.54 cm. long steel cylinders sandwiched between two optically flat steel surfaces was measured in the experiments instead of the resistance of an actual roller bearing assembly.

Several researchers have investigated the thermal resistance, in vacuum, of a wire mesh sandwiched between two flat surfaces (e.g., [19, 20]) as mentioned in Section I.1.2. Cividino [20] attempted to model the conductance of a wire screen but had limited success, with differences between experimental and theoretical results as great as 70% being reported (differences were usually in the 20-50% range).

Schneider and Yovanovich measured the thermal resistances of several hollow and solid metallic O-rings sandwiched between flat surfaces and correlated the results [63]. Again, this study was performed to obtain information required in spacecraft heat flow analyses. An analytical model of an O-ring/flat contact could not be developed because it was impossible to achieve uniform circumferential contact between the O-rings and the flats when commercially available O-rings were used.

I.2.3 Non-Conforming Contacts in Gases

I.2.3.1 Thermal Models and Experimental Studies

Since the boundary conditions around a contact in a gas are considerably more complex than those present in vacuum, an exact solution for the heat transfer through such a contact is usually difficult to obtain without making major simplifications. In spite of this, numerical techniques have rarely been used in the past to solve such problems, probably because results obtained from numerical solutions are not easily disseminated unless presented in the form of publishable correlations and charts.

As mentioned in Section I.1.2, heat transfer between contacting spheres in a stagnant gas has been studied by several researchers because of its significance in packed bed heat transfer and other applications. Descriptions of some of the more relevant studies follow.

Wakao and Vortmeyer [64] performed a study of heat transfer in a packed bed containing a stagnant gas in which they developed a numerical model (finite-difference) for evaluating heat transfer rates between two contacting spheres in a gas. The two-dimensional (axisymmetric) finite-difference grid used in the analysis was not graded and was too coarse near the contact in the author's opinion. A plot was included in the paper which, after appropriate algebraic manipulations, showed the resistance (predicted numerically) of the sphere-sphere contact without gas conduction as a function of the radius of the contact patch. This functional relationship is given to a first

approximation by the resistance equation (R_{ct}^T) for contact 1 in Table I.2.1 for $a \ll R$, where a is the contact radius and R is the sphere radius. The resistance in Table I.2.1 must be doubled for two spheres in contact, of course,

$$\text{i.e., } R_{ct}^T = \frac{1}{4 k_s a} \times 2 = \frac{1}{2 k_s a} \text{ for two spheres (contacts).}$$

Comparison of this expression with the numerically predicted results in [64] shows that while the results differ by approximately 35% at an a/R ratio of 0.10, the agreement deteriorates as a/R decreases until, at an a/R ratio of 0.001, the results differ by almost two orders of magnitude. This comparison supports the author's contention that the finite-difference grid used by Wakao and Vortmeyer was much too coarse to accurately model the macroscopic constriction resistance associated with the contact.

A positive feature of the analysis in [64] was that it used a non-continuum gas conductivity model near the contact patch. Such a model is needed because the effective thermal conductivity of a gas between two surfaces decreases below its continuum value when the gap between the surfaces becomes of the same order of magnitude as the mean free path of the gas molecules. This effect was included in the analysis by Wakao and Vortmeyer, in which local gas conductivities were evaluated using local gap sizes.

Yovanovich and Kitscha [57] developed analytical models for predicting the thermal resistance of a smooth sphere-smooth flat contact both in a gas and in vacuum. The resistance of the contact in vacuum was, as mentioned previously, assumed equal to (twice) the

macroscopic resistance (R_{ct}^T) of contact 1 in Table I.2.1 (or contact 2 with $\psi^T = 1.0$), with the contact radius evaluated using Hertzian theory. The vacuum-related contact model was modified to account for the presence of a gas by assuming that the temperatures on the contacting surfaces near the contact with gas conduction would be negligibly different from those prevailing without gas conduction. The latter temperatures were easily calculated and used in conjunction with a non-continuum gas conductivity model (identical to that used in the previously discussed model of Wakao and Vortmeyer) to predict the gas conduction contribution to the total heat flow between the contact surfaces. The analytical model just discussed was compared with experimental results in [16] for steel contact specimens in air and in argon for a wide range of gas pressures and contact loads. Agreement between the analytical and experimental results was excellent. Gas conduction was found to be very significant in some of the tests.

The analytical model of Yovanovich and Kitscha was improved slightly by Ogniewicz [65] by incorporating the effect of the contact-related elastic deformation of the contiguous surfaces on gas conduction into the model. This refinement was worthwhile since the gas conductance immediately adjacent to a contact (where elastic strains are significant) is very high. Ogniewicz presented more experimental results in [65] which once again verified the adequacy of the proposed analytical model. He then used the basic sphere-sphere contact model to predict the effective thermal conductivity of a packed bed of spherical particles with a stagnant interstitial gas present. Simple cubic and body and face-centered cubic packing states were considered.

Peddicord and Ades [12] used the finite-difference method to estimate the effective thermal conductivity of Sphere-Pac reactor fuel, which consists of contacting hard carbide fuel spheres surrounded by stagnant helium gas. The fuel spheres and interstitial helium gas are contained in cylindrical sheaths (i.e., fuel pins) and coolant flowing outside of the pins removes the heat generated in the fuel. The two-dimensional (axisymmetric) finite-difference model developed by Peddicord and Ades for a unit cell of the Sphere-Pac fuel array neglected solid-solid conduction between the contacting spheres, as well as radiative heat transfer, but used a non-continuum gas conductivity model to evaluate conduction in the gas phase. Little additional information on the analytical technique is given in [12], but experimental results are presented which appear to agree quite well with analytical predictions.

Other types of non-conforming contact heat transfer problems (i.e., problems involving other than sphere-sphere contacts) have been reported in the literature. Hwang and Wang [21], for instance, performed an analysis to determine the heat flow through the spacer rods separating the inner and outer hexcans of a LMFBR (liquid metal fast breeder reactor) fuel channel during a LOPI (loss of piping integrity) accident. This analysis was discussed in section I.1.2.

Heat transfer between parallel contacting tubes carrying hot and cold fluids was investigated by Simpson in connection with a proposed method for detecting coolant channel blockages in CANDU reactors [22, 66, 67]. As mentioned in section I.1.2, the proposed method involved monitoring the temperatures of DN (delayed neutron) lines (tubes) carrying

small bleed flows of the coolant from the end of each fuel channel outside of the reactor core for the purpose of monitoring radiation levels. In general, the fluid in a tube leading from a blocked channel would be hotter than the fluid in other tubes, thus creating the possibility of easy blockage detection. The coolant-carrying tubes would have been bundled together for a distance of 15-30 m. before reaching the temperature monitor, however, so heat exchange between the tubes would have attenuated the temperature difference between the fluid in a "hot" tube coming from a blocked channel and the fluid in other tubes. Simpson's study was directed towards evaluating the rate of heat transfer between the tubes.

Simpson's investigation was an experimental one in which two lengths of commercially-available drawn tubing, one of which carried a hot fluid and the other a cold fluid, were clamped together in a 3.35 m. (11'ft.) long clamping press. Heat exchange between the hot and cold fluid streams was measured for several tube materials, tube sizes and wall thicknesses over a wide range of contact loads. The tubes in any given contact pair were always identical. A simple thermal model, proposed by Rogers [68, 69] neglected axial conduction in the tube walls and exploited the symmetry of the remaining two-dimensional problem by dividing the tubes in half about the chordal diameters passing through the line of contact. The two tube walls were then modelled as fins, one of which was exposed to a hot fluid and the other to a cold fluid (on one side only) connected by a resistance which represented the combined effect of gas conduction and the macroscopic and microscopic constriction resistances of the contact.

The three-dimensional heat flow problem was, in this manner, represented by a one-dimensional model. This model was used to separate the conductance (inverse of the resistance) between the fins from the experimental data. The conductance was then correlated as a function of six dimensionless groups representing material mechanical and thermal properties, tube wall thicknesses, etc. The correlation exhibited considerable scatter ($\pm 40\%$) unlike most previously published results for non-conforming contacts, although most of the earlier studies compared experimental results to analytical models and did not involve statistically derived correlations.

Comparison of Simpson's results to predictions obtained with a computer program described in Part IV indicated that axial spreading of the contact between the tubes was the most probable explanation for the conductance-load relationship observed by Simpson. In other words, the tubes were probably not in line contact along their entire lengths but rather were only in partial contact axially. It will be remembered that Schneider and Yovanovich had been unable to achieve uniform contact circumferentially between flats and metallic O-rings in their study of the conductance of an O-ringed joint in vacuum.

I.2.4 Mechanical Models

State of the art reviews of the literature dealing with mechanical models for rough surfaces in contact are given in references [69a] and [69b] so the review given here will be brief. The recent (1977) review and bibliography prepared by Thomas and King [69a] is especially

comprehensive since it contains summaries of 651 papers devoted to rough surface characterization, rough surface interaction, and contact resistance. Only a selected sampling of the most relevant references will be discussed in this section, although other references will be referred to in later sections as they are needed.

Mechanical models for predicting the real area of contact between rough contiguous surfaces along with the density of microcontacts and their size distribution as a function of load have been developed for contacts formed between nominally flat (i.e., non-wavy) surfaces by Cooper et al [1], Mikic [2, 3, 33], Thomas and Probert [69c], Greenwood and Williamson [62], Whitehouse and Archard [69d] and others. Most studies have assumed that the mode of deformation of the asperities is either totally elastic or totally plastic, although a mixture of the two modes of deformation is most likely. Greenwood and Williamson [62], Whitehouse and Archard [69d] and Mikic [3] have all derived similar "plasticity criteria" for predicting, from the shapes of the asperities and the mechanical properties (i.e., hardness, Young's modulus and Poisson's ratio) of the surface material, when appreciable plastic deformation of the tips of the asperities will occur on loading. The aforementioned plasticity criteria differ in the parameters used to characterize the shapes of the asperities but all yield the result that the mode of deformation of the asperities should not depend on the applied load (i.e., they all indicate that asperities which deform plastically will do so at any applied load). The plasticity criterion

developed by Mikic [3] will be described in Section I.6.5.

Mikic [3], Cooper et al. [1], Jones et al. [69e] and others have used the results of mechanical analyses of contact between rough non-wavy surfaces to derive theoretical expressions for the contact conductance (h_c) between such surfaces in vacuo. Such expressions are generally derived from the density of microcontacts and their average size by assuming (1) that the thermal resistances of the microcontacts act in parallel; (2) that the microcontacts themselves are roughly circular and (3) that the resistance of each individual microcontact can be obtained from the constriction resistance expression for contact 1 or contact 2 in Table I.2.1.

Models of contact between rough curved surfaces, particularly spheres, have been developed by several researchers. Burde [58, 59], for instance, developed a mechanical model for predicting the distribution of microcontacts within the contour area of a smooth sphere/rough flat contact by assuming that the asperities on the rough surface would deform plastically while the mean lines of the two contact surfaces deformed elastically. The results of that analysis were used to predict the resistance (in vacuo) of the contact in the manner described in Section I.2.2.2. The mechanical model developed by Burde was similar to that developed by Roca [4, 69f] in an earlier study, although Roca also considered the case where all of the asperities deform elastically instead of plastically. A mechanical model of rough spheres in completely elastic contact was also developed by Greenwood and Tripp [61].

Kagami et al [69g] recently developed a model of contact between a smooth sphere and a rough flat in which mixed asperity deformation (i.e., elastic and plastic) was considered.

Yip [29, 60] developed mechanical models for predicting the real areas of contact between contiguous rough spheres, cylinders (in line contact) and annuli. It was assumed in developing the models that the asperities and surface mean lines would deform plastically and elastically respectively.

The models of Burde, Yip and Roca will be discussed in greater detail in Section 1.3.5 when the solution of the model contact problem with rough surfaces is considered. For reasons that will be discussed at that time, it was assumed in all of the aforementioned analyses that either the surface heights or the asperity peak heights on the surfaces of interest would be normally distributed.

1.2.5 Summary

Most of the non-conforming contacts investigated to date have been sphere-sphere or sphere-flat contacts. Simple analytical models for determining the resistances of such contacts in gases and in vacuum have been developed by several researchers for contacts with smooth contacting surfaces. Hertzian theory has invariably been used in these analyses to predict contact sizes. The agreement between proposed

analytical models and experimental results has, in general, been excellent. It has been shown that the resistance of a sphere-sphere or sphere-flat contact is characterized by a total absence of any microscopic constriction component when the contact surfaces are smooth and that conduction through a gas around such a contact can significantly reduce its thermal resistance. Mechanical and thermal models for a rough sphere in contact with either another sphere or a flat in vacuum have been developed and verified experimentally.

No accurate experimental results have ever been obtained for an isolated non-circular contact, in particular for a contact of high ellipticity, and the author knows of no published references (other than those of Simpson) reporting measurements of the resistance of an isolated contact (circular or otherwise) with rough contact surfaces in the presence of a conducting fluid (e.g., air). Also, numerical methods have rarely been used in the past to model heat transfer problems in which small contact patches are present and the numerical models that have been developed have generally not been properly tested for accuracy either by comparison with accurate experimental data and/or by comparison with more accurate analytical models. The author is also unaware of a numerical model having ever been used to analyse heat transfer between rough curved surfaces, either in a gas or in vacuum. This thesis addresses all of these topics.

CHAPTER 1.3

PROBLEM ANALYSIS

I.3.1 Introduction

The model contact problem was described briefly in Section I.1.3.3. As mentioned in that section, the resistance of the contact shown in Figure I.1.5 was measured with the contact in air and in vacuo. Analytical models were developed for predicting the resistance of the contact under those conditions although several assumptions were made in order to simplify the analyses, primarily to reduce the extent of the region actually modelled. It is the intent of this chapter to describe and justify those assumptions and simplifications, as well as to introduce some background material required for the analysis.

An obvious prerequisite to the solution of any non-conforming contact problem is the evaluation of contact dimensions and surface deformations. Surface deformations outside of the contact area are required for the gas conduction analysis (if a gas is present) and contact dimensions are required to compute the metal-to-metal conduction contribution to heat transfer between the contacting bodies. In view of these considerations, the analysis in this chapter commences with a brief summary of Hertz's theory of elastic contact. A description of the assumptions and resulting simplifications made in the derivation of the thermal model for the contact in vacuum follows the description of Hertz's theory. The thermal model of the contact in vacuum is then modified to obtain a model applicable to the contact in a gas. Contact between rough surfaces is considered in the last section of the chapter.

1.3.2 Elastic Contact Between Solids

Hertz's theory of elastic contact is described in detail in several references (e.g., [30], [70]) so only results relevant to the present analysis will be given here, where the theory will be used principally to evaluate contact dimensions and surface deformations adjacent to a contact. The theory permits those parameters to be calculated for an elastic contact formed between two large solid bodies, such as those shown in Figure I.1.2 and described briefly in Section I.1.3.1, provided the assumptions mentioned in Section I.1.3.) are satisfied. The theory predicts that the contact patch formed between the contacting bodies upon application of a load P normal to the plane of contact and co-linear with an axis passing through the center of the contact (i.e., the z -axis shown in Figure I.1.2) will be elliptical in shape and provides expressions for predicting the semi-major and semi-minor diameters of the patch, designated a and b respectively, as functions of P , R_1 , R_2 , R_1' , R_2' , γ and material properties. The semi-major and semi-minor axes of the contact patch are designated x and y respectively, as shown in Figure I.1.2.

The contact dimensions a and b are determined by first calculating the geometric parameters C_1 and C_2 from the equations,

$$C_1 = C_3 - C_4 \quad (I.3.1)$$

and,

$$C_2 = C_3 + C_4 \quad (I.3.2)$$

where,

$$C_3 = 1/4 (1/R_1 + 1/R_2 + 1/R_1' + 1/R_2') \quad (1.3.3)$$

and

$$C_4 = 1/4 \sqrt{\left[\left(\frac{1}{R_1} - \frac{1}{R_1'}\right) + \left(\frac{1}{R_2} - \frac{1}{R_2'}\right)\right]^2 - 4 \left(\frac{1}{R_1} - \frac{1}{R_1'}\right) \left(\frac{1}{R_2} - \frac{1}{R_2'}\right) \sin^2 \gamma} \quad (1.3.4)$$

Bodies 1 and 2 must be numbered so that $C_1 \leq C_2$. Defining $k' = b/a$,

the value of k' for the contact is determined by solving the transcendental equation,

$$\frac{C_2}{C_1} = \left[\frac{1}{k'^2} E(k) - K(k) \right] / [K(k) - E(k)] \quad (1.3.5)$$

in which,

$$k = \sqrt{1 - k'^2} = \sqrt{1 - b^2/a^2} \quad (1.3.6)$$

$K(k)$ = the complete elliptic integral of the first kind with modulus

$$K = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

and,

$E(k)$ = the complete elliptic integral of the second kind with modulus k

$$E = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta$$

The integrals representing $K(k)$ and $E(k)$ cannot be integrated explicitly but numerically determined values of $K(k)$ and $E(k)$ can be found tabulated as functions of k in a number of references (eg., [71]).

The value of k' (and hence k) satisfying Equation (1.3.5) must be determined by trial and error since the equation cannot be rewritten to

however, the contact dimensions a and b follow immediately from the relationships,

$$b = G_b L_h \quad (I.3.7)$$

and,

$$a = b/k' \quad (I.3.8a)$$

$$= G_a L_h \quad (I.3.8b)$$

where,

$$G_b = \left(\frac{3k' E(k)}{2\pi} \right)^{1/3} \quad (I.3.9)$$

$$L_h = \left(\frac{P_0}{c_1 + c_2} \right)^{1/3} \quad (I.3.10)$$

and,

$$G_a = G_b/k' \quad (I.3.11)$$

Also,

$$\Omega = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \quad (I.3.12)$$

where E is Young's modulus and ν is Poisson's ratio for body 1 or body 2 (subscripts 1 and 2).

Since $\gamma=90^\circ$, R_1^∞ and R_2^∞ for the contact configuration shown in Figure I.1.5, the expressions for the geometric parameters C_1 and C_2 reduce to the equations,

$$C_1 = \frac{1}{2R_2} \quad (I.3.13)$$

and

$$C_2 = \frac{1}{2R_1} \quad (I.3.14)$$

in this analysis.

Since the foregoing procedure for determining the contact dimensions a and b is somewhat awkward because k' must be determined by trial and error from Equation (I.3.5), the results of Hertz's theory are often presented in graphical or tabular form (see reference [30] for instance). The simplest relationships known to the author for calculating a and b are given by Dyson [73], however, who expresses these quantities as,

$$a = G'_a L_h \quad (I.3.15)$$

and,

$$b = G'_b L_h \quad (I.3.16)$$

where,

$$G'_a = C_a \left(\frac{C_1}{C_2}\right)^n \quad (I.3.17)$$

and,

$$G'_b = C_b \left(\frac{C_1}{C_2}\right)^m \quad (I.3.18)$$

The coefficients C_a and C_b and the indices n and m seen in Equations (I.3.17) and (I.3.18) are given in Table I.3.1 along with the maximum errors incurred by using Dyson's approximations. The errors quoted are likely no greater than the errors that would result from using graphs to determine a and b , in particular for values of C_1/C_2 lying in the range $10^{-5} \leq C_1/C_2 \leq 0.03$ when the contact dimensions a and b calculated with

Dyson's formulae would be in error by no more than 2.3% and 1.6% respectively. It can be shown, in particular, that G_a and G'_a (i.e., the exact and approximate coefficients of L_h seen in Equations (I.3.8b) and (I.3.15) respectively) differ by only 0.02% and that G_b and G'_b differ by only 0.04% when $C_1/C_2 = 0.01253$, which is the value that this ratio assumes for the model contact described in Section I.1.3.3. Since Dyson's equations for a and b are very accurate for contact patches with large aspect ratios (which correspond to small values of C_1/C_2), they were used extensively in this study.

Table I.3.1: Constants for Dyson's Formulae

Range of C_1/C_2	C_a	n	Max. Error in a , %	C_b	m	Max. Error in b , %
$10^{-5} \leq \frac{C_1}{C_2} \leq 3 \times 10^{-2}$	0.976	-0.372	2.3	0.702	+0.186	1.6
$3 \times 10^{-2} \leq \frac{C_1}{C_2} \leq 1$	0.908	-0.380	2.2	0.894	+0.265	4.0

Development of an accurate gas conduction model requires that local values of the separation distance between the contiguous surfaces near the contact be calculated in a manner that includes the elastic deformation of the surfaces in addition to the separation dictated by geometric constraints. The latter separation (which is the separation present when the bodies are in point contact with no contact load) is defined by the expression [70],

$$s(x,y) = C_1 x^2 + C_2 y^2 \quad (\text{I.3.19a})$$

near to the contact (where the contact surfaces are approximated as paraboloids in Hertz's theory). When elastic deformation of the contact surfaces is taken into account, it is shown in Appendix I.A that,

$$\delta(x,y) = \frac{-3P\eta}{4\pi} \left[\frac{2}{a} F(\phi, k) - x^2 \left(\frac{2}{a(a^2-b^2)} (F(\phi, k) - E(\phi, k)) \right) \right. \\ \left. + \frac{2}{a^2} \sqrt{\frac{\lambda}{(\lambda+b^2)(\lambda+a^2)}} - y^2 \left(\frac{2a}{b^2(a^2-b^2)} (E(\phi, k) - \frac{b^2}{a^2} F(\phi, k)) \right) \right] \quad (I.3.19b)$$

where

$$\lambda = (x^2 + y^2 - a^2 - b^2 + \sqrt{(a^2 + b^2 - x^2 - y^2)^2 - 4(a^2 b^2 - y^2 a^2 - x^2 b^2)})/2$$

$F(\phi, k)$ = incomplete elliptic integral of the first kind with argument ϕ and modulus k .

$$= \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

$E(\phi, k)$ = incomplete elliptic integral of the second kind with argument ϕ and modulus k .

$$= \int_0^\phi \sqrt{1 - k^2 \sin^2 \theta} d\theta$$

$$k = \sqrt{1 - \frac{b^2}{a^2}}$$

and,

$$\phi = \sin^{-1} \sqrt{\frac{\lambda}{\lambda + b^2}}$$

Values for $F(\phi, k)$ and $E(\phi, k)$ are tabulated in [71] as functions of ϕ and k . In the case of long, thin elliptical contacts, where $k \rightarrow 1$,

it is shown in Appendix I.A that $F(\phi, k)$ and $E(\phi, k)$ can be expressed approximately as,

$$F(\phi, 1) = \ln \left(\frac{\sqrt{\lambda} + \sqrt{\lambda + b^2}}{b} \right) \text{ for } \begin{matrix} \phi \leq 85^\circ \text{ if } \theta^* \leq 85^\circ \\ \phi \leq \theta^* \text{ if } \theta^* > 85^\circ \end{matrix} \quad (\text{I.3.20})$$

$$F(\phi, 1) = \ln \left(\frac{4a}{b} \right) - \ln \left(\frac{a + \sqrt{a^2 + \lambda}}{\sqrt{\lambda}} \right) \text{ for } \begin{matrix} \phi > 85^\circ \text{ if } \theta^* \leq 85^\circ \\ \phi > \theta^* \text{ if } \theta^* > 85^\circ \end{matrix} \quad (\text{I.3.21})$$

and

$$E(\phi, 1) = \sin \phi = \sqrt{\frac{\lambda}{\lambda + b^2}} \quad (\text{I.3.22})$$

where

$$\theta^* = \sin^{-1} \sqrt{\frac{1 - b/a}{1 - (b/a)^2}}$$

Equations (I.3.20) to (I.3.22), which can be used approximate the elliptic integrals $F(\phi, k)$ and $E(\phi, k)$ in Equation (I.3.19b) if $k = 1$, were used in the present study because $k = 0.998$ for the contact produced by the contact specimens described in Section I.1.3.3 (for which $b/a = 1/16$).

I.3.3 Thermal Analysis of the Contact in Vacuum

It should be apparent from the literature review given in Chapter I:2 that the boundaries and boundary conditions in regions remote from a non-conforming contact of the type shown in Figure I.1.5, do not have to be represented exactly in a thermal model of the contact. The insensitivity of a contact's thermal resistance to boundary shapes

and boundary conditions in regions remote from the contact stems from the highly localized nature of the contact's constriction resistance. Yovanovich [74] has shown that over 90% of the constriction resistance of a circular contact on an infinite half-space (contact 1 Table I.2.1) occurs within a zone, termed the constriction zone, with boundaries not more than ten contact radii from the center of the contact. It should be noted that the expression for the constriction resistance of an isothermal circular contact on an infinite half-space (i.e., $R_{ct}^T = 1/4 k_s a$) was used successfully by several researchers (e.g., [54, 57, 65]) to predict the resistances of sphere-sphere and sphere-flat contacts in vacuo even though this expression contains no geometric information other than the contact radius (a). The highly localized nature of the constriction resistance causes other parameters (e.g., sphere radii) to be of secondary importance in such problems provided the dimensions of the contacts, and hence the associated constriction zones, are small relative to the dimensions of the contacting bodies.

From the foregoing observations, one might expect that the resistance equation (R_{ct}^T) for contact 2 in Table I.2.1 could be used to predict the resistance,* in vacuum, of the contact configuration shown in Figure I.1.5.** This did not prove to be the case in this investigation, however, primarily because the dimensions (particularly the semi-major diameters) of the elliptical contacts produced in the

* The resistance of contact 2 would have to be doubled to be applicable to the geometry of Figure I.1.5, of course, since there are two contacting bodies.

** With smooth surfaces.

experiments were not small relative to the dimensions of the contact specimens. Differences between resistances measured in the experiments with smooth surfaces and values of R_{ct}^T for contact 2 in Table I.2.1 ranged from roughly 6% of the measured value at the lowest contact load to 23% of the measured value at the highest load. Analytical and experimental results showing this are presented in Chapter I.6, where the comparison is discussed in greater detail.

One of the factors responsible for the widespread applicability of the resistance equation for contact 1 in Table I.2.1 can be used to advantage in this analysis even if the analogous equation for a general elliptical contact (i.e., the equation for contact 2) is not directly applicable (at least if high accuracy is desired). The former equation predicts the resistance of a contact on the flat surface of an insulated

Figure I.3.1: Heat Flow Approaching Contact

2

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One of the factors responsible for the widespread applicability of the resistance equation for contact 1 in Table I.2.1 can be used to advantage in this analysis even if the analogous equation for a general elliptical contact (i.e., the equation for contact 2) is not directly applicable (at least if high accuracy is desired). The former equation predicts the resistance of a contact on the flat surface of an insulated

Figure I.3.1: Heat Flow Approaching Contact

half-space yet, as mentioned previously, it has been used successfully to predict the resistances of contacts on curved surfaces. The reason for this success becomes apparent upon examination of Figure I.3.1, which shows the anticipated heat flow net in the vicinity of a contact on a flat surface. The cross-hatched areas in Figure I.3.1 represent low heat flux zones, the removal (or addition) of which should only marginally affect the shape of the heat flow net and hence the resistance of the contact. Hence, the actual shape of the bounding surface adjacent to a contact is likely of little importance provided the surface does not cause the heat flow to be further constricted.

The preceding remarks apply to the contact surfaces shown in Figure I.1.5. The heat flow nets in the upper and lower contact specimens should be very similar, so it is assumed that the thermal problem can be considered symmetrical about the contact interface even though the surface radii differ greatly. With this assumption, only one of the contact specimens needs to be modelled and the contact patch can be considered isothermal in the analysis of the contact with smooth surfaces. It is also assumed that the shape of the contact surface on the modelled specimen is not critical for the reasons discussed previously. Provided the aforementioned assumptions are approximately correct, and results will be given later which show that they must be, the bar shown in Figure I.3.2 qualifies as a suitable substitute for either of the original contact specimens.

Appropriate boundary conditions for the model of the contact with smooth surfaces in vacuum are also shown in Figure I.3.2. Only one-quarter of the substitute specimen actually needs to be modelled, of course, since the thermal problem is symmetrical about planes passing through the

Figure I.3.2: Substitute Contact Specimen

major (x) and minor (y) axes of the contact patch. The line of intersection of these planes of symmetry coincides with the z -axis.

To compare the contact resistance calculated theoretically for the substitute specimen to the experimental resistance defined by the Equation (I.1.1), the predicted contact resistance (R'_c) must be defined as:

$$R'_c = 2 \left[\frac{T_1 - T_0}{Q_c} - \frac{L}{k_s t w} \right] \quad (I.3.23)$$

where the dimensions L , t and w are indicated in Figure I.3.2. The first term contained within the square brackets in Equation (I.3.23) represents the thermal resistance between the isothermal plane at temperature T_1 and the elliptical contact at temperature T_0 , while the second term is the resistance of a bar of cross-sectional area $t \times w$ and length L . To emphasize the significance of the second term, Equation (I.3.23) can be rewritten as:

$$R'_c = 2 \left(\left(T_1 - \frac{Q_c L}{k_s t w} \right) - T_0 \right) \frac{1}{Q_c}$$

or as,

$$R'_c = \frac{\Delta T_c}{Q_c}$$

The dimensions t and w of the substitute contact specimen can be made the same as the corresponding dimensions of the real contact specimens. The length (L) of the substitute specimen should be made as large as possible, however, to ensure that the heat flow at a plane (perpendicular to the specimen's longitudinal axis) located this distance away from the interface in either of the real contact specimens would be undisturbed by the presence of the contact and hence that the plane at that location would be approximately isothermal, as assumed. Since conduction in the substitute specimen was modelled numerically in this study, it was necessary to select a length for the substitute specimen which allowed solutions to be obtained at reasonable cost. Consequently, L was made equal to w (where $w > t$) in the first thermal model (described

in Chapter I.4), to which the substitute specimen shown in Figure I.3.2 applies. Results obtained by previous contact conductance researchers (e.g., [28]) indicated that this specimen length would be sufficient to produce accurate resistance predictions.

Metal-to-metal conduction through the contact patch was the only mode of heat transfer accounted for in the thermal model of the contact in vacuum. While radiative heat transfer could easily have been included in the model, it is not hard to show that at the lowest contact load in vacuum, where radiation would have been most significant relative to conduction, the heat transferred by radiation between the specimens would have been less than 1.5%* of that transferred by conduction through the contact. Radiation was relatively insignificant in the experiments because the mean interface temperature was maintained at a low value (typically 60°C) and one or both of the contact surfaces were highly polished.

I.3.4 Modification of the Vacuum Contact Model to Account for the Presence of a Gas

When a gas is present around the contact, heat transfer between the contact specimens can occur by conduction through the gas and solid phases, by radiation, and conceivably, by free convection. In the experiments performed in this study, the hot contact surface was located above the cold surface so free convection would have been suppressed.

* Calculated conservatively by using a view factor of unity for radiative heat exchange between the contact surfaces.

Radiative heat transfer would have been even less important with a gas present than it was when the contact was in vacuo because the overall heat transfer rate between the contact specimens was significantly increased by gas conduction (i.e., the contact resistance was much lower than it was in vacuum). Thus the radiative resistance would have been even larger, relative to the overall contact resistance, than it was with the contact in vacuum. Heat transfer by radiation was ignored for that reason.

The vacuum-related contact model can be modified relatively easily to account for gas conduction if it is assumed that heat transfer in the gas from one contact surface to the other occurs through parallel heat flow channels normal to the contact plane. Thus the heat flow through the gas is assumed to be one-dimensional, as shown in Figure I.3.3.

Figure I.3.3: One-Dimensional Heat Flow in the Gas Gap Adjacent to the Contact

In reality, of course, conduction in the gas is not one-dimensional except in the unrealistic case in which the solid and gas thermal conductivities are equal. However, conduction between infinite, isothermal, parallel surfaces is one-dimensional and for those regions immediately adjacent to a non-conforming contact,* where gas conductances are very high due to the close proximity of the contiguous surfaces, the surfaces are approximately parallel and the gap is very small. The assumption of one-dimensional heat flow in the gas should be approximately correct in that region, where it is assumed that most of the gas conduction occurs.

The local gas conductance, h_g , between the contact surfaces at a position (x,y) in the contact-centered coordinate system (Figure I.1.2) can be expressed as:

$$h_g(x,y) = k_g / \delta(x,y) \quad (I.3.24)$$

where $\delta(x,y)$ is the local gap between the contact surfaces, defined by Equation (I.3.19b), and k_g is the thermal conductivity of the gas. Since non-continuum effects cause k_g to decrease when δ is of the same order of magnitude (or smaller than) the mean free path of the gas molecules (L_m), as mentioned in Chapter I.2, k_g must be expressed as a function of the local Knudsen number (Kn), where

$$Kn = L_m / \delta$$

* Excluding non-conforming contacts formed by surfaces with discontinuous slopes (e.g., wedges).

Gas behaviour is traditionally classified as either continuum, slip, transition or free-molecular according to values of the Knudsen number [57, 75]. Knudsen numbers are lowest in the continuum regime, where molecular interactions predominate, and highest in the free-molecular regime where molecule-surface collisions occur much more frequently than molecule-molecule collisions. Expressions relating the effective thermal conductivity of a gas trapped between infinite, isothermal, parallel surfaces to the Knudsen number are available for all of the regimes mentioned, but Prasolov [76] has shown that the expression normally used in the slip and transition regimes can, in fact, be used to predict $k_g(Kn)$ satisfactorily at all values of Kn .

That expression is:

$$k_g = \frac{k_{og}}{1 + [4\gamma Kn(\alpha_1 + \alpha_2 - \alpha_1\alpha_2) / Pr \alpha_1\alpha_2 (\gamma + 1)]} \quad (I.3.25)$$

where

γ = ratio of specific heats for the gas ($=C_p/C_v$)

Pr = Prandtl number of the gas

k_{og} = gas conductivity under continuum conditions

α_1, α_2 = accommodation coefficients of surfaces 1 and 2 [75].

For air at the temperatures of interest in this investigation (i.e., 20°C - 70°C), Equation (I.3.25) simplifies to:

$$k_g = \frac{k_{og}}{[1 + 1.67 Kn \cdot (\frac{2-\alpha_1}{\alpha_1} + \frac{2-\alpha_2}{\alpha_2})]} \quad (I.3.26)$$

Kitscha and Yovanovich [57] and Ogniewicz [65] have successfully incorporated Equation (I.3.25) into gas conduction models applicable to sphere-sphere and sphere-flat contacts. Loyalka [77, 78] has recently derived an alternate equation for predicting the effective thermal conductivity of a gas over the entire range of Knudsen numbers but, while Loyalka's expression is reportedly more accurate than Equation (I.3.25), it is also more complicated. Since Equation (I.3.25) had been successfully applied by other contact conductance researchers, it was chosen for use in this study.

Equation (I.3.25) is strictly only applicable to cases where the heat conducting gas is contained between infinite, isothermal parallel surfaces (i.e., where the heat flow in the gas is one-dimensional). It can be used as a first approximation in the analysis of gas conduction between contacting, non-conforming surfaces, however, since, as mentioned previously, the heat flow through the gas is expected to be approximately one-dimensional very near the contact where gas conduction is most significant.

Values of the local gas conductance (h_g), defined by Equation (I.3.24), can be used in a simple manner in the analysis if it is assumed that heat flow in the real contact model is symmetrical about a fictitious isothermal surface coinciding with the contact interface within the boundaries of the contact patch and located midway between the contact surfaces outside of the contact patch. This simplification, which simply extends the isothermal surface used within the contact patch in the vacuum-related contact model to regions outside of the contact patch, effectively halves the solution domain as shown in Figure I.3.4. In reality, the fictitious

Figure I.3.4 Simplified Boundary Conditions for the Model Contact in Air

surface constructed as described above is only isothermal when the contact problem is truly symmetric about the contact plane, in which case the fictitious surface coincides with the contact plane both inside and outside of the contact patch. In problems where a gas is present in the gap adjacent to the contact, gas conduction is highest very near the contact due to the close proximity of the contiguous surfaces in that region. Since the contact surfaces are approximately parallel near the contact, where most of the heat flow occurs, the contact plane and the fictitious isothermal surface should be roughly coincident in that region, making the assumption of approximate thermal symmetry about the isothermal surface a valid one.

The introduction of the fictitious isothermal surface discussed previously makes it possible to treat the local gas conductances (h_g) as heat transfer coefficients on the single contact surface of the substitute contact specimen shown in Figure I.3.2. These heat transfer coefficients (multiplied by a factor of two, of course) connect the temperatures of surface elements on the contact surface to the fictitious isothermal surface. Since the heat flow in the neighbourhood of the contact will be severely constricted even when a gas is present (assuming the solid/gas conductivity ratio is large), the shape of the contact surface on the substitute specimen should, once again, be of secondary importance.

I.3.5 Rough Surface Contact Model

I.3.5.1 Surface Characterization

When the stylus of a stylus profilometer [78a] is moved across a rough surface, a surface profile such as that shown in

Figure I.3.5 is obtained. With reference to that profile, the following

Figure I.3.5 Surface Profile

geometric parameters will be used in the forthcoming analysis of contact between rough surfaces.

- (1) Surface mean line - this line is located at the ordinate $\bar{z}$, where

$$\bar{z} = 1/L_p \int_0^{L_p} z(x) dx \quad (I.3.27)$$

and $z(x)$ is measured from a reference line parallel to the mean line.

- (2) RMS roughness (σ) - this parameter is defined by the expression,

$$\sigma = (1/L_p \int_0^{L_p} (z(x) - \bar{z})^2 dx)^{1/2} \quad (I.3.28)$$

and is effectively the standard deviation of the surface heights (in the limit as $L_p \rightarrow \infty$).

- (3) Average absolute asperity slope ($\bar{m}$) - this parameter is defined by the equation,

$$\bar{m} = 1/L_p \int_0^{L_p} \left| \frac{dz}{dx} \right| dx \quad (I.3.29)$$

I.3.5.2 Mechanical Analysis (First Loading)

For reasons that will be discussed later, it will be assumed in the forthcoming analysis that plastic deformation or interpenetration of the surface asperities occurred when the model contact with rough surfaces was first loaded. When the asperities on two rough but otherwise conforming surfaces whose mean planes are separated by a distance Z deform or interpenetrate plastically, it is commonly assumed in studies of friction and contact heat transfer that the true area of contact can be obtained by superimposing profiles of the surfaces in the manner shown in Figure I.3.6 and summing the interference lengths ℓ_i ; $i=1,2,\dots$ [1,2,3]. If the ratio of the true area of contact to the apparent area is designated ϵ , then it follows that,

Figure I.3.6 Superimposed Roughness Profiles

$$\epsilon = \frac{\sum \ell_i}{L_p} \quad (I.3.30)$$

if L_p is the length over which the profiles are superimposed. Although this procedure ignores the effect of displaced material on ϵ , that effect is probably small for apparent contact pressures (p_a) which are significantly

less than the indentation hardness (e.g., Vickers hardness) of the softer surface [3]. While the contact area fraction ϵ can be obtained as a function of Z by digitizing the surface profiles and calculating the interference lengths (or areas in the case of two-dimensional representations of the surface topographies) with a digital computer [78a], this complicated procedure is unnecessary if the distribution of surface heights on the surfaces can be described mathematically as, for instance, a uniform, exponential or normal (Gaussian) distribution, since $\epsilon(Z)$ can be obtained statistically in that case. For a Gaussian distribution of surface heights, for example, it can be shown that [29, 33, 69b],

$$\epsilon = \frac{1}{2} \operatorname{erfc} \left(\frac{Z}{\sqrt{2} \sigma_e} \right) \quad (I.3.31)$$

where erfc is the complementary error function and,

$$\sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2} \quad (I.3.32)$$

This parameter σ_e is termed the total rms roughness while σ_1 and σ_2 are the rms roughnesses of surfaces 1 and 2 respectively.

It has been determined that a normal distribution accurately describes the distribution of surface heights found on a surface, generated by a random process, such as bead blasting, peening or flame spray deposition [62, 78a]. The rough surface tests of the model contact that were referred to in Section I.1.3.3 were performed with roughness produced by bead blasting, so it is assumed that the surface heights were normally distributed and hence that Equation (I.3.31) can, after being modified slightly in a manner that will be described

shortly, be used to determine ϵ for a given mean plane separation (Z) and total rms roughness (σ_e). When asperity deformation or interpenetration is plastic (as assumed), as it probably is for bead-blasted (and most other) surfaces on first loading [58]), it follows that [1, 3, 29],

$$\epsilon = p_a/H \quad (I.3.33)$$

where H is the Vickers microhardness (preferably the pyramid hardness [78b]) of the softer surface. If a measurement of H is not available, it can be assumed that $H \approx 3S_y$ [78c], where S_y is the yield strength in uniaxial tension or frictionless compression of the contact material with the lower S_y .

Equation (I.3.31) was derived by assuming that the material underneath each deformed asperity (i.e., the substrate) does not change position with respect to the surface mean line and hence ignores elastic deformation of the substrate (relative to the mean line) under the contact points. When the microcontacts are far enough apart for the elastic deformation underneath each contact point to be considered independently of the deformation underneath neighbouring contact points, Mikic [3] has shown that Equation (I.3.31) should be rewritten as,

$$\epsilon = \frac{\phi_1}{2} \operatorname{erfc} \left(\frac{Z}{\sqrt{2} \sigma_e} \right) \quad (I.3.34)$$

where

$$\phi_1 = \frac{1}{1+2.5 B^{1.2}} \quad (I.3.35)$$

and,

$$\beta = H\Omega/\bar{m}_e \quad (I.3.36)$$

with,

$$\bar{m}_e = \sqrt{\bar{m}_1^2 + \bar{m}_2^2} \quad (I.3.37)$$

The surface slopes of the asperities must be normally distributed in order to apply Equation (I.3.37) [1]*. If the surface slopes are not normally distributed, Mikic et al [2] recommend that $\bar{m}_e$ be taken to be the larger of $\bar{m}_1$ and $\bar{m}_2$.

Equation (I.3.34) will be used in this analysis to calculate ϵ instead of Equation (I.3.31) since it was used by Burde in his mechanical analysis of a rough sphere-smooth flat contact with good results [58, 59]. Equations (I.3.33) and (I.3.34) can be combined to express the apparent contact pressure as;

$$p_a = \frac{H \phi_1}{2} \operatorname{erfc} \left(\frac{Z}{\sqrt{2} \sigma_e} \right) \quad (I.3.38)$$

If it is assumed that the mean lines of non-conforming curved surfaces deform elastically when the surfaces contact each other (even if the asperities deform plastically), it is possible in theory to calculate the contact pressure distribution that would cause the normal displacements of each surface mean line to satisfy both Equation (I.3.38) and the equations of linear elasticity in each contacting body simultaneously. It would be required that the integrated contact pressure distribution match the contact load, of course. Such a solution was obtained by Roca [4, 69f] for contacting rough spheres by using Terewaza's solution [78c] for the surface deformations produced

* If the surface heights on a surface are normally distributed, the slopes are as well [69d].

by a specified pressure distribution acting within a circular area on the surface of an elastic half-space. The same solution was later used by Burde [58] in his analysis of heat transfer through a rough sphere-smooth flat contact. To obtain such a coupled solution for the geometry of the model contact would be more difficult, so it will be assumed for simplicity that the surface deformations obtained at a given contact load with roughness present would be the same as the deformations predicted by Hertz's theory for the same contact load (i.e., the "smooth surface" deformations). This assumption, which was used by Yip [29, 60] to estimate the size of contacts formed between rough spheres, cylinders (in line contact) and annuli, would probably be good for fairly smooth surfaces but poor for very rough ones.

With the assumption of Hertzian deformation of the contact surfaces, the separation between the surface mean lines (Z) at a given load P can be obtained from the relations,

$$Z(x,y) = \delta^*, \text{ when } \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \quad (I.3.39)$$

and

$$Z(x,y) = \delta^* + \delta(x,y), \text{ when } \frac{x^2}{a^2} + \frac{y^2}{b^2} > 1 \quad (I.3.40)$$

The parameters a and b are the dimensions of the contact patch obtained using Hertz's theory of elastic contact with the contact load P and $\delta(x,y)$ is defined by Equation (I.3.19b). The parameter δ^* is a gap that is assumed to exist between the contact surfaces within the Hertzian contact area, as shown in Figure I.3.7.

Figure I.3.7 Gap (δ^*) Between Deformed Contact Surfaces (Radii Illustrated are Radii of Undeformed Surfaces).

The value of δ^* can be found by locating the root of the equation,

$$g(\delta^*) = P - \iint_{A_c} p_a(x, y, \delta^*) dx dy = 0 \quad (I.3.41)$$

when p_a is obtained from Equation (I.3.38) with Equations (I.3.39) and (I.3.40) substituted for Z . For convenience, the contour area A_c over which p_a is integrated is assumed to be the region within which $Z/\sigma_e \leq 3$ since $\epsilon \approx 1.1 \times 10^{-5}$ for $Z/\sigma_e > 3$ according to Equation (I.3.31) (if the surface heights are normally distributed, as assumed). The integral in Equation (I.3.41) and the value of δ^* satisfying that equation were both evaluated numerically in this study.

I.3.5.3 Contact Conductance Evaluation

Once the initial gap δ^* has been determined in the manner described in the preceding section, Equations (I.3.39) and (I.3.40) can be substituted into Equation (I.3.34) to obtain the contact area fraction ϵ at any point (x,y) within the contour area. As before, the contour area is assumed to be the region within which $Z/\sigma_e \leq 3$. Defining $n=Z/\sigma_e$, it is assumed that the local contact conductance in vacuum ($h_{cv}(x,y)$) can now be obtained from the expression [3],

$$h_{cv}(x,y) = \frac{k_s \bar{m}_e \phi_2 \exp(-n^2/2)}{2\sqrt{2\pi} \sigma_e (1-\sqrt{\epsilon})^{1.5}} \quad (I.3.42)$$

where,

$$\phi_2 = \frac{1}{1+1.058} \quad (I.3.43)$$

and,

$$k_s = 2k_{s1} k_{s2} / (k_{s1} + k_{s2})$$

when k_{s1} and k_{s2} are the thermal conductivities of contacting solids 1 and 2 respectively.

Equation (I.3.42) was derived theoretically by Mikic* [3] for macroscopically conforming (i.e., nominally flat) surfaces in contact in a vacuum. It was stipulated that the surfaces be macroscopically conforming because the macroscopic constriction effects and non-uniform clustering of contact points that would result from surface waviness were not accounted for in the equation's derivation. Such effects are modelled directly if Equation (I.3.42) is used to define the average contact conductance acting over each surface element within the contour

* The manner in which Equation (I.3.42) was derived was discussed briefly in Section I.2.4. This expression applies to surfaces with random isotropic roughness and normally distributed surface heights.

area in a numerical model, however, so the fact that the equation was derived for macroscopically conforming contacts does not disallow its use in this application. In effect, each surface element is assumed to be a small "macroscopically conforming" (because of its smallness) contact with its own statistically "most probable" values of n , ϵ and h_{cv} .

Equation (I.3.42) represents the contact conductance for conduction through the metal-to-metal contact points only and hence must be modified when a gas is present. If it is assumed that the gas phase and solid phase resistances at a given point (x,y) within the contour area act in parallel and that the effective gap thickness for gas conduction at that point is $Z(x,y)$, the total contact conductance ($h_c(x,y)$) can be written as,

$$h_c(x,y) = h_{cv}(x,y) + (1-\epsilon) k_g(x,y)/Z(x,y) \quad (I.3.44)$$

when $k_g(x,y)$ is obtained from Equation (I.3.25) (using $Z(x,y)$ to evaluate Kn). The factor $(1-\epsilon)$ seen in front of the second term in Equation (I.3.44) is the ratio of the heat flow area for gas conduction to the apparent area (since ϵ is the fraction of the apparent contact area in actual contact).

Boundaries and boundary conditions for the analytical contact model have been described for the cases in which the contact occurs with smooth or rough surfaces in a gas or in vacuum. Although the heat flow within the substitute contact specimen (i.e., in the solid) remains to be modelled, the complexity of the boundary conditions (particularly those associated with the model of the contact in a gas or of the contact with rough surfaces) necessitates using an approximate analytical technique for that purpose. The numerical model that was used to model conduction in the substitute specimen itself is described in the next chapter.

CHAPTER I.4

ELLIPSOIDAL FINITE DIFFERENCE MODEL

1.4.1 Introduction

It has been shown by Strong et al. [46] that curvilinear coordinate systems can be used to advantage in thermal problems with spatially non-uniform temperature gradients if the coordinate surfaces coincide approximately with the flow net of the thermal problem. The ellipsoidal coordinate system satisfies this requirement when used for non-conforming contact heat transfer problems involving elliptical contacts. Yovanovich [15] demonstrated that the problem of evaluating the constriction resistance of an isothermal elliptical contact patch on an otherwise insulated infinite half-space (i.e., contact 2 in Table I.2.1) is one-dimensional when formulated in the ellipsoidal coordinate system. If posed in any other coordinate system this problem is exceedingly complex because of the mixed boundary conditions on the surface of the half-space.

It is logical to assume that the advantages enjoyed by the ellipsoidal coordinate system for the problem just discussed also render it the most appropriate coordinate system to use for problems with more complex boundary conditions, such as that of the model contact problem with gas conduction. This problem is three-dimensional even in the ellipsoidal coordinate system and is virtually impossible to solve using classical mathematical techniques. However, a numerical solution to this

problem, formulated in the ellipsoidal coordinate system, is described in this chapter.

This chapter commences with a description of the ellipsoidal coordinate system, followed by a brief discussion of finite-difference modelling in orthogonal curvilinear coordinate systems. It will be shown that the finite-difference method cannot be applied directly to the discretization of the heat conduction equation in the ellipsoidal coordinate system because the equation has several singular points.

An outline of the numerical technique selected to analyse the model contact problem is given in section I.4.4, followed by a discussion of how boundary conditions are treated in the numerical model. Computational aspects of the solution are described in Appendix I.C., where the computer program developed to implement the numerical model is documented.

I.4.2 The Ellipsoidal Coordinate System

The three orthogonal coordinate surfaces of the ellipsoidal coordinate system are confocal conicoids obtained when the parameter θ assumes different values in the equation

$$\frac{x^2}{a^2 + \theta} + \frac{y^2}{b^2 + \theta} + \frac{z^2}{c^2 + \theta} = 1 \quad (I.4.1)$$

where $a > b > c > 0$ and $-a^2 < \theta < \infty$. For the range $-c^2 < \theta < \infty$,

every term in equation (I.4.1) is positive so the equation represents an ellipsoid. Values of θ in this range are designated λ . As $\lambda \rightarrow \infty$ the ellipsoid approaches a sphere of radius $\sqrt{\lambda}$ and as $\lambda \rightarrow -c^2$, the ellipsoid is flattened to an elliptical disk lying in the x - y plane. This elliptical disk has semi-major and semi-minor diameters of magnitudes a and b respectively. As θ passes from $(-c^2)^+$ to $(-c^2)^-$ (i.e., from θ slightly greater than $-c^2$ to θ slightly less than $-c^2$) the points represented by equation (I.4.1) move from the region of the x - y plane lying inside the elliptical disk to the region outside the disk. When θ lies in the range $-b^2 < \theta < -c^2$, equation (I.4.1) represents a one-sheeted hyperboloid, as shown in Figure I.4.1. Values of θ in this range are designated μ . The one-sheeted hyperboloid is flattened onto the x - z plane when $\mu = -b^2$. In this case, the flattened hyperboloid covers that region of the x - z plane which includes the z axis and lies between the hyperbolas cutting the x -axis at:

$$x = \pm \sqrt{a^2 - b^2} \quad (I.4.2)$$

Figure I.4.1: One-Sheeted Hyperboloid

As θ passes from $(-b^2)^+$ to $(-b^2)^-$, the points represented by equation (I.4.1) move from the region of the x - z plane lying between the hyperbolas defined by equation (I.4.2) to the region lying outside the hyperbolas. This is the limiting case in which the third coordinate surface, a hyperboloid of two sheets (Figure I.4.2) defined by equation (I.4.1) when $-a^2 < \theta < -b^2$, is flattened onto the x - z plane. Values of θ in this range are designated v .

Another limiting case occurs when $v = -a^2$, at which point the two sheets of the hyperboloid coalesce in the z - y plane.

It is evident that each coordinate surface sweeps through all points in space as θ assumes values between the appropriate limits for

Figure I.4.2: Two-Sheeted Hyperboloid

λ , μ and ν (i.e., $-c^2 \leq \lambda < \infty$, $-b^2 \leq \mu \leq -c^2$ and $-a^2 \leq \nu \leq -b^2$).

It should be noted that changes in the coordinate values in an orthogonal curvilinear coordinate system generally do not correspond directly to changes in physical lengths. For instance, a change in the λ coordinate, $\Delta\lambda$, in the ellipsoidal coordinate system does not, by itself, define a length. In a general orthogonal curvilinear coordinate system with coordinates x_1 , x_2 and x_3 , physical arc lengths (ds_i) on the coordinate lines (x_i) are related to differential changes in coordinate values (dx_i) by the "metric coefficients" (g_i) of the coordinate system. The functional relationship is given by [79]:

$$ds_i = \sqrt{g_i} dx_i \quad i = 1, 2, 3 \quad (I.4.3)$$

Also

$$\sqrt{g} = \sqrt{g_1 g_2 g_3} \quad (I.4.4)$$

so the volume of a differential element (dV) can be expressed as

$$dV = \sqrt{g} dx_1 \cdot dx_2 \cdot dx_3 \quad (I.4.5)$$

and differential areas become,

$$dA_i = \sqrt{g_j g_k} dx_j dx_k \quad \begin{array}{l} i, j, k = 1, 2, 3 \\ i \neq j \neq k \end{array} \quad (I.4.6)$$

where the elemental area, dA_i , is normal to the coordinate direction x_i .

In general, it is possible to relate the cartesian coordinates (x, y, z) to the curvilinear coordinates through relationships of the form:

$$x = X(x_1, x_2, x_3) \quad (I.4.7)$$

$$y = Y(x_1, x_2, x_3) \quad (I.4.8)$$

and,

$$z = Z(x_1, x_2, x_3) \quad (I.4.9)$$

from which the metric coefficients can be obtained by evaluating the expression,

$$g_i = \left(\frac{\partial x}{\partial x_i}\right)^2 + \left(\frac{\partial y}{\partial x_i}\right)^2 + \left(\frac{\partial z}{\partial x_i}\right)^2 \quad i = 1, 2, 3 \quad (I.4.10)$$

In the case of the cartesian and ellipsoidal coordinate systems, equations (I.4.7) - (I.4.9) can be written as,

$$x = \sqrt{\frac{(a^2 + \lambda)(a^2 + \mu)(a^2 + \nu)}{(b^2 - a^2)(c^2 - a^2)}} \quad (I.4.11)$$

$$y = \sqrt{\frac{(b^2 + \lambda)(b^2 + \mu)(b^2 + \nu)}{(b^2 - c^2)(b^2 - a^2)}} \quad (I.4.12)$$

and,

$$z = \sqrt{\frac{(c^2 + \lambda)(c^2 + \mu)(c^2 + \nu)}{(c^2 - a^2)(c^2 - b^2)}} \quad (I.4.13)$$

and then substituted into equation (I.4.10) (with $x_1 = \lambda$, $x_2 = \mu$ and $x_3 = \nu$) to obtain,

$$g_1 = \frac{(\lambda - \mu)(\lambda - \nu)}{4(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)} \quad (I.4.14)$$

$$g_2 = \frac{(\mu - \lambda)(\mu - \nu)}{4(a^2 + \mu)(b^2 + \mu)(c^2 + \mu)} \quad (I.4.15)$$

and,

$$g_3 = \frac{(\nu - \mu)(\nu - \lambda)}{4(a^2 + \nu)(b^2 + \nu)(c^2 + \nu)} \quad (I.4.16)$$

as the expressions for the metric coefficients in the ellipsoidal coordinate system.

It will be shown in the next section that the heat conduction equation can be conveniently expressed in a general form applicable to any orthogonal curvilinear coordinate system by using the metric coefficients.

For the purposes of this analysis, it is convenient to let $c=0$ in equation (I.4.1), in which case the limits of λ become, $0 \leq \lambda < \infty$. As $\lambda \rightarrow 0$, the ellipsoid degenerates to an elliptical disk on the x-y plane, as mentioned previously. The problem of determining the constriction resistance of an isothermal elliptical contact on an otherwise

insulated infinite half-space, discussed briefly in section I.4.1, is a one-dimensional (with respect to λ) Dirichlet problem (i.e., $T(\lambda)=T_0$ at $\lambda = 0$; $T(\lambda)=0$ at $\lambda = \infty$) in the ellipsoidal coordinate system.

I.4.3 Finite-Difference Modelling in Orthogonal Curvilinear Coordinate Systems

Steady-state heat conduction in a medium with an internal heat generation rate, q_v , per unit volume and thermal conductivity, k_s , is governed by the heat conduction equation, $\nabla \cdot k_s \nabla T + q_v = 0$, which reduces to Laplace's equation, $\nabla^2 T = 0$, if k_s is constant (i.e., independent of position and temperature) and $q_v = 0$. The heat conduction equation in a general orthogonal curvilinear coordinate system with coordinates x_1, x_2 and x_3 can be written in terms of the metric coefficients (g_i) for those coordinates as [80];

$$\frac{1}{\sqrt{g}} \left[\frac{\partial}{\partial x_1} \left(\frac{k_s \sqrt{g}}{g_1} \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\frac{k_s \sqrt{g}}{g_2} \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\frac{k_s \sqrt{g}}{g_3} \frac{\partial T}{\partial x_3} \right) \right] + q_v = 0 \quad (\text{I.4.17})$$

Each term contained within the square brackets in equation (I.4.17) can also be expressed in expanded form as:

$$\frac{\partial}{\partial x_i} \left(\frac{k_s \sqrt{g}}{g_i} \frac{\partial T}{\partial x_i} \right) = \frac{k_s \sqrt{g}}{g_i} \frac{\partial^2 T}{\partial x_i^2} + \frac{\partial}{\partial x_i} \left(\frac{k_s \sqrt{g}}{g_i} \right) \frac{\partial T}{\partial x_i}; \quad i=1,2,3 \quad (\text{I.4.18})$$

A finite-difference solution to equation (I.4.17) (with prescribed boundary conditions) is obtained by approximating the partial derivatives in that equation as finite-differences at a finite number of discrete points (nodes) within the solution domain. The most systematic method available for approximating the partial derivatives at a point requires

expanding the temperature at the point as a truncated Taylor series involving temperatures at neighbouring points [81]. The resulting approximations are only valid at a given point if the partial derivatives neglected in the truncated Taylor series expansion are bounded at the point, however, and they are definitely not valid at or near singular points. Singular points can be produced by the boundary conditions of a particular problem or they may be present in the governing differential equation itself. With regard to the present analysis, the finite-difference method is not applicable in its normal form at or near points (in the x_i coordinate) where the inverse of the factor

$$\sqrt{g}/g_i \text{ (i.e., } g_i/\sqrt{g}\text{)}$$

in equation (I.4.18) is infinite. This observation can be verified by considering a one-dimensional (with respect to x_i) heat flow problem with uniform thermal conductivity and no heat generation. In this case

$$\frac{d}{dx_i} \left(\frac{\sqrt{g}}{g_i} \frac{dT}{dx_i} \right) = 0 \quad (I.4.26)$$

so integrating once with respect to x_i yields

$$\frac{dT}{dx_i} = \frac{Ag_i}{\sqrt{g}} \quad (I.4.27)$$

where A is a constant determined by the boundary conditions of the one-dimensional problem. It is evident that a singular value of $g_i/\sqrt{g}$ corresponds to an unbounded first derivative of $T(x_i)$ if A is non-zero. These comments only apply if the singular point of equation (I.4.26) belongs to the x_i coordinate of course.

Singular points in the heat conduction (and other) differential equations* usually occur at points which are mathematical but not physical boundaries. The heat conduction equation is singular at the origin (i.e., at $r=0$) of the cylindrical and spherical coordinate systems, for example, where a mathematical, but not a physical, boundary exists. In the case of the cylindrical coordinate system, where $x_1 = r$, $x_2 = \theta$, $x_3 = z$ and $g_1 = 1$, $g_2 = r^2$, $g_3 = 1$, the singularity is easily detected by observing that $g_1/\sqrt{g} = 1/r = \infty$ at $r = 0$. For problems in the cylindrical coordinate system which exhibit circular symmetry, the singularity at $r = 0$ can be removed by enforcing the symmetry condition; i.e., $\partial T/\partial r = 0$ at $r = 0$. The symmetry condition allows a finite-difference analog of the Laplacian operator to be obtained at the origin, as described in many references (e.g., [80, 82]). The accuracy of a finite-difference solution is poor near a singular point of the governing differential equation, however, even if the singular point is excluded from the solution domain. This observation can be verified by considering a simple two-point Dirichlet problem for which the governing differential equation is

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0 \quad \begin{array}{l} \text{(i.e., Laplace's equation for radial} \\ \text{heat flow in the cylindrical coordinate} \\ \text{system)} \end{array} \quad (I.4.28)$$

$$\text{or } \frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = 0$$

* For the heat conduction equation in n -dimensions ($n=2$ or 3), any point which is a singular point of one of the n ordinary differential equations which govern one-dimensional heat flows in the n coordinate directions can be regarded as a possible singular point of the complete equation.

and the prescribed boundary conditions are:

$$T = T_1 \text{ at } r = r_1, r_1 > 0$$

$$T = T_2 \text{ at } r = r_2, r_2 > r_1$$

The exact solution for the radial temperature distribution between r_1 and r_2 is:

$$T(r) = \frac{(T_2 - T_1)}{\ln(r_2/r_1)} \ln\left(\frac{r}{r_1}\right) + T_1 \quad (\text{I.4.29})$$

so the singularity at $r = 0$ has no influence on the exact solution for $r_1 > 0$. In a finite-difference solution, the derivatives in equation (I.4.28) are replaced by the following expressions at the i th node:

$$\left. \frac{dT}{dr} \right|_{r_i} = \frac{T_{i+1} - T_{i-1}}{2\Delta r} (+ R'_n) \quad (\text{I.4.30})$$

$$\left. \frac{d^2T}{dr^2} \right|_{r_i} = \frac{T_{i+1} + T_{i-1} - 2T_i}{\Delta r^2} (+ R''_n) \quad (\text{I.4.31})$$

$$\text{where } R'_n = -\frac{1}{4\Delta r} \left[\int_{r_i}^{r_i+\Delta r} \frac{d^3T}{dr^3} ((r_i+\Delta r) - r)^2 dr \right. \quad (\text{I.4.32})$$

$$\left. + \int_{r_i-\Delta r}^{r_i} \frac{d^3T}{dr^3} ((r_i-\Delta r) - r)^2 dr \right]$$

and

$$R''_n = -\frac{1}{6\Delta r^2} \left[\int_{r_i}^{r_i+\Delta r} \frac{d^4T}{dr^4} ((r_i+\Delta r) - r)^3 dr \right. \quad (\text{I.4.33})$$

$$r_i = \int_{r_i - \Delta r}^{r_i} \frac{d^4 T}{dr^4} ((r_i - \Delta r) - r)^3 dr]$$

Equations (I.4.30) and (I.4.31) are second-order accurate finite-difference approximations for the first and second radial derivatives at node i .

The truncation errors associated with these approximations are given in integral form [83] by equations (I.4.32) and (I.4.33). Since the exact solution for the radial temperature distribution is known (i.e.,

equation (I.4.29)), we can evaluate the first to fourth derivatives found in equations (I.4.30) - (I.4.33). To this end, it is easily shown that,

$$\frac{d^n T}{dr^n} = \frac{(-1)^{n+1} (n-1)! A}{r^n} \quad \text{for } n = 1, 2, \dots \quad (\text{I.4.34})$$

where

$$A = (T_2 - T_1) / \ln(r_2/r_1) \quad (\text{I.4.35})$$

If equation (I.4.34) with $n=3$ and $n=4$ is substituted into equations (I.4.32) and (I.4.33) respectively and the resulting integrals evaluated, we obtain:

$$R'_n = \frac{A}{2\Delta r} \left(\frac{2\Delta r}{r_i} - \ln \left(\frac{r_i/\Delta r + 1}{r_i/\Delta r - 1} \right) \right) \quad (\text{I.4.36})$$

and

$$R''_n = -\frac{A}{\Delta r^2} \left(-\frac{2\Delta r}{r_i} + \ln \left(\frac{r_i/\Delta r + 1}{r_i/\Delta r - 1} \right) \right) \quad (\text{I.4.37})$$

Using equations (I.4.34) - (I.4.37), we can define error measures ϵ' and ϵ'' as,

$$\epsilon' = |R'_n / \frac{dT}{dr}| = \frac{1}{2} \left(\frac{r_i}{\Delta r} \right) \ln \left(\frac{r_i/\Delta r + 1}{r_i/\Delta r - 1} \right) - 1 \quad (I.4.38)$$

and

$$\epsilon'' = |R''_n / \frac{d^2T}{dr^2}| = \left(\frac{r_i}{\Delta r} \right)^2 \left(-\frac{2\Delta r}{r_i} + \ln \left(\frac{r_i/\Delta r + 1}{r_i/\Delta r - 1} \right) \right) \quad (I.4.39)$$

The error measures ϵ' and ϵ'' are the ratios of the truncation errors associated with the finite-difference representations of dT/dr and d^2T/dr^2 (i.e., R'_n and R''_n respectively) to the actual values of those derivatives. Low values of these error measures are desired of course. As is evident from equations (I.4.38) and (I.4.39), both error measures are functions of $r_i/\Delta r$. The functional relationships between the error measures and $r_i/\Delta r$ are displayed in Figure I.4.3. Both error measures approach infinity as $\Delta r \rightarrow r_i$ (i.e., as $r_i/\Delta r \rightarrow 1$), reflecting the fact that the radius of convergence* of the Taylor series expansion of $T(r)$ about a point r_i is simply r_i itself. As seen in Figure I.4.3, the errors associated with the finite-difference representations of dT/dr and d^2T/dr^2 increase as r_i decreases if Δr is fixed. This occurs because the higher order derivatives neglected in the truncated Taylor series expansion of $T(r)$ about r_i increase more rapidly than the retained terms as r_i decreases.

The unfavourable error characteristics seen in Figure I.4.3 for the simple two-point Dirichlet problem in cylindrical coordinates can be

* The radius of convergence of the Taylor series expansion of a function about a point u_i is simply the distance from u_i to the nearest singularity of the function.

Figure I.4.3 Truncation Error Measures as a Function of $r_i/\Delta r$

completely eliminated by substituting $\xi = \ln(r)$ into equation (I.4.28) to obtain the equivalent equation

$$\frac{d^2 T}{d\xi^2} = 0 \quad (I.4.40)$$

which, it is easily shown, can be differenced exactly using a second order accurate difference equation, (i.e., equation (I.4.31) with ξ substituted for r) for the two-point Dirichlet problem because T is a linear function of ξ (see equation (I.4.29)). Differential equations with singular points can often be transformed into equivalent, more easily differenced, equations, as shown in the next section for the heat conduction equation in the ellipsoidal coordinate system.

I.4.4 Differencing the Heat Conduction Equation in the Ellipsoidal Coordinate System

The governing equation for one-dimensional heat flow in the λ direction of the ellipsoidal coordinate system (with $c = 0$) is,

$$\frac{d}{d\lambda} \left[\sqrt{(a^2 + \lambda)(b^2 + \lambda)} \lambda \frac{dT}{d\lambda} \right] = 0 \quad (\text{if } k_s \text{ is constant})$$

or

$$\frac{d}{d\lambda} \left(\sqrt{(a^2 + \lambda)(b^2 + \lambda)} \lambda \frac{dT}{d\lambda} \right) + \sqrt{(a^2 + \lambda)(b^2 + \lambda)} \lambda \frac{d^2 T}{d\lambda^2} = 0 \quad (I.4.41)$$

If the prescribed boundary conditions are, $T = T_0$ at $\lambda = 0$ and $T = T_1 (\neq T_0)$ at $\lambda = \lambda_1 (\neq 0)$, the constant A in the partially integrated form of equation (I.4.41) (i.e., $\frac{dT}{d\lambda} = \frac{A}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)} \lambda}$) will be non-zero.

The derivatives $\frac{d^n T}{d\lambda^n}$; $n=1, \infty$ will all be infinite* in that case at $\lambda = 0$, which is therefore a singular point, and truncation error characteristics similar to those shown in Figure I.4.3 for finite-difference analogs of the derivatives $\frac{dT}{dr}$ and $\frac{d^2 T}{dr^2}$ will apply to finite-difference representations of the derivatives in equation (I.4.41). A finite-difference solution will therefore be completely invalid at, or anywhere near, the singularity at $\lambda = 0$ (i.e., at the contact).

By using the metric coefficients for the ellipsoidal coordinate system, equations (I.4.14) - (I.4.16), it can be shown (by forming the ratios $g_i/\sqrt{g}$, $i = 1, 2, 3$) that the heat conduction equation for the ellipsoidal coordinate system has other singular points at $\mu = -b^2$, 0 and $\nu = -a^2, -b^2$. The presence of these singularities precludes differencing the derivatives in the ellipsoidal heat conduction equation

* Note that an infinite value of $dT/d\lambda$ does not necessarily produce an infinite heat flux within the elliptical contact at $\lambda=0$ because the heat flux at that point depends on g_λ . Noting that normals to the upper surface of the ellipsoid at $\lambda=0$ are parallel to and in the same direction as the positive z axis at all points on the surface, it follows that $\partial\lambda/\partial z = 1/\sqrt{g_\lambda}$ at $\lambda=0$ and hence that,

$$\left. \frac{\partial T}{\partial z} \right|_{\lambda=0} = \frac{1}{\sqrt{g_\lambda}} \frac{dT}{d\lambda} = \frac{A}{\sqrt{(a^2+\lambda)(b^2+\lambda)\lambda}} \sqrt{\frac{4(a^2+\lambda)(b^2+\lambda)\lambda}{(\lambda-\mu)(\lambda-\nu)}} = \frac{2A}{\sqrt{\mu\nu}}$$

This expression shows that $\partial T/\partial z$ (and hence the surface heat flux) is finite at $\lambda=0$ except when $\mu=0$ (i.e., at the edge of the elliptical patch), where the heat flux is infinite.

directly, but it is possible to transform that equation into an equivalent differential equation which has no singular points. To this end, consider the transformations:

$$\underline{x} = \frac{a}{2} \int_0^{\lambda} \frac{d\lambda'}{\sqrt{(a^2 + \lambda') (b^2 + \lambda') \lambda'}} = F(\phi, k) \quad (I.4.42)$$

$$\underline{y} = \frac{a}{2} \int_{-b^2}^{\mu} \frac{d\mu'}{\sqrt{(a^2 + \mu') (b^2 + \mu') (-\mu')}} = F(\beta, k') \quad (I.4.43)$$

and,

$$\underline{z} = \frac{a}{2} \int_{-a^2}^v \frac{dv'}{\sqrt{(a^2 + v') (b^2 + v') v'}} = F(\gamma, k) \quad (I.4.44)$$

where $k = \sqrt{1 - b^2/a^2}$

$$k' = \sqrt{1 - k^2}$$

$$\phi = \sin^{-1} \sqrt{\frac{\lambda}{\lambda + b^2}}$$

$$\beta = \sin^{-1} \left(\frac{a}{b} \sqrt{\frac{\mu + b^2}{\mu + a^2}} \right)$$

$$\gamma = \sin^{-1} \sqrt{\frac{v + a^2}{a^2 - b^2}}$$

and F is the incomplete elliptic integral of the first kind. Equations (I.4.42) - (I.4.44) were obtained by integrating the one-dimensional form of Laplace's equation in each of the coordinate directions (λ , μ and v). In the λ direction, for instance, equation (I.4.41) can be integrated once to give:

$$\frac{dT}{d\lambda} = \frac{A}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)\lambda}}$$

and then a second time to obtain:

$$\begin{aligned} T(\lambda) &= A \int_0^\lambda \frac{d\lambda'}{\sqrt{(a^2 + \lambda')(b^2 + \lambda')\lambda'}} + B \\ &= \frac{2A}{a} F(\phi, k) + B \\ &= C F(\phi, k) + B \end{aligned} \quad (I.4.45)$$

where A , B and $C (=2A/a)$ are constants. For a two-point Dirichlet problem, with $T = T_0$ at $\lambda = \lambda_0$ and $T = T_1$ at $\lambda = \lambda_1$, the two constants in equation (I.4.45) can be determined from the two specified boundary conditions, although we are not interested in evaluating those constants at this time. The other two transformations (i.e., equations (I.4.43) - (I.4.44)) were obtained by performing similar integrations of Laplace's equation in the μ and ν directions:

If equations (I.4.42) - (I.4.44) are substituted into the homogeneous form of equation (I.4.17) and the resulting equation simplified, we obtain:

$$\frac{\partial}{\partial \underline{x}} (k_s(\mu - \nu) \frac{\partial T}{\partial \underline{x}}) + \frac{\partial}{\partial \underline{y}} (k_s(\lambda - \nu) \frac{\partial T}{\partial \underline{y}}) + \frac{\partial}{\partial \underline{z}} (k_s(\lambda - \mu) \frac{\partial T}{\partial \underline{z}}) = 0 \quad (I.4.46)$$

The factors $(\mu - \nu)$, $(\lambda - \nu)$ and $(\lambda - \mu)$ in equation (I.4.46) should, strictly speaking, be expressed in terms of $\underline{x}$, $\underline{y}$ and $\underline{z}$; but the inverse transformations corresponding to equations (I.4.42) - (I.4.44) are, in

general, impossible to obtain explicitly. When $k \approx 1$, however, explicit expressions for λ , μ and ν in terms of x , y and z can be obtained because the elliptic integral (F) can be approximated by logarithmic and trigonometric functions, as explained in Appendices I.A and I.B. For this special case, it can be shown that the following relationships apply:

$$\lambda = \frac{(be^x)^2}{4} + \frac{b^4}{4(be^x)^2} - \frac{b^2}{2} \text{ for } x \leq 2.09 \ (\phi \leq 76^\circ) \quad (I.4.47)$$

$$\lambda = 16a^2 / \left(\frac{64a^2}{(be^x)^2} + \frac{(be^x)^2}{4a^2} - 8 \right) \text{ for } x > 2.09 \ (\phi > 76^\circ) \quad (I.4.48)$$

$$\mu = \frac{b \cos y)^2}{\left(\frac{b}{a} \sin y \right)^2 - 1} \text{ for } y \leq 0.786 \ (\beta \leq 45^\circ) \quad (I.4.49)$$

$$\mu = - \left(b \sin \left(\frac{\pi}{2} \left(1 + \frac{b^2}{4a^2} \right) - y \right) \right)^2 \text{ for } y > 0.786 \ (\beta > 45^\circ) \quad (I.4.50)$$

$$\nu = -b^2 + \frac{4e^{2z} (b^2 - a^2)}{(1 + e^{2z})^2} \text{ for } z \leq 2.09 \ (\gamma \leq 76^\circ) \quad (I.4.51)$$

and

$$\nu = \frac{-a^2 \left(\frac{16a^2}{e^{2z}} + b^2 \right)^2}{\left(\frac{16a^2}{e^{2z}} - b^2 \right)^2 + \frac{64a^4}{e^{2z}}} \text{ for } z > 2.09 \ (\gamma > 76^\circ) \quad (I.4.52)$$

The ranges of applicability given for equations (I.4.47) - (I.4.52) apply to a contact with $a/b=16$ - ranges of applicability for other aspect ratios can be determined from equations given in Appendix I.B.

Since equations (I.4.47) - (I.4.52) are quite cumbersome, the correct, expanded form of equation (I.4.46), which is obtained by expressing the variables λ , μ and ν in terms of $\underline{x}$, $\underline{y}$ and $\underline{z}$ respectively, will not be written out explicitly.

For heat conduction in an isotropic homogeneous medium without heat generation, equation (I.4.46) can be rewritten as:

$$\frac{\partial}{\partial \underline{x}} \left(k_{\underline{x}} \frac{\partial T}{\partial \underline{x}} \right) + \frac{\partial}{\partial \underline{y}} \left(k_{\underline{y}} \frac{\partial T}{\partial \underline{y}} \right) + \frac{\partial}{\partial \underline{z}} \left(k_{\underline{z}} \frac{\partial T}{\partial \underline{z}} \right) = 0 \quad (\text{I.4.53})$$

where

$$k_{\underline{x}} = k_s (\mu - \nu) = \text{the "effective" thermal conductivity in the } \underline{x} \text{ direction.}$$

$$k_{\underline{y}} = k_s (\lambda - \nu) = \text{the "effective" thermal conductivity in the } \underline{y} \text{ direction.}$$

and

$$k_{\underline{z}} = k_s (\lambda - \mu) = \text{the "effective" thermal conductivity in the } \underline{z} \text{ direction.}$$

Equation (I.4.53) is identical to the heat conduction equation in the Cartesian coordinate system. By using equations (I.4.42) - (I.4.44) to transform the heat conduction equation in ellipsoidal coordinates to an equivalent equation in Cartesian coordinates, a problem such as that shown in Figure I.4.4 can be transformed to the equivalent problem shown in Figure I.4.5. The problem shown in Figure I.4.4 requires evaluating the heat flow through the surface of a half-space when the boundary conditions on the surface are:

Figure I.4.4 Heat Flow Problem in Ellipsoidal Coordinates

Figure I.4.5 Heat Flow Problem in Transformed Coordinates

$$T(x,y,0) = T_0 \text{ over } r = \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$$

and,

$$k_s \left. \frac{\partial T}{\partial z} \right|_{z=0} = h_g (T(x,y,0) - T_0) \text{ outside } r$$

Also, at $r = \sqrt{x^2 + y^2 + z^2} = \infty$, $T(x,y,z) = T_1$

The transformed boundary conditions for the equivalent problem are shown in Figure I.4.5. The limits of the variables $\underline{x}$, $\underline{y}$ and $\underline{z}$ in the transformed solution domain are: $0 \leq \underline{x} \leq K(k)$, $0 \leq \underline{y} \leq K(k')$ and $0 \leq \underline{z} \leq K(k)$, where $K(k)$ and $K(k')$ are the complete and associated complete elliptic integrals of the first kind respectively. As $k \rightarrow 1$ (i.e., for long, thin, elliptical contacts), $K(k) \rightarrow \ln \left(\frac{4a}{b} \right)$ and $K(k') \rightarrow \frac{\pi}{2} \left(1 + \frac{k'^2}{4} \right)$. The problem shown in Figure I.4.5 and governed by equation (I.4.53) is one-dimensional with respect to $\underline{x}$ when $h_g = 0$ (i.e., for a contact in vacuo). The temperature gradient, $dT/d\underline{x}$, is constant from $\underline{x} = 0$ to $\underline{x} = K(k)$ in that case, of course.

The transformed heat conduction equation, equation (I.4.46) (or equation (I.4.53)), does not have any singular points. In consequence, finite-differences can be used to represent the derivatives in equation (I.4.46) at all points in the transformed solution domain. Since the effective thermal conductivity in a particular coordinate direction is a function of the values of the remaining coordinates in the $\underline{x}$, $\underline{y}$, $\underline{z}$, coordinate system, a finite-difference representation of equation (I.4.53) at a mesh point i, j, k (located at $\underline{x}_i, \underline{y}_j, \underline{z}_k$) is most appropriately derived by applying the "box integration" method [84, 85, 86]. In this method, Gauss' divergence theorem is used to transform the integral of equation (I.4.53) over the volume of a "box"

surrounding a mesh point to the integral of the normal flux over the surfaces (boundaries) of the box. Referring to Figure I.4.6, the edges of the box surrounding the mesh point, i, j, k are aligned with the x , y and z axes and each side of the box bisects the distance between node i, j, k and one of its nearest neighbours. Side 1, for instance, is parallel to the $y - z$ plane and intersects the line joining nodes i, j, k and $i+1, j, k$ at its midpoint.

Figure I.4.6: Box (Cell) Surrounding Mesh Point i, j, k

Applying Gauss' divergence theorem to the box surrounding node i, j, k , we can express the volume integral of equation (I.4.53) within the box as:

$$\begin{aligned}
 & \int_{z_1}^{z_2} \int_{y_1}^{y_2} \int_{x_1}^{x_2} \left[\frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) \right] dx dy dz \\
 &= \int_{y_1}^{y_2} \int_{x_1}^{x_2} k_z \frac{\partial T}{\partial z} \Big|_{z_2} dx dy + \int_{y_1}^{y_2} \int_{x_1}^{x_2} k_z \left(- \frac{\partial T}{\partial z} \right) \Big|_{z_1} dx dy \\
 &+ \int_{z_1}^{z_2} \int_{x_1}^{x_2} k_y \frac{\partial T}{\partial y} \Big|_{y_2} dx dz + \int_{z_1}^{z_2} \int_{x_1}^{x_2} k_y \left(- \frac{\partial T}{\partial y} \right) \Big|_{y_1} dx dz \\
 &+ \int_{z_1}^{z_2} \int_{y_1}^{y_2} k_x \frac{\partial T}{\partial x} \Big|_{x_2} dy dz + \int_{z_1}^{z_2} \int_{y_1}^{y_2} k_x \left(- \frac{\partial T}{\partial x} \right) \Big|_{x_1} dy dz \\
 &= 0
 \end{aligned} \tag{I.4.54}$$

$$\text{where } x_1 = \frac{x(i,j,k) + x(i-1,j,k)}{2}, \quad x_2 = \frac{x(i,j,k) + x(i+1,j,k)}{2}$$

and y_1, y_2, z_1, z_2 are similarly defined by averaging the values of y and z at node (i,j,k) and nodes $(i,j-1,k)$, $(i,j+1,k)$, $(i,j,k-1)$ and $(i,j,k+1)$ respectively.

The partial derivative in each surface integral on the right-hand side of equation (I.4.54) can be expressed in central-difference form, using an equation similar to equation (I.4.30), and then moved outside

of the integral sign. If the last surface integral in equation (I.4.54) is treated in this manner, for example, we obtain:

$$\begin{aligned} \int_{z_1}^{z_2} \int_{y_1}^{y_2} k_x \left(- \frac{\partial T}{\partial x} \right) \bigg|_{x_1} dy dz &= \frac{T(i-1, j, k) - T(i, j, k)}{x(i, j, k) - x(i-1, j, k)} \int_{z_1}^{z_2} \int_{y_1}^{y_2} k_x dy dz \\ &= k_s \left(\frac{T(i-1, j, k) - T(i, j, k)}{x(i, j, k) - x(i-1, j, k)} \right) \int_{z_1}^{z_2} \int_{y_1}^{y_2} (\mu - \nu) dy dz \\ &\quad \text{if } k_s \neq f(y, z) \end{aligned} \quad (I.4.55)$$

The integral on the right-hand side of equation (I.4.55) can be evaluated since μ is related to y by equation (I.4.43) and ν is related to z by equation (I.4.44).

The box integration method was used to obtain the finite-difference analog of equation (I.4.53) because the effective thermal conductivity in each coordinate direction in the x , y , z coordinate system is a function of the values of the other two coordinates, as stated previously. The surface integrals in equation (I.4.54) account for this dependence. Equation (I.4.46) could have been expressed in the form:

$$k_s (\mu - \nu) \frac{\partial^2 T}{\partial x^2} + k_s (\lambda - \nu) \frac{\partial^2 T}{\partial y^2} + k_s (\lambda - \mu) \frac{\partial^2 T}{\partial z^2} = 0 \quad (k_s \text{ assumed constant}) \quad (I.4.56)$$

and differenced by applying an equation similar to equation (I.4.31) to each second-order partial derivative. This approach would have been equivalent to evaluating each surface integral in equation (I.4.54) in an approximate manner by multiplying the appropriate effective conductivity (k_x , k_y , or k_z) at x_i , y_j , z_k by the area of the surface in question

(i.e., $\Delta x \Delta y$, $\Delta x \Delta z$ or $\Delta y \Delta z$). The integral on the right-hand side of equation (I.4.55) would be approximated by $(u(i,j,k) - v(i,j,k)) \Delta y \Delta z$ in this case, for example. While this procedure would probably produce acceptable results when applied to a very fine mesh, the surface integrals should be evaluated in a more accurate fashion if coarse coordinate subdivisions are used.

An additional benefit of using the box integration method to derive difference equations is that the coefficient matrix for the resulting system of simultaneous nodal equations is diagonally dominant and symmetric [84], with only positive entries on the diagonal and non-positive entries off of the diagonal. From these properties it follows that the coefficient matrix is positive-definite [86]. These properties, which are not always obtained with other differencing schemes, facilitate solution of the system of simultaneous nodal equations by almost any method (e.g., LU factorization, successive over-relaxation, etc.). As noted by Forsythe and Wasow [85], the coefficient matrix obtained from a finite-difference discretization of the heat conduction equation should be positive-definite because the equation itself is positive-definite (and self-adjoint).*

It is convenient at this time to use equations (I.4.42) - (I.4.44) to transform the right-hand side of equation (I.4.55), with the derivatives expressed in central-difference form as described previously, to an equivalent expression in terms of the ellipsoidal coordinates λ , μ and ν . If this is done, it can easily be shown that each term in the transformed equation is simply an expression for the overall conductance

* Provided the equation is linear.

between the mesh point i,j,k and a neighbouring node (e.g., $i-1,j,k$), multiplied by the corresponding temperature difference (e.g., $T(i-1,j,k) - T(i,j,k)$). Considering the right-hand side of equation (I.4.55), for example, the transformation to the ellipsoidal coordinate system can be accomplished by applying the identities:

$$\begin{aligned} \underline{x}_{i,j,k} - \underline{x}_{i-1,j,k} &= F(\phi_{i,j,k}, k) - F(\phi_{i-1,j,k}, k) \\ &= \frac{a}{2} \int_{\lambda(i-1,j,k)}^{\lambda(i,j,k)} \frac{d\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)\lambda}} \end{aligned} \quad (I.4.57)$$

$$d\underline{y} = \frac{a}{2} \frac{d\mu}{\sqrt{(a^2+\mu)(b^2+\mu)(-\mu)}} \quad (I.4.58)$$

and,

$$d\underline{z} = \frac{a}{2} \frac{dv}{\sqrt{(a^2+v)(b^2+v)v}} \quad (I.4.59)$$

The limits of integration in equation (I.4.57) can be found by using equation (I.4.47) (or (I.4.48)) to transform $\underline{x}(i,j,k)$ and $\underline{x}(i-1,j,k)$ to $\lambda(i,j,k)$ and $\lambda(i-1,j,k)$ respectively.

Substituting equations (I.4.57) - (I.4.59) into equation (I.4.55), we obtain

$$k_s \frac{(T(i-1,j,k) - T(i,j,k))}{\frac{z_2}{z_1} \frac{y_2}{y_1}} \int_{z_1}^{z_2} \int_{y_1}^{y_2} (\mu - \nu) dy dz$$

$$= a(T(i-1,j,k) - T(i,j,k)) \cdot [k_s \left[\frac{1}{2} \right.$$

$$\left. \int_{\nu_1}^{\nu_2} \int_{\mu_1}^{\mu_2} \frac{(\mu - \nu) d\mu d\nu}{\sqrt{(a^2 + \mu)(b^2 + \mu)(- \mu)(a^2 + \nu)(b^2 + \nu)\nu}} \frac{\lambda(i,j,k)}{\lambda(i-1,j,k)} \frac{d\lambda'}{\sqrt{(a^2 + \lambda')(b^2 + \lambda')\lambda'}} \right]]$$

(I.4.60)

where μ_1 and μ_2 are obtained from z_1 and z_2 respectively by applying equation (I.4.49) (or (I.4.50)). Similarly, ν_1 and ν_2 are obtained from z_1 and z_2 by applying equation (I.4.51) (or (I.4.52)). The factor a (i.e., the semi-major diameter of the elliptical contact) can be removed from equation (I.4.60) since it can be shown that it is common to all of the expressions on the right-hand side of equation (I.4.54) (after transforming to λ, μ, ν coordinates). The expression contained within the outer set of square brackets on the right-hand side of equation (I.4.60) can be shown to be equivalent to the conductance between nodes $i-1, j, k$ and i, j, k , which we shall designate $K(i,j,k)_{i-1}$, if we use a generalized conduction shape factor expression derived by Yovanovich in [79]. According to Yovanovich, the shape factor for heat conduction in a general, orthogonal, curvilinear coordinate system (x_1, x_2, x_3) between isothermal surfaces at x_{1a} and x_{1b} within the heat flow channel bounded by x_{2a}, x_{2b}, x_{3a} and x_{3b} is,

$$U_{x_{1a} \rightarrow x_{1b}} = \int_{x_{3a}}^{x_{3b}} \int_{x_{2a}}^{x_{2b}} \frac{dx_2 dx_3}{\int_{x_{1a}}^{x_{1b}} \frac{g_1}{\sqrt{g}} dx_1} \quad (I.4.61)$$

Equivalent expressions for conduction in the x_2 and x_3 directions can be obtained by cyclically interchanging the subscripts 1, 2 and 3.

Equation (I.4.61), evaluated for conduction in the λ direction using the metric coefficients for the ellipsoidal coordinate system (equations (I.4.14) - (I.4.16)), yields an expression that is identical to the expression contained within the inner set of square brackets on the right-hand side of equation (I.4.60), thereby proving that

$K(i,j,k)_{i-1} (=k_s U_{\lambda_{i-1} \rightarrow \lambda_i} = k_s U(i,j,k)_{i-1})$ is simply the conductance between nodes $(i-1,j,k)$ and (i,j,k) .

If the internodal conductances between node (i,j,k) and its neighbours $(i+1,j,k)$, $(i-1,j,k)$, $(i,j+1,k)$, $(i,j-1,k)$, $(i,j,k+1)$ and $(i,j,k-1)$ are designated $K(i,j,k)_{i+1}$, $K(i,j,k)_{i-1}$, $K(i,j,k)_{j+1}$, $K(i,j,k)_{j-1}$, $K(i,j,k)_{k+1}$ and $K(i,j,k)_{k-1}$ respectively, the finite-difference equation for node (i,j,k) , which relates the temperature of that node to the temperatures of its neighbours, can be written as follows:

$$\begin{aligned} & K(i,j,k)_{i+1} (T(i,j,k) - T(i+1,j,k)) + K(i,j,k)_{i-1} (T(i,j,k) - T(i-1,j,k)) \\ & + K(i,j,k)_{j+1} (T(i,j,k) - T(i,j+1,k)) + K(i,j,k)_{j-1} (T(i,j,k) - T(i,j-1,k)) \\ & + K(i,j,k)_{k+1} (T(i,j,k) - T(i,j,k+1)) + K(i,j,k)_{k-1} (T(i,j,k) - T(i,j,k-1)) \\ & = 0 \end{aligned} \quad (I.4.62)$$

An equation similar to equation (I.4.62) can be written for each node in the finite-difference grid, yielding a system of simultaneous equations relating the nodal temperatures. The shape factors (i.e., $U(i,j,k)_{i+1}$, $U(i,j,k)_{i-1}$, etc.) corresponding to the conductances in equation (I.4.62) are evaluated in Appendix I.B. These shape factors, which are expressed in general form by equation (I.4.61), reduce to combinations of elliptic integrals in the ellipsoidal coordinate system. Each integral is expressed in terms of the Legendre-Jacobi fundamental integrals (i.e., elliptic integrals of the first, second and third kinds) in Appendix I.B, and approximations to the fundamental integrals are presented which apply when $k \approx 1$ (i.e., for long, thin elliptical contacts). This is the case of interest in this study.

The manner in which the boundary conditions for the problem are incorporated into the thermal model is discussed in the next section.

I.4.5 Representation of Boundary Conditions

In analyses of the contact with smooth surfaces in the presence of a gas, points on the single contact surface of the substitute contact specimen which are outside of the contact communicate with the fictitious isothermal surface described in Section I.3.3 through gas conductances obtained from Equation (I.3.24) (after multiplying those conductances by 2 to account for the fact that Equation (I.3.24) uses $\delta(x,y)$ to calculate h_g instead of the distance to the fictitious isothermal surface, $\delta(x,y)/2$). If no gas is present, the surface outside of the contact is assumed to be insulated, of course. In analyses of the

contact with rough surfaces, surface elements within the contour area exchange heat with the fictitious isothermal surface just mentioned through contact conductances calculated using Equation (I.3.44) (after multiplying those conductances by 2, as before). Since it is assumed that the shape of the contact surface on the substitute specimen is not critical for the reasons discussed in Section I.3.3, the contact surface in the ellipsoidal finite-difference model is represented by a portion of the x-y plane and hence is flat, as shown in Figure I.3.2. In the ellipsoidal coordinate system, the x-y plane coincides with the surface at $\lambda = -c^2$ (i.e., at $\lambda = 0$ if $c = 0$) within the elliptical contact and with the surface at $\mu = -c^2$ (i.e., at $\mu = 0$) outside of the elliptical contact.

In analyses of the contact with smooth surfaces in the presence of a gas, the temperature of each node located inside of the elliptical contact is set to some temperature T_0 while the temperature of each node i, j, k (located at λ_i, μ_j, ν_k) outside of the contact is related to the temperatures of neighbouring nodes by an equation of the form,

$$\begin{aligned} & K(i, j, k)_{i+1} [T(i, j, k) - T(i+1, j, k)] + K(i, j, k)_{i-1} [T(i, j, k) \\ & - T(i-1, j, k)] + K(i, j, k)_{j-1} [T(i, j, k) - T(i, j-1, k)] \\ & + K(i, j, k)_{k+1} [T(i, j, k) - T(i, j, k+1)] + K(i, j, k)_{k-1} [T(i, j, k) \\ & - T(i, j, k-1)] + 2 \cdot K_g [T(i, j, k) - T_0] = 0 \end{aligned} \quad (\text{I.4.63})$$

in which K_g is the integral of h_g (Equation (I.3.24)) over the surface

element associated with node i, j, k , viz.,

$$K_g = \int_{\lambda_1}^{\lambda_2} \int_{v_1}^{v_2} \frac{k_g(\delta)}{\delta} \sqrt{g_v \cdot g_\lambda} \, dv d\lambda \quad (I.4.64)$$

The limits of integration λ_1 , λ_2 , v_1 and v_2 are the boundaries of the surface element belonging to node i, j, k (which is found at $\mu=0$).

In analyses of heat transfer through the contact in vacuo, $K_g=0$ of course.

In analyses of heat transfer through the contact with rough surfaces, Equations (I.4.63) and (I.4.64) can be used without modification at nodes which are outside of the contour area. Those equations can also be used at nodes which are inside of the contour area but outside of the Hertzian area by replacing $k_g(\delta)/\delta$ in Equation (I.4.64) by h_c (obtained from Equation (I.3.44)). In addition to the replacement of k_g/δ by h_c , Equation (I.4.64) must be further modified by replacing λ by μ in order to use it at nodes which are inside of the Hertzian area. The form of the finite-difference equation that is used at each such node (i.e., at $\lambda=0$ or $i=1$) is similar to that of Equation (I.4.63) (but not identical because node $i-1, j, k$ does not exist at $\lambda=0$ whereas node $i, j+1, k$, which is not used in Equation (I.4.63), does) and hence will not be given here.

The inner and outer integrals in equation (I.4.64) are integrated numerically using three-point Gauss quadrature [87] in the computer program implementing the ellipsoidal finite-difference model, found documented in Appendix I.C.

The remaining boundary conditions associated with the substitute contact model are difficult to incorporate into a numerical model formulated in the ellipsoidal coordinate system. While the boundary conditions applied to the substitute model should match the real (i.e., experimental) boundary conditions as closely as possible, as discussed in section I.3.3, the coordinate surfaces of the ellipsoidal coordinate system obviously do not coincide with the boundaries of the "ideal" substitute specimen, shown contained within a roughly hemispherical* semi-ellipsoid (λ) in Figure I.4.7, very well at all. The segment of the surface of the large ($\sqrt{\lambda} \gg w > t$) semi-ellipsoid intersected by the boundaries of the rectangular contact specimen passing through that surface (i.e., the region for which $|x| \leq \frac{w}{2}$, $|y| \leq \frac{t}{2}$) satisfies, to a first approximation, the criteria established for the second isothermal surface discussed in section I.3.3 (i.e., the segment is remote from the contact, roughly planar, and approximately perpendicular to the sides of the specimen), however, so this segment, shown with cross-hatching in the top view of Figure I.4.7, serves as the second isothermal surface in the ellipsoidal finite-difference model. The adiabatic boundaries of the

* For values of $\lambda \gg a^2$, an ellipsoid (λ) approximates a sphere of radius $\sqrt{\lambda}$.

Figure I.4.7 Substitute Contact Specimen Contained Within a Semi-Ellipsoid.

substitute specimen, shown as dashed lines in Figure I.4.7, cannot be matched exactly by coordinate surfaces in the ellipsoidal coordinate system but they can be represented in an average sense in the numerical model by decoupling the nodes within the boundaries of the substitute specimen from the nodes outside of the specimen. In other words, only those nodes actually contained within the specimen are used to represent it. The adiabatic boundaries produced in this manner are not flat, as in the real case, but rather conform to the particular shapes of the finite-difference subvolumes straddling the boundaries, as shown in Figure I.4.8. The coincidence of the approximate and real adiabatic boundaries improves as the spatial density of mesh points increases (i.e., as the subvolumes decrease in size).

The requirement that a sufficient number of mesh points be used to adequately approximate the shape of the contact specimen affects the choice of coordinate subdivisions in the λ direction since values of λ , corresponding to uniform increments in the x coordinate produce very coarse subdivisions at large values of λ . In consequence, exploiting the "natural" grading of the transformed (x, y, z) coordinate system would cause the boundaries of the substitute contact specimen to be poorly approximated in regions remote from the contact (i.e., at large λ). The computer program described in Appendix I.C accounts for this detrimental effect by allowing smaller values of Δx to be used in regions remote from the contact than in regions near it.

SHADED REGIONS LIE INSIDE OF THE
APPROXIMATED SUBSTITUTE CONTACT SPECIMEN

Figure I.4.8: Approximate Representation of the Substitute Contact Specimen in the Ellipsoidal Finite-Difference Model

Details related to the size of ellipsoid used to contain the substitute specimen, which in essence determined the length (L) of the specimen, and the number of nodes used in the finite-difference model will be discussed in section I.6.2.

The computer program written to implement the ellipsoidal finite-

difference model was tested by using it to solve several problems with known solutions. Specifically, it was used to perform:

(1) one-dimensional Dirichlet analyses in the λ , μ and ν directions. These problems, while one-dimensional, were solved using a three-dimensional grid with several nodes inserted in each coordinate direction. Resistances in each direction were calculated manually from the shape factor equations given in Appendix I.B (elliptic integral values were obtained from tables) and compared to the computed results. The temperatures of nodes on surfaces which should have been isothermal (e.g., surfaces of constant λ for one-dimensional heat flow in the λ direction) were checked to verify that the surfaces were, in fact, isothermal.

(2) a three-dimensional analysis of the heat flow between a circular isothermal disc of radius $\sqrt{\lambda_d}$ located on the otherwise insulated equatorial plane of a large, isothermal, hemisphere of radius R and the hemispherical surface itself. This analysis was three-dimensional instead of just one-dimensional because the isothermal circular disc in the x - y plane (i.e., at $z = 0$ or $\mu = 0$) contained all those nodes with coordinates $\mu = 0$, $-a^2 < \nu < -b^2$ and $\lambda \leq \lambda_d$, where $\lambda_d \gg a^2 > b^2$. For $\lambda_d \gg a^2 > b^2$ and $z = 0$, equation (I.4.1) (with $\theta = \lambda$ and $c = 0$) reduces to the expression,

$$1 = \frac{x^2}{\lambda_d + a^2} + \frac{y^2}{\lambda_d + b^2} = \frac{x^2}{\lambda_d} + \frac{y^2}{\lambda_d}$$

which defines a circle of radius $\sqrt{\lambda_d}$. The computed resistance of the circular disc differed by less than 1% from the exact result (R_{ex}), given

by the expression^{*}:

$$R_{ex} = \frac{\phi}{2\pi k_s \sqrt{\lambda_d}} \quad (I.4.65)$$

where $\phi = \sin^{-1} \sqrt{\frac{R^2}{R^2 + \lambda_d}}$

* Equation (I.4.65) can be obtained from equation (I.6.2a) by setting $k = 0$ (for a circular contact) since $F(\phi, 0) = \phi$ [71].

CHAPTER I.5

EXPERIMENTAL PROGRAM

1.5.1 Introduction

The essential features of the experimental program were discussed in section I.1.3.2. As was noted in that section, the object of the experimental program was to determine the resistance, defined by equation (I.1.1), between two cylindrical contacting surfaces. The axes of the cylindrical surfaces, which had widely different radii of curvature, were orthogonal so the contact patch produced at the interface between the surfaces upon application of a load normal to the interface was elliptical in shape. A steady flow of heat was established in the contact specimens and measurement of the temperature gradients in the specimens permitted calculation of the resistance at the interface using equation (I.1.1).

The basic apparatus used to load and support the contact specimens, create a steady heat flow in the specimens and monitor required temperatures, loads, etc., is described in section I.5.2. The contact specimens themselves are described in section I.5.3. A detailed discussion of the thermal measurements performed in the experiments is included in section I.5.4. Subsequent sections deal with the experimental procedure and the exact manner in which contact resistances were calculated from the experimental data.

1.5.2 Experimental Apparatus

The experimental apparatus used in this investigation was required to perform three principal functions, specifically it had to:

- (a) be capable of evacuating the region surrounding the contact in order to eliminate gas conduction between the contact surfaces. Tests in air were also required of course.
- (b) apply a known load to the contact and support the contact specimens in the proper orientation. The contact patch had to be centrally located on the contact surface of each specimen and the planes of curvature of the surfaces had to be orthogonal.
- (c) establish a steady heat flow of known magnitude through the contact interface. In addition, the contact assemblies had to be fabricated in a manner that permitted evaluation of the extrapolated temperature drop (ΔT_c in Figure I.1.5) at the interface. This requirement meant, of course, that regions of one-dimensional heat flow had to be provided on each side of the interface.

The experimental apparatus designed to perform these functions is shown in Figure I.5.1 and Plates 1 and 2. The first task (a) was

Figure I.5.1 Experimental Apparatus

Plate 1 Experimental Apparatus

Plate 2 Vacuum System

accomplished by enclosing the test column and its supporting structure in a bell-jar type vacuum system. The available bell-jar, which had an inside diameter of 26.7 cm (10 1/2") and a height of 44.5 cm (17.5"), was not high enough to accommodate a satisfactory test apparatus so the 20.3 cm (8") high aluminum extension shown positioned underneath the bell-jar in Plate 2 was fabricated to increase the height. The vacuum system, which was equipped with a mechanical vacuum pump and a diffusion pump, had a minimum operating pressure of 10^{-6} mm Hg. A thermocouple gauge and an ion gauge were used for measuring chamber pressures above 10^{-2} mm Hg. and below 10^{-3} mm Hg. respectively. The pressure at which gas conduction between contacting surfaces becomes negligible is a function of the type of gas present and the characteristics (e.g., waviness) of the contacting surfaces, but any pressure below 10^{-3} mm Hg is normally considered a "hard" vacuum (i.e., a vacuum in which gas conduction is negligible) by contact conductance researchers, at least when air is the gas present. Kitscha [16] observed that gas conduction between a 2.54 cm (1") diameter sphere and a flat surface decreased only slightly as the pressure of the air around the contact was lowered from 740 mm Hg. to ≈ 10 mm Hg., but then dropped rapidly to a negligible value as the pressure was further reduced from 10 mm Hg. to $\approx 5 \times 10^{-2}$ mm Hg. No further increase in the resistance of the sphere/flat contact was observed as the gas pressure was reduced from 5×10^{-2} mm Hg to 10^{-5} mm Hg. In view of these observations, it is believed that the chamber pressure used in the present investigation (i.e., $10^{-5} \rightarrow 10^{-6}$ mm Hg) was low enough to eliminate gas conduction.

Various methods have been used in contact conductance experiments to apply loads to contacts located in vacuum chambers. Deadweight systems, where the load is applied either by weights located in the vacuum chamber itself [58] or by externally located weights acting through linkages which penetrate the chamber walls [16], are perhaps the most popular loading arrangements. The former arrangement requires a large vacuum chamber in order to accommodate the loading plates, however, and does not allow the contact load to be changed while the system is under vacuum. If the load on the contact is not measured directly with a load cell, friction in the mechanism (e.g., loading pin and sleeve) used to transmit the loading force can create difficulties since it causes the applied and transmitted loads to differ. The second loading arrangement, where the weights are externally located, suffers from the same problems with friction. Other loading systems using hydraulic cylinders, springs, and bellows containing compressed air have been reported in the literature. Each of these systems has its own advantages and disadvantages. In the present investigation, the small size of the available bell-jar necessitated using a compact loading system. In addition, the ability to change the load on the contact while the vacuum chamber was evacuated was considered a desirable feature. For these reasons, the spring loading system shown in Figure I.5.1 was selected as the most appropriate loading arrangement. A brief description of the operation of the contact loading/alignment system is given below.

As seen in Figure I.5.1, a shaft, threaded at the upper end and prevented from moving vertically by a thrust bearing located in the upper support plate, was rotated by a rotary feedthrough

penetrating the baseplate. A nut on the threaded part of the rotatable shaft pressed against one end of a steel loading beam, the other end of which was connected to a spring with a spring constant of approximately 13.4 kg/cm (75 lb./in.). The center of the beam was supported (on a pivot) by a loading pin which was free to move up and down in a closely fitting sleeve. The sleeve was bolted to the upper support plate. A flat on the upper part of the otherwise circular (in cross-section) loading pin interfered with a matching protrusion (i.e., guide) on a rotatable plate attached to the sleeve. The rotatable plate, which could be locked in any position, was used to align the upper and lower contact specimens so that the axes of curvature of the contact surfaces were orthogonal. Alignment of the contact specimens in other planes was accomplished by adjusting the nuts clamping the upper support plate to the threaded support rods and, in addition, by adjusting the position of the lower contact assembly (shown in Figure I.5.1) on the lower support plate.

An Interface SM-250 load cell (strain-gauge type) attached to the bottom of the loading pin was used to monitor the load on the contact. The load cell measured the actual load on the contact less the weight of the items between the load cell and the interface. The thick-walled Tygon tubing supplying cooling water to the upper heat sink was preformed to the proper shape with a hot-air gun in order to minimize any loads the bent tubing might exert on the upper contact assembly.

The items located in the test column below the load cell were all associated with task c) in that they were required to establish and measure the heat flow through the contact interface. From top to bottom, the principal items in the thermally active part of the column were:

(1) a water-cooled copper heat sink, which was fabricated by silver-soldering a coil of 6.35 mm (1/4") O.D. copper tubing to a 5 cm (1") diameter x 5 cm long copper cylinder. This heat sink permitted the load cell to be maintained at a more or less constant temperature (although the load cell was temperature compensated) and, in addition, prevented the loading pin from overheating and binding in its sleeve.

(2) four 1.25 cm (1/2") diameter x 2.5 cm (1") long ceramic spacers. The ceramic spacers restricted the amount of heat flowing from the heat source (item 7) to the upper heat sink (item 9).

(3) a heat source, which was fabricated by silver-soldering two Chromalox C-201C cartridge heaters into 0.952 cm (3/8") diameter holes in a 5 cm (2") diameter x 2.86 cm (1.125") long copper cylinder.

(4) the upper contact assembly composed of an Armco-iron (i.e., commercially pure iron) heat flow metering bar and the upper contact specimen.

(5) the lower contact assembly composed of an Armco-iron heat flow metering bar and the lower contact specimen.

(6) a water-cooled copper heat sink similar to item 9.

The heat source was located above the interface in order to suppress free convection across the interface. The use of two heat sinks (i.e., one above and one below the heat source) caused the heat from the heat

source to be contained within the contact assembly and prevented it from flowing into the supporting structure. By isolating the "active" part of the apparatus in this manner, the time required by the apparatus to reach steady-state conditions after a change in heater power level, contact load, etc., was minimized. This was considered to be a desirable feature since the rate of response of test columns of the type described here to changes in experimental conditions is quite slow, particularly under vacuum conditions.

All of the thermocouples in the Armco-iron heat flow bars and the contact specimens were connected to a rotary switch located inside the vacuum chamber, thus only one pair of thermocouple wires penetrated the walls of the chamber. The rotary switch was operated from outside the vacuum chamber by means of a rotary feedthrough that penetrated the baseplate of the chamber. The placement of the thermocouples in the heat flow bars and contact specimens will be described in section I.5.4.

The test column was insulated with fiberglass wool and aluminum foil, as shown in Plate 2. In order to keep the fiberglass wool away from the contact, the contact was encased in a sheath of thin polyester film before the fiberglass was applied. The polyester film was held in place by the elastic bands seen above and below the contact in Plate 1. A small slit was cut in the film to allow air to enter and escape from the contact area.

Cooling water for the two heat sinks was supplied by the constant-temperature circulating bath visible on the right-hand side of Plate 2. The temperature of the cooling water could be regulated to within 0.2°C .

with this system. Power supplies (DC) for the load cell and heat source, and digital voltmeters for monitoring thermocouple and load cell voltages are also shown in Plate 2. A list of the equipment used in the experiments is given in Table I.5.1.

I.5.3 Contact Specimens

Both the upper and lower contact specimens were fabricated from oil-hardening tool steel (AISI type O-1), the nominal chemical composition of which is given in Table I.5.2.

Table I.5.2: Principal Alloying Elements in O-1 Steel

<u>Element</u>	<u>%</u>
C	0.95
Mn	1.20
W	0.65
Cr	0.55
V	0.15

This steel, which is widely used as die material, was selected for use in this investigation because of its good hardenability and good oxidation resistance. The contact material was required to have the former characteristic since it was anticipated that a hard contact surface would be needed to obtain the very low surface roughness desired in the preliminary experiments. The hardness in the as-received, annealed, condition ($R_C 17$) was found to be sufficient, however.

Table I.5.1: List of Equipment

Item	Manufacturer	Model	Function
Digital voltmeter	Hewlett-Packard	3440A with 3443A high gain/auto range unit	Monitoring load cell output and excitation voltages and the heat source supply voltage.
Digital voltmeter (DC)	Fluke	8300A (μ V resolution)	Monitoring thermocouple emf's.
Load Cell	Interface	SM-250 ($\pm$ 250 lbs.)	Contact load measurement.
Power Supply (DC)	Hewlett-Packard	6824A ($\pm$ 50V, $\pm$ 1A)	Power supply for two CHROMALOX C-201C cartridge heaters used in the heat source.
Power Supply (DC)	Hewlett-Packard	6213A (0-10V)	Load cell excitation power supply.
Surface Roughness Meter	Micrometrical Manufacturing (MM) Co.	Type A.D. with Type Q amplifier (model 8)	Contact surface roughness measurement.
Thermocouple Selector Switch	Omega Engineering	28 position for copper-constantan thermocouples	Thermocouple selection.
Vacuum System	High Vacuum Equipment Corp.		Vacuum system (10^{-6} mm Hg minimum operating pressure) for performing contact resistance measurements in vacuo.

The upper contact specimen, shown in Figure I.5.2, had a cylindrical contact surface of 76.04 cm (29.94") radius. The cylindrical surface was prepared by first grinding the surface to a 61 cm (24") radius with a 100 grit grinding wheel. A tool grinder was used to perform the grinding operation and a 61 cm boom, pivoted at one end and equipped with a machinist's vice at the other, was used to guide the contact specimen in its path across the face of the grinding wheel. The contact specimen was clamped in the machinist's vice at the free end of the boom. After completion of the grinding operation, the contour of the ground surface was checked with a dial gauge on a surface plate to ensure that the radius of the surface was uniform and that it was approximately 61 cm. Subsequent polishing of the surface with 0.05 micron alumina powder on a metallographic specimen polishing machine produced a surface with a roughness of approximately 0.05 microns (2μ in rms). A constant-velocity roughness meter was used to measure the roughness of the surface. The wavelength cutoff was set at 0.0254 cm (0.010") for the measurements.

While it was recognized that the radius of curvature of the contact surface would be altered by the polishing process, it was expected that the radius of curvature would remain fairly uniform in the region in which contact would occur (i.e., in the central part of the surface). The radius of curvature of the finished surface was measured interferometrically at six different locations in the central area of the contact surface using the equipment shown in Figure I.5.3. As seen in that figure, the interferometer consisted of a monochromatic light

Figure I.5.2: Upper Contact Specimen

source in the form of a sodium* lamp (yellow light), a splitter plate, a travelling microscope, and an optical flat. A flame-polished microscope slide served as the optical flat.

* Yellow sodium light is not truly monochromatic since it consists of two wavelengths, specifically the two sodium D lines at 5890Å and 5896Å. In most applications, particularly in those where the gap being measured is just a few fringes in thickness as in the present case, the departure from monochromaticity does not create problems and the two sodium D lines form a broadened line with a mean wavelength of $\approx 5893\text{Å}$. At gap thicknesses > 500 fringes, the 6Å separation between the sodium D lines cannot be ignored, however.

Figure I.5.3: Interferometer Used to Measure Radius of Upper Contact Surface

When viewed under the microscope, the fringe pattern produced by the interference of light waves passing through the optical flat with waves reflected from the polished surface of the contact specimen appeared as shown in Figure I.5.4. The fringes on the left and right-hand sides of the center of the pattern were numbered as shown in Figure I.5.4 and the separation distance, d_{ii} , between the i th fringe on the left-hand side and the i th fringe on the right-hand side of the pattern was measured using the travelling microscope, for $i = 1, N$.

Figure I.5.4: Fringe Pattern

By plotting d_{ij}^2 versus i , a straight line (least-squares fit) with a slope (S_λ) of $4 \lambda_{Na} R_2$, where λ_{Na} = the wavelength of yellow sodium light ($\approx 5893\text{\AA}$), was obtained. A typical plot of d_{ij}^2 versus i obtained from the interferometric examination of the upper contact surface is shown in Figure I.5.5.

The values of the radii measured at the six locations on the contact surface are given in Table I.5.3 along with the standard deviations of the individual measurements. The standard deviation of each radius measurement, σ_R , was calculated from the standard deviation of the slope of the corresponding d_{ij}^2 vs. i graph (σ_S) by using the relationship*

* The uncertainty in λ_{Na} was assumed to be negligible.

Figure I.5.5 Results of Interferometric Measurement of R_2 (Trial 4)

Table I.5.3: Results of Interferometric Radius Measurements

Trial	R_2 (cm)	σ_R (cm)	$\sigma_R/R_2 \times 100$ %
1	76.06	0.25	0.33
2	77.04	0.41	0.54
3	71.70	0.34	0.48
4	79.04	0.37	0.47
5	76.66	0.67	0.87
6	75.77	0.24	0.32

* The plot of fringe spacing vs fringe number shown in Figure I.5.5 is for trial 4.

$\sigma_R = \sigma_S / 4\lambda_{Na}$. The standard deviations of the individual measurements were very low, as seen in Table I.5.3, so the simple interferometric technique described previously was judged to be highly satisfactory for the purpose of measuring local values of the contact surface radius. Also, the distance over which the contact radius was sampled (i.e., the largest fringe separation, d_{NN}) in a single measurement was of the same order of magnitude as the major diameter of the contact patch produced in the experiments* so the measured values of R_2 should have corresponded closely to the "effective" values of R_2 seen by the contact surfaces within the contact region.

As seen in Table I.5.3, the measured surface radii ranged from 71.70 cm (28.23") to 79.04 cm (31.12"). The mean value of the six radii was 76.04 cm, and the standard deviation of the six measurements was 2.42 cm ($n-1$ weighting), or 3.18% of the mean value. The mean value of the six radius measurements was used to obtain analytical predictions of the contact resistance and the uncertainty in the value of R_2 used in the predictions was taken to be the uncertainty in the six measurements (i.e., 2.42 cm for a 1σ variation).

The lower contact specimen consisted of a 9.52 mm radius x 25.4 mm long section of 0-1 steel shafting and a block of 0-1 steel with a semi-cylindrical cavity machined in one end. An exploded view of the lower contact specimen is shown in Figure I.5.6a and its two parts are shown dimensioned in Figure I.5.6b. The section of steel shafting, which was

* The value of d_{NN} was typically 0.40 cm while the major diameter of the contact at the largest contact load was 0.42 cm (according to Hertz's theory).

Figure I.5.6a Exploded View of Lower Contact Specimen

Figure I.5.6b Dimensioned Views of Lower Contact Specimen

polished to a surface roughness of 0.05 microns (2μ in) in a lathe, was silver-soldered into the semi-cylindrical cavity at the end of the steel mounting block. A low-melting point (210°C) cadmium-silver solder was used in this, and all other, soldering operations in order to minimize oxidation of the polished contact surfaces. The contact surfaces were also coated with vacuum grease before the soldering operations to further reduce the likelihood of oxidation.

As mentioned previously, the resistance of the contact was measured with both smooth and rough contact surfaces. Only the upper contact surface (i.e., the large radius surface) was roughened for the rough surface tests, however. The lower contact surface was left in the polished condition. The upper contact surface was roughened to a surface roughness of 1.0 - 1.12 microns ($40\text{-}45 \mu$ in) by-bead blasting it, momentarily, with glass beads ranging from 0.074 mm to 0.125 mm in diameter.

Since the microhardness of the softer contact surface was required to apply the rough surface contact model described in Section I.3.5, the microhardnesses of both contact surfaces were measured with a Leitz Wetzlar Miniload (Vickers) hardness tester when both surfaces were in the polished condition. At least five indentations were made on each contact surface with a load of 100 grams and four additional measurements of the microhardness of the large radius contact surface were made with a load of 50 grams. The average pyramid microhardnesses obtained in those tests are given below.

Small radius contact surface (100 gm. load) - $H = 1670 \text{ MPa}$

Large radius contact surface (100 gm. load) - $H = 1610 \text{ MPa}$

Large radius contact surface (50 gm. load) - $H = 1690 \text{ MPa}$

It is significant to note that the measured microhardnesses are all very similar and that they are all approximately 3 times the yield stress (S_y) of annealed 0-1 steel, given in Appendix I.D as 550 MPa ($3 \times 550 \text{ MPa} = 1650 \text{ MPa}$). This result supports using $3S_y$ instead of H if measurements of H are not available (this possibility was mentioned in Section I.3.5).

Since the hardness of the large radius contact surface would have increased due to strain-hardening when it was bead blasted, it was assumed that the small radius contact surface would have been the softer surface in the rough surface resistance tests. As a result, H was assumed to be 1670 MPa in order to obtain analytical predictions of the rough surface contact resistances.

Values for the average absolute asperity slope ($\bar{m}$) and the rms roughness (σ) of the bead-blasted contact surface were also required to apply the rough surface contact model described in Section I.3.5. The rms roughness was measured and found to be approximately $1.06 \mu\text{m}$ (on average), as mentioned previously, but a value had to be assumed for $\bar{m}$ because the specialized equipment required to measure it was not available. The value of $\bar{m}$ for a bead-blasted surface is generally much less than 10° (0.17 radians) according to Burde [58], who himself tested five bead-blasted surfaces with values of $\bar{m}$ ranging from 0.036 radians to 0.104 radians. Values of $\bar{m}$ and σ are usually correlated to some extent in that they usually increase together [87a] (obviously $\bar{m}=0$ when $\sigma=0$), as is evident from the plot of $\bar{m}$ vs. σ shown in Figure I.5.7 for the surfaces tested by Burde, although one would expect the bead size to have some influence on the form of the correlation for bead-blasted surfaces. Since the beads used to prepare

the surfaces tested by Burde were 0.075 mm - 0.25 mm in diameter [58] and hence similar in size to the beads used to roughen the large radius contact surface in the present study, the data shown in Figure I.5.7 was used to estimate that the value of $\bar{m}$ for that surface (for which $\sigma = 1.06 \mu\text{m}$) was probably approximately 0.06 radians. Predicted results will be given in Section I.6.5 for this value of $\bar{m}$ as well as for values which are twice and one-half of it (i.e., 0.03 radians and 0.12 radians respectively).

Figure I.5.7 Plot of Mean Slope ($\bar{m}$) vs. Roughness (σ) for Bead-Blasted Surfaces Tested by Burde [58].

I.5.4 Thermal Measurements

Heat flow rates in the upper and lower contact assemblies were evaluated by measuring one-dimensional temperature gradients in Armco-iron metering bars of known thermal conductivity and known cross-sectional area. Since Armco iron is commonly used as a reference material in heat flow measurements, its thermal conductivity has been measured over a wide temperature range by several laboratories. From a summary prepared by Lucks [88] of most of the available conductivity data, the thermal conductivity of Armco-iron (k_i) can be expressed as:

$$k_i = 74.86 - 0.0718 T(^{\circ}\text{C}) \text{ (W/mK) for } 0 \leq T \leq 150^{\circ}\text{C} \quad (\text{I.5.1})$$

Thermal conductivities predicted using equation (I.5.1) differ negligibly from the National Standard Reference Data System (NSRDS) recommended values of k_i for the temperature range indicated, given in the Journal of Physical and Chemical Reference Data [89]. Lucks gives the average coefficient of variation* for the values of k_i obtained at temperatures below 973 K as 1.5%, so the 2 σ uncertainty in k_i at temperatures below 973 K is ~ 3%.

The two Armco-iron heat flow metering bars were both 2.54 cm (1") x 1.715 cm (0.675") in cross-section and 7.84 cm (3.08") long. Seven #30 gauge copper-constantan thermocouples were used in each bar to measure the temperature gradient. The thermocouple attachment points on a single metering bar are shown in Figure I.5.8. All of the thermocouples were silver-soldered into holes 0.5 mm in diameter and 2.5 mm deep using the low melting point cadmium-silver alloy mentioned previously. Each thermocouple wire was epoxied to the Armco-iron for a short distance from its attachment

* The coefficient of variation (c.v.) of a population is its standard deviation expressed as a percentage of the population mean (μ), i.e.,

$$\text{c.v.} = \frac{\sigma}{\mu} \times 100.$$

Figure I.5.8 Thermocouple Attachment Points on Armco-Iron Bar

Figure I.5.9 Apparatus for Measuring Thermal Conductivity of O-1 Steel

attachment point, as shown in Figure I.5.8, to allow the temperature of the wire to equilibrate with that of the metering bar before the attachment point.

The contact specimens themselves, which were described in Section I.5.3, were silver-soldered to the Armco-iron metering bars. This method of attachment was found to be satisfactory by Kitscha and Yovanovich [16], who demonstrated that a well-prepared silver-soldered junction has a negligible thermal resistance. The upper contact specimen in the present investigation was equipped with four thermocouples spaced 6.35 mm apart, as shown in Figure I.5.2, so it was possible for the author to verify Kitscha et al.'s observation by extrapolating the temperature gradients in the upper metering bar and the upper contact specimen to the soldered junction between them. No temperature drop was observed at the junction, as will be demonstrated later for a typical set of temperature measurements.

The analytical model of the contact required an accurate value for the thermal conductivity of the contact material. For this reason, the thermal conductivity of 0-1 steel (annealed) was measured by sandwiching an 11.4 cm (4.5") long section of steel between the two Armco-iron metering bars. The steel specimen had the same cross-sectional dimensions as the Armco-iron bars. The Armco-iron bars were silver-soldered to the steel specimen and the experimental apparatus shown in Figure I.5.9 was used to establish a steady heat flow in the three-tiered test assembly. As seen in Figure I.5.9, six 30-gauge copper constantan thermocouples, spaced 1.27 cm (0.5") apart, were used to measure the temperature gradient in the steel specimen. The first and last thermocouples in the steel specimen were located 2.5 cm (1") from the upper Armco-iron/steel junction and lower iron/steel junction respectively. The thermocouples were attached to the steel specimen in the manner described previously for the heat flow metering bars.

The test assembly shown in Figure I.5.9 was insulated with fiber-glass wool and aluminum foil and temperature gradients in the metering bars and the steel specimen were measured in vacuo. Since the thermal conductivity of the Armco-iron was known, the thermal conductivity of the steel was easily calculated from the measured temperature gradients. The heat flow rate in each Armco-iron metering bar was evaluated using the thermal conductivity of Armco-iron at the average temperature of the bar (i.e., the temperature at the midpoint of the heat flow metering length) and the heat flow rates in the upper and lower bars were, in turn, averaged to obtain the heat flow rate in the steel specimen. The average heat flow rate ($\bar{Q}$) and the temperature gradient* (dT/dx) in the steel specimen were used to calculate the thermal conductivity of the steel (k_{st}) according to the relationship

$$k_{st} = \frac{\bar{Q}}{A_{st} \frac{dT}{dx}} \quad (I.5.2)$$

where A_{st} was the cross-sectional area of the steel specimen. The value of k_{st} obtained in this manner was assumed to be the steel's conductivity at the average temperature (T_{st}) of the steel bar (i.e., at the average of the six thermocouple readings in the bar). The thermal conductivity of the steel was evaluated at four average specimen temperatures in this manner, producing the results shown in Table I.5.4 and Figure I.5.10.

A least-squares fit to the tabulated data gives the following expression for k_{st} :

$$k_{st} = 36.205 + 0.01447 T(^{\circ}\text{C}), \text{ W/mK} \quad (I.5.3)$$

The correlation coefficient for equation (I.5.3) is 0.983.

* A least-squares fit to the thermocouple readings was used to determine this, and all other, temperature gradients.

Figure 1.5.10 Thermal Conductivity of 0-1 Steel

Table I.5.4: Measured Values of k_{st}

T_{st} °C	k_{st} W/mK
33.8	36.67
54.4	37.06
65.0	37.12
75.6	37.28

It should be mentioned that the heat flow rate in the upper Armco-iron bar was greater than that in the lower bar in all of the thermal conductivity tests, with the difference ranging from 1.7% of $\bar{Q}$ at $T_{st} = 33.8^\circ\text{C}$ to 5.6% of $\bar{Q}$ at $T_{st} = 75.6^\circ\text{C}$ ($\bar{Q}$ was typically 4.5-5.0 W). Guard heaters would have been required to obtain better agreement than this, although in retrospect, the heat loss could probably have been significantly reduced by using an O-1 steel test sample shorter than 11.4 cm. It is estimated that the values of k_{st} calculated using equation (I.5.3) are accurate to within approximately $\pm 2\%$, relative to k_i^* , at the 95% confidence level (i.e., the uncertainty in k_{st}/k_i is $\pm 2\%$ of the nominal value of k_{st}/k_i).

The author was unable to find any published thermal conductivity data for a steel with exactly the same composition as the test steel to compare equation (I.5.3) against, but laboratory measurements of the thermal conductivity of a similar oil-hardening tool steel are reported in Vol.1 of the Thermophysical Properties of Matter reference series [90].

* The 2σ uncertainty in k_i itself is $\sim 3\%$, as mentioned previously

The oil-hardening steel in question, which was tested by Hattori [91], contained C (0.79%), Cr (0.64%) and Mn (0.59%) as the principal alloying elements. The steel did not contain any tungsten (W), but in other respects its composition corresponded more closely to that of O-1 steel than any other steel for which the author was able to find accurate thermal conductivity data. The thermal conductivity of this steel in the annealed condition, designated k_{st} , was not measured below 98°C, but it was measured at three temperatures between 98°C and 152°C. A least-squares fit to those measurements gave k_{st} as,

$$k_{st} = 36.673 + 0.01467 T(^{\circ}\text{C}) \text{ W/mK} \quad (\text{I.5.4})$$

with a correlation coefficient of ~ 1.0 . If equation (I.5.4) is compared to equation (I.5.3), the agreement is seen to be excellent. At a temperature of 58°C, which was typical of the mean interface temperature that prevailed in the contact resistance experiments, k_{st} and k_{st}' differ by only 1.28%. Hattori tested another steel which was also similar in composition to the O-1 steel used in this study in that it contained C (1.03%), W (1.02%), Mn (0.98%), Si (0.21%), Ni (0.11%) and Cr (0.10%) as the principal alloying elements. The thermal conductivity of this steel in the annealed condition, designated k_{st}'' , was measured at three temperatures between 98°C and 145°C. A least-squares fit to those measurements gave k_{st}'' as,

* Thermal conductivity data for several different tempers are given in [90 and 91].

$$k_{st}'' = 37.014 + 0.01666 T(^{\circ}\text{C}) \text{ W/mK}$$

with a correlation coefficient of 0.857. At the typical mean interface temperature of 58°C mentioned previously, k_{st} and k_{st}'' differ by 2.5%. The good agreement obtained between thermal conductivities predicted with equation (I.5.3) and thermal conductivities published in the literature for steels similar to AISI 0-1 serves as proof of the accuracy of the thermal conductivity measurements performed in this study.

I.5.5 Experimental Procedure

The activities in each set of experiments were performed in the following sequence:

- (1) The test column was assembled and adjustments were made as required to properly align the contact specimens.
- (2) Thermocouple wires, cooling water lines, etc., were connected to the experimental apparatus.
- (3) The contact surfaces were thoroughly cleaned, first with acetone and then with alcohol.
- (4) Insulation was applied to the test column.
- (5) The first contact load was set with the spring loading system.
- (6) The circulating water bath was started with the regulated cooling water temperature set at approximately 15°C. Sufficient electrical (DC) power was supplied to the heat source in the test column to produce an extrapolated temperature drop (ΔT_c) of 33-55°C (60-100°F) at the contact interface. When the vacuum chamber was evacuated, power

levels of ≈ 5 W and ≈ 11 W were required at the lowest (≈ 26 N) and highest (≈ 454 N) contact loads respectively to achieve this temperature drop, while the corresponding power levels in air were ≈ 13 W (lowest load) and ≈ 18 W (highest load).

(7) The temperatures of the thermocouples in the contact assemblies were monitored until they stabilized, at which time it was assumed that steady-state conditions prevailed and all of the thermocouple voltages, the chamber pressure and the contact load (i.e., load cell output voltage) were recorded. The time required to reach steady-state ranged from ≈ 30 min. at the highest loads in air to ≈ 3 hr. at the lightest loads in vacuum. These figures refer only to the times required to achieve steady-state conditions after modest changes in contact load, heater power level, etc. The time required to reach equilibrium from a cold start was much longer, typically 12-18 hrs in vacuum.

(8) The test chamber was evacuated using the mechanical vacuum pump followed by the diffusion pump. The power to the heat source was reduced since the power required to produce an extrapolated temperature drop of $33-55^\circ$ at the contact interface was much lower in vacuum than it was in air.

(9) The necessary data (i.e., thermocouple voltages, chamber pressure and contact load) were recorded after equilibrium was reached.

(10) The contact load was increased and more data were recorded after equilibrium was reached.

(11) Air was allowed to enter the test chamber, the necessary data were recorded after the thermocouple readings had stabilized, and the

contact load was increased.

(12) Steps 9-11 were repeated until data at all of the test loads (measurements were made at ten loads ranging from 26N to 454N) had been obtained.

The test procedure described above was repeated twice with the smooth contact surfaces, providing two sets of resistance vs. contact load data. That data was complete in the sense that resistance measurements were made both in air and in vacuum at each contact load. A third series of measurements was made at all contact loads in air only. The position of the contact on the contact surfaces was shifted slightly after each set of tests in case the surfaces had sustained some permanent damage in the previous loading sequences, although it is shown in Appendix I.D that yielding of the contact surfaces would not have occurred, according to von Mises yield criterion, at the highest contact load used (454N). Both contact surfaces were lightly polished after each set of tests, primarily to remove any oxide film that might have formed.

The resistance of the contact with the upper contact surface roughened by bead blasting was measured for one complete loading and unloading cycle. In other words, the contact was loaded in steps up to the maximum load and then unloaded in steps until the minimum load was reached. The resistance of the contact was measured in air and in vacuum at each load in the loading cycle, and in vacuum only at each load in the unloading cycle. The resistance measurements were performed in the unloading cycle to see if the contact resistance vs. load data exhibited any hysteresis.

I.5.6 Calculation of Contact Resistance

The resistance of the contact (R_c) was defined in Chapter I.1 as,

$$R_c = \frac{\Delta T_c}{Q_c} \quad (I.1.1)$$

In order to calculate R_c , the parameters ΔT_c and Q_c had to be evaluated for each set of test conditions. The former quantity was expressed as,

$$\Delta T_c = T_{ch} - T_{cc} \quad (I.5.4)$$

where T_{ch} and T_{cc} were the temperatures obtained by extrapolating the undisturbed (by the contact) temperature gradients in the "hot" and "cold" contact specimens to the contact interface, as shown in Figure I.1.5. The value of T_{ch} was determined relatively easily in the present investigation because the upper (hot) contact specimen was equipped with four thermocouples, as mentioned previously, the nearest of which to the contact interface was located 1.57 cm (0.62") away from it. Since temperature gradients measured using the four thermocouples were reasonably linear, as will be seen shortly, it was assumed that these thermocouples were in a region where the heat flow could, to a first approximation, be considered "undisturbed" by the presence of the contact. Consequently, the value of T_{ch} was determined by extrapolating the temperature gradient in the upper contact specimen, measured using the four thermocouples, to the contact interface.

The lower (cold) contact specimen was not equipped with thermocouples so T_{cc} was calculated from the relationship,

$$T_{cc} = T_s + \left(\frac{k_i}{k_{st}} \right) \left. \frac{dT}{dx} \right|_i L_s \quad (I.5.5)$$

where T_s was the temperature at the silver-soldered junction between the lower contact specimen and the Armco-iron metering bar, obtained by extrapolating the temperature gradient in the metering bar to the junction (where $x = x_s$), $\left. \frac{dT}{dx} \right|_i$ was the temperature gradient in the lower metering bar, and L_s was the length of the lower contact specimen (2.91 cm). The thermal conductivity of the iron (k_i) was evaluated at the average temperature of the heat flow metering length in the Armco-iron bar and k_{st} was evaluated at the average temperature of the lower contact specimen in a particular test. An example of these calculations will be given later.

It was assumed that the heat flow rate through the contact (Q_c) was the same as the heat flow rate in the lower (cold) metering bar, Q_{ci} , given by:

$$Q_{ci} = k_i A_i \left. \frac{dT}{dx} \right|_i \quad (I.5.6)$$

Heat transfer to the surroundings through the insulation surrounding the test column caused the heat flow rate in the upper (hot) metering bar (Q_{hi}) to be greater than Q_{ci} in all of the tests. In vacuum, Q_{hi} was $\approx 7\%$ greater than Q_{ci} at the highest contact load and $\approx 13\%$ greater than Q_{ci} at the lowest contact load. In air, Q_{hi} was 12-15% greater than Q_{ci} . The resistance of the contact was calculated using Q_{ci}

because it was presumably the heat flow that actually came through the contact. The average temperature of the lower contact assembly (composed of the lower contact specimen and heat flow metering bar) was approximately equal to or, in some cases, slightly greater than the ambient temperature so heat flow between the lower contact assembly and the environment should have been very small. Most of the heat loss from the test column, must have occurred, therefore, from the upper (hot) contact assembly, i.e., before the contact was reached. It is standard practice to use Q_{ci} to evaluate R_c in experiments of this nature [58]. Better agreement between Q_{ci} and Q_{hi} can be achieved in experiments of this type by using guard heaters and/or by reducing the number of thermocouples used in the temperature measurements, since this reduces the heat lost by conduction along the thermocouple wires.

As an example of how R_c was evaluated, consider the experimental data shown in Table I.5.5. The temperature measurements shown in Table I.5.5 were obtained at a contact load of 300N in vacuo. The x-axis, which has its origin at the bottom of the lower Armco-iron bar, is shown in Figure I.1.5 and the measured temperatures given in Table I.5.5 are shown plotted vs. x in Figure I.5.11. Note that the temperature gradient in the upper contact specimen appears very linear, indicating that the heat flow in the upper contact specimen must have been approximately one-dimensional in the section spanned by thermocouples 8-11. Also, note that the temperature profile is continuous at the silver-soldered interface between the upper contact specimen and the upper Armco-iron bar. The continuous temperature profile indicates, as mentioned previously,

Table I.5.5: Experimental Data for Resistance Measurement
at 300N in Vacuum

	Thermocouple #	Thermocouple Position, x (cm)	Temperature (°C)
Upper Armco-Iron Bar	18	20.17	96.59
	17	18.90	95.20
	16	18.26	94.55
	15	17.63	93.75
	14	16.99	92.95
	13	16.36	92.29
	12	15.72	91.57
Upper Contact Specimen	11	14.24	89.30
	10	13.60	87.93
	9	12.97	86.55
	8	12.33	85.26
Lower Armco-Iron Bar	7	6.99	26.00
	6	6.35	25.41
	5	5.72	24.79
	4	5.08	24.18
	3	4.45	23.51
	2	3.81	22.91
	1	2.54	21.67

Figure I.5.11 Temperature Gradients in Upper and Lower Contact Assemblies

that the thermal resistance of such a junction is negligible. This conclusion is important because the value of T_{cc} was determined by extrapolating the temperature gradient (multiplied by k_i/k_{st}) in the lower Armco-iron bar through two such interfaces, specifically the interface between the lower Armco-iron bar and the lower contact specimen, and the interface between the steel cylinder and the steel mounting block, shown in Figure I.5.6a. If the thermal resistances of these interfaces had not been negligible, it would not have been possible to calculate the value of T_{cc} .

Least-squares fits (linear) to the temperature measurements in the upper and lower Armco-iron bars and the upper contact specimen give the following expressions for the temperatures in those assemblies.

Lower Armco-Iron Bar

$$T = 0.97905 x + 19.1821 \text{ (}^\circ\text{C)} \quad (\text{I.5.7})$$

Upper Contact Specimen

$$T = 2.12735 x + 59.0016 \text{ (}^\circ\text{C)} \quad (\text{I.5.8})$$

Upper Armco-Iron Bar

$$T = 1.14079 x + 73.6311 \text{ (}^\circ\text{C)} \quad (\text{I.5.9})$$

At $x = x_s = 7.84$ cm, equation (I.5.7) gives $T_s = 26.85^\circ\text{C}$. The temperature at the middle of the heat flow metering length in the lower Armco-iron bar is 23.8°C , so equation (I.5.1) gives $k_i = 73.15$ W/mK for the lower bar. The average temperature of the lower contact specimen

is 29.7°C (this temperature must be found by trial and error) so $k_{st} = 36.63 \text{ W/mK}$ from equation (I.5.3). Substituting these results and $\left.\frac{dT}{dx}\right|_i = 97.9 \text{ W/m}$ into equation (I.5.5), we obtain $T_{cc} = 32.56^{\circ}\text{C}$. Introducing the value of x corresponding to the position of the contact interface (10.76 cm) into equation (I.5.8) gives $T_{ch} = 81.88^{\circ}\text{C}$ so $\Delta T_c = 81.88 - 32.56 = 49.32^{\circ}\text{C}$. The cross-sectional area of the lower Armco-iron bar (A_i) is $4.34 \times 10^{-4} \text{ m}^2$ so $Q_c = Q_{ci} = 73.15 \times 4.34 \times 10^{-4} \times 97.9 = 3.104 \text{ W}$. From equation (I.1.1), we obtain $R_c = 49.32^{\circ}\text{C}/3.104 \text{ W} = 15.89^{\circ}\text{C/W}$.

For comparison purposes, the heat flow rate in the upper Armco-iron bar (Q_{hi}) is easily calculated as 3.367 W for this test. Thus, Q_{hi} is 0.263 W or 7.8% (of Q_{hi}) greater than Q_{ci} in this instance. Also, if the position of the upper Armco-iron/contact specimen interface ($x = 14.87 \text{ cm}$) is substituted into equations (I.5.8) and (I.5.9), the resulting temperatures are found to differ by only 0.04°C (i.e., the difference is negligible so the interface's resistance is also negligible).

The results of all of the experiments are given in Chapter I.6, where they are compared to the theoretical results. A rigorous error analysis and sensitivity study, which considers how uncertainties in all of the experimental parameters propagate into both the experimental and theoretical results, is given in Appendix I.E. Uncertainties in parameters used only in the analytical model (e.g., k_{og} , R_1 , R_2 , etc.) are included in the analysis along with the uncertainties in $\left.\frac{dT}{dx}\right|_i$, T_{cc} , etc. The error analysis applies only to the smooth surface results.

CHAPTER I.6

RESULTS AND DISCUSSION

I.6.1 Experimental Results

Experimental results for the smooth surface tests are presented in Table I.6.1. The resistances of the contact in vacuum and in air are designated R_{cv} and R_{ca} respectively in that table. As mentioned in Chapter I.5, the resistance of the contact with smooth contact surfaces was measured for two different loading cycles in vacuum (i.e., trials 2 and 3 in Table I.6.1) and three loading cycles in air (i.e., trials 1, 2, and 3). The mean interface temperature (i.e., $(T_{cc} + T_{ch})/2$), designated $\bar{T}$, is also given in Table I.6.1 for each test. The mean interface temperature did not vary significantly from test to test for a particular contact load and test condition, as seen in Table I.6.1, so the results of the tests at a given contact load were combined to produce an average contact resistance in vacuum ($\bar{R}_{cv}$) and an average resistance in air ($\bar{R}_{ca}$) for that load. The average resistances are tabulated in Table I.6.1 and plotted vs. contact load in Figure I.6.1.

Figure I.6.1 shows that the thermal resistance of the contact (with smooth surfaces) in air was significantly less than it was in vacuum, ranging from about 50% of the value in vacuum at the highest contact load to about 25% of the vacuum value at the lowest contact load. This result, which is in agreement with the work reported by Kitscha

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Table I.6.1: Resistance of Contact With Smooth Surfaces

Contact Load	Results of Tests in Vacuum				Average Resistance in Vacuum	Results of Tests in Air						Average Resistance in Air		
	Trial 2		Trial 3			Trial 1		Trial 2		Trial 3				
	R_{cv} °C/W	T °C	R_{cv} °C/W	T °C		R_{ca} °C/W	T °C	R_{ca} °C/W	T °C	R_{ca} °C/W	T °C			
P														
N														
26.0	41.71	55	42.28	57.5	41.99	10.24	54.5	10.40	53	10.32	58.5	10.32	58.5	10.32
45.7	35.76	55	34.55	60	35.16	10.06	52.5	9.74	55.5	9.84	62	9.84	62	9.88
70.5	28.80	55	29.28	60	29.04	9.77	52.5	9.44	55.5	9.53	62	9.53	62	9.58
98.9	26.27	57	25.48	59.5	25.88	9.41	52	9.08	55.5	9.25	61.5	9.25	61.5	9.25
132.1	23.59	56.5	22.57	59.5	23.08	8.88	52	8.74	55.5	8.91	61.5	8.91	61.5	8.85
175.8	20.95	56.5	20.70	61	20.83	8.64	52	8.39	56	8.64	61	8.64	61	8.56
215.9	18.68	58	18.80	61	18.74	8.36	52	8.26	56	8.43	61	8.43	61	8.35
299.8	15.89	57	17.04	63.5	16.47	8.01	55.5	7.73	54.5	7.85	61	7.85	61	7.86
382.1	14.41	57	15.13	62.5	14.77	7.72	57.5	7.63	55.5	7.57	61	7.57	61	7.64
453.9	13.71	56.5	14.33	63.5	14.02	7.44	57.5	7.15	57.5	7.41	61	7.41	61	7.33

Figure I.6.1 Resistance of Contact With Smooth Surfaces at Different Contact Loads

Figure I.6.2 Logarithmic Plot of Contact Resistance (R_c) vs. Contact Load (P)

and Yovanovich [16] and by Ogniewicz and Yovanovich [92] for sphere-flat and sphere-sphere contacts, demonstrates the importance of gas conduction outside the actual contact region on the overall contact resistance. This conclusion is emphasized by the relative insensitivity of the contact resistance in air to contact load, as seen in Figure I.6.1 and also in Figure I.6.2, where the results are plotted logarithmically. The slope of the $\log R_{cv}$ vs. $\log P$ trace shown in Figure I.6.2, and hence also the power of P to which R_{cv} is proportional, is approximately -0.39 ,* while the corresponding power for R_{ca} is -0.10 , so the large fraction of heat transfer occurring by gas conduction when a gas is present significantly reduces the dependence of R_c on P . This reduction in dependence occurs, presumably, because the size, shape, and hence thermal resistance, of the gas layer outside the contact region are all relatively insensitive to contact load.

The amount by which the dependence of the contact resistance on load, and also the contact resistance itself, is reduced in the presence of a gas will depend in general on the solid/gas conductivity ratio, with gas conduction becoming increasingly important as this ratio decreases. The solid/gas conductivity ratio for the contact in question was approximately 1300. Since the corresponding value of this ratio in most nuclear applications will be much lower than this, due to the relatively low thermal conductivities of most reactor structural materials

* Note that this value is not too different from the exponent of $-1/3$ that applies to an isothermal elliptical contact on an insulated elastic half-space. The result that $R_c \propto P^{-1/3}$ for such a contact follows from the fact that $R_{ct} = \psi^T / 4k_s a$ (see the resistance equation for contact 2 in Table I.2.1) and $a \propto P^{1/3}$ (see Equation (I.3.8b)). The factor ψ^T in the expression for R_{ct} is a function only of a/b and hence is independent of load according to Hertz's theory.

(e.g., Zircaloy, which typically has a conductivity about one-third that of 0-1 steel) and the relatively high temperatures, and hence conductivities, of participating gases (e.g., N_2 or CO_2 in the case of the gas gap between the pressure and calandria tubes in a CANDU reactor), gas conduction will be even more significant in those situations than it was in the experiments performed in this study. Radiation will also be important in those situations, of course.

The results of tests performed with a rough surface on the large radius contact specimen are shown in Table I.6.2 and the smooth and rough surface test results are compared in Figure I.6.3. As seen in that figure, roughening one of the contact surfaces produced virtually no change in the resistance of the contact in air, except at low loads where the measured resistance increased slightly. In contrast, the rough contact surface produced a definite reduction ($\approx 5-20\%$) in the thermal resistance of the contact in vacuum, with the reduction decreasing with increasing contact load and pressure. The resistance reduction in vacuum was undoubtedly caused by the crushing of surface asperities as well as, to some extent, the penetration of the smooth surface by asperities on the rough surface, since both of these interactions would have increased the effective area of the contact patch and thereby reduced the macroscopic constriction resistance. It will be seen later that results obtained with the analytical model described in Section I:3.5 support this explanation.

Table I.6.2: Resistance of Contact With Large Radius Surface Roughened

Contact Load	Tests in Vacuum		Tests in Air	
	Loading Cycle	Unloading Cycle	Loading Cycle	
P N	R_{CV} °C/W	R_{CV} °C/W	R_{ca} °C/W	T °C
26.0	36.80	34.16	11.01	60.5
45.7	28.04	27.87	10.66	60.5
70.5	24.74	24.90	9.68	63.5
98.9	21.40	22.17	9.34	63
132.1	19.64	20.23	8.87	63
175.8	17.68	18.08	8.57	62.5
215.9	16.74	16.82	8.36	63
299.8	14.99	15.26	7.98	62.5
382.1	13.87	14.13	7.60	63
453.9	13.43	13.43	7.49	63.5

Figure I.6.3 Comparison of Smooth and Rough Surface Experimental Results

While the presence of roughness on one or both of the surfaces of a contact always increases the size of the contour area above the corresponding smooth surface size and hence decreases the macroscopic constriction resistance, it may also introduce a non-negligible microscopic constriction resistance component into the total contact resistance. Thus, it cannot always be stated with certainty that the overall resistance of a contact, which, in vacuum can be considered (at least qualitatively) to be the sum of the microscopic and macroscopic constriction resistances, will decrease when the contact surfaces are roughened. Burde [58] measured the thermal resistance in vacuum of a contact formed between a smooth steel sphere (spheres with surface radii ranging from 14.3 to 38.1 mm were used) and a rough steel flat, with roughnesses on the flat ranging from 0.625 microns to 2.588 microns (rms), and found that at the higher roughnesses the contact resistance was greater in some cases and less in others than the resistance obtained analytically using the Hertzian contact size (i.e., the "smooth surface" resistance). At high loads, the analytical resistances calculated using the Hertzian contact size were in good agreement with the measured values, even for the roughest surfaces, whereas at low loads those resistances differed by as much as 35% (of the measured resistance value). The large discrepancy referred to (i.e., 35%) was obtained with the flat roughened to 2.44 microns and, as in the present study, the experimental resistance was lower than the analytical result obtained using the Hertzian contact size. The flat was roughened for that test by blasting it with aluminium oxide grit which, because of

the sharp edges of the oxide particles, produced asperities with steep flanks (i.e., high asperity surface slopes). The average value of the absolute slope of the asperities ($\bar{m}$) was estimated to be 0.217 radians from profilometer measurements of the surface profile.

In another test performed with the flat roughened to a similar nominal roughness value (2.562 microns rms) by bead blasting with spherical glass beads, the measured contact resistance was higher at low loads than the analytical result obtained using the Hertzian contact size. The spherical glass beads produced asperities with relatively small surface slopes (average absolute asperity slope = 0.08 radians), causing Burde to speculate that the different average absolute asperity slopes of the aluminum oxide grit and the glass bead-blasted surfaces had caused the observed differences in contact resistance. Basically, it was argued that the microcontacts produced at a given contact load by the peaky asperities on the aluminum oxide blasted surface would have been much more numerous but only slightly smaller in diameter (on average) than the microcontacts produced at the same load by the less steeply-sloped asperities on the glass bead blasted surface. Microcontact number generally has a greater effect on contact resistance than does microcontact size [58], so the surface which produced the greatest number of microcontacts (i.e., the aluminum oxide blasted surface) should have produced the lowest contact resistance, as actually observed in the experiments. Burde's analytical model, which was discussed briefly in Sections I.2.2.1 and I.3.5.2, more or

less corroborated this explanation. As one might expect from the direct proportionality between h_{cv} and $\bar{m}_e$ seen in Equation (I.3.42), contact resistances obtained with the rough surface model described in Section I.3.5 display the same dependence on $\bar{m}_e$, as will be seen later.

As seen in Table I.6.2, the thermal resistances measured in the tests with the roughened contact surface in vacuum were always slightly greater in the unloading cycle than they were in the loading cycle ($\leq 3\%$ difference), except at very low contact loads. If the surface asperities on the rough surface had been flattened themselves or had at least indented the smooth surface during the loading cycle, as suggested by visual examination of both contact surfaces after the tests, the contact patch at a given load would have been smaller in the unloading cycle than in the loading cycle. Thus, the macroscopic resistance of the contact at a given load would have increased while, because of the flattening of the asperities, the microscopic resistance would have decreased. It would appear from the results obtained with the roughened contact surface in vacuum that the crushing of asperities during the loading cycle resulted in a slightly greater increase in macroscopic resistance than decrease in microscopic resistance during the unloading cycle.

The loading and unloading resistance data would not have exhibited any hysteresis (as was almost the case in the experiments) if the asperities had deformed elastically instead of plastically as assumed in Section I.3.5. The asperity deformation and smooth surface

indentation that was observed when the contact surfaces were examined after the tests supported the assumption that the asperity deformation and/or smooth surface indentation had been plastic instead of elastic, however. This point will be discussed further in Section I.6.5.

I.6.2 Smooth Surface Theoretical Results

As discussed in Section I.4.5, the substitute contact specimen in the ellipsoidal finite-difference model was obtained by modelling only the appropriate regions of a hemisphere (really a semi-ellipsoid) of radius L , specifically only those parts of the hemisphere lying within the boundaries of a rectangular bar of cross-sectional dimensions $w \times t$, as shown in Figure I.4.8. Nodes on the hemispherical surface segment at the top of the bar and nodes within the elliptical contact on the bottom of the bar were assigned temperatures of T_1 and T_0 respectively ($T_1 > T_0$), while nodes on the bottom of the bar but outside of the contact were linked to the sink temperature (T_0) by appropriate gas conductances for thermal analyses of the contact in a gas or otherwise were considered to be insulated. The boundary value problem just described was solved for the total heat flow rate, Q_c , between the isothermal surfaces at temperatures T_1 and T_0 , following which the resistance of the substitute contact specimen (R_{sc}) was calculated from the relationship

$$R_{sc} = (T_1 - T_0)/Q_c$$

To obtain the resistance of the contact based on the extrapolated temperature drop at the interface (R'_c), the resistance of the bar contained within the semi-ellipsoid was computed with all of the nodes on the bottom of the bar set to T_0 and the result, designated R_b , taken to represent the second term seen within the square brackets on the right-hand side of Equation (I.3.23) (i.e., $L/k_s tw$). R_{sc} represents the first term seen within the brackets in Equation (I.3.23), of course, so Equation (I.3.23) can be rewritten as

$$R'_c = 2(R_{sc} - R_b)$$

The resistance of the bar with isothermal top and bottom surfaces (i.e., R_b) was always computed with the grid that was used to calculate R_{sc} since it was reasoned that errors produced by the approximate representation of the bar in the ellipsoidal finite-difference model would affect R_b and R_{sc} more or less equally. An infinite number of nodes would have been required to model the flat sides of the bar exactly in the ellipsoidal coordinate system, but errors in R_b and R_{sc} resulting from the approximate representation of the bar (due to the use of a finite number of nodes) would have been approximately equal if most of the imperfections in the representation of the bar had occurred near the upper isothermal surface (i.e., remote from the contact). This condition was satisfied for the bar in the ellipsoidal finite-difference model because the natural grading of the coordinate surfaces caused the density of mesh points in the bar, and hence the fidelity with which the bar was modelled, to decrease with increasing distance from the contact.

The cross-sectional dimensions, w and t , of the contact specimens used in the experiments were approximately 2.54 cm and 1.72 cm respectively so the radius of the hemisphere containing the substitute specimen (L) was assigned a value of 2.54 cm in all of the contact resistance calculations to obtain an upper isothermal surface for the substitute specimen which would satisfy the criteria mentioned in Section I.3.3 (i.e., be approximately flat and in a region of the bar essentially undisturbed by the presence of the contact). A larger hemisphere could have been used but the resulting finite-difference model would have contained more nodes and hence have been more expensive to use.*

It was expected that the ellipsoidal finite-difference model would produce accurate results with relatively few nodes because heat flow would occur primarily in the x direction, particularly in thermal analyses of the contact in vacuum, and this, in fact, proved to be the case. Table I.6.3 shows the results of a study performed at an intermediate contact load (175.8N) to determine the number of subdivisions needed on the $\bar{x}$, $\bar{y}$ and $\bar{z}$ axes to obtain results which would not change significantly when the mesh was further refined. Miscellaneous material properties used in the study, all of which correspond to a mean interface temperature of 57°C since this value is roughly the average of the interface temperatures given in Table I.6.1 for 175.8N, are listed in Table I.6.4. Several significant observations can be made about the results shown in Table I.6.3. First, and perhaps most importantly,

* Results showing that insignificant error would have resulted from using a substitute contact specimen with $L=2.54$ cm. instead of one of greater length will be given in Part II.

Table I.6.3 Results Of Mesh Size Sensitivity Analysis At A Contact Load Of 176 N.

NX	NY	NZ	N1	N2	Total Number of Nodes In Model of Bar	R_b °C/W	$2R_b$ °C/W	Vacuum			Air			Q_p/Q_c
								R_{scv} °C/W	$2R_{scv}$ °C/W	R'_{cv} °C/W	R_{sca} °C/W	$2R_{sca}$ °C/W	R'_{ca} °C/W	
34	9	9	21	13	2567	1.690	3.380	12.510	25.02	21.64	6.221	12.443	9.063	0.329
34	7	7	21	13	1649	2.300	4.600	13.125	26.25	21.65	6.838	13.676	9.076	0.329
34	4	4	21	13	646	2.715	5.430	13.515	27.03	21.60	7.287	14.575	9.145	0.335
24	11	11	11	13	2260	1.680	3.360	12.605	25.21	21.85	6.246	12.492	9.132	0.327
24	7	7	11	13	1009	2.265	4.530	13.175	26.35	21.82	6.830	13.661	9.131	0.328
24	4	4	11	13	396	2.695	5.390	13.610	27.22	21.83	7.305	14.611	9.221	0.334

Variable Definitions

User-Specified
Variables

NX
NY
NZ
N1
N2
 R_b

- Number of subdivisions on $\bar{x}$ axis within a hemisphere of 2.54 cm. radius.
- Number of equi-sized subdivisions on $\bar{y}$ axis.
- Number of equi-sized subdivisions on $\bar{z}$ axis.
- Number of equi-sized subdivisions on x axis within a hemisphere of 1 cm. radius.
- Number of subdivisions on x axis between hemispheres with radii of 1 cm. and 2.54 cm. (i.e., $N2=NX-N1$).
- Resistance of substitute specimen with isothermal top and bottom surfaces.

R_{scv} , R_{sca}

- R_{scv} , R_{sca} - R_{sc} for vacuum and air calculations respectively. R_{sc} is the total resistance of the substitute contact specimen and hence is not based on the extrapolated temperature drop.

R'_{cv} , R'_{ca}

- R'_{cv} , R'_{ca} - R'_c for vacuum and air calculations respectively. R'_c is the resistance of the model contact based on the extrapolated temperature drop.

Q_p/Q_c

- Ratio of heat flow through contact patch (Q_p) to total heat flow (Q_c).

Note: $R'_{cv} = 2R_{scv} - 2R_b$.

$R'_{ca} = 2R_{sca} - 2R_b$.

Table I.5.4: Material Properties Used in Analyses
Performed at a Contact Load of 175.8N
(57°C Mean Interface Temperature)

PROPERTY	VALUE	SOURCE
Steel Thermal Conductivity	37.03 W/m°C	Equation (I.5.3)
Air Continuum Thermal Conductivity	0.0284 W/m°C	Reference [93]
Air/Steel Accommodation Coefficient	0.90	Reference [75]
Mean Free Path of Air Molecules	7.6×10^{-6} cm	Reference [75]
Young's Modulus of Steel	2.036×10^5 MPa	
Poisson's Ratio	0.30

Table I.6.3 shows contact resistances predicted with grids containing a few and as many as 396 and 2567 nodes respectively and yet, despite this more than sixfold variation in the number of nodes used, the predicted resistances vary by less than 1.2% in vacuum and 1.8% in air. This comparison demonstrates that the ellipsoidal finite-difference model can yield accurate solutions to the model contact problem with surprisingly few nodes. Secondly, the value of R_b can be seen to depend strongly on the number of nodes used in the model since this affects the fidelity with which the boundaries of the substitute specimen are modelled, but R_{scv} (i.e., R_{sc} for the contact in vacuum) and R_{sca} (i.e., R_{sc} for the contact in air) vary by almost exactly the same amount as R_b when the grid is refined. As a result, the resistance of the contact based on the extrapolated temperature drop (i.e., R'_{cv} in vacuum and R'_{ca} in air) remains essentially constant, particularly at constant NX. Comparing the results at constant NX (i.e., at either NX = 24 or NX = 34) but different values of NY and NZ, it can be seen that the variations in NY and NZ produce moderately large variations in R_b , R_{scv} and R_{sca} (or $2R_b$, $2R_{scv}$ and $2R_{sca}$) but have little effect on R'_{ca} and essentially no effect on R'_{cv} . These results indicate that errors produced by the approximate representation of the flat sides of the substitute contact specimen in the ellipsoidal finite-difference model affect R_{sc} and R_b more or less equally, as postulated previously.

Small changes in R'_{cv} and R'_{ca} occurred when NX was increased from 24 to 34, as seen in Table I.6.3, but these changes could not have been

caused by improvements in the representation of the bar boundaries because the increase in NX from 24 to 34 was not obtained by refining the mesh in regions near the flat sides of the bar. The increase in NX was obtained by refining the mesh within a hemisphere (centered at the contact) of 1 cm radius (i.e., by increasing N1 as shown in Table I.6.3), a change which would have had essentially no effect on the density of mesh points near the flat sides of the substitute specimen because the 1 cm radius hemisphere would have barely reached the sides of the specimen. Remember that the bar measured 2.54 cm x 1.72 cm in cross section so the increase in N1 (and hence NX) would have significantly affected the mesh point density on the bottom, but not on the sides, of the bar. Note that the values of R_b obtained at the same values of NY and NZ, but different values of N1, were essentially identical (e.g., $R_b = 2.695^\circ\text{C/W}$ with NX=24, N1=11, NY=4 and NZ=4 while $R_b = 2.715^\circ\text{C/W}$ with NX=34, N1=21, NY=4 and NZ=4). This result suggests that the contact resistances (R'_{ca} and R'_{cv}) obtained with N1=21 must be slightly more accurate than the resistances obtained with N1=11 because the two sets of resistances are slightly different and the difference could not have been caused by a change in the fidelity with which the flat sides of the substitute specimen were modelled, because that parameter didn't vary.

As a point of interest, the lowest value of R_b listed in Table I.6.3, 1.68°C/W , is still 13.9% greater than the author's best estimate of the true value of R_b , 1.48°C/W , obtained by assuming that conduction in the substitute contact specimen with isothermal ends (the upper

isothermal end is a segment of a hemisphere) occurs through "channels" parallel to the longitudinal axis of the specimen.

The grid with $NX=34$, $NY=7$ and $NZ=7$ provided results which were essentially the same as those obtained with $NX=34$, $NY=9$ and $NZ=9$, as seen in Table I.6.3, only with less computational effort. For this reason the $34 \times 7 \times 7$ grid should probably be considered the best of those tried. The same conclusion was reached in sensitivity analyses performed with the same grids at the lowest and highest contact loads of 26N and 454N respectively. Contact resistances were only predicted at contact loads of 26, 176 and 454N because it was felt that additional comparisons would not provide significantly more information. The final resistance predictions provided by the ellipsoidal finite-difference model are given in the next section, where they are compared to the experimental results.

I.6.3 Comparison of Smooth Surface Experimental and Theoretical Results

The average resistances obtained in the smooth surface tests of the contact at loads of 26N, 176N and 454N are compared to the ellipsoidal finite-difference model predictions in Table I.6.5 for the contact in air and Table I.6.6 for the contact in vacuum. The experimental and theoretical results are also compared graphically in Figure I.6.4. In addition to the predicted contact resistances, Table I.6.5 also

Table I.6.5: Comparison of Predicted and Measured Contact Resistances in Air (Smooth Surfaces)

Contact Load N	R_{ca} °C/W	Predicted Results		
		$R_{ca}, °C/W$	Q_p/Q_c	$(R_{ca} - R_{ca})/R_{ca}, \%$
26.0	10.32	10.79	0.18	4.5
175.8	8.56	9.08	0.33	6.0
453.9	7.33	7.85	0.42	7.1

Table I.6.6: Comparison of Predicted and Measured Contact Resistances in Vacuum (Smooth Surfaces)

Contact Load N	R_{cv} °C/W	Predicted Results		
		$R_{cv}, °C/W$	$(R_{cv} - R_{cv})/R_{cv}, \%$	
26.0	41.99	42.56	1.3	
175.8	20.83	21.65	3.9	
453.9	14.02	15.31	9.2

Figure I.6.4 Comparison of Experimental and Theoretical Results for Smooth Surface Tests.

shows, for each contact load, the ratio of the predicted heat flow rate through the contact patch itself (Q_p) to the total predicted heat flow rate between the contact specimens (Q_c).

Temperature dependent material properties used in the analysis were evaluated at the mean interface temperatures given in Table I.6.1 for each contact load and test condition. Material properties were taken from the sources cited in Table I.6.4.

As seen in Tables I.6.5 and I.6.6 and Figure I.6.4, the analytical model provides results for the contact in air calculations which are in good agreement with the experimental results, particularly at low contact loads. The results of the contact in vacuum calculations are not quite so good, however, in that the predicted resistance at a contact load of 454N differs from the measured value by 9.2%. It should be noted, however, that the predicted result lies within the 95% confidence interval determined in Appendix I.E by considering the uncertainties in both the predicted and measured contact resistances. Much better agreement between the predicted and measured resistances in vacuum is achieved at lower contact loads, with differences of only 1.3% and 3.9% appearing at contact loads of 26N and 176N respectively. It is not known why a relatively large discrepancy between the experimental and theoretical results was obtained at the highest contact load (454N) in vacuum, (although the discrepancy lies within the error band as mentioned previously) but it would appear from the results that the source of the

error affected both the vacuum-related and air-related resistance comparisons in that the largest difference between theory and experiment appears at the highest contact load in each case. The error, if not experimental in nature, could possibly have been caused by violation of the "small contact" assumption in Hertz's theory. In other words, it is possible that the theory underpredicted the contact dimensions at high contact loads because the contact major diameter, which was 16.5% of the width (w) of either contact specimen at the highest contact load, was therefore not "small" relative to the cross-sectional dimensions of the specimens. Although it might have been possible to obtain a better estimate of the contact dimensions numerically using, for instance, a computer program written by Paul and Hashemi [94, 95] to predict contact dimensions and stresses (elastic) for railway wheel/railhead contacts (which badly violate Hertz's "small contact" assumption), such an analysis was not undertaken because the agreement obtained between theory and experiment in this study was considered to be adequate for most engineering purposes.

One final point worthy of mention is that the resistance of the contact in air at an intermediate contact load of 176N was recalculated with the gap between the contact surfaces obtained from Equation (I.3.19a) (i.e., without accounting for elastic deformation of the contact surfaces) instead of from Equation (I.3.19b). The calculation was performed with the ellipsoidal finite-difference model and the $34 \times 7 \times 7$ grid referred to in Table I.6.3. The value of R'_{ca} increased by 7.1% from 9.076°C/W

to 9.720°C/W , in that analysis while Q_p/Q_c increased by 15.6%, from 0.32 to 0.37. These results demonstrate that elastic surface deformations should be included in the gap size calculations for best results (as actually done).

I.6.4 Comparison of Measured Resistances With Constriction Formulas (Vacuum) and Quasi-Coupled Model (Gas)

In addition to the comparisons of theoretical and experimental results discussed previously, the smooth surface contact resistances measured in vacuum were compared to the predictions of two simple constriction formulas described in the next section. Also, the measured contact resistances in air were compared to results obtained with a "quasi-coupled" model of conduction in the gas and solid. A description of the quasi-coupled model follows the constriction formula comparisons.

I.6.4.1 Constriction Formula Comparisons

The first constriction formula for predicting the resistance of the contact in vacuum expresses an estimate of the contact resistance, designated $R_{c\infty}$ in this case, as

$$R_{c\infty} = 2R_{ct}^T \quad (\text{I.6.1})$$

where R_{ct}^T is the resistance of an isothermal elliptical contact on an otherwise insulated infinite half-space, as defined by the resistance

equation for contact 2 in Table I.2.1. The factor of 2 in Equation (I.6.1) appears because there are two contacting bodies and hence two constrictions in the model contact, of course.

The measured contact resistances in vacuum are compared to $R_{c\infty}$ in Table I.6.7 and Figure I.6.5 for all of the contact loads. The results obtained with the ellipsoidal finite-difference model are also shown in Figure I.6.5. It can be seen that $R_{c\infty}$ is greater than the measured resistance ($\bar{R}_{cv}$) at all loads, with the overprediction ranging from less than 5% at low loads to over 20% (of the measured resistance) at the highest loads. The deterioration in predictive accuracy with increasing contact load occurs because $R_{c\infty}$ is not based on an extrapolated temperature drop and hence includes the thermal resistance of all of the material contained within the infinite semi-ellipsoid to which the resistance equation for contact 2 in Table I.2.1 applies. While this inconsistency between the analytical and experimental resistance definitions introduces little error at low contact loads, where the contact is small and most of the resistance to heat flow is localized to a region very near the contact, it causes $R_{c\infty}$ to be significantly in error at high loads where the resistance of the contact is relatively low and less localized in nature. It should be remembered, however, that the measured resistances in vacuum were 1.3 to 9.2% lower than the benchmark predictions obtained using the ellipsoidal finite-difference model so the values of $R_{c\infty}$ compare much more favourably with the benchmark values, as seen in Figure I.6.5. At 454N, for instance, $R_{c\infty}$ and R_{cv}

Table I.6.7: Comparison of Experimental Results (Smooth Surface Tests) With Constriction Formulas (Vacuum) and Quasi-Coupled Model (Gas)

Contact Load- N	Results in Vacuum					Results in Air		
	$\bar{R}_{cv}$ °C/W	$R_{C^{\infty}}$ °C/W	$\frac{(R_{C^{\infty}} - \bar{R}_{cv})}{\bar{R}_{cv}}$, %	R_{ca} °C/W	$\frac{(R_{ca} - \bar{R}_{cv})}{\bar{R}_{cv}}$, %	R_{ca} °C/W	R_{qc} °C/W	$\frac{(R_{qc} - \bar{R}_{ca})}{\bar{R}_{ca}}$, %
26.0	41.99	43.98	4.7	43.22	2.9	10.32	9.17	-11.1
45.7	35.16	36.41	3.6	35.65	1.4	9.88		
70.5	29.04	31.52	8.6	30.52	5.1	9.58		
98.9	25.88	28.14	8.7	27.21	5.1	9.25		
132.1	23.08	25.56	10.7	24.64	6.8	8.85		
175.8	20.83	23.25	11.6	22.22	6.7	8.56	8.16	-4.7
215.9	18.74	21.71	15.8	20.80	11.0	8.35		
299.8	16.50	19.46	17.9	18.55	12.4	7.86		
382.1	14.77	17.94	21.5	17.04	15.4	7.64		
453.9	14.02	16.95	20.9	16.05	14.5	7.33	7.38	+0.7

Figure I.6.5 Comparison of Experimental Results With Constriction Formulas (Vacuum) and Quasi-Coupled Model (Gas) and Comparison of Latter Results With Those of Ellipsoidal Finite-Difference Model

differ by 20.9% while $R_{c\infty}$ and R'_{cv} differ by only 10.7%.

While it is difficult to define an "extrapolated" temperature drop for use with contact 2 in Table I.2.1 because the heat flow in the infinite semi-ellipsoid is radial (i.e., not normal to the contact plane) in regions remote from the contact, somewhat better results than those obtained with $R_{c\infty}$ can be obtained by reducing the "radius" of the isothermal semi-ellipsoidal surface (used in the derivation of R_{ct}^T for contact 2 in Table I.2.1) from infinity to some finite value. One-half of the smallest cross-sectional dimension of the contact specimens (i.e., $t/2$) was chosen, somewhat arbitrarily, as the "radius" (R) of the finite semi-ellipsoid since a semi-ellipsoid of larger radius than this would interfere with (i.e., intersect) the sides of the specimens and hence not be a complete semi-ellipsoid. Setting $\lambda = (t/2)^2$, where $t/2 = 0.86$ cm, and designating the resistance of the contact on the equatorial plane of the semi-ellipsoid of "radius" $\sqrt{\lambda}$ by R_λ , it can easily be shown that,

$$R_\lambda = \frac{1}{2\pi k_s a} F(\phi, k) \quad (I.6.2a)$$

$$= \frac{1}{2\pi k_s a} \left[\ln \left(\frac{4a}{b} \right) - \ln \left(\frac{a + \sqrt{a^2 + \lambda}}{\sqrt{\lambda}} \right) \right] \text{ when } k = 1 \quad (I.6.2b)$$

R_λ must be doubled to obtain the total resistance of the actual contact, designated $R_{c\lambda}$ (i.e., $R_{c\lambda} = 2R_\lambda$). $R_{c\lambda}$ is compared to the measured resistances of the contact in vacuum in Table I.6.7 and Figure I.6.5, where it can be seen that the difference between $R_{c\lambda}$ and R_{cv} ranges from approximately 2% at low loads to 15% at high loads. Values of $R_{c\lambda}$

compare more favourably with the benchmark resistance predictions (R'_{cv}) of course, in that $R_{c\lambda}$ exceeds R'_{cv} by less than 5% at all of the loads for which R'_{cv} is available (i.e., 26, 176 and 454N). The maximum difference of 4.8% is found at the highest load, 454N, where the resistance of the contact is lowest and where, in consequence, the error introduced by not using an extrapolated temperature difference to obtain the theoretical result will be largest.

I.6.4.2 Quasi-Coupled Model Comparisons

The measured contact resistances in air were compared to results obtained with a quasi-coupled model of conduction in the gas and solid, as mentioned previously. The quasi-coupled model, proposed originally by Kitscha and Yovanovich [57], assumes that the temperatures on the contiguous surfaces near the contact with gas conduction are negligibly different from those prevailing without gas conduction for the purpose of evaluating the heat flow rate through the gas (Q_g). The latter quantity is calculated by integrating the product of the local temperature difference between the surfaces, $\Delta T(\lambda)$, and the local gas conductance, h_g , over the area of the contact surface (A_{cs}), viz,

$$Q_g = \int_{A_{cs}} h_g \Delta T dA_{cs} \quad (I.6.3)$$

The thermal resistance of the solid is assumed to be R_{cs} and the corresponding temperature difference between the contact surfaces is calculated from the relation,

$$\frac{\Delta T(\lambda)}{\Delta T_{\infty}} = \frac{F(\phi, k)}{K(k)} \quad (I.6.4)$$

where ΔT_{∞} is twice the absolute value of the difference between the contact temperature (i.e., $T(\lambda)$ at $\lambda = 0$) and the temperature at $\lambda = \infty$. The argument ϕ in the incomplete elliptic integral of the first kind ($F(\phi, k)$) is defined by equation (I.A.11). Setting $\Delta T_{\infty} = 1$ for convenience, the resistance of the gas phase (R_g) can be defined as:

$$\begin{aligned} R_g &= 1/Q_g \\ &= K(k) / \int_{A_{cs}} h_g F(\phi, k) dA_{cs} \end{aligned} \quad (I.6.5)$$

so the resulting quasi-coupled contact resistance (R_{qc}) becomes,

$$R_{qc} = R_g R_{c\infty} / (R_g + R_{c\infty}) \quad (I.6.6)$$

when the gas and solid phase resistances are combined in parallel.

R_{qc} is calculated by the ellipsoidal finite-difference computer program described in Appendix I.C when R'_{ca} is computed. Since R'_{ca} was only evaluated at contact loads of 26, 176 and 454N, R_{qc} is only available for those loads. Values of R_{qc} are compared to measured resistances in Table I.6.7 and Figure I.6.5, where it can be seen that the quasi-coupled model underpredicts the resistance of the contact in air by approximately 11% at the lowest contact load but provides a very accurate prediction of the contact resistance at the highest load. It will be remembered, however, that the measured contact resistances in air were 4.5% and 7% less than the benchmark predictions obtained using

the ellipsoidal finite-difference model (R'_{ca}) at the lowest and highest contact loads respectively (see Table I.6.5), so the quasi-coupled resistances do not compare so favourably with the latter values, as seen in Figure I.6.5. R_{qc} and R'_{ca} differ by 15% and 6% at the lowest and highest contact loads respectively. The improved agreement at the high load is not surprising because the quasi-coupled model will be most accurate when the ratio of the heat flowrate through the solid to the heat flowrate through the gas (i.e., Q_p/Q_g) is largest, i.e., at the highest contact load.

I.6.5 Comparison of Rough Surface Experimental and Theoretical Results

Theoretical results obtained with the rough surface contact model described in Section I.3.5 are compared to the loading cycle experimental results in Figures I.6.6 - I.6.7 and Tables I.6.8 - I.6.9. The theoretical results shown in Figure I.6.6 were obtained with $\bar{m}_e = 0.06$ radians, which is assumed to be that parameter's most probable value for the reasons discussed in Section I.5.3., while the results shown in Figure I.6.7 were obtained with $\bar{m}_e = 0.03$ and $\bar{m}_e = 0.12$ radians. Theoretical results were only obtained for contact loads of 26N, 175.8N and 453.9N, as before. In addition to the contact resistance obtained for each value of $\bar{m}_e$ and contact load, Tables I.6.8 and I.6.9 show the ratio of the heat flow rate through the Hertzian contact area (Q_H) to the total heat flow rate between the contact surfaces (Q_c) along with the dimensionless gap

Table I.6.8: Comparison of Experimental and Theoretical Results for the Contact With Rough Surfaces in Vacuum

Load P (N)	Measured Resistance R_{CV} ($^{\circ}\text{C/W}$)	Assumed Asperity Slope m_e (radians)	Predicted Resistance R'_{CV} ($^{\circ}\text{C/W}$)	Error $\frac{R'_{CV} - R_{CV}}{R_{CV}}$ (%)	Q_H/Q_C	ϵ_H (ϵ in Hertzian Area)	a_r/a	δ^*/σ
26.0	36.80	0.03	56.00	52.0	0.25	0.035	2.24	1.465
175.8	17.68	0.03	20.20	14.2	0.36	0.108	1.64	0.769
453.9	13.43	0.03	13.50	0.4	0.40	0.174	1.44	0.364
26.0	36.80	0.06	41.20	12.0	0.24	0.036	2.12	1.621
175.8	17.68	0.06	17.59	-0.5	0.34	0.111	1.61	0.984
453.9	13.43	0.06	12.38	-7.8	0.37	0.181	1.42	0.632
26.0	36.80	0.12	32.31	-12.2	0.22	0.037	2.09	1.707
175.8	17.68	0.12	15.87	-10.2	0.30	0.113	1.57	1.099
453.9	13.43	0.12	11.61	-13.6	0.33	0.184	1.40	0.770

1. Loading Cycle.

Table I.6.9: Comparison of Experimental and Theoretical Results for the Contact With Rough Surfaces In Air

Load P (N)	Measured Resistance ¹ R_{ca} (°C/W)	Assumed Asperity Slope m_e (radians)	Predicted Resistance R'_{ca} (°C/W)	Error $\frac{R'_{ca} - R_{ca}}{R_{ca}}$ (%)	Q_H/Q_C	ϵ_H (ϵ in Hertzian Area)	a_r/a	δ^*/σ
26.0	11.01	0.03	12.24	11.2	0.05	0.035	2.24	1.465
175.8	8.57	0.03	9.30	8.5	0.15	0.108	1.64	0.769
453.9	7.49	0.03	7.72	3.1	0.21	0.174	1.44	0.364
26.0	11.01	0.06	11.75	6.7	0.06	0.036	2.12	1.621
175.8	8.57	0.06	8.92	4.1	0.16	0.111	1.61	0.984
453.9	7.49	0.06	7.48	-0.1	0.21	0.181	1.42	0.632
26.0	11.01	0.12	11.19	1.6	0.07	0.037	2.09	1.707
175.8	8.57	0.12	8.59	0.2	0.15	0.113	1.57	1.099
453.9	7.49	0.12	7.27	-2.9	0.20	0.184	1.40	0.770

¹ Loading Cycle.

Figure I.6.6 Comparison of Rough Surface Experimental and Theoretical Results ($\bar{m}_e = 0.06$ radians)

Figure I.6.7 Comparison of Rough Surface Experimental and Theoretical Results ($\bar{m}_e = 0.03$ and $\bar{m}_e = 0.12$ radians)

between the contact surfaces within the Hertzian contact area (δ^*/σ) and the corresponding contact area fraction (ϵ_H). Also shown is the ratio of the x-coordinate at which $Z/\sigma_e = 3$, designated a_r , to the semi-major diameter of the Hertzian contact area, a . The ratio a_r/a is a measure of the extent to which the contact patch is lengthened by the presence of surface roughness.

The predicted contact resistances shown in Figures I.6.6 - I.6.7 and Tables I.6.8 - I.6.9 were obtained with the material properties listed in Table I.6.4, $H = 1670$ MPa, $\sigma_e = 1.06$ μm and the value of $\bar{m}_e$ noted. The $34 \times 7 \times 7$ grid referred to in Table I.6.3 was used in all of the analyses.

As seen in Figure I.6.6 and Tables I.6.8 - I.6.9, the predicted contact resistances obtained with $\bar{m}_e = 0.06$ differ from the measured resistances by no more than 12%. The agreement between the theoretical and experimental results is better for the contact in air than it is for the contact in vacuum, as one might expect from the reduced significance of conduction through the metal-to-metal contact points in air. The theoretical results for the contact in air are less sensitive to variations in $\bar{m}_e$ for the same reason, as seen in Figures I.6.6 - I.6.7 and Table I.6.9. In particular, while the error in the value of R'_{CV} obtained with $\bar{m}_e = 0.03$ at a contact load of 26N is quite large at 52%, as seen in Table I.6.8, the error in the value of R'_{ca} predicted with the same value of $\bar{m}_e$ at the same contact load is only 11.2%, as seen in Table I.6.9. These results show that an accurate

model of metal-to-metal conduction between non-conforming surfaces in contact may not be required for contacts occurring in the presence of a gas because of the significance of gas conduction through and around the contour area.

It is significant to note that while the measured rough surface resistances in vacuum were only 5-20% less than the smooth surface resistances, the theoretical results obtained with $\bar{m}_e = 0.06$ radians indicate that the observed rough surface resistances in vacuum could have been obtained with only 24-37% (approximately) of the total heat flow between the contact surfaces passing through the Hertzian contact area. Thus the fact that the resistances in vacuum changed only slightly when one of the contact surfaces was roughened does not mean that changes in the heat flow rate through the Hertzian contact area were also small. A large increase in the overall size of the contact and hence reduction in Q_H could have occurred without changing the contact resistance by a correspondingly large amount if the resulting reduction in R_{mac} had been largely balanced by an increase in R_{mic} .^{*} The theoretical results suggest that this is what happened in the experiments.

* The concept of the total contact resistance consisting of the sum of R_{mic} and R_{mac} is being used in a qualitative sense here. These resistances do not have to be defined and evaluated separately when the total contact resistance is calculated numerically, of course.

It is also significant to note that the values of ϵ_H and a_r/a tabulated in Tables I.6.8 and I.6.9 display very little sensitivity to $\bar{m}_e$. These parameters are insensitive to $\bar{m}_e$ because the real contact area (A_r) at a given contact load P is defined by the expression,

$$A_r = \frac{P}{H}$$

if the asperities deform plastically and the pressure over each deformed asperity is H (as assumed in Section I.3.5.2). Thus A_r is not related to $\bar{m}_e$ and $\bar{m}_e$ can only affect the manner in which the microcontacts (whose areas sum to equal A_r) are distributed. The value of $\bar{m}_e$ can affect that distribution because the elastic deformation of the substrate under each microcontact is related to it [3], but its overall influence is small because the elastic deformations of the substrate are small compared to the plastic deformations of the asperities. Since the values of ϵ_H at a contact load of 175.8N are 0.108 and 0.113 (i.e., almost identical) for $\bar{m}_e = 0.03$ and $\bar{m}_e = 0.12$ radians respectively, for instance, it follows that the corresponding increase in δ^*/σ (from 0.769 to 1.099) was due primarily to elastic deformation of the substrate under each contact point in the Hertzian contact area. The resulting change in δ^* (i.e., $(1.099 - 0.769) = 0.33\sigma$) would thus have been only a small fraction of the height of the highest asperities, assumed to be 3σ .

Since the physical characteristics of the contour area were insensitive to variations in $\bar{m}_e$, the observed sensitivity of R'_{cv} to $\bar{m}_e$ must have resulted from its effect on h_{cv} (as defined by Equation (I.3.42)).

The rough surface contact analysis that was described in Section I.3.5 was predicated on the assumption that the contact surfaces would interact plastically, either through plastic deformation of the asperities themselves or through the indentation of one contact surface by the asperities on the other. Visual examination of both contact surfaces after the rough surface tests indicated that significant plastic interaction had occurred, as mentioned previously, but the question of whether asperity deformation will be elastic or plastic generally has to be answered by applying one of the "plasticity criteria" mentioned in Section I.2.4. According to the criterion derived by Mikic [3], for example, asperity deformation and interpenetration will be predominantly plastic when $\beta \leq 1/3$ and predominantly elastic when $\beta \geq 3$, where β is defined by Equation (I.3.36). It can be shown that $\beta = 0.25$ for the model contact when $\bar{m}_e = 0.06$, so asperity deformation and interpenetration should have been predominantly plastic (assuming $\bar{m}_e = 0.06$, of course) in the rough surface tests according to the Mikic criterion.

It was assumed in Section I.5.3 that the hardness of the large radius contact surface would have increased when it was bead-blasted because of strain-hardening and hence have become greater than that of the smooth surface. This would not necessarily have caused the asperities on the rough surface to indent the smooth surface, however, because several studies (e.g., [69e], [95], [95a], [95b], [95c]) have shown that for small deformations at least, the pressure that can be supported by the

deformed area on a partially flattened asperity (i.e., the flow pressure) is probably closer to $H/2$ than it is to H . Pullen and Williamson [95d], Uppal and Probert [95b] and others have shown that the flow pressure ($=P/A_r$) of asperities increases with load (even in the absence of strain-hardening) because of asperity interaction, however, and that it can be much greater than H when the asperities are highly deformed (i.e., at high loads). If the flow pressure of the asperities on a given surface is less than the indentation hardness (H) for that surface at low loads and greater at high loads as the aforementioned studies suggest, then using H to calculate ϵ in Equation (I:3.33) is a compromise that will cause the real area of contact to be underestimated (and hence the contact resistance to be overestimated) at low loads and overestimated at high loads. These errors will only occur if the asperities on the rough surface(s) do not indent the other surface but flatten when they contact it and hence would only be expected if the hardnesses of the contact surfaces were similar (as would have been the case in the experiments performed in this study) or if the rougher surface was also the softer surface. When the rougher surface is much harder than the other, the asperities on the former surface should indent the latter and, at low loads at least, presumably no error will result from using the indentation hardness of the softer surface to calculate ϵ . At high loads, the flow pressure over each indentation probably increases above the value of H for the softer surface due to interaction of the zones of deformation [95c].

Ignoring the effect of displaced material on ϵ by obtaining ϵ from Equation (I.3.31) or Equation (I.3.34) is another compromise because for small deformations at least, the real area of contact would probably be greater than that predicted by Equation (I.3.31) if the contact surface being plastically deformed did not strain-harden readily and might actually be less if it did. These trends would be expected from the behaviour of materials in indentation hardness tests, where it has been observed that materials which do not strain-harden readily often form an elevated ridge around the indenter [78b], thereby increasing the area of contact with the indenter above that which would exist without the ridge. Materials with a high capacity for strain-hardening often "sink in" around the sides of the indenter (and produce a surface bulge well away from it) [78b], thereby reducing the area of contact with the indenter below that which would exist without sinking.

The real area of contact between rough surfaces would have to increase above that predicted by Equation (I.3.31) at very high pressures, however, because the contact area fraction (ϵ) should approach unity as $Z \rightarrow 0$ instead of the value of $1/2$ obtained with Equation (I.3.31). A method for correcting Equation (I.3.31) so that it yields the correct result (i.e., that $\epsilon \rightarrow 1$) when $Z \rightarrow 0$ is described by Mikic [3].

The analytical results for the contact in vacuum display considerable sensitivity to $\bar{m}_e$, as seen previously. Unfortunately, it is difficult to determine this parameter accurately because the finite tip width of the stylus on a stylus profilometer (typically $2.5 - 4 \mu\text{m}$ in the

direction of motion for the commonly used Talysurf instrument [69b]) effectively filters out the short wavelength irregularities on a surface and hence makes it impossible to measure $\bar{m}$ as it is defined in Equation (I.3.29). The value of $\bar{m}$ for a given surface is obtained in practice (with a stylus profilometer at least; optical methods have also been used with some success [69b]) by sampling its height at regular intervals and approximating the slope at each sampling point as a central-difference (usually) involving the surface heights at neighbouring points. The absolute values of the local slopes are then averaged to obtain $\bar{m}$. The product of this procedure is an average absolute surface slope which is strongly dependent on the chosen sampling interval, as seen in Figure I.6.8 for a bead-blasted surface whose mean slope was measured at different sampling intervals by Al-Astrabadi et al [95c]. There is no procedure reported in the literature for extrapolating a curve such as that shown in Figure I.6.8 from the lowest sampling interval used (which was $2.82 \mu\text{m}$ in the case of Figure I.6.8 because smaller sampling intervals were beyond the capability of the measuring instrument) to a sampling interval of zero, although guidelines for selecting a sampling interval appropriate to a particular application are given in references [69b], [69d], [69e], and [95c].

Although further research into the mechanics of rough surface interaction would be required to improve the accuracy with which contact resistances in vacuum could be predicted, the results of this study show that the presence of a conducting fluid around a non-conforming contact may reduce the sensitivity of the results of interest to uncertainties in the measured and predicted contact parameters (i.e., $\bar{m}$, ϵ , h_c , etc.) to the point where quite high accuracy can be obtained irrespective of

the shortcomings of existing rough surface contact models. The sensitivity of the results of interest in a particular problem to uncertainties in the conditions over the contact area can only be ascertained by modelling the problem and performing sensitivity analyses, of course, as will be demonstrated for the problem of a pressure tube contacting a calandria tube in a CANDU reactor in Part III.

Figure I.6.8 Mean Slope ($\bar{m}$) versus Sampling Interval for a Grit-Blasted Surface with $\sigma_{\text{rms}} = 4.2 \mu\text{m.}$ (from [95c]).

CHAPTER I.7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

As mentioned in Chapters I.1 and I.2, most of the previous studies of heat transfer between contacting non-conforming surfaces dealt with circular contacts, such as would be produced by contacting spheres. To the author's knowledge, the present study provides the first accurate experimental resistance data for an isolated non-circular contact.* Elliptical contacts, such as the one investigated in the present study, arise naturally from the contact of cylindrical surfaces or surfaces which have different radii of curvature in different planes. Analytical models that could be extended to predict the thermal resistances of such contacts (with smooth surfaces) in a gas and in vacuum had been developed by Yovanovich [15], Kitscha and Yovanovich [57] and Ogniewicz [65] before this study was undertaken, but their accuracy for other than circular contacts had not been tested. The experimental resistance data and the predictions of the ellipsoidal finite-difference model developed in this study were used to demonstrate the adequacy of the simple analytical models mentioned previously, at least for contacts with geometries and solid/gas thermal conductivity ratios similar to that of the model contact.

* Excluding contacts involving wedge-shaped or conical indenters.

The resistance of the model contact was measured with both smooth and rough contact surfaces in air and in vacuum. Measurements of the resistance of a smooth sphere/rough flat contact in vacuum had been reported previously by Kitscha [16] and Burde [58] but no similar measurements of the resistance in air (or any other gas) of an isolated contact formed between rough non-conforming surfaces have been reported in the heat transfer literature to the author's knowledge. While roughening the large radius contact surface to a roughness of 1 micron in the present study produced a definite reduction ($\approx 5-20\%$) in the thermal resistance of the contact in vacuum, it produced virtually no change in the resistance of the contact in air. This result indicates that the presence of roughness may be unimportant in many instances, specifically when conduction (and/or radiation) through the gas between the contact surfaces is the primary mode of heat transport. The gas conduction component of the heat transferred between non-conforming contacting surfaces, at least surfaces similar to those used in this study, appears to be relatively unaffected by surface roughness, as one might expect.

Good agreement was obtained between the measured resistances of the contact with smooth surfaces in vacuum and all of the theoretical models considered when it was assumed that the contact size at any load was that given by Hertz's theory and that the contact conductance within the contact patch was infinite. These results are consistent

with those obtained previously by other researchers [16, 65] for sphere-sphere and sphere-flat contacts, of course. Since roughening the surface of the large radius contact specimen to a roughness of 1 micron only reduced the resistance of the contact in vacuum by 5-20%, the analytical models predicated on the assumptions mentioned previously could even be used to estimate the resistance in vacuum of a non-conforming contact with moderately rough surfaces if a very accurate result was not required. Burde [58] observed discrepancies of as much as 35% between analytical contact resistances calculated using the Hertzian contact size and measured resistances in his tests of smooth sphere/rough flat contacts in vacuum, but even errors of this magnitude could be acceptable in some problems.

The resistance measurements in air were of greater interest in the present study than the measurements in vacuum because the contacts in the reactor-related problems which provided the impetus for this work would occur in the presence of gases (vapors). The maximum difference between the measured resistance of the contact (with smooth surfaces) in air and the resistance predicted using the ellipsoidal finite-difference model was 7.1% (of the measured resistance) while the maximum difference between theory and experiment for Kitscha's quasi-coupled model was approximately 11%. It was assumed in all of the analytical models that the contact problem was symmetrical about a fictitious isothermal plane located midway between the contact surfaces and that the shape of the contact surface on the substitute contact

specimen was of secondary importance. These assumptions, which are not absolutely essential in a numerical analysis but nevertheless simplify the problem considerably, caused the real contact surface geometries to be most highly modified in the lateral (y) direction since the curvatures of the true contact surfaces were most dissimilar in that direction.* It was argued that the assumptions were justifiable because most of the heat transfer through the gas probably occurs in a region very near the contact, where the contact surfaces are approximately parallel and symmetrical about the contact plane. The good agreement achieved between theory and experiment validates this argument to some extent, but further proof is presented in Figure I.7.1 where the rate of heat transfer between the contact surfaces through a heat flow channel bounded by the contact's longitudinal centerline and a plane located a distance y away from the centerline (Q_y) is expressed as a fraction of the total contact heat flow (Q_c) and plotted against y/b and y/R_1 . At the contact load of 176N to which the results apply, $b=0.00961$ cm (according to Hertz's theory) and $R_1=0.952$ cm as mentioned previously. It can be seen from the results, which were obtained using the ellipsoidal finite-difference model for the contact with smooth surfaces, that over 80% of the total heat flow between the contact surfaces occurs within ten contact half-widths from the contact's

* It will be remembered that the radius of curvature of the upper contact surface in the y direction was infinite while that of the lower contact surface in the same direction was 0.952 cm.

Figure I.7.1 Contact Heat Flow Fraction vs. y/b and y/R_1 .

longitudinal centerline or, equivalently, within one-tenth of the radius of the semi-cylindrical contact specimen from the centerline. The heat flow between the contact surfaces is therefore severely constricted even when augmented by gas conduction, making the shape of the substitute contact specimen's contact surface of secondary importance for the reasons given in Chapter I.3.

Kitscha's quasi-coupled model [57] for predicting the resistance of a contact in a gas, in which it is assumed that the temperature distribution around a contact with gas conduction is negligibly different from that prevailing without gas conduction, is obviously best suited to contact problems with large solid/gas thermal conductivity ratios, of which a contact in vacuo is a limiting case. The numerical model developed in the present study should give accurate results over a wider range of solid/gas thermal conductivity ratios than Kitscha's quasi-coupled model, however, because it accounts for the coupling between conduction in the solid and gas phases. It should be recognized, however, that the intended applications of Kitscha's quasi-coupled model of heat transfer through a sphere-sphere or sphere-flat contact in a gas (e.g., the analysis of heat transfer in packed beds) were such that a numerical model for predicting the resistance of the contact was neither desirable nor necessary, whereas the intended applications of the model developed in this study necessitated its being numerical in nature. The comment just made about Kitscha's model applies to all of the models developed to predict the resistance of a sphere-sphere or sphere-flat contact,

such as the models that Burde [58] and Yip [60] developed to predict the resistances of such contacts with rough surfaces in vacuo.

Since the ellipsoidal finite-difference model described previously would be difficult to apply to either of the complex reactor-related problems which provided the impetus for this work, the application of other more practical and widely applicable numerical techniques to the solution of three-dimensional problems in which small elliptical contacts are present will be considered in the remainder of the thesis. The alternate numerical models will be used in Part II to analyze heat transfer through the model contact with smooth surfaces (when it is assumed that the contact area is a perfect ellipse) since the ellipsoidal finite-difference model can provide an extremely accurate reference solution for that case.

Recommendations for Future (Fundamental) Research

The recommendations for future research are as follows:

- (1) Experimental and analytical studies should be performed to obtain a more accurate relationship between the apparent contact pressure (p_a) and contact area fraction (ϵ) than that used in Section I.3.5, where it was assumed, as is commonly done, that $p_a = H \epsilon$. This relationship may be approximately correct at low dimensionless pressures (p_a/H) if all of the asperities on one surface indent the other (softer) surface in a manner similar to that used to

measure H in a standard indentation test, but it is probably not correct at high pressures (which are often achieved in non-conforming contacts because the contour areas for such contacts are typically quite small) or when the asperities flatten, as discussed in Section I.6.5. Materials which strain-harden readily should be considered, although not initially.

- (2) Research aimed at obtaining a more realistic relationship between ϵ and η ($=Z/\sigma$) than Equation (I.3.31) (or Equation (I.3.34)) should be initiated (although one such relationship has been proposed by Mikic [3], as mentioned previously). Experiments to obtain more accurate relationships between (1) p_a and ϵ and (2) ϵ and η should be performed with nominally flat (i.e., non-wavy) rather than curved surfaces to facilitate interpretation of the results.
- (3) A procedure for specifying $\bar{m}$ in the rough surface contact theories which use that parameter should be developed and clearly defined. Jones et al [69a] performed experiments to determine the sampling interval that gave the best agreement between optically measured and theoretically predicted microcontact densities (for microcontacts within a certain size range) - other studies of that type might be worthwhile.

APPENDICES
(PART I)

APPENDIX I.A

SURFACE DEFORMATION ADJACENT TO AN ELASTIC CONTACT

It is not the intent of this appendix to present Hertz's theory of elastic contact, from which the separation distance formula (equation (I.3.19b)) is derived, in its entirety since that theory is discussed in detail by Love in reference [30]. The specific form of equation (I.3.19b) will be derived here from a general equation, derived by Hertz, for the contact-related separation distance, $\delta(x,y)$ shown in Figure I.1.2.

According to Hertz's theory, the separation distance, $\delta(x,y)$, between the surfaces of two paraboloids in elastic contact is, in regions adjacent to the contact, given by the following formula.

$$\delta(x,y) = -\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right) \frac{3P}{4\pi} \int_0^\lambda \left(1 - \frac{x^2}{a^2+\psi} - \frac{y^2}{b^2+\psi}\right) \frac{d\psi}{\sqrt{(a^2+\psi)(b^2+\psi)\psi}} \quad (\text{I.A.1})$$

where λ is the positive root of the equation

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1 \quad (\text{I.A.2})$$

Solving for λ we obtain

$$\lambda = \frac{(x^2+y^2-a^2-b^2) + \sqrt{(a^2+b^2-x^2-y^2)^2 - 4(a^2b^2-y^2a^2-x^2b^2)}}{2} \quad (\text{I.A.3})$$

The single integral on the right-hand side of equation (I.A.1) can be separated into three integrals, as shown below:

$$\delta(x,y) = -\left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right) \frac{3P}{4\pi} (I_1 - x^2 I_2 - y^2 I_3) \quad (\text{I.A.4})$$

where

$$I_1 = \int_0^\lambda \frac{d\psi}{\sqrt{(a^2+\psi)(b^2+\psi)\psi}} \quad (\text{I.A.5})$$

$$I_2 = \int_0^\lambda \frac{d\psi}{(a^2+\psi) \sqrt{(a^2+\psi)(b^2+\psi)\psi}} \quad (\text{I.A.6})$$

and

$$I_3 = \int_0^\lambda \frac{d\psi}{(b^2+\psi) \sqrt{(a^2+\psi)(b^2+\psi)\psi}} \quad (\text{I.A.7})$$

The solution of the first integral (I_1) is given by Byrd and Friedman([71], equation (237))as:

$$I_1 = \frac{2}{a} F(\phi, k) \quad (\text{I.A.8})$$

where

$F(\phi, k)$ = incomplete integral of the first kind with modulus k and argument ϕ .

$$= \int_0^\phi \frac{d\theta}{\sqrt{1-k^2 \sin^2 \theta}} \quad (\text{I.A.9})$$

and

$$k = \sqrt{1 - \frac{b^2}{a^2}} \quad (\text{I.A.10})$$

$$\phi = \sin^{-1} \sqrt{\frac{\lambda}{\lambda+b^2}} \quad (\text{I.A.11})$$

The integral I_1 can also be expressed in an alternate form as

$$I_1 = I_1' - I_1'' \quad (\text{I.A.12})$$

where

$$I_1' = \int_0^{\infty} \frac{d\psi}{\sqrt{(a^2 + \psi)(b^2 + \psi)\psi}} \quad (\text{I.A.13})$$

and

$$I_1'' = \int_{\lambda}^{\infty} \frac{d\psi}{\sqrt{(a^2 + \psi)(b^2 + \psi)\psi}} \quad (\text{I.A.14})$$

I_1' can be evaluated by substituting ∞ for λ in equation (I.A.11), in which case $\phi \rightarrow \frac{\pi}{2}$ and $F(\phi, k) \rightarrow K(k)$, where K is the complete elliptic integral of the first kind. The solution of I_1'' is ([71], equation (238)),

$$I_1'' = \frac{2}{a} F(\xi, k) \quad (\text{I.A.15})$$

where

$$\xi = \sin^{-1} \sqrt{\frac{a^2}{\lambda + a^2}} \quad (\text{I.A.16})$$

and k is given by equation (I.A.10).

Thus, I_1 can be expressed as:

$$I_1 = \frac{2}{a} (K(k) - F(\xi, k)) \quad (\text{I.A.17})$$

From equations (I.A.8) and (I.A.17), it is apparent that,

$$F(\phi, k) = K(k) - F(\xi, k) \quad (\text{I.A.18a})$$

Equation (I.A.18) is valid in general if two arguments ϕ and ξ^* satisfy

* The arguments ϕ and ξ are used in a generic (non-specific) sense in equation (I.A.18b) to denote any two angles.

the relationship

$$\cot \phi = k' \tan \xi \quad ([71], \text{equation (117.01)}) \quad (\text{I.A.18b})$$

which is true in this case. The parameter k' , designated the complementary modulus, is defined as:

$$k' = \sqrt{1-k^2} \quad (\text{I.A.19})$$

The solution of the second integral (I_2) is: ([71], equation (237.13))

$$I_2 = \frac{2}{a(a^2-b^2)} (F(\phi, k) - E(\phi, k)) + \frac{2}{a^2} \sqrt{\frac{\lambda}{(\lambda+a^2)(\lambda+b^2)}} \quad (\text{I.A.20})$$

where k and ϕ are given by equations (I.A.10) and (I.A.11) respectively and

$E(\phi, k)$ = the incomplete elliptic integral of the second kind

$$= \int_0^\phi \sqrt{1-k^2 \sin^2 \theta} d\theta \quad (\text{I.A.21})$$

The solution of the third integral (I_3) is: ([71], equation (237.12))

$$I_3 = \frac{2a}{b^2(a^2-b^2)} (E(\phi, k) - \frac{b^2}{a^2} F(\phi, k)) \quad (\text{I.A.22})$$

Combining equations (I.A.4), (I.A.20) and (I.A.22) we can express the separation distance, $\delta(x, y)$, as follows:

$$\begin{aligned} \delta(x, y) = C & \left[\frac{2}{a} F(\phi, k) - x^2 \left(\frac{2}{a(a^2-b^2)} (F(\phi, k) - E(\phi, k)) + \frac{2}{a^2} \sqrt{\frac{\lambda}{(\lambda+a^2)(\lambda+b^2)}} \right) \right. \\ & \left. - y^2 \left(\frac{2a}{b^2(a^2-b^2)} (E(\phi, k) - \frac{b^2}{a^2} F(\phi, k)) \right) \right] \quad (\text{I.A.23}) \end{aligned}$$

where

$$C = -\left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right) \frac{3P}{4\pi} \quad (\text{I.A.24})$$

For elliptical contacts with small semi-minor to semi-major diameter ratios, $k \rightarrow 1$. In such cases the following approximations can be used for the elliptic integrals F and E [71].

$$E(\phi, 1) = \sin \phi \quad (\text{I.A.25})$$

$$F(\phi, 1) = \ln (\tan \phi + \sec \phi) \quad (\text{I.A.26})$$

Substituting equation (I.A.11) into equations (I.A.25) and (I.A.26) yields,

$$E(\phi, 1) = \sqrt{\frac{\lambda}{\lambda + b^2}} \quad (\text{I.A.27})$$

and

$$F(\phi, 1) = \ln \left(\frac{\sqrt{\lambda} + \sqrt{\lambda + b^2}}{b} \right) \quad (\text{I.A.28})$$

Strictly speaking, equations (I.A.25) and (I.A.26) are only valid approximations to $E(\phi, k)$ and $F(\phi, k)$ when the elliptic modulus (k) has a value of unity. In most cases, however, these approximations provide sufficiently accurate estimates of $E(\phi, k)$ and $F(\phi, k)$ when k is close to, but not necessarily equal to, unity. Equation (I.A.26), when used with values of ϕ approaching 90° is the one exception to this observation since $F(\phi, 1) \rightarrow \infty$ as $\phi \rightarrow 90^\circ$ while $F(\phi, 1-\epsilon)$ remains finite as $\phi \rightarrow 90^\circ$ if ϵ is non-zero. For example, $F(90, 1) = \infty$ while $F(90, 0.99985) = 5.435$ (i.e., $F(90, 1)$ and $F(90, k)$ with $k \approx 1$ are in poor agreement in this case). However, when $\phi = 85^\circ$, $F(85, 1) = 3.131$ = equation (I.A.26) and $F(85, 0.99985) = 3.122$ (i.e., $F(85, 1)$ and $F(85, k)$, when $k \approx 1$, are in good

agreement). It is apparent from these examples that equation (I.A.26) should not be used to approximate $F(\phi, k)$ when k is not exactly equal to unity and $\phi \rightarrow 90^\circ$ (say $\phi > 85^\circ$ for convenience).

While it would appear that we will be unable to use a simple approximation for $F(\phi, k)$ when $\lambda \gg b$ since $\phi \rightarrow 90^\circ$ in that case, it should be noted that the argument in I_1'' (i.e., ξ) decreases as λ increases. At large values of λ , we can approximate $F(\xi, k)$, but not $F(\phi, k)$, by equation (I.A.26). In this case, as $k \rightarrow 1$, $F(\xi, k)$ becomes:

$$F(\xi, 1) = \ln (\tan \xi + \sec \xi) \quad (\text{I.A.29})$$

$$= \ln \left(\frac{a + \sqrt{\lambda + a^2}}{\sqrt{\lambda}} \right) \quad (\text{I.A.30})$$

The complete elliptic integral ($K(k)$) can be approximated in a simple manner as $k \rightarrow 1$, as shown below.

$$\lim_{k \rightarrow 1} K(k) = \ln \left(\frac{4a}{b} \right) \quad [71], \text{ equation (112.01)} \quad (\text{I.A.31})$$

Substituting equations (I.A.30) and (I.A.31) into equation (I.A.18), we obtain:

$$F(\phi, k) = \ln \left(\frac{4a}{b} \right) - \ln \left(\frac{a + \sqrt{\lambda + a^2}}{\sqrt{\lambda}} \right) \text{ for } k = 1 \quad (\text{I.A.32})$$

It would appear, therefore, that $F(\phi, k)$ can be approximated by equation (I.A.28) (if $k = 1$), for $\phi \leq 85^\circ$ and by equation (I.A.32) for $\phi > 85^\circ$. Since we can only use equation (I.A.32) if $\xi \leq 85^\circ$ (for the same reason that we can only use equation (I.A.28) if $\phi \leq 85^\circ$), it must be demonstrated that the ranges of λ in which equations (I.A.28) and (I.A.32)

are applicable overlap to a certain extent. If $\phi = 85^\circ$ at λ_1 and $\xi = 85^\circ$ at λ_2 , we must demonstrate that $\lambda_1 \geq \lambda_2$. If $\lambda_1 < \lambda_2$, then the range of λ in which equation (I.A.28) is applicable ($0 < \lambda \leq \lambda_1$) and the range in which equation (I.A.32) is applicable ($\lambda_2 \leq \lambda < \infty$) do not overlap and an approximation to $F(\phi, k)$ will be unavailable for $\lambda_1 < \lambda < \lambda_2$. Since we require $\phi \leq 85^\circ$ and $\xi \leq 85^\circ$, equations (I.A.11) and (I.A.16) can be rearranged to give:

$$\frac{\lambda}{\lambda+b} \leq (\sin 85^\circ)^2 \quad (\text{I.A.33})$$

$$\frac{a^2}{\lambda+a} \leq (\sin 85^\circ)^2 \quad (\text{I.A.34})$$

Equations (I.A.33) and (I.A.34) are valid for $\lambda \leq \lambda_1$ and $\lambda \geq \lambda_2$ respectively. The actual values of λ_1 and λ_2 are functions of a and b of course. For reasons discussed previously, we also require that the condition $\lambda_1 \geq \lambda_2$ be satisfied. For given values of a and b there is a minimum value of θ^* (where $\theta^* = \phi$ or ξ), designated $\theta_{\min}^*$, above which the values of λ from equation (I.A.11) are greater than the values obtained from equation (I.A.16). The angle $\theta_{\min}^*$ is indicated in Figure I.A.1, where the arguments ϕ and ξ are shown as functions of λ .

The value of λ at which the ϕ and ξ curves intersect is designated λ^* . Two hypothetical cases are also shown in Figure I.A.1. In case A, $\theta_{\min}^* < 85^\circ$ and $\lambda_1 > \lambda_2$ while in case B, $\theta_{\min}^* > 85^\circ$ and $\lambda_1 < \lambda_2$. Since we require $\lambda_1 > \lambda_2$, case A (i.e., $\theta_{\min}^* < 85^\circ$) is the only acceptable situation. For this reason, it is worthwhile deriving a general expression for $\theta_{\min}^*$, which will be a function only of the contact dimensions a and b .

so that we can determine easily if $\theta_{\min}^* < 85^\circ$ for a particular contact. If this criterion is not satisfied, it will be necessary to increase the maximum allowable value of the argument (presently at 85°) to

Figure I.A.1: ϕ and ξ vs. λ

a value greater than $\theta_{\min}^*$. As mentioned previously, however, the accuracy with which equation (I.A.26) approximates $F(\phi, k)$ deteriorates as $\phi \rightarrow 90^\circ$ if k is not exactly unity.

Since $\theta_{\min}^*$ is the angle at which ϕ and ξ are equal (for the same value of λ , specifically λ^*), it can be obtained by setting $\phi = \xi = \theta_{\min}^*$ in equation (I.A.18b). In this case we have,

$$\cot \theta_{\min}^* = k \tan \theta_{\min}^*$$

or

$$\theta_{\min}^* = \tan^{-1} \sqrt{\frac{1}{k}} \quad (\text{I.A.35})$$

An expression for λ^* can be obtained by writing equation (I.A.11) as,

$$(\sin \theta_{\min}^*)^2 = \frac{\lambda^*}{\lambda^* + b^2} \quad (\text{I.A.36})$$

or

$$\lambda^* = \frac{b^2 \sin^2 \theta_{\min}^*}{1 - \sin^2 \theta_{\min}^*} \quad (\text{I.A.37})$$

Substituting equation (I.A.35) into equation (I.A.37) yields,

$$\lambda^* = \frac{b^2}{k} = ab \quad (\text{I.A.38})$$

As an example of how equation (I.A.35) is used, consider a contact for which $b/a = 1/16$. This is, in fact, the b/a ratio of the contact investigated experimentally and analytically in this study. In this case we obtain,

$$\begin{aligned} \theta_{\min}^* &= \tan^{-1} \sqrt{\frac{1}{k}} = \tan^{-1} \sqrt{\frac{a}{b}} \\ &= 75.96^\circ \end{aligned}$$

Since $\theta_{\min}^*$ is less than 85° , we can obviously satisfy equations (I.A.33) and (I.A.34) and still have $\lambda_1 > \lambda_2$. If we consider a contact with $a = 2.5$ mm and $b = 2.5/16 = 0.1562$ mm, we can solve equations (I.A.33) and (I.A.34) for λ_1 and λ_2 to obtain:

$$\lambda_1 = 3.1896$$

$$\lambda_2 = 0.0478$$

Equation (I.A.28) can be used to approximate $F(\phi, k)$ (if $k = 1$) in the range $0 < \lambda \leq 3.1896$ while equation (I.A.32) can be used in the range $0.478 < \lambda < \infty$. In this manner $F(\phi, k)$ can be approximated at all values of λ (i.e., $0 < \lambda < \infty$). The ranges of λ in which equations (I.A.28) and (I.A.32) are applicable overlap so either equation can be used in the range $0.0478 < \lambda < 3.1896$. In order to demonstrate that this observation is valid, consider the contact with $a = 2.5$ mm and $b = 0.1562$ mm. Evaluating equation (I.A.28) at $\lambda = \lambda^* = 0.390$ (this value lies in the overlap region), we obtain,

$$F(\phi, 1) = 2.0939$$

while substitution of $\lambda^* = 0.390$ into equation (I.A.32) yields,

$$F(\phi, 1) = 2.0634$$

These two values of $F(\phi, 1)$ differ by only 1.4%. The actual* value of $F(\phi, k)$ at $\lambda^* = 0.390$ (i.e., at $\phi = \phi_{\min}^* = 75.96^\circ$) and $k = \sqrt{1 - (1/16)^2} = 0.99804$ is 2.0900 so all three values of $F(\phi, k)$ are in good agreement.

* The actual value quoted here was obtained from a table by interpolation and thus is not really exact.

It can be shown using equation (I.A.35) that $b/a \approx 1/131$ when $\theta_{\min}^* = 85^\circ$. At b/a ratios smaller than this value, it is not possible to have $\lambda_1 > \lambda_2$ unless the maximum allowable argument is increased from 85° to a value greater than $\theta_{\min}^*$.

A final modification can be made to the equation for $\delta(x,y)$ (equation (I.A.23)). The constant C (equation (I.A.24)), which at present is a function of P , v_1 , v_2 , E_1 and E_2 , can be expressed, alternately, as a function of R_1 , R_2 , a and b , where R_1 and R_2 are shown in Figure I.1.5. The modified form of C can be obtained in the following manner. From Hertz's theory we have the relationship [70],

$$b^3 = \frac{3b}{2\pi a} \frac{E(k)}{(C_1 + C_2)} \cdot P \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right) \quad (\text{I.A.39})$$

where C_1 and C_2 are given by equations (I.3.13) and (I.3.14) as,

$$C_1 = \frac{1}{2R_2}$$

$$C_2 = \frac{1}{2R_1}$$

and $E(k)$ is the complete elliptic integral of the second kind. As $k \rightarrow 1$, $E(k) \rightarrow 1$ so equation (I.A.39) can be rearranged to give:

$$\frac{3P}{4\pi} \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right) = \frac{ab^2 (R_1 + R_2)}{4 R_1 R_2} \quad (\text{I.A.40})$$

or

$$C = - \frac{ab^2 (R_1 + R_2)}{4 R_1 R_2} \quad (\text{I.A.41})$$

Equation (I.A.41) is the new form of C (which is valid when $k \approx 1$).

APPENDIX I.B

SHAPE FACTOR DERIVATIONS

In the following derivations, initial and final values of coordinates are identified by the subscripts 1 and 2 respectively. The general shape factor expression (equation (I.4.61)) can be written for conduction in the λ direction as:

$$U_{\lambda_1 \rightarrow \lambda_2} = \int_{\nu_1}^{\nu_2} \int_{\mu_1}^{\mu_2} \frac{d\mu \, d\nu}{\int_{\lambda_1}^{\lambda_2} \frac{g_\lambda \, d\lambda}{\sqrt{g_\lambda \cdot g_\mu \cdot g_\nu}}} \quad (\text{I.B.1})$$

where the metric coefficients g_λ , g_μ and g_ν are given by equations (I.4.14) - (I.4.16). Substitution of the metric coefficients into equation (I.B.1) yields the following expression:

$$U_{\lambda_1 \rightarrow \lambda_2} = \frac{1}{2} \int_{\nu_1}^{\nu_2} \int_{\mu_1}^{\mu_2} \frac{(\mu - \nu) \, d\mu \, d\nu}{\sqrt{(a^2 + \mu)(b^2 + \mu)(-\mu)(a^2 + \nu)(b^2 + \nu)}} \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)}} \quad (\text{I.B.2})$$

which can be further simplified by moving terms out of integrals in which they are not involved, thereby obtaining the following relatively simple form of equation (I.B.2).

$$\begin{aligned}
 U_{\lambda_1 \rightarrow \lambda_2} &= \frac{1}{\lambda_1^2 \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)\lambda}}} \\
 &\times \left[\int_{v_1}^{v_2} \frac{dv}{\sqrt{(a^2+v)(b^2+v)v}} \int_{\mu_1}^{\mu_2} \frac{\mu d\mu}{\sqrt{(a^2+\mu)(b^2+\mu)(-\mu)}} \right. \\
 &\quad \left. - \int_{v_1}^{v_2} \frac{v dv}{\sqrt{(a^2+v)(b^2+v)v}} \int_{\mu_1}^{\mu_2} \frac{d\mu}{\sqrt{(a^2+\mu)(b^2+\mu)(-\mu)}} \right] \quad (\text{I.B.3})
 \end{aligned}$$

Thus the double integral in equation (I.B.2) is replaced by a combination of single integrals. For convenience, let,

$$A = \int_0^{\lambda^*} \frac{d\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)\lambda}} \quad (\text{I.B.4})$$

$$B = \int_0^{\lambda^*} \frac{\lambda d\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)\lambda}} \quad (\text{I.B.5})$$

$$C = \int_{-b^2}^{\mu^*} \frac{d\mu}{\sqrt{(a^2+\mu)(b^2+\mu)(-\mu)}} \quad (\text{I.B.6})$$

$$D = \int_{-b^2}^{\mu^*} \frac{\mu d\mu}{\sqrt{(a^2+\mu)(b^2+\mu)(-\mu)}} \quad (\text{I.B.7})$$

$$G = \int_{-a^2}^{v^*} \frac{dv}{\sqrt{(a^2+v)(b^2+v)v}} \quad (\text{I.B.8})$$

and,

$$H = \int_{-a^2}^{v^*} \frac{v \, dv}{\sqrt{(a^2+v)(b^2+v)}v} \quad (\text{I.B.9})$$

Equation (I.B.3) can now be expressed more compactly as,

$$U_{\lambda_1 \rightarrow \lambda_2} = \frac{1}{2(A_2 - A_1)} [(G_2 - G_1)(D_2 - D_1) - (H_2 - H_1)(C_2 - C_1)] \quad (\text{I.B.10})$$

where the subscript on each term in equation (I.B.10) is the same as the subscript on the upper limit of integration in the corresponding integral. The integral produced by substituting λ_2 for λ^* in equation (I.B.4) would be designated A_2 in this case, for example.

Expressions similar to equation (I.B.10) can be written for the shape factors associated with conduction in the μ and v directions, as shown below

$$U_{\mu_1 \rightarrow \mu_2} = \frac{1}{2(C_2 - C_1)} [(G_2 - G_1)(B_2 - B_1) - (H_2 - H_1)(A_2 - A_1)] \quad (\text{I.B.11})$$

$$U_{v_1 \rightarrow v_2} = \frac{1}{2(G_2 - G_1)} [(C_2 - C_1)(B_2 - B_1) - (D_2 - D_1)(A_2 - A_1)] \quad (\text{I.B.12})$$

Solutions to the integrals in equations (I.B.4) - (I.B.9) can be found in Byrd and Friedman [71]. Those solutions are given below, along with the number of the corresponding equation in [71].

$$A = \frac{2}{a} F(\phi, k) = \frac{2}{a} \underline{x} \quad [\text{71}], \text{equation (237)} \quad (\text{I.B.13})$$

$$B = 2a \left[\sqrt{\frac{(\lambda^* (\lambda^* + a^2))}{(\lambda^* + b^2) a^2}} - E(\phi, k) \right] \quad [71], \text{equation (237.03)} \quad (\text{I.B.14})$$

$$C = \frac{2}{a} F(\beta, k') = \frac{2}{a} \underline{y} \quad [71], \text{equation (235)} \quad (\text{I.B.15})$$

$$D = 2a [E(\theta, k') - (k'^2 \sin \theta \cos \theta / \sqrt{1 - k'^2 \sin^2 \theta}) - F(\beta, k')] \quad [71], \text{equation (235.16)} \quad (\text{I.B.16})$$

$$G = \frac{2}{a} F(\gamma, k) = \frac{2}{a} \underline{z} \quad [71], \text{equation (233)} \quad (\text{I.B.17})$$

$$H = -2a E(\gamma, k) \quad [71], \text{equation (233.17)} \quad (\text{I.B.18})$$

where

$$k = \text{the elliptic modulus} = \sqrt{1 - b^2/a^2} \quad (\text{I.B.19})$$

$k' = \text{the complementary elliptic modulus}$

$$= \sqrt{1 - k^2} = b/a \quad (\text{I.B.20})$$

$$\phi = \sin^{-1} \sqrt{\frac{\lambda^*}{\lambda^* + b^2}} \quad (\text{I.B.21})$$

$$\beta = \sin^{-1} \frac{a}{b} \sqrt{\frac{\mu^* + b^2}{\mu^* + a^2}} \quad (\text{I.B.22})$$

$$\gamma = \sin^{-1} \sqrt{\frac{v^* + a^2}{a^2 - b^2}} \quad (\text{I.B.23})$$

ϕ , β and γ are the arguments of the elliptic integrals F and E , which are known as the elliptic integrals of the first and second kinds respectively. If the argument in one of these integrals is $\frac{\pi}{2}$, that integral is called complete; otherwise it is incomplete.

Each of the fundamental elliptic integrals (i.e., F and E) in equations (I.B.13) - (I.B.18) can be expressed in an alternate form since, according to equation (117.01) in [71], the following identities apply provided equation (I.A.18b) (i.e. $\cot \phi = k' \tan \xi$, where ϕ and ξ are any two arguments) is satisfied.

$$F(\phi, k) + F(\xi, k) = K(k) \quad (\text{I.B.24})$$

$$E(\phi, k) + E(\xi, k) = E(k) + k^2 \sin \phi \sin \xi \quad (\text{I.B.25})$$

The elliptic integrals $K(k)$ and $E(k)$ in equations (I.B.24) and (I.B.25) are the complete elliptic integrals of the first and second kinds respectively. Using equations (I.B.24) and (I.B.25), the alternate forms of the elliptic integrals in equations (I.B.13) - (I.B.18) can be written as:

$$F(\phi, k) = K(k) - F(\xi, k) \quad (\text{I.B.26})$$

$$E(\phi, k) = E(k) + k^2 \sin \phi \sin \xi - E(\xi, k) \quad (\text{I.B.27})$$

$$E(\beta, k') = K(k') - F(\zeta, k') \quad (\text{I.B.28})$$

$$E(\beta, k') = E(k') + k'^2 \sin \beta \sin \zeta - E(\zeta, k') \quad (\text{I.B.29})$$

$$F(\gamma, k) = K(k) - F(\lambda, k) \quad (\text{I.B.30})$$

$$E(\gamma, k) = E(k) + k^2 \sin \gamma \sin \lambda - E(\lambda, k) \quad (\text{I.B.31})$$

where

$$\xi = \sin^{-1} \sqrt{\frac{a^2}{\lambda^2 + a^2}} \quad (\text{I.B.32})$$

$$\zeta = \sin^{-1} \sqrt{\frac{-\mu^*}{b^2}} \quad (\text{I.B.33})$$

$$\lambda = \sin^{-1} \sqrt{\frac{a^2(b^2 + v^*)}{(a^2 - b^2)v^*}} \quad (\text{I.B.34})$$

The argument pairs, (ϕ, ξ) , (β, ζ) and (γ, λ) all satisfy equation (I.A.18b). The alternate form of $F(\phi, k)$ given by equation (I.B.26), was also derived in Appendix I.A, where it appeared as equation (I.A.18a).

For elliptical contacts with small semi-minor to semi-major diameter ratios, $k \rightarrow 1$ and $k' \rightarrow 0$. In such cases the following approximations can be used for the elliptic integrals F and E ([71], pp.10-11).

$$E(\phi, 1) = \sin \phi \quad (\text{I.B.35})$$

$$F(\phi, 1) = \ln (\tan \phi + \sec \phi) \quad (\text{I.B.36})$$

$$E(\phi, 0) = \phi \quad (\text{I.B.37})$$

$$F(\phi, 0) = \phi \quad (\text{I.B.38})$$

In addition, the complete integrals (i.e., $K(k)$, $K(k')$, $E(k)$ and $E(k')$) appearing in equations (I.B.26) - (I.B.31) can be approximated by applying the following limit relationships.

$$\lim_{k \rightarrow 1} K(k) = \ln \left(\frac{4a}{b} \right) \quad (\text{I.B.39})$$

$$\lim_{k' \rightarrow 0} K(k') = \frac{\pi}{2} \left(1 + \frac{k'^2}{4} \right) \quad (\text{I.B.40})$$

$$\lim_{k \rightarrow 1} E(k) = \frac{1 + \frac{k'^2}{2} \ln \left(\frac{4a}{b} \right)}{\left(1 + \frac{k'^2}{4} \right)} \quad (\text{I.B.41})$$

$$\lim_{k' \rightarrow 0} E(k') = \frac{\pi}{2} \left(1 - \frac{k'^2}{4} \right) \quad (\text{I.B.42})$$

In equations (I.B.35) - (I.B.38), ϕ is simply a generic angle - obviously the appropriate angles in the different integrals (i.e., ϕ , ϵ , β , ζ , γ , or λ) should be used in the approximation of those integrals. Strictly speaking equations (I.B.35) - (I.B.38) are only valid when the elliptic modulus (k or k') has the value indicated (i.e., 1 or 0). When k is slightly less than 1, or k' is slightly greater than 0, these approximations are all fairly accurate when the value of the argument (ϕ) is low. At high values of the argument, in particular as $\phi \rightarrow 90^\circ$ all of the approximations become less accurate, although only the approximation to $F(\phi, k)$ (i.e., equation (I.B.36)) becomes totally inadequate, as discussed in Appendix I.A. It will be remembered, however, that two expressions are available for each elliptic integral in equations (I.B.13) - (I.B.18). It can be shown that, in all cases, the arguments in the two alternate expressions for each elliptic integral vary in opposite senses as the limit of integration (i.e., λ^* , u^* or v^*) varies. In the case of β and ζ , for example, β is 0° and ζ is 90° when $u^* = -b^2$, while β is 90° and ζ is 0° when $u^* = 0$. From these observations, it should be apparent that a particular elliptic integral (F or E) can be approximated over the entire range of definition of its variable limit of integration by switching to the alternate form of that integral when the argument in the original form becomes excessive. Specifically, we

would like to switch to the alternate form of an integral when the argument in the alternate form, for the same value of the variable limit of integration (i.e., λ^* , μ^* or ν^*), is lower in value than the argument in the integral's original form.* In order to do this, we have to find the value of the variable limit of integration, or equivalently, the original argument, at which the arguments in the original and alternate forms of a particular elliptic integral are equal. Denoting this value of the original argument by ψ , we can find it by setting $\phi = \psi$ and $\xi = \psi$ in equation (I.A.18b), viz.

$$\cot \psi = k' \tan \psi \quad (\text{I.B.43})$$

from which we obtain,

$$\psi = \tan^{-1} \sqrt{\frac{1}{k}} \quad (\text{I.B.44})$$

This result was also obtained in Appendix I.A (as equation (I.A.35)).

It should be recognized that the parameter k' in equation (I.B.44) is equal to the complement of the modulus appearing in the elliptic integrals in equation (I.B.24) or equation (I.B.25). In the case of equation (I.B.29), for example, the complement of the modulus appearing in the integrals in that equation is k so, for this specific case, equation (I.B.44) needs to be rewritten as:

$$\psi = \tan^{-1} \sqrt{\frac{1}{k}} \quad (\text{I.B.45})$$

* This matter is discussed in greater detail in Appendix I.A.

When $k \approx 1$, equation (I.B.45) gives $\psi = 45^\circ$ so equation (I.B.29) should only be used to represent $E(\beta, k')$ for those values of μ^* at which $\beta > 45^\circ$ since, at those values of μ^* , $\zeta < 45^\circ$. Since ζ is lower in value than β when β is greater than 45° , the approximation to $E(\beta, k')$ in its alternate form (equation (I.B.29)), viz.

$$\begin{aligned} E(\beta, k') &= E(k') + k'^2 \sin \beta \sin \zeta - E(\zeta, k') \\ &= \frac{\pi}{2} \left(1 - \frac{k'^2}{4}\right) + k'^2 \sin \beta \sin \zeta - \zeta \end{aligned} \quad (\text{I.B.46})$$

should* be more accurate than the approximation to $E(\beta, k')$ in its original form (i.e., $E(\beta, k') = E(\beta, 0) = \beta$). Equation (I.B.37) was used to approximate $E(\zeta, k')$ in equation (I.B.46) while equation (I.B.42) was used to approximate $E(k')$. There is actually little difference between the approximations for $E(\beta, k')$ in its original and alternate forms when k' is very small, and the difference vanishes completely when k' is zero (equation (I.B.37) is exact in that case, of course). While such differences might appear insignificant, it is important that the elliptic integrals in equations (I.B.13) - (I.B.18) be approximated in a fashion which recognizes that k is not exactly unity ($k=1$ corresponds to a line contact) and k' is not exactly zero. If the elliptic integrals in the

* The superiority of equation (I.B.46) over the simple approximation $E(\beta, k') = E(\beta, 0) = \beta$, for $\beta > 45^\circ$ can be proven by comparing the two approximations to tabulated values of E . For $k' = 1/16$ and $\beta = 70^\circ$, for example, the actual (tabulated) value of E is 1.22083 while equation (I.B.46) yields $E = 1.22082$ and the simple expression ($E = \beta$) yields $E = 1.22173$.

shape factor expressions are not approximated accurately, negative shape factors (i.e., negative internodal conductances) may result.

The final approximations to equations (I.B.13) - (I.B.18) are given below along with their ranges of applicability. The range of applicability of each approximation is given in terms of:

- (a) permissible values of the argument in the original integral (i.e., ϕ , β , or γ), expressed in general form (equation (I.B.44)) with the appropriate complementary modulus, as well as in degrees for a contact with $a/b = 16$, for which $k = .99804$ and $k' = .0625$.
- (b) the value of the appropriate transformed coordinate (i.e., x , y or z) for a contact with $a/b = 16$. The transformed coordinates are defined in equations (I.4.42) - (I.4.44).

$$\left. \begin{aligned} A &= \frac{2}{a} \ln \left(\frac{\sqrt{\lambda^*} + \sqrt{\lambda^* + b^2}}{b} \right) \\ B &= 2a \left[\sqrt{\frac{\lambda^* (\lambda^* + a^2)}{(\lambda^* + b^2) a^2}} - \sqrt{\frac{\lambda^*}{\lambda^* + b^2}} \right] \end{aligned} \right\} \begin{aligned} \phi &\leq \tan^{-1} \sqrt{\frac{1}{k}} \leq 76^\circ & (I.B.47) \\ x &\leq 2.091 & (I.B.48) \end{aligned}$$

$$\left. \begin{aligned} A &= \frac{2}{a} \left(\ln \left(\frac{4a}{b} \right) - \ln \left(\frac{a + \sqrt{\lambda^* + a^2}}{\sqrt{\lambda^*}} \right) \right) \\ B &= 2a \left[\sqrt{\frac{\lambda^* (\lambda^* + a^2)}{(\lambda^* + b^2) a^2}} - \frac{(1 + \frac{k'^2}{2} \ln \left(\frac{4a}{b} \right))}{(1 + \frac{k'^2}{4})} \right] \end{aligned} \right\} \begin{aligned} \phi &> \tan^{-1} \sqrt{\frac{1}{k}} > 76^\circ & (I.B.49) \\ x &> 2.091 \end{aligned}$$

$$-k^2 \sqrt{\frac{\lambda^* a^2}{(\lambda^* + b^2)(\lambda^* + a^2)}} + \sqrt{\frac{a^2}{\lambda^* + a^2}} \quad (I.B.50)$$

$$C = \frac{2}{a} \sin^{-1} \left(\frac{a}{b} \sqrt{\frac{\mu^* + b^2}{\mu^* + a^2}} \right)$$

$$D = -2 \sqrt{\frac{-\mu^* (\mu^* + b^2)}{\mu^* + a^2}}$$

$$C = \frac{2}{a} \left(\frac{\pi}{2} \left(1 + \frac{k'^2}{4} \right) - \sin^{-1} \sqrt{\frac{\mu^*}{b^2}} \right)$$

$$D = \frac{-\pi b^2}{2a}$$

$$G = \frac{2}{a} \ln \left(\frac{\sqrt{\mu^* + a^2} + \sqrt{a^2 - b^2}}{\sqrt{-(\mu^* + b^2)}} \right)$$

$$H = -2a \sqrt{\frac{\mu^* + a^2}{a^2 - b^2}}$$

$$\beta \leq \tan^{-1} \sqrt{\frac{1}{k}} \leq 45^\circ$$

(I.B.51)

$$\underline{y} \leq 0.786$$

(I.B.52)

$$\beta > \tan^{-1} \sqrt{\frac{1}{k}} > 45^\circ$$

(I.B.53)

$$\underline{y} > 0.786$$

(I.B.54)

$$\gamma \leq \tan^{-1} \sqrt{\frac{1}{k}} \leq 76^\circ$$

(I.B.55)

$$\underline{z} \leq 2.091$$

(I.B.56)

$$G = \frac{2}{a} \left(\ln \left(\frac{4a}{b} \right) - \ln \left(\frac{\sqrt{-a^2(b^2 + \mu^*)} + \sqrt{(b^2 - a^2)\mu^*}}{b \sqrt{\mu^* + a^2}} \right) \right)$$

(I.B.57)

$$\gamma > \tan^{-1} \sqrt{\frac{1}{k}} > 76^\circ$$

$$H = -2a \left[\frac{\left(1 + \frac{k'^2}{2} \ln \left(\frac{4a}{b} \right) \right)}{\left(1 + \frac{k'^2}{4} \right)} + \frac{1}{a} \sqrt{\frac{(a^2 + \mu^*)(b^2 + \mu^*)}{\mu^*}} - \frac{1}{k} \sqrt{\frac{(b^2 + \mu^*)}{\mu^*}} \right]$$

$$\underline{z} > 2.091$$

(I.B.58)

Substitution of equations (I.B.47) - (I.B.58) into equations (I.B.10) - (I.B.12) with the appropriate limits of integration yields the shape factors for the finite-difference model in ellipsoidal coordinates.

APPENDIX I.C

COMPUTER IMPLEMENTATION OF THE ELLIPSOIDAL FINITE-DIFFERENCE MODEL

I.C.1 Introduction

The ellipsoidal finite-difference model is implemented in a Fortran IV computer program named RECELL (RESISTANCE OF CONTACT-ELLIPSOIDAL MODEL). Input variables for RECELL are defined in Table I.C.1 and the computations performed in each routine of the program are described in section I.C.2. Calculations performed in RECELL that are not connected directly with the implementation of the ellipsoidal finite-difference model are discussed in section I.C.3. Key variables used in the program are defined in section I.C.4. A listing of the program is included at the end of the Appendix along with the output listings for sample calculations of the resistance of the contact* in air and in vacuum at a load of 175.8N. The input data used in the sample calculations are listed on the printouts of the results.

I.C.2 Descriptions of Routines In RECELL

Activities in each routine are presented in the order in which they occur.

* With smooth surfaces.

I.C.2.1 Main Program

Input variables 1-15 inclusive (see Table I.C.1) are read by the main program. Subroutine HERTZ, where the contact dimensions A and B are calculated, is called by the main program and the contact dimensions are returned to the main program. The elliptic modulus k , (variable XKCON) and the complementary modulus, k' , (variable XKCC) are calculated along with the limiting values of the complete elliptic integrals, $K(0)$, $K(1)$, $E(0)$ and $E(1)$, designated XKKO, XKKI, EKO and EK1 respectively in the program. Expressions for the limiting values of these integrals were given in Appendix I.B (equations (I.B.39) - (I.B.42)). Subroutine COORD, which calculates the coordinates of all of the mesh points in both the λ , μ , ν and $\underline{x}$, $\underline{y}$, and $\underline{z}$ coordinate systems, is called by the main program. The coordinates λ_i , μ_j and ν_k (where $i = 1, IX$; $j = 1, JX$ and $k = 1, KX$) are returned to the main program in the arrays XLMDP, XMUP and XNUP, while the corresponding transformed coordinates $\underline{x}_i$, $\underline{y}_j$ and $\underline{z}_k$ are returned in the arrays X, Y and Z. The values of

$\frac{(\underline{x}_i + \underline{x}_{i+1})}{2}$, $\frac{(\underline{y}_j + \underline{y}_{j+1})}{2}$ and $\frac{(\underline{z}_k + \underline{z}_{k+1})}{2}$, which are needed to define the boundaries of the finite-difference "cells" (boxes) around the mesh points, are also calculated in subroutine COORD and returned to the main program in the arrays XAV, YAV and ZAV respectively. The corresponding values of λ , μ and ν are returned in the arrays XLMAV, XMUAV and XNUAV respectively.

Subroutine NNUMB, where nodes located inside the substitute contact specimen are identified and numbered, is called by the main program.

The total number of nodes contained within the substitute specimen (NEQ) and information indicating the addresses of these nodes (i, j, k is the address of a node located at λ_i, μ_j, ν_k) are returned to the main program.

The total number of nodes in the model and the number of storage spaces in fast memory available for equation solving (MTOT) are used to calculate the information required to store the conductance matrix and right-hand side vector for the finite-difference equations in blocks (i.e., groups of equations) on a scratch file. The number of blocks (NBLOCK), the number of equations per block (NEQB), and the total number of entries in each block (NAV) are needed to arrange the transfer. The total number of entries in each block is calculated from $NAV = NEQB \times (MA + NV)$, where MA is the maximum half-bandwidth of the conductance matrix and NV is the number of right-hand side vectors (NV=1 in this case).

Equations (I.B.10) - (I.B.12) are used to obtain conductance equations for all of the nodes located within the substitute contact specimen. The integrals contained within the square brackets in equations (I.B.10) - (I.B.12) (i.e., A_1, A_2, B_1, B_2 , etc.) are designated A1, A2, B1, B2, etc., in the computer program. Equations (I.B.47) - (I.B.58) are used to approximate these integrals, with the limits of integration $\lambda_1, \lambda_2, \mu_1, \mu_2, \nu_1$ and ν_2 , at a particular mesh point being denoted XLM1, XLM2, XMU1, XMU2, XNU1 and XNU2 respectively. The denominators of equations (I.B.10) - (I.B.12) (i.e., $2(A_2 - A_1), 2(C_2 - C_1), 2(G_2 - G_1)$) are designated DX12, DY12 and DZ12 respectively in the program.

Internodal conductances are stored, temporarily, in the vector C, where the elements of this vector are related to the internodal conductances, K , ($K=k_s U$) in equation (I.4.62) as follows:

$$C(1) \rightarrow K(i,j,k)_{i+1}$$

$$C(2) \rightarrow K(i,j,k)_{i-1}$$

$$C(3) \rightarrow K(i,j,k)_{j+1}$$

$$C(4) \rightarrow K(i,j,k)_{j-1}$$

$$C(5) \rightarrow K(i,j,k)_{k+1}$$

$$C(6) \rightarrow K(i,j,k)_{k-1}$$

If a particular node (i,j,k) is located within the substitute contact specimen itself, but has a neighbour located outside of the specimen, the conductance between the neighbour and node i,j,k is set to zero. The nodes located inside the substitute specimen are decoupled from the nodes located outside the specimen in this manner. The mechanics of the decoupling process are discussed in greater detail in the description of subroutine NNUMB.

The coefficients (conductances) contained within the right-hand bandwidth of each nodal equation are stored in vector D. Right-hand side constants are also stored in D, as are all of the zero elements within the band. D is used to transfer blocks of equations to a scratch file on tape or disc. Elements in the conductance and right-hand side matrices (i.e., K and Q when the finite-difference equations are written

as $KT=Q$) are related to elements in D in the manner shown in Figure I.C.1 for the first block. It should be recognized that the conductance matrices associated with three-dimensional problems have, in general, very large bandwidths and, in addition, contain a large number of zero elements within their bands. These zero elements are stored in D along with the non-zero elements.

The vector D for the first block of equations is written onto file NSTIF. It is then used to store coefficients and right-hand side constants for the next NEQB equations, which are subsequently written onto the scratch file, NSTIF, as before. The process of assembling and storing blocks of equations continues until the entire system of nodal equations has been transferred to the scratch file. Further details of this procedure are given by Wilson et al. in reference [96].

In the process of assembling the nodal equations, subroutine TKGAS is called by the main program each time a gas conductance must be evaluated (i.e., each time at node at $\mu = 0$, or $J = JX$, and for which $\lambda > 0$, or $I > 1$, is encountered). Subroutine TKGAS is only called if the resistance of the contact with gas conduction is requested (i.e., if $INSUL=0$), of course.

Conductances in the λ direction between nodes located at $\lambda = 0$ and $\lambda = \lambda_1$ (i.e., between nodes at $I=1$ and $I=2$) and between nodes at $\lambda = \lambda_{IX}$ and $\lambda = \lambda_{IX-1}$ (i.e., between nodes at $I = IX$ and $I = IX-1$) are stored in the arrays CQF1 and CQF2 respectively for use later in evaluating the heat flow through the contact patch and the total heat flow through the contact specimen.

Diagram illustrating the relationship between matrices K , Q , and D for the first block of NEQB equations.

Matrix K is symmetric and contains elements k_{ij} . The elements shown are $k_{11}, k_{12}, k_{13}, k_{14}$ in the first row; $k_{22}, k_{23}, k_{24}, k_{25}$ in the second row; and $k_{n,n}, k_{n,n+1}, k_{n,n+2}, k_{n,n+3}$ in the n -th row. The matrix is labeled "SYMMETRIC" and "n = NEQB".

Matrix Q is a column vector containing elements $Q_1, Q_2, \dots, Q_n$.

Matrix D is a row vector containing elements $D_1, D_2, \dots, D_n$.

The relationship is given by the equation:

$$K \cdot Q = D$$

etc.

For the first block of NEQB equations shown above,

$$D = (k_{11}, k_{22}, \dots, k_{n,n}, k_{12}, k_{23}, \dots, k_{n,n+1}, k_{13}, k_{24}, \dots, k_{n,n+2}, k_{14}, k_{25}, \dots, k_{n,n+3}, Q_1, Q_2, \dots, Q_n)$$

Figure I.C.1 Relationship Between Elements in Matrices K and Q and Elements in D for the First Block of NEQB Equations.

Subroutine SESOL, which solves the system of simultaneous equations stored on file NSTIF, is called by the main program and the solution vector (T) is returned, block by block, from that subroutine on scratch file NL. The solution blocks are stored on file NL in reverse order. The order is corrected and the temperatures are stored, properly ordered, in the vector TI. The heat flow rate through the contact patch (QF1) and the heat flow rate through the isothermal surface at $\lambda = \lambda_{IX}$ (QF2) are calculated. QF2, which is the total heat flow rate through the substitute specimen, is used to calculate the overall resistance of the contact. Since only one-quarter of the substitute contact specimen is modelled, the total resistance (RES) of the contact, based on the extrapolated interfacial temperature drop, is calculated from the equation

$$RES = 2\left(\frac{T1 - T0}{4QF2}\right) - RESIST$$

where

T1 = temperature of isothermal surface at $\lambda = \lambda_{max}$.

T0 = temperature of isothermal surface at $\lambda = 0$ and $\mu = 0$.

RESIST = the second term in equation (I.3.7) (i.e., $\frac{2L}{k_s tw}$).

The variable RESIST must be specified (as input variable #12) before the resistance of the contact can be calculated. The value of RESIST itself can be obtained by running RECELL initially with NRES=1, INSUL=1, and RESIST=0.0 (see Table I.C.1).

I.C.2.2 Subroutine HERTZ

Input variables 16-20 inclusive are read in via subroutine HERTZ. Dyson's formulae (see section I.3.2) are used to calculate the contact dimensions in this subroutine.

I.C.2.3 Subroutine TKGAS

This subroutine calculates the gas conductance, defined by equation (I.4.64), for nodes located on the contact surface outside the elliptical contact patch (i.e., for nodes at $\mu = 0, \lambda > 0$). Three-point Gauss quadrature is used to evaluate the inner and outer integrals in equation (I.4.64). Equation (I.3.19b), in conjunction with the simplification expressed by equation (I.A.41), is used to evaluate local values of the separation distance between the contact surfaces. Equations (I.3.20) to (I.3.22) are used to approximate the elliptic integrals in equation (I.3.19b).

I.C.2.4 Subroutine NNUMB

This subroutine is used to determine which nodes lie within the laterally-insulated boundaries of the substitute contact specimen, located at $x = \pm XDIM$ cm. and $y = \pm YDIM$ cm., and, in addition, to number those nodes sequentially with the nodal subscripts (i.e., i, j, k for a node located at λ_i, μ_j, ν_k) varying from least to most rapidly in the order i, j, k . The three subscripts vary between the limits shown below

$$\lambda = 0 \text{ to } \lambda = \lambda_{\max} \rightarrow I = 1, IX$$

$$\mu = -b^2 \text{ to } \mu = 0 \rightarrow J = 1, JX$$

$$v = -a^2 \text{ to } v = -b^2 \rightarrow K = 1, KX$$

The location of each node (i, j, k) in the ellipsoidal coordinate system (i.e., λ_i, μ_j, v_k) is converted to the corresponding locations in the cartesian coordinate system (i.e., $(x, y, z)_{ijk}$) using equations (I.4.11) to (I.4.13). The values of x_{ijk} and y_{ijk} obtained in this manner are checked to see if $|x_{ijk}| > XDIM$ cm and $|y_{ijk}| > YDIM$ cm. respectively. If this check indicates that the node is located within the substitute specimen, the number of the node is stored as $NNUM(I, J, K)$. Note that I, J, K is the address of the node (i.e., the node is located at λ_i, μ_j, v_k) while $NNUM(I, J, K)$ is the number of the equation associated with that node. If a particular node lies outside of the substitute specimen, then $NNUM(I, J, K)$ is assigned a value of zero at that node. The total number of nodes contained within the substitute specimen is designated $NNEQ$ in subroutine $NNMB$ while the required number of nodal equations is designated NEQ . The latter variable (NEQ) is smaller than the former ($NNEQ$) by the number of nodes located on the isothermal surfaces at $I = 1$ ($\lambda=0$) and $I = IX$ ($\lambda=\lambda_{\max}$). Since the temperatures of these nodes are specified, their corresponding equations need not be included in the system of nodal equations.

The array $NNUM(I, J, K)$ is used in the main program to identify the nodes contained within the substitute specimen. At each value of I, J, K the value of $NNUM(I, J, K)$ is checked to see if it is zero. If $NNUM(I, J, K)$

= 0, the node is skipped and the program proceeds to the next value of I,J,K. If $NNUM(I,J,K) \neq 0$, the values of $NNUM(I \pm 1, J, K)$, $NNUM(I, J \pm 1, K)$ and $NNUM(I, J, K \pm 1)$ are checked to see if any of the neighbouring nodes (i.e., nodes at $I \pm 1, J, K$; $I, J \pm 1, K$; and $I, J, K \pm 1$) lie outside of the substitute specimen. If any of them do, the conductance between that node and node I,J,K is set equal to zero in the nodal equation for node I,J,K, as mentioned in the discussion of the main program. The nodes located inside the substitute specimen are decoupled from the nodes located outside of the specimen in this manner.

I.C.2.5 Subroutine COORD

Input variables 21 to 25 are read by subroutine COORD. The values of RHSP(1), RHSP(2) and RHSP(3) are converted to values of $\underline{x}$ by squaring them (i.e., by converting them to values of λ) and substituting the resulting squared values into the expression:

$$\underline{x} = F(\phi, k) = \ln\left(\frac{4a}{b}\right) - \ln\left(\frac{a + \sqrt{\lambda + a^2}}{\sqrt{\lambda}}\right)$$

which is valid when $k = 1$. The values of $\underline{x}$ produced in this manner are stored in the array XM and subsequently used, along with LMC1, LMC2 and IX, to calculate the mesh spacings on the $\underline{x}$ axis, DX1, DX2 and DX3. DX1 is the spacing between adjacent nodes in the range $1 \leq I \leq LMC1$, while DX2 and DX3 are the mesh spacings in the ranges $LMC1 \leq I \leq LMC2$ and $LMC2 \leq I \leq IX$ respectively. The mesh spacings DX1, DX2 and DX3 are used to calculate the values of $\underline{x}_i$ ($i=1, IX$) and the latter values are converted to values of λ_i using equations (I.4.47) and (I.4.48). The

$\underline{x}_i$ are stored in the array X and the corresponding λ_i are stored in the array XLMDP. Values of $(\underline{x}_i + \underline{x}_{i+1})/2$, which are needed to define the boundaries of the "cells" (boxes) around the mesh points, are also evaluated and stored in the array XAV(I), $I=1, IX-1$. The corresponding values of λ are stored in the array XLMAV.

The maximum value of $\underline{y}$ (i.e., $K(k')$) is calculated using equation (I.B.40) and divided by $(JX-1)$ to obtain the uniform mesh spacing, DY, on the $\underline{y}$ axis. The values of $\underline{y}_j$, ($j=1, JX$) are calculated and stored in the array Y. The $\underline{y}_j$ are also transformed, using equations (I.4.49) and (I.4.50), to values of μ_j which are stored in the array XMUP. The values of $(\frac{\underline{y}_j + \underline{y}_{j+1}}{2})$ and the corresponding values of μ are evaluated and stored in the arrays YAV and XMUAV respectively.

The maximum value of $\underline{z}$ (i.e., $K(k)$) is calculated using equation (I.B.39) and divided by $(KX-1)$ to obtain the uniform mesh spacing, DZ, on the $\underline{z}$ axis. Values of $\underline{z}_k$, v_k , $(\frac{\underline{z}_k + \underline{z}_{k+1}}{2})$ and the corresponding v are calculated for $k=1, KX$ and stored in the arrays Z, XNUP, ZAV and XNUAV respectively. Equations (I.4.51) and (I.4.52) are used to transform values of $\underline{z}$ to values of v .

I.C.2.6 Subroutine SESOL (Simultaneous Equation Solver)

This subroutine solves the system of simultaneous equations $KT=Q$ by first computing the LDL^T factorization of K and then solving the triangular system $L V=Q$ by forward substitution and the system $DL^T T=V$ by back substitution. The equations are stored in blocks on file NSTIF and T is

stored, block by block (with the blocks arranged in reverse order), on file NL. Further details on subroutine SESOL are given by Wilson, Bathe and Doherty [96].

I.C.3 Additional Features of RECELL

In addition to computing the resistance of the contact in air using the model described in Chapters I.3 and I.4, RECELL also computes two other approximations to that resistance using Kitscha and Yovanovich's "decoupled" and "quasi-coupled" models [57]. The decoupled model is similar to the quasi-coupled model (described in section I.6.42) in that the resistances of the gas and solid phases are assumed to act in parallel, only the temperature difference across the gap is assumed to be ΔT_∞ in the decoupled model instead of $\Delta T(\lambda)$ (see equation (I.6.4)). The decoupled resistance (R_{dc}) is calculated in RECELL by assuming that the solid resistance is R_{cs} and that the gas resistance (R_g) is equal to $1/K_g$, where K_g is obtained by integrating equation (I.4.64) over the area of the contact surface (A_{cs}). By evaluating R_g in this manner, we are implicitly assuming that the contact surface is isothermal. With the approximations described above, R_{dc} can be expressed as,

$$R_{dc} = \frac{1}{\frac{1}{R_c} + \frac{1}{R_g}} = \frac{1}{\frac{\pi k_s a}{K(k)} + \int_{A_{cs}} \int \frac{k_g}{\delta} \sqrt{g_v g_\lambda} dv d\lambda} \quad (I.C.i)$$

where $K(k) \rightarrow \ln\left(\frac{4a}{b}\right)$ as $k \rightarrow 1$.

RECELL computes R_{dc} and R_{qc} whenever the resistance of the contact in air is requested (i.e., when $INSUL=0$). The integrals in equations (I.C.1) and (I.6.5) are evaluated by summing contributions (calculated in subroutine TKGAS) from all of the surface elements.

I.C.4 Dictionary of Key Variables in RECELLMain Program

<u>REAL VARIABLE</u>	<u>DESCRIPTION</u>
A	a, semi-major diameter of elliptical contact
B	b, semi-minor diameter of elliptical contact
A1, A2, ... H1, H2	Elliptic integrals found in equations (I.B.10) - (I.B.12) evaluated using approximations given by equations (I.B.47) - (I.B.58).
C(L), L=1, 6	Internodal conductances at any given node (see section I.C.2.1).
CN, CS	see Table I.C.1.
CQF1(L), L=1, NNP	Internodal conductances (in the λ direction) between nodes on surfaces at $\lambda=0$ and $\lambda=\Delta\lambda$; i.e., between nodes at $I=1$ and $I=2$. These conductances are used to calculate the heat flow rate through the contact patch.
CQF2(L), L=1, NNP	Internodal conductances (in the λ direction) between nodes on surfaces at $\lambda=\lambda_{\max}$ and $\lambda=\lambda_{\max}-\Delta\lambda$; i.e., between nodes at $I=IX$ and $I=IX-1$. These conductances are used to calculate the total heat flow rate through the substitute contact specimen. Note that there will not, in general, be NNP nodes on the surface at $\lambda=\lambda_{\max}$ since only a segment of this surface is used in the model.
D(L), L=1, NAV	Vector containing nodal equations in block form, as described in section I.C.2.1.
DX12	Denominator of equation (I.B.10).
DY12	Denominator of equation (I.B.11).
DZ12	Denominator of equation (I.B.12).

EK0	Complete elliptic integral of the second kind $E(k)$, with $k=0$, (i.e., $E(0)$).
EK1	$E(k)$ with $k=1$.
KO'	See Table I.C.1.
MFP	See Table I.C.1.
QF1	Heat flow rate through contact patch.
QF2	Heat flow rate through contact specimen (total).
QVECT	Right-hand side constant in a nodal equation.
RCY1, RCY2	See Table I.C.1.
RES	Resistance of contact based on extrapolated interfacial temperature drop.
RESIST	See Table I.C.1.
SIGC	Diagonal coefficient in a nodal equation.
TI(L), L=1, NEQ	Nodal temperatures.
TKG	Integrated gas conductance for a surface element, as defined by equation (I.4.64).
TO, T1	See Table I.C.1.
X(I), I=1, IX	Array containing values of x_i .
Y(J), J=1, JX	Array containing values of y_j .
Z(K), K=1, KX	Array containing values of z_k .
XAV(I), I=1, IX-1	Array containing values of $(\frac{x_i + x_{i+1}}{2})$.
YAV(J), J=1, JX-1	Array containing values of $(\frac{y_j + y_{j+1}}{2})$.
ZAV(K), K=1, KX-1	Array containing values of $(\frac{z_k + z_{k+1}}{2})$.

XLMDP, XMUP, XNUP

Arrays containing values of λ_i , μ_j , ν_k corresponding to values of $\underline{x}_i$, $\underline{y}_j$, $\underline{z}_k$ stored in arrays X, Y and Z.

XLMAV, XMUAV, XNUAV

Arrays containing values of λ , μ and ν corresponding to values of $\underline{x}$, $\underline{y}$ and $\underline{z}$ stored in arrays XAV, YAV and ZAV.

X1, X2, Y1, Y2, Z1, Z2

Limits of integration used in evaluating the integrals A1, A2, ... H1, H2.

XDIM, YDIM

See Table I.C.1.

INTEGER VARIABLEDESCRIPTION

INSUL

See Table I.C.1.

IX

See Table I.C.1.

JX

See Table I.C.1.

KX

See Table I.C.1.

MA

Maximum right-hand bandwidth of conductance matrix. ($=JX \times KX + 1$).

MTOT

Number of high speed storage locations available for solution of system of simultaneous equations (see Table I.C.1).

NAV

Number of entries in each block of equations ($=NEQB \times (MA + 1)$).

NBLOCK

Number of equation blocks stored on file NSTIF.

NEQ

Number of nodes used in model, excluding those located on surfaces at $\lambda=0$ and $\lambda=\lambda_{\max}$. NEQ is, therefore, equal to the number of nodal equations that must be solved simultaneously.

NEQB

Number of equations/block.

NL

File on which solution vector is returned, block by block, to the main program. The blocks are arranged in reverse order on file NL.

NNP

Maximum number of nodes located on a single surface of constant λ (i.e., $NNP = JX \times KX$).

NNUM (I,J,K)	Number of nodal equation for node located at λ_i, μ_j, ν_k (see section I.C.2.1 for further details).
NR	Working file.
NRED	Working file.
NRES	See Table I.C.1.
NSTIF	File on which nodal equations are stored in block form.

SUBROUTINE TKGAS

Variables which are used in SUBROUTINE TKGAS, but have not been defined previously, are described below.

<u>VARIABLE</u>	<u>DESCRIPTION</u>
COEFF	Coefficient (C), defined by equation (I.A.41).
FTHIK	$F(\phi, k)$, given by equation (I.A.28) or by equation (I.A.32).
SEPN	Gap between contact surfaces (δ) given by equation (I.3.19b).
TC	Effective gas conductivity (k_g) given by equation (I.3.25).
W(I), I=3	Weighting factors for three-point Gauss quadrature.
X(I), I=1,3	Zeros of third-order Legendre polynomial on normalized interval of integration (i.e., $-1 \leq x \leq 1$).
XML(J), J=1,3	Actual values of λ at zeros of third-order Legendre polynomial.
XN(J), J=1,3	Actual values of ν at zeros of third-order Legendre polynomial.
XNUP	Value of λ at which expression for $F(\phi, k)$ changes from equation (I.A.28) to equation (I.A.32).

XP2	x^2 , where x is given by equation (I.4.11) with $\lambda = XLM(I)$, $v = XN(J)$, $\mu = 0$ and $C = 0$.
YP2	y^2 , where y is given by equation (I.4.12) with $\lambda = XLM(I)$, $v = XN(J)$, $\mu = 0$ and $C = 0$.

SUBROUTINE NNUMB

Variables which are used in SUBROUTINE NNUMB, but have not been defined previously, are described below.

<u>VARIABLE</u>	<u>DESCRIPTION</u>
XP	x , given by equation (I.4.11) with $C = 0$.
YP	y , given by equation (I.4.12) with $C = 0$.
NNEQ	Number of nodes used in the model, including those nodes located on the surfaces at $\lambda = 0$ and $\lambda = \lambda_{\max}$ (i.e., at $I = 1$ and $I = IX$).

SUBROUTINE HERTZ

<u>VARIABLE</u>	<u>DESCRIPTION</u>
AZZ	A , as given by equation (I.3.15).
BZZ	B , as given by equation (I.3.16).
CAF	C_a , as used in equation (I.3.17) and as defined in Table I.3.1.
CBF	C_b , as used in equation (I.3.18) and as defined in Table I.3.1.
EXPM	m , as used in equation (I.3.18) and as defined in Table I.3.1.
EXPN	n , as used in equation (I.3.17) and as defined in Table I.3.1.

P	Contact load (see Table I.C.1).
POISS	Poisson's ratio (see Table I.C.1).
THII	Ω , as given by equation (I.3.12).
YMOD	Young's modulus of elasticity (see Table I.C.1).

SUBROUTINE COORD

Variables which are used in SUBROUTINE COORD, but have not been defined previously, are described below.

<u>VARIABLE</u>	<u>DESCRIPTION</u>
LMC1, LMC2	See Table I.C.1.
RHSP(I), I=1,3	See Table I.C.1.
XM(I), I=1,3	Array containing values of x corresponding to $\lambda = \text{RHSP}(I)^{**2}$, (I=1,3).
DX1	Spacing between nodes on x axis in the range $1 \leq I \leq \text{LMC1}$.
DX2	Spacing between nodes on x axis in the range $\text{LMC1} \leq I \leq \text{LMC2}$.
DX3	Spacing between nodes on x axis in the range $\text{LMC2} \leq I \leq \text{IX}$.
XMLX (LL), LL=1,2	Variable which, at a particular value of I, assumes the value of XLMDP(I) when LL=1 and XLMAV(I) when LL=2.
XMUX (LL), LL=1,2	Variable which, at a particular value of J, assumes the value of XMUP(J) when LL=1 and XMUAV(J) when LL=2.
XNUX (LL), LL=2	Variable which, at a particular value of K, assumes the value of XNUP(K) when LL=1 and XNUAV(K) when LL=2.

TABLE I.C.1: INPUT VARIABLES FOR RECELL

The input variables defined below are presented in the order in which they must be input. The format for each variable and the number of the data card on which it should appear are also given.

VARIABLE #	VARIABLE NAME	FORMAT	DATA CARD #	DEFINITION/FUNCTION
1	INSUL	I5	1	<p>= 1, Resistance of the contact calculated without gas conduction (i.e., with the contact surface insulated outside of the elliptical contact itself).</p> <p>= 0, Resistance of the contact calculated with gas conduction.</p>
2	NRES	I5	1	<p>= 1, Resistance of contact specimen calculated with isothermal boundary condition prescribed over entire contact surface. The resistance of the contact specimen calculated in this manner (i.e., RESIST) is subtracted from the resistance of the contact calculated in the normal manner (i.e., with an isothermal elliptical contact and either gas conductances or no-flux condition prescribed over the remainder of the contact surface) to obtain the resistance based on the extrapolated interfacial temperature drop.</p> <p>Note: When NRES=1, INSUL must also = 1 and RESIST must = 0.0.</p> <p>= Q, Resistance of the contact calculated in the normal manner.</p>

VARIABLE #	VARIABLE NAME	FORMAT	DATA CARD #	DEFINITION/FUNCTION
3	IX	I3	2	Number of surfaces of constant λ or, equivalently, constant $\underline{x}$ in the transformed coordinate system - includes surfaces at $\lambda=0$ and $\lambda=\lambda_{\max}$.
4	JX	I3	2	Number of surfaces of constant μ or, equivalently, constant $\underline{y}$ in the transformed coordinate system - includes surfaces at $\mu=-b^2$ and $\mu=0$. These surfaces are equi-spaced on the $\underline{y}$ axis.
5	KX	I3	2	Number of surfaces of constant ν or, equivalently, constant $\underline{z}$ in the transformed coordinate system - includes surfaces at $\nu=-a^2$ and $\nu=-b^2$. These surfaces are equi-spaced on the $\underline{z}$ axis.
6	K0	F10.5	3	Thermal conductivity of gas between contact surfaces under continuum conditions ($W/m^\circ C$). Use thermal conductivity at appropriate temperature and pressure.
7	MFP	E10.4	3	Mean free path of gas molecules (cm).
8	CN	F10.5	3	Constant associated with non-continuum gas conductivity model, defined by: $CN = 4 \gamma (\alpha_1 + \alpha_2 - \alpha_1 \alpha_2) / (Pr \alpha_1 \alpha_2 (\gamma + 1))$ where γ = ratio of specific heats Pr = Prandtl number of gas α_1, α_2 = accommodation coefficients of surfaces.

VARIABLE #	VARIABLE NAME	FORMAT	DATA CARD #	DEFINITION/FUNCTION
9	CS	F10.5	3	Solid thermal conductivity, (W/m°C).
10	T0	F10.5	3	Temperature (°C) of isothermal plane adjacent to contact surface (i.e., temperature of elliptical contact).
11	T1	F10.5	3	Temperature (°C) of isothermal plane at opposite end of contact specimen from contact surface.
12	RESIST	F10.5	3	Resistance of contact specimen calculated with isothermal boundary condition prescribed over entire contact surface. RESIST, which is obtained by running program with RESIST=0.0, NRES=1 and INSUL=1, is actually twice the resistance of a single contact specimen (i.e., it is the resistance of a continuous length of contact specimen that is twice as long as the substitute contact specimen). RESIST should be evaluated with a particular grid before that grid is used to predict contact resistances.
13	MTOT	I6	4	<p>Number of storage spaces (words) available in fast memory for equation solving. MTOT is effectively the fast memory available to SESOL and hence is equal to the total fast memory available on a particular computer less the memory occupied by RECELL's object program and non-subscripted variables (approximately 5,000 words) and the memory occupied by arrays not used directly by SESOL. SESOL uses the arrays designated D, F and MAXA in the main program (the corresponding arrays in SESOL itself are A, B and MAXA). The array NNUM, which should be assigned the dimensions IX, JX, KX, and the vector TI, which has a maximum of M (=IX*JX*KX) entries, are by far the largest arrays not used directly by SESOL so, to a first approximation, MTOT = TOTAL FAST MEMORY - 5000 - 2M (words).</p>

VARIABLE #	VARIABLE NAME	FORMAT	DATA CARD #	DEFINITION/FUNCTION
14	XDIM	F10.5	5	XDIM is one-half of the width of the substitute contact specimen (cm), where the width of the specimen is the dimension parallel to the major (x) axis of the contact patch.
15	YDIM	F10.5	5	YDIM is one-half of the thickness of the substitute contact specimen, where the specimen's thickness is the dimension parallel to the minor (y) axis of the contact patch.
16	P	F10.5	6	Contact load (kg).
17	RCY1	F10.5	6	Radius of small-radius contact surface (cm).
18	RCY2	F10.5	6	Radius of large-radius contact surface (cm).
19	YMOD	E10.4	6	Young's modulus of contact material (kgf/cm ²).
20	POISS	F10.5	6	Poisson's ratio for contact material.
21	LMC1	I5	7	Number of node on λ (or x) axis at which $\lambda = RHSP(1)**2$, (i.e., $\lambda_i = RHSP(1)**2$ when $i = LMC1$). Mesh spacings on the x axis change from DX1 to DX2 at this node.
22	LMC2	I5	7	$\lambda_i = RHSP(2)**2$ when $i = LMC2$. Mesh spacings on the x axis change from DX2 to DX3 at this node.
23	RHSP(1)	F10.5	7	Specified "radius" of "hemispherical" surface at which mesh spacings on the x axis change from DX1 to DX2 (cm).

VARIABLE #	VARIABLE NAME	FORMAT	DATA CARD #	DEFINITION/FUNCTION
24	RHSP(2)	F10.5	7	Specified "radius" of "hemispherical" surface at which mesh spacings on the $\underline{x}$ axis change from DX2 to DX3 (cm).
25	RHSP(3)	F10.5	7	Radius of "hemisphere" used to enclose the substitute contact specimen (discussed in section I.4.4) (cm). The upper isothermal surface of the contact specimen is a segment of this hemisphere. The variables RHSP(I), I=1,3 and LMC1, LMC2, IX provide the user of RECELL with some control over the density of mesh points used in each region of the substitute specimen. This control is required because the density of mesh points, which governs how well the adiabatic boundaries of the substitute specimen are approximated, would be very low at large values of λ if the nodes on the $\underline{x}$ -axis were equi-spaced on that axis.

Listing I.C.1 RECELL

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0001 *****
0002 THIS PROGRAM COMPUTES THE RESISTANCE OF A CONTACT INVESTIGATED BY
0003 T. CURRIE IN CONNECTION WITH HIS PH.D. RESEARCH. CONTACT OCCURS
0004 BETWEEN TWO CYLINDRICAL SURFACES OF DIFFERENT RADII WITH ORTHOG.
0005 PLANES OF CURVATURE. THE DIMENSIONS OF THE ASSOCIATED ELLIPTICAL
0006 CONTACT PATCH (SEMI-MAJOR DIAMETER - A I SEMI-MINOR DIAMETER - B )
0007 ARE PREDICTED USING HERTZIAN THEORY. THE PROGRAM CAN PREDICT THE
0008 RESISTANCE OF THE CONTACT WITH AND WITHOUT GAS CONDUCTION.
0009
0010 ***** INPUT VARIABLES REQUIRED IN MAIN PROGRAM *****
0011
0012 INSUL=1 - RESISTANCE PREDICTED WITHOUT GAS CONDUCTION
0013 INSUL=0 - RESISTANCE PREDICTED WITH GAS CONDUCTION
0014
0015 MRES=1 - RESISTANCE OF CONTACT SPECIMENS CALCULATED WITH
0016 SPECIMENS CONTINUOUS AT INTERFACE : NOTE - WHEN MRES
0017 =1, INSUL MUST ALSO =1.
0018 MRES=0 - RESISTANCE OF CONTACT CALCULATED IN NORMAL MANNER.
0019
0020 IX - NUMBER OF SURFACES OF CONSTANT LAMBDA (IE. ELLIPSOIDS )
0021 OR, EQUIVALENTLY, X (TRANSFORMED COORDINATES).
0022
0023 JX - NUMBER OF SURFACES OF CONSTANT MU ( IE. ONE-SHEETED
0024 HYPERBOLOIDS) OR EQUIVALENTLY, Y (TRANSFORMED COORDINATES)
0025
0026 KX - NUMBER OF SURFACES OF CONSTANT MU ( IE. TWO-SHEETED
0027 HYPERBOLOIDS) OR EQUIVALENTLY, Z (TRANSFORMED COORDINATES)
0028
0029 KD - GAS CONTINUUM THERMAL CONDUCTIVITY (W/M.DEG.C)
0030
0031 MFP - MEAN FREE PATH OF GAS MOLECULES (CM.)
0032
0033 CN - CONSTANT ASSOCIATED WITH NON-CONTINUUM GAS CONDUCTIVITY
0034 MODEL ( SEE APPENDIX I.C IN THIS IS )
0035
0036 CS - SOLID THERMAL CONDUCTIVITY (W/M.DEG.C.)
0037
0038 TO,TI - TEMPERATURES (DEG.C) OF ISOTHERMAL PLANES BETWEEN WHICH
0039 RESISTANCE IS CALCULATED. TI SHOULD BE GREATER THAN TO.
0040
0041 RESIST - RESISTANCE (DEG.C/W) OF CONTINUOUS CONTACT
0042 SPECIMENS. RESIST CAN BE OBTAINED BY RUNNING THE
0043 PROGRAM INITIALLY WITH MRES=1.
0044
0045 NTOI - NUMBER OF STORAGE SPACES IN CORE MEMORY AVAILABLE FOR
0046 EQUATION SOLVING.
0047
0048 XBIN - ONE-HALF OF CROSS-SECTIONAL DIMENSION OF SUBSTITUTE
0049 SPECIMEN THAT IS ALIGNED WITH MAJOR AXIS OF CONTACT
0050 PATCH (CM) (RECTANGULAR X-SECTION ASSUMED)
0051
0052 YBIN - ONE-HALF OF CROSS-SECTIONAL DIMENSION OF SUBSTITUTE
0053 SPECIMEN THAT IS ALIGNED WITH MINOR AXIS OF CONTACT
0054 PATCH (CM).
0055
0056 ***** INPUT VARIABLES REQUIRED IN SUBROUTINE HERTZ *****
0057
0058 P - CONTACT LOAD (KG)
0059
0060 RCV1,RCV2 - RADII OF CONTACTING SURFACES. RCV2 SHOULD BE GREAT-
0061 ER THAN RCV1. (CM)
0062
0063 YMOD - YOUNG'S MODULUS OF CONTACTING MATERIALS (KGF/CM2).
0064
0065 POISS - POISSON'S RATIO.
0066
0067 ***** INPUT VARIABLES REQUIRED IN SUBROUTINE COORD *****
0068
0069 LNC1 - NUMBER OF NODE ON LAMBDA (OR X) AXIS AT WHICH
0070 LAMBDA=RHP(1)DEG.
0071
0072 LNC2 - NUMBER OF NODE ON LAMBDA (OR X) AXIS AT WHICH
0073 LAMBDA=RHP(2)DEG.
0074
0075 RHP(1) - SPECIFIED "RADIUS" OF "Hemispherical" SURFACE
0076 AT WHICH MESH SPACINGS ON THE X AXIS CHANGE FROM
0077 BX1 TO BX2 (CM).
0078
0079 RHP(2) - SPECIFIED "RADIUS" OF "Hemispherical" SURFACE
0080 AT WHICH MESH SPACINGS ON THE X AXIS CHANGE FROM
0081 BX2 TO BX3 (CM).
0082
0083 RHP(3) - "RADIUS" OF "Hemisphere" USED TO ENCLOSE THE
0084 SUBSTITUTE CONTACT SPECIMEN (CM). THE UPPER
0085 ISOTHERMAL SURFACE OF THE CONTACT SPECIMEN IS A
0086 SEGMENT OF THIS HEMISPHERE.
0087
0088 *****
0089
0090 IMPLICIT DOUBLE PRECISION (A-H,O-Z)
0091 REAL KD,MFP
0092 DIMENSION X(25),XAV(25),XLAMP(25),XLMU(25),Y(14),YAV(14)
0093 ,XHP(14),XHPAV(14),Z(13),ZAV(13),XHP(13),XHPAV(13)
0094 DIMENSION T(144),C(8),B(1024),F(1024),MAXA(228),
0095 ,NMAX(25,13,12),CBF1(157),CBF2(157),TI(3000)
0096 COMMON /BRI/XBIN,XLMU
0097 COMMON /BRS/EX1,XPR1
0098 COMMON /BSPOL/XBIN,YBIN
0099
0100 NTOI=0
0101
0102 NTO=NTOI+1
0103
0104 19 FORMAT ('***** THERMAL MODEL FORMULATED IN ELLIPSOIDAL COORD *****
0105
0106

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0106 READ (105,15) INSUL,MRES
0107 FORMAT (Z15)
0108 IF(INSUL.EQ.1) WRITE (108,16)
0109 IF(INSUL.NE.1) WRITE (108,17)
0110 16 FORMAT ('***** RUN WITHOUT GAS CONDUCTION *****')
0111 17 FORMAT ('***** RUN WITH GAS CONDUCTION *****')
0112 READ(JOS,2) IX,JX,KX
0113 2 FORMAT (I3)
0114 OUTPUT IX,JX,KX
0115 READ (105,1) KO,MFP,CN,CS,TOT,T1,RESIST
0116 1 FORMAT (F10.5,E10.4,F10.5)
0117 READ (105,3) MTOT
0118 3 FORMAT (I6)
0119 OUTPUT MTOT
0120 READ(105,4) XDIM,YDIM
0121 4 FORMAT (2F10.5)
0122 OUTPUT XDIM,YDIM
0123 WRITE (108,21) OS
0124 21 FORMAT (IX,'SOLID CONDUCTIVITY =','F10.5,' W/M.DEG.C., '/')
0125 IF(INSUL.NE.1) WRITE (108,22) KO,MFP,CN
0126 22 FORMAT (IX,'GAS CONTINUUM CONDUCTIVITY =','F10.5,' W/M.DEG.C., '/')
0127 1,2 //IX,'MEAN FREE PATH OF GAS MOLECULES =','E10.4,' CM., ',IX,'CM'
0128 1 //IX,'F10.5,/')
0129 KO=KO/100.0
0130 CS=CS/100.0
0131 C CALL HERTZ (A,B,RCY1,RCY2)
0132 CALL CALCULATE ELLIPTIC MODULUS (XKCOM), COMPLEMENTARY ELLIPTIC
0133 MODULUS (XCCE) AND COMPLETE ELLIPTIC INTEGRALS OF THE FIRST
0134 (XKXO AND XKXI) AND SECOND (EXO AND EXI) KINDS.
0135 C
0136 XPR2=BRR2.0/ARR2.0
0137 XPR1=1.0-XPR2
0138 XKCOM=BSRT(XPR1)
0139 XCCE=B/A
0140 XKXO=1.5707968(1.0+XPR2/4.0)
0141 XKXI=XLODB(4.0BA/B)
0142 EXO=1.5707968(1.0-XPR2/4.0)
0143 EXI=(1.0+XPR2/XKX1/2.0)/(1.0+XPR2/4.0)
0144 C
0145 JX=JX-1
0146 KX=KX-1
0147 C
0148 CALL COORD (IX,JX,KX,XKX,XCCE,A,B,X,XAV,XLMBP,XLMW,Y,YAV
0149 X,XUP,XDMM,Z,ZAU,XMUP,XMAU)
0150 C
0151 CALCULATE MAXIMUM HALF-BANDWIDTH (MA) OF CONDUCTANCE MATRIX.
0152 C
0153 MNP=JXKX
0154 MA=MNP+1
0155 RX=2.0ALODB(4.0BA/B)/(3.14159BCSA)
0156 OS=(T1-T0)/RX
0157 TXP=T0
0158 TCON=0.0
0159 OTOT=0.0
0160 XLNDA=0.0
0161 NRODE=0
0162 C
0163 CALL MNUP (IX,XKX,XLMBP,MNP,XUP,MEO,MNUM,A,B)
0164 C
0165 SPECIFY NUMBER OF RIGHT-HAND SIDES (NV) AND ASSIGN SCRATCH
0166 FILE NUMBERS (NSTIF,NREB,NL,NR)
0167 C
0168 NV=1
0169 NSTIF=1
0170 NREB=2
0171 NL=1
0172 NR=4
0173 C
0174 COMPUTE INFORMATION REQUIRED TO STORE THE CONDUCTANCE MATRIX ON
0175 A SCRATCH FILE (IE, NUMBER OF BLOCKS AND NUMBER OF EQUATIONS PER
0176 BLOCK). ALSO COMPUTE DIMENSIONS FOR ARRAYS IN SUBROUTINE SEND.
0177 C
0178 NK=(NEB(MA+NVP)+NVP)/NBEB((MA-2)/NEB+2)+NEB+MA-1
0179 IF(NTOT.NE.NK) GO TO 10
0180 NK1=(NTOT-NA)/(2*(MA+NVP)+1)
0181 NK2=(NTOT-NA-NVP(MA-2))/(3*MA+MA+1)
0182 NEB=NINB(NK1,NK2)
0183 NBLOCK=(NEB-1)/NEB+1
0184 MAV=NEB(MA+NVP)
0185 NI=NEB+MA-1
0186 GO TO 6
0187 10 NEB=NEB
0188 NBLOCK=1
0189 MAV=NEB(MA+NVP)
0190 NI=NEB+MA-1
0191 C
0192 DO 7 L=1,NAV
0193 B(L)=0.0
0194 REWIND NSTIF
0195 LNAX=0
0196 LNIZ=0
0197 JLLOC=1
0198 C
0199 ZERO CONDUCTANCES BELONGING TO NODES AT LAMBDA=0 AND LAMBDA=FINAL
0200 VALUE .
0201 C
0202 DO 8 L=1,MNP
0203 CBF1(L)=0.0
0204 CBF2(L)=0.0
0205 CONTINUE
0206 IXX=IX-1
0207 C
0208 SWEEP THROUGH ELLIPSOIDS (IE, VALUES OF LAMBDA)
0209 C
0210 DO 20 I=2,IXX
0211 IZ=1
0212 C
0213 CALCULATE ELLIPTIC INTEGRALS A1,A2,B1,B2 REQUIRED IN INTERNAL
0214 CONDUCTANCE EXPRESSIONS (SEE THESIS).
0215 C
0216 XLAI=XLAU(I-1)
0217 XLBI=XLAU(I)
0218 AI=2.0XLAI(I-1)/A
0219 BI=2.0XLBI(I)/A
0220

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00224 IF(XAU(I-1),LE,2,3) B1=2.848*(EP1-EO1)
00227 IF(XAU(I),LE,2,3) B2=2.848*(EP2-EO2)
00228 EO1=A/SQRT(XLM1+AS2,0)
00229 EO2=A/SQRT(XLM2+AS2,0)
00230 IF(XAU(I-1),GT,2,3) B1=2.848*(EP1-EK1-XPR18EO18EO1)
00231 IF(XAU(I),GT,2,3) B2=2.848*(EP2-EK1-XPR18EO28EO2)
00232
00233 SWEET THROUGH ONE-SHEETED HYPERBOLOIDS (IE. VALUES OF MU)
00234 DO 30 J=1,N
00235
00236 CALCULATE ELLIPTIC INTEGRALS C1,C2,D1,D2 REQUIRED IN INTERMODAL
00237 CONDUCTANCE EXPRESSIONS (SEE THESIS)
00238
00239 XMU1=XMUP(J)
00240 XMU2=XMUP(J)
00241 IF(J.NE.1) XMU1=XMUAU(J-1)
00242 IF(J.NE.N) XMU2=XMUAU(J)
00243 Y1=Y(J)
00244 Y2=Y(J)
00245 IF(J.NE.1) Y1=YAU(J-1)
00246 IF(J.NE.N) Y2=YAU(J)
00247 C1=2.08Y1/A
00248 C2=2.08Y2/A
00249 IF(Y1.LE,0,79) D1=-2.08SQRT((-XMU1)*(XMU1+B22,0)/
00250 (XMU1+AS2,0))
00251 IF(Y2.LE,0,79) D2=-2.08SQRT((-XMU2)*(XMU2+B22,0)/
00252 (XMU2+AS2,0))
00253 IF(Y1.GT,0,79) D1=-1.5797948*(B22,0)/A
00254 IF(Y2.GT,0,79) D2=-1.5797948*(B22,0)/A
00255
00256 SWEET THROUGH TWO-SHEETED HYPERBOLOIDS (IE. VALUES OF MU).
00257 DO 40 K=1,KX
00258
00259 CALCULATE ELLIPTIC INTEGRALS G1,G2,M1,M2 REQUIRED IN INTERMODAL
00260 CONDUCTANCE EXPRESSIONS (SEE THESIS).
00261
00262 Z1=Z(K)
00263 Z2=Z(K)
00264 IF(K.NE.1) Z1=ZAU(K-1)
00265 IF(K.NE.KX) Z2=ZAU(K)
00266 G1=2.08Z1/A
00267 G2=2.08Z2/A
00268 XMU1=XMUP(K)
00269 XMU2=XMUP(K)
00270 IF(K.NE.1) XMU1=XMUAU(K-1)
00271 IF(K.NE.KX) XMU2=XMUAU(K)
00272 F2=SQRT((XMU2+AS2,0)/(AS2,0-B22,0))
00273 F1=SQRT((XMU1+AS2,0)/(AS2,0-B22,0))
00274 IF(Z1.LE,2,3) M1=-2.08AS1
00275 IF(Z2.LE,2,3) M2=-2.08AS2
00276 G01=0.0
00277 IF(K.NE.1) G01=SQRT((AS2,0+XMU1)*(B22,0+XMU1)/XMU1)/A
00278 IF(K.NE.KX) G02=SQRT((AS2,0+XMU2)*(B22,0+XMU2)/XMU2)/A
00279 G01=SQRT((B22,0+XMU1)/XMU1)/XKCON
00280 G02=SQRT((B22,0+XMU2)/XMU2)/XKCON
00281 IF(K.NE.KX) G002=SQRT((B22,0+XMU2)/XKCON)
00282 IF(Z1.GT,2,3) M1=-2.08AS1*(EK1+G01-G001)
00283 IF(Z2.GT,2,3) M2=-2.08AS2*(EK1+G02-G002)
00284
00285 CHECK IF NODE LIES WITHIN THE BOUNDARIES OF THE SUBSTITUTE
00286 SPECIMEN, IF NOT, PROCEED TO NEXT NODE.
00287 IF(MIN(X1,J,K),EQ,0) GO TO 40
00288 LMAX=LMAX+1
00289 LMAX=LMAX+1
00290 ME=MEOS
00291 SISC=0.0
00292
00293 CALCULATE (SUCCESSIVELY) THE SIX (AT MOST) CONDUCTANCES ASSOCIATED
00294 WITH EACH NODE.
00295 DO 45 L=1,6
00296 C(L)=0.0
00297
00298 CHECK IF NODE IS BOUNDARY NODE. ALSO CHECK EACH CONDUCTANCE TO SEE
00299 IF APPROPRIATE NEIGHBORING NODE LIES WITHIN THE BOUNDARIES OF THE
00300 SUBSTITUTE SPECIMEN.
00301 IF(MIN(X1-1,J,K),EQ,0,AND,L,EO,2) GO TO 45
00302 IF(J,EO,1,AND,L,EO,4) GO TO 45
00303 IF(J,EO,1) GO TO 31
00304 IF(MIN(X1,J-1,K),EQ,0,AND,L,EO,4) GO TO 45
00305 IF(K,EO,1,AND,L,EO,4) GO TO 45
00306 IF(K,EO,1) GO TO 32
00307 IF(MIN(X1,J,K-1),EQ,0,AND,L,EO,4) GO TO 45
00308 IF(MIN(X1+1,J,K),EQ,0,AND,L,EO,1) GO TO 45
00309 IF(J,EO,N,AND,L,EO,3) GO TO 45
00310 IF(J,EO,N) GO TO 33
00311 IF(MIN(X1,J+1,K),EQ,0,AND,L,EO,3) GO TO 45
00312 IF(K,EO,N,AND,L,EO,5) GO TO 45
00313 IF(K,EO,N) GO TO 34
00314 IF(MIN(X1,J,K+1),EQ,0,AND,L,EO,5) GO TO 45
00315 IF(L,NE,1,AND,L,EO,2) GO TO 50
00316
00317 CALCULATE CONDUCTANCES IN THE LAMBDA DIRECTION.
00318 X1=X(I)
00319 X2=X(I+1)
00320 IF(L,EO,2) X1=X(I-1)
00321 XK12=2.08(X2-X1)/A
00322 C(L)=0.5828*(B2-G1)*(B2-G1)-(M2-M1)*(C2-C1)/BX12
00323 IF(L,EO,2,AND,J,EO,1,AND,I,LT,IX,AND,K,EO,1) TXL=TXP+(0.881,008X12
00324 /3.1415926)
00325 IF(L,EO,2,AND,J,EO,1,AND,I,LT,IX,AND,K,EO,1) TXP=TXL
00326 IF(L,NE,3,AND,L,EO,4) GO TO 50
00327
00328 CALCULATE CONDUCTANCES IN THE MU DIRECTION.
00329 Y1=Y(J)
00330 Y2=Y(J)
00331 IF(L,EO,3) Y2=Y(J+1)
00332 IF(L,EO,4) Y1=Y(J-1)

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0346 C CALCULATE CONDUCTANCES IN THE MU DIRECTION. 0035.500
0347 C 0035.400
0348 Z1=Z(K) 0035.700
0349 Z2=Z(K) 0035.800
0350 IF(L.EQ.5) Z2=Z(K+1) 0035.900
0351 IF(L.EQ.6) Z2=Z(K-1) 0036.000
0352 DZ12=2.0*(Z2-Z1)/A 0036.100
0353 C(L)=0.58CS8*(C2-C1)*(B2-B1)-(B2-D1)*(A2-A1)/DZ12 0036.200
0354 C 0036.300
0355 C STORE CONDUCTANCES IN BLOCK TO BE TRANSFERRED TO SCRATCH FILE. 0036.400
0356 C 0036.500
0357 60 IF(L.EQ.5) D(LMAX+ME)=-C(L) 0036.600
0358 IF(L.EQ.3) MMXC=MMUM(I,J+1,K)-MMUM(I,J,K) 0036.700
0359 IF(L.EQ.3) D(LMAX+MMXCME)=-C(L) 0036.800
0360 IF(L.EQ.IXX) GO TO 66 0036.900
0361 IF(L.EQ.1) MMXD=MMUM(I+1,J,K)-MMUM(I,J,K) 0037.000
0362 IF(L.EQ.1) D(LMAX+MMXDME)=-C(L) 0037.100
0363 C 0037.200
0364 C STORE CONDUCTANCES (IN THE LAMBDA DIRECTION) BELONGING TO MODES AT 0037.300
0365 LAMBDA=0 AND LAMBDA=LAMBDA FINAL.THESE CONDUCTANCES ARE USED 0037.400
0366 C LATER TO EVALUATE HEAT FLOWS. 0037.500
0367 C 0037.600
0368 64 IF(I.EQ.2,AND,L.EQ.2) COF1(LMAXX)=C(L) 0037.700
0369 IF(I.EQ.IXX,AND,L.EQ.1,AND,MMUM(I,J,K).NE.0,AND,MMUM(I+1,J,K).NE.0 0037.800
0370 ) MMODE=MMODE+1 ; COF2(MMODE)=C(L) 0037.900
0371 SIGC=SIGC+C(L) 0038.000
0372 QVECT=Q.0 0038.100
0373 C 0038.200
0374 C INCLUDE BOUNDARY CONDITIONS. 0038.300
0375 C 0038.400
0376 IF(I.EQ.2) QVECT=QVECT+C(2)*TO 0038.500
0377 IF(I.EQ.IXX) QVECT=QVECT+C(1)*T1 0038.600
0378 IF(MRES.EQ.1,AND,J.EQ.IX) SIGC=SIGC*(10.0**12) ; QVECT=SIGC*TO 0038.700
0379 IF(INSUL.EQ.1) GO TO 39 0038.800
0380 IF(J.EQ.IX,AND,I.OT.1) GO TO 46 0038.900
0381 GO TO 39 0039.000
0382 44 XMU1=XMUP(K) 0039.100
0383 XMU2=XMUP(K) 0039.200
0384 IF(K.NE.1) XMU1=XMAV(K-1) 0039.300
0385 IF(K.NE.KX) XMU2=XMAV(K) 0039.400
0386 CALL TKBAS (XMAV(I-1),XMAV(I),XMU1,XMU2,A,B,KO,HFP, 0039.500
0387 ) CM,TKB,R,RCY1,RCY2,KX,I2) 0039.600
0388 IF(TKB.LT.0.0) OUTPUT I,J,K,XMAV(I-1),XMAV(I),XMU1, 0039.700
0389 ) XMU2,TKB 0039.800
0390 SIGC=SIGC+2.0*TKB 0039.900
0391 QVECT=QVECT+2.0*TKB*TO 0040.000
0392 TQCON=TQCON+2.0*TKB 0040.100
0393 QTOT=QTOT+((TXL-TO)*2.0*TKB) 0040.200
0394 D(LMAX)=SIGC 0040.300
0395 IF(SIGC.LE.0.0) OUTPUT I,J,K,SIGC,LMAX 0040.400
0396 D(LMAX+MMSE)=-QVECT 0040.500
0397 C 0040.600
0398 C CHECK IF BLOCK OF MODAL EQUATIONS IS COMPLETE. 0040.700
0399 C 0040.800
0400 IF(LMAX.EQ.MEGB,OR,LMAXX.EQ.MEGB) GO TO 42 0040.900
0401 GO TO 40 0041.000
0402 LMAX=0 0041.100
0403 C 0041.200
0404 C TRANSFER BLOCK OF MODAL EQUATIONS TO SCRATCH FILE. 0041.300
0405 C 0041.400
0406 WRITE (MTIF) (B(IL),IL=1,MAV) 0041.500
0407 JBLDC=JBLOC+1 0041.600
0408 DO 54 LI=1,MAV 0041.700
0409 B(LI)=0.0 0041.800
0410 C 0041.900
0411 C PROCEED TO NEXT MODE. 0042.000
0412 C 0042.100
0413 40 CONTINUE 0042.200
0414 30 CONTINUE 0042.300
0415 20 CONTINUE 0042.400
0416 END FILE 0042.500
0417 OTTOT=QTOT+Q8 0042.600
0418 TRES=(T1-TO)/OTTOT 0042.700
0419 TRES=TRES/2.0 0042.800
0420 TQC=TQCON/(1.0/RX) 0042.900
0421 RTOT=0.5/TQC 0043.000
0422 C 0043.100
0423 C TRANSFER CONTROL TO EQUATION SOLVING SUBROUTINE. 0043.200
0424 C 0043.300
0425 CALL SESOL (B,F,MAXA,MEGB,MA,MV,NBLOK,MEGB,MAV,MI,MTIF,MRED,ML,MR 0043.400
0426 ) 0043.500
0427 C 0043.600
0428 C TEMPERATURE VECTOR (T) IS RETURNED FROM SESOL WITH ELEMENTS IN 0043.700
0429 C REVERSE ORDER. CORRECT THE ORDER AND PLACE MODAL TEMPERATURES IN 0043.800
0430 C ARRAY T1. 0043.900
0431 C 0044.000
0432 REMIND ML 0044.100
0433 DO 125 LI=1,NBLOK 0044.200
0434 ME=MEGB 0044.300
0435 IF(L.EQ.1) ME=MEGB-(NBLOK-1)*MEGB 0044.400
0436 READ (ML) (T(IL),IL=1,MEGB) 0044.500
0437 DO 126 II=1,ME 0044.600
0438 ME=(NBLOK-L)*MEGB+II 0044.700
0439 T(ME)=T(II) 0044.800
0440 CONTINUE 0044.900
0441 125 CONTINUE 0045.000
0442 C 0045.100
0443 C CALCULATE HEAT FLOWS AT LAMBDA=0 AND LAMBDA=LAMBDA FINAL. 0045.200
0444 C 0045.300
0445 QF1=0.0 0045.400
0446 QF2=0.0 0045.500
0447 MMODE=0 0045.600
0448 L1=0 0045.700
0449 DO 150 J=1,JX 0045.800
0450 DO 140 K=1,KX 0045.900
0451 L1=L1+1 0046.000
0452 QF1=QF1+COF1(L1)*T1(L1)-TO 0046.100
0453 IF(MMUM(IXX,J,K).NE.0,AND,MMUM(I,J,K).NE.0) MMODE=MMODE+1 ; 0046.200
0454 QF2=QF2+COF2(MMODE)*T1-T1(MMUM(IXX,J,K)-MMUP) 0046.300
0455 CONTINUE 0046.400
0456 150 CONTINUE 0046.500
0457 C 0046.600
0458 C CALCULATE RESISTANCE OF CONTACT. 0046.700
0459 C 0046.800
0460 RES=((T1-TO)/QF2)/4.0)*82.0-RESIST 0046.900
0461 IF(MRES.EQ.1) RESIST=RES 0047.000
0462 WRITE (100,200) RESIST 0047.100
0463 200 FORMAT (IX,'RESISTANCE OF SUBSTITUTE SPECIMEN WITH BOTH ENDS ISOTM 0047.200
0464 ) FPRAL = 'E10.3,' DEB.C/M ' 0047.300
0465 IF(MRES.NE.1) WRITE (100,202) RES 0047.400

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0444 202 FORMAT (1X,'RESISTANCE OF CONTACT CALCULATED USING SHAPE FACTOR TE 0047.500
0447 1 CHWIQUE = 'F10.5,') 0047.600
0448 IF (INSL.NE.1) WRITE (100,201) RIOT,TRES 0047.700
0449 201 FORMAT (1X,'RESISTANCE OF CONTACT WITH GAS AND SOLID RESISTANCES I 0047.800
0470 1 N PARALLEL = 'F10.5,' DEG.C/W ' //1X,' RESISTANCE OF CONTACT CA 0047.900
0471 1 CALCULATED USING YOVANOVICH'S METHOD = 'F10.5,' DEG.C/W ' //) 0048.000
0472 RATIO=OF1/OF2 0048.100
0473 WRITE (100,203) RATIO 0048.200
0474 203 FORMAT (1X,'RATIO OF HEAT FLOW THROUGH CONTACT PATCH TO TOTAL HEAT 0048.300
0475 1 FLOW= 'F10.5,') 0048.400
0476 IF (NTOL.LT.6) GO TO 756 0048.500
0477 STOP 0048.600
0478 END 0048.700

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0001 SUBROUTINE SESOL (A,B,MAXA,NEQ,MA,MV,NBLOCK,NEQB,MAV,MI,NSTIF,NRED 0048.800
0002 1 ,ML,NR) 0048.900
0003 1 0049.000
0004 ***** 0049.100
0005 THIS SUBROUTINE CALCULATES THE SOLUTION OF THE SIMULTANEOUS 0049.200
0006 LINEAR EQUATIONS 0049.300
0007 0049.400
0008 K X = R 0049.500
0009 0049.600
0010 WHERE 0049.700
0011 0049.800
0012 K = POSITIVE DEFINITE (OR DIAGONALLY DOMINANT) MATRIX 0049.900
0013 R = SUPPLIED RIGHT HAND SIDE VECTORS 0050.000
0014 X = CALCULATED SOLUTION VECTORS 0050.100
0015 0050.200
0016 THE MATRIX K AND THE RIGHT HAND SIDE VECTORS R ARE READ IN BLOCKS 0050.300
0017 FROM SCRATCH FILE NSTIF. THE SOLUTION VECTORS X ARE STORED IN 0050.400
0018 BLOCKS IN REVERSE ORDER ON SCRATCH FILE ML. 0050.500
0019 0050.600
0020 ***** INPUTS ***** 0050.700
0021 0050.800
0022 NEQ = NUMBER OF EQUATIONS 0050.900
0023 0051.000
0024 MA = MAXIMUM HALF BANDWIDTH OF K (INCL. DIAGONAL) 0051.100
0025 0051.200
0026 MV = NUMBER OF RHS. VECTORS 0051.300
0027 0051.400
0028 NBLOCK = NUMBER OF BLOCKS 0051.500
0029 0051.600
0030 NEQB = NUMBER OF EQUATIONS IN EACH BLOCK 0051.700
0031 0051.800
0032 MAV = NEQB*(MA+MV) 0051.900
0033 0052.000
0034 MI = MA+NEQB-1 0052.100
0035 0052.200
0036 NSTIF,NRED,ML,NR = SCRATCH FILE NUMBERS 0052.300
0037 0052.400
0038 A , B , MAXA = STORAGE ARRAYS 0052.500
0039 ***** 0052.600
0040 0052.700
0041 0052.800
0042 IMPLICIT DOUBLE PRECISION (A-H,O-Z) 0052.900
0043 DIMENSION A(MAV),B(MAV),MAXA(MI) 0053.000
0044 0053.100
0045 NM=1 0053.200
0046 MA2=MA-2 0053.300
0047 IF (MA2.EQ.0) MA2=1 0053.400
0048 INC=NEQB-1 0053.500
0049 MMA=NEQB*MA 0053.600
0050 NTB=(MA-2)/NEQB+1 0053.700
0051 NEB=NTB*NEQB 0053.800
0052 NEBT=NEB+NEQB 0053.900
0053 MAV=NEQB*MV 0054.000
0054 MAVU=NEBT*MV 0054.100
0055 NI=ML 0054.200
0056 N2=NR 0054.300
0057 REWIND NSTIF 0054.400
0058 REWIND NRED 0054.500
0059 REWIND MI 0054.600
0060 REWIND N2 0054.700
0061 0054.800
0062 C MAIN LOOP OVER ALL BLOCKS 0054.900
0063 C 0055.000
0064 DO 400 NJ=1,NBLOCK 0055.100
0065 IF (NJ.NE.1) GO TO 10 0055.200
0066 READ (NSTIF) A 0055.300
0067 IF (NEQ.GT.1) GO TO 100 0055.400
0068 MAXA(1)=1 0055.500
0069 WRITE (NRED) A,MAXA 0055.600
0070 IF (A(1)) 1,174,3 0055.700
0071 1 KK=1 0055.800
0072 WRITE (100,1010) KK,A(1) 0055.900
0073 DO 5 L=1,MV 0056.000
0074 A(L+1)=A(L+1)/A(1) 0056.100
0075 KR=1+MV 0056.200
0076 WRITE (ML) (A(KK),KK=2,KR) 0056.300
0077 RETURN 0056.400
0078 IF (NTB.EQ.1) GO TO 100 0056.500
0079 REWIND N1 0056.600
0080 REWIND N2 0056.700
0081 READ (MI) A 0056.800
0082 C FIND COLUMN HEIGHTS 0056.900
0083 C 0057.000
0084 KU=1 0057.100
0085 KM=NINO(MA,NEQB) 0057.200
0086 MAXA(1)=1 0057.300
0087 DU 110 N=2,MI 0057.400
0088 IF (N-MA) 120,120,130 0057.500
0089 KU=KU+NEQB 0057.600
0090 KK=KU 0057.700
0091 NM=NINO(N,KM) 0057.800
0092 GO TO 140 0057.900
0093 130 KU=KU+1 0058.000
0094 KK=KU 0058.100
0095 IF (N-NEQB) 140,140,134 0058.200
0096 NM=NM-1 0058.300
0097 140 DO 160 K=1,NM 0058.400
0098 IF (A(KK)) 110,140,110 0058.500
0099 KK=KA-INC 0058.600
0100 MAXA(N)=KK 0058.700
0101 IF (A(1)) 172,174,174 0058.800
0102 174 KK=(NJ-1)*NEQB+1 0058.900

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0103      IF(KK.GT.MEQ) GO TO 590
0104      WRITE (108,1000) KK
0105      STOP
0106      KK=(N1-1)*MEQB+1
0107      WRITE (108,1010) KK,A(1)
0108      C
0109      FACTORIZE LEADING BLOCK
0110      C
0111      DO 200 N=2,MEQB
0112      MM=MAXA(N)
0113      IF(MM-N) 200,200,210
0114      210      KL=N+INC
0115      K=N
0116      D=0.0
0117      DO 220 KK=KL,MM,INC
0118      K=K-1
0119      C=A(KK)/A(K)
0120      D=D+C*A(KK)
0121      A(KK)=C
0122      A(K)=A(N)-D
0123      IF(A(N)) 222,224,230
0124      KK=(N1-1)*MEQB+N
0125      IF(KK.GT.MEQ) GO TO 590
0126      WRITE (108,1000) KK
0127      STOP
0128      KK=(N1-1)*MEQB+N
0129      WRITE (108,1010) KK,A(N)
0130      IC=MEQB
0131      DO 240 J=1,MA2
0132      NJ=MAXA(N+J)-IC
0133      IF(NJ-N) 240,240,280
0134      KU=N+INC(NJ,NH)
0135      KN=N+IC
0136      C=0.0
0137      DO 300 KK=KL,KU,INC
0138      C=C+A(KK)*A(KN+IC)
0139      B(KN)=A(KN)-C
0140      IC=IC+MEQB
0141      K=N+MM
0142      DO 430 L=1,MV
0143      LJ=L
0144      C=0.0
0145      DO 440 KK=KL,MM,INC
0146      KJ=KJ-1
0147      C=C+A(KK)*A(KJ)
0148      A(K)=A(K)-C
0149      K=K+MEQB
0150      200      CONTINUE
0151      C
0152      CARRY OVER INTO TRAILING BLOCKS
0153      C
0154      DO 400 NK=1,MTB
0155      IF((NK+N1).GT.NBLOCAT) GO TO 400
0156      NI=N1
0157      IF((N1.EQ.1).OR.(NK.EQ.MTB)) NI=NSTIF
0158      REAR(N1) B
0159      NL=NR*MEQB+1
0160      MR=N1+((NK+1)*MEQB,NI)
0161      IF(MA.EQ.1) ML=MR
0162      MD=NI-NL
0163      KL=MEQB+(NK-1)*MEQB+MEQB
0164      N=1
0165      DO 500 M=ML,MR
0166      MM=MAXA(M)
0167      KL=KL+MEQB
0168      IF(MM-KL) 505,510,510
0169      K=MEQB
0170      D=0.0
0171      DO 520 KK=KL,MM,INC
0172      C=A(KK)/A(K)
0173      D=D+C*A(KK)
0174      A(KK)=C
0175      K=K-1
0176      B(N)=B(N)-D
0177      IF(MD) 580,580,530
0178      IC=MEQB
0179      DO 540 J=1,ND
0180      NJ=MAXA(M+J)-IC
0181      IF(NJ-KL) 540,550,550
0182      KU=N+INC(NJ,NH)
0183      KN=N+IC
0184      C=0.0
0185      DO 575 KK=KL,KU,INC
0186      C=C+A(KK)*A(KN+IC)
0187      B(KN)=B(KN)-C
0188      IC=IC+MEQB
0189      KN=N+MM
0190      K=MEQB+MM
0191      DO 610 L=1,MV
0192      LJ=L
0193      C=0.0
0194      DO 620 KK=KL,MM,INC
0195      C=C+A(KK)*A(KJ)
0196      KJ=KJ-1
0197      B(KN)=B(KN)-C
0198      KN=KN+MEQB
0199      K=K+MEQB
0200      NB=NB-1
0201      N=N+1
0202      IF(MTB.NE.1) GO TO 560
0203      WRITE (NRD) A,MAXA
0204      DO 570 J=1,MMV
0205      A(I)=B(I)
0206      GO TO 600
0207      540      WRITE (N2) B
0208      400      CONTINUE
0209      N=N1
0210      M1=M2
0211      N2=N
0212      590      WRITE (NRD) A,MAXA
0213      600      CONTINUE
0214      C
0215      VECTOR BACKSUBSTITUTION
0216      C
0217      DO 700 K=1,MMV
0218      B(K)=0.0
0219      REWIND NL
0220      DO 800 N=1,NBLOCAT
0221      BACKSPACE NRD
0222      READ (NRD) A,MAXA

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0059.000
0059.100
0059.200
0059.300
0059.400
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0070.500
0070.600
0070.700
0070.800
0070.900

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0223      BACKSPACE NREB
0224      K=NEBT
0225      DO 810 L=1,MV
0226      DO 820 I=1,NEB
0227      B(K)=B(K-NEOB)
0228      K=K-1
0229      L=K+NEBT+NEB
0230      K=0
0231      K=K+MVA
0232      NDIF=NEOB
0233      IF (NJEQ.1) NDIF=NEOB-(NMBLOCK+NEOB-NEO)
0234      DO 855 L=1,MV
0235      DO 850 K=1,NDIF
0236      B(K+K)=B(K+K)/A(K)
0237      K=K+NEOB
0238      855 IF (NAEQ.1) GO TO 915
0239      ML=NEOB+1
0240      KL=NEOB
0241      DO 840 M=ML,M1
0242      AL=KL+NEOB
0243      KU=MAXA(M)
0244      IF (KU-KL) 840,870,870
0245      K=NEOB
0246      K=M
0247      DO 880 L=1,MV
0248      K=K
0249      DO 890 KK=KL,KU,INC
0250      B(KJ)=B(KJ)-A(KK)B(KM)
0251      K=KJ-1
0252      K=K+NEBT
0253      K=K+NEBT
0254      880 CONTINUE
0255      840 N=NEOB
0256      DO 910 I=2,NEOB
0257      AL=N+INC
0258      KU=MAXA(M)
0259      IF (KU-KL) 910,920,920
0260      K=N
0261      DO 930 L=1,MV
0262      K=K
0263      DO 940 KK=KL,KU,INC
0264      K=KJ-1
0265      B(KJ)=B(KJ)-A(KK)B(K)
0266      K=K+NEBT
0267      N=N-1
0268      K=0
0269      K=0
0270      DO 950 L=1,MV
0271      DO 940 K=1,NEOB
0272      KK=KK+1
0273      A(KK)=B(KK+K)
0274      K=K+NEBT
0275      WRITE (ML) (A(K),K=1,MV)
0276      CONTINUE
0277      FORMAT (//44H STOP *** ZERO DIAGONAL ENCOUNTERED DURING
0278      1 1BN EQUATION SOLUTION, /
0279      1 1X,1BN EQUATION NUMBER =,16 )
0280      FORMAT (/ 50H WARNING *** NEGATIVE DIAGONAL ENCOUNTERED DURING
0281      1 1BN EQUATION SOLUTION, /
0282      1 1X,1BN EQUATION NUMBER =,16, 5X, 7HVALUE =, E20.8 )
0283      RETURN
0284      END
0285

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0001      SUBROUTINE TKGAS (XLM1,XLM2,XMU1,XMU2,A,B,KD,MFP,CN,TKD,K,
0002      1 RY1,RY2,AX,I2)
0003      1 RY1,RY2,AX,I2)
0004      *****
0005      *****
0006      THIS SUBROUTINE COMPUTES THE GAS CONDUCTANCE ASSOCIATED WITH EACH
0007      MODE LOCATED ON THE SURFACE AT MU=0. THE CONDUCTANCE IS
0008      EVALUATED BY NUMERICALLY INTEGRATING LOCAL VALUES OF THE GAS
0009      CONDUCTANCE OVER THE AREA OF THE SURFACE ELEMENT. THREE-POINT GAUS
0010      QUADRATURE IS USED TO EVALUATE THE INNER AND OUTER INTEGRALS.
0011      HERTZIAN THEORY IS USED TO EVALUATE LOCAL SEPARATION DISTANCES
0012      BETWEEN THE CONTACTING SURFACES.
0013      *****
0014      *****
0015      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
0016      REAL KD,MFP
0017      DIMENSION XLM(3),XM(3),X(3),M(3)
0018      COMMON /DBGS/EX1,XPR1
0019      IF (I2.EQ.2) XLM1=2.0E-8
0020      *****
0021      *****
0022      STORE WEIGHTING FACTORS AND ZEROS OF THIRD ORDER LEGENDRE
0023      POLYNOMIAL ON NORMALIZED INTERVAL OF INTEGRATION.
0024      *****
0025      X(1)=0.0
0026      M(1)=.88888
0027      X(2)=.774597
0028      M(2)=.55555
0029      X(3)=-X(2)
0030      M(3)=M(2)
0031      *****
0032      CONVERT ZEROS ON NORMALIZED INTERVALS OF INTEGRATION TO ACTUAL
0033      VALUES OF LAMBDA AND MU.
0034      *****
0035      DO 10 J=1,3
0036      XLM(J)=((XLM1+XLM2)/2.0)+(((XLM2-XLM1)/2.0)*X(J))
0037      XM(J)=((XMU1+XMU2)/2.0)+(((XMU2-XMU1)/2.0)*X(J))
0038      CONTINUE
0039      SUM2=0.0
0040      *****
0041      SWEEP THROUGH ZEROS OF OUTER INTEGRAL (IE. INTEGRAL WITH LAMBDA
0042      AS THE VARIABLE OF INTEGRATION).
0043      *****
0044      DO 15 I=1,3
0045      SUM=0.0
0046      *****
0047      SWEEP THROUGH ZEROS OF INNER INTEGRAL (IE. INTEGRAL WITH MU
0048      AS THE VARIABLE OF INTEGRATION).
0049      *****
0050      DO 20 J=1,3
0051      *****
0052      CONVERT ELLIPSOIDAL COORDINATES TO CARTESIAN COORDINATES.

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0053 C
0054
0055 XZ=(A822.0+XLN(I))*A822.0+XN(J)/(A822.0-B822.0)
0056 YP2=(B822.0+XLN(I))*B822.0+XN(J)/(B822.0-A822.0)
0057 XMUS=XLN(I)
0058 XNUP=ASB
0059 IF(XMUS.LE.XNUP) FTHIX=ALOG((SORT(XMUS)+SORT(XMUS+B822.0))/B)
0060 IF(XMUS.GT.XNUP) FTHIX=ALOG(4.08A/B)-ALOG(SORT((A822.0/XMUS))+SOR
0061 T((XNUS+A822.0)/XMUS))
0062 RS=SORT(XMUS/(XMUS+B822.0))
0063 IF(XMUS.LE.XNUP) R3=RS
0064 IF(XMUS.GT.XNUP) R3=EX1+XPR18A8R5/SORT(A822.0+XMUS)-(A/
0065 S(SORT(A822.0+XMUS)))
0066 COEFF=-1.08A8(B822.0)*(RCY1+RCY2)/(4.08RCY1RCY2)
0067
0068 EVALUATE LOCAL SEPARATION DISTANCE.
0069
0070 SEPN=COEFF*((2.0/A)*FTHIX)-(XP28*((2.0/(A*(A822.0-B822.0)))*FTHI
0071 K-R3))+((2.0/A822.0)*SORT(1.0/(XMUS+A822.0)*RS))-((YP22.08A8(R3-(
0072 B822.0/A822.0)*FTHIX))/((B822.0)*(A822.0-B822.0)))
0073
0074 EVALUATE EFFECTIVE GAS CONDUCTIVITY.
0075
0076 TC=KD/(1.0+((HFP/SEPN)*ECN))
0077
0078 EVALUATE LOCAL GAS CONDUCTANCE.
0079
0080 R4=TC*(XLN(I)-XN(J))/(SEPN*SORT((A822.0+XN(J))*(B822.0-XN(J))*A8
0081 22.0+XLN(I))*B822.0+XLN(I))*R4.0)
0082
0083 MULTIPLY LOCAL GAS CONDUCTANCE BY APPROPRIATE WEIGHTING FACTOR
0084 AND ADD TO INNER INTEGRAL.
0085
0086 SUM=SUM+R48N(J)
0087 CONTINUE
0088 SUM=SUM*(XN22-XN11)/2.0
0089
0090 MULTIPLY INNER INTEGRAL BY WEIGHTING FACTOR AND ADD TO OUTER
0091 INTEGRAL.
0092
0093 SUM2=SUM2+SUM8N(I)
0094 CONTINUE
0095 TAB=SUM2*(XLN2-XL11)/2.0
0096 RETURN
0097 END

```

```

0001 SUBROUTINE NNUMB (IX,JX,KX,XLNDP,XNUP,XNUP,MEQ,NUMN,A,B)
0002
0003 *****
0004
0005 THIS SUBROUTINE DETERMINES WHICH MODES LIE WITHIN THE SUBSTITUTE
0006 CONTACT SPECIMEN, THE BOUNDARIES OF WHICH ARE LOCATED AT X=XBIN
0007 CH. Y=XBIN CH. Y=XBIN CH. THESE MODES ARE
0008 NUMBERED SEQUENTIALLY WITH THE NUMBER OF EACH MODE (I,J,K) BEING
0009 GIVEN BY NUMN(I,J,K). NUMN(I,J,K)=0 FOR THOSE MODES LYING OUTSIDE
0010 THE BOUNDARIES OF THE SUBSTITUTE SPECIMEN.
0011 *****
0012
0013 IMPLICIT DOUBLE PRECISION (A-H,O-Z)
0014 DIMENSION NUMN(15,13,12)
0015 DIMENSION XLNDP(15),XNUP(14),XNUP(13)
0016 COMMON /DAPDS/XBIN,YBIN
0017 DO 5 I=1,IX
0018 DO 10 J=1,JX
0019 DO 15 K=1,KX
0020 NUMN(I,J,K)=0
0021
0022 15 CONTINUE
0023 10 CONTINUE
0024 5 CONTINUE
0025 MEQ=0
0026 MED=0
0027
0028 SHEEP THROUGH GRID.
0029
0030 DO 20 I=1,IX
0031
0032 CALCULATE VALUE OF LAMBDA.
0033
0034 XLNDA=XLNDP(I)
0035 DO 30 J=1,JX
0036
0037 CALCULATE VALUE OF MU.
0038
0039 XMU=XNUP(J)
0040 DO 40 K=1,KX
0041
0042 CALCULATE VALUE OF MU.
0043
0044 XMU=XNUP(K)
0045
0046 CONVERT ELLIPSOIDAL COORDINATES TO CARTESIAN COORDINATES.
0047
0048 IF(K.EQ.1) XP=0.0 I 80 TO 44
0049 XP=SORT(((A822+XLNDA)*(A822+XMU)*(A822+XNUP))/((A822-B822)*(A822)))
0050 IF(J.EQ.1,OR,K.EQ.KX) YP=0.0 I 80 TO 44
0051 YP=SORT(((B822+XLNDA)*(B822+XMU)*(B822+XNUP))/((B822-A822)*(B822)))
0052 CONTINUE
0053
0054 CHECK IF THE MODE LIES WITHIN THE BOUNDARIES OF THE SUBSTITUTE
0055 CONTACT SPECIMEN. IF IT DOES, STORE THE MODE NUMBER NMEQ IN THE
0056 APPROPRIATE LOCATION OF THE ARRAY NUMN.
0057
0058 IF(XP.LT.XBIN.AND.YP.LT.YBIN) NMEQ=NMEQ+1 I NUMN(I,J,K)=NMEQ
0059 IF(XP.LT.XBIN.AND.YP.LT.YBIN.AND.I.NE.1.AND.I.NE.IX) NMEQ=NMEQ+1
0060 CONTINUE
0061 CONTINUE
0062 CONTINUE
0063 WRITE (100,45) NMEQ
0064 45 FORMAT (IX,'TOTAL NUMBER OF MODES USED TO REPRESENT SPECIMEN =',I5
0065 ,/)
0066 OUTPUT MED
0067 RETURN
0068 END

```

```

0001      SUBROUTINE HERTZ (A,B,RCY1,RCY2)
0002      C
0003      *****
0004      C
0005      THIS SUBROUTINE COMPUTES THE SEMI-MAJOR (A) AND SEMI-MINOR (B)
0006      DIAMETERS OF AN ELLIPTICAL CONTACT PATCH USING HERTZIAN THEORY.
0007      SIMPLIFIED FORMULAE ATTRIBUTABLE TO DYSON REPLACE THE RELATIVELY
0008      COMPLICATED FORMULAE DERIVED BY HERTZ.
0009      C
0010      *****
0011      C
0012      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
0013      READ (105,2) P,RCY1,RCY2,YNOB,POISS
0014      2      FORMAT (F10.5,E10.4,F10.5)
0015      RCY=RCY2
0016      IF (RCY1.GT.RCY2) RCY2=RCY1 IRCY1=RCY
0017      AZZ=1.0/(2.0*RCY2)
0018      B/2=1.0/(2.0*RCY1)
0019      ADIVB=AZZ/BZZ
0020      TH11=2.0*(1.0-(POISS**2))/YNOB
0021      C
0022      DETERMINE COEFFICIENTS AND EXPONENTS FOR DYSONS FORMULAE.
0023      C
0024      IF (ADIVB.LE.0.03) CBF=0.974 IEXPH=-0.372 ICBF=0.702 IEXPH=0.184
0025      IF (ADIVB.GT.0.03) CBF=0.908 IEXPH=-0.380 ICBF=0.894 IEXPH=0.265
0026      IF (ADIVB.LT.0.00001) WRITE (106,5) ADIVB ISTOP
0027      5      FORMAT (1X,'ADIVB=',E12.5,10X,'OUTSIDE RANGE OF DYSONS FORMULAE')
0028      CONFAC=(PTH11/(AZZ+BZZ))**88.3333
0029      A=CBF*CONFAC*(ADIVB**EXPH)
0030      B=ICBF*CONFAC*(ADIVB**EXPH)
0031      WRITE (106,10) P,RCY1,RCY2,YNOB,POISS,A,B
0032      10      FORMAT (1X,'CONTACT LOAD =',F10.5,' KG. ',1X,'SMALL CONTACT RADI
0033      US =',F10.5,' CM. ',1X,'LARGE CONTACT RADIUS =',F10.5,' CM.
0034      ',1X,'YOUNGS MOD =',E10.4,' KG/CM**2',1X,'POISSONS RATIO =',F10
0035      5,1X,'SEMI-MAJOR DIAMETER =',F10.5,' CM. ',1X,'SEMI-MINOR D
0036      IAMETER =',F10.5,' CM. ')
0037      RETURN
0038      END
0091.700
0091.800
0091.900
0094.000
0094.100
0094.100
0094.300
0094.400
0094.500
0094.600
0094.700
0094.800
0094.900
0095.000
0095.100
0095.200
0095.300
0095.400
0095.500
0095.600
0095.700
0095.800
0095.900
0096.000
0096.100
0096.200
0096.300
0096.400
0096.500
0096.600
0096.700
0096.800
0096.900
0097.000
0097.100
0097.200
0097.300
0097.400

0001      SUBROUTINE COORD (IX,JX,KX,XJX,XKX,A,B,X,XAV,XLMBP
0002      ,XLMMV,Y,YAV,XMLP,XLMMV,Z,ZAV,XMSP,XLMMV)
0003      C
0004      *****
0005      C
0006      THIS SUBROUTINE CALCULATES THE COORDINATES OF ALL OF THE MESH
0007      POINTS IN BOTH THE X,Y,Z AND LAMBDA,MU,MU COORDINATE SYSTEMS.
0008      C
0009      *****
0010      C
0011      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
0012      DIMENSION X(35),XAV(35),XLMBP(35),XLMMV(35),XJX(2),Y(14),
0013      XMLMMV(13),XMLX(2)
0014      DIMENSION RMSP(3),XM(3)
0015      READ (105,1) LNC1,LNC2,RMSP(1),RMSP(2),RMSP(3)
0016      1      FORMAT (2I5,F10.5)
0017      OUTPUT LNC1,LNC2,RMSP(1),RMSP(2),RMSP(3)
0018      XIX=IX
0019      XLNC1=LNC1
0020      XLNC2=LNC2
0021      C
0022      CALCULATE THE VALUES OF LAMBDA AND X FOR THE SPECIFIED 'HEMISPHERI
0023      SURFACES. RMSP(1)=1.
0024      C
0025      DO 2 IH=1,3
0026      2      XLMS=RMSP(IH)**2.0
0027      XN(IH)=ALOG(4.0*A/B)-ALOG((A+SQRT(XLMS+A**2.0))/SQRT(XLMS))
0028      C
0029      CALCULATE THE THREE DIFFERENT MESH SPACINGS ON THE X AXIS.
0030      DX1,DX2,DX3.
0031      C
0032      DX1=XN(1)/(XLNC1-1.0)
0033      DX2=(XN(2)-XN(1))/(XLNC2-XLNC1)
0034      DX3=(XN(3)-XN(2))/(XIX-XLNC2)
0035      C
0036      CALCULATE THE VALUES OF LAMBDA AND Y AT ALL OF THE MESH POINTS,
0037      AS WELL AS THEIR VALUES AT POINTS LOCATED MIDWAY BETWEEN ADJACENT
0038      NODES ON THE X AXIS.
0039      C
0040      X(1)=0.0
0041      Y(1)=0.0
0042      C
0043      XAV(1)=DX1/2.0
0044      XLMBP(1)=0.0
0045      BEX=B*EXP(DX1/2.0)
0046      XLMMV(1)=(BEX**2.0)/4.0-(B**2.0)/2.0+(B**4.0)/(4.0*(BEX**2.0))
0047      DO 5 I=2,IX
0048      5      X(I)=X(I-1)+DX
0049      DX=DX1
0050      IF (1.0E.LNC1.AND.1.0E.LNC2) DX=DX2
0051      IF (1.0E.LNC2) DX=DX3
0052      IF (1.0E.IX) XAV(I)=X(I)+DX/2.0
0053      DO 10 LL=1,2
0054      10      IF (LL.EQ.2.AND.1.0E.IX) GO TO 10
0055      IF (LL.EQ.1) XLL=X(I)
0056      IF (LL.EQ.2) XLL=XAV(I)
0057      BEX=B*EXP(XLL)
0058      IF (XLL.LE.2.3) XLX(LL)=(BEX**2.0)/4.0-(B**2.0)/2.0+(B**4.0)
0059      /4.0*(BEX**2.0))
0060      IF (XLL.GT.2.3) XLX(LL)=16.0*(A**2.0)/((64.0*(A**2.0)/BEX**2
0061      0)+(BEX**2.0)/(4.0*(A**2.0))-8.0)
0062      CONTINUE
0063      XLMBP(I)=XLX(1)
0064      XLMMV(I)=XLX(2)
0065      CONTINUE
0066      C
0067      CALCULATE THE VALUES OF MU AND Y AT ALL OF THE MESH POINTS,
0068      AS WELL AS THEIR VALUES AT POINTS LOCATED MIDWAY BETWEEN
0069      ADJACENT NODES ON THE Y AXIS.
0070      C
0071      Y(1)=0.0
0072      XPR2=B**2.0/A**2.0
0101.100
0101.200
0101.300
0101.400
0101.500
0101.600
0101.700
0101.800
0101.900
0102.000
0102.100
0102.200
0102.300
0102.400
0102.500
0102.600
0102.700
0102.800
0102.900
0103.000
0103.100
0103.200
0103.300
0103.400
0103.500
0103.600
0103.700
0103.800
0103.900
0104.000
0104.100
0104.200
0104.300
0104.400
0104.500
0104.600

```

```

0073      XAKO=1.5707968(1.0+XPR2/4.0)
0074      BY=XAKO/XJX
0075      YAU(1)=BY/2.0
0076      XMLP(1)=-(BS22.0)
0077      XMMU(1)=(BSCDS(BY/2.0)BS2.0)/((BSIN(BY/2.0)/A)BS2.0)-1.0)
0078      DO 20 J=2,JX
0079      Y(J)=Y(J-1)+BY
0080      IF(J.NE.JX) YAU(J)=Y(J)+BY/2.0
0081      DO 25 LL=1,2
0082      IF(LL.EQ.2.AND.J.EQ.JX) GO TO 25
0083      IF(LL.EQ.1) YLL=Y(J)
0084      IF(LL.EQ.2) YLL=YAU(J)
0085      XMLL(1)=(BSCDS(YLL)BS2.0)/((BSIN(YLL)/A)BS2.0)-1.0)
0086      IF(YLL.GT.0.79) XMLL(1)=-(BSIN(XAKO-YLL)BS2.0)
0087      CONTINUE
0088      XMLP(J)=XMLL(1)
0089      XMMU(J)=XMLL(2)
0090      CONTINUE
0091
0092      C
0093      C
0094      C
0095      C
0096      Z(1)=0.0
0097      BZ=ALOG(4.084/B)/XAX
0098      ZAU(1)=BZ/2.0
0099      XMLP(1)=-(AS22.0)
0100      EX=EXP(BZ)
0101      XMMU(1)=-(BS22.0)+(4.08EX8(BS22.0-AS22.0)/((1.0+EX)BS2.0))
0102      DO 30 K=2,KX
0103      Z(K)=Z(K-1)+BZ
0104      IF(K.NE.KX) ZAU(K)=Z(K)+BZ/2.0
0105      DO 35 LL=1,2
0106      IF(LL.EQ.2.AND.K.EQ.KX) GO TO 35
0107      IF(LL.EQ.1) ZLL=Z(K)
0108      IF(LL.EQ.2) ZLL=ZAU(K)
0109      EX=EXP(2.0*ZLL)
0110      COEF=14.08(AS22.0)/EX
0111      IF(ZLL.LE.2.3) XMLL(1)=-(BS22.0)+(4.08EX8(BS22.0-AS22.0)
0112      /((1.0+EX)BS2.0))
0113      IF(ZLL.GT.2.3) XMLL(1)=-(AS(COEF+BS22.0)BS2.0)/((COEF-
0114      BS22.0)BS2.0+4.08COEF8(AS22.0))
0115      CONTINUE
0116      XMLP(K)=XMLL(1)
0117      XMMU(K)=XMLL(2)
0118      CONTINUE
0119      RETURN
0120      END

```

```

8 ACTUAL PROGRAM SIZE:
8 PROGRAM 1 INITIALIZED VARIABLES = 4826 (WORDS)
8 NON-INITIALIZED VARIABLES = 57047 (WORDS)
8 TOTAL = 61873 (WORDS)

```

***** THERMAL MODEL FORMULATED IN ELLIPSOIDAL COORDINATE SYSTEM *****

***** RUN WITH GAS CONDUCTION *****

IX = 35

JX = 8

KX = 8

MMT = 22000

XMP = 1.2700000000000000

YDM = .8572000000000000

SOLID CONDUCTIVITY = 37.03000 W/M.DEG.C.

GAS CONTINUUM CONDUCTIVITY = .02840 W/M.DEG.C.

MEAN FREE PATH OF GAS MOLECULES = .7600E-05 CM.

CN = 4.082EC

CONTACT LOAD = 17.94000 MG.

SMALL CONTACT RADIUS = .95270 CM.

LARGE CONTACT RADIUS = 16.04000 CM.

YOUNG'S MOD. = .2079E+07 KG/CM**2

POISSON'S RATIO = .30000

SEMI-MAJOR DIAMETER = .15395 CM.

SEMI-MINOR DIAMETER = .00961 CM.

LX1 = 22

LX2 = 29

RES(1) = 1.0000000000000000

RES(2) = 1.0000000000000000

RES(3) = 2.5400000000000000

TOTAL NUMBER OF NODES USED TO REPRESENT SPECIMEN = 1049.

NEQ = 1583

RESISTANCE OF SUBSTITUTE SPECIMEN WITH BOTH ENDS ISOTHERMAL = 4.60000 DEG.C/W

RESISTANCE OF CONTACT CALCULATED USING SHAPE FACTOR TECHNIQUE = .07603

RESISTANCE OF CONTACT WITH GAS AND SOLID RESISTANCES IN PARALLEL = 6.00095 DEG.C/W

RESISTANCE OF CONTACT CALCULATED USING YOVANOVICH'S METHOD = 3.15656 DEG.C/W

RATIO OF HEAT FLOW THROUGH CONTACT PATCH TO TOTAL HEAT FLOW = .32857

STOP

Listing I.C.2
Sample Output for Resistance
Analysis Performed With Gas
Conduction at a Contact Load
of 175.8 N Using the 34x7x7
Grid Referred to in Table 1.6.3.

***** THERMAL MODEL FORMULATED IN ELLIPSOIDAL COORDINATE SYSTEM *****

***** RUN WITHOUT GAS CONDUCTION *****

IV = 35

JX = 8

KX = 8

MTMT = 22000

XDIM = 1.270000000000000

YDIM = .857200000000000

SOLID CONDUCTIVITY = 37.03000 W/M.DEG.C.

Listing I.C.3 Sample Output for Resistance
Analysis Performed Without Gas
Conduction at a Contact Load
of 175.8 N Using the 34x7x7
Grid Referred to in Table 1.6.3.

CONTACT LOAD = 17.94000 KG.

SMALL CONTACT RADIUS = .95270 CM.

LARGE CONTACT RADIUS = 76.24000 CM.

YOUNGS MOD = .2C/BE+07 KG/CM**2

POISSONS RATIO = .30000

SEMI-MAJOR DIAMETER = .19395 CM.

SEMI-MINOR DIAMETER = .00961 CM.

LXC1 = 22

LXC2 = 29

RUSP(1) = 1.000000000000000

RUSP(2) = 1.800000000000000

RUSP(3) = 2.540000000000000

TOTAL NUMBER OF NODES USED TO REPRESENT SPECIMEN = 1649

NEW = 1583

RESISTANCE OF SUBSTITUTE SPECIMEN WITH BOTH ENDS ISOHERMAL = .600000 DEG.C/W

RESISTANCE OF CONTACT CALCULATED USING SHAPE FACTOR TECHNIQUE = 21.65512

RATIO OF HEAT FLOW THROUGH CONTACT PATCH TO TOTAL HEAT FLOW = 1.00000

-STOP-

EXECUTE TIME = 0218.04 SECONDS

APPENDIX I.D

CONTACT SURFACE YIELD ANALYSIS

It can be shown using results of Hertz's theory of elastic contact presented graphically by Seely [70] that contact pressures within the patch would have produced a maximum octahedral shear stress (τ_G)* of ≈ 215 MPa (31 ksi) in both contact specimens at the maximum test load of 454N used in the experiments described in Chapter I.5. That stress would have occurred at the point $x=0$, $y=0$ and $z=0.0102$ cm (i.e., below the contact surface) in the contact-centered coordinate system. Assuming von Mises failure theory can be used to predict the onset of yielding in the specimens, yielding would have occurred for $\tau_G > \frac{\sqrt{2}}{3} S_y$, where S_y is the uniaxial yield strength (in tension or frictionless compression) of the contact material. Yield strength data given by Roberts et al. [97] for 0-1 steel indicates that it has a yield strength of ≈ 550 MPa (80 ksi) in the annealed condition, so

$$\frac{\sqrt{2}}{3} S_y = \frac{\sqrt{2}}{3} (550) = 259 \text{ MPa}$$

in this case. Since $\tau_G = 215$ MPa in this analysis, $\tau_G < \frac{\sqrt{2}}{3} S_y$ and yielding of the contact specimens should not have occurred according to von Mises criterion.

* $\tau_G = \frac{1}{3} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2}$, where σ_x , σ_y and σ_z are principal stresses.

APPENDIX I.E

ERROR AND SENSITIVITY ANALYSIS (SMOOTH SURFACE RESULTS)

I.E.1 Uncertainty Analysis for Contact Resistance in Air

It is assumed that the resistance of the contact with smooth surfaces is functionally related to several variables, as expressed below,

$$R_c = f(P, R_1, R_2, k_{st}, k_{og}, E, \nu, L_m, \alpha_1, \alpha_2, Pr) \quad (I.E.1)$$

and that the independent variables in the relationship are known by virtue of having been measured directly or inferred from measurements of related quantities in the experiments. The mean interface temperature measured in an experiment would be used to obtain k_{og} from published tables of the thermal conductivity of air vs. temperature, for instance. It is furthermore assumed that the analytical model described in Chapters I.3 and I.4 can be used to predict the value of R_c from specified values of the independent variables in equation (I.E.1), although the validity of this assumption should be tested by comparing predicted and measured values of the contact resistance. Those values differed at all of the test conditions examined in the experiments, of course, as shown in Chapter I.6, so an error analysis is required to determine whether there is a reasonable probability that the observed differences could have been produced completely by experimental errors. If the error analysis proves this to be the case, further development of the analytical model would probably not be warranted without first obtaining more accurate experimental data.

Since we wish to determine whether there is a reasonable probability that the difference between the experimental and predicted contact resistances for a given test (i.e., R_C and R'_C respectively) could have been produced completely by experimental errors, it is convenient to formally define that difference as,

$$\bar{E} = R'_C - R_C \quad (\text{I.E.2})$$

The experimental resistance, R_C , is calculated from the equation,

$$R_C = \frac{\Delta T_C}{Q_C} \quad (\text{I.E.3a})$$

$$= \frac{\Delta T_C}{k_i A_i \left. \frac{dT}{dx} \right|_i} \quad (\text{I.E.3b})$$

which can be derived in its second form (i.e., as equation (I.E.3b)) by substituting equation (I.5.6) (with Q_C taken as Q_{C1}) into equation (I.E.3a). There would be no error in R_C for a given test (i.e., R_C would be exact) if the parameters ΔT_C , k_i , A_i and $\left. \frac{dT}{dx} \right|_i$ in equation (I.E.3b) were determined exactly in that test, of course. Similarly, the exact values of the independent variables in equation (I.E.1) (i.e., P , R_1 , R_2 , etc.), in other words the values that those variables had actually assumed in the test in question, would be required to obtain the exact value of R'_C for that test. If it were possible to determine the exact values of the variables on the right-hand side of equation (I.E.3b) as well as the exact values of the independent variables in equation (I.E.1) in a given test, there would be no error in the value of $\bar{E}$ calculated

using equation (I.E.2) and we could be definitive about the error in the analytical prediction.

The independent variables in equations (I.E.1) and (I.E.3b), and hence also the values of R_C , R'_C and E obtained directly or indirectly from them, were not determined exactly, of course, so the best that can be done is to calculate an interval around our estimate of E within which we would expect, at a specified level of confidence, the true value of E to lie.

If it is assumed that our estimates of R_C and R'_C are in error by amounts ΔR_C and $\Delta R'_C$ respectively, the resulting error in E , designated ΔE , is,

$$\Delta E = \Delta R'_C - \Delta R_C \quad (\text{I.E.4})$$

The errors in R_C and R'_C are, in turn, functions of measurement and/or sampling errors in the independent variables seen in equations (I.E.1) and (I.E.3b). If (I.E.4) is expanded in terms of the errors in those variables, we obtain:

$$\begin{aligned} \Delta E = & \frac{\partial R'_C}{\partial P} \Delta P + \frac{\partial R'_C}{\partial R_1} \Delta R_1 + \frac{\partial R'_C}{\partial R_2} \Delta R_2 + \frac{\partial R'_C}{\partial k_{st}} \Delta k_{st} + \frac{\partial R'_C}{\partial k_{og}} \Delta k_{og} \\ & + \frac{\partial R'_C}{\partial E} \Delta E - \frac{\partial R'_C}{\partial (\Delta T_C)} \Delta (\Delta T_C) - \frac{\partial R'_C}{\partial \left(\frac{dT}{dx} \right)_1} \Delta \left(\frac{dT}{dx} \right)_1 - \frac{\partial R'_C}{\partial k_1} \Delta k_1 \\ & + \left[\frac{\partial R'_C}{\partial v} \Delta v + \frac{\partial R'_C}{\partial L_m} \Delta L_m + \frac{\partial R'_C}{\partial \alpha_1} \Delta \alpha_1 + \frac{\partial R'_C}{\partial \alpha_2} \Delta \alpha_2 + \frac{\partial R'_C}{\partial Pr} \Delta Pr \right] \quad (\text{I.E.5}) \end{aligned}$$

It will be shown later that the terms contained with the square brackets in equation (I.E.5) have an insignificant effect on the uncertainty in E . Disregarding those terms and squaring both sides of equation (I.E.5) gives,

$$\begin{aligned}
 (\Delta E)^2 = & \left(\frac{\partial R_c}{\partial P}\right)^2 (\Delta P)^2 + \left(\frac{\partial R_c}{\partial R_1}\right)^2 (\Delta R_1)^2 + \left(\frac{\partial R_c}{\partial R_2}\right)^2 (\Delta R_2)^2 + \left(\frac{\partial R_c}{\partial k_{st}}\right)^2 (\Delta k_{st})^2 \\
 & + \left(\frac{\partial R_c}{\partial k_{og}}\right)^2 (\Delta k_{og})^2 + \left(\frac{\partial R_c}{\partial E}\right)^2 (\Delta E)^2 + \left(\frac{\partial R_c}{\partial (\Delta T_c)}\right)^2 (\Delta (\Delta T_c))^2 \\
 & + \left(\frac{\partial R_c}{\partial \left(\frac{dT}{dx}\right)_i}\right)^2 \left(\Delta \left(\frac{dT}{dx}\right)_i\right)^2 + \left(\frac{\partial R_c}{\partial k_i}\right)^2 (\Delta k_i)^2 + \text{terms involving}
 \end{aligned}$$

products of errors, e.g.,

$$\Delta P \Delta R_1, \Delta k_s \Delta k_i, \Delta (\Delta T_c) \Delta \left(\frac{dT}{dx}\right)_i, \text{ etc.} \quad (\text{I.E.6})$$

If we have a large number of observations of the independent variables (P , R_1 , R_2 , etc.) and hence of E , the mean value of $(\Delta E)^2$ over all of the observations is the variance of E , denoted $V(E)$.^{*} Similarly, the mean value of $(\Delta q_i)^2$ over all observations, where q_i is one of the independent variables, is the variance of q_i , i.e., $V(q_i)$.

* It is assumed that all of the variables are normally distributed. As will be discussed later, the values of R_2 listed in Table I.5.3 were tested for normality because the results of the error analysis showed that the uncertainty in R_2 was the largest source of uncertainty in E . The normality test indicated that there was a reasonable probability that the measured values of R_2 came from a normal distribution.

The mean values of products of errors (e.g., $\frac{\sum \Delta P \Delta R_1}{N}$, for N observations) termed covariances, should all be zero on average if the independent variables are uncorrelated [101]. If that proviso is satisfied, $V(E)$ is related to the variances of the independent variables by the following expression.

$$\begin{aligned}
 V(E) = & \left(\frac{\partial R_c}{\partial P}\right)^2 V(P) + \left(\frac{\partial R_c}{\partial R_1}\right)^2 V(R_1) + \left(\frac{\partial R_c}{\partial R_2}\right)^2 V(R_2) + \left(\frac{\partial R_c}{\partial k_{st}}\right)^2 V(k_{st}) \\
 & + \left(\frac{\partial R_c}{\partial k_{og}}\right)^2 V(k_{og}) + \left(\frac{\partial R_c}{\partial E}\right)^2 V(E) + \left(\frac{\partial R_c}{\partial (\Delta T_c)}\right)^2 V(\Delta T_c) \\
 & + \left(\frac{\partial R_c}{\partial \left(\frac{dT}{dx}\right)_i}\right)^2 V\left(\frac{dT}{dx}\right)_i + \left(\frac{\partial R_c}{\partial k_i}\right)^2 V(k_i) \quad (I.E.7)
 \end{aligned}$$

In the present case, the covariances involving the products $\Delta k_i \Delta k_{st}$ and $\Delta(\Delta T_c) \Delta\left(\frac{dT}{dx}\right)_i$ cannot be neglected because: (1) k_{st} was determined from k_i in the experiment described in Chapter I.5, and; (2) the value of $\frac{dT}{dx}_i$ was used to calculate ΔT_c . Thus k_i and k_{st} , and similarly ΔT_c and $\frac{dT}{dx}_i$ are correlated variables and as such will not, in general, have zero covariances.

Since k_{st} was determined from k_i , the error in k_{st} (i.e., Δk_{st}) should be describable in terms of the error in k_i (i.e., Δk_i) and a measurement error associated with the manner in which k_{st} was calculated from k_i . Basically the experiment described in Chapter I.5 established the value of a factor C which, when multiplied by k_i , yielded k_{st} ; i.e., $k_{st} = C k_i$. The factor C in the expression $k_{st} = C k_i$ is a function of

temperature and has associated with it an error ΔC , which is also a function of temperature. The total error in k_{st} (i.e., Δk_{st}) can thus be written as:

$$\Delta k_{st} = C \Delta k_i + k_i \Delta C \quad (\text{I.E.8})$$

Introducing equation (I.E.8) into equation (I.E.5) and rearranging terms yields:

$$\begin{aligned} \Delta E = & \frac{\partial R_C}{\partial P} \Delta P + \frac{\partial R_C}{\partial R_1} \Delta R_1 + \frac{\partial R_C}{\partial R_2} \Delta R_2 + \left(\frac{\partial R_C}{\partial k_{st}} C - \frac{\partial R_C}{\partial k_i} \right) \Delta k_i \\ & + \frac{\partial R_C}{\partial k_{st}} k_i \Delta C + \frac{\partial R_C}{\partial k_{og}} \Delta k_{og} + \frac{\partial R_C}{\partial E} \Delta E - \frac{\partial R_C}{\partial (\Delta T_C)} \Delta (\Delta T_C) - \frac{\partial R_C}{\partial \left(\frac{dT}{dx} \right)_i} \Delta \left(\frac{dT}{dx} \right)_i \end{aligned} \quad (\text{I.E.9})$$

If both sides of equation (I.E.9) are squared, the covariance involving the product $\Delta k_i \Delta C$ disappears because the variables C and k_i are independent.

The second combination of correlated variables, $\Delta T_C (= T_{ch} - T_{cc})$ and $\left. \frac{dT}{dx} \right|_i$ is classified as such because T_{cc} was evaluated from $\left. \frac{dT}{dx} \right|_i$ in the experiments, as seen in equation (I.5.5). Equation (I.5.5) is not ideal for use in an error analysis because it gives the error in T_{cc} as,*

* It is assumed that the factor L_s in equation (I.5.5) (i.e., the length of the lower contact specimen) is known exactly.

$$\Delta T_{cc} = \frac{\partial T_{cc}}{\partial T_s} \Delta T_s + \frac{\partial T_{cc}}{\partial C} \Delta C + \frac{\partial T_{cc}}{\partial \left(\frac{dT}{dx}\right)_i} \Delta \left(\frac{dT}{dx}\right)_i \quad (\text{I.E.10})$$

If equation (I.E.10) is squared to obtain the variance of T_{cc} , the product of ΔT_s and $\left(\frac{dT}{dx}\right)_i$ (i.e., the covariance of those terms) will not be zero on average because T_s and $\frac{dT}{dx}_i$ are correlated variables. The temperature T_s is related to $\frac{dT}{dx}_i$ by the expression,

$$T_s = \bar{T} + \frac{dT}{dx}_i (x_s - \bar{x}) \quad (\text{I.E.11})$$

in which $\bar{T}$ and $\bar{x}$ are the means of the thermocouple temperatures and positions in the lower Armco-iron bar respectively and x_s is the position of the interface between the lower Armco-iron bar and the lower contact specimen. Equation (I.E.11) is the fundamental form of the linear regression of thermocouple temperature on position in the lower Armco-iron bar, since $\bar{T}$ and $\frac{dT}{dx}_i$ are uncorrelated variables [10]. If equation (I.E.11) is substituted into equation (I.5.5), T_{cc} can be written as,

$$T_{cc} = \bar{T} + \frac{dT}{dx}_i \left(x_s - \bar{x} + \frac{L_s}{C}\right) \quad (\text{I.E.12})$$

The variance in T_{cc} can now be expressed in terms of the variances of $\bar{T}$, $\frac{dT}{dx}_i$ and C only (x_s and $\bar{x}$ are assumed exact) because all of these variables are uncorrelated (i.e., covariances can be neglected).

Introducing equation (I.E.12) and the expression $\Delta T_c = T_{ch} - T_{cc}$ into equation (I.E.3b) we obtain,

$$R_c = \frac{1}{k_i A_i} \left[\frac{(T_{ch} - T)}{\frac{dT}{dx}|_i} - x_s + \bar{x} - \frac{L_s}{C} \right] \quad (I.E.13)$$

The error in R_c (i.e., ΔR_c) arising from errors in the independent variables in equation (I.E.13) can now be written as;

$$\begin{aligned} \Delta R_c = & \frac{\partial R_c}{\partial k_i} \Delta k_i + \frac{\partial R_c}{\partial T_{ch}} \Delta T_{ch} + \frac{\partial R_c}{\partial T} \Delta T + \frac{\partial R_c}{\partial (\frac{dT}{dx}|_i)} \Delta (\frac{dT}{dx}|_i) \\ & + \frac{\partial R_c}{\partial C} \Delta C \end{aligned} \quad (I.E.14)$$

The terms involving the errors ΔA_i , Δx_s and $\Delta \bar{x}$ are neglected in equation (I.E.14) since it can be shown that they are negligible compared to the retained terms. The last two terms in equation (I.E.9) can now be replaced by the last three terms in equation (I.E.14) to give:

$$\begin{aligned} \Delta \bar{E} = & \frac{\partial R_c}{\partial P} \Delta P + \frac{\partial R_c}{\partial R_1} \Delta R_1 + \frac{\partial R_c}{\partial R_2} \Delta R_2 + \left(\frac{\partial R_c}{\partial k_{st}} \Delta C - \frac{\partial R_c}{\partial k_i} \Delta k_i \right. \\ & + \left(k_i \frac{\partial R_c}{\partial k_{st}} - \frac{\partial R_c}{\partial C} \right) \Delta C + \frac{\partial R_c}{\partial k_{og}} \Delta k_{og} + \frac{\partial R_c}{\partial \bar{E}} \Delta \bar{E} - \frac{\partial R_c}{\partial T_{ch}} \Delta T_{ch} \\ & \left. - \frac{\partial R_c}{\partial T} \Delta T - \frac{\partial R_c}{\partial (\frac{dT}{dx}|_i)} \Delta (\frac{dT}{dx}|_i) \right) \end{aligned} \quad (I.E.15)$$

Since all of the independent variables in equation (I.E.15) (i.e., P , R_1 , R_2 , k_i , C , $\dots$, T , $\frac{dT}{dx}|_i$) are uncorrelated, the variance of $\bar{E}$ can be expressed in terms of the variances of those variables as,

$$\begin{aligned}
V(E) = & \left(\frac{\partial R_C}{\partial P}\right)^2 V(P) + \left(\frac{\partial R_C}{\partial R_1}\right)^2 V(R_1) + \left(\frac{\partial R_C}{\partial R_2}\right)^2 V(R_2) \\
& + \left(\frac{\partial R_C}{\partial k_{st}}\right)^2 C + \left(\frac{\partial R_C}{\partial k_i}\right)^2 V(k_i) + \left(k_i \frac{\partial R_C}{\partial k_{st}} - \frac{\partial R_C}{\partial C}\right)^2 V(C) \\
& + \left(\frac{\partial R_C}{\partial k_{og}}\right)^2 V(k_{og}) + \left(\frac{\partial R_C}{\partial E}\right)^2 V(E) + \left(\frac{\partial R_C}{\partial T_{ch}}\right)^2 V(T_{ch}) \\
& + \left(\frac{\partial R_C}{\partial T}\right)^2 V(T) + \left(\frac{\partial R_C}{\partial \left(\frac{dT}{dx}\right)_i}\right)^2 V\left(\frac{dT}{dx}\right)_i \quad (I.E.16)
\end{aligned}$$

If we take the square root of both sides of equation (I.E.16) we obtain an expression for the standard deviation of E (i.e., $\sigma(E)$) in terms of the variances (or equally, standard deviations) of the independent variables. If the standard deviation of each independent variable is known exactly (i.e., calculated from an infinite number of observations), we can assert with 68% confidence that the true value of independent variable q_i lies between the limits $q_i + \sigma(q_i)$ and $q_i - \sigma(q_i)$ and similarly that the true value of E lies between the limits $E + \sigma(E)$ and $E - \sigma(E)$. Thus $q_i \pm \sigma(q_i)$ are the limits of q_i for a 68% confidence interval and $\sigma(q_i)$ is the uncertainty in q_i at the 68% confidence level. Generally we wish to give the limits for a variable at a larger confidence level, say 95%, so in this case we would write the limits as $q_i + \delta q_i$ and $q_i - \delta q_i$, where δq_i is the uncertainty in q_i at the 95% confidence level. The uncertainty in E at the 95% confidence level (i.e., δE) can be calculated from equation (I.E.16) by taking the square

root of both sides and substituting $(\delta P)^2$, $(\delta R_1)^2$, etc., for $V(P)$, $V(R_1)$, etc., respectively.

If this is done, we obtain the following expression for δE in terms of the uncertainties in the independent variables at the 95% confidence level (i.e., δP , δR_1 , etc.).

$$\begin{aligned} \delta E = & \left[\left(\frac{\partial R_c}{\partial P} \delta P \right)^2 + \left(\frac{\partial R_c}{\partial R_1} \delta R_1 \right)^2 + \left(\frac{\partial R_c}{\partial R_2} \delta R_2 \right)^2 + \left(\frac{\partial R_c}{\partial T_{ch}} \delta T_{ch} \right)^2 \right. \\ & + \left(\frac{\partial R_c}{\partial T} \delta T \right)^2 + \left(\frac{\partial R_c}{\partial \left(\frac{dT}{dx} \right)_i} \delta \left(\frac{dT}{dx} \right)_i \right)^2 + \left(\left(\frac{\partial R_c}{\partial k_{st}} C - \frac{\partial R_c}{\partial k_i} \right) \delta k_i \right)^2 \\ & \left. + \left(\left(k_i \frac{\partial R_c}{\partial k_{st}} - \frac{\partial R_c}{\partial C} \right) \delta C \right)^2 + \left(\frac{\partial R_c}{\partial k_{og}} \delta k_{og} \right)^2 + \left(\frac{\partial R_c}{\partial E} \delta E \right)^2 \right]^{1/2} \quad (I.E.17) \end{aligned}$$

The significance of the coefficient multiplying δk_i (i.e., $\frac{\partial R_c}{\partial k_{st}} C - \frac{\partial R_c}{\partial k_i}$) should be explained. To simplify the explanation, the case of the contact in vacuum will be examined although some of the comments will also apply to the contact in air analysis provided the assumed value of k_i is not seriously in error. As seen in equation (I.E.3b), the measured value of R_c is inversely proportional to k_i (i.e., $R_c \propto k_i^{-1}$) so $\frac{\partial R_c}{\partial k_i} = -\frac{R_c}{k_i}$. The predicted resistance of the contact in vacuum, where metallic conduction is the only heat transfer mode possible (neglecting radiation), is inversely proportional to k_{st} so

$\frac{\partial R_c}{\partial k_{st}} = -\frac{R_c}{k_{st}}$. Evaluating $\left(\frac{\partial R_c}{\partial k_{st}} C - \frac{R_c}{k_i} \right)$ in this case gives:

$$\frac{\partial R'_c}{\partial k_{st}} C - \frac{\partial R_c}{\partial k_i} = \frac{R_c - R'_c}{k_i} \quad \text{since } C = \frac{k_{st}}{k_i} \quad (\text{I.E.18})$$

If $R_c = R'_c$, the contribution to δE from the uncertainty in k_i will be very small or zero regardless of the size of k_i . This is as it should be since k_{st} , which is used to evaluate R'_c , was calculated from k_i . For instance, if the value assumed for k_i is one-half of its actual value, k_{st} will also be one-half of its actual value since it is determined from k_i . The value of R_c determined from equation (I.E.3b) will be twice its actual value in this case, as will be R'_c . Thus E will contain almost no contribution from the large error in k_i if $R_c = R'_c$. In the case of the contact in air, k_{og} is not referenced to k_i so it is important that the assumed value of k_i be reasonably accurate.

The partial derivatives of R_c seen in equation (I.E.17) can be obtained from equation (I.E.13) but the partial derivatives of R'_c must be obtained as finite-differences by noting the changes in R'_c produced by varying the independent variables one at a time.

Equation (I.E.17) can be used to predict the uncertainty in a single resistance measurement (or, more accurately, a single value of E) from the uncertainties in the independent variables. Since more than one resistance measurement was made at each set of test conditions, some of the independent variables in equation (I.E.17) were sampled more than once and the additional information thus provided should be incorporated into the error analysis. It is assumed, in particular, that the variables P , R_1 , R_2 , T_{ch} , T and $\frac{dT}{dx}_i$ were randomly sampled in each

repeated resistance measurement at a particular nominal contact load. Different values of R_1 and R_2 would have been obtained in different tests because the position of the contact on each contact surface was changed slightly from one trial to the next. The standard deviation of the repeat measurements of R_c at a particular contact load, multiplied by an appropriate value of Student's "t" confidence coefficient represents, in this case, an estimate of the uncertainty produced by the first six terms in the square brackets on the right-hand side of equation (I.E.17). These terms do not therefore, have to be evaluated separately. Initially, however, each resistance measurement will be treated as a single sample experiment and equation (I.E.17) will be applied in its full form to estimate δE from δP , δR_1 , δR_2 , etc.

It was decided to perform an error analysis on the data obtained in the resistance measurement at a load of 454N in air since the range of the repeat resistance measurements (i.e., 7.44°C/W - 7.15°C/W = 0.29°C/W =4% of the mean resistance of 7.29°C/W) at that load was larger, relative to the mean resistance, than that at any other load (see Table I.6.1). Of the three resistance measurements in air at 454N, in which the resistances shown in Table I.6.1 were obtained, the data obtained in trial 2 were selected for scrutiny because the measured value of R_c was very low in that test. The uncertainties needed in equation (I.E.17) to obtain δE for that test are evaluated below.

δp

Calibration of the load cell indicated that δP was approximately 4N at a contact load of 454N. The effects of excitation voltage drift and load cell output voltage measuring error (i.e., voltmeter precision error) are included in this uncertainty figure.

δR_1

While repeat measurements of the diameter of the 0-1 steel shaft from which the small contact radius specimen was fabricated produced $\delta R_1 = 0.014$ mm, such measurements would not have detected small wavelength variations in the curvature of the contact surface. For this reason, δR_1 was arbitrarily assigned a value of 0.10 mm. Sensitivity analyses indicated that even this value of δR_1 contributed negligibly to the uncertainty in E .

 δR_2

The six interferometric measurements of R_2 produced values ranging from 71.70 cm to 79.04 cm, with a mean value of 76.04 cm and a standard deviation of 2.42 cm (N-1 weighting). For 5 degrees of freedom and a 95% confidence interval, Student's "t" confidence coefficient = 2.57 so $\delta R_2 = 2.57 \times 2.42 \approx 6.22$ cm. The distribution of R_2 values was checked for normality by applying the W test [98], which is especially good for small samples, because, as will be seen later, most of the uncertainty in E is contributed by δR_2 . The probability that the observed values of R_2 could have arisen by chance while belonging to a normal distribution was found to be 35% so there did not appear to be any reason to reject the normality assumption.

 δT_{ch}

It can be shown from a statistical analysis of the temperature measurements in the upper contact specimen for the test in question that

$\delta T_{ch} = 0.456^\circ\text{C}$. The standard least-squares regression technique, which assumes that the values of the controlled variable (i.e., the thermocouple locations in this case) are known exactly, was used to obtain δT_{ch} . The thermocouple positions were actually not known exactly because of locational tolerances but arguments presented by Berkson [99] and Lindley [100] suggest that the standard least-squares technique is still applicable because the thermocouple positions were fixed in advance of the experiments. The value of δT_{ch} mentioned previously was calculated from the expression [101],

$$\delta T_{ch} = t \sqrt{\frac{\sum_{i=1}^n (T_i - T_{r_i})^2}{n-2} \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \right)} \quad (\text{I.E.19})$$

where T_i = the temperature measured by the i th thermocouple

x_i = the position of the i th thermocouple

n = the total number of thermocouples (4 in this case)

T_{r_i} = the regression estimate at x_i

$\bar{x} = (\sum_{i=1}^n x_i)/n$

x_0 = the x -coordinate at which the uncertainty in T_r is required
(the contact interface in this case)

t = Student's "t" confidence coefficient for $n-2$ degrees of freedom and a 95% confidence interval.

δT

The value of δT was calculated from the equation [101],

$$\delta T = t \sqrt{\frac{\sum_{i=1}^n (T_i - T_{r_i})^2}{n(n-2)}} \quad (\text{I.E.20})$$

where T_i , T_{r_i} , n ($=7$ for the lower Armco-iron bar) and t were defined previously for equation (I.E.19). Applying equation (I.E.20) to the temperature measurements in the lower Armco-iron bar for the resistance measurement in question produced $\delta T = 0.033^\circ\text{C}$.

$\delta \left(\frac{dT}{dx} \right)_i$

The value of $\delta \left(\frac{dT}{dx} \right)_i$ was calculated from the equation [101],

$$\delta \left(\frac{dT}{dx} \right)_i = t \sqrt{\frac{\sum_{i=1}^n (T_i - T_{r_i})^2}{(n-2) \sum_{i=1}^n (x_i - \bar{x})^2}} \quad (\text{I.E.21})$$

and found to be 2.344°C/m for the temperature measurements in the lower Armco-iron bar.

δk_i

It was estimated from uncertainty information given by Lucks in [88] and other information given in the Journal of Physical and Chemical Reference Data [89] that $\delta k_i = 2.14 \text{ w/mK}$ at 27°C , which was the average temperature of the metering length in the lower Armco-iron bar for the resistance measurement in question.

δC

With the exception of the thermal conductivity measurement performed at the lowest average specimen temperature (33.8°C) in the 0-1 steel thermal conductivity experiments, the heat flow rates in the upper and lower Armco-iron metering bars differed by 5.4-5.6%. Since the heat flow rate in the steel specimen was taken to be the average of the heat flow rates in the upper and lower Armco-iron bars ($\bar{Q}$) in the thermal conductivity measurements, the actual heat flow rate in the steel specimen could have been as much as 2.8% (i.e., $5.6\%/2$) different from $\bar{Q}$ depending on how the heat loss was distributed over the test column. The entire heat loss would have had to occur on only one side of the heat flow metering length in the steel specimen to have obtained such a large difference, of course, and an error of 2.8% is considered extremely unlikely for that reason. It is nevertheless estimated that the uncertainty in C for a 95% confidence interval is probably about 2% in the $50^\circ\text{C} - 60^\circ\text{C}$ temperature range, which corresponds to the range of mean interface temperatures encountered in the contact resistance experiments.

δk_{og}

It was estimated from uncertainty information given in the Journal of Physical and Chemical Reference Data [102] that $\delta k_{og} = 5.66 \times 10^{-4}$ W/mK at the mean interface temperature of 57.5°C recorded in trial 2 at 454N (in air).

δE

The value of Young's modulus for 0-1 steel will almost certainly lie between 200.1 GPa and 207.0 GPa ($29-30 \times 10^6$ psi), so for the purposes of this analysis, its value will be taken as 203.5 ± 3.5 GPa at a 95% confidence level, i.e., $\delta E = 3.5$ GPa.

I.E.1.1 Evaluation of δE

All of the uncertainties required to estimate δE from equation (I.E.17) have now been determined. The partial derivatives of R_c with respect to the retained independent variables in equation (I.E.1) were evaluated by computing R_c (at a nominal load of 454N) with the ellipsoidal finite-difference model (described in Chapter I.4) while varying the independent variables one at a time. Two values of each variable were used, one a given increment above the nominal value and the other the same increment below the nominal value. The appropriate partial derivative was then central-differenced. The values obtained for the partial derivatives in this manner are shown in Table I.E.1 (written as $\Delta R_c / \Delta q_i$ instead of $\partial R_c / \partial q_i$) along with the nominal value of each parameter, its

Table I.E.1: Results of Uncertainty Analysis

Variable q_i	Nominal Value	$\frac{\Delta R_C}{\Delta q_i} (\equiv \pm \frac{\partial E}{\partial q_i})$	S.R.	δq_i	$(\frac{\Delta R_C}{\Delta q_i} \delta q_i)^2 (\frac{^\circ C}{W})^2$
P	454 N	$-2.39 \times 10^{-3} \frac{^\circ C}{WN}$	0.142	4N	9.1×10^{-5}
R_1	9.527×10^{-3} m.	$-311.2 \frac{^\circ C}{Wm}$	0.388	1×10^{-4} m.	9.685×10^{-4}
R_2	0.7604 m.	$-2.783 \frac{^\circ C}{Wm}$	0.278	6.22×10^{-2} m.	2.996×10^{-2}
T_{ch}	75.25 $^\circ C$	$0.2038 W^{-1}$	2.15	0.456 $^\circ C$	8.636×10^{-3}
T	26.88 $^\circ C$	$-0.2038 W^{-1}$	0.769	0.0331 $^\circ C$	4.55×10^{-5}
$\frac{dT}{dx} _i$	155.6 $^\circ C/m$	$-0.06335 \frac{m}{W}$	1.384	2.344 $\frac{^\circ C}{m}$	2.205×10^{-2}
k_i^*	72.9 $\frac{W}{mK}$	$0.03658 \frac{^\circ C}{W/mK}$	0.373	2.14 $\frac{W}{mK}$	6.128×10^{-3}
C^{**}	0.5230	$-12.168 \frac{^\circ C}{W}$	0.894	1.05×10^{-2}	1.62×10^{-2}
k_{og}	0.0284 $\frac{W}{mK}$	$-119.0 \frac{^\circ C}{W/mK}$	0.442	5.66×10^{-4} W/mK	4.537×10^{-3}
E	203.5 GPa	$5.8 \times 10^{-3} \frac{^\circ C/W}{GPa}$	0.157	3.5 GPa	3.952×10^{-4}
ν	0.30	$0.657 \frac{^\circ C}{W}$	0.026	-	-
L_m	0.076 μm	$1.74 \frac{^\circ C}{W\mu m}$	0.017	-	-
α	0.90	$-0.242 \frac{^\circ C}{W}$	0.028	-	-
A_f	4.34×10^{-4} m ²	$-1.641 \times 10^4 \frac{^\circ C}{Wm^2}$	1	5.5×10^{-7} m ²	8.146×10^{-5}
					$\Sigma \delta E^2 = 8.91 \times 10^{-2}$

* The value of $\Delta R_C / \Delta q_i$ given is $(\frac{\partial R'_C}{\partial k_{st}} C - \frac{\partial R_C}{\partial k_i})$, which is actually $\frac{\partial E}{\partial k_i}$.

** The value of $\Delta R_C / \Delta q_i$ given is $(\frac{\partial R'_C}{\partial k_{st}} k_i - \frac{\partial R_C}{\partial C})$ which is actually $\partial E / \partial C$.

uncertainty and sensitivity ratio (S.R.). The sensitivity ratio for variable q_i is defined as

$$S.R. = \left| \left(\frac{\Delta R_c}{R_c} \right) / \left(\frac{\Delta q_i}{q_i} \right) \right|.$$

A sensitivity ratio less than unity indicates that q_i does not propagate its error into R_c (or R_c) as strongly as its own variation, whereas a sensitivity ratio greater than unity shows that Δq_i has a greater relative effect on the value of R_c (the result) than it does on the value of q_i .

The value of $\left(\frac{\partial R_c}{\partial q_i} \delta q_i \right)^2$ for each parameter (q_i) is also shown in Table I.E.1 so the dominant uncertainties can be identified. As is evident from the results shown, the uncertainties in R_2 , $\frac{dT}{dx}|_i$ and C dominate the uncertainty in $\bar{E}$, which can be obtained by taking the square root of the sum of the terms in the last column of Table I.E.1. In this example, $\delta \bar{E} = \sqrt{8.91 \times 10^{-2}} = 0.30^\circ\text{C/W}$.

The sensitivity ratios for the parameters v , L_m and α (α_1, α_2) are very small, as seen in Table I.E.1. Since the likely variations in these parameters are certainly not more than 15% of their nominal values, their contribution to the uncertainty in $\bar{E}$ should be negligible. The uncertainty in $\bar{E}$ contributed by the uncertainty in P_r is also expected to be negligible, since P_r is not strongly temperature dependent and is known fairly accurately.

While the results presented in Table I.E.1 apply to the second resistance measurement in air at 454N (i.e., trial 2), most of the

information presented is also applicable to trials 1 and 3 at 454N. Neglecting small changes in δC , δk_{og} and δk_i caused by small changes in the mean interface temperature and the average temperature of the lower Armco-iron bar, the only uncertainties and partial derivatives which change in uncertainty analyses of trials 1 and 3 are those associated with the resistance measurement itself (i.e., those associated with δT_{ch} , $\delta \bar{T}$ and $\delta (\frac{dT}{dx})_i$). If those uncertainties and partial derivatives are evaluated, it is found that the resulting change in $\delta \bar{E}$ is negligible, thus $\delta \bar{E}$ remains effectively 0.30°C/W for trials 1 and 3. The root-mean square value of $\delta \bar{E}$ for the three trials, designated δE_r , is also 0.30°C/W in this case. The significance of δE_r is that 95% of the individual observations of $\bar{E}$ should, if our estimate of δE_r is accurate, lie between $\bar{E} \pm \delta E_r$, where $\bar{E}$ here refers to the true value of $\bar{E}$ (i.e., the value defined by equation (I.E.2)) with "exact" values of R_c and R_c .

Another estimate of the value of $\delta \bar{E}_r$ cannot be obtained from the variance of $\bar{E}$ for the three resistance measurements performed in air at 454N because some of the significant independent variables in Table I.E.1, specifically k_i , C , k_{og} , $\bar{E}$ and A_i , were not actually measured or sampled more than once in the experiments. It is assumed, however, that the variables P , R_1 , R_2 , T_{ch} , $\bar{T}$ and $(\frac{dT}{dx})_i$ were randomly sampled in each test, as mentioned previously, and hence that the standard deviation of the repeat measurements of R_c represents, to a first approximation, the uncertainty in $\bar{E}$ contributed by those variables. The standard deviation of the repeat measurements of R_c , multiplied by an appropriate confidence coefficient, and the square root of the sum of the first six entries in the last column of Table I.E.1 will be designated δR_{exp} and δR_c^+ .

respectively for convenience. The values of δR_{exp} and δR_C^+ should be comparable, as indicated previously.

The standard deviation of the three values of R_C obtained in trials 1, 2 and 3 at 454N in air is 0.16°C/W so $\delta R_{\text{exp}} = 4.3 \times 0.16 = 0.686^\circ\text{C/W}$ since $t = 4.3$ for two degrees of freedom and a 95% confidence interval. The value of $(\delta R_C^+)^2$ ranges from $5.95 \times 10^{-2} (^\circ\text{C/W})^2$ in trial 3 to $6.18 \times 10^{-2} (^\circ\text{C/W})^2$ in trial 2, with its average value in the three trials being $6.08 \times 10^{-2} (^\circ\text{C/W})^2$, so the rms value of δR_C^+ is 0.246°C/W (i.e., $\sqrt{6.08 \times 10^{-2}}$). Thus $\delta R_{\text{exp}} = 0.686^\circ\text{C/W}$ while $\delta R_C^+ = 0.246^\circ\text{C/W}$. These two values are obviously in poor agreement, but it must be recognized that the value of δR_{exp} , which was estimated from a very small sample size (i.e., only three measurements of R_C), is itself very uncertain. A much more precise estimate of δR_{exp} can be obtained if it is assumed that the resistance measurements in air are distributed with approximately the same variance at all loads since this allows all of the resistance measurements made in air (30 in total) to be combined into a single data base for the purpose of estimating δR_{exp} . It is actually reasonable to assume that the error variance is not a strong function of load because the contact resistance itself is not a strong function of load (in air). The equal variance assumption was tested by estimating the variance of the three resistance measurements at each contact load and then applying the F ratio test [101] to the high and low estimates. The probability that the observed high/low variance ratio could have been obtained when the variances were actually equal was found to be greater than 20% so there did not appear to be any reason to reject the equal variance assumption.

It can be shown that the average value of the variance estimates at the ten contact loads is $0.0176 (^{\circ}\text{C}/\text{W})^2$. The average variance, which represents the best estimate of the true error variance if the error variance is not a function of T_{oad} as assumed, is based on 20 degrees of freedom (i.e., $30-10$) since the total number of resistance measurements is 30 and the number of means estimated from these measurements is 10, so $\delta R_{\text{exp}} = 2.09 \times \sqrt{0.0176 (^{\circ}\text{C}/\text{W})^2} = 0.277^{\circ}\text{C}/\text{W}$ since the confidence coefficient (t) is 2.09 for 20 degrees of freedom and a 95% confidence interval. The agreement between δR_{exp} ($=0.277^{\circ}\text{C}/\text{W}$) and δR_{C}^{+} ($=0.246^{\circ}\text{C}/\text{W}$) is now much better than before and most of the improvement in agreement was achieved by lowering the confidence coefficient (t) for the estimate of δR_{exp} . Since the agreement between δR_{exp} and δR_{C}^{+} is now fairly good, it will be assumed that $\delta R_{\text{C}}^{+} = \delta R_{\text{exp}} = 0.277^{\circ}\text{C}/\text{W}$. The rms value of δE at 454N, which was $0.30^{\circ}\text{C}/\text{W}$ previously, becomes $0.32^{\circ}\text{C}/\text{W}$ with this change.

Since three resistance measurements were made at a contact load of 454N, the corresponding values of E can be averaged to obtain E_{m} , which should be a better estimate of the true value of E than the individual values. The uncertainty in an individual observation of E (i.e., δE), is known but the mean values of samples of three resistance measurements will be more closely grouped about the true resistance than the individual observations. In other words, the uncertainty in E_{m} , denoted δE_{m} , should be less than the uncertainty in an individual measurement (δE). The uncertainty contributed to δE_{m} by the last eight sources of uncertainty listed in Table I.E.1 remains the same as for an individual observation, however, because those sources were not sampled in the repeat measurements of R_{C} . The uncertainty contributed to δE_{m} by the first six sources of

uncertainty listed in Table I.E.1, which is $(\delta R_{\text{exp}})^2$ for the single observation case, is decreased by a factor of three if the mean resistance is calculated from three resistance measurements. Thus, if δE is calculated from:

$$\delta E = [(\delta R_{\text{exp}})^2 + \left(\frac{\partial R_c}{\partial k_{\text{st}}} C - \frac{\partial R_c}{\partial k_i}\right)^2 \delta k_i^2 + \left(\frac{\partial R_c}{\partial k_{\text{st}}} k_i \delta C\right)^2 + \left(\frac{\partial R_c}{\partial k_{\text{og}}} \delta k_{\text{og}}\right)^2 + \left(\frac{\partial R_c}{\partial E} \delta E\right)^2]^{1/2} \quad (\text{I.E.22})$$

then δE_m is given by:

$$\delta E_m = \left[\frac{(\delta R_{\text{exp}})^2}{3} + \left(\frac{\partial R_c}{\partial k_{\text{st}}} C - \frac{\partial R_c}{\partial k_i}\right)^2 \delta k_i^2 + \left(\frac{\partial R_c}{\partial k_{\text{st}}} k_i \delta C\right)^2 + \left(\frac{\partial R_c}{\partial k_{\text{og}}} \delta k_{\text{og}}\right)^2 + \left(\frac{\partial R_c}{\partial E} \delta E\right)^2 \right]^{1/2} \quad (\text{I.E.23})$$

At a contact load of 454N, it can easily be shown that $\delta E_m = 0.23^\circ\text{C/W}$.

Since it has already been assumed that δR_{exp} did not vary significantly with contact load, the first term in the square brackets in equation (I.E.23) will be the same at all contact loads. The values of the remaining terms in equation (I.E.23) will probably vary slightly with contact load* but it is doubtful whether there is much to be gained

* $\frac{\partial R_c}{\partial k_{\text{og}}}$ will increase in value as the contact load is decreased and gas conduction becomes more important, but this increase will be offset by a corresponding decrease in the uncertainty contributed by δC .

by re-evaluating these terms at each contact load, particularly since the value of

$$\frac{(\delta R_{\text{exp}})^2}{3}$$

is, itself, equal to the combined value of the remaining terms in the square brackets (at least at 454N). Thus it will be assumed that $\delta E_m = 0.23^\circ\text{C/W}$ at all contact loads. This uncertainty is plotted as an error band on the mean resistance curve (for tests in air) shown in Figure I.6.4, but it should be recognized that δE_m is actually the uncertainty in E , as defined in equation (I.E.2), rather than the uncertainty in the mean value of the measured contact resistance at a particular load.

I.E.2 Uncertainty Analysis for Contact Resistance in Vacuum

As was the case in the contact resistance in air uncertainty analysis, the values of E and δE for the contact resistance in vacuum measurements can be computed and compared. The value of E can be computed from equation (I.E.2), as before, while δE can be calculated from equation (I.E.17) if the term containing the uncertainty δk_{og} is omitted. All of the uncertainties in equation (I.E.17) (i.e., δP , δR_1 , ... δE) are calculated as described in section I.E.1 and, except for the uncertainties δT_{ch} , δT and $\delta \left(\frac{dT}{dx} \right)_f$, they all have the values given in Table I.E.1. Some sort of analytical model must be used to calculate the partial derivatives.

$$\frac{\partial R_c}{\partial P}, \frac{\partial R_c}{\partial R_1}, \frac{\partial R_c}{\partial R_2}, \frac{\partial R_c}{\partial k_{st}} \text{ and } \frac{\partial R_c}{\partial E}$$

so, for the purposes of this analysis, it is assumed that a valid model can be obtained by combining Yovanovich's simple expression for the constriction resistance of an elliptical contact (i.e., resistance equation 2 in Table I.2.1) with Dyson's formulae for estimating contact dimensions (equations (I.3.15) and (I.3.16)). Contact resistances predicted using Yovanovich's model are compared to the measured resistances in Figure I.6.5. If Dyson's formulae are introduced into Yovanovich's constriction resistance equation, an explicit expression for R_c ($P, R_1, R_2, k_{st}, E, \nu$) is obtained from which the partial derivatives required in equation (I.E.17) can be evaluated. That expression, and the associated partial derivatives, are somewhat clumsy so they will not be given here.

It was decided to perform an uncertainty analysis on the resistance measurements obtained at 299.8N in vacuum since the difference between the two resistances obtained at that load was larger, relative to the mean resistance, than that at any other load. Resistances of 15.89°C/W and 17.04°C/W, which differ by 1.15°C/W or 7.0% of their average value, were obtained in trials 2 and 3 respectively at 299.8N. The experimental data for trial 2 are given in Table I.6.1.

The results of the uncertainty analysis for trials 2 and 3 at 299.8N are given in Tables I.E.2 and I.E.3 respectively. As is evident from the values of

$$\left(\frac{\partial R_c}{\partial q_i} \delta q_i \right)^2$$

Table I.E.2: Uncertainty Analysis for Trial 2 at a Contact Load of 299.8 N in Vacuum

Variable q_i	Nominal Value	$\frac{\partial R_C}{\partial q_i} (\equiv \pm \frac{\partial E}{\partial q_i})$	S.R.	δq_i	$(\frac{\partial R_C}{\partial q_i} \delta q_i)^2, (\frac{^\circ C}{W})^2$
P	299.8N	$-0.0218 \frac{^\circ C}{WN}$	0.377	4N	7.6×10^{-3}
R_1	9.527×10^{-3} m.	$-187.37 \frac{^\circ C}{Wm}$	0.103	1×10^{-4} m.	3.51×10^{-4}
R_2	0.7604 m.	$-6.229 \frac{^\circ C}{Wm}$	0.273	6.22×10^{-2} m.	0.1501
T_{ch}	81.86°C	$0.322 W^{-1}$	1.661	0.2704 °C	7.58×10^{-3}
$\bar{T}$	24.07°C	$-0.322 W^{-1}$	0.488	0.0198 °C	4.04×10^{-5}
$\left. \frac{dT}{dx} \right _i$	$98.08 \frac{^\circ C}{m}$	$-0.1895 \frac{m}{W}$	1.171	$1.394 \frac{^\circ C}{m}$	6.98×10^{-2}
k_i^*	$73.13 \frac{W}{mK}$	$-0.0278 \frac{^\circ C/W}{W/mK}$	0.128	$2.15 \frac{W}{mK}$	3.58×10^{-3}
C^{**}	0.521	$-33.25 \frac{^\circ C}{W}$	1.091	1.04×10^{-2}	0.1196
E	203.5 GPa	$2.778 \times 10^{-2} \frac{^\circ C/W}{GPa}$	0.333	3.5 GPa	9.46×10^{-3} $\varepsilon = 0.368$ $\delta E = 0.606$

* The value of $\frac{\partial R_C}{\partial q_i}$ given is $(\frac{\partial R_C'}{\partial k_{st}} C - \frac{\partial R_C}{\partial k_i})$, which is actually $\frac{\partial E}{\partial k_i}$.

** The value of $\frac{\partial R_C}{\partial q_i}$ given is $(\frac{\partial R_C'}{\partial k_{st}} k_i - \frac{\partial R_C}{\partial C})$, which is actually $\frac{\partial E}{\partial C}$.

Table I.E.3: Uncertainty Analysis for Trial 3 At 299.8 N in Vacuum

Variable, q_i	Nominal Value	$\frac{\partial R_c}{\partial q_i} (\equiv \pm \frac{\partial E}{\partial q_i})$	S.R.	δq_i	$(\frac{\partial R_c}{\partial q_i} \delta q_i)^2, (\frac{^\circ C}{W})^2$
P	299.8 N	$-0.0218 \frac{^\circ C}{WN}$	0.377	4N	7.6×10^{-3}
R_1	9.527×10^{-3} m	$-187.37 \frac{^\circ C}{Wm}$	0.103	1×10^{-4} m	3.51×10^{-4}
R_2	0.7604 m	$-6.229 \frac{^\circ C}{Wm}$	0.273	6.22×10^{-2} m	0.1501
T_{ch}	90.12°C	$0.3173 W^{-1}$	1.697	0.325°C	1.06×10^{-2}
T	28.53°C	$-0.3173 W^{-1}$	0.537	0.0212°C	4.52×10^{-5}
$\frac{dT}{dx}_i$	99.31°C/m	$-0.68 m/W$	1.160	1.496°C/m	8.668×10^{-2}
k_i^*	72.8 W/mK	$-0.015 \frac{^\circ C/W}{W/mK}$	0.065	2.14 W/mK	1.03×10^{-3}
C^{**}	0.526	$-36.39 \frac{^\circ C}{W}$	1.0	1.05×10^{-2}	0.1460
E	203.5 GPa	$2.778 \times 10^{-2} \frac{^\circ C/W}{GPa}$	0.333	3.5 GPa	9.46×10^{-3}
					$\Sigma = 0.404$
					$\delta E = 0.636 \text{ } ^\circ C/W$

* The value of $\frac{\partial R_c}{\partial q_i}$ given is $(\frac{\partial R_c}{\partial k_{st}} C - \frac{\partial R_c}{\partial k_i})$, which is actually $\frac{\partial E}{\partial k_i}$

** The value of $\frac{\partial R_c}{\partial q_i}$ given is $(\frac{\partial R_c}{\partial k_{st}} k_i - \frac{\partial R_c}{\partial C})$, which is actually $\frac{\partial E}{\partial C}$

given in the last columns of those tables, the uncertainty in E (δE) contributed by the uncertainty in R_2 was much larger in vacuum than it was in air. Also, the contributions made by all of the other uncertainties, except δC , are relatively small.* The values of δE obtained in trials 2 and 3 were 0.606°C/W and 0.636°C/W respectively, while the value of δR_C^+ (i.e., the uncertainty contributed by the first six terms in Tables I.E.2 and I.E.3) was 0.485°C/W in trial 2 and 0.498°C/W in trial 3. The rms value of δR_C^+ for trials 2 and 3 was 0.491°C/W . δR_{exp} , which should be comparable in value to δR_C^+ , can be obtained from the repeat resistance measurements, as was done in the resistance in air uncertainty analysis, but it would obviously be unrealistic to assume that the error variance of the resistance measurements in vacuum is approximately the same at all contact loads. This assumption, while easily justified in the air uncertainty analysis, cannot be used in the vacuum analysis because the contact resistance itself changes by a factor of three in vacuum from the largest contact load to the smallest. If an estimate of the error variance at a given load is obtained from the two resistance measurements at that load, however, the estimate will be based on only one degree of freedom and hence will be very imprecise. For the purpose of obtaining a larger data

* It can be shown that this comment applies at high contact loads in vacuum, while at low loads (i.e., below 175.8N) the uncertainties contributed by δR_2 and $\delta(\frac{dT}{dx})_1$ are almost equal and at 26N the uncertainty in P dominates the uncertainty in E .

base from which to estimate δR_{exp} , it was decided to average the error variances for the last five contact loads (i.e., from 175.8N to 454N) and use the resulting average variance to estimate δR_{exp} for those loads. Similarly, it was assumed that an estimate of δR_{exp} for the first five contact loads (i.e., from 26N to 132.1N) could be obtained by averaging the error variances at those loads. Since two resistance measurements were made at each contact load, an estimate of the average value of δR_{exp} for five contact loads will be based on five degrees of freedom (i.e., $10-5=5$) since 10 resistance measurements were made in total and five means were estimated from those measurements. Only two series of resistance measurements were made (i.e., the position of the contact on the contact surface was only changed once), however, so the contact radii R_1 and R_2 were each sampled only twice. In reality, each contact radius was sampled somewhat more than twice because part of the variation in the contact radii was sampled as the contact load, and hence contact size, changed within each load range. Since the uncertainty in R_2 dominates the uncertainty in E , at least at the last five contact loads, it seems somewhat over-optimistic to calculate δR_{exp} for those loads using a confidence coefficient appropriate to five degrees of freedom. If R_2 was only sampled twice, and the remaining five variables contributing to δR_{exp} were measured without error (or with negligible error), a confidence coefficient appropriate to one degree of freedom would have to be used to calculate δR_{exp} . Since both of the preceding provisos are not satisfied exactly in this case, it is assumed that $t=5$ for the mean error variance at the last five contact loads since this value is less than t 's value for one d.o.f. (12.7) but greater than its value for five d.o.f. (2.57).

It should be apparent from the range of possible values of t , however, that our estimate of δR_{exp} , and hence δE , will be very imprecise.

Averaging the error variance estimates for the last five contact loads and multiplying the result by $t(=5)$ yields a value of 2.2°C/W for δR_{exp} at those loads. Since the average value of δR_C^+ for trials 2 and 3 at 299.8N is 0.49°C/W , the agreement between δR_C^+ and δR_{exp} (2.2°C/W) is very poor in this instance. It can be shown that the rms value of δR_{exp} for the last five contact loads is 0.50°C/W (i.e., only slightly higher than the value at 299.8N) so the agreement between δR_C^+ and δR_{exp} is not improved by considering more tests. Under the circumstances (i.e., with such a small data base), this agreement is probably the best that can be achieved. For the purposes of this analysis, it is assumed that $\delta R_C^+ = \delta R_{\text{exp}} = 2.2^\circ\text{C/W}$ at loads in the 299.8N to 454N range. Since the uncertainties in k_1 , C and E contribute very little to the uncertainty in E , at least relative to the uncertainty contributed by δR_{exp} , it is also assumed that $\delta E = 2.2^\circ\text{C/W}$ in this load range.

The value of δE is the uncertainty in an individual measurement of E , so the uncertainty in the mean of two measurements of E (i.e., δE_m) is given by

$$\begin{aligned}\delta E_m &= \sqrt{\frac{\delta E^2}{2}} \\ &= \sqrt{\frac{2.2^2}{2}} = 1.55^\circ\text{C/W} \quad (\text{I.E.24})\end{aligned}$$

This value of δE_m is plotted as an error interval about the mean resistance curve for the last five contact loads (in vacuum) in Figure I.6.4.

As mentioned previously, it can be shown that the uncertainty in R_2 does not dominate the uncertainty in $\bar{E}$ at the first five contact loads to the extent that it does at higher loads because, at low loads, the uncertainties in P and $\left. \frac{dT}{dx} \right|_1$ also contribute significantly. Thus the fact that R_2 was not sampled randomly in each of the ten resistance measurements in the 26N-132.1N load range is not so important and it is reasonable to assume that our estimate of δR_{exp} for the first five loads will be based, more or less, on five d.o.f. Averaging the error variance estimates for the first five contact loads and multiplying the result by a confidence coefficient (t) for five d.o.f. and a 95% confidence interval in this case yields a value of 1.55°C/W for δR_{exp} at those loads. If we assume that $\delta \bar{E} = \delta R_{\text{exp}} = 1.55^\circ\text{C/W}$ then

$$\delta \bar{E}_m = \sqrt{1.55^2/2} = 1.20^\circ\text{C/W}.$$

This uncertainty is plotted as an error interval about the mean resistance curve for the first five contact loads (in vacuum) in Figure I.6.4.

PART II

ALTERNATE NUMERICAL ALGORITHMS FOR THE ANALYSIS
OF THREE-DIMENSIONAL HEAT TRANSFER THROUGH CONTACTS

CHAPTER II.1

INTRODUCTION

This part of the thesis considers other numerical methods for solving the model contact problem, specifically methods which will be more generally applicable than the ellipsoidal finite-difference model already described. The analyses will be three-dimensional for greatest accuracy and generality, although a two-dimensional analysis might provide results of acceptable accuracy in many cases of practical interest. Predicted heat fluxes through and around the contact patch and predicted contact resistances for each of the new models will be compared to the corresponding predictions of the ellipsoidal finite-difference model to permit a complete assessment of their accuracy. Solutions obtained with the ellipsoidal finite-difference model will thus serve as benchmark solutions to evaluate the other models.

II.1.1 Choice of Algorithms

The characteristically small dimensions of a contact patch necessitate using a very high degree of mesh refinement in its vicinity to minimize discretization errors in that region as well as to permit the contact's size and shape to be accurately represented in the numerical model.* A grid incorporating a high degree of local refinement is difficult to construct (in practice) in three dimensions with most numerical solution procedures, however, and also causes the system of algebraic equations defining the discrete solution to be unavoidably large and hence expensive to solve. Using a mesh which is highly refined locally (or globally) can also produce finite-element or finite-difference equations which are ill-conditioned [106].

* Although it might be possible to develop a hybrid method which used the ellipsoidal finite-difference model in regions near to the contact and a conventional finite-element [103] or finite-difference [104] discretization in regions remote from it, the development of such a model was not attempted in this study.

as demonstrated by Fried [105, 107], and adversely affects the convergence rate of the classical iterative technique for solving those equations, successive overrelaxation [108, 109]. The algebraic equations defining the discrete solution could be solved directly (using Gaussian elimination, for instance) instead of iteratively provided they were sufficiently well conditioned, of course, but direct equation solving algorithms of the type commonly used in finite-element programs require large computer memory allocations and can be prohibitively slow when applied to the very large systems of equations obtained in three-dimensional analyses. This matter is discussed in greater detail in Appendix II.A where it is shown that Carleton University's Xerox Sigma 9 computer, on which all of the analyses described in this thesis were performed, would likely require more than 90 hours to solve the finite-difference equations for a $30 \times 30 \times 30$ grid (i.e., a grid with 30 nodes in each of the three coordinate directions) if the solution was obtained with a banded equation solving routine of the sort commonly employed in finite-element programs. In addition, approximately 24×10^6 words of computer memory (i.e., a large amount even for modern computers) would be required to store the banded coefficient matrix for that analysis. Since the Carleton computer mentioned previously has a usable high-speed core capacity of only about 70,000 words (32 bit word length), out-of-core storage devices would have to be used extensively to solve such a large problem on that computer. The use of out-of-core storage devices would likely increase computation times (and hence costs) and would certainly complicate the programming of the solution.

The discussions given above and in Appendix II.A of some of the practical and cost-related aspects of three-dimensional finite-element and finite-difference analyses were included to demonstrate that such analyses need large computer memory allocations and can be expensive to perform using

methods which are quite acceptable for two-dimensional or small three-dimensional analyses. To avoid the expense and difficulty of performing a true three-dimensional analysis of the model contact problem with an algorithm that would also be applicable to other heat transfer problems involving large aspect ratio contacts, the author made the simplifying assumption that most such analyses would have a strong two-dimensional character owing to the much larger lateral than longitudinal constriction of the heat flow approaching the contact. Three-dimensional effects were still modelled in an approximate manner, however, by applying a numerical technique which linearly combines two-dimensional (e.g., x-y) solutions (obtained from the problem of interest) in the third (e.g., z) dimension to produce an approximate three-dimensional solution. The two-dimensional solutions, termed trial functions, are obtained in the dimensions in which it is assumed that most of the heat flow occurs. In heat transfer problems involving large aspect ratio contacts, two-dimensional finite-difference (or finite-element) solutions obtained at different points along the x-axis (i.e., major axis) of the contact-centered coordinate system using the y-z boundary conditions at those points serve as the trial functions that are linearly combined in the x-direction to obtain the approximate three-dimensional solution. The two-dimensional (y-z) trial functions are obtained at points (on the x-axis) both inside and outside of the contact patch.

The method just described, which is known as the synthesis method, permits three-dimensional effects to be accounted for in an approximate manner at reasonable cost and with almost no more computer memory than that required to obtain one of the two-dimensional trial functions.

Approximate solutions to extremely large three-dimensional problems can be obtained on small computers with this method. The computer program

that was written to implement the synthesis method in this study was actually developed by modifying a two-dimensional finite-difference program that the author had originally written to analyse the results of Simpson's study [22, 66, 67] of heat transfer between parallel tubes in line contact, described in Chapter I.2. The modification, which was accomplished quite easily, was undertaken in an attempt to permit the approximate modelling of three-dimensional effects in the model contact problem, as mentioned previously. It will be seen in later sections of this thesis that the synthesis method was fairly successful in that regard even though three-dimensional effects are quite significant in the model contact problem, particularly for the case of the contact in vacuum.

The synthesis method is described in considerable detail in Chapter II.2 where it is formulated in a manner that is not found elsewhere. The resulting model is applied to the model contact problem in Chapter II.3.

It will be seen later that the synthesis method, as it is implemented in this study, is really an approximate method for solving the finite-difference equations obtained from a three-dimensional finite-difference discretization. Although it would obviously be preferable to solve those equations exactly, that can be a prohibitively expensive proposition with large systems of equations for the reasons given previously. Iterative methods would appear to offer the greatest promise for solving large systems of finite-difference equations because they do not require storage for, or operate on, zero elements in the

equations, but the most widely used iterative method, successive over-relaxation, converges prohibitively slowly (in the author's experience) when used to solve the finite-difference equations for a problem such as that of the model contact. A relatively new iterative method known as multigrid (MG) overcomes many of the shortcomings of successive overrelaxation, however, and also has a number of capabilities which make it particularly attractive for application to three-dimensional analyses requiring a high degree of local mesh refinement, such as analyses involving contacts. Although multigrid has shortcomings of its own at the present time, the author believes that even its current capabilities warrant its consideration in this study as an alternative to the synthesis method for solving certain types of problems. Multigrid is particularly well-suited to the analysis of the model contact problem, for instance, as will be demonstrated later. Multigrid will not be described in detail in this report since the design of a multigrid algorithm depends strongly on the specific details of the problem being analyzed (e.g., the problem geometry and boundary conditions), but the basic operation of the algorithm and features that need to be understood to apply it to the model contact problem are outlined in Chapter II.4. The results of the analysis of the model contact problem are then discussed in Chapter II.5 and a comparison of the advantages and disadvantages of the synthesis method and multigrid is given in Chapter II.6.

CHAPTER II.2

THE SYNTHESIS METHOD

II.2.1 Introduction

As mentioned previously, the large system of simultaneous equations associated with a complex, three-dimensional finite-difference or finite-element analysis can often only be solved at great expense, if indeed sufficient computer memory is available to solve the equations at all. Two-dimensional problems, on the other hand, are quite tractable and can be solved easily on most computer installations. For this reason, analysts frequently attempt to acquire some knowledge about a three-dimensional temperature distribution by solving related two-dimensional problems with different boundary conditions, heat generation rates, etc. In such situations the analyst expects that the two-dimensional solutions in some sense bracket the true, three-dimensional, solution. In some instances, however, this approach may be inadequate, in which case it is worthwhile examining the possibility of extracting information more effectively from the two-dimensional "bracketing" solutions, perhaps obtaining quantitative rather than qualitative information about the three-dimensional temperature distributions from those solutions. It is for this purpose that the synthesis method, which is described in the rest of this chapter, is useful.

II.2.2 Continuous Synthesis

In the synthesis method, two-dimensional "bracketting" solutions obtained from the problem of interest are linearly combined in the third dimension, using variational or weighted-residual techniques, to produce an approximation to the true three-dimensional temperature distribution. The two-dimensional bracketting solutions, termed trial functions, are solutions which the analyst expects will, in some sense, bracket the actual temperature distribution in those dimensions. In the present case, a three-dimensional temperature distribution, $T(r, \theta, z)$, in the cylindrical coordinate system* is approximated with the synthesis method by expressing it in separated form as,

$$T'(r, \theta, z) = \sum_{n=1}^N T_n(r, \theta) Z_n(z) \quad (II.2.1)$$

where the prime on the synthesized temperature distribution identifies it as an approximation, $T_n(r, \theta)$ is the n th trial function, $Z_n(z)$ is the n th axial "mixing" or "blending" function, and N is the total number of trial functions used in the approximation. Any number (N) of trial functions can be used, the only restriction being that they must be linearly independent.

The procedure of combining two-dimensional trial functions to obtain three-dimensional solutions (i.e., the synthesis method) has been used extensively to solve the neutron group diffusion equations in reactor cores [112], and the method was developed initially for that

* The synthesis method is applied to the pressure/calandria tube contact problem in Part III of the thesis so the method is formulated in the cylindrical (r - θ - z) coordinate system in this chapter for convenience.

purpose by Kaplan [113]. Yasinsky extended the method to steady-state heat transfer in references [114] and [115] but no other instances of the method being applied to heat transfer problems, at least as a numerical technique, are reported in the literature. In Yasinsky's solution for the unknown Z_n ($n=1,N$), the temperature approximation given by equation (II.2.1) is substituted into an appropriate form of the variational (quadratic) functional for heat conduction, which is provided in Appendix II.B, whereupon stationary conditions defining the Z_n are produced by requiring that the first variation of the functional vanish. The stationary conditions take the form of a set of coupled ordinary differential equations in z .

In an alternate method for obtaining the unknown Z_n , the expression obtained by substituting equation (II.2.1) into the heat conduction equation (i.e., equation (I.4.17)) is multiplied by N linearly independent weighting functions to give N equations which are then integrated over the solution domain. This procedure, which is an application of the method of weighted-residuals [116], produces N coupled second order ordinary differential equations defining the Z_n . Kaplan et al [117] used the method of weighted residuals to derive the equations defining the unknown Z_n for the neutron diffusion problem, but they did not attempt to relate the technique to any of the standard, global* trial function methods. The key ingredients of a weighted-residual

* A global trial function method is one in which the trial functions are defined and used at all points in the solution domain. The finite-element method is not a global method by this definition since each trial function is defined and used in only one element.

solution, namely the trial, residual and weighting functions, were not clearly defined over the entire solution domain in the derivation of Kaplan et al. The author will correct that deficiency in his description of the synthesis technique, which will also be based on the method of weighted-residuals.

In practical applications of the synthesis method, both the trial functions (T_n) and the mixing functions (Z_n) are usually represented by discrete (i.e., point-wise) rather than continuous functions since derivatives can be expressed as finite-differences in that case. As an example of this, consider the three-dimensional cylindrical grid shown in Figure II.2.1. The indices i , j and k ($i=1, NR$, $j=1, NT$ and $k=1, NZ$) in Figure II.2.1 are nodal addresses identifying mesh points at constant values of r , θ and z respectively (i.e., node i, j, k is located at r_i, θ_j, z_k). The synthesized temperature distribution given by equation (II.2.1) can be redefined for the discrete case as;

$$T(i,j,k) = \sum_{n=1}^N T_n(i,j) Z_n(k) \quad (\text{II.2.2})$$

so the problem now consists of finding values for the $Z_n(k)$ ($k=1, NZ$; $n=1, N$) instead of obtaining expressions for the N continuous functions $Z_n(z)$ ($n=1, N$). The trial functions, $T_n(i,j)$, are easily obtained from finite-difference (as was the case in this analysis) or finite-element solutions. These solutions would normally be obtained using the r - θ boundary conditions, heat generation rates, etc., at different axial (z) locations in the problem of interest.

The $Z_n(k)$ in equation (II.2.2) can be obtained very simply if the

Figure II.2.1 Cylindrical Grid

finite-difference equations for all of the nodes in a finite-difference grid are obtained first. Those equations can be derived in a number of ways, one of which, the box integration method, was described in Chapter I.4. Second order accurate (at best) finite-difference equations can also be derived by applying the "heat balance" method, in which neighbouring nodes in a finite-difference grid are considered to be connected by appropriate internodal conductances [118] and a heat balance is performed on each node.* The conductance between nodes i, j, k and $i, j, k-1$ (designated $K(i,j,k)_{k-1}$ as in Section I.4.4) in Figure II.2.1, for instance, can be expressed as:

$$K(i,j,k)_{k-1} = k_s r_i \Delta\theta\Delta r/\Delta z \quad (\text{II.2.3})$$

Similarly, the surface conductance $K(i,j,k)_s$ between a convectively cooled (or heated) surface node i,j,k and a fictitious node at the bulk temperature (T_∞) of the surrounding fluid can be written as,

$$K(i,j,k)_s = h_f r_i \Delta\theta\Delta z \quad (\text{II.2.4})$$

where h_f is the film coefficient. Equations similar to equations (II.2.3) and (II.2.4) can be defined for all of the internodal conductances in a finite-difference grid, as shown in Appendix A of Part IV of the thesis for a graded cylindrical grid. If the same conductance designations are used in the present case as were used in Section I.4.4 (i.e., $K(i,j,k)_{i+1}$ refers to the conductance between nodes i, j, k and $i+1, j, k$, etc.),

* This method is closely related to the box integration method.

then performing a heat balance on an interior node i, j, k yields an equation which is identical to equation (I.4.62). The nodal equations for boundary nodes, although similar to the equations for interior nodes, are nevertheless slightly different in that they generally have some additional terms and lack certain others. Since different equations are obtained by performing heat balances on different types of nodes (e.g., interior, boundary), it is convenient to define a general equation that is applicable to all nodes. A suitable equation is:

$$\begin{aligned}
 & -K(i,j,k)_{i+1} T(i+1,j,k) - K(i,j,k)_{i-1} T(i-1,j,k) - K(i,j,k)_{j+1} T(i,j+1,k) \\
 & -K(i,j,k)_{j-1} T(i,j-1,k) - K(i,j,k)_{k+1} T(i,j,k+1) - K(i,j,k)_{k-1} T(i,j,k-1) \\
 & + (\Sigma K)T(i,j,k) - K(i,j,k)_s T_\infty + q'' A_s(i,j,k) = 0 \quad (II.2.5)
 \end{aligned}$$

where ΣK is the sum of the different conductances, $A_s(i,j,k)$ is the surface area of a boundary node and q'' is a specified surface heat flux defined by,

$$\nabla T \cdot \hat{n} = \frac{-q''}{k_s} \quad (II.2.6)$$

where $\hat{n}$ is the outward normal to the surface. Some of the terms in the general nodal equation (i.e., equation (II.2.5)) are inapplicable at certain nodes and hence would be set to zero at those nodes. At an interior node, for instance, we must have $K(i,j,k)_s = 0$ and $q'' = 0$ since the boundary-related terms must drop out of the equation. The terms $K(i,j,k)_s$ and q'' cannot both be non-zero at a given boundary node of course.

It should be mentioned that the synthesis method can, in general, only be used to solve problems with boundary conditions of the second (i.e., $\nabla T \cdot \hat{n} = -q''/k_s$) and/or third (i.e., $\nabla T \cdot \hat{n} = \frac{h_f}{k_s} (T_\infty - T)$) kinds. A specified boundary temperature, (i.e., $T = T_s$ on S_1 , where T_s is a specified temperature and S_1 is a boundary segment) must be treated as a boundary condition of the third kind by specifying $\nabla T \cdot \hat{n} = \frac{h_f}{k_s} (T_s - T)$ on S_1 and setting h_f to a very large value ($T \rightarrow T_s$ as $\frac{h_f}{k_s} \rightarrow \infty$). It will be shown in Section II.2.5 that Dirichlet conditions can, in special circumstances, be treated in an alternate manner.

If equation (II.2.5) is evaluated at every node in the grid shown in Figure II.2.1, $NR \times NT \times NZ (=N_e)$ nodal equations are obtained relating the temperatures at all of the nodes to one another. If the nodal temperatures and right-hand side terms in the nodal equations are stored in the N_e -dimensional column vectors T and Q respectively, the system of finite-difference equations can be written compactly as;

$$KT = Q \quad (II.2.8)$$

where K is the $N_e \times N_e$ "conductance matrix" containing all of the internodal conductances. As argued in Chapter I.4, K should be symmetric and positive-definite for a well-posed heat conduction problem because the heat conduction equation itself is self-adjoint and positive definite.* For a finite-difference grid which is continuous in the axial direction, such as that shown in Figure II.2.1, K has non-zero diagonal entries and six off-diagonal "stripes" of non-zero elements, as shown in Figure II.2.2. If the nodes in the grid are numbered sequentially with the subscript k .

* Provided the equation is linear.

X - NON-ZERO ELEMENT

=

Figure 11.2.2 K Matrix in Partitioned Form For Grid With $NR=4$, $NT=2$ and $NZ=4$

varying least rapidly, the unbroken stripes farthest from the diagonal contain the conductances for conduction in the axial direction (i.e., $K(i,j,k)_{k-1}$ and $K(i,j,k)_{k+1}$). The broken stripes nearer to the diagonal contain the remaining conductances (i.e., those associated with conduction in the r and θ directions). Because the mesh in question is axially continuous, the matrix K can be partitioned into NZ rows and NZ columns of $L \times L$ submatrices (where $L = NR \times NT$), as indicated by the dashed lines in Figure II.2.2. If the partitioned form of K is designated $\bar{K}$, then each $L \times L$ submatrix in K becomes an element of the $NZ \times NZ$ partitioned matrix, $\bar{K}$. This relationship is shown diagrammatically in Figure II.2.2. $\bar{K}$ is a tridiagonal matrix and K itself is block-tridiagonal.

We are now in a position to start deriving the governing equations for the $Z_n(k)$ in the synthesis technique. As a first step in the derivation, we attempt to obtain an approximate solution to equation (II.2.8) by applying the discrete weighted-residual method [119]. This method requires approximating the N_e -dimensional vector T in equation (II.2.8) as a linear combination of known, N_e -dimensional trial vectors (T_i), as shown below

$$T' = \sum_{i=1}^V C_i T_i \quad (\text{II.2.9})$$

The vector T' in equation (II.2.9) is the approximate solution to equation (II.2.8), while the C_i are unknown constants which must be evaluated in some manner. If the column vectors T_i ($i=1, V$) are placed side by side in the columns of an $N_e \times V$ rectangular matrix, W , and the C_i are stacked on top of each other in a column vector, C , then equation

(II.2.9) can be expressed compactly in matrix form as,

$$T' = WC \quad (II.2.10)$$

where

$$T' = \begin{bmatrix} T'(1) \\ T'(2) \\ \vdots \\ T'(N_e) \end{bmatrix} \quad (II.2.11)$$

$$W = \begin{bmatrix} T_1 & T_2 & T_3 & \cdots & T_V \end{bmatrix} \quad (II.2.12)$$

$$= \begin{bmatrix} T_1(1) & T_2(1) & T_3(1) & \cdots & T_V(1) \\ T_1(2) & T_2(2) & T_3(2) & \cdots & T_V(2) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ T_1(N_e) & T_2(N_e) & T_3(N_e) & \cdots & T_V(N_e) \end{bmatrix} \quad (II.2.13)$$

and

$$C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_V \end{bmatrix} \quad (II.2.14)$$

It should be recognized that, in general, the number of linearly-independent trial vectors used in the approximate solution will be much less than the number of unknown nodal temperatures (i.e., $V \ll N_e$).

with $\bar{W}$, it can easily be shown that the N -dimensional subvector $\bar{C}_k$ contains the elements $Z_n(k)$; $n=1, N$, found in equation (II.2.2). For this reason it is convenient to refer to the subvector $\bar{C}_k$ as $\bar{Z}_k$ (i.e., $\bar{C}$ becomes $\bar{Z}$) and to the vector C as Z . The vectors $\bar{Z}_k$ and Z are shown below.

$$\bar{C}_k \equiv \bar{Z}_k = \begin{bmatrix} Z_1(k) \\ Z_2(k) \\ \vdots \\ Z_{N-1}(k) \\ Z_N(k) \end{bmatrix} \quad (\text{II.2.18})$$

$$C \equiv Z = \begin{bmatrix} Z_1(1) \\ \vdots \\ Z_N(1) \\ Z_1(2) \\ \vdots \\ Z_N(2) \\ \vdots \\ Z_1(NZ) \\ \vdots \\ Z_N(NZ) \end{bmatrix} = \begin{bmatrix} \bar{Z}_1 \\ \bar{Z}_2 \\ \vdots \\ \bar{Z}_{NZ-1} \\ \bar{Z}_{NZ} \end{bmatrix} \quad (\text{II.2.19})$$

Using equations (II.2.10) - (II.2.19), we can rewrite equation (II.2.2) in matrix form as,

$$T = WZ \quad (\text{II.2.20})$$

The vector Z can now be obtained by applying the discrete weighted-residual method in the normal manner. First, equation (II.2.20) is substituted into

equation (II.2.8) to obtain the residual vector, designated E , where

$$E = KT - Q = KWZ - Q \quad (II.2.21)$$

The residual vector (E) is then forced to be zero, at least in an average sense, by orthogonalizing it with respect to $V (=N \times NZ)$ "weighting" vectors. This procedure produces V simultaneous equations which can be solved for the V unknowns (i.e., the $Z_n(k)$, $n=1, N$; $k=1, NZ$). While any set of linearly independent, real vectors could serve as the weighting vectors, an obvious choice is to make the weighting vectors the same as the trial vectors. The resulting formulation, known as Galerkin's method, is very popular since it produces results which, for a positive-definite, self-adjoint problem (such as a linear heat conduction problem), are identical to the results obtained by applying the classical Ritz method derived from variational principles [103].

Since each column of the trial matrix W , defined by equation (II.2.12), is a trial vector, we can apply Galerkin's method to the approximate solution of equation (II.2.8) by premultiplying equation (II.2.21) by W^T and setting the result to zero to obtain,

$$W^T E = W^T (KWZ - Q) = 0 \quad (II.2.22a)$$

or

$$W^T KWZ = W^T Q \quad (II.2.22b)$$

Equation (II.2.22b) represents a set of V simultaneous equations for which $W^T KW$ is the coefficient matrix, Z is the vector of unknowns, and $W^T Q$ is the right-hand side vector of known constants. Denoting the product

$W^T K W$ by D , it can easily be shown that D has certain desirable properties. First, $(W^T K W)^T = W^T K^T W$ by the transposition rule for matrix products (i.e., $(AB)^T = B^T A^T$) so, if $K=K^T$ (i.e., if the conductance matrix, K , is symmetric), then $D^T=D$ and D is symmetric. It can also be demonstrated that D is positive-definite (i.e., $x^T D x > 0$ for real $x \neq 0$) if K is positive-definite since, if we let $Wx=y$, then $y^T = x^T W^T$ and $x^T D x = x^T W^T K W x = y^T K y = +ve$ for real $y \neq 0$ and positive-definite K . If D is positive-definite, equation (II.2.22b) can be solved for the vector Z using direct (elimination) methods without resorting to pivoting.

Another weighted-residual formulation that is very popular is the least-squares method, in which the quadratic form $E^T E$ is minimized. The weighting matrix for this method, which can be found by differentiating $E^T E$ with respect to Z and setting the result to zero, is $W^T K$ so equation (II.2.22b) is replaced by the expression

$$W^T K K W Z = W^T K Q \quad (II.2.22c)$$

when the least-squares method is applied. Equation (II.2.22c) defines the "normal equations" for the least-squares solution. Although it is well-known that the normal equations are often poorly conditioned and hence difficult to solve directly, other methods with greater numerical stability are available for obtaining the least-squares solution, as discussed by Stewart [106]. Those methods will not be considered here, however, since only the Galerkin formulation of the synthesis method was used in this study.

The coefficient matrix, D , has a very simple structure in the Galerkin version of the synthesis method because the diagonal form of $\bar{W}$ causes each element of the partitioned conductance matrix, $\bar{R}$, (shown in Figure II.2.2) to be premultiplied by F^T and postmultiplied by F during the formation of $W^T K W$ (i.e., D). If the $V \times V$ matrix D is partitioned into NZ rows and NZ columns of $N \times N$ submatrices, then the partitioned form of D , designated $\bar{D}$, is a tridiagonal matrix (and hence D itself is block tridiagonal) in which the diagonal elements, $\bar{D}_{k,k}$ ($k=1, NZ$), are given by the relation,

$$\bar{D}_{k,k} = F^T \bar{R}_{kk} F \quad (\text{II.2.23})$$

and the off-diagonal terms (i.e., $\bar{D}_{k,k-1}$ and $\bar{D}_{k,k+1}$) are

$$\bar{D}_{k,k-1} = F^T \bar{R}_{k,k-1} F \quad (\text{II.2.24})$$

and

$$\bar{D}_{k,k+1} = F^T \bar{R}_{k,k+1} F \quad (\text{II.2.25})$$

The entries $\bar{D}_{k,k-1}$ and $\bar{D}_{k,k+1}$ do not exist at $k=1$ and $k=NZ$ respectively, of course.

It is convenient to let $R = W^T Q$, where $W^T Q$ is found on the right-hand side of equation (II.2.22b). If R is partitioned into NZ columns of N -dimensional subvectors to obtain $\bar{R}$, each equation in the partitioned system $\bar{D} \bar{Z} = \bar{R}$ can be written as,

$$\bar{D}_{k,k-1} \bar{Z}_{k-1} + \bar{D}_{k,k} \bar{Z}_k + \bar{D}_{k,k+1} \bar{Z}_{k+1} = \bar{R}_k \quad (\text{II.2.26})$$

The partitioned system of equations defining the "mixing" vector $\bar{Z}$ is

shown in Figure II.2.3 for a problem in which $NZ=4$. As mentioned previously, $\bar{D}$ is tridiagonal so D is banded with a maximum half-bandwidth of $2N$.

N COLUMNS

N ROWS	$D_{1,1}$	$D_{1,2}$			Z_1	R_1
	$D_{2,1}$	$D_{2,2}$	$D_{2,3}$		Z_2	R_2
		$D_{3,2}$	$D_{3,3}$	$D_{3,4}$	Z_3	R_3
			$D_{4,3}$	$D_{4,4}$	Z_4	R_4

BLANK BOXES CONTAIN ZERO ELEMENTS

Figure II.2.3: System of Simultaneous Equations Defining the Mixing Functions Expressed in Partitioned Form

For completeness, explicit expressions for the entries $\bar{D}_{k,k-1}(ii,jj)$, $\bar{D}_{k,k}(ii,jj)$ and $\bar{D}_{k,k+1}(ii,jj)$ where $ii=1,N$ and $jj=1,N$ in each matrix, will be given for the grid shown in Figure II.2.1. The general nodal equation (equation (II.2.5)) allows these entries to be expressed relatively simply as,

$$\bar{D}_{k,k-1}(ii,jj) = - \sum_{j=1}^{NT} \sum_{i=1}^{NR} T_{jj}(i,j) T_{ii}(i,j) K(i,j,k)_{k-1} \quad (II.2.27)$$

$$\begin{aligned}
\bar{D}_{k,k}(ii,jj) = & - \sum_{j=1}^{NT} \sum_{i=1}^{NR} [K(i,j,k)_{i+1} (T_{jj}(i+1,j) - T_{jj}(i,j)) \\
& + K(i,j,k)_{i-1} (T_{jj}(i-1,j) - T_{jj}(i,j)) + K(i,j,k)_{j+1} (T_{jj}(i,j+1) - T_{jj}(i,j)) \\
& + K(i,j,k)_{j-1} (T_{jj}(i,j-1) - T_{jj}(i,j)) - K(i,j,k)_{k+1} T_{jj}(i,j) \\
& - K(i,j,k)_{k-1} T_{jj}(i,j) - K(i,j,k)_s T_{jj}(i,j)] T_{ii}(i,j) \quad (II.2.28)
\end{aligned}$$

and

$$\bar{D}_{k,k+1}(ii,jj) = - \sum_{j=1}^{NT} \sum_{i=1}^{NR} T_{jj}(i,j) T_{ii}(i,j) K(i,j,k)_{k+1} \quad (II.2.29)$$

Similarly, element $\bar{R}_k(ii)$ is given by:

$$\bar{R}_k(ii) = - \sum_{S_b} (q'' A_{sz}(i,j,k) - K(i,j,k)_s T_{\infty}) T_{ii}(i,j) \quad (II.2.30)$$

where S_b is the set of boundary nodes on the r - θ plane at k . Note that the vector $\bar{R}_k$ contains only known quantities.

Equation (II.2.26) is not valid at $k=1$ and $k=NZ$ since the nodes at $k-1$ and $k+1$ respectively do not exist (i.e., the nodes on the planes at $k=1$ and $k=NZ$ are boundary nodes). If the boundary nodes at $k=1$ have surface areas $A_{sz}(i,j,1)$, where the subscript SZ identifies quantities evaluated on planes normal to the z -axis, and communicate with a fluid at bulk temperature T_{∞} through conductances $K(i,j,1)_{SZ}$, then we can set $K(i,j,1)_{k-1} = K(i,j,1)_{SZ}$ and $T(i,j,k-1) = T_{\infty}$ and rewrite equation (II.2.26) as:

$$\bar{D}_{1,1} \cdot Z_1 + \bar{D}_{1,2} \cdot Z_2 = R_1 \quad (\text{II.2.31})$$

where $\bar{D}_{1,1}$ is given by equation (II.2.28) with the term $K(i,j,1)_{SZ}$ substituted for $K(i,j,1)_{k-1}$. Element $R_1(ii)$ is given by,

$$\begin{aligned} R_k(ii) = & - \sum_{S_b} (q'' A_s(i,j,k) - K(i,j,k)_s T_\infty) T_{1i}(i,j) \\ & - \sum_{j=1}^{NT} \sum_{i=1}^{NR} (q'' A_{SZ}(i,j,k) - K(i,j,k)_{SZ} T_\infty) T_{1i}(i,j) \text{ with } k=1 \end{aligned} \quad (\text{II.2.32})$$

The other matrices in equation (II.2.31) remain the same.

The corresponding equation at $k=NZ$ can be written as follows:

$$\bar{D}_{NZ,NZ-1} \cdot Z_{NZ-1} + \bar{D}_{NZ,NZ} \cdot Z_{NZ} = R_{NZ} \quad (\text{II.2.33})$$

where $\bar{D}_{NZ,NZ}$ is given by equation (II.2.28) with the term $K(i,j,NZ)_{SZ}$ substituted for $K(i,j,k)_{k+1}$. Element $R_{NZ}(ii)$ is given by equation (II.2.32) with $k=NZ$.

The preceding formulation of the synthesis technique, termed continuous synthesis for reasons discussed below, is very powerful and can be used to obtain accurate solutions to three-dimensional heat transfer problems at a fraction of the cost of pure finite-difference solutions. This formulation does have certain deficiencies however. Consider an application of the synthesis technique in which the $r-\theta$ temperature distribution exhibits large variations axially. In such a problem, a large number of trial functions may be required to adequately approximate the true $r-\theta$ temperature distribution at all axial locations. This requirement will increase the computational effort and thus the cost of

obtaining a solution of course. It is likely, however, that each trial function will contribute significantly to the temperature synthesis at only a few axial locations in the region of interest. One would obviously prefer to use trial functions at axial locations where they are likely to be useful, since to do otherwise increases the computational effort unnecessarily. Ideally, different sets of trial functions would be used in different axial regions. The trial functions used in a particular axial region would probably be two-dimensional solutions obtained in that region. Unfortunately, the continuous synthesis technique uses all of the available trial functions at each axial location and it is not possible to discard trial functions in regions where they are not appropriate. Stated another way, the trial functions, or more correctly the associated mixing functions (Z_n), are required to be axially continuous, hence the designation "continuous synthesis".

Another restriction imposed by the continuous synthesis formulation is that the finite-difference/synthesis mesh must be axially continuous. This requirement limits the applicability of the method since problems involving geometries which are axially non-uniform cannot be treated.

Both of the deficiencies associated with continuous synthesis can be avoided if an alternate formulation of the synthesis technique, termed discontinuous synthesis, is used. Discontinuous synthesis accepts meshes which are axially discontinuous and in addition does not require that all of the trial functions be used to synthesize the temperature at each axial location. This capability obviously lessens the computational effort required to obtain a solution.

The discontinuous synthesis method will be developed in the next section by modifying the governing equations for the continuous synthesis technique in an almost trivial manner. Yasinsky's formulation of the discontinuous synthesis method, which is described in detail in references [114] and [115], will also be discussed briefly and compared to the formulation derived by the author.

II.2.3 Discontinuous Synthesis

In the discrete formulation of the discontinuous synthesis method, the approximate solution is still given by equation (II.2.20) but the F matrix, defined in equation (II.2.17), is allowed to be different at different values of k . To account for this, we can simply write F with a subscript (i.e., $F \rightarrow F_k$). The number of trial vectors, and hence the number of columns in F_k , will be designated N_k so the subvector $\bar{Z}_k$, defined in equation (II.2.18), now has N_k elements. It is assumed, for convenience, that the finite-difference grid in question is still axially continuous and hence that the conductance matrix, K , can still be partitioned as shown in Figure II.2.2.

If the F matrix and the $\bar{Z}_k$ vector used to define the governing equations for the continuous synthesis method are modified as described above, all of the equations derived for the continuous synthesis method apply to discontinuous synthesis. In particular, we obtain,

$$\bar{D}_{k,k} = F_k^T \bar{K}_{k,k} F_k \quad (\text{II.2.34})$$

$$\bar{D}_{k,k-1} = F_k^T R_{k,k-1} F_{k-1} \quad (\text{II.2.35})$$

and

$$\bar{D}_{k,k+1} = F_k^T R_{k,k+1} F_{k+1} \quad (\text{II.2.36})$$

as the equations defining the diagonal and off-diagonal entries in the partitioned coefficient matrix ($\bar{D}$) in the discontinuous synthesis method and,

$$\bar{R}_k = F_k^T Q_k \quad (\text{II.2.37})$$

as the expression defining the right-hand side subvector ($\bar{R}_k$). The coefficient matrix ($\bar{D}$) is still block-tridiagonal in structure, symmetric and positive-definite, although its maximum half-bandwidth is now $\max (N_k + N_{k+1})$; $k=1, 2, \dots, NZ-1$, or simply the maximum value of $(N_k + N_{k+1})$ found in the range of k specified. It should be mentioned that while D is still a block-tridiagonal matrix, the "blocks" (submatrices) are no longer necessarily all the same size. The submatrix $\bar{D}_{k,k-1}$ is now an $N_k \times N_{k-1}$ matrix, $\bar{D}_{k,k+1}$ is an $N_k \times N_{k+1}$ matrix, and $\bar{D}_{k,k}$ is an $N_k \times N_k$ matrix.

It is worthwhile comparing the governing equation for discontinuous synthesis derived by the author to that derived by Yasinsky since the two equations do not agree whereas the author's and Yasinsky's equations for continuous synthesis are the same. In Yasinsky's derivation of discontinuous synthesis, all of the required functions were not expressed in discrete form at the outset, as done by the author, but rather were only discretized after the governing equations for the method had been obtained. Actually, only the mixing functions (i.e., the $Z_n(k)$) were expressed in discrete form in Yasinsky's final expression - the trial functions (T_n) were left as continuous functions. The terms involving ∇T_n in Yasinsky's

final expression can be expressed in finite-difference form, however, to make Yasinsky's solution suitable for comparison with the author's. If that is done it can be shown that Yasinsky's formulation gives,

$$\bar{D}_{k,k} = F_k^T (R_{k,k} + R_{k,k-1}) F_k - F_k^T R_{k,k-1} F_{k-1} (F_{k-1}^T R_{k,k-1} F_{k-1})^{-1} F_{k-1}^T R_{k,k-1} F_k \quad (\text{II.2.38a})$$

$$= F_k^T [R_{k,k} + R_{k,k-1} - (R_{k,k-1} F_{k-1} (F_{k-1}^T R_{k,k-1} F_{k-1})^{-1} F_{k-1}^T) R_{k,k-1}] F_k \quad (\text{II.2.38b})$$

$$\bar{D}_{k,k-1} = F_k^T R_{k,k-1} F_{k-1} \quad (\text{II.2.39})$$

and,

$$\bar{D}_{k,k+1} = F_k^T R_{k,k+1} F_{k+1} \quad (\text{II.2.40})$$

as the expressions that correspond to equations (II.2.34) - (II.2.36) derived by the author. Both Yasinsky and the author define $\bar{R}_k$ by equation (II.2.37).

It is apparent that Yasinsky's expressions for the off-diagonal entries in $\bar{D}$ are the same as the author's but that his expression for the diagonal entry, $\bar{D}_{k,k}$, is different. Yasinsky noted that equation (II.2.38) reduces to the equivalent expression for the continuous synthesis method (i.e., $\bar{D}_{k,k} = F_k^T \bar{R}_{k,k} F_k$) if $F_{k-1} = F_k = F$, however, since the term $(F_{k-1}^T R_{k,k-1} F_{k-1})^{-1} F_{k-1}^T R_{k,k-1} F_k$ becomes $(F^T R_{k,k-1} F)^{-1} F^T R_{k,k-1} F = I$ in that case, allowing equation (II.2.38) to be written as,

$$\begin{aligned} \bar{D}_{k,k} &= F_k^T (R_{k,k} + R_{k,k-1} - R_{k,k-1}) F_k \\ &= F_k^T R_{k,k} F_k \end{aligned}$$

Yasinsky's formulation of discontinuous synthesis is deficient in certain respects, as can be demonstrated by noting that equation (II.2.38) is not symmetric with respect to terms evaluated at axial nodes $k-1$ and $k+1$. Since equation (II.2.38) contains terms evaluated at axial node $k-1$ but not at node $k+1$, two apparently different equations relating Z_k to Z_{k-1} and Z_{k+1} can be obtained by varying the order in which the nodes on the z -axis are visited. To illustrate this point, it is assumed that $F_k \neq F_{k-1}$ and $F_k = F_{k+1}$, in which case we obtain,

$$F_{k,k,k-1}^T F_{k-1}^T Z_{k-1} + F_k^T [R_{k,k} + R_{k,k-1} - R_{k,k-1} F_{k-1} (F_{k-1}^T R_{k,k-1} F_{k+1})^{-1} F_{k-1}^T R_{k,k-1}] F_k Z_k + F_{k,k,k+1}^T F_{k+1} Z_{k+1} = F_k^T Q_k \quad (\text{II.2.42})$$

as the synthesis equation for axial node k when the axial nodes are visited in the order $1, 2, \dots, k-1, k, k+1, \dots, NZ$, and,

$$F_{k,k,k-1}^T F_{k-1} Z_{k-1} + F_{k,k,k}^T F_k Z_k + F_{k,k,k+1}^T F_{k+1} Z_{k+1} = F_k^T Q_k \quad (\text{II.2.43})$$

as the equation relating Z_k to Z_{k-1} and Z_{k+1} when the axial nodes are visited in reverse order. Node $k-1$ does not contribute any terms to the expression for $D_{k,k}$ when it is visited after node k because it effectively assumes the role of node $k+1$ in equations (II.2.38) - (II.2.40) in that case (i.e., the subscript k in equations (II.2.38) - (II.2.40) is assumed to refer to the order in which the nodes are visited whereas node $k-1$ in equation (II.2.42) is the same as node $k-1$ in equation (II.2.43)).

Since equations (II.2.42) and (II.2.43) both relate Z_k to Z_{k-1} and Z_{k+1} and also differ only in the diagonal term, these equations must

actually be identical if Z is unique and does not depend on the order in which the nodes on the z -axis are visited. Since equation (II.2.43) is the equation that would be obtained by applying the author's formulation of discontinuous synthesis, it follows that Yasinsky's formulation must be the same as the author's if the former formulation provides a mixing vector (Z) which is independent of the order in which the axial nodes are visited. Numerical experiments performed by the author with Yasinsky's method revealed that the latter proviso is not satisfied in general, however, and hence that Yasinsky's formulation is different from the author's as originally stated. It should be noted that Yasinsky does not mention in either reference [114] or [115] that results obtained with his formulation of discontinuous synthesis will exhibit a nodal ordering dependence. An explanation for that dependence is given in Appendix II.C, where the essential differences between the author's and other derivations of discontinuous synthesis are described. The reason that the author was able to accomplish the transition from continuous to discontinuous synthesis much more easily than in previous formulations is also discussed in considerable detail in the appendix.

The application of discontinuous synthesis to problems with axially discontinuous geometries is very straightforward but will not be described here because that capability was not required in this study. The method's application to transient and/or non-linear analyses is described in Part III of the thesis, where it is used to obtain solutions to the CANDU pressure-calandria tube contact problem discussed in Chapter I.1.

II.2.4 Improving the Numerical Stability of the Synthesis Method

As mentioned in Section II.2.2, the trial functions that are used in the synthesis method must be linearly independent. If trial functions are chosen that are linearly dependent, the coefficient matrix (D) for the system of simultaneous equations defining the blending vector (Z) will be singular or, of greater significance, if the chosen trial functions are close to being linearly dependent then the simultaneous equations defining Z will be very ill-conditioned. Since the synthesis method can, at best, provide the exact solution to the system of finite-difference equations $KT=Q$, it seems logical to assume that the conditioning of the system of equations $DZ=R$ will depend, in some manner, on the conditioning of the original system $KT=Q$. If the original system of equations is ill-conditioned, for example, as it could be if the finite-difference equations were obtained for a highly refined grid, then the system of equations $DZ=R$ might be ill-conditioned even if the trial functions were not even close to being linearly dependent. Nevertheless, it is fairly obvious that the trial functions should be as different as possible to avoid worsening the situation and, in this context, it can be shown that the trial vectors should be orthonormal for maximum numerical stability. Since the trial vector matrix W is block diagonal, the columns of W (i.e., the trial vectors, C_i) can be made orthonormal by orthonormalizing the columns of each F_k , $k=1,2,\dots,NZ$ using, for instance, the modified Gram-Schmidt algorithm with column pivoting [87]. The orthonormal vectors obtained by applying this algorithm span the same space as the original vectors so the solution is, in theory, not affected by the orthonormalization process. In practice, orthonormalizing the trial vectors before using them almost

guarantees that the conditioning of the system of equations $DZ=R$ will be no worse than the conditioning of the original system of equations, $KT=Q$. A proof of this statement is given in Appendix II.D. In the author's experience, orthonormalizing the trial vectors before using them increases the overall cost of a synthesis solution by a negligible amount, typically by less than 2%, and can increase the numerical stability of the method greatly.*

II.2.5 Application of the Finite-Difference/Synthesis Method to the Model Contact Problem

II.2.5.1 Substitute Contact Specimen and Boundary Conditions

The synthesis method was applied to the model contact problem by using it to obtain an approximate solution to the finite-difference equations for a highly graded three-dimensional grid superimposed over the semi-cylindrical substitute contact specimen shown in Figure II.2.4. As in the case of ellipsoidal finite-difference model, it was assumed that the shape of the contact surface in the lateral direction would be of secondary importance because the heat flow would be severely constricted in that direction. The radius of the semi-cylinder was made equal to the radius of the small-radiused contact specimen used in the experiments (i.e., 0.953 cm.), and its length (w) was made equal to the width of that specimen (i.e., 2.54 cm.).

Since boundary conditions of the first kind must, in general, be re-expressed as boundary conditions of the third kind in a synthesis

* The benefit accruing from trial vector orthonormalization is not described elsewhere to the author's knowledge.

Figure II.2.4 Semi-Cylindrical Substitute Contact Specimen

analysis, as mentioned in Section II.2.2, a very large contact conductance (specifically $1136 \text{ W/cm}^2 \text{ K}$ or $2 \times 10^6 \text{ BTU/hr ft}^2 \text{ }^\circ\text{F}$) was prescribed over the contact area in all of the synthesis analyses to force the contact area to be approximately isothermal. Gas conductances, defined by Equation (I.3.24), connected the nodes outside of the contact patch to the isothermal surface at temperature T_0 in the analyses performed with gas conduction. The temperature of the diametral plane parallel to the contact plane was set to T_1 , as shown in Figure II.2.4, and the ends of the semi-cylinder were considered to be insulated. The boundary condition on the diametral plane shown at temperature T_1 did not have to be modelled as a boundary condition of the third kind for reasons that will be discussed later.

The analytical resistance of the contact based on the extrapolated temperature drop at the contact plane was calculated from the expression

$$R_c = 2 \left[\frac{T_1 - T_0}{Q_c} - \frac{1}{2k_s w} \right] \quad (\text{II.2.44})$$

with the semi-cylindrical contact specimen. The significance of each term in Equation (II.2.44) is the same as that of the corresponding term in Equation (I.3.23).

The semi-cylindrical contact specimen shown in Figure II.2.4 was less than ideal in one respect in that the isothermal plane at temperature T_1 was located only a distance R_s away from the contact. While that plane should really have been located further away from the contact to ensure that the heat flow at its location in one of the real contact specimens would have been undisturbed by the presence of

the contact, errors resulting from this deficiency would have been mitigated by the highly localized nature of the contact's constriction resistance. It was estimated using the multigrid model described in Chapter II.4, for instance, that the resistance of the contact in air and in vacuum would have been overestimated by less than 0.2% at an intermediate contact load of 176N as a result of using the semi-cylindrical contact specimen instead of a specimen with greater height.

As mentioned previously, boundary conditions of the first kind must, in general, be modelled as boundary conditions of the third kind in a synthesis analysis. The nodes on the isothermal surface at temperature T_1 in the contact model would, in this case, be modelled as nodes communicating with T_1 through a large surface conductance. In Figure II.2.1, the nodes in question would be those with subscripts i, NT, k ($i=1, NR; k=1, NZ$) if only the right-half of the cylindrical grid shown in that figure was used to model the substitute specimen. If the nodal equation for an arbitrary node i, NT, k is written as,

$$\begin{aligned}
 & -K(i, NT, k)_{i+1} T(i+1, NT, k) - K(i, NT, k)_{i-1} T(i-1, NT, k) \\
 & -K(i, NT, k)_{j-1} T(i, NT-1, k) - K(i, NT, k)_{k+1} T(i, NT, k+1) \\
 & -K(i, NT, k)_{k-1} T(i, NT, k-1) + (\Sigma K) T(i, NT, k) = K(i, NT, k)_s T_1
 \end{aligned}
 \tag{II.2.45}$$

where $K(i, NT, k)_s = h_f A_s(i, NT, k)$, we can force $T(i, NT, k)$ to be approximately equal to T_1 by making h_f very large. Alternately, we can exclude the nodes at $j=NT$ from the model altogether, since their temperatures are known, and write the nodal equation for an arbitrary

node $i, NT-1, k$ ($i=1, NR; k=1, NZ$) as,

$$\begin{aligned}
 & -K(i, NT-1, k)_{i-1} T(i-1, NT-1, k) - K(i, NT-1, k)_{i+1} T(i+1, NT-1, k) \\
 & -K(i, NT-1, k)_{j-1} T(i, NT-2, k) - K(i, NT-1, k)_{k+1} T(i, NT-1, k+1) \\
 & -K(i, NT-1, k)_{k-1} T(i, NT-1, k-1) + (\Sigma K) T(i, NT-1, k) \\
 & = K(i, NT-1, k)_{j+1} T_1
 \end{aligned}
 \tag{II.2.46}$$

The isothermal surface at temperature T_1 is now modelled exactly and, as an added benefit, the finite-difference grid now contains fewer nodes because the nodes at $j=NT$ are no longer required. The nodes at $j=NT-1$ are the new boundary nodes.

The substitution of Equation (II.2.46) for Equation (II.2.45) is permitted in this case because the isothermal surface at temperature T_1 extends from $z=0$ to $z=w$ or, equivalently, from $k=1$ to $k=NZ$. Thus the nodes i, NT, k ($i=1, NR$) can be excluded from the synthesis model at all values of k to leave a mesh which is still axially continuous. If the isothermal surface at temperature T_1 had extended from $z=0$ to $z=w/2$, on the other hand, with an insulated boundary condition replacing the isothermal boundary condition for $w/2 < z \leq w$, the use of Equation (II.2.46) for $0 \leq z \leq w/2$ and Equation (II.2.45) (with $h_f=0$) for $w/2 < z \leq w$ would cause the mesh to be discontinuous in the axial direction at $z=w/2$. The nodes i, NT, k ($i=1, NR$) would be required for those values of k lying in the range $w/2 < z \leq w$ and would be excluded from the $r-\theta$ integrations at the remaining values of k . While axially

discontinuous meshes of this nature can, in theory, be handled without difficulty by the discontinuous synthesis method, implementing mesh modifications in a general synthesis program nevertheless requires considerable programming effort. Dirichlet conditions prescribed on the end surfaces of a body (i.e., at $k=1$ and $k=NZ$) can always be treated without difficulty, however, by using the nodes at $k=2$ and $k=NZ-1$ as the new boundary nodes on the z -axis. The specified temperatures at $k=1$ and $k=NZ$ would be incorporated into the nodal equations at $k=2$ and $k=NZ-1$ in a manner analogous to that used to obtain Equation (II.2.46).

II.2.5.2 Grid Construction

A grid similar to that shown in Figure II.2.5 was used to model the r - θ geometry of the semi-cylindrical contact specimen. The finite-difference equations for the nodes at which the grid was refined or coarsened were obtained using a method formulated by MacNeal [124] for non-orthogonal grids, as described in greater detail in Appendix A of Part IV of the thesis.

The mesh increments in the radial (r) and circumferential (θ) directions, designated Δr and $\Delta \theta$ respectively, are functions of both r and θ in the grid shown in Figure II.2.5 (i.e., $\Delta r=f(r,\theta)$ and $\Delta \theta=g(r,\theta)$). While a three-dimensional grid could theoretically be similarly graded with Δr , $\Delta \theta$ and Δz all specified as functions of r , θ and z (i.e., with $\Delta r=f(r,\theta,z)$; $\Delta \theta=g(r,\theta,z)$; $\Delta z=h(r,\theta,z)$), such a grid is difficult to construct in practice, particularly when using the finite-difference method.*

* Except when multigrid is used to solve the finite-difference equations, as will be seen in Chapter II.4.

Figure II.2.5: Graded Finite-Difference Grid

A much simpler but less node-conserving grid can be constructed by extending the grid shown in Figure II.2.5 in the z direction using axial subdivisions which are functions of axial position only (i.e., $\Delta z = h(z)$), as shown pictorially in Figure II.2.6. Orthogonal views of the same grid showing the mesh refinement in the contact region more clearly are given in Figure II.2.7. This type of grid was chosen to analyze the model contact problem.

The grid shown in Figures II.2.6 and II.2.7 is obviously not ideal for modelling problems in which the contact patch has a finite length since the detailed r - θ grid must be used at all axial positions,

Figure II.2.6 Graded Finite-Difference Grid Extended in the Axial (z) Direction.

CONTACT

BOTTOM VIEW
OF GRID

Figure II.2.7 3-D Finite-Difference Grid.

including positions remote from the contact where a much coarser grid would suffice. The penalty, in terms of increased computation time and computer memory requirement, incurred by using a detailed two-dimensional (in this case $r-\theta$) grid at all axial locations instead of a more complex but more node-conserving three-dimensional grid is minimized when the synthesis technique is used to solve the resulting finite-difference equations, however, since the synthesis method is very fast and requires almost no more computer memory than that needed to obtain a two-dimensional solution, as mentioned previously. It should also be recognized that the synthesis method is most easily applied when the grid structure is the same at all axial locations so the simple grid shown in Figures II.2.6 and II.2.7 is the best from this point of view as well.

II.2.5.3 Contact Representation

The elliptical contact patch on the contact surface of the semi-cylindrical contact specimen cannot be modelled exactly using the grid shown in Figures II.2.6 and II.2.7 but there are at least three ways in which it can be modelled approximately. The first and simplest of these is to prescribe the contact-related boundary condition, that is a boundary condition of the third kind with an extremely large surface conductance, at each node on the contact surface whose coordinates lie within the boundaries of the contact patch. For convenience, this contact approximation technique will be referred to as method 1 in the remainder of the thesis. The second way to approximate the elliptical contact patch is to have the problem analyst decide how many nodes should be used to model the lateral extent of the contact at each axial mesh.

point within its length. The lateral and longitudinal subdivision sizes around the contact are chosen to represent the contact patch as accurately as possible in this method. If it is desired to have an elliptical contact (or really one-quarter of it) span six nodes laterally (i.e., on the minor axis) and thirteen nodes longitudinally (i.e., on the major axis) as shown in Figure II.2.8, for instance, the lateral subdivision size is chosen so that the boundary of the contact at its widest point lies midway between the sixth and seventh lateral nodes

Figure II.2.8: Elliptical Contact Representation (Method 2)

since that location coincides with the outer boundary of the finite-difference cell for the sixth lateral node. Similarly, the longitudinal subdivision size is chosen so that the end of the contact lies midway between the thirteenth and fourteenth longitudinal nodes. If the number of nodes spanned by the synthesized contact (i.e., the assemblage of rectangular surface elements) in the lateral direction at a particular axial mesh point k is designated $N_x(k)$, then it can be seen for the example shown in Figure II.2.8 that $N_x(k) = 6$ for $k = 1-6$, $N_x(k) = 5$ for $k = 7-9$, $N_x(k) = 4$ for $k = 10-11$; $N_x(k) = 3$ for $k = 12$ and $N_x(k) = 2$ for $k = 13$. Appropriate values of $N_x(k)$ are selected by the problem analyst in this method of approximating a contact, which will hereafter be referred to as method 2, although it is likely that the contact representation selected by the analyst would usually be similar to the representation that would be obtained with the first contact approximation technique discussed. The main advantage that accrues from having the analyst specify the number of nodes that are to be used to represent the lateral extent of the contact at each axial mesh point (k) within its length is that it facilitates the task of assigning trial functions to those mesh points. Since the problem analyst usually decides which trial functions will be used at each axial node in a synthesis analysis, it is convenient for him to choose the $N_x(k)$ that will be used to represent the contact as well, so that the trial functions can be matched to the contact representation. The two-dimensional solution to a problem involving a line contact spanning 6 nodes laterally (within its half-width) would logically be used as a trial function at axial nodes (k) for which $N_x(k) = 6$, for example.

A third way to approximate an elliptical contact on a rectilinear grid such as that shown in Figure II.2.7 (bottom view) is to make the discrete contact width between the finite-difference cell boundaries at $z = (z_k + z_{k-1})/2$ and $z = (z_k + z_{k+1})/2$ for the k th axial node equal the width of the elliptical contact at the midpoint of that interval, as shown in Figure II.2.9. If $z_{k+1} - z_k = z_k - z_{k-1}$ (i.e., if Δz is the same on both sides of axial node k , which will not always be the case because $\Delta z = h(z)$), this procedure would be equivalent to making the width of the approximated contact at axial node k equal to the width of the elliptical contact at that point. That width would be assumed to apply to the axial segment between $z = (z_k + z_{k-1})/2$ and $z = (z_k + z_{k+1})/2$, of course.

Figure II.2.9: Elliptical Contact Approximated Using Contact Widths At Axial Subdivision Midpoints

Although the last contact approximation technique discussed approximates the boundary of the elliptical contact more smoothly than either of the first two methods discussed and hence would be expected to be more accurate, particularly if the mesh refinement in the lateral (y) direction was more coarse than that shown in Figure II.2.8, a procedure for approximating a line contact whose edge does not lie midway between two lateral surface nodes (i.e., whose edge does not coincide with a finite-difference cell boundary) is needed in order to implement the method. A line contact whose edge lies midway between two surface nodes can be approximated accurately by simply applying the contact boundary condition to the nodes contained within the width of the contact, as shown in Appendix II.E. In the case of a line contact which extends only partway across the surface element for the n th lateral node, as shown in Figure II.2.10, the procedure just mentioned would cause the resistance of a contact with $b \geq y_n$ to be underestimated and the resistance of a contact with $b < y_n$ to be overestimated.

Figure II.2.10: Contact Extending Partway Across the Surface Element of the n th ($n=4$ in this example) Lateral Node

Several procedures for approximating a line contact covering only part of the n th lateral surface element in a finite-difference analysis are considered in Appendix II.E. Although methods for approximating an isothermal line contact receive the greatest attention in the appendix, methods for approximating a contact with a uniform contact conductance h_c prescribed over its area are also considered in somewhat less detail. Several methods for approximating a contact in vacuum are derived and incorporated into finite-difference analyses of the heat flow through an isothermal contact on a semi-cylinder with an isothermal diametral plane. The resistance of such a contact is given by the expression defining R_{ct}^T for contact 6 in Table I.2.1. The numerically predicted contact resistances and contact heat fluxes are compared to the corresponding analytical solutions in order to assess the accuracy of each technique. The approximation of contacts in gases with the techniques is also examined in Appendix II.E.

Although three of the contact approximation techniques considered in Appendix II.E are shown to predict contact resistances accurately, all three of those methods are based on an expression which defines the difference between the resistances (in vacuum) of two narrow line contacts with different widths. That expression can be derived by noting that the equation defining R_{ct}^T for contact 6 in Table I.2.1 can be written in the form,

$$R_{ct}^T = \ln(2(1+\cos\alpha)/\sin\alpha)/\pi k_s \quad (\text{II.2.47})$$

for small contacts [42a]. Introducing the approximation $\sin(\pi b/2t) \approx \pi b/2t$ into Equation (II.2.47), we get,

$$R_{ct}^T = \ln(4R/b)/\pi k_s \quad (II.2.48)$$

Denoting the y -coordinate of the outermost boundary of the surface element for the n th lateral node shown in Figure II.2.10 by $y_{n+1/2}$ ($= (y_n + y_{n+1})/2$), the difference between the resistance of a contact with a half-width of $y_{n+1/2}$ and another with a half-width of b , designated ΔR_c for convenience, can now be written as,

$$\begin{aligned} \Delta R_c &= \frac{1}{\pi k_s} [\ln(4R/b) - \ln(4R/y_{n+1/2})] \\ &= \ln(y_{n+1/2}/b)/\pi k_s, \quad (b \leq y_{n+1/2}) \end{aligned} \quad (II.2.49)$$

As shown in Appendix II.E, Equation (II.2.49) could have been obtained in the same manner from the expression defining R_c^T for contact 4 in Table I.2.1 or even from the expression defining R_c^Q for that contact.*

The incremental resistance ΔR_c can be used to define contact conductances which, when prescribed over the surface elements for lateral nodes $j=1, \dots, n$ (i.e., over the surface segment for which $0 \leq y \leq y_{n+1/2}$) in the numerical model, cause the resulting total resistance of the surface segment of half-width $y_{n+1/2}$ with those conductances prescribed over to its area to be approximately equal to the resistance of an isothermal line contact of half-width b . The three contact approximation techniques which are based on Equation (II.2.49) all use it to define contact conductance distributions for that purpose. In the first and

* R_c^Q is defined using the area-average contact temperature ($\bar{T}_c$), as noted in Chapter I.2.

simplest of those techniques, ΔR_c is equated to the resistance of a uniform contact conductance h_{cu} acting over the surface segment of width $2y_{n+1/2}$ in the finite-difference model to obtain,

$$\Delta R_c = \frac{1}{2y_{n+1/2} h_{cu}} \quad (\text{II.2.50})$$

or,

$$h_{cu} = \pi k_s / (2y_{n+1/2} \ln(y_{n+1/2}/b)) \quad (\text{II.2.51})$$

Although the surface segment of half-width $y_{n+1/2}$ is not isothermal (as assumed in the derivation of Equation (II.2.49)) when the contact conductance h_{cu} is prescribed over its area, the resistance of an isothermal line contact of half-width b can nevertheless be predicted very accurately with this technique, as shown in Appendix II.E. The contact heat fluxes are underestimated when the contact is approximated in this manner, however, because the heat flow is distributed over a surface segment which is larger than the actual contact width (i.e., $2y_{n+1/2}$ instead of $2b$).

Other contact approximation techniques can be derived from Equation (II.2.49) if it is assumed that the resistance of a line contact of a given size will be approximately the same for any contact flux (and hence temperature) distribution when it is defined using the area-average contact temperature (i.e., T_c) in each case. This assumption is at least partially justified in Appendix II.E by noting that R_c^T and R_c^q for contact 4 in Table I.2.1 differ by only a few percent for small contacts. The difference between these resistances is less than

3% when $b/t < 0.02$, for example. The assumption just referred to can be used to explain why the contact approximation technique which replaces ΔR_c by the conductance h_{cu} acting over the surface segment of half-width $y_{n+1/2}$ is accurate even though that segment is no longer isothermal when that is done. To this end, it is apparent that the temperature of the "contact" of half-width $y_{n+1/2}$ will be $Q' \Delta R_c$ if;

- (1) the contacts with half-widths b and $y_{n+1/2}$ are both isothermal, as assumed in the derivation of Equation (II.2.49)*
- (2) $T_0 = 0$, where T_0 is the temperature of the contact of half-width b ; and,
- (3) the heat flow rate through both contacts (per unit length) is Q' .

Since we have assumed that the resistance of a line contact of a given size will be approximately the same for any contact flux distribution if it is defined using the area-average contact temperature, it follows that the heat flow rate through the contact of half-width $y_{n+1/2}$ will be Q' if its area-average contact temperature at that heat flow rate is $Q' \Delta R_c$ (i.e., this heat flow rate and area-average contact temperature are compatible if the aforementioned assumption is valid). It is easily verified that the area-average temperature of the contact of half-width $y_{n+1/2}$ will be $Q' \Delta R_c$ when the heat flow rate through the contact is Q' even when the conductance h_{cu} is prescribed over its area since,

* It is assumed that the incremental resistance ΔR_c is inserted between these contacts.

$$\begin{aligned}
 Q' &= 2h_{cu} \int_0^{y_{n+1/2}} (T_c(y) - T_0) dy \\
 &= 2h_{cu} \int_0^{y_{n+1/2}} T_c(y) dy \quad (\text{since } T_0 = 0) \\
 &= 2h_{cu} y_{n+1/2} \bar{T}_c \quad (II.2.52)
 \end{aligned}$$

In that case, where,

$$\bar{T}_c = \frac{1}{y_{n+1/2}} \int_0^{y_{n+1/2}} T_c(y) dy$$

Solving for $\bar{T}_c$ from Equation (II.2.52), we get,

$$\bar{T}_c = \frac{Q'}{2h_{cu} y_{n+1/2}} = Q' \Delta R_c \quad (\text{from Equation (II.2.50)})$$

In the second contact approximation technique derived from Equation (II.2.49), only the n th surface element has a contact conductance (h_n) prescribed over its area and the segment of the contact for which $0 \leq y \leq y_{n-1/2}$ (for one-half of the contact) is set equal to T_0 . By requiring that the contact of half-width $y_{n+1/2}$ have an area-average temperature of $Q' \Delta R_c$, we obtain,

$$\begin{aligned}
 \bar{T}_c = Q' \Delta R_c &= \frac{1}{y_{n+1/2}} \left[\int_0^{y_{n-1/2}} T_0 dy + \int_{y_{n-1/2}}^{y_{n+1/2}} T_c(y) dy \right] \\
 &= \frac{q_n}{y_{n+1/2} h_n} \quad (\text{for } T_0 = 0) \quad (II.2.53).
 \end{aligned}$$

as an expression that can be solved for h_n once an equation defining the heat flow rate through the n th surface element, q_n , has been provided. Such an expression can be obtained by assuming that the heat flux distribution over the surface segment for which $0 \leq y \leq y_{n-1/2}$ will be approximately the correct one, viz.,

$$q''(y) = \frac{Q'}{\pi(b^2 - y^2)} \quad (\text{II.2.54})$$

since

$$\begin{aligned} q_n &= \frac{Q'}{2} - \int_0^{y_{n-1/2}} q''(y) dy \\ &= \frac{Q'}{2} \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) \right) \end{aligned} \quad (\text{II.2.55})$$

in that case (Equation (II.2.54) is derived in Appendix II.E).

Substituting Equation (II.2.55) into Equation (II.2.53) we obtain,

$$h_n = \frac{1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right)}{\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b} \right)} \quad (\text{II.2.56})$$

as the desired expression for h_n . Note that $h_n \rightarrow \infty$ as $b \rightarrow y_{n+1/2}$ and $h_n \rightarrow 0$ as $b \rightarrow y_{n-1/2}$ according to Equation (II.2.56). These results are correct, of course, because the contact covers all of the n th surface element when $b = y_{n+1/2}$ and does not cover any of it when $b = y_{n-1/2}$.

The foregoing contact approximation technique must be modified when the contact of half-width b has a uniform contact conductance h_c prescribed over its area. If the contact conductance h_c is prescribed at lateral nodes $j=1, \dots, n-1$ in the finite-difference model, the expression that

should be satisfied to obtain the correct area-average contact temperature is,

$$\begin{aligned} \bar{T}_c &= Q'(\Delta R_c + \frac{1}{2h_c b}) = \frac{1}{y_{n+1/2}} \left[\int_0^{y_{n-1/2}} T_c(y) dy + \int_{y_{n-1/2}}^{y_{n+1/2}} T_c(y) dy \right] \\ &= \frac{1}{y_{n+1/2}} \left[\frac{1}{h_c} \int_0^{y_{n-1/2}} q''(y) dy + \frac{1}{h_n} \int_{y_{n-1/2}}^{y_{n+1/2}} q''(y) dy \right] \end{aligned} \quad (II.2.57)$$

If it is assumed that Equation (II.2.54) defines the heat fluxes at lateral (y) coordinates in the range $0 \leq y \leq y_{n-1/2}$ (this assumption should be approximately correct for large h_c), as before, Equation (II.2.57) can be solved for h_n to obtain,

$$h_n = \frac{1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right)}{\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b} \right) + \frac{1}{h_c} \left(\frac{y_{n+1/2}}{b} - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) \right)} \quad (II.2.58)$$

The contact conductances h_c and h_n are prescribed at lateral nodes $j=1, \dots, n-1$ and n respectively in this technique. Note that Equation (II.2.58) reduces to Equation (II.2.56) if $h_c \rightarrow \infty$ (i.e., if the contact is isothermal), as it should. Also, $h_n \rightarrow h_c$ as $b \rightarrow y_{n+1/2}$ and $h_n \rightarrow 0$ as $b \rightarrow y_{n-1/2}$ according to Equation (II.2.58), which are again correct results.

The resistance of an isothermal contact covering part of the n th surface element is predicted very accurately when h_n is calculated from Equation (II.2.56), as shown in Appendix II.E. The results given in Appendix II.E indicate that the contact heat flux distribution obtained with this technique corresponds to the heat flux distribution for a contact which is slightly smaller than the actual contact, however, so a third contact approximation technique is developed by taking the half-width of the slightly smaller contact referred to previously to be the intended half-width and using the larger contact half-width to calculate h_n . To avoid obtaining the resistance of the contact with the larger half-width with this procedure, a contact conductance h_{lu} defined by the expression,

$$h_{lu} = \pi k_s / (2b_l \ln (b_l / b)) \quad (\text{II.2.59})$$

is prescribed over the width of the larger contact, $2b_l$. Except for the substitution of b_l for $y_{n+1/2}$, Equation (II.2.59) is identical to Equation (II.2.51) and fulfills a similar purpose.

Since the contact conductance h_{lu} is prescribed over the contact of half-width b_l (where $b \leq b_l \leq y_{n+1/2}$), that conductance is prescribed at lateral nodes $j=1, \dots, n-1$ in the finite-difference model and the corresponding value of h_n is calculated from Equation (II.2.58) by substituting h_{lu} and b_l for h_c and b in that equation respectively. Following arguments presented in Appendix II.E, the resistance $1/h_n \Delta y$ (where $\Delta y = y_{n+1/2} - y_{n-1/2}$) is assumed to be equal to the

resistance resulting from constriction of the heat flow over the n th surface element alone in order to obtain an expression defining b_l . Designating the latter resistance by $\Delta R'_C$, we obtain the definition,

$$\Delta R'_C = \frac{2}{\pi k_s} \ln \left(\frac{\Delta y}{b - y_{n-1/2}} \right) \quad (\text{II.2.60})$$

by replacing the terms $y_{n+1/2}$ and b in Equation (II.2.49) by Δy and $b - y_{n-1/2}$ respectively (since the heat flow is constricted from the width of the surface element, Δy , to the width of the contact segment on the element, $b - y_{n-1/2}$) and inserting the factor of 2 to account for the fact that the contact segment on the n th surface element is located to one side of the element rather than at its center. The equation defining $\Delta R'_C$ (i.e., Equation (II.2.49)) was derived for a contact geometry that is symmetrical about the center of the contact, of course. Arguments supporting the use of Equation (II.2.60) in this application are given in Appendix II.E.

Since it is assumed that $1/h_n \Delta y = \Delta R'_C$ when h_n is obtained by substituting the terms h_{lu} and b_l for the terms h_c and b in Equation (II.2.58) respectively, the parameter b_l can be obtained by solving the equation,

$$\begin{aligned} f(b_l) &= \Delta R'_C - 1/h_n \Delta y \\ &= \frac{2}{\pi k_s} \left[\ln \left(\frac{\Delta y}{b - y_{n-1/2}} \right) - \frac{(y_{n+1/2} \ln \left(\frac{y_{n+1/2}}{b} \right) - \frac{2b_l}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_l} \right) \ln \left(\frac{b}{b_l} \right))}{\Delta y \left(1 - \frac{2}{\pi} \sin \left(\frac{y_{n-1/2}}{b_l} \right) \right)} \right] \\ &= 0 \end{aligned} \quad (\text{II.2.61})$$

for its root in the interval $b \leq b_l \leq y_{n+1/2}$. The proof that Equation (II.2.61) will always have a root in this interval is given in Appendix II.E.

To summarize the last contact approximation technique described, which is explained in greater detail in Appendix II.E, the parameter b_l is obtained from Equation (II.2.61) and then substituted into Equation (II.2.59) to evaluate h_{lu} . The contact conductance h_{lu} is prescribed at lateral nodes $j=1, \dots, n-1$ in the finite-difference model. The contact conductance that is prescribed at node n (i.e., h_n) is then obtained from Equation (II.2.58) by setting $h_c = h_{lu}$ and $b = b_l$ or by noting that,

$$\begin{aligned} h_n &= 1/\Delta R'_c \Delta y && \text{(from Equation (II.2.61))} \\ &= \pi k_s / (2\Delta y \ln \left(\frac{\Delta y}{b - y_{n-1/2}} \right)) && \text{(II.2.62)} \end{aligned}$$

If $n=1$ (i.e., if the contact does not extend past the boundary of the 1st surface element), h_n is calculated from the equation defining h_{cu} (i.e., Equation (II.2.51)).

Results demonstrating the accuracy of the last contact approximation technique described are given in Appendix II.E. The resistance of a contact as well as surface heat fluxes at lateral (y) coordinates in the range $0 \leq y \leq y_{n-1/2}$ and the average heat flux over the n th surface element are all predicted very accurately with this technique. Although $\Delta R'_c$ is assumed to represent the resistance resulting from constriction of the heat flow over the n th surface element alone, as mentioned previously,

resistance estimates obtained by prescribing h_n at lateral node n but without using h_{lu} (i.e., with the contact conductance set to infinity at lateral nodes $j=1, \dots, n-1$) are much less accurate than those obtained with h_{lu} set to its proper value, as shown in Appendix II.E. The physical significance of h_{lu} can be explained qualitatively by referring to the two contacts shown in Figures II.2.11 and II.2.12. The contact shown in Figure II.2.11 is unbroken and extends halfway across the surface element for the n th ($=4$ th in this example) lateral node. The contact shown in Figure II.2.12 is unbroken up to $y_{n-1/2}$ but covers the second half of the n th surface element instead of the first half. Since both contacts are isothermal over the same total width (b) and the fraction of the n th surface element covered by the contact is the same for the two cases, it follows that a numerical model which used a resistance at node n calculated by considering the constriction of heat flow over the surface element for that node alone would yield the same resistance for the two contacts. The resistance of the unbroken contact must actually be greater than that of the broken contact, however, because the heat flow through the n th surface element crowds or constricts the heat flow through the rest of the contact (and vice-versa) to a greater extent when the contact is continuous than when it is broken. The thermal resistance of each elemental flux tube to the left of the coordinate $y_{n-1/2}$ is increased by this additional pinching so it is only reasonable to expect that it will be necessary to prescribe a finite contact conductance (i.e., h_{lu}) over the segment of the contact for which $0 \leq y \leq y_{n-1/2}$ (to account for the increased resistance to heat flow in that region), in addition to the contact

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Figure II.2.11 Unbroken Contact of Half-Width b Covering One-Half of the Surface Element for the n th Lateral Node ($n=4$ in This Example)

Figure II.2.12 Broken Contact of Total Half-Width b Covering the Second Half of the Surface Element for the n th Lateral Node ($n=4$ in This Example)

conductance prescribed over the n th surface element (i.e., h_n), in order to obtain accurate resistance predictions.

To apply the last contact approximation technique considered to a contact with a surface conductance h_c prescribed over its area, h_n is redefined as,

$$h_n = 1/\left[\Delta y \left(\frac{2}{\pi k_s} \ln \left(\frac{\Delta y}{b-y_{n-1/2}}\right) + h_c \left(\frac{1}{b-y_{n-1/2}}\right)\right)\right] \quad (\text{II.2.63})$$

and h_{lu} is replaced by h'_{lu} , where

$$h'_{lu} = 1/\left[\frac{2b_l}{\pi k_s} \ln \left(\frac{b_l}{b}\right) + \frac{b_l}{h_c b}\right], \text{ if } b_l \geq b$$

$$= h_c \quad \text{if } b_l < b \quad (\text{II.2.64})$$

The parameter b_l is the root of the equation,

$$g(b_l) = \frac{2}{\pi k_s} \ln \left(\frac{\Delta y}{b-y_{n-1/2}}\right) + \frac{1}{h_c (b-y_{n-1/2})}$$

$$- \frac{\left[\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b_l}\right) + \frac{1}{h'_{lu}} \left(\frac{y_{n+1/2}}{b_l} - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_l}\right)\right)\right]}{\Delta y \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_l}\right)\right)} = 0 \quad (\text{II.2.65})$$

found in the interval $y_{n-1/2} \leq b_l \leq y_{n+1/2}$. Equations (II.2.63) and (II.2.64), which reduce to Equations (II.2.62) and (II.2.59) respectively if $h_c = \infty$ (i.e., if the contact is isothermal), are derived in Appendix II.E where it is shown that $b_l \rightarrow b$ and hence $h'_{lu} \rightarrow h_c$ when $b \rightarrow y_{n-1/2}$ or $b \rightarrow y_{n+1/2}$. These results are correct, of course. It is easily

seen from Equation (II.2.63) that $h_n \rightarrow 0$ as $b \rightarrow y_{n-1/2}$ and $h_n \rightarrow h_c$ as $b \rightarrow y_{n+1/2}$, which are again correct results.

As discussed in Appendix II.E, the contact approximation techniques derived from Equation (II.2.49) can be applied to contacts in gases because the half-width of the surface segment used to represent the contact in the numerical model (i.e., $y_{n+1/2}$) is never very different from b (note that $y_{n+1/2} - \Delta y \leq b \leq y_{n+1/2}$) and so only the temperature distribution in the immediate vicinity of the contact is affected by the approximation. The results of numerical studies demonstrating the applicability of the last contact approximation technique described to contacts in gases are provided in the appendix. The use of the methods derived from Equation (II.2.49) in approximating elliptical contacts in the manner illustrated in Figure II.2.9 is also discussed in the appendix since that is the purpose for which the methods were developed. It is concluded from the analysis of their use in that application that it should be possible to apply the techniques in the manner proposed when the aspect (a/b) ratio of the elliptical contact being approximated is greater than three or four. This conclusion is not proven rigorously in the analysis described in Appendix II.E but results will be given later which corroborate it.

Synthesis analyses of the model contact problem were performed using:

- (1) The contact approximation technique which uses user-specific $N_\ell(k)$ to define the region covered by the contact (i.e., method 2).
- (2) The contact approximation technique in which the contact conductances h_c and h_n (defined by Equation (II.2.58)) are prescribed over surface

elements that are completely covered and only partly covered by the contact respectively; and,

- (3) The contact approximation technique in which the contact conductances h'_{lu} and h_n (defined by Equation (II.2.63)) are prescribed over the completely covered and partly covered surface elements respectively.

For convenience, the contact approximation techniques referred to in (2) and (3) will be called methods 3 and 4 in the remainder of the thesis. The results of the analyses performed with the different contact approximation techniques are discussed in the next chapter.

CHAPTER II.3

COMPARISON OF RESULTS OBTAINED WITH THE SYNTHESIS METHOD TO BENCHMARK RESULTS OBTAINED WITH THE ELLIPSOIDAL FINITE-DIFFERENCE MODEL

II.3.1 Introduction

Synthesis analyses of the model contact problem were performed using the computer program SYNCON, which was written by the author and is described in Part IV of the thesis. Results were only obtained for an intermediate contact load of 176N since it was felt that analyses at other contact loads would not provide significantly more information about the applicability of the synthesis method to the model contact problem.

II.3.2 Finite-Difference Grid Details

A finite-difference grid similar to that shown in Figure II.2.7 was chosen to analyze the model contact problem for the reasons discussed in Section II.2.5. The grid selected used 397 nodes to model the $r-\theta$ geometry of the semi-cylindrical contact specimen and 40 axial nodes so the entire model contained 15,880 ($= 397 \times 40$) mesh points. At a contact load of 176N, when the contact semi-minor and semi-major diameters would have been 9.61×10^{-3} cm. and 0.154 cm. respectively according to Hertz's theory, the contact patch or, more correctly, one-quarter of it, spanned fifteen nodes longitudinally and six nodes

laterally with the chosen grid, as seen in Figure II.3.1 where the grid is shown superimposed on the contact. The finite-difference cell boundaries (which were displayed as dashed lines in Figures II.2.8 and II.2.9) are shown in Figure II.3.1 instead of the grid lines themselves and the lateral scale is expanded relative to the longitudinal scale for visualization purposes. The lateral and longitudinal coordinates y and x in the contact centered coordinate system have been normalized with respect to the contact semi-minor diameter (b) and semi-major diameter (a) respectively.

The elliptical contact was approximated by the shaded area shown in Figure II.3.1 in analyses performed with the second contact approximation technique described in Section II.2.5 (i.e., method 2). The area of the approximated contact differed from that of the real contact by only 1.5% with the representation shown in Figure II.3.1, for which $N_x(k) = 6$ for $k = 1-3$, $N_x(k) = 5$ for $k = 4-6$, $N_x(k) = 4$ for $k = 7-10$, $N_x(k) = 3$ for $k = 11-13$ and $N_x(k) = 2$ for $k = 14-15$. The shaded area shown in Figure II.3.2 defines the contact representation used in the analyses performed with the last contact approximation technique described in Section II.2.5.3, which is the technique to which the approximation methods derived from equation (II.2.49 (i.e., methods 3 and 4) apply. The same finite-difference grid was used with both contact representations. It can be seen in either Figure II.3.1 or Figure II.3.2 that the fifteen nodes used on the semi-major axis were more closely spaced near the end of the contact (to model flux peaking in that region) than in the middle. The six nodes used on the semi-minor axis of the contact were equally

Figure II.3.1 Contact Representation Used With Contact Approximation Technique 2 (Lateral Scale Expanded Relative to Longitudinal Scale).

Figure II.3.2 Contact Representation Used With Contact Approximation Techniques 3 and 4 (Lateral Scale Expanded Relative to Longitudinal Scale).

spaced on that axis, however. The edge of the contact was located midway between the sixth and seventh lateral nodes at its widest point.

Since $r-\theta$ temperature distributions obtained by performing $r-\theta$ heat transfer analyses at different axial locations within and outside of the contact are more varied when the contact is in air than when it is in vacuum, it is somewhat easier to obtain trial functions for the former case than for the latter. In view of this observation, the thermal analysis of the contact in air will be considered before the analysis in vacuum.

II.3.3 Contact In Air Analysis

II.3.3.1 Trial Function Selection

For the purpose of specifying trial functions that can be used at axial nodes within the length of the contact, it is useful to think of the elliptical contact as being approximated by the shaded region shown in Figure II.3.1 even when the representation shown in Figure II.3.2 is the one actually used. The reason for this is that far fewer discrete patch widths are used in the former representation than in the latter so far fewer sets of $r-\theta$ boundary conditions are needed to characterize the $r-\theta$ boundary conditions found within the approximated contact. There are only five discrete contact widths used in the approximated contact shown

in Figure II.3.1, for instance, so only five significantly different trial functions could be obtained using the $r-\theta$ boundary conditions found at different axial locations within the contact. A different trial function would be obtained for each discrete contact width, of course, probably using the $r-\theta$ boundary conditions found at the midpoint of the axial segment in which each width was used. Denoting the m th axial segment in which the width of the approximated contact is constant by δz_m and designating the width of the contact in that segment by w_m , it is reasonable to assume that some of the trial functions used at axial nodes in δz_m should be two-dimensional ($r-\theta$) solutions to problems involving line contacts of width w_m . The axial segments δz_m are the constant-width shaded segments seen in Figure II.3.1, not those seen in Figure II.3.2. The edge of each contact of width w_m lies midway between two lateral surface nodes and hence does not require special approximation.

By deploying the solutions to the problems involving line contacts of width w_m in axial segments of the contact where they are likely to be useful in the temperature synthesis, they can serve as trial functions even when the contact patch is approximated in the manner shown in Figure II.3.2. A two-dimensional solution obtained using the $r-\theta$ boundary conditions at $x=0$ (where the contact spans six nodes laterally) would likely be most useful in the temperature synthesis at the first three axial nodes, for instance, and hence would be used at those nodes even when the contact patch was approximated in the manner shown in Figure II.3.2. Similarly, trial functions obtained with five nodes within the contact half-width would be used at axial nodes 4, 5 and 6

with either contact representation since Figure II.3.1 shows that those trial functions would probably be useful at those nodes.

Since the shaded region shown in Figure II.3.1 contains five discrete patch widths, five trial functions were obtained by computing an $r-\theta$ solution for each w_m using the boundary conditions at the midpoint of the corresponding axial segment, δz_m . Other trial functions were obtained at different axial locations outside of the contact, with the axial locations being chosen to provide a wide range of $r-\theta$ temperature distributions. It was assumed for the purpose of obtaining a measure of the temperature variation over a given $r-\theta$ plane that the temperature distribution or variation (ϵ_T) within the plane at axial position z could be roughly characterized by the ratio,

$$\epsilon_T(z) = \frac{T_1 - T_{\min}(z)}{T_1 - T_0} \quad (\text{II.3.1})$$

where T_1 and T_0 represent the source and sink temperatures respectively and $T_{\min}(z)$ was the minimum temperature in the plane. The latter temperature was always found at the bottom of the semi-cylinder (i.e., nearest to the isothermal surface at temperature T_0), of course. For axial locations within the contact, $T_{\min} = T_0$ and hence $\epsilon_T = 1$ while at axial locations remote from the contact, $T_{\min} \rightarrow T_1$ and $\epsilon_T \rightarrow 0$. The value of ϵ_T obtained using the boundary conditions immediately beyond the end of the contact, specifically at $x=0.16$ cm. (or $x/a=1.039$), was found to be approximately 0.70 so the variation of ϵ_T with z , determined from $r-\theta$ analyses performed at different axial locations, was examined to find the values of z for which ϵ_T was approximately 0.5 and 0.25. The corresponding $r-\theta$ temperature distributions were

then used as trial functions. A constant temperature,* for which $\xi_T = 0$, was used as the final trial function, giving a total of four trial functions with $\xi_T < 1$ in addition to the five trial functions obtained at different axial locations within the contact. It was felt that the total range of possible values of ξ_T (i.e., $0 \leq \xi_T \leq 1$) would be adequately bracketted by the few discrete values mentioned previously.

The set of trial functions just described, which will be summarized later, was not the only set tried. Since it was recognized that three-dimensional effects would cause the contact heat fluxes at a given axial location within the contact to be greater than those obtained in a two-dimensional ($r-\theta$) analysis at the same axial location, it was reasoned that the best results would be obtained if the set of trial functions included ones which could account for those heat flux increases. In other words, trial functions which would provide higher contact heat fluxes than the trial functions obtained using the boundary conditions at the midpoint of each axial segment δz_m were required for each discrete patch width, w_m . Such trial functions could have been obtained in a number of ways, but the most obvious of these and the one which was actually used was to compute the temperature distribution for each w_m in the same manner as before only without gas conduction. Since the contact heat fluxes for each value of w_m were much higher without gas conduction than with it (for the same value of $T_1 - T_0$), it was reasoned that trial functions obtained using the former boundary condition would

* When a constant is used as a trial function, its absolute value is unimportant because the $Z_n(k)$ by which it is multiplied in the synthesis technique are free to vary in value and hence scale as necessary.

be of some benefit in accounting for contact heat flux increases caused by axial conduction.

All of the trial functions that have been discussed so far are listed and numbered in Table II.3.1. Trial functions, 1, 3, 5, 7 and 9 were the trial functions obtained with gas conduction for the five discrete patch widths visible in the shaded area in Figure II.3.1 and trial functions 2, 4, 6, 8 and 10 were the corresponding trial functions obtained without gas conduction. Trial functions 11, 12, and 13 were obtained using the r - θ boundary conditions at different axial locations outside of the contact and trial function 14 was a constant, as indicated. The material properties given previously in Table I.6.4 were used to obtain these trial functions. In addition to the actual value of ξ_T for each trial function, Table II.3.1 gives the resistance (R_{2D}) of the semi-circular r - θ plane between the isothermal surfaces at temperatures T_1 and T_0 in each case. It should be noted that the values of R_{2D} obtained with and without gas conduction for a line contact of a given width (e.g., as for trial functions 1 and 2) are not as different as one might expect from the experimental results presented in Part I. The value of R_{2D} obtained with gas conduction ranges from 83% of the vacuum value for a contact half-width of 2.62×10^{-3} cm. (trial functions 9 and 10) to 91% of the vacuum value for a half-width of 9.61×10^{-3} cm. (trial functions 1 and 2). Since the resistance of the contact in air was only 25-50% of the resistance measured in vacuum in the experiments, most of the conduction through the gas must have occurred beyond the ends of the elliptical contact instead of beside it. This result emphasizes, once again, that the shape of the contact surface in the lateral direction is of secondary importance, as assumed.

Table II.3.1: Trial Function Data for 176N Contact Load

Trial Function No.	Axial Location		Gas Conduction?	Number of Nodes With- in Contact Half-Width	Local Contact Half- Width (cm.)	ϵ_T	R_{2D}^*
	z (cm.)	$\frac{z}{a}$ (or $\frac{x}{a}$)					$\frac{^\circ\text{C-cm.}}{W}$
1	0.0	0.0	Yes	6	9.61×10^{-3}	= 1	4.66
2	0.0	0.0	No	6	9.61×10^{-3}	1	5.13
3	0.088	0.57	Yes	5	7.86×10^{-3}	1	4.76
4	0.088	0.57	No	5	7.86×10^{-3}	1	5.30
5	0.117	0.76	Yes	4	6.11×10^{-3}	1	4.87
6	0.117	0.76	No	4	6.11×10^{-3}	1	5.52
7	0.137	0.89	Yes	3	4.37×10^{-3}	1	5.02
8	0.137	0.89	No	3	4.37×10^{-3}	1	5.82
9	0.148	0.96	Yes	2	2.62×10^{-3}	1	5.20
10	0.148	0.96	No	2	2.62×10^{-3}	1	6.28
11	0.160	1.04	Yes	0		0.69	6.36
12	0.210	1.36	Yes	0		0.47	8.30
13	0.350	2.27	Yes	0		0.25	13.33
14	CONSTANT					0.0	

* R_{2D} is the actual two-dimensional resistance of the semi-cylinder between T_1 and T_0 for the stated boundary conditions, not the resistance based on the extrapolated temperature drop at the interface. To obtain the latter resistance, subtract 1.35°C-cm/W from R_{2D} . A contact conductance (h_c) of $1136 \text{ W/cm}^2 \text{ K}$ was applied over the contact area in all the analyses but values of R_{2D} for the analyses without gas conduction do not include $R_{mic} (=1/h_c A_c)$.

GEOMETRICAL DATA USED IN ANALYSES

$a = 0.154 \text{ cm.}$

$b = 0.00961 \text{ cm.}$

Semi-cylinder radius = 0.952 cm.

Semi-cylinder length = 2.54 cm.

MATERIAL PROPERTIES USED IN ANALYSES

(see Table I.6.4)

II.3.3.2 Trial Function Deployment

The trial functions listed in Table II.3.1 were deployed at different axial mesh points in the 397 x 40 grid described previously according to the two schedules shown in Table II.3.2. The major difference between the two schedules was that schedule 1 did not use any of the trial functions computed without gas conduction (i.e., trial functions 2, 4, 6, 8 and 10) whereas schedule 2 always used the most appropriate of those trial functions at any axial node within the length of the contact. The most appropriate trial functions in this context were trial functions obtained with patch widths similar to the width of the elliptical contact at the axial location in question. Trial function 2 was used at the first three axial nodes, for instance, because the width of the elliptical contact at those points was similar to the patch width used to obtain it (i.e., the contact semi-minor diameter).

As seen in Table II.3.2, trial functions other than those obtained using the appropriate number of nodes within the lateral width of the contact (i.e., 2, 3, 4, 5 or 6 nodes) were used at axial nodes within the contact to provide additional $r-\theta$ temperature variation for the temperature synthesis. At the first axial mesh point ($k=1$), for instance, where trial functions 1, 2, 3, 5 and 11 were used in schedule 2 to obtain the synthesized temperature distribution, only trial functions 1 and 2 were solutions to problems involving line contacts of the proper half-width (6 lateral nodes). The other trial functions (3, 5, and 11) were inserted to provide additional $r-\theta$ variation, although it was recognized that they would be less useful than trial functions 1 and 2 in the temperature synthesis at that location.

Table II.3.2: Trial Function Deployment Schedules For Synthesis
Analyses Performed at a Contact Load of 176N

Axial Node (k)	z (cm.)	$N_x(k)$	Trial Functions (Schedule 1)	Trial Functions (Schedule 2)
1	0.0	6	1,3,5,9,11	1,2,3,5,11
2	0.0280	6	1,3,5,9,11	1,2,3,5,11
3	0.0560	6	1,3,5,9,11	1,2,3,5,11
4	0.0700	5	1,3,5,9,11	1,3,4,5,11
5	0.0840	5	1,3,5,9,11	1,3,4,5,11
6	0.0980	5	1,3,5,9,11	3,4,5,6,11
7	0.1073	4	1,3,5,9,11	3,4,5,6,11
8	0.1166	4	1,3,5,9,11	3,5,6,7,11
9	0.1260	4	1,5,7,9,11	3,5,6,7,11
10	0.1306	4	1,5,7,9,11	5,6,7,8,11
11	0.1353	3	1,5,7,9,11	5,6,7,8,11
12	0.1400	3	1,5,7,9,11	5,7,8,9,11
13	0.1439	3	1,5,7,9,11	5,7,8,9,11
14	0.1479	2	1,5,9,10,11	7,8,9,10,11
15	0.1519	2	1,5,9,10,11	7,8,9,10,11
16	0.1559	0	1,5,9,11	7,9,10,11,12
17	0.1599	0	1,5,9,11	7,9,10,11,12
18	0.1639	0	5,9,11,12	9,10,11,12,13
19	0.1679	0	5,9,11,12	9,10,11,12,13
20	0.1726	0	9,11,12,13	9,10,11,12,13
21	0.1773	0	9,11,12,13	10,11,12,13,14
22	0.1819	0	9,11,12,13	10,11,12,13,14
23	0.1913	0	9,11,12,13	11,12,13,14
24	0.2006	0	11,12,13,14	11,12,13,14
25	0.2099	0	11,12,13,14	11,12,13,14
26	0.2239	0	11,12,13,14	11,12,13,14
27	0.2519	0	11,12,13,14	11,12,13,14
28	0.2799	0	11,12,13,14	11,12,13,14
29	0.3079	0	11,12,13,14	11,12,13,14
30	0.3359	0	11,12,13,14	11,12,13,14
31	0.3919	0	12,13,14	12,13,14
32	0.4478	0	12,13,14	12,13,14
33	0.5038	0	12,13,14	12,13,14
34	0.5598	0	12,13,14	12,13,14
35	0.6782	0	12,13,14	12,13,14
36	0.7965	0	12,13,14	12,13,14
37	0.9149	0	12,13,14	12,13,14
38	1.0333	0	12,13,14	12,13,14
39	1.1516	0	12,13,14	12,13,14
40	1.2700	0	12,13,14	12,13,14

There are two characteristics of the trial function deployment schedules shown in Table II.3.2 which are especially important. First, it should be noted that although different sets of trial functions are used at different axial locations, no more than one trial function is changed, in general, when proceeding from one axial node to the next. That is, no more than one trial function is dropped and/or added at one time. Also, each set of trial functions is usually used at a minimum of two (consecutive) axial nodes before a switch to another set of trial functions is made. The author has found, by experience, that these two characteristics are required to obtain synthesized temperature distributions which vary reasonably smoothly in the axial direction. Dropping and/or adding two or more trial functions at a time when proceeding from one axial node to the next usually affects the smoothness of the axial temperature profile at those nodes adversely, particularly if the trial functions in question contribute significantly to the temperature synthesis before they are dropped or after they are added. Trial functions which do not contribute significantly to the temperature synthesis in a particular axial region can be dropped and/or added in that region without affecting the smoothness of the axial temperature distribution significantly.

One final point worthy of mention is that the trial functions listed in the schedules shown in Table II.3.2 had to be orthonormalized before they were used in order to obtain solutions that were unaffected by round-off errors when the computations were performed in single precision. Solutions unaffected by round-off errors could be obtained without orthonormalizing the trial vectors when the computations were performed

in double precision, however. The system of equations defining the blending vector (Z) was poorly conditioned unless the trial vectors were orthonormalized because the trial functions obtained using the $r-\theta$ boundary conditions at different axial locations within the length of the contact were very similar and hence almost linearly dependent. The values of R_{2D} for trial functions, 1, 3 and 5, which were all used to obtain the synthesized temperature distributions at axial nodes 1-7 and 1-5 in schedules 1 and 2 respectively, were all very close, for instance, as seen in Table II.3.1.

II.3.3.3 Results of the Synthesis Analyses for the Contact in Air

The trial functions listed in Table II.3.1 and deployed according to the schedules given in Table II.3.2 were used to predict the resistance of the contact in air. Results were obtained with the contact patch approximated by methods 2, 3 and 4 with a contact conductance (h_c) of $1136 \text{ W/cm}^2\text{K}$ prescribed over the contact area, as mentioned previously. The same contact conductance had been used to obtain the trial functions, as noted in the text underneath Table II.3.1. The material properties used in the analyses were the same as those given previously in Table I.6.4. All of the input and output data

for the analysis performed with trial function schedule 2 and the contact patch approximated with method 3 are given in Part IV, where the analysis is performed as a sample problem.

The results of the synthesis analyses performed with trial function schedules 1 and 2 are tabulated in Tables II.3.3 and II.3.4 respectively. The benchmark results with which the predictions of the synthesis model are compared were obtained by re-applying the ellipsoidal finite-difference model to the analysis of the model contact problem with the isothermal contact boundary condition that was used originally replaced by the boundary condition of the third kind that was used in the synthesis analyses. The value of R_{ca} predicted by the ellipsoidal finite-difference model increased by only 0.6%, from 9.06 °C/W to 9.12 °C/W, when the ellipsoidal finite-difference analysis was repeated with the latter boundary condition.

As seen in Tables II.3.3 and II.3.4, the different contact approximation techniques used provided essentially the same results but the results obtained with trial function schedule 1 were less accurate than those obtained with schedule 2. The values of Q_p/Q_c obtained with schedule 1 were particularly inaccurate. The heat fluxes obtained on the minor and major axes of the contact with schedule 1 were also quite inaccurate, as seen in Figures II.3.3 and II.3.4 respectively where those heat fluxes are compared to the heat fluxes obtained in the ellipsoidal finite-difference analysis performed with the same contact boundary condition. Similar comparisons of the heat fluxes obtained with schedule 2 to the ellipsoidal finite-difference results are shown

Table II.3.3: Results of Contact Resistance Calculations
Performed With Trial Function Schedule 1

Contact Approximation Technique	R'_{ca} ($^{\circ}\text{C}/\text{W}$)	Error ¹ in R'_{ca} (%)	Q_p/Q_c	Error ¹ in Q_p/Q_c (%)
2	8.73	-4.3	0.409	28.6
3	8.77	-3.8	0.407	28.0
4	8.77	-3.8	0.407	28.0

Table II.3.4: Results of Contact Resistance Calculations
Performed With Trial Function Schedule 2

Contact Approximation Technique	R'_{ca} ($^{\circ}\text{C}/\text{W}$)	Error ¹ in R'_{ca} (%)	Q_p/Q_c	Error ¹ in Q_p/Q_c (%)
2	8.85	-3.0	0.327	2.8
3	8.87	-2.7	0.327	2.8
4	8.87	-2.7	0.326	2.5

1. The % error in a particular synthesis result was calculated from the expression,

$$\% \text{ error in synthesis result} = 100 \frac{(\text{Synthesis result} - \text{Benchmark value})}{\text{Benchmark Value}}$$

Benchmark Values

$$R'_{ca} = 9.12^{\circ} \text{ C/W}, \quad Q_p/Q_c = 0.318$$

Figure II.3.3 Heat Fluxes on the Minor Axis of the Contact in Air - Synthesis Results Obtained With Trial Function Schedule 1

Figure II.3.4 Heat Fluxes on the Major Axis of the Contact in Air - Synthesis Results Obtained With Trial Function Schedule 1

in Figures II.3.5 and II.3.6. Although the heat fluxes shown for the synthesis analyses were obtained with the contact approximated by method 3, the heat fluxes obtained with the other contact approximation techniques were very similar.

The ellipsoidal finite-difference analysis that provided the benchmark heat fluxes shown in Figures II.3.3 - II.3.6 was performed with the geometric and material property data used in the analyses described in Chapter I.6 and a total source-to-sink temperature difference of 100K. The total heat flow rate between the contact surfaces was 3.64W with that temperature difference. Since the ellipsoidal finite-difference and synthesis analyses were performed with different boundary conditions and specimen shapes, etc., the total heat flow rate between the contact surfaces was held constant, at 3.64W, to obtain the heat fluxes attributed to the synthesis model in Figures II.3.3 - II.3.6. Since the total heat flow rate between the contact surfaces was held constant to obtain the comparisons shown in Figures II.3.3 - II.3.6, the comparisons really only indicate whether the heat fluxes obtained in the synthesis analyses are distributed correctly, not whether their magnitudes are correct. The favorable contact resistance comparisons suggest that the synthesis method predicted at least the average heat flux between the contact surfaces accurately however.

Before considering the heat flux comparisons any further, it should be mentioned that even the heat fluxes shown for the ellipsoidal finite-difference model may be slightly in error, not only because the model is inherently approximate when applied to analyses which are not one-dimensional with respect to one of the coordinate directions, but

Figure II.3.5 Heat Fluxes on the Minor Axis of the Contact in Air - Synthesis Results Obtained With Trial Function Schedule 2

Figure II.3.6 Heat Fluxes on the Major Axis of the Contact in Air - Synthesis Results Obtained With Trial Function Schedule 2

also because of the approximations made in calculating the internodal shape factors in Appendix I.B. Since the ellipsoidal finite-difference model should predict the exact heat flux distribution over an isothermal elliptical contact exchanging heat with a semi-ellipsoidal surface when the equation defining the contact shape and that defining the semi-ellipsoid are both obtained from Equation (I.4.1) (because the heat flow in that case is one-dimensional with respect to the λ coordinate), a rough estimate of the errors produced by at least the second source mentioned previously was obtained by applying the finite-difference model to the problem just mentioned and comparing the resulting contact heat fluxes to the exact values, which were easily calculated. It was estimated from that comparison that most of the heat fluxes attributed to the ellipsoidal finite-difference model in Figures II.3.3 - II.3.6 are accurate to within 1 or 2%, although the errors may be larger than this at the extreme edge of the contact. This accuracy was considered to be adequate for the purpose of assessing the accuracy of other solutions.

As seen in Figure II.3.5, the heat flux distribution inside of the contact patch on the minor axis was predicted with reasonable fidelity in the synthesis analysis performed with trial function schedule 2 in that local heat flux estimates were within 6% of those obtained with the ellipsoidal finite-difference model over most of the width of the contact. The accuracy of the synthesis solution deteriorated at the outer edge of the contact, however, because the finite-difference grid was not sufficiently refined in that region. The same

deterioration in accuracy occurred at the end of the contact for the same reason, as seen in Figure II.3.6. The heat flux "spike" which occurs at the boundary of a contact would require an extremely fine mesh to resolve and, in the case of an isothermal contact on a large body (without gas conduction), could not be resolved at all with a conventional finite-difference (or finite-element) discretization because the solution at the boundary would be singular. It is doubtful whether there would be any need to predict the size of the heat flux spike accurately in most practical applications, however, because the high local heat fluxes would diffuse rapidly in the material on either side of the contact. In the case of the pressure/calandria tube contact problem mentioned in Chapter I.1, for instance, any heat flux peaking which occurred at the boundary of the contact patch would be markedly attenuated by the time the heat flow reached the calandria tube outer surface, which is the only place it would be of any interest because of the risk of dryout. Heat flux peaking would also be reduced by surface roughness, of course, since the intermeshing of asperities would make the boundary of the contact less well-defined.

The heat fluxes obtained in a two-dimensional finite-difference analysis at $x=0$ are shown in Figure II.3.5 to illustrate the extent to which the contact heat fluxes are increased by axial conduction, in this case by about 50% at $y=0$. Trial function 1 in Table II.3.1 provided the two-dimensional results plotted in Figure II.3.5, which are based on the same total source-to-sink temperature difference as the three-dimensional results attributed to the synthesis model. Since the two-dimensional and three-dimensional results for the semi-cylindrical contact specimen are based on the same value of $(T_1 - T_0)$, they are directly comparable. The 3-D (synthesis) result for the heat flux at $y=0$ is 53% greater than the corresponding 2-D result.

The heat flux distribution on the minor axis of the contact, but outside of the contact patch itself, was not predicted as accurately as the distribution inside of the contact in the synthesis analysis performed with schedule 2, as seen in Figure II.3.5. Note that different scales are used to display the heat fluxes at points inside and outside of the contact and that the heat fluxes outside of the contact are much lower than those inside. Although the heat fluxes immediately beyond the edge of the contact were badly overpredicted by the synthesis model, those heat fluxes would probably have been overpredicted to a certain extent even if the finite-difference equations for the 397×40 grid had been solved exactly. That result would be expected because the surface temperatures immediately beyond the edge of the contact were overpredicted in the two-dimensional analyses described in Appendix II.E for contacts in vacuum (see Table II.E.1, for instance). Since the values of R_{2D} given in Table II.3.1 indicate that conduction through the gas beside the contact is not too significant, it is likely that the surface temperature overpredictions obtained in the resistance calculations for contacts in vacuum would also be obtained for contacts in air. The heat fluxes beside the contact would be overpredicted if the surface temperatures were overpredicted, of course.

As seen in Figure II.3.6, the heat flux distribution on the major axis of the contact was predicted reasonably accurately in the synthesis analysis performed with trial function schedule 2 in that local heat estimates were within 10% of those obtained with the ellipsoidal finite-difference model both within and outside of the

contact. Note that a logarithmic scale is used for heat fluxes above 2000 W/cm^2 .

As seen in Figures II.3.3 - II.3.4, the heat flux distributions obtained with trial function schedule 1 were very inaccurate compared to those obtained with schedule 2, thus demonstrating the utility of including the two-dimensional solutions obtained with different contact widths but without gas conduction as trial functions. Although this example illustrates the importance of choosing good trial functions, it is worth noting that the contact resistances obtained with trial function schedule 1 were almost as accurate as those obtained with schedule 2. It was found in numerical studies performed with other trial functions and trial function schedules that the contact resistance was almost always predicted fairly accurately even if the contact heat fluxes were not, suggesting that the choice of trial functions is less important when only average heat fluxes are required than it is when precise local values are needed. It was also found that heat fluxes which varied smoothly in the axial direction within the contact (i.e., on the contact's major axis) were usually more accurate than those which did not. The heat fluxes seen in Figure II.3.6 vary somewhat more smoothly within the contact than those shown in Figure II.3.4 and are also more accurate, for instance.

II.3.4 Contact in Vacuum Analysis

In the case of the contact in vacuum, it is obvious that performing two-dimensional analyses with the limited range of $r-\theta$

boundary conditions available at different axial locations within the contact specimen will not provide enough significantly different trial functions to synthesize temperature distributions at axial locations beyond the end of the contact but nevertheless close enough to it to be affected by its presence. Because two-dimensional solutions obtained without gas conduction at different axial locations within the contact patch (i.e., with different discrete patch widths) were already used as trial functions in the contact in air analyses performed with trial function schedule 2, it was decided to apply schedule 2 to the thermal analysis of the contact in vacuum as well. Trial functions 11-14 in Table II.3.1 would, it was reasoned, provide the r - θ temperature variation needed to synthesize temperature distributions at axial locations beyond the end of the contact. Results were obtained with the contact approximated by methods 2, 3 and 4 with a contact conductance of $1136 \text{ W/cm}^2 \text{ K}$ prescribed over its area, as in the case of the contact in air analyses.

Similar contact resistances were obtained with the different contact approximation techniques, as seen in Table II.3.5 where those resistances are tabulated. The resistances given in the table do not include the contact's microscopic resistance (i.e., $1/h_c A_c$), which was $0.379 \text{ }^\circ\text{C/W}$, and hence are macroscopic constriction resistances. A benchmark resistance of $21.64 \text{ }^\circ\text{C/W}$, obtained with the ellipsoidal finite-difference model, was used to calculate the errors quoted in Table II.3.5.

Although the resistance estimates obtained with the synthesis model were in error by 8-9%, as noted in Table II.3.5, it should be

Table II.3.5: Results of Contact Resistance in Vacuum
Calculations Performed With Trial
Function Schedule 2

Contact Approximation Technique	R'_{cv} (°C/W)	Error ¹ in R'_{cv} (%)
2	19.64	-9.2
3	19.82	-8.4
4	19.85	-8.3

1. The % error in R'_{cv} was calculated from the expression

$$\% \text{ error in } R'_{cv} = 100 \left(\frac{R'_{cv} - \text{Benchmark Resistance}}{\text{Benchmark Resistance}} \right)$$

Benchmark Resistance - 21.64°C/W.

recognized that the contact in vacuum analysis represents a severe test of the synthesis technique because axial conduction near the contact in vacuum is quite substantial. If the two-dimensional resistances of the different width axial segments visible in the shaded region in Figure II.3.1 are calculated from the values of R_{2D} listed in Table II.3.1 (for trial functions 2, 4, 6, 8 and 10) and the length of each segment and then combined in parallel to obtain an estimate of the resistance of the contact without axial conduction, for instance, the resulting resistance of 33.86 °C/W (based on the extrapolated temperature difference at the interface) is 56.5% greater than the benchmark value of 21.64 °C/W. Stated somewhat differently, it can be shown using the resistance figure of 33.86 °C/W just mentioned that axial conduction increases the total heat flow rate through the contact by approximately 54% (over the "two-dimensional" value) for fixed values of T_1 and T_0 . With an axial heat flow component of this magnitude, it is encouraging that the synthesis method worked as well as it did with a fairly straightforward set of trial functions.

The heat fluxes obtained on the minor and major axes of the contact in vacuum with the contact approximated by method 3 are compared to the corresponding heat fluxes obtained with the ellipsoidal finite-difference model in Figures II.3.7 and II.3.8 respectively. As in the case of the heat flux comparisons for the contact in air, the total heat flow rate through the contact for the synthesis analysis was matched to the total heat flow rate for the ellipsoidal finite-difference analysis to obtain the comparisons so it is really the heat flux distributions that are being compared, not the actual heat flux magnitudes.

Figure II.3.7 Heat Fluxes on the Minor Axis of the Contact in Vacuum.

Figure II.3.8 Heat Fluxes on the Major Axis of the Contact in Vacuum.

 heat flux distribution on the minor axis of the contact was predicted quite well except at the contact's outer edge where, as in the case of the synthesis analysis of the contact in air, the accuracy deteriorated because the finite-difference grid was not sufficiently refined. The heat fluxes obtained on the minor axis in a two-dimensional ($r-\theta$) analysis (trial function 2) are shown in Figure II.3.7 along with the synthesis and ellipsoidal finite-difference results to illustrate the significance of axial conduction in this case. As before, the heat fluxes attributed to the 2-D and 3-D (synthesis) models are based on the same value of $T_1 - T_0$. The 3-D heat flux at $y=0$ is almost 70% greater than the 2-D heat flux at the same location, indicating, once again, the significance of axial conduction in vacuum.

As seen in Figure II.3.8, the heat flux distribution on the major axis of the contact was predicted very accurately in that all of the heat fluxes predicted with the synthesis model were within 6% of those obtained with the ellipsoidal finite-difference model. Note that a logarithmic scale is used for heat fluxes above 3000 W/cm^2 .

Although the heat flux distributions plotted in Figures II.3.7 and II.3.8 were obtained with the contact approximated by method 3, the heat fluxes obtained with method 4 were virtually identical. The heat fluxes obtained with the contact approximated by method 2 were less accurate, however, in that they did not vary quite as smoothly in the axial direction (i.e., on the contact's major axis) as the heat fluxes calculated with methods 3 and 4. As demonstrated in Chapter II.5, where the different contact approximation techniques are

compared further, the superior accuracy of methods 3 and 4 becomes increasingly important as the lateral grid refinement around the contact is coarsened.

CHAPTER II.4

THE MULTIGRID METHOD

II.4.1 Introduction

The synthesis method was applied to the three-dimensional analysis of the model contact problem for the reasons already discussed in Chapters II.1 - II.3, but it is obvious that a less approximate method would be preferred in general if solutions could still be obtained at reasonable cost and with an algorithm of comparable simplicity. It will be shown in the next two chapters that the multiple grid method, which is commonly referred to in abbreviated form as multigrid (MG), satisfies both of the provisos just mentioned when applied to the model contact problem. Although presented in its most basic form by Federenko [125] in 1964, multigrid has only recently become well-known as a method for solving partial differential equations and its capabilities in that context are still being extended by Brandt [126, 127, 128], Hemker [129] and others. Almost all of the development of the method has so far been done by mathematicians, but it has nevertheless been applied to some very challenging problems, primarily in fluid dynamics, such as solving the full potential compressible flow equation in three-dimensions for transonic flow around an axisymmetric jet engine intake at a non-zero angle of attack [130].

It has been rigorously proven theoretically and verified by numerical experiments that multigrid can solve a system of N_e algebraic

equations obtained from the finite-element or finite-difference discretization of a linear elliptic problem in $O(N_e)$ arithmetic operations [126]. Further numerical experiments have shown that the $O(N_e)$ operations count order still holds for non-linear elliptic problems and even for a wide variety of linear and non-linear non-elliptic problems. The $O(N_e)$ operations count order is generally considered to be optimal [131] and there is, at present, no other technique for solving finite-element or finite-difference equations that can match it, as demonstrated by Mol [132] in a recent survey of all known techniques for that purpose. The low operations count order of multigrid makes it a natural choice for three-dimensional analyses, although the transonic flow analysis mentioned previously is the only example that has been reported in the literature (to the author's knowledge) of the method being applied to a three-dimensional problem.

Only the essential features of multigrid and features relevant to the present analysis will be described in this chapter since interested readers can refer to articles by Brandt [126-128], Hemker [129], Mol [132, 133], Nicolaides [131, 134] and others for further details and applications. Two basic multigrid algorithms, specifically the Correction Storage (CS) algorithm, which is applicable only to linear problems (unless combined with Newton iteration or some similar procedure) and the Full Approximation Storage (FAS) algorithm, which can be applied to both linear and non-linear problems, are considered in the next section.

II.4.2 Basic Multigrid Algorithms

II.4.2.1 The Correction Storage (CS) Algorithm

Consider the elliptic partial differential equation,

$$\mathcal{L}U(\tilde{x}) = F(\tilde{x}), \text{ for } \tilde{x} = (x_1, \dots, x_d) \quad (\text{II.4.1a})$$

defined in a domain $D \subseteq \mathbb{R}^d$ and subject to the boundary conditions,

$$\mathcal{L}U(\tilde{x}) = \phi(\tilde{x}) \text{ on the boundary } S_D \quad (\text{II.4.1b})$$

If equations (II.4.1a) and (II.4.1b) are discretized (using, for instance, the finite-element or finite-difference technique) on a hierarchy of grids of varying coarseness ranging from G^0 , the coarsest grid, to G^M , the finest grid, it is possible (with multigrid) to use all of the grids interactively to obtain the fine grid (G^M) solution very rapidly. It is assumed for the purpose of demonstrating this that the finite-difference or finite-element equations on the n th grid (G^n) can be written as,

$$L^n U^n = F^n \quad n=0,1,\dots,M \quad (\text{II.4.2})$$

where the superscript, n , refers to the discretization level and U^n is the exact solution to equation (II.4.2).

The essential features of the multigrid method for solving equation (II.4.2) on G^M (i.e., the finest grid) can be demonstrated by assuming that we have an approximation u^M to the exact solution U^M for which we can define a correction, v^M , as,

$$v^M = U^M - u^M \quad (\text{II.4.3})$$

Substituting equation (II.4.3) into equation (II.4.2) we obtain (for $n=M$),

$$L^M(V^M + U^M) = F^M \quad (\text{II.4.4})$$

or for linear systems,

$$\begin{aligned} L^M V^M &= F^M - L^M U^M \\ &= f^M \end{aligned} \quad (\text{II.4.5})$$

Solving equation (II.4.5) for V^M would be no easier than solving equation (II.4.2) directly for U^M were it not for the fact that relaxation techniques, such as Gauss-Seidel iteration [108], can be used to smooth V^M so that it can accurately be represented on a coarser grid (G^{M-1}) where the solution to equation (II.4.5) (or its replacement on G^{M-1}) is relatively inexpensive to obtain because of the reduced number of nodes. High frequency components of the error (V^M) are the components that cannot be represented on a more coarse grid but it is easily shown that a relaxation technique, such as Gauss-Seidel iteration, is very efficient at removing precisely those components (i.e., at smoothing the error) but considerably less efficient at removing the low frequency components. A qualitative explanation of why this is so is related to the "localness" of relaxation. Consider, for example, the 5-point difference replacement for the two-dimensional Laplacian operator on a square mesh with subdivisions of size Δ , viz.,

$$\nabla^2 U = \frac{U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} - 4U_{i,j}}{\Delta^2} \quad (\text{II.4.6})$$

which, when used to solve Laplace's equation, is written as

$$\bar{u}_{i,j} = \frac{u_{i+1,j} + \bar{u}_{i-1,j} + u_{i,j+1} + \bar{u}_{i,j-1}}{4} \quad (\text{II.4.7})$$

if the finite-difference equations are solved by Gauss-Seidel iteration. It is assumed that the nodes i,j are scanned lexicographically* with the subscript j varying most rapidly and "old" (i.e., before the sweep) and "new" (i.e., after the sweep) values of u are written without and with an overbar respectively. Iteration using equation (II.4.7) is obviously just a local averaging (smoothing) process in that the value of u at each node i,j is set equal to the average of the current values of its neighbours.

The smoothing properties of a relaxation sweep can be defined more formally by expanding the errors at a given node i,j before and after the sweep (i.e., $u_{i,j} - \bar{u}_{i,j}$ and $u_{i,j} - \bar{u}_{i,j}$ respectively) as complex Fourier series and determining the amplification factors for the different Fourier components (frequencies). Most discussions of multigrid describe the Fourier error analysis (e.g., see Brandt [126] or Mol [133]) so it will not be discussed any further here.

The smoothing properties of a given relaxation scheme are characterized by its smoothing factor, which is essentially the largest (i.e., worst), amplification factor for any Fourier error component that cannot be represented on the coarse grid used to obtain an approximate solution to equation (II.4.5). The Fourier error components that cannot be represented on the coarse grid can only be eliminated by performing (relatively expensive)

* Lexicographic ordering is consecutive ordering with the indices i,j for nodes located at x_i, y_j varying from least to most rapidly in, for instance, the order just mentioned.

relaxation sweeps on the fine grid so the best relaxation schemes are ones with small smoothing factors since they reduce those error components most quickly (i.e., with the fewest sweeps). It should be mentioned that the amplification factor for any error component, which is defined as the ratio of the amplitude of the component after the sweep to the amplitude of the component before the sweep, must be less than unity in order to reduce (and eventually largely eliminate) that component.

The smoothing factor for a given relaxation scheme applied to a particular finite-element or finite-difference discretization of equation (II.4.1) is a function of the ratio of the fine grid mesh size, say Δ_M , to the coarse grid mesh size, Δ_{M-1} . Designating this ratio β , we have the definition,

$$\beta = \frac{\Delta_M}{\Delta_{M-1}} \quad (\text{II.4.8})$$

The best smoothing factors are obtained for symmetric elliptic equations when the subdivisions on each grid are equi-sized in the different coordinate directions (i.e., $\Delta x_M = \Delta y_M = \Delta z_M = \Delta_M$ and $\Delta x_{M-1} = \Delta y_{M-1} = \Delta z_{M-1} = \Delta_{M-1}$ on a 3-D Cartesian grid) but this characteristic is not an absolute requirement. Brandt [126] has shown that, in addition, the most efficient smoothing is usually obtained when $\beta=1/2$, that is when the mesh sizes on the coarse grid are twice those on the fine grid, so it will be assumed in the following discussion that this mesh size ratio applies unless stated otherwise. Smoothing factors for several different relaxation schemes applied to a number of commonly encountered elliptic operators are given by Brandt in reference [126].

Provided a suitable relaxation scheme is chosen, high frequency error components on G^M can usually be reduced by one order of magnitude or more in just a few relaxation sweeps (on G^M). Once those components have been largely eliminated, the correction (V^M) can be obtained on a more coarse grid, G^{M-1} . A possible procedure for solving the G^M equations, therefore, would be to:

- (1) Smooth the error $V^M = U^M - u^M$ on the fine grid G^M by performing a few relaxation sweeps.
- (2) Go to a more coarse grid G^{M-1} (say with $\Delta_{M-1} = 2\Delta_M$) and solve the equation

$$L^{M-1} V^{M-1} = I_M^{M-1} (F^M - L^M u^M) = I_M^{M-1} f^M \quad (\text{II.4.9})$$

(where I_M^{M-1} is an averaging or restriction operator) exactly. This solution could be obtained iteratively or directly (e.g., by using Gaussian elimination).

- (3) Since $U^M = V^M + u^M$, add $I_{M-1}^M V^{M-1} (= V^M)$ to u^M to obtain a better approximation to U^M

$$\text{i.e., } u^M \leftarrow u^M + I_{M-1}^M V^{M-1} \quad (\text{II.4.10})$$

The operator I_{M-1}^M is termed the interpolation or prolongation operator and is usually some sort of polynomial interpolation.

- (4) Go to step (1) unless the solution has converged.

The algorithm described by steps 1-4 is by no means complete in that the operators I_M^{M-1} , I_{M-1}^M and L^{M-1} remain to be specified, as does the

convergence test referred to in step (4). In addition, some rationale must be developed for specifying how many relaxation sweeps are to be performed on G^M in step (1) in order to smooth the error. Defining the operators I_M^{M-1} , I_{M-1}^M and L^{M-1} can only be done by analyzing the coarse grid correction cycle (CGC), which consists of transferring the residual vector (f^M) to the coarse grid (using I_M^{M-1}), solving the corresponding residual problem on G^{M-1} , and then interpolating V^{M-1} back to G^M to serve as a correction to u^M . Such an analysis is performed by Brandt in [126] by examining the extent to which different Fourier components of the error are amplified by the cycle, where, for simplicity, it is assumed in the analysis that L^{M-1} is derived from L in the same way as L^M and that I_M^{M-1} is an averaging operator defined (for $d=2$) by the expression;

$$f^{M-1}\left(\frac{i}{2}, \frac{j}{2}\right) = I_M^{M-1} f^M = \sum_{\alpha=-J}^J C_{\alpha} (f^M(i+\alpha, j) + f^M(i, j+\alpha)) \quad (\text{II.4.11})$$

for some small integer J and real constants C_{α} . The integer J is usually either 0 or 1 in multigrid algorithms. Grid indices on G^M and G^{M-1} are related in the manner shown in Figure II.4.1, where segments of each grid are superimposed and grid lines are numbered. It is assumed that the set of nodes on G^{M-1} , designated S^{M-1} , is a subset of the set of nodes on G^M , denoted S^M (i.e., $S^{M-1} \subset S^M$).

By making the additional assumption in the analysis of the CGC cycle that the interpolation operation I_{M-1}^M would be an m th order polynomial, Brandt was able to conclude:

Figure II.4.1: Grids G^M and G^{M-1} when $S^{M-1} \subset S^M$

- (1) that the optimal averaging operator (I_M^{M-1}) for symmetric second-order elliptic equations is obtained when $J=0$ and $C_0=1/2$ in equation (II.4.11). Substituting these values for J and C_0 into equation (II.4.11) yields the relation,

$$f^{M-1}\left(\frac{i}{2}, \frac{j}{2}\right) = f^M(i, j) \quad \text{for } i=0, 1, \dots, N_i, \quad j=0, 1, \dots, N_j \quad (\text{II.4.12})$$

for grids G^M and G^{M-1} when $S^{M-1} \subset S^M$. The operator I_M^{M-1} is termed the "injection" operator for this choice of J and C_α ($\alpha=-J, \dots, J$). since the residuals for nodes i, j on G^M are simply "injected" into coincident nodes on G^{M-1} , as seen in Figure II.4.1. For non-symmetric second order equations or equations with rapidly varying coefficients, "half" and "full" weighting schemes represented symbolically (on G^M) by the stencils,

$$\frac{1}{8} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xleftarrow{I_M^{M-1}} \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Half-Weighting Full-Weighting

are recommended in two-dimensions (i.e., $d=2$). Analogous weighting schemes can be used in three-dimensions.

- (2) that the order of the interpolating polynomial used in the coarse-to-fine grid interpolation I_{M-1}^M should be $\geq l-1$; where l is the order of the differential equation. Thus linear interpolation (at least) should be used for second order equations, such as the heat conduction equation. With linear interpolation, the replacement operation $u^M \leftarrow u^M + I_{M-1}^M v^{M-1}$ for nodes i, j on G^M surrounding and coincident with the node $i/2, j/2$ on G^{M-1} becomes,

$$\begin{aligned} u^M(i, j) &\leftarrow u^M(i, j) + v^{M-1}(i/2, j/2) \\ u^M(i \pm 1, j) &\leftarrow u^M(i \pm 1, j) + \frac{1}{2}(v^{M-1}(i/2, j/2) + v^{M-1}((i \pm 2)/2, j/2)) \\ u^M(i, j \pm 1) &\leftarrow u^M(i, j \pm 1) + \frac{1}{2}(v^{M-1}(i/2, j/2) + v^{M-1}(i, (j \pm 2)/2)) \\ u^M(i \pm 1, j \pm 1) &\leftarrow u^M(i \pm 1, j \pm 1) + \frac{1}{4}(v^{M-1}(i/2, j/2) + v^{M-1}((i \pm 2)/2, j/2) \\ &\quad + v^{M-1}(i/2, (j \pm 2)/2) + v^{M-1}((i \pm 2)/2, (j \pm 2)/2)) \end{aligned} \quad (\text{II.4.13})$$

Numerical experiments performed on problems with Dirichlet boundary conditions confirmed the validity of Brandt's theoretical analysis and showed quite clearly, for instance, that injection works quite satisfactorily

for the fine-to-coarse mesh residual transfers in such analyses.

Injection is only effective when not only the correction (V^M) but also the residual (f^M) is smooth, however, and while a smooth V^M implies a smooth f^M when the boundary conditions are of the Dirichlet type and F is smooth, Neumann boundary conditions (i.e., specified $\partial U / \partial \hat{n}$, where $\hat{n}$ is the outward-facing surface normal) or Robbins boundary condition (i.e., $\partial U / \partial \hat{n} = C(U_{\infty} - U)$ for specified C and U_{∞}) can produce very large residuals at the boundary nodes even if V^M is very smooth [135].

Boundary conditions were ignored in the local (interior) Fourier mode analysis performed by Brandt to determine optimal forms of the restriction and prolongation operators so it is perhaps not surprising that early attempts to solve problems with Neumann and Robbins boundary conditions using injection for the fine-to-coarse mesh residual transfers were not very successful in that convergence rates were much slower than expected [136]. Better results are obtained when half or full-weighting of the residuals is used at boundary points where non-Dirichlet boundary conditions are prescribed (injection can usually be retained at the interior nodes), but even this practice does not completely cure the problem with these points because relaxation does not smooth the error (V^M) at such boundary nodes as well as it does at interior nodes. The author has found, by performing numerical experiments, that satisfactory convergence rates can be obtained in problems involving Neumann and/or Robbins boundary conditions by relaxing the boundary nodes before the interior nodes. When the relaxations are performed in this manner, the boundary values of u remain fixed when the interior nodes are relaxed and normal interior smoothing rates apply. Relaxing the boundary points before those in the interior and then holding the boundary values of u

constant while the interior nodes are relaxed provides smoothing rates comparable to the rates obtained with only Dirichlet boundary conditions because the non-Dirichlet boundary conditions are effectively transformed to those of Dirichlet type for the interior node relaxations.

The relaxation procedure just described is implemented in subroutine RELAX in the Fortran IV computer program MULTIGRID written to solve the model contact problem. The documentation for MULTIGRID and a listing of the program are given in Appendix II.G, so readers interested in specific details of the relaxation procedure should refer to the listing of subroutine RELAX. The listing is liberally interspersed with comment statements. Most of the discussions of multigrid found in the literature do not consider non-Dirichlet boundary conditions and the author does not know of any reference which discusses the treatment of such boundary conditions satisfactorily.

Assuming that high frequency errors on G^M can be smoothed by relaxation, we now turn our attention to the solution of equation (II.4.9) on G^{M-1} . The system of simultaneous equations defined by equation (II.4.9) may still be too large to solve quickly, but this problem can be resolved by recognizing that the two level (i.e., two grid) multigrid algorithm described previously, which involved only G^M and G^{M-1} , can be applied recursively in the manner shown in Figure II.4.2. The algorithm shown in Figure II.4.2 starts with some approximation u_0^M being given on G^M , where u_0^M could have been obtained by solving the given problem on a more coarse mesh and interpolating the results up to G^M (such an algorithm is termed "Full Multigrid" if the initial approximation on G^n ($n=1,2,\dots,M$)

Figure II.4.2 Multigrid Algorithm For Linear Problems (Correction Storage).

is obtained by interpolating a solution from G^{n-1}). The equations that are solved on G^M are the original ones, viz.,

$$L^M U^M = F^M$$

but the equations that are solved on G^n ($0 \leq n \leq M-1$) are the equations defining the correction V^n which, once obtained, is interpolated up to G^{n+1} to serve as an approximation to V^{n+1} (where $V^{n+1} = U^M$ if $n=M-1$). Denoting the approximation to V^n by v^n , the equation that is satisfied (approximately) by v^n is:

$$L^n V^n = f^n \tag{II.4.14}$$

where f^n is the residual left by v^{n+1} , that is

$$f^n = I_{n+1}^n (f^{n+1} - L^{n+1} v^{n+1}) \tag{II.4.15}$$

As mentioned previously, v^n is an approximation to V^n . When v^n is changing in the algorithm (causing V^{n-1} to change), V^n remains fixed.

A detailed discussion of the algorithm shown in Figure II.4.2, and several others as well, is given by Brandt in [126]. In particular, methods for determining when convergence is "slow" on the n th grid, at which time relaxations on the n th grid should be discontinued (unless $n=0$) and the correction computed on G^{n-1} , and methods for determining when the G^n problem has converged, are described in detail by Brandt.

II.4.2.2 The Full Approximation Storage (FAS) Algorithm

The multigrid algorithm described in the preceding section can only solve linear finite-difference or finite-element equations and hence can only be applied to non-linear problems if it is used as the linear equation solver in, for instance, Newton-Raphson's method. Non-linear equations can be solved by iterative methods such as successive over-relaxation if non-linear terms are simply updated each iteration using the most recent values of the unknowns, but the "corrections" in multigrid are difficult to use for that purpose. To avoid complicated linearizations, the "Full Approximation Storage" (FAS) version of multigrid operates with the full values of the unknowns on the coarse grids instead of just the corrections. The FAS algorithm offers other advantages over the CS algorithm, as will be seen shortly. The FAS multigrid algorithm can be formulated by reconsidering equation (II.4.4), repeated here as,

$$L^M u^M = L^M (V^M + u^M) = F^M \quad (\text{II.4.4})$$

Equation (II.4.4) cannot be rewritten in the form of equation (II.4.5) if the problem is non-linear, but by subtracting $L^M u^M$ from both sides of equation (II.4.4) we can obtain the equation,

$$\begin{aligned} L^M (V^M + u^M) - L^M u^M &= F^M - L^M u^M \\ &= r^M \end{aligned} \quad (\text{II.4.16})$$

which we can now solve on a more coarse grid (G^{M-1}) as,

$$L^{M-1} (\hat{I}_M^{M-1} u^M + V^{M-1}) - L^{M-1} (\hat{I}_M^{M-1} u^M) = \hat{I}_M^{M-1} r^M \quad (\text{II.4.17})$$

or as,

$$L^{M-1}(\hat{I}_M^{M-1} u^M + v^{M-1}) = I_M^{M-1} f^M + L^{M-1}(\hat{I}_M^{M-1} u^M) \quad (\text{II.4.18})$$

Note that $\hat{I}_M^{M-1} u^M + v^{M-1}$ is the current approximation to U^M (i.e., the exact solution) on G^{M-1} . By designating this quantity u^{M-1} , we can re-write equation (II.4.18) as,

$$\begin{aligned} L^{M-1} u^{M-1} &= I_M^{M-1} f^M + L^{M-1}(\hat{I}_M^{M-1} u^M) \\ &= F^{M-1}, \text{ say} \end{aligned} \quad (\text{II.4.19})$$

On grid G^M we had,

$$L^M u^M = F^M$$

so the FAS algorithm gives exactly the same type of equation on each grid. If equation (II.4.19) is solved exactly on G^{M-1} , thus providing u^{M-1} , then from the original definition of u^{M-1} (i.e., $u^{M-1} = \hat{I}_M^{M-1} u^M + v^{M-1}$) we have*,

$$v^{M-1} = u^{M-1} - \hat{I}_M^{M-1} u^M \quad (\text{II.4.20})$$

so we can set,

$$u^M \leftarrow u^M + I_{M-1}^M v^{M-1} \quad (\text{II.4.21})$$

or

$$u^M \leftarrow u^M + I_{M-1}^M (u^{M-1} - \hat{I}_M^{M-1} u^M) \quad (\text{II.4.22})$$

* u^{M-1} is the exact solution to equation (II.4.19) here, not the exact solution to equation (II.4.2) (with $n=M-1$).

to obtain the next approximation to U^M .

Numerical experiments performed by Brandt [128] and others have shown that the I_M^{M-1} operators written with a hat (i.e., as $\hat{I}_M^{M-1}$) must be the same but that I_M^{M-1} and $\hat{I}_M^{M-1}$ can be different. Usually, $\hat{I}_M^{M-1}$ is taken to be the injection operator, but I_M^{M-1} can be either the injection operator or some sort of averaging operator (e.g., the half or full weighting operator given previously).

The two-level FAS algorithm just described can be applied recursively, as was the case with the CS algorithm, and a flowchart for accomplishing this is given in Figure II.4.3. The FAS and CS algorithms are very similar, as close examination of Figures II.4.2 and II.4.3 readily shows.

The multigrid algorithms shown in Figures II.4.2 and II.4.3 are known as "cycling" algorithms because they start on the finest grid (G^M) and cycle up and down over all of the grids until the G^M solution has converged. Another type of algorithm is the "Full Multigrid" (FMG) algorithm mentioned previously in which the initial approximation on G^{n+1} ($n=0, \dots, M$ or $n=1, \dots, M$) is obtained by interpolating a solution obtained previously on G^n . "Full Multigrid" algorithms are generally slightly more efficient than cycling ones [135]. "Cycling" and "Full" multigrid algorithms can operate according to either an "accommodative" or a "fixed" strategy, where an "accommodative" strategy is one in which the decision to switch to either a finer or a coarser level depends on tests which indicate if relaxation has "slowed down" or if convergence has been achieved, and a fixed strategy is one in which grids interact in a predetermined, fixed manner. The CS and FAS multigrid algorithms

The sequence of operations in the fixed algorithm flowcharted in Figure II.4.4 is determined completely by the fixed numbers p , q and r . In general, only the value of q (i.e., the number of relaxation sweeps performed on the coarsest grid during each visit to that grid) would be at all problem-specific. Fixed algorithms work very well when applied to some problems, such as those governed by Poisson's equation, because even an accommodative algorithm usually behaves as though it was fixed in such analyses. Additional information on fixed algorithms can be found in articles by Brandt [128] and Hemker [129].

The FAS multigrid algorithm offers several advantages over the CS algorithm even when applied to linear problems since it;

- (1) permits highly-graded grids to be constructed very easily.
- (2) provides a natural criterion for deciding when the fine grid solution has converged.
- (3) permits the truncation errors (τ) at the fine grid nodes to be estimated and then used in a technique known as τ -extrapolation [127] to improve the accuracy of the final solution (under certain circumstances), and;
- (4) permits transient solutions to be obtained rapidly by exploiting the very different rates of change of the high and low frequency components of such solutions (the high frequency components converge to a steady-state more quickly than those of low frequency).

The first of the capabilities mentioned above is probably the most important in the present application and hence is discussed in the next section. The other capabilities mentioned (of which only (2) and (3)

were implemented in this study) are outlined in Appendix II.F.

II.4.2.3 Composite (Graded) Grids

Probably the most significant advantage of the FAS multigrid algorithm over the CS algorithm, at least for solving problems in which the grid must be much finer in some regions than in others, is that the FAS algorithm permits highly graded but nevertheless node-conserving grids to be constructed in an almost trivial manner [126]. To obtain such a grid with the FAS multigrid algorithm, one simply uses grids (G^n ; $n=0, \dots, M$) which are not co-extensive or, more precisely, it is no longer required that each grid G^n ($n>0$) completely cover the next coarser grid, G^{n-1} , as illustrated in Figure II.4.5 in two-dimensions. Referring to the superposition of a coarse and fine grid shown in Figure II.4.6, where the fine grid (G^n) nodes are marked with circles ("internal boundary" nodes) and crosses (all other nodes) and the coarse grid (G^{n-1}) nodes are located at grid line intersections, the solution at the G^n nodes marked with circles and squares (which also belong to G^{n-1}) should satisfy the G^{n-1} difference equations, such as the 5-point relations indicated by squares. The solution at "internal boundary" nodes of G^n which do not belong to G^{n-1} is always obtained by interpolating from G^{n-1} . Nodes on G^{n-1} that are coincident with G^n inner points (crosses) are used to obtain the corrections for G^n inner points (i.e., equation (II.4.19) is solved at these G^{n-1} nodes) but the remaining nodes on G^{n-1} serve as fine grid nodes.

Figure II.4.5 Generating a Composite (Graded) Grid With Multigrid.

Figure II.4.6 Fine Grid Superimposed on Coarse Grid

The solution at each node on G^n automatically satisfies the correct relationship if all of the nodes except the internal boundary nodes (marked with circles in Figure II.4.6) are treated as they would normally be in all of the phases of a multigrid solution. The internal boundary nodes do not participate in anything but the coarse-to-fine grid correction interpolations - they are excluded from the relaxation sweeps and are not used in the fine-to-coarse grid residual transfers. Provided the initial approximation used on G^n (i.e., u_0^n) satisfies the chosen interpolation relations between G^{n-1} and G^n at internal boundary nodes of G^n , subsequent FAS interpolation operations automatically preserve those relations.

It may be undesirable in some applications to introduce progressively finer grids that are refined more or less equally in all of the coordinate directions. Analyzing the heat flow through a long thin elliptical contact is a good example of such an application since the grid refinement required to model the lateral constriction of the heat flow is much greater than that needed to model the longitudinal constriction except, perhaps, right at the end of the contact. Thus one would prefer to use a grid which was more highly refined in the y and z directions (in the contact-centered coordinate system) at the contact than in the x direction. This requirement is easily accommodated in a multigrid analysis since progressively finer grids do not have to be refined in all of the coordinate directions, as noted by Brandt [139]. Introducing grids (G^n) which are finer than their predecessors in, say, the y and z directions but not in the x direction ultimately produces grids for which $\Delta y = \Delta z \ll \Delta x$, which is the desired structure of course, but it can be

shown that relaxation sweeps, while still effective at smoothing the error in the y and z directions, are much less effective at smoothing the error in the x direction in this case. This shortcoming does not matter, however, because the only purpose of the relaxation sweeps is to smooth the error so that it can be approximated on a coarser grid. Smoothing the error in a particular direction is not important if the next coarser grid is not coarser in that direction, which is the case with grids created by partial refinement.

The grid refinement and truncation error measuring capabilities of the FAS multigrid algorithm (the latter capability is discussed in Section II.F.1) can be used interactively to automatically refine a grid in any region where large truncation errors are detected. The set of grids is left open-ended so that additional grids can be added by the algorithm to reduce truncations errors everywhere to a level specified by the analyst. The process by which the multigrid algorithm itself decides when and where to refine the grid is known as adaptive discretization and, while not used by the author in this study, it is a capability which has been studied in depth by Brandt [126-128].

II.4.3 Applying Multigrid To The Model Contact Problem

A Fortran IV computer program was written to apply the FAS multigrid algorithm to the model contact problem. The program solves the heat conduction equation in a rectangular bar of length L , thickness t and width w and permits any number of nested grids to be used anywhere within the

bar or on its boundary. Each grid can be obtained from its predecessor by either halving the mesh subdivisions in all directions or, as discussed in Section II.4.2.3, only in the y and z directions in the contact-centered coordinate system. Only one-quarter of the elliptical contact is modelled, using a grid such as that shown in Figure II.4.7, because of symmetry considerations. Nodes on the contact surface are linked to the sink temperature (T_0) by gas conductances, defined by equation (I.3.24), in analyses performed with gas conduction and the contact patch can be modelled either as being isothermal or with a large contact conductance (h_c) prescribed over its area. Any one of the contact approximation techniques designated methods 1, 3 and 4 in Section II.2.5.3 can be used to approximate the shape of the contact. The contact is modelled by prescribing the desired boundary condition at each node on the contact surface whose coordinates lie within the boundaries of the contact patch in method 1 and by using a representation such as that shown in Figure II.2.9 in methods 3 and 4. The nodal coordinates are computed and compared to those of the ellipse boundary by the program, which then prescribes boundary conditions appropriate to the chosen approximation technique.

The multigrid program, which is documented in Appendix II.G, uses the 7-point "star" difference approximation to the three-dimensional (x,y,z) heat conduction operator. Gauss-Seidel relaxation with "red-black" or "checkerboard" ordering, in which "odd" grid points (i.e., points x_i, y_j, z_k for which $i+j+k$ is an odd number) are relaxed before "even"

Figure II.4.7 Graded Grid Superimposed Over Substitute Contact Specimen.

grid points (i.e., points x_i, y_j, z_k for which $i+j+k$ is an even number), is used as the smoothing scheme since it is highly recommended by Brandt [140] and Stuben et al. [141]. The smoothing factor for this scheme is 0.444 (with $\beta=1/2$) when it is applied to finite-difference equations obtained using the 7-point difference approximation to the Laplacian operator [140], so all of the components of the error (V^n) on G^n that cannot be represented on G^{n-1} are reduced by more than one order of magnitude in three relaxation sweeps on G^n (since $0.444^3=0.088<0.1$). The smoothing factor for Gauss-Seidel relaxation performed with lexicographic (i.e., normal) ordering is 0.567 for the same difference equations [126] by way of comparison, so it is slightly less effective than checkerboard relaxation.

The difference equations for boundary nodes at which non-Dirichlet boundary conditions are prescribed are relaxed before the equations applying to interior nodes for the reasons discussed in Section II.4.2.1. The program operates like a "fixed" algorithm (see Section II.4.2.2) in some respects in that only one relaxation sweep is performed on each grid before and after its CGC (i.e., $p=r=1$ in the fixed algorithm discussed previously). This strategy was found to give better performance than the alternate strategy in which relaxation sweeps are performed on each grid (both before and after the CGC) until relaxation "slows down".

The program uses full residual weighting at the boundary points where non-Dirichlet boundary conditions are prescribed but half-weighting elsewhere for the fine-to-coarse grid residual transfers. Since the residual at each "even" node is zero after a relaxation sweep performed

with checkerboard ordering, half-weighting of the residuals is accomplished in practice by "injecting" one-half of the residual at each odd grid point on the fine grid into the coincident node on the coarse grid [140,141]. Note that the coarse grid nodes coincide with "odd" nodes on the fine grid (initial values of i, j and k are unity) - "even" nodes on the fine grid do not appear on the coarse grid. Since half-weighting of the residuals is equivalent to injection with the factor $1/2$ inserted when checkerboard ordering is used, the residuals need to be calculated at far fewer nodes than would otherwise be the case (thus leading to significant reductions in computer time).

The multigrid program described in Appendix II.G can operate as either an "accommodative" or a "fixed" algorithm. The fixed algorithm follows the sequence of operations indicated in Figure II.4.4 ($p=r=1$ in the program but q must be specified by the user) while the accommodative algorithm can operate in either a "cycling" or a "full" mode. A flowchart for the accommodative FMG algorithm is given in Appendix II.G. Although Brandt [127] recommends that FMG algorithms use cubic interpolation to obtain the initial approximation on G^{n+1} from the solution on G^n for best results, the author's program uses only trilinear interpolation for that purpose for simplicity. Both Brandt's and the author's algorithms use bilinear (for $d=2$) or trilinear (for $d=3$) interpolation for all of the coarse-to-fine grid correction transfers, however.

The natural stopping criterion described in Section II.F.1 is used to stop the iterations in the program. Provision is made for using τ -extrapolation (described in Section II.F.2) to improve the accuracy of a fine grid solution by either computing the fine grid solution first and then solving the G^{M-1} problem obtained by replacing τ_M^{M-1} by $4/3 \tau_M^{M-1}$, as discussed in Section II.F.2, or by incorporating the extrapolation procedure into the fixed algorithm shown in Figure II.4.4.

CHAPTER II.5

COMPARISON OF RESULTS OBTAINED WITH THE MULTIGRID METHOD TO BENCHMARK RESULTS OBTAINED WITH THE ELLIPSOIDAL FINITE-DIFFERENCE MODEL

II.5.1 Grid and Contact Specimen Details

As was the case with the synthesis model, numerical results were only obtained for the intermediate contact load of 176N with the multi-grid method. The cross-sectional dimensions of the contact specimen used in the analyses were made the same as those of the lower contact specimen in the experiments, in other words 1.71 x 2.54 cm. (these dimensions were halved to take advantage of planes of symmetry), and the length (L) of the specimen was taken to be 2.54 cm. The contact specimen used in the analyses had the same nominal dimensions, therefore, as the contact specimen that was modelled (approximately) in the ellipsoidal finite-difference analyses.

The heat flow in the contact specimen was modelled with the contact approximated by methods 1, 3 and 4. For convenience, a contact conductance (h_c) of $1136 \text{ W/cm}^2 \text{ K}$ was prescribed over the contact area since that contact boundary condition had been used to obtain the synthesis and ellipsoidal finite-difference solutions compared previously. Results were obtained with two grids which differed only in the degree of refinement used near the contact in the y-z plane of the contact-centered coordinate system. In one grid, designated grid A, 12 nodes were

contained within the half-width of the contact on its minor axis and the edge of the contact was located midway between the twelfth and thirteenth lateral nodes. In the other grid, designated grid B, the contact's semi-minor diameter extended only 38% of the way across the surface element for the fourth lateral node so the lateral grid refinement was much coarser than that of grid A. Defining $n' = b/\Delta y$, where Δy is the mesh increment used in the lateral (y) direction within the contact, the values of n' for grids A and B were 11.5 and 2.88 respectively. The mesh increments used in the z-direction near the contact were approximately equal to those used in the y-direction in each case (i.e., $\Delta y = \Delta z$ near the contact). Both grids used 24 equally-spaced nodes within the semi-major diameter of the contact.

The coarsest (G^0) grid employed in the multigrid analysis with grid A or grid B used 2, 3, and 6 subdivisions across the 0.855 cm. (= 1.71 cm./2), 1.27 cm. (= 2.54 cm./2) and 2.54 cm. dimensions of the rectangular contact specimen respectively, which were its dimensions in the y, x and z directions of the contact-centered coordinate system after taking advantage of planes of symmetry. A total of 10 grids was used in the multigrid algorithm with grid A, although grids 8-10 did not provide additional refinement in the x-direction. Subdivisions were halved in the directions of refinement when finer meshes were introduced, so the finest subdivisions used in the x, y and z directions with grid A were:

$$\text{x-direction} \rightarrow \Delta x_{10} = \Delta x_9 = \Delta x_8 = \Delta x_7 = \frac{1.27 \text{ cm.}}{3 \times 2^6} = 6.615 \times 10^{-3} \text{ cm.}$$

$$\text{y-direction} \rightarrow \Delta y_{10} = \frac{0.855 \text{ cm.}}{2 \times 2^9} = 8.350 \times 10^{-4} \text{ cm.}$$

$$\text{z-direction} \rightarrow \Delta z_{10} = \frac{2.54 \text{ cm.}}{6 \times 2^9} = 8.268 \times 10^{-4} \text{ cm.}$$

The subscripts on the mesh increments Δx , Δy and Δz refer to grid numbers.

The composite grid referred to as grid A contained a total of 20,703 nodes. Each node was counted only once, even if it appeared on several grids, to obtain this figure. All of the input variables used to define grid A, from which additional information about the grid can be derived, are listed on a sample printout included in Appendix II.G.

Grid B was obtained from grid A by omitting grids 9 and 10 in the multigrid algorithm. With this modification, the finest subdivisions used in the x, y and z directions with grid B were:

$$\text{x-direction} \rightarrow \Delta x_8 = \Delta x_7 = \frac{1.27 \text{ cm.}}{3 \times 2^6} = 6.615 \times 10^{-3} \text{ cm.}$$

$$\text{y-direction} \rightarrow \Delta y_8 = \frac{0.855 \text{ cm.}}{2 \times 2^7} = 3.340 \times 10^{-3} \text{ cm.}$$

$$\text{z-direction} \rightarrow \Delta z_8 = \frac{2.54 \text{ cm.}}{6 \times 2^7} = 3.307 \times 10^{-3} \text{ cm.}$$

With each node in the composite grid counted only once, as before, grid B contained a total of 8987 nodes. Since the semi-minor diameter of the elliptical contact produced at a contact load of 175.8N in the experiments would have been 9.61×10^{-3} cm. according to Hertz's theory, $n' = b/\Delta y = 9.61 \times 10^{-3} \text{ cm.} / 3.34 \times 10^{-3} \text{ cm.} = 2.88$ for grid B, as mentioned previously.

II.5.2 Results for the Contact in Vacuum

Resistances obtained for the contact in vacuum using contact approximation techniques 1, 3 and 4 and grids A and B are tabulated in Table II.5.1. The tabulated results were obtained with the accommodative multigrid algorithm without using τ -extrapolation. The solution was assumed to have converged in each case when the 2-norm of the residuals on the finest grid was less than one-tenth of the 2-norm of the truncation errors on that grid, in other words when the inequality obtained by dividing the right-hand side of Equation (II.F.3) by ten was satisfied. The results would not have been significantly different if the right-hand side of Equation (II.F.3) had not been divided by ten, however, since a resistance of 21.84°C/W was obtained with grid A and contact approximation technique 1 when Equation (II.F.3) was used without modification. Since a resistance of 21.64°C/W was obtained with that grid and contact approximation technique when the norm of the residuals was further reduced by a factor of ten, as seen in Table II.5.1, the additional reduction in the residual norm changed the resistance by less than 1%. Similarly, a resistance of 21.49°C/W was obtained with grid A and contact approximation technique 3 when Equation (II.F.3) was used without modification as the convergence criterion. Since a resistance of 21.47°C/W was obtained with that grid and contact approximation technique when the residual norm was further reduced by an order of magnitude, as seen in Table II.5.1, the additional reduction in the residual norm changed the resistance by less than 0.1% in this example. These examples demonstrate the effectiveness of the convergence test described in Section II.F.1.

Table II.5.1: Resistances Obtained for the Contact in Vacuum With the Multigrid Model

Grid	Contact Approximation Technique	R'_{cv} °C/W	Error ¹ in R'_{cv} (%)
A	1	21.64	0.0
A	3	21.47	-0.8
A	4	21.49	-0.7
B	1	22.59 ²	4.4
B	3	21.50	-0.6
B	4	21.58	-0.3

1. The % error in R'_{cv} was calculated from the expression,

$$\% \text{ error in } R'_{cv} = 100 \frac{(R'_{cv} - \text{Benchmark Resistance})}{\text{Benchmark Resistance}}$$

Benchmark Resistance - 21.64°C/W.

2. This solution was obtained with the residual norm reduced to one-fifth of the norm of the truncation errors instead of one-tenth. Further reductions in the residual norm could not be achieved.

The resistances calculated with the different contact approximation techniques were very similar when the solutions were obtained with grid A, as seen in Table II.5.1. This result is as one would expect, of course, since the different contact approximation techniques should produce similar results when used with a grid which is highly refined around the contact. It is significant to note, however, that while the resistances obtained with the contact approximated by methods 3 and 4 changed very little when the model contact problem was solved with the coarser grid (i.e., grid B) instead of grid A, the resistance obtained with the contact approximated by method 1 increased by 4.4% when the coarser grid was used. Method 1 became less accurate when the grid was coarsened because the boundaries of the contact were not represented as accurately, of course.

The heat flux distributions obtained on the minor and major axes of the contact in all of the analyses performed with grid A were virtually identical to those obtained with the ellipsoidal finite-difference model and hence will not be shown. The heat flux distributions obtained on the minor and major axes of the contact in the analyses performed with grid B are compared to the corresponding distributions obtained with the ellipsoidal finite-difference model in Figures II.5.1 - II.5.3, however, although results are only shown for contact approximation techniques 3 and 4. As in the case of the comparisons of heat fluxes obtained with the synthesis and ellipsoidal finite-difference models, the total heat flow rate between the contact surfaces is held constant (at 1.862 W) in the comparisons shown in Figures II.5.1 - II.5.3. Also, since the contact extended only 38%

Figure II.5.1 Heat Fluxes on the Minor Axis of the Contact in Vacuum - Multigrid Results Obtained With Grid B and Contact Approximation Techniques 3 and 4

Figure II.5.2 Heat Fluxes on the Major Axis of the Contact in Vacuum - Multigrid Results Obtained With Grid B and Contact Approximation Technique 3.

Figure II.5.3 Heat Fluxes on the Major Axis of the Contact in Vacuum - Multigrid Results Obtained With Grid B and Contact Approximation Technique 4.

of the way across the surface element for the fourth lateral node on the minor axis in the analyses performed with grid B, the heat flows through that surface element were averaged over 38% of its area to obtain the heat fluxes shown plotted at $y=0.009$ cm. in Figure II.5.1. That coordinate is located 19% (i.e., $38\% / 2$) of the way across the surface element for the fourth lateral node and hence lies at the center of the contact segment on that element.

The heat flux distribution on the minor axis of the contact was predicted more accurately with contact approximation technique 4 than with method 3, as seen in Figure II.5.1. The heat flux at the third lateral node (i.e., at $y=0.0067$ cm.) was overestimated with the latter method, as could have been predicted from the results that were obtained in Appendix II.E with the corresponding technique for approximating a line contact. A line contact which extends partway across the surface element for the n th lateral node and has a contact conductance h_c prescribed over its area is approximated by prescribing h_c over the surface elements for lateral nodes $j = 1, \dots, n-1$ and h_n (defined by Equation (II.2.58)) at lateral node n in the technique just referred to. Results given in Appendix II.E for isothermal contacts (i.e., for contacts with $h_c = \infty$) show that the heat flux at lateral node $n-1$ (i.e., at the third lateral node in this example) is always overestimated with this technique.

The deficiency of contact approximation technique 3 which caused the heat flux at the third lateral node on the minor axis to be overestimated also produced the heat flux overpredictions seen in

Figure II.5.2 at the fourth, third and second to last nodes on the contact's major axis. The boundary of the elliptical contact extended partway across the surface element for the second lateral node at each of the axial nodes just mentioned, whose x-coordinates are 0.132 cm., 0.139 cm. and 0.146 cm. respectively, so the (n-1)st node, where the aforementioned deficiency of contact approximation technique 3 would have caused the heat flux to be overestimated, was the first lateral node (i.e., the node on the contact's major axis) in each case.

The heat flux distribution on the contact's major axis was predicted relatively accurately in the analysis performed with the contact approximated by method 4, as seen in Figure II.5.3.

A resistance of 22.40 °C/W was obtained for the contact in vacuum when the accommodative multigrid algorithm was combined with τ -extrapolation to solve the model contact problem on grid A with contact approximation technique 1. The norm of the residuals on the finest grid was reduced until it was less than one-tenth of the norm of the truncation errors on that grid to obtain the aforementioned result. The resistance of 22.40 °C/W differs from the benchmark resistance of 21.64 °C/W by 3.5% and is less accurate than the resistance of 21.64 °C/W that was obtained with the same grid and contact approximation technique without using τ -extrapolation. Since analyses performed with other grid/contact approximation combinations gave the same result (i.e., that resistances obtained with τ -extrapolation are less accurate than those obtained without it), it appears that τ -extrapolation causes a decrease in accuracy in this application.

Brandt [127] states that τ -extrapolation will not provide an improvement in accuracy unless the exact solution is fairly smooth on the finest grid, however, so the failure of τ -extrapolation in this application can probably be attributed to the singularity which exists in the exact solution at the edge of the contact. It is possible that τ -extrapolation could be made to work better than it did in the analysis of the model contact problem by changing the averaging operator used in the fine-to-coarse grid residual transfers at nodes near the edge of the contact; for instance, but the author did not attempt to modify the algorithm to achieve that end since the results obtained without τ -extrapolation were considered to be very satisfactory.

A resistance of 21.80 °C/W was obtained for the contact in vacuum in an analysis performed using the fixed multigrid algorithm shown in Figure II.4.4 with $p=r=1$ and $q=8$. This result, which differs from the benchmark resistance of 21.64 °C/W by only 0.7%, was obtained with grid A and the contact approximated by method 1. The heat fluxes obtained on the major and minor axes with the fixed algorithm were essentially identical to those obtained with the accommodative algorithm in the analyses performed with grid A and hence will not be shown. The fixed algorithm was slightly slower than the accommodative algorithm, however, in that it required 131 seconds of CPU time to compute the resistance of the contact in vacuum with grid A compared to 113 seconds for the accommodative algorithm. The latter solution time applies to the analysis in which the contact was approximated with method 3 (on grid A) and the iterations were stopped when Equation (II.F.3) was satisfied - the solution times for the analyses in which the residual

norm was reduced by an additional order of magnitude are all tabulated in Table II.5.2. It should be noted that the solution times for the more accurate contact approximation techniques, methods 3 and 4, were lower than the solution times for method 1 and that the times required to obtain solutions on grids A and B with a given contact approximation technique approximately satisfied the $O(N_e)$ operations count order claimed for multigrid. The latter observation is easily verified by noting that the ratio of the number of nodes in grid A to the number of nodes in grid B was roughly 2.3 and that the analysis with grid A and contact approximation technique 3, for instance, took approximately 2.5 times as long to complete as the analysis with grid B and the same contact approximation technique. Although the operations count order for multigrid is not necessarily of $O(N_e)$ when the algorithm is applied to grids created by partial refinement, the solution times shown in Table II.5.2 exhibit approximately this dependence on N_e .

Table II.5.2: Solution Times for the Accommodative Multigrid Analyses - Contact in Vacuum Calculations

Grid	Contact Approximation Technique	CPU Time (Secs.)
A	1	177
A	3	134
A	4	132
B	1	88
B	3	54
B	4	53

II.5.3 Results for the Contact in Air

Resistances obtained for the contact in air using contact approximation techniques 1, 3 and 4 and grids A and B are tabulated in Table II.5.3. All of the analyses of the contact in air were performed with the property data listed in Table I.6.4. As in the case of the analyses of the contact in vacuum, the results shown in Table II.5.3 were obtained with the accommodative multigrid algorithm without using τ -extrapolation. As before, the norm of the residuals on the finest grid was reduced until it less than one-tenth of the norm of the truncation errors on that grid in each analysis. Since a resistance of 8.96°C/W was obtained with grid A and contact approximation technique 1 when Equation (II.F.3) was used without modification to judge when convergence had been achieved and a resistance of 8.89°C/W was obtained when the residual norm was further reduced by an order of magnitude, as seen in Table II.5.3, the contact resistances shown in Table II.5.3 would not have been appreciably different if Equation (II.F.3) had been used without modification as the convergence criterion, as noted previously for the contact in vacuum analyses.

The resistances obtained with the different grids and contact approximation techniques were all very similar, as seen in Table II.5.3. All of the resistances differed from the benchmark resistance of 9.12°C/W by no more than $(-)2.7\%$, although it is interesting to note that the resistances obtained with grid A were either equal to or only slightly different from the resistance of 8.87°C/W obtained in the synthesis analysis performed with schedule 2. It is possible, therefore,

Table II.5.3: Resistances Obtained For the Contact in Air With the Multigrid Model

Grid	Contact Approximation Technique	R'_{ca} (°C/W)	Error ¹ in R'_{ca} (%)	Q_p/Q_c	Error ¹ in Q_p/Q_c (%)
A	1	8.89	-2.5	0.309	-2.8
A	3	8.87	-2.7	0.312	-1.9
A	4	8.89	-2.5	0.312	-1.9
B	1	8.93	-2.1	0.298	-6.3
B	3	8.91	-2.3	0.317	-0.3
B	4	8.92	-2.2	0.316	-0.6

1. The % error in a particular multigrid result was calculated from the expression,

$$\% \text{ error in multigrid result} = 100 \frac{(\text{Multigrid result} - \text{Benchmark value})}{\text{Benchmark value}}$$

Benchmark values

$$R'_{ca} - 9.12^\circ \text{ C/W}$$

$$Q_p/Q_c - 0.318$$

that the error in the synthesis result for the resistance of the contact in air should be attributed to differences between the ellipsoidal finite-difference model and the finite-difference model of the semi-cylindrical contact specimen rather than to a failure of the synthesis method to solve the finite-difference equations for the latter model more accurately. The error in the synthesis result was very small whatever its cause, of course.

The most inaccurate result of the multigrid analyses performed with the different grids and contact approximation techniques was the value of Q_p/Q_c obtained in the analysis with grid B and contact approximation technique 1, as seen in Table II.5.3. Although the value of Q_p/Q_c obtained in that analysis was in error by (-)6.3%, the corresponding value of R_{ca} was relatively accurate and in close agreement with the resistances calculated in the other analyses.

The heat flux distributions obtained on the minor and major axes of the contact in the analyses performed with grid A were in almost perfect agreement with those provided by the ellipsoidal finite-difference model irregardless of the contact approximation technique used. The distributions obtained on the minor and major axes of the contact with contact approximation technique 1 (grid A) are compared to the corresponding distributions obtained with the ellipsoidal finite-difference model in Figures II.5.4 and II.5.5 respectively to demonstrate this. The input file and output listing for this analysis are included in Appendix II.G, where the analysis is performed as a sample problem. The heat flux distributions obtained with the other

Figure II.5.4 Heat Fluxes on the Minor Axis of the Contact in Air - Multigrid Results Obtained With Grid A and Contact Approximation Technique 1

Figure II.5.5 Heat Fluxes on the Major Axis of the Contact in Air - Multigrid Results Obtained With Grid A and Contact Approximation Technique 1

contact approximation techniques (on grid A) would be indistinguishable from those shown in Figures II.5.4 and II.5.5. As with all the other heat flux comparisons considered so far, the total heat flow rate between the contact surfaces is held constant (at 3.64 W) in the comparisons shown in those figures.

The heat flux distributions obtained on the minor axis of the contact in the analyses performed with grid B and contact approximation techniques 3 and 4 are compared to the distribution obtained with the ellipsoidal finite-difference model in Figure II.5.6. Heat fluxes outside of the contact were predicted quite accurately with both contact approximation techniques but the heat flux at the third lateral node within the contact was overestimated in the analysis performed with the contact approximated by method 3 for the same reason that it was overestimated with that method in the thermal analysis of the contact in vacuum. The heat fluxes calculated with the contact approximated by method 4 were quite accurate, as before.

The heat flux distributions obtained on the major axis of the contact in the analyses performed with grid B and contact approximation techniques 3 and 4 are compared to the distribution obtained with the ellipsoidal finite-difference model in Figures II.5.7 and II.5.8a respectively. The heat fluxes at the third and second to last nodes within the contact on its major axis were overpredicted slightly in the analysis performed with the contact approximated by method 3 for the same reason that the heat fluxes at those nodes were overestimated with that contact approximation technique in the analysis of the contact in vacuum. The heat fluxes at the remaining nodes within

Figure 11.5.6 Heat Fluxes on the Minor Axis of the Contact in Air - Multigrid Results Obtained With Grid 8 and Contact Approximation Techniques 3 and 4

Figure II.5.7 Heat Fluxes on the Major Axis of the Contact in Air - Multigrid Results Obtained With Grid B and Contact Approximation Technique 3

Figure II.5.8a Heat Fluxes on the Major Axis of the Contact in Air - Multigrid Results Obtained With Grid B and Contact Approximation Technique 4

and outside of the contact were predicted quite accurately with contact approximation technique 3, however, as seen in Figure II.5.7.

Although heat fluxes beyond the end of the contact were predicted very accurately with contact approximation technique 4, as seen in Figure II.5.8a the heat fluxes obtained within the contact with that technique were less accurate. The cause of the hump which occurred in the heat flux distribution at an x-coordinate of approximately 0.08 cm. is easily explained by referring to Figure II.5.8b which shows calculated values of the heat transfer coefficient h_{lu} , the surface temperature $T_c(x)$ (in dimensionless form) and the heat flux q'' at each node in the interval $0 \leq x \leq 0.12$ cm. ($= 0.78a$). The peak in h_{lu} which occurred at an x-coordinate of approximately 0.08 cm. caused surface temperatures to decrease and surface heat fluxes to increase at x-coordinates near to it. The results given in Table II.5.3 show that these variations had little effect on the accuracy of either the predicted contact resistance or the predicted value of Q_p/Q_c , however.

Values of 8.94 °C/W and 0.312 were obtained for R_{ca} and Q_p/Q_c respectively in an analysis performed using the fixed multigrid algorithm shown in Figure II.4.4 with $p=r=1$ and $q=8$. These results were obtained with grid A and contact approximation technique 1. The resistance of 8.94 °C/W differs from the benchmark value of 9.12 °C/W by 2.0% while the aforementioned value of Q_p/Q_c is in error by (-)1.9%. The heat fluxes obtained on the contact's minor and major axes with the fixed algorithm were virtually identical to those shown in Figures II.5.4 and II.5.5 respectively.

Figure II.5.8b Calculated Values of h'_{lu} , T_c and q'' at Nodes on the Major Axis of the Contact in Air - Results Obtained With Contact Approximation Technique 4 for the Interval $0 \leq x \leq 0.12$ cm.

About 8 seconds more CPU time was required to compute the resistance of the contact in air with grid A and a particular contact approximation technique than was required to compute the contact's resistance in vacuum with the same grid and contact approximation technique. Similarly, solutions obtained with grid B for the contact in air took approximately 4 seconds longer to compute than the corresponding contact in vacuum solutions. The solution times for all of the contact in vacuum analyses were given previously in Table II.5.2. The solutions for the contact in air took slightly longer to compute than the solutions for the contact in vacuum because the gap sizes and heat transfer coefficients at nodes on the contact surface had to be evaluated in the former case and not in the latter. Unlike most iterative procedures for solving finite-difference and finite-element equations, multigrid convergence rates (and hence solution times) are not strongly boundary condition dependent.

All of the analyses described in this chapter were performed in single-precision on Carleton University's Xerox Sigma 9 computer.

II.5.4 Numerical Studies With a Contact of Relatively Low Ellipticity

The accommodative multigrid algorithm was used to predict the resistance of a contact with an aspect ratio of only 3.27 to further assess the accuracy of the different contact approximation techniques. The elliptical contact was located on the bottom of the rectangular contact specimen shown in Figure II.4.7 in the analyses, which were performed with and without gas conduction. A contact conductance of

1136 W/cm² K was prescribed over the contact area in all of the analyses. The contact semi-minor diameter (b) and small contact radius (R_1) were assigned values of 0.00961 cm. and 0.9527 cm. respectively, which were the same values that those parameters had assumed in the analyses of the model contact problem, while the contact semi-major diameter (a) and large contact radius (R_2) were changed to 0.03142 cm. and 5.81 cm. respectively. The large contact radius was selected (using Hertz's theory of elastic contact) so that the contact surface radii would be compatible with the contact's aspect ratio (i.e., 3.27). The analyses were performed with the material property values listed in Table I.6.4.

The contact problem just described was analyzed with two grids which differed in the degree of refinement used in the y-z plane of the contact-centered coordinate system, as before. In one grid, designated grid C, 12 nodes were contained within the half-width of the contact on its minor axis and the edge of the contact was located midway between the twelfth and thirteenth lateral nodes, as with grid A described previously. In the other grid, designated grid D, the contact's semi-minor diameter extended only 38% of the way across the surface element for the fourth lateral node, as it had with grid B. The mesh increments used in the z-direction near the contact were approximately equal to those used in the y-direction in each case, as with grids A and B. Grids C and D both used 10 equally-spaced nodes within the semi-major diameter of the contact.

The coarsest (6°) grid employed in the multigrid analyses with grids C and D was the same as that used with grids A and B. A total of 10 grids was used in the multigrid algorithm with grid C, although grids 9 and 10 did not provide additional refinement in the x-direction. While the finest subdivisions used in the y and z directions in grid C were the same as those found in grid A, G^8 provided additional refinement in the x-direction in grid C but not in grid A so the finest subdivision used in the x-direction in the former grid was one-half of that used in the latter. With each node counted only once, the composite grid designated grid C contained a total of 13,984 nodes.

Grid D was obtained from grid C in the same manner that grid B was obtained from grid A, in other words by omitting grids 9 and 10 in the multigrid algorithm. Grid D contained 7,332 nodes.

Resistances obtained for the contact in vacuum with contact approximation techniques 1, 3 and 4 and grids C and D are tabulated in Table II.5.4. The results shown were obtained without using τ -extrapolation. As before, the solution was assumed to have converged in each case when the norm of the residuals on the finest grid was less than one-tenth of the norm of the truncation errors on that grid.

The highest and lowest contact resistances obtained with grid C differed by less than 0.5%, as seen in Table II.5.4. Since the primary reason for performing the analyses with grids C and D was to determine whether contact approximation techniques 3 and 4 could replace high lateral refinement of the grid around the contact as a means of effectively representing the shape of the contact, the error in each

Table II.5.4: Resistances Obtained With the Multigrid Model for a Contact With $a/b=3.27$ in Vacuum

Grid	Contact Approximation Technique	R'_{cv} (°C/W)	Error ¹ in R'_{cv}
C	1	71.11	-
C	3	70.77	-
C	4	70.86	-
D	1	76.65 ²	8.2
D	3	71.16	0.5
D	4	71.62	1.1

1. The % error in R'_{cv} was calculated from the expression;

$$\% \text{ error in } R'_{cv} = 100 \frac{(R'_{cv} - \text{Benchmark Resistance})}{\text{Benchmark Resistance}}$$

Benchmark Resistance - 70.82°C/W

2. This solution was obtained with the residual norm reduced to one-fifth of the norm of the truncation errors instead of one-tenth. Further reductions in the residual norm could not be achieved.

resistance obtained with grid D was evaluated with respect to the average of the resistances obtained with grid C and contact approximation techniques 3 and 4. While the resulting average resistance of 70.82 °C/W would not have been exact, the author expects that further increases in just the lateral grid refinement used near the contact would have had little effect on the solution. More accurate resistances could undoubtedly have been obtained by using more nodes to model the length of the contact, of course, but it was the effect of lateral refinement on the solution that was of greatest interest, as noted previously. The resistance of 70.82 °C/W was probably quite accurate in any case since $R_{c\lambda} = 2R_\lambda = 70.42$ °C/W for this contact (R_λ is defined by Equation (I.6.2a)) when $\lambda = (t/2)^2$ and $t = 1.71$ cm. The resistances of 70.82 °C/W and 70.42 °C/W differ by less than 0.6%.

While the resistances obtained with grid C and contact approximation techniques 3 and 4 changed by only 0.5% and 1.1% respectively when they were recalculated on grid D, the corresponding change in the resistance obtained with contact approximation technique 1 was 7.8%. These changes are similar to the errors listed in Table II.5.4 because the resistance of 70.82 °C/W used to calculate these errors was less than 0.4% different from any of the resistances calculated with grid C. The superiority of contact approximation techniques 3 and 4 over method 1 is clearly demonstrated by the results shown in Table II.5.4, as is the applicability of methods 3 and 4 to contacts with aspect ratios as low as 3-4.

The heat fluxes obtained at each node on the contact's minor and major axes in the analyses performed with grid C and the different contact approximation techniques differed by less than 4% in all cases and were typically within 1% of each other. Significantly different heat fluxes were obtained when the different contact approximation techniques were used with grid D, however, as seen in Figures II.5.9 and II.5.10 where the heat fluxes obtained on the minor and major axes of the contact with grid C and contact approximation technique 3 are compared to heat fluxes calculated with grid D and contact approximation techniques 3 and 4. As seen in those figures, heat fluxes on the minor axis of the contact were predicted most accurately by contact approximation technique 4 while those on the major axis were predicted most accurately by method 3. The values of $h'_{\ell U}$ used at the nodes on the major axis with contact approximation technique 4 (and grid D) are shown in Figure II.5.10 to demonstrate, once again, that the errors in the heat fluxes obtained on the major axis with method 4 are correlated to the variations in $h'_{\ell U}$. The over-predicted heat fluxes seen at x-coordinates of 0.0165 cm. and 0.0265 cm. coincide with peaks in $h'_{\ell U}$, for instance.

The heat fluxes shown in Figures II.5.9 and II.5.10, as well as all of the heat fluxes that will be shown in forthcoming comparisons, were obtained with a source-to-sink temperature difference ($T_1 - T_0$) of 100 K.

Resistances obtained for the contact in air with contact approximation techniques 1, 3 and 4 and grids C and D are tabulated in

Figure II.5.9 Heat Fluxes on the Minor Axis of the Low Aspect Ratio Contact in Vacuum - Comparison of Multigrid Results Obtained With Grid C and Contact Approximation Technique 3 to Results Obtained With Grid D and Contact Approximation Techniques 3 and 4

Figure II.5.10 Heat Fluxes on the Major Axis of the Low Aspect Ratio Contact in Vacuum - Comparison of Results Obtained With Grid C and Contact Approximation Technique 3 to Results Obtained With Grid D and Contact Approximation Techniques 3 and 4

Table II.5.5. The results shown were obtained without using τ -extrapolation. The norm of the residuals on the finest grid was reduced until it was less than one-tenth of the truncation errors on that grid, as before.

Since the resistances obtained with grid C were all within 0.08% of 25.78 °C/W, this resistance was used to evaluate the errors in the resistances calculated with grid D. Similarly, the correct value of Q_p/Q_c was taken to be 0.275 because this value was obtained in two of the analyses performed with grid C and the value obtained in the third analysis (i.e., 0.273) was only slightly different.

The least accurate result obtained in the analyses with grid D was the value of Q_p/Q_c calculated with contact approximation technique 1, as seen in Table II.5.5. The contact resistance itself was predicted quite accurately in all of the analyses.

As in the contact in vacuum analyses, the heat fluxes obtained at each node on the contact's minor and major axes in the analyses performed with grid C and the different contact approximation techniques differed by less than 4% in all cases and were typically within 1% of each other. The heat fluxes obtained when the different contact approximation techniques were used with grid D differed significantly, however, as seen in Figures II.5.11 and II.5.12 where the heat fluxes obtained on the minor and major axes of the contact with grid C and contact approximation technique 3 are compared to heat fluxes obtained with grid D and contact approximations techniques 3 and 4. Although heat fluxes outside of the contact were predicted accurately with both

Table II.5.5: Resistances Obtained With the Multigrid Model
For a Contact With $a/b=3.27$ in Air

Grid	Contact Approximation Technique	R'_{ca} (°C/W)	Error ¹ in R'_{ca} %	Q_p/Q_c	Error ¹ in Q_p/Q_c %
C	1	25.78	-	0.273	-
C	3	25.77	-	0.275	-
C	4	25.80	-	0.275	-
D	1	25.97 ²	0.7	0.259	-5.8
D	3	25.84	0.2	0.282	-2.5
D	4	25.88	0.4	0.280	-1.8

1. The % error in a particular multigrid result was calculated from the expression,

$$\% \text{ error in multigrid result} = 100 \frac{(\text{Multigrid result} - \text{Benchmark value})}{\text{Benchmark value}}$$

Benchmark Values

$$R'_{ca} - 25.78^\circ\text{C/W}$$

$$Q_p/Q_c - 0.275$$

Figure II.5.11 Heat Fluxes on the Minor Axis of the Low Aspect Ratio Contact in Air - Comparison of Results Obtained With Grid C and Contact Approximation Technique 3 to Results Obtained With Grid D and Contact Approximation Techniques 3 and 4.

Figure II.5.12 Heat Fluxes on the Major Axis of the Low Aspect Ratio Contact in Air - Comparison of Results Obtained With Grid C and Contact Approximation Technique 3 to Results Obtained With Grid D and Contact Approximation Techniques 3 and 4.

contact approximation techniques, the errors in the heat fluxes obtained within the contact with grid D and contact approximation techniques 3 and 4 resemble those seen in Figures II.5.9 and II.5.10 for the contact in vacuum. As in the contact in vacuum analyses, heat fluxes obtained with contact approximation techniques 3 and 4 were most accurate on the major and minor axes of the contact respectively.

The results presented in this and the preceding sections of this chapter demonstrate the utility of the methods developed in Appendix II.E (and summarized in Section II.2.5.3) for approximating a line contact whose edge does not lie midway between two surface nodes. The main advantage that accrues from using those methods to approximate an elliptical contact in the manner shown in Figure II.2.9 is that the lateral grid refinement within and around the contact can be made more coarse than it could be with, for instance, method 1.

CHAPTER II.6

CONCLUSIONS

As seen in Chapter II.3, the synthesis method predicted the resistances of the contact in air and in vacuum with errors (relative to results obtained with the ellipsoidal finite-difference model) of only 2.7% and 8.3% respectively in the analyses performed with trial function schedule 2 and contact approximation technique 4. The method also predicted the heat flux distributions on the contact's major and minor axes quite accurately and predicted the value of Q_p/Q_c in the analysis of the contact in air with an error of only 2.5%. Since the heat flow through the contact was shown to be highly three-dimensional in air or in vacuum, it is encouraging that the synthesis method worked as well as it did with a fairly straightforward set of trial functions. The same trial functions were used in the thermal analyses of the contact in air and in vacuum.

In the author's opinion, the synthesis method was completely appropriate for the purpose for which it was chosen since it permitted fairly accurate 3-D results to be obtained inexpensively and easily from 2-D solutions to problems involving line contacts. The accuracy of the method depends on the choice of trial functions, however, as demonstrated in Chapter II.3. While no attempt is made in any of the discussions of the synthesis method found in the literature to identify any criteria that the trial functions should satisfy, the

author mentioned two such criteria in Section II.3.3.2, specifically that a given set of trial functions should generally be used at a minimum of two (consecutive) axial nodes and that no more than one trial function should be dropped and/or added when proceeding from one axial node to the next. These criteria were found to be important by performing analyses with a wide variety of trial function schedules.

Yasinsky derives an upper bound for the L_2 norm of the error in the synthesized temperature distribution (T') in reference [114]. The L_2 norm of the error in the approximate solution, designated $||T' - T||_2$, is defined by the expression

$$||T' - T||_2 = \sqrt{\iint_D (T' - T)^2 dD}$$

in which T is the exact temperature distribution. The upper bound to $||T' - T||_2$ given in reference [114] is of limited usefulness in practice because it does not provide any information about pointwise errors or the accuracy of the solution in different regions of D .

As seen in Chapter II.5, the resistances of the contact in air and in vacuum were predicted with errors of only 2.5% and 0.7% respectively in the multigrid analyses performed with the finer grid (i.e., grid A) and contact approximation technique 4. The multigrid method also predicted the heat flux distributions on the contact's major and minor axes very accurately and predicted the value of Q_p/Q_c in the analysis of the contact in air with an error of only 1.9%. Essentially the same results were obtained with the other contact approximation techniques (i.e., methods 1 and 3) in the analyses performed with grid A.

Multigrid analyses of the model contact problem were performed with a coarser grid, grid B, to further assess and compare the different contact approximation techniques. Grids A and B differed only in the degree of refinement used within and near the contact in the y-z plane of the contact-centered coordinate system. Although the accuracy of the solutions obtained with the contact approximated by method 1 decreased when the grid was coarsened, the solutions calculated with contact approximation techniques 3 and 4 remained largely unchanged. Although the heat fluxes obtained on the major and minor axes of the contact in air with contact approximation techniques 3 and 4 became slightly less accurate when the grid was coarsened, the author doubts that the observed loss in accuracy would be very important in most practical applications because local errors in the heat flux through the contact would be smoothed out as the heat flow diffused in the material on either side of the contact plane. It would usually be more important to obtain an accurate estimate of the heat flow rate, or equivalently, the average heat flux, through a contact than it would be to calculate local heat fluxes within it accurately.

Although contact approximation techniques 3 and 4 are based on methods developed to approximate a line contact whose edge does not lie midway between two surface nodes, the use of those methods in approximating an elliptical contact was at least partially justified in Section II.E.5. Since one of the main conclusions of the analysis described in Section II.E.5 was that contact approximation techniques 3 and 4 should be applicable to contacts with aspect ratios as low as 3 or 4, those approximation techniques and technique 1 were used to

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approximate an elliptical contact with an aspect ratio of only 3.27 in Section II.5.4. The multigrid model was used to calculate the resistance of that contact in air and in vacuum with two grids which, as before, differed only in the degree of refinement used within and near the contact in the y-z plane of the contact-centered coordinate system. The solutions obtained with the fine and coarse grids differed only slightly when the contact was approximated by method 3 or method 4, thereby proving the applicability of those methods to contacts with low aspect ratios. Although the resistance of the contact was predicted accurately in air and in vacuum with both contact approximation techniques, as was the value of Q_p/Q_c for the contact in air, heat fluxes were predicted more accurately with method 4 than with method 3 on the contact's minor axis and vice-versa on the major axis. Both contact approximation techniques yielded more accurate results than method 1 in the analyses performed with the coarser grid.

A worthwhile reduction in the number of nodes needed to model an elliptical contact accurately can be obtained by using either contact approximation technique 3 or technique 4 instead of method 1.* The model contact problem was solved as accurately with grid B as it was with grid A when methods 3 and 4 were used to approximate the contact even though the latter grid contained roughly 2.3 times the number of nodes used in the former, for instance. Although the methods developed by the author to approximate a line contact whose edge does

* Or method 2 in the case of a synthesis analysis.

not lie midway between two surface nodes could possibly be replaced by a method such as that of Motz [104], which uses the first few terms of a series solution for the temperature distribution around a singularity to modify the finite-difference equations used at nodes in that region, the methods developed by the author in Appendix II.E preserve the symmetry of the conductance matrix, which the method of Motz and similar methods do not, and do not require that any finite-difference equations be modified. Since a contact is approximated by prescribing appropriate boundary conditions over surface elements within and around the contact with contact approximation techniques 3 and 4, those methods are very easy to implement in either a finite-difference or a finite-element analysis. Although the analyses will not be described here, numerical experiments performed with finite-element discretizations constructed from four node quadrilaterals with bilinear shape functions were used to verify that contact approximation techniques 3 and 4 could be used with the finite-element method.

Contact approximation techniques 3 and 4 were derived to approximate an isothermal contact or a contact with a large uniform surface conductance (h_c) prescribed over its area. Although it would probably be possible to adapt one or both of those techniques to approximate contour areas formed between surfaces with low roughness (contact conductances obtained with the rough surface contact model described in Part I are non-uniform outside of the Hertzian contact area), contour areas formed between rough surfaces generally will have to be modelled with contact approximation technique 1 (or 2).

The multigrid analysis of the model contact problem served to demonstrate that the method has some remarkable capabilities, not the least of which is the facility with which highly-graded grids can be constructed. This

capability was very useful in the analysis of the model contact problem. Although multigrid has most often been used with finite-difference discretizations, its use with finite-element discretizations has been studied by Brandt [127], Nicolaides [134], Poling [142], Bank [143] and others. Other multigrid algorithm design considerations are dealt with in recent articles by Brandt [144], Mol [132, 133], Hemker [145] and McCormick [146].

As mentioned in Section II.5.2, the accommodative multigrid algorithm required only 113 seconds of CPU time to compute the resistance of the contact in vacuum with the grid containing 20,703 nodes (i.e., grid A) when Equation (II.F.3) was satisfied at convergence. The contact was approximated by method 3 in that analysis. The multigrid algorithm was used without modification in all of the analyses described in Chapter II.5 since multigrid convergence rates do not depend on the precise specification of problem-specific overrelaxation or acceleration parameters, unlike the convergence rates of most iterative methods (e.g., successive overrelaxation).

By way of comparison, the synthesis solutions obtained with trial function schedules 1 and 2 required approximately 200 seconds of CPU time to complete with the 397×40 grid, which contained 15,880 nodes. While longer than the solution times quoted for the multigrid algorithm, the CPU times for the synthesis method could undoubtedly have been reduced considerably by optimizing the synthesis program (described in Part IV) for the model contact problem. Also, the synthesis solutions were still obtained much more quickly than most of the ellipsoidal finite-difference solutions, since the CPU time

for the ellipsoidal finite-difference analysis performed with the 25 x 11 x 11 grid referred to in Table I.6.3 was 1110 seconds, for instance, as already noted in Appendix II.A.

All of the synthesis and multigrid solutions to the model contact problem were obtained without using more than the roughly 70,000 words of memory available in the high-speed core of the Xerox Sigma 9 computer on which the analyses were performed. The ellipsoidal finite-difference model used out-of-core storage devices extensively, however, as noted in Appendix II.A. A factor contributing to the large memory allocation needed to perform the ellipsoidal finite-difference analyses was that those analyses had to be performed in double precision.

Although the multigrid method has the advantage of being less approximate than the synthesis method, it also has some disadvantages relative to the latter method. First, the synthesis method can be applied to much larger problems than the multigrid method because there is no direct connection between model size (i.e., number of nodes) and memory requirement for the former method whereas there is for the latter. There is no direct connection between model size and memory requirement for the synthesis method because while most of the additional memory required to obtain a 3-D synthesis solution instead of a 2-D finite-difference solution is needed to store the trial

functions, the number of such functions used in a particular analysis is not directly related to the model size. In the case of the multigrid method, however, at least two memory spaces are required for each node on each grid used in the algorithm because the function value and right-hand side "residual" at each node must be stored.

The multigrid method is also at a disadvantage in applications where it is difficult to avoid using widely different mesh increments in the different coordinate directions (unless partial refinement is used). The reason for this is that there is, at present, no relaxation scheme or other error smoothing method that does not lose effectiveness when applied to grids which have different mesh increments in the different coordinate directions (to the author's knowledge)*. As mentioned in Section II.4.2.1, the best smoothing rate is obtained for a second order symmetric elliptic equation when $\Delta x = \Delta y = \Delta z$. Since Δx , Δy and Δz were 0.4233 cm., 0.4275 cm. and 0.4233 cm. respectively on G^0 in the multigrid analyses of the model contact problem, the multigrid algorithm operated almost optimally in those analyses because the mesh increments used in the different coordinate directions on the remaining grids (i.e., G^1 , G^2 , etc.) were effectively equal (except on the grids created by partial refinement, where the lack of equality did not matter for the reasons discussed in Section II.4.2.3). To assess the effect of using values of Δx , Δy and Δz which are not approximately equal, the multigrid analysis performed with grid A and contact approximation technique 1 was repeated with the height (i.e., the z-dimension) of the rectangular contact specimen doubled so that Δz_0 was 0.8466 cm. instead of 0.4233 cm. The values of Δz_1 , Δz_2 , etc.

* A possible exception is Stone's Strongly Implicit Algorithm [153].

would also have been doubled by this change. The CPU time required to obtain the solution with the contact specimen height doubled was approximately three times what it had been previously, proving the importance of using roughly equal mesh increments in the different coordinate directions.

There are applications in which it is either impractical or impossible to use roughly equal mesh increments in the different coordinate directions. It is impossible to construct a three-dimensional cylindrical (i.e., r - θ - z) grid in which all of the finite-difference control volumes are approximately cubic in shape unless the object being modelled has an outer/inner radius ratio that is close to unity, for example. Although McCarthy and Reyhner [130] formulated a three-dimensional model of transonic flow around an axisymmetric engine intake at a non-zero angle of attack in the cylindrical coordinate system, they used the same number of azimuthal nodes on all of the grids employed in the multigrid algorithm in order to obtain satisfactory convergence rates. Since each grid (past the first) was finer than its predecessor in the r - z plane only, McCarthy and Reyhner obtained satisfactory convergence rates by using partial refinement.

Although there are a large number of error smoothing schemes and algorithmic design possibilities that can be used to apply multigrid successfully to a given problem, those smoothing schemes and algorithm designs are described in the references mentioned previously and hence will not be discussed here. It should be recognized, however, that even a sub-optimal multigrid algorithm would often be a better solution

procedure than any of the alternatives, in particular in three-dimensional, non-linear analyses. Numerical experiments performed by Brandt [126] and others have shown that non-linear problems can often be solved almost as rapidly as linear ones with multigrid.

Multigrid was only applied to the model contact problem in this study. The finite-difference/synthesis model was developed further to permit it to solve non-linear and/or transient problems, however, as discussed in Part III of the thesis where it is used to perform simplified analyses of the accident scenario in which a pressure tube in a CANDU reactor overheats and sags onto the calandria tube which encloses it during a loss of coolant accident (LOCA) combined with a loss of emergency core coolant (LOECC).

APPENDICES
(PART II)

APPENDIX II.A

EQUATION SOLVING LIMITATIONS IN THREE-DIMENSIONAL FINITE-DIFFERENCE AND FINITE-ELEMENT ANALYSES

The purpose of this appendix is to illustrate some of the problems that arise when a direct equation solving algorithm, specifically Gaussian elimination or some variant thereof, is used to solve the simultaneous equations obtained in a three-dimensional finite-difference or finite-element analysis. It will be shown that the most commonly-used direct equation solving routines would be impractical to apply in anything but a fairly small three-dimensional analysis.

Most of the finite-element and finite-difference programs that use direct equation solving routines (this would include the vast majority of finite-element programs) employ "band solvers", that is routines which apply Gaussian elimination to banded systems of equations. Although a banded finite-difference or finite-element coefficient matrix generally contains a large proportion of zero entries within the band, in particular in a three-dimensional analysis, all of the entries contained within the bandwidth of the matrix have to be stored because they generally become non-zero during the elimination process even if they are zero initially. Bandwidths of coefficient matrices for three-dimensional finite-difference and finite-element analyses can easily become large enough to preclude storing the coefficient matrix (in banded form) in the high speed core memory of even a large computer, however, since to store the banded

coefficient matrix for a three-dimensional finite-difference grid with 30 grid lines in each of the x, y, and z directions would require $(30 \times 30 + 1) \times 30^3 = 24.44 \times 10^6$ memory spaces (i.e., a large amount even for modern computers) if the coefficient matrix was symmetric and almost twice the figure just mentioned if it wasn't, for example. The memory requirement just mentioned was obtained by multiplying the semi-bandwidth of the coefficient matrix, which would be $(30 \times 30 + 1) = 901$ for nodes numbered lexicographically (i.e., with the nodes i, j, k located at x_i, y_j, z_k numbered consecutively with, for instance, the indices i, j, k varying from most to least rapidly in that order), by the total number of nodes (i.e., $30^3 = 27,000$). It can be shown in the same manner that it would not be possible to store the banded coefficient matrix for even a $10 \times 10 \times 10$ finite-difference grid in the roughly 70,000 word usable core memory of Carleton University's Xerox Sigma 9 computer, on which all of the analyses described in this thesis were performed. Note that finite-difference equations obtained using the second-order accurate seven-point "star" difference approximation to the Laplacian operator contain only seven non-zero entries so, of the 901 entries stored for each equation in the $30 \times 30 \times 30$ finite-difference grid mentioned previously, only four (because only the semi-band is stored if the matrix is symmetric) of those entries would be non-zero.

A band solver, written by Wilson, Bathe and Doherty (see reference [96]) for use in finite-element programs,* was used to solve the

* This equation solver is virtually identical to that used in the state-of-the-art commercial finite-element program ADINA [110].

simultaneous equations for the ellipsoidal finite-difference model described in Part I. Although Wilson et al's routine permits very large systems of equations to be solved by partitioning the coefficient matrix and right-hand side vector into blocks and storing only one block at a time in the high speed core (the remaining blocks are stored in scratch files), the routine exhibited the rapid increase in computation time with increasing problem size that is characteristic of such solvers. The computation time for such a solver, while only directly proportional to the total number of unknowns, is, however, proportional to the square of the bandwidth (or semi-bandwidth) so computation times for three-dimensional problems, which have characteristically large bandwidths, can be prohibitively high. To illustrate this point, the total computation (CPU) time for the analysis with the 24x4x4 grid* referred to in Table I.6.3, which had a total of 396 nodes, was approximately 20 seconds while that for the 24x11x11 grid, which had 2260 nodes, was 1100 seconds. Since the ratio of the number of nodes in the 24x11x11 grid to the number of nodes in the 24x4x4 grid was roughly 5.7 and the ratio of the corresponding computation times was 55, the computation time per mesh point for the large grid was almost an order of magnitude greater than that for the small grid. The quoted times include the time required to calculate the finite-difference coefficient matrix and right-hand side vector but this time would have varied in direct proportion to the number of nodes, and, for the 24x11x11 grid at least, would have been a relatively small part of the total.

* The numbers given refer to the number of subdivisions used in each direction so the number of grid lines would have been 25x5x5.

As mentioned previously, the equation solver to which the computation times just mentioned apply uses out-of-core storage devices to permit the solution of very large systems of equations. Systems of equations that are small enough to be solved completely in-core can be solved more quickly with much simpler routines, but it is still relatively easy to show that the equations obtained from finite-element and finite-difference discretizations of three-dimensional problems will be expensive to solve using direct methods even if the solution is obtained completely in-core. To illustrate this point, Carleton's Xerox Sigma 9 computer required approximately 74 seconds to solve the finite-difference equations for a $9 \times 9 \times 9$ grid* superimposed on a cubic block using an efficient variable bandwidth Cholesky reduction routine given by Jennings in reference [11]. Using the fact that the time required to compute the solution to a banded system of equations is proportion to $N_e M_b^2$, where N_e is the total number of equations and M_b is the bandwidth (or half bandwidth), it is easily estimated that the finite-difference equations for the $30 \times 30 \times 30$ grid mentioned previously would take approximately 92 hours to solve with Jennings's routine even if they could be solved without using out-of-core storage devices. This simple example illustrates why it is generally believed that all but very small three-dimensional finite-element and finite-difference analyses are prohibitively expensive to perform.

It was assumed in the examples just described that the solution to the finite-difference or finite-element equations would be obtained using

* This grid which contained only 729 nodes, was the largest that could be handled in-core.

a banded equation solver, that is a routine which essentially applies Gaussian elimination to equations with banded coefficient matrices. In practice, finite-element equations are often solved using the relatively complex frontal algorithm [103], which reduces the finite-element equations (using Gaussian elimination) as soon as they are formed and so eliminates the need to store the entire unreduced coefficient matrix. Provided the high speed memory required during the reduction phase of the solution doesn't exceed the available high speed storage capacity, a frontal solver is at least as efficient as, and usually more efficient than a band solver. According to Bathe [103], the efficiency of the algorithm is drastically reduced if that proviso is not satisfied, however. The author expects that, as in the case of a band algorithm, the frontal algorithm would be expensive to apply to all but the smallest three-dimensional problems.

APPENDIX II.B

WEAK GALERKIN AND VARIATIONAL STATEMENTS FOR HEAT CONDUCTION

Consider a linear heat conduction problem governed by the heat conduction equation,

$$\nabla \cdot k_S \nabla T + q_v = 0 \quad \text{in } D \quad (\text{II.B.1})$$

and subject to the boundary condition,

$$k_S \nabla T \cdot \hat{n} = h_f (T_\infty - T) - q'' \quad \text{on } S_b, \quad (\text{II.B.2})$$

where q_v is the heat generation rate in D and Equation (II.B.2) is a generalized boundary condition which reduces to a boundary condition of the second or third kind if $h_f = 0$ or $q'' = 0$ respectively. The variational functional which is minimized when Equations (II.B.1) and (II.B.2) are satisfied is [120],

$$\pi(T) = \int_D (\nabla T \cdot k_S \nabla T - 2T q_v) dD + \int_{S_b} (h_f (T - T_\infty)^2 + 2q'' T) dS_b \quad (\text{II.B.3})$$

Also, if an approximate solution of the form $T' = \sum_{i=1}^N c_i T_i$ is proposed, the Galerkin orthogonality condition for the i th trial function is,

$$\int_D T_i (\nabla \cdot k_S \nabla T' + q_v) dD = 0 \quad (\text{II.B.4})$$

Equation (II.B.4) can be integrated by parts using Green's first identity [120] to obtain,

$$\begin{aligned} \int_D \int_D \int_D T_i (\nabla \cdot k_s \nabla T' + q_v) dD = & - \int_D \int_D (\nabla T_i \cdot k_s \nabla T' - T_i q_v) dD \\ & + \int_{S_b} T_i k_s \nabla T' \cdot \hat{n} dS_b = 0 \end{aligned} \quad (II.B.5)$$

Introducing Equation (II.B.2) into the surface integral on the right-hand side of Equation (II.B.5) provides the "weak Galerkin" statement,

$$\int_D \int_D (\nabla T_i \cdot k_s \nabla T' - T_i q_v) dD = \int_{S_b} T_i (h_f(T_\infty - T) - q'') dS_b \quad (II.B.6)$$

Equation (II.B.6) is preferred over Equation (II.B.4) because (1) the differentiability requirements for the approximating functions (T_i) are lower (i.e., only first order instead of second order) and (2) Equation (II.B.6) incorporates the boundary conditions directly. Also, if the approximation $T' = \sum_{i=1}^V C_i T_i$ is substituted into Equation (II.B.3) $\pi(T')$ is stationary (i.e., $\partial \pi / \partial T' = 0$) when $\frac{\partial \pi}{\partial C_j} = 0$; $j=1,2,\dots,V$. It can be shown that the expression that is obtained by requiring that $\frac{\partial \pi}{\partial C_i} = 0$ is the same as Equation (II.B.6) [116], thus proving that the Galerkin and Ritz methods (Equation (II.B.3) is minimized in the latter method) are equivalent when applied to a linear heat conduction problem.

APPENDIX II.C

DISCONTINUOUS SYNTHESIS FORMULATIONS

Yasinsky's formulation of the discontinuous synthesis method for heat transfer problems was obtained by modifying a method that had been developed previously to solve the neutron group diffusion equations in reactor cores [114]. While only Yasinsky proposed equations which applied specifically to heat transfer, several other investigators derived competing discontinuous synthesis procedures for the neutron diffusion problem [112]. Those formulations were, for the most part, much more complicated than their continuous synthesis counterparts whereas, in contrast, the author obtained the governing equations for discontinuous synthesis very easily from the corresponding continuous synthesis equations. The author was able to accomplish the transition from continuous to discontinuous synthesis relatively easily because: (a) the trial vectors used to approximate the i, j, k (i.e., r, θ, z) temperature distribution were assumed from the outset to be functions of all three coordinates (i.e., i, j, k or r, θ, z), as seen in equation (II.2.9), and (b) the trial and weighting functions were considered to be discrete, rather than continuous, functions from the beginning. Most derivations of the discontinuous synthesis method eventually discretize at least the mixing functions, for reasons that will be discussed briefly later, but this is generally not done at an early stage in the formulation

of the method. If all of the variables are discretized as early as possible in the derivation, the mathematics which follows is simplified because integrations and differentiations are replaced by algebraic operations which, in turn, can be described very economically using matrices. In addition, any N_e -dimensional real vector (i.e., a vector belonging to the real Euclidean space, E^{N_e}) can be classified as an "admissible" trial vector (T_i) in the discrete method of weighted residuals because the vector of nodal temperatures, T , also belongs to E^{N_e} (i.e., $T, T_i \in E^{N_e}$)*, whereas the class of admissible trial functions in a "continuous" (i.e., not discrete) weighted-residual or variational formulation includes only those functions whose derivatives, up to a certain order, are continuous. The "weak Galerkin" and standard variational statements for a heat conduction problem (given in Appendix II.B), for example, admit only those trial functions whose first derivatives are continuous, at least in a piecewise sense [120], because the highest order derivatives appearing in the domain integrals in those statements are first derivatives. The trial functions are also required to satisfy boundary conditions of the first kind identically, but a similar restriction is imposed on the trial vectors in the discrete formulation, as previously noted. The requirement that a function have

* If the values of any of the elements of T are prescribed, those values have to be satisfied identically by the T_i . This requirement restricts the class of admissible T_i in a manner that is very inconvenient, so boundary conditions which require that certain elements of T be fixed in value (i.e., boundary conditions of the first kind) should be replaced by boundary conditions of the third kind, as mentioned previously.

at least piecewise continuous first derivatives in order to be classified as admissible in either the weak Galerkin or variational formulation is equivalent to requiring that the function itself be continuous within the solution domain. This restriction causes problems in the non-discrete formulation of the discontinuous synthesis method if the trial functions are assumed to be truly axially discontinuous because the functions are inadmissible in the weak Galerkin and variational statements in that case. Previous derivations of the discontinuous synthesis method, for instance Yasinsky's [114] and Henry's [121], have resolved this dilemma by discarding the standard variational functional in favor of an extended variational principle, derived by Selengut [122], which can be modified to accept discontinuous trial functions. Both the derivation of this principle and its application to discontinuous synthesis are quite complicated and the reader is referred to the references mentioned previously for details. The essential point being made here is that the discrete formulation of the discontinuous synthesis method offers advantages over the continuous formulation because discrete functions are, by their very nature, spatially discontinuous.

It was mentioned previously that the governing equations for the discontinuous synthesis method were obtained in an almost trivial manner from the corresponding equations for continuous synthesis because the trial vectors were defined as functions of i , j , and k (i.e., r , θ and z) at an early stage in the derivation, as seen in equation (II.2.16). While we have seen that each N_e -dimensional trial vector used in the synthesis technique (either the continuous or discontinuous version) is only non-zero at a particular value of k , this is exactly what is called for in

discontinuous synthesis.

If the discontinuous synthesis method is not derived using discrete functions, the synthesized r, θ, z temperature distribution, $T'(r, \theta, z)$ must be written as [115],

$$T'(r, \theta, z) = \sum_{n=1}^{N_z} T_n^z(r, \theta) Z_n(z) \quad (\text{II.C.1})$$

where the $T_n^z(r, \theta)$ are known r - θ trial functions which are "axially discontinuous" (i.e., used at some values of z but not at others) and N_z is the number of trial functions used at a particular axial location. Equation (II.C.1), although customarily used to define the synthesized temperature (or neutron flux) distribution in the discontinuous synthesis method, is not particularly convenient because the trial functions, while ostensibly "axially discontinuous", are not written explicitly as functions of z . The discontinuous synthesis temperature approximation is, in fact, more meaningful if it is written as,

$$T'(r, \theta, z) = \sum_{i=1}^V C_i T_i(r, \theta, z) \quad (\text{II.C.2})$$

where the $T_i(r, \theta, z)$ are known functions of r, θ and z , the C_i are unknown constants, and V is the total number of trial functions used in the approximation. Equation (II.C.2) is the continuous analog of equation (II.2.9), which applied in the discrete case. Carrying the analogy with equation (II.2.9) one step further, each C_i in equation (II.C.2) will be associated with the temperature synthesis over only a small portion of the z -axis, say for $z_k - \frac{\Delta z}{2} \leq z \leq z_k + \frac{\Delta z}{2}$, if the corresponding T_i is non-zero

only in that range of z . If each T_i is defined to be non-zero over only a small part of the z -axis, the C_i in equation (II.C.2) are the $Z_n(z)$ in equation (II.C.1) defined in discrete form. The three-dimensional trial functions (T_i) in the discontinuous synthesis method are, therefore, usually obtained (e.g., see [114, 121]) by multiplying the $T_n(r, \theta)$ by axial "step" functions which have a value of unity for $z_k - \frac{\Delta z}{2} \leq z \leq z_k + \frac{\Delta z}{2}$ and are zero elsewhere. If NZ step functions covering the entire axial region of interest are used, a total of $V = \sum_{k=1}^{NZ} N_k$ three-dimensional trial functions is obtained. Formally, the synthesized temperature distribution is written in this case as [115],

$$T(r, \theta, z) = \sum_{k=1}^{NZ} \sum_{n=1}^{N_k} T_n^k(r, \theta) S_k(z) Z_n(k) \quad (\text{II.C.3})$$

where the $S_k(z)$ are step functions defined by,

$$\begin{aligned} S_k(z) &= 1 \text{ for } z_k - \frac{\Delta z}{2} \leq z \leq z_k + \frac{\Delta z}{2} \\ &= 0 \text{ otherwise} \end{aligned} \quad (\text{II.C.4})$$

and the $T_n^k(r, \theta)$ are the trial functions used at a particular value of k .

Comparing equation (II.C.3) to equation (II.C.2), we see that

$$T_i(r, \theta, z) = T_n^k(r, \theta) S_k(z), \text{ where } i=1, V; V = \sum_{k=1}^{NZ} N_k. \text{ Because the trial}$$

functions given by equation (II.C.3) are truly axially discontinuous,

they cannot be used to obtain a weighted-residual or variational solution

directly, as discussed previously. They can be used if certain constraints

or "side conditions" are enforced explicitly in the formulation, however.

In the case of discontinuous synthesis, the constraints are that

the temperature and the axial heat flux component be continuous, at least in an average sense, across internal "surfaces" separating regions, in which different sets of trial functions are used.* Selengut's extended variational principle embodied these constraints but it was necessary to approximate not only the temperature distribution, but also the axial heat flux component, as linear combinations of the trial functions [115]. This requirement complicated the discontinuous synthesis derivation considerably. Yasinsky approximated the axial heat fluxes between nodes k and $k-1$ on the z -axis by linearly combining the trial functions assigned to node $k-1$ and it is for that reason that his formulation of discontinuous synthesis gives results which depend on the order in which the nodes are visited. While the axial heat fluxes between nodes k and $k-1$ are approximated by linearly combining the columns of F_{k-1} when node $k-1$ is visited before node k , they are obtained from F_k when node k is visited before $k-1$.

It is possible to derive a discontinuous synthesis formulation which uses continuous trial functions relatively simply if each $T_i(r, \theta, z)$

* If a volume integral is evaluated over a surface (i.e., as $\epsilon \rightarrow 0$ in the integral

$$\int_{z_1-\epsilon}^{z_1+\epsilon} \int_{y_1}^{y_2} \int_{x_1}^{x_2} f(x,y,z) \, dx \, dy \, dz$$

the integral should vanish because a surface has no volume. The constraints mentioned above guarantee that the domain integrals in the weighted-residual and variational statements vanish over the internal "surfaces" mentioned above.

used in equation (II.C.2) is defined in a manner that makes it axially continuous but, nevertheless, non-zero over only a small segment of the z -axis. Trial functions satisfying these requirements can be obtained by multiplying the two-dimensional trial functions (i.e., the $T_n(r, \theta)$) by the well-known "tent" (or "hat") functions, $\phi_k(z)$, proposed originally by Harrington [123] and defined by the relationships,

$$\begin{aligned} \phi_k(z) &= 1 - \frac{|z - z_k|}{\Delta z} \quad \text{for } |z - z_k| \leq \Delta z \\ &= 0 \text{ otherwise} \quad (k=1, NZ) \end{aligned} \quad (\text{II.C.5})$$

The $\phi_k(z)$ defined by equation (II.C.5) are overlapping tent-shaped functions with the apex of each tent (triangle) located at the corresponding z_k . Three of these functions are shown in Figure II.C.1.

Figure II.C.1: Tent Functions

The $T_i(r, \theta, z)$ required in equation (II.C.2) are defined formally by,

$$T_i(r, \theta, z) = T_n^k(r, \theta) \phi_k(z) \quad \text{where } i=1, V; V = \sum_{k=1}^{NZ} N_k; n=1, N_k; k=1, NZ \quad (\text{II.C.6})$$

Since the trial functions defined by equation (II.C.6) are axially continuous and have piecewise continuous first derivatives, they are admissible trial functions for either a weighted-residual or variational statement (for heat conduction). A system of V simultaneous equations defining the C_i in equation (II.C.2) (which are equivalent to the $Z_n(k)$ in chapter II.2) can be obtained by substituting each of the V trial functions (T_i) defined by equation II.C.6 into equation II.B.6 (with T' defined by equation II.C.2) and integrating the resulting expressions with respect to z . This procedure is straightforward and hence will not be described here.

APPENDIX II.D

PROOF OF THE UTILITY OF TRIAL VECTOR ORTHONORMALIZATION IN THE SYNTHESIS METHOD

The system of equations defining the blending vector Z in the Galerkin formulation of the synthesis method can be written in the form,

$$W^T KWZ = W^T Q \quad (II.D.1)$$

as shown in Chapter II.2. The synthesis method provides an approximate solution to the overdetermined system of equations,

$$KWZ = Q \quad (II.D.2)$$

or

$$AZ = Q \quad (II.D.3)$$

where $A = KW$. Since the approximate solution to this system of equations can be written as,

$$Z = (W^T KW)^{-1} W^T Q \quad (II.D.4)$$

it is useful to define a pseudo-inverse of A , designated A' , as

$$A' = (W^T KW)^{-1} W^T \quad (II.D.5)$$

since then equation (II.D.4) becomes

$$Z = A' Q \quad (II.D.6)$$

The pseudo-inverse A' is similar to the Moore-Penrose generalized inverse $A'' (= (A^T A)^{-1} A^T)$ defined for the least-squares approximation to equation (II.D.3) [106]. It is well known that the conditioning of the least-squares problem (i.e., the sensitivity of the solution to perturbations in A or Q) depends on the condition number $\|A\|_2 \|A''\|_2$ and, similarly, the conditioning of the Galerkin solution to equation (II.D.3) can be expected to depend on the condition number $\|A\|_2 \|A'\|_2$. This observation is easily proven for perturbations in Q . To this end, let δQ be the error in Q and let $\underline{Q} (= Q + \delta Q)$ be the resulting approximation to Q . If $\underline{Z}$ is the solution to the expression

$$W^T K W \underline{Z} = W^T \underline{Q} \quad (\text{II.D.7})$$

or

$$\underline{Z} = A' \underline{Q}, \quad (\text{II.D.8})$$

then for a linear system we can subtract equation (II.D.6) from equation (II.D.8) to obtain,

$$\underline{Z} - Z = A' (\underline{Q} - Q) \quad (\text{II.D.9})$$

or

$$\delta Z = A' \delta Q \quad (\text{II.D.10})$$

where $\delta Z (= \underline{Z} - Z)$ is the change in Z resulting from the perturbation (δQ) in Q . For consistent norms $\|\cdot\|$, equation (II.D.10) yields the inequality,

$$\|\delta Z\| \leq \|A'\| \|\delta Q\| \quad (\text{I.D.11})$$

and similarly,

$$||Z|| \geq \frac{||Q||}{||A||} \quad (\text{I.D.12})$$

follows from equation (II.D.3). Equations (II.D.11) and (II.D.12) can be combined to give the result

$$\frac{||\delta Z||}{||Z||} \leq ||A|| ||A'|| \frac{||\delta Q||}{||Q||} \quad (\text{I.D.13})$$

which indicates that the relative error in Q (i.e., $(||\delta Q||/||Q||)$) can be magnified by as much as a factor of $||A|| ||A'||$ in the solution. The condition number $||A|| ||A'||$ will be designated κ for convenience.

Equation (II.D.13) indicates that the relative error in Z is related to the relative error in Q by the condition number κ . To prove that an error in A could also be magnified by a factor as large as κ (or some function involving positive powers of κ) in the solution would probably be more difficult since a similar proof for the least-squares problem is very complex (see [106]), so the existence of such a dependence will be assumed rather than proven here. The task that remains, therefore, is to prove that the minimum value of κ is obtained when W has orthonormal columns.

It is assumed in the following proof that the conductance matrix (K) is positive-definite since, as discussed in Chapter I.4.1, it should be for finite-difference and finite-element approximations to the heat conduction equation. If K is positive-definite, it is always possible to find (using Cholesky decomposition) a lower triangular matrix L such that $K=LL^T$. With this result, the identity

$$A' = (W^T K W)^{-1} W^T$$

can be written as,

$$A' = (W^T L L^T W)^{-1} W^T L L^{-1} \quad (\text{II.D.14})$$

from which the inequality

$$\|A'\|_2 \leq \|(W^T L L^T W)^{-1} W^T L\|_2 \|L^{-1}\|_2 \quad (\text{II.D.15})$$

or equivalently,

$$\|A'\|_2^2 \leq \|(W^T L L^T W)^{-1} W^T L\|_2^2 \|L^{-1}\|_2^2 \quad (\text{II.D.16})$$

follows from the consistency property of the 2-norm.*

Another useful property of the 2-norm is that [106],

$$\|B^T B\|_2 = \|B B^T\|_2 = \|B\|_2^2 \quad (\text{II.D.17})$$

since this property allows us to rewrite the term $\|L^{-1}\|_2^2$ in equation (II.D.16) as,

$$\|L^{-1}\|_2^2 = \|L^{-1 T} L^{-1}\|_2 = \|(L L^T)^{-1}\|_2 = \|K^{-1}\|_2 \quad (\text{II.D.18})$$

Also, if $B = (W^T L L^T W)^{-1} W^T L$, then

$$\begin{aligned} \|B\|_2^2 &= \|(W^T L L^T W)^{-1} W^T L\|_2^2 \\ &= \|B B^T\|_2 \\ &= \|(W^T L L^T W)^{-1} W^T L L^T W (W^T L L^T W)^{-1 T}\|_2 \end{aligned}$$

* The 2-norm is used in this proof since it has several useful properties, as noted above.

$$= ||(W^T K W)^{-1^T}||_2$$

$$= ||(W^T K W)^{-1}||_2 \quad (\text{since } ||B^T||_2 = ||B||_2 \text{ [106]}) \quad (\text{II.D.19})$$

Substituting the results given by equations (II.D.18) and (II.D.19) into equation (II.D.16) we obtain,

$$||A'||_2^2 \leq ||(W^T K W)^{-1}||_2 ||K^{-1}||_2 \quad (\text{II.D.20})$$

Also, postmultiplying both sides of equation (II.D.5) by W gives,

$$A'W = (W^T K W)^{-1} (W^T W) \quad (\text{II.D.21})$$

so

$$A'W(W^T W)^{-1} = (W^T K W)^{-1} \quad (\text{II.D.22})$$

since $W^T W$ is a square invertible matrix. The inequality,

$$||(W^T K W)^{-1}||_2 \leq ||A'||_2 ||W||_2 ||(W^T W)^{-1}||_2 \quad (\text{I.D.23})$$

follows from equation (II.D.22) and the consistency property of the 2-norm, so combining equations (II.D.20) and (II.D.23) gives the result that,

$$||A'||_2^2 \leq ||A'||_2 ||W||_2 ||(W^T W)^{-1}||_2 ||K^{-1}||_2$$

or

$$||A'||_2 \leq ||W||_2 ||(W^T W)^{-1}||_2 ||K^{-1}||_2 \quad (\text{I.D.24})$$

Now $A = KW$ by definition, so

$$||A||_2 \leq ||K||_2 ||W||_2 \quad (\text{I.D.25})$$

Combining equations (II.D.24) and (II.D.25) yields the inequality,

$$||A||_2 ||A^T||_2 \leq ||W||_2^2 ||(W^T W)^{-1}||_2 ||K||_2 ||K^{-1}||_2$$

which can be rewritten in the form,

$$||A||_2 ||A^T||_2 \leq ||W^T W||_2 ||(W^T W)^{-1}||_2 ||K||_2 ||K^{-1}||_2 \quad (\text{I.D.26})$$

since $||W||_2^2 = ||W^T W||_2$. Defining the spectral condition numbers (i.e., condition numbers based on the 2-norm) of A , $W^T W$ and K as $\kappa_2(A)$, $\kappa_2(W^T W)$ and $\kappa_2(K)$ respectively where

$$\kappa_2(A) = ||A||_2 ||A^T||_2$$

$$\kappa_2(W^T W) = ||W^T W||_2 ||(W^T W)^{-1}||_2$$

and

$$\kappa_2(K) = ||K||_2 ||K^{-1}||_2,$$

allows us to rewrite Equation (II.D.26) as,

$$\kappa_2(A) \leq \kappa_2(W^T W) \kappa_2(K) \quad (\text{II.D.27})$$

This result shows that the conditioning of the synthesis solution will depend on the conditioning of the original system of finite-difference equations, $KT=Q$ (as measured by $\kappa_2(K)$) as well as on the condition number of $W^T W$ (i.e., $\kappa_2(W^T W)$). No matrix has a lower condition number than the identity matrix I , for which $\kappa_2=1$, and since $W^T W=I$ when W has orthonormal columns the utility of trial vector orthonormalization in the synthesis method is proven.

APPENDIX II.E

CONTACT APPROXIMATION

II.E.1 Introduction

Methods for properly representing the width of an isothermal line contact in a numerical heat transfer analysis are considered in this appendix. Methods for approximating a line contact with a surface conductance (h_c) prescribed over its area are also considered, only in less detail. The techniques developed to approximate an isothermal contact are compared by incorporating them into finite-difference analyses of the heat flow through a narrow isothermal line contact on a semi-cylinder with an isothermal diametral plane. The most accurate contact approximation technique is determined by comparing numerically predicted results for the resistance of the aforementioned contact in vacuum to the appropriate analytical solution, which is the solution for contact 6 in Table I.2.1. Numerical results for the heat fluxes within the contact and the surface temperatures around the contact are also compared to the corresponding analytical solutions. The latter solutions are derived in the next section of this appendix.

The contact approximation technique that is found to be best for approximating contacts in vacuum is demonstrated to be applicable to contacts in gases and to contacts such as those shown in Figure II.2.9 (i.e., line contacts used to approximate an ellipse) in the last two sections of the appendix.

II.E.2 Analytical Solutions

Since the exact solution for the resistance of contact 6 in Table I.2.1 will be used to assess the accuracy of the different contact approximation techniques, that solution is repeated here for convenience as,

$$R_{ct}^T = K(k')/2k_s K(k) \quad (I.E.1)$$

where $k = \sin \alpha$ and $k' = \sqrt{1-k^2}$. The parameter α is the contact half-angle shown in Table I.2.1, and K is the complete elliptic integral of the first kind. Equation (II.E.1), which was derived by Schneider [42] can be approximated with high accuracy by the equation,

$$R_{ct}^T = \ln (2(1+\cos \alpha)/\sin \alpha)/\pi k_s \quad (I.E.2)$$

when α is small. Equation (II.E.2), which was derived in an approximate analysis performed by Yovanovich and Coutanceau [42a], is accurate to within 0.24% when $\alpha \leq 45^\circ$ and 0.04% when $\alpha \leq 5^\circ$.

The appraisal of the different contact approximation techniques will be based not only on the accuracy with which they predict the resistance of contact 6 in Table I.2.1 but also on the accuracy with which they predict local contact heat fluxes. An expression for the heat flux distribution across the contact is not available in the published literature for the case of interest (to the author's knowledge) but can be easily derived from the results of the approximate analysis of Yovanovich and Coutanceau mentioned previously. In the first step of that analysis, the conformal transformation,

$$w = u + iv = \frac{2 \frac{R}{r} \sin \theta + i \left[\left(\frac{R}{r} \right)^2 - 1 \right]}{\left(\frac{R}{r} \right)^2 + 2 \frac{R}{r} \cos \theta + 1} \quad (\text{II.E.3})$$

is used to map the cylinder with diametrically opposed isothermal contacts shown in Figure II.E.1a onto the complex w -plane shown in Figure II.E.1b. The locations indicated by the capital letters (i.e., O, C, D, etc.) in Figure II.E.1a transform to the locations indicated by the small letters (i.e., o, c, d, etc.) in Figure II.E.1b.

Figure II.E.1a Diametrically Opposed Isothermal Contacts on a Cylinder

Figure II.E.1b Cylinder Mapped Onto the Complex w -Plane

The diameter $C'OC$ in Figure II.E.1a is isothermal for reasons of symmetry so the semi-circular arc $c'oc$ seen in Figure II.E.1b, which has a radius of unity, is also isothermal. Yovanovich and Coutanceau obtain an approximate expression for the thermal resistance between the line segment $e'e$ (of half-width b') and the semi-circular arc $c'oc$ in the complex w -plane by assuming that $b' \ll 1$ and hence that the semi-circular arc $c'oc$ approximates a semi-ellipse with focal points at e and e' and semi-major and semi-minor diameters both approximately equal to unity. Since the heat flow between the isothermal line segment $e'e$ and the isothermal semi-elliptical arc just mentioned is one-dimensional with respect to the η coordinate in the elliptic cylinder coordinate system, which has the confocal ellipses (η) and hyperbolas (ψ).

Figure II.E.2: Coordinate Surfaces of the Elliptic Cylinder Coordinate System

shown in Figure II.E.2 as its coordinate surfaces, the resistance between ee' and the semi-elliptical arc is easily calculated. The coordinates u and v are related to n and ψ by the expressions [40],

$$u = b' \cosh n \cos \psi \quad (\text{II.E.4})$$

$$v = b' \sinh n \sin \psi \quad (\text{II.E.5})$$

while the metric coefficients g_n , g_ψ and g_z (the z -axis is normal to the n - ψ plane) of the elliptic cylinder coordinate system are defined by the equations [79],

$$g_n = g_\psi = b'^2 (\cosh^2 n - \cos^2 \psi) \quad (\text{II.E.6})$$

and,

$$g_z = 1$$

The governing equation for heat flow which is one-dimensional with respect to the n coordinate can be obtained by substituting the metric coefficients for the elliptic cylinder coordinate system into Equation (I.4.18) (with $x_i = n$) to get,

$$\frac{d}{dn} \left[\frac{\sqrt{g}}{g_n} \frac{dT}{dn} \right] = 0 \quad (\text{II.E.7})$$

Integrating Equation (II.E.7) once with respect to n (with k_s assumed constant) gives,

$$\frac{dT}{dn} = C g_n / \sqrt{g} = C \quad (\text{II.E.8})$$

where C is a constant of integration. The heat flux distribution across the isothermal line segment $e'e$ can now be obtained easily by noting that normals to the upper surface of the "ellipse" at $n=0$ are parallel to and in the same direction as the positive v axis at all points on the surface.

Thus $\partial\eta/\partial v = 1/\sqrt{g_\eta}$ at $\eta=0$ and,

$$\begin{aligned}\left.\frac{\partial T}{\partial v}\right|_{\eta=0} &= \frac{1}{\sqrt{g_\eta}} \left.\frac{dT}{d\eta}\right|_{\eta=0} \\ &= \frac{C}{\sqrt{b'^2 (\cosh^2 \eta - \cos^2 \psi)}} \Big|_{\eta=0} \\ &= \frac{C}{\sqrt{b'^2 - u^2}} \quad (\text{II.E.9})\end{aligned}$$

since $\cosh(0) = 1$ and $b'^2 \cos^2 \psi = \frac{u^2}{\cosh^2 \eta} = u^2$ from Equation (II.E.4).

The constant of integration (C) can now be related to the total heat flow rate Q' through the line segment e'e (per unit length in the z-direction)

since,

$$\begin{aligned}Q' &= 2 \int_0^{b'} k_s \left.\frac{\partial T}{\partial v}\right|_{v=0} du \\ &= 2 k_s C \int_0^{b'} \frac{du}{\sqrt{b'^2 - u^2}} \\ &= 2 k_s C \left[\sin^{-1} \left(\frac{u}{b'} \right) \right]_0^{b'} \\ &= \pi k_s C\end{aligned}$$

Thus,

$$C = \frac{Q'}{\pi k_s}$$

and,

$$\frac{\partial T}{\partial v} = \frac{Q'}{\pi k_s \sqrt{b'^2 - u^2}} \quad (\text{II.E.10})$$

The heat flux distribution across one of the isothermal contacts on the cylinder shown in Figure II.E.1a can now be obtained from the

transformations,

$$u = \frac{2 \frac{R}{r} \sin \theta}{\left(\frac{R}{r}\right)^2 + 2 \frac{R}{r} \cos \theta + 1} \quad (\text{II.E.11})$$

$$v = \frac{\left(\frac{R}{r}\right)^2 - 1}{\left(\frac{R}{r}\right)^2 + 2 \frac{R}{r} \cos \theta + 1} \quad (\text{II.E.12})$$

and the expression,

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial v} \frac{\partial v}{\partial r} + \frac{\partial T}{\partial u} \frac{\partial u}{\partial r} \quad (\text{II.E.13})$$

where $\frac{\partial T}{\partial u} = 0$ within the contact, of course. For small angles α and θ , when $\sin \alpha \approx \alpha$, $\sin \theta \approx \theta$, $\cos \alpha \approx 1$ and $\cos \theta \approx 1$, we can write,

$$u = \frac{\theta}{2} \quad (\text{at } r=R) \quad (\text{II.E.14})$$

$$b' = \frac{\alpha}{2} \quad (\text{II.E.15})$$

and,

$$\begin{aligned} \frac{\partial v}{\partial r} &= \frac{-2 \left(\frac{R^2}{r^3}\right)}{\left(\frac{R}{r}\right)^2 + 2 \frac{R}{r} \cos \theta + 1} + \frac{\frac{2R}{r^2} \left(\left(\frac{R}{r}\right)^2 - 1\right) \left(\frac{R}{r} + 1\right)}{\left(\left(\frac{R}{r}\right)^2 + 2 \frac{R}{r} \cos \theta + 1\right)^2} \\ &= -\frac{1}{2R} \quad (\text{at } r=R) \quad (\text{II.E.16}) \end{aligned}$$

so,

$$\begin{aligned} \frac{\partial T}{\partial r} &= \frac{\partial T}{\partial v} \frac{\partial v}{\partial r} \\ &= -\frac{Q'}{\pi k_s R \sqrt{\alpha^2 - \theta^2}} \quad (\text{II.E.17}) \end{aligned}$$

within the contact. Setting $y = R\theta$ and $b = R\alpha$, we can rewrite Equation (II.E.17) as,

$$q''(y) = \frac{Q'}{\pi\sqrt{b^2 - y^2}} \quad (\text{II.E.18})$$

or as,

$$q''(y) = \frac{Q}{\pi\ell\sqrt{b^2 - y^2}} \quad (\text{II.E.19})$$

where $q'' (= -k_s \frac{\partial T}{\partial r})$ is the local heat flux within the contact, ℓ is the length of the cylinder and $Q(=Q'\ell)$ is the total heat flow rate through the contact.

The heat flux distribution defined by Equation (II.E.19) is quite commonly used to predict local values of the surface heat flux within a small isothermal contact on a relatively large body (e.g., see Mikic [33]) and is usually assumed to apply to contacts on surfaces with different shapes (e.g., cylindrical, flat) and even to circular contacts if the lateral coordinate y is replaced by the distance from the center of the contact, b by the contact radius (a), and the factor $Q/\pi\ell$ by $Q/2\pi a$.

The last analytical result that will be used in the forthcoming comparison of numerical and analytical solutions to the thermal problem for contact 6 in Table I.2.1 is the temperature distribution on the outer surface of the semi-cylinder in regions very close to the contact since it is assumed that a numerical model that cannot predict this distribution with reasonable fidelity will be unable to provide an accurate solution for the heat flow around a contact with a gas present, at least at high solid-to-gas thermal conductivity ratios. The temperature distribution adjacent to the contact in vacuum can be obtained very easily by

substituting Equations (II.E.11) and (II.E.15) into the equation,

$$\frac{T_0 - T}{T_0 - T_1} = \frac{\cosh^{-1} (u/b' \cos \psi)}{\cosh^{-1} (1/b' \cos \psi)} \quad (\text{II.E.20})$$

which defines the temperature distribution between the contact at temperature T_0 and the isothermal semi-elliptical arc (which approximates the semi-circular arc $c'oc$) at temperature T_1 in the complex w -plane shown in Figure II.E.16. Since $\psi=0$ when $v=0$ and $v=0$ when $r=R$ we have,

$$\frac{T_0 - T}{T_0 - T_1} = \frac{\cosh^{-1} (u/b')}{\cosh^{-1} (1/b')} \quad (\text{II.E.21})$$

as the equation that will provide the temperature distribution on the outer surface of the cylinder shown in Figure II.E.1a when the transformations defined by Equations (II.E.14) and (II.E.15) are substituted for u and b' . Equation (II.E.20) was derived by Yovanovich [40], who also showed that

$$T_0 - T_1 = \frac{Q'}{\pi k_s} \cosh^{-1} (1/b') \quad (\text{II.E.22})$$

Substituting Equation (II.E.22) into Equation (II.E.21) and setting $T_0 = 0$ in the resulting equation (for convenience), we obtain

$$T(u,0) = \frac{Q'}{\pi k_s} \cosh^{-1} (u/b') \quad (u \geq b') \quad (\text{II.E.23})$$

so the result,

$$T(R,\theta) = \frac{Q'}{\pi k_s} \cosh^{-1} (\theta/\alpha) \quad (\theta \geq \alpha) \quad (\text{II.E.24})$$

follows from Equations (II.E.14) and (II.E.15). The heat flow rate (Q') seen in Equations (II.E.17) and (II.E.23) is obtained by dividing

the difference between the isothermal contact and diametral plane temperatures by R_{ct}^T , of course, where R_{ct}^T is defined by Equation (II.E.1) (or by Equation (II.E.2) for small α).

II.E.3 Numerical Studies Without Gas Conduction

Techniques for approximating a line contact in vacuum are considered in this section of the appendix. The easiest way to approximate such a contact is to size the lateral subdivisions used within it so that its edge coincides with the boundary of a finite-difference "cell" or surface element, as shown in Figure II.E.3 for one-half of a contact.

Figure II.E.3: Contact Whose Edge Coincides With the Boundary of a Finite-Difference "Cell" (i.e., the cell for the 4th lateral node in this example).

The outermost boundary of the finite-difference cell for the n th lateral node is assumed to be located midway between lateral nodes $j=n$ and $j=n+1$ so,

$$b = (n-1/2) \Delta y \quad (\text{II.E.25})$$

if Δy is the lateral subdivision size shown in Figure II.E.3 and one-half of the contact spans n nodes widthwise. The resistance of the contact can be predicted accurately by simply applying the contact boundary condition at all of the surface nodes located within the width of the contact when the lateral subdivisions are sized in this manner, as proven by the comparison of numerically predicted and analytical resistances given in Table II.E.1 for an isothermal contact of half-angle α on a semi-cylinder with an isothermal diametral plane. The theoretical resistances shown in Table II.E.1 were calculated with Equation (II.E.2) (and hence are not exact) because the largest contact half-angle for which numerical results were obtained was only 0.578° and hence was small enough for Equation (II.E.2) to be used with negligible error.

The results shown in Table II.E.1 were obtained for the $r-\theta$ geometry of the semi-cylindrical contact specimen described in Section II.2.5.1 (i.e., with a semi-cylinder radius of 0.953 cm.). The significance of the largest value of α seen in Table II.E.1 (i.e., 1.009×10^{-2} radians = 0.578°) is that it corresponds to the angle subtended by the semi-minor diameter of the elliptical contact produced at a contact load of 175.8N. The semi-minor diameter of the contact at that load would have been 9.61×10^{-3} cm. according to Hertz's

Table II.E.1: Comparison of Numerical and Analytical Results Obtained in Analyses of the Heat Flow Through an Isothermal Contact of Half-Angle α on a Semi-Cylinder

Contact Half-Angle α (radians) $\times 10^3$	Number of Nodes With- in Contact Half-Width n	$n' (=n-1/2)$	Resistance From Equation (II.E.2) $R_{ct} k_s$	Numerically Predicted Resistance $R'_{ct} k_s$	Error in Numerical Resistance (%)	Error in Heat Flux at $y=0$	Error in Heat Flux			Error in Temperature	
							At Node $n-1$	At Node n	At Node $n+1$	At Node $n+2$	At Node $n+2$
2.751	2	1.5	2.3180	2.3243	0.27	0.40	0.40	38.2	16.4		7.0
4.585	3	2.5	2.1553	2.1576	0.11	2.79	0.89	38.2	15.1		6.0
6.419	4	3.5	2.0483	2.0456	-0.13	2.90	0.19	38.1	14.8		5.8
8.253	5	4.5	1.9683	1.9633	-0.25	2.56	-0.26	38.1	14.6		5.6
10.09	6	5.5	1.9044	1.8979	-0.34	2.20	-0.56	38.2	14.6		5.6

¹ The % error in a particular numerical result is obtained from the expression

$$\% \text{ error in numerical result} = 100 \times \frac{(\text{numerical result} - \text{analytical result})}{\text{analytical result}}$$

² Results assume $T_0 = 0$.

theory so $\alpha = b/R = 9.61 \times 10^{-3} \text{ cm.}/0.953 \text{ cm.} = 1.009 \times 10^{-2} \text{ radians}$ for that patch half-width, as previously stated.

The $r-\theta$ finite-difference grid that was used to obtain the numerical results tabulated in Table II.E.1 was the same as the grid that was used in the synthesis analyses to model the $r-\theta$ geometry of the semi-cylindrical substitute contact specimen shown in Figure II.2.4. The structure of the grid in question, which contained 397 nodes, was similar to that shown in Figure II.2.5. All of the data that was supplied to SYNCON in order to construct the grid can be found in a sample input file (for a synthesis analysis of the resistance of the model contact) included in Part IV as Listing 4 but it is probably sufficient for the purposes at hand to note that the azimuthal subdivisions ($\Delta\theta$) used within and near to the contact were 1.834×10^{-3} radians ($=0.105^\circ$) in size and that the radial subdivisions (Δr) used in the same region were $1.8 \times 10^{-3} \text{ cm.}$ The finite-difference "cells" used in the vicinity of the contact were, therefore, approximately square since $R\Delta\theta = 0.953 \text{ cm.} \times 1.834 \times 10^{-3} \text{ radians} = 1.75 \times 10^{-3} \text{ cm.} = \Delta r$. Each value of α seen in Table II.E.1 can be obtained by substituting the corresponding value of n , which is also indicated in the table, into the equation,

$$\alpha = (n - 1/2)\Delta\theta \quad (\text{II.E.26a})$$

$$= n' \Delta\theta \quad (\text{II.E.26b})$$

$$= 1.834 \times 10^{-3} n' \text{ radians}$$

where $n' = \alpha/\Delta\theta$. For $n=6$ (i.e., with 6 nodes used within the half-width of the contact) we have $\alpha = 5.5 \times 1.834 \times 10^{-3} = 1.009 \times 10^{-2} \text{ radians}$,

for example, as noted in Table II.E.1. The contact half-angle (α) was varied by changing n (or n') instead of $\Delta\theta$ since that is the manner in which the different patch widths found at different axial locations within an elliptical contact must be approximated if the $r-\theta$ geometry of the grid (in particular the value of $\Delta\theta$ used near to the contact) is fixed (i.e., independent of z).

Table II.E.1 compares the numerically predicted contact resistances (in the form $R'_{ct} k_s$) to ~~resistances~~ resistances calculated using Equation (II.E.2). Also shown are the errors in the numerically predicted heat fluxes at the center of the contact (i.e., at $y = R\theta = 0$ or at the first lateral node) as well as the corresponding errors at lateral nodes $n-1$ and n .

The errors in the numerically predicted surface temperatures at lateral nodes $n+1$ and $n+2$ are also given in Table II.E.1. The errors in the numerically predicted heat fluxes at nodes between the first lateral node and node $n-1$ are not shown since they were comparable in size to the errors obtained at the two nodes just mentioned. The errors in the numerically predicted surface temperatures at nodes beyond node $n+2$ are also not shown because the errors at nodes $n+1$ and $n+2$ were the largest in each case.

The numerical model provided extremely accurate contact resistance estimates in that the largest error obtained in any of the analyses was only 0.34%, as seen in Table II.E.1. The heat flux at the first lateral node (i.e., at $j=1$) was predicted with less than 3% error in all of the analyses while the error in the heat flux obtained

at node $n-1$ never exceeded 0.89%. The surface temperatures at nodes $n+1$ and $n+2$ were also predicted reasonably accurately. The predicted heat flux at lateral node n was consistently 38.1 - 38.2% too high, however, making this result the least accurate of those shown in Table II.E.1. Recognizing that the heat fluxes at lateral nodes between the first and the $(n-1)$ st. were predicted quite accurately and that the resistance of the contact was also predicted accurately, it follows that the heat flux that was predicted at node n must actually have been the average heat flux over the surface element for that node, designated $\bar{q}_n''$, rather than the value of q'' obtained by substituting the value of y at node n (i.e., y_n) into Equation (II.E.18). If the heat flux distribution given by Equation (II.E.18) is used to calculate an approximate value for $\bar{q}_n''$, we obtain the expression,

$$\begin{aligned}\bar{q}_n'' &= \frac{Q'}{\pi \Delta y} \int_{b-\Delta y}^b \frac{dy}{\sqrt{b^2 - y^2}} \\ &= \frac{Q'}{\pi \Delta y} \left(\frac{\pi}{2} - \sin^{-1} \left(1 - \frac{\Delta y}{b} \right) \right)\end{aligned}\quad (\text{II.E.27})$$

from which the ratio,

$$\frac{\bar{q}_n''}{q''(y_n)} = \left(\frac{\pi}{2} - \sin^{-1} \left(1 - \frac{\Delta y}{b} \right) \right) (\sqrt{b^2 - y_n^2}) / \Delta y \quad (\text{II.E.28})$$

follows from Equation (II.E.18). It can be shown that the value of this ratio remains essentially constant as $\Delta y/b$ is varied over quite a large range and, furthermore, that the constant value just mentioned approximates the value observed in the numerical contact resistance calculations (i.e., 1.38). To this end, consider the approximation,

$$\sin\left(\frac{\pi}{2} - \phi\right) = \sin\frac{\pi}{2} - \phi \cos\frac{\pi}{2} - \frac{\phi^2}{2} \sin\left(\frac{\pi}{2}\right) + O(\phi^4) = 1 - \frac{\phi^2}{2} \quad (\text{II.E.29})$$

obtained by expanding $\sin\left(\frac{\pi}{2} - \phi\right)$ as a truncated Taylor series about $\frac{\pi}{2}$. Equation (II.E.29) can be rewritten as,

$$\sin^{-1}\left(1 - \frac{\phi^2}{2}\right) \approx \frac{\pi}{2} - \phi \quad (\text{II.E.30})$$

or as,

$$\sin^{-1}\left(1 - \frac{\Delta y}{b}\right) \approx \frac{\pi}{2} - \sqrt{2} \sqrt{\frac{\Delta y}{b}} \quad (\text{II.E.31})$$

by setting $\frac{\phi^2}{2} = \frac{\Delta y}{b}$. Now,

$$y_n = b - \frac{\Delta y}{2}$$

so,

$$\begin{aligned} \sqrt{b^2 - y_n^2} &= \sqrt{b^2 - \left(b - \frac{\Delta y}{2}\right)^2} \\ &= \sqrt{b\Delta y - \frac{\Delta y^2}{4}} \\ &= \sqrt{b\Delta y} \end{aligned}$$

if Δy is small. Hence,

$$\begin{aligned} \sqrt{\frac{\Delta y}{b}} &= \frac{\Delta y}{\sqrt{b\Delta y}} \\ &= \frac{\Delta y}{\sqrt{b^2 - y_n^2}} \end{aligned}$$

so Equation (II.E.31) can be rewritten as,

$$\sin^{-1}\left(1 - \frac{\Delta y}{b}\right) \approx \frac{\pi}{2} - \frac{\sqrt{2} \Delta y}{\sqrt{b^2 - y_n^2}} = \frac{\pi}{2} - \frac{\sqrt{2} (\Delta y / b)}{\sqrt{1 - (1 - \frac{\Delta y}{2b})}} \quad (\text{II.E.32})$$

It can be shown that this approximation to $\sin^{-1}\left(1 - \frac{\Delta y}{b}\right)$ is accurate to within 1.6% when $\frac{\Delta y}{b} = 1/3$ and to within 10% when $\frac{\Delta y}{b} = 2/3$. The latter

value of $\frac{\Delta y}{b}$ corresponds to the value this parameter would have had in the resistance calculations performed with $n=2$ (i.e., with the smallest contact half-angle). Introducing Equation (II.E.32) into Equation (II.E.28) and simplifying we get,

$$\frac{\bar{q}_n''}{q''(y_n)} = \sqrt{2} = 1.414$$

for the heat flux ratio which had a value of ≈ 1.38 in the numerical contact resistance calculations. These two results are in good agreement.

The results given in Table II.E.1 show that a line contact is not difficult to represent in a finite-difference analysis if the outermost boundary of the contact is located midway between two surface nodes. This relationship cannot generally be satisfied at all of the axial nodes in a three-dimensional grid that is used to model an elliptical contact, however, as seen in Figure II.2.9, so a contact approximation technique that can handle the more general case in which the outermost boundary of the contact extends only partway across the surface element for the n th lateral node is required. The situation of interest is shown in Figure II.2.10. Note that the parameter n' ($=\alpha/\Delta\theta$) seen in Equation (II.E.26b) is no longer equal to $n-1/2$ in general, as verified by the fact that $n=4$ and $n'=3$ for the contact shown in Figure II.2.10. The contact boundary is still located midway between two nodes when the relationship $n'=n-1/2$ is satisfied, of course.

The first procedure that will be considered for representing a contact such as that shown in Figure II.2.10 in a finite-difference analysis is derived by introducing the approximations $\sin \alpha = \alpha$ and $\cos \alpha = 1$ into Equation (II.E.2) to obtain,

$$R_{ct} = \ln(4R/b)/\pi k_s \quad (\text{II.E.33})$$

for small contacts. Denoting the y -coordinate of the outermost boundary of the surface element for the n th node by $y_{n+1/2} (= (y_n + y_{n+1})/2)$, the difference between the resistance of a contact with a half-width of $y_{n+1/2}$ and one with a half-width of b , designated ΔR_c for convenience, is simply,

$$\begin{aligned} \Delta R_c &= \left(\ln \left(\frac{4R}{b} \right) - \ln \left(\frac{4R}{y_{n+1/2}} \right) \right) / \pi k_s \\ &= \ln (y_{n+1/2}/b) / \pi k_s, \quad (b \leq y_{n+1/2}) \end{aligned} \quad (\text{II.E.34})$$

If this incremental resistance is equated to the resistance of a uniform contact conductance h_{cu} acting over the contact of width $2y_{n+1/2}$ we obtain,

$$\Delta R_c = \frac{1}{2y_{n+1/2} h_{cu}}$$

or,

$$h_{cu} = \pi k_s / (2y_{n+1/2} \ln (y_{n+1/2}/b)) \quad (\text{II.E.35})$$

The resistance of a contact of half-width b can be predicted very accurately by prescribing h_{cu} over the surface elements for lateral nodes $j=1, \dots, n$, as proven by the comparison of numerically predicted

and analytical resistances given in Table II.E.2' for $n' = 1.75-5.25$.

The analytical resistances were calculated using Equation (II.E.2)

and values of h_{cu} were calculated from the equation,

$$h_{cu} = \frac{\pi k_s}{2y_{n+1/2} \ln \left(\frac{n-1/2}{n'} \right)}$$

which is easily shown to be equivalent to Equation (II.E.35) because

$$y_{n+1/2} = R(n-1/2)\Delta\theta \text{ and } b = R\alpha = Rn'\Delta\theta.$$

Comparing the errors in the resistance predictions given in Table II.E.2 to those given in Table II.E.1, we see that the errors quoted in the former table are even less than those quoted in the latter. The errors in the surface temperature predictions are also very low but the heat flux predictions are relatively inaccurate, particularly for the analyses in which the contact covered only one-quarter of the surface element for the n th node (i.e., for $n'=1.75, 2.75, 3.75$ and 4.75). It is not surprising that the numerically predicted contact heat fluxes were too low, of course, because the contact half-width used in the numerical model (i.e., $y_{n+1/2}$) was larger than the actual patch half-width (b) so the same heat flow would have been distributed over a larger area. The "exact" heat fluxes with which the numerical values are compared were predicted using the actual patch half-width b rather than $y_{n+1/2}$.

A contact approximation technique which predicts all of the results of interest accurately will be described shortly but first it should be mentioned that Equation (II.E.34) could have been derived

Table II.E.2: Comparison of Numerical Results (Obtained With the Uniform Contact Conductance Defined by Equation (II.E.35) Prescribed Over the Contact Area) to the Corresponding Analytical Results for Heat Flow Through a Contact Patch Extending Partway Across the Surface Element for the n th Lateral Node

Contact Half- Angle α (radians $\times 10^3$)	n	n'	Resistance From Equation (II.E.2) $R_{ct}^1 k_s$	Numerically Predicted Resistance $R_{ct}^1 k_s$	Error in Numerical Resistance (%)	$100 \times \frac{\Delta R_c}{R_{ct}}$	Error in Heat Flux			Error in Temperature ²		
							At $y = 0$			At Node		
							Ab	$n-1$	$n+1$	At	$n+2$	$n+2$
					(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
3.209	3	1.75	2.2688	2.2737	0.21	5.0	-13.6	-22.3	-0.5	2.6		
3.668	3	2.00	2.2264	2.2295	0.14	3.2	-6.1	-10.0	2.0	3.2		
4.126	3	2.25	2.1888	2.1901	0.06	1.5	-0.5	-1.4	6.5	4.6		
5.043	4	2.75	2.1250	2.1266	0.08	3.6	-7.1	-21.6	-1.4	2.2		
5.502	4	3.00	2.0973	2.0972	0.00	2.3	-2.7	-9.8	1.3	3.0		
5.960	4	3.25	2.0718	2.0691	-0.13	1.1	0.7	-1.7	5.8	4.4		
6.877	5	3.75	2.0263	2.0265	0.00	2.9	-4.1	-21.6	-2.0	2.0		
7.336	5	4.00	2.0057	2.0037	-0.10	1.9	-1.1	-10.0	0.8	2.9		
7.794	5	4.25	1.9865	1.9816	-0.25	0.9	1.1	-1.9	5.7	4.0		
8.711	6	4.75	1.9510	1.9498	-0.07	2.4	-2.5	-21.8	-2.3	1.7		
9.170	6	5.00	1.9347	1.9314	-0.17	1.6	-0.3	-10.2	0.6	2.7		
9.628	6	5.25	1.9192	1.9131	-0.31	0.8	1.2	-2.2	5.5	4.0		

¹ The % error in a particular numerical result is obtained from the expression,

$$\% \text{ error in numerical result} = 100 \times \frac{(\text{numerical result} - \text{analytical result})}{\text{analytical result}}$$

² Results assume $T_0 = 0$.

using Veziroglu's expression for the resistance of an isothermal contact of width $2b$ on a rectangular flux tube of width $2t$ as a starting point instead of Equation (II.E.2). Veziroglu's expression is the equation which predicts the resistance R_C^T for contact 4 in Table I.2.1 and is repeated here for convenience as,

$$R_C^T = \frac{1}{\pi k_s} \ln \left(\frac{1}{\sin(\pi b/2t)} \right) \quad (\text{II.E.36})$$

For cases where $b \ll t$ we can approximate $\sin(\frac{\pi b}{2t})$ by $(\frac{\pi b}{2t})$ and then rewrite Equation (II.E.36) in the form,

$$R_C^T = \frac{1}{\pi k_s} \ln \left(\frac{2t}{\pi b} \right) = \frac{1}{\pi k_s} \ln (0.63662t/b) \quad (\text{II.E.37})$$

The incremental resistance ΔR_C can now be obtained from the equation,

$$\Delta R_C = (\ln \left(\frac{2t}{\pi b} \right) - \ln \left(\frac{2t}{\pi y_{n+1/2}} \right)) / \pi k_s = \ln (y_{n+1/2}/b) / \pi k_s$$

which is the same as Equation (II.E.34). The equation defining h_{cu} (i.e., Equation (II.E.35)) follows immediately from ΔR_C , as before.

Equation (II.E.34) (and hence Equation (II.E.35)) could even have been obtained from the equation defining R_C^q (i.e., the contact resistance for a uniform contact heat flux) for contact 4 in Table I.2.1 since it can be shown that the approximation,

$$R_C^q = \frac{1}{\pi k_s} \ln (0.71032t/b) \quad (\text{II.E.38})$$

predicts the uniform flux resistance of contact 4 with less than 1.2% error when $b/t \leq 0.20$ and less than 0.10% error when $b/t \leq 0.02$.

Equation (II.E.38) was obtained by the author by fitting an equation of the form $R_C^q k_s = \frac{1}{\pi} \ln (\text{constant} \times t/b)$ to exact values of $R_C^q k_s$ calculated

using the equation given in Table I.2.1. It is easily verified that Equation (II.E.38) gives the same equation as was obtained previously for ΔR_c (and hence h_{cu}) when it is used to calculate the difference between the uniform flux resistances of contacts with half-widths of b and $y_{n+1/2}$. The resistance R_c^q is based on the area-average contact temperature (T_c), as mentioned in Chapter I.2.

It can be shown that there is very little difference between the uniform flux and uniform temperature resistances of a line contact of a given size when ϵ ($=b/t$) is $\ll 1$. In the case of a contact with $b=0.00961$ cm. and $t=0.9527$ cm., for instance, which are dimensions corresponding to the semi-minor diameter of the elliptical contact patch produced at a contact load of 175.8N in the experiments (according to Hertz's theory) and the radius of the small-radiused contact specimen respectively, $\epsilon = 0.01009$ and R_c^q and R_c^T differ by only 2.6% according to Equations (II.E.37) and (II.E.38). The close agreement between R_c^q and R_c^T for narrow line contacts suggests that it is reasonable to assume that the resistance of a small contact of given ϵ will not be substantially different for different contact heat flux and temperature distributions provided it is based on the area-average contact temperature. This assumption will be used in the forthcoming analysis to derive a contact approximation technique which prescribes a non-uniform contact conductance over the "contact" of half-width $y_{n+1/2}$ (instead of h_{cu}) in order to obtain more accurate predictions of the contact heat flux.

Since the resistance of a contact of half-width b can be obtained from the resistance of a contact of half-width $y_{n+1/2}$ by adding ΔR_c

when both b and $y_{n+1/2}$ are small, it follows that the area-average temperature of the contact of half-width $y_{n+1/2}$ (i.e., $\bar{T}_C$) will be $Q' \Delta R_C$ if the contact of half-width b is at zero temperature and the heat flow rate through both contacts is Q' , as shown in Figure II.E.4. By now invoking the assumption that the resistance of a line contact of given ϵ is approximately constant when it is based on the area-average contact temperature ($\bar{T}_C$), we can replace the incremental resistance ΔR_C by any distribution of contact conductances over the contact

Figure II.E.4: Heat Flow Through Isothermal Contacts Joined by the Incremental Resistance ΔR_C

of half-width $y_{n+1/2}$ which leaves its area-average surface temperature unchanged (since this will leave Q' and hence the total contact resistance also unchanged). In particular, it might seem reasonable to make the contact conductance infinite at nodes $j=1, \dots, n-1$ (in order to make at least that segment of the contact isothermal) and to then choose a finite conductance at node n , designated h_n , such that the relationship,

$$\begin{aligned} \bar{T}_c &= Q' \Delta R_c = \frac{Q'}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b} \right) = \frac{1}{y_{n+1/2}} \int_0^{y_{n+1/2}} T_c(y) dy \\ &= \frac{1}{y_{n+1/2}} \int_{y_{n-1/2}}^{y_{n+1/2}} T_c(y) dy \end{aligned} \quad (\text{II.E.39a})$$

$$\begin{aligned} &= \frac{1}{y_{n+1/2}} \int_{y_{n-1/2}}^{y_{n+1/2}} \frac{q''(y)}{h_n} dy \\ &= \frac{q_n}{h_n y_{n+1/2}} \end{aligned} \quad (\text{II.E.39b})$$

is satisfied. The parameter q_n seen in Equation (II.E.39b) is the heat flow rate through the surface element for the n th node. It was possible to change the lower limit of integration in the surface temperature integral from 0 to $y_{n-1/2}$ because $T_c(y) = T_0 = 0$ for $0 \leq y \leq y_{n-1/2}$.

Equation (II.E.39b) can be rearranged to give the relationship,

$$h_n = \frac{q_n}{\frac{Q' y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b} \right)} \quad (\text{II.E.40})$$

which can be solved for h_n once an expression for q_n has been provided. Such an expression can be obtained by assuming that the heat flux distribution over the surface segment for which $0 \leq y \leq y_{n-1/2}$ will be approximately the correct one, viz.,

$$q''(y) = \frac{Q'}{\pi \sqrt{b^2 - y^2}}$$

since

$$\begin{aligned} q_n &= \frac{Q'}{2} - \int_0^{y_{n-1/2}} q''(y) dy \\ &= \frac{Q'}{2} \left(1 - \frac{2}{\pi} \int_0^{y_{n-1/2}} \frac{dy}{\sqrt{b^2 - y^2}} \right) \\ &= \frac{Q'}{2} \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) \right) \end{aligned} \quad (\text{II.E.41})$$

in that case. Substituting Equation (II.E.41) into Equation (II.E.40) we obtain,

$$h_n = \frac{1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right)}{\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b} \right)} \quad (\text{II.E.42})$$

as a tentative equation for h_n . Note that,

- (1) $h_n \rightarrow \infty$ as $b \rightarrow y_{n+1/2}$; and,
- (2) $h_n \rightarrow 0$ as $b \rightarrow y_{n-1/2}$.

according to Equation (II.E.42). These results are correct because (1) and (2) are the limiting cases in which the contact extends to the outermost boundaries of the surface elements for lateral nodes n and $n-1$ respectively.

Numerical results obtained with h_n calculated using Equation (II.E.42) are compared to the corresponding analytical results for contacts with $1.75 \leq n' \leq 5.25$ in Table II.E.3. The numerically predicted contact resistances are again very accurate in that the largest error seen in Table II.E.3 for any value of n' is only (-)0.44%. While the surface temperatures at nodes $n+1$ and $n+2$ were also predicted reasonably accurately, as before, the numerically predicted heat fluxes are seen to be consistently too high, particularly at node $n-1$ where the error in the heat flux estimate ranges from 26.9% (at $n' = 2.25$) to 48.4% (at $n' = 5.0$). Although not shown, the difference between the numerically predicted heat flux at node $n-1$ (i.e., q''_{n-1}) and the average heat flux over the $(n-1)$ st surface element (i.e., $\bar{q}''_{n-1}$) was generally almost as large as the error given in Table II.E.3 for the former result, which was calculated relative to $q''(y_{n-1})$. The heat flux q''_{n-1} was 35.1% greater than $\bar{q}''_{n-1}$ at $n' = 5.25$ and 35.3% greater at $n' = 4.75$ for instance, whereas the results given in Table II.E.3 show that the errors in q''_{n-1} for the same two values of n' were 44.4% and 37.9% respectively (relative to $q''(y_{n-1})$).

The numerical result for the heat flow rate through the surface element for the n th node was as much as 51% less than the assumed value calculated using Equation (II.E.41), as can be seen from the errors in q_n listed in Table II.E.3. The relatively low values of q_n obtained in the numerical resistance calculations coupled with the relatively high heat fluxes obtained at lateral nodes $j=1, \dots, n-1$ suggest that the heat flux distributions predicted numerically probably corresponded more closely to the heat flux distributions for contact half-widths

Table II.E.3: Comparison of Numerical Results (Obtained With h_n Defined by Equation (II.E.42)) to the Corresponding Analytical Results for Heat Flow Through A Contact Patch Extending Partway Across the Surface Element for the n th Lateral Node

Contact Half- Angle (Radians) $\times 10^3$	n	n'	Resistance From Equation (II.E.2) $R_{ct}^T k_s$	Numerically Predicted Resistance $R_{ct}^N k_s$	Error ¹ in Numerical Resistance (%)	Error ¹ in Heat Flux		Error ¹ in q_n (%)	Error ¹ in Temperature ²	
						At y=0 (%)	At Node n-1 (%)		At Node n+1 (%)	At Node n+2 (%)
3.209	3	1.75	2.2688	2.2588	-0.44	0.1	34.2	-35.3	1.6	1.9
3.668	3	2.00	2.2264	2.2271	0.03	4.8	36.2	-27.7	6.3	4.7
4.126	3	2.25	2.1888	2.1935	0.21	6.5	26.9	-16.2	11.1	6.7
5.043	4	2.75	2.1250	2.1177	-0.34	2.8	39.3	-42.2	3.1	2.8
5.502	4	3.00	2.0973	2.0966	-0.03	5.8	41.8	-33.9	8.0	5.6
5.960	4	3.25	2.0718	2.0731	0.06	6.6	31.5	-20.8	12.6	7.2
6.877	5	3.75	2.0263	2.0183	-0.39	3.2	42.3	-47.3	4.1	3.3
7.336	5	4.00	2.0057	2.0036	-0.11	5.4	45.6	-38.4	9.2	6.1
7.794	5	4.25	1.9865	1.9862	-0.01	5.9	35.0	-24.3	13.3	7.5
8.711	6	4.75	1.9510	1.9430	-0.41	3.0	44.4	-51.0	4.8	3.8
9.170	6	5.00	1.9347	1.9320	-0.14	4.9	48.4	-42.0	10.1	6.6
9.628	6	5.25	1.9192	1.9180	-0.06	5.3	37.9	-27.2	14.7	8.1

¹ The % error in a particular numerical result is obtained from the expression,

$$\% \text{ error in numerical result} = 100 \times \frac{(\text{numerical result} - \text{analytical result})}{\text{analytical result}}$$

² Results assume $T_0 = 0$.

which were slightly smaller than the intended values than they did to the heat flux distributions for the intended values themselves. In the case of the analysis performed with $n' = 4.00$, for instance, when the intended value of b was 6.99×10^{-3} cm., the value of q_n/Q' obtained numerically would have been the correct value for a contact half-width of 6.42×10^{-3} cm. (which corresponds to $\alpha = 6.740 \times 10^{-3}$ radians or $n' = 3.675$) according to Equation (II.E.41). If 6.42×10^{-3} cm. is substituted for b in Equation (II.E.18) instead of 6.99×10^{-3} cm., the errors in the numerically predicted heat fluxes at the first and $(n-1)$ st lateral nodes decrease to -3.2% and 16.9% respectively from their former values (given in Table II.E.3) of 5.4% and 45.6% respectively and the difference between q_{n-1}'' and $\bar{q}_{n-1}''$ decreases from 40.9% to only 7.5% . It should be noted, however, that the value of $R_{ct}^T k_s$ for the smaller contact (for which $\alpha = 6.74 \times 10^{-3}$ radians, as mentioned previously) is 2.0327 or 1.44% different from the value of 2.0036 obtained for $R_{ct}^T k_s$ in the analysis performed with $n' = 4.00$, so the numerical result corresponds much more closely to the resistance of a contact with the intended patch half-width (i.e., $n' = 4.00$), with which it differs by only $(-)0.11\%$ as seen in Table II.E.3, than it does to the resistance of the smaller contact with $n' = 3.675$.

Hence while the heat flux distribution obtained in the numerical solution with $n' = 4.00$ is in good agreement with the heat flux distribution for a smaller contact, the corresponding resistance agrees more closely with the resistance of a contact with the intended patch half-width.

It was assumed in the foregoing discussion that the larger contact half-width of 6.99×10^{-3} cm. was the intended half-width (b) and that the only significance of the smaller contact half-width of 6.42×10^{-3} cm. was that the heat flux distribution for this contact size corresponded fairly closely to the heat flux distribution obtained in the numerical solution with the intended contact size. If it is assumed instead that the smaller contact half-width of 6.42×10^{-3} cm. is the intended half-width (b), then it follows that the heat flux distribution obtained in a numerical analysis of the heat flow through a contact of this size would be approximately correct if h_n (given by Equation (II.E.42)) was evaluated using the larger contact half-width of 6.99×10^{-3} cm. instead of b . The larger contact half-width will be denoted by b_ℓ in the following discussion for convenience. The contact resistance which would be obtained in a numerical calculation performed with b replaced by b_ℓ would be too low since it would be the resistance corresponding to b_ℓ , of course, but the correct resistance could presumably then be obtained by prescribing a contact conductance, defined by Equation (II.E.35) with $y_{n+1/2}$ replaced by b_ℓ , over the contact of width $2b_\ell$. Designating that contact conductance by $h_{\ell u}$, we have the definition,

$$h_{\ell u} = \frac{\pi k_s}{2b_\ell \ln\left(\frac{b_\ell}{b}\right)} \quad (\text{II.E.43})$$

A method for approximating the contact of half-width b_ℓ with the contact conductance $h_{\ell u}$ prescribed over its area must be found in order to implement the foregoing procedure. It is assumed that the contact of half-width b_ℓ extends partway across the surface element for the n th lateral node, as shown in Figure II.E.5. The problem shown in Figure II.E.5

Figure II.E.5 Contact of Half-Width b_l With a Contact Conductance h_{lu} Prescribed Over its Area.

is obviously similar to that shown in Figure II.2.10 except that we must now account for the presence of the contact conductance h_{lu} . This can be done by noting that the area-average temperature of the contact of half-width b_l , designated $\bar{T}'_c$, will equal $Q'/(2h_{lu} b_l)$ if $T_0 = 0$ and the heat flow rate through the contact is Q' since,

$$\begin{aligned} Q' &= 2h_{lu} \int_0^{b_l} (T'_c(y) - T_0) dy \\ &= 2h_{lu} \int_0^{b_l} T'_c(y) dy \quad (\text{since } T_0 = 0) \\ &= 2h_{lu} b_l \bar{T}'_c \end{aligned} \quad (\text{II.E.44})$$

in that case. The area-average temperature of the "contact" of half-width $y_{n+1/2}$, designated $\bar{T}_c$ as before, can now be obtained from the relation,

$$\begin{aligned} \bar{T}_c &= (\bar{T}_c - \bar{T}'_c) + \bar{T}'_c \\ &= Q' \Delta R_c + \frac{Q'}{2h_{lu} b_l} \\ &= Q' \left(\Delta R_c + \frac{1}{2h_{lu} b_l} \right) \end{aligned} \quad (\text{II.E.45})$$

when the heat flow rate through the contact is still Q' and ΔR_c is defined by Equation (II.E.34).

The contact shown in Figure II.E.5 is now approximated by prescribing the contact conductance h_{lu} over the surface elements for lateral nodes $j=1, \dots, n-1$ and then determining h_n such that the area-average temperature of the contact is equal to the value of $\bar{T}_c$ obtained from Equation (II.E.45).

Following the method used to derive an expression defining h_n for an isothermal contact, we can write

$$\begin{aligned}\bar{T}_c &= \frac{1}{y_{n+1/2}} \int_0^{y_{n+1/2}} T_c(y) dy \\ &= \frac{1}{y_{n+1/2} h_{lu}} \int_0^{y_{n+1/2}} q''(y) dy + \frac{q_n}{h_n y_{n+1/2}}\end{aligned}$$

and then assume,

$$q''(y) = \frac{Q'}{\pi \sqrt{b_\ell^2 - y^2}}$$

to get,

$$\bar{T}_c = \frac{Q'}{y_{n+1/2}} \left[\sin^{-1} \left(\frac{y_{n+1/2}}{b_\ell} \right) / \pi h_{lu} + \frac{1}{2h_n} \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n+1/2}}{b_\ell} \right) \right) \right] \quad (\text{II.E.46})$$

Equating Equations (II.E.45) and (II.E.46) and solving the resulting equation for h_n , we obtain,

$$h_n = \frac{1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n+1/2}}{b_\ell} \right)}{\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b_\ell} \right) + \frac{1}{h_{lu}} \left[\frac{y_{n+1/2}}{b_\ell} - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n+1/2}}{b_\ell} \right) \right]} \quad (\text{II.E.47})$$

Note that,

- (1) $h_n \rightarrow h_{lu}$ as $b_\ell \rightarrow y_{n+1/2}$; and,
- (2) $h_n \rightarrow 0$ as $b_\ell \rightarrow y_{n-1/2}$.

according to Equation (II.E.47). These results are correct because (1) and (2) are the limiting cases in which the contact extends to

the outermost boundaries of the surface elements for lateral nodes n and $n-1$ respectively. Also note that Equation (II.E.47) reduces to Equation (II.E.42) if $h_{lu} = \infty$ (i.e., if the contact of half-width b_l is isothermal), as it should.

Returning now to the example discussed previously of the contact for which $b = 6.42 \times 10^{-3}$ cm. and $b_l = 6.99 \times 10^{-3}$ cm., we have the additional dimensions,

$$\begin{aligned} y_{n+1/2} &= R(n-1/2)\Delta\theta = 0.953 \times (5 - 1/2) \times 0.001834 \\ &= 7.862 \times 10^{-3} \text{ cm.} \end{aligned}$$

and,

$$\begin{aligned} y_{n-1/2} &= R(n - 1 \frac{1}{2})\Delta\theta = 0.953 \times (5 - 1 \frac{1}{2}) \times 0.001834 \\ &= 6.115 \times 10^{-3} \text{ cm.} \end{aligned}$$

so,

$$h_{lu}/k_s = 2652 \text{ cm.}^{-1} \text{ from Equation (II.E.43)}$$

and,

$$h_n/k_s = 424.5 \text{ cm.}^{-1} \text{ from Equation (II.E.47)}$$

The value of h_{cu}/k_s for a contact with the given values of $y_{n+1/2}$ and b would be 986 cm.^{-1} (i.e., less than h_{lu}/k_s but greater than h_n/k_s) by way of comparison.

The results of a finite-difference analysis performed with the aforementioned value of h_{lu} prescribed at lateral nodes 1-4 and h_n prescribed at lateral node 5 (since $n=5$ in this example) are given in Table II.E.4. The error in the numerically predicted contact resistance was only $(-)0.13\%$, as seen in the table, and the heat fluxes at lateral

Table II.E.4: Results Obtained in the Sample Analysis
 Performed with h_{lu} Defined by Equation
 (II.E.43) and h_n Defined by Equation
 (II.E.47)

Contact half-angle (α)	6.74×10^{-3} radians
n	5
n^*	3.675
Analytical resistance ($R_{ct}^T k_s$)	2.0327
Numerical Resistance ($R_{ct}^I k_s$)	2.0300
Error in the numerical resistance	-0.13%
Error in the numerically predicted heat flux at node:	
1	2.8%
2	2.6%
3	1.2%
4. (=n-1)	5.8%
$100 \frac{(q_{n-1}'' - \bar{q}_{n-1}'')}{\bar{q}_{n-1}''}$	-2.8%
Error in the numerical result for the heat flow rate through the surface element for the nth (=5th) node	1.3%

nodes 1-4 were all predicted quite accurately. The error in the finite-difference result for the heat flux at node $n-1$ ($=4$) was only 5.8%, in particular, and the difference between q''_{n-1} and $\bar{q}''_{n-1}$ was even less at $(-)2.8\%$. The numerical result for the heat flow rate through the surface element for the n th ($=5$ th) node was only 1.3% greater than the correct value for the contact width of b (given by Equation (II.E.41)).

The results of the foregoing analysis demonstrate that local values of the surface heat flux within a line contact that only partially covers the surface element for the n th lateral node in a finite-difference grid can be predicted quite accurately by prescribing a contact conductance at lateral node n which is smaller than that prescribed at nodes 1, ..., $n-1$. Such a procedure effectively replaces the original isothermal contact of half-width b by a larger non-isothermal contact of half-width $y_{n+1/2}$ without significantly affecting either the total heat flow rate through the contact (Q) or local values of the surface heat flux at lateral (y) coordinates for which $-y_{n-1/2} \leq y \leq y_{n-1/2}$, as shown in Figures II.E.6a and II.E.6b for one-half of a contact. The larger contact communicates with T_0 through the contact conductance h_{2u} when $0 \leq |y| \leq |y_{n-1/2}|$ and through the conductance h_n when $|y_{n-1/2}| \leq |y| \leq |y_{n+1/2}|$, of course. The heat flow line separating the heat flow passing through the segment of the contact for which $0 \leq y \leq y_{n-1/2}$ (for one-half of the contact) from the remainder of the heat flow should be in approximately the same position for the contacts of width b and $y_{n+1/2}$ shown in Figures II.E.6a and II.E.6b respectively because Q is the same both both contacts and the heat fluxes at y

Figure II.E.6a Isothermal Contact of Half-Width b

$\equiv$

Non-isothermal contact with area-average temperature T_c (segment between $y_{n-1/2}$ and $y_{n+1/2}$ is assumed isothermal)

Figure II.E.6b Non-Isothermal Contact of Half-Width $y_{n+1/2}$

Figure II.E.7 Flux Tube Carrying the Heat Flow q_n to an Isothermal Contact of Width e

coordinates less than $y_{n-1/2}$ are the same, so the resistance $1/h_n (y_{n+1/2} - y_{n-1/2}) (=1/h_n \Delta y)$ associated with the conductance h_n acting over the surface segment between $y_{n-1/2}$ and $y_{n+1/2}$ should be approximately equal to the difference between the resistance of one contact with a width of $b-y_{n-1/2}$ and another with a width of Δy when both contacts are located on the bottom of the flux tube which carries the heat flow q_n to the contacts shown in Figures II.E.6a and II.E.6b. The flux tube just referred to is shown with cross-hatching in Figure II.E.7. The contact on the bottom of the flux tube is shown as having a width e in Figure II.E.7, where $e = b-y_{n-1/2}$ for the contact shown in Figure II.E.6a and $e = y_{n+1/2} - y_{n-1/2} = \Delta y$ for the contact shown in Figure II.E.6b.

The contact shown in Figure II.E.6b is assumed to be isothermal between $y_{n-1/2}$ and $y_{n+1/2}$ because this segment of the contact coincides with the surface element for the n th node in the finite-difference model. The contact shown in Figure II.E.6a is completely isothermal, of course, so the resistance of a contact of width $b-y_{n-1/2}$ located on the bottom of the flux tube shown with cross-hatching in Figure II.E.7 will be greater than the resistance of another contact of width Δy by an amount $\Delta R'_C$, where it is assumed that to a first approximation,

$$\Delta R'_C = \frac{2}{\pi k_s} \ln \left(\frac{\Delta y}{b-y_{n-1/2}} \right) \quad (\text{II.E.48})$$

Equation (II.E.48) represents the difference between the resistances of contacts with widths of Δy and $b-y_{n-1/2}$ when the contacts are located on one side of the bottom of a large flux tube, as shown in Figure II.E.8. The incremental resistance $\Delta R'_C$ is twice the incremental resistance ΔR_C .

Figure II.E.8: Contacts of Width Δy and $b - y_{n-1/2}$ Located On One Side of the Bottom of A Large Flux Tube

defined by Equation (II.E.34) (the contact widths Δy and $b - y_{n-1/2}$ in Equation (II.E.48) are analogous to $y_{n+1/2}$ and b in Equation (II.E.34)) because Equation (II.E.34) was derived for contacts which are centrally located on the bottom of a large flux tube. Dividing such a contact in half about the line of symmetry passing through its center would produce two contacts similar to those shown in Figure II.E.8.

The boundary conditions near to the contacts shown in Figure II.E.8 are not exactly the same as the corresponding boundary conditions for the contact of width e at the bottom of the cross-hatched flux tube shown in Figure II.E.7, of course, because the heat flow line defining

the left-hand boundary of the flux tube shown in Figure II.E.7 is curved whereas the left-hand boundaries of the flux tube cutouts shown in Figure II.E.8 are straight. All of these boundaries intersect their respective contacts at right-angles, however, so it is assumed for simplicity that the left-hand boundary of the cross-hatched flux tube shown in Figure II.E.7 will be approximately normal to the contact in a large enough region around it for Equation (II.E.48) to be approximately applicable.

The significance of $\Delta R'_C$ is that it can be equated to the resistance $1/h_n \Delta y$, with h_n defined by Equation (II.E.47), to yield an expression for b_ℓ . For convenience, Equation (II.E.47) will be rewritten as,

$$h_n = \frac{1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_\ell} \right)}{\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b} \right) - \frac{4b_\ell}{\pi^2 k_s} \sin^{-1} \left(\frac{y_{n-1/2}}{b_\ell} \right) \ln \left(\frac{b_\ell}{b} \right)} \quad (\text{II.E.49})$$

by substituting Equation (II.E.43) for $h_{\ell u}$ in Equation (II.E.47).

Equating $1/h_n \Delta y$ to $\Delta R'_C$ now gives the expression,

$$\begin{aligned} f(b_\ell) &= \Delta R'_C - 1/h_n \Delta y \\ &= \frac{2}{\pi k_s} \ln \left(\frac{\Delta y}{b - y_{n-1/2}} \right) \\ &\quad - \frac{\left(\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b} \right) - \frac{4b_\ell}{\pi^2 k_s} \sin^{-1} \left(\frac{y_{n-1/2}}{b_\ell} \right) \ln \left(\frac{b_\ell}{b} \right) \right)}{\Delta y \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_\ell} \right) \right)} \\ &= 0 \end{aligned} \quad (\text{II.E.50})$$

which can be solved numerically for the root lying between $b_l = b$ and $b_l = y_{n+1/2}$. There will always be a root in this interval because it is easily verified that,

- (1) $f(b_l)$ is positive when $b_l = y_{n+1/2}$; and
- (2) $f(b_l)$ is negative when $b_l = b$.

Considering first the proof that $f(b_l)$ is positive when $b_l = y_{n+1/2}$, we can rewrite $f(b)$ for this case as,

$$\begin{aligned} f(y_{n+1/2}) &= \frac{2}{\pi k_s} \left(\ln \left(\frac{\Delta y}{b - y_{n-1/2}} \right) - \frac{y_{n+1/2}}{\Delta y} \ln \left(\frac{y_{n+1/2}}{b} \right) \right) \\ &= \frac{2}{\pi k_s} \ln \left[\left(\frac{\Delta y}{b - y_{n-1/2}} \right) \left(\frac{b}{y_{n+1/2}} \right)^{\frac{y_{n+1/2}}{\Delta y}} \right] \quad (\text{II.E.51}) \end{aligned}$$

The quantity $f(y_{n+1/2})$ will be positive if the term contained within the square brackets in Equation (II.E.51) is ≥ 1 . The second term in the square brackets can be written in the form,

$$\left(\frac{b}{y_{n+1/2}} \right)^{\frac{y_{n+1/2}}{\Delta y}} = \left(1 + \frac{b - y_{n+1/2}}{y_{n+1/2}} \right)^{\frac{y_{n+1/2}}{\Delta y}}$$

and then expressed as a Taylor series expansion of the function

$g(u) = u^{\frac{y_{n+1/2}}{\Delta y}}$ about the point $u=1$ as,

$$\begin{aligned} \left(\frac{b}{y_{n+1/2}} \right)^{\frac{y_{n+1/2}}{\Delta y}} &= 1 + \frac{b - y_{n+1/2}}{\Delta y} + \frac{(y_{n+1/2})(y_{n-1/2})}{2\Delta y^2} \left(\frac{b - y_{n+1/2}}{y_{n+1/2}} \right)^2 \frac{(\frac{y_{n+1/2}}{\Delta y} - 2)}{\xi} \\ &\quad (\text{II.E.52}) \end{aligned}$$

where ξ is some number between 1 and $b/y_{n+1/2}$. Since $\xi > 0$ and $y_{n-1/2} \geq 0$, the remainder in Equation (II.E.52) is positive and,

$$\left(\frac{b}{y_{n+1/2}}\right)^{\frac{y_{n+1/2}}{\Delta y}} \geq 1 + \left(\frac{b-y_{n+1/2}}{\Delta y}\right) \quad (\text{II.E.53})$$

Multiplying both sides of Equation (II.E.53) by $\Delta y/(b-y_{n-1/2})$ we obtain the inequality,

$$\begin{aligned} \left(\frac{\Delta y}{b-y_{n-1/2}}\right) \left(\frac{b}{y_{n+1/2}}\right)^{\frac{y_{n+1/2}}{\Delta y}} &\geq \left(\frac{\Delta y}{b-y_{n-1/2}}\right) \left(1 + \frac{b-y_{n+1/2}}{\Delta y}\right) \\ &\geq \frac{\Delta y + b - y_{n+1/2}}{b - y_{n-1/2}} \\ &\geq 1 \quad (\text{since } \Delta y = y_{n+1/2} - y_{n-1/2}) \end{aligned}$$

which now can be substituted into Equation (II.E.51) to prove that

$f(b_e) \geq 0$ when $b_e = y_{n+1/2}$.

When $b_e = b$, Equation (II.E.50) simplifies to the expression,

$$f(b) = \frac{2}{\pi k_s} \left[\ln \left(\frac{\Delta y}{b-y_{n-1/2}} \right) - \frac{\frac{y_{n+1/2}}{\Delta y} \ln \left(\frac{y_{n+1/2}}{b} \right)}{1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right)} \right] \quad (\text{II.E.54})$$

which indicates that we must have,

$$\frac{\frac{y_{n+1/2}}{\Delta y} \ln \left(\frac{y_{n+1/2}}{b} \right)}{1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right)} \geq \ln \left(\frac{\Delta y}{b-y_{n-1/2}} \right) \quad (\text{I.E.55})$$

in order for $f(b)$ to be negative. It is easily verified that,

$$\ln \left(\frac{\Delta y}{b-y_{n-1/2}} \right) = \ln \left(1 + \frac{y_{n+1/2} - b}{b-y_{n-1/2}} \right) \leq \frac{y_{n+1/2} - b}{b-y_{n-1/2}} \quad (\text{II.E.56})$$

and that,

$$\sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) \geq \frac{\pi}{2} \left(1 - \sqrt{1 - \frac{y_{n-1}}{b}} \right) \quad (\text{II.E.57})$$

so the proof that Equation (II.E.55) is valid will have been obtained if we can show that the inequality obtained by substituting Equations (II.E.56) and (II.E.57) into Equation (II.E.55) is satisfied. Introducing Equations (II.E.56) and (II.E.57) into Equation (II.E.55) and simplifying we get,

$$\ln \left(\frac{y_{n+1/2}}{b} \right) \geq \left(1 - \frac{y_{n-1/2}}{y_{n+1/2}} \right) \sqrt{\frac{y_{n+1/2}}{b}} - 1 \quad (\text{II.E.58})$$

as the inequality that we must prove is true. Now,

$$\frac{y_{n-1/2}}{y_{n+1/2}} = \frac{(n - \frac{1}{2})\Delta y}{(n+1/2)\Delta y} \geq \frac{(2 - \frac{1}{2})}{(2 + 1/2)} \approx 1/3 \quad (\text{II.E.59})$$

since we are only interested in determining b , for $n \geq 2$ (only one node is located within the contact when $n=1$). It can be shown in a similar manner that $\frac{y_{n+1/2}}{b}$ must lie in the range $1 \leq \frac{y_{n+1/2}}{b} \leq 3$ (the upper limit follows from Equation (II.E.59) because $b \geq y_{n-1/2}$) so Equation (II.E.58) will be satisfied if the inequality,

$$\ln \left(\frac{y_{n+1/2}}{b} \right) \geq \frac{2}{3} \sqrt{\frac{y_{n+1/2}}{b}} - 1, \quad 1 \leq \frac{y_{n+1/2}}{b} \leq 3 \quad (\text{II.E.60})$$

is valid. It is easily verified by direct comparison that Equation (II.E.60) is satisfied since $\ln \left(\frac{y_{n+1/2}}{b} \right) \approx 1.098 \dots$, and $\frac{2}{3} \sqrt{\frac{y_{n+1/2}}{b}} - 1 = 0.942 \dots$, when $\frac{y_{n+1/2}}{b} = 3$, for instance.

Since $\Delta R'_C \rightarrow \infty$ as $b \rightarrow y_{n-1/2}$, we must have $h_n \rightarrow 0$ or, equivalently, $b_l \rightarrow b$ as $b \rightarrow y_{n-1/2}$ in order for Equation (II.E.50) to remain satisfied. Thus $h_n \rightarrow 0$ and $h_{lu} \rightarrow \infty$ (since $b_l \rightarrow b$) as $b \rightarrow y_{n-1/2}$, which is the correct result, of course, because the contact does not cover any of the surface element for the n th node when $b = y_{n-1/2}$. Similarly, $\Delta R'_C \rightarrow 0$ when $b \rightarrow y_{n+1/2}$ so we must have $h_n \rightarrow \infty$ or, equivalently, $b_l \rightarrow b$ and $h_{lu} \rightarrow \infty$, as $b \rightarrow y_{n+1/2}$ in order to satisfy Equation (II.E.50). This result is also correct because the n th surface element is completely covered by the contact when $b = y_{n+1/2}$.

A value of 7.17×10^{-3} cm. is obtained for b_l when Equation (II.E.50) is solved using the data given previously for the contact with $b = 6.42 \times 10^{-3}$ cm. This figure is in reasonable agreement with the value of 6.99×10^{-3} cm. that was assigned to b_l to obtain the results given in Table II.E.4.

Results showing the accuracy of the contact approximation technique just described are given in Table II.E.5. Results are provided for the values of n' used in the previous comparisons of analytical and numerical results as well as for the contact with $n' = 3.675$ (or $b = 6.42 \times 10^{-3}$ cm.). In addition to the errors in the numerically predicted contact resistances for the contact approximation technique just described, Table II.E.5 gives the errors in resistances obtained with h_n defined by the relation,*

$$h_n = \frac{1}{\Delta R'_C \Delta y} \quad (\text{II.E.61})$$

* Equation (II.E.61) also defines h_n in the full technique which obtains h_{lu} from Equation (II.E.43), of course, because Equation (II.E.50) is satisfied when $1/h_n \Delta y = \Delta R'_C$.

Table II.E.5: Comparison of Numerical Results (Obtained With h_n Defined by Equation (II.E.49)) to the Corresponding Analytical Results for Heat Flow Through a Contact Patch Extending Partway Across the Surface Element for the n th Lateral Node

Contact Half- Angle α (radians) $\times 10^3$	n	n'	Resistance From Equation (II.E.2) $R_{ct}^T k_s$	Numerically Predicted Resistance $R_{ct}^N k_s$	Error in Numerical Resistance (%)	Error in Numerical Resistance Obtained With $h_n = 1/\Delta R_{ct}^N \Delta y$ $h_n = 1, n-1$ (%)	Error in Heat Flux		Error in q_n (%)	Error in Temperature	
							At $y = 0$	At Node $n-1$		At Node $n+1$	At Node $n+2$
3.209	3	1.75	2.2689	2.2694	0.02	-1.59	-1.4	-0.9	8.2	1.3	3.0
3.668	3	2.00	2.2264	2.2305	0.19	-0.96	-0.4	1.3	1.9	3.6	4.1
4.126	3	2.25	2.1889	2.1903	0.06	-0.45	0.5	0.9	0.7	7.2	5.1
5.043	4	2.75	2.1250	2.1264	0.06	-1.39	-0.3	0.0	9.6	2.1	3.6
5.502	4	3.00	2.0973	2.0985	0.06	-0.93	0.4	0.2	2.9	3.6	4.2
5.960	4	3.25	2.0718	2.0699	-0.09	-0.56	1.4	1.1	0.0	6.9	4.7
6.740	5	3.675	2.0327	2.0316	-0.06	-1.40	1.7	-1.0	15.8	4.8	4.9
6.877	5	3.75	2.0263	2.0277	0.07	-1.29	0.7	-1.0	10.4	2.3	3.8
7.336	5	4.00	2.0057	2.0052	-0.03	-0.94	1.0	-0.5	3.0	3.4	4.1
7.794	5	4.25	1.9868	1.9830	-0.17	-0.62	1.8	1.2	-0.5	6.7	4.5
8.711	6	4.75	1.9510	1.9512	0.01	-1.20	1.1	-2.6	12.2	2.5	3.9
9.170	6	5.00	1.9347	1.9326	-0.11	-0.91	1.2	-1.5	3.6	3.4	3.9
9.628	6	5.25	1.9192	1.9144	-0.25	-0.68	1.8	1.3	-0.9	7.5	4.5

but with h_{lu} set to infinity at lateral nodes $j=1, \dots, n-1$. The only results given in Table II.E.5 for the latter contact approximation technique are the errors in the contact resistance predictions - the remaining results in the table apply to the full contact approximation technique in which the contact conductance h_{lu} is prescribed at lateral nodes $1, \dots, n-1$ and h_n is prescribed at node n .

As seen in Table II.E.5, the error in the numerically predicted contact resistance was $\leq 0.25\%$ in all of the analyses performed with the contact properly approximated and less than 0.10% in the majority of those analyses. The heat fluxes at nodes within the contact were also predicted very accurately, typically with less than 2% error, as were the surface temperatures outside of the contact. The heat flow through the n th surface element (q_n) was predicted reasonably accurately in all of the analyses in that the largest error in that result was only 15.8% , as seen in Table II.E.5. The errors in q_n were largest in the analyses in which the contact covered only one-quarter of the surface element for the n th node (or less in the case of the analysis with $n = 3.675$), which is as one would expect because,

- (a) the values of q_n were comparatively small in those analyses so absolute errors of a given size would have resulted in greater relative errors; and,
- (b) the left-hand boundary of the flux tube shown with cross-hatching in Figure II.E.7 would have been most poorly represented by the left hand boundaries of the flux tube cutouts shown in Figure II.E.8 when the contact covered only a small fraction of the width of the n th surface

element (i.e., when $(b-y_{n-1/2})/\Delta y$ was small). High curvature (near the contact) of the flux line defining the left-hand boundary of the flux tube shown with cross-hatching in Figure II.E.7 would have caused the real value of $\Delta R'_C$ to be greater than that given by Equation (II.E.48) in such situations, causing q_n to be over-estimated (as actually observed in all of the analyses with small values of $(b-y_{n-1/2})/\Delta y$).

The errors in the resistance predictions obtained with h_n calculated using Equation (II.E.61) and h_{LU} set to infinity ranged from (-)0.45% to (-)1.59%, as seen in Table II.E.5, making these resistance predictions the least accurate of those considered so far. While even these errors are not very large in absolute terms, it should be recognized that any method for approximating a contact covering part of the n th surface element should at the very least be able to predict that the resistance of the contact will lie somewhere between the resistances of contacts extending to the innermost and outermost boundaries of that element. Those resistances would be the resistances of contacts with n and $n-1$ nodes within their half-widths, of course. The difference between the resistances of any two such contacts was typically only a few percent in the present study, as can be seen from the results given in Table II.E.1, so even a poor contact approximation technique would be unlikely to predict resistances which were in error by more than one or two percent. The resistances of the contacts with $n' = 3.5$ and $n' = 4.5$ differ by only 4%, for instance, so the errors of (-)0.62% to (-)1.40% obtained in the analyses with $n=5$

(i.e., with $n' = 3.675 - 4.25$) and $h_{lu} = \infty$ were significant in comparison. The contact resistances would have been underestimated even more if the incremental resistance used to calculate h_n (i.e., $\Delta R'_C$) had been obtained from Veziroglu's constriction formula (i.e., Equation (II.E.36)) after multiplying that formula by a factor of 2 (to account for the fact that the segment of the contact on the n th surface element is located to one side of the element rather than at its center) and replacing b and t by $(b - y_{n-1/2})$ and Δy respectively because the resulting resistance would have been even smaller than $\Delta R'_C$ (since $\sin(\pi(b - y_{n-1/2})/2\Delta y) \geq (b - y_{n-1/2})/\Delta y$). These comparisons illustrate the accuracy of the proposed contact approximation technique when compared with more obvious techniques.

Although the method of approximating a contact which uses the two contact conductances h_n and h_{lu} was derived for the case of an isothermal contact, it can also be used to approximate a contact over which a large contact conductance h_c is prescribed by first replacing $\Delta R'_C$ by $\Delta R''_C$, where

$$\Delta R''_C = \frac{2}{\pi k_s} \ln \left(\frac{\Delta y}{b - y_{n-1/2}} \right) + \frac{1}{h_c (b - y_{n-1/2})} \quad (\text{II.E.62})$$

and then combining in series the resistances associated with the contact conductances h_c and h_{lu} acting over b and b_l respectively to obtain an equivalent contact conductance acting over b_l . The equivalent contact conductance, designated h'_{lu} , must satisfy the relationship,

$$\frac{1}{h'_{lu} b_l} = \frac{1}{h_{lu} b_l} + \frac{1}{h_c b}$$

so,

$$\begin{aligned} h'_{lu} &= 1 / \left[\frac{1}{h_{lu}} + \frac{b_l}{h_c b} \right] \\ &= 1 / \left[\frac{2b_l}{\pi k_s} \ln \left(\frac{b_l}{b} \right) + \frac{b_l}{h_c b} \right] \end{aligned} \quad (\text{II.E.63})$$

The contact is approximated by prescribing h'_{lu} at lateral nodes $j=1, \dots, n-1$ and h_n at node n , where h_n is obtained from Equation (II.E.47) by substituting h'_{lu} for h_{lu} . It is also assumed that $1/h_n \Delta y = \Delta R''_c$ (or equivalently that $h_n = 1/\Delta R''_c \Delta y$), as before, so b_l is obtained by solving the equation,

$$\begin{aligned} g(b_l) &= \Delta R''_c - \frac{1}{h_n \Delta y} \\ &= \frac{2}{\pi k_s} \ln \left(\frac{\Delta y}{b - y_{n-1/2}} \right) + \frac{1}{h_c (b - y_{n-1/2})} \\ &\quad - \left[\frac{\frac{2y_{n+1/2}}{\pi k_s} \ln \left(\frac{y_{n+1/2}}{b_l} \right) + \frac{1}{h'_{lu}} \left(\frac{y_{n+1/2}}{b_l} - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_l} \right) \right)}{\Delta y \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_l} \right) \right)} \right] \end{aligned} \quad (\text{II.E.64})$$

$$= 0$$

for the root lying between $y = y_{n-1/2}$ and $y = y_{n+1/2}$. It is easily verified that $g(b_l)$ will always be positive when $b_l = y_{n+1/2}$, as was the case with Equation (II.E.50), but $g(b_l)$ will no longer necessarily be negative when $b_l = b$. The proof that $g(b_l)$ must be negative when $b_l = y_{n-1/2}$ is easily obtained when $b > y_{n-1/2}$ since the first two terms in Equation (II.E.64) are finite in that case and the third

(subtracted) term is infinite.

Since $\Delta R_C'' \rightarrow \infty$ as $b \rightarrow y_{n-1/2}$, we must have $h_n \rightarrow 0$ or, equivalently, $b_l \rightarrow b$, as $b \rightarrow y_{n-1/2}$ in order for Equation (II.E.64) to remain satisfied. Thus, $h_n \rightarrow 0$ and $h'_{lu} \rightarrow h_c$ (since $b_l \rightarrow b$) as $b \rightarrow y_{n-1/2}$, which is the correct result, of course. Similarly, $\Delta R_C'' \rightarrow 1/h_c \Delta y$ as $b \rightarrow y_{n+1/2}$ so we must have $h_n \rightarrow h_c$, or equivalently $b_l \rightarrow b$ and $h'_{lu} \rightarrow h_c$ as $b \rightarrow y_{n+1/2}$ in order to satisfy Equation (II.E.64). These results are also correct, indicating that the contact approximation technique just described yields the correct results when $b = y_{n+1/2}$ and $b = y_{n-1/2}$.

It can be shown that the value of b_l will always drop below that of b (when $b > y_{n-1/2}$) if h_c is made sufficiently small because it becomes impossible to satisfy the relationships,

$$h_n = 1/\Delta R_C'' \Delta y \quad (\text{II.E.65})$$

$$\bar{T}_c = Q' \left(\frac{\ln \left(\frac{y_{n+1/2}}{b} \right)}{\pi k_s} + \frac{1}{2h_c b} \right) \quad (\text{II.E.66})$$

and,

$$\bar{T}_c = \frac{Q'}{y_{n+1/2}} \left[\sin^{-1} \left(\frac{y_{n-1/2}}{b_l} \right) / \pi h'_{lu} + \frac{1}{2h_n} \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b_l} \right) \right) \right] \quad (\text{II.E.67})$$

simultaneously (with h'_{lu} defined by Equation (II.E.63)) and still have $b_l \geq b$. The problem that arises is that the value of $\bar{T}_c$ obtained from Equation (II.E.67) (with Equation (II.E.65) substituted for h_n) eventually exceeds the required value given by Equation (II.E.66) when h_c is made small enough, even if $b_l = b$. Reducing b_l below b causes

the heat flow rate through the n th surface element to decrease, thereby lowering the value of T_c obtained from Equation (II.E.67) and restoring equality with Equation (II.E.66). It is possible to verify that the value of T_c given by Equation (II.E.66) will be less than that given by Equation (II.E.67) (with $b_l = b$) if h_c is made sufficiently small by letting $b_l = b$ and $h_c \rightarrow 0$ since then,

$$\begin{aligned} h'_{lu} &= h_c \\ h_n &= h_c \left(\frac{b - y_{n-1/2}}{\Delta y} \right) \\ T_c &= \frac{Q'}{2h_c b} \quad \text{from Equation (II.E.66)} \end{aligned} \quad (\text{II.E.68})$$

and,

$$T_c = \frac{Q'}{y_{n+1/2} h_c} \left[\sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) / \pi + \frac{\Delta y}{2(b - y_{n-1/2})} \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) \right) \right] \quad (\text{II.E.69})$$

from Equation (II.E.67). The value of T_c predicted by Equation (II.E.69) will be greater than that predicted by Equation (II.E.68) if,

$$\frac{1}{y_{n+1/2}} \left[\sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) / \pi + \frac{\Delta y}{2(b - y_{n-1/2})} \left(1 - \frac{2}{\pi} \sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) \right) \right] \geq \frac{1}{2b}$$

or if,

$$\frac{\sin^{-1} \left(\frac{y_{n-1/2}}{b} \right)}{\pi} \left(1 - \frac{\Delta y}{b - y_{n-1/2}} \right) + \frac{\Delta y}{2(b - y_{n-1/2})} \geq \frac{y_{n+1/2}}{2b} \quad (\text{II.E.70})$$

Since it is easily verified that,

$$\sin^{-1} \left(\frac{y_{n-1/2}}{b} \right) \leq \frac{\pi}{2} - \sqrt{2} \sqrt{1 - \frac{y_{n-1/2}}{b}} \quad (\text{II.E.71})$$

the proof that Equation (II.E.70) is valid will have been obtained if we can prove that the inequality obtained by introducing Equation (II.E.71)

into Equation (II.E.70) is satisfied (note that the factor

$(1 - \frac{\Delta y}{b-y_{n-1/2}})$ is a negative quantity because $\Delta y \geq b-y_{n-1/2}$).

Performing this substitution we get,

$$(\frac{1}{2} - \frac{\sqrt{2}}{\pi} \sqrt{1 - \frac{y_{n-1/2}}{b}}) (1 - \frac{\Delta y}{b-y_{n-1/2}}) + \frac{\Delta y}{2(b-y_{n-1/2})} \geq \frac{y_{n+1/2}}{2b}$$

as the inequality that we must prove is true. This inequality can be rewritten as,

$$\frac{1}{2} - \frac{\sqrt{2}}{\pi} \sqrt{\frac{b-y_{n-1/2}}{b}} (\frac{b-y_{n+1/2}}{b-y_{n-1/2}}) \geq \frac{y_{n+1/2}}{2b}$$

and then as,

$$\sqrt{1 - \frac{y_{n-1/2}}{b}} \leq \frac{2\sqrt{2}}{\pi} = 0.9003 \quad (\text{II.E.72})$$

after simplifying. Since we must have $\frac{y_{n-1/2}}{b} \geq \frac{1}{3}$ (because

$\frac{y_{n-1/2}}{b} \geq \frac{y_{n-1/2}}{y_{n+1/2}} \geq \frac{1}{3}$ as shown previously), Equation (II.E.72) will be

satisfied if the inequality

$$\sqrt{1 - \frac{1}{3}} = .8165 \leq 0.9003$$

is valid, which it obviously is. Thus the value of T_c obtained from Equation (II.E.67) with $b_c = b$ will always be greater than that given by Equation (II.E.66) if h_c is made sufficiently small. Equality between the two values of T_c can be restored by reducing b_c below b , as mentioned previously.

It was assumed in the derivation of the technique for approximating a contact with the contact conductance h_c prescribed over its area that the heat flux distribution over the contact would approximate the distribution over an isothermal contact. An obvious deficiency of that

technique is that the contact heat flux distribution will become increasingly uniform as h_c is reduced, thereby violating the assumption just referred to and causing the value of b_l needed to obtain equality between Equations (II.E.66) and (II.E.67) to be underestimated. One deleterious consequence of b_l decreasing below b in value is that the value of h'_{lu} predicted by Equation (II.E.63) exceeds h_c in that case. Since $R_{mjc} + R_{mac} + R_{mic} = 1/2h_c b$ when $h_c \rightarrow 0$, it follows that a parallel combination of the resistances $1/2h'_{lu} y_{n-1/2}$ and $1/2h_n \Delta y$ should equal $1/2h_c b$ when $h_c \rightarrow 0$ if the contact approximation technique is correct. Now $h_n \rightarrow h_c \left(\frac{b - y_{n-1/2}}{\Delta y} \right)$ when $h_c \rightarrow 0$ according to Equation (II.E.65) so the value of h'_{lu} needed to obtain the correct value of R_{mic} (i.e., $1/2h_c b$) can be calculated from the expression,

$$\begin{aligned} 2h_c b &= 2h'_{lu} y_{n-1/2} + 2h_n \Delta y \\ &= 2h'_{lu} y_{n-1/2} + 2h_c (b - y_{n-1/2}) \end{aligned}$$

as,

$$h'_{lu} = h_c$$

Since this value of h'_{lu} is obtained when $b_l = b$ in Equation (II.E.63), values of $b_l < b$ or equivalently $h'_{lu} > h_c$ are assumed to be physically unrealistic.

It was shown in the foregoing analysis that prescribing the contact conductance h_c at lateral nodes $j=1, \dots, n-1$ and h_n (obtained from Equation (II.E.65)) at lateral node n produces the correct contact resistance if h_c is infinitesimally small. This result suggests that errors incurred by applying the last contact approximation technique

described to problems with more uniform contact heat flux distributions than that used to derive the technique could be minimized by setting $b_L = b$ when the method predicted a value of b_L less than b . Results demonstrating the improvement in predictive accuracy that results when the original technique is modified in this manner are given in Table II.E.6 for a contact with $n' = 4$ (i.e., $\alpha = 7.336 \times 10^{-3}$ radians or $b = 6.989 \times 10^{-3}$ cm.) and values of R_{mac}/R_{mic} ranging from 1.5 to 86. The table also shows the errors in the resistances obtained with the unmodified technique, termed method A for convenience, along with the errors obtained when the contact conductance h_n (defined by Equation (II.E.65)) is prescribed at lateral node n and b_L is set equal to b (or, equivalently, h'_{Lu} is set equal to h_c). The latter technique is designated method B for convenience. The errors for the modified technique, termed method C, correspond to the errors for method A when $b_L/b \geq 1$ and to those for method B when $b_L/b < 1$.

All of the errors listed in Table II.E.6 represent the errors in the total resistance of the contact (i.e., $R_{mic} + R_{mac}$) rather than just R_{mac} alone.* Also, the "exact" resistances used to calculate the different errors were actually resistances obtained numerically by adjusting the value of $\Delta\theta$ used near the contact so that $b (= 6.989 \times 10^{-3}$ cm.) was equal to $y_{n+1/2}$ for $n=4$. The resulting $\Delta\theta$ of 2.096×10^{-3} radians caused the edge of the contact to be located midway between the 4th and 5th lateral nodes. The uniform temperature resistance of the

* Errors in R_{mac} alone would be greater than those quoted, of course, particularly at low values of R_{mac}/R_{mic} .

Table II.E.6: Errors in Resistances Obtained With Different Methods of Approximating a Contact with $n'=4$
(or $\alpha=7.336 \times 10^{-3}$ Radians)

R_{mat}/R_{mic}	$\frac{b_1}{b}$ ¹	Error in Resistance Obtained With Unmodified Technique (Method A) (%)	Error in Resistance Obtained With h'_{tu} Set Equal to h_c (Method B) (%)	Error in Resistance Obtained With the Modified Technique (Method C) (%)
86	1.068	0.09	-0.69	0.09
38	1.041	-0.12	-0.62	-0.12
23	1.017	-0.30	-0.54	-0.30
15	0.998	-0.59	-0.52	-0.52
7.6	0.967	-1.1	-0.45	-0.45
3.8	0.943	-1.9	-0.31	-0.31
1.5	0.928	-3.2	-0.22	-0.22

¹ The values of b_1/b quoted apply to Method A. For comparison purposes, $\frac{y_{n-1/2}}{b} = 0.8749$ and $\frac{y_{n+1/2}}{b} = 1.1248$ for this contact.

contact was predicted with only $(-)0.03\%$ error by this grid so it is assumed that the resistances it provided for contacts with $R_{\text{mac}}/R_{\text{mic}} < \infty$ were also quite accurate.

As seen in Table II.E.6, the unmodified contact approximation technique predicted the resistance of the contact with the largest value of $R_{\text{mac}}/R_{\text{mic}}$ (i.e., 86) with only 0.09% error while the corresponding error for method B was somewhat greater at $(-)0.69\%$. These results are similar to those given in Table II.E.5* for the isothermal contact with $n' = 4$, as one might expect because the heat flux distribution across the contact with $R_{\text{mac}}/R_{\text{mic}} = 86$ would probably have corresponded closely to the distribution for an isothermal contact. The results given in Table II.E.6 show that the accuracy of the unmodified contact approximation technique deteriorated as h_c was reduced whereas the accuracy of method B improved as h_c decreased. The accuracy of the former technique was particularly poor at the lowest values of $R_{\text{mac}}/R_{\text{mic}}$ used because the technique was predicting values of h'_{lu} that were significantly higher than h_c in those analyses. The value of h'_{lu} obtained in the analysis with $R_{\text{mac}}/R_{\text{mic}} = 1.5$ was approximately 10% greater than the value of h_c used, for instance, so it is not surprising that the value of $R_{\text{mac}} + R_{\text{mic}}$ was underpredicted by 3.2% since it is easily verified that a 10% increase in h_c over the entire contact half-width (instead of just the segment for which $0 \leq y \leq y_{n-1/2}$) would reduce the value of $R_{\text{mac}} + R_{\text{mic}}$ by 4%.

* The resistances listed in Table II.E.5 for analyses performed with h_j set to infinity at lateral nodes $j=1, \dots, n-1$ are effectively resistances obtained by applying method B with $h_c = \infty$.

As seen in Table II.E.6, method C yields accurate resistance predictions irrespective of the value of h_c used, as expected. The method is obviously most accurate when R_{mic} is negligible, however, since it was really derived for contacts with that characteristic.

The results of other resistance calculations performed with method A are given in Table II.E.7 to further demonstrate the method's accuracy in applications where R_{mic} is small relative to R_{mac} . Since the ratio b_c/b never dropped below unity in those analyses, method A was used without modification. A contact conductance (h_c) of $1136 \text{ W/cm}^2\text{K}$ ($2 \times 10^6 \text{ BTU/hr ft}^2\text{°F}$) was used in all of the resistance calculations since h_c was assigned this value in all of the synthesis and multigrid analyses of the model contact problem. The thermal conductivity of the semi-cylinder (k_s) was assigned a value of 0.3703 W/cm K in the calculations, as it was in all of the synthesis and multigrid analyses just mentioned.

The analytical resistances used to calculate the errors in the numerically predicted values of R_{mac} were obtained from Equation (II.E.2) even though that equation applies strictly to isothermal contacts. The good agreement obtained between resistances predicted using Equation (II.E.2) and the numerically predicted resistances of the contacts with $n' = 1.5, 2.5, \dots, 5.5$ (which did not require special approximation because b coincided with $y_{n+1/2}$ in those analyses) suggest that Equation (II.E.2) was used with negligible error in this application, however. The resistances quoted in Table II.E.7 are macroscopic constriction resistances, not total resistances.

Table II.E.7: Errors in Resistances Calculated Numerically for Contacts With Different Half-Angles on a Semi-Cylinder

Data used in analyses: $h_c = 1136 \text{ W/cm}^2 \text{ K}$; $k_s = 0.3703 \text{ W/cm K}$; $R = 0.953 \text{ cm}$.

Contact Half-Angle (radians) $\alpha \times 10^3$	n	n'	R_{mac}/R_{mfc}	b_t/b	Resistance From Equation (II.E.2) $R_{ct}^T k_s$	Numerically Predicted Resistance $R_{ct}^* k_s$	Error in Numerical Resistance (%)
2.751	2	1.5	37.3	1.000	2.3180	2.3258	0.34
3.209	3	1.75	42.6	1.0743	2.2688	2.2587	-0.45
3.668	3	2.00	47.7	1.1040	2.2264	2.2288	0.11
4.126	3	2.25	52.8	1.0647	2.1888	2.1922	0.15
4.585	3	2.50	57.8	1.000	2.1554	2.1561	0.03
5.043	4	2.75	62.6	1.0653	2.1250	2.1203	-0.22
5.502	4	3.00	67.5	1.0807	2.0973	2.0994	0.10
5.960	4	3.25	72.2	1.0505	2.0718	2.0729	0.05
6.419	4	3.50	76.9	1.000	2.0483	2.0456	-0.13
6.877	5	3.75	81.5	1.0604	2.0263	2.0238	-0.12
7.336	5	4.00	86.0	1.0684	2.0057	2.0068	0.06
7.794	5	4.25	90.5	1.0423	1.9865	1.9857	-0.04
8.253	5	4.50	95.0	1.000	1.9683	1.9640	-0.22
8.711	6	4.75	99.3	1.0543	1.9510	1.9494	-0.09
9.170	6	5.00	103.7	1.0578	1.9347	1.9355	0.04
9.628	6	5.25	108.0	1.0342	1.9192	1.9169	-0.12
10.087	6	5.50	112.3	1.000	1.9044	1.8989	-0.29

All of the contact resistances were predicted very accurately by the proposed contact approximation technique; as can be seen from the results given in Table II.E.7. The largest error obtained in any of the analyses in which the contact covered only part of the n th surface element was $(-)0.45\%$ and the next largest was $(-)0.22\%$. An error of 0.10% or less was obtained in more than half of the analyses. Although errors in the numerically predicted contact heat fluxes are not given because a suitable analytical expression for predicting the heat flux distribution with a contact conductance prescribed was not available, the analysis with $n' = 4.0$ was repeated with $\Delta\theta$ reduced by a factor of two within and near the contact in order to obtain alternate heat flux estimates for comparison purposes. The heat fluxes obtained at the lateral (y) coordinates of the 1st and $(n-1)$ st nodes in the first analysis changed by only 1.6% and 0.04% respectively in the second analysis (i.e., in the analysis performed with the finer grid), suggesting that the former heat flux estimates were quite accurate.

All of the contact approximation techniques considered so far have accounted for differences between b and $y_{n+1/2}$ by prescribing contact conductance(s) over the surface elements for lateral nodes 1 to n . If the fraction of the area of the n th surface element which is covered by the contact is denoted by f (i.e., $f = (b - y_{n-1/2})/\Delta y$), a fairly obvious way to attempt to approximate the contact is to multiply the radial conductance $K(NR-1, n, k)_{i+1}$ (i.e., the conductance between nodes $NR-1, n, k$ and NR, n, k , where NR is the outermost

radial node) by f . In the case of a contact with a contact conductance h_c prescribed over its area, the surface conductance $K(NR, n, k)_s$ ($= h_c A_s(NR, n, k)$) would also be multiplied by f in this technique.

Results obtained by applying the technique just described to isothermal contacts with values of n' ranging from 1.75 to 5.25 are given in Table II.E.8. As seen in the table, the technique consistently underestimated the resistance of the contact. The numerically predicted resistances shown in Table II.E.8 are the most inaccurate seen so far, thus demonstrating, once again, the comparatively high accuracy of the contact approximation techniques derived from Equation II.E.34.

Table II.E.8: Errors in Resistances Calculated Numerically for Isothermal Contacts With Different Half-Angles On A Semi-Cylinder - Resistances Obtained By Replacing $K(NR-1, n, k)_{i+1}$ by $f \times K(NR-1, n, k)_{i+1}$: ($f = (b-y_{n-1/2})/\Delta y$)

Contact Half-Angle α (radians) $\times 10^3$	n	n'	Resistance From Equation (II.E.2) $R_{ct}^T k_s$	Numerically Predicted Resistance $R_{ct}' k_s$	Error In Numerical Resistance (%)
3.209	3	1.75	2.2688	2.2280	-1.80
3.668	3	2.00	2.2264	2.1939	-1.46
4.126	3	2.25	2.1888	2.1699	-0.86
8.711	6	4.75	1.9510	1.9243	-1.37
9.170	6	5.00	1.9347	1.9115	-1.20
9.628	6	5.25	1.9192	1.9024	-0.88

II.E.4 Numerical Studies With Gas Conduction

It was shown in the preceding section that a contact which extends only partway across the surface element for the n th node can be approximated very accurately by prescribing the contact conductance h_n at lateral node n and h_{lu} (or h'_{lu}) at lateral nodes $j=1, \dots, n-1$. Although this method was only demonstrated to be accurate for approximating a contact in vacuum, the method can also be used to approximate a contact in the presence of a gas because the difference between the half-width of the actual contact (i.e., b) and the segment used to represent it in the numerical model (i.e., $y_{n+1/2}$) will always be quite small. These half-widths must satisfy the relationship $y_{n+1/2} - \Delta y \leq b \leq y_{n+1/2}$, of course. The closeness of b and $y_{n+1/2}$ is important because it guarantees that the heat flow nets for the contacts with those widths will be similar outside of a small region around the contact. Provided most of the heat flow entering that region reaches the contact (i.e., is not conducted across the gas gap), the proposed contact approximation technique will be applicable because it could be derived without considering the material beyond the isotherm defining the outer boundary of the affected region. It is assumed that the isotherm just referred to would resemble one of the elliptical surfaces of constant η shown in Figure II.E.2. The resistance between the elliptical isotherm and the contact of half-width b , denoted R_{ct}^T , is given by the expression [40].

$$\begin{aligned}
 R_{ct}^T &= \frac{1}{\pi k_s} \cosh^{-1} (\hat{y}/b) \\
 &= \frac{1}{\pi k_s} \ln (\hat{y}/b + \sqrt{(\hat{y}/b)^2 - 1})
 \end{aligned}
 \quad (II.E.73)$$

where $\hat{y}$ is identified in Figure II.E.9. The heat flux distribution over the contact is defined by Equation (II.E.9) if w and b' in that equation are replaced by $\hat{y}$ and b respectively. If $\hat{y}/b > 2$, Equation (II.E.73) can be approximated by the expression,

$$R_{ct}^T = \frac{1}{\pi k_s} \ln \left(\frac{2\hat{y}}{b} \right) \quad (II.E.74)$$

with less than 5% error. The difference between the resistance of the contact with a half-width of b and another with a half-width of $y_{n+1/2}$ would obviously be given by Equation (II.E.34), as before, if R_{ct}^T was defined by Equation (II.E.74). The rest of the derivation of the proposed contact approximation technique would be identical to that already described.

Figure II.E.9 Region Affected by Contact Approximation

The validity of the foregoing qualitative explanation of why the proposed contact approximation technique should be applicable to contacts in gases will depend on the magnitude of the gas conduction component in some small region around the contact. Although the method will likely not be accurate if that component is large relative to the heat flow rate through the contact itself, it can be argued that the importance of accurately representing the width of the contact in the numerical model will be diminished in that case anyway. This observation is supported by the results of resistance calculations performed with different gas conductivities, as seen in Table II.E.9. All of the results shown in Table II.E.9 were obtained with the semi-cylindrical substitute contact specimen (used in all of the other analyses considered in this appendix) using a contact conductance (h_c) of $1136 \text{ W/cm}^2 \text{ K}$ and a solid conductivity of 0.3703 W/cm K (as in the analyses whose results are summarized in Table II.E.7).

The results given in Table II.E.9 can be interpreted by noting that there are six groups of data each containing the results for three similarly-sized contacts. All of the results in each group were obtained with the same continuum gas conductivity. The values of n' for the first two sets of results in each group satisfy the relationship $n' = n - 1/2$ for the value of n given, and hence apply to contacts whose outer boundaries coincide with the lateral coordinate $y_{n+1/2}$ (and thus do not require special approximation) while the last value of n' in each group can be seen to be the average of the first two values. Thus the last set of results in each group applies to a contact whose half-width is the average of the contact half-widths for the two

Table II.E.9: Results Obtained in Finite-Difference Analyses of the Heat Flow Through An Isothermal Contact On a Semi-Cylinder With a Gas Present

Contact Half-Angle α (radians) $\times 10^3$	n	n'	Axial Location in Contact x (cm.)	Continuum Gas Conductivity k_{og} (W/cm K) $\times 10^4$	Numerically Predicted Resistance R'_{ct} (cm° C/W)	Average of n-1/2 & n-3/2 Resistances $\bar{R}_{ct}$ (cm° C/W)	Error in Numerical Resistance (%)
2.751	2	1.5	0.148	2.84	10.3944		
4.585	3	2.5	0.148	2.84	10.0842		
3.668	3	2.0	0.148	2.84	10.2376	10.2393	-0.02
2.751	2	1.5	0.148	5.68	9.2714		
4.585	3	2.5	0.148	5.68	9.1394		
3.668	3	2.0	0.148	5.68	9.2140	9.2054	0.09
2.751	2	1.5	0.148	14.2	7.7679		
4.585	3	2.5	0.148	14.2	7.7451		
3.668	3	2.0	0.148	14.2	7.7615	7.7565	0.06
8.253	5	4.5	0.0886	2.84	9.5169		
10.087	6	5.5	0.0886	2.84	9.3588		
9.170	6	5.0	0.0886	2.84	9.4609	9.4379	0.24
8.253	5	4.5	0.0886	5.68	8.7927		
10.087	6	5.5	0.0886	5.68	8.7106		
9.170	6	5.0	0.0886	5.68	8.7697	8.7517	0.20
8.253	5	4.5	0.0886	14.2	7.6103		
10.087	6	5.5	0.0886	14.2	7.5907		
9.170	6	5.0	0.0886	14.2	7.6088	7.6005	0.11

Data used in analyses: $h_c = 1136.0 \text{ W/cm}^2 \text{ K}$; $k_g = 0.3703 \text{ W/cm K}$; $R = 0.9527 \text{ cm}$.

preceding sets of results. The first two lines in the table contain the results for contacts with $n' = 1.5$ and $n' = 2.5$ respectively while the results in the third line apply to a contact with a value of n' equal to the average of the two values just mentioned (i.e., $(1.5 + 2.5)/2 = 2.0$), for instance

The results for integral values of n were obtained by prescribing the contact conductance h'_{lu} (defined by Equation (II.E.63)) at lateral nodes $j=1, \dots, n-1$ and h_n (defined by Equation (II.E.65)) at node n . The gas conductances that were prescribed at lateral nodes $j > n$ were obtained using the non-continuum gas conductivity model described in Section I.3.4 with the continuum gas conductivities listed in Table II.E.9. The lowest gas conductivity used (i.e., 2.84×10^{-4} W/cm K) corresponds to the thermal conductivity of air at 60°C , which was the average mean interface temperature encountered in the experiments described in Chapter I.6, while the thermal conductivities of 5.68×10^{-4} W/cm K and 14.2×10^{-4} W/cm K are simply twice and five times the lowest conductivity used respectively. The mean free path of the gas molecules and the surface accommodation coefficients were assigned the values indicated in Table I.6.4.

The gap size that was used to calculate the gas conductance at each lateral node $j > n$ was obtained from Equation (I.3.20) using the surface radii of the contact specimens tested in the experiments and contact dimensions for a contact load of 175.8N. The contact semi-major (a) and semi-minor (b) diameters would have been 0.154 cm. and 0.00961 cm. respectively at that load according to Hertz's theory.

The gap size at each lateral node $j > n$ was calculated from its y-coordinate and the x-coordinate listed in Table II.E.9. The x-coordinate for each group of three sets of results was the coordinate where the half-width of the elliptical contact with the contact dimensions mentioned previously equalled the half-width of the smallest contact in the group (i.e., the line contact to which the first set of results in the group applied).

The correct resistance of each contact that covered only one-half of the surface element for the n th node (i.e., a contact for which $n' = n-1$) was assumed to be the average of the resistances obtained for the contacts with $n' = n-1/2$ and $n' = n-3/2$. An estimate of the correct resistance of the contact with $n' = 2.0$ (and $n=3$) was obtained by averaging the resistances of the contacts with $n' = 1.5$ and $n' = 2.5$, for instance. The resulting average resistance of $10.2393 \text{ cm}^{\circ} \text{ C/W}$ was then used to calculate the error in the resistance obtained numerically for the contact with $n' = 2.0$, which was $10.2376 \text{ cm}^{\circ} \text{ C/W}$.

The error introduced by averaging the resistances of contacts with $n' = n-1/2$ and $n' = n-3/2$ to obtain an estimate of the resistance of the contact with $n' = n-1$ was probably negligible since it is easily verified that this approximation yields good results even when applied to contacts in vacuum. Since the values of $R_{ct}^T k_s$ for contacts with $n' = 1.5$, 2.0 and 2.5 are 2.3180 , 2.2264 and 2.1553 respectively (in vacuum) according to Tables II.E.1 and II.E.2, the average value of $R_{ct} k_s$ for the contacts with $n' = 1.5$ and $n = 2.5$ is

2.2366 or 0.46% different from the value of $R_{ct}^T k_s$ for the contact with $n' = 2.0$, for instance. Better agreement between the average value of $R_{ct}^T k_s$ for contacts with $n' = n-1/2$ and $n' = 3/2$ and its value for the contact with $n' = n-1$ is obtained for larger n , of course. These results differ by only 0.08% when $n=6$, for instance.

The proposed contact approximation technique predicted the resistances of the contacts with integral values of n' very accurately, as seen in Table II.E.9. While the accuracy of the technique did not appear to degrade as k_{og} was increased, it should be noted that the difference between the resistances of the contacts with $n' = n-1/2$ and $n' = n-3/2$ decreased with increasing k_{og} until it was only 0.29% for $n=3$ and 0.26% for $n=6$ with $k_{og} = 14.2 \times 10^{-4}$ W/cm K. Since any method for calculating the resistance of a contact extending only partway across the n th lateral surface element should be able to predict that its value will lie somewhere between the resistances of contacts extending to the innermost and outermost boundaries of that element, it is not surprising that the resistance of the contact with $n' = 5.0$ was predicted with only 0.11% error. This error is not insignificant compared to the 0.26% difference between the resistances of the contacts with $n' = 4.5$ and $n' = 5.5$, of course, but it is nevertheless extremely small in absolute terms and hence quite satisfactory. This comparison shows that the proposed contact approximation technique can be applied to virtually any problem irrespective of the significance of gas conduction.

II.E.5 Three-Dimensional Applications

Although derived by considering two-dimensional heat flow through a line contact, the contact approximation techniques derived from Equation (II.E.34) were developed so that an elliptical contact could be approximated in the manner shown in Figure II.2.9. While it is difficult to estimate how accurate the methods will be when used for that purpose, it is reasonable to assume that their accuracy will be quite good for contacts with large aspect ratios because the heat flow through such contacts is strongly two-dimensional in character. Some indication of what constitutes a large aspect ratio in this application can be obtained by noting that the expression,

$$R_{ct}^T = \ln(4a/b)/2\pi k_s a \quad (II.E.75)$$

defines the resistance of an isothermal elliptical contact on an infinite half-space with less than 1% error when $a/b \geq 4$ [74]. If the difference between the resistances of two contacts with the same semi-major diameter (a) but different semi-minor diameters b_1 and b_2 is designated δR_c , then,

$$\delta R_c = \frac{1}{2\pi k_s a} \left(\ln \left(\frac{4a}{b_1} \right) - \ln \left(\frac{4a}{b_2} \right) \right) = \ln \left(\frac{b_2}{b_1} \right) / 2\pi k_s a \quad (II.E.76)$$

when the aspect ratios of both contacts are large enough to use Equation (II.E.75) to estimate their resistances. It is assumed that $b_2 \geq b_1$, as shown in Figure II.E.10.

Figure II.E.10: Elliptical Contacts 1 and 2 With the Same Semi-Major Diameter (a) and Semi-Minor Diameters of b_1 and b_2 Respectively

Equation (II.E.76) can be derived in an alternate manner by dividing the contact major diameter $2a$ into an infinite number of strips of infinitesimal thickness dx and inserting the resistance,

$$dR_c(x) = \ln(s_2(x)/s_1(x))/\pi k_s dx \quad (\text{II.E.77})$$

between each strip of half-width $s_1(x)$ on contact 1 and the coincident strip of half-width $s_2(x)$ on contact 2. The contact half-widths $s_1(x)$ and $s_2(x)$ are defined by the expressions,

$$s_1(x) = b_1 \sqrt{1 - (x/a)^2} \quad (\text{II.E.78})$$

and,

$$s_2(x) = b_2 \sqrt{1 - (x/a)^2} \quad (\text{II.E.79})$$

so Equation (II.E.77) can be rewritten as,

$$dR_c = \ln(b_2/b_1)/\pi k_s dx \quad (\text{II.E.80})$$

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Since contacts 1 and 2 are both isothermal, the resistances defined by Equation (II.E.80) can be combined in parallel to obtain an equivalent resistance $\delta R'_C$ satisfying the expression,

$$\begin{aligned} \frac{1}{\delta R'_C} &= 2 \int_0^a \frac{\pi k_s dx}{\ln(b_2/b_1)} \\ &= \frac{2\pi k_s a}{\ln(b_2/b_1)} \end{aligned} \quad (\text{II.E.81})$$

Hence,

$$\delta R'_C = \ln(b_2/b_1)/2\pi k_s a \quad (\text{II.E.82})$$

and,

$$\delta R'_C = \delta R_C$$

If the conductance between each strip of half-width $s_1(x)$ on contact 1 and the coincident strip of half-width $s_2(x)$ on contact 2 is designated dK_C , then,

$$dK_C = 1/dR_C = \pi k_s dx / \ln(s_2/s_1)$$

and,

$$\frac{dK_C}{dx} = \pi k_s / \ln(s_2/s_1) = \pi k_s / \ln(b_2/b_1) \quad (\text{II.E.83})$$

Applying the constant conductance (per unit length) dK_C/dx over the contact of total length $2a$ would obviously yield a total conductance (K_C) of $2\pi k_s a / \ln(b_2/b_1)$ ($= 1/\delta R_C$). The fact that $\delta R'_C = \delta R_C$ when the linear conductance dK_C/dx is defined by Equation (II.E.83) does not guarantee that prescribing this linear conductance over the length of contact 2 would cause the resulting total resistance to be that of contact 1, however, since there are an infinite number of definitions for dK_C/dx that would give the same result (i.e., that $\delta R'_C = \delta R_C$).

It is easily verified that $\delta R'_C = \delta R_C$ when,

$$\frac{dK_C}{dx} = \frac{2k_s a}{\ln(b_2/b_1) \sqrt{a^2 - x^2}}$$

for instance, since,

$$\begin{aligned} \frac{1}{\delta R'_C} &= 2 \int_0^a \frac{dK_C}{dx} dx \\ &= \frac{4k_s a}{\ln(b_2/b_1)} \int_0^a \frac{dx}{\sqrt{a^2 - x^2}} \\ &= \frac{4k_s a}{\ln(b_2/b_1)} \sin^{-1} \left(\frac{x}{a} \right) \Big|_0^a \\ &= \frac{2\pi k_s a}{\ln(b_2/b_1)} \end{aligned}$$

or,

$$\delta R'_C = \ln(b_2/b_1)/2\pi k_s a = \delta R_C.$$

in that case.

The difference between the temperatures of contacts 1 and 2 will be $Q\delta R_C$ if the total heat flow rate through contact 2 is Q . Since contact 2 is assumed to be isothermal, the heat flow rate through contact 2, per unit length in the x direction, designated Q_x , can be calculated (as a function of x) from the expression defining the heat fluxes through an isothermal contact on an infinite half-space and then used to determine the linear conductance dK_C/dx for which,

$$\frac{Q_x}{dK_c/dx} = Q\delta R_c \quad (\text{II.E.84})$$

at all axial (x) locations within the contact. Equation (II.E.84) must be satisfied at all axial locations within the contact because contact 1 was also assumed to be isothermal in the calculation of $\delta R'_c$. Equation (II.E.84) will define dK_c/dx uniquely at all values of x in the range $-a \leq x \leq a$.

Heat flow through an isothermal elliptical contact on the surface of an infinite half-space is one-dimensional with respect to the λ coordinate in the ellipsoidal coordinate system, as discussed in Chapter I.4. The temperature gradient normal to the surface of such a contact is defined by the expression,

$$\begin{aligned} \frac{\partial T}{\partial n} &= - \left. \frac{\partial T}{\partial z} \right|_{\lambda=0} = - \frac{2A}{\sqrt{(\lambda-\mu)(\lambda-\nu)}} \\ &= - \frac{2A}{\sqrt{\mu\nu}} \quad (\text{since } \lambda = 0 \text{ at the contact}) \end{aligned} \quad (\text{II.E.85})$$

as shown in Chapter I.4 so the corresponding surface heat flux distribution will be,

$$q''(\mu, \nu) = \frac{2k_s A}{\sqrt{\mu\nu}} \quad (\text{II.E.86})$$

At $z=0$, the μ coordinate is related to x and y by the expression,

$$\frac{x^2}{a^2 + \mu} + \frac{y^2}{b^2 + \mu} = 1 \quad (\text{II.E.87})$$

so solving for μ with the quadratic formula we get,

$$\mu = \frac{(x^2 + y^2 - a^2 - b^2) \pm \sqrt{(a^2 + b^2 - x^2 - y^2)^2 - 4(a^2 b^2 - x^2 b^2 - y^2 a^2)}}{2} \quad (\text{II.E.88})$$

Equation (II.E.88) should provide the result that $\mu=0$ when $x=a$ and $y=0$ so the positive sign in front of the second term is correct. The right-hand side of Equation (II.E.88) also defines v when $z=0$ except that the negative sign in front of the second term is correct in that case because $v=-b^2$ when $x=a$ and $y=0$.

Replacing μ and v in Equation (II.E.86) by the right-hand side of Equation (II.E.88) with positive and negative signs in front of the second term respectively, we get,

$$q''(x,y) = \frac{A}{a\sqrt{s^2 - y^2}} \quad (\text{II.E.89})$$

after simplifying. The parameter s in Equation (II.E.89) is simply the half-width of the contact at x , viz.,

$$s = b\sqrt{1 - \left(\frac{x}{a}\right)^2} \quad (\text{II.E.90})$$

The constant A in Equation (II.E.89) can be related to the total heat flow rate through the contact (Q) since,

$$\begin{aligned} Q &= 4 \int_0^a \int_0^s q''(x,y) dy dx \\ &= \frac{4A}{a} \int_0^a \left[\int_0^s \frac{dy}{\sqrt{s^2 - y^2}} \right] dx \\ &= \frac{4A}{a} \int_0^a \left(\sin^{-1} \left(\frac{y}{s} \right) \Big|_0^s \right) dx \end{aligned}$$

$$= \frac{2\pi A}{a} \int_0^a dx$$

$$= 2\pi A$$

and hence,

$$A = \frac{Q}{2\pi}$$

Thus,

$$q''(x,y) = \frac{Q}{2\pi a \sqrt{s^2 - y^2}} \quad (\text{II.E.91})$$

The heat flow rate per unit length in the x direction (i.e., Q_x) can now be obtained from the expression,

$$\begin{aligned} Q_x &= 2 \int_0^s q''(x,y) dy \\ &= \frac{Q}{\pi a} \int_0^s \frac{dy}{\sqrt{s^2 - y^2}} \\ &= \frac{Q}{\pi a} \sin^{-1} \left(\frac{y}{s} \right) \Big|_0^s \\ &= \frac{Q}{2a} \end{aligned} \quad (\text{II.E.92})$$

Introducing Equation (II.E.92) into Equation (II.E.84) with

$\delta R_c = \ln(b_2/b_1)/2\pi k_s a$ we obtain,

$$\frac{dK_c}{dx} = \frac{Q_x}{Q \delta R_c} = \frac{\pi k_s}{\ln(b_2/b_1)}$$

as the equation which defines dK_c/dx uniquely. This expression is the same as Equation (II.E.83). Since contacts 1 and 2 are both isothermal and differ in temperature by the required amount (i.e., $Q\delta R_c$) when dK_c/dx and Q_x are defined by Equations (II.E.83) and (II.E.92) respectively, it follows that prescribing the linear conductance dK_c/dx over the length of contact 2 will cause the resulting total resistance to be that of contact 1.

Since prescribing the linear conductance dK_c/dx over the length of contact 2 is equivalent to inserting dR_c (defined by Equation (II.E.77)) between each strip of thickness dx on contact 2 and the sink temperature (T_0), the latter procedure would also provide the correct resistance for contact 2. Noting that Equation (II.E.77) can be obtained from Equation (II.E.34) by substituting s_2 and s_1 for $y_{n+1/2}$ and b respectively and including the term dx in the denominator to account for the fact that Equation (II.E.77) applies to a contact of length dx instead of unity, it follows that the incremental resistance (dR_c) that must be assigned to each slice of thickness dx in contact 2 in order to approximate contact 1 could be obtained by treating the portions of those contacts within each slice as though they were line contacts with half-widths of s_2 and s_1 respectively. The similarity between the heat flow through a slice of thickness dx in an elliptical contact and the heat flow through a line contact of the same half-width (i.e., s) even extends to include the heat flux distribution since Equation (II.E.91) can be rewritten as,

$$q''(x,y) = \frac{Q_x}{\pi \sqrt{s^2 - y^2}} \quad (\text{II.E.93})$$

by substituting Q_x for $Q/2a$. Except for the substitution of s for b and Q_x for Q' , Equation (II.E.93) is identical to Equation (II.E.18) defining the heat flux distribution within an isothermal line contact.

The results of the foregoing analysis suggest that the methods which were derived from Equation (II.E.34) to approximate line contacts can be used to approximate the discrete contact widths seen within the elliptical contact in Figure II.2.9. Since the foregoing analysis applied to elliptical contacts with aspect ratios greater than three or four because of the limitations of Equation (II.E.75), it is expected that the contact approximation techniques developed previously would lose accuracy when applied to elliptical contacts with aspect ratios smaller than this. It was demonstrated that contact 1 could be approximated by assigning the incremental resistance dR_c to each slice of thickness dx in a larger elliptical contact with the same major diameter (i.e., contact 2) but it is assumed that the larger contact could equally well be the surface segment bounded by the different values of $y_{n+1/2}$ for the different discrete contact widths seen in Figure II.2.9.

Results showing the accuracy that is achieved when an elliptical contact is approximated in the manner shown in Figure II.2.9 are given in Chapter II.5 for contacts with aspect ratios of 16 (i.e., the model contact) and 3.27.

APPENDIX II.F

ADDITIONAL CAPABILITIES OF THE FAS MULTIGRID ALGORITHM

As mentioned in Section II.4.2.2, the FAS multigrid algorithm offers several advantages over the CS algorithm even when applied to linear problems. Some of the more important advantages of the FAS algorithm are outlined in the sections which follow.

II.F.1 Natural Convergence Criterion

The FAS multigrid algorithm provides a natural criterion for deciding when the fine grid solution has converged, in other words when u^M is sufficiently close to U^M to warrant stopping the iterations in an accommodative algorithm. Two quantities are required to define the natural stopping criterion, these being;

- (1) The G^M truncation error, τ^M , defined by the expression,

$$\tau^M = L^M U - F^M \quad (\text{II.F.1})$$

where U is the exact solution to the original boundary value problem governed by equations (II.4.1a) and (II.4.1b) and,

- (2) The G^M static residual, f^M , already defined (in equation (II.4.5))

as

$$f^M = F^M - L^M u^M \quad (\text{II.F.2})$$

It is generally agreed [126] that an iterative solution to finite-difference or finite-element equations should be regarded as having

converged when the static residual (r^M) is smaller than the truncation error (τ^M) since further reductions in the size of the static residual are not meaningful, at least insofar as approximating the exact solution (U) is concerned. Formally, the iterations should be stopped when,

$$||r^M|| < ||\tau^M|| \quad (II.F.3)$$

where $||\cdot||$ is any appropriate vector norm (e.g., the 2-norm, $||\cdot||_2$).

Although equation (II.F.3) represents a natural stopping criterion for any iterative procedure applied to finite-difference or finite-element equations, it usually cannot be applied because τ^M cannot be estimated. The FAS multigrid algorithm, however, permits τ^M to be estimated (and hence equation (II.F.3) applied) because the expression

$$\tau_M^{M-1} = L^{M-1} U^{M-1} - F^{M-1} \quad (II.F.4a)$$

$$= F^{M-1} - F^{M-1} \quad (\text{by equation (II.4.19)}) \quad (II.F.4b)$$

defines the G^M approximation to the G^{M-1} truncation error, denoted τ_M^{M-1} [135]. The quantity F^{M-1} in equation (II.F.4b) is simply the right-hand side vector for the G^{M-1} equations used to obtain the V^M correction (see equation (II.4.19)) and hence is always computed in a multigrid solution, while F^{M-1} is obtained by evaluating the right-hand side of equation (II.4.1a) at the G^{M-1} grid points.

It should be mentioned that the values of u at coincident nodes on all grids are equal* when an FAS solution has converged, so the values of u in the vectors U^{M-1} (which, in equation (II.F.4a), is the exact solution to equation (II.4.19)) and U^M are equal* at grid points which are common.

* Approximately.

to both grids (i.e., $u^{M-1}(i/2, j/2) = u^M(i, j)$ at convergence).

Although τ_M^{M-1} is an estimate of the truncation error on G^{M-1} , the truncation error on G^M (i.e., τ^M) can be estimated from τ_M^{M-1} if the approximation order of the finite-difference or finite-element discretization is known. If the truncation error of the discrete operator is $O(\Delta^g)$ (where $g = 2$ for central-difference representations of the Laplacian operator), we can compute $||\tau^M||$ approximately as,

$$\begin{aligned} ||\tau^M|| &= 2^{-g} ||\tau_M^{M-1}|| \quad (\text{for } g = 1/2) \\ &= \frac{1}{4} ||\tau_M^{M-1}|| \quad (\text{for } g = 2) \end{aligned} \quad (\text{II.F.5})$$

as shown by Brandt and Dinar [135]. This estimate of $||\tau^M||$ is usually compared to the norm of the dynamic residuals [127], rather than the static residuals, to determine when the iterations should be stopped because the dynamic residuals, which are calculated during the relaxation sweeps, are less expensive to compute.

II.F.2 High Order Approximations

The G^M estimate of the G^{M-1} truncation error, denoted τ_M^{M-1} in the preceding section, can be used to obtain a fourth-order accurate solution on G^{M-1} even if the finite-difference equations used on all of the grids are only second order accurate (i.e., have truncation errors which are $O(\Delta^2)$ in size). To demonstrate this, note that equation (II.F.4b) can be substituted into equation (II.4.19) to obtain,

$$L^{M-1} u^{M-1} = F^{M-1} + \tau_M^{M-1} \quad (II.F.6)$$

where, according to Brandt [127],

$$\tau_M^{M-1} = \tau^{M-1} - \tau^M \quad (II.F.7)$$

When the equation that is satisfied on G^{M-1} is written in the form of equation (II.F.6), it is obvious that the quantity τ_M^{M-1} is a correction which makes the solution of the G^{M-1} equations coincide with the solution of the G^M equations, at least at points which are common to both grids. It is actually possible to find a correction that is closer to τ^{M-1} , however, by noting that τ^{M-1} and τ^M must both obey a relationship of the form,

$$\tau^{\Delta}(\bar{x}) = C(\bar{x})\Delta^2 + O(\Delta^4) \quad \bar{x} = (x_1, \dots, x_d) \quad (II.F.8)$$

if second-order accurate difference equations are used to approximate $\mathcal{L}$ on a grid of mesh size Δ . Brandt [128] states that the factor $C(\bar{x})$ in equation (II.F.8) is independent of Δ , in which case equation (II.F.7) can be written as,

$$\tau_M^{M-1} = C(\bar{x}) (\Delta_{M-1}^2 - \Delta_M^2) + O(\bar{\Delta}^4), \text{ where } \Delta_M \leq \bar{\Delta} \leq \Delta_{M-1} \quad (II.F.9)$$

and hence,

$$C(\bar{x}) = \frac{\tau_M^{M-1}}{(\Delta_{M-1}^2 - \Delta_M^2)} + O(\bar{\Delta}^4) \quad (I.F.10)$$

Introducing equation (II.F.10) into equation (II.F.8) and evaluating the resulting equation for $\Delta = \Delta_{M-1}$, we obtain,

$$\begin{aligned}\tau^{M-1} &= \frac{\tau_M^{M-1}}{(1-\beta^2)} + O(\Delta_{M-1}^4) \\ &= \frac{4}{3} \tau_M^{M-1} + O(\Delta_{M-1}^4) \quad (\text{for } \beta = \frac{1}{2})\end{aligned}\quad (\text{II.F.11})$$

as the improved estimate of τ^{M-1} mentioned previously. Replacing τ_M^{M-1} in equation (II.F.6) by the improved estimate in equation (II.F.11) is called "local truncation extrapolation" or " τ -extrapolation". The replacement can be made after the G^M solution has converged, in which case the resulting (new) G^{M-1} problem is solved using the normal multigrid algorithm until a convergence criterion similar to that discussed in Section II.F.1 is satisfied, or it can be incorporated into a fixed algorithm, as discussed by Brandt and Dinar [135]. In either case, the high accuracy G^{M-1} solution is interpolated up to G^M as a correction but a final relaxation sweep on G^M is not performed since such a sweep would lower the accuracy of the final solution.

Numerical results obtained by applying multigrid with τ -extrapolation to a geothermal reservoir flow problem are reported by Gary in reference [137] and compared to results obtained using another high order multigrid technique proposed by Lindberg [138]. The multigrid algorithms used by Gary were all fixed algorithms since τ -extrapolation can be combined with these very effectively.

II.F.3 Time-Dependent Problems

While the author did not apply multigrid to time-dependent heat conduction problems in this study, a brief discussion of how such problems can be solved with the method is nevertheless warranted because the method's capabilities in this area are somewhat unusual. The most obvious way to apply multigrid to time-dependent heat conduction problems would be to use it to solve the simultaneous finite-difference equations that are obtained when the time-derivative in the heat equation is expressed as anything other than a forward-difference, but that procedure would not take full advantage of multigrid's ability to distinguish between high and low frequency components of the solution. The highest frequency components are resolved on the finest grid while lower frequency components are adequately resolved on the coarser grids, of course.

According to Brandt [127], multigrid can be used to exploit the very different rates of change of high-frequency and low-frequency components in a time-dependent problem, where high-frequency components of the solution always converge to a steady-state much more quickly than those of low frequency. Since the high frequency components contribute less and less to the changes in the solution as the transient progresses, those changes can be approximated on increasingly coarser grids. Brandt states that the changes in error components with wavelengths smaller than $O(\Delta^n)$ can be neglected, without neglecting the components themselves, by freezing the correction term τ_{n+1}^n in the coarse grid correction equation for G^{n+1} written in the form of Equation (II.F.6) (with M replaced by $n+1$) since this freezes the high frequency part of the solution but retains

its influence on the coarse grid equations. When τ_{n+1}^n is frozen, only the grids G^r for which $r \leq n$ are visited in the multigrid computations, of course. Visits to the finest grid are made infrequently to readjust the high-frequency components.

Brandt notes that the technique just described, known as the "frozen- τ " technique, may permit the time derivative in the heat equation to be forward-differenced without requiring that prohibitively small time steps be used* because the finite-difference equations are solved on coarse grids over most of the transient. Further details on this and other aspects of multigrid applied to time-dependent problems are given in [127] and [140].

* As discussed in greater detail in Section III.2, stability considerations limit the time step that can be used when the time derivative is forward-differenced in a time-dependent heat transfer analysis to a value which is $O(\Delta^2)$ in size. Relatively large time steps can be used on coarse grids for that reason.

APPENDIX II.G

DESCRIPTION OF THE FORTRAN IV COMPUTER PROGRAM "MULTIGRID" WRITTEN TO ANALYZE THE MODEL CONTACT PROBLEM

II.G.1 Introduction

The Fortran IV computer program "MULTIGRID" was developed from a simple 146 line multigrid program given by Brandt in Appendix B of reference [126]. Brandt's and the author's programs are similar in structure but nevertheless very different in detail. Brandt's program solves Poisson's equation in a two-dimensional rectangular domain with Dirichlet boundary conditions using the 5-point "star" difference operator, Gauss-Seidel iteration with lexicographic ordering as the smoothing scheme, injection for the fine-to-coarse mesh residual transfers and bilinear interpolation for the coarse-to-fine grid correction transfers. Brandt's program also operates in the "Correction Storage" (C.S.) mode and hence does not have any of the capabilities discussed in Sections II.4.2.3, II.F.1 and II.F.2. The program MULTIGRID on the other hand, solves the heat conduction equation in a three-dimensional rectangular domain with the boundary conditions applying to the model contact problem. Analyses of the latter problem can be performed with or without gas conduction and with either an isothermal boundary condition or a boundary condition of the third kind prescribed over the contact area. MULTIGRID uses half-weighting of the residuals at interior nodes and full-weighting at boundary nodes where non-Dirichlet boundary conditions are prescribed, the 7-point "star" difference approximation to the three-dimensional heat conduction operator,

Gauss-Seidel iteration with "checkerboard" ordering (boundary points are relaxed before interior points) as the smoothing scheme, and trilinear interpolation for the coarse-to-fine mesh correction interpolations. The program operates as either a "cycling" or a "Full Multigrid" (FMG) accommodative algorithm or, alternatively, as a fixed FMG algorithm, and uses trilinear interpolation to obtain the first approximation on G^n ($n=1, \dots, M$) from the solution obtained previously on G^{n-1} . The program also operates in the "Full Approximation Storage" mode and has all of the capabilities discussed in Sections II.4.2.3, II.F.1 and II.F.2. In particular, grids which are not co-extensive can be nested anywhere within the solution domain or on its boundary. The grids are always nested at one corner of the rectangular bar used as the contact specimen in analyses of the model contact problem because of symmetry considerations, but the program is not limited to grids nested in that manner.

II.6.2 Input Data

The input variables required by MULTIGRID are listed and defined at the top of the program listing, included as Listing II.6.1. The first three input variables read in, XLEN, YLEN and ZLEN, are the dimensions of the rectangular body being modelled in the x, y and z coordinate directions respectively. The x, y, z coordinate system referred to here is not the contact-centered system but rather is a system in which the x and z axes are parallel to the contact minor and major axes respectively and the y axis is perpendicular to the contact plane.

The next three variables read in, NX0, NY0 and NZ0, are the number

of subdivisions used on the coarsest grid in the x, y and z directions respectively. A total of M grids is used in the multigrid model, where grid 1 is the coarsest and grid M is the finest.

The variables TC, TCGAS, A, B, RCY1 and RCY2 require no explanation other than that given at the top of the program listing. Any set of units can be used provided that the units are consistent.

The variable CCNN is a parameter used in the non-continuum gas conductivity model. This variable is the same as the input variable CCNN used in the synthesis program described in Part IV and hence will not be defined here.

The input variable REDFAC is the ratio of the residual norm to the truncation error norm at which the iterations are to be stopped. Hence the iterations are stopped when,

$$||f^M|| \leq \frac{||\tau^M||}{\text{REDFAC}} = \frac{||\tau_M^{M-1}||}{4 \cdot \text{REDFAC}} \quad (\text{II.G.1})$$

Equation (II.G.1) reduces to equation (II.F.3) when REDFAC=1 and provides additional reductions in $||f^M||_2$ when REDFAC>1.

The variable WMAX is an upper limit for the number of work units performed by the program, where a work unit (WU) is defined (by Brandt [126]) as the computational work required to perform one relaxation sweep on the finest grid, where the finest grid in this context is the final, composite grid, an example of which is shown in Figure II.4.5. Although the final, composite grid consists of several subgrids, no mesh point is counted more than once for the purpose of obtaining the WU even if the mesh point is repeated several times on several grids. Program

execution is normally stopped when the residuals (dynamic) on the finest grid satisfy equation (II.G.1), but in the event that the algorithm is not converging for some reason, the computations are stopped when the work unit count (kept by the program) exceeds WMAX. Setting WMAX to 25 usually ensures that program execution is not halted prematurely.

The variables M, INSUL, NEXTR, NCYCAL and NSPSW require no explanation other than that given at the top of the program listing, except perhaps to note that results are obtained both with and without τ -extrapolation when the accommodative algorithm is used with NEXTR=1. The solution without τ -extrapolation is obtained and printed out before the one with τ -extrapolation in that case.

The variables NZDIS(I) (I=1,2,...10) indicate which grids (other than the first) are to be obtained from the next coarser grid by halving the subdivisions on that grid only in the x and y directions instead of in all of the coordinate directions. Specifically, only the subdivisions in the x and y directions are halved to obtain the Ith grid if NZDIS(I)=1 while the subdivisions in all directions are halved to obtain the Ith grid if NZDIS(I)≠1. If M>10, the upper limit for the implied DO loop in the READ statement for NZDIS(I) should be adjusted accordingly along with the corresponding FORMAT statement.

The variables HPATCH, TBASE and NH are adequately defined at the top of the program listing.

The variables NIIC(NG), NFIC(NG), NIJC(NG), NFJC(NK), NIKC(NG) and NFKC(NG) are grid lines on the (NG-1)st grid which define the locations of the boundaries for the next finer grid, NG. The first and last nodes in the x direction on grid NG coincide with the x-axis nodes NIIC(NG) and

NFIC(NG) on grid NG-1, for instance. The y-axis nodes NIJC(NG) and NFJC(NG) on grid NG-1 coincide with the first and last nodes (respectively) on grid NG in the y direction and the z-axis nodes NIKC(NG) and NFKC(NG) similarly serve to define the boundaries of grid NG in the z direction. An example of the manner in which the grid limits are specified is given in Figure II.G.1 (for an x-y grid).

II.G.3 Grid Data Storage

The grid function values and finite-difference equation right-hand sides are stored in the manner recommended by Brandt [136]. For each grid G^n (where $1 \leq n \leq M$ in the program), both v^n and f^n are stored sequentially in a large vector Q which thus stores these arrays in the order $v^1, f^1, v^2, f^2, \dots, v^M, f^M$, as shown below.

The first entry in v^n is stored as $Q(NST(n))$ and the first entry in f^n is stored as $Q(NST(n+M))$, as illustrated. Entries in the pointer vector

Figure II.G.1 Example of Procedure For Specifying G^{NG} Boundary Limits Using G^{NG-1} Grid Lines.

NST are defined in subroutine GRDFN which, along with subroutine KEY, defines most of the grid-specific parameters. Most of the routines in the program, such as subroutines RELAX and INTADD which perform the relaxation sweeps and the coarse-to-fine grid correction interpolations respectively, are written to be used with all grids so, to use grid n for example, the statement `CALL KEY (n,IST,KBST,IMAX,JMAX,KMAX,DXX,DYY,DZZ)` retrieves the grid parameters $IMAX, JMAX, \dots, DZZ$ and defines an array $IST(i,k)$ such that $v_{i,j,k}^n = Q(IST(i,k)+j)$ (for grid points, x_i, y_j, z_k ; $i = 1, \dots, IMAX, j=1, \dots, JMAX$ and $k=1, \dots, KMAX$). Another statement, `CALL KEY (n+M, IRC, \dots, DZZ)` defines the entries in $IRC(i,k)$ so that $f_{i,j,k}^n = Q(IRC(i,k)+j)$. Thus the array f^n is effectively stored as v^{n+M} . The variables DXX, DYY and DZZ in the call list for subroutine KEY are the subdivision sizes used in the x, y and z directions respectively and $KBST(K)$ is defined (by KEY) so that $h_{i,k}^n = HX(KBST(k)+i)$, where $h_{i,k}^n$ is the heat transfer coefficient on the contact surface (where $j = 1$) at the point x_i, z_k on grid n .

The main advantage of storing the arrays v^n and f^n ($n=1,2,\dots, M$) sequentially in Q in the manner illustrated previously is that new grids can be added in an adaptive discretization (see Section II.4.2.3) without disturbing the data already stored for other grids and without defining any new arrays.

II.G.4 Routine Descriptions

Main Program (right-hand line numbers 1-29.5)

The main program both reads all of the input data required by MULTIGRID and calculates parameters describing the M grids, such as the

volume and number of nodes in each grid, denoted $VG(n)$ and $NPT(n)$ ($n=1,2,\dots,M$) respectively. The grid volumes are used in subroutine MULTIG in a manner discussed in the description of that routine. The total number of nodes in the composite grid, $NPOINT$, is also calculated, without counting repeated nodes (i.e., nodes which appear on more than one grid) more than once.

Six fairly important variables, specifically $NIBI$, $NJBI$, $NKBI$, $NIBF$, $NJBF$ and $NKBF$, are defined in the main program. The variable $NIBI$ represents the first (coarsest) grid for which the y - z plane at $i=1$ is not a boundary plane (i.e., face) of the rectangular body being modelled while $NIBF$ represents the first grid for which the y - z plane at $i=IMAX$ is not a boundary plane. If $NIBI=3$ and $NIBF=4$, for instance, then the first x -axis nodes on grids 1 and 2 and the last x -axis nodes on grids 1-3 would be boundary nodes but the first x -axis nodes on grids 3, ..., M and the last x -axis nodes on grids 4, ..., M would not lie on the boundary of the rectangular body being modelled. Variables $NJBI$ and $NJBF$ serve the same purpose as $NIBI$ and $NIBF$ respectively, only for nodes on the y -axis instead of the x -axis, while $NKBI$ and $NKBF$ are similarly related to the locations (i.e., boundary or interior) of the nodes for which $k=1$ and $k=KMAX$ respectively.

The x , y and z coordinates of the first x -axis, y -axis and z -axis nodes in the NG th grid, designated $XI(NG)$, $YI(NG)$ and $ZI(NG)$ respectively, are also calculated in the main program for $NG=1, \dots, M$. These coordinates permit the x , y and z coordinates of any node I, J, K in the NG th grid to be calculated very easily, as seen in lines 31.7-31.9 in the program listing (where $NN=NG$ in those statements).

SUBROUTINE MULTIG (DXO, DYO, DZO, WMAX, U1, F, NXO, NYO, NZO, M, XLEN,
YLEN, ZLEN)

This subroutine controls the overall flow of the multigrid algorithm. Two versions of the subroutine are provided, one of which applies to the "accommodative" algorithm where various tests control the sequence of operations, and another that implements the "fixed" algorithm, in which grids interact in a predetermined, fixed manner. A listing of subroutine MULTIG for the accommodative algorithm is found between lines 34.1 and 48.1 in Listing II.G.1 while the code for the subroutine applying to the fixed version of the algorithm is provided as Listing II.G.2. The latter routine can simply be substituted for the former to apply the fixed algorithm instead of the accommodative one.

MULTIG For the Accommodative Algorithm

The accommodative program operates either as a "cycling" algorithm, which starts on the finest mesh and cycles up and down over the coarser levels until the fine grid solution has converged (as shown in Figure II.4.3), or as a "Full Multigrid" algorithm which starts on the second to coarsest grid and works its way up to the finest. The solution is iterated on each level until the residuals are less than the truncation errors, at which point the solution is interpolated up to the next finer level where the procedure is repeated. A flowchart of the FMG algorithm is shown in Figure II.G.2. The currently finest level is denoted level k in the flowcharted algorithm and ML in the program. Similarly, the current working level is denoted level n in Figure II.G.2 and NG in the

program. Thus $n \leq 2$ at all times. Levels 1 and M are the coarsest and finest grids respectively.

The iterations in the FMG algorithm start on level 2, so u^2 must be equated to some initial approximation u_0^2 , as seen in the first box in Figure II.G.2. In the cycling algorithm, u^M must be equated to u_0^M if all of the grids are coextensive, otherwise initial values of u must be assigned to all of the nodes on G^{NG} that are not found on G^{NG+1} ($NG=1, \dots, M-1$). Similarly, initial values of u should be assigned to all of the nodes on G^1 that are not found on G^2 if G^1 and G^2 are not coextensive in the FMG algorithm. The right-hand side (F) terms must also be defined at all of these nodes. The DO loop which starts at line 49 and ends at line 52* in subroutine MULTIG (Listing II.G.1) performs all of these initializations and assignments for the cycling and FMG algorithms and, in addition, calls subroutine HX0 to define the contact surface heat transfer coefficients for the same grids. Line 50 calls subroutine PUTF to initialize the temperatures on the NGth grid while line 51 calls the same subroutine to assign values to the right-hand side terms. The initial approximations and right-hand side terms are obtained from functions defined in the FUNCTION G and FUNCTION F subroutines respectively.

When the FMG algorithm is being used, the currently finest level, ML, is equated to 1 in line 58 and then incremented by 1 in line 62 to make

* Unless otherwise stated, the line numbers referred to in the descriptions of routines are the left-hand line numbers.

its first value 2. The currently finest level, ML, is equated to M-1 in line 59 in the cycling algorithm, however, since it then becomes M, as required, in line 62. The working level, NG, is equated to ML in line 65 since the iterations which ultimately produce a solution on the currently finest level always start on that level. This operation is equivalent to the operation $n \leftarrow x$ seen in box 2 in Figure II.G.2. The operation $\epsilon_x \leftarrow \epsilon_x^0$ in the same box is performed in line 66, where ϵ_x is designated EPS(ML) in the program and $\epsilon_x^0 (=1.E-6)$ is an error tolerance that is so small that the comparison seen in box 4 will never be satisfied after the first relaxation sweep on level ML. Thus at least one CGC will always be performed for level ML.

Subroutine PUTF is called by line 72 to initialize the right-hand side vector for the currently finest level (ML), an operation which is equivalent to the assignment $f^L \leftarrow F^L$ seen in box 2. Line 73 then calls HXO to define the contact surface heat transfer coefficients on G^{ML} .

As indicated in box 3 in Figure II.G.2 and line 77 in MULTIG, a relaxation sweep on level NG is performed next. The work unit count (WU) is subsequently incremented in line 80 (the work unit increment for grid NG, designated WUINC(NG), is defined as $NPT(NG)/NPOINT$ in line 27.6 (right-hand line number) in the main program) after which the working level (NG), the dynamic residual just calculated in subroutine RELAX for that level (ERR2), and the current work unit count (WU) are printed out by line 81. The dynamic residual ERR2 ($=e_n$ in Figure II.G.2) is compared to the convergence parameter EPS(NG) ($=\epsilon_n$) in line 94 in MULTIG (box 4 in Figure II.G.2), after which;

(1) line 104 returns control to the main program if the convergence check is satisfied and $NG=M$ since the solution has been obtained in this case.

(2) line 106 causes lines 137-140 to be executed (i.e., subroutine INTADD is called to perform the coarse-to-fine mesh correction interpolations, the working level is increased by 1, and a relaxation sweep on the new working level is performed) if $NG \neq ML$ (i.e., $n \neq 1$) and convergence on level NG has been achieved. These operations are shown in boxes 5, 6 and 3 in the algorithm flowchart.

(3) subroutine INTERP is called by line 109 to interpolate an initial approximation up to G^{ML+1} if convergence has been achieved and $NG=ML(<M)$. The currently finest level (ML) is incremented by 1 in line 62 and iterations start on the new finest level. Boxes 13, 12 and 2 indicate this sequence of operations in the algorithm flowchart.

(4) line 112 causes another relaxation sweep to be performed (i.e., line 77 is re-executed) if $NG=1$ and convergence has not been achieved because the solution on the coarsest grid must always be iterated to convergence. If convergence has not been achieved and $NG \neq 1$, the working level is reduced by 1 in line 118 (box 7), subroutine PUTZ is called in line 119 to perform the operation indicated in box 8, and subroutine RESCAL is called in line 120 to perform the fine-to-coarse grid residual transfers (box 9). Line 124 then calculates the convergence parameter for the G^{NG} problem (i.e., $EPS(NG)$) as a fraction of the last dynamic residual (ERR2) on level $NG+1$, as indicated by the operation $\epsilon_n = \delta \epsilon_{n+1}$

($\delta = .15 \sqrt{V_g^n / V_g^{n+1}}$, where V_g^n is the volume of the n th grid) shown in box 9

in Figure II.G.2. The factor involving the square roots of the grid volumes is required to keep the dynamic residual norms (the 2-norm is used) on grids with different volumes compatible and the factor 0.15 is a constant that is not problem specific and hence will never need to be changed. If $NG=ML-1$, line 127 redefines the convergence parameter to be $1/4$ of the G^{ML} estimate of the G^{ML-1} truncation error, designated $RESERR(NG-1)$ in the program and τ_{ML}^{ML-1} in Section II.F.1, as noted in box 10 of the algorithm flowchart. The factor $REDFAC$ is included in the denominator of line 127 (i.e., DEN) to permit further reductions in the residual norm.

Box 11 in Figure II.G.2 is only executed when τ -extrapolation is used to obtain a high accuracy solution on G^{M-1} . The G^M estimate of the G^{M-1} truncation error is extrapolated when $NEX=1$ in subroutine $MULTIG$, but NEX is not set to 1 until the G^M solution has converged (see lines 95-103). The user-specified variable $NEXTR$ must be unity in order for τ -extrapolation to be applied, of course. The actual extrapolation itself is performed in subroutine $RESCAL$.

MULTIG for the Fixed Algorithm (Listing II.G.2)

The version of subroutine $MULTIG$ applying to the fixed multigrid algorithm shown in Figure II.4.4 should be fairly easy to understand after studying the routine for the accommodative algorithm and hence will not be discussed in detail. The algorithm starts on the second level, solves the G^2 problem until the residual norm is less than the norm of the truncation errors, and then performs the fixed sequence of operations

indicated in Figure II.4.4 until the G^M solution has been obtained. As in the version of MULTIG for the accommodative algorithm, the currently finest level and the working level are designated ML and NG respectively. The variable NCYCLE has a value of 1 during the first CGC for level ML and a value of 2 during the second CGC for that level. After two sweeps and two CGC's on level ML, the algorithm proceeds to the next finer level (i.e., ML is incremented by 1), as seen in Figure II.4.4. The variable NSWEEP represents the number of sweeps (plus one) that have been performed on the coarsest grid (where NG=1) and hence is 1 in value during a CGC until the algorithm has worked its way down to that level. A total of NSPSW relaxation sweeps are performed on level 1 in each CGC. When NSWEEP=1 after a relaxation sweep on any level (NG), NG is reduced by 1 (i.e., lines 54-62 in Listing 2 are executed) whereas if NSWEEP $\neq$ 1 after a relaxation sweep and NG $\neq$ ML, NG is incremented by 1 (i.e., lines 63-66 are executed). A variable NXX, which is initialized to 0 in line 26 and subsequently set to 1 in line 45 when level 1 is visited, is used in line 31 to determine when the second CGC on level ML has started. Subroutine INTERP is called in line 52 to interpolate the G^{ML} solution up to G^{ML+1} (unless ML=M) if NCYCLE=2 and NG=ML. Provision is made for using τ -extrapolation (when NEXTTR=1) in the G^{ML} to G^{ML-1} residual transfers when the second CGC is started (i.e., when NCYCLE=2).

FUNCTION F(X, Y, Z, XLEN, YLEN, ZLEN) (Lines 32.5-33.2)

This subroutine calculates the right-hand side function F at the coordinates x, y and z in a rectangular body of dimensions XLEN x YLEN x ZLEN. The program user must specify the functional relationship in terms of these variables.

FUNCTION G(X, Y, Z, XLEN, YLEN, ZLEN) (Lines 33.3-34)

The function G serves to define the first approximation on level 2 in the multigrid algorithm in terms of the indicated variables. The function also defines the Dirichlet boundary conditions in the model contact problem, although the temperatures of the nodes in the isothermal contact patch must be reset to the contact temperature (assumed 0) after each interpolation for reasons mentioned in the discussion of subroutine INTADD.

SUBROUTINE GRDFN (N, IMAX, JMAX, KMAX, DXX, DYY, DZZ, M) (Lines 120.3-122.6)

The Nth entry in the pointer vector NST (i.e., NST(N)) is set equal to IQ (which is the next available location in the Q vector) in this routine, which thus allocates space in Q for each v^n and f^n as discussed previously. The number of grid points in each direction and the subdivision sizes are also stored for each grid along with a variable NBST(N), which is the location in the vector of contact surface heat transfer coefficients (HX) of the heat transfer coefficient for the first node on the contact surface (at which $i=j=k=1$) of the Nth grid.

SUBROUTINE KEY (NG, IST, KBST, IMAX, JMAX, KMAX, DXX, DYY, DZZ) (Lines 117.1-120.2)

Subroutine KEY is called by any routine which requires details about a particular grid, such as the values of IMAX, ..., DZZ and the entries in the arrays IST and KBST, both of which have been discussed previously.

When the value of NG is $\leq M$, subroutine KEY defines entries in the array IST(i,k) so that $v_{i,j,k}^{NG} = Q(IST(i,k)+j)$. If NG > M, entries in IST(i,k) are defined so that $f_{i,j,k}^{NG-M} = Q(IST(i,k)+j)$.

SUBROUTINE HXO(NG) (Lines 129.6-135.0)

The routine defines contact surface heat transfer coefficients for grid NG. Coefficients for nodes outside of the contact patch are calculated using an expression derived in Appendix I.A for the gap (SEPN) between the contact surfaces and a non-continuum gas conductivity model, as in subroutine TKGAS in RECELL (see Appendix I.C), while coefficients for nodes inside of the patch are set to the user-specified contact conductance HPATCH in the version of subroutine HXO found in Listing II.G.1. That version of HXO applies to the contact approximation technique designated method 1 in Section II.2.5.3 - versions applying to contact approximation techniques 3 and 4 are provided as Listings II.G.3 and II.G.4 respectively. All versions of HXO use approximations to elliptic integrals which are only valid for large aspect ratio contacts, say for contacts with $A/B > 4$.

SUBROUTINE INTADD (NGC, NGF) (Lines 52.1-62.7)

Trilinear interpolation is used in this subroutine to transfer the FAS corrections from grid NGC to grid NGF (see box 6 in Figure II.G.2). The corrected temperatures of all of the nodes in grid NGF are obtained first, after which the temperatures of nodes within the contact patch on the fine grid are set to the sink temperature, assumed to be zero, if an isothermal contact boundary condition is being used. The temperatures of nodes within the isothermal contact must be reset to the correct values after the correction interpolations because some of them, specifically those nearest the boundary of the contact, are modified by the interpolations.

SUBROUTINE INTERP (NGC, NGF) (Lines 122.7 - 129.5)

Trilinear interpolation is used in this subroutine to transfer the nodal temperatures from grid NGC to grid NGF, where they serve as a first approximation to the temperature distribution on that grid (see box 8 in Figure II.G.2). This subroutine is almost identical to subroutine INTADD, with the only difference being that the FAS replacement operation performed in the latter subroutine, viz,

$$u^{NGF} \leftarrow u^{NGF} + I_{NGC}^{NGF} (u^{NGC} - I_{NGF}^{NGC} u^{NGF})$$

is replaced by the simpler operation,

$$u^{NGF} = I_{NGC}^{NGF} u^{NGC}$$

SUBROUTINE PUTF (NG, F, XLEN, YLEN, M) (Lines 29.6-32.4)

This subroutine initializes the right-hand side vector (F^{ML}) for the currently finest grid (ML), as indicated by the operation $f^L \leftarrow F^L$ shown in box 2 in Figure II.G.2. The right-hand side function, F, is defined in the FUNCTION F subroutine. When the FMG algorithm is being used, line 41.2 (right-hand line number) in subroutine MULTIG (Listing 1) calls subroutine PUTF for the purpose just mentioned but PUTF is also called by line 39.0 (right-hand line number) in MULTIG to set the temperatures of the nodes on G^2 , which is the grid on which the FMG algorithm starts, to the first approximation defined in the FUNCTION G subroutine.

SUBROUTINE PUTZ (NGC, NGF) (Lines 48.2-52.0)

This subroutine performs the operation,

$$U_{NGC} \leftarrow I_{NGC} U_{NGF}$$

indicated in box 8 in Figure II.G.2.

SUBROUTINE RELAX (NG, NGRHS, ERR1, ERR2, M) (Lines 62.8-79.3)

This subroutine performs a relaxation sweep on grid NG. To permit boundary points at which non-Dirichlet boundary conditions are prescribed to be relaxed before interior nodes and to permit "odd" grid points to be relaxed before "even" ones, all of the relaxations are performed within a DO loop which causes the variable NSW to assume values between 1 and 4. The DO loop in question starts at line 46 in RELAX and ends at line 161. A different type of node is relaxed for each value of NSW, as noted below.

<u>NSW</u>	<u>TYPE OF NODE RELAXED</u>
1	Odd boundary (not "internal" boundary)
2	Even boundary (not "internal" boundary)
3	Odd interior
4	Even interior

Nodes on "internal" boundaries of grids which do not cover the entire solution domain are not relaxed at all, so the temperatures of those nodes are not changed by the relaxation sweep. The values of NIB1, NJBI, NKBI, NIBF and NKBF (see the discussion of the main program for descriptions of these variables) are compared to the value of NG in

lines 54-57 and 83 to determine which of the nodes on the boundary of the grid are also nodes on the boundary of the rectangular body being modelled since boundary nodes of the grid (i.e., nodes 1 and II in the x-direction, nodes 1 and JJ in the y-direction, and nodes 1 and KK in the z-direction) that do not also lie on the boundary of the body are not relaxed. Nodes at $J=JJ$ are never relaxed because their temperatures are always known. Note that $I=1$, $J=1$ and $K=1$ at the center of the contact patch, where $x_i = y_j = z_k = 0$, so the boundary plane at $J=JJ$ coincides with the isothermal surface at temperature T_1 in Figure II.4.7 when grid NG extends to the top of the contact specimen.* When grid NG does not extend to the top of the contact specimen (i.e., when $NG \geq NJBF$), the boundary plane at $J=JJ$ is an "internal" boundary plane and, as such, is still not visited in the relaxation sweep.

Boundary nodes are relaxed in lines 102-148 and interior nodes are relaxed in lines 151 and 152. The dynamic residual at each node is squared and added to ERR2 as the nodes are relaxed, as seen in lines 147 and 155. To ensure that the 2-norm of the dynamic residuals is properly calculated, the dynamic residual at each boundary node is multiplied by the factor $DXX * DYY * DZZ/DVOL$ because the "volume" of a boundary node is not the same as that of an interior node. The 2-norm of the dynamic residuals is calculated in line 164 (and stored as ERR2) after all of the nodes have been relaxed.

* The x-axis in Figure II.4.7 corresponds to the y-axis in the program.

SUBROUTINE RESCAL (NGF, NGC, NGRF, NGRC, M, NEX) (Lines 79.4-117.0)

This subroutine transfers residuals from the fine grid NGF to the coarse grid NGC. When NEX=1 and NGF=M, τ -extrapolation is used in the transfers, as seen in lines 47 and 48 in RESCAL where the factor of $4/3$ that multiplies τ_M^{M-1} in that case is set. This factor is designated FACT in the program.

Nodes on the fine grid are scanned by three nested DO loops, each of which sweeps through one of the grid indices IF, JF or KF. The outer DO loop, which starts at line 63 and ends at line 371, sweeps over the axial nodes (KF) on the fine grid while the middle and inner DO loops between lines 72 and 140 sweep over the grid indices IF and JF (for nodes located at x_{IF} , y_{JF}) respectively. The tests indicated in lines 97-100 determine whether the residual at node IF, JF, KF is actually needed because the right-hand side entry at an interior node on the coarse grid is obtained by "injecting" one-half of the residual from the coincident node on the fine grid. Thus residuals do not have to be calculated at interior fine grid nodes which do not coincide with interior nodes on the coarse grid, in other words, "even" nodes (i.e., nodes for which $IF + JF + KF$ is an even number) if NZDIS(NGF)=1 or nodes for which $(IF+JF)$ is odd if NZDIS(NGF)≠1.

Line 104 checks whether node IF, JF, KF is a boundary node and if it is, lines 115-138 calculate its residual, designated QFRT (IF, JF). If node IF, JF, KF is an interior node, on the other hand, its residual is calculated by lines 109-111. The residuals for all of the nodes on the x-y plane at $k = KF$ are calculated and stored in the appropriate

locations in the array QFRT(IF,JF) by the program segment between lines 72 and 140.

If NZDIS(NGF)≠1, which means that grids NGC and NGF have the same axial mesh structure because grid NGC was not refined in the axial direction to obtain grid NGF, the program segment between lines 149 and 239 calculates the right-hand side term $(f_{i,j,k}^{NGC})$ for each coarse grid node on the x-y plane at $k=KF$. The DO loop which starts at line 95 and ends at line 157 sweeps through the x-axis nodes (IC) on that plane while the DO loop which starts at line 182 and ends at line 238 sweeps through the y-axis nodes (JC). The operation indicated in box 9 is performed by lines 195-198 for an interior node and by lines 207-237 for a boundary node. The resulting right-hand side term for a coarse grid node IC, JC, KC is stored as Q(ICR+JC), where $ICR=IRC(IC,KC)$ (see line 173) and the array IRC is defined in subroutine KEY (called by line 25). The variable RESERR seen in lines 199 and 235 represents a running sum of the nodal values of $(\tau_M^{M-1})^2 \Delta V$, where ΔV is the volume of a node's finite-difference control-volume (normalized with respect to the volume of an interior node). The variable RESERR is later converted to the 2-norm of the G^{NGF} estimate of the G^{NGC} truncation errors (i.e., $||\tau_{NGF}^{NGC}||_2$, designated RESER(NGC), in line 375.

If NZDIS(NGF)=1 and $KF \neq KIN$, where KIN is the initial value of KF, line 241 directs the program to line 367 where the entries in the array QFRT are stored in another array, QFCT. The value of KF is then incremented by 1 in the outer DO loop which starts at line 63 and QFRT is recalculated for the new value of KF. Lines 258-359 then calculate

the right-hand side term for each coarse grid node on the x-y plane at $z=z_{KF-1}$ provided, of course, that the fine grid axial node $KF-1$ coincides with an axial node on the coarse grid. The residuals at nodes on the x-y plane at $z=z_{KF-1}$ are stored in the array QFCT so the operation $I_{NGF}^{NGC} (f^{NGF} - L^{NGF} u^{NGF})$ shown in box 9 in Figure II.G.2 is performed at an interior node I, J (of grid NGC) on that x-y plane by "injecting" one-half of the residual $(=f^{NGF} - L^{NGF} u^{NGF})$ at the coincident fine grid node, stored as QFCT (I,J), in the manner shown in line 300. Lines 301-303 then add the term $L^{NGC} u^{NGC}$ to the "injected" fine grid residual, as indicated in box 9 in Figure II.G.2. Lines 312-357 calculate the right-hand side term for a coarse grid node on the boundary of the region being modelled and lines 312-338, in particular, calculate $I_{NGF}^{NGC} (f^{NGF} - L^{NGF} u^{NGF})$ for the "full-weighting" residual transfer operator I_{NGF}^{NGC} . If the coarse grid z-axis node KC is coincident with the fine grid z-axis node KF, then the residual for the I, Jth fine grid node at $K=KF$ is stored as QFCT(I,J) and the residuals for the I, Jth nodes at $K=KF-1$ and $K=KF+1$ are stored as QFLT(I,J) and QFRT(I,J) respectively. All of these fine grid residuals may be needed to calculate the weighted (averaged) residual at the nearest coarse grid node.

Lines 339-354 add the term $L^{NGC} u^{NGC}$ to the term $I_{NGF}^{NGC} (f^{NGF} - L^{NGF} u^{NGF})$ just calculated for each boundary node on the coarse grid (when NZDIS(NGF)=1). The right-hand side term for a coarse grid node IC, JC, KC is stored as Q(ICR+JC), as before, and squared truncation errors are added to RESERR. Entries in the arrays QFLT and QFCT are replaced by those in QFCT and QFRT respectively in lines 364-369, after which KF is incremented by 1 (unless KF has reached its final value, KKF), QFRT is recalculated, and the entire procedure is repeated.

SUBROUTINE RESIST (M) (Lines 135.1-141.8)

The nodal temperatures obtained by the multigrid algorithm are used to calculate the resistance of the contact (based on the extrapolated temperature drop at the interface) in this subroutine. The DO loop which starts at line 13 in RESIST and ends at line 54 sweeps through all of the grids in the model (from NG=1 to NG=M) and inner DO loops starting at lines 24 and 25 sweep through the x-axis nodes (I=1,II) and the z-axis nodes (K=1,KK) on the NGth grid respectively. The temperature of the I, Kth node on the contact surface (i.e., at J=1) of the NGth grid is calculated in line 33 and then used in lines 35-45 to calculate the heat flux (HFLUX) and heat flow (HFLO) at the node if an isothermal contact boundary condition is being used (i.e., if NH=1) and if node I,K lies within the contact. If NH=1 or if node I,K is located outside of the contact patch, the heat flux is calculated by line 47. Total contact and contact patch heat flows are accumulated as HFLOW and HFLOWP respectively and the contact resistance (RES) is calculated by lines 55-60.

II.G.5 Sample Output

The output for the FMG accommodative analysis of the model contact problem with the contact in air is provided as Listing II.G.5 to illustrate the convergence characteristics of the multigrid algorithm. Contact approximation technique 1 was used in that analysis. All of the input data used in the analysis are given on the output listing along with calculated grid data, such as the total number of nodes contained in the composite grid (NPOINT). Also given are the grid (level) number; residual

norm and work unit count after each relaxation sweep. Contact resistances, heat flows, etc. are given for analyses performed with and without τ -extrapolation. The results shown were obtained with the input variable REDFAC set to 10, so the residual norm for the converged solution was one order of magnitude less than the norm of the truncation errors on the finest grid.

Note that the relaxation sweeps started on level 2 (as shown in Figure II.G.2). The algorithm worked its way up from one level to the next until the finest level (10) was reached, after which it cycled up and down over the coarser grids until the level 10 solution had converged. Since the work unit count and residual norm on level 10 were 12.04 and 32.5 respectively after the first relaxation sweep on that level and 20.53 and 0.0722 respectively after the last sweep, the factor $\bar{\mu}$ satisfying the expression,

$$\bar{\mu}^{-(20.53-12.04)} = \bar{\mu}^{-8.49} = \frac{0.0722}{32.50} \quad (\text{II.G.2})$$

represents the factor by which the residual norm on level 10 was reduced per WU while the algorithm was cycling on that level. Equation (II.G.2) can be solved for the parameter $\bar{\mu}$, termed the overall multigrid convergence factor (126), to obtain,

$$\bar{\mu} = \left(\frac{0.0722}{32.50} \right)^{1/8.49} = 0.487$$

The overall multigrid convergence factor for a given multigrid algorithm can be predicted (approximately) from the results of Brandt's local mode analysis (126) and used to determine whether the algorithm in question is functioning optimally. It can be shown that $\bar{\mu}$ should lie in the range 0.4-0.5 for the algorithm used in MULTIGRID, for instance, so the observed convergence factor of 0.487 is proof of the proper operation of MULTIGRID.

Listing II.G.1 Fortran IV Program "MULTIGRID" Written to Solve the Model Contact Problem

```

0001 C ***** MULTIGRID *****
0002
0003 Programmed by: T. C. Currie
0004
0005 This program uses multigrad to solve the model contact problem.
0006 A rectangular 3-D contact specimen is used in the model and a
0007 boundary condition of the first or third kind is applied over the
0008 area of the contact patch. The program uses the 7-point star
0009 approximation to the heat conduction operator and a grid which is
0010 highly graded in the neighbourhood of the contact is constructed
0011 by using non-coextensive nested grids in the multigrad algorithm.
0012
0013 ***** INPUT VARIABLES *****
0014
0015 TITLE(I),I=1,8 - Job title (must be less than or equal to 32 char.)
0016 XLEN - Length of rectangular body being modelled in x-dir.
0017 YLEN - Length of rectangular body in y-dir.
0018 ZLEN - Length of rectangular body in z-dir.
0019 NYO - Number of subdivisions in x-dir. on coarsest grid.
0020 NZO - Number of subdivisions in y-dir. on coarsest grid.
0021 TC - Thermal conductivity of solid.
0022 TEGAS - Thermal conductivity of gas in gas gap.
0023 A - One-half of major diameter of contact patch.
0024 R - One-half of minor diameter of contact patch.
0025 RCY1 - Small contact radius.
0026 RCY2 - Large contact radius.
0027 CCON - Parameter used in non-continuous gas conductivity
0028 model.
0029 REDFAC - Additional factor (greater than 1.) by which
0030 residual norm is reduced below truncation error
0031 norm. Set REDFAC=1 unless additional reduction is
0032 desired.
0033 NMAX - Work unit limit.
0034 N - Total number of grids used in multigrad algorithm.
0035 INSUL - = 0 for run with gas conduction.
0036 = 1 for run without gas conduction.
0037 MEXTR - = 0 for run without tau extrapolation.
0038 = 1 for run with tau extrapolation.
0039 MCTCAL - = 0 for run performed using full multigrad
0040 algorithm.
0041 = 1 for run performed using coarsening algorithm.
0042 Fixed multigrad operates as a full algorithm
0043 regardless of the value of MCTCAL.
0044 NSFSW - Number of relaxation sweeps performed on the
0045 coarsest grid (each time it is visited) when the
0046 fixed multigrad algorithm is used.
0047 NZRIS(I),I=1,10 - = 0 if subdivisions in all directions are halved to
0048 obtain the Ith grid from the (I-1)st.
0049 = 1 if subdivisions are halved in only the x and y
0050 directions to obtain the Ith grid from the (I-1)st.
0051 HPATCH - Contact conductance within contact patch (only
0052 used if NM=1).
0053 TBASE - Temperature with which the nodes on the contact
0054 surface communicate through surface conductances
0055 (needed even if NM=0 if run with gas conduction is
0056 being performed).
0057 NM - = 0 if isothermal contact boundary condition is
0058 used.
0059 = 1 if boundary condition of the third kind (with
0060 contact conductance HPATCH) is applied over the
0061 contact area.
0062 NJIC(NG) - X-axis node on grid NG-1 that coincides with the
0063 first x-axis node on grid NG.
0064 NFIC(NG) - X-axis node on grid NG-1 that coincides with the
0065 last x-axis node on grid NG.
0066 NJC(NG) - Y-axis node on grid NG-1 that coincides with the
0067 first y-axis node on grid NG.
0068 NFJ(NG) - Y-axis node on grid NG-1 that coincides with the
0069 last y-axis node on grid NG.
0070 NIKC(NG) - Z-axis node on grid NG-1 that coincides with the
0071 first z-axis node on grid NG.
0072 NFKC(NG) - Z-axis node on grid NG-1 that coincides with the
0073 last z-axis node on grid NG.
0074
0075 ***** MAIN PROGRAM *****
0076
0077 The main program reads all of the input variables described above
0078 and calculates a number of grid-specific parameters.
0079
0080 ***** VARIABLE DESCRIPTIONS *****
0081
0082 DX - Subdivision size in the x-direction on grid NG.
0083 DY - Subdivision size in the y-direction on grid NG.
0084 DZ - Subdivision size in the z-direction on grid NG.
0085 DXO - Subdivision size in the x-direction on the coarsest grid.
0086 DYO - Subdivision size in the y-direction on the coarsest grid.
0087 DZO - Subdivision size in the z-direction on the coarsest grid.
0088 Grid number.
0089 NJB1 - Number of the first (coarsest) grid whose 1st. x-axis
0090 node is not also a boundary node.
0091 NJB2 - Number of the coarsest grid whose 1st. y-axis node
0092 is not also a boundary node.
0093 NJB3 - Number of the coarsest grid whose 1st. z-axis node
0094 is not also a boundary node.
0095 NJB4 - Number of the coarsest grid whose last x-axis node
0096 is not also a boundary node.
0097 NJB5 - Number of the coarsest grid whose last y-axis node
0098 is not also a boundary node.
0099 NJB6 - Number of the coarsest grid whose last z-axis node
0100 is not also a boundary node.
0101 NPOINT - Number of nodes in the composite grid; counts only
0102 once those nodes that are repeated on several grids.
0103 NPT(NG) - Number of nodes in grid NG.

```



```

00226      IF(MIIC(MG).GT.1.AND.NIRI.EQ.M1) NIRI=MG      0023.300
00227      IF(MIJC(MG).GT.1.AND.NJRI.EQ.M1) NJRI=MG      0023.400
00228      IF(MIKC(MG).GT.1.AND.NKRI.EQ.M1) NKRI=MG      0023.500
00229      INCI=2*(INCI+MIIC(MG)-1)      0023.600
00230      XI(MG)=INCIRDX      0023.700
00231      NDIVX=2*(MFIIC(MG)-MIIC(MG))      0023.800
00232      ITOT=INCI+NDIVX      0023.900
00233      IF(ITOT.LT.JRED.AND.NIBF.EQ.M1) NIBF=MG      0024.000
00234      INCJ=2*(INCJ+MIJC(MG)-1)      0024.100
00235      YI(MG)=INCJRDY      0024.200
00236      MDIVY=2*(MFJC(MG)-MIJC(MG))      0024.300
00237      JTOT=INCJ+MDIVY      0024.400
00238      IF(JTOT.LT.JRED.AND.NJBF.EQ.M1) NJBF=MG      0024.500
00239      C      0024.600
00240      C Check if the value of MKBF needs to be corrected.      0024.700
00241      IF(MZDIS(MG)-1) 35,30,35      0024.800
00242      30      KREQ=2*KREQ      0024.900
00243      INCK=2*(INCK+MIKC(MG)-1)      0025.000
00244      MDIVZ=2*(MFKC(MG)-MIKC(MG))      0025.100
00245      DZ=DZ/2.      0025.200
00246      GO TO 40      0025.300
00247      35      INCK=INCK+MIKC(MG)-1      0025.400
00248      MDIVZ=MFKC(MG)-MIKC(MG)      0025.500
00249      40      KTOT=INCK+MDIVZ      0025.600
00250      ZI(MG)=INCKSDZ      0025.700
00251      IF(KTOT.LT.KREQ.AND.MKBF.EQ.M1) MKBF=MG      0025.800
00252      C      0025.900
00253      C Call subroutine GRDFN to allocate space for the MGth. grid temp.      0026.000
00254      C and right-hand sides in the Q vector, and to store the subdivision      0026.100
00255      C sizes and node limits.      0026.200
00256      CALL GRDFN (MG,NDIVX+1,NDIVY+1,NDIVZ+1,DX,DY,DZ,M)      0026.300
00257      CALL GRDFN (MG+M,NDIVX+1,NDIVY+1,NDIVZ+1,DX,DY,DZ,M)      0026.400
00258      C      0026.500
00259      C Calculate the number of nodes in the MGth. grid.      0026.600
00260      NPT(MG)=(NDIVX+1)*(NDIVY+1)*(NDIVZ+1)      0026.700
00261      IF(MG.EQ.M) MPOINT=MPOINT+NPT(MG)      0026.800
00262      20      CONTINUE      0026.900
00263      XNP=MPOINT      0027.000
00264      C      0027.100
00265      C Use the values of NPT(MG) and MPOINT to define the work unit      0027.200
00266      C increment for the MGth. grid (MG=1,...,M).      0027.300
00267      DO 45 MG=1,M      0027.400
00268      XNPT=NPT(MG)      0027.500
00269      MUINC(MG)=XNPT/XNP      0027.600
00270      WRITE (106,103)      0027.700
00271      103      FORMAT ('000000 CALCULATED GRID DATA =====')      0027.800
00272      OUTPUT( MPOINT,MUINC,MPT,MIBI,MJBI,MKBI,NIBF,NJBF,MKBF,UG,XI,YI,ZI      0027.900
00273      WRITE (106,104)      0028.000
00274      104      FORMAT ('000000 REGIONAL WORKS DURING ITERATIONS =====')      0028.100
00275      C      0028.200
00276      C Call subroutine MULTIG to apply the multigrid algorithm.      0028.300
00277      CALL MULTIG (DXO,DYO,DZO,TOL,UMAX,G,F,MXO,MYO,MZO,M,XLEN,YLEN,      0028.400
00278      ZLFN)      0028.500
00279      IF(MEXTR.NE.1) WRITE (106,104)      0028.600
00280      IF(MEXTR.EQ.1) WRITE (106,105)      0028.700
00281      104      FORMAT ('000000 RESULTS OBTAINED WITHOUT TAU EXTRAPOLATION =====')      0028.800
00282      //      0028.900
00283      105      FORMAT ('000000 RESULTS OBTAINED WITH TAU EXTRAPOLATION =====')      0029.000
00284      C      0029.100
00285      C Call subroutine RESIST to calculate the resistance of the contact.      0029.200
00286      CALL RESIST(M)      0029.300
00287      15      STOP      0029.400
00288      END      0029.500

```

```

0001      SUBROUTINE PUTF(MG,F,XLEN,YLEN,ZLEN,M)      0029.600
0002      C      0029.700
0003      C This subroutine sets the first approximation on the MGth. grid      0029.800
0004      C when MG .LE. M and assigns values to the right-hand side terms      0029.900
0005      C on the MGth. grid when MG .GT. M.      0030.000
0006      C      0030.100
0007      DIMENSION IST(19,33),KPST(35)      0030.200
0008      COMMON /DBW/Q(56500)/DBCRD/XI(10),YI(10),ZI(10)      0030.300
0009      S /DBIST/IST      0030.400
00010      C      0030.500
00011      C Call subroutine KEY to obtain IST,KPST,...,DZ for the MGth. grid.      0030.600
00012      CALL KEY(MG,IST,KPST,IMAX,JMAX,KMAX,DX,DY,DZ)      0030.700
00013      MN=MG      0030.800
00014      IF(MG.GT.M) MN=MG-M      0030.900
00015      C      0031.000
00016      C Sweep through the nodes on the MGth. grid.      0031.100
00017      DO 1 K=1,KMAX      0031.200
00018      DO 1 J=1,JMAX      0031.300
00019      DO 1 I=1,IMAX      0031.400
00020      C      0031.500
00021      C Calculate the coordinates of the I,J,Kth. node on grid MG (ie.,X,Y,Z)      0031.600
00022      X=XI(MN)+I-18DX      0031.700
00023      Y=YI(MN)+J-18DY      0031.800
00024      Z=ZI(MN)+K-18DZ      0031.900
00025      C      0032.000
00026      C Assign a value to the temperature or the rhs. term at node I,J,K.      0032.100
00027      Q(IST(I,K)+J)=F(X,Y,Z,XLEN,YLEN,ZLEN)      0032.200
00028      RETURN      0032.300
00029      END      0032.400

```

```

0001      FUNCTION F(X,Y,Z,XLEN,YLEN,ZLEN)      0032.500
0002      C      0032.600
0003      C This subroutine defines the right-hand side function in terms of the      0032.700
0004      C variables X,Y,...,ZLEN.      0032.800
0005      C      0032.900
0006      F=0.      0033.000
0007      RETURN      0033.100
0008      END      0033.200

```

```

0001      FUNCTION G(X,Y,Z,XLEN,YLEN,ZLEN)      0033.300
0002      C      0033.400
0003      C This subroutine defines the first approximation (and Dirichlet      0033.500
0004      C boundary conditions) in terms of the variables X,Y,...,ZLEN.      0033.600
0005      C      0033.700

```

```

0006      G=(Y/YLEN)
0007      RETURN
0008      END
0033.800
0033.900
0034.000

0001      SUBROUTINE MULTIG(DX0,DY0,DZ0,IDL,UMAX,UI,F,MX0,MY0,MZ0,M,XLEN,
0002      & YLEN,ZLEN)
0003
0004      This subroutine controls the sequence of operations in a 'full'
0005      or 'cyclic' accommodative multigrid algorithm. If the 'full'
0006      multigrid algorithm is used (obtained when NCYCAL=0), the algorithm
0007      starts on the second to coarsest grid (ie., MG=2) and works its way
0008      up to the finest whereas the iterations start on the finest (Mth)
0009      grid when the cyclic algorithm is used (obtained when NCYCAL=0).
0010      In either case, the solution on the currently finest level (ML) is
0011      not considered to have converged until the 2-norm of the residuals
0012      (ERR2) is less than the 2-norm of the truncation errors
0013      (RESERR(ML-1)/4.) by a factor REDFAC.
0014      Tau extrapolation is used after the fine grid solution has been
0015      obtained if MEX=1.
0016
0017      ***** VARIABLE DESCRIPTIONS *****
0018
0019      IEN - Denominator by which RESERR(ML-1) is divided to obtain
0020      convergence parameter for grid ML.
0021      EPS(MG) - Norm of dynamic residuals on grid MG (ERR2) must be less
0022      than EPS(MG) in order for the algorithm to return to the next finer
0023      grid. MU=1.
0024      ERR2 - 2-norm of dynamic residuals.
0025      M - Number of finest grid.
0026      ML - Currently finest grid in the algorithm.
0027      MEX - * 0 when transfer of residuals from level M to level
0028      M-1 is to be performed without tau extrapolation.
0029      * 1 when transfer of residuals from level M to level
0030      M-1 is to be performed with tau extrapolation.
0031      MG - Number of grid on which a relaxation sweep is currently
0032      being performed (ie., the working level).
0033      RESERR(MG) - Estimated norm of truncation errors on grid MG.
0034      MU - Work unit count.
0035
0036      EXTERNAL UI,F
0037      DIMENSION EPS(10)
0038      COMMON /DBDZ/NZDIS(10)/DBUG/UG(10)/MUNIC(10)
0039      & /DBEX/MEXTR/DBPAR/A,B,C,DBM/MPATCH,TRASE,MH
0040      & /DBERR/RESERR(10)/DBRED/REDFAC,NCYCAL,MSPSW
0041      & /DBERR3/ERR3(10)
0042      & /DBIST/IST(19,33)/DBO/Q(56500)
0043      MU=2
0044      IF(NCYCAL.EQ.1) MU=M
0045
0046      Set first approximations, right-hand side terms, and contact
0047      surface heat transfer coefficients on all grids up to and
0048      including the initially finest.
0049      DO I=1,MU
0050      CALL PUTF(MG,UI,XLEN,YLEN,ZLEN,M)
0051      CALL PUTF(MG,F,XLEN,YLEN,ZLEN,M)
0052      CALL HXO(MG)
0053
0054      Zero the work unit count (MU) and set the currently finest level.
0055      ML
0056      MU=0
0057      MEX=0
0058      ML=1
0059      IF(NCYCAL.EQ.1) ML=M-1
0060
0061      Increment the currently finest level by 1.
0062      ML=ML+1
0063
0064      Set the working level (MG).
0065      MG=ML
0066      EPS(MG)=1.E-6
0067      DEN=4.
0068      IF(ML.EQ.M) DEN=4.*REDFAC
0069
0070      Assign values to the rhs. terms and the contact surface heat transfer
0071      coefficients on the currently finest level.
0072      CALL PUTF(MG,F,XLEN,YLEN,ZLEN,M)
0073      CALL HXO(MG)
0074
0075      Call subroutine RELAX to perform a relaxation sweep on grid MG.
0076      The dynamic residual norm (ERR2) is returned.
0077      CALL RELAX(MG,MGMH,ERR2,ML)
0078
0079      Increment the work unit count
0080      MU=MU+MUNIC(MG)
0081      WRITE (108,4) MG,ERR2,MU
0082      FORMAT ('LEVEL=',I2,5X,'RESIDUAL NORM=',E12,5X,'WORK=',F10.5)
0083
0084      check
0085      Check whether the solution for the coarse grid corrections on grid
0086      MG has converged. If it has, interpolate the corrections up to
0087      the next finer grid (unless MG=M in which case the fine grid soln.
0088      has been obtained) with subroutine INTADD if MG=ME, ML or
0089      interpolate the first approximation up to the next finer grid with
0090      subroutine INTERP if MG=ML. If it hasn't, compute the
0091      correction on the next coarser grid unless MG=1, in which case
0092      further relaxation sweeps must be performed until convergence has
0093      been achieved.
0094      IF(ERR2.GE.EPS(MG)) GO TO 6
0095      IF(MG.EQ.M.AND.MEXTR.EQ.1) GO TO 10
0096      GO TO 19
0097      MEX=1
0098      DEN=4.
0099      WRITE (108,100)
0100      FORMAT ('***** RESULTS OBTAINED WITHOUT TAU EXTRAPOLATION *****')
0101      //
0102      CALL RESIST(M)
0103      GO TO 6
0104      IF(MG.EQ.M) RETURN
0105      IF(MEX.EQ.1.AND.MG.EQ.M-1) CALL INTADD(MG,MG+1) ; RETURN
0106      IF(MG.ME.ML) GO TO 2
0107
0108      Interpolate the first approximation for the next finer level.
0109      CALL INTERP(MG,MG+1)
0110      GO TO 10
0111      IF(MU.GE.UMAX) RETURN
0112      IF(MG.EQ.1) GO TO 5

```

```

0113 C
0114 C Go to the next coarser level to compute the corrections. Subr.
0115 PUTZ injects the temperatures from the fine grid to the coarse
0116 grid while subr. RESCAL performs the fine-to-coarse grid residual
0117 transfers.
0118 C
0119 C NG=NG-1
0120 C CALL PUTZ(NG,NG+1)
0121 C CALL RESCAL (NG+1,NG,NG+1,NG+1,ML, MEX)
0122 C
0123 C Calculate the convergence parameter for the coarse grid correction
0124 calculations.
0125 EPS(NG)=.158ERK2SQRT(V6(NG)/V6(NG+1))
0126 C
0127 C Calculate the convergence parameter for the currently finest level.
0128 IF(NG+1.EQ.ML) EPS(NG+1)=KESEK(NG)/DEN
0129 C
0130 C Calculate the convergence parameter for the solution on grid M-1
0131 when tau extrapolation is used.
0132 IF(NG+1.EQ.M-1.AND.MEX.EQ.1) EPS(NG+1)=RESERR(NG)/DEN
0133 IF(NG+1.EQ.M.AND.MEX.EQ.1) EPS(NG)=1.E-4
0134 GO TO 5
0135 C
0136 C Transfer the corrections to the next finer level and let it be
0137 the new working level.
0138 CALL INTADD (NG,NG+1)
0139 MERR=0
0140 NG=NG+1
0141 GO TO 5
END

```

```

0001 SUBROUTINE PUTZ(NGC,NGF)
0002 C
0003 C This subroutine transfers temperatures from nodes on grid NGF to
0004 coincident nodes on grid NGC
0005 C
0006 C DIMENSION IUC(19,33),IUF(19,33),KBSTC(35),KBSTF(35)
0007 C COMMON /DBD/Q(56500)/DBGR/NIIC(10),NFIC(10),MIJC(10),MFJC(10),
0008 C NIKC(10),MKC(10)/DBU2/NZDIS(10)
0009 C /BBIST/IUC/DBISTF/IUF
0010 C
0011 C Call subroutine KEY to define IUC,KBSTC,....,DZC for grid NGC and
0012 IUF,KBSTF,....,DZF for grid NGF.
0013 CALL KEY (NGC,IUC,KBSTC,IIC,JJC,KKC,BXC,DYC,DZC)
0014 CALL KEY (NGF,IUF,KBSTF,IIF,JJF,KKF,BXF,DYF,DZF)
0015 C
0016 C Obtain the coarse grid nodes that define the boundaries of the
0017 fine grid.
0018 KIN=NIKC(NGF)
0019 KFIN=MKC(NGF)
0020 IIN=MIJC(NGF)
0021 IFIN=MFJC(NGF)
0022 JIN=MIJC(NGF)
0023 JFIN=MFJC(NGF)
0024 C
0025 C Sweep through the coarse grid nodes KC, IC, JC to obtain the coincident
0026 nodes on the fine grid (KF, IF, JF) and compute the nodal tears.
0027 C in Q.
0028 DO 1 KC=KIN,KFIN
0029 KF=KC-KIN+1
0030 IF(NZDIS(NGF).EQ.1) KF=2*(KC-KIN+1)-1
0031 DO 1 IC=IIN,IFIN
0032 IF=2*(IC-IIN+1)-1
0033 ICO=IUC(IC,KC)
0034 IFO=IUF(IF,KF)
0035 DO 1 JC=JIN,JFIN
0036 JF=2*(JC-JIN+1)-1
0037 Q(ICO+JC)=Q(IFO+JF)
0038 RETURN
0039 END

```

```

0001 SUBROUTINE INTADD(NGC,NGF)
0002 C
0003 C This subroutine interpolates the FAS corrections from grid NGC to
0004 grid NGF using trilinear interpolation formulae.
0005 C
0006 C DIMENSION ISTC(19,33),ISTF(19,33),OB(19,19),OP(19,19)
0007 C ,KBSIC(35),KBSTF(35)
0008 C COMMON /DBD/Q(56500)/DBPAR/A,B,IC,DBDIR/NDIRCH
0009 C /DBU2/NZDIS(10)/DBM/MATCH,IBASE,MH
0010 C /DBGR/NIIC(10),NFIC(10),MIJC(10),MFJC(10),NIKC(10),MKC(10)
0011 C /BBIST/ISTC/DBISTF/ISTF
0012 C
0013 C Call subroutine KEY to obtain the coarse and fine grid particulars.
0014 CALL KEY (NGC,ISTC,KBSTC,IIC,JJC,KKC,BXC,DYC,DZC)
0015 CALL KEY (NGF,ISTF,KBSTF,IIF,JJF,KKF,BXF,DYF,DZF)
0016 C
0017 C Define the coarse grid x-axis nodes I1 and I2 that locate the initial
0018 and final boundaries respectively of the fine grid on the x-axis.
0019 I1=MIJC(NG)
0020 I2=MFJC(NGF)
0021 C
0022 C Define the coarse grid y-axis nodes J1 and J2 that locate the initial
0023 and final boundaries respectively of the fine grid on the y-axis.
0024 J1=MIJC(NGF)
0025 J2=MFJC(NGF)
0026 C
0027 C Define the coarse grid z-axis nodes K1 and K2 that locate the initial
0028 and final boundaries respectively of the fine grid on the z-axis.
0029 K1=NIKC(NGF)
0030 K2=MKC(NGF)
0031 C
0032 C Sweep through the coarse grid nodes KC, IC, JC to obtain the
0033 coincident nodes KF, IF, JF on the fine grid and linearly
0034 interpolate the corrections from grid NGC to grid NGF.
0035 DO 10 KC=K1,K2
0036 KF=KC-K1+1
0037 IF(NZDIS(NGF).EQ.1) KF=2*(KC-K1+1)-1
0038 DO 1 IC=I1,I2
0039 IF=2*(IC-I1+1)-1
0040 IF(IC.EQ.I1) GO TO 6
0041 IFN=ISTF(IF-1,KF)
0042 IFM2=ISTF(IF-2,KF)

```



```

0043      ICM=ISTC(IC-1,KC)
0044      IFQ=ISTF(IF,KF)
0045      ICD=ISTC(IC,KC)
0046      IF(KC.EQ.K1) GO TO 8
0047      IKFM=ISTF(IF,KF-1)
0048      IF(IF.NE.1) INFR2=ISTF(IF-1,KF-1)
0049      IF(WDIS(MG),NE.1) GO TO 9
0050      IKFM=ISTF(IF,KF-2)
0051      IF(IC.NE.1) IKFM=ISTF(IF-2,KF-2)
0052      IKCM=ISTC(IC,KC-1)
0053      IF(IC.NE.1) IKCM=ISTC(IC-1,KC-1)
0054      DO 1 JC=J1,J2
0055      JF=2*(JC-J1)+1
0056      OG(IF,JF)=O(ICD+JC)+O(ICD+JC-1)-O(IFO+JF)-O(IFO+JF-2)
0057      IF(JC.NE.J1) A1=.5*(O(ICD+JC)+O(ICD+JC-1)-O(IFO+JF)-O(IFO+JF-2))
0058      OG(IF,JF-1)=O(IFO+JF-1)+A1
0059      IF(JC.NE.J1.AND.IC.NE.1) A2=.5*(O(ICM+JC)+O(ICM+JC-1)-O(IFN2+JF)-O(IFN2+JF-2))
0060      OG(IF-1,JF-1)=O(IFN+JF-1)+.5*(A1+A2)
0061      IF(IC.NE.1) A3=.5*(O(ICD+JC)+O(ICM+JC)-O(IFO+JF)-O(IFN2+JF))
0062      OG(IF-1,JF)=O(IFN+JF)+A3
0063      IF(WDIS(MG),NE.1) GO TO 1
0064      IF(KC.NE.K1) A4=.5*(O(ICD+JC)+O(ICM+JC)-O(IFO+JF)-O(IFO+JF-2))
0065      OG(IFO+JF)=O(IFO+JF)+A4
0066      IF(KC.NE.K1.AND.JC.NE.J1) A5=.5*(O(ICM+JC)+O(ICM+JC-1)-O(IFO+JF)-O(IFO+JF-2))
0067      OG(IFO+JF-1)=O(IFO+JF-1)+.5*(A1+A5)
0068      IF(KC.NE.K1.AND.IC.NE.1) A6=.5*(O(ICM+JC)+O(ICM+JC-1)-O(IFO+JF)-O(IFO+JF-2))
0069      OG(IFO+JF-1)=O(IFO+JF-1)+.5*(A3+A6)
0070      IF(KC.NE.K1.AND.IC.NE.1.AND.JC.NE.J1) A7=.5*(O(ICM+JC)+O(ICM+JC-1)-O(IFO+JF)-O(IFO+JF-2))
0071      OG(IFO+JF-1)=O(IFO+JF-1)+.5*(A3+A6)
0072      A7=A1+A2+A5+A7
0073      CONTINUE
0074      IF(WDIS(MG),NE.1) GO TO 11
0075      DO 12 IL=1,11F
0076      IKD=ISTF(IL,KF)
0077      IF(KC.NE.K1) IKD=ISTF(IL,KF-2)
0078      DO 12 JL=1,JJF
0079      IF(KC.EQ.K2) W(IKD+JL)=OG(IL,JL)
0080      IF(KC.EQ.K1) GO TO 13
0081      O(IKD+JL)=OG(IL,JL)
0082      DP(IL,JL)=OG(IL,JL)
0083      CONTINUE
0084      GO TO 10
0085      DO 14 IL=1,11F
0086      IKD=ISTF(IL,KF)
0087      DO 14 JL=1,JJF
0088      O(IKD+JL)=OG(IL,JL)
0089      CONTINUE
0090      IF(MH.EQ.1) GO TO 16
0091      C
0092      C Set the temperatures of the nodes within the contact patch on the
0093      C finw grid to zero if an isothermal contact boundary condition is
0094      C being used (i.e., if MH.NE.1)
0095      NI=B/DZF+1
0096      NK=A/DZF+1
0097      IF(NI.GT.11F) NI=11F
0098      IF(NK.GT.KKF) NK=KKF
0099      DO 15 K=1,NK
0100      DO 15 I=1,NI
0101      IFO=ISTF(I,K)
0102      XPOS=(I-1)*DZF
0103      ZPOS=(K-1)*DZF
0104      XPATCH=BSORT(1,-(ZPOS*2))/(A*2)
0105      IF(XPOS.LE.XPATCH) O(IFO+1)=0.
0106      RETURN
0107      END

```

```

0001      SUBROUTINE RELAX (MG,NGRHS,ERR2,K)
0002      C
0003      C This subroutine performs a relaxation sweep on grid MG.
0004      C Checkerboard ordering is used in the sweep (i.e., nodes for which
0005      C I+J+K is odd are relaxed before those nodes for which it is even if
0006      C NZDIS(MG)=1 or nodes for which I+J is even are relaxed before those
0007      C nodes for which it is odd if NZDIS(MG)=0) and boundary points at
0008      C which non-Dirichlet boundary conditions are prescribed are relaxed
0009      C before interior points.
0010      C
0011      DIMENSION IST(19,13),IRMS(19,33),KBST(35)
0012      COMMON /DPR/W(24500)/DPRAR/A,R,TC/DBDIR/NDIRCH
0013      S /DBH/NPATCH,IBASE,MH/DBDZ/NZDIS(10)/DBGR/NLIC(10)/NFIC(10),
0014      S NI,K(10),NR,K(10),NIKE(10),NFKC(10)
0015      S /DBH/NIRI,NJBI,NKBI,NIBF,NJBF,NKBF
0016      S /DBH/RS/ERRS(10)/DBH/NK(2300)
0017      S /DBIST/IST/DBISTF/IRMS
0018      C
0019      C Call subroutine KEY to define the entries in the array IRMS, which
0020      C locates the right-hand side terms for grid MG in the B vector.
0021      C The right-hand side term for node I,J,K is stored as O(IRMS(I,K)+J).
0022      C CALL KEY (NGRHS,IRMS,KBST,11,JJ,KK,DX,DY,DZ)
0023      C
0024      C Call subroutine KEY to define the variables IST,KBST,....,DZ for
0025      C grid MG. The temperature of node I,J,K is stored as O(IST(I,K)+J).
0026      C CALL KEY (MG,IST,KBST,11,JJ,KK,DX,DY,DZ)
0027      I1=11-1
0028      J1=JJ-1
0029      K1=KK-1
0030      DX2=1./DX**2
0031      DY2=1./DY**2
0032      DZ2=1./DZ**2
0033      CDEFF=1./(2.*(DX2+DY2+DZ2))
0034      DVOL=DX*DY*DZ
0035      C
0036      C Zero the residual norm (ERR2).
0037      ERR2=0.
0038      C
0039      C Perform four sweeps over all of the nodes on grid MG, where the
0040      C variable NSW assumes a different value (between 1 and 4) in each
0041      C sweep. When NSW=1, relax the 'odd' boundary nodes (nodes where
0042      C non-Dirichlet boundary conditions are prescribed). When NSW=2,
0043      C relax the 'even' boundary nodes where non-Dirichlet bc's are
0044      C prescribed. When NSW=3 and NSW=4, relax the 'odd' and 'even'
0045      C interior points respectively.
0046      DO 5 NSW=1,4
0047      C
0048      C Determine the limiting nodes on the z-axis: KIN and KFIN, and the
0049      C limiting nodes on the x-axis: IIN and IFIN.
0050      KIN=

```

```

0051      KFIN=KK
0052      IIN=1
0053      IFIN=11
0054      IF(MG.GE.NKBI) KIN=2
0055      IF(MG.GE.NKBF) KFIN=K1
0056      IF(MG.GE.NIBI) IIN=2
0057      IF(MG.GE.NIBF) IFIN=11
0058      IF(MSW.GE.3) KIN=2 ; KFIN=K1 ; IIN=2 ; IFIN=11
0059      C
0060      C Sweep through the z-axis nodes (K).
0061      DO 10 K=KIN,KFIN
0062      DZ=DZ
0063      IF(K.EQ.1.OR.K.EQ.KK) DZZ=DZ/2.
0064      C
0065      C Sweep through the x-axis nodes (I).
0066      DO 10 I=IIN,IFIN
0067      DXX=DX
0068      IF(I.EQ.1.OR.I.EQ.II) DXX=DX/2.
0069      IO=IST(I,K)
0070      IR=IRMS(I,K)
0071      IF(I.NE.1) IM=IST(I-1,K)
0072      IF(I.NE.II) IP=IST(I+1,K)
0073      IF(K.NE.1) IK=IST(I,K-1)
0074      IF(K.NE.KK) IKP=IST(I,K+1)
0075      IR=IRK
0076      I2=IK/2
0077      I22=2*I2
0078      C
0079      C Determine the first y-axis node (JIM). This value is selected so
0080      C that the resulting value of I+J+K (or I+J if NZDIS(MG)=0) is either
0081      C odd or even depending on the value of MSW.
0082      JIN=1
0083      IF(MSW.GE.3.OR.MG.GE.NJBT) JIN=3
0084      IF(NZDIS(MG).NE.1) GO TO 31
0085      IF((MSW.EQ.2.OR.MSW.EQ.4).AND.IK.EQ.I22).OR((MSW.EQ.1.OR.
0086      MSW.EQ.3).AND.IK.NE.I22)) JIN=2
0087      GO TO 32
0088      31 I2=I/2
0089      I22=2*I2
0090      IF((MSW.EQ.1.OR.MSW.EQ.3).AND.I.EQ.I22).OR((MSW.EQ.2.OR.MSW.EQ.
0091      4).AND.I.NE.I22)) JIN=2
0092      C
0093      C Sweep through the odd or even y-axis nodes. Note that nodes are
0094      C jumped 2 at a time.
0095      DO 20 J=JIN,J12
0096      C
0097      C Go to statement 25 for interior node relaxations and statement 30
0098      C for boundary node relaxations.
0099      IF(MSW.GE.3) GO TO 25
0100      IF(I.EQ.1.OR.I.EQ.II.OR.J.EQ.1.OR.K.EQ.1.OR.K.EQ.KK) GO TO 30
0101      GO TO 10
0102      30 DYT=DY
0103      IF(J.NE.1) GO TO 35
0104      MP=MK(KST(K)+1)
0105      DYT=DY/2.
0106      XPUS=(I-1)*DX
0107      ZPOS=(K-1)*DZ
0108      C
0109      C Determine if node I,J,K lies within the contact patch area. If it
0110      C does and an isothermal contact bc. is being used, go to the next
0111      C node (ie., skip the current one).
0112      IF(ZPOS.GT.A.OK.XPOS.GT.B) GO TO 35
0113      XPATCH=BSORT(1,-((ZPOS**2)/(A**2)))
0114      IF(XPOS.LE.XPATCH.AND.MP.NE.1) GO TO 20
0115      C
0116      C Relax the finite-difference equation at boundary node I,J,K
0117      C using the Gauss-Seidel method.
0118      35 CTOT=DXXDZ/DY
0119      CTOT=CTOT*(IO+J+1)
0120      IF(J.EQ.1) GO TO 36
0121      CN=DXXDZ/DY
0122      CTOT=CTOT+CN
0123      CTOT=CTOT+CN*(IO+J-1)
0124      IF(I.EQ.1) GO TO 37
0125      CN=DYXDZ/DX
0126      CTOT=CTOT+CN
0127      CTOT=CTOT+CN*(IM+J)
0128      IF(I.EQ.II) GO TO 38
0129      CN=DYXDZ/DX
0130      CTOT=CTOT+CN
0131      CTOT=CTOT+CN*(IP+J)
0132      IF(K.EQ.1) GO TO 39
0133      CN=DXXDZ/DZ
0134      CTOT=CTOT+CN
0135      CTOT=CTOT+CN*(IK+J)
0136      IF(K.EQ.KK) GO TO 40
0137      CN=DXXDZ/DZ
0138      CTOT=CTOT+CN
0139      CTOT=CTOT+CN*(IKP+J)
0140      IF(J.NE.1) GO TO 41
0141      CN=DXXDZ*HF/TC
0142      CTOT=CTOT+CN
0143      CTOT=CTOT+CN*TRASE
0144      UT=(-(Q*(IR+J)+DXXDZ*DYXDZ/TC + CTOT)/CTOT)
0145      C
0146      C Add the square of the dynamic residual to ERR2
0147      ERR2=ERR2+DXXDZ*DYXDZ*((Q*(IO+J)+HS2)/DVOL)
0148      GO TO 45
0149      C
0150      C Relax the finite-difference equation for the interior node I,J,K
0151      25 Q1=COEFF*(-(Q*(IR+J)/TC+DX2*(Q*(IM+J)+Q*(IP+J))+DY2*(Q*(IO+J)+
0152      Q*(IO+J-1))+DZ2*(Q*(IK+J)+Q*(IKP+J)))
0153      C
0154      C Add the square of the dynamic residual to ERR2.
0155      ERR2=ERR2+(Q1-Q*(IO+J))**2
0156      C
0157      C Update the nodal temperature Q(IST(I,J)+J) (=Q(IO+J)).
0158      45 U(IO+J)=Q1
0159      CONTINUE
0160      10 CONTINUE
0161      11 CONTINUE
0162      C
0163      C Calculate the 2-norm of the residuals.
0164      ERR2=SQRT(ERR2*DVOL)/COEFF
0165      RETURN
0166      END

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0001      SUBROUTINE RESCAL(MGF,MGC,MGRF,MGRG,M, MEX)
0002
0003      This subroutine transfers the residuals at nodes on grid MGF to
0004      nodes on grid MGC. The residuals are 'half-weighted' at interior
0005      nodes and 'full-weighted' at boundary nodes where non-Dirichlet
0006      boundary conditions are prescribed. When MGF is the finest grid
0007      (M) and MEX=1, tau extrapolation is applied by multiplying the
0008      estimated truncation error on grid M-1 by the factor 4/3.
0009
0010      DIMENSION IUF(19,33),IRF(19,33),IRC(19,33),IUC(19,33),OFCT(19,19)
0011      $ OFCT(19,19)=OFCT(19,19)+KBSSTC(33)*KBSSTF(33)
0012      COMMON /BAG/DBAG(10,5400)/DBPAR/A,B,C/DDBIR/MBIRCH
0013      $ /DBDZ/NZDIS(10)/DBM/MPATCH/IBASE/M
0014      $ /DBGR/NIGC(10),NIFC(10),NIFJ(10),NIFK(10),NIFL(10),NIFM(10)
0015      $ /DBMB/NIBI,NJBI,NKBI,NLBI,NMFI,NJFI,NKFI,NLFI,NMFI
0016      $ /DBEX/NEXTX
0017      $ /DBERR/RESER(10)/DBMX/MX(2300)
0018      $ /DBIST/IUC/DBISTF/IUF
0019
0020      Call subroutine KEY to define the coarse and fine grid parameters
0021      , in particular the arrays IRF and IRC that are used to locate
0022      the rhs. terms for the nodes on grids MGF and MGC respectively in
0023      the Q vector.
0024      CALL KEY (MGRF,IRF,KBSSTF,IIF,JJF,KKF,DXF,DYF,DZF)
0025      CALL KEY (MGRG,IRC,KBSSTC,IIC,JJC,KKC,DXC,DYC,DZC)
0026
0027      Call subroutine KEY to define the arrays IUC and IUF that are used
0028      to locate the temperatures of nodes on grids MGC and MGF respectively
0029      in the Q vector.
0030      CALL KEY (MGC,IUC,KBSSTC,IIC,JJC,KKC,DXC,DYC,DZC)
0031      CALL KEY (MGF,IUF,KBSSTF,IIF,JJF,KKF,DXF,DYF,DZF)
0032      IIC=IIC-1
0033      JJC=JJC-1
0034      IIF=IIF-1
0035      JJF=JJF-1
0036      KKF=KKF-1
0037      DX2=1./DXF**2
0038      DY2=1./DYF**2
0039      DZ2=1./DZF**2
0040      DXC2=1./DXC**2
0041      DYC2=1./DYC**2
0042      DZC2=1./DZC**2
0043      RESERR=0.
0044      DVOLC=DXC*DYC*DZC
0045
0046      C Set the factor of 4/3 for tau extrapolation (if required).
0047      FACT=1.
0048      IF(MGF.EQ.M.AND.MEX.EQ.1) FACT=1.333333
0049
0050      C Define the fine grid node limits, IIN,IFIN,JIN,KIN,KFIN.
0051      IIN=1
0052      IFIN=IIF
0053      IF(MGF.GE.NIBI) IIN=2
0054      IF(MGF.GE.NIMF) IFIN=IIF+1
0055      JIN=1
0056      IF(MGF.GE.NJBI) JIN=2
0057      KIN=1
0058      KFIN=KKF
0059      IF(MGF.GE.NKBI) KIN=2
0060      IF(MGF.GE.NKFI) KFIN=KKF+1
0061
0062      C Sweep through the z-axis nodes (KF) on the fine grid.
0063      DO 10 KF=KIN,KFIN
0064      K2=KF/2
0065      K22=2*K2
0066      NSK=0
0067      IF(MZDIS(MGF).EQ.1.AND.K22.EQ.KF) NSK=1
0068      DZDF=DZF
0069      IF(KF.EQ.1.OR.KF.EQ.KKF) DZDF=DZF/2.
0070
0071      C Sweep through the x-axis nodes (IF) on the fine grid.
0072      DO 15 IF=IIN,IFIN
0073      I2=IF/2
0074      I22=2*I2
0075      NSI=0
0076      IF(I22.EQ.IF) NSI=1
0077      DXF=DXF
0078      IF(IF.EQ.1.OR.IF.EQ.IIF) DXF=DXF/2.
0079      IFO=IUF(IF,KF)
0080      IFR=IRF(IF,KF)
0081      IF(IF.NE.1) IFM=IUF(IF-1,KF)
0082      IF(IF.NE.IIF) IFP=IUF(IF+1,KF)
0083      IF(KF.NE.1) IFAN=IUF(IF,KF-1)
0084      IF(KF.NE.KKF) IFKP=IUF(IF,KF+1)
0085
0086      C Sweep through the y-axis nodes (JF) on the fine grid.
0087      DO 20 JF=JIN,JFIN
0088      J2=JF/2
0089      J22=2*J2
0090
0091      C Check whether the residual at node IF,JF,KF is required. The residual
0092      is not required at nodes IF,JF,KF more than one node away from the
0093      boundary if :
0094      IF :
0095      a) NZDIS(MGF)=1 and IF+JF+KF is an even number.
0096      b) NZDIS(MGF)=0 and IF+JF is an odd number.
0097      $ IF(NSK.EQ.1.AND.KF.NE.2.AND.NF.NE.KKF1.AND.JF.GT.2.AND.IF.GT.
0098      $ 2.AND.IF.LT.IIF1) GO TO 15
0099      $ IF(NSA.EQ.0.AND.KF.NE.1.AND.NF.NE.KKF.AND.JF.GT.2.AND.IF.GT.
0100      $ 2.AND.IF.LT.IIF1.AND(NSI.EQ.1.OR.J22.EQ.JF)) GO TO 20
0101
0102      C If node IF,JF,KF is a boundary node, go to statement 30 to calculate
0103      the residual.
0104      $ IF(IF.EQ.1.OR.IF.EQ.IIF.OR.JF.EQ.1.OR.KF.EQ.1.OR.KF.EQ.KKF)
0105      $ GO TO 30
0106
0107      C Calculate the residual for interior node IF,JF,KF and store it as
0108      OFCT(IF,JF).
0109      $ OFCT(IF,JF)=O(IFR+JF)-TC*(DX2*(O(IFM+JF)+O(IFP+JF))-2.*O(IFO+JF)
0110      $ )+DY2*(O(IFO+JF1)+O(IFO+JF-1))-2.*O(IFO+JF))+DZ2*(O(IFKN+JF)
0111      $ +O(IFKP+JF))-2.*O(IFO+JF))
0112      $ GO TO 20
0113
0114      C Calculate the residual for the boundary node IF,JF,KF.
0115      $ DYF=DYF
0116      $ IF(JF.NE.1) GO TO 35
0117      $ HF=HX(KBSSTF(KF)+IIF)
0118      $ UYF=DYF/2.
0119      $ XPOSF=(IF-1)*DXF

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0120      ZPOSF=(KF-1)*DZ
0121
0122      C If node IF,JF,KF lies within the boundaries of the contact patch
0123      and an isothermal contact boundary condition is prescribed (i.e.,
0124      NH.NE.1), go to the next node.
0125      IF(ZPOSF.GT.A.OR.XPOSF.GT.B) GO TO 35
0126      XPACH=RESORT(1,-((ZPOSF+2)/(A+2)))
0127      IF(XPOSF.LE.XPACHF.AND.NH.NE.1) OFRT(IF,JF)=0.1 GO TO 20
0128
0129      C Sum the contributions to the residual for the boundary node IF,JF,KF
0130      in OTOT and store the final result as OFRT(IF,JF).
0131      35      OTOT=0(IFR+JF)-TC*(Q(IFD+JF+1)-Q(IFD+JF))/(DYF+DYFF)
0132      1-(JF.NE.1) OTOT=OTOT-TC*(Q(IFD+JF-1)-Q(IFD+JF))/(DYF+DYFF)
0133      IF(IF.NE.1) OTOT=OTOT-TC*(Q(IFR+JF)-Q(IFD+JF))/(DXF+DXXF)
0134      IF(IF.NE.1) OTOT=OTOT-TC*(Q(IFR+JF)-Q(IFD+JF))/(DXF+DXXF)
0135      IF(KF.NE.1) OTOT=OTOT-TC*(Q(IFR+JF)-Q(IFD+JF))/(DZF+DZFF)
0136      IF(KF.NE.1) OTOT=OTOT-TC*(Q(IFR+JF)-Q(IFD+JF))/(DZF+DZFF)
0137      IF(JF.NE.1) OTOT=OTOT-HP*(BASE-Q(IFD+JF))/DYF
0138      OFRT(IF,JF)=OTOT
0139
0140      20      CONTINUE
0141      15      CONTINUE
0142      IF(NZDIS(MGF).NE.1) GO TO 16
0143      GO TO 17
0144
0145      C Calculate the rhs. terms for grid MUC when grid MNC is coarser
0146      than grid MGF only in the x and y directions (i.e., when NZDIS(MGF)
0147      .NE.1)
0148      C First define the coarse grid node limits for the x-axis; ICIN and
0149      ICFIN.
0150      16      ICIN=1
0151      ICFIN=IIC
0152      DZC=DZC
0153      1-(KF.NE.1.OR.KF.EQ.KKF) DZC=DZC/2.
0154      IF(MGF.GE.NIB1) ICIN=IIC(MGF)+1
0155      1-(MGF.GE.NIBF) ICFIN=IIC(MGF)-1
0156
0157      C Sweep through the x-axis nodes (IC) on the coarse grid.
0158      DO 18 IC=ICIN,ICFIN
0159
0160      C Calculate the fine grid node I that is coincident with IC.
0161      I=2*(IC-ICIN+1)-1
0162      IFR=IRF(I,KF)
0163      DXC=DXC
0164      IF(IC.EQ.1.OR.IC.EQ.IIC) DXC=DXC/2.
0165
0166      C Calculate the coarse grid z-axis node KR that is coincident with
0167      KF.
0168      KR=KF+NIAC(MGF)-1
0169      ICD=IUC(IC,KR)
0170      IF(IC.NE.1) ICD=IUC(IC-1,KR)
0171      IF(IC.NE.IIC) ICP=IUC(IC,KR)
0172      IF(KR.NE.1) ICK=IUC(IC,KR-1)
0173      IF(KR.NE.KKC) ICKP=IUC(IC,KR+1)
0174      ICR=IRC(IC,KR)
0175
0176      C Calculate the coarse grid y-axis node limits, JGIN and JCFIN.
0177      JGIN=1
0178      JCFIN=JJC1
0179      IF(MGF.GE.NJB1) JGIN=NIJC(MGF)+1
0180      IF(MGF.GE.NJBF) JCFIN=NIJC(MGF)-1
0181
0182      C Sweep through the coarse grid y-axis nodes (JC).
0183      DO 19 JC=JGIN,JCFIN
0184
0185      C Calculate the fine grid y-axis node (J) that is coincident with JC.
0186      J=2*(JC-JGIN+1)-1
0187
0188      C If node IC,JC,KR is a boundary node, go to statement 21 to calculate
0189      its right-hand side terms.
0190      IF(JC.EQ.1.OR.IC.EQ.1.OR.IC.EQ.IIC.OR.KR.EQ.1.OR.KR.EQ.KKC)
0191      GO TO 21
0192
0193      C Calculate the rhs. terms for the interior node IC,JC,KR and store
0194      it as O(ICR+JC). Also add the square of the truncation error
0195      to RESEMR.
0196      21      OICR=SEOFRT(I,J)
0197      +TC*(DXC2*(Q(ICM+JC)+Q(ICP+JC))-2.*Q(ICD+JC))+DYC2*(Q(ICD+JC+1)
0198      +Q(ICD+JC-1))-2.*Q(ICD+JC))+DZC2*(Q(ICK+JC)+Q(ICKP+JC))-2.*
0199      Q(ICK+JC))
0200      RESEMR=RESEMR+(OICR-O(IFR+J))**2
0201      O(ICR+JC)=FACTS*(OICR-O(IFR+J))+O(IFR+J)
0202      GO TO 19
0203
0204      C Calculate the rhs. terms for the boundary node IC,JC,KR, first by
0205      using the residuals stored in OFR1 to obtain the full-weighted
0206      residual and then by adding the appropriate coarse grid terms to
0207      that result.
0208      21      OT=0.
0209      OTT=0.
0210      IF(IC.NE.1) OTT=OTT+OFRT(I,J)+OFRT(I,J+1)+OFRT(I-1,J)
0211      +OFRT(I-1,J+1)
0212      1-OT=OTT+4.
0213      IF(IC.NE.1.AND.JC.NE.1) OTT=OTT+OFRT(I,J)+OFRT(I,J-1)+OFRT(I-1,J)
0214      +OFRT(I-1,J-1) 1-OT=OTT+4.
0215      IF(IC.NE.IIC) OTT=OTT+OFRT(I,J)+OFRT(I,J+1)+OFRT(I+1,J)
0216      +OFRT(I+1,J+1) 1-OT=OTT+4.
0217      IF(IC.NE.1.IIC.AND.JC.NE.1) OTT=OTT+OFRT(I,J)+OFRT(I+1,J)
0218      +OFRT(I+1,J+1)+OFRT(I+1,J-1) 1-OT=OTT+4.
0219      RES=OTT/OT
0220      DYVC=DYVC
0221      1-(JC.NE.1) GO TO 57
0222      HP=MX(NBSTC(KF)+IC)
0223      DYVC=DYVC/2.
0224      XPOSF=(IC-1)*DXC
0225      ZPOSF=(KF-1)*DZC
0226      IF(ZPOSF.GT.A.OR.XPOSF.GT.B) GO TO 57
0227      XPACH=RESORT(1,-((ZPOSF+2)/(A+2)))
0228      IF(XPOSF.LE.XPACHF.AND.NH.NE.1) GO TO 19
0229      RES=RES+TC*(Q(ICD+JC+1)-Q(ICD+JC))/(DYC+DYVC)
0230      IF(JC.NE.1) RES=RES+TC*(Q(ICD+JC)-Q(ICD+JC))/(DYC+DYVC)
0231      IF(IC.NE.1) RES=RES+TC*(Q(ICM+JC)-Q(ICD+JC))/(DXC+DXXC)
0232      IF(IC.NE.IIC) RES=RES+TC*(Q(ICM+JC)-Q(ICD+JC))/(DXC+DXXC)
0233      IF(KF.NE.1) RES=RES+TC*(Q(ICM+JC)-Q(ICD+JC))/(DZC+DZVC)
0234      IF(KF.NE.NF) RES=RES+TC*(Q(ICM+JC)-Q(ICD+JC))/(DZC+DZVC)
0235      IF(JC.NE.1) RES=RES+HP*(BASE-Q(ICD+JC))/DYVC
0236      RESEMR=RESEMR+DXC*DZVC*(RES-O(IFR+J))**2
0237      1-//IVOLC
0238      O(ICR+JC)=FACTS*(RES-O(IFR+J))+O(IFR+J)
0239      CONTINUE
0240      18      CONTINUE
0241      17      CONTINUE

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0240      GO TO 10
0241 17      IF (NF.EQ.KIN) GO TO 40
0242      IF (KF.EQ.KKF) GO TO 45
0243      C
0244      C Check whether the fine grid z-axis node KF-1 coincides with a
0245      coarse grid z-axis node. If it does, calculate the rhs. terms
0246      for the coarse grid nodes at that coarse grid z-axis node (KC).
0247      If it doesn't, go to statement 45 to setup the entries in
0248      DFLT to those in DFLT and the entries in OFCT to those in OFRT.
0249      The residuals for the fine grid nodes at KF-1, KF-2 and KF
0250      are stored in the arrays DFLT, DFLT and OFRT respectively.
0251      KC=KF/2
0252      KC2=2*KC
0253      IF (KF.NE.KC2) GO TO 45
0254      KC=KC+NIKC(NGF)-1
0255      IF (KF.EQ.KKF) KC=KKC
0256      C
0257      C Define the first and last coarse grid x-axis nodes: ICIN and ICFIN.
0258      ICIN=1
0259      ICFIN=IIC
0260      DZC=DZC
0261      IF (KC.EQ.1.OR.KC.EQ.KKC) DZC=DZC/2.
0262      IF (NGF.NE.NJBI) ICIN=NIIC(NGF)+1
0263      IF (NGF.GE.NJBF) ICFIN=NFIC(NGF)-1
0264      C
0265      C Sweep through the coarse grid x-axis nodes (IC).
0266      DO 50 IC=ICIN,ICFIN
0267      C
0268      C Calculate the fine grid x-axis node I that coincides with IC.
0269      I=2*(IC-ICIN)+1
0270      IFR=IRF(I,KF-1)
0271      DXIC=DXC
0272      IF (IC.EQ.1.OR.IC.EQ.IIC) DXIC=DXC/2.
0273      ICO=IUC(IC,KC)
0274      IF (IC.NE.1) ICN=IUC(IC-1,KC)
0275      IF (IC.NE.IIC) ICP=IUC(IC+1,KC)
0276      IF (KC.NE.1) ICAN=IUC(IC,KC-1)
0277      IF (KC.NE.KKC) ICKP=IUC(IC,KC+1)
0278      ICN=IRC(IC,KC)
0279      C
0280      C Define the first and last coarse grid y-axis nodes: JCIN and JCFIN.
0281      JCIN=1
0282      JCFIN=JJC
0283      IF (NDIRCH.EQ.1) JCIN=2
0284      IF (NGF.GE.NJBI) JCIN=NIJC(NGF)+1
0285      IF (NGF.GE.NJBF) JCFIN=NFJC(NGF)-1
0286      C
0287      C Sweep through the coarse grid y-axis nodes (JC).
0288      DO 55 JC=JCIN,JCFIN
0289      C
0290      C Calculate the fine grid y-axis node J that coincides with JC.
0291      J=2*(JC-JCIN)+1
0292      C
0293      C Check whether node IC,JC,KC is a boundary node. If it is, so to
0294      statement 70 to calculate its rhs. term.
0295      IF (KC.EQ.1.OR.KC.EQ.KKC.OR.JC.EQ.1.OR.IC.EQ.1.OR.IC.EQ.IIC)
0296      GO TO 70
0297      C
0298      C Calculate the rhs. term for the interior node IC,JC,KC and store
0299      it as D(ICK+JC). Add the square of the truncation error to RESERR.
0300      QICK=.5*OFCT(I,J)
0301      +TCR(DXC2*(Q(ICN+JC)+Q(ICP+JC)-2.*Q(ICO+JC))+BYC2*(Q(ICD+JC+1)
0302      +Q(ICD+JC-1)-2.*Q(ICO+JC))+DZC2*(Q(ICKN+JC)+Q(ICKP+JC)-2.*
0303      *Q(ICK+JC)))
0304      RESERR=RESERR+(QICK-Q(IFR+J))**2
0305      Q(ICK+JC)=FACTS(QICK-Q(IFR+J))+Q(IFR+J)
0306      GO TO 55
0307      C
0308      C Calculate the rhs. term for the boundary node IC,JC,KC: first by
0309      transferring the residuals at the nearest fine grid nodes with
0310      full-weightings and then by adding the appropriate coarse grid
0311      terms to that result.
0312 70      QTO=0
0313      QTT=0.
0314      IF (KC.NE.1.AND.IC.NE.1) QTT=QTT+OFCT(I,J)+OFCT(I-1,J)+OFCT(I-1,
0315      J+1)+OFCT(I-1,J+1)+OFCT(I-1,J+1)+OFCT(I-1,J+1)+OFCT(I-1,J+1)
0316      + QTO+QTO+8.
0317      IF (KC.NE.1.AND.IC.NE.1.AND.JC.NE.1) QTT=QTT+OFCT(I,J)+OFCT(I-1,J)
0318      +OFCT(I-1,J-1)+OFCT(I-1,J-1)+OFCT(I-1,J-1)+OFCT(I-1,J-1)+OFCT(I-1,J-1)
0319      +OFCT(I-1,J-1) + QTO+QTO+8.
0320      IF (KC.NE.1.AND.IC.NE.IIC) QTT=QTT+OFCT(I,J)+OFCT(I+1,J)+OFCT(I,
0321      J+1)+OFCT(I+1,J+1)+OFCT(I+1,J+1)+OFCT(I+1,J+1)+OFCT(I+1,J+1)
0322      + QTO+QTO+8.
0323      IF (KC.NE.1.AND.IC.NE.IIC.AND.JC.NE.1) QTT=QTT+OFCT(I,J)+OFCT(I+1,
0324      J)+OFCT(I+1,J-1)+OFCT(I+1,J-1)+OFCT(I+1,J-1)+OFCT(I+1,J-1)+OFCT(I+1,J-1)
0325      +OFCT(I+1,J-1) + QTO+QTO+8.
0326      IF (KC.NE.KKC.AND.IC.NE.1) QTT=QTT+OFCT(I,J)+OFCT(I-1,J)+OFCT(I-1,
0327      J+1)+OFCT(I-1,J+1)+OFCT(I-1,J+1)+OFCT(I-1,J+1)+OFCT(I-1,J+1)
0328      + QTO+QTO+8.
0329      IF (KC.NE.KKC.AND.IC.NE.1.AND.JC.NE.1) QTT=QTT+OFCT(I,J)+OFCT(I-1,J)
0330      +OFCT(I-1,J-1)+OFCT(I-1,J-1)+OFCT(I-1,J-1)+OFCT(I-1,J-1)+OFCT(I-1,J-1)
0331      +OFCT(I-1,J-1) + QTO+QTO+8.
0332      IF (KC.NE.KKC.AND.IC.NE.IIC) QTT=QTT+OFCT(I,J)+OFCT(I+1,J)+OFCT(I,
0333      J+1)+OFCT(I+1,J+1)+OFCT(I+1,J+1)+OFCT(I+1,J+1)+OFCT(I+1,J+1)
0334      + QTO+QTO+8.
0335      IF (KC.NE.KKC.AND.IC.NE.IIC.AND.JC.NE.1) QTT=QTT+OFCT(I,J)+OFCT(I+1,
0336      J)+OFCT(I+1,J-1)+OFCT(I+1,J-1)+OFCT(I+1,J-1)+OFCT(I+1,J-1)+OFCT(I+1,J-1)
0337      +OFCT(I+1,J-1) + QTO+QTO+8.
0338      RES=QTT/QT
0339      D(ICK+JC)=DYC+DYC
0340      IF (JC.NE.1) GO TO 56
0341      MP=MX(KBST(KC)+IC)
0342      DYC=MYC/2.
0343      XPOSC=(IC-1)*DXC
0344      ZPOSC=(KC-1)*DZC
0345      IF (ZPOSC.GT.A.OR.XPOSC.GT.B) GO TO 56
0346      XPACMC=MSORT(1,-((ZPOSC**2)/(A**2)))
0347      IF (XPACMC.LE.XPACMC.AND.MH.NE.1) GO TO 55
0348      RES=RES+TCL*(Q(ICO+JC)-Q(ICO+JC))/(DYC*DYC)
0349      IF (JC.NE.1) RES=RES+TCL*(Q(ICK+JC)-Q(ICK+JC))/(DXC*DXC)
0350      IF (IC.NE.IIC) RES=RES+TCL*(Q(ICP+JC)-Q(ICP+JC))/(DXC*DXC)
0351      IF (IC.NE.1) RES=RES+TCL*(Q(ICAN+JC)-Q(ICAN+JC))/(DZC*DZC)
0352      IF (KC.NE.KKC) RES=RES+TCL*(Q(ICKP+JC)-Q(ICKP+JC))/(DZC*DZC)
0353      IF (JC.EQ.1) RES=RES+MP*(TRASE-Q(ICO+JC))/DYC
0354      RESERR=RESERR+DXC*DYC*DZC*(RES-Q(IFR+J))**2
0355      /DZC
0356      Q(ICK+JC)=FACTS(RES-Q(IFR+J))+Q(IFR+J)
0357      CONTINUE
0358 55      CONTINUE
0359 50      CONTINUE

```

```

0340      IF(NF.EQ.KNF) GO TO 10
0341      C
0342      C Equate the entries in QFLT to those in DFCT and the entries in
0343      C DFCT to those in OFRT.
0344      45      DO 60 I=1,IIF
0345              DO 60 J=1,JJF1
0346              QFLT(I,J)=DFCT(I,J)
0347      60      DO 65 I=1,IIF
0348              DO 65 J=1,JJF1
0349              OFCT(I,J)=OFRT(I,J)
0350      65      IF(KF.EQ.KKF) GO TO 75
0370      10      CONTINUE
0371      C
0372      C Calculate the 2-norm of the truncation errors on grid MGC (estimated)
0373      C designated RESER(MGC).
0374      RESER(MGC)=SORT(DVULCRESER)
0375      RETURN
0376      END
0377

```

```

0001      SUBROUTINE KEY (NG,IST,KBST,IMAX,JMAX,KMAX,DX, DYY,DZZ)
0002      C
0003      C This subroutine defines the entries in the array IST so that
0004      C it locates the temperatures of the nodes on grid NG in the Q
0005      C vector when NG .GT. N. The temperature of the rhs. belongs to
0006      C grid NG-M when NG .GT. N. The temperature of the rhs. belongs to
0007      C node I,J,K is stored as Q(IST(I,K)+J). The subroutine
0008      C also defines the variables KBST,IMAX,...,DZZ for grid NG if
0009      C NG .LE. N or for grid NG-M if NG .GT. N. The array KBST locates
0010      C the contact surface heat transfer coefficients for the Mth.
0011      C grid in the vector HX since HX(KBST(K)+1) represents the heat
0012      C transfer coefficient at node I,K on the contact surface (where J=1).
0013      C
0014      DIMENSION IST(19,33),KBST(35)
0015      COMMON /GRD/NST(20),IMX(20),JMX(20),KMX(20),BX(20),BY(20),DZ(20)
0016      $
0017      NST(20)
0018      IMAX=IMX(NG)
0019      JMAX=JMX(NG)
0020      KMAX=KMX(NG)
0021      IS=NST(NG)-JMAX-1
0022      NS=NST(NG)-IMAX-1
0023      DO 1 K=1,KMAX
0024      NS=NS+IMAX
0025      KBST(K)=NS
0026      DO 1 I=1,IMAX
0027      IS=IS+JMAX
0028      IST(I,K)=IS
0029      1      DX=BX(NG)
0030      DYY=BY(NG)
0031      DZZ=DZ(NG)
0032      RETURN
0033      END
0034

```

```

0001      SUBROUTINE GKUFN (N,IMAX,JMAX,KMAX,DX, DYY,DZZ,M)
0002      C
0003      C This subroutine allocates space in the Q vector for the nodal
0004      C temperatures on grid N if N .LE. M or for the rhs. terms on grid
0005      C N-M if N .GT. M. The variable IQ points to the next available
0006      C location in Q, which is stored as NST(N). The variable IN points
0007      C to the next available location in the vector of contact surface
0008      C heat transfer coefficients, HX.
0009      C
0010      COMMON /GRD/NST(20),IMX(20),JMX(20),KMX(20),BX(20),BY(20),DZ(20)
0011      $
0012      NST(20)
0013      DAT=IQ,IM/1,1/
0014      NST(N)=IQ
0015      IMX(N)=IMAX
0016      JMX(N)=JMAX
0017      KMX(N)=KMAX
0018      DX(N)=DX
0019      DY(N)=DYY
0020      DZ(N)=DZZ
0021      IF(N.LE.M) IM=IM+IMAX*KMAX
0022      IQ=IQ+IMAX*JMAX*KMAX
0023      RETURN
0024      END

```

```

0001      SUBROUTINE INTERP(NGC,MGF)
0002      C
0003      C This subroutine uses trilinear interpolation to obtain the first
0004      C approximation on grid MGF from the converged solution on grid NGC.
0005      C The subroutine is identical to subr. INTADD except that the nodal
0006      C temperatures are interpolated from grid NGC to grid MGF instead of
0007      C the FAS corrections.
0008      C
0009      DIMENSION ISTC(19,33),ISTF(19,33),KBSTC(35),KBSTF(35)
0010      COMMON /DBDZ/DBDIS(10),DBP/PA(8),A-IC/DBDIR/WDIRCH
0011      $
0012      $ /DBDZ/DBDIS(10),NFKC(10),NFKC(10),NFKC(10),NFKC(10)
0013      $ /DBDZ/ISTC(10),NFKC(10),NFKC(10),NFKC(10),NFKC(10)
0014      CALL KEY (NGC,ISTC,KBSTC,IIC,JJC,KKC,DXC,DYC,DZC)
0015      CALL KEY (MGF,ISTF,KBSTF,IIF,JJF,KKF,DXF,DYF,DZF)
0016      I1=NFKC(NGF)
0017      I2=NFKC(NGF)
0018      J1=NFKC(NGF)
0019      J2=NFKC(NGF)
0020      K1=NFKC(NGF)
0021      K2=NFKC(NGF)
0022      DO 10 KC=K1,K2
0023      IF(KC-K1)
0024      IF(WDIS(NGF).EQ.1) KF=2*(KC-K1+1)-1
0025      DO 1 IC=I1,I2
0026      IF=3*(IC-I1+1)-1
0027      IF(IC.EQ.I1) GO TO 6
0028      IFM=ISTF(IF-1,KF)
0029      ICM=ISTC(IC-1,KC)
0030      IFO=ISTF(IF,KF)
0031      ICO=ISTC(IC,KC)

```

```

0032 IF(KC.EQ.K1) GO TO 8
0033 IKFM=ISTF(1F,KF-1)
0034 IF(1F.NE.1) IKFM2=ISTF(1F-1,KF-1)
0035 IF(NZUJS(NGF).NE.1) GO TO 9
0036 IKCM=ISTC(IC,KC-1)
0037 IF(IC.NE.1) IKCA2=ISTC(IC-1,KC-1)
0038 DO 1 JC=J1,J2
0039 JF=2*(JC-J1+1)-1
0040 Q(1F+JF)=Q(1C+JC)
0041 IF(JC.NE.J1) A1=.5*(Q(1C+JC)+Q(1C+JC-1))
0042 IF(JC.NE.J1) A1=.5*(Q(1C+JC)+Q(1C+JC-1))
0043 IF(JC.NE.J1) A1=.5*(Q(1C+JC)+Q(1C+JC-1))
0044 IF(JC.NE.J1) A1=.5*(Q(1C+JC)+Q(1C+JC-1))
0045 IF(JC.NE.J1) A1=.5*(Q(1C+JC)+Q(1C+JC-1))
0046 IF(NZDIS(NGF).NE.1) GO TO 1
0047 IF(KC.NE.K1) A4=.5*(Q(1C+JC)+Q(1C+JC-1))
0048 IF(KC.NE.K1) A4=.5*(Q(1C+JC)+Q(1C+JC-1))
0049 IF(KC.NE.K1) A4=.5*(Q(1C+JC)+Q(1C+JC-1))
0050 IF(KC.NE.K1) A4=.5*(Q(1C+JC)+Q(1C+JC-1))
0051 IF(KC.NE.K1) A4=.5*(Q(1C+JC)+Q(1C+JC-1))
0052 IF(KC.NE.K1) A4=.5*(Q(1C+JC)+Q(1C+JC-1))
0053 IF(KC.NE.K1) A4=.5*(Q(1C+JC)+Q(1C+JC-1))
0054 CONTINUE
0055 CONTINUE
0056 IF(NK.EQ.1) GO TO 16
0057 NI=8/DZF+1
0058 NK=4/DZF+1
0059 IF(NI.NE.1) NI=1
0060 IF(NK.NE.1) NK=1
0061 DO 15 K=1,NK
0062 DO 15 I=1,NI
0063 IFQ=ISTF(1,K)
0064 XPOS=(I-1)*DZF
0065 ZPOS=(K-1)*DZF
0066 XPATCH=BSORT(1,-(ZPOS*2)/(A*2))
0067 IF(XPOS.LE.XPATCH) Q(1F+1)=0.
0068 RETURN
0069 END

```

```

0001 SUBROUTINE MXD(NG)
0002
0003 Contact surface heat transfer coefficients are defined for grid NG
0004 in this subroutine using an expression obtained from Hertz's
0005 theory of elastic contact for the gap (SEPN) between the contact
0006 surfaces and a non-continuum gas conductivity model. The subroutine
0007 uses approximations to elliptic integrals which are only valid for
0008 large aspect ratio contacts; see for contacts with A/B.GT. 4.
0009
0010 DIMENSION IST(19,33),KBST(35),NIW(40)
0011 COMMON /DBAR/A,B,TC,BRIST/IST/DBMX/HX(2300)
0012 /DBH/HPATCH,IBASE,NIW
0013 /DBRAD/RCY1,RCY2,TCGAS,CCNM,INSUL/DBNIW/NIW
0014 CALL KEY (NG,IS,KBST,IPAX,IMAX,IMAX,IX,IX,DZ)
0015
0016 Sweep through the x-axis nodes (I) and calculate the x-coordinate of
0017 each node ( XPOS ).
0018 DO 1 I=1,IMAX
0019 XPOS=(I-1)*BX
0020
0021 Sweep through the z-axis nodes (K) and calculate the z-coordinate of
0022 each node ( ZPOS ).
0023 DO 1 K=1,IMAX
0024 NIW(K)=0
0025 ZPOS=(K-1)*BZ
0026
0027 If node I,K lies outside the contact, go to statement 10 to
0028 calculate the gas conductance. If it doesn't, set the surface
0029 conductance HX(KBST(K)+1)=HPATCH and go to statement 1.
0030 IF(XPOS.GT.A.OR.XPOS.GT.B) GO TO 10
0031 XPATCH=BSORT(1,-(ZPOS*2)/(A*2))
0032 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0033 IF(INSUL.EQ.1) HX(KBST(K)+1)=0.
0034 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0035 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0036 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0037 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0038 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0039 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0040 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0041 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0042 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0043 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0044 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0045 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0046 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0047 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0048 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0049 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0050 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0051 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0052 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0053 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0054 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0055 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH
0056 IF(XPOS.LE.XPATCH) HX(KBST(K)+1)=HPATCH

```

```

0001 SUBROUTINE RESIST(N)
0002
0003 The resistance of the contact (RES) is calculated in this
0004 subroutine
0005
0006 DIMENSION KBST(35),NIW(40)
0007 COMMON /DBR/D(56500)/DBAR/A,B,TC,BRIST/IST(19,33)/DBH/HPATCH,
0008 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0009 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0010 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0011 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0012 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0013 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0014 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0015 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),
0016 /DBH/HPATCH,IBASE,NIW(40),HKE(10),HKE(10),HKE(10),HKE(10),

```

```

0017 CALL KEY(MG,IST,KBS1,II,JJ,KK,DX,DY,DZ) 0136.600
0018 NI1=1 0136.700
0019 MK1=1 0136.800
0020 IF(MG.EQ.N) GO TO 2 0136.900
0021 NI1=II+1 0137.000
0022 MK1=KK+1 0137.100
0023 IF(MG1.GE.NIMF) NI1=MFIC(MG1) 0137.200
0024 IF(MG1.GE.NKSF) MK1=MFIC(MG1) 0137.300
0025 DO 3 I=1,II 0137.400
0026 DO 3 K=1,KK 0137.500
0027 IF(I.LT.NI1.AND.K.LT.MK1) GO TO 3 0137.600
0028 DXX=DX 0137.700
0029 IF((I.EQ.NI1.AND.K.LE.MK1).OR.I.EQ.1.OR.I.EQ.II) DXX=DX/2. 0137.800
0030 XPOS=(I-1)*DX 0137.900
0031 DZZ=DZ 0138.000
0032 IF((K.EQ.MK1.AND.I.LE.NI1).OR.K.EQ.1.OR.K.EQ.KK) DZZ=DZ/2. 0138.100
0033 ZPOS=(K-1)*DZ 0138.200
0034 I=0(IST(I,K)+1) 0138.300
0035 IF(MH.EQ.1) GO TO 5 0138.400
0036 IF(ZPOS.GT.A.OR.XPOS.GT.B) GO TO 5 0138.500
0037 XPACHF=BSORT(1,-((ZPOS**2)/(A**2))) 0138.600
0038 IF(XPOS.GT.XPACHF) GO TO 5 0138.700
0039 T=0(IST(I,K)+1) 0138.800
0040 HFLO=DXX*DZZ*(0(IST(I,K)+2)-T)/DY + .5*DY*DZZ*(0(IST(I+1,K)+1) 0138.900
0041 -T)/DX + .5*DY*DXX*(0(IST(I,K+1)+1)-T)/DZ 0139.000
0042 IF(I.NE.1) HFLO=HFLO + .5*DY*DZZ*(0(IST(I-1,K)+1)-T)/DX 0139.100
0043 IF(K.NE.1) HFLO=HFLO + .5*DY*DXX*(0(IST(I,K-1)+1)-T)/DZ 0139.200
0044 HFLOW=HFLOW+HFLO 0139.300
0045 HFLOMP=HFLOW+HFLO 0139.400
0046 HFLOX=HFLO/(DXX*DZZ) 0139.500
0047 GO TO 8 0139.600
0048 HFLOX=HX(KBST(K)+1)*(T-TBASE) 0139.700
0049 HFLOW=HFLOW+HFLOX*DXX*DZZ 0139.800
0050 IF(HX(KBS1(K)+1).EQ.HPATCH.OR.(I.LE.NIM(K).AND.MG.EQ.N)) HFLOMP 0139.900
0051 =HFLOMP+HFLOX*DXX*DZZ 0140.000
0052 IF(I.EQ.1.OR.K.EQ.1) WRITE (108,4) MG,I,K,XPOS,ZPOS,HFLOX,T 0140.100
0053 FORMAT (1X,'MG=',I3,' I=',I3,' K=',I3,' XPOS=',F10.7, 0140.200
0054 ' ZPOS=',F10.7,' HFLOX=',F10.5,' T=',F10.7) 0140.300
0055 CONTINUE 0140.400
0056 RESTOT=1./HFLOW 0140.500
0057 RSPEC=YLEN/(TC*XLENSZLEN) 0140.600
0058 IF(MH.NE.1) GO TO 10 0140.700
0059 RMICRO=1./(0./BS48*XBSPATCH) 0140.800
0060 IF(LIMSLA.EQ.1) RSPEC=RSPEC+RMICRO 0140.900
0061 RES=(RESTOT-RSPEC)/2. 0141.000
0062 RATIO=HFLOMP/HFLOW 0141.100
0063 WRITE (108,100) HFLOW,HFLOMP,RATIO,RES 0141.200
0064 FORMAT ('0 TOTAL HEAT FLOW RATE =',E12.5/4X, 0141.300
0065 ' HEAT FLOW RATE THROUGH CONTACT PATCH =',E12.5/4X, 0141.400
0066 ' RATIO OF HEAT FLOW THROUGH CONTACT PATCH TO TOTAL HT. FLOW =', 0141.500
0067 F10.5/4X,' RESISTANCE OF CONTACT =',F10.5) 0141.600
0068 RETURN 0141.700
0069 END 0141.800
0070

```


Listing II.G.2 Subroutine MULTIG for the Fixed Multigrid Algorithm.

```

SUBROUTINE MULTIG(DX0,DY0,DZ0,TOL,UMAX,U1,F,NX0,NY0,NZ0,M,XLEN, MFIX0000
$YLEN,ZLEN) MFIX0001
C MFIX0002
C This subroutine controls the sequence of operations performed in the MFIX0003
C 'fixed' version of the multigrid algorithm. The solution is MFIX0004
C obtained initially on the second to coarsest grid (NG=2) using the MFIX0005
C accommodative algorithm and then on subsequent (finer) grids MFIX0006
C with the fixed algorithm. MFIX0007
C MFIX0008
EXTERNAL U1,F MFIX0009
DIMENSION EPS(10) MFIX0010
COMMON /DRGZ/NDIS(10)/DRUG/VG(10),WUINC(10) MFIX0011
$/DREX/NEXTR/DEFAK/A,B,TC/DBH/HATCH,HOUT,TRASE,NH MFIX0012
$/DBERR/RESERR(10)/DBRED/REDFAC,NCYCAL,NSPSW MFIX0013
$/DBERR3/ERR3(10) MFIX0014
$/DBIST/IST(19,33)/DBQ/Q(56500) MFIX0015
NEX=0 MFIX0016
DO 1 NG=1,2 MFIX0017
CALL PUTF(NG,U1,XLEN,YLEN,M) MFIX0018
1 CALL HXO(NG) MFIX0019
LU=0. MFIX0020
ML=1 MFIX0021
10 ML=ML+1 MFIX0022
NG=ML MFIX0023
EPS(NG)=1.E-6 MFIX0024
NCYCLE=1 MFIX0025
NXX=0 MFIX0026
NSWEEP=1 MFIX0027
DEN=4. MFIX0028
CALL PUTF(NG+M,F,XLEN,YLEN,M) MFIX0029
CALL HXO(NG) MFIX0030
5 IF(NXX.EQ.1.AND.NCYCLE.EQ.1.AND.NG.EQ.ML) NCYCLE=2 ; NSWEEP=1 MFIX0031
CALL RELAX(NG,NG+M,ERR1,ERR2,ML) MFIX0032
WU=WU+WUINC(NG) MFIX0033
WRITE (108,4) NG,ERR2,WU MFIX0034
4 FORMAT ('LEVEL=',I2,5X,'RESIDUAL NORM=',E12.3,5X,'WORK=',F10.5) MFIX0035
IF(LU.GE.UMAX) RETURN MFIX0036
IF(ML-2) 7,7,6 MFIX0037
7 IF(ERR2.GE.EPS(NG)) GO TO 6 MFIX0038
IF(NG.NE.ML) GO TO 2 MFIX0039
CALL INTERP(NG,NG+1) MFIX0040
GO TO 10 MFIX0041
6 IF(NG-1) 11,12,11 MFIX0042
12 IF(ML.EQ.2) GO TO 5 MFIX0043
IF(NSWEEP.LT.NSPSW) NSWEEP=NSWEEP+1 ; GO TO 5 MFIX0044
NXX=1 MFIX0045
GO TO 2 MFIX0046
11 IF(ML.GT.2.AND.NSWEEP.NE.1) GO TO 14 MFIX0047
GO TO 15 MFIX0048
14 IF(NCYCLE.EQ.2.AND.NG.EQ.ML) GO TO 16 MFIX0049
GO TO 2 MFIX0050
16 IF(ML.EQ.M) RETURN MFIX0051
CALL INTERP(NG+1,NG+2) MFIX0052
GO TO 10 MFIX0053
15 CALL PUT2(NG-1,NG) MFIX0054
IF(NEXTR.EQ.1) NEX=NXX MFIX0055
CALL RESCAL (NG,NG-1,NG+M,NG+M-1,ML,RESERR,NEX) MFIX0056
IF(NG.EQ.ML) EPS(NG)=RESERR(NG-1)/DEN MFIX0057
EPS(NG-1)=.15*ERR2*SQRT(VG(NG-1)/VG(NG)) MFIX0058
IF(NG.EQ.M-1.AND.NEX.EQ.1) EPS(NG)=RESERR(NG-1)/DEN MFIX0059
IF(NG.EQ.4.AND.NEX.EQ.1) EPS(NG-1)=1.E-6 MFIX0060
NG=NG-1 MFIX0061
GO TO 5 MFIX0062
2 CALL INTADD (NG,NG+1) MFIX0063
NERR=C MFIX0064
NG=NG+1 MFIX0065
GO TO 5 MFIX0066
END MFIX0067

```

Listing II.G.3 Subroutine HXO For Contact Approximation Technique 3.

```

SUBROUTINE HXO(MG)
  *** VERSION USED WITH CONTACT APPROXIMATION TECHNIQUE 3 ***

  Contact surface heat transfer coefficients are defined for grid MG
  in this subroutine using an expression obtained from Hertz's
  theory of elastic contact for the gap (SEPN) between the contact
  surfaces and a non-continuum gas conductivity model. The subroutine
  uses approximations to elliptic integrals which are only valid for
  large aspect ratio contact, say for contacts with A/B .GT. 4.

  DIMENSION IST(19,33),KBST(33),NIW(40)
  COMMON /DMPAR/A,B,TC,DBIST/IST/DBHX/HX(2300)
  /DBH/DBH,TRASE,NH
  /DBRAD/RCT1,KC12,TCGAS,CCNH,INSUL
  /DBHIV/NIW
  CALL IET(MG,IST,KBST,INAX,JMAX,KMAX,DX,DY,DZ)

  Summ through the x-axis nodes (I) and calculate the x-coordinate of
  each node ( XPOS ).
  DO 2 KX=1,KMAX
    NIW(KX)=0
    DO 1 I=1,IMAX
      XPOS=(I-1)*DX
      XPOS=0.0
      IF(I.NE.1) XPOS=XPOS-.5*DX
      XPOS=(IMAX-1)*DX
      IF(I.NE.IMAX) XPOS=XPOS+.5*DX

  Summ through the z-axis nodes (K) and calculate the z-coordinate of
  each node ( ZPOS ).
  DO 3 K=1,KMAX
    ZPOS=(K-1)*DZ

  If node I,K lies outside the contact, go to statement 10 to
  calculate the gas conductance. If it doesn't, determine the
  appropriate contact conductance for contact approximation technique
  3 and store that conductance as HX(KBST(K)+1).
  IF(ZPOS.GT.A.OR.XPOSA.GT.B) GO TO 10
  XPACHF=B*SQRT(1-((ZPOS**2)/(A**2)))
  IF(XPACHF.LE.XPOS) GO TO 10
  IF(XPACHF.GE.XPOS) HX(KBST(K)+1)=HPATCH 1 GO TO 1
  NIW(K)=1
  HGRID=1./(XPOS*(XPACHF)+XPOS*B*.63662*ALOG(XPOS*(XPACHF)/
  10 1TC))
  IF(I.EQ.1) HX(KBST(K)+1)=HGRID 1 GO TO 1
  AS=.6366ASIN(XPOS*A/XPACHF)
  HPATCH=(1.-AS)/(1./HGRID-AS/HPATCH)
  KATIO=(XPACHF-XPOS)/DX
  IF(XPACHF.LT.XPOS.AND.INSUL.NE.1) GO TO 10
  HX(KBST(K)+1)=HPATCH
  GO TO 1

  10 IF(INSUL.EQ.1) HX(KBST(K)+1)=0.0 1 GO TO 1
  RH=(A**2.0)/((B**2.0)-(ZPOS**2.0)-(XPOS**2.0))
  CC=((A**2**2.0)-((ZPOS**2.0)-(XPOS**2.0))
  1TC=TCGAS**2.0
  XPOS=XPOS**2.0
  XNUS=(-B*SQRT(B**2.0-4.0*CC))/2.0
  XNUP=ASB
  IF(XNUS.LE.XNUP) FTHI=ALOG((SQRT(XNUS)+SQRT(XNUS+B**2.0))/B)
  IF(XNUS.GT.XNUP) FTHI=ALOG(4.0*B/B)-ALOG(SQRT(A**2.0/XNUS))+SQK
  1TC=SQRT(XNUS/(XNUS+B**2.0))
  COEFF=-1.0*AS(B**2.0)*(RCT1+RCT2)/(4.0*RCY1*RCY2)

  Calculate the gap between the contact surfaces (SEPN).
  SEPN=COEFF*(1.0/(2.0)*FTHI)-(ZP2*((2.0)/(A*(A**2.0-B**2.0)))*FTHI
  1TC-R3))+(2.0/A**2.0)*SQRT(1.0/(XNUS+B**2.0)*R3)) -(XP2**2.0*AS*(R3-
  1TC*(R3**2.0/A**2.0)*FTHI)/((B**2.0)*(A**2.0-B**2.0)))
  SEPN=XP2/(2.0*RCY1)+ZP2/(2.0*RCY2)

  Calculate the contact surface heat transfer coefficient for node
  I,K as: HX(KBST(K)+1).
  HX(KBST(K)+1)=2.0*TCGAS/(1.0+(CCNH/SEPN))/SEPN
  IF(NIW(K).EQ.1) HX(KBST(K)+1)=(1.-KATIO)*HX(KBST(K)+1)+HPATCH
  CONTINUE
  RETURN
END

```

Listing II.6.4 Routines Used With
Contact Approximation
Technique 4.

```

SUBROUTINE MXO(MG)
  ***** VERSION USED WITH CONTACT APPROXIMATION TECHNIQUE 4 *****
  Contact surface heat transfer coefficients are defined for grid MG
  in this subroutine using an expression obtained from Hertz's
  theory of elastic contact for the gap (SEPN) between the contact
  surfaces and a non-continuous see conductivity model. The subroutine
  uses approximations to elliptic integrals which are only valid for
  large aspect ratio contacts. See for contacts with A/B .GT. 4.

  DIMENSION IST(19,33),KBST(35),NIU(40)
  COMMON /BPAR/A,B,TC,DBIST/IST/DBMX/MX(2300)
  /DBM/MPATCH,TRABE,MH
  /DBRAI/RCY1,RCY2,TCBAS,CCMH,INSUL
  /DBNIU/NIU
  CALL KEY (MG,IST,KBST,IMAX,JMAX,KMAX,DX,DY,DZ)

  Sweep through the x-axis nodes (I) and calculate the x-coordinate of
  each node ( XPOS ).
  DO 2 KK=1,KMAX
    NIU(KK)=0
    DO 1 I=1,IMAX
      XPOS=(I-1)*DX
      XPOS=0.0
      IF(1,NE,1) XPOS=XPOS-.5*DX
      XPOS=(IMAX-1)*DX
      IF(1,NE,IMAX) XPOS=XPOS+.5*DX

  Sweep through the z-axis nodes (K) and calculate the z-coordinate of
  each node ( ZPOS ).
  DO 1 KK=1,KMAX
    ZPOS=(K-1)*DZ

  If node I,K lies outside the contact, go to statement 10 to
  calculate the its conductance. If it doesn't, determine the
  appropriate contact conductance for contact approximation technique
  4 and store that conductance as MX(KBST(K)+1).
  IF(ZPOS.GT.0,OR,XPOS.GT.B) GO TO 10
  XPCNF=RSORT(1,-((ZPOS**2)/(A**2)))
  IF(XPCNF.LE,XPOS) GO TO 10
  IF(XPCNF.GE,XPOS) MX(KBST(K)+1)=MPATCH ; GO TO 1
  NIU(KK)=1
  IF(1.EQ.1) MCONR=MPATCH ; PIC=XPCNF ; GO TO 20
  CALL WIDTH (XPOS,XPOS,XPCNF,IC,MPATCH,HA,PIC)
  MCONR=MPATCH*(MCONR+XPCNF)/(MPATCH+XPCNF+HA+PIC)
  MGRID=1-(XPOS/(MCONR+PIC)+XPOS/(.63662*ALOG(XPOS)/PIC)/TC)
  IF(1.EQ.1) MX(KBST(K)+1)=MGRID ; GO TO 1
  AS=.63662*ASIN(XPOS/PIC)
  MPATT=(1-AS)/(1+MGRID-AS/MCONR)
  PATIO=(XPCNF-XPOS)/DX
  NIF=1-1
  DO 2 I=1,NIF
    MX(KBST(K)+1)=MCONR
    IF(XPCNF.LT,XPOS,AND,INSUL,NE,1) GO TO 10
    MX(KBST(K)+1)=MPATT
  GO TO 1
10 IF(INSUL.EQ.1) MX(KBST(K)+1)=0.0 ; GO TO 1
  BB=(A**2.0)+(B**2.0)-(ZPOS**2.0)-(XPOS**2.0)
  CC=(A*B*B**2.0)-(XPOS**2.0)-(ZPOS**2.0)-(XPOS**2.0)
  ZPC=XPOS**2.0
  XNUS=(-BB+SQRT(BB**2.0-4.0*CC))/2.0
  XNUP=AB
  IF(XNUS.LE,XNUP) FTHIK=ALOG((SQRT(XNUS)+SQRT(XNUS+BB**2.0))/B)
  IF(XNUS.GT,XNUP) FTHIK=ALOG(4.0*A/B)-ALOG((SQRT((A**2.0/XNUS))+SQRT((XNUS+BB**2.0)/XNUS)))
  K3=SQRT(XNUS/(XNUS+BB**2.0))
  COEFF=-1.0*A*(BB**2.0)*(RCY1+RCY2)/(4.0*RCY1*RCY2)
  Calculate the gap between the contact surfaces (SEPN).
  SEPN=COEFF*((2.0/A)*FTHIK)-(ZPC**((2.0/(A*(A**2.0-BB**2.0)))*FTHIK)
  9A-R3)+((2.0/A**2.0)*SQRT(1.0/(XNUS+BB**2.0)))*R3)-((XPC**2.0*A*(R3-
  1)*BB**2.0/A**2.0)*FTHIK)/((BB**2.0)*(A**2.0-BB**2.0)))
  Calculate the contact surface heat transfer coefficient for node
  I,K as MX(KBST(K)+1)
  MX(KBST(K)+1)=2.0*(TCBAS/(1.0+(CCMH/SEPN)))/SEPN
  IF(NIU(KK).EQ.1) MX(KBST(K)+1)=(1-PATIO)*MX(KBST(K)+1)+MPATT
  CONTINUE
  RETURN
END

SUBROUTINE WIDTH(YA,YB,P11,CK2,CCOND,HA,PIC)
  THIS SUBROUTINE IS USED TO DETERMINE DATA FOR
  APPROXIMATING A CONTACT WHICH EXTENDS PARTWAY ACROSS A SURFACE
  ELEMENT. IT USES THE HALF-INTERVAL SEARCH METHOD TO FIND
  THE ROOT OF F(A) BETWEEN YA AND YB, WHICH ARE THE INNER AND
  OUTER BOUNDARIES OF THE SURFACE ELEMENT THAT THE CONTACT EXTENDS
  PARTWAY ACROSS. ONCE FOUND, THE DESIRED ROOT IS STORED AS PIC
  AND THE HEAT TRANSFER COEFF. THAT IS PRESCRIBED OVER THE SURFACE
  SEGMENT OF HALF-WIDTH P11 TO OBTAIN THE RESISTANCE IF THE ACTUAL
  CONTACT OF HALF-WIDTH P11 (WHERE P11,LE,PIC) IS STORED AS HA.

  F(A)=C11-(1-((.63662*ASIN(YA/A)))/(C3-.63662*ASIN(YA/A)
  9A.63662*ALOG(A/P11)/CK2-.63662*ASIN(YA/A)*(A/P11)/CCOND)
  9A-YA
  C11=PI*(.63662*ALOG(DX/(P11-YA))/CK2+1/(CCOND*(P11-YA)))
  C11=1./C1
  C2=CCOND*P11
  C3=YB*(.63662*ALOG(YB/P11)/CK2+1./C2)
  9A-YB+.0001*(YB-YA)
  IF(YA.GE,YB) FIC=P11 ; HA=1.E+12 ; RETURN
  YL=YL
  AR=YB
  YF=F(AR)
  IF(YL.LT.0.0,AND,YR.LT.0.0,OR,(YL.GT.0.0,AND,YR.GT.0.0)) PIC=P11
  SHA=1.E+12 ; RETURN
  A=IALAS/2.
  YA=F(A)
  IF(YL) 10,25,30
  40 AR=A
  YB=Y
  DO 10 35
  30 AL=A
  10 YL=Y
  10 C1=C.01*(YB-YA)
  C2=ABS(AR-AL)
  IF(C2P.LT,TOL) GO TO 25
  GO TO 20
  HA=1./((.63662*ALOG(A/P11)/CK2)
  25 FIC=
  IF(PIC.LT,P11) HA=1.E+12 ; FIC=P11
  RETURN
END

```

Listing II.G.5 Output Listing for an Accommodative Multigrid Analysis.

***** RESISTANCE IN AIR CALCULATION *****

***** INPUT DATA *****

XLEN= .85720 YLEN= 2.54000 ZLEN= 1.27000 NX0= 2 NY0= 6 NZ0= 3
TE= .37030 TCGAS= .28400E-03 CCHN= .31000E-04
B= .153950 R= .009610 RCV1= .95270 RCV2= 76.03999
REFDFA= 10.00000 WMAX= 30.00000 M= 10 INSUL= 0 NEXTR= 1 NCYCAL= 0 NPSW= 0

NZIC = 1

1
1
1
1
1
0
0
0
0

HPATCH= 1136.000 TBASE= .00000 MH= 1

NG = 2
N1IC(NG) = 1
N1IC(NG) = 3
N1JC(NG) = 1
N1JC(NG) = 7
N1IC(NG) = 1
N1IC(NG) = 4
NG = 3
N1IC(NG) = 1
N1IC(NG) = 4
N1JC(NG) = 1
N1JC(NG) = 6
N1IC(NG) = 1
N1IC(NG) = 5
NG = 4
N1IC(NG) = 1
N1IC(NG) = 5
N1JC(NG) = 1
N1JC(NG) = 6
N1IC(NG) = 1
N1IC(NG) = 4
NG = 5
N1IC(NG) = 1
N1IC(NG) = 5
N1JC(NG) = 1
N1JC(NG) = 5
N1IC(NG) = 1
N1IC(NG) = 7
NG = 7
N1IC(NG) = 1
N1IC(NG) = 5
N1JC(NG) = 1
N1JC(NG) = 5
N1IC(NG) = 1
N1IC(NG) = 10
NG = 7
N1IC(NG) = 1
N1IC(NG) = 5
N1JC(NG) = 1
N1JC(NG) = 5
N1IC(NG) = 1
N1IC(NG) = 17
NG = 8
N1IC(NG) = 1
N1IC(NG) = 6
N1JC(NG) = 1
N1JC(NG) = 8
N1IC(NG) = 1
N1IC(NG) = 31
NG = 9
N1IC(NG) = 1
N1IC(NG) = 8
N1JC(NG) = 1
N1JC(NG) = 8
N1IC(NG) = 1
N1IC(NG) = 29
NG = 10
N1IC(NG) = 1
N1IC(NG) = 10
N1JC(NG) = 1
N1JC(NG) = 10
N1IC(NG) = 1
N1IC(NG) = 27

***** CALCULATED GRID DATA *****

NFOINT = 20703
WUINC = 1.057381E-03
1.97749E-03
1.347341E-02
5.260107E-03
5.086219E-03
1.433701E-03
1.59112
10.181
215172
470801

NFT = 84

85
86
87
88
89
90
91
92
93
94
95
96
97
98
99

N1IC = 11
N1IC = 11
N1IC = 11
N1IC = 3
N1IC = 3
N1IC = 3
UG = 2.74515
2.74516
5.76074
1.29015
1.440185E-02

LEVEL=1	RESIDUAL NORM=	355E-01	WORK=	40438
LEVEL=1	RESIDUAL NORM=	335E-01	WORK=	40944
LEVEL=1	RESIDUAL NORM=	315E-01	WORK=	41250
LEVEL=1	RESIDUAL NORM=	294E-01	WORK=	41623
LEVEL=1	RESIDUAL NORM=	192E+00	WORK=	41883
LEVEL=1	RESIDUAL NORM=	279E-01	WORK=	42459
LEVEL=1	RESIDUAL NORM=	417E-01	WORK=	44465
LEVEL=1	RESIDUAL NORM=	573E-01	WORK=	45070
LEVEL=1	RESIDUAL NORM=	547E-01	WORK=	45476
LEVEL=1	RESIDUAL NORM=	525E-01	WORK=	45882
LEVEL=1	RESIDUAL NORM=	505E-01	WORK=	46286
LEVEL=1	RESIDUAL NORM=	485E-01	WORK=	46693
LEVEL=1	RESIDUAL NORM=	467E-01	WORK=	47099
LEVEL=1	RESIDUAL NORM=	449E-01	WORK=	47505
LEVEL=1	RESIDUAL NORM=	432E-01	WORK=	47911
LEVEL=1	RESIDUAL NORM=	416E-01	WORK=	48316
LEVEL=1	RESIDUAL NORM=	400E-01	WORK=	48722
LEVEL=1	RESIDUAL NORM=	379E-01	WORK=	49127
LEVEL=1	RESIDUAL NORM=	357E-01	WORK=	49534
LEVEL=1	RESIDUAL NORM=	344E-01	WORK=	49939
LEVEL=1	RESIDUAL NORM=	331E-01	WORK=	50344
LEVEL=1	RESIDUAL NORM=	319E-01	WORK=	50749
LEVEL=1	RESIDUAL NORM=	307E-01	WORK=	51155
LEVEL=1	RESIDUAL NORM=	296E-01	WORK=	51562
LEVEL=1	RESIDUAL NORM=	285E-01	WORK=	51968
LEVEL=1	RESIDUAL NORM=	117E+00	WORK=	52374
LEVEL=1	RESIDUAL NORM=	122E-01	WORK=	52779
LEVEL=1	RESIDUAL NORM=	127E-01	WORK=	53185
LEVEL=1	RESIDUAL NORM=	130E-01	WORK=	53591
LEVEL=1	RESIDUAL NORM=	128E-01	WORK=	53997
LEVEL=1	RESIDUAL NORM=	127E-01	WORK=	54403
LEVEL=1	RESIDUAL NORM=	126E-01	WORK=	54809
LEVEL=1	RESIDUAL NORM=	125E-01	WORK=	55215
LEVEL=1	RESIDUAL NORM=	124E-01	WORK=	55621
LEVEL=1	RESIDUAL NORM=	123E-01	WORK=	56027
LEVEL=1	RESIDUAL NORM=	122E-01	WORK=	56433
LEVEL=1	RESIDUAL NORM=	121E-01	WORK=	56839
LEVEL=1	RESIDUAL NORM=	120E-01	WORK=	57245
LEVEL=1	RESIDUAL NORM=	119E-01	WORK=	57651
LEVEL=1	RESIDUAL NORM=	118E-01	WORK=	58057
LEVEL=1	RESIDUAL NORM=	117E-01	WORK=	58463
LEVEL=1	RESIDUAL NORM=	116E-01	WORK=	58869
LEVEL=1	RESIDUAL NORM=	115E-01	WORK=	59275
LEVEL=1	RESIDUAL NORM=	114E-01	WORK=	59681
LEVEL=1	RESIDUAL NORM=	113E-01	WORK=	60087
LEVEL=1	RESIDUAL NORM=	112E-01	WORK=	60493
LEVEL=1	RESIDUAL NORM=	111E-01	WORK=	60899
LEVEL=1	RESIDUAL NORM=	110E-01	WORK=	61305
LEVEL=1	RESIDUAL NORM=	109E-01	WORK=	61711
LEVEL=1	RESIDUAL NORM=	108E-01	WORK=	62117
LEVEL=1	RESIDUAL NORM=	107E-01	WORK=	62523
LEVEL=1	RESIDUAL NORM=	106E-01	WORK=	62929
LEVEL=1	RESIDUAL NORM=	105E-01	WORK=	63335
LEVEL=1	RESIDUAL NORM=	104E-01	WORK=	63741
LEVEL=1	RESIDUAL NORM=	103E-01	WORK=	64147
LEVEL=1	RESIDUAL NORM=	102E-01	WORK=	64553
LEVEL=1	RESIDUAL NORM=	101E-01	WORK=	64959
LEVEL=1	RESIDUAL NORM=	100E-01	WORK=	65365
LEVEL=1	RESIDUAL NORM=	99E-01	WORK=	65771
LEVEL=1	RESIDUAL NORM=	98E-01	WORK=	66177
LEVEL=1	RESIDUAL NORM=	97E-01	WORK=	66583
LEVEL=1	RESIDUAL NORM=	96E-01	WORK=	66989
LEVEL=1	RESIDUAL NORM=	95E-01	WORK=	67395
LEVEL=1	RESIDUAL NORM=	94E-01	WORK=	67801
LEVEL=1	RESIDUAL NORM=	93E-01	WORK=	68207
LEVEL=1	RESIDUAL NORM=	92E-01	WORK=	68613
LEVEL=1	RESIDUAL NORM=	91E-01	WORK=	69019
LEVEL=1	RESIDUAL NORM=	90E-01	WORK=	69425
LEVEL=1	RESIDUAL NORM=	89E-01	WORK=	69831
LEVEL=1	RESIDUAL NORM=	88E-01	WORK=	70237
LEVEL=1	RESIDUAL NORM=	87E-01	WORK=	70643
LEVEL=1	RESIDUAL NORM=	86E-01	WORK=	71049
LEVEL=1	RESIDUAL NORM=	85E-01	WORK=	71455
LEVEL=1	RESIDUAL NORM=	84E-01	WORK=	71861
LEVEL=1	RESIDUAL NORM=	83E-01	WORK=	72267
LEVEL=1	RESIDUAL NORM=	82E-01	WORK=	72673
LEVEL=1	RESIDUAL NORM=	81E-01	WORK=	73079
LEVEL=1	RESIDUAL NORM=	80E-01	WORK=	73485
LEVEL=1	RESIDUAL NORM=	79E-01	WORK=	73891

PART III

ADVANCED APPLICATIONS OF THE
FINITE-DIFFERENCE/SYNTHESIS MODEL

CHAPTER III.1

INTRODUCTION

The synthesis method was described for linear static analyses in Part II and applied to the model contact problem with encouraging results. The synthesis method can be applied to problems which are much more complex than that considered in Part II, however, as will be demonstrated in subsequent sections where its application to transient and non-linear problems will be described. Simplified analyses of one of the non-conforming contact heat transfer problems cited as a motivation for this study, specifically the problem in which a pressure tube in a CANDU reactor overheats and sags onto the surrounding calandria tube during a LOCA accompanied by a loss of ECC flow (LOECC), will be described to demonstrate the applicability of the synthesis method to such problems. The pressure/calandria tube contact problem was not expected to be, and nor did it turn out to be, very three-dimensional. The aforementioned LOECC analyses were performed primarily to demonstrate the potential of the synthesis method and, as such, were also not intended to be rigorous in every respect. The methods used to determine the slumped shape of the pressure tube and the contact dimensions, in particular, were very approximate.

The simultaneous linear equations defining the blending vector (Z) in a synthesis analysis, given in Chapter II.2 for the linear static case, are modified in Chapters III.2 and III.3 for use in transient

and non-linear analyses. The finite-difference grid that was used to represent a CANDU fuel channel in the sample LOECC analyses is described in Section III.4.1, although only briefly because it is described in greater detail in Part IV. The sample transient and non-linear LOECC analyses themselves are described in Sections III.4.2 and III.4.3 respectively.

CHAPTER III.2

COMBINED SPACE-TIME SYNTHESIS

The governing equation for transient heat conduction is,

$$\nabla \cdot k_s \nabla T + q_v = \rho_m c_m \frac{\partial T}{\partial t} \quad (\text{III.2.1})$$

where ρ_m and c_m are the density and specific heat respectively of the conducting medium. If the spatial derivatives in equation (III.2.1) are differenced using, for instance, the simple heat balance method [118], the general nodal equation, given previously as equation (II.2.5) can be rewritten for a transient analysis (with $q_v=0$) as:

$$\begin{aligned} \rho_m c_m V(i,j,k) \frac{dT}{dt}(i,j,k) = & K(i,j,k)_{i+1} T(i+1,j,k) \\ & + K(i,j,k)_{i-1} T(i-1,j,k) + K(i,j,k)_{j+1} T(i,j+1,k) \\ & + K(i,j,k)_{j-1} T(i,j-1,k) + K(i,j,k)_{k+1} T(i,j,k+1) \\ & + K(i,j,k)_{k-1} T(i,j,k-1) - (\Sigma K) T(i,j,k) \\ & + K(i,j,k)_s T_\infty - q'' A_s(i,j,k) \end{aligned} \quad (\text{III.2.2})$$

where $V(i,j,k)$ is the volume of the finite-difference cell surrounding node i,j,k and $\rho_m c_m V(i,j,k)$ is the "capacitance" of the node. If equation (III.2.2) is evaluated at all of the nodes in a finite-difference grid, the assemblage of nodal equations can be written in matrix form as:

$$C\dot{T} = Q - KT \quad (\text{III.2.3a})$$

or as

$$\dot{T} = C^{-1}(Q - KT) \quad (\text{III.2.3b})$$

where C is a diagonal capacitance matrix containing the nodal capacitances (i.e., $C_{\ell,\ell} = \rho_m c_m V(\ell)$ and $C_{\ell,\ell}^{-1} = 1/\rho_m c_m V(\ell)$ for the ℓ th node in the grid), $\dot{T}$ is a vector containing the time derivatives of the nodal temperatures, Q is a "heat flow" vector containing the constant terms in each nodal equation, K is the conductance matrix described in Section II.2.2, and T is the vector of nodal temperatures. Equation (III.2.3), which applies at any instant in time, represents a system of N_e ordinary differential equations which must be integrated with respect to time, starting with the initial conditions $T = T_{in}$ at $t=0$, to obtain the desired transient solution. Equation (III.2.3) could be integrated with respect to time by applying one of the standard integration methods (e.g., Euler, Runge-Kutta, predictor-corrector, etc.), but the integrations are usually performed in heat transfer analyses by expressing the time-derivatives as finite-differences. A general expression which includes all of the commonly used time-integration procedures can be obtained by expressing the average value of $\dot{T}$ between times t and $t+\Delta t$ as a weighted average of the forward-difference, $\dot{T}^t (=C^{-1}(Q-KT^t))$, and backward-difference, $\dot{T}^{t+1} (=C^{-1}(Q-KT^{t+1}))^*$ representations of $\dot{T}$. The resulting expression is,

$$C \frac{(T^{t+1} - T^t)}{\Delta t} = \theta(Q - KT^{t+1}) + (1-\theta)(Q - KT^t) \quad (\text{III.2.4})$$

* The superscript $t+1$ refers to a quantity evaluated at time $t+\Delta t$.

where θ ($0 \leq \theta \leq 1$) is a weighting parameter. Pure forward and backward difference schemes, which are often referred to as the explicit (or Euler) and fully implicit methods respectively, are obtained by specifying $\theta=0$ and $\theta=1$ respectively in equation (III.2.4). The Crank-Nicholson and Galerkin integration procedures are obtained for $\theta=1/2$ and $\theta=2/3$ respectively [147].

Inspection of equation (III.2.4) reveals that a system of simultaneous equations must be solved to obtain T^{t+1} when $\theta > 0$. While the explicit formulation with $\theta=0$ would appear to be preferable to the formulations with $\theta > 0$ for that reason, it is well known that the methods with $1/2 \leq \theta \leq 1$ are stable for any size time step (Δt) whereas the methods with $0 \leq \theta < 1/2$ are not [148]. The maximum stability-limited time step for the explicit method is roughly $O(\Delta_{\min}^2)^*$ in size and hence can be prohibitively small for a highly graded grid with a small $\Delta_{\min}$. An example of this will be given later for the highly graded grid used to analyse the pressure/calandria tube contact problem.

It should also be mentioned that although all of the versions of equation (III.2.4) with $1/2 \leq \theta \leq 1$ are stable for any size time step, bounded numerical oscillations can appear in the solution with all of the methods except the fully implicit scheme.

* $\Delta_{\min}$ is the smallest mesh spacing found in a non-uniform mesh.

The synthesis method can be applied to the solution of equation (III.2.3a) by setting $T' = WZ$ and $\dot{T}' = W\dot{Z}$ and defining the error (E) in the synthesis solution at any instant in time as,

$$\begin{aligned} E &= CT' - (Q - KT) \\ &= CWZ - (Q - KWZ) \end{aligned} \quad (\text{III.2.5})$$

Orthogonalizing E with respect to each of the trial vectors in W yields the expression,

$$W^T CWZ = W^T (Q - KWZ) \quad (\text{III.2.6})$$

which, by analogy with equation (III.2.4), can be approximated as,

$$\begin{aligned} W^T CW \frac{(Z^{t+1} - Z^t)}{\Delta t} &= \theta W^T (Q - KWZ^{t+1}) \\ &\quad + (1 - \theta) W^T (Q - KWZ^t) \end{aligned} \quad (\text{III.2.7})$$

When $\theta = 0$,

$$\begin{aligned} Z^{t+1} &= [I - \Delta t (W^T CW)^{-1} W^T KW] Z^t + \Delta t (W^T CW)^{-1} W^T Q \\ &= D_e Z^t + Q_e \end{aligned} \quad (\text{III.2.8})$$

where $D_e = (I - \Delta t (W^T CW)^{-1} W^T KW)$ and $Q_e = \Delta t (W^T CW)^{-1} W^T Q$.

When $\theta = 1$, Z^{t+1} is given implicitly by the equation,

$$W^T \left(\frac{C}{\Delta t} + K \right) WZ^{t+1} = W^T Q + \frac{1}{\Delta t} W^T CWZ^t \quad (\text{III.2.9a})$$

or

$$W^T K' WZ^{t+1} = W^T Q' \quad (\text{III.2.9b})$$

where $K' = \frac{C}{\Delta t} + K$ and $Q' = Q + \frac{C}{\Delta t} T^t$.

Equation (III.2.8) exhibits only conditional stability (i.e., it is only stable if Δt is small enough), as shown in Appendix III.A where its stability characteristics are examined briefly*. In contrast, equation (III.2.9) must be stable for any size time step because it reduces to the equation governing the steady-state synthesis solution (i.e., $W^T K W Z = W^T Q$) as $\Delta t \rightarrow \infty$. Because it is unconditionally stable, only the time-dependent synthesis formulation governed by equation (III.2.9) will be considered further.

Since equation (III.2.9) represents a system of simultaneous equations defining the nodal temperatures at time $t + \Delta t$, all of the equations derived in Sections II.2.2 and II.2.3 for a static synthesis analysis apply to a transient analysis if the matrices K and Q in those equations are replaced by K' and Q' respectively. In the system of simultaneous equations defining the $Z_n(k)$ (i.e., $DZ = R$), only the submatrices on the diagonal of the partitioned coefficient matrix (D) and the elements of the right-hand side vector (R) have to be altered to apply to a transient analysis. Adding the term $(-\rho_m c_m V(i,j,k)/\Delta t) T_{jj}(i,j)$ to the expression contained within the square brackets in equation (II.2.28) yields the new expression for the matrix element $D_{k,k}(ii,jj)$ and adding the term $\sum_{j=1}^{NT} \sum_{i=1}^{NR} (-\rho_m c_m V(i,j,k)/\Delta t) T^t(i,j,k)$ (where $T^t(i,j,k) = \sum_{n=1}^N Z_n(k) T_n(i,j)$) to the right-hand side of equation (II.2.30) yields the new expression for the vector component, $R_k(ii)$. If the new matrix entries are designated $D'_{k,k}(ii,jj)$ and $R'_k(ii)$, they can be written in terms of the corresponding static synthesis entries as;

* The time integration procedure that is obtained when $\theta = 0$ in Equation (III.2.7) is, by definition, not an explicit method because $W^T C W$ is not a diagonal matrix (in general).

$$\begin{aligned}
 D'_{k,k}(ii,jj) &= D_{k,k}(ii,jj) + \frac{1}{\Delta t} \sum_{j=1}^{NT} \sum_{i=1}^{NR} \rho_m c_m V(i,j,k) T_{jj}(i,j) T_{ii}(i,j) \\
 &= D_{k,k}(ii,jj) + R_{k,k}(ii,jj)/\Delta t \quad (III.2.10)
 \end{aligned}$$

$$\text{where } R_{k,k}(ii,jj) = \sum_{j=1}^{NT} \sum_{i=1}^{NR} \rho_m c_m V(i,j,k) T_{jj}(i,j) T_{ii}(i,j),$$

and

$$R'_k(ii) = R_k(ii) + \sum_{j=1}^{NT} \sum_{i=1}^{NR} \frac{\rho_m c_m V(i,j,k)}{\Delta t} T^t(i,j,k) T_{ii}(i,j) \quad (III.2.11)$$

As seen in equations (III.2.10) and (III.2.11), the right-hand side vector (R) has to be updated at the end of each time step using the new nodal temperatures but the D matrix only has to be altered when the time step (Δt) is changed.* If the matrix H is computed and stored separately when D is computed at the start of a transient analysis, it becomes a trivial matter to recompute the submatrices $D'_{k,k}$, ($k=1,NZ$) when the time step is changed because the $r-\theta$ (i.e., $i-j$) integrations indicated in equations (II.2.10) and (II.2.11) do not have to be repeated. If the time step is changed from Δt_1 to Δt_2 , for instance, a new submatrix $D'_{k,k}|_{\Delta t_2}$, can be calculated from its predecessor, $D'_{k,k}|_{\Delta t_1}$, by means of the relationship,

$$D'_{k,k}|_{\Delta t_2} = D'_{k,k}|_{\Delta t_1} + \left(\frac{1}{\Delta t_2} - \frac{1}{\Delta t_1} \right) R_{k,k} \quad (III.2.12)$$

The computer program SYNCON (described in Part IV) uses equation (III.2.12) to modify the D' matrix when the time step is changed in a transient (linear) synthesis analysis.

* Provided the problem is linear.

CHAPTER III.3

NON-LINEAR SYNTHESIS ANALYSES

Although the synthesis method yields approximate solutions to linear systems of equations, it can also be applied to non-linear systems by using it as the linear equation solver in, for instance, the iterative Newton-Raphson procedure [103]. As a first step in the demonstration of this, the Newton-Raphson (N-R) method for obtaining the solution to a non-linear system of finite-difference equations will be outlined and then modified by applying the synthesis method to the iterative linear equation solutions.

Consider the system of finite-difference equations given by,

$$KT = Q \quad (\text{III.3.1a})$$

or

$$E(T) = KT - Q = 0 \quad (\text{III.3.1b})$$

If K is a function of temperature (i.e., $K=f(T)$), equations (III.3.1a) and (III.3.1b) are non-linear and must be solved iteratively. These equations can be solved with the N-R method by iterating with the recursive equation,

$$J(T^n) [T^{n+1} - T^n] = Q - KT^n$$

or

$$\begin{aligned} J(T^n) T^{n+1} &= Q - KT^n + J(T^n) T^n \\ &= Q_{NR} \end{aligned} \quad (\text{III.3.2})$$

where T^n and T^{n+1} are the n th and $(n+1)$ st estimates of the solution vector (T) and $J(T^n)$ is the Jacobian matrix defined by the expression,

$$J(T^n) = \left. \frac{\partial E}{\partial T} \right|_{T^n} \quad (\text{III.3.3})$$

Using equation (III.3.3), the matrix element J_{ij} can be written as

$$J_{ij} = \partial E_i / \partial T_j \quad (\text{III.3.4})$$

The recursive equation for the N-R method (equation (III.3.2)) reduces to equation (III.3.1a) if $K \neq f(T)$ since $J(T^n) = K$ and hence $Q_{NR} = Q$ in that case. It can also be shown that an entry K_{ij} in the conductance matrix is the same as the corresponding entry J_{ij} in the Jacobian if the K_{il} ; $l = 1, N_e$ ($K \in R^{N_e \times N_e}$) are not functions of T_j . Thus rows of K which do not contain any non-linear entries remain unchanged in J .

If we let $T^{n+1} = WZ^{n+1}$, we can apply Galerkin's method to the approximate solution of equation (III.3.2) by premultiplying both sides of that equation by W^T , viz

$$W^T J(T^n) WZ^{n+1} = W^T [Q - KT^n + J(T^n) T^n] \quad (\text{III.3.5a})$$

or

$$W^T \left. \frac{\partial E}{\partial T} \right|_{T^n} WZ^{n+1} = W^T Q - W^T K WZ^n + W^T \left. \frac{\partial E}{\partial T} \right|_{T^n} WZ^n \quad (\text{III.3.5b})$$

Equation (III.3.5b) can also be derived by applying the N-R method directly to the non-linear system of equations given by,

$$\begin{aligned} F(Z) &= W^T E \\ &= W^T K WZ - W^T Q = 0 \end{aligned} \quad (\text{III.3.6})$$

(where $K=f(Z)$) for which the N-R recursive equation is,

$$J(Z^n) Z^{n+1} = W^T Q - W^T K W Z^n + J(Z^n) Z^n \quad (\text{III.3.7})$$

with $J(Z^n) = \left. \frac{\partial F(Z)}{\partial Z^T} \right|_{Z^n} = W^T \left. \frac{\partial E}{\partial Z^T} \right|_{Z^n}$ since $W^T = f(Z)$. But $\frac{\partial E}{\partial Z^T} = \frac{\partial E}{\partial T^T} \frac{\partial T}{\partial Z^T}$
 $= \frac{\partial E}{\partial T^T} W$ since $\frac{\partial T}{\partial Z^T} = W$, so, equation (III.3.7) can be rewritten as,

$$W^T \left. \frac{\partial E}{\partial T^T} \right|_{T^n} W Z^{n+1} = W^T Q - W^T K W Z^n + W^T \left. \frac{\partial E}{\partial T^T} \right|_{T^n} W Z^n \quad (\text{III.3.8})$$

which is the same as equation (III.3.5b).

The manner in which equation (III.3.5b) (or equation (III.3.8)) is used in practice will be demonstrated by considering the contacting pressure/calandria tube problem mentioned in Section I.1.1 in connection with a LOCA plus LOECC accident in a CANDU reactor. For simplicity, specifically to avoid modelling heat transfer in the fuel bundles, it is assumed that heat flow in the fuel bundles is purely radial and that, in consequence, the heat input to the pressure tube inner periphery (from the fuel bundles) is uniform circumferentially. With this simplification, the remainder of the analysis is non-linear because,

- (1) radiative heat transfer, which varies according to T^4 , is the dominant mode of heat transfer between the pressure and calandria tubes in regions remote from the contact.
- (2) the thermal conductivity of the gas (N_2) in the gap between the pressure and calandria tubes is strongly temperature dependent.
- (3) the thermal conductivities of the Zircaloy alloys from which the pressure and calandria tubes are fabricated are temperature dependent.

- (4) natural convection on the calandria tube exterior varies according to $\Delta T^{5/4}$ (laminar) or $\Delta T^{4/3}$ (turbulent), where ΔT is the difference between the calandria tube outer surface and moderator temperatures. Nucleate boiling, if present, varies according to approximately $\Delta T_s^3 \rightarrow \Delta T_s^4$, where ΔT_s is the excess temperature (i.e., the difference between the calandria tube outer surface and moderator saturation temperatures).
- (5) natural convection between the pressure and calandria tubes varies according to $\Delta T^{5/4}$ (assumed laminar), where ΔT in this case is the difference between the pressure tube outer wall and calandria tube inner wall temperatures. This heat transfer mode, if present, will be most significant at the top of the fuel channel (i.e., at the 12 o'clock position) where the gap between the pressure and calandria tubes is greatest.

The relative importance of each non-linear heat transfer mode identified above, will, for fixed surface emissivities, accommodation coefficients, etc., depend on the magnitude of the heat input to the pressure tube inner surface. The importance of radiation, for example, will increase, particularly in regions remote from the contact, as the heat input to the pressure tube, and hence component temperatures, increase. Since the primary purpose of the present analysis is to demonstrate how non-linearities can be handled in a synthesis analysis by applying Newton linearization, only the first two non-linearities listed above (i.e., types 1 and 2) will be considered initially. With this simplification, only the finite-difference equations for nodes on the pressure tube exterior and calandria tube interior surfaces (outside

of the contact patch) will contain non-linear terms and the Jacobian (J) and conductance (K) matrices mentioned previously will be identical except for the rows corresponding to those equations. Natural convection between the pressure and calandria tubes could also be considered without introducing non-linear terms into additional equations but that heat transfer mechanism will be neglected for simplicity. The analysis will be performed initially with the thermal conductivities of the Zircaloy alloys from which the pressure and calandria tubes are fabricated held constant. Kirchoff's transformation [80] will be used later to account for this non-linearity. The calandria tube/moderator heat transfer coefficient will, likewise, be maintained at a constant, uniform value.

It is assumed in the numerical model of the contacting pressure and calandria tubes that heat transferred by conduction through the eccentric gas gap between the tubes flows in a direction normal to the pressure tube outer surface. The effective length of the heat flow path at given circumferential and axial positions ($\delta(\theta, z)$) is, in this case, equal to the radial distance from the outer surface of the pressure tube to the inner surface of the calandria tube as measured on a ray emanating from the pressure tube center. Thus the overall gas conductance (K'_g) between a node I,J,K on the pressure tube outer surface, shown in Figure III.3.1, and the opposing node I+1,J,K on the calandria tube inner surface is given by;

$$K'_g = \frac{k_g \bar{A}_{I \rightarrow I+1}}{\delta_{J,K}} \quad (\text{III.3.9})$$

where $\bar{A}_{I \rightarrow I+1}$ is the average heat flow area between nodes I,J,K

Figure III.3.1 Pressure-Calandria Tube Gas Gap Geometry

and $I+1, J, K$, k_g is the local gas conductivity ($=f(\delta, T)$) and $\delta_{J,K}$ is the value of δ evaluated at circumferential mesh point J and axial mesh point K . The local gas conductivity, k_g , is given by the expression,

$$k_g = \frac{k_{og}}{1+C/\delta} \quad (\text{III.3.10})$$

which allows for non-continuum effects near the contact. The continuum gas conductivity (k_{og}) is assumed for simplicity to be a linear function of temperature*, viz,

$$k_{og} = \xi + \chi T \quad (\text{III.3.11})$$

while the parameter C ($= 4\gamma L_m [\alpha_1 + \alpha_2 - \alpha_1 \alpha_2] / [P_r \alpha_1 \alpha_2 (\gamma + 1)]$ **) is assumed constant. The latter parameter is not really constant because L_m (i.e., the mean free path of the gas molecules), in particular, is a function of temperature. The non-continuum correction in equation (III.3.10) is only significant very near the contact (i.e., at very small gap thicknesses), however, where it is assumed that an average temperature can be used to evaluate L_m satisfactorily. The temperature (T) appearing in equation (III.3.11) is taken to be the average of the temperatures of nodes I, J, K and $I+1, J, K$ (i.e., $T = (T(I, J, K) + T(I+1, J, K))/2$), so with this substitution and equations (III.3.10) and (III.3.11), we can rewrite equation (III.3.9) as:

$$k_g = \frac{[\xi + \chi(T(I, J, K) + T(I+1, J, K))/2]}{(1+C/\delta_{J,K})\delta_{J,K}} A_{I \rightarrow I+1}$$

* This assumption should be fairly accurate over a limited temperature range.

** See Equation (I.3.25).

$$= \epsilon B_J + x \frac{(T(I,J,K) + T(I+1,J,K))}{2} B_J \quad (\text{III.3.12})$$

where

$$B_J = \frac{A_{I \rightarrow I+1}}{(1+C/\delta_{J,K}) \delta_{J,K}} \quad (\text{III.3.13})$$

The parameter B_J is assumed to be independent of temperature and hence is constant at a particular value of J .

With the overall gas conductance defined by equation (III.3.12), the rate of heat transfer by gas conduction (q_g) from node I,J,K to node $I+1,J,K$ can now be written as:

$$\begin{aligned} q_g &= K_g (T(I,J,K) - T(I+1,J,K)) \\ &= B_J \epsilon (T(I,J,K) - T(I+1,J,K)) \\ &\quad + \frac{B_J x}{2} (T(I,J,K)^2 - T(I+1,J,K)^2) \end{aligned} \quad (\text{III.3.14})$$

For reasons of computational expediency and simplicity, radiative heat transfer across the gap at any angle θ is calculated by assuming that the local temperature at θ on one of the radiating surfaces (i.e., either the pressure tube outer surface or the calandria tube inner surface) represents, to a first approximation, the average radiation temperature seen by the other surface at θ . Thus it is assumed, for instance, that the average radiation temperature seen by the surface element at node I,J,K in Figure III.3.1 is approximately equal to $T(I+1,J,K)$ and, similarly, that the average radiation temperature seen by node $I+1,J,K$ is approximately equal to $T(I,J,K)$. In reality, each surface element on the pressure tube "sees" several elements on the calandria tube inner surface and each surface element on the latter surface sees

several elements on both its own and the pressure tube outer surfaces. The simplifying assumption described previously, specifically that the average radiation temperature seen by a surface element at θ on one of the radiating surfaces is adequately represented by the local temperature at θ on the other surface, is not expected to introduce serious errors, however, because;

- (1) the calandria tube is maintained at a more or less constant temperature by the moderator so the temperatures seen by surface elements on the pressure tube will be fairly uniform circumferentially.
- (2) although the pressure tube temperature will be circumferentially and axially non-uniform near the contact, heat transfer by gas conduction through the thin gas gap between the pressure and calandria tubes in that region will be much more significant than radiation. Thus the simplifications inherent in the radiation model will have little effect on the heat flow near the contact. The simplified radiation model will also be realistic in regions remote from the contact where the pressure tube temperature can be expected to be fairly uniform.* The adequacy of the simplified radiation model is analyzed in greater detail (for $r-\theta$ heat flow) in reference [24].

The shape factor for a surface element on the pressure tube outer surface viewing the calandria tube is equal to unity in the simplified radiation model while the shape factor for an element on the calandria

* Results given in [24] show that the pressure tube temperature is essentially constant for $\theta \geq 45^\circ$.

tube inner surface viewing the pressure tube is calculated from the reciprocity theorem, i.e., the latter shape factor is taken to be the ratio of the pressure tube outer surface area to the calandria tube inner surface area or, equivalently, the ratio of the pressure tube outer radius (r_1) to the calandria tube inner radius (r_2). These radiation shape factors, which are strictly only correct when the pressure tube outer and calandria tube inner surfaces are isothermal, are nevertheless defined in a manner which is consistent with the use of only two temperatures to evaluate the radiative heat flux at a given circumferential position since the latter simplification implicitly assumes that circumferential wall temperature variations are negligible.

With the foregoing provisions, the radiative heat transfer rate (q_r) between nodes I,J,K and I+1,J,K can be expressed as:

$$q_r = \frac{\sigma A_s(I,J,K) (T(I,J,K)^4 - T(I+1,J,K)^4)}{\frac{1}{\epsilon_1} + \frac{r_1}{r_2} \left(\frac{1}{\epsilon_2} - 1 \right)}$$

$$= R_s(I,J,K) (T(I,J,K)^4 - T(I+1,J,K)^4) \quad (\text{III.3.15})$$

where $R_s(I,J,K) = \sigma A_s(I,J,K) / \left(\frac{1}{\epsilon_1} + \frac{r_1}{r_2} \left(\frac{1}{\epsilon_2} - 1 \right) \right)$, σ is the Stefan-Boltzmann constant, $A_s(I,J,K)$ is the surface area of node I,J,K, and ϵ_1 and ϵ_2 are the surface emissivities of the pressure tube outer and calandria tube inner surfaces respectively. These emissivities would depend on temperature in an actual accident but will be held constant in this analysis for simplicity.

With equations (III.3.15) and (III.3.14) defining the radiative

and conductive (gas) heat transfer rates respectively between nodes at the same circumferential position on the pressure tube outer and calandria tube inner surfaces,, the complete finite-difference equation for a node i,j,k on the outer surface of the pressure tube can now be written (for a steady-state analysis) as;

$$\begin{aligned}
 & - K(i,j,k)_{i-1} T(i-1,j,k) - K(i,j,k)_{j+1} T(i,j+1,k) \\
 & - K(i,j,k)_{j-1} T(i,j-1,k) - K(i,j,k)_{k+1} T(i,j,k+1) - K(i,j,k)_{k-1} T(i,j,k-1) \\
 & - [B_j \epsilon T(i+1,j,k) + \frac{B_j X}{2} T(i+1,j,k)^2 + R_s T(i+1,j,k)^4] \\
 & + [(\Sigma K + B_j \epsilon) T(i,j,k) + \frac{B_j X}{2} T(i,j,k)^2 + R_s T(i,j,k)^4] \\
 & = \epsilon(i,j,k) = 0
 \end{aligned} \tag{III.3.16}$$

where, for the synthesis procedure, we have $T \leftarrow T' = WZ$ and $\epsilon(i,j,k) \neq 0$ in general. The nodal residual $\epsilon(i,j,k)$ is the i,j,k th entry, in the vector $E (=KT' - Q$ in the synthesis method).

The equation analogous to equation (III.3.16) for a node i,j,k on the calandria tube inner surface can be obtained by interchanging the subscripts $i-1$ and $i+1$ in equation (III.3.16).

If the left-hand coefficient matrix for the system of equations defining the vector Z^{n+1} (i.e., equation (III.3.5b) or (III.3.8)) is designated D , as in the linear case, then it can be seen from equation (III.3.5b) or (III.3.8) that,

$$D = W^T J (T'^n) W = W^T \frac{\partial E}{\partial T} \bigg|_{T'^n} W \tag{III.3.17}$$

Comparing equation (III.3.17) to the corresponding expression for D in the linear case (i.e., $D = W^T K W$, as seen in equation (II.2.22b)), it is apparent that $J(T'^n) (= \frac{\partial E}{\partial T} \bigg|_{T'^n})$ simply replaces K. The entries in

the rows of J corresponding to finite-difference equations which do not contain any non-linear terms will be the same as the corresponding entries in K, as mentioned previously. For the non-linear finite-difference equations encountered at nodes on the pressure tube outer surface (outside of the contact patch) we have, from equation (III.3.16),

$$\frac{\partial \epsilon(i, j, k)}{\partial T(i-1, j, k)} = -K(i, j, k)_{i-1} \quad (\text{III.3.18})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial T(i, j+1, k)} = -K(i, j, k)_{j+1} \quad (\text{III.3.19})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial T(i, j-1, k)} = -K(i, j, k)_{j-1} \quad (\text{III.3.20})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k+1)} = -K(i, j, k)_{k+1} \quad (\text{III.3.21})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k-1)} = -K(i, j, k)_{k-1} \quad (\text{III.3.22})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial T(i+1, j, k)} \bigg|_{T'^n} = -(B_{j\epsilon} + B_{jx} T'^n(i+1, j, k) + 4R_s T'^n(i+1, j, k)^3) \quad (\text{III.3.23})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k)} \bigg|_{T'^n} = (\Sigma K + B_{j\epsilon}) + B_{jx} T'^n(i, j, k) + 4R_s T'^n(i, j, k)^3 \quad (\text{III.3.24})$$

The corresponding derivatives for a node i, j, k on the calandria tube inner surface can be obtained by interchanging the subscripts $i-1$ and $i+1$ in equations (III.3.18) and (III.3.23).

Expressions for the entries $D_{k,k-1}(i, j, j)$, $D_{k,k}(i, j, j)$ and $D_{k,k+1}(i, j, j)$ in the partitioned form of D were given in Chapter II.2 as equations (II.2.27), (II.2.28) and (II.2.29) respectively for a linear analysis. Those expressions can be applied to both linear and non-linear analyses if they are rewritten more generally as,

$$D_{k,k-1}(i, j, j) = \sum_{j=1}^{NT} \sum_{i=1}^{NR} T_{jj}(i, j) T_{ii}(i, j) \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k-1)} \right|_{T_n} \quad (\text{III.3.25})$$

$$\begin{aligned} D_{k,k}(i, j, j) = & \sum_{j=1}^{NT} \sum_{i=1}^{NR} \left[\left. \frac{\partial \epsilon(i, j, k)}{\partial T(i+1, j, k)} \right|_{T_n} T_{jj}(i+1, j) \right. \\ & + \left. \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i-1, j, k)} \right|_{T_n} T_{jj}(i-1, j) \right. \\ & + \left. \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j+1, k)} \right|_{T_n} T_{jj}(i, j+1) + \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j-1, k)} \right|_{T_n} T_{jj}(i, j-1) \right. \\ & + \left. \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k)} \right|_{T_n} T_{jj}(i, j) \right] T_{ii}(i, j) \quad (\text{III.3.26}) \end{aligned}$$

and

$$D_{k,k+1}(i, j, j) = \sum_{j=1}^{NT} \sum_{i=1}^{NR} T_{jj}(i, j) T_{ii}(i, j) \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k+1)} \right|_{T_n} \quad (\text{III.3.27})$$

It is easily verified that equations (II.2.27) - (II.2.29) and equations

(III.3.25) - (III.3.27) are equivalent if $K \neq f(T)$. Equations (II.2.27) and (II.2.29) can still be used in the present case to evaluate $D_{k,k-1}$ and $D_{k,k+1}$ because the axial internodal conductances ($K(i,j,k)_{k-1}$ and $K(i,j,k)_{k+1}$) are not functions of temperature, but $D_{k,k}(i,j)$ must be evaluated using equation (III.3.26). The identities

$$\frac{\partial \epsilon(i,j,k)}{\partial T(i+1,j,k)} = -K(i,j,k)_{i+1}, \quad \frac{\partial \epsilon(i,j,k)}{\partial T(i-1,j,k)} = -K(i,j,k)_{i-1} \dots \text{etc.}$$

hold at nodes other than those on the pressure tube outer and calandria tube inner surfaces (outside of the contact patch), where equations (III.3.18) - (III.3.24) must be used to evaluate the partial derivatives of $\epsilon(i,j,k)$ with respect to the temperatures of the neighbouring nodes. Initial estimates of $T'^n(I,J,K)$ and $T'^n(I+1,J,K)$ must be provided to evaluate equations (III.3.23) and (III.3.24) for nodes I,J,K and $I+1,J,K$ on the first iteration ($n=1$) but values for those temperatures in subsequent iterations ($n=2,3,\dots$) can be obtained from the relation $T'^n = WZ^n$.

The foregoing procedure provides the left-hand coefficient matrix needed in equation (III.3.5b) to compute Z^{n+1} (or T'^{n+1}) from Z_n (or T'^n). The right-hand side of equation (III.3.5b), designated R as in the linear case, can be easily computed if the expression contained within the square brackets in equation (III.3.5a), designated Q'_{NR} (i.e., $Q'_{NR} = Q - KT'^n + J(T'^n)T'^n$), is written as

$$Q'_{NR} = A - E \tag{III.3.28}$$

where $E = KT'^n - Q$, as before, and $A = J(T'^n)T'^n$. The entry in A

corresponding to node i, j, k , designated $\alpha(i, j, k)$, is given by the expression,

$$\begin{aligned} \alpha(i, j, k) = & \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i+1, j, k)} \right|_{T^n} T'^n(i+1, j, k) \\ & + \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i-1, j, k)} \right|_{T^n} T'^n(i-1, j, k) + \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j+1, k)} \right|_{T^n} T'^n(i, j+1, k) \\ & + \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j-1, k)} \right|_{T^n} T'^n(i, j-1, k) + \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k+1)} \right|_{T^n} T'^n(i, j, k+1) \\ & + \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k-1)} \right|_{T^n} T'^n(i, j, k-1) + \left. \frac{\partial \epsilon(i, j, k)}{\partial T(i, j, k)} \right|_{T^n} T'^n(i, j, k) \end{aligned} \quad (\text{III.3.29})$$

where $\left. \frac{\partial \epsilon(i, j, k)}{\partial T(i+1, j, k)} \right|_{T^n} = -K(i, j, k)_{i+1}$, $\left. \frac{\partial \epsilon(i, j, k)}{\partial T(i-1, j, k)} \right|_{T^n} = -K(i, j, k)_{i-1}$..., etc.

if the internodal conductances are not functions of temperature, as we have seen previously. With the entry in E corresponding to node i, j, k given by $\epsilon(i, j, k)$, as before, the expression for the i th entry in the subvector R_k , given previously by equation (II.2.30) for the linear case, can be written for a non-linear analysis as,

$$R_k(i) = \sum_{j=1}^{NT} \sum_{k=1}^{NR} [\alpha(i, j, k) - \epsilon(i, j, k)] T'_{ij}(i, j) \quad (\text{III.3.30})$$

When the finite-difference equation for a given node i, j, k does not contain any non-linear terms,

$$\begin{aligned} \epsilon(i,j,k) = & -K(i,j,k)_{i+1} T'^n(i+1,j,k) - K(i,j,k)_{i-1} T'^n(i-1,j,k) \\ & - \dots - K(i,j,k)_{k-1} T'^n(i,j,k-1) + \epsilon K \end{aligned} \quad (\text{III.3.31})$$

and it is easily verified that $\epsilon(i,j,k) = \alpha(i,j,k)$. Thus in the present analysis the bracketted term in equation (III.3.30) vanishes at interior nodes so those nodes can be excluded from the summations in that equation. Equation (III.3.30) can, in this case, be rewritten as,

$$R_k(i) = \sum_{S_1+S_2+S_3+S_4} [\alpha(i,j,k) - \epsilon(i,j,k)] T_{ij}(i,j) \quad (\text{III.3.32})$$

where S_1 , S_2 , S_3 and S_4 are the sets of nodes on the pressure tube inner and outer and calandria tube inner and outer surfaces (at axial mesh point k) respectively. The nodes within the contact patch, over which a constant value of h_c is prescribed, are excluded from S_2 and S_3 because the finite-difference equations for those nodes do not contain any non-linear terms (i.e., $\alpha(i,j,k) = \epsilon(i,j,k)$ at those nodes).

The finite-difference equations for nodes on S_1 contain constant terms arising from the prescribed heat flux (q'') but no non-linear terms so

$$\alpha(i,j,k) - \epsilon(i,j,k) = q'' A_s(i,j,k) - K(i,j,k)_s T_\infty \quad \text{for } i,j,k \in S_1 \quad (\text{III.3.33})$$

as in equation (II.2.30). Note that $K(i,j,k)_s = 0$ on S_1 in the present analysis. Equation (II.2.30) also applies to the nodes on the calandria tube outer surface (i.e., $i,j,k \in S_4$), only $q''=0$ at those nodes and $K(i,j,k)_s = h_f A_s(i,j,k)$, where h_f is the heat transfer coefficient (assumed constant for simplicity) between the calandria tube and the moderator in this instance.

For nodes on the pressure tube outer surface (i.e., $i, j, k \in S_2$) equation (III.3.16) and equations (III.3.18) - (III.3.24) can be combined to give,

$$\begin{aligned} \alpha(i, j, k) - \epsilon(i, j, k) &= \left(-\frac{B_j}{2}\right) T'^n(i, j, k)^2 + 3R_s T'^n(i, j, k)^4 \\ &= \left(-\frac{B_j}{2}\right) T'^n(i+1, j, k)^2 + 3R_s T'^n(i+1, j, k)^4 \quad (\text{III.3.34}) \end{aligned}$$

Equation (III.3.34) also applies to nodes on the calandria tube inner surface (i.e., $i, j, k \in S_3$) when the subscript $i-1$ is substituted for $i+1$.

Substituting equations (III.3.33) and (III.3.34) into equation (III.3.32) provides the desired expression for $R_k(ii)$. It should be recognized that while the preceding derivations of expressions for entries in the D and R matrices did not identify the trial functions with specific axial mesh points by writing them, for instance, with a superscript k, this identification was omitted for simplicity and all of the derived equations are applicable to the general case in which different sets of trial functions are used at different axial mesh points. It should also be recognized that the summations performed in the computation of $D_{k,k}(ii, jj)$ (where $k=1, NZ$; $ii=1, N$, $jj=1, N$) do not have to be entirely repeated in the second and subsequent iterations because only the non-linear terms in the finite-difference equations for nodes on S_2 and S_3 need to be updated. The computer program described in Part IV performs this limited updating and thus avoids reassembling the finite-difference equations for all of the interior nodes after the first iteration. Convergence of the iterations to the correct solution vector is assumed to have occurred when all of the nodal

temperatures vary by less than some user-specified amount from one iteration to the next.

The preceding analysis illustrated how radiation and temperature dependent gas conduction between the pressure and calandria tubes in a CANDU reactor can be incorporated into a synthesis analysis by modifying only the finite-difference equations for nodes on the pressure tube outer and calandria tube inner surfaces. Natural convection and/or boiling heat transfer between the calandria tube and the moderator could be incorporated in a similar manner by modifying the finite-difference equations for nodes on the calandria tube outer surface. While it might appear that the finite-difference equations for all of the nodes in the numerical model would have to be modified to account for the temperature dependence of the thermal conductivity of Zircaloy, this need not be done if Kirchoff's transformation [80] is used to linearize the heat conduction equation since the finite-difference equations for interior nodes become linear in that case. The unknown variable at node i,j,k in the finite-difference grid is $\int_{T_r}^{T_{i,j,k}} k_s(\hat{T}) d\hat{T}$ (where T_r is a convenient reference temperature) instead of $T_{i,j,k}$ when Kirchoff's transformation is applied. The manner in which the transformation is incorporated into the analysis is described in Appendix III.B.

CHAPTER III.4

EXAMPLES OF TRANSIENT AND NON-LINEAR

SYNTHESIS ANALYSES

The finite-difference/synthesis method was used to analyze heat transfer between contacting pressure and calandria tubes in a Bruce B CANDU reactor fuel channel to demonstrate its applicability to transient and non-linear problems, as mentioned previously. As discussed in greater detail in the next section, a finite-difference grid that was highly graded near the contact was used to model the fuel channel between its two most central garter springs, where contact is most likely to occur. To minimize computing costs, the sample transient analysis was treated as being linear and the sample non-linear analysis was performed for steady-state conditions, as discussed in Sections III.4.2 and III.4.3 respectively.

III.4.1 A CANDU Fuel Channel Model

Complete details of the fuel channel model that was used in the synthesis analyses are given in Part IV of the thesis, which contains the documentation for the computer program SYNCON, so only a brief summary of the model's features will be given here.

Garter springs are inserted between the pressure and calandria tubes in a CANDU reactor at 1-2 m. intervals to prevent them from contacting over the design life of the reactor due to creep. It was

assumed for simplicity in developing the fuel channel model that the garter springs would prevent complete contact of the pressure and calandria tubes during a LOECC accident by maintaining gaps equal to their diameters at the points where they were located. The garter springs would likely indent the pressure tube to a certain extent in a real LOECC accident, of course, so the gaps at the garter springs would probably be less than the garter spring diameters in reality. It was also assumed that initial contact between the pressure and calandria tubes would occur midway between the two most central garter springs and that the deflected shape of the pressure tube between those springs would approximate that of a uniformly loaded (elastic) beam. The ends of the beam, located at the garter springs, were assumed fixed as shown in Figure III.4.1 since it is likely that the pressure and

y = Gap between bottom of P.T. outer surface and bottom of C.T. inner surface

$$= \frac{16D_s z^2}{L_{sp}^4} \left(\frac{L_{sp}^2}{2} - z^2 \right)$$

Figure III.4.1 Slumped Pressure Tube Approximated as a Uniformly Loaded,

calandria tubes would remain approximately parallel at those points due to the approximate symmetry of the loading conditions about the springs. It was also assumed for simplicity that the pressure and calandria tubes would remain circular after contact, although some distortion of particularly the pressure tube would undoubtedly occur in reality.

The bowed radius of the pressure tube at the contact was evaluated from the deflection curve for a uniformly loaded, fixed end beam (given in Figure III.4.1) and used along with the pressure tube outer radius and the calandria tube inner radius to estimate the dimensions of the contact patch. It was assumed for simplicity that the pressure/calandria tube contact would form elastically since this allowed Hertz's theory to be used to obtain a crude estimate of the contact size.* The load on the contact was assumed to be one-half of the total weight of the pressure tube and the fuel bundles between the two most central garter springs. Preliminary analyses indicated that the pressure tube outer and calandria tube inner surface temperatures at the contact would both be approximately 473K (200°C)** for the analyses of interest (which were all performed with a moderator temperature of 353K) so Young's modulus for the Zircaloy-2 contact surfaces was evaluated at that temperature. The resulting contact semi-minor and semi-major diameters were 0.063 cm. and 0.550 cm. respectively, giving a semi-major to semi-minor diameter ratio of 8.8

* The effect of surface roughness on the contact size was not considered.

** The preliminary analyses indicated that the pressure tube outer and calandria tube inner surface temperatures at the point of contact would both reach approximately this value very shortly after contact.

and a mean pressure over the contact area of 53 MPa (7750 psi). The aforementioned semi-minor and semi-major diameters are approximately 46% and 400% of the calandria tube wall thickness respectively and 15% and 135% of the pressure tube thickness respectively.

As noted in Appendices B and C of Part IV, where the contact size calculations are described in more detail, Hertz's theory is only applicable to contacts which are very small compared to the dimensions of the contacting bodies so the pressure/calandria tube contact dimensions mentioned previously would be inexact even if the elastic deformation assumption was valid. Two-dimensional $r-\theta$ sensitivity analyses described by Rogers and Currie in references [24] and [25] showed that variations in contact width and contact conductance have almost no effect on any of the results of either a steady-state or transient LOECC analysis, however, including the heat flux on the calandria tube outer surface at the contact, so it is probably not necessary to obtain a precise estimate of the contact size in this application.

Only the section of the fuel channel between one of the garter springs nearest to the channel center and the channel center itself, which is a section 51 cm. long in a Bruce B reactor, was modelled in the LOECC analysis. The $r-\theta$ plane at the garter spring was assumed to be adiabatic. Sixty-five unevenly spaced axial nodes were used in the finite-difference model of the fuel channel segment and 454 nodes were used on each $r-\theta$ plane (i.e., at each axial node). The complete finite-difference model contained 29,510 ($= 65 \times 454$) nodes. The $r-\theta$ grid was highly graded, as shown in Figure III.4.2, to accurately model the heat

Figure III.4.2 Highly Graded Grid Used For Thermal Analysis of Contacting Pressure and Calandria Tubes.

flow in the vicinity of the contact. Only one-half of the fuel channel cross-section was modelled because of symmetry. Eight equi-sized radial subdivisions were used across the thickness of the calandria tube wall at the contact and the same size subdivisions were used in neighbouring parts of the pressure tube, as seen in Figure III.4.2. The 180° sector of the fuel channel cross-section was subdivided into 38 unequally sized circumferential subdivisions, with 0.12° subdivisions being used within and near the contact.

The contact approximation technique designated method 1 in Section II.2.5.3 was used to approximate the contact with the representation defined by the shaded area shown in Figure II.4.3. Six subdivisions were used to model the contact half-width and ten subdivisions were used on the major axis to model one-half of the length of the contact. The contact approximation techniques designated methods 3 and 4 in Section II.2.5.3 were not used because the $r-\theta$ sensitivity analyses described in references [24] and [25] indicated that it was not necessary to approximate the width of the contact accurately in this application, as mentioned previously. Since contact approximation techniques 3 and 4 were derived for steady state conditions, or at least quasi-steady state conditions around the contact, they would not have been applicable to the period immediately after contact in the transient analysis anyway.

A more detailed description of the fuel channel model can be found in Part IV.

Figure III.4.3: Contact Representation Used in LOECC Analyses

III.4.2 Sample Transient Analyses

The synthesis technique was used to analyze transient heat transfer between contacting pressure and calandria tubes in a CANDU reactor, as mentioned previously. Since the primary purpose of the analysis was to demonstrate the applicability of the synthesis technique to transient problems, the pressure-to-calandria tube heat transfer analysis was treated as being linear to minimize computing costs. The thermal

* SYNCON can perform non-linear, transient, LOECC analyses without difficulty but such analyses are, nevertheless, more costly than linear ones.

conductivities of the pressure and calandria tube materials and the gas in the gap between the pressure and calandria tubes were held constant, as was the radiative heat transfer coefficient between the tubes. The radiative heat transfer coefficient was considered to be circumferentially uniform as well as temperature independent, simplifications which probably did not introduce serious errors because the coefficient was assigned a value appropriate to the regions in which radiation would have been important (i.e., regions remote from the contact).

A reference solution was required to assess the accuracy of the synthesis solution. A reference solution would have been almost impossible or at least very expensive to obtain for a full 3-D LOECC analysis, however, so the analysis was initially restricted to only two-dimensions, specifically to the $r-\theta$ plane at the center of the fuel channel. The pressure and calandria tubes were assumed to be in contact at that location. A two-dimensional ($r-\theta$) transient (linear) finite-difference solution was obtained at the center of the fuel channel for use as the reference solution and the synthesis solution to the same problem was obtained using exactly the same grid, time step sizes, etc.

Since the synthesis technique produces an approximate solution to a system of finite-difference equations, the exact solution to those equations is obviously the reference against which the synthesis solution should be compared.

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The 2-D LOECC analysis was performed for the $r-\theta$ cross-section at the center of a CANDU fuel channel using the fuel channel model described briefly in the preceding section. The contact half-width was set to the semi-minor diameter calculated for the 3-D analysis (i.e., 0.0628 cm.). The pressure tube temperature at the time of slumping was assumed to be approximately 1000°C [149] and it was assumed, conservatively, that cooling of the pressure tube during the slumping period would be insignificant until contact with the calandria tube actually occurred. The circumferentially uniform radiative heat transfer coefficient used in the analysis was the value corresponding to a pressure tube temperature of 1000°C, assuming pressure tube outer and calandria tube inner surface emissivities of 0.8 and 0.2 respectively [24]. The moderator temperature was assumed to be 353K (80°C), as mentioned previously, while the heat transfer coefficient between the calandria tube and the moderator (h_f) was set to $7 \text{ W/cm}^2 \text{ K}$ to limit the calandria tube outer surface temperature at the 6 o'clock position (i.e., at the contact) to the maximum value of 130-140°C observed in the transient analysis of pressure-to-calandria tube heat transfer described in reference [24]. The value of h_f used in that analysis was calculated for either natural convection or sub-cooled (nucleate) pool-boiling conditions, depending on the local surface heat flux, and hence was not circumferentially uniform. Although the uniform coefficient model would not have been as realistic as a variable coefficient one, analyses performed by Rogers [150] with the calandria tube outer surface temperature held at that of the moderator (i.e., with $h_f = \infty$) instead of with a variable h_f showed that the results of

a LOECC analysis are very insensitive to the boundary conditions applied to the calandria tube outer surface.

Other data used in the synthesis and finite-difference analyses of transient pressure-to-calandria tube heat transfer are given in Part IV, where the synthesis analysis is performed as a sample problem.

The thermal history of the pressure/calandria tube assembly was obtained for a period of 260 seconds after contact with the finite-difference and synthesis methods. A time step size of 18 msec. was used for the first 50 time steps (i.e., for the first 0.9 seconds of the transient) to accurately model the initial, very rapid release of energy stored in the pressure tube. The time step size was gradually increased as the transient progressed and temperatures changed more slowly. The largest time step size, 18 seconds, was used for times after 149 seconds. A total of 160 time steps was used to model the period before 149 seconds and 6 time steps were used to model the rest of the transient (i.e., up to 260 seconds).

Eleven trial functions were used in the synthesis analysis to obtain the pressure and calandria tube temperature distributions at times up to 260 seconds after contact. The first trial function was obtained by solving for the steady-state temperatures in the pressure and calandria tubes before contact (the desired initial pressure tube temperature of approximately 1000°C was obtained by prescribing

a heat flux of 3.914 W/cm^2^* over the inner surface of the pressure tube). The next nine trial functions were transient trial functions obtained by recording temperature distributions at different times in a finite-difference analysis of the period following contact. The initial pressure and calandria tube temperature distributions for the analysis were taken from the first trial function and the time step sizes used were those mentioned previously. Trial functions were recorded at the following times (after contact) in the transient.

<u>Trial Function</u>	<u>Time (After Contact), secs.</u>
2	0.360
3	0.738
4	1.242
5	1.962
6	3.402
7	5.202
8	14.202
9	26.800
10	46.602

The finite-difference solution from which trial functions 2-10 were taken later served as the reference solution against which the results of the synthesis analysis were compared. The eleventh and final trial function used in the synthesis analysis was the steady-state solution to the pressure calandria tube contact problem with a heat flux of 3.914 W/cm^2 supplied to the pressure tube inner surface. The

* This heat flux corresponds to a linear heating rate (i.e., heating rate per unit length of fuel channel) of 127 W/cm , as is easily verified by multiplying it by the inner circumference of the pressure tube (32.5 cm).

final trial function was, therefore, the long time (i.e., steady-state) solution to the transient problem from which trial functions 2-10 were obtained.*

Only some of the available trial functions (i.e., trial functions 1-11) were used to synthesize the temperature distribution in the pressure/calandria tube assembly at any given time in the transient. Each trial function was only used at times when it was expected that it would be useful in the temperature synthesis, in other words at times near to the time at which it was recorded. To this end, trial functions 1-5 were used at times less than 1.46 seconds, trial functions 4-8 were used at times between 1.46 seconds and 10.60 seconds, and trial functions 7-11 were used at times greater than 10.60 seconds. With the trial functions deployed in this manner, only five trial functions were used in the temperature synthesis at any one time and only five simultaneous equations had to be solved at each time step to obtain the synthesized temperature distribution. The left-hand side coefficient matrix for the system of equations defining the blending vector (Z) only had to be recalculated when the trial functions were changed, which meant that it was recalculated three times during the course of the analysis because the trial functions were changed three times.

Some of the results of the synthesis and finite-difference solutions to the pressure/calandria tube contact problem, with the initial conditions (temperatures) taken from trial function 1, are shown in Figure III.4.4. The second to uppermost curve in that figure shows the temperature

* With the pressure tube inner surface heat flux held constant. This heat flux would decrease with time in a real LOECC accident.

of the pressure tube inner surface at the 6 o'clock position as a function of time for both the finite-difference and synthesis solutions. The synthesis and finite-difference results for the temperature at that point differed by less than 2K at all times in the transient. The times at which trial functions were recorded in the finite-difference analysis are marked on the second to uppermost curve in Figure III.4.4 to show that the trial functions spanned the total temperature variation of the pressure tube inner surface at the 6 o'clock position quite uniformly. The times at which trial functions were recorded were chosen to achieve this. The wiggly lines shown in Figure III.4.4 mark the times at which the trial functions were changed in the synthesis calculations.

The lower solid curves in Figure III.4.4 show the peak heat flux on the calandria tube outer surface as a function of time for the synthesis and finite-difference solutions. The peak heat flux on the calandria tube outer surface, which is important because it determines whether local dryout occurs, was always found at the contact, i.e., at the 6 o'clock position, of course. Because the first transient trial function was recorded at a time of 0.36 seconds in the finite-difference analysis, the synthesis result for the peak heat flux on the calandria tube outer surface was very inaccurate at times less than approximately 0.25 seconds into the transient. At times greater than approximately 0.25 seconds, the finite-difference and synthesis results for the peak heat flux were in almost perfect agreement.

All of the input data and the output listing for the foregoing analysis can be found in Part IV, where it is performed as a sample problem.

To determine whether three-dimensional effects would significantly increase the peak heat flux on the calandria tube outer surface, the synthesis method was used to obtain a three-dimensional solution to the transient pressure/calandria tube contact problem. The problem was again treated as being linear to permit comparison with the two-dimensional solution obtained previously and all of the material properties, the radiative heat transfer coefficient between the pressure and calandria tubes, time step sizes, etc., were left unchanged. The r - θ - z grid used in the three-dimensional transient analysis was the same as that described in Section III.4.1 except that only the first 11.2 cm. of the fuel channel segment between the channel center and the first garter spring (at $z=51.0$ cm.) were modelled. Regions more remote from the contact were not modelled because it was expected that their presence would have little effect on conditions near to the contact, particularly during the time period of interest (i.e., less than 260 seconds after contact). A total of 45 axial nodes was used in the 11.2 cm. section of the fuel channel modelled, so, with 454 nodes on each r - θ plane, the 3-D finite-difference grid contained 20,430 nodes. The contact dimensions used were those mentioned in Section III.4.1.

A total of thirty-seven trial functions was used in the 3-D transient synthesis analysis to account for r - θ temperature variations in the axial (z) dimension as well as with time. Trial functions

1-11 were the same as those used in the 2-D transient synthesis analysis described previously while trial functions 12-37 were obtained by performing transient and steady-state two-dimensional finite-difference analyses at axial (z) locations of 0.487 cm., 1.50 cm. and 4.30 cm. These axial coordinates were selected by studying two-dimensional steady-state solutions obtained at different axial locations (in the range $0 \leq z \leq 11$ cm.) to determine coordinates where the solutions were different enough to span the range of steady-state r - θ temperature distributions found between $z=0$ and $z=11$ cm. fairly uniformly. Trial function 1, which was the steady state solution for the pressure and calandria tube temperatures before contact (with a heat flux of 3.914 W/cm^2 supplied to the pressure tube inner surface) provided the initial temperature distribution for the transient analyses performed at the different axial locations. Trial functions 12-20 were transient trial functions obtained at $z=0.487$ cm. (i.e., within the length of the contact), at the times given previously for trial functions 2-10. Trial function 21 was the steady-state solution at $z=0.487$ cm. Trial functions 22-30 were transient trial functions obtained at $z=1.50$ cm. at the times listed below,

<u>Trial Function</u>	<u>Time (After Contact), secs.</u>
22	0.54
23	1.17
24	1.962
25	3.402
26	5.202
27	8.802
28	16.002
29	34.002
30	57.402

while trial function 31 was the steady-state solution at $z=1.50$ cm. Of the remaining trial functions, which were all obtained at $z=4.30$ cm., numbers 32-36 were recorded in a transient analysis at the times listed below,

<u>Trial Function</u>	<u>Time (After Contact), secs.</u>
32	1.296
33	3.744
34	8.784
35	19.584
36	44.784

while the final trial function, 37 was the corresponding steady-state solution.

Three trial function schedules were used during the course of the transient to avoid using trial functions at times when it was expected that they would not be useful. Trial function schedule changes were made at times of 1.46 seconds and 10.60 seconds, as in the 2-D analysis. The schedules for the three periods are given in Table III.4.1, from which it can be seen that the trial functions used at axial nodes 1-6 (i.e., for $0 \leq z \leq 0.375$ cm.) in each time period were the same as those used in the 2-D synthesis analysis. Trial function 1 was used at all axial locations for times less than 1.6 seconds because it served as the initial ($t=0$) temperature distribution in the analysis.

The peak heat flux obtained on the calandria tube outer surface in the 3-D transient synthesis analysis is plotted against time (t)

Table III.4.1: Trial Function Schedules for Different Time Periods in the 3-D Transient Synthesis Analysis

Trial Functions for		Zone of Application		Corresponding Axial Nodes
		Initial z (cm.)	Final z (cm.)	
$t < 1.458s$	$1.458s < t < 10.602s$			
1,2,3,4,5	4,5,6,7,8	0.0	0.375	1-6
1,12,13,14,15	14,15,16,17,18	0.375	1.3	7-20
1,22,23,24,25	23,24,25,26,27	1.3	3.40	21-30
1,32,33,34	32,33,34,35	3.40	11.2	31-45

in Figure III.4.4 where it is compared to the results of the 2-D finite-difference and synthesis analyses. The 3-D and 2-D synthesis solutions were in perfect agreement during the initial stages of the transient (up to about 2 seconds after contact). While neither solution agreed with the finite-difference solution before approximately 0.25 seconds into the transient, the fact that they agreed so well with each other indicates that axial conduction probably does not increase the magnitude of the heat flux "spike" that occurs at the 6 o'clock position on the calandria tube outer surface immediately after contact. The 3-D result for the peak calandria tube heat flux gradually rises above the corresponding 2-D result as time progresses, however, as can be seen in Figure III.4.4 and as will be shown again in the next section where heat fluxes obtained in 2-D and 3-D steady-state analyses are compared. The insignificance of 3-D effects in the early stages of the transient justifies the neglect of these effects by Rogers and Currie in their assessment of the likelihood and consequences of calandria tube dryout at the contact [24-27].

All of the analyses described previously in this section were performed with a contact conductance (h_c) of $56.75 \text{ W/cm}^2\text{K}$ ($100,000 \text{ BTU/hr ft}^2 \text{ }^\circ\text{F}$) prescribed over the contact area. The two-dimensional results were obtained with a contact half-width of 0.063 cm. , as mentioned previously. To investigate the sensitivity of the results to the assumed contact conductance and patch width, other two-dimensional analyses were performed with:

- (1) the original patch half-width of 0.063 cm. but a contact conductance of $0.5675 \text{ W/cm}^2\text{K}$ ($1000 \text{ BTU/hr ft}^2 \text{ }^\circ\text{F}$), and;
- (2) the original contact conductance of $56.75 \text{ W/cm}^2\text{K}$ but a patch half-width twice that used previously (i.e., $2 \times 0.063 \text{ cm.} = 0.126 \text{ cm.}$).

The peak heat flux on the calandria tube exterior is plotted against the time after contact in Figure III.4.5 for the original and alternate boundary conditions (1) and (2). It can be seen from that figure that doubling the contact width had no effect on the peak heat flux and that reducing the contact conductance from $56.75 \text{ W/cm}^2\text{K}$ to $0.5675 \text{ W/cm}^2\text{K}$ caused the peak heat flux to decrease by approximately 20%, from 428 W/cm^2 to 340 W/cm^2 . These results show that the assumed contact conductance and patch width are not particularly important in this application, as concluded previously in references [24] and [25] (for slightly different contact boundary conditions) from the results of steady-state and transient analyses respectively.

To put the calandria tube heat fluxes shown in Figure III.4.5 into perspective, 240 W/cm^2 represents a conservative (lower bound) estimate of the critical heat flux (CHF) under steady-state conditions for the moderator subcooling at the time of contact, estimated to be approximately 30°C [24]. Although the aforementioned CHF applies to steady-state conditions, experimental results indicate that CHF under rapid transient conditions is at least as high as, if not higher than, CHF under quasi steady-state conditions [151]. Additional arguments justifying the selection of 240 W/cm^2 as a lower bound estimate of the CHF in the present application are given by Rogers and Currie in references [24] and [25].

Figure III.4.5 Comparison of Peak Calandria Tube Heat Fluxes Obtained With Different Contact Conductances and Patch Widths

Although the peak heat fluxes shown in Figure III.4.5 exceed estimated lower bound CHF of 240 W/cm^2 by as much as 78%, the heat flux curves for the three contact conditions investigated all fall below the estimated CHF within less than 2 seconds after contact. By modelling heat transfer from the calandria tube to the moderator in the nucleate, transition and film boiling regimes of the boiling curve in a detailed fashion, Rogers and Currie [27] were able to demonstrate that any drypatch formed as a result of CHF being exceeded would cover less than 10 degrees of the calandria tube outer surface and would, in addition, rewet within less than 10 seconds after contact. A minimum film boiling temperature of 350°C was used to obtain those predictions—results for a less probable minimum film boiling temperature of 250°C are given in reference [27] along with predicted calandria tube temperatures, fuel temperatures, etc.

It should be mentioned that the thermal conductivity of the nitrogen gas in the gas gap between the pressure and calandria tubes and the thermal conductivity of the Zircaloy pressure and calandria tube material were both allowed to be temperature dependent in the analyses whose results are shown in Figure III.4.5. Also, radiative heat transfer coefficients between opposing nodes on the pressure tube outer and calandria tube inner surfaces were evaluated using local surface temperatures instead of being held constant, as in the previous analyses. The highest heat flux shown in Figure III.4.5 for the original contact conductance and patch width (i.e., 56.75 W/cm^2 and 0.063 cm , respectively) is not the same as the corresponding 2-D result shown in Figure III.4.4 for that reason, although the results differ by only about 12% (i.e.,

428 W/cm² in Figure III.4.5 vs. 373 W/cm² in Figure III.4.4).

One final point worthy of mention is that although the time step (Δt) used immediately after contact was very small, only 18 msec as mentioned previously, the stability-limited time step for an explicit time integration scheme applied to the original difference equations (i.e., Equation (III.2.4) with $\theta=0$) would have been even smaller because the finite-difference grid described in Section III.4.1 was so finely discretized near the contact. It can be shown that the latter time step would have been less than 0.5 msec., indicating why the explicit time integration scheme is rarely used in problems requiring fine spatial discretization. A scheme which used Equation (III.2.7) with $\theta=0$ (i.e., Equation (III.2.8)) might be more practical, however, because such a scheme could be formulated to permit the use of larger time steps than Equation (III.2.4) with $\theta=0$, as discussed in greater detail in Appendix III.A.

III.4.3 Sample Non-Linear Analyses

A non-linear steady-state thermal analysis of pressure and calandria tubes in contact is described in this section. The nonlinearities considered in the analysis were the same as those discussed in Section III.3 and Appendix III.B.

Numerical results were obtained with heat flow rates of 570 W/cm and 127 W/cm supplied to the pressure tube inner surface. The first heating rate, 570 W/cm., corresponds to the decay heat generated at 16

minutes after reactor trip by a fuel bundle with an initial (i.e., before reactor trip) outer fuel element power rating of 48 W/cm. when the decay power (P) is related to the initial power (P_0) by the expression [24],

$$\frac{P}{P_0} = 0.10 t^{-0.18} \quad (\text{for } t > 19 \text{ sec.}) \quad (\text{III.4.1})$$

where t is the time (in seconds) after reactor trip. The r-o LOECC analysis described by Rogers and Currie in reference [24], which ignores heat generated by the Zircaloy-water reaction at high pressure tube and fuel sheath temperatures on the basis that, without ECC flow, there will be no steam supply to maintain the reaction, shows that quasi-steady state conditions prevail at approximately 16 minutes after reactor trip (or roughly 13 minutes after the end of the blowdown process*) for the aforementioned initial outer fuel element power rating and power run-down curve. The bundle heat generation rate at that time, 570 W/cm, was prescribed over the pressure tube inner surface from the center of the fuel channel to the first garter spring in the 3-D synthesis analysis. Although, in reality, the axial profile of the heat input rate would approximate a chopped cosine. The heat generation rate at the first garter spring (51 cm. from the channel center) would actually be about 96.5% of the channel center value because of the chopped cosine power distribution, but this variation was neglected for simplicity.

The second linear heat flow rate used, 127 W/cm, has no special significance except that it produces a pressure tube temperature of

* For blowdown resulting from a large (~25%) rupture of a reactor inlet header.

approximately 1000°C under steady-state conditions when the pressure and calandria tubes are concentric.* It has been established that contact between the pressure and calandria tubes during a LOECC accident occurs when the pressure tube temperature reaches approximately 1000°C [149]. A linear heat flow rate of 127 W/cm was used in the sample transient LOECC analysis, described in the preceding section, because it produced the correct initial conditions for the analysis (i.e., an initial pressure tube temperature of 1000°C at the time of contact) in the simplest possible manner.

Thirteen trial functions, obtained as 2-D (r- θ) solutions at the axial (z) coordinates listed below, were used in the 3-D synthesis calculations performed with a heating rate of 570 W/cm.

TRIAL FUNCTION	AXIAL COORDINATE (z) cm.	T_{PT} , K
1	0.0 (channel center)	601
2	0.3125	604
3	0.4250	606
4	0.4875	608
5	0.5250	609
6	1.0	643
7	1.5	696
8	2.0	756
9	3.0	880
10	4.3	1034
11	6.0	1211
12	11.0	1555
13	51.0 (1st Garter spring)	1902

The temperature given for each trial function, T_{PT} , was the temperature of the pressure tube inner surface at the 6 o'clock position. This

* Assuming pressure tube outer surface and calandria tube inner surface emissivities of 0.8 and 0.2 respectively.

temperature was relatively constant for trial functions 1-5, since they were all obtained at axial coordinates within the length of the contact patch, but increased monotonically for the succeeding trial functions up to the maximum value of 1902K obtained at the first garter spring. The trial function evaluation coordinates (z) were actually chosen so that the total range of values of T_{PT} was more finely resolved at the low end of the range than at the high end to accurately synthesize the r - θ - z temperature distribution near the contact. The values of T_{PT} for trial functions 5 and 6 differed by only 34K, for example, whereas the values of T_{PT} for trial functions 12 and 13 differed by 347K. The value of T_{PT} was used to characterize the r - θ temperature distributions of the trial functions because axial variations in the temperature of the calandria tube and the temperature of the pressure tube at the 12 o'clock position were both relatively insignificant.

Trial functions 1-13, as well as the 3-D synthesis solution itself, were obtained with a moderator temperature of 353K and a calandria tube-to-moderator heat transfer coefficient of $1.2 \text{ W/cm}^2 \text{ K}$.

The 3-D synthesis solution was obtained with the trial function schedule shown in Table III.4.2. A comparison of the 3-D results to the results of the 2-D analyses performed at different axial locations indicated that the peak heat flux on the calandria tube outer surface near and at the center of the fuel channel was the only result that was significantly affected by axial conduction. The peak heat flux obtained at each axial position (z) on the calandria tube outer surface is shown plotted versus z in Figure III.4.6. The solid line

Table III.4.2: Trial Function Schedule Used in the Steady-State LOECC Analysis Performed With A Heat Input Rate of 570 W/cm.

Trial Functions	Zone of Application		Corresponding Axial Nodes
	Initial z (cm.)	Final z (cm.)	
1,6,7,8,9	0.0	0.225	1-3
2,6,7,8,9	0.225	0.375	4-6
3,6,7,8,9	0.375	0.475	7-8
4,6,7,8,9	0.475	0.5166	9
5,6,7,8,9	0.5166	0.550	10*
4,6,7,8,9	0.550	2.10	11-24
6,7,8,9,10	2.10	3.40	25-30
7,8,9,10,11	3.40	4.40	31-33
8,9,10,11	4.40	5.40	34-36
9,10,11,12	5.40	7.00	37-39
10,11,12,13	7.00	16.00	40-50
11,12,13	16.00	51.00	51-65

* Last node within the contact

Figure III.4.6 Peak Heat Flux on Calandria Tube Outer Surface vs. Axial Position (z) - 570 W/cm Heat Input

Figure III.4.7 Peak Heat Flux on Calandria Tube Outer Surface vs. Axial Position (z) - 127 W/cm Heat Input

represents the 3-D result while the peak heat flux obtained in the 2-D solution at the channel center is indicated by the dashed line. The maximum heat fluxes obtained on the calandria tube outer surface in the 3-D and 2-D analyses were 76.9 W/cm^2 and 70.2 W/cm^2 respectively, so axial conduction increased the peak heat flux by only 9.5%.

Initial conditions for the Newton-Raphson iterative solution were taken from trial function #13. The peak calandria tube heat flux at $z=0$ is shown in Figure III.4.6 for each iteration up to the fourth, when convergence was achieved. The solution was assumed to have converged on the fourth iteration because all of the nodal temperatures varied by less than 2K (2K was the specified convergence parameter) from the third iteration to the fourth.

The analysis just described was repeated with the heat input to the pressure tube inner surface changed from 570 W/cm to 127 W/cm. The trial functions used in this analysis were obtained at the axial locations given previously for the analysis with the 570 W/cm heating rate and the trial functions were deployed in the same manner (i.e., according to the schedule given in Table III.4.2). The peak heat flux on the calandria tube outer surface at and near the contact was increased much more by axial conduction in this analysis than it was in the analysis with the higher heating rate, as seen in Figure III.4.7 where the peak calandria tube heat flux (3-D result) for the lower heating rate analysis is plotted versus axial position. The peak heat flux obtained in the corresponding 2-D analysis at $z=0$ is also indicated.

in the figure. The maximum heat fluxes obtained on the calandria tube outer surface in the 3-D and 2-D analyses at the 127 W/cm heating rate were 28.0 W/cm^2 and 23.4 W/cm^2 respectively, so axial conduction increased the peak heat flux by 20% in this case. The corresponding increase obtained at a heating rate of 570 W/cm was only 9.5% as mentioned previously. The increased significance of axial conduction near the contact in the lower heating rate analysis can probably be attributed to a reduction in the significance of radiation from the pressure tube to the calandria tube as a heat transfer mechanism. While lowering the heating rate from 570 W/cm to 127 W/cm caused the difference between the maximum temperature of the pressure tube (found at the 12 o'clock position) and the average contact temperature ($\approx 450\text{K}$) to drop from approximately 1500K to 850K (a 43% decrease), the corresponding reduction in the radiative heat transfer coefficient in regions remote from the contact would have been about 70%. The radiative conductance between the pressure and calandria tubes would therefore have sustained a much larger reduction than the temperature differences in the problem. Since the conductive resistances in the pressure tube would not have changed significantly when the heating rate was lowered, conduction in the pressure tube towards the contact would have increased in significance.

To further assess the importance of 3-D effects under steady-state conditions, another steady-state analysis of pressure-to-calandria tube heat transfer was performed with an even lower heat input rate of 25 W/cm. This heat input rate was obtained by reducing the second heating rate of 127 W/cm by the same factor that the original heating

rate of 570 W/cm had been reduced to obtain the second value, in other words by a factor of about five. As in the case of the analyses described previously, the synthesis solution for the 25 W/cm heating rate was obtained with a total of thirteen trial functions. The axial coordinate at which each trial function was obtained and the values of T_{PT} for the trial functions are listed below. The analyses were performed with a moderator temperature of 353K and a calandria tube-to-moderator heat transfer coefficient of $1.2 \text{ W/cm}^2 \text{ K}$, as before.

TRIAL FUNCTION	AXIAL COORDINATE	T_{PT} , K
1	0.0	378
2	0.3125	379
3	0.425	379
4	0.4875	379
5	0.525	379
6	1.4	389
7	2.0	398
8	3.0	418
9	4.3	445
10	6.0	482
11	8.0	524
12	11.0	584
13	51.0	789

Except for trial functions 6 and 8, the trial functions listed above were obtained at the same axial coordinates as the trial functions used in the steady-state analyses described previously. As before, the trial function evaluation coordinates were selected so that the total range of values of T_{PT} would be more finely resolved at the low end of the range than at the high end to accurately synthesize the r- θ -z temperature distribution near the contact.

Trial functions 1-13 were deployed according to the schedule given in Table III.4.2.

The maximum heat fluxes obtained on the calandria tube outer surface in the 3-D and 2-D analyses performed with a heating rate of 25 W/cm were 7.58 W/cm² and 5.92 W/cm² respectively, giving a heat flux increase due to axial conduction of 28%. That heat flux increase was larger than the increase of 20% obtained with a heating rate of 127 W/cm, undoubtedly for the same reason that the heat flux increase in the latter analysis was greater than it had been with a heating rate of 570 W/cm.

The peak heat flux obtained at each axial position (z) on the calandria tube outer surface with the 25 W/cm heating rate is shown plotted versus z in Figure III.4.8.

The sensitivity of the 3-D results obtained in the 127 W/cm and 25 W/cm heating rate analyses to trial function changes was tested by repeating those analyses with fewer trial functions in total and with fewer trial functions used at most of the axial nodes. The new set of trial functions was obtained from the original set by dropping trial functions 2, 4 and 6 in each case. The remaining trial functions were deployed according to the schedules given in Tables III.4.3 and III.4.4 for the 127 W/cm and 25 W/cm analyses respectively. Comparing Tables III.4.2 and III.4.3, it can be seen that the number of trial functions used at each axial node in the repeat analysis performed with a heating rate of 127 W/cm was one less than the number used in the original analysis, except at axial nodes 34-39 where four trial functions were used in each analysis and nodes 51-65 where only three trial functions were used in each case. Similarly, the number of trial functions used at each of axial nodes 1-33 in the original

Figure III.4.8 Peak Heat Flux on Calandria Tube Outer Surface vs Axial Position (z) - 25 W/cm Heat Input

Table III.4.3: Trial Function Schedule Used in the Repeat Analysis With A Heating Rate of 127 W/cm.

Trial Functions	Zone of Application		Corresponding Axial Nodes
	Initial z (cm.)	Final z (cm.)	
1,7,8,9	0.0	0.375	1-6
3,7,8,9	0.375	0.5166	7-9
5,7,8,9	0.5166	2.10	10-24
7,8,9,10	2.10	3.40	25-30
9,10,11,12	3.40	7.00	31-39
11,12,13	7.00	51.0	40-65

Table III.4.4: Trial Function Schedule Used in the Repeat Analysis With A Heating Rate of 25 W/cm.

Trial Functions	Zone of Application		Corresponding Axial Nodes
	Initial z (cm.)	Final z (cm.)	
1,7,8,9	0.0	0.375	1-6
3,7,8,9	0.375	0.5166	7-9
5,7,8,9	0.5166	2.70	10-27
7,8,9,10	2.70	4.60	28-33
8,9,10,11	4.60	5.80	34-36
9,10,11,12	5.80	7.0	37-39
10,11,12,13	7.0	16.0	40-50
11,12,13	16.0	51.0	51-65

analysis with the 25 W/cm heating rate was reduced by one in the repeat analysis with that heating rate. The trial functions which were dropped at a given axial node were ones which would probably have contributed significantly to the temperature synthesis at that node. Trial function 6 (i.e., the first trial function obtained beyond the end of the contact) was used at axial nodes 1-30 in the original analysis with either heating rate and was not used at all in the repeat analyses, for instance. Trial function 6 would have contributed significantly to the temperature synthesis at axial nodes within the length of the contact (i.e., at nodes 1-10), in particular, because (1) only one of the trial functions used at each such node was obtained using r=0 boundary conditions found within the contact itself and, (2) the other trial functions used at those nodes were obtained using r=0 boundary conditions found further away from the contact.

The results of the repeat analyses were virtually identical to the results of the original analyses. With a heating rate of 127 W/cm, for instance, the peak heat flux on the calandria tube outer surface was 27.99 W/cm^2 in the repeat analysis compared to 27.95 W/cm^2 in the original. Similarly, the peak heat flux on the calandria tube outer surface was 7.55 W/cm^2 in the original analysis with a heating rate of 25 W/cm and 7.60 W/cm^2 in the repeat analysis with the same heating rate.

It should be mentioned that the heat flux at the 6 o'clock position on the calandria tube outer surface always peaked near the end of the contact instead of at $z=0$, in the 3-D analyses. That result is as one would expect, of course, and lends credibility to the analyses. Also,

although the data points used to draw the heat flux vs. axial position curves given in Figures III.4.6-III.4.8 are not shown on the figures, those points exhibited essentially no scatter about the curves. Since the smoothness with which synthesized surface heat fluxes vary in the axial direction is usually a good indicator of their accuracy when they are relatively high (in the author's experience), as mentioned in Section II.3, the lack of scatter just referred to provides additional evidence that the heat fluxes shown in Figures III.4.6-III.4.8 are accurate. In addition, it should be remembered that the heat fluxes obtained at the contact center in the synthesis analyses of the model contact problem were quite accurate even though they were more than 50% greater than the heat fluxes obtained at the same location in 2-D analyses, so it would be surprising if the heat fluxes calculated in the 3-D LOECC analyses, which were at most only 28% greater than their 2-D counterparts, were less accurate.

All of the input data used in the repeat analysis with a heat input rate of 25 W/cm (i.e., the analysis to which Table III.4.4 applies) are given in Part IV where the analysis is performed as a sample problem. A listing of the output for the analysis is provided as well.

CHAPTER III.5

CONCLUSIONS

The analyses described in Part III prove that realistic transient and/or non-linear heat transfer problems can be analyzed with the synthesis method and demonstrate that it is particularly useful as a method for assessing the significance of 3-D effects in problems where that significance is uncertain. Heat transfer problems involving large aspect ratio contacts should be included in that category since, as seen in the steady-state LOECC analyses described in the preceding section, the significance of 3-D effects in such problems depends not only on their geometry but also on the prescribed boundary conditions (and the time in a transient analysis). Since a synthesis analysis provides 2-D results in addition to the synthesized 3-D solution, the significance of axial conduction can be readily assessed by comparing the different solutions. Such comparisons were performed for transient and steady-state LOECC analyses in Sections III.4.2 and III.4.3 respectively, for instance.

Although the synthesis method loses accuracy and generally becomes increasingly sensitive to the choice of trial functions as axial conduction increases in importance, the results provided by the method should at the very least be useful in determining whether a less approximate solution would be worth obtaining. That information would be particularly valuable if the problem of interest would be extremely difficult and/or expensive to solve with a less approximate procedure, as would probably be the case for the steady-state non-linear contacting

pressure/calandria tube problem that was analyzed in Section III.4.3. That analysis, which was performed with a grid that contained almost 30,000 nodes, required approximately 15 minutes of CPU time to complete on Carleton University's Xerox Sigma 9 computer with the synthesis method and was performed completely in-core with a total memory requirement of less than 70,000 words. Of the roughly 15 minutes of CPU time required to solve the problem, approximately one-half of that time was expended in calculating the 2-D trial functions and one-half was used for the remainder of the analysis. Although the contacting pressure/calandria tube problem was only weakly three-dimensional in the cases analyzed, most solution procedures would not have been able to utilize that characteristic of the problem to facilitate its solution.

The three-dimensional linear transient LOECC analysis described in Section III.4.2 was completed with a grid containing over 20,000 nodes in approximately 52 minutes of CPU time, again without exceeding the high speed core capacity of the Xerox Sigma 9 computer on which the analysis was performed. Since 156 time steps were used in the analysis, approximately 20 seconds of CPU time were required per time step. Although the solution was obtained extremely rapidly considering the size of the problem, the analysis probably still took much longer than necessary to complete because all of the entries in the matrices $W^T Q$ and $W^T C W$ seen on the right-hand side of Equation (III.2.9a) were recalculated each time step. Those matrices do not change in a linear analysis, of course, and hence really only need to be calculated once at the beginning of such an analysis. The program described in Part IV, SYNCON, recalculates $W^T Q$ and $W^T C W$ at each time step for

generality, however, because $W^T Q$ always needs to be recalculated at each time step in a non-linear analysis and $W^T C W$ needs to be recalculated at each time step if material specific heats are permitted to be temperature dependent.

It was assumed in this study that the size and shape of the contact patch would be available for the thermal analysis of the problem of interest whereas in reality, both of those parameters may be difficult to estimate with any accuracy because of uncertainties in the contact load, the effective contact radii, etc. The sensitivity of the results of interest in a given problem to variations in contact-related parameters should be determined before much effort is expended in evaluating those parameters, however, because the results of interest may be quite insensitive to their values. This situation was observed in the thermal analysis of a pressure tube in contact with a calandria tube in a CANDU reactor, as discussed in Section III.4.2 and references [24] and [25].

III.5.1 Summary of Accomplishments

This thesis has described the results of heat transfer experiments performed with contacts of high ellipticity, such as might be produced by a bowed tube or cylinder contacting a relatively flat surface, and has considered numerical techniques which offer distinct advantages over other techniques for solving problems involving such contacts. It is in the emphasis that has been placed on contacts of high ellipticity and the numerical solution of realistic heat transfer problems involving

such contacts that this work differs from previous related studies which, almost without exception, dealt with circular contacts and methods for determining their thermal resistances.

This thesis has reported and described the first measurements of the thermal resistance of an isolated elliptical but non-circular contact as well as the first measurements of the resistance of an isolated contact (of any shape) with rough contact surfaces in the presence of a gas. If such measurements were performed previously, they were not reported in the heat transfer literature.

A finite-difference model formulated in the ellipsoidal coordinate system was developed to analyze heat transfer through an elliptical contact in the presence of a gas or in vacuum. It was shown that although the heat conduction equation in the ellipsoidal coordinate system possesses several singular points which preclude differencing it directly, a finite-difference solution can still be obtained by transforming the equation to one which does not have any singular points. The author is not aware of any previous attempt to develop a finite-difference model of heat transfer in the ellipsoidal coordinate system.

It was concluded by comparing resistances obtained experimentally and numerically for the model contact with smooth surfaces in vacuum that the contact's microscopic constriction resistance must have been negligible at even the lowest contact loads. It was found, moreover, that roughening one of the contact surfaces caused the resistance of the contact in vacuum to decrease, in direct contrast to the resistance increase that results

when nominally flat surfaces in contact are roughened. Explanations for these results were provided and it was shown that the steady-state resistance of the model contact in air or in vacuum could be predicted quite accurately, without recourse to contact-specific empirical data, by using techniques and models similar to those used by other researchers in the analysis of heat transfer through circular sphere-sphere and sphere-flat contacts.

The synthesis method was shown to be applicable to the analysis of heat flow between surfaces in contact over a highly elliptical area because of the strong two dimensional character of that heat flow. A unique algebraic formulation of the synthesis method was presented and compared to the formulation derived previously by Yasinsky. A deficiency of the latter was discussed. It was proven fairly rigorously that the numerical stability of the synthesis method could be improved by orthonormalizing the trial vectors before using them. The matrix formulation of the synthesis method derived by the author greatly facilitated that proof.

The rationale that was used to choose the trial functions for the synthesis analysis of the model contact problem was explained in considerable detail and guidelines for choosing trial functions were presented. The discussion of the method's application was far more detailed than any discussion found in the reactor physics or heat transfer literature. In fact, the author knows of no other instance of the synthesis method even being applied to a three-dimensional heat transfer analysis

since the simple test problems that Yasinsky performed to demonstrate the method's capabilities in references [114] and [115] were two-dimensional rather than three-dimensional.

The multigrid method was also applied successfully to the analysis of the model contact problem. Although all of the features of the method that were used in the analysis (e.g., partial refinement, composite grids, etc.) had been described previously in the literature, the specific algorithm design that was used was unique. The design was robust in the sense that it provided excellent convergence rates with boundary conditions of the second and third kinds. The relaxation of boundary points before interior points and the use of full-weighting of the fine grid residuals at the boundary nodes as opposed to half-weighting at the interior nodes were the features of the algorithm that were mainly responsible for its robustness. The description given in the thesis of the application of the multigrid method to the model contact problem should be very helpful to those wishing to apply the method to other heat transfer problems. There are still very few discussions of applications of the method in the engineering literature and none in the heat transfer literature that the author is aware of.

Methods for approximating a line contact whose edge (does not lie midway between two surface nodes in a finite-difference grid were described in the thesis and used to develop methods for approximating elliptical contacts with aspect ratios greater than three or four on a rectilinear grid. The multigrid model was used to show that the heat flow through such a contact can be calculated very accurately with the

proposed methods without using a high degree of lateral grid refinement around the contact. It was shown that worthwhile reductions in model size can be obtained by approximating a contact with one of the proposed methods.

Lastly, the manner in which the synthesis method can be applied to realistic non-linear and/or transient heat transfer problems was described and demonstrated by using it to perform simplified non-linear steady-state and linear transient analyses of heat transfer between contacting pressure and calandria tubes in a CANDU reactor fuel channel. Those capabilities of the method had not been discussed by Yasinsky. The analyses of the contacting pressure and calandria tube problem showed that axial conduction in the pressure and calandria tubes could be neglected without affecting any of the results of interest and that accurate knowledge of the contact's size and microscopic resistance was not required.

The results, techniques and models described in this thesis should be extremely useful to analysts wishing to solve heat transfer problems involving large aspect ratio contacts in CANDU reactor-related (and other) applications.

APPENDICES
(PART III)

APPENDIX III.A

STABILITY ANALYSIS FOR THE FORWARD-DIFFERENCE FORMULATION OF THE TIME-DEPENDENT SYNTHESIS METHOD

The stability characteristics of equation (III.2.8) can be determined by applying the matrix method of stability analysis [84] since it is a well-known result of that method that equation (III.2.8) is stable when the spectral radius of D_e , designated $\rho(D_e)$, is ≤ 1 . This is a necessary and sufficient condition for stability, but it only applies if D_e is symmetric or at least similar to a symmetric matrix, that is if a matrix P can be found such that the matrix S_D produced by the similarity transformation,

$$\begin{aligned} S_D &= P (I - \Delta t (W^T C W)^{-1} W^T K W) P^{-1} \\ &= P D_e P^{-1} \end{aligned} \quad \text{(III.A.1)}$$

is symmetric. The matrix D_e will generally not be symmetric in the present instance but it is easily shown that it must be similar to a symmetric matrix. To this end, note that $P D_e P^{-1}$ will be symmetric if $P B P^{-1}$ is symmetric, where $B = (W^T C W)^{-1} W^T K W$. Also, $W^T K W$ will be symmetric if K is symmetric and $W^T C W$ must be positive-definite because C , which is a diagonal matrix with positive diagonal elements, is itself positive-definite. Since $W^T C W$ is positive-definite, Cholesky's algorithm [106] can be used to find a lower triangular matrix L such that $W^T C W = L L^T$. Now,

$$(W^T C W)^{-1} W^T K W = (L L^T)^{-1} W^T K W = (L^T)^{-1} L^{-1} W^T K W$$

so setting $P=L^T$ gives

$$PBP^{-1} = L^{-1} W^T K W (L^{-1})^T \quad (\text{III.A.2})$$

Since,

$$[L^{-1} W^T K W (L^{-1})^T]^T = L^{-1} W^T K W (L^{-1})^T \quad (\text{if } K \text{ is symmetric})$$

by the transposition rule for matrix products, both PBP^{-1} and $PD_e P^{-1}$ are symmetric and D_e is, therefore, similar to a symmetric matrix. Moreover, PBP^{-1} is positive-definite if $W^T K W$ is positive-definite (as it should for reasons discussed in Section II.2.2), in which case both PBP^{-1} and B have positive eigenvalues, $\lambda(B)_i$; $i=1,2,\dots,I$ ($B \in R^{I \times I}$).

Since $\rho(D_e) = \max_i |\lambda(D_e)_i|$, where $\lambda(D_e)_i$ are the eigenvalues of D_e , the stability condition mentioned previously (i.e., that $\rho(D_e) \leq 1$) means that we must have,

$$|\lambda(D_e)_i| \leq 1 \quad \text{for } i = 1, 2, \dots, I$$

or equivalently

$$|1 - \Delta t \lambda(B)_i| \leq 1 \quad \text{for } i=1, 2, \dots, I \quad (\text{III.A.3})$$

for stability.

Equation (III.A.3) is satisfied when,

$$0 \leq \Delta t \lambda(B)_i \leq 2 \quad \text{for } i=1, 2, \dots, I \quad (\text{III.A.4})$$

Since $\lambda(B)_i \geq 0$ for all i , as mentioned previously, the inequality $\Delta t \lambda(B)_i \geq 0$ is satisfied when $\Delta t \geq 0$. The other inequality in equation (III.A.4) is satisfied when,

$$\Delta t \leq \frac{2}{\lambda(B)_i} \quad \text{for } i=1,2,\dots,I \quad (\text{III.A.5})$$

so equation (III.2.8) will be stable when,

$$\Delta t \leq \frac{2}{\rho(B)} \quad (\text{III.A.6})$$

since $\rho(B) = \max_i |\lambda(B)_i|$. The precise value of $\rho(B)$ would be expensive to compute in practice but the inequality,

$$\rho(B) \leq \max_{\ell=1,I} \left(\sum_{m=1}^I |B_{\ell m}| \right), \quad (\text{III.A.7})$$

follows from Gerschgorin's first theorem [148]. Substituting equation (III.A.7) into equation (III.A.6), we obtain

$$\Delta t \leq \frac{2}{\max_{\ell=1,I} \left(\sum_{m=1}^I |B_{\ell m}| \right)} \quad (\text{III.A.8})$$

as an easily computed, conservative estimate of the stability-limited time step for equation (III.2.8). Note that the inversion of the block diagonal matrix $W^T C W$ (needed to obtain B) can be easily accomplished by inverting the diagonal blocks.

The forward-difference formulation of the time-dependent synthesis method might be of some practical value if it could be used with reasonably large time steps. While the actual value of the stability-limited time step will depend on the trial functions, it is fairly easy to show that it is guaranteed to be greater than or equal to the stability-limited time step for the explicit time integration procedure applied to the original finite-difference equations (i.e., Equation (III.2.4) with $\theta=0$) when the columns of the matrix $C^{1/2}W$ are orthonormalized

by applying the Gram-Schmidt algorithm [87] (the matrix $C^{1/2}$ is a diagonal matrix obtained by taking the square roots of the nodal capacitances ($\rho_m C_m V(l)$ for node l) found on the diagonal of C). To this end, it can be shown by following the procedure used to determine the stability-limited time step for equation (III.2.8) that the relation,

$$\Delta t \leq \frac{2}{\rho(C^{-1}K)} \quad (\text{III.A.9})$$

defines the range of stable time steps for equation (III.2.4) where $\theta=0$.

Also, the matrix $C^{-1}K$ is similar to the matrix $C^{-1/2}KC^{-1/2}$ since $PC^{-1}KP^{-1} = C^{-1/2}KC^{-1/2}$ when $P=C^{1/2}$, so equation (III.A.9) can be rewritten as

$$\Delta t \leq \frac{2}{\rho(C^{-1/2}KC^{-1/2})} \quad (\text{III.A.10})$$

since $C^{-1}K$ and $C^{-1/2}KC^{-1/2}$ have the same eigenvalues. Since $C^{-1/2}KC^{-1/2}$ is a symmetric matrix (assuming K is symmetric), the identity

$$\rho(C^{-1/2}KC^{-1/2}) = \|C^{-1/2}KC^{-1/2}\|_2 \quad (\text{III.A.11})$$

applies [106]. Similarly,

$$\rho(B) = \|L^{-1}W^TKW(L^{-1})^T\|_2 \quad (\text{III.A.12})$$

since B and $L^{-1}W^TKW(L^{-1})^T$ are similar (see equation (III.A.2)) and the latter matrix is symmetric. Since $L^{-1}W^TKW(L^{-1})^T = L^{-1}W^TC^{1/2}C^{-1/2}KC^{-1/2}C^{1/2}W(L^{-1})^T$ the inequality,

$$\begin{aligned} \rho(B) &= \|L^{-1}W^TKW(L^{-1})^T\|_2 \leq \|L^{-1}\|_2 \|W^TC^{1/2}\|_2 \|C^{-1/2}KC^{-1/2}\|_2 \|C^{1/2}W\|_2 \\ &\quad \cdot \|(L^{-1})^T\|_2 = \|L^{-1}(L^{-1})^T\|_2 \|W^TC^{1/2}C^{1/2}W\|_2 \rho(C^{-1/2}KC^{-1/2}) \\ &= \|(W^TCW)^{-1}\|_2 \|W^TCW\|_2 \rho(C^{-1}K) \end{aligned} \quad (\text{III.A.13})$$

follows from the consistency property of the 2-norm, the fact that $||YY^T||_2 = ||Y^TY||_2 = ||Y||_2^2 = ||Y^T||_2^2$,* and equations (III.A.9) - (III.A.12). If the columns of $C^{1/2}W$ are orthonormal, then $W^TCW = (W^TCW)^{-1} = I$ (where $||I||_2=1$) and equation (III.A.13) simplifies to the inequality,

$$\rho(B) \leq \rho(C^{-1}K) \quad (\text{III.A.14})$$

which indicates that the stability-limited time step defined by equation (III.A.5) is guaranteed to be equal to or larger than that defined by equation (III.A.9).

When the columns of $C^{1/2}W$ are orthonormalized to produce a new matrix, say X , then the equation,

$$W' = C^{-1/2}X$$

defines the new "trial matrix", designated W' (i.e., W' replaces W).

It can be shown that the new trial vectors defined by the columns of W' span the same space as the original trial vectors (as they must if this modification is to be useful), since a vector Z_2 satisfying the relationship,

$$C^{1/2}WZ_1 = XZ_2 \quad (\text{III.A.15})$$

can be found for any Z_1 because the column space of $C^{1/2}W$ is the same as that of X . Hence the relationship,

$$WZ_1 = C^{-1/2}XZ_2 \quad (\text{III.A.16})$$

is satisfied for any Z_1 (i.e., any linear combination of the columns of W) so the column space of $C^{-1/2}X$ includes that of W . Equation (III.A.16)

* For any real matrix Y .

is also satisfied for any Z_2 so $C^{-1/2}X$ and W must have the same column space.

If the columns of $C^{1/2}W$ are orthonormal, equation (III.2.8) simplifies to the expression,

$$Z^{t+1} = [I - \Delta t W'^T K W'] Z^T + \Delta t W'^T Q \quad (\text{III.A.17})$$

APPENDIX III.B

USING KIRCHOFF'S TRANSFORMATION TO LINEARIZE TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY IN A SYNTHESIS ANALYSIS

The thermal conductivity (k_s) of the pressure and calandria tube wall material (assumed the same for both components) can be modelled by applying Kirchoff's transformation, written as $\phi = \int_{T_r}^T k_s(\hat{T}) d\hat{T}$. Letting $T_r = 0$ (giving $\phi = \int_0^T k_s d\hat{T}$) and $k_s = e + fT$, the finite-difference equation for a node i, j, k on the pressure tube outer surface outside of the contact patch, given formerly by equation (III.3.16), can be rewritten as:

$$\begin{aligned} & -K(i,j,k)_{i-1} \phi(i-1,j,k) - K(i,j,k)_{j+1} \phi(i,j+1,k) \\ & -K(i,j,k)_{j-1} \phi(i,j-1,k) - K(i,j,k)_{k+1} \phi(i,j,k+1) - K(i,j,k)_{k-1} \phi(i,j,k-1) \\ & + (\Sigma K) \phi(i,j,k) + B_j \xi (T(i,j,k) - T(i+1,j,k)) \\ & + \frac{B_j X}{2} (T(i,j,k)^2 - T(i+1,j,k)^2) + R_s (T(i,j,k)^4 - \\ & T(i+1,j,k)^4) = \epsilon(i,j,k) = 0 \end{aligned} \tag{III.B.1}$$

where the parameters $K(i,j,k)_{i-1}$, $K(i,j,k)_{j+1}$, etc. are now internodal shape factors instead of internodal conductances as they were previously. The internodal shape factors are simply the conductances with the thermal conductivity factor omitted (i.e., set to unity).

Equation (III.B.1) is obviously equivalent to equation (III.3.16) when k_s is constant since $\phi = k_s T$ in that case. Its validity in the general case when k_s is not constant is easily established [152] and hence will not be proven here. An equation analogous to equation (III.B.1) for a node i, j, k on the calandria tube inner surface can be obtained by interchanging the subscripts $i-1$ and $i+1$ in equation (III.B.1).

Since $k_s = e + fT$,

$$\phi = \int_0^T k_s dT = eT + \frac{fT^2}{2} \quad (\text{III.B.2})$$

and

$$T = \frac{-e + \sqrt{e^2 + 2f\phi}}{f} \quad (\text{III.B.3})$$

the nodal temperatures in equation (III.B.1) (i.e., $T(i, j, k)$ and $T(i+1, j, k)$) can be expressed in terms of the corresponding values of ϕ . The values of ϕ at the nodes in the grid are the unknowns in the solution vector so the non-zero entries in the Jacobian matrix ($J(\phi^n)$) for a node i, j, k on the pressure tube outer surface, given formerly by equations (III.3.18 - III.3.24), must be re-expressed as,

$$\frac{\partial \epsilon(i, j, k)}{\partial \phi(i-1, j, k)} = -K(i, j, k)_{i-1} \quad (\text{III.B.4})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial \phi(i, j+1, k)} = -K(i, j, k)_{j+1} \quad (\text{III.B.5})$$

$$\frac{\partial \epsilon(i, j, k)}{\partial \phi(i, j-T, k)} = -K(i, j, k)_{j-1} \quad (\text{III.B.6})$$

$$\frac{\partial \epsilon(i,j,k)}{\partial \phi(i,j,k+1)} = -K(i,j,k)_{k+1} \quad (\text{III.B.7})$$

$$\frac{\partial \epsilon(i,j,k)}{\partial \phi(i,j,k-1)} = -K(i,j,k)_{k-1} \quad (\text{III.B.8})$$

$$\begin{aligned} \frac{\partial \epsilon(i,j,k)}{\partial \phi(i+1,j,k)} \Big|_{\phi'^n} &= \frac{dT(i+1,j,k)}{d\phi(i+1,j,k)} \frac{\partial \epsilon(i,j,k)}{\partial T(i+1,j,k)} \Big|_{\phi'^n} \\ &= \left(\frac{1}{k_s} \otimes T'^n(i+1,j,k) \right) \frac{\partial \epsilon(i,j,k)}{\partial T(i+1,j,k)} \Big|_{T'^n} \end{aligned} \quad (\text{III.B.9})$$

$$\begin{aligned} \frac{\partial \epsilon(i,j,k)}{\partial \phi(i,j,k)} \Big|_{\phi'^n} &= \frac{dT(i,j,k)}{d\phi(i,j,k)} \frac{\partial \epsilon(i,j,k)}{\partial T(i,j,k)} \Big|_{\phi'^n} \\ &= \left(\frac{1}{k_s} \otimes T'^n(i,j,k) \right) \frac{\partial \epsilon(i,j,k)}{\partial T(i,j,k)} \Big|_{T'^n} \end{aligned} \quad (\text{III.B.10})$$

where $\phi' = WZ$ and the trial functions stored as columns in W provide nodal values of ϕ instead of T . The terms

$$\frac{\partial \epsilon(i,j,k)}{\partial T(i+1,j,k)} \Big|_{T'^n} \quad \text{and} \quad \frac{\partial \epsilon(i,j,k)}{\partial T(i,j,k)} \Big|_{T'^n}$$

in equations (III.B.9) and (III.B.10) are given by equations (III.3.23) and (III.3.24) respectively and the terminology $k_{s \otimes T'^n}(i,j,k)$ (or

$k_s @ T^n(i+1, j, k)$ indicates that $k_s (=e+fT)$ is to be evaluated at the temperature $T^n(i, j, k)$ (or $T^n(i+1, j, k)$). The derivatives corresponding to equations (III.B.4) - (III.B.10) for a node i, j, k on the calandria tube inner surface can be obtained by interchanging the subscripts $i-1$ and $i+1$ in equations (III.B.4) and (III.B.9).

The application of Kirchoff's transformation causes the finite-difference equations for nodes on the calandria tube outer surface and nodes within the contact patch on the pressure tube outer and the calandria tube inner surfaces to become non-linear even though they were linear originally. The residual equation for a node i, j, k on the pressure tube outer surface (but within the contact patch) can be written as,

$$\begin{aligned}
 & -K(i, j, k)_{i-1} \phi(i-1, j, k) - K(i, j, k)_{j+1} \phi(i, j+1, k) \\
 & -K(i, j, k)_{j-1} \phi(i, j-1, k) - K(i, j, k)_{k+1} \phi(i, j, k+1) \\
 & -K(i, j, k)_{k-1} \phi(i, j, k-1) + (\Sigma K) \phi(i, j, k) \\
 & + h_c A_s(i, j, k) (T(i, j, k) - T(i+1, j, k)) = \epsilon(i, j, k) = 0 \quad \text{(III.B.11)}
 \end{aligned}$$

where h_c is the contact conductance within the patch. Except for

$$\frac{\partial \epsilon(i, j, k)}{\partial \phi(i+1, j, k)} \quad \text{and} \quad \frac{\partial \epsilon(i, j, k)}{\partial \phi(i, j, k)},$$

the non-zero entries in the Jacobian matrix ($J(\phi^n)$) for this residual equation are the same as those given previously for equation (III.B.1).

The derivatives

$\frac{\partial \epsilon(i,j,k)}{\partial \phi(i+1,j,k)}$ and $\frac{\partial \epsilon(i,j,k)}{\partial \phi(i,j,k)}$ for equation (III.B.11) can be

written as

$$\begin{aligned} \left. \frac{\partial \epsilon(i,j,k)}{\partial \phi(i+1,j,k)} \right|_{\phi,n} &= \frac{dT(i+1,j,k)}{d\phi(i+1,j,k)} \left. \frac{\partial \epsilon(i,j,k)}{\partial T(i+1,j,k)} \right|_{\phi,n} \\ &= - \frac{h_c A_s(i,j,k)}{k_s \theta T^n(i+1,j,k)} \end{aligned} \quad (III.B.12)$$

and

$$\begin{aligned} \left. \frac{\partial \epsilon(i,j,k)}{\partial \phi(i,j,k)} \right|_{\phi,n} &= \frac{dT(i,j,k)}{d\phi(i,j,k)} \left. \frac{\partial \epsilon(i,j,k)}{\partial T(i,j,k)} \right|_{\phi,n} \\ &= \frac{h_c A_s(i,j,k)}{k_s \theta T^n(i,j,k)} \end{aligned} \quad (III.B.13)$$

For a node on the pressure tube outer surface within the contact patch, equations (III.B.12) and (III.B.13) replace equations (III.B.9)

and (III.B.10) given previously for a node outside of the patch.

Equations (III.B.4) - (III.B.8) and (III.B.11) - (III.B.13) apply to a node i,j,k on the calandria tube inner surface within the contact patch

if the subscripts $i+1$ and $i-1$ are interchanged. Equations (III.B.4) -

(III.B.8), (III.B.11) and (III.B.13) also apply to a node i,j,k on the

calandria tube outer surface if the moderator temperature, T_m , is

substituted for $T(i+1,j,k)$ and h_f , the calandria tube/moderator heat

transfer coefficient, is substituted for h_c . Equation (III.B.12) is not

needed at a calandria tube outer surface node.

Entries in the left-hand coefficient matrix, $J(\phi^n)$, for the n th iteration of the Newton-Raphson solution for ϕ can be obtained by replacing the nodal temperatures in the general expressions given previously for $D_{k,k-1}(ii,jj)$, $D_{k,k}(ii,jj)$ and $D_{k,k+1}(ii,jj)$ (i.e., equations (III.3.25) - (III.3.27)) by the nodal values of ϕ . The derivatives with respect to the nodal temperatures become derivatives with respect to the nodal values of ϕ , of course, as defined by equations (III.B.4) - (III.B.10) and equations (III.B.12) - (III.B.13) for nodes with non-linear finite-difference equations. Equations such as equations (III.B.4) - (III.B.8) define the partial derivatives with respect to ϕ for linear residuals.

The foregoing procedure provides the left-hand coefficient matrix for the system of equations defining ϕ^{n+1} . Entries in the right-hand side vector R can be determined in a manner similar to that described previously for the analysis with constant thermal conductivity and non-linear gap conductances. The expression for the ii th entry in the subvector R_k , given previously by equation (III.3.32) for the analysis with constant thermal conductivity, can be rewritten for a variable thermal conductivity analysis as,

$$R_k(ii) = \sum_{S_1+S_2+S_3} [\alpha(i,j,k) - \epsilon(i,j,k)] \phi_{ij}(i,j) + S_4+S_5+S_6 \quad (III.B.14)$$

where S_1 , S_2 , S_3 , S_4 , S_5 and S_6 are the sets of nodes defined below.

S_1 - Pressure tube inner surface nodes

S_2 - Pressure tube outer surface nodes outside of the contact patch

S_3 - Calandria tube inner surface nodes outside of the contact patch

S_4 - Calandria tube outer surface nodes

S_5 - Pressure tube outer surface nodes within the contact patch

S_6 - Calandria tube inner surface nodes within the contact patch

Node sets $S_1 - S_4$ are the same as those used previously in the constant thermal conductivity analysis.

The finite-difference equations for nodes on S_1 contain constant terms arising from the prescribed heat flux (q'') in a LOECC analysis so,

$$\alpha(i,j,k) - \epsilon(i,j,k) = q'' A_s(i,j,k) \text{ for } i,j,k \in S_1 \quad (\text{III.B.15})$$

For $i,j,k \in S_2$ we have,

$$\begin{aligned} \alpha(i,j,k) - \epsilon(i,j,k) = & \left(\frac{B_j \epsilon + B_j x T'^n(i,j,k) + 4R_s T'^n(i,j,k)^3}{k_s \theta T'^n(i,j,k)} \right) \phi'^n(i,j,k) \\ & - \left(\frac{B_j \epsilon + B_j x T'^n(i+1,j,k) + 4R_s T'^n(i+1,j,k)^3}{k_s \theta T'^n(i+1,j,k)} \right) \phi'^n(i+1,j,k) \\ & - B_j \epsilon (T'^n(i,j,k) - T'^n(i+1,j,k)) \\ & - \frac{B_j x}{2} (T'^n(i,j,k)^2 - T'^n(i+1,j,k)^2) \\ & - R_s (T'^n(i,j,k)^4 - T'^n(i+1,j,k)^4) \quad (\text{III.B.16}) \end{aligned}$$

Equation (III.B.16) can be applied to nodes on the calandria tube inner surface outside of the contact patch (i.e., to $i,j,k \in S_3$) by substituting the subscript $i-1$ for $i+1$.

For $i, j, k \in S_4$,

$$\begin{aligned} \alpha(i, j, k) - \epsilon(i, j, k) &= \frac{h_f A_s(i, j, k)}{k_s \theta T^n(i, j, k)} \phi'^n(i, j, k) \\ &\quad - h_f A_s(i, j, k) (T'^n(i, j, k) - T_\infty) \end{aligned} \quad (\text{III.B.17})$$

while for $i, j, k \in S_5$,

$$\begin{aligned} \alpha(i, j, k) - \epsilon(i, j, k) &= \frac{h_c A_s(i, j, k)}{k_s \theta T^n(i, j, k)} \phi'^n(i, j, k) \\ &\quad - \frac{h_c A_s(i, j, k)}{k_s \theta T^n(i+1, j, k)} \phi'^n(i+1, j, k) \\ &\quad - h_c A_s(i, j, k) (T'^n(i, j, k) - T'^n(i+1, j, k)) \end{aligned} \quad (\text{III.B.18})$$

Equation (III.B.18) can be applied to nodes on the calandria tube inner surface within the contact patch (i.e., to $i, j, k \in S_6$) by substituting the subscript $i-1$ for $i+1$.

Substituting equations (III.B.15) - (III.B.18) into equation (III.B.14) yields the desired expression for $R_k(ii)$. As in the constant thermal conductivity analysis, different trial functions $\{\phi_n\}$ can be used at different axial mesh points.

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HEAT TRANSFER BETWEEN CONTACTING NON-
CONFORMING SURFACES - EXPERIMENTAL
AND NUMERICAL STUDIES WITH SPECIAL
EMPHASIS ON PROBLEMS INVOLVING
CONTACT PATCHES OF HIGH ELLIPTICITY

PART IV

POOR PRINT
Epreuve illisible

SYNCON - A Fortran IV Program for Applying the Finite-Difference and
Synthesis Methods to Two and Three-Dimensional Heat Transfer
Problems in Cylindrical Geometries with Special Emphasis on
Problems Involving Parallel Cylinders and Tubes in Partial
or Complete Axial Contact

Programmed by: T.C. Currie, Carleton University

Implemented on: Xerox Sigma 9 computer with Fortran IV-H compiler

Memory Requirement: ~ 14,000 words (for program only - excludes arrays)

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1.0 GENERAL DESCRIPTION OF PROGRAM STRUCTURE AND CAPABILITIES

The Fortran IV program SYNCON can be used to solve heat transfer problems in cylindrical geometries in two or three space dimensions.

Two-dimensional r - θ - z analyses are performed using either;

- 1) the finite-difference method if the problem size is not excessive (for the available computer installation), or;
- 2) the synthesis method if a pure finite-difference solution would be impossible or impractical to obtain.

Approximate three-dimensional solutions are obtained in the synthesis method by linearly combining two-dimensional (r - θ) solutions, obtained from the problem of interest, in the third (z) dimension, as described in Parts II and III.

SYNCON can be used to perform either steady-state analyses or transient analyses with forward-differenced time derivatives. The program can also be used in its present form to solve certain specific problems, namely;

- 1) that of determining the resistance of an elliptical contact patch formed between two contiguous semi-cylindrical surfaces with orthogonal axes of curvature, either in a gas or in vacuum. This problem is referred to as the "model contact problem".
- 2) that of analysing heat transfer between contacting pressure and calandria tubes in a CANDU nuclear reactor fuel channel following a loss-of-coolant accident (LOCA) combined with a loss-of-emergency core coolant (LOECC) or, at least, impaired ECC flow. The pressure tube overheats and sags onto the calandria tube in this accident

scenario. The program does not model heat transfer in the fuel bundle so the user must specify the fuel bundle-to-pressure tube heat flux at each node on the pressure tube inner surface. This problem is referred to as a "LOECC analysis".

- 3) that of determining the temperature distribution in a nuclear fuel element with a non-uniform heat transfer coefficient prescribed over its exterior. Contact-related boundary conditions can be considered, as can radiative heat transfer from the fuel element surface to the surroundings. The surroundings are assumed to be at a fixed (specified) temperature.
- 4) that of evaluating the temperature distribution in a tube exchanging heat, by convection, with an internal coolant at a fixed temperature. The tube can be modelled with an isothermal line contact on its surface, such as would be produced by contact with a similar (parallel) tube.

General heat transfer problems involving tubes or solid cylinders (with or without contact-related boundary conditions) can be analysed with SYNCON, although modification of the program to incorporate specific features of the problem of interest will likely be required in most instances. The program could be easily modified to handle problems in rectangular geometries by replacing the internodal conductance expressions currently used, which are valid for the cylindrical coordinate system, by equivalent expressions appropriate to the cartesian coordinate system. Alternately, an object with a rectangular cross-section could be approximated as a hollow cylindrical

segment with large inner and outer radii and a small included angle.

The user of SYNCON is required to select one of three different grid options, specifically;

- a) a simple grid in which all of the mesh lines are continuous, as shown in Fig. 1.1 (for an r - θ grid). In this grid, $\Delta r=f(r)$, $\Delta \theta=g(\theta)$ and $\Delta z=h(z)$ and no restrictions are placed on the functional relationships f , g and h .
- b) a more complicated grid in which circumferential mesh lines can be discontinuous, as shown in two-dimensions in Fig. 1.2. In this grid, $\Delta r=f(r, \theta)$, $\Delta \theta=g(\theta)$ and $\Delta z=h(z)$, but the functional relationships f and g cannot be specified completely independently because the angle β shown in Fig. 1.2 must be less than 90° (i.e., the criterion $r\Delta\theta > \Delta r_1$ must be satisfied) at each fine/coarse mesh transition for reasons that are discussed in Appendix A. Also, the radial mesh spacing (Δr) can only be doubled at each transition (i.e., $\Delta r_2 = 2\Delta r_1$) and Δr must get larger, not smaller, as r decreases (as required for an external contact). Basically, the grid must be similar in structure to that shown in Fig. 1.2.
- c) a node-conserving grid in which both radial and circumferential mesh lines can be discontinuous, as shown in Fig. 1.3. In this grid, $\Delta r=f(r, \theta)$, $\Delta \theta=g(r, \theta)$, $\Delta z=h(z)$ and, as in the case of mesh b), the angle β must be acute at each fine-coarse mesh transition, whether it be in the r or θ direction. Mesh spacings can only be doubled at each transition.

Fig. 1.1 Graded Grid with $\Delta r = f(\theta)$, $\Delta \theta = g(\theta)$

Fig. 1.2 Graded Grid with $\Delta r = f(r, \theta)$, $\Delta \theta = g(\theta)$

Fig. 1.3 Graded Grid with $\Delta r = f(r, \theta)$, $\Delta \theta = g(r, \theta)$.

Grid option c) is not necessarily the best choice for all contact problems. A problem involving a contact on the external surface of a thin-walled tube could be analysed effectively with a type b) grid, for example, because such a grid would contain relatively few "unnecessary" nodes, that is nodes that could be eliminated by switching to a type c) grid. Heat flow through a contact on the surface of a cylinder with a solid core (eg., a fuel element) would be analysed most efficiently with grid option c), however, because that option would permit the small circumferential mesh spacing ($\Delta \theta$)

used near the contact to be increased at radii remote from the contact.

The allowable grid options are restricted for certain types of problems. In particular, an analysis of the heat flow in a complete cylinder, in contrast to a cylindrical segment such as that shown in Fig. 1.2 or Fig. 1.3, must be performed using grid option a) because the coding required to perform the analysis with grid types b) and c) is not included in the program. Also, heat transfer between contacting pressure and calandria tubes in a CANDU reactor fuel channel must be analysed using grid option c). Grid option restrictions do not exist for problems other than these.

The coding exists in SYNCON to obtain finite-difference solutions to transient and steady-state problems in which the thermal conductivity of the region being modelled varies with temperature, but other material thermal properties (i.e., specific heat, density and surface emissivity) are assumed constant for simplicity. Both the thermal conductivity of the pressure/calandria tube material* and the thermal conductivity of the gas between the pressure and calandria tubes in a CANDU reactor are allowed to vary with temperature in a transient or steady-state LOECC analysis performed using either the finite-difference/synthesis method or the finite-difference method alone, but this is the only

*It is assumed that a single thermal conductivity vs temperature relationship applies to both the pressure and calandria tube materials, which are similar zirconium alloys in all modern CANDU reactors.

type of synthesis analysis in which temperature-dependent thermal conductivities can be accommodated.

The gas and solid conductivities are linearized in a finite-difference pressure-calandria tube heat transfer analysis by recomputing them in each iteration or time step using the most recent nodal temperatures. In the case of a synthesis analysis, the temperature-dependent gas conductivity is linearized by applying the Newton-Raphson (N-R) method and Kirchoff's transformation is used to linearize the thermal conductivity of the pressure/calandria tube material (if temperature dependent), as described in Appendix III-B.

The use of Kirchoff's transformation is very beneficial in large 3-D problems solved with the synthesis method because the finite-difference equations at interior nodes, which are initially non-linear if the thermal conductivity of the region being modelled varies with temperature, become linear when the transformation is applied. The transformation causes the boundary node equations to become non-linear even if they are initially linear (which they usually aren't), but the boundary node equations are usually far fewer in number than the interior node equations and hence are less expensive to update with new temperatures.

Radiative heat transfer is linearized either by recomputing the radiative heat transfer coefficient in each iteration or time step or by applying the N-R method. The former method is used in two-dimensional finite-difference LOECC analyses to linearize radiative heat transfer between the pressure and calandria tubes because it

preserves the symmetry of the conductance matrix whereas the standard N-R method does not. This consideration is important because Cholesky decomposition (1), which cannot be applied to a non-symmetric matrix, is used to solve the finite-difference equations for the nodal temperatures. Finite-difference analyses involving radiation to the surroundings at a fixed temperature are linearized using the N-R method, however, because all of the non-linear terms encountered in those analyses appear on the diagonal of the Jacobian matrix and in the right-hand side vector. Thus the Jacobian matrix is symmetric and Cholesky decomposition can be used to solve for the nodal temperatures.

Radiative heat transfer in three-dimensional synthesis analyses is always linearized by applying the N-R method because, while the Jacobian matrix for the non-linear equations defining the mixing functions will be non-symmetric if the corresponding Jacobian for the finite-difference equations is non-symmetric, a non-symmetric band solver (i.e., an equation solver which can handle non-symmetric, banded coefficient matrices) is used to obtain the mixing coefficients in those analyses. With this equation solver, it does not matter whether the Jacobian is symmetric or not.

As mentioned previously, Cholesky decomposition is used in SYNCON to solve for the nodal temperatures in a finite-difference analysis. Since this method is very efficient for solving multiple systems of equations with a common coefficient matrix because the coefficient matrix only has to be factored once, it can be used to

advantage in solving the finite-difference equations for an implicitly formulated linear transient heat transfer problem because the coefficient matrix for those equations does not change unless the time increment (Δt) is changed. Hence the coefficient matrix (K) is factored in SYNCON at the start of such an analysis, or whenever the time increment is altered, but otherwise only the right-hand side vector (Q) is changed (using the most recent temperature vector, T^t) at each time step. The nodal temperatures at time $t+\Delta t$ (i.e., T^{t+1}) are obtained from the updated right-hand side vector at each time step by performing forward and backward substitutions in the normal manner.

As a three-dimensional synthesis solver, SYNCON offers advanced capabilities in several areas. First, three-dimensional steady-state analyses can be performed using axially discontinuous trial functions, that is two-dimensional trial functions which are not used at all axial mesh points in the region of interest but rather are only used where they are likely to be useful. Also, the trial functions used in transient analyses can be both axially (spatially) discontinuous and discontinuous in time. Trial functions which are discontinuous in time can be dropped or added at any time during the transient. Certain trial functions may only be useful at the start of a transient, for instance, in which case they can be dropped after that period.

It is, in general, very difficult to obtain accurate synthesized solutions to transient problems if only steady-state two-dimensional solutions are used as trial functions, so SYNCON can be instructed to use temperature distributions obtained at user-specified times in

a two-dimensional transient analysis as trial functions in a synthesis analysis. Any number of these "transient trial functions" can be recorded during a single two-dimensional transient analysis. Several two-dimensional transient analyses, performed at different axial locations in the region of interest, would normally be required to produce enough trial functions for a single three-dimensional transient synthesis analysis, of course.

The coding exists in the program to orthonormalize the trial vectors in a linear, steady-state analysis before they are used but the trial vectors cannot be orthonormalized in transient and/or non-linear analyses. The program would have to be modified to permit this to be done. Orthonormalizing the trial vectors before using them improves the numerical stability of the synthesis method, as discussed in Sec II.2.4.

This completes the initial discussion of the capabilities of SYNCON. Subsequent sections of this document give detailed descriptions of the calculations performed in the program as well as instructions for its use. A summary of the input variables required by SYNCON is given in Table 2.1 and sample "resistance of contact" and pressure tube-calandria tube heat transfer analyses are performed in Sec 5. The basic finite-difference equations used in the program at different types of nodes in graded and ungraded grids are derived in Appendix A. Listings of SYNCON, miscellaneous routines and input/output files for the sample problems can be found after the appendices. The listings are referenced by number as noted in the table of contents.

2.0 DETAILED PROGRAM DESCRIPTION AND FLOWCHART

A flowchart describing the computations performed in SYNCON is given in Fig. 2.1. When possible the name of the routine in which the task referred to in a particular box (or diamond) is performed is given in the box, usually at the bottom. Routine names are capitalized and underlined to distinguish them from variable names, which are only capitalized.

At the beginning of any job, regardless of whether it is a pure finite-difference or a finite-difference/synthesis analysis, several variables which describe the job in general terms are read by lines 15.2-16.4 of the main program*, as seen in box 1 of the program flowchart. These variables, which are listed as variables 1a-30a in Table 2.1a; indicate the dimensionality of the problem (2 or 3-D), the type of problem, if specific (eg. resistance of contact analysis, LOECC analysis), the solution technique to be used (i.e., the synthesis method, which uses the finite-difference technique to obtain the trial functions, or the finite-difference method by itself) and the type of grid to be used in the analysis. Grid types a), b) and c) are obtained with NGROPT=0, 1 and 2 respectively, where NGROPT is variable 2a in Table 2.1a. For specific problems, geometrical data (eg., tube diameters, contact dimensions, etc.) are also read at this time.

*Unless otherwise noted, line numbers referred to in this section are the right-hand line numbers in Listing 1.

The program next calls some of the routines listed at the bottom of box 2 to define the r - θ grid geometry, specifically the r - θ coordinates of each node in the grid, the radial subdivisions before and after each radial node (Δr_a and Δr_b), the circumferential subdivisions before and after each circumferential node ($\Delta \theta_a$ and $\Delta \theta_b$) and, finally, variables needed to determine which nodes in the grid communicate with each other. The r - θ grid definition routines called at this time depend on the type of grid being used (i.e., a), b) or c)), as discussed in Sec 3.0 where the grid definition process is explained in detail. The grid type also affects the r - θ grid definition variables that the user must provide, which are summarized in Tables 2.1b-2.1f.

Detailed descriptions of the grid definition variables are given in Sections 3.1-3.3. If the analysis being performed is a LOECC analysis, some alterations have to be made to the r - θ grid defined in box 2, as noted in box 4 and as explained in detail in Sec 3.4.

If the information read in box 1 indicates that the analysis being performed is a 3-D finite-difference analysis, in contrast to a 2-D finite-difference analysis or a 3-D finite-difference/synthesis analysis, the program calls subroutine DZ to read axial grid definition data and to define the axial grid geometry, as noted in box 8. Two-dimensional finite-difference solutions (trial functions) have to be obtained in a 3-D finite difference/synthesis analysis before the 3-D computations can be performed so the axial grid geometry is not defined for such an analysis at this time. The computations performed in subroutine DZ are described in detail in Sec 3.1.3 and the input

variables read by that subroutine are summarized in Table 2.1g. As mentioned previously, those variables are not required at this time in a 3-D synthesis analysis.

With the grid geometry defined, finite-difference solutions to specific problems can now be obtained using that grid. These solutions may or may not be used later as trial functions in a synthesis analysis and their ultimate use is not important at this time. Input variables describing the first specific problem, which will likely also be the last unless a synthesis analysis is being performed, are read (as noted in box 11) in subroutine DATA1, which is described in detail in Sec 4.1.2. The input variables read by subroutine DATA1 are summarized in Table 2.1h. Subroutine DATA1 reads and/or calculates the $r-\theta$ boundary conditions at all of the axial nodes in the grid, of which there will only be one if the specific problem being analyzed at this time is 2-D. In a contact analysis (i.e., resistance of contact or LOECC analysis), subroutine DATA1 calls subroutine HGAS to evaluate gap sizes, and then the corresponding gas conductances, in regions near to the contact (where it is assumed that Hertz's theory is applicable). In a LOECC analysis, subroutine DATA1 uses formulas derived in Appendix B to calculate the gap between a sagged pressure tube and the calandria tube which encloses it as a function of angular and axial position in regions remote from the contact.

In a 2-D LOECC analysis, the user specifies the axial position in the fuel channel at which a solution is desired and subroutines DATA1 and HGAS calculate the appropriate $r-\theta$ boundary conditions.

Several such solutions, obtained at different axial positions in a fuel channel, would normally be used as trial functions in a synthesis analysis. A similar procedure is followed in analyses of the model contact problem where, once again, subroutines DATA1 and HGAS evaluate the $r-\theta$ boundary conditions at different axial locations in the contact specimen.

Once the input variables for the specific problem being analysed have been read in by subroutine DATA1, the program proceeds to obtain the problem's solution. If the problem in question is a transient one, which will be the case if TRANS=1 (TRANS is input variable 1b), boxes 13-18 (which correspond to lines 23.4-25.4 in the main program) are executed whereas, if TRANS $\neq$ 1, the program proceeds directly to box 19 and sets the axial node index (K)=1. The program actually uses three nested DO loops to sweep through all of the radial, circumferential and axial nodes in the grid, identified by the indices J, I and K respectively, so the assignment indicated in box 19 is really performed the first time the outer DO loop, which sweeps through the axial nodes (K), is executed. The DO loop for the axial nodes is found in the main program between lines 29.4 and 32.1 while the DO loops for the radial (J) and circumferential nodes (I) are found in subroutine GRQPT0 if NGROPT=0, subroutine GRQPT1 if NGROPT=1, or subroutine GRQPT2 if NGROPT=2. If the problem being performed is only 2-D, which will be the case if the number of axial nodes (KX)=1, the DO loop in the main program for the axial node index (K) is only executed once.

After the DO loop for the axial node index (K) has assigned a value to K, the program proceeds to subroutine GROPT0, GROPT1 or GROPT2 depending on whether the specified grid type is a), b) or c), as noted above. The DO loops which sweep through the radial (J) and circumferential (I) nodes are executed in those routines. Some of the combinations of I and J may represent nodes that are not actually included in the finite-difference model if grid type b) or c) is being used, as discussed in greater detail in Sects 3.2 and 3.3, so the nodes that are actually used are identified and numbered sequentially in the order in which they are visited by the nested DO loops for I and J. A node number indicator, L, which is set=1 at the first node, is incremented by 1 each time the values of I and J correspond to a node that is actually used.

Subroutines GROPT0, GROPT1 and GROPT2 perform the computations needed to determine whether a particular combination of I and J should be included in the finite-difference model or excluded. These subroutines also determine whether a given node I, J is a normal node of the type found at orthogonal grid line intersections or a node at which either the radial or circumferential mesh structure is altered. Each combination of I and J represents a node of the former type when grid type a) (obtained with NGROPT=0) is used because no discontinuous grid lines are present in that case. Normal nodes found at orthogonal grid line intersections are referred to as type 0 nodes in the program. Of the two types of special nodes needed to connect discontinuous grids (described in Appendix A), special nodes

used to couple circumferentially discontinuous nets are referred to as node types 1 and 2 in the program and nodes used to couple radially discontinuous nets are designated types 3 and 4.

Only circumferentially discontinuous grids need to be coupled together when grid type b) (obtained with NGROPT=1) is used so only node types 0, 1 and 2 are used with that type of grid. A type c) grid uses all five possible node types, however.

When subroutine GROPT0, GROPT1 or GROPT2 has determined the category of node L it calls one of the routines listed in box 22 to evaluate at least the linear coefficients in the finite-difference equation for that node. Finite-difference equations for the five node types are derived in Appendix A and five different subroutines, one for each node type, are used to calculate the terms in those equations, as seen in box 22.

The program can follow one of two possible paths after box 22 depending on whether it is solving a finite-difference problem, which may or may not be used later as a trial function in a synthesis analysis, or it has reached the substitute/weight/integrate stage in a synthesis analysis when previously computed trial functions are substituted into all of the finite-difference equations to obtain residuals that are then multiplied by weighting functions (i.e., the trial functions in Galerkin's method) and summed as appropriate. The substitution, weighting and summation operations are performed in subroutine SUBST1 in SYNCON. A control variable, NSYN, is assigned a value of 1 at the start of a synthesis analysis and re-assigned a

value of 2 when all of the trial functions have been computed and the substitute/weight/integrate phase of the analysis has been reached. Thus the program checks the value of NSYN, as seen in box 23, at the end of the subroutine used in box 22 to decide whether it should proceed to subroutine SUBST1 or continue with the finite-difference analysis.

Boxes 24-36 describe the computations performed in subroutine SUBST1. Boxes 24-33 describe the manner in which initial conditions are determined for non-linear and/or transient analyses and perform the necessary linearizations. When all of the terms in the finite-difference equation for node L, K have been computed, the trial functions are substituted into the equation to obtain a residual which is then weighted and added to the appropriate entries in the left-hand side coefficient matrix and the right-hand side vector of constants in the system of equations defining the blending vector (Z), as described in box 34. If the analysis being performed is non-linear and/or transient in nature the entries in the left-hand side coefficient matrix (stored as subdiagonal, diagonal and superdiagonal band matrices AL, CB and CRI respectively) that change in value, either when the time increment is changed or when linearized coefficients are updated using new nodal temperatures, are stored in a second matrix DIASUM. The terms in DIASUM are subtracted from the appropriate entries in AL, CB and CRI, recomputed using a new time increment and/or new nodal temperatures, and then added back onto those entries in subsequent iterations or time steps to avoid having to recompute the entire

left-hand side coefficient matrix (see boxes 87-91). Further details of the computations performed in subroutine SUBST1 can be found in the description of that subroutine given in Sec 4.1.12.

As seen in boxes 24, 25, 27 and 31, nodal temperatures are set equal to TMPIN (input variable 5h) at the beginning of the substitute/weight/integrate phase of a transient synthesis analysis when grid type a) or b) is used. An alternative to setting all of the nodal temperatures equal to TMPIN at the start of a transient analysis is available when grid type c) is used, however, in that it is then possible to obtain the initial temperature distribution from one of the previously obtained trial functions (i.e., trial function NTFIC, where NTFIC is input variable 4h), as seen in box 33. A previously obtained trial function must be used to obtain the initial temperature distribution for a transient or non-linear steady-state LOECC analysis. A LOECC analysis can only be performed with grid type c) as mentioned previously.

The nodal temperatures in trial function NTFIC are used in subroutine SUBST1 to linearize non-linear terms in the first time step of a transient LOECC analysis or in the first iteration of a non-linear steady-state LOECC analysis. The Newton-Raphson method is used to linearize radiative and conductive (gas) heat transfer terms in the finite-difference equations for the pressure tube outer and calandria tube inner surface nodes while Kirchoff's transformation is used to linearize the temperature dependence of the pressure/calandria tube thermal conductivity, as mentioned in Sect 1.0.

In the second and subsequent time steps or iterations, nodal temperatures from the previous time step or iteration are used to update non-linear terms and advance in time (transient analysis). Subroutine SUBST1 is not used to perform the linearization and substitute/weight/integrate operations in a synthesis analysis for time steps or iterations past the first, however, when the matrix DIASUM is used to modify the coefficient matrices AL, CB and CRI, as mentioned previously and as indicated in boxes 86-91 and 109-110.

The radiative boundary condition on the outer surface of a radiating cylinder is linearized using the temperature TRADIN (input variable 4h) in a steady-state analysis or TMPIN in a transient analysis. The initial temperature TMPIN is used for linearization purposes in a radiating cylinder analysis even if the initial temperature distribution in the cylinder is obtained from trial function NTFIC, as it could be with grid type c). The Newton-Raphson method is used in subroutine SUBST1 to linearize radiative heat transfer terms in the finite-difference equations for the radiating cylinder outer surface nodes in the substitute/weight/integrate phase of a synthesis analysis. The thermal conductivity of the cylinder material is assumed constant (i.e., temperature independent) for simplicity.

If the check performed in box 23 indicates that only a normal finite-difference solution is required, the program finishes computing the terms in the finite-difference equation for node L, K and inserts them into the appropriate locations in the left-hand side conductance matrix (CX) and the right-hand side heat flow vector (Q), as seen in

boxes 37-51. Boxes 37-49 describe how initial conditions are determined in transient and/or non-linear analyses and how non-linear terms are linearized in a LOECC or radiating cylinder (eg., fuel element) analysis. The subroutine * referred to in some of the boxes is the subroutine used in box 22 (i.e., NORNOD, BIGRAD, SMARAD, SMACIR or BIGCIR) and hence depends on the type of node being processed while the subroutine ** is GROPT0 if NGROPT=0, GROPT1 if NGROPT=1 or GROPT2 if NGROPT=2.

As seen in boxes 39 and 40, nodal temperatures are set equal to TMPIN at the beginning of a transient finite-difference analysis when grid type a) or b) is used. The initial temperature distribution for such an analysis can also be obtained from a previous 2-D solution (i.e., trial function #NTFIC) when grid type c) is used, as seen in boxes 41 and 42 and as described previously for the substitute/weight/integrate phase of a synthesis analysis. This option cannot be used if no such solution is available (i.e., if the solution being obtained is the first), of course, and NTFIC must be assigned a value of zero in that case. The initial temperature distribution for a non-linear transient LOECC analysis must be obtained from a previous solution. The nodal temperatures in trial function #NTFIC are used in subroutine HXIO to evaluate the non-linear conductive (gas) and radiative contributions to the gap conductance at pressure tube outer and calandria tube inner surface nodes for the first time step in a transient finite-difference LOECC analysis. Gap conductances are calculated from user-supplied values of the gas conductivity (i.e., input

variable 30h, TCGAS) and radiative heat transfer coefficient (i.e., input variable 22h, HRADIN) in the first iteration of a non-linear steady-state LOECC analysis (obtained with input variables NLIN (17h) and IMPECC (14a) both assigned values of 1), as seen in box 45, or in all of the time steps of a linear transient LOECC analysis (obtained with NLIN=0 and IMPECC=1). The constant thermal conductivities CK1 and CK2, which apply to the pressure and calandria tube materials respectively (see input variables 33h and 34h), are always used to evaluate inter-nodal conductances in the first time step of a transient analysis (even if NTFIC > 0) or in the first iteration of a steady-state analysis, as seen in boxes 45 and 49.

It is not assumed automatically that a LOECC analysis is non-linear so the program user must specify the types of non-linearities that are to be considered. This observation applies to both a finite-difference analysis and to the second (i.e., substitute/weight/integrate) phase of a synthesis analysis in which a user-specified 3-D problem is being analysed. If NLIN=0 and NVCON=0 (NVCON is input variable 27h), the problem is considered to be linear and the gas conductivity in the pressure/calandria tube gas gap, the radiative heat transfer coefficient across the gap, and the pressure and calandria tube conductivities are assumed to be equal to the constant, user-specified values of TCGAS, HRADIN, CK1 and CK2 respectively. If NLIN=1 and NVCON=0, the pressure and calandria tube conductivities are assumed to be equal to the constant values of CK1 and CK2 respectively but the gas conductivity and radiative heat transfer coefficient

across the pressure/calandria tube gap are linearized using the most recent temperatures in every time step or iteration. The gas conductivity and radiative heat transfer coefficient are updated in subroutine HXIO in finite-difference analyses, as seen in box 48. If NLIN=1 and NVCON=1, the problem is treated as being fully non-linear and gas and solid thermal conductivities, along with radiative heat transfer coefficients, are linearized in the second and subsequent time steps or iterations using the nodal temperatures from the preceding time step or iteration. The pressure or calandria tube conductivity in a given internodal conductance expression is updated in subroutine * just before that expression is evaluated (in a finite-difference analysis).

In a radiating cylinder analysis, TMPIN is used to linearize the radiative boundary condition on the cylinder's outer surface for the first time step even if initial nodal temperatures are taken from a previously computed 2-D solution. Nodal temperatures from the preceding time step are used to linearize the radiative boundary condition in the second and subsequent time steps. The radiative boundary condition is linearized using the user-specified temperature TRADIN (input variable 21h) in the first iteration of a steady-state analysis, as seen in box 49, and then linearized in subsequent iterations using the most recent boundary temperatures, as seen in box 48. The Newton linearization of the radiative boundary condition on the cylinder's outer surface is performed in subroutine RADIAT in all analyses.

Boxes 21-51 are executed for each node (L, K) on the r- θ plane at axial mesh point K. The program then checks whether the final axial node (KX) has been reached, as seen in box 54. If it hasn't, the program increments the value of K by 1 (i.e., proceeds to the next axial node) and then returns to box 20 to start assembling finite-difference equations for nodes on the new r- θ plane. If the check performed in box 54 indicates that all of the nodes in the grid have been visited, the program proceeds either to box 57 if a finite-difference analysis is being performed or to box 85 if NSYN=2.

If a finite-difference analysis is being performed, Cholesky's algorithm is used to factor the coefficient matrix for the system of finite-difference equations into the form LL^T , as seen in box 57. The nodal temperatures, stored in the vector T, are subsequently obtained by performing the forward and backward substitution phases of the equation solution, as seen in box 58. Box 59 indicates that the nodal temperatures, boundary heat fluxes, etc., are printed out at this time but it should be mentioned that they are not printed out in a steady-state non-linear analysis unless the solution has converged or the user-specified maximum iteration count (input variable 19h, MAXIT) has been exceeded. The results are also only printed out in a synthesis analysis when OUTRY=1, where OUTRY is input variable 36h. The trial function print option (i.e., OUTRY=0 or 1) was included because a user may not always want a listing of the trial function nodal temperatures, boundary heat fluxes, etc., in a synthesis analysis.

Program execution will stop at this point in a finite-difference transient analysis if the user-specified final time (input variable 6h, TFINAL) or final time step number (input variable 11h, NFTS) has been reached and if, in addition, a synthesis analysis is not being performed, as seen in boxes 60, 64, 69 and 70. If the final time or time step number has been reached but a synthesis analysis is being performed, the program will either read the data for and compute, if required, another trial function or start the second (substitute/weight/integrate) phase of the synthesis analysis. If NEXT=1, where NEXT is input variable 37h, the program returns to box 10 where it increments the trial function count (NTRF) and proceeds to compute another trial function. If NEXT $\neq$ 1, the program may, if NCONST=1 (NCONST is input variable 7a), record a constant (input variable 8a, CONST) as the final trial function before starting the second phase of the analysis. These operations are shown in boxes 79-82.

At the end of each time step in a transient finite-difference analysis being performed to produce trial functions for a synthesis analysis, the current time step number (NTS) is examined to determine if it is one at which a trial function should be recorded, as seen in box 65. The time step numbers at which trial functions should be recorded are designated NTFTS (N) ($N=1,2,\dots,NTRTF$) (see input variable 13h). Boxes 66-68 store the nodal temperatures as a trial function, if necessary.

If the user-specified final time or time step number has been reached the program either computes another trial function (with new

input data) or proceeds to the second phase of the synthesis analysis, as described above. If the end of the analysis has not been reached the program either returns to box 17 to completely recalculate all of the finite-difference equations, as seen in box 71, or recalculates only the right-hand sides of the equations in subroutine QFORM, as seen in box 76. The equations are completely recalculated only if the analysis is non-linear, in which case the most recent nodal temperatures are used to update the non-linear terms, or if the time increment for the next time step differs from the one currently used. It should be noted that the program does not iterate until convergence is achieved in each time step in a non-linear transient analysis so small time steps should be used in problems with strong non-linearities to ensure that they are modelled accurately.

In linear transient finite-difference analyses only the right-hand sides of the finite-difference equations need to be recalculated for each time step if the time increment is not changed, as mentioned previously. The new nodal temperatures can be obtained very rapidly for each time step in this case because only the forward and backward substitution phases of the equation solution process need to be repeated. Thus the program returns to box 58 to perform the forward and backward substitutions after the finite-difference equation right-hand sides have been recomputed in subroutine QFORM (box 76).

Transient finite-difference analyses that are not being performed to obtain trial functions for a synthesis analysis follow the sequence of events described previously for the trial function analyses except

that boxes 65-68, which cause transient trial functions to be recorded, are bypassed.

Non-linear steady-state finite-difference solutions are iterated until convergence is achieved or until the maximum permissible iteration count (MAXIT) is exceeded. All of the finite-difference equations are completely recomputed, using the most recent nodal temperatures to update non-linear terms, in each iteration. If the maximum permissible iteration count is exceeded the program prints a message to that effect but does not stop program execution*, as seen in box 63. The final temperature distribution obtained in a steady-state finite-difference analysis is recorded as a trial function, if required, in box 78. If data for another trial function is provided, which is the case if NEXT=1, box 79 returns the program to box 10 to read the data for and compute the new trial function. If data for another trial function is not provided but the check performed in box 80 indicates that a constant (CONST) is to be recorded as the last trial function, this is done before the program starts the second phase of the synthesis analysis in box 83. If a constant is not to be recorded as the last trial function the program proceeds directly from box 80 to box 83, where NSYN is set to 2 to indicate that the substitute/weight/integrate phase of the synthesis analysis has started.

The matrices AL, CB and CRI comprising the left-hand side coefficient matrix for the system of equations defining the blending vector (Z) in a synthesis analysis, the corresponding right-hand side matrix RES (or RD), and the matrix DIASUM (see box 36) are zeroed in

*The program stops iterating, however, and proceeds to the next task.

subroutine ZERO at the start of the second phase of a synthesis analysis, as seen in box 84. The program then returns to box 8 to start the 3-D analysis. Axial grid definition data are read in subroutine DZ (box 8), which also defines axial and geometric parameters. Input data describing the 3-D problem are subsequently read by subroutine DATA1, as seen in box 11. Subroutine DATA1 reads and/or calculates the $r-\theta$ boundary conditions at all of the axial nodes in the grid (i.e., for $K=1,2,\dots,KX$), as discussed previously, and in the second phase of a synthesis analysis, reads the numbers of the trial functions that are to be used at each axial mesh point as well. The user must specify how many trial functions are to be used at each axial node K , designated $NTRI(K)$, along with their numbers which are stored as $NTRK(K, NK)$, $NK=1,2,\dots,NTRI(K)$. The trial function deployment schedule is thus stored in the matrix $NTRK(K, NK)$ ($K=1,2,\dots,KX$; $NK=1,2,\dots,NTRI(K)$). The time steps at which the user intends to specify new trial function deployment schedules are also read as variables $NTSTFC(NXT)$; $NXT=1,2,\dots,NTRCH$.

After reading the 3-D job data, the program proceeds to assemble the matrices AL , CB , CRI , $DIASUM$ and RES (or RD), as already described. The left-hand side coefficient matrices (AL , CB and CRI) are then stored in a second set of arrays ($AL1$, $CB1$ and $CRI1$) if the problem is non-linear or transient in nature, as seen in box 86, because the original matrices are destroyed when the blending vector (Z) is obtained in subroutine BNDEL (box 93). The time increment for the next time step in a linear transient analysis is computed and compared to the current

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time increment, as seen in box 91, to determine whether the left-hand coefficient matrices, AL, CB and CRI, will have to be modified for the next time step. If the time increment for the next time step differs from the one currently used the entries in DIASUM are multiplied by the factor shown in box 91 and added to the appropriate entries in AL1, CB1 and CR11 to obtain matrices that can be used with the new time increment. These modifications are described in greater detail in Sec III.2 of the thesis. The matrices AL, CB and CRI are later equated to the modified matrices AL1, CB1 and CR11 respectively before computing the blending vector (Z) for the next time step (see box 110).

The entries in DIASUM are subtracted from the appropriate entries in AL1, CB1 and CR11 if the analysis is non-linear, as seen in box 88. The system of simultaneous equations defining the blending vector (Z) is then solved for Z in subroutine BNDEL, as seen in box 93. The nodal temperatures are subsequently calculated from the trial functions and Z in subroutine OUTSYN and used, in a non-linear analysis, to recalculate the matrix DIASUM. The nodal temperatures are also used in non-linear and/or transient analyses to recalculate the right-hand side vector, RD, for the next iteration (non-linear steady-state analysis) or time step (transient analysis), as seen in box 94.

Boundary heat fluxes and specific results (eg. the resistance of a contact) are printed out by subroutine OUTSYN along with the temperatures of user-specified nodes. The coordinates (I, J, K) of those nodes are read as input variables 28a-30a. The program does not print out the temperatures of all of the nodes in the grid because

most of them would likely be of little interest to the user.

The outer surface temperatures obtained in each iteration of a steady-state radiating cylinder analysis are compared in subroutine OUTSYN to the corresponding temperatures obtained in the preceding iteration to determine if the solution has converged. In a steady-state non-linear LOECC analysis, only the pressure tube outer surface temperatures are tested for convergence, again in subroutine OUTSYN. If the convergence test used in a particular non-linear steady-state analysis indicates that the solution has not converged and if, in addition, the iteration count check performed in box 106 indicates that the maximum permissible iteration count (MAXIT) has not been exceeded, the program performs another iteration. If the solution has not converged after MAXIT iterations, the program prints a message to that effect and stops execution. To perform an additional iteration in a steady-state non-linear analysis or to proceed from one time step to the next in a non-linear transient analysis, the program simply adds the entries in DIASUM, which were recomputed in subroutine OUTSYN using the most recent nodal temperatures, to the appropriate entries in AL1, CB1 and CR11, as seen in box 109. All additions or subtractions of entries in DIASUM to entries in AL1, CB1 and CR11 are performed in subroutine MATCH. The matrices AL, CB and CR1 are then equated to AL1, CB1 and CR11, as seen in box 110, producing a new set of left-hand side coefficient matrices which are subsequently used with the updated right-hand side vector (RD), computed previously in subroutine OUTSYN, to obtain a new blending vector (Z).

The preceding description of the manner in which the program proceeds from one time step to the next applies unless the test performed in box 101 indicates that the trial function schedule should be changed (i.e., unless the time step number, NTS, equals one of the user-specified values of NTSTFC (NXT); $NXT=1,2,\dots,NTFCH$). If an affirmative answer is obtained to the query in box 101 the program stores the existing trial function schedule contained in the matrix NTRK (K, NK) in another matrix, NTRKP (K, NK), for use later in advancing from time step NTS to NTS+1 (i.e., the nodal temperatures at time step NTS, which can only be obtained from the trial functions and Z if the trial function schedule is known, are needed to advance to time step NTS+1). Subroutine TFREAD then reads a new trial function schedule, as seen in box 104, for use between time steps NTSTFC (NXT) and NTSTFC (NXT+1). Subroutine ZERO subsequently re-zeroes the matrices AL, CB, CRI, DIASUM, RD and RES before the program returns to box 19 to recompute all of these matrices using the new trial function schedule. The synthesis solution then carries on as it did before. Note that only the trial function schedule, which assigns specific trial functions to each axial node, is changed at time step NTSTFC (NXT) - the trial functions themselves remain the same.

A flowchart indicating the order in which input variables read by Tables 2.1a-2.1i should be supplied to SYNCON is given as Fig. 2.2.

Fig. 2.1 Flowchart for SYNCON.

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

Fig. 2.1 (cont'd)

TABLE 2.2 Input Variables Read by the Main Program (lines 1-53.5 in Listing 1)
(READ statement line numbers are the left-hand line numbers in the
main program.)

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
1a	TITLE(I) I=1,8	A4	1a	143	Job title - may be up to 32 alphanumeric characters long.
2a	NGROPT	I5	2a	144	Grid option number, = 0 for grid type a) = 1 for grid type b) = 2 for grid type c) Grid types a, b and c are shown in Figs. 1.1, 1.2 and 1.3 respectively and are described in detail in Sec 3.
3a	MBW	I5	2a	144	Number greater than the left-hand bandwidth of any row in the finite-difference coefficient matrix. Radial node numbers vary most rapidly in the grid, then circumferential node numbers, and finally axial node numbers. In 3-D F-D analyses, therefore, the maximum left-hand row bandwidth will be equal to the number of nodes on each r- θ plane and MBW must exceed that value. For simplicity, MBW should be assigned a value greater than the number of r- θ nodes in 2-D (r- θ) analyses also.
4a	N2D	I5	2a	144	Dimensionality indicator, = 0 for 3-D analyses performed using the finite-difference (not synthesis) method, = 1 otherwise. Note that N2D = 1 for a 3-D analysis performed using the synthesis method because the finite-difference solutions required initially (i.e., the trial functions) are 2-D.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
5a	NCCYL	I5	2a	144	Complete cylinder analysis option, = 1 if a complete cylinder is to be analysed, = 0 if only a segment of a cylinder is to be analysed. This option is only available with grid type a), thus if NCCYL = 1, NGROPT (variable #2a) <u>must</u> equal 0.
6a	SYN	I5	3a	145	Synthesis option, = 1 if synthesis analysis is to be performed, = 0 otherwise.
7a	NCONST	I5	3a	145	Constant trial function option, = 1 if a constant is to be used as a trial function in a synthesis analysis, = 0 otherwise. Value not required if SYN = 0.
8a	CONST	F10.5	3a	145	Value of the constant to be used as a trial function (needed only if NCONST = 1). Actual value is not important.
9a	RESIST	I5	4a	146	Resistance of contact analysis option, = 1 if the resistance of a contact of specified size on a semi-cylinder is to be computed, = 0 otherwise. This option can be requested in 2-D and 3-D analyses. Values of variables 10-13 following are not required if RESIST = 0.
10a	INSUL	I5	4a	146	Insulated boundary option, = 1 if the resistance of a contact on a semi-cylinder is to be computed without gas conduction, = 0 if the analysis is to be performed with gas conduction.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
11a	XLOAD	F10.5	4a	146	Contact load in resistance of contact analysis (N).
12a	RESIS	F10.5	4a	146	Resistance of segment of contact specimen with a length equal to the semi-cylinder diameter and cross-sectional dimensions equal to the semi-cylinder's diameter and length (°C/W). RESIS is subtracted from the overall resistance of the contact to obtain the resistance based on the extrapolated temperature drop at the interface.
13a	T0	F10.5	4a	146	Temperature of diametral plane of semi-cylinder in resistance of contact analysis (°C). The contact temperature is read later (see variable #47h).
14a	IMECC	I5	5a	147	LOECC analysis option, = 1 if a LOECC analysis is to be performed, = 0 otherwise. LOECC analyses can only be performed with grid type c), so if IMPECC = 1, NGROPT must equal 2. Values of variables 15-20 following are not required if IMPECC = 0.
15a	PTID	F10.5	5a	147	Pressure tube inner diameter.
16a	PTOD	F10.5	5a	147	Pressure tube outer diameter.
17a	CTID	F10.5	5a	147	Calandria tube inner diameter.
18a	CTOD	F10.5	5a	147	Calandria tube outer diameter.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
19a	DGART	F10.5	5a	147	Spacing between garter springs in CANDU fuel channel.
20a	SPRDIA	F10.5	5a	147	Garter spring diameter.
21a	A	F10.5	6a	148	Semi-major diameter of elliptical contact patch (needed if RESIST = 1 or IMPECC = 1).
22a	B	F10.5	6a	148	Semi-minor diameter of elliptical contact patch (needed if RESIST = 1 or IMPECC = 1).
23a	RCY1	F10.5	6a	148	Small contact radius if RESIST = 1 or small effective contact radius (see App. B) if IMPECC = 1. Value not required for other analyses.
24a	RCY2	F10.5	6a	148	Large contact radius if RESIST = 1 or large effective contact radius if IMPECC = 1. Value not required for other analyses.
25a	TUBE	I5	7a	149	Tube analysis option, = 1 for analysis of heat flow in a tube containing an internal fluid at bulk temperature BT (see input variable #26a) which exchanges heat with the tube wall through a film coefficient HI (see input variable #32h). The tube can be modelled with an isothermal line contact on its surface, such as would be produced if it was in line contact with a similar tube.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
26a	BT	F10.5	7a	149	Bulk temperature of the fluid in the tube. Value only required if TUBE = 1.
27a	NTPRNT	I5	8a	150	Number of nodal temperatures to be printed out in a synthesis analysis. A synthesis analysis is usually performed with a very large number of nodes so the analyst must identify the nodes at which temperatures are really required. The total number of these nodes is NTPRNT and the location of the LLth such node in the grid, found at $r_{JCN(LL)}$, $\theta_{ICN(LL)}$, $z_{KCN(LL)}$, is calculable from the values of ICN(LL), JCN(LL) and KCN(LL), read as input variables 28a, 29a and 30a respectively.
28a	ICN(LL)	I5	9a	155	Azimuthal node index of the LLth node whose temperature is to be printed out.
29a	JCN(LL)	I5	9a	155	Radial node index of the LLth node whose temperature is to be printed out.
30a	KCN(LL)	I5	9a	155	Axial node index of the LLth node whose temperature is to be printed out.
<p>Note that NTPRNT values of ICN(LL), JCN(LL) and KCN(LL) must be read so NTPRNT data cards similar to card #9a must be provided. The cards must be arranged so that the values of ICN, JCN and KCN vary from least to most rapidly in that order. If the analysis being performed is not a synthesis analysis or no nodal temperatures are required, set NTPRNT = 0 and omit data card #9a.</p>					

TABLE 2.1b Input Variables Read by Subroutine DTH01 (lines 53.6-64 in Listing 1)
(READ statement line number are the left-hand line numbers in subroutine DTH01.)

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
1b	IX	I5	1b	31	Total number of circumferential (azimuthal) nodes.
2b	ATOT	F10.5	1b	31	Total included angle between the first and last azimuthal nodes, located at $I = 1$ and $I = IX$ respectively. If $NCCYL = 0$, ATOT represents the total included angle of the cylindrical segment being modelled. If $NCCYL = 1$, $2\pi - ATOT = DTHA(1) = DTHB(IX)$.
3b	DTH	F10.5	2b	44	Size of angular subdivision.
4b	IN(N)	I5	2b	44	Azimuthal node at which use of the angular subdivision size read previously (DTH) is discontinued.
5b	NEXT	I5	2b	44	Control variable; NEXT = 0 signals that no more values of DTH, IN(N) and NEXT follow while NEXT = 1 signals that the statement which reads DTH, IN(I) and NEXT (line 44) should be re-executed because more values of those variables are supplied. Thus data card #2b will generally be repeated several times with different values of DTH and IN(N) and only the last card will have NEXT = 0.

TABLE 2.1c Input Variables Read by Subroutine RADO (lines 64.1-73.7 in Listing 1)
(READ statement line numbers are the left-hand line numbers in subroutine RADO)

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
1c	RI	F10.5	1c	34	Inner radius of cylindrical segment being modelled. RI must be greater than zero even if the core of the cylinder being modelled is solid, although it should be set to a small value in that case.
2c	MX	I5	1c	34	Number of the radial node on the outer surface of a cylinder contained within a larger cylinder (tube) with which is exchanges heat through a heat transfer coefficient HI (in general - see input variable #32h) or HX(I,K) (if IMPECC = 1). MX would be the number of the radial node on the outer surface of the fuel pellet at the pellet/sheath interface in a fuel element heat transfer analysis or the number of the radial node on the outer surface of the pressure tube in a LOECC analysis. When calculating MX, note that radial node numbers start at 2, rather than 1, at the innermost node (at radius RI).
3c	DR	F10.5	2c	45	Size of radial mesh subdivision.
4c	M(J)	I5	2c	45	Number of the radial node at which use of the radial subdivision size read previously (DR) is discontinued.
5c	NEXT	I5	2c	45	Control variable; NEXT = 0 signals that no more values of DR, M(J) and NEXT follow while NEXT = 1 signals that the statement which reads DR, M(J) and NEXT should be re-executed because another set of values for those variables

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
					is supplied. Thus data card #2c will generally be repeated several times with different values of DR and M(J) and only the last card will have NEXT = 0.

TABLE 2.1d Input Variables Read by Subroutine RAD12 (lines 80.7-93.9 in Listing 1)
(READ statement line numbers are the left-hand line numbers in subroutine RAD12.)

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
1d	NR	I5	1d	48	Total number of radial grid lines (i.e., lines of constant θ) at which radial mesh size alterations are made. NR is also equal to the number of times the smallest variable radial subdivision, which must be used at the outermost radial nodes starting at the first radial grid line (i.e., at $\theta=0$), is increased (doubled) in size at subsequent radial grid lines.
2d	MX	I5	1d	48	See the description of input variable #2c.
3d	JX	I5	1d	48	
4d	M(N), N=1,2,... NR	I5	2d	50	Number of outermost radial node or, equivalently, the total number, plus one, of radial nodes used at $\theta=0^\circ$. The addition of one to the total number of radial nodes is needed because radial node numbers, in the grid start at 2, rather than 1, at the innermost radial node.
5d	RI	F10.5	3d	53	Radial nodes at which radial subdivisions change in size, with M(1) and M(NR) being the innermost and outermost nodes in that group respectively. Note that data card 2d will be repeated NR times because NR values of M(N) must be supplied.
6d	RO	F10.5	3d	53	Inner radius of cylindrical segment being modelled (must be greater than zero).
					Outer radius of cylindrical segment being modelled.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
7d	DR(NR+1)	F10.5	3d	53	Size of the smallest variable radial subdivision. This subdivision size must be used at the outermost radial nodes (i.e., between radial nodes M(NR) and JX) starting at the first radial grid line (i.e., at $\theta=0^\circ$). A contact, if present, should be centered about the first radial grid line.

TABLE 2.1e Input Variables Read by Subroutine ISIJ (lines 94.1-102.9 in Listing 1)
 (READ statement line numbers are the left-hand line numbers in
 subroutine ISIJ)

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/function
1e	JSC(N), N=1,2,..., NR	I5	1e	26	Radial (constant θ) grid lines at which the radial mesh structure is altered. JSC(NR) is the first radial grid line at which radial subdivisions are increased (doubled) in size and JSC(1) is the last. Note that data card 1e will be repeated NR times because NR values of JSC(N) must be supplied.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
1f	IX	I5	1f	57	Total number of circumferential (azimuthal) nodes.
2f	ATOT	F10.5	1f	57	Total included angle (in radians) of the cylindrical segment being modelled.
3f	DTH(1)	F10.5	1f	57	Size of the smallest variable angular subdivision (in radians). DTH(1) must be used at the outermost radial nodes starting at the first radial grid line. The values of the variables DTH(1) and MZ1 (see input variable 49h) can be adjusted to model one-half of a contact patch of a given width. The contact conductance HEX (see input variable 48h) is used at the circumferential nodes (I) lying in the range $1 \leq I \leq MZ1$.
4f	NI	I5	1f	57	Number of circumferential (constant radius) grid lines at which the circumferential mesh structure is altered.
5f	IN(II) II=1,NI	I5	2f	71	Radial grid lines (I) at which the angular subdivisions on the outermost circumferential grid line (at radial node JX) change (double) in size. IN(NI) is actually the radial grid line separating the sector in which angular subdivisions depend on both angular position (θ) and radius (r) from the sector in which angular subdivisions depend only on θ .

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
6f	ISC(II) II=1, NI	I5	2f	71	Circumferential (constant radius) grid lines at which the circumferential mesh structure is altered: ISC(1) and ISC(NI) are the outermost and innermost grid lines in that group respectively. Note that data card 2f must be repeated NI times. Each card should contain the IIth value of IN and the IIth value of ISC.
7f	DTH(N+1)	F10.5	3f		Angular subdivision size used between radial grid lines IN(N-1) and IN(N).
8f	IN(N)	I5	3f		Radial grid line (i.e., circumferential node) at which the use of DTH(N+1) is discontinued.
9f	NEXT	I5	3f		Control variable; NEXT = 0 signals that no more values of DTH, IN and NEXT follow while NEXT = 1 signals that the statement which reads those variables should be re-executed because more values are supplied. Note that data card #3f may be repeated several times (i.e., until NEXT is assigned a value of 0 by the user).

TABLE 2.1g Input Variables Read by Subroutine DZ (lines 73.8-80.6 in Listing 1)
(READ statement line numbers are the left-hand line numbers in
subroutine DZ)

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
1g	DZ	F10.5	1g	31	Size of axial subdivision.
2g	KCH(K), K=1,2,...	I5	1g	31	Number of axial nodes at which use of the subdivision size DZ read previously is discontinued.
3g	NEXT	I5	1g	31	Control variable: NEXT = 0 signals that no more values of DZ, KCH(K) and NEXT follow while NEXT = 1 indicates that the statement which reads DZ, KCH(K) and NEXT should be re-executed because additional values of these variables are provided. Note that data card #1g may be repeated several times (i.e., until NEXT is assigned a value of 0 by the user).

TABLE 2.1h Input Variables Read by Subroutine DATA1 (lines 137.1-174.5 in Listing 1)
(READ statement line numbers are the left-hand line numbers in subroutine DATA1)

Variable #	Variable Name	Format	Data Card.#	Read statement line number	Definition/Function
1h	TRANS	I5	1h	169	Transient analysis option, = 1 if a transient analysis is to be performed, = 0 otherwise.
2h	NTRTF	I5	1h	169	Number of transient trial functions to be recorded in a 2-D transient analysis being performed to obtain trial functions for a synthesis analysis. Leave blank if inapplicable.
3h	NTFCH	I5	1h	169	Number of times the trial function deployment schedule is to be changed in the second phase of a transient synthesis analysis. Can be left blank if the input data being read in does not apply to the second phase synthesis calculations. Enter value only if applicable.
4h	NTFIC	I5	1h	169	Number of a previously computed trial function from which initial conditions (temperatures) for a transient and/or non-linear analysis are to be taken. Initial conditions will only be taken from trial function #NTFIC if NGROPT = 2 (NGROPT is input variable 2a). Initial conditions for a transient 2-D finite-difference LOECC analysis or for the second phase calculations of either a transient or non-linear steady-state LOECC analysis (synthesis solution) must be taken from a previously computed trial function (i.e., NTFIC must be greater than zero for these analyses).

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
5h	TMPIN	F10.5	2h	177	Temperature to which all of the nodal temperatures are equated at the start of a transient analysis. Not used if NGROPT = 2 and NTFIC > 0.
6h	TFINAL	F10.5	2h	177	Final time to which a transient analysis is to be taken.
7h	DENS1	F10.5	2h	177	Density of material at radial nodes (J) ≤ MX.
8h	DENS2	F10.5	2h	177	Density of material at radial nodes (J) ≥ NIS. It is assumed that the cylindrical regions to which DENS1 and DENS2 apply are separated by an interface or gap. The radial nodes MX and NIS coincide with the outer surface of the inner cylinder and the inner surface of the outer cylinder respectively.
9h	SPHT1	F10.5	2h	177	Specific heat of material at radial nodes (J) ≤ MX.
10h	SPHT2	F10.5	2h	177	Specific heat of material at radial nodes (J) ≥ NIS.
11h	NFTS	I5	2h	177	Number of the time step at which a transient analysis is to stop (i.e., the number of the final time step). A transient analysis stops when either the time exceeds TFINAL or the time step number (NTS) exceeds NFTS. Note: Variables 5h-11h are only read if TRANS = 1.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
12h	NTSFC(NXT) NST = 1, NTFCH	I5	3h	190	Numbers of the time steps at which the trial function deployment schedule is to be changed in the second phase calculations of a transient synthesis analysis. Data card 3h is repeated NTFCH times since only one value of NTSFC is read on each card. Note: The variables NTSFC(NXT) (NXT = 1, 2, ..., NTFCH) are not read unless data for the second phase calculations of a synthesis analysis are being read in (i.e., unless NSYN = 2) and NTFCH > 0.
13h	NTFTS (ITIM) ITIM=1, NTRTF	I5	4h	203	Numbers of the time steps at which trial functions are to be recorded in a 2-D transient analysis being performed to obtain trial functions for a synthesis analysis. Note: The variables NTFTS(ITIM) (ITIM = 1, 2, ..., NTRTF) are only read when input data for a 2-D trial function analysis are being read in (i.e., when SYN = 1 and NSYN ≠ 2).
14h	DT(NBT)	F10.5	5h	212	NDTth time step size.
15h	NTC(NDT)	I5	5h	212	Number of the time step at which the use of time step size DT(NDT) is discontinued. Time step size DT(1) is used for time steps (NTS) lying in the range $1 \leq NTS < NTC(1)$. Time step size DT (NDT), where $NDT > 1$, is used for time steps in the range $NTC(NDT-1) \leq NTS < NTC(NDT)$.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
16h	NEXT	I5	5h	212	Control variable: NEXT = 1 indicates that the statement which reads DT(NDT), NTC(NDT) and NEXT should be re-executed with the value of NDT incremented by 1 because more values of those variables are supplied: NEXT = 0 indicates that the last set of values for DT, NTC and NEXT has been read in. Note that variables 14h-16h are read for NDT=1, 2, ... so data card 5h may be repeated several times. Variables 14h-16h are only read if TRANS = 1.
17h	NLIN	I5	6h	221	Non-linear gap conductance option: = 1 if a LOECC analysis with non-linear gap conductances is to be performed, = 0 otherwise. Note: When NLIN=1, IMPECC (variable 14a) must = 1.
18h	RADBND	I5	6h	221	Radiating cylinder option: = 1 if heat transfer in a cylinder radiating to the surrounding is to be analysed, = 0 otherwise.
19h	MAXIT	I5	6h	221	Maximum permissible iteration count for a non-linear steady-state analysis.
20h	EPSLN	F10.5	6h	221	Convergence parameter for a non-linear steady-state analysis. Solution is assumed to have converged when all of the nodal temperatures vary by an amount less than EPSLN from one iteration to the next.
21h	TRADIN	F10.5	6h	221	Initial boundary temperature estimate for a steady-state radiating cylinder analysis.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
22h	HRADIN	F10.5	6h	221	Pressure to calandria tube radiative heat transfer coefficient - used in linear (NLIN=0) LOECC analyses and in the 1st iteration of a steady-state non-linear (NLIN=1) finite-difference LOECC analysis. HRADIN is defined by the expression: $HRADIN = REXCO * (T_{PT} + T_{CT}) * (T_{PT} + T_{CT})$ where REXCO is defined below (variable 26h), and T_{PT} and T_{CT} are the estimated pressure and calandria tube temperatures respectively.
23h	BETA	F10.5	6h	221	Underrelaxation factor used in the linearization of gap conductances in finite-difference LOECC analyses. Recommend value in the 0.5-0.7 range.
24h	TCINTG	E12.4	7h	221	Intercept(a) of thermal conductivity vs temperature regression (of form $k=a+bT$) for the gas between the pressure and calandria tubes in a CANDU reactor.
25h	SLOPG	E12.4	7h	221	Slope(b) of regression for which TCINTG is the intercept.
26h	REXCO	E12.4	7h	221	Radiation exchange coefficient defined by the expression, $REXCO = \frac{1}{\frac{1}{\epsilon_{PT}} + \frac{PTOR}{CTOR} \left(\frac{T_{PT} - T_{CT}}{T_{PT} + T_{CT}} \right)}$ for radiative heat transfer between the pressure and calandria tubes in a CANDU reactor.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
27h	NVCON	I5	8h	224	<p>reactor. In this expression, ϵ_{PT} and ϵ_{CT} are the emissivities of the pressure tube outer and calandria tube inner surfaces respectively.</p> <p>Variable thermal conductivity option: = 1 if the thermal conductivity of the pressure and calandria tube material in a CANDU reactor is to be treated as temperature dependent in a LOECC analysis, = 0 otherwise. NVCON can actually be equated to 1 in any transient finite-difference analysis or in steady-state finite-difference analyses for which NLIN=1 or RADBN=1.</p>
28h	TCINTS	E12.5	8h	224	Intercept(a) of thermal conductivity vs temperature regression (of form $k=a+bT$) for the cylinder material (used if NVCON=1).
29h	SLOPS	E12.5	8h	224	Slope(b) of linear thermal conductivity vs temperature regression.
30h	TCGAS	E10.4	9h	228	Thermal conductivity of gas - used in "resistance of contact" analyses. Also used in the first iteration of a steady-state, non-linear, finite-difference LOECC analysis.
31h	CCNN	E10.4	9h	228	<p>Variable defined by the expression</p> $CCNN = 4vL_m \frac{(\alpha_1 + \alpha_2 - \alpha_1\alpha_2)}{\alpha_1\alpha_2 Pr(v+1)}$ <p>where α_1 and α_2 are accommodation coefficients of contact surfaces, v = ratio of specific</p>

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
32h	H1	F10.5	10h	236	heats for gas between contact surfaces, L_m = mean free-path of gas molecules, Pr = Prandtl number of gas. CCNN is used in a non-continuum gas conductivity model.
33h	CK1	F10.5	10h	236	Heat transfer coefficient between inner and outer cylinders in an analysis involving two (concentric) cylinders. When IMPECC = 1 (i.e., in a LOECC analysis), H1 is the heat transfer coefficient applied to the calandria tube outer surface.
34h	CK2	F10.5	10h	236	Thermal conductivity of inner cylinder in a two cylinder analysis.
					Thermal conductivity of outer cylinder in a two cylinder analysis. In a LOECC analysis, CK1 and CK2 apply to the pressure and calandria tubes respectively unless NVCON = 1, in which case the thermal conductivities of these components are defined by the regression for which ICINTS and SLOPS are the intercept and slope respectively. If NVCON = IMPECC = 1 and data for the 2nd phase of a synthesis analysis are being read in (i.e., NSYN=2), set CK1=CK2=1.0.
35h	CCOND	F10.5	10h	236	Contact conductance used within the contact patch in a LOECC or "resistance of contact" analysis.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
36h	OUTRY	I5	10h	236	Trial function print option: OUTRY = 1 causes trial function results (i.e., nodal temperatures, boundary heat fluxes) to be printed out after they have been computed in a synthesis analysis: OUTRY = 0 suppresses the printing of trial function results.
37h	NEXT	I5	10h	236	Additional trial function option: NEXT = 1 indicates that input data for another trial function are provided: NEXT = 0 indicates otherwise.
38h	NZFLO	I5	11h	240	Axial boundary condition option: NZFLO = 1 indicates that the temperature distribution stored in trial function #NTFBC is to be applied over an r- θ plane located a distance DZBC from the plane on which nodal temperatures are being calculated; NZFLO = 0 indicates that an axial boundary condition is not to be applied.
39h	NTFBC	I5	11h	240	Number of trial function from which axial boundary temperatures are to be taken.
40h	DZBC	F10.5	11h	240	Axial distance between the boundary plane on which the temperature distribution in trial function #NTFBC is prescribed and the r- θ plane on which nodal temperatures are being computed.

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
41h	NAX	I5	12h	244	Calculated r- θ boundary condition option: NAX = 1 causes the r- θ boundary conditions outside of the contact at axial nodes K(=1, 2, ..., KX) to be calculated from the appropriate geometric parameters in "resistance of contact" and LOECC analyses. If KX=1 (i.e., if the analysis being performed is 2-0) the boundary conditions are evaluated at the axial coordinate XPP. If KX>1, the boundary conditions at a given axial node K are evaluated at that node's axial coordinate (AXPOS(K)). If NAX=0, the circumferentially uniform heat transfer coefficient HOUT (input variable 46h) is used outside of the contact.
42h	XPP	F10.5	12h	244	Axial location (z) at which r- θ boundary conditions are to be evaluated in a "resistance of contact" or LOECC analysis (not needed if NAX=0 and/or KX>1).
43h	HTGEN(K)	F10.5	13h	255	Volumetric heat generation rate at axial node K.
44h	HFLZ(K)	F10.5	13h	255	Heat flux applied to the innermost radial nodes at axial node K (i.e., to the nodes I, 2, K; I=1, IX).
45h	TOUT	F10.5	13h	255	Sink temperature for circumferential nodes (I) > NZ1 (outer surface).

Variable #	Variable Name	Format	Data Card #	Read statement line number	Definition/Function
46h	HOUT	F10.5	13h	255	Heat transfer coefficient applied to the outer surface of the cylindrical segment being modelled at circumferential nodes (I)>MZ1.
47h	TEX	F10.5	13h	255	Sink temperature for circumferential (outer surface) nodes (I)≤MZ1.
48h	HEX	F10.5	13h	255	Heat transfer coefficient applied to the outer surface of the cylindrical segment being modelled at circumferential nodes (I)≤MZ1.
49h	MZ1	I5	13h	255	Circumferential node at the edge of the arc over which the heat transfer coefficient HEX is prescribed (usually the edge of a contact patch). Note: HOUT is calculated as a function of circumferential position (θ) in "resistance of contact" and LOECG analyses (if MAX=1). The value of HOUT supplied by the user is not used in those analyses.
50h	NTRI(K)	I5	14h	265	Number of trial functions used at the Kth axial node (not read unless NSYN=2).
51h	NT2(NK), NK=1,2,...,10	I5	14h	265	Trial functions used at the Kth axial node (not read unless NSYN=2). With the existing input format, a maximum of 10 trial function numbers can be read in for each axial node. Note: Variables 43h-51h are read for K=1, 2, ..., KX.

TABLE 2.1i Input Variables Read by Subroutine TIREAD (lines 359.7-363.2 in Listing 1)

Variable #	Variable Name	Format	Data Card #	Read statement line number	Définition/Function
1i	NTRI(K)	I5	1i	19	See input variable 50h.
2i	NT2(NK), NK= 1, 2, ..., 10	I5	1i	19	See input variable 51h.

Fig. 2.2 Order in which input variables read by Tables 2.1a - 2.1i should be supplied to SYNCON (table numbers are indicated in boxes)

3.0' GRID DEFINITION

As mentioned in Sec 1.0, SYNCON can be used with one of three different grid options. Grid types a), b) and c), described briefly in Sec 1.0, are selected by the program user by assigning the input variable NGROPT values of 0, 1 and 2 respectively. The user is then required to supply grid definition data which depend, to a large extent, on the grid type selected.

The variable NGROPT is read by the main program but the variables needed to define the grid structure in the r , θ and z directions are read by a series of subroutines, each of which is responsible for defining the mesh structure in a single coordinate direction. Specifically, these subroutines are responsible for computing the mesh increments (i.e., Δr_a , Δr_b , $\Delta \theta_a$, $\Delta \theta_b$, Δz_a , Δz_b) at all of the nodes in the mesh, as well as the positions (i.e., θ_i , r_j , z_k) of the nodes. Internodal conductances are computed in other subroutines from the mesh increments and nodal positions.

Some of the grid definition subroutines are common to more than one grid type, as seen in Fig. 3.1 where the grid definition subroutines used by each of the three grid types are listed. It will be noted that some of the subroutines, specifically those beginning with the letters RAD (used in the r direction) and DTH (used in the θ direction), are followed by numbers (i.e., 0, 01, 12, 2). The numbers at the end of a particular subroutine name refer to the grid options which use that subroutine, thus RADO is used solely by grid option 0 because

$\Delta r=f(r)$ for this grid option alone, RAD12 is used by grid options 1 and 2 because $\Delta r=f(r,\theta)$ for both of these options, etc. Specific details of the computations performed in each subroutine will be described later.

Since grid types a), b) and c) vary from least to most complex in that order, the grid definition data required by grid type a) will be described first, followed by the data required by grid type b) and then that needed by grid type c).

3.1 GRID TYPE a)

Grid type a), in which $\Delta r=f(r)$, $\Delta \theta=g(\theta)$ and $\Delta z=h(z)$, is selected by setting NGROPT=0, where NGROPT is read by the main program, as mentioned previously. Control is then transferred to subroutines DTH01 and RADO, in that order, where variables needed to define the grid structure in the circumferential (θ) and radial (r) directions respectively are read in and then used to evaluate mesh subdivisions and nodal coordinates. Subroutine DZ is subsequently called, in three-dimensional analyses, to define the axial mesh structure. Those subroutines, and the grid definition data required by them, are described below.

Fig. 3.1: GRID DEFINITION SUBROUTINES

3.1.1 Subroutine DTH01 (right-hand line numbers 53.6-64.0 in Listing 1)

The values of $\Delta\theta_a$ and $\Delta\theta_b$ at each circumferential mesh point I, designated DTHA(I) and DTHB(I) respectively when NGROPT=0 or 1, are defined in subroutine DTH01 for I=1, 2, ..., IX. The value of θ at each circumferential mesh point I, designated AN(I), is also defined in the subroutine.

The first variable read by subroutine DTH01 is IX, which is the total number of circumferential mesh points in the grid. The second variable read, ATOT is the total included angle measured in the + θ direction, between the first and last radial grid lines (constant θ) grid lines (found at I=1 and I=IX respectively) as shown in Fig. 3.2. When only a segment of a cylinder is being modelled, which is the case in Fig. 3.2, ATOT is also the total included angle of the segment since the grid lines at I=1 and I=IX bound the segment. When a complete cylinder is being modelled, which will be the case when NCCYL=1 (see input variable #5a in Table 2.1a), only the original definition of ATOT applies, as seen in Fig. 3.3 where a complete cylinder is illustrated. For the complete cylinder, $2\pi - \text{ATOT} = \text{DTHA}(I) = \text{DTHB}(IX)$.

The READ statement in subroutine DTH01 for the variables IX and ATOT is line #31* in the subroutine listing found in Listing 1. After reading IX and ATOT the subroutine initializes a counter, N, in line 37 and subsequently reads three more variables, DTH, IN(N)

*Unless otherwise noted, line numbers referred to in the description of a routine are the left-hand line numbers in that routine.

Fig. 3.2 Circumferential Mesh Structure with NGROPT=0 or 1

Fig. 3.3 Circumferential Mesh Structure for a Complete Cylinder.

(where $N=1$ at this point) and NEXT, in line 44. The variable DTH is an angular subdivision size which, when $N=1$, is used between circumferential nodes 1 and $IN(1)$ (i.e., from $I=1$ to $I=IN(1)$), as seen in Fig. 3.2. Lines 51-66 then assign appropriate values of $DTHA(I)$ and $DTHB(I)$ to the nodes (I) in that range and calculate the value of θ ($AN(I)$) at each node. A check on the value of NEXT follows in line 70 and, if $NEXT=1$, the value of the counter (N) is increased by one and new values of DTH $IN(N)$ and NEXT are read by line 44, as before. Lines 51-66 are then re-executed and the new subdivision size, DTH, is used to define values of $DTHA(I)$, $DTHB(I)$ and $AN(I)$ for I between $IN(N-1)$ and $IN(N)$ (note that the circumferential nodes at which the use of DTH begins and ends are designated NIP and IN2 respectively in the program). If $NEXT=1$ again, the whole procedure is repeated.

The process described above continues until NEXT is assigned a value other than 1 by the user (see line 70). If the accumulated value of θ at the last value of $IN(N)$, designated ANG, is greater than ATOT an error message is printed and program execution is stopped, otherwise a final angular subdivision size for use at nodes (I) between the last value of $IN(N)$ and $I=IX$ is evaluated in line 81 by dividing the total remaining angle ($ATOT-ANG$) by the number of remaining subdivisions ($IX-IN(N)$). Lines 86-93 then define $DTHA(I)$, $DTHB(I)$ and $AN(I)$ at the remaining nodes between the last value of $IN(N)$ and IX . If a complete cylinder is being analysed (indicated by $NCCYL=1$), line 97 sets $DTHA(I)=DTHB(IX)=2\pi-ATOT$.

The values of DTH read by subroutine DTH01 can be chosen by the user to model a line contact on the outer surface (at radius $R0$) of a cylindrical segment exactly if the contact half-width (b) spans the first $MZ1$ circumferential nodes ($MZ1$ is input variable 49h) on the surface, as illustrated in Fig. 3.4. Assuming the surface element associated with node $MZ1$ extends halfway between nodes $MZ1$ and $MZ1+1$ (i.e., to $\theta_b = (AN(MZ1) + AN(MZ1+1))/2$) the contact half-width (arc length) will be

$$b = \theta_b R0 \quad 3.1$$

from which we obtain,

$$\theta_b = \frac{b}{R0} \quad 3.2$$

The values of $R0$ and b will be known so the values of DTH and $MZ1$ can be selected to give θ_b the required value. If $MZ1$ is chosen to be less than $IN(1)$, the contact half-width will span nodes separated by the first value of DTH only, making it possible to express the required value of DTH as,

$$DTH = b / (MZ1 - 1/2) R0 \quad 3.3$$

It is generally desirable to choose $MZ1 < IN(1)$ so that the entire contact is modelled using the smallest angular subdivision (assuming the first value of DTH read in is the smallest - this is not mandatory).

Fig. 3.4 Contact Patch Representation with $MZ1=4$ and $IN(1)=4$

3.1.2 Subroutine RADO (lines 64.1-73.7)

The values of Δr_a and Δr_b at each radial node JJ, designated DRA(JJ) and DRB(JJ) respectively when NGROPT=0, are defined in subroutine RADO for JJ=2, 3, ..., JX, where JX is the largest radial node number, along with the radius at each node, designated RAD(JJ).

The first input variable read by subroutine RADO, viz RI, is the inner radius of the cylindrical segment, or complete cylinder, being modelled. This variable must be assigned a value greater than zero even if the core of the cylinder is solid, although it should be set at an arbitrarily small value if that is the case. The center of a solid cylinder is not modelled because the heat conduction equation in the cylindrical coordinate system is singular at the origin ($r=0$).

The second variable read by subroutine RADO, MX, is only significant in analyses where an inner cylinder is contained within an outer cylinder (tube) with which it exchanges heat radially across a gap or interface. In such analyses MX is the number of the radial node on the outer surface of the inner cylinder. This surface would be the surface of the fuel pellet at the pellet/sheath interface in a nuclear fuel element heat transfer analysis or the pressure tube outer surface in a LOECC analysis.* When calculating MX it should be noted that radial node numbers start at 2, rather than 1, at the innermost node (at radius RI), as seen in Fig. 3.5. In analyses which do not involve inner and outer cylinders separated by an interface

*LOECC analyses can only be performed with grid type c) (i.e., with NGROPT=2)

Fig. 3.5 Radial Mesh Structure with $NGROPT=0$

or gap, MX should be assigned a value larger than the largest radial node number (i.e. larger than JX, shown in Fig. 3.5) since this will cause the entire cylinder to be modelled as a solid object. When analyzing heat transfer in a tube carrying a heat exchanging fluid (i.e., when input variable #25a in Table 2.1a, TUBE, =1), set MX=1 and number the radial nodes in the tube in the normal fashion (i.e., starting at 2 on the tube's inner surface).

The input variables RI and MX described previously are read by line 34 in subroutine RADO. The subroutine then uses MX to compute the number of the node on the inner surface of the outer cylinder, NIS, (=MX+1), in line 39, initializes a counter, J, in line 44 and subsequently reads three more variables, DR, M(J) (where J=1 at this point) and NEXT, in line 45. The variable DR is a radial subdivision size which, when J=1, is used between radial nodes 2 and M(1) (i.e., from JJ=2 to JJ=M(1)), as seen in Fig. 3.5. Lines 50-66 subsequently assign appropriate values of DRA(JJ) and DRB(JJ) to the nodes (JJ) in that range and calculate the radius (RAD(JJ)) at each node. The value of the counter, J, is then increased by 1 and the value of NEXT is examined in line 74. If NEXT=1, the program returns to statement 10 (line 45) to read new values for DR, M(J) and NEXT as before. Lines 50-66 are then re-executed and the new subdivision size, DR, is used to define DRA(JJ), DRB(JJ) and RAD(JJ) for JJ between M(J-1) and M(J) (note that the radial nodes at which the use of DR begins and ends are designated MP and JJF respectively in the program). If NEXT=1 again, the whole procedure is repeated.

The process described above continues until NEXT is assigned a value other than 1 by the user. The last value of M(J) entered is, at that time, used in line 77 to define JX, which is the largest radial node number and hence the number of the outermost radial node in the grid. The radius at node JX (RAD(JX)) is recorded as the outer radius, RO, of the cylinder being modelled in line 81.

3.1.3 Subroutine DZ (lines 73.8-80.6 in Listing 1)

The values of Δz_a , Δz_b and z at each axial node KK, designated DZA(KK), DZB(KK) and AXPOS(KK) respectively in the program, are defined in subroutine DZ for KK=1, 2, ..., KX, where KX is the largest axial node number. The manner in which the axial mesh structure is defined in subroutine DZ is similar to the way in which the circumferential mesh structure was defined in subroutine DTHO1 and the radial mesh structure was defined in subroutine RADO, as described below.

Computations in subroutine DZ start with the initialization of a counter, K, in line 27 and the reading of three variables, DZ, KCH(K) (where K=1 at this point) and NEXT in line 31. The variable DZ is an axial subdivision size which, when K=1, is used between axial nodes 1 and KCH(1) (i.e., from KK=1 to KK=KCH(1)), as seen in Fig. 3.6. Lines 37-50 subsequently assign appropriate values of DZA(KK) and DZB(KK) to the nodes (KK) in that range and calculate the axial position (AXPOS(KK)) of each node. The value of the counter, K, is then increased by 1 and the value of NEXT is examined in line 58.

Fig. 3.6 Axial Mesh Structure.

If NEXT=1, the program returns to statement 10 (line 31) where it reads new values for DZ, KCH(K) and NEXT as before. Lines 37-50 are then re-executed and the new subdivision size, DZ, is used to define values of DZA(KK), DZB(KK) and AXPOS(KK) for KK between KCH(K-1) and KCH(K) (note that the axial nodes at which the use of DZ begins and ends are designated NKP and KKF respectively in the program). If NEXT=1 again, the whole procedure is repeated.

The process described above continues until NEXT is assigned a value other than 1 by the user. The last value of KCH(K) entered is, at that time, used to define the largest axial node number, KX, in line 61.

3.2 GRID TYPE b)

Grid type b), in which $\Delta r=f(r,\theta)$, $\Delta \theta=g(\theta)$ and $\Delta z=h(z)$, is selected by setting the input variable NGR0PT=1, as mentioned previously. The r- θ mesh structure for a grid of this type is defined in subroutine DTH01, RAD12, ISIJ and DRAB, which are called by the main program in that order, while the axial mesh structure, if needed, is defined in subroutine DZ. Subroutines DTH01 and DZ have already been described in connection with grid type a) so only the three remaining subroutines RAD12, ISIJ and DRAB, are described below.

3.2.1 Subroutine RAD12 (lines 80.7-93.9 in Listing 1)

Subroutine RAD12, which is used when NGROPT=1 or 2, defines the radii at radial nodes $(J)=2, 3, \dots, JX$, where the radius at node J is designated $RAD(J)$, as well as certain specific information required by the program to determine which nodes in the graded finite-difference network communicate with each other. The latter task is easily accomplished in a finite-difference grid with continuous grid lines, of course, but it is much more difficult to accomplish automatically, with minimal user-supplied data, when the grid lines are discontinuous.

Line 48 in subroutine RAD12 reads three input variables, NR, MX and JX. The first of these variables, NR, is the total number of radial grid lines (i.e., lines of constant θ) at which the radial grid structure is altered. In the sample grid (of type b)) shown in Fig. 3.7, NR=3 and the radial grid structure is altered at circumferential nodes $(I)=4, 8$ and 11. The values of I at which the radial grid structure changes, designated $JSC(1), JSC(2), \dots, JSC(NR)$, are not required in subroutine RAD12 but they are read as input variables by subroutine ISIJ, which will be the next subroutine described. The other two input variables read by line 48, in subroutine RAD12, specifically MX and JX, were defined in the description of subroutine RADO and they are used in the same context here.

With the input variables NR, MX and JX defined, subroutine RAD12 then reads values for $M(1), M(2), \dots, M(NR)$ in line 50. These variables are the radial nodes (J) at which radial subdivisions change

Fig. 3.7 Graded Grid for One-Piece Cylindrical Segment (NGROPT=1)

(double) in size, as seen in Fig. 3.7, with $M(1)$ and $M(NR)$ being the innermost and outermost nodes in that group respectively. $M(1)$ is actually the radial node which separates the cylindrical region in which $\Delta r = f(r, \theta)$ from the region in which Δr is fixed, as seen in Fig. 3.7, and it is not required that $DR(1)$ and $DR(2)$ be different. If the cylindrical region being modelled is split into two parts (eg., a fuel pellet and a fuel sheath) by an interface, the two nodes on either side of the interface (i.e., MX and NIS) are both numbered in sequence, as seen in Fig. 3.8. The two grid lines representing the cylindrical surfaces at the interface must both be circumferentially continuous.

After reading the values of $M(1)$, $M(2)$, etc., subroutine $RAD12$ reads the inner and outer radii of the cylindrical segment being modelled, designated RI and RO respectively, and the size of the smallest radial subdivision, $DR(NR+1)$, (shown as $DR(4)$ in Fig. 3.7) in line 53. The last variable, $DR(NR+1)$, is the radial subdivision used between radial nodes $M(NR)$ and JX and circumferential nodes 1 and $JSC(NR)$. As mentioned in the description of subroutine $RAD0$, RI must be assigned a value greater than zero even if the cylinder being modelled has a solid core.

In the rest of the computations performed in subroutine $RAD12$, the input variables read previously in lines 48, 50 and 53 are used to define the radius ($RAD(J)$) at each radial node J ($=2, 3 \dots JX$) along with two vectors, $NTW(J)$ and $NWTT(N)$ ($N=1, 2, \dots, NR+1$), used later in subroutine $ISIJ$ to establish an addressing sequence for the nodes in the grid. Also computed are the radial subdivisions, $DR(N)$

Fig. 3.8 Graded Grid for Cylindrical Segment With Interface.
(NGROPT=1).

(N=1, 2, ..., NR), used in different parts of the grid, which are defined by the relations;

$$DR(NR) = 2*DR(NR+1) \quad 3.4$$

$$DR(NR-1) = 2*DR(NR) \quad 3.5$$

$$DR(2) = 2*DR(3) \quad 3.6$$

and,

$$DR(1) = \frac{(RAD(M(1)) - RI)}{(M(1) - LC)} \quad \begin{matrix} (LC=3 \text{ if } MX < M(1)) \\ =2 \text{ otherwise} \end{matrix} \quad 3.7$$

where $DR(NR+1)$ is used between nodes $M(NR)$ and JX , as mentioned previously, $DR(N)$ ($N=NR, NR-1, \dots, 2$) is used between nodes $M(N-1)$ and $M(N)$ and $DR(1)$ is used between nodes 2 and $M(1)$. It should be mentioned that the range of applicability just mentioned for each radial subdivision is its range of applicability at $\theta=0^\circ$ (i.e., at $I=1$ or, more precisely, at $I \leq JSC(NR)$). The upper limit in the range of applicability of radial subdivision $DR(N)$ ($N=NR, NR-1, \dots, 2$) changes from $M(N)$ to JX when I exceeds $JSC(N)$ but this variation is not taken into account until the values of Δr_a and Δr_b are calculated at each node (I, J) in subroutine DRAB.

The radial subdivisions $DR(NR), DR(NR-1), \dots, DR(2)$ are computed sequentially in line 79 of subroutine RAD12 while $DR(1)$ is calculated in line 117. It should be noted that the denominator of the equation defining $DR(1)$ (i.e., Eq 3.7 given previously or line 117) is simply the number of radial subdivisions between the innermost node ($J=2$) and node $M(1)$ while the numerator is the corresponding radial distance.

It can be seen from Equ's 3.4-3.7 that the radial subdivisions $DR(NR)$, $DR(NR-1)$, ..., $DR(2)$ are even multiples of $DR(NR+1)$ and that only $DR(1)$ depends on the dimensions of the cylinder, the total number of radial nodes, etc. The radial grid spacing used between nodes $M(1)$ and JX at $I > JSC(1)$ is not provided with its own designation and when required in computations, is simply expressed as $2DR(2)$, as seen in Fig. 3.7.

The radii of the nodes between JX and $M(1)$ are computed in descending order in lines 69-103 of subroutine `RAD12` and the radii of the remaining nodes (i.e., between 2 and $M(1)$) are computed in ascending order in lines 121-124. The program segment between lines 69 and 103 is actually repeated NR times since the counter N , defined initially in line 52 and modified later in line 69, assumes values of NR , $NR-1$, ..., 1 as the program segment is executed for the first, second, ..., $(NR-1)$ st and NR th times respectively. After NR executions of the program segment, N assumes a value of 1 and line 109 transfers control outside of the loop. Within the aforementioned program segment, the DO loop between lines 97 and 103 computes $RAD(J)$ at J between $M(N+1)$ (or JX if $N=NR$) and $M(N)$ by subtracting the radial subdivision used between those limits (i.e., $DR(N+1)$) from the last radius calculated (i.e., $RAD(J+1)$), since, as mentioned previously, the radii are calculated in descending order. The value of $DR(N+1)$ is calculated in line 79 each time the program segment is executed.

For the purpose of being able to locate the nodes in the grid that are actually used in the finite-difference model, the smallest radial subdivision $DR(NR+1)$ is assigned a "weight" of unity and the

(N+1)st subdivision, $DR(N+1)$, ($N+1=NR, NR-1, \dots, 2$) is assigned a "weight" of 2^{NR-N} . Thus $DR(NR)$, which is twice the size of $DR(NR+1)$, is assigned a weight of 2 and $DR(NR-1)$, which is four times the size of $DR(NR+1)$, is assigned a weight of 4, etc. The weight of the (N+1)st subdivision size is stored as $NWTT(N+1)$, as seen in lines 76 and 80, and the sum of the subdivision weights for all of the subdivisions between a given node J ($\geq M(1)$) and the outermost node JX is stored at $NTW(J)$. This sum is computed in lines 99 and 100 for each radial node (J) by adding the weight NWT ($=NWTT(N+1)$) corresponding to the subdivision size used between nodes J and $J+1$ (i.e., $DR(N+1)$) to the sum computed previously ($=NTWT=NTW(J+1)$) for the ($J+1$)st node. Thus the values of $NTW(J)$ at nodes (J) between JX and $M(NR)$ would be given by the relation

$$NTW(J) = JX - J$$

while for J between $M(NR)$ and $M(NR-1)$ the values would be

$$NTW(J) = (JX - M(NR)) + 2 (M(NR) - J)$$

for instance. The variables $NWTT$ and NTW are combined with the values of $JSC(1), JSC(2), \dots, JSC(NR)$ in subroutine $ISIJ$ to determine how much the radial node index (J) should be incremented when proceeding from one of the nodes that is actually used to the next such node on a given radial grid line (I). To proceed from the node at $I=12, J=10$ to the node at $I=12, J=17$ in Fig. 3.7, for instance, the radial node index must be increased by 7. This increment would be stored as $IS(12,10)$ in subroutine $ISIJ$ (i.e., $IS(12,10)=7$).

The description of subroutine RAD12 will be terminated with a brief discussion of certain criteria the values of $M(1)$, $M(2)$, ..., $M(NR)$ must satisfy. Since the radial subdivisions between nodes $M(N)$ and JX are doubled in size and hence halved in number between radial grid lines $JSC(N)$ and $(JSC(N)+1)$, there must be an even number of these subdivisions available for combination at $JSC(N)$ to avoid having one remaining after the mesh size alteration. At the first such alteration in Fig. 3.7, for instance, where all of the subdivisions of size $DR(4)$ are changed to size $DR(3)$, all of the original subdivisions of size $DR(4)$ are eliminated because there is an even number (6) of them to start with. Similarly, all subdivisions of size $DR(3)$ are eliminated between radial grid lines $JSC(2)$ and $(JSC(2)+1)$ because an even number (6) of them are available for combination at $JSC(2)$. It is possible to determine whether given values of $M(1)$, $M(2)$, ..., $M(NR)$ are compatible in the sense just described by sketching the graded grid before using it, and this is always recommended, and/or by applying the following simple test. To start with, it is obvious that the value of $(JX - M(NR))$, which will be designated n_{NR+1} , must be an even number since it is equal to the number of subdivisions of size $DR(NR+1)$. The number of subdivisions of size $DR(NR)$ available for combination at $JSC(NR-1)$, designated n_{NR} , can now be calculated from the recursive relationship,

$$n_{NR} = \frac{n_{NR+1}}{2} + (M(NR) - M(NR-1)) \quad 3.8$$

and more generally we can write,

$$\eta_N = \frac{\eta_{N+1}}{2} + (M(N) - M(N-1)) \quad 3.9$$

for the number of subdivisions of size $DR(N)$ available for combination at $JSC(N-1)$. The values of η_N , ($N=NR+1, NR, \dots, 2$) must all be even in order for the values of $M(1), M(2), \dots, M(NR)$ to be compatible. Equation 3.9 is actually incorrect when MX lies between $M(1)$ and JX (i.e., when an interface is present between these nodes) but it can still be applied if the two interface nodes, MX and NIS , are counted as only one node or, equivalently, if the difference $(M(N) - M(N-1))$ in Eq. 3.9 is interpreted as being the number of radial subdivisions between $M(N)$ and $M(N-1)$. This interpretation will produce a number which is one less than the actual value of $(M(N) - M(N-1))$ if MX lies between $M(N)$ and $M(N-1)$.

As mentioned in Sec 1.0, the angle β shown in Fig. 1.2 must always be acute. The onus is on the user of SYNCON to ensure that the radial and circumferential subdivision sizes used in the grid satisfy this requirement at all of the transition nodes. The value of β at each of these nodes is checked by the program, however, and an error message is printed out and program execution stopped if an obtuse angle is encountered.

3.2.2 Subroutine ISIJ (lines 94.0-102.9 in Listing 1)

Nodes on a given r - θ plane, such as that shown in Fig. 3.7, are ordered with the radial node index (J) varying more rapidly than the circumferential node index (I) or, equivalently, all of the radial nodes (J) on the I th radial (constant θ) grid line are numbered in ascending order before proceeding to the $(I+1)$ st line. Some radial nodes are dropped at each transition grid line $JSC(N)$ ($N=1, 2, \dots, NR$), however, so the program must be able to identify the nodes that are actually used in the finite-difference model. This requirement is satisfied in SYNCON by calculating radial node number increments, designated $IS(I,J)$, which, when added to the radial node index (J) of a given node I, J , give the radial node index of the next node (that is actually used) on grid line I . Thus a program segment which would assemble the finite-difference equations for all of the nodes in the r - θ grid in the required order would be

```
DO 1    I=1, IX
```

```
  JI=2
```

```
  IF(NGROPT.EQ. 2) JI=JIN(I)
```

```
DO 2    J=JI, JX, ISP
```

```
  :   Lines calculating the coefficients and the
  :   right-hand side term in the finite-
  :   difference equation for node I, J
```

```
ISP = IS (I, J)
```

```
CONTINUE
```

```
1 CONTINUE
```


The initial value of J in the inner DO loop (i.e., $J1$) is seen to be 2 unless grid type c), (NGROPT=2) is being used, in which case it is $JIN(I)$. Values of $JIN(I)$ are defined for $I=1, 2, \dots, IX$ in subroutine JINI when NGROPT=2. The radial node number increment, ISP , ($=IS(I, J)$), must be calculated for each node I, J in the finite-difference grid so that unwanted radial nodes can be skipped. These increments are computed in subroutine ISIJ.

Values of $JSC(N)$ ($N=1, 2, \dots, NR$) are read in line 26 of subroutine ISIJ. These variables are the radial (constant θ) grid lines at which the radial mesh structure is altered, as mentioned previously in the description of subroutine RAD12 and as shown in Fig. 3.7 for a grid with $NR=3$. The subroutine then enters a pair of DO loops, similar to those shown above, in which $IS(I, J)$ is determined for each node (I, J) in the grid.

The variables $IS(I, J)$ and ISP are assigned values of 1 initially in lines 42 and 43, after which checks are performed in lines 47-52 to see if these are the correct values for node I, J . If the checks indicate that the radial node increment for node I, J should be 1, assumed, the program proceeds to the next value of J (i.e., $J+1$), otherwise the correct value of $IS(I, J)$ is calculated in lines 59-84. The radial node increment $IS(I, J)$ should be 1 if $J < M(NR)$ when $I \leq JSC(NR-1)$ or, more generally, if $J < M(II+1)$ (or JX if $II=NR$) when $I \leq JSC(II)$ ($II=NR, NR-1, \dots, 2$). With reference to the grid shown in Fig. 3.7, these conditions are satisfied for nodes (I, J) with $J < M(3)$ and $I \leq JSC(2)$, and for $J < M(2)$ with $I \leq JSC(1)$. The DO loop between

lines 48 and 52 checks to see if a given node (I,J) satisfies these conditions and a normal exit from the DO loop is obtained if it doesn't.

The manner in which the program computes the value of IS(I,J) at nodes where IS(I,J) $\neq 1$ is fairly straightforward. It will be remembered from the description of subroutine RAD12 that to each radial subdivision size DR(N) (N=NR+1, NR, ..., 2) there corresponds an integer variable, NWTT(N), which is effectively equal to the ratio of DR(N) to the smallest subdivision size, DR(NR+1) (i.e., $NWTT(N) = DR(N)/DR(NR+1) = 1, 2, 3, \dots$ for N=NR+1, NR, NR-1, ... etc.). From the values of JSC(N) and M(N) (N=1, 2, ..., NR) read in subroutines ISIJ and RAD12 respectively it is possible to determine the subdivision size (DR(N)) used between a given node (I,J) and the next node on the same radial gridline (I) or, equally, the corresponding value of NWTT(N), designated NDB in subroutine ISIJ. Since NTW(J), calculated in subroutine RAD12, is the accumulated sum of the values of NWTT(N) for all of the radial subdivisions between radial nodes J and JX (where NTW(JX)=0), the next radial node (on radial grid line I) that is used in the model, designated JJ, is the one at which $NTW(JJ) = NTW(J) - NDB$. Once this node (JJ) has been located we can set $IS(I,J) = JJ - J$. Lines 59-67 in subroutine ISIJ determine the value of NDB for the subdivision size employed between node I,J and the next node used on grid line I (i.e., node I,JJ) and lines 75 to 84 determine the value of JJ by finding the one node between J and JX which satisfies the expression $NTW(JJ) = NTW(J) - NDB$. The value of IS(I,J) is calculated in line 84 and the whole procedure is then repeated for the next node in the grid.

3.2.3 Subroutine DRAB (Lines 103.0-107.7 in Listing 1)

The radial subdivisions Δr_a and Δr_b at each node I,J in the finite-difference grid, designated $DRA(I,J)$ and $DRB(I,J)$ respectively when $NGROPT=1$ or 2, are defined in subroutine DRAB for circumferential nodes (I) between 1 and IX and radial nodes (J) between JI (=2 if $NGROPT=1$, =JIN(I) if $NGROPT=2$) and JX. Radial nodes that are not actually used in the finite-difference model are skipped. The values of I and J are compared to the values of JSC(1), JSC(2), ..., JSC(NR), and M(1), M(2), ..., M(NR) respectively in lines 25-39 of the subroutine to determine the appropriate values of $DRA(I,J)$ and $DRB(I,J)$. These subdivisions are computed in each case from the appropriate radial subdivision size $DR(N)$ ($N=1, 2, \dots, NR+1$) defined previously in subroutine RAD12. The coding in the subroutine is essentially self-explanatory and hence will not be discussed further.

3.3 GRID TYPE c)

Grid type c), in which $\Delta r=f(r,\theta)$, $\Delta \theta=g(r,\theta)$ and $\Delta z=h(z)$, is selected by setting the input variable $NGROPT=2$, as mentioned previously. The r- θ mesh structure for a grid of this type is defined in subroutines RAD12, DTH2, JINI, ISIJ and DRAB, which are called by the main program in that order.

The axial mesh structure, if needed, is defined in subroutine DZ. Subroutines RAD12, ISIJ and DRAB have already been described in connection with grid type b) so only the two remaining subroutines, DTH2 and JINI, are described below.

3.3.1 Subroutine DTH2 (lines 107.8-126.4 in Listing 1)

The angular subdivisions $\Delta\theta_a$ and $\Delta\theta_b$ at each node I, J in the finite-difference grid, designated $DTHA(I, J)$ and $DTHB(I, J)$ respectively when $NGROPT=2$, are defined in subroutine DTH2 for $I=1, 2, \dots, IX$ and $J=2, 3, \dots, JX$ along with the angular position (θ) of each radial grid line (I), designated $AN(I)$. Two additional variables, $INWT(II)$ and $ITWT(I)$, are also defined in subroutine DTH2 for use later in subroutine JINI where the first (innermost) node on each radial grid line (I), designated $JIN(I)$, is calculated. The variables $INWT(II)$ and $ITWT(I)$ are entirely analogous to the variables $NWTT(N)$ and $NTW(J)$ respectively, which were defined in subroutine RAD12 and then used in subroutine ISIJ to evaluate a radial node number increment, $IS(IJ)$, for each node (I, J) in the grid.

The first two variables read by subroutine DTH2 are IX and $ATOT$, which are the total number of circumferential nodes used in the model and the total included angle of the cylindrical segment* being modelled respectively. Both variables are shown in Fig. 3.9. The next variable read is $DTH(1)$, which is the smallest variable angular subdivision size. This subdivision size must be used at the outermost radial nodes starting at the first radial grid line (i.e., at $I=1$ or $\theta=0^\circ$) as seen in Fig. 3.9.

*Only grid type a, (i.e. $NGROPT=0$), can be used to model a complete cylinder.

Fig. 3.9 Circumferential Mesh Structure With NGROPT=2

Variables IX, ATOT, DTH(1) and NI are read by line 57 in subroutine DTH2. The last variable, NI, is the number of circumferential (constant r) grid lines at which the circumferential mesh structure is altered. For the grid shown in Fig. 3.9, NI=3 and the circumferential grid lines (J) at which the circumferential mesh structure is altered are 4, 6 and 9. These values of J (radial nodes) are designated ISC(3), ISC(2) and ISC(1) respectively.

After reading IX, ATOT, DTH(1) and NI, subroutine DTH2 assigns the first angular subdivision size, DTH(1), a "weight" of unity in line 63 and stores the weight as INWT(1). The subroutine then enters a DO loop in line 68 and reads variables IN(1), IN(2), ..., IN(NI) and ISC(1), ISC(2), ..., ISC(NI) before leaving the loop. The variables IN(1), IN(2), ..., IN(NI-1) are the first, second, ..., and (NI-1)st radial grid lines (I) at which the angular subdivisions between circumferential nodes at $J=JX$ change (double) in size while IN(NI) is the radial grid line separating the sector in which $\Delta\theta=f(r,\theta)$ from the sector in which $\Delta\theta=g(\theta)$. In the grid shown in Fig. 3.9, IN(1)=7, IN(2)=10 and IN(3)=13. The variables ISC(1), ISC(2), ..., ISC(NI) are the circumferential grid lines at which the circumferential mesh structure is changed, as already described.

If one-half of a line contact of half-width b spans the first MZ1 circumferential nodes (MZ1 is input variable 49h) at $r=R0(J=JX)$, as illustrated in Fig. 3.4, and MZ1 and IN(1) are chosen so that $MZ1 < IN(1)$, the value of DTH(1) is easily obtained from the relationship,

$$DTH(1) = b/(MZ1-1/2)R0$$

If $1N(1) \leq MZ1 < 1N(2)$, we obtain

$$DTH(1) = b/(2.*MZ1-1N(1))R0$$

and similar expressions can be derived for larger values of $MZ1$.

Subroutine DTH2 calculates $DTH(2) (=2DTH(1))$, $DTH(3) (=2DTH(2) = 4DTH(1))$, ..., $DTH(NI) (=2DTH(NI-1))$ and the corresponding subdivision weights $INWT(2) (=2)$, $INWT(3) (=4)$, ..., $INWT(NI) (=2^{NI-1})$ in the DO loop starting at line 68. The last subdivision weight, $DTH(NI+1)$, which is used between radial grid lines 1 and $1N(NI)$ at radial nodes (J) lying in the range $2 \leq J \leq 1SC(NI)$, is calculated in line 78 and the corresponding weight of the (NI+1)st subdivision size is calculated in line 79. Appropriate values of $DTHA(I,J)$ and $DTHB(I,J)$ are assigned to each node (I,J) in the sector of the grid in which $\Delta\theta=f(r,\theta)$ (i.e., for $I=1, 2, \dots, 1N(NI)$ and $J=2, 3, \dots, JX$), even if the node is not actually used in the finite-difference model, in lines 88-119. The value of θ at each radial grid line (I) (i.e., $AN(I)$) is defined in the same program segment for $I=1, 2, \dots, 1N(NI)$ along with the sum of the weights of the circumferential subdivisions (used at $J=JX$) between the first and Ith grid lines (i.e., $ITWT(I)$). The values of $ITWT(I)$ ($I=1, 2, \dots, 1N(I)$) and $INWT(II)$ ($II=1, 2, \dots, NI+1$) are used in subroutine JINI to determine the innermost node on each radial grid line (I), as mentioned previously.

The segment of subroutine DTH2 following line 119 defines the values of $DTHA(I,J)$, $DTHB(I,J)$ and $AN(I)$ at nodes where $\Delta\theta=g(\theta)$ (i.e., for $I > 1N(NI)$). This segment of the subroutine is essentially

identical to the portion of subroutine DTH01 following line 37 and hence will not be described in detail. A counter for the number of subdivision sizes used, N , ($N=NI$ initially) is incremented by 1 in line 125 and variables $DTH(N+1)$, $IN(N)$ and $NEXT$ are read in line 131. $DTH(N+1)$ is the angular subdivision size used between radial grid lines $IN(N-1)$ and $IN(N)$, $IN(N)$ is the radial grid line (I) at which the use of $DTH(N+1)$ is discontinued as just described and $NEXT$ is a control variable. If $NEXT=1$ the subroutine will, after executing lines 134-152, return to line 131 to read more values of $DTH(N+1)$, $IN(N)$ and $NEXT$ (with N increased in value by 1) whereas if $NEXT \neq 1$, the program will not return to re-execute the loop between lines 125 and 153.

The values of $AN(I)$, $DTHA(I,J)$ and $DTHB(I,J)$ are defined in the DO loops between lines 140 and 152 for all of the nodes (I,J) between radial grid line $IN(N-1)$ and $IN(N)$. A check on the value of $NEXT$ follows in line 153 and, if $NEXT=1$, the program returns to line 125 where N is increased in value by 1 and new values of $DTH(N+1)$, $IN(N)$ and $NEXT$ are read. If $NEXT=0$, the final subdivision size, $DTH(N+2)$, is calculated in line 162 from the included angle of the remaining segment, $(ATOT-AN(IN(N)))$, and the number of remaining subdivisions, $(IX-IN(N))$. This subdivision size is used to define $DTHA(I,J)$, $DTHB(I,J)$ and $AN(I)$ at the remaining nodes in the grid in lines 169-176.

As mentioned in Sec 1.0, the angle β identified in Fig. 1.2 must always be acute. The onus is on the user of SYNCON to ensure that the radial and circumferential subdivision sizes used in the grid

satisfy this requirement at all of the transition nodes. The value of β at each of these nodes is checked by the program, however, and an error message is printed out and program execution stopped if an obtuse angle is encountered.

3.3.2 Subroutine JINI (lines 126.5-130.7 in Listing 1)

As mentioned in the description of subroutine DTH2, subroutine JINI calculates the radial node index (J) of the first (innermost) node on each radial grid line (I), designated $JIN(I)$. For the grid shown in Fig. 3.9, for example, $JIN(I)=2$ when $I=1, 8, 11, 13$ and 14 ; $JIN(I)=5$ when $I=5, 10$ and 12 ; $JIN(I)=7$ when $I=3, 7$ and 9 ; and $JIN(I)=10$ when $I=2, 4$ and 6 . As seen in Fig. 3.9, the value of $JIN(I)$ at any radial grid line (I) must be either 2 or one of the values $ISC(NI)+1, ISC(NI-1)+1, \dots, ISC(1)+1$. The variables $ITWT(I)$ ($I=1, 2, \dots, IN(NI)$) and $INWT(N)$ ($N=1, 2, \dots, NI+1$), defined in subroutine DTH2, are used to determine the values of $JIN(I)$ for $I < IN(NI)$ in much the same manner as $NTW(J)$ and $NWTT(N)$ ($N=1, 2, \dots, NR+1$) were used in subroutine ISIJ to evaluate a node number increment, $IS(I,J)$, for each node (I,J) in the grid. When $I \geq IN(NI)$, $JIN(I) = 2$ of course. Since the coding in subroutine JINI is similar to that used in subroutine ISIJ, it will not be described further.

3.4 GRID MODIFICATIONS FOR A PRESSURE-CALANDRIA TUBE HEAT TRANSFER ANALYSIS

The pressure and calandria tubes in a CANDU fuel channel are discretized in a finite-difference/synthesis analysis by first generating a grid of type c) for the pressure tube surrounded by a concentric "sheath" with the same wall thickness as the calandria tube but having an inner diameter equal to the pressure tube outer diameter. Such a grid is shown in Fig. 3.10 for the pressure and calandria tubes in a Bruce reactor unit, which have wall thicknesses of 0.405cm and 0.135cm respectively.

The wall thickness of the pressure tube is three times that of the calandria tube in this reactor, a feature which permits the use of three equi-sized radial subdivisions across the pressure tube wall and one such subdivision across the calandria tube wall at circumferential locations remote from the pressure/calandria tube contact, located at the six o'clock position in Fig. 3.10. The grid shown in Fig. 3.10 is highly refined at the pressure/calandria tube contact, with the smallest radial subdivision ($DR(4)$) being one-eighth of the calandria tube wall thickness. All of the parameters required to generate the grid shown in Fig. 3.9 for the pressure and calandria tubes in a Bruce fuel channel are given in Table 3.1.

The grid depicted in Fig. 3.10 represents the pressure tube accurately but the radii of the nodes in the calandria tube (i.e., $RAD(J)$, $J=MX+1, MX+2, \dots, JX$) are incorrect. The radial subdivisions Δr_a and Δr_b in the calandria tube (i.e., $DRA(I,J)$, $DRB(I,J)$; $I=1, 2, \dots, IX$;

Fig. 3.10 Grid Type c) Representation of a CANDU Pressure Tube Surrounded by a "Sheath" with a Thickness Equal to that of the Calandria Tube Wall.

TABLE 3.1 GRID DEFINITION DATA FOR CONTACTING PRESSURE AND CALANDRIA TUBES

VARIABLE	LOCATION OF READ STATEMENT	VALUE
NR	RAD12 48	3
MX	RAD12 48	14
JX	RAD12 48	23
M(1)	RAD12 50	2
M(2)	RAD12 50	4
M(3)	RAD12 50	10
RI	RAD12 53	5.17(cm)
RO	RAD12 53	5.7135(cm)
DR(4)	RAD12 53	0.01687(cm)
IX	DTH2 57	39
ATOT	DTH2 57	3.14159
DTH(1)	DTH2 57	0.002048
NI	DTH2 57	3
IN(1)	DTH2 71	9
ISC(1)	DTH2 71	9
IN(2)	DTH2 71	13
ISC(2)	DTH2 71	5
IN(3)	DTH2 71	15
ISC(3)	DTH2 71	3
DTH(5)	DTH2 131	0.0192
IN(4)	DTH2 131	19
NEXT	DTH2 131	1
DTH(6)	DTH2 131	0.0384
IN(5)	DTH2 131	23
NEXT	DTH2 131	1
DTH(7)	DTH2 131	0.0768
IN(6)	DTH2 131	27
NEXT	DTH2 131	1
DTH(8)	DTH2 131	0.1536
IN(7)	DTH2 131	31
NEXT	DTH2 131	0
JSC(1)	ISIJ 26	25
JSC(2)	ISIJ 26	18
JSC(3)	ISIJ 26	11

$J=MX+1, MX+2, \dots, JX$)) defined in subroutine DRAB when $NGROPT=2$, are correct, however, because the 'sheath' surrounding the pressure tube was modelled with a thickness equal to that of the calandria tube wall. The radii of the nodes in the calandria tube can be easily corrected, a task which is performed by subroutine CALAND, but the angular subdivisions used in the calandria tube (i.e., $DTHA(I,J)$, $DTHB(I,J)$; $I=1, 2, \dots, IX$; $J=MX+1, MX+2, \dots, JX$) should also be modified so that nodes on the pressure tube outer and calandria tube inner surfaces line up with each other after the nodal radii have been adjusted, as shown in Fig. 3.11. As an example of what happens if the angular subdivisions in the calandria tube are not corrected, consider the two nodes on the pressure tube outer and calandria tube inner surfaces at the first radial grid line (i.e., nodes $(I,J)=1, MX$ and $1, NIS$, where $NIS=MX+1$). It is assumed that these nodes lie within the contact patch. The finite-difference solution will predict the rate of heat flow between nodes $1, MX$ and $1, NIS$ but the resulting heat flux across the contact interface will be discontinuous at the interface unless the nodes have the same surface areas. Since the calandria tube inner radius is larger than the pressure tube outer radius the nodes $1, MX$ and $1, NIS$ will not have equal surface areas unless the angular subdivisions used near the contact in the calandria tube are smaller than those used in the pressure tube in the same region. The angular subdivisions in the calandria tube must be modified for this reason.

It is assumed that heat transferred by conduction or radiation

Fig. 3.11 Correct Grid for Thermal Analysis of Contacting Pressure and Calandria Tubes.

across the gap between the pressure and calandria tubes flows in a direction normal to the pressure tube outer surface, as discussed in Sec III 3, so a point at circumferential position θ on the pressure tube outer surface communicates with a point at circumferential position ϕ on the calandria tube inner surface, as shown in Fig. 3.12.

It is easily shown that

$$\phi = \theta - \sin^{-1} \left(\frac{ECC \sin \theta}{CTIR} \right) \quad 3.10$$

where ECC is the distance between the pressure and calandria tube centers (i.e., the pressure tube eccentricity) and CTIR is the calandria tube inner radius. From this relationship we obtain the factor,

$$\frac{d\phi}{d\theta} = \frac{ECC \cos \theta}{CTIR \sqrt{1 - \left(\frac{ECC \sin \theta}{CTIR} \right)^2}} \quad 3.11$$

which, when multiplied by the values of DTHA(I,J) and DTHB(I,J) for nodes (I,J) in the calandria tube wall, yields the correct angular subdivisions for that region. The value of θ at the midpoint of an azimuthal subdivision is used to evaluate $d\phi/d\theta$ for that subdivision.

It should be noted that $\frac{d\phi}{d\theta} = 1 - \frac{ECC}{CTIR} = \frac{PTOR}{CTIR}$ when $\theta=0^\circ$ and the

pressure and calandria tubes are in contact (PTOR is the outer radius of the pressure tube), so differential arc lengths on the pressure tube outer and calandria tube inner surfaces, designated ds_{PT} and ds_{CT} respectively, are given by

$$ds_{PT} = PTOR d\theta \quad 3.12$$

$$ds_{CT} = CTIR \frac{d\phi}{d\theta} d\theta$$

$$= CTIR \frac{PTOR}{CTIR} d\theta = PTOR d\theta, \text{ at } \theta=0^\circ \quad 3.13$$

Fig. 3.12 Pressure and Calandria Tube Circumferential Coordinates θ and ϕ Respectively

under those conditions. These arc lengths are equal, as they must be if the heat flux across the contact interface is to remain continuous at the interface.

The radii and angular subdivisions for the calandria tube nodes are modified in subroutine CALAND, described briefly below. While the angular subdivisions in the calandria tube should, strictly speaking, be changed at each axial mesh point in a three-dimensional analysis of pressure and calandria tubes in contact because the eccentricity (ECC) depends on axial position, they are held constant at the values used at the contact (where $ECC = CTIR - PTOR$) for simplicity. This simplification causes heat fluxes on the calandria tube outer surface at low and high values of θ to be slightly overestimated and underestimated respectively in axial regions remote from the contact, where $ECC \ll CTIR - PTOR$.

3.4.1 Subroutine CALAND (lines 130.8-137.0 in Listing 1)

The coding in subroutine CALAND is largely self-explanatory and hence will only be described briefly. The variables CGI and XCH, calculated in lines 21 and 22 respectively, are used in subroutine DATA1 to calculate the gap between the pressure and calandria tubes as a function of axial position. They are not used for any purpose in subroutine CALAND. The radial subdivision size used in the calandria tube wall, designated DC in subroutine CALAND, is calculated in line 30 and subsequently used in line 37 to calculate the radius at each radial

node (J) in the tube wall. The values of θ at which $\frac{d\phi}{d\theta}$ (given by Eq. 3.11) is evaluated when correcting DTHA(I,J) and DTHB(I,J) are calculated as THIA1 and THIA2 respectively in lines 45 and 46. Those values are subsequently used, along with Eq. 3.11, to modify the values of DTHA(I,J) and DTHB(I,J) in lines 51 and 53 respectively.

4.0 DESCRIPTIONS OF REMAINING ROUTINES

Routines not connected with the grid definition process and hence not yet considered are described in this section. Alternate versions of both the main program and subroutines SUBST1 and OUTSYN have to be substituted for the versions described in Sec 4.1 in order to use orthonormalized trial vectors, as discussed in Sec 4.2.

4.1 Routines in Listing 1

4.1.1 Main Program (lines 1.0-53.5)

The main program directs the sequence of operations performed in SYNCON. It directs the trial function assembly process and the subsequent 3-D computations in a linear or non-linear, steady-state or transient synthesis analysis, directs the grid definition process, controls output operations, etc.

Lines 143-155* in the main program read the input variables defined in Table 2.1a. Note that the read statement for the circumferential, radial and axial indices of the LLth node at which the temperature is to be printed out in a 3-D synthesis analysis, designated ICN(LL), JCN(LL) and KCN(LL) respectively, is executed NTPRNT times (NTPRNT is the number of such nodes) so NTPRNT data cards giving values of ICN(LL),

*Left-hand line numbers are used in the descriptions of routines in SYNCON.

JCN(LL) and KCN(LL) must be provided.

Line 178 checks the grid option number, NGROPT, to determine which routines are required to define the r - θ geometry of the finite-difference grid. The required r - θ grid definition routines, shown for different values of NGROPT in Fig. 3.1, are called by lines 179-180 if NGROPT=0, by lines 182-185 if NGROPT=1 or by lines 187-191 if NGROPT=2. If NGROPT=2 and the analysis being performed is a LOECC analysis, line 192 calls subroutine CALAND to modify the radii of the calandria tube nodes and the angular increments used in the calandria tube, as described in Sec 3.4. Line 198 calls subroutine DZ to define the axial grid geometry if a 3-D finite-difference (not finite-difference/synthesis) analysis is being performed.

The trial function count (NTRF) is initially set =0 in line 201 and then incremented by 1 in line 206. Subroutine DATA1 is called in line 214 to read the input variables defined in Table 2.1h for the first analysis, which will also be the last unless a synthesis analysis is being performed. The r - θ boundary conditions at each axial node (K) are read and/or computed for all of the axial nodes ($K=1, 2, \dots, KX$) in the grid in subroutine DATA1. The value of KX was set =1 in line 194 and KX will still have that value unless subroutine DZ, in which it is redefined, was called to compute the axial grid geometry for 3-D computations. Subroutine DZ is not called in a 3-D synthesis analysis until all of the 2-D trial functions have been computed.

If the analysis being performed is a transient analysis, lines 225-227 initialize the counters for the number (-1) of transient trial

functions recorded so far in analysis #NTRF (i.e., ITIM), the number (-1) of times the trial function deployment schedule has been changed in a transient synthesis analysis (i.e., NXT), and the total number of time steps performed so far in a transient analysis (i.e., NTS). Line 234, which the program returns to after every time step in a transient finite-difference analysis, increments the time step count and lines 238-242 change the time step size (DTAU), initially set at DT(1) in line 231, if the time step number equals one of the user-specified time steps at which a change should be made (i.e., NTC(NDT), NDT=1, 2, ..., NTD). The time step sizes (i.e., DT(N), N=1, 2, ..., NDT+1) and the values of NTC(NDT), NDT=1, 2, ..., NDT are read in subroutine DATA1. The time in the analysis (TIME) is calculated in line 243 if the time step size has changed or in line 245 otherwise. In the second and subsequent time steps of a linear, finite-difference analysis, lines 252-255 call subroutine QFORM to re-evaluate the right-hand side heat flow vector using the most recent nodal temperatures. This operation, which is indicated in box 76 of the program flowchart (Fig. 2.1), is only done if the time step size has not changed because the finite-difference left-hand coefficient matrix will be unchanged from the one already assembled and factored in that case. Line 256 then directs the program to statement 64 (line 435) which calls subroutine SUB to perform the forward and backward substitutions needed to calculate the vector of new nodal temperatures.

The iteration counter for non-linear problems (NITER), which is initialized in line 210 for finite-difference solutions, is initialized

in line 269 for the substitute/weight/integrate of a synthesis analysis. In a steady-state synthesis analysis the program returns to statement 5 (line 267) to start the substitute/weight/integrate phase of the analysis whereas in a transient synthesis analysis the program returns to statement 4 (line 225) to start this phase. The solution is not iterated to convergence at each time step in a non-linear transient analysis.

The DO loop which starts at line 285 and ends at line 312 sweeps through all of the axial nodes in the grid. Line 292 defines the number of trial functions used at axial node K in the second (substitute/weight/integrate) phase of a synthesis analysis, designated NTRF (=NTRI(K)), while line 293 defines the number of trial functions used at axial node K with the previous trial function deployment schedule (i.e., NTRFP). The value of NTRFP is only needed when the trial function deployment schedule has just been changed in a transient synthesis analysis.

Line 295 defines the axial subdivision size (DZTOT) used at axial node K in a 3-D analysis while line 296 assigns this variable a value of unity for a 2-D analysis. Lines 300-305 call the appropriate subroutine for sweeping through the r- θ nodes in the grid at axial node K. The variable NDIA, set =0 in line 267 and incremented by NTRF in line 310 at each axial node, is the location in the blending vector (Z) of the multiplier for the first trial function used at axial node K (actually, NDIA+1 is that location). The multipliers for the second, third, ..., etc. (up to NTRF) trial

functions used at axial node K are found in the locations NDIA+2, NDIA+3, ..., NDIA+NTRF respectively in Z. The variable NDIAP, defined in line 311, performs the same function for the multipliers applying to a preceding trial deployment schedule when the schedule has just been changed in a transient synthesis analysis. The variables NDIA and NDIAP are needed to calculate the temperatures of all of the nodes in the 3-D grid for the purpose of advancing to the next time step.

Lines 320-427 in the main program are only executed in the second phase of a synthesis analysis. Within that program segment, lines 330-336 store the subdiagonal, diagonal and superdiagonal matrices making up the left-hand side coefficient matrix for the system of simultaneous equations defining the blending vector (Z), designated AL, CB and CRI respectively, as a second set of matrices, AL1, CB1 and CRI1. This operation is indicated in box 86 of the program flowchart (Fig. 2.1). The columns of a two-dimensional array, RES, each of which contains the NTRI(K) entries belonging to the Kth subvector of the right-hand side vector for the system of equations defining Z (i.e., RD), are stacked on top of each other in lines 341-349 to obtain RD. In a non-linear analysis, line 354 calls subroutine MATCH to subtract the entries in the matrix DIASUM, which contains terms in AL, CB and CRI which change from one iteration or time step to the next in a non-linear analysis, from the appropriate entries in AL1, CB1 and CRI1. This operation is indicated in box 88 of the program flowchart. Since the only non-linear conductances found in SYNCON lie on r- θ planes, the only entries in AL1, CB1 and CRI1 that

are modified in subroutine MATCH are those arising from the r - θ integrations. This matter is discussed further in the description of subroutine MATCH.

Lines 361-365 in the main program compute the size of the next time step for a transient analysis. The time step size, DTAU, remains unchanged unless $NTS = NTC(NDT)$. If the time step size does change and the analysis being performed is linear, so that AL1, CB1 and CR11 will not already have been modified in subroutine MATCH, line 373 defines the factor mentioned in box 91 of the program flowchart, designated XMF. Line 374 then calls subroutine MATCH, which multiplies the entries in DIASUM by XMF and adds them to the appropriate entries in AL1, CB1 and CR11 to obtain matrices suitable for use with the new time increment.

A non-symmetric banded equation solving routine, subroutine BNDEL, is called by line 378 of the main program to solve for the blending vector, Z. Subroutine OUTSYN is subsequently called in line 383 to print out nodal temperatures, boundary heat fluxes, etc. In a transient and/or non-linear analysis, subroutine OUTSYN also recalculates the entries in RES (i.e., RD) and DIASUM using the new nodal temperatures. Line 393 checks whether the time step number, NTS, is one at which the trial function deployment schedule should be changed and, if it is, subroutines NTRP and TFPREAD are called to store the existing trial function deployment schedule and to read the new schedule respectively. Subroutine ZERO is also called to zero the matrices AL, CB, CR1, RES, RD and DIASUM. The counter for the

number of trial function deployment schedules used (i.e., NXT) is incremented by 1, after which the program returns to statement 5 (line 267) to recompute the matrices AL, CB, CRI, RES and DIASUM with the new trial function deployment schedule.

If the trial function deployment schedule is not changed and the analysis being performed is non-linear, subroutine MATCH is called by line 401 to add the entries in DIASUM, computed in subroutine OUTSYN, to the appropriate entries in AL, CB and CRI. These matrices can now be applied to the next time step or iteration. This operation is indicated in box 109 of the program flowchart.

Lines 407-413 equate AL, CB and CRI to AL1, CB1 and CRI1 respectively, as indicated in box 110 of the program flowchart because the latter set of matrices is applicable to the next time step or iteration. After incrementing the iteration count (NITER) by 1 in line 416, the program returns to statement 55 (line 341) to repeat the program segment between lines 341 and 427 for the next time step or iteration. That program segment is repeated for the second and subsequent iterations or time steps in the second phase of a synthesis analysis unless the trial function deployment schedule is changed (transient analysis), in which case the program returns (from line 395) to statement 5 (line 267), as mentioned previously. Program execution is stopped at line 419 in a non-linear steady-state analysis when the number of iterations (NITER) exceeds the user-specified maximum permissible iteration count (MAXIT). A transient analysis stops when the check performed in line 423 indicates that either the user-

specified final time (TFINAL) or the user-specified final time step number (NFTS) has been exceeded.

Statement 60 (line 433) is executed after the simultaneous equations defining the nodal temperatures in a finite-difference analysis have been assembled. Subroutine CHLSKI, called by statement 60, uses Cholesky's algorithm to factor the symmetric conductance matrix into the product of a lower triangular matrix and its transpose. Subroutine SUB, called by statement 64 (line 434), performs the forward and backward substitution phases of the equation solution. Line 443 then directs the program back to statement 6 (line 257) to perform another iteration if the analysis being performed is steady-state and non-linear and if, in addition, the continuation variable, NCONT, has a value of 1. The continuation variable is set to zero (in line 264) at the start of each iteration and it is only assigned a value of 1 if the convergence test performed in either subroutine RADIAT (radiating cylinder analysis) or subroutine HXIO (LOECC analysis) fails. If the maximum permissible number of iterations (MAXIT) is exceeded in a steady-state non-linear analysis, line 445 prints a message to that effect. Line 449 or line 452 calls subroutine OUTPUT to print out the nodal temperatures and boundary heat fluxes unless the analysis being performed is a synthesis analysis (i.e., unless the nodal temperatures belong to a trial function) and OUTRY $\neq$ 1 (OUTRY is input variable 36h). Subroutine OUTPUT is not called in a non-linear steady-state analysis until the iterating stops, either because the solution has converged or because the maximum permissible iteration count has been exceeded.

Line 462 calls subroutine TRIAL to record the temperature vector (T) as a trial function (#NTRF) if a synthesis analysis is being performed, although line 455 causes line 462 to be bypassed if the analysis is transient in nature and the time step number (NTS) is not one at which a transient trial function should be recorded. A transient trial function is recorded when NTS equals one of the user-specified values of NTFTS (ITIM) (ITIM=1, 2, ..., NTRTF), read in subroutine DATA1. Line 470 increments the trial function count (NTRF) and the counter for the number (-1) of trial functions recorded in a transient analysis (ITIM) when NTS = NTFTS(ITIM). Line 471 directs the program back to statement 3 (line 234) in a transient analysis to compute the temperature vector for the next time step unless the final time or final time step number has been reached. If the end of the analysis has been reached and input data for another trial function is provided (this is the case when NEXT=1), line 476 directs the program back to statement 1 (line 206) to obtain the additional trial function.

If NEXT#1, line 483 checks whether a synthesis analysis is being performed. If a synthesis analysis is not being performed the analysis is finished and so program execution stops - otherwise the variable NSYN is assigned a value of 2 in line 485 to indicate that the second (substitute/weight/integrate) phase of the synthesis analysis has been reached. The variable N2D is set =0 (this variable would have been assigned a value of 1 by the user initially) in this case since future computations will be 3-D. Subroutine DZ is called to define

the axial grid geometry and redefine the value of KX (i.e., the total number of axial nodes) in line 486, following which subroutine DATA1 is called to read input data for the 3-D analysis. The r - θ boundary conditions at all of the axial nodes in the grid (i.e., for $K=1, 2, \dots, KX$) are read and/or calculated in subroutine DATA1. Subroutine ZERO is subsequently called to zero the matrices in which weighted residuals are summed (i.e., AL, CB, CRI, RES, RD and DIASUM) in the second phase of a synthesis analysis. If a transient synthesis analysis is to be performed, line 496 directs the program back to statement 4 (line 225) to start the analysis; otherwise the program returns to statement 5.

4.1.2 Subroutine DATA1 (lines 137.1-174.5)

Subroutine DATA1 reads data needed to perform a specific 2-D or 3-D analysis. The input variables read by the routine are summarized in Table 2.1h. The subroutine reads and/or calculates the r - θ boundary conditions at all of the axial nodes (K) in a finite-difference grid and, in the case of a LOECC or resistance of contact analysis, the routine can be instructed to calculate the r - θ boundary conditions at a user-specified axial location for use in a two-dimensional analysis at that location. Several such solutions, obtained

at different axial locations, could serve as the trial functions in a three-dimensional synthesis analysis.

Line 169 in subroutine DATA1 reads the variables TRANS, NTRTF, NTFCH and NTFIC. The first variable in this group, TRANS, is assigned a value of 0 for a steady-state analysis and a value of 1 for a transient analysis. The variable NTRTF is the number of transient trial functions that are to be recorded in a transient analysis, NTFCH is the number of times the trial function deployment schedule is to be changed in the substitute/weight/integrate phase of a transient synthesis analysis and the last variable, NTFIC, is the number of a previously computed trial function from which initial conditions (temperatures) are to be taken. If TRANS $\neq$ 1, NTRTF and NTFCH should both be set to zero (i.e., left blank). The variable NTFIC should be set to zero if NGROPT $\neq$ 2 because initial conditions can only be taken from a previously computed trial function when grid type c) is used. When TRANS=0, NTFIC should be set to zero unless data for the second phase of a non-linear (NLIN=1) steady-state synthesis analysis of pressure-calandria tube heat transfer are being read in, in which case NTFIC must be specified because the initial conditions for the analysis must be taken from a previously computed trial function. Initial conditions for the second phase of a steady-state radiating cylinder synthesis analysis are also taken from trial function #NTFIC when NGROPT=2 and NTFIC>0.

When NGROPT $\neq$ 2 and/or NTFIC=0, the cylinder's boundary temperatures are equated to TRADIN for the first iteration in the analysis. When

NTFIC>0 and TRANS=1, the initial conditions for any type of analysis are taken from trial function #NTFIC provided NGROPT=2, as mentioned previously.

If transient trial functions are not required, set NTRTF=0. Similarly, set NTFCH=0 if data for the second phase of a synthesis analysis are not being read in or if the trial function deployment schedule is not to be changed in the second phase calculations of a synthesis analysis.

Lines 176-220, and the READ statements found in that program segment, are only executed if TRANS=1 (i.e., in a transient analysis). The first READ statement in that program segment, found in line 177, reads the variables TMPIN, TFINAL, DENS1, DENS2, SPHT1, SPHT2 and NFTS. The first variable in that group, TMPIN, is the temperature to which all of the nodes in a finite-difference grid are equated at the beginning of a transient analysis (i.e., it is the initial temperature). The value of TMPIN is not used if NGROPT=2 and NTFIC>0, however, since the initial temperatures are taken from trial function #NTFIC in that case. The next variable, TFINAL, is the final time to which the analysis is to be taken. The variables DENS1 and SPHT1 are the density and specific heat respectively of the material found between the innermost radius (RI) of the cylindrical object being modelled and the radial node MX while DENS2 and SPHT2 are the density and specific heat of the material found outside the inner region. In a thermal analysis of a nuclear fuel element, for example, DENS1 and SPHT1 would be the density and specific heat

respectively of the fuel while DENS2 and SPHT2 would be the density and specific heat of the sheath material. In a LOECC analysis, DENS1 and SPHT1 would apply to the pressure tube material while DENS2 and SPHT2 would apply to the calandria tube material.

The final variable read by line 177, NFTS, is the number of the time step at which the analysis is to stop (i.e., it is the number of the final time step). The variables TFINAL and NFTS perform much the same function and a transient analysis stops when either the time exceeds TFINAL or the time step number exceeds NFTS (i.e., the first condition satisfied stops the analysis).

The next READ statement, located in line 190, is only executed when a transient synthesis analysis is in its second phase and the data needed in the 3-D computations are being read in. The READ statement located in line 190 reads the numbers of the time steps at which trial function deployment schedule changes are to be made, designated NTSTFC(NXT), $NXT=1, 2, \dots, NTFCH$. A new trial function deployment schedule is read by subroutine TFREAD at each of these time steps.

The READ statement in line 203 is only executed in a transient finite-difference analysis being performed to obtain transient trial functions for a synthesis analysis since it reads values of NTFTS(ITIM), ($ITIM=1, 2, \dots, NTRTF$) which are the numbers of the time steps at which trial functions are to be recorded. When $NTS=NTFTS(ITIM)$ (NTS is the time step count), the vector of nodal temperatures (T) is recorded as a trial function by subroutine TRIAL.

The READ statement found in line 212, which reads the variables DT(NDT), NTC(NDT) and NEXT, is executed in all transient analyses. The first variable, DT(NDT), is the NDTth time step size, NTC(NDT) is the time step (NTS) at which the use of DT(NDT) is discontinued and NEXT is a variable used to indicate whether more values of DT(NDT), NTC(NDT) and NEXT are provided. If NEXT=1, the check performed in line 216 causes the READ statement in line 212 to be re-executed. The first time step size, DT(1), is used for time steps (NTS) lying in the range $1 \leq \text{NTS} < \text{NTC}(1)$ while the NDTth time step size ($\text{NDT} > 1$) is used when $\text{NTC}(\text{NDT}-1) \leq \text{NTS} < \text{NTC}(\text{NDT})$. Time steps for which $\text{NTS} \geq \text{NTDT}$ (NTDT is defined in line 217) use the time step size DT(NTDT+1), which is the last time step size read in (i.e., the one for which NEXT=0).

The READ statement in line 221 is executed in both transient and steady-state analyses, as are all of the remaining READ statements in subroutine DATA1. The first variable read by that statement, NLIN, is assigned a value of 1 if a pressure-calandria tube heat transfer analysis is being performed with non-linear gap conductances and a value of 0 otherwise. If IMPECC=1 and NLIN=0, a pressure-calandria tube heat transfer analysis is performed with linear gap conductances, specifically with gap conductances calculated using a constant value (TCGAS) for the gas conductivity and a circumferentially uniform and temperature independent radiative heat transfer coefficient (HRADIN).

The second variable read by line 221, RADBND, is assigned a value of 1 if heat transfer in a cylinder radiating to the surroundings

is being analysed. For all other analyses, RADBNB must be assigned a value of 0. The third variable read, MAXIT, is the maximum permissible iteration count for a non-linear (i.e., NLIN=1 or RADBNB=1) steady-state heat transfer analysis and EPSLN, the fourth variable, is the convergence parameter for such an analysis. A steady-state non-linear analysis is assumed to have converged when each nodal temperature varies by an amount less than EPSLN from one iteration to the next. TRADIN is the boundary temperature used to linearize the radiative boundary condition in the first iteration of a steady-state radiating cylinder analysis, although it is not used if NTFIC > 0 and NGROPT=2 because the boundary temperatures in trial function #NTFIC are used for that purpose in that case. HRADIN is a radiative heat transfer coefficient used across the gas gap between the pressure and calandria tubes in the first iteration of a steady-state, non-linear, (NLIN=1) finite-difference LOECC analysis or in all of the time steps of a linear (NLIN=0) transient LOECC analysis. HRADIN can also be used in a linear steady-state LOECC analysis, which is solved without iterating of course. BETA is an underrelaxation factor (i.e., $0 < \text{BETA} < 1$) used in line 31 of subroutine HXIO (right-hand line #371.5) to update the gap conductances in either a transient or a steady-state, non-linear finite-difference LOECC analysis. The updated gap conductances in each iteration or time step are calculated by adding BETA times the gap conductances calculated using the most recent nodal temperatures to (1-BETA) times the gap conductances used in the previous time step or iteration.

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BETA is only used in finite-difference LOECC analyses because Newton-Raphson's method is used to linearize the gap conductances in the second phase of a synthesis analysis. TCINTG and SLOPG are the intercept and slope respectively of a linear thermal conductivity vs temperature relationship for the gas in the gap between the pressure and calandria tubes and REXCO is the radiation exchange coefficient, defined by the expression

$$\text{REXCO} = \frac{\sigma}{\frac{1}{\epsilon_{PT}} + \frac{\text{PTOR}}{\text{CTIR}} \left(\frac{1}{\epsilon_{CT}} - 1 \right)} \quad 4.1$$

for radiative heat transfer across the gap. The variables σ , ϵ_{PT} and ϵ_{CT} in Eq. 4.1 are the Stefan-Boltzmann constant ($5.669 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$) and the emissivities of the pressure tube outer and calandria tube inner surfaces respectively. REXCO must also be specified in a radiating cylinder analysis (i.e., when RADBN=1), when it is defined by an expression analogous to Eq. 4.1. If the cylinder is radiating to the surroundings, $\text{REXCO} = \epsilon$, where ϵ is the emissivity of the cylinder's outer surface.

The first variable read by the READ statement in line 224, NVCON, is assigned a value of 1 if the thermal conductivity of the material in the cylindrical object being modelled is to be considered temperature dependent. If a constant thermal conductivity is to be used, NVCON must be set to zero. The variables TCINTS and SLOPS are the intercept and slope respectively of the solid's thermal conductivity vs temperature relationship, which is assumed linear. TCINTS and

SLOPS should be left blank if NVCON=0. If NVCON=1, the cylindrical object being modelled must consist of a single material because the conductivity vs temperature relationship defined by the variables TGINTS and SLOPS is applied to the entire object.

The constant conductivities CK1 and CK2, read by line 236, can be used in analyses involving two distinctly different materials separated by a cylindrical interface at radial node MX. NVCON should not be assigned a value of 1 when input variables for the second phase of a synthesis analysis are being read in and the analysis being performed is not a LOECC analysis (i.e. when IMPECC=0) because the coding for other cases has not been included in the program.

Similarly, NVCON should not be assigned a value of 1 in steady-state finite-difference analyses unless NLIN=1 or RADBN=1 (i.e., unless a LOECC analysis with non-linear gap conductances or a radiating cylinder analysis is being performed) because there is no provision for iterating and performing convergence tests in analyses other than these. These capabilities could be added very easily however. Any transient finite-difference analysis can be performed with NVCON=1 because the nodal temperatures from one time step are used to calculate internodal thermal conductivities for the next time step. The constant conductivities, CK1 and CK2, are used in the first time step or iteration of a finite-difference analysis to evaluate internodal conductances. In a synthesis analysis, NVCON must be assigned the same value in all of the analyses making up the complete analysis, i.e., it must be assigned the same value in all of the first phase

(trial function) and second phase (3-D) computations. Thus trial functions obtained with NVCON=0 cannot be used in second phase calculations for which NVCON=1 (or vice-versa).

The first variable read by line 228, TCGAS, is a gas conductivity used in linear LOECC and "resistance of contact" analyses. It is also used in the first iteration of a steady-state non-linear finite-difference LOECC analysis to evaluate gas conduction contributions to the pressure-calandria tube gap conductances. Gas conductances in LOECC and "resistance of contact" analyses are calculated using a non-continuum gas conductivity model, described in Ch I.3, which expresses the effective thermal conductivity (k_g) of a gas in a gap of thickness GGAP as,

$$k_g = \frac{k_{og}}{1 + [4\nu L_m (\alpha_1 + \alpha_2 - \alpha_1 \alpha_2) / GGAP \alpha_1 \alpha_2 Pr (\nu + 1)]} \quad 4.2$$

where ν = ratio of specific heats (≈ 1.4 for air at room temperature)

Pr = Prandtl number of gas (≈ 0.7 for air)

L_m = mean free path of gas molecules

k_{og} = gas conductivity under continuum conditions (i.e., TCGAS or gas conductivity calculated using TCINTG and SLOPG)

α_1, α_2 = accommodation coefficients of surfaces 1 and 2

Defining CCNN, which is the second variable read in line 228, by the expression,

$$CCNN = \frac{4\nu L_m (\alpha_1 + \alpha_2 - \alpha_1 \alpha_2)}{\alpha_1 \alpha_2 Pr (\nu + 1)} \quad 4.3$$

allows Eq. 4.2 to be rewritten as,

$$k_g = \frac{k_{og}}{T + \frac{CCNN}{GGAP}}$$

$$= \frac{k_{og}}{T + KP}$$

where $KP = \frac{CCNN}{GGAP}$

The READ statement found in line 236 of subroutine DATA1 reads the variables H1, CK1, CK2, CCOND, OUTRY and NEXT. The variable H1 is the interfacial contact conductance between an inner cylinder (eg., a fuel pellet) and an outer concentric sheath in all but LOECC and tube (TUBE=1) heat transfer analyses, when it is the heat transfer coefficient on the calandria tube outer surface or the heat transfer coefficient on the tube inner surface respectively. In a tube heat transfer analysis, the tube inner surface exchanges heat with a coolant at bulk temperature BT through the film coefficient H1. For simplicity, H1 is assumed constant and circumferentially uniform in all analyses. The thermal conductivities CK1 and CK2, read next, apply to the inner and outer cylinders respectively in a two cylinder problem, as mentioned previously. If NVCON=1 and input data for the second phase of a synthesis (LOECC) analysis are being read in, set CK1=CK2=1.0. CCOND is the contact conductance used within the contact patch in a LOECC or "resistance of contact" analysis.

When OUTRY, which is the next variable read by line 236, is assigned a value of 1 in an analysis being performed to obtain a trial

function for a synthesis analysis, the trial function nodal temperatures and boundary heat fluxes are printed out at the end of the 2-D analysis. The trial function results are not printed out when OUTRY is set to zero. The last variable read by line 236, NEXT, is assigned a value of 1 if a synthesis analysis is being performed and another trial function is required, and a value of 0 otherwise. The program will return to subroutine DATA1 after completing the present analysis to read the input data for another trial function if NEXT=1.

The data card for the READ statement found in line 240 can be left blank if a previously computed trial function is not to be used to define axial boundary conditions in a 2-D analysis. If NZFLO=1, where NZFLO is the first variable read by line 240, the r - θ temperature distribution in trial function #NTFBC is applied to an r - θ plane located a distance DZBC from the plane on which the r - θ temperature distribution is being calculated, as shown in Fig. 4.1. Applying axial boundary conditions in 2-D analyses increases the choice of trial function boundary conditions available to the user above the normal choice afforded by varying only those on r - θ boundaries.

The lines in subroutine DATA1 following and including line 255 read and/or calculate the r - θ boundary conditions at every axial node (K) in the grid (i.e., for $K=1, 2, \dots, KX$). If NAX=0, where NAX is the first variable read by line 244, the r - θ boundary conditions at each axial node must be completely specified by the user. A DO loop, which starts at line 252 and ends at line 334, causes the input variables seen in line 255 to be read in for each axial node. The

Fig. 4.1 Axial Boundary Condition

first of these variables, $HTGEN(K)$, is the volumetric heat generation rate at axial node K while the second variable, $HFLZ(K)$, is the heat flux prescribed at axial node K over the inner surface of the cylindrical object being modelled. In a LOECC analysis, for example, $HFLZ(K)$ would be the heat flux supplied to the pressure tube inner surface at axial node K . The remaining variables read by line 255, specifically $TOUT$, $HOUT$, TEX , HEX and $MZ1$, define very simple $r-\theta$ boundary conditions for the K th axial node. As seen in Fig. 4.2, the outer surface nodes (I,K) for which $I \leq MZ1$ (where $MZ1=6$ in Fig. 4.2) exchange heat with a sink at temperature TEX through a heat transfer coefficient HEX . The value of $MZ1$ and the size of the angular subdivisions used between circumferential nodes 1 and $MZ1$ are adjusted in a contact analysis so that the length of the circular arc over which HEX is prescribed equals, if possible, the contact half-width at axial node K . The value of HEX should be the same in this case as the contact conductance, $CCOND$, read by line 236. If the contact does not extend to the K th axial node, set $MZ1=0$.

It is assumed that the remaining outer surface nodes (i.e., those for which $I > MZ1$) exchange heat with a sink at temperature $TOUT$ through a circumferentially uniform heat transfer coefficient $HOUT$. In most analyses, $TOUT$ and TEX will be equal. In a radiating cylinder analysis ($RADBND=1$), $TOUT$ and TEX apply to the surroundings.

The simple $r-\theta$ boundary conditions defined completely by the variables TEX , HEX , $TOUT$, $HOUT$ and $MZ1$ must obviously be modified in a LOECC or "resistance of contact" analysis. Interfacial conditions

Fig. 4.2 R-θ Boundary Condition Definition.

within the contact and the lateral extent of the contact continue to be defined by the variables HEX and MZ1 in those analyses, but the other boundary conditions are altered. The heat transfer coefficient HOUT becomes the pressure/calandria tube gap conductance in a LOECC analysis and the temperatures TEX and TOUT are both set equal to the moderator temperature. The heat transfer coefficient HOUT is the gas conductance between the contact specimens in a resistance of contact analysis and TEX and TOUT are set equal to the temperature of a fictitious isothermal plane coinciding with the contact plane within the contact patch and located midway between the surfaces in regions outside of the patch. The program is instructed to calculate the gap conductance (HOUT) as a function of circumferential position (θ) at each axial node K (i.e., HOUT is calculated at each combination of I and K) in LOECC and resistance of contact analyses by setting NAX=1, where NAX is read by line 244. The second variable read by line 244, XPP, is the axial (z) position at which $HOUT(=f(\theta))$ is to be evaluated if the analysis being performed is two-dimensional. If the analysis being performed is 3-D, which will be the case if $KX > 1$, line 253 assigns XPP the correct value for the Kth axial node (i.e., AXPOS(K)). The user-specified value of XPP is not used in that case. Several 2-D solutions can be obtained in this way, using the r- θ boundary conditions at different axial locations (i.e., obtained with different values of XPP), for eventual use as the trial functions in a 3-D LOECC or resistance of contact analysis.

The READ statement in line 265, which is only executed when input data for the second phase of a synthesis analysis are being read in, reads the number of trial functions that are to be used at the Kth axial node, designated $NTRI(K)$, and the numbers of those trial functions, stored temporarily as $NT2(NK)$ ($NK=1, 2, \dots, 10$). Note that a maximum of ten trial function numbers can be read by line 265. The trial function numbers in the vector $NT2$ are reassigned to their permanent storage locations in the matrix $NTRK(K, NK)$ ($K=1, 2, \dots, KX; NK=1, 2, \dots, 10$) in lines 268-270. Lines 277-278 perform calculations to determine the semi-bandwidth (MBA) of the left-hand coefficient matrix for the system of equations defining the blending vector (Z) in a synthesis analysis. The total number of equations, designated $NEQU$, is incremented, as appropriate, in line 283.

A DO loop starting at line 291 and ending at line 316 sweeps through the circumferential nodes (I) lying in the range $MZ1+1 \leq I \leq IX$. If $NAX \neq 1$, the heat transfer coefficients at the outer surface nodes (I, K) designated $HX(I, K)$, are equated in line 313 to the circumferentially uniform value of $HOUT$ read in line 255. The sink temperatures at the outer surface nodes, designated $TC(I, K)$, are equated to the user-specified temperature $TOUT$ in line 315. If $NAX=1$ and a LOECC analysis is being performed, lines 299-304 calculate the gap (GGAP) between the pressure and calandria tubes and the corresponding gas conductance ($HOUT$) when the node I, K is in a region remote from the contact, in other words when $\theta \geq 0.2$ and $z \geq XCH (=0.2 * DGART)$, where $DGART$ is L_{SP} , the spacing between the two most central garter springs, as discussed in

Appendix B. Subroutine HGAS is called by line 309 to evaluate the same parameters when the node I,K is near the contact. The distance y shown in Fig. B.1 and defined by Eq. B.2 is designated GAPZ in subroutine DATA1, where it is calculated in line 299. The gap $\delta(\theta, z)$ defined by Eq. B.3 is calculated as GGAP in line 301. The total gap conductance at node I,K is calculated in line 311 by adding the user-specified radiative heat transfer coefficient, HRADIN, to the previously calculated gas conductance. The overall heat transfer coefficient $HX(I,K)$ is equated to the result of this addition in line 313.

If $NAX=1$ and a "resistance of contact" analysis is being performed (i.e., $RESIST=1$), subroutine HGAS is called to evaluate the gap size and the corresponding gas conductance between the contact specimens at each surface node I,K. The heat transfer coefficient $HX(I,K)$ is equated to the gas conductance in line 313, as before.

Lines 323-326 in subroutine DATA1 equate the heat transfer coefficients $HX(I,K)$ and sink temperatures $TC(I,K)$ at surface nodes (I,K) with $1 \leq I \leq MZ1$ to the user-supplied heat transfer coefficient HEX and sink temperature TEX respectively. Lines 331-333 initialize the boundary temperatures and boundary heat fluxes for a radiating cylinder analysis.

Many of the variables read by subroutine DATA1 may not be applicable in a particular analysis. Except as otherwise noted, those variables can be left blank.

The version of subroutine DATA1 described above, termed version 1, implements the contact approximation technique designated method 2 in Ch. II.2. Versions of subroutine DATA1 implementing the contact approximation techniques designated methods 3 and 4 in Ch. II.2 are included in Listing 2 as versions 2 and 3 respectively. All versions of subroutine DATA1 read the same input data.

4.1.3. Subroutine GROPT0 (lines 174.6-181.3)

This subroutine contains nested DO loops which sweep through all of the circumferential nodes I (=1, IXX) and radial nodes J (=2, JX) on the r- θ plane at the Kth axial node when NGROPT=0. Subroutine NORNOD is called at each node I,J,K to calculate terms in the node's finite-difference equation. The coding in subroutine GROPT0 is completely self-explanatory with the comment statements provided and hence will not be explained further.

4.1.4. Subroutine GROPT1 (lines 181.4-193.8)

This subroutine contains nested DO loops which sweep through all of the circumferential nodes I(=1, IXX) and radial nodes J (=2, JX) on the r- θ plane at the Kth axial node when NGROPT=1. One of subroutines NORNOD, SMARAD and BIGRAD is called at each node I,J,K to calculate terms in the node's finite-difference equation. The coding in subroutine GROPT1 is self-explanatory with the comment statements provided and hence will not be explained further.

4.1.5. Subroutine GROPT2 (lines 193.9-213.1)

Subroutine GROPT2 contains nested DO loops which sweep through all of the circumferential nodes I (=1, IXX) and radial nodes J (=JI, JX) on the r- θ plane at the Kth axial node when NGROPT=2. One of subroutines NORNOD, SMARAD, BIGRAD, SMACIR and BIGCIR is called at each

node to calculate terms in the node's finite-difference equation.

The subscripts (I) for the circumferential nodes are varied by the outer DO loop, which starts at line 34 and ends at line 191. The inner DO loop starting at line 39 and ending at line 190, sweeps through the radial nodes (J) on the Ith radial grid line. The first radial node that is actually used on the Ith radial grid line is designated JIN(I) - hence this is the first node visited by the inner DO loop, as seen in line 39 (note that JI is equated to JIN(I) in line 36). The loop increment, ISP, seen in line 39 is equated to IS(I,J) (in line 80) within the DO loop so that radial nodes which are not actually used at a particular circumferential node (I) can be skipped. Entries in the array IS(I,J) are defined in subroutine ISIJ before subroutine GROPT2 is called.

Lines 45-77 define variables IA, IB, IC and ID that are used in subroutines NORNOOD, SMARAD, etc., to determine the radial grid lines on which nodes which communicate with node L are located. When node L is located at the point of intersection of orthogonal grid lines I and J, neighbouring grid lines are referenced in the manner shown in Fig. 4.3. The variables IC and ID locate neighbouring radial grid lines in situations such as that shown in Fig. 4.4.

The values of $\overline{\Delta\theta}$ and $\overline{\Delta r}$ used in Eqs. A.2-A.7, designated ANTOT and DTOT respectively in the program, are defined in lines 86 and 87 respectively. The thermal conductivity (ϵK) of the material at the I, Jth node is defined in lines 88-90 along with the product of the material's specific heat and density, designated THCAP. The node

Fig. 4.3 Grid line numbering at a type 0 node.

Fig. 4.4 Radial grid line numbering at node types 3 and 4.

number count (L) is incremented by 1 in line 93, after which the temperature of node L (i.e., $T(L)$) is set to an initial temperature TMPIN in line 100, if required. The initial temperature can also be obtained from trial function #NTFIC, as seen in line 101.

Entries in the node number difference vectors JL and JP, which are used in subroutines NORNOD, SMARAD, etc. are zeroed in lines 106-109. Variables JA and JB, which represent the number of radial nodes (J) between node L and nodes L-1 and L+1 respectively, are defined in lines 113-118 (see Fig. 4.4).

Lines 123-132 use temperatures obtained from trial function #NTFIC to initialize various temperatures and surface heat fluxes when NTFIC>0 (input variable 4h). When NSYN=2 (i.e., in the substitute/weight/integrate phase of a synthesis analysis), the temperature of the L, Kth node is designated TMPIN while TMP2 is the temperature of node L+1, K when J=MX and that of node L-1, K when J=NIS. Lines 141-148 define nodal temperatures in the second phase of a transient synthesis analysis when the trial function schedule has just been changed. Variables TMPIN and TMP2 have the same meanings as before. In a radiating cylinder analysis (i.e., when RADBN=1), lines 149-150 initialize the temperatures of outer surface nodes (i.e., nodes at J=JX) and the surface heat fluxes.

Lines 163-189 determine which of the subroutines NORNOD, SMARAD, etc., should be used to calculate terms in the finite-difference equation for r- θ node L, and call that subroutine..

4.1.6 Subroutine NORNOD (lines 213.2-231.1)

Subroutine NORNOD calculates terms in finite-difference equations for normal nodes of the type found at orthogonal grid line intersections, whether in the interior or on the boundary of the object being modelled. Terms in the general nodal equation (Eq. A.15) are calculated only as required.

Lines 24-27 in subroutine NORNOD determine the heat transfer coefficient on the outer surface of the cylinder being modelled, designated HX2, and the gap conductance between inner and outer cylinders (eg., the pressure and calandria tubes in a CANDU reactor) in a two cylinder analysis, designated HX1. Lines 36-65 then calculate the interior r- θ conductances for the node in question, designated C(1), ..., C(4), where

$$C(1) \equiv K(i,j,k)_{i-1}$$

$$C(2) \equiv K(i,j,k)_{i+1}$$

$$C(3) \equiv K(i,j,k)_{j-1}$$

$$C(4) \equiv K(i,j,k)_{j+1}$$

and lines 69-80 calculate the axial conductances C(5) ($=K(i,j,k)_{k+1}$) and C(6) ($=K(i,j,k)_{k-1}$). These conductances are defined by Eqs. A.2-

A.7 when

$$\Delta\theta_a \leftarrow \text{THA}$$

$$\Delta\theta_b \leftarrow \text{THB}$$

$$\Delta r_a \leftarrow \text{DA}$$

$$\Delta r_b \leftarrow \text{DB}$$

$$\Delta z_a \leftarrow \text{DZA}$$

$\Delta z_b \leftarrow DZB$
 $\overline{\Delta \theta} \leftarrow ANTOT$
 $\overline{\Delta r} \leftarrow DTOT$
 $\overline{\Delta z} \leftarrow DZTOT$
 $r_i \leftarrow R$

and

$k_s \leftarrow CK$

In a finite-difference analysis with temperature-dependent thermal conductivity, $NVC=1$ after the first time step or iteration.* Lines such as line 56 calculate the thermal conductivity (k_s or CK) of the cylinder material at the average of the temperatures of node I,J,K ($=L$) and its appropriate neighbour in that case.

As seen in lines 53-60, where the conductance $K(i,j,k)_{j-1}$ ($=C(3)$) is calculated, a typical conductance calculation is started by setting the conductance in question ($C(3)$ in this case) to zero.* A check is then performed to see if the conductance in question actually exists at node I,J,K and the conductance is not calculated if it doesn't. This check is performed in line 54 during the calculation of $C(3)$, since $C(3)$ should not be calculated if $J=NIS$ (i.e., if node I,J,K lies on the inner surface of the outer cylinder, eg., fuel sheath, in a two cylinder analysis) or if $J=2$ (i.e., if node J is the innermost radial node). Line 56 then calculates an average thermal conductivity with which to calculate $C(3)$ if $NVC=1$, as mentioned previously, after which line 57 calculates $C(3)$ using Eq. A.5. The number of nodes

*NVC is assigned a value by lines 280-281 in the main program.

between node I,J,K and node I,J-1,K (i.e., 1, since the radial nodes are visited most rapidly) is set equal to JP(2) in line 58 and -C(3) is stored as CBW (MBW-JP(2)) (or as CBW(MBW-1) since JP(2)=1) in line 60 if a finite-difference analysis is being performed. If NSYN=2 (i.e., if the substitute/weight/integrate phase of a synthesis analysis has been reached), CBW(MBW-JP(2)) is not assigned a value since the conductance matrix is not required in that case and hence is not stored. In a finite-difference analysis, only those entries that lie to the left of and on the diagonal of the conductance matrix are stored, or stated in another way, conductances between a given node I,J,K and nodes visited after it (i.e., node I+1, J,K; I,J+1,K and I,J,K+1) are not stored since it is assumed that the conductance matrix is symmetric. Hence $K(i,j,k)_{j-1}$ (=C(3)) is stored temporarily in the vector CBW for later transferral to the conductance matrix CX in subroutine CONMAG, but $K(i,j,k)_{j+1}$ (=C(4)) is not stored in CBW, for instance. Note that the diagonal entry for each finite-difference equation is stored at the end of CBW (i.e., as CBW(MBW), where MBW is input variable #3a) and that all of the other entries (conductances) are stored before it.

Although the conductance between a given node I,J,K and any neighbouring node visited after it is not stored in CBW, it is still necessary to know the value of the conductance because it contributes to the diagonal coefficient for node I,J,K. In addition, both the conductance and the subscript of the neighbouring node must be known in the substitute/weight/integrate phase of a synthesis analysis

(i.e., when NSYN=2) in order to calculate the residual for node I,J,K ($\equiv$ L,K). The subscript of any node visited before node L (or L,K in a synthesis analysis) is calculated as L-JP(n), where JP(n) is defined when the conductance between node L and the neighbour in question is calculated while the subscript of any node visited after node L is expressed as L+JL(n) if NSYN=2 (see line 65 for example). As in the case of JP(n), JL(n) is defined when the conductance between nodes L and L+JL(n) is calculated. The subscript (location) of each node I,J,K is stored as NNUM(I,J,K), so values of JL(n) and JP(n) are readily calculated in the manner shown in lines 48 and 39. The variables IA and IB both equal 1 if NGROPT=0 or 1 (see lines 172-173 in the main program), but are assigned values which may be different from 1 when NGROPT=2 (see subroutine GROPT2).

If necessary, lines 85-99 in subroutine NORNOD calculate the gap conductance (area integrated value) between inner and outer cylinders in a two cylinder analysis, designated C(7). Subroutine HXIO is called by line 93 to evaluate the heat transfer coefficient between pressure and calandria tubes in a CANDU reactor if a finite-difference non-linear LOECC analysis is being performed.

Lines 103-108 calculate the convective boundary conductance (C(8)) for a node located on the outer boundary of the cylindrical object being modelled. The surface area of the boundary node (I,K) is stored as SAR(I,K).

Line 110 calculates the cross-sectional area of the finite-difference cell for node L in the r- θ plane and stores its value

as AREA(L). The nodal heat generation rate is calculated by line 115 if $J \leq MX$ (i.e., if node L is located in the inner cylinder in a two cylinder analysis) since nodes for which $J > MX$ would be found in the sheath (where $q_v = 0$) in a fuel element heat transfer analysis.

All of the internodal conductances ($C(1)$, ..., $C(8)$) are summed in line 119 to yield the diagonal coefficient for the Lth finite-difference equation, stored temporarily as CTOT. The nodal capacitance $\rho_m c_m V / \Delta t$ (=HGEN) is added to the diagonal coefficient in a transient analysis, as seen in lines 125-126, following which terms needed to apply the boundary conditions are added as necessary to the right-hand side entry for the Lth node (QQ or Q(L)).

Subroutine SUBST1 is called by line 112 to substitute the trial functions into the finite-difference equation for the L,Kth node in the substitute/weight/integrate phase of a synthesis analysis. The resulting residual is also weighted by the trial functions and added to the appropriate entries in the matrices constituting the system of equations defining the blending vector, Z, in subroutine SUBST1.

Subroutine RADIAT is called by line 170 to compute terms needed to linearize the radiative boundary condition in a radiating cylinder analysis using the Newton-Raphson method. The diagonal coefficient CTOT is subsequently stored as CBW(MBW) in line 171, following which subroutine CONMAG is called to transfer the contents of CBW to the conductance matrix CX in a finite-difference analysis.

4.1.7 Subroutine BIGRAD (lines 231.2-253.1)

This subroutine calculates the terms in finite-difference equations for type 1 nodes. Node type designations and the applicable finite-difference expressions are described in Appendix A. Subroutine BIGRAD is similar to subroutine NORNOD in structure and hence will only be described briefly.

Lines 39-115 in subroutine BIGRAD calculate the r - θ conductances for node I,J,K, designated C(1), C(2), ..., C(9), where

$$C(1) \equiv K_{A-B} \text{ (Fig. 4.5, node A is node I,J,K)}$$

$$C(2) \equiv K_{A-D}$$

C(3) = gap conductance between inner and outer cylinders in a two cylinder analysis

$$C(4) \equiv K_{6-8} \text{ (Fig. A.6 and Eq. A.16, node 6 is node I,J,K)}$$

$$C(5) \equiv K_{6-2} \text{ (Fig. A.6 and Eq. A.16, node 6 is node I,J,K)}$$

$$C(6) \equiv K_{6-5} \text{ (Fig. A.6 and Eq. A.16, node 6 is node I,J,K)}$$

$$C(7) \equiv K_{6-1} \text{ (Fig. A.6 and Eq. A.16, node 6 is node I,J,K)}$$

$$C(8) \equiv K_{6-7} \text{ (Fig. A.6 and Eq. A.16, node 6 is node I,J,K)}$$

and $C(9) \equiv K_{6-3} \text{ (Fig. A.6 and Eq. A.16)}$

The conductances C(1) and C(2) do not become non-zero unless the comparisons performed in line 42 indicate that node I,J,K is similar to node A in Fig. 4.5. Also, C(5), C(8) and C(9) only become non-zero when the comparisons performed in lines 81 and 104 indicate that node I,J,K is not similar to node A (i.e., node I,J,K must be similar to node 6 in Fig. A.6 for C(5), C(8) and C(9) to be calculated).

Fig. 4.5 Innermost radial node at which the grid is circumferentially discontinuous ($N=1, NR$)

Lines 123-124 calculate the r - θ area of the finite-difference cell for the L th node, designated $AREI$ and $AREA(L)$. Lines 129-140 then use that area to calculate the axial conductances $C(10)$ ($=K(i,j,k)_{k-1}$) and $C(11)$ ($=K(i,j,k)_{k+1}$). If node I,J,K is an outer surface node, the convective boundary conductance, $C(12)$, is calculated by lines 144-149.

The remaining code in subroutine BIGRAD is similar to that found in subroutine NORNOD and hence will not be explained further.

4.1.8 Subroutine SMARAD (lines 253.2-267.3)

This subroutine calculates the terms in finite-difference equations for type 2 nodes. Lines 38-73 calculate the r- θ conductances for node I,J,K designated C(1), C(2), ..., C(5), where,

$$C(1) \equiv K_{3-4}$$

$$C(2) \equiv K_{3-2}$$

$$C(3) \equiv K_{3-9}$$

$$C(4) \equiv K_{3-7}$$

$$\text{and, } C(5) \equiv K_{3-6}$$

for the conductances K_{3-4} , K_{3-2} , etc., used in Eq. A.17. Line 52 checks that the angle β shown in Fig. A.6 is acute and stops program execution if it is not,

Lines 77-88 calculate the axial conductances $C(6) (=K(i,j,k)_{k-1})$ and $C(7) (=K(i,j,k)_{k+1})$. The remaining code in subroutine SMARAD is similar to that found in subroutine NORNOD and hence will not be explained further.

4.1.9 Subroutine SMACIR (lines 267.4-281.0)

This subroutine calculates the terms in finite-difference equations for type 3 nodes and is similar to subroutine SMARAD. Lines 34-63 calculate the r- θ conductances for node I,J,K, designated C(1), C(2), ..., C(6), where

$$C(1) \equiv K_{3-4}$$

$$C(2) \equiv K_{3-2}$$

$$C(3) \equiv K_{3-9}$$

$$C(4) \equiv K_{3-7}$$

$$C(5) \equiv K_{3-6}$$

for the conductances K_{3-4} , K_{3-2} , etc., used in Eq. A.17 and,

$C(6)$ = gap conductance between inner and outer cylinders in a two cylinder analysis.

Line 49 checks that the angle β shown in Fig. A.6 is acute and stops program execution if it is not.

Lines 72-83 calculate the axial conductances $C(7)$ ($=K(i,j,k)_{k-1}$) and $C(8)$ ($=K(i,j,k)_{k+1}$). The remaining code in subroutine SMACIR is similar to that found in subroutine NORNOD and hence will not be explained further.

4.1.10 Subroutine BIGCIR (lines 281.1-298.2)

This subroutine calculates the terms in finite-difference equations for type 4 nodes and is similar to subroutine BIGRAD. Lines 34-89 calculate the r- θ conductances for node I,J,K, designated $C(1)$, $C(2)$, ..., $C(9)$, where

$$C(1) \equiv K_{A-B} \text{ (Fig. 4.6, node A is node I,J,K)}$$

$$C(2) \equiv K_{A-D} \text{ (Fig. 4.6, node A is node I,J,K)}$$

$C(3)$ = gap conductance between inner and outer cylinders in a two cylinder analysis

$C(4) \equiv K_{6-8}$ (Fig. A.6 and Eq. A.19, node 6 is node I,J,K)
 $C(5) \equiv K_{6-3}$ (Fig. A.6 and Eq. A.19, node 6 is node I,J,K)
 $C(6) \equiv K_{6-7}$ (Fig. A.6 and Eq. A.19, node 6 is node I,J,K)
 $C(7) \equiv K_{6-2}$ (Fig. A.6 and Eq. A.19, node 6 is node I,J,K)
 $C(8) \equiv K_{6-5}$ (Fig. A.6 and Eq. A.19, node 6 is node I,J,K)
 and, $C(9) \equiv K_{6-1}$ (Fig. A.6 and Eq. A.19, node 6 is node I,J,K)

The conductances $C(1)$ and $C(2)$ do not become non-zero unless the comparisons performed in line 37 indicate that node I,J,K is similar to node A in Fig. 4.6. Also, $C(7)$, $C(8)$ and $C(9)$ only become non-zero when the comparisons performed in line 77 indicate that node I,J,K is not similar to node A.

Fig. 4.6 Last circumferential node at which the grid is radially discontinuous ($N=I, NI$)

Lines 97-99 calculate the $r-\theta$ area of the finite-difference cell for the L th node, designated $ARE1$ and $AREA(L)$. Lines 103-114 then use that area to calculate the axial conductances $C(10) (=K(i,j,k)_{k-1})$ and $C(11) (=K(i,j,k)_{k+1})$.

The remaining code in subroutine $BIGCIR$ is similar to that found in subroutine $NORNOD$ and hence will not be explained further.

4.1.11 Subroutine CONMAG (lines 298.3-302.0)

Subroutine $CONMAG$, transfers elements of the vector CBW , which contains finite-difference coefficients found within the left-hand bandwidth of a particular row (L) in the conductance matrix, to a singly-subscripted form of the banded, symmetric, conductance matrix (CX). Only the elements from the first non-zero entry in each row up to and including the diagonal term are stored and the rows are lined up end to end in CX . The N th entry in a pointer vector, $NDIAG$, indicates the location of the diagonal coefficient for the N th finite-difference equation in CX . This storage mode, termed profile storage, is described in greater detail by Jennings (1).

The finite-difference coefficients found within the left-hand bandwidth of each row in the conductance matrix are computed in one of subroutines $NORNOD$, $BIGRAD$, $SMARAD$, $BIGCIR$ or $SMACIR$, depending on the node type. The coefficients for the L th node are stored at the end of CBW rather than at the beginning (i.e., the diagonal coefficient for the node is stored as $CBW(MBW)$, where MBW is input

variable #3a) because it is not known when the computation of terms in the Lth finite-difference equation starts what the left-hand bandwidth of the equation will be. If the diagonal coefficient is stored as CBW(MBW), the coefficient to the left of the diagonal will not extend beyond the beginning of CBW unless the user has underestimated MBW.

Lines 20-24 in subroutine CONMAG determine the left-hand bandwidth (-1) of the Lth finite-difference equation, designated NBW, from the values of JP(JJ) defined in the subroutine used to compute the finite-difference coefficients (eg., NORNOD). The location of the first non-zero entry in CBW (entry LM) is subsequently calculated from the values of MBW and NBW in line 25. Lines 29-32 then insert the entries in CBW between and including the entries LM and MBW into the conductance vector, CX. The location in CX of the diagonal element for the Lth row is stored as NDIAG(L) in line 36.

4.1.12 Subroutine SUBST1 (Lines 302.1-323.1)

Trial functions are substituted into the finite-difference equation for the L,Kth node to obtain its residual and that residual is then weighted by the trial functions in subroutine SUBST1. Subroutine SUBST1 also adds the resulting terms to the appropriate entries in the matrices AL, CB, CRI and RES, the first three of which comprise the left-hand side coefficient matrix for the system of equations defining Z and the last of which contains the entries in the right-hand side "residual" vector in doubly-subscripted form. Entries in the

left-hand side coefficient matrix which change from one iteration or time step to the next in a non-linear and/or transient analysis are stored in a separate matrix DIASUM as well as in the coefficient matrix itself so that the latter matrix can be modified in later time-steps or iterations without having to completely recompute all of its entries.

The conductivity of the solid at the temperature of node L,K designated SCOND, is initially set to 1.0 in line 37 since its actual value is not required unless NVCON=1. If NVCON=1, the "temperature" of node L,K, designated TMPIN, is actually Kirchoff's transformed variable, viz $\int_0^{T(L,K)} k_s dT$. This variable is stored as THII by line 45 and T(L,K) is then calculated from TMPIN in line 48 (if NVCON=1). The variable TMPIN is now T(L,K) and THII is Kirchoff's transformed variable. The variable TMP2 is initially $T(L^1, K)$ if NVCON $\neq$ 1 or $\int_0^{T(L^1, K)} k_s dT$ if NVCON=1, where $L^1 = L+1$ if J=MX and $L^1 = L-1$ if J=NIS. As in the case of TMPIN, the variable TMP2 is stored as THII2 by line 46 and then recomputed in line 49 (where it becomes $T(L^1, K)$) if NVCON=1. The variable SCOND2 is the thermal conductivity of the solid at temperature $T(L^1, K)$ if NVCON=1 and simply a factor of 1.0 otherwise. The thermal conductivity SCOND assumes its proper value at the temperature T(L,K) in line 53 if NVCON=1.

Non-linear terms are incorporated into the right-hand side residual (RR) and the diagonal coefficient (CTOT) for the finite-difference equation applying to the L,Kth node in lines 67-86. The variables SCOND and SCOND2 seen in those expressions equal unity

unless NVCON=1. Also, the variable C5 seen in some of those expressions is only non-zero when J=MX or J=NIS (i.e., when node L,K is located on one of the surfaces at the interface between inner and outer cylinders in an analysis involving two cylinders), when it is the surface area of the node at J=MX. Hence expressions in which C5 is a factor only modify RR or CTOT when J=MX or J=NIS. The variable C4 seen in other expressions is only non-zero when J=JX (i.e., when node L,K is an outer surface node), when it is the surface area of node L,K. Hence expressions in which C4 is a factor only modify RR or CTOT when J=JX. Lastly, C(NN5) is the gap conductance between nodes at J=MX and J=NIS and hence is only non-zero when J=MX or J=NIS.

A DO loop which starts at line 91 and ends at line 156 substitutes each of the NTRF trial functions used at axial node K into the left-hand side of Eq. A.15* (or its equivalent for a type 1, 2, 3 or 4 node) one at a time. The residual that is obtained when this substitution is performed with trial function NN is stored as FD(1) and then weighted by each of the NTRF trial functions in turn in the inner DO loop starting at line 142 and ending at line 155. Each weighted residual is added to the appropriate entry in AL, CB or CRI in the inner DO loop. The variable N1 seen in lines 94-115 is the node type, so FD(1) for a type 0 node (N1=0) would be calculated by lines 100-102 because of the comparison performed in line 94, for instance.

*Without the terms $K(i,j,k)_{k+1} T(i,j,k+1)$ and $K(i,j,k)_{k-1} T(i,j,k-1)$.

Terms on the left-hand side of the finite-difference equation for node L,K that change from one iteration or time step to the next in a transient and/or non-linear analysis are calculated by lines 124-133 and stored as DSUM. DSUM is then multiplied (weighted) by trial function NWT (=1, NTRF) in the inner DO loop and added to the appropriate entry in the matrix DIASUM.

In the DO loop starting at line 164 and ending at line 181, line 169 computes FD(3) as the product of C(N4) ($=K(i,j,k)_{k-1}$) and the NKth trial function used at axial node K-1, which is trial function NN (see line 165). The resulting value of FD(3) is then weighted by each of the trial functions used at axial node K in turn in the inner DO loop starting at line 175 and ending at line 180. Each weighted result is added to the appropriate entry in AL by line 179 in the inner DO loop.

Line 191 computes FD(2) as the product of C(N3) ($=K(i,j,k)_{k+1}$) and the NKth trial function used at axial node k+1, which is trial function NN (see line 187). The inner DO loop starting at line 197 and ending at line 202 multiplies FD(2) by each of the trial functions used at axial node K in turn and adds each result to the appropriate entry in CRI.

Lines 206-209 multiply the residual (RR) by each of the trial functions used at axial node K and add each result to the appropriate entry in RES. The matrix RES is later transformed into the vector RD in lines 341-349 of the main program.

4.1.13 Subroutine OUTSYN (lines 323.2-351.8)

Subroutine OUTSYN outputs the results of a synthesis analysis. It also recomputes the right-hand side vector (RES) for the system of equations defining the blending vector (Z) in each time step of a transient synthesis analysis and recomputes the matrix (DIASUM) containing non-linear terms in the left-hand side coefficient matrix in each time step of a transient analysis or in each iteration of a non-linear steady-state analysis.

Lines 35 and 36 initialize the convergence indicator NCONT for the first and subsequent iterations of a non-linear steady-state analysis respectively. If the convergence test performed later in the subroutine indicates that convergence has not been achieved, NCONT is assigned a value of 1. The variables SCOND and SCOND2, defined initially in lines 42 and 43, are redefined later in lines 116 and 118 if NVCON=IMPECC=1, when they become thermal conductivities needed in boundary node finite-difference equations because of the application of Kirchoff's transformation.

Three nested DO loops sweep through all of the nodes in the 3-D finite-difference grid. The outer DO loop, which starts at line 61 and ends at line 260, sweeps through all of the circumferential nodes (I) while the center DO loop, which starts at line 68 and ends at line 259, sweeps through the radial nodes (J). The inner DO loop beginning at line 81 and ending at line 258 sweeps through the axial nodes (K). The variable L2, which is initialized in line 57 and incremented by 1 in line 76 for each r- θ node, is the r- θ node number counter.

The temperature of the I,J,Kth (or L2,Kth) node, designated TSYN, is initialized in line 87. The DO loop between lines 95 and 101 then adds the product of the L2th entry in the NKth trial function used at axial node K (i.e., TRF(L2,NN)) and its coefficient in the blending vector (i.e., Z(L)) to TSYN for all of the trial functions used at the Kth axial node (i.e., for NK=1, 2, ..., NTRI(K)). The final result is the synthesized temperature of the L2, Kth node.

The variable TSYN2, which is only calculated at pressure tube outer and calandria tube inner surface nodes, is the temperature of the opposing node on the other surface (i.e., the surface to which TSYN does not apply). Both temperatures are needed simultaneously to linearize the gap conductance.

If NVCON=1, the "temperature" TSYN is actually the value of $\int_0^{T^1} k dT$, where T^1 is the temperature of the L2,Kth node. The "temperature" TSYN2 is similarly obtained when the appropriate temperature is substituted for T^1 . Since the real temperatures are needed to linearize boundary node equations, the conductivity integrals TSYN and TSYN2 are stored as TH11 and TH12 respectively in lines 109 and 110 and TSYN and TSYN2 are replaced by the corresponding real temperatures in lines 114 and 116. If $\phi = \int_0^{T^1} k_s dT$ and $k_s = TCINTS + SLOPS * T$, it is easily shown that

$$T^1 = \frac{-TCINTS + \sqrt{TCINTS^2 + 2\phi SLOPS}}{SLOPS} \quad 4.4$$

Equation 4.4 is used to calculate the real temperatures corresponding

to the conductivity integrals (ϕ) TSYN and TSYN2 in lines 114 and 116. The thermal conductivities at the real temperatures TSYN and TSYN2, designated SCOND and SCOND2 respectively, are also calculated.

Lines 136-162 calculate the right-hand side residual (RR) for the finite-difference equation at the L2,Kth node when the analysis being performed is a non-linear (NLIN=1) LOECC analysis while lines 169-185 perform that function for a radiating cylinder analysis (RADBN=1). The right-hand side residual at the L2,Kth node in a linear transient analysis is calculated in lines 191-194. The residual is weighted by the NKth trial function in line 207 (NK=1, NTRI(K)) and added to the appropriate entry in the matrix RES, which later becomes the right-hand side vector (RD) for the system of simultaneous equations defining Z (see lines 341-349 in the main program). Lines 209-219 recompute non-linear terms in the left-hand side of the residual equation for the L2,Kth node (this is the equation obtained by substituting a trial function into the finite-difference equation for the L2,Kth node) and store the sum of these terms as DSUM. The DO loop between lines 224 and 227 weights DSUM by each trial function used at the Kth axial node and adds the result to the appropriate entry in DIASUM. Subroutine MATCH later uses DIASUM to update the left-hand side coefficient matrix for the system of simultaneous equations defining Z, thus avoiding complete recomputation of the coefficient matrix at each iteration or time step.

Convergence tests for steady-state non-linear LOECC and radiating cylinder analyses are performed in lines 143-144 and lines 173-174

respectively. Only the temperature variations from one iteration to the next of nodes on the pressure tube outer surface are compared to the convergence parameter, EPSLN, in LOECC analyses. Similarly, only the temperatures of outer surface nodes are tested for convergence in radiating cylinder analyses. The temperatures of the nodes tested for convergence are stored as $TRAD(I,K)$ ($I=1, 2, \dots, IX; K=1, 2, \dots, KX$) in each case for use in the convergence tests in the next iteration.

If $NTPRNT \neq 0$, indicating that some nodal temperatures are to be output, the DO loop between lines 245 and 247 compares the current values of I,J and K to the user-specified values of $ICN(LL)$, $JCN(LL)$ and $KCN(LL)$ respectively to determine if the temperature of node I,J,K should be printed. Line 254 prints the temperature of the node if required, and line 255 calculates and prints the boundary heat flux if the node is an outer surface node (i.e., at $J=JX$).

Lines 270-276 calculate and print the results of a "resistance of contact" analysis.

4.1.14 Subroutine OUTPUT (lines 351.9-359.9)

This subroutine outputs the results of a finite-difference analysis. Lines 20-48 print the temperature of each node I,J,K and lines 52-71 calculate and print boundary heat fluxes. The latter program segment also calculates and prints out the heat flow rate at each boundary node.

Subroutine OUTPUT can be modified to print out specific results, if required.

4.1.15 Subroutine TFREAD (lines 360.0-363.5)

This subroutine reads new trial function schedules, when required, in the second phase of a transient synthesis analysis. A DO loop starting at line 14 and ending at line 32 sweeps through the axial nodes (K) and line 19 reads the number of trial functions to be used at K, designated NTRI(K), along with their numbers, stored as NT2(NK); NK=1,10. These variables are the same as those read by line 265 in subroutine DATA1 (see input variables 50h and 51h). Lines 22-23 then store the contents of NT2 at axial node K as NTRK(K,NK) and lines 29-30 calculate the semi-bandwidth of the synthesis coefficient matrix (MBA). Line 31 calculates the number of equations defining Z (i.e., NEQU). Once completed, NTRK contains the new trial function schedule.

4.1.16 Subroutine MATCH (lines 363.6-368.7)

Subroutine MATCH adds XMF times the entries in DIASUM to the appropriate entries in AL1, CB1 and CR11. It also modifies entries in DIASUM when the time step size is changed in a linear transient analysis so that the entries will be applicable to the new time step. That modification is performed in line 36 after the entry that is modified has been used to change the appropriate entry in AL1, CB1 or CR11. Lines 45 and 46 zero the entries in DIASUM (after they have been used) in a non-linear analysis so that they can be recomputed in subroutine OUTSYN.

4.1.17 Subroutine HXIO (lines 368.8-372.1)

Subroutine HXIO calculates the radiative and gas conductances designated HRAD and HOUT respectively, and then the combined conductance, $HX(I,K)$, between opposing nodes on the pressure tube outer and calandria tube inner surfaces in a CANDU reactor fuel channel. These conductances are calculated using the pressure tube outer and calandria tube inner surface temperatures from the preceding time step in a transient analysis, or the preceding iteration in a steady-state analysis, for the second and subsequent time steps or iterations. The user-specified radiative heat transfer coefficient, HRADIN, (input variable 22h) and gas conductivity, TCGAS, (input variable 30h) are used in subroutine DATA1 to evaluate gap conductances in the first iteration of a non-linear (NLIN=1) steady-state LOECG analysis so subroutine HXIO is not called for that iteration. Pressure tube outer and calandria tube inner surface temperatures in the first time step of a transient analysis are obtained from a previously computed 2-D finite-difference solution, stored as trial function #NTFIC (NTFIC is input variable 4h). The pressure tube outer and calandria tube inner surface temperatures on radial grid line I, designated TMP and TNEIGH respectively in the subroutine, are assigned their correct values in lines 19-21. Subroutine HXIO is only called when the finite-difference equation for a node (with r- θ node number L) on the pressure tube outer surface is being assembled so the current temperature of the opposing node on the calandria tube inner

surface is $T(L+1)$. Thus $TMP=T(L)$ (or $TRF(L,NTFIC)$) and $TNEIGH=T(L+1)$ (or $TRF(L+1,NTFIC)$) always. The gap conductance calculated for the pressure tube outer surface node at radial grid line I is also used in the finite-difference equation for the calandria tube inner surface node at the same radial grid line.

Subroutine HXIO compares the absolute value of the difference between pressure tube outer surface temperatures obtained in consecutive iterations of a steady-state LOECC analysis to a user-specified convergence parameter, EPSLN (input variable 20h) to determine if the solution has converged, as seen in lines 22-23. The temperature ($T(L)$) of the pressure tube outer surface node at each radial grid line (I) in a given iteration is stored as $TRAD(I)$ for use in the convergence test in the next iteration, as seen in line 24. The variable $NCONT$, which is assigned a value of zero at the start of each iteration, is set to 1 in line 23 if the convergence test involving the temperatures $T(L)$ and $TRAD(I)$, which are the temperatures of the pressure tube outer surface node at radial grid line I from the current and preceding iterations respectively, fails.

The continuum thermal conductivity ($CKGG$) of the gas between the pressure and calandria tubes at radial grid line I is calculated in line 26 from the average of the pressure tube outer and calandria tube inner surface temperatures at that radial grid line (TAV). The gas conductance ($HOUT$) across the gap at radial grid line I , where the gap size is $GGAP(I,K)$, is calculated using a non-continuum gas conductivity expression in line 29 of subroutine HXIO.

The gas conductance is added in line 30 to the radiative conductance (HRAD), to obtain the total conductance across the gap, HNEW. Using HNEW as the new gap conductance at radial grid line I gives solutions which fail to converge at high pressure tube temperatures (when the radiative heat transfer coefficient is large relative to the gas conductance) so the gap conductance must be underrelaxed in the manner shown in line 31. The underrelaxation factor (BETA), which is used to weight the new and old gap conductances (HNEW and HX(I,K)), is read in subroutine DATA1 as input variable 23h. Convergence is usually obtained if $0.5 \leq \text{BETA} \leq 0.7$ (values closer to unity can be used at low pressure tube temperatures).

4.1.18 Subroutine HGAS (lines 372.2-376.3)

(Subroutine HGAS uses Hertz's theory to calculate the gap sizes and corresponding gas conductances in regions adjacent to an elastically formed contact. It is assumed that each contact surface is cylindrical (i.e., that it is curved in only one plane) and that the planes of curvature to which the surface radii RCY1 and RCY2 ($\text{RCY1} < \text{RCY2}$) apply are orthogonal. The radii RCY1 and RCY2 are calculated as effective radii in a LOECC analysis, as described in Appendix B.

Subroutine HGAS uses approximations to the incomplete elliptic integrals of the first and second kinds that only apply to large aspect ratio contacts, say to contacts for which $A/B > 4$, as described in Appendix I.A.

4.1.19 Subroutine RADIAT (lines 376.4-379.5)

Subroutine RADIAT calculates terms needed for the Newton linearization of a radiative boundary condition on the outer surface of a radiating cylinder in a finite-difference analysis. The code is essentially self-explanatory and hence will not be discussed.

4.1.20 Subroutine QFORM (lines 379.6-386.3)

The right-hand side heat flow vector (Q) is computed at each time step in a linear transient finite-difference analysis in subroutine QFORM. The coding in the subroutine is self-explanatory with the comment statements provided and hence will not be discussed further.

4.1.21 Subroutine BNDEL (lines 386.4-394.4)

Subroutine BNDEL uses Gaussian elimination to solve systems of simultaneous equations with banded, not necessarily symmetric, coefficient matrices. The Jth equation in such a system of equations is written in the form,

$$\begin{aligned} &A(J,M)*X(J-M) + A(J,M-1)*X(J-(M-1)) + \dots + A(J,2)*X(J-2) + \\ &A(J,1)*X(J-1) + B(J)*X(J) + C(J,1)*X(J+1) + C(J,2)*X(J+2) + \\ &\dots + C(J,M-1)*X(J+(M-1)) + C(J,M)*X(J+M) = D(J) \end{aligned}$$

where $1 \leq J \leq N$ and $N \geq M$. The variable M is the half-bandwidth of the coefficient matrix and N is the total number of equations. X is the vector of unknowns.

Subroutine BNDEL does not use pivoting. If a zero pivot element is encountered in line 390.8, a message "SINGULAR MATRIX" is printed by line 78 and program execution is stopped.

4.1.22 Subroutines CHLSKI and SUB (lines 394.5-401.0)

Subroutine CHLSKI uses Cholesky's algorithm to factor a positive-definite matrix, in this case the left-hand side conductance matrix for a system of finite-difference equations (CX), into the product of a lower triangular matrix (L) and the transpose of that matrix (L^T) (i.e., $CX=LL^T$). The conductance matrix (CX) is actually designated A in subroutines CHLSKI and SUB. The conductance matrix is stored as a vector (i.e., singly-subscripted array) in the profile storage mode described by Jennings (1). Only the elements from the first non-zero entry in each row up to and including the diagonal term are stored and the rows are lined up end to end in the coefficient vector. A pointer vector, KDIAG (or NDIAG in subroutine CONMAG) provides the location of each diagonal element in the coefficient vector. The coefficient and pointer vectors are stored in COMMON block DB15 so the size of the block will, of course, equal the sum of the dimensions of A and KDIAG (or CX and NDIAG). The same COMMON block is used to store the band matrices CB, AL, CRI and the bandwidth, MBA, of AL and CRI in the second phase of a synthesis analysis so the user should adjust the dimensions of CB, AL, CRI, A and KDIAG to keep the size of COMMON block DB15 constant. Since the matrices A and KDIAG are only

used in finite-difference analyses (eg., when trial functions are being computed for use in a synthesis analysis) and the variables CB, AL, CRI and MBA are only used in the substitute/weight/integrate phase of a synthesis analysis, these sets of variables can be stored in the same COMMON block to conserve memory.

Subroutine SUB performs the forward and backward substitutions needed to obtain the solution vector (T) from the factored conductance matrix (A) and the right-hand side heat flow vector (Q, or B in subroutine SUB).

Subroutines CHLSKI and SUB were both written by Jennings and are described in more detail in (1).

4.1.23 Subroutine TRIAL (lines 401.1-407.1)

Finite-difference solutions to two-dimensional (r- θ) problems are recorded as trial functions for a synthesis analysis in subroutine TRIAL. Inner and outer DO loops sweep through the radial and circumferential nodes respectively in the r- θ grid and equate entries in the trial function matrix (TRF) to entries in the r- θ temperature vector (T). The temperature of the Lth r- θ node in trial function #NTRF, which is stored as TRF (L,NTRF), is equated to the Lth entry in the r- θ temperature vector (T(L)) in line 42. If NVCON=1 (i.e., if the thermal conductivity of the object being modelled is considered temperature dependent), the conductivity integral corresponding to the temperature of the Lth node (i.e., $\int_0^{T(L)} k_s dT$) is calculated in line 37

and stored as $TRF(L, NTRF)$ instead of $T(L)$ itself. This conversion is needed because Kirchoff's transformation is used in the second phase of a synthesis analysis, but not in finite-difference analyses, to linearize the heat conduction equation with temperature dependent thermal conductivity. When Kirchoff's transformation is used, the conductivity integrals at the nodes replace the temperatures at the nodes as the unknowns.

4.1.24 Subroutine NTRP (lines 407.2-408.6)

Subroutine NTRP stores variables NTRI and NTRK as NTRIP and NTRKP respectively in preparation for changing the trial function schedule in a transient synthesis analysis. The entries in NTRI and NTRK are overwritten by new values in subroutine TFPREAD when the new trial function scheduling information is read in.

4.1.25 Subroutine ZERO (lines 408.7-412.0)

Subroutine ZERO zeroes entries in the matrices AL, CB, CRI, RES, RD and DIASUM before they are used to accumulate weighted residuals in the second (substitute/weight/integrate) phase of a synthesis analysis. The code in subroutine ZERO is self-explanatory and hence will not be discussed.

4.2 Routines to be Used When Trial Vector Orthonormalization is Required

As mentioned in Sec 1.0, the coding needed to use orthonormalized trial vectors in other than linear, steady-state synthesis analyses does not exist in SYNCON. Trial vector orthonormalization is not permitted as an option with the regular version of SYNCON, but rather is obtained by substituting alternate versions of the main program, subroutine OUTSYN and subroutine SUBST1 for the regular versions of those routines already described. Subroutine GRMSCH, which uses the modified Gram-Schmidt algorithm with column pivoting (2) to orthonormalize the trial vectors, must also be added to the program. The modified routines and subroutine GRMSCH are included as Listing 3. Brief descriptions of those routines follow.

4.2.1 Main Program

The version of the main program that is used with orthonormalized trial vectors contains several program segments that are not found in the regular version, although the routines are identical up to and after lines 282 and 312 in the original routine respectively.

Since the trial vectors used at axial nodes $K-1, K$ and $K+1$ are needed simultaneously in subroutine SUBST1 (when $K \neq 1$ and $K \neq KX$) to calculate entries in the coefficient matrix and right-hand side vector for the system of equations defining Z , the trial vectors designated for use at those axial nodes are stored in matrices TORKM1,

TOR and TORKP1 respectively and then orthonormalized. Those matrices are subsequently used in subroutine SUBST1 to perform the trial vector substitution and weighting operations at axial node K. When K is subsequently incremented by 1, the entries in TORKP1 and TOR replace those in TOR and TORKM1 respectively and a new version of TORKP1 is computed, either by orthonormalizing the trial vectors designated for use at axial node K+1, or by equating the entries in TORKP1 to those in TOR. TORKP1 is obtained in the second manner if a comparison of the trial function numbers designated for use at axial nodes K and K+1 indicates that the same trial functions are used at both locations.

Lines 289-293 in the listing of the main program found in Listing 3 store the trial vectors for the first axial node ($K=1$) as the columns of TOR. Line 294 then calls subroutine GRMSCH to orthonormalize the columns of TOR and hence the trial vectors. The DO loop starting at line 297 then sets K to 1 and lines 316-318 compare the trial functions designated for use at K+1 to those used at K. If the comparison indicates that the same trial functions are used at both axial nodes, lines 319-321 equate the entries in TORKP1 to those in TOR. If the trial vectors used at K+1 are not the same as those used at K, lines 323-326 store the trial vectors for axial node K+1 as the columns of TORKP1. Subroutine GRMSCH is called by line 327 to orthonormalize the columns of TORKP1 and hence the trial vectors used at axial node K+1.

Lines 329-343 in the modified version of the main program are the same as lines 295-309 in the original version. Lines 353-358 equate the entries in TORKM1 and TOR to those in TOR and TORKP1 respectively in preparation for advancing to the next axial node. If $K \neq KX$, lines 298-358 are re-executed for the next value of K .

4.2.2 Subroutine SUBST1

The version of subroutine SUBST1 intended for use with orthonormalized trial vectors is simpler than the regular version because all of the lines used to calculate terms for non-linear and/or transient analyses have been deleted. Except for the substitution of TOR, TORKM1 or TORKP1 for TRF and the omission of lines used to calculate terms for non-linear and/or transient analyses, the modified and original versions of subroutine SUBST1 are identical.

4.2.3 Subroutine OUTSYN

As in the case of subroutine SUBST1, lines used to calculate terms for non-linear and/or transient analyses have been omitted from the modified version of subroutine OUTSYN.

A DO loop starting at line 25 and ending at line 104 assigns a value to the axial node index K . If $K=1$, lines 38-41 store the trial vectors designated for use at K as the columns of TOR and subroutine GRMSCH is called to orthonormalize its columns. If $K \neq 1$

lines 34-36 compare the trial functions used at axial node K to those used at K-1 to determine if TOR needs to be changed. If the trial functions used at axial node K are not the same as those used at K-1, the former trial functions are stored as the columns of TOR by lines 38-41 and subroutine GRMSCH is called by line 42 to perform the orthonormalizations, as before.

Outer and inner DO loops starting at lines 49 and 55 respectively sweep through the azimuthal nodes (I) and radial nodes (J). The synthesized temperature of node L2,K (L2 is the r- θ node number), designated TSYN, is calculated by lines 63-66 from the trial vectors in TOR and the blending vector Z. If node L2,K is an outer surface node (i.e., if J=JX), line 73 calculates the surface conductance C0 and line 74 increments the total heat flow rate through the surface (QTOT).

If the resistance of a contact is being computed, line 79 increments the heat flow through the contact patch (QPAT) if node I,K is located within the contact patch (i.e., if J=JX and $I \leq NIW(K)$) and line 80 calculates and prints out the heat flux at the node if it is a surface node (i.e., if J=JX) located on the contact-major axis (where I=1) or minor axis (where K=1).

Lines 88-90 check whether the values of I, J and K correspond to any of the user-supplied values of ICN(LL), JCN(LL) and KCN(LL), LL=1, NTPRNT (see input variables 27a-30a). When SYNCON is used with orthonormalized trial vectors, the data cards containing values of ICN, JCN and KCN should be ordered so that the values of those

variables vary from least to most rapidly in the order KCN, ICN, JCN.

Lines 114-120 print out the results of a "resistance of contact" analysis.

4.2.4 Subroutine GRMSCH

Subroutine GRMSCH uses the modified Gram-Schmidt algorithm with column pivoting (2) to orthonormalize the columns of TOR. The variables NROW and NCOL are the number of rows and the number of columns in TOR respectively. Subroutine GRMSCH should never need to be altered and hence will not be described further.

5.0 SAMPLE ANALYSES

5.1 Analysis of the Model Contact in Air

The input file and the output listing for a synthesis analysis performed to determine the resistance of the model contact in air at a load of 176N are provided as Listings 4 and 5 respectively. The analysis was performed with grid option c) and a total of 14 trial functions, which were orthonormalized before being used. The characteristics of the finite-difference grid and the trial functions used in the analysis are summarized in Sec II.3 of the thesis, where the results of the analysis are discussed in detail.

Each line of the input file ends with a number and a letter (eg., 1a, 2d, etc.) referring to a data card # in Table 2.1. With the data card # for each line known, Table 2.1 can be consulted to determine the input variables defined in each line. The data card # for the second line in the file is 2a, for instance, so the variables defined in that line must be NGROPT (=2), MBW (=200), N2D (=1) and NCCYL (=0) according to Table 2.1a. The data card # at the end of each line in the input file is not read by SYNCON, of course.

Lengths are defined in cm., temperatures in $^{\circ}\text{C}$ and thermal conductivities in $\text{W}/\text{cm}^{\circ}\text{C}$ in the input file.

5.2 Non-Linear Steady-State Pressure-Calandria Tube Heat-Transfer Analysis

The input file and the output listing for a non-linear synthesis analysis of steady-state heat transfer between contacting pressure and calandria tubes in a CANDU reactor are provided as Listings 6 and 7 respectively. A heat flux of 0.770 W/cm^2 was prescribed over the pressure tube inner surface in the analysis, producing a heating rate of 25 W. per cm. length of the channel (since $0.770 \text{ W/cm}^2 \times \pi \times \text{PTID} = 25 \text{ W/cm}$). A 51cm length of the fuel channel between the channel center and one of the centermost garter springs was modelled in the analysis, which was performed with a total of 10 trial functions. The trial functions were not orthonormalized before use. The characteristics of the trial functions and the results of the analysis are discussed in detail in Sec III.4.3 of the thesis.

The thermal conductivity of the nitrogen gas in the gap between the pressure and calandria tubes was permitted to be temperature-dependent in the analysis, as was the radiative heat transfer coefficient between the tubes and the thermal conductivity of the pressure/calandria tube material (Zircaloy).

As was the case with the input file for the analysis of the model contact problem, each line in the input file for the steady-state LOECC analysis ends with a number and a letter referring to a data card # in Table 2.1.

Lengths are defined in cm, temperatures in Kelvins (K), thermal conductivities in W/cmK , and heat fluxes in W/cm^2 in the input file

and output listing.

Trial functions 1 and 10 were r- θ solutions obtained at $z=0$ (i.e., at the channel center) and $z=51\text{cm}$ (i.e., at the first garter spring) respectively. Since azimuthal node $I=1$ is located at the 6 o'clock position (i.e., at the pressure/calandria tube contact) and radial node $J=23$ coincides with the calandria tube outer surface, it can be seen from the printed results for trial function 1 that the heat flux on the calandria tube outer surface at the contact (i.e., at $I=1, J=23$ and $K=1$) was 5.92 W/cm^2 . By way of comparison, a peak calandria tube outer surface heat flux of 7.60 W/cm^2 was obtained at the seventh axial node (i.e., at $K=7$ or $z=0.4\text{cm}$.) in the 3-D synthesis analysis (see the heat flux at $I=1, J=23, K=7$ for the fourth and final iteration). These 2-D and 3-D results indicate that axial heat flow increased the peak calandria tube outer surface heat flux by 28.4%.

Contact dimensions used in the analysis were those calculated in Appendix C. A contact conductance of 56.75 W/cm^2 was prescribed over the contact area.

5.3 Transient (Linear) LOECC Analysis

The input file and the output listing for a linear transient synthesis analysis of heat transfer between contacting pressure and calandria tubes in a CANDU reactor for the period immediately following contact are included as Listings 8 and 9 respectively. This analysis

is essentially the "two dimensional" linear transient LOECC analysis described in Sec III.4.2 since the 3-D finite-difference grid used in the analysis contained only three axial nodes. With an axial subdivision size of 0.05cm and three axial nodes, the model did not include axial locations (z) outside of the segment for which $0 \leq z \leq 0.10\text{cm}$. The width of the contact patch within that axial segment was assumed to be uniform and equal to the contact semi-minor diameter (B) calculated in Appendix C (i.e., 0.0628cm). The analysis was really only 2-D because the r - θ boundary conditions would have been effectively the same at the three axial nodes used.

A total of 11 trial functions was used in the analysis, of which trial functions 1 and 11 were steady-state r - θ solutions obtained at $z=51\text{cm}$ (i.e., at the first garter spring) and $z=0$ respectively. Trial functions 2-10 were recorded at times of 0.36, 0.74, 1.24, 1.96, 3.40, 5.20, 14.20, 26.80 and 46.60 seconds after contact respectively in an r - θ transient analysis performed at the channel center (i.e., at $z=0$). A heat flux of 3.914 W/cm^2 was prescribed over the pressure tube inner surface in all of the analyses, as in the steady-state analysis described in the preceding section. This heat flux produces a pressure tube temperature of approximately 1000°C under steady-state conditions when the pressure and calandria tubes are concentric and have emissivities of 0.8 and 0.2 respectively. The user-specified radiative heat transfer coefficient ($HRADIN$) was assigned a value consistent with these emissivities and pressure tube temperature. The moderator temperature was assumed to be 80°C .

The initial conditions for the analysis were taken from trial function 1 (i.e., the analysis performed at the first garter spring) since the pressure tube temperature for that trial function was approximately uniform and $\sim 1000^{\circ}\text{C}^*$. Trial function 1 provided the initial conditions for both the transient finite-difference analysis, which produced trial functions 2-10, and the transient synthesis analysis. The time increments and number of time steps performed with each increment were the same for both the transient finite-difference and transient synthesis analyses.

Although a total of 11 trial functions was used in the synthesis analysis, only 5 trial functions (which were the same at the three axial nodes) were used at any one time. Trial functions 1-5 were used at times (t) less than 1.46 seconds, trial functions 4-8 were used for $1.46\text{s} \leq t < 10.60\text{s}$ and trial functions 7-11 were used for $t \geq 10.60\text{s}$.

Further details of this analysis can be found in Sec III.4.2 of the thesis where a full 3-D linear transient synthesis analysis of pressure-calandria tube heat transfer is also described.

Lengths are defined in cm, temperatures in Kelvins (K), thermal conductivities in W/cmK , and heat fluxes in W/cm^2 in the input file and output listing.

Each line in the input file ends with a number and a letter referring to a data card # in Table 2.1, as with the input files for the other sample analyses.

*It was assumed that the pressure tube sags into contact with the calandria tube when its temperature reaches $\sim 1000^{\circ}\text{C}$.

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APPENDICES

APPENDIX A: DERIVATION OF DIFFERENCE EQUATIONS

A.1 Normal Nodes Located at Orthogonal Grid Line Intersections

The difference equations used in SYNCON at normal nodes of the type shown in Fig. A.1, where orthogonal grid lines oriented parallel to the r , θ and z directions intersect, are obtained using the heat balance method (3). In this method, a heat balance is performed on the r - θ - z cell shown in two views in Fig. A.1 and centered around

Fig. A.1 Cylindrical coordinate system nodal control volume.

the mesh point i, j, k (located at θ_i, r_j, z_k) to give,

$$\begin{aligned}
 & -K(i, j, k)_{i+1} T(i+1, j, k) - K(i, j, k)_{i-1} T(i-1, j, k) - K(i, j, k)_{j+1} \\
 & T(i, j+1, k) - K(i, j, k)_{j-1} T(i, j-1, k) - K(i, j, k)_{k+1} T(i, j, k+1) \\
 & - K(i, j, k)_{k-1} T(i, j, k-1) + (\Sigma K) T(i, j, k) = q_v V(i, j, k)
 \end{aligned}
 \tag{A.1}$$

where q_v is the volumetric heat generation rate within the cell

enclosing node i, j, k and $V(i, j, k)$ is the volume of the cell. The conductances between node i, j, k and its neighbours $i \pm 1, j, k$; $i, j \pm 1, k$ and $i, j, k \pm 1$ are designated $K(i, j, k)_{i \pm 1}$, $K(i, j, k)_{j \pm 1}$ and $K(i, j, k)_{k \pm 1}$ respectively in Eq. A.1 and ΣK is the sum of those conductances. With the geometric parameters $\Delta r_a, \Delta r_b, \Delta \theta_a, \Delta \theta_b, \Delta z_a, \Delta z_b$ identified in Fig A.1, the internodal conductances between node i, j, k and its neighbours can be written as:

$$K(i, j, k)_{i+1} = k_s \bar{\Delta r} \bar{\Delta z} / r_j \Delta \theta_b \quad A.2$$

$$K(i, j, k)_{i-1} = k_s \bar{\Delta r} \bar{\Delta z} / r_j \Delta \theta_a \quad A.3$$

$$K(i, j, k)_{j+1} = k_s (r_j + \Delta r_b) \bar{\Delta \theta} \bar{\Delta z} / \Delta r_b \quad A.4$$

$$K(i, j, k)_{j-1} = k_s (r_j - \Delta r_a) \bar{\Delta \theta} \bar{\Delta z} / \Delta r_a \quad A.5$$

$$K(i, j, k)_{k+1} = k_s \bar{\Delta r} r_j \bar{\Delta \theta} / \Delta z_b \quad A.6$$

$$K(i, j, k)_{k-1} = k_s \bar{\Delta r} r_j \bar{\Delta \theta} / \Delta z_a \quad A.7$$

where k_s is the thermal conductivity of the solid and,

$$\bar{\Delta r} = (\Delta r_a + \Delta r_b) / 2 \quad A.8$$

$$\bar{\Delta \theta} = (\Delta \theta_a + \Delta \theta_b) / 2 \quad A.9$$

$$\bar{\Delta z} = (\Delta z_a + \Delta z_b) / 2 \quad A.10$$

The volume of the finite-difference "cell" associated with node i, j, k , (i.e., $V(i, j, k)$) is,

$$V(i, j, k) = r_j \bar{\Delta r} \bar{\Delta \theta} \bar{\Delta z} \quad A.11$$

Equation A.1, which is valid for a steady-state analysis, can be applied to a transient analysis if the term $\frac{\partial}{\partial t} V(i,j,k)T(i,j,k)$ is added to the right-hand side of the equation and the term $\frac{\partial}{\partial t} V(i,j,k) \cdot T^{t+1}(i,j,k)$ is added to the coefficient of $T(i,j,k)$ (i.e., ΣK) on the left-hand side, where $T^t(i,j,k)$ and $T^{t+1}(i,j,k)$ are the temperatures of node i,j,k at times t and $t+\Delta t$ respectively. If the time derivatives of the nodal temperatures are backward differenced in the transient analysis, as they are in SYNCON, the nodal temperatures on the left-hand side of Eq. A.1 should all be written with the superscript $t+1$ to indicate that they are temperatures at time $t+\Delta t$.

If the temperature of a node adjacent to the mesh point (i,j,k) is specified, the term associated with that node in Eq. A.1 is constant and can be transferred to the right-hand side of the equation (i.e., it joins the Q vector for the system of finite-difference equations $KT=Q$). Similarly, the finite-difference equation for a node (i,j,k) located on an external cylindrical surface (of radius r_0) and exchanging heat with a fluid at temperature T_∞ through a film coefficient h_f , as shown in Fig. A.2, can be written as,

$$\begin{aligned} & -K(i,j,k)_{j-1}T(i,j-1,k) - K(i,j,k)_{j+1}T(i,j+1,k) - K(i,j,k)_{i-1} \cdot \\ & T(i-1,j,k) - K(i,j,k)_{k+1}T(i,j,k+1) - K(i,j,k)_{k-1}T(i,j,k-1) \\ & + (\Sigma K)T(i,j,k) = K(i,j,k)_s T_\infty \end{aligned} \quad \text{A.12}$$

where $K(i,j,k)_s$ = conductance between surface node i,j,k and fluid at T_∞

$$= h_f r_j \Delta \theta \Delta z \quad \text{A.13}$$

Fig. A.2 Boundary Node - Convective Boundary Condition.

$$\text{and } K(i,j,k)_{i+1} = k_s \Delta r_a \overline{\Delta z} / 2(r_j - \Delta r_a) \frac{\Delta \theta_b}{4} \quad \text{A.14}$$

$\approx$ Eq. A.2 with $\Delta r_b = 0$

Similarly $K(i,j,k)_{i-1} \approx$ Eq. A.3 with $\Delta r_b = 0$. The remaining conductances are identical to those defined previously for Eq. A.1. If the node is insulated, $h_f = 0$ (i.e., $\frac{\partial T}{\partial r}|_{r_0} = 0$). An equation similar to

Eq. A.12 can be derived for a boundary node located on an internal cylindrical surface or, alternately, for the fuel or sheath node at the fuel/sheath interface in a nuclear fuel element.

If the boundary node (i,j,k) in Fig. A.2 has a heat flux q'' prescribed over its external surface area $A_s(i,j,k)$ ($= r_j \overline{\Delta \theta} \overline{\Delta z}$) instead of a convective boundary condition, the appropriate finite-difference equation can be obtained from Eq. A.12 by setting $K(i,j,k)_s = 0$ and replacing that equation's right-hand side, previously $K(i,j,k)_s T_\infty$, by $-q'' A_s(i,j,k)$.

It should be clear from the examples given above that similar, but nevertheless different, nodal equations are obtained by performing heat balances on different types of nodes (i.e., interior, boundary). A general equation that is applicable to all nodes can be written as:

$$\begin{aligned} & -K(i,j,k)_{i+1} T(i+1,j,k) - K(i,j,k)_{i-1} T(i-1,j,k) - K(i,j,k)_{j+1} T(i,j+1,k) \\ & - K(i,j,k)_{j-1} T(i,j-1,k) - K(i,j,k)_{k+1} T(i,j,k+1) - K(i,j,k)_{k-1} T(i,j,k-1) \\ & T(i,j,k) + (\Sigma K) T(i,j,k) = K(i,j,k)_s T_\infty - q'' A_s(i,j,k) + q_v V(i,j,k) \end{aligned} \quad \text{A.15}$$

for a steady-state analysis. Equation A.15 can be used in a transient

analysis if terms are added to the left and right-hand sides of the equation as discussed previously for Eq. A.1. It should be understood that terms in Eq. A.15 which were not applicable at a certain node would be set to zero at that node. At an interior node, for instance, we must have $K(i,j,k)_s = 0$ and $q'' = 0$ since the boundary-related terms must drop out of the equation. The terms $K(i,j,k)_s$ and q'' cannot both be non-zero at a given boundary node, of course.

All of the finite-difference equations given previously can be represented by Eq. A.15 if the relevant internodal conductances defined by Eq.'s A.2-A.7 and A.13 are calculated using appropriate values for the geometric parameters Δr_a , Δr_b , $\Delta \theta_a$, $\Delta \theta_b$, Δz_a and Δz_b . If $q_v = q'' = \Delta r_b = K(i,j,k)_{j+1} = 0$, for instance, Eq. A.15 reduces to Eq. A.12. Similarly, setting $q_v = q'' = \Delta r_a = K(i,j,k)_{j-1} = 0$ yields an equation applicable to a convectively cooled (or heated) boundary node on an internal cylindrical surface.

The finite-difference equation for each normal node located at an orthogonal grid line intersection, either at an interior point as shown in Fig. A.1 or at a boundary point such as that shown in Fig. A.2, is computed in subroutine NORNOD (acronym for normal node) in SYNCON. Subroutine NORNOD computes the coefficients and right-hand side term for each normal node finite-difference equation in steady-state and transient, linear and non-linear, analyses. The computations performed in subroutine NORNOD are described in detail, with reference to the coding actually used in the subroutine, in Sec 4.1.6.

A.2 Mesh Size Alteration

The geometric parameters in the conductance equations given in Sec A.1 are variable and can be specified independently. The radial subdivision sizes, Δr_a and Δr_b , can be different at a given node, for instance. Note, however, that Δr_a and Δr_b are both functions of radial position (r) only, since the cylindrical surfaces at different values of r must be continuous with respect to θ and z . This is clearly shown in Fig. 1.1. Similarly, $\Delta\theta_a$ and $\Delta\theta_b$ are functions of θ only while Δz_a and Δz_b are functions of z only.

A more flexible finite-difference grid than that shown in Fig. 1.1 would allow the nodal spacings in each coordinate direction to be functions of all three coordinates (i.e. r , θ and z) but such a mesh is extremely difficult to construct in practice. It is not difficult, however, to grade a two-dimensional r - θ grid such that $\Delta r_a = f_1(r, \theta)$, $\Delta r_b = f_2(r, \theta)$, $\Delta\theta_a = g_1(r, \theta)$ and $\Delta\theta_b = g_2(r, \theta)$, as shown in Fig. 1.3. This r - θ grid can be extended in the axial (z) direction using axial subdivisions that are functions of z only (i.e., $\Delta z_a = h_1(z)$, $\Delta z_b = h_2(z)$).

When the grid parameters Δr_a , Δr_b , $\Delta\theta_a$ and $\Delta\theta_b$ are made functions of both coordinate positions (r and θ), discontinuous grid lines are introduced into the finite-difference mesh in both the r and θ directions. Discontinuous nets are coupled together in SYNCON by applying first order accurate representations of the heat conduction operator developed by MacNeal (4) for use with triangular grids.

Referring to the network of triangles shown in Fig. A.3, in which obtuse angles are prohibited, MacNeal's method requires that perpendicular bisectors of the sides of the triangles be constructed to form polygons around the nodes.

The conductance between two nodes A and B can then be expressed quite simply as,

$$K_{AB} = k_s \frac{b_{12}}{l_{ab}} \quad (\text{for a unit length in the axial direction}) \quad A.16$$

where b_{12} and l_{ab} are identified in Fig. A.3. A similar expression applies to the conductance between each pair of adjacent nodes in the triangular grid.

Fig. A.3 Triangular Grid

The area associated with a node in the triangular network is the area inside the polygon defined by the perpendicular bisectors of grid lines emanating from the node in question. Thus the area associated with node B in Fig. A.3 is the area inside the dashed line polygon surrounding B.

MacNeal's method, which preserves the symmetry of the conductance matrix because $K_{AB} = K_{BA}$, is used in SYNCON to double the subdivision size at each grid discontinuity in a graded r - θ grid of the type shown in Fig. 1.3. Enlarged views of the two types of grid discontinuities found in such a grid are shown in Figs. A.4 and A.5. Nodes 3 and 6 in each figure are representative of the two types of exceptional nodes found at interfaces between fine and coarse nets.

Fig. A.4 Radial Mesh Spacing Alteration

Fig. A.5 Circumferential Mesh Spacing Alteration

If the cell (control volume) associated with each exceptional node is small, circular arcs can be replaced by straight lines and the two types of discontinuous nets shown in Figs. A.4 and A.5 can be replaced by the equivalent net shown in Fig. A.6. Non-parallel grid lines are shown as being parallel for simplicity.

Fig. A.6 COUPLING DISCONTINUOUS MESHES

As mentioned previously, obtuse angles in the triangles identifiable in Fig. A.6 are prohibited. The angle β is the only angle capable of violating this restriction, and it is evident that $\beta \leq 90^\circ$ when $h_1 \geq h_2$, where h_1 and h_2 are nodal spacings identified in Fig. A.6.

Neglecting conductances in the axial direction, the finite-difference equation for node 3 can be written as;

$$-K_{3-9}T_9 - K_{3-2}T_2 - K_{3-4}T_4 - K_{3-6}T_6 - K_{3-7}T_7 + (\Sigma K)T_3 = 0 \quad A.17$$

where K_{3-2} and K_{3-4} are given by Eqs. A.4 and A.5 respectively when the net is discontinuous in the circumferential direction and by Eqs. A.2 and A.3 respectively when the net is discontinuous in the radial direction. The conductances K_{3-6} and K_{3-7} are identical and can be evaluated by constructing a polygon composed of perpendicular bisectors of grid lines emanating from point 3, as shown in Fig. A.6.

In this case, K_{3-6} and K_{3-7} are given by,

$$K_{3-6} = K_{3-7} = k_s \times h_2 \times \frac{(\Delta z_a + \Delta z_b)}{2} / 2h_1 \quad A.18$$

where $(\Delta z_a + \Delta z_b)/2$ is the axial length of the control volume.

Similarly, the nodal equation for node 6 can be written as;

$$-K_{6-1}T_1 - K_{6-2}T_2 - K_{6-3}T_3 - K_{6-5}T_5 - K_{6-7}T_7 - K_{6-8}T_8 + (\Sigma K)T_6 = 0 \quad A.19$$

where K_{6-2} and K_{6-8} are given by Eq's A.3 and A.2 respectively when the net is discontinuous in the circumferential direction and by Eq's A.4 and A.5 respectively when the net is discontinuous in the radial direction. K_{6-1} and K_{6-3} are both given by Eq. A.18, while K_{6-7} can be calculated from the relationship,

$$K_{6-7} = k_s \times \frac{(\Delta z_a + \Delta z_b)}{2} \times \frac{(3h_1^2 - (h_1^2 + h_2^2))}{2h_1} / 2h_2 \quad A.20$$

The areas and volumes associated with nodes 3 and 6 are;

$$A_3 = h_1 \times h_2^3 / 4h_1 \quad A.21$$

$$V_3 = A_3 \times \frac{(\Delta z_a + \Delta z_b)}{2} \quad A.22$$

$$\text{and, } A_6 = 2h_1 h_2 - \frac{h_2^3}{4h_1} \quad A.23$$

$$V_6 = A_6 \times \frac{(\Delta z_a + \Delta z_b)}{2} \quad A.24$$

Equations A.18 and A.20 can be used to calculate the corresponding inter-nodal conductances in the nets shown in Figs. A.4 and A.5 by replacing the grid spacings h_1 and h_2 by the appropriate arc lengths ($r\Delta\theta$) and radial increments (Δr) for each node. Each arc length ($r\Delta\theta$) is calculated using the average value of the radii at the two nodes between which the conductance (K_{ij}) is being evaluated to preserve the symmetry of the K matrix.

Since two types of exceptional nodes are found at each radial grid discontinuity (i.e., nodes 3 and 6 in Fig. A.4) and two exceptional node types are found at each circumferential grid discontinuity (i.e., nodes 3 and 6 in Fig. A.5), SYNCON must be able to evaluate the finite-difference equations at a total of four atypical types of nodes. The finite-difference equations for normal nodes found at orthogonal grid line intersections, which are referred to as type 0 nodes in SYNCON, are evaluated in subroutine NORNOD, as mentioned previously, but the finite-difference equations for the four exceptional node types are evaluated in separate subroutines. Those subroutines and the type designations for the four atypical nodes are listed below.

FIG #	NODE #	TYPE DESIGNATION IN SYNCON	SUBROUTINE IN WHICH F-D EQUATION IS EVALUATED
A.4	6	1	BIGRAD
A.4	3	2	SMARAD
A.5	3	3	SMACIR
A.5	6	4	BIGCIR

APPENDIX B: PRESSURE-CALANDRIA TUBE GAP GEOMETRY CALCULATIONS

The pressure and calandria tubes in a CANDU reactor have garter springs, spaced typically 1-2m apart, inserted between them to prevent them from contacting (due to creep of the pressure tube under normal operating conditions) over the design life of the reactor. It is assumed for simplicity in this analysis that the garter springs would prevent complete contact of the pressure and calandria tubes during a LOECC accident by maintaining gaps equal to their diameters at the points where they were located. The garter springs would likely indent the pressure tube to a certain extent in a real LOECC accident, of course, so the gaps at the garter springs would probably be less than the garter spring diameters in reality. It is also assumed that initial contact between the pressure and calandria tubes would occur midway between the two most central garter springs and that the deflected shape of the pressure tube between those springs would approximate that of a uniformly loaded (elastic) beam. The ends of the beam, located at the garter springs, are assumed fixed as shown in Fig. B.1 since it is likely that the pressure and calandria tubes would remain approximately parallel at those points due to the approximate symmetry of the loading conditions about the springs. It is also assumed for simplicity that the pressure and calandria tubes would remain circular after contact although some distortion of the pressure tube, in particular, would undoubtedly occur in reality.

Fig. B.1 SLUMPED PRESSURE TUBE APPROXIMATED AS A UNIFORMLY LOADED, FIXED END BEAM BETWEEN THE TWO MOST CENTRAL GARTER SPRINGS

With the assumptions described previously, the gap (y) between the bottom of the pressure tube outer surface and the bottom of the calandria tube inner surface, shown in Fig. B.1, can be expressed as,

$$y = \frac{16D_s}{4L_{sp}} z^2 \left(\left(\frac{L_{sp}^2}{2} \right) - z^2 \right) \quad B.1$$

where D_s is the garter spring diameter, L_{sp} is the distance between the garter springs and z is the distance from the center of the fuel channel to the axial location at which y is being evaluated. Equation B.1 can be rewritten more compactly as,

$$y = Cz^2 \left(\left(\frac{L_{sp}^2}{2} \right) - z^2 \right) \quad B.2$$

where $C = 16D_s / L_{sp}^4$.

The gap between the pressure tube outer surface and the calandria tube inner surface at given circumferential and axial positions θ and z respectively (i.e. $\delta(\theta, z)$) can now be calculated from the relationship,

$$\delta(\theta, z) = -e \cos \theta + \sqrt{(e \cos \theta)^2 - e^2 + CTIR^2} - PTOR \quad B.3$$

where CTIR is the calandria tube inner radius, PTOR is the pressure tube outer radius and e is the eccentricity of the pressure tube ($= (CTIR - PTOR)$, $y = ECC - y$) at axial position z . The gap between the pressure and calandria tubes is measured on a radial line emanating from the center of the pressure tube (at circumferential position θ), as shown in Fig. B.2. Equation B.3 is only valid if the pressure and calandria tubes remain circular after contact, of course, as assumed.

Equation B.3 provides the gap size dictated by purely geometric constraints, that is the gap size that would be obtained if the pressure and calandria tubes were in contact with no contact load. The pressure/calandria tube contact would not be completely unloaded in reality, of course, so a contact patch would form and the pressure and calandria tube surfaces would deform slightly in the region around the contact. For simplicity, it is assumed in SYNCON that the surface deformations within and around the contact would be elastic and, furthermore, that they can be predicted using Hertzian theory. The former assumption may be invalid and the latter assumption would undoubtedly not be accurate even if the contact was formed elastically because typical (elastic) contact dimensions (see Appendix C) are large relative to the calandria and pressure tube

Fig. B.2 Pressure-Calandria Tube Gap Geometry on an r-θ Plane.

wall thicknesses. Hertz's theory is only applicable to contacts which are very small relative to the dimensions of the contacting bodies so it is used only to obtain a rough estimate of the contact size in this application.

Contacting bodies are approximated as paraboloids in Hertz's theory for the purpose of evaluating contact dimensions and surface deformations near the contact so the theory cannot be used to predict gap sizes in regions remote from a contact unless, of course, the contacting bodies are really paraboloids. The latter condition is not satisfied in the present case so Hertz's theory is only used to predict gap sizes in a somewhat arbitrarily defined region around the contact, specifically at values of θ and z for which $0 \leq \theta < 0.2$ (radians) and $0 \leq z < 0.2L_{sp}$. Gap sizes predicted using Hertzian theory at the upper limits cited for θ and z were found to be in good agreement with gap sizes predicted using Eq. B.3 at the same locations so the paraboloidal surface approximations must be fairly realistic in the ranges of θ and z specified. Gap sizes at locations outside of the specified region are calculated in lines 299-302 of subroutine DATA1 in SYNCON by applying Eq. B.3 while gap sizes at locations within the specified region are calculated, using Hertzian theory, in subroutine HGAS. Subroutine HGAS is called by subroutine DATA1 if the checks on the values of θ and z performed in lines 297 and 298 in subroutine DATA1 indicate that $0 \leq \theta < 0.2$ and $0 \leq z < 0.2L_{sp}$.

The principal radii of the pressure and calandria tubes, which will be referred to in the following analysis as bodies 1 and 2

respectively, must be known at the contact in order to predict the dimensions of the contact and the gap sizes around it. If the smaller principal radii of the pressure tube outer and calandria tube inner surfaces are designated R_1 and $(-)R_2$ respectively, then $R_1 = \text{PTOR}$ and $R_2 = \text{CTIR}^*$. The larger principal radius of the calandria tube inner surface, designated R_2' , is infinite, while the larger principal radius of the pressure tube outer surface at the contact (i.e., the bowing/sagging-related radius of curvature), designated R_1' , is given by the expression,

$$R_1' = \frac{1}{d^2 y / dz^2 |_{z=0}} = L_{sp}^2 / 16 D_s \quad \text{B.4}$$

when y is defined by Eq. B.1. The principal radii R_1 , R_1' , R_2 and R_2' are used in Hertz's theory to evaluate additional variables, C_3 and C_4 , defined by the expressions,

$$C_3 = \frac{1}{4} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_1'} + \frac{1}{R_2'} \right) \quad \text{B.5}$$

and,

$$C_4 = \frac{1}{4} \sqrt{\left[\left(\frac{1}{R_1} - \frac{1}{R_1'} \right) + \left(\frac{1}{R_2} - \frac{1}{R_2'} \right) \right]^2 - 4 \left(\frac{1}{R_1} - \frac{1}{R_1'} \right) \left(\frac{1}{R_2} - \frac{1}{R_2'} \right) \sin^2 \gamma} \quad \text{B.6}$$

when γ is the angle between the planes of curvature containing R_1 and R_2 , or equivalently, R_1' and R_2' since the principal planes of

* R_2 is negative because the calandria tube center is on the same side of the contact as the pressure tube center.

curvature (i.e., the planes containing the principal radii of curvature) in each body are orthogonal. For the pressure/calandria tube contact $\gamma = 0^\circ$ so, with $R_2 = \infty$, Eq's B.5 and B.6 simplify to,

$$C_3 = \frac{1}{4} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_1} \right) \quad \text{B.7}$$

and,

$$C_4 = \frac{1}{4} \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_1} \right) \quad \text{B.8}$$

With two additional variables C_1 and C_2 determined from the relationships

$$C_1 = C_3 - C_4 \quad \text{B.9}$$

and,

$$C_2 = C_3 + C_4 \quad \text{B.10}$$

the contact semi-major and semi-minor diameters, designated A and B respectively in SYNCON can be obtained (approximately) from Dyson's formulae, given in Sec I.3.2 and repeated here for convenience as,

$$A = C_a \left(\frac{P\Omega}{C_1 + C_2} \right)^{1/3} \cdot \left(\frac{C_1}{C_2} \right)^n \quad \text{B.11}$$

and,

$$B = C_b \left(\frac{P\Omega}{C_1 + C_2} \right)^{1/3} \cdot \left(\frac{C_1}{C_2} \right)^m \quad \text{B.12}$$

The variables C_a , C_b , m and n in Eq's B.11 and B.12 are tabulated as functions of C_1/C_2 in Table B.1. Also, P is the contact load, and

$$\Omega = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

(ν is Poisson's ratio and E is Young's modulus).

RANGE OF C_1/C_2	C_a	n	C_b	m
$10^{-5} \leq C_1/C_2 \leq 3 \times 10^{-2}$	0.976	-0.372	0.702	0.186
$3 \times 10^{-2} \leq C_1/C_2 \leq 1$	0.908	-0.380	0.894	0.265

TABLE B.1 CONSTANTS FOR CONTACT DIMENSION FORMULAE

Subroutine HGAS was originally written to calculate gas conductances in the model contact problem where contact occurred between cylindrical surfaces with orthogonal planes of curvature. Each cylindrical surface had only one plane of curvature (i.e., only one principal radius was finite on each surface) in the model contact and hence only two radii, R_{s1} and R_{s2} ($R_{s2} \geq R_{s1}$), were needed to characterize the geometry of the contact, specifically to calculate the parameters C_1 and C_2 which, for that contact, were given by the expressions,

$$C_1 = \frac{1}{2R_{s2}} \quad \text{B.13}$$

and

$$C_2 = \frac{1}{2R_{s1}} \quad \text{B.14}$$

Hertz's results for the dimensions of a contact and gap thicknesses adjacent to a contact depend only on the values of C_1 and C_2 (for fixed P , E_1, E_2 , ν_1, ν_2) and hence are the same for contact configurations which have the same values of C_1 and C_2 even if the principal radii are quite different in the different cases. Subroutine HGAS assumes that C_1 and C_2 are defined by Eq's B.13 and B.14 respectively and uses R_{s1}

and R_{s2} in lieu of C_1 and C_2 to define the geometry of the contact, as discussed in Appendix I.A, because the radii are more convenient. Subroutine HGAS can, however, be applied to other contact configurations if the values of C_1 and C_2 for those configurations are used to define two equivalent or effective contact radii, designated R_{1e} and R_{2e} respectively ($R_{2e} \geq R_{1e}$), from the relationships

$$R_{1e} = \frac{1}{2C_2} \quad \text{B.15}$$

and,

$$R_{2e} = \frac{1}{2C_1} \quad \text{B.16}$$

Equations B.15 and B.16 are obviously equivalent to Equations B.13 and B.14 with R_{s1} and R_{s2} replaced by R_{1e} and R_{2e} . The geometric parameters C_1 and C_2 can be determined from Eq's B.7-B.10 for a pressure/calandria tube contact and then used to evaluate R_{1e} and R_{2e} from Eq's B.15 and B.16. The effective contact radii, R_1 and R_2 , are read as RCY1 and RCY2 respectively by SYNCON.

APPENDIX C: CALCULATION OF CONTACT PATCH DIMENSIONS FOR CONTACTING PRESSURE AND CALANDRIA TUBES

The following geometric data apply to a CANDU fuel channel of Bruce B design.

$$R_1 = \text{pressure tube outer radius} = 5.575\text{cm}$$

$$R_2 = -(\text{calandria tube inner radius}) = -6.450\text{cm}$$

$$R_2' = \infty$$

$$L_{sp} = 102\text{cm}$$

Bruce B, units 10^{-4}

$$D_s = 0.56\text{cm}$$

For the stated values of L_{sp} and D_s , Eq. B.4 gives,

$$R_1' = 1161\text{cm}$$

as the bowing/sagging related radius of curvature of the pressure tube.

Substituting the different radii given above into Eq's B.7 and B.8

we obtain,

$$C_3 = 6.299 \times 10^{-3}\text{cm}^{-1}$$

$$C_4 = 5.868 \times 10^{-3}\text{cm}^{-1}$$

and then,

$$C_1 = 4.307 \times 10^{-4}\text{cm}^{-1}$$

and,

$$C_2 = 1.2167 \times 10^{-2}\text{cm}^{-1}$$

from Eq's B.9 and B.10. Introducing these values of C_1 and C_2 into

Eq's B.15 and B.16 yields,

$$R_{1e} = 41.1\text{cm}$$

and,

$$R_{2e} = 1161.0\text{cm}$$

as the effective contact radii.

The contact load (P) and the material properties ν_1 , ν_2 , E_1 and E_2 must be known in order to estimate the contact dimensions A and B . Since the pressure and calandria tubes are fabricated from similar zirconium alloys, we can assume that $\nu_1 \approx \nu_2 \approx \nu$ and $E_1 \approx E_2 \approx E$ if ν and E are evaluated at an average contact temperature. At an estimated average contact temperature of 150°C - 200°C , $E \approx 85\text{GPa}$ and,

$$\nu \approx 0.3$$

for zircaloy -2 tube material (4). Hence $\Omega \approx 2.141 \times 10^{-7} \text{cm}^2/\text{N}$, where Ω is the material dependent parameter seen in Eq's B.11 and B.12.

It can be shown that the combined weight of the fuel bundles and the pressure tube is approximately 569N/m , so the total weight of the fuel bundles and the pressure tube between the two most central garter springs is $1.02\text{m} \times 569\text{N/m} = 580\text{N}$. If it is assumed (arbitrarily) that one-half of this weight is supported by the contact and that the two garter springs each support one-quarter of the weight, the resulting contact load of 290N can be used along with the values of C_1 , C_2 and Ω given previously to evaluate Eq's B.11 and B.12 for the contact dimensions A and B respectively. For $C_1/C_2 = 3.54 \times 10^{-2}$, $C_a = 0.908$, $C_b = 0.894$, $m = 0.265$ and $n = -0.38$ from Table B.1 so,

$$A = 0.908 \left[\frac{290\text{N} \times 2.141 \times 10^{-7} \text{cm}^2/\text{N}}{(4.307 \times 10^{-4} + 1.2167 \times 10^{-2}) \text{cm}^{-1}} \right]^{1/3} (3.54 \times 10^{-2})^{-0.38}$$

$$= 0.550\text{cm}$$

from Eq. B.11 and,

201.

$$B = 0.894 \left(\frac{(290N \times 2.141 \times 10^{-7} \text{ cm}^2/N)}{(4.307 \times 10^{-4} + 1.2167 \times 10^{-2}) \text{ cm}^{-1}} \right)^{1/3} (3.54 \times 10^{-2})^{0.265}$$
$$= 0.0628 \text{ cm}$$

from Eq. B.12.

The aspect ratio of the pressure/calandria tube contact is
 $0.550/0.0628=8.76$ in this example.

APPENDIX D: DICTIONARY OF KEY VARIABLES USED IN SYNCON.

Most of the variables listed below are used in more than one routine in SYNCON. Since COMMON blocks in SYNCON are labelled DBn, where $n = 1, 2, 3, \dots$, etc., the value of n for any variable communicated via a COMMON block is included in brackets after the definition of that variable. Input variables are defined in Tables 2.1a-2.1i.

Real Variables

A	- input variable 21a (77)
AL	- matrix containing subdiagonal elements of the left-hand side coefficient matrix for the system of equations defining Z in a synthesis analysis (15)
AL1	- second matrix containing the contents of AL (34)
AN(I)	- azimuthal coordinate (θ) of the I th azimuthal node (26)
ANTOT	- $\Delta\theta$ in Eq's A.2-A.7 (11)
AREA(L)	- cross-sectional area of $r-\theta$ node L in the $r-\theta$ plane (35)
AXPOS(K)	- axial (z) coordinate of axial node K (7)
B	- input variable 22a (77)
BETA	- input variable 23h (20)
BT	- input variable 26a (2)
C(N)	- the N th internodal conductance calculated in one of subroutines NORNOD, SMARAD, BIGRAD, SMACIR or BIGCIR

- CB - vector containing diagonal elements of the left-hand side coefficient matrix for the system of equations defining Z in a synthesis analysis (15)
- CB1 - second vector containing the contents of CB (34)
- CBW - vector containing the finite-difference coefficients for a single node (4)
- CCNN - input variable 31h (33)
- CCOND - input variable 35h (18)
- CG1 - constant used to calculate the pressure tube-calandria tube gap (37)
- CK - thermal conductivity of solid
- CK1 - input variable 33h
- CK2 - input variable 34h
- CONST - input variable 8a (19)
- CRI - matrix containing superdiagonal elements of the right-hand side coefficient matrix for the system of equations defining Z in a synthesis analysis (15)
- CRI1 - second matrix containing the contents of CRI (34)
- CTID - input variable 17a (37)
- CTOD - input variable 18a (37)
- CTOT - diagonal coefficient in a finite-difference equation
- CX - finite-difference coefficient matrix in vector form (15)

C4	- surface area of outer surface node (at J=JX) (25)
C5	- surface area of node at J=MX
DA	- Δr_a in Eq's A.2-A.7
DB	- Δr_b in Eq's A.2-A.7
DGART	- input variable 19a (37)
DIASUM	- matrix containing terms in AL, CB and CRI that change from one iteration or time step to the next in a non-linear and/or transient synthesis analysis (32)
DR(N)	- Nth radial subdivision size - input variable 3c (NGROPT=0) or 7d (NGROPT=1 or 2) (53)
DRA(J)	- Δr_a at radial node J (singly-subscripted form used when NGROPT=0) (62)
DRA(I,J)	- Δr_a at azimuthal node I and radial node J (doubly-subscripted form used when NGROPT=1 or 2) (59)
DRB(J)	- Δr_b at radial node J (62)
DRB(I,J)	- Δr_b at azimuthal node I and radial node J (59)
DT(N)	- input variable 14h (31)
DTAU	- time step size (Δt)
DTH	- azimuthal subdivision size ($\Delta \theta$) - input variable 3b (NGROPT=0 or 1) or 3f, 7f (NGROPT=2)
DTHA(I)	- $\Delta \theta_a$ (in Eq's A.2-A.7) at azimuthal node I (singly-subscripted form used when NGROPT=0 or 1) (63)
DTHA(I,J)	- $\Delta \theta_a$ at azimuthal node I and radial node J (doubly-subscripted form used when NGROPT=2) (72)

DTHB(I)	- $\Delta\theta_b$ (in Eq's A.2-A.7) at azimuthal node I (63)
DTHB(I,J)	- $\Delta\theta_b$ at azimuthal node I and radial node J (72)
DTOT	- $\overline{\Delta r}$ in Eq's A.2-A.7 (11)
DZA	- Δz_a in Eq's A.2-A.7 (27)
DZB	- Δz_b in Eq's A.2-A.7 (27)
DZTOT	- $\overline{\Delta z}$ in Eq's A.2-A.7 (11)
ECC	- pressure tube eccentricity at fuel channel center (37)
EPSLN	- input variable 20h (44)
G	- volumetric heat generation rate (q_v)
GGAPP(I,K)	- pressure-calandria tube gap at azimuthal node I and axial node K (20)
HEX	- input variable 48h
HFLZ(K)	- input variable 44h (18)
HGEN	- nodal capacitance ($\rho c V_g \Delta t$) (i.e., $THCAP * AREA(L) * DZTOT/DTAU$) (25)
HOUT	- input variable 46h
HRADIN	- input variable 22h
HTGEN(K)	- input variable 43h (27)
HX(I,K)	- outer surface heat transfer coefficient at azimuthal node I and axial node K when IMPECC#1 (24) - pressure/calandria tube gap conductance at azimuthal node I and axial node K when IMPECC=1 (24)
HX1	- H1 when IMPECC#1 - HX(I,K) when IMPECC=1

HX2	- HX(I,K) when IMPECC#1 - H1 when IMPECC=1
HXX	- HX(I,K)
H1	- input variable 32h (9)
PTID	- input variable 15a (37)
PTOD	- input variable 16a (37)
Q	- right-hand side (heat flow) vector for a system of finite-difference equations (10)
QFLX(I,K)	- radiative heat flux at outer surface node I,K (located at azimuthal node I and axial node K) - used when RADBN=1 (45)
QQ	- right-hand side term in a finite-difference equation (=Q(L) in a finite-difference analysis)
R	- radius
RAD(J)	- radius at radial node J (COMMON block 62 when NGROPT=0, block 52 otherwise)
RCY1	- input variable 23a (77)
RCY2	- input variable 24a (77)
RD	- right-hand side vector for the system of equations defining Z in a synthesis analysis
RES	- doubly-subscripted matrix containing the entries in RD. RD is obtained from RES by stacking the columns of RES on top of each other (13)
RESIS	- input variable 12a (3)
REXCO	- input variable 26h (43)

RI	- inner radius of cylinder-input variable 1c (NGROPT=0) or 5d (NGROPT=1 or 2)
RO	- outer radius of cylinder - input variable 6d when NGROPT=1 or 2 (90)
SAR(I,K)	- surface area of outer surface node I,K (located at azimuthal node I, axial node K) (46)
SEPN	- gap between contact surfaces
SLOPDZ	- SLOPS/2 (78)
SLOPG	- input variable 25h (30)
SLOPS	- input variable 29h (78)
SPRDIA	- input variable 20a (37)
T	- vector of nodal temperatures in a finite-difference analysis (23)
TC(I,K)	- sink temperature with which outer surface node I,K (located at azimuthal node I, axial node K) communicates (21)
TCGAS	- input variable 30h (33)
TCINTG	- input variable 24h (30)
TCINTS	- input variable 38h (78)
TEFINAL	- input variable 6h (31)
THA	- $\Delta\theta_a$ in Eq's A.2-A.7
THB	- $\Delta\theta_b$ in Eq's A.2-A.7
THCAP	- product of specific heat and density (31)
THCAP1	- product of DENS1 (input variable 7h) and SPHT1 (input variable 9h) (36)

- THCAP2 - product of DENS2 (input variable 8h) and SPHT2 (input variable 10h) (36)
- THII - Kirchhoff's transformed variable for node I,J,K
 $T(I,J,K)$
 (i.e., $\int_0^T k_s dT$, where $k_s = TCINTS + SLOPS * T$)
- THII2 - Kirchhoff's transformed variable for node I, J-1,K when J=NIS (=MX+1) or for node I,J+1,K when J=MX
- TIME - time from start of transient in a transient analysis
- TO - input variable 13a (1)
- TOR - doubly-subscripted matrix containing orthonormalized trial vectors used at axial node K (i.e., the current axial node) as its columns (COMMON block DBTOR)
- TORKM1 - equivalent of TOR for trial vectors used at axial node K-1 (COMMON block DBTOR)
- TORKP1 - equivalent of TOR for trial vectors used at axial node K+1 (COMMON block DBTOR)
- TRAD(I,K) - temperature of outer surface node I,K (located at azimuthal node I and axial node K) (45)
- TRADIN - input variable 21h (42)
- TRF - doubly-subscripted matrix containing trial vectors as its columns (14)
- XCH - 0.2*DBART (37)

- XLOAD - input variable 11a (3)
- XMF - ((old time step/new time step)-1)
- XPP - input variable 42h
- YP2 - lateral coordinate of outer surface node (y-coordinate in contact centered coordinate system)
- Z - vector of coefficients multiplying trial functions in a synthesis analysis (i.e., the mixing or blending vector) (16)

Integer Variables

- I - azimuthal node index (9)
- IA, IB, IC, ID - variables used to determine azimuthal indices of nodes adjacent to a given node I,J,K (see description of subroutine GROPT2) (51,76)
- ICN - input variable 28a (3)
- IMPECC - input variable 14a (38)
- IN - input variable 4b (NGROPT=0 or 1) or input variable 5f (NGROPT=2) (71)
- INSUL - input variable 10a
- INWT - see the description of subroutine DTH2 (75)
- IS(I,J) - integer by which the radial node index J must be increased (at r=0 node I,J) to obtain the radial node index of the next real node on the Ith radial grid line (i.e., if node I,J is the Lth

- node in the finite-difference grid, then node $I, J+IS(I, J)$ is the $(L+1)$ st if $J \neq JX$) (67)
- ISC - input variable 6f (71)
- ISP - $IS(I, J)$
- ITWT - see the description of subroutine DTH2 (75)
- IX - input variable 1b (NGROPT=0 or 1) or input variable 1f (NGROPT=2) (6)
- IXX - IX if RESIST $\neq$ 1
- IX-1 if RESIST=1 (60)
- J - radial node index (9)
- JA, JB - variables used to determine radial indices of nodes adjacent to a given node I, J, K (JA = $IS(I, J-1)$; JB = $IS(I, J)$) (65)
- JCN - input variable 29a (3).
- JIN(I) - radial index of first (innermost) node visited on radial grid line I (needed when NGROPT=2) (70)
- JL - vector containing numbers which, when added to the sequence number L of a given node I, J, K , indicate the sequence numbers of nodes which are neighbours of node I, J, K but are visited after it (4)
- JP - vector containing numbers which, when added to the sequence number L of a given node I, J, K , indicate the sequence numbers of nodes which are neighbours of node I, J, K but are visited before it (4)
- JSC - input variable 1e (55)

JX	- number of outermost radial node (input variable 3d when NGROPT=1 or 2) (6)
K	- axial node index (9)
KCN	- input variable 30a (3)
KL	- running sum of the finite-difference equation semi-bandwidths and hence the number of entries stored in CX at any given time (5)
KX	- total number of axial nodes (6)
L	- node sequence number (5)
LMAX	- total number of nodes in a finite-difference analysis
M(N)	- input variable 4c (NGROPT=0) or input variable 4d (NGROPT=1 or 2) (58)
MAXIT	- input variable 19h (42)
MBA	- maximum half-bandwidth of the left-hand side coefficient matrix for the system of equations defining Z in a synthesis analysis (15)
MBW	- input variable 3a (4)
MMBA	- MBA (66)
MX	- input variable 2c (NGROPT=0) or input variable 2d (NGROPT=1 or 2) (56)
NCCYL	- input variable 5a (8)
NCONST	- input variable 7a (48)
NCONT	- control variable which is set to zero at the start of each iteration in a non-linear analysis.

but is later set to 1 if convergence tests performed in one of routines OUTSYN, HXIO and RADIAT, indicate that convergence has not been achieved and hence that another iteration should be performed (50)

- NDIA - = 0 if $K=1$
 = $\sum_{n=1}^{K-1} \text{NTRI}(n)$ if $K>1$ (73)
- NDIAP - NDIA for previous (old) trial function schedule
 (needed when trial function schedules are changed in a transient synthesis analysis) (61)
- NDT - number of the time step size (i.e., $\text{DT}(\text{NDT})$)
 currently being used
- NEQU - number of equations in the system of equations
 defining Z in a synthesis analysis (i.e., $\sum_{K=1}^{KX} \text{NTRI}(K)$) (17)
- NEXT - input variable 37h
- NFTS - input variable 11h (42)
- NGROPT - input variable 2a (68)
- NI - input variable 4f (71)
- NIS - $\text{MX}+1$ (9)
- NITER - number of iterations performed in a non-linear
 analysis (28)
- NJL - control variable which =0 if entries in JL do not
 need to be calculated and which =1 otherwise
 (defined in lines 279 and 282 of the main program) (78)

NLIN	- input variable 17h (57)
NNUM(I,J,K)	- node sequence number (L) for node I,J,K (12)
NN5	- entry in C representing the gap conductance in a LOECC analysis (i.e., C(NN5) is the gap conductance)
NR	- input variable 1d (58)
NSYN	- =2 in the substitute/weight/integrate phase of a synthesis analysis = 1 otherwise
NTC	- input variable 15h (31)
NTDT	- number of entries in NTC (or DT), less one (31)
NTFCH	- input variable 3h
NTFIC	- input variable 4h (49)
NTFTS	- input variable 13h (42)
NTPRNT	- input variable 27a (3)
NTRF	- NTRI(K) when the current axial node number is K (14)
NTRFP	- NTRIP(K) when the current axial node number is K (61)
NTRI	- input variable 50h or 1i (69)
NTRIP	- NTRI for the previous (old) trial function schedule (needed when trial function schedules are changed in a transient synthesis analysis) (64)
NTRK NK)	- the number of the NKth trial function used at axial node K in a synthesis analysis (69)
NTRKP	- NTRK for the previous (old) trial function schedule (needed when trial function schedules are changed in a transient synthesis analysis) (64)

NTRTF	- input variable 2h (42)
NTS	- time step number (40)
NTSTFC	- input variable 12h (47)
NTW	- see the description of subroutine RAD12 (54)
NVC	- =1 when the thermal conductivity in an internodal conductance expression is to be evaluated at the average of the temperatures of the participating nodes = 0 otherwise (78)
NVCOR	- input variable 27h (78)
NXT	- counter for the number of trial function schedules used in a transient synthesis analysis (47)
NWTT	- see the description of subroutine RAD12 (54)
NZFLO	- input variable 38h (80)
N2D	- input variable 4a
OUTRY	- input variable 36h (19)
RADBND	- input variable 18h (41)
RESIST	- input variable 9a (1)
SYN	- input variable 6a (19)
TRANS	- input variable 1h (31)
TUBE	- input variable 25a (2)

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COMPUTER LISTINGS

Listing 1 SYNCON

```

0001      0001.000
0002      0001.100
0003      0001.200
0004      0001.300
0005      0001.400
0006      0001.500
0007      0001.600
0008      0001.700
0009      0001.800
0010      0001.900
0011      0002.000
0012      0002.100
0013      0002.200
0014      0002.300
0015      0002.400
0016      0002.500
0017      0002.600
0018      0002.700
0019      0002.800
0020      0002.900
0021      0003.000
0022      0003.100
0023      0003.200
0024      0003.300
0025      0003.400
0026      0003.500
0027      0003.600
0028      0003.700
0029      0003.800
0030      0003.900
0031      0004.000
0032      0004.100
0033      0004.200
0034      0004.300
0035      0004.400
0036      0004.500
0037      0004.600
0038      0004.700
0039      0004.800
0040      0004.900
0041      0005.000
0042      0005.100
0043      0005.200
0044      0005.300
0045      0005.400
0046      0005.500
0047      0005.600
0048      0005.700
0049      0005.800
0050      0005.900
0051      0006.000
0052      0006.100
0053      0006.200
0054      0006.300
0055      0006.400
0056      0006.500
0057      0006.600
0058      0006.700
0059      0006.800
0060      0006.900
0061      0007.000
0062      0007.100
0063      0007.200
0064      0007.300
0065      0007.400
0066      0007.500
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0069      0007.800
0070      0007.900
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0072      0008.100
0073      0008.200
0074      0008.300
0075      0008.400
0076      0008.500
0077      0008.600
0078      0008.700
0079      0008.800
0080      0008.900
0081      0009.000
0082      0009.100
0083      0009.200
0084      0009.300
0085      0009.400
0086      0009.500
0087      0009.600
0088      0009.700
0089      0009.800
0090      0009.900
0091      0010.000
0092      0010.100
0093      0010.200
0094      0010.300
0095      0010.400
0096      0010.500
0097      0010.600
0098      0010.700
0099      0010.800
0100      0010.900
0101      0011.000
0102      0011.100
0103      0011.200
0104      0011.300
0105      0011.400

0001      ***** PROGRAM SYNCON *****
0002
0003      THIS PROGRAM, DEVELOPED BY T. CURRIE AS PART OF HIS PH.D. RESEARCH,
0004      IS CAPABLE OF SOLVING STEADY-STATE AND TRANSIENT PROBLEMS IN
0005      CYLINDRICAL COORDINATES IN TWO (R-THETA) OR THREE (R-THETA-Z)
0006      DIMENSIONS. THE PROGRAM FEATURES A FINITE-DIFFERENCE GRID WITH
0007      VARIABLE NODAL SPACINGS WHICH MAY BE USED TO SOLVE PROBLEMS
0008      INVOLVING SOLID-SOLID CONTACTS ON THE EXTERNAL SURFACES OF
0009      CYLINDERS (EG. FUEL RODS). THE PROGRAM CAN ALSO BE USED TO ANALYSE
0010      HEAT TRANSFER BETWEEN A SAMPED PRESSURE TUBE AND A CALANDRIA
0011      TUBE IN A CANDU REACTOR DURING A LOECC (LOSS OF EMERGENCY CORE
0012      COOLANT) ACCIDENT. 2-D PROBLEMS ARE SOLVED USING THE FINITE
0013      DIFFERENCE METHOD (FDM) WHILE 3-D PROBLEMS MAY BE SOLVED USING
0014      EITHER THE FINITE-DIFFERENCE METHOD OR THE SYNTHESIS TECHNIQUE.
0015      THE LATTER TECHNIQUE IS ESSENTIALLY A GLOBAL TRIAL FUNCTION METHOD
0016      IN WHICH 2-D TRIAL FUNCTIONS (IN THIS CASE FIN. DIFF. R-THETA
0017      SOLUTIONS) ARE LINEARLY COMBINED AT EACH AXIAL (Z) STATION OF THE
0018      GRID TO PRODUCE AN APPROXIMATION TO THE TRUE 3-D TEMPERATURE
0019      DISTRIBUTION. TRANSIENT ANALYSES MAY BE PERFORMED USING EITHER THE
0020      FINITE-DIFFERENCE OR THE SYNTHESIS TECHNIQUE. IN BOTH CASES THE
0021      IMPLICIT FORMULATION OF THE HEAT CONDUCTION EQUATION IS USED.
0022      NEWTON LINEARIZATION IS USED IN BOTH TRANSIENT AND STEADY-STATE
0023      ANALYSES (SYNTHESIS OR FDM) TO TREAT BOUNDARY CONDITIONS INVOLVING
0024      RADIATION HEAT TRANSFER AND/OR TEMPERATURE DEPENDENT GAS
0025      CONDUCTION, EXCEPT IN THE CASE OF A FINITE-DIFFERENCE LOECC
0026      ANALYSIS WHERE THE METHOD OF SUCCESSIVE APPROXIMATIONS IS
0027      USED TO HANDLE NON-LINEARITIES. IF THE THERMAL CONDUCTIVITY OF
0028      THE PRESSURE AND CALANDRIA TUBE MATERIAL IS CONSIDERED
0029      TEMPERATURE DEPENDENT IN A LOECC ANALYSIS, KIRCHOFF'S
0030      TRANSFORMATION IS USED TO LINEARIZE THE CONDUCTIVITY.
0031
0032      ***** INPUT VARIABLES READ BY THE MAIN PROGRAM *****
0033
0034      TITLE(1):J=1-8 - JOB TITLE-MAY BE UP TO 32 CHARACTERS LONG.
0035      NROPT - NUMBER OF GRID OPTION. THIS VARIABLE MAY ASSUME ONE
0036      OF THE THREE VALUES GIVEN BELOW.
0037      NROPT=0 - GRID SPACINGS 'DR', 'DTHETA', AND 'DZ' ARE FUNCTIONS OF
0038      R (RADIUS), THETA (CIRCUMFERENTIAL POSITION), AND Z
0039      (AXIAL POSITION) RESPECTIVELY.
0040      NROPT=1 - 'DR' IS A FUNCTION OF R AND THETA BUT 'DTHETA' IS ONLY A
0041      FUNCTION OF THETA AND 'DZ' IS ONLY A FUNCTION OF Z.
0042      NROPT=2 - 'DR' AND 'DTHETA' ARE FUNCTIONS OF BOTH R AND THETA BUT
0043      'DZ' IS A FUNCTION OF Z ALONE.
0044
0045      ROW - 'MAXIMUM BANDWIDTH' THE VALUE OF THIS VARIABLE MUST
0046      BE GREATER THAN THE LEFT-HAND BANDWIDTH OF ANY ROW IN
0047      THE CONDUCTANCE MATRIX (IE. MUST BE GREATER THAN THE
0048      LARGEST LEFT-HAND ROW BANDWIDTH).
0049      MZD=1 - 2-D FINITE-DIFFERENCE ANALYSIS OR 3-D SYNTHESIS
0050      ANALYSIS. THE LATTER ANALYSIS COMMENCES WITH 2-D FINITE
0051      DIFFERENCE ANALYSES, HENCE MZD=1 DESPITE THE FACT THAT
0052      THE ANALYSIS IS ULTIMATELY 3-D.
0053      MZD=0 - 3-D FINITE-DIFFERENCE ANALYSIS.
0054      NCYL - COMPLETE CYLINDER ANALYSIS OPTION: NCYL=1 INDICATES
0055      THAT THE ANALYSIS INVOLVES A COMPLETE CYLINDER (IE. 360
0056      DEGREES); NCYL=0 INDICATES THAT A SECTION OF A CYLINDER
0057      (IE. LESS THAN 360 DEGREES) IS TO BE ANALYZED.
0058
0059      SYN=1 - SYNTHESIS SOLUTION.
0060      SYN=0 - NO SYNTHESIS SOLUTION.
0061      NCONST - CONSTANT TRIAL FN. OPTION: =1 IF A CONSTANT IS TO BE USED
0062      AS THE LAST TRIAL FN. IN A SYNTHESIS ANALYSIS. =0 OTHERWISE.
0063      CONST - VALUE OF THE CONSTANT TO BE USED AS A TRIAL FN. (NEEDED
0064      ONLY IF NCONST=1).
0065      RESIST - RESISTANCE OF CONTACT ANALYSIS OPTION: =1 IF THE
0066      RESISTANCE OF A CONTACT OF SPECIFIED SIZE ON A SEMI-
0067      CYLINDER IS TO BE COMPUTED; =0 OTHERWISE. VALUES OF
0068      VARIABLES 'INSL' TO 'TO' FOLLOWING ARE NOT REQUIRED IF
0069      RESIST=0.
0070      INSL - ISOLATED BOUNDARY OPTION: =1 IF THE CONTACT RESISTANCE
0071      IS TO BE COMPUTED WITHOUT GAS CONDUCTION; =0 IF THE
0072      ANALYSIS IS TO BE PERFORMED WITH GAS CONDUCTION.
0073      XLOAD - CONTACT LOAD IN RESISTANCE OF CONTACT ANALYSIS (N.).
0074      RESIS - RESISTANCE OF SEGMENT OF CONTACT SPECIMEN WITH A LENGTH
0075      EQUAL TO THE SEMI-CYLINDER DIAMETER AND CROSS-SECTIONAL
0076      DIMENSIONS EQUAL TO THE SEMI-CYLINDER'S DIAMETER AND
0077      LENGTH (DEB. C/U). RESIS IS SUBTRACTED FROM THE OVERALL
0078      RESISTANCE OF THE CONTACT TO OBTAIN THE RESISTANCE BASED
0079      ON THE EXTRAPOLATED TEMPERATURE DROP AT THE INTERFACE.
0080      TO - TEMP. OF FLAT BISECTING PLANE OF SEMI-CYLINDER IN RES.
0081      OF CONTACT ANALYSIS (DEB. C).
0082      IMPECC - LOECC ANALYSIS OPTION: =1 IF A LOECC ANALYSIS IS TO BE
0083      PERFORMED; =0 OTHERWISE. LOECC ANALYSES CAN ONLY BE
0084      PERFORMED WITH NROPT=2. VALUES OF VARIABLES PTID TO
0085      SPRDIA FOLLOWING ARE NOT REQUIRED IF IMPECC=0.
0086      PTID - PRESSURE TUBE INNER DIAMETER.
0087      PTOD - PRESSURE TUBE OUTER DIAMETER.
0088      CTID - CALANDRIA TUBE INNER DIAMETER.
0089      CTOD - CALANDRIA TUBE OUTER DIAMETER.
0090      DGART - SPACING BETWEEN TWO MOST CENTRAL GARTER SPRINGS IN CANDU
0091      FUEL CHANNEL.
0092      SPRDIA - GARTER SPRING DIAMETER.
0093      A - SEMI-MAJOR DIAMETER OF ELLIPTICAL CONTACT PATCH (NEEDED IF
0094      RESIST=1 OR IMPECC=1).
0095      B - SEMI-MINOR DIAMETER OF ELLIPTICAL CONTACT PATCH (NEEDED IF
0096      RESIST=1 OR IMPECC=1).
0097      RCV1 - SMALL CONTACT RADIUS IF RESIST=1 OR SMALL EFFECTIVE CONTACT
0098      RADIUS IF IMPECC=1. VALUE NOT REQUIRED FOR OTHER ANALYSES.
0099      RCV2 - LARGE CONTACT RADIUS IF RESIST=1 OR LARGE EFFECTIVE CONTACT
0100      RADIUS IF IMPECC=1. VALUE NOT REQUIRED FOR OTHER ANALYSES.
0101      TUBE - TUBE ANALYSIS OPTION: =1 FOR ANALYSIS OF HEAT FLOW IN
0102      A TUBE CONTAINING AN INTERNAL FLUID AT BULK TEMPERATURE
0103      BT.
0104      BT - BULK TEMPERATURE OF FLUID IN TUBE. VALUE ONLY REQUIRED IF
0105      TUBE=1.

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0106 C NTPRNT - NUMBER OF MODAL TEMPERATURES TO BE PRINTED OUT IN A
0107 C SYNTHESIS ANALYSIS.
0108 C ICH(LL),JCN(LL),KCN(LL) - LOCATION (CIRCUMFERENTIAL, RADIAL AND
0109 C AXIAL MODE NUMBERS RESPECTIVELY) OF LLTH MODE WHOSE
0110 C TEMPERATURE IS TO BE PRINTED OUT AFTER A SYNTHESIS
0111 C ANALYSIS. NTPRNT VALUES OF THESE VARIABLES MUST BE
0112 C SUPPLIED WITH DATA CARDS (EACH OF WHICH CONTAINS VALUES
0113 C OF ICH, JCN AND KCN FOR A PARTICULAR MODE) ARRANGED SO
0114 C THAT THE VALUES OF ICH, JCN AND KCN VARY FROM LEAST TO MOST
0115 C RAPIDLY IN THAT ORDER.
0116 C
0117 C INTEGER TUNE,SYN,RESIST,OUTRY,TRANS,RADBNB
0118 C DOUBLE PRECISION RES,RO,AL,CRI,CB,DIASUM,AL1,CRI1,CB1
0119 C DIMENSION OFLC( 1, 1),TRAD(39,45),TITLE(8),ICH(43),JCN(43),
0120 C KCN(43),DZA(43),DZB(43),HX(39,45),TC(39,45),HTGEN(45),RES(5,45),
0121 C RB(240),T(470),AL(240,7),AL1( 240,7),CRI(240,7),CRI1( 240,7),
0122 C CB(240),CB1(240),DT(10),NTL(10),TRF(470,13),NTFTS(10),NTRI(45),
0123 C NTRK(45,10),DIASUM( 5, 5,45),NTSTFC(10),JL(9),JP(9),CBW(250),
0124 C NTRIP(45),NTRKP(45,10)
0125 C COMMON /DB1/RESIST,TO /DB2/TUNE,BT /DB3/RESIS,XLOAD,NTPRNT,ICH,
0126 C JCN,KCN /DB4/CBW,HMW,JL,JP /DB5/KL,L /DB6/IX,JX,KX /DB8/MCCYL
0127 C /DB9/I,J,K,MIS,MI /DB11/ANTOT,DZTOT,DTOT /DB13/RES /DB14/NTRF,
0128 C TRF /DB15/CB,AL,CRI,HBA /DB17/NEBU /DB19/SYN,OUTRY,CONST /DB21/TC
0129 C /DB23/T /DB24/HX /DB27/DZA,DZB,HTGEN /DB28/NITER /DB31/TRANS,
0130 C TNPIN,TFINAL,THCAP,DT,NTST,NTC /DB32/DIASUM /DB33/TCBAS,CCW
0131 C COMMON /DB34/AL1,CRI1,CB1 /DB36/THCAP1,THCAP2 /DB37/PTIB,PTOB,
0132 C CTID,CTOB,DGART,SPRDBA,ECC,CB1,KCN /DB38/INPECC /DB39/NTOL
0133 C /DB40/NTS /DB41/RADBNB /DB42/TRADIN,MAXIT,NTRIF,NFTS,NTFTS
0134 C /DB43/REXCO /DB45/OFLX,TRAD /DB47/NTSTFC,NXT /DB48/NCONST
0135 C /DB49/NTFIC /DB50/NCONY /DB51/IA,IB /DB54/HX /DB57/NLIN /DB60/IXX
0136 C /DB61/NTRFP,NBIAP /DB64/NTRIP,NTRKP /DB66/HBA /DB68/NOROPT
0137 C /DB69/NTRI,NTRK /DB73/HBA /DB77/A,B,RCY1,RCY2
0138 C COMMON /DB78/TCINTS,SLOPS,SLOPD2,HJL,HAC,HVCON
0139 C /DB80/NZFLO,NZFC,NZBC
0140 C
0141 C READ INPUT DATA.
0142 C
0143 C READ (105,995) (TITLE(I),I=1,8)
0144 C READ (105,991) NOROPT,HMW,NZB,MCCYL
0145 C READ (105,996) SYN,NCONST,CONST
0146 C READ (105,992) RESIST,INSL,XLOAD,RESIS,TO
0147 C READ (105,999) INPECC,PTIB,PTOB,CTID,CTOB,DGART,SPRDBA
0148 C READ (105,997) A,B,RCY1,RCY2
0149 C READ (105,993) TUNE,BT
0150 C READ (105,998) NTPRNT
0151 C
0152 C IF(NTPRNT-1) 20,25,25
0153 C DO 15 LL=1,NTPRNT
0154 C
0155 C READ (105,994) ICH(LL),JCN(LL),KCN(LL)
0156 C
0157 C CONTINUE
0158 C CONTINUE
0159 C 20 WRITE (106,987) (TITLE(I),I=1,8)
0160 C IF(SYN-1) 21,22,21
0161 C 22 WRITE (106,981)
0162 C IF(RESIST.EB.1.AND.INSL.EB.1) WRITE (106,982)
0163 C IF(RESIST.EB.1.AND.INSL.NE.1) WRITE (106,983)
0164 C IF(INPECC.EB.1) WRITE (106,988) PTIB,PTOB,CTID,CTOB,DGART,SPRDBA
0165 C ,A,B,RCY1,RCY2
0166 C
0167 C GO TO 23
0168 C 21 WRITE (106,984)
0169 C 23 WRITE (106,985) NOROPT
0170 C
0171 C NTRF=1
0172 C THCAP1=0.0
0173 C THCAP2=0.0
0174 C IA=1
0175 C IB=1
0176 C
0177 C CALL ROUTINES TO DEFINE THE R-THETA GRID GEOMETRY. ROUTINES CALLED
0178 C DEPEND ON THE VALUE OF NOROPT.
0179 C
0180 C IF(NOROPT-1) 11,12,13
0181 C 11 CALL BTH01 (IX)
0182 C CALL RABO (RI,JX,RO )
0183 C GO TO 14
0184 C 12 CALL BTH01 (IX)
0185 C CALL RAB12
0186 C CALL ISTJ
0187 C CALL BRAB
0188 C GO TO 14
0189 C 13 CALL RAB12
0190 C CALL BTH2
0191 C CALL JINI
0192 C CALL ISTJ
0193 C CALL BRAB
0194 C IF(INPECC.EB.1) CALL CALAND
0195 C NIS=IX+1
0196 C KX=1
0197 C
0198 C CALL ROUTINE (B2) TO DEFINE THE AXIAL GRID GEOMETRY IF NECESSARY.
0199 C
0200 C IF(NZB.NE.1) CALL B2 (BZA,BZB,KX)
0201 C
0202 C INITIALIZE THE TRIAL FN. COUNTER (NTRF).
0203 C NTRF=0
0204 C IXX=IX
0205 C IF(RESIST.EB.1) IXX=IX-1
0206 C
0207 C INCREMENT THE TRIAL FN. COUNTER (NTRF) BY 1.
0208 C NTRF=NTRF+1
0209 C IF(STN.EB.1) WRITE (106,986) NTRF
0210 C
0211 C INITIALIZE THE ITERATION COUNTER (NITER).
0212 C NITER=1
0213 C
0214 C CALL ROUTINE (BATAL) TO READ SPECIFIC JOB DATA AND TO READ AND/OR
0215 C CALCULATE BOUNDARY CONDITIONS.
0216 C CALL BATAL (HX,TC ,MI,CK1,CK2,NEXT,NMTH)
0217 C NTS=0
0218 C
0219 C CHECK IF A TRANSIENT ANALYSIS IS REQUIRED.
0220 C IF(TRANS.NE.1) GO TO 4
0221 C
0222 C INITIALIZE COUNTERS FOR THE NUMBER OF TIME STEPS (NTS) PERFORMED,
0223 C THE NUMBER OF TRANSIENT TRIAL FNS. RECORDED IN A PARTICULAR 2-D
0224 C TRANSIENT ANALYSIS (ITN), AND THE NUMBER OF TRIAL FN. DEPLOYMENT
0225 C SCHEDULE CHANGES MADE IN THE 2ND. PHASE CALCS. OF A
0226 C TRANSIENT SYNTHESIS ANALYSIS (NXT).
0227 C ITN=1
0228 C

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0226      NXT=1
0227      NTS=0
0228
0229      C INITIALIZE THE TIME IN THE ANALYSIS.
0230      TIME=0.0
0231      DTAU=DT(1)
0232
0233      C INCREMENT THE TIME STEP COUNT (NTS) BY 1.
0234      NTS=NTS+1
0235
0236      C CHECK WHETHER THE TIME STEP SIZE (DTAU) CHANGES AT THIS VALUE OF
0237      C NTS AND CHANGE IT IF NECESSARY. ADVANCE THE TIME IN THE ANALYSIS.
0238      DO 7 NDT=1,NTDT
0239      IF(NTS.EQ.NTC(NDT)) GO TO 8
0240      7 CONTINUE
0241      GO TO 9
0242      8 DTAU=DT(NDT+1)
0243      TIME=TIME+DTAU
0244      GO TO 6
0245      9 TIME=TIME+DTAU
0246
0247      C CHECK WHETHER ONLY THE RHS. VECTOR NEEDS TO BE RECALCULATED TO
0248      C ADVANCE TO THE NEXT TIME STEP (FIN.-DIFF. ANALYSIS). ONLY THE RHS.
0249      C VECTOR NEEDS TO BE RECALCULATED FOR THE SECOND AND SUBSEQUENT TIME
0250      C STEPS OF A LINEAR TRANSIENT F-D ANALYSIS. THE RHS. VECTOR IS
0251      C RECALC. IN OFORN.
0252      IF(MSYN.EQ.2.OR.NLIN.EQ.1) GO TO 6
0253      IF(NTS.EQ.1) GO TO 6
0254      NITER=1
0255      CALL OFORN (THCAP,DTAU)
0256      GO TO 64
0257      6 KL=0
0258
0259      C INITIALIZE THE CONVERGENCE INDICATOR FOR A STEADY-STATE
0260      C FINITE-DIFFERENCE ANALYSIS (NCONT).
0261      C NCONT IS ASSIGNED A VALUE OF 1 IN CONVERGENCE TESTS PERFORMED IN SUB-
0262      C ROUTINES KXID AND RADIAT IF CONVERGENCE HAS NOT BEEN ACHIEVED AND
0263      C ANOTHER ITERATION IS REQUIRED.
0264      NCONT=0
0265      IF(NITER.LE.2) NCONT=1
0266      L=0
0267      5 NRIA=0
0268      NRIAP=0
0269      IF(MSYN.EQ.2) NITER=1
0270
0271      C IF INTERNAL THERMAL CONDUCTIVITIES ARE TO BE CALCULATED FROM
0272      C NODAL TEMPERATURES IN THIS TIME STEP OR ITERATION, SET MNC=1.
0273      C IF ENTRIES IN THE VECTOR OF NODAL TEMPERATURES PAST THE ENTRY
0274      C CORRESPONDING TO THE TEMPERATURE OF THE NODE FOR WHICH THE F-D
0275      C EQUATION IS BEING ASSEMBLED ARE REQUIRED FOR SOME REASON, SET
0276      C NLA=1. LATER ENTRIES IN THE TEMP. VECTOR ARE REQUIRED IN THE
0277      C 2ND PHASE CALCS. OF A SYNTHESIS ANALYSIS AND TO CALCULATE
0278      C INTERNAL THERMAL CONDUCTIVITIES WHEN MNC=1.
0279      NLA=0
0280      MNC=0
0281      IF(MACON.EQ.1.AND.(NITER.GT.1.OR.NTS.GT.1)) MNC=1
0282      IF(MSYN.EQ.2.OR.NITER.GT.1.OR.NTS.GT.1) NLA=1
0283
0284      C SWEEP THROUGH THE AXIAL NODES IN THE GRID. (K=1,2,...,KX)
0285      DO 10 K=1,KX
0286      IF(MSYN=2) 17,16,17
0287
0288      C IN THE 2ND. PHASE CALCS. OF A SYNTHESIS ANALYSIS, ASSIGN VALUES TO
0289      C THE NUMBER OF TRIAL FNS. USED AT AXIAL NODE K IN THE CURRENT AND
0290      C PREVIOUS TRIAL FN. DEPLOYMENT SCHEDULES (NTRF AND NTRFP RESPECTIVELY
0291      C ) AND INITIALIZE THE R-THETA NODE NUMBER COUNTER (L).
0292      16 NTRF=NTRI(K)
0293      NTRFP=NTRIP(K)
0294      L=0
0295      17 IF(NZB.NE.1) NZTOT=(BZA(K)+BZB(K))/2.0
0296      IF(NZB.EQ.1) NZTOT=1.0
0297      IF(NZFLO.EQ.1) NZTOT=BZBC/2.0
0298
0299      C CALL ROUTINE TO SWEEP THROUGH THE R-THETA NODES AT AXIAL NODE K.
0300      IF(NROPT.EQ.0) CALL GROPT0 (DTAU,HTGEN(K),BZA(K),BZB(K),MSYN,
0301      5 CK1,CK2,XX,NTS)
0302      IF(NROPT.EQ.1) CALL GROPT1 (DTAU,HTGEN(K),BZA(K),BZB(K),MSYN,
0303      5 CK1,CK2,XX,NTS)
0304      IF(NROPT.EQ.2) CALL GROPT2 (DTAU,HTGEN(K),BZA(K),BZB(K),MSYN,
0305      5 CK1,CK2,XX,NTS)
0306
0307      C CALCULATE THE NUMBER OF EQUATIONS (IN THE SYSTEM OF EQUATIONS
0308      C DEFINING THE BLENDING VECTOR (Z)) OBTAINED SO FAR IN THE 2ND.
0309      C PHASE CALCS. OF A SYNTHESIS ANALYSIS. (NBIA).
0310      IF(MSYN.EQ.2) NBIA=NBIA+NTRF
0311      IF(MSYN.EQ.2) NBIAP=NBIAP+NTRFP
0312      10 CONTINUE
0313      IF(NITER.EQ.1.AND.NTS.EQ.1) OUTPUT KL
0314      LMAX=L
0315
0316      C UNLESS THE 2ND. PHASE CALCULATIONS OF A SYNTHESIS ANALYSIS ARE
0317      C BEING PERFORMED, GO TO STATEMENT 60 WHERE ROUTINES WHICH SOLVE
0318      C THE F-D EQUATIONS ARE CALLED.
0319      IF(MSYN=2) 60,50,60
0320      50 NSTOP=0
0321      IF(TRANS.NE.1.AND.RABND.NE.1.AND.NLIN.NE.1) NSTOP=1
0322      IF(NSTOP.EQ.1) GO TO 55
0323
0324      C STORE ENTRIES IN THE Banded MATRICES CB,AL AND CRI (WHICH, TAKEN
0325      C TOGETHER, DEFINE THE LHS. COEFFICIENT MATRIX FOR THE SYSTEM OF
0326      C EQU. S DEFINING Z) IN A SECOND SET OF MATRICES CB1,AL1 AND CRI1.
0327      C THIS IS DONE BECAUSE THE ORIGINAL MATRICES, WHICH CAN BE UPDATED
0328      C AND USED IN ADDITIONAL TIME STEPS OR ITERATIONS, ARE DESTROYED
0329      C WHEN Z IS OBTAINED IN SUBROUTINE BNDEL.
0330      DO 95 N1=1,NBIA
0331      CB1(N1)=CB(N1)
0332      DO 100 N2=1,NBA
0333      AL1(N1,N2)=AL(N1,N2)
0334      CRI1(N1,N2)=CRI(N1,N2)
0335      100 CONTINUE
0336      95 CONTINUE
0337      TIME=TIME
0338
0339      C TRANSFER THE RMS. WEIGHTED RESIDUALS (FOR THE SYSTEM OF EQUATIONS
0340      C DEFINING Z) STORED IN RES TO THE VECTOR RS.
0341      NC=0
0342      DO 65 K=1,KX
0343      NRI=NTRI(K)
0344      DO 70 NNT=1,NT1
0345      NC=NC+1

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0346          RD(MC)=RES(MNT,K)
0347          RES(MNT,K)=0.0
0348          CONTINUE
0349          70 CONTINUE
0350          IF(MSTOP.EQ.1) GO TO 112
0351          C
0352          IN A NON-LINEAR ANALYSIS, CALL MATCH TO SUBTRACT THE ENTRIES IN
0353          C THE MATRIX DIASUM FROM THE APPROPRIATE ENTRIES IN AL1,CB1 AND CR11.
0354          IF(RABND.EQ.1.OR.MLIN.EQ.1) CALL MATCH (MNT,DT,KX,NTRF
0355          ,MBA,-1.0,TRANS)
0356          IF(TRANS.NE.1) GO TO 112
0357          C
0358          IN A TRANSIENT ANALYSIS, INCREMENT THE TIME STEP COUNT (NTS) AND
0359          C CALCULATE THE NEXT TIME STEP SIZE.
0360          NTS=NTS+1
0361          DO 101 MNT=1,MNTDT
0362          IF(MTS.EQ.MTC(MNT)) GO TO 105
0363          101 CONTINUE
0364          GO TO 110
0365          105 DTAU=DT(MNT+1)
0366          IF(RABND.EQ.1.OR.MLIN.EQ.1) GO TO 110
0367          C
0368          IF THE TIME STEP SIZE HAS CHANGED AND THE ANALYSIS BEING PERFORMED
0369          C IS LINEAR, CALCULATE A FACTOR XMF FOR USE IN CHANGING AL1,CB1 AND
0370          C CR11 TO THE NEW TIME STEP SIZE. CALL MATCH TO ADD XMF TIMES THE
0371          C ENTRIES IN DIASUM TO THE APPROPRIATE ENTRIES IN AL1,CB1 AND CR11
0372          C TO ACHIEVE THE CHANGE.
0373          XMF=((DT(MNT)/DT(MNT+1))-1.0)
0374          CALL MATCH (MNT,DT,KX,NTRF,MBA,XMF,TRANS)
0375          110 TINI=TINI+DTAU
0376          C
0377          CALL THE ROUTINE (BNDL) TO OBTAIN Z FROM AL,CB,CRI AND RB.
0378          CALL BNDL (RB)
0379          C
0380          CALL OUTSYN TO PRINT THE RESULTS OF THE SYNTHESIS ANALYSIS AND
0381          C TO RECOMPUTE, IF NECESSARY, THE ENTRIES IN DIASUM AND RES (IE.,RB)
0382          C USING THE NEW NODAL TEMPERATURES.
0383          CALL OUTSYN (IXX,TC,MTCEN,BZA,BZB,KX,MSTOP,DTAU,TRCAP,TIME,TRANS
0384          ,M1,INSUL,LMAX)
0385          C
0386          IF THE TIME STEP NUMBER (NTS) IS ONE AT WHICH THE TRIAL FN.
0387          C DEPLOYMENT SCHEDULE SHOULD BE CHANGED IN THE 2ND PHASE CALCS.
0388          C OF A TRANSIENT SYNTHESIS ANALYSIS, CALL NTRP TO STORE THE EXISTING
0389          C SCHEDULE, CALL TFREAD TO READ THE NEW SCHEDULE, CALL ZERO TO
0390          C RE-ZERO THE MATRICES AL,CB,CRI,DIASUM AND RES AND THEN RETURN
0391          C TO STATEMENT 5 TO REPEAT THE SUBSTITUTE/WEIGHT/INTEGRATE PHASE OF
0392          C THE ANALYSIS WITH THE NEW SCHEDULE.
0393          IF(TRANS.EQ.1.AND.MTS.EQ.MTSTFC(MNT)) TIME=TINI ; CALL NTRP(KX)
0394          ; CALL TFREAD (TIME,KX) ; MBA=MBA ; CALL ZERO (KX,NTRF,RB,
0395          ,TRANS) ; NKT=MNT+1 ; GO TO 5
0396          C
0397          IF THE ANALYSIS BEING PERFORMED IS NON-LINEAR, CALL MATCH TO ADD
0398          C THE ENTRIES IN DIASUM (WHICH WERE RECOMPUTED IN SUB. OUTSYN USING
0399          C THE NEW NODAL TEMPERATURES) TO THE APPROPRIATE ENTRIES IN AL1,CB
0400          C AND CR11.
0401          IF((RABND.EQ.1.OR.MLIN.EQ.1).AND.MSTOP.NE.1) CALL MATCH (
0402          ,MNT,DT,KX,NTRF,MBA,1.0,TRANS)
0403          IF(MSTOP.EQ.1) GO TO 80
0404          C
0405          PREPARE THE MATRICES AL,CB AND CRI FOR THE NEXT TIME STEP OR
0406          C ITERATION BY EQUATING THEM TO THE UPDATED MATRICES AL1,CB1 AND CR11.
0407          DO 115 M1=1,NBDA
0408          CB(M1)=CB1(M1)
0409          DO 120 M2=1,NBA
0410          AL(M1,M2)=AL1(M1,M2)
0411          CRI(M1,M2)=CRI1(M1,M2)
0412          120 CONTINUE
0413          115 CONTINUE
0414          C
0415          INCREMENT THE ITERATION COUNT (NITER) BY 1.
0416          NITER=NITER+1
0417          C
0418          CHECK WHETHER MAX. PERMISSIBLE ITER. COUNT (MAXIT) HAS BEEN EXCEEDED.
0419          IF(TRANS.NE.1.AND.NITER.GT.MAXIT) OUTPUT*
0420          , 'CONVERGENCE NOT ACHIEVED' ; GO TO 80
0421          IF(TRANS.NE.1) GO TO 55
0422          TIME=TINI
0423          IF(TIAL.GT.TFINAL.OR.MTS.GT.MFTS) MSTOP=1
0424          C
0425          RETURN TO STATEMENT 55 TO CALCULATE Z FOR THE NEXT TIME STEP OR
0426          C ITERATION.
0427          GO TO 55
0428          C
0429          CALL CHLSKI TO FACTOR THE F-0 COEFFICIENT MATRIX INTO THE PRODUCT
0430          C OF A LOWER TRIANGULAR MATRIX AND ITS TRANSPOSE AND THEN CALL SUB
0431          C TO PERFORM THE FORWARD AND BACKWARD SUBSTITUTION PHASES OF THE
0432          C SOLN. FOR T.
0433          60 CALL CHLSKI(LMAX)
0434          CALL SUB (LMAX,T)
0435          C
0436          INCREMENT THE ITERATION COUNT (NITER) BY 1.
0437          NITER=NITER+1
0438          C
0439          IF THE ANALYSIS BEING PERFORMED IS A NON-LINEAR STEADY-STATE
0440          C F-0 ANALYSIS WHICH HAS NOT CONVERGED AND THE MAX. PERMISSIBLE ITER.
0441          C COUNT HAS NOT BEEN EXCEEDED, RETURN TO STATEMENT 6 TO PERFORM
0442          C ANOTHER ITERATION.
0443          IF((RABND.EQ.1.OR.MLIN.EQ.1).AND.TRANS.NE.1.AND.MCONT.
0444          ,EQ.1.AND.NITER.LE.MAXIT) GO TO 6
0445          IF((MLIN.EQ.1.OR.RABND.EQ.1).AND.MCONT.EQ.1.AND.NITER.GT.
0446          ,MAXIT) OUTPUT 'CONVERGENCE NOT ACHIEVED'
0447          C
0448          CALL OUTPUT TO PRINT THE RESULTS OF THE ANALYSIS.
0449          IF(SYN.EQ.1.AND.OUTPUT.EQ.1.AND.TRANS.EQ.1.AND.MTS.EQ.MTFTS(ITIN))
0450          , WRITE (100,999) NTRF ; CALL OUTPUT (MNT,T,TIME,TRANS,SYN,M1)
0451          IF(SYN.EQ.1.AND.TRANS.EQ.1) GO TO 61
0452          IF(SYN.NE.1.OR.(SYN.EQ.1.AND.OUTPUT.EQ.1)) CALL OUTPUT (MNT,T,TIME,
0453          ,TRANS,SYN,M1)
0454          61 IF(SYN=1) 61,62,61
0455          62 IF(TRANS.EQ.1.AND.MTS.NE.MTFTS(ITIN)) GO TO 61
0456          C
0457          IN A SYNTHESIS ANALYSIS, CALL TRIAL TO RECORD THE TEMPERATURE VECTOR
0458          C AS A TRIAL FN. THIS STATEMENT WILL ONLY HAVE BEEN REACHED IN A
0459          C TRANSIENT ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FNS. FOR A ...
0460          C SYNTHESIS ANALYSIS IF MTS=MTFTS(ITIN) (IE., IF THE TIME STEP NUMBER
0461          C (NTS) IS ONE AT WHICH A TRIAL FN. SHOULD BE RECORDED).
0462          CALL TRIAL (NEXT,MX,CONST,T)
0463          C
0464          IF A TRANSIENT ANALYSIS IS BEING PERFORMED AND MORE TRANSIENT
0465          C TRIAL FUNCTIONS ARE REQUIRED, INCREMENT THE COUNTERS FOR THE

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0466 C NUMBER OF TRIAL FNS. RECORDED (NTRF) AND THE NUMBER OF TRANSIENT
0467 TRIAL FNS. RECORDED IN THIS PARTICULAR ANALYSIS (ITIM). IF
0468 THE FINAL TIME OR TIME STEP NUMBER HAS NOT BEEN EXCEEDED, RETURN
0469 C TO STATEMENT 3 TO COMPUTE THE TEMPS. FOR THE NEXT TIME STEP.
0470 IF (TRANS.EQ.1.AND.NTS.LT.NTFIS(NTRF)) NTRF=NTRF+1 ITIM=ITIM+1
0471 61 IF (TRANS.EQ.1.AND.TIME.LT.TFINAL.AND.NTS.LT.NTFIS) GO TO 3
0472
0473 C CHECK WHETHER ANOTHER TRIAL FUNCTION IS REQUIRED (IE., WHETHER NEXT=1
0474 ). RETURN TO STATEMENT 1 TO READ THE DATA FOR AND COMPUTE THE NEXT
0475 TRIAL FN. IF NEXT=1.
0476 IF (NEXT.EQ.1) GO TO 1
0477
0478 C IF A SYNTHESIS ANALYSIS IS BEING PERFORMED, START THE 2ND. PHASE
0479 CALCULATIONS BY ASSIGNING NSYN A VALUE OF 2, CALLING DZ TO READ
0480 AND DEFINE THE AXIAL GRID GEOMETRY AND CALLING DATA1 TO READ
0481 THE 3-D JOB DATA AND TO READ AND/OR CALCULATE THE BOUNDARY
0482 CONDITIONS (OR GAP CONDUCTANCES).
0483 IF (SYN=1) GO TO 75
0484 75 NSYN=2
0485 CALL DZ (DZA,DZB,KX)
0486 CALL DATA1 (HX,TC,HI,CK1,CK2,NEXT,NSYN)
0487 HBA=HABA
0488
0489 C CALL ZERO TO ZERO THE ENTRIES IN THE LHS. COEFF. MATRIX AND RHS.
0490 C VECTOR FOR THE SYSTEM OF EQUATIONS DEFINING THE BLENDING VECTOR (Z).
0491 CALL ZERO (KX,NTRF,RO,TRANS)
0492
0493 C RETURN TO STATEMENT 4 OR STATEMENT 5 TO START THE SUBSTITUTIONS
0494 , WEIGHTINGS AND INTEGRATIONS OF THE 2ND. PHASE CALCULATIONS.
0495 82 IF (TRANS.EQ.1) GO TO 4
0496 GO TO 5
0497
0498 80 STOP
0499 901 FORMAT (4I3)
0500 992 FORMAT (2I3,3F10.5)
0501 993 FORMAT (15,F10.5)
0502 994 FORMAT (3I3)
0503 995 FORMAT (80A4)
0504 996 FORMAT (2I3,F10.5)
0505 997 FORMAT (4F10.5)
0506 998 FORMAT (15)
0507 999 FORMAT (15,6F10.5)
0508 888 FORMAT ('000000000000 SYNTHESIS SOLUTION 000000000000'//)
0509 882 FORMAT ('1X,000000000000 RUN WITHOUT GAS CONDUCTION 000000000000'//)
0510 883 FORMAT ('1X,000000000000 RUN WITH GAS CONDUCTION 000000000000'//)
0511 884 FORMAT ('000000000000 FINITE-DIFFERENCE SOLUTION 000000000000'//)
0512 885 FORMAT ('1X,GRID OPTION NUMBER =',I3,/)
0513 886 FORMAT ('000000000000 TRIAL FUNCTION NO.,',I3,' 000000000000'//)
0514 887 FORMAT ('100000000000',80A4,' 000000000000'//)
0515 888 FORMAT ('000000000000 IMPEDANCE ANALYSIS DATA 000000000000'//,1X,
0516 1 'PRESSURE TUBE INNER DIAMETER =',F10.5,10X,
0517 1 'PRESSURE TUBE OUTER DIAMETER =',F10.5,10X,
0518 1 'CALAMBRIA TUBE INNER DIAMETER =',F10.5,10X,
0519 1 'CALAMBRIA TUBE OUTER DIAMETER =',F10.5,10X,
0520 1 'SPACING BETWEEN LAMINATE SPRINGS =',F10.5,10X,
0521 1 'SPRING DIAMETER =',F10.5,10X, 'PATCH SEMI-MAJOR DIAMETER =',
0522 1 'F10.5,14X, 'PATCH SEMI-MINOR DIAMETER =',F10.5,10X,
0523 1 'SMALL EFFECTIVE CONTACT RADIUS =',F10.5,10X,
0524 1 'LARGE EFFECTIVE CONTACT RADIUS =',F10.5,10X,
0525 889 FORMAT ('000000000000 RESULTS FOR TRIAL FN. NO. ',I2,' FOLLOW 000000000000'//)
0526 END

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0001 SUBROUTINE BTM01 (IX)
0002 C
0003 THIS SUBROUTINE COMPUTES THE ANGULAR MESH INCREMENTS
0004 BEFORE AND AFTER EACH CIRCUMFERENTIAL MESH POINT
0005 DESIGNATED DTHA(1) AND DTHB(1) RESPECTIVELY FOR GRID
0006 OPTIONS 0 AND 1 (IE., KROPT=0 OR 1). THE VALUE OF
0007 THETA AT EACH CIRCUMFERENTIAL MESH POINT I, DESIGNATED
0008 AN(I), IS ALSO COMPUTED.
0009
0010 0000 INPUT VARIABLES READ BY SUB. BTM01 0000
0011
0012 IX - TOTAL NUMBER OF CIRCUMFERENTIAL MESH POINTS.
0013 ATOT - TOTAL INCLUDED ANGLE BETWEEN FIRST AND LAST
0014 CIRCUMFERENTIAL MESH POINTS (I=1 AND I=IX
0015 RESPECTIVELY).
0016 DTH - SIZE OF ANGULAR SUBDIVISION.
0017 IN(N) - CIRCUMFERENTIAL MESH POINT AT WHICH USE OF
0018 SUBDIVISION SIZE READ PREVIOUSLY (DTH) IS
0019 DISCONTINUED.
0020 NEXT - CONTROL VARIABLE 1 NEXT=0 SIGNALS THAT NO MORE
0021 VALUES OF DTH AND IN(N) FOLLOW WHILE NEXT=1
0022 SIGNALS THAT THE STATEMENT WHICH READS VALUES
0023 OF DTH, IN(N), AND NEXT SHOULD BE RE-EXECUTED
0024 BECAUSE MORE VALUES ARE SUPPLIED.
0025
0026 DIMENSION IN(10),AN(39),DTHA(39),DTHB(39)
0027 COMMON /DB4/MCCTL/DB26/AN/DB43/DTHA,DTHB
0028 WRITE (100,001)
0029
0030 READ (105,991) IX,ATOT
0031
0032 WRITE (100,002) IX,ATOT
0033 INITIALIZE VARIABLES.
0034 AN(1)=0.0
0035 N=0
0036
0037 INCREMENT COUNTER (N) FOR NUMBER OF DIFFERENT SETS OF
0038 VALUES FOR DTH,IN(N) AND NEXT PROVIDED.
0039 N=N+1
0040
0041 READ (105,992) DTH,IN(N),NEXT
0042
0043 WRITE (100,003) N,DTH,IN(N),NEXT
0044
0045 DETERMINE CIRCUMFERENTIAL MESH POINT WHERE USE OF
0046 ANGULAR SUBDIVISION DTH BEGINS (NIP).
0047 NIP=1
0048 IF (N.NE.1) NIP=IN(N-1)
0049
0050 DETERMINE CIRCUMFERENTIAL MESH POINT WHERE USE OF

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0055 C      ANGULAR SUBDIVISION DTH ENDS (IN2)
0056      IN2=IN(N)
0057
0058      SET VALUES OF DTHA(I) AT I BETWEEN NIP+1 AND IN2 AND
0059      VALUES OF DTHB(I) AT I BETWEEN NIP AND IN2-1 TO DTH.
0060      CALCULATE AN(I).
0061      DO 15 I=NIP,IN2
0062      IF(I.NE.NIP) AN(I)=ANG+DTH
0063      ANG=AN(I)
0064      IF(I.NE.NIP) DTHA(I)=DTH
0065      IF(I.NE.IN(N)) DTHB(I)=DTH
0066      CONTINUE
0067
0068      CHECK TO SEE IF MORE VALUES OF DTH,IN(N) AND NEXT ARE
0069      PROVIDED.
0070      IF(NEXT.EQ.1) GO TO 10
0071
0072      CALCULATE THE INCLUDED ANGLE OF THE CYLINDRICAL SEGMENT
0073      REMAINING BY SUBTRACTING THE LAST VALUE OF IN(N) FROM
0074      ATOT AND THEN DIVIDE THE ANGLE REMAINING BY THE NUMBER
0075      OF SUBDIVISIONS REMAINING (=IX-IN(N)) TO OBTAIN THE
0076      SUBDIVISION SIZE TO BE USED IN THE REST OF THE SEGMENT:
0077      IF(IN(N).EQ.IX) GO TO 25
0078      IF(ANG-ATOT) 16,17,17
0079      OUTPUT 'TOTAL ANGLE EXCEEDED'
0080      STOP
0081      16 DTH=(ATOT-ANG)/(IX-IN(N))
0082      OUTPUT DTH
0083
0084      ASSIGN VALUES OF DTHA(I) AND DTHB(I) TO THE REMAINING
0085      VALUES OF I.
0086      IN1=IN(N)
0087      DO 20 I=IN1,IX
0088      IF(I.NE.IN(N)) AN(I)=AN(I-1)+DTH
0089      IF(I.NE.IN(N)) DTHA(I)=DTH
0090      DTHB(I)=DTH
0091      CONTINUE
0092      20 DTHA(I)=0.0
0093      DTHB(I)=0.0
0094
0095      IF COMPLETE CYLINDER IS BEING MODELLED, REDEFINE
0096      DTHA(I) AND DTHB(I).
0097      IF(MCCYL.EQ.1) DTHA(I)=4.28318-ATOT ; DTHB(I)=DTHA(I)
0098      RETURN
0099      991 FORMAT (15,F10.5)
0100      992 FORMAT (F10.5,2I5)
0101      993 FORMAT ('***** DATA READ IN SUBROUTINE DTHO1 *****')
0102      994 FORMAT (1X,'IX=',I3,10X,'ATOT=',F10.5,/)
0103      995 FORMAT (1X,'N=',I3,10X,'DTH=',F10.5,10X,'IN(N)=' ,I3,10X,'NEXT=' ,I3,/)
0104      996
0105      END

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0001 SUBROUTINE RADO (RI,JX,RO)
0002
0003 THIS SUBROUTINE COMPUTES THE RADIAL MESH SUBDIVISIONS
0004 BEFORE AND AFTER EACH RADIAL NODE JJ, DESIGNATED
0005 DRB(JJ) AND DRB(JJ) RESPECTIVELY AS WELL AS THE RADIAL
0006 COORDINATE AT JJ, RAD(JJ). THE SUBDIVISIONS DRB(JJ)
0007 AND DRB(JJ) ARE FUNCTIONS OF RADIAL POSITION ONLY.
0008
0009 ***** INPUT VARIABLES READ IN SUB. RADO *****
0010
0011 RI - INNER RADIUS OF CYLINDRICAL SEGMENT TO BE
0012 MODELLED (MUST BE GREATER THAN ZERO).
0013 RX - NUMBER OF NODES ON FUEL PELLET BOUNDARY AT FUEL
0014 PELLET/SHEATH INTERFACE (SET TO A VALUE GREATER
0015 THAN THE MAXIMUM VALUE OF JJ IF THERE IS NO
0016 INTERFACE IN THE PELLET BEING MODELLED). WHEN
0017 ANALYZING HEAT TRANSFER IN A TUBE CARRYING A
0018 HEAT EXCHANGING FLUID (IE., IF INPUT VARIABLE
0019 TUBE, READ IN THE MAIN PROGRAM, =1), SET RX=1.
0020 NOTE THAT RADIAL MESH NUMBERS START AT 2, NOT 1.
0021 DR - SIZE OF RADIAL MESH SUBDIVISION.
0022 N(J) - NUMBER OF RADIAL NODE AT WHICH USE OF
0023 SUBDIVISION DR READ PREVIOUSLY IS DISCONTINUED.
0024 NEXT - CONTROL VARIABLE ; NEXT =0 SIGNALS THAT NO MORE
0025 VALUES OF DR, N(J) AND NEXT FOLLOW WHILE NEXT=1
0026 INDICATES THAT THE READ STATEMENT WHICH READS DR
0027 ,N(J) AND NEXT SHOULD BE RE-EXECUTED BECAUSE
0028 ADDITIONAL VALUES OF THESE VARIABLES ARE
0029 PROVIDED.
0030
0031 DIMENSION RAD(25),DRB(25),DRB(25),N(10)
0032 COMMON /RADO/DRB,DRB/DRB/RX
0033 WRITE (100,991) RI,RX
0034 READ (105,991) RI,RX
0035 WRITE (100,992) RI,RX
0036
0037 CALCULATE NUMBER OF NODE AT SHEATH OR TUBE INNER
0038 SURFACE (NIS). SET RADIUS OF FIRST NODE (RAD(2)) TO RI.
0039 NIS=RX+1
0040 RAD(2)=RI
0041
0042 INITIALIZE COUNTER FOR NUMBER OF DIFFERENT SETS OF
0043 VALUES OF DR, N(J) AND NEXT PROVIDED. (J)
0044 J=1
0045 READ (105,992) DR,N(J),NEXT
0046 WRITE (100,993) J,DR,N(J),NEXT
0047
0048 CALCULATE NODE AT START OF REGION IN WHICH SUBDIVISION
0049 DR IS USED (JP), NODE AT END OF REGION IS JJP (=N(J)).
0050 JP=2
0051 IF(J.NE.1) JP=N(J-1)
0052 JJP=N(J)
0053
0054 SET DRB(JJ)=DR AT JJ BETWEEN JP+1 AND JJP UNLESS JJ
0055 NIS (IE., NODE IS ON SHEATH OR TUBE INNER SURFACE)
0056 BECAUSE DRB=0.0 IN THAT CASE. ALSO SET DRB(JJ)=DR AT JJ
0057 BETWEEN JP AND JJP-1 UNLESS JJ=NIS (IE., NODE IS ON
0058 PELLET OUTER SURFACE) BECAUSE DRB=0.0 IN THAT CASE.
0059 DO 5 J=JP,JJP
0060 IF(JJ.NE.NIS.AND.JJ.NE.JP) DRB(JJ)=DR
0061 IF(JJ.NE.NIS.AND.JJ.NE.JX.AND.JJ.NE.N(J)) DRB(JJ)=DR
0062
0063 CALCULATE RADIUS AT JJ (RAD(JJ))
0064 IE (JJ.NE.JP.AND.JJ.NE.NIS) RAD(JJ)=RAD(JJ-1)+DR

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0065 IF (JJ.EQ.MIS) RAD(JJ)=RAD(JJ-1) 0070.500
0066 CONTINUE 0070.400
0067 0070.700
0068 INCREMENT COUNTER FOR NUMBER OF SETS OF VALUES FOR 0070.800
0069 DR, R(J) AND NEXT READ. 0070.900
0070 J=J+1 0071.000
0071 0071.100
0072 CHECK WHETHER MORE INPUT DATA ARE PROVIDED. IF ANSWER 0071.200
0073 IS AFFIRMATIVE, RETURN TO STATEMENT 10. 0071.300
0074 IF (NEXT.EQ.1) GO TO 10 0071.400
0075 0071.500
0076 STORE LARGEST NODE NUMBER AS JX. 0071.600
0077 JX=M(J-1) 0071.700
0078 0071.800
0079 SET THE OUTER RADIUS OF THE CYLINDRICAL SEGMENT (RO) 0071.900
0080 TO RAD(JX). 0072.000
0081 RO=RAD(JX) 0072.100
0082 0072.200
0083 ASSIGN PROPER VALUES OF DRA AND DRB TO NODES ON INNER 0072.300
0084 AND OUTER SURFACES OF CYLINDER, PELLETT, SHEATH, ETC. 0072.400
0085 DRB(JX)=0.0 0072.500
0086 DRB(MX)=0.0 0072.600
0087 DRA(2)=0.0 0072.700
0088 DRA(MIS)=0.0 0072.800
0089 OUTPUT RO 0072.900
0090 RETURN 0073.000
0091 991 FORMAT(F10.5,I5) 0073.100
0092 992 FORMAT (F10.5,I5) 0073.200
0093 881 FORMAT ('088888 DATA READ IN SUBROUTINE RADO 88888',/) 0073.300
0094 882 FORMAT (1X,'R1=',F10.5,10X,'M1=',I5,/) 0073.400
0095 883 FORMAT (1X,'J=',I3,10X,'DR=',F10.5,10X,'M(J)=' ,I3,10X,'NEXT=' ,I3,/) 0073.500
0096 0073.600
0097 END 0073.700

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0001 SUBROUTINE DZ (DZA,DZB,KX) 0073.800
0002 0073.900
0003 THIS SUBROUTINE COMPUTES THE AXIAL NODE SPACINGS BEFORE 0074.000
0004 AND AFTER EACH AXIAL NODE KX, DESIGNATED DZA(KX) AND 0074.100
0005 DZB(KX) RESPECTIVELY, AS WELL AS THE AXIAL COORDINATE 0074.200
0006 AT KX, AXPOS(KX). THE SUBDIVISIONS DZA(KX) AND DZB(KX) 0074.300
0007 ARE FUNCTIONS OF AXIAL POSITION ONLY. 0074.400
0008 0074.500
0009 8888 INPUT VARIABLES READ IN SUB. DZ 8888 0074.600
0010 0074.700
0011 DZ - SIZE OF AXIAL MESH SUBDIVISION. 0074.800
0012 KCH(K) - NUMBER OF AXIAL NODE AT WHICH USE OF 0074.900
0013 SUBDIVISION DZ READ PREVIOUSLY IS DISCONTINUED. 0075.000
0014 NEXT - CONTROL VARIABLE ; NEXT=0 SIGNALS THAT NO MORE 0075.100
0015 VALUES OF DZ, KCH(K) AND NEXT FOLLOW WHILE 0075.200
0016 NEXT=1 INDICATES THAT THE READ STATEMENT WHICH 0075.300
0017 READS DZ, KCH(K) AND NEXT SHOULD BE RE- 0075.400
0018 EXECUTED BECAUSE ADDITIONAL VALUES OF THESE 0075.500
0019 VARIABLES ARE PROVIDED. 0075.600
0020 0075.700
0021 DIMENSION DZA(45),DZB(45),AXPOS(45),KCH(11) 0075.800
0022 COMMON /DB7/AXPOS 0075.900
0023 WRITE (100,881) 0076.000
0024 0076.100
0025 INITIALIZE COUNTER FOR NUMBER OF DIFFERENT SETS OF 0076.200
0026 VALUES OF DZ, KCH(K) AND NEXT PROVIDED.(K) 0076.300
0027 K=1 0076.400
0028 AXLTH=0.0 0076.500
0029 0076.600
0030 READ INPUT VARIABLES. 0076.700
0031 READ (105,991) DZ,KCH(K),NEXT 0076.800
0032 WRITE (100,882) K,DZ,KCH(K),NEXT 0076.900
0033 0077.000
0034 CALCULATE NUMBER OF NODE AT START OF REGION IN WHICH 0077.100
0035 SUBDIVISION DZ IS USED (NKP). NODE AT END OF REGION IS 0077.200
0036 KCF (=KCH(K)). 0077.300
0037 NKP=1 0077.400
0038 IF (K.NE.1) NKP=KCH(K-1) 0077.500
0039 KCF=KCH(K) 0077.600
0040 0077.700
0041 SET DZA(KX)=DZ AT KX BETWEEN NKP+1 AND KCF AND SET 0077.800
0042 DZB(KX)=DZ AT KX BETWEEN NKP AND KCF-1. 0077.900
0043 DO 5 KX=NKP,KCF 0078.000
0044 IF (KX.NE.NKP) DZA(KX)=DZ 0078.100
0045 IF (KX.NE.KCH(K)) DZB(KX)=DZ 0078.200
0046 0078.300
0047 CALCULATE AXIAL POSITION AT KX (AXPOS(KX)). 0078.400
0048 IF (KX.NE.NKP) AXLTH=AXLTH+DZ 0078.500
0049 AXPOS(KX)=AXLTH 0078.600
0050 CONTINUE 0078.700
0051 0078.800
0052 INCREMENT COUNTER FOR NUMBER OF SETS OF VALUES FOR 0078.900
0053 DZ, KCH(K) AND NEXT READ. 0079.000
0054 K=K+1 0079.100
0055 0079.200
0056 CHECK WHETHER MORE INPUT DATA ARE PROVIDED. RETURN TO 0079.300
0057 STATEMENT 10 IF ANSWER IS AFFIRMATIVE. 0079.400
0058 IF (NEXT.EQ.1) GO TO 10 0079.500
0059 0079.600
0060 STORE LARGEST NODE NUMBER AS KX. 0079.700
0061 KX=KCH(K-1) 0079.800
0062 DZA(1)=0.0 0079.900
0063 DZB(KX)=0.0 0080.000
0064 RETURN 0080.100
0065 991 FORMAT (F10.5,I5) 0080.200
0066 881 FORMAT ('088888 AXIAL GRID DATA READ IN SUBROUTINE DZ 88888',/) 0080.300
0067 882 FORMAT (1X,'K=' ,I3,10X,'DZ=' ,F10.7,10X,'KCH(K)=' ,I3,10X,'NEXT=' ,I2,/) 0080.400
0068 0080.500
0069 END 0080.600

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0001 SUBROUTINE RAD12 0080.700
0002 0080.800
0003 THIS SUBROUTINE COMPUTES THE RADIUS AT EACH RADIAL 0080.900
0004 NODE J, DESIGNATED RAD(J), AS WELL AS INDEXING 0081.000
0005 VARIABLES NTU(J) AND MUTT(N) (N=NR+1,NR,...,42) USED 0081.100
0006 IN SUBROUTINE IS1 TO DEFINE A RADIAL NODE NUMBER 0081.200
0007 INCREMENT, IS(I,J), FOR EACH NODE (I,J) IN THE FINITE- 0081.300
0008 DIFFERENCE GRID WHEN THE RADIAL SUBSCRIPT, J, IS 0081.400

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0009      VARIED MORE RAPIDLY THAN THE CIRCUMFERENTIAL SUBSCRIPT,
0010      I. SUBROUTINE RAD12 ALSO DEFINES RADIAL SUBDIVISION
0011      SIZES DR(N) (N=NR+1,NR,...,2) USED IN SUBROUTINE
0012      DRAB TO DEFINE THE RADIAL SUBDIVISIONS BEFORE AND
0013      AFTER EACH NODE I,J, DESIGNATED DR(I,J) AND
0014      DR(I,J) RESPECTIVELY WHEN NGROPT=1 OR 2.
0015
0016      ***** INPUT VARIABLES REQUIRED IN SUB. RAD12 *****
0017
0018      NR - TOTAL NUMBER OF RADIAL GRID LINES
0019           (IE., LINES OF CONSTANT THETA) AT WHICH THE
0020           RADIAL GRID STRUCTURE IS ALTERED.
0021      NX - NODE ON THE OUTER SURFACE OF THE INNER CYLINDER
0022           (EG., FUEL PELLET) IN A PAIR OF CONCENTRIC
0023           (USUALLY) CYLINDERS. SET NX TO A VALUE GREATER
0024           THAN JX IF THERE IS NO INTERFACE IN THE OBJECT
0025           BEING MODELLED. NOTE THAT RADIAL NODE NUMBERS
0026           START AT 2, NOT 1.
0027      JX - NUMBER OF OUTERMOST RADIAL NODE (IE., LARGEST
0028           RADIAL NODE NUMBER).
0029      M(N), (N=1,2,...,NR) - RADIAL NODES AT WHICH RADIAL
0030           SUBDIVISIONS CHANGE IN SIZE, WITH M(1) AND
0031           M(NR) BEING THE INNERMOST AND OUTERMOST NODES
0032           IN THAT GROUP RESPECTIVELY.
0033      RI - INNER RADIUS OF CYLINDRICAL SEGMENT BEING
0034           MODELLED (MUST BE GREATER THAN ZERO).
0035      RO - OUTER RADIUS OF CYLINDRICAL SEGMENT BEING
0036           MODELLED.
0037      DR(NR+1) - SIZE OF SMALLEST VARIABLE RADIAL SUBDIVISION
0038           (MUST BE USED AT OUTERMOST RADIAL NODES
0039           STARTING AT FIRST RADIAL GRID LINE
0040           AT THETA=0.0 DEGREES).
0041
0042      DIMENSION M(10),DR(11),RAD(25),NWT(11),NTW(25)
0043      COMMON /DBS2/RAD,DBS8/NR,N,DBS3/DR,DBS4/NTW,NWT,DB4/IX,JX,KX,DBS6
0044      /NX,DB9/RO
0045      WRITE (108,881)
0046
0047      READ INPUT VARIABLES.
0048      READ (105,991) NR,NX,JX
0049      WRITE (108,882) NR,NX,JX
0050      READ (105,992) (M(N),N=1,NR)
0051      OUTPUT, (M(N),N=1,NR)
0052      N=NR+1
0053      READ (105,993) RI,RO,DR(N)
0054      WRITE (108,883) RI,RO,DR(N)
0055      NIS=NX+1
0056      RAD(2)=RI
0057      RAD(JX)=RO
0058
0059      INITIALIZE SUBDIVISION WEIGHT TOTAL (NWT).
0060      NWT=0
0061
0062      DEFINE SUBDIVISION WEIGHT TOTAL FOR OUTERMOST NODE JX
0063      (NTW(JX)).
0064      NTW(JX)=0
0065      DEFINE SUBDIVISION SIZE INDICATOR (N) WHICH HAS A VALUE
0066      OF NR AFTER STATEMENT 10 HAS BEEN EXECUTED ONCE AND IS
0067      SUBSEQUENTLY REDUCED BY 1 EACH TIME STATEMENT 10 IS
0068      RE-EXECUTED UNTIL IT REACHES ITS FINAL VALUE OF 1.
0069      N=NR-1
0070
0071      DEFINE THE 'WEIGHT' OF THE (N+1)ST SUBDIVISION SIZE
0072      IE., THE RATIO OF THAT SIZE TO THE SMALLEST SUBDIVISION
0073      SIZE DR(NR+1). THE 'WEIGHT' IS DESIGNATED NWT OR
0074      NWT(N+1).
0075      IF(N.EQ.NR) NWT=1
0076      IF(N.NE.NR) NWT=2*NR/N
0077
0078      CALCULATE THE (N+1)ST SUBDIVISION SIZE.
0079      IF(N.NE.NR) DR(N+1)=2.0*DR(N+2)
0080      NWT(N+1)=NWT
0081
0082      CALCULATE THE INITIAL AND FINAL NODES BRACKETING
0083      THE REGION OF THE R AXIS IN WHICH SUBDIVISION SIZE
0084      DR(N+1) IS USED. THESE NODES ARE DESIGNATED NI AND
0085      NF RESPECTIVELY.
0086      NI=2
0087      IF(N.GT.0) NI=M(N)
0088      NF=NI+1
0089      IF(N.NE.NR) NF=M(N+1)
0090
0091      CALCULATE RAD(J) FOR RADIAL NODES J BETWEEN NF-1 AND
0092      NI. THE RADII ARE CALCULATED IN DESCENDING ORDER AND
0093      RAD(J) AT A GIVEN NODE J IS CALCULATED BY SUBTRACTING
0094      DR(N+1) FROM RAD(J+1). ALSO CALCULATE THE SUBDIVISION
0095      WEIGHT SUM AT J (IE., NTV(J)).
0096      DO 15 J=NF,NF
0097      J=NF-(J-J-1)
0098      IF(J.NE.NX) NTV=NTV+NTW
0099      NTV(J)=NTV
0100      IF(J.NE.NX) RAD(J)=RAD(J+1)-DR(N+1)
0101      IF(J.EQ.NX) RAD(J)=RAD(J+1)
0102      CONTINUE
0103
0104      CHECK TO SEE IF RAD(J) HAS BEEN CALCULATED AT ALL OF
0105      THE RADIAL NODES DOWN TO AND INCLUDING M(1). IF NOT,
0106      RETURN TO STATEMENT 10 AND CALCULATE RADII USING THE
0107      NEXT (LARGER) SUBDIVISION SIZE.
0108      IF(N.GT.1) GO TO 10
0109
0110      CALCULATE THE FINAL SUBDIVISION SIZE, DR(1), USED AT
0111      THE INNERMOST RADIAL NODES (BETWEEN NODES 2 AND N(1)).
0112      DR(1)=0.0
0113      IF(M(1).EQ.2) GO TO 21
0114      LC=2
0115      IF(NX.LT.M(1)) LC=3
0116      DR(1)=(RAD(M(1))-RI)/(M(1)-LC)
0117      NO=M(1)-1
0118
0119      CALCULATE RAD(J) AT RADIAL NODES (J) 3 TO M(1)-1.
0120      DO 20 J=3,NO
0121      IF(J.EQ.NIS) RAD(J)=RAD(J-1)
0122      IF(J.NE.NIS) RAD(J)=RAD(J-1)+DR(1)
0123
0124      CONTINUE
0125      CONTINUE
0126      FORMAT (3I5)
0127      FORMAT (15)
0128      FORMAT (3F10.5)

```

```

0129 881 FORMAT ('000000 DATA READ IN SUBROUTINE RAD12 000000'//)
0130 882 FORMAT (IX,'NR',I3,10X,'MX',I3,10X,'JX',I3,10X)
0131 883 FORMAT (IX,'RI',F10.3,10X,'RO',F10.3,10X,'DR(N)',F10.5,10X)
0132 RETURN
0133 END
0093.500
0093.600
0093.700
0093.800
0093.900

0001 SUBROUTINE ISIJ
0002
0003 THIS SUBROUTINE COMPUTES THE RADIAL MODE NUMBER
0004 INCREMENT, DESIGNATED IS(I,J), WHICH MUST BE ADDED TO
0005 THE RADIAL MODE NUMBER J AT A GIVEN NODE I,J TO OBTAIN
0006 THE RADIAL MODE NUMBER AT THE NEXT NODE ON THE SAME
0007 RADIAL GRID LINE I. THE RADIAL MODE NUMBER
0008 AT THE NEXT NODE (I,JJ) IS DESIGNATED JJ SO JJ=J+
0009 IS(I,J) (JJ IS GREATER THAN J).
0010
0011 ***** INPUT VARIABLES READ IN SUB. ISIJ *****
0012
0013 JSC(N), N=1,2,...,NR - RADIAL GRID LINES AT
0014 WHICH THE RADIAL MESH STRUCTURE IS ALTERED.
0015 JSC(NR) IS THE 1ST RADIAL GRID LINE AT WHICH
0016 RADIAL SUBDIVISIONS ARE INCREASED (DOUBLED) IN
0017 SIZE AND JSC(1) IS THE LAST.
0018
0019 DIMENSION JIN(39)
0020 DIMENSION N(10),NMTT(11),NTW(25),JSC(10),IS(39,25)
0021 COMMON /DBS4/NTW,NMTT,DBS5/NR,N,DBS5/JSC,DBA/IX,JX,KX,DBA7/IS,DBS4
0022 /MX,DB70/JIN,DBA8/NROPT
0023 WRITE (100,881)
0024
0025 READ INPUT VARIABLES.
0026 READ (105,991) (JSC(N),N=1,NR)
0027 OUTPUT, (N,JSC(N),N=1,NR)
0028 FORMAT (IS)
0029
0030 SWEEP THROUGH CIRCUMFERENTIAL NODES (I).
0031 DO 1 I=1,IX
0032 ISP=1
0033
0034 DETERMINE 1ST RADIAL MODE ON RADIAL GRID LINE I.
0035 JI=2
0036 IF(NROPT.EQ.2) JI=JIN(I)
0037
0038 SWEEP THROUGH RADIAL NODES (J) ON RADIAL GRID LINE I.
0039 DO 2 J=JI,JX,ISP
0040
0041 ASSIGN VALUES OF UNITY TO IS(I,J) AND ISP INITIALLY.
0042 IS(I,J)=1
0043 ISP=1
0044
0045 CHECK WHETHER ASSUMED VALUE OF IS(I,J) IS CORRECT. IF
0046 IT IS, PROCEED TO THE NEXT NODE.
0047 IF(J.LT.N(1).OR.J.EQ.MX.OR.J.EQ.JX) GO TO 2
0048 DO 3 II=1,NR
0049 MII=JX
0050 IF(II.EQ.NR) MII=N(II+1)
0051 IF(J.LT.MII.AND.I.LE.JSC(II)) GO TO 2
0052 CONTINUE
0053
0054 VALUE OF IS(I,J) ISN'T UNITY SO CALCULATE WHAT IT
0055 SHOULD BE. FIRST DETERMINE SUBDIVISION WEIGHT (NMTT)
0056 FOR SUBDIVISION BETWEEN RADIAL MODE J AND NEXT RADIAL
0057 MODE THAT IS USED IN MODEL (RADIAL MODE JJ) AND STORE
0058 WEIGHT AS NDB.
0059 IF(J.GE.N(NR).AND.I.LE.JSC(NR)) NDB=NMTT(NR+1)
0060 NX=NR-1
0061 DO 5 NN=1,NX
0062 NN=NR-NN
0063 IF(J.GE.N(NN).AND.J.LT.N(NN+1).AND.I.LE.JSC(NN)) NDB=NMTT(NN+1)
0064 IF(J.GE.N(NN+1).AND.I.GE.(JSC(NN+1)+1).AND.I.LE.JSC(NN)) NDB=
0065 NMTT(NN+1)
0066 CONTINUE
0067 IF(J.GE.N(1).AND.I.GE.(JSC(1)+1)) NDB=2*NMTT(2)
0068
0069 CALCULATE WHAT THE VALUE OF NTW(JJ) SHOULD BE AND
0070 STORE THIS VALUE AS NREN, NTW(JJ), WHICH WAS CALCULATED
0071 FOR ALL J BETWEEN N(1) AND JX IN SUB. RAD12. IS THE
0072 ACCUMULATED SUM OF THE VALUES OF NMTT (CALC. IN SUB.
0073 RAD12 ALSO) FOR ALL OF THE RADIAL SUBDIVISIONS
0074 BETWEEN J AND JX.
0075 NREN=NTW(J)-NDB
0076 FIND THE VALUE OF JJ.
0077 DO 10 JJ=J,JX
0078 IF(NREN-NTW(JJ)) 10,11,12
0079 CONTINUE
0080 OUTPUT 'VALUES OF N ARE NOT VALID'
0081 STOP
0082
0083 CALCULATE IS(I,J).
0084 IS(I,J)=JJ-J
0085 ISP=IS(I,J)
0086 CONTINUE
0087 CONTINUE
0088 RETURN
0089 FORMAT ('000000 DATA READ IN SUBROUTINE ISIJ 000000'//)
0090 END
0094.000
0094.100
0094.200
0094.300
0094.400
0094.500
0094.600
0094.700
0094.800
0094.900
0095.000
0095.100
0095.200
0095.300
0095.400
0095.500
0095.600
0095.700
0095.800
0095.900
0096.000
0096.100
0096.200
0096.300
0096.400
0096.500
0096.600
0096.700
0096.800
0096.900
0097.000
0097.100
0097.200
0097.300
0097.400
0097.500
0097.600
0097.700
0097.800
0097.900
0098.000
0098.100
0098.200
0098.300
0098.400
0098.500
0098.600
0098.700
0098.800
0098.900
0099.000
0099.100
0099.200
0099.300
0099.400
0099.500
0099.600
0099.700
0099.800
0099.900
0100.000
0100.100
0100.200
0100.300
0100.400
0100.500
0100.600
0100.700
0100.800
0100.900
0101.000
0101.100
0101.200
0101.300
0101.400
0101.500
0101.600
0101.700
0101.800
0101.900
0102.000
0102.100
0102.200
0102.300
0102.400
0102.500
0102.600
0102.700
0102.800
0102.900

0001 SUBROUTINE DRAB
0002
0003 THIS SUBROUTINE COMPUTES THE RADIAL SUBDIVISIONS BEFORE
0004 AND AFTER EACH NODE I,J, DESIGNATED DBA(I,J) AND
0005 DBA(I,J) RESPECTIVELY WHEN NROPT=1 OR 2.
0006
0007 DIMENSION JSC(10),N(10),DBA(11),DBA(39,25),DRB(39,25)
0008 JIN(39),IS(39,25)
0009 COMMON /DBS4/NR,N,DBS5/JSC,DBS7/DBA,DBS3/DRB,DBA/IX,JX,KX,DBS6/
0010 MX,DBA7/IS,DBA8/NROPT,DB70/JIN
0011 NIS=NX+1
0012
0013 SWEEP THROUGH CIRCUMFERENTIAL NODES (I).
0014 DO 10 I=1,IX
0015
0103.000
0103.100
0103.200
0103.300
0103.400
0103.500
0103.600
0103.700
0103.800
0103.900
0104.000
0104.100
0104.200
0104.300
0104.400

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0014 C      DETERMINE 1ST RADIAL NODE ON RADIAL GRID LINE I (JI).
0015 JI=2
0016 IF (NGROPT.EQ.2) JI=JIN(I)
0017 ISP=1
0018
0019 C      SNEEP THROUGH RADIAL NODES (J) ON RADIAL GRID LINE I.
0020 DO 20 J=JI,JX,ISP
0021
0022 C      DETERMINE DRA(I,J) AND DRB(I,J) FOR NODE I,J.
0023
0024 C      DRA(I,J)=DR(I)
0025 DRB(I,J)=DR(I)
0026 IF (J.GT.N(NR).AND.I.LE.JSC(NR)) DRA(I,J)=DR(NR+1)
0027 IF (J.GE.N(NR).AND.I.LE.JSC(NR)) DRB(I,J)=DR(NR+1)
0028
0029 NX=NR-1
0030 DO 25 NR=1,NX
0031   NH=NR-NH
0032   IF (J.GE.N(NH).AND.J.LT.N(NH+1).AND.I.LE.JSC(NH)) DRB(I,J)=
0033     DR(NH+1)
0034   IF (J.GE.N(NH+1).AND.I.GE.(JSC(NH+1)+1).AND.I.LE.JSC(NH))
0035     DRB(I,J)=DR(NH+1)
0036   IF (J.GT.N(NH).AND.J.LE.N(NH+1).AND.I.LE.JSC(NH)) DRA(I,J)=
0037     DR(NH+1)
0038   IF (J.GT.N(NH+1).AND.I.GE.(JSC(NH+1)+1).AND.I.LE.JSC(NH))
0039     DRA(I,J)=DR(NH+1)
0040 25 CONTINUE
0041 IF (J.GE.N(1).AND.I.GE.(JSC(1)+1)) DRB(I,J)=2.0*DR(2)
0042 IF (J.GT.N(1).AND.I.GE.(JSC(1)+1)) DRA(I,J)=2.0*DR(2)
0043 IF (J.EQ.KX.OR.J.EQ.JX) DRB(I,J)=0.0
0044 IF (J.EQ.NIS.OR.J.EQ.2) DRA(I,J)=0.0
0045 20 CONTINUE
0046 10 CONTINUE
0047 RETURN
0048 END

```

```

0001 SUBROUTINE DTH2
0002
0003 C      THIS SUBROUTINE COMPUTES THE ANGULAR SUBDIVISIONS
0004 C      BEFORE AND AFTER EACH NODE I,J, DESIGNATED DTHA(I,J)
0005 C      AND DTHB(I,J) RESPECTIVELY WHEN NGROPT=2, AS WELL AS
0006 C      THE ANGULAR POSITION OF EACH RADIAL GRID LINE,
0007 C      DESIGNATED AN(I). TWO ADDITIONAL VARIABLES, INUT(II)
0008 C      AND ITWT(I), ARE ALSO DEFINED FOR USE LATER IN
0009 C      SUBROUTINE JINI WHERE THE FIRST (INNERMOST) NODE ON
0010 C      EACH RADIAL GRID LINE (I), DESIGNATED JIN(I), IS
0011 C      CALCULATED.
0012
0013 C      ***** INPUT VARIABLES READ IN SUB. DTH2 *****
0014
0015 IX - TOTAL NO. OF CIRCUMFERENTIAL NODES.
0016 ATOT - TOTAL INCLUDED ANGLE (IN RADIANS) OF
0017 C      CYLINDRICAL SEGMENT BEING MODELLED.
0018 DTH(1) - SIZE OF SMALLEST VARIABLE ANGULAR
0019 SUBDIVISION (MUST BE USED AT OUTERMOST
0020 RADIAL NODES STARTING AT THE 1ST RADIAL GRID
0021 LINE (IE., AT I=1)).
0022 NI - NO. OF CIRCUMFERENTIAL (CONSTANT RADIUS) GRID
0023 LINES AT WHICH THE CIRCUMFERENTIAL MESH
0024 STRUCTURE IS ALTERED.
0025 IN(II), II=1,2,...,NI - RADIAL GRID LINES (I) AT WHICH
0026 THE ANGULAR SUBDIVISIONS BETWEEN CIRC. NODES
0027 AT J=JX CHANGE (DOUBLE) IN SIZE. IN(II) IS
0028 ACTUALLY THE RADIAL GRID LINE SEPARATING THE
0029 SECTOR IN WHICH ANGULAR SUBDIVISIONS DEPEND
0030 ON BOTH ANGULAR POSITION AND RADIUS FROM THE
0031 SECTOR IN WHICH ANGULAR SUBDIVISIONS DEPEND
0032 ONLY ON ANGULAR POSITION.
0033 ISC(II), II=1,2,...,NI - CIRCUMFERENTIAL (CONSTANT
0034 RADIUS) GRID LINES AT WHICH THE CIRC. MESH
0035 STRUCTURE IS ALTERED. ISC(1) AND ISC(NI) ARE
0036 THE OUTERMOST AND INNERMOST GRID LINES IN THAT
0037 GROUP RESPECTIVELY.
0038 DTH(N+1) - ANGULAR SUBDIVISION SIZE USED BETWEEN RADIAL
0039 GRID LINES IN(I-1) AND DTH(N).
0040 IN(N) - RADIAL GRID LINE AT WHICH THE USE OF DTH(N+1)
0041 IS DISCONTINUED.
0042 NEXT - CONTROL VARIABLE. NEXT=0 SIGNALS THAT NO MORE
0043 VALUES OF DTH, IN, AND NEXT FOLLOW WHILE NEXT=
0044 1 SIGNALS THAT THE STATEMENT WHICH READS THOSE
0045 VARIABLES SHOULD BE RE-EXECUTED BECAUSE MORE
0046 VALUES ARE SUPPLIED.
0047
0048 DIMENSION ISC(10), IN(10), DTHA(39,25), DTHB(39,25), DTH(10), ITWT(39),
0049 INUT(11), AN(39)
0050 COMMON /DB4/IX,JX,KX /DB26/AN /DB71/ISC,IN,NI /DB72/DTHA,DTHB
0051 /DB75/ITWT,INUT
0052 WRITE (106,881)
0053
0054 READ INPUT VARIABLES.
0055
0056 READ (105,991) IX,ATOT,DTH(1),NI
0057
0058 WRITE (106,882) IX,ATOT,DTH(1),NI
0059
0060 ASSIGN 'WEIGHT' OF UNITY TO 1ST SUBDIVISION SIZE.
0061 INUT(1)=1
0062
0063 READ INPUT VARIABLES IN(II) AND ISC(II) (II=1,2,...,NI)
0064 AND COMPUTE SUBDIVISION SIZES DTH(II) AND WEIGHTS
0065 INUT(II) FOR II=2,3,...,NI+1.
0066 DO 5 II=1,NI
0067
0068 READ (105,992) IN(II),ISC(II)
0069
0070 WRITE (106,883) II,IN(II),ISC(II)
0071 IF (II.NE.1) DTH(II)=2.0*DTH(II-1)
0072 IF (II.NE.1) INUT(II)=2*INUT(II-1)
0073 CONTINUE
0074 DTH(NI+1)=2.0*DTH(NI)
0075 INUT(NI+1)=2*INUT(NI)
0076 ITWT(1)=0
0077 AN(1)=0.0
0078 NNI=IN(NI)
0079
0080
0081
0082

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0083      SNEEP THROUGH CIRCUMFERENTIAL NODES (I) IN REGION IN
0084      WHICH DTHA(I,J) AND DTHB(I,J) DEPEND ON BOTH ANGULAR
0085      POSITION AND RADIUS (IE., BETWEEN CIRC. NODES I AND
0086      IN(NI)).
0087      DO 10 I=1,NMI
0088      SNEEP THROUGH RADIAL NODES (J).
0089      DO 20 J=2,JX
0090      ASSIGN APPROPRIATE VALUES OF DTHA(I,J) AND DTHB(I,J)
0091      TO NODE I,J AND CALCULATE THE ANGULAR POSITION OF
0092      RADIAL GRID LINE I (IE., AN(I)). ALSO CALCULATE THE
0093      SUM OF THE WEIGHTS OF THE CIRC. SUBDIVISIONS (USED AT
0094      J=JX) BETWEEN THE 1ST AND ITH GRID LINES (IE., ITWT(I)).
0095      DTHA(I,J)=0.0
0096      DTHB(I,J)=0.0
0097      DO 30 II=1,NI
0098      INII=1
0099      IF(II.NE.1) INII=IN(II-1)
0100      IF(I.LE.IN(II).AND.I.GT.INII/AMB.J.EQ.JX) ITWT(I)=ITWT(I-1)+
0101      INII
0102      IF(J.GT.ISC(II).AND.I.LT.IN(II).AND.I.GE.INII) DTHB(I,J)=
0103      DTHB(II)
0104      IF(J.GT.ISC(II).AND.I.LE.IN(II).AND.I.GT.INII) DTHA(I,J)=
0105      DTHA(II)
0106      IF(II.EQ.1) GO TO 30
0107      IF(J.GT.ISC(II).AND.J.LE.ISC(II-1).AND.I.LT.IN(II)) DTHB(I,J)=
0108      DTHB(II)
0109      IF(J.GT.ISC(II).AND.J.LE.ISC(II-1).AND.I.LE.IN(II).AND.I.GT.1)
0110      DTHA(I,J)=DTHA(II)
0111      CONTINUE
0112      IF(J.LE.ISC(NI).AND.I.LT.IN(NI)) DTHB(I,J)=DTHB(NI+1)
0113      IF(J.LE.ISC(NI).AND.I.GT.1) DTHA(I,J)=DTHA(NI+1)
0114      IF(I.NE.1.AND.J.EQ.JX) AN(I)=AN(I-1)+DTHA(I,J)
0115      CONTINUE
0116      N=N+1
0117      IF(IN(NI).EQ.IX) GO TO 40
0118      INCREMENT COUNTER (N) FOR NO. OF DIFFERENT ANGULAR
0119      SUBDIVISION SIZES USED.
0120      N=N+1
0121      READ NEXT SUBDIVISION SIZE (DTH(N+1)) AND LAST CIRC.
0122      NODE AT WHICH IT IS USED (IN(N)).
0123      READ (105,993) DTH(N+1),IN(N),NEXT
0124      WRITE (106,994) N,IN(N),DTH(N+1),NEXT
0125      NI=IN(N-1)
0126      N2=IN(N)
0127      SNEEP THROUGH CIRC. NODES (I) IN REGION IN WHICH
0128      DTH(N+1) IS USED.
0129      DO 7 I=NI,N2
0130      CALCULATE ANGULAR POSITION OF CIRC. NODE I.
0131      IF(I.NE.IN(N-1)) AN(I)=AN(I-1)+DTH(N+1)
0132      SNEEP THROUGH RADIAL NODES (J).
0133      DO 8 J=2,JX
0134      CALCULATE DTHA(I,J) AND DTHB(I,J) AT NODE I,J.
0135      IF(I.NE.N1) DTHA(I,J)=DTH(N+1)
0136      IF(I.NE.N2) DTHB(I,J)=DTH(N+1)
0137      CONTINUE
0138      IF(NEXT.EQ.1) GO TO 4
0139      IF(AN(IN(N)).GT.ATOT) OUTPUT 'TOTAL ANGLE EXCEEDED'
0140      IF(IN(N).GE.IX) GO TO 45
0141      IF(AN(IN(N)).GT.ATOT) STOP
0142      CALCULATE FINAL SUBDIVISION SIZE (DTH(N+2)) FROM
0143      INCLUDED ANGLE OF REMAINING SEGMENT (IE., ATOT-
0144      AN(IN(N))) AND NUMBER OF REMAINING SUBDIVISIONS (IE.,
0145      IX-IN(N)).
0146      DTH(N+2)=(ATOT-AN(IN(N)))/(IX-IN(N))
0147      OUTPUT DTH(N+2)
0148      N2=IN(N)
0149      CALCULATE THE ANGULAR POSITIONS OF THE REMAINING
0150      RADIAL GRID LINES (I) AND THE VALUES OF DTHA(I,J) AND
0151      DTHB(I,J) AT THE REMAINING NODES (I,J).
0152      DO 52 I=N2,IX
0153      IF(I.NE.N2) AN(I)=AN(I-1)+DTH(N+2)
0154      DO 55 J=2,JX
0155      IF(I.NE.N2) DTHA(I,J)=DTH(N+2)
0156      IF(I.NE.IX) DTHB(I,J)=DTH(N+2)
0157      IF(I.EQ.IX) DTHB(I,J)=0.0
0158      CONTINUE
0159      RETURN
0160      FORMAT (15,2F10.5,15)
0161      FORMAT (215)
0162      FORMAT (F10.5,215)
0163      FORMAT ('***** DATA READ IN SUBROUTINE DTH2 *****')
0164      FORMAT ('X','IX=',I3,0X,'ATOT=',F10.5,0X,'DTH(1)=' ,F10.6,0X,'NI=',I
0165      3,/)
0166      FORMAT ('X','II=',I3,0X,'IN(II)=' ,I3,0X,'ISC(II)=' ,I3,/)
0167      FORMAT ('X','N=' ,I3,0X,'IN(N)=' ,I3,0X,'DTH(N+1)=' ,F10.5,0X,'NEXT
0168      =' ,I3,/)
0169      END
0170
0171      SUBROUTINE JINT
0172      THIS SUBROUTINE CALCULATES THE NUMBER OF THE FIRST
0173      (INNERMOST) RADIAL NODE ON EACH RADIAL GRID LINE (I),
0174      DESIGNATED JINT(I). THE VALUE OF JINT(I) AT ANY RADIAL
0175      GRID LINE (I) MUST BE EITHER 2 OR ONE OF THE VALUES
0176      ISC(NI)+1, ISC(NI-1)+1,.....,ISC(1)+1 (NI IS AN INPUT
0177      VARIABLE READ IN SUBR. DTH2).
0178      DIMENSION ITWT(39),INNT(11),ISC(10),JIN(39),IN(10)

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0011 COMMON /DBA/IX,JX,KX /DB70/JIN /DB71/ISC,IN,NI /DB75/ITVT,INVT
0012
0013 CALCULATE JIN(I) AT RADIAL GRID LINES (I) LESS THAN
0014 IN(NI) (IE., IN THE SECTOR OF THE GRID WHERE ANGULAR
0015 SUBDIVISIONS DEPEND ON BOTH RADIUS AND ANGULAR POSITION
0016 )
0017 DO 5 N=1,NI
0018   ISC1=2
0019   IF(N.NE.NI) ISC1=ISC(N+1)+1
0020   IN1=IN(N)
0021   IN1=INVT(N+1)
0022   NI2=INVT(N)
0023   NREN1=ITVT(IN1)-NI2
0024   NREN2=ITVT(IN1)-NI2
0025   DO 20 I1=2,IN1
0026     I2=IN1-I1+1
0027     IF(ITVT(I2).EQ.NREN2) JIN(I2)=ISC(N)+1
0028     IF(ITVT(I2)-NREN1) 25,30,20
0029   CONTINUE
0030   JIN(I2)=ISC1
0031   IN1=I2
0032   IF(I2.GT.1) GO TO 15
0033   GO TO 5
0034   OUTPUT 'VALUES OF IN ARE INCORRECT'
0035   STOP
0036 CONTINUE
0037 INN=IN(NI)
0038
0039 ASSIGN A VALUE OF 2 TO JIN(I) FOR I .GE. IN(NI).
0040 DO 40 I=INN,IX
0041   JIN(I)=2
0042 RETURN
0043 END

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0001 SUBROUTINE CALAND
0002
0003 THIS SUBROUTINE IS USED IN A LINEC ANALYSIS TO MODIFY
0004 INCORRECT GEOMETRIC GRID DATA COMPUTED PREVIOUSLY FOR
0005 THE CALANDRIA TUBE NODES. THE RADII OF NODES IN THE
0006 CAL. TUBE ARE CORRECTED AND THE ANGULAR SUBDIVISIONS
0007 ARE ADJUSTED SO THAT NODES ON THE CAL. TUBE INNER
0008 SURFACE LINE UP WITH THOSE ON THE PRESSURE TUBE OUTER
0009 SURFACE.
0010
0011 DIMENSION RAD(25),AN(39),BTNA(39,25),BTHB(39,25)
0012 COMMON /DB52/RAB,DB54/RX,DBA/IX,JX,KX/DB72/BTNA,BTHB/DB2A/AN
0013 /DB37/PTID,PTOB,CTID,CTOB,DBART,SPRDLA,ECC,CB1,XCN
0014
0015 CALCULATE THE PRESSURE TUBE ECCENTRICITY (ECC).
0016 ECC=(CTID-PTOB)/2.
0017
0018 CALCULATE THE VARIABLES CB1 AND XCN USED IN SUBROUTINE
0019 DATA1 TO DETERMINE THE GAP BETWEEN THE PRESSURE AND
0020 CAL. TUBES AT DIFFERENT AXIAL LOCATIONS.
0021 CB1=14.05SPRDLA/(DBART*24)
0022 XCN=0.25DBART
0023
0024 CALC. THE NUMBER OF THE RADIAL NODE ON THE CAL. TUBE
0025 INNER SURFACE (NIS).
0026 NIS=RX+1
0027
0028 CALC. THE SIZE OF THE RADIAL SUBDIVISION USED IN THE
0029 CAL. TUBE (BC).
0030 BC=(CTOB-CTID)/(2.04(JX-NIS))
0031 CTIR=CTID/2.0
0032
0033 SWEEP THROUGH THE RADIAL NODES (J) IN THE CAL. TUBE.
0034 DO 10 J=NIS,JX
0035
0036 CALCULATE THE RADIUS AT RADIAL NODE J (RAD(J)).
0037 RAD(J)=CTIR+(J-NIS)*BC
0038
0039 SWEEP THROUGH THE CIRCUMFERENTIAL NODES (I).
0040 DO 15 I=1,IX
0041
0042 CALCULATE THE ANGULAR POSITIONS OF THE MIDPOINTS OF
0043 THE ANGULAR SUBDIVISIONS PRECEDING AND FOLLOWING
0044 CIRC. NODE I, DESIGNATED THIA1 AND THIA2 RESPECTIVELY.
0045 THIA1=AN(I)-BTHB(I,RX)/2.0
0046 THIA2=AN(I)+BTHB(I,RX)/2.0
0047
0048 CALCULATE THE CORRECT VALUES OF BTHA(I,J) AND BTHB(I,J)
0049 FOR THE NODES IN THE CAL. TUBE WALL ON RADIAL GRID LINE
0050 I.
0051 BTHAC=((1.0-(ECC*CB1*(THIA1)/(CTIR*2.0*(1.0-(ECC*CB1*(THIA1)/
0052 CTIR*2.0))))*BTHA(I,RX)
0053 BTHBC=((1.0-(ECC*CB1*(THIA2)/(CTIR*2.0*(1.0-(ECC*CB1*(THIA2)/
0054 CTIR*2.0))))*BTHB(I,RX)
0055 IF(I.EQ.1) BTHAC=0.0
0056 IF(I.EQ.IX) BTHBC=0.0
0057 DO 20 J=NIS,JX
0058   BTHA(I,J)=BTHAC
0059   BTHB(I,J)=BTHBC
0060 CONTINUE
0061 CONTINUE
0062 RETURN
0063 END

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0001 SUBROUTINE DATA1 (RX,TC,HI,CK1,CK2,NEST,NSTN)
0002
0003 THIS SUBROUTINE READS DATA FOR SPECIFIC 2-D AND 3-D ANALYSES.
0004 DATA INDICATING WHETHER THE PROBLEM IS TRANSIENT OR STEADY-STATE,
0005 LINEAR OR NON-LINEAR ARE READ ALONG WITH MATERIAL PROPERTIES,
0006 TIME STEP SIZES, ETC. THE R-THETA BOUNDARY CONDITIONS AT ALL OF
0007 THE AXIAL NODES (K) IN THE GRID (IE., FOR K=1,2,...,KX) ARE READ
0008 AND/OR CALCULATED.
0009
0010 ***** INPUT VARIABLES READ BY SUBROUTINE DATA1 *****
0011
0012 TRANS - CONTROL VARIABLE: TRANS=1 INDICATES THAT A TRANSIENT
0013 ANALYSIS IS REQUIRED; TRANS=0 INDICATES THAT THE
0014 ANALYSIS WILL BE STEADY-STATE.

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0015	NTRTF	- NUMBER OF TRANSIENT TRIAL FUNCTIONS TO BE RECORDED	0130.500
0016		IN A 2-B TRANSIENT ANALYSIS.	0130.600
0017	NTRFCH	- NUMBER OF TIMES THE TRIAL FN. DEPLOYMENT SCHEDULE IS	0130.700
0018		TO BE CHANGED IN THE 2ND. PHASE OF A TRANSIENT	0130.800
0019		SYNTHESIS ANALYSIS.	0130.900
0020	NTFIC	- NUMBER OF A PREVIOUSLY COMPUTED TRIAL FN. FROM WHICH	0130.000
0021		INITIAL CONDITIONS (TEMPERATURES) FOR A TRANSIENT	0130.100
0022		AND/OR NON-LINEAR ANALYSIS ARE TO BE TAKEN (CAN ONLY	0130.200
0023		BE USED IF NROPT=2).	0130.300
0024	TMPTN	- TEMPERATURE TO WHICH ALL OF THE NODAL TEMPS. ARE EQUATED	0130.400
0025		AT THE START OF A TRANSIENT ANALYSIS. NOT USED IF	0130.500
0026		NROPT=2 AND NTFIC .GT. 0.	0130.600
0027	TFINAL	- FINAL TIME TO WHICH A TRANSIENT ANALYSIS IS TO BE TAKEN.	0130.700
0028	DEMS1	- DENSITY OF MATERIAL AT RADIAL NODES (J) .LE. KX	0130.800
0029	DEMS2	- DENSITY OF MATERIAL AT RADIAL NODES (J) .GT. KX	0130.900
0030		IT IS ASSUMED THAT THE CYLINDRICAL REGIONS TO WHICH	0140.000
0031		DEMS1 AND DEMS2 APPLY ARE SEPARATED BY AN INTERFACE	0140.100
0032		OR GAP.	0140.200
0033	SPMT1	- SPECIFIC HT. OF MATERIAL AT RADIAL NODES (J) .LE. KX	0140.300
0034	SPMT2	- SPECIFIC HT. OF MATERIAL AT RADIAL NODES (J) .GT. KX	0140.400
0035	NFTS	- NUMBER OF THE TIME STEP AT WHICH A TRANSIENT ANALYSIS	0140.500
0036		IS TO STOP.	0140.600
0037	NSTFC(NXT)	- NXT=1,2,...,NTRFCH - NUMBERS OF THE TIME STEPS AT	0140.700
0038		WHICH THE TRIAL FN. DEPLOYMENT SCHEDULE IS TO BE	0140.800
0039		CHANGED IN THE SECOND PHASE CALCS. OF A TRANSIENT	0140.900
0040		SYNTHESIS ANALYSIS (NOT READ UNLESS NSTN=2 AND NTRFCH	0141.000
0041		.GT. 0)	0141.100
0042	NFTTS(ITIN)	- ITIN=1,2,...,NTRTF - NUMBERS OF THE TIME STEPS AT	0141.200
0043		WHICH TRIAL FNS. ARE TO BE RECORDED IN A 2-B TRANSIENT	0141.300
0044		ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FNS. FOR A	0141.400
0045		SYNTHESIS ANALYSIS (NOT READ UNLESS STN=1 AND TRANS	0141.500
0046		.EQ. 1 AND NSTN .NE. 2).	0141.600
0047	BT(NBT)	- NBT IN TIME STEP SIZE.	0141.700
0048	NTC(NBT)	- NUMBER OF THE TIME STEP AT WHICH THE USE OF TIME STEP	0141.800
0049		SIZE BT(NBT) IS DISCONTINUED.	0141.900
0050	NEXT	- CONTROL VARIABLE : NEXT=1 INDICATES THAT THE STATEMENT	0142.000
0051		WHICH READS BT(NBT), NTC(NBT) AND NEXT SHOULD BE	0142.100
0052		RE-EXECUTED WITH THE VALUE OF NBT INCREMENTED BY 1.	0142.200
0053		BECAUSE MORE VALUES OF THOSE VARIABLES ARE SUPPLIED ;	0142.300
0054		NEXT=0 INDICATES THAT THE LAST SET OF VALUES FOR BT,	0142.400
0055		NTC AND NEXT HAS BEEN READ.	0142.500
0056			0142.600
0057		NOTE : VARIABLES FROM TMPTN TO NEXT INCLUSIVE ARE NOT READ IF	0142.700
0058		TRANS=0	0142.800
0059			0142.900
0060	NLIN	- NON-LINEAR GAP CONDUCTANCE OPTION : NLIN=1 INDICATES	0143.000
0061		THAT A LOECC ANALYSIS WITH NON-LINEAR GAP CONDUCTANCES	0143.100
0062		IS TO BE PERFORMED ; NLIN=0 INDICATES OTHERWISE.	0143.200
0063	RADBN0	- RADIATING CYLINDER OPTION : RADBN0=1 INDICATES THAT	0143.300
0064		HEAT TRANSFER IN A CYLINDER RADIATING TO THE	0143.400
0065		SURROUNDINGS IS TO BE ANALYSED ; RADBN0=0 INDICATES	0143.500
0066		OTHERWISE.	0143.600
0067	MAXIT	- MAXIMUM PERMISSIBLE ITERATION COUNT FOR A NON-LINEAR	0143.700
0068		STEADY-STATE ANALYSIS.	0143.800
0069	EPSLN	- CONVERGENCE PARAMETER FOR A NON-LIN. STEADY-STATE ANAL.	0143.900
0070	TRADIN	- INITIAL BOUNDARY TEMP. ESTIMATE FOR A STEADY-STATE	0144.000
0071		RADIATING CYLINDER ANALYSIS.	0144.100
0072	HRADIN	- PRESSURE TO CAL. TUBE RADIATIVE HEAT TRANSFER COEFF.-	0144.200
0073		USED IN LINEAR (NLIN=0) LOECC ANALYSES AND IN THE 1ST	0144.300
0074		ITER. OF A STEADY-STATE NON-LIN. (NLIN=1) FIN.-DIFF.	0144.400
0075		LOECC ANALYSIS.	0144.500
0076	BETA	- UNDERRELAXATION FACTOR USED IN THE LINEARIZATION OF GAP	0144.600
0077		CONDUCTANCES IN FIN.-DIFF. LOECC ANALYSES (RECOMMEND	0144.700
0078		BETA=0.5)	0144.800
0079	TCINTB	- INTERCEPT OF LINEAR THERMAL CONDUCTIVITY VS. TEMP.	0144.900
0080		REGRESSION FOR GAS IN PRESSURE/CAL. TUBE GAP.	0145.000
0081	SLOPB	- SLOPE OF REGRESSION FOR WHICH TCINTB IS THE INTERCEPT.	0145.100
0082	REXCD	- RADIATION EXCHANGE COEFF. FOR RADIATION BETWEEN THE	0145.200
0083		PRESSURE AND CAL. TUBES. (USUAL FORMULA FOR CONCENTRIC	0145.300
0084		CYLINDERS) OR TO SURROUNDINGS IF RADBN0=1.	0145.400
0085	MVCON	- VARIABLE THERMAL CONDUCTIVITY OPTION : MVCON=1 CAUSES	0145.500
0086		COND. OF PRESSURE AND CAL. TUBE MAT'L TO BE TREATED AS	0145.600
0087		TEMP. DEPENDENT IN A LOECC ANALYSIS. WHEN MVCON=1, SET	0145.700
0088		CK1=CK2=1.0	0145.800
0089	TCINTS	- INTERCEPT OF THERMAL COND. VS. TEMP. REGRESSION (ASSUMED	0145.900
0090		LINEAR) FOR PL. AND CT. MAT'L.	0146.000
0091	SLOPS	- SLOPE OF REGRESSION FOR WHICH TCINTS IS THE INTERCEPT.	0146.100
0092	TCGAS	- THERMAL COND. OF GAS - USED IN 'RESISTANCE OF CONTACT'	0146.200
0093		AND LINEAR (NLIN=0) LOECC ANALYSES. ALSO USED IN THE	0146.300
0094		1ST ITER. OF A ST.-STATE NON-LIN. FIN.-DIFF. LOECC ANAL.	0146.400
0095	CCON	- NON-CONTINUUM GAS CONDUCTIVITY PARAMETER (NEEDED IN	0146.500
0096		LOECC AND 'RESISTANCE OF CONTACT' ANALYSES.).	0146.600
0097	H1	- HT. TRANS. COEFF. BETWEEN INNER AND OUTER CYLINDERS IN	0146.700
0098		AN ANALYSIS INVOLVING TWO (CONCENTRIC) CYLINDERS. IN A	0146.800
0099		LOECC ANALYSIS, H1 IS THE HT. TRANSFER COEFF. ON THE	0146.900
0100		CAL. TUBE OUTER SURFACE.	0147.000
0101	CK1	- THERMAL COND. OF INNER CYL. IN A TWO CYLINDER ANALYSIS.	0147.100
0102	CK2	- THERMAL COND. OF OUTER CYL. IN A TWO CYLINDER ANALYSIS.	0147.200
0103	CCOND	- CONTACT CONDUCTANCE USED WITHIN THE CONTACT PATCH IN A	0147.300
0104		LOECC OR 'RESISTANCE OF CONTACT' ANALYSIS.	0147.400
0105	OUTRY	- TRIAL FUNCTION PRINT OPTION : OUTRY=1 CAUSES TRIAL FN.	0147.500
0106		RESULTS TO BE PRINTED WITH OUTRY=0, RESULTS ARE NOT	0147.600
0107		OUTPUT.	0147.700
0108	NEXT	- ADDITIONAL TRIAL FN. OPTION : NEXT=1 INDICATES THAT INPUT	0147.800
0109		DATA FOR ANOTHER TRIAL FN. ARE PROVIDED ; NEXT=0	0147.900
0110		INDICATES OTHERWISE.	0148.000
0111	NZFLO	- AXIAL BOUNDARY CONDITION OPTION : NZFLO=1 INDICATES THAT	0148.100
0112		THE TEMP. DISTRIBUTION STORED IN TRIAL FN. NTRFCH IS TO	0148.200
0113		BE APPLIED OVER AN R-THETA PLANE LOCATED A DISTANCE BZBC	0148.300
0114		FROM THE PLANE ON WHICH NODAL TEMPS. ARE BEING CALCULATED	0148.400
0115		. WHEN NZFLO=0, AN AXIAL BOUNDARY COND. IS NOT APPLIED.	0148.500
0116	NTRFBC	- NUMBER OF TRIAL FN. FROM WHICH AXIAL BOUNDARY TEMPS.	0148.600
0117		ARE TO BE TAKEN.	0148.700
0118	BZBC	- AXIAL DISTANCE SEPARATING BOUNDARY PLANE AND	0148.800
0119		COMPUTATIONAL PLANE.	0148.900
0120	MAX	- CALCULATED R-THETA BOUNDARY CONDITION OPTION : MAX=1	0149.000
0121		CAUSES THE D.C.'S OR GAP CONDUCTANCES AT AXIAL NODES	0149.100
0122		K (K=1,2,...,KX) TO BE CALC. FROM THE APPROPRIATE	0149.200
0123		GEOMETRIC PARAMETERS IN 'RESISTANCE OF CONTACT' AND	0149.300
0124		LOECC ANALYSES. IN A 2-B ANALYSIS (KX=1), THE D.C.'S	0149.400
0125		ARE EVAL. AT THE AXIAL COORD. XPP (SEE BELOW). IF MAX=0,	0149.500
0126		THE CIRCUMFERENTIALLY UNIFORM HT. TRANSFER COEFF. HOUT	0149.600
0127		(SEE BELOW) IS USED OUTSIDE OF THE CONTACT.	0149.700
0128	XPP	- AXIAL LOCATION AT WHICH R-THETA D.C.'S ARE TO BE EVAL.	0149.800
0129		IN A 'RESISTANCE OF CONTACT' OR LOECC ANALYSIS (NOT	0149.900
0130		NEEDED IF MAX=0 AND/OR KX .GT. 1).	0150.000
0131	HTGEN(K)	- VOLUMETRIC HEAT GENERATION RATE AT AXIAL NODE K.	0150.100
0132	HFL2(K)	- HEAT FLUX APPLIED TO THE INNERMOST RADIAL NODES (J=2) AT	0150.200
0133		AXIAL NODE K.	0150.300
0134	TOUT	- SINK TEMP. FOR CIRCUMFERENTIAL (OUTER SURFACE) NODES	0150.400

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0135      (1),GT,RZ1
0136      - OUTER SURFACE HT. TRANSFER COEFF. USED AT CIRCUMFERENTIAL
0137      NODES (1),GT,RZ1
0138      TEX - SINK TEMP. FOR CIRCUMFERENTIAL (OUTER SURFACE) NODES (1)
0139      - LE, RZ1
0140      HEX - OUTER SURFACE HT. TRANSFER COEFF. USED AT
0141      CIRCUMFERENTIAL NODES (1), LE, RZ1
0142      RZ1 - CIRCUMFERENTIAL NODE AT THE EDGE OF THE ARC OVER WHICH
0143      THE HT. TRANSFER COEFF. HEX IS PRESCRIBED (USUALLY THE
0144      EDGE OF A CONTACT PATCH).
0145      NTRI(K) - NUMBER OF TRIAL FMS. USED AT THE KTH AXIAL NODE (NOT
0146      READ UNLESS NMTN=2).
0147      NT2(NK) - (NK-1,2,...,10) TRIAL FMS. USED AT THE KTH. AXIAL NODE
0148      (NOT READ UNLESS NMTN=2)
0149
0150      NOTE : VARIABLES NTRIEN(K) TO NT2 INCLUSIVE ARE READ FOR K=1,2,...,KX
0151
0152      INTEGER RABOND,TRANS,RESIST,SYN,OUTRY
0153      DIMENSION AXPOS(45),HX(39,45),TC(39,45),NTRIEN(45),AM(39)
0154      1 NTFTS(10),NT2(10),GRAPP(39,1),NTRI(45),NTRX(45,10),BT(10),
0155      1 NTC(10),NTSTFC(10),DPLX( 1, 1),TRAB(39,45),NFLZ(45),NTRIP(45),
0156      1 NTRAP(45,10),DZ(45),N7B(45),NIM(45)
0157      COMMON /B01/RESIST, /B04/IX,J,KX, /B07/AXPOS, /B07/NEBU
0158      1 /B018/NFLZ,CCOMB, /B019/SYN,OUTRY,COMST, /B020/BETA,GRAPP, /B026/AM
0159      1 /B030/TCINTS,SLOPS,BASC, /B031/TRANS,THPIN,TFINAL,THCAP,BT,NTBT,
0160      1 NTC, /B032/TCBAS,CCOM, /B034/THCAP1,THCAP2, /B037/PTIS,PTOB,CTID,
0161      1 CTOS, /B041/SPRIDA,ECC,CAL,CCM, /B050/INPECC, /B041/RABOND
0162      1 /B042/TRABIN,MAXIT,NTSTF,NTFS,NTFTS, /B043/NECCO, /B044/EPBLN
0163      1 /B045/DPLX,TRAB, /B047/NTSTF,NTFT, /B049/NTFC, /B057/NLIN
0164      1 COMMON /B064/NTRI,NTRIP, /B064/NBA, /B065/NTRI,NTRX, /B076/
0165      1 TCINTS,SLOPS,SLOPS2,NLA,NAC,NACON, /B076/NT
0166      1 /B080/NZFLD,NTFC,NZBC, /B027/DZ,N,B76,NTGEN, /B010/NIM
0167      1 WRITE (100,801)
0168
0169      C
0170      READ (105,994) TRANS,NTRIF,NTFCN,NTFIC
0171
0172      C
0173      WRITE (100,890) TRANS,NTRIF,NTFCN,NTFIC
0174
0175      C
0176      READ DATA FOR A TRANSIENT ANALYSIS.
0177
0178      C
0179      IF(TRANS-1) 5,25,5
0180
0181      C
0182      READ (105,996) THPIN,TFINAL,BENS1,BENS2,SPMT1,SPMT2,NTFS
0183
0184      C
0185      WRITE (100,802) THPIN,TFINAL,BENS1,BENS2,SPMT1,SPMT2,NTFS
0186      1 THCAP1=BENS1,SPMT1, THCAP2=BENS2,SPMT2
0187      1 IF(SYN,NE.1,OR,(NMTN,EQ.2,AND,NTFCN,EQ.0)) GO TO 40
0188      1 IF(MEYN,NE.2) GO TO 42
0189      1 WRITE (100,893)
0190
0191      C
0192      READ NUMBERS OF TIME STEPS AT WHICH THE TRIAL FM. DEPLOYMENT
0193      SCHEDULE IS TO BE CHANGED.
0194
0195      C
0196      DO 44 NKT=1,NTFCN
0197
0198      C
0199      READ (105,995) NTSTFC(NKT)
0200
0201      C
0202      WRITE (100,894) NKT,NTSTFC(NKT)
0203
0204      C
0205      CONTINUE
0206      GO TO 40
0207
0208      C
0209      WRITE (100,891)
0210
0211      C
0212      READ NUMBERS OF TIME STEPS AT WHICH TRIAL FMS. ARE TO BE RECORDED
0213      IN A TRANSIENT 2-3 ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FMS.
0214      FOR A SYNTHESIS ANALYSIS.
0215
0216      C
0217      DO 43 ITIN=1,NTRIF
0218
0219      C
0220      READ (105,995) NTFTS(ITIN)
0221
0222      C
0223      WRITE (100,892) ITIN,NTFTS(ITIN)
0224
0225      C
0226      CONTINUE
0227
0228      C
0229      READ TIME STEP SIZES.
0230
0231      C
0232      NBT=1
0233
0234      C
0235      READ (105,997) BT(NBT),NTC(NBT),NEXT
0236
0237      C
0238      WRITE (100,803) NBT,BT(NBT),NTC(NBT),NEXT
0239      NBT=NBT+1
0240      IF(NEXT,EQ.1) GO TO 45
0241      NBT=NBT-2
0242
0243      C
0244      READ GENERAL DATA.
0245
0246      C
0247      READ (105,990) NLIN,RABOND,MAXIT,EPBLN,TRABIN,NRABIN,BETA,
0248      1 TCINTS,SLOPS,NECCO
0249
0250      C
0251      READ (105,990) NACON,TCINTS,SLOPS
0252
0253      C
0254      SLOPS2=0.36SLOPS
0255
0256      C
0257      READ (105,999) TCBAS,CCOM
0258
0259      C
0260      IF((INPECC,EQ.1,AND,NLIN,EQ.1,AND,NMTN,EQ.2) TCBAS=TCINTS
0261      1 WRITE (100,804) NLIN,RABOND,MAXIT,EPBLN,TRABIN,NRABIN,BETA,
0262      1 TCINTS,SLOPS,NECCO,TCBAS,CCOM
0263      1 WRITE (100,896) NACON,TCINTS,SLOPS
0264      1 IF(RABOND,EQ.1,AND,TRANS,EQ.1) TRABIN=THPIN
0265
0266      C
0267      READ (105,991) N1,CK1,CK2,CCOMB,OUTRY,NEXT
0268
0269      C
0270      WRITE (100,805) N1,CK1,CK2,CCOMB,OUTRY,NEXT
0271
0272      C
0273      READ (105,901) NZFLD,NTFCN,NZBC
0274
0275      C
0276      WRITE (100,895) NZFLD,NTFCN,NZBC
0277
0278      C
0279      READ (105,993) MAX,XPP
0280
0281      C
0282      WRITE (100,806) MAX,XPP
0283      NBA=0
0284      NEQU=0
0285
0286      C
0287      SWEEP THROUGH THE AXIAL NODES (K=1,2,...,KX)
0288
0289      C
0290      DO 10 K=1,KX
0291      1 IF(KX,GT.1) XPP=AXPOS(K)
0292
0293      C

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0233      READ (105,992) HTOEN(K),HFLZ(K),TOUT,HOUT,TEX,HEX,HZ1
0234      WRITE (106,997) K,HTOEN(K),TOUT,HOUT,TEX,HEX,HZ1
0235      OUTPUT HFLZ(K)
0236
0237      READ TRIAL FN. DEPLOYMENT DATA FOR 2ND. PHASE OF SYNTHESIS ANALYSIS.
0238
0239      IF (NSTN.NE.2) GO TO 24
0240      WRITE (106,998)
0241
0242      READ (105,999) NTRI(K),(NT2(NK),NK=1,10)
0243
0244      NTRIP(K)=NTRI(K)
0245      DO 14 NK=1,10
0246      IF (NT2(NK).GT.0) WRITE (106,999) NT2(NK)
0247      NTRK(K,NK)=NT2(NK)
0248
0249      14
0250      CALCULATE THE MAXIMUM BANDWIDTH (HBA) OF THE LEFT-HAND COEFF. MATRIX
0251      FOR THE SYSTEM OF EQUATIONS DEFINING THE BLENDING VECTOR (Z) IN A
0252      SYNTHESIS ANALYSIS.
0253
0254      IF (K.EB.1) GO TO 27
0255      HBA=NTRI(K)+NTRI(K-1)-1
0256      IF (HBA.GT.HBA) HBA=HBA
0257
0258      INCREASE THE COUNTER FOR THE NUMBER OF EQUATIONS IN THE COEFF.
0259      MATRIX (NEQU).
0260      NEQU=NEQU+NTRI(K)
0261      HZ1=HZ1+1
0262
0263      SWEEP THROUGH THE CIRCUMFERENTIAL NODES (I) OUTSIDE OF THE CONTACT
0264      AND ASSIGN HEAT TRANSFER COEFFICIENTS (HX(I,K)) AND SINK
0265      TEMPERATURES (TC(I,K)) TO THOSE NODES. IF MAX=1, THE HEAT TRANSFER
0266      COEFFICIENTS WILL BE CALCULATED.
0267
0268      DO 15 I=MAX,IX
0269      IF (MAX=1) 16,17,17
0270      RTUB=H0
0271      IF (INTEC.EB.1) RTUB=PTOBS.5
0272      IF (INTEC.NE.1) GO TO 50
0273      IF (XPP=H0N) 50,51,51
0274      IF (AN(I).GT.0) 50,51,51
0275      GAP2=CB(8,XPP*RTUB*8(0.58*(BARTS2)-(XPP*RTUB)))
0276      ECC1=ECC-GAP2
0277      GBAF=ECC1*CB(AN(I))+SORT((ECC1*CB(AN(I)))*RTUB-ECC1*RTUB)
0278      (CTIB/2.0)*RTUB
0279      HX=GBAF/GBAF
0280      HOUT=TCBAS/((1.0+H0)*GBAF)
0281      GO TO 21
0282
0283      CALL SUBR. H0AS TO CALCULATE THE GAS CONDUCTANCE (HOUT) AND GAP
0284      THICKNESS (GBAF) AT LATERAL COORDINATE YP2 AND AXIAL COORDINATE XPP.
0285      CALL H0AS (XPP,YP2,HOUT,GBAF)
0286
0287      IF (NLI.NE.1 AND NSTN.EB.2) GO TO 22
0288      IF (INTEC.EB.1) HOUT=HOUT*H0ASIN
0289      IF (INTEC.NE.1) HOUT=HOUT*CB(AN(I))
0290      HX(I,K)=HOUT
0291      IF (NSTN.NE.2) GBAF(I,K)=GBAF
0292      TC(I,K)=TOUT
0293
0294      15 CONTINUE
0295      IF (NZ1.LT.1) GO TO 23
0296
0297      SWEEP THROUGH CIRCUMFERENTIAL NODES (I) INSIDE OF THE CONTACT AND
0298      ASSIGN HEAT TRANSFER COEFFICIENTS (HX(I,K)) AND SINK TEMPERATURES
0299      (TC(I,K)) TO THOSE NODES.
0300
0301      DO 20 I=1,NZ1
0302      HX(I,K)=HEX
0303      TC(I,K)=TEX
0304
0305      20 CONTINUE
0306
0307      INITIALIZE BOUNDARY CONDITIONS FOR A RADIATING CYLINDER ANALYSIS.
0308
0309      IF (RABND=1) 10,30,10
0310      DO 35 I=1,IX
0311      QFLX(I,K)=H0XCB((TRADIN04)-(TC(I,K)*EB4))
0312      TRAD(I,K)=TRADIN
0313
0314      35 CONTINUE
0315      FORMAT (15,2E12.5)
0316      FORMAT (15,F10.5)
0317      FORMAT (15,4F10.5,2E12.4)
0318      FORMAT (4F10.5,2E15)
0319      FORMAT (4F10.5,15)
0320      FORMAT (15,F10.5)
0321      FORMAT (15)
0322      FORMAT (4F10.5,15)
0323      FORMAT (4E15)
0324      FORMAT (F10.5,2E15)
0325      FORMAT (11E15)
0326      FORMAT (2E10.4)
0327      FORMAT ('***** DATA READ IN SUBROUTINE DATA1 *****')
0328      FORMAT (1X,'TMIN='F10.5,10X,'TMAX='F10.5,10X,'BEM1='F10.5,10X,
0329      'OK','BEM2='F10.5,10X,'BMT1='F10.5,10X,'BMT2='F10.5,10X,
0330      'HTS='F15,10X,
0331      'HT1='F10.5,10X,'HT(NBT)'F10.5,10X,'HTC(NBT)'F10.5,10X,
0332      'NEXT='F15,10X,
0333      'NLI='F10.5,10X,'RABND='F10.5,10X,'MAXIT='F10.5,10X,
0334      'EPM='F10.5,10X,'TRADIN='F10.5,10X,'H0ASIN='F10.5,10X,
0335      'BETA='F10.5,10X,'TCINTS='F10.5,10X,'SLOPS='F10.5,10X,
0336      'H0XCB='F10.5,10X,'TCBAS='F10.5,10X,
0337      'CCBN='F10.5,10X,
0338      'H0='F10.5,10X,'CK1='F10.5,10X,'CK2='F10.5,10X,'CCBNB'
0339      'F10.5,10X,'OUTRT='F10.5,10X,'NEXT='F10.5,10X,
0340      'OK','MAX='F10.5,10X,'XPP='F10.5,10X,
0341      'F7.2,5X,'TEX='F7.2,5X,'HEX='F10.5,10X,'NZ1='F10.5,10X,
0342      'F10X,'TRIAL FUNCTIONS')
0343      FORMAT (14X,12)
0344      FORMAT (1X,'TRAD='F15,10X,'NTRIP='F15,10X,'NTRF='F15,10X,
0345      'NTRIC='F15,10X,
0346      'TIME STEPS AT WHICH TRIAL FNS. ARE RECORDED FOLLOW')
0347      FORMAT (1X,'NTRIS(12)'F15,10X,
0348      'TIME STEPS AT WHICH TRIAL FNS. ARE CHANGED FOLLOW')
0349      FORMAT (1X,'NTRIFC(12)'F15,10X,
0350      'NZFLO='F10.5,10X,'NTRFC='F10.5,10X,'BZBC='F10.5,10X,
0351      'H0XCB='F10.5,10X,'TCINTS='F10.5,10X,'SLOPS='F10.5,10X,
0352      'E12.5,10X,
0353      RETURN

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0375

END

0174.500

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0001 SUBROUTINE GPROPT (BTAN,B,BZA,BZB,NSYN,CK1,CK2,IXX,NTS) 0174.600
0002 THIS SUBROUTINE DIRECTS THE FINITE-DIFFERENCE EQUATION 0174.700
0003 ASSEMBLY PROCESS ON EACH R-THETA PLANE (K) IN THE 0174.800
0004 FIB.-DIFF. GRID WHEN NSOPT=0. THE NODES IN THE RADIAL 0174.900
0005 AND CIRCUMFERENTIAL (R & THETA) DIRECTIONS ARE PROVIDED 0175.000
0006 WITH ADDRESSES J AND I RESPECTIVELY AND SUBROUTINE 0175.100
0007 GPROPT CONTROLS THE SWEEP THROUGH ALL VALUES OF I AND 0175.200
0008 J. IT ALSO DEFINES SOME OF THE GEOMETRIC PARAMETERS AND 0175.300
0009 MATERIAL PROPERTIES USED AT EACH NODE. 0175.400
0010 0175.500
0011 0175.600
0012 0175.700
0013 INTEGER TRANS,SYN,OUTRY 0175.800
0014 DIMENSION BTHA(39),BTHB(39),BRA(25),BRB(25),RAD(25),RX(39,65),CDB( 0175.900
0015 250),NRUN(39,25,1),JL(9),JP(9),T(470),BT(10),NTC(10) 0176.000
0016 COMMON /B04/CDB,NR,N,J,P /B05/RX,L /B06/IX,JX,KX /B07/I,J,K,NIS, 0176.100
0017 N1 /B08/ANTOT,BZTOT,BTOT /B09/NSYN /B10/STN,OUTRY,CONST 0176.200
0018 /B23/1 /B24/RX /B31/TRANS,THPIN,TFINAL,THCAP,BT,NTBT,NTC 0176.300
0019 /B34/THCAP1,THCAP2 /B35/RX /B42/RAS,BRA,BRS /B43/BTHA,BTHB 0176.400
0020 0176.500
0021 SWEEP THROUGH NODES ON THE THETA AXIS, IDENTIFIED BY 0176.600
0022 THE SUBSCRIPT I. 0176.700
0023 DO 15 I=1,IXX 0176.800
0024 ANTOT=(BTHA(I)+BTHB(I))/2.0 0176.900
0025 0177.000
0026 SWEEP THROUGH NODES ON THE R AXIS, IDENTIFIED BY THE 0177.100
0027 SUBSCRIPT J. 0177.200
0028 DO 20 J=2,JX 0177.300
0029 BTOT=(BRA(J)+BRB(J))/2.0 0177.400
0030 0177.500
0031 CALCULATE THE MATERIAL THERMAL CONDUCTIVITY (CK) AND 0177.600
0032 DENSITY SPECIFIC HEAT (THCAP) AT RADIAL NODE J. 0177.700
0033 CK=CK2 0177.800
0034 THCAP=THCAP2 0177.900
0035 IF(J.LE.RX) CK=CK1 THCAP=THCAP1 0178.000
0036 0178.100
0037 INCREMENT R-THETA NODE COUNT (L). 0178.200
0038 L=L+1 0178.300
0039 0178.400
0040 DEFINE INITIAL TEMP. OF NODE L FOR TRANSIENT ANALYSIS. 0178.500
0041 IF(TRANS.EB.1.AND.NTS.EB.1) T(L)=THPIN 0178.600
0042 0178.700
0043 ZERO NODE NUMBER DIFFERENCES J AND JP. THESE 0178.800
0044 DIFFERENCES ARE LATER RECALC. AS THE INTERNAL 0178.900
0045 CONDUCTANCES ARE COMPUTED IN SUB. NRUN. 019.000
0046 DO 25 JL=1,4 019.100
0047 J(JL)=0 019.200
0048 JP(JL)=0 019.300
0049 CONTINUE 019.400
0050 IF(NSYN=2) 22,40,22 019.500
0051 019.600
0052 ASSIGN THE CORRECT NODE NUMBER TO THE POINT WHERE 019.700
0053 THE ITH, JTH AND KTH GRID LINES INTERSECT. THE CORRECT 019.800
0054 NODE NUMBER IS STORED AS NRUN(I,J,K). 019.900
0055 NRUN(I,J,K)=L 019.000
0056 019.100
0057 ZERO THE VECTOR IN WHICH THE INTERNAL CONDUCTANCES 019.200
0058 FOR A SINGLE ROW (NODE) ARE STORED (CDB). 019.300
0059 DO 30 NR=1,NR 019.400
0060 CDB(NR)=0.0 019.500
0061 019.600
0062 PROCEED TO THE ROUTINE THAT CALCULATES THE TERMS IN THE 019.700
0063 F-D EQUATION FOR NODE L. 019.800
0064 CALL NRUN(RAD(J),BTHA(I),BTHB(I),BRA(J),BRB(J),BZA,BZB, 019.900
0065 RX(I),K,CK,NSYN,BTAN) 019.000
0066 CONTINUE 019.100
0067 RETURN 019.200
0068 END 019.300

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0001 SUBROUTINE GPROPT1 (BTAN,B,BZA,BZB,NSYN,CK1,CK2,IXX,NTS) 0181.400
0002 THIS SUBROUTINE CONTAINS NESTED DO LOOPS WHICH SWEEP THROUGH ALL 0181.500
0003 OF THE CIRCUMFERENTIAL NODES I (=1,IXX) AND RADIAL NODES J (=2,JX) 0181.600
0004 ON THE R-THETA PLANE AT THE KTH AXIAL NODE WHEN NSOPT=1. EACH 0181.700
0005 NODE I, J IS CHECKED TO SEE IF IT IS ACTUALLY USED AND SKIPPED IF 0181.800
0006 IT ISN'T. THE SUBROUTINE DETERMINES WHICH OF SUBROUTINES NRUN, 0181.900
0007 SWRAB AND SIGRAB SHOULD BE CALLED TO EVALUATE TERMS IN THE 0182.000
0008 FINITE-DIFFERENCE EQUATION FOR A GIVEN NODE AND CALLS THAT 0182.100
0009 SUBROUTINE. SUBROUTINE GPROPT1 ALSO DEFINES SOME OF THE GEOMETRIC 0182.200
0010 PARAMETERS AND MATERIAL PROPERTIES USED AT EACH NODE AND INITIALIZES 0182.300
0011 VARIOUS TEMPERATURES IN TRANSIENT AND/OR NON-LINEAR ANALYSES. 0182.400
0012 0182.500
0013 0182.600
0014 0182.700
0015 0182.800
0016 0182.900
0017 0183.000
0018 0183.100
0019 0183.200
0020 0183.300
0021 0183.400
0022 0183.500
0023 0183.600
0024 0183.700
0025 0183.800
0026 0183.900
0027 0184.000
0028 0184.100
0029 0184.200
0030 0184.300
0031 0184.400
0032 0184.500
0033 0184.600
0034 0184.700
0035 0184.800
0036 0184.900
0037 0185.000
0038 0185.100
0039 0185.200
0040 0185.300
0041 0185.400

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0042 C CALCULATE 'BR' FOR NODE I,J.
0043 STOT=(BR(I,J)+BR(I,J))/2.0
0044
0045 C DETERMINE THE THERMAL CONDUCTIVITY OF THE MATERIAL AT NODE I,J (CK)
0046 ALONG WITH THE PRODUCT OF DENSITY AND SPECIFIC HEAT AT THAT POINT
0047 (THCAP).
0048 CK=CK2
0049 THCAP=THCAP2
0050 IF(J.LE.NX) CK=CK1 THCAP=THCAP1
0051
0052 C INCREMENT THE NODE SEQUENCE COUNTER, L
0053 L=L+1
0054
0055 C INITIALIZE T(L) IN THE 1ST. TIME STEP OF A TRANSIENT F-B ANALYSIS
0056 IF(TRANS.EG.1.AND.NTS.EG.1.AND.NSTN.NE.2) T(L)=TMIN
0057
0058 C ZERO THE NODE NUMBER DIFFERENCE VECTORS JP AND J. ENTRIES IN THESE
0059 VECTORS ARE REDEFINED IN SUBROUTINES NORMOD,SNARAB, ETC.
0060 DO 25 JJ=1,8
0061 J(JJ)=0
0062 JP(JJ)=0
0063
0064 25 CONTINUE
0065
0066 C DEFINE VARIABLES JA AND JB, WHICH REPRESENT THE NUMBER OF RADIAL
0067 NODES (J) BETWEEN NODE L AND NODES L-1 AND L+1 RESPECTIVELY.
0068 JB=1
0069 IF(I.NE.1) JA=IPR(I-1,J)
0070 IF(I.NE.1) JB=IS(I-1,J)
0071 IF(J.EQ.NIS) JA=0
0072 IF(J.EQ.NX.OR.J.EQ.JX) JB=0
0073 IF(NSTN.EG.2.AND.TRANS.EG.1.AND.NTS.EG.1) GO TO 94
0074 GO TO 97
0075
0076 C THE PROGRAM REACHES THIS POINT WHEN THE TRIAL FN. SCHEDULE HAS JUST
0077 BEEN CHANGED IN A TRANSIENT SYNTHESIS ANALYSIS. THE FOLLOWING
0078 PROGRAM SEGMENT DETERMINE THE TEMP. OF NODE L,K (THP(L,K)) ALONG WITH
0079 THAT OF NODE L+1,K IF J=NX OR THAT OF NODE L-1,K IF J=NIS,
0080 DESIGNATED THP2.
0081 THP1=0.0
0082 THP2=0.0
0083 NT=NTIP(K)
0084 DO 96 NK=L,NT
0085 THP=THP(K,NK)
0086 IF(J.EQ.NX) THP2=THP2+THP(L+1,NK)*Z(NBIP+HK)
0087 IF(J.EQ.NIS) THP2=THP2+THP(L-1,NK)*Z(NBIP+HK)
0088 THP1=THP1+THP(L,NK)*Z(NBIP+HK)
0089 96 IF(RABND.EG.1.AND.J.EQ.JX) THP(L,K)=THP1+OFLX(I,K)=
0090 REXCO(THP1*84-TC(I,K)*84)
0091 IF(NSTN=2) 22,40,22
0092
0093 C STORE THE NUMBER OF NODE I,J,K AS NOUN(I,J,K).
0094 NOUN(I,J,K)=L
0095
0096 C ZERO THE VECTOR CDM USED TO STORE THE FIN.-DIFF. COEFFICIENTS FOR
0097 NODE L.
0098 DO 30 NB=1,NBN
0099 CDM(NB)=0.0
0100 30
0101
0102 C DETERMINE WHICH OF THE SUBROUTINES NORMOD, BIORAB, ETC., SHOULD
0103 BE USED TO CALCULATE TERMS IN THE FIN.-DIFF. EQUATION FOR R-THETA
0104 NODE L, AND CALL THAT SUBROUTINE.
0105 DO 45 N=1,NR
0106 IF(I.EQ.(JBC(N)+1).AND.J.EQ.N(N)) GO TO 55
0107 IF(I.EQ.JBC(N).AND.J.EQ.N(N)) GO TO 60
0108 45 CONTINUE
0109 CALL NORMOD (RAB(J),BTHA(I),BTHB(I),BRA(I,J),BRB(I,J),
0110 BZA,BZB,HX(I,K),CK,6,NSTN,BTAU)
0111 GO TO 20
0112 55 CALL BIORAB (RAB(J),BTHA(I),BTHB(I),BRA(I,J),BRB(I,J),
0113 BZA,BZB,HX(I,K),CK,6,NSTN,BTAU)
0114 GO TO 20
0115 60 IF(J.NE.NIS) NSID=NSID+1
0116 IS2=NSID/2
0117 ZSI(2)=NSID
0118 Z2=ZSI(2)/2.0
0119 IF(I.EQ.BZ) GO TO 50
0120 CALL BIORAB (RAB(J),BTHA(I),BTHB(I),BRA(I,J),BRB(I,J),
0121 BZA,BZB,HX(I,K),CK,6,NSTN,BTAU)
0122 20 CONTINUE
0123 RETURN
0124 END
0125
0193.500
0193.600
0193.700
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0152 102      MMUN(I,J,K)=L
0153 C
0154 C ZERO THE VECTOR CMU USED TO STORE THE FIN.-DIFF. COEFFICIENTS FOR
0155 C MODE L.
0156      DO 100 NM=1,MMU
0157      CMU(NM)=0.0
0158      CONTINUE
0159 C
0160 C DETERMINE WHICH OF THE SUBROUTINES NORMOD, SHARAD, ETC., SHOULD
0161 C BE USED TO CALCULATE TERMS IN THE FINITE-DIFFERENCE EQUATION FOR
0162 C R-THETA MODE L, AND CALL THAT SUBROUTINE.
0163 104      DO 110 M=1,MM
0164      IF(J.EQ.(JSC(N)+1).AND.J.GE.M(N)) GO TO 115
0165      IF(J.EQ.JSC(M).AND.J.GT.M(N)) GO TO 120
0166      CONTINUE
0167 110      DO 125 NM=1,MM
0168      IF(J.EQ.(JSC(NM)+1).AND.JIN(I).EQ.J) GO TO 130
0169      IF(J.EQ.JSC(NM).AND.I.LE.IN(NM)) GO TO 135
0170      CONTINUE
0171 125      CALL NORMOD (RAD(J),DTMA(I,J),BTHB(I,J),DRA(I,J),DRB(I,J),
0172      ,BZA,BZB,MX(I,K),CK,G,MSYN,DTAU)
0173      GO TO 20
0174 115      CALL SHARAD (RAD(J),BTHA(I,J),BTHB(I,J),DRA(I,J),DRB(I,J),
0175      ,BZA,BZB,MX(I,K),CK,G,MSYN,DTAU)
0176      GO TO 20
0177 120      IF(J.NE.MIS) MSIG=MSIG+1
0178      MSIG=MSIG/2
0179      MSIG=MSIG
0180      BZ=MSIG/2.0
0181      IF(MSIG.EQ.2) GO TO 140
0182      CALL SHARAD (RAD(J),BTHA(I,J),BTHB(I,J),DRA(I,J),DRB(I,J),
0183      ,BZA,BZB,MX(I,K),CK,G,MSYN,DTAU)
0184      GO TO 20
0185 130      CALL SHARAD (RAD(J),BTHA(I,J),BTHB(I,J),DRA(I,J),DRB(I,J),
0186      ,BZA,BZB,CK,G,MSYN,DTAU)
0187      GO TO 20
0188 135      CALL SHARAD (RAD(J),BTHA(I,J),BTHB(I,J),DRA(I,J),DRB(I,J),
0189      ,BZA,BZB,CK,G,MSYN,DTAU)
0190      GO TO 20
0191      CONTINUE
0192      RETURN
0193      END

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0001      SUBROUTINE NORMOD (R,THA,THB,DA,DB,BZA,BZB,MX,CK,G,MSYN,DTAU)
0002
0003      THIS SUBROUTINE CALCULATES INTERNODAL CONDUCTANCES AT NORMAL NODES
0004      OF THE TYPE FOUND AT ORTHOGONAL GRID LINE INTERSECTIONS.
0005
0006      INTEGER RESIST,TRANS,SYN,OUTRY,TUBE,RADNDS
0007      DOUBLE PRECISION C,CG,CTOT
0008      DIMENSION BT(10),NTC(10),CMU(250),JP(9),Q(470),C(9),MMUN(37,25,1)
0009      ,AREA(470),T(470),JL(9),SAR(39,45),TC(39,45),HFLZ(45)
0010      ,TRF(470,13)
0011      COMMON /B01/RESIST,TO,DS2/TUBE,BT,DD4/CMU,MMU,R,JP,DS5/TL,L
0012      ,DB4/IX,J,K,KX,DD9/I,J,K,MIS,M1,DD10/O,DD11/ANTOT,BTOT,DTOT
0013      ,DD12/MMUN,DD19/SYN,OUTRY,CONST,DD21/TC,DD23/T,DD25/CA,MMEN
0014      ,DD28/MITER,DD31/TRANS,THPIN,TFINAL,THCAP,BT,MTST,NTC,DD33/
0015      ,AREA,DD38/INPECC,DD39/NTC,DD41/RADNDS,DD44/SAR,DD51/IA,IB
0016      ,DD54/IX,DD57/HLIN,DD60/IX,DD78/TCINTS,BLOP,BLOP2,MJL,MJC,
0017      ,MMCON,DD18/HFLZ,CG,DD88/MCCTL
0018      ,DD90/MZFLG,MTFIC,BZBC,DD14/MTF,TRF
0019
0020 C
0021      MM=7
0022      KK=K
0023      NCOM=(IX-1)*J*(J-1)
0024      IF(MSYN.EQ.2) KK=1
0025      KK1=M1
0026      IF(INPECC.EQ.1) KK1=MM
0027      KK2=MM
0028      IF(INPECC.EQ.1) KK2=M1
0029
0030 C
0031      CALCULATE CONDUCTANCES C(1),C(2),C(3),C(4),C(7) BETWEEN NODE I,J,K
0032      AND NEIGHBOURING NODES ON THE SAME R-THETA PLANE (K). IN F-B
0033      ANALYSIS, CONDUCTANCES TO THE LEFT OF THE DIAGONAL IN THE F-B
0034      COEFF. MATRIX ARE STORED IN THE TEMPORARY ARRAY CMU. CONDUCTANCES
0035      TO THE RIGHT OF THE DIAGONAL ARE NOT STORED BECAUSE THE
0036      COEFF. MATRIX IS SYMMETRIC.
0037
0038 C(1)=0.0
0039 IF(L.EQ.1.AND.MCCTL.NE.1) GO TO 10
0040 IF(MCCTL.EQ.1.AND.L.EQ.1) JP(1)=NCOM GO TO 9
0041 JP(1)=MMUN(I,J,KK)-MMUN(I-1,J,KK)
0042 IF(MVC.EQ.1) CX=TCINTS+BLOP2S(T(L)+T(L-JP(1)))
0043 C(1)=CX*BZTOT*BTOT/(BSTMA)
0044 IF(MCCTL.EQ.1.AND.L.EQ.1) GO TO 10
0045 IF(MSYN.EQ.2) GO TO 10
0046 CMU(NM-JP(1))=C(1)
0047
0048 10 C(2)=0.0
0049 IF(L.EQ.IX.AND.MCCTL.NE.1) GO TO 15
0050 IF(MCCTL.EQ.1.AND.L.EQ.IX) R(1)=NCOM GO TO 11
0051 IF(MJL.EQ.1) JL(1)=MMUN(I,IB,J)-MMUN(I,J,1)
0052 IF(MVC.EQ.1) CX=TCINTS+BLOP2S(T(L)+T(L+JL(1)))
0053 C(2)=CX*BZTOT*BTOT/(BSTMB)
0054 IF(MCCTL.EQ.1.AND.L.EQ.IX.AND.MSYN.NE.2) CMU(NM+JL(1))=C(2)
0055 JL(1)=JL(2)
0056
0057 15 C(3)=0.0
0058 IF(J.EQ.MIS.OR.J.EQ.2) GO TO 20
0059 IF(DA.EQ.0.0) OUTPUT I,J
0060 IF(MVC.EQ.1) CX=TCINTS+BLOP2S(T(L)+T(L-1))
0061 C(3)=CX*BZTOT*BTOT/(R-BA/2.0)/BA
0062 JP(2)=1
0063 IF(MSYN.EQ.2) GO TO 20
0064 CMU(NM-1)=C(3)
0065
0066 20 C(4)=0.0
0067 IF(J.EQ.MX.OR.J.EQ.JK) GO TO 25
0068 IF(MVC.EQ.1) CX=TCINTS+BLOP2S(T(L)+T(L+1))
0069 C(4)=CX*BZTOT*BTOT/(R+BA/2.0)/BA
0070 IF(MSYN.EQ.2) JL(2)=1
0071
0072 C
0073      CALCULATE AXIAL CONDUCTANCES C(5) AND C(6).
0074
0075 C(5)=0.0
0076 IF(K.EQ.KX) GO TO 30
0077 IF(MSYN.NE.2.AND.MVC.EQ.1) JL=MMUN(I,J,K+1)-MMUN(I,J,K)
0078 CX=TCINTS+BLOP2S(T(L)+T(L+JL))
0079 C(5)=CX*(R+((DB-BA)/4.0))*BZTOT*BTOT/BZB

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0074 30 C(4)=0.0 0220.500
0075 IF(K.EQ.1) GO TO 35 0220.600
0076 IF(MSYN.NE.2) JP(3)=MMUM(I,J,K)-MMUM(I,J,K-1) 0220.700
0077 IF(MNC.EQ.1) CX=TCINTS+SLOPDZ(T(L)+T(L-JP(3))) 0220.800
0078 C(6)=CX*(R+((DB-DA)/4.0))*SANTOTSDTOT/DZA 0220.900
0079 IF(MSYN.EQ.2) GO TO 35 0221.000
0080 CBM(NBU-JP(3))=C(6) 0221.100
0081 0221.200
0082 C CALCULATE THE GAP CONDUCTANCE (C(7)) AT THE INTERFACE OR GAP 0221.300
0083 BETWEEN INNER AND OUTER CYLINDERS (IF NECESSARY). 0221.400
0084 0221.500
0085 35 IF(J.EQ.NIS.AND.TUBE.NE.1) GO TO 36 0221.600
0086 C(7)=0.0 0221.700
0087 CS=0.0 0221.800
0088 IF(J.NE.MX.AND.J.NE.NIS)GO TO 40 0221.900
0089 IF(INPECC.NE.1) GO TO 37 0222.000
0090 IF(MSYN.EQ.2) GO TO 37 0222.100
0091 IF(MN.NE.1) GO TO 37 0222.200
0092 IF(TRANS.NE.1.AND.NITER.EQ.2) GO TO 37 0222.300
0093 IF(MX1.NE.CCOND) CALL MX10(MX1,TRANS) 0222.400
0094 C(7)=MX1RSANTOTSDTOT 0222.500
0095 CS=RSANTOTSDTOT 0222.600
0096 36 IF(J.EQ.NIS) JP(4)=1 0222.700
0097 IF(J.EQ.MX.AND.MSYN.EQ.2) JP(4)=-1 0222.800
0098 IF(MSYN.EQ.2) GO TO 40 0222.900
0099 IF(J.EQ.NIS) CBM(NBU-1)=-C(7) 0223.000
0100 0223.100
0101 C CALCULATE CONVECTIVE BOUNDARY CONDUCTANCE (C(8)). 0223.200
0102 0223.300
0103 40 C(8)=0.0 0223.400
0104 CA=0.0 0223.500
0105 IF(J.NE.JX)GO TO 45 0223.600
0106 CA=RSANTOTSDTOT 0223.700
0107 SAR(I,K)=CA 0223.800
0108 C(8)=MX2RSANTOTSDTOT 0223.900
0109 45 C(9)=0.0 0224.000
0110 AREA(L)=(R+((DB-DA)/4.0))*SANTOTSDTOT 0224.100
0111 IF(J.GT.MX) GO TO 50 0224.200
0112 0224.300
0113 C CALCULATE NORMAL HEAT GENERATION RATE (C(9)). 0224.400
0114 0224.500
0115 C(9)=GBSDTOT*(R+((DB-DA)/4.0))*SANTOTSDTOT 0224.600
0116 0224.700
0117 C SUM CONDUCTANCES TO YIELD DIAGONAL ELEMENT IN CONDUCTANCE MATRIX. 0224.800
0118 0224.900
0119 50 CTOT=C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8) 0225.000
0120 IF(TRANS.NE.1) GO TO 54 0225.100
0121 0225.200
0122 C ADD HEAT STORAGE TERM (HGEN) TO DIAGONAL ELEMENT FOR TRANSIENT 0225.300
0123 ANALYSIS. 0225.400
0124 0225.500
0125 HGEN=THECAPAREA(L)*SDTOT/DTAU 0225.600
0126 CTOT=CTOT+HGEN 0225.700
0127 GO=C(9) 0225.800
0128 0225.900
0129 C IF NECESSARY, INCORPORATE AXIAL BOUNDARY CONDITIONS IN A 2-D 0226.000
0130 ANALYSIS. 0226.100
0131 0226.200
0132 IF(MZFLD.EQ.1) CZFLO=CKSAREA(L)/BZBC ; CTOT =CTOT+CZFLO 0226.300
0133 ; GO=GO+CZFLOTRF(L,NTFBC) 0226.400
0134 IF(J.EQ.2) GO=GO+HFLZ(K)*RSANTOTSDTOT 0226.500
0135 TI=TMPIN 0226.600
0136 IF(MSYN.NE.2) TI=T(L) 0226.700
0137 0226.800
0138 C ADD PRODUCT OF HEAT STORAGE TERM AND PREVIOUS NORMAL TEMP. TO 0226.900
0139 APPROPRIATE ELEMENT IN RIGHT-HAND SIDE HEAT FLOW VECTOR. 0227.000
0140 0227.100
0141 IF(TRANS.EQ.1) GO=GO+HGEN*TI 0227.200
0142 0227.300
0143 C ADD TERM ARISING FROM CONVECTIVE BOUNDARY CONDITION TO 0227.400
0144 APPROPRIATE ELEMENT IN RIGHT-HAND SIDE HEAT FLOW VECTOR. 0227.500
0145 IF(J.EQ.JX) GO=GO+C(8)*TC(I,K) 0227.600
0146 0227.700
0147 C INCORPORATE BOUNDARY CONDITION FOR TUBE ANALYSIS (IE. TUBE=1) 0227.800
0148 0227.900
0149 IF(TUBE.EQ.1.AND.J.EQ.NIS) GO=GO+C(7)*ST ; C(7)=0.0 0228.000
0150 0228.100
0151 C INCORPORATE BOUNDARY CONDITION FOR RESISTANCE ANALYSIS (IE. RESIST 0228.200
0152 =1). 0228.300
0153 IF(RESIST.EQ.1.AND.I.EQ.IXX) GO=GO+C(2)*ST ; C(2)=0.0 0228.400
0154 0228.500
0155 0228.600
0156 0228.700
0157 C IF SUBSTITUTE/INTEGRATE STAGE OF SYNTHESIS ANALYSIS HAS BEEN 0228.800
0158 REACHED (INDICATED BY MSYN=2) ; TRANSFER CONTROL TO SUBROUTINE 0228.900
0159 SUBST1, WHERE THE SUBSTITUTIONS AND INTEGRATIONS ARE PERFORMED. 0229.000
0160 0229.100
0161 IF(MSYN=2) 35,65,55 0229.200
0162 45 CALL SUBST1(CS,MNC,C,GO,JP,J,L,CTOT,0.5,4) 0229.300
0163 RETURN 0229.400
0164 G(L)=GO 0229.500
0165 0229.600
0166 35 0229.700
0167 C IF BOUNDARY CONDITIONS INVOLVE RADIATIVE HEAT TRANSFER, TRANSFER 0229.800
0168 CONTROL TO SUBROUTINE RADIAT WHERE TERMS REQUIRED FOR NEWTON 0229.900
0169 LINEARIZATION ARE COMPUTED. 0230.000
0170 IF(RABND.EQ.1.AND.J.EQ.JX) CALL RADIAT (C4,TRANS,CTOT) 0230.100
0171 CBM(NBU)=CTOT 0230.200
0172 0230.300
0173 C TRANSFER CONTROL TO SUBROUTINE CONNAS WHERE ELEMENTS CONTAINED 0230.400
0174 WITHIN LEFT-HAND SUBMATRIX OF ROW L OF CONDUCTANCE MATRIX (IE. ROW 0230.500
0175 ASSOCIATED WITH NODE L) ARE TRANSFERRED TO CONDUCTANCE VECTOR CX 0230.600
0176 FROM TEMPORARY VECTOR CBM. 0230.700
0177 0230.800
0178 CALL CONNAS (4) 0230.900
0179 RETURN 0231.000
0180 END 0231.100

0001 SUBROUTINE BIGNAB (R,THA,THB,DA,DB,BZA,BZB,HGX,CX,G,MSYN,DTAU) 0231.200
0002 0231.300
0003 C TWO TYPES OF EXCEPTIONAL NODES ARE REQUIRED TO LINK FINITE- 0231.400
0004 DIFFERENCE NETS WITH DIFFERENT RADIAL GRID SPACINGS. ONE OF THESE 0231.500
0005 TRANSITION NODES IS LOCATED IN THE COARSE NET WHILE THE OTHER IS 0231.600
0006 LOCATED IN THE FINE NET. SUBROUTINE BIGNAB (ACRONYM FOR 'BIG 0231.700
0007 RADIAL SPACING') CALCULATES CONDUCTANCES FOR TRANSITION NODES IN 0231.800
0008 THE COARSE NET. 0231.900

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0009 C INTEGER RESIST, TRANS, SYN, OUTRY, TUBE, RADBNB
0010 DOUBLE PRECISION C, GO, CTOT
0011 DIMENSION JSC(10), H(10), C(13), J(9), DT(10), NTC(10), CBN(250),
0012 J(9), Q(470), NNUM(39,25,1), AREA(470), T(470), SAR(39,65), TC(39,65),
0013 HFLZ(45), TRF(470,13)
0014 COMMON /DB1/RESIST, TO /DB2/TUBE, BT /DB4/CBN, NNU, J, JP /DB5/KL, L
0015 /DB6/IX, JX, KX /DB9/I, J, K, NIS, H1 /DB10/Q /DB11/AMTOT, BZTOT, DTOT
0016 /DB12/NUM /DB18/HFLZ, CCOMD /DB19/SYN, OUTRY, COMST /DB21/TC
0017 /DB23/T /DB25/C4, MGEN/DB28/ITER/DB31/TRANS, TWPIN, TFINAL, THCAP, DT,
0018 NTD, NTC /DB35/AREA /DB38/IMPECC /DB41/RADBNB /DB46/SAR /DB51/IA,
0019 IS /DB55/JSC /DB56/HX /DB57/MLIN /DB58/NR, H /DB60/IXX /DB65/JA, JB
0020 /DB78/TCINTS, SLOPS, SLOPD2, NLA, MLC, MLCOM
0021 /DB80/NZFLO, NTFBC, DZBC /DB14/NTRF, TRF
0022
0023 C
0024 HX1=H1
0025 IF(IMPECC.EQ.1) HX1=HXX
0026 HX2=HXX
0027 IF(IMPECC.EQ.1) HX2=H1
0028 KK=K
0029 NMS=3
0030 IF(MSYN.EQ.2) KK=1
0031
0032 C
0033 CALCULATE CONDUCTANCES (C(1), C(2), ..., C(9)) BETWEEN NODE I, J, K
0034 AND NEIGHBOURING NODES ON THE SAME R-THETA PLANE (K).
0035 IN F-D ANALYSES, CONDUCTANCES TO THE LEFT OF THE DIAGONAL IN THE
0036 F-D COEFF. MATRIX ARE STORED IN THE TEMPORARY ARRAY CBW.
0037 CONDUCTANCES TO THE RIGHT OF THE DIAGONAL ARE NOT STORED BECAUSE
0038 THE COEFF. MATRIX IS SYMMETRIC.
0039
0040 C(1)=0.0
0041 C(2)=0.0
0042 DO 10 N=1, NR
0043 IF(I.EQ.(JSC(N)+1).AND.J.EQ.N(N)) GO TO 20
0044 CONTINUE
0045 GO TO 25
0046 CONTINUE
0047 JP(1)=NUM(I, J, KK)-NUM(I-1, J, KK)
0048 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L-JP(1)))
0049 C(2)=CX*BZTOT*(DB/4.0)+(DA/2.0)/(R*THA)
0050 IF(MSYN.EQ.2) GO TO 26
0051 CBW(NNU-JP(1))=-C(2)
0052 IF(J.EQ.2) GO TO 25
0053 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L-1))
0054 C(1)=CX*BZTOT*(R-DA/2.0)/DA
0055 JP(2)=1
0056 IF(MSYN.EQ.2.AND.TUBE.NE.19) GO TO 25
0057 CBW(NNU-1)=-C(1)
0058
0059 C
0060 CALCULATE THE GAP CONDUCTANCE (C(3)) AT THE GAP OR INTERFACE
0061 BETWEEN INNER AND OUTER CYLINDERS (IF NECESSARY).
0062
0063 C
0064 IF(J.EQ.NIS) GO TO 29
0065 C(3)=0.0
0066 CS=0.0
0067 IF(J.NE.NIS.AND.J.NE.HX) GO TO 30
0068 IF(IMPECC.NE.1) GO TO 28
0069 IF(MSYN.EQ.2) GO TO 28
0070 IF(MLIN.NE.1) GO TO 28
0071 IF(TRANS.NE.1.AND.ITER.EQ.1) GO TO 28
0072 IF(HX1.NE.CCOMD) CALL HX10 (HX1, TRANS)
0073 C(3)=HX1*RSANTOT*BZTOT
0074 CS=RSANTOT*BZTOT
0075 IF(J.EQ.NIS) JP(3)=1
0076 IF(J.EQ.HX.AND.MSYN.EQ.2) JP(3)=-1
0077 IF(MSYN.EQ.2) GO TO 30
0078 IF(J.EQ.NIS) CBW(NNU-1)=-C(3)
0079 IF(MVC.EQ.1) J(1)=NUM(I+1, J, 1)-NUM(I, J, 1)
0080 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L+J(1)))
0081 C(4)=CX*BZTOT*(R*THA)
0082 C(5)=0.0
0083 DO 35 N=1, NR
0084 IF(I.EQ.(JSC(N)+1).AND.J.EQ.N(N)) GO TO 40
0085 CONTINUE
0086 JP(4)=NUM(I, J, KK)-NUM(I-1, J, KK)
0087 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L-JP(4)))
0088 C(5)=CX*BZTOT*(2.0/R*THA)
0089 IF(MSYN.EQ.2) GO TO 40
0090 CBW(NNU-JP(4))=-C(5)
0091
0092 C
0093 C(6)=0.0
0094 C(7)=0.0
0095 IF(J.EQ.HX.OR.J.EQ.IX) GO TO 45
0096 YS=((R+DB/2.0)*THA)*S2.0+((DB/2.0)*S2.0)/(2.0*(R+DB/2.0)*THA)
0097 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L+1))
0098 C(6)=CX*BZTOT*((R+DB/2.0)*(THA/2.0)+THA)-YS/DB
0099 JP(5)=NUM(I, J, KK)-NUM(I-1, J, KK)
0100 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L-JP(5)))
0101 C(7)=(DB/2.0)*BZTOT*(2.0*(R+DB/4.0)*THA)
0102 IF(MSYN.EQ.2) J(2)=1
0103 IF(MSYN.EQ.2) GO TO 45
0104 CBW(NNU-JP(5))=-C(7)
0105
0106 C
0107 C(8)=0.0
0108 C(9)=0.0
0109 IF(J.EQ.NIS.OR.J.EQ.2) GO TO 60
0110 DO 55 N=1, NR
0111 IF(I.EQ.(JSC(N)+1).AND.J.EQ.N(N)) GO TO 60
0112 CONTINUE
0113 Y6=((R-DA/2.0)*THA)*S2.0+((DA/2.0)*S2.0)/(2.0*(R-DA/2.0)*THA)
0114 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L-1))
0115 C(8)=CX*BZTOT*((R-DA/2.0)*(THA/2.0)+THA)-Y6/DA
0116 JP(7)=NUM(I, J, KK)-NUM(I-1, J, KK)
0117 IF(MVC.EQ.1) CX=TCINTS+SLOPD2*(T(L)+T(L-JP(7)))
0118 C(9)=CX*BZTOT*(DA/2.0)/(2.0*(R-DA/4.0)*THA)
0119 JP(6)=1
0120 IF(MSYN.EQ.2) GO TO 60
0121 CBW(NNU-1)=-C(8)
0122 CBW(NNU-JP(7))=-C(9)
0123
0124 C
0125 DA2=DA
0126 DO 45 N=1, NR
0127 IF(I.EQ.(JSC(N)+1).AND.J.EQ.N(N)) DA2=0.0
0128 CONTINUE
0129
0130 C
0131 CALCULATE AREA (ARE1) OF NODAL CELL IN R-THETA PLANE.
0132 ARE1=(R+DB/4.0-DA/4.0)*SANTOT*BZTOT-(((DB/2.0)*S2.0)/(8.0*(R+DB/4.0
0133 *THA))-((DA/2.0)*S2.0)/(8.0*(R-DA/4.0)*THA))
0134 ARE1(L)=ARE1
0135
0136 C
0137 CALCULATE AXIAL CONDUCTANCES C(10) AND C(11).
0138

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0129      C(10)=0.0
0130      IF(K.EQ.1) GO TO 70
0131      IF(MSYN.NE.2) JP(8)=MMUM(I,J,K)-MMUM(I,J,K-1)
0132      IF(MVC.EQ.1) CX=TCINTS+SLOPD28(T(L)+T(L-JP(8)))
0133      C(10)=CX*ARE1/DZA
0134      IF(MSYN.EQ.2) GO TO 70
0135      CBW(MBV-JP(8))=C(10)
0136      C(11)=0.0
0137      IF(K.EQ.KX) GO TO 75
0138      IF(MSYN.NE.2.AND.MVC.EQ.1) JLZ=MMUM(I,J,K+1)-MMUM(I,J,K)
0139      IF(MVC.EQ.1) CX=TCINTS+SLOPD28(T(L)+T(L+JLZ))
0140      C(11)=CX*ARE1/DZB
0141
0142      CALCULATE CONVECTIVE BOUNDARY CONDUCTANCE (C(12)).
0143
0144      75      C(12)=0.0
0145      C4=0.0
0146      IF(J.NE.JX) GO TO 80
0147      C4=RSANTOT*SZTOT
0148      SAR(I,K)=C4
0149      C(12)=C4*HGX2
0150
0151      CALCULATE MODAL HEAT GENERATION RATE (C(13)).
0152
0153      80      C(13)=0.0
0154      IF(J.LE.MX) C(13)=GBARE*SZTOT
0155
0156      SUM CONDUCTANCES TO YIELD DIAGONAL ELEMENT IN CONDUCTANCE MATRIX.
0157
0158      CTOT=C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)+C(9)+C(10)+C(11)+C(12)
0159
0160      IF(TRANS.NE.1) GO TO 84
0161
0162      ADD HEAT STORAGE TERM (HGEN) TO DIAGONAL ELEMENT FOR TRANSIENT
0163      ANALYSIS.
0164
0165      HGEN=THCAPTARE*SZTOT/DTAU
0166      CTOT=CTOT+HGEN
0167      84      GO=C(13)
0168
0169      IF NECESSARY, INCORPORATE AXIAL BOUNDARY CONDITIONS IN A 2-D
0170      ANALYSIS.
0171
0172      IF(NZFLO.EQ.1) CZFLO=CX*AREAL/DZBC+CTOT-CTOT+CZFLO
0173      1      GO=GO+CZFLO*TRF(L,NTRFBC)
0174      IF(J.EQ.2) GO=GO+HFLZ(K)*RRANTOT*SZTOT
0175      TI=TMPIN
0176      IF(MSYN.NE.2) TI=T(L)
0177
0178      ADD PRODUCT OF HEAT STORAGE TERM AND PREVIOUS MODAL TEMP. TO
0179      APPROPRIATE ELEMENT IN RIGHT-HAND SIDE HEAT FLOW VECTOR.
0180
0181      IF(TRANS.EQ.1) GO=GO+HGEN*TI
0182
0183      ADD TERM ARISING FROM CONVECTIVE BOUNDARY CONDITION TO
0184      APPROPRIATE ELEMENT IN RIGHT-HAND SIDE HEAT FLOW VECTOR.
0185
0186      IF(J.EQ.JX) GO=GO+C(12)*TC(I,K)
0187
0188      INCORPORATE BOUNDARY CONDITION FOR TUBE ANALYSIS (IE, TUBE=1)
0189
0190      IF(TUBE.EQ.1.AND.J.EQ.NIS) GO=GO+C(3)*ST+C(3)=0.0
0191
0192      INCORPORATE BOUNDARY CONDITION FOR RESISTANCE ANALYSIS (IE, RESIST
0193      =1)
0194
0195      IF(RESIST.EQ.1.AND.I.EQ.IXX) GO=GO+C(4)*ST+C(4)=0.0
0196
0197      IF SUBSTITUTE/INTEGRATE STAGE OF SYNTHESIS ANALYSIS HAS BEEN
0198      REACHED (INDICATED BY MSYN=2), TRANSFER CONTROL TO SUBROUTINE
0199      SUBST1, WHERE THE SUBSTITUTIONS AND INTEGRATIONS ARE PERFORMED.
0200
0201      IF(MSYN=2) 85,75,85
0202      75      CALL SUBST1(CS,MVS,C,GO,JP,JL,L,CTOT,I,11,10)
0203      RETURN
0204      85      GO=GO
0205
0206      IF BOUNDARY CONDITIONS INVOLVE RADIATIVE HEAT TRANSFER, TRANSFER
0207      CONTROL TO SUBROUTINE RADIAT WHERE TERMS REQUIRED FOR NEWTON
0208      LINEARIZATION ARE COMPUTED.
0209
0210      IF(RADWB.EQ.1.AND.J.EQ.JX) CALL RADIAT(C4,TRANS,CTOT)
0211      CBW(MBV)=CTOT
0212
0213      TRANSFER CONTROL TO SUBROUTINE CONWAS WHERE ELEMENTS CONTAINED
0214      WITHIN LEFT-HAND BANDWIDTH OF ROW L OF CONDUCTANCE MATRIX (IE, ROW
0215      ASSOCIATED WITH MODE L) ARE TRANSFERRED TO CONDUCTANCE VECTOR CX
0216      FROM TEMPORARY VECTOR CBW.
0217
0218      CALL CONWAS(8)
0219      RETURN
0220      END

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0001 SUBROUTINE SPWRAB (R,THA,THB,DA,DB,DZA,DZB,HGX,CX,G,MSYN,DTAU)

0002 TWO TYPES OF EXCEPTIONAL NODES ARE REQUIRED TO LINK FINITE-
0003 DIFFERENCE NETS WITH DIFFERENT RADIAL GRID SPACINGS. ONE OF THESE
0004 TRANSITION NODES IS LOCATED IN THE COARSE NET WHILE THE OTHER IS
0005 LOCATED IN THE FINE NET. SUBROUTINE SPWRAB (ACRONYM FOR 'SPALL
0006 RADIAL SPACING') CALCULATES CONDUCTANCES FOR TRANSITION NODES IN
0007 THE FINE NET.

0008
0009
0010
0011 INTEGER TUBE,RESIST,TRANS,SYN,OUTRY
0012 DOUBLE PRECISION C,GO,CTOT
0013 DIMENSION BT(10),NTC(10),CBW(250),JP(9),G(470),C(13),MMUM(39,25,1)
0014 1 AREA(470),T(470),JL(9),TC(39,45),TRF(470,13)
0015 COMMON /BB1/RESIST,TO /BB4/CBW,MMUM,JL,JP /BB5/KL,L /BB6/IX,JX,KX
0016 /BB2/TUBE,BT /BB7/I,J,K,NIS,N1 /BB10/B /BB11/AMTOT,BTOT,BTOT
0017 /BB12/MMUM /BB19/SYN,OUTRY,CONST /BB21/TC /BB23/T /BB25/C4,HGEN
0018 /BB31/TRANS,TMPIN,TFINAL,THCAP,DT,NTDT,NTC /BB35/AREA /BB41/
0019 RADWB /BB51/IA,IB /BB54/MX /BB60/IXX /BB65/IA,JB /BB70/TCINTS,
0020 SLOPD,SLOPD2,NJL,MVC,MVCON /BB80/NZFLO,NTRFBC,DZBC
0021 /BB14/NTRF,TRF

0022 C4=0.0
0023 CS=0.0

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0024      NWS=1                                0253.500
0025      KK=K                                0253.600
0026      IF(NSYN.EQ.2) KK=1                 0253.700
0027      IF(J.EQ.NX.OR.J.EQ.NIS.OR.J.EQ.JX) GO TO 10 0253.800
0028      GO TO 20                             0253.900
0029      WRITE (100,991) I,J                 0254.000
0030      991 FORMAT (1X, 'CHECK GRID - MODE TYPE 2 ENCOUNTERED AT FUEL-SHEATH 0254.100
0031      OR SHEATH-COOLANT INTERFACE',//, 'I =',I5,X, 'J =',I5) 0254.200
0032      STOP                                  0254.300
0033      GO TO 30                             0254.400
0034      C                                     0254.500
0035      CALCULATE CONDUCTANCES C(1),C(2),...,C(6) BETWEEN NODE I,J,K 0254.600
0036      AND NEIGHBOURING NODES ON THE SAME R-THETA PLANE (K) 0254.700
0037      C                                     0254.800
0038      20 JP(1)=1                           0254.900
0039      IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L-1)) 0257.000
0040      C(1)=CK*BTOT*(R-DA/2.0)*SANTOT/DA 0257.100
0041      IF(MJL.EQ.1) JL(1)=NMUM(I,J+JB,1)-NMUM(I,J,1) 0257.200
0042      IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L+JL(1))) 0257.300
0043      C(2)=CK*BTOT*(R+DB/2.0)*SANTOT/DB 0257.400
0044      JP(2)=NMUM(I,J,1)-NMUM(I-1,J,1) 0257.500
0045      IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L-JP(2))) 0257.600
0046      C(3)=CK*BTOT*(R+DTOT/(R*THA)) 0257.700
0047      C                                     0257.800
0048      CHECK FOR OBTUSE VERTEX ANGLE (NOT PERMITTED). 0257.900
0049      C                                     0258.000
0050      DLDA=(R-DA/2.0)*THB 0258.100
0051      DLDB=(R+DB/2.0)*THB 0258.200
0052      IF(DA*BT.DLDA*OR.DB*DT.DLDB) WRITE (100,992) I,J ; STOP 0258.300
0053      992 FORMAT (1X, '***** CHECK GRID - OBTUSE VERTEX ANGLE ENCOUNTERED AT 0258.400
0054      TRANSITION NODE I=',I3, ' J=',I3, ' IN SUBROUTINE SARRAD *****') 0258.500
0055      IF(MJL.EQ.1) JL(2)=NMUM(I+1,J-JA,1)-NMUM(I,J,1) 0258.600
0056      JL(3)=NMUM(I+1,J+JB,1)-NMUM(I,J,1) 0258.700
0057      IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L+JL(2))) 0258.800
0058      C(4)=CK*BTOT*(R+DA/2.0)*DLDA 0258.900
0059      IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L+JL(3))) 0259.000
0060      C(5)=CK*BTOT*(R+DB/2.0)*DLDB 0259.100
0061      C                                     0259.200
0062      CALCULATE AREA (ARE1) OF NODAL CELL IN R-THETA PLANE. 0259.300
0063      C                                     0259.400
0064      ARE1=SANTOT*BTOT+((DB*BT.0)/(4.0*BT*THB)) 0259.500
0065      AREA(L)=ARE1 0259.600
0066      IF(NSYN.EQ.2) GO TO 35 0259.700
0067      C                                     0259.800
0068      PLACE CONDUCTANCES TO THE LEFT OF THE DIAGONAL IN THE TEMPORARY 0259.900
0069      ARRAY CBW. SINCE THE CONDUCTANCE MATRIX IS SYMMETRIC, CONDUCTANCES 0260.000
0070      TO THE RIGHT OF THE DIAGONAL ARE NOT STORED. 0260.100
0071      C                                     0260.200
0072      CBW(NBW-1)=C(1) 0260.300
0073      CBW(NBW-JP(2))=-C(3) 0260.400
0074      C                                     0260.500
0075      CALCULATE AXIAL CONDUCTANCES C(6) AND C(7). 0260.600
0076      C                                     0260.700
0077      35 C(6)=0.0 0260.800
0078      IF(K.EQ.1) GO TO 40 0260.900
0079      IF(NSYN.NE.2) JP(3)=NMUM(I,J,K)-NMUM(I,J,K-1) 0261.000
0080      IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L-JP(3))) 0261.100
0081      C(6)=CK*ARE1/DZA 0261.200
0082      IF(NSYN.EQ.2) GO TO 40 0261.300
0083      CBW(NBW-JP(3))=-C(6) 0261.400
0084      C(7)=0.0 0261.500
0085      IF(K.EQ.KX) GO TO 45 0261.600
0086      IF(NSYN.NE.2.AND.MVC.EQ.1) JLZ=NMUM(I,J,K+1)-NMUM(I,J,K) ; 0261.700
0087      CK=TCINTS+SLOPD2*(T(L)+T(L+JLZ)) 0261.800
0088      C(7)=CK*ARE1/DZB 0261.900
0089      C                                     0262.000
0090      CALCULATE NODAL HEAT GENERATION RATE C(8). 0262.100
0091      C                                     0262.200
0092      45 C(8)=0.0 0262.300
0093      IF(J.LE.PX) C(8)=GSARE1*BTOT 0262.400
0094      C                                     0262.500
0095      SUM CONDUCTANCES TO YIELD DIAGONAL ELEMENT IN CONDUCTANCE MATRIX. 0262.600
0096      C                                     0262.700
0097      CTOT=C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7) 0262.800
0098      IF(TRANS.NE.1) GO TO 50 0262.900
0099      ADD HEAT STORAGE TERM (HGEN) TO DIAGONAL ELEMENT FOR TRANSIENT 0263.000
0100      ANALYSIS. 0263.100
0101      HGEN=THCA*ARE1*BTOT/STAU 0263.200
0102      CTOT=CTOT+HGEN 0263.300
0103      GO=C(8) 0263.400
0104      C                                     0263.500
0105      IF NECESSARY, INCORPORATE AXIAL BOUNDARY CONDITIONS IN 2-D 0263.600
0106      ANALYSIS. 0263.700
0107      C                                     0263.800
0108      IF(NZFLO.EQ.1) CZFLO=CK*ARE1/L/BZC ; CTOT=CTOT+CZFLO 0263.900
0109      ; GO=GO+CZFLO*STRF(L,NTFBC) 0264.000
0110      TI=THPIN 0264.100
0111      IF(NSYN.NE.2) TI=T(L) 0264.200
0112      C                                     0264.300
0113      ADD PRODUCT OF HEAT STORAGE TERM AND PREVIOUS NODAL TEMP. TO 0264.400
0114      APPROPRIATE ELEMENT IN RIGHT-HAND SIDE HEAT FLOW VECTOR. 0264.500
0115      IF(TRANS.EQ.1) GO=GO+HGEN*TI 0264.600
0116      C                                     0264.700
0117      INCORPORATE BOUNDARY CONDITION FOR RESISTANCE ANALYSIS (IE. RESIST 0264.800
0118      =1) 0264.900
0119      IF(RESIST.EQ.1.AND.I.EQ.IXX) GO=GO+(C(4)+C(5))*STOIC(4)=0.01C(5)=0. 0265.000
0120      C                                     0265.100
0121      IF SUBSTITUTE/INTEGRATE STAGE OF SYNTHESIS ANALYSIS HAS BEEN 0265.200
0122      REACHED (INDICATED BY NSYN=2) ; TRANSFER CONTROL TO SUBROUTINE 0265.300
0123      SUBST1, WHERE THE SUBSTITUTIONS AND INTEGRATIONS ARE PERFORMED. 0265.400
0124      C                                     0265.500
0125      IF(NSYN=2) 55,40,55 0265.600
0126      CALL SUBST1 (CS,NWS,C,GO,JP,JL,L,CTOT,2,7,6) 0265.700
0127      RETURN 0265.800
0128      55 G(L)=GO 0265.900
0129      CBW(NBW)=CTOT 0266.000
0130      C                                     0266.100
0131      TRANSFER CONTROL TO SUBROUTINE CONWAS WHERE ELEMENTS CONTAINED 0266.200
0132      WITHIN LEFT-HAND BANDWIDTH OF ROW L OF CONDUCTANCE MATRIX (IE. ROW 0266.300
0133      ASSOCIATED WITH NODE L) ARE TRANSFERRED TO CONDUCTANCE VECTOR CX 0266.400
0134      FROM TEMPORARY VECTOR CBW. 0266.500
0135      C                                     0266.600
0136      CALL CONWAS (3) 0266.700
0137      RETURN 0266.800
0138      30 END 0266.900
0139      C                                     0267.000
0140      0267.100
0141      0267.200
0142      0267.300

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0001      SUBROUTINE SPACIR (R,THA,THB,DA,DB,DZA,DZB,CK,G,MSYN,BTAU)
0002      C
0003      C TWO TYPES OF EXCEPTIONAL NODES ARE REQUIRED TO LINK FINITE-
0004      C DIFFERENCE NETS WITH DIFFERENT CIRCUMFERENTIAL GRID SPACINGS. ONE
0005      C OF THESE TRANSITION NODES IS LOCATED IN THE COARSE NET WHILE THE
0006      C OTHER IS LOCATED IN THE FINE NET. SUBROUTINE SPACIR CALCULATES
0007      C CONDUCTANCES FOR TRANSITION NODES IN THE FINE NET.
0008      C
0009      C INTEGER RESIST,TRANS,SYN,OUTRY
0010      C DOUBLE PRECISION C,CG,CTOT
0011      C DIMENSION C(13),JL(9),DT(10),NTC(10),CBW(250),JP(9),Q(470),NRUM(39
0012      C ,ZS(1),AREA(470),T(470),TRF(470,13)
0013      C COMMON /DB31/TRANS,TMPIN,TFINAL,THCAP,BT,NTDT,NTC,DB1/RESIST,TO,DB
0014      C 4/CBW,NBU,JL,JP,DB5/KL,L,DB9/I,J,K,NIS,H1,DB10/Q,DB11/AMTOT,DZTOT
0015      C ,DTOT /DB12/NRUM /DB1/IA,IB /DB56/MX /DB60/IXX/DB25/C4,HGEN /DB35
0016      C /AREA,DB23/T,DB19/SYN,OUTRY,CONST
0017      C -DB78/TCINTS,SLOPS,SLOPD2,NJL,MVC,MVCON
0018      C /DB80/NZFLO,NTFBC,DZBC /DB14/NTRF,TRF
0019      C C4=0.0
0020      C C5=0.0
0021      C HNS=1
0022      C KK=1
0023      C IF(MSYN.EQ.2) KK=1
0024      C
0025      C CALCULATE CONDUCTANCES (C(1),C(2),...,C(6)) BETWEEN NODE I,J,K
0026      C AND NEIGHBOURING NODES ON THE SAME R-THETA PLANE (K).
0027      C IN F-D ANALYSES, CONDUCTANCES TO THE LEFT OF THE DIAGONAL IN THE
0028      C F-D COEFF. MATRIX ARE STORED IN THE TEMPORARY ARRAY CBW.
0029      C CONDUCTANCES TO THE RIGHT OF THE DIAGONAL ARE NOT STORED BECAUSE
0030      C THE COEFF. MATRIX IS SYMMETRIC.
0031      C
0032      C
0033      C
0034      C JP(1)=NRUM(I,J,KK)-NRUM(I-1A,J,KK)
0035      C IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L-JP(1)))
0036      C C(1)=-CK*DTOT*SBZTOT/(R*THA)
0037      C IF(NJL.EQ.1) JL(1)=NRUM(I+1B,J,1)-NRUM(I,J,1)
0038      C IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L+JL(1)))
0039      C C(2)=-CK*SBZTOT*SBZTOT/(R*THB)
0040      C IF(MSYN.EQ.2) GO TO 10
0041      C CBW(NBU-JP(1))=-C(1)
0042      C C(3)=0.0
0043      C IF(JL.EQ.KX) GO TO 15
0044      C IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L+1))
0045      C C(3)=-CK*SBZTOT*(R+DB/2.0)*AMTOT/DB
0046      C IF(MSYN.EQ.2) JL(2)=1
0047      C DLTHA=(R-DA/2.0)*STHA
0048      C DLTHB=(R-DA/2.0)*STHB
0049      C IF(DLTHA.GT.DA.OR.DLTHB.GT.DA) WRITE (100,880) I,J I STOP
0050      C 880 FORMAT (1X,'***** CHECK GRID - OBTUSE VERTEX ANGLE ENCOUNTERED AT
0051      C TRANSITION NODE I=',I3,' J=',I3,' IN SUBROUTINE SPACIR *****')
0052      C JP(2)=NRUM(I,J,KK)-NRUM(I-1A,J-1,KK)
0053      C IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L-JP(2)))
0054      C C(4)=-CK*SBZTOT*DLTHA/(2.0*DBA)
0055      C IF(NJL.EQ.1) JL(3)=NRUM(I+1B,J-1,1)-NRUM(I,J,1)
0056      C IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L+JL(3)))
0057      C C(5)=-CK*SBZTOT*DLTHB/(2.0*DBA)
0058      C IF(MSYN.EQ.2) GO TO 20
0059      C CBW(NBU-JP(2))=-C(4)
0060      C C(6)=0.0
0061      C IF(JL.EQ.KX) GO TO 22
0062      C C(6)=-HNS*AMTOT*SBZTOT
0063      C IF(MSYN.EQ.2) JL(4)=1
0064      C
0065      C CALCULATE AREA (ARE1) OF NODAL CELL IN R-THETA PLANE.
0066      C
0067      C 22 ARE1=RSANTOT*SBZTOT+((R-DA/2.0)*STHA)*S3.0)/(4.0*DBA)
0068      C AREA(L)=ARE1
0069      C
0070      C CALCULATE AXIAL CONDUCTANCES C(7) AND C(8).
0071      C
0072      C C(7)=0.0
0073      C IF(K.EQ.1) GO TO 25
0074      C IF(MSYN.EQ.2) JP(3)=NRUM(I,J,K)-NRUM(I,J,K-1)
0075      C IF(MVC.EQ.1) CK=TCINTS+SLOPD2*(T(L)+T(L-JP(3)))
0076      C C(7)=-CK*ARE1/DBZ
0077      C IF(MSYN.EQ.2) GO TO 25
0078      C CBW(NBU-JP(3))=-C(7)
0079      C C(8)=0.0
0080      C IF(K.EQ.KX) GO TO 30
0081      C IF(MSYN.EQ.2 AND MVC.EQ.1) JL(2)=NRUM(I,J,K+1)-NRUM(I,J,K)
0082      C C(8)=-CK*ARE1/DBZ
0083      C
0084      C
0085      C CALCULATE NODAL HEAT GENERATION RATE (C(9)).
0086      C
0087      C 30 C(9)=0.0
0088      C IF(JL.EQ.KX) C(9)=-GBARE*SBZTOT
0089      C
0090      C SUM CONDUCTANCES TO YIELD DIAGONAL ELEMENT IN CONDUCTANCE MATRIX.
0091      C
0092      C CTOT=C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)
0093      C IF(TRANS.ME.1) GO TO 35
0094      C
0095      C ADD HEAT STORAGE TERM (HGEN) TO DIAGONAL ELEMENT FOR TRANSIENT
0096      C ANALYSIS.
0097      C
0098      C HGEN=THCAP*ARE1*SBZTOT/BTAU
0099      C CTOT=CTOT+HGEN
0100      C CG=C(9)
0101      C
0102      C IF NECESSARY, INCORPORATE AXIAL BOUNDARY CONDITIONS IN A 2-D
0103      C ANALYSIS.
0104      C
0105      C IF(NZFLO.EQ.1) CZFLO=CK*ARE1/L/DBZC /CTOT=CTOT+CZFLO
0106      C /DBZC=CTFLO*GTRF(L,NTFBC)
0107      C TI=TMPIN
0108      C IF(MSYN.ME.2) TI=T(L)
0109      C
0110      C ADD PRODUCT OF HEAT STORAGE TERM AND PREVIOUS NODAL TEMP. TO
0111      C APPROPRIATE ELEMENT IN RIGHT-HAND SIDE HEAT FLOW VECTOR.
0112      C
0113      C IF(TRANS.EQ.1) QD=CG+HGEN*TI
0114      C
0115      C INCORPORATE BOUNDARY CONDITION FOR RESISTANCE ANALYSIS (IE. RESIST

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0116 C =1) 0278.900
0117 C 0279.000
0118 C IF (RESIST.EQ.1.AND.I.EQ.IXX) Q0=Q0+(C(2)+C(5))*TOIC(2)=0.1C(5)=0.0 0279.100
0119 C 0279.200
0120 C IF SUBSTITUTE/INTEGRATE STAGE OF SYNTHESIS ANALYSIS HAS BEEN 0279.300
0121 C REACHED (INDICATED BY NSYN=2), TRANSFER CONTROL TO SUBROUTINE 0279.400
0122 C SUBST1, WHERE THE SUBSTITUTIONS AND INTEGRATIONS ARE PERFORMED. 0279.500
0123 C 0279.600
0124 C IF (NSYN=2) 40,45,40 0279.700
0125 45 CALL SUBST1 (CS,MNS,C,Q0,JP,J,L,CTOT,3,8,7) 0279.800
0126 C RETURN 0279.900
0127 40 C(L)=Q0 0280.000
0128 C CM(CM)=CTOT 0280.100
0129 C 0280.200
0130 C TRANSFER CONTROL TO SUBROUTINE CONNAG WHERE ELEMENTS CONTAINED 0280.300
0131 C WITHIN LEFT-HAND BANDWIDTH OF ROW L OF CONDUCTANCE MATRIX (IE. ROW 0280.400
0132 C ASSOCIATED WITH NODE L) ARE TRANSFERRED TO CONDUCTANCE VECTOR CX 0280.500
0133 C FROM TEMPORARY VECTOR CM. 0280.600
0134 C 0280.700
0135 C CALL CONNAG (3) 0280.800
0136 C RETURN 0280.900
0137 C END 0281.000

0001 SUBROUTINE BIGCIR (R,THA,THB,DA,DB,DZA,DZB,CX,G,NSYN,DTAU) 0281.100
0002 C 0281.200
0003 C TWO TYPES OF EXCEPTIONAL NODES ARE REQUIRED TO LINK FINITE- 0281.300
0004 C DIFFERENCE NETS WITH DIFFERENT CIRCUMFERENTIAL GRID SPACINGS. ONE 0281.400
0005 C OF THESE TRANSITION NODES IS LOCATED IN THE COARSE NET WHILE THE 0281.500
0006 C OTHER IS LOCATED IN THE FINE NET. SUBROUTINE BIGCIR CALCULATES 0281.600
0007 C CONDUCTANCES FOR TRANSITION NODES IN THE COARSE NET. 0281.700
0008 C 0281.800
0009 C INTEGER RESIST,TRANS,SYN,OUTRY,TUBE 0281.900
0010 C DOUBLE PRECISION C,Q0,CTOT 0282.000
0011 C DIMENSION C(13),J(9),BT(10),NFC(10),CM(250),JP(9),Q(470),NRUM(39 0282.100
0012 C ,ZS(1),AREA(470),T(470),ISC(10),IN(10),TRF(470,13) 0282.200
0013 C COMMON /DB31/TRANS,TRFIN,TFINAL,THCAP,BT,NTBT,NTC,DB1/RESIST,TO,DB 0282.300
0014 C ,4/CM,NRM,J,JP,DB5/XL,L/DB9/I,J,K,MIS,H1/DB10/G/DB11/ANTOT,BZTOT 0282.400
0015 C ,BTOT/DB12/NRM/DB13/IA,IB/DB14/MX/DB15/IX/DB16/CA,NRM/DB17 0282.500
0016 C /AREA/DB23/T/DB19/SYN,OUTRY,CONST/DB71/ISC,IN,NI 0282.600
0017 C COMMON /DB74/IC,IB/DB2/TUBE,BT/DB4/IX,JX,KX 0282.700
0018 C /DB78/TCINTS,SLOP5,SLOP2,NJL,NVC,NVCON 0282.800
0019 C /DB80/MZFLQ,NTRFC,BZDC/DB14/NTRF,TRF 0282.900
0020 C4=0.0 0283.000
0021 C5=0.0 0283.100
0022 C MNS=1 0283.200
0023 C KX=K 0283.300
0024 C IF (NSYN.EQ.2) KX=1 0283.400
0025 C 0283.500
0026 C CALCULATE CONDUCTANCES C(1),C(2),...,C(9) BETWEEN NODE I,J,K 0283.600
0027 C AND NEIGHBOURING NODES ON THE SAME R-THETA PLANE (K). 0283.700
0028 C IN F-B ANALYSES, CONDUCTANCES TO THE LEFT OF THE DIAGONAL IN THE 0283.800
0029 C F-B COEFF. MATRIX ARE STORED IN THE TEMPORARY ARRAY CM. 0283.900
0030 C CONDUCTANCES TO THE RIGHT OF THE DIAGONAL ARE NOT STORED BECAUSE 0284.000
0031 C THE COEFF. MATRIX IS SYMMETRIC. 0284.100
0032 C 0284.200
0033 C 0284.300
0034 C(1)=0.0 0284.400
0035 C(2)=0.0 0284.500
0036 IF (J.EQ.ISC(11)).AND.I.EQ.IN(11)) GO TO 15 0284.600
0037 CONTINUE 0284.700
0038 GO TO 20 0284.800
0039 IF (I.EQ.IX) GO TO 17 0284.900
0040 IF (N.J.EQ.1) J(1)=NRUM(I+1),J(1)=NRUM(I,J,1) 0285.000
0041 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L+J(1))) 0285.100
0042 C(1)=CX*BZTOT/STOT/(R*THB) 0285.200
0043 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L+1)) 0285.300
0044 C(2)=CX*BZTOT*(R+DB/2.0)*(THA/4.0)+(THB/2.0)/DB 0285.400
0045 IF (NSYN.EQ.2) J(2)=1 0285.500
0046 IF (J.EQ.MIS) GO TO 25 0285.600
0047 C(3)=MIS*ANTOT*BZTOT 0285.700
0048 JP(1)=1 0285.800
0049 IF (NSYN.EQ.2) GO TO 25 0285.900
0050 CM(NRM-1)=C(3) 0286.000
0051 C(4)=0.0 0286.100
0052 IF (J.EQ.MIS) GO TO 30 0286.200
0053 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L-1)) 0286.300
0054 C(4)=CX*BZTOT*(R-DA/2.0)*ANTOT/DA 0286.400
0055 JP(2)=1 0286.500
0056 IF (NSYN.EQ.2) GO TO 30 0286.600
0057 CM(NRM-1)=C(4) 0286.700
0058 C(5)=0.0 0286.800
0059 IF (I.EQ.1) GO TO 35 0286.900
0060 JP(3)=NRUM(I,J,KX)-NRUM(I-IC,J+1,KX) 0287.000
0061 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L-JP(3))) 0287.100
0062 C(5)=CX*BZTOT*(R+DB/2.0)*(THA/2.0)/(2.0*DB) 0287.200
0063 X5=((DBS2.0)+((R*THB/2.0)*S2.0))/(2.0*DB) 0287.300
0064 JP(4)=NRUM(I,J,KX)-NRUM(I-IA,J,KX) 0287.400
0065 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L-JP(4))) 0287.500
0066 C(6)=CX*BZTOT*(DB+(DA/2.0)-X5)/(R*THA) 0287.600
0067 IF (NSYN.EQ.2) GO TO 35 0287.700
0068 CM(NRM-JP(3))=C(5) 0287.800
0069 CM(NRM-JP(4))=C(6) 0287.900
0070 C(7)=0.0 0288.000
0071 C(8)=0.0 0288.100
0072 C(9)=0.0 0288.200
0073 DO 40 I=1,NI 0288.300
0074 IF (J.EQ.ISC(11)).AND.I.EQ.IN(11)) GO TO 45 0288.400
0075 CONTINUE 0288.500
0076 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L+1)) 0288.600
0077 C(7)=CX*BZTOT*(R+DB/2.0)*ANTOT/(2.0*DB) 0288.700
0078 IF (NSYN.EQ.2) J(3)=1 0288.800
0079 IF (I.EQ.1) GO TO 45 0288.900
0080 X4=((DBS2.0)+((R*THB/2.0)*S2.0))/(2.0*DB) 0289.000
0081 IF (N.J.EQ.1) J(4)=NRUM(I+1B,J,1)-NRUM(I,J,1) J(5)= 0289.100
0082 NRUM(I+1B,J+1,1)-NRUM(I,J,1) 0289.200
0083 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L+J(4))) 0289.300
0084 C(8)=CX*BZTOT*(DB+(DA/2.0)-X4)/(R*THB) 0289.400
0085 IF (NVC.EQ.1) CX=TCINTS+SLOP2*(T(L)+T(L+J(5))) 0289.500
0086 C(9)=CX*BZTOT*(R+DB/2.0)*(THB/2.0)/(2.0*DB) 0289.600
0087 DA3=(R+DB/2.0)*THB 0289.700
0088 DO 50 I=1,NI 0289.800
0089 IF (J.EQ.ISC(11)).AND.I.EQ.IN(11)) DA3=0.0 0289.900
0090 CONTINUE 0290.000
0091 50 0290.100
0092 0290.200
0093 0290.300

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0094 C
0095 C CALCULATE AREA (ARE1) OF NODAL CELL IN R-THETA PLANE.
0096 C
0097 ARE1=RIANTOTSDTOT-(((DAJ/2.0)*83.0)/(8.0888))-(((R+DB/2.0)*THA/
0098 2.0)*83.0)/(8.0888))
0099 AREA(L)=ARE1
0100 C
0101 C CALCULATE AXIAL CONDUCTANCES C(10) AND C(11).
0102 C
0103 C(10)=0.0
0104 IF(K.EQ.1) GO TO 55
0105 IF(MSYN.NE.2) JP(5)=NRUM(I,J,K)-NRUM(I,J,K-1)
0106 IF(MVC.EQ.1) CX=TCINTS+SLOPD28(T(L)+T(L-JP(5)))
0107 C(10)=CXARE1/DZA
0108 IF(MSYN.EQ.2) GO TO 55
0109 CBW(NBW-JP(5))=C(10)
0110 55 C(11)=0.0
0111 IF(K.EQ.KX) GO TO 60
0112 IF(MSYN.NE.2.AND.MVC.EQ.1) JLZ=NRUM(I,J,K+1)-NRUM(I,J,K)
0113 CX=TCINTS+SLOPD28(T(L)+T(L+JLZ))
0114 C(11)=CXARE1/DZB
0115 C
0116 C CALCULATE NODAL HEAT GENERATION RATE (C(12)).
0117 C
0118 C(12)=0.0
0119 IF(J.LE.MX) C(12)=BSARE1SDTOT
0120 C
0121 C SUM CONDUCTANCES TO YIELD DIAGONAL ELEMENT IN CONDUCTANCE MATRIX.
0122 C
0123 CTOT=C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)+C(9)+C(10)+C(11)
0124 IF(TRANS.NE.1) GO TO 65
0125 C
0126 C ADD HEAT STORAGE TERM (HGEN) TO DIAGONAL ELEMENT FOR TRANSIENT
0127 ANALYSIS.
0128 C
0129 HGEN=THCAPSARE1SDTOT/DTAU
0130 CTOT=CTOT+HGEN
0131 65 C(12)=C(12)
0132 C
0133 IF NECESSARY, INCORPORATE AXIAL BOUNDARY CONDITIONS IN A 2-B
0134 ANALYSIS.
0135 C
0136 IF(KZFLO.EQ.1) CZFLO=CXARE1(L)/BZIC+CTOT-CTOT+CZFLO
0137 1 GO=GO+CZFLOSTRF(L,NTFBC)
0138 TI=TWPIN
0139 IF(MSYN.NE.2) TI=T(L)
0140 C
0141 C ADD PRODUCT OF HEAT STORAGE TERM AND PREVIOUS NODAL TEMP. TO
0142 APPROPRIATE ELEMENT IN RIGHT-HAND SIDE HEAT FLOW VECTOR.
0143 C
0144 IF(TRANS.EQ.1) GO=GO+HGEN*TI
0145 C
0146 INCORPORATE BOUNDARY CONDITION FOR TUBE ANALYSIS (IE. TUBE=1)
0147 C
0148 IF(TUBE.EQ.1.AND.J.EQ.NIS) GO=GO+C(3)*BT+C(3)=0.0
0149 C
0150 INCORPORATE BOUNDARY CONDITION FOR RESISTANCE ANALYSIS (IE. RESIST
0151 =1)
0152 C
0153 IF(RESIST.EQ.1.AND.I.EQ.IXX) GO=GO+C(1)*TO+C(1)=0.0
0154 C
0155 IF SUBSTITUTE/INTEGRATE STAGE OF SYNTHESIS ANALYSIS HAS BEEN
0156 REACHED (INDICATED BY MSYN=2), TRANSFER CONTROL TO SUBROUTINE
0157 SUBST1, WHERE THE SUBSTITUTIONS AND INTEGRATIONS ARE PERFORMED.
0158 C
0159 IF(MSYN=2) 70,75,70
0160 75 CALL SUBST1(CS,NBS,C,GO,JP,JL,CTOT,4,11,10)
0161 RETURN
0162 70 B(L)=GO
0163 CBW(NBW)=CTOT
0164 C
0165 C TRANSFER CONTROL TO SUBROUTINE CONMAS WHERE ELEMENTS CONTAINED
0166 WITHIN LEFT-HAND BANDWIDTH OF ROW L OF CONDUCTANCE MATRIX (IE. ROW
0167 ASSOCIATED WITH NODE L) ARE TRANSFERRED TO CONDUCTANCE VECTOR CX
0168 FROM TEMPORARY VECTOR CBW.
0169 C
0170 CALL CONMAS(5)
0171 RETURN
0172 END

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0290.400
0290.500
0290.600
0290.700
0290.800
0290.900
0291.000
0291.100
0291.200
0291.300
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0291.500
0291.600
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0296.900
0297.000
0297.100
0297.200
0297.300
0297.400
0297.500
0297.600
0297.700
0297.800
0297.900
0298.000
0298.100
0298.200

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0001 SUBROUTINE CONMAS (JPMAX)
0002 C
0003 C SUBROUTINE CONMAS (ACRONYM FOR 'CONDUCTANCE MATRIX GENERATION')
0004 TRANSFERS ELEMENTS OF THE VECTOR CBW, WHICH CONTAINS ENTRIES
0005 FOUND WITHIN THE LEFT-HAND BANDWIDTH OF A PARTICULAR ROW (L) OF
0006 THE CONDUCTANCE MATRIX, TO A SINGLE-SUBSCRIPTED FORM OF THE Banded
0007 CONDUCTANCE MATRIX (CX). A POINTER VECTOR (NBIA) LOCATES DIAGONAL
0008 ELEMENTS WITHIN CX. THE WITH ELEMENT OF NBIA IDENTIFIES THE
0009 LOCATION OF THE WITH DIAGONAL ELEMENT IN THE ARRAY CX. ELEMENTS
0010 CONTAINED WITHIN THE LEFT-HAND BANDWIDTH OF A PARTICULAR ROW IN
0011 THE CONDUCTANCE MATRIX ARE STORED AT THE END OF CBW RATHER THAN
0012 AT THE BEGINNING (IE. THE DIAGONAL ELEMENT OF THE ROW IS FOUND IN
0013 THE LOCATION CBW(NBIA)).
0014 C
0015 DIMENSION JP(9),CBW(250),CX(7331),NBIA(470),JL(9)
0016 COMMON /B4/CBW,NBW,JL,JP,BBS,KL,L/8015/CX,NBIA
0017 C
0018 C CALCULATE (LEFT-HAND BANDWIDTH - 1) OF ROW L (NBW) BY FINDING THE
0019 LARGEST JP(JJ).
0020 NBW=1
0021 IF(L.EQ.1) NBW=0
0022 DO 10 JJ=1,JPMAX
0023 IF(JP(JJ).GT.NBW) NBW=JP(JJ)
0024 10 CONTINUE
0025 LX=NBW-NBW
0026 LX=0
0027 C
0028 C TRANSFER ELEMENTS IN CBW TO CX.
0029 DO 20 JJ=LX,NBW
0030 LX=LX+1
0031 CX(KL+LX)=CBW(JJ)
0032 20 CONTINUE
0033 KL=KL+LX
0034 C
0035 C STORE POSITION ( IN CX ) OF DIAGONAL ELEMENT OF ROW L AS NBIA(L).
0036 NBIA(L)=KL

```

0298.300
0298.400
0298.500
0298.600
0298.700
0298.800
0298.900
0299.000
0299.100
0299.200
0299.300
0299.400
0299.500
0299.600
0299.700
0299.800
0299.900
0300.000
0300.100
0300.200
0300.300
0300.400
0300.500
0300.600
0300.700
0300.800
0300.900
0301.000
0301.100
0301.200
0301.300
0301.400
0301.500
0301.600
0301.700
0301.800


```

0037      RETURN                                0301.900
0038      END                                    0302.000

0001      SUBROUTINE SUBST1 (CS,MMS,C,QQ,JP,J,L,CTOT,M1,M3,M4) 0302.100
0002      THIS SUBROUTINE PERFORMS SUBSTITUTIONS AND INTEGRATIONS AS 0302.200
0003      REQUIRED IN A SYNTHESIS ANALYSIS. TRIAL FUNCTIONS ARE SUBSTITUTED 0302.300
0004      INTO FINITE-DIFFERENCE ANALOGS OF THE LAPLACIAN OPERATOR. THE 0302.400
0005      RESULTING RESIDUALS ARE THEN MULTIPLIED BY EACH OF THE TRIAL 0302.500
0006      FUNCTIONS (IE, THE TRIAL FUNCTIONS SERVE AS THE WEIGHTING 0302.600
0007      FUNCTIONS IN GALERKIN'S METHOD). THESE WEIGHTED RESIDUALS ARE 0302.700
0008      INTEGRATED (IE, SUMMED) OVER EACH R-THETA PLANE AND STORED IN 0302.800
0009      APPROPRIATE LOCATIONS IN THE MATRICES AL,CB,CRI,RES. THESE 0302.900
0010      MATRICES REPRESENT A SYSTEM OF SIMULTANEOUS EQUATIONS WHICH 0303.000
0011      DEFINES THE MIXING MATRIX. 0303.100
0012      0303.200
0013      0303.300
0014      INTEGER RADBNB,TRAMS 0303.400
0015      DOUBLE PRECISION DIASUM,DSUM,XI1,XI2,XMULT,RES,FD,AL,CRI,CB,C, 0303.500
0016      GO,CTOT,RR 0303.600
0017      DIMENSION NTRI(45),NTRK(45,10),DT(10),NTC(10),HX(39,45), 0303.700
0018      DFLX(1,1),TRAD(39,45),TRF(470,13),C(13),TC(39,45),FD(3), 0303.800
0019      RES(5,45),JP(9),JL(9),AL(240,7),CRI(240,7),CB(240), 0303.900
0020      DIASUM(5,5,45),XI1(4,4),XI2(4,4),XMULT(4,4) 0304.000
0021      ,HFLZ(45) 0304.100
0022      COMMON /DB4/IX,JX,KX /DB9/I,J,K,MIS,M1 /DB13/RES /DB14/NTRF,TRF 0304.200
0023      /DB15/CB,AL,CRI,MBA /DB24/HX /DB25/C4,HGEN /DB29/TMP2 /DB30/ 0304.300
0024      TCINT6,SLOPS,BASC /DB31/TRAMS,TMPIN,TFINML,THCAP,BT,MTOT,NTC 0304.400
0025      /DB32/DIASUM /DB39/MTOL /DB41/RADBNB /DB43/REXCO /DB45/GFLX,TRAD 0304.500
0026      /DB57/MLIN /DB60/IXX /DB68/HOROPT /DB69/NTRI,NTRK /DB73/NBIA 0304.600
0027      /DB78/TCINT6,SLOPS,SLOPD2,NJL,HVC,MVCOM 0304.700
0028      /DB79/XI1,XI2,XMULT /DB18/HFLZ,CCOND 0304.800
0029      0304.900
0030      INITIALIZE THERMAL CONDUCTIVITIES SCOMB AND SCOMB2. THESE VARIABLES 0305.000
0031      ARE ASSIGNED PROPER VALUES LATER IF KIRCHOFF'S TRANSFORMATION IS 0305.100
0032      USED. WHEN KIRCHOFF'S TRANS. IS USED, SCOMB IS THE THERMAL 0305.200
0033      CONDUCTIVITY OF THE MAT'L AT NODE L,K WHILE SCOMB2 IS THE THERMAL 0305.300
0034      COND. OF THE MAT'L AT NODE1 0305.400
0035      1) L+1,K IF J=MX. 0305.500
0036      2) L-1,K IF J=MIS (=MX+1) 0305.600
0037      SCOMB=1.0 0305.700
0038      SCOMB2=1.0 0305.800
0039      0305.900
0040      IF KIRCHOFF'S TRANS. IS BEING USED, THE NORMAL 'TEMPS.' TMPIN AND TMP2 0306.000
0041      ARE ACTUALLY THERMAL CONDUCTIVITY INTEGRALS. THE REAL TEMPS. ARE 0306.100
0042      REQUIRED IN SUBSEQUENT CALCS. SO STORE THE INTEGRALS TMPIN AND 0306.200
0043      TMP2 AS TH11 AND TH12 AND CALCULATE THE REAL TEMPS. FROM THE 0306.300
0044      INTEGRALS. STORE THE REAL TEMPS. AS THPIN AND THP2. 0306.400
0045      TH11=TMPIN 0306.500
0046      TH12=TMP2 0306.600
0047      IF(MVCOM) 26,26,27 0306.700
0048      THPIN=(-TCINT6+SORT(TCINT6*2+.2*OSLOPS*TMPIN))/SLOPS 0306.800
0049      IF(J.EQ.MX.OR.J.EQ.MIS) THP2=(-TCINT6+SORT(TCINT6*2+.2*OSLOPS 0306.900
0050      *TMP2))/SLOPS SCOMB2=TCINT6+SLOPS*THP2 0307.000
0051      0307.100
0052      CALCULATE THE REAL THERMAL COND. SCOMB FROM THE TEMPERATURE THPIN. 0307.200
0053      SCOMB=TCINT6+SLOPS*THPIN 0307.300
0054      0307.400
0055      26 0307.500
0056      MX=MIS-1 0307.600
0057      JPO=0 0307.700
0058      IF(J.EQ.MX) JPO=-1 0307.800
0059      IF(J.EQ.MIS) JPO=1 0307.900
0060      MX1=MX(I,K) 0308.000
0061      0308.100
0062      CALCULATE RIGHT-HAND SIDE RESIDUAL (RR). 0308.200
0063      RR=QQ 0308.300
0064      0308.400
0065      INCLUDE LINEARIZATION TERMS FOR RADIATION AND TEMPERATURE-DEPEND. 0308.500
0066      THERMAL CONDUCTIVITY IN RIGHT-HAND SIDE RESIDUAL (RR) AND DIAGONAL 0308.600
0067      ELEMENT (CTOT). ALSO MODIFY THE PRESSURE-CALAMARIA TUBE GAS GAP 0308.700
0068      CONDUCTANCE (CNMS) AS NECESSARY. 0308.800
0069      IF(RADBNB.EQ.1) RR=RR+GFLX(I,K)*C4-(4.08REXCOC48*(TRAD(I,K)*84)) 0308.900
0070      IF(MVCOM.EQ.1.AND.(J.EQ.MX.OR.J.EQ.MIS)) RR=RR+C(MIS)*S(TMPIN- 0309.000
0071      TMP2-TH11/SCOMB2+TH12/SCOMB) 0309.100
0072      IF(MLIN.EQ.1.AND.HX1.NE.CCOND.AND.(J.EQ.MX.OR.J.EQ.MIS)) RR=RR- 0309.200
0073      4.08REXCOC48*(TMPIN*83)+TH11/SCOMB-(TMP2*83)+TH12/SCOMB2+ 0309.300
0074      REXCOC48*(TMPIN*84.-TMP2*84.)+SLOPS*CNMS*(TMPIN*82.-TMP2*82.) 0309.400
0075      /(2.0*TCINT6)-SLOPS*CNMS*(TMPIN*TH11/SCOMB-TMP2*TH12/SCOMB2) 0309.500
0076      /TCINT6 0309.600
0077      CNMS=C(MMS) 0309.700
0078      IF(MVCOM.EQ.1) CTOT=CTOT+HISC48(1./SCOMB-1.) 0309.800
0079      IF(MVCOM.EQ.1.AND.(J.EQ.MX.OR.J.EQ.MIS)) CTOT=CTOT+CNMS*(1./ 0309.900
0080      SCOMB-1.) / C(MMS) 0310.000
0081      IF(MLIN.EQ.1.AND.HX1.NE.CCOND.AND.(J.EQ.MX.OR.J.EQ.MIS))CTOT=CTOT 0310.100
0082      +4.08REXCOC48*(TMPIN*83)/SCOMB+SLOPS*CNMS*(TMPIN/(SCOMB*TCINT6) 0310.200
0083      + C(MMS)*CNMS)+4.08REXCOC48*(TMP2*83)/SCOMB2+SLOPS*CNMS*(TMP2/ 0310.300
0084      (TCINT6*SCOMB2) 0310.400
0085      IF(RADBNB.EQ.1) CTOT=CTOT+4.08REXCOC48*(TRAD(I,K)*83) 0310.500
0086      IF(TRANS.EQ.1.AND.MVCOM.EQ.1) CTOT=CTOT+HGEN*(1.0/SCOMB-1.0) / 0310.600
0087      RR=RR+HGEN*TH11/SCOMB 0310.700
0088      0310.800
0089      SELECT TRIAL FUNCTIONS SUCCESSIVELY. TRIAL FUNCTIONS ARE 0310.900
0090      DESIGNATED TRF(L,MN) WHERE L IS A NODE NUMBER AND MN IS A TRIAL 0311.000
0091      FUNCTION NUMBER. 0311.100
0092      DO 35 NK=1,NTRF 0311.200
0093      MN=NTRK(K,NK) 0311.300
0094      DSUM=0.0 0311.400
0095      IF(M1-1) 34,37,38 0311.500
0096      0311.600
0097      SUBSTITUTE TRIAL FUNCTION NO. MN INTO R-THETA FINITE-DIFFERENCE 0311.700
0098      EQUATION AND COMPUTE RESIDUAL FB(1). M1 IS A VARIABLE USED TO 0311.800
0099      INDICATE THE TYPE OF NODE UNDER CONSIDERATION (EG. NODE PROCESSED 0311.900
0100      BY SUBROUTINE HORNBO, SUBROUTINE BIORAB, ETC.). 0312.000
0101      34 FB(1)=C(1)*TRF((L-JP(1)),MN)+C(2)*TRF((L+JL(1)),MN)+C(3)*TRF((L-JP 0312.100
0102      (2)),MN)+C(4)*TRF((L+JL(2)),MN)+C(7)*TRF((L-JP(4)),MN)-CTOT*TRF(L, 0312.200
0103      MN) 0312.300
0104      GO TO 37 0312.400
0105      37 FB(1)=C(1)*TRF((L-JP(2)),MN)+C(2)*TRF((L-JP(3)),MN)+C(3)*TRF((L-JP 0312.500
0106      (3)),MN)+C(4)*TRF((L+JL(1)),MN)+C(5)*TRF((L-JP(4)),MN)+C(6)*TRF((L 0312.600
0107      +JL(2)),MN)+C(7)*TRF((L-JP(5)),MN)+C(8)*TRF((L-JP(6)),MN)+C(9)*TRF 0312.700
0108      ((L-JP(7)),MN)-CTOT*TRF(L,MN) 0312.800
0109      GO TO 39 0312.900
0110      38 IF(M1.EQ.2) FB(1)=C(1)*TRF((L-JP(1)),MN)+C(2)*TRF((L+JL(1)),MN)+C( 0313.000
0111      3)*TRF((L-JP(2)),MN)+C(4)*TRF((L+JL(2)),MN)+C(5)*TRF((L+JL(3)),MN) 0313.100
0112      -CTOT*TRF(L,MN) 0313.200
0113      IF(M1.EQ.3) FB(1)=C(1)*TRF((L-JP(1)),MN)+C(2)*TRF((L+JL(1)),MN)+C( 0313.300

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0114      $ -CTOTSTRF(L,MN)
0115      IF(M1.EQ.4) FD(1)=C(1)*STRF((L+JL(1)),MN)+C(2)*STRF((L+JL(2)),MN)+C(
0116      $ 3)*STRF((L+JP(1)),MN)+C(4)*STRF((L+JP(2)),MN)+C(5)*STRF((L+JP(3)),MN)
0117      $ +C(6)*STRF((L+JP(4)),MN)+C(7)*STRF((L+JL(3)),MN)+C(8)*STRF((L+JL(4)),
0118      $ MN)+C(9)*STRF((L+JL(5)),MN)-CTOTSTRF(L,MN)
0119
0120      C CALCULATE TERM (DSUM) WHICH IS USED TO MODIFY MATRICES AL, CB AND
0121      CRI IN TRANSIENT AND NON-LINEAR ANALYSES, THEREBY ELIMINATING
0122      NEED FOR COMPLETE RECOMPUTATION OF THESE MATRICES IN EACH
0123      ITERATION OR TIME STEP.
0124      IF(TRANS.EQ.1) DSUM=-HGENSTRF(L,MN)/SCOMB
0125      IF(RADBNB.EQ.1) DSUM=DSUM-4.0*REXCOC48*(TRAB(1,K)*83)*STRF(L,MN)
0126      IF(MLTN.EQ.1.AND.NX1.NE.CCOND.AND.(J.EQ.NX.OR.J.EQ.NIS))
0127      $ DSUM=DSUM-(4.0*REXCOC
0128      $ SC5*(THPIN*83)/SCOMB+SLOP8CS*HX1*THPIN/(SCOMB*TCINTG))*STRF(L,MN)
0129      $ +(4.0*REXCOC*SC5*(THP2*83)/SCOMB2+SLOP8CS*HX1*THP2/(SCOMB2*TCINTG)
0130      $ )*STRF((L-JP0),MN)
0131      IF(MVCON.EQ.1) DSUM=DSUM-(HSC4/SCOMB)*STRF(L,MN)
0132      IF(MVCON.EQ.1.AND.(J.EQ.NX.OR.J.EQ.NIS)) DSUM=DSUM-(CMNS/SCOMB)*
0133      $ TRF(L,MN)+(CMNS/SCOMB2)*STRF((L-JP0),MN)
0134      NZ1=NDIA+MK
0135
0136      C MULTIPLY FD(1) BY NTRF WEIGHTING FNS. (IE., THE TRIAL FNS. USED AT
0137      C AXIAL NODE K) AND ADD RESULTS TO APPROPRIATE ENTRIES IN AL, CB AND
0138      CRI. THE MATRICES AL, CB AND CRI CONTAIN THE SUBDIAGONAL, DIAGONAL,
0139      C AND SUPERDIAGONAL ENTRIES IN THE SYNTHESIS COEFFICIENT MATRIX
0140      RESPECTIVELY. ALSO MULTIPLY DSUM BY WEIGHTING FNS. AND ADD RESULTS
0141      C TO DIASUM.
0142      DO 31 MK=1,NTRF
0143      $ NMT=NTRK(K,MK)
0144
0145      C MULTIPLY DSUM BY WEIGHTING FUNCTION (TRF(L,MNT)) AND ADD TO
0146      C DIASUM(MN,MNT,K)
0147      IF(TRANS.EQ.1.OR.RADBNB.EQ.1.OR.MLTN.EQ.1) DIASUM(MN,MK,K)=
0148      $ DIASUM(MN,MK,K)+DSUM*STRF(L,MNT)
0149      NC=NDIA+MK
0150      IF(NZ1.EQ.NC) CB(MC)=CB(MC)+FD(1)*STRF(L,MNT)
0151      NBF1=NC-NZ1
0152      IF(NBF1.GT.0) AL(MC,NBF1)=AL(MC,NBF1)+FD(1)*STRF(L,MNT)
0153      IF(NBF1.LT.0) NDF1=-NDF1; CRI(MC,NDF1)=CRI(MC,NDF1)+FD(1)*
0154      $ TRF(L,MNT)
0155      CONTINUE
0156      C(NMS)=CMNS
0157
0158      C CALCULATE WEIGHTED-RESIDUALS FOR AXIAL NODES K-1 AND K+1.
0159      IF(K.EQ.1) GO TO 40
0160      NTR1=NTRI(K-1)
0161
0162      C SELECT TRIAL FNS. USED AT AXIAL NODE K-1 SEQUENTIALLY.
0163      DO 45 NK=1,NTR1
0164      $ NM=NTRK(K-1,NK)
0165
0166      C CALCULATE 'RESIDUAL' FOR AXIAL NODE K-1 (FD(3)) WITH THE NKTH TRIAL
0167      C FN. USED AT THAT AXIAL NODE.
0168      FD(3)=C(M4)*STRF(L,MN)
0169      NZ2=NDIA-NTRI(K-1)+NK
0170
0171      C MULTIPLY FD(3) BY NTRF WEIGHTING FNS. (IE., THE TRIAL FNS. USED AT
0172      C AXIAL NODE K) AND ADD RESULTS TO APPROPRIATE ENTRIES IN AL, CB, AND
0173      C CRI.
0174      DO 44 MK=1,NTRF
0175      $ NMT=NTRK(K,MK)
0176      $ NC=NDIA+MK
0177      $ NBF2=NC-NZ2
0178      $ AL(MC,NBF2)=AL(MC,NBF2)+FD(3)*STRF(L,MNT)
0179      CONTINUE
0180      IF(K.EQ.NX) GO TO 61
0181      NTR2=NTRI(K+1)
0182
0183      C SELECT TRIAL FNS. USED AT AXIAL NODE K+1 SEQUENTIALLY.
0184      DO 50 NK=1,NTR2
0185      $ NM=NTRK(K+1,NK)
0186
0187      C CALCULATE 'RESIDUAL' FOR AXIAL NODE K+1 (FD(2)) WITH THE NKTH
0188      C TRIAL FN. USED AT THAT AXIAL NODE.
0189      FD(2)=C(N3)*STRF(L,MN)
0190      NZ3=NDIA+NTRI(K)+NK
0191
0192      C MULTIPLY FD(2) BY NTRF WEIGHTING FNS. (IE., THE TRIAL FNS. USED AT
0193      C AXIAL NODE K) AND ADD RESULTS TO APPROPRIATE ENTRIES IN AL, CB, AND
0194      C CRI.
0195      DO 51 MK=1,NTRF
0196      $ NMT=NTRK(K,MK)
0197      $ NC=NDIA+MK
0198      $ NBF3=NC-NZ3
0199      $ CRI(MC,NBF3)=CRI(MC,NBF3)+FD(2)*STRF(L,MNT)
0200      CONTINUE
0201      CONTINUE
0202
0203      C MULTIPLY RIGHT-HAND SIDE RESIDUAL BY WEIGHTING FUNCTIONS.
0204      DO 40 NK=1,NTRF
0205      $ NMT=NTRK(K,MK)
0206      $ RES(MK,K)=RES(MK,K)+RSTRF(L,MNT)
0207      CONTINUE
0208      RETURN
0209      END
0210
0211
0001      SUBROUTINE OUTSYN (IXX,TC,HTGEN,BZA,BZD,NX,NSTOP,BTAU,TMCAP,TIME,T
0002      $ RANS,N1,IMUL,LMAX)
0003
0004      C THIS SUBROUTINE PRINTS THE RESULTS OF A SYNTHESIS ANALYSIS.
0005      C IT ALSO RECOMPUTES THE RHS VECTOR (RES) FOR THE SYSTEM OF EQUATIONS
0006      C DEFINING THE BLENDING VECTOR (1) IN A TRANSIENT SYNTHESIS ANALYSIS
0007      C AT EACH TIME STEP AND RECOMPUTES THE MATRIX (DIASUM) CONTAINING
0008      C NON-LINEAR TERMS IN THE LHS. COEFFICIENT MATRIX AT EACH
0009      C TIME STEP IN A TRANSIENT ANALYSIS OR FOR EACH ITERATION IN A
0010      C NON-LINEAR STEADY-STATE ANALYSIS.
0011
0012      INTEGER RADBNB,RESIST,TRANS
0013      DOUBLE PRECISION RES,Z,RR,TSYN,DIASUM
0014      DIMENSION SAN(39,65),OFLX(1,1),TRAB(39,65),DIASUM(5,5,65)
0015      $ ,HTGEN(65),AREA(470),BZA(65),BZD(65),RES(5,65),IS(39,25),
0016      $ HFLZ(65),HTSTFC(10),NTRF(65),NTRK(65,10),JIN(39),NX(39,65),
0017      $ TRF(470,13),Z(260),TC(39,65),ICN(63),JCN(63),KCN(63),AXPOS(65),

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0001 COMMON /DTM/RESIST,TO /DB3/RESIS,XLOAD,NTPRNT,ICH,JCN,KCN /DB4/
0002 $ IX,JX,KX /DB7/AXPOS /DB13/RES /DB14/NTRF,TRF /DB16/Z /DB18/HFLZ,
0003 $ CCMD /DB24/HX /DB28/MITER /DB30/TCINTB,SLOPS,GASC /DB32/DIASUM
0004 $ /DB35/AREA /DB36/THCAP1,THCAP2 /DB38/IMPECC /DB40/NTS /DB41/
0005 $ RABBD /DB43/REXCO /DB44/EPSLN /DB45/DFLX,TRAD /DB46/SAR /DB47/
0006 $ NTSFPC,NXT /DB52/RAD /DB57/MLIN /DB67/IS /DB68/MGROPT /DB69/
0007 $ NTR1,NTRK /DB70/JIN /DB72/DTHA,DTHB /DB77/A,B,RCY1,RCY2
0008 $ /DB78/NTS1,NFS,SLOPPD,NL,NLC,WCOGA,DBNIN/NIW
0009 884 IF (TRANS.EQ.1) WRITE (100,884) TIME
0010 FORMAT ('0:10X: TIME=' /10.3/)
0011 IF (MLIN.EQ.1.OR.RABBD.EQ.1) WRITE (100,884) NITER
0012
0013 INITIALIZE THE CONVERGENCE INDICATOR NCONT. IF THE CONVERGENCE TEST
0014 PERFORMED LATER IN THE SUBROUTINE INDICATES THAT CONVERGENCE HAS
0015 NOT BEEN ACHIEVED, NCONT IS ASSIGNED A VALUE OF 1.
0016
0017 NCONT=0
0018 IF (NITER.EQ.1) NCONT=1
0019
0020 INITIALIZE THERMAL CONDUCTIVITIES SCMDND AND SCMDN2. THESE VARIABLES
0021 ARE ASSIGNED PROPER VALUES LATER IF KIRCHHOFF'S TRANSFORMATION IS
0022 USED.
0023
0024 SCMDND=1.0
0025 SCMDN2=1.0
0026
0027 INITIALIZE THE TOTAL HEAT FLOW RATE THROUGH THE CONTACT SURFACE
0028 (QTOT) AND THE TOTAL HEAT FLOW RATE THROUGH THE CONTACT PATCH
0029 (QTTOT) FOR A 'RESISTANCE OF CONTACT' ANALYSIS.
0030
0031 OPAT=0.0
0032 QTOT=0.0
0033 QTTOT=0.0
0034 NIS=NX+1
0035 LI=1
0036
0037 INITIALIZE THE R-THETA NODE NUMBER COUNTER (L2)
0038
0039 L2=0
0040
0041 SWEEP THROUGH THE CIRCUMFERENTIAL NODES (L2).
0042
0043 DO 10 I=1,IXX
0044 JI=2
0045 IF (NGROPT.EQ.2) JI=JIN(I)
0046 ISP=1
0047
0048 SWEEP THROUGH THE RADIAL NODES (J).
0049
0050 DO 15 J=JI,JX,ISP
0051 JP=0
0052 IF (J.EQ.NX) JP=-1
0053 IF (J.EQ.NIS) JP=1
0054 IF (NGROPT.NE.0) ISP=IS(I,J)
0055
0056 INCREMENT THE R-THETA NODE COUNT (L2) BY 1.
0057
0058 L2=L2+1
0059 L=0
0060
0061 SWEEP THROUGH THE AXIAL NODES (K).
0062
0063 DO 20 K=1,KX
0064
0065 INITIALIZE THE TEMPERATURE OF THE I,J,KTH (OR L2,KTH) NODE (TSYN)
0066 AND THE TEMPERATURE OF THE OPPOSING NODE ON THE PRESSURE TUBE
0067 OUTER OR THE CALAMBRIA TUBE INNER SURFACE (TSYN2).
0068
0069 TSYN=0.0
0070 TSYN2=0.0
0071 NT=NTRK(K)
0072
0073 CALCULATE THE TEMPERATURES TSYN AND TSYN2 BY RUNNING THE APPROPRIATE
0074 PRODUCTS OF TRIAL FUNCTION (TRF) ENTRIES AND BLENDING VECTOR (Z)
0075 ENTRIES FOR THE TRIAL FNS. USED AT AXIAL NODE K.
0076
0077 DO 25 NN=1,NT
0078 NN=NTRK(K,NK)
0079 L=1
0080 TSYN=TSYN+TRF(L2,NN)*Z(L)
0081 IF (J.EQ.NX) TSYN2=TSYN2+TRF(L2+1,NN)*Z(L)
0082 IF (J.EQ.NIS) TSYN2=TSYN2+TRF(L2-1,NN)*Z(L)
0083 CONTINUE
0084
0085 IF KIRCHHOFF'S TRANSFORMATION IS BEING USED, THE NODAL 'TEMPERATURES'
0086 ARE ACTUALLY THERMAL CONDUCTIVITY INTEGRALS. THE REAL TEMPERATURES
0087 ARE REQUIRED IN SUBSEQUENT CALCULATIONS SO STORE THE INTEGRALS
0088 TSYN AND TSYN2 AS TH11 AND TH12 AND CALCULATE THE REAL TEMPERATURES
0089 FROM THE INTEGRALS. STORE THE REAL TEMPERATURES AS TSYN AND TSYN2.
0090
0091 TH11=TSYN
0092 TH12=TSYN2
0093 IF (TSYN.LT.0.0) OUTPUT 'TSYN NEG.' / STOP
0094 IF (TSYN2.LT.0.0) OUTPUT 'TSYN2 NEG.' / STOP
0095 IF (NACOM) 23,23,24
0096 TSYN=(-TCINTB*SORT(TCINTB*SZ2+2.0*SLOPS*TSYN))/SLOPS
0097 SCMDND=TCINTB+SLOPS*TSYN
0098 IF (J.EQ.NX.OR.J.EQ.NIS) TSYN2=(-TCINTB*SORT(TCINTB*SZ2+2.0*
0099 SLOPS*TSYN2))/SLOPS / SCMDN2=TCINTB+SLOPS*TSYN2
0100 OR=0.0
0101 CD=0.0
0102 KX2=NX(I,K)
0103 IF (IMPECC.EQ.1) KX2=N1
0104 IF (J.EQ.JX) CD=CD+ZABN(I,K)
0105 IF (TRANS.NE.1.AND.RABBD.NE.1.AND.MLIN.NE.1) GO TO 27
0106 IF (TRANS.EQ.1.AND.NTS.EQ.NTSFPC(NXT)) GO TO 27
0107 NTS=NTGEN(K)
0108 THCAP=THCAP1
0109 IF (J.EQ.NX) NTS=0.0 / THCAP=THCAP2
0110 CD=0.0
0111 IF (J.EQ.JX) CS=SAR(I,K)
0112 IF (MLIN) 31,31,32
0113
0114 IF THE ANALYSIS BEING PERFORMED IS A NON-LINEAR LOECC ANALYSIS,
0115 CALCULATE THE RMS. RESIDUAL (RR) FOR THE F-B EQUATION AT THE I,J,KTH
0116 (OR L2,KTH) NODE.
0117
0118 CS=0.0
0119 IF (J.EQ.NX.OR.J.EQ.NIS) CS=RAB(NX)

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[illegible]

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0258 20      CONTINUE                                0348.900
0259 15      CONTINUE                                0349.000
0260 10      CONTINUE                                0349.100
0261          IF((RADND.EQ.1.OR.MLIN.EQ.1).AND.TRANS.NE.1.AND.NCONT.EQ.0) 0349.200
0262          $ NSTOP=1                                0349.300
0263          WRITE (108,882) QTOT                     0349.400
0264 882      FORMAT (1X,'TOTAL HEAT FLOW =',F13.5,' W',/) 0349.500
0265          IF(RESIST-1) 40,41,40                    0349.600
0266          C                                         0349.700
0267          C CALCULATE AND PRINT THE RESULTS OF A 'RESISTANCE OF CONTACT' 0349.800
0268          C ANALYSIS.                                0349.900
0269          C                                         0350.000
0270 41      TLOW=TC(1,1)                              0350.100
0271          IF(INSUL.EQ.1) TLOW=TLOW+(QTOT/(0.785488*DH*(1,1))) 0350.200
0272          OUTPUT TLOW                               0350.300
0273          QPDOT=QPAT/QTOT                           0350.400
0274          RESS=((10-TLOW)/QTOT)/2.0-RESIS           0350.500
0275          WRITE (108,883) XLOAD,A,B,RCY1,RCY2,QTOT,RESS 0350.600
0276          WRITE (108,888) QPDOT                     0350.700
0277 885      FORMAT (1X,'CONTACT LOAD =',F10.5,' W',/ 0350.800
0278          $ 1X,'SEMI-MAJOR DIAMETER (A) =',F10.5,' CM',/ 0350.900
0279          $ 1X,'OR DIAMETER (B) =',F10.5,' CM',/ 0351.000
0280          $ 1X,'SMALL CONTACT RADIUS =',F10.5,' CM',/ 0351.100
0281          $ 1X,'LARGE CONTACT RADIUS =',F10.5,' CM',/ 0351.200
0282          $ 1X,'HEAT FLOW THROUGH CONTACT PATCH =',F10.5,' W',/ 0351.300
0283          $ 1X,'RESISTANCE OF CO 0351.400
0284 886      HTACT =',F10.5,' DEG.C/W',/ 0351.500
0285 888      FORMAT (1X,'RATIO OF HEAT FLOW THROUGH PATCH TO TOTAL HEAT FLOW=' 0351.600
0286          $ ,F10.5) 0351.700
0287          RETURN 0351.800
0288          END

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0001      SUBROUTINE OUTPUT (HX,T,TIME,TRANS,SYN,M1) 0351.900
0002  C                                         0352.000
0003  C THIS SUBROUTINE OUTPUTS THE RESULTS OF A FINITE-DIFFERENCE ANALYSIS. 0352.100
0004  C                                         0352.200
0005      INTEGER TRANS,SYN,RADND,RESIST              0352.300
0006      DIMENSION T(470),HX(39,65),TC(39,65),IS(39,25),JIN(39) 0352.400
0007      DIMENSION SAR(39,65),DPLX( 1, 1),TRAB(39,65),NRUM(39,25,1) 0352.500
0008      COMMON /DB21/TC,DB38/INPECC,DB70/JIN,DB47/IS,DB68/MGROPT,DB43/ 0352.600
0009      REXCO,DB41/RADND,DB46/SAR,DB45/DPLX,TRAB,DB4/IX,J1,KX,DB12/NRUM 0352.700
0010      /DB1/RESIST,TO,DB57/MLIN,DB28/WITER 0352.800
0011      /DB40/IX 0352.900
0012      IF(TRANS.EQ.1) WRITE (108,883) TIME 0353.000
0013 883      FORMAT ('0',10X,'TIME=',F10.5/) 0353.100
0014      WITER=WITER-1 0353.200
0015      IF(MLIN.EQ.1.OR.RADND.EQ.1).AND.TRANS.NE.1) WRITE (108,884) 0353.300
0016      $ WITER 0353.400
0017 884      FORMAT ('000000 NUMBER OF ITERATIONS PERFORMED =',I3,' 00000' /) 0353.500
0018  C                                         0353.600
0019  C INITIALIZE THE NODE COUNT (L). 0353.700
0020      L=0 0353.800
0021      LI=0 0353.900
0022  C                                         0354.000
0023  C SWEEP THROUGH THE AXIAL NODES (K). 0354.100
0024      DO 10 K=1,KX 0354.200
0025      C 0354.300
0026  C SWEEP THROUGH THE CIRCUMFERENTIAL NODES (I). 0354.400
0027      DO 20 I=1,IXX 0354.500
0028      JI=2 0354.600
0029      IF(MGROPT.EQ.2) JI=JIN(I) 0354.700
0030      ISP=1 0354.800
0031  C 0354.900
0032  C SWEEP THROUGH THE RADIAL NODES (J). 0355.000
0033      DO 30 J=JI,JX,ISP 0355.100
0034      IF(MGROPT.NE.0) ISP=IS(I,J) 0355.200
0035      C 0355.300
0036      C INCREMENT THE NODE COUNT. 0355.400
0037      L=L+1 0355.500
0038      IF(I.EQ.1.OR.I.EQ.IX) GO TO 34 0355.600
0039      GO TO 30 0355.700
0040      IF(J.EQ.2.OR.J.EQ.JX) GO TO 31 0355.800
0041      GO TO 30 0355.900
0042  C 0356.000
0043  C PRINT THE NODE ADDRESS AND TEMPERATURE. 0356.100
0044      WRITE (108,881) I,J,K,T(L) 0356.200
0045 881      FORMAT (1X,'I=',I5,4X,'J=',I5,4X,'K=',I5,4X,'TEMP =', 0356.300
0046      30 0356.400
0047      CONTINUE 0356.500
0048      CONTINUE 0356.600
0049      QTOT=0.0 0356.700
0050  C 0356.800
0051  C SWEEP THROUGH THE AXIAL AND CIRCUMFERENTIAL NODES (K AND I) 0356.900
0052      DO 40 K=1,KX 0357.000
0053      DO 45 I=1,IXX 0357.100
0054      C 0357.200
0055      C CALCULATE AND PRINT OUT THE HEAT FLUX AND HEAT FLOW AT OUTER 0357.300
0056      C SURFACE NODE I,J,K. ALSO SUM THE NODAL HEAT FLOWS (HTFLUX) TO 0357.400
0057      C OBTAIN THE TOTAL HEAT FLOW RATE (QTOT). 0357.500
0058      HX1=HX(I,K) 0357.600
0059      IF(INPECC.EQ.1) HX1=M1 0357.700
0060      HTFLUX=HX1*(NRUM(I,J,K)-TC(I,K)) 0357.800
0061      IF(RADND.EQ.1.AND.TRANS.EQ.1) DPLX(I,K)-REXCO*(NRUM(I, 0357.900
0062      J,K))*884-TC(I,K)*884) 0358.000
0063      IF(RADND.EQ.1) HTFLUX=HTFLUX+DPLX(I,K) 0358.100
0064      HTFLOW=SAR(I,K)*HTFLUX 0358.200
0065      QTOT=QTOT+HTFLOW 0358.300
0066      IF(I.NE.1) GO TO 45 0358.400
0067      WRITE(108,882) I,K,HTFLUX,HTFLOW 0358.500
0068 882      FORMAT (1X,'I=',I5,4X,'K=',I5,4X,'HEAT FLUX=',F15.5,5X,'HEAT FLOW=' 0358.600
0069      $ ,F15.5) 0358.700
0070      CONTINUE 0358.800
0071      CONTINUE 0358.900
0072      OUTPUT QTOT 0359.000
0073      IF(QTOT.EQ.0.0) GO TO 50 0359.100
0074  C 0359.200
0075  C IN A 'RESISTANCE OF CONTACT' ANALYSIS, CALCULATE AND PRINT OUT THE 0359.300
0076  C CONTACT RESISTANCE (RESS). 0359.400
0077      IF(RESIST-1) 50,55,50 0359.500
0078      RESS=(10-TC(1,1))/QTOT 0359.600
0079      OUTPUT RESS 0359.700
0080      RETURN 0359.800
0081      END 0359.900

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0001      SUBROUTINE TFRAD (TIME,KX)                                0360.000
0002      C                                                         0360.100
0003      C THIS SUBROUTINE READS NEW TRIAL FUNCTION SCHEDULES IN THE SECOND  0360.200
0004      C PHASE OF A TRANSIENT SYNTHESIS ANALYSIS (WHEN REQUIRED). THE CODE IS  0360.300
0005      C ESSENTIALLY A REPRODUCTION OF PART OF SUBROUTINE DATA1.          0360.400
0006      C                                                         0360.500
0007      C DIMENSION NTRI(65),NTRK(65,10),NT2(10)                   0360.600
0008      C COMMON /DB44/RBA/DB17/NEOU/DB49/NTRI,NTRK                 0360.700
0009      C OUTPUT TIME                                               0360.800
0010      C NBA=0                                                     0360.900
0011      C NEOU=0                                                    0361.000
0012      C                                                         0361.100
0013      C SWEEP THROUGH THE AXIAL MODES (K).                        0361.200
0014      C DO 10 K=1,KX                                             0361.300
0015      C   OUTPUT K                                               0361.400
0016      C                                                         0361.500
0017      C READ THE NUMBER OF TRIAL FUNCTIONS TO BE USED AT AXIAL MODE K  0361.600
0018      C (NTRI(K)) ALONG WITH THEIR NUMBERS (NT2(NK),NK=1,10).      0361.700
0019      C   READ (105,990) NTRI(K),NT2(NK),NK=1,10)              0361.800
0020      C   WRITE (108,800) NT2(1),NT2(2),NT2(3),NT2(4),NT2(5),NT2(6),  0361.900
0021      C   NT2(7),NT2(8),NT2(9),NT2(10)                          0362.000
0022      C   DO 15 NK=1,10                                         0362.100
0023      C     NTRK(K,NK)=NT2(NK)                                  0362.200
0024      C   15                                                    0362.300
0025      C CALCULATE THE HALF-BANDWIDTH OF THE COEFFICIENT MATRIX FOR THE  0362.400
0026      C SYSTEM OF EQUATIONS DEFINING Z (NBA) ALONG WITH THE TOTAL NUMBER OF  0362.500
0027      C EQUATIONS (NEOU).                                         0362.600
0028      C   IF(K.EQ.1) GO TO 20                                    0362.700
0029      C   NMB=NTRI(K)+NTRI(K-1)-1                                0362.800
0030      C   IF(NMBA.GT.NBA) NBA=NMB                                0362.900
0031      C   NEOU=NEOU+NTRI(K)                                       0363.000
0032      C   20 CONTINUE                                           0363.100
0033      C   FORMAT (11I5)                                           0363.200
0034      C   990 FORMAT (1X,'NEW TRIAL FUNCTIONS ARE',10I5,/)      0363.300
0035      C   RETURN                                                  0363.400
0036      C   END                                                    0363.500

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0001      SUBROUTINE MATCH (NDT,BT,KX,NTRF,NBA,XNF,TRANS)          0363.600
0002      C                                                         0363.700
0003      C THIS SUBROUTINE ADDS XNF TIMES THE ENTRIES IN DIASUM TO THE  0363.800
0004      C APPROPRIATE ENTRIES IN AL1, CB1 AND CR11. IT ALSO MODIFIES ENTRIES IN  0363.900
0005      C DIASUM WHEN THE TIME STEP SIZE IS CHANGED IN A LINEAR TRANSIENT  0364.000
0006      C ANALYSIS SO THAT THE ENTRIES WILL BE APPLICABLE TO THE NEW TIME STEP.  0364.100
0007      C                                                         0364.200
0008      C INTEGER TRANS,RADBNB                                       0364.300
0009      C DOUBLE PRECISION AL1,CR11,CB1,DIASUM                     0364.400
0010      C DIMENSION BT(10),CB1(260),AL1(260,7),CR11(260,7),DIASUM( 5,5,65)  0364.500
0011      C   NTRI(65),NTRK(65,10)                                   0364.600
0012      C   COMMON /DB32/DIASUM /DB34/AL1,CR11,CB1 /DB41/RADBNB /DB57/NLIN  0364.700
0013      C   /DB48/NTRI,NTRK                                         0364.800
0014      C   NDBA1=0                                                  0364.900
0015      C   NC=0                                                     0365.000
0016      C                                                         0365.100
0017      C SWEEP THROUGH THE AXIAL MODES (K).                        0365.200
0018      C DO 10 K=1,KX                                             0365.300
0019      C                                                         0365.400
0020      C REPRODUCE STATEMENTS USED IN SUBROUTINE SUBST1 IN ORDER TO MODIFY  0365.500
0021      C ENTRIES IN AL1, CB1 AND CR11 USING THOSE IN DIASUM.      0365.600
0022      C   NTRF=NTRI(K)                                           0365.700
0023      C   DO 15 NK=1,NTRF                                         0365.800
0024      C     NZ1=NDBA1+NK                                           0365.900
0025      C     DO 20 NKK=1,NTRF                                       0366.000
0026      C       NC=NDBA1+NKK                                         0366.100
0027      C       IF(NZ1.EQ.NC) CB1(NC)=CB1(NC)+XNFB*DIASUM(NK,NKK,K)  0366.200
0028      C       NBF1=NC-NZ1                                           0366.300
0029      C       IF(NBF1.GT.0) AL1(NC,NBF1)=AL1(NC,NBF1)+XNFB*DIASUM(NK  0366.400
0030      C       ,NKK,K)                                              0366.500
0031      C       IF(NBF1.LT.0) NBF1=-NBF1+CR11(NC,NBF1)=CR11(NC,NBF1)  0366.600
0032      C       +XNFB*DIASUM(NK,NKK,K)                              0366.700
0033      C   15                                                    0366.800
0034      C IN A LINEAR TRANSIENT ANALYSIS, MODIFY THE ENTRIES IN DIASUM SO  0366.900
0035      C THAT THEY WILL BE APPLICABLE TO THE NEW TIME STEP SIZE.  0367.000
0036      C   IF(TRANS.EQ.1.AND.RADBNB.NE.1.AND.NLIN.NE.1) DIASUM(  0367.100
0037      C   NK,NKK,K)=DIASUM(NK,NKK,K)*XNF+1.0)                   0367.200
0038      C   20 CONTINUE                                           0367.300
0039      C IF THE ENTRIES IN DIASUM HAVE BEEN SUBTRACTED FROM THE APPROPRIATE  0367.400
0040      C ENTRIES IN AL1,CB1 AND CR11 IN A NON-LINEAR ANALYSIS, ZERO THEM SO  0367.500
0041      C THAT THEY CAN BE RECOMPUTED IN SUBROUTINE OUTSYN USING THE MOST  0367.600
0042      C RECENT NODAL TEMPERATURES. THEY WILL THEN BE ADDED BACK ONTO THE  0367.700
0043      C ENTRIES IN AL1,CB1 AND CR11 (BY SETTING XNF=+1.0), THEREBY  0367.800
0044      C UPDATING THOSE MATRICES.                                   0367.900
0045      C   IF(NLIN.EQ.1.AND.XNF.LT.0.0) DIASUM(NK,NKK,K)=0.0     0368.000
0046      C   IF(RADBNB.EQ.1.AND.XNF.LT.0.0) DIASUM(NK,NKK,K)=0.0     0368.100
0047      C   20 CONTINUE                                           0368.200
0048      C   15 CONTINUE                                           0368.300
0049      C   NDBA1=NDBA1+NTRF                                         0368.400
0050      C   10 CONTINUE                                           0368.500
0051      C   RETURN                                                  0368.600
0052      C   END                                                    0368.700

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0001      SUBROUTINE HX10 (HX1,TRANS)                              0368.800
0002      C                                                         0368.900
0003      C THIS SUBROUTINE CALCULATES THE RADIATIVE AND GAS CONDUCTANCES,  0369.000
0004      C DESIGNATED HRAD AND HGAS RESPECTIVELY, AND THEN THE COMBINED  0369.100
0005      C CONDUCTANCE, HX1(K), BETWEEN OPPOSING NODES ON THE PRESSURE TUBE  0369.200
0006      C OUTER AND CALANDRIA TUBE INNER SURFACES IN A CANDU FUEL CHANNEL  0369.300
0007      C (AT AZIMUTHAL NODE I AND AXIAL NODE K). ALSO, NODAL TEMP. CHANGES  0369.400
0008      C ARE COMPARED TO THE USER-SPECIFIED CONVERGENCE PARAMETER EPSLN  0369.500
0009      C TO TEST FOR CONVERGENCE. THIS SUBROUTINE IS NOT USED IN THE  0369.600
0010      C SECOND PHASE OF A SYNTHESIS ANALYSIS, WHEN NON-LINEARITIES ARE  0369.700
0011      C HANDLED WITH NEWTON-RAPHSON'S METHOD.                      0369.800
0012      C                                                         0369.900
0013      C INTEGER TRANS                                             0370.000
0014      C DIMENSION TRF(470,13),T(470),GGAP(39,1),TRAB(39),HX(39,65)  0370.100
0015      C COMMON /DB23/T/DB7/I,J,K,NIS,N1/DB40/NTS/DB49/NTFIC/DB5/R/L/L  0370.200
0016      C   /DB24/HX /DB26/BETA,GGAP /DB44/EPSLN /DB50/NCNTY /DB43/REXED  0370.300
0017      C   /DB30/TCINTS,SLOPG,GASC /DB14/NTRF,TRF /DB28/NITER /DB33/TCGAS,  0370.400
0018      C   CCMN                                                    0370.500
0019      C   TNEIGH=T(L+1)                                           0370.600
0020      C   THP=T(L)                                                 0370.700
0021      C   IF(NTS.EQ.1) TNEIGH=TRF(L+1,NTFIC)+THP=TRF(L,NTFIC)    0370.800

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0022 IF(TRANS.NE.1.AND.NITER.GT.2) DIFF=ABS(T(L)-TRAB(I)) 0370.900
0023 IF(TRANS.NE.1.AND.NITER.GT.2.AND.DIFF.GT.EPSLM) NCONT=1 0371.000
0024 IF(TRANS.NE.1) TRAB(I)=T(L) 0371.100
0025 TAU=(TNEIGH+TNP)/2.0 0371.200
0026 CKGG=TCINTG+SLOPGSTAV 0371.300
0027 HRAD=REXCD8(TNEIGH82+TNP82)*(TNEIGH+TNP) 0371.400
0028 KP=CCNN/CGAP(I,K) 0371.500
0029 HOUT=CKGG/((1.0+KP)*GGAP(I,K)) 0371.600
0030 HNEW=HOUT+HRAD 0371.700
0031 HX(I,K)=BETAS*HNEW+(1.0-BETA)*HX(I,K) 0371.800
0032 HX1=HX(I,K) 0371.900
0033 RETURN 0372.000
0034 END 0372.100

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0001 SUBROUTINE HGAS (XPP,YP2,HOUT,SEPN) 0372.200
0002 C 0372.300
0003 THIS SUBROUTINE USES HERTZ'S THEORY TO CALCULATE THE GAP (SEPN) 0372.400
0004 BETWEEN SURFACES IN ELASTIC CONTACT. THE GAP IS CALCULATED AT A 0372.500
0005 POINT WITH LATERAL AND LONGITUDINAL COORDINATES YP2 AND XPP 0372.600
0006 RESPECTIVELY, WHERE THE LATERAL AND LONGITUDINAL AXES REFERRED TO 0372.700
0007 ARE PARALLEL TO THE CONTACT'S MINOR AND MAJOR AXES RESPECTIVELY. 0372.800
0008 THE POINT AT COORDINATES YP2 AND XPP MUST BE OUTSIDE OF THE CONTACT 0372.900
0009 PATCH (ASSUMED ELLIPTICAL WITH CONTACT SEMI-MAJOR AND SEMI-MINOR 0373.000
0010 DIAMETERS A AND B RESPECTIVELY). THE CONTACT SURFACES ARE ASSUMED TO 0373.100
0011 BE CYLINDRICAL WITH SURFACE RADIi RY1 AND RY2 (RY1 LT. RY2). 0373.200
0012 THE GAS CONDUCTANCE AT YP2,XPP, DESIGNATED HOUT, IS CALCULATED 0373.300
0013 FROM SEPN WITH A NON-CONTINUUM GAS CONDUCTIVITY MODEL. 0373.400
0014 THIS SUBROUTINE SHOULD NOT BE USED UNLESS A/B .GT. 4. 0373.500
0015 C 0373.600
0016 INTEGER RESIST 0373.700
0017 COMMON /DB33/TCBAS,CCNN /DB39/NTOL /DB77/A,B,RCY1,RCY2 0373.800
0018 * /DB1/RESIST,TO 0373.900
0019 BB=(ASS2.0)+(B822.0)-(XPP822.0)-(YP2822.0) 0374.000
0020 CC=((ASS)822.0)-((XPP8)822.0)-((YP28)822.0) 0374.100
0021 XP2=XPP822.0 0374.200
0022 YP2=YP2822.0 0374.300
0023 XNUS=(-BB+SQRT(BB822.0-4.0*CC))/2.0 0374.400
0024 XNUP=ASS 0374.500
0025 C 0374.600
0026 CALCULATE APPROXIMATION TO ELLIPTIC INTEGRAL OF FIRST KIND (FTHIK). 0374.700
0027 IF(XNUS.LE.XNUP) FTHIK=ALOG((SQRT(XNUS)+SQRT(XNUS+B822.0))/B) 0374.800
0028 IF(XNUS.GT.XNUP) FTHIK=ALOG(4.084/B)-ALOG(SQRT((ASS2.0/XNUS))+SOR 0374.900
0029 * T((XNUS+ASS2.0)/XNUS)) 0375.000
0030 R3=SQRT(XNUS/(XNUS+B822.0)) 0375.100
0031 COEFF=-1.0848(B822.0)*RCY1+RCY2)/(4.08RCY18RCY2) 0375.200
0032 C 0375.300
0033 EVALUATE LOCAL GAP SIZE (SEPN). 0375.400
0034 SEPN=COEFF*((12.0/A)*FTHIK)-(XP28(((2.0/(A*(ASS2.0-B822.0)))*FTHI 0375.500
0035 * N-R3))+((2.0/ASS2.0)*SQRT(1.0/(XNUS+ASS2.0)*R3))-((YP282.0848(R3- 0375.600
0036 * (B822.0/ASS2.0)*FTHIK))/((B822.0)*(ASS2.0-B822.0))) 0375.700
0037 C 0375.800
0038 EVALUATE LOCAL GAS CONDUCTANCE (HOUT) 0375.900
0039 HOUT=(TCBAS/(1.0+(CCNN/SEPN)))/SEPN 0376.000
0040 IF(RESIST.EQ.1) HOUT=2.08HOUT 0376.100
0041 RETURN 0376.200
0042 END 0376.300
0001 SUBROUTINE RABAT (C4,TRANS,CTOT) 0376.400
0002 C 0376.500
0003 THIS SUBROUTINE CALCULATES TERMS REQUIRED FOR NEWTON 0376.600
0004 LINEARIZATION OF A RADIATIVE BOUNDARY CONDITION. ELEMENTS IN THE 0376.700
0005 HEAT FLOW VECTOR (Q(L), WHERE L IS THE NODE NUMBER) AND DIAGONAL 0376.800
0006 ELEMENTS (CTOT) OF THE CONDUCTANCE MATRIX MUST BE MODIFIED IN 0376.900
0007 THE LINEARIZATION PROCESS. THIS SUBROUTINE IS NOT USED WITH 0377.000
0008 SYNTHESIS SOLUTIONS. TERMS REQUIRED FOR LINEARIZATION IN SYNTHESIS 0377.100
0009 SOLUTIONS ARE COMPUTED IN SUBROUTINES SUBST1 (1ST ITER. OR TIME 0377.200
0010 STEP) AND OUTSWN. 0377.300
0011 C 0377.400
0012 INTEGER TRANS 0377.500
0013 DOUBLE PRECISION CTOT 0377.600
0014 DIMENSION OFLX(1,1),TRAB(39,45),Q(470),T(470),TC(39,45) 0377.700
0015 COMMON /DB21/T/DB3/CL,L/DB10/B/DB9/I,J,K,NIS,HI/DB21/TC/DB28/WITER 0377.800
0016 * /DB43/REXCD/DB44/EPSLM/DB50/NCONT/DB45/OFLX,TRAB 0377.900
0017 IF(NITER.EB.1) GO TO 5 0378.000
0018 IF(TRANS-1) 10,15,10 0378.100
0019 C 0378.200
0020 CHECK IF BOUNDARY TEMPERATURE HAS CONVERGED (STEADY-STATE 0378.300
0021 ANALYSIS) 0378.400
0022 10 DIFF=ABS(T(L)-TRAB(I,K)) 0378.500
0023 IF(DIFF.GT.EPSLM) NCONT=1 0378.600
0024 OFLX(I,K)=REXCD8(T(L)884-TC(I,K)884) 0378.700
0025 TRAB(I,K)=T(L) 0378.800
0026 C 0378.900
0027 INCORPORATE LINEARIZATION TERMS INTO APPROPRIATE ELEMENT OF HEAT 0379.000
0028 FLOW VECTOR (IE.Q(L)) AND DIAGONAL ELEMENT OF CONDUCTANCE MATRIX. 0379.100
0029 15 Q(L)=Q(L)-OFLX(I,K)*C4*((4.08REXCD848(TRANS(I,K)884)) 0379.200
0030 CTOT=CTOT+4.08REXCD848(TRANS(I,K)883) 0379.300
0031 RETURN 0379.400
0032 END 0379.500

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0001 SUBROUTINE QFORM(THCAP,BTAU) 0379.600
0002 C 0379.700
0003 THIS SUBROUTINE CALCULATES THE RMS. HEAT FLOW VECTOR (Q) AT EACH 0379.800
0004 TIME STEP IN A LINEAR TRANSIENT FINITE-DIFFERENCE ANALYSIS UNLESS 0379.900
0005 THE TIME STEP SIZE (BTAU) HAS JUST BEEN CHANGED. 0380.000
0006 C 0380.100
0007 INTEGER TUBE 0380.200
0008 DIMENSION BZA(45),BZB(45),JIN(39),BTHA(39,25),BTHB(39,25) 0380.300
0009 * BTHA(39),BTHB(39),B(39,25),HTREN(45),HX(39,45), 0380.400
0010 * BAR(39,45),AREA(470),T(470),Q(470),TC(39,45),NFLZ(45), 0380.500
0011 * RAB(25) 0380.600
0012 COMMON /DB72/BTHA,BTHB/DB52/RAB/DB18/NFLZ,CCNN/DB33/AREA/DB38/ 0380.700
0013 * INPECC/DB44/BAR/DB47/I8/DB24/HX/DB4/IX,JX,KX/DB23/T 0380.800
0014 * /DB10/B /DB21/TC/DB9/I,J,K,NIS,HI /DB27/BZA,BZB,HTOEN 0380.900
0015 * /DB70/JIN/DB48/HTOEN/DB54/HX 0381.000
0016 * /DB43/BTHA,BTHB /DB34/THCAP1,THCAP2 /DB2/TUBE,BT 0381.100
0017 BZTOT=1.0 0381.200
0018 C 0381.300

```

```

0019 C INITIALIZE THE NODE SEQUENCE NUMBER (L).
0020 L=0
0021 C
0022 C SWEEP THROUGH THE AXIAL MODES (K).
0023 DO 5 K=1,KX
0024 C
0025 C CALCULATE 'DZ' (DZTOT) AT AXIAL MODE K.
0026 IF(KX.GT.1) DZTOT=(DZA(K)+DZB(K))/2.0
0027 C
0028 C SWEEP THROUGH THE CIRCUMFERENTIAL MODES (I).
0029 DO 10 I=1,IX
0030 JI=2
0031 IF(NGROPT.EQ.2) JI=JIN(I)
0032 C
0033 C CALCULATE 'DTHETA' (ANTOT) AT CIRCUMFERENTIAL MODE I IF NGROPT.NE.2.
0034 IF(NGROPT.NE.2) ANTOT=(DTHA(I)+DTHB(I))/2.0
0035 ISP=1
0036 C
0037 C SWEEP THROUGH THE RADIAL MODES (J).
0038 DO 15 J=JI,JX,ISP
0039 C
0040 C DETERMINE THE PRODUCT OF SPECIFIC HEAT AND DENSITY AT NODE I,J,K.
0041 (THCAP).
0042 THCAP=THCAP1
0043 IF(J.GT.NX) THCAP=THCAP2
0044 C
0045 C CALCULATE 'DTHETA' (ANTOT) AT R-THETA NODE I,J IF NGROPT=2.
0046 IF(NGROPT.EQ.2) ANTOT=(DTHA(I,J)+DTHB(I,J))/2.0
0047 IF(NGROPT.NE.0) ISP=IS(I,J)
0048 C
0049 C INCREMENT THE NODE NUMBER COUNTER (L).
0050 L=L+1
0051 C
0052 C DETERMINE THE HEAT GENERATION RATE (HTG) AND THE SURFACE CONDUCTANCE
0053 (CO) AT NODE I,J,K (IF APPLICABLE) AND THEN CALCULATE Q(L).
0054 HTG=HTGEN(K)
0055 IF(J.BT.NX) HTG=0.0
0056 CO=0.0
0057 HX2=HX(I,K)
0058 IF(TWPEC.EQ.1) HX2=H1
0059 IF(J.EQ.IX) CO=CO288AR(I,K)
0060 Q(L)=((THCAP*(L)/DTAU)+HTG)*RAREA(L)*DZTOT+CO*TC(I,K)
0061 IF(J.EQ.2) Q(L)=Q(L)+HFLZ(K)*RAD(J)*ANTOT*DZTOT
0062 IF(TURE.EQ.1.AND.J.EQ.NIS) Q(L)=Q(L)+HISRAD(J)*ANTOT*
0063 DZTOT*BT
0064 15 CONTINUE
0065 10 CONTINUE
0066 5 CONTINUE
0067 RETURN
0068 END

```

```

0001 SUBROUTINE BNDEL (D)
0002 C
0003 C THIS SUBROUTINE SOLVES SYSTEMS OF SIMULTANEOUS EQUATIONS WITH
0004 BANDED LEFT HAND SIDE COEFFICIENT MATRICES. THE COEFFICIENT
0005 MATRICES NEED NOT BE SYMMETRIC. SIMULTANEOUS EQUATIONS ARE WRITTEN
0006 IN THE FORM:
0007
0008 
$$A(J,M)X(J-M) + A(J,M-1)X(J-M-1) + \dots + A(J,2)X(J-2) + A(J,1)$$

0009 
$$X(J-1) + B(J)X(J) + C(J,1)X(J+1) + C(J,2)X(J+2) + \dots$$

0010 
$$C(J,M-1)X(J+M-1) + C(J,M)X(J+M) = B(J)$$

0011
0012 FOR 1 LE. J LE. N WITH N GE. M.
0013
0014 WHERE J IS THE EQUATION NUMBER AND M IS THE LEFT OR RIGHT HAND
0015 MATRIX BANDWIDTH. THESE BANDWIDTHS ARE ASSUMED EQUAL. N
0016 REPRESENTS THE TOTAL NUMBER OF UNKNOWN IN X (IE. TOTAL NUMBER OF
0017 EQUATIONS).
0018
0019 DOUBLE PRECISION A,B,C,D,X,SUM1,SUM2,SUM3,SUM4,SUM5
0020 DIMENSION A(260,7),B(260),C(260,7),D(260),X(260)
0021 COMMON /DB15/B,A,C,N,DB16/X,DB17/M
0022 C
0023 C REDUCE LEFT-HAND COEFFICIENT MATRIX TO UPPER TRIANGULAR FORM.
0024 DO 10 J=1,N
0025 SUM2=0.0
0026 IF(J.EQ.1) GO TO 31
0027 NM=M
0028 IF(J.LE.M) NM=J-1
0029 DO 20 I=1,NM
0030 K=M-N+1
0031 SUM1=0.0
0032 KK=K+1
0033 IF(K.EQ.NM) GO TO 20
0034 DO 30 NP=KK,NM
0035 SUM1=SUM1+A(J,NP)*C(J-NP,NP-K)
0036 A(J,K)=A(J,K)-SUM1
0037 SUM2=0.0
0038 DO 35 NP=1,NM
0039 SUM2=SUM2+A(J,NP)*C(J-NP,NP)
0040 B(J)=B(J)-SUM2
0041 IF(B(J).EQ.0.0) GO TO 100
0042 IF(J.EQ.M) GO TO 46
0043 NM=M
0044 MX=M-J+1
0045 IF(MX.LE.N) NM=MX-1
0046 DO 40 K=1,NM
0047 KK=K+1
0048 SUM3=0.0
0049 IF(J.EQ.1) GO TO 41
0050 IF(K.EQ.N) GO TO 41
0051 DO 45 NP=KK,N
0052 JJ=J-(NP-K)
0053 SUM3=SUM3+A(J,NP-K)*C(JJ,NP)
0054 C(J,K)=(C(J,K)-SUM3)/B(J)
0055 40 CONTINUE
0056 SUM4=0.0
0057 IF(J.EQ.1) GO TO 51
0058 DO 50 NP=1,NM
0059 SUM4=SUM4+A(J,NP)*D(J-NP)
0060 D(J)=(D(J)-SUM4)/B(J)
0061 50 CONTINUE

```



```

0044 C
0045 C PERFORM BACK SUBSTITUTION.
0046 DO 55 JJ=1,N
0047 J=N-JJ+1
0048 SUMS=0.0
0049 MM=0
0050 IF(JJ.LE.N) MM=JJ-1
0051 IF(JJ.EQ.1) GO TO 55
0052 DO 60 MP=1,MM
0053 SUMS=SUMS+C(J,MP)*X(J+MP)
0054 X(J)=B(J)-SUMS
0055 RETURN
0056 WRITE (108,105)
0057 FORMAT (10X,'SINGULAR MATRIX')
0058 STOP
0059 END

```

```

0392.900
0393.000
0393.100
0393.200
0393.300
0393.400
0393.500
0393.600
0393.700
0393.800
0393.900
0394.000
0394.100
0394.200
0394.300
0394.400

```

```

0001 SUBROUTINE CHLSK(N)
0002 C
0003 C THIS SUBROUTINE USES CHOLESKY'S ALGORITHM TO FACTOR A MATRIX 'A'
0004 C INTO THE PRODUCT OF A LOWER TRIANGULAR MATRIX AND ITS TRANSPOSE.
0005 C THE MATRIX 'A', WHICH MUST BE POSITIVE-DEFINITE, IS STORED AS A
0006 C VECTOR IN JENNING'S PROFILE STORAGE MODE. ONLY THE ELEMENTS FROM
0007 C THE 1ST NON-ZERO ENTRY IN EACH ROW UP TO AND INCLUDING THE DIAGONAL
0008 C TERM ARE STORED AND THE ROWS ARE LINED UP END TO END IN 'A'. A
0009 C POINTER VECTOR KDIAG PROVIDES THE LOCATION OF EACH DIAGONAL ELEMENT
0010 C IN 'A'. THE LOWER TRIANGULAR MATRIX OBTAINED FROM THE FACTORIZATION
0011 C OF 'A' OVERWRITES 'A' IN THIS SUBROUTINE.
0012 C
0013 DIMENSION KDIAG(470),A( 7331)
0014 COMMON /DB15/A,KDIAG
0015 A(1)=SORT(A(1))
0016 DO 1 I=2,N
0017 KI=KDIAG(I)-1
0018 L=KDIAG(I-1)-KI+1
0019 DO 2 J=L,I
0020 X=A(KI+J)
0021 KJ=KDIAG(J)-J
0022 IF(J.EQ.1) GO TO 2
0023 LBAR=KDIAG(J-1)-KJ+1
0024 LBAR=MAX0(L,LBAR)
0025 IF(LBAR.EQ.J) GO TO 2
0026 MM=J-1
0027 DO 3 K=LBAR,MM
0028 X=X-A(KI+K)*A(KJ+K)
0029 A(KI+J)=X/A(KJ+J)
0030 A(KI+1)=SORT(X)
0031 RETURN
0032 END

```

```

0394.500
0394.600
0394.700
0394.800
0394.900
0395.000
0395.100
0395.200
0395.300
0395.400
0395.500
0395.600
0395.700
0395.800
0395.900
0396.000
0396.100
0396.200
0396.300
0396.400
0396.500
0396.600
0396.700
0396.800
0396.900
0397.000
0397.100
0397.200
0397.300
0397.400
0397.500
0397.600

```

```

0001 SUBROUTINE SUB (N,T)
0002 C
0003 C THIS SUBROUTINE PERFORMS THE FORWARD AND BACKWARD SUBSTITUTIONS
0004 C NEEDED TO OBTAIN THE SOLUTION VECTOR (T) FROM THE FACTORED
0005 C COEFFICIENT MATRIX 'A' AND THE RIGHT-HAND SIDE VECTOR 'B'.
0006 C
0007 DIMENSION KDIAG(470),A( 7331),B(470),T(470)
0008 COMMON /DB15/A,KDIAG,DB10/B
0009 B(1)=B(1)/A(1)
0010 DO 4 I=2,N
0011 KI=KDIAG(I)-1
0012 L=KDIAG(I-1)-KI+1
0013 X=B(I)
0014 IF(L.EQ.1) GO TO 4
0015 MM=I-1
0016 DO 5 J=L,MM
0017 X=X-A(KI+J)*B(J)
0018 B(I)=X/A(KI+1)
0019 DO 4 IT=2,N
0020 I=N+2-IT
0021 KI=KDIAG(I)-1
0022 X=B(I)/A(KI+1)
0023 B(I)=X
0024 L=KDIAG(I-1)-KI+1
0025 IF(L.EQ.1) GO TO 4
0026 MM=I-1
0027 DO 7 K=L,MM
0028 B(K)=B(K)-X*A(KI+K)
0029 CONTINUE
0030 B(1)=B(1)/A(1)
0031 DO 8 K=1,N
0032 T(K)=B(K)
0033 RETURN
0034 END

```

```

0397.700
0397.800
0397.900
0398.000
0398.100
0398.200
0398.300
0398.400
0398.500
0398.600
0398.700
0398.800
0398.900
0399.000
0399.100
0399.200
0399.300
0399.400
0399.500
0399.600
0399.700
0399.800
0399.900
0400.000
0400.100
0400.200
0400.300
0400.400
0400.500
0400.600
0400.700
0400.800
0400.900
0401.000

```

```

0001 SUBROUTINE TRIAL (NEXT,NX,CONST,T)
0002 C
0003 C THIS SUBROUTINE RECORDS F-D SOLUTIONS TO 2-B PROBLEMS FOR USE AS
0004 C TRIAL FUNCTIONS IN A SYNTHESIS ANALYSIS. THE TEMPERATURE OF THE LTH
0005 C R-THETA NODE IN TRIAL FN. NTRF, WHICH IS STORED AS TRF(L,NTRF),
0006 C IS EQUATED TO THE LTH ENTRY IN THE R-THETA SOLUTION VECTOR T
0007 C (IE.,T(L)) FOR L=1,2,... (IE.,FOR ALL OF THE R-THETA NODES).
0008 C
0009 DIMENSION TRF(470,13),MX(39,65),T(470),NMUN(39,25,1)
0010 $ I(39,25),JIN(39)
0011 COMMON /DB4/IX,JX,KX /DB12/NMUN /DB14/NTRF,TRF /DB48/CONST
0012 $ /DB47/IL,DB48/NROPT /DB70/JIN /DB78/TCINTS,SLOPS,SLOPD2,NJL
0013 $ NVC,NUCOM
0014 C
0015 C INITIALIZE THE R-THETA NODE COUNTER (L).
0016 L=0
0017 C
0018 C SWEEP THROUGH THE CIRCUMFERENTIAL NODES (I).
0019 DO 10 I=1,IX
0020 JI=2
0021 IF(NROPT.EQ.2) JI=JIN(I)
0022 ISP=1
0023 C

```

```

0401.100
0401.200
0401.300
0401.400
0401.500
0401.600
0401.700
0401.800
0401.900
0402.000
0402.100
0402.200
0402.300
0402.400
0402.500
0402.600
0402.700
0402.800
0402.900
0403.000
0403.100
0403.200
0403.300

```

```

0024 C SWEEP THROUGH THE RADIAL NODES (J).
0025 DO 15 J=J1,JX,ISP
0026 IF (NGROPT.NE.0) ISP=IS(I,J)
0027 C
0028 C INCREMENT THE R-THETA NODE COUNT (L) BY 1.
0029 L=L+1
0030 C
0031 C IF THERMAL CONDUCTIVITY IS BEING CONSIDERED TEMPERATURE DEPENDENT,
0032 THE THERMAL CONDUCTIVITY INTEGRAL DEFINED BY KIRCHOFF'S
0033 TRANSFORMATION IS CALCULATED AND SUBSTITUTED FOR T(L) BECAUSE
0034 KIRCHOFF'S TRANSFORMATION IS USED IN THE 2ND PHASE OF A SYNTHESIS
0035 ANALYSIS, BUT NOT IN A FINITE-DIFFERENCE ANALYSIS, TO LINEARIZE THE
0036 HEAT CONDUCTION EQUATION.
0037 IF (MVCON.EQ.1) TKIRCH=TCINTS*(L)+.5*SLOPS*(T(L)*.2)
0038 C
0039 C RECORD T(L) OR TKIRCH AS THE TEMPERATURE OF THE LTH R-THETA NODE IN
0040 TRIAL FN. NTRF (IE, AS TRF(L,NTRF)).
0041 IF (MVCON.EQ.1) TRF(L,NTRF)=TKIRCH
0042 IF (MVCON.NE.1) TRF(L,NTRF)=T(L)
0043 C
0044 C RECORD THE USER-SPECIFIED CONSTANT 'CONST' AS THE LAST TRIAL FN.
0045 IF MCONST=1 AND NO MORE TRIAL FNS. ARE TO BE COMPUTED (INDICATED
0046 BY NEXT=0).
0047 IF (NEXT.NE.1.AND.MCONST.EQ.1) TRF(L,NTRF+1)=CONST
0048 IF (NEXT.NE.1) NMUM(I,J,1)=L
0049 15 CONTINUE
0050 10 CONTINUE
0051 OUTPUT L
0052 C
0053 C IF MCONST=1 AND NO MORE TRIAL FNS. ARE TO BE COMPUTED, INCREMENT
0054 THE TOTAL TRIAL FN. COUNT BY 1 TO ACCOUNT FOR THE CONSTANT
0055 C RECORDED AS THE LAST TRIAL FN.
0056 IF (NEXT.NE.1.AND.MCONST.EQ.1) NTRF=NTRF+1
0057 IF (NEXT.NE.1.AND.MCONST.EQ.1) WRITE (108,881) NTRF,CONST
0058 881 FORMAT ('CONSTANT TEMPERATURE FOR FINAL TRIAL FUNCTION (NO.,13,
0059 ) =',F10.5,'//,1X,'***** 3-D DATA FOLLOWS *****',/)
0060 RETURN
0061 END

```

```

0001 SUBROUTINE NTRP(KX)
0002 C
0003 C THIS SUBROUTINE STORES VARIABLES NTRI AND NTRK AS NTRIP AND NTRKP
0004 RESPECTIVELY IN PREPARATION FOR CHANGING THE TRIAL FUNCTION
0005 SCHEDULE IN A TRANSIENT SYNTHESIS ANALYSIS.
0006 C
0007 DIMENSION NTRI(65),NTRK(65,10),NTRIP(65),NTRKP(65,10)
0008 COMMON /DB49/NTRI,NTRK,DB44/NTRIP,NTRKP
0009 DO 10 K=1,KX
0010 NTRIP(K)=NTRI(K)
0011 DO 15 NK=1,10
0012 NTRKP(K,NK)=NTRK(K,NK)
0013 15 CONTINUE
0014 RETURN
0015 END
0001 SUBROUTINE ZERO (KX,NTRF,RO,TRANS)
0002 C
0003 C THIS SUBROUTINE ZEROES ENTRIES IN THE MATRICES AL,CB,CRI,RES,RO AND
0004 DIASUM BEFORE THEY ARE USED TO ACCUMULATE WEIGHTED RESIDUALS IN THE
0005 2ND (SUBSTITUTE/WEIGHT/INTEGRATE) PHASE OF A SYNTHESIS ANALYSIS.
0006 C
0007 INTEGER RADNDB,TRANS
0008 DOUBLE PRECISION RES,RO,AL,CB,CRI,CB,Z,DIASUM
0009 DIMENSION NTRI(65),NTRK(65,10),DIASUM(5,5,65)
0010 DIMENSION CB(260),RD(260),Z(260),AL(260,7),CRI(260,7),RES(5,65)
0011 COMMON /DB13/RES /DB15/CB,AL,CRI,CB,Z /DB14/Z /DB17/RES
0012 /DB32/DIASUM /DB41/RADNDB /DB49/NTRI,NTRK
0013 DO 10 I1=1,NEOU
0014 CB(I1)=0.0
0015 RD(I1)=0.0
0016 DO 15 J1=1,NBA
0017 AL(I1,J1)=0.0
0018 CRI(I1,J1)=0.0
0019 15 CONTINUE
0020 10 CONTINUE
0021 DO 20 J1=1,KX
0022 NK=NTRI(J1)
0023 DO 25 I1=1,NK
0024 RES(I1,J1)=0.0
0025 25 CONTINUE
0026 20 CONTINUE
0027 IF (TRANS.NE.1.AND.RADNDB.NE.1) GO TO 35
0028 DO 30 I1=1,KX
0029 NK=NTRI(I1)
0030 DO 30 I2=1,NK
0031 DO 30 I3=1,NK
0032 DIASUM(I3,I2,I1)=0.0
0033 30 RETURN
0034 END

```

```

* ACTUAL PROGRAM SIZE:
* PROGRAM * INITIALIZED VARIABLES = 13926 (WORDS)
* NON-INITIALIZED VARIABLES = 55400 (WORDS)
* TOTAL = 69326 (WORDS)

```

COMPILE TIME = 0036.64 SECONDS

Listing 2 Versions 2 and 3 of Subroutine DATA1 ; Subroutine WIDTH
(Version 3 of Subroutine DATA1 requires Subroutine WIDTH)

```

SUBROUTINE DATA1 (MX,TC,HT,1,CH,1,CH,2,NEXT,MSYN)
      DATA 000
      DATA 001
      DATA 002
      DATA 003
      DATA 004
      DATA 005
      DATA 006
      DATA 007
      DATA 008
      DATA 009
      DATA 010
      DATA 011
      DATA 012
      DATA 013
      DATA 014
      DATA 015
      DATA 016
      DATA 017
      DATA 018
      DATA 019
      DATA 020
      DATA 021
      DATA 022
      DATA 023
      DATA 024
      DATA 025
      DATA 026
      DATA 027
      DATA 028
      DATA 029
      DATA 030
      DATA 031
      DATA 032
      DATA 033
      DATA 034
      DATA 035
      DATA 036
      DATA 037
      DATA 038
      DATA 039
      DATA 040
      DATA 041
      DATA 042
      DATA 043
      DATA 044
      DATA 045
      DATA 046
      DATA 047
      DATA 048
      DATA 049
      DATA 050
      DATA 051
      DATA 052
      DATA 053
      DATA 054
      DATA 055
      DATA 056
      DATA 057
      DATA 058
      DATA 059
      DATA 060
      DATA 061
      DATA 062
      DATA 063
      DATA 064
      DATA 065
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      DATA 067
      DATA 068
      DATA 069
      DATA 070
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      DATA 072
      DATA 073
      DATA 074
      DATA 075
      DATA 076
      DATA 077
      DATA 078
      DATA 079
      DATA 080
      DATA 081
      DATA 082
      DATA 083
      DATA 084
      DATA 085
      DATA 086
      DATA 087
      DATA 088
      DATA 089
      DATA 090
      DATA 091
      DATA 092
      DATA 093
      DATA 094
      DATA 095
      DATA 096
      DATA 097
      DATA 098
      DATA 099
      DATA 100
      DATA 101
      DATA 102
      DATA 103
      DATA 104
      DATA 105
      DATA 106
      DATA 107
      DATA 108
      DATA 109
      DATA 110
      DATA 111
      DATA 112
      DATA 113
      DATA 114

      ***** VERSION 2 *****

      THIS SUBROUTINE READS DATA FOR SPECIFIC 2-D AND 3-D ANALYSES.
      DATA INDICATING WHETHER THE PROBLEM IS TRANSIENT OR STEADY-STATE,
      LINEAR OR NON-LINEAR ARE READ ALONG WITH MATERIAL PROPERTIES,
      TIME STEP SIZES, ETC. THE R-THETA BOUNDARY CONDITIONS AT ALL OF
      THE AXIAL NODES (K) IN THE GRID (IE., FOR K=1,2,...,KX) ARE READ
      AND/OR CALCULATED.

      ***** INPUT VARIABLES READ BY SUBROUTINE DATA1 *****

      TRANS - CONTROL VARIABLE: TRANS=1 INDICATES THAT A TRANSIENT
      ANALYSIS IS REQUIRED. TRANS=0 INDICATES THAT THE
      ANALYSIS WILL BE STEADY-STATE.
      NTRTF - NUMBER OF TRANSIENT TRIAL FUNCTIONS TO BE RECORDED
      IN A 2-D TRANSIENT ANALYSIS.
      NTFCH - NUMBER OF TIMES THE TRIAL FN. DEPLOYMENT SCHEDULE IS
      TO BE CHANGED IN THE 2ND. PHASE OF A TRANSIENT
      SYNTHESIS ANALYSIS.
      NTFIC - NUMBER OF A PREVIOUSLY COMPUTED TRIAL FN. FROM WHICH
      INITIAL CONDITIONS (TEMPERATURES) FOR A TRANSIENT
      AND/OR NON-LINEAR ANALYSIS ARE TO BE TAKEN (CAN ONLY
      BE USED IF NGROPT=2).
      TAPIN - TEMPERATURE TO WHICH ALL OF THE NODAL TEMPS. ARE EQUATED
      AT THE START OF A TRANSIENT ANALYSIS. NOT USED IF
      NGROPT=2 AND NTFIC .GT. 0.
      TFINAL - FINAL TIME TO WHICH A TRANSIENT ANALYSIS IS TO BE TAKEN.
      DENS1 - DENSITY OF MATERIAL AT RADIAL NODES (J) .LE. MX
      DENS2 - DENSITY OF MATERIAL AT RADIAL NODES (J) .GT. MX
      IT IS ASSUMED THAT THE CYLINDRICAL REGIONS TO WHICH
      DENS1 AND DENS2 APPLY ARE SEPARATED BY AN INTERFACE
      OR GAP.
      SPHT1 - SPECIFIC HT. OF MATERIAL AT RADIAL NODES (J) .LE. MX
      SPHT2 - SPECIFIC HT. OF MATERIAL AT RADIAL NODES (J) .GT. MX
      NPTS - NUMBER OF THE TIME STEP AT WHICH A TRANSIENT ANALYSIS
      IS TO STOP.
      NTSFCH(NXT) - NXT=1,2,...,NTFCH - NUMBERS OF THE TIME STEPS AT
      WHICH THE TRIAL FN. DEPLOYMENT SCHEDULE IS TO BE
      CHANGED IN THE SECOND PHASE CALCS. OF A TRANSIENT
      SYNTHESIS ANALYSIS (NOT READ UNLESS MSYN=2 AND NTFCH
      .GT. 0).
      NPTS(ITIN) - ITIN=1,2,...,NTRTF - NUMBERS OF THE TIME STEPS AT
      WHICH TRIAL FNS. ARE TO BE RECORDED IN A 2-D TRANSIENT
      ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FNS. FOR A
      SYNTHESIS ANALYSIS (NOT READ UNLESS SYN=1 AND TRANS
      .EQ. 1 AND MSYN .NE. 2).
      DT(NDT) - NDT TH TIME STEP SIZE.
      NTC(NDT) - NUMBER OF THE TIME STEP AT WHICH THE USE OF TIME STEP
      SIZE DT(NDT) IS DISCONTINUED.
      NEXT - CONTROL VARIABLE: NEXT=1 INDICATES THAT THE STATEMENT
      WHICH READS DT(NDT), NTC(NDT) AND NEXT SHOULD BE
      RE-EXECUTED WITH THE VALUE OF NDT INCREMENTED BY 1
      BECAUSE MORE VALUES OF THOSE VARIABLES ARE SUPPLIED;
      NEXT=0 INDICATES THAT THE LAST SET OF VALUES FOR DT,
      NTC AND NEXT HAS BEEN READ.

      NOTE: VARIABLES FROM TAPIN TO NEXT INCLUSIVE ARE NOT READ IF
      TRANS=0.

      MLIN - NON-LINEAR GAP CONDUCTANCE OPTION: MLIN=1 INDICATES
      THAT A LOECC ANALYSIS WITH NON-LINEAR GAP CONDUCTANCES
      IS TO BE PERFORMED; MLIN=0 INDICATES OTHERWISE.
      RADBN0 - RADIATING CYLINDER OPTION: RADBN0=1 INDICATES THAT
      HEAT TRANSFER IN A CYLINDER RADIATING TO THE
      SURROUNDINGS IS TO BE ANALYSED; RADBN0=0 INDICATES
      OTHERWISE.
      MAXIT - MAXIMUM PERMISSIBLE ITERATION COUNT FOR A NON-LINEAR
      STEADY-STATE ANALYSIS.
      EPSLN - CONVERGENCE PARAMETER FOR A NON-LIN. STEADY-STATE ANAL.
      TRADIN - INITIAL BOUNDARY TEMP. ESTIMATE FOR A STEADY-STATE
      RADIATING CYLINDER ANALYSIS.
      HRADIN - PRESSURE TO C.H. TUBE RADIATIVE HEAT TRANSFER COEFF.-
      USED IN LINEAR (MLIN=0) LOECC ANALYSES AND IN THE 1ST
      ITER. OF A STEADY-STATE NON-LIN. (MLIN=1) FIN.-DIFF.
      LOECC ANALYSIS.
      BETA - UNDERRELAXATION FACTOR USED IN THE LINEARIZATION OF GAP
      CONDUCTANCES IN FIN.-DIFF. LOECC ANALYSES (RECOMMEND
      BETA=0.5).
      TCINTG - INTERCEPT OF LINEAR THERMAL CONDUCTIVITY VS. TEMP.
      REGRESSION FOR GAS IN PRESSURE/CAL. TUBE GAP.
      SLOPB - SLOPE OF REGRESSION FOR WHICH TCINTG IS THE INTERCEPT.
      REXCO - RADIATION EXCHANGE COEFF. FOR RADIATION BETWEEN THE
      PRESSURE AND CAL. TUBES (USUAL FORMULA FOR CONCENTRIC
      CYLINDERS) OR TO SURROUNDINGS IF RADBN0=1.
      MVCON - VARIABLE THERMAL CONDUCTIVITY OPTION: MVCON=1 CAUSES
      COND. OF PRESSURE AND CAL. TUBE MAT'L TO BE TREATED AS
      TEMP. DEPENDENT IN A LOECC ANALYSIS. WHEN MVCON=1, SET
      CX1-CX2=1.0.
      TCINTS - INTERCEPT OF THERMAL COND. VS. TEMP. REGRESSION (ASSUMED
      LINEAR) FOR PT. AND CT. MAT'L.
      SLOPS - SLOPE OF REGRESSION FOR WHICH TCINTS IS THE INTERCEPT.
      TCGAS - THERMAL COND. OF GAS USED IN 'RESISTANCE OF CONTACT'
      AND LINEAR (MLIN=0) LOECC ANALYSES. ALSO USED IN THE
      1ST ITER. OF A ST.-STATE NON-LIN. FIN.-DIFF. LOECC ANAL.
      CCNN - NON-CONTINUUM GAS CONDUCTIVITY PARAMETER (NEEDED IN
      LOECC AND 'RESISTANCE OF CONTACT' ANALYSES.).
      HI - HT. TRANS. COEFF. BETWEEN INNER AND OUTER CYLINDERS IN
      AN ANALYSIS INVOLVING TWO (CONCENTRIC) CYLINDERS. IN A
      LOECC ANALYSIS, HI IS THE HT. TRANSFER COEFF. ON THE
      CAL. TUBE OUTER SURFACE.
      CX1 - THERMAL COND. OF INNER CYL. IN A TWO CYLINDER ANALYSIS.
      CX2 - THERMAL COND. OF OUTER CYL. IN A TWO CYLINDER ANALYSIS.
      CCND - CONTACT CONDUCTANCE USED WITHIN THE CONTACT PATCH IN A
      LOECC OR 'RESISTANCE OF CONTACT' ANALYSIS.
      OUTRY - TRIAL FUNCTION PRINT OPTION: OUTRY=1 CAUSES TRIAL FN.
      RESULTS TO BE PRINTED WITH OUTRY=0, RESULTS ARE NOT
      OUTPUT.
      NEXT - ADDITIONAL TRIAL FN. OPTION: NEXT=1 INDICATES THAT INPUT DATA
      DATA FOR ANOTHER TRIAL FN. ARE PROVIDED; NEXT=0
      INDICATES OTHERWISE.
      NZFLO - AXIAL BOUNDARY CONDITION OPTION: NZFLO=1 INDICATES THAT
      THE TEMP. DISTRIBUTION STORED IN TRIAL FN. NTFCH IS TO
      BE APPLIED OVER AN R-THETA PLANE LOCATED A DISTANCE DZNC

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C      FROM THE PLANE ON WHICH NODAL TEMPS. WFE BEING CALCULATED DATA 115
C      : WHEN NZFLD=0, AN AXIAL BOUNDARY COND. IS NOT APPLIED. DATA 116
C      NTFBC - NUMBER OF TRIAL FN. FROM WHICH AXIAL BOUNDARY TEMPS. DATA 117
C      ARE TO BE TAKEN. DATA 118
C      DZBC - AXIAL DISTANCE SEPARATING BOUNDARY PLANE AND DATA 119
C      COMPUTATIONAL PLANE. DATA 120
C      MAX - CALCULATED R-THETA BOUNDARY CONDITION OPTION : MAX=1 DATA 121
C      CAUSES THE B.C.'S OR GAP CONDUCTANCES AT AXIAL NODES DATA 122
C      K (K=1,2,...,KX) TO BE CALC. FROM THE APPROPRIATE DATA 123
C      GEOMETRIC PARAMETERS IN RESISTANCE OF CONTACT AND DATA 124
C      LOECC ANALYSES. IN A 2-D ANALYSIS (K=1), THE R.C. S DATA 125
C      ARE EVAL. AT THE AXIAL COORD. XPP (SEE BELOW). IF MAX=0, DATA 126
C      THE CIRCUMFERENTIALLY UNIFORM HT. TRANSFER COEFF. HOUT DATA 127
C      (SEE BELOW) IS USED OUTSIDE OF THE CONTACT. DATA 128
C      XPP - AXIAL LOCATION AT WHICH R-THETA B.C.'S ARE TO BE EVAL. DATA 129
C      IN A RESISTANCE OF CONTACT OR LOECC ANALYSIS (NOT DATA 130
C      NEEDED IF MAX=0 AND/OR KX.GT. 1). DATA 131
C      HTGEN(K) - VOLUMETRIC HEAT GENERATION RATE AT AXIAL NODE K. DATA 132
C      HFLZ(K) - HEAT FLUX APPLIED TO THE INNERMOST RADIAL NODES (J=2) AT DATA 133
C      AXIAL NODE K. DATA 134
C      TOUT - SINK TEMP. FOR CIRCUMFERENTIAL (OUTER SURFACE) NODES DATA 135
C      (I).GT. MZ1 DATA 136
C      HOUT - OUTER SURFACE HT. TRANSFER COEFF. USED AT CIRCUMFERENTIAL DATA 137
C      NODES (I).GT. MZ1 DATA 138
C      TEX - SINK TEMP. FOR CIRCUMFERENTIAL (OUTER SURFACE) NODES (I) DATA 139
C      .LE. MZ1 DATA 140
C      HEX - OUTER SURFACE HT. TRANSFER COEFF. USED AT DATA 141
C      CIRCUMFERENTIAL NODES (I).LE. MZ1 DATA 142
C      MZ1 - CIRCUMFERENTIAL NODE AT THE EDGE OF THE ARC OVER WHICH DATA 143
C      THE HT. TRANSFER COEFF. HEX IS PRESCRIBED (USUALLY THE DATA 144
C      EDGE OF A CONTACT PATCH). DATA 145
C      NTRI(K) - NUMBER OF TRIAL FNS. USED AT THE KTH AXIAL NODE (NOT DATA 146
C      READ UNLESS NSYN=2). DATA 147
C      NT2(MK) - (MK=1,2,...,10) TRIAL FNS. USED AT THE KTH. AXIAL NODE DATA 148
C      (NOT READ UNLESS NSYN=2) DATA 149
C      NOTE : VARIABLES HTGEN(K) TO NT2 INCLUSIVE ARE READ FOR K=1,2,...,KX DATA 150
C      DATA 151
C      INTEGER RADBN,TRANS,RFSIST,SYN,OUTRY DATA 152
C      DIMENSION AXPOS(40),HX(39,40),TC(39,40),HTGEN(40),AN(39) DATA 153
C      ,NFTS(10),NT2(10),GGAPP(39,1),NTRI(40),NTRK(40,10),DT(10), DATA 154
C      ,NTC(10),NTSTFC(10),QFLX(1,1),TRAD(39,40),HFLZ(40),NTRIP(40), DATA 155
C      ,NTRKP(40,10),DZAC(40),DZB(40) DATA 156
C      ,NTRW(40) DATA 157
C      COMMON /DB1/RESIST,TO,DR4/IX,IX,KX /DB7/AXPOS /DB17/NEQU DATA 158
C      ,/DB18/HFLZ,CCOND /DB19/SYN,OUTRY,CONST /DB20/BETA,GGAPP /DB26/AN DATA 159
C      ,/DB30/TCINTG,SLOPG,GASC /DB31/TRANS,THPIN,TFINAL,THCAP,DT,NTDT, DATA 160
C      ,NTRC /DB33/TCGAS,CCOM /DB36/THCAP1,THCAP2 /DB37/PTID,PTOD,CTID, DATA 161
C      ,CTOD,DGART,SPRDIA,ECC,CGI,XCM /DB38/IMPECC /DB41/RADBN DATA 162
C      ,/DB42/TRADIN,MAXIT,NTRTF,NFTS,NTFTS /DB43/REXCO /DB44/EPSLN DATA 163
C      ,/DB45/QFLX,TRAD /DB47/NTSTFC,NXT /DB49/NTFC /DB57/MLIN DATA 164
C      ,COMMON /DB64/NTRIP,NTRKP /DB66/HBA /DB69/NTRI,NTRK /DB78/ DATA 165
C      ,STCINTS,SLOPS,SLOPD2,ML,MUCON,MUCON /DB90/RO DATA 166
C      ,/DB98/NZFLD,NTFBC,DZBC,DB27/DZA,DZB,HTGEN DATA 167
C      ,/DB77/A,B,RCY1,RCY2,DBN1W/N1W DATA 168
C      WRITE (108,881) DATA 169
C      DATA 170
C      READ (105,994) TRANS,NTRTF,NTFCH,NTFIC DATA 171
C      DATA 172
C      WRITE (108,890) TRANS,NTRTF,NTFCH,NTFIC DATA 173
C      DATA 174
C      READ DATA FOR A TRANSIENT ANALYSIS. DATA 175
C      DATA 176
C      IF(TRANS-1) 5,25,5 DATA 177
C      DATA 178
C      READ (105,996) THPIN,TFINAL,DENS1,DENS2,SPMT1,SPMT2,NFTS DATA 179
C      DATA 180
C      WRITE (108,882) THPIN,TFINAL,DENS1,DENS2,SPMT1,SPMT2,NFTS DATA 181
C      ,THCAP1=DENS1*SPMT1,THCAP2=DENS2*SPMT2 DATA 182
C      ,IF(SYN.NE.1.OR.(NSYN.EQ.2.AND.NTFCH.EQ.0)) GO TO 40 DATA 183
C      ,IF(NSYN.NE.2) GO TO 42 DATA 184
C      ,WRITE (108,893) DATA 185
C      DATA 186
C      READ NUMBERS OF TIME STEPS AT WHICH THE TRIAL FN. DEPLOYMENT DATA 187
C      SCHEDULE IS TO BE CHANGED. DATA 188
C      DATA 189
C      DO 44 NXT=1,NTFCH DATA 190
C      DATA 191
C      READ (105,995) NTSTFC(NXT) DATA 192
C      DATA 193
C      WRITE (108,894) NXT,NTSTFC(NXT) DATA 194
C      DATA 195
C      CONTINUE DATA 196
C      GO TO 40 DATA 197
C      WRITE (108,891) DATA 198
C      DATA 199
C      READ NUMBERS OF TIME STEPS AT WHICH TRIAL FNS. ARE TO BE RECORDED DATA 200
C      IN A TRANSIENT 2-D ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FNS. DATA 201
C      FOR A SYNTHESIS ANALYSIS. DATA 202
C      DATA 203
C      DO 43 ITIN=1,NTRTF DATA 204
C      DATA 205
C      READ (105,995) NTFTS(ITIN) DATA 206
C      DATA 207
C      WRITE (108,892) ITIN,NTFTS(ITIN) DATA 208
C      DATA 209
C      CONTINUE DATA 210
C      READ TIME STEP SIZES. DATA 211
C      DATA 212
C      NDT=1 DATA 213
C      DATA 214
C      READ (105,997) DT(NDT),NTC(NDT),NEXT DATA 215
C      DATA 216
C      WRITE (108,883) NDT,DT(NDT),NTC(NDT),NEXT DATA 217
C      ,NDT=NDT+1 DATA 218
C      ,IF(NEXT.EQ.1) GO TO 45 DATA 219
C      ,NTDT=NDT-2 DATA 220
C      DATA 221
C      READ GENERAL DATA. DATA 222
C      DATA 223
C      READ (105,990) MLIN,RADBN,MAXIT,EPSLN,TRADIN,HRADIN,BETA, DATA 224
C      ,STCINTG,SLOPG,REXCO DATA 225
C      DATA 226
C      READ (105,980) MUCON,TCINTS,SLOPS DATA 227
C      DATA 228
C      SLOPD2=0.5*SLOPS DATA 229
C      DATA 230
C      READ (105,999) TCGAS,CCOM DATA 231
C      DATA 232

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IF (IMPECC.EQ.1.AND.MLIN.EQ.1.AND.HSYN.EQ.2) TCGAS=TCINTG
WRITE (108,884) MLIN,RADBND,MAXIT,EPSLN,TRADIN,HRADIN,BETA,
TCINTG,SLOPS,REXCO,TCGAS,CONM
WRITE (108,894) MUCON,TCINTS,SLOPS
IF (RADBND.EQ.1.AND.TRANS.EQ.1) TRADIN=TRPIN
C
READ (105,991) M1,CK1,CK2,CCOND,OUTRY,NEXT
C
WRITE (108,885) M1,CK1,CK2,CCOND,OUTRY,NEXT
C
READ (105,981) N7FLO,MTFBC,DZBC
C
WRITE (108,895) N7FLO,MTFBC,DZBC
C
READ (105,993) MAX,XPP
C
WRITE (108,886) MAX,XPP
MBA=0
MEQU=0
C
SWEEP THROUGH THE AXIAL NODES (K=1,2,...,AX)
C
DO 10 K=1,AX
IF (KX.GT.1) XPP=AXPOS(K)
NIM(K)=0
C
READ (105,992) HTGEN(K),HFLZ(K),TOUT,HOUT,TEX,HEX,MZI
C
WRITE (108,887) K,HTGEN(K),TOUT,HOUT,TEX,HEX,MZI
OUTPUT HFLZ(K)
C
READ TRIAL FN. DEPLOYMENT DATA FOR 2ND. PHASE OF SYNTHESIS ANALYSIS.
C
IF (MSYN.NE.2) GO TO 26
WRITE (108,888)
C
READ (105,998) NTRI(K),(NT2(NK),NK=1,10)
C
NTRIP(K)=NTRI(K)
DO 14 NK=1,10
IF (NT2(NK).GT.0) WRITE (108,889) NT2(NK)
NTRK(K,NK)=NT2(NK)
14
C
CALCULATE THE AXIAL COORDINATE AT THE CENTER OF THE SURFACE ELEMENT
FOR NODE K (XMID).
C
XMID=XPP+.25*OZB(K)-.25*OZA(K)
C
CALCULATE THE Y-COORDINATE OF THE EDGE OF THE ELLIPTICAL CONTACT
AT AXIAL LOCATION XMID, DESIGNATED PII.
C
IF (XMID.LT.A) PII=BSORT(1.0-(XMID/A)**2.0)
C
CALCULATE THE MAXIMUM BANDWIDTH (MBA) OF THE LEFT-HAND COEFF. MATRIX
FOR THE SYSTEM OF EQUATIONS DEFINING THE BLENDING VECTOR (Z) IN A
SYNTHESIS ANALYSIS.
C
IF (K.EQ.1) GO TO 27
MBA=NTRI(K)+NTRI(K-1)-1
IF (NMBA.GT.MBA) MBA=NMBA
C
INCREASE THE COUNTER FOR THE NUMBER OF EQUATIONS IN THE COEFF.
MATRIX (MEQU).
C
MEQU=MEQU+NTRI(K)
MMZ=MZ+1
IF (MSYN.EQ.2) MMZ=1
C
ASSIGN HEAT TRANSFER COEFFICIENTS (HX(I,K)) AND SINK TEMPERATURES
(TC(I,K)) TO OUTER SURFACE NODES. IF MAX=1, THE HEAT TRANSFER
COEFFICIENTS AT NODES OUTSIDE OF THE CONTACT ARE CALCULATED IN
SUBROUTINE MBAS. OTHERWISE THEY ARE SET TO HOUT. IF MSYN=2, ALL
OF THE CIRCUMFERENTIAL NODES (I) ARE VISITED. OTHERWISE ONLY THE
NODES OUTSIDE OF THE CONTACT (IE., WITH I.GT.MZI) ARE VISITED.
C
AMVAMP=0.0
DO 15 I=MMZ,IX
IF (MSYN.NE.2) AMNI=AN(I) ; GO TO 75
IF (MSYN.EQ.2.AND.XMID.GT.A) AMNI=AN(I) ; GO TO 75
AMVAMP=AN(I)
IF (I.NE.IX) AMVAMP=(AN(I)+AM(I+1))/2.
C
CALCULATE THE Y-COORDINATES OF THE INNER AND OUTER BOUNDARIES OF
THE SURFACE ELEMENT FOR LATERAL NODE I, DESIGNATED YA AND YB
RESPECTIVELY. ALSO CALCULATE THE Y-COORDINATE OF NODE I ITSELF,
DESIGNATED YI.
C
YA=ROSAVAMP
YB=ROBAVAMP
YI=ROBAN(I)
C
CHECK WHETHER THE SURFACE ELEMENT FOR NODE I IS COMPLETELY OUTSIDE
OF THE CONTACT AND GO TO STATEMENT 75 IF IT IS.
C
IF (PII.LE.YA) GO TO 75
C
IF THE SURFACE ELEMENT FOR NODE I IS COMPLETELY INSIDE OF THE
CONTACT, SET HX(I,K)=HEX AND TC(I,K)=TEX AND GO TO STATEMENT 15.
C
IF (PII.GE.YB) HX(I,K)=HEX ; TC(I,K)=TEX ; GO TO 15
C
THE SURFACE ELEMENT FOR NODE I IS PARTLY COVERED BY THE CONTACT
SO SET NIM(K)=1 AND CALCULATE HX(I,K).
C
NIM(K)=1
MGRID=1.0/(YB/(CCONB*PII) + .634428*YB*ALOG(YB/PII)/CX2)
IF (I.EQ.1) HX(I,K)=MGRID ; TC(I,K)=TEX ; GO TO 15
AS=.634484*SIN(YA/PII)
HX(I,K)=(1.-AS)/(1./MGRID-AS/CCONB)
TC(I,K)=TEX
GO TO 15
IF (MAX-1) 16,17,17
C
CALCULATE THE SURFACE CONDUCTANCE AT SURFACE NODE I, A, WHICH IS
OUTSIDE OF THE CONTACT.
C
RTUB=RO
IF (IMPECC.EQ.1) RTUB=PTOD*0.5

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      YP2=RTUB*8IN(AM(I))
      IF(IMPECC.NE.1) GO TO 50
      IF(XPP-XCH) 59,51,51
      IF(AM(I)-0.2) 50,51,51
59      GAP7=CR18(XPP882)*8(0.58(DGART882)-(XPP882))
51      ECC1=ECC-GAP7
      GGAP=-ECC1*8COS(AM(I))+8SORT((ECC1*8COS(AM(I)))*882-ECC1*8)
      C11D2,01882)-RTUB
      AP=CCNN/GGAP
      HOUT=TCGAS/((1.0+AP)*8GGAP)
      GO TO 21
C
C CALL SUBR. HGAS TO CALCULATE THE GAS CONDUCTANCE (HOUT) AND GAP
C THICKNESS (GGAP) AT LATERAL COORDINATE YP2 AND AXIAL COORDINATE XPP.
50      CALL HGAS (XPP,YP2,ICUT,GGAP)
21      IF(MLIN.EQ.1.AND.NSYN.EQ.2) GO TO 22
      IF(IMPECC.EQ.1) HOUT=HOUT+HRADIN
22      IF(RESIST.EQ.1) HOUT=HOUT*8COS(AM(I))
16      HX(I,K)=HOUT
      IF(NSYN.NE.2) GGAP(I,K)=GGAP
      TC(I,K)=TOUT
15      AVANGP=AVANG
      IF(NSYN.EQ.2) GO TO 23
85      IF(MZ1.LT.1) GO TO 23
C
C SWEEP THROUGH CIRCUMFERENTIAL MODES (I) INSIDE OF THE CONTACT AND
C ASSIGN HEAT TRANSFER COEFFICIENTS (HX(I,K)) AND SINK TEMPERATURES
C (TC(I,K)) TO THOSE MODES (IF NECESSARY).
C
      DO 20 I=1,MZ1
      HX(I,K)=HMX
      TC(I,K)=TEX
20      CONTINUE
C
C INITIALIZE BOUNDARY CONDITIONS FOR A RADIATING CYLINDER ANALYSIS.
C
23      IF(RADBND-1) 10,30,10
30      DO 35 I=1,IX
      QFAX(I,K)=REXCOS((TRADIN884)-(TC(I,K)884))
35      TRAD(I,K)=TRADIN
10      CONTINUE
980      FORMAT (15,2E12.5)
981      FORMAT (215,F10.5)
990      FORMAT (315,4F10.5/3E12.4)
991      FORMAT (4F10.5,215)
992      FORMAT (6F10.5,15)
993      FORMAT (13,F10.5)
995      FORMAT (13)
996      FORMAT (6F10.5,15)
994      FORMAT (415)
997      FORMAT (F10.5,215)
998      FORMAT (1115)
999      FORMAT (2E10.4)
881      FORMAT ('***** DATA READ IN SUBROUTINE DATA1 *****',/)
882      FORMAT (1X,'TMIN='F10.5,10X,'TFINAL='F10.5,10X,'DENS1='F10.5,10X,
10X,'DENS2='F10.5,10X,'SPHT1='F10.5,10X,'SPHT2='F10.5,10X,
10X,'NTFS='F10.5,10X,
883      FORMAT (1X,'NBT='F10.5,10X,'DT(NBT)'F10.5,10X,'NTC(NBT)'F10.5,10X,
10X,'NEXT='F10.5,10X,
884      FORMAT (1X,'MLIN='F10.5,10X,'RADBND='F10.5,10X,'MAXIT='F10.5,10X,
10X,'EPSLN='F10.5,10X,'TRADIN='F10.5,10X,'HRADIN='F10.5,10X,
10X,'BETA='F10.5,10X,'TCINTB='F10.5,10X,'SLOPB='F10.5,10X,
10X,'REXCOS='F10.5,10X,'TCGAS='F10.5,10X,
10X,'CCNN='F10.5,10X,
885      FORMAT (1X,'M1='F10.5,10X,'CK1='F10.5,10X,'CK2='F10.5,10X,'CCONDDATA'
10X,'F10.5,10X,'OUTRY='F10.5,10X,'NEXT='F10.5,10X,
886      FORMAT (1X,'MAX='F10.5,10X,'XPP='F10.5,10X,
887      FORMAT (1X,'OK='F10.5,10X,'HTGEN(K)'F10.5,10X,'TOUT='F10.5,10X,
10X,'F7.2,5X,'TEX='F7.2,5X,'HMX='F10.5,10X,'MZ1='F10.5,10X,
888      FORMAT (10X,'TRIAL FUNCTIONS',/)
889      FORMAT (16X,12)
890      FORMAT (1X,'TRANS='F10.5,10X,'NTRF='F10.5,10X,'NTFC='F10.5,10X,
10X,'NTFC='F10.5,10X,
891      FORMAT (1X,'TIME STEPS AT WHICH TRIAL FNS. ARE RECORDED FOLLOW',/)
892      FORMAT (1X,'NTFTS('F10.5,10X,'F10.5,10X,
893      FORMAT (1X,'TIME STEPS AT WHICH TRIAL FNS. ARE CHANGED FOLLOW',/)
894      FORMAT (1X,'NTSTFC('F10.5,10X,'F10.5,10X,
895      FORMAT (1X,'NZFLO='F10.5,10X,'NTFBC='F10.5,10X,'BZBC='F10.5,10X,
896      FORMAT (1X,'MVCOM='F10.5,10X,'TCINTS='F10.5,10X,'SLOPB='F10.5,10X,
10X,'E12.5,10X,
      RETURN
      END

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SUBROUTINE DATA1 (HX,TC,MI,CK1,CP,NTXT,MSYN)

***** VERSION 3 *****

THIS SUBROUTINE READS DATA FOR SPECIFIC 2-D AND 3-D ANALYSES. DATA INDICATING WHETHER THE PROBLEM IS TRANSIENT OR STEADY-STATE, LINEAR OR NON-LINEAR ARE READ ALONG WITH MATERIAL PROPERTIES, TIME STEP SIZES, ETC. THE R-THETA BOUNDARY CONDITIONS AT ALL OF THE AXIAL MODES (K) IN THE GRID (I.E., FOR K=1,2,...,KX) ARE READ AND/OR CALCULATED.

***** INPUT VARIABLES READ BY SUBROUTINE DATA1 *****

TRANS - CONTROL VARIABLE: TRANS=1 INDICATES THAT A TRANSIENT ANALYSIS IS REQUIRED; TRANS=0 INDICATES THAT THE ANALYSIS WILL BE STEADY-STATE.
 NTRTF - NUMBER OF TRANSIENT TRIAL FUNCTIONS TO BE RECORDED IN A 2-D TRANSIENT ANALYSIS.
 NTFCH - NUMBER OF TIMES THE TRIAL FN. DEPLOYMENT SCHEDULE IS TO BE CHANGED IN THE 2ND. PHASE OF A TRANSIENT SYNTHESIS ANALYSIS.
 NTFIC - NUMBER OF A PREVIOUSLY COMPUTED TRIAL FN. FROM WHICH INITIAL CONDITIONS (TEMPERATURES) FOR A TRANSIENT AND/OR NON-LINEAR ANALYSIS ARE TO BE TAKEN (CAN ONLY BE USED IF MROPT=2).
 TNPIN - TEMPERATURE TO WHICH ALL OF THE NODAL TEMPS. ARE EQUATED AT THE START OF A TRANSIENT ANALYSIS. NOT USED IF MROPT=2 AND NTFIC .GT. 0.
 TFINAL - FINAL TIME TO WHICH A TRANSIENT ANALYSIS IS TO BE TAKEN.
 DENS1 - DENSITY OF MATERIAL AT RADIAL MODES (J) .I.E., HX
 DENS2 - DENSITY OF MATERIAL AT RADIAL MODES (J) .GT. HX. IT IS ASSUMED THAT THE C/LINDRICAL REGIONS TO WHICH DENS1 AND DENS2 APPLY ARE SEPARATED BY AN INTERFACE OR GAP.
 SPMT1 - SPECIFIC HT. OF MATERIAL AT RADIAL MODES (J) .LE. HX
 SPMT2 - SPECIFIC HT. OF MATERIAL AT RADIAL MODES (J) .GT. HX
 NTFB - NUMBER OF THE TIME STEP AT WHICH A TRANSIENT ANALYSIS IS TO STOP.
 NTSTFC(NXT) - NXT=1,2,...,NTFCH - NUMBERS OF THE TIME STEPS AT WHICH THE TRIAL FN. DEPLOYMENT SCHEDULE IS TO BE CHANGED IN THE SECOND PHASE CALCS. OF A TRANSIENT SYNTHESIS ANALYSIS (NOT READ UNLESS MSYN=2 AND NTFCH .GT. 0)
 NTFST(ITIM) - ITIM=1,2,...,NTRTF - NUMBERS OF THE TIME STEPS AT WHICH TRIAL FNS. ARE TO BE RECORDED IN A 2-D TRANSIENT ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FNS. FOR A SYNTHESIS ANALYSIS (NOT READ UNLESS SYN=1 AND TRANS .EQ. 1 AND MSYN .NE. 2).
 DT(MDT) - NOT IN TIME STEP SIZE.
 NTC(MDT) - NUMBER OF THE TIME STEP AT WHICH THE USE OF TIME STEP SIZE DT(MDT) IS DISCONTINUED.
 NEX - CONTROL VARIABLE: NEX=1 INDICATES THAT THE STATEMENT WHICH READS DT(MDT), NTC(MDT) AND NEX SHOULD BE RE-EXECUTED WITH THE VALUE OF DT INCREMENTED BY 1 BECAUSE MORE VALUES OF THOSE VARIABLES ARE SUPPLIED; NEX=0 INDICATES THAT THE LAST SET OF VALUES FOR DT, NTC AND NEX HAS BEEN READ.

NOTE: VARIABLES FROM TNPIN TO NEX INCLUSIVE ARE NOT READ IF TRANS=0

MLIN - NON-LINEAR GAP CONDUCTANCE OPTION: MLIN=1 INDICATES THAT A LOECC ANALYSIS WITH NON-LINEAR GAP CONDUCTANCES IS TO BE PERFORMED; MLIN=0 INDICATES OTHERWISE.
 RADMB - RADIATING CYLINDER OPTION: RADMB=1 INDICATES THAT HEAT TRANSFER IN A CYLINDER RADIATING TO THE SURROUNDINGS IS TO BE ANALYSED; RADMB=0 INDICATES OTHERWISE.
 MAXIT - MAXIMUM PERMISSIBLE ITERATION COUNT FOR A NON-LINEAR STEADY-STATE ANALYSIS.
 EPSLN - CONVERGENCE PARAMETER FOR A NON-LIN. STEADY-STATE ANAL.
 TPRADIN - INITIAL BOUNDARY TEMP. ESTIMATE FOR A STEADY-STATE RADIATING CYLINDER ANALYSIS.
 HRADIN - PRESSURE TO CAL. TUBE RADIATIVE HEAT TRANSFER COEFF.- USED IN LINEAR (MLIN=0) LOECC ANALYSES AND IN THE 1ST ITER. OF A STEADY-STATE NON-LIN. (MLIN=1) FIN.-DIFF. LOECC ANALYSIS.
 BETA - UNDERRELAXATION FACTOR USED IN THE LINEARIZATION OF GAP CONDUCTANCES IN FIN.-DIFF. LOECC ANALYSES (RECOMMEND BETA=.5)
 TCINTS - INTERCEPT OF LINEAR THERMAL CONDUCTIVITY VS. TEMP. REGRESSION FOR GAS IN PRESSURE/CAL. TUBE GAP.
 SLOPS - SLOPE OF REGRESSION FOR WHICH TCINTS IS THE INTERCEPT.
 REXCO - RADIATION EXCHANGE COEFF. FOR RADIATION BETWEEN THE PRESSURE AND CAL. TUBES. (USUAL FORMULA FOR CONCENTRIC CYLINDERS) OR TO SURROUNDINGS IF RADMB=1.
 MVCON - VARIABLE THERMAL CONDUCTIVITY OPTION: MVCON=1 CAUSES COND. OF PRESSURE AND CAL. TUBE MAT'L TO BE TREATED AS TEMP. DEPENDENT IN A LOECC ANALYSIS. WHEN MVCON=1, SFT CK1=CK2=1.0
 TCINTS - INTERCEPT OF THERMAL COND. VS. TEMP. REGRESSION (ASSUMED LINEAR) FOR PT. AND CT. MAT'L.
 SLOPS - SLOPE OF REGRESSION FOR WHICH TCINTS IS THE INTERCEPT.
 TCGAS - THERMAL COND. OF GAS - USED IN 'RESISTANCE OF CONTACT' AND LINEAR (MLIN=0) LOECC ANALYSES. ALSO USED IN THE 1ST ITER. OF A ST.-STATE NON-LIN. FIN.-DIFF. LOECC ANAL.
 CCNN - NON-CONTINUUM GAS CONDUCTIVITY PARAMETER (NEEDED IN LOECC AND 'RESISTANCE OF CONTACT' ANALYSES.).
 H1 - HT. TRANS. COEFF. BETWEEN INNER AND OUTER CYLINDERS IN AN ANALYSIS INVOLVING TWO (CONCENTRIC) CYLINDERS. IN A LOECC ANALYSIS, H1 IS THE HT. TRANSFER COEFF. ON THE CAL. TUBE OUTER SURFACE.
 CK1 - THERMAL COND. OF INNER CYL. IN A TWO CYLINDER ANALYSIS.
 CK2 - THERMAL COND. OF OUTER CYL. IN A TWO CYLINDER ANALYSIS.
 CCONB - CONTACT CONDUCTANCE USED WITHIN THE CONTACT PATCH IN A LOECC OR 'RESISTANCE OF CONTACT' ANALYSIS.
 OUTRY - TRIAL FUNCTION PRINT OPTION: OUTRY=1 CAUSES TRIAL FN. RESULTS TO BE PRINTED WITH OUTRY=0, RESULTS ARE NOT OUTPUT.
 NEX - ADDITIONAL TRIAL FN. OPTION: NEX=1 INDICATES THAT INPUT DATA FOR ANOTHER TRIAL FN. ARE PROVIDED; NEX=0 INDICATES OTHERWISE.
 NZFLO - AXIAL BOUNDARY CONDITION OPTION: NZFLO=1 INDICATES THAT THE TEMP. DISTRIBUTION STORED IN TRIAL FN. NTFB IS TO BE APPLIED OVER AN R-THETA PLANE LOCATED A DISTANCE DZBC

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FROM THE PLANE ON WHICH MODAL TEMPS. ARE BEING CALCULATED
: WHEN NZFLO=0, AN AXIAL BOUNDARY COND. IS NOT APPLIED.
NTFBC - NUMBER OF TRIAL FNS. FROM WHICH AXIAL BOUNDARY TEMPS.
      ARE TO BE TAKEN.
DZBC - AXIAL DISTANCE SEPARATING BOUNDARY PLANE AND
      COMPUTATIONAL PLANE.
MAX - CALCULATED R-THETA BOUNDARY CONDITION OPTION: MAX=1
      CAUSES THE B.C.'S OR GAP CONDUCTANCES AT AXIAL MODES
      K (K=1,2,...,KX) TO BE CALC. FROM THE APPROPRIATE
      GEOMETRIC PARAMETERS IN "RESISTANCE OF CONTACT" AND
      LOECC ANALYSES. IN A 2-D ANALYSIS (KX=1) THE B.C.'S
      ARE EVAL. AT THE AXIAL COORD. XPP (SEE BELOW). IF MAX=0,
      THE CIRCUMFERENTIALLY UNIFORM HT, TRANSFER COEFF. HOUT
      (SEE BELOW) IS USED OUTSIDE OF THE CONTACT.
XPP - AXIAL LOCATION AT WHICH R-THETA B.C.'S ARE TO BE EVAL.
      IN A "RESISTANCE OF CONTACT" OR LOECC ANALYSIS (NOT
      NEEDED IF MAX=0 AND/OR KX .GT. 1).
HTGEN(K) - VOLUMETRIC HEAT GENERATION RATE AT AXIAL MODE K.
HFLZ(K) - HEAT FLUX APPLIED TO THE INNERMOST RADIAL MODES (J=2) AT
      AXIAL MODE K.
TOUT - SINK TEMP. FOR CIRCUMFERENTIAL (OUTER SURFACE) MODES
      (I) .GT. NZ1
HOUT - OUTER SURFACE HT, TRANSFER COEFF. USED AT CIRCUMFERENTIAL
      MODES (I) .GT. NZ1
TEX - SINK TEMP. FOR CIRCUMFERENTIAL (OUTER SURFACE) MODES (I)
      .LE. NZ1
HEX - OUTER SURFACE HT, TRANSFER COEFF. USED AT
      CIRCUMFERENTIAL MODES (I) .LE. NZ1
NZ1 - CIRCUMFERENTIAL MODE AT THE EDGE OF THE ARC OVER WHICH
      THE HT, TRANSFER COEFF. HEX IS PRESCRIBED (USUALLY THE
      EDGE OF A CONTACT PATCH).
NTRI(K) - NUMBER OF TRIAL FNS. USED AT THE KTH AXIAL MODE (NOT
      READ UNLESS NSYN=2).
NT2(NK) - (NK=1,2,...,10) TRIAL FNS. USED AT THE KTH. AXIAL MODE
      (NOT READ UNLESS NSYN=2)

NOTE : VARIABLES HTGEN(K) TO NT2 INCLUSIVE ARE READ FOR K=1,2,...,KX

      INTEGER RADND,TRANS,RESIST,SYN,OUTRY
      DIMENSION AXPOS(40),HX(39,40),TC(39,40),HTGEN(40),AM(39)
      $NMTFS(10),NMT2(10),GGAPP(39,1),NTRI(40),NTRK(40,10),DT(10),
      $NTRC(10),NTRTFC(10),OFLX( 1, 1),TRAD(39,40),HFLZ(40),NTRIP(40),
      $NTRKP(40,10),DZA(40),DZB(40)
      $NIM(40)
      COMMON /DB1/RESIST,TO /DB4/IX,JX,KX /DB7/AXPOS /DB17/MEQU
      $/DB18/HFLZ,CCOND /DB19/SYN,OUTRY,COMST /DB20/BETA,GGAPP /DB26/AM
      $/DB30/TCINTG,SLOPB,GASC /DB31/TRANS,TMPIN,TFINAL,THCAP,DT,MYDT,
      $NTRC /DB33/TCGAS,CCOM /DB36/THCAP1,THCAP2 /DB37/PTID,PTOD,CTID,
      $CTOD,DSART,SPRDI,ECC,CGI,XCH /DB38/IMPECC /DB41/RADND
      $/DB42/TRADIN,MAXIT,NTRTF,NTRF,NTRFS /DB43/REXCO /DB44/EPSLN
      $/DB45/OFLX,TRAD /DB47/NTRTFC,NXT /DB49/NTRIC /DB57/NLIN
      COMMON /DB64/NTRIP,NTRKP /DB66/HBA /DB69/NTRI,NTRK /DB78/
      $TCINTS,SLOPS,SLOPD2,NL,NVC,MVCOM /DB90/RO
      $/DB80/NZFLO,NTFBC,DZBC,DB27/DZA,DZB,HTGEN
      $/DB77/A,B,RCY,RCY2,DBNIM/NIM
      WRITE (108,881)

      READ (105,994) TRANS,NTRTF,NTRFN,NTRIC
      WRITE (108,890) TRANS,NTRTF,NTRFN,NTRIC

      READ DATA FOR A TRANSIENT ANALYSIS.
      IF(TRANS-1) 5,25,5
      READ (105,996) TMPIN,TFINAL,DENS1,DENS2,SPMT1,SPMT2,NFTS
      WRITE (108,882) TMPIN,TFINAL,DENS1,DENS2,SPMT1,SPMT2,NFTS
      THCAP1=DENS1$SPMT1 + THCAP2=DENS2$SPMT2
      IF(SYN.NE.1.OR.(NSYN.EQ.2.AND.NTRFN.EQ.0)) GO TO 40
      IF(NSYN.NE.2) GO TO 42
      WRITE (108,893)

      READ NUMBERS OF TIME STEPS AT WHICH THE TRIAL FNS. DEPLOYMENT
      SCHEDULE IS TO BE CHANGED.
      DO 44 NXT=1,NTRFN
      READ (105,995) NTRTFC(NXT)
      WRITE (108,894) NXT,NTRTFC(NXT)
      CONTINUE
      GO TO 40
      WRITE (108,891)

      READ NUMBERS OF TIME STEPS AT WHICH TRIAL FNS. ARE TO BE RECORDED
      IN A TRANSIENT 2-D ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FNS.
      FOR A SYNTHESIS ANALYSIS.
      DO 43 ITIN=1,NTRTF
      READ (105,997) NTRFS(ITIN)
      WRITE (108,892) ITIN,NTRFS(ITIN)
      CONTINUE

      READ TIME STEP SIZES.
      NBT=1
      READ (105,997) DT(NBT),NTRC(NBT),NEXT
      WRITE (108,883) NBT,DT(NBT),NTRC(NBT),NEXT
      NBT=NBT+1
      IF(NEXT.EQ.1) GO TO 45
      NBT=NBT-2

      READ GENERAL DATA.
      READ (105,998) NLIN,RADND,MAXIT,EPSLN,TRADIN,MRADIN,BETA,
      $TCINTG,SLOPB,REXCO
      READ (105,998) MVCOM,TCINTS,SLOPS
      SLOPD2=0.5$SLOPS
      READ (105,999) TCGAS,CCOM

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      IF (IMPECC.EQ.1.AND.MLIN.EQ.1.AND.MSYN.EQ.2) TCRAS=TCINTG
      WRITE (108,884) MLIN,RADBD,MAXIT,EPSSLN,TRADIN,HRADIN,BETA,
      TCINTG,SLOPG,REXCN,TCGAS,CCON
      WRITE (108,896) MVCON,TCINTS,SLOPS
      IF (RADBD.EQ.1.AND.TFANS.FG.1) TFADIN=THPIN
C
      READ (105,991) MI,CK1,CK2,CCOND,OUTRY,NEXT
C
      WRITE (108,885) MI,CK1,CK2,CCOND,OUTRY,NEXT
C
      READ (105,981) NZFLO,MTFBC,DZBC
C
      WRITE (108,895) NZFLO,MTFBC,DZBC
C
      READ (105,993) MAX,XPP
C
      WRITE (108,886) MAX,XPP
      HBA=0
      NEQU=0
C
C SWEEP THROUGH THE AXIAL NODES (K=1,2,...,KX)
C
      DO 10 K=1,KX
      IF (X.GT.1) XPP=AXPOS(K)
      NIW(K)=0
C
      READ (105,992) HTGEN(K),HFLZ(K),TOUT,HOUT,TEX,HEX,MZ1
C
      WRITE (108,887) K,HTGEN(K),TOUT,HOUT,TEX,HEX,MZ1
      OUTPUT HFLZ(K)
C
      READ TRIAL FN. DEPLOYMENT DATA FOR 2ND. PHASE OF SYNTHESIS ANALYSIS.
C
      IF (MSYN.EQ.2) GO TO 26
      WRITE (108,888)
C
      READ (105,998) NTRI(K),(NT2(NK),NK=1,10)
C
      NTRIP(K)=NTRI(K)
      DO 14 NK=1,10
      IF (NT2(NK).GT.0) WRITE (108,889) NT2(NK)
      NTRK(K,NK)=NT2(NK)
14
C
      CALCULATE THE AXIAL COORDINATE AT THE CENTER OF THE SURFACE ELEMENT
      FOR NODE K (XNID).
C
      XNID=XPP+.25*QZB(K)-.25*DZA(K)
C
      CALCULATE THE Y-COORDINATE OF THE EDGE OF THE ELLIPTICAL CONTACT
      AT AXIAL LOCATION XNID, DESIGNATED PII.
C
      IF (XNID.LT.A) PII=BSORT(1.0-(XNID/A)**2.0)
C
      CALCULATE THE MAXIMUM BANDWIDTH (HBA) OF THE LEFT-HAND COEFF. MATRIX
      FOR THE SYSTEM OF EQUATIONS DEFINING THE BLENDING VECTOR (Z) IN A
      SYNTHESIS ANALYSIS.
C
      IF (K.EQ.1) GO TO 27
      HBA=NTRI(K)+NTRI(K-1)-1
      IF (HBA.GT.HBA) HBA=HBA
C
      INCREASE THE COUNTER FOR THE NUMBER OF EQUATIONS IN THE COEFF.
      MATRIX (NEQU).
C
      NEQU=NEQU+NTRI(K)
27
      HNZ=MZ1+1
      IF (MSYN.EQ.2) HNZ=1
C
      ASSIGN HEAT TRANSFER COEFFICIENTS (HX(I,K)) AND SINK TEMPERATURES
      (TC(I,K)) TO OUTER SURFACE NODES. IF MAX=1, THE HEAT TRANSFER
      COEFFICIENTS AT NODES OUTSIDE OF THE CONTACT ARE CALCULATED IN
      SUBROUTINE HBA8; OTHERWISE THEY ARE SET TO HOUT. IF MSYN=2, ALL
      OF THE CIRCUMFERENTIAL NODES (I) ARE VISITED; OTHERWISE ONLY THE
      NODES OUTSIDE OF THE CONTACT (IE., WITH I.GT.MZ1) ARE VISITED.
C
      AVANBP=0.0
      DO 15 I=HNZ,IX
      IF (MSYN.EQ.2) ANNI=AN(I) GO TO 75
      IF (MSYN.EQ.2.AND.XNID.GT.A) ANNI=AN(I) GO TO 75
      AVANB=AN(I)
      IF (I.NE.IX) AVANB=(AN(I)+AN(I+1))/2.
C
      CALCULATE THE Y-COORDINATES OF THE INNER AND OUTER BOUNDARIES OF
      THE SURFACE ELEMENT FOR LATERAL NODE I, DESIGNATED YA AND YB
      RESPECTIVELY. ALSO CALCULATE THE Y-COORDINATE OF NODE I ITSELF,
      DESIGNATED YI.
C
      YA=ROB*AVANBP
      YB=ROB*AVANB
      YI=ROB*AN(I)
C
      CHECK WHETHER THE SURFACE ELEMENT FOR NODE I IS COMPLETELY OUTSIDE
      OF THE CONTACT AND GO TO STATEMENT 75 IF IT IS.
C
      IF (PII.LE.YA) GO TO 75
C
      IF THE SURFACE ELEMENT FOR NODE I IS COMPLETELY INSIDE OF THE
      CONTACT, SET TC(I,K)=TEX AND GO TO STATEMENT 15.
C
      IF (PII.GE.YB) TC(I,K)=TEX GO TO 15
C
      THE SURFACE ELEMENT FOR NODE I IS PARTLY COVERED BY THE CONTACT
      SO SET NIW(K)=1 AND CALCULATE HX(I,K).
C
      NIW(K)=1
      IF (I.EQ.1) HCONB=CCOND; PIC=PII GO TO 80
C
      CALL SUBROUTINE NIWTH TO CALCULATE THE LARGER CONTACT SIZE (PIC)
      AND THE CONTACT CONDUCTANCE THAT MUST BE PRESCRIBED OVER IT (HAI)
      TO SIMULATE THE ACTUAL (SMALLER) CONTACT OF HALF-WIDTH PII.
C
      CALL NIWTH (YA,YB,PII,CK2,CCOND,HAI,PIC)
C
      CALCULATE THE CONTACT CONDUCTANCE THAT MUST BE PRESCRIBED OVER THE
      SURFACE ELEMENTS FOR LATERAL NODES UP TO BUT NOT INCLUDING NIW(K)
      (IE., HCONB).
C
      HCONB=CCOND*HAI/((CCOND*PII+HAI*PIC)
      HBRID=1.0/(YB/(HCONB*PIC) + .63662*YB*ALOG(YB/PIC)/CK2)
      BO

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      IF(I.EQ.1) HX(I,K)=HGRID ; TC(I,K)=TEX ; GO TO 15
      AS=.6366ASIN(YA/FIC)
      HX(I,K)=(1.-AS)/(1./HGRID-AS/HCONB)
      TC(I,K)=TEX
      GO TO 15
      IF(MAX-1) 16,17,17
75  CALCULATE THE SURFACE CONDUCTANCE AT SURFACE NODE I,K, WHICH IS
      OUTSIDE OF THE CONTACT.
17  RTUB=RO
      IF(IMPECC.EQ.1) RTUB=PTOD*0.5
      YP2=RTUB*ASIN(AN(I))
      IF(IMPECC.NE.1) GO TO 50
      IF(XPP-XCM) 59,51,51
59  IF(AN(I)-0.2) 50,51,51
51  GAP=CG1(XPP*2)*0.58(DGART*2)-(XPP*2)
      ECC=ECC-GAP
      GGAP=-ECC*18COS(AN(I))*SORT((ECC*18COS(AN(I))))*ECC*2
      (CTID/2.0)*2-RTUB
      KP=CONW/GAP
      HOUT=TCGAS/((1.0+KP)*GGAP)
      GO TO 21
      CALL SUBR, HGAS TO CALCULATE THE GAS CONDUCTANCE (HOUT) AND GAP
      THICKNESS (GGAP) AT LATERAL COORDINATE YP2 AND AXIAL COORDINATE XPP.
50  CALL HGAS, XPP, YP2, HOUT, GGAP
21  IF(MLIN.EQ.1.AND.NSYN.EQ.2) GO TO 22
      IF(IMPECC.EQ.1) HOUT=HOUT*HRADIN
22  IF(RESIST.EQ.1) HOUT=HOUT*CDOS(AN(I))
16  HX(I,K)=HOUT
      IF(NSYN.NE.2) GGAP(I,K)=GGAP
      TC(I,K)=TOUT
15  AVAMP=HAMP
      IF(NSYN.NE.2) GO TO 85
      IF(NIM(K).LE.1) GO TO 23
      SET HX(I,K)=HCONB AT LATERAL NODES (II) .LT. NIM(K) (THESE
      NODES ARE ALL WITHIN THE CONTACT).
      NIF=NIM(K)-1
      DO 70 I=1,NIF
70  HX(I,K)=HCONB
      GO TO 23
      IF(MZ1.LT.1) GO TO 23
      SWEEP THROUGH CIRCUMFERENTIAL NODES (I) INSIDE OF THE CONTACT AND
      ASSIGN HEAT TRANSFER COEFFICIENTS (HX(I,K)) AND SINK TEMPERATURES
      (TC(I,K)) TO THOSE NODES (IF NECESSARY).
      DO 20 I=1,MZ1
20  HX(I,K)=HEX
      TC(I,K)=TEX
      CONTINUE
      INITIALIZE BOUNDARY CONDITIONS FOR A RADIATING CYLINDER ANALYSIS.
      IF(RADIND-1) 10,30,10
      DO 35 I=1,IX
35  DFLX(I,K)=REXCON((TRADIN*4)-(TC(I,K)*4))
      TRAD(I,K)=TRADIN
      CONTINUE
10  FORMAT (15,2E12.5)
980  FORMAT (215,F10.5)
981  FORMAT (315,4F10.5/3E12.4)
990  FORMAT (4F10.5,215)
991  FORMAT (4F10.5,15)
992  FORMAT (15,F10.5)
993  FORMAT (15)
994  FORMAT (4F10.5,15)
995  FORMAT (415)
996  FORMAT (F10.5,215)
997  FORMAT (1115)
998  FORMAT (2E10.4)
999  FORMAT ('***** DATA READ IN SUBROUTINE DATA1 *****')
881  FORMAT (1X, 'TIN=' ,F10.5,10X, 'TFIN=' ,F10.5,10X, 'DEN1=' ,F10.5,
882  10X, 'DEN2=' ,F10.5,10X, 'SPMT1=' ,F10.5,10X, 'SPMT2=' ,F10.5,10X,
      10X, 'NTS=' ,15,/)
883  FORMAT (1X, 'NBT=' ,13,10X, 'BT(NBT)' ,F10.5,10X, 'NTC(NBT)' ,14,10X,
      10X, 'NXT=' ,13,/)
884  FORMAT (1X, 'MLN=' ,12,7X, 'RABND=' ,12,7X, 'MAXIT=' ,13,7X,
      10X, 'EPSLN=' ,F10.5,7X, 'TRADIN=' ,F10.5,10X, 'HRADIN=' ,F10.5,7X,
      10X, 'BETA=' ,F10.5,7X, 'TCINTB=' ,E12.4,7X, 'SLOP6=' ,E12.4,10X,
      10X, 'REXCO=' ,E12.4,7X, 'TCBAS=' ,E10.4,7X,
      10X, 'CCON=' ,E10.4,/)
885  FORMAT (1X, 'M=' ,F10.2,10X, 'CK1=' ,F10.5,10X, 'CK2=' ,F10.5,9X, 'CCONDATA' ,F10.5,
      10X, 'OUTRY=' ,12,10X, 'NXT=' ,12,/)
886  FORMAT (1X, 'MAX=' ,12,10X, 'XPP=' ,F10.5,/)
887  FORMAT ('OK=' ,13,5X, 'NTDEN(K)' ,F12.1,5X, 'TOUT=' ,F7.2,5X, 'HOUT=' ,
      10X, 'TEX=' ,F7.2,5X, 'HEX=' ,F10.1,5X, 'MZ1=' ,13,/)
888  FORMAT (10X, 'TRIAL FUNCTIONS' ,/)
889  FORMAT (14X,12)
890  FORMAT (1X, 'TRANS=' ,15,7X, 'NTRTF=' ,15,7X, 'NTFCN=' ,13,7X,
      10X, 'NTFIC=' ,13,/)
891  FORMAT (1X, 'TIME STEPS AT WHICH TRIAL FNS. ARE RECORDED FOLLOW' ,/)
892  FORMAT (1X, 'NTF1=' ,12,10X, 'NTF2=' ,13,/)
893  FORMAT (1X, 'TIME STEPS AT WHICH TRIAL FNS. ARE CHANGED FOLLOW' ,/)
894  FORMAT (1X, 'NTFPC(' ,12,10X, 'NTF1=' ,13,/)
895  FORMAT (1X, 'MZPLO=' ,12,10X, 'NTFBC=' ,13,10X, 'BZBC=' ,F10.5,/)
      SET IZ=1
      RETURN
      END

```

DATA0351
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 DATA0449
 DATA0450
 DATA0451
 DATA0452
 DATA0453

NAME = WIDTH

SUBROUTINE WIDTH(YA,YB,PII,CK2,CCOND,HA,PIC)

THIS SUBROUTINE IS USED TO DETERMINE DATA FOR
 APPROXIMATING A CONTACT WHICH EXTENDS PARTWAY ACROSS A SURFACE
 ELEMENT. IT USES THE HALF-INTERVAL SEARCH METHOD TO FIND
 THE FOOT OF F(A) BETWEEN YA AND YB, WHICH ARE THE INNER AND
 OUTER BOUNDARIES OF THE SURFACE ELEMENT THAT THE CONTACT EXTENDS
 PARTWAY ACROSS. ONCE FOUND, THE DESIRED ROOT IS STORED AS PIC
 AND THE HEAT TRANSFER COEFF. THAT IS PRESCRIBED OVER THE SURFACE
 SEGMENT OF HALF-WIDTH PIC TO OBTAIN THE RESISTANCE OF THE ACTUAL
 CONTACT OF HALF-WIDTH PII (WHERE PII.LE.PIC) IS STORED AS HA.

```

C      F(A)=C11-(1.-.636628ASIN(YA/A))/(C3-.636628ASIN(YA/A))
C      S8=.636628ALOG(A/PII)/CK2-.636628ASIN(YA/A)*A/PII/CCOND
C      DX=YB-YA
C      C1=BX8(.636628ALOG(DX/(PII-YA))/CK2+1./((CCOND*(PII-YA))))
C      C11=1./C1
C      C2=CCOND*PII
C      C3=YB8(.636628ALOG(YB/PII)/CK2+1./C2)
C      AL=YA+.0001*(YB-YA)
C      IF(AL.GE.YB) PIC=PII ; HA=1.E+12 ; RETURN
C      YL=F(AL)
C      AR=YB
C      YR=F(AR)
C      IF((YL.LT.0.0.AND.YR.LT.0.0).OR.(YL.GT.0.0.AND.YR.GT.0.0)) PIC=PII
20    HA=1.E+12 ; RETURN
C      A=(AL+AR)/2.
C      Y=F(A)
C      IF(YL*Y) 10,25,30
10    AR=A
C      YR=Y
C      GO TO 35
30    AL=A
C      YL=Y
C      TOL=0.01*(YB-YA)
C      COMP=ABS(AR-AL)
C      IF(COMP.LT.TOL) GO TO 25
C      GO TO 20
25    HA=1./(.636628ALOG(A/PII)/CK2)
C      PIC=A
C      RETURN
C      END

```

Listing 3 Routines Used With Orthonormalized Trial Vectors
(Main Program, Subroutines SUBST1, OUTSYN and GRMSCH).

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*** VERSION OF MAIN PROGRAM USED WITH ORTHONORMALIZED TRIAL VECTORS ***MAIN 000
THIS PROGRAM, DEVELOPED BY T. CURRIE AS PART OF HIS PH.D. RESEARCH, MAIN 001
IS CAPABLE OF SOLVING STEADY-STATE AND TRANSIENT PROBLEMS IN MAIN 002
CYLINDRICAL COORDINATES IN TWO (R-THETA) OR THREE (R-THETA-Z) MAIN 003
DIMENSIONS. THE PROGRAM FEATURES A FINITE-DIFFERENCE GRID WITH MAIN 004
VARIABLE NODAL SPACINGS WHICH MAY BE USED TO SOLVE PROBLEMS MAIN 005
INVOLVING SOLID-SOLID CONTACTS ON THE EXTERNAL SURFACES OF MAIN 006
CYLINDERS (EG. FUEL RODS). THE PROGRAM CAN ALSO BE USED TO ANALYSE MAIN 007
HEAT TRANSFER BETWEEN A SAGGED PRESSURE TUBE AND A CALANDRIA MAIN 008
TUBE IN A CANDU REACTOR DURING A LOECC (LOSS OF EMERGENCY CORE MAIN 009
COOLANT) ACCIDENT. 2-D PROBLEMS ARE SOLVED USING THE FINITE MAIN 010
DIFFERENCE METHOD (FDM) WHILE 3-D PROBLEMS MAY BE SOLVED USING MAIN 011
EITHER THE FINITE-DIFFERENCE METHOD OR THE SYNTHESIS TECHNIQUE. MAIN 012
THE LATTER TECHNIQUE IS ESSENTIALLY A GLOBAL TRIAL FUNCTION METHOD MAIN 013
IN WHICH 2-D TRIAL FUNCTIONS (IN THIS CASE FIN.-DIFF. R-THETA MAIN 014
SOLUTIONS) ARE LINEARLY COMBINED AT EACH AXIAL (Z) STATION OF THE MAIN 015
GRID TO PRODUCE AN APPROXIMATION TO THE TRUE 3-D TEMPERATURE MAIN 016
DISTRIBUTION. TRANSIENT ANALYSES MAY BE PERFORMED USING EITHER THE MAIN 017
FINITE-DIFFERENCE OR THE SYNTHESIS TECHNIQUE. IN BOTH CASES THE MAIN 018
IMPLICIT FORMULATION OF THE HEAT CONDUCTION EQUATION IS USED. MAIN 019
NEWTON LINEARIZATION IS USED IN BOTH TRANSIENT AND STEADY-STATE MAIN 020
ANALYSES (SYNTHESIS OR FDM) TO TREAT BOUNDARY CONDITIONS INVOLVING MAIN 021
RADIATION HEAT TRANSFER AND/OR TEMPERATURE DEPENDENT GAS MAIN 022
CONDUCTION, EXCEPT IN THE CASE OF A FINITE-DIFFERENCE LOECC MAIN 023
ANALYSIS WHERE THE METHOD OF SUCCESSIVE APPROXIMATIONS IS MAIN 024
USED TO HANDLE NON-LINEARITIES. IF THE THERMAL CONDUCTIVITY OF MAIN 025
THE PRESSURE AND CALANDRIA TUBE MATERIAL IS CONSIDERED MAIN 026
TEMPERATURE DEPENDENT IN A LOECC ANALYSIS, KIRCHOFF'S MAIN 027
TRANSFORMATION IS USED TO LINEARIZE THE CONDUCTIVITY. MAIN 028
***** INPUT VARIABLES READ BY THE MAIN PROGRAM ***** MAIN 029
TITLE(I), I=1,8 - JOB TITLE-MAY BE UP TO 32 CHARACTERS LONG. MAIN 030
NGROPT - 'NUMBER OF GRID OPTION'; THIS VARIABLE MAY ASSUME ONE MAIN 031
OF THE THREE VALUES GIVEN BELOW. MAIN 032
NGROPT=0 - GRID SPACINGS 'DR', 'DTHETA', AND 'DZ' ARE FUNCTIONS OF MAIN 033
R (RADIUS), THETA (CIRCUMFERENTIAL POSITION), AND Z MAIN 034
(AXIAL POSITION) RESPECTIVELY. MAIN 035
NGROPT=1 - 'DR' IS A FUNCTION OF R AND THETA BUT 'DTHETA' IS ONLY A MAIN 036
FUNCTION OF THETA AND 'DZ' IS ONLY A FUNCTION OF Z. MAIN 037
NGROPT=2 - 'DR' AND 'DTHETA' ARE FUNCTIONS OF BOTH R AND THETA BUT MAIN 038
'DZ' IS A FUNCTION OF Z ALONE. MAIN 039
KBU - 'MAXIMUM BANDWIDTH'; THE VALUE OF THIS VARIABLE MUST MAIN 040
BE GREATER THAN THE LEFT-HAND BANDWIDTH OF ANY ROW IN MAIN 041
THE CONDUCTANCE MATRIX (IE, MUST BE GREATER THAN THE MAIN 042
LARGEST LEFT-HAND ROW BANDWIDTH). MAIN 043
M2D=1 - 2-D FINITE-DIFFERENCE ANALYSIS OR 3-D SYNTHESIS MAIN 044
ANALYSIS. THE LATTER ANALYSIS COMMENCES WITH 2-D FINITE MAIN 045
DIFFERENCE ANALYSES, HENCE M2D=1 DESPITE THE FACT THAT MAIN 046
THE ANALYSIS IS ULTIMATELY 3-D. MAIN 047
M2D=0 - 3-D FINITE-DIFFERENCE ANALYSIS. MAIN 048
MCCYL - COMPLETE CYLINDER ANALYSIS OPTION; MCCYL=1 INDICATES MAIN 049
THAT THE ANALYSIS INVOLVES A COMPLETE CYLINDER (IE, 360 MAIN 050
DEGREES); MCCYL=0 INDICATES THAT A SECTOR OF A CYLINDER MAIN 051
(IE, LESS THAN 360 DEGREES) IS TO BE ANALYZED. MAIN 052
SYN=1 - SYNTHESIS SOLUTION. MAIN 053
SYN=0 - NO SYNTHESIS SOLUTION. MAIN 054
MCONST - CONSTANT TRIAL FN. OPTION; =1 IF A CONSTANT IS TO BE USED MAIN 055
AS THE LAST TRIAL FN. IN A SYNTHESIS ANALYSIS, =0 OTHERWISE. MAIN 056
CONST - VALUE OF THE CONSTANT TO BE USED AS A TRIAL FN. (NEEDED MAIN 057
ONLY IF MCONST=1). MAIN 058
RESIST - RESISTANCE OF CONTACT ANALYSIS OPTION; =1 IF THE MAIN 059
RESISTANCE OF A CONTACT OF SPECIFIED SIZE ON A SEMI- MAIN 060
CYLINDER IS TO BE COMPUTED, =0 OTHERWISE. VALUES OF MAIN 061
VARIABLES 'INSUL' TO 'TO' FOLLOWING ARE NOT REQUIRED IF MAIN 062
RESIST=0. MAIN 063
INSUL - INSULATED BOUNDARY OPTION; =1 IF THE CONTACT RESISTANCE MAIN 064
IS TO BE COMPUTED WITHOUT GAS CONDUCTION, =0 IF THE MAIN 065
ANALYSIS IS TO BE PERFORMED WITH GAS CONDUCTION. MAIN 066
XLOAD - CONTACT LOAD IN RESISTANCE OF CONTACT ANALYSIS (N.). MAIN 067
RESIS - RESISTANCE OF SEGMENT OF CONTACT SPECIMEN WITH A LENGTH MAIN 068
EQUAL TO THE SEMI-CYLINDER DIAMETER AND CROSS-SECTIONAL MAIN 069
DIMENSIONS EQUAL TO THE SEMI-CYLINDER'S DIAMETER AND MAIN 070
LENGTH (DEB, C/M). RESIS IS SUBTRACTED FROM THE OVERALL MAIN 071
RESISTANCE OF THE CONTACT TO OBTAIN THE RESISTANCE BASED MAIN 072
ON THE EXTRAPOLATED TEMPERATURE DROP AT THE INTERFACE. MAIN 073
TO - TEMP. OF FLAT BISECTING PLANE OF SEMI-CYLINDER IN RES. MAIN 074
OF CONTACT ANALYSIS (DEG. C). MAIN 075
INPECC - LOECC ANALYSIS OPTION; =1 IF A LOECC ANALYSIS IS TO BE MAIN 076
PERFORMED, =0 OTHERWISE. LOECC ANALYSES CAN ONLY BE MAIN 077
PERFORMED WITH NGROPT=2. VALUES OF VARIABLES PTID TO MAIN 078
SPRDIA FOLLOWING ARE NOT REQUIRED IF INPECC=0. MAIN 079
PTID - PRESSURE TUBE INNER DIAMETER. MAIN 080
PTOD - PRESSURE TUBE OUTER DIAMETER. MAIN 081
CTID - CALANDRIA TUBE INNER DIAMETER. MAIN 082
CTOD - CALANDRIA TUBE OUTER DIAMETER. MAIN 083
DGART - SPACING BETWEEN TWO MOST CENTRAL GARTER SPRINGS IN CANDU MAIN 084
FUEL CHANNEL. MAIN 085
SPRDIA - SEMI-MAJOR DIAMETER OF ELLIPTICAL CONTACT PATCH (NEEDED IF MAIN 086
RESIST=1 OR INPECC=1). MAIN 087
B - SEMI-MINOR DIAMETER OF ELLIPTICAL CONTACT PATCH (NEEDED IF MAIN 088
RESIST=1 OR INPECC=1). MAIN 089
RCY1 - SMALL CONTACT RADIUS IF RESIST=1 OR SMALL EFFECTIVE CONTACT MAIN 090
RADIUS IF INPECC=1. VALUE NOT REQUIRED FOR OTHER ANALYSES. MAIN 091
RCY2 - LARGE CONTACT RADIUS IF RESIST=1 OR LARGE EFFECTIVE CONTACT MAIN 092
RADIUS IF INPECC=1. VALUE NOT REQUIRED FOR OTHER ANALYSES. MAIN 093
TUBE - TUBE ANALYSIS OPTION; =1 FOR ANALYSIS OF HEAT FLOW IN MAIN 094
A TUBE CONTAINING AN INTERNAL FLUID AT BULK TEMPERATURE MAIN 095
BT. MAIN 096
BT - BULK TEMPERATURE OF FLUID IN TUBE. VALUE ONLY REQUIRED IF MAIN 097
TUBE=1. MAIN 098
NTPRNT - NUMBER OF NODAL TEMPERATURES TO BE PRINTED OUT IN A MAIN 099
SYNTHESIS ANALYSIS. MAIN 100
ICN(LL), JCN(LL), KCN(LL) - LOCATION (CIRCUMFERENTIAL, RADIAL AND MAIN 101
AXIAL NODE NUMBERS RESPECTIVELY) OF LLTH NODE WHOSE MAIN 102
TEMPERATURE IS TO BE PRINTED OUT AFTER A SYNTHESIS MAIN 103
ANALYSIS. NTPRNT VALUES OF THESE VARIABLES MUST BE MAIN 104
SUPPLIED WITH DATA CARDS (EACH OF WHICH CONTAINS VALUES MAIN 105
OF ICN, JCN AND KCN FOR A PARTICULAR NODE) ARRANGED SO MAIN 106
THAT THE VALUES OF ICN, JCN AND KCN VARY FROM LEAST TO MOST MAIN 107
RAPIDLY IN THAT ORDER. MAIN 108
MAIN 109
MAIN 110
MAIN 111
MAIN 112
MAIN 113
MAIN 114

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INTEGER TUBE,SYN,RESIST,OUTRY,TRANS,RADBNB
DOUBLE PRECISION RES,RI,AL,CRI,CB,DIASUM,AL1,CRI1,CB1
DIMENSION OFLX(1,1),TRAD(39,40),TITLE(8),ICN(63),JCN(63),
$KCN(63),DZA(40),DZRT(40),HX(39,40),TC(39,40),MTGEN(40),RES(5,40),
$RD(220),T(520),AL(220,13),AL1(1,1),CRI(220,13),CRI1(1,1),
$CB(220),CB1(220),DT(10),NTC(10),TRF(520,14),NTFTS(10),NTRI(40),
$NTRF(40,10),DIASUM(1,1),NTSTFC(10),JL(9),JP(9),CBW(250),
$NTRIP(40),NTRKP(40,10)
$TOR(400,5),TORKP1(400,5),TORKM1(400,5)
COMMON /DBTOR/TOR,TORKM1,TORKP1
COMMON /DB1/RESIST,TO /DB2/TUBE,BT /DB3/RESIS,XLOAD,NTPRNT,ICN,
$JCN,KCN /DB4/CBW,MBA,JL,JP /DB5/KL,L /DB6/IX,JX,KX /DB8/MCCYL,
$DB9/I,J,K,MIS,M1 /DB11/MTOT,DZTOT,DTOT /DB13/RES /DB14/NTRF,
$TRF /DB15/CB,AL,CRI,MBA /DB17/NEQU /DB19/SYN,OUTRY,CONST /DB21/TC
$ /DB23/T /DB24/HX /DB27/DZA,DZB,MTGEN /DB28/NITER /DB31/TRANS,
$TRPIN,TF,IMAL,THCAP,DT,NTOT,NTC /DB32/DIASUM /DB33/TCGAS,CCNH
COMMON /DB34/AL1,CRI1,CB1 /DB36/THCAP1,THCAP2 /DB37/PTID,PTOD,
$CTID,CTOD,DGART,SPRDIA,ECC,CG1,KCH /DB38/IMPECC /DB39/NTOL
$ /DB40/NTS /DB41/RADBNB /DB42/TRADIN,NAXIT,NTRTF,NTFTS,NTFTS
$ /DB43/REXCO /DB45/OFLX,TRAD /DB47/NTSTFC,NXT /DB48/MCONST
$ /DB49/NTRFIC /DB50/MCONT /DB51/IA,IB /DB54/HX /DB57/MLIN /DB60/IXX
$ /DB61/NTRF,NTRIP /DB64/NTRIP,NTRKP /DB66/MBA /DB68/MGROPT
$ /DB69/NTRI,NTRA /DB73/NDIA /DB77/A,B,RCY1,RCY2
COMMON /DB78/TCINTS,SLOPS,SLOPD2,NJL,MVC,MVCON
$ /DB80/NZFLO,NTFBC,DZBC
NTOL=0
NTOL=NTOL+1
READ INPUT DATA.
READ (105,995) (TITLE(I),I=1,8)
READ (105,991) MGROPT,MBA,MZD,MCCYL
READ (105,996) SYN,MCONST,CONST
READ (105,992) RESIST,INSUL,XLOAD,RESIS,TO
READ (105,999) IMPECC,PTID,PTOD,CTID,CTOD,DGART,SPRDIA
READ (105,997) A,B,RCY1,RCY2
READ (105,993) TUBE,BT
READ (105,998) NTPRNT
IF(NTPRNT-1) 20,25,25
DO 15 LL=1,NTPRNT
READ (105,994) ICN(LL),JCN(LL),KCN(LL)
CONTINUE
CONTINUE
WRITE (108,887) (TITLE(I),I=1,8)
IF(SYN-1) 21,22,21
WRITE (108,881)
IF(RESIST.EQ.1.AND.INSUL.EQ.1) WRITE (108,882)
IF(RESIST.EQ.1.AND.INSUL.EQ.1) WRITE (108,883)
IF(IMPECC.EQ.1) WRITE (108,888) PTID,PTOD,CTID,CTOD,DGART,SPRDIA
$A,B,RCY1,RCY2
GO TO 23
WRITE (108,884)
WRITE (108,885) MGROPT
MSYN=1
THCAP1=0.0
THCAP2=0.0
IA=1
IB=1
CALL ROUTINES TO DEFINE THE R-THETA GRID GEOMETRY. ROUTINES CALLED
DEPEND ON THE VALUE OF MGROPT.
IF(MGROPT-1) 11,12,13
CALL DTH01 (IX)
CALL RADO (RI,JX,RO T)
GO TO 14
CALL DTH01 (IX)
CALL RAD12
CALL ISTJ
CALL DRAB
GO TO 14
CALL RAD12
CALL DTH2
CALL JINI
CALL ISTJ
CALL DRAB
IF(IMPECC.EQ.1) CALL CALAB
NIS=HX+1
KX=1
CALL ROUTINE (BZ) TO DEFINE THE AXIAL GRID GEOMETRY IF NECESSARY.
IF(MZD.NE.1) CALL BZ (BZA,BZB,KX)
INITIALIZE THE TRIAL FN. COUNTER (NTRF).
NTRF=0
IXX=IX
IF(RESIST.EQ.1) IXX=IX-1
INCREMENT THE TRIAL FN. COUNTER (NTRF) BY 1.
NTRF=NTRF+1
IF(SYN.EQ.1) WRITE (108,886) NTRF
INITIALIZE THE ITERATION COUNTER (NITER).
NITER=1
CALL ROUTINE (DATA1) TO READ SPECIFIC JOB DATA AND TO READ AND/OR
CALCULATE BOUNDARY CONDITIONS.
CALL DATA1 (HX,TC,M1,CK1,CK2,NEXT,MSYN)
NTS=0
CHECK IF A TRANSIENT ANALYSIS IS REQUIRED.
IF(TRANS.NE.1) GO TO 4
INITIALIZE COUNTERS FOR THE NUMBER OF TIME STEPS (NTS) PERFORMED,
THE NUMBER OF TRANSIENT TRIAL FNS. RECORDED IN A PARTICULAR 2-D
TRANSIENT ANALYSIS (ITIM), AND THE NUMBER OF TRIAL FN. DEPLOYMENT
SCHEMES & CHANGES MADE IN THE 2ND. PHASE CALCS. OF A
TRANSIENT SYNTHESIS ANALYSIS (NXT).
ITIM=1
NXT=1
NTS=0
INITIALIZE THE TIME IN THE ANALYSIS.
TIME=0.0

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      DTAU=DT(1)
      INCREMENT THE TIME STEP COUNT (NTS) BY 1.
      NTS=NTS+1
      CHECK WHETHER THE TIME STEP SIZE (DTAU) CHANGES AT THIS VALUE OF
      NTS AND CHANGE IT IF NECESSARY. ADVANCE THE TIME IN THE ANALYSIS.
      DO 7 NDT=1,NTDT
      IF(NTS.EQ.NTC(NDT)) GO TO 8
      CONTINUE
      GO TO 9
      DTAU=DT(NDT+1)
      TIME=TIME+DTAU
      GO TO 6
      TIME=TIME+DTAU
      CHECK WHETHER ONLY THE RHS. VECTOR NEEDS TO BE RECALCULATED TO
      ADVANCE TO THE NEXT TIME STEP (FIN.-DIFF. ANALYSIS). ONLY THE RHS.
      VECTOR NEEDS TO BE RECALCULATED FOR THE SECOND AND SUBSEQUENT TIME
      STEPS OF A LINEAR TRANSIENT F-D ANALYSIS. THE RHS. VECTOR IS
      RECALC. IN QFORM.
      IF(NSYN.EQ.2.OR.NLIN.EQ.1) GO TO 6
      IF(NTS.EQ.1) GO TO 6
      NITER=1
      CALL QFORM (THCAP,DTAU)
      GO TO 64
      KL=0
      INITIALIZE THE CONVERGENCE INDICATOR FOR A STEADY-STATE
      FINITE-DIFFERENCE ANALYSIS (MCONT).
      MCONT IS ASSIGNED A VALUE OF 1 IN CONVERGENCE TESTS PERFORMED IN SUB-
      ROUTINES HX10 AND RAD1AT IF CONVERGENCE HAS NOT BEEN ACHIEVED AND
      ANOTHER ITERATION IS REQUIRED.
      MCONT=0
      IF(MITER.LE.2) MCONT=1
      L=0
      NDIA=0
      NDIAP=0
      IF(NSYN.EQ.2) NITER=1
      IF INTERMODAL THERMAL CONDUCTIVITIES ARE TO BE CALCULATED FROM
      MODAL TEMPERATURES IN THIS TIME STEP OR ITERATION, SET MVC=1.
      IF ENTRIES IN THE VECTOR OF MODAL TEMPERATURES PAST THE ENTRY
      CORRESPONDING TO THE TEMPERATURE OF THE NODE FOR WHICH THE F-D
      EQUATION IS BEING ASSEMBLED ARE REQUIRED FOR SOME REASON, SET
      MJL=1. LATER ENTRIES IN THE TEMP. VECTOR ARE REQUIRED IN THE
      2ND PHASE CALCS. OF A SYNTHESIS ANALYSIS AND TO CALCULATE
      INTERMODAL THERMAL CONDUCTIVITIES WHEN MVC=1.
      MJL=0
      MVC=0
      IF(MVCOR.EQ.1.AND.(MITER.GT.1.OR.NTS.GT.1)) MVC=1
      IF(NSYN.EQ.2.OR.NITER.GT.1.OR.NTS.GT.1) MJL=1
      IF(NSYN=2) 170,171,170
      STORE THE TRIAL FNS. USED AT THE FIRST AXIAL NODE AS THE COLUMNS
      OF TOR AND CALL SUB. GRNSCH TO ORTHONORMALIZE THOSE COLUMNS.
      NTRF=NTRI(1)
      DO 173 NTRIAL=1,NTRF
      NTF=NTRK(1,NTRIAL)
      DO 173 LL=1,LMAX
      TOR(LL,NTRIAL)=TRF(LL,NTF)
      CALL GRNSCH(TOR,LMAX,NTRF)
      SWEET THROUGH THE AXIAL NODES IN THE GRID. (K=1,2,...,KX)
      DO 170 K=1,KX
      IF(NSYN=2) 17,16,17
      IN THE 2ND. PHASE CALCS. OF A SYNTHESIS ANALYSIS, ASSIGN VALUES TO
      THE NUMBER OF TRIAL FNS. USED AT AXIAL NODE K IN THE CURRENT AND
      PREVIOUS TRIAL FN. DEPLOYMENT SCHEDULES (NTRF AND NTRFP RESPECTIVELY)
      AND INITIALIZE THE R-THETA NODE NUMBER COUNTER (L).
      NTRF=NTRI(K)
      NTRFP=NTRIP(K)
      L=0
      IF(K.EQ.KX) GO TO 17
      KP1=K+1
      NTRFP=NTRI(KP1)
      CHECK WHETHER THE TRIAL FNS. USED AT AXIAL NODE K+1 ARE THE SAME AS
      THOSE USED AT K. IF THEY ARE, EQUATE THE ENTRIES IN TOR(KP1) TO THOSE
      IN TOR. IF THEY AREN'T, STORE THE TRIAL FNS. USED AT AXIAL NODE
      K+1 AS THE COLUMNS OF TOR(KP1) AND THEN CALL SUBR. GRNSCH TO
      ORTHONORMALIZE THOSE COLUMNS.
      DO 26 NTRIAL=1,10
      IF(NTRK(K,NTRIAL).NE.NTRK(KP1,NTRIAL)) GO TO 27
      CONTINUE
      DO 28 NTRIAL=1,NTRFP
      DO 28 LL=1,LMAX
      TOR(KP1(LL,NTRIAL)=TOR(LL,NTRIAL)
      GO TO 17
      DO 18 NTRIAL=1,NTRFP
      NTF=NTRK(KP1,NTRIAL)
      DO 18 LL=1,LMAX
      TOR(KP1(LL,NTRIAL)=TRF(LL,NTF)
      CALL GRNSCH(TOR(KP1,LMAX,NTRF)
      CONTINUE
      IF(NZD.NE.1) BZTOT=(DZA(K)+DZB(K))/2.0
      IF(NZD.EQ.1) BZTOT=1.0
      IF(NZFLD.EQ.1) BZTOT=BZDC/2.0
      CALL ROUTINE TO SWEET THROUGH THE R-THETA NODES AT AXIAL NODE K.
      IF(GROPT.EQ.0) CALL GROPT0 (DTAU,HTGEN(K),DZA(K),DZB(K),NSYN,CK1,
      &CK2,IX,NTS)
      IF(GROPT.EQ.1) CALL GROPT1 (DTAU,HTGEN(K),DZA(K),DZB(K),NSYN,
      &CK1,CK2,IX,NTS)
      IF(GROPT.EQ.2) CALL GROPT2 (DTAU,HTGEN(K),DZA(K),DZB(K),NSYN,CK1,
      &CK2,IX,NTS)
      CALCULATE THE NUMBER OF EQUATIONS (IN THE SYSTEM OF EQUATIONS
      DEFINING THE BLENDING VECTOR (Z)) OBTAINED SO FAR IN THE 2ND.
      PHASE CALCS. OF A SYNTHESIS ANALYSIS. (NDIA).
      IF(NSYN=2) 10,176,10
      NDIA=NDIA+NTRF
      NDIAP=NDIAP+NTRFP
      IF(K.EQ.KX) GO TO 10
      TRANSFER THE CONTENTS OF TOR AND TOR(KP1) TO ARRAYS TORKM1 AND TOR
      RESPECTIVELY IN PREPARATION FOR ADVANCING TO THE NEXT AXIAL NODE.

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C TORKM1 THEN CONTAINS THE (ORTHONORMALIZED) TRIAL VECTORS
C USED AT A-1.
DO 177 NTRIAL=1,NTRF
DO 177 LL=1,LMAX
177 TORKM1(LL,NTRIAL)=TOP(LL,NTRIAL)
DO 178 NTRIAL=1,NTRFP
DO 178 LL=1,LMAX
178 TOR(LL,NTRIAL)=TORKM1(LL,NTRIAL)
10 CONTINUE
IF(NITER.EQ.1.AND.NTS.EQ.1) OUTPUT KL
OUTPUT KL
LMAX=L
C
C UNLESS THE 2ND. PHASE CALCULATIONS OF A SYNTHESIS ANALYSIS ARE
C BEING PERFORMED, GO TO STATEMENT 60 WHERE ROUTINES WHICH SOLVE
C THE F-D EQUATIONS ARE CALLED.
IF(NSTP=2) GO TO 60
50 IF(TRANS.NE.1.AND.RADBND.NE.1.AND.MLIN.NE.1) NSTOP=1
IF(NSTOP.EQ.1) GO TO 55
C
C STORE ENTRIES IN THE BANDED MATRICES CB,AL AND CRI (WHICH, TAKEN
C TOGETHER, DEFINE THE LHS. COEFFICIENT MATRIX FOR THE SYSTEM OF
C EQUATIONS DEFINING Z) IN A SECOND SET OF MATRICES CB1,AL1 AND CRI1.
C THIS IS DONE BECAUSE THE ORIGINAL MATRICES, WHICH CAN BE UPDATED
C AND USED IN ADDITIONAL TIME STEPS OR ITERATIONS, ARE DESTROYED
C WHEN Z IS OBTAINED IN SUBROUTINE BNDEL.
DO 95 N1=1,NEDU
CB1(N1)=CB(N1)
DO 100 N2=1,NBA
AL1(N1,N2)=AL(N1,N2)
CRI1(N1,N2)=CRI(N1,N2)
100 CONTINUE
95 CONTINUE
TIM1=TIME
C
C TRANSFER THE RMS. WEIGHTED RESIDUALS (FOR THE SYSTEM OF EQUATIONS
C DEFINING Z) STORED IN RES TO THE VECTOR RD.
55 NC=0
DO 65 K=1,KX
NT1=NTRI(K)
DO 70 NWT=1,NT1
NC=NC+1
RD(NC)=RES(NWT,K)
70 RES(NWT,K)=0.0
65 CONTINUE
CONTINUE
IF(NSTOP.EQ.1) GO TO 112
C
C IN A NON-LINEAR ANALYSIS, CALL MATCH TO SUBTRACT THE ENTRIES IN
C THE MATRIX DIASUM FROM THE APPROPRIATE ENTRIES IN AL1,CB1 AND CRI1.
IF(RADBND.EQ.1.OR.MLIN.EQ.1) CALL MATCH (NDT,DT,KX,NTRF
,MBA,-1.0,TRANS)
IF(TRANS.NE.1) GO TO 112
C
C IN A TRANSIENT ANALYSIS, INCREMENT THE TIME STEP COUNT (NTS) AND
C CALCULATE THE NEXT TIME STEP SIZE.
NTS=NTS+1
DO 101 NBT=1,NTDT
IF(NTS.EQ.NTC(NDT)) GO TO 105
101 CONTINUE
GO TO 110
105 DTAU=DT(NDT+1)
IF(RADBND.EQ.1.OR.MLIN.EQ.1) GO TO 110
C
C IF THE TIME STEP SIZE HAS CHANGED AND THE ANALYSIS BEING PERFORMED
C IS LINEAR, CALCULATE A FACTOR XNF FOR USE IN CHANGING AL1,CB1 AND
C CRI1 TO THE NEW TIME STEP SIZE. CALL MATCH TO ADD XNF TIMES THE
C ENTRIES IN DIASUM TO THE APPROPRIATE ENTRIES IN AL1,CB1 AND CRI1
C TO ACHIEVE THE CHANGE.
XNF=((DT(NDT)/DT(NDT+1))-1.0)
CALL MATCH (NDT,DT,KX,NTRF,MBA,XNF,TRANS)
110 TIM1=TIM1+DTAU
C
C CALL THE ROUTINE (BNDEL) TO OBTAIN Z FROM AL,CB,CRI AND RD.
112 CALL BNDEL (RD)
C
C CALL OUTSYN TO PRINT THE RESULTS OF THE SYNTHESIS ANALYSIS AND
C TO RECOMPUTE, IF NECESSARY, THE ENTRIES IN DIASUM AND RES (IE.,RD)
C USING THE NEW NODAL TEMPERATURES.
CALL OUTSYN (IXX,TC,NTGEN,DZA,DZB,HX,NSTOP,DTAU,THCAP,TIME,TRANS
,M1,INSUL,LMAX)
C
C IF THE TIME STEP NUMBER (NTS) IS ONE AT WHICH THE TRIAL FN.
C DEPLOYMENT SCHEDULE SHOULD BE CHANGED IN THE 2ND. PHASE CALCS.
C OF A TRANSIENT SYNTHESIS ANALYSIS, CALL NTRP TO STORE THE EXISTING
C SCHEDULE, CALL TFREAD TO READ THE NEW SCHEDULE, CALL ZERO TO
C RE-ZERO THE MATRICES AL,CB,CRI,DIASUM AND RES AND THEN RETURN
C TO STATEMENT 5 TO REPEAT THE SUBSTITUTE/WEIGHT/INTEGRATE PHASE OF
C THE ANALYSIS WITH THE NEW SCHEDULE.
IF(TRANS.EQ.1.AND.NTS.EQ.NTSFC(NXT)) TIME=TIM1 ; CALL NTRP(KX)
; CALL TFREAD (TIME,KX) ; MBA=MBA ; CALL ZERO (KX,NTRF,RD,
;TRANS) ; NXT=NXT+1 ; GO TO 5
C
C IF THE ANALYSIS BEING PERFORMED IS NON-LINEAR, CALL MATCH TO ADD
C THE ENTRIES IN DIASUM (WHICH WERE RECOMPUTED IN SUB. OUTSYN USING
C THE NEW NODAL TEMPERATURES) TO THE APPROPRIATE ENTRIES IN AL1,CB
C AND CRI1.
IF((RADBND.EQ.1.OR.MLIN.EQ.1).AND.NSTOP.NE.1) CALL MATCH (
,NDT,DT,KX,NTRF,MBA,1.0,TRANS)
IF(NSTOP.EQ.1) GO TO 80
C
C PREPARE THE MATRICES AL,CB AND CRI FOR THE NEXT TIME STEP OR
C ITERATION BY EQUATING THEM TO THE UPDATED MATRICES AL1,CB1 AND CRI1.
DO 115 N1=1,NEDU
CB(N1)=CB1(N1)
DO 120 N2=1,NBA
AL(N1,N2)=AL1(N1,N2)
CRI(N1,N2)=CRI1(N1,N2)
120 CONTINUE
115 CONTINUE
C
C INCREMENT THE ITERATION COUNT (NITER) BY 1.
NITER=NITER+1
C
C CHECK WHETHER MAX. PERMISSIBLE ITER. COUNT (MAXIT) HAS BEEN EXCEEDED.
IF(TRANS.NE.1.AND.NITER.GT.MAXIT) OUTPUT
;CONVERGENCE NOT ACHIEVED ; GO TO 80

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      IF TRANS.NE.1 GO TO 55
      TIME=TIME1
      IF (TIME1.GT.TFINAL.OR.NTS.GT.NFTS) NSTOP=1
      RETURN TO STATEMENT 55 TO CALCULATE Z FOR THE NEXT TIME STEP OR
      ITERATION.
      GO TO 55
      CALL CHLSKI TO FACTOR THE F-D COEFFICIENT MATRIX INTO THE PRODUCT
      OF A LOWER TRIANGULAR MATRIX AND ITS TRANSPOSE AND THEN CALL SUB
      TO PERFORM THE FORWARD AND BACKWARD SUBSTITUTION PHASES OF THE
      SOLN. FOR T.
      CALL CHLSKI(MAX)
      CALL SUB (LAMBDA,T)
      INCREMENT THE ITERATION COUNT (NITER) BY 1.
      NITER=NITER+1
      IF THE ANALYSIS BEING PERFORMED IS A NON-LINEAR STEADY-STATE
      F-D ANALYSIS WHICH HAS NOT CONVERGED AND THE MAX. PERMISSIBLE ITER.
      COUNT HAS NOT BEEN EXCEEDED, RETURN TO STATEMENT 6 TO PERFORM
      ANOTHER ITERATION.
      IF ((RADBNQ.EQ.1.OR.MLIN.EQ.1).AND.TRANS.NE.1.AND.NCONT.
      EQ.1.AND.NITER.LE.MAXIT) GO TO 6
      IF ((MLIN.EQ.1.OR.RADBNQ.EQ.1).AND.NCONT.EQ.1.AND.NITER.GT.
      MAXIT) OUTPUT 'CONVERGENCE NOT ACHIEVED'
      CALL OUTPUT TO PRINT THE RESULTS OF THE ANALYSIS.
      IF (SYN.EQ.1.AND.OUTPUT.EQ.1.AND.TRANS.EQ.1.AND.NTS.EQ.NFTS(ITIM))
      WRITE (100,889) NTRF ; CALL OUTPUT (HX,T,TIME,TRANS,SYN,HI) ;
      GO TO 63
      IF (SYN.NE.1.OR.(SYN.EQ.1.AND.OUTPUT.EQ.1)) CALL OUTPUT (HX,T,TIME,
      TRANS,SYN,HI)
      IF (SYN=1) 61,62,61
      IF (TRANS.EQ.1.AND.NTS.NE.NFTS(ITIM)) GO TO 61
      IN A SYNTHESIS ANALYSIS, CALL TRIAL TO RECORD THE TEMPERATURE VECTOR
      AS A TRIAL FN. THIS STATEMENT WILL ONLY HAVE BEEN REACHED IN A
      TRANSIENT ANALYSIS BEING PERFORMED TO OBTAIN TRIAL FNS. FOR A
      SYNTHESIS ANALYSIS IF NTS=NFTS(ITIM) (IE., IF THE TIME STEP NUMBER
      (NTS) IS ONE AT WHICH A TRIAL FN. SHOULD BE RECORDED).
      CALL TRIAL (NEXT,HX,CONST,T)
      IF A TRANSIENT ANALYSIS IS BEING PERFORMED AND MORE TRANSIENT
      TRIAL FUNCTIONS ARE REQUIRED, INCREMENT THE COUNTERS FOR THE
      NUMBER OF TRIAL FNS. RECORDED (NTRF) AND THE NUMBER OF TRANSIENT
      TRIAL FNS. RECORDED IN THIS PARTICULAR ANALYSIS (ITIM). IF
      THE FINAL TIME OR TIME STEP NUMBER HAS NOT BEEN EXCEEDED, RETURN
      TO STATEMENT 3 TO COMPUTE THE TEMPS. FOR THE NEXT TIME STEP.
      IF (TRANS.EQ.1.AND.NTS.LT.NFTS(NTRF)) NTRF=NTRF+1 ; ITIM=ITIM+1
      IF (TRANS.EQ.1.AND.TIME.LT.TFINAL.AND.NTS.LT.NFTS) GO TO 3
      CHECK WHETHER ANOTHER TRIAL FUNCTION IS REQUIRED (IE., WHETHER NEXT=1
      ). RETURN TO STATEMENT 1 TO READ THE DATA FOR AND COMPUTE THE NEXT
      TRIAL FN. IF NEXT=1.
      IF (NEXT.EQ.1) GO TO 1
      IF A SYNTHESIS ANALYSIS IS BEING PERFORMED, START THE 2ND. PHASE
      CALCULATIONS BY ASSIGNING NSYN A VALUE OF 2, CALLING DZ TO READ
      AND DEFINE THE AXIAL GRID GEOMETRY AND CALLING DATA1 TO READ
      THE 3-D JOE DATA AND TO READ AND/OR CALCULATE THE BOUNDARY
      CONDITIONS (OR GAP CONDUCTANCES).
      IF (SYN=1) 60,75,60
      N2D=0
      NSYN=2
      CALL DZ (DZA,DZB,KX)
      CALL DATA1 (HX,TC,H1,CK1,CK2,NEXT,NSYN)
      MDA=MNDA
      CALL ZERO TO ZERO THE ENTRIES IN THE LHS. COEFF. MATRIX AND RHS.
      VECTOR FOR THE SYSTEM OF EQUATIONS DEFINING THE BLENDING VECTOR (Z).
      CALL ZERO (KX,NTRF,RB,TRANS)
      RETURN TO STATEMENT 4 OR STATEMENT 5 TO START THE SUBSTITUTIONS
      ,WEIGHTINGS AND INTEGRATIONS OF THE 2ND. PHASE CALCULATIONS.
      IF (TRANS.EQ.1) GO TO 4
      GO TO 5
      IF (MTOL.LT.1) GO TO 2
      STOP
      991 FORMAT (4I5)
      992 FORMAT (2I5,3F10.3)
      993 FORMAT (I5,F10.5)
      994 FORMAT (3I5)
      995 FORMAT (8A4)
      996 FORMAT (2I5,F10.5)
      997 FORMAT (4F10.5)
      998 FORMAT (I5)
      999 FORMAT (I5,4F10.5)
      881 FORMAT ('0000000000 SYNTHESIS SOLUTION 0000000000',/)
      882 FORMAT (1X,'000000 RUN WITHOUT GAS CONDUCTION 000000',/)
      883 FORMAT (1X,'000000 RUN WITH GAS CONDUCTION 000000',/)
      884 FORMAT ('0000000000 FINITE-DIFFERENCE SOLUTION 0000000000',/)
      885 FORMAT (1X,'GRID OPTION NUMBER =',I3,/)
      886 FORMAT ('0000000000 TRIAL FUNCTION NO.',I3,' 0000000000')
      887 FORMAT ('180000',B4,' 0000',/)
      888 FORMAT ('000000 INPECE ANALYSIS DATA 000000',/,1X,
      $'PRESSURE TUBE INNER DIAMETER =',F10.5,10X,
      $'PRESSURE TUBE OUTER DIAMETER =',F10.5,/,
      $'CALANDRIA TUBE INNER DIAMETER =',F10.5,10X,
      $'CALANDRIA TUBE OUTER DIAMETER =',F10.5,/,
      $'SPACING BETWEEN GARTER SPRINGS =',F10.5,9X,
      $'SPRING DIAMETER =',F10.5,/, 'PATCH SEMI-MAJOR DIAMETER =',
      $'F10.5,14X, 'PATCH SEMI-MINOR DIAMETER =',F10.5,/,
      $'SMALL EFFECTIVE CONTACT RADIUS =',F10.5,10X,
      $'LARGE EFFECTIVE CONTACT RADIUS =',F10.5,/)
      889 FORMAT ('000000 RESULTS FOR TRIAL FN. NO.',I2,' FOLLOW 000000',/)
      END

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SUBROUTINE SUBST1 (CS,NMS,C,QQ,JP,JL,L,CTOT,N1, N3,NA)
C
C ***** VERSION USED WITH ORTHONORMALIZED TRIAL VECTORS *****
C
C THIS SUBROUTINE PERFORMS SUBSTITUTIONS AND INTEGRATIONS AS
C REQUIRED IN A SYNTHESIS ANALYSIS. TRIAL FUNCTIONS ARE SUBSTITUTED
C INTO FINITE-DIFFERENCE ANALOGS OF THE LAPLACIAN OPERATOR. THE
C RESULTING RESIDUALS ARE THEN MULTIPLIED BY EACH OF THE TRIAL
C FUNCTIONS (IE. THE TRIAL FUNCTIONS SERVE AS THE WEIGHTING
C FUNCTIONS IN GALERKIN'S METHOD). THESE WEIGHTED RESIDUALS ARE
C INTEGRATED (IE. SUMMED) OVER EACH K-THETA PLANE AND STORED IN
C APPROPRIATE LOCATIONS IN THE MATRICES AL,CB,CRI,RES. THESE
C MATRICES REPRESENT A SYSTEM OF SIMULTANEOUS EQUATIONS WHICH
C DEFINES THE MIXING MATRIX.
C
C DOUBLE PRECISION RES,FD,AL,CRI,CB,C,QQ,CTOT,RR
C DIMENSION NTRI(40),NTRK(40),HX(39,40),C(13),TC(39,40),FD(3),
C RES(5,40),JP(9),JL(9),AL(220,13),CRI(220,13),CB(220),
C TOR(400,5),TORKM1(400,5),TORKP1(400,5),TRF(520,14)
C COMMON /DB4/IX,JX,KX /DB9/1,J,A,NIS,M1 /DB13/RES /DB14/NTRF,TRF
C /DB15/CB,AL,CRI,MBA /DB24/HX /DB39/NTRK /DB60/IXX /DB68/NGROPT
C /DB69/NTRI,NTRK /DB73/NDIA /DBTOR/TOR,TORKM1,TORKP1
C
C CALCULATE RIGHT-HAND SIDE RESIDUAL (RR).
C RR=-QQ
C
C SELECT TRIAL FUNCTIONS SUCCESSIVELY. TRIAL FUNCTIONS ARE
C DESIGNATED TOR(L,NN) WHERE L IS A NODE NUMBER AND NN IS A TRIAL
C FUNCTION NUMBER.
C DO 35 NN=1,NTRF
C IF(N1-1) 36,37,38
C
C SUBSTITUTE TRIAL FUNCTION NO. NN INTO R-THETA FINITE-DIFFERENCE
C EQUATION AND COMPUTE RESIDUAL. FD(1), N1 IS A VARIABLE USED TO
C INDICATE THE TYPE OF NODE UNDER CONSIDERATION (EG. NODE PROCESSED
C BY SUBROUTINE NORMOD, SUBROUTINE BTGRAD, ETC.)
C
C 36 FD(1)=C(1)*TOR((L-JP(1)),NN)+C(2)*TOR((L+JL(1)),NN)+C(3)*TOR((L-JP
C (2)),NN)+C(4)*TOR((L+JL(2)),NN)+C(7)*TOR((L-JP(4)),NN)-CTOT*TOR(L,
C NN)
C GO TO 39
C
C 37 FD(1)=C(1)*TOR((L-JP(2)),NN)+C(2)*TOR((L-JP(1)),NN)+C(3)*TOR((L-JP
C (3)),NN)+C(4)*TOR((L+JL(1)),NN)+C(5)*TOR((L-JP(4)),NN)+C(6)*TOR((L
C +JL(2)),NN)+C(7)*TOR((L-JP(5)),NN)+C(8)*TOR((L-JP(6)),NN)+C(9)*TOR
C ((L-JP(7)),NN)-CTOT*TOR(L,NN)
C GO TO 39
C
C 38 IF(N1.EQ.2) FD(1)=C(1)*TOR((L-JP(1)),NN)+C(2)*TOR((L+JL(1)),NN)+C
C (3)*TOR((L-JP(2)),NN)+C(4)*TOR((L+JL(2)),NN)+C(5)*TOR((L+JL(3)),NN)
C -CTOT*TOR(L,NN)
C IF(N1.EQ.3) FD(1)=C(1)*TOR((L-JP(1)),NN)+C(2)*TOR((L+JL(1)),NN)+C
C (3)*TOR((L+JL(2)),NN)+C(4)*TOR((L-JP(2)),NN)+C(5)*TOR((L+JL(3)),NN)
C -CTOT*TOR(L,NN)
C IF(N1.EQ.4) FD(1)=C(1)*TOR((L+JL(1)),NN)+C(2)*TOR((L+JL(2)),NN)+C
C (3)*TOR((L-JP(1)),NN)+C(4)*TOR((L-JP(2)),NN)+C(5)*TOR((L-JP(3)),NN)
C +C(6)*TOR((L-JP(4)),NN)+C(7)*TOR((L+JL(3)),NN)+C(8)*TOR((L+JL(4)),
C NN)+C(9)*TOR((L+JL(5)),NN)-CTOT*TOR(L,NN)
C
C 39 NZ1=NDIA*NN
C
C MULTIPLY FD(1) BY NTRF WEIGHTING FMS. (IE., THE TRIAL FMS. USED AT
C AXIAL NODE K) AND ADD RESULTS TO APPROPRIATE ENTRIES IN AL, CB AND
C CRI. THE MATRICES AL, CB AND CRI CONTAIN THE SUBDIAGONAL, DIAGONAL
C AND SUPERDIAGONAL ENTRIES IN THE SYNTHESIS COEFFICIENT MATRIX
C RESPECTIVELY.
C DO 31 MNT=1,NTRF
C NC=NDIA+MNT
C IF(NZ1.EQ.NC) CB(NC)=CB(NC)+FD(1)*TOR(L,MNT)
C NDF1=NC-NZ1
C IF(NDF1.GT.0) AL(NC,NDF1)=AL(NC,NDF1)+FD(1)*TOR(L,MNT)
C IF(NDF1.LT.0) NDF1=-NDF1 + CRI(NC,NDF1)=CRI(NC,NDF1)+FD(1)*TOR(L,M
C NNT)
C
C 31 CONTINUE
C 35 CONTINUE
C
C CALCULATE WEIGHTED-RESIDUALS FOR AXIAL NODES K-1 AND K+1.
C IF(K.EQ.1) GO TO 60
C NTR3=NTRI(K-1)
C
C SELECT TRIAL FUNCTIONS USED AT AXIAL NODE K-1 SEQUENTIALLY.
C DO 45 NN=1,NTR3
C
C CALCULATE 'RESIDUAL' FOR AXIAL NODE K-1 (FD(3)) WITH THE NNTH
C TRIAL FM. USED AT THAT AXIAL NODE.
C FD(3)=C(N4)*TORKM1(L,NN)
C NZ2=NDIA-NTRI(K-1)+NN
C
C MULTIPLY FD(3) BY NTRF WEIGHTING FMS. (IE., THE TRIAL FMS. USED AT
C AXIAL NODE K) AND ADD RESULTS TO APPROPRIATE ENTRIES IN AL, CB
C AND CRI.
C DO 46 MNT=1,NTRF
C NC=NDIA+MNT
C NDF2=NC-NZ2
C AL(NC,NDF2)=AL(NC,NDF2)+FD(3)*TOR(L,MNT)
C
C 46 CONTINUE
C 45 CONTINUE
C IF(K.EQ.KX) GO TO 61
C NTR2=NTRI(K+1)
C
C SELECT TRIAL FMS. USED AT AXIAL NODE K+1 SEQUENTIALLY
C DO 50 NN=1,NTR2
C
C CALCULATE 'RESIDUAL' FOR AXIAL NODE K+1 (FD(2)) WITH THE NNTH TRIAL
C FM. USED AT THAT AXIAL NODE.
C FD(2)=C(N3)*TORKP1(L,NN)
C NZ3=NDIA+NTRI(K)+NN
C
C MULTIPLY FD(2) BY NTRF WEIGHTING FMS. (IE., THE TRIAL FMS. USED AT
C AXIAL NODE K) AND ADD RESULTS TO APPROPRIATE ENTRIES IN AL,CB
C AND CRI.
C DO 51 MNT=1,NTRF
C NC=NDIA+MNT
C NDF3=NC-NZ3
C CRI(NC,NDF3)=CRI(NC,NDF3)+FD(2)*TOR(L,MNT)
C
C 51 CONTINUE
C 50 CONTINUE
C
C MULTIPLY RIGHT-HAND SIDE RESIDUAL BY WEIGHTING FUNCTIONS.
C DO 60 MNT=1,NTRF
C RES(MNT,K)=RES(MNT,K)+RR*TOR(L,MNT)
C
C 60 CONTINUE
C RETURN
C END

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      JUPROTIME OUTS(N (IX,TC,HTGEN,DZA,DZB,IX,NSTOP,DTAU,THCAP,TIME,T OUT 000
      TRANS,H1,INSIL,LAAX) OUT 001

      ***** VERSION USED WITH ORTHONORMALIZED TRIAL VECTORS ***** OUT 002
      THIS SUBROUTINE PRINTS THE RESULTS OF A SYNTHESIS ANALYSIS. OUT 003
      INTEGR. RESIST, TRANS OUT 004
      DOUBLE PRECISION RES,Z,RR,TSYN OUT 005
      DIMENSION SAR(39,40),HTGEN(40),DZA(40),DZB(40),RES(5,40), OUT 006
      $IS(39,25),NTR1(40),NTR(40,10),JIN(39),HX(39,40),TRF(40,14), OUT 007
      $Z(260),TC(39,40),ICN(63),JCN(63),ACR(63),AXPOS(40),RAD(25) OUT 008
      DIMENSION NIW(40),RATIO(40) OUT 009
      COMMON /DPNIW/RATIO,NIW OUT 010
      COMMON /DB1/RESIST,TO/DB3/RESIS,XLOAD,NTPRNT,ICN,JCN,KCN OUT 011
      $,DB6/IX,JF,AX/DB7/AXFOS/DB13/RES/DB14/NTRF,TRF/DB16/Z OUT 012
      $,DB24/M/DB46/SAR,IRSC,RAU/DB67/IS,DB68/MGROPT OUT 013
      $,DB69/NTRF,NTRF,IR70/JIN,IR77/A,K,RCY1,RCY2 OUT 014
      $,DB70/TOR(400,5),TORANI(400,5),TORAP1(400,5) OUT 015
      IF(KES'5T.EQ.1) WRITE (108,883) OUT 016
      803 IF(RAT (CDB CONFUTED SURFACE CONDUCTANCE (H) AND HEAT FLUX AT EAC OUT 017
      $H NODE ON THE CONTACT MINOR AND MAJOR AXIS IS //) OUT 018
      OPAT=0.0 OUT 019
      DTCT=0.0 OUT 020
      L=0 OUT 021
      C SWEEP THROUGH THE AXIAL NODES (K). OUT 022
      DO 10 K=1,AX OUT 023
      $K=NTRF(K) OUT 024
      $IF(K.EQ.1) GO TO 12 OUT 025
      CHECK WHETHER THE TRIAL FMS. USED AT AXIAL NODE K ARE THE SAME AS OUT 026
      THOSE USED AT K-1. IF THEY ARE, REUSE THE MATRIX CONTAINING THE OUT 027
      ORTHONORMALIZED TRIAL VECTORS (TOR). IF THEY AREN'T, STORE THE TRIAL OUT 028
      FMS. USED AT AXIAL NODE K AS THE COLUMNS OF TOR AND THEN CALL SUBR. OUT 029
      GRMSCH TO ORTHONORMALIZE THOSE COLUMNS. OUT 030
      DO 13 NTRIAL=1,10 OUT 031
      $IF(NTR(K,NTRIAL).NE.NTRK(K-1,NTRIAL)) GO TO 12 OUT 032
      13 CONTINUE OUT 033
      GO TO 14 OUT 034
      12 DO 11 NTRIAL=1,NT OUT 035
      $NTR=NTRK(K,NTRIAL) OUT 036
      DO 11 LL=1,LMAX OUT 037
      $TOR(LL,NTRIAL)=TRF(LL,NTR) OUT 038
      CALL GRMSCH(TOR,LMAX,NT) OUT 039
      LL=1 OUT 040
      14 INITIALIZE THE R-THETA NODE COUNTER (L2). OUT 041
      L2=0 OUT 042
      C SWEEP THROUGH THE CIRC. NODES (I). OUT 043
      DO 15 I=1,IXX OUT 044
      $I=2 OUT 045
      $IF(MGROPT.EQ.2) JI=JIN(I) OUT 046
      $ISP=1 OUT 047
      C SWEEP THROUGH THE RADIAL NODES (J). OUT 048
      DO 20 J=1,JX,ISP OUT 049
      $IF(MGROPT.NE.0) ISP=IS(I,J) OUT 050
      INCREMENT THE R-THETA NODE COUNT (L2). OUT 051
      L2=L2+1 OUT 052
      CALCULATE THE TEMPERATURE OF NODE I,J,K (TSYN) FROM THE NT TRIAL OUT 053
      FMS. USED AT AXIAL NODE K AND THEIR MULTIPLIERS IN Z. OUT 054
      TSYN=0. OUT 055
      DO 25 NN=1,NT OUT 056
      $LSP=L+NN OUT 057
      $TSYN=TSYN+TOR(L2,NN)*Z(LSP) OUT 058
      25 CALCULATE THE SURFACE CONDUCTANCE (CO) AT OUTER SURFACE NODE I,J,K OUT 059
      AND INCREASE THE TOTAL HEAT FLOW RATE THROUGH THE SURFACE (QTOT) OUT 060
      BY AN APPROPRIATE AMOUNT (IE.,CDB(TSYN-TC(I,K))). OUT 061
      CO=0.0 OUT 062
      HX2=HX(I,K) OUT 063
      IF(J.EQ.JX) CO=HX2*SAR(I,K) OUT 064
      IF(J.EQ.JX) QTOT=QTOT+CDB(TSYN-TC(I,K)) OUT 065
      INCREASE THE TOTAL HEAT FLOW RATE THROUGH THE CONTACT PATCH (OPAT) OUT 066
      BY AN APPROPRIATE AMOUNT AND CALCULATE THE SURFACE HEAT FLUX OUT 067
      (HFLUPA) (IF NECESSARY). OUT 068
      IF(J.EQ.JX.AND.I.LE.NIW(K)) OPAT=OPAT+CDB(TSYN-TC(I,K)) OUT 069
      IF(J.EQ.JX.AND.(K.EQ.1.OR.I.EQ.1)) HFLUPA=HX2*(TSYN-TC(I,K)) OUT 070
      887 WRITE (108,887) I,K,HX(I,K),TSYN,HFLUPA OUT 071
      $FORMAT(IX,'I=',I3,' K=',I3,' H=',F12.4,' T=',F10.5, OUT 072
      $ HEAT FLUX=',F10.3) OUT 073
      $IF(NTPRNT.EQ.0) GO TO 20 OUT 074
      CHECK WHETHER THE TEMPERATURE OF NODE I,J,K SHOULD BE PRINTED. OUT 075
      DO 30 LL=1,NTPRNT OUT 076
      $IF(ICN(LL).EQ.I.AND.JCN(LL).EQ.J.AND.KCN(LL).EQ.K) GO TO 35 OUT 077
      30 CONTINUE OUT 078
      GO TO 20 OUT 079
      35 LL=LL+1 OUT 080
      PRINT THE TEMPERATURE OF NODE I,J,K IF NECESSARY. IF IT IS A OUT 081
      BOUNDARY NODE, CALCULATE AND PRINT THE SURFACE HEAT FLUX (HFLUX). OUT 082
      WRITE (108,881) I,J,K,AXPOS(K),TSYN OUT 083
      IF(J.EQ.JX) HFLUX=HX2*(TSYN-TC(I,K)) ; OUTPUT HFLUX OUT 084
      $FORMAT(IX,'I=',I3,' J=',I3,' K=',I3,' T=',F10.5, OUT 085
      $ AXIAL POSITION =',F10.5,' TEMP =',F10.5) OUT 086
      20 CONTINUE OUT 087
      15 CONTINUE OUT 088
      $L=INT OUT 089
      10 CONTINUE OUT 090
      $IF(RESIST.EQ.1) WRITE (108,884) OUT 091
      884 FORMAT ('***** CONTACT RESISTANCE AND HEAT FLOW *****') OUT 092
      WRITE (108,882) QTOT OUT 093
      $FORMAT(IX, TOTAL HEAT FLOW =',F13.5,' W') OUT 094
      $IF(RESIST=1) 40,41,40 OUT 095
      CALCULATE AND PRINT THE RESULTS OF A 'RESISTANCE OF CONTACT' OUT 096
      ANALYSIS. OUT 097
      OUT 098
      OUT 099
      OUT 100
      OUT 101
      OUT 102
      OUT 103
      OUT 104
      OUT 105
      OUT 106
      OUT 107
      OUT 108
      OUT 109
      OUT 110
      OUT 111
      OUT 112

```

```

C
41 TLOW=TC(1,1) OUT 113
IF INSUL.EQ.1 TLOW=TIOW+(OTOT/(0.785*PI*HX(1,1))) OUT 114
OUTPUT TLOW OUT 115
OPDOT=OPAT/OTOT OUT 116
RESS=((10-TLOW)/OTOT*(2.0)-RESIS OUT 117
4R1TF (108,BB5) XLOAD,A,B,ACY1,RTY2,OPAT,RESS OUT 118
WRITE (108,BB8) OPDOT OUT 119
585 FORMAT (1X,'CONTACT LOAD =',F10.5,' N',/, OUT 120
1 1X,'SEMI-MAJOR DIAMETER (A) =',F10.5,' CM',/,1X,'SEMI-MIN OUT 121
1OR DIAMETER (B) =',F10.5,' CM',/,1X,'SMALL CONTACT RADIUS =',F10. OUT 122
50,' CM',/,1X,'LARGE CONTACT RADIUS =',F10.5,' CM',/,1X,'HEAT FLO OUT 123
SW THROUGH CONTACT PATCH =',F10.5,' W',/,1X,' RESISTANCE OF CO OUT 124
588 INTACT =',F10.5,' DEG.C/W',/, OUT 125
FORMAT (1X,'RATIO OF HEAT FLOW THROUGH PATCH TO TOTAL HEAT FLOW=' OUT 126
,F10.5, OUT 127
40 RETURN OUT 128
END OUT 129
OUT 130

```

```

SUBROUTINE GRMSCH (TOR,NROW,NCOL) GRMS 000
C THIS SUBROUTINE USES THE MODIFIED GRAM-SCHMIDT ALGORITHM WITH GRMS 001
C COLUMN PIVOTING TO ORTHONORMALIZE THE COLUMNS OF TOR. THE VARIABLES GRMS 002
C NROW AND NCOL ARE THE NUMBER OF ROWS AND THE NUMBER OF COLUMNS IN GRMS 003
C TOR RESPECTIVELY GRMS 004
C GRMS 005
C GRMS 006
DOUBLE PRECISION TSUM,R,RKI,DOTPR,XNORM GRMS 007
DIMENSION TOR(400,5),R(10),XNORM(10) GRMS 008
DO 10 K=1,NCOL GRMS 009
IF(K.EQ.1) GO TO 14 GRMS 010
TSUM=0. GRMS 011
DO 15 L=1,NROW GRMS 012
TSUM=TSUM+TOR(L,K)**2 GRMS 013
15 R(K)=SQRT(TSUM) GRMS 014
GO TO 14 GRMS 015
14 XNORM=0. GRMS 016
DO 12 LC=K,NCOL GRMS 017
DOTPR=0. GRMS 018
DO 13 LR=1,NROW GRMS 019
DOTPR=DOTPR+TOR(LR,LC)*TOR(LR,K)**2 GRMS 020
13 XNORM(LC)=SQRT(DOTPR) GRMS 021
IF(XNORM(LC).LE.XNORM) GO TO 12 GRMS 022
XNORM=XNORM(LC) GRMS 023
NEWIV=LC GRMS 024
CONTINUE GRMS 025
IF(NEWIV.EQ.K) R(K)=XNORM(K) ; GO TO 16 GRMS 026
DO 17 LR=1,NROW GRMS 027
TT=TOR(LR,K) GRMS 028
TOR(LR,K)=TOR(LR,NEWIV) GRMS 029
TOR(LR,NEWIV)=TT GRMS 030
R(K)=XNORM(NEWIV) GRMS 031
17 R(K)=1./R(K) GRMS 032
DO 20 L=1,NROW GRMS 033
TOR(L,K)=TOR(L,K)*R(K) GRMS 034
20 IF(K.EQ.NCOL) GO TO 50 GRMS 035
L=L+1 GRMS 036
DO 25 L=L,NCOL GRMS 037
DOTPR=0. GRMS 038
DO 30 M=1,NROW GRMS 039
DOTPR=DOTPR+TOR(M,K)*TOR(M,L) GRMS 040
25 R(L)=DOTPR GRMS 041
DO 40 L=L,NCOL GRMS 042
DO 40 M=1,NROW GRMS 043
TOR(M,L)=TOR(M,L)-R(L)*TOR(M,K) GRMS 044
40 CONTINUE GRMS 045
10 RETURN GRMS 046
50 END GRMS 047

```

2	200	1			
1	1	195.0			
1	0	175.8	1.0632	200.0	
0.15395	0.00961	0.9527	76.04	.000284	3.10000E-5

6 26 24

11				
13				
15				
17				
18				
35	0.09	0.9527	0.0018	
9	1.5708	0.00184		4
13	13			
17	11			
21	9			
	0.03	24	1	
	0.06	27	1	
	0.12	30		

27
24
21
17
13
8

2.8400E-4	3.1000E-5					
0.0	.3703	.3703	1134.0	1	1	
1						
0.0	0.0	100.0	0.0	100.0	1134.0	6

2.8400E-4	3.1000E-5					
0.0	.3703	.3703	1134.0	1	1	
0.0	0.0	100.0	0.0	100.0	1134.0	6

2.8400E-4	3.1000E-5						
0.0	.1703	.3703	1134.0	1	1		
1	.088						
0.0	0.0	100.0	0.0	100.0	1134.0	3	

2.8400E-4	3.1000E-5					
0.0	.3703	.3703	1134.0	1	1	
0.0	0.0	100.0	0.0	100.0	1134.0	5

2.8400E-4	3.1000E-3						
0.0	.3703	.3703	1134.0	1	1		
1	.117						
0.0	0.0	100.0	0.0	100.0	1134.0	.4	

2.8400E-4	3.1000E-5						
0.0	.3703	.3703	1134.0	1	1		
0.0	0.0	100.0	0.0	100.0	1134.0	4	

2.8400E-4	3.1000E-3						
0.0	.3703	.3703	1134.0	1	1		
1	.137						
0.0	0.0	100.0	0.0	100.0	1134.0	3	

2.8400E-4	3.1000E-5					
0.0	.3703	.3703	1134.0	1	1	
0.0	0.0	100.0	0.0	100.0	1134.0	3

2.8400E-4	3.1000E-5						
0.0	.3703	.3703	1134.0	1	1		
1	0.168						
0.0	0.0	100.0	0.0	100.0	1134.0	2	

[illegible]

[illegible][illegible][illegible][illegible][illegible]

271.

3	12	13	14	100.0
3	12	13	14	100.0
3	12	13	14	100.0
3	12	13	14	100.0
3	12	13	14	100.0
3	12	13	14	100.0

14h
13h
14h
13h
14h
13h
14h
13h
14h

Listing 5 Output Listing for the Sample Analysis of the Model Contact in Air.

***** RESISTANCE OF CONTACT ANALYSIS *****

***** SYNTHESIS SOLUTION *****

***** RUN WITH GAS CONDUCTION *****

GRID OPTION NUMBER = 2

***** DATA READ IN SUBROUTINE RAD12 *****

IF= 6 MX= 26 JX= 24

N = 1
N(1) = 7
N = 2
N(2) = 11
N = 3
N(3) = 13
N = 4
N(4) = 15
N = 5
N(5) = 17
N = 6
N(6) = 18
N2 = .09000 R0 = .95270 DR(N) = .00180

***** DATA READ IN SUBROUTINE DTH2 *****

IX= 35 ATOT= 1.57080 DTH(1)= .001834 NI= 4
II= 1 IN(II)= 9 ISC(II)= 15
II= 2 IN(II)= 13 ISC(II)= 13
II= 3 IN(II)= 17 ISC(II)= 11
II= 4 IN(II)= 21 ISC(II)= 9
N= 5 IN(N)= 24 DTH(N+1)= .03000 NEXT= 1
N= 6 IN(N)= 27 DTH(N+1)= .06000 NEXT= 1
N= 7 IN(N)= 30 DTH(N+1)= .12000 NEXT= 0
DTH(N+2) = .164685

***** DATA READ IN SUBROUTINE ISIJ *****

N = 1
JSC(N) = 27
N = 2
JSC(N) = 24
N = 3
JSC(N) = 21
N = 4
JSC(N) = 17
N = 5
JSC(N) = 13
N = 6
JSC(N) = 9

***** TRIAL FUNCTION NO. 1 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 0 RADBNB = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
MRADIN = .00000 BETA = .00000 TCINTB = .0000E+00 SLOPB = .0000E+00
REXCD = .0000E+00 TCCAS = .2840E-03 CCMN = .3100E-04
MYCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00
NI= .00 CX1= .37030 CK2= .37030 CCOND=1134.00000
OUTRY = 1 NEXT = 1
NZFLD = 0 NTFIC = 0 DZBC = .00000
MAX= 1 XPP= .00000

K= 1 HTGEN(K)= .0 TOUT= 100.00 HOUT= .00 TEX= 100.00 HEX= 1134.0 NZ1= 6

HFLZ(K) = .000000

KL = 6668

K	J	K	TEMP	HEAT FLUX	HEAT FLOW
1	24	1	100.49561	543.00781	.49186
2	24	2		572.09082	.99939
3	24	3		601.29858	1.05042
4	24	4		663.05937	1.15853
5	24	5		807.48633	1.41088
6	24	6		1444.54492	2.52398
7	24	7		113.45882	.19824
8	24	8		126.01614	.22018
9	24	9		114.68512	.30038
10	24	10		85.41142	.29847
11	24	11		84.49033	.22534
12	24	12		50.44001	.17433
13	24	13		49.71933	.21344
14	24	14		28.35399	.19818
15	24	15		21.04157	.14720
16	24	16		14.41377	.11472
17	24	17		13.21830	.13858
18	24	18		9.17449	.12824
19	24	19		6.79910	.09504
20	24	20		5.27043	.07347
21	24	21		4.22231	.06993
22	24	22		2.88044	.08233
23	24	23		2.10509	.04017
24	24	24		1.61758	.04913
25	24	25		1.03843	.03737
26	24	26		.72834	.04164
27	24	27		.54033	.04633
28	24	28		.33028	.03774
29	24	29		.22110	.02528
30	24	30		.15405	.02114
31	24	31		.10107	.01584

```

I= 32 K= 1 HEAT FLUX= .06584 HEAT FLOW= .01033
I= 33 K= 1 HEAT FLUX= .04034 HEAT FLOW= .00633
I= 34 K= 1 HEAT FLUX= .01946 HEAT FLOW= .00305
TOT = 10.7333
RESS = 9.31771
L = 397

```

***** TRIAL FUNCTION NO. 2 *****

***** DATA READ IN SUBROUTINE DATA *****

```

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MIRN = 0 RADBNB = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
MRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPS = .0000E+00
RFXCO = .0000E+00 TCGAS = .2840E 03 CCNN = .3100E-04
MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00
H1= .00 CK1= .37030 CK2= .37030 CCEND=1136.00000
OUTRY = 1 NEXT = 1
NZFLQ = 0 NTFBC = 0 NZBC = .00000
NAX = 0 XPP = .00000

```

```

F= 1 HTGEN(K)= .0 TOUT= 100.00 HOUT= .00 TEX= 100.00 HEX= 1136.0 HZ1= 0

```

HFLZ(K) = .000000

NL = 6668

```

I= 1 J= 24 K= 1 TEMP = 100.60085
I= 1 K= 1 HEAT FLUX= 682.56030 HEAT FLOW= .59630
I= 2 K= 1 HEAT FLUX= 694.83276 HEAT FLOW= 1.21405
I= 3 K= 1 HEAT FLUX= 734.80493 HEAT FLOW= 1.28389
I= 4 K= 1 HEAT FLUX= 819.60278 HEAT FLOW= 1.43205
I= 5 K= 1 HEAT FLUX= 1020.79563 HEAT FLOW= 1.78771
I= 6 K= 1 HEAT FLUX= 1970.18945 HEAT FLOW= 3.35505
I= 7 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 8 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 9 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 10 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 11 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 12 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 13 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 14 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 15 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 16 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 17 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 18 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 19 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 20 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 21 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 22 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 23 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 24 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 25 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 26 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 27 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 28 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 29 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 30 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 31 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 32 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 33 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000
I= 34 K= 1 HEAT FLUX= .00000 HEAT FLOW= .00000

```

OTOT = 9.64406

RESS = 10.3476

L = 397

***** TRIAL FUNCTION NO. 3 *****

***** DATA READ IN SUBROUTINE DATA *****

```

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MIRN = 0 RADBNB = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
MRADIN = .00000 BETA = .00000 TCINTS = .0000E+00 SLOPS = .0000E+00
RFXCO = .0000E+00 TCGAS = .2840E 03 CCNN = .3100E-04
MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00
H1= .00 CK1= .37030 CK2= .37030 CCEND=1136.00000
OUTRY = 1 NEXT = 1
NZFLQ = 0 NTFBC = 0 NZBC = .00000
NAX = 1 XPP = .08800

```

```

F= 1 HTGEN(K)= .0 TOUT= 100.00 HOUT= .00 TEX= 100.00 HEX= 1136.0 HZ1= 5

```

HFLZ(K) = .000000

NL = 6668

```

I= 1 J= 24 K= 1 TEMP = 100.57510
I= 1 K= 1 HEAT FLUX= 433.31787 HEAT FLOW= .37076
I= 2 K= 1 HEAT FLUX= 668.88379 HEAT FLOW= 1.16871
I= 3 K= 1 HEAT FLUX= 722.84448 HEAT FLOW= 1.26299
I= 4 K= 1 HEAT FLUX= 866.57788 HEAT FLOW= 1.51413
I= 5 K= 1 HEAT FLUX= 1528.40474 HEAT FLOW= 2.67051
I= 6 K= 1 HEAT FLUX= 120.34367 HEAT FLOW= .10327
I= 7 K= 1 HEAT FLUX= 135.86258 HEAT FLOW= .11739
I= 8 K= 1 HEAT FLUX= 124.87636 HEAT FLOW= .10837
I= 9 K= 1 HEAT FLUX= 109.18790 HEAT FLOW= .09415
I= 10 K= 1 HEAT FLUX= 81.56738 HEAT FLOW= .07804
I= 11 K= 1 HEAT FLUX= 62.53817 HEAT FLOW= .05461
I= 12 K= 1 HEAT FLUX= 49.56683 HEAT FLOW= .04321
I= 13 K= 1 HEAT FLUX= 40.36380 HEAT FLOW= .03458
I= 14 K= 1 HEAT FLUX= 28.38387 HEAT FLOW= .02437
I= 15 K= 1 HEAT FLUX= 21.26329 HEAT FLOW= .01861
I= 16 K= 1 HEAT FLUX= 16.67505 HEAT FLOW= .01419
I= 17 K= 1 HEAT FLUX= 13.41418 HEAT FLOW= .01162
I= 18 K= 1 HEAT FLUX= 9.32744 HEAT FLOW= .00808
I= 19 K= 1 HEAT FLUX= 6.91837 HEAT FLOW= .00594
I= 20 K= 1 HEAT FLUX= 5.36007 HEAT FLOW= .00462
I= 21 K= 1 HEAT FLUX= 4.79713 HEAT FLOW= .00413
I= 22 K= 1 HEAT FLUX= 2.92494 HEAT FLOW= .00260
I= 23 K= 1 HEAT FLUX= 2.13513 HEAT FLOW= .00182
I= 24 K= 1 HEAT FLUX= 1.63383 HEAT FLOW= .00140
I= 25 K= 1 HEAT FLUX= 1.05044 HEAT FLOW= .00095

```



```

I= 26  A= 1  HEAT FLUX= .73574  HEAT FLOW= .04206
I= 27  A= 1  HEAT FLUX= .54502  HEAT FLOW= .04673
I= 28  A= 1  HEAT FLUX= .33255  HEAT FLOW= .03802
I= 29  A= 1  HEAT FLUX= .22732  HEAT FLOW= .02542
I= 30  A= 1  HEAT FLUX= .15673  HEAT FLOW= .02125
I= 31  A= 1  HEAT FLUX= .10139  HEAT FLOW= .01591
I= 32  A= 1  HEAT FLUX= .06599  HEAT FLOW= .01035
I= 33  A= 1  HEAT FLUX= .04039  HEAT FLOW= .00634
I= 34  A= 1  HEAT FLUX= .01947  HEAT FLOW= .00305

```

OTOT = 10.5086
 RESS = 9.51298
 L = 397

***** TRIAL FUNCTION NO. 4 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 0 RABND = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
 HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPS = .0000E+00
 REACO = .000E+00 TCGAS = .2840E-03 CCNN = .3100E-04

MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

M1 = .00 CK1 = .37030 CK2 = .37030 CCND = 1136.00000

OUTRY = 1 NEXT = 1

NZFLO = 0 NTFBC = 0 DZBC = .00000

NAX = 0 TPP = .00000

N = 1 NTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1136.0 NZI = 5

HFLZ(K) = .000000

KL = 6668

```

I= 1  J= 24  K= 1  TEMP = 100.71613
I= 1  A= 1  HEAT FLUX= 813.51855  HEAT FLOW= .71071
I= 2  A= 1  HEAT FLUX= 835.23804  HEAT FLOW= 1.45937
I= 3  A= 1  HEAT FLUX= 911.23022  HEAT FLOW= 1.59715
I= 4  A= 1  HEAT FLUX= 1115.52856  HEAT FLOW= 1.94911
I= 5  A= 1  HEAT FLUX= 2070.37109  HEAT FLOW= 3.61746
I= 6  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 7  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 8  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 9  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 10  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 11  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 12  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 13  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 14  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 15  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 16  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 17  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 18  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 19  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 20  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 21  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 22  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 23  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 24  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 25  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 26  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 27  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 28  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 29  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 30  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 31  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 32  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 33  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000
I= 34  A= 1  HEAT FLUX= .00000  HEAT FLOW= .00000

```

OTOT = 9.32880

RESS = 10.7195

L = 397

***** TRIAL FUNCTION NO. 5 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 0 RABND = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
 HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPS = .0000E+00
 REACO = .0000E+00 TCGAS = .2840E-03 CCNN = .3100E-04

MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

M1 = .00 CK1 = .37030 CK2 = .37030 CCND = 1136.00000

OUTRY = 1 NEXT = 1

NZFLO = 0 NTFBC = 0 DZBC = .00000

NAX = 1 XPP = .11700

N = 1 NTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1136.0 NZI = 4

HFLZ(K) = .000000

KL = 6668

```

I= 1  J= 24  K= 1  TEMP = 100.69095
I= 1  A= 1  HEAT FLUX= 784.91748  HEAT FLOW= .68572
I= 2  A= 1  HEAT FLUX= 816.25732  HEAT FLOW= 1.42621
I= 3  A= 1  HEAT FLUX= 951.51440  HEAT FLOW= 1.66253
I= 4  A= 1  HEAT FLUX= 1641.23364  HEAT FLOW= 2.86763
I= 5  A= 1  HEAT FLUX= 130.19386  HEAT FLOW= .22748
I= 6  A= 1  HEAT FLUX= 150.09714  HEAT FLOW= .26226
I= 7  A= 1  HEAT FLUX= 139.78300  HEAT FLOW= .24424
I= 8  A= 1  HEAT FLUX= 122.78444  HEAT FLOW= .21454
I= 9  A= 1  HEAT FLUX= 106.31613  HEAT FLOW= .27864
I= 10  A= 1  HEAT FLUX= 80.08621  HEAT FLOW= .27986
I= 11  A= 1  HEAT FLUX= 62.10233  HEAT FLOW= .21702
I= 12  A= 1  HEAT FLUX= 49.61833  HEAT FLOW= .17359
I= 13  A= 1  HEAT FLUX= 40.57533  HEAT FLOW= .21369
I= 14  A= 1  HEAT FLUX= 28.81030  HEAT FLOW= .20136
I= 15  A= 1  HEAT FLUX= 21.45488  HEAT FLOW= .15136
I= 16  A= 1  HEAT FLUX= 16.96190  HEAT FLOW= .11855
I= 17  A= 1  HEAT FLUX= 13.69669  HEAT FLOW= .14359
I= 18  A= 1  HEAT FLUX= 9.52730  HEAT FLOW= .13317
I= 19  A= 1  HEAT FLUX= 7.05984  HEAT FLOW= .09868

```

I= 20	K= 1	HEAT FLUX=	5.46742	HEAT FLOW=	.07647
I= 21	K= 1	HEAT FLUX=	4.37421	HEAT FLOW=	.09308
I= 22	K= 1	HEAT FLUX=	2.97587	HEAT FLOW=	.08505
I= 23	K= 1	HEAT FLUX=	2.14898	HEAT FLOW=	.06199
I= 24	K= 1	HEAT FLUX=	1.65752	HEAT FLOW=	.07106
I= 25	K= 1	HEAT FLUX=	1.06346	HEAT FLOW=	.06079
I= 26	K= 1	HEAT FLUX=	.74364	HEAT FLOW=	.04251
I= 27	K= 1	HEAT FLUX=	.55014	HEAT FLOW=	.04717
I= 28	K= 1	HEAT FLUX=	.33302	HEAT FLOW=	.03830
I= 29	K= 1	HEAT FLUX=	.17363	HEAT FLOW=	.02577
I= 30	K= 1	HEAT FLUX=	.15748	HEAT FLOW=	.02136
I= 31	K= 1	HEAT FLUX=	.10174	HEAT FLOW=	.01594
I= 32	K= 1	HEAT FLUX=	.06615	HEAT FLOW=	.01038
I= 33	K= 1	HEAT FLUX=	.04045	HEAT FLOW=	.00635
I= 34	K= 1	HEAT FLUX=	.01948	HEAT FLOW=	.00306

QTOT = 10.2780

RESS = 9.74853

L = 397

***** TRIAL FUNCTION NO. 6 *****

***** DATA READ IN SUBROUTINE DATA *****

TFMS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 0 RADRND = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPG = .0000E+00
REXCO = .0000E+00 TCGAS = .7840E-03 CCNM = .3100E-04

NUCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

HJ = .00 CK1 = .37030 CK2 = .37030 CCOND = 1136.00000

OUTRY = 1 NEX = 1

NZFLO = 0 NTFBC = 0 DZBC = .00000

MAX = 0 XPP = .00000

K = 1 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1136.0 NZ1 = 4

HFLZ(K) = .000000

KL = 6668

I= 1	J= 24	K= 1	TEMP =	100.89354	
I= 1	K= 1	HEAT FLUX=	1015.06079	HEAT FLOW=	.88678
I= 2	K= 1	HEAT FLUX=	1060.66650	HEAT FLOW=	1.85325
I= 3	K= 1	HEAT FLUX=	1259.53931	HEAT FLOW=	2.20073
I= 4	K= 1	HEAT FLUX=	2285.84985	HEAT FLOW=	3.99395
I= 5	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 6	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 7	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 8	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 9	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 10	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 11	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 12	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 13	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 14	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 15	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 16	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 17	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 18	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 19	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 20	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 21	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 22	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 23	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 24	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 25	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 26	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 27	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 28	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 29	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 30	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 31	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 32	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 33	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000
I= 34	K= 1	HEAT FLUX=	.00000	HEAT FLOW=	.00000

QTOT = 8.93472

RESS = 11.1923

L = 397

***** TRIAL FUNCTION NO. 7 *****

***** DATA READ IN SUBROUTINE DATA *****

TFMS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 0 RADRND = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPG = .0000E+00
REXCO = .0000E+00 TCGAS = .7840E-03 CCNM = .3100E-04

NUCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

HJ = .00 CK1 = .37030 CK2 = .37030 CCOND = 1136.00000

OUTRY = 1 NEX = 1

NZFLO = 0 NTFBC = 0 DZBC = .00000

MAX = 1 XPP = .13700

K = 1 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1136.0 NZ1 = 3

HFLZ(K) = .000000

KL = 6668

I= 1	J= 24	K= 1	TEMP =	100.88451	
I= 1	K= 1	HEAT FLUX=	1007.04982	HEAT FLOW=	.87780
I= 2	K= 1	HEAT FLUX=	1102.85742	HEAT FLOW=	1.92697
I= 3	K= 1	HEAT FLUX=	1825.00854	HEAT FLOW=	3.18875
I= 4	K= 1	HEAT FLUX=	141.97417	HEAT FLOW=	.24804
I= 5	K= 1	HEAT FLUX=	165.95407	HEAT FLOW=	.28996
I= 6	K= 1	HEAT FLUX=	156.82520	HEAT FLOW=	.27401
I= 7	K= 1	HEAT FLUX=	139.00865	HEAT FLOW=	.24288
I= 8	K= 1	HEAT FLUX=	120.88577	HEAT FLOW=	.21122
I= 9	K= 1	HEAT FLUX=	104.84000	HEAT FLOW=	.17483
I= 10	K= 1	HEAT FLUX=	79.89474	HEAT FLOW=	.12719
I= 11	K= 1	HEAT FLUX=	62.48305	HEAT FLOW=	.10135
I= 12	K= 1	HEAT FLUX=	50.28528	HEAT FLOW=	.07572
I= 13	K= 1	HEAT FLUX=	41.39784	HEAT FLOW=	.05197

276.

I	K	HEAT FLUX	HEAT FLOW
14	K=1	29.54405	.26548
15	K=1	22.75821	.15336
16	K=1	17.44638	.12193
17	K=1	14.08951	.14770
18	K=1	9.78136	.13688
19	K=1	7.24472	.10129
20	K=1	5.40432	.07834
21	K=1	4.47771	.09328
22	K=1	3.03911	.08686
23	K=1	2.21063	.06318
24	K=1	1.68651	.07230
25	K=1	1.07925	.06189
26	K=1	.75316	.04305
27	K=1	.55629	.04770
28	K=1	.33797	.03864
29	K=1	.22521	.02575
30	K=1	.15836	.02148
31	K=1	.10715	.01603
32	K=1	.06433	.01041
33	K=1	.04052	.00436
34	K=1	.01950	.00306

QDOT = 9.94567
RESS = 10.0334
L = 397

***** TRIAL FUNCTION NO. 8 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MIN = 0 RADBNB = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPS = .0000E+00
REXCO = .0000E+00 TCGAS = .2840E-03 CDM = .3100E-04

MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

HI = .00 CK1 = .37030 CK2 = .37030 CCONB = 1134.00000

OUTRY = 1 NEX = 1

NZFLO = 0 NTFSC = 0 DZBC = .00000

MAX = 0 XPP = .00000

K = 1 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1134.0 HZ1 = 3

HFLZ(K) = .000000

KL = 6668

I	J	K	TEMP	HEAT FLUX	HEAT FLOW
1	24	K=1	101.20544	1369.38477	1.19633
2	K=1			1515.70874	2.64867
3	K=1			2630.41479	4.37600
4	K=1			.000000	.000000
5	K=1			.000000	.000000
6	K=1			.000000	.000000
7	K=1			.000000	.000000
8	K=1			.000000	.000000
9	K=1			.000000	.000000
10	K=1			.000000	.000000
11	K=1			.000000	.000000
12	K=1			.000000	.000000
13	K=1			.000000	.000000
14	K=1			.000000	.000000
15	K=1			.000000	.000000
16	K=1			.000000	.000000
17	K=1			.000000	.000000
18	K=1			.000000	.000000
19	K=1			.000000	.000000
20	K=1			.000000	.000000
21	K=1			.000000	.000000
22	K=1			.000000	.000000
23	K=1			.000000	.000000
24	K=1			.000000	.000000
25	K=1			.000000	.000000
26	K=1			.000000	.000000
27	K=1			.000000	.000000
28	K=1			.000000	.000000
29	K=1			.000000	.000000
30	K=1			.000000	.000000
31	K=1			.000000	.000000
32	K=1			.000000	.000000
33	K=1			.000000	.000000
34	K=1			.000000	.000000

QDOT = 8.44100
RESS = 11.8469
L = 397

***** TRIAL FUNCTION NO. 9 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MIN = 0 RADBNB = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPS = .0000E+00
REXCO = .0000E+00 TCGAS = .2840E-03 CDM = .3100E-04

MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

HI = .00 CK1 = .37030 CK2 = .37030 CCONB = 1134.00000

OUTRY = 1 NEX = 1

NZFLO = 0 NTFSC = 0 DZBC = .00000

MAX = 1 XPP = .14800

K = 1 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1134.0 HZ1 = 2

HFLZ(K) = .000000

KL = 6668

I	J	K	TEMP	HEAT FLUX	HEAT FLOW
1	24	K=1	101.31750	1496.48335	1.30754
2	K=1			2169.34814	3.79040
3	K=1			143.93179	.28443
4	K=1			174.84491	.34049
5	K=1			186.83354	.32648
6	K=1			166.81677	.29147
7	K=1			145.26205	.25381

I= 8	K= 1	HEAT FLUX=	125.71607	HEAT FLOW=	.21966
I= 9	K= 1	HEAT FLUX=	109.00876	HEAT FLOW=	.28570
I= 10	K= 1	HEAT FLUX=	83.23117	HEAT FLOW=	.29085
I= 11	K= 1	HEAT FLUX=	55.34511	HEAT FLOW=	.2335
I= 12	K= 1	HEAT FLUX=	52.64093	HEAT FLOW=	.18395
I= 13	K= 1	HEAT FLUX=	43.34783	HEAT FLOW=	.22719
I= 14	K= 1	HEAT FLUX=	30.91826	HEAT FLOW=	.21609
I= 15	K= 1	HEAT FLUX=	23.75772	HEAT FLOW=	.16255
I= 16	K= 1	HEAT FLUX=	18.19673	HEAT FLOW=	.12718
I= 17	K= 1	HEAT FLUX=	14.46736	HEAT FLOW=	.15377
I= 18	K= 1	HEAT FLUX=	10.16137	HEAT FLOW=	.14204
I= 19	K= 1	HEAT FLUX=	7.49906	HEAT FLOW=	.10481
I= 20	K= 1	HEAT FLUX=	5.78434	HEAT FLOW=	.08085
I= 21	K= 1	HEAT FLUX=	4.61170	HEAT FLOW=	.09813
I= 22	K= 1	HEAT FLUX=	3.11929	HEAT FLOW=	.08915
I= 23	K= 1	HEAT FLUX=	2.26277	HEAT FLOW=	.06467
I= 24	K= 1	HEAT FLUX=	1.72248	HEAT FLOW=	.07385
I= 25	K= 1	HEAT FLUX=	1.09866	HEAT FLOW=	.06780
I= 26	K= 1	HEAT FLUX=	.76481	HEAT FLOW=	.04372
I= 27	K= 1	HEAT FLUX=	.56381	HEAT FLOW=	.04834
I= 28	K= 1	HEAT FLUX=	.34157	HEAT FLOW=	.03905
I= 29	K= 1	HEAT FLUX=	.22713	HEAT FLOW=	.02597
I= 30	K= 1	HEAT FLUX=	.15945	HEAT FLOW=	.02162
I= 31	K= 1	HEAT FLUX=	.10264	HEAT FLOW=	.01611
I= 32	K= 1	HEAT FLUX=	.06656	HEAT FLOW=	.01044
I= 33	K= 1	HEAT FLUX=	.04061	HEAT FLOW=	.00637
I= 34	K= 1	HEAT FLUX=	.01952	HEAT FLOW=	.00306

OTOT = 9.62287
 RESS = 10.3919
 L = 397

***** TRIAL FUNCTION NO. 10 *****

***** DATA READ IN SUBROUTINE DATA *****

TRANS = 0 NTRIF = 0 NTFCH = 0 NTFIC = 0

MLIN = 0 PADBNR = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
 HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPS = .0000E+00
 REXCO = .0000E+00 TCGAS = .2840E-03 CCNW = .3100E-04

MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

HI = .00 CK1 = .37030 CK2 = .37030 CCOWD = 1136.00000
 OUTRY = 1 NEXT = 1

NZFLO = 0 NTFBC = 0 DZBC = .00000

MAX = 0 XPP = .00000

K= 1 NTGEN(K)= .0 TOUT= 100.00 MOUT= .00 TEX= 100.00 MEX= 1136.0 NZI= 2

HFLZ(K) = .000000

KL = 6668

I	J	K	TEMP	HEAT FLUX	HEAT FLOW
1	24	1	101.95401	2219.75537	1.93923
2	1	1		3327.51831	5.81401
3	1	1		.000000	.00000
4	1	1		.000000	.00000
5	1	1		.000000	.00000
6	1	1		.000000	.00000
7	1	1		.000000	.00000
8	1	1		.000000	.00000
9	1	1		.000000	.00000
10	1	1		.000000	.00000
11	1	1		.000000	.00000
12	1	1		.000000	.00000
13	1	1		.000000	.00000
14	1	1		.000000	.00000
15	1	1		.000000	.00000
16	1	1		.000000	.00000
17	1	1		.000000	.00000
18	1	1		.000000	.00000
19	1	1		.000000	.00000
20	1	1		.000000	.00000
21	1	1		.000000	.00000
22	1	1		.000000	.00000
23	1	1		.000000	.00000
24	1	1		.000000	.00000
25	1	1		.000000	.00000
26	1	1		.000000	.00000
27	1	1		.000000	.00000
28	1	1		.000000	.00000
29	1	1		.000000	.00000
30	1	1		.000000	.00000
31	1	1		.000000	.00000
32	1	1		.000000	.00000
33	1	1		.000000	.00000
34	1	1		.000000	.00000

OTOT = 7.73325
 RESS = 12.8978
 L = 397

***** TRIAL FUNCTION NO. 11 *****

***** DATA READ IN SUBROUTINE DATA *****

TRANS = 0 NTRIF = 0 NTFCH = 0 NTFIC = 0

MLIN = 0 PADBNR = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000
 HRADIN = .00000 BETA = .00000 TCINTG = .0000E+00 SLOPS = .0000E+00
 REXCO = .0000E+00 TCGAS = .2840E-03 CCNW = .3100E-04

MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

HI = .00 CK1 = .37030 CK2 = .37030 CCOWD = 1136.00000
 OUTRY = 1 NEXT = 1

NZFLO = 0 NTFBC = 0 DZBC = .00000

MAX = 1 XPP = .16000

K= 1 NTGEN(K)= .0 TOUT= 100.00 MOUT= .00 TEX= 100.00 MEX= .0 NZI= 0

HFLZ(K) = .000000

KL = 6668

I	J	K	TEMP	HEAT FLUX	HEAT FLOW
1	24	1	131.77731	476.37769	.41618

```

J= 2  K= 1  HEAT FLUX= 460.19946  HEAT FLOW= .80408
J= 3  K= 1  HEAT FLUX= 418.06396  HEAT FLOW= .73046
J= 4  K= 1  HEAT FLUX= 363.63892  HEAT FLOW= .63537
J= 5  K= 1  HEAT FLUX= 308.73486  HEAT FLOW= .53944
J= 6  K= 1  HEAT FLUX= 259.66455  HEAT FLOW= .45370
J= 7  K= 1  HEAT FLUX= 218.37336  HEAT FLOW= .38156
J= 8  K= 1  HEAT FLUX= 184.58719  HEAT FLOW= .32252
J= 9  K= 1  HEAT FLUX= 157.71289  HEAT FLOW= .27503
J= 10 K= 1  HEAT FLUX= 137.00752  HEAT FLOW= .23888
J= 11 K= 1  HEAT FLUX= 119.97310  HEAT FLOW= .21144
J= 12 K= 1  HEAT FLUX= 105.14679  HEAT FLOW= .18862
J= 13 K= 1  HEAT FLUX= 92.47184  HEAT FLOW= .16831
J= 14 K= 1  HEAT FLUX= 81.12386  HEAT FLOW= .15043
J= 15 K= 1  HEAT FLUX= 71.57355  HEAT FLOW= .13469
J= 16 K= 1  HEAT FLUX= 63.25342  HEAT FLOW= .12092
J= 17 K= 1  HEAT FLUX= 55.88401  HEAT FLOW= .10895
J= 18 K= 1  HEAT FLUX= 49.25700  HEAT FLOW= .09858
J= 19 K= 1  HEAT FLUX= 43.89123  HEAT FLOW= .08957
J= 20 K= 1  HEAT FLUX= 39.26574  HEAT FLOW= .08169
J= 21 K= 1  HEAT FLUX= 35.33351  HEAT FLOW= .07479
J= 22 K= 1  HEAT FLUX= 32.04420  HEAT FLOW= .06888
J= 23 K= 1  HEAT FLUX= 29.34329  HEAT FLOW= .06385
J= 24 K= 1  HEAT FLUX= 27.19096  HEAT FLOW= .05953
J= 25 K= 1  HEAT FLUX= 25.49894  HEAT FLOW= .05581
J= 26 K= 1  HEAT FLUX= 24.2468  HEAT FLOW= .05264
J= 27 K= 1  HEAT FLUX= 23.40726  HEAT FLOW= .04997
J= 28 K= 1  HEAT FLUX= 22.8990  HEAT FLOW= .04774
J= 29 K= 1  HEAT FLUX= 22.686  HEAT FLOW= .04591
J= 30 K= 1  HEAT FLUX= 22.6493  HEAT FLOW= .04443
J= 31 K= 1  HEAT FLUX= 22.7825  HEAT FLOW= .04326
J= 32 K= 1  HEAT FLUX= 23.0773  HEAT FLOW= .04236
J= 33 K= 1  HEAT FLUX= 23.5106  HEAT FLOW= .04164
J= 34 K= 1  HEAT FLUX= 24.0762  HEAT FLOW= .04108

```

```

QTDI = 7.85901
RESS = 12.7242
L = 397

```

```
***** TRIAL FUNCTION NO. 12 *****
```

```
***** DATA READ IN SUBROUTINE DATA *****
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```

TRANS = 0      MTRTF = 0      MTFCH = 0      MTFIC = 0
MLIN = 0      RADBN0 = 0      MAXIT = 0      EPSLN = .00000      TRADIN = .00000
HRADIN = .00000      BETA = .00000      TCINTG = .0000E+00      SLOPG = .0000E+00
REXCO = .0000E+00      TCGAS = .2840E 03      CCNN = .3100F-04
MVCON = 0      TCINTS = .00000E+00      SLOPS = .00000E+00
H1 = .00      CK1 = .37030      CK2 = .37030      CCNND=1136.00000
OUTRY = 1      MEXT = 1
NZFLO = 0      MTFBC = 0      DZBC = .00000
MAX = 1      XPP = .11000

```

```

K= 1  HTGEN(K)= .0  TOUT= 100.00  HOUT= .00  TEX= 100.00  HEX= .0  NZ1= 0

```

```
HFLZ(K) = .000000
```

```
NL = 6668
```

```

J= 1  K= 24  K= 1  TEMP = 152.93587
J= 1  K= 1  HEAT FLUX= 209.33057  HEAT FLOW= .18288
J= 2  K= 1  HEAT FLUX= 207.20929  HEAT FLOW= .18205
J= 3  K= 1  HEAT FLUX= 201.10428  HEAT FLOW= .18138
J= 4  K= 1  HEAT FLUX= 191.72121  HEAT FLOW= .18099
J= 5  K= 1  HEAT FLUX= 180.01920  HEAT FLOW= .18145
J= 6  K= 1  HEAT FLUX= 166.99236  HEAT FLOW= .18178
J= 7  K= 1  HEAT FLUX= 153.50771  HEAT FLOW= .18222
J= 8  K= 1  HEAT FLUX= 140.22548  HEAT FLOW= .18450
J= 9  K= 1  HEAT FLUX= 127.58868  HEAT FLOW= .18349
J= 10 K= 1  HEAT FLUX= 105.15334  HEAT FLOW= .18746
J= 11 K= 1  HEAT FLUX= 86.76476  HEAT FLOW= .18320
J= 12 K= 1  HEAT FLUX= 72.10477  HEAT FLOW= .18197
J= 13 K= 1  HEAT FLUX= 60.48953  HEAT FLOW= .17107
J= 14 K= 1  HEAT FLUX= 43.87721  HEAT FLOW= .16467
J= 15 K= 1  HEAT FLUX= 33.03995  HEAT FLOW= .16392
J= 16 K= 1  HEAT FLUX= 25.69777  HEAT FLOW= .17960
J= 17 K= 1  HEAT FLUX= 20.53044  HEAT FLOW= .21523
J= 18 K= 1  HEAT FLUX= 13.95076  HEAT FLOW= .19500
J= 19 K= 1  HEAT FLUX= 10.09521  HEAT FLOW= .14111
J= 20 K= 1  HEAT FLUX= 7.64977  HEAT FLOW= .10493
J= 21 K= 1  HEAT FLUX= 6.00231  HEAT FLOW= .12773
J= 22 K= 1  HEAT FLUX= 3.93214  HEAT FLOW= .11296
J= 23 K= 1  HEAT FLUX= 2.80431  HEAT FLOW= .08013
J= 24 K= 1  HEAT FLUX= 2.09603  HEAT FLOW= .08986
J= 25 K= 1  HEAT FLUX= 1.29779  HEAT FLOW= .07431
J= 26 K= 1  HEAT FLUX= .88332  HEAT FLOW= .05062
J= 27 K= 1  HEAT FLUX= .51154  HEAT FLOW= .05501
J= 28 K= 1  HEAT FLUX= .34874  HEAT FLOW= .04130
J= 29 K= 1  HEAT FLUX= .24489  HEAT FLOW= .02823
J= 30 K= 1  HEAT FLUX= .17059  HEAT FLOW= .02313
J= 31 K= 1  HEAT FLUX= .10791  HEAT FLOW= .01693
J= 32 K= 1  HEAT FLUX= .06894  HEAT FLOW= .01082
J= 33 K= 1  HEAT FLUX= .04152  HEAT FLOW= .00451
J= 34 K= 1  HEAT FLUX= .01972  HEAT FLOW= .00309

```

```

QTDI = 4.02302
RESS = 16.6030
L = 397

```

```
***** TRIAL FUNCTION NO. 13 *****
```

```
***** DATA READ IN SUBROUTINE DATA *****
```

```

TRANS = 0      MTRTF = 0      MTFCH = 0      MTFIC = 0
MLIN = 0      RADBN0 = 0      MAXIT = 0      EPSLN = .00000      TRADIN = .00000
HRADIN = .00000      BETA = .00000      TCINTS = .0000E+00      SLOPS = .0000E+00
REXCO = .0000E+00      TCGAS = .2840E 03      CCNN = .3100E-04
MVCON = 0      TCINTS = .00000E+00      SLOPS = .00000E+00
H1 = .00      CK1 = .37030      CK2 = .37030      CCNND=1136.00000
OUTRY = 1      MEXT = 0
NZFLO = 0      MTFBC = 0      DZBC = .00000
MAX = 1      XPP = .35000

```

N = 1 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = .0 HZ1 = 0

HFLZ(K) = .000000

KL = 6668

I	J	K	TEMP	HEAT FLUX	HEAT FLOW
1	1	24	175.1178	66.03195	.05769
1	2	K=1		65.87502	.11510
1	3	K=1		65.40808	.11428
1	4	K=1		64.64444	.11295
1	5	K=1		63.60481	.11113
1	6	K=1		62.31653	.10888
1	7	K=1		60.81239	.10625
1	8	K=1		59.12695	.10331
1	9	K=1		57.29492	.10017
1	10	K=1		53.34502	.18441
1	11	K=1		49.20576	.17195
1	12	K=1		45.08644	.15755
1	13	K=1		41.12851	.21559
1	14	K=1		34.00841	.23768
1	15	K=1		28.10095	.19640
1	16	K=1		23.34489	.16316
1	17	K=1		17.55334	.29499
1	18	K=1		14.10813	.19720
1	19	K=1		10.15404	.14752
1	20	K=1		8.15144	.11394
1	21	K=1		6.46713	.13762
1	22	K=1		4.30268	.12297
1	23	K=1		3.05998	.08746
1	24	K=1		2.28470	.09795
1	25	K=1		1.40955	.08057
1	26	K=1		.95378	.05452
1	27	K=1		.68450	.05886
1	28	K=1		.40076	.04582
1	29	K=1		.25874	.02958
1	30	K=1		.17730	.02404
1	31	K=1		.11107	.01743
1	32	K=1		.07034	.01104
1	33	K=1		.04204	.00660
1	34	K=1		.01984	.00311

QTOT = 3.74773

RFSS = 26.6686

L = 397

CONSTANT TEMPERATURE FOR FINAL TRIAL FUNCTION (NO. 14) = 195.00000

***** 3-D DATA FOLLOWS *****

***** AXIAL GRID DATA READ IN SUBROUTINE DZ *****

K	DZ	KCH(K)	NEXT
K= 1	DZ = .0279900	KCH(K)= 3	NEXT= 1
K= 2	DZ = .0139950	KCH(K)= 6	NEXT= 1
K= 3	DZ = .0091300	KCH(K)= 9	NEXT= 1
K= 4	DZ = .0046650	KCH(K)= 12	NEXT= 1
K= 5	DZ = .0039986	KCH(K)= 19	NEXT= 1
K= 6	DZ = .0046650	KCH(K)= 22	NEXT= 1
K= 7	DZ = .0091300	KCH(K)= 25	NEXT= 1
K= 8	DZ = .0139950	KCH(K)= 26	NEXT= 1
K= 9	DZ = .0279900	KCH(K)= 30	NEXT= 1
K= 10	DZ = .0559800	KCH(K)= 34	NEXT= 1
K= 11	DZ = .1183670	KCH(K)= 40	NEXT= 0

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 0 RADIND = 0 MAXIT = 0 EPSLN = .00000 TRADIN = .00000

MRABIN = .00000 BETA = .00000 TCINTS = .00000E+00 SLOPS = .00000E+00

RFCD = .00000E+00 TCBAS = .2840E-03 CCMN = .3700E-04

MVCOM = 0 TCINTS = .00000E+00 SLOPS = .00000E+00

H1 = .00 CK1 = .37030 CK2 = .37030 CCOMB = 1134.00000

OUTRY = 1 NTFBC = 0 DZBC = .00000

NZFLO = 0 NTFBC = 0 DZBC = .00000

MAX = 1 XPP = .00000

K= 1 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1136.0 HZ1 = 6

HFLZ(K) = .000000
TRIAL FUNCTIONS

1
2
3
5
11

K= 2 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1136.0 HZ1 = 6

HFLZ(K) = .000000
TRIAL FUNCTIONS

1
2
3
5
11

K= 3 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = 100.00 HEX = 1136.0 HZ1 = 6

HFLZ(K) = .000000
TRIAL FUNCTIONS

1
2

```

      3
      11
K= 4  HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 5
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      1
      3
      4
      5
      11
K= 5  HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 5
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      1
      3
      4
      5
      11
K= 6  HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 5
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      3
      4
      5
      6
      11
K= 7  HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 4
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      3
      4
      5
      6
      11
K= 8  HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 4
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      3
      5
      6
      7
      11
K= 9  HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 4
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      3
      5
      6
      7
      11
K= 10 HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 4
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      5
      6
      7
      8
      11
K= 11 HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 3
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      5
      6
      7
      8
      11
K= 12 HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 3
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      5
      7
      8
      9
      11
K= 13 HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 3
HFLZ(K) = .000000
      TRIAL FUNCTIONS
      5
      7
      8
      9
      11
K= 14 HTGEN(K)=      .0  TOUT= 100.00  HOUT=      .00  TEX= 100.00  HEX= 1136.0  NZ1= 2

```

I = 39 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = .00 HEX = .0 MZ1 = 0

HFL7(K) = .000000
TRIAL FUNCTIONS

12

13

14

I = 40 HTGEN(K) = .0 TOUT = 100.00 HOUT = .00 TEX = .00 HEX = .0 MZ1 = 0

HFL7(K) = .000000
TRIAL FUNCTIONS

12

13

14

AL = 6668

*** COMPUTED SURFACE CONDUCTANCE (H) AND HEAT FLUX AT EACH NODE ON THE CONTACT MINOR AND MAJOR AXIS **

I = 1	K = 1	H =	1134.0000	T = 100.75723	HEAT FLUX =	860.216
I = 2	K = 1	H =	1134.0000	T = 100.76719	HEAT FLUX =	871.525
I = 3	K = 1	H =	1134.0000	T = 100.80194	HEAT FLUX =	911.004
I = 4	K = 1	H =	1134.0000	T = 100.88042	HEAT FLUX =	1000.383
I = 5	K = 1	H =	1134.0000	T = 101.03799	HEAT FLUX =	1179.151
I = 6	K = 1	H =	1079.9788	T = 101.87620	HEAT FLUX =	2047.854
I = 7	K = 1	H =	14.9719	T = 113.51275	HEAT FLUX =	228.662
I = 8	K = 1	H =	12.7143	T = 120.07977	HEAT FLUX =	255.301
I = 9	K = 1	H =	.2901	T = 174.52678	HEAT FLUX =	227.857
I = 10	K = 1	H =	5.3024	T = 130.22689	HEAT FLUX =	160.274
I = 11	K = 1	H =	3.1677	T = 134.78730	HEAT FLUX =	115.471
I = 12	K = 1	H =	2.3208	T = 137.41867	HEAT FLUX =	86.843
I = 13	K = 1	H =	1.6763	T = 139.97107	HEAT FLUX =	67.803
I = 14	K = 1	H =	1.0215	T = 143.85679	HEAT FLUX =	44.801
I = 15	K = 1	H =	.6823	T = 146.94590	HEAT FLUX =	32.033
I = 16	K = 1	H =	.4895	T = 149.47246	HEAT FLUX =	24.218
I = 17	K = 1	H =	.3684	T = 151.61080	HEAT FLUX =	19.025
I = 18	K = 1	H =	.2310	T = 155.00510	HEAT FLUX =	12.706
I = 19	K = 1	H =	.1584	T = 157.77472	HEAT FLUX =	9.152
I = 20	K = 1	H =	.1154	T = 160.08284	HEAT FLUX =	6.936
I = 21	K = 1	H =	.0879	T = 162.06133	HEAT FLUX =	5.453
I = 22	K = 1	H =	.0553	T = 163.31732	HEAT FLUX =	3.612
I = 23	K = 1	H =	.0380	T = 167.99484	HEAT FLUX =	2.582
I = 24	K = 1	H =	.0277	T = 170.24382	HEAT FLUX =	1.944
I = 25	K = 1	H =	.0155	T = 173.79074	HEAT FLUX =	1.219
I = 26	K = 1	H =	.0109	T = 176.67562	HEAT FLUX =	.839
I = 27	K = 1	H =	.0077	T = 179.08337	HEAT FLUX =	.612
I = 28	K = 1	H =	.0044	T = 182.86370	HEAT FLUX =	.366
I = 29	K = 1	H =	.0028	T = 185.96300	HEAT FLUX =	.241
I = 30	K = 1	H =	.0019	T = 188.58607	HEAT FLUX =	.168
I = 31	K = 1	H =	.0012	T = 191.67428	HEAT FLUX =	.107
I = 32	K = 1	H =	.0007	T = 194.40004	HEAT FLUX =	.069
I = 33	K = 1	H =	.0004	T = 196.89868	HEAT FLUX =	.042
I = 34	K = 1	H =	.0002	T = 199.26413	HEAT FLUX =	.020
I = 1	K = 2	H =	1134.0000	T = 100.76776	HEAT FLUX =	872.176
I = 1	K = 3	H =	1134.0000	T = 100.79639	HEAT FLUX =	904.702
I = 1	K = 4	H =	1134.0000	T = 100.83320	HEAT FLUX =	946.513
I = 1	K = 5	H =	1134.0000	T = 100.89993	HEAT FLUX =	1022.317
I = 1	K = 6	H =	1134.0000	T = 100.86253	HEAT FLUX =	979.832
I = 1	K = 7	H =	1134.0000	T = 100.95294	HEAT FLUX =	1062.538
I = 1	K = 8	H =	1134.0000	T = 101.08536	HEAT FLUX =	1232.972
I = 1	K = 9	H =	1134.0000	T = 101.23036	HEAT FLUX =	1397.691
I = 1	K = 10	H =	1134.0000	T = 101.17115	HEAT FLUX =	1330.431
I = 1	K = 11	H =	1134.0000	T = 101.37857	HEAT FLUX =	1546.060
I = 1	K = 12	H =	1134.0000	T = 101.54479	HEAT FLUX =	1754.877
I = 1	K = 13	H =	1134.0000	T = 101.77890	HEAT FLUX =	2020.835
I = 1	K = 14	H =	1134.0000	T = 102.15568	HEAT FLUX =	2448.854
I = 1	K = 15	H =	1134.0000	T = 104.45535	HEAT FLUX =	7333.273
I = 1	K = 16	H =	17.3728	T = 122.23831	HEAT FLUX =	386.342
I = 1	K = 17	H =	15.0239	T = 128.16742	HEAT FLUX =	423.185
I = 1	K = 18	H =	12.9279	T = 131.63704	HEAT FLUX =	407.550
I = 1	K = 19	H =	11.1985	T = 134.01577	HEAT FLUX =	380.925
I = 1	K = 20	H =	9.5907	T = 136.53269	HEAT FLUX =	350.375
I = 1	K = 21	H =	8.3193	T = 138.37772	HEAT FLUX =	319.276
I = 1	K = 22	H =	7.7998	T = 140.50171	HEAT FLUX =	295.456
I = 1	K = 23	H =	5.7854	T = 143.83036	HEAT FLUX =	253.583
I = 1	K = 24	H =	4.7790	T = 146.78414	HEAT FLUX =	218.878
I = 1	K = 25	H =	3.9583	T = 148.46489	HEAT FLUX =	191.838
I = 1	K = 26	H =	3.1356	T = 151.32312	HEAT FLUX =	160.930
I = 1	K = 27	H =	2.1418	T = 155.93940	HEAT FLUX =	119.813
I = 1	K = 28	H =	1.5760	T = 159.58474	HEAT FLUX =	93.904
I = 1	K = 29	H =	1.1778	T = 162.58443	HEAT FLUX =	74.179
I = 1	K = 30	H =	.9778	T = 165.98069	HEAT FLUX =	63.580
I = 1	K = 31	H =	.6695	T = 170.17206	HEAT FLUX =	44.781
I = 1	K = 32	H =	.4916	T = 173.40716	HEAT FLUX =	34.089
I = 1	K = 33	H =	.3777	T = 174.11217	HEAT FLUX =	28.746
I = 1	K = 34	H =	.2999	T = 178.39706	HEAT FLUX =	23.512
I = 1	K = 35	H =	.1990	T = 182.16521	HEAT FLUX =	16.348
I = 1	K = 36	H =	.1470	T = 184.92813	HEAT FLUX =	12.059
I = 1	K = 37	H =	.1046	T = 186.94320	HEAT FLUX =	9.265
I = 1	K = 38	H =	.0830	T = 188.35359	HEAT FLUX =	7.333
I = 1	K = 39	H =	.0645	T = 189.25265	HEAT FLUX =	5.733
I = 1	K = 40	H =	.0545	T = 189.58954	HEAT FLUX =	4.881

***** CONTACT RESISTANCE AND HEAT FLOW *****

TOTAL HEAT FLOW = 3.07579 W

TLOW = 100.000

CONTACT LOAD = 175.80000 N

SEMI-MAJOR DIAMETER (A) = .15395 CH.

SEMI-MINOR DIAMETER (B) = .00961 CH.

SMALL CONTACT RADIUS = .95270 CH.

LARGE CONTACT RADIUS = 76.03999 CH.

HEAT FLOW THROUGH CONTACT PATCH = 1.44443W

RESISTANCE OF CONTACT = 8.84572 DBB.C/W

RATIO OF HEAT FLOW THROUGH PATCH TO TOTAL HEAT FLOW = .32659

EXECUTE TIME = 0210.33 SECONDS

Listing 6 Input File for the Sample Non-Linear Steady-State Pressure-Calandria Tube Heat Transfer Analysis.

INFECT ANALYSIS-STEADY STATE

1	10.34	11.15	12.90	13.17	102.0	0.36
0.5502	0.0628	41.1	1160.9			

62	1
1	2
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1	98
1	99
1	100

10	5.17	3.7135	.01687
39	3.14159	.002048	3
9	9		
13	5		
15	3		
	0.0192	19	1
	0.0384	23	1
	0.0768	27	1
	0.1536	31	

8.2270E-5	9	2.0		.00334		0.50
1	3.9544E-7	1.2043E-12				
1	.40000E-1	.15700E-3				
0000E-4	4.9043E-5					
1.2	0.150	0.150	54.75	1	1	
1	0.0	0.770	333.0	0.0	333.0	54.75
1	9	2.0		.00334		0.50
8.2270E-5	3.9544E-7	1.2043E-12				
1	.40000E-1	.15700E-3				
0000E-4	4.9043E-5					
1.2	0.150	0.150	54.75	1	1	
1	0.0	0.770	333.0	0.0	333.0	54.75
1	9	2.0		.00334		0.5
8.2270E-5	3.9544E-7	1.2043E-12				
1	.40000E-1	.15700E-3				
0000E-4	4.9043E-5					
1.2	0.150	0.150	54.75	1	1	
1	0.0	0.770	333.0	0.0	333.0	54.75
1	9	2.0		.00334		0.5

(continued)

8.2270E-5	5.9564E-7	1.2043E-12						7h
1	.40000E-1	.15700E-3						8h
5.0000E-4	4.9045E-5							9h
1.2	0.150	0.150	56.75	1	1			10h
1	0.0	2.0						11h
	0.770	353.0	0.0	353.0	56.75	0		12h
1	0.0	2.0						13h
	0.770	353.0	0.00336	0.5				14h
8.2270E-5	5.9564E-7	1.2043E-12						15h
1	.40000E-1	.15700E-3						16h
5.0000E-4	4.9045E-5							17h
1.2	0.150	0.150	56.75	1	1			18h
1	0.0	3.0						19h
	0.770	353.0	0.0	353.0	56.75	0		20h
1	0.0	3.0						21h
	0.770	353.0	0.00336	0.5				22h
8.2270E-5	5.9564E-7	1.2043E-12						23h
1	.40000E-1	.15700E-3						24h
5.0000E-4	4.9045E-5							25h
1.2	0.150	0.150	56.75	1	1			26h
1	0.0	4.3						27h
	0.770	353.0	0.0	353.0	56.75	0		28h
1	0.0	4.3						29h
	0.770	353.0	0.00336	0.5				30h
8.2270E-5	5.9564E-7	1.2043E-12						31h
1	.40000E-1	.15700E-3						32h
5.0000E-4	4.9045E-5							33h
1.2	0.150	0.150	56.75	1	1			34h
1	0.0	6.0						35h
	0.770	353.0	0.0	353.0	56.75	0		36h
1	0.0	6.0						37h
	0.770	353.0	0.00336	0.50				38h
8.2270E-5	5.9564E-7	1.2043E-12						39h
1	.40000E-1	.15700E-3						40h
5.0000E-4	4.9045E-5							41h
1.2	0.150	0.150	56.75	1	1			42h
1	0.0	8.0						43h
	0.770	353.0	0.0	353.0	56.75			44h
1	0.0	8.0						45h
	0.770	353.0	0.00336	0.5				46h
8.2270E-5	5.9564E-7	1.2043E-12						47h
1	.40000E-1	.15700E-3						48h
5.0000E-4	4.9045E-5							49h
1.2	0.150	0.150	56.75	1	1			50h
1	0.0	11.0						51h
	0.770	353.0	0.0	353.0	56.75	0		52h
1	0.0	11.0						53h
	0.770	353.0	0.00336	0.5				54h
8.2270E-5	5.9564E-7	1.2043E-12						55h
1	.40000E-1	.15700E-3						56h
5.0000E-4	4.9045E-5							57h
1.2	0.150	0.150	56.75	1				58h
1	0.0	51.0						59h
	0.770	353.0	0.0	353.0	56.75	0		60h
0.10	3	1						61h
0.05	9	1						62h
0.0333	12	1						63h
0.05	16	1						64h
0.10	20	1						65h
0.20	30	1						66h
0.40	40	1						67h
0.80	50	1						68h
1.60	55	1						69h
2.78	65	1						70h
1	0.0	3.0						71h
	0.770	353.0	0.00336	0.5				72h
8.2270E-5	5.9564E-7	1.2043E-12						73h
1	.40000E-1	.15700E-3						74h
5.0000E-4	4.9045E-5							75h
1.2	0.150	0.150	56.75	1				76h
1	0.0	0.770	353.0	0.0	353.0	56.75	6	77h
4	0.0	0.770	353.0	0.0	353.0	56.75	6	78h
4	0.0	0.770	353.0	0.0	353.0	56.75	6	79h
4	0.0	0.770	353.0	0.0	353.0	56.75	6	80h
4	0.0	0.770	353.0	0.0	353.0	56.75	5	81h
4	0.0	0.770	353.0	0.0	353.0	56.75	5	82h
4	0.0	0.770	353.0	0.0	353.0	56.75	5	83h
4	0.0	0.770	353.0	0.0	353.0	56.75	4	84h
4	0.0	0.770	353.0	0.0	353.0	56.75	4	85h
4	0.0	0.770	353.0	0.0	353.0	56.75	3	86h
4	0.0	0.770	353.0	0.0	353.0	56.75	2	87h
4	0.0	0.770	353.0	0.0	353.0	56.75		88h
4	0.0	0.770	353.0	0.0	353.0	56.75		89h
4	0.0	0.770	353.0	0.0	353.0	56.75		90h
4	0.0	0.770	353.0	0.0	353.0	56.75		91h
4	0.0	0.770	353.0	0.0	353.0	56.75		92h
4	0.0	0.770	353.0	0.0	353.0	56.75		93h
4	0.0	0.770	353.0	0.0	353.0	56.75		94h
4	0.0	0.770	353.0	0.0	353.0	56.75		95h
4	0.0	0.770	353.0	0.0	353.0	56.75		96h
4	0.0	0.770	353.0	0.0	353.0	56.75		97h
4	0.0	0.770	353.0	0.0	353.0	56.75		98h
4	0.0	0.770	353.0	0.0	353.0	56.75		99h
4	0.0	0.770	353.0	0.0	353.0	56.75		100h

[illegible]

Listing 7 Output Listing for the Sample Non-Linear Steady-State
Pressure-Calandria Tube Heat Transfer Analysis.

***** IMPCC ANALYSIS-STEADY STATE *****

***** SYNTHESIS SOLUTION *****

***** IMPCC ANALYSIS DATA *****

PRESSURE TUBE INNER DIAMETER = 10.14000 PRESSURE TUBE OUTER DIAMETER = 11.15000
CALANDRIA TUBE INNER DIAMETER = 12.90000 CALANDRIA TUBE OUTER DIAMETER = 13.17000
SPACING BETWEEN GARTER SPRINGS = 102.00000 SPRING DIAMETER = 5.0000
PATCH SPRI-MAJOR DIAMETER = .50020 PATCH SENT-PINOR DIAMETER = .06280
SMALL EFFECTIVE CONTACT RADIUS= 41.10001 LARGE EFFECTIVE CONTACT RADIUS=1160.89990
GRID OPTION NUMBER = 2

***** DATA READ IN SUBROUTINE RAD12 *****

NR= 3 MA= 14 JA= 23
N= 1
M(N)= 2
N= 2
M(N)= 4
N= 3
M(N)= 10
R1= 5.17000 RO= 5.71350 PR(N)= .01687

***** DATA READ IN SUBROUTINE DTH7 *****

IX= 39 ATOT= 3.14159 DTH(1)= .002048 NI= 3
II= 1 IN(II)= 9 ISC(II)= 9
II= 2 IN(II)= 13 ISC(II)= 5
II= 3 IN(II)= 15 ISC(II)= 3
N= 4 IN(N)= 19 DTH(N+1)= .01920 NEXT= 1
N= 5 IN(N)= 23 DTH(N+1)= .03840 NEXT= 1
N= 6 IN(N)= 27 DTH(N+1)= .07680 NEXT= 1
N= 7 IN(N)= 31 DTH(N+1)= .15360 NEXT= 0
DTH(N+2)= .242555

***** DATA READ IN SUBROUTINE ISIJ *****

N= 1
JSC(N)= 25
N= 2
JSC(N)= 18
N= 3
JSC(N)= 11

***** TRIAL FUNCTION NO. 1 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 1 RADIND = 0 MAXIT = 9 EPSLN = 2.00000 TRABIN = .00000
HRAIN = .00336 BETA = .50000 TCINTS = .8227E-04 SLOPS = .3954E-04
REXCO = .1204E-11 TCGAS = .5000E 03 CCMN = .4904E-04
MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03
NI= 1.20 CK1= .15000 CK2= .15000 CCOND= 56.75000
OUTRY = 1 NEXT = 1
NZFLO = 0 NTFBC = 0 D7BC = .00000
MAX = 1 XPP = .00000

K= 1 NTGEN(K)= .0 TOUT= 333.00 HOUT= .00 TEX= 333.00 HEX= 56.8 NZ1= 4
WFLZ(K)= .770000

***** NUMBER OF ITERATIONS PERFORMED = 0 *****

I= 1 J= 2 K= 1 TFP = 378.25195
I= 1 J= 23 K= 1 TEMP = 357.93359
I= 39 J= 2 K= 1 TEMP = 798.53418
I= 39 J= 23 K= 1 TEMP = 353.35864
I= 1 J= 1 HEAT FLUX= 5.92031 HEAT FLOW= .03451
OTOT = 12.5177
L = 454

***** TRIAL FUNCTION NO. 2 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 1 RADIND = 0 MAXIT = 9 EPSLN = 2.00000 TRABIN = .00000
HRAIN = .00336 BETA = .50000 TCINTS = .8227E-04 SLOPS = .3954E-04
REXCO = .1204E-11 TCGAS = .5000E 03 CCMN = .4904E-04
MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03
NI= 1.20 CK1= .15000 CK2= .15000 CCOND= 56.75000
OUTRY = 1 NEXT = 1
NZFLO = 0 NTFBC = 0 D7BC = .00000
MAX = 1 XPP = .42500

K= 1 NTGEN(K)= .0 TOUT= 333.00 HOUT= .00 TEX= 333.00 HEX= 56.8 NZ1= 4
WFLZ(K)= .770000

***** NUMBER OF ITERATIONS PERFORMED = 0 *****

I= 1 J= 2 K= 1 TFP = 378.83228

```

I= 1 J= 23 K= 1 TEMP = 357.96436
I= 19 J= 2 K= 1 TEMP = 797.83984
I= 39 J= 23 K= 1 TEMP = 353.35791
I= 1 J= 2 K= 1 HEAT FLUX= 5.95722 HEAT FLOW= .03472
QTOT = 12.5088
L = 454

```

***** TRIAL FUNCTION NO. 3 *****

***** DATA READ IN SUBROUTINE DATA *****

```

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 1 RADBNB = 0 MAXIT = 9 EPSLN = 2.00000 TRADIN = .00000
HRADIN = .00334 BETA = .50000 TCINTB = .8227E-04 SLOPB = .5956E-06
REXCO = .1204E-11 TCAS = .5000E-03 CCNH = .4904E-04
MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03
MI= 1.20 CK1= .15000 CK2= .15000 CCONB= 56.75000
OUTRY = 1 NEXT = 1
NZFLO = 0 NTFBC = 0 DZBC = .00000
MAX= 1 XPP= .52500

```

```

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 MZ1= 2
MFLZ(K) = .770000

```

***** NUMBER OF ITERATIONS PERFORMED = 8 *****

```

I= 1 J= 2 K= 1 TEMP = 379.23755
I= 1 J= 23 K= 1 TEMP = 357.95166
I= 19 J= 2 K= 1 TEMP = 798.60322
I= 39 J= 23 K= 1 TEMP = 353.35864
I= 1 J= 2 K= 1 HEAT FLUX= 5.94199 HEAT FLOW= .03463
QTOT = 12.5181
L = 454

```

***** TRIAL FUNCTION NO. 4 *****

***** DATA READ IN SUBROUTINE DATA *****

```

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 1 RADBNB = 0 MAXIT = 9 EPSLN = 2.00000 TRADIN = .00000
HRADIN = .00334 BETA = .50000 TCINTB = .8227E-04 SLOPB = .5956E-06
REXCO = .1204E-11 TCAS = .5000E-03 CCNH = .4904E-04
MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03
MI= 1.20 CK1= .15000 CK2= .15000 CCONB= 56.75000
OUTRY = 1 NEXT = 1
NZFLO = 0 NTFBC = 0 DZBC = .00000
MAX= 1 XPP= 2.00000
K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 MZ1= 0
MFLZ(K) = .770000

```

***** NUMBER OF ITERATIONS PERFORMED = 8 *****

```

I= 1 J= 2 K= 1 TEMP = 398.39746
I= 1 J= 23 K= 1 TEMP = 354.80176
I= 19 J= 2 K= 1 TEMP = 798.71973
I= 39 J= 23 K= 1 TEMP = 353.35938
I= 1 J= 2 K= 1 HEAT FLUX= 4.54211 HEAT FLOW= .02659
QTOT = 12.5116
L = 454

```

***** TRIAL FUNCTION NO. 5 *****

***** DATA READ IN SUBROUTINE DATA *****

```

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 1 RADBNB = 0 MAXIT = 9 EPSLN = 2.00000 TRADIN = .00000
HRADIN = .00334 BETA = .50000 TCINTB = .8227E-04 SLOPB = .5956E-06
REXCO = .1204E-11 TCAS = .5000E-03 CCNH = .4904E-04
MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03
MI= 1.20 CK1= .15000 CK2= .15000 CCONB= 56.75000
OUTRY = 1 NEXT = 1
NZFLO = 0 NTFBC = 0 DZBC = .00000
MAX= 1 XPP= 3.00000

```

```

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 MZ1= 0
MFLZ(K) = .770000

```

***** NUMBER OF ITERATIONS PERFORMED = 8 *****

```

I= 1 J= 2 K= 1 TEMP = 417.70435
I= 1 J= 23 K= 1 TEMP = 354.16626
I= 19 J= 2 K= 1 TEMP = 799.28857
I= 39 J= 23 K= 1 TEMP = 353.36660
I= 1 J= 2 K= 1 HEAT FLUX= 3.79951 HEAT FLOW= .02214
QTOT = 12.5142
L = 454

```

***** TRIAL FUNCTION NO. 6 *****

***** DATA READ IN SUBROUTINE DATA *****

```

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 1 RADBNB = 0 MAXIT = 9 EPSLN = 2.00000 TRADIN = .00000
HRADIN = .00334 BETA = .50000 TCINTB = .8227E-04 SLOPB = .5956E-06
REXCO = .1204E-11 TCAS = .5000E-03 CCNH = .4904E-04
MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03

```

H1= 1.20 CK1= .15000 CK2= .15000 CCOND= 56.75000
 OUTRY = 1 NEXT = 1
 NZFLO = 0 NTFBC = 0 DZBC = .00000
 MAX= 1 XPP= 4.30000

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 0
 HFLZ(K) = .770000

***** NUMBER OF ITERATIONS PERFORMED = 8 *****

I= 1	J= 2	K= 1	TEMP =	445.27100	
I= 1	J= 23	K= 1	TEMP =	355.58887	
I= 39	J= 2	K= 1	TEMP =	800.38159	
I= 39	J= 23	K= 1	TEMP =	353.36328	
I= 1	K= 1		HEAT FLUX=	3.10664	HEAT FLOW= .01811

DTOT = 12.5107
 L = 454

***** TRIAL FUNCTION NO. 7 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 1	RADBNB = 0	MAXIT = 9	EPSLN = 2.00000	TRABIN = .00000
HRADIN = .00336	BETA = .50000	TCINTG = .8227E-04	SLOPS = .5956E-06	
REXCO = .1204E 11	TCGAS = .5000E 03	CCNN = .4906E-04		
MYCOM = 1	TCINTS = .60000E-01	SLOPS = .15700E-03		

H1= 1.20 CK1= .15000 CK2= .15000 CCOND= 56.75000
 OUTRY = 1 NEXT = 1
 NZFLO = 0 NTFBC = 0 DZBC = .00000
 MAX= 1 XPP= 6.00000

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 0
 HFLZ(K) = .770000

***** NUMBER OF ITERATIONS PERFORMED = 8 *****

I= 1	J= 2	K= 1	TEMP =	482.42114	
I= 1	J= 23	K= 1	TEMP =	355.08643	
I= 39	J= 2	K= 1	TEMP =	803.74023	
I= 39	J= 23	K= 1	TEMP =	353.34743	
I= 1	K= 1		HEAT FLUX=	2.50371	HEAT FLOW= .01459

DTOT = 12.5302
 L = 454

***** TRIAL FUNCTION NO. 8 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 1	RADBNB = 0	MAXIT = 9	EPSLN = 2.00000	TRABIN = .00000
HRADIN = .00336	BETA = .50000	TCINTG = .8227E-04	SLOPS = .5956E-06	
REXCO = .1204E 11	TCGAS = .5000E 03	CCNN = .4906E-04		
MYCOM = 1	TCINTS = .60000E-01	SLOPS = .15700E-03		

H1= 1.20 CK1= .15000 CK2= .15000 CCOND= 56.75000
 OUTRY = 1 NEXT = 1
 NZFLO = 0 NTFBC = 0 DZBC = .00000
 MAX= 1 XPP= 8.00000

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 0
 HFLZ(K) = .770000

***** NUMBER OF ITERATIONS PERFORMED = 8 *****

I= 1	J= 2	K= 1	TEMP =	524.15039	
I= 1	J= 23	K= 1	TEMP =	354.69019	
I= 39	J= 2	K= 1	TEMP =	805.69263	
I= 39	J= 23	K= 1	TEMP =	353.37254	
I= 1	K= 1		HEAT FLUX=	2.02822	HEAT FLOW= .01182

DTOT = 12.5378
 L = 454

***** TRIAL FUNCTION NO. 9 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0

MLIN = 1	RADBNB = 0	MAXIT = 9	EPSLN = 2.00000	TRABIN = .00000
HRADIN = .00336	BETA = .50000	TCINTG = .8227E-04	SLOPS = .5956E-06	
REXCO = .1204E 11	TCGAS = .5000E 03	CCNN = .4906E-04		
MYCOM = 1	TCINTS = .60000E-01	SLOPS = .15700E-03		

H1= 1.20 CK1= .15000 CK2= .15000 CCOND= 56.75000
 OUTRY = 1 NEXT = 1
 NZFLO = 0 NTFBC = 0 DZBC = .00000
 MAX= 1 XPP= 11.00000

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 0
 HFLZ(K) = .770000

***** NUMBER OF ITERATIONS PERFORMED = 8 *****

I= 1	J= 2	K= 1	TEMP =	583.66675	
I= 1	J= 23	K= 1	TEMP =	354.34104	
I= 39	J= 2	K= 1	TEMP =	809.87427	

I= 39 J= 23 K= 1 TEMP = 353.38037
 I= 1 J= 23 K= 1 HEAT FLUX= 1.66928 HEAT FLOW= .00938
 OTOT = 12.6377
 L = 454

***** TRIAL FUNCTION NO. 10 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
 MLIN = 1 RADAND = 0 MAXIT = 9 EPSLN = 2.00000 TRADIN = .00000
 HRADIN = .00336 BETA = .50000 TCINTG = .8227E-04 SLOPG = .5954E-04
 REXCO = .1704E 11 TGCAS = .5000E-03 CCNN = .4904E-04
 MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03
 H1= 1.20 CA1= .15000 CK1= .15000 CCONB= 56.75000
 OUTRY = 1 NEXT = 0
 NZFLN = 0 NTFBC = 0 D7BC = .00000
 MAX= 1 XPP= 51.00000

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 0
 HFLZ(K) = .770000

***** NUMBER OF ITERATIONS PERFORMED = 7 *****

I= 1 J= 2 K= 1 T=HP = 788.56787
 I= 1 J= 23 K= 1 TEMP = 353.39692
 I= 39 J= 2 K= 1 TEMP = 818.13672
 I= 39 J= 23 K= 1 TEMP = 353.42773
 I= 1 J= 23 K= 1 HEAT FLUX= .71631 HEAT FLOW= .00417
 OTOT = 12.5525
 L = 454

***** AXIAL GRID DATA READ IN SUBROUTINE DZ *****

K= 1 DZ= .1000000 KCH(K)= 3 NEXT= 1
 K= 2 DZ= .0500000 KCH(K)= 9 NEXT= 1
 K= 3 DZ= .0333000 KCH(K)= 12 NEXT= 1
 K= 4 DZ= .0500000 KCH(K)= 16 NEXT= 1
 K= 5 DZ= .1000000 KCH(K)= 20 NEXT= 1
 K= 6 DZ= .2000000 KCH(K)= 30 NEXT= 1
 K= 7 DZ= .4000000 KCH(K)= 40 NEXT= 1
 K= 8 DZ= .8000000 KCH(K)= 50 NEXT= 1
 K= 9 DZ= 1.6000004 KCH(K)= 55 NEXT= 1
 K= 10 DZ= 2.7799997 KCH(K)= 65 NEXT= 0

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 10
 MLIN = 1 RADAND = 0 MAXIT = 6 EPSLN = 3.00000 TRADIN = .00000
 HRADIN = .00336 BETA = .50000 TCINTG = .8227E-04 SLOPG = .5954E-04
 REXCO = .1204E 11 TGCAS = .8227E-04 CCNN = .4904E-04
 MVCON = 1 TCINTS = .60000E-01 SLOPS = .15700E-03
 H1= 1.20 CK1= 1.00000 CK2= 1.00000 CCONB= 56.75000
 OUTRY = 0 NEXT = 0
 NZFLN = 0 NTFBC = 0 D7BC = .00000
 MAX= 1 XPP= .00000

K= 1 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 6
 HFLZ(K) = .770000

TRIAL FUNCTIONS

1
4
5
6

K= 2 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 6

HFLZ(K) = .770000
 TRIAL FUNCTIONS

1
4
5
6

K= 3 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 6

HFLZ(K) = .770000
 TRIAL FUNCTIONS

1
4
5
6

K= 4 HTGEN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 HZ1= 5

HFLZ(K) = .770000
 TRIAL FUNCTIONS

1
4
5
6

K= 5	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= 353.00	HEX=	56.8	NZ1= 5
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
1									
4									
5									
6									
K= 6	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= 353.00	HEX=	56.8	NZ1= 5
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
1									
4									
5									
6									
K= 7	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= 353.00	HEX=	56.8	NZ1= 4
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
2									
4									
5									
6									
K= 8	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= 353.00	HEX=	56.8	NZ1= 4
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
2									
4									
5									
6									
K= 9	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= 353.00	HEX=	56.8	NZ1= 3
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
2									
4									
5									
6									
K= 10	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= 353.00	HEX=	56.8	NZ1= 2
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
3									
4									
5									
6									
•									
•									
•									
•									
•									
•									
K= 43	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= .00	HEX=	.0	NZ1= 0
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
8									
9									
10									
K= 44	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= .00	HEX=	.0	NZ1= 0
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
8									
9									
10									
K= 45	HTGEN(K)=	.0	TOUT= 353.00	HOUT=	.00	TEX= .00	HEX=	.0	NZ1= 0
HFLZ(K) = .770000									
TRIAL FUNCTIONS									
8									
9									
10									
SERIES NUMBER OF ITERATIONS = 1 SERIES									
I= 1	J= 2	K= 1	AXIAL POSITION =	.00000	TEMP =	371.56152			
I= 1	J= 2	K= 5	AXIAL POSITION =	.30000	TEMP =	373.44502			
I= 1	J= 2	K= 10	AXIAL POSITION =	.53330	TEMP =	372.31643			


```

I= 1 J= 23 K= 13 AXIAL POSITION = .64990 TEMP = 359.21875
NFLUX = 7.46750
I= 1 J= 23 K= 15 AXIAL POSITION = .74990 TEMP = 359.08643
NFLUX = 7.30371
I= 1 J= 23 K= 17 AXIAL POSITION = .89990 TEMP = 358.92236
NFLUX = 7.10683
I= 1 J= 23 K= 19 AXIAL POSITION = 1.09990 TEMP = 358.67896
NFLUX = 6.81474
I= 1 J= 23 K= 21 AXIAL POSITION = 1.39990 TEMP = 358.29590
NFLUX = 6.35308
I= 1 J= 23 K= 25 AXIAL POSITION = 2.19990 TEMP = 357.48438
NFLUX = 5.38125
I= 1 J= 23 K= 30 AXIAL POSITION = 3.19990 TEMP = 356.54956
NFLUX = 4.25947
I= 1 J= 23 K= 35 AXIAL POSITION = 5.19989 TEMP = 355.44360
NFLUX = 2.93232
I= 1 J= 23 K= 40 AXIAL POSITION = 7.19989 TEMP = 354.93945
NFLUX = 2.32734
I= 1 J= 23 K= 45 AXIAL POSITION = 11.19989 TEMP = 354.73071
NFLUX = 2.07685
I= 1 J= 23 K= 50 AXIAL POSITION = 15.19989 TEMP = 354.05688
NFLUX = 1.26826
I= 1 J= 23 K= 55 AXIAL POSITION = 23.19984 TEMP = 353.74707
NFLUX = .894184
I= 1 J= 23 K= 60 AXIAL POSITION = 37.09984 TEMP = 353.61670
NFLUX = .740039
I= 1 J= 23 K= 65 AXIAL POSITION = 50.99983 TEMP = 353.60400
NFLUX = .724805
I= 3 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.49707
NFLUX = 7.79648
I= 6 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.39429
NFLUX = 7.67114
I= 9 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 384.70410
I= 9 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.23511
NFLUX = 7.48213
I= 12 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 358.75122
NFLUX = 6.90146
I= 15 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 388.80981
I= 15 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 357.59619
NFLUX = 5.51543
I= 18 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 402.50732
I= 18 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 354.82983
NFLUX = 2.17588
I= 21 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 430.52490
I= 21 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 354.01465
NFLUX = 1.21758
I= 24 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 475.63086
I= 24 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.89648
NFLUX = 1.07578
I= 27 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 538.51465
I= 27 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.73975
NFLUX = .887695
I= 30 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 639.88306
I= 30 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.64673
NFLUX = .776074
I= 34 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 752.09473
I= 34 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.62671
NFLUX = .752051
I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 800.20825
I= 39 J= 10 K= 10 AXIAL POSITION = .53330 TEMP = 800.04812
I= 39 J= 2 K= 20 AXIAL POSITION = 1.19990 TEMP = 799.86450
I= 39 J= 2 K= 30 AXIAL POSITION = 1.19990 TEMP = 801.12085
I= 39 J= 2 K= 50 AXIAL POSITION = 15.19989 TEMP = 814.00391
I= 39 J= 2 K= 65 AXIAL POSITION = 50.99983 TEMP = 817.98433
I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.64526
NFLUX = .774316
I= 39 J= 23 K= 20 AXIAL POSITION = 1.19990 TEMP = 353.47974
NFLUX = .575683
I= 39 J= 23 K= 45 AXIAL POSITION = 11.19989 TEMP = 353.32593
NFLUX = .391113
I= 39 J= 23 K= 65 AXIAL POSITION = 50.99983 TEMP = 353.42163
NFLUX = .385957
TOTAL HEAT FLOW = 646.53809 W

```

***** NUMBER OF ITERATIONS = 3 *****

```

I= 1 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 384.25806
I= 1 J= 2 K= 5 AXIAL POSITION = .30000 TEMP = 384.50024
I= 1 J= 2 K= 10 AXIAL POSITION = .53330 TEMP = 385.26904
I= 1 J= 2 K= 15 AXIAL POSITION = .74990 TEMP = 387.12988
I= 1 J= 2 K= 25 AXIAL POSITION = 2.19990 TEMP = 409.17480
I= 1 J= 2 K= 35 AXIAL POSITION = 5.19989 TEMP = 468.37646
I= 1 J= 2 K= 45 AXIAL POSITION = 11.19989 TEMP = 580.19604
I= 1 J= 2 K= 55 AXIAL POSITION = 23.19984 TEMP = 709.95306
I= 1 J= 2 K= 65 AXIAL POSITION = 50.99983 TEMP = 784.92822
I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.14624
NFLUX = 7.37549
I= 1 J= 23 K= 3 AXIAL POSITION = .20000 TEMP = 359.16504
NFLUX = 7.37905
I= 1 J= 23 K= 5 AXIAL POSITION = .30000 TEMP = 359.19165
NFLUX = 7.42998
I= 1 J= 23 K= 7 AXIAL POSITION = .40000 TEMP = 359.35791
NFLUX = 7.62949
I= 1 J= 23 K= 9 AXIAL POSITION = .50000 TEMP = 359.33838
NFLUX = 7.60605
I= 1 J= 23 K= 10 AXIAL POSITION = .53330 TEMP = 359.26099
NFLUX = 7.51318
I= 1 J= 23 K= 11 AXIAL POSITION = .56660 TEMP = 359.24927
NFLUX = 7.49912
I= 1 J= 23 K= 13 AXIAL POSITION = .64990 TEMP = 359.17212
NFLUX = 7.40654
I= 1 J= 23 K= 15 AXIAL POSITION = .74990 TEMP = 359.10156
NFLUX = 7.32187
I= 1 J= 23 K= 17 AXIAL POSITION = .89990 TEMP = 358.91284
NFLUX = 7.09541
I= 1 J= 23 K= 19 AXIAL POSITION = 1.09990 TEMP = 358.64331
NFLUX = 6.77197
I= 1 J= 23 K= 21 AXIAL POSITION = 1.39990 TEMP = 358.29102
NFLUX = 6.34922
I= 1 J= 23 K= 25 AXIAL POSITION = 2.19990 TEMP = 357.52515
NFLUX = 5.43017
I= 1 J= 23 K= 30 AXIAL POSITION = 3.19990 TEMP = 356.26416
NFLUX = 3.91699
I= 1 J= 23 K= 35 AXIAL POSITION = 5.19989 TEMP = 355.45288
NFLUX = 2.94346
I= 1 J= 23 K= 40 AXIAL POSITION = 7.19989 TEMP = 354.87769
NFLUX = 2.25322
I= 1 J= 23 K= 45 AXIAL POSITION = 11.19989 TEMP = 354.25684
NFLUX = 1.50820

```

```

I= 1 J= 23 K= 50 AXIAL POSITION = 15.19989 TEMP = 353.97559
HFLUX = 1.17070
I= 1 J= 23 K= 55 AXIAL POSITION = 23.19984 TEMP = 353.74438
HFLUX = .897262
I= 1 J= 23 K= 60 AXIAL POSITION = 37.09984 TEMP = 353.61792
HFLUX = .741504
I= 1 J= 23 K= 65 AXIAL POSITION = 50.99983 TEMP = 353.60083
HFLUX = .709996
I= 3 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.12964
HFLUX = 7.35557
I= 6 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.02490
HFLUX = 7.27988
I= 9 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 384.78345
I= 9 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 358.84863
HFLUX = 7.01836
I= 12 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 358.59248
HFLUX = 6.37098
I= 15 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 388.70068
I= 15 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 357.03882
HFLUX = 4.84658
I= 18 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 401.61743
I= 18 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 354.65820
HFLUX = 1.98984
I= 21 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 428.54199
I= 21 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.92505
HFLUX = 1.11006
I= 24 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 473.46680
I= 24 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.69507
HFLUX = .834082
I= 27 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 537.04150
I= 27 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.51538
HFLUX = .618457
I= 30 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 639.85889
I= 30 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.41870
HFLUX = .502441
I= 34 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 753.52930
I= 34 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.41187
HFLUX = .494238
I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 799.79932
I= 39 J= 2 K= 10 AXIAL POSITION = .53330 TEMP = 800.04592
I= 39 J= 2 K= 20 AXIAL POSITION = 1.19990 TEMP = 799.85400
I= 39 J= 2 K= 30 AXIAL POSITION = 3.19990 TEMP = 801.55713
I= 39 J= 2 K= 50 AXIAL POSITION = 15.19989 TEMP = 813.04590
I= 39 J= 2 K= 65 AXIAL POSITION = 50.99983 TEMP = 818.02148
I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.42725
HFLUX = .512695
I= 39 J= 23 K= 20 AXIAL POSITION = 1.19990 TEMP = 353.49144
HFLUX = .589746
I= 39 J= 23 K= 45 AXIAL POSITION = 11.19989 TEMP = 353.36279
HFLUX = .433531
I= 39 J= 23 K= 65 AXIAL POSITION = 50.99983 TEMP = 353.42139
HFLUX = .503644
TOTAL HEAT FLOW = 641.68677 W

```

```

===== NUMBER OF ITERATIONS = 24 =====

```

```

I= 1 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 384.20239
I= 1 J= 2 K= 5 AXIAL POSITION = .30000 TEMP = 384.36401
I= 1 J= 2 K= 10 AXIAL POSITION = .53330 TEMP = 385.32715
I= 1 J= 2 K= 15 AXIAL POSITION = .74990 TEMP = 387.30811
I= 1 J= 2 K= 25 AXIAL POSITION = 2.19990 TEMP = 409.21191
I= 1 J= 2 K= 35 AXIAL POSITION = 5.19989 TEMP = 448.37085
I= 1 J= 2 K= 45 AXIAL POSITION = 11.19989 TEMP = 580.25172
I= 1 J= 2 K= 55 AXIAL POSITION = 37.19984 TEMP = 709.96436
I= 1 J= 2 K= 65 AXIAL POSITION = 50.99983 TEMP = 784.78125
I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.12769
HFLUX = 7.35322
I= 1 J= 23 K= 3 AXIAL POSITION = .20000 TEMP = 359.17114
HFLUX = 7.40537
I= 1 J= 23 K= 5 AXIAL POSITION = .30000 TEMP = 359.20093
HFLUX = 7.44111
I= 1 J= 23 K= 7 AXIAL POSITION = .40000 TEMP = 359.33496
HFLUX = 7.46195
I= 1 J= 23 K= 9 AXIAL POSITION = .50000 TEMP = 359.30054
HFLUX = 7.54044
I= 1 J= 23 K= 10 AXIAL POSITION = .53330 TEMP = 359.18237
HFLUX = 7.41885
I= 1 J= 23 K= 11 AXIAL POSITION = .56660 TEMP = 359.16602
HFLUX = 7.39922
I= 1 J= 23 K= 13 AXIAL POSITION = .64990 TEMP = 359.10449
HFLUX = 7.32539
I= 1 J= 23 K= 15 AXIAL POSITION = .74990 TEMP = 358.98535
HFLUX = 7.18242
I= 1 J= 23 K= 17 AXIAL POSITION = .89990 TEMP = 358.84229
HFLUX = 7.01074
I= 1 J= 23 K= 19 AXIAL POSITION = 1.09990 TEMP = 358.63916
HFLUX = 6.74699
I= 1 J= 23 K= 21 AXIAL POSITION = 1.39990 TEMP = 358.28613
HFLUX = 6.34336
I= 1 J= 23 K= 25 AXIAL POSITION = 2.19990 TEMP = 357.45386
HFLUX = 5.34463
I= 1 J= 23 K= 30 AXIAL POSITION = 3.19990 TEMP = 356.41943
HFLUX = 4.10332
I= 1 J= 23 K= 35 AXIAL POSITION = 5.19989 TEMP = 355.42993
HFLUX = 2.91592
I= 1 J= 23 K= 40 AXIAL POSITION = 7.19989 TEMP = 354.80737
HFLUX = 2.16885
I= 1 J= 23 K= 45 AXIAL POSITION = 11.19989 TEMP = 354.21021
HFLUX = 1.45225
I= 1 J= 23 K= 50 AXIAL POSITION = 15.19989 TEMP = 353.94507
HFLUX = 1.13408
I= 1 J= 23 K= 55 AXIAL POSITION = 23.19984 TEMP = 353.74316
HFLUX = .891797
I= 1 J= 23 K= 60 AXIAL POSITION = 37.09984 TEMP = 353.61841
HFLUX = .742090
I= 1 J= 23 K= 65 AXIAL POSITION = 50.99983 TEMP = 353.60840
HFLUX = .730878
I= 3 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.11133
HFLUX = 7.33359
I= 6 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 359.00732
HFLUX = 7.20879
I= 9 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 384.72705
I= 9 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 358.83228
HFLUX = 6.99873
I= 12 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 358.28149
HFLUX = 6.33779
I= 15 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 388.64233
I= 15 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 357.03711
HFLUX = 4.84453

```

```

I= 18 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 401.56348
I= 18 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 354.65259
MFLUX = 1.98310
I= 21 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 428.50977
I= 21 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.91406
MFLUX = 1.09688
I= 24 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 473.44604
I= 24 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.68579
MFLUX = .822949
I= 27 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 537.01538
I= 27 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.50659
MFLUX = .607910
I= 30 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 639.80933
I= 30 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.41016
MFLUX = .492137
I= 34 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 753.44556
I= 34 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.40308
MFLUX = .483691
I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 799.78929
I= 39 J= 2 K= 10 AXIAL POSITION = .53330 TEMP = 799.98668
I= 39 J= 2 K= 20 AXIAL POSITION = 1.19990 TEMP = 799.84180
I= 39 J= 2 K= 30 AXIAL POSITION = 3.19990 TEMP = 801.07446
I= 39 J= 2 K= 50 AXIAL POSITION = 15.19989 TEMP = 812.68823
I= 39 J= 2 K= 65 AXIAL POSITION = 50.99983 TEMP = 817.93579
I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.41870
MFLUX = .502441
I= 39 J= 23 K= 20 AXIAL POSITION = 1.19990 TEMP = 353.48950
MFLUX = .587402
I= 39 J= 23 K= 45 AXIAL POSITION = 11.19989 TEMP = 353.36768
MFLUX = .441211
I= 39 J= 23 K= 65 AXIAL POSITION = 50.99983 TEMP = 353.42212
MFLUX = .506543
TOTAL HEAT FLOW = 642.06519 W

```

STOP

EXECUTE TIME = 0056.91 SECONDS

· Listing 8

[illegible][illegible]

Listing 9 Output Listing for the Sample Transient (Linear) Pressure-Calandria Tube Heat Transfer Analysis.

***** LIM. TRANSIENT LOECC ANALYSIS *****

***** SYNTHESIS SOLUTION *****

***** IMPEL.C ANALYSIS DATA *****

PRESSURE TUBE INNER DIAMETER = 10.34000	PRESSURE TUBE OUTER DIAMETER = 11.15000
CALANDRIA TUBE INNER DIAMETER = 12.90000	CALANDRIA TUBE OUTER DIAMETER = 13.17000
SPACING BETWEEN BARTER SPRINGS = 102.00000	SPRING DIAMETER = .56000
PATCH SEMI-MAJOR DIAMETER = .55020	PATCH SEMI-MINOR DIAMETER = .06280
SMALL EFFECTIVE CONTACT RADIUS = 41.10001	LARGE EFFECTIVE CONTACT RADIUS = 1100.89990
GRID OPTION NUMBER = 2	

***** DATA READ IN SUBROUTINE RAD12 *****

NR= 3 MX= 14 JX= 23
 N = 1
 M(N) = 2
 N = 2
 M(N) = 4
 N = 3
 M(N) = 10
 RI= 5.17000 RD= 5.71350 DR(N)= .01687

***** DATA READ IN SUBROUTINE DTH2 *****

IX= 39 ATOT= 3.14159 DTH(1)= .002048 NI= 3
 II= 1 IN(II)= 9 ISC(II)= 9
 II= 2 IN(II)= 13 ISC(II)= 5
 II= 3 IN(II)= 15 ISC(II)= 3
 N= 4 IN(N)= 19 DTH(N+1)= .01920 NEXT= 1
 N= 5 IN(N)= 23 DTH(N+1)= .03840 NEXT= 1
 N= 6 IN(N)= 27 DTH(N+1)= .07680 NEXT= 1
 N= 7 IN(N)= 31 DTH(N+1)= .15360 NEXT= 0
 DTH(N+2) = .24255

***** DATA READ IN SUBROUTINE ISI *****

N = 1
 JSC(N) = 23
 N = 2
 JSC(N) = 18
 N = 3
 JSC(N) = 11

***** TRIAL FUNCTION NO. 1 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
 ALIN = 0 RADINS = 0 MAXIT = 0 EPSLN = .00000 TRABIN = .00000
 HRABIN = .00334 BETA = .00000 TCINTS = .00000E+00 SLOPS = .00000E+00
 RECD = .00000E+00 TC8AB = .4800E-03 CCNN = .7800E-04
 MVCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00
 NI= 7.00 CK1= .15000 CK2= .15000 CCONB= 56.75000
 DUTRY = 1 NEXT = 1
 NZFLD = 0 NTFBC = 0 BZNC = .00000
 MAX= 1 XPP= 51.00000

K= 1 NTFEN(K)= .0 TOUT= 353.00 MOUT= .00 TEX= 353.00 MEX= 54.8 NZ1= 0
 HFLZ(K) = 3.91440

I= 1	J= 2	K= 1	TEMP = 1255.05713
I= 1	J= 23	K= 1	TEMP = 173.51443
I= 39	J= 23	K= 1	TEMP = 173.51443
I= 1	J= 1	K= 1	HEAT FLUX= 3.60254
			HEAT FLOW= .02100

 GTDT = 43.7061
 L = 454

***** TRIAL FUNCTION NO. 2 *****

***** DATA READ IN SUBROUTINE DATA1 *****

TRANS = 1 NTRTF = 9 NTFCH = 0 NTFIC = 1
 TFINL= 1.00000 TFINAL= 260.00000 DENB1= 6.57000 DENB2= 6.57000
 BPWT1= .35000 SPWT2= .35000
 NPTS = 115

TIME STEPS AT WHICH TRIAL FNS. ARE RECORDED FOLLOW

NTFTS(1) = 20
 NTFTS(2) = 41
 NTFTS(3) = 54
 NTFTS(4) = 60
 NTFTS(5) = 64
 NTFTS(6) = 69
 NTFTS(7) = 74
 NTFTS(8) = 81
 NTFTS(9) = 90

```

DT= 1      DT(NDT)= .01800      NTC(NDT)= 50      NEXT= 1
NDT= 2      DT(NDT)= .07200      NTC(NDT)= 60      NEXT= 1
NDT= 3      DT(NDT)= .36000      NTC(NDT)= 70      NEXT= 1
NDT= 4      DT(NDT)= 1.80000      NTC(NDT)= 90      NEXT= 1
NDT= 5      DT(NDT)= 5.40000      NTC(NDT)= 110     NEXT= 1
NDT= 6      DT(NDT)= 18.00000     NTC(NDT)= 160     NEXT= 0

MLIN = 0      RADBNB = 0      MAXIT = 0      EPSLN = .00000      TRADIN = .00000
HRADIN = .00336      BETA = .00000      TCINT0 = .0000E+00      SLOPG = .0000E+00
REXCO = .0000E+00      CGAS = .4800E-03      CCMH = .7800E-04
MVCOM = 0      TCINTS = .00000E+00      SLOPS = .00000E+00

M1= 7.00      CK1= .15000      CK2= .15000      CCOMB= 56.75000
OUTRY = 1      NEXT = 1
NZFLO = 0      NTFBC = 0      DZBC = .00000
MAX= 1      XPP= .00000

```

```

K= 1      HTGEN(K)= .0      TOUT= 353.00      HOUT= .00      TEX= 353.00      HEX= 56.8      NZI= 6
NFLZ(K)= 3.91440
KL = 7305

```

***** RESULTS FOR TRIAL FN. NO. 2 FOLLOW *****

```

TIME= .36000
I= 1      J= 2      K= 1      TEMP = 1213.28101
I= 1      J= 23     K= 1      TEMP = 404.44702
I= 39     J= 2      K= 1      TEMP = 1317.76445
I= 39     J= 23     K= 1      TEMP = 353.37061
I= 1      K= 1      HEAT FLUX= 360.12915      HEAT FLOW= 2.09893
OTOT = 235.801
L = 454

```

***** RESULTS FOR TRIAL FN. NO. 3 FOLLOW *****

```

TIME= .73800
I= 1      J= 2      K= 1      TEMP = 1120.35034
I= 1      J= 23     K= 1      TEMP = 396.96045
I= 39     J= 2      K= 1      TEMP = 1317.76440
I= 39     J= 23     K= 1      TEMP = 353.36914
I= 1      K= 1      HEAT FLUX= 307.72314      HEAT FLOW= 1.79350
OTOT = 226.169
L = 454

```

***** RESULTS FOR TRIAL FN. NO. 4 FOLLOW *****

```

TIME= 1.24200
I= 1      J= 2      K= 1      TEMP = 1016.22559
I= 1      J= 23     K= 1      TEMP = 390.46899
I= 39     J= 2      K= 1      TEMP = 1317.81348
I= 39     J= 23     K= 1      TEMP = 353.36914
I= 1      K= 1      HEAT FLUX= 282.28296      HEAT FLOW= 1.52866
OTOT = 209.155
L = 454

```

***** RESULTS FOR TRIAL FN. NO. 5 FOLLOW *****

```

TIME= 1.96199
I= 1      J= 2      K= 1      TEMP = 910.21875
I= 1      J= 23     K= 1      TEMP = 384.34302
I= 39     J= 2      K= 1      TEMP = 1317.90698
I= 39     J= 23     K= 1      TEMP = 353.36914
I= 1      K= 1      HEAT FLUX= 219.40112      HEAT FLOW= 1.27873
OTOT = 191.276
L = 454

```

***** RESULTS FOR TRIAL FN. NO. 6 FOLLOW *****

```

TIME= 3.40199
I= 1      J= 2      K= 1      TEMP = 776.84353
I= 1      J= 23     K= 1      TEMP = 376.73247
I= 39     J= 2      K= 1      TEMP = 1318.09180
I= 39     J= 23     K= 1      TEMP = 353.36914
I= 1      K= 1      HEAT FLUX= 166.12866      HEAT FLOW= .94824
OTOT = 167.016
L = 454

```

***** RESULTS FOR TRIAL FN. NO. 7 FOLLOW *****

```

TIME= 5.20199
I= 1      J= 2      K= 1      TEMP = 679.20313
I= 1      J= 23     K= 1      TEMP = 371.17310
I= 39     J= 2      K= 1      TEMP = 1318.12200
I= 39     J= 23     K= 1      TEMP = 353.36914
I= 1      K= 1      HEAT FLUX= 127.21167      HEAT FLOW= .74143
OTOT = 147.561
L = 454

```

***** RESULTS FOR TRIAL FN. NO. 8 FOLLOW *****

```

TIME= 14.20199
I= 1      J= 2      K= 1      TEMP = 521.37878
I= 1      J= 23     K= 1      TEMP = 362.21191
I= 39     J= 2      K= 1      TEMP = 1318.24895
I= 39     J= 23     K= 1      TEMP = 353.36914
I= 1      K= 1      HEAT FLUX= 44.48340      HEAT FLOW= .37583
OTOT = 110.090

```

L = 454

***** RESULTS FOR TRIAL FN. NO. 9 FOLLOW *****

TIME= 26.80191

```

I= 1 J= 2 K= 1 TEMP = 444.42573
I= 1 J= 23 K= 1 TEMP = 358.99731
I= 39 J= 2 K= 1 TEMP = 1320.89746
I= 39 J= 23 K= 1 TEMP = 353.37061
I= 1 K= 1 HEAT FLUX= 41.98120 HEAT FLOW= .24468
QTOT = 92.8055
L = 454

```

***** RESULTS FOR TRIAL FN. NO. 10 FOLLOW *****

TIME= 46.60181

```

I= 1 J= 2 K= 1 TEMP = 437.54639
I= 1 J= 23 K= 1 TEMP = 357.46484
I= 39 J= 2 K= 1 TEMP = 1323.03711
I= 39 J= 23 K= 1 TEMP = 353.37109
I= 1 K= 1 HEAT FLUX= 31.25391 HEAT FLOW= .18216
QTOT = 82.3315
L = 454

```

***** TRIAL FUNCTION NO. 11 *****

***** DATA READ IN SUBROUTINE DATA1 *****

```

TRANS = 0 NTRTF = 0 NTFCH = 0 NTFIC = 0
MLIN = 0 RADMB = 0 MAXIT = 0 EPSLN = .00000 TRABIN = .00000
MRABIN = .00334 BETA = .00000 TCINTS = .00000E+00 SLOPS = .00000E+00
REXCO = .00000E+00 TCBAS = .4800E-03 CCNM = .7800E-04
MUCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00
HI= 7.00 CK1= .15000 CK2= .15000 CCONB= 56.75000
OUTRY = 1 NEXT = 0
NZFLD = 0 NTFBC = 0 DZBC = .00000
MAX = 1 XPP = .00000

```

```

K= 1 NTRBN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 NZ1= 6

```

NFLZ(K) = 3.91440

```

I= 1 J= 2 K= 1 TEMP = 409.33618
I= 1 J= 23 K= 1 TEMP = 355.87109
I= 39 J= 2 K= 1 TEMP = 1308.26904
I= 39 J= 23 K= 1 TEMP = 353.34548
I= 1 K= 1 HEAT FLUX= 20.09746 HEAT FLOW= .11713
QTOT = 43.4549
L = 454

```

***** AXIAL GRID DATA READ IN SUBROUTINE DZ *****

```

K= 1 DZ = .0500000 KCH(K)= 3 NEXT = 0

```

***** DATA READ IN SUBROUTINE DATA1 *****

```

TRANS = 1 NTRTF = 0 NTFCH = 2 NTFIC = 1
TWIN= 1.00000 TFINAL= 15.00000 DEMS1= 6.57000 DEMS2= 6.57000
SPHT1= .35000 SPHT2= .35000
NFTS = 75

```

TIME STEPS AT WHICH TRIAL FNS. ARE CHANGED FOLLOW

NTSTFC(1) = 57

NTSTFC(2) = 72

```

NBT= 1 BT(NBT)= .01800 NTC(NBT)= 50 NEXT= 1
NBT= 2 BT(NBT)= .07200 NTC(NBT)= 60 NEXT= 1
NBT= 3 BT(NBT)= .36000 NTC(NBT)= 70 NEXT= 1
NBT= 4 BT(NBT)= 1.80000 NTC(NBT)= 90 NEXT= 1
NBT= 5 BT(NBT)= 5.40000 NTC(NBT)= 110 NEXT= 1
NBT= 6 BT(NBT)= 18.00000 NTC(NBT)= 160 NEXT= 0

```

```

MLIN = 0 RADMB = 0 MAXIT = 0 EPSLN = .00000 TRABIN = .00000
MRABIN = .00334 BETA = .00000 TCINTS = .00000E+00 SLOPS = .00000E+00
REXCO = .00000E+00 TCBAS = .4800E-03 CCNM = .7800E-04
MUCON = 0 TCINTS = .00000E+00 SLOPS = .00000E+00
HI= 7.00 CK1= .15000 CK2= .15000 CCONB= 56.75000
OUTRY = 1 NEXT = 0
NZFLD = 0 NTFBC = 0 DZBC = .00000
MAX = 1 XPP = .00000

```

```

K= 1 NTRBN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 NZ1= 6

```

NFLZ(K) = 3.91440

TRIAL FUNCTIONS

```

1
2
3
4
5

```

```

K= 2 NTRBN(K)= .0 TOUT= 353.00 HOUT= .00 TEX= 353.00 HEX= 56.8 NZ1= 6

```

NFLZ(K) = 3.91440

TRIAL FUNCTIONS


```

      1
      2
      3
      4
      5
K= 3      HTGEN(K)=      .0      TOUT= 333.00      HOUT=      .00      TEX= 333.00      HEX=      56.8      MZ1= 6
MFLZ(K) = 3.91440
      TRIAL FUNCTIONS
      1
      2
      3
      4
      5
NL = 0
      TIME=      .01800

I= 1      J= 2      K= 2      AXIAL POSITION =      .05000      TEMP = 1259.45744
I= 1      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 409.56918
MFLUX = 395.984
I= 1      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 409.56373
MFLUX = 395.940
I= 1      J= 23      K= 3      AXIAL POSITION =      .10000      TEMP = 409.35644
MFLUX = 395.895
I= 39      J= 2      K= 1      AXIAL POSITION =      .00000      TEMP = 1317.11072
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 1317.11078
I= 39      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 333.17282
MFLUX = 1.20975
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 333.17284
MFLUX = 1.20984
TOTAL HEAT FLOW =      15.38117 W

      TIME=      .03600

I= 1      J= 2      K= 2      AXIAL POSITION =      .05000      TEMP = 1259.77728
I= 1      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 416.24516
MFLUX = 442.716
I= 1      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 416.24282
MFLUX = 442.700
I= 1      J= 23      K= 3      AXIAL POSITION =      .10000      TEMP = 416.23627
MFLUX = 442.654
I= 39      J= 2      K= 1      AXIAL POSITION =      .00000      TEMP = 1317.07607
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 1317.07611
I= 39      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 333.16231
MFLUX = 1.13616
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 333.16232
MFLUX = 1.13623
TOTAL HEAT FLOW =      17.22168 W

      TIME=      .05400

I= 1      J= 2      K= 2      AXIAL POSITION =      .05000      TEMP = 1259.16095
I= 1      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 416.25379
MFLUX = 442.778
I= 1      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 416.25204
MFLUX = 442.764
I= 1      J= 23      K= 3      AXIAL POSITION =      .10000      TEMP = 416.24644
MFLUX = 442.725
I= 39      J= 2      K= 1      AXIAL POSITION =      .00000      TEMP = 1317.12344
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 1317.12347
I= 39      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 333.17722
MFLUX = 1.24057
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 333.17723
MFLUX = 1.24062
TOTAL HEAT FLOW =      18.02663 W

      TIME=      .07200

I= 1      J= 2      K= 2      AXIAL POSITION =      .05000      TEMP = 1258.07176
I= 1      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 415.40357
MFLUX = 434.825
I= 1      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 415.40177
MFLUX = 434.812
I= 1      J= 23      K= 3      AXIAL POSITION =      .10000      TEMP = 415.39644
MFLUX = 434.775
I= 39      J= 2      K= 1      AXIAL POSITION =      .00000      TEMP = 1317.17744
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 1317.17747
I= 39      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 333.19413
MFLUX = 1.35888
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 333.19413
MFLUX = 1.35893
TOTAL HEAT FLOW =      18.63663 W

      TIME=      .09000

I= 1      J= 2      K= 2      AXIAL POSITION =      .05000      TEMP = 1254.60483
I= 1      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 414.48043
MFLUX = 430.345
I= 1      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 414.47889
MFLUX = 430.332
I= 1      J= 23      K= 3      AXIAL POSITION =      .10000      TEMP = 414.47372
MFLUX = 430.316
I= 39      J= 2      K= 1      AXIAL POSITION =      .00000      TEMP = 1317.22817
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 1317.22820
I= 39      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 333.20995
MFLUX = 1.44945
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 333.20996
MFLUX = 1.44949
TOTAL HEAT FLOW =      19.17049 W

      TIME=      .10800

I= 1      J= 2      K= 2      AXIAL POSITION =      .05000      TEMP = 1254.80239
I= 1      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 413.58877
MFLUX = 424.123
I= 1      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 413.58727
MFLUX = 424.111
I= 1      J= 23      K= 3      AXIAL POSITION =      .10000      TEMP = 413.58219
MFLUX = 424.075
I= 39      J= 2      K= 1      AXIAL POSITION =      .00000      TEMP = 1317.27428
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 1317.27430
I= 39      J= 23      K= 1      AXIAL POSITION =      .00000      TEMP = 333.22440
MFLUX = 1.44945
I= 39      J= 23      K= 2      AXIAL POSITION =      .05000      TEMP = 333.22440
MFLUX = 1.44949
TOTAL HEAT FLOW =      19.17049 W

```

MFLUX = 1.57078
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.22440
 MFLUX = 1.57082
 TOTAL HEAT FLOW = 19.64857 W

TIME = .12600

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1252.69786
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 412.73949
 MFLUX = 418.178
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 412.73802
 MFLUX = 418.144
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 412.73302
 MFLUX = 418.131
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.31618
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.31821
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.23753
 MFLUX = 1.66272
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.23754
 MFLUX = 1.66277
 TOTAL HEAT FLOW = 20.07753 W

TIME = .14400

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1250.31992
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 411.93153
 MFLUX = 412.521
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 411.92988
 MFLUX = 412.509
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 411.92496
 MFLUX = 412.475
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.35423
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.35425
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.24946
 MFLUX = 1.74424
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.24947
 MFLUX = 1.74429
 TOTAL HEAT FLOW = 20.46179 W

TIME = .16200

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1247.69556
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 411.16175
 MFLUX = 407.132
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 411.16013
 MFLUX = 407.121
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 411.15528
 MFLUX = 407.087
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.38877
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.38879
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.26030
 MFLUX = 1.82207
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.26030
 MFLUX = 1.82212
 TOTAL HEAT FLOW = 20.80518 W

TIME = .18000

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1244.84933
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 410.42743
 MFLUX = 401.993
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 410.42603
 MFLUX = 401.982
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 410.42125
 MFLUX = 401.949
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.42011
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.42014
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.27013
 MFLUX = 1.89089
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.27013
 MFLUX = 1.89093
 TOTAL HEAT FLOW = 21.11118 W

TIME = .19800

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1241.80374
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 409.72643
 MFLUX = 397.086
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 409.72505
 MFLUX = 397.075
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 409.72033
 MFLUX = 397.042
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.44854
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.44858
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.27904
 MFLUX = 1.95331
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.27905
 MFLUX = 1.95335
 TOTAL HEAT FLOW = 21.38310 W

TIME = .21600

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1238.57944
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 409.05440
 MFLUX = 392.395
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 409.05404
 MFLUX = 392.384
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 409.05018
 MFLUX = 392.351
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.47437
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.47439
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.28713
 MFLUX = 2.00990
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.28714
 MFLUX = 2.00995
 TOTAL HEAT FLOW = 21.62375 W

TIME = .23400

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1235.19542
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 408.41478
 MFLUX = 387.904
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 408.41324

MFLUX = 387.893
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 408.40864
 MFLUX = 387.860
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.49779
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 1317.49781
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.29446
 MFLUX = 2.04120
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.29446
 MFLUX = 2.04125
 TOTAL HEAT FLOW = 21.83601 W

TIME = .25200

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1231.66900
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 407.79977
 MFLUX = 383.598
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 407.79825
 MFLUX = 383.588
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 407.79370
 MFLUX = 383.556
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.51904
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.51907
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.30110
 MFLUX = 2.10768
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.30110
 MFLUX = 2.10773
 TOTAL HEAT FLOW = 22.02213 W

TIME = .27000

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1228.01610
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 407.20953
 MFLUX = 379.467
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 407.20803
 MFLUX = 379.456
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 407.20353
 MFLUX = 379.425
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.53833
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.53836
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.30711
 MFLUX = 2.14977
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.30712
 MFLUX = 2.14982
 TOTAL HEAT FLOW = 22.18462 W

TIME = .28800

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1224.25132
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 406.64238
 MFLUX = 375.497
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 406.64089
 MFLUX = 375.486
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 406.63444
 MFLUX = 375.455
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.55585
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.55588
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.31255
 MFLUX = 2.18788
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.31256
 MFLUX = 2.18793
 TOTAL HEAT FLOW = 22.32542 W

TIME = .30400

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1220.38801
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 406.09673
 MFLUX = 371.677
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 406.09526
 MFLUX = 371.667
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 406.09086
 MFLUX = 371.636
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.57176
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.57179
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.31748
 MFLUX = 2.22238
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.31749
 MFLUX = 2.22243
 TOTAL HEAT FLOW = 22.44455 W

TIME = .32400

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1216.43840
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 405.57115
 MFLUX = 367.998
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 405.56970
 MFLUX = 367.988
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 405.56534
 MFLUX = 367.957
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.58622
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.58625
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.32194
 MFLUX = 2.25360
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.32195
 MFLUX = 2.25365
 TOTAL HEAT FLOW = 22.54962 W

TIME = .34200

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1212.41349
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 405.04432
 MFLUX = 364.490
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 405.04288
 MFLUX = 364.480
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 405.03856
 MFLUX = 364.410
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.59738
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.59741
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.32598
 MFLUX = 2.28185
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.32599
 MFLUX = 2.28190
 TOTAL HEAT FLOW = 22.63643 W

TIME = .36000

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1208.32413
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 404.57499
MFLUX = 361.025					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 404.57356
MFLUX = 361.015					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 404.56928
MFLUX = 360.985					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1317.61137
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1317.61140
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.32963
MFLUX = 2.30741					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.32964
MFLUX = 2.30747					
TOTAL HEAT FLOW =	22.70845 W				

TIME = .37800

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1204.17909
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 404.10203
MFLUX = 357.714					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 404.10062
MFLUX = 357.704					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 404.09638
MFLUX = 357.675					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1317.62229
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1317.62232
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.33293
MFLUX = 2.33054					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.33294
MFLUX = 2.33060					
TOTAL HEAT FLOW =	22.76678 W				

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MFLUX = 267.628					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 391.22939
MFLUX = 267.606					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1317.80945
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1317.80947
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37100
MFLUX = 2.59698					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37100
MFLUX = 2.59702					
TOTAL HEAT FLOW =	21.13835 W				

TIME = 1.24200

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1015.61616
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 390.44941
MFLUX = 262.146					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 390.44835
MFLUX = 262.138					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 390.44519
MFLUX = 262.116					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1317.82269
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1317.82270
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37729
MFLUX = 2.60606					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37230
MFLUX = 2.60609					
TOTAL HEAT FLOW =	20.93521 W				

TIME = 1.31400

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1002.93722
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 389.69968
MFLUX = 256.898					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 389.69864
MFLUX = 256.890					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 389.69554
MFLUX = 256.869					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1317.83299
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1317.83300
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37354
MFLUX = 2.61476					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37354
MFLUX = 2.61479					
TOTAL HEAT FLOW =	20.73860 W				

TIME = 1.38599

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 990.71898
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 388.98207
MFLUX = 251.874					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 388.98105
MFLUX = 251.867					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 388.97800
MFLUX = 251.846					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1317.84920
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1317.84922
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37469

HFLUX = 2.62284
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.37470
 HFLUX = 2.62289
 TOTAL HEAT FLOW = 20.54831 W

TIME = 1.45799

K = 1
 NEW TRIAL FUNCTIONS ARE 4 5 6 7 8 0 0 0 0 0

K = 2
 NEW TRIAL FUNCTIONS ARE 4 5 6 7 8 0 0 0 0 0

K = 3
 NEW TRIAL FUNCTIONS ARE 4 5 6 7 8 0 0 0 0 0

TIME = 1.45799

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 979.30285
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 388.32027
 HFLUX = 247.242
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 388.32039
 HFLUX = 247.243
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 388.32042
 HFLUX = 247.244
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.85384
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.85380
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.37285
 HFLUX = 2.60995
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.37284
 HFLUX = 2.60989
 TOTAL HEAT FLOW = 20.36592 W

TIME = 1.52999

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 967.98980
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 387.66601
 HFLUX = 242.662
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 387.66613
 HFLUX = 242.663
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 387.66636
 HFLUX = 242.664
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.86391
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.86386
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.37303
 HFLUX = 2.61119
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.37302
 HFLUX = 2.61111
 TOTAL HEAT FLOW = 20.17924 W

TIME = 1.60199

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 957.07104
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 387.03557
 HFLUX = 238.249
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 387.03569
 HFLUX = 238.250
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 387.03591
 HFLUX = 238.251
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.87407
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.87402
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.37323
 HFLUX = 2.61261
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.37322
 HFLUX = 2.61252
 TOTAL HEAT FLOW = 19.99852 W

TIME = 1.96199

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 910.38414
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 384.35381
 HFLUX = 219.477
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 384.35393
 HFLUX = 219.477
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 384.35413
 HFLUX = 219.479
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.92137
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.92133
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.37336
 HFLUX = 2.61349
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.37335
 HFLUX = 2.61342
 TOTAL HEAT FLOW = 19.19531 W

TIME = 2.32199

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 870.10785
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 382.04952
 HFLUX = 203.347
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 382.04962
 HFLUX = 203.347
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 382.04981
 HFLUX = 203.349
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1317.96732
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1317.96728
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.37318
 HFLUX = 2.61227
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.37317
 HFLUX = 2.61220
 TOTAL HEAT FLOW = 18.48129 W

TIME = 2.68199

I= 1 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 835.07978
 I= 1 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 380.05099
 HFLUX = 189.337
 I= 1 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 380.05108
 HFLUX = 189.338
 I= 1 J= 23 K= 3 AXIAL POSITION = .10000 TEMP = 380.05126
 HFLUX = 189.339
 I= 39 J= 2 K= 1 AXIAL POSITION = .00000 TEMP = 1318.01184
 I= 39 J= 2 K= 2 AXIAL POSITION = .05000 TEMP = 1318.01180
 I= 39 J= 23 K= 1 AXIAL POSITION = .00000 TEMP = 353.37269
 HFLUX = 2.60882
 I= 39 J= 23 K= 2 AXIAL POSITION = .05000 TEMP = 353.37268

MFLUX = 2.60874
TOTAL HEAT FLOW = 17.84088 W

TIME = 3.04199

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 804.39019
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 378.30299
MFLUX = 177.121					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 378.30308
MFLUX = 177.122					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 378.30325
MFLUX = 177.123					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1318.05350
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1318.05546
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37199
MFLUX = 2.60396					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37198
MFLUX = 2.60388					
TOTAL HEAT FLOW =					17.26338 W

TIME = 3.40199

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 777.32035
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 376.76255
MFLUX = 166.338					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 376.76263
MFLUX = 166.338					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 376.76279
MFLUX = 166.340					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1318.09888
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1318.09883
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37126
MFLUX = 2.59882					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37125
MFLUX = 2.59874					
TOTAL HEAT FLOW =					16.74129 W

TIME = 3.76199

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 753.29710
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 375.39585
MFLUX = 156.771					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 375.39592
MFLUX = 156.771					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 375.39608
MFLUX = 156.773					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1318.14260
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1318.14256
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37063
MFLUX = 2.59441					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37062
MFLUX = 2.59433					
TOTAL HEAT FLOW =					16.26875 W

TIME = 4.12199

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 731.85903
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 374.17601
MFLUX = 148.232					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 374.17608
MFLUX = 148.233					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 374.17623
MFLUX = 148.234					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1318.18215
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1318.18210
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37022
MFLUX = 2.59153					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37021
MFLUX = 2.59145					
TOTAL HEAT FLOW =					15.84096 W

TIME = 4.48199

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 712.63126
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 373.08143
MFLUX = 140.570					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 373.08150
MFLUX = 140.570					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 373.08164
MFLUX = 140.571					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1318.23282
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1318.23277
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37010
MFLUX = 2.59068					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37009
MFLUX = 2.59060					
TOTAL HEAT FLOW =					15.45356 W

TIME = 4.84199

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 693.30450
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 372.09455
MFLUX = 133.642					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 372.09462
MFLUX = 133.642					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 372.09475
MFLUX = 133.643					
I = 39	J = 2	K = 1	AXIAL POSITION =	.00000	TEMP = 1318.27274
I = 39	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 1318.27274
I = 39	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 353.37036
MFLUX = 2.59210					
I = 39	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 353.37029
MFLUX = 2.59202					
TOTAL HEAT FLOW =					15.18258 W

TIME = 5.20199

I = 1	J = 2	K = 2	AXIAL POSITION =	.05000	TEMP = 679.63069
I = 1	J = 23	K = 1	AXIAL POSITION =	.00000	TEMP = 371.20098
MFLUX = 127.407					
I = 1	J = 23	K = 2	AXIAL POSITION =	.05000	TEMP = 371.20104
MFLUX = 127.407					
I = 1	J = 23	K = 3	AXIAL POSITION =	.10000	TEMP = 371.20117

MFLUX = 127.408
 I = 39 J = 2 K = 1 AXIAL POSITION = .00000 TEMP = 1318.32801
 I = 39 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 1318.32797
 I = 39 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 353.37083
 MFLUX = 2.59578
 I = 39 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 353.37082
 MFLUX = 2.59571
 TOTAL HEAT FLOW = 14.78427 W

TIME = 7.00199

I = 1 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 675.89234
 I = 1 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 348.13385
 MFLUX = 105.937
 I = 1 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 348.13391
 I = 1 J = 23 K = 3 AXIAL POSITION = .10000 TEMP = 348.13401
 MFLUX = 105.938
 I = 39 J = 2 K = 1 AXIAL POSITION = .00000 TEMP = 1318.37898
 I = 39 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 1318.37894
 I = 39 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 353.37590
 MFLUX = 2.63133
 I = 39 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 353.37590
 MFLUX = 2.63129
 TOTAL HEAT FLOW = 13.67112 W

TIME = 8.80199

I = 1 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 587.61190
 I = 1 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 345.95210
 MFLUX = 90.6447
 I = 1 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 345.95215
 MFLUX = 90.6450
 I = 1 J = 23 K = 3 AXIAL POSITION = .10000 TEMP = 345.95223
 MFLUX = 90.6454
 I = 39 J = 2 K = 1 AXIAL POSITION = .00000 TEMP = 1318.82219
 I = 39 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 1318.82217
 I = 39 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 353.37932
 MFLUX = 2.65661
 I = 39 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 353.37931
 MFLUX = 2.65659
 TOTAL HEAT FLOW = 12.83027 W

TIME = 10.6020

K = 1
NEW TRIAL FUNCTIONS ARE 7 8 9 10 11 0 0 0 0 0

K = 2
NEW TRIAL FUNCTIONS ARE 7 8 9 10 11 0 0 0 0 0

K = 3
NEW TRIAL FUNCTIONS ARE 7 8 9 10 11 0 0 0 0 0

TIME = 10.60199

I = 1 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 540.50847
 I = 1 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 344.41331
 MFLUX = 79.8932
 I = 1 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 344.41332
 MFLUX = 79.8933
 I = 1 J = 23 K = 3 AXIAL POSITION = .10000 TEMP = 344.41335
 MFLUX = 79.8934
 I = 39 J = 2 K = 1 AXIAL POSITION = .00000 TEMP = 1319.11372
 I = 39 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 1319.11371
 I = 39 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 353.34987
 MFLUX = 2.44909
 I = 39 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 353.34987
 MFLUX = 2.44909
 TOTAL HEAT FLOW = 11.73349 W

TIME = 12.40199

I = 1 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 538.31331
 I = 1 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 343.17816
 MFLUX = 71.2471
 I = 1 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 343.17817
 MFLUX = 71.2472
 I = 1 J = 23 K = 3 AXIAL POSITION = .10000 TEMP = 343.17819
 MFLUX = 71.2474
 I = 39 J = 2 K = 1 AXIAL POSITION = .00000 TEMP = 1319.38149
 I = 39 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 1319.38148
 I = 39 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 353.37572
 MFLUX = 2.43004
 I = 39 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 353.37572
 MFLUX = 2.43004
 TOTAL HEAT FLOW = 11.54446 W

TIME = 14.20199

I = 1 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 521.09352
 I = 1 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 342.28427
 MFLUX = 64.4439
 I = 1 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 342.28428
 MFLUX = 64.4439
 I = 1 J = 23 K = 3 AXIAL POSITION = .10000 TEMP = 342.28430
 MFLUX = 64.4441
 I = 39 J = 2 K = 1 AXIAL POSITION = .00000 TEMP = 1319.32087
 I = 39 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 1319.32086
 I = 39 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 353.38084
 MFLUX = 2.64403
 I = 39 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 353.38084
 MFLUX = 2.64402
 TOTAL HEAT FLOW = 11.16135 W

TIME = 16.00199

I = 1 J = 2 K = 2 AXIAL POSITION = .05000 TEMP = 507.02983
 I = 1 J = 23 K = 1 AXIAL POSITION = .00000 TEMP = 341.43791
 MFLUX = 59.0593
 I = 1 J = 23 K = 2 AXIAL POSITION = .05000 TEMP = 341.43792
 MFLUX = 59.0591
 I = 1 J = 23 K = 3 AXIAL POSITION = .10000 TEMP = 341.43794
 MFLUX = 59.0593
 I = 39 J = 2 K = 1 AXIAL POSITION = .00000 TEMP = 1319.48230

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