Fatigue Analysis of Laser-Welded Aircraft Structures

By

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Abstract

The motivation of cost reduction and performance improvement requires that future aircraft structures not only have higher fatigue life, higher damage tolerance capacity but also can minimize the maintenance and service costs. The laser welded integral structures provide an alternative solution to the problem.

In this research, the fatigue analysis of the improved pad design of laser welded stringers for aircraft is addressed. The research starts with the benefits of the pad design stringer given through the analyses based on the regular T-plate structure and the pad design was optimized by stress state analyses on the potential crack plane using the finite element method.

The solutions of stress intensity factor for a single edge crack originating from the weld toe on the stringer were derived using both the finite element method and the weight function method. The two methods gave the stress intensity factor solutions for the range of relative crack depths (a/t) between 0.05 and 0.8 under pure tension, three point bending and four point bending. The derived weight function solutions were obtained from the assumed approximated general weight function and two reference stress intensity factors, and were verified against the finite element method solutions.

The derived weight function solutions were utilized to predict the fatigue crack propagation behavior under remote tension and bending for a single edge crack originating from the weld toe of a pad design stringer. The thickness effects of stringers under remote tension and bending were also discussed using the derived weight function solution. The solutions derived in this thesis were proven suitable for the fracture assessment and fatigue crack propagation analysis for a single edge crack originating from the weld toe of a pad design stringer. The fatigue strength improvements of the laser welded stringers under the specified loading conditions by the pad design were verified by comparing with regular T-plate structures.

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NOMENCLATURE

ν	Poisson's ratio
θ	weld throat angle
Γ	an arbitrary counter-clockwise path around the crack tip
Γ_0	the contour surrounding the crack tip
Γ_-	the lower crack face
Γ_+	the upper crack face
σ_0	nominal stress on the potential crack plane
$\sigma_{\max}$	maximum nominal stress
$\sigma_{\min}$	minimum nominal stress
σ_{ij}	stress field component
$\sigma_{r1}(x)$	constant crack face pressure stress field
$\sigma_{r2}(x)$	linearly increasing crack face pressure stress field
$\sigma(x)$	stress distribution
$\Delta\sigma$	stress range
ξ, η	local coordinates for isoparametric element
a	crack length
a_0	initial crack length
A	cracked area
a/t	relative crack depth
b	stringer length

B	specimen thickness
$C \text{ \& } n$	material constants
C_{ijn}	Irwin-Williams solution parameter
da/dN	fatigue crack propagation rate
E	modulus of elasticity
$f_{ij}(b)$	universal functions
FEM	finite element method
FSW	friction stir welding
G	strain energy release rate
H	general elastic modulus
I	second moment of area
J	J -Integral value
$[J]$	Jacobian matrix
K	stress intensity factor
$K_{\max}$	stress intensity factors corresponding to $\sigma_{\max}$
$K_{\min}$	stress intensity factors corresponding to $\sigma_{\min}$
K_r, K_{ru}, K_{rl}	reference stress intensity factors
K_{r1}, K_{r2}, K_{r3}	reference stress intensity factors
$\Delta K, \Delta K_{th}$	stress intensity factor range
l	the nodal point
L	weld leg length
LBW	laser beam welding

$m(x,a)$	weight function of stress intensity factor
$M_{applied}$	applied bending moment
$M_i (i = 1,2,3)$	weight function parameters
n_i	the unit vectors normal to Γ_0
N_i	the shape function corresponding to node i
N	fatigue cycle numbers
P	splitting forces applied to the crack face
q	shape function
r	notch root radius at the weld toe
R	stress ratio
R	pad edge profile radius
S	external load
SIF	stress intensity factor
t	cross section thickness
t_j	crack face traction
T	kinetic energy densities; traction vector
T_1	skin thickness
T_2	attachment thickness
T_p	pad thickness
TANGO	Technology Application to the Near-Term Business Goals and Objectives of the Aerospace Industry
u	the displacement vector component

$u(x,a), u_r(x,a).$	crack opening displacement field
U	the potential energy of deformation
w	the pad width
W	the component overall width
y	distance from neutral axis
Y, Y_u, Y_l	normalized stress intensity factors

Chapter 1

Introduction

In recent years, competition in the aerospace industry has increased pressure to reduce costs in all areas of aircraft design and manufacture. Major aircraft manufacturers are looking into new materials and new processing methods to reduce cost and improve performance of their aircraft [Collins, 2003]. This demands that the structure should have a long life, low weight and minimum maintenance costs. The traditional riveted structure construction has reached its optimum and new ultra-light materials for large scale replacement are also not available. Alternatively, the use of innovative joining techniques like laser beam welding (LBW) or friction stir welding (FSW) coupled with new alloys is likely to provide an immediate solution [Vaidya et al, 2003]. These welded integral structures offer the prospect of reduced cost and weight, along with reduced maintenance due to the elimination of structural joints, and the resultant suppression of manufacturing defects during assembly. However, the widespread application of large-scale integral manufacture to damage tolerance critical areas of the primary structure has been prevented by concerns over a perceived lack of damage tolerance [Collins, 2003].

In the present thesis, the laser welded integral stringer on aircraft is the main focus. On typical laser welded T-joint structures (Figure 1.1), many failures are due to the fatigue cracks initiating and developing from the weld toes where large stress concentrations are present. The structure fatigue life can be determined by the fatigue crack propagation on the critical potential crack plane at the weld toe. The stress concentration at the weld toe and the stress distribution on the potential crack plane are primarily influenced by local geometrical effects. These, in turn, will influence the stress

intensity factor for a crack originating from the weld toe and form the baseline of structure fatigue strength. Thus, the pad design structure (Figure 1.2) based on the regular T-plate was brought up to reduce the stress concentration at the weld toe and improve stress distribution on the potential crack plane, so that fatigue strength improvement of the structure can be achieved. The pad geometrical parameters that likely can affect the stress distribution on the potential crack plane at the weld toe are the pad width (w) and pad thickness (T_p). The stress distribution analysis using the finite element method (FEM) will be conducted to obtain an optimized geometrical solution for the pad design.

On the structures, cracks might be present as small flaws in the material; they may arise during fabrication or may be the result of damage to the completed structure. Since cracks can't be eliminated, procedures must be devised to quantify and predict the behavior of cracked structures under service conditions. Fracture mechanics has become the primary tool to analyze brittle fracture and fatigue failures in structures. The fundamental postulate of linear elastic fracture mechanics (LEFM) is that the behavior of crack (i.e. whether they grow or not, and how fast they grow) is determined solely by the value of the stress intensity factor (K). In linear elastic fracture mechanics, the stress intensity factor characterizes the stress field in the vicinity of a crack tip in a linear elastic material. Therefore it is necessary to evaluate the stress intensity factor in order to be able to predict the behavior of cracked structures.

The stress intensity factors for a single edge crack originating from the weld toe of a stringer depend on the crack geometry and the applied loading conditions (Figure 1.3). In two dimensional plane strain conditions which are the typical stringer service condition in aircraft structures, the geometrical parameter that affect the stress intensity factor solutions is the relative crack depth, a/t , where, a is the crack length and t is the potential crack plane thickness. The details of the geometry for a single edge crack at the weld toe of a stringer are shown in Figure 1.3. Considering the typical load in the service

conditions, the loads applied in the study are characterized as: remote tension load, pure tension load, three points bending load and four points bending load (detailed descriptions will be given in Chapter 3).

Due to the complexity of the problem, exact solutions for stress intensity factors are not available, and approximate methods must be used. Although there are many approximate methods which can be used to solve this problem, the most accepted stress intensity factor solutions for the present problem were obtained using the finite element method [Aliabade and Rooke, 1991]

However, the stress intensity factors also depend on the loading conditions, and the finite element method can only solve one loading condition at a time. This means that the computation has to be performed repeatedly at multiple crack length steps and loading conditions, leading to high costs in terms of both manpower and computational resources, especially when the load is complex and a high degree of accuracy is required. The efficient method to obtain a large number of stress intensity factor solutions for complex loading is the weight function method.

The weight function method was developed for calculating stress intensity factors by Bueckner [Bueckner, 1970] and Rice [Rice, 1972] in the early 1970's. Mathematically, the weight function is the Green function for the stress intensity factor problem, which can be interpreted as the stress intensity factor corresponding to a unit point load applied to the crack surface. The weight function only depends on the crack geometry. The stress intensity factor can be calculated by the integration of the product of the weight function and the stress distribution acting on the prospective crack plane. Once the weight function is known for a particular geometry, the stress intensity factors for various loading conditions can be calculated.

There are several ways to obtain a weight function. Rice [Rice, 1972] found that the weight function could be derived from a reference stress intensity factor solution for a specific geometry and corresponding crack opening displacement fields. To overcome the difficulty of obtaining crack opening displacement fields, many authors proposed approximate weight forms from which the weight function can be obtained from reference stress intensity factor solutions alone. Recently, Shen and Glinka [Shen and Glinka, 1991], found general forms that can be used to approximate weight functions for a variety of two dimensional cracks. Based on the general form, the weight function for a given geometry can be obtained by using two reference stress intensity factor solutions and one boundary condition. In the present thesis, this approach is applied to derive the weight functions for a single edge crack originating from the weld toe of a two dimensional stringer. The derived weight function is then applied to calculate the stress intensity factors in the fatigue crack propagation analysis.

In the present thesis, the fatigue analysis of the stringer is another focus of the work. Fatigue is the process of cumulative damage caused by repeated fluctuating loads, which typically occurs in aircraft structures. After a certain number of load fluctuations, the accumulated damage causes the initiation and subsequent propagation of a crack or cracks in the plastically damaged regions. The fluctuation of the stress intensity factor is the primary driving force for fatigue crack propagation. The rate of fatigue crack propagation is governed by the local stress intensity factor fluctuation. The relationship is given by the Paris Law [Paris, 1962].

Many parameters affect the fatigue performance of structural components. They include parameters related to stress (load), geometry, properties of the component and external environment [Barsom and Rolfe, 1987]. The primary factor that affects the fatigue behavior of structural components is the fluctuation in the localized stress or strain. Consequently, the most effective methods for increasing the fatigue life

significantly are usually accomplished by decreasing the severity of the stress concentration and the magnitude of the applied nominal stress. In the case considered in the thesis, the decrease in the severity of stress concentration was accomplished by adding the pad on the regular T-joint structure.

Moreover, in the geometrical parameters, the effect of thickness has been noted experimentally as a decrease in fatigue strength by an increase in plate thickness [Gurney, 1979]. The thickness effect is primarily caused by the magnitude of the stress concentration at the weld toe and the stress gradient in the plane of crack growth. In the current research, the effects of the pad addition on the fatigue strength of welded stringer in the different configurations under the various loadings were investigated using the derived weight function and Paris Law. The results demonstrated that the pad design of regular T-plate significantly improve the fatigue strength of the structure. Finally, the thickness effect of pad design stringer was analyzed using the same approach.

1.1 Thesis Objective

The main objective of this thesis is to investigate the structural behavior of improved pad design of a regular T-plate using both the numerical and analytical approaches. Two-dimensional finite element analyses will be conducted to optimize the pad geometry and to determine the stress intensity factor for a single edge crack originating from the weld toe of a stringer under various loading conditions. The weight function over the geometric region $0 \leq a/t \leq 0.8$ of the crack will be derived and verified using the finite element solutions. The derived weight function will be applied to investigate the structure fatigue behavior of the chosen stringer under the different loading conditions. Finally, the thickness effects of the stringer will be presented.

1.2 Outline of Thesis

The thesis is divided into six chapters. Chapter 1 is the introduction of the thesis. In Chapter 2, an overview of fracture mechanics, calculation of the stress intensity factor and fatigue crack propagation behavior is presented. In Chapter 3, two-dimensional finite element analyses are used to optimize the pad design of T-plate joint. Chapter 4 shows the method to calculate the stress intensity factor using the finite element method and the weight function approach. In Chapter 5, the structure fatigue behavior and thickness effect of pad design stringer are discussed. The conclusions and recommendations are given in Chapter 6.

Figure 1.1 Regular T-joint structure

(all dimensions in mm)

Figure 1.2 Improved pad design of the stringer

Chapter 2

Background and Literature Review

2.1 Welded Integral Structure in Aerospace Industry

Commercial pressures to reduce aircraft manufacturing and operating costs are pushing manufacturers to consider alternative ways of fabricating aircraft structures. The introduction of novel materials and new manufacturing technologies is the primary means by which airframe manufacturers such as Airbus are able to improve structural reliability and enhance product competitiveness.

Advanced materials, manufacturing processes and joining techniques offer significant cost and weight savings, a reduction in the need for expensive tooling, and an increase in the overall aircraft performance [Collins, 2003, Rendigs, 1997].

The European Union has initiated the TANGO program (Technology Application to the Near-Term Business Goals and Objectives of the Aerospace Industry), which aims to improve the structural efficiency and reduce the manufacturing cost by a large scale application of new technologies to achieve low-cost, low-weight primary structures [Schmidt et al, 2001].

Within Airbus, a wide range of research initiatives are currently being considered, including [Collins, 2003, Boetsch, 1999, Schmidt, 1999]:

- the use of new lighter, stronger materials, e.g. advanced aluminium alloys
- the adoption of innovative structural concepts, e.g. integral components

- the further application of unconventional fabrication methods, e.g welding (including laser beam, electron beam and friction stir welding)

Since one of the more expensive and traditional process employed in aircraft construction is riveting, weight reduction of airframes is nearing a saturation stage. The riveted construction has reached its optimum and new ultra-light materials for large scale replacement are also not available. Alternatively, the use of innovative joining techniques coupled with new alloys to produce rivet-free welded airframe is likely to provide an immediate solution. By utilizing laser beam welding (LBW) and alloys like 6013 and 6056 to produce welded airframe, not only a substantial weight reduction, but also reduction in manufacturing cost can be achieved [Rendigs, 1997]. Recently, a new welding technique, called friction stir welding (FSW) which was invented and patented by TWI [Thomas et al], is rapidly gaining acceptance in the airframe industries [Williams, 2001]. It is a relatively new solid phase welding process. The motivations for the application of FSW including:

- the potential reduction in costs
- minimizing the number of conventional joints;
- narrow extrusion panels may be assembled by FSW to wider panels;
- furthermore, due to the nature of the technique, FSW is applicable to 2000 and 7000 series aluminium alloys, which are regarded as difficult materials to fusion weld.
- also according to W. V. Vaidya et al [Vaidya et al, 2003], FSW shows reasonable good fatigue performance in the high cycle regime. And, the resistance to Fatigue Crack Propagation (FCP) is comparable or slightly better than that of the base material at high stress levels.

According to Shepherd [Shepherd, 2003], friction stir welding (FSW) offers high quality welds with low distortion and shrinkage, and improved mechanical properties. The FSW could be applied to manufacture longitudinal and circumferential skin joints in panels (Figure 2.1a and 2.1b).

The laser beam welding (LBW) technology is distinguished by concentrated heat coupling, narrow weld seams, a high welding rate (max. 10m/min) and a high automation potential. It is most suitable for welding T-joints, e.g. skin-to-stringer (Figure 2.2) or skin-to-clip-joints and eventually for joining skin panels (Figure 2.3).

These processes permit gap-free connections which improve the corrosion properties of the fuselage very considerably as compared to riveted structures with innumerable rivet holes and skin/stringer gaps as well to bonded structures with possible voids between the joints. Recently, Airbus introduced these emerging technologies into its aircraft production:

- In 2002, Airbus family's A318 took off for the first time, with the first forward fuselage panel with laser beam welded stringers [Albugues and Rodrigo, 2003].
- February 2003 the first double curved welded stiffened panel was produced for the A380 [Albugues and Rodrigo, 2003].

However, to date the widespread application of large-scale welded integral structure to damage tolerance critical areas of the primary structure has been prevented by concerns over a perceived lack of damage tolerance. One of the challenges of substituting welding for riveting in aircraft structures is a cluster of design and damage tolerance issues which need to be resolved if the welded structure is to match the riveted ones in terms of weight and structural integrity. This is why the knowledge and the prediction of structural fracture and fatigue behavior of welded stringer become major preoccupations. Therefore improved stringers are under investigation to increase the residual strength without reducing the allowable skin stress.

To replace the traditional riveted stringer with a welded one, more confidence is needed for the welded stringer structure, especially in larger scale application. Therefore, a closer look and better understanding of the welded stringer in terms of its structural behavior is definitely a prerequisite. In the present thesis, fracture mechanics based approaches are used to investigate the improved design of the stringer and its fracture and

fatigue behavior.

Since the present work undertaken is entirely based on the fracture mechanics, it is necessary to take a look at some fundamental aspects of fracture mechanics first.

2.2 Fracture Mechanics Overview

A cracked body can be loaded in any one or combinations of the three displacement modes shown in Figure 2.4.

- Mode I (opening mode): where the principal load is applied normal to the crack plane, and tends to open the crack. The displacements of the crack surface are perpendicular to the plane of the crack.
- Mode II (sliding mode): corresponds to shear loading in the plane and tends to slide one crack face with respect to the other. The displacement of the crack surfaces is in the plane of the crack and perpendicular to the leading edge of the crack.
- Mode III (tearing mode): associated with out-of-plane shear, and also involves relative sliding of the crack faces. Crack surface displacements are in the plane of the crack and parallel to the leading edge of the crack.

Mode I is technically the most important and primary mode in most cracking analysis of engineering interests. So, in this thesis, only Mode I will be discussed.

Under Mode I, there could be a plane strain or plane stress condition. The plane strain condition exists at the center of thick plates and the plane stress condition exists in thin plates and near the edge surfaces of thick plates as shown in Figure 2.5. It can be seen that plastic zone size is smaller under plane strain than the plane stress conditions.

2.2.1 Crack Tip Stress Field Analysis

Williams [Williams, 1957] resolved the stress field near a crack front of modes I and II from knowledge of the solution for a V corner when the V angle approaches 0. Solution of mode I was also given by Irwin [Irwin, 1957]. The general solution, also called the Irwin-Williams solution, is expressed as the expansion of a series whose first terms are written for mode I as [Miannay, 1998]:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{1i}\delta_{1j} + C_{ij2}(\theta)r^{\frac{1}{2}} + \dots + C_{ijn}(\theta)r^{\frac{n-1}{2}} + \dots \quad (2.1)$$

where, (r, θ) is the polar coordinate system as shown in Figure 2.6. The first term figures the $r^{-1/2}$ singularity when r approaches 0. The magnitude of the singularity term, K , is called “Stress Intensity Factor” (SIF). The $f_{ij}(\theta)$ are universal functions. The second term is a constant, called “transverse stress” or “ T -stress”. The higher-order terms are negligible.

The stress intensity factor K defines the amplitude of the crack tip singularity. That is, stresses near the crack tip increase in proportion to K . Moreover, the stress intensity factor defines the crack tip conditions; if K is known, it is possible to obtain all components of stress, strain and displacement as a function of r and θ at the vicinity of the crack tip (Eq.(2.1)). Then, the information of the state of stress can be used to reliably predict the character of the crack propagation.

The calculation and application of the stress intensity factor are one of the main focus in this research. The stress intensity factor can be determined directly from the local crack tip stress field, Eq. (2.1), or the corresponding strain and displacement field. It can also be obtained indirectly through other fracture parameters such as the strain energy release rate, G , or the J -integral, which are uniquely related to the stress intensity factor for linear elastic materials.

2.2.2 Strain Energy Release Rate and J-integral

The energy release rate G , the J -integral and the stress intensity factor are all equivalent fracture parameters within the confines of small-scale yielding. Consequently, when any one of the three attains its critical value, the others must also reach simultaneously their critical values. Since they are all inter-related, and one of them is known, the others can be obtained from the relationship between them.

The strain energy release rate, G , proposed by Griffith [Griffith, 1920], and defined as the rate of change in potential energy of a cracked body with an increase in crack area for linear elastic materials, is shown in Figure 2.7, and can be expressed as:

$$G = -\frac{dU}{dA} \quad (2.2)$$

where, U is the potential energy of deformation contained in the member, and A is the cracked area.

The change in crack area dA , is Bda , where a and B are the crack length and specimen thickness respectively. Thus, G characterizes the energy per unit crack area available to form an addition crack. Irwin established the relationship between K and G [Irwin, 1957]:

$$G = K^2/H \quad (2.3)$$

where, H is the general elastic modulus:

$$H = E \quad (\text{Plane stress})$$

$$H = E/(1 - \nu^2) \quad (\text{Plane strain})$$

with E , Modulus of Elasticity, and ν , Poisson's ratio.

In the case where there is appreciable plasticity, G is no longer applicable since it is based on the linear elastic behavior [Forman, 1965]. A generalization to nonlinear elastic materials leads to the path independent J -integral, which provides a means to determine an energy release rate for cases where plasticity effects are not negligible.

Eshelby [Eshelby, 1974] has defined a number of contour integrals which are path independent by virtue of the theorem of energy conservation [Knowles and Sternberg, 1972]. Cherapanov [Cherepanov, 1967] and Rice [Rice, 1968] applied this integral to crack problems. By considering the two-dimensional deformation of the nonlinear elastic body shown in Figure 2.8, the J -integral is given by

$$J = \int_{\Gamma} (W dy - T \frac{\partial u}{\partial x} ds) \quad (2.4)$$

with

$$W = W(x, y) = W(\varepsilon) = \int_0^{\varepsilon} \sigma_{i,j} d\varepsilon_{i,j} \quad (2.5)$$

where, Γ is an arbitrary counter-clockwise path around the crack tip, W is the strain energy per unit volume, T is the traction vector perpendicular to Γ in the outside direction, u is the displacement vector component, and ds is increment of length along the contour Γ .

Rice [Rice, 1968] has shown that the value of the J -integral is independent of the path of integration around the crack tip. Also, this integral can be considered as an energy release rate for nonlinear elastic materials. Equation (2.2) defines the energy release rate for linear elastic behavior. The same definition holds for nonlinear elastic behavior, except that G is replaced by J :

$$J = -\frac{dU}{dA} \quad (2.6)$$

If the material considered is linear elastic, the J -integral is equal to the strain energy release rate, G :

$$J=G \quad (\text{linear elastic material}) \quad (2.7)$$

Thus, the J -integral is related to the stress intensity factor by Eq. (2.3)

2.3 Calculation of Stress Intensity Factor

Over the last four decades, many methods of obtaining stress intensity factor solutions have been developed. These methods can be divided into three categories depending on their degree of sophistication and the time required to obtain a solution [Aliabade and Rooke, 1991]:

Category one: Handbooks

Category two: Superposition, Stress Concentration, Stress Distribution,
Green's Function, Approximate Weight Function and Compounding

Category three: Collocation (Mapping), Integral Transformation, Body Force
Method, Alternating Technique, Line Spring Method,
Boundary Element Method (BEM), Finite Element Method (FEM)

The stress intensity factor depends on the geometrical configuration of a cracked body and the loading condition. For simple geometrical configurations and loading conditions, it is possible to use available handbooks [Sih, 1973, Tada et al, 1985, Rooke and Cartwright, 1976]. Where a solution can not be obtained directly from a handbook, one of the relatively simple methods in Category Two may be adequate. They seldom require more than a few hours to obtain stress intensity factor values. The accuracy of the methods in the category depends on the accuracy of the method itself and the accuracy of

the reference stress intensity factors used. However, for complicated problems, where accurate stress intensity factors are important, numerical approaches in Category three are needed. Among them, the finite element method (FEM) is most commonly applied.

The combination of finite element method and the weight function approach is a good compromise. Normally the weight function solution of stress intensity factor can be derived from the reference stress intensity factors calculated by the finite element method for simple loading conditions. In this research, the combination of finite element and weight function results are used to develop the stress intensity factor solutions.

The application of finite element method and weight function approach will be discussed in the following two sections

2.3.1 Numerical Approach of Calculating Stress Intensity Factor

The finite element method (FEM) has emerged over the last two decades as one of the most powerful numerical tools for the solution of crack problems in fracture mechanics. The method has great versatility: it allows the analysis of complicated engineering geometries (e.g. bolted or welded structures), it enables treatment of three-dimensional problems, and it permits the use of elastic-plastic elements to include crack tip plasticity. Since the behavior of the displacement field in common finite elements is at most quadratic or cubic, the stress field is at most linear or quadratic [Zienkiewicz and Taylor, 1991]. But the stresses at the neighborhood of a crack tip in a linear elastic material are singular. Therefore, in order to properly model the stresses in the region adjacent to the crack tip, singular quarter-point, isoparametric elements were employed in this research. The stress intensity factor must be extracted from either the stress field or displacement field or by an energy based method. In this research, the stress intensity factor was extracted from the J -integral, which is obtained using the domain integral method.

Crack Tip Stress Field Modeling

The model using conventional elements requires a very fine mesh at the crack tip to model the singular behavior of stress in the vicinity of the crack tip. Henshell and Shaw [Henshell and Shaw, 1975] noted that if the mid-side nodes of an isoparametric element in the vicinity of the crack tip were moved to the quarter points, shown in Figure 2.9, then the element shape functions gave a $1/\sqrt{r}$ singularity along these edges. About the same time, Barsoum [Barsoum, 1976] discovered that if the isoparametric element was collapsed along one side so that three nodes were positioned at the crack tip and mid-side nodes were moved to the quarter points, shown in Figure 2.10, then the element gives the $1/\sqrt{r}$ singularity along any ray emanating from the crack tip node. The element formulation is very simple and gives very good accuracy. For two dimensional edge cracks in this research, eight-node singular element (collapsed rectangular) were employed at the crack tip.

The information of the isoparametric elements is well documented [Zienkiewicz and Taylor, 1991]. For a two-dimensional eight-node isoparametric element as shown in Figure 2.11, the geometry of the element is mapped into the normalized square space (ξ, η) , $(-1 \leq \xi \leq 1, -1 \leq \eta \leq 1)$ through the following transformation:

$$x = \sum_{i=1}^8 N_i(\xi, \eta) x_i \quad (2.8)$$

$$y = \sum_{i=1}^8 N_i(\xi, \eta) y_i \quad (2.9)$$

where N_i is the shape function corresponding to node i , whose co-ordinates are (x_i, y_i) in the x - y coordinate system and (ξ, η) in the transformed ξ - η coordinate system. $\xi, \eta = \pm 1$ for corner points and zero for mid-side node.

After the displacement field in the member is obtained, the stress field can be

derived by using the stress-strain relationship, i.e., Hook's law for elastic materials.

Since $N_i(\xi, \eta)$ are polynomials and hence $\partial N_i / \partial \xi$ and $\partial N_i / \partial \eta$ are non-singular. For conventional elements, the determinant of Jacobian $[J]$, $\det[J]$ can not be zero anywhere in the element [Zienkiewicz and Taylor, 1991], and the stress and strain fields are linear. Therefore, the conventional element can not be used directly to model the stress singularity at the crack tip. By the way of Henshell & Shaw [Henshell and Shaw, 1975] and Barsoum [Barsoum, 1976] mentioned above, $\det[J]$ becomes zero at crack tip, so the desired $1/\sqrt{r}$ singularity in strains and stresses can be achieved.

Extracting Stress Intensity Factors

Basically, two different approaches can be followed in employing finite element procedures to obtain the required stress intensity factors. One is the direct method in which K follows from the stress field or displacement field [Chan et al, 1970]. The second is an indirect method in which K is determined via its relation with other quantities such as the elastic energy or the J -integral. In this research, J -integral is used to extract the stress intensity factors.

For linear elastic materials, the stress intensity factor K can be obtained by using the following relationships between K and J ,

$$K = \sqrt{\frac{JE}{(1 - \nu^2)}} \quad (\text{Plane strain}) \quad (2.10)$$

$$K = \sqrt{JE} \quad (\text{Plane stress}) \quad (2.11)$$

The J -integral numerical analysis was found by Shih [Shih et al, 1986] using the energy domain integral methodology which can be applied to both quasi-static and dynamic problems with elastic or elastic-plastic material and can accommodate crack face traction and thermal loading. Moreover, the domain integral formulation [Shih et al,

1986] is relatively simple to implement numerically, and it is very efficient.

Eq.(2.4) is limited to quasi-static conditions, and requires the absence of body forces and crack faces free of traction. In order to remove these limitations, Shih [Shih et al, 1986] proposed the generalized expression of J -integral where the contour surrounding the crack tip, Γ_0 , is vanishingly small:

$$J = \lim_{\Gamma_0 \rightarrow 0} \int_{\Gamma_0} \left[(w + T)n_1 - \sigma_{ij}n_i \frac{\partial u_j}{\partial x_1} \right] ds \quad (2.12)$$

where w and T are the stress work and kinetic energy densities, respectively, σ_{ij} and u_i are the components of the stress and displacement, n_i are the unit vectors normal to Γ_0 , and ds is the arc length as shown in Figure 2.12.

Under quasi-static conditions ($T=0$), Eq. (2.12) becomes

$$J = \lim_{\Gamma_0 \rightarrow 0} \int_{\Gamma_0} \left[w\delta_{li} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} \right] n_i ds \quad (2.13)$$

where $i=1,2,3$ represent the directions of axis x , y and z , while l represents the nodal point in each element.

Eq. (2.13) can be used in a material with the plastic deformation and its body forces, thermal strains and crack face tractions being ignored. This equation is path independent which means that its value does not depend on the crack tip contour Γ_0 and Eq. (2.13) becomes (2.4). If any of these conditions are not satisfied, Eq.(2.13) must be used.

Because the form of Eq.(2.13) is not directly suitable for finite element calculation, it is converted to the following equation and used in the finite element calculation to obtain the J -integral in two dimensions.

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_l} - w \delta_{li} \right] \frac{\partial q}{\partial x_i} dA - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j} \frac{\partial u_j}{\partial x_l} q ds \quad (2.14)$$

where, Γ_- and Γ_+ are the lower and upper crack faces; the inner contour, Γ_0 is taken as the crack tip, in which A^* corresponds to the area inside Γ_l ; q is an arbitrary but smooth function that is equal to unity on Γ_0 (crack tip) and zero on Γ_l (the outer boundary).

Γ_l should coincide with element boundaries. The q function must be specified at all nodes within the area. The shape of the q function is arbitrary, as long as q has the correct values on the domain boundaries.

The discretized form of the domain integrals are as follows:

$$J = \sum_{\text{all elements within } A^*} \sum_{p=1}^n \left\{ \left[\sigma_{ij} \frac{\partial u_j}{\partial x_l} - w \delta_{li} \right] \frac{\partial q}{\partial x_i} \right\}_p w_p - \sum_{\text{all edges on } \Gamma_+ + \Gamma_-} \left\{ t_j \frac{\partial u_j}{\partial x_l} q \right\} w \quad (2.15)$$

the quantities within $\{ \}_p$ are evaluated at the Gauss integration points $p=1, n$, and w_p and w are the respective weights at the points. The second term in Eq.(2.15) is the contribution due to crack face traction t_j . Eq.(2.15) is the numerical formulation for J used in the two dimensional finite element calculation.

2.3.2 Analytical Approach of Calculating Stress Intensity Factors

In this research, the weight function method will be used to determine the stress intensity factor solutions under arbitrary loading system.

Theoretical Background

Most of the existing methods of calculating stress intensity factors require separate analysis of each load and geometry configuration. The weight function method developed

by Bueckner [Bueckner, 1970] and Rice [Rice, 1972] simplifies considerably the determination of stress intensity factors. The important feature of the weight function is that it depends only on the geometry of the cracked body. If the weight function is known for a given cracked body, the stress intensity factor due to any load system applied to that body can be determined by using the same weight function. Thus, if the weight function is available, the stress intensity factor can be obtained by multiplying the weight function, $m(x,a)$ and the internal stress distribution, $\sigma(x)$, in the prospective crack plane, and integrating the product along the crack length 'a'.

$$K = \int_0^a \sigma(x)m(x,a)dx \quad (2.16)$$

The success of the weight function technique for calculating stress intensity factors lies in the possibility of using superposition. It can be shown [Broek, 1988] that the stress intensity factor for a cracked body (Figure. 2.13) subjected to the external loading, S , is the same as the stress intensity factor in a geometrically identical body with the local stress field $\sigma(x)$ applied to the crack faces. The local stress field, $\sigma(x)$, induced in the prospective crack plane by the external load, S , is determined for uncracked body.

Mathematically the weight function $m(x,a)$ is a generalization of Green's function [Hult and McClintock, 1957] for the stress intensity factor problem. The weight function is the stress intensity factor for a pair of unit point loads on the crack surface. Bueckner [Bueckner, 1970] found that the weight function could be interpreted as normalized displacement fields as well, which satisfy the equations of linear elasticity. The determination of weight function, $m(x,a)$, can be simplified by using the following relationship derived by Rice [Rice, 1972]:

$$m(x,a) = \frac{H}{K_r} \frac{\partial u_r(x,a)}{\partial a} \quad (2.17)$$

where H is the generalized elastic modulus, which equals E for plane stress or $E/(1-\nu^2)$ for plane strain.

In order to determine the weight function from the Eq.(2.17), a reference stress intensity factor, K_r , for a specific geometry and applied stress field has to be known in addition to the corresponding crack opening displacement field, $u_r(x, a)$. Following this methodology, different weight functions can be derived.

The general principle of using weight functions for calculating stress intensity factors is shown in (Figure 2.14a), where the through thickness crack is used as an example. The weight function, $m(x, a)$, can be interpreted as the stress intensity factor that results from a pair of splitting forces, P , applied to the crack face at position x . Since the stress intensity factors depend linearly on the applied load, the contributions from multiple splitting forces applied along the crack surface can be superimposed and the resultant stress intensity factor can be calculated as the sum of all individual load contributions (Figure 2.14b). This results in the integration, Eq.(2.16), of the product of the weight function, $m(x, a)$, and the stress function, $\sigma(x)$, for a continuously distributed stress field.

In this research, weight functions for plane strain, one dimensional Mode I stress distribution was investigated. Details of derivation will be discussed in Chapter 4.

Universal Weight Functions for One-Dimensional Stress Fields

The weight function is dependent on the geometry only and in principle it should be derived individually for each geometrical configuration. Several analytical and numerical weight functions for cracks in mode I can be found in technical journals [Niu and Glinka, 1990, Xu, 1991] and stress intensity factor handbooks. However, Glinka and Shen [Glinka and Shen, 1991] have found that the following general weight function expression can be used to approximate weight functions for a variety of geometrical crack configurations subjected to one-dimensional stress fields of mode I.

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a} \right)^{\frac{1}{2}} + M_2 \left(1 - \frac{x}{a} \right) + M_3 \left(1 - \frac{x}{a} \right)^{\frac{3}{2}} \right] \quad (2.18)$$

where $M_i (i = 1, 2, 3)$ are parameters which depend only on the geometrical configuration of the cracked body.

The system of coordinates and the notation for symmetrical through and edge crack given are in Figure 2.15 as an example.

Because the mathematical form of the weight function, Eq.(2.18), is the same for all cracks, the same integration routine can be used for calculating stress intensity factors from Eq.(2.16) regardless of the geometrical configuration of a cracked body.

Derivation of Weight Function

In order to determine the weight function, $m(x,a)$, for a particular cracked body, it is necessary to determine the three parameters M_1, M_2 and M_3 in Eq (2.18). They can be determined from Eq.(2.16) providing that three reference stress intensity factors K_{r1}, K_{r2} and K_{r3} corresponding to three different stress distributions σ_{r1}, σ_{r2} and σ_{r3} respectively are known. The stress distribution expressions and the general weight functions of Eq.(2.18) can be substituted for $\sigma(x)$ and $m(x,a)$ in Eq. (2.16). This leads to three simultaneous equations sufficient for the determination of the three unknown parameters M_1, M_2 and M_3 .

$$K_{ri} = \int_0^a \frac{2\sigma_{ri}(x)}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a} \right)^{\frac{1}{2}} + M_2 \left(1 - \frac{x}{a} \right) + M_3 \left(1 - \frac{x}{a} \right)^{\frac{3}{2}} \right] dx \quad (2.19)$$

where $i=1, 2, 3$

Solution of the set of three equations given by Eq.(2.19) for M_1, M_2 and M_3 enables the three unknown parameters to be determined. The accuracy of the three parameters and consequently, accuracy of the weight function depends on the quality of the reference stress intensity factors. The obvious limitation of this method is the number of reference stress intensity factors that has to be known and which could be difficult to find. However, the number of necessary reference stress intensity factors can be decreased by using certain characteristic properties of weight functions.

As shown in Eq.(2.17), the weight function is uniquely related to the crack opening displacement field $u(x,a)$. Thus, the first derivative of the weight function with respect to the coordinate x can be written in the form

$$\frac{\partial m(x,a)}{\partial x} = \frac{H}{K} \frac{\partial}{\partial x} \left[\frac{\partial u(x,a)}{\partial a} \right] = \frac{H}{K} \frac{\partial}{\partial a} \left[\frac{\partial u(x,a)}{\partial x} \right] \quad (2.20)$$

Fett [Fett et al, 1987] has noted that the slope of the crack surface of central through cracks (Figure 2.16a) and double edge cracks (Figure 2.16b) under symmetrical loading should be zero at $x=0$, i.e.,

$$\left. \frac{\partial u(x,a)}{\partial x} \right|_{x=0} = 0 \quad (2.21)$$

Consequently Eq.(2.20) and (2.21) indicate that the first derivative of the weight function for these cases should be zero at $x=0$.

$$\left. \frac{\partial m(x,a)}{\partial x} \right|_{x=0} = 0 \quad (2.22)$$

It can be shown that condition (2.22) also applies to a single edge crack in a semi-infinite plate. In practical terms, it means that condition (2.22) also holds for short single edge crack of relative length $a/t < 0.5$. However, due to the rotation of the cracked

section in a finite thickness body Eq.(2.22) becomes inaccurate for single deep edge cracks. Therefore, Fett [Fett et al, 1987] has suggested that, in the case of single edge cracks, relation (2.23) is the more appropriate one.

$$\left. \frac{\partial^2 u(x, a)}{\partial x^2} \right|_{x=0} = 0 \quad (2.23)$$

The meaning of Eq.(2.23) is that the curvature of the crack surface at the mouth (free corner) is zero due to the fact that the surfaces near the crack mouth are stress free. So the second derivative of the weight function for such a crack should be zero at $x=0$:

$$\left. \frac{\partial^2 m(x, a)}{\partial x^2} \right|_{x=0} = 0 \quad (2.24)$$

Eq.(2.22) or Eq.(2.24) can be used as the additional conditions needed for derivation of weight functions. This will enable the reduction of the number of necessary reference stress intensity factors. Eq.(2.18) in combination with Eq.(2.24) results in a constant value of the parameter $M_2 = 3$ regardless of the crack geometry. This means only M_1 and M_3 are used to fit the weight function of Eq.(2.18) to a particular set of the reference data when the condition of Eq.(2.24) is used.

2.4 Crack Growth Analysis

The determination of the fatigue crack propagation behavior is an essential part of the fracture mechanics design approach. To establish the minimum fatigue life of structural components, it is reasonable to assume that the component contains the largest discontinuity that can not be detected by the inspection method. The initial crack length associated with these cracks can be estimated conservatively based on Nondestructive Inspection (NDI) method. The useful life of these structural components is determined by

the fatigue-crack-growth behavior of the material. Therefore, to predict the minimum fatigue life of structural components and to establish safe inspection intervals, an understanding of the rate of fatigue crack propagation is required. The most successful approach to the study of the fatigue crack propagation is based on fracture mechanics concepts.

2.4.1 Crack Growth and the Stress Intensity Factor

In the elastic case, the stress intensity factor is a sufficient parameter to describe the whole stress field at the tip of a crack. When the size of the plastic zone at the crack tip is small compared to the crack length, the stress intensity factor may still give a good indication of the stress environment of the crack tip. If two different cracks have the same stress environment, i.e. the same stress intensity factor, they behave in the same manner and show equal rates of growth. Paris [Paris, 1962] and Paris, Gomez & Anderson [Paris et al, 1961] pointed out that the rate of fatigue crack propagation per cycle is governed by the stress intensity factor range, it leads to a power law proposed by Paris [Paris, 1962]

$$\frac{da}{dN} = C(\Delta K)^n \quad (2.25)$$

where, da/dN is the fatigue crack growth per cycle, C and n are material constants which are determined experimentally, ΔK is the stress intensity factor range, which can be expressed:

$$\Delta K = K_{\max} - K_{\min} \quad (2.26)$$

where $K_{\max}$ and $K_{\min}$ are the stress intensity factors corresponding to the maximum and minimum nominal stress $\sigma_{\max}$ and $\sigma_{\min}$. By using the above relation it is possible to predict fatigue crack propagation in any configuration for which the stress intensity factor is known.

Based on the stress intensity factor range ΔK , the fatigue crack propagation behavior for metals can be divided into three regions in general (Figure 2.17). The behavior in region I exhibits a “fatigue-threshold” cyclic stress intensity factor fluctuation ΔK_{th} , below which cracks do not propagate under cyclic stress fluctuation. Region II represents the fatigue crack propagation behavior above ΔK_{th} , which can be represented by Eq.(2.25). In region III the fatigue crack growth per cycle is higher than predicted for region II, the rate of fatigue crack growth accelerates much faster until the final failure.

In this research, fatigue crack propagation of the improved designed welded stiffener with an initial crack under constant-amplitude load fluctuation will be investigated.

2.4.2 Factors Affecting Fatigue Crack Propagation in Laser Welded Structures

Fatigue crack propagation is affected by a large number of parameters. The influence of the environment is the most conspicuous. It is usually assumed that the environmental effect is associated with the small, but systematic effect of cycling frequency [Bradshaw and Wheeler, 1969, Hartman and Schijve, 1970, Schijve and Broek, 1966, Schijve et al, 1972].

Among the other factors that affect crack propagation, the following should be taken into consideration for crack growth prediction:

- Thickness
- Type of product
- Heat treatment
- Cold deformation
- Temperature
- Manufacture
- Batch-to-batch variation

- Environment and frequency

For the factors lower in this list it is less likely that they can be properly accounted for in a fracture mechanics approach.

It is well known that increasing the size of a given type of specimen while maintaining all the other parameters will, in general, cause a decrease in fatigue strength. This so called size effect may be given several interpretations. One which has received much attention is the statistical effect, which stems from the fact that fatigue is a weakest link process, nucleating at the location where stresses, geometry, defects and material properties combine to form the optimum conditions for fatigue crack initiation and growth. Increasing the size of a specimen will statistically produce locations which are more vulnerable to fatigue failures. In welded joints there is also an effect of plate thickness. This effect has been noted experimentally as a decrease in fatigue strength by increasing the plate thickness. It was well addressed in [Berge, 1984, Gurney, 1979, Johnston, 1980]. Candidate explanations for the thickness effect include: weld technology and residual stresses, geometry effect.

In the present work, fatigue failures developing from weld toes are considered. This is the most typical fatigue failure in sound welds, i.e. welds produced to normal quality standards. A useful model for describing the thickness effect in welded toe fatigue failure is obtained from the following assumptions:

- Welded joints of the same type in various plate thickness are geometrically similar
- Initial conditions of fatigue crack growth are independent of plate thickness.
(initial crack length $a_0 = \text{const}$)

Under these assumptions, the effect of thickness may be understood from a simple geometric model, as shown in Figure 2.18. The first assumption leads to a steeper stress gradient in the thinner joint. Hence, under the second assumption, the initial crack in the thinner plate will experience a smaller stress than the initial crack of the same length in the thicker plate, causing a smaller initial crack growth rate in the former case. The difference in initial crack growth rate overmatches the difference in crack length to cause

fracture; hence the thinner joint will have a longer fatigue life.

The thickness effect is thus caused by the following parameters:

- The magnitude of the stress concentration at the weld toe;
- The gradient of the stress in the plane of crack growth;
- The number of cycles in crack growth through the region of steep stress gradient, relative to the total number of cycles to failure;

Therefore, any local geometrical modification on the laser welded structure which can lead to changing one of the above parameters might affect the fatigue strength of the structure. Alternatively, the pad design on regular T-plate to change the stress concentration at the weld toe and stress distribution on the potential crack plane starting at the weld toe is considered in the present work. Two parameters, pad width and pad thickness, will be investigated in this work. Obviously, the pad thickness will play a dominant role in the thickness effect study, although the pad width could also have an influence on the fatigue life through changing the stress distribution in the welded structure.

Investigation will be focused on how the pad addition can effectively lower down the stress concentration at the weld toe and stress gradient in the potential crack plane, thus to improve the fatigue strength of the laser welded structure. Thickness effect on the fatigue strength will be also investigated.

Figure 2.1a Manufacture of friction stir welded wing ribs [Shepherd, 2003]

Figure 2.1b Welded integrally stiffened extruded wing skin panels [Shepherd, 2003]

Figure 2.2 Welded skin-to-stringer joint [Rendigs, 1997]

(a) current manufacture: skin panel machined from one billet

**(b) future manufacture: skin panel machined from
two smaller billets welded together**

Figure 2.3 Welded skin panel concepts [Collins, 2003]

Figure 2.4 Loading modes [Broek, 1988]

Figure 2.5 Plastic zone size [Miannay, 1998]

Figure 2.6 Elastic stresses near the crack tip [Miannay, 1998]

Figure 2.7 Strain energy release rate

Figure 2.8 Arbitrary contour around the tip of a crack

Henshell
& Shaw

Figure 2.9 An isoparametric element with moved mid-side node

Barsoum

Figure 2.10 The isoparametric element collapsed along one side and moved mid-side node

a)

b)

a). global co-ordinates

b). local co-ordinates

Figure 2.11 Eight-node isoparametric element

Figure 2.12 Inner and outer contours

Nomenclature and the concept of superposition; a) Cracked body subjected to the load system S , b) Geometrically identical un-cracked body with the stress field $\sigma(x)$ in the crack plane, induced by the loading system S , c) Geometrically identical cracked body with the crack subjected to the stress field $\sigma(x)$

Figure 2.13 Concept of superposition [Wu and Carlsson, 1991]

Weight Function - stress intensity factor, K, for a pair of splitting forces applied to crack surfaces

For a through crack in infinite plate:

$$m_A(x, a) = \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a+x}{a-x}}$$

$$m_B(x, a) = \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a-x}{a+x}}$$

Stress intensity factors for crack crack tips A and B respectively:

$$K_A = m_A(x, a, P) = \frac{P}{\sqrt{\pi a}} \sqrt{\frac{a+x}{a-x}}$$

$$K_B = m_B(x, a, P) = \frac{P}{\sqrt{\pi a}} \sqrt{\frac{a-x}{a+x}}$$

Figure 2.14a. Weight function and stress intensity factors for a through crack in an infinite plate

Superposition in the case of two forces:

$$K_A = m_A(x_1, a, P_1) + m_A(x_2, a, P_2)$$

$$K_A = \frac{P_1}{\sqrt{\pi a}} \sqrt{\frac{x_1 + a}{a - x_1}} + \frac{P_2}{\sqrt{\pi a}} \sqrt{\frac{x_2 + a}{a - x_2}}$$

In the case of multiple forces

$$K_A = m(a, x_1, P_1) + m(a, x_2, P_2) + \dots + m(a, x_i, P_i)$$

For a continuous stress distribution $\sigma(x)$:

$$K_A = \int_{-a}^a \sigma(x) m_A(x, a) dx$$

$$K_B = \int_{-a}^a \sigma(x) m_B(x, a) dx$$

Figure 2.14b Weight function and stress intensity factors for a through crack in a plate subjected to multiple forces or/ and continuously distributed stress

Figure 2.15 Parameters and co-ordinates system

Figure 2.16 Through and edge cracks under symmetrical stress systems: (a) Through central crack in a finite width plate; (b) Double edge crack in a finite width plate

[Fett et al, 1987]

Figure 2.17 Sigmoidal behavior of fatigue crack growth [Barsom, 1987]

Chapter 3

Welded Joint Configuration Optimization Using Stress Analysis by FEM

As shown in the previous chapter, the current trend in commercial aircraft manufacture is for the replacement of conventional built-up structures by welded and integral structures. To get a longer service life to fatigue crack initiation, reducing the local stress concentration at the critical area on the component becomes essential. Therefore improved stringers are under investigation [Schmidt et al, 2001] to increase the residual strength without reducing the allowable skin stresses.

It is well known that the regular T-plate joint has a high stress concentration at the vicinity of weld toe, which makes the region vulnerable to crack initiation, and subjected to high stress range. This means the stress state on the potential crack plane at the weld toe will cause a shorter crack initiation and propagation life along the plane, which defines the baseline fatigue strength of welded structures. To improve the fatigue strength of T-welded structure, several changes could be considered, including: increase the critical crack size by lowering design stress or increasing the fracture toughness of the material; reduce the initial crack size by improving the fabrication and inspection quality; lower the stress range ($\Delta\sigma$) to decrease the rate of crack growth, thereby increasing the number of cycles required for the crack to grow to the critical size. Noted by Paris Law (Chapter 2, section 2.4.1) that the rate of crack growth is a power function of $\Delta\sigma$, lowering the stress range slightly can have a significant effect on the structure fatigue life. Therefore, the improved pad design (Figure 3.1) on the basis of T-plate was brought up for the following reasons:

- Increase the overall thickness of the stringer, so reduce the stress level on the potential crack plane at the weld toe region;
- Change the stress state at the weld toe by adding additional bending effect through the pad (see the detail discussions in the following sections), therefore lower the stress concentration at the weld toe, which could lead to a longer crack initiation life;
- Provide a longer path for the crack propagation;

Since the introduction of the pad on T-plate will change loading condition on the potential crack plane at weld toe, four different loading conditions (see the detail descriptions in the following section) on the plane will be investigated: remote tension (Figure 3.2a), pure tension (Figure 3.2b), three-point-bending (Figure 3.2c) and four-point-bending (Figure 3.2d).

In this chapter, analyses were carried out to investigate the effects of the pad width (w) and thickness (T_p) on the stress distribution on the potential crack plane at the weld toe, and pad configuration (pad width- w and thickness- T_p) optimization was achieved for the stringer.

3.1 Stress Distribution Variations with Varying Pad Width

In this section, the effect of pad width (w) (Figure 3.1) on the stress distribution on the potential crack plane of a pad design stringer is investigated. The geometry shape is defined by the following parameters (Figure 3.1):

Weld throat angle (θ): 45°

Skin thickness (T_1): 5mm

Attachment thickness (T_2): 5mm

Notch root radius at the weld toe (r): 0.5mm

Weld leg length (L): 2.5mm

Overall width (W): 30mm

Overall height (H): 20mm
Stringer length (b): unit length
Pad edge profile radius (R): 2mm
Pad width (w): w=15, 10, 5 and 3 mm
Pad thickness (T_p): 2mm

In this case, the normal stress σ_{11} is the predominant stress component for crack initiation and propagation. Further, due to the fabrication nature of the stringer, the potential crack plane or the critical area on the member is located at the weld notch tip, where the highest peak stress usually occurs. So, the σ_{11} distribution along the potential crack plane under four different loading conditions (remote tension, pure tension, four points bending and three points bending, Figure 3.2a, 3.2b, 3.2c and 3.2d correspondingly) was used to study the geometrical effects. Four different configurations with four different pad widths ($w = 15\text{mm}$, 10mm , 5mm and 3mm) were analyzed to see the local effects of the dimensions on the σ_{11} stress distribution along the potential crack plane.

The loading conditions applied in the model are as follows:

- Remote tension: unit uniform distributed tension load is applied at the remote free end (Figure 3.2a). Under the remote tension load, the stress distribution on the potential crack plane was a combination of tension and bending due to the pad causing the shift of neutral axis (Figure 3.3). The stress on the potential crack plane can be found by the superposition of the stress due to tension and stress due to bending (Figure 3.4);
- Pure tension: an additional moment is introduced by the concentrated force at the free end to balance the bending moment generated by the distributed tension load on the free end, therefore only the tension loading situation can be applied on the potential crack plane (Figure 3.2b). The sketch of the stress distribution modification is given by Figure 3.5;
- Three points bending: unit uniform distributed tension load is applied at the attachment end (Figure 3.2c);
- Four points bending: unit bending moment is applied at the remote free end

(Figure 3.2d);

As the typical application in the industry, the stringer will be long enough, so the plane strain state prevails in the member (unit length is used in the model). ABAQUS (version 6.4) was used in the finite element analysis with 8-node isoparametric 2D solid elements, plane strain and reduced integration. The comparisons on several mesh models were performed. A good solution was achieved from the typical mesh shown in Figure 3.6. Due to the geometrical symmetry of the component, only one half of the component was modeled.

The result stresses were normalized by the nominal stress σ_0 on the potential crack surface cross section as follows:

- In case of remote tension and pure tension:

$$\sigma_0 = \frac{\text{skin thickness } (T_1)}{\text{skin thickness } (T_1) + \text{pad thickness } (T_p)} \times \sigma_{\text{applied}} \quad (3.1)$$

where, σ_{applied} is the stress applied at the remote end

- In case of three points bending:

$$\sigma_0 = \frac{\text{applied bending moment } (M_{\text{applied}}) \times \text{distance from neutral axis } (y)}{\text{second moment of area } (I)} \quad (3.2)$$

$$\text{where, } M_{\text{applied}} = \sigma_{\text{applied}} \times \frac{T_2}{2} \times (W - \frac{T_2}{2} - L)$$

$$y = \frac{T_1 + T_p}{2}$$

$$I = \frac{b \times (T_1 + T_p)^3}{12}, (b=1)$$

- In case of four points bending:

$$\sigma_0 = \frac{M_{\text{applied}} (M = 1) \times \text{distance from neutral axis (y)}}{\text{second moment of area (I)}} \quad (3.3)$$

$$\text{where, } y = \frac{T_1 + T_p}{2}$$

$$I = \frac{b \times (T_1 + T_p)^3}{12}, b=1$$

Table A-1 (see Appendix) and Figure 3.7a,b show the normalized σ_{11} distributions on the potential crack plane for the models with $w=15\text{mm}$, 10mm , 5mm and 3mm under remote tension (a shows the overall stress distribution, b is the close-up plot around the crack tip);

The normalized σ_{11} distribution results for stringers with $w=15\text{mm}$, 10mm , 5mm and 3mm under pure tension are given in Table A-2 and Figure 3.8a,b;

For the models with $w=15\text{mm}$, 10mm , 5mm and 3mm under three-point-bending, the normalized σ_{11} distributions can be found in Table A-3 and Figure 3.9a,b;

Last, for four-point-bending load, Table A-4 and Figure 3.10a,b give the normalized σ_{11} distribution results for the model with $w=15\text{mm}$, 10mm , 5mm and 3mm ;

3.2 Configuration Optimization by Pad Width

From the normalized σ_{11} distributions on the potential crack plane under different loading conditions (Figure 3.7, 3.8, 3.9 and 3.10), it can be shown that the pad width (w) will only affect the stress distributions at the weld toe vicinity ($x/t \leq 0.2$) to some various extents (Figure 3.7b, 3.8b, 3.9b, 3.10b). In case of remote tension, the addition of the pad on a regular T-plate will shift the neutral axis up, which will cause the bending effect with respect to the shifted neutral axis due to the remote tension load (Figure 3.3). So, the

stress distribution is actually a combination of tension and bending effect. The stress distribution data (Table A-1) and figures (Figure 3.7) in this case show that $w = 5\text{mm}$ is a critical pad width for this particular configuration. By increasing the pad width (w) from 5mm to 15mm, the normalized stress at the weld toe only changed about 1.7% (normalized stress at weld toe changed from 0.1572 to 0.1599 respectively, table A-1, Figure 3.7b). So, any further increment of pad width (w) above 5mm will not result in any significant change of the stress distribution on the cross section. By decreasing the pad width (w) from 5mm to 3mm, the stress at the weld toe can be decreased by 30% (normalized stress at weld toe changed from 0.15 to 0.10, Table A-1, Figure 3.17b). The same pad width variation effect can also be found in the pure tension load case (Figure 3.8): the weld toe stress changed less than 1% when pad width (w) is modified from 5mm to 15mm (the weld toe normalized stress changed from 1.777 to 1.789 respectively, Table A-2, Figure 3.8b); however, as the pad width is changed from 5mm to 3mm, the stress at weld toe shows 15% differences (the normalized stress at weld toe changed from 1.777 to 1.524 respectively, Table A-2, Figure 3.8b)

In case of a load-carrying weld (three-point bending) and non-load-carrying weld (four-point bending), the results gave quite the same stress distribution profile after normalization (at the weld toe, the difference is only about 2%). It means the effect of bending can be characterized by either three-point-bending (Figure 3.9) or four-point-bending (Figure 3.10). They will result in the same normalized stress state. The critical pad width ($w=5\text{mm}$) effect can also be observed in the bending case: By increasing the pad width (w) from 5mm to 15mm, the normalized stress at the weld toe only changed about 1% (the normalized stress at the weld toe changed from 1.9444 to 1.9553 in three-point bending, Table A-3, Figure 3.9b, from 1.8901 to 1.9010 in four-point-bending, Table A-4, Figure 3.10b). So, this will not bring any major change to the stress distribution by increasing the pad width above 5mm. Decreasing the pad width from 5mm to 3mm can reduce the weld toe stress by about 13% (the normalized stress at the weld toe changed from 1.9 to 1.6, Table A-3, Figure 3.9b and Table A-4, Figure 3.10b)

Therefore, for the stringer dimensions chosen above, 5mm pad width can represent

a whole class of stringers with a pad width (w) more than 5mm, and the critical stress state is at the weld toe which is the potential crack plane position. For those stringers whose pad width (w) are less than 5mm, since the stress state at the weld toe is significantly lowered down, the critical stress region on the stringer is no longer at the weld toe, but moves to the region on the skin beside the pad (Figure 3.11). When the potential crack plane is across the skin instead of the pad, the fatigue strength of the stringer would be the same as the regular T-plate. This will lose the whole benefit of the pad design mentioned above. From the point of view of industrial application, the stringer with 5mm pad width will give the most material saving and higher fatigue strength structure. So, the stringer with 5mm pad width is the optimized design for the dimensions listed above, and it will be used for the following analysis.

3.3 Stress Distribution Variations with Varying Pad Thickness

In this section, the pad thickness (T_p) was analyzed by following the same methodology as mentioned above. The geometry shape of the models is defined by the following parameters (Figure 3.1):

Weld throat angle (θ): 45°

Skin thickness (T_1): 5mm

Attachment thickness (T_2): 5mm

Notch root radius at the weld toe (r): 0.5mm

Weld leg length (L): 2.5mm

Overall width (W): 30mm

Overall height (H): 20mm

Stiffener length (b): unit length

Pad width (w): 5mm

Pad edge profile radius (R): $R=0.5, 1.0, 2.0$ and 2.5 mm

Pad thickness (T_p): $T_p=0.5, 1.0, 2.0$ and 2.5 mm

The models in different pad thickness ($T_p=0.5\text{mm}$, 1.0mm , 2.0mm and 2.5mm) were applied under the remote tension, pure tension or bending load conditions. From the previous analysis, the results showed the three points bending and four points bending gave the same normalized stress state. So in this section, only the four points bending was applied. And the same nominal stress σ_0 (Eq.(3.1) and Eq.(3.3)) are used.

Result:

First, the σ_{11} distributions for the models with $T_p=0.5\text{mm}$, 1.0mm , 2.0mm and 2.5mm under remote tension are shown in Table A-5 and Figure 3.12;

Second, Table A-6 and Figure 3.13 give the σ_{11} distributions for the models with $T_p=0.5\text{mm}$, 1.0mm , 2.0mm and 2.5mm under pure tension;

Last, for the models with $T_p=0.5\text{mm}$, 1.0mm , 2.0mm and 2.5mm under four-point-bending the σ_{11} distribution results are given in Table A-7 and Figure 3.14;

3.4 Configuration Optimization by Pad Thickness

From the normalized σ_{11} distribution variations under different loading conditions (Figure 3.12, 3.13 and 3.14), it can be shown that the pad thickness (T_p) made two totally different contributions in two different loading conditions: when the member is subjected to bending load, the pad thickness did not cause any significant changes of normalized σ_{11} stress state by varying its thickness (the normalized weld toe stress changed from 1.77 to 1.88 when the pad thickness changed from 0.5mm to 2.5mm, the difference is only about 5%, Table A-7, Figure 3.14). But, when the stringer is subjected to remote tension, because of the bending effect (Figure 3.3), the pad thickness can dramatically change the normalized σ_{11} stress distribution on the whole cross section where the weld toe is located (Figure 3.12). By changing the pad thickness from 0.5mm to 2.5mm, the weld toe

normalized stress can be changed from 1.1935 to -0.1109 respectively; at the same time, the normalized stress at the bottom of skin will change from 1.3 to 2.0 respectively (Table A-5). The additional bending moment caused by the pad addition is the main reason of the change, which can be further verified by the normalized stress distribution on the potential crack plane under pure tension load (Figure 3.13). The maximum weld toe stress difference among the four configurations with different pad thickness is only 3% (the normalized weld toe stress is 1.74 and 1.68 when the pad thickness is 0.5mm and 2.5mm respectively, Table A-6). It indicates that when the bending effect is excluded under the pure tension conditions, the various thicknesses of the pad didn't cause any substantial differences for the stress state on the potential crack plane.

Therefore, in terms of the normalized stress state, the stress distribution is insensitive to the pad thickness (T_p) variations under the bending load conditions, and it is dependent on the pad thickness (T_p) under the remote tension loading situations. Under the remote tension load, the weld toe stress state is decreasing with increasing pad thickness, getting close to zero when the thickness is 2mm and becomes negative (compression) when the thickness is larger than 2.5mm. So, a stringer whose dimensions are defined as above but has a pad thicker than 2.5mm would have a very good crack initiation resistance.

Based on the discussions above, for the particular dimensions (section 3.1, 3.3), the optimized pad design can be achieved by using 5mm pad width and at least 2mm pad thickness to obtain higher fatigue strength of welded structure.

In order to analyze the fatigue crack propagation behavior, the stress intensity factor (SIF) will be calculated on the model with 5mm pad width and various pad thickness (larger than 2mm) using the numerical approach (Finite Element Method) and the analytical approach (Weight Function Method) in the next chapter.

(all dimensions in mm)

Figure 3.1 Geometry of the stringer

Figure 3.2a Remote tension load

Figure 3.2b Pure tension condition on the potential cracking surface modified from the remote tension condition

Figure 3.2c Three points bending load

Figure 3.2d Four points bending load

Figure 3.3 Bending effect caused by the shift of neutral axis

Figure 3.4 Stress superposition of tension and bending on potential crack plane under remote tension load

Figure 3.5 Pure tension stress state on the potential crack plane after adding balancing moment

Figure 3.6 Finite Element mesh of a stringer

Figure 3.7a Normalized σ_{11} distributions under remote tension

Figure 3.7b Close up of normalized σ_{11} distributions at the weld toe vicinity under remote tension

Figure 3.8a Normalized σ_{II} distributions under pure tension

Figure 3.8b Close up of normalized σ_{11} distributions at the weld toe vicinity under pure tension

Figure 3.9a Normalized σ_{11} distributions under three points bending

Figure 3.9b Close up of normalized σ_{11} distributions at the weld toe vicinity under three points bending

Figure 3.10a Normalized σ_{11} distributions under four points bending

Figure 3.10b Close up of σ_{11} distributions at the weld toe vicinity
under four points bending

Figure 3.11 Critical stress plane on the stringer skin

Figure 3.12 σ_{11} distributions on the potential crack plane for the stringer with different pad thickness under remote tension

Figure 3.13 σ_{11} distributions on the potential crack plane for the stringer with different pad thickness under pure tension

Figure 3.14 σ_{11} distributions on the potential crack plane for the stringer with different pad thickness under bending

Chapter 4

Stress Intensity Factor Calculation

It has been generally recognized that welded structures contain defects, either built in during fabrication or initiated early under service conditions. The welded structures with crack or crack-like flaws require a thorough fracture and fatigue analysis in order to assess their load carrying capacity, remaining service life or level of safety. The analyses are usually carried out by using fracture mechanics models, requiring an accurate calculation of stress intensity factors.

In Chapter 3, the stress distribution variations caused by the pad width and thickness were discussed. As it has been shown in Chapter 3, three different loading conditions (remote tension, pure tension and bending) were considered; and in the case of remote tension, the stress distribution is a combination of pure tension and bending. In order to calculate the stress intensity factor (K) in every individual cracked body configuration, the pure tension or bending load conditions will be applied. The stress intensity factors for remote tension can be obtained using the superposition method.

In this chapter, the finite element method and weight function method will be applied to calculate the stress intensity factor (K) for a single edge crack originating from the weld toe of the chosen stringer. First, the stress intensity factor for a single edge crack originating from the weld toe on a T-joint will be obtained using the finite element method and validated against the results from Niu & Glinka's weight function [Shen and Glinka, 1991]. Then, the weight function for a single edge crack originating from the weld toe on the T-joint will be derived and verified against the finite element method solutions. Last, the stress intensity factors for a single edge crack originating from the weld toe on a pad design stringer will be obtained using the finite element method and the

derived weight function, and the weight function solutions will be verified by comparing with the finite element method solutions.

4.1 Calculation of the Stress Intensity Factor for a Single Edge Crack in a T-Joint Using the Finite Element Method

In this section, two dimensional finite element analyses for a single edge crack originating from the weld toe on a T-joint (Figure 4.1) under pure tension or bending have been conducted. The weld toe radius on the T-joint is zero. The stress intensity factors (K) were determined for the edge cracks with relative depth, $a/t=0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 0.8$, where, a is the crack length and t is the plate thickness.

4.1.1 Finite Element Model

The finite element analysis was conducted using ABAQUS standard (version 6.4) [ABAQUS Manual, 2003] with 8-node isoparametric 2D solid elements, plane strain and reduced integration. The member containing a one-dimensional edge crack originating from the weld toe was modeled. In the vicinity of the crack tip, two dimensional prism elements with one mid-side node at the quarter point and coalescing nodes along one side were used (as discussed in chapter 2, section 2.3.1) to model the square root singularity at the crack tip. In order to model the vicinity of crack tip more accurately, sub-modeling technique [ABAQUS Manual, 2003] of Finite Element Method was used, so that the mesh at crack tip vicinity could be refined substantially.

The stress intensity factor (K) for Mode I was calculated from the J -integral which was calculated using the energy domain integral methodology (section 2.3.1). The analyses were made with a linear elastic material model with a Young's modulus, $E=70$ GPa and Poisson's ratio, $\nu=0.33$. The typical finite element mesh of the T-plate for $a/t=0.3$ is shown in Figure 4.2a and 4.2b.

4.1.2 Verification of the FEM Model

The general solution to the problem of a crack originating from an angular corner in a finite thickness plate (Figure 4.3) has been given by Glinka & Niu's weight function [Shen and Glinka, 1991].

The stress intensity factor can be calculated by integrating the product of weight function $m(x,a)$ and stress distribution $\sigma(x)$ on the potential crack plane:

$$K = \int_0^a \sigma(x)m(x,a)dx \quad (2.16)$$

In order to verify the finite element model, the stress intensity factor solutions for a single edge crack on the T-plate (Figure 4.1) under remote tension or bending obtained using the current finite element model were compared with the results from Niu & Glinka's weight function. The comparison is listed in Table 4.1.

The stress intensity factors (SIF) in the comparison are given in terms of the dimensionless normalized factors by the following relation:

$$Y = K/\sigma_0 \sqrt{\pi a} \quad (4.1)$$

where, K is the stress intensity factor calculated by FEM or Niu & Glinka's weight function; σ_0 is the maximum nominal stress on the potential crack face; a is the crack length.

Since the Niu & Glinka's weight function is only valid for the relative crack depths $a/t \leq 0.5$, the comparisons were made only for $a/t=0.02, 0.05, 0.1, 0.2, 0.3$ and 0.5 . The maximum difference is 7.2%.

The inspection of Table 4.1 indicates that the present finite element models were suitable for the analysis of a single edge crack originating from the weld toe of a T-joint

problem.

4.1.3 Simplification of the Finite Element Model

The finite element mesh shown in Figure 4.2 can be used to obtain the stress intensity factors for the unsymmetrical model with a single edge crack. In order to simplify the modeling process, an analysis was also conducted using FEM to study the two different modeling approaches: modeling the whole component (Figure 4.2) and modeling the half component (Figure 4.4). In other words, the analysis was conducted to see if there is a significant effect by introducing a symmetrical crack on the other half part of the component (when the geometry is symmetric, modeling the half side of the component becomes possible).

Since $a/t=0.5$ is the most severe situation, it was chosen to use this ratio to investigate the effect of a symmetrical edge crack. A comparison was made in table 4.2 between the stress intensity factor for one single edge crack and two symmetric single edge cracks.

The results have shown the difference is small enough to be negligible. Therefore it can be concluded that for this kind of geometry, modeling the half component wouldn't introduce any noticeable effect on the stress intensity factor.

4.2 Calculation of the Stress Intensity Factor for a Single Edge Crack in a T-Joint Using the Weight Function Method

As have been discussed in Chapter 2, the weight function is only dependent on geometry. Once the weight function is known for a particular geometry, it enables the stress intensity factors for a variety of loading conditions to be calculated by simple integration (2.16) of the weight function $m(x,a)$ and stress distribution $\sigma(x)$ expression.

$$K = \int_0^a \sigma(x)m(x,a)dx \quad (2.16)$$

However, the weight function developed by Niu and Glinka [Niu and Glinka, 1990] is for the problem of a crack originating from an angular corner in a finite thickness plate, and has only two terms for the approximation. Moreover, it is only valid for the relative depth $(a/t) \leq 0.5$.

To obtain more accurate and more widely applicable weight function solutions, in this section, weight function for a single crack originating from the weld toe of a T-joint subjected to mode I loading is derived following the method discussed in section 2.3.2. The method used requires two reference stress intensity factors to be known. The general weight function expression and its characteristic properties are used as the complementary information necessary for the determination of the unknown weight function parameters.

The weight function for a T-joint with a crack of relative depth (a/t) up to 0.8 originating from the weld toe has been derived by this method. The derived weight function was verified by comparing the stress intensity factors from weight function and the results from the finite element method for various loading conditions.

4.2.1 Derivation of the Weight Function for a Single Edge Crack in a T-Joint

For a single edge crack originating from the weld toe on a T-joint, and the following boundary condition, expressed in Eq.(2.24) of Chapter 2 is used:

$$\left. \frac{\partial^2 m(x,a)}{\partial x^2} \right|_{x=0} = 0 \quad (2.24)$$

This leads to the constant $M_2 = 3$, so only the parameters M_1 and M_3 needed to be determined for the weight function, Eq.(2.18), from a set of the reference data.

$$m(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a} \right)^{1/2} + M_2 \left(1 - \frac{x}{a} \right)^1 + M_3 \left(1 - \frac{x}{a} \right)^{3/2} \right] \quad (2.18)$$

In the present work, two reference SIF solutions are employed. And the weight function has been obtained after the parameters M_1 and M_3 are determined by performing the following tasks:

First, two reference loading conditions are applied on the crack front face on the T-plate: constant crack face pressure stress field (Figure 4.5a) and linearly increasing crack face pressure stress field (Figure 4.5b):

$$\text{Constant crack face pressure stress field: } \sigma_{r1}(x) = \sigma_0 \quad (4.2)$$

$$\text{Linearly increasing crack face pressure stress field: } \sigma_{r2}(x) = \sigma_0 \left(\frac{x}{a} \right) \quad (4.3)$$

where, σ_0 is the nominal stress

The stress intensity factors (K) for these two loading cases are obtained using the finite element method, and used as the reference SIF.

Second, the obtained FEM results are normalized as follows :

$$Y = \frac{K_r}{\sigma_0 \sqrt{\pi a}} \quad (4.1)$$

where, Y is the normalized stress intensity factor, K_r is the reference stress intensity factor obtained by FEM, and σ_0 is the nominal stress on the potential crack plane for each loading condition. The normalized stress intensity factors are presented in Table 4.3.

Then, the normalized stress intensity factor solutions for the constant and linear crack face pressure for $a/t=0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7$ and 0.8 are approximated by the least square fitting curves. The choice of equation is based on the engineering judgment. The difference between the prediction of fitted formula and the FEM data is less than 1%.

The polynomial function of the normalized stress intensity factor (Y_u) for the constant crack face pressure loading (Eq.(4.2)) is:

$$Y_u = \frac{K_m}{\sigma_0 \sqrt{\pi a}} = 0.8843 + 4.3274 \left(\frac{a}{t} \right) - 39.4056 \left(\frac{a}{t} \right)^2 + 284.5721 \left(\frac{a}{t} \right)^3 - 1038.1899 \left(\frac{a}{t} \right)^4 + 2116.4717 \left(\frac{a}{t} \right)^5 - 2218.4035 \left(\frac{a}{t} \right)^6 + 955.5433 \left(\frac{a}{t} \right)^7 \quad (4.4)$$

The polynomial function of the normalized stress intensity factor (Y_l) for the linearly increasing crack face pressure loading (Eq.(4.3)) is:

$$Y_l = \frac{K_{rl}}{\sigma_0 \sqrt{\pi a}} = 0.5854 + 1.8116 \left(\frac{a}{t} \right) - 16.4166 \left(\frac{a}{t} \right)^2 + 116.5429 \left(\frac{a}{t} \right)^3 - 421.5994 \left(\frac{a}{t} \right)^4 + 848.8765 \left(\frac{a}{t} \right)^5 - 876.9786 \left(\frac{a}{t} \right)^6 + 370.8612 \left(\frac{a}{t} \right)^7 \quad (4.5)$$

By substituting Equation (4.2-4.3), (2.18) and (4.4-4.5) into Equation (2.16) and using the boundary condition expressed in Eq.(2.24), a system of three simultaneous equations can be written as follows:

$$Y_u \cdot \sqrt{\pi a} = \int_0^a \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a} \right)^{1/2} + M_2 \left(1 - \frac{x}{a} \right) + M_3 \left(1 - \frac{x}{a} \right)^{3/2} \right] dx \quad (4.6)$$

$$Y_l \cdot \sqrt{\pi a} = \int_0^a \frac{2(x/a)}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a} \right)^{1/2} + M_2 \left(1 - \frac{x}{a} \right) + M_3 \left(1 - \frac{x}{a} \right)^{3/2} \right] dx \quad (4.7)$$

$$M_2 = 3 \quad (4.8)$$

Solving Eq.(4.6-4.8), the parameters of M_1, M_2 and M_3 for the weight function can be obtained:

$$\begin{aligned} M_1 &= \sqrt{2\pi}(3Y_I - Y_{II}) - 4.8 \\ M_2 &= 3 \\ M_3 &= 3\sqrt{2\pi}(Y_{II} - 2Y_I) + 1.6 \end{aligned} \quad (4.9)$$

The weight function for a single edge crack originating from the weld toe on a T-joint can now be determined directly in the form of Eq.(2.18).

4.2.2 Validations

As shown in Chapter 2, using the derived weight function $m(x,a)$ (Eq.(2.18)) and the stress distribution $\sigma(x)$ on the potential crack plane, the stress intensity factors can be obtained by integrating Eq.(2.16):

$$K = \int_0^a \sigma(x)m(x,a)dx \quad (2.16)$$

In order to check the validity of the derived weight function in section 4.2.1, the comparisons are made between the stress intensity factor solutions based on the weight function approach and the solutions from the FEM approach for a single edge crack originating from the weld toe on a T-joint.

Using the Stress Distribution on the Potential Crack Plane of a T-plate Under Remote Tension and Bending

The stress intensity factors for a single edge crack originating from the weld toe on

a T-plate (Figure 4.1) under remote tension and bending are predicted using the derived weight function $m(x,a)$ (Eq.(4.9)) and the stress distribution $\sigma(x)$ (Table A-8) on the potential crack plane at the weld toe. The solutions are normalized following Eq.(4.1):

The normalized stress intensity factor solutions were then compared with the FEM results. The comparison was summarized in Table 4.4 and illustrated in Figure 4.6. The maximum difference between the normalized stress intensity factors is 2.12% for remote tension and 1.56% for bending. The results indicate that the derived weight function can be used confidently to predict the stress intensity factors for a single edge crack originating from the weld toe of a T-joint.

4.3 Calculation of Stress Intensity Factor for Single Edge Crack on the Chosen Pad Design Stringers Using the Two Approaches

In the previous sections, two approaches of obtaining the stress intensity factors for a single edge crack on a T-joint - Finite Element Method and Weight Function Method - were derived and verified.

In this section, the finite element approach will be first applied to obtain the stress intensity factors for a single edge crack originating from the weld toe of the chosen pad design stringers. Then the derived weight function in section 4.2 will be used to calculate the stress intensity factors for the same corresponding problem. It will be demonstrated that the weight function derived for T-joint can be used for the padded stringer joints.

4.3.1 The Model Geometry of Pad Design Stringers and Applied Loading Conditions

Two different geometrical models (Figure 4.7a and 4.7b) were investigated in the

present work. The geometry of the models is defined by the following parameters:

Weld throat angle (θ): 45°

Skin thickness (T_1): 5mm

Attachment thickness (T_2): 5mm

Notch root radius at the weld toe (r): 0.5mm

Weld leg length (L): 2.5mm

Overall width (W): 30mm

Overall height (H): 20mm

Stiffener length (b): unit length

Pad width (w): 5 (mm)

Pad edge curve radius (R): 2mm (Figure 4.7a), and 4mm (Figure 4.7b)

Pad thickness (T_p): 2mm (Figure 4.7a), and 4mm (Figure 4.7b)

Thickness of potential crack plane (t): 7mm (Figure 4.7a), and 9 mm (Figure 4.7b)

Length of crack: a

Relative depth (a/t): 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7 and 0.8

From chapter 3, it has been shown that due to the effect of the pad addition, for the case of remote tension, the stress distribution on the potential crack face plane is actually the combination of pure tension and bending effect (Chapter 3, Section 3.1, Figure 3.3 and 3.4). Thus the remote tension case can be obtained by superposition of the pure tension and bending cases. To find the stress intensity factor (K) for the single edge crack subjected to the pure tension condition, the pure tension loading condition was applied (see Chapter 3, Section 3.1, Figure 3.2b and 3.5). And three point bending and four point bending load was also considered.

4.3.2 Finite Element Method Approach

Finite Element Mesh

Based on the discussions in section 4.1.3, only half of the member considered was modeled. A typical mesh at crack tip vicinity for the present analysis contains eight

contour paths for the J -integral calculation. The typical meshes of the component and crack tip vicinity were plotted in Figure 4.8a and 4.8b for $a/t=0.2$.

Stress Intensity Factor Solutions of the Finite Element Method for a Single Edge Crack Originating from the Weld Toe of the Pad Design Stringers

The stress intensity factors (K) of opening mode (Mode I) for single edge cracks in a stringer under pure tension, three-point-bending or four-point-bending loads along the cracking face with various relative depth ratios ($a/t=0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7$ and 0.8) have been determined.

The stress intensity factors have been normalized following Eq.(4.1), with σ_0 being the nominal theoretical tensile or maximum bending stress on the crack face.

The results are summarized in Table 4.5 and Figure 4.9 for the model with the Pad thickness: $T_p=2.0\text{mm}$, and thickness of the potential crack plane (t): $t=7\text{mm}$ (Figure 4.7a);

Table 4.6 and Figure 4.10 show the results for the model with the Pad thickness: $T_p=4.0\text{mm}$, and thickness of the potential crack plane (t): $t=9\text{mm}$ (Figure 4.7b);

4.3.3 Weight Function Method Approach

As have been discussed in Chapter 2, section 2.3.2, using the stress distribution $\sigma(x)$ (Table A-9, A-10) on the potential crack plane of an uncracked stringer under pure tension or bending and the derived weight function $m(x,a)$ (Eq.(4.10)), the stress intensity factors for a single edge crack originating from the weld toe of the chosen stringers with the pad thickness $T_p=2\text{mm}$ (Figure 4.7a) and 4mm (Figure 4.7b) are predicted for the various relative depth ratios ($a/t=0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7$ and 0.8)

The finite element method results (Table 4.5 and 4.6, Figure 4.9 and 4.10) show that

the differences of normalized solutions for the three points bending and four point bending load conditions are really negligible. So, only the four point bending load is chosen as the applied bending load in the weight function approach.

Comparisons of Stress Intensity Factors

The same equation, Eq.(4.1), was used to normalize the solutions. Comparisons of the results from the weight function approach against the FEM solutions are made. They are shown in Table 4.7 and Figure 4.11 for $T_p=2\text{mm}$ and Table 4.8 and Figure 4.12 for $T_p=4\text{mm}$ respectively.

For the both configurations ($T_p=2\text{mm}$ and 4mm), the very shallow crack situation ($a/t=0.02$) has a 7% difference in both pure tension and bending case. It is due to the fact that the stress gradient is very large at the vicinity of the weld toe (Figure 3.13 and 3.14, Table A-9 and A-10), the slight difference between the two stress distribution approximations would cause a larger percentage difference. At the remaining relative depth ratios (a/t), the comparison gives 2% maximum difference, and less than 1% difference for the relative crack depth ratio $a/t>0.2$.

4.4 Conclusions

The numerical and analytical approaches of calculating the stress intensity factor for a single edge crack originating from the weld toe of a member have been carried out. The stress intensity factors for the various relative crack depth ratio ($a/t=0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7$ and 0.8) were determined using the finite element method. They were then validated (for $a/t \leq 0.5$) using the Niu & Glinka's two term weight function. The results from the two methods show the similar trend for the normalized stress intensity factors when the relative depth ratio a/t gets larger. The differences were mainly caused by the limitations of Niu & Glinka's weight function, which was derived for a single edge crack

in a finite thickness plate with an angular corner (Figure 4.3), and it only has two terms for the approximation. This would limit its approximation accuracy.

Then an accurate weight function for a single edge crack originating from the weld toe of a T-plate was derived based on the universal form of weight function. The weight function was verified using the finite element method solutions under various loading conditions. The comparisons showed that the derived weight function can make the predictions quite close to the finite element solutions. The two verified approaches were then applied to the chosen pad design stringers to obtain the stress intensity factors for a single edge crack originating from the weld toe of the structures. The comparisons against the finite element solutions once again showed that the derived weight function gave quite accurate results. Through these analyses, it has been demonstrated that the derived weight function is suitable for the stress intensity factor calculations for any complex loading conditions on the chosen stringers.

	Normalized SIF $K/\sigma_0 \sqrt{\pi a}$					
	Tension			bending		
a/t	FEM	N&G	difference	FEM	N&G	difference
0.02	1.666	1.549	7.0%	1.785	1.656	7.2%
0.05	1.389	1.339	3.6%	1.409	1.343	4.7%
0.1	1.292	1.214	6.1%	1.196	1.122	6.2%
0.2	1.393	1.311	5.9%	1.103	1.032	6.4%
0.3	1.667	1.556	6.7%	1.143	1.073	6.1%
0.5	2.826	2.646	6.4%	1.499	1.398	6.8%

Table 4.1 Comparison of the normalized stress intensity factors (SIF) for a single edge crack originating from the angular corner of a T-plate using the Finite Element Method (FEM) and Niu & Glinka's (N&G) weight function

	Full Modeling	Half Modeling
Tension: $K/\sigma_0 \sqrt{\pi a}$	2.826	2.827
Bending: $K/\sigma_0 \sqrt{\pi a}$	1.499	1.500

Table 4.2 Comparisons of the normalized stress intensity factors for one single edge crack and two symmetric edge cracks originating from the corner of a T-plate

a/t	Uniform tension stress $\left(\frac{K_{ru}}{\sigma_0 \sqrt{\pi a}} \right)$	Linearly varying stress $\left(\frac{K_{lu}}{\sigma_0 \sqrt{\pi a}} \right)$
0.02	0.9572	0.6160
0.05	1.032	0.6471
0.1	1.123	0.6845
0.2	1.337	0.7691
0.3	1.645	0.8877
0.5	2.824	1.327
0.7	6.359	2.595
0.8	11.96	4.551

Table 4.3 The normalized reference stress intensity factors for uniform tension stress field and linearly varying stress field acting on the crack front face on a T-plate

	Tension			Bending		
a/t	F.E.M	Weight function	Difference	F.E.M	Weight function	Difference
0.02	1.666	1.631	2.12%	1.785	1.757	1.56%
0.05	1.389	1.395	-0.40%	1.409	1.422	-0.93%
0.1	1.292	1.288	0.29%	1.196	1.191	0.44%
0.2	1.393	1.389	0.26%	1.103	1.100	0.26%
0.3	1.667	1.665	0.13%	1.143	1.140	0.30%
0.5	2.827	2.816	0.38%	1.499	1.484	1.06%
0.7	6.356	6.381	-0.39%	2.725	2.696	1.05%
0.8	11.961	12.073	-0.93%	4.677	4.621	1.20%

Table 4.4 The normalized stress intensity factor results comparison of the derived weight function approach and the finite element method approach for a single edge crack originating from the weld toe of a T-plate under tension and bending

Normalized K for model with $T_p=2\text{mm}$, $t=7\text{mm}$			
a/t	Pure Tension	3-point-bending	4-point-bending
	$K/\sigma_0 \sqrt{\pi a}$	$K/\sigma_0 \sqrt{\pi a}$	$K/\sigma_0 \sqrt{\pi a}$
0.02	1.536	1.651	1.605
0.05	1.350	1.389	1.355
0.1	1.270	1.199	1.177
0.2	1.371	1.095	1.084
0.3	1.650	1.133	1.128
0.5	2.822	1.496	1.496
0.7	6.359	2.725	2.727
0.8	11.952	4.676	4.676

Table 4.5 The normalized stress intensity factors (K) for a single edge crack of a stringer with $T_p=2\text{mm}$ and $t=7\text{mm}$ under pure tension or bending by the FEM approach

Normalized K for model with $T_p=4\text{mm}$, $t=9\text{mm}$			
a/t	Pure Tension	3-point-bending	4-point-bending
	$K/\sigma_0 \sqrt{\pi a}$	$K/\sigma_0 \sqrt{\pi a}$	$K/\sigma_0 \sqrt{\pi a}$
0.02	1.422	1.566	1.498
0.05	1.256	1.320	1.267
0.1	1.166	1.119	1.082
0.2	1.287	1.034	1.012
0.3	1.590	1.090	1.078
0.5	2.802	1.483	1.482
0.7	6.360	2.728	2.729
0.8	11.963	4.678	4.681

Table 4.6 The normalized stress intensity factors (K) for a single edge crack of a stringer with $T_p=4\text{mm}$ and $t=9\text{mm}$ under pure tension or bending by the FEM approach

	Pure tension			Bending		
a/t	F.E.M approach	Weight function approach	Difference	F.E.M approach	Weight function approach	Difference
0.02	1.536	1.418	7.66%	1.605	1.480	7.75%
0.05	1.350	1.321	2.09%	1.355	1.326	2.17%
0.1	1.270	1.251	1.48%	1.177	1.159	1.50%
0.2	1.371	1.369	0.15%	1.084	1.082	0.22%
0.3	1.650	1.650	0.00%	1.128	1.127	0.06%
0.5	2.822	2.818	0.14%	1.496	1.490	0.39%
0.7	6.359	6.362	-0.05%	2.727	2.717	0.37%
0.8	11.952	11.971	-0.16%	4.676	4.653	0.49%

Table 4.7 The normalized stress intensity factor results comparison of the derived weight function approach and the finite element method approach for a single edge crack originating from the weld toe of a stringer ($T_p=2\text{mm}$) under pure tension and bending

	Pure tension			bending		
a/t	F.E.M approach	Weight function approach	Difference	F.E.M approach	Weight function approach	Difference
0.02	1.422	1.314	7.60%	1.498	1.385	7.52%
0.05	1.256	1.227	2.32%	1.267	1.237	2.40%
0.1	1.166	1.153	1.09%	1.082	1.071	1.00%
0.2	1.287	1.285	0.14%	1.012	1.01	0.15%
0.3	1.590	1.59	0.01%	1.078	1.077	0.09%
0.5	2.802	2.811	-0.30%	1.482	1.478	0.24%
0.7	6.360	6.371	-0.17%	2.729	2.714	0.54%
0.8	11.963	11.983	-0.16%	4.681	4.654	0.59%

Table 4.8 The normalized stress intensity factor results comparison of the derived weight function approach and the finite element method approach for a single edge crack originating from the weld toe of a stringer ($T_p=4\text{mm}$) under pure tension and bending

(all dimensions in mm)

Figure 4.1 Geometry of T-plate

Figure 4.2a T-plate finite element mesh ($a/t=0.3$)

Figure 4.2b Detailed T-plate finite element mesh around the crack tip ($a/t=0.3$)

Figure 4.3 Crack in a finite thickness plate with an angular corner
[Shen and Glinka, 1991]

Figure 4.4 Symmetrical half modeling of the T-plate with edge crack

Figure 4.5a Constant crack face pressure stress field on the T-plate

Figure 4.5b Linearly increasing crack face pressure stress field on the T-plate

Figure 4.6 The normalized stress intensity factor comparison of derived weight function solutions and FEM solutions for a single edge crack on T-plate under tension and bending

Figure 4.7a Model geometry and coordinate system ($T_p=2\text{mm}$, $t=7\text{mm}$)

Figure 4.7b Model geometry and coordinate system ($T_p=4\text{mm}$, $t=9\text{mm}$)

Figure 4.8a The typical FEM mesh of the stringer ($a/t=0.2$)

Figure 4.8b The detailed FEM mesh at the vicinity of crack tip on the stringer ($a/t=0.2$)

Figure 4.9 The normalized stress intensity factors (K) for a single edge crack of a stringer with $T_p=2\text{mm}$ and $t=7\text{mm}$ under tension or bending by the FEM approach

Figure 4.10 The normalized stress intensity factors (K) for a single edge crack of a stringer with $T_p=4\text{mm}$ and $t=9\text{mm}$ under tension or bending by the FEM approach

Figure 4.11 The normalized stress intensity factor comparison of the derived weight function solutions and FEM solutions for a single edge crack originating from the weld toe of a stringer ($T_p=2\text{mm}$) under pure tension and bending

Figure 4.12 The normalized stress intensity factor comparison of the derived weight function solutions and FEM solutions for a single edge crack originating from the weld toe of a stringer ($T_p=4\text{mm}$) under pure tension and bending

Chapter 5

Fatigue Crack Propagation Analysis

In Chapter 4, the weight function was derived for a single edge crack originating from the weld toe of a stringer. Using these derived weight functions solutions, one can calculate the stress intensity factors for different loading conditions. Therefore, the fatigue crack propagation behavior can be investigated. In this chapter, fatigue crack propagation analyses of the chosen stringers are conducted to predict the crack growth behavior under various loading conditions. One of the important factors influencing structure fatigue behavior – the thickness effect is also investigated.

As has already been shown in Chapter 2, the stress intensity factor, K , is a single-term parameter which describes the stress conditions adjacent to the tip of a crack and it describes the crack growth behavior as long as the bulk of the material is elastic and plastic deformation is limited to a small region at the crack tip. It was therefore suggested by Paris et al. [Paris et al, 1961] that, since fatigue crack propagation is clearly a phenomenon which must depend on those stresses, the stress intensity concept could also be used as a basis for defining quantitative values of crack propagation rate. As a result of a reanalysis of crack propagation data obtained by several investigators they suggested that the relevant parameter was the range of stress intensity factor, ΔK , where

$$\Delta K = K_{\max} - K_{\min} \quad (2.26)$$

$K_{\max}$ and $K_{\min}$ are the values of K at the maximum and minimum stresses of the loading cycle. The use of ΔK ensures that the relationship between crack growth rate and ΔK can be regarded as a law applicable to any geometry of the same material. The crack growth

law approximates to a linear logarithmic relationship (usually referred to as the Paris law):

$$\frac{da}{dN} = C(\Delta K)^n \quad (2.25)$$

where, a : crack length

N : number of cycles

C and n are material constant, which have to be defined experimentally for any particular material.

In the present work, $da/dN = \text{m/cycle}$, $\Delta K = \text{MPa}\sqrt{\text{m}}$, the values of C and n are obtained from the reference [Carpinteri, 1994] for a typical aluminum alloy commonly used in the aerospace industry ($C = 7.54 \times 10^{-9}$ and $n = 2.7$).

5.1 Fatigue Life Prediction of a Single Edge Crack Originating from Weld Toe on a Stringer

In this section, the fatigue life of a single edge crack originating from the weld toe of a stringer is predicted using the derived weight function. Two types of loading - remote tension and four point bending - are considered.

In order to verify the fatigue strength improvement of pad design compared with the regular T-plate joint, one regular T-plate joint (Figure 5.1) and three pad design stringers with different pad thickness (Figure 5.2, $T_p = 1\text{mm}, 2\text{mm}$ and 3mm) are chosen.

5.1.1 Stress Intensity Factor Calculation

The first step to conduct the fatigue crack propagation analysis is to find the range

of stress intensity factor ΔK . The stress intensity factor K used in the analysis can be obtained by the weight function approach discussed in the previous chapters.

The stress intensity factor (K) can be calculated using the expression in terms of the weight function form $m(x,a)$ as discussed in Chapter 4,

$$K = \int_0^a \sigma(x)m(x,a)dx \quad (2.16)$$

where, $\sigma(x)$ is the stress distribution at the potential crack plane, and the weight function is:

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a} \right)^{1/2} + M_2 \left(1 - \frac{x}{a} \right)^1 + M_3 \left(1 - \frac{x}{a} \right)^{3/2} \right] \quad (2.18)$$

where, M_1, M_2 and M_3 were determined in chapter 4 with the range of $0.02 \leq a/t \leq 0.8$.

Once the crack plane stress distribution $\sigma(x)$ is known, K can be obtained from equation (2.16) directly. The potential crack plane stress distribution $\sigma(x)$ can be obtained by the finite element method as discussed in chapter 3 (the typical stress distribution on the potential crack plane can be found: Figure 3.12 for remote tension and Figure 3.14 for bending). The detailed results are presented in the Appendix (Table A-5 and A-7 respectively).

5.1.2 Fatigue Life Prediction

Using the Paris law for the fatigue crack propagation proposed by Paris et al [Paris et al, 1961], the fatigue crack growth rate da/dN can be expressed by the Eq.(2.25):

$$\frac{da}{dN} = C(\Delta K)^n \quad (2.25)$$

The range of stress intensity factor (ΔK), as discussed in Chapter 4, can be found by integrating the equation (2.16) after substituting the weight function $m(x,a)$ and the stress distribution ($\sigma_{\max}$ and $\sigma_{\min}$). In the present work, $\sigma_{\min}$, therefore $K_{\min}$ was taken as zero.

Loading Conditions:

As have been discussed in Chapter 4, the stress intensity factor depends not only on the structural crack geometry, but also on the stress level. Therefore, the effect of two different stress levels on the fatigue behavior is investigated.

First, in order to evaluate the structural fatigue life at the same stress level, the loads were applied in such a fashion that: the maximum nominal stress at the potential crack plane at the weld toe on the four models were kept at the same amplitude (100MPa in the cases considered) in both remote tension and bending conditions.

Second, considering the real applications in the industry where the loads applied at the ends of the stringer were determined by the overall structure, i.e. fuselage, once the conditions on the overall structure are fixed, the loads at the ends of stringers are also fixed. This will result in the constant loads at the remote ends. So the constant tension load 100N (stress at cross skin section: 100MPa) was applied at remote ends in the remote tension case; the constant remote bending moment $416.67 \times 10^{-3} \text{ Nm}$ (stress at cross skin section: 100MPa) was applied at the remote ends in the bending case.

Initial and Final Crack Conditions:

The initial crack length at the weld toe is 0.05mm for T-joint and stringers under bending conditions. In the case of remote tension, as shown in Chapter 3, when the stringers are subjected to remote tension, the stresses at the weld toe change from tensile to compressive as the pad thickness increases (Figure 3.12). It is well known that compressive stress wouldn't cause crack propagation. To study the fatigue crack propagation of the stringer with pad thickness=3mm in remote tension, the initial crack

length was increased to 1.3mm, so that the crack tip could experience tensile stress which is just slightly higher than zero. The remaining models under remote tension still kept the 0.05mm initial crack length. The structural final failure is assumed to happen at $a/t=0.8$.

Results:

Once the range of stress intensity factor ΔK and the initial crack length values are known, the fatigue crack propagation of a single edge crack at the weld toe on a member can be found by using the equation (2.25). The results were presented in the form of relative crack length (a/t) vs. number of fatigue life cycles.

For the T-joint and pad design stringers under the same nominal stress level on the potential crack plane, the results were presented in Figure 5.3 (remote tension) and Figure 5.4 (bending) respectively.

Figure 5.5 and 5.6 give the results for the T-joints and pad design stringers under constant remote tension and constant bending conditions respectively.

5.1.3 Discussions

For the structures with the same nominal stress at the potential crack plane, Figures 5.3 and 5.4 show the results of typical fatigue crack growth curves. For the case of remote tension (Figure 5.3), the fatigue life from the initial crack length to the final failure (defined as $a/t=0.8$, where, a is the crack length, t is the potential crack plane thickness) shows quite significant improvements due to the pad design. Under 100MPa nominal stress on the potential crack plane, the T-joint structure shows the fatigue life about 12000 cycles. Due to the stress distribution variations brought by the pad design, as discussed in Chapter 3, the stringer with pad thickness=1mm under the same nominal stress on the potential crack plane can withstand 59000 fatigue cycles to the final failure, which is almost 5 times longer. When the pad thickness is increased to 2mm, the fatigue life to failure is about 780000 cycles, which is more than 65 times of the T-joint fatigue life. The pad effect on the fatigue life is even more obvious when the pad thickness is increased to

3mm. Even though with a more than 20 times longer initial crack length, the stringer's fatigue life is about 2470000 cycles, which represents 200 times longer fatigue life.

The fatigue life improvements brought about by the pad design can be further verified by the bending case (Figure 5.4). In this case, all models have the same initial crack length (0.05mm) and the same nominal bending stress at the potential crack plane. The fatigue life is 18400 cycles for the T-joint, 18900 cycles for the pad design stringer with pad thickness =1mm, 20000 cycles for the pad design stringer with pad thickness =2mm and 22000 cycles for the pad design stringer with pad thickness =3mm. It indicates that the pad design stringer also gives higher fatigue strength than the T-joint under the applied conditions even though the difference is quite small. The difference is due to the reason that under the similar stress states on the potential crack plane of the different models, the pad provides a longer path for the crack to propagate before the final failure, which also explains why the thicker pad stringer has a longer fatigue life than the thinner one.

The pad design benefits can also be found in the results for structure under the constant loading conditions. Under the constant remote tension, the similar substantial improvements can be found (Figure 5.5), i.e. the stringer with pad thickness=3mm gives a fatigue life 800 times longer than the T-joint structure.

For the structures under the constant bending conditions, the similar trend can be found (Figure 5.6). But the improvements are much more obvious. When the pad thickness is 3mm, the structure's fatigue life is about 60000 cycles. Comparing with 3700 cycles for the T-joint structure, it indicates more than 16 times longer fatigue life. Under this loading condition, the bigger improvements are due to the facts that the pad not only provides a longer crack propagating path but also lowers down the stress level on the potential crack plane on the stringers.

The above discussion indicates that, compared with the regular T-joint, the pad design will always improve the fatigue strength for the stringer subjected to the remote tension and bending conditions by various extents. Therefore, the pad design stringers

will have higher fatigue strength than the T-plate under all the considered loading conditions. Moreover, under both the same nominal stress on the potential crack plane and the constant loading conditions, the fatigue strength will always increase with the increment of the pad. So, the fatigue strength of thicker pad stringer is higher than the one of thinner pad stringer. The improvement of fatigue strength due to the increment of the pad is much higher in the remote tension condition (Figure 5.3 and 5.5) than in the bending condition (Figure 5.4 and Figure 5.6); it is also higher in the constant loading conditions (Figure 5.5 and 5.6) than in the same nominal stress at the potential crack plane conditions (Figure 5.3 and 5.4).

5.2 Thickness Effects of the Laser Welded Stringers

In the previous section, the fatigue life improvements of different pad thickness designs for the given skin thickness were investigated. In practice, the industrial applications of the structure may face different dimensions. Therefore, the knowledge of thickness effect on the stringer's fatigue life becomes necessary for a wide range application of the welded pad design stringer.

5.2.1 Background

As discussed in Chapter 2, the thickness effect has been noted experimentally as a decrease in fatigue strength by an increase in plate thickness. Candidate explanations for the thickness effect are weld technology and residual stress, which tend to be more adverse for heavy section welds. However, according to Berge [Berge, 1984], plate thickness primarily influences the fatigue strength of welds through effects of geometry, which act independently of any other effects of plate thickness.

In the present study, fatigue cracks developing from the weld toes are considered. Statistical methods may be applied in order to simulate the effects of random variables like initial crack size and weld shape. In general, data are not available for such analyses

or at best very uncertain. The initiation problem may be circumvented by a calibration procedure. A parameter which lends itself to calibration is the size of the initial defect.

Also following the assumptions mentioned in Chapter 2:

- Welded joints of the same type in various plate thickness are geometrically similar
- Initial conditions of fatigue crack growth are independent of plate thickness (initial crack length is constant)

The stringer models set up for describing the thickness effect in the weld toe fatigue failure are geometrically similar to each other and have the same initial crack length. The constant initial crack length assumption is justified by the fact that the initial defects are produced by the last weld pass at the weld toe.

The effect of plate thickness was analyzed using a fracture mechanics approach; i.e. a crack initiation stage was neglected. The neglect of an initiation stage is believed not to imply any larger errors in the analysis, for the following reasons. In any manual weld of this type of stringer, there will be initial defects formed along the weld toe, such as undercut, slag, intrusion or microcracks. Assuming crack initiation or nucleation to be located at these defects, the local stress field governing the cyclic strains will be subject to the stress gradient in a manner similar to that for crack growth. Hence, a thickness effect should be also apparent in an initiation stage although possibly with a somewhat different functional dependency.

5.2.2 Model Geometry

In the present study, two different skin thicknesses of stringer, 5mm and 2.2mm, were investigated. Based on the skin thickness, four geometrically similar pairs of stringers were set up. In order to verify the validity of the results, a pair of geometrically similar regular T-plate was used.

The pad design stringer model shapes are defined by the following parameters (in mm) (Figure 5.7):

	Skin thickness (T_1)	Pad thickness (T_p)	Pad width (w)	Attachment thickness (T_2)	Weld leg length (L)
Pair #1	5	0.5	5	5	2.5
	2.2	0.22	2.2	2.2	1.1
Pair #2	5	1	5	5	2.5
	2.2	0.44	2.2	2.2	1.1
Pair #3	5	2	5	5	2.5
	2.2	0.88	2.2	2.2	1.1
Pair #4	5	2.5	5	5	2.5
	2.2	1.1	2.2	2.2	1.1

Table 5.1 Pad design stringer geometry

The dimensions for regular T-joints are (in mm) (Figure 5.8):

Skin thickness (T_1)	Attachment thickness (T_2)	Weld leg length (L)
5	5	2.5
2.2	2.2	1.1

Table 5.2 T-joint geometry

5.2.3 Loading Conditions

Considering the real applications in the industry, the constant loading (remote tension and constant bending) was chosen. It was applied in such a manner that it will generate axial maximum nominal stress range $\Delta\sigma = 100\text{MPa}$, 150MPa , 200MPa and 225MPa at a stress ratio $R=0$ ($R = \sigma_{\min} / \sigma_{\max}$) on the skin cross section plane at the

remote ends.

5.2.4 Calculation and Results

In the present study, the initial crack length is 0.05mm and final fracture failure happens when $a/t=0.8$. At each stress range level, the fatigue life was predicted for each geometry. The calculation process follows the exact same way as in section 5.1:

- First, the stress distribution $\sigma(x)$ on the potential crack plane at the weld toe is obtained using the finite element method
- Then, using the derived weight function $m(x, a)$ for a single edge crack at the weld toe, the range of stress intensity factor was determined by integrating Eq.(2.16).
- Last, the structure fatigue life can be determined by integrating Eq.(2.25) with the crack length at the initial and final failure conditions.

The results (Table 5.3a and 5.3b) obtained were the number of fatigue cycles N to fracture at every constant stress range for each stringer model. The thickness effects were presented by the S-N curve on the log-log scale (Figure 5.9 and 5.10). It should be noted that in the case of remote tension, it has been found, in Chapter 3, that the stress state becomes compressive at the weld toe when the pad thickness $T_p \geq 2.5mm$, which will not cause small crack to propagate. Therefore, under remote tension, with the particular initial crack (0.05mm), the fatigue life is infinite, no thickness effect can be found for the stringer pair #4 under remote tension.

5.2.5 Discussions and Conclusions

Table 5.3a and figure 5.9 shows the results for the structures under the remote constant tension conditions. As have in many investigations, the T-joint gave the regular thickness effect: the thinner T-joint structure has longer fatigue life than the thicker one. The pad design stringers show the results in two different directions. Pair #1 shows the similar trend as the T-joint structure, which is the regular thickness effect: the stringer with 0.44mm pad on 2.2mm skin has higher fatigue strength than the stringer with 0.5mm pad on 5mm skin. However, for the second pair, the structure with 1mm thick pad on

5mm skin has a slightly longer fatigue life than the geometrically similar one with 0.44mm pad on 2.2mm skin. In other words, the thicker skin structure has higher fatigue strength than the thinner one. This phenomenon is even greater on the third pair: one with 2mm pad on 5mm skin, one with 0.88mm pad on 2.2mm skin. The thicker component has 80% higher fatigue strength than the thinner one.

The main reason for this negative thickness effect is the change of stress distribution on the potential crack plane caused by the pad addition. As discussed in Chapter 3, when the structures subject to remote tension, adding pad on the skin can generate an additional bending effect in the structure. This bending effect can significantly reduce the stress concentration at the vicinity of weld toe. So, in the remote tension condition, the thicker the pad is, the bigger the bending moment, which can result in the lower tensile stress state at the weld toe. It is the substantially reduced tensile stress state that gives the structure a much longer fatigue life. When the pad becomes thick enough (thicker than 2.5mm for the remaining given dimensions) the stress state at the weld toe becomes compressive, which would not cause the initial small crack to grow.

Another effect can be found from the remote tension results (Table 5.3a and Figure 5.9) is that all the pad design stringers have higher fatigue strength than the regular T-joint. Considering the same skin thickness structures, it can also be observed that the thicker pad design stringer always has higher fatigue strength than the thinner one. The fatigue strength improvement can be as high as 260 times for the 5mm skin pad design stringers and 140 times for the 2.2mm pad design stringers.

From Table 5.3b and Figure 5.10 it may be observed that the structures subject to constant bending show quite clear the thickness effect. Between the two geometrically similar stringers all four pair structures give the similar trend: the component with thinner skin thickness always has a longer fatigue life or higher fatigue strength than the thicker skin structure. This trend is verified by the T-joint results, which present the similar trend that can be found in many sources [Berge, 1984, Gurney, 1979, Johnston, 1980]. By decreasing the skin thickness from 5mm to 2.2mm, the fatigue strength improvements for the four pair stringers are about 16% to 20%. The result is close to the T-joint's

performance: 18%.

At the same time, it is particularly interesting to note that, similar to the remote tension case, under the constant bending conditions all the pad design stringers have higher fatigue strength than the regular T-plate (Table 5.3b and Figure 5.10). And the thicker the pad becomes, the higher fatigue strength the structure is. Considering the same skin thickness, for the 5mm skin thickness group, by increasing the pad thickness from 0.5mm to 2.5mm, the fatigue strength can be improved by 5.4 times. For the 2.2mm skin thickness group, when the pad thickness is changed from 0.22mm to 1.1mm, the fatigue strength becomes 5.6 times higher.

Based on the above discussions, the following conclusions can be obtained: the pad design stringers show the regular thickness effect under the constant bending conditions. When stringer subject to the constant remote tension conditions, the thickness effect will depend on the pad thickness: for the given dimensions on the 5mm skin structures, the stringers with 1mm or thicker pad (0.44mm or thicker pad on the 2.2mm skin stringer) will show a negative thickness effect: increase in fatigue strength by an increase of plate thickness. Below that value of pad thickness, the stringer will show a normal thickness effect, although the effect is very small for the given conditions.

From the point of view of pad design, compared with the T-plate, the pad design stringer can always bring fatigue strength improvement to the structure; moreover, for the same skin stringer, the thicker the pad is designed, the bigger fatigue strength improvement it can bring.

Nominal Stress (MPa)	T-joint		Stringer Pair #1		Stringer Pair #2		Stringer Pair #3		Stringer Pair #4	
	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1
	5mm	2.2mm	5mm	2.2mm	5mm	2.2mm	5mm	2.2mm	5mm	2.2mm
	T_p	T_p	T_p	T_p	T_p	T_p	T_p	T_p	T_p	T_p
	0	0	0.5mm	0.22mm	1mm	0.44mm	2mm	0.88mm	2.5mm	1.1mm
100	41934	44047	149862	152169	517616	501168	38802777	21363596	N/A	N/A
150	14032	14739	50147	50920	173205	167702	12984236	7148736	N/A	N/A
200	6453	6778	23063	23418	79660	77129	5971685	3287825	N/A	N/A
225	4695	4932	16780	17039	57959	56117	4344857	2392134	N/A	N/A

Table 5.3a Fatigue life cycles for T-joint and stringer under constant remote tension

Nominal Stress (MPa)	T-joint		Stringer Pair #1		Stringer Pair #2		Stringer Pair #3		Stringer Pair #4	
	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1
	5mm	2.2mm	5mm	2.2mm	5mm	2.2mm	5mm	2.2mm	5mm	2.2mm
	T_p	T_p	T_p	T_p	T_p	T_p	T_p	T_p	T_p	T_p
	0	0	0.5mm	0.22mm	1mm	0.44mm	2mm	0.88mm	2.5mm	1.1mm
100	49651	58492	99825	115471	155510	184859	365482	433945	535980	647117
150	16614	19573	33404	38640	52037	61858	122298	145208	179350	216540
200	7640	9001	15361	17769	23930	28446	56241	66775	82478	99578
225	5562	6553	11183	12936	17421	20709	40944	48613	60044	72494

Table 5.3b Fatigue life cycles for T-joint and stringer under constant bending

(all dimensions in mm)

Figure 5.1 Regular T-joint with a single crack originating from the weld toe

Figure 5.2 The chosen pad design stringer

Figure 5.3 Fatigue crack propagation with the same nominal tension stress at the potential crack plane under remote tension condition

Figure 5.4 Fatigue crack propagation with the same nominal bending stress at the potential crack plane under bending condition

Figure 5.5 Fatigue crack propagation under constant remote tension condition

Figure 5.6 Fatigue crack propagation under constant bending condition

(all dimensions in mm)

Figure 5.7 Geometry of pad design stringer

Figure 5.8 Geometry of regular T-joint

Figure 5.9 S-N curve for T-joint and stringers under remote tension

Figure 5.10 S-N curve for T-joint and stringers under constant bending

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

Based on the typical T-joint structure from aerospace industry, a laser welded pad design stringer was brought up and optimized using the stress analysis on the potential crack plane at the weld toe with two-dimensional finite element analyses for the given dimensions. The optimized pad width and thickness were obtained for the structure under the remote tension and bending conditions. Through the analyses, the pad design stringer substantially reduces the stress concentration at the weld toe and improves the stress state on the potential crack plane of the stringer under various loadings.

Two-dimensional finite element analyses for a single edge crack originating from the weld toe of the T-plate under uniaxial loadings have been conducted. The stress intensity factors were obtained for the edge cracks with relative depth, $a/t=0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7$ and 0.8 , under the remote tension, three point bending and four point bending conditions. The finite element solutions were verified against the solutions from the available Niu and Glinka's weight function [Niu and Glinka, 1990]. A good agreement was achieved.

The weight function of stress intensity factor for a single edge crack originating from the weld toe of a T-plate over the geometric region, $0 \leq a/t \leq 0.8$, was derived. The reference stress intensity factor solutions and the appropriate boundary conditions on the weight function were used to derive the weight function from the universal form of weight function for one dimensional stress field [Glinka and Shen, 1991]. The weight

function was verified using the finite element solutions for a single edge crack originating from the weld toe of a T-plate under remote tension and bending. Excellent accuracy was achieved.

The method of calculating the stress intensity factors for a single edge crack originating from the weld toe of the chosen pad design stringers using both the finite element method and the weight function method was presented. The comparisons were made for the solutions from the finite element method and the weight function method for the pure tension, three point bending and four point bending conditions. Excellent agreement was obtained. The derived weight function of T-plate is therefore applicable for the cracks on stringers.

Using the weight function based stress intensity factor solutions, the crack growth behavior analyses on the chosen pad design stringers under the remote tension and bending were conducted. The results were compared with the T-joint behavior under the corresponding loading conditions. It was demonstrated that fatigue life improvements to a various extent could be achieved on the pad design stringers. The pad design stringers show higher fatigue strength than the regular T-plate under various loading conditions.

Last, the related thickness effect was analyzed for the pad design stringer under the remote tension and constant bending conditions. The thickness effects of the pad design stringers under bending are consistent with the observations on the regular T-joint structures, but show the different behavior for the pad design stringers under remote tension.

6.2 Recommendations

In this thesis, the stress intensity factors and the weight functions were considered for a single edge crack on the stringers in two-dimensional configurations. Three-dimensional analyses are needed to give more accurate solutions for the real

engineering problems.

In the present research, the pad design stringer optimization was based on one given skin and attachment thickness (5 mm). In practice, other thickness applications are being used in the industry. So more finite element analyses are required to obtain the solutions for the other skin and attachment thickness stringers, i.e. 1.8mm, 2mm, 2.2mm or 3mm.

The stress intensity factor solutions obtained in the present work are for a single edge crack on the stringer. In a real engineering problem, it is possible to have multiple cracks originating from the vicinity of weld toe on the stringer as well. Further analysis is required to find the solutions for this kind of problem.

The present research analyzed the fatigue crack propagation and thickness effect based on the assumption that the material along the potential crack plane is uniform and homogeneous. But in practice, the material production process and welding process can introduce different defects and material inhomogeneity into the material. A further analysis which could take the effects of material defect and inhomogeneity into account is required to find a more accurate fatigue prediction model. And as a natural extension to this point, more fatigue experimental data is needed to verify the accuracy of the fatigue crack propagation analysis model.

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Appendix:

In this appendix, the detail results of normalized stress (or stress) distributions are presented.

Here, σ_0 - the nominal stress at the potential crack plane, x - coordinates of the point along the potential crack plane, t – overall thickness

Table A-1 to A-4 show the normalized stress (σ_{11}/σ_0) distributions at the potential crack plane of the stringer with different pad width (w) (See section 3.1 and figure 3.1)

Table A-5 to A-7 show the normalized stress (σ_{11}/σ_0) distributions at the potential crack plane of the stringer with different pad thickness (T_p) (See section 3.1 and figure 3.1)

Table A-8 shows the stress distribution (σ_0) on the potential crack plane on a T-plate (Figure 4.1)

Table A-9 shows the stress distribution (σ_0) on the potential crack plane on a pad design stringer (Figure 4.7a)

Table A-10 shows the stress distribution (σ_0) on the potential crack plane on a pad design stringer (Figure 4.7b)

w = 15mm		w = 10mm		w = 5mm		w = 3mm	
x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0
0	0.1599	0	0.1594	0	0.1572	0	0.1089
0.0158	0.1592	0.0157	0.1584	0.0104	0.1548	0.0113	0.1174
0.0316	0.1812	0.0315	0.1837	0.0207	0.1545	0.0226	0.1266
0.0482	0.2123	0.0479	0.2124	0.0318	0.1755	0.0349	0.1468
0.0647	0.2398	0.0643	0.2397	0.0430	0.1931	0.0473	0.1664
0.0817	0.2714	0.0813	0.2712	0.0543	0.2128	0.0603	0.1913
0.0986	0.3026	0.0983	0.3024	0.0657	0.2324	0.0734	0.2160
0.1158	0.3344	0.1155	0.3342	0.0773	0.2537	0.0862	0.2419
0.1329	0.3661	0.1327	0.3660	0.0889	0.2751	0.0991	0.2676
0.1502	0.3976	0.1500	0.3975	0.1006	0.2971	0.1116	0.2933
0.1674	0.4290	0.1673	0.4290	0.1124	0.3191	0.1241	0.3190
0.1847	0.4601	0.1847	0.4601	0.1242	0.3413	0.1370	0.3459
0.2020	0.4912	0.2020	0.4912	0.1360	0.3635	0.1500	0.3729
0.2457	0.5685	0.2512	0.5778	0.1975	0.4786	0.1890	0.4548
0.3004	0.6636	0.3004	0.6634	0.2591	0.5922	0.2348	0.5505
0.3659	0.7764	0.3659	0.7760	0.3412	0.7403	0.3004	0.6830
0.4096	0.8510	0.4315	0.8877	0.4232	0.8845	0.3659	0.8083
0.4643	0.9438	0.4940	0.9938	0.4940	1.0056	0.4315	0.9267
0.5089	1.0194	0.5238	1.0441	0.5238	1.0556	0.4643	0.9833
0.5536	1.0949	0.5536	1.0944	0.5536	1.1053	0.4940	1.0334
0.6205	1.2080	0.5982	1.1699	0.5982	1.1789	0.5387	1.1060
0.6429	1.2457	0.6131	1.1950	0.6131	1.2033	0.5536	1.1299
0.6875	1.3212	0.6280	1.2202	0.6280	1.2276	0.5685	1.1532
0.7321	1.3967	0.6429	1.2454	0.6429	1.2520	0.5833	1.1765
0.7768	1.4724	0.7024	1.3461	0.7024	1.3486	0.6131	1.2222
0.8214	1.5481	0.7619	1.4470	0.7619	1.4452	0.6429	1.2673
0.8661	1.6243	0.8214	1.5483	0.8214	1.5424	0.7321	1.3994
0.9107	1.7005	0.8810	1.6500	0.8810	1.6412	0.8214	1.5307
0.9554	1.7774	0.9405	1.7523	0.9405	1.7424	0.9107	1.6668
0.9777	1.8159	0.9702	1.8038	0.9702	1.7946	0.9554	1.7402
1	1.8544	1	1.8553	1	1.8468	1	1.8138

Table A-1 Normalized σ_{11} distributions on the potential crack plane for the stringers
with different pad width under remote tension

w = 15mm		w = 10mm		w = 5mm		w = 3mm	
x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0
0	1.7893	0	1.7881	0	1.7773	0	1.5243
0.0112	1.5487	0.0157	1.4660	0.0104	1.5392	0.0113	1.3045
0.0225	1.3200	0.0315	1.2590	0.0207	1.3262	0.0226	1.1630
0.0347	1.2289	0.0479	1.1371	0.0318	1.2086	0.0349	1.0871
0.0468	1.1312	0.0643	1.0801	0.0430	1.1305	0.0473	1.0174
0.0591	1.0874	0.0813	1.0441	0.0543	1.0875	0.0603	0.9710
0.0714	1.0395	0.0983	1.0067	0.0657	1.0435	0.0734	0.9202
0.0844	1.0136	0.1155	0.9863	0.0773	1.0163	0.0862	0.8993
0.0975	0.9868	0.1327	0.9656	0.0889	0.9889	0.0991	0.8821
0.1103	0.9718	0.1500	0.9543	0.1006	0.9720	0.1116	0.8768
0.1232	0.9565	0.1673	0.9428	0.1124	0.9550	0.1241	0.8714
0.1362	0.9473	0.1847	0.9368	0.1242	0.9448	0.1370	0.8733
0.1493	0.9379	0.2020	0.9307	0.1360	0.9340	0.1500	0.8752
0.1625	0.9323	0.2184	0.9278	0.1565	0.9257	0.1630	0.8823
0.1757	0.9266	0.2348	0.9251	0.1770	0.9182	0.1760	0.8893
0.1889	0.9234	0.2512	0.9242	0.1975	0.9182	0.1890	0.8994
0.2020	0.9202	0.2676	0.9233	0.2180	0.9181	0.2020	0.9095
0.2512	0.9168	0.3004	0.9242	0.2591	0.9262	0.2348	0.9391
0.3004	0.9192	0.3331	0.9270	0.3001	0.9382	0.2676	0.9694
0.3659	0.9284	0.3987	0.9363	0.3822	0.9657	0.3332	1.0229
0.4151	0.9378	0.4479	0.9456	0.4438	0.9837	0.3823	1.0529
0.4479	0.9447	0.4792	0.9520	0.4792	0.9923	0.4151	1.0678
0.5089	0.9585	0.5387	0.9651	0.5387	1.0030	0.4792	1.0840
0.5536	0.9691	0.5833	0.9753	0.5833	1.0081	0.5238	1.0859
0.6131	0.9836	0.6429	0.9894	0.6429	1.0115	0.5833	1.0767
0.6429	0.9908	0.7024	1.0035	0.7024	1.0119	0.6131	1.0680
0.7143	1.0080	0.7619	1.0172	0.7619	1.0110	0.6429	1.0576
0.7500	1.0162	0.7917	1.0237	0.7917	1.0107	0.6875	1.0377
0.7857	1.0244	0.8214	1.0302	0.8214	1.0105	0.7321	1.0169
0.8571	1.0395	0.8810	1.0421	0.8810	1.0123	0.8214	0.9718
0.8929	1.0460	0.9107	1.0471	0.9107	1.0153	0.8661	0.9537
0.9286	1.0525	0.9405	1.0523	0.9405	1.0181	0.9107	0.9356
0.9643	1.0573	0.9702	1.0561	0.9702	1.0237	0.9554	0.9286
1	1.0621	1	1.0598	1	1.0465	1	0.9221

Table A-2 Normalized σ_{11} distributions on the potential crack plane for the stringers
with different pad width under pure tension

w = 15mm		w = 10mm		w = 5mm		w = 3mm	
x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0
0	1.9553	0	1.9538	0	1.9444	0	1.6603
0.0158	1.4798	0.0157	1.4754	0.0104	1.5707	0.0113	1.3915
0.0237	1.3572	0.0315	1.2605	0.0207	1.3606	0.0226	1.2034
0.0316	1.2684	0.0479	1.1086	0.0318	1.2250	0.0349	1.0982
0.0399	1.1774	0.0643	1.0046	0.0430	1.1297	0.0473	0.9933
0.0482	1.1163	0.0813	0.9015	0.0543	1.0428	0.0603	0.9177
0.0564	1.0538	0.0983	0.8272	0.0657	0.9651	0.0734	0.8420
0.0647	1.0013	0.1155	0.7633	0.0773	0.9054	0.0862	0.7887
0.0690	0.9507	0.1327	0.6990	0.0889	0.8455	0.0991	0.7348
0.0732	0.9301	0.1500	0.6470	0.1006	0.7975	0.1116	0.6955
0.0817	0.9089	0.1673	0.5948	0.1124	0.7494	0.1241	0.6561
0.0901	0.8671	0.1847	0.5501	0.1242	0.7095	0.1370	0.6240
0.1072	0.7936	0.2184	0.4674	0.1565	0.6114	0.1630	0.5657
0.1243	0.7299	0.2512	0.3946	0.1975	0.5082	0.1890	0.5170
0.1415	0.6721	0.2840	0.3268	0.2386	0.4206	0.2184	0.4693
0.1674	0.5944	0.3167	0.2627	0.2796	0.3421	0.2512	0.4206
0.1847	0.5498	0.3495	0.2013	0.3206	0.2690	0.2840	0.3737
0.2020	0.5050	0.3823	0.1420	0.3617	0.1989	0.3168	0.3268
0.2457	0.4054	0.4315	0.0557	0.4232	0.0958	0.3659	0.2545
0.3113	0.2736	0.4792	-0.0255	0.4792	0.0019	0.4151	0.1772
0.3550	0.1919	0.5089	-0.0756	0.5089	-0.0486	0.4479	0.1227
0.4206	0.0752	0.5536	-0.1501	0.5536	-0.1254	0.4940	0.0418
0.4643	0.0002	0.5833	-0.1994	0.5833	-0.1774	0.5238	-0.0133
0.5089	-0.0750	0.6131	-0.2487	0.6131	-0.2300	0.5536	-0.0703
0.5536	-0.1494	0.6429	-0.2980	0.6429	-0.2832	0.5833	-0.1292
0.5982	-0.2235	0.7024	-0.3971	0.7024	-0.3913	0.6131	-0.1898
0.6429	-0.2974	0.7619	-0.4973	0.7619	-0.5013	0.6429	-0.2514
0.6875	-0.3720	0.7917	-0.5484	0.7917	-0.5569	0.6875	-0.3471
0.7321	-0.4465	0.8214	-0.5995	0.8214	-0.6125	0.7321	-0.4433
0.7768	-0.5229	0.8512	-0.6519	0.8512	-0.6685	0.7768	-0.5419
0.8214	-0.5993	0.8810	-0.7044	0.8810	-0.7245	0.8214	-0.6405
0.8661	-0.6788	0.9107	-0.7587	0.9107	-0.7805	0.8661	-0.7386
0.9107	-0.7583	0.9405	-0.8131	0.9405	-0.8366	0.9107	-0.8368
0.9554	-0.8429	0.9702	-0.8700	0.9702	-0.8927	0.9554	-0.9315
1	-0.9277	1	-0.9271	1	-0.9386	1	-1.0259

Table A-3 Normalized σ_{11} distributions on the potential crack plane for the stringers
with different pad width under three-point-bending

w = 15mm		w = 10mm		w = 5mm		w = 3mm	
x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0
0	1.9010	0	1.9002	0	1.8901	0	1.6518
0.0158	1.5260	0.0157	1.5255	0.0104	1.6151	0.0113	1.3849
0.0237	1.3289	0.0315	1.2545	0.0207	1.3670	0.0226	1.2091
0.0316	1.2518	0.0479	1.0788	0.0318	1.2453	0.0349	1.0970
0.0399	1.1539	0.0643	0.9805	0.0430	1.0937	0.0473	0.9928
0.0482	1.0761	0.0813	0.9017	0.0543	1.0204	0.0603	0.9097
0.0564	1.0364	0.0983	0.8217	0.0657	0.9463	0.0734	0.8416
0.0647	0.9768	0.1155	0.7608	0.0773	0.8897	0.0862	0.7670
0.0732	0.9378	0.1327	0.6995	0.0889	0.8328	0.0991	0.7169
0.0817	0.8988	0.1500	0.6496	0.1006	0.7874	0.1116	0.6808
0.0901	0.8591	0.1673	0.5995	0.1124	0.7418	0.1241	0.6445
0.1072	0.7893	0.2020	0.5127	0.1360	0.6655	0.1500	0.5860
0.1243	0.7287	0.2348	0.4386	0.1770	0.5573	0.1760	0.5388
0.1415	0.6736	0.2676	0.3696	0.2180	0.4683	0.2020	0.4986
0.1674	0.5990	0.3167	0.2729	0.2796	0.3532	0.2512	0.4320
0.1847	0.5559	0.3495	0.2113	0.3206	0.2819	0.2840	0.3889
0.2020	0.5125	0.3823	0.1513	0.3617	0.2126	0.3168	0.3449
0.2457	0.4150	0.4315	0.0636	0.4232	0.1094	0.3659	0.2753
0.3113	0.2842	0.4792	-0.0194	0.4792	0.0140	0.4151	0.1987
0.3550	0.2021	0.5089	-0.0707	0.5089	-0.0378	0.4479	0.1438
0.3987	0.1229	0.5387	-0.1216	0.5387	-0.0904	0.4792	0.0883
0.4206	0.0839	0.5536	-0.1470	0.5536	-0.1169	0.4940	0.0612
0.4643	0.0075	0.5833	-0.1976	0.5833	-0.1708	0.5238	0.0043
0.5089	-0.0695	0.6131	-0.2481	0.6131	-0.2254	0.5536	-0.0550
0.5536	-0.1459	0.6429	-0.2986	0.6429	-0.2806	0.5833	-0.1165
0.5982	-0.2219	0.7024	-0.3997	0.7024	-0.3929	0.6131	-0.1799
0.6205	-0.2598	0.7321	-0.4506	0.7321	-0.4497	0.6280	-0.2124
0.6875	-0.3739	0.7917	-0.5530	0.7917	-0.5636	0.6875	-0.3451
0.7321	-0.4500	0.8214	-0.6044	0.8214	-0.6206	0.7321	-0.4463
0.7768	-0.5273	0.8512	-0.6568	0.8512	-0.6772	0.7768	-0.5492
0.8214	-0.6045	0.8810	-0.7092	0.8810	-0.7338	0.8214	-0.6520
0.8661	-0.6840	0.9107	-0.7630	0.9107	-0.7893	0.8661	-0.7525
0.9107	-0.7634	0.9405	-0.8167	0.9405	-0.8450	0.9107	-0.8531
0.9554	-0.8466	0.9702	-0.8724	0.9702	-0.8994	0.9554	-0.9469
1	-0.9299	1	-0.9281	1	-0.9337	1	-1.0404

Table A-4 Normalized σ_{11} distributions on the potential crack plane for the stringers
with different pad width under four-point-bending

$T_p=0.5\text{mm}$		$T_p=1.0\text{mm}$		$T_p=2.0\text{mm}$		$T_p=2.5\text{mm}$	
x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0
0	1.1935	0	0.8075	0	0.1572	0	-0.1109
0.0052	1.1277	0.0066	0.7608	0.0055	0.1560	0.0067	-0.0880
0.0136	1.0619	0.0132	0.7141	0.0110	0.1548	0.0133	-0.0652
0.0223	0.9848	0.0199	0.6711	0.0219	0.1543	0.0200	-0.0427
0.0344	0.9082	0.0265	0.6282	0.0341	0.1765	0.0266	-0.0202
0.0470	0.8568	0.0339	0.6179	0.0463	0.1973	0.0332	0.0012
0.0593	0.8229	0.0413	0.6077	0.0590	0.2218	0.0399	0.0226
0.0715	0.8009	0.0562	0.5820	0.0717	0.2459	0.0465	0.0443
0.0836	0.7863	0.0713	0.5825	0.0847	0.2708	0.0531	0.0660
0.0958	0.7768	0.0864	0.5814	0.0976	0.2956	0.0675	0.1048
0.1110	0.7699	0.1021	0.5895	0.1105	0.3200	0.0819	0.1437
0.1231	0.7674	0.1179	0.5969	0.1233	0.3443	0.0974	0.1819
0.1353	0.7669	0.1436	0.6175	0.1362	0.3685	0.1130	0.2201
0.1505	0.7683	0.1693	0.6384	0.1491	0.3926	0.1286	0.2561
0.1748	0.7742	0.2207	0.6885	0.1755	0.4416	0.1594	0.3257
0.1933	0.7785	0.2464	0.7153	0.1888	0.4659	0.1746	0.3592
0.2026	0.7819	0.2721	0.7420	0.2020	0.4903	0.1898	0.3920
0.2489	0.8040	0.3364	0.8109	0.2512	0.5794	0.2357	0.4896
0.3045	0.8360	0.3750	0.8526	0.3004	0.6673	0.2788	0.5790
0.3323	0.8533	0.4010	0.8809	0.3332	0.7253	0.3003	0.6230
0.3786	0.8835	0.4401	0.9232	0.3823	0.8112	0.3650	0.7537
0.4157	0.9084	0.4661	0.9514	0.4151	0.8679	0.4081	0.8395
0.4621	0.9400	0.5052	0.9936	0.4643	0.9521	0.4727	0.9664
0.5084	0.9721	0.5443	1.0356	0.5089	1.0275	0.5374	1.0916
0.5568	1.0043	0.5833	1.0775	0.5536	1.1022	0.6020	1.2150
0.6136	1.0438	0.6875	1.1883	0.6131	1.2008	0.6944	1.3900
0.6477	1.0676	0.7396	1.2436	0.6429	1.2498	0.7500	1.4949
0.7159	1.1149	0.7917	1.2989	0.7143	1.3670	0.8056	1.6003
0.7500	1.1463	0.8177	1.3267	0.7500	1.4257	0.8333	1.6532
0.7841	1.1697	0.8438	1.3545	0.7857	1.4844	0.8611	1.7068
0.8523	1.2162	0.8958	1.4103	0.8571	1.6031	0.9167	1.8150
0.8977	1.2468	0.9219	1.4385	0.8929	1.6637	0.9306	1.8423
0.9659	1.2918	0.9740	1.4951	0.9643	1.7869	0.9722	1.9256
1	1.3138	1	1.5234	1	1.8495	1	1.9816

Table A-5 Normalized σ_{11} distributions on the potential crack plane for the stringers
with different pad thickness under remote tension

$T_p=0.5\text{mm}$		$T_p=1.0\text{mm}$		$T_p=2.0\text{mm}$		$T_p=2.5\text{mm}$	
x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0
0	1.6774	0	1.7184	0	1.6946	0	1.7403
0.0136	1.4737	0.0132	1.4912	0.0120	1.4218	0.0133	1.4770
0.0253	1.3244	0.0265	1.2785	0.0240	1.3489	0.0266	1.2391
0.0406	1.1962	0.0413	1.1894	0.0479	1.0582	0.0399	1.1595
0.0563	1.1111	0.0562	1.0902	0.0605	1.0431	0.0531	1.0584
0.0715	1.0564	0.0713	1.0499	0.0731	1.0280	0.0675	1.0178
0.0867	1.0178	0.0864	1.0065	0.0856	0.9906	0.0819	0.9744
0.1019	0.9901	0.1021	0.9836	0.0982	0.9532	0.0974	0.9521
0.1170	0.9695	0.1179	0.9592	0.1240	0.9385	0.1130	0.9294
0.1444	0.9446	0.1436	0.9415	0.1498	0.9161	0.1443	0.9095
0.1687	0.9311	0.1693	0.9249	0.1759	0.9131	0.1594	0.9065
0.1933	0.9202	0.1950	0.9203	0.1890	0.9106	0.1898	0.9041
0.2211	0.9144	0.2207	0.9152	0.2020	0.9081	0.2204	0.9081
0.2489	0.9122	0.2464	0.9159	0.2348	0.9130	0.2357	0.9113
0.2767	0.9123	0.2721	0.9163	0.2512	0.9155	0.2788	0.9253
0.2952	0.9134	0.2979	0.9198	0.2676	0.9179	0.3003	0.9336
0.3230	0.9162	0.3236	0.9231	0.2840	0.9227	0.3219	0.9420
0.3508	0.9201	0.3493	0.9281	0.3004	0.9274	0.3650	0.9590
0.3786	0.9248	0.3750	0.9330	0.3332	0.9367	0.3865	0.9669
0.4065	0.9300	0.4010	0.9387	0.3659	0.9469	0.4081	0.9747
0.4250	0.9338	0.4271	0.9445	0.3987	0.9570	0.4296	0.9815
0.4528	0.9398	0.4531	0.9505	0.4315	0.9662	0.4512	0.9882
0.4806	0.9460	0.4792	0.9565	0.4643	0.9753	0.4727	0.9936
0.5084	0.9525	0.5052	0.9625	0.4978	0.9826	0.4943	0.9990
0.5568	0.9625	0.5573	0.9744	0.5647	0.9951	0.5589	1.0093
0.5795	0.9679	0.5833	0.9803	0.5982	1.0003	0.5805	1.0117
0.6364	0.9815	0.6354	0.9915	0.6317	1.0037	0.6236	1.0141
0.6818	0.9923	0.6875	1.0025	0.6652	1.0071	0.6944	1.0138
0.7386	1.0133	0.7396	1.0129	0.6987	1.0094	0.7222	1.0131
0.7955	1.0258	0.7917	1.0232	0.7321	1.0115	0.8056	1.0105
0.8409	1.0353	0.8438	1.0331	0.7991	1.0166	0.8333	1.0101
0.8977	1.0462	0.8958	1.0430	0.8661	1.0215	0.9167	1.0158
0.9432	1.0539	0.9479	1.0523	0.9330	1.0334	0.9444	1.0194
1	1.0618	1	1.0615	1	1.0458	1	1.0329

Table A-6 Normalized σ_{11} distributions on the potential crack plane for the stringers with different pad thickness under pure tension

$T_p=0.5\text{mm}$		$T_p=1.0\text{mm}$		$T_p=2.0\text{mm}$		$T_p=2.5\text{mm}$	
x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0	x/t	σ_{11}/σ_0
0	1.7743	0	1.8218	0	1.8901	0	1.8817
0.0108	1.5590	0.0132	1.5541	0.0110	1.6138	0.0067	1.7120
0.0223	1.3701	0.0265	1.3007	0.0219	1.3518	0.0133	1.5422
0.0344	1.2206	0.0339	1.2321	0.0341	1.2147	0.0266	1.2593
0.0470	1.1025	0.0413	1.1634	0.0463	1.0683	0.0332	1.1981
0.0593	1.0111	0.0562	1.0163	0.0590	0.9855	0.0399	1.1369
0.0715	0.9369	0.0713	0.9348	0.0717	0.8992	0.0465	1.0646
0.0836	0.8739	0.0864	0.8502	0.0847	0.8400	0.0531	0.9924
0.0988	0.8067	0.1021	0.7883	0.0976	0.7800	0.0675	0.9130
0.1110	0.7594	0.1179	0.7246	0.1105	0.7349	0.0747	0.8719
0.1231	0.7165	0.1307	0.6864	0.1233	0.6894	0.0819	0.8307
0.1353	0.6771	0.1436	0.6481	0.1362	0.6519	0.0974	0.7702
0.1505	0.6317	0.1693	0.5729	0.1491	0.6143	0.1130	0.7093
0.1626	0.5979	0.1757	0.5581	0.1623	0.5813	0.1286	0.6634
0.1748	0.5658	0.1821	0.5432	0.1755	0.5482	0.1364	0.6404
0.2026	0.4983	0.2079	0.4835	0.2020	0.4888	0.1594	0.5808
0.2304	0.4358	0.2464	0.4012	0.2348	0.4223	0.1898	0.5121
0.2674	0.3589	0.2721	0.3488	0.2676	0.3609	0.2204	0.4509
0.3045	0.2867	0.2979	0.3006	0.3004	0.3026	0.2573	0.3839
0.3323	0.2348	0.3236	0.2524	0.3332	0.2463	0.3003	0.3107
0.3601	0.1843	0.3621	0.1837	0.3659	0.1911	0.3434	0.2401
0.3972	0.1186	0.4010	0.1157	0.3987	0.1363	0.3865	0.1702
0.4435	0.0384	0.4531	0.0264	0.4479	0.0538	0.4404	0.0817
0.4713	-0.0091	0.4792	-0.0180	0.4792	0.0010	0.4727	0.0272
0.5084	-0.0718	0.5052	-0.0621	0.5089	-0.0499	0.5158	-0.0471
0.5568	-0.1532	0.5573	-0.1502	0.5536	-0.1271	0.5805	-0.1623
0.6136	-0.2484	0.6354	-0.2829	0.6131	-0.2324	0.6667	-0.3230
0.6477	-0.3055	0.6875	-0.3716	0.6429	-0.2857	0.7222	-0.4293
0.7159	-0.4203	0.7396	-0.4614	0.7143	-0.4157	0.7778	-0.5365
0.7500	-0.4782	0.7656	-0.5064	0.7500	-0.4814	0.8056	-0.5898
0.8295	-0.6151	0.8177	-0.5971	0.8214	-0.6128	0.8611	-0.6956
0.8977	-0.7355	0.8958	-0.7344	0.8929	-0.7434	0.9167	-0.7992
0.9659	-0.8599	0.9479	-0.8290	0.9643	-0.8723	0.9722	-0.8995
1	-0.9242	1	-0.9237	1	-0.9363	1	-0.9487

Table A-7 Normalized σ_{11} distributions on the potential crack plane for the stringers with different pad thickness under bending

x (mm)	Remote Tension ($\sigma_0 = 100\text{MPa}$) (MPa)	Bending ($\sigma_0 = 100\text{MPa}$) (MPa)
0.000	412.60	463.72
0.009	319.30	357.11
0.029	223.87	246.91
0.040	195.06	213.50
0.062	167.90	181.43
0.073	158.51	170.26
0.096	145.96	154.94
0.130	133.74	139.51
0.175	124.14	126.67
0.239	115.56	114.30
0.279	111.77	108.44
0.317	108.91	103.76
0.353	106.62	99.81
0.416	103.45	93.93
0.466	101.49	89.98
0.539	99.18	84.84
0.612	97.36	80.30
0.686	95.94	76.25
0.710	95.55	75.05
0.781	94.53	71.61
0.919	93.07	65.60
1.138	91.68	57.30
1.531	90.84	44.64
2.013	91.20	31.13
2.538	92.40	17.60
3.063	94.01	4.68
3.500	95.51	-5.86
4.025	97.40	-18.37
4.550	99.32	-30.86
5.075	101.20	-43.41
5.600	103.00	-56.08
6.125	104.64	-69.03
6.650	106.06	-82.31
7.000	106.80	-91.50

Table A-8 σ_{11} stress distributions on the potential crack plane at the weld toe of the
T-plate under remote tension and bending

x (mm)	Bending ($\sigma_0 = 100\text{MPa}$) (MPa)	Pure Tension ($\sigma_0 = 100\text{MPa}$) (MPa)
0.000	185.86	175.09
0.037	172.20	162.69
0.056	165.42	156.83
0.074	158.47	150.80
0.093	152.69	146.01
0.151	137.06	133.34
0.208	125.19	124.35
0.419	97.36	105.86
0.610	82.56	98.37
0.801	71.93	94.44
1.012	62.75	92.21
1.204	55.76	91.28
1.414	48.99	90.99
1.618	43.06	91.19
1.822	37.55	91.71
2.026	32.31	92.42
2.230	27.25	93.25
2.434	22.30	94.14
2.638	17.41	95.04
2.842	12.53	95.91
3.046	7.65	96.74
3.250	2.73	97.50
3.438	-1.82	98.14
3.625	-6.43	98.71
3.813	-11.08	99.20
4.000	-15.79	99.64
4.609	-31.40	100.64
5.031	-42.42	101.09
6.063	-69.54	102.25
7.000	-93.65	104.73

Table A-9 σ_{11} stress distributions on the potential crack plane at the weld toe of the stringer ($T_p = 2\text{mm}$) under pure tension and bending

x (mm)	Bending ($\sigma_0 = 100\text{MPa}$) (MPa)	Pure Tension ($\sigma_0 = 100\text{MPa}$) (MPa)
0.000	187.54	175.16
0.030	175.28	163.90
0.063	162.40	152.48
0.098	151.13	142.78
0.169	132.74	127.56
0.208	125.09	121.51
0.404	99.25	102.85
0.605	84.23	94.10
0.800	74.46	89.83
1.011	66.61	87.62
1.415	55.78	87.04
1.823	47.72	88.96
2.212	41.29	91.88
2.601	35.37	95.22
3.010	29.25	98.70
3.252	25.53	100.62
3.434	22.67	101.94
3.615	19.70	103.14
3.858	15.58	104.52
4.039	12.37	105.40
4.221	9.03	106.09
4.402	5.57	106.65
4.645	0.76	107.11
4.887	-4.27	107.29
5.008	-6.86	107.27
5.500	-17.91	106.51
6.000	-29.82	104.80
6.500	-42.20	102.45
7.000	-54.76	99.87
7.500	-67.20	97.51
8.000	-79.17	95.85
8.500	-90.35	95.38
9.000	-100.44	96.55

Table A-10 σ_{11} stress distributions on the potential crack plane at the weld toe of the stringer ($T_p = 4\text{mm}$) under pure tension and bending