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THE INTERFERENCE, DIFFRACTION AND
SCATTERING OF ACOUSTIC WAVES IN
A TURBULENT ATMOSPHERE

by
GILLES DAIGLE

A thesis submitted to the faculty of graduate studies in
partial fulfillment of the requirements for the degree of
Doctor of Philosophy.

Carleton University

Ottawa Ontario

1981

The undersigned recommend to the Faculty of Graduate Studies acceptance of the thesis "THE INTERFERENCE, DIFFRACTION AND SCATTERING OF ACOUSTIC WAVES IN A TURBULENT ATMOSPHERE", submitted by GILLES DAIGLE, M.Sc., in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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ABSTRACT

The existing theory to predict the interference of sound waves above a hard boundary allows for partial coherence between direct and ground-reflected sound but is limited by the need for one adjustable parameter. The theory is reexamined to include, in addition to the transverse autocorrelation, the longitudinal autocorrelation of the interfering waves but, when recompared with previous measurements over asphalt, is still limited by the need for the same adjustable parameter. The theory is further extended to include propagation over ground of finite acoustic impedance for both short and longer distances of propagation where, in addition to phase covariance, covariance of sound amplitude is important while phase-amplitude covariance can always be neglected. The new theory provides an explanation for measurements of jet noise over grass where the sound pressure levels are up to 10 dB higher than predicted by coherent acoustic theory.

Measurements of traffic noise attenuated by a barrier alongside a highway are described. They show two characteristic discrepancies when compared with classical diffraction theory. To investigate these discrepancies an idealised experiment was performed using a point source and a model barrier erected on a flat asphalt surface. The sound pressure levels measured behind the barrier are, when compared with two formulations of classical diffraction theory, up to 10 dB higher than predicted. An empirical correction to the classical line-integral theory, as well as an alternative diffraction theory, show better agreement with the measured results but does not resolve the other discrepancy in sound levels that occurs mainly at the higher frequencies. A calculation is described to estimate the amount of sound energy scattered by atmospheric turbulence above the barrier edge. The sound pressure levels calculated from the logarithmic sum of the scattered energy and the energy from diffraction at the barrier edge show better agreement with the results than just diffraction theory alone, especially at the larger distances.

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INTRODUCTION

Noise is now recognized as one of the major pollutants of industrialized societies. The control and reduction of noise in the community depend upon the understanding of how sound propagates outdoors, and in particular near the earth's surface. The principal mechanisms determining the sound field a few meters above the ground aside from geometrical spreading may include molecular absorption, the finite impedance of the ground, diffraction, atmospheric turbulence, and refraction by wind and temperature gradients. Each of these mechanisms has received some attention in recent years. However many situations occur where more than one mechanism interact to determine the sound field. These situations have received less attention and are thus not well understood. Two typical situations of this type which are of practical importance are studied in this thesis.

The sound field above a plane ground surface is the result of the interference between a direct wave and a wave reflected from the ground. It is known that atmospheric turbulence influences the propagation of acoustic waves in free space, however, its effects on the interfering sound field a few meters above the ground are still not well understood. Some earlier work has shown how the interference phenomenon is sensitive to atmospheric turbulence. A more recent study by the author (M.Sc. thesis "The Effect of Atmospheric Turbulence

on the Interference of Acoustic Waves Due to the Presence of a Boundary" submitted to the Physics Dept. of Carleton University), which removed some of the limitations of the earlier work by extending the theory and making more complete measurements, has shown the importance of the partial coherence between the direct and reflected sound. However the existing theory is still limited by the need for an adjustable parameter and is only valid for sound fields above a hard boundary and short distances of propagation. Chapter 1 is a summary of the work by previous authors as well as the author's previous extension. Chapter 2 is the author's new extension of the existing theory in three respects. The partial coherence between the direct sound and sound reflected from the ground is reexamined to include, in addition to the transverse covariance, the longitudinal covariance between the interfering waves. The theory above hard ground is further extended to include the interference from reflection from a finite impedance ground in a turbulent atmosphere. Finally the new theory now also considers both short and longer distances of propagation. Chapter 3 is a review of the relevant properties of "line-of-sight" propagation in a turbulent free space. Chapter 4 then compares the new theory described in Chapter 2 with previous measurements above hard ground by the author and measurements above soft ground by other authors.

In recent years the use of noise barriers has become a common measure of environmental protection by intercepting the line-of-sight from source to receiver. Thus, there has

been a relatively large number of studies on the diffraction of acoustic waves at the edge of simple barrier shapes. Some of these studies also consider the specific situation of diffraction by noise barriers placed on ground of finite impedance. However serious discrepancies between sound levels measured behind noise barriers and those predicted by theory are still often reported in the literature. Many of these discrepancies are attributed to atmospheric turbulence since the effects of atmospheric turbulence have not previously been considered when predicting the attenuation of sound by barriers. Chapter 5 is a brief review of the previous work on noise barriers. Chapter 6 is a summary of diffraction theory which includes the classical Kirchhoff-Fresnel theory commonly used by many authors. Chapter 7 is a review of scattering of acoustic waves in a turbulent atmosphere while Chapter 8 describes the author's utilisation of this scattering theory. Chapters 9, 10 and 11 describe the author's experimental measurements. These include measurements of highway noise behind the Queensway barrier in Ottawa and an idealised experiment of diffraction at a thin screen placed on a flat hard surface (asphalt) with measurements of relevant meteorological variables. In Chapter 12 the Queensway barrier results are first discussed. The precise measurements from the model barrier are then compared with diffraction theory alone. Finally these same measurements are compared with a theory that includes both diffraction at the barrier edge and scattering by atmospheric turbulence.

PART I

I N T E R F E R E N C E

Chapter 1.

PREVIOUS WORK

1-1 Coherent and Incoherent Wave Theory

The derivation of the wave equation satisfied by the pressure or other parameter of a compressible fluid can be found in any elementary text on acoustics. In the case of acoustic spherical waves, after some simplifying assumptions regarding small amplitudes of density change, condensation, particle displacement and particle velocity, the pressure disturbances satisfy

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0 \quad (1)$$

where c is the velocity of propagation of the wave. Furthermore in the derivation of Eq. (1) the medium is assumed to be homogeneous, isotropic, and perfectly elastic.

A solution of Eq. (1) is represented in complex form by

$$p = \frac{A}{r} e^{i(kr - \omega t)} \quad (2)$$

where A , ω and k are respectively the amplitude, angular frequency and wave number of the acoustic wave; the latter two being related by $\omega = kc$.

In the case of a gaseous medium, such as the atmosphere, the velocity of propagation c is, in part, a function of the absolute temperature of the gas. Furthermore a sound wave is an alternating disturbance of the propagation medium and c describes the velocity of propagation relative to the medium itself. Sound waves in air are convected by movements of the medium and hence the velocity of propagation relative to some instrument, fixed

with respect to the ground, is also a function of the component of wind velocity in the direction of propagation. Hence we write

$$c = c_0 \sqrt{\frac{T}{T_0}} + \hat{n} \cdot \vec{V} \quad (3)$$

where T and $\vec{V}$ are respectively the absolute temperature and the wind velocity. The unit vector $\hat{n}$ is in the direction of propagation and T_0 is the reference temperature at which the velocity of propagation has the value c_0 .

However wind velocity and temperature undergo random fluctuations in space and time, resulting in atmospheric turbulence. Thus the assumption of an isotropic and homogeneous medium is too stringent a restriction for acoustic waves propagating outdoors. The velocity of propagation c is in reality a random function of space and time resulting in a fluctuating amplitude and phase for p in Eq. (2).

Recent measurements have shown that these amplitude and phase fluctuations are significant in the case of the propagation of acoustic waves in the audio frequency range.

Theoretical treatment of waves propagating in a turbulent atmosphere in the absence of a boundary has been extensively treated in recent years by several authors^{2,3,4,5}. Using statistical treatments of the atmosphere these give the magnitude of the phase and amplitude fluctuations, as well as their correlations, of plane and spherical waves as a function of measurable meteorological and acoustical parameters.

In the presence of a boundary where the sound field

is the sum of direct and reflected waves, even weak turbulence can produce important effects on the resulting interference spectrum. This has been shown, first by Ingard and Maling⁶, and then by Daigle et al.⁷ in the case of a perfectly reflecting boundary. The effects of atmospheric turbulence on the sound field above a finite impedance boundary have also been considered briefly by Chessell⁸.

1-2 Existing Analysis of the Sound Field Above a Boundary

a) Hard Ground

The work by Ingard and Maling⁶ was the first attempt to explain the effects of atmospheric turbulence on the field above a hard boundary. They investigated the interference between the direct sound and that reflected from a hard ground, in the presence of turbulence, at distances of propagation up to about 75 m.

However their theoretical treatment was simplified by the assumption of zero correlation between the direct and reflected waves and by the use of a theory⁹ for plane waves to predict the fluctuations in phase and amplitude. Furthermore, they did not measure any meteorological variables but used them as a constant adjusted for best fit to measured sound levels.

Recently more detailed theory⁷ and experiments have removed the simplifications in the work of Ingard and Maling. The theory was extended to include the partial correlation between direct and reflected waves that is expected, since it was also found experimentally⁷ that the correlation length (~ 1.1 m) of the meteorological fluctuations is comparable to the separation between

the interfering sound paths.. Further, a theory for the fluctuations of phase and amplitude in spherical waves was used, because experimentally a point source is used. Finally, measurements of the fluctuating wind and temperature were described; these were made to confirm the validity of certain assumptions in the theory of atmospheric structure above the boundary, and to quantify the relevant meteorological parameters for comparison between predicted and measured sound levels. However, this latter theory², when compared with simultaneous meteorological and acoustical measurements to distances of propagation up to 45 m, is still limited by the need for one adjustable parameter.

The more recent work³ uses a theoretical analysis³ of sound waves propagating in a turbulent atmosphere which is directly applicable only in the absence of a boundary. The limitation enters when trying to adapt this analysis to the situation of near-horizontal propagation of sound close to the ground, where the sound field is the result of the interference between the direct sound and that reflected from the ground surface. The specific problem of most concern is the quantification of the amount of partial correlation between the direct and ground reflected sound. In free space, the transverse autocorrelation of the amplitude and phase fluctuations of sound from a point source S at two receivers R_1 and R_2 separated laterally by a distance l (see Fig. 1) has been calculated by Karavainikov³. When adapting his result to predict the partial correlation between direct and ground reflected sound it was found³, by using l as an adjustable

parameter, that satisfactory agreement was achieved between predicted and measured sound levels when a value of l equal to half the maximum path separation is used.

In order to summarise the mathematical analysis of the previous work we restrict this discussion to the analysis of Daigle et al. since the result of Ingard and Maling is then simply the limiting case of zero partial correlation.

The sound field at R due to a point source at S above a hard, perfectly reflecting ground plane, (see Fig. 2) is given by

$$\dot{p} = \frac{A_d}{r_d} \exp[i(k_d r_d - \omega t)] + \frac{A_r}{r_r} \exp[i(k_r r_r - \omega t)] \quad (4)$$

where r_d is the distance from the source to receiver, r_r is the

FIG. 1 Schematic diagram showing a point source S and two receivers R₁ and R₂ on a given wave front and separated laterally by a distance l .

path length of the reflected waves, and the quantities A_d , A_r and k_d , k_r are, respectively, the fluctuating amplitudes of the direct and reflected waves and the corresponding fluctuating wavenumbers.

We assume that both amplitudes and both wavenumbers fluctuate about some mean value and write

$$A_d = 1 + a_d, \quad A_r = 1 + a_r \quad (5)$$

where the mean value of the amplitude has been normalized to unity at unit distance, and similarly,

$$k_d r_d = k_r r_d + \delta_d, \quad k_r r_r = k_r r_r + \delta_r \quad (6)$$

where k is the wavenumber in the absence of turbulence. Implicitly one has $\langle a_d \rangle = \langle a_r \rangle = 0$ and $\langle \delta_d \rangle = \langle \delta_r \rangle = 0$, where the bracket

FIG. 2 Schematic diagram showing source S and receiver R at heights h_s and h_r , respectively, above a hard, perfectly reflecting ground plane. The path lengths of the direct and reflected waves are r_d and r_r , respectively.

indicates the time average over a statistically large number of turbulent fluctuations.

The mean-square sound pressure at R is, from Eq. (4),

$$\bar{p}^2 = A_d^2/r_d^2 + A_r^2/r_r^2 + (2A_d A_r / r_d r_r) \cos[k(r_r - r_d) + (\delta_r - \delta_d)] \quad (7)$$

Equation (7) expresses the time-dependent mean-square pressure

$\bar{p}^2$ in terms of the fluctuating quantities A_d , A_r and δ_d , δ_r ; it is implicit that the averaging time for $\bar{p}^2$ is shorter than the time constants of A_d , A_r and δ_d , δ_r .

The long-time average value, denoted by $\langle \bar{p}^2 \rangle$, can be found from Eq. (7) in terms of the statistical properties of the fluctuating quantities. Restricting the analysis to the case $r_d/r_r \approx 1$ we can assume that $\langle a_d^2 \rangle = \langle a_r^2 \rangle = \langle a^2 \rangle$. We further assume that the quantities a , δ_r , and δ_d are normally distributed with standard deviations of $\langle a^2 \rangle^{1/2}$, σ_r , and σ_d , respectively.

Writing $\phi = k(r_r - r_d)$ the time average of the cosine term in Eq. (7) can be evaluated by allowing for a specified amount of partial correlation ρ between the direct and reflected sound as follows:

$$\begin{aligned} \langle \cos(\phi + \delta_r - \delta_d) \rangle &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos(\phi + \delta_r - \delta_d) \\ &\quad \times m(\delta_d, \delta_r; \sigma_d, \sigma_r) d\delta_d d\delta_r \\ &= \cos \phi \exp[-\sigma_d^2(1 - \rho)] \end{aligned} \quad (8)$$

where, furthermore, it is assumed that $\sigma_r = \sigma_d$ and

$$m(\delta_d, \delta_r; \sigma_d, \sigma_r) = \frac{1}{2\pi \sigma_d \sigma_r (1 - \rho^2)^{1/2}} \exp \left[-\frac{1}{2(1 - \rho^2)} \left(\frac{\delta_r^2}{\sigma_r^2} - \frac{2\rho \delta_r \delta_d}{\sigma_r \sigma_d} + \frac{\delta_d^2}{\sigma_d^2} \right) \right] \quad (9)$$

The integrals in Eq. (8) can be evaluated using any standard mathematical tables.

Using Eq. (8) the long-time average of Eq. (7) can be obtained as

$$\langle \bar{p}^2 \rangle = \frac{A^2}{2r_r} \left\{ \frac{\langle a^2 \rangle}{2} \left(\frac{r_r}{r_d} + \frac{r_d}{r_r} \right) + \frac{r_r}{2r_d} \left(1 - \frac{r_d}{r_r} \right)^2 + 1 + \cos \phi \cdot \exp[-\sigma_d^2 (1 - \rho)] \right\} \quad (10)$$

In the limiting case of zero correlation between the two interfering waves, $\rho = 0$, Eq. (10) reduces to the expression obtained by Ingard and Maling.

b) Soft Ground:

The first attempt to explain the effects of atmospheric turbulence on the field above a finite impedance boundary is due to Chessell⁸. However his analysis is oversimplified by having neglected amplitude fluctuations and by assuming zero correlation between the direct and reflected waves. He also uses a theory² for plane waves to predict the fluctuations in phase of both waves. Finally, he did not compare his predicted curves with measured sound levels.

The result obtained by Chessell can be easily derived by an analysis similar to that in the previous section. The sound field at R due to a point source S at a height h_s above the plane boundary between two homogeneous media, each described by a finite impedance (see Fig. 3) is given by

$$p = \frac{A_d}{r_d} \exp[i(k_d r_d - \omega t)] + Q \frac{A_r}{r_r} \exp[i(k_r r_r - \omega t)] \quad (11)$$

where all the quantities are as defined before and Q is a complex number describing the apparent strength of the image source I in Fig. 3. The specific form of Q can vary depending on the model chosen for the ground impedance, hence a more detailed discussion of this point will be delayed to Chapter 3.

The long-time average value, $\langle \bar{p}^2 \rangle$, is obtained in a similar way to that of the previous section but with the differing assumptions of $\langle a^2 \rangle = 0$ and $\rho = 0$ in the evaluation of the average of the cosine term. A straightforward calculation then leads to

$$\langle \bar{p}^2 \rangle = \frac{2}{r_d r_r} \left[\frac{r_r}{2 r_d} \left(1 - |Q| \frac{r_d}{r_r} \right)^2 + |Q| + |Q| \cos(\phi + \gamma) \exp(-\sigma_d^2) \right] \quad (12)$$

where we have written $Q = |Q| e^{i\gamma}$. Equation (12) is equivalent to Chessell's result (Ref. 8, Eq. (27) by setting $r_r'/r_d = r'$, $\gamma = \theta$ and $\sigma_d^2 = \frac{1}{2} \sigma_{dpp}^2$).

FIG. 3 Schematic diagram showing point source S and receiver R at heights h_s and h_r , respectively, above a flat ground of impedance Z_2 . The path lengths of the direct and reflected waves are r_d and r_r , respectively.

Chapter 2 AUTHOR'S EXTENSION OF THE THEORY

2-1 Introduction

As discussed in the previous chapter, although the theory of Ingard and Maling has been extended by Daigle et al.⁷, the newer theory is still limited by the use of one adjustable parameter.

Therefore this chapter examines in more detail the specific problem of the partial correlation between the direct sound and that reflected from a hard ground. The theory is then further extended to include explicitly, in addition to the transverse autocorrelation, the longitudinal autocorrelation of the phase fluctuations along each path. In Chapter 4 the new extended theory is compared with previous simultaneous measurements of sound levels and meteorological variables over asphalt.

The chapter then discusses the field resulting from the interference between direct sound and sound reflected from a finite impedance ground. The earlier theory, first described by Chessell⁸, is extended to include, in addition to the transverse and longitudinal autocorrelation of the phase fluctuations, the corresponding autocorrelations of the amplitude fluctuations. Two cases corresponding to short and longer distances of propagation are distinguished in the analysis. The new theory will be compared with measurements of sound propagated to large distances up to 1100 m in Chapter 4.

2-2 Analysis of the Sound Field Above a Hard Ground

We remember that the sound field at R due to a point source at S above a hard, perfectly reflecting ground plane (see Fig. 2) is given by Eq. (4),

$$p = \frac{A_d}{r_d} \exp[i(k_d r_d - \omega t)] + \frac{A_r}{r_r} \exp[i(k_r r_r - \omega t)] \quad (4)$$

We first consider the fluctuating quantities A_d , A_r , k_d , and k_r . Deviating from the procedure in the previous chapter we split the direct and reflected sound path such that $r_d = r_{d1} + r_{d2}$ and $r_r = r_{r1} + r_{r2}$ (see Fig. 4). We can then write

$$k_d r_d = k r_{d1} + \delta_{d1} + k r_{d2} + \delta_{d2}$$

and

$$k_r r_r = k r_{r1} + \delta_{r1} + k r_{r2} + \delta_{r2}$$

FIG. 4. Schematic diagram showing point source S and receiver R above a flat ground where the maximum path separation between direct and reflected sound is l . The total length of the direct and reflected sound path is written $r_d = r_{d1} + r_{d2}$ and $r_r = r_{r1} + r_{r2}$, respectively.

where k is again the wavenumber in the absence of turbulence.

$$\text{Implicitly one has } \langle \delta_{d1} \rangle = \langle \delta_{d2} \rangle = \langle \delta_{r1} \rangle = \langle \delta_{r2} \rangle = 0. \quad (13)$$

The mean square sound pressure at R is, from Eq. (4)

$$\bar{p}^2 = \frac{A_d^2}{r_d^2} + \frac{A_r^2}{r_r^2} + \frac{2A_d A_r}{r_d r_r} \cos[\phi + (\delta_{r1} + \delta_{r2} - \delta_{d1} - \delta_{d2})]$$

where $\phi = k(r_r - r_d)$, and it is implicit that the averaging for $\bar{p}^2$ is shorter than the time constants of A_d , A_r , and δ_{d1} , δ_{d2} , δ_{r1} , δ_{r2} .

In calculating the long time average, $\langle \bar{p}^2 \rangle$ we first restrict the analysis to relatively shorter distances. It has been shown⁷ that only the phase covariance between direct and reflected sound is important in this case. In fact, the partial covariance between direct and reflected sound above hard ground is most important only for shorter distances of propagation.

At larger distances, because of the smaller path length difference, the interference minima (the parts of the spectrum sensitive to the degree of partial coherence) occur at higher frequencies. Since the variations due to turbulence increase rapidly with distance and with frequency, partial covariance becomes progressively less important (see Fig. 5).

We can thus write as before, $A_d = 1 + a_d$ and $A_r = 1 + a_r$ where $\langle a_d \rangle = \langle a_r \rangle = 0$. In performing the calculation we again assume that $\langle a_d^2 \rangle = \langle a_r^2 \rangle = \langle a^2 \rangle$ since $r_d/r_r \approx 1$ for the case considered here. The quantities a , δ_{d1} , δ_{d2} , δ_{r1} and δ_{r2} are assumed to be normally distributed with standard deviations

of $\langle a^2 \rangle^{1/2}$, σ_{d1} , σ_{d2} , σ_{r1} and σ_{r2} respectively.

The time average of the cosine term can be calculated as follows:

$$\begin{aligned} \langle \cos(\phi + \delta_{r1} + \delta_{r2} - \delta_{d1} - \delta_{d2}) \rangle &= \iiint_{-\infty}^{\infty} \cos(\phi + \delta_{r1} + \delta_{r2} - \delta_{d1} - \delta_{d2}) \\ &\times m(\delta_{r1}, \delta_{r2}, \delta_{d1}, \delta_{d2}) d\delta_{r1} d\delta_{r2} d\delta_{d1} d\delta_{d2} \end{aligned} \quad (14)$$

where

$$m(\delta_{r1}, \delta_{r2}, \delta_{d1}, \delta_{d2}) = C \exp \left[-\frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 A_{ij} x_i x_j \right] \quad (15)$$

is the multivariate distribution of the random variables by identifying $x_1 = \delta_{d1}$, $x_2 = \delta_{r1}$, $x_3 = \delta_{d2}$ and $x_4 = \delta_{r2}$. In Eq. (15) A_{ij} are elements of a 4×4 matrix, which can be obtained from the inverse of a variance-covariance matrix (see Appendix A) of elements $\langle x_i x_j \rangle$, and the normalization constant is

$$C = [(2\pi)^4 \det(A_{ij})]^{-1/4} \quad (16)$$

We now construct the variance-covariance matrix.

If it is assumed that $\sigma_{d1} \approx \sigma_{d2} \approx \sigma_{r1} \approx \sigma_{r2}$ (equality holds only when $d_1 = d_2$ and $r_1 = r_2$) the elements $\langle x_i x_j \rangle$ are simply

$$\langle \delta_{d1}^2 \rangle = \langle \delta_{d2}^2 \rangle = \langle \delta_{r1}^2 \rangle = \langle \delta_{r2}^2 \rangle = \sigma_{d1}^2 \quad (17)$$

For the elements $\langle x_i x_j \rangle$, we assume that

$$\langle \delta_{d1} \delta_{r1} \rangle = \langle \delta_{d2} \delta_{r2} \rangle = \sigma_{d1}^2 \rho_\delta \quad (18)$$

where we identify ρ as the transverse autocorrelation of the phase fluctuations (see Figs. 1 and 4). We further assume that

$$\langle \delta_{d1} \delta_{d2} \rangle = \langle \delta_{r1} \delta_{r2} \rangle = \sigma_{d1}^2 R_\delta \quad (19)$$

where we can identify R_δ as the longitudinal autocorrelation of the phase fluctuations. The remaining terms $\langle \delta_{d1} \delta_{r2} \rangle$ and $\langle \delta_{d2} \delta_{r1} \rangle$ are a function of both ρ_δ and R_δ as discussed in Appendix B (see Eq. (B7) and the subsequent discussion). However the inversion of the 4x4 variance-covariance matrix in this general case, to obtain the elements A_{ij} and the subsequent evaluation of the integral in Eq. (14), are prohibitively complicated. Fortunately Chernov² and Karavainikov³ find that the longitudinal autocorrelation, contrary to the transverse autocorrelation, extends over distances many times larger than the correlation distance of the medium. Thus for shorter distances of propagation $R_\delta \approx 1$, and we can safely assume that

$$\langle \delta_{d1} \delta_{r2} \rangle = \langle \delta_{d2} \delta_{r1} \rangle \approx \sigma_{d1}^2 \rho_\delta \quad (20)$$

Inverting the variance-covariance matrix constructed from Eqs. (17) through (20) yields the elements A_{ij} in Eq. (15) and then the integration required by Eq. (14) leads to the

following result (see Appendix C)

$$\begin{aligned} \langle \cos(\phi + \delta_{ra} - \delta_{d1} - \delta_{d2}) \rangle \\ = \cos \phi \exp \left[-\sigma_d^2 \left(1 - \frac{2\rho_s}{1+R_s} \right) \right] \end{aligned} \quad (21)$$

where, furthermore, we have written $\sigma_d^2 = \langle \delta_d^2 \rangle = \langle (\delta_{d1} + \delta_{d2})^2 \rangle = 2\sigma_{d1}^2(1+R_s)$ since $\delta_d = \delta_{d1} + \delta_{d2}$. In the limiting case of perfect transverse and longitudinal covariance, $\rho_s = R_s = 1$, Eq. (21) reduces to $\cos \phi^0 = \cos \phi$. We also note that for $\rho_s = R_s = 0$ the exponential reduces to the expected value of $\exp(-\sigma_d^2)$.

The long-time average of Eq. (13) can now be written by using the result evaluated in Eq. (21) and one obtains,

$$\begin{aligned} \langle \bar{\rho}^2 \rangle = \frac{2}{r_d r_r} \left\{ \frac{\langle a^2 \rangle}{2} \left(\frac{r_r}{r_d} + \frac{r_d}{r_r} \right) \right. \\ \left. + \frac{r_r}{2r_d} \left(1 - \frac{r_d}{r_r} \right)^2 + 1 \right. \\ \left. + \cos \phi \exp \left[-\sigma_d^2 \left(1 - \frac{2\rho_s}{1+R_s} \right) \right] \right\} \end{aligned} \quad (22)$$

Eq. (22) differs from the previously found result⁷, discussed in Chapter 1, in the exponential term. It was expected that the addition of both the transverse and longitudinal covariances between the interfering waves would eliminate the need for an adjustable parameter when comparing Eq. (22) with experimental results. This, unfortunately, is not the case as will be shown in Chapter 4. In fact since we have assumed $R_g = 1$, we have $2/(1+R_g) = 1$ and the exponential term reduces to the previously found result of $\exp[-\sigma_d^2(1-\rho_g)]$.

It is instructive to consider here the integration of Eq. (14) when the matrix elements in Eqs. (19) and (20) are set to zero. The remaining non-zero elements of Eqs. (17) and (18) are used to obtain the elements A_{ij} of Eq. (15) and the subsequent integration then leads to the previously found result⁷ of $\cos\phi \exp[-\sigma_d^2(1-\rho)]$. A situation of practical importance, namely large distances over soft ground, is considered in the following section where this point will be discussed in more detail.

2.3 Analysis of the Sound Field Above a Soft Ground

We first consider briefly the theory⁹ of sound propagation in near-grazing directions over a flat boundary between two homogeneous media, each described by a finite impedance -- for

example, propagation in air over flat ground covered with grass.

The sound field at R due to a point source S, at a height h_s above the plane boundary between the two media (see Fig. 3) is given by Eq. (11),

$$p = \frac{A_s}{r_s} \exp[i(k_s r_s - \omega t)] + Q \frac{A_r}{r_r} \exp[i(k_r r_r - \omega t)] \quad (11)$$

We now specify the apparent strength of the image source I as $Q = R_p + (1 - R_p)F$. For simplicity and numerical convenience we use the formulation of Rudnick¹⁰ who assumed two semi-infinite media. The plane-wave reflection coefficient can be written as

$$R_p = \frac{\sin \psi - Z_1/Z_2 (1 - k_1^2/k_2^2 \cos^2 \psi)^{1/2}}{\sin \psi + Z_1/Z_2 (1 - k_1^2/k_2^2 \cos^2 \psi)^{1/2}} \quad (23)$$

where $Z_1 = \rho c$ is the characteristic acoustic impedance of the air, $Z_2 = (R + iX)$ is the specific normal impedance of the ground and $k_1, k_2 = (\alpha + i\beta)$ are the respective wavenumbers in the two media. The function $F(w)$ describes the interaction of a curved wavefront with a surface of finite impedance and is given by

$$F(w) = 1 + 2i w^{1/2} \exp(-w) \int_{-iw^{1/2}}^{\infty} \exp(-u^2) du \quad (24)$$

where w is the numerical distance

$$w = \frac{2i k_1 r_s}{(1 - R_p) \cos^2 \psi} \left(\frac{Z_1}{Z_2} \right)^2 \left(1 - \frac{k_1^2}{k_2^2} \cos^2 \psi \right) \quad (25)$$

The function $F(w)$ is often described as a ground wave, see the discussion in Section 4-3.

To compare with experimental results both Z_2 and k_2 must be known as a function of frequency. We use the model of Delany and Bazley¹¹ for the acoustic properties of a fibrous porous medium, in which the characteristic impedance Z_2 is given by

$$\begin{aligned} R/\rho c &= 1 + 9.08(f/\sigma)^{-0.75} \\ X/\rho c &= 11.9(f/\sigma)^{-0.73} \end{aligned} \quad (26)$$

and the propagation constant k_2 by

$$\begin{aligned} \alpha/(\frac{\omega}{c}) &= 1 + 10.8(f/\sigma)^{-0.70} \\ \beta/(\frac{\omega}{c}) &= 10.3(f/\sigma)^{-0.59} \end{aligned} \quad (27)$$

Here $\omega = 2\pi f$ and $\sigma (\text{g}\cdot\text{cm}^{-3})$ is the specific flow resistance per unit distance in the direction of flow, i.e. the flow resistivity. The value of σ is determined empirically¹² by a fit to measurements¹² of the impedance of grass covered surfaces.

The mean square sound pressure at R above soft ground is written from Eq. (11). Following the notation of the previous section, we find

$$\bar{p}^2 = \frac{A_d^2}{r_d^2} + Q^2 \frac{A_r^2}{r_r^2} + 2Q \frac{A_d A_r}{r_d r_r} \cos[\phi + (\delta_{r1} + \delta_{r2} - \delta_{d1} - \delta_{d2})] \quad (28)$$

In the evaluation of the long-time average, $\langle \bar{p}^2 \rangle$, the transverse and longitudinal autocorrelations of the phase fluctuations are considered as before. Furthermore it will be shown in Chapter 4 that above soft ground, correlation between the direct and reflected path remains important to longer distances (above hard ground it has been explained that correlation is only important at shorter distances). Therefore, for soft ground, we also include

consideration of the transverse and longitudinal autocorrelation of the amplitude fluctuations, ρ_a and R_a , respectively. It can be shown that other covariance terms have a $\sin \psi$ (see Fig. 3) dependence, and consequently, can be neglected. (This point will be discussed further in Chapter 4).

We write $A_d = 1 + a_{d1} + a_{d2}$ and $A_r = 1 + a_{r1} + a_{r2}$, as we did for the phase terms, in order to show explicitly both ρ_a and R_a . In the evaluation of the long-time average of Eq. (28) the terms $\langle a_{d1} a_{r2} \rangle$ and $\langle a_{d2} a_{r1} \rangle$ that appear are analogous to the corresponding phase terms and can also be approximated from the discussion in Appendix B.

In order to continue we must now restrict the discussion to two specific cases.

a) Short Distances of Propagation:

At short distances we can neglect amplitude covariance, and assume $R_s \approx 1$, in order to cope with the matrix terms of Eq. (20). The evaluation of $\langle \bar{p}^2 \rangle$ is then similar to the previous analysis and a straightforward calculation leads to

$$\begin{aligned} \langle \bar{p}^2 \rangle = \frac{2}{r_a r_r} \left\{ \frac{\langle a^2 \rangle}{2} \left(\frac{r_r}{r_a} + |Q|^2 \frac{r_a}{r_r} \right) + \frac{r_r}{2 r_a} \left(1 - |Q| \frac{r_a}{r_r} \right)^2 \right. \\ \left. + |Q| + |Q| \cos(\phi + \gamma) \exp \left[-\sigma_a^2 \left(1 - \frac{2 \rho_s}{1 + R_s} \right) \right] \right\} \quad (29) \end{aligned}$$

where $Q = |Q|e^{i\gamma}$. In the evaluation of Eq. (29) it has again been assumed that $\sigma_d^2 = 2\sigma_a^2(1+R_s)$ and also, implicitly, that $\langle a_d^2 \rangle = \langle a_r^2 \rangle = \langle a^2 \rangle$. Furthermore we have considered a non-fluctuating reflection coefficient and numerical distance by setting $k_1 = k$ in Eqs. (23) and (25).

b) Large Distances of Propagation:

At large distances, Chernov² and Karavainikov³ show that the longitudinal autocorrelations of both the amplitude and phase fluctuations go to zero. This is contrary to the transverse autocorrelation which remains, because the path separation between direct and reflected sound is still of the same order as the correlation length of the medium. Therefore at distances where $R_a = R_s = 0$ the long time average of Eq. (28) is reevaluated by setting to zero the matrix elements in Eqs. (19), (20) and in the corresponding amplitude terms that appear. One then obtains

$$\begin{aligned} \langle \bar{p}^2 \rangle = & \frac{2}{r_a r_r} \left\{ \frac{\langle a^2 \rangle}{2} \left(\frac{r_r}{r_a} + |Q|^2 \frac{r_a}{r_r} \right) + \frac{r_r}{2r_a} \left(1 - |Q| \frac{r_a}{r_r} \right)^2 \right. \\ & + |Q| + |Q|(1 + \langle a^2 \rangle \rho_a) \cos(\phi + \gamma) \\ & \left. \times \exp[-\sigma_a^2(i - \rho_s)] \right\} \end{aligned} \quad (30)$$

where we have again used the assumptions that follow Eq. (29).

Chapter 3 REVIEW OF LINE-OF-SIGHT PROPAGATION**/

3-1 Introduction

Waves which are propagated in a medium with random inhomogeneities are scattered by the inhomogeneities of the medium. The scattered waves, when they are superimposed upon the primary waves, cause fluctuations of the amplitude and phase of the resultant field.

It will be shown in Chapter 7 that the inhomogeneities of the atmosphere produce scattering predominantly in the forward direction. That is, all the eddies scatter energy mainly through angles negligibly different from zero. For this reason, an analysis based on scattering theory (implicitly, large angle scattering) does not provide a good model for propagation through turbulence. Chernov¹ has shown for plane waves and Karavainikov² for spherical waves that a method of small perturbation in the manner proposed by Rytov³ can be

** Some of the material of this chapter has appeared in the M.Sc. thesis "The Effect of Atmospheric Turbulence on the Interference of Acoustic Waves Due to the Presence of a Boundary" submitted to the Physics Department of Carleton University by the author. For completeness and clarity this material has been integrated with the new material to form a coherent chapter for the benefit of the reader.

used for the problem of propagation through turbulence. Furthermore it has been shown⁷ that this relatively simpler theory, which assumes homogeneous isotropic turbulence, can be justified experimentally and provides an adequate explanation for the interfering sound field in turbulence a few meters above a boundary. The theory provides the dependence of $\langle a^2 \rangle$, σ_d^2 , ρ_a , ρ_b , R_a and R_b on such measurable parameters as distance, frequency, and strength and scale of wind and temperature fluctuations.

The more popular model of Tatarskii^{4,5}, which allows for turbulence that is only locally homogeneous and isotropic will be considered in Chapter 7 in another context.

3-2 Phase and Amplitude Fluctuation of Spherical Waves in the Absence of a Boundary

Most of the following analysis is due to Karavainikov³ with additions due to Daigle et al.⁷. We wish to calculate the magnitude of the phase and amplitude fluctuations of a spherical wave from a point source in free space, i.e. in the absence of any boundaries.

The origin of a Cartesian coordinate system is placed at the source and we fix a receiver R at the point $(r, 0, 0)$. From Eq. (3) we see that any random fluctuations in temperature and wind speed produce in turn random fluctuations in the velocity of sound propagation c . We can thus write that the index of refraction, $n = c_0/c$, fluctuates around an average

value of unity, i.e.,

$$n = 1 + \mu$$

(31)

where $|\mu| \ll 1$.

The pressure disturbances will now satisfy Eq. (1) written in the form

$$\frac{(1+\mu)^2}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0$$

(32)

We must look for a solution of Eq. (32) of the form

$$p = \frac{A(r)}{r} \exp\{i[S(r) - \omega t]\}$$

(33)

where $A(r)$ and $S(r)$ are unknown functions.

The essence of Rytov's method consists of replacing the function p by another function ψ which is related to the first one by the following expression

$$p = \frac{A_0}{r} \exp\{i[\psi(r) - \omega t]\}$$

(34)

where A_0 is the amplitude at unit distance from the source, and $p_0 = A_0/r \exp[i(S_0 - \omega t)]$ is the pressure disturbance for a homogeneous medium.

Comparing Eq. (34) with Eq. (33) we see that the function $\psi(r)$ is

$$\psi(r) = S(r) - i/n \frac{A(r)}{A_0}$$

(35)

Replacing the function p by ψ in Eq. (32) on the basis of Eq. (34) Karavainikov obtains, after several algebraic manipulations, expressions for the mean-square phase fluctuations, $\langle \delta_\alpha^2 \rangle = \langle (S - S_0)^2 \rangle$, and log-amplitude fluctuations $\langle [\ln(1+a)]^2 \rangle$.

In his derivation a theory of homogeneous isotropic turbulence is assumed where the fluctuating acoustical index of refraction is assumed to have a spatial correlation function of Gaussian form

$$\langle \mu, \mu_2 \rangle = \langle \mu^2 \rangle \exp(-r^2/L^2) \quad (36)$$

where L is a measure of the scale of the turbulence.

The analytical expressions for $\langle [\ln(1+a)]^2 \rangle$ and $\langle \delta_d^2 \rangle = \sigma_d^2$ published by Karavainikov are incorrect, and so correct versions previously derived by Daigle et al.⁷ are given here as the results of the above analysis. These are, for the phase and amplitude fluctuations of spherical waves at a point R in free space

$$\sigma_d^2 = \frac{1}{2}(I_1 + I_2) \quad (37)$$

$$\langle [\ln(1+a)]^2 \rangle = \frac{1}{2}(I_1 - I_2) \quad (38)$$

In the above,

$$I_1 = \sqrt{\pi} \langle \mu^2 \rangle k^2 r_d L \quad (39)$$

$$I_2 = \sqrt{\pi} \langle \mu^2 \rangle k^2 r_d L \cdot \frac{1}{\Delta^2(\Omega+1)\sqrt{2\Omega}} \left(\frac{\Omega\Delta}{2} \ln \frac{1+\Delta(2\Omega)^{1/2}}{1-\Delta(2\Omega)^{1/2}} + \operatorname{arctg} \frac{\Omega\Delta}{1-\Delta(2\Omega)^{1/2}} - \operatorname{arctg} \frac{\Omega\Delta}{1+\Delta(2\Omega)^{1/2}} \right) \quad (40)$$

where $\Omega = (1 + 1/\Delta^2)^{1/2} - 1$ and $\Delta = r_d/kL^2$.

Equation (38) gives the value of $\alpha = \langle [\ln(1+a)]^2 \rangle$ while in Eqs. (22), (29) and (30) it is necessary to know $\langle a^2 \rangle$. Expanding the natural logarithm and expressing the higher-order moments in terms of second-order moments, one finds

$$\langle a^2 \rangle \approx \begin{cases} \frac{x}{1 + \frac{11}{4}x}, & x \leq 1 \\ 0.27 x^{0.33}, & x > 1 \end{cases} \quad (41)$$

3-3 Transverse and Longitudinal Correlations of Acoustic Waves

In addition to finding the mean-square phase and log-amplitude fluctuations using Rytov's method, it is also possible to obtain the transverse and longitudinal autocorrelations of the amplitude and phase fluctuations in a spherical wave.

The transverse autocorrelation of the fluctuations is calculated by considering two receivers R_1 and R_2 placed respectively at $(r, 0, 0)$ and $(r, 0, \lambda)$. The origin of the coordinate system is again placed at the point source S . At these two receivers in free space separated by a distance λ (see Fig. 1) Karavainikov³ finds that the transverse autocorrelation of the phase fluctuations, ρ_ϕ , is expressed as

$$\rho_\phi = \frac{\bar{\Phi}(\lambda/L)}{(\lambda/L)} \quad (42)$$

where $\Phi(l/L) = \int_0^{l/L} e^{-u^2} du$ and L is again a measure of the scale of the turbulence. For the cases of source-receiver separations considered here, $\rho_a = \rho_s$, where ρ_a is the transverse autocorrelation of the amplitude fluctuations (see Ref. 2).

In using Eq. (42) to estimate the covariance between direct and ground-reflected sound, for insertion into Eqs. (22), (29) or (30), it is expected that l (see Fig. 4) represents the maximum path separation for propagation above both hard and soft boundaries and for all distances. This is contrary to previous analysis⁷ where l had to be chosen as half the maximum path separation if satisfactory agreement between measurements of acoustical and meteorological parameters and theory was to be obtained.

The longitudinal autocorrelation of the fluctuations is calculated by considering, for spherical waves from a point source S , two receiving points; R_1 at $(r_1, 0, 0)$ and R_2 at $(r_2, 0, 0)$. Thus, for a point source S placed at the origin of the coordinate system, S , R_1 and R_2 lie on a straight line with R_1 and R_2 separated by $\Delta r = r_2 - r_1$.

The longitudinal autocorrelation of the phase and amplitude fluctuations, R_ϕ and R_a respectively, has been calculated by Karavainikov for spherical waves. However for short distances of propagation and corresponding to cases where $R_a \approx 1$ and $R_\phi \approx 1$, the expressions of Chernov in the case of plane waves yield numerical results similar to the expressions of Karavainikov. Therefore, for convenience, we use the simpler expression of Chernov which is

$$R_a = R_\phi = [1 + (2\Delta r/kL^2)^2]^{-1} \quad (43)$$

To use Eq. (43) to estimate the covariance along the direct sound path, for example, we use $\Delta r = r_d - r_{dl}$ (see Fig. 4).

We need not consider any other expression for R_a and R_g since the only other case studied explicitly here is $R_a = R_g = 0$.

3-4 Calculation of the Fluctuating Index of Refraction and the Effective Scale of Turbulence L from Meteorological Measurements

In Eqs. (39) and (40) it is necessary to express $\langle \mu^2 \rangle$ in terms of measurable meteorological variables. Writing $T = T_0 + \delta T$ and $V = V_0 + \delta V$ in Eq. (3) one has

$$c = c_0 \left(1 + \frac{\delta T}{T_0}\right)^{1/2} + V_0 \cos \theta \left(1 + \frac{\delta V}{V_0}\right) \quad (44)$$

where θ is the angle between the average direction of the wind and the direction of sound propagation. Squaring and averaging the fluctuating part of Eq. (44) one finds

$$\langle \delta c^2 \rangle = \langle \delta V^2 \rangle \cos^2 \theta + \frac{c_0}{T_0} \cos \theta \langle \delta V \delta T \rangle + \frac{1}{4} \left(\frac{c_0}{T_0}\right)^2 \langle \delta T^2 \rangle \quad (45)$$

It has previously been found⁷ that the fluctuations of both wind and temperature are approximately normally distributed. One can therefore describe these variables by their standard deviations. Since $\langle \delta V \rangle = \langle \delta T \rangle = 0$, one can write $\langle \delta V^2 \rangle = \sigma_V^2$, $\langle \delta T^2 \rangle = \sigma_T^2$, and can define the cross correlation between wind and temperature fluctuations by

$$R = \frac{\langle \delta V \delta T \rangle}{\sqrt{\langle \delta V^2 \rangle \langle \delta T^2 \rangle}} \quad (46)$$

Eq. (45) then becomes

$$\langle \mu^2 \rangle = \left(\frac{\sigma_v \cos \theta}{c_o} \right)^2 + \frac{\sigma_v \sigma_T R \cos \theta}{c_o T_o} + \left(\frac{1}{2} \frac{\sigma_T}{T_o} \right)^2 \quad (47)$$

which enables the fluctuating index of refraction to be calculated as a function of measurable meteorological variables.

It was assumed above that the fluctuations in the index of refraction had a Gaussian spatial correlation function of the form given in Eq. (36). The scale of turbulence L must be estimated from the corresponding correlation functions of the wind and temperature fluctuations. The spatial correlation functions of both wind and temperature have been found individually to be of approximate Gaussian form⁷. From a calculation similar to that used in obtaining Eq. (47), but by neglecting all cross terms between wind and temperature, one obtains

$$\langle \mu_1 \mu_2 \rangle \approx \left(\frac{\sigma_v \cos \theta}{c_o} \right)^2 e^{-r^2/L_v^2} + \left(\frac{\sigma_T}{2 T_o} \right)^2 e^{-r^2/L_T^2} \quad (48)$$

where L_v and L_T are the characteristic lengths of the wind and temperature correlation functions. From Eq. (48), if $L_T = L_v$ then evidently $L = L_T = L_v$; if $L_T \neq L_v$ then L is weighted towards L_T or L_v depending on the relative importance of the corresponding variance terms.

Chapter 4 COMPARISON OF THEORY AND MEASUREMENTS

4-1 Previous Measurements over Hard Ground

Measurements of sound levels over a large relatively flat asphalt surface (a runway at Rockcliffe Airport, Ottawa) have been described previously⁷. These measurements are summarized briefly here.

A point source of sound was placed at a height of 1.2 m above the asphalt surface while two receivers were placed variously at heights of 1.2 m and 0.6 m and at horizontal distances of 15, 30, and 45 m from the source. The sound frequency was varied over a series of discrete values, each maintained for a period of about 3 minutes, between 1 and 6 kHz. The acoustical signals were recorded on magnetic tape to be subsequently analysed for mean sound levels.

Simultaneously with the acoustic measurements, wind velocity and temperature, as well as their fluctuations, were measured at two points with sensitive thermoanemometers. The electrical signals from each of the two thermoanemometers were recorded on tape to be subsequently analysed to obtain means, standard deviations and correlation functions for wind velocity and temperature. These are used in Eqs. (47) and (48) to obtain, respectively, values of $\langle \mu^2 \rangle$ and the correlation length of the medium, L .

4-2 Comparison of New Theory with Previous Measurements over Hard Ground

In the case of propagation at short distances above a hard ground, Eq. (22) differs in the exponential term from the previously found Eq. (10). The previous result, Eq. (10), contained the exponential $\exp[-\sigma_d^2(1-\rho_g)]$, where ρ_g expressed the partial correlation between the phase fluctuations of the direct and reflected sound. When ρ_g was estimated from Eq. (42) it was found[†] that satisfactory agreement between theory and experiment could be obtained only if l was chosen to be half the maximum path separation.

These same experimental results are now compared with the new theory as expressed by Eq. (22), with R_g obtained from Eq. (43) and ρ_g from Eq. (42). In Fig. 5(a) the dashed curve are the sound levels predicted from Eq. (22) with l in Eq. (42) representing the maximum path separation. The agreement with measured values (points) is not much better than with the prediction of incoherent acoustic theory ($\rho_g = R_g = 0$). This can be seen by the value of the exponential term which gives $\exp(-0.84\sigma_d^2) \approx \exp(-\sigma_d^2)$. However if l is again arbitrarily chosen to be half the maximum path separation, as before[†], the solid curve is obtained. Figures 5(b) through 5(f) compare the theory with more measured values for different heights of receiver and distances from the source, and for different values of $\langle \mu^2 \rangle$. The solid curves show essentially identical agreement as before[†]. In Fig. 5(e) since Eq. (43) yields $R_g = 0.5$ at 1400 Hz it is better to use the previous result[†] Eq. (10) which corresponds to the case $R_g \approx 0$ (see page 19).

It is very unfortunate that the new theory is still limited by the need for the same adjustable parameter. The reasons for this are not known at the present time. However, this further shows the rather complex nature of the problem and again, strongly indicates the need for more study. It is possible that the difficulty of this problem lies in the non-Gaussian nature of Eq. (20), as explained in Appendix B.

FIG. 5 Comparison of measured sound levels above asphalt with the theory valid above hard ground and shorter distances of propagation. The partial coherence of the interfering waves is estimated from the correlation length of the medium $L \approx 1.1$ m, the transverse covariance ρ_t , and the longitudinal covariance R_L . The dashed curve above is obtained by calculating ρ_t using l equal to the maximum path separation while the solid curve by using l equal to half the maximum separation. In the case above, $h_s = h_r = 1.2$ m, $r_s = 15$ m, $\langle \mu^2 \rangle = 7.7 \times 10^{-6}$, and on the following page, (b) $h_s = 1.2$ m, $h_r = 0.6$ m, $r_s = 15$ m, $\langle \mu^2 \rangle = 7.7 \times 10^{-6}$. (c) $h_s = h_r = 1.2$ m, $r_s = 15$ m, $\langle \mu^2 \rangle = 10.8 \times 10^{-6}$. (d) $h_s = h_r = 1.2$ m, $r_s = 15$ m, $\langle \mu^2 \rangle = 4.8 \times 10^{-6}$. (e) $h_s = h_r = 1.2$ m, $r_s = 30$ m, $\langle \mu^2 \rangle = 10.8 \times 10^{-6}$. (f) $h_s = h_r = 1.2$ m, $r_s = 45$ m, $\langle \mu^2 \rangle = 4.8 \times 10^{-6}$.

FIG. 5 Continued

4-3 Comparison of New Theory with Results of Parkin and Scholes over Soft Ground

Many practical situations arise where sound propagates in near-grazing directions over ground of finite impedance for distances of a few hundred meters. The resulting sound transmission spectra are dominated by a broad minimum centered at a frequency of about 500 Hz, this minimum being due to the interaction of the "zero path-length difference" interference dip with the ground and surface waves⁹. Experimental measurements made by Parkin and Scholes¹⁴ for the propagation of jet noise across grass at an airport clearly illustrate this phenomenon.

An attempt to predict the sound levels in the spectra of Ref. 14 under neutral atmospheric conditions (no wind or temperature gradients) has previously been made¹ using the theory of Donato¹⁶. The relative importance of a ground wave as well as a surface wave at frequencies below 300 Hz is illustrated in the example shown in Fig. 6 where the points are the measured sound levels. The excess attenuation refers to the sound level relative to the level from a point source in a free field with inverse square law and molecular absorption removed⁹. The curved labeled $D + R + G$ is obtained from Eqs. (11) through (25) by setting $A_d = A_r = A$ and $k_d = k_r = k_l = k$ where A and k are, respectively, the amplitude and wavenumber in the absence of turbulence. The impedance of the ground is described by $\sigma = 300$ cgs units^{12,8} in Eqs. (26) and (27). The theory of Donato, in essence, adds a surface wave term to Eq. (11). This term, illustrated by the curve

FIG. 6 Excess attenuation for propagation from a point source over mown grass, $h_s = 1.8 \text{ m}$, $h_r = 1.5 \text{ m}$. The calculated curves show the contributions from the various waves - direct D, reflected R, ground G, and surface S. The points are measurements of jet noise from Ref. 14. The excess attenuation is relative to that for a point source in a free field with inverse square law and molecular absorption removed.

labeled S in Fig. 6, describes a cylindrically spreading wave whose amplitude decreases exponentially with height above the ground. The discrepancy above 300 Hz, which is progressively more apparent at larger distances, between the curve labeled $D + R + G + S$ and the measured points has been attributed to the use of coherent acoustic theory which fails to take into account the effect of turbulence in the atmosphere.

We must note here that, as mentioned above, the theory of propagation in a turbulent medium described in Chapter 2 does not include the possibility of a surface wave. Hence in all the curves in subsequent figures the surface wave is calculated, in the spectra below ~ 300 Hz, from the theory of Donato¹⁵ which assumes a non-turbulent medium. The previous section shows that the regions of the maxima in interference spectra are relatively insensitive to atmospheric turbulence. Furthermore the fluctuations of phase and amplitude decrease strongly with decreasing frequency¹. Hence turbulence is not expected to alter the shape of these spectra below 300 Hz.

In Fig. 7 the solid curve predicted by coherent acoustic theory is compared with Eq. (30) (semi-solid), valid for large distances of propagation, for the particular configuration $h_s = 1.8$ m, $h_r = 1.5$ m, $r_d = 200$ m and 1100 m. The strength of turbulence is characterized by typical values⁷ of $\langle \mu^2 \rangle$ and L in Eqs. (37) through (42). The relative importance of correlation of phase and amplitude can be easily seen. First, if the amplitude covariance is neglected by setting $\rho_a = 0$ in Eq. (30), the dashed curve is obtained.

Neglecting the phase covariance by setting $\rho_\phi = 0$ in Eq. (30) results in essentially the dashed curve with only a slight deviation of a few decibels at frequencies near 5 kHz (dotted curve).

Finally for the case $L \rightarrow \infty$ resulting in $\rho_a = \rho_\phi = 1$, the results differ by at most 0.5 dB from the solid curve (at 1100 m). This difference is not significant and thus supports the assumption that the phase-amplitude covariance can be neglected in calculating Eq. (30). It also confirms that the predictions of propagation in a turbulent medium become those of coherent acoustic theory when the fluctuations in the interfering waves are completely correlated.

It is interesting to note the small dependence of the results on the amplitude-phase covariance despite the relatively large amplitude-amplitude and phase-phase covariances brought about by the small path separation¹⁶. This is presumably due to the fact that different parts of the acoustic path between source and receiver are important for amplitude and phase fluctuations respectively¹⁷.

Chessell⁸, in obtaining Eq. (12), neglected amplitude fluctuations and only considered uncorrelated phase fluctuations. In the case of larger distances of propagation his predictions lie close to the dotted curve in Fig. 7, but his analysis is oversimplified.

The dependence of the spectra on the turbulence parameters is illustrated in the following two figures. In Fig. 8

FIG. 7 Comparison of the excess attenuation predicted by coherent acoustic theory, — (solid) and that predicted assuming fluctuations, — (semisolid) for two larger distances of propagation over soft ground ($h_s = 1.8\text{ m}$, $h_r = 1.5\text{ m}$, and $\sigma = 300\text{ cgs units}$). The other two curves show the relative importance of the amplitude and phase correlation by first setting $\rho_s = 0$, --- (dashed) and then $\rho_s = 0$, ... (dotted).

FIG. 8 Comparison of the excess attenuation predicted for different values of partial coherence by varying the correlation length of the medium, L , over any probable range, --- $L = 0.8\text{ m}$, — $L = 2.8\text{ m}$ ($h_s = 1.8\text{ m}$, $h_r = 1.5\text{ m}$, $\sigma = 300\text{ cgs units}$).

FIG. 9. The excess attenuation predicted for different values of $\langle \mu^2 \rangle$ ($h_s = 1.8\text{ m}$, $h_r = 1.5\text{ m}$, $\sigma = 300\text{ cgs units}$) — $\langle \mu^2 \rangle = 0$, — — — $\langle \mu^2 \rangle = 0.1 \times 10^{-6}$, — — — $\langle \mu^2 \rangle = 1 \times 10^{-6}$, — · — $\langle \mu^2 \rangle = 8 \times 10^{-6}$.

$\langle \mu^2 \rangle$ is kept constant while L is varied over any probable range of value (0.8 m to 2.8 m). Since L determines mainly ρ_a and ρ_s from Eq. (42), it is noted that the dependence on correlation is small at 200 m and only moderate at 1100 m. This, fortunately, also shows that the adjustable parameter l is less important above soft ground. However, if L is kept constant while $\langle \mu^2 \rangle$ is varied, a strong dependence is found, as shown in Fig. 9.

We now compare the curves described above with the results of Parkin and Scholes¹⁴. Since they report no measurements of meteorological fluctuations from which values of $\langle \mu^2 \rangle$ or L can be deduced, these parameters have been adjusted for best fit using typical values measured elsewhere⁷ under weather conditions similar to those described in Ref. 14.

Since Fig. 8 shows that the curves are relatively insensitive to reasonable changes in the value of L , a previous result measured at Ottawa⁷ of $L = 1.1$ m is used.

Parkin and Scholes classify the particular results being studied here as being taken with zero vector wind and under neutral temperature conditions (no lapse or inversion). If this does in fact represent a calm day then the value $\langle \mu^2 \rangle$ should be relatively small. A value of $\langle \mu^2 \rangle \approx 1 \times 10^{-6}$, (between 0.6×10^{-6} and 1.5×10^{-6} , corresponding to l equal to the maximum path separation or half the maximum separation, respectively) representative of a calm day⁷, and an impedance of the ground described by $\sigma = 300$ cgs units, as discussed earlier, show (solid curve) good agreement with the experimental results (points) in Fig. 10. We can speculate that the discrepancy at 1100 m may result either from the theory not considering probable saturation effects or that slightly

refractive meteorological conditions may have existed; in either case any discrepancies would then be most noticeable at the greatest distance. The dashed curve is the spectrum predicted from coherent acoustic theory.

As a final comment we note that at a distance of 100 m the discrepancy produced by turbulence is barely apparent in Fig. 10. Hence for even shorter distances of propagation the discrepancy in the region of the "zero path-length difference" interference dip progressively disappears. However at distances under 100 m, the interference minima due to finite path-length differences have moved down the spectrum to frequencies of a few kHz^{8,9}. At these shorter distances Eq. (29) is valid and predicts that the interference minima are degraded in a way similar to that predicted by Eq. (22) over hard ground. The absence of suitable experimental results, however, places a more detailed discussion of this point beyond the scope of this thesis.

FIG. 10. Comparison of measured sound levels (points) with values predicted from the theory ($h_s = 1.8$ m, $h_r = 1.5$ m, $\sigma = 300$ cgs units). — $\langle \mu^2 \rangle \approx 1 \times 10^{-6}$, $L = 1.1$ m, --- coherent acoustic theory, ($\langle \mu^2 \rangle = 0$).

PART II

D I F F R A C T I O N
A N D
S C A T T E R I N G

5-1 Noise Reduction by Barriers

In recent years sound barriers have been widely used to protect listeners against transportation and other noise sources. The barriers are constructed between the noise source and listener alongside the highway, as shown in Fig. 11, and reduce the sound levels by blocking the "line-of-sight", so that sound reaching the listener does so by being diffracted over the top, or around the end, of the barrier.

Much investigation has been carried out for the purpose of predicting sound levels behind barriers. The work of many authors characteristically develops and extends classical optical diffraction theory. Hence, although atmospheric turbulence influences many aspects of sound propagation, its effects have not previously been considered when predicting the reduction of sound by barriers. Therefore scattering of sound by turbulence and its effects on the reduction of sound by barriers will be discussed in the following chapters.

Further, some of the basic assumptions in optical diffraction theory are now found to be too restrictive in the case of diffraction of acoustic waves. Therefore, although one of the main goals of this thesis is the study of the effects of atmospheric turbulence on diffraction, it was found that the limiting assumptions needed to be discussed

FIG. 11. A typical situation where highway noise is attenuated by diffraction at a barrier before reaching the listener.

in detail in Chapter 6 and quantified in Chapter 12 by comparing the theory and some experimental results.

5-2 Brief Review of Existing Work on Noise Reduction by Barriers

The first papers on noise control by barriers go back to 1940 by Redfern¹⁸ and to 1951 by Fehr¹⁹. These early attempts were followed by a more complete and detailed study by Maekawa^{20,21} in 1962. In these papers Maekawa developed his well-known curve for the attenuation provided by a barrier, by correcting empirically the theoretical values calculated by Kirchhoff's diffraction theory to allow for the presence of the ground. The attenuation is shown as a function of Fresnel number N , which is the difference between the shortest path over the barrier and the direct path, divided by a half wavelength. More details on Maekawa's results are found in Chapter 6.

The last decade (1970-1979) has seen a large number of studies on noise reduction by barriers. Amongst these we find the work of Kurze et al.²² for an incoherent line source, also in terms of Fresnel number. Some authors, like Kawai et al.²³, have developed simple approximate expressions based on Macdonald's²⁴ solution, again using the Fresnel number. It is worth mentioning that several studies, including Kawai et al.²³, Fujiwara et al.²⁵, Yuzawa²⁶, Pierce²⁷ and Isei²⁸, have considered the effectiveness of absorbent material on the barrier surface.

In addition, diffraction theory for different shaped obstacles, such as a wedge or a thick barrier, has been considered by Pierce²⁷ and Fujiwara et al.²⁹. An excellent summary of the work prior to 1974 on noise reduction by barriers can be found in the review paper by Kurze³⁰. However most of these prediction schemes are based on the idea of a semi-infinite screen, which does not allow for interference due to reflection from the ground.

Some of the more recent work allows for the presence of the ground, and especially interference due to waves reflected from the ground. Scholes et al.³¹ and Isei et al.³² have observed experimentally interference patterns due to reflection at the ground. Both Jonasson³³ and Thomasson³⁴ have proposed methods for calculating the noise reduction of a barrier on ground of finite impedance. In the most recent work by Isei et al.³⁵, the attenuation by barriers on the ground, as predicted by five different theories, were compared with each other and with measured results. In addition, the effects of the ground on the reduction of sound levels behind a barrier for both point and line sources, as well as the effects for absorptive barriers, were considered. One of the theoretical models, developed by the authors³⁶, uses a line integral formulation³⁶ of Kirchhoff's diffraction theory. This formulation will be discussed further in Chapter 6. Finally none of the models discussed above allows for the effects of atmospheric turbulence.

Except for a few words in some papers, there is in general very little discussion on the effects of atmospheric turbulence on the attenuation by barriers. The most extensive discussion is found in the review paper by Kurze³⁰ who points out that barrier attenuation measured outdoors is often much lower than predicted by theory or measured with models in anechoic rooms. Possible mechanisms for the explanation of this behavior are offered and include temperature and wind stratification of the atmosphere and turbulence. However most of the discussion considers refraction by wind with only a qualitative discussion on scattering of sound by turbulence. Some experimental results reported by Scholes et al.³¹ are discussed, but it is not possible to separate the effects of scattering by turbulence and refraction by wind. Based on these, and other experimental results, however, an empirical correction is offered to include atmospheric effects when predicting barrier attenuation. Finally Kurze concludes that the influence of atmospheric conditions on the attenuation performance of barriers is not well understood.

6-1 Introduction

Diffraction was defined by Sommerfeld as "... any deviation of light rays from rectilinear path which cannot be interpreted as reflection or refraction". Thus, in essence, the phenomenon of diffraction is the penetration of "radiation" into the geometrical "shadows" of opaque obstacles or of apertures in an opaque screen.

The first recorded observation of diffraction was made by Grimaldi in 1665. Contrary to the prediction of the corpuscular theory of light, Grimaldi noted that the transition from light to shadow was gradual rather than abrupt.

The first mathematical formulation of diffraction was given empirically by Fresnel in 1818 by modifying Huygens' principle of secondary wavelets. Specifically, at each observation point, Fresnel used Young's principle of interference by superposing the contributions of all the Huygens' wavelets emanating from points on the given wavefront, with due regard for their respective phases since the transit time to the observation point varies with the location of the Huygen source. Mathematically, we consider a closed surface Σ . The wave disturbance at $d \in \Sigma$ a distance r_s from a point source S can be written (see Fig. 12)

$$\psi = \frac{A}{r_s} e^{i(kr_s - \omega t)} \quad (49)$$

By supposing that all points on Σ constitute secondary sources of waves it is plausible to postulate that the contribution from any element of area $d\Sigma$ will be proportional to

- (1) The value of Ψ at the surface Σ , $(A/r_s)e^{i(kr_s - \omega t')}$, where the prime on t distinguishes the time at which the Huygen's wavelets leave points on the surface Σ in order to reach R at the desired time.
- (2) The area $d\Sigma$.
- (3) A spherical-wave factor $(1/r_r)e^{i[kr_r - \omega(t-t')]}$, where r_r is the distance from $d\Sigma$ to R and $t-t'$ is the time required for the wave to travel from $d\Sigma$ to R .
- (4) Some unknown function $f(\theta_s, \theta_r)$, called the obliquity factor, that takes into account the dependence of the amplitude of the secondary wave on the angles θ_s and θ_r as defined in Fig. 12.

Accordingly, the wavelet that arrives at R from the secondary source element $d\Sigma$ may be written

$$\begin{aligned}
 d\Psi &= K f(\theta_s, \theta_r) \left(\frac{A}{r_s} e^{i(kr_s - \omega t')} \right) \left(\frac{1}{r_r} e^{i[kr_r - \omega(t-t')]} \right) d\Sigma \\
 &= K f(\theta_s, \theta_r) \frac{A}{r_s r_r} e^{i[k(r_s + r_r) - \omega t]} d\Sigma
 \end{aligned} \tag{50}$$

where K is a normalization constant. Moreover, K must have the dimensions of inverse length in order that $d\psi$ have the same dimensions as ψ in Eq. (49). The entire disturbance at R is found by summing the complex contributions from each $d\Sigma$ of $\Delta\Sigma$, where $\Delta\Sigma$ is the part of Σ that is not obstructed by obstacles between S and R . That is,

$$\psi(R) = KAe^{-i\omega t} \int_{\Delta\Sigma} f(\theta_s, \theta_r) \frac{e^{ik(r_s + r_r)}}{r_s r_r} d\Sigma \quad (51)$$

FIG. 12. Schematic diagram used to illustrate the Huygens-Fresnel principle. The angles θ_s and θ_r are defined with respect to the normal to $d\Sigma$ on Σ .

Equation (51) is thus the mathematical formulation of the Huygens-Fresnel principle of diffraction. It enabled Fresnel to account quantitatively for most of the diffraction effects of importance in optics.

6-2 Kirchhoff-Fresnel Formulation

The first formal statement of diffraction is due to Kirchhoff in 1882. Kirchhoff arrived at his formulation by invoking Green's theorem.

a) Green's theorem:

This theorem, which is a generalisation of Gauss' theorem, equates an integral, involving two scalar fields ψ_1 and ψ_2 , over a closed surface Σ , to an integral over the volume V , enclosed by Σ . Mathematically,

$$\oint_{\Sigma} (\psi_1 \nabla \psi_2 - \psi_2 \nabla \psi_1) \cdot \hat{n} d\Sigma = \int_V (\psi_1 \nabla^2 \psi_2 - \psi_2 \nabla^2 \psi_1) dV \quad (52)$$

where $\hat{n}$ is an outward unit vector on Σ .

Applied to diffraction, the closed surface Σ is taken to enclose the receiver R , whereas in the previous section it was taken to enclose the source point S . In order to avoid the discontinuity at R , Eq. (52) is applied to a volume V surrounding, but excluding, the receiver R . Thus V is bounded by two closed surfaces Σ and Σ' (see Fig. 13) and the surface integral in Eq. (52) is broken into two portions.

$$\oint_{\Sigma} \rightarrow \oint_{\Sigma} + \oint_{\Sigma'} \quad (53)$$

For the two functions appearing in Green's theorem we first choose

$$\psi = \psi(r, t) \quad (54)$$

where $\psi(r, t)$ is a solution of the source-free wave equation inside the surface Σ , which implies that the sources that generate ψ are all located outside Σ . That is, inside Σ ,

$$\nabla^2 \psi + k^2 \psi = 0 \quad (55)$$

Secondly we take ψ_2 to be the so called "free-space" Green function.

$$\psi_2 = \frac{1}{r} e^{i(kr - \omega t)} \quad (56)$$

Likewise, Eq. (56) satisfies Eq. (55) except at R , which has been excluded from the volume V by the small surface Σ' .

Consequently, the integrand on the RHS of Eq. (52) vanishes everywhere throughout V ,

$$\int_V (\psi \nabla^2 \psi_2 - \psi_2 \nabla^2 \psi) dV = 0 \quad (57)$$

We are left with the two surface integrals of Eq. (53)

$$\oint_{\Sigma} (\psi \nabla \psi_2 - \psi_2 \nabla \psi) \cdot \hat{n} d\Sigma + \oint_{\Sigma'} (\psi \nabla \psi_2 - \psi_2 \nabla \psi) \cdot \hat{n} d\Sigma = 0 \quad (58)$$

Taking Σ' to be a spherical surface of radius ϵ centered on R we find

$$\oint_{\Sigma'} \left[\psi \nabla \left(\frac{1}{r} e^{i(kr - \omega t)} \right) - \left(\frac{1}{r} e^{i(kr - \omega t)} \right) \nabla \psi \right] \cdot \hat{n} d\Sigma \quad (59)$$

$$\xrightarrow{\epsilon \rightarrow 0} 4\pi \psi(R, t) e^{-i\omega t}$$

where $\Psi(R, t)$ is the value of $\Psi(r, t)$ at the receiving point.

This is the quantity we wish to know, thus

$$\Psi(R, t) = -\frac{1}{4\pi} \oint_{\Sigma} \left[\Psi \nabla \left(\frac{1}{r} e^{ikr} \right) - \left(\frac{1}{r} e^{ikr} \right) \nabla \Psi \right] \cdot \hat{n} d\Sigma \quad (60)$$

Eq. (60) enables one to find the wave amplitude Ψ at a receiving point R from the value of Ψ and $\hat{n} \cdot \nabla \Psi$ over some enclosing surface Σ . Further progress depends upon finding reasonable approximation to these values on Σ .

FIG. 13. Surfaces Σ and Σ' enclosing the receiver R . $\hat{n}$ is an outward normal to $d\Sigma$ on Σ .

b) Kirchhoff Boundary Conditions:

We consider a receiving point R entirely surrounded by an opaque screen Σ except for one or more apertures of finite extent, $\Delta\Sigma$ (see Fig. 14). At S , a distance r_s from $d\Sigma$ on $\Delta\Sigma$, there is a point source, hence the field ψ at $d\Sigma$ is

$$\psi = \frac{A}{r_s} e^{i(kr_s - \omega t)} \quad (61)$$

FIG. 14. Geometry involved in the Fresnel-Kirchhoff formulation.

In order to evaluate Eq. (60) one traditionally makes the following assumptions:

- At the aperture $\Delta\Sigma$,

$$\psi = \frac{A}{r_s} e^{i(kr_s - \omega t)} \quad (62)$$

$$\hat{n} \cdot \nabla \psi = \frac{\partial \psi}{\partial n} = \left(\frac{1}{r_s} - ik \right) \frac{A}{r_s} e^{i(kr_s - \omega t)} \cos \theta_s$$

- At the screen Σ other than at the aperture,

$$\psi = \frac{\partial \psi}{\partial n} = 0 \quad (63)$$

These assumptions are known as the "Kirchhoff boundary conditions". They are merely physically plausible approximations. In fact, it can be shown that Eq. (63) overdetermines the boundary conditions, and that moreover the assumptions in Eq. (63) are mathematically inadmissible. There is a well known theorem which states that if ψ vanishes together with its normal derivative along a finite curve segment, then ψ vanishes identically in the whole plane. However, so long as the aperture dimensions and the distances of R and S from the screen are large compared with the wave length - The limit known as geometrical optics - the Kirchhoff assumptions give satisfactory accuracy.

Using Eqs. (62) and (63) in Eq. (60) we obtain in the limit $r_s, r_r \gg \lambda$, as discussed above

$$\psi(R, t) = - \frac{iA}{\lambda} e^{-i\omega t} \int_{\Delta\Sigma} \left(\frac{\cos \theta_s + \cos \theta_r}{2} \right) \frac{e^{ik(r_s + r_r)}}{r_s r_r} d\Sigma \quad (64)$$

Eq. (61) is known as the Kirchhoff-Fresnel diffraction formula.

By comparing Eq. (64) and (51) we can identify the proportionality coefficient

$$K = -\frac{i}{\lambda} \quad (65)$$

and the obliquity factor

$$f(\theta_s, \theta_r) = \frac{\cos \theta_s + \cos \theta_r}{2} \quad (66)$$

c) Solution in the Case of a Semi-infinite screen:

If, in addition to the limitations $r_s, r_r \gg \lambda$, other approximations are introduced in Eq. (64), it is possible to obtain simple solutions to a few diffraction problems. The restriction most commonly introduced is the paraxial approximation, according to which the aperture $\Delta\Sigma$, in an opaque screen separating source from receiver, is assumed to be small compared with the distances r_s and r_r . In essence this approximation ensures that the obliquity factor is approximately constant over the aperture and thus permits geometrical simplifications in the integrand of Eq. (64).

The diffraction at the edge of a thin, semi-infinite, half plane, as illustrated in Fig. 15, is an example. If Eq. (64) is simplified according to the paraxial approximation the field at R in Fig. 15 can be written³⁷

$$\psi = -\frac{iA}{2} \frac{e^{ik(r_s + r_r)}}{(X_s + X_r)} (1+i) \left[\frac{1+i}{2} - \int_0^{\infty} e^{i(\pi/4)v^2} dv \right] \quad (67)$$

where

$$\mathcal{J} = (2N)^{1/2} = [4(r_s + r_r - d)/\lambda]^{1/2} \quad (68)$$

Prediction schemes like the curves of Maekawa or Kurze for noise reduction by barriers, which rely on Kirchhoff-Fresnel diffraction theory are based on Eq. (64). The curve in Fig. 16 is the attenuation predicted by Eq. (64) for a semi-infinite screen and shows the characteristic fall of 10 dB per decade of frequency for $N > 1$.

FIG. 15. Schematic diagram showing a point source S, a receiver R and an infinite thin screen. The Fresnel number $N = 2[(r_s + r_r) - d]/\lambda$ where $(r_s + r_r)$ is the path length between the sound ray diffracted over the edge of the screen and d the direct ray from the source to the receiver when the screen is absent.

6-3 Young-Rubinowicz Line-integral Formulation

Some of the limitations in the evaluation of Eq. (60) can be removed. In 1917, Rubinowicz showed that Kirchhoff's surface integral could be transformed into an equivalent line integral around the edge of the aperture.

FIG. 16. Graph showing the characteristic fall of 10 dB per decade of frequency for Fresnel number $N > 1$ of sound diffracted at the edge of a thin semi-infinite screen.

Again assuming ψ given by Eq. (61) we write Eq. (60) as

$$\psi = -\frac{A}{4\pi} e^{-i\omega t} \oint_{\Sigma} \left[\frac{e}{r_s} \nabla \left(\frac{e}{r_r} \right) - \frac{e}{r_r} \nabla \left(\frac{e}{r_s} \right) \right] \cdot \hat{n} d\Sigma \quad (69)$$

We let the given aperture $\Delta\Sigma$ in an opaque screen define a conical surface $\Delta\Sigma'$ by extending generators from the point source through the aperture boundary, as shown in Fig. 17. The aperture $\Delta\Sigma$, the truncated cone $\Delta\Sigma'$, and a spherical cap at infinity constitute a closed mathematical surface, over which the

FIG. 17. Special surface of integration to establish the Young-Rubinowicz formula which is a line integral formulation of the Kirchhoff-Fresnel diffraction theory.

integral, Eq. (69), can be evaluated. Symbolically

$$\int_{\text{aperture}} \dots d\mathcal{E} + \int_{\text{cone}} \dots d\mathcal{E} + \int_{\text{cap}} \dots d\mathcal{E} = \begin{cases} (A/r) e^{i(kr - \omega t)} & , R \text{ in direct beam} \\ 0 & , R' \text{ in geometrical shadow} \end{cases} \quad (70)$$

It can be shown that the integral over the cap vanishes. The integral over the aperture is precisely that of the Fresnel-Kirchhoff theory, using the approximate Kirchhoff boundary conditions Eqs. (62) and (63). Thus Eq. (70) is rearranged to obtain

$$\Psi(R, t) \xrightarrow[\text{approx.}]{\text{Kirchhoff}} \int_{\text{aperture}} \dots d\mathcal{E} = \begin{cases} \frac{A}{r} e^{i(kr - \omega t)} \\ 0 \end{cases} - \int_{\text{cone}} \dots d\mathcal{E} \quad (71)$$

It is then not difficult to show that the integral over the conical surface $\Delta\mathcal{E}'$ can be transformed to a line integral around the aperture boundary (see for example Elmore and Heald²⁰). Thus we find (see Fig. 18)

$$\Psi(R, t) = \begin{cases} (A/r) e^{i(kr - \omega t)} & , R \text{ in direct field} \\ 0 & , R' \text{ in geometrical shadow} \end{cases} - \frac{A}{4\pi} e^{-i\omega t} \oint_{\substack{\text{aperture} \\ \text{boundary}}} \left[\frac{\cos(\rho_r, n) \sin(\rho_s, \delta l)}{1 + \cos(\rho_s, \rho_r)} \right] \times \frac{e^{ik(\rho_s + \rho_r)}}{\rho_s \rho_r} dl \quad (72)$$

where r is the distance from source S to receiver R , ρ_s and ρ_r are the distances from source and receiver respectively, to the line element of the aperture boundary δl , and $\hat{n}$ is the outward normal to δl and ρ_s .

Eq. (72) is the Young-Rubinowicz diffraction formula. This formulation is entirely equivalent to the Fresnel-Kirchhoff formula, and is subject to the same limitations inherent in Kirchhoff's approximate boundary conditions on the surface of the screen, viz. both $\psi = 0$ and $\partial\psi/\partial n = 0$. However the Young-

FIG. 18. Schematic diagram showing source and receiver on opposite sides of an opaque screen containing an aperture. The sound field can be expressed as an integral along line elements of the boundary of the aperture, δl . The unit vector $\hat{n}$ is the outwardly drawn normal to δl and to the vector from S .

Rubinowicz formula easily lends itself to numerical integration and therefore by not requiring a constant obliquity factor, i.e. the paraxial approximation, provides a more accurate formulation.

This formulation has been recently exploited by Embleton³⁴ in the case of a thin, semi-infinite screen. Let the aperture around which the line integral is required be composed of the infinite half-plane above the barrier edge, and let the opaque screen be the barrier itself. Hence the line integral is along the straight edge of the barrier, of infinite length, plus the semi-circular arc at infinity that connects the two ends of the barrier.

FIG. 19. Schematic diagram from Ref. 36 showing the coordinate system and parameters used to express the sound field at receiver R due to a point source at S . Distances r_s , r_r , and angle θ define the given barrier configuration. Distances ρ_s and ρ_r , and angle β change with the integration variable l .

It is evident that the integral of Eq. (72) with respect to dl is zero around the arc at infinity owing to the product $\rho_s \rho_r$ in the denominator and that the integrand tends to zero at the ends of the edge of the barrier because $\sin(\rho_s, \delta l) \rightarrow 0$. Figure 18 is thus transformed into the familiar geometry of a straight edge sound barrier, see Fig. 19. The line element δl at L is a distance l along the edge of the barrier from B . The lengths ρ_s and ρ_r are defined in Fig. 18 and vary through the integration. The lengths r_s and r_r are constants for a fixed source, receiver, and barrier geometry and with the angle θ specify the source and receiver locations relative to the edge of the barrier. Applying a few elementary geometrical relations the line integral of Eq. (72) along the edge of the barrier becomes

$$\psi(R, t) = -\frac{A}{2\pi} e^{i\omega t} r_s \sin \theta \int_0^{\pi/2} \frac{\exp[ik \sec \beta (r_r + \rho)] \cos^2 \beta d\beta}{\rho(r_r \sin^2 \beta + \rho - r_s \cos \theta \cos^2 \beta)} \quad (73)$$

where β is defined in Fig. 19 and

$$\rho = (r_s^2 \cos^2 \beta + r_r^2 \sin^2 \beta)^{1/2}$$

Although Eq. (73) is limited by the Kirchhoff boundary conditions, the correct obliquity factor is included implicitly in the integrand during any numerical evaluation. Hence Eq. (73) appears to be one of the more rigorous solutions obtained from Kirchhoff-Fresnel theory for the problem of diffraction at a thin, semi-infinite barrier. The predictions of Eq. (73) will be compared with those of Eq. (64) in Chapter 12.

6-4 Layer Potential Theory and Acoustic Diffraction

It will be shown experimentally in Chapter 12 that Kirchhoff-Fresnel diffraction theory is not adequate to predict precisely the attenuation of acoustic waves by a sound barrier when source and receiver are only a few meters from the barrier. This is because of the limitations introduced by Kirchhoff's boundary conditions discussed in the previous section.

In recent years more rigorous mathematical concepts have been developed which are expected to overcome some of the limitations of diffraction theory applied to acoustics. In particular it has been recently shown by Filippi and Dumery³⁹ that acoustic diffraction problems can be solved by so called "layer potential theory" which is based on concepts of distribution theory. These concepts have been used successfully by Cassot⁴⁰ and Daumas⁴¹ in respective diffraction problems.

Up till now we have only considered in detail the diffraction of acoustic waves by a thin semi-infinite screen where both source and receiver are in free space. In reality, for noise reduction as discussed in Chapter 5, a barrier of finite height is placed on a ground surface which can be acoustically hard or of finite impedance. The source and receiver are then on, or a few meters above the ground surface, as illustrated in Fig. 20.

Consider the case where both the source S and receiver R are a few meters above a hard perfectly reflecting plane surface. It has been shown³⁹ mathematically that, in this case,

FIG. 20. Schematic diagram showing a thin screen σ_0 above a ground surface and between a source S and receiver R .

FIG. 21. Schematic diagram showing how diffraction at a thin screen σ_0 above a hard, perfectly reflecting surface is mathematically equivalent to diffraction at a thin strip $\sigma = 2\sigma_0$. The source above the surface is replaced by the source S and its image I .

the problem illustrated in Fig. 20 is rigorously and exactly equivalent to that illustrated in Fig. 21 of a source S and its image I in the presence of a thin semi-infinite strip; $\sigma = 2\sigma_0$. It is further shown that the diffraction field measured at R in Fig. 21 can be written

$$\psi(R) = \int_{\sigma} \mu(P) \frac{\partial G(R,P)}{\partial_n(P)} d\sigma(P), \quad P \in \sigma \quad (74)$$

where $\mu(P)$ is the density of the double layer potential on $\sigma = 2\sigma_0$ and G is an elementary solution that satisfies

$$(\nabla^2 + k^2)G = \delta \quad (75)$$

Equation (74) must be solved by numerical methods by first finding a series representation for $\mu(P)$ from the boundary conditions and then evaluating the integral over the surface σ of the screen. Consequently the difficulty and complexity involved in evaluating Eq. (74) places this task beyond the scope and purpose of this thesis. (The reader is referred to Ref. 39 for more details).

However, the work of Daumas⁴ in this respect will be useful in Chapter 12. In the case of an incident cylindrical wave Daumas has obtained from Eq. (74) the solid curve illustrated in Fig. 22 for S on the ground, and at a distance 27.94 λ from the screen σ of height 5.88 λ . The dashed curve in Fig. 22 is obtained from Kirchhoff-Fresnel diffraction theory by substituting an incident cylindrical wave in Eq. (60), instead of the incident spherical wave of Eq. (61), and invoking the paraxial approximation. The presence of the ground is considered by the method discussed

FIG. 22. Schematic diagram and graph illustrating the model experiment of Ref. 41. The solid curve and dashed curve in the graph show the different prediction of double layer potential diffraction theory and classical Kirchhoff-Fresnel diffraction theory, respectively.

FIG. 23. Schematic diagram showing the classical approximation of Kirchhoff-Fresnel diffraction theory to the problem of diffraction at a thin screen σ_0 above a hard perfectly reflecting surface. The diffraction at a strip, $\sigma = 2\sigma_0$, is approximated by the sum of diffraction at two semi-infinite screens.

by Isei et al.³⁵ or, equivalently by splitting the strip of Fig. 21 into two semi-infinite barriers as illustrated in Fig. 23, where the two barriers are thin semi-infinite screens with S and I approaching each other if the source is on the ground. The points are results obtained by Daumas from a model experiment and will not be considered further in this thesis. In the case of incident cylindrical waves, Fig. 22 shows that the sound levels along D_1 on hard ground predicted by Eq. (74) are higher than the levels predicted by Eq. (60), especially at distances of a few wavelengths from the barrier.

If the diffraction at a thin barrier σ placed on a flat, perfectly reflecting surface is rigorously equivalent to the diffraction of sound from the source S and its image I at a thin strip; $\sigma = 2\sigma$ (see Figs. 20 and 21), then, the solution provided by the picture illustrated in Fig. 23 of the sum of diffractions at two thin, semi-infinite half-planes is only a reasonable approximation to the problem of Fig. 20. The two approaches differ in the possibility of one edge of the strip interacting with the field diffracted by the other edge of the strip. According to the Young-Rubinowicz formulation of Kirchhoff-Fresnel theory the evaluation of Eq. (72) in both the cases illustrated in Figs. (21) and (23) leads to the same result. Indeed elementary symmetry considerations show that the solution in the two cases is the sum of identical terms given by Eq. (73). This will be discussed further in Chapter 12.

6-5 Author's Empirical Correction to Classical Theory

Eq. (74) shows the relevance and importance of describing the details of the incident field over the surface of the barrier. Although the Kirchhoff-Fresnel line integral formulation allows for the spherical curvature of the incident field along the edge of the barrier, because of the approximations in the Kirchhoff boundary conditions it cannot account for this curvature in a direction perpendicular to the edge, and thus over the whole surface of the barrier. Hence it should be expected that the Kirchhoff-Fresnel formulation may fail to predict sound levels when r_s and r_r , defined in Fig. 19, are small. These are the cases where the curvature of the incident wavefront in a plane perpendicular to the barrier surface is most pronounced.

It is therefore postulated that the sound levels predicted by Eq. (73) can be corrected empirically by adding to the levels a term of the form

$$\Delta = K \left(\frac{1}{r_s} + \frac{1}{r_r} \right) \quad (76)$$

Eq. (76) will be useful when comparing experimental results with diffraction theory in Chapter 12 by using K as a constant adjusted for best fit.

Chapter 7 SURVEY OF SCATTERING BY TURBULENCE

7-1 Introduction

For acoustic waves the atmosphere is a continuous medium with random variations of pressure, density, temperature and velocity that locally affect the phase velocity of acoustical propagation. A convenient description of observed phenomena is that some of the incident sound energy is scattered by the inhomogeneities of the medium. It will be shown in a later section of this chapter that waves are scattered predominantly in the forward direction. When superimposed upon the primary waves, the scattered waves produce fluctuations of amplitude and phase in the resultant field. Therefore it is convenient to divide the discussion of this problem into "scattering" of a wave and "line-of-sight" wave propagation.

Line-of-sight propagation theory seeks to evaluate the magnitudes and correlations of the phase and amplitude fluctuations in the resultant field. This aspect of wave propagation was considered in Chapters 1 through 4 using Rytov's method, and applied to the problem of interference between a direct and ground reflected sound field.

However, consider a situation where a barrier, or opaque screen, is placed between source and receiver as illustrated in Fig. 15. Although fluctuations are observed along r_s and r_r , the field at the top of the barrier is not simply bent by refraction, but is diffracted. Hence "line-of-sight" methods as

they are known are not valid in this case.

On the other hand, scattering allows one to evaluate the total energy (or field) which is scattered from a volume of the random medium. It is reasonable to assume that the total energy received at R is the sum of the energy resulting from the diffraction at the barrier edge and the energy scattered by the inhomogeneities of the atmosphere above the barrier.

There is an extensive literature on scattering by turbulence in the atmosphere. This chapter summarizes the relevant concepts and equations which are needed in subsequent chapters.

7-2 The Scattering Cross Section

We consider a volume V of the random medium irradiated by an incident field Ψ . Consider an elementary volume δV in V , a distance r from a receiver R as illustrated in Fig. 24. We assume that the randomness of the medium is sufficiently slight that the wave incident on δV is almost equal to the incident wave in the absence of randomness. This constitutes the single scattering approximation.

We further assume that the elementary volume δV is small enough that the incident wave can be considered a plane wave throughout δV . At the same time, we take δV to be large enough so that the dimension D ($\delta V \sim D^3$) is much greater than the correlation length L of the medium. This corresponds to the Fraunhofer approximation and requires that D is smaller than the Fresnel size $\sqrt{\lambda r}$. However, it has been established⁵ that calculations

done in this way remain valid under the weaker condition $L \ll \sqrt{\lambda r}$.

Under this latter assumption, the scattered fields from different elementary volumes are uncorrelated and therefore scattered energies rather than scattered pressure fields from the different volume elements can be added. Thus the total received scattered energy is

$$E_s = \int_V \psi^2 \frac{\sigma(\theta)}{r^2} dV \quad (77)$$

where $\sigma(\theta)$ is the scattering cross section per unit scattering

FIG. 24. Schematic diagram illustrating an arbitrary volume V of turbulent atmosphere. θ is the scattering angle through which some of the incident sound ψ , incident on an elementary volume δV , is scattered to R .

volume per unit solid angle.

The scattering cross section for a random medium is well known. The classical derivation can be found in Tatarskii⁵ or, with more rigorous theory, in Wenzel and Keller^{4a}. The classical derivation consists of solving the differential equation of sound propagation in a turbulent medium obtained from Euler's equation of motion. If u_i is the given velocity of turbulent motion and, taking into account the fluctuating temperature by writing $T = T_0 + T'$, it is possible to obtain

$$\nabla^2 \varphi + k^2 \varphi = - \frac{\partial}{\partial x_i} \left(\frac{T'}{T_0} \frac{\partial \varphi}{\partial x_i} \right) - \frac{2}{i\omega} \frac{\partial^2}{\partial x_i \partial x_j} \left(u_i \frac{\partial \varphi}{\partial x_j} \right) \quad (78)$$

Equation (78) is solved in the single scattering approximation (first order or Born approximation) by writing

$$\varphi = \varphi_0 + \varphi_s \quad (79)$$

where φ_0 is the incident wave satisfying the equation $\nabla^2 \varphi_0 + k^2 \varphi_0 = 0$, and φ_s is the scattered field. The solution of Eq. (78) is

$$\varphi_s(r) = \frac{1}{4\pi} \int_V d^3 r' \frac{e^{ik|r-r'|}}{|r-r'|} \frac{\partial}{\partial x'_i} \left[\frac{T'(r')}{T_0} \frac{\partial \varphi_0(r')}{\partial x'_i} + \frac{2}{i\omega} \frac{\partial}{\partial x'_i} \left(u_i(r') \frac{\partial \varphi_0(r')}{\partial x'_j} \right) \right] \quad (80)$$

Since we have assumed that the field incident on δV is a plane wave, $\varphi_0(r') = A e^{ikr'}$. Solving Eq. (80) yields the scattering cross section

$$\sigma(\theta) = 2\pi k^4 \cos^2 \theta \Phi_n(k) \quad (81)$$

where θ is the scattering angle, Φ_n is the spectral density of the fluctuations of acoustic refractive index and $k = 2k \sin(\theta/2)$.

We recognize in Eq. (81) the familiar k^4 power of Rayleigh scattering.

It is now necessary to express the spectral density Φ_n in terms of measurable parameters. This depends on the model assumed for the turbulent atmosphere.

7-3 Homogeneous and Isotropic Turbulence

In the case of homogeneous isotropic turbulence the fluctuating acoustical index of refraction, Eq. (31), is assumed to have a spatial correlation function of Gaussian form as given by Eq. (36)

$$\langle \mu, \mu_2 \rangle = \langle \mu^2 \rangle \exp(-\tau^2/L^2) \quad (36)$$

Then according to the Wiener-Khinchin theorem, the spectral density Φ_n is the Fourier transform of the correlation function

$$\Phi_n(k) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \langle \mu, \mu_2 \rangle \exp(i \vec{k} \cdot \vec{\tau}) d^3 \tau \quad (82)$$

Substituting Eq. (36) into Eq. (82) we find

$$\Phi_n(k) = \frac{\langle \mu^2 \rangle L^3}{8\pi\sqrt{\pi}} \exp\left[-\frac{(kL)^2}{4}\right] \quad (83)$$

where, as above, $k = 2k \sin(\theta/2)$. The scattering cross section, Eq. (81) then becomes

$$\sigma(\theta) = \frac{\langle \mu^2 \rangle}{4\sqrt{\pi}} k^4 L^3 \cos^2 \theta \exp\left[-(kL \sin \frac{\theta}{2})^2\right] \quad (84)$$

Although the model of homogeneous isotropic turbulence can sometimes be adequately used and justified in the case of line-of-sight fluctuations, Eq. (84) cannot fully explain detailed characteristics of the actual scattering phenomena in the

atmosphere. For example Eq. (84) does predict small angle, forward scattering, but also predicts that the scattering is a maximum at a frequency $f_m = ck_m/2\pi$ where

$$k_m^2 = \frac{a}{L^2 \sin^2 \theta/a} \quad (85)$$

Eq. (85) is not observed experimentally. Furthermore, Eq. (84) does not provide an explanation for the experimental results to be discussed in Chapter 12.

7-4 Locally Homogeneous and Isotropic Turbulence

The inadequacies in Eq. (84) are not surprising since homogeneous and isotropic turbulence provides only a crude approximation to real meteorological fields. For example the mean wind velocity and mean temperature in a turbulent atmosphere vary as a function of height above the ground. Furthermore these mean values experience slow and smooth changes such that they can be considered constant only over a limited time.

However, when we consider the velocities or temperatures at two different points in the medium and take their difference, this difference is almost homogeneous over a wide range of spatial distances. If we denote a non-homogeneous random function of this type by $f(\vec{r})$, then the function $[f(\vec{r} + \vec{r}_d) - f(\vec{r})]$ is homogeneous even though $f(\vec{r})$ itself is not homogeneous.

Mathematically, such a random function is called a random process with stationary increments as a function of time, and a locally homogeneous random function as a function of position.

Such random processes can be described most conveniently in terms of "structure functions".

Consider a locally homogeneous random scalar field $f(\vec{r})$. Then the structure function depends only on $\vec{r} = \vec{r}_2 - \vec{r}_1$ and is given by

$$D_f(\vec{r}) = \langle |f(\vec{r}_1 + \vec{r}) - f(\vec{r}_1)|^2 \rangle \quad (86)$$

The spectral representation of the locally homogeneous random process can also be obtained. The result can be written

$$D_f(\vec{r}) = 2 \iiint_{-\infty}^{\infty} (1 - \cos \vec{k} \cdot \vec{r}) \Phi_f(\vec{k}) d^3k \quad (87)$$

where Φ_f is the spectral density. The inverse of Eq. (87) can be obtained by taking its gradient and inverting the resulting Fourier integral.

If the locally homogeneous process is also locally isotropic, then the structure function depends only on the magnitude of $\vec{r}$, not its direction:

$$D_f(|\vec{r}|) = D_f(r) = \langle |f(\vec{r}_1 + \vec{r}) - f(\vec{r}_1)|^2 \rangle \quad (88)$$

Similarly the spectral density Φ_f becomes a function of $|\vec{k}|$ only.

The spectral density $\Phi_f(k)$ is a three-dimensional spectrum. For a locally isotropic field we may also introduce a one-dimensional spectrum $V_f(k)$ by writing the spectral representation of the structure function along one of the coordinate axes, say the x axis:

$$D_f(r) = D_f(x) = 4 \int_0^{\infty} (1 - \cos kx) V_f(k) dk \quad (89)$$

It is not difficult to show that

$$\Phi_f(k) = - \frac{1}{2\pi k} \frac{\partial V_f(k)}{\partial k} \quad (90)$$

In the case of a locally homogeneous random vector field $\vec{v}(\vec{r})$ the structure $D_{ik}(\vec{r})$ becomes a tensor:

$$D_{ik}(\vec{r}) = \langle [v_i(\vec{r} + \vec{r}_1) - v_i(\vec{r}_1)] [v_k(\vec{r} + \vec{r}_1) - v_k(\vec{r}_1)] \rangle \quad (91)$$

with spectral representation

$$D_{ik}(\vec{r}) = 2 \iiint_{-\infty}^{\infty} (1 - \cos \vec{k} \cdot \vec{r}) \left(\delta_{ik} - \frac{k_i k_k}{k^2} \right) \Phi_v(\vec{k}) d^3 k \quad (92)$$

With the additional assumption of local isotropy Eqs. (91) and (92) can be simplified in ways similar to the case of a scalar field. However the tensor nature of these equations obviously makes the mathematics more elaborate. For further details the reader is referred to Tatarskii⁶. It suffices here to discuss specifically the spectral densities of the temperature and wind velocity fields to be used in Eq. (81) for the case of locally homogeneous turbulence.

a) Kolmogorov Spectrum:

In 1941, Kolmogorov advanced an important theory of local structure of turbulence from which the spectral densities of the temperature and velocity fields can be modeled.

According to Kolmogorov, the energy is injected into the turbulence as a result of instabilities in the wind shear and temperature gradient. For example, near the ground, the average horizontal wind velocity varies as a function of height

(being essentially zero at the ground surface) and this variation creates turbulence of a size approximately equal to the height. The size at which the energy enters into the turbulence is called the outer scale of turbulence and is designated by L_0 . The eddies of sizes greater than L_0 are generally anisotropic. The spectrum in this range depends on how the turbulence is created for the particular case, and thus there is no general formula describing the turbulence characteristics in this range.

In the inertial range of the spectrum, where the eddy size is smaller than L_0 , the kinetic energy is very much greater

FIG. 25. Schematic graph illustrating the three characteristic ranges of the spectral density of the turbulent atmosphere. L_0 and l_0 are respectively the outer and inner scale of turbulence and $K = 2\pi/L$.

than the dissipation due to viscosity. Since the dissipation is negligible almost all the kinetic energy can be transferred to eddies of smaller size. Thus, the energy transfer can be visualized as a process of eddy fragmentation where large-scale eddies cascade into eddies of ever decreasing size.

However, as the eddy size becomes smaller, the dissipation increases relative to the available kinetic energy. Eventually the smallest size l_0 of the eddies is reached where the kinetic energy is of the same order as the dissipation of energy by viscosity. At this size l_0 , virtually all the energy is dissipated into heat and practically no energy is left for eddies of size smaller than l_0 . This size l_0 is called the inner scale of turbulence and the spectrum range of eddy size smaller than l_0 is called the dissipation range (see Fig. 25).

In the inertial range ($L_0 > \text{eddy size} > l_0$) the influences of the initial conditions disappear, the fluid motion is almost completely random and irregular, and its characteristics can be described only in statistical terms. Kolmogorov's hypothesis is that the small scale structure of motion has the property of being locally homogeneous and isotropic. From this beginning Kolmogorov and Obukhov were able to deduce that the structure function of the temperature and velocity fluctuations, $D_T(r)$ and $D_V(r)$ respectively, should have the form

$$D_T(r) = \langle |T(\vec{r}_1 + \vec{r}) - T(\vec{r}_1)|^2 \rangle = C_T^2 r^{2/3} \quad (93)$$

and, similarly

$$D_V(r) = C_V^2 r^{2/3} \quad (94)$$

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Eqs. (93) and (94) are known as the "two-thirds law" and the proportionality constants, C_T^2 and C_V^2 , as the temperature and velocity structure constants respectively.

Since the temperature is a scalar field the spectral density of the temperature field, $\Phi_T(k)$ can be obtained by inverting Eq. (87) and substituting Eq. (93) to yield

$$\Phi_T(k) = 0.033 C_T^2 k^{-4/3} \quad (95)$$

Similarly using Eq. (94) and the theory for a locally homogeneous and isotropic vector field the spectral density of the velocity field can be obtained:

$$\Phi_V(k) = 0.061 C_V^2 k^{-4/3} \quad (96)$$

When solving Eq. (80) in the case of locally homogeneous and isotropic turbulence the spectral density $\Phi_n(k)$ in Eq. (81) is identified with Eq. (95) and (96) and is given by

$$\Phi_n(k) = \left(\frac{1}{c_o^2}\right) \Phi_V(k) \cos^2(\theta/2) + \left(\frac{1}{4T_o}\right) \Phi_T(k) \quad (97)$$

Thus the cross section Eq. (81) can be written

$$\sigma(\theta) = 0.38 k^{1/3} \frac{\cos^2 \theta}{(2 \sin \theta/2)^{1/3}} \left[\frac{C_V^2}{c_o^2} \cos^2(\theta/2) + 0.13 \frac{C_T^2}{T_o} \right] \quad (98)$$

where c_o and T_o are the mean values of the sound velocity (m/s) and the temperature ($^{\circ}$ K) respectively. Equation (98) has a different frequency dependence from Eq. (84) but still predicts the strong forward nature of the scattering. In fact for very small scattering angles Eq. (98) very quickly exceeds unity, and when $\theta \rightarrow 0$, $\sigma(\theta) \rightarrow \infty$. This is a consequence of the particular model of single scattering approximation as discussed earlier in this section. Thus Eq. (98) should only be used for

scattering angles where $\sigma(\theta) \ll 1 \text{ m}^{-1}$.

b) Taylor's Frozen Turbulence Hypothesis:

In Eq. (98) it is necessary to know the structure constants of the temperature and velocity fields, C_T^a and C_V^a respectively. In principle, these can be obtained directly by measuring the respective structure functions as in Eqs. (93) and (94). Unfortunately these measurements are usually difficult.

Eqs. (95) and (96) on the other hand provide an indirect means of obtaining C_T^a and C_V^a . However the direct measurement of these spectral densities involves a spatial sampling of the temperature and velocity fields in three-dimensions which is prohibitive. For locally isotropic fields it would suffice to measure the spatial structure in one direction to obtain the one-dimensional spectral densities and then use Eq. (90). However even this measurement is impractical and difficult.

For this reason Taylor advanced his "frozen turbulence" hypothesis, according to which the entire spatial pattern of a random field is transported with the mean wind velocity $\langle V \rangle$. Thus the one-dimensional spatial spectrum $V(k)$ of a locally isotropic field is related to its time (frequency) spectrum $W(\omega)$ according to

$$V(k) = \langle V \rangle W(\omega) \quad (99)$$

where $\omega = \langle V \rangle k$. Consequently the structure functions of the turbulence can be obtained by a measurement as a function of time at one point in space.

If $V_T(k)$ is the one-dimensional spectrum of the fluctuating temperature field we obtain from Eqs. (95) and (90),

$$V_T(k) = \frac{6\pi}{5} (0.033 C_T^2) k^{-5/3} \quad (100)$$

From Eq. (99) we then find

$$W_T(\omega) = \frac{6\pi}{5} (0.033 C_T^2) \langle V \rangle^{2/3} \omega^{-5/3} \quad (101)$$

Similarly from Eq. (96) we obtain the time (frequency) spectrum $W_V(\omega)$ of the fluctuating wind velocity field,

$$W_V(\omega) = \frac{6\pi}{5} (0.061 C_V^2) \langle V \rangle^{2/3} \omega^{-5/3} \quad (102)$$

It can be shown that Taylor's hypothesis is adequately justified in the inertial range. Furthermore there is experimental verification of the "frozen field" hypothesis (see Ref. 5 for example). Thus measurements based on Eqs. (101) and (102) are used to calculate the two structure constants C_t^2 and C_v^2 as described below in Chapter 11.

Chapter 8 AUTHOR'S EVALUATION OF THE SCATTERING

VOLUME INTEGRAL

8-1 Limits of Integration

As discussed in Chapter 7 we wish to estimate the amount of sound energy scattered by the atmospheric inhomogeneities above a sound barrier using Eq. (77).

In Fig. 26 the region abc above the barrier defines the effective scattering volume V. In three dimensions bc is a spherical cap on which the scattering angle θ is 90° . According to Eq. (81) there is no scattering at 90° and it was found that the backscattered energy from the region above bc produces a negligible contribution at R. The two surfaces ac and ab are semi-circular segments of plane surfaces bounded by the spherical cap bc and the barrier edge.

The plane surface ac lies along the line-of-sight between barrier edge and receiver. Any energy scattered below the surface ac must be subsequently diffracted by the barrier to reach the receiver and can thus be neglected.

As we go through the plane surface ab which lies along the line-of-sight between barrier edge and source (along the dashed line illustrated in Fig. 26(a) for example) we pass from the irradiated region into the shadow of the barrier. The field as a function of distance along the dashed line, calculated from Eq. (73), is shown in the sketch on the RHS of Fig. 26. Therefore, because the field is attenuated by diffraction in

FIG. 26. Schematic diagram showing the specific scattering volume, bounded by the three surfaces between a, b, and c, from which a significant amount of sound energy is scattered by the atmospheric turbulence to R. The sketch on the upper RHS illustrates the variation of the incident field (re free field) along a typical line through the surface ab.

the shadow, scattering from this region is negligible and the surface ab thus provides a natural limit for the integration.

We must however remark that according to Eq. (98) the amount of scattered energy increases rapidly for small angles of scattering. Hence it may be expected that some of the scattered energy comes from the region below the surface ab , where θ is smallest, even though energy incident on scattering elements in this region of space has been much reduced by the process of diffraction into the shadow of the barrier. But for such small scattering angles, the cross section Eq. (98) is close to, or greater than, unity and the single scattering hypothesis which Eq. (98) embodies is no longer valid (see the discussion following Eq. (98)). It must be expected however that the turbulence in this region will produce line-of-sight fluctuations. Secondly, it follows from the above discussion that most of the scattered energy comes from the portion of the scattering volume just above the edge of the barrier where θ is smallest. Therefore to respect the validity of Eq. (98) we are restricted to diffraction angles greater than about 20° . This restriction ensures that $\sigma(\theta)$ is much smaller than 1 m^{-1} for realistic values of C_T^a and C_V^a and for all other scattering angles.

8-2 Numerical Evaluation of the Total Scattered Energy

After substituting the scattering cross section Eq. (98) into Eq. (77), the evaluation of the resulting integrals

over the volume V defined in Fig. 26 is prohibitively complex by analytical methods. Therefore a numerical evaluation of Eq. (77) was performed on a medium sized (PDP 11-34) computer.

The basis of the numerical evaluation is to choose elementary volumes δV in V and to sum for all δV 's. This can be done in many ways and the following is an example of the main evaluation used in this thesis.

The angle γ in Fig. 27 is divided into N equal parts

$$\Delta\gamma = \gamma/N. \quad (103)$$

forming N "orange slices" in V . These "orange slices" are further divided into wedges in the following way (see Fig. 27). Consider a length δl along the barrier edge a distance ρ_s from the source S . The element of surface $abb'a'$ subtended by $\Delta\alpha$ and limited by δl and the spherical cap cb of Fig. 26 is projected (by rotation at the edge of the barrier) onto the N^{th} slice, forming the "wedge". This wedge is then sectioned into n elementary volumes such that

$$\delta V_n = \Delta S_n \cdot \Delta L_n. \quad (104)$$

If δV_n is a distance $r_n = n\Delta r$ from the barrier edge then the elementary area ΔS_n is given by

$$\begin{aligned} \Delta S_n &= \pi[(n+1)\Delta r]^2 \frac{\Delta\gamma}{2\pi} - \pi(n\Delta r)^2 \frac{\Delta\gamma}{2\pi} \\ &= (2n+1)\Delta S_0. \end{aligned} \quad (105)$$

where $\Delta S_0 = \Delta r^2 (\Delta\gamma/2)$. In addition it is evident that

$$\Delta L_n = \Delta\alpha (\rho_s + r_n) \quad (106)$$

FIG. 27. Schematic diagram illustrating how the elementary scattering volumes δV can be calculated from

If we assume that $\delta l \approx \rho_s \Delta\alpha$, Eq. (106) becomes

$$\Delta L_n = \delta l + r_n \frac{\delta l}{\rho_s} \quad (107)$$

Thus Eq. (77) can be written

$$E_s \approx \sum_{n, \delta l, \Delta\alpha} \psi^2 \frac{\sigma(\theta)}{\rho_s^2} \delta V_n \quad (108)$$

where $r_r = r$ is the distance from δV_n to R (see Fig. 27) and from Eqs. (105) and (107)

$$\delta V_n = (2n + \frac{1}{2}) \Delta S_0 \left(\delta l + \frac{r_n \delta l}{\rho_s} \right) \quad (109)$$

Hence the sum in Eq. (108) is performed for all n sections in the first wedge. Then by stepping δl along the barrier edge the sum over n is repeated for each wedge in the first "orange slice" of angle $\Delta\gamma$ according to Eq. (103). Finally this process is repeated for each of the N "orange slices".

In the case of a point source S the field ψ incident on most δV_n 's is

$$\psi = \frac{A}{r_s} e^{i(kr_s - \omega t)} \quad (110)$$

where r_s is the distance from S to each δV_n . However for the δV_n 's close to the surface ab , the incident field is no longer given by Eq. (110) as illustrated in Fig. 26(b). For these δV_n 's the incident field ψ can be calculated from Eq. (72). We note that when δV_n and S lie on a plane perpendicular to the barrier, Eq. (72) has the form of Eq. (73). When δV_n and S lie on a plane oblique to the barrier the alternative expression can be found in Ref. 36. Finally, we remark that for most cases

considered here it was found that the error introduced by using Eq. (110) for all δV_n is negligible. This finding reduces the computing time considerably.

The angle γ and all other such angles can be obtained from the initial distances XS , XR , r_s and r_r . The distances r_s and r_r in Fig. 27 can be calculated for all δV_n using elementary trigonometry. Then the scattering angle θ can be obtained from

$$(XS + XR)^2 = r_s^2 + r_r^2 + 2 r_s r_r \cos \theta \quad (111)$$

A computer print-out of one program, as well as other details, can be found in Appendix D.

The integer N in Eq. (103) and the increments Δr and δl were chosen to optimize the conflict between accuracy and computing time. Typical values of N range between 10 and 20 while Δr and δl range between 10 cm and 50 cm.

For example, for δV_n close to the barrier edge where θ is small, greater accuracy is required and the smaller values of Δr and δl were chosen. However close to the spherical cap where $\theta \rightarrow 90^\circ$ the dimension of δV_n can be larger without loss of accuracy. Finally, although θ is defined as the scattering angle in the middle of the volume δV_n , it was found that any convenient point in δV_n can be chosen to calculate θ from r_s and r_r .

The result of a calculation is shown in Fig. 28 for various source and receiver distances, XS and XR respectively. In the case $XS = 10$ m and $XR = 30$ m the diffraction angle is

about 20° . As discussed earlier, because $\sigma(\theta)$ obtained from Eq. (98) approaches unity for diffraction angles smaller than this value of about 20° , larger source and receiver distances are not considered. This also avoids contributions from refractive and saturation effects* which may become important at larger distances

In Chapter 7 it was shown that in order to add scattered energies the condition $L < \sqrt{\lambda}r$ must be respected. The correlation length $L \approx 1.1$ m has been previously measured*. For $r = 30$ m the Fresnel zone $\sqrt{\lambda}r$ is smaller than 1 m for frequencies above 10 000 Hz. However for $r = 10$ m, $\sqrt{\lambda}r$ is smaller than 1 m for frequencies above about 3500 Hz. Hence at the shorter distances the curves in Fig. 28 probably overestimate the amount of scattered energy at the higher frequencies. This is because there is a 180° phase difference between sound from adjacent Fresnel zones. Hence if there is more than one Fresnel zone in a correlation length L , there should be some phase cancellation between the sound scattered from the different zones (despite the turbulence, strong partial correlation is maintained, see Part I). Thus the overall scattered sound level is expected to be lower than predicted by purely energy considerations. However this problem is encountered only at the shorter distances where scattering is of lesser importance.

* saturation: The effects of atmospheric turbulence on line-of-sight propagation initially increase with distance but reach limiting values at distances of typically 50 m for frequencies of a few kHz.

FIG. 28. Results of a typical calculation to obtain the amount of energy scattered by the atmospheric turbulence above the edge of the barrier.

Chapter 9

MEASUREMENTS OF HIGHWAY NOISE

BEHIND A BARRIER

9-1 Queensway Barrier and Measurements

During the summer of 1979 measurements of traffic noise attenuated by a barrier along a highway were made. The barrier is located along the Queensway in Ottawa between Rochester St. and Island Park Drive. It is constructed from 10 cm x 10 cm wooden beams placed one on top of the other to a height of 3 m. The beams are held in place by guide posts.

Two microphones were placed behind the barrier at a height of about 1 m above an asphalt surface and at horizontal distances of about 15 m and 30 m from the barrier, respectively. A third microphone was placed above the barrier to measure the reference sound field, i.e. the direct sound field from the source. This configuration was repeated on three different sites; Spadina Ave., Edgar St., and Breeze Hill. Details of these sites are illustrated in Fig. 29.

The Queensway in this area has three eastbound lanes and three westbound lanes. Traffic counts made during the measurements yielded a fairly constant rate of about 50 eastbound and 50 westbound vehicles per minute. The traffic consisted mostly of passenger cars and small trucks with some 5 to 10% of tractor trailers or buses.

High quality magnetic tape recordings were made of the traffic noise measured by the three microphones for periods

FIG. 29. Scaled drawing of the three sites where noise attenuated by a barrier was measured. The two receivers are 1 m above asphalt, respectively about 15 m and 30 m from the barrier.

ranging between 10 minutes and 30 minutes.

The meteorological conditions during all the measurements were similar and can be described as typical warm summer days (temperature between 20°C and 25°C) with few clouds and little wind.

9-2 Field Equipment

The equipment used for the field measurements is portable and battery powered. The three receivers were General Radio piezoelectric microphone-preamplifier units (type 1560-P40) with 10 cm diameter foam wind-shields, oriented to receive sound at grazing incidence to obtain a flat frequency response. The electrical signal from each receiver was fed into a switched gain amplifier (B & K sound level meter type 2203).

A General Radio calibrator (type 1562-A) was used for calibration and when coupled to the microphone produces a sound level of 114 dB in an enclosed cavity of known volume at either 125, 250, 500, 1000 or 2000 Hz. The sound level meter can then be adjusted to produce the correct reading (and correct corresponding output voltage) using the C-weighted network. The C-weighting as defined by the appropriate Canadian Standards, ANSI or IEC is used to ensure an essentially flat frequency response over the frequency range of interest, 100 to 8000 Hz.

Finally the electrical output of the sound level meters is recorded on tape using a RACAL Store 4 FM tape recorder with Ampex precision magnetic tape type 766-15D111. The frequency

response of this recorder is flat to within 1 dB from 0 to 10 kHz at a tape speed of 30 ips.

9-3 Analysis of Field Measurements

Each tape recorded in the field was first transferred onto paper using a B & K level recorder type 2305 to obtain a visual record of the sound level as a function of time. Since the traffic flow was constant, the chart thus obtained shows noise levels that are fairly regular over a long period. However the noise history is occasionally changed abruptly by the passage of a tractor trailer or similar noisy vehicle. This is seen as a sharp rise and fall in the noise level over a period of a few seconds. The passage of such a vehicle can also be clearly heard on playback.

During the time analysis of the tapes, these periodic passages were eliminated from the record. Therefore the analysis was made on a spectrum of traffic noise which is as uniform as possible with time. A typical portion of a noise history used in the analysis is illustrated in Fig. 30.

For our purposes here, the most meaningful way to present the analysis of the traffic noise is as a graph of the time-averaged sound level as a function of frequency. This permits a direct comparison with certain aspects of diffraction theory.

The output of the RACAL tape recorder was therefore fed into a modified B & K 1/3-octave analyser type 1612/S1A. It

is possible to obtain from this analyser simultaneous and parallel outputs from each 1/3-octave filter.

The parallel outputs from the filter were fed into the A/D converter of a DEC PDP-1134 computer, digitized, and the average sound level in each 1/3-octave calculated. A standard program for this purpose was written which optimized the sampling of the wave form and time required to obtain the necessary accuracy (see Appendix E).

The A/D converter accepted and analysed up to 16 parallel inputs. Therefore the sixteen 1/3-octaves between 160 Hz and 5 kHz were chosen. We are mainly interested in the higher frequencies; however, above 5 kHz the spectrum of traffic noise is weak and the signal to noise ratio is not sufficient to permit a reliable analysis.

FIG. 30. Typical noise history of Queensway traffic to be analysed by 1/3-octave band filters.

Chapter 10

MODEL EXPERIMENTS

10-1 Construction of Barrier

In order to conduct a more searching test of diffraction theory a model barrier was constructed to perform controlled experiments. The first model barrier was erected during the summer of 1979. However the validity of these preliminary experiments is doubtful for various reasons including a questionable choice of site because of the proximity of buildings, causing reflections, and large trees whose leaves can also scatter sound. Furthermore it is suspected that the construction of this first barrier was inadequate and that transmission through or beneath the barrier was not sufficiently suppressed.

However these preliminary experiments in 1979 indicated the need for, and method of construction of, a suitable barrier which was built and used for a successful series of experiments during the summer of 1980. In order to suppress transmission through the structure a sandwich construction was chosen for the barrier. The sandwich consisted of a sheet of 2 cm thick plywood and one of 0.6 cm separated by a fixed spacing. The space between the two was filled partly with butyl rubber caulking compound in a random pattern, the remainder of the space being an air gap. This construction eliminated the main transmission problem of flexural modes in the plywood sheets because this phenomenon occurs at different frequencies for different bending stiffnesses, i.e. different thicknesses of plywood.

Adjacent sandwiches were butt-jointed to produce a barrier 17 m long which was erected on a relatively flat asphalt surface (a parking lot across the road from the Physics Division, NRC and sufficiently isolated from buildings, trees and shrubs). The height of the resulting barrier was 2.44 m. The structure was held upright by guy wires after bolting the bottom edge of the barrier to a 4 cm x 4 cm beam anchored to the asphalt. The guy wires also made it possible to straighten the top edge of the barrier.

All joints between the sandwiches, and the top edge, were sealed with waterproof tape ("Ductape" cloth tape). All air leaks between the bottom edge of the barrier and the asphalt were carefully sealed with butyl rubber caulking compound.

10-2 Equipment for the Acoustical Measurements

The sound generating system consists of a Bruel and Kjaer (B & K) beat frequency oscillator type 1014 whose adjustable audio signal was applied to the input of one channel of a Dynaco Stereo 120 power amplifier. The output of this amplifier was connected to a University UA-30 horn driver driving one end of an ebonite pipe 2.54 cm internal diameter and 75 cm long. It was found in an anechoic chamber that the other end, which is the point source, obeys classical theory for radiation from a pipe. In order that this point source be sufficiently omnidirectional up to a frequency of 10 kHz a brass nozzle tapered to a diameter of about 1 cm was fitted to the point

source end of the pipe. Finally the power amplifier was placed relatively close to the sound source rather than to the oscillator (which was on a table to the side of the barrier) to avoid a long high-current cable and thus reduce electromagnetic pickup by the receiving microphones or their cables.

Two sets of receivers were used in the course of the measurements: 3 General Radio piezoelectric microphone-preamplifier units (type 1560-P40) each with 10 cm diameter foam wind shield or 3 B & K 1.2 cm diameter condenser microphones with 10 cm diameter foam wind shields, each connected to a B & K Microphone Power Supply type 2801 (or its two-channel version type 2807).

The electrical signal was filtered either through a narrow band B & K tracking filter type 1623 or through a standard B & K octave filter set type 1613. In either case, each filter was connected to, and the signal was measured by, a B & K sound level meter type 2203. These sound level meters have switched input attenuators (10 dB steps) and an analogue meter (see also Section 9-2). One of the tracking filters doubled as a frequency counter.

The receivers were first calibrated at 1000 Hz by coupling a General Radio calibrator to each microphone in turn and adjusting the sound level meter to produce the correct reading (see Section 9-2). The microphones are then intercompared under controlled conditions at fixed frequencies of 600, 800, 1000, 1500, 2000, 3000, 4000, 5000, 6000, 7000, 8000 and 9000 Hz, to obtain their relative calibrations - the main quantity of interest for this work.

The output of one of the sound level meters was fed through a Noise Average Meter type CEL-150 to obtain an instrument-averaged value for the energy-equivalent sound level.

Finally, most of the equipment is DC powered by the appropriate dry cells. AC power was obtained for the remainder of the equipment from a nearby source.

10-3 Performance Test

The amount of sound energy transmitted through the barrier structure was measured several times during the course of the summer. The source was placed on the ground a few metres in front of the barrier. Two microphones were placed a few centimetres behind the barrier, one on the ground and the other about 1 m above the ground. A third microphone was placed 0.8 m above the barrier edge to obtain a reference level for the sound field.

The levels of sound due primarily to leaks and transmission through the barrier were measured at a series of discrete frequencies. The open circles in Fig. 31 are the results of the first measurements. The results of the microphone on the ground, Fig. 31(b), indicate that the seal between the barrier and the asphalt was not adequate (this is also the reason why the results obtained during the summer of 1979 at high frequencies are questionable). After resealing, subsequent measurements on separate occasions yielded the results shown by the points and crosses in Fig. 31. The solid curve in Fig. 31(a) illustrates the relative levels typically obtained by diffraction (see Chapter 12). Thus the levels due to leaks and transmission are sufficiently lower than the diffracted levels at all frequencies.

FIG. 31. The points in this graph are the measurements of sound due primarily to leaks and transmission through the barrier. The source S is on the ground. In (a) the receiver is a few cm behind the barrier and about 1 m above the ground while in (b) the receiver, also a few cm behind the barrier, is on the ground. We note by the excessive levels on May 16 (o) in (b) that a proper seal between barrier and asphalt is needed. After resealing subsequent measurements were made on May 21 (•) and further checked on July 7 (x).

10-4 Acoustical Measurements and Analysis

The source and receivers are placed on the ground to avoid contamination of this study of diffraction and scattering phenomena by the interference between direct and ground reflected waves. Furthermore the ground surface was restricted to asphalt which could be assumed, as in previous work, to behave as a perfect reflector. The source was placed variously at distances of 2 m, 4 m and 10 m "in front of" the barrier and the receivers at distances between 3 m and 30 m "behind" the barrier. A third microphone was again placed 0.8 m above the barrier edge as a reference measurement of the sound field incident on the barrier that is relatively uncontaminated by diffraction.

Meteorological measurements were made, sometimes simultaneously and sometimes sequentially with the acoustical measurements, and are described in the following chapter.

From May through July General Radio microphones were used, and oriented to receive the incident sound at normal incidence. The microphones on the ground with their image constitute a receiving aperture whose effective diameter is greater than the physical diameter of 2.54 cm. Hence although the microphones were calibrated by intercomparison one against the other, the exact interrelation of experimental measurements between the microphone in free space above the barrier edge and the ones on the ground could not be simulated during calibration. Therefore the results were corrected empirically

to account for the different diffraction effects around the receiving aperture. The measurements were therefore repeated in August with smaller B & K microphones, type 4133 having a diameter of 1.2 cm. Used at grazing incidence these microphones have a negligible diffraction correction in the frequency range of interest here - below 10 kHz - and in fact up to nearly 40 kHz. Hence, once calibrated one against the other, these microphones provide the calibrated difference in response also between free field and ground receiver positions.

Some of the measured results were obtained by filtering the signal through a 12% or 23% bandwidth of the tracking filters. However it was found that octave filters provided adequate and more convenient filtering for most of the results.

The scattering of sound by turbulence is a random process. Furthermore both the scattered field as well as the diffracted field are subjected to "line-of-sight" fluctuations before reaching the receiver. Hence the sound level meters receive a fluctuating signal and the average sound level must be obtained. This is done with the Noise Average Meter (see Section 10-2) which calculates the sound level L_{eq} of the equivalent average sound pressure squared. This takes the following mathematical form,

$$L_{eq} = 10 \log \frac{1}{T_s} \int_0^{T_s} \frac{p^2(t)}{p_0^2} dt \quad (112)$$

where T_s is the total measurement period, and $p(t)$ is the instantaneous sound pressure relative to the reference pressure

P_0 (0.00002 N/m²). L_{eq} is called the Equivalent Continuous Level. The total measurement period is determined by the time required for a statistically valid number of independently scattering eddies to be convected through the scattering volume (see Chapter 11).

Because only one averager was available, all sound level meters had to be averaged by eye. However it was found by frequent monitoring that for the same microphone, the readings obtained by eye differed by at most 1.0 dB from those obtained with the averager.

Finally for each configuration of source, barrier and receivers the measurements were repeated several times for each frequency. Typically each reported sound level is the average of eight to ten measured values. On a few occasions source and receiver positions were interchanged sequentially for a given geometry. Since no difference was observed in the measured sound levels this verified that refraction by wind produced a negligible effect.

Chapter 11

METEOROLOGICAL MEASUREMENTS

11-1 Experimental Apparatus

The fluctuating temperature and wind velocity field was measured simultaneously with, or immediately before or after, most of the acoustical measurements. A fast response (<1 ms) TSI 1050 series system was used. This was composed of a power supply model 1051-2, constant temperature linearized-anemometer model 1054A, a variable decade resistor model 1056, and a probe with a quartz coated hot film sensor model 1210-60.

The sensor is the variable resistance component of a Wheatstone bridge circuit operating as a resistance thermometer or as a hot-wire anemometer, but not both simultaneously. As an anemometer the system is calibrated to measure both the absolute and fluctuating component of wind velocities from 0 to 6 m/s. As a thermometer the system yields only the fluctuating component of the temperature field. The mean temperature was obtained either from a mercury thermometer or Wallac thermo-anemometer used in early work⁷. The response of the system is optimized for stability and frequencies sufficiently high (>1 kHz) to measure the full spectrum of turbulent fluctuations.

The probe and sensor is mounted above the barrier edge to sample the turbulence which is responsible for the scattering of sound (see Chapter 8). The probe is omnidirectional in a plane perpendicular to the sensor element. It is therefore mounted so that the element is vertical and hence equally

sensitive to horizontal wind from any direction.

Measurements of the wind velocity and temperature field are recorded on a RACAL Store-4 FM magnetic tape recorder.

11-2 Analysis of the Meteorological Measurements

The meteorological tape recordings are analysed with a DEC*PDP/1134 computer. The output of the RACAL tape recorder is connected to the input of the computer's A/D converter. A standard program developed by the Acoustics Section of NRC is used to obtain the autocorrelation function and frequency spectrum of the fluctuating temperature and wind velocity field.

If $v(t)$ is the output voltage from the tape recorder corresponding to the temperature or wind velocity field, the program calculates the spectrum in the following mathematical form:

$$W_v(\omega) = \int \cos(\omega \tau) R(\tau) d\tau \quad (113)$$

where

$$R(\tau) = \int e^{i\omega\tau} A_v(\omega) A_v^*(\omega) d\omega \quad (114)$$

is the correlation function if

$$A_v(\omega) = \int e^{-i\omega t} v(t) dt \quad (115)$$

After the spectra are normalized according to their respective calibrations, the structure constants C_T^2 and C_V^2 can be obtained from Eqs. (101) and (102) respectively.

Typical spectra of the temperature and wind velocity

fluctuations are shown in Fig. 32. On a decibel scale both spectra should have a slope of $-5/3$ in the inertial range according to the Kolmogorov model for the spectra of turbulence. Most of the wind velocity spectra measured here have a slope of $-5/3$ but most of the temperature spectra have a steeper slope close to -2 . However the overall error in using Eq. (101) to obtain C_T^2 is negligible.

The dashed portion of curves in Fig. 32 indicates the transition into the input range of the spectrum. In the inertial range (solid portion) the region of the spectrum which is responsible for the scattering of an acoustic wave of wavenumber k through an angle θ is given by the so-called "Bragg condition"

$$K = \frac{\omega}{\langle V \rangle} = 2k \sin \theta/2 \quad (116)$$

where $\langle V \rangle$ is the average wind velocity. For the typical scattering angles and frequencies considered here this corresponds to the region of the spectrum having values of $\omega/2\pi$ greater than those indicated by the arrow. We note here that it is the size of these eddies and the magnitude of the wind velocity that determine the averaging time necessary when measuring the fluctuating sound levels. A simple calculation yields an averaging time that ranges from a few seconds to over a minute in order to allow for a statistically significant number of eddies to be swept through the scattering volume.

FIG. 32. Typical spectra of the temperature and wind fluctuations measured simultaneously with some of the acoustical measurements. The curve is obtained from the analysis, programmed on a DEC PDP/1134 computer, of tape recordings of measurements. The dashed portion of the curve shows the transition from the inertial range into the input range. The spatial frequencies below the arrow are the typical frequencies responsible for the scattering of sound above the barrier according to the Bragg condition.

Chapter 12 COMPARISON OF THEORY AND EXPERIMENTS

12-1 Discussion of Queensway Barrier Measurements

The results of the measurements described in Chapter 9 for the sites illustrated in Fig. 29 are plotted in Fig. 33. The sound levels are plotted relative to the level (after correction to allow for distance) measured by the microphone above the edge of the barrier. The error bars are an indication of the standard deviation in the fluctuating 1/3-octave band sound levels over the entire 3-minute time of the analysed recordings.

It is expected from diffraction theory that the sound levels should decrease by 10 dB per decade increase in frequency. The solid curves in Fig. 33 are the lines of "best fit" with a slope of -10 dB per decade which can be drawn through the lower frequency points. Priority is given to the lower frequencies because these are the ones which are least influenced by atmospheric turbulence.

According to the line integral formulation of Kirchhoff's diffraction theory, Eq. (73), the sound levels at two receivers in positions equivalent to the two illustrated in Fig. 29 for each site should differ by about 4 dB. In Fig. 33, a difference this large is not observed at any of the three sites between the sound levels measured by the two receivers that are about 15 m and 30 m from the barrier.

Further, the sound levels, especially the ones measured at 30 m from the barrier, do not decrease as steeply as the

FIG. 33. The points are the results of measurements of highway noise (Queensway - Ottawa) attenuated by a barrier. XR is the distance between barrier and receiver; the latter being about 1 m above asphalt. The curves are the lines of "best fit" with a slope of -10 dB per decade of frequency which can be drawn through the lower frequency points. Sound pressure levels are relative to the levels at the top of the barrier corrected for distance.

solid curve at higher frequencies. This evidence indicates a contribution from energy scattered by the atmospheric turbulence above the barrier edge.

These two differences and phenomena are elucidated more clearly with measurements from the idealised experiment in the next two sections.

It is suspected that, in addition to scattering by atmospheric turbulence, another mechanism particular to the site is responsible for the excessively high sound levels measured on Spadina Avenue. Finally, no further attempt was made to compare these results with the models of Maekawa^{20,21}, Kurze³⁰ or Isei et al²⁵ because this would not yield any information additional to that which has already been obtained by the above analysis.

12-2 Comparison of Results with Diffraction Theory

The results obtained with the model barrier during the summer of 1980 at NRC are plotted in Fig. 34. The sound levels are relative to the level that would be measured in free field without the barrier for the same source and receiver positions. XS is the horizontal distance between source and barrier while XR is the corresponding distance between receiver and barrier.

The dashed curves are the prediction of the line integral formulation of Kirchhoff's diffraction theory, Eq. (73). The different symbols indicate results measured on different days (see Table 1).

The General Radio microphone placed on the ground with its 10 cm diameter wind screen constitutes a receiving aperture whose center is a few centimetres above the surface.

FIG. 34. The points (identified in Table 1) are the measured sound pressure levels from a point source diffracted at the edge of a barrier 2.44 m high. XS and XR are respectively the horizontal distances from source to barrier and receiver to barrier (on hard ground - asphalt). The dashed curve is the prediction of the line integral formulation of Kirchhoff-Fresnel diffraction theory. The solid curve is the proposed empirical correction to the line integral theory.

FIG. 34 Continued

FIG. 34 Continued

Because of path-length differences between direct sound rays from the edge of the barrier and those reflected on the ground surface just ahead of the receiver, there will be phase cancellation at certain frequencies (see Part I). A simple calculation shows that the first destructive interference occurs below 10 kHz for the two geometrical configurations corresponding to $XS = 2 \text{ m}$, $XR = 3 \text{ m}$ and $XS = 4 \text{ m}$, $XR = 5 \text{ m}$; - also barrier height of 2.44 m in both cases. The two corresponding diffraction curves are therefore calculated allowing for interference in the manner described by Isei et al.³⁶ using the line integral formulation of Kirchhoff's diffraction theory. (However the degradation of the interference by atmospheric turbulence in a manner described in Part I is not considered here.) For all other configurations (including the results at $XS = 4 \text{ m}$, $XR = 5 \text{ m}$ measured with the smaller B & K microphones; open circles) this first path-length interference occurs at a frequency well above 10 kHz.

The results measured at $XS = 4 \text{ m}$ and $XR = 10 \text{ m}$ are in each case plotted on two graphs to separate the measurements obtained earlier in the summer from those obtained later in the summer. The later measurements, mostly taken with the smaller diameter microphones, have less experimental scatter and are considered more accurate.

In general all the measured points show sound levels higher than predicted by the dashed curves, especially at the shorter distances. An analysis of this discrepancy is the subject of the next five paragraphs.

In Fig. 35(a) the dashed curve represents the sound levels predicted by Eq. (73) for a frequency of 950 Hz plotted, for convenience, as a function of Fresnel number. This frequency is chosen for comparison with the scaled model experiments of Daumas⁴. In Fig. 22, a source-barrier distance of $X_S = 10 \text{ m} = 27.94\lambda$ yields $c/\lambda \approx 950 \text{ Hz}$. This also gives a barrier height $5.88\lambda = 2.1 \text{ m}$, which is the best correspondence obtainable between Daumas' work⁴ and the barrier height of 2.44 m used in our own model experiments. The points in Fig. 35(a) are obtained from the solid curve in Fig. 22, which is the prediction of double layer potential theory but for the case of an incident cylindrical wave. In Fig. 22, "EFF" refers to sound levels relative to the levels without the barrier but in the presence of the hard ground. Therefore the levels in Fig. 22 must be corrected by 6 dB before replotting in Fig. 35 where the levels are relative to the free field.

The dotted curve in Fig. 35(a) is obtained from the Kirchhoff-Fresnel formulation of diffraction theory, Eq. (67), in the case of an incident spherical wave. Essentially the same curve is obtained in the case of an incident cylindrical wave (dashed curve in Fig. 22). This is not surprising since both waves have a spherical front in a plane perpendicular to the barrier.

The solid curve in Fig. 35(a) is obtained empirically from Eq. (76) by choosing $K = 10 \text{ dB}\cdot\text{m}$ and shows good agreement with the prediction of double layer potential theory.

The solid curve and the dashed curve of Fig. 35(a) are reproduced in Fig. 35(b). The points in Fig. 35(b) are the

FIG. 35. (a) The dashed curve (---) is the prediction of the line integral formulation of Kirchhoff-Fresnel diffraction theory as a function of Fresnel number. The dotted curve (---) is the prediction of the more limited classical formulation of Kirchhoff-Fresnel theory. The solid curve (—) is an empirical correction to the line integral theory. The points ◇ are the predictions of double layer potential theory obtained from Ref. 41. (b) The points are obtained from Fig. 34 at 1000 Hz and correspond to the geometries ○ $X_S = 2$ m, $X_R = 3$ m; ▲ $X_S = 4$ m, $X_R = 5$ m; ▽ $X_S = 4$ m, $X_R = 10$ m; ▼ $X_S = 4$ m, $X_R = 15$ m; * $X_S = 10$ m, $X_R = 10$ m; △ $X_S = 10$ m, $X_R = 15$ m; □ $X_S = 10$ m, $X_R = 20$ m; × $X_S = 10$ m, $X_R = 25$ m; ● $X_S = 10$ m, $X_R = 30$ m; ◆ results obtained in 1979.

experimental results of Fig. 34 for a frequency of 1000 Hz (i.e. approximately 950 Hz). The different symbols correspond to the different geometrical configurations as described in the figure caption. The measured points clearly show better agreement with the empirically corrected line integral theory (solid curve) than with the uncorrected line integral theory (dashed curve).

We also note from Fig. 35 that the prediction of Eq. (67) (dotted curve) which is the Fresnel-Kirchhoff formulation provides better agreement with experimental measurements than the more exact line integral formulation. This anomaly arises because two unrelated approximations made in the Fresnel-Kirchhoff formulation, namely that the obliquity factor Eq. (66) can be set equal to unity and that curvature of the wave fronts in the plane perpendicular to the barrier's edge can be neglected, lead to compensating errors.

The prediction of the solid curve in Fig. 35(b) is plotted by corresponding solid curves in Fig. 34 for all frequencies and geometrical configurations.

Finally, the discrepancy between the solid curves in Fig. 34 and the measured points at higher frequencies and longer distances will be discussed in the following section.

Returning to the Queensway measurements (Fig. 33) we noted earlier that, according to the line integral formulation, the sound levels at receiver positions equivalent to the two illustrated in Fig. 29 should differ by about 4 dB. The Fresnel

numbers corresponding to the two receiver positions at about 15 m and 30 m from the Queensway barrier are approximately 6.5 and 4 respectively. Therefore the 4 dB difference can be obtained from the dashed curve in Fig. 35. From the solid curve of Fig. 35 however the difference in sound level between $N = 4$ and $N = 6.5$ is about 1.5 dB. This agrees much more closely with the difference in sound level observed in the Queensway measurements at the two different distances. Thus the limitations of classical diffraction theory, elucidated by careful measurements in controlled geometrical configurations, appear to offer a satisfactory explanation for similar discrepancies in the case of a practical highway noise barrier.

12-3 Comparison of Results with Diffraction and Scattering Theories

At the larger source-receiver distances it is observed that the sound levels at higher frequencies are generally higher than predicted by diffraction theory alone. The experimental points of Fig. 34 are replotted separately for each of the larger distances in Fig. 36. The error bars indicate the standard deviation in the measurements.

The dashed curves in Fig. 36 correspond to the solid curves in Fig. 34. On certain days simultaneous meteorological measurements were made. The values of the temperature and wind velocity structure constants, C_T^2 and C_V^2 respectively, obtained from these measurements and Eqs. (101) and (102) are

listed in Table 1. Where the values of C_T^2 and C_V^2 are directly available the dotted curves in Fig. 36 are obtained from Eqs. (77) and (98) (and see Fig. 28. As discussed in Section 7-3, only Eq. (98) is considered here since Eqs. (84) and (85) predict that the scattered energy is a maximum at frequencies below 400 Hz for $\theta > 20^\circ$, contrary to what is observed here). The solid curves in Fig. 36 are therefore the logarithmic sum of the energies predicted by the dashed curve (diffraction) and the dotted curve (scattering). In general the solid curves show better agreement with the measured sound levels than diffraction alone (dashed curve) especially at the larger distances.

For source and receiver distances from the barrier smaller than $XS = 4$ m and $XR = 15$ m, the scattered energy calculated from the simultaneously measured values of C_T^2 and C_V^2 is sufficiently small that diffraction theory alone shows satisfactory agreement with the measured points as illustrated by the results at $XS = 4$ m and $XR = 10$ m.

We note from Table 1 that a comparison between daytime and nighttime meteorological and acoustical measurements is possible. The values of C_T^2 and C_V^2 are plotted in Fig. 37 as a function of the time at which the measurements were made. We see that the value of C_T^2 is much smaller during the night but that the measured values of C_V^2 are similar. For this reason, little difference is observed in the acoustical measurements between day and night. It was not possible to

obtain a set of acoustical measurements at night, on the site at NRC, where both the values of C_T^2 and C_V^2 were smaller by an order of magnitude than the daytime values as commonly reported in the literature.

Finally we note that the discrepancy between the measured points at the higher frequencies and the solid curve in Fig. 33 is similar to the difference between the dashed curve (diffraction alone) and the solid curve (diffraction and scattering) in Fig. 36. Thus a theory that considers both diffraction at the barrier edge and scattering by atmospheric turbulence appears to offer a satisfactory explanation for similar discrepancies in the case of a practical highway noise barrier.

It is often reported that barriers are less effective than expected. The experimental measurements reported here have shown that classical Kirchhoff-Fresnel diffraction theory overestimates the performance of barriers and that, in addition, their performance is limited due to scattering by atmospheric turbulence.

FIG. 36. The points are the same measured sound pressure levels as Fig. 34 (and identified in Table 1). The dashed curve (---) reproduces the prediction of the corrected line-integral theory (solid curve in Fig. 34). The dotted curve (---) is the calculated scattered energy when the values of C_v^2 (in m^2/s^2) and C_T^2 (in $^\circ\text{K}/\text{m}^2$) are available. The solid curve is the logarithmic sum of the energies predicted by diffraction (dashed curve) and scattering (dotted curve).

FIG. 36 Continued

FIG. 36 Continued

FIG. 36 Continued

FIG. 36 Continued

FIG. 36 Continued

FIG. 36 Continued

TABLE 1. Summary of meteorological measurements made simultaneously with some of the acoustical measurements. XS and XR are respectively the distance between source to barrier and receiver to barrier while C_V^2 and C_T^2 are the respective structure functions of the wind and temperature fluctuations.

Date (1980) of acoustical measurements	Symbols of SPL in Figs. 34 and 35	Description of weather conditions	XS,XR (m)	C_V^2 ($m^{1/3}/s^4$)	C_T^2 ($^{\circ}K/m^{1/3}$)
May 22	.	Wind from SW (3 to 4.5 m/s). Temperature 27°C. No clouds.	4,10	-	-
May 27	x	Brisk wind from NW. Temperature 13°C. No clouds.	2,3 4,10	-	-
June 4	*	Wind from NE (3 m/s). Temperature 20°C. 40% clouds.	4,10	-	-
June 5	+	No wind. Temperature 20°C. No clouds.	2,3	-	-
June 12	▲	Wind from E (3 m/s). Temperature 20°C. No clouds.	4,5 4,15	-	-
June 17	Δ	Wind from NW (1.5 to 3 m/s). Temperature 21°C.	10,10 10,15	~1	~1
July 3	▼	Light winds gradually increasing to moderate from E (0 and 1.5 to 3 m/s). Temperature 27°C. No clouds.	4,10 10,20 10,30	-	-
July 7	▽	Wind from NE (1.5 m/s). Temperature 25°C. Partly hazy and cloudy.	4,5 10,20	-	-
July 9	■	Wind about 1.5 m/s). Temperature 21°C. Cloudy with sunny intervals.	4,15 10,25 10,30	0.5	~5

Date (1980) of acoustical measurements	Symbols of SPL in Figs. 34 and 35	Description of weather conditions	XS, XR (m)	C_V^2 (m^2/s^2)	C_T^2 ($K/m^2/s$)
July 25	□	Steady breeze from SW (1 m/s). Temperature 20°C down to 15°C. Night between midnight and 3 a.m.	10,10 10,15 10,20 10,25 10,30	1.2 1.1 1.0 0.9 0.9	0.2 0.2 0.4 0.5 1.0
August 7	●	Wind from NW (1.5 m/s). Temperature 29°C. 20% cirrus clouds.	10,10 10,15 10,20	1	5
August 11	○	Wind from SW (0.5 to 1.6 m/s). Temperature 24°C. No clouds.	4,5 4,10 4,10 4,15 10,15 10,25 10,30	1.5 1.5 0.2 2.0 1.5 0.8 0.4	10.0 10.0 2.5 10.0 10.0 5.0 2.5

FIG. 37. The variation in time of the wind and temperature structure constants, C_V^2 and C_T^2 respectively. July 24-25 x, August 11 o.

CONCLUSION

The existing theory to predict the effects of atmospheric turbulence on the interference of acoustic waves above a hard boundary, which is limited to short distances of propagation and by the need for an adjustable constant, is extended in three respects in Part I. The theory is reexamined to include, in addition to the previously considered transverse autocorrelation between the direct and reflected path, the longitudinal autocorrelation along each path. The new theory above hard ground is also extended to include the interference above a finite impedance boundary and further extended to include both short and longer distances of propagation. In addition to phase covariance it is also found that covariance of sound amplitude is relatively more important at long distances while phase-amplitude covariance is negligible at all distances.

The new theory is compared with the author's previous simultaneous acoustical and meteorological measurements above hard ground and shows essentially the same agreement as before but, unfortunately, still requires the same adjustable parameter. The new theory is also compared with measurements (by other authors) above a soft ground (grass) and longer distances of propagation by using the meteorological parameters (these were not reported) as constants adjusted for best fit. This

comparison also shows good agreement.

Measurements of traffic noise attenuated by a barrier, described in Part II, show two characteristic discrepancies when compared with classical diffraction theory. To elucidate these discrepancies a model experiment was conducted using a point source to study the diffraction at a thin barrier (17 m long and 2.44 m high) erected above a flat asphalt surface and scattering by atmospheric turbulence.

When compared with theory, the precise sound levels measured behind the model barrier are higher than the more precise line-integral formulation of the Kirchhoff-Fresnel diffraction theory by 2 to 10 dB. The largest discrepancies occur at the shorter source and receiver distances from the barrier. These discrepancies are a result of the limitations imposed by the approximate Kirchhoff boundary conditions. Smaller discrepancies (1 to 3.5 dB) are observed between the same measurements and the more limited classical formulation of Kirchhoff-Fresnel diffraction theory but for fortuitous reasons. An empirical correction to the line-integral theory, which accounts for the spherical curvature of the incident sound field in a plane perpendicular to the barrier, is offered to overcome the limitations of the Kirchhoff boundary conditions. An alternative diffraction theory based on double layer potential theory is also discussed. The corrected theory shows good agreement with all the results, except at the higher frequencies of the larger source-receiver distances where the measured sound

pressure levels are still higher than predicted by up to 3 dB.

A numerical calculation using a single scattering theory is described to estimate the amount of sound energy scattered by atmospheric turbulence above the barrier edge. The calculated sound pressure levels resulting from the logarithmic sum of the scattered energy and the energy from diffraction at the barrier edge show better agreement with all the results than just diffraction theory alone, especially at the larger distances.

Because of the discrepancies observed between measured sound levels and Kirchhoff-Fresnel diffraction theory it is recommended that double layer potential theory, or some other similar more rigorous theory, be further developed and compared with suitable acoustic measurements. Further, because the theory of scattering by atmospheric turbulence is limited by its single scattering nature, a multiple scattering theory is needed to consider diffraction in turbulence at larger distances. These larger distances are frequent in real-life noise barrier problems.

It is often reported in the literature that sound levels measured behind highway barriers are higher than those predicted by theory, that is, noise barriers are less effective than expected. The idealised experiment has shown that measured sound levels are higher than those predicted by classical Kirchhoff-Fresnel diffraction theory and that sound energy is scattered behind the barrier, by atmospheric turbulence, thus limiting,

its performance. Because these two phenomena are also observed in the analysis of the Queensway measurements, it is expected that they can be responsible for similar discrepancies reported with many noise barriers.

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APPENDIX A: Multivariate Analysis

The mean or "expectation value", denoted by $\langle \rangle$, of a continuous function $f(x)$, of a random variable x , is by definition

$$\langle f(x) \rangle = \int f(x) m(x) dx \quad (A1)$$

where $m(x)$ is called the probability density function. Usually $m(x)$ is normalized such that

$$\int_{x_{\min}}^{x_{\max}} m(x) dx = 1 \quad (A2)$$

A well known example is the Gaussian or normal probability density function

$$m(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} \quad (A3)$$

where μ is its mean value and σ^2 its variance,

$$\sigma^2 = \langle (x - \mu)^2 \rangle \quad (A4)$$

Two random variables are said to have a bivariate normal distribution if their joint probability distribution is

$$m(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2} \frac{1}{1-\rho^2} \left[\frac{(x_1-\mu_1)^2}{\sigma_1^2} - 2\rho \frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} \right] \right\} \quad (A5)$$

where ρ is the correlation coefficient between x_1 and x_2 .

An example of Eq. (A5) was used in Eq. (8) where

$$x_1 = \delta_r \text{ and } x_2 = \delta_d.$$

To generalize, if n random variables $x_1, x_2, \dots, x_n$

have the joint probability distribution function

$$m(x_1, \dots, x_n) = C \exp \left[-\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n A_{ij} (x_i - \mu_i)(x_j - \mu_j) \right] \quad (A6)$$

then they are said to have a multivariate normal distribution.

C is a normalization constant.

The A_{ij} form a symmetric $n \times n$ matrix $\underline{A}$. The matrix $\underline{A}$ has an inverse $\underline{A}^{-1}$, and it can be demonstrated that

$$\sigma_{ij} = (\underline{A}^{-1})_{ij} \quad (A7)$$

where σ_{ij} are the elements of a variance-covariance matrix $\underline{\sigma}$.

The elements $\sigma_{ij} = \sigma_{ji}$ are the variances and σ_{ij} the covariances.

The normalization constant is

$$C = [(2\pi)^{-n} (\det \underline{A})]^{1/2} \quad (A8)$$

where $(\det \underline{A})$ is the determinant of the square matrix $\underline{A}$. An example is used in Eq. (14) and the details of the calculation can be found in Appendix C.

APPENDIX B: Transverse-longitudinal Correlation

In order to estimate the two terms $\langle \delta_{d1} \delta_{r2} \rangle$ and $\langle \delta_{d2} \delta_{r1} \rangle$ appearing in the variance-covariance matrix, Eq. (A7), we consider the correlation of the amplitude and phase fluctuations at two receiving points where the coordinates of the two receivers are $(r_1, 0, 0)$ and $(r_2, 0, l)$.

We follow an analysis identical to that of Chernov² who used Rytov's method in the case of phase waves. It is then possible to write (see pages 89 and 95 in Chernov) the following expression for the correlation function of the phase and amplitude fluctuations

$$\langle \delta(r_1, 0, 0) \delta(r_2, 0, l) \rangle = \langle \delta, \delta \rangle = \frac{1}{2} \langle \mu^2 \rangle (I_1 + I_2) \quad (B1)$$

$$\langle \ln(1 + a_1) \ln(1 + a_2) \rangle = \frac{1}{2} \langle \mu^2 \rangle (I_1 - I_2) \quad (B2)$$

where

$$I_1 = \int_0^{r_1'} dx \int_{-\infty}^{\infty} d\tau \iint \frac{1}{2\pi[(r_2' - r_1') - \tau]} \sin \frac{\eta^2 + (\zeta + l')^2}{2[(r_2' - r_1') - \tau]} \times \exp\left(-\frac{\tau^2 + \eta^2 + \zeta^2}{L'^2}\right) d\eta d\zeta \quad (B3)$$

$$I_2 = \int_0^{r_1'} dx \int_{-\infty}^{\infty} d\tau \iint \frac{1}{2\pi[(r_2' + r_1') - 2x]} \sin \frac{\eta^2 + (\zeta + l')^2}{2[(r_2' + r_1') - 2x]} \times \exp\left(-\frac{\tau^2 + \eta^2 + \zeta^2}{L'^2}\right) d\eta d\zeta \quad (B4)$$

The integrations with respect to x , η and τ in I_1 are straightforward and lead to the result

$$I_1 = r_1' \int_{-\infty}^{\infty} \exp\left(-\frac{\tau^2}{L'^2}\right) \exp(\gamma_1') \frac{\cos \gamma_1' + \frac{2(\Delta r' - \tau)}{L'^2} \sin \gamma_1''}{1 + \frac{4(\Delta r' - \tau)^2}{L'^4}} d\tau \quad (B5)$$

where

$$\gamma_1' = -\frac{\ell'^2/L'^2}{1 + \frac{4(\Delta r' - \tau)^2}{L'^4}}, \quad \gamma_1'' = \frac{(\ell'^2/L'^2) 2(\Delta r' - \tau)/L'^2}{1 + \frac{4(\Delta r' - \tau)^2}{L'^4}}$$

and furthermore $\Delta r' = r_2' - r_1'$. In the case $k\Delta r \gg kL \gg 1$ the integration in Eq. (B5) is greatly simplified and leads to

$$I_1 = \sqrt{\pi} k^2 r_1 L \frac{1}{1 + \left(\frac{2\Delta r}{kL^2}\right)^2} \exp\left[\frac{-\ell^2/L^2}{1 + \left(\frac{2\Delta r}{kL^2}\right)^2}\right] \quad (B6)$$

It can be shown that I_2 is smaller than I_1 and thus, for the purpose of this calculation, I_2 can be neglected in Eq. (B1) leading to the result

$$\langle \delta_1 \delta_2 \rangle \approx \frac{\sqrt{\pi}}{2} \langle \mu^2 \rangle k^2 r_1 L \frac{1}{1 + \left(\frac{2\Delta r}{kL^2}\right)^2} \exp\left[\frac{-\ell^2/L^2}{1 + \left(\frac{2\Delta r}{kL^2}\right)^2}\right] \quad (B7)$$

In general the error in neglecting I_2 in Eq. (B2) is more significant². However, again for the purpose of this calculation, we can see that Eq. (B2) leads to a result of the same form as Eq. (B7) since I_1 is still the dominant term.

One must beware that this analysis considers plane waves. In comparing Eq. (B7) with Eq. (43) and remembering that for plane waves, $\rho_s \approx \exp(-l^2/L^2)$, we can expect, on physical grounds alone, that a similar analysis for spherical waves will lead to the dependence shown in Eq. (B7). However difficulties in such an analysis place this task beyond the scope and purpose of this calculation.

APPENDIX C: Evaluation of Variance-covariance Matrix

The variance-covariance matrix constructed from Eqs. (17) through (20) has the form

$$\tilde{\sigma} = \sigma_{dl}^2 \begin{pmatrix} 1 & \rho_s & R_s & \rho_s \\ \rho_s & 1 & \rho_s & R_s \\ R_s & \rho_s & 1 & \rho_s \\ \rho_s & R_s & \rho_s & 1 \end{pmatrix} \quad (C1)$$

Inverting this matrix yields the matrix $\tilde{A}$

$$\tilde{A} = \begin{pmatrix} X & -Y & Z & -Y \\ -Y & X & -Y & Z \\ Z & -Y & X & -Y \\ -Y & Z & -Y & X \end{pmatrix} \quad (C2)$$

where

$$X = \frac{1}{\sigma_{dl}^2} \frac{(1 + R_s - 2\rho_s^2)}{(1 - R_s)[(1 + R_s)^2 - 4\rho_s^2]} \quad (C3)$$

$$Y = \frac{1}{\sigma_{dl}^2} \frac{\rho_s}{[(1 + R_s)^2 - 4\rho_s^2]} \quad (C4)$$

and

$$Z = \frac{1}{\sigma_{dl}^2} \frac{[2\rho_s^2 - R_s(1 + R_s)]}{(1 - R_s)[(1 + R_s)^2 - 4\rho_s^2]} \quad (C5)$$

APPENDIX C: Evaluation of Variance-covariance Matrix

The variance-covariance matrix constructed from Eqs. (17) through (20) has the form

$$\Sigma = \sigma_{dl}^2 \begin{pmatrix} 1 & \rho_s & R_s & \rho_s \\ \rho_s & 1 & \rho_s & R_s \\ R_s & \rho_s & 1 & \rho_s \\ \rho_s & R_s & \rho_s & 1 \end{pmatrix} \quad (C1)$$

Inverting this matrix yields the matrix $\tilde{A}$

$$\tilde{A} = \begin{pmatrix} X & -Y & Z & -Y \\ -Y & X & -Y & Z \\ Z & -Y & X & -Y \\ -Y & Z & -Y & X \end{pmatrix} \quad (C2)$$

where

$$X = \frac{1}{\sigma_{dl}^2} \frac{(1 + R_s - 2\rho_s^2)}{(1 - R_s)[(1 + R_s)^2 - 4\rho_s^2]} \quad (C3)$$

$$Y = \frac{1}{\sigma_{dl}^2} \frac{\rho_s}{[(1 + R_s)^2 - 4\rho_s^2]} \quad (C4)$$

and

$$Z = \frac{1}{\sigma_{dl}^2} \frac{[2\rho_s^2 - R_s(1 + R_s)]}{(1 - R_s)[(1 + R_s)^2 - 4\rho_s^2]} \quad (C5)$$

Using the identity

$$\langle \cos \theta \rangle = \frac{1}{2} \langle e^{i\theta} \rangle + \text{c.c.} \quad (\text{C6})$$

Eq. (14) can be written as

$$\langle \cos(\phi + x_2 + x_4 - x_1 - x_3) \rangle = \frac{1}{2} e^{i\phi} \iiint_{-\infty}^{\infty} e^{i(x_2 + x_4 - x_1 - x_3)} m(x_1, x_2, x_3, x_4) dx_1 dx_2 dx_3 dx_4 + \text{c.c.} \quad (\text{C7})$$

where

$$m(x_1, x_2, x_3, x_4) = C \exp \left[-\frac{1}{2} \left(X x_1^2 - 2Y x_1 x_2 + 2Z x_1 x_3 - 2Y x_1 x_4 \right. \right. \\ \left. \left. + X x_2^2 - 2Y x_2 x_3 + 2Z x_2 x_4 \right. \right. \\ \left. \left. + X x_3^2 - 2Y x_3 x_4 + X x_4^2 \right) \right] \quad (\text{C8})$$

and we have again identified $x_1 = \delta_{d1}$, $x_2 = \delta_{r1}$, $x_3 = \delta_{d2}$ and $x_4 = \delta_{r2}$.

Using the identity

$$\int_{-\infty}^{\infty} e^{-\rho^2 x^2 + q x} dx = \sqrt{\frac{\pi}{\rho}} \exp\left(\frac{q^2}{4\rho^2}\right), \quad \rho > 0 \quad (\text{C9})$$

all the integration in Eq. (C7) can be performed. Integration with respect to x_1 yields

$$\langle \cos(\phi + x_2 + x_4 - x_1 - x_3) \rangle = \frac{1}{2} C e^{i\phi} \sqrt{\frac{2\pi}{X}} \iiint_{-\infty}^{\infty} e^{i(x_2 + x_4 - x_3)} \\ \times \exp \left[-\frac{1}{2} \left(X x_2^2 - 2Y x_2 x_3 + 2Z x_2 x_4 + X x_3^2 - 2Y x_3 x_4 + X x_4^2 \right. \right. \\ \left. \left. + \frac{1}{2X} \left(i - Y x_2 + Z x_3 - Y x_4 \right)^2 \right) \right] dx_2 dx_3 dx_4 + \text{c.c.} \quad (\text{C10})$$

After the integration with respect to x_2 we obtain

$$\begin{aligned}
 \langle \cos(\phi + x_2 + x_4 - x_1 - x_3) \rangle &= \frac{1}{2} \int_{-\infty}^{\infty} e^{i\phi} \sqrt{\frac{2\pi}{X}} \cdot \sqrt{\frac{\pi}{\delta}} \iint_{-\infty}^{\infty} e^{i(x_4 - x_3)} \\
 &\times \exp\left[-\frac{1}{2} (X x_3^2 - 2Y x_3 x_4 + X x_4^2)\right] \\
 &\times \exp\left[-\frac{1}{2X} (1 - Z^2 x_3^2 - Y^2 x_4^2 - i 2Z x_3 + i 2Y x_4 + 2Z Y x_3 x_4)\right] \\
 &\times \exp\left[\frac{1}{4Y} (c^2 + 2abc x_3 + 2ac x_4 + 2ab x_3 x_4 + b^2 x_3^2 + a^2 x_4^2)\right] \\
 &\times d\alpha_3 d\alpha_4 + \text{c.c.} \quad (C11)
 \end{aligned}$$

where

$$\begin{aligned}
 \gamma &= \left(\frac{X}{2} - \frac{Y^2}{2X} \right), & c &= \frac{i}{X} (X - Y) \\
 b &= \frac{Y}{X} (X - Z) & a &= \frac{1}{X} (Y^2 - ZX)
 \end{aligned}$$

Next we integrate with respect to x_3 to find

$$\begin{aligned}
\langle \cos(\phi + x_2 + x_4 - x_1 - x_3) \rangle &= \frac{1}{2} C e^{i\phi} \sqrt{\frac{2\pi}{X}} \cdot \sqrt{\frac{\pi}{Y}} \cdot \sqrt{\frac{\pi}{S}} \\
&\times \int_{-\infty}^{\infty} e^{ix_4} \exp\left[-\frac{1}{2} X x_4^2\right] \\
&\times \exp\left[-\frac{1}{2X} (1 - Y^2 x_4^2 + i 2Y x_4)\right] \\
&\times \exp\left[\frac{1}{4Y} (c^2 + 2acx_4 + a^2 x_4^2)\right] \\
&\times \exp\left[\frac{1}{4S} (A^2 x_4^2 + 2AB x_4 + B^2)\right] dx_4 + c.c.
\end{aligned}$$

(C12)

where

$$S = \left(\frac{X}{2} - \frac{Z^2}{2X} - \frac{b^2}{4Y} \right)$$

$$A = b \left(1 + \frac{a}{2Y} \right), \quad B = - \frac{cbX}{2SY}$$

Finally the last integration yields

$$\langle \cos(\phi + x_2 + x_4 - x_1 - x_3) \rangle = \frac{1}{2} C e^{i\phi} \sqrt{\frac{2\pi}{X}} \cdot \sqrt{\frac{\pi}{Y}} \cdot \sqrt{\frac{\pi}{S}} \cdot \sqrt{\frac{\pi}{\mu}}$$

(C13)

$$\times \exp\left[-\frac{1}{2X} + \frac{c^2}{4Y} + \frac{B^2}{4S} + \frac{1}{4\mu} \left(i - i \frac{Y}{X} + \frac{ac}{2Y} + \frac{AB}{2S} \right)^2\right] + c.c.$$

where

$$\mu = \left[\frac{1}{2X} (X^2 - Y^2) - \frac{a^2}{4Y} - \frac{A^2}{4S} \right]$$

After some laborious substitutions and algebraic manipulations, Eq. (C13) reduces to Eq. (21).

We add that because of the two covariance terms $\langle \delta_{d1} \delta_{ra} \rangle$ and $\langle \delta_{r1} \delta_{da} \rangle$ encountered in the variance-covariance matrix σ it is not obvious that the multivariate density Eq. (15) is normal. This follows because it is shown in Appendix B that these two covariance terms are a complicated function of both the transverse and longitudinal autocorrelation of the interfering path (see Eq. (B7) and the following discussion with, in addition, Eq. (43)). Further it is known that there exist non-normal multivariate densities with normal marginal densities^{43,44}.

However after making the approximation Eq. (20) and rewriting Eqs. (14) and (15) in matrix notation we have

$$\langle \cos(\phi - x_1 + x_2 - x_3 + x_4) \rangle = \text{Re} \left[e^{i\phi} \int d^4 \underline{x} \left(e^{i \underline{B} \cdot \underline{x} - \frac{1}{2} \underline{x}^T \underline{A} \underline{x}} \right) \right] \quad (\text{C14})$$

where Re is the real part operator, $\underline{B}$ is the column matrix of the coefficients of the x 's in the argument of the cosine, and superscript T indicates a transpose. Since we assume $\underline{A} = \underline{\sigma}^{-1}$ the integral can be identified* as the characteristic function of a multivariate-normal distribution⁴⁴. Thus we obtain

$$\langle \cos(\phi - x_1 + x_2 - x_3 + x_4) \rangle = \cos \phi e^{-\frac{1}{2} \underline{B}^T \underline{\sigma} \underline{B}} \quad (\text{C15})$$

* S. F. Clifford and S. M. Wandzura, Private communication.

If we use Eq. (C1) and

$$\tilde{B} = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

(C1)

in Eq. (C16) we obtain Eq. (21). This seems to indicate that under the approximation Eq. (20), giving σ in Eq. (C1), the multivariate density Eq. (15) is normal and that the approximation Eq. (20) can be made before or after the integration.

APPENDIX D: Program to Evaluate Total Scattered Energy

The following is an example of a program used to obtain the amount of scattered energy behind the model barrier. The program sums the elementary scattering volumes according to Eq. (108) in the limits of the volume defined by abc in Fig. 27(a). Other versions of this program calculate the incident field near the surface ab from Eq. (72) by calling a subroutine written for Ref. 36.

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```

      C      PROGRAMME SCATSA
      C
      C      LE PROGRAMME CALCULE L'ENERGIE ACOUSTIQUE TRANSMISE
      C      PAR UN ECRAN PAR LA SOMMATION DE VOLUMES ELEMENTAIRES
      C
0001      REAL XR,XS,SR,DR,H
      C
      C      FACTEURS GEOMETRIQUES: DISTANCES SOURCE-BARRIERE
      C      XS, BARRIERE-RECEPTEUR XR, SOURCE-RECEPTEUR SR, HAUTEUR
      C      H, ELEMENT DR
      C
0002      REAL CV,CT,FREQ,FMULT,FLAST,TMAX,DIRFSQ,PI,EN,SL,DIFF,RMIN
0003      REAL TMIN
      C
      C      FACTEURS DE TURBULENCE: CV POUR LE VENT, CT POUR LA
      C      TEMPERATURE
      C      FACTEURS DE FREQUENCE: FREQ, FMULT, FLAST
      C      ANGLES OPTIMA: TMAX,TMIN
      C      ENERGIE EN, NIVEAU DE SON, SL
      C
0004      INTEGER Z,N,K,ZM,INDICE,INDIC,KMAX
0005      COMMON/REEL/XS,XR,H,DR,CV,CT,FREQ,TMAX,EN,PI,SR,AK,DIRFSQ
0006      COMMON/SUITE/DIFF,RMIN,RSO,RSI,NDR
0007      COMMON/ENTIER/ZM,INDICE,INDIC,KMAX
      C
      C      LECTURE DES DIFFERENTS PARAMETRES
      C
0008      ACCEPT 1, XS,XR,H,DR,CV,CT,FREQ,FMULT,FLAST
0009      1      FORMAT(9(F6.3))
0010      ACCEPT 8, RMIN,TMAX
0011      8      FORMAT(2(F6.3))
0012      ACCEPT 9, ZM,KMAX
0013      9      FORMAT(I2,1X,I3)
0014      WRITE(7,2) XS,XR,H
0015      WRITE(7,3) CV,CT
0016      WRITE(7,11) TMAX
0017      11     FORMAT(1X,'ANGLE OPTIMUM=',F6.3,'RADIANS')
0018      2      FORMAT('1','XS= ',F6.3,'M XR= ',F6.3,'M H= ',F6.3,'M')
0019      3      FORMAT('0','CV= ',F6.3,2X,'CT= ',F6.3)
0020      WRITE(7,10) KMAX
0021      10     FORMAT(' KMAXIMUM=',I3)
      C
0022      SR=XS+XR
0023      DIRFSQ=1./SR**2.
0024      PI=3.141592
0025      5      CONTINUE
0026      EN=0.
0027      AK=2.*PI*FREQ/332.
0028      SL=-500
0029      INDICE=1
0030      INDIC=1
0031      CALL INTGR
0032      SL=10.*ALOG10(EN/DIRFSQ)+3.

```

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```
0033      WRITE(7,6)
0034      5  FORMAT('0', 'FREQ', T10, 'NIVEAU(DB)')
0035      WRITE(7,7) FREQ, SL
0036      7  FORMAT(' ', F6.0, T10, F6.2)
0037      FREQ=FREQ*FMULT
0038      IF(FREQ.LE.FLAST) GO TO 5
0040      STOP
0041      END
```

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1000 1000

```

0001      SUBROUTINE INTGR
0002      REAL XS,XR,H,DR,CV,CT,FREQ,TMAX,SR,RSO,ALPHA0,RRO,BETA0,
1EN,AK,ALPHA1,RSI,RR1,CVCT,SIGMA,TETA1,PI,RSO,RRO,GAMMA,
1DGAMMA,TETA,NDR,DALPHA,RSQ,DS,DSO,KDR,DIFF,DIRFSQ,RMIN
0003      REAL TMIN,DRN
0004      INTEGER Z,N,K,ZM,INDICE,INDIC,KMAX
0005      COMMON/REEL/XS,XR,H,DR,CV,CT,FREQ,TMAX,EN,PI,SR,AK,DIRFSQ
0006      COMMON/SUITE/DIFF,RMIN,RSO,RSI,NDR
0007      COMMON/ENTIER/ZM,INDICE,INDIC,KMAX
          C
0008      RSO=(XS**2+H**2)
0009      RRO=(XR**2+H**2)
0010      RSD=RSO**0.5
0011      RRO=RRO**0.5
0012      K=1
0013      TMIN=0.75
          C
          C      DEBUT DE LA SOMMATION SUR K (POSITION SUR L'ECRAN)
          C
0014      101  CONTINUE
0015      DELT=((SR**2-RSO**2-RRO**2)/(2*RRO*RSO))*(-1)
0016      CALL ARCCOS(DELT,GAMMA)
0017      IF(INDICE.EQ.2) GAMMA=PI-GAMMA
0019      DGAMMA=GAMMA/ZM
0020      DSO=DR**2*DGAMMA/2.
0021      DELT=XS/RSO
0022      CALL ARCCOS(DELT,ALPHA0)
0023      DELT=XR/RRO
0024      CALL ARCCOS(DELT,BETA0)
0025      DIFF=1
          C
          C      DEBUT DE LA SOMMATION SUR L'ANGLE (COMPTEUR Z)
          C
0026      Z=1
0027      104  CONTINUE
          C
          C      DEBUT DE LA SOMMATION SUR LE RAYON (COMPTEUR N)
          C
0028      N=1
0029      IF(INDICE.EQ.1) TETA=PI-GAMMA+Z*DGAMMA
0031      IF(INDICE.EQ.2) TETA=PI-Z*DGAMMA
0033      102  CONTINUE
0034      NDR=N*DR
0035      RSI=DELTA(RSO,NDR,TETA)
0036      DELT=(1-(NDR/RSI*SIN(TETA))**2)**0.5
0037      CALL ARCCOS(DELT,DALPHA)
0038      IF(INDICE.EQ.1) ALPHA1=ALPHA0+DALPHA
0040      IF(INDICE.EQ.2) ALPHA1=ALPHA0-DALPHA
0042      RSI=DELTA(RSI,SR,ALPHA1)
0043      IF(INDICE.EQ.2.AND.RR1.LT.RMIN) GO TO 103
0045      IF(INDICE.EQ.1) DELT*=(SR**2-RSI**2-RR1**2)/(2.*RSI*RR1)
0047      IF(INDICE.EQ.2) DELT*=(RRO**2-NDR**2-RSI**2)/(2.*NDR*RSI)
0049      IF(DELT.LT.1) CALL ARCCOS(DELT,TETA1)
0051      IF(DELT.GE.1) TETA1=0.

```

[illegible]

100

100

VERIFICATION DE LA CONDITION: TETA1 PLUS PETIT QUE TMAX
OU PLUS GRAND QUE TMIN

```
0053 IF(TETAI.GE.TMAX.OR.TETAI.LE.TMIN) GO TO 103
0055 CVCT=CV* $\cos(TETAI/2.)^{**2/332.}^{**2+0.13*CT/280.}*+2$ 
```

$$\frac{C}{C}$$

LA TEMPERATURE EST DE 280 K

```
0056      SIGMA=0.38*AK**0.3333*CDS(TETAI)**2*CVCI/(2.*  
1SIN(TETAI/2.))**3.6667
```

```
0057      DS=(2*N-1)*DS0
0058      DRN=DR+(N*DR**2/RSQ)
```

```
0059      IF (INDICE.EQ.1.AND.Z.EQ.ZM)
1CALL FRESN(INDICE)
```

```
0061      IF (INDICE.EQ.2) CALL FBRESN(INDICE)
```

```
0063      IF(DIFF.EQ.1.) RSQ=(RSI*ERI)/%V
```

```
0065 IF (DIFF.NE.1.) RSQ=RSI**2
```

```
0067      EN=EN+(SIGMA*DS*PRN/RSQ)*DIFF
```

0068	105	CONTINUE
------	-----	----------

```

0069      N=N+1

```

0070 GO TO 102

0070	00	TS	IV
0071	103	CONTINUE	

```
0071      105      CONTINUE
0072      IF (TETA1.LE.TMIN.AND.INDICE.EQ.1) GO TO 105
```

0074 IF (TETA1,LE.TMIN,AND,INDICE,EG,2) GO TO 120

```
0076      IF(N.EQ.1.AND.INDICE.EQ.1) GO TO 110
```

```

0078      Z=Z+1

```

```

0078      Z=Z+1
0079      IF(Z,LE,ZM) GO TO 104

```

```
0081      IF(Z.GT.ZM.AND.INDICE.EQ.1) GO TO 111
```

0083 120 CONTINUE

0084 K=K+1

```
0085      IF (INDICE.EQ.2.AND.INDIC.EQ.2.AND.K.GT.KMAX)
```

100 TO 106

0087 KDR=((K-1)*DR)**2/

```
0088      RSO=(KDR+RSO)**0.5
```

```
0089      RRO=(KPR+RRO)**0.5
```

```
0090      IF (INDIC.NE.2) INDICE=1
```

0092 GO TO 101

0093 110 CONTINUE

0094 INDIC=2

0095 111 CONTINUE

0096 INDICE=2

0097 GAMMA=PI-GAMMA

0098 DGAMMA=GAMMA/ZM

```
0099      DS0=DR**2*DGAMMA/2.
```

0100 Z=1

0101 GO TO 104

0102 106 CONTINUE

0103 RETURN

0104 END

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```
0001      REAL FUNCTION DELTA(RRX,D,ANGLE)
0002      DELTA=(RRX**2+D**2-2.*RRX*D*COS(ANGLE))**.5
0003      RETURN
0004      END
```


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```
0001      SUBROUTINE ARCCOS(X,ACOS)
0002      REAL X,ACOS
0003      COMPLEX Y
0004      Y=CLOG(CMPLX(X,(1-X**2)**0.5))
0005      ACOS=AIMAG(Y)
0006      RETURN
0007      END
```

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```
0001      SUBROUTINE FRESN(INDICE)
0002      REAL NDR,RSO,RSI,DELTA0,FREQ,FR,SLO,DIFF,DIRFSQ,AK
0003      INTEGER INDICE
0004      COMMON/REEL/XS,XR,H,DR,CV,CT,FREQ,TMAX,EN,FI,SS,AK,DIRFSQ
0005      COMMON/SUITE/DIFF,RMIN,RSO,RSI,NDR
0006      IF(INDICE.EQ.1) DELTA0=0.
0008      IF(INDICE.EQ.2) DELTA0=RSO+NDR-RSI
0010      FR=2*FREQ*DELTA0/332.
0011      IF(FR.LT.1) SLO=(10*FR+6)*(-0.1)
0013      IF(FR.GE.1) SLO=(1.111*FR+14.889)*(-0.1)
0015      DIFF=DIRFSQ*10.**SLO
0016      RETURN
0017      END
```

APPENDIX E: Program to Sample and Average 1/3-Octave
 Bands of Noise

The following is the program used to analyse the sound pressure levels measured behind the Queensway barrier in Ottawa. The program uses standard subroutines available on DEC computers.

```

C      PROGRAM TO READ DATA FROM ANALOG TAPE AND CALCULATE
C      1 SL IN 1/3 OCTAVE.
      INTEGER K,TNO,DAY,MONTH,YR,CNO,FTOCT,RNO,CNO1,CNO2
      CALL ASSIGN(1,'KH:/C')
      TYPE 10
10     FORMAT(' ','TYPE TAPE NO AND CHANNEL NO',3X,'DAY,MONTH,YEAR',/
      1' ','RUN NO',4X,'COUNTER NO1 AND NO2',4X,'FIRST 1/3 OCT',/
      1' ','IN FORMAT 9I4',4X,'PUSH RETURN')
      ACCEPT 11,TNO,CNO,DAY,MONTH,YR,RNO,CNO1,CNO2,FTOCT
      WRITE(1,20)TNO,CNO,DAY,MONTH,YR,RNO,CNO1,CNO2,FTOCT
      TYPE 12
12     FORMAT(' ','TYPE HOW MANY LOOPS, K, IN I4'/
      1' ','(ONE LOOP CALCULATES 100 SAMPLES OF EACH 1/3 OCT'/
      1' ','WHICH REPRESENTS ABOUT 1 SEC OF RECORD)'/
      1' ','START TAPE, RETURN')
      ACCEPT 13,K
      TYPE 14,K
      CALL BANDS(K)
      CALL SETR(-1,,,)
11     FORMAT(9I4)
13     FORMAT(I4)
14     FORMAT(I4)
20     FORMAT('0','TAPE NO',I4,5X,'CHANNEL NO',I4,5X,I2,'/',I2,'/',I2
      1/'0','RUN NO',I6,5X,'COUNTER FROM',I5,' TO',I5/
      1'0','FIRST 1/3 OCT',I6)
      STOP
      END
      SUBROUTINE BANDS(K)
      INTEGER K
      PROGRAM TO READ 16 CHANNELS IN REPEATED SEQUENCE
      INTEGER*2 IBUF(1600),IBUF2(1600)
      REAL*4 L,SL(16),DBSL(16),DR,ATT
      COMMON IBUF2,SL
      DO 8 I=1,16
8       SL(I)=0.
      DO 9 J=1,1600
9       IBUF2(J)=2048
      DO 1 I=1,K+1
      SET PARAMETERS FOR SAMPLING A/D CONVERTER
      SAMPLE AND LOAD INTO 'IBUF'
      ICMF=0
      CALL RTS(IBUF,1600,,100,0,8,1,6,ICMF,IBEF)
      PRESET=1000.
      ICMF0=0
      CALL SETR(2,1,PRESET,ICMF0)
      ACCUMULATE RMS SUM OF WAVWFORM FOR EACH BAND
      II=0
      DO 3 JL=1,100
      DO 4 M=1,16
      II=II+1
      L=IBUF2(II)-2048
      SL(M)=SL(M)+ABS(L)*ABS(L)
4       CONTINUE
3       CONTINUE
      CALL LWAIT(ICMF,0)
      DO 5 JJ=1,1600
      IBUF2(JJ)=IBUF(JJ)
5       CONTINUE
      CALL SETR(-1,,,)
      CONTINUE
      CALCULATE RMS VALUE
      TYPE 15
15     FORMAT(' ','ATT=?')
      ACCEPT 16,ATT
      WRITE(1,17)ATT
      DO 7 I=1,16
      DB=(SL(I)/(K*100))/2048.**2
      DBSL(I)=10.*ALOG10(DB)-ATT
7       CONTINUE
      WRITE(1,21)(DBSL(J),J=1,16)
21     FORMAT('0',8F8.2/'0',8F8.2)
16     FORMAT(F5.2)
17     FORMAT('0',10X,'ATT=',F5.2)
      RETURN
      END

```

END

3	0	H	0	8	H	8	2
---	---	---	---	---	---	---	---

FIN