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TITLE OF THESIS/TITRE DE LA THESE INTERFERENCE BETWEEN RIGID
STRIP FOUNDATIONS RESTING
ON COHESIONLESS SOILS
UNIVERSITY/UNIVERSITE CARLETON
DEGREE FOR WHICH THESIS WAS PRESENTED/
GRADE POUR LEQUEL CETTE THESE FUT PRESENTÉE PH.D.
YEAR THIS DEGREE CONFERRED/ANNÉE D'OBTENTION DE CE DEGRÉ 1981
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INTERFERENCE BETWEEN RIGID STRIP
FOUNDATIONS RESTING ON COHESIONLESS SOILS

by

El-Sayed A. Rabbaa

A thesis submitted to
The Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for
the degree of
DOCTOR OF PHILOSOPHY
in
CIVIL ENGINEERING

Faculty of Engineering
Carleton University
Ottawa, Ontario
June 1981

To My Parents,

To My Wife,

and

To Sherif, Samah, Ameera and Muhammad

The undersigned recommends to the Faculty of Graduate
Studies and Research acceptance of the thesis:

"INTERFERENCE BETWEEN RIGID STRIP FOUNDATIONS RESTING
ON COHESIONLESS SOILS".

by El-Sayed A. Rabbaa, in partial fulfillment of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

Ats. Selvadurai

Thesis Supervisor

John A. G. Reilly
Chairman,
Department of Civil Engineering

Carleton University

ABSTRACT

This thesis is concerned with certain problems related to the interference between two rigid strip foundations resting on a layer of sand. The layer of sand rests on a rigid base with rough or smooth boundary conditions. The structural design of such foundations should take into account the influence of interference on the following characteristics:

- (a) Contact stress distribution,
- (b) Settlement, and
- (c) Bearing capacity.

Among several variables affecting these characteristics, the following are investigated:

- (1) Spacing between the two foundations,
- (2) Ratio between the layer thickness and the foundation width,
- (3) Roughness conditions of the interface between the foundation and the soil and the interface between the soil and the rigid base, and
- (4) Load and unload cycling.

All the other variables are fixed throughout the work. The results of the investigation are as follows.

The spacing between foundations has a significant influence on the contact stress distribution. Beneath a single foundation,

the contact stress distribution is symmetrical. As the two foundations approach each other, it becomes asymmetrical. The degree of asymmetry of the contact stress distribution depends on the spacing between the two foundations.

The settlement is strongly affected by variation of the spacing. The settlement reaches a minimum value when the two foundations touch each other and becomes maximum at spacing equal to about six times the width of one of the two foundations. At this particular spacing, each foundation acts separately. The settlement also decreases as the ratio of the layer thickness to the foundation width decreases. The roughness conditions of the interface between the foundation and the soil and the interface between the soil and the rigid base have minor effects on the settlement. The shape of the load-settlement curve is influenced by the number of loading and unloading cycles. Appreciable total and inelastic settlements occur during the first cycle. The rate of increase in settlements decreases by increasing the number of cycles.

The bearing capacity of the two interfering foundations depends, to a great extent, on the spacing between the two foundations. The bearing capacity increases as the spacing decreases, reaching a maximum value when the two foundations are in contact. The influence of the ratio of the layer thickness to the foundation width on the bearing capacity depends on the roughness conditions of the interface between the foundation and the soil and the interface between the soil and the rigid base. For a smooth interface the bearing capacity

decreases with the decrease of this ratio; but for rough interface, it increases steadily.

Numerical and empirical solutions for the problem of the interference between surface foundations are presented. The numerical solutions are based on the nonlinear finite element technique developed at the University of California, Berkeley (ISBILD program). The theoretical load-settlement curves agree closely with those obtained from experiments. But they were found to be sensitive to the input nonlinear stress-strain parameters. The empirical solution is obtained by adopting Schmertmann's method. It is found that, by introducing certain modifications, Schmertmann's method can be adopted to predict the settlement of two interfering foundations.

ACKNOWLEDGEMENTS

This work was carried out under the supervision of Professor A. P. S. Selvadurai.

The author would like to express his deep appreciation and gratitude to Professor Selvadurai for his helpful guidance and continuous support throughout this work.

The author wishes to thank Professors J. Adjelejan, W. H. Bowes, for their appreciated support throughout this work.

Acknowledgements are due to Ismail Naguib, Terry Nicholas, Mostafa El-Kafrawy and Bernie Moore of the Civil Engineering Department, Carleton University.

Thanks are also given to Les Rice, John Herler of the technical staff at Carleton University.

Financial support from the Natural Sciences and Engineering Research Council of Canada (Research Grant A3866) is gratefully acknowledged.

The care and attention of Miss Diane Dodds in typing the manuscript is appreciated.

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LIST OF SYMBOLS

B	Foundation Width
B_e	Width of Single Foundation Experiencing Same Settlement as That of Two Interfering Foundations; Each of Width B
C	Cohesion
D	Foundation Depth
D_r	Relative Density
e	Eccentricity
E	Young's Modulus
E_i	Initial Young's Modulus
E_t	Tangential Young's Modulus
H	Depth of Compressible Layer from the Foundation Level to Rigid Base
I	Moment of Inertia
I_z	Vertical Strain Influence Factor
K	Modulus Number
K_r	Relative Rigidity of Soil and Foundation
L	Length of Foundation
M	Bending Moment
n	Exponent Determining Rate of Variation of E_i with σ_3
N_c, N_q, N_γ	Bearing Capacity Factors
N'_c, N'_q, N'_γ	Modified Bearing Capacity Factors to Take Into Account Effect of Rigid Base
q	Contact Stress
q_0	Average Measured Contact Stress

q_1	Maximum Contact Stress Beneath Edge of Foundation
q_2	Minimum Contact Stress Beneath Edge of Foundation
q_a	Average Applied Pressure
q_c	Static, Dutch Cone Bearing Capacity
q_g	Bearing Capacity of Foundation in Group
q_{gh}	Bearing Capacity of Foundation in Group Resting on Half Space Medium
q_s	Bearing Capacity of Single Foundation
q_{sh}	Bearing Capacity of Single Foundation Resting on Half Space Medium
q_{sr}	Bearing Capacity of Single Foundation Resting on Compressible Layer in Rough Contact With Rigid Base
q_{ss}	Bearing Capacity of Single Foundation Resting on Compressible Layer in Smooth Contact with Rigid Base
q_u	Ultimate Bearing Capacity
q_x	Measured Contact Stress
P	Applied Load
P_a	Atmospheric Pressure
R_f	Failure Ratio
S	Centre to Centre Spacing
S_e	Edge to Edge Spacing
t	Time
T	Shearing Force
T_r	Shearing Force Developed on Sheet Roughened With Sand
T_s	Shearing Force Developed on Sheet of Stainless Steel

u	Displacement Component in x-Direction
v	Displacement Component in y-Direction
V	Pull-out Force
V_r	Pull-out Force of Rough Plate
V_s	Pull-out Force of Stainless Steel Plate
x	Horizontal Distance from Centre of Strip Foundation
z	Vertical Distance Below Foundation Level
α	$(45^\circ - \phi/2)$
γ	Unit Weight
δ, δ_h	Vertical and Horizontal Displacements
δ_s	Settlement of Single Foundation
δ_g	Settlement of Foundation in Group
ψ	$(45^\circ + \phi/2)$
ϕ	Angle of Internal Friction
ϕ_p	Angle of Internal Friction Determined From Plane Strain Tests
ϕ_t	Angle of Internal Friction Determined From Triaxial Tests
ϕ_{tc}	Angle of Internal Friction Determined From Triaxial Tests and Corrected to Take into Account Plane Strain Conditions
ϵ	Strain
ϵ_l or ϵ_a	Axial Strain
ϵ_z	Vertical Strain
ν	Poisson's Ratio
ν_i	Initial Poisson's Ratio

ν_t	Tangential Poisson's Ratio
λ	Angle of Friction
λ_r	Angle of Friction of Rough Plate
λ_s	Angle of Friction of Stainless Steel Plate
η_q	Ratio of Ultimate Bearing Capacity of Foundation in Group to that of Single Foundation
η_{qr}	Ratio of Ultimate Bearing Capacity of Foundation Resting on Shallow Layer of Thickness $H < 2B$ to Bearing Capacity of Similar Foundation Resting on Deep Layer of Thickness $H \geq 2B$
η_s	Ratio of Settlement of Foundation in Group to Settlement of Single Foundation
η_{sr}	Settlement Reduction Factor to Take Into Account Influence of Rigid Base
ξ	Ratio of Slope of Load-Settlement Curve of Foundation in Group to that of Single Foundation
ξ_r	Ratio of Slope of Load-Settlement Curve of Foundation Resting on Shallow Layer ($H/B < 2.0$) to that of Foundation Resting on Deep Layer ($H/B \geq 2.0$)
σ_1, σ_3	Major and Minor Principal Stresses
τ	Shear Stress
w	Displacement Component in z-Direction

Chapter 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

The study of the interaction between structural foundations and the supporting soil media is of fundamental importance to both geotechnical and structural engineering. Information regarding contact stress distribution, settlement and bearing capacity is required for an adequate design of the foundation. The mutual interference of foundations in a group has a significant influence on these design factors.

The influence of grouping of piles on the load capacity and settlement characteristics of a pile group has been appreciated for a long time. As for the foundations near the surface, the problem of interference between these foundations has received no attention until publications concerning the problem were presented by Stuart and Hanna (1961) and Stuart (1962). Since then, the problem of interference between surface foundations has received momentum; and much research on the subject has been presented. Most of this research has been concerned mainly with the influence of interference between foundations on the ultimate bearing capacity. A few researchers have studied this influence on the settlement. As far as the author knows, the influence of interference between foundations on the contact stress distribution beneath these

foundations has not been investigated as yet.

Also, all previous research concerning the problem of interference between foundations, has dealt with soil media which could be considered as deep layers (in most of this research, the ratio of the thickness of the layer to the width of the foundation model was in the order of eight or more). In practice, the foundation may be located on a layer of soil of shallow depth and low rigidity and strength resting on a rigid base. Many researchers have investigated this problem associated with a single foundation. Research of the problem associated with a group of foundations has not received the same amount of attention.

In view of this, the main objective of the present work is to present an experimental and theoretical investigation of the influence of the interference between foundations on their settlement and bearing capacity characteristics. An emphasis will also be placed on the study of this influence on the contact stress distribution beneath these foundations. The particular problem being examined is intended to model the behaviour of the two adjacent identical rigid strip foundations. The two foundations are located on the surface of a granular medium resting on a rigid base.

1.2 Literature Review

1.2.1 Interference Between Adjacent Foundations

This section presents detailed accounts of the theoretical and experimental research that has been carried out to investigate the interference effect of structural foundations. Each work is discussed in chronological order and a summary of the important features of each investigation is presented in Appendix A.

-- Stuart and Hanna (1961)

It is evident that the earliest study of the problem of interference between surface foundations was presented by Stuart and Hanna (1961). They investigated, both theoretically and experimentally, the effect of the interference between two rigid strip foundations in sand on their ultimate bearing capacity. The approximate theoretical solution used by Stuart and Hanna is based on Meyerhof's Theory (1951). The experimental investigation was carried out by using pairs of smooth and rough models of strip foundations of dimensions 0.5 in. (1.25 cm.) by 9.0 in. (22.9 cm.). Effects of the spacing ratio S/B (where S is the centre to centre spacing, and B is the width of one of the two foundations) and the foundation depth were investigated. Stuart and Hanna used the term "efficiency" to express the ratio of the ultimate load on a pair of adjacent footings to twice the ultimate load on a single footing. Stuart and Hanna found that the efficiency increased as the spacing ratio decreased. The increase in efficiency depended

on the interface conditions between the footing and the soil medium, and depended as well on the foundation level. As for smooth footings, on or near the surface, the maximum efficiency (about 175%) occurred at spacing ratio $S/B=1.0$ (the two foundations touched). Then, the efficiency decreased gradually as the spacing ratio increased (see Figure 1.1). When these smooth footings were located at depths of $8B$ or more, their behaviour was completely different. The efficiency was about 90% at spacing ratio $S/B=1.0$ and then increased as the spacing increased to reach a maximum value of about 130% at spacing ratio $S/B=2$ to 3 (Figure 1.1). As for rough foundations near the surface, the maximum efficiency occurred at spacing ratio $S/B=1.5$, then, decreased as this ratio increased (see Figure 1.1). The results predicted from the theoretical solution were, in general, higher than those obtained in the experiment.

-- Stuart (1962)

Stuart (1962) presented a more rigorous theoretical treatment for the problem of interference between two strip foundations near the surface of a cohesionless medium. Stuart's solution was based on modifications of the familiar Terzaghi and Hill isolated foundation failure mechanisms (see Figure 1.2 to Figure 1.4). Stuart modified Terzaghi's result for the bearing capacity of a single foundation resting at a depth, or in the surface, of a granular medium by introducing (modifying) coefficients which took

FIGURE 1.1 EXPERIMENTAL RELATIONSHIP BETWEEN EFFICIENCY AND SPACING FOR TWO STRIP FOUNDATIONS IN SAND (AFTER STUART AND HANNA, 1961)

FIGURE 1.2 THE PRANDTL MECHANISM FOR SMOOTH-BASED FOUNDATIONS ON COHESIONLESS SOIL, AND TERZAGHI MODIFICATION FOR ROUGH-BASED FOUNDATIONS (STUART, 1962)

FIGURE 1.3. HILL'S MECHANISM MODIFIED FOR COHESIONLESS SOIL:
 (a) SMOOTH-BASED, AND (b) ROUGH-BASED FOUNDATIONS
 (STUART, 1962)

FIGURE 1.4 THE DEVELOPMENT OF THE FAILURE SURFACES AS TWO ROUGH-BASED FOUNDATIONS APPROACH EACH OTHER ON THE SURFACE OF COHESIONLESS SOIL (AFTER STUART, 1962)

into account the influence of adjacent foundations. The bearing capacity equation of a single foundation resting on a granular soil is:

$$q_u = q_0 N_q + \frac{1}{2} \gamma B N_\gamma \quad (1.1)$$

The modified bearing capacity equation proposed by Stuart takes the form:

$$q_u = q_0 \eta_q N_q + \frac{1}{2} \gamma B \eta_\gamma N_\gamma \quad (1.2)$$

where q_0 = surcharge pressure

γ = unit weight of soil

B = width of the foundation

N_q, N_γ = bearing capacity coefficients which are functions of the soil's angle of shearing resistance, ϕ and

η_q, η_γ = bearing capacity efficiency factors which represent the ratios of the "interfering" to "isolated" values of the bearing capacity coefficients.

For foundations on the surface ($q_0=0$), Equation (1.2) becomes:

$$q_u = \frac{1}{2} \gamma B \eta_\gamma N_\gamma \quad (1.3)$$

Stuart found that the bearing capacity efficiency factors η_q and η_γ depend on many variables such as spacing between the two foundations (S), the angle of shearing resistance (ϕ), the interface conditions between the foundation and the soil; rough or smooth, and the foundation depth (D).

Stuart also carried out a series of tests, using smooth and rough strip foundation models of dimensions 0.5 in. (1.25 cm.) x 9.0 in. (22.9 cm.) and 1 in. (2.54 cm.) x 13 in. (32.1 cm.). The models were placed on the surface of compacted fine sand. Stuart's experimental results confirmed those obtained by Stuart and Hanna (1961). The experimental results were lower than the theoretical results with greater differences occurring at lower spacing ratios.

Mandel (1965)

Mandel (1965) investigated, theoretically, the influence of interference between foundations on their bearing capacity. The study was concerned with a row of continuous footings, rough or smooth, resting on a granular medium. Using the theory of limit equilibrium, Mandel presented numerical results in a graphical form for soils with $\phi = 0^\circ, 15^\circ, 30^\circ$ and 45° . Mandel's results confirmed those obtained earlier by Stuart (1962). The increase in the bearing capacity is strongly influenced by the change of distance between the adjacent foundations and the change of the angle of internal friction of the soil. The minimum spacing necessary so that the two foundations do not interfere with each other is also dependent on the angle of internal friction. For instance, this state of affairs occurs at spacing ratio of $S/B=7$ for $\phi = 45^\circ$ and at $S/B=4$ for $\phi = 30^\circ$.

-- West and Stuart (1965)

West and Stuart (1965) investigated experimentally the influence of the interference between two strip foundations located at the surface of a layer of sand on the ultimate bearing capacity. They also investigated the effect of the spacing, between the foundations, on inclination and eccentricity of the soil reaction (see Figure 1.5). The models used in experiments were aluminium "I" sections of width 1.75 in. (4.5 cm.) and with rough bottom surfaces. These researchers determined the eccentricity and the inclination of the soil reaction. They showed that the bending stresses were determined at two different locations on each web of the two "I" sections. The total load was assumed to be halved between the two foundations. Then, by using simple static relationships, the eccentricity and the inclination were obtained.

West and Stuart used the term "efficiency" as the ratio of the ultimate bearing capacity of two adjacent foundations to twice the ultimate bearing capacity of a single foundation. They found that a maximum efficiency of 180% had occurred at a spacing ratio of $S/B=1.3$. When the two foundations touched, the efficiency dropped to 150%. The maximum inclination determined was about 17° to the vertical. It occurred at the same spacing ratio ($S/B=1.3$) as the maximum efficiency.

FIGURE 1.5 GENERAL INTERFERENCE MECHANISMS FOR
SHALLOW ROUGH-BASED STRIP FOUNDATIONS
(AFTER WEST AND STUART, 1965)

-- Murthy (1970)

Murthy (1970) carried out an experimental study of two interfering strip foundations. One of the two foundations already carried its working load (which would represent the existence of a foundation). The other was subjected to an increasing load up to its failure load. Two pairs of wooden models, 5 cm. and 7.6 cm. wide, were used to simulate rough strip foundations. The models were placed on the surface of a compacted coarse sand stratum. Murthy (1970) found that the settlement decreased as the spacing decreased. As for the bearing capacity, Murthy's results were similar to those recorded by previous researchers.

-- Myslivec and Kysela (1973)

Myslivec and Kysela (1973) presented an experimental investigation related to the problem of the interference between foundations. Myslivec and Kysela directed their attention to studying the effect of the interference on the bearing capacity of two adjacent foundations of different widths. These foundations were located in a compacted sand stratum at different depths. The models used were 10 cm. in length and had widths of 1 cm. 1.5 cm. and 2 cm. The surface of each foundation model had different degrees of roughness. Myslivec and Kysela found that if two adjacent foundations were located at different depths, the ultimate bearing capacity of the shallower one would be considerably lower than that of a single foundation. As for the deeper foundation, its bearing would be higher. Myslivec and Kysela's results confirmed those obtained by the previous researchers as to the significant role

of the foundation depth. The amount of increase in the bearing capacity, due to the interference between two foundations located at the same level, depended on the foundation depth. A maximum increase in the bearing capacity in the order of about 100% occurred for foundations on the surface. For foundations located at depths of $D=6B$, the maximum increase in the bearing capacity was only about 45%.

-- Singh, et al. (1973)

Singh, et al. (1973) carried out a series of tests to study the effects of the interference between a group of square footings. Square models of dimensions 4 cm. x 4 cm., 4.9 cm. and 6 cm. x 6 cm. were used. The models were located on the surface of a layer of sand. Singh, et al. (1973) examined the influence of the interference on the settlement as well as on the bearing capacity. These researchers found that the settlement decreased and the bearing capacity increased as the spacing ratio (S/B) decreased (B is the side of each square). About 50% decrease in settlement and about 100% increase in bearing capacity were observed when the two foundations were in contact. No interference was observed at spacing ratio of $S/B \geq 5$.

-- Khadilkar and Varma (1977)

Khadilkar and Varma (1977) presented a theoretical solution for the problem of interference between two strip foundations resting on a cohesionless medium by using nonlinear finite element analyses. The study was mainly concerned with the examination of

the effect of the interference on the total settlement and tilts of both rigid and flexible foundations. They found that the average settlement decreased as the spacing decreased up to 3B. There was little change when the two foundations were brought still closer. Friction at the foundation-soil interface had a significant influence in determination of tilts and in estimation of heave around the foundations. The pattern of tilt changed depending on the width of foundation, the spacing and the intensity of the load.

-- Deshmukh (1978)

Deshmukh (1978) performed an experimental study for the behaviour of a group of adjacent rough foundations of different shapes resting on the surface of a layer of dense sand. The foundation models were wooden blocks with square (10.16 cm. x 10.16 cm.), circular (10.16 cm. diameter), rectangular (8.90 cm. x 17.80 cm.), and strip (5.08 cm. x 25.40 cm.) plan shapes. Both settlement and bearing capacity of these foundations were investigated. Deshmukh (1978) compared his experimental results related to the bearing capacity with those obtained from theories of Terzaghi (1943), Meyerhof (1951) and Balla (1962). The theoretical results were obtained by introducing the bearing capacity group efficiency factors suggested earlier by Stuart (1962) into the equations of the bearing capacity of an isolated foundation. The experimental results agreed well with the values obtained from Balla's theory. (For

foundations on the surface of a cohesionless medium, Balla's theory gives bearing capacity values almost twice those obtained by Meyerhof or Terzaghi's theory). Deshmukh (1978) also found that for any shape of the foundations used in the experiments, the bearing capacity increased as the spacing between the foundations decreased. As for the settlement at a constant specific load, it decreased with the decrease in spacing for all types of the foundations used, but at failure, the settlement increased as the spacing decreased.

1.2.2 Settlement and Bearing Capacity of a Soil Stratum Resting on a Rigid Base

This section presents a brief review of some research concerning the settlement and bearing capacity of a soil medium resting on top of a rigid base. This research is discussed chronologically as in the previous section. The discussion is divided into two parts: the first concerns the settlement and the second concerns the bearing capacity.

(A) Settlement of a Soil Stratum Resting on a Rigid Base

-- Steinbrenner (1934)

Steinbrenner (see Terzaghi, 1943) presented an approximate method of computing the settlement of an elastic isotropic homogeneous layer on a rigid base. Steinbrenner computed the settlement (δ)

of the corners of a uniformly loaded rectangular area on the horizontal surface of a semi-infinite mass. Then, he computed the vertical displacement ($\Delta\delta$) of the points located at a depth (H) below these corners and assumed that the settlement ($\Delta\delta_H$) of the corners of the loaded area on the surface of an elastic layer, of the thickness H, could be obtained from the following equation:

$$\begin{aligned}\Delta\delta_H &= \delta - \Delta\delta \\ &= q \frac{B}{E} [(1-\nu^2) F_1 + (1-\nu-2\nu^2) F_2]\end{aligned}\quad (1.4)$$

$$\Delta\delta_H = q \frac{B}{E} \cdot I_p \quad (1.5)$$

where: q = the load intensity

B = the width of the loaded area

E = the elastic modulus, ν = Poisson's ratio

F_1 and F_2 = dimensionless numbers dependent on the ratio H/B and the plan aspect ratio (L/B) of the loaded area.

Steinbrenner (1934) gave the values of F_1 and F_2 in a graphical form for different values of H/B (0 to 10) and different values of L/B (1, 2, 5, 10 and ∞). In the case of a strip loaded area (plane strain problem), Steinbrenner's method can be used for computation of the settlement, using the values of F_1 and F_2 corresponding to the ratio $L/B = \infty$ in Equation (1.4).

Steinbrenner's solution is approximate for two reasons: (i) the solution did not consider the change in both normal and shear stress distributions along the depth (H) due to the rigid base; and (ii) the solution did not take into account the friction conditions

on the interface between the soil stratum and the rigid base.

-- Terzaghi (1943)

Terzaghi (1943) discussed the influence of the rigid base on the shear and normal stresses produced by a strip loaded area on the surface of an elastic layer. Terzaghi explained that the distribution of normal stresses on a rigid base of an elastic layer and the distribution of shear stresses along a vertical section through the edges of the loaded area depend on the friction conditions between the elastic layer and the rigid base (see Figure 1.6). Also the maximum normal pressure (p_H), acting on the rigid base, which occurs beneath the centre of the loaded area, depends on the ratio H/B . For example, $p_H = 0.900 q$ for $H/B = 0.5$ and $p_H = 0.743 q$ for $H/B = 0.84$.

-- Burmister (1956)

Burmister (1956) presented a mathematical solution for the stresses and displacements in a layer on top of a rough rigid base. The layer is loaded by a point load on the surface. The charts and tables given in the paper can be used to compute stresses and displacements for any shape of surface loading. The solution is valid for $\nu = 0.2$ and 0.4 .

-- Harr (1966)

Harr (1966) presented influence factors in a tabular form (modified from Egorov, 1958) that can be used to compute the settlement

FIGURE 1.6 INTENSITY AND DISTRIBUTION OF NORMAL STRESSES ON RIGID BASE OF ELASTIC LAYER BENEATH FLEXIBLE STRIP LOAD AND OF SHEARING STRESSES ON VERTICAL SECTIONS THROUGH EDGES OF LOADED STRIP.
(MODIFIED FROM TERZAGHI, 1943)

at the centre and the midpoint of the long side of a uniformly loaded area. The load is located on the surface of a compressible layer resting on top of a rigid base.

-- Poulos (1967)

Using Burmister's theory (1956), Poulos (1967) evaluated a set of influence factors for computing the stresses and the surface displacements of an elastic layer on top of a rough rigid base. The solution was first obtained for a point load on the surface of the layer for different values of Poisson's ratio (0, 0.2, 0.4 and 0.5). Then, the solution for the distributed loads was evaluated by an integration process. Specific influence factors were given for loadings in the form of line, strip and sector plan shapes.

(B) Bearing Capacity of a Soil Stratum on a Rigid Base

-- Shield (1955)

Shield (1955) presented a theoretical solution for the bearing capacity of a pure cohesive soil resting on a rough rigid base. The solution was obtained by using limit equilibrium techniques and was based on the following assumptions:

- i) The soil is an elastic-perfectly plastic material.
- ii) The soil obeys Tresca's yield criterion.

The solution was obtained for rigid smooth footings with square, circular, rectangular, and strip plan shapes. Shield found that the rigid base would have no effect on the bearing capacity of the

soil stratum if the ratio B/H was less than 3, in the case of strip footings, and 5, in the case of square footings.

-- Livneh (1965)

Livneh (1965) theoretically investigated the bearing capacity of soils on rock foundation. The solution is given for the general case of soils with any value of the angle of internal friction from 0° to 40° . The soil stratum is loaded by a strip load on the surface. The solution is based on the following assumptions:

- i) The soil medium is governed by the Coulomb-Mohr strength law.
- ii) The soil is weightless, so that the solution relates only to the value of the bearing capacity coefficient (N_c).
- iii) The length of the loading strip is taken to be infinite.

The solution was given for soils in smooth or rough contact with the rigid base. Livneh found that the bearing capacity of a soil stratum in smooth contact with the rigid base is smaller than that calculated according to the failure mechanism of Prandtl for a deep stratum. For example, with $\phi = 30^\circ$ and $H/B=6$, the value of the bearing capacity is only 50 percent of the value obtained from Prandtl's failure mechanism. Increasing the ratio H/B , the bearing capacity increases, reaching the Prandtl mechanism value at a certain limiting depth (the minimum depth of a soil layer on a rigid base at which the bearing capacity of this layer is equal to that obtained from Prandtl's solution for a deep layer). The limiting depth depends on the angle of internal friction. Livneh also found that the

presence of friction between the soil and the rock increases the value of the bearing capacity. The limiting depth is smaller in this case.

-- Mandel and Salencon (1969)

Mandel and Salencon (1969) applied limit equilibrium techniques to examine the bearing capacity of a soft ground layer on a rigid base. The solution is for a plane strain problem and for a material obeying Coulomb's yield criterion. For computing the bearing capacity of a strip footing resting on a $c-\phi$ soil, the following equation proposed by Terzaghi (1943) can be used:

$$q_u = CN_c + q_0 N_q + 0.5 \gamma B N_\gamma \quad (1.6)$$

in which: q_u = the ultimate bearing capacity

C = the cohesion of the soil

q_0 = the surcharge

γ = the unit weight of the soil

B = the width of the strip footing

N_c , N_q and N_γ = the bearing capacity coefficients which are functions of the angle of internal friction (ϕ) of the soil.

Mandel and Salencon (1969) proposed the modification of Equation (1.6) into the following form to take into account the effect of the rigid base:

$$q_u = C F_c N_c + q_0 F_q N_q + 0.5 \gamma B F_\gamma N_\gamma \quad (1.7)$$

$$\text{or} \quad q_u = C N'_c + q_0 N'_q + 0.5 \gamma B N'_\gamma \quad (1.8)$$

where: N'_c , N'_q and N'_γ = the modified bearing capacity coefficients that depend on H/B , ϕ and the interface conditions (for a layer of soil with $\phi = 0$ and $H/B=0.707$, $N'_c = 5.14$, $N'_q=1.0$ and $N'_\gamma = 0$).

The correction factors F_c , F_q and F_γ depend also on the friction conditions on the interface between the soil stratum and the rigid base. For example, the values of F_c and F_q ($\phi=30^\circ$ and $H/B=0.5$) are equal to 0.562 and 0.586 in the case of smooth rigid base and respectively equal to 2.5 and 2.42 in the case of rough rigid base.

-- Mandel and Salencon (1972)

Mandel and Salencon (1972) extended their previous study to include the calculation of the bearing capacity for different values of ϕ ranges between 0° to 40° . They presented the values of the correction factors F_c , F_q and F_γ mentioned above in both tabular and graphical forms as a function of H/B , ϕ and the interface conditions between the footing and the soil, and the soil and the rigid base. When a perfectly rough contact is assumed the bearing capacity increases steadily with a decrease in the ratio H/B . When a perfectly smooth contact is assumed, the bearing capacity starting from the classical value (e.g. it is equal to 5.14 C for a purely cohesive soil with $\phi=0$), decreases as the ratio H/B decreases. After reaching a minimum value, the bearing capacity increases with a decrease in the ratio H/B (see Figure 1.7).

FIGURE 1.7 RELATIONSHIP BETWEEN THE RATIO H/B AND THE BEARING CAPACITY OF A PURELY COHESION SOIL, ($\phi=0$) IN SMOOTH CONTACT WITH A RIGID BASE (AFTER MANDEL AND SALENCON, 1972)

-- Suklje and Vidmar (1973)

Suklje and Vidmar (1973) investigated the bearing capacity of a cohesionless layer of limited thickness subjected to a rigid strip load. They carried out a series of experiments, using cylindrical steel rods to simulate the cohesionless medium. Their results confirmed the theoretical results obtained by Mandel and Salencon (1969) and (1972).

-- Meyerhof (1974)

Meyerhof (1974) examined the bearing capacity of a soft layer on a rigid base. Solutions were developed for circular and rectangular footings with rough contact surfaces. The footings were assumed to be located on the surface of a layer of sand resting on a rough rigid base. Meyerhof proposed the following equations for the ultimate bearing capacity of rectangular/circular foundations:

$$q_u = S_q' \gamma D N_q' + S_Y' \gamma B N_Y' \quad (\text{circular}) \quad (1.9)$$

$$q_u = \left[1 - (1 - S_q') \frac{B}{L} \right] \gamma D N_q' + \left[1 - (1 - S_Y') \frac{B}{L} \right] \gamma B N_Y' \quad (\text{rectangular}) \quad (1.10)$$

where: N_q' and N_Y' are modified bearing capacity coefficients, which depend on ϕ and H/B and the degree of roughness of the rigid base.

S_q' and S_Y' are modified shape factors for circular footings.

Meyerhof gave the values of N_q' , N_γ' , S_q' and S_γ' in a graphical form as functions of ϕ and H/B . Meyerhof (1974) also carried out a number of tests, using rough strip and circular footing models. The models were placed on the surface, and at a shallow depth in sand layers resting on a rough rigid base. For layers of ratios $H/B > 0.5$, the bearing capacity values obtained from the experiment were in general higher than those obtained from the theory. For layers of ratios $H/B < 0.5$, the bearing capacity values obtained from the experiment were less.

-- Chen (1975)

Chen (1975) presented a solution for the bearing capacity of a strip footing located on the surface of a shallow clay stratum resting on a rough rigid base. The solution was obtained by the finite element techniques. The agreement between the finite element solution for the limit load and the theoretical limit load was found to be satisfactory.

-- Tournier and Milovic (1977)

Tournier and Milovic (1977) studied the bearing capacity of a layer of sand resting on a rough rigid base. They performed a series of tests, using rough rigid strip footing models. The models were of widths $B=20, 35$ and 50 cm. Several values of the ratio H/B , between 0.5 and 6.8 , were studied. For all sizes, the bearing capacity increased with the decrease in the thickness of the sand layer. Tournier and Milovic compared their results with those obtained from the theory of Mandel and Salencon (1972). The

comparison confirmed the findings of Meyerhof (1974) mentioned earlier. The experimental results were higher than the theoretical for $H/B > 0.5$ and lower for $H/B < 0.5$. Tournier and Milovic's results indicated that the influence of the rigid base can be still appreciable until a ratio of $H/B=2$. The theoretical results of Mandel and Salencon (1972) reveals that the rigid base would have no influence on the bearing capacity if the ratio H/B was greater than about 1.2.

-- Pfeifle and Das (1979)

Pfeifle and Das (1979) investigated the problem of the bearing capacity of a cohesionless layer on a rough rigid base. Rough rigid steel models of width 5.08 cm. and lengths 5.08, 10.16, 15.24 and 30.48 cm. were used. Their results confirmed those of Tournier and Milovic (1977). Pfeifle and Das found that the modified shape factors proposed by Meyerhof (1974) agreed closely with the experimental findings.

1.2.3 General Comments on Literature Review

From the review of the literature in the area of soil foundation interaction presented above, it is possible to draw the following conclusions:

- 1 - Most research related to the interference between foundations is concerned with the determination of the bearing capacity of the individual foundation or the foundations in a group.

2 - A limited number of these investigations examined the effect of the interference between surface foundations on the load-settlement response of the foundations in a group.

3 - Despite the fact that there are many theoretical and experimental studies that are concerned with the contact stress distribution beneath foundations, these studies are related to a single foundation. (Detailed accounts of the current state of knowledge in the area of the contact stress distribution beneath foundations are well documented by Selvadurai, 1979).

4 - Experiments carried out for study of the problem of the interference between surface foundations were done by using small scale models resting on a deep soil layer. Most experiments were carried out by using strip footing models of width $B=1$ cm. to 4 cm. The reliability of the results obtained from such types of experiments is questionable. Also, these models were placed on the surfaces of cohesionless layers of thickness $H=8B$ or more. Thus, it can be said that the influence of the rigid base on the settlement and the bearing capacity of a soil layer loaded by a group of foundations has not yet been investigated.

1.3 Scope and Objectives of the Present Work

The main objective of this work can be summarized as follows:

- (a) To conduct an experimental investigation which will examine the group behaviour of interfering surface foundations tested under plane strain conditions. The experimental techniques attempt to investigate the influence of interference between foundations, on the contact stress distribution and the settlement.
- (b) To examine the effect of some selected parameters on the behaviour of two adjacent rigid strip foundations. The two foundations are located on the surface of a layer of sand resting on a rigid base. The parameters that have been investigated include:
 - i) the spacing between the two foundations,
 - ii) the layer thickness
 - iii) the interface conditions (rough or smooth) between the foundation and the soil, and the soil and the rigid base.
 - iv) load cycling (loading and unloading)

Other factors such as the foundation depth, shape of the foundation, density of the sand, loading sequence are kept constant throughout the experiments.

- (c) To develop numerical and empirical solutions to the interference problem by using nonlinear finite element analyses and an empirical method such as that proposed by Schmertmann (1970).

Chapter 2

EXPERIMENTAL PROGRAMME

2.1 Modelling of the Plane Strain Problem

The particular problem examined in this thesis attempts to model the behaviour of two interfering rigid strip footings. The two footings are located on the surface of a layer of dense sand resting on top of a rigid base. A review of the experimental investigations, already conducted in the area, indicated that most of the experiments were conducted in small scale tests. For example, the strip footing models used by Stuart and Hanna (1961), Stuart (1962), Myslivec and Kysela (1969) and (1973) had dimensions which ranged from 1 cm. to 2.5 cm. The results obtained with this type of small scale models are questionable. In the present experimental program, strip footing models of dimensions 15 cm. and 38 cm. are used.

The problem modelled in the experimental investigation is shown in Figure 2.1. The problem can be described as follows: to consider two strip foundations which are located on the surface of a stratum of sand. The two strip foundations are loaded in such a way that the central plane between the two foundations acts as a plane of symmetry. In this situation a single half of the group can be examined. Assuming plane strain conditions to exist, a finite section of the strip foundation can be examined. The plane

FIGURE 2.1. MODELLING THE PROBLEM OF TWO INTERFERING STRIP FOUNDATIONS
(PLANE STRAIN PROBLEM)

strain conditions are characterized by the displacement field:

$$u = u(x,y); \quad v = v(x,y); \quad w=0$$

where u , v and w are the displacement components in the x , y and z directions respectively. The shear stresses τ_{xz} and τ_{yz} on any plane xy are zeros. Also, by virtue of the symmetry of the problem, the shear stress is zero on the plane of symmetry ($x=0$).

Hence, the experimental programme can concentrate on a single half of the symmetric strip configuration, in which the relevant planes are replaced by frictionless surfaces.

Now, the strip footings can be subjected to several types of loading configuration and/or displacement conditions consistent with their practical uses. One particular type of loading configuration assumes that each strip footing is subjected to a concentrated load located at the centre. In this situation the strip foundation is capable of rotation due to the asymmetrical nature of the interference between foundations (Figure 2.2.b). In other types of loading configurations the strip foundations are rigidly connected to the loading shafts. Such situations can occur in strip foundations which are located by closely spaced wall-column systems. In this instance the stiffness of the structure prevents rotation at the foundation base. Consequently, there is uniform settlement of the interfering foundations (Figure 2.2.a). The problem investigated in the experimental programme has the loading configuration shown in Figure 2.2.a. The loading shaft was rigidly connected to the

FIGURE 2.2.a
TEST PLATE IS TO
SETTLE UNIFORMLY

FIGURE 2.2.b
TEST PLATE IS
ALLOWED TO TILT

FIGURE 2.2 UNIFORM SETTLEMENT OR TILT OF THE FOOTING ACCORDING TO THE CONNECTION
BETWEEN THE FOOTING AND THE LOAD SHAFT

foundation model such that no horizontal displacement nor rotation should occur.

In the following sections, the description of the plane strain apparatus, the properties of the tested material and the schedule of the experimental programme will be discussed in detail.

2.2 The Plane Strain Apparatus

2.2.1 Test Tank

All the tests were performed in a rigid steel tank with inside dimensions of 1.52 m. x 0.38 m. x 0.40 m. (Figure 2.3 and Plate 1). The basic design of the test tank was developed by Professor Selvadurai and it was constructed in the Civil Engineering Laboratories at Carleton University in Ottawa. The base and the ends of the tank were made of C381x60 channels. The side walls were made of 12.5 mm. thick steel sheets. Each wall was fastened with lengths of C76x6 channels. Additional rigidity was incorporated by two lengths of 152 mm. x 102 mm. x 9.5 mm. rectangular hollow steel sections. In each test, these removable sections were placed where the maximum lateral pressures were to be expected. The lateral displacements of the side walls of the tank were prevented by constraining each rectangular hollow steel section against the main steel frame by means of a screw jack (Figure 2.3.a and Plate 2). When the maximum load was applied, measurements taken for the lateral displacements of the side walls indicated that this method had been fairly successful in minimizing these displacements.

FIGURE 2.3.a. THE PLANE STRAIN APPARATUS (CROSS SECTION)

FIGURE 2.3.b THE PLANE STRAIN APPARATUS (LONGITUDINAL SECTION)

PLATE 1

THE PLANE STRAIN APPARATUS

(SIDE VIEW)

PLATE 2
THE PLANE STRAIN APPARATUS
(FRONT VIEW)

2.2.2 Study of Effect of the Side Friction

Friction on the interior faces of the side walls of the test tank was minimized by using sheets of polished No.16 gauge stainless steel. The effectiveness of the stainless steel in minimizing the friction was established in two different ways. The first technique involved the determination of the coefficient of friction of the stainless steel. The second technique involved the determination of the effect of the friction on the test results.

The following is a brief description of the test carried out for this purpose:

(a) Determination of Coefficient of Friction

To determine the coefficient of the friction of the stainless steel, two types of tests were carried out: the shear box test and the pull-out test.

(i) Shear Box Tests - In this series of tests, a block of stainless steel was sheared against the sand. The stainless steel block had the same inside dimensions as the upper half of the shear box. The bottom surface of the block was polished No.16 gauge. The lower half of the shear box was filled with sand by raining, using the same technique described in section 2.3.2. This method provided sand with a relative density D_r of $90\% \pm 2\%$. The surface of the sand filling the lower half of the shear box was carefully levelled. The two halves of the shear box were fitted together as shown in Figure 2.4. The stainless steel block, working as a cap, was loaded and sheared against the sand. Several tests were performed using different values of the normal stress (σ_n). For

FIGURE 2.4 SHEAR BOX WITH HALF SAMPLE OF SAND

FIGURE 2.5 RELATIONSHIP BETWEEN SHEAR AND NORMAL STRESSES OF THE STAINLESS STEEL SURFACE

each normal stress the corresponding shear strength (τ) was determined. The relationship between τ and σ_n is shown in Figure 2.5. The angle of friction of the stainless steel (λ_s) was found to be about 8° . However, there are some limitations in determining the angle of friction of the stainless steel by this method; these are:

- (1) that due to the scale effects, the half sample of sand in the lower half of the shear box may have a relative density which is different from that obtained from raining the sand in the test tank.
- (2) that the levelling process causes disturbance of the top surface of the sand.
- (3) that there is a possibility of locking of grains of sand between the stainless steel block and the shear box.

(ii) Pull Out Test - To overcome these limitations in the determination of the angle of the stainless steel by shear box, another series of tests was carried out. The purpose of this series of tests was to determine the ratio between the angle of friction of a plate of stainless steel (λ_s) and the angle of friction of another plate roughened with sand (λ_r). This ratio was used to determine the effect of the angle of friction of the stainless steel on the test results obtained for the test tank with interior walls lined with sheets of stainless steel, as will be explained in the next section. The two plates were of dimensions 38 cm. x 36 cm. x 0.1 cm. Each plate was vertically placed at the centre of the test tank and then the sand was rained until the tank

was completely filled (see Plate 3). Pull out loads were applied to each plate and the vertical displacements were recorded. The pull out load-displacement curves of the two cases are shown in Figure 2.6. The maximum vertical pull out forces in both cases were determined. In the case of the stainless plate, the relationship between the maximum pullout force (V_s) and the maximum shearing resistance force (T_s) is as follows:

$$V_s = 2T_s = 2P \tan \lambda_s \quad (2.1)$$

in which: P = the total force of the horizontal earth pressure

λ_s = the angle of friction of the stainless steel plate

In the case of the rough plate, the similar relationship between the maximum pull out force (V_r) and the maximum shearing resistance force (T_r) takes the following form:

$$V_r = 2T_r = 2P \tan \lambda_r \quad (2.2)$$

in which: P is as defined above

λ_r = the angle of friction of the plate roughened with sand.

from Equation (2.1) and Equation (2.2):

$$\frac{V_s}{V_r} = \frac{T_s}{T_r} = \frac{\tan \lambda_s}{\tan \lambda_r} \quad (2.3)$$

from experiments (see Figure 2.6):

$$\frac{V_s}{V_r} = \frac{0.1 \gamma B^3}{0.72 \gamma B^3} = 0.133$$

$$\therefore \frac{\tan \lambda_s}{\tan \lambda_r} = 0.133 \quad (2.4)$$

FIGURE 2.6 RELATIONSHIP BETWEEN NORMALIZED PULL-OUT FORCE AND NORMALIZED DISPLACEMENT FOR ROUGH AND SMOOTH PLATES

PLATE 3

LOAD APPLICATION IN THE PULL OUT TESTS

(b) Influence of the Side Friction on the Test Results

In order to investigate the effect of friction of the interior walls of the test tank on the load-settlement relationship and the ultimate load, three different series of tests were performed. The first was carried out with both interior walls lined with sheets of stainless steel. The second was carried out with the two walls lined with sheets roughened by sand. The third was carried out with one of the two walls lined with a polished stainless steel sheet and the other lined with a rough sheet. The footing was placed first at the centre of the tank and then close to the edge.

The results of these tests are shown in Figure 2.7.a and Figure 2.7.b. From these results and by using the results of the pull out test mentioned before, it was found that the effect of the angle of friction of the stainless steel was to increase the ultimate load by 4% (see Appendix B). It could also be noted that the side friction had an influence on the settlement of the footing. But this influence is negligible as long as the load level is within the working load limits.

The friction produced on the surface of the stainless sheets can also be appreciated by comparing the two failure surfaces shown in Plate 4 and Plate 5. The failure surface shown in Plate 4 was obtained with the interior walls of the test tank lined with sheets of stainless steel. The second failure surface, shown in Plate 5, was obtained with the interior walls lined with rough sheets. From the two plates, it is evident that the failure surface is almost straight in the case of the stainless steel sheets and

FIGURE 2.7.a EFFECT OF ROUGHNESS OF THE SIDE WALLS OF THE TEST TANK ON LOAD-DISPLACEMENT CURVES (FOOTING AT CENTRE OF THE TEST TANK)

FIGURE 2.7.b EFFECT OF ROUGHNESS OF THE SIDE WALLS OF THE TEST TANK ON LOAD-DISPLACEMENT CURVES (FOOTING AT EDGE OF THE TEST TANK)

PLATE 4
SURFACE PROFILE OF FAILURE SURFACE
(SMOOTH CASE)

PLATE 5
SURFACE PROFILE OF FAILURE SURFACE
(ROUGH CASE)

curved in the case of the rough sheets. This reflects the effect of friction in each case.

2.2.3 Modelling of the Footings

Two sizes of footing models were used in most experiments performed in this program; a rigid square steel plate, and a rigid rectangular aluminium plate (Plate 6).

(i) Square Steel Plate

A square steel plate of dimensions 378 mm. x 378 mm. x 51 mm. and with a smooth bottom surface was used as a model of a smooth rigid strip footing. The plate was used in the first series of tests which was carried out for the purpose of investigating the effect of the interference between foundations on the contact stress distribution. The plate was designed by Kempthorne (1978) in connection with a research project which involved the examination of the plane strain behaviour of a rigid strip footing resting on a compacted bed of clay. The complete details of this plate may be found in Kempthorne (1978). For the present purposes, a brief description will be given.

The plate has sixteen pressure cells as shown in Figure 2.8. The locations of the sixteen pressure cells were chosen as follows. Six transducers were located along the centre line of the plate to provide a relatively accurate contact stress distribution. Three transducers were located very close to each edge of the plate; parallel to the centre line, to check the assumption of equivalent contact stress distribution along the axis of the strip foundation. The last four transducers were located on

the diagonals of the square plate to check the equivalent contact stresses along the two diagonals in case the plate was to be used in a three dimensional problem. For the present purposes, for which a plane strain problem is investigated, it was decided to use only twelve transducers; six for measuring the contact stresses along the section A-A of the centre line and six for measuring the contact stresses at the two sections; B-B and C-C, close to the edges (see Figure 2.8).

To avoid any effect on the calibration constants of the pressure transducers by the mounting process in the plate, the transducers were mounted, and then calibrated in place. Each transducer was held in its location by means of a brass collar which ensured that the face of the transducer was flush with the bottom surface of the plate. The method of calibration involved placing a pore water pressure chamber over the face of each transducer. The chamber was connected to the triaxial system and all the connections were filled with water. Based on the method outlined by Bishop and Henkel (1962), the pressure was accurately applied by a system of mercury pots in the Soil Mechanics Laboratory. For each pressure increment the output voltage was recorded. From the relationship between the applied pressure and the output voltage, the calibration constant was obtained.

The output of the pressure cells were recorded by means of a multi-channel digital strain indicator (Plate 7). The system

FIGURE 2,8 DIAGRAM OF RIGID STEEL SQUARE PLATE

PLATE 7

"B & F" 40-CHANNEL DATA ACQUISITION SYSTEM

PLATE 6

THE RECTANGULAR ALUMINIUM PLATE,
THE SQUARE STEEL PLATE, THE LOAD CELL
AND THE HYDRAULIC JACK

was manufactured by B & F Instruments Ltd. and supplied by Intertechnology Ltd. This system has up to 40 channels available for recording. It was necessary to convert the output of this system to the required units by application of the appropriate conversion constants. Details of the operational procedures attributed to the B & F system are given by Kempthorne (1978).

(ii) Rectangular Aluminium Plate

A rectangular aluminium plate of dimensions 378 mm. x 152 mm. x 51 mm. was used in most of the tests carried out in this experimental program. The bottom surface of the plate was lined with a sheet of polished No.16 gauge stainless steel in order that the plate simulated a footing with a smooth surface. The rigidity of the plate was checked by estimation the amount of differential deflection of the plate during application of the maximum pressure, causing failure of sand. Calculations were also made to estimate the relative rigidity of the plate as defined by Borowicka (1938). These calculations are given in Appendix C, and indicate that the plate can be considered to be rigid.

Other sizes of steel plates with rough or smooth surfaces were also used in the subsequent series of tests as listed in Table 2.1.

2.2.4 Measurements of Displacements

The vertical displacements of the test plate were measured by means of four dial gauges with an accuracy of 0.001 in., located at each corner of the plate. The four readings indicated that the plate was settling uniformly. The settlement of the plate was taken as the average of the four readings. Surface displacements on either side of the footing were measured by means of a number of displacement transducers and dial gauges (see Plate 8).

2.2.5 Loading Devices

The loading rig consisted of a steel frame, a hydraulic jack and a load cell, as shown in Figure 2.3. The steel frame, which was designed and fabricated at Carleton University, had 155 cm. x 110 cm. outside dimensions. The frame had two columns consisting of two W105 x 19 I-sections. The two columns were connected with two horizontal beams; one at the top and the other in the middle. The beam at the top was a MC 152 x 24 channel and the one in the middle was a C 229 x 22 channel. The two columns of the frame were bolted to two MC 152 x 24 channels which were welded together to form the base of the frame. Despite the fact that the frame was able to stand firmly under its own weight, and the weight of the test tank provided extra support, the frame was anchored to the floor by means of two inclined struts. Each strut consisted of a C 76 x 6 channel that was welded to a steel plate anchored firmly to the floor (see Figure 2.3.b). In general the frame was designed to hold the

PLATE 8
THE MEASUREMENT OF SURFACE DISPLACEMENTS

maximum vertical reaction of the hydraulic jack safely.

The load was applied by means of a manually operated pump and hydraulic jack. The upper end of the hydraulic jack was attached to the MC 152 x 24 channel at the top of the steel frame, and the lower end was connected to a steel shaft 38 mm. in diameter. Throughout the experiments, the shaft was kept vertically aligned by two precision ball bushings enclosed in two pillow blocks. These pillow blocks were in turn mounted on a C 229 x 22 channel positioned over the plate and tank, by the steel frame (see Figure 2.3.a).

The steel shaft was attached rigidly to a 45 kN capacity load cell by a threaded connection. This connection was designed to minimize tilting of the test plate and horizontal deflection.

The purpose of the load cell was to measure the load increment applied directly to the rigid test plate by the hydraulic jack. The calibration chart for the load cell is given in Figure 2.9. The system was capable of measuring a minimum applied load of 0.1 kN. The design of the hydraulic load application system was not capable of unloading in an incremental fashion. (It was not possible to reverse the flow of the hydraulic fluid without releasing the incipient pressure and the corresponding load). Nevertheless, it was possible to maintain a high initial load of about 25 kN without appreciable loss of load.

The entire loading apparatus was designed to ensure that the

FIGURE 2.9 LOAD CELL CALIBRATION CHART

load could be applied vertically to the centre of the footing so that no horizontal movement or rotation to the footing would occur.

2.3 Materials Used in the Experimental Program

2.3.1 General Properties of Ottawa Sand

The material which was used in this study was flint grade sand supplied by the Ottawa Silica Company, Ottawa, Illinois. The flint grade sand (Ottawa sand) was a medium white quartz sand with rounded to subrounded particles. The sand was poorly graded as indicated by a uniformity coefficient of 1.7, and the grain size distribution shown in Figure 2.10. By using the method outlined by Bowles (1978), it was found that the minimum and maximum densities were 15.3 kN/m^3 and 18.0 kN/m^3 respectively.

2.3.2 Method of Placement of Sand in Test Tank

It is known that both settlement and ultimate bearing capacity are highly dependent on the density of soil. Since investigation of the effect of some other variables was the concern of this study, it was necessary to keep the density of the sand constant throughout the entire work. This was done by using a specific technique for placement of the sand in the test tank.

The technique that was employed, has been used successfully by several researchers: Atefi (1975), Kempthorne (1978), and

FIGURE 2.10 GRAIN SIZE DISTRIBUTION OF OTTAWA SAND

Moore (1979). This technique entailed raining the sand into the test tank at a predetermined rate of deposition, using a sand hopper which traversed the tank at a constant velocity (see Plate 9). Although it was possible to control the rate of deposition, it was not possible to move the manually operated hopper at a constant velocity. The rate of deposition was controlled by allowing the sand to rain through a mesh of openings 1.6 mm. in diameter located at the base of the hopper.

In order to determine the density of the deposited sand, containers, 76 mm. in diameter and 51 mm. in height were placed at different locations inside the test tank, during the raining process of the sand. It was found that the densities obtained were almost the same, with values of $1760 \pm 10 \text{ kg/m}^3$. This gave a relative density (D_r) of $90\% \pm 2\%$.

To achieve a level surface of deposited sand, a straight-edged metal rule was used. During the levelling process, the metal rule was supported against the straight edges of the test tank; this ensured a perfectly levelled surface.

Great care was taken during the process of placing the square steel plate (55 kgm in weight) on the surface of the sand layer. By means of rope, the plate was suspended from a hydraulic jack supported on top of the steel frame. After the surface of the sand had been levelled, the plate was gently lowered in place.

PLATE 9

METHOD OF PLACEMENT OF
SAND IN THE TEST TANK

2.3.3 Shear Strength and Deformability Characteristics of Ottawa Sand

A series of triaxial compression tests was carried out in order to determine the angle of internal friction ϕ and the stress-strain properties of the sand. These properties were required as input parameters in the nonlinear finite element investigation of the footing problem (see Chapter 6).

The samples for triaxial compression testing were prepared by following the procedures outlined by Bishop and Henkel (1962) and Bowles (1978). Seven tests were performed using different values of confining pressure (σ_3). All the tests were performed on samples having almost the same relative density of $D_r=90\%$. The samples were built by allowing the sand to rain into the specimen mould, using the method outlined in Section 2.3.2. For each sample, the density was found to be reproducible.

The stress-strain curves of the seven tests with different confining pressures are shown in Figure 2.11. The angle of internal friction ϕ was determined from the q - p diagram shown in Figure 2.12. The value of ϕ , as obtained by using the line of best fit, was approximately 41° .

2.3.4 A Plain Strain Test for Determination of ϕ

The analyses of the bearing capacity of strip footings, as developed by Terzaghi (1943), Meyerhof (1951) and others, assume the plane

FIGURE 2.11 · STRESS-STRAIN RELATIONSHIP OF OTTAWA SAND AT DIFFERENT CONFINING PRESSURES

FIGURE 2.12 FAILURE ENVELOPE OF OTTAWA SAND

strain deformation of the soil medium. These analyses also assume that the angle of internal friction ϕ is an invariant material property i.e., independent of the method of testing. It is known that the angle of internal friction, as determined under an axially symmetrical triaxial compression state, is several degrees less than that determined under plane strain conditions (Bishop, 1961; Cornforth, 1964; Lee, 1970; and others). According to Bishop (1961), the value of ϕ obtained from plane strain tests for dense sand is higher by 4° . Figure 2.13, from Lee (1970), also leads to the same conclusion.

Based on the above comments it is reasonable to modify the angle of internal friction of the Ottawa sand. The triaxial value of ϕ of 41° corrected to plane strain effect can be taken as 45° . However, in order to obtain an independent estimate for ϕ , it was decided to conduct a separate series of tests which included the plane strain assumption. The basic test consisted of the passive action of a retaining wall with a smooth vertical back. The retaining wall supported a cohesionless soil medium with horizontal surface. The equation describing the maximum passive force (at failure) in such a case was given by Rankine (1857). This equation is as follows:

$$P = \frac{\gamma H^2}{2} K_p$$

or

$$P = \frac{\gamma H^2}{2} \frac{1 + \sin \phi}{1 - \sin \phi} \quad (2.4)$$

FIGURE 2.13 COMPARISON OF RESULTS FROM DRAINED TRIAXIAL AND PLANE STRAIN TESTS ON SAND - AFTER BISHOP AND CORNFORTH (FROM LEE, 1970).

in which: P = the maximum passive force per length of the retaining wall,

γ = the unit weight of the soil medium,

H = the height of the soil supported by the retaining wall, and

ϕ = the angle of internal friction

The value of ϕ can be determined by using Equation (2.4) if the other variables were known. The main features of the testing technique used to achieve this purpose were as follows:

(a) Retaining Wall Model:

A steel plate of dimensions 390 mm. x 378 mm. x 25.4 mm. lined with a sheet of polished No.16 gauge stainless steel was used to model the rigid retaining wall with a smooth back. The rigidity of the plate was checked by estimating the amount of differential deflection which would occur at the failure load condition. This estimation is given in Appendix D. The results indicated that the plate could be considered to be rigid. In order to minimize the friction on the lower edge of the plate, the edge was bevelled and a stiff rubber strip of a thickness of 2 mm. was inserted as shown in Figure 2.15. However, the friction correction was determined by pushing the retaining wall model horizontally, while the test tank was empty of sand. Permanent contact between the sides of the plate and the stainless steel sheets was ensured by lightly spring loaded sealing strips of low friction (see Figure 2.15). The technique followed in modelling the plate had been used previously

FIGURE 2.14 RETAINING WALL MODEL

(All Dimensions in mm)

FIGURE 2.15 DETAILS OF THE RETAINING WALL MODEL

2

PLATE 10
THE RETAINING WALL EXPERIMENTS
(RETAINING WALL MODEL-LOADING SYSTEM)

PLATE 11

RETAINING WALL EXPERIMENT

(MEASUREMENT OF SURFACE DISPLACEMENTS)

by Arthur, et al. (1964) in a similar test.

(b) Loading System:

The load was applied horizontally by means of the hydraulic jack and the manually operated pump described previously. The load cell described earlier, was attached to the rigid steel plate and connected to the hydraulic jack (Figure 2.14). Precautions were taken to ensure that the load would be applied horizontally such that no upward or downward forces on the plate could occur. However, the upward vertical movements of the plate were monitored by two dial gauges (see Plate 10).

(c) Displacement Measurements:

The horizontal displacements of the plate were recorded by using two dial gauges of 0.001 in. accuracy. The surface displacements in front of the plate were measured by means of a number of dial gauges (see Plate 11).

(d) Results:

Two series of experiments were carried out using two heights of sand (H); the first with $H=38$ cm. and the second with $H=20$ cm. For each height, the test was repeated three times. The load-displacement curves and the surface displacements of the two tests are shown in Figure 2.16 and Figure 2.17 respectively. Despite the fact that sand used in the experiment was of relative density of 90%; i.e., it was considered to be compacted to a dense state, the load-displacement curve did not follow the usual pattern of

FIGURE 2.16 LOAD-DISPLACEMENT CURVES OF RETAINING WALL MODEL

FIGURE 2.17 SURFACE DISPLACEMENTS AT FAILURE FOR TWO DIFFERENT HEIGHTS OF SAND SUPPORTED BY A RETAINING WALL MODEL

curves of dense sand with a clearly defined peak load. However, the maximum load was obtained by replotting the load expressed in the ratio $P/\gamma H^2 B$ versus the displacement expressed in the ratio δ/H on a semi-log scale. The ratio $P/\gamma H^2 B$ at failure, that is equal to the coefficient of passive earth pressure (K_p), depends only on the angle of friction (ϕ). This was the case in these experiments, where the values of that ratio obtained from using different heights were almost the same (see Figure 2.16.b).

Using Equation (2.4), it was possible to determine the value of ϕ appropriate for the plane strain tests. Calculations are given in Appendix D. A value of 45.5° was obtained assuming completely smooth conditions with zero friction angle (this value of 45.5° may recall the 45° mentioned before as the triaxial value of ϕ corrected to the plane strain effect). If the angle of friction of the stainless steel ($\lambda=8^\circ$) is taken into account, a value of 41 degrees will be obtained for ϕ under plane strain conditions (see Appendix D).

2.4 Schedule of Experiments

Eight different series of experiments were performed. The schedule of the experimental program is given in Table 2.1. In each series of experiments, the spacing (i.e., the distance between the test footing and its imaginary counterpart) was changed. This was done indirectly by keeping the footing stationary and moving the test tank by means of rollers.

TABLE 2.1 SCHEDULE OF EXPERIMENTS

TEST SERIES	FOOTING SIZE			SPACING (S) CENTRE TO CENTRE	LAYER DEPTH H	FOOTING BASE CONDITION	RIGID BASE CONDITION	TANK SIDES CONDITION	NO. OF REPEATED TESTS	TOTAL NO. OF TESTS	PURPOSE OF INVESTIGATION
	LENGTH L CM.	WIDTH B CM.	THICK. t CM.								
A	37.8	37.8	5.1	1B, 2B, 3B, 4B	B	SMOOTH	SMOOTH	SMOOTH	6	24	CONTACT STRESS DISTRIBUTION
B	37.8	15.2	5.1	1B, 2B, 4B, 6B, 10B	2.5B	SMOOTH	SMOOTH	SMOOTH	3	15	EFFECT OF SPACING AND LAYER DEPTH TO RIGID BASE
C	37.8	15.2	5.1	1B, 2B, 4B, 6B, 10B	1B	SMOOTH	SMOOTH	SMOOTH	3	15	
D*	37.8	15.2	5.1	1B, 2B, 4B, 6B, 10B	0.5B	SMOOTH	SMOOTH	SMOOTH	3	8	
E	37.8	15.2	5.1	1B, 2B, 4B, 6B, 10B	1B	ROUGH	ROUGH	SMOOTH	3	15	EFFECT OF INTERFACE CONDITIONS
F	37.8	15.2	5.1	AT THE CENTRE OF THE BOX	1B	SMOOTH	SMOOTH	SMOOTH	3	15	EFFECT OF LATERAL SPACING
G	30.5 22.5 15.2 7.6				2.5B				3	15	
H	37.8				2.5B				3	15	
				AT CENTRE AND EDGE OF THE TEST TANK				TWO SIDES ROUGH ONE SIDE ROUGH TWO SIDES SMOOTH	3 3 3	15	EFFECT OF THE SIDE FRICTION OF THE TEST TANK

This series of tests was stopped after 8 tests.

The details of the eight different series of experiments are as follows:

(i) In series A of the tests, the group effect on the contact stress distribution was examined. The square steel plate shown in Figure 2.8 was used for this purpose. The plate was placed on the surface of a layer of sand. The thickness of the layer was equal to the footing width; B of 37.8 cm. The centre to centre spacing was changed by $1B$, $2B$, $3B$ and $4B$. To ensure repeatability of the results, each test was carried out six times.

(ii) In series B, C and D of tests, the effects of spacing and layer depth to the rigid base both on settlement within the working load and the ultimate bearing capacity were examined. The aluminium plate described previously was used as a footing model in this series of tests. The plate of width (B) equals to 15.2 cm. was rested on layers of sand of thickness H equals to $2.5B$, $1B$ and $0.5B$ in series B, C and D respectively, in the three series of tests. The interface conditions between the footing and the soil and between the soil and the rigid base were considered to be smooth since the base of the footing and the rigid base were lined with sheets of polished stainless steel. In each series of tests, centre to centre spacings were changed by $1B$, $2B$, $4B$, $6B$ and $10B$. For each spacing, the test was repeated three times. Good repeatability was obtained in series B and C. As for series D of tests performed on a layer of thickness of 7.5 cm., the half width of the footing, it was not possible to reproduce the tests and the results had a wide range of scatter (See Figure 2.18). This scatter of the results may be attributed to the fact that the

FIGURE 2.18 NORMALIZED LOAD VERSUS NORMALIZED DISPLACEMENT FOR A LAYER OF SAND OF THICKNESS TO FOOTING WIDTH RATIO $H/B=0.5$

technique followed in depositing the sand by raining was too sensitive to a thickness as small as 7.5 cm. Therefore, this series of tests was stopped and was not completed.

(iii) In series E of tests, the effect of the interface conditions was investigated. Having investigated the effect in the extreme case of smooth boundaries in series B and C of tests, the other extreme case of rough boundaries was examined in this series. Ottawa sand was bonded to the bottom surface of a rigid steel plate of dimensions 378 mm. x 152 mm. x 51 mm. The same sand was also bonded to a thin sheet of galvanized steel which was carefully placed on the bed of the test tank in order to simulate a rough rigid base. The plate of width (B) equal to 152 mm. was placed on the surface of a layer of Ottawa sand of thickness equal to the footing width. The spacings were changed and the tests were repeated as in series B and C.

(iv) In series F and G of tests, five rigid steel plates of different sizes (see Table 2.1) were placed on the surface of two layers of sand of thicknesses B and 2.5 B respectively. In the two series, the plates were always placed at the centre of the test tank and the interfaces between the footings and the sand and between the sand and the rigid base were modelled to simulate smooth conditions.

(v) Series H of tests was carried out to investigate the effect of the side friction of the test tank on the results. These tests and their results were discussed, in detail, early in this Chapter.

Chapter 3

INFLUENCE OF SPACING ON PERFORMANCE OF FOUNDATIONS-PLANE STRAIN PROBLEM

In this chapter, the influence of the spacing between two adjacent rigid strip foundations on the contact stress distribution, the settlement and the bearing capacity characteristics of the two foundations, will be discussed.

3.1 Influence of Spacing on the Contact Stress Distribution

The rigid steel plate of 16 pressure cells which is shown in Figure 2.8. was used to measure the contact stress distribution underneath one of the two interfering foundations. As mentioned at the beginning of Chapter 2, only one foundation was modelled due to the symmetry of the problem. The spacing between that foundation model and its imaginary counterpart was changed four times using ratio (S/B) of the order of 1, 2, 3 and 4, where (S) is the centre to centre spacing and (B) is the width of one of the two foundations. For each spacing ratio, the test was repeated six times. The maximum load applied was 98 kPa which is about one third of the ultimate bearing capacity as computed by the classical theory of Terzaghi. Four cycles of loading and unloading were performed. The contact stresses were measured when the maximum load (98 kPa) was being applied. It was observed that the contact stress values measured at the end of the first cycle were less than

those measured at the end of the next three cycles, while the contact stresses measured at the end of these three cycles were almost the same. This indicated that the cycling process had no influence on the values of the contact stress after the second cycle. However, since the differences observed at the first cycle may have resulted from bedding effects, the readings taken at this cycle were neglected and the readings of only the last three cycles were taken into account. The reliable value of the contact stress measured by a certain pressure transducer was taken as the average of these three readings.

The contact stress distributions corresponding to spacing ratios of $S/B = 1, 2, 3$ and 4 are shown in Figure 3.1.a to Figure 3.1.d respectively. In each figure, three distribution diagrams of the contact stresses produced beneath one of the two interfering foundations are shown. The first distribution was obtained by the six pressure transducers located along the centre line of the test plate (section A-A, Figure 2.8). The second and the third distributions were obtained by the three transducers located near each edge of the plate (section B-B and C-C, Figure 2.8). If the plane strain conditions are ideally fulfilled, the three distributions along sections A-A, B-B and C-C should be the same. Nevertheless, the average values of the contact stress distributions along the two sections near the edges were about 5% to 15% less than that value along the centre line. These differences may have resulted either from the lateral yield of the side walls of the test tank, or

FIGURE 3.1.a CONTACT STRESS DISTRIBUTION BENEATH ONE OF TWO INTERFERING RIGID STRIP FOUNDATIONS (SPACING RATIO $S/B = 1$)

FIGURE 3.1.b CONTACT STRESS DISTRIBUTION BENEATH ONE OF TWO INTERFERING RIGID STRIP FOUNDATIONS (SPACING RATIO $S/B = 2$)

Range of 6 Tests
Average Value of 6 tests

• Pressure Cell

q_x = measured contact stress

q_0 = average contact stress along axis a-a

Applied pressure = 98 kPa

$B = 38$ cm.

FIGURE 3.1.c CONTACT STRESS DISTRIBUTION BENEATH TWO INTERFERING RIGID STRIP FOUNDATIONS (SPACING RATIO $S/B = 3$)

Range of 6 Tests
Average Value of 6 Tests

• Pressure Cell

q_x = measured contact stress

q_0 = Average contact stress along axis a-a

Applied pressure = 98 kPa

$B = 38$ cm.

FIGURE 3-1.d CONTACT STRESS DISTRIBUTION BENEATH TWO
 INTERFERING RIGID STRIP FOUNDATIONS
 (SPACING RATIO $S/B = 4$)

from variation of the structure and the density of the deposited sand due to the edge effect. The first possible cause was almost eliminated by restraining the side walls against the steel frame by means of screw jacks as described previously in Chapter 2. Therefore, the edge effect may have been the possible reason for that discrepancy. However, the contact stresses at the central region along section A-A, were free from that effect. Therefore, the average contact stress along that section (q_0) was considered the more reliable value. The measured contact stress (q_x) was normalized and expressed in terms of q_0 .

The contact stress distribution curves were convex with lower values near the free edge (far from the conjugate foundation). This type of distribution differs from the concave shape proposed by Schultz (1961) for a deep stratum subjected to the same level of loading ($\frac{q_u}{3}$). The contact stress distributions right under the edges of footing model were not possible to be measured (due to the location geometry of the pressure cells, see Figure 2.8). Therefore, they are indicated on the figures by dashed lines. Similar contact stress distribution with convex shape and measured under a 30.5 cm. wide rigid strip foundation resting on the surface of crushed quartz sand, were observed by Bauer, et al. (1979).

Referring to Figure 3.1.a through to Figure 3.1.d, it appears that the shape of the contact stress distribution depends on the spacing between the two interfering foundations; it graduates from asymmetry for spacing ratio $S/B=1$ to symmetry for spacing

ratio $S/B=4$. When the spacing ratio is equal to 1, the test plate in this position, acts as half of a foundation of width $2B$.

The contact stress distribution measured under the test plate represents, in this case, half of the complete distribution of width $2B$, and hence, the measured contact stress distribution is asymmetrical. As the spacing ratio becomes greater than one, the test plate will act as one of two conjugate foundations with interfering stress zones. Due to the interference, the confining pressures within these zones becomes higher, or in other words, the rigidity and strength of the soil under the curtailed edge of the test plate (close to the conjugate foundation) becomes higher than it is under the free edge (far from the conjugate foundation). The test plate under such conditions tends to tilt outward and to experience larger settlements under the free edge, but since the plate, as explained in Chapter 2, is designed to settle uniformly without tilting, a bending moment will be produced causing the contact stress distribution to be asymmetrical.

When the spacing had increased to $4B$, the plate was right at the centre of the test tank. The plate, in this position, came under the influence of two imaginary adjacent foundations located on each side, on the same distance (see Figure 3.2). Therefore, the contact stress distribution turned out to be symmetrical.

FIGURE 3.2 TEST PLATE AND ITS COUNTERPART AT DIFFERENT SPACINGS

Figure 3.3 shows the average contact stress distributions corresponding to the spacing ratios from 1 to 4. These distributions were based on the calculated average values of the contact stresses along the sections: A-A, B-B and C-C shown in Figure 3.1.a to Figure 3.1.d. As mentioned above, the measured contact stress distributions are produced under the combined effect of the concentrated load (P) applied at the centre of the plate and the bending moment resulting from preventing the plate from tilting or rotating. This system of concentric load (P) and the bending moment (M) can be replaced with an equivalent system of the same load (P) acting at a distance (e) from the centre (where $e = \frac{M}{P}$). The eccentricity (e) corresponding to each spacing was computed by using the contact stress distributions shown in Figure 3.3. The relationship between the eccentricity (e) expressed in terms of the foundation width (B) and the spacing ratio (S/B) is shown in Figure 3.4. This relationship can be described by the following equation:

$$e/B = 0.1 - 0.025 (S/B) \quad 1 \leq S/B \leq 4 \quad (3.1)$$

An approximate distribution of the contact stresses beneath one of two interfering rigid strip foundations resting on a layer of sand of limited thickness and located at spacing (S) from each other, could be obtained as follows:

$$q_1 = \frac{q_a}{2} \left(1 \pm \frac{6e}{B} \right) \quad (3.2)$$

Test footing

Counterpart

q_x = Measured contact stress

q_0 = Average contact stress along axis a-a

FIGURE 3.3 EFFECT OF SPACING ON CONTACT STRESS DISTRIBUTION

FIGURE 3.4 RELATIONSHIP BETWEEN SPACING AND ECCENTRICITY DUE TO INTERFERENCE OF TWO RIGID STRIP FOUNDATIONS

FIGURE 3.5 RELATIONSHIP BETWEEN SPACING AND EDGE CONTACT STRESSES BENEATH AN INTERFERING FOUNDATION

and from Equation (3.1):

$$q_1 = q_a \left[1 \pm \left(0.6 - 0.15 \frac{S}{B} \right) \right] \quad (3.3)$$

in which: q_1 = the contact stress beneath the confined edge;
close to the conjugate foundation.

q_2 = the contact stress beneath the free edge;
far from the conjugate foundation.

q_a = average applied pressure

The measured values of q_1 and q_2 are plotted versus the spacing ratio in Figure 3.5. Values of q_1 and q_2 as predicted from Equation 3.3 are also shown. Similar observations about the eccentricity and inclination of the soil reaction due to the interference between two rigid strip foundations were recorded by West and Stuart (1965).

3.2 Influence of Spacing on the Settlement

Three series of tests: B, C and E were performed in order to investigate the effects of spacing, layer thickness and interface conditions on settlement and bearing capacity characteristics of two interfering rigid strip foundations. In series B and C, the interface conditions between the footing and the soil and between the soil and the rigid base were smooth. The ratio of the layer thickness to the footing width (H/B) was equal to 2.5 and 1.0, respectively. In series E, the interface conditions were rough and the ratio (H/B) was equal to 1. In each series, the centre to centre spacing was changed from 1B to 2B, 4B, 6B and 10B. Details

FIGURE 3.6.a LOAD-SETTLEMENT CURVES OF TWO INTERFERING RIGID STRIP FOOTINGS LOCATED AT DIFFERENT SPACINGS (SMOOTH INTERFACE - $H/B = 2.5$)

FIGURE 3.6.b LOAD-SETTLEMENT CURVES OF TWO INTERFERING RIGID STRIP FOOTINGS LOCATED AT DIFFERENT SPACINGS (SMOOTH INTERFACE - $H/B = 1.0$)

FIGURE 3.6.c LOAD-SETTLEMENT CURVES OF TWO INTERFERING RIGID STRIP FOOTINGS LOCATED AT DIFFERENT SPACINGS (ROUGH INTERFACE - $H/B=1.0$)

of each series have been mentioned in section 2.4. Typical load-settlement curves obtained from series B, C and E are shown in Figure 3.6.a, Figure 3.6.b, and Figure 3.6.c respectively. The trends of these curves are similar regardless of variation of the layer thickness or the interface conditions; as the centre to centre spacing (S) decreases, the settlement decreases. The interference between the stressed zones causes the confining pressure to increase. Consequently, the elastic modulus of the soil increases causing a decrease of settlements. The relationship between the settlements at the maximum applied load (98 kPa), for the three different problems investigated in series B, C and E, and the spacing ratio S/B is shown in Figure 3.7. Referring to that figure, it appears that as the spacing increases, the effect of interference gradually decreases, to approximately vanish at a spacing ratio of $S/B = 6$. At such a spacing ratio the two foundations could be considered as acting separately. Similar observations were recorded by Murthy (1970), Singh, et al., (1973), and Deshmukh (1978).

Based on Figure 3.7 the interference efficiency factor for settlement η_s is introduced. This factor is defined as the ratio of the settlement of an interfering footing at a given intensity of applied pressure to that of an identical individual one (corresponding to $S/B=10$). The intensity of pressure should be within the working stress limits. The relationship between η_s and the spacing ratio S/B is shown in Figure 3.8 for the three problems mentioned before. It is noted

FIGURE 3.7 RELATIONSHIP BETWEEN SETTLEMENTS AND THE SPACING RATIO

FIGURE 3.8 RELATIONSHIP BETWEEN SETTLEMENT EFFICIENCY FACTOR AND SPACING RATIO

FIGURE 3.9 RELATIONSHIP BETWEEN SETTLEMENT EFFICIENCY FACTOR AND SPACING AS OBTAINED BY SINGH, ET AL. (1973) AND BY THE PRESENT WORK

in that figure, that the efficiency factor (η_s) is almost independent of the layer thickness and the interface conditions. It depends only on the spacing. The relationships shown in Figure 3.8 can be approximated fairly well by a straight line. It starts from the spacing ratio $S/B=1$ where the two foundations touch, to the spacing ratio $S/B=6$ where the two foundations act individually. The equation of that line is as follows:

$$\eta_s = 0.52 + 0.08 (S/B) \quad 1 \leq S/B \leq 6 \quad (3.4)$$

The settlement of two interfering strip foundations resting on a cohesionless medium could be predicted by using the following equation:

$$\delta_g = \eta_s \delta_s \quad (3.5)$$

where: δ_g = settlement of the foundation in a group

δ_s = settlement of a single foundation which could be obtained by using any reliable formula

The relationship between the efficiency factor η_s and the spacing (S/B) expressed by Equation (3.4) compares to that obtained by Singh, et al., (1973) from similar experiments using square footings instead of the strip footings used in the present work. The two relationships are shown in Figure 3.9. Referring to that figure, it can be said that the shape of the foundation has almost no effect on the settlement efficiency factor η_s .

The relationships between the slopes of the load-settlement curves (obtained from the three different series of tests) and the spacing ratio are shown in Figure 3.10. It can be noted from that figure that the slope of the load-settlement curve depends not only on the spacing ratio (S/B) but on the layer thickness and the interface conditions as well. As the spacing ratio decreases, this slope increases due to the influence of the interference between the two foundations which causes the soil to be more confined as the spacing decreases.

Figure 3.11 shows the relationship between ξ and the spacing. The term (ξ) is defined as the ratio of the slope of the load-settlement curve of an interfering foundation, to that of an identical individual one. Referring to that figure, it can be noted that the term ξ , similar to the settlement efficiency factor η_s , is almost independent of the layer thickness and the interface conditions. The relationship between (ξ) and the spacing ratio (S/B) shown in Figure 3.11 can be replotted using a log-log scale as shown in Figure 3.12. The relationship shown in Figure 3.12 can be described by the following equation:

$$\log \xi = \log K - n \log (S/B)$$

or

$$\xi = K(S/B)^{-n} \quad (3.6)$$

where K and n are equal to 1.8 and 0.3 respectively.

The settlement, at a given pressure, decreases as the slope of the load-settlement curve increases. Therefore, the relationship between the settlement efficiency factor η_s and the term ξ can be expressed as follows:

$$\begin{aligned}\eta_s &= \frac{1}{\xi} \\ &= \frac{1}{K} (S/B)^n = \frac{1}{1.8} (S/B)^{0.3} \\ \eta_s &= \frac{1}{2} (S/B)^{1/3} \quad 1 \leq S/B \leq 6 \quad (3.7)\end{aligned}$$

For the sake of comparison, values of η_s obtained from Equation (3.4), Equation (3.7) and those values obtained from the equation given by Singh, et al., (1973) are shown in Table 3.1. It appears that the three equations give almost the same results.

Table 3.1 Settlement Efficiency Factor η_s

S/B	Equation (3.4)	Equation (3.7)	Singh, et al. (1973)
1	0.60	0.50	0.50
2	0.68	0.63	0.60
3	0.76	0.72	0.70
4	0.84	0.79	0.80
5	0.92	0.85	0.90
6	1.00	0.91	1.00

FIGURE 3.10 EFFECT OF SPACING RATIOS ON SLOPE OF LOAD-SETTLEMENT CURVE

FIGURE 3.11 RELATIONSHIP BETWEEN THE RATIO ξ AND THE SPACING RATIO (S/B)

FIGURE 3.12 ξ VERSUS S/B (LOG-LOG SCALE)

3.3 Influence of Spacing on the Bearing Capacity

In the three series of tests, B, C and E mentioned previously in Section 3.2, the load was applied until it reached about one half of the ultimate bearing capacity as determined by Terzaghi's theory, and then it was removed. Eleven cycles of loading and unloading were performed. In the twelfth cycle, the load was increased until failure occurred.

The load-settlement curves for the three different problems investigated in Series B, C and E are shown in Figure 3.13.a to Figure 3.13.c. All load-settlement curves shown a definite peak, or failure point which indicates the ultimate load. It is observed from these figures that the ultimate load expressed in terms of the ratio $(P/\gamma B^3)$ increases as the spacing decreases. The relationship between the ultimate load and the spacing ratio (S/B) is shown in Figure 3.14. It is noted from that figure that the ratio of the layer thickness to the footing width (H/B) has a significant influence on the ultimate bearing capacity. But it is also important to note that the influence of the roughness or smoothness of the interface conditions between the foundation and soil and between the soil and the rigid base on the bearing capacity is much greater. It is also noted that the two interfering foundations would act separately if the spacing reached $6B$ or more, as was previously observed from the relationship between the settlement and the spacing ratio.

FIGURE 3.13.a LOAD-SETTLEMENT CURVES (AT FAILURE) OF TWO INTERFERING RIGID STRIP FOOTINGS (SMOOTH INTERFACE, $H/B=2.5$)

FIGURE 3.13.b LOAD-SETTLEMENT CURVES (AT FAILURE) OF TWO
 INTERFERING RIGID STRIP FOOTINGS
 (SMOOTH INTERFACE, $H/B=1.0$)

FIGURE 3.13.c LOAD-SETTLEMENT CURVES (AT FAILURE) OF TWO INTERFERING RIGID STRIP FOOTING (ROUGH INTERFACE, $H/B=1.0$)

The effect of the interference between foundations on bearing capacity can be described by the term (η_q) which is defined as the ratio between the ultimate bearing capacity of an interfering foundation (q_g) and that of a similar individual foundation (q_s). If the value of q_s were taken to be one corresponding to a spacing ratio $S/B=6$, where the two adjacent foundations act individually, the bearing capacity efficiency factor η_q would be dependent on the spacing ratio, as well as the ratio (H/B) and the interface conditions. In order to explain the effect of these factors on the bearing capacity efficiency factor η_q , the following definitions are introduced:

q_{sr} = the ultimate bearing capacity of a single rough strip footing located on a layer of sand of thickness to foundation width ratio $H/B=1.0$, resting on a rough rigid base (from Figure 3.14, $q_{sr}=430$ kPa).

q_{ss} = the ultimate bearing capacity of a single smooth strip footing located on a layer of sand of thickness to foundation width ratio $H/B=2.5$, resting on a smooth rigid base (from Figure 3.14, $q_{ss} = 275$ kPa).

q_{sh} = the ultimate bearing capacity of a single strip footing resting on a half space medium of soil.

(From Terzaghi's theory, $q_{sh} = 0.5 \gamma B N_\gamma = 170$ kPa)

q_{gh} = the ultimate bearing capacity of one of two interfering strip footing resting on a half space medium of soil.

Now, should there be a difference between (q_{gh}) and (q_{sh}), it will be because of the influence of the interference. The difference in this case depends only on the spacing ratio (S/B). On the other hand, because of the influence of the rough rigid base, there should be a difference between (q_{sh}) and (q_{sr}); and because of the influence of the smooth rigid base, there should be a difference between (q_{sh}) and (q_{ss}). If the value of η_q is calculated with the effect of the rigid base included, it will be must less than the value calculated with the effect of the rigid base excluded. The following example explains this more fully.

In the case of rough interface and spacing ratio $S/B=2$, the ultimate bearing capacity of one of the two interfering foundations resting on a layer of thickness $H=B$ is equal to 540 kPa and that of a single foundation resting on a layer of the same thickness (q_{sr}) is equal to 430 kPa. then

$$\eta_q = \frac{540}{430} \times 100 = 126\%.$$

As mentioned before, the ultimate bearing capacity of a single foundation resting on a half space soil medium $q_{sh} = 170$ kPa. The effect of the rough rigid base in this case will be:

$$q_{sr} - q_{sh} = 430 - 170 = 260 \text{ kPa}$$

If that effect is omitted, and the efficiency bearing capacity factor η_q is recalculated, a different value will be obtained as follows:

$$\eta_q = \frac{540 - 260}{170} \times 100 = 165\%$$

FIGURE 3.14 ULTIMATE BEARING CAPACITY VERSUS SPACING RATIO

FIGURE 3.15 EFFECT OF THE LAYER THICKNESS TO THE RIGID BASE AND INTERFACE CONDITIONS ON THE RATIO η_q

From this example, it appears that the effect of the interference, when the rigid base is present is to increase the bearing capacity by about 26%; while for a deep layer, or in the absence of the rigid base, the interference causes an increase of about 65% in the bearing capacity.

The relationship between n_q calculated by the two ways mentioned above and the spacing ratio (S/B) is shown in Figure 3.15. It should be pointed out that Figure 3.15 is only qualitative and not quantitative. The relationship shown in the figure is based on using the bearing capacity factor N_Y given by Caquot and Kerisel (1953) in calculating the value of q_{sh} . There exists in the literature a great variety of theories which give different values of N_Y . These values range from about one third to double the values given by Caquot and Kerisel (Vesic, 1973). However, Caquot-Kerisel's factors are the most reliable factors available at present (Vesic, 1973).

In conclusion, it can be said that the influence of the interference depends on many factors rather than on the spacing ratio alone. Among these factors are the interface conditions, the layer thickness, the angle of internal friction of the soil, and the foundation depth. Myslevic and Kysela (1968) observed that the bearing capacity efficiency factor decreased from 200% to less than 150% by increasing the foundation depth from $0.2B$ to $6.2B$. Briefly, it can be said that any factor causing the ultimate bearing capacity of the single footing to be of high value, will significantly

minimize the influence of the interference between foundations. In the case of two adjacent foundations resting directly on a layer of infinite rigidity and infinite strength, the influence of the interference will not appear.

The surface displacements measured just before failure in series B of tests are shown in Figure 3.16. The sand between the two interfering foundations heaved. The amount of heave and its length were dependent on the spacing ratio. The heave extended to more than $5B$ (the width of foundation) when the spacing ratio was equal to 1.0, and to about $4B$ when the spacing was increased to $6B$ or more. Also, for a spacing ratio $S/B \leq 6B$ the displacements on both sides of the foundation were asymmetrical, but when the spacing reached $10B$, the displacements became symmetrical. At spacing ratio $S/B=6$ there was still a slight interference between the two foundations which completely vanished at spacing ratio $S/B=10$ as can be seen from the comparison of the surface displacements. This again confirms the result previously drawn from Figure 3.8 and Figure 3.14; that the two foundations can be considered to be acting individually if the spacing reaches $6B$ or more.

3.4 Influence of Lateral Spacing: Correlation With the Plane Strain Problem

In order to study the effect of the lateral spacing between a group of footings of equal dimensions on their settlement and

FIGURE 3.16 SURFACE DISPLACEMENTS AROUND TWO INTERFERING RIGID STRIP FOOTINGS RESTING ON A LAYER OF SAND OF FINITE DEPTH (JUST BEFORE FAILURE)

bearing capacity characteristics, two series of tests: F and G (see Section 2.4 and Table 2.1) were carried out. Five different sizes of rigid footings were used. The footings were of equal widths and had a length to width ratio L/B of 2.5, 2, 1.5, 1.0 and 0.5 corresponding to an edge to edge lateral spacing (S_e) = 0, 0.5B, 1B, 1.5B and 2B. These footings are placed in one row (see Figure 3.17). In series F and G of tests, the ratio (H/B) of the layer thickness to the footing width was 1.0 and 2.5 respectively.

The load-settlement curves of the five footings resting on the surface of a layer of thickness ratio $H/B=1.0$ and 2.5 are shown in Figure 3.18.a and Figure 3.18.b respectively. It can be noted that the shape of the load-settlement curve as well as the ultimate load are, to a great extent, dependent on the lateral spacing ratio (S_e/B). The relationship between the ultimate load and the lateral spacing ratio (S_e/B) is shown in Figure 3.19. From this figure it can be noted that the bearing capacity decreases as the ratio S_e/B decreases. This can be explained by the influence of the conjugate footings. This influence causes the soil between the real footing and the conjugate footing to be more confined as the ratio S_e/B decreases. Consequently, the bearing capacity increases. As this ratio increases the influence of the conjugate footings decreases until finally vanishes at ratio $S_e/B \geq 2.0$ where the real footing can be considered acting as an isolated footing. From Figure 3.19, it also appears that the ultimate load depends not only on the lateral spacing but also, on the ratio of the layer thickness to the foundation

(H/B) As the lateral spacing ratio (S_e/B) increases, the effect

PLAN OF FIVE ROWS OF ADJACENT FOUNDATIONS

FIGURE 3.17. DIFFERENT LATERAL SPACINGS AND DIFFERENT PLAN ASPECT RATIOS USED IN SERIES F AND G OF TESTS.

FIGURE 3.18.a LOAD-SETTLEMENT CURVES OF A GROUP OF RIGID STRIP FOOTINGS OF DIFFERENT PLAN ASPECT RATIOS ($H/B=1.0$)

FIGURE 3.18.b LOAD-SETTLEMENT CURVES OF A GROUP OF RIGID STRIP FOOTINGS OF DIFFERENT PLAN ASPECT RATIOS ($H/B=2.5$)

FIGURE 3.19 EFFECT OF LATERAL SPACING ON THE ULTIMATE LOAD
A ROW OF IDENTICAL STRIP FOOTINGS

FIGURE 3.20 EFFECT OF THE PLAN ASPECT RATIO ON THE ULTIMATE
BEARING CAPACITY

of the ratio (H/B) decreases to almost vanish at a spacing ratio $S_e/B = 2.0$.

Figure 3.20 shows the relationship between the ratio of the ultimate load of a strip foundation of an infinite length and of a width B , to that of a similar foundation of the same width B but of a definite length L . Referring to that figure, it appears that the plan aspect ratio (L/B) has a significant effect on the bearing capacity. It can also be said that a row of footings with equal widths (B) and equal lengths (L) can be treated as a single strip footing of width (B) if the lateral spacing ratio S_e/B is equal to, or less than 0.5.

Chapter 4

INFLUENCE OF RIGID BASE

The review of the literature, mentioned in Chapter 1, concerning the settlement and bearing capacity behaviour of the foundation located on the surface of a layer of soil resting on a rigid base, indicates that this behaviour depends on many factors. The ratio of the layer thickness (H) to the foundation width (B) and the roughness or smoothness of the rigid base are among the important factors. Other factors such as the foundation shape and the shear strength parameters of the soil, also have significant influences on that behaviour.

The main concern of this Chapter is to present an experimental investigation of the effect of the rigid base on the settlement and the bearing capacity characteristics of two interfering rigid strip foundations. Two main variables have been investigated; the ratio of the layer thickness to the foundation width (H/B), and the interface conditions between the foundation and the soil and between the soil and the rigid base. Three different problems were investigated. The first problem had two interfering foundations resting on a layer of sand of ratio $H/B=2.5$. The interface conditions were smooth. The second problem was similar to the first one but the ratio H/B was changed from 2.5 to 1.0. The third problem had a ratio of $H/B=1.0$ and rough interface conditions. Details of the

testing technique followed in experiments have been previously mentioned in Chapter 2.

4.1 Influence of Rigid Base on the Settlement

Shown in Figure 4.1.a and Figure 4.1.b are the load-settlement curves of the three problems mentioned above for spacing ratios of $S/B=1$ and 10 respectively. As can be seen from the two figures, at a given load the settlement decreases as the thickness of the soil, expressed by the ratio H/B , decreases. The settlement also decreases when the interface conditions are changed from smooth to rough. The level of decrease in the settlement depends on the spacing between the two interfering foundations. As was mentioned in Chapter 3, the settlement decreases gradually from that of a single foundation as the spacing decreases. The settlement reaches a minimum value when the two foundations touch each other. In this case the two foundations act as a single foundation of width $2B$. Also, the two foundations would act individually if the spacing ratio (S/B) became equal to 6 or more.

Therefore, it can be said that the interfering foundation is equivalent to any other individual one with width B_e . The equivalent width B_e ranges from B to $2B$ depending on the spacing ratio (S/B). It is equal to B when $S/B=6$ or more and equal to $2B$ when $S/B=1.0$. An experimental relationship between the equivalent width (B_e) and the spacing ratio (S/B) can be expressed as follows:

FIGURE 4.1.a EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON LOAD-SETTLEMENT RELATIONSHIP OF TWO INTERFERING RIGID STRIP FOOTINGS (SPACING RATIO $S/B=1$).

FIGURE 4.1.b EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON LOAD-SETTLEMENT RELATIONSHIP OF TWO INTERFERING RIGID STRIP FOOTINGS (SPACING RATIO $S/B=10$)

$$B_e = B[2.2 - 0.2 (S/B)] \quad 1 \leq S/B \leq 6 \quad (4.1)$$

in which: B_e = the width of an individual foundation experiencing the same settlement-bearing capacity behaviour as one of two interfering foundations.

S and B as defined before.

From the practical point of view, it can be said that the rigid base would have no influence on the load-settlement behaviour of the above soil layer if the ratio (H/B) of the layer thickness to the foundation width was greater than 2.0 (Schmertmann, 1970; Schultze and Sherif, 1973). As the ratio H/B decreases the settlement decreases and consequently the slope of the load-settlement curve increases. The term (ξ_r) can be introduced to define the influence of the rigid base on the slope of the load-settlement curve. This term expresses the ratio of the slope of the load-settlement curve of a layer of sand with thickness ratio $H/B < 2.0$, to that of an identical layer but with thickness ratio $H/B \geq 2.0$.

Equation (4.1) was used to compute the equivalent width (B_e) corresponding to the different values of the spacing ratio (S/B) that were used in experiments. The relationship between the term ξ_r as defined above and the ratio H/B_e is shown in Figure 4.2.a. This relationship can be replotted on a log-log scale as shown in Figure 4.2.b. A straight line can be fitted through the data points. The equation of this line can be described as follows:

$$\log \epsilon_r = \log 1.55 - 0.65 \log \left(\frac{H}{B_e} \right)$$

which leads to:

$$\epsilon_r = 1.55 \left(\frac{B_e}{H} \right)^{0.65} \quad (4.2)$$

in which: ϵ_r = the ratio between the slope of the load-settlement curve of a layer of sand of ratio $H/B_e < 2.0$ to that of another layer of ratio $H/B_e \geq 2.0$ and H and B_e , as defined before.

By the definition, the term ϵ_r describes the increase in the stiffness modulus of a layer of sand as the ratio H/B decreases. The variation in the settlement with the variation in the term (ϵ_r) is linear. Therefore, the relationship between the settlement (δ_r) of a layer of sand with thickness $H < 2B$ and the settlement (δ) of another layer with thickness $H \geq 2B$ can be as follows:

$$\delta_r = \frac{1}{\epsilon_r} \delta \quad (4.3)$$

or

$$\delta_r = n_{sr} \delta \quad (4.4)$$

where: $n_{sr} = \frac{1}{\epsilon_r}$

From Equation (4.2):

$$n_{sr} = \frac{2}{3} \left(\frac{H}{B_e} \right)^{2/3} \quad 0.5 \leq H/B_e \leq 2 \quad (4.5)$$

Schultze and Sherif (1973), based on settlement measurements of 48 building and industrial constructions in sand, suggested using a

FIGURE 4.2,a ϵ_r VERSUS (H/B_e)
(ARITHMETIC SCALE)

FIGURE 4.2.b ξ_r VERSUS (H/B_e) (LOG-LOG SCALE)

reduction factor for settlements of foundations on compressible layers of ratio $H/B < 2$ to take into account the varied effects of the rigid base, as the ratio H/B changes. For the sake of comparison, in Table 4.1 reduction factors proposed by Schultze and Sherif are given together with those predicted by Equation (4.5).

Table 4.1 Reduction Factor of Settlement (η_{sr})

H/B	Equation (4.4)	Schultze & Sherif (1973) $L/B = 5$
2.0	1.06	1.0
1.5	0.87	0.87
1.0	0.67	0.65
0.5	0.42	0.43

Equation (4.5) is applicable only for layers of ratio $0.5 \leq H/B \leq 2$. For layers of ratio $H/B > 2$, η_{sr} is taken as one, and for layers of ratio $H/B < 0.5$, Equation (4.5) may not be applicable. Layers of such a small ratio have not been investigated in this work. In practice, if a rigid layer is located below a soft one of a thickness ratio $H/B < 0.5$, it may be advisable to remove the soft layer and to place the foundation directly on the rigid layer.

4.2 Influence of Rigid Base on the Bearing Capacity

As mentioned above, eleven cycles of loading and unloading were performed with a maximum applied pressure equal to one half the ultimate load. At the twelfth cycle the load was increased until

failure. Shown in Figure 4.3.a and 4.3.b are the load-settlement curves (after 12 cycles) for different ratios of H/B_e and different interface conditions. As can be seen from the two figures, for smooth interface between the foundation and the soil and between the soil and the rigid base, as the ratio (H/B_e) decreases the ultimate load decreases. A decrease of the ratio (H/B_e) from 1.25 to 0.5 and from 2.5 to 1.0 caused a decrease in the ultimate load in the order of 30% and 20% respectively. For layers of ratios of $H/B=1.0$, changing the interface conditions from smooth to rough caused an increase in the ultimate load in the order of about 100%. These results confirm the theoretical findings by Mandel and Salençon (1972).

It should be pointed out that the ratio (H/B_e) depends on the spacing ratio S/B . As was mentioned in the previous section, the load-settlement behaviour of a foundation interfering with another can be considered to be equivalent to a single foundation of a wider width (B_e). The equivalent width (B_e) depends on the spacing ratio (S/B) according to the experimental relationship given in Equation (4.1). For example, at spacing ratio $S/B=6$ and 1, the equivalent width B_e will be equal to $1B$ and $2B$ respectively. For the sand layer of thickness $H=37.5$ cm. and the foundation model of width $B=15$ cm., the ratios of H/B_e corresponding to spacing ratios of $S/B=6$ and 1 are 2.5 and 1.25 respectively. By following similar procedures the

FIGURE 4.3.a EFFECT OF THE LAYER THICKNESS AND THE INTERFACE CONDITIONS ON THE ULTIMATE LOAD CAPACITY OF TWO INTERFERING RIGID STRIP FOOTINGS (SPACING RATIO $S/B=1.0$)

FIGURE 4.3.b EFFECT OF THE LAYER THICKNESS AND THE INTERFACE CONDITIONS ON THE ULTIMATE LOAD CAPACITY OF TWO INTERFERING RIGID STRIP FOOTINGS (SPACING RATIO $S/B=6$)

equivalent width B_e corresponding to each spacing ratio (S/B) was computed and consequently the ratio (H/B_e) . The term η_{qr} will be used herein in order to describe the ratio of the ultimate load of a layer of limited thickness $H < 2B$ to that of another identical layer of relatively deep thickness $H \geq 2B$.

The relationship between η_{qr} and the ratio (H/B) for rough interface conditions is shown in Figure 4.4.a. For sake of comparison, the experimental results obtained by Meyerhof (1974) and Torhler and Miloyic (1977) are shown on the same figure. As can be seen from the results shown in Figure 4.4.a, the ultimate bearing capacity in the case of rough interface conditions increases steadily as the ratio H/B decreases. For layers of ratio $H/B > 1.5$, the effect of the rigid base on the bearing capacity can be neglected. On the other hand as soon as this ratio decreases to 0.5 or less, a rapid increase in the ultimate bearing capacity will take place.

The relationship shown in Figure 4.4.a. can be replotted by using a log-log scale as shown in Figure 4.4.b. Now, this relationship between η_{qr} and (H/B) can be described by a straight line having the following equation:

$$\log \eta_{qr} = \log 1.5 - 0.5 \log (H/B)$$

which can be rewritten in the form:

$$n_{qr} = 1.5 (B/H)^{0.5} \quad (4.6)$$

in which: n_{qr} = the ratio of the bearing capacity of a layer of limited thickness ($H/B < 2$) in rough contact with a rigid base to the bearing capacity of a deep layer ($H/B \geq 2$)

H and B as defined before.

Figure 4.5.a to 4.5.f show the surface displacements measured just before failure of layers of sand of different ratios (H/B) and in smooth or rough contact with the rigid base. From these figures it appears that the amount of heave and the extension of the slip surface depend on the ratio H/B as well as the interface conditions. In the case of smooth interface conditions the amount of heave was less, and the extension of the slip surface was greater, than those in the case of rough interface conditions for the same ratio H/B . On the other hand, in the smooth case when the ratio H/B decreased from 2.5 to 1.0, the extension of the slip surface around the footing was also decreased.

From these observations it can be said that the extension of failure zones in the case of a thin layer of sand resting on a rough rigid base will be less than it will be in the case of a smooth rigid base. This may be attributed to the shearing resistance developed between the slip surfaces and the rigid base in the case of rough interface conditions. The shearing resistance constrains the extension of the slip surfaces causing a higher bearing capacity. This explains the results obtained and which were discussed earlier.

FIGURE 4.4.a EFFECT OF THE THICKNESS OF A LAYER OF SAND IN ROUGH CONTACT WITH A RIGID BASE ON THE ULTIMATE BEARING CAPACITY OF A ROUGH RIGID STRIP FOUNDATION (ARITHMETIC SCALE)

FIGURE 4.4.b EFFECT OF THE THICKNESS OF A LAYER OF SAND IN ROUGH CONTACT WITH A RIGID BASE ON THE ULTIMATE BEARING CAPACITY OF A RIGID STRIP FOUNDATION (LOG-LOG SCALE)

FIGURE 4.5 EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON SURFACE DISPLACEMENTS OF RIGID STRIP FOUNDATIONS

FIGURE 4.5 CONT.

Chapter 5

INFLUENCE OF LOAD AND UNLOADING CYCLING

There are many research papers which deal with the effect of repeated dynamic loads such as those resulting from vibration, blasting, earthquake, etc., on the behaviour of soils and especially the cohesionless soil. In practice there are some structures which are subjected to repeated static loads, rather than dynamic, such as silos, oil and water storage tanks, etc. As these structures are filled and emptied, a series of loading and unloading cycles occurs. This process of load cycling has effects on the load-settlement characteristics of such structures. Some researchers have investigated the effect of reloading on the elastic modulus of the soil, by using the standard triaxial test; Crawford and Burn (1962), Ladd (1964), Makhlof and Stewart (1969), Soderman, et al. (1958), Pyke (1978) and others.

In this study, the effect of repeated static loads on settlement characteristics has been investigated by using the rigid strip foundation models described in the previous chapters. Effects on total and inelastic settlement as well as the slope of load and unload - settlement curves are discussed.

In order to investigate the effect of cycling on the loading and unloading process on settlement characteristics, the load was applied to a maximum value about one half the ultimate load as

determined from Terzaghi's theory, then, the load was removed. Eleven cycles of loading and unloading were performed. At the twelfth cycle, the load was increased until failure occurred. The reload settlement curve at each cycle was measured, but unfortunately, it was not possible to measure the load and the corresponding displacement during the unloading process. Since, as mentioned in Chapter 2, the loads were applied by means of a hydraulic jack, it was not possible to reverse the flow of the hydraulic fluid without releasing the incipient pressure and consequently the corresponding load. However, the unloading curves are shown in the following figures by dashed lines to indicate that they were not measured.

5.1 Influence of Cycling on the Total and Inelastic Displacements

The load and unload settlement curves for three different problems are shown in Figure 5.1.a to Figure 5.1.c. These three problems, as mentioned in the previous chapters are as follows. The first and the second concern a rigid strip footing resting on a layer of sand of thickness ratios $H/B=2.5$ and $H/B=1.0$. The interface conditions between the footing and the sand and between the sand and the rigid base were smooth. The third problem is as the second but with rough interface conditions. As can be seen from Figure 5.1.a and Figure 5.1.b, the layer depth to the rigid base has an influence on the reloading-displacement relationship. For the deeper layer ($H/B=2.5$) the relationship was in shape of a

curve until the fourth cycle, and then, starting from the fifth cycle it took a straight line shape. For a relatively thin layer ($H/B=1.0$), the reloading-displacement relationship took the straight line shape starting from the second cycle. Changing the interface conditions from smooth to rough caused the total and inelastic displacements to be smaller, the hysteresis loop to be narrower and the slopes of the reload-displacement curves to be higher. In all of the tests performed, most of the inelastic deformation occurred in the first cycle of loading and unloading, and increased with additional cycles at a decreasing rate. The width of the hysteresis loop was also greater in the first cycle and decreased with subsequent cycles.

The relationships between the total and inelastic displacements and the number of cycles for the three problems mentioned above are shown in Figure 5.2.a to Figure 5.2.c. Regardless of the layer thickness or the interface conditions, the change in the inelastic displacement with the number of cycles is almost the same as that of the total settlement. Figure 5.3 shows the percentage increase in the total displacement at first cycle with the increase in the cycle number. As can be seen from this figure, the displacements continued increasing with increase of cycles with an indication that this increase will not stop until a few cycles after the twelve cycles already performed. However, the rate of the increase in displacements started to decrease after about eight cycles. From Figure 5.3, it can also be noted that the percentage of

FIGURE 5.1.a EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT
RELATIONSHIP OF A RIGID STRIP FOOTING RESTING
ON A LAYER OF SAND (SMOOTH INTERFACE, $H/B=2.5$)

FIGURE 5.1.b EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT RELATIONSHIP OF A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND (SMOOTH INTERFACE, $H/B=1.0$)

FIGURE 5.1.c EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT
 RELATIONSHIP OF STRIP FOOTING ON A LAYER OF
 SAND (ROUGH INTERFACE, $H/B=1.0$)

FIGURE 5.2.a RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES (SMOOTH INTERFACE, $H/B=2.5$)

FIGURE 5.2.b RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES (ROUGH INTERFACE, $H/B=1.0$)

FIGURE 5.2.c RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES (SMOOTH INTERFACE, $H/B=1.0$)

FIGURE 5.3 INCREASE IN TOTAL DISPLACEMENT WITH INCREASE IN NUMBER OF CYCLES

increase in total displacements depends on the layer thickness, but the interface conditions seems to have a slight effect.

5.2 Influence of Cycling on the Load-Displacement Relationship

5.2.1 Influence of Cycling on the Slopes of Load-Displacement Curves

Figure 5.4 shows the relationship between the slopes of the load and the reload, displacement curves and the number of cycles, for the three different problems mentioned before. As can be seen from that figure, the slope of the reload-displacement curve increases with the increase of number of cycles regardless of the layer thickness or the interface conditions. This increase was significant until about the sixth cycle and after that the rate of increase noticeably decreased. For example, in the case of a layer of thickness to foundation width ratio $H/B=2.5$, the slope of the load-displacement curve at the first cycle increased by more than 100% by increasing the cycle number to six, but adding another six cycles more caused an increase of only about 15%. The same trend can also be noted in the two other cases. However, the slopes of reload-displacement curves in the case of a layer of thickness ratio $H/B=1.0$ and rough interface have the biggest values, followed by those in the case of a layer with the same thickness but with smooth interface. The reload-displacement curves in the case of a layer of thickness ratio $H/B=2.5$ and smooth interface have slopes of the smallest values.

FIGURE 5.4 EFFECT OF NUMBER OF CYCLES ON SLOPE OF LOAD-DISPLACEMENT CURVE

FIGURE 5.5 EFFECT OF NUMBER OF CYCLES ON SLOPE OF UNLOAD-DISPLACEMENT CURVE

5.2.2 Influence of Cycling on the Slopes of Unloading-Displacement Curves

Although the paths of the unload-displacement curves in all tests were not measured and consequently the exact slope of these curves were not known, reasonable values of these slopes can be predicted by assuming that these curves are straight lines as shown by the dashed lines in Figure 5.1.a to Figure 5.1.b. This assumption is based on the fact that all reloading-displacement curves are almost straight lines at the first cycles and exactly straight lines during the rest of cycles. It was assumed that this straight line relationship held for the unloading-displacement curve and its slope was calculated. The relationship between this slope and the number of cycles is shown in Figure 5.5. It can be noted that irrespective of the layer thickness or the interface conditions, the number of cycles has no influence on the slope of the unload-displacement curve.

Chapter 6

NUMERICAL AND EMPIRICAL SOLUTIONS

This chapter examines certain numerical and empirical solutions which may be used to examine, theoretically, the displacements (settlements) of the interfering rigid strip foundations and the contact stresses that are developed at the sand-foundation interface. These developments follow two lines of investigations:

- (i) a finite element technique which takes into account the nonlinear character of the stress-strain properties of the sand and
- (ii) an empirical method (similar to Schmertmann's method).

In both these investigations the effect of the spacing and the influence of the rigid layer on the settlement of the foundation will be examined.

6.1 Nonlinear Finite Element Analysis

6.1.1 Nonlinear Stress-Strain Relationship

The stress-strain characteristics of soils and rocks are influenced by a number of factors such as density, water content, structure, drainage conditions, stress history, etc. These factors make the stress-strain behaviour highly complex and nonlinear. Any realistic treatment for the problems of soils and rocks, therefore, should take into consideration the nonlinear stress-strain

behaviour of the geotechnical medium. Using the finite element method, it is possible to approximate nonlinear behaviour in stress analysis. In order to perform such an analysis, however, it is necessary to be able to describe the nonlinear stress-strain behaviour of soils and of rocks in quantitative terms, and to develop techniques for incorporating this behaviour into the analysis. There are several such techniques (see Desai and Abel 1972 and Desai and Christian, 1977).

One of these techniques which is widely used in analysis of nonlinear problems in geotechnical engineering is due to Duncan and Chang (1970). They have used the hyperbolic function proposed by Kondner (1963) for simulation of stress-strain curves in finite element analysis. Kondner has shown that the nonlinear stress-strain curves of both sand and clay can be represented by hyperbolae to a reasonable degree of accuracy. Figure 6.1.a illustrates such a relationship which can be stated in equation form:

$$(\sigma_1 - \sigma_3) = \frac{\epsilon}{a + b\epsilon} \quad (6.1)$$

in which: σ_1 and σ_3 = the major and minor principal stresses;

ϵ = the axial strain; and

a and b = constants

Referring to Figure 6.1.a, it is indicated that, at very small strains, $E_i = \frac{1}{a}$ = initial Young's modulus; and $(\sigma_1 - \sigma_3)_{ult} = \frac{1}{b}$ = asymptotic values of stress difference which the stress-strain curves approach at infinite strains.

FIGURE 6.1.a HYPERBOLIC STRESS-STRAIN CURVE

FIGURE 6.1.b TRANSFORMED HYPERBOLIC STRESS-STRAIN CURVE

Equation (6.1) can be transformed into a linear relationship of the form:

$$\frac{\epsilon}{\sigma_1 - \sigma_3} = a + b\epsilon \quad (6.2)$$

This form of equation plots a straight line (Figure 6.1.b).

Such a plot, with the transformed axes $\frac{\epsilon}{\sigma_1 - \sigma_3}$ and ϵ , can be used to check whether the data from a test do fit a hyperbola, or to find the parameters of the hyperbola from the test data.

Since the compressive strength $(\sigma_1 - \sigma_3)_f$ can be reached before the curve becomes asymptotic, the ratio (R_f) between $(\sigma_1 - \sigma_3)_f$ and $(\sigma_1 - \sigma_3)_{ult}$ is less than 1.0 and it is usually between 0.7 and 0.95. Thus

$$R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} = b(\sigma_1 - \sigma_3)_f \quad (6.3)$$

Equation (6.1) can also be solved for ϵ :

$$\epsilon = \frac{a(\sigma_1 - \sigma_3)}{1 - b(\sigma_1 - \sigma_3)} \quad (6.4)$$

The tangent modulus at any level of stress or strain is:

$$E_t = \frac{d}{d\epsilon} (\sigma_1 - \sigma_3) = \frac{a}{(a + b\epsilon)^2} = \frac{1}{a} [1 - b(\sigma_1 - \sigma_3)]^2 \quad (6.5)$$

Using the results $a = \frac{1}{E_i}$ and $b = \frac{1}{(\sigma_1 - \sigma_3)_{ult}}$, Equation (6.5)

can be rewritten in the form:

$$E_t = E_i \left[1 - R_f \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f} \right]^2 \quad (6.6)$$

For a Mohr-Coulomb material at failure:

$$(\sigma_1 - \sigma_3)_f = \frac{2\sigma_3 \sin \phi + 2c \cos \phi}{1 - \sin \phi} \quad (6.7)$$

The tangent modulus now becomes:

$$E_t = E_i \left[1 - \frac{R_f (\sigma_1 - \sigma_3) (1 - \sin \phi)}{2\sigma_3 \sin \phi + 2C \cos \phi} \right]^2 \quad (6.8)$$

Experimental studies by Janbu (1963) indicated that the relationship between the initial tangent modulus and the confining pressure can be expressed by the following empirical equation:

$$E_i = K P_a \left(\frac{\sigma_3}{P_a} \right)^n \quad (6.9)$$

in which; E_i = the initial tangent modulus;

σ_3 = the minor principal stress;

P_a = the atmospheric pressure (expressed in the same units as E_i and σ_3);

K = a modulus number; and

n = an exponent determining the rate of variation of E_i with σ_3 .

The constants K and n for any particular material can be determined by representing the Equation (6.9) in a logarithmic form:

$$\log \left(\frac{E_i}{P_a} \right) = \log K + n \log \left(\frac{\sigma_3}{P_a} \right) \quad (6.10)$$

Using Equation (6.9), the tangent modulus in Equation (6.8) can take the form:

$$E_t = K P_a \left(\frac{\sigma_3}{P_a} \right)^n \left[1 - \frac{R_f (\sigma_1 - \sigma_3) (1 - \sin \phi)}{2\sigma_3 \sin \phi + 2C \cos \phi} \right]^2 \quad (6.11)$$

For development of the full stiffness, a second elastic parameter is required. Kulhawy, et al. (1969) concluded from examination of many tests on sands and clays that the tangential value of Poisson's ratio satisfies:

$$\nu_t = \frac{G - F \log \left(\frac{\sigma_3}{P_a} \right)}{(1 - A)^2} \quad (6.12)$$

in which $A = \frac{d}{E_t} (\sigma_1 - \sigma_3)$

where d, F and G are constants to be determined experimentally.

6.1.2 Incremental Stress Analysis Procedure

In finite element analyses, the nonlinear stress-strain behaviour of the material can be incorporated by using one of three techniques; namely incremental procedure, iterative or Newton methods and step-iterative or mixed procedures (Desai and Abel, 1972). Since the computer program used in this work utilizes the incremental procedure, it is pertinent to briefly discuss the features of the procedure. A more complete treatment of the other techniques is given by Desai and Abel (1972).

The incremental procedure is schematically indicated in Figure 6.2. The applied load (Q) is divided into small partial loads or increments. At the beginning of each increment of loading, appropriate values for the tangent modulus and Poisson's ratio are

FIGURE 6.2 BASIC INCREMENTAL PROCEDURE

FIGURE 6.3 MIDPOINT RUNGE-KUTTA INCREMENTAL SCHEME

(FROM DESAI AND ABEL 1972)

selected for each element on the basis of the state of stress and strain in the element. The solution for each increment of loading is obtained as an increment of the displacement (q). These displacement increments are accumulated to give the total displacement at any stage of loading, and the incremental process is repeated until the total load has been reached.

The midpoint Runge-Kutta scheme is one common modification of the incremental method. Here two cycles of analysis are performed for each load of increment. In the first cycle half of the increment is applied and a temporary displacement increment is computed from

$$[K_{i-1}] \{\Delta q_{i-1/2}\} = \{\Delta Q_i\}/2 \quad (6.13.a)$$

The stiffness matrix corresponding to $\{q_{i-1/2}\}$ and $\{Q_{i-1/2}\}$ is evaluated and utilized to compute an approximation to the full displacement increment, as follows:

$$[K_{i-1/2}] \{\Delta q_i\} = \{\Delta Q_i\} \quad (6.13.b)$$

The midpoint Runge-Kutta scheme is illustrated in Figure 6.3.

6.1.3 Prediction of the Nonlinear Behaviour of Soils

The stress-strain relationships described in Section 6.1.1 and the incremental procedure discussed in Section 6.1.2 can be used for prediction of the nonlinear behaviour of the soil. The loading to be analyzed is divided into a number of increments, and for purposes of analysis the soil is assumed to behave linearly under

each increment of load. The incremental strains for each load increment are related to the incremental stress changes for that increment by the generalized Hooke's law which, for an isotropic material under plane strain condition, may be expressed in the form:

$$\begin{bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{bmatrix} = \frac{E_t}{(1+\nu_t)(1-2\nu_t)} \begin{bmatrix} 1-\nu_t & \nu_t & 0 \\ \nu_t & 1-\nu_t & 0 \\ 0 & 0 & \frac{1-2\nu_t}{2} \end{bmatrix} \begin{bmatrix} \Delta\epsilon_x \\ \Delta\epsilon_y \\ \Delta\gamma_{xy} \end{bmatrix} \quad (6.14)$$

in which E_t and ν_t are the tangent modulus and Poisson's ratio respectively. The values of E_t and ν_t can be obtained from Equation (6.11) and Equation (6.12), by means of the experimentally determined parameters.

6.1.4 Input Parameters

It has been found that the parameters that affect the nonlinear stress-strain behaviour of the soil significantly are K , n , R_f , C and ϕ (Duncan and Chang, 1970). For the purpose of evaluating these parameters, a series of triaxial compression tests was carried out using the Ottawa sand (see Chapter 2). The Figure 6.4 presents typical plot of the conventional curve of the deviator stress $(\sigma_1 - \sigma_3)$ versus the axial strain ϵ_1 . The experimental data of the Figure 6.4 can be replotted by using the transformed axes $\epsilon_1/(\sigma_1 - \sigma_3)$ and ϵ_1 as shown in Figure 6.5. Duncan and Chang (1970) observed

that the tests data deviate somewhat from a straight line at both low and high values of strains. The deviation depends on the relative density of the sand. For dense sand, the test data tend to deviate to above the straight line plot; and for loose sand, they tend to deviate to below. It can be seen from Figure 6.5, that the data are deviating to above the straight line plot. The sand used in the triaxial tests was compacted to a dense state (relative density $D_r=90\%$), this result, therefore, confirms Duncan and Chang's observations.

As discussed in Chapter 2, seven triaxial tests were carried out by using seven different values of confining pressure (σ_3). In all the seven tests, the sand was compacted to the same relative density of $90\% \pm 2\%$. For each test, the values of E_i and R_f were determined by using the line of best fit (see Figure 6.5). The variation of (E_i/P_a) with (σ_3/P_a) using a log-log scale is shown in Figure 6.6. The values of the two parameters K and n can be obtained from the Figure 6.6.

Table 6.1 gives a summary of the stress-strain parameters for the Ottawa sand used in the present study. These parameters are characterized in terms of the material parameters used in the ISBILD program.

FIGURE 6.4 TYPICAL PLOT OF DEVIATOR STRESS VERSUS AXIAL STRAIN.

FIGURE 6.5 TYPICAL STRESS-STRAIN RELATIONSHIP IN TRANSFORMED HYPERBOLIC FORM.

FIGURE 6.6. VARIATION OF THE INITIAL TANGENT MODULUS WITH THE CONFINING PRESSURE

Table 6.1 Summary of Stress-Strain Parameters
of Ottawa Sand

D_r	ϕ_{tc}	ϕ_p	K	n	R_f
90% $\pm$ 2%	45°	41°	1700	0.40 $\pm$ 0.05	0.90 $\pm$ 0.02

Note: ϕ_p = the angle of friction obtained from the earth
pressure plane strain tests.

ϕ_{tc} = the angle of friction obtained from the triaxial
tests, corrected to take into account the plane
strain conditions of the problem under investigation.
The value of ϕ as obtained from triaxial tests was
41°. This value was increased to 45°. Justification
for that increase is given in Chapter 2.

It should be pointed out that the values of the parameters
K, n, and R_f listed in Table 6.1 and which were obtained from
triaxial tests, might change if they were determined from plane
strain tests. Since the problem investigated herein is a plane
strain problem, it may be ideal to determine these values by using
the plain strain testing technique. However, the use of the values
obtained from the triaxial testing technique may be justified for
the following reasons (Duncan and Chang, 1970):

- (i) The triaxial testing technique is much easier and more
conventional than the plane strain.
- (ii) The stress-strain relationships described previously in

section 6.1.1 have been derived on the basis of data obtained by using the standard triaxial testing technique.

As for the Poisson's ratio parameters d , G and F incorporated in Equation 6.12, they can be determined from triaxial test results by using a hyperbolic relationship between the axial and the radial strains similar to that given by Equation 6.1 and by following the same procedures described above. But since these parameters for Ottawa sand were determined by many research workers (see e.g. Kulhawy, et al., 1969; Selvadurai, et al., 1979) the values given in these references were used in the present analysis and no effort has been made to establish them. These values of d , G and F were taken as 3.0, 0.3 and 0.2 respectively which resulted in an initial Poisson's ratio of value 0.33.

6.1.5 The Computer Program "ISBILD"

The computer program used in the present analysis is called "ISBILD". This program was developed by Ozawa and Duncan (1973) and published in a report by the University of California. The program employs the hyperbolic stress-strain relationship described previously. The modified incremental procedure discussed in section 6.1.2 is used in the program for approximating nonlinear behaviour of soil. The program can be used for analyzing the problems under plane strain conditions utilizing the incremental stress-strain relationship given by Equation (6.14).

The elastic properties of soil are to be represented in the program by a number of soil parameters which are used for

determination of the initial stresses. The stresses, strains and displacements for each element and under any increment of loading are calculated by using the procedures developed by Duncan and Chang (1970) and Kulhawy, et al. (1969).

Finite element discretizations of triangular or rectangular elements can be used in the program ISBILD. The rigidity of a foundation can be simulated by imposing equal displacements to the nodal points underneath the foundation using a special boundary code.

6.1.6 Plane Strain Finite Element Discretizations of the Footing Problem

The geometry of the foundation problem which is to be analyzed herein is shown in Figure 6.7.a. The foundation is a rigid strip of a base which is to be smooth or rough. The foundation rests on the surface of a compressible layer of finite depth on top of a rigid base. The surface of the rigid base is to be smooth or rough.

The problem can be modelled for the finite element analysis as shown in Figure 6.7.b or Figure 6.7.c; according to the interface conditions. Due to symmetry, one half of the problem is considered. The nodal points beneath the foundation can be constrained to move equally in the vertical direction (y-displacements) to simulate the rigidity of the foundation. The horizontal displacements (x-displacements) of the nodal points beneath the foundation and

FIGURE 6.7.a. RIGID STRIP FOUNDATION RESTING ON A LAYER OF SAND ON TOP OF A RIGID BASE

FIGURE 6.7.b FINITE ELEMENT MODEL (SMOOTH INTERFACE)

FIGURE 6.7.c FINITE ELEMENT MODEL (ROUGH INTERFACE)

FIGURE 6.7 FINITE ELEMENT MODELLING OF A STRIP FOUNDATION RESTING ON A LAYER OF SAND ON TOP OF A RIGID BASE

3 3

OF / DE

FIGURE 6.8.b FINITE ELEMENT DISCRETIZATION FOR AN ELASTIC LAYER OF FINITE DEPTH
(TRIANGULAR ELEMENTS)

FIGURE 6.9. LOAD-DISPLACEMENT CURVE AS OBTAINED BY USING RECTANGULAR AND TRIANGULAR ELEMENTS - ISBILD PROGRAM

on the bottom boundary in contact with the rigid base, can be allowed zero or free movement thereby simulating interface conditions with full friction or zero friction respectively.

The nodal points along the vertical boundaries are to be constrained to move only in the vertical direction.

As for the discretizations, there are two possible elements that can be used. These are either 4 noded rectangular elements or 3 noded triangular elements. Typical discretizations of the two types of elements are shown in Figure 6.8.a and 6.8.b. Investigations carried out indicate that both finite element meshes tend to give almost the same results, (see Figure 6.9). The finite element mesh with rectangular elements was used throughout this investigation because it is simpler and more economical than that of triangular elements.

6.1.7 Calibration of the Program "ISBILD"

To obtain some measure of confidence in the results obtained from the finite element program "ISBILD" used in the present analysis, the program was used to solve a problem with known results. The problem concerns the plain strain load-displacements of a rigid strip footing in sand as examined by Duncan and Chang (1970) using a nonlinear finite element technique.

In order to reanalyze the problem by using the program "ISBILD" the stress-strain parameters given by Duncan and Chang were used.

As for the subdivisions, Duncan and Chang represented the rigid foundation by a single row of elements of a large elastic modulus value compared with the elastic modulus of the soil. They also represented the three columns of elements above the foundation (Figure 6.10.a) by what was described as air elements. These air elements were assigned very small moduli compared with the modulus of the soil.

In the program "ISBILD", the rigidity of the foundation is simulated by imposing uniform downward settlements of the nodes where the loads are applied. Two analyses; A and B, were performed. In analysis A the three column elements above the foundation were taken as soil elements whereas in analysis B they were eliminated as shown in Figure 6.10.b and Figure 6.10.c respectively.

It was expected that the results derived by Duncan and Chang should lie very close to the results given in the solution of problem B, but as can be seen from Figure 6.11, there is a slight difference between the two results at the first part of the curves. For higher load levels, the discrepancy becomes prominent. However, the two solutions obtained from using the program ISBILD for problems A and B are very close to the experimental results given by Duncan and Chang.

A third analysis C for the problem as given in Figure 6.10.d was performed to see the effect of the soil above the foundation levels. As shown in Figure 6.11, that effect is quite significant.

- Air element
- Soil element
- Foundation element

FIGURE 6.10 FINITE ELEMENT DISCRETIZATIONS USED TO CALIBRATE THE PROBLEM "ISBILD"

FIGURE 6.11 COMPARISON BETWEEN THE LOAD-SETTLEMENT-CURVES GIVEN BY DUNCAN AND CHANG (1970) AND THOSE OBTAINED BY USING THE PROGRAM "ISBILD"

6.1.8 Comparison Between Experimental and Theoretical Results

(A) Strip Foundation Model

Referring to Table 6.1, it can be noted that the parameter (n) has a value of 0.4 ± 0.05 and that the angle of friction (ϕ) can vary from 41° to 45° . Therefore, it was decided to investigate the effect of variations in n and ϕ on the load-displacement curves of five different problems. These problems were chosen to investigate the effect of spacing, the width to depth ratio, and the interface conditions. For each problem a separate finite element mesh was used, but the general layout of these discretizations is as shown in Figure 6.8.a. Based on the possible values of n and ϕ mentioned above, each problem was analyzed four times, by using the four different sets of stress-strain parameters listed in Table 6.2.

Table 6.2 Parameter Sets Used in Analysis

Parameter	Set 1	Set 2	Set 3	Set 4
K	1700	1700	1700	1700
n	0.35	0.45	0.40	0.40
ϕ	45	45	45	41

As mentioned previously, the nonlinear stress-strain analysis is based on a concept of approximation of the actual stress-strain

curve by a hyperbola. Since the discrepancy between that approximation and the actual curve becomes more pronounced in the case of a dense sand and at relatively high levels of stresses, it was decided to confine the analysis to a maximum stress of approximately one third the ultimate stress. The ultimate stress was estimated by using the classical theory of Terzaghi.

The analysis involved the development of load-displacement curves and the contact stress distributions for the various soil-footing interaction problems.

(i) Load-Displacement Curves

The experimental load-displacement curves of the five different cases are shown in Figure 6.12.a to Figure 6.12.e together with the theoretical results obtained from the nonlinear finite element analysis. As can be seen from these figures, the amount of displacement for a given load depends on the parameter (n) as well as on the angle of friction (ϕ) . Variation of n from 0.35 to 0.45 increases the displacement by approximately 15% to 20%. On the other hand, increasing the value of ϕ from 41° to 45° decreases the displacement by approximately 20% to 25%. This indicates that displacements are quite sensitive to the variation of these parameters. However, despite the wide scatter in the theoretical results, the general shapes of both theoretical and experimental curves are similar. Also, it can be noted that the

initial parts of both theoretical and experimental curves are concave. This concave shape is in contrast to the convex one common for a half space soil medium. This difference may be attributed to the effect of the rigid base and the plane strain conditions which cause the confining pressure to increase as the load increases. Consequently, the initial tangent modulus increases giving rise to the concave shape. This increase of the initial tangent modulus continues only for a small level of loading, then, the slope of the load-displacement curve becomes constant until the load reaches one third to one half of the ultimate load. Further increase of loads is expected to cause a decrease in the slope of the load displacement curve resulting in failure of the soil medium.

This interpretation can be confirmed by considering the load-displacement curves obtained from the finite element analysis and which are shown in Figure 6.13. These curves are for three different cases in which the rigid base lies at depths of $H=2.5B$, $5B$ and $10B$. When the rigid base is on a depth of $2.5B$, the initial part of the load-displacement curve is concave. This result indicates that the effect of the rigid base is appreciable even for a depth ratio $H/B=2.5$. The results of Schmertman, et al. (1978), which are based on the nonlinear finite element analyses, indicate that the depth of a soil medium affected by the loads applied on a strip foundation may extend to about four times the width of the foundations. For a layer of thickness $H \geq 5B$, the load-settlement curve exhibited a convex shape.

FIGURE 6.12.a LOAD-DISPLACEMENT CURVE - COMPARISON BETWEEN
 EXPERIMENT AND NONLINEAR FINITE ELEMENT ANALYSIS
 ($S/B=4$, SMOOTH INTERFACE)

FIGURE 6.12.b LOAD-DISPLACEMENT CURVE-COMPARISON BETWEEN EXPERIMENT AND NONLINEAR FINITE ELEMENT ANALYSIS (S/B=10, SMOOTH INTERFACE)

FIGURE 6.12.c LOAD-DISPLACEMENT CURVE-COMPARISON BETWEEN EXPERIMENT AND NONLINEAR FINITE ELEMENT ANALYSIS (S/B=1.0, SMOOTH INTERFACE)

FIGURE 6.12.d LOAD-DISPLACEMENT CURVE-COMPARISON BETWEEN EXPERIMENT AND NONLINEAR FINITE ELEMENT ANALYSIS (S/B=10.0, ROUGH INTERFACE)

FIGURE 6.12.e LOAD-DISPLACEMENT CURVE-COMPARISON BETWEEN EXPERIMENT AND NONLINEAR FINITE ELEMENT ANALYSIS ($\delta/B=1$, ROUGH INTERFACE)

FIGURE 6.13 EFFECT OF RIGID BASE ON SHAPE OF LOAD-DISPLACEMENT CURVE AS OBTAINED FROM THE NONLINEAR FINITE ELEMENT ANALYSIS.

(ii) Contact Stress Distribution

The experimental and the theoretical contact stress distributions under two adjacent rigid strip foundations placed at spacing ratios of $S/B = 4$ and $S/B = 1$ are shown in Figure 6.14.a and Figure 6.14.b, respectively. The measured contact stress distributions differ completely from the theoretical predictions based on nonlinear finite element analysis.

For the sake of comparison, the contact stress distribution beneath a smooth rigid strip foundation resting on a soil medium of depth $H=B$ (where B is the width of the foundation) as predicted from nonlinear finite element analysis, is shown in Figure 6.15 together with that of a similar foundation but resting on a half space soil medium as obtained from the following equation given by Boussinesq (1885):

$$q(x) = \frac{P}{\pi} \cdot \frac{1}{[a^2 - x^2]^{1/2}} \quad (6.12)$$

in which: P = applied load/m

$$a = B/2$$

x = distance from centre line

As can be seen from Figure 6.15, the two solutions are in agreement and exhibit the same discrepancy with the experiments. This indicates that the two solutions do not give a realistic contact stress distribution for Ottawa sand used in the present investigation. Bauer, et al. (1979b) have observed similar discrepancies between the convex shape of the normal stress distribution as obtained from

Exper.

FEM:

	K	n	ϕ
○	1700	0.35	45°
●	1700	0.45	45°
▲	1700	0.40	45°
▲	1700	0.40	41°

$B = 38 \text{ cm.}$
 $q_a = 100 \text{ kPa}$

Smooth

 $\frac{q}{q_a}$
(a) $S/B = 4$ (b) $S/B = 1.0$

FIGURE 6.14 THEORETICAL AND EXPERIMENTAL CONTACT STRESS DISTRIBUTIONS BENEATH TWO INTERFERING RIGID STRIP FOUNDATIONS

FIGURE 6.15 CONTACT STRESS DISTRIBUTION UNDER A RIGID STRIP FOUNDATION AS OBTAINED FROM THE NONLINEAR FINITE ELEMENT ANALYSIS AND BOUSSINESQ'S SOLUTION

experiments and the concave shape as obtained by Schultze's method (1973) which is again based on Boussinesq's solution.

(B) Retaining Wall Model.

The problem of the retaining wall model shown in Figure 2.14 was analysed by using the program ISBILD. The four sets of soil parameters listed in Table 6.2 were used in the analysis. The problem was analyzed twice; firstly by assuming the back of the retaining wall model to be completely smooth, and secondly by assuming it to be completely rough. The load-displacement curves obtained from the finite element analysis are shown together with the experimental curve in Figure 6.16.

As mentioned in Chapter 2, the experiments were performed under smooth conditions. As can be seen from Figure 6.16, good agreement between the experimental curve and the theoretical one was obtained for the smooth case. This agreement continues until a point close to the ultimate load, as determined experimentally from Figure 2.16.b. After that point the two curves diverge from each other. As for the theoretical curve obtained for the rough case, it is almost completely different from the experimental curve.

FIGURE 6.16 COMPARISON BETWEEN LOAD-DISPLACEMENT CURVE OF A RETAINING WALL MODEL OBTAINED FROM EXPERIMENT AND THOSE CURVES OBTAINED FROM NONLINEAR FINITE ELEMENT ANALYSIS

6.2 Empirical Methods

6.2.1 General

Because no rational theoretical method for prediction of settlements due to distortion and compressibility of cohesionless soils is available, a variety of empirical and semi-empirical methods have been employed. Such empirical methods have been proposed by Terzaghi and Peck (1948), Gibbs and Holtz (1957), Alpan (1964), Meyerhof (1965), Peck and Bazaraa (1969), Peck (1974), DeBeer (1965), D'Appolonia (1970) and Schmertmann (1970). All these methods have been based on either the Standard Penetration Test or the static Dutch cone test. Several researchers including Schmertmann (1970), Sutherland (1974) and Jordan (1977) have reviewed and discussed these methods. Of all of these methods, it has been found that the method proposed by Schmertmann (1970) leads to results which are compatible with field measurements (Perloff, 1975).

All the methods mentioned above are concerned with prediction of settlements of isolated footings. Therefore, in the present study concerning the group behaviour of foundations, the applicability of Schmertmann's method for prediction of settlements of the foundation in a group will be discussed.

6.2.2 Schmertmann's Method

Schmertmann (1978) modified his method to take into account the plane strain conditions that can exist under long foundations ($L/B > 10$). Despite the fact that the details of the original method and its modifications are available in many papers (Schmertmann, 1970; Perloff, 1975; Jordan, 1977; Schmertmann, et al., 1978; Oweis, 1979; and others). For the convenience of the reader, the main features are mentioned in Appendix E.

Before proceeding, two points should be raised; the first concerns the applicability of the method for prediction of settlements of two adjacent foundations, the second concerns the extension of the method to accommodate the effect of a rigid layer.

(i) Effect of Spacing

As for the adjacent foundation, Schmertmann (1970) stated that "if the two foundations are far enough apart any interaction will be negligible. This will be the case if 45° lines from the edges intersect at a depth greater than $2B_2$, when a second loading of width B_2 is placed next to an existing foundation of greater width B_1 as shown in Figure 6.17.a. For a 45° intersection depth also greater than B_1 , the two foundations are assumed to be independent regardless of load sequence (see Figure 6.17.b). If adjacent foundations are close enough to interact without question, as in the case shown in Figure 6.17.c, where the edge to edge spacing S_e between them is less than B of the smallest, and they are loaded

(a) $S_e > 4B_2$ Loads in sequence (Interference Negligible)

(b) $S_e > 2B_1$ Regardless Load Sequence (Interference Negligible)

(c) $S_e < B_2$ Loading simultaneously (Interference Considered)

FIGURE 6.17 DIFFERENT CASES OF INTERFERENCE BETWEEN TWO ADJACENT FOUNDATIONS DISCUSSED BY SCHMERTMANN (1970)

simultaneously, then Schmertmann would treat them as a single foundation with some appropriate, equivalent width. Intermediate situations should fall within these boundaries". Schmertmann has mentioned nothing about how that appropriate equivalent width could be estimated. However, he stated that more research is needed to formulate design rules for this problem, and that the present state of knowledge requires the engineer to use conservative judgement.

(ii) Effect of Rigid Base

As to the foundations which rest on a layer of sand of limited thickness (less than twice the width of the foundation; B) underlain by a rigid base, Schmertmann (1970) mentioned that the simple triangular distribution of the strain factor I_z would remain the same. The soil below the rigid boundary to the depth $2B$ is assumed to have a very high modulus of elasticity. The vertical strains below such a boundary, then, become negligible. Schmertmann (1970) also mentioned that the suggested simple cut-off procedure appeared successful in the treatment of case records with rigid boundaries located at depths $1B$, $0.72B$ and $0.23B$. Schmertmann also carried out a series of tests using a circular rigid plate 2.3 in. diameter, resting on the surface of a layer of sand underlain by a rigid base and found that the effect of the rigid base on settlements as obtained from these tests was very similar to that predicted from the suggested cut-off procedure.

In this thesis, a theoretical study was carried out to investigate the influence of a rigid base on the vertical strain distribution beneath a rigid strip footing resting on a compressible layer of limited thickness. The investigation was carried out by using both linear and nonlinear finite element techniques. The linear finite element technique was employed by the program "SP23" developed by Bowes (1975), and the nonlinear technique by the program "ISBILD" described at the beginning of this chapter.

Shown in Figure 6.18.a is the vertical strain distribution obtained from the linear finite element analysis for compressible layers of sand of thickness to foundation width ratios of $H/B=1.0$, 2.5 and 5.0. Figure 6.18.b shows this distribution as obtained from the nonlinear finite element analysis for layers of thickness to foundation width ratios of $H/B=2.5$, 5.0 and 10.0. As can be noted from both figures that:

- (i) the position where the maximum vertical strain occurs, is dependent on the ratio H/B . The maximum strain occurs at ratio of $Z/B = 1.0$ for the layers which are relatively deep (e.g. $H/B=5.0$), and it occurs at ratio $Z/B = 0.5$ for the layers which are of shallow depths (e.g. $H/B=1.0$).
- (ii) the magnitude of the maximum vertical strain depends on the ratio H/B as well as it decreases as this ratio decreases.

Now, the approximate vertical strain distribution as proposed by Schmertmann (1970) and modified by Schmertmann, et al. (1978),

is for a deep compressible layer unaffected by a rigid base:

Since as explained above, the vertical strain distribution is dependent on the ratio H/B , or in other words, on the position of the rigid base, it is pertinent to compare the actual strain distribution of shallow layers ($H/B < 4$) and the truncated shape of this distribution suggested by Schmertmann (1970) for such layers. The comparison is shown in Figure 6.19.a and Figure 6.19.b for a compressible layer of thickness to foundation width ratio $H/B=2.5$ and 1.0 respectively. As can be seen from these figures, the actual distribution of the vertical strain, whether obtained from linear or nonlinear finite element analysis, is completely different from the truncated distribution as suggested by Schmertmann (1970). However the areas of the actual distribution of the vertical strain as obtained from the nonlinear finite element analysis are almost the same as those obtained from Schmertmann's distribution. This may justify using the truncated shape in computing the settlement following the procedures proposed by Schmertmann (1970) and which have been discussed at the beginning of this section.

Based on these observations, it was decided to use Schmertmann's method in computation of the settlement of two adjacent rigid strip foundations resting on a layer of sand underlain by a rigid base; and to compare the results obtained with the experimental results.

FIGURE 6.18.a. EFFECT OF THE RIGID BASE ON THE DISTRIBUTION OF THE VERTICAL STRAIN BENEATH RIGID STRIP FOUNDATIONS AS OBTAINED FROM LINEAR FINITE ELEMENT ANALYSIS

FIGURE 6.18.b EFFECT OF THE RIGID BASE ON THE VERTICAL STRAIN DISTRIBUTION BENEATH RIGID STRIP FOUNDATIONS AS OBTAINED FROM NONLINEAR FINITE ELEMENT ANALYSES

FIGURE 6.19.a DISTRIBUTION OF THE VERTICAL STRAIN INFLUENCE FACTOR
 ALONG THE CENTRE LINE OF A RIGID STRIP FOUNDATION
 RESTING ON A LAYER OF SAND ON TOP OF A RIGID BASE
 ($H/B=2.5$)

FIGURE 6.19.b DISTRIBUTION OF THE VERTICAL STRAIN INFLUENCE FACTOR ALONG THE CENTRE LINE OF A RIGID STRIP FOUNDATION RESTING ON A LAYER OF SAND ON TOP OF A RIGID BASE ($H/B=1.0$)

6.2.3 Application of Schmertmann's Method for Prediction of Settlement of Two Interfering Strip Foundations

As mentioned in Chapter 3, the experiments showed that adjacent foundations would act separately, each as a single foundation of width B , if the spacing between them reached $6B$ or more. Also, the two foundations would act as a single foundation of width $B_e = 2B$ if the spacing was equal to B (the two foundations touch). For intermediate spacing ratios ($1 < S/B < 6$), Equation (4.1) can be used to obtain the width B_e of a single foundation experiencing the same settlement as the foundation in a group. Equation (4.1) is as follows:

$$B_e = B[2.2 - 0.2(S/B)] \quad 1 \leq S/B \leq 6 \quad (4.1)$$

where: B_e = the width of the equivalent foundation

B = width of a single foundation

S = centre to centre spacing

The values of B_e corresponding to spacing ratios of $S/B=1, 2, 4$ and 6 were determined from Equation (4.1). The settlements of two interfering strip foundations resting on a layer of thickness $H=2.5B$ were computed by using the truncated distribution of the modified vertical strain influence factor proposed by Schmertmann, et al. (1978) (See Appendix E). The settlement of the two interfering foundations is assumed to be equivalent to that of a single foundation of width B_e . The settlement corresponding to each spacing ratio was computed for an average pressure of 98 kPa and

and different values of the elastic modulus (E).

The relationship between the settlement efficiency factor (η_s) as defined in Chapter 3 and the spacing ratio S/B as obtained by Schmertmann's method (using the equivalent width, B_e) are shown in Figure 6.20.a together with the experimental results. As can be seen, there is a discrepancy between the computed and the experimental results. This discrepancy may be due to the fact that the rigid base has a different response for each location of the two interfering foundations. To take into account the varied effect of the rigid base on the settlement as the spacing ratio (S/B) changes, the reduction factor, given in Equation (4.5) is included in the calculation. The reduction factor (η_{sr}) as given by Equation (4.5) is as follows:

$$\eta_{sr} = \frac{2}{3} \left(\frac{H}{B} \right)^{2/3} \quad (4.5)$$

where: η_{sr} = the settlement reduction factor to take into account the influence of the rigid base

H = the thickness of the compressible layer from the foundation level to the rigid base

B = the width of the foundation

By reducing the final settlement obtained by Schmertmann's method according to this equation, the results have improved as shown in Figure 6.20.b. Numerical examples explaining the procedures followed in the calculation are given in Appendix F.

FIGURE 6.20.a RELATIONSHIP BETWEEN THE SETTLEMENT EFFICIENCY FACTOR AND THE SPACING AS OBTAINED FROM EXPERIMENTS AND BY SCHMERTMANN'S METHOD MODIFIED BY Eq.4.1

FIGURE 6.20.b RELATIONSHIP BETWEEN THE SETTLEMENT EFFICIENCY FACTOR AND THE SPACING AS OBTAINED FROM EXPERIMENTS AND BY SCHMERTMANN'S METHOD MODIFIED BY Eq.4.1 AND Eq.4.5

FIGURE 6.21.a LOAD-SETTLEMENT RELATIONSHIP-COMPARISON
BETWEEN EXPERIMENT AND SCHMERTMANN'S
METHOD ($S/B=1$)

FIGURE 6.21.b LOAD-SETTLEMENT RELATIONSHIP-COMPARISON BETWEEN EXPERIMENT AND SCHMERTMANN'S METHOD ($S/B=6$)

Figure 6.21a and Figure 6.21.b show the experimental load-settlement curves together with those predicted from Schmertmann's method following the procedures outlined by the two examples given in Appendix F. The computation of the settlement was carried out using different values of the elastic modulus (E) which cover the post probable value of the elastic modulus of the soil used in the experiments. As can be seen from these figures, the results obtained from Schmertmann's method agree closely with those obtained from experiments. Based on the above observations, Schmertmann's method can be adopted to predict the settlement of two interfering strip foundations following the procedures mentioned above.

Chapter 7

CONCLUSIONS AND RECOMMENDATIONS7.1 Summary and Conclusions

The review of literature related to the problem of interference between adjacent foundations has shown that research workers in the area were mainly concerned with estimation of the bearing capacity of the interfering foundation. In this thesis, an emphasis is placed on studying the effect of the interference between the foundations on contact stress distribution and settlement. More specifically, the problem investigated herein is concerned with the interference between two rigid strip foundations resting on the surface of a layer of sand underlain by a rigid base. The two foundations are of equal width and are subjected to central loads of equal magnitude. Owing to the symmetry of the geometry of the two interfering foundations and the symmetry of the loading configuration, the problem can be effectively reduced to the examination of a single foundation. This requires that the side walls of the test tank be rigid and smooth enough such that no lateral displacements nor friction should occur. To achieve this end, the side walls of the test tank were stiffened by channel and rectangular steel sections. The side walls were lined with sheets of polished no.16 gauge stainless steel. Several factors which influence the behaviour of the foundation in a group were investigated. These factors, namely, were the spacing between the foundations,

the ratio of the thickness of the compressible layer to the width of the foundation, the roughness of the interface between the foundation and the soil and the interface between the soil and the rigid base, and load and unload cycling.

Numerical and empirical solutions to the problem of interference between foundations were presented. The numerical solution was based on the nonlinear finite element technique developed at the University of California, Berkeley (ISBILD program). The input parameters were determined from the results of a series of triaxial tests on Ottawa sand; the material used in the experiments. The empirical solution was obtained by using Schmertmann's method. The following conclusions can be drawn:

- (a) i - The contact stress distribution beneath the foundation in a group is asymmetrical and differs substantially from that beneath an individual foundation.
- ii - The degree of asymmetry of the contact stress distribution depends on the spacing between foundations.
- iii - The contact stress distribution, measured at loading level equal to one third the ultimate load, is of a convex shape. This type of distribution beneath foundations resting on a layer of limited thickness differs from the concave shape distribution proposed by Schultz (1961) for deep layers subjected to the same level of loading.
- iv - After the second cycle of loading and unloading, the contact stress distribution is no longer influenced by the loading

and unloading process.

- v - An equation was proposed to obtain an approximate linear distribution beneath two interfering foundations resting on the surface of a layer of sand. The equation is based on the experimental observations and can be expressed as follows:

$$q_1 = q_a \left[1 \pm \left(0.6 - 0.15 \frac{S}{B} \right) \right]$$

where: q_1 = the contact stress beneath the confined edge of the interfering foundation,

q_2 = the contact stress beneath the free edge of the interfering foundation.

q_a = average applied pressure

S = the centre to centre spacing

B = the width of the foundation

- (b) i - The settlement is significantly influenced by the variation of the spacing between the two interfering foundations. The settlement is maximum when the two foundations act separately at spacing ratio $S/B \geq 6.0$ and becomes minimum at spacing ratio $S/B=1.0$ (the two foundations touch each other).
- ii - The relationship between the settlement efficiency factor (i.e. the ratio (η_s) between the settlement of the foundation in a group and that of an individual foundation) and the spacing ratio as obtained from experiments can be described by the following equation:

$$\eta_s = 52 + 8\left(\frac{S}{B}\right) \% \quad 1 < S/B < 6$$

- iii - The settlement also decreases as the ratio (H/B) between the layer thickness and the foundation width decreases. Based on the experimental results, the relationship between the decrease in the settlement expressed by the reduction factor (η_{sr}) and the ratio (H/B) is:

$$\eta_{sr} = \frac{2}{3} \left(\frac{H}{B}\right)^{2/3} \quad 0.5 < \frac{H}{B} < 2$$

where: η_{sr} = the reduction factor to take into account the influence of a rigid base.

H = the thickness of the compressible layer.

B = the width of the foundation.

- iv - The roughness conditions of the interface between the foundation and the soil and the interface between the soil and the rigid base have minor effects on the settlement.
- v - The shape of the load-settlement curve is influenced by the number of loading and unloading cycles. Most of the total and inelastic settlements occur at the first cycle and the rate of increase in settlements decreases as the number of cycles increases.
- (c) i - The bearing capacity is strongly affected by the spacing ratio. The bearing capacity starting from a minimum value for a single foundation ($S/B \geq 6$) reaches a maximum value when the two foundations touch each other ($S/B=1$).

- ii - The influence of interference between foundations, measured by the efficiency bearing capacity factor; η_q (i.e. the ratio of the ultimate bearing capacity of the foundation in a group to that of an individual foundation) depends on the layer thickness and the roughness conditions. The efficiency bearing capacity factor decreases as the layer thickness decreases. Changing the interface conditions from smooth to rough has a similar effect.
- iii - The influence of the ratio (H/B) of the layer thickness to the foundation width on the bearing capacity of a foundation depends on the roughness conditions of the interface between the foundation and the soil and the interface between the soil and the rigid base. For smooth interface, the bearing capacity decreases as this ratio decreases (for the range of H/B between 1.0 and 2.5, used in experiments). But for rough interface, the bearing capacity increases steadily with the decrease in the ratio H/B. The relationship between the ratio H/B and the bearing capacity as obtained from experiments can be described by the following equation:

$$\eta_{qr} = 1.5 \left(\frac{B}{H} \right)^{1/2} \quad 0.5 < \frac{H}{B} < 2$$

where: η_{qr} = the ratio of the bearing capacity of the foundation resting on a layer of thickness $H < 2B$ to the bearing capacity of a similar foundation resting on a relatively deep layer of thickness $H > 2B$.

H and B are as defined above.

- (d) i - The theoretical load-settlement curves predicted from the nonlinear finite element technique employed in the program "ISBILD" agreed closely with those obtained from experiments. But they showed to be sensitive to the input nonlinear stress-strain parameters.
- ii - The theoretical contact stress distribution obtained from the nonlinear finite element analysis disagreed with the observed distribution. This disagreement indicates that the nonlinear finite element technique employed in the program "ISBILD" does not give realistic contact stress distribution for the particular granular material used in experiments.
- (e) i - Using the linear finite element technique employed in the program "SP23" (developed by Bowes, 1975), it was found that the vertical strain distribution under a rigid strip foundation resting on a compressible layer underlain by a rigid base was significantly influenced by the ratio (H/B) as defined above. The position and the magnitude of the maximum strain depended on this ratio. Similar results were obtained from the nonlinear finite element analysis by using the program "ISBILD".
- ii - Comparing these results with the truncated shape of the vertical strain influence factor proposed by Schmertmann (1970) and (1978) for layers of shallow depth underlain by a rigid base, it was found that they were different.

The discrepancy increased as the ratio H/B decreased.

- iii - Simple procedures have been proposed which make Schmertmann's method applicable for prediction of the settlement under two interfering rigid strip foundations.

7.2 Recommendations for Future Work

The present work has raised the following points:

1. This study has attempted to investigate a number of factors which are of significant influence on the behaviour of two interfering foundations resting on a cohesionless soil medium. To date there is no experiments which have been carried out with model footings resting on cohesive soil media. Further research is necessary to investigate the influence of the interference between foundations resting on a cohesive soil medium and to compare the results with those obtained from the present work.
2. All the research that has been done until now in this area is concerned with the problem of interference between two foundations. It is suggested to investigate the influence of the interference between a group of foundations; more than two.
3. Further research is required to establish the influence of interference between foundations on the contact stress distribution.
4. It is suggested that the influence of interference between foundations of different sizes be investigated.

5. The influence of a rigid base on the settlement and bearing capacity of a compressible layers was found to be strongly dependent on the ratio of the layer thickness to the foundation width as well as the roughness conditions of the interface between the foundation and the soil and the interface between the soil and the rigid base. More research is recommended to establish the findings of the present work in this respect.
6. It is recommended to investigate the influence of the variation in the relative density on the behaviour of the interfering foundations concerning the settlement and the bearing capacity.

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APPENDICES

Appendix A

Summary of Research Concerning Interference Between Adjacent Surface Foundations

Article	Study	Model			Soil	Embedment D/B	Purpose of study; Investigation of effect of interference on:
		Width B	Length L	Bottom Surface			
Stuart & Hanna, 1961	Theor. Exp.	1/2" 1/2"	9" 9"	Smooth Rough	Sand	0, 8, 16 and 20	Bearing Capacity
Stuart, 1962	Theor. Exp.	1/2" 1"	9" 13"	Smooth Rough	Sand	0.2, 0.5	Bearing Capacity
Mandel, 1965	Theor.	-	-	-	Sand	Surface	Bearing Capacity
West & Stuart, 1965	Theor. Exp.	1.75"	24"	Rough	Sand	0.5	Bearing Capacity
Myslivec & Kysela, 1968	Exp.	1 cm.	10 cm.	Rough	Sand	0.2 to 6.2	Bearing Capacity
Murthy, 1970	Exp.	3" 2"	7.8" 7.8"	Rough	Sand	Surface	Bearing Capacity Settlement
Myslivec & Kysela, 1973	Exp.	1 cm. 1.5 cm. 2 cm.	10 cm.	Rough	Sand	0 to 4	Bearing Capacity

Appendix A (contd.)

Article	Study	Model				Soil	Embedment D/B	Purpose of study: Investigation of effect of Interference on:
		Width B	Length L	Bottom Surface	Rigidity			
Singh, et al., 1973	Exp.	4 cm. 4.9 cm. 6 cm.	4 cm. 4.9 cm. 6 cm.	Smooth	Rigid	Sand	Surface	Bearing Capacity Settlement
Siva Reddy & Mogalish, 1976	Theor.	-	Strip	Smooth Rough	Rigid	Clay	Surface	Bearing Capacity
Khadilkar & Varma, 1977	Theor.	-	Strip	Smooth Rough	Rigid Flexible	Sand	Surface	Settlement
Deshmukh, 1978	Exp.	10 cm. 10 cm. 5 cm. 8.9 cm.	(diam.) 25 cm. 17.8 cm.	Rough	Rigid	Sand	Surface	Bearing Capacity Settlement

Appendix B

Study of Friction on the Interior Walls of the Test Tank

In order to study the effect of friction of the interior walls of the test tank, let us assume that:

P_s = the ultimate load obtained with both interior walls of the test tank lined with sheets of stainless steel of angle of friction λ_s ,

P_r = the ultimate load obtained with both interior walls of the test tank lined with rough sheets of angle of friction λ_r ,

P_{sr} = the ultimate load obtained with one of the interior walls lined with a stainless steel sheet and the other lined with a rough sheet,

P_0 = the ultimate load which is supposed to be obtained with both interior walls perfectly smooth,

ΔP_s = the increase in the ultimate load due to the friction on one side of the test tank lined with a sheet of stainless steel and

ΔP_r = the increase in the ultimate load due to the friction on one side of the test tank lined with a sheet roughened with sand.

By definition:

$$P_{s_r} = P_0 + 2\Delta P_s \quad (1)$$

$$P_r = P_0 + 2\Delta P_r \quad (2)$$

$$P_{sr} = P_0 + \Delta P_s + \Delta P_r \quad (3)$$

The values of P_s , P_r and P_{sr} as obtained from experiments and expressed in terms of (γB^3) are 255, 319 and 286 respectively (see Figure 2.7.a). Equations (1) to (3) can be rewritten as follows:

$$P_0 + 2\Delta P_s = 255 \quad (4)$$

$$P_0 + 2\Delta P_r = 319 \quad (5)$$

$$P_0 + \Delta P_s + \Delta P_r = 286 \quad (6)$$

$$\text{but } \frac{\Delta P_s}{\Delta P_r} = \frac{\tan \lambda_s}{\tan \lambda_r} \quad (7)$$

from the results of the pull out test, Equation (2.4) is as follows (see Section 2.2.2):

$$\frac{\tan \lambda_s}{\tan \lambda_r} = 0.133 \quad (8)$$

$$\therefore \frac{\Delta P_s}{\Delta P_r} = 0.133 \quad (9)$$

From Equation (4) and Equation (5);

$$\Delta P_r - \Delta P_s = 32 \quad (10)$$

and from Equation (9) and Equation (10):

$$\Delta P_r = 0.133 \Delta P_r = 32$$

$$\therefore \Delta P_r = 36.9$$

from Equation (9):

$$\Delta P_s = 0.13 \Delta P_r$$

$$\therefore \Delta P_s = 4.9$$

and from Equation (4):

$$P_0 = 255 - 2\Delta P_s$$

$$= 255 - 2 \times 4.9$$

$$P_0 = 245.2$$

As a check for Equation (6):

$$P_{sr} = P_0 + \Delta P_s + \Delta P_r$$

$$= 245.2 + 4.9 + 36.9$$

$$= 287 \text{ compared to } 286 \text{ from experiments.}$$

Now, the percentage of increase in the ultimate load due to the friction angle λ_s of the stainless steel is:

$$\frac{2\Delta P_s}{P_0} \times 100 = \frac{2 \times 4.9}{245.2} \times 100 = 4 \%$$

Appendix C

Rigidity of the Aluminium Plate

(A) The differential displacement of the aluminium plate can be obtained by simplifying the problem to the following configuration:

For a strip foundation resting on the surface of a cohesionless layer:

$$q_{ult} = 0.5 \gamma B N_\gamma$$

for sand of angle of friction $\phi = 41^\circ$,

$$N_\gamma = 130$$

$$q_{ult} = 0.5 \times 17.6 \times 0.156 \times 130 = 172 \text{ kPa}$$

assume the maximum applied pressure is equal to q_{ult} and assuming that

$$E_{alum} = 70 \times 10^6 \text{ kPa}$$

$$I = \frac{bt^3}{12} = (1) \times \frac{(0.051)^3}{12} = 11 \times 10^{-6} \text{ m}^4$$

At the end of the plate, the maximum displacement $\delta_{max} = \frac{ql^4}{8EI}$

$$\delta_{max} = \frac{172(0.115)^4}{8 \times 70 \times 10^6 \times 11 \times 10^{-6}} = 5 \times 10^{-6} \text{ m}$$

$$\delta_{max} = 0.005 \text{ mm}$$

Relative to the total plate settlement of 4 to 5 mm. the differential displacement is small, and the plate can be considered to settle uniformly.

(B) Relative Rigidity, K_r

According to Borowicka (1938):

$$K_r = \frac{4}{3} \frac{(1-\nu_s^2)}{(1-\nu_b^2)} \frac{E_b}{E_s} \left(\frac{t}{B}\right)^3$$

For the sand compacted to a dense state

$$\nu_s = 0.25$$

$$E_s = 30 \times 10^3 \text{ kPa}$$

For the aluminium plate:

$$\nu_b = 0.3$$

$$E_b = 70 \times 10^6 \text{ kPa}$$

$$t = 0.051 \text{ m.}$$

$$B = 0.152 \text{ m.}$$

$$\begin{aligned} \text{Then } K_r &= \frac{4}{3} \frac{(1-0.25^2)}{(1-0.3^2)} \left(\frac{70 \times 10^6}{30 \times 10^3} \right) \left(\frac{0.051}{0.152} \right)^3 \\ &= 121 \end{aligned}$$

From that, it appears that the plate can be considered to be rigid.

Appendix D

Retaining Wall Model

(i) Rigidity of the Retaining Wall Model

In order to determine the maximum deflection which can occur at failure load, the vertical back of the model lined with a sheet of stainless steel was

assumed to be perfectly smooth. Then the maximum passive pressure can be obtained from the following equation:

$$P_{\max} = \frac{\gamma H^2 K_p}{2} \quad (1)$$

where: $K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$ (for $\phi = 41^\circ$, $K_p = 4.5$)

Now at the lower end of the plate, the deflection is:

$$\delta = \frac{\omega l^4}{8EI} \quad (2)$$

where: $E = 200 \times 10^6 \text{ kN/m}^2$, $l = 0.12 \text{ m}$

$$I = \frac{1 \times t^3}{12} = \frac{(0.025)^3}{12} = 0.49 \times 10^{-6} \text{ m}^4$$

$$\omega = \gamma H_2 K_p = (17.3 \times 0.32 \times 4.5) = 25 \text{ kN/m}^2$$

$$\therefore \delta = \frac{25(0.12)^4}{8 \times 2 \times 10^8 \times 49 \times 10^{-8}} = 7 \times 10^{-6} \text{ m} = .007 \text{ mm}$$

This amount of deflection is negligible compared with the

horizontal displacement of the order of about 10 mm.

(ii) The Angle of Internal Friction ϕ

From experiments (see Figure 2.16.b)

$$\frac{P_{\max}}{\gamma H^2 B} = 3 \quad (3)$$

and from Equation (1):

$$\frac{P_{\max}}{\gamma H^2 B} = \frac{K_p}{2}$$

$$K_p = 6$$

Assuming the vertical back of the retaining wall model to be smooth, the value of K_p is given by the equation (Figure D.1.a):

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = 6$$

from which $\phi = 45.5$ degree

If the angle of friction of the stainless steel λ of 7° was taken into account, then the case would be as shown in Figure D.1.b.

For small values of λ (e.g. $\lambda < \phi/3$), the slip surface can be assumed to be plane and the value of K_p is given by:

$$K_p = \left[\frac{\sin(\psi + \phi)}{\sin(\psi + \lambda) - \frac{\sin(\phi + \lambda) \cdot \sin(\phi + \beta)}{\sin(\psi - \beta)}} \right]^2 \quad (4)$$

Now:

$$K_p = 6, \psi = 90^\circ, \lambda = 7^\circ \text{ and } \beta = 0$$

$$\therefore \phi = 41^\circ$$

$$P_{\max} = \frac{\gamma H^2}{2} K_p \quad K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$$

(a) A Retaining Wall With a Smooth Vertical Back

$$P_{\max} = \frac{\gamma H^2}{2} K_p \quad \& \quad K_p = \left[\frac{\sin(\psi + \phi)}{\sin(\psi + \lambda) - \sqrt{\frac{\sin(\phi + \lambda) \sin(\phi + \beta)}{\sin(\psi - \beta)}}} \right]^2$$

(b) A Retaining Wall With a Rough Inclined Back

FIGURE D.1 PASSIVE EARTH PRESSURE ACTING ON A RETAINING WALL

Appendix E

Schmertmann's Method for Prediction of Settlements of Cohesionless Soils

Schmertmann (1970) presented a new method of prediction of settlements under rigid foundations resting on the surface of a homogeneous, isotropic half space. In the following, the main features will be described.

Based on the theory of elasticity the vertical strain ϵ_z at any depth z under the loaded area may be expressed as:

$$\epsilon_z = \frac{P}{E} I_z \quad (E.1)$$

in which: P = load intensity

E = elastic modulus

I_z = the influence strain factor that depends on the location of the point considered and on Poisson's ratio (ν).

From the theoretical, model study and experimental and computer simulation results, as shown in Figure E.1, Schmertmann found that the vertical strain distribution defined by the strain influence factor I_z , could be approximated, for practical work, by the triangle distribution shown in Figure E.1.

FIGURE E.1 VERTICAL STRAIN DISTRIBUTION FOR SETTLEMENT OF COHESIONLESS MASSES. (a) DISTRIBUTION IN ELASTIC MEDIUM; (b) DISTRIBUTION IN COHESIONLESS MASS FROM MODEL TESTS AND ANALYSIS; (c) ASSUMED 2B-0.6 DISTRIBUTION (MODIFIED FROM SCHMERTMANN, 1970)

Now defining ϵ_z as:

$$\epsilon_z = \frac{db}{dz}$$

or

$$db = \epsilon_z dz$$

Then

$$\begin{aligned} \delta &= \int_0^{\infty} \epsilon_z dz \\ &= \int_0^{2B} \epsilon_z dz \end{aligned}$$

from Equation (1):

$$\delta = \int_0^{2B} \left(\frac{\Delta P}{E_s} I_z \right) dz$$

or

$$\delta = \Delta P \int_0^{2B} \frac{I_z}{E_s} dz$$

$$\delta = \Delta P \sum_0^{2B} \frac{I_z}{E_s} \Delta z \quad (E.2)$$

where: δ = settlement under a rigid footing.

To take into account some of the strain relief due to embedment,

Schmertmann proposed to introduce the following correction into

Equation (E.2):

$$C_1 = 1 - 0.5 \frac{P_0}{\Delta P} \quad C_1 \leq 0.5 \quad (E.3)$$

in which: P_0 = overburden pressure at the foundation level

ΔP = net foundation pressure increase $\equiv P - P_0$

On the other hand, to consider the time rate of development of settlement (creep) another correction C_2 has been proposed:

$$C_2 = 1 + 0.2 \log \left(\frac{t_{yr}}{0.1} \right) \quad (E.4)$$

The general form of Equation (E.2) is:

$$\delta = C_1 C_2 \Delta P \sum_0^{2B} \frac{I_z}{E_s} \Delta z \quad (E.5)$$

Schmertmann recommended to use the following simple equation to calculate the elastic modulus:

$$E_s = 2 q_c \quad (E.6)$$

where: q_c = the Dutch Cone bearing capacity

Schmertmann, et al. (1978) recommended improvements to the method as follows:

1. Using separate strain factor distribution with the modified shape for square and long footings; as shown in Figure E.2
2. Using a variable peak value of strain factor I_z according to the following equation:

$$I_{zp} = 0.5 + 0.1 \left(\frac{\Delta P}{P_{vp}} \right)^{1/2} \quad (E.7)$$

where: P_{vp} = Effective overburden pressure at the depth where the peak strain occurs (at $B/2$ and B for square and long footings respectively).

FIGURE E.2 RECOMMENDED MODIFIED VALUES FOR STRAIN INCLUDENCE FACTOR DIAGRAMS (AFTER SCHMERTMANN, ET AL. (1978))

3. Increasing the equivalent sand modulus E_s as follows:

For square footings:

$$E_s = 2.5 q_c \quad (E.8)$$

For long footings ($L/B > 10$):

$$E_s = 3.5 q_c \quad (E.9)$$

Appendix F

Schmertmann's Method: Numerical Examples

The following two examples illustrate the procedures proposed in this thesis in order to use Schmertmann's method in computing the settlement under two interfering strip foundations.

Example F.1

The two strip foundations shown schematically in Figure F.1 are to be constructed at the groundwater table, which is 2.0 m. below the surface of a medium dense sand stratum of considerable depth. The Dutch cone bearing capacity profile is shown on the same figure. Determine the settlement to be expected five years after construction.

Solution:

Calculation of settlement is carried out in the following sequence:

- 1 - Calculate the value of I_{zp} at depth B below the foundation level using Equation (E.7) as follows:

- The initial overburden pressure at the foundation level, P_0 , is

$$P_0 = 1.6 \times 2 = 3.2 \quad \text{t/m}^2 = 0.32 \text{ kg/cm}^2$$

- The net foundation pressure Δp , is:

$$\Delta P = 1.82 - 0.32 = 1.5 \text{ kg/cm}^2$$

EXAMPLE ONE

FIGURE F.1 TWO ADJACENT RIGID STRIP FOUNDATION ON A RELATIVELY DEEP LAYER OF SAND

- The initial overburden pressure at depth B below the foundation level P_{vp} , is

$$P_{vp} = 1.6 \times 2 + 0.9 \times 2.5 = 5.5 \text{ t/m}^2 = 0.55 \text{ kg/cm}^2$$

$$I_{zp} = 0.5 + 0.1 \sqrt{\frac{\Delta P}{P_{vp}}} = 0.5 + 0.1 \sqrt{\frac{1.5}{0.55}} = 0.67$$

- 2 - Plot the strain influence factor distribution for the plane strain case as shown in Figure (E.2).
- 3 - Based on the q_c profile and the I_z distribution, divide the 4B depth into a convenient number of layers, as shown in Figure E.2 and columns 1 and 2 of Table E.1.
- 4 - Determine the average q_c for each layer, and then calculate the corresponding E from Equation (E.9) as indicated in columns 3 and 4 of Table E.1.
- 5 - Locate the depth of the middle of each layer (column 5) and determine the I_z value for this depth from the I_z distribution. The values are given in column 6.
- 6 - Determine C_1 from Equation (E.3)

$$C_1 = 1 - 0.5 \left(\frac{P_0}{\Delta P} \right)$$

$$= 1 - \frac{0.5 \times 0.32}{1.5} = 0.89$$

- 7 - Determine C_2 from Equation (E.4) for the time of interest,

$$C_2 = 1 + 0.2 \log \frac{t}{0.1}$$

$$= 1 + 0.2 \log \frac{5}{0.1} = 1.34$$

8 - Calculate the settlement of an individual foundation (δ_s)

from Equation (E.5)

$$\delta_s = C_1 C_2 p \sum_0^{4B} \frac{I_z}{E} \Delta z$$

$$= 0.89 \times 1.34 \times 1.5 \times 2.27 = 4.06 \text{ cm.}$$

9 - Calculate the settlement efficiency factor η_s from Equation (3.4)

$$\eta_s = 0.52 + 0.08 (S/B)$$

$$= 0.52 + \frac{0.08 \times 7.5}{2.5} = 0.76$$

10 - Multiply the settlement obtained in step 8 by η_s to take into the account the group interaction effect

$$\delta_g = \eta_s \delta_s = 0.76 \times 4.06 = 3.09 \text{ cm.}$$

Table F.1 Data for Settlement Estimate of Example F.1
(Modified from Foundation Engineering Handbook)

(1) Layer	(2) Δz cm.	(3) q_c kg/cm ²	(4) $E_s = 3.5 q_c$ kg/cm ²	(5) z cm.	(6) I_z	(7) $\frac{I_z \cdot \Delta z}{E}$ cm. ³ /kg
1	100	25	87.5	50	0.290	0.33
2	150	35	122.5	175	0.515	0.63
3	50	35	122.5	275	0.628	0.26
4	50	70	245.0	325	0.585	0.12
5	100	30	105.0	400	0.520	0.50
6	50	85	297.5	475	0.455	0.08
7	150	170	595.0	575	0.368	0.09
8	100	60	210.0	700	0.260	0.12
9	100	100	350.0	800	0.173	0.5
10	150	40	140.0	900	0.087	0.9
$\sum \frac{I_z}{E} \Delta z$						2.27

Table F.2 Data for Settlement Estimate of Example F.2

(1) Layer No.	(2) Δz cm.	(3) q_c kg/cm ²	(4) $E_s = 3.5 q_c$ kg/cm ²	(5) z cm.	(6) I_z	(7) $\frac{I_z}{E} \Delta z$
1	50	25	87.5	50	0.28	0.15
2	100	50	175	100	0.35	0.19
3	100	125	437.5	200	0.50	0.11
4	50	75	262.5	275	0.62	0.12
5	50	150	525	325	0.67	0.06
6	50	150	525	375	0.60	0.06
7	80	200	700	440	0.57	0.07
$\sum \frac{I_z}{E} \Delta z$						0.76

Example F.2

The two bridge piers shown in Figure F.2.a are to be constructed at 1.5 m. below the groundwater table which coincides with the ground surface. The foundation soil consists of a layer of dense sand of thickness $H=4.8$ m. below the foundation level followed by another layer of rock. The Dutch cone bearing capacity is shown on the same figure. Determine the settlement to be expected ten years after construction.

Solution:

- 1 - Calculate the equivalent width B_e from Equation (4.1)

$$B_e = [2.2 - 0.2(S/B)]B$$

$$= (2.2 - \frac{0.2 \times 6}{2})2 = 3.2\text{m.}$$

- 2 - Consider the problem as if you have only one individual foundation of width $B_e=3.2\text{m}$ as shown in Figure F.2.b.

- 3 - Calculate the value of I_{zp} at depth B below the foundation level:

$$P_0 = 0.9 \times 1.5 = 1.35 \text{ t/m}^2 = 0.14 \text{ kg/cm}^2$$

$$\Delta P = 2.14 - 0.14 = 2.0 \text{ kg/cm}^2$$

$$P_{vp} = 0.9 \times 4.7 = 4.2 \text{ t/m}^2 = 0.42 \text{ kg/m}^2$$

$$I_{zp} = 0.5 + 0.1 \sqrt{\frac{2.0}{0.42}} = 0.69$$

- 4 - Follow the steps from 2 to 5 outlined in the previous example.

- 5 - Determine C_1

$$C_1 = 1 - 0.5 \frac{P_0}{\Delta P}$$

$$= 1 - \frac{0.5 \times 0.14}{2} = 0.96$$

FIGURE F.2.a TWO BRIDGE PIERS

FIGURE F.2.b EQUIVALENT FOUNDATION

FIGURE F.2 TWO BRIDGE PIERS ON A SHALLOW LAYER OF SAND ON TOP OF A RIGID LAYER

6 - Determine C_2

$$C_2 = 1 + 0.2 \log \frac{10}{0.1} = 1.4$$

7 - Calculate the settlement of an individual foundation (δ_s)
from Equation (E.5)

$$\delta_s = C_1 C_2 p \sum_0^{1.5B} e \frac{I_z}{E} \Delta z$$

$$= 0.96 \times 1.4 \times 2 \times 0.76 = 2.04 \text{ cm.}$$

8 - Calculate the settlement reduction factor (η_{sr}) to take
into account the influence of the rigid base

$$\eta_{sr} = \frac{2}{3} \left(\frac{H}{B} \right)^{2/3}$$

$$= \frac{2}{3} \left(\frac{4.8}{3.2} \right)^{2/3} = 87\%$$

9 - Now, the settlement (σ_q) of the interfering foundation
can be obtained as follows:

$$\sigma_q = \eta_{sr} \delta_s = 0.87 \times 2.04 = 1.78 \text{ cm.}$$

END

0	3	H	0	9	H	8	2
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FIN