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Time Domain Macromodels for High Speed Interconnects

by

Anestis Dounavis

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Engineering**

**Ottawa-Carleton Institute for Electrical Engineering,
Department of Electronics,
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January, 2000

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Studies and Research acceptance of the thesis.

**“Time Domain Macromodels for High Speed
Interconnects”**

submitted by Anestis Dounavis, B.Eng. in partial
fulfillment of the requirements for the degree
of Master of Engineering

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January 2000

Abstract

A passive closed form model for interconnect analysis is presented. The interconnect line parameters can be lossy, coupled and frequency dependent. The proposed model is suitable for inclusion in general circuit simulators such as SPICE and overcomes the mixed frequency/time simulation difficulties encountered during nonlinear transient analysis. The coefficients describing the discrete macromodel are computed a priori and analytically, using a closed form Padé approximation of an exponential matrix that describes Telegrapher's equations. The method offers an efficient means to discretize interconnects compared to the conventional lumped segmentation model, while preserving the passivity of the interconnect. In addition, the proposed algorithm is compatible with passive model reduction techniques based on Krylov space methods.

Acknowledgment

I wish to extend my appreciation and gratitude to Professor Michel Nakhla for his knowledge, experience and kindness. His invaluable advice during supervision made the work of this thesis less difficult and more enjoyable.

I would like to acknowledge and thank the following colleagues: X. Li and R. Achar for the many useful discussions we had about the interconnect model; R. Khazaka for answering many of my OPTEMID and FRAMEMAKER questions, as well as proof reading my thesis; I. Erdin for his IFFT program which was used in this thesis to verify the transient responses of linear circuits; R. Griffith, P. Kumar, N. Soveiko, E. Gad, M. Cannon and C. Oregan for their support when I needed help. I would also like to thank the students, professors and staff at the department of electronics for making Carleton an enjoyable experience.

My final thoughts are with my parents, Thomas and Catina, my brother Vasilios and my sister Anthoula. Without their encouragement and endless support, I would not have had the opportunity to complete this study.

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List of Symbols

$\mathbf{0}$	Zero matrix.
ψ	Number of coupled interconnects.
$\mathbf{a}$	Matrix formed by $(\mathbf{R} + s\mathbf{L})\mathbf{d}$.
$\text{adj}(\)$	Adjoint of matrix.
$\mathbf{b}$	Matrix formed by $(\mathbf{G} + s\mathbf{C})\mathbf{d}$.
$\mathbf{C}$	Per unit length capacitance matrix of transmission line.
$\mathbf{C}_0$	MNA matrix composed of memory elements.
$\mathbf{d}$	Length of transmission line.
$\det(\)$	Determinant of matrix.
$\mathbf{e}_0(t)$	MNA matrix composed of independent voltage and current sources.
$\mathbf{G}$	Per unit length conductance matrix of transmission line.
$\mathbf{G}_0$	MNA matrix composed of memoryless elements.
$\mathbf{I}(0, s), \mathbf{I}(d, s)$	Terminal current vectors of transmission lines.
$\mathbf{i}(x, t)$	Current vector.
$\mathbf{L}$	Per unit length inductance matrix of transmission line.

M	Order of $Q_{M,N}(Z)$ polynomial (everywhere except Chap. 2).
N	Order of $P_{N,M}(Z)$ polynomial (everywhere except Chap. 2).
$P_{N,M}(Z)$	Polynomial matrix of Padé expression given by the denominator.
$P_{N,M_{ij}}$	Polynomial matrix that forms $P_{N,M}(Z)$ (defined in (3.7)).
$[P_{n,n}(Z)]_i$	Polynomial matrix derived from the zeros of $P_{N,M}(Z)$.
$Q_{M,N}(Z)$	Polynomial matrix of Padé expression given by the numerator.
$Q_{M,N_{ij}}$	Polynomial matrix that forms $Q_{M,N}(Z)$ (defined in (3.7)).
$[Q_{n,n}(Z)]_i$	Polynomial matrix derived from the zeros of $Q_{M,N}(Z)$.
R	Per unit length resistance matrix of transmission line
s	Laplace variable ($j\omega$).
t	Time.
U	Unity matrix.
$V(0, s), V(d, s)$	Terminal voltage vectors of transmission lines.
$v(x, t)$	Voltage vector.
x	Position variable.
X_ϕ	MNA matrix composed of node voltages appended by independent voltage source currents and linear inductor currents.
$Y_{11}, Y_{12}, Y_{21}, Y_{22}$	Y-parameters.
Z	Matrix of transmission line parameters (defined in (3.3)).

Abbreviations

AWE	Asymptotic waveform evaluation.
CFH	Complex frequency hopping.
IFFT	Inverse fast fourrier transform.
MCM	Multichip modules.
MNA	Modified nodal admittance.
PCB	Printed circuit board.
RL	Resistor, inductor network.
RLC	Resistor, inductor, capacitor network.
TDR	Time domain reflectometry.
TEM	Transverse electromagnetic.

Chapter 1

Introduction

1.1 Background and Motivation

The phenomenal growth in density, operating speeds and complexity of modern integrated circuits has made interconnect analysis a requirement for all state-of-the-art circuit simulators. Interconnect effects such as ringing, signal delay, distortion, attenuation and crosstalk can severely degrade signal integrity [1]. As frequency increases, the length of the interconnects become a significant fraction of the operating wavelength and the geometries of the interconnects become more dominant in delaying and distorting signals. For many high speed electrical networks, overall performance may depend mostly on the delay of the interconnects rather than the delay of devices [2]. Consequently, circuit designers must consider interconnect analysis at early stages of the design cycle to ensure circuit performance and reliability.

Circuit simulators such as SPICE have difficulties modeling interconnects in the presence of nonlinear elements. This is due to the fact that distributed interconnects are usually characterized in the frequency domain, whereas nonlinear elements such as drivers and receivers are described in the time domain only. Approaches based on conventional lumped segmentation of interconnects provides a brute force solution to the problem of the mixed frequency/time simulation [3]. However, at high frequencies these methods lead to large circuit matrices, rendering the method inefficient.

Over the past twenty years, a variety of interconnect models have been proposed for nonlinear circuit simulation [3]-[24]. Their development were proposed to overcome two major difficulties: the modeling of interconnects in an efficient and accurate manner and the simulation of interconnects within the context of nonlinear elements. The accuracy and efficiency of these interconnect models are attributed to one or more of the following factors: the physical properties of the interconnects, the bandwidth of interest to specific transient simulation or the specific design application [20]. For example, the lumped segmentation model has difficulties representing interconnects whose electrical length is a significant fraction of the signal wavelength. Another example is the method of characteristics. It is ideal for lossless interconnects, however, the method has difficulty modeling lossy interconnects.

There are two modeling strategies for high speed interconnects. One approach is based on model reduction techniques that reduce large linear interconnect networks into smaller networks. The other is based on developing a macromodel for each interconnect.

Model reduction techniques such as asymptotic waveform evaluation (AWE) [9] and complex frequency hopping (CFH) [13], [16] were proposed as an efficient means to reduce large linear interconnect networks. These techniques are based on Padé synthesis. To

overcome the ill-conditioning associated with direct Padé synthesis. Krylov space methods were developed [14], [17], [18]. Efficient schemes based on multi-port/multi-point congruent transformation for passive reduction of large linear systems were also reported [17], [21]-[24]. Model reduction techniques are attractive in simulating large linear interconnect networks, due to the efficiency gains they are able to achieve over conventional modeling methods.

Macromodeling techniques attempt to represent each distributed transmission line as ordinary differential equations. The description of a passive general purpose interconnect model is the subject of this thesis. The goal here is to develop a compact model suitable for nonlinear simulation. This approach is extremely important for the analysis of electrical networks since it enables circuit designers to efficiently evaluate their design within the familiar framework of a circuit simulation environment. This enables the circuit simulator to view interconnects as regular circuit components, regardless of what the electrical parameters might be. The availability of a general passive interconnect model can become an important tool for the verification of other models, such as the ones obtained by model reduction techniques [9], [13], [16]-[18], [21]-[24]. In addition, accurate passive models are highly desirable by passive reduction techniques that require discretization of interconnects [17], [21], [22], [24].

1.2 Objective

An interconnect model based on a closed form Padé rational function approximation was introduced in [25], [26]. The method offered an efficient means to discretize distributed transmission lines compared to the conventional lumped segmentation model, while preserving the passivity of the macromodel.

The objective of this thesis is to extend the Padé interconnect model to the time domain by developing several techniques to convert the Padé rational function into ordinary differential equations. A time domain representation described by ordinary differential equations is necessary in order to make the Padé algorithm useful to nonlinear circuit simulators. The proposed techniques obtain the ordinary differential equations analytically, in terms of the per unit length parameters and predetermined constants given by the Padé approximation.

The next objective is to incorporate distributed transmission lines with frequency dependent parameters directly into the Padé macromodel, while preserving the macromodel's passivity. The proposed method uses positive real rational functions to represent the frequency dependent parameters and to preserve passivity.

1.3 Contributions

The main contributions made by this thesis are:

1. Several time domain representations are developed to convert the Padé rational functions to ordinary differential equations. The time domain macromodels are formulated analytically in terms of per unit length parameters and predetermined constants obtained by the Padé approximation [27], [28].
2. A technique to simulate interconnects with frequency dependent parameters is described for the Padé macromodel. The proposed method uses positive real rational functions to represent the frequency dependent parameters. Such structures can be proven mathematically to preserve passivity [28], [29]. A curve fitting technique is proposed to obtain positive real rational functions that match the 'measured data'

values of the frequency dependent parameters to the impedance of a passive circuit network.

3. In this thesis a new proof of passivity is presented for the Padé interconnect model. The proof demonstrates that the macromodel is passive when the order of the numerator and denominator of the Padé approximation are equal. In addition, a method of restoring the macromodel's passivity is described for cases when the order of the numerator differs from the denominator.
4. A proof is given that demonstrates that the Padé macromodel can be represented in a form compatible with passive model reduction techniques based on Krylov space methods [30], [31]. The proof shows that the modified nodal admittance (MNA) [32] matrices of the interconnect model are nonnegative definite. Nonnegative definite matrices are required by Krylov space techniques to preserve the passivity of the reduced network.

1.4 Organization of the Thesis

The organization of the thesis is as follows. Chapter 2 briefly discusses how distributed transmission lines are modeled and the difficulties the models cause to nonlinear circuit simulators. The chapter also presents several numerical strategies to simulate interconnects in the presence of nonlinear elements. In Chapter 3, several time domain representations are derived for the Padé interconnect model. Next a passivity proof for the interconnect model is presented. In addition, a proof demonstrating that the interconnect model is compatible with passive model reduction techniques based on Krylov space methods is given. Numerical examples are provided to demonstrate the validity and efficiency of the interconnect model. In Chapter 4, a technique to incorporate interconnects with frequency dependent parameters is presented. The conditions necessary to preserve

the passivity of the Padé macromodel are investigated. From these conditions a method of modeling frequency dependent parameters as rational functions is proposed and the time domain macromodel is derived. Numerical examples are given to demonstrate the validity of the proposed method. The conclusion and suggestions for further research are presented in Chapter 5.

Chapter 2

Interconnect Simulation

2.1 Introduction

The simulation of interconnects requires the selection of an appropriate model and a solution algorithm. The complexity of the model depends on the frequency of operation and the structure of the interconnect. Various interconnect models offer different challenges to circuit simulators. The goal of this chapter is to review some of the interconnect models and numerical techniques that are used for interconnect analysis.

2.2 Interconnect Models

Interconnects occur at various levels of the design hierarchy, such as on the chip, packaging structures, multichip modules (MCM), printed circuit board (PCB), backplane ca-

bles and PCB to backplane connectors, as shown in Figure 2.1. An example of the top cross-sectional view of an interconnect is shown in Figure 2.2. Interconnect models are based on quasi-transverse electromagnetic (TEM) or full wave assumptions. For interconnect structures that can not be modeled analytically, linear subnetworks characterized by measured data have been proposed.

Figure 2.1: Interconnect hierarchy

Figure 2.2: Top view and cross-sectional view of an interconnect system

2.2.1 Quasi-Transverse Electromagnetic Models

Transverse electromagnetic (TEM) waves exist for interconnects with homogeneous mediums and perfect conductors [33]. Under these conditions interconnects produce electric and magnetic fields that are transverse or perpendicular to one another and to the direction of propagation.

Interconnects with inhomogeneous mediums produce electromagnetic waves with many velocities. As well, interconnects that have imperfect conductors produce an electric field along the surface conductor. Such structures violate the TEM characteristics, since TEM waves propagate with only one velocity and have no electric field along the surface conductor. Nevertheless, for most practical cases, the resulting field structure is similar to the TEM structure. Such field structures can still be approximated by TEM waves, which are more precisely referred to as quasi-TEM waves [33].

Quasi-TEM assumptions remain the dominant trend for analyzing lossy multiconductor interconnect lines, since the approximation is valid for most practical interconnects

and offers relative ease and low CPU cost compared to full wave approaches [34]. The voltages and currents for the quasi-TEM distributed models are described by partial differential equations known as Telegrapher's equations. The simplest distributed model is the delay-line or lossless line. More complicated models include per unit length losses of the conductor and the dielectric substrate, as well as coupling between adjacent lines. Certain distributed models may even require nonuniform per unit length parameters to accurately model interconnects. The difficulty associated with quasi-TEM distributed models stems from the fact that they are formulated by partial differential equations, which are best described in the frequency domain. Nonlinear elements on the other hand are described in the time domain only as nonlinear ordinary differential equations. A method of representing the quasi-TEM distributed equations as ordinary differential equations must be developed to overcome the mixed frequency-time problem.

The simplest solution to the mixed frequency-time problem is to discretize the line using the conventional lumped segmentation model [3]. However, for high speed interconnects the lumped model introduces many variables making the method inefficient and in some cases non-convergent. In addition to the lumped segmentation model, there exists other more sophisticated algorithms such as the method of characteristics [7], [35] optimization techniques [15], [19] and Chebyshev polynomials [6], [20]. These methods are able to efficiently capture the frequency response of the interconnects as transfer functions that can be converted to ordinary differential equations. Even though these methods are efficient when compared to the lumped segmentation model, there are problems associated with these algorithms. For example, the method of characteristics is very vulnerable to numerical time domain stability problems caused by the delay of the interconnect in the integration formula. Optimization techniques require predefined configurations that are problem specific and are susceptible to convergence problems. The Chebyshev algorithms described in [6], [20] are not passive, therefore the stability of the overall network can not

be guaranteed when the macromodel is terminated to other passive elements. From the above discussion, it becomes obvious that a general and passive interconnect model that can provide broad-band accuracy without compromising efficiency is still a topic of intense research.

2.2.2 Full Wave Models

If the cross-sectional dimensions of interconnects become a significant fraction of the circuit's operating wavelength, the interconnect's field components in the direction of propagation can no longer be ignored [36]. Under these conditions, quasi-TEM assumptions become inadequate in describing interconnects and full wave models are required. These models are able to account for all possible field components and satisfy all boundary conditions required to accurately model the high frequency effects of interconnects.

Even though full wave models provide better accuracy when compared to quasi-TEM models, they are generally not used by circuit simulators due to the CPU requirements [37]. The cost of full wave analysis associated with each interconnect at a particular frequency point is extremely high. Usually, high speed interconnects require thousands of frequency points to accurately model the response. The cost of the evaluation of the full wave model combined with the computational cost of the overall circuit makes the method prohibitively expensive to simulate. Another problem associated with full wave methods is the manner in which the macromodel is represented in the circuit simulator. The information provided by wave full analysis is in terms of field parameters such as propagation constants, characteristic impedances, current eigenvectors, etc. Circuit simulators require information in terms of voltages, currents and impedances. An interface linking the two representations must be developed to incorporate full wave methods into circuit simulators.

2.2.3 Measured Subnetworks

For interconnects that have geometric inhomogeneity and associated discontinuities, it may not be possible to obtain accurate analytical models. To overcome this difficulty, modeling techniques based on measured data have been proposed [38], [39]. The interconnects are modeled by measured frequency dependent scattering parameters or by time domain terminal measurements. The time domain measurements can be obtained by time domain reflectometry (TDR) methods [40] or by numerical solution of the electromagnetic field problem [41], [42]. It should be mentioned that measured data obtained by these techniques are susceptible to noise contamination. To reduce the impact of noise large data sets are required, otherwise single sets of measurements may lead to inaccurate results.

2.3 Interconnect Simulation Issues

Quasi-TEM distributed models are described by Telegrapher's equations.

$$\frac{\partial}{\partial x} \mathbf{v}(x, t) = -\mathbf{R}\mathbf{i}(x, t) - \mathbf{L} \frac{\partial}{\partial t} \mathbf{i}(x, t) \quad (2.1a)$$

$$\frac{\partial}{\partial x} \mathbf{i}(x, t) = -\mathbf{G}\mathbf{v}(x, t) - \mathbf{C} \frac{\partial}{\partial t} \mathbf{v}(x, t) \quad (2.1b)$$

where $\mathbf{R} \in \mathbb{R}^{\Psi \times \Psi}$, $\mathbf{L} \in \mathbb{R}^{\Psi \times \Psi}$, $\mathbf{C} \in \mathbb{R}^{\Psi \times \Psi}$ and $\mathbf{G} \in \mathbb{R}^{\Psi \times \Psi}$ are the per-unit-length parameters of the interconnect; $\mathbf{v}(x, t) \in \mathbb{R}^{\Psi}$ and $\mathbf{i}(x, t) \in \mathbb{R}^{\Psi}$ represent the voltage and current vectors as a function of position x and time t ; Ψ is the number of coupled lines. The solution of (2.1) can be expressed in the frequency domain as,

$$\mathbf{A}(s)\mathbf{V}(s) + \mathbf{B}(s)\mathbf{I}(s) = \mathbf{0} \quad (2.2)$$

where $V(s)$ and $I(s)$ are the terminal voltages and currents; $A(s)$ and $B(s)$ are matrices described by the per unit length parameters. The difficulty with (2.2) is that it can not be expressed in the time domain as ordinary differential equations, which makes it difficult to include with nonlinear circuit simulators. In this section, several simulation strategies are examined that convert Telegrapher's equations into ordinary differential equations.

2.3.1 Lumped Segmentation

This method uses lumped equivalent circuits to approximate the partial differential equations. Applying Euler's method [33] to (2.1) yields

$$v_{\sigma+1}(t) - v_{\sigma}(t) = -\Delta x R i_{\sigma}(t) - \Delta x L \frac{\partial}{\partial t} i_{\sigma}(t) \quad (2.3a)$$

$$i_{\sigma+1}(t) - i_{\sigma}(t) = -\Delta x G v_{\sigma+1}(t) - \Delta x C \frac{\partial}{\partial t} v_{\sigma+1}(t) \quad (2.3b)$$

where $\sigma = [1, \dots, \eta]$, $\Delta x = d/\eta$ and η is the number of sections and d is the length of the interconnect. Equations (2.3) can be implemented by lumped equivalent circuits composed of resistors, inductors and capacitors. Figure 2.3 shows the general lumped component model for a single transmission line.

The lumped segmentation model provides a direct method to discretize interconnects, however the approximation is only valid if Δx is chosen to be a small fraction the wavelength. (i.e. $\ll \lambda = v/f$, where λ is the wavelength, v is the velocity, f is the frequency of interest). If the frequency of interest is high or if the interconnect is electrically long many lumped elements are required. This leads to large circuit matrices, rendering the method inefficient.

Figure 2.3: Lumped transmission line model

2.3.2 Method of Characteristics

The method of characteristics or Branin's method [35] is able to represent lossless interconnects as ordinary differential equations. The Laplace domain solution of (2.1) for the single reference transmission line [43] is.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{Z_0(1 - e^{-2\gamma d})} \begin{bmatrix} 1 + e^{-2\gamma d} & -2e^{-\gamma d} \\ -2e^{-\gamma d} & 1 + e^{-2\gamma d} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (2.4a)$$

$$\gamma = \sqrt{(R + sL)(G + sC)} \quad (2.4b)$$

$$Z_0 = \sqrt{\frac{(R + sL)}{(G + sC)}} \quad (2.4c)$$

where γ is the propagation constant and Z_0 is the characteristic impedance. If the interconnect is lossless, γ and Z_0 are reduced to

$$\gamma = s\sqrt{LC} \qquad Z_0 = \sqrt{\frac{L}{C}} \quad (2.5)$$

This restriction makes γ purely imaginary and Z_0 a real constant. The time domain representation of (2.4a) is obtained by taking the Laplace inverse which replaces $e^{-\gamma d}$ and $e^{-2\gamma d}$ with time shifts (or delays), expressed as

$$Z_0 \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} - \begin{bmatrix} i_1(t-2\gamma d) \\ i_2(t-2\gamma d) \end{bmatrix} = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} + \begin{bmatrix} v_1(t-2\gamma d) - 2v_2(t-\gamma d) \\ -2v_1(t-\gamma d) + v_2(t-2\gamma d) \end{bmatrix} \quad (2.6)$$

Equation (2.6) can easily be incorporated into nonlinear circuit simulators as the stamp of the lossless transmission line. The difficulty with this algorithm stems from the delay, which effects the numerical stability of the integration formula. This causes the step size of the integration formula to decrease, thus increasing CPU time. The method of characteristics is most practical for lossless lines since the time domain solution for lossy interconnects can not be directly obtained. Numerical techniques to incorporate lossy and coupled lines have been proposed for the method of characteristics [7], however, these methods are also susceptible to numerical stability problems caused by the delay of the interconnect.

2.3.3 Least Square Optimization

Least square optimization obtains transfer functions for Telegrapher's equations that can be converted to ordinary differential equations. The frequency response of interconnects can be approximated by rational functions, written as

$$\tilde{H}(s) = H_\infty + \sum_{m=0}^M \frac{\tilde{r}_m}{1 + s/\tilde{p}_m} \quad (2.7)$$

where H_∞ , $\tilde{r}_m$ and $\tilde{p}_m$ represent the quotient, residues and poles, respectively. The least square optimization method fits data samples to the complex rational function at arbitrary

frequency points $[0, \omega_1, \dots, \omega_K]$. The following steps were proposed in [15], [19] to solve (2.7). To obtain poles in the left half plane, the real part of (2.7) (i.e. the even part of $\tilde{H}(s)$) is fitted to the real data samples. Let the real part of (2.7) be defined as

$$Re(\tilde{H}(s)) = \frac{\sum_m^M c_m(\omega^2)^m}{\sum_m^M b_m(\omega^2)^m} \quad (2.8)$$

The solution of (2.8) is obtained by solving the following system of linear equations

$$\overbrace{\begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & \omega_1^2 & \dots & \omega_1^{2M} \\ \vdots & \vdots & & \vdots \\ 1 & \omega_K^2 & \dots & \omega_K^{2M} \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} 0 & \dots & 0 \\ -\omega_1^2 Re(H(\omega_1)) & \dots & -\omega_1^{2M} Re(H(\omega_1)) \\ \vdots & & \vdots \\ -\omega_K^2 Re(H(\omega_K)) & \dots & -\omega_K^{2M} Re(H(\omega_K)) \end{bmatrix}}^{\mathbf{X}} = \overbrace{\begin{bmatrix} Re(H(0)) \\ Re(H(\omega_1)) \\ \vdots \\ Re(H(\omega_K)) \end{bmatrix}}^{\mathbf{B}} \quad (2.9)$$

where the least square solution of (2.9) is

$$\mathbf{A}'\mathbf{A}\mathbf{x} = \mathbf{A}'\mathbf{B} \quad (2.10)$$

Since (2.8) is an even function, the poles obtained belong to $\tilde{H}(s)$ and $\tilde{H}(-s)$, where the left hand poles belong to $\tilde{H}(s)$. This ensures that no unstable poles are produced for $\tilde{H}(s)$. If any poles are purely imaginary, they are rejected. The next step is to find the quotient and residues of (2.7). This is done by matching the real and imaginary parts of (2.7) with the sampled data, written as

$$\begin{array}{c} \text{A} \\ \left[\begin{array}{cccc} 1 & 1 & \dots & 1 \\ 1 & \frac{1}{1 + (\omega_1/\tilde{p}_0)^2} & \dots & \frac{1}{1 + (\omega_1/\tilde{p}_M)^2} \\ \vdots & \vdots & & \vdots \\ 1 & \frac{1}{1 + (\omega_K/\tilde{p}_0)^2} & \dots & \frac{1}{1 + (\omega_K/\tilde{p}_M)^2} \\ 0 & \frac{-\omega_1/\tilde{p}_0}{1 + (\omega_1/\tilde{p}_0)^2} & \dots & \frac{-\omega_1/\tilde{p}_M}{1 + (\omega_1/\tilde{p}_M)^2} \\ \vdots & \vdots & & \vdots \\ 0 & \frac{-\omega_K/\tilde{p}_0}{1 + (\omega_K/\tilde{p}_0)^2} & \dots & \frac{-\omega_K/\tilde{p}_M}{1 + (\omega_K/\tilde{p}_M)^2} \end{array} \right] \end{array} \begin{array}{c} \text{X} \\ \left[\begin{array}{c} H_\infty \\ r_0 \\ r_1 \\ \vdots \\ r_m \end{array} \right] \end{array} = \begin{array}{c} \text{B} \\ \left[\begin{array}{c} Re(H(0)) \\ Re(H(\omega_1)) \\ \vdots \\ Re(H(\omega_K)) \\ Im(H(\omega_1)) \\ \vdots \\ Im(H(\omega_1)) \end{array} \right] \end{array} \quad (2.11)$$

where the solution of (2.11) is obtained by solving (2.10). Once the coefficients of (2.7) are determined, the rational function can be converted into ordinary differential equations to obtain the time domain stamp of the transmission line.

Least square optimization provides great flexibility in modeling all types of interconnect configurations. However the solution of (2.7) is ill-conditioned. This makes the method very susceptible to convergence problems. In addition the algorithm is not passive, therefore the stability of the overall network can not be guaranteed when the macromodel is terminated to other passive elements.

2.3.4 Chebyshev Polynomials

Algorithms based on Chebyshev polynomials have been proposed to convert Telegrapher's equations into ordinary differential equations [6], [20]. For the single transmission line case, the variations in space for the voltage $v(x, t)$ and current $i(x, t)$ are expressed as

$$v(x, t) = \sum_{n=0}^N a_n(t) T_n(x) \quad (2.12a)$$

$$i(x, t) = \sum_{n=0}^N b_n(t) T_n(x) \quad (2.12b)$$

where $T_n(x)$ is the n th degree Chebyshev polynomial; $a_n(t)$ and $b_n(t)$ are the unknown variables. The derivatives of $v(x, t)$ and $i(x, t)$ with respect to x are also expanded as Chebyshev polynomials

$$\frac{\partial}{\partial x} v(x, t) = \sum_{n=0}^N \hat{a}_n(t) T_n(x) \quad (2.13a)$$

$$\frac{\partial}{\partial x} i(x, t) = \sum_{n=0}^N \hat{b}_n(t) T_n(x) \quad (2.13b)$$

where $a_n(t)$, $\hat{a}_n(t)$, $b_n(t)$ and $\hat{b}_n(t)$ have the following relations [6],

$$a_n(t) = \frac{1}{2n} (\hat{a}_{n-1}(t) - \hat{a}_{n+1}(t)) \quad (2.14a)$$

$$b_n(t) = \frac{1}{2n} (\hat{b}_{n-1}(t) - \hat{b}_{n+1}(t)) \quad (2.14b)$$

Inserting (2.12) and (2.13) into (2.1) yields

$$\sum_{n=0}^N \hat{a}_n(t) T_n(x) = -R \sum_{n=0}^N b_n(t) T_n(x) - L \sum_{n=0}^N \frac{\partial}{\partial t} b_n(t) T_n(x) \quad (2.15a)$$

$$\sum_{n=0}^N \hat{b}_n(t) T_n(x) = -C \sum_{n=0}^N a_n(t) T_n(x) - G \sum_{n=0}^N \frac{\partial}{\partial t} a_n(t) T_n(x) \quad (2.15b)$$

Equations (2.15) can be expressed as

$$\mathbf{C}_{Cheb} \dot{\mathbf{X}}_{Cheb}(t) + \mathbf{G}_{Cheb} \mathbf{X}_{Cheb}(t) = \mathbf{0} \quad (2.16)$$

where $\mathbf{X}_{Cheb}(t)$ are the unknown variables of the Chebyshev model; $\mathbf{C}_{Cheb}$ and $\mathbf{G}_{Cheb}$ are matrices constructed from the equations in (2.15). The algorithm can also be applied for interconnects with non-uniform line parameters by expanding the line parameters as Chebyshev polynomials with respect to position x [6].

The Chebyshev models are able to achieve better accuracy with fewer variables when compared to the lumped segmentation model. However, the algorithms described in [6], [20] are not passive and have difficulty preserving the stability of the overall network when connected to other passive elements.

Chapter 3

Development of a New Time Domain Interconnect Model

3.1 Introduction

Transient analysis of lossy coupled distributed interconnects in the presence of nonlinear elements has become an important factor in designing high-speed electronic circuits. In this chapter, a new passive and efficient interconnect model is introduced. The interconnect model is based on a closed-form Padé rational function approximation that replaces the exponential matrix derived from Telegrapher's equation. Once the exponential matrix is expressed as rational functions, the time domain macromodel can be represented as ordinary differential equations.

The development of the new interconnect model is briefly reviewed in section 3.2.

Next, section 3.3 presents several alternative techniques to obtain the time domain macro-model from the Padé approximation. A new passivity proof is given for the interconnect model in section 3.4. Section 3.5 provides an error criteria for selecting the order of the approximation. Numerical examples are presented in section 3.6 to demonstrate the validity of the proposed method and to illustrate its application to a variety of interconnect structures.

3.2 Development of the Interconnect Model

Interconnects are described by a set of partial differential equations (2.1), which can be written in the frequency domain as

$$\frac{\partial}{\partial x} V(x, s) = -(R + sL)I(x, s) \quad (3.1a)$$

$$\frac{\partial}{\partial x} I(x, s) = -(G + sC)V(x, s) \quad (3.1b)$$

The solution of (3.1) can be expressed as an exponential matrix function

$$\begin{bmatrix} V(d, s) \\ I(d, s) \end{bmatrix} = e^z \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} \quad (3.2)$$

where

$$Z = (D + sE)d; \quad D = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix}; \quad E = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix} \quad (3.3)$$

V, I represent the Laplace-domain terminal voltage and current vectors of the multiconductor transmission line and d is the length of the line.

Equation (3.2) does not have a direct representation in the time-domain, which makes it difficult to interface with nonlinear simulators. The objective of the proposed algorithm, is to represent (3.2) as ordinary differential equations, while retaining the passivity of the structure. The matrix exponential of e^Z can be approximated using matrix rational function as

$$\mathbf{P}_{N,M}(\mathbf{Z})e^Z \approx \mathbf{Q}_{M,N}(\mathbf{Z}) \quad (3.4)$$

where $\mathbf{P}_{N,M}(\mathbf{Z})$, $\mathbf{Q}_{M,N}(\mathbf{Z})$ are polynomial matrices representing the numerator and denominator of the Padé rational functions. The coefficients of $\mathbf{P}_{N,M}(\mathbf{Z})$ and $\mathbf{Q}_{M,N}(\mathbf{Z})$ are expressed as [32]

$$\mathbf{P}_{N,M}(\mathbf{Z}) = \sum_{i=0}^N \frac{(M+N-i)!N!}{(M+N)!i!(N-i)!} (-\mathbf{Z})^i \quad (3.5a)$$

$$\mathbf{Q}_{M,N}(\mathbf{Z}) = \sum_{i=0}^M \frac{(M+N-i)!M!}{(M+N)!i!(M-i)!} \mathbf{Z}^i \quad (3.5b)$$

It is to be noted that the coefficients of (3.5) have a closed form relation. This provides immense advantage for the proposed algorithm over previous techniques which rely on some form of numerical optimization. The matrix rational function of (3.4) can also be represented as

$$\begin{bmatrix} \mathbf{P}_{N,M_{11}} & \mathbf{P}_{N,M_{12}} \\ \mathbf{P}_{N,M_{21}} & \mathbf{P}_{N,M_{22}} \end{bmatrix} e^Z \equiv \begin{bmatrix} \mathbf{Q}_{M,N_{11}} & \mathbf{Q}_{M,N_{12}} \\ \mathbf{Q}_{M,N_{21}} & \mathbf{Q}_{M,N_{22}} \end{bmatrix} \quad (3.6)$$

where the coefficients of (3.6) are expressed as

$$\mathbf{P}_{N,M_{11}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 + (-1)^i) (\mathbf{a}\mathbf{b})^{i/2} \right] \quad (3.7a)$$

$$P_{N, M_{12}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 - (-1)^i) (ab)^{\frac{i-1}{2}} a \right] \quad (3.7b)$$

$$P_{N, M_{21}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 - (-1)^i) (ba)^{\frac{i-1}{2}} b \right] \quad (3.7c)$$

$$P_{N, M_{22}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 + (-1)^i) (ba)^{i/2} \right] \quad (3.7d)$$

$$Q_{M, N_{11}} = \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 + (-1)^i) (ab)^{i/2} \right] \quad (3.7e)$$

$$Q_{M, N_{12}} = - \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 - (-1)^i) (ab)^{\frac{i-1}{2}} a \right] \quad (3.7f)$$

$$Q_{M, N_{21}} = - \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 - (-1)^i) (ba)^{\frac{i-1}{2}} b \right] \quad (3.7g)$$

$$Q_{M, N_{22}} = \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 + (-1)^i) (ba)^{i/2} \right] \quad (3.7h)$$

where $a = (R + sL)d$ and $b = (G + sC)d$. Alternatively, there also exist several recursive relationships for $P_{N, M}(Z)$ and $Q_{M, N}(Z)$ defined as

$$P_{N, M}(Z) = P_{N-1, M}(Z) + Z \left(\frac{-M}{(M+N)(M+N-1)} \right) P_{N-1, M-1}(Z) \quad (3.8a)$$

$$P_{N, M}(Z) = P_{N, M-1}(Z) + Z \left(\frac{N}{(M+N)(M+N-1)} \right) P_{N-1, M-1}(Z) \quad (3.8b)$$

$$N \geq 1, M \geq 1$$

$$Q_{M, N}(Z) = Q_{M-1, N}(Z) + Z \left(\frac{N}{(M+N)(M+N-1)} \right) Q_{M-1, N-1}(Z) \quad (3.8c)$$

$$\mathcal{Q}_{M,N}(\mathbf{Z}) = \mathcal{Q}_{M,N-1}(\mathbf{Z}) + \mathbf{Z} \left(\frac{-M}{(M+N)(M+N-1)} \right) \mathcal{Q}_{M-1,N-1}(\mathbf{Z}) \quad (3.8d)$$

The advantages of the proposed technique is that it has a closed form relation. As a result the method does not suffer from any ill-conditioning problems for obtaining the Padé rational functions [13] and requires no optimization or numerical techniques. Once the exponential matrix of (3.2) is expressed as rational functions, ordinary differential equations can be obtained.

3.3 Development of the Time Domain Macromodel

In order to make the proposed interconnect model useful to nonlinear circuit simulators, the model must be expressed as ordinary differential equations. In this section several alternative techniques to obtain ordinary differential equations from the Padé approximation of (3.4) are presented.

There are many methods of expressing rational functions as ordinary differential equations. Usually differential equations are written in either the controllable canonical, observable canonical or Jordan form [44], [45]. The time domain macromodel presented here converts (3.6) into the Y-parameters and the equations are expressed in the Jordan form. The Jordan form is chosen because great insight is provided in the location of the poles and it is suitable for proving stability. In addition to the Jordan form, ordinary differential equations can be obtained by using the polynomial coefficients of (3.6) directly. The advantages of this method when compared to the Jordan form is that the macromodel is realized with fewer variables for multiconductor interconnects. Alternatively, a third approach expresses the polynomials as product of factors formed by the pole-zero pairs of the Padé function. Such a formulation allows the model to be viewed as a series of subcircuits. Using this formulation, a method of representing each subcircuit in terms of resistors, capacitors and inductors is presented.

3.3.1 Time Domain Realization in Jordan Form

The time domain macromodel is obtained by converting (3.6) into the Y-parameters. From the Y-parameters, the rational functions are represented in terms of quotients, poles and residues calculated analytically in terms of per unit length parameters and predetermined constants given by the Padé macromodel. The discussion will focus on how to obtain the predetermined constants and how to formulate the differential equations.

For the sake of simplicity of presentation, the discussion will pertain to single transmission lines and then extend to multiconductor transmission lines. The Y-parameters for a single ground reference line are defined as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (3.9)$$

Converting (3.6) to the Y-parameters yields

$$Y_{11} = \frac{Q_{M,N_{11}}P_{N,M_{22}} - Q_{M,N_{21}}P_{N,M_{12}}}{Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}} \quad (3.10a)$$

$$Y_{12} = \frac{P_{N,M_{21}}P_{N,M_{12}} - P_{N,M_{11}}P_{N,M_{22}}}{Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}} \quad (3.10b)$$

$$Y_{21} = \frac{Q_{M,N_{11}}Q_{M,N_{22}} - Q_{M,N_{21}}Q_{M,N_{12}}}{Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}} \quad (3.10c)$$

$$Y_{22} = \frac{P_{N,M_{21}}Q_{M,N_{12}} - Q_{M,N_{22}}P_{N,M_{11}}}{Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}} \quad (3.10d)$$

For the case when $M=N=n$, the rational functions of (3.10) can be expressed as

$$Y_{11} = Y_{22} = K_{11}b + \frac{\sum_{i=0}^{n-1} \gamma_i (ab)^i}{a \left(\sum_{i=0}^{n-1} \mu_i (ab)^i \right)} \quad (3.11a)$$

$$Y_{12} = Y_{21} = K_{12}b + \frac{\sum_{i=0}^{n-1} \delta_i (ab)^i}{a \left(\sum_{i=0}^{n-1} \mu_i (ab)^i \right)} \quad (3.11b)$$

where γ_i , δ_i , μ_i , K_{11} and K_{12} are predetermined constants obtained by the Padé macro-model. The quotients of (3.11) satisfy the relationship

$$K_{11}b = K_{11}Cs + K_{11}G \quad (3.12a)$$

$$K_{12}b = K_{12}Cs + K_{12}G \quad (3.12b)$$

To obtain the poles of the macromodel, the denominator of (3.11) is represented as a product of factors,

$$a \left(\sum_{i=0}^{n-1} \mu_i (ab)^i \right) = a \prod_{i=1}^{n-1} (ab + \zeta_i) \quad (3.13)$$

where ζ_i are the poles of ab . The poles of the macromodel can then be written as

$$p_0 = \frac{-R}{L} \quad p_i = \frac{-(LG + CR)}{2LC} \pm \frac{\sqrt{(LG + CR)^2 + 4LC(RG + \zeta_i)}}{2LC} \quad (3.14)$$

where p_0 represents the pole of $a = sL + R$ and p_i represent the poles of $ab + \zeta_i = (sL + R)(sC + G) + \zeta_i$. The residues of the poles are calculated by partial fraction expansion, expressed as

$$r_{11_i} = (Y_{11} - K_{11}b)(s + p_i)|_{s=-p_i} \quad (3.15a)$$

$$r_{12_i} = (Y_{12} - K_{12}b)(s + p_i)|_{s=-p_i} \quad (3.15b)$$

Equation (3.15) can be reduced to

$$r_{11_0} = \frac{1}{L} \quad r_{11_i} = \eta_{11_i} \frac{(LG - CR)}{L} \frac{1}{\sqrt{(LG + CR)^2 - 4LC(RG + \zeta_i)}} \quad (3.16a)$$

$$r_{12_0} = \frac{-1}{L} \quad r_{12_i} = \eta_{12_i} \frac{(LG - CR)}{L} \frac{1}{\sqrt{(LG + CR)^2 - 4LC(RG + \zeta_i)}} \quad (3.16b)$$

where η_{11_i} and η_{12_i} are predetermined constants, obtained from

$$\eta_{11_i} = \frac{\sum_{k=0}^{n-1} \gamma_k (-\zeta_i)^k}{2\mu_{N-1} \zeta_i \prod_{\substack{k=1 \\ k \neq i}}^{n-1} (\zeta_k - \zeta_i)} \quad (3.17a)$$

$$\eta_{12_i} = \frac{\sum_{k=0}^{n-1} \delta_k (-\zeta_i)^k}{2\mu_{N-1} \zeta_i \prod_{\substack{k=1 \\ k \neq i}}^{n-1} (\zeta_k - \zeta_i)} \quad (3.17b)$$

A method of calculating the quotients, poles and residues in terms of predetermined constants and per unit length parameters has been developed. Table 3.1 lists the predetermined constants of the macromodel for various orders.

Order	K_{11}	K_{12}	ζ_i	$\eta_{11,i}$	$\eta_{12,i}$
1/1	1/12	-1/12			
2/2	1/24	1/24	12.000000	1.500000	1.500000
3/3	1/40	-1/40	60.000000 10.000000	2.500000 1.041667	-2.500000 1.041667
4/4	1/60	1/60	170.12490 42.000000 9.875098	3.997662 1.225000 1.002338	3.997662 -1.225000 1.002338
5/5	1/84	-1/84	380.23513 102.13025 39.764868 9.869750	5.967175 1.566589 1.032824 1.000077	-5.967175 1.566589 -1.032824 1.000077

Table 3.1: Lists of predetermined constants obtained by the Padé macromodel

Once quotients, poles and residues are calculated, the differential equations can be expressed as

$$\begin{bmatrix} \dot{X}_0 \\ \dot{X}_1 \\ \vdots \\ \dot{X}_{N-1} \end{bmatrix} = \begin{bmatrix} P_0 & 0 & \dots & 0 \\ 0 & P_1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & P_{N-1} \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} + \begin{bmatrix} B_0 \\ B_1 \\ \vdots \\ B_{N-1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (3.18)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} C_0 & C_1 & \dots & C_{N-1} \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} + \begin{bmatrix} CK_{11} & CK_{12} \\ CK_{12} & CK_{11} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} + \begin{bmatrix} GK_{11} & GK_{12} \\ GK_{12} & GK_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where

$$\begin{aligned}
P_0 &= \begin{bmatrix} p_0 & 0 \\ 0 & p_0 \end{bmatrix} & P_i &= \begin{bmatrix} \operatorname{Re}(p_i) & 0 & \operatorname{Im}(p_i) & 0 \\ 0 & \operatorname{Re}(p_i) & 0 & \operatorname{Im}(p_i) \\ -\operatorname{Im}(p_i) & 0 & \operatorname{Re}(p_i) & 0 \\ 0 & -\operatorname{Im}(p_i) & 0 & \operatorname{Re}(p_i) \end{bmatrix} \\
B_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & B_i &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
C_0 &= \begin{bmatrix} r_{11_0} & r_{12_0} \\ r_{12_0} & r_{11_0} \end{bmatrix} & C_i &= \begin{bmatrix} \operatorname{Re}(r_{11_i}) & \operatorname{Re}(r_{12_i}) & \operatorname{Im}(r_{11_i}) & \operatorname{Im}(r_{12_i}) \\ \operatorname{Re}(r_{12_i}) & \operatorname{Re}(r_{11_i}) & \operatorname{Im}(r_{12_i}) & \operatorname{Im}(r_{11_i}) \end{bmatrix} \\
X_0 &= [X_{0_1} \ X_{0_2}]^T & X_i &= [X_{i_1} \ X_{i_2} \ X_{i_3} \ X_{i_4}]^T \quad i = [1 \ \dots \ N-1]
\end{aligned} \tag{3.19}$$

It should be mentioned that the analytical solution for calculating the poles of the Y-parameters suggests that the macromodel is stable. Since $\xi > 0$ (table 3.1), the poles are located in the left half plane and adjacent to the imaginary axis as illustrated by Figure 3.1. Section 3.4 will show that the proposed macromodel is not only stable but also passive.

Figure 3.1: Location of the poles given by the Padé macromodel for the Y-parameters

The time domain macromodel for coupled interconnects can also be represented in the Jordan form. However, since the per unit length parameters are matrices the calculations of the poles and residues are slightly more involved. The quotient, pole and residue matrices of the coupled lines are calculated in a similar manner to that of the single line, expressed as,

$$\text{Quotients} \Rightarrow \begin{array}{c} K_{11}Cs + K_{11}G \\ K_{12}Cs + K_{12}G \end{array}$$

$$\text{Poles} \Rightarrow p_0 = -RL^{-1}$$

$$p_i = -\frac{1}{2}(LG + CR)(LC)^{-1} \pm \frac{1}{2}((LG + CR)^2 + 4LC(RG + \zeta_i U))^{\frac{1}{2}}(LC)^{-1} \quad (3.20)$$

$$\text{Residues} \Rightarrow r_{11_0} = L^{-1}$$

$$r_{11_i} = \eta_{11_i}(LG - CR)L^{-1}((LG + CR)^2 - 4LC(RG + \zeta_i U))^{-\frac{1}{2}}$$

$$r_{12_0} = -L^{-1}$$

$$r_{12_i} = \eta_{12_i}(LG - CR)L^{-1}((LG + CR)^2 - 4LC(RG + \zeta_i U))^{-\frac{1}{2}}$$

where the square root of the matrix is calculated in terms of eigenvalue/eigenvector analysis and U is the unity matrix. To represent the differential equations as (3.18), inversions of polynomial matrices must be performed for each pole and residue, written as

$$r_{11_i}(sU - p_i)^{-1} = \frac{r_{11_i} \text{adj}(sU - p_i)}{\det(sU - p_i)} \quad (3.21a)$$

$$r_{12_i}(sU - p_i)^{-1} = \frac{r_{12_i} \text{adj}(sU - p_i)}{\det(sU - p_i)} \quad (3.21b)$$

The rational function matrices of (3.21) need to be expressed in terms of poles and residues. Let the poles and residues of (3.21) be defined as p_i^j , r_{11}^j and r_{12}^j , where p_i^j represents the poles, r_{11}^j , r_{12}^j represent the residues and $j = [1 \dots \psi]$. The form of the differential equations for the matrix case is then described as

$$\begin{bmatrix} \dot{X}_0 \\ \dot{X}_1 \\ \vdots \\ \dot{X}_{N-1} \end{bmatrix} = \begin{bmatrix} P_0 & 0 & \dots & 0 \\ 0 & P_1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & P_{N-1} \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} + \begin{bmatrix} B_0 \\ B_1 \\ \vdots \\ B_{N-1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (3.22)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} C_0 & C_1 & \dots & C_{N-1} \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} + \begin{bmatrix} CK_{11} & CK_{12} \\ CK_{12} & CK_{11} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} + \begin{bmatrix} GK_{11} & GK_{12} \\ GK_{12} & GK_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The variables of (3.22) are defined as

$$P_i = \begin{bmatrix} P_i^1 & 0 & \dots & 0 \\ 0 & P_i^2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & P_i^\psi \end{bmatrix} \quad B_0 = [U]_{\psi \times \psi} \quad (3.23)$$

$$C_i = \begin{bmatrix} C_i^1 & C_i^2 & \dots & C_i^\psi \end{bmatrix} \quad B_i = \begin{bmatrix} 2U \\ 0 \end{bmatrix}_{\psi \times 2\psi}$$

$$X_i = \begin{bmatrix} X_i^1 & X_i^2 & \dots & X_i^\psi \end{bmatrix}^i$$

where

$$P_0^j = \begin{bmatrix} p_i^j & 0 & \dots & 0 \\ 0 & p_i^j & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & p_i^j \end{bmatrix}_{2\psi \times 2\psi} \quad C_0^j = \begin{bmatrix} r_{11}^j & r_{12}^j \\ r_{12}^j & r_{11}^j \end{bmatrix}_{2\psi \times 2\psi} \quad (3.24)$$

$$X_0^j = [X_{i_1}^j \ X_{i_2}^j \ \dots \ X_{i_{2\psi}}^j]^t$$

for the real poles, and

$$P_i^j = \begin{bmatrix} Re(p_i^j) & 0 & Im(p_i^j) & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & Re(p_i^j) & 0 & Im(p_i^j) & \dots & 0 & 0 & 0 & 0 \\ -Im(p_i^j) & 0 & Re(p_i^j) & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & -Im(p_i^j) & 0 & Re(p_i^j) & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & Re(p_i^j) & 0 & Im(p_i^j) & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & Re(p_i^j) & 0 & Im(p_i^j) \\ 0 & 0 & 0 & 0 & \dots & -Im(p_i^j) & 0 & Re(p_i^j) & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -Im(p_i^j) & 0 & Re(p_i^j) \end{bmatrix}_{4\psi \times 4\psi} \quad (3.25)$$

$$C_i^j = \begin{bmatrix} Re(r_{11}^j) & Re(r_{12}^j) & Im(r_{11}^j) & Im(r_{12}^j) \\ Re(r_{12}^j) & Re(r_{11}^j) & Im(r_{12}^j) & Im(r_{11}^j) \end{bmatrix}_{4\psi \times 2\psi} \quad X_i^j = [X_{i_1}^j \ X_{i_2}^j \ \dots \ X_{i_{4\psi}}^j]^t$$

for the complex poles.

The advantages of representing the time domain macromodel in the Jordan form is that great insight is provided in the location of the poles and it is easy to show that the macromodel is stable. However, for the case of coupled lines inversions of polynomial matrices are required. This increases the complexity of the formulation. By avoiding inversions of

polynomial matrices, the complexity of the formulation is reduced and the macromodel can be realized with fewer equations. This will be shown in the next section. The number of new variables required to implement the macromodel in the Jordan form is

$$2\psi^2(2n - 1) + 2\psi \quad (3.26)$$

3.3.2 Time Domain Realization of Padé Function

Representing the time domain macromodel in the Jordan form requires inversions of polynomial matrices to obtain ordinary differential equations for coupled interconnects. The method described in this section uses the coefficients of (3.6) directly to acquire the time domain macromodel. For the case when $N=M=n$, the polynomials of (3.6) can be expressed as,

$$\begin{bmatrix} P_{N,M_{11}} & P_{N,M_{12}} \\ P_{N,M_{21}} & P_{N,M_{22}} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^n F_i s^i & \sum_{i=0}^n H_i s^i \\ \sum_{i=0}^n J_i s^i & \sum_{i=0}^n F_i s^i \end{bmatrix} \quad (3.27a)$$

$$\begin{bmatrix} Q_{M,N_{11}} & Q_{M,N_{12}} \\ Q_{M,N_{21}} & Q_{M,N_{22}} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^n F_i s^i & -\sum_{i=0}^n H_i s^i \\ -\sum_{i=0}^n J_i s^i & \sum_{i=0}^n F_i s^i \end{bmatrix} \quad (3.27b)$$

Using (3.2) and (3.27), the time domain macromodel of the interconnect becomes

$$\begin{bmatrix} [p_a] & [q_a] \end{bmatrix} \begin{bmatrix} \dot{x}_2 \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} [p_b] & [q_b] \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = 0 \quad (3.28)$$

The matrices of (3.28) are defined as

$$\begin{aligned}
 [p_a] &= \begin{bmatrix} U_X \\ 0 \\ M \end{bmatrix} & [p_b] &= \begin{bmatrix} U_Y \\ 0 \\ S \end{bmatrix} \\
 [q_a] &= \begin{bmatrix} 0 \\ U_X \\ N \end{bmatrix} & [q_b] &= \begin{bmatrix} 0 \\ U_Y \\ T \end{bmatrix} \\
 x_2 &= [v_2 \ i_2 \ \gamma_2^0 \ \rho_2^0 \ \dots \ \gamma_2^{N-1} \ \rho_2^{N-1}]' & x_1 &= [v_1 \ i_1 \ \gamma_1^0 \ \rho_1^0 \ \dots \ \gamma_1^{N-1} \ \rho_1^{N-1}]'
 \end{aligned} \tag{3.29}$$

where $[\gamma_1^0 \ \rho_1^0 \ \dots \ \gamma_1^{N-1} \ \rho_1^{N-1}]$ and $[\gamma_2^0 \ \rho_2^0 \ \dots \ \gamma_2^{N-1} \ \rho_2^{N-1}]$ are extra variables needed for the realization. The variables M, N, S, T, U_X and U_Y are defined as

$$M = \begin{bmatrix} F_1 & H_1 & F_2 & H_2 & \dots & F_{n-1} & H_{n-1} & F_n & H_n \\ J_1 & F_1 & J_2 & F_2 & \dots & J_{n-1} & F_{n-1} & J_n & F_n \end{bmatrix} \tag{3.30a}$$

$$N = \begin{bmatrix} -F_1 & H_1 & -F_2 & H_2 & \dots & -F_{n-1} & H_{n-1} & -F_n & H_n \\ J_1 & -F_1 & J_2 & -F_2 & \dots & J_{n-1} & -F_{n-1} & J_n & -F_n \end{bmatrix} \tag{3.30b}$$

$$S = \begin{bmatrix} F_0 & H_0 & 0 & 0 & \dots & 0 & 0 \\ J_0 & F_0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{3.30c}$$

$$T = \begin{bmatrix} -F_0 & H_0 & 0 & 0 & \dots & 0 & 0 \\ J_0 & -F_0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{3.30d}$$

$$U_x = \begin{bmatrix} U & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & U & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & U & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & U & 0 & 0 \end{bmatrix} \quad (3.30e)$$

$$U_y = \begin{bmatrix} 0 & 0 & -U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -U & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -U & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & -U & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & -U \end{bmatrix} \quad (3.30f)$$

The time domain macromodel described by (3.28) is fairly simple to implement since the polynomial coefficients of (3.27) are directly stenciled into differential equations. In addition, the method allows for the realization of the macromodel with fewer variables when compared with the Jordan form for coupled interconnects. The number of variables required to realize (3.28) is

$$2\psi(2n - 1) \quad (3.31)$$

3.3.3 Time Domain Realization of Padé Function as Subsections

In the previous section, the polynomial coefficients of the Padé approximation were used directly to obtain the time domain macromodel. Alternatively, (3.4) can be represented as a product of lower order polynomials derived from the poles and zeros of the Padé function [27], [28]. This method allows the macromodel to be viewed as a series of subcircuits, where each subcircuit is derived from per unit length parameters and predetermined

constants given by the Padé approximation.

The polynomial matrices of (3.4) can be represented in terms of poles and zeros. For $M=N=n$ the form of the function is

$$[P_{n,n}(Z)]^{-1}Q_{n,n}(Z) = \prod_{i=1}^{\theta=n/2} [(a_i U - Z)(a_i^* U - Z)]^{-1} [(a_i U + Z)(a_i^* U + Z)] \quad (3.32)$$

for even values of n , and

$$[P_{n,n}(Z)]^{-1}Q_{n,n}(Z) = [a_0 U - Z]^{-1} [a_0 U + Z] \prod_{i=1}^{\theta=(n-1)/2} [(a_i U - Z)(a_i^* U - Z)]^{-1} [(a_i U + Z)(a_i^* U + Z)] \quad (3.33)$$

for odd values of n , where $a_i = x_i + jy_i$ are the complex roots for $i > 0$ and a_0 is a real root. The symbol $*$ represents the complex conjugate operation.

The pole-zero pairs given by (3.32) and (3.33) allows the macromodel to be viewed as series of subsections, where each subsection is composed of predetermined constants a_0 , $a_i = x_i + jy_i$, and the per unit length parameters. The hybrid stencil for such an i^{th} subsection can be written as

$$[P_{n,n}(Z)]_i \begin{bmatrix} V_{i+1} \\ I_{i+1} \end{bmatrix} = [Q_{n,n}(Z)]_i \begin{bmatrix} V_i \\ I_i \end{bmatrix} \quad (3.34)$$

A graphical representation of (3.34) is given in Figure 3.2, where $V(0,s)$, $I(0,s)$, $V(d,s)$ and $I(d,s)$ represent the terminal voltages and currents. $V_1 \dots V_i \dots V_{n-1}$ and $I_1 \dots I_i \dots I_{n-1}$ represent the internal voltages and currents created by the Padé macromodel. Each subsection consists of two complex pole-zero pairs or one real pole-zero pair. The matrices $[P_{n,n}(Z)]_i$ and $[Q_{n,n}(Z)]_i$ are expressed as

$$[P_{n,n}(Z)]_i = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad [Q_{n,n}(Z)]_i = \begin{bmatrix} B_{11} & -B_{12} \\ -B_{21} & B_{22} \end{bmatrix} \quad (3.35)$$

where

$$\begin{aligned} B_{11} &= LCd^2s^2 + (LG + RC)d^2s + RGd^2 + (x_i^2 + y_i^2)U \\ B_{12} &= 2x_i(Ls + R)d \\ B_{21} &= 2x_i(Cs + G)d \\ B_{22} &= LCd^2s^2 + (LG + RC)d^2s + RGd^2 + (x_i^2 + y_i^2)U \end{aligned} \quad (3.36)$$

for the subsections consisting of complex pole-zero pairs, and

$$\begin{aligned} B_{11} &= a_0U \\ B_{12} &= (Ls + R)d \\ B_{21} &= (Cs + G)d \\ B_{22} &= a_0U \end{aligned} \quad (3.37)$$

for the subsection consisting of a real pole-zero pair.

Figure 3.2: Padé interconnect model represented by pole-zero sections

Converting frequency-domain expression (3.34) into time-domain we obtain

$$\begin{bmatrix} [p_a]_i & [q_a]_i \end{bmatrix} \begin{bmatrix} \dot{x}_{i+1} \\ \dot{x}_i \end{bmatrix} + \begin{bmatrix} [p_b]_i & [q_b]_i \end{bmatrix} \begin{bmatrix} x_{i+1} \\ x_i \end{bmatrix} = 0 \quad (3.38)$$

where the variables of (3.38) are computed using (3.34)-(3.37) as follows. For the complex pole subsections described by (3.34)-(3.36), we have

$$\begin{aligned} [p_a]_i &= \begin{bmatrix} U_x \\ 0 \\ M_i \end{bmatrix} & [p_b]_i &= \begin{bmatrix} U_y \\ 0 \\ S_i \end{bmatrix} \\ [q_a]_i &= \begin{bmatrix} 0 \\ U_x \\ N_i \end{bmatrix} & [q_b]_i &= \begin{bmatrix} 0 \\ U_y \\ T_i \end{bmatrix} \\ x_{i+1} &= [v_{i+1} \ i_{i+1} \ \gamma_{i+1} \ \rho_{i+1}]' & x_i &= [v_i \ i_i \ \gamma_i \ \rho_i]' \end{aligned} \quad (3.39)$$

where γ_i, ρ_i are extra variables needed for the realization and

$$M_i = \begin{bmatrix} (LG + RC)d^2 & 2x_i Ld & LCd^2 & 0 \\ 2x_i Cd & (LG + RC)d^2 & 0 & LCd^2 \end{bmatrix} \quad (3.40a)$$

$$N_i = \begin{bmatrix} -(LG + RC)d^2 & 2x_i Ld & -LCd^2 & 0 \\ 2x_i Cd & -(LG + RC)d^2 & 0 & -LCd^2 \end{bmatrix} \quad (3.40b)$$

$$S_i = \begin{bmatrix} RGd^2 + (x_i^2 + y_i^2)U & 2x_i Rd & 0 & 0 \\ 2x_i Gd & RGd^2 + (x_i^2 + y_i^2)U & 0 & 0 \end{bmatrix} \quad (3.40c)$$

$$T_i = \begin{bmatrix} -(RGd^2 + (x_i^2 + y_i^2)U) & 2x_i Rd & 0 & 0 \\ 2x_i Gd & -(RGd^2 + (x_i^2 + y_i^2)U) & 0 & 0 \end{bmatrix} \quad (3.40d)$$

$$U_X = \begin{bmatrix} U & 0 & 0 & 0 \\ 0 & U & 0 & 0 \end{bmatrix} \quad U_Y = \begin{bmatrix} 0 & 0 & -U & 0 \\ 0 & 0 & 0 & -U \end{bmatrix} \quad (3.40e)$$

For the real pole subsection described by (3.34),(3.35) and (3.37), we have

$$\begin{aligned} [p_a]_0 = [M_0] &= \begin{bmatrix} 0 & Ld & 0 & 0 \\ Cd & 0 & 0 & 0 \end{bmatrix} & [p_b]_0 = [S_0] &= \begin{bmatrix} a_0 U & Rd & 0 & 0 \\ Gd & a_0 U & 0 & 0 \end{bmatrix} \\ [q_a]_0 = [N_0] &= \begin{bmatrix} 0 & Ld \\ Cd & 0 \end{bmatrix} & [q_b]_0 = [T_0] &= \begin{bmatrix} -a_0 U & Rd \\ Gd & -a_0 U \end{bmatrix} \\ x_1 = [v_1 \ i_1 \ \gamma_1 \ \rho_1]^T & & x_0 = [v_0 \ i_0]^T \end{aligned} \quad (3.41)$$

The differential equations obtained from each subsection can be assembled together to form the overall stamp of the transmission line. Using (3.38) - (3.41), the new discrete model for the transmission line becomes

$$\begin{bmatrix} U_X & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & U_X & 0 & \dots & 0 & 0 & 0 \\ M_\theta & N_\theta & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & U_X & \dots & 0 & 0 & 0 \\ 0 & M_{\theta-1} & N_{\theta-1} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & U_X & 0 \\ 0 & 0 & 0 & \dots & M_1 & N_1 & 0 \\ 0 & 0 & 0 & \dots & 0 & M_0 & N_0 \end{bmatrix} \begin{bmatrix} \dot{x}_{\theta+1} \\ \dot{x}_\theta \\ \dot{x}_{\theta-1} \\ \vdots \\ \vdots \\ \dot{x}_2 \\ \dot{x}_1 \\ \dot{x}_0 \end{bmatrix} + \begin{bmatrix} U_Y & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & U_Y & 0 & \dots & 0 & 0 & 0 \\ S_\theta & T_\theta & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & U_Y & \dots & 0 & 0 & 0 \\ 0 & S_{\theta-1} & T_{\theta-1} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & U_Y & 0 \\ 0 & 0 & 0 & \dots & S_1 & T_1 & 0 \\ 0 & 0 & 0 & \dots & 0 & S_0 & T_0 \end{bmatrix} \begin{bmatrix} x_{\theta+1} \\ x_\theta \\ x_{\theta-1} \\ \vdots \\ \vdots \\ x_2 \\ x_1 \\ x_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.42)$$

It should be mentioned that the analytical relation of (3.42) makes the computation of the macromodel just as efficient as the previous method. In addition (3.42) requires fewer variables to realize when compared to (3.28). The number of variables required to implement (3.42) are

$$\begin{cases} 2\psi & n = 1 \\ (2n + 2)\psi & n > 1 \end{cases} \quad (3.43)$$

3.3.4 Time Domain Realization in Terms of Circuit Elements

The time domain macromodel described in the previous section can also be represented in terms of RLC circuit elements. The representation is derived by converting each subsection of (3.34) into the Y-parameters. From the Y-parameters a circuit equivalent model is derived, composed of resistors, capacitors and inductors.

Each subsection described by (3.34)-(3.37) can be represented in terms of the Y-parameters. The complex pole subsection is written as

$$Y_{11} = Y_{22} = \frac{(x_i^2 + y_i^2)}{4x_id}(R + sL)^{-1} + \frac{(G + sC)d}{4x_i} + x_i \left((R + sL)d + \frac{(x_i^2 + y_i^2)}{d}(G + sC)^{-1} \right)^{-1} \quad (3.44)$$

$$Y_{12} = Y_{21} = \frac{(x_i^2 + y_i^2)}{4x_id}(R + sL)^{-1} + \frac{(G + sC)d}{4x_i} - x_i \left((R + sL)d + \frac{(x_i^2 + y_i^2)}{d}(G + sC)^{-1} \right)^{-1}$$

and the real pole subsection is

$$Y_{11} = Y_{22} = \frac{a_0}{2d}(R + sL)^{-1} + \frac{d}{2a_0}(G + sC) \quad (3.45)$$

$$Y_{12} = Y_{21} = -\frac{a_0}{2d}(R + sL)^{-1} + \frac{d}{2a_0}(G + sC)$$

A circuit equivalent model that represents the Y-parameters of (3.44) and (3.45) is shown in Figure 3.3a and 3.3b for a single transmission line.

Figure 3.3a: Circuit equivalent model of complex pole subsection

Figure 3.3b: Circuit equivalent model of real pole subsection

The MNA stamp of each subsection derived by (3.44) and (3.45) is

$$C_{\phi}^i \ddot{X}_{\phi}^i + G_{\phi}^i \dot{X}_{\phi}^i = 0 \quad (3.46)$$

where the variables C_{ϕ}^i , G_{ϕ}^i and X_{ϕ}^i are defined as

$$G_{\phi}^i = \begin{bmatrix} \text{Node1} & \text{Node2} & \text{Node3} & \text{Node4} & \text{Node5} & I_{L1} & I_{L2} \\ \left(\frac{x_i}{d} + \frac{x_i^2 + y_i^2}{4x_i d}\right)R^{-1} + \frac{d}{4x_i}G & \frac{-x_i}{d}R^{-1} & 0 & \frac{d}{4x_i}G & \frac{x_i^2 + y_i^2}{4x_i d}R^{-1} & 0 & 0 \\ \frac{-x_i}{d}R^{-1} & \frac{x_i}{d}R^{-1} & 0 & 0 & 0 & U & 0 \\ 0 & 0 & \frac{x_i d}{x_i^2 + y_i^2}G & \frac{-x_i d}{x_i^2 + y_i^2}G & 0 & -U & 0 \\ \frac{d}{4x_i}G & 0 & \frac{-x_i d}{x_i^2 + y_i^2}G & \left(\frac{x_i d}{x_i^2 + y_i^2} + \frac{d}{4x_i}\right)G & 0 & 0 & U \\ \frac{x_i^2 + y_i^2}{4x_i d}R^{-1} & 0 & 0 & 0 & \frac{x_i^2 + y_i^2}{4x_i d}R^{-1} & 0 & -U \\ 0 & -U & U & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -U & U & 0 & 0 \end{bmatrix} \quad (3.47)$$

$$C_{\phi}^i = \begin{bmatrix} \text{Node1} & \text{Node2} & \text{Node3} & \text{Node4} & \text{Node5} & I_{L1} & I_{L2} \\ \frac{d}{4x_i}C & 0 & 0 & \frac{d}{4x_i}C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{x_i d}{x_i^2 + y_i^2}C & \frac{-x_i d}{x_i^2 + y_i^2}C & 0 & 0 & 0 \\ \frac{d}{4x_i}C & 0 & \frac{-x_i d}{x_i^2 + y_i^2}C & \left(\frac{x_i d}{x_i^2 + y_i^2} + \frac{d}{4x_i}\right)C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{d}{x_i}L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{4x_i d}{x_i^2 + y_i^2}L \end{bmatrix} \quad X_{\phi}^i = \begin{bmatrix} X_{v1_i} \\ X_{v2_i} \\ X_{v3_i} \\ X_{v4_i} \\ X_{v5_i} \\ X_{I1_i} \\ X_{I2_i} \end{bmatrix}$$

for the second order subsection and

$$G_{\circ}^i = \begin{array}{c} \begin{array}{cccc} \text{Node1} & \text{Node2} & \text{Node3} & I_{L1} \\ \left[\begin{array}{cccc} \frac{d}{2a_0}G & 0 & \frac{d}{2a_0}G & U \\ 0 & \frac{a_0}{2d}R^{-1} & \frac{-a_0}{2d}R^{-1} & -U \\ \frac{d}{2a_0}G & \frac{-a_0}{2d}R^{-1} \left(\frac{a_0}{2d}R^{-1} + \frac{d}{2a_0}G \right) & 0 & 0 \\ U & -U & 0 & 0 \end{array} \right] \end{array} \end{array} \quad (3.48)$$

$$C_{\circ}^i = \begin{array}{c} \begin{array}{cccc} \text{Node1} & \text{Node2} & \text{Node3} & I_{L1} \\ \left[\begin{array}{cccc} \frac{d}{2a_0}C & 0 & \frac{d}{2a_0}C & 0 \\ 0 & 0 & 0 & 0 \\ \frac{d}{2a_0}C & 0 & \frac{d}{2a_0}C & 0 \\ 0 & 0 & 0 & \frac{2d}{a_0}L \end{array} \right] \end{array} \end{array} \quad X_{\circ}^i = \begin{bmatrix} X_{v1_i} \\ X_{v2_i} \\ X_{v3_i} \\ X_{I1_i} \end{bmatrix}$$

for the first order subsection.

One of the main advantages of (3.46) is that it can easily be implemented since the equations are described in terms of a circuit elements (i.e. resistors, inductors and capacitors). In fact the form is very similar to the conventional lumped segmentation model. The time domain representations of (3.22), (3.28) and (3.42) can also be expressed as circuit elements, however, the representations are more involved. Another advantage of (3.46) is that it is compatible with passive reduction techniques based on Krylov space methods [30]. Section 3.4.3 will show that the matrices of (3.47) and (3.48) are nonnegative definite. Nonnegative matrices are required by Krylov space techniques to preserve the passivity of the reduced network. The number of new variables that are introduced by (3.46) is

$$\psi(3n - 1) \quad (3.49)$$

3.4 Proof of Passivity

One of the key issues of modeling transmission lines is the preservation of passivity. A passive macromodel implies that a network cannot generate more energy than it absorbs; in other words no passive termination of the network will cause the system to go unstable [46]-[48]. Passivity is an important property to satisfy because stable, but not passive macromodels can produce unstable systems when connected to other passive loads [22]. The loss of passivity can be a serious problem for general circuit simulators because the overall network may encounter artificial oscillations.

The Padé macromodel was shown to be passive [27], [28] when the order of the numerator and denominator of (3.4) are equal. In this thesis, an alternative proof of passivity is given for the Padé macromodel. In addition, a method of obtaining a passive macromodel is described for cases when the order of the numerator differs from the denominator. Section 3.4.1 will review some of the basic properties of positive real functions. From this discussion, a new passivity proof is given.

3.4.1 Passivity Concepts

A passive network implies that the overall network will remain asymptotically stable for any passive termination, however, a stable network does not imply passivity. A linear n -port network with an admittance matrix $Y(s)$ is said to be passive for all signals of z , if and only if [17], [46]-[48],

1. $Y(s^*) = Y^*(s)$ where $*$ is the complex conjugate operator.

2. $Y(s)$ is a positive real matrix. That is the product $z^* [Y'(s^*) + Y(s)] z \geq 0$ for all possible values of s satisfying $\text{Re}(s) > 0$ and any arbitrary value of z .

The first condition implies, that the coefficients of the rational function matrix generated by the proposed macromodel must be real. The second condition implies, that $Y(s)$ must be a positive real matrix for all $\text{Re}(s) > 0$, since $\text{Real}(Y(s)) = \frac{1}{2} [Y'(s^*) + Y(s)]$. The coefficients generated by (3.4) are real values, therefore the first condition is always satisfied. The task that remains is to show that the macromodel satisfies the second condition.

The following properties of nonnegative definite and positive real matrices are used to prove passivity.

Theorem 3.1

- The sum of two nonnegative definite matrices of similar dimensions is nonnegative definite [49].

Theorem 3.2

- Let $A \in \Re^{\Psi \times \Psi}$ be a matrix that is nonnegative definite and $X \in \Re^{\Psi \times \Psi}$ be a nonsingular real matrix. The matrix formed by AXX' results in another nonnegative definite matrix [49].

Theorem 3.3

- The sum of two positive real matrices of similar dimensions is positive real [50].

Theorem 3.4

- The inverse of a positive real matrix if the inverse exists, is positive real [50].

Theorem 3.5

- Let $A(s) \in \mathfrak{R}^{\Psi \times \Psi}$ be a matrix that is positive real and $X \in \mathfrak{R}^{\Psi \times \Psi}$ be a nonsingular real matrix. The matrix formed by $XA(s)X'$ results in another positive real matrix.

The proof of theorem 3.5 is based on theorem 3.2. If s is replaced by any $\sigma + j\omega$ value where $\sigma \geq 0$, the resulting matrix $Re(A((\sigma + j\omega)))$ is nonnegative definite since $A(s)$ is positive real. Therefore $XRe(A(\sigma + j\omega))X'$ will also be nonnegative definite according to theorem 3.2. Since $XRe(A(\sigma + j\omega))X'$ is nonnegative definite for every value of $\sigma + j\omega$ where $\sigma \geq 0$, the matrix $XA(s)X'$ must be positive real.

To show that the macromodel is passive, the passivity proof relies on the fact that the polynomial matrix $P_{N,M}(Z)$ of (3.4) is a strict Hurwitz polynomial [51]. A strict Hurwitz polynomial possesses zeros only in the left half-plane. To identify if a polynomial is a strict Hurwitz polynomial, the function is separated into even and odd parts as

$$\begin{aligned}\rho(s) &= \alpha_n s^n + \alpha_{n-1} s^{n-1} + \dots + \alpha_0 \\ M(s) &= \alpha_n s^n + \dots + \alpha_0 \\ N(s) &= \alpha_{n-1} s^{n-1} + \dots + \alpha_1 s\end{aligned}\tag{3.50}$$

where $\rho(s)$ is a strict Hurwitz polynomial, $M(s)$ and $N(s)$ represent the even and odd functions of $\rho(s)$, respectively. The rational function formed from the even and odd polynomials can be expressed as a continued fraction expansion, written as

$$\frac{M(s)}{N(s)} = \kappa_n s + \frac{1}{\kappa_{n-1} s + \frac{1}{\kappa_{n-2} s + \dots \kappa_0 s}} \quad (3.51)$$

The necessary and sufficient conditions for $\rho(s)$ to be a strict Hurwitz polynomial are the coefficients κ_i for $i = 1, 2, \dots, n$ must be strictly positive [46], [54]. This result leads to the following theorem.

Theorem 3.6

- If $\rho(s)$ is a strict Hurwitz polynomial, then the rational function created by the even and odd polynomials (3.51) is an odd and positive real rational function [54].

Sometimes it becomes extremely difficult to determine if a function satisfies the second condition of the passivity definition, that is, whether $Y(s)$ is positive real for all possible values of s where $\text{Re}(s) > 0$. An alternative set of equivalent and necessary conditions to test for positive real functions are listed in theorem 3.7.

Theorem 3.7

- A rational function $A(s)$ is positive real if and only if it satisfies all of the following conditions [46]:
 1. $A(s)$ is real for all real values of s .
 2. The denominator polynomial of $A(s)$ is a Hurwitz polynomial or may have zeros on the j axis; that is $A(s)$ must be analytic in the right half-plane.

3. If $A(s)$ has poles on the j axis, these poles must be simple and have real and positive residues.
4. The real part of $A(s)$ is nonnegative along the j axis, or expressed analytically, $Re(A(j\omega)) > 0$ for all ω .

3.4.2 Passivity Proof

The proof described in this section relies on two facts; the line parameters R, L, G and C are nonnegative definite symmetric matrices [7], [33] and the polynomial $P_{N,M}(Z)$ is a strict Hurwitz polynomial [51]. To understand the properties of the closed form Padé exponential matrix of (3.4), the scalar form of the equation is examined. The scalar form of (3.4) is

$$e^{-s} \approx \frac{q_{M,N}(s)}{p_{N,M}(s)} \quad (3.52)$$

where $p_{N,M}(s)$ and $q_{M,N}(s)$ are scalar polynomials defined as

$$\begin{aligned} p_{N,M}(s) &= \sum_{j=0}^N \frac{(M+N-j)!N!}{(M+N)!j!(N-j)!} (s)^j \\ q_{M,N}(s) &= \sum_{j=0}^M \frac{(M+N-j)!M!}{(M+N)!j!(M-j)!} (-s)^j \end{aligned} \quad (3.53)$$

The polynomials of (3.53) can be separated into even and odd parts as

$$\begin{aligned} p_{N,M}(s) &= p_{N,M_{EVEN}}(s) + p_{N,M_{ODD}}(s) \\ q_{M,N}(s) &= q_{M,N_{EVEN}}(s) + q_{M,N_{ODD}}(s) \end{aligned} \quad (3.54)$$

where

$$p_{N, M_{EVEN}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 + (-1)^i) s^i \right] \quad (3.55a)$$

$$p_{N, M_{ODD}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 - (-1)^i) s^i \right] \quad (3.55b)$$

$$q_{M, N_{EVEN}} = \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 + (-1)^i) s^i \right] \quad (3.55c)$$

$$q_{M, N_{ODD}} = - \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 - (-1)^i) s^i \right] \quad (3.55d)$$

It should be noted that the form of (3.55) is very similar to that of (3.7). In fact a relationship exists between the two forms. This relationship will be exploited to prove the passivity of the macromodel.

The polynomials generated by $p_{N, M_{EVEN}}(s) + p_{N, M_{ODD}}(s)$ and $q_{M, N_{EVEN}}(s) - q_{M, N_{ODD}}(s)$ are strict Hurwitz polynomials when $N=M$ [51]. The following rational functions generated by (3.55) are odd positive real rational functions (theorem 3.6), written as

$$\frac{p_{N, M_{EVEN}}(s)}{p_{N, M_{ODD}}(s)} = \chi_N s + \frac{1}{\chi_{N-1} s + \frac{1}{\chi_{N-2} s + \dots \frac{1}{\chi_1 s + \frac{1}{\chi_0 s}}}} \quad (3.56a)$$

$$\frac{q_{M, N_{EVEN}}(s)}{-q_{M, N_{ODD}}(s)} = \varphi_N s + \frac{1}{\varphi_{N-1} s + \frac{1}{\varphi_{N-2} s + \dots \frac{1}{\varphi_1 s + \frac{1}{\varphi_0 s}}}} \quad (3.56b)$$

where χ_i and φ_i for $i = 1, 2, \dots, n$ are positive values. Replacing the scalar s with the matrix $\mathbf{a}$ for every even coefficient $[\chi_0 \ \chi_2 \ \chi_4 \ \dots]$, $[\varphi_0 \ \varphi_2 \ \varphi_4 \ \dots]$ and replacing the scalar s with the matrix $\mathbf{b}$ for every odd coefficient $[\chi_1 \ \chi_3 \ \chi_5 \ \dots]$, $[\varphi_1 \ \varphi_3 \ \varphi_5 \ \dots]$ yields new rational function matrices

$$\frac{P^{ab_{EVEN}}}{P^{ab_{ODD}}} = \chi_N \mathbf{b} + (\chi_{N-1} \mathbf{a} + (\chi_{N-2} \mathbf{b} + (\dots + (\chi_1 \mathbf{b} + (\chi_0 \mathbf{a})^{-1})^{-1})^{-1})^{-1})^{-1} \quad (3.57a)$$

$$\frac{Q^{ab_{EVEN}}}{-Q^{ab_{ODD}}} = \varphi_N \mathbf{b} + (\varphi_{N-1} \mathbf{a} + (\varphi_{N-2} \mathbf{b} + (\dots + (\varphi_1 \mathbf{b} + (\varphi_0 \mathbf{a})^{-1})^{-1})^{-1})^{-1})^{-1} \quad (3.57b)$$

where $\mathbf{a}$ and $\mathbf{b}$ are positive real matrices since the line parameters are nonnegative matrices (i.e. $\mathbf{a} = (\mathbf{R} + s\mathbf{L})d$, $\mathbf{b} = (\mathbf{G} + s\mathbf{C})d$). The rational function matrices of (3.57) are positive real, since $\mathbf{a}$ and $\mathbf{b}$ are positive real (theorem 3.3 and 3.4). The equations formed by (3.57) are equivalent to (3.7). This is illustrated by Table 3.2, which compares (3.56a) with (3.57a). The last row of table 3.2 shows the closed form relationship of the numerator and denominator of (3.57a) to be that of (3.7). The following equalities hold true between (3.7) and (3.57)

$$\begin{aligned} P_{N, M_{11}} &= P_{N, M_{22}} = P^{ab_{EVEN}} & P_{N, M_{12}} &= P^{ab_{ODD}} \\ Q_{M, N_{11}} &= Q_{M, N_{22}} = Q^{ab_{EVEN}} & Q_{M, N_{12}} &= Q^{ab_{ODD}} \end{aligned} \quad (3.58)$$

This means that the following rational functions matrices formed by the polynomials of (3.7) are positive real.

$$P_{N, M_{11}}[P_{N, M_{12}}]^{-1}, \quad P_{N, M_{22}}[P_{N, M_{12}}]^{-1} \quad Q_{M, N_{11}}[-Q_{M, N_{12}}]^{-1}, \quad Q_{M, N_{22}}[-Q_{M, N_{12}}]^{-1} \quad (3.59)$$

Order N	$p_{N,M_{EVEN}}(s), p_{N,M_{ODD}}(s)$	$P_{ab_{EVEN}}, P_{ab_{ODD}}$
1	$p_{1,M_{EVEN}}(s) = 1$ $p_{1,M_{ODD}}(s) = \chi_0 s$	$P_{ab_{EVEN}}^1 = U$ $P_{ab_{ODD}}^1 = \chi_0 a$
2	$p_{2,M_{EVEN}}(s) = \chi_1 \chi_0 s^2 + 1$ $p_{2,M_{ODD}}(s) = \chi_0 s$	$P_{ab_{EVEN}}^2 = \chi_1 \chi_0 ab + U$ $P_{ab_{ODD}}^2 = \chi_0 a$
3	$p_{3,M_{EVEN}}(s) = \chi_1 \chi_0 s^2 + 1$ $p_{3,M_{ODD}}(s) = \chi_2 \chi_1 \chi_0 s^3 + (\chi_2 + \chi_0)s$	$P_{ab_{EVEN}}^3 = \chi_1 \chi_0 ab + U$ $P_{ab_{ODD}}^3 = \chi_2 \chi_1 \chi_0 a^2 b + (\chi_2 + \chi_0)a$
...	...	...
n (even)	$p_{n,M_{EVEN}}(s) = \chi_{n-1} s p_{n-1,M_{ODD}}(s)$ $\quad + p_{n-1,M_{EVEN}}(s)$ $p_{n,M_{ODD}}(s) = p_{n-1,M_{ODD}}(s)$	$P_{ab_{EVEN}}^n = \chi_{n-1} b P_{ab_{ODD}}^{n-1}$ $\quad + P_{ab_{EVEN}}^{n-1}$ $P_{ab_{ODD}}^n = P_{ab_{ODD}}^{n-1}$
n+1(odd)	$p_{n+1,M_{EVEN}}(s) = p_{n,M_{EVEN}}(s)$ $p_{n+1,M_{ODD}}(s) = \chi_n s p_{n,M_{EVEN}}(s)$ $\quad + p_{n,M_{ODD}}(s)$	$P_{ab_{EVEN}}^{n+1} = P_{ab_{EVEN}}^n$ $P_{ab_{ODD}}^{n+1} = \chi_n a P_{ab_{EVEN}}^n$ $\quad + P_{ab_{ODD}}^n$
	$p_{N,M_{EVEN}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1+(-1)^i) s^i \right]$ $p_{N,M_{ODD}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1-(-1)^i) s^i \right]$	$P_{ab_{EVEN}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1+(-1)^i) (ab)^{i/2} \right]$ $P_{ab_{ODD}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1-(-1)^i) (ab)^{\frac{i-1}{2}} a \right]$

Table 3.2: Comparison of (3.56a) with (3.57a)

Similarly, if (3.56) is modified by replacing the scalar s with the matrix b for every even coefficient $[\chi_0 \ \chi_2 \ \chi_4 \ \dots]$, $[\varphi_0 \ \varphi_2 \ \varphi_4 \ \dots]$ and replacing the scalar s with the matrix a for every odd coefficient $[\chi_1 \ \chi_3 \ \chi_5 \ \dots]$, $[\varphi_1 \ \varphi_3 \ \varphi_5 \ \dots]$ then the rational functions formed by

$$P_{N,M_{11}}[P_{N,M_{21}}]^{-1}, \quad P_{N,M_{22}}[P_{N,M_{21}}]^{-1} \quad Q_{M,N_{11}}[-Q_{M,N_{21}}]^{-1}, \quad Q_{M,N_{22}}[-Q_{M,N_{21}}]^{-1} \quad (3.60)$$

can also be shown to be positive real.

To show that the macromodel is passive, (3.6) is converted to the Y-parameters. The Y-parameters of the macromodel for the coupled interconnect case are

$$Y_{11} = [Q_{M,N_{11}}P_{N,M_{22}} - Q_{M,N_{21}}P_{N,M_{12}}][Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}]^{-1} \quad (3.61a)$$

$$Y_{12} = [P_{N,M_{21}}P_{N,M_{12}} - P_{N,M_{11}}P_{N,M_{22}}][Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}]^{-1} \quad (3.61b)$$

$$Y_{21} = [Q_{M,N_{11}}Q_{M,N_{22}} - Q_{M,N_{21}}Q_{M,N_{12}}][Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}]^{-1} \quad (3.61c)$$

$$Y_{22} = [P_{N,M_{21}}Q_{M,N_{12}} - Q_{M,N_{22}}P_{N,M_{11}}][Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}]^{-1} \quad (3.61d)$$

When the order of the Padé macromodel is $M=N$, the admittance equations of (3.61) are reduced to

$$Y_{11} = Y_{22} = \frac{1}{2}P_{N,M_{11}}[P_{N,M_{12}}]^{-1} + \frac{1}{2}P_{N,M_{21}}[P_{N,M_{11}}]^{-1} \quad (3.62a)$$

$$Y_{12} = Y_{21} = -\frac{1}{2}P_{N,M_{11}}[P_{N,M_{12}}]^{-1} + \frac{1}{2}P_{N,M_{21}}[P_{N,M_{11}}]^{-1} \quad (3.62b)$$

Y_{11} and Y_{22} are positive real rational function matrices since (3.59) and (3.60) are positive real (theorem 3.3). To show that the overall matrix is positive real the admittance matrix of (3.62) is expressed as

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \Psi_1 + \Psi_2 \quad (3.63)$$

where

$$\Psi_1 = \begin{bmatrix} \frac{1}{2} \mathbf{P}_{N, M_{11}} [\mathbf{P}_{N, M_{12}}]^{-1} & -\frac{1}{2} \mathbf{P}_{N, M_{11}} [\mathbf{P}_{N, M_{12}}]^{-1} \\ -\frac{1}{2} \mathbf{P}_{N, M_{11}} [\mathbf{P}_{N, M_{12}}]^{-1} & \frac{1}{2} \mathbf{P}_{N, M_{11}} [\mathbf{P}_{N, M_{12}}]^{-1} \end{bmatrix} \quad (3.64a)$$

$$\Psi_2 = \begin{bmatrix} \frac{1}{2} \mathbf{P}_{N, M_{21}} [\mathbf{P}_{N, M_{11}}]^{-1} & \frac{1}{2} \mathbf{P}_{N, M_{21}} [\mathbf{P}_{N, M_{11}}]^{-1} \\ \frac{1}{2} \mathbf{P}_{N, M_{21}} [\mathbf{P}_{N, M_{11}}]^{-1} & \frac{1}{2} \mathbf{P}_{N, M_{21}} [\mathbf{P}_{N, M_{11}}]^{-1} \end{bmatrix} \quad (3.64b)$$

To prove that Ψ_1 and Ψ_2 are positive real, the matrices are written in terms of diagonal matrices,

$$\begin{aligned} \Psi_1 &= \mathbf{W}_\alpha' \text{diag} \left(\frac{1}{2} \mathbf{P}_{N, M_{11}} [\mathbf{P}_{N, M_{12}}]^{-1}, \mathbf{0} \right) \mathbf{W}_\alpha \\ \Psi_2 &= \mathbf{W}_\beta' \text{diag} \left(\frac{1}{2} \mathbf{P}_{N, M_{21}} [\mathbf{P}_{N, M_{11}}]^{-1}, \mathbf{0} \right) \mathbf{W}_\beta \end{aligned} \quad (3.65)$$

where

$$\mathbf{W}_\alpha = \begin{bmatrix} \mathbf{U} & -\mathbf{U} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \quad \mathbf{W}_\beta = \begin{bmatrix} \mathbf{U} & \mathbf{U} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \quad (3.66)$$

Since the matrices $\text{diag} \left(\frac{1}{2} \mathbf{P}_{N, M_{11}} [\mathbf{P}_{N, M_{12}}]^{-1}, \mathbf{0} \right)$ and $\text{diag} \left(\frac{1}{2} \mathbf{P}_{N, M_{21}} [\mathbf{P}_{N, M_{11}}]^{-1}, \mathbf{0} \right)$ are positive real, Ψ_1 and Ψ_2 are also positive real (theorem 3.5). This means that the admittance matrix of the macromodel is positive real when $M=N$.

For the case when $M \neq N$, $p_{N, M_{\text{EVEN}}}(s) + p_{N, M_{\text{ODD}}}(s)$ and $q_{M, N_{\text{EVEN}}}(s) - q_{M, N_{\text{ODD}}}(s)$ may not be Hurwitz polynomials. Extensive work has been reported on the locations of the poles and zeros of a Padé approximation of e^{-s} [52], [53]. For example, if $M+1=N$ then the polynomials $p_{N, M_{\text{EVEN}}}(s) + p_{N, M_{\text{ODD}}}(s)$ and $q_{M, N_{\text{EVEN}}}(s) - q_{M, N_{\text{ODD}}}(s)$ retain their Hurwitz

characteristics. Even though the Padé polynomials are Hurwitz for $M+1=N$, the Padé macromodel is still not passive. The symmetry of the Y-parameters is broken since the order used to approximate $P_{N,M}(Z)$ differs from $Q_{M,N}(Z)$. This causes the order of Y_{12} to be different than Y_{21} . Both Y_{12} and Y_{21} of (3.61) attempt to approximate the same curve. However, since the order of Y_{12} differs from Y_{21} , the curves eventually diverge as frequency increases. Replacing Y_{12} with Y_{21} or vice versa restores the symmetry of the Y-parameters, but does this result in a passive macromodel? It turns out that the passivity of the macromodel is restored when the higher order approximation is replaced with the lower order approximation. For example, for the case when $M < N$, replacing the higher order approximation of Y_{12} with the lower order approximation of Y_{21} gives passive Y-parameters. To demonstrate this point the new Y-parameters become

$$Y_{11} = Y_{22} = [Q_{M,N_{11}}P_{N,M_{22}} - Q_{M,N_{21}}P_{N,M_{12}}][Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}]^{-1} \quad (3.67a)$$

$$Y_{12} = Y_{21} = [Q_{M,N_{11}}Q_{M,N_{22}} - Q_{M,N_{21}}Q_{M,N_{12}}][Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}]^{-1} \quad (3.67b)$$

To show that Y_{11} and Y_{22} are positive real, (3.67a) is rewritten as

$$Y_{11} = Y_{22} = [P_{N,M_{12}}[P_{N,M_{11}}]^{-1} - Q_{M,N_{12}}[Q_{M,N_{11}}]^{-1}]^{-1} + [P_{N,M_{11}}[P_{N,M_{21}}]^{-1} - Q_{M,N_{11}}[Q_{M,N_{21}}]^{-1}]^{-1} \quad (3.68)$$

Equation (3.68) is positive real since it is formed from the positive real matrices of (3.59) and (3.60) (theorem 3.3 and 3.4). To show that the Y-parameter matrix is passive, (3.67) is expressed as

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} + \begin{bmatrix} \gamma & \delta \\ \delta & \gamma \end{bmatrix} = \Psi_1 + \Psi_2 \quad (3.69)$$

where

$$\alpha = Q_{M,N_{11}}P_{N,M_{22}}[Q_{M,N_{22}}P_{N,M_{12}} - Q_{M,N_{12}}P_{N,M_{22}}]^{-1} \quad (3.70a)$$

$$\beta = \mathcal{Q}_{M, N_{11}} \mathcal{Q}_{M, N_{22}} [\mathcal{Q}_{M, N_{22}} \mathbf{P}_{N, M_{12}} - \mathcal{Q}_{M, N_{12}} \mathbf{P}_{N, M_{22}}]^{-1} \quad (3.70b)$$

$$\gamma = -\mathcal{Q}_{M, N_{21}} \mathbf{P}_{N, M_{12}} [\mathcal{Q}_{M, N_{22}} \mathbf{P}_{N, M_{12}} - \mathcal{Q}_{M, N_{12}} \mathbf{P}_{N, M_{22}}]^{-1} \quad (3.70c)$$

$$\delta = -\mathcal{Q}_{M, N_{21}} \mathcal{Q}_{M, N_{12}} [\mathcal{Q}_{M, N_{22}} \mathbf{P}_{N, M_{12}} - \mathcal{Q}_{M, N_{12}} \mathbf{P}_{N, M_{22}}]^{-1} \quad (3.70d)$$

The rational function matrices of α and γ are positive real since they are formed from the rational matrices of (3.68). To show that Ψ_1 and Ψ_2 are positive real, the matrices are represented in terms of their real parts. Let (3.70) be expressed as

$$\alpha = [\mathbf{m}_\alpha(s) + s\mathbf{n}_\alpha(s)][\mathbf{m}(s) + s\mathbf{n}(s)]^{-1} \quad (3.71a)$$

$$\beta = [\mathbf{m}_\beta(s) + s\mathbf{n}_\beta(s)][\mathbf{m}(s) + s\mathbf{n}(s)]^{-1} \quad (3.71b)$$

$$\gamma = [\mathbf{m}_\gamma(s) + s\mathbf{n}_\gamma(s)][\mathbf{m}(s) + s\mathbf{n}(s)]^{-1} \quad (3.71c)$$

$$\delta = [\mathbf{m}_\delta(s) + s\mathbf{n}_\delta(s)][\mathbf{m}(s) + s\mathbf{n}(s)]^{-1} \quad (3.71d)$$

where $\mathbf{m}_\alpha(s)$, $\mathbf{m}_\beta(s)$, $\mathbf{m}_\gamma(s)$, $\mathbf{m}_\delta(s)$, $\mathbf{m}(s)$ represent the even parts of the polynomial matrices and $s\mathbf{n}_\alpha(s)$, $s\mathbf{n}_\beta(s)$, $s\mathbf{n}_\gamma(s)$, $s\mathbf{n}_\delta(s)$, $s\mathbf{n}(s)$ represent the odd parts of the polynomial matrices. The real parts of (3.71) are written as,

$$\mathbf{Re}(\alpha) = [\mathbf{m}_\alpha(s)\mathbf{m}(s) - s^2\mathbf{n}_\alpha(s)\mathbf{n}(s)][\mathbf{m}(s)^2 - s^2\mathbf{n}(s)^2]^{-1} \quad (3.72a)$$

$$\mathbf{Re}(\beta) = [\mathbf{m}_\beta(s)\mathbf{m}(s) - s^2\mathbf{n}_\beta(s)\mathbf{n}(s)][\mathbf{m}(s)^2 - s^2\mathbf{n}(s)^2]^{-1} \quad (3.72b)$$

$$\mathbf{Re}(\gamma) = [\mathbf{m}_\gamma(s)\mathbf{m}(s) - s^2\mathbf{n}_\gamma(s)\mathbf{n}(s)][\mathbf{m}(s)^2 - s^2\mathbf{n}(s)^2]^{-1} \quad (3.72c)$$

$$\mathbf{Re}(\delta) = [\mathbf{m}_\delta(s)\mathbf{m}(s) - s^2\mathbf{n}_\delta(s)\mathbf{n}(s)][\mathbf{m}(s)^2 - s^2\mathbf{n}(s)^2]^{-1} \quad (3.72d)$$

Replacing s with $j\omega$ yields,

$$\mathbf{Re}(\alpha) = [\mathbf{m}_\alpha(\omega)\mathbf{m}(\omega) + \omega^2\mathbf{n}_\alpha(\omega)\mathbf{n}(\omega)][\mathbf{m}(\omega)^2 + \omega^2\mathbf{n}(\omega)^2]^{-1} \quad (3.73a)$$

$$\mathbf{Re}(\beta) = [\mathbf{m}_\beta(\omega)\mathbf{m}(\omega) + \omega^2\mathbf{n}_\beta(\omega)\mathbf{n}(\omega)][\mathbf{m}(\omega)^2 + \omega^2\mathbf{n}(\omega)^2]^{-1} \quad (3.73b)$$

$$Re(\gamma) = [m_\gamma(\omega)m(\omega) + \omega^2 n_\gamma(\omega)n(\omega)][m(\omega)^2 + \omega^2 n(\omega)^2]^{-1} \quad (3.73c)$$

$$Re(\delta) = [m_\delta(\omega)m(\omega) + \omega^2 n_\delta(\omega)n(\omega)][m(\omega)^2 + \omega^2 n(\omega)^2]^{-1} \quad (3.73d)$$

It should be mentioned that the denominator of (3.73) is always positive for all values of ω since $Re(\alpha)$ and $Re(\gamma)$ are positive real. The differences between α and β as seen from (3.70) are from the numerator. The variable α has $P_{N,M_{\pm}}$ on the numerator while β has $Q_{M,N_{\pm}}$. Since the order of the Padé approximation is $N > M$, the coefficients generated by $P_{N,M_{\pm}}$ (3.7d) are greater than the coefficients generated by $Q_{M,N_{\pm}}$ (3.7h). As a result, the following inequalities hold true for all ω ,

$$|m_\alpha(\omega)m(\omega)| \geq |m_\beta(\omega)m(\omega)| \quad (3.74a)$$

$$|n_\alpha(\omega)n(\omega)| \geq |n_\beta(\omega)n(\omega)| \quad (3.74b)$$

Similarly, the differences between γ and δ are also from the numerator. The variable γ has $P_{N,M_{12}}$, while δ has $Q_{M,N_{12}}$. Since $N > M$, the coefficients generated by $P_{N,M_{12}}$ (3.7b) are greater than the coefficients generated by negative $Q_{M,N_{12}}$ (3.7f). Therefore, the following inequalities also hold true for all ω ,

$$|m_\gamma(\omega)m(\omega)| \geq |m_\delta(\omega)m(\omega)| \quad (3.75a)$$

$$|n_\gamma(\omega)n(\omega)| \geq |n_\delta(\omega)n(\omega)| \quad (3.75b)$$

The real parts of Ψ_1 and Ψ_2 matrices are represented in terms of diagonal matrices expressed as

$$\begin{aligned} \Psi_1 &= W_\alpha' \text{diag}(Re(\alpha), Re(\alpha) - Re(\beta)Re(\alpha)^{-1}Re(\beta)) W_\alpha \\ \Psi_2 &= W_\beta' \text{diag}(Re(\gamma), Re(\gamma) - Re(\delta)Re(\gamma)^{-1}Re(\delta)) W_\beta \end{aligned} \quad (3.76)$$

where

$$W_\alpha = \begin{bmatrix} U & Re(\alpha)^{-1} Re(\beta) \\ 0 & U \end{bmatrix} \quad W_\beta = \begin{bmatrix} U & Re(\gamma)^{-1} Re(\delta) \\ 0 & U \end{bmatrix} \quad (3.77)$$

If the diagonal matrices of (3.76) are positive real then Ψ_1 and Ψ_2 are also positive real (theorem 3.5). This implies that

$$\begin{aligned} Re(\alpha) - Re(\beta) Re(\alpha)^{-1} Re(\beta) &\geq 0 \\ Re(\gamma) - Re(\delta) Re(\gamma)^{-1} Re(\delta) &\geq 0 \end{aligned} \quad (3.78)$$

The polynomial matrices of $Re(\alpha)$, $Re(\beta)$, $Re(\gamma)$ and $Re(\delta)$ are all symmetric since the line parameters are symmetric. Since the matrices of (3.78) are symmetric and $Re(\alpha)$ and $Re(\gamma)$ are positive real, the inequalities of (3.78) can be rewritten as

$$\begin{aligned} Re(\alpha)^2 - Re(\beta)^2 &\geq 0 \\ Re(\gamma)^2 - Re(\delta)^2 &\geq 0 \end{aligned} \quad (3.79)$$

The relations of (3.79) are always satisfied since the relation of (3.74) and (3.75) hold true. This proves that the Y-parameter matrix of (3.67) is greater or equal to zero for all values of ω . However this does not imply that (3.67) is positive real unless all conditions of theorem 3.7 are satisfied. It was mentioned that the coefficients of the Padé approximation are real, therefore the macromodel is real for all real values of s . The denominator of (3.67) is analytic in the right half-plane since Y_{11} and Y_{22} are shown to be positive real. The poles of the Y-parameters are not on the j axis as shown by (3.14) and (3.20). All four conditions of theorem 7 are satisfied, therefore the Y-parameters of (3.67) are passive.

3.4.3 Passivity of MNA Stamp of Circuit Equivalent Model

This section demonstrates that the MNA stamp of (3.46) is compatible with passive reduction techniques based on Krylov space methods. Let the MNA equations of the linear

network be expressed as

$$C_\phi \dot{X}_\phi + G_\phi X_\phi = e_\phi \quad (3.80)$$

The conditions that (3.80) must satisfy to implement passive model reduction using Krylov techniques are [22], [30]

1. G_ϕ is of the form of $\begin{bmatrix} N_\phi & E_\phi \\ -E_\phi' & 0 \end{bmatrix}$, where N_ϕ is nonnegative definite.
2. C_ϕ is a nonnegative definite matrix.

The predetermined constants x_i , $x_i^2 + y_i^2$ and a_0 of (3.47) and (3.48) are all positive values since $P_{N,M}$ is a strict Hurwitz polynomial [51]. To show that MNA matrices of (3.47) satisfy the two conditions, G_ϕ^i is expressed as

$$G_\phi^i = \begin{bmatrix} N_\phi^i & E_\phi \\ -E_\phi' & 0 \end{bmatrix} \quad (3.81)$$

where

$$N_\phi^i = \begin{bmatrix} \left(\frac{x_i}{d} + \frac{x_i^2 + y_i^2}{4x_id}\right)R^{-1} + \frac{d}{4x_i}G & \frac{-x_i}{d}R^{-1} & 0 & \frac{d}{4x_i}G & \frac{x_i^2 + y_i^2}{4x_id}R^{-1} \\ \frac{-x_i}{d}R^{-1} & \frac{x_i}{d}R^{-1} & 0 & 0 & 0 \\ 0 & 0 & \frac{x_id}{x_i^2 + y_i^2}G & \frac{-x_id}{x_i^2 + y_i^2}G & 0 \\ \frac{d}{4x_i}G & 0 & \frac{-x_id}{x_i^2 + y_i^2}G & \left(\frac{x_id}{x_i^2 + y_i^2} + \frac{d}{4x_i}\right)G & 0 \\ \frac{x_i^2 + y_i^2}{4x_id}R^{-1} & 0 & 0 & 0 & \frac{x_i^2 + y_i^2}{4x_id}R^{-1} \end{bmatrix} \quad (3.82a)$$

$$E_\phi' = \begin{bmatrix} 0 & U & -U & 0 & 0 \\ 0 & 0 & 0 & U & -U \end{bmatrix} \quad (3.82b)$$

To show that $N_{\circ}^i$ is nonnegative definite, the matrix is written as a summation of nonnegative matrices

$$N_{\circ}^i = N_{\circ 1}^i + N_{\circ 2}^i + N_{\circ 3}^i + N_{\circ 4}^i \quad (3.83)$$

where

$$N_{\circ 1}^i = \begin{bmatrix} \frac{x_i^2 + y_i^2}{4x_id} R^{-1} & 0 & 0 & 0 & \frac{x_i^2 + y_i^2}{4x_id} R^{-1} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{x_i^2 + y_i^2}{4x_id} R^{-1} & 0 & 0 & 0 & \frac{x_i^2 + y_i^2}{4x_id} R^{-1} \end{bmatrix} \quad (3.84a)$$

$$N_{\circ 2}^i = \begin{bmatrix} \frac{x_i}{d} R^{-1} & -\frac{x_i}{d} R^{-1} & 0 & 0 & 0 \\ -\frac{x_i}{d} R^{-1} & \frac{x_i}{d} R^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.84b)$$

$$N_{\circ 3}^i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{x_id}{x_i^2 + y_i^2} G & -\frac{x_id}{x_i^2 + y_i^2} G & 0 \\ 0 & 0 & -\frac{x_id}{x_i^2 + y_i^2} G & \frac{x_id}{x_i^2 + y_i^2} G & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.84c)$$

$$N_{\phi^4}^i = \begin{bmatrix} \frac{d}{4x_i}G & 0 & 0 & \frac{d}{4x_i}G & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{d}{4x_i}G & 0 & 0 & \frac{d}{4x_i}G & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.84d)$$

The general form of (3.84) is

$$\Theta_1 = \begin{bmatrix} T_1 & \dots & T_1 \\ \vdots & & \vdots \\ T_1 & \dots & T_1 \end{bmatrix} \quad \Theta_2 = \begin{bmatrix} T_2 & \dots & -T_2 \\ \vdots & & \vdots \\ -T_2 & \dots & T_2 \end{bmatrix} \quad (3.85)$$

where T_1 and T_2 are nonnegative definite since the predetermined constants and the per unit length parameters are nonnegative definite. To simplify the proof, a permutation of the matrices Θ_1 and Θ_2 are considered, which have the following structure

$$\Theta_1 = P_{\Theta_1} \Theta_{P_1} P_{\Theta_1}' \quad \Theta_2 = P_{\Theta_2} \Theta_{P_2} P_{\Theta_2}' \quad (3.86)$$

where P_{Θ_1} and P_{Θ_2} are some nonsingular real matrices and

$$\Theta_{P_1} = \begin{bmatrix} T_1 & T_1 & 0 & \dots & 0 \\ T_1 & T_1 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad \Theta_{P_2} = \begin{bmatrix} T_2 & -T_2 & 0 & \dots & 0 \\ -T_2 & T_2 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (3.87)$$

To demonstrate that Θ_{P_1} and Θ_{P_2} are nonnegative definite, they are expressed as diagonal matrices,

$$\Theta_{P_1} = W_\gamma' \text{diag}(T_1, \mathbf{0}, \dots, \mathbf{0}) W_\gamma \quad (3.88a)$$

$$\Theta_{P_2} = W_\delta' \text{diag}(T_2, \mathbf{0}, \dots, \mathbf{0}) W_\delta \quad (3.88b)$$

where

$$W_\gamma = \begin{bmatrix} U & U & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & U & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & U & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & U \end{bmatrix} \quad W_\delta = \begin{bmatrix} U & -U & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & U & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & U & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & U \end{bmatrix} \quad (3.89)$$

The diagonal matrices of (3.88) are nonnegative definite since T_1 and T_2 are nonnegative definite. This implies that $N_{\phi 1}^i$, $N_{\phi 2}^i$, $N_{\phi 3}^i$ and $N_{\phi 4}^i$ are nonnegative definite according theorem 3.2. The summation of $N_{\phi 1}^i$, $N_{\phi 2}^i$, $N_{\phi 3}^i$ and $N_{\phi 4}^i$ is also nonnegative definite (theorem 3.1), therefore N_ϕ^i satisfies the conditions for passive reduction using Krylov space techniques.

To show that C_ϕ^i is nonnegative definite, the matrix is also represented as a summation of nonnegative definite matrices,

$$C_\phi^i = C_{\phi 1}^i + C_{\phi 2}^i + C_{\phi 3}^i \quad (3.90)$$

where

$$C_{\phi 1}^i = \begin{bmatrix} \frac{d}{4x_i}C & 0 & 0 & \frac{d}{4x_i}C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{d}{4x_i}C & 0 & 0 & \frac{d}{4x_i}C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.91a)$$

$$C_{\phi 2}^i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{x_i d}{x_i^2 + y_i^2}C & \frac{-x_i d}{x_i^2 + y_i^2}C & 0 & 0 & 0 \\ 0 & 0 & \frac{-x_i d}{x_i^2 + y_i^2}C & \frac{x_i d}{x_i^2 + y_i^2}C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.91b)$$

$$C_{\phi 3}^i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{d}{x_i}L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{4x_i d}{x_i^2 + y_i^2}L \end{bmatrix} \quad (3.91c)$$

The matrix $C_{\phi 3}^i$ is nonnegative definite since all the diagonal elements are nonnegative definite. The form of $C_{\phi 1}^i$ and $C_{\phi 2}^i$ are similar to (3.85), which can be expressed as diagonal matrices proven to be nonnegative definite (3.85)-(3.89). Therefore C_{ϕ}^i also satisfies the conditions for passive reduction (theorem 1 and 2). Similar arguments can be made for

the first order subsection of (3.48) to show that the MNA matrices are nonnegative definite. This demonstrates that the time domain representation of (3.46) is compatible with passive model reduction techniques based on Krylov space methods.

3.5 Criteria for Selecting the Order of Approximation

Accuracy of the proposed model depends on the order of the approximation (N, M) . Since all the elements in (3.4) are computed in a closed-form manner, the required order (N, M) can be easily estimated using the following error criterion

$$\left\| \frac{e^z - (P_{N,M}^{-1}(Z)Q_{N,M}(Z))}{e^z} \right\| < \varepsilon \quad (3.92)$$

where ε is the predefined percentage error tolerance. If the error tolerance is not satisfied over the frequency range of interest, the order of the macromodel can be increased either by the closed-form relations of (3.5) or (3.8). Alternatively, the difference between a higher order approximation and a lower order approximation [32] derived as,

$$\left\| \frac{P_{N+1,M+1}^{-1}(Z)Q_{N+1,M+1}(Z) - P_{N,M}^{-1}(Z)Q_{N,M}(Z)}{P_{N+1,M+1}^{-1}(Z)Q_{N+1,M+1}(Z)} \right\| < \varepsilon \quad (3.93)$$

can also be used to estimate the error tolerance.

3.6 Numerical examples

Five examples are presented in this section to demonstrate the validity and efficiency of the proposed macromodel. The simulations results are compared with the lumped segmentation model and/or 'exact' analysis of the distributed transmission line equations. Within the context of this section, 'exact' analysis refers to solving the equations of the

linear subnetwork analytically through eigenvalue/eigenvector analysis. Whenever possible, the number of variables introduced by the Padé and lumped models are compared to analyze their accuracy and efficiency. All comparisons with the lumped model are made with (3.46) since both forms are expressed in terms of RLC components.

3.6.1 Example 3.1

The simple MCM interconnect (Figure 3.4) is used to show the accuracy of the proposed macromodel. Figure 3.5 shows the comparison of the output frequency domain using three different approaches; proposed macromodel, conventional lumped segmentation model and 'exact' analysis. The proposed macromodel cuts the line into four equal parts, each of order (9/10), while the lumped segmentation model uses 40 sections. Both approximations add equal number of variables to the MNA equations (119 variables). In terms of accuracy, the proposed model matches the 'exact' frequency response up to 2.75GHz, while the lumped segmentation model deviates from the exact response at 0.5GHz. The transient response corresponding to an input pulse with rise/fall times of 0.35ns and a pulse width of 1ns is shown in Figure 3.6. Once again the proposed method obtains better accuracy when compared with the lumped segmentation model for their given approximations.

Figure 3.4: MCM interconnect example

Figure 3.5: Frequency response of the circuit in example 3.1

Figure 3.6: Transient response of the circuit in example 3.1

3.6.2 Example 3.2

The coupled interconnect configuration proposed in [16] is shown in Figure 3.7. The output frequency response obtained using the proposed model (order 8/8) is compared to the 'exact' response in Figure 3.8. The transient response corresponding to an input pulse with rise/fall times 0.1ns and a pulse width of 0.8ns is shown in Figure 3.9. Both the frequency and transient responses are in agreement when compared to the exact response.

$$R = \begin{bmatrix} 3.448 & 0 & 0 \\ 0 & 3.448 & 0 \\ 0 & 0 & 3.448 \end{bmatrix} \frac{\Omega}{cm} \quad L = \begin{bmatrix} 4.976 & 0.765 & 0.152 \\ 0.765 & 4.976 & 0.765 \\ 0.152 & 0.765 & 4.976 \end{bmatrix} \frac{nH}{cm} \quad C = \begin{bmatrix} 1.082 & -0.197 & -0.006 \\ -0.197 & 1.082 & -0.197 \\ -0.006 & -0.197 & 1.082 \end{bmatrix} \frac{pF}{cm}$$

Figure 3.7: Circuit with coupled interconnects

Figure 3.8: Frequency response of the circuit in example 3.2

Figure 3.9: Transient response of the circuit in example 3.2

3.6.3 Example 3.3

The nonlinear circuit shown in Figure 3.10 was proposed in [55]. Figure 3.11 shows the comparison of Y_{II} of the linear subnetwork obtained using the proposed method (order 7,7) and the 'exact' response. The nonlinear transient response corresponding to a 5V input pulse with rise/fall times of 0.02ns and pulse width of 3ns, using the proposed macromodel and the 'U' model (150 lumped segments) of HSPICE is shown in Figure 3.12. In terms of accuracy the proposed model matches the time domain response given in [55]. The HSPICE 'U' model introduces 1347 new variables to the MNA matrix, whereas the proposed macromodel introduces only 60 new variables (95% savings).

Figure 3.10: Nonlinear circuit with coupled interconnect

Figure 3.11: Frequency response $|Y_{11}|$ of linear subcircuit in example 3.3

Figure 3.12: Transient response of the nonlinear circuit in example 3.3

3.6.4 Example 3.4

In this example, a seven transmission line circuit (Figure 3.13) with nonlinear terminations is considered. Figure 3.12 compares the Y_{II} of the linear subnetwork using the proposed method (order 5/5), lumped segmentation model (33 sections) and the 'exact' analysis. Both the proposed and the lumped segmentation model match the 'exact' response up to 6GHz with their respective approximations. The output transient response of the nonlinear circuit is shown in Figure 3.14. Both models give similar results. The lumped segmentation model introduces 98 new variables for each interconnect, whereas the proposed method introduces only 14 new variable for each interconnect (86% savings).

Figure 3.13: Seven transmission line example with nonlinear terminations

Figure 3.14: Frequency response $|Y_{11}|$ of linear subcircuit in example 3.4

Figure 3.15: Transient response of the nonlinear circuit in example 3.4

3.6.5 Example 3.5

The circuit of Figure 3.13, is reproduced seven times to form the tree structure circuit of Figure 3.16. The overall circuit is composed of 15 nonlinear inverters, 49 interconnects, 70 resistors, 63 inductors and 98 capacitors. The transient response of the circuit is simulated using the proposed method (order 5/5) and lumped segmentation model (33 sections). The simulation results are given in Figure 3.17, which shows the transient responses of both models matching. On a Sun-ultra-5 computer, it takes HSPICE 103 seconds to simulate the results with the lumped segmentation model while the proposed method only requires 23 seconds which is about 4.5 times faster.

Figure 3.16: Large nonlinear circuit example

Figure 3.17: Transient response of the nonlinear circuit in example 3.5

Chapter 4

Frequency Dependent Parameters

4.1 Introduction

Analysis of interconnects with frequency dependent parameters is very important for the design of high speed applications. It is shown in [56], [57] that neglecting these effects leads to significant errors in simulation results. The goal of this chapter is to incorporate frequency dependent parameters to the Padé macromodel while preserving the macromodel's passivity. Some background information is provided in section 4.2 to the physical reasons why interconnect parameters vary with frequency. Section 4.3 establishes the necessary conditions required to preserve passivity for interconnects with frequency dependent parameters. From these conditions, a method of characterizing the interconnect parameters in terms of rational functions is presented in section 4.4. Next, the time domain macromodel is established in section 4.5. Section 4.6 provides numerical examples

to show the validity of the Padé macromodel and to illustrate its application to a variety of interconnect structures.

4.2 Frequency Dependent Interconnects

As frequency increases, an electric field is induced which causes the current distribution of conductors to vary. This causes the per unit length resistance and inductance to change as frequency increases. These changes are classified into three parts: proximity effect, edge effect and skin effect.

4.2.1 Edge and Proximity Effects

The edge and proximity effects influence the per unit length resistance and inductance at the medium frequency range. At low frequencies the per unit length resistance and inductance are practically constant. As frequency increases, the edge effect causes current to concentrate at the sharp edges of the conductor, thus increasing the resistance. This effects both the signal and ground conductor, however the current concentration is more pronounced on the signal conductors. The proximity effect causes current to concentrate in sections of the ground plane that are near the signal conductors. This reduces the magnetic field between the ground and signal conductor which causes the inductance to drop. The proximity effect also increases the resistance of the interconnect since more current is crowded in the ground plane near the signal conductors.

4.2.2 Skin Effect

The skin effect occurs at the high frequency region and influences both ground and signal conductors. As frequency increases, current begins to concentrate in a thin layer at the

conductor surface. The thickness of the layer is related to the skin depth, and decreases as frequency increases. This causes the resistance to be directly proportional to the square root of frequency. The magnetic fields inside the ground and signal conductors are also reduced at high frequencies. This causes the inductance of the interconnect to drop even further. At higher frequencies, the inductance contribution due to the magnetic fields inside the ground and signal conductors becomes negligible and L essentially remains constant.

4.2.3 Typical Behaviour of R and L

The frequency plots of R and L of a stripline in Figure 4.1 are shown in Figure 4.2 and 4.3. These plots show the typical behavior of R and L . Both R and L are essentially constants at low frequencies. As frequency increases, the edge and proximity effects cause the resistance to increase and the inductance to decrease. At higher frequencies the skin effect causes the resistance to be directly proportional to the square root of frequency, while the inductance approaches a constant value.

Figure 4.1: Cross-sectional geometry and dimensions of stripline interconnect

Figure 4.2: Resistance with respect to frequency

Figure 4.3: Inductance with respect to frequency

4.3 Passivity Issues

Before preceding with the development of the time domain macromodel, the issue of how to preserve passivity for frequency dependent parameters must first be considered. Frequency dependent parameters are often characterized as rational functions [16], [58]-[61]. Such structures can easily be incorporated into the proposed macromodel since the model itself is in a rational form.

Let the frequency dependent line parameters be modeled as

$$\mathbf{a} = \mathbf{R}(s) + s\mathbf{L}(s) = \frac{\mathbf{N}_Z}{D_Z} = \frac{\mathbf{f}_k s^k + \dots + \mathbf{f}_0}{b_{k-1} s^{k-1} + \dots + 1} \quad (4.1a)$$

$$\mathbf{b} = \mathbf{G}(s) + s\mathbf{C}(s) = \frac{\mathbf{N}_Y}{D_Y} = \frac{\mathbf{g}_k s^k + \dots + \mathbf{g}_0}{c_{k-1} s^{k-1} + \dots + 1} \quad (4.1b)$$

where N_Z and N_Y are polynomial matrices; D_Z and D_Y are scalar polynomials. In the previous chapter, the macromodel is shown to be passive by examining the scalar form of the Padé approximation with the matrix form. The proof relied on the fact that the polynomials generated by $p_{N, M_{EVEN}}(s) + p_{N, M_{ODD}}(s)$ and $q_{M, N_{EVEN}}(s) - q_{M, N_{ODD}}(s)$ of (3.55) are strict Hurwitz polynomials. Thus, the rational functions formed by

$$\frac{p_{N, M_{EVEN}}(s)}{p_{N, M_{ODD}}(s)} = \chi_N s + \frac{1}{\chi_{N-1} s + \frac{1}{\chi_{N-2} s + \dots \frac{1}{\chi_1 s + \frac{1}{\chi_0 s}}}} \quad (4.2a)$$

$$\frac{q_{M,N_{EVEN}}(s)}{-q_{M,N_{ODD}}(s)} = \varphi_N s + \frac{1}{\varphi_{N-1} s + \frac{1}{\varphi_{N-2} s + \dots \frac{1}{\varphi_1 s + \frac{1}{\varphi_0 s}}}} \quad (4.2b)$$

are positive real, where the constants χ_i and φ_i for $i = 1, 2, \dots, n$ are all positive. Then s is replaced by the matrix $\mathbf{a}$ for every even coefficient $[\chi_0 \ \chi_2 \ \chi_4 \ \dots]$, $[\varphi_0 \ \varphi_2 \ \varphi_4 \ \dots]$ and s is replaced by the matrix $\mathbf{b}$ for every odd coefficient $[\chi_1 \ \chi_3 \ \chi_5 \ \dots]$, $[\varphi_1 \ \varphi_3 \ \varphi_5 \ \dots]$ to yield new rational function matrices

$$\frac{\mathbf{P}_{ab_{EVEN}}}{\mathbf{P}_{ab_{ODD}}} = \chi_N \mathbf{b} + (\chi_{N-1} \mathbf{a} + (\chi_{N-2} \mathbf{b} + (\dots + (\chi_1 \mathbf{b} + (\chi_0 \mathbf{a})^{-1})^{-1})^{-1})^{-1})^{-1} \quad (4.3a)$$

$$\frac{\mathbf{Q}_{ab_{EVEN}}}{-\mathbf{Q}_{ab_{ODD}}} = \varphi_N \mathbf{b} + (\varphi_{N-1} \mathbf{a} + (\varphi_{N-2} \mathbf{b} + (\dots + (\varphi_1 \mathbf{b} + (\varphi_0 \mathbf{a})^{-1})^{-1})^{-1})^{-1})^{-1} \quad (4.3b)$$

If the rational functions of (4.1) are positive real then the rational functions of (4.3) are also positive real (theorem 3.3 and 3.4). Using this fact, the functions formed by the polynomials of (3.7)

$$\mathbf{P}_{N,M_{11}}[\mathbf{P}_{N,M_{12}}]^{-1}, \quad \mathbf{P}_{N,M_{21}}[\mathbf{P}_{N,M_{22}}]^{-1} \quad \mathbf{Q}_{M,N_{11}}[-\mathbf{Q}_{M,N_{12}}]^{-1}, \quad \mathbf{Q}_{M,N_{21}}[-\mathbf{Q}_{M,N_{22}}]^{-1} \quad (4.4a)$$

$$\mathbf{P}_{N,M_{11}}[\mathbf{P}_{N,M_{21}}]^{-1}, \quad \mathbf{P}_{N,M_{21}}[\mathbf{P}_{N,M_{11}}]^{-1} \quad \mathbf{Q}_{M,N_{11}}[-\mathbf{Q}_{M,N_{21}}]^{-1}, \quad \mathbf{Q}_{M,N_{21}}[-\mathbf{Q}_{M,N_{11}}]^{-1} \quad (4.4b)$$

are shown to be positive real. Ensuring that the functions of (4.4) remain positive real results in a passive macromodel. The proof of chapter 3 relied on this fact to prove the macromodel's passivity. This implies that (4.1) must be positive real to make the macromodel passive. The issue of how to obtain positive real rational functions is considered next in section 4.4.

4.4 Modelling of Frequency Dependent Parameters

The modeling of frequency dependent line parameters can be done analytically or by curve fitting techniques [16], [58]-[62]. For example, analytical methods can model the skin effect as the square root of frequency by forming passive circuit networks composed of resistors and inductors [58], [59]. The rational functions created by these passive elements can be incorporated with the proposed macromodel to simulate the skin effect of the interconnect. The limitation of analytical methods is that they only model specific characteristics. The methods of [58] and [59] accurately model the skin effect as the square root of frequency, however, other effects such as the edge and proximity effects are ignored. Frequency dependent parameters are most accurately characterized by measured or simulated data than by closed form functions [61]. In this section, a curve fitting technique is proposed that characterizes the frequency dependent line parameters as positive real rational functions.

4.4.1 The Modelling of R and L

The proposed curve fitting technique uses linear constraint optimization to obtain positive real rational functions. The data used in the fitting algorithm can be obtained from measurements, empirical formula or electromagnetic simulation [56], [63]-[65]. The tabulated data for R and L, regardless of how it is obtained is referred to as 'measured data' in this thesis.

Directly forcing the rational function of (4.1) to be positive real requires nonlinear constraint optimization. However, nonlinear constraint optimization is usually difficult to implement and should be avoided if possible [16]. Networks composed of passive circuit elements have often been used to characterize positive real rational functions [46]-[48],

[50], [58], [59], [66]. Representing rational functions by passive circuit elements simplifies the formulation to a linear constraint optimization problem. In addition, most commercial circuit simulators are capable of optimizing linear networks using sensitivity analysis [67], [68].

For the sake of simplicity of presentation, the discussion will focus on single interconnect structures and then extend to multiconductor lines. In the past, the circuit configurations of Figure 4.4 have been proposed to model the skin effect of interconnects [58], [66]. The proposed algorithm matches the input impedance of the circuit with the 'measured data' values of $R(s) + sL(s)$. By ensuring each element of Figure 4.4 is nonnegative, the resulting network remains passive and the rational function formed by the input impedance is positive real [46]-[48], [50].

Figure 4.4a: RL canonical circuit configuration

Figure 4.4b: RL tank circuit configuration

For multiconductor interconnects, the circuit topology is composed of resistors, inductors and ideal transformers. As an example, Figure 4.5 shows the circuit topology for the three conductor case that represents $R(s)+sL(s)$. The general form of the rational function matrix becomes

$$\mathbf{a}(s) = \begin{bmatrix} Z_{11} + Z_{00} + \sum_{j=2}^{\psi} T_{1j}^2 Z_{1j} & T_{12}T_{21}Z_{12} + Z_{00} & \dots & T_{1\psi}T_{\psi 1}Z_{1\psi} + Z_{00} \\ T_{12}T_{21}Z_{12} + Z_{00} & Z_{22} + Z_{00} + \sum_{\substack{j=1 \\ j \neq 2}}^{\psi} T_{2j}^2 Z_{2j} & \dots & T_{2\psi}T_{\psi 2}Z_{2\psi} + Z_{00} \\ \vdots & \vdots & \ddots & \vdots \\ T_{1\psi}T_{\psi 1}Z_{1\psi} + Z_{00} & T_{2\psi}T_{\psi 2}Z_{1\psi} + Z_{00} & \dots & Z_{\psi\psi} + Z_{00} + \sum_{j=1}^{\psi-1} T_{\psi j}^2 Z_{\psi j} \end{bmatrix} \quad (4.5)$$

where T_{ij} is the turn ratio of the ideal transformer and Z_{ij} represents the impedance of the circuit given by Figure 4.5b or Figure 4.5c. The matrix configuration of (4.5) represents the Z parameters of the circuit for the general case. Imposing the constraints that all resistors and inductors are nonnegative makes (4.5) positive real [46]-[48], [50].

The circuit topologies of the RL canonical and RL tank configurations (Figure 4.4 and 4.5) were implemented using HSPICE [67]. Both forms fitted the ‘measured data’ of $R(s)+sL(s)$. For the examples given in this thesis, the RL canonical configuration matched the data with greater accuracy than the RL tank configuration. The RL tank circuit represents the function as a summation of smaller positive real rational functions formed by each RL tank. This restriction is not imposed by the RL canonical representation which provides greater flexibility to match the measured data values. The rational function approximations given in this thesis are from the RL canonical form since it provided greater accuracy.

Figure 4.5a: Circuit topology for $R(s)+sL(s)$ for three conductor interconnect case

Figure 4.5b: Circuit representation of Z_{ij} , RL canonical configuration

Figure 4.5c: Circuit representation of Z_{ij} , RL tank configuration

4.5 Development of the Time Domain Macromodel

The development of the time domain macromodel for frequency dependent parameter case is very similar to that of the frequency independent parameter case. The discussion will focus on how to incorporate (4.1) into the Padé approximation. The time domain macromodel given in this section focuses on how to represent the ordinary differential equations in terms of subsections obtained by the pole-zero pairs of the Padé function. From this formulation, a method of representing each subsection in terms of circuit elements (i.e. resistors, inductors, capacitors and ideal transformers) is shown.

4.5.1 Time Domain Realization of Padé Function as Subsections

As discussed in section 3.3.3, the polynomial matrices of (3.4) can be expressed as products of lower order polynomials derived from the poles and zeros of the Padé function. This allows the model to be viewed as series of subcircuits formed by the per unit length parameters and predetermined constants. The hybrid stencil for each subsection i is written as

$$[P_{n,n}(Z)]_{i+1} \begin{bmatrix} V_{i+1} \\ I_{i+1} \end{bmatrix} = [Q_{n,n}(Z)]_i \begin{bmatrix} V_i \\ I_i \end{bmatrix} \quad (4.6)$$

Using (3.32), (3.33) and (4.1), the matrices $[P_{n,n}(Z)]_i$ and $[Q_{n,n}(Z)]_i$ for the complex pole-zero pairs become

$$\begin{aligned}
[\mathbf{P}_{n,n}(\mathbf{Z})]_i &= \begin{bmatrix} N_Z N_Y + (x_i^2 + y_i^2) D_Z D_Y U & 2x_i N_Z D_Y \\ 2x_i N_Y D_Z & N_Z N_Y + (x_i^2 + y_i^2) D_Z D_Y U \end{bmatrix} \\
&= \begin{bmatrix} \sum_{\lambda=0}^{2k} \mathbf{F}_{\lambda}^i S^{\lambda} & \sum_{\lambda=0}^{2k-1} \mathbf{H}_{\lambda}^i S^{\lambda} \\ \sum_{\lambda=0}^{2k-1} \mathbf{J}_{\lambda}^i S^{\lambda} & \sum_{\lambda=0}^{2k} \mathbf{F}_{\lambda}^i S^{\lambda} \end{bmatrix} \quad (4.7a)
\end{aligned}$$

$$\begin{aligned}
[\mathbf{Q}_{n,n}(\mathbf{Z})]_i &= \begin{bmatrix} N_Z N_Y + (x_i^2 + y_i^2) D_Z D_Y U & -2x_i N_Z D_Y \\ -2x_i N_Y D_Z & N_Z N_Y + (x_i^2 + y_i^2) D_Z D_Y U \end{bmatrix} \\
&= \begin{bmatrix} \sum_{\lambda=0}^{2k} \mathbf{F}_{\lambda}^i S^{\lambda} & \sum_{\lambda=0}^{2k-1} -\mathbf{H}_{\lambda}^i S^{\lambda} \\ \sum_{\lambda=0}^{2k-1} -\mathbf{J}_{\lambda}^i S^{\lambda} & \sum_{\lambda=0}^{2k} \mathbf{F}_{\lambda}^i S^{\lambda} \end{bmatrix} \quad (4.7b)
\end{aligned}$$

For the subsection consisting of a real pole-zero pair, the matrices $[\mathbf{P}_{n,n}(\mathbf{Z})]_i$ and $[\mathbf{Q}_{n,n}(\mathbf{Z})]_i$ are given as

$$[\mathbf{P}_{n,n}(\mathbf{Z})]_0 = \begin{bmatrix} a_0 D_Z U & N_Z \\ N_Y & a_0 D_Y U \end{bmatrix} = \begin{bmatrix} \sum_{\lambda=0}^{k-1} \mathbf{K}_{\lambda} S^{\lambda} & \sum_{\lambda=0}^k \mathbf{f}_{\lambda} S^{\lambda} \\ \sum_{\lambda=0}^k \mathbf{g}_{\lambda} S^{\lambda} & \sum_{\lambda=0}^{k-1} \mathbf{W}_{\lambda} S^{\lambda} \end{bmatrix} \quad (4.8a)$$

$$[\mathbf{Q}_{n,n}(\mathbf{Z})]_0 = \begin{bmatrix} a_0 D_Z U & -N_Z \\ -N_Y & a_0 D_Y U \end{bmatrix} = \begin{bmatrix} \sum_{\lambda=0}^{k-1} \mathbf{K}_{\lambda} S^{\lambda} & \sum_{\lambda=0}^k -\mathbf{f}_{\lambda} S^{\lambda} \\ \sum_{\lambda=0}^k -\mathbf{g}_{\lambda} S^{\lambda} & \sum_{\lambda=0}^{k-1} \mathbf{W}_{\lambda} S^{\lambda} \end{bmatrix} \quad (4.8b)$$

Using (4.6)-(4.8), the time domain macromodel for each subsections can be expressed as,

$$\begin{bmatrix} [p_a]_i & [q_a]_i \end{bmatrix} \begin{bmatrix} \dot{x}_{i+1} \\ \dot{x}_i \end{bmatrix} + \begin{bmatrix} [p_b]_i & [q_b]_i \end{bmatrix} \begin{bmatrix} x_{i+1} \\ x_i \end{bmatrix} = 0 \quad (4.9)$$

For the subsections consisting of complex pole-zero pairs (4.7), the coefficients of (4.9) are given as,

$$\begin{aligned} [p_a]_i &= \begin{bmatrix} U_x \\ 0 \\ M_i \end{bmatrix} & [p_b]_i &= \begin{bmatrix} U_y \\ 0 \\ S_i \end{bmatrix} \\ [q_a]_i &= \begin{bmatrix} 0 \\ U_x \\ N_i \end{bmatrix} & [q_b]_i &= \begin{bmatrix} 0 \\ U_y \\ T_i \end{bmatrix} \end{aligned} \quad (4.10)$$

$$x_{i+1} = [v_{i+1} \ i_{i+1} \ \gamma_{i+1}^0 \ \rho_{i+1}^0 \ \dots \ \gamma_{i+1}^{2k-1} \ \rho_{i+1}^{2k-1}]^t \quad x_i = [v_i \ i_i \ \gamma_i^0 \ \rho_i^0 \ \dots \ \gamma_i^{2k-1} \ \rho_i^{2k-1}]^t$$

where $[\gamma_i^0 \ \rho_i^0 \ \dots \ \gamma_i^{2k-1} \ \rho_i^{2k-1}]$ are extra variables needed for the realization and

$$M_i = \begin{bmatrix} F_1^i & H_1^i & F_2^i & H_2^i & \dots & F_{2k-1}^i & H_{2k-1}^i & F_{2k}^i & 0 \\ J_1^i & F_1^i & J_2^i & F_2^i & \dots & J_{2k-1}^i & F_{2k-1}^i & 0 & F_{2k}^i \end{bmatrix}_{2 \times 4k} \quad (4.11a)$$

$$N_i = \begin{bmatrix} -F_1^i & H_1^i & -F_2^i & H_2^i & \dots & -F_{2k-1}^i & H_{2k-1}^i & -F_{2k}^i & 0 \\ J_1^i & -F_1^i & J_2^i & -F_2^i & \dots & J_{2k-1}^i & -F_{2k-1}^i & 0 & -F_{2k}^i \end{bmatrix}_{2 \times 4k} \quad (4.11b)$$

$$S_i = \begin{bmatrix} F_0^i & H_0^i & 0 & 0 & \dots & 0 & 0 \\ J_0^i & F_0^i & 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{2 \times 4k} \quad (4.11c)$$

$$T_i = \begin{bmatrix} -F_0^i & H_0^i & 0 & 0 & \dots & 0 & 0 \\ J_0^i & -F_0^i & 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{2 \times 4k} \quad (4.11d)$$

$$U_X = \begin{bmatrix} U & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & U & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & U & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & U & 0 & 0 \end{bmatrix}_{(4k-2) \times 4k} \quad (4.11e)$$

$$U_Y = \begin{bmatrix} 0 & 0 & -U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -U & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -U & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & -U & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & -U \end{bmatrix}_{(4k-2) \times 4k} \quad (4.11f)$$

For the subsection consisting of a real pole-zero pair (4.8), the coefficients of (4.9) are given as

$$\begin{aligned} [p_a]_0 &= [M_0] = \begin{bmatrix} M_{M0} & 0 \\ 0 & 0 \end{bmatrix} & [p_b]_0 &= [S_0] = \begin{bmatrix} S_{S0} & 0 \\ 0 & 0 \end{bmatrix} \\ [q_a]_0 &= [N_0] = \begin{bmatrix} N_{N0} \\ U_X \end{bmatrix} & [q_b]_0 &= [T_0] = \begin{bmatrix} T_{T0} \\ U_Y \end{bmatrix} \\ x_1 &= [v_1 \ i_1 \ \gamma_1^0 \ \rho_1^0 \ \dots \ \gamma_1^{2k-1} \ \rho_1^{2k-1}]^t & x_0 &= [v_0 \ i_0 \ \gamma_0^0 \ \rho_0^0 \ \dots \ \gamma_0^{k-1} \ \rho_0^{k-1}]^t \end{aligned} \quad (4.12)$$

where $[\gamma_0^0 \ \rho_0^0 \ \dots \ \gamma_0^{k-1} \ \rho_0^{k-1}]$ are extra variables needed for the realization and

$$M_{M0} = \begin{bmatrix} K_1 & f_1 & K_2 & f_2 & \dots & K_{k-1} & f_{k-1} & 0 & f_k \\ g_1 & W_1 & g_2 & W_2 & \dots & g_{k-1} & W_{k-1} & g_k & 0 \end{bmatrix}_{2 \times 2k} \quad (4.13a)$$

$$N_{N0} = \begin{bmatrix} -K_1 & f_1 & -K_2 & f_2 & \dots & -K_{k-1} & f_{k-1} & 0 & f_k \\ g_1 & -W_1 & g_2 & -W_2 & \dots & g_{k-1} & -W_{k-1} & g_k & 0 \end{bmatrix}_{2 \times 2k} \quad (4.13b)$$

$$S_{S0} = \begin{bmatrix} K_0 & f_0 & 0 & 0 & \dots & 0 & 0 \\ g_0 & W_0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{2 \times 2k} \quad (4.13c)$$

$$T_{T0} = \begin{bmatrix} -K_0 & f_0 & 0 & 0 & \dots & 0 & 0 \\ g_0 & -W_0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{2 \times 2k} \quad (4.13d)$$

$$U_X = \begin{bmatrix} U & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & U & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & U & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & U & 0 & 0 \end{bmatrix}_{(k-2) \times 2k} \quad (4.13e)$$

$$U_Y = \begin{bmatrix} 0 & 0 & -U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -U & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -U & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & -U & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & -U \end{bmatrix}_{(k-2) \times 2k} \quad (4.13f)$$

The differential equations obtained by (4.9)-(4.13) can be assembled together to form the overall MNA stamp of the transmission line, written as

$$\begin{bmatrix}
U_X & 0 & 0 & \dots & 0 & 0 & 0 \\
0 & U_X & 0 & \dots & 0 & 0 & 0 \\
M_\theta & N_\theta & 0 & \dots & 0 & 0 & 0 \\
0 & 0 & U_X & \dots & 0 & 0 & 0 \\
0 & M_{\theta-1} & N_{\theta-1} & \dots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \dots & 0 & U_X & 0 \\
0 & 0 & 0 & \dots & M_1 & N_1 & 0 \\
0 & 0 & 0 & \dots & 0 & M_0 & N_0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_{\theta+1} \\
\dot{x}_\theta \\
\dot{x}_{\theta-1} \\
\vdots \\
\vdots \\
\vdots \\
\dot{x}_2 \\
\dot{x}_1 \\
\dot{x}_0
\end{bmatrix}
+
\begin{bmatrix}
U_Y & 0 & 0 & \dots & 0 & 0 & 0 \\
0 & U_Y & 0 & \dots & 0 & 0 & 0 \\
S_\theta & T_\theta & 0 & \dots & 0 & 0 & 0 \\
0 & 0 & U_Y & \dots & 0 & 0 & 0 \\
0 & S_{\theta-1} & T_{\theta-1} & \dots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \dots & 0 & U_Y & 0 \\
0 & 0 & 0 & \dots & S_1 & T_1 & 0 \\
0 & 0 & 0 & \dots & 0 & S_0 & T_0
\end{bmatrix}
\begin{bmatrix}
x_{\theta+1} \\
x_\theta \\
x_{\theta-1} \\
\vdots \\
\vdots \\
\vdots \\
x_2 \\
x_1 \\
x_0
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
\vdots \\
\vdots \\
\vdots \\
0 \\
0 \\
0
\end{bmatrix} \quad (4.14)$$

It should be noted that the differential equations of (4.14) are obtained analytically in terms of polynomial coefficients of (4.1) and predetermined constants of the macromodel. Since all the coefficients of (4.14) are known a priori in terms of per unit length parameters and predetermined constants, (4.14) can easily be stenciled into a circuit simulator as the stamp of the interconnect.

4.5.2 Time Domain Realization in Terms of Circuit Elements

The time domain macromodel of each subsection can also be represented in terms of circuit elements. The circuit representation for frequency dependent parameter case is obtained by converting each subsection of (4.6) into the Y-parameters. From the Y-parameters a circuit model is derived that represents each subsection in terms of resistors, inductors, capacitors and ideal transformers.

Using (3.44)-(3.45) and (4.1), the Y-parameters of the subsections for the frequency dependent case are given as

$$Y_{11} = Y_{22} = \frac{(x_i^2 + y_i^2)}{4x_id}(a(s))^{-1} + \frac{a(s)d}{4x_i} + x_i \left(a(s)d + \frac{(x_i^2 + y_i^2)}{d}(b(s))^{-1} \right)^{-1} \quad (4.15)$$

$$Y_{12} = Y_{21} = \frac{(x_i^2 + y_i^2)}{4x_id}(a(s))^{-1} + \frac{b(s)d}{4x_i} - x_i \left(a(s)d + \frac{(x_i^2 + y_i^2)}{d}(b(s))^{-1} \right)^{-1}$$

for the complex pole-zero subsection and

$$Y_{11} = Y_{22} = \frac{a_0}{2d}(a(s))^{-1} + \frac{d}{2a_0}(b(s)) \quad (4.16)$$

$$Y_{12} = Y_{21} = -\frac{a_0}{2d}(a(s))^{-1} + \frac{d}{2a_0}(b(s))$$

for the real pole subsection. The circuit representations of (4.15) and (4.16) are obtained by incorporating the network representation of the line parameters (Figure 4.4 or 4.5), with the circuit representation of the Padé macromodel (Figure 3.3). Replacing the R and L elements of Figure 3.3 with the circuit topology of Figure 4.4 or 4.5 realizes the Y-parameters of (4.15) and (4.16). For the single conductor case, the subsections are shown in Figures 4.6 and 4.7. Figure 4.6 shows the topology of the complex pole-zero subsection and Figure 4.7 shows the topology of the real pole-zero subsection. The MNA stamp of each subsection is written as

$$C_o^i \ddot{X}_o^i + G_o^i \dot{X}_o^i = 0 \quad (4.17)$$

Figure 4.6a: Circuit model of complex pole-zero subsection for single conductor line

Figure 4.6b: $Z_{\alpha i}$ circuit of Figure 4.6a (RL canonical representation)

Figure 4.6c: $Z_{\beta i}$ circuit of Figure 4.6a (RL canonical representation)

Figure 4.7a: Circuit model for real pole-zero subsection for single conductor line

Figure 4.7b: Z_0 circuit of Figure 4.7a (RL canonical representation)

Equations (4.18) and (4.19) describe C_o^i and G_o^i of (4.17), for a second order approximation of $R(s) + sL(s)$, for the case of a single reference interconnect. Equation (4.18) describes the complex pole subsection (Figure 4.6) and (4.19) describes the real pole subsection (Figure 4.7).

$$G_o^i = \begin{array}{c|cccc|cccc|cccc} \text{Node } V_i & \text{Internal Nodes} & & & & \text{Node } V_{i+1} & & & & \text{Nodes due to L} \\ \hline \frac{dG}{4x_i} & 0 & 0 & 0 & 0 & 0 & \frac{dG}{4x_i} & 1 & 0 & 1 & 0 \\ 0 & \frac{x_i}{dR_1} & 0 & \frac{-x_i}{dR_1} & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & \frac{x_i}{dR_2} & \frac{-x_i}{dR_2} & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{-x_i}{dR_1} & \frac{-x_i}{dR_2} & \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{x_i}{d} + \frac{x_i dG}{x_i^2 + y_i^2} & 0 & 0 & \frac{-x_i dG}{x_i^2 + y_i^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{x_i^2 + y_i^2}{4x_i dR_1} & 0 & \frac{x_i^2 + y_i^2}{4x_i dR_1} & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{x_i^2 + y_i^2}{4x_i dR_2} & \frac{x_i^2 + y_i^2}{4x_i dR_2} & 0 & 0 & 0 & -1 \\ \frac{dG}{4x_i} & 0 & 0 & \frac{-x_i dG}{x_i^2 + y_i^2} & \frac{x_i^2 + y_i^2}{4x_i dR_1} & \frac{x_i^2 + y_i^2}{4x_i dR_2} & \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{x_i^2 + y_i^2}{4x_i d} + \frac{x_i dG}{x_i^2 + y_i^2} + \frac{dG}{4x_i} & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}$$

(4.18)

$$C_o^i = \begin{array}{c|cccc|cccc|cccc} \text{Node } V_i & \text{Internal nodes} & & & & \text{Node } V_{i+1} & & & & \text{Nodes due to L} \\ \hline \frac{dC}{4x_i} & 0 & 0 & 0 & 0 & 0 & \frac{dC}{4x_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_i dC}{x_i^2 + y_i^2} & 0 & 0 & \frac{-x_i dC}{x_i^2 + y_i^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{dC}{4x_i} & 0 & 0 & \frac{-x_i dC}{x_i^2 + y_i^2} & 0 & 0 & \frac{x_i dC}{x_i^2 + y_i^2} + \frac{dC}{4x_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{dL_1}{x_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dL_2}{x_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4x_i dL_1}{x_i^2 + y_i^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4x_i dL_1}{x_i^2 + y_i^2} & 0 \end{array}$$

$$X_o^i = \begin{bmatrix} X_{V1} \\ X_{V2} \\ X_{V3} \\ X_{V4} \\ X_{V5} \\ X_{V6} \\ X_{V7} \\ X_{I1} \\ X_{I2} \\ X_{I3} \\ X_{I4} \end{bmatrix}$$

$$G_{\phi}^i = \begin{array}{c} \begin{array}{cccccc} \text{Node } V_i & \text{Internal nodes} & & \text{Node } V_{i+1} & \text{Nodes due to L} \\ \hline \frac{d}{2a_0}G & 0 & 0 & \frac{d}{2a_0}G & 1 & 0 \\ 0 & \frac{a_0}{2d}R_1^{-1} & 0 & \frac{-a_0}{2d}R_1^{-1} & -1 & 1 \\ 0 & 0 & \frac{a_0}{2d}R_2^{-1} & \frac{-a_0}{2d}R_2^{-1} & 0 & -1 \\ \frac{d}{2a_0}G & \frac{-a_0}{2d}R_1^{-1} & \frac{-a_0}{2d}R_2^{-1} & \frac{a_0}{2d}(R_1^{-1} + R_2^{-1}) + \frac{d}{2a_0}G & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{array} \end{array} \quad (4.19)$$

$$C_{\phi}^i = \begin{array}{c} \begin{array}{cccccc} \text{Node } V_i & \text{Internal nodes} & \text{Node } V_{i+1} & \text{Nodes due to L} \\ \hline \frac{d}{2a_0}C & 0 & 0 & \frac{d}{2a_0}C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{d}{2a_0}C & 0 & 0 & \frac{d}{2a_0}C & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2d}{a_0}L_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2d}{a_0}L_2 \end{array} \end{array} \quad X_{\phi}^i = \begin{bmatrix} X_{V1} \\ X_{V2} \\ X_{V3} \\ X_{V4} \\ X_{I1} \\ X_{I2} \end{bmatrix}$$

The MNA matrices for the frequency dependent parameter case are similar to that of the frequency independent parameter case. The inductor elements are all nonnegative since the predetermined constants and the optimized inductor values are nonnegative. The general stamps for the resistor and capacitor elements of (4.18) and (4.19) are

$$\Theta_1 = \begin{bmatrix} T_1 & \dots & T_1 \\ T_1 & \dots & T_1 \end{bmatrix} \quad \Theta_2 = \begin{bmatrix} T_2 & \dots & -T_2 \\ -T_2 & \dots & T_2 \end{bmatrix} \quad (4.20)$$

where T_1 and T_2 are also nonnegative due to the predetermined constants and per unit length parameter components. It was shown in Chapter 3 that the MNA matrices formed by the stamps of (4.20) are nonnegative definite. This demonstrates that the time domain macromodel for the frequency dependent parameter case is compatible with passive reduction techniques based on Krylov space methods [31].

The time domain macromodel for multiconductor interconnects with frequency dependent parameters is realized with circuit elements composed of resistors, inductors, capacitors and ideal transformers. As an example, the circuit topology for a two conductor line is given in Figure 4.8 and 4.9. Figure 4.8 shows the topology of the complex pole subsection and Figure 4.9 shows the topology of the real pole subsection. The MNA stamps for the inductors, resistors and capacitors are similar to those of the scalar case. The stamp of the ideal transformer is

$$\begin{bmatrix} \mathbf{0} & \vdots & E_{T_r} \\ \vdots & \ddots & \vdots \\ -E_{T_r}^t & \vdots & \mathbf{0} \end{bmatrix} = \begin{bmatrix} & \bar{v}_j & \bar{v}_{j+1} & \bar{v}_k & \bar{v}_{k+1} & \\ & & & & & 1 \\ & & & & & -1 \\ & & & & & -T_r \\ & & & & & T_r \\ -1 & 1 & T_r & -T_r & & \end{bmatrix} \quad (4.21)$$

where $E_{T_r}^t = \begin{bmatrix} 1 & -1 & -T_r & T_r \end{bmatrix}$, $\bar{v}_j, \bar{v}_{j+1}, \bar{v}_k, \bar{v}_{k+1}$ are the terminal node voltages and T_r is the turn ratio of the transformer. Equation (4.21) satisfies the Krylov conditions as defined in section 3.4.3, therefore, the MNA matrices formed by the multiconductor interconnect case can also be incorporated with Krylov space reduction.

Figure 4.8a: Circuit model for complex pole-zero subsection for two conductor line

Figure 4.8b: $Z_{\alpha jk}$ circuit of Figure 4.8a (RL canonical representation)

Figure 4.8c: $Z_{\beta jk}$ circuit of Figure 4.8a (RL canonical representation)

Figure 4.8d: $Y_{\alpha jk}$ circuit of Figure 4.8a

Figure 4.8e: $Y_{\beta jk}$ circuit of Figure 4.8a

Figure 4.9a: Circuit model of real pole-zero subsection for two conductor line

Figure 4.9b: Z_{jk} circuit of Figure 4.9a (RL canonical representation)

Figure 4.9c: Y_{jk} circuit of Figure 4.9a

4.6 Numerical Examples

Four examples are presented in this section to demonstrate the validity and efficiency of the proposed macromodel. The simulation results of the Padé macromodel are compared with ‘exact’ analysis. Within the context of this section, ‘exact’ analysis refers to using the ‘measured data’ values to represent the frequency dependent line parameters and solving the equations of the linear subnetwork analytically through eigenvalue/eigenvector analysis.

4.6.1 Example 4.1

The frequency dependent parameters of the seven transmission line circuit (example 3.4) is considered. The interconnect dimensions are shown in Figure 4.1. The ‘measured data’ values for R and L are given in Table 4.1. These values were obtained using an electromagnetic simulator [65]. Figure 4.10 and 4.11 compare the rational function approximation (RL canonical order 4/3) with the ‘measured data’ for R and L respectively. The frequency response of Y_{II} for the linear subnetwork is shown in Figure 4.12. The proposed model (order 4/4) matches the ‘exact’ response up to 6GHz. The output transient response of the nonlinear circuit is shown in Figure 4.13. Comparing the simulation results with example 3.4 (section 3.6.4) reveals that neglecting the frequency dependency of the interconnect parameters gives in different simulation results.

F(GHz)	1e-5	4.1e-5	6.9e-4	2.8e-3	1.2e-2	4.7e-2	0.19	0.79	1.05	1.39	1.84	2.44	3.24	4.29	5.68	7.54	10.0
R(Ω /m)	2.35	2.47	2.63	2.81	3.28	4.86	9.52	19.1	22.1	25.3	29.1	33.5	38.6	44.5	51.1	59.1	68.0
L(nH/m)	469	467	398	381	369	363	359	356	356	356	355	355	355	355	354	354	354

Table 4.1: R and L as functions of frequency for Figure 4.1

Figure 4.10: R as a function of frequency (example 4.1)

Figure 4.11: L as a function of frequency (example 4.1)

Figure 4.12: Frequency response $|Y_{11}|$ of linear subcircuit in example 4.1

Figure 4.13: Transient response of the nonlinear circuit in example 4.1

Figure 4.15: R_{22} as a function of frequency (example 4.2)

Figure 4.16: L_{22} as a function of frequency (example 4.2)

Figure 4.17: Frequency response of the circuit in example 4.2

Figure 4.18: Transient response of the circuit in example 4.2

4.6.3 Example 4.3

The transmission line circuit of Figure 4.19 is considered. The interconnect dimensions are shown in Figure 4.20. The ‘measured data’ values for the R and L matrices (Table 4.2) are obtained using an electromagnetic simulator [65]. The rational function approximations for R_{12} , L_{12} , R_{11} and L_{11} are compared with the ‘measured data’ values in Figure 4.21, 4.22, 4.23 and 4.24, respectively. The output frequency response given by the proposed model (order 8/8) is compared with ‘exact’ analysis in Figure 4.25. The proposed model matches the ‘exact’ response up to 7GHz. The transient response corresponding to a unit step with rise time 0.5ns is shown in Figure 4.26. The transient response of the proposed model matches the IFFT response.

Figure 4.19: Coupled interconnects with frequency dependent parameters

Figure 4.20: Cross-sectional geometry and dimensions of four conductor overlay

F(GHz)	1e-5	1.7e-4	6.9e-4	2.8e-3	1.2e-2	4.7e-2	0.19	0.79	1.05	1.39	1.84	2.44	3.24	4.29	5.68	7.54
$R_{11}(\Omega/\text{m})$	14.2	14.7	15.1	15.4	15.9	19.6	35.0	69.5	79.8	91.6	105	120	139	159	183	211
$R_{12}(\Omega/\text{m})$	0.27	0.29	0.38	0.47	0.68	1.47	3.06	6.34	7.30	8.42	9.71	11.2	13.0	15.0	17.2	19.8
$R_{13}(\Omega/\text{m})$	0.27	0.29	0.36	0.40	0.40	0.52	1.00	1.82	2.04	2.29	2.58	2.91	3.40	3.85	4.95	5.64
$R_{14}(\Omega/\text{m})$	0.27	0.28	0.33	0.34	0.30	0.25	0.42	0.57	0.59	0.61	0.63	0.65	0.76	0.81	0.85	0.89
$R_{22}(\Omega/\text{m})$	14.2	14.7	15.1	15.4	16.1	20.2	36.4	72.6	83.4	95.9	110	127	146	168	192	221
$R_{23}(\Omega/\text{m})$	0.27	0.29	0.38	0.48	0.88	1.71	3.60	7.53	8.70	10.1	11.6	13.4	15.6	17.9	20.7	23.8
$L_{11}(\text{nH}/\text{m})$	744	736	706	686	678	668	656	644	642	641	639	638	637	636	635	634
$L_{12}(\text{nH}/\text{m})$	358	352	325	307	303	299	299	297	297	297	296	296	296	296	296	296
$L_{13}(\text{nH}/\text{m})$	217	213	193	180	179	179	179	179	179	179	179	179	179	179	179	179
$L_{14}(\text{nH}/\text{m})$	137	136	125	117	117	119	119	119	119	119	119	119	119	119	119	119
$L_{22}(\text{nH}/\text{m})$	740	734	706	686	678	664	650	638	636	634	633	632	630	629	628	628
$L_{23}(\text{nH}/\text{m})$	356	351	325	307	303	298	295	294	294	294	294	294	294	294	293	293

Table 4.2: R and L as functions of frequency for Figure 4.20

Figure 4.21: R_{12} as a function of frequency (example 4.3)

Figure 4.22: L_{12} as a function of frequency (example 4.3)

Figure 4.23: R_{11} as a function of frequency (example 4.3)

Figure 4.24: L_{11} as a function of frequency (example 4.3)

Figure 4.25: Frequency response of the circuit in example 4.3

Figure 4.26: Transient response of the circuit in example 4.3

4.6.4 Example 4.4

The transmission line circuit of Figure 4.27 is considered. The interconnect dimensions of the two conductor lines are shown in Figure 4.28. The ‘measured data’ values for the R and L matrices (Table 4.3) are obtained by [65]. The rational function approximations for R_{12} , L_{12} , R_{11} and L_{11} are compared with the ‘measured data’ values in Figure 4.29, 4.30, 4.31 and 4.32, respectively. The frequency response of the linear subnetwork (Figure 4.33) is obtained by applying a unit voltage source at P1 and measuring the voltage at P2. The proposed macromodel (order 8/8) matches the exact frequency response up to 8 GHz. The transient response of the entire nonlinear circuit is shown in Figure 4.34.

Figure 4.27: Nonlinear circuit with frequency dependent interconnect parameters

Figure 4.28: Cross-sectional geometry and dimensions of two conductor microstrip

F(GHz)	1e-5	1e-4	1.1e-3	1.1e-2	0.12	0.38	0.76	1.21	1.94	2.45	3.10	3.91	4.95	6.26	7.91	10.0
$R_{11}(\Omega/\text{m})$	4.63	4.75	5.24	6.47	15.6	28.1	39.9	50.5	63.8	71.7	80.6	90.6	102	115	129	145
$R_{12}(\Omega/\text{m})$	0.74	0.76	0.55	0.44	1.48	2.72	3.88	4.90	6.18	6.94	7.80	8.75	9.83	11.0	12.4	13.9
$L_{11}(\text{nH}/\text{m})$	337	335	285	230	209	203	200	199	197	197	196	195	195	195	194	194
$L_{12}(\text{nH}/\text{m})$	58.4	57.2	13.1	17.2	15.0	14.4	14.1	13.9	13.8	13.7	13.7	13.6	13.6	13.6	13.5	13.5

Table 4.3: R and L as functions of frequency for Figure 4.28

Figure 4.29: R_{12} as a function of frequency (example 4.4)

Figure 4.30: L_{12} as a function of frequency (example 4.4)

Figure 4.31: R_{11} as a function of frequency (example 4.4)

Figure 4.32: L_{11} as a function of frequency (example 4.4)

Figure 4.33: Frequency response of linear circuit in example 4.4

Figure 4.34: Transient response of nonlinear circuit in example 4.4

Chapter 5

Conclusions and Future Research

5.1 Conclusion

An interconnect macromodel has been presented that is suitable for inclusion in non-linear circuit simulators such as SPICE. The proposed method is based on computing the coefficients of the macromodel using a closed form Padé approximation of the exponential matrix that describes Telegrapher's equations. In this thesis, the following advantages have been demonstrated for the proposed algorithm:

1. The time domain macromodel is obtained analytically in terms of the per unit length parameters and predetermined constants. This fact provides substantial advantage for the proposed algorithm over previous techniques, which rely on some form of numerical and/or optimization technique to compute the required coefficients. In

addition, a circuit equivalent network composed of RLC elements is presented for linking the Padé model with nonlinear circuit simulators.

2. A technique to incorporate frequency dependent parameters has been developed that guarantees the passivity of the Padé macromodel. The method represents the frequency dependent parameters as rational functions obtained by passive circuit networks composed of resistors, inductors and ideal transformers.
3. The proposed interconnect model obtains better accuracy with fewer variables when compared to the lumped segmentation model. Numerical examples have shown significant savings in the size of the macromodel and in CPU time.
4. An important property that an interconnect model must satisfy is passivity. In this thesis, a new passivity proof has been presented. The proof demonstrates that the macromodel is passive when the order of the numerator and denominator of the Padé approximation are equal. In addition, a method of restoring the macromodel's passivity is shown when the order of the numerator differs from the denominator.
5. When dealing with large linear networks that contain interconnects it is often desirable to use model reduction techniques to represent the overall network. It is shown that the Padé macromodel is compatible with passive reduction techniques based on Krylov space methods. Interconnects of large linear networks can be modelled using the proposed algorithm. The overall linear network can then be reduced by Krylov techniques while preserving passivity of the reduced system [30], [31].
6. A method is given for selecting the order of the Padé approximation. This method is based on comparing the macromodel's approximation with the matrix exponential over the frequency bandwidth of interest.

5.2 Suggestions for Future Research

In this section, some suggestions for further research are listed for the proposed interconnect model:

1. In many situations sensitivity analysis is often required for the optimization of circuits. If the Padé interconnect model is able to support sensitivity analysis, it becomes an important tool for interconnect optimization. Since the coefficients of the interconnect model are obtained analytically in terms of per unit length parameters and predetermined constants, the proposed interconnect model should be able to support sensitivity analysis with respect to any interconnect parameter.
2. Model reduction techniques based on the Krylov space methods have been developed to preserve the passivity of the reduced network [22]. These methods are limited to systems described by RLC components. Handling interconnects using these techniques requires discretization which becomes a source of significant error at high frequencies. It has been demonstrated that the Padé model is compatible with Krylov space methods. In addition, numerical examples have shown that the proposed method is more efficient in accuracy when compared to the conventional lumped segmentation model. Using the Padé model instead of the lumped segmentation model to represent interconnect networks and applying Krylov reduction may achieve in more accurate simulation results.
3. Recently, a new interconnect model has been developed based on a congruent transformation [24]. This interconnect model was shown to be passive and efficient for interconnects with frequency independent parameters. A comparison between the Padé macromodel and the congruent transform macromodel [24] can be done to analyse the accuracy and efficiency of the two methods.

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