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EARLY CRETACEOUS HEDBERGELLA (FORAMINIFERIDA)
FROM DARWIN, AUSTRALIA: A BIOMETRICAL
APPROACH

by

• CHEN-WAH LEE, B.Sc.

A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
Master of Science

Carleton University

Ottawa, Ontario.

August, 1979

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ABSTRACT

Multivariate morphometric analysis has been carried out on the morphologically similar Cretaceous planktonic foraminiferal species Hedbergella delrioensis (Carsey), Hedbergella planispira (Tappan) and Hedbergella infracretacea (Glaessner) from one Aptian core sample from Darwin Area, Australia. Previous non-biometric studies did not allow clear interspecific differentiation and intraspecific variations were neglected. The result of the stepwise discriminant analysis in this study points to the existence of three distinct forms in the sample and the essential morphologic differences are related to the height of the spire and the size of the last chamber. The result of the principal component analysis shows that the first two principal components can account for most of the morphological variations within each species; the first component can be explained as a variation in size of the test and the second as a variation in shape of the test as influenced by the variation of the size of the last chamber.

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CHAPTER 1

INTRODUCTION

1.1 General

The paleontological species concept defines species as a group of individuals or populations with the same or similar morphological characters (Mayr, 1942). However, the limits of morphological species, "are subjectively defined and can be varied to suit the whims of the taxonomist" (Sylvester-Bradley, 1956). These subjective definitions and assessment of variations are inadequate for scientific communication, and as a result there are either many species or else a great amount of variation.

While working on the Cretaceous Claystone samples of Darwin Area, Australia, the problem arose as to how the three morphological groups of Hedbergella differ and what is the morphological variation within each group.

Three species have been identified from routine identification procedures. They are: 1) Hedbergella delrioensis (Carsey), 2) H. planispira (Tappan), and 3) H. infracretacea (Glaessner).

The morphological characters of the first and the second species are very similar and grade into each other, and the designation of the intermediate forms to either

species is difficult. The third species has distinct characters which differs from those of the first two species. However, the descriptions of H. delrioensis and H. infracretacea by different authors are inconsistent and as a result often misleading in meaning.

A review of the literature and an examination of the morphology of a large number of individuals in the samples suggested that three morphological groups are recognizable. The question then is whether the differences of these species are objectively supported by quantitative assessment of characters. Another question is how the morphological characters vary within each designated species.

These studies require the measurements of the external morphological characters by which the three groups have been distinguished. As in some instances groups can be distinguished by using fewer characters than have been measured, one of the purposes of the study is to find the minimum number of measured bio-characters necessary to discriminate among the three groups. Stepwise discriminant function analysis was employed to find these most important discriminating characters. The second purpose of the study is to describe the morphological variation within each species,

in terms of fewer new variables than that have been measured. Principal component analysis which can summarize most of the variances within the measured characters with fewer new dimensions was employed.

These multivariate morphometric methods appear to offer powerful tools for solving taxonomic problems, and it is hoped that this study will also show the potential of these methods in the classification of other geological samples.

1.2 Geology of the area and the material for study

Three long and two short boreholes with a total of 170 samples of Cretaceous Claystone from Gunn Point, Darwin Area, Northern Australia were provided to K. Hooper through courtesy of the chief geologist, Mines Branch, N.T. Administration, Commonwealth of Australia. The locations of the holes are shown on Figure 1. The distribution of the samples is shown in Table 1.

Core	Range of core samples
DDH1	100 ft. - 142 ft.
M8	20.00 m. - 26.10 m.
M9	22.19 m. - 27.00 m.
M17	26.31 m. - 69.42 m.
M18	30.06 m. - 53.54 m.

Table 1. Distribution of core samples

The boreholes formed part of a drilling project performed by the Mines Branch in 1972 for a pottery clay investigation. Based on the marine microplankton from parts of the sedimentary sequence, D. Burger

Fig. 1. General location map. (From Lau, 1973)

(1977, personal communication) of the Bureau of Mineral Resources, Commonwealth of Australia suggested that the Cenomanian strata overlies Aptian strata and that Albian strata are absent. The profile of the three deepest cores (DDH1, M17, M18) and the ages of part of the core samples determined by Burger is given in Figure 2.

Sections in the deeper holes (M17, M18) can be correlated because a marker bed occurs in each. The depth of the marker bed in core M17 is at 52.05 - 52.10 m. and is at 47.90 - 48.00 m. in core M18. The sample under study was taken from core M17 at 52.08 m. (K.H. field record no. 76660). It lies within the marker bed and was Aptian in age, according to Burger.

The sections above the marker bed contain grey silty claystone and the sections below the marker bed contain grey calcareous radiolarian siltstone-mudstone (Lau, 1973). The marker bed (includes the present sample) composed of "intramicrite with about 50% dark grey glauconite-quartz-collophane intraclasts in about 50% light grey calcitic? secondary microsparite matrix" (Lau, 1973).

Foraminifera are rare in every core sample available except in the samples lying within the marker bed, i.e. 52.08 m. of M17 and 47.76 m. of M18, in which they are abundant in both planktonic and

Fig. 2. Profile of cores DDH1, M17, M18.

benthonic foraminifera. The foraminifera fauna is similar in both samples but is more abundant in the first sample. Thus the first sample (52.08 m. of M17) was chosen for further analysis.

1.3 Foraminifera and Paleoenvironment

In the sample under study, benthonic foraminifera are dominated by Gavelinella intermedia - Gaudryina dividens association, which, together with the presence of Lingulogavelinella albiensis aptica, indicates a non-tropical, rather cool, shallow water, normal salinity environment (Sliter, 1972, Scheibnerova, 1972, 1974, 1976).

Planktonic foraminifera are represented by Hedbergella delrioensis, H. planispira and H. infracretacea. They are non-tropical and epipelagic planktonic species, occupying shelf waters in Early/Middle Cretaceous time (Scheibnerova, 1976).

In addition, based on the clay testing results, Lau (1973) suggests that Gunn Point sediments were deposited in a moderately reducing alkaline, neritic marine environment, bordering a low relief land mass.

CHAPTER 2

TAXONOMY

2.1 Introduction

The Cretaceous planktonic foraminifera were included in the Superfamily Globigerinacea by Banner and Blow (1959) and Loeblich and Tappan (1964, 1974), and were recently reclassified with the Superfamily Hedbergelloidea, new superfamily, by Longoria and Gamper (1975), based on phyletic relationships. Longoria and Gamper believed that Tertiary planktonic foraminifera are descendents of Cretaceous forms and thus the superfamilies should include taxa with close phyletic relationships. They grouped in this superfamily planktonic, distinctly perforated calcareous forms with double walls of lamellar radial calcite. The criteria used for the classification of the Hedbergelloidea at the family level are the position of the primary aperture and the type of apertural structure. The families are subdivided into subfamilies on bases of the modification of the type of coiling and the presence of unique type of ornamentation. Genera are separated within the family on the bases of presence or absence of keels in the last whorl and by the characteristic type of sutures on the umbilical side.

The new superfamily Hedbergelloidea includes all the Cretaceous trochospiral and planispiral planktonic foraminifera. The origin of the Hedbergelloidea is still quite uncertain. However, according to Longoria and Gamper (1975) it is possible that they originated from the Discorboidea during mid-Jurassic times. All members of the Hedbergelloidea are connected to each other during their phylogenesis forming a natural group. Their evolution at the generic level is shown in Figure 3.

Fig. 3. Phylogenesis of the trochoidal Hedbergelloidea at the generic level. (From Longoria and Gamper, 1975)

2.2 Systematics

2.2a Descriptions of Taxa

Subfamily Hedbergellinae, Loeblich and Tappan, 1961.

Emended Definition (Longoria, 1974, p.51) -

Tests coiled in a low to high trochospire;
periphery often with poreless margin; chambers
spherical, ovoid to clavate; primary aperture
extraumbilical to spiroumbilical, bordered by
an imperforated flap; relict apertures frequently
present on the spiral side.

Remarks. - The Hedbergellinae, as emended by
Longoria, include forms which commonly have
relict apertures on the spiral side and with
extraumbilical to spiroumbilical primary aperture.
The family Hedbergellidae differs from the
Loeblichellidae in having inflated rather than
laterally compressed chambers in the last whorl.

The Hedbergellinae include two genera: Hedbergella
Bronnimann and Brown and Praeglobotruncana Bermudez.
They are separated on basis of their imperforate
peripheral margin, which is present in Praeglobotruncana
and absent in Hedbergella.

Range: - Hauterivian to Campanian.

Occurrence: - World-wide.

Genus *Hedbergella* Bronnimann and Brown, 1958, emended,
Longoria, 1974.

Diagnosis. - The genus Hedbergella as emended here includes forms with 1) globular, rounded to elongate chambers lacking imperforate peripheral bands, 2) an interiomarginal, extra-umbilical to spiroumbilical primary aperture, bordered by imperforate flaps and 3) relict apertures often present on the spiral side.

Emended Definition. - Test coiled in a low to high trochospire, biconvex, spiroconvex or umbilicoconvex; periphery circular, elliptical, lobate to stellate with no indication of imperforate peripheral band, chambers globular to clavate; sutures radial, depressed, straight to slightly curved; primary aperture an interiomarginal arch, extraumbilical to spiroumbilical in position, bordered above by an imperforate porticus which is typically triangular or spatulate; relict apertures sometimes present on the spiral side; umbilical area small to wide, shallow to deep, forms with wide umbilicus frequently show successive apertural flaps, smooth to hispid, sometimes ornamented with fine pustules.

2.2b Systematic Paleontology

Systematic Paleontology

Superfamily HEDBERGELLOIDEA Longoria and Gamper, 1975

(Partim GLOBIGERINACEA Carpenter, Parker and

Jones, Loeblich and Tappan, 1964, p. C652)

Family HEDBERGELLIDAE Loeblich and Tappan, 1961

Subfamily HEDBERGELLINAE Loeblich and Tappan, 1961,

emended Longoria, 1974

Genus HEDBERGELLA Bronnimann and Brown, 1958,

emended Longoria, 1974

Hedbergella delrioensis (Carsey), 1926

(Plate I, figs. 1-3)

Globigerina cretacea d'Orbigny var. del rioensis n. var.

Carsey, 1926, p. 43, no fig.

Globigerina cretacea d'Orbigny, Tappan, 1940, p. 121,

pl. 19, fig. 11. ---1943, p. 512, pl. 82, fig. 16-17.

Praeglobotruncana (Hedbergella) delrioensis (Carsey),

Banner and Blow, 1959, p. 8, (no illustration).

Praeglobotruncana gautierensis (Bronnimann), Jones, 1960,

p. 102, pl. 15, figs. 1-9, text fig. 1.

Hedbergella delrioensis (Carsey), Loeblich and Tappan,

1961, p. 275, pl. 2, figs. 11-13.

?Hedbergella delrioensis (Carsey), Todd and Low, 1964,
p.402, pl.1, fig.2.

Hedbergella delrioensis (Carsey), Eicher, 1966, p.27,
pl.5, figs.12-13.

Hedbergella delrioensis (Carsey), Pessagno, 1967,
p.383-283, pl.48, figs.1-5.

Hedbergella delrioensis (Carsey), Eicher and Worstell,
1970, p.302, pl.9, figs.10-11.

non: Hedbergella sp. cf. H. delrioensis (Carsey), Kuhry,
1971, p.229, pl.1, figs.7-8.

Hedbergella delrioensis (Carsey), Longoria, 1974, p.54-
55, pl.10, figs.7-9.

Hedbergella delrioensis (Carsey), Scheibnerová, 1976,
p.86-87, pl.47, figs.1-2.

Hedbergella delrioensis (Carsey), Price, 1977, p.519-
525, pl.61, figs.4-6.

Test of medium size, trochospiral (spire ranges from low spiroconvex, planispiral to depressed), formed by three whorls; peripheral margin subcircular, lobate; five globular chambers, increasing rapidly in size as added, ultimate chamber is more inflated than the penultimate, chamber subcircular to spherical in both spiral and umbilical views; spherical in peripheral view, sutures radial, straight, strongly depressed on both spiral and umbilical sides; primary apertures a

low arch, extraumbilical-spiroumbilical in position,
bordered on top by an imperforate flap; umbilicus small,
circular; surface of the test finely hispid to smooth.

Hedbergella planispira (Tappan), 1940
(Plate I, figs. 4-6)

Globigerina planispira Tappan, 1940, p.122, pl.19,
fig.12.

Globigerina planispira Tappan, Tappan, 1943, p.513,
pl.83, fig.3.

Praeglobotruncana planispira (Tappan), Bolli, Loeblich,
and Tappan, 1957, p.40, pl.9, fig.3.

Praeglobotruncana modesta Bolli, 1959, p.267, pl.22,
fig.2.

Praeglobotruncana planispira (Tappan), Bolli, 1959,
p.267, pl.22, figs.3-4.

Praeglobotruncana infracretacea (Glaessner), Bolli,
1959, p.266, pl.21, fig.9.

Hedbergella planispira (Tappan), Loeblich and Tappan,
1961, p.267-268, pl.5, figs.4-11.

Hedbergella trocoidea (Gandolfi), Loeblich and Tappan,
1961, p.277-278, pl.5, figs.1-2.

Not Hedbergella planispira (Tappan), Olsson, 1964,
p.161-162, pl.1, figs.4-5.

Hedbergella trocoidea (Gandolfi), Loeblich and Tappan,
1964, p.C659, fig.527:1.

Hedbergella trocoidea (Gandolfi), Todd and Low, 1964,
p.395-414, pl.2, figs.1a-c.

Hedbergella planispira (Tappan), Marianos and Zingula,
p.335, pl.37, figs.6a-c.

Hedbergella planispira (Tappan), Pessagno, 1967, p.283-
284, pl.51, fig.1, pl.53, figs.1-4.

Hedbergella planispira (Tappan), Douglas, 1969, p.168,
pl.5, fig.1.

Hedbergella planispira (Tappan), Eicher and Worstell,
1970, p.302-303, pl.9, figs.12-13.

non: Hedbergella sp. aff. H. planispira (Tappan), Kuhry,
1971, p.231, pl.21, figs.5a-c.

Hedbergella planispira (Tappan), Kuhry, 1971, p.230,
pl.2, figs.4a-c.

Hedbergella planispira (Tappan), Longoria, 1974, p.64-
65, pl.11, figs.4-6, pl.20, fig.4, pl.23, figs.
1-2, 3-4, 5-7, 11-13, 17-18, pl.24, fig.10.

Hedbergella planispira (Tappan), Scheibnerova, 1974,
p.86, pl.47, figs.3-4, pl.50, figs.1-5.

Hedbergella planispira (Tappan), Price, 1977, p.520-
525, pl.61, figs.10-12.

Test small sized; formed by three whorls, all coiled in one plane; peripheral margin subcircular, lobate; six to seven chambers in the last whorl, increasing gradually in size as added; chambers spherical on both spiral and umbilical sides, spherical in peripheral view; sutures radial, slightly curved, strongly depressed on both spiral and umbilical sides; umbilicus wide shallow; relict apertural flaps frequently visible; surface pustulose to smooth.

Hedbergella infracretacea (Glaessner), 1937
(Plate I, figs. 7-9)

Globigerina infracretacea Glaessner, 1937, in Subbotina, 1971, p.56, pl.1, figs.5a,b,c-10a,b,c.

Hedbergella infracretacea (Glaessner), Kuhry, 1971, p.230, pl.2, figs.3a-c.

Hedbergella delrioensis (non Carsey), Longoria, 1974, p.54-55, pl.10, figs.1-6, 10-12.

Hedbergella infracretacea (Glaessner), Longoria, 1974, p.59-60, pl.13, fig.9.

Hedbergella infracretacea (Glaessner), Scheibnerova, 1976, p.87-88, pl.46, figs.1-2, pl.52, fig.1, text fig.102.

Hedbergella infracretacea (Glaessner), Price, p.519-525, pl.61, figs.7-9.

Hedbergella infracretacea (Glaessner)

Test medium size, high trochospiral, formed by three whorls; peripheral margin subcircular, lobate; five to six chambers in the last whorl, increasing slowly in size as added, ultimate chamber protrudes towards the umbilicus, generally smaller than the penultimate chamber as seen from umbilical view; chamber ovoid on both spiral and umbilical sides; spherical in peripheral view; umbilicus narrow, deep.

2.2c Differentiation of the Species

The morphology of Hedbergella species found in the sample is quite diverse and many characters grade into each other. Reliable identification of species was made more difficult because of the inconsistent definitions found in the literature. Furthermore, nomenclatural confusion characterizes the literature of these species which often have been grouped under various names by several authors. The summary of their differences which follows is based upon the work of Loeblich and Tappan (1961), Pessagno (1967), Longoria (1974), Scheibnerová (1976) and Price (1977).

Hedbergella delrioensis differs from H. planispira and H. infracretacea 1) by the more rapid increase in size of its chambers in the last whorl, 2) by the more variation of the height of the spire (in H. delrioensis the early whorl varies in height from slightly elevated above to slightly depressed below the final whorl; in H. planispira, the spiral side is exclusively flat, and in H. infracretacea, the spiral side is medium trocho-spiral), 3) by the more inflated final chamber.

Hedbergella delrioensis differs further from H. planispira 1) by the larger size of the test, 2) by the smaller number of chambers in the last whorl, 3) by the smaller and deeper umbilicus, and 4) by the thicker nature of the test.

Hedbergella delrioensis differs further from H. infracretacea 1) by the smaller size of the test, 2) by its more evolute coiling, 3) by its last chamber not protruding toward the umbilicus.

Hedbergella planispira differs from H. delrioensis and H. infracretacea 1) by its smallest size among the three species, 2) by its exclusively flat spiral side, 3) by its parallel outline on both sides of the spire, 4) by its larger number of chambers in the final whorl, 5) by its gradually increasing size of chambers in the final whorl, 6) by the thinner nature of the test, 7) by its wider umbilicus.

Hedbergella infracretacea differs from H. delrioensis and H. planispira 1) by its largest size among the three species, 2) by its exclusively high to medium trochospire, 3) by the nearly equal size of its last two chambers in the final whorl, 4) by its last chamber which is incurved toward the umbilicus, 5) by its smallest number of chambers in the last whorl, 6) by its greatest height among the three species.

2.2d Problems in Species Identification

The greatest problems in establishing the correct taxonomy of these Hedbergella species in Early Cretaceous time are:

- 1) their close phyletic relationships,
- 2) the gradation of characters between species,
- 3) the inconsistent definition of species.

1. There has been a tendency to group similar forms of this genus into a single species, namely H. (Globigerina) infracretacea (Glaessner). Longoria (1974) states that H. infracretacea (Glaessner) has served as a 'waste basket' name for almost thirty years and nearly every Lower Cretaceous species of unknown affinity has been referred to this species. For example, according to Scheibnerova (1976) Ludbrook in 1966 included all planktonic forms (including H. planispira and H. delrioensis) occurring in the Great Australian Basin into this species.

It has been pointed out by Hofker (1961) and Price (1977) that H. infracretacea formed the root stock and primitive hedbergellid parent from which H. delrioensis arose in the Aptian - Albian. Price (1977) further reported H. planispira shows a close affinity with early forms of H. delrioensis and therefore it is possible

that H. planispira arose from H. delrioensis within Aptian - Albian. So these three species should be referred to by three different species names.

2. These three species have very close morphologic characters and many of them grade into each other. The greatest difficulty in classification of the forms is the wide morphologic variation within the intermediate-form H. delrioensis, which links its ancestor H. infracretacea to its descendent H. planispira. The differences between H. infracretacea and H. planispira are very distinct. The morphologic similarities, differences and gradation of characters of the three species have been discussed in the previous section (Section 2.2c).

3. The third difficulty in taxonomy is the inconsistent definition of the species. According to Price (1977) the descriptions of H. infracretacea and H. delrioensis of Glaessner 1966 and of Price 1977 differ from those of Longoria 1974. Longoria (1974) states "... in H. infracretacea the last chamber is ovoid, generally smaller than the penultimate in umbilical view and is in the same plane as other chambers of the last whorl; whereas with H. delrioensis

the last chamber is spherical and protrudes towards the umbilicus...". The descriptions of H. delrioensis and H. planispira by Glaessner and Price are essentially similar to those of Longoria. Price concurred that the last chamber of H. delrioensis is very slightly inturned towards the umbilicus and added that the ultimate chamber is more inflated than the penultimate.

In the present sample, the last two chambers of the specimens with the high trochospire have nearly the same size and the last chamber is incurved towards the umbilicus. These forms are classified as H. infracretacea, following Glaessner and Price. This differs from Longoria's definition only on the position of the last chamber, which is not actually in the same plane as other chambers of the last whorl. In H. delrioensis, most of the chambers are planispirally or low trochospirally arranged except that in some cases the last chamber is either slightly depressed or elevated. The last chamber of this species is always more inflated than the penultimate, and is not protruding towards the umbilicus. The specimens with the last chamber protruding towards the umbilicus were regarded as H. infracretacea by Price (1977) and this distinction is adopted here.

CHAPTER 3

MULTIVARIATE MORPHOMETRICS

3.1 Introduction

In the study of foraminifera, we may encounter problems in identification of many species, particularly in planktonic forms which always show a great amount of intraspecific variation and interspecific gradation (Pessagno, 1964). The establishment of a foraminiferal taxon, in almost all cases, is based on subjective criteria which cannot lead to quantitative evaluation and statistical analysis. As the paleontological species has been defined as a group of individuals or populations with the same or similar morphological characters (Mayr, 1942), it is the population that can define a species and the population variability that can provide the taxonomic information (Scott, 1974). Objective assessment of the variation of each morphological character within the population is prerequisite for understanding the intraspecific variation and interspecific gradation. A form as complex as the planktonic foraminiferal test requires many characters for its quantitative representation (Kennett, 1976), and as all these characters constituted the integrated structure of the test, so these characters need to be considered simultaneously (Gould, 1970). Here lies the appeal of multivariate

methods of analysis. Multivariate methods in biology have been summarized by Rao (1952), Seal (1966) and Sneath and Sokal (1973). Blackith and Reyment (1971) dealt specially with their use in paleontology.

3.2 Methods

3.2a Choosing of Multivariate Methods

The routine qualitative classification of the species Hedbergella delrioensis, H. planispira and H. infracretacea in the previous chapter initiates some questions to be answered:

1. Are there significant multivariate differences between the three species?
2. Is it possible to differentiate among the three species using fewer biocharacters?
3. Is there any misclassification of the species established by qualitative appraisal?
4. What is the morphological variation within each species?

All these questions concern one basic problem: given a set of points in n-dimensional space, what can be learnt by defining their geometrical position. The solution involves a rotation of the original axes to new positions and projection of all points upon the new axes. The appropriate rotation depends upon the aim of the study.

Questions 1 to 3, which are similar to the problems discussed by Klován and Billings (1967), can be considered as a problem of discriminating between groups and can be solved by the multivariate method of Stepwise Discriminant Analysis. Discriminant analysis begins with the desire to distinguish statistically between two or more groups of specimens. This requires the measurement of the morphological variables (biocharacters) of each specimen in the group. Discriminant analysis is performed based on the variable measured. Stepwise method can further select the most useful variables that can satisfactorily discriminate the groups. The discriminant variables established can be used to reclassify the original set of specimens in order to check how many are correctly classified by the variables being used. Finally, we would like to display the group differences on a plane that can maximize the distance of group means - this is a plot of canonical variates. This can be formed by rotating the original axes such that the first axis maximizes the separation of the predetermined groups as projected upon it.

Question 4 involves the description of variance and the problem can be solved by Principal Component Analysis (Reyment, 1972). This technique can maximize most of the original variation in a multivariate system in fewer new variables. The rotation requires that the first new axis maximizes the variance of a point projected upon it.

3.2b Sampling Method for Biometrics

All the specimens studied were obtained from one Aptian core sample. The specimens selected were all greater than 0.1 mm. in greatest diameter of the tests and those with greatest diameter less than this value were not included in the population for analysis. The specimens in the population were then subjectively subdivided into four morphological groups: 1) specimens with distinct characters as described in Chapter 2 as H. delrioensis, 2) specimens with distinct characters as described as H. planispira, 3) specimens with distinct characters as described as H. infracretacea, and 4) forms with indistinct characters that could not be placed easily in any of the three species.

Specimens in group 4 were temporarily allocated to the "most likely" of group 1, 2, or 3 and the number of specimens in each morphological group was as follows: H. delrioensis: 758, H. planispira: 343, H. infracretacea: 120. An arbitrary number of specimens from each group was then selected at random utilizing a table of random numbers. The number of specimens thus selected from each group was: H. delrioensis: 63, H. planispira: 48, H. infracretacea: 37 and the total was 148.

3.2c Choosing of Biocharacters

The following morphological characters generally are used to distinguish clusters that are interpreted as species, particularly on planktonic foraminifera (Pessagno, 1967, Berger, 1969).

1. Size of the test.
2. Relative proportion of test.
3. Basic shape of the chambers.
4. Ratio between successive chambers.
5. General shape of the test as defined by the relative chamber increase.

In order to differentiate the three species and account for their morphological variations within each species, the following biocharacters are chosen for biometric analysis. Illustrations of the variables are given in Figure 4 and all the measured data are given in Appendix I.

- L Length of the test
- W Width of the test
- N Number of chambers in the final whorl
- T1 Thickness of the last chamber
- T2 Thickness of the second last (N-1) chamber
- T3 Thickness of the N-2 chamber
- T4 Thickness of the N-3 chamber
- T5 Thickness of the N-4 chamber

Fig. 4. Illustration of variables

U Width of the umbilicus

H Height of the spire

All the values of the above variables (biocharacters) except H are always measured in positive sign. The value of H may be positive or negative; the elevation of the last chamber to the early whorl plane is measured in positive sign, the depression in negative sign. The unit of measurement used is mm.

Based on the above measurements some ratios of the variables are also obtained:

U/L Relative size of the umbilicus

L/W Elongation of the test

T1/T2 Expansion rate of the N to (N-1) chamber

T2/T3 Expansion rate of the (N-1) to (N-2) chamber

T3/T4 Expansion rate of the (N-2) to (N-3) chamber

T1/T5 Expansion rate of the N to (N-4) chamber

These characters or the combinations of these characters can give a whole spectrum of morphologic differentiation between the three species and morphological variations within each species.

3.3 The Problem of Homogeneity

Although it might seem desirable to carry out the test of the homogeneity of dispersions whenever a multivariate analysis of variance is to be performed, in fact the test is often omitted because its interpretation presents difficulties. Furthermore, the multivariate analysis of variance can survive a certain amount of heterogeneity among the dispersion of the group (Hope, 1968, p.23). For these reasons the preliminary test of homogeneity is not investigated.

CHAPTER 4

STEPWISE DISCRIMINANT ANALYSIS

4.1 Introduction

Computer program BMDP7MLM (Dixon, 1975) was used for stepwise discriminant analysis. To distinguish between the three species the writer selects 14 variables that measure characteristics on which the groups are expected to differ. These variables are: L, W, N, T1, L/W, U/L, T1/T2, T2/T3, T3/T4, T1/T5, U, T2, T5, H.

The program is used to find the subset of variables that maximizes group differences. Variables are entered into the classification function one at a time until the group separation ceases to improve notably. Further analysis is performed using only the selected variables. A geometrical interpretation of discriminant analysis can be given by plotting each specimen as a point in a plane selected so that the groups are farthest apart. This is a good visual representation of the distinctness of the groups and is a plot of the canonical variables. The mathematical treatment and computer print out of this program are given in Appendix II and III respectively.

4.2 The Method and the Program

The stepwise method: - In this program, variables will be selected for entry into the analysis on the basis of their discriminating power. In many instances the full set of variables contains excess information about the group differences, or perhaps some of the variables may not be very useful in discriminating among the groups. Sequential selection of the "next best" discriminator at each step, produces a reduced set of variables which is almost as good as the full set, and lends itself to more efficient and economical analysis. For example: Malmgren and Kennett (1977) discriminated Recent Globigerina bulloides from G. falconensis using only 3 out of a total of 13 dimensions by using this stepwise method.

At each step, the program uses a one-way analysis of variance F statistic (F - to - enter) to determine which variable should join the function next. At step zero, the standard univariate analysis of variance test is made for each of the variables. The variable for which the means differ most is entered first into the classification. After step zero, the initial variable selected is then paired with each of the other available

Means					
	Group =	Del	Plan	Infra	All Gps. *
Variable					
1	HED	1.000	2.000	3.000	1.824
2	L	.184	.151	.206	.179
3	W	.152	.130	.185	.153
4	N	5.436	5.947	5.221	5.548
5	T1	.099	.072	.099	.090
6	LN	1.206	1.166	1.113	1.170
7	UL	.116	.147	.115	.126
8	T12	1.148	1.041	1.048	1.088
9	T23	1.143	1.110	1.093	1.120
10	T34	1.193	1.137	1.147	1.164
11	T15	1.837	1.486	1.480	1.634
12	U	.021	.022	.023	.022
13	T2	.086	.069	.095	.083
14	T5	.054	.048	.068	.056
15	H	.002	-.000	-.028	-.006
Counts		63	48	37	148

Table 2. Means of each variable.

variables, one at a time, and the F - to - enter value is computed. The new variable which in conjunction with the initial variable produces the best criterion score (F - to - enter value) is selected as the second variable to "enter the equation". This procedure of locating the next variable that would yield the best criterion score, given the variables already selected, continues until all variables are selected or the F - to - enter value yielded is smaller than a specified value.

4.3 Basic Statistics Results

The means of all variables of each species are computed before the stepwise analysis. It gives an outline of the differences between these three species. These means are given in Table 2.

Hedbergella infracretacea is the largest species and H. planispira is the smallest. H. planispira has the largest number of chambers in the last whorl. H. delrioensis has the most elongated test and H. planispira has the widest umbilicus diameter relative to the size of its test. The expansion rate of each succeeding pair of chambers $T1/T2$, $T2/T3$, $T3/T4$ as well as the expansion rate of the last (N) to the (N-4) chamber $T1/T5$ of H. delrioensis are the greatest among the three species. During most of the growth

of a foraminifer a steady increase occurs in size of successive new chambers and chamber ratios are greater than one (Kennett, 1976). An interesting discovery is that the expansion rate of the succeeding chamber in each species is decreasing, i.e., $T_1/T_2 < T_2/T_3 < T_3/T_4$, with the only exception in the T_1/T_2 of H. delrioensis which is slightly greater than T_2/T_3 . This does not mean the succeeding chamber is decreasing in size but that the ratio of expansion is not as high as those preceding. H. infracretacea has the highest trochospire, the negative sign represents the depression of the last chamber to the spiral plane. These data support the qualitative observations on the species as described in Chapter 2.

4. The Stepwise Results

The choosing of variables is according to the F - to - enter method described. The stepwise procedure stops at the end of step 4, as none of the F - to - enter values of the remaining variables is greater than the critical values. The critical value of the F distribution with 1% significance and degrees of freedom of 2 and 141 is 4.75. Table 3 is a list of the F - to - enter values of the remaining variables at the end of step 4.

Variable entered	F to remove	Variable remained	F to enter
	DF= 2 142		DF=2 141
3 W	9.512	2 L	.073
5 T1	29.968	4 N	1.855
7 U/L	6.382	6 L/W	.213
15 H	68.230	8 T1/T2	.130
		9 T2/T3	.226
		10 T3/T4	3.695
		11 T1/T5	3.598
		12 U	.894
		13 T2	.207
		14 T5	2.416

Table 3. F - to - enter values of the remaining variables at the end of step 4.

The sequence of variables entered is H, Tl, W, U/L and their F - to - enter values, U statistics and Approximate F - statistics are printed in Summary Table (Table 4). U statistic value is a measure of the inequality of the group means and can vary from zero to unity. The larger the inequality of group means the smaller is the value (Hope, 1968, Tatsuoaka, 1971). After step 4, the U statistic value is so low (0.0992) that we have evidence that the groups are drawn from populations which differ in the mean values of their variables.

Step Number	Variable Entered	F Value To Enter	U-Statistic	Approx. F-Statistic
1	15 H	189.034	.277	189.035
2	5 Tl	91.753	.121	134.233
3	3 W	9.145	.108	97.336
4	7 U/L	6.382	.099	77.241

Table 4. Summary table.

4.5 Classification of Specimens

The original purpose of classification is to test the adequacy of the derived discriminant functions. By classifying the specimens used to derive the functions in the first place and comparing predicted group membership with original group membership, one can empirically measure the success in discrimination by observing the proportion of specimens correctly classified. As we have proved from the stepwise method that it is possible to discriminate the groups by using the derived discriminant functions, so we can reclassify the original group members according to the predicted group membership. /

Classification Functions

Variable	Group =	Del	Plan	Infra
3	W	163.889	201.570	294.889
5	Tl	805.011	557.520	651.272
7	U/L	379.817	416.605	345.144
15	H	176.233	170.253	-248.615
Constant		-75.961	-64.963	-84.474

Classification Matrix

Group	Percent Correct	Number Of Cases Into Group -	Cases Classified	
		Del	Plan	Infra
Del	90.5	57	6	0
Plan	89.6	5	43	0
Infra	91.9	2	1	34
Total	90.5	64	50	34

Table 5. Classification Functions at the end of step 4.

The predicted group membership is computed as follows: The classification functions at the end of step 4 (Table 5) are to be multiplied by the raw variables, summed together and added onto a constant to arrive at a discriminant score. As there is a separate equation for each group so there are three scores for each specimen. These scores are converted into probabilities of group membership and then the specimen can be classified into the group with the greatest probability of membership. The classification matrix which shows the correct classification of the original grouping is printed (Table 5). The average percentage of correct grouping in the original classification is above 90%.

4.6 Plots of Discriminant Scores on Canonical Axes

We have already proved that three species can be discriminated by using four discriminant variables H, Tl, W, and U/L. The classification functions derived have been used to reclassify the original specimens to their correct groups. As we still have four discriminant variables, it is not possible for us to visualize the geometrical relationship between the respective groups and their individuals in a four dimensional space. A plane is thus selected for plotting each specimen a point on it so that the group means are farthest apart - this is a plot of the canonical variables.

Eigenvalues	3.446	1.268
Canonical Correlations	.880	.747
Variable	Coefficients for canonical variables	
3 W	25.388	-21.073
5 Tl	-2.143	96.905
7 U/L	-13.081	-11.025
15 H	-98.150	27.078
Constant	-2.677	-3.958
Group	Group means	
Del	-.768	1.207
Plan	-1.416	-1.360
Infra	3.146	-.291

Table 6. Results of Canonical Analysis

Fig. 5. Plots of the group means and the specimens of each group along the first two canonical axes.

The calculation for canonical variates involves a rotation of axes in the original multidimensional space. The first axis is inclined in the direction of greatest variability between the means of the three species. The second axis is perpendicular to the first and inclined in the direction of next greatest variability, and so on for the subsequent axes. The coefficients for the first two canonical variables are printed (Table 6). These are multiplied by the raw variable values, summed together and added to a constant to arrive at a score on the respective canonical axis. By averaging the scores for the specimens within a particular group, the group mean on the respective canonical axis can be obtained. Plots of the group means and the specimens of each group along the first two axes are given (Figure 5).

The ability of the first two canonical variables to display the group differences can be tested by the eigenvalues and canonical correlation (Tatsuoka, 1971). The sum of the eigenvalues is a measure of the total variance existing in the canonical variables. For the first two variables, the sum is 4.6 ($3.4 + 1.2$), i.e. it accounts for 46% of the total variance (Table 6).

This means that the plot can display 46% of the variability between the means of the 3 groups. Canonical correlation is a measure of the association between the canonical variables and the set of $(g-1)$ dummy variables which define the g - group membership (g is the total number of groups and equal to 3 here). It tells us how closely the canonical variables and the group variables are related. The first two coefficients of correlation are 0.88 and 0.75 (Table 6), so we can say the first two canonical variables are each rather highly correlated with the groups. Canonical variables are useful in discriminating more than two groups. For example, Buzas (1966) has successfully discriminated three Elphidium species by using the first two canonical variables.

A plot can make the group and specimen relationships easier to visualize and can show the degree of overlap of each group. In the plot (Fig.5), the first axis can distinguish specimens of Hedbergella infracretacea from those of the other two species. H. delrioensis and H. planispira cannot be separated on the first axis, but they are distinguishable on the second axis. The specimens located closest to the means of each group were chosen as type specimens of the species.

4.7 Discussion

Before the stepwise discriminant analysis was performed on the three species established by qualitative appraisal, the mean of each variable (biocharacter) of each species was computed. A comparison of the mean of each variable between each species supports the qualitative observation of each species. These quantitative values can give an exact amount of differences between each variable of each species. The descriptions as "... H. delrioensis has the most elongated test and H. planispira has the widest umbilicus diameter relative to the size of its test..." can then become valid as they are based on statistical data.

Choice of type specimen of a species was always a problem in conventional taxonomy and the type specimen set up may not represent the group to which it belonged. The type specimen was consciously or unconsciously established on the form which had maximum morphologic differences from the type specimen of the other species. The type specimen chosen in this manner cannot represent the group to which it belongs, it is only one of the end members of the group.

As stepwise discriminant analysis has chosen the most important variables that can discriminate the

groups and the canonical variables have located the maximum distance between the specimens of each group, this is the mean of the group that can represent the group itself as well as the between group differences. So the type specimen of each species should then be chosen from the specimen located closest to the mean of that group. This method of establishing type specimens is followed in this study.

CHAPTER 5

PRINCIPAL COMPONENT ANALYSIS

5.1 Introduction

The computer program for Principal Component Analysis is modified from Davis (1973, Program 7.10) and comprises the following computations: standardized data matrix (Subprogram STAND), similarity matrix (Subprogram RCOEF), eigenvalues and eigenvectors (Subprogram EIGENJ) and scores on the principal component (Subprogram MMULT). Subprogram for the plot of the scores on the principal component is adapted from Blackith and Reyment (1971, Subroutine PLT21).

These computations are performed on each species data set one at a time. Each species includes the specimens that have been correctly reclassified through discriminant analysis. Ten variables that can account for the morphologic variation within a species have been selected for this analysis. They are: L, W, L/W, T1/T2, T1/T5, U, T2, T5, H. The similarity matrix shows the relationships between variables. Eigenvalues extracted can be used to explain the morphologic variation. The first two principal components can be expected to account for most of the variation and in each specimen a point plotted on the plane formed by these two axes will present a good visual display of the species variation. The mathematical treatment and computer print out are given in Appendix IV and Appendix V respectively.

5.2 The Method

Principal component analysis is a multivariate statistical technique which attempts to explain variation in a multidimensional set of data in terms of a minimum number of few, mutually independent variables termed the principal components. These are obtained by rotating the coordinate axes of the original variables to new positions, so that the variance associated with each axis is maximized. The first principal component corresponds to the axis with the largest amount of variance, and the remaining are ranked in order of decreasing values. The principal components are expressed as linear combinations of the original variables. Much of the variability can be summarized by the use of a few principal components that absorb most variance, as indicated by the percentage contribution of their respective eigenvalues. This enables linear combinations with small values to be rejected and highlights those with large variances. The best positions of the axes are obtained by computing the characteristic values (eigenvalues) and vectors (eigenvectors) of a covariance or a correlation matrix. In the present investigation, the principal components were extracted from the correlation matrix based on the variables selected.

5.3 Correlation Matrix and Development of Chambers

Because the original data were measured in differing units, it is necessary to standardize values before computing the variance-covariance matrix (Davis, 1973). This variance-covariance matrix of standardized variables is nothing more than the correlation matrix. Correlation coefficient is a ratio ranging in value from +1 to -1. A correlation of +1 indicates a perfect direct relationship between two variables; i.e. the value of one variable increases as that of the other increases. A correlation of -1 indicates that one variable changes inversely in relation to the other; i.e. the value of one variable decreases as that of the other increases. Between the two extremes is a spectrum of less-than-perfect relationships, including zero which indicates the lack of any sort of linear relationship. The correlation matrices of Hedbergella delrioensis, H. planispira and H. infracretacea are given in Table 7, Table 8 and Table 9 respectively.

	L	T1	L/N	T1/T2	T1/T5	C	T2	T5	H
L	1.00								
T1	.92	1.00							
T2	.85	.82	1.00						
T5	-.01	-.39	-.10	1.00					
H	-.22	-.23	.01	.07	1.00				
C	-.06	-.07	.20	.08	.20	1.00			
T1/T2	.27	.30	.16	-.10	.02	-.05	1.00		
T1/T5	.85	.84	.87	-.13	-.48	.07	.13	1.00	
T2/T5	.70	.68	.58	-.12	-.16	-.65	.19	.60	1.00
H	.14	.08	.21	.13	-.11	.17	-.24	.24	.01

Table 7. Similarity Matrix of H. delrioensis

	T1	1/4	1/12	1/15	T2	T5
1	1.00					
2	.91	1.00				
3	.70	.65	1.00			
4	-.05	-.06	-.06	1.00		
1/2	-.10	-.21	.31	.27	1.00	
1/5	-.12	-.17	.43	.16	.67	1.00
5	.56	.62	.27	-.32	-.11	1.00
2	.75	.77	.77	-.24	-.34	.34
5	.76	.77	.49	-.23	-.38	.36
5	-.22	-.20	-.11	.01	.19	.15
					-.27	-.28
						1.00

Table 8. Similarity Matrix of H. planispira

	I	T1	I/T1	T1/T2	T1/T5	T2	I
I	1.00						
T1	.84	1.00					
T2	.64	.61	1.00				
I/T1	-.13	-.64	-.21	1.00			
T1/T2	-.05	-.17	.37	.26	1.00		
T1/T5	-.31	-.38	.16	.29	.27	1.00	
T2	.47	.59	.19	-.40	-.37	1.00	✓
I	.67	.72	.61	-.38	-.09	.31	1.00
T5	.74	.76	.56	-.35	.02	.45	.52
I	-.33	-.47	-.25	.39	.17	-.23	-.32
							1.00

Table 9. Similarity matrix of H. infracretacea

Studies of gross shell form using three dimensions: length, width, and height of the test in planktonic foraminifera have shown that correlations among them are positive and high (Gradstein, 1971, Hecht, 1974, Jones, 1960, Scott, 1972). In all the three species studied, the length L and the width W of the test are always highly correlated and this indicates that the size factors increase proportional to each other as a result of growth. The height of the last chamber T1, second last chamber T2, and the N-4 chamber T5 are highly correlated with L and W respectively in all species. This indicates that the size of the test increases proportionally to the size of the chambers in the final whorl. T1 and T2 are highly correlated to each other, indicating that the size of the second last chamber normally controls the size of the succeeding chamber (last chamber). T1 and T5 are moderately correlated with each other, indicating the size of the first chamber has some control on the size of the last chamber in all species. In other examples of planktonic foraminifera, the sizes of these two chambers may have no correlation (Lindenberg, 1969).

Rates of chamber expansion, particularly in the final whorl, are important taxonomic characteristics in planktonic foraminifera (Berger, 1969). These rates will affect the shape and the size of the test. In the present study, chamber expansion rates $T1/T2$ and $T1/T5$ have very little correlation with L and W , demonstrating that these expansion rates are not related to their respective test size. In the study of the intraspecific variation in recent populations of Globigerinoides ruber and G. trilobus, Hecht (1974) also found that the expansion rate is poorly correlated with the size measurement and was assumed to be independent of size. The two expansion rates $T1/T2$ and $T1/T5$ themselves have little correlation with each other in H. delrioensis and H. infracretacea but have rather high correlation in H. planispira. This high correlation indicates that H. planispira has regular rates of increase in chamber size in the final whorl. The width of the umbilicus U has little correlation with test size factors L and W in H. delrioensis but it is rather high in H. planispira and H. infracretacea. The height of the spire H has no or very little correlation with any other dimension of the test in all three species.

5.4 Principal Component Analysis

The approach of this method will be to summarize the maximum possible amount of information within the least possible number of dimensions. In each of the three species analysed, about 60% or more of the variation can be accounted for by the first two principal components (Table 10, 11, and 12). These two principal components can be explained as follows:

The first principal component - The first principal component is correlated highly with the length L, width W of the test, thickness of the last chamber T1, second last chamber T2, and the N-4 chamber T5, in each of the three species (Table 10, 11, and 12). The correlation among these variables is positive and as the sizes of the chambers T1, T2 and T5 can affect the size of the test as indicated in the correlation matrix, this component is therefore identifiable with size variation in the specimens. That the first principal component often has the character of a size vector in foraminifera has been demonstrated by many authors (Berggren et al. 1967, Fordham and Geoffrey, 1978, Malmgren, 1976, deMeuter and Symons, 1975, Reyment, 1966, and Scott, 1972).

	I	II
Eigenvalue	4.35	1.64
Percent of total variance	43.5	16.4
Cumulative percentage	43.5	59.9
Eigenvectors		
L	.45	.06
J	.45	-.01
T1	.42	.25
I/I	-.10	.23
T1/T2	-.15	.08
T1/T5	-.06	.65
T	.13	-.24
T2	.44	.18
T5	.37	-.34
H	.07	.46

Table 10. First two principal components of H. delrioensis

	I	II
Eigenvalue	4.52	2.14
Percent of total variance	45.26	21.43
Cumulative percentage	45.26	66.69
Eigenvectors		
1	.42	.13
2	.44	.04
3	.31	.47
4	-.17	.18
5	-.14	.55
6	-.11	.60
7	.28	-.01
8	.41	.09
9	.41	-.15
10	-.16	.11

Table 11. First two principal components of H. planispira

	I	II
Eigenvalue	4.67	1.68
Percent of total variance	46.71	16.86
Cumulative percentage	46.71	63.57
L	.39	.20
W	.44	.02
T1	.29	.56
L/W	-.26	.26
T1/T2	-.09	.59
T1/T5	-.22	.43
U	.29	-.17
T2	.36	.03
T5	.39	.04
H	-.24	.04

Table 12. First two principal components of H. infracretacea

The second principal component - The second principal component demonstrates the most dominating patterns of shape variation occurring in the samples, which are not associated with general size variation. In H. delrioensis, the component shows a positive correlation with $T1/T5$ (the expansion rate of the last to the N-4 chamber), and with H (elevation of the last chamber to the inner whorl plane). The variation of the chamber expansion rates and the height of the spire will affect the shape of the test. The height of the spire is also influenced by the size and position of the last chamber in the final whorl. So this component can be explained as the size and position of the last chamber in the final whorl.

In H. planispira, the component also shows a positive correlation with $T1/T2$ and $T1$ in addition. This component can be explained as the variation in the size of the last chamber $T1$, which consequently will affect the expansion rates $T1/T5$ and $T1/T2$, for $T1$ is the numerator in both of these two ratios. H. infracretacea has a similar correlation coefficient pattern in which $T1/T5$, $T1/T2$ and $T1$ show positive correlation with this component (Table 12). So this

component can be explained as a variation in shape of the test as influenced by the variation of the size of the last chamber. Variations in increment rates between successive chambers in planktonic foraminifera have been found to contribute to shape variation by Banner and Blow (1965), Blow (1956), Malmgren (1976), and Scott (1967).

5.5 Plots of the First Two Principal Component Scores

The first two principal components of each species have been explained as above and a geometric interpretation is necessary in order to give a good visual presentation of the results. Principal component scores of each species are plotted on a plane to show their respective positions and these plots summarize most of the morphologic variation within the respective species (Fig. 6, 7, and 8). In all the three plots, a general pattern of the specimen distribution can be visualized, that is; gross size of the test increases along the X-axis from west to east, and expansion rate of the last to the preceding chambers increases along the Y-axis from south to north.

Fig. 6. Plots of the first two principal component scores of *Hedbergella delrioensis*. Specimens numbered 1-5 are illustrated in Plate I, figs. 10-14 respectively.

Fig. 7. Plots of the first two principal component scores of *Hedbergella planispira*. Specimens numbered 1-5 are illustrated in Plate I, figs. 15-19 respectively.

Fig. 8. Plots of the first two principal component scores of *Hedbergella infractetacea*. Specimens numbered 1-5 are illustrated in Plate I, figs. 20-24 respectively.

5.6 Discussion

For simplicity of discussions that follow, the plot of the scores of the first two principal components of each species (Fig. 6, 7, 8) is divided into four quadrants represented by four "phases" as in the following diagram:

The principal component I is the size component, with the specimen size increasing along the axis from west to east. Principal component II is the component of chamber expansion rate, with the rate increasing along the axis from south to north. The specimens are distributed in each phase as follows:

- 1) Phase I - Specimens with larger test size and rapid chamber expansion rate within that species.
- 2) Phase II - Specimens with smaller test size and rapid chamber expansion rate within that species.
- 3) Phase III - Specimens with smaller test size and slow chamber expansion rate within that species.

- 4) Phase IV - Specimens with larger test size and slow chamber expansion rate within that species.

Based on the basic statistical data (Table 2) of each species, it is known that the mean of the test size of H. infracretacea is largest, and that of H. planispira is smallest and that of H. delrioensis is medium. In H. delrioensis, the mean chamber expansion rate is higher than that in H. infracretacea and H. planispira. Their morphologic variations can be compared by placing their phase diagrams as follows:

This diagram can explain the gradation of characters between the species. The relative position between each phase of different species explains why the form of

larger specimen of H. planispira with rapid chamber expansion rate (phase I of H. planispira) is similar to the form of the smaller specimen of H. delrioensis with less rapid expansion rate (phase III of H. delrioensis). As shown by canonical analysis (section 4.6), these two species can only be discriminated by their different expansion rates and if their rates are close to each other, the two forms cannot be easily distinguished. On the contrary, H. infracretacea can be distinguished from the other two species by its height of the spire, so the differentiation is distinct despite its gradation of size and chamber expansion rate compared with the other two species.

The specimens in phase I of H. delrioensis, phase III of H. planispira and phase IV of H. infracretacea have maximum morphologic distances between them and these specimens would most likely be chosen as type specimens by conventional taxonomic procedure. As have been discussed in section 4.7, they are only end members of the groups to which they belong. The morphologic variations within each species and morphologic gradation between each species as have been discussed in this section are illustrated in Figures 10-24, Plate I.

CHAPTER 6

SUMMARY

Three morphologically similar Cretaceous planktonic foraminiferal species Hedbergella delrioensis, H. planispira and H. infracretacea from one Aptian core sample from Darwin, Australia were analyzed morphometrically in order to determine their between-species differences and within-species variations.

These three species were first subjectively established through conventional qualitative appraisal of characters and intermediate forms with indistinct characters were temporarily allocated to the "most likely" of one of the species. Sixty-three specimens of H. delrioensis, forty-eight specimens of H. planispira and thirty-seven specimens of H. infracretacea were selected at random for morphometric analysis. These analyses were based on measurements of morphological characters of the test that were able to distinguish groups interpreted as species. The measurements included the length and width of the test, number of chambers in the final whorl, thickness of the last five chambers in the final whorl, width of the umbilicus and height of the spire. Ratios of some of the above measurements were also employed in order to show the relative size of the umbilicus, elongation ratio of the test and expansion rate of succeeding chambers.

The means of each variable of each species show that Hedbergella infracretacea is the largest species, and has the highest trochospire. H. planispira is the smallest species, and has the largest number of chamber in the final whorl. H. delrioensis is intermediate in size, has the most elongated test and most rapid expansion rate of each succeeding pair of chambers. These statistical data support the qualitative observations on the species.

Stepwise discriminant analysis was performed on the three species based on the 14 variables measured. Four of them were selected through use of the stepwise method as the most important characters that can significantly discriminate the species. They are the height of spire, the size of the last chamber, the width of the test and the relative size of the umbilicus.

Discriminant functions of each species were established by the linear combination of these four variables and each specimen was then tested with its probability as a group (species) member in each of the three groups. Each specimen was then reclassified into the species in which it has the greatest probability of membership. The analysis showed that more than 90% of the original species had been correctly classified through qualitative appraisal.

As the group and specimen relationships in four dimensional space with axes represented by the four discriminant variables was still impossible to visualize, a plane was selected for plotting each specimen a point on it so that the group means were farthest apart and group and specimen relationships were displayed. This plot of canonical variables shows that Hedbergella infracretacea can be distinguished from the other two species on the first canonical axis and H. delrioensis and H. planispira can be distinguished on the second axis. The specimens located closest to the means of each group were chosen as type specimens as they can most represent their respective groups.

Further analysis was performed on each species one at a time in order to investigate the relationships between each variable within a species. As chambers are an integrated part of the test, the size of the chambers in the final whorl is highly correlated with the gross size factor of the test. This means that the larger the test, the larger are its chambers. The expansion rate of chambers, however, is quite independent of test size, as the correlation between them is low. This means that the rapid or slow expansion rates of chambers in any species are not controlled by the size of the test. Similar results were obtained on each species.

Principal component analysis shows that in each of the three species analysed, about 60% or more of the morphologic variation can be accounted for by the first two principal components and the explanations of them are essentially similar in each species. The first principal component correlates highly with the size of the test and the thickness of the chambers in the final whorl, and so it is identifiable with size variation in the specimens. The second principal component correlates with the size of the last chamber and the expansion rate of chambers, so it is explained as a variation in shape of the test as influenced by the variation in the size of the last chamber.

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PLATE I

- 1-3 Hedbergella delrioensis (Carsey)
1, umbilical view; x150
2, apertural view; x150
3, spiral view; x150
Hypotype based on Discriminant Function Analysis
value, CUCWL1018.
- 4-6 Hedbergella planispira (Tappan)
1, umbilical view; x200
2, apertural view; x200
3, spiral view; x200
Hypotype based on Discriminant Function Analysis
value, CUCWL2028.
- 7-9 Hedbergella infracretacea (Glaessner)
1, umbilical view; x150
2, apertural view; x150
3, spiral view; x150
Hypotype based on Discriminant Function Analysis
value, CUCWL3023.
- 10-14 Hedbergella delrioensis (Carsey)
(All in umbilical views); 10, CUCWL1005, x150;
11, CUCWL1056, x180; 12, CUCWL2012, x170,
13, CUCWL1004, x130; 14, CUCWL1003, x160.
- 15-19 Hedbergella planispira (Tappan)
(All in umbilical views); 15, CUCWL2024, x160;
16, CUCWL2046, x230; 17, CUCWL2040, x250;
18, CUCWL3029, x170; 19, CUCWL2011, x200.
- 20-24 Hedbergella infracretacea (Glaessner)
(All in umbilical views); 20, CUCWL3007, x130;
21, CUCWL3037, x160; 22, CUCWL3017, x160;
23, CUCWL3001, x140; 24, CUCWL3031, x140.

APPENDIX I

RAW DATA SET

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14
.02500	.10125	.55000	.11250	1.24130	.11111	1.05802	1.00250	1.33333	1.00000	.02500	.10625	.06250	.01250
.17500	.15000	.55000	.09375	1.16666	.14285	1.15384	1.08333	1.25000	1.07500	.02500	.08125	.05000	.00000
.10125	.15000	.55000	.10625	1.20833	.10344	1.21428	1.16666	1.20000	1.00000	.01875	.08750	.06250	.00000
.22500	.10125	.55000	.11250	1.21379	.11111	1.05802	1.06250	1.14285	1.00000	.02500	.10625	.08125	.00000
.20000	.10250	.55000	.11250	1.23076	.12500	1.20000	1.25000	1.20000	2.25000	.02500	.09375	.05000	.02500
.19375	.15625	.50000	.10000	1.24000	.09677	1.14285	1.15384	1.20000	2.00000	.01875	.08750	.05000	.00000
.19375	.16875	.55000	.10625	1.14814	.09677	1.13333	1.15384	1.30000	2.12500	.01875	.09375	.05000	.01250
.10750	.15000	.55000	.11250	1.25000	.10000	1.20000	1.25000	1.00000	2.00000	.01875	.09375	.05625	.00000
.16875	.13750	.55000	.08750	1.22727	.11111	1.16666	1.00000	1.20000	1.00000	.01875	.07500	.05000	.00000
.16975	.13750	.50000	.09375	1.22721	.11111	1.25000	1.09090	1.22222	1.07500	.01875	.07500	.05000	.00000
.10250	.15000	.50000	.08750	1.08333	.11538	1.16666	1.20000	1.25000	1.00000	.01875	.07500	.05000	.00000
.10250	.13750	.55000	.08750	1.10101	.11538	1.16666	1.09090	1.10000	1.00000	.01875	.07500	.05000	.00000
.21250	.17500	.55000	.11875	1.21428	.11764	1.18750	1.14285	1.16666	1.00000	.02500	.10000	.06250	.00000
.10250	.13750	.55000	.09375	1.10101	.11538	1.25000	1.20000	1.25000	2.12500	.02500	.09375	.05000	.00000
.10750	.15625	.55000	.09375	1.20000	.13333	1.15384	1.00000	1.30000	1.00000	.02500	.08125	.05625	.00000
.20000	.16250	.55000	.11250	1.23077	.09375	1.28571	1.07692	1.08333	2.00000	.01875	.08750	.05625	.01250
.10125	.15000	.50000	.10000	1.20833	.13793	1.14285	1.07692	1.08333	1.00000	.02500	.08750	.08250	.00000
.19375	.15000	.50000	.10000	1.29166	.12903	1.14285	1.16666	1.20000	2.00000	.02500	.08750	.05000	.00000
.22500	.10750	.50000	.11250	1.20000	.11111	1.12500	1.14285	1.16666	1.00000	.02500	.10000	.06250	.00000
.10750	.10250	.55000	.10000	1.15384	.13333	1.14285	1.16666	1.20000	2.00000	.02500	.08750	.05000	.00000
.21250	.16250	.55000	.10000	1.30769	.11764	1.23076	1.08333	1.09090	1.00000	.02500	.08125	.06250	.01250
.10750	.10250	.55000	.10000	1.15384	.10000	1.14285	1.07692	1.30000	1.00000	.01875	.08750	.06250	.00000
.10250	.10125	.50000	.09375	1.10000	.09000	1.00000	1.30000	1.25000	1.00000	.01250	.08125	.05000	.01250

APPENDIX I Cont'd

24	.15000	.13750	0.00000	.08125	1.09091	.16666	1.08333	1.20000	1.25000	1.42500	.02500	.07500	.05000	.01250
25	.18750	.16250	5.00000	.10000	1.15384	.13333	1.14285	1.07692	1.30000	1.77777	.02500	.06750	.05625	.01250
26	.18750	.14375	5.00000	.10000	1.30434	.10000	1.14285	1.00000	1.16666	1.60000	.01875	.06750	.06250	.01250
27	.16250	.13750	5.00000	.09375	1.18181	.11538	1.25000	1.09090	1.10000	1.87500	.01875	.07500	.05000	.00000
28	.16875	.13750	0.00000	.08750	1.22727	.14814	1.16666	1.20000	1.11111	2.00000	.02500	.07500	.04375	.00000
29	.16250	.13750	5.50000	.08750	1.18181	.11538	1.27272	1.00000	1.10000	1.55555	.01875	.06875	.05625	.00000
30	.16875	.13750	5.50000	.10000	1.22727	.11111	1.14285	1.16666	1.20000	2.00000	.01875	.06750	.05000	.00000
31	.21250	.18125	5.00000	.11875	1.17281	.11764	1.18750	1.06666	1.25000	1.70000	.02500	.10000	.06250	.00000
32	.15625	.13750	5.50000	.08125	1.13636	.12000	1.08333	1.09090	1.10000	1.44444	.01875	.07500	.05625	.01250
33	.16875	.14375	5.50000	.08750	1.17391	.14814	1.07692	1.30000	1.11111	1.75000	.02500	.06125	.05000	.00000
34	.20000	.15625	5.00000	.10625	1.28000	.12500	1.06250	1.23076	1.18181	1.88888	.02500	.10000	.05625	.00000
35	.20625	.16875	5.50000	.11250	1.22222	.06060	1.12500	1.14285	1.27272	2.00000	.01250	.10000	.05625	.02500
36	.20000	.16250	5.00000	.11250	1.23076	.09375	1.12500	1.14285	1.40000	2.25000	.01875	.10000	.05000	.00000
37	.18750	.15625	5.50000	.10000	1.20000	.13333	1.14285	1.07692	1.44444	2.00000	.02500	.08750	.05000	.01250
38	.22500	.19375	5.50000	.11250	1.16129	.13688	1.12500	1.00000	1.06666	1.50000	.03125	.10000	.07500	.00000
39	.21250	.16250	5.00000	.11250	1.30769	.05882	1.12500	1.23076	1.06333	1.80000	.01250	.10000	.06250	.00000
40	.21250	.18750	5.00000	.11875	1.13333	.11764	1.11764	1.21428	1.16666	1.70000	.02500	.10625	.06250	.00000
41	.19375	.16250	5.50000	.10625	1.19230	.12903	1.21428	1.07692	1.30000	1.70000	.02500	.06750	.06250	.00000
42	.20625	.18125	5.50000	.11250	1.13793	.09090	1.05882	1.21428	1.27272	1.80000	.01875	.10625	.06250	.03500
43	.18750	.15000	5.00000	.09375	1.25000	.06666	1.07142	1.07692	1.18181	1.66666	.01250	.08750	.05625	.01250
44	.16250	.13750	5.50000	.10000	1.18181	.11538	1.33333	1.20000	1.11111	2.00000	.01875	.07600	.05000	.00000
45	.17500	.13750	5.50000	.10000	1.27272	.10714	1.14285	1.27272	1.22422	2.00000	.01875	.08750	.05000	.01250
46	.17500	.15000	5.50000	.09375	1.16666	.10714	1.07142	1.27272	1.22422	1.87500	.01875	.08750	.05000	.00000
47	.15000	.13125	5.50000	.08125	1.14285	.12500	1.08333	1.20000	1.11111	1.55714	.01875	.07500	.04375	.00000
48	.15625	.13125	0.00000	.09375	1.19047	.12000	1.15384	1.18181	1.37500	1.87500	.01875	.06125	.05000	.00000
49	.17500	.14375	0.00000	.09375	1.21719	.14285	1.15384	1.18181	1.10000	1.66666	.02500	.04125	.05625	.01250

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50	.10125	.13750	5.00000	.09375	1.51018	.10340	1.07142	1.27272	1.10000	1.87500	.01875	.08750	.05000	.01250
51	.17500	.13750	5.00000	.09375	1.27272	.10714	1.15304	1.10101	1.10000	1.86666	.01875	.08125	.05025	.00000
52	.15000	.13125	5.00000	.08125	1.14285	.12500	1.09333	1.09090	1.37500	1.85714	.01875	.07500	.04375	.00000
53	.10750	.10250	5.00000	.10000	1.15304	.13333	1.00000	1.33333	1.00000	1.77777	.02500	.10000	.05625	.00000
54	.10750	.15000	6.00000	.10000	1.25000	.16666	1.14285	1.00000	1.27272	2.00000	.03125	.08750	.05000	.01250
55	.15000	.13750	6.00000	.08125	1.00000	.12500	1.08333	1.20000	1.11111	1.95714	.01875	.07500	.04375	.00000
56	.10250	.12500	6.00000	.08750	1.30000	.11538	1.16666	1.20000	1.25000	2.33333	.01875	.07500	.03750	.00000
57	.20000	.16250	5.50000	.10625	1.23076	.09375	1.21428	1.16666	.92307	1.70000	.01875	.08750	.06250	.01250
58	.10750	.15000	6.00000	.08750	1.25000	.13333	1.16666	.92307	1.10101	1.70000	.02500	.07500	.05000	.01250
59	.17500	.15000	5.50000	.08750	1.16666	.10714	1.07692	1.00000	1.01010	1.75000	.01875	.08125	.05000	.01250
60	.16250	.12500	6.00000	.09375	1.30000	.15304	1.25000	1.20000	1.11111	1.67500	.02500	.07500	.05000	.00000
61	.19375	.15625	5.50000	.10000	1.24000	.12903	1.14285	1.27272	1.37500	2.00000	.02500	.08750	.05000	.00000
62	.20000	.16250	5.00000	.11250	1.23076	.09375	1.28570	1.16666	1.20000	2.00000	.01875	.08750	.05625	.00000
63	.21250	.18125	5.00000	.11250	1.17241	.11764	1.12500	1.14285	1.16666	1.80000	.02500	.10000	.06250	.00000
64	.10875	.15625	6.00000	.08125	1.08000	.14814	1.08333	1.09090	1.22222	1.82500	.02500	.07500	.05000	.00000
65	.17500	.15000	6.00000	.08250	1.16666	.14285	.93333	1.00000	1.10000	1.25000	.02500	.07500	.05000	.00000
66	.14375	.13125	6.00000	.05625	1.09523	.17391	.90000	1.00000	1.11111	1.12500	.02500	.06250	.05000	.00000
67	.13125	.11250	6.00000	.05625	1.16666	.14285	1.00000	1.12500	1.14285	1.28571	.01875	.05625	.04375	.00000
68	.17500	.16250	5.00000	.08875	1.15000	.17391	1.10000	1.11111	1.12500	1.71428	.02500	.08250	.04375	.00000
69	.18500	.11250	6.00000	.06250	1.11111	.15000	1.11111	1.00000	1.12500	1.86666	.01875	.05625	.03750	.00000
70	.12500	.10625	6.00000	.06250	1.17667	.15000	1.00000	1.25000	1.00000	1.82857	.01875	.06250	.04375	.00000
71	.19025	.17750	5.50000	.08125	1.13333	.12000	1.00000	1.10101	1.10000	1.84444	.01875	.08125	.05625	.01250
72	.13750	.11075	6.00000	.05625	1.15769	.13636	.90000	1.00000	1.25000	1.12500	.01875	.06250	.05000	.00000
73	.11075	.10625	6.00000	.06250	1.11764	.21052	1.11111	1.12500	1.33333	2.00000	.02500	.05625	.03125	.00000
74	.17500	.13750	6.00000	.08875	1.13636	.18000	1.10000	1.00000	1.11111	1.37500	.02500	.06250	.05000	.00000

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75	.17500	.13750	0.00000	.08750	1.27272	.14285	1.16666	1.09090	1.10000	1.40000	.02500	.07500	.06450	.00000
76	.14375	.11875	0.00000	.06250	1.21052	.13083	1.10000	1.00000	1.25000	1.25000	.01875	.06250	.05000	.00000
77	.14375	.11875	0.00000	.06250	1.21052	.17391	1.00000	1.11111	1.12500	1.42857	.02500	.06250	.04375	.00000
78	.16875	.13750	0.00000	.06250	1.22727	.14814	1.00000	1.00000	1.11111	1.45000	.02500	.06250	.05000	.00000
79	.13125	.11875	0.00000	.06250	1.10526	.14285	1.00000	1.11111	1.12500	1.45000	.01875	.06250	.05000	.00000
80	.17500	.16875	0.00000	.07500	1.03703	.16285	.05714	1.07692	1.08333	1.40000	.02500	.08750	.06450	.00000
81	.13125	.11250	0.50000	.06675	1.16666	.14285	1.00000	1.22222	1.12500	1.57142	.01875	.06875	.04375	.00000
82	.15000	.12500	0.50000	.06875	1.20000	.16666	1.10000	1.11111	1.00000	1.57500	.02500	.06250	.05000	.00000
83	.15000	.14375	0.00000	.07500	1.02367	.16666	1.00000	1.09090	1.22222	1.50000	.02500	.07500	.05000	.00000
84	.14375	.12500	0.00000	.06875	1.15000	.17391	1.10000	1.00000	1.25000	1.57142	.02500	.06250	.04375	.00000
85	.16875	.14375	0.00000	.07500	1.17391	.14814	1.09090	1.00000	1.00000	1.53333	.02500	.06875	.05625	.00000
86	.12500	.10625	0.50000	.06250	1.17647	.15000	1.00000	1.25000	1.14285	1.66666	.01875	.06250	.03750	.00000
87	.19375	.17500	0.50000	.09375	1.10714	.16129	1.07142	1.07692	1.10000	1.50000	.03125	.08750	.06450	.00000
88	.16875	.14375	0.00000	.08125	1.17391	.14814	1.00000	1.18181	1.22222	1.44444	.02500	.08125	.05625	.00000
89	.16250	.14375	0.50000	.06250	1.13043	.15384	1.00000	1.11111	1.12500	1.42857	.02500	.06250	.04375	.00000
90	.16250	.15000	0.00000	.08125	1.08333	.15384	1.08333	1.20000	1.11111	1.62500	.02500	.07500	.05000	.00000
91	.15000	.11875	0.00000	.06875	1.26315	.12500	1.00000	1.22222	1.12500	1.57500	.01875	.06875	.05000	.00000
92	.14375	.13125	0.00000	.06875	1.09523	.13083	1.00000	1.10000	1.11111	1.57500	.01875	.06875	.05000	.00000
93	.13125	.11250	0.50000	.05625	1.16666	.14285	.90000	1.11111	1.00000	1.26571	.01875	.06250	.04375	.00000
94	.15000	.11875	0.00000	.08750	1.26315	.16666	1.16666	1.20000	1.11111	2.00000	.02500	.07500	.04375	.00000
95	.16875	.13750	0.50000	.08125	1.22727	.11111	1.18181	1.10000	.83333	1.50000	.01875	.06875	.06250	.00000
96	.13750	.11250	0.00000	.06250	1.22222	.09090	.83333	1.09090	1.22222	1.45000	.01250	.07500	.05000	.00000
97	.13125	.11250	0.00000	.06250	1.16666	.14285	1.11111	1.12500	1.14285	1.42857	.01875	.05625	.04375	.00000
98	.16125	.15000	0.00000	.08750	1.20833	.13793	1.07692	1.18181	1.10000	1.55555	.02500	.08125	.05625	.01250
99	.14375	.13750	0.50000	.06875	1.04865	.17391	1.00000	1.22222	1.12500	1.57500	.02500	.06875	.05000	.00000
100	.15000	.12500	0.00000	.06875	1.20000	.16666	1.00000	1.00000	1.22222	1.57500	.02500	.06875	.05000	.00000

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101	.18750	.18250	5.00000	.10000	1.15388	.13333	1.14285	1.27272	1.10000	1.77777	.02500	.08750	.05625	.00000
102	.14375	.11250	5.00000	.07500	1.27777	.13043	1.07090	1.10000	1.25000	2.09000	.01875	.06875	.03750	.00000
103	.12500	.11250	7.00000	.05000	1.11111	.20000	1.00000	1.00000	1.14285	1.14285	.02500	.05000	.04175	-.00625
104	.14375	.12500	5.00000	.08125	1.15000	.13043	1.18181	1.22222	1.12500	1.85714	.01875	.06875	.04375	.00000
105	.15625	.11875	5.00000	.08125	1.31578	.16000	1.18181	1.10000	1.11111	1.85714	.02500	.06875	.04175	.00000
106	.17500	.15625	5.00000	.08750	1.12000	.17857	.87500	1.14285	1.16666	1.40000	.03125	.10000	.06250	-.01250
107	.18250	.13750	5.00000	.08750	1.18181	.11538	1.16666	1.00000	1.33333	1.75000	.01875	.07500	.05000	.00000
108	.16875	.13750	5.00000	.08750	1.22727	.14814	1.16666	1.09090	1.10000	1.75000	.02500	.07500	.05000	.00000
109	.13125	.10625	5.00000	.06875	1.23529	.09523	1.22222	1.12500	1.14285	1.83333	.01250	.05625	.03750	.00000
110	.15000	.13125	5.50000	.07500	1.16285	.12500	1.10000	1.09090	1.10000	1.33333	.01875	.07500	.05625	.00000
111	.16250	.13125	5.00000	.08125	1.23009	.11538	1.08333	1.20000	1.11111	1.62500	.01875	.07500	.05000	.00000
112	.24375	.23125	5.00000	.10000	1.05405	.12820	.94117	1.21428	.87500	1.09090	.03125	.10625	.09175	-.06250
113	.22125	.21875	5.50000	.11875	1.05714	.13513	1.05555	1.12500	1.14285	1.45153	.03125	.11250	.08125	-.02500
114	.20625	.17500	5.00000	.10000	1.17857	.12121	1.06666	1.07142	1.27272	1.60000	.02500	.09175	.06250	-.02500
115	.23750	.21250	5.50000	.11250	1.11764	.10526	1.12500	.94117	1.06450	1.28571	.02500	.10000	.08750	-.03750
116	.22750	.20000	5.00000	.10000	1.18750	.10326	.84210	1.18750	1.00000	1.43750	.02500	.11875	.08875	-.04375
117	.20000	.20000	5.50000	.11875	1.00000	.09375	1.11764	1.06250	1.14285	1.98333	.01875	.10625	.07500	-.05000
118	.23125	.20000	5.50000	.12500	1.15625	.10810	1.42857	1.14666	1.00000	1.89666	.02500	.08750	.07500	-.03750
119	.18125	.18250	5.00000	.08750	1.11550	.10384	1.10000	1.16888	1.20000	1.40000	.01875	.08750	.08250	-.03125
120	.19375	.16250	5.50000	.09375	1.19230	.09677	1.25000	.92307	1.18181	1.58165	.01875	.07500	.06875	-.02500
121	.25625	.24375	5.00000	.11250	1.05128	.09756	1.00000	1.00000	1.00000	1.28571	.02500	.11250	.08750	-.03750
122	.20625	.18750	5.50000	.10625	1.11000	.09000	1.08250	1.14285	1.18000	1.55555	.01675	.10000	.08875	-.03750
123	.21250	.17500	5.50000	.10000	1.21428	.11764	1.06666	1.07142	1.16000	1.77777	.02500	.09375	.05625	-.03750
124	.19375	.16875	5.50000	.10000	1.14814	.12903	1.14285	1.07692	1.08333	1.80000	.02500	.08750	.06450	-.02500
125	.22875	.20000	5.50000	.11250	1.09375	.08571	1.10000	1.28571	1.18000	1.85000	.01875	.11250	.08875	-.02500

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126	.21875	.20625	5.00000	.10000	1.06060	.11428	.94117	1.00000	1.06250	1.55333	.02500	.10625	.07500	-.02500
127	.18750	.16875	5.50000	.08750	1.11111	.10000	1.00000	1.16666	1.33333	1.55555	.01875	.08750	.05625	-.03125
128	.18750	.16875	5.50000	.08125	1.11111	.13333	1.08333	1.00000	1.00000	1.18181	.02500	.07500	.06875	-.03125
129	.20625	.19375	5.00000	.10625	1.06451	.12121	1.06250	1.00000	1.06066	1.41666	.02500	.10000	.07500	-.03125
130	.21875	.19375	5.00000	.10625	1.12903	.11428	1.05882	1.06250	1.14285	1.26571	.02500	.10625	.08750	-.01250
131	.20000	.21875	5.00000	.10625	.91828	.15625	1.06250	1.23076	.92857	1.54545	.03125	.10000	.06875	-.03750
132	.20625	.20000	5.50000	.10000	1.03125	.12121	.88888	1.12500	1.33333	1.60000	.02500	.11250	.08250	-.04375
133	.21250	.19375	5.00000	.08750	1.09677	.11764	1.00000	1.07692	1.00000	1.16666	.02500	.08750	.07500	-.02500
134	.20000	.17500	5.00000	.10000	1.16285	.12500	1.06666	1.07142	1.16666	1.53333	.02500	.09375	.07500	-.03125
135	.19375	.18125	5.50000	.08750	1.06896	.12903	1.00000	1.07692	1.18181	1.40000	.02500	.08750	.08250	-.04375
136	.20625	.18750	5.00000	.09375	1.10860	.12121	1.07142	1.00000	1.16666	1.50000	.02500	.08750	.06250	-.03125
137	.19375	.17500	5.00000	.10000	1.10714	.12903	1.00000	1.14285	1.00000	1.45454	.02500	.10000	.06875	-.02500
138	.18750	.16875	5.50000	.08750	1.11111	.10000	.93333	1.25000	1.20000	1.40000	.01875	.09375	.08250	-.02500
139	.18750	.16250	5.00000	.09375	1.15384	.10000	1.15384	1.08333	1.20000	1.86666	.01875	.08125	.05625	-.02500
140	.17500	.16250	5.00000	.08125	1.07692	.14285	.81250	1.53333	1.20000	1.18181	.02500	.10000	.06875	-.01250
141	.19375	.17500	9.20000	.08750	1.10714	.12903	1.00000	1.00000	1.27272	1.40000	.02500	.08750	.08250	-.03125
142	.21250	.18125	6.00000	.10000	1.17241	.11764	1.00000	1.00000	1.06666	1.53333	.02500	.10000	.07500	-.1075
143	.19375	.16875	5.50000	.08750	1.16814	.12903	1.00000	1.16666	1.20000	1.55555	.02500	.08750	.05625	-.01250
144	.20625	.16875	5.00000	.10000	1.22222	.09090	1.14285	1.07692	1.30000	1.60000	.01875	.08750	.06250	-.01250
145	.21250	.18750	5.50000	.11875	1.13333	.11764	1.18750	1.00000	1.23076	2.11111	.02500	.10000	.05625	-.01250
146	.18750	.16250	5.00000	.10000	1.15384	.13333	1.14285	1.07692	1.18181	1.60000	.02500	.08750	.06250	-.01250
147	.20000	.18750	6.00000	.08750	1.06666	.12500	1.00000	1.00000	1.40000	1.55555	.02500	.08750	.05625	-.01250
148	.18750	.15000	6.00000	.09375	1.25000	.10000	1.07142	1.07692	1.30000	1.87500	.01875	.08750	.05000	-.02500

INPUT DATA MATRIX - COLUMNS = VARIABLES, ROWS = CASES.

EXIT

APPENDIX II STEPWISE DISCRIMINANT ANALYSIS

COMPUTATION PROCEDURE*

Notation p = number of variables
 g = number of groups used for the analysis
 t = total number of groups
 n_m = number of cases in group m
 n = total number of cases
 x_{mki} = value of variable i for case k
of group m

Assume for simplicity that the first g of the t groups are used for the analysis.

Step 1. The data are read and the following are formed:

$$\text{Means } \bar{x}_i = \frac{1}{n} \sum_{m=1}^g \sum_{k=1}^{n_m} x_{mki} \quad i=1,2,\dots,p$$

$$\text{Group means } \bar{x}_{mi} = \frac{1}{n_m} \sum_{k=1}^{n_m} x_{mki}, \quad \begin{matrix} i=1,2,\dots,p \\ m=1,2,\dots,p \end{matrix}$$

Group standard deviations

$$s_{mi} = \sqrt{\frac{1}{n_m - 1} \sum_{k=1}^{n_m} (x_{mki} - \bar{x}_{mi})^2}$$

$i=1,2,\dots,p$
 $m=1,2,\dots,p$

*Computational procedure adopted from Dixon, 1975;
BMDP biomedical computer programs, pp. 239-244.

Within and total cross-product matrices

$$W = \{w_{ij}\}; \quad w_{ij} = \sum_{m=1}^g z_m \sum_{k=1}^{n_m} (x_{mki} - \bar{x}_{mi})(x_{mkj} - \bar{x}_{mj})$$

$$T = \{t_{ij}\}; \quad t_{ij} = \sum_{m=1}^g \sum_{k=1}^{n_m} (x_{mki} - \bar{x}_i)(x_{mkj} - \bar{x}_j)$$

where z_m is the covariance weight of group m ,

$$i=1,2,\dots,p$$

$$j=1,2,\dots,p$$

Step 2 At each step of the procedure the variables are divided into two disjoint sets; those included in the discriminant functions and those not included. Assume for simplicity that the first r are included.

$$\text{Let } W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \quad \text{and} \quad T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

where W_{11} and T_{11} are $r \times r$.

$$\text{Let } A = \begin{bmatrix} W_{11}^{-1} & W_{11}^{-1} W_{12} \\ W_{21} W_{11}^{-1} & W_{22} - W_{21} W_{11}^{-1} W_{12} \end{bmatrix} = \{a_{ij}\}$$

$$\text{and } B = \begin{bmatrix} T_{11}^{-1} & T_{11}^{-1} T_{12} \\ T_{21} T_{11}^{-1} & T_{22} - T_{21} T_{11}^{-1} T_{12} \end{bmatrix} = \{b_{ij}\}$$

The following statistics are computed:

- a) Coefficients and constant terms of the classification functions

$$c_{ki} = (n-g) \sum_{j=1}^r \bar{x}_{kj} a_{ij} \quad \begin{matrix} i=1,2,\dots,r \\ k=1,2,\dots,g \end{matrix}$$

$$c_{k0} = -\frac{1}{2} \sum_{i=1}^r c_{ki} \bar{x}_{ki} \quad k=1,2,\dots,g$$

- b) The square of the Mahalanobis distance between each pair of the groups

$$D_{ml}^2 = \sum_{i=1}^r (c_{mi} - c_{li})(\bar{x}_{mi} - \bar{x}_{li}) \quad m, l=1,2,\dots,g$$

- c) The F values for testing differences between each pair of groups

$$F_{ml} = \frac{(n-g-r+1) n_m n_l}{r(n-g)(n_m + n_l)} D_{ml}^2 \quad m, l=1,2,\dots,g$$

with r and $n-g-r+1$ degrees of freedom

- d) F value for each variable

(1) If variable j has been entered

$$F_j = \frac{a_{jj} - b_{jj}}{b_{jj}} \frac{n-r-g+1}{g-1}$$

with degrees of freedom $g-1$ and $n-r-g+1$

(2) If variable j has not been entered

$$F_j = \frac{b_{jj} - a_{jj}}{a_{jj}} \frac{n-r-g}{g-1}$$

with degrees of freedom $g-1$ and $n-g-r$

e) Wilks' Λ to test equality of group means

$$U = \text{Det}(W_{11}) / \text{Det}(T_{11})$$

with degrees of freedom $(r, g-1, n-g)$

f) Approximate F statistic to test equality of group means

$$F = \frac{1 - U}{U^{1/s}} \cdot \frac{ms + 1 - rq/2}{rq}$$

$$\text{where } s = \sqrt{\frac{r^2 q^2 - 4}{r^2 + q^2 - 5}}, \text{ if } r^2 + q^2 \neq 5$$

$$s = 1, \text{ if } r^2 + q^2 = 5$$

$$m = n - \frac{r+q+3}{2}$$

$$q = g-1$$

Its degrees of freedom are rq and $ms+1-rq/2$.

If either r or q is 1 or 2, the approximation is exact.

g) Tolerance values

$$w_i = a_{ii} / t_{ii} \quad i=r+1, \dots, p$$

Step 3 To move from one step to the next, one variable is added or removed from the discriminating set according to one of the following rules:

- a) If there are one or more variables which have entered and have a control value of 1 and an F value less than "F to remove", the one with the smallest F will be deleted.
- b) If no variable satisfies a), then from among those variables which have not been included, which pass the tolerance test and have greatest control value, one which has greatest F-to-enter is selected.

Step 4 When the number of variables entered is equal to one of the numbers indicated on the Subproblem card and after the last step the following are computed for $l=1,2,\dots,t$; $m=1,2,\dots,g$; $k=1,2,\dots,n_l$:

- a) Value of the m^{th} classification function evaluated at case k of group l

$$s_{lmk} = c_{m0} + \sum_{j=1}^r c_{mj} x_{lkj}$$

- b) Posterior probability of case k in group l having come from group m

$$P_{lmk} = \frac{p_m \exp(s_{lmk})}{\sum_{i=1}^g p_i \exp(s_{lik})}$$

where p_m is the prior probability of group m.

- e) Square of Mahalanobis distance of case k in group m from group 1

$$D_{lmk}^2 = (n-g) \sum_{i=1}^r \sum_{j=1}^r (x_{mki} - \bar{x}_{1i}) a_{ij} (x_{mkj} - \bar{x}_{1j})$$

This may be used as a chi-square variable with r degrees of freedom for classification purposes.

Step 5 At this point let p denote the number of variables which are included after the last step and let W and T be their within and total sum of product matrices. Let $B = T - W$. The eigenvalue problem

$$Bu_i = \lambda_i Wu_i \quad i=1, 2, \dots, p$$

is solved to find coefficients, u_i , of canonical variables and the amount of dispersion λ_i explained by each canonical variable.

The vectors are normalized so that

$$u_i' Wu_j = \delta_{ij}$$

The canonical correlations $\rho_1, \rho_2, \dots, \rho_p$ relative to the groups are then computed

$$\rho_i = (\lambda_i / (1 + \lambda_i))^{1/2}$$

For each case the first three canonical variables are computed

$$z_{mki} = \sum_{j=1}^r u_{ji} (x_{mkj} - \bar{x}_j) \quad \begin{array}{l} m=1, \dots, g \\ k=1, \dots, n_m \\ i=1, 2, 3 \end{array}$$

The first two of these are plotted on a scattergram. The cutpoints used are the average of adjacent values.

APPENDIX III

STEPWISE DISCRIMINANT ANALYSIS

BMOP7M - STEPWISE DISCRIMINANT ANALYSIS,
HEALTH SCIENCES COMPUTING FACILITY
UNIVERSITY OF CALIFORNIA, LOS ANGELES

PROGRAM REVISED JULY 7, 1975
MANUAL DATE -- 1975

IN THIS VERSION OF BMOP7M

-- SUBPROBLEMS ARE NOT PERMITTED.

-- GROUP CODES ON CUTPOINTS MUST BE STATED.

PROGRAM CONTROL INFORMATION

PROBLEM TITLE IS 'STEPWISE DISCRIMINANT ANALYSIS'.
INPUT VARIABLES ARE 15.

CASES ARE 148.

UNIT IS 5.

FORMAT IS '(F1.0,F7.5,YF8.5/4F8.5)'.
VARIABLE NAMES ARE MED,L,M,N,1,LM,UL,T12,T23,T34,T15,U,T2,T5,M.

GROUPING IS MED.
GROUP CODES(1) ARE 1,2,3.

NAMES(1) ARE DELIPLAN,INFHA.
END/

PROBLEM TITLE STEPWISE DISCRIMINANT ANALYSIS

NUMBER OF VARIABLES TO READ IN. . . . 15
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. . . . 0

TOTAL NUMBER OF VARIABLES 15

NUMBER OF CASES TO READ IN. . . . 148

CASE LABELING VARIABLES TRANSFORMATIONS

LIMITS AND MISSING VALUE CHECKED AFTER TRANSFORMATIONS

BLANKS ARE ZEROS

INPUT UNIT NUMBER 3

REMIING INPUT UNIT PRIOR TO READING. . DATA. . . . NO

INPUT FORMAT

(F1.0,F7.5,YF8.5/4F8.5)

VARIABLES TO BE USED

1 MED

6 LM

11 T15

2 L

UL

12 U

3 M

8 T12

13 T2

4 N

9 T23

14 T5

5 T1

10 T34

15 M

VARIABLE		AFTER TRANSFORMATION		INTERVAL RANGE	
NO.	NAME	MINIMUM	MAXIMUM	CATEGORY	CATEGORY NAME
		LIMIT	LIMIT	CODE	
1	MAXIMUM FORCED LEVEL	0	30	33333	DEL
1	MAXIMUM NUMBER OF STEPS	0	30	33333	PLAN
1	PRIOR PROBABILITIES	0	30	33333	INFNA

[illegible]

APPENDIX III Cont'd

PAGE

MEANS

VARIABLE	GROUP = DEL	PLAN	INFRA	ALL GPS
1 MED	1.00900	2.00000	3.00000	1.02432
2 L	.18452	.15143	.20608	.17918
3 M	.15298	.13021	.18581	.15380
4 N	5.45651	5.94792	5.22162	5.54865
5 T1	.04921	.07201	.09049	.09046
6 LM	1.20677	1.16614	1.11350	1.17015
7 UL	.11602	.14740	.11583	.12649
8 T12	1.14816	1.04145	1.04806	1.08553
9 T23	1.14375	1.11056	1.09330	1.12117
10 T34	1.19397	1.13728	1.14762	1.16400
11 T15	1.03719	1.48620	1.48052	1.63419
12 U	.02143	.02227	.02382	.02230
13 T2	.06061	.06927	.09561	.08323
14 T5	.05446	.04896	.06824	.05612
15 M	.00238	-.00039	-.02589	-.00653
COUNTS	65.	48.	37.	148.

STANDARD DEVIATIONS

VARIABLE	GROUP = DEL	PLAN	INFRA	ALL GPS
1 MED	.00000	.00000	.00000	.00000
2 L	.02088	.01833	.01864	.01954
3 M	.01483	.01731	.02128	.01819
4 N	.34186	.29607	.42091	.35222
5 T1	.01080	.01139	.01104	.01106
6 LM	.05553	.06373	.06471	.06062
7 UL	.02407	.02356	.01620	.02129
8 T12	.08621	.09726	.11090	.09005
9 T23	.08982	.07936	.09510	.08796
10 T34	.09759	.08974	.11055	.10086
11 T15	.19091	.23380	.20514	.20917
12 U	.08416	.08006	.00354	.00348
13 T2	.01003	.00959	.01081	.01019
14 T5	.00737	.00722	.01025	.00821
15 M	.00436	.00326	.01163	.00818

APPENDIX III Cont'd

PAGE

MEANS/ST.DEVS.

GROUP = DEL PLAN INFR ALL GPS

VARIABLE	DEL	PLAN	INFR	ALL GPS
1 MED	.0000	.0000	.0000	.0000
2 L	8.63404	8.25923	11.04270	9.17053
3 M	9.08807	7.52064	8.73212	8.45707
4 N	15.90252	20.04981	12.17416	15.75358
5 T1	9.18442	6.32411	8.99485	8.18178
6 LN	21.72725	18.24889	17.20662	19.30370
7 UL	5.29418	6.25495	7.14986	5.94073
8 T12	16.63234	10.70775	9.45053	12.08786
9 T23	12.73399	13.99322	11.49645	12.73784
10 T34	12.23403	12.67292	9.68018	11.54082
11 T15	9.62553	6.35679	7.21630	7.81277
12 U	5.15475	5.48699	6.69157	5.59677
13 T2	8.03571	7.00475	8.84246	8.17176
14 T5	7.19860	6.78387	6.66025	6.83443
15 M	.28490	-.11970	-2.48318	-.77439

STEP NUMBER	0	38
VARIABLE	F TO FORCE REMOVE LEVEL	F TO FORCE ENTER LEVEL
	DF= 2 146	DF= 2 145
2 L	85.824	1.000000
3 M	17.770	1.000000
4 N	49.981	1.000000
5 T1	98.438	1.000000
6 LN	27.572	1.000000
7 UL	38.265	1.000000
8 T12	24.110	1.000000
9 T23	4.275	1.000000
10 T34	4.954	1.000000
11 T15	51.669	1.000000
12 U	4.193	1.000000
13 T2	75.861	1.000000
14 T5	57.884	1.000000
15 M	189.035	1.000000

APPENDIX III Cont'd

STEP NUMBER 1 VARIABLE ENTERED 15 M		F TO FORCE REMOVE LEVEL OF= 2 145		VARIABLE		F TO FORCE ENTER LEVEL OF= 2 144		TOLERANCE		PAGE
VARIABLE										
15 M				2 L		54.715	1	.996004		
				3 W		40.158	1	.974520		
				4 N		35.195	1	.985187		
				5 Y		91.753	1	.998975		
				6 LW		5.942	1	.970565		
				7 UL		34.180	1	.985920		
				8 T12		18.466	1	.997031		
				9 T23		2.143	1	.999834		
				10 T34		4.486	1	.975616		
				11 Y15		35.810	1	.971594		
				12 U		.394	1	.972803		
				13 T2		53.924	1	.999977		
				14 T5		16.890	1	.975146		
U=STATISTIC OR MILKS' LAMBDA .2772096 DEGREES OF FREEDOM 1 2 145										
APPROXIMATE P-STATISTIC 189.035 DEGREES OF FREEDOM 2.00 145.00										
F - MATRIX DEGREES OF FREEDOM = 1 145										
PLAN DEL PLAN										
PLAN 3.13										
INFRA 340.56 253.54										
CLASSIFICATION FUNCTIONS										
VARIABLE GROUP = DEL PLAN INFRA										
15 M				35.58906		-5.83801		-431.69702		
CONSTANT				-1.14097		-1.09975		-7.33342		

APPENDIX III Cont'd

PAGE

STEP NUMBER 2
VARIABLE ENTERED 5 T1

VARIABLE	F TO FORCE REMOVE LEVEL	VARIABLE	F TO FORCE ENTER LEVEL	TOLERANCE
5 T.	DF= 2 144	2 L	DF= 2 145	.385970
15 M	91.755 1	3 M	5.092 1	.486752
	177.612 1	4 N	9.146 1	.890264
		6 L	4.157 1	.962612
		7 UL	3.255 1	.924237
		8 T12	6.011 1	.861114
		9 T23	4.457 1	.993601
		10 T34	1.053 1	.967490
		11 T15	3.663 1	.856928
		12 U	7.860 1	.910889
		13 T2	4.321 1	.425266
		14 T5	4.592 1	.673456
			7.292 1	

U-STATISTIC OR WILKS' LAMBDA .1218850 DEGREES OF FREEDOM 2 2 145
APPROXIMATE F-STATISTIC 134.233 DEGREES OF FREEDOM 4.00 288.00

F = MATRIX DEGREES OF FREEDOM = 2 144

PLAN DEL PLAN
INFRA 82.80
169.36 195.99

CLASSIFICATION FUNCTIONS

GROUP = DEL PLAN INFRA
VARIABLE
5 T1 811.62134 589.83569 825.04858
15 M .87113 -31.33568 -467.39067
CONSTANT -41.35815 -22.34036 -48.89232

APPENDIX III Cont'd

PAGE 2

STEP NUMBER 3
VARIABLE ENTERED 3 M

VARIABLE	F TO FORCE REMOVE LEVEL	*	VARIABLE	F TO FORCE ENTER LEVEL	TOLERANCE
	OF= 2 143	*		DF= 2 142	
3 M	9.146 1	*	2 L	.220 1	.154632
5 T1	46.245 1	*	4 N	4.154 1	.889843
15 H	68.786 1	*	6 LM	.035 1	.643215
		*	7 UL	6.382 1	.906860
		*	8 T12	.362 1	.594723
		*	9 T23	.212 1	.961384
		*	10 T34	3.670 1	.951378
		*	11 T15	2.530 1	.582660
		*	12 U	5.740 1	.742774
		*	13 T2	.368 1	.300653
		*	14 T5	1.784 1	.447652

U-STATISTIC OR WILKS' LAMBDA .1080627 DEGREES OF FREEDOM 5 2 145
APPROXIMATE F-STATISTIC 97.336 DEGREES OF FREEDOM 6.00 266.00

F MATRIX DEGREES OF FREEDOM 5 143

PLAN DEL PLAN
INFRA 130.89 139.39

CLASSIFICATION FUNCTIONS

VARIABLE	GROUP	DEL	PLAN	INFRA
3 M	247.41301	293.73193	371.24194	
5 T1	526.86626	252.21550	398.33716	
15 H	100.78061	87.49287	-317.18018	
CONSTANT	-46.30530	-29.28511	-59.98582	

APPENDIX III Cont'd

PAGE

STEP NUMBER 4
VARIABLE ENTERED 7 UL

VARIABLE	F TO FORCE REMOVE LEVEL	DF	VARIABLE	F TO FORCE ENTER LEVEL	DF	TOLERANCE
3 M	9.512	1	2 L	.073	1	.151084
5 T1	29.966	1	4 N	1.855	1	.797699
7 UL	0.382	1	6 LM	.213	1	.605505
15 M	66.230	1	8 T12	.130	1	.578800
			9 T23	.226	1	.960985
			10 T34	3.695	1	.950011
			11 T15	3.598	1	.561692
			12 U	.894	1	.086695
			13 T2	.207	1	.293855
			14 T5	2.416	1	.436731

U-STATISTIC OR WILKS' LAMBDA .0991501 DEGREES OF FREEDOM 4 2 145
APPROXIMATE F-STATISTIC 77.241 DEGREES OF FREEDOM 8.00 284.00

F - MATRIX DEGREES OF FREEDOM = 4 142

PLAN DEL PLAN
INFRA 100.30 114.32

CLASSIFICATION FUNCTIONS

VARIABLE	GROUP = DEL	PLAN	INFRA
3 M	163.88459	201.57024	294.88916
5 T1	805.01172	557.52051	651.27271
7 UL	379.81787	416.60596	345.14478
15 M	176.23335	170.25349	-248.61574
CONSTANT	-75.96121	-64.96399	-84.87440

CLASSIFICATION MATRIX

GROUP	PERCENT CORRECT	NUMBER OF CASES CLASSIFIED INTO GROUP
DEL	90.5	DEL PLAN INFRA
PLAN	89.6	57 6 0
INFRA	91.9	2 1 34

PAGE

JACKKNIFED CLASSIFICATION					PAGE
GROUP	PERCENT CORRECT	NUMBER OF CASES CLASSIFIED INTO GROUP -			
		DEL	PLAN	INFRA	
DEL	87.3	55	8	0	
PLAN	85.4	7	41	0	
INFRA	91.9	2	1	34	
TOTAL	87.8	64	50	34	

APPENDIX III Cont'd

PAGE

SUMMARY TABLE

STEP NUMBER	VARIABLE ENTERED	VARIABLE REMOVED	F VALUE TO ENTER OR REMOVE	NUMBER OF VARIABLES INCLUDED	U-STATISTIC	APPROXIMATE P-STATISTIC
1	15 M		189.0309	1	.2772	189.035
2	5 T1		91.7535	2	.1214	134.233
3	3 M		9.1455	3	.1081	97.336
4	7 UL		6.3821	4	.0992	77.241

APPENDIX III Cont'd

INCORRECT CLASSIFICATIONS

MAHALANOBIS D-SQUARE FROM AND POSTERIOR PROBABILITY FOR GROUP -

GROUP DEL	DEL	PLAN	INFRA
CASE			
1	4.7 .996	16.7 .002	27.2 .000
2	1.6 .785	4.1 .214	18.0 .000
3	1.6 .997	13.2 .003	19.1 .000
4	2.5 .995	14.3 .003	14.3 .003
5	10.0 .996	22.5 .002	48.9 .000
6	1.1 .986	9.8 .013	14.7 .001
7	3.4 .996	14.3 .004	26.2 .000
8	4.0 .999	19.0 .001	23.2 .000
9	1.6 .800	4.3 .199	17.2 .000
10	1.1 .949	6.9 .050	18.6 .000
11	2.0 .681	3.5 .318	14.6 .001
12	1.4 .774	3.8 .226	17.3 .000
13	3.5 .999	18.4 .001	19.3 .000
14	4.1 .796	6.8 .204	24.6 .000
15	1.1 .804	3.9 .196	15.2 .001
16	3.3 .999	18.0 .001	19.5 .000
17	1.4 .954	7.5 .046	19.4 .000
18	.6 .966	7.4 .034	18.1 .000
19	3.7 .991	15.1 .003	13.9 .006
20	1.0 .936	6.4 .062	15.4 .001
21	3.8 .713	9.9 .031	6.1 .197
22	1.3 .979	9.2 .019	13.5 .002
23	11.7 .626	14.0 .199	14.3 .175
24	9.1 .107	4.9 .893	37.3 .000
25	3.7 .821	9.0 .058	7.5 .121
26	2.1 .991	11.6 .009	29.8 .000
27	1.0 .981	6.5 .058	18.8 .000
28	2.8 .507	2.9 .493	21.0 .000
29	1.4 .774	3.8 .226	17.3 .000
30	2.0 .989	10.9 .011	21.4 .000
31	3.6 .998	18.0 .001	17.8 .001
32	6.0 .304	4.9 .602	9.7 .055
33	2.7 .888	2.3 .552	19.2 .000
34	1.1 .992	10.7 .008	18.5 .000
35	14.5 1.000	31.3 .000	47.3 .000
36	2.5 .999	17.0 .001	18.0 .000
37	3.8 .892	9.6 .050	9.3 .058
38	6.5 .983	15.4 .012	17.0 .005
39	6.1 .998	25.2 .000	21.2 .001
40	4.1 .997	14.1 .001	14.7 .002

APPENDIX III Cont'd

41	1.0	.688	9.9	.012	17.1	.000
42	12.7	.999	26.4	.001	44.4	.000
43	6.6	.985	17.0	.015	30.4	.000
44	2.0	.987	10.6	.013	21.7	.000
45	2.5	.991	11.9	.009	32.3	.000
46	.8	.931	6.0	.068	14.8	.001
47	2.7	.393	1.9	.606	19.0	.000
PLAN						
48	2.0	.945	7.7	.055	21.8	.000
49	3.5	.833	6.7	.167	32.2	.000
50	2.3	.964	8.9	.036	29.9	.000
51	1.2	.956	7.4	.044	18.5	.000

APPENDIX III Cont'd

PAGE

GROUP	PLAN	DEL	PLAN	INFRA
52	PLAN	2.7 .393	1.9 .606	19.0 .000
53		1.0 .938	0.4 .062	15.4 .001
54		8.4 .085	12.5 .115	39.0 .000
55	PLAN	2.8 .339	1.5 .661	17.5 .000
56		2.7 .846	6.1 .154	21.9 .000
57		2.9 .997	14.5 .003	27.1 .000
58		4.5 .461	4.6 .449	7.8 .040
59		4.6 .758	6.9 .242	27.3 .000
60		7.2 .862	10.8 .138	30.9 .000
61		4 .957	6.7 .042	16.2 .000
62		2.5 .999	17.0 .001	18.0 .000
63		2.5 .994	13.9 .003	14.8 .002
64	CASE	6.7 .097	2.2 .902	18.0 .000
65		20.1 .002	7.3 .998	27.0 .000
66		21.7 .000	4.9 1.000	33.7 .000
67		15.5 .001	2.4 .999	50.3 .000
68		11.8 .006	1.7 .994	29.3 .000
69		11.6 .005	1.0 .995	28.8 .000
70		11.9 .006	1.8 .994	30.7 .000
71		4.9 .399	4.1 .601	29.8 .000
72		16.4 .001	3.3 .999	29.1 .000
73		25.5 .001	11.0 .999	48.5 .000
74		11.2 .006	1.1 .994	24.4 .000
75	DEL	2.3 .555	2.7 .425	20.1 .000
76		11.5 .008	1.9 .992	25.6 .000
77		14.7 .002	1.9 .998	31.9 .000
78		15.3 .002	3.0 .998	25.8 .000
79		11.5 .005	1.9 .995	26.5 .000
80		16.2 .017	8.1 .982	22.0 .001
81		8.2 .030	1.3 .970	26.8 .000
82		10.5 .008	1.9 .992	27.5 .000
83		9.5 .018	1.6 .982	23.5 .000
84		11.8 .006	1.7 .994	29.3 .000
85		7.3 .036	1.7 .984	20.0 .000
86		11.9 .006	1.8 .994	30.7 .000
87		8.0 .419	7.3 .579	19.1 .002
88		4.3 .147	1.8 .852	18.9 .000
89		17.8 .001	4.5 .999	27.0 .000
90		5.8 .100	1.4 .900	19.2 .000
91		7.8 .045	1.6 .955	23.5 .000
92		8.6 .024	1.2 .976	21.4 .000
93		15.5 .001	2.4 .994	30.3 .000
94	DEL	9.4 .511	0.0 .000	14.5 .000

APPENDIX III Cont'd

95	---	3.4	.061	3.1	.539	24.2	.000
96		15.8	.043	9.6	.957	17.1	.000
97		11.2	.007	1.2	.993	28.9	.000
98		4.7	.431	4.4	.497	27.4	.000
99		13.2	.004	2.1	.996	8.3	.0/2
100		10.5	.008	.9	.992	27.4	.000
101	DEL	1.0	.938	6.4	.062	27.5	.000
102		6.0	.187	3.1	.813	15.4	.001
103		28.8	.000	8.3	1.000	25.7	.000
104		3.2	.402	2.4	.598	40.3	.000
105		7.4	.223	4.9	.777	21.5	.000
						29.4	.000

APPENDIX III Cont'd

GROUP	INFRA	DEL	PLAN	INFRA
106		11.0 .432	7.2 .856	15.7 .012
107	DEL	1.4 .774	3.8 .228	17.3 .000
108	DEL	2.8 .507	2.9 .493	21.0 .000
109		11.4 .184	8.5 .816	28.4 .000
110		5.1 .121	1.1 .879	19.4 .000
111		3.0 .480	2.8 .520	18.9 .000
CASE				
112		80.1 .000	80.0 .000	23.0 1.000
113		21.6 .000	31.6 .000	5.9 1.000
114		11.9 .004	17.0 .000	1.0 .995
115		31.8 .000	41.3 .000	3.2 1.000
116		38.8 .000	43.0 .000	4.3 1.000
117		47.1 .000	61.3 .000	12.3 1.000
118		30.1 .000	46.5 .000	8.6 1.000
119		20.6 .000	21.7 .000	2.6 1.000
120		13.9 .004	16.7 .000	2.7 .946
121		50.9 .000	58.6 .000	13.6 1.000
122		28.3 .000	37.6 .000	3.3 1.000
123		24.3 .000	29.5 .000	2.5 1.000
124		11.4 .013	16.4 .001	2.6 .986
125		18.9 .000	31.0 .000	2.9 1.000
126		22.7 .000	26.0 .000	3.1 1.000
127		22.4 .000	23.2 .000	2.5 1.000
128		22.5 .000	17.8 .001	3.0 .949
129		19.8 .000	26.6 .000	6 1.000
130		8.4 .132	15.9 .003	4.7 .865
131		36.3 .000	38.5 .000	7.7 1.000
132		37.1 .000	40.1 .000	3.7 1.000
133		24.9 .000	22.6 .000	5.0 1.000
134		17.3 .000	22.1 .000	1.4 1.000
135		36.2 .000	33.9 .000	4.9 1.000
136		22.1 .000	23.1 .000	7 1.000
137		11.9 .006	16.4 .001	1.5 .994
138		17.0 .001	18.0 .000	2.4 .949
139		13.4 .004	18.0 .000	2.4 .946
140	PLAN	9.5 .075	4.8 .782	8.3 .143
141		20.9 .000	19.1 .000	1.4 1.000
142		9.1 .023	14.1 .002	1.6 .975
143		7.7 .187	6.6 .315	5.7 .497
144		6.3 .404	14.2 .006	5.5 .589
145	DEL	6.9 .726	20.8 .001	4.9 .274
146	DEL	3.7 .621	9.0 .058	7.5 .121
147		13.5 .025	13.4 .074	6.3 .900
148		13.4 .020	14.8 .001	5.4 .974

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APPENDIX III. Cont'd

EIGENVALUES

5.44641 1.26828

CANONICAL CORRELATIONS

.86040 .74776

VARIABLE COEFFICIENTS FOR CANONICAL VARIABLES

3 M 25.36887 -21.07391
5 T1 -2.14305 96.90567
7 UL -15.08128 -11.02536
15 H -98.15025 27.07876

CONSTANT -2.67795 -3.95837

GROUP CANONICAL VARIABLES EVALUATED AT GROUP MEANS

DEL -7.6863 1.20792
PLAN -1.41028 -1.36034
INFRA 3.14611 -2.29195

POINTS TO BE PLOTTED			
GROUP	MEAN	SYMBOL	SYMBOL
COORDINATES FOR CASES FOR MEAN			
DEL	-0.77	1.21	D
PLAN	-1.42	-1.36	P
INFRA	3.15	-0.24	I

APPENDIX III Cont'd

PAGE 1

POINTS TO BE PLOTTED

GROUP MEAN SYMBOL SYMBOL
COORDINATES FOR CASES FOR MEAN

DEL 1 1.21 0 1
PLAN 1.02 -1.36 P 2
INFRA 3.15 -0.29 I 3

GROUP DEL

CASE	X	Y	CASE	X	Y	CASE	X	Y	CASE	X	Y	CASE	X	Y	CASE	X	Y
1	-1.00	2.24	11	-0.57	0.09	21	.42	.67	31	.13	2.43	41	-.47	1.49	51	-.79	1.05
2	-.94	.39	12	-.88	.35	22	-.07	1.21	32	.30	-.64	42	-1.96	2.80	52	-1.16	-.23
3	-.25	2.04	13	-.03	2.56	23	.66	-.07	33	-1.15	-.14	43	-1.17	1.57	53	-.51	.84
4	.23	1.90	14	-1.40	.53	24	-2.77	-.48	34	-.57	1.67	44	-.91	1.56	54	-2.49	1.07
5	-2.88	2.82	15	-.66	.36	25	.72	.50	35	-1.88	3.40	45	-2.03	1.94	55	-1.00	-.36
6	-.19	1.37	16	-1.25	2.82	26	-1.78	1.94	36	-.02	2.49	46	-.47	.78	56	-1.20	.61
7	-1.11	2.05	17	-.89	1.05	27	-.90	.96	37	.56	.63	47	-1.16	-.23	57	-1.23	2.22
8	-.42	2.68	18	-.77	1.15	28	-1.31	-.01	38	.18	1.33	48	-1.12	1.04	58	.43	-.45
9	-.83	.40	19	.39	1.77	29	-.88	.35	39	.44	2.87	49	-2.32	.86	59	-1.69	.52
10	-.84	1.00	20	-.51	.84	30	-.85	1.61	40	.29	2.30	50	-1.97	1.43	60	-1.72	.80

GROUP PLAN

61 -0.61 1.02
62 -0.02 2.49
63 .14 1.85

GROUP INFRA

CASE	X	Y	CASE	X	Y	CASE	X	Y	CASE	X	Y	CASE	X	Y	CASE	X	Y
64	-.82	-1.01	74	-1.43	-1.96	84	-1.93	-1.85	94	-2.03	.16	104	-1.36	-.16	114	-.83	.59
65	-.87	-2.66	75	-1.24	.05	85	-1.13	-1.35	95	-.81	-.21	105	-1.93	-.35	115	-.12	.12
66	-1.74	-3.19	76	-1.50	-1.84	86	-2.08	-1.79	96	-1.14	-1.27	106	-.01	-1.08	116	-.35	.35
67	-1.61	-2.45	77	-2.07	-2.32	87	-.55	-.34	97	-1.82	-1.85	107	-.88	.35	117	-.50	.50
68	-1.93	-1.85	78	-1.26	-2.43	88	-1.14	-.75	98	.37	-.50	108	-1.31	-.01	118	-.86	.86
69	-1.92	-.73	79	-1.87	-.98	89	-1.17	-2.83	99	-1.81	2.11	109	-1.37	.59	119	-.83	.83
70	-2.08	-1.74	80	-.82	-1.82	90	-1.06	-.94	100	-1.83	-1.77	110	-1.14	-.83	120	-.50	.50
71	-2.16	.03	81	-1.84	-1.24	91	-1.45	-1.18	101	-.51	.84	111	-1.03	-.12	121	-.86	.86
72	-1.57	-2.51	82	-1.83	-1.77	92	-1.20	-1.50	102	-1.69	-.50	112	-1.93	-.86	122	-.86	.86
73	-2.87	-2.46	83	-1.37	-1.56	93	-1.81	-2.45	103	-1.93	-3.86	113	-1.93	-3.86	123	-.86	.86

APPENDIX III Cont'd

CASE	X	Y	CASE	X	Y	CASE	X	Y	CASE	X	Y
112	7.44	-2.25	122	4.35	.37	132	4.89	-1.00	142	2.01	.11
113	3.31	.77	123	3.69	-.27	133	2.97	-1.54	143	.96	-.80
114	2.42	.03	124	2.16	.08	134	2.98	-.18	144	1.43	.84
115	4.78	.29	125	3.49	1.11	135	4.34	-1.91	145	1.52	1.96
116	5.10	-.81	126	3.30	-.55	136	3.36	-1.01	146	.72	.50
117	5.83	.95	127	3.18	-.98	137	2.32	-.06	147	1.49	-1.15
118	4.40	1.73	128	2.76	-1.96	138	2.56	-.81	148	2.08	.19
119	2.97	-.89	129	3.49	.07	139	2.39	-.06			
120	2.43	-.04	130	1.75	.66	140	.63	-1.42			
121	5.67	-.28	131	4.28	-1.01	141	2.96	-1.44			

PAGE

OVERLAP OF DIFFERENT GROUPS IS INDICATED BY *

[illegible]

CANONICAL VARIATION

APPENDIX IV PRINCIPAL COMPONENT ANALYSIS

COMPUTATION PROCEDURE

Notation p = number of variables

n = number of cases

Let x_{ij} be the value of the j^{th} variable and i^{th} case, where $(i = 1, 2, \dots, n), (j = 1, 2, \dots, p)$

1. The data are standardized:

$$W = (w_{ij}) = (x_{ij} - x_{\cdot j}) / \left\{ \sum_{i=1}^n (x_{ij} - x_{\cdot j})^2 \right\}^{\frac{1}{2}}$$

$$\text{where } x_{\cdot j} = 1/n \sum_{i=1}^n x_{ij}$$

2. The correlation matrix C of the values w_{ij} is computed

$$C = (c_{jk}) = \frac{\sum_{i=1}^n (x_{ij} \cdot x_{ik}) - \left(\sum_{i=1}^n x_{ij} \cdot \sum_{i=1}^n x_{ik} \right) / n}{\left(\sum_{i=1}^n x_{ij}^2 - \left(\sum_{i=1}^n x_{ij} \right)^2 / n \right) \left(\sum_{i=1}^n x_{ik}^2 - \left(\sum_{i=1}^n x_{ik} \right)^2 / n \right)}$$

3. The eigenvalues and corresponding eigenvectors of C are found, solving the system

$$(C - \lambda I) b = 0$$

where I = identity matrix

Let λ_j be the j^{th} eigenvalue and

$B = (b_{kj})$ the matrix whose columns are the correspond eigenvectors.

4. The matrix of multiplication is performed

$$Z = (z_{ij}) = WB = \sum_{k=1}^p (w_{ik})(b_{kj})$$

which transforms the W 's into their orthogonal components.

APPENDIX V. PRINCIPAL COMPONENT ANALYSIS

SERVICES COORDINATEUR

MINISTRE DES COMMUNICATIONS

PRINCIPAL COMPONENT ANALYSIS

NUMBER OF VARIABLES TO BE USED IN 10

NUMBER OF CASES TO BE USED IN

FOR H. DELRIOUENSIS 64

FOR H. PLANISPIRA 50

FOR H. INFRACHETACEA 34

VARIABLES TO BE USED

1 L

2 W

3 T1

4 L/W

5 T1/T2

6 T3/T5

7 U

8 T2

9 T5

10 H

COMPUTER SERVICES

DEPARTMENT OF COMMUNICATIONS

APPENDIX V Cont'd

MINISTRE DES COMMUNICATIONS										
SERVICES D'ORDONNATEUR										
1	2	3	4	5	6	7	8	9	10	
1	1,00000	92495	85339	01556	22837	06472	27880	85831	70719	14001
2	92495	1,00000	82843	39085	23774	07758	30561	84167	68670	08024
3	85339	82843	1,00000	10424	00540	20578	16443	87249	58546	21195
4	01556	39085	10424	1,00000	07436	08513	10899	13080	12196	13544
5	22837	23774	00540	07436	1,00000	20246	02698	46050	16700	11404
6	06472	07758	20578	08513	20246	1,00000	05208	07372	65865	17150
7	27880	30561	16443	10899	02698	05208	1,00000	13367	19802	24260
8	85831	84167	87249	13080	46050	07372	13367	1,00000	60178	24666
9	70719	68670	58546	12196	16700	65865	19802	60178	1,00000	01752
10	14001	08024	21195	13544	11404	17150	24260	24666	01752	1,00000

SIMILARITY MATRIX

SPECIES - M. DELMIDENSIS

COMPUTER SERVICES

DEPARTMENT OF COMMUNICATIONS

APPENDIX V Cont'd

SERVICES D'ORDONNATEUR

MINISTÈRE DES COMMUNICATIONS

	1	2	3
1	4.34889	43.49925	43.49925
2	1.68045	16.40460	59.90587
3	1.30489	13.04904	72.95291
4	1.03859	10.38595	83.33885
5	.86681	8.66680	92.00706
6	.63130	6.31309	98.32014
7	.15507	1.55071	99.87086
8	.01022	.10218	99.97304
9	.00166	.01662	99.98964
10	.00103	.01035	99.99998

COLUMN 1 = EIGENVALUES COLUMN 2 = PERCENT OF TRACE

COLUMN 3 = CUMULATIVE PERCENT OF TRACE

SPECIES = M. DELRIOENSIS

COMPUTER SERVICES

DEPARTMENT OF COMMUNICATIONS

APPENDIX V Cont'd

SERVICES D'ORDONNATEUR

MINISTRE DES COMMUNICATIONS

	1	2	3	4	5	6	7	8	9	10
1	.45475	.06685	.03514	.17035	.08455	.09017	.56605	.06008	.28036	.58370
2	.05836	.01884	.13568	.12732	.08592	.01064	.49165	.04614	.29492	.64529
3	.02173	.25794	.017565	.12827	.15218	.13809	.40837	.55549	.53916	.28678
4	.10627	.23084	.25704	.74227	.45170	.17665	.06064	.04250	.10851	.24202
5	.15356	.08226	.51713	.49130	.57226	.06158	.02887	.09478	.31635	.14030
6	.06793	.65933	.37979	.19738	.16445	.13181	.04595	.57359	.03267	.02952
7	.13799	.24933	.50776	.10058	.53136	.59123	.13483	.02533	.00095	.01024
8	.44527	.18186	.10085	.12479	.13975	.13908	.58911	.21295	.65397	.28216
9	.37711	.34833	.17434	.28335	.21416	.02025	.50412	.69434	.05575	.04230
10	.07889	.46414	.41855	.03235	.22487	.74230	.04659	.00145	.00504	.00394

PRINCIPAL AXIS MATRIX - COLUMNS = EIGENVECTORS, ROWS = VARIABLES

SPECIES - H. DELKIOENSIS

COMPUTER SERVICES

DEPARTMENT OF COMMUNICATIONS

APPENDIX V - Cont'd

SERVICES COORDINATOR MINISTRE DES COMMUNICATIONS

	1	2	3	4	5	6	7	8	9	10
1	3,75334	6,7671	1,00808	1,6808	1,11388	6,1608	2,9416	1,0885	1,0702	0,0272
2	-1,00362	0,46845	-0,78191	-0,92297	0,23032	0,63278	-0,07532	-0,01750	0,00468	-0,03053
3	0,20935	-0,54887	0,17054	0,77938	-0,131025	-0,47581	0,09201	0,00345	0,03430	-0,01144
4	4,79504	-2,42251	1,49905	0,6135	0,31099	-0,06304	0,0307	0,46041	1,0019	0,08139
5	1,12150	3,30938	-0,66208	0,28105	-0,01976	1,05190	-0,05234	0,07506	0,02965	-0,05427
6	-0,23384	0,97897	0,06412	-0,10772	0,33799	-0,06435	-0,55453	0,02803	-0,00373	-0,00951
7	0,92489	1,92153	-0,08347	-1,66996	-0,69151	0,28081	-0,33729	0,09701	0,01619	0,01450
8	0,42323	1,22369	-0,29325	0,78453	-0,76054	1,05362	0,77896	-0,05613	0,03034	-0,02737
9	-2,34386	-0,50573	0,57588	0,5712	-0,19786	0,15526	-0,34878	-0,04479	0,00203	0,00348
10	-2,12248	-0,18186	-0,4492	0,4565	-0,92840	-0,24566	0,09994	-0,02323	-0,04990	0,02451
11	-1,06374	-1,19618	-0,22843	-2,16082	-1,56423	0,35711	0,34936	0,02125	0,00182	-0,01940
12	-2,39664	-0,73424	0,36001	-0,65194	-0,62849	0,02948	0,11105	-0,02668	-0,00332	-0,02186
13	2,87418	0,37365	-1,07354	0,40461	-0,05871	-0,27881	0,52493	-0,09540	0,03496	-0,04128
14	-2,58067	0,77581	-2,08900	-0,42146	-0,15520	0,62852	0,14333	0,15264	0,05914	-0,01139
15	-0,22476	-1,27640	-0,13451	0,07055	0,20148	-0,61597	0,44736	-0,04452	0,01290	-0,01329
16	7,1938	1,85167	-0,43911	1,10567	-1,72202	0,22054	0,26915	0,02500	0,01908	-0,02542
17	0,35807	-1,51146	0,28529	0,06468	0,13213	0,49632	0,64820	-0,00040	0,03040	-0,01842
18	-0,29827	0,84692	-0,39134	0,83245	1,62448	-0,15050	0,23579	-0,03593	0,00264	-0,01475
19	3,48894	-0,23903	-0,46088	0,6143	0,37512	-0,19993	0,62718	-0,05713	-0,00964	0,00815
20	0,14924	0,18432	-1,16516	-1,29742	0,31241	0,34290	0,24557	0,02506	0,01003	-0,00891
21	0,60914	-1,65781	-0,75925	2,71925	0,54555	-0,93038	0,94849	-0,01647	-0,02805	-0,00741
22	0,73977	-1,37155	0,6714	-0,5634	1,18117	-0,24071	0,02455	0,06862	0,03483	0,01786
23	-4,59352	-3,70636	2,35676	-2,03824	0,54715	-2,03343	0,13989	-0,03339	0,04310	-0,02984

COMPUTER SERVICES IN PARTMENT OF COMMUNAL AFFAIRS

APPENDIX V Cont'd

24	.44830	-1.54926	-1.20580	.88171	.27925	-.59535	.03938	.01270	-.00070
25	.05211	.01867	2.18365	1.78607	-.08458	.36018	.0340	.04488	.02134
26	-2.37539	-.04637	-.66049	-.06255	-1.35849	-.06113	.13572	-.00517	-.00055
27	-2.54295	.25641	-.81804	-.26381	.99046	.54640	.34335	.06078	-.01005
28	-2.54204	-1.66234	-.02974	.67074	-2.00246	.36006	-.19085	-.02992	.03838
29	-1.28638	.85524	.18289	-.37282	.20890	-.69311	.72072	.01095	-.01422
30	3.12746	.17623	-1.33303	.05623	-.45919	-.13724	.19711	-.07750	.00547
31	1.40886	.42640	.51580	.33646	2.03660	-.38911	.42957	-.13171	-.00556
32	1.90088	2.91562	2.05659	-.29282	-1.33105	.10669	-.42957	-.00614	-.00723
33	1.13791	2.32022	-.47526	-.61618	.60613	-1.39773	.16183	.08535	-.01328
34	-.20628	-.28956	-1.50629	-.63546	1.08765	-.91621	-.05345	.00148	-.01674
35	4.64358	-2.39557	-.58761	.39163	.18329	1.05185	-.11140	.05810	.06991
36	1.86016	.98345	1.81597	1.38301	-.19429	-2.26926	-.08588	-.04212	-.00900
37	3.81898	.01654	-.95279	-.1.16286	-.12869	-.16129	.44162	-.09288	.06653
38	1.07227	-.96641	-.79011	.71762	-.66276	.40031	.19462	-.01587	-.01339
39	3.43756	1.20599	1.90671	-.1.58795	-1.01763	1.27380	.10254	-.04150	.08526
40	-.31344	.40596	3.03160	-.12431	-.43336	-.38957	-.52206	-.00345	.01166
41	-2.35412	.64148	-1.68091	.52677	-2.08841	-.15175	.58249	.01636	.02074
42	-1.11817	1.77454	1.02571	.38425	.30475	.22704	.41566	-.00500	.00522
43	-.73669	-.08541	.70445	-.1.78047	.26070	-.42024	.00349	.00800	-.01741
44	-2.19600	-.03024	.27702	-.75829	-.33644	-.23711	.77226	-.00734	-.03639
45	-.76606	-.53588	.69118	.35092	.02850	1.72500	.18151	-.06217	-.02618
46	-1.11501	1.33039	2.16653	.59470	1.35964	.14429	-.07218	-.06213	.03517
47	-1.36948	-.59396	1.15749	.99396	.08797	-.45248	.16043	-.05530	.02239
48	.46519	-.81238	MINISTER OF COMMERCE	-2.07913	1.38833	MINISTER OF COMMERCE	.51256	.09457	.03682
49	-.01845	-.05249	-.71003	.37711	1.00001	2.01403	.13854	-.00680	-.00140

[illegible]

PRINCIPAL COMPONENT SCORES • COLUMNS = EIGENVECTORS, ROWS = CASES
SPECIES • H. DELHIENSIS

9,9999

SCALE FACTOR ON X IS	9.99999	SCALE FACTOR ON Y IS	9.99999
.350			
.275			
.200			
.125			
.050			
.025			
.100			
.175			
.250			
.310			
.400			
.500			

X=AXI COMPONENT 1, Y=AXIS COMPONENT 2

DEPARTMENT OF COMMUNICATIONS

SPECTER - H. DELRIENST

APPENDIX A Cont'd

SERVICES COORDINATOR MINISTRE DES COMMUNICATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.00000	.91130	.70599	.05919	.10089	.12238	.56375	.75320	.76210	.122758
2	.91130	1.00000	.65308	.46195	.21018	.17597	.62836	.77934	.77193	.20609
3	.70599	.65308	1.00000	.06227	.31842	.43782	.27166	.77592	.49556	.11853
4	.05919	.46195	.06227	1.00000	.27531	.16914	.52499	.24692	.23016	.01583
5	.10089	.21018	.31842	.27531	1.00000	.67491	.11657	.34128	.38200	.19157
6	.12238	.17597	.43782	.16914	.67491	1.00000	.12431	.00537	.55100	.15527
7	.56375	.62836	.27166	.32499	.11657	.12431	1.00000	.54538	.56292	.19643
8	.75320	.77934	.77592	.24692	.34128	.00537	.54538	1.00000	.74384	.27751
9	.76210	.77193	.49556	.23016	.38200	.55100	.56292	.74384	1.00000	.28864
10	.122758	.20609	.11853	.01583	.19157	.15527	.19643	.27751	.28864	1.00000

SIMILARITY MATRIX

SPECIES - M. PLANISPINA

APPENDIX V Cont'd

SERVICES D'ORDONNATEUR

MINISTÈRE DES COMMUNICATIONS

1	2	3
1	4,52601	45,26045
2	2,14376	21,43771
3	1,08788	10,87885
4	56177	5,61761
5	78723	7,47233
6	45019	4,50198
7	17055	1,70551
8	.00880	.08801
9	.00322	.03215
10	.00092	.00921

COLUMN 1 = EIGENVALUES COLUMN 2 = PERCENT OF TRACE
COLUMN 3 = CUMULATIVE PERCENT OF TRACE

SPECIES - H; PLANISPIRA

APPENDIX V - Cont'd

GENES DORDONNEUR

MINISTRE DES COMMUNICATIONS

	1	2	3	4	5	6	7	8	9	10
1	.42507	.13639	.13250	.09399	.29422	.02234	.53151	.07006	.10224	.62180
2	.44881	.04467	.17942	.02481	.07097	.08976	.48739	.10136	.13898	.69537
3	.31991	.47149	.06620	.10587	.11869	.06955	.28535	.63848	.53320	.14491
4	.17200	.18701	.72929	.16804	.44215	.30676	.03551	.04329	.05540	.28322
5	.14862	.55557	.05778	.12371	.25595	.61450	.10526	.10934	.42156	.08212
6	.11601	.60560	.12790	.16191	.26384	.35088	.12130	.50692	.53569	.01084
7	.28932	.01040	.34463	.43571	.57632	.34526	.59684	.00462	.00814	.00515
8	.41469	.09940	.11552	.15262	.37621	.32771	.24509	.12592	.65993	.14715
9	.41096	.15630	.19671	.23619	.03669	.37663	.58527	.53906	.36050	.00172
10	.16077	.11093	.47275	.80059	.25581	.15714	.08132	.01998	.01390	.00842

PRINCIPAL AXIS MATRIX - COLUMNS = EIGENVECTORS, ROWS = VARIABLES

SPECIES - H. PLANISPIRA

APPENDIX V Cont'd

	MINISTÈRE des COMMUNICATIONS									
	SERVICES D'ORIGINEUR									
	1	2	3	4	5	6	7	8	9	10
1	1,94240	1,14395	1,30763	2,3255	0,2123	3,1451	0,0266	0,6220	0,2192	0,4698
2	1,68316	1,186327	0,8531	4,2685	7,7446	1,37563	1,17459	0,00345	0,3573	0,06113
3	0,12134	2,73669	0,96706	0,21972	5,2000	0,09583	0,03433	0,01844	0,09943	0,00723
4	2,40887	1,56566	1,1042	0,1831	1,3857	2,8746	1,2107	0,5942	0,2867	0,1232
5	0,93494	4,7289	0,69619	0,72253	0,67598	0,0057	1,9409	0,1174	0,5478	0,01098
6	2,92223	2,63318	0,97687	0,71356	0,64596	4,6079	1,18230	0,03972	0,6782	0,04477
7	2,38428	0,88274	3,0710	0,5787	4,2578	1,5475	5,0384	0,7122	0,0622	0,2342
8	0,89013	4,6219	0,94918	3,28754	0,24521	0,12554	4,4268	0,00384	0,2105	0,04352
9	1,23171	2,61034	4,0171	5,7209	0,7654	0,6084	0,05870	0,00691	0,09467	0,06647
10	3,32457	1,21719	1,76180	1,84348	2,4968	9,4549	1,1173	1,31073	1,12912	0,04934
11	1,8730	0,07756	0,62230	0,33674	1,09956	7,3888	0,13488	0,02840	0,04955	0,00901
12	1,25074	1,19032	0,99194	5,9663	4,3348	2,9981	0,00056	0,01609	0,01652	0,01985
13	1,23333	0,41187	1,9578	3,9960	1,09216	0,81800	1,9668	0,4194	0,1699	0,2096
14	2,6553	0,90646	0,65532	1,3606	1,96053	0,24924	0,69123	0,04077	0,00969	0,03431
15	1,26312	1,61385	0,36812	2,3892	0,55013	8,4755	0,32599	0,0776	0,03785	0,01190
16	4,11065	1,78698	1,10623	8,1215	5,8546	0,6144	4,4813	2,1362	0,2430	0,8274
17	1,66345	0,12776	0,20299	1,3218	0,86290	4,4817	0,57049	0,7356	0,00246	0,0092
18	0,47636	0,3562	0,22519	0,21615	1,40150	3,5971	4,5509	0,02471	0,4260	0,0277
19	1,24864	0,25940	1,67942	1,3353	0,63839	0,5425	1,1419	0,6384	0,0606	0,3613
20	0,93494	4,7289	0,69619	0,72253	0,67598	0,00574	1,9409	0,1174	0,5478	0,01098
21	1,38075	0,20875	1,6236	2,4053	1,33035	7,3216	0,02321	0,07582	0,03390	0,02262
22	2,84788	1,3369	0,1136	1,0575	7,2573	8,2913	0,5636	0,0712	0,3123	0,1380
23	5,08840	1,49859	0,89400	1,4674	1,07320	1,4933	0,5177	0,4020	0,06187	0,05652

DEPARTMENT OF COMMUNICATIONS

COMPUTER SERVICES

APPENDIX V Cont'd

24	2.16156	.36385	.33030	.53105	.59167	-.38186	.28095	.03263	-.02012	-.03477
25	-.11559	-.64322	-.88447	-.4766	-.94050	-.30405	-.14970	-.02651	-.03659	-.00963
26	1.61263	1.08797	-1.25000	-.26762	-.13199	.27284	-.22732	.07156	.01113	.03357
27	-.86296	-.30593	1.71504	.84481	.44475	-.31504	.09123	.01783	.00425	.03330
28	-.21245	-.83963	-.48275	.36919	-.83569	.60693	-.17162	.03729	.00923	-.01056
29	-2.00207	-2.10129	.27129	.24194	-.36020	-.56635	.08743	.10252	-.02871	.01240
30	1.03031	.98476	1.56974	1.21713	1.00422	1.96813	.18877	.16512	-.07257	-.02883
31	-1.29111	-2.07732	1.92417	1.66143	-1.41391	.74149	-.12925	.04161	.02864	.00833
32	-2.46065	-.24497	.03596	-.16311	.16705	.84056	.10360	.01038	.12063	-.00322
33	3.01276	1.30695	2.04935	-1.67418	.13507	.28735	-.29375	.13841	-.06828	-.03259
34	.54111	-.99256	-1.67926	-.43161	-.27841	.36037	.23569	.02832	-.01607	.02130
35	-.07155	-.50001	.35006	.00748	.90273	-.49410	.48872	.01838	-.01477	.00284
36	-2.17457	2.38684	1.21747	-.04544	-.21745	-1.30522	-.22297	-.14738	-.08307	.05523
37	-2.10466	-2.67842	-.47041	-2.03155	.65584	.63522	.29349	.08749	-.00133	.00597
38	-1.01485	2.35439	-.22020	-.13430	-.76795	.53792	.01773	.01119	.01148	-.01443
39	-.89148	2.93156	1.39921	-.29426	1.52125	-.87929	.51969	-.02561	.02424	.05635
40	5.20724	-.57264	.92422	-1.76805	-.72183	-1.12207	.89909	.03060	.11852	-.03368
41	-3.79484	2.09750	1.04146	.12763	-.54945	.84689	-.52586	-.15893	.16289	.02650
42	.61766	-.53441	.43483	.91774	-.63967	.58439	.32759	-.01623	-.05625	-.01416
43	.23156	1.53110	1.33653	.80176	.02433	-.07665	-.21253	.08034	.01139	-.02080
44	.49576	1.29258	-2.46065	1.92915	.37674	-.25962	.15393	.03309	.02970	-.01277
45	1.41932	.27114	1.60454	-1.44281	-1.26723	1.51049	-.06976	.08562	-.07545	.01869
46	1.73801	2.03457	-.00343	.08361	.11170	.60882	.05079	.12324	-.01358	-.02636
47	-.26988	1.87203	-.19375	.11195	-1.17959	-.24493	-.32950	.04587	-.03173	-.02008
48	.26988	1.87203	-.19375	.11195	-1.17959	-.24493	-.32950	.04587	-.03173	-.02008
49	.04234	1.72859	-.88327	-.02087	-1.53146	.05037	-.53523	.04559	-.02795	-.00631
50	-.00406	-2.03085	-.15711	-.8357	-1.67978	-.28403	.27379	.26610	.07589	-.01168

PRINCIPAL COMPONENT SCORES - COLUMNS = EIGENVECTORS, ROWS = CASES

SPECIES - A-H, PLANISPIRA

MINISTÈRE des COMMUNICATIONS

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X=AXZ COMPONENT 1, Y=AYZ COMPONENT 2

SPECIES - M. PLANISPINA

DEPARTMENT OF COMMUNICATIONS

APPENDIX V Cont'd

MINISTRE DES COMMUNICATIONS									
SERVICES D'ORDINATEUR									
1	2	3	4	5	6	7	8	9	10
1	1,00000	.84329	.64230	.13167	.05550	.31373	.47457	.67181	.74733
2	.84329	1,00000	.61986	.64015	.17606	.38500	.59124	.72616	.76229
3	.64230	.61986	1,00000	.21731	.37924	.16515	.19287	.61027	.56771
4	.13167	.64015	.21731	1,00000	.26200	.29869	.40958	.38708	.55388
5	.05550	.17606	.37924	.26200	1,00000	.27636	.16481	.49088	.02481
6	.31373	.38500	.16515	.29869	.27636	1,00000	.37543	.09998	.70116
7	.47457	.59124	.19287	.40958	.16481	.37543	1,00000	.51735	.45998
8	.67181	.72616	.61027	.38708	.49088	.09998	.51735	1,00000	.52893
9	.74733	.76229	.56771	.55388	.02481	.70116	.45998	.52893	1,00000
10	.33908	.47552	.25967	.39381	.11776	.17525	.33256	.32381	1,00000

SIMILARITY MATRIX

SPECIES - H. INERACRETACEA

APPENDIX V Cont'd

MINISTÈRE des COMMUNICATIONS

SERVICES D'ORDINATEUR

	1	2	3
1	4,67166	46,71716	46,71716
2	1,68616	16,86169	63,57887
3	1,18861	11,88619	75,46506
4	,93463	9,34640	84,81146
5	,70933	7,09334	91,90982
6	,58304	5,83041	97,73521
7	,20641	2,06413	99,79935
8	,01294	,12940	99,92874
9	,00582	,05818	99,98589
10	,00131	,01308	99,99998

COLUMN 1 = EIGENVALUES COLUMN 2 = PERCENT OF TRACE
COLUMN 3 = CUMULATIVE PERCENT OF TRACE

SPECIES = H. INFRACRETACEA

APPENDIX V Cont'd

MINISTÈRE DES COMMUNICATIONS										
SERVICES D'ORDONNATEUR										
1	2	3	4	5	6	7	8	9	10	
1	.39052	.20487	.07405	.33110	.09788	.26159	.53772	.14703	.01213	.54852
2	.44609	.02730	.04843	-.05738	.10380	-.04129	.50967	.15665	-.00851	.70552
3	.29295	.56382	.11310	-.02872	.06939	.18257	.51933	.29223	.54410	.06470
4	.26911	.26001	.18814	.56742	.31148	.47356	.15357	.05121	.03098	.39264
5	-.09943	.59087	.44159	-.36847	.07243	-.02915	-.03664	.16040	.52545	.03079
6	-.22326	.43056	.56214	-.08795	.23992	.20728	.09211	.55897	.03490	.13548
7	.29110	.17735	.18362	-.09308	.61524	.60979	.50523	.01665	.00080	.00251
8	.36034	.03149	.46026	.33127	-.02144	.13048	.34404	.20303	.60679	.04811
9	.39558	.04940	.38647	.10259	.16687	.23813	.29805	.69719	.01843	.14626
10	.20383	.04804	.19354	.54024	.64081	.42784	.12385	.00425	.00208	.00105

PRINCIPAL AXIS MATRIX = COLUMNS = EIGENVECTORS, ROWS = VARIABLES

SPECIES = H. INFRAURETACEA

APPENDIX V Cont'd

SERVICES COORDINATEUR

MINISTRE DES COMMUNICATIONS

	1	2	3	4	5	6	7	8	9	10
1	5.15577	-1.68306	-1.01332	-58936	-1.57141	1.36034	.04223	.27246	-.00301	-.00346
2	3.41965	.91455	.04978	.49542	1.73642	.09592	.44337	-.06779	.08089	.01727
3	-1.02580	.61620	.14440	.55008	.46678	.78514	.22303	-.03464	.05725	.00383
4	2.78265	1.07891	-1.30446	-23183	-.68505	-.03584	-.15434	.05007	.11819	-.03749
5	1.98322	-.58084	1.65606	1.87217	-1.21586	1.42426	.05022	-.26660	-.14225	.01459
6	1.51146	1.41530	1.46354	-1.94325	-1.12786	-1.46581	.46578	.02932	.05317	-.00685
7	.92627	.437251	-1.39452	-1.31201	.14934	.75569	-.25846	-.16284	-.06525	-.04372
8	-2.14538	-1.20491	.06804	-.36848	-.95708	-.71164	.22098	-.02916	.03125	.00475
9	-2.51359	.91464	-2.16744	-.37538	-.92419	-.44696	-.12605	-.09379	-.15633	.08109
10	.42197	.39701	-.08133	.74605	.43537	-.40994	1.16450	.04231	.03711	.05690
11	.02109	.79410	.85423	-.33422	-1.01968	-.72677	-.03980	-.02136	.04623	-.02184
12	-1.22354	1.17846	1.04037	.20109	-.11695	1.96627	-.00942	.08421	.05110	-.02217
13	-1.57382	.76450	.31151	.37152	.70477	.45549	.44623	-.00496	-.00644	-.01175
14	.81546	1.21757	1.77161	.94779	-.17032	-1.23162	-.31008	-.03863	.01022	-.01828
15	1.70317	-.93652	.22051	.76253	.42495	-.64498	-.22497	-.04878	-.01824	.01523
16	-2.30075	-.79028	.79428	-.44848	-.62680	-.33785	-.38093	.04109	.01090	-.03416
17	-1.51392	.184757	-2.11114	-.94357	-.26251	.19477	.08541	-.07324	-.02624	.00850
18	1.04876	.10374	-.09261	-.37622	.29936	-.49805	-.42177	-.02059	.04181	-.01701
19	1.48591	.30930	-1.26756	1.70459	.63011	-1.10450	.51297	.25103	-.23304	-.04114
20	2.26982	-.55783	.85839	-.242083	2.10231	-.13041	.00486	.05866	-.05906	.09495
21	1.27827	-.96082	2.47167	-.50241	.02248	.25290	.14200	-.04663	-.10891	-.00774
22	.36975	.167713	-1.57528	.70320	-.00552	-.23104	-.55470	.00035	.05549	-.00803
23	-.06206	-.21734	-.93906	.08451	-.26345	.08467	.89186	-.03429	.06656	.01287

DEPARTMENT OF COMMUNICATIONS

COMPUTER SERVICES

APPENDIX V Cont'd

20	-0.5350	-1.57003	0.9751	-1.35022	-0.4053	0.6175	-0.0415	-0.06521	0.0084	-0.02670
25	-0.56252	-0.27033	-0.16277	-0.59202	-0.36038	0.50857	-0.51457	-0.04897	0.0015	-0.03150
26	-0.31599	-0.08965	0.23942	0.27447	0.50819	-0.26028	0.89035	-0.04811	0.02971	-0.00979
27	-1.73820	-1.07661	0.09789	0.41929	-0.63453	-0.96232	0.02539	-0.04333	0.02795	0.00540
28	-3.05895	0.85568	0.06976	-0.56432	-0.17978	-0.21202	-0.52146	0.09483	-0.07227	-0.01915
29	-1.09434	-1.38158	-0.24590	-0.40380	0.11139	0.41429	0.05408	-0.07230	0.02937	-0.00961
30	0.20337	-0.20046	-0.74093	1.57884	0.23227	-0.08035	0.52060	-0.06792	0.08416	0.02498
31	-0.24052	-0.79473	0.02047	0.74175	1.029562	0.31494	-0.17423	0.00733	0.03437	-0.00945
32	-0.37500	1.54651	-0.48267	1.25338	-0.01001	-0.35809	-0.40407	-0.01279	0.03985	0.05289
33	-1.36941	-1.01790	0.27351	0.11291	1.73263	-0.20912	-0.99031	0.03198	-0.00669	-0.05710
34	-3.92862	1.24768	1.27931	0.59557	-0.37150	0.83255	0.13015	0.33872	0.02189	0.06257

PRINCIPAL COMPONENT SCORES - COLUMNS = EIGENVECTORS, ROWS = CASES

SPECIES - H. INFRACRETACEA

END

12-12-80

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