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Performance Analysis Of Space-Based Radar Tracking Techniques

by

Richard Jérôme

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering,
Faculty of Engineering,
Department of Systems and Computer Engineering,
Carleton University,
Ottawa, Ontario, Canada

July 6, 1990

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Thesis Supervisor

Chairman,
Department of Systems and
Computer Engineering

Department of Systems and Computer Engineering
Faculty of Engineering
Carleton University
July 6, 1990

Abstract

The performances of the Nearest Neighbour Standard Filter (NNSF) and of the Probabilistic Data Association Filter (PDAF) were analyzed in the context of a Space-Based Radar (SBR) application. Computer simulation experiments where the SBR / target geometry and the clutter environment were allowed to vary were performed. The presence of false measurements in the early stages of tracking significantly affects the performance of the NNSF, whereas the PDAF achieves adequate performance. This is particularly relevant to the case where tracking is initiated at low grazing angles. Typical track position estimate RMS errors are of the order of the kilometre due to the large target range values involved. Extrapolation of the track estimate over a long coverage gap time, as may be the case in the SBR application, significantly affects the tracking accuracy of both filters and may lead to the loss of the track.

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2.2 SBR System Environment

The system under consideration consists of a satellite-based radar whose mission is to detect targets flying near the surface of the earth within a given surveillance area and to subsequently track the detected targets. This multi-mission operation requires the SBR system to: [20]

- i) Monitor the entire surveillance area and detect all targets present in the area.
- ii) Initiate tracks on detected targets within the surveillance area in order to obtain an initial target state estimate.
- iii) Maintain track on initiated tracks within the surveillance area with the accuracy required to achieve the objectives of the mission.

A generic SBR system configuration is depicted in Figure 2.2-1. The main functions of the system are as follows :

- Detection : this function processes the radar returns and creates a new radar measurement whenever the presence of a target (i.e. true target or false alarm) is detected;
- Surveillance Area Scanning : this function schedules beam scanning of the surveillance area in order to detect new targets;
- Track Initiation : this function processes the radar measurements in order to generate new tracks along with their initial estimates;
- Track Maintenance : this function maintains the system track base by correlating new radar measurements with existing tracks;

List of Symbols

x'	- the transpose of x
$E[.]$	- the expectation operator
$E[x \mid z]$	- the expectation of x conditioned upon z
$\hat{x}$	- the estimate of x
$\hat{x}(i \mid j)$	- the estimate of x at time i conditioned upon the past up to time j
$\tilde{z}$	- the error between z and the estimate of z
$\dot{x}$	- the first derivative of x with respect to time
$\ddot{x}$	- the second derivative of x with respect to time
$\bar{x}$	- the third derivative of x with respect to time
$\wedge$	- the vector product operator

Chapter 1

Introduction

1.1 General

Over the past twenty years, the use of space-based radars (SBR) has been envisioned in a variety of applications ranging from remote sensing of the earth through global air traffic surveillance and control, to military missions such as fleet surveillance and air defence of the North American continent.

Techniques used to support the above applications have been discussed in the scientific literature [1 - 5]. For military applications, the advantages of satellite-based radars over ground-based or airborne radars include the provision of a very large field-of-view and of an essentially unrestricted line-of-sight to any point near the surface of the earth.

Furthermore, with the use of an electronically steered antenna, a space-based radar provides the capability to perform a wide variety of missions including surveillance of large areas on the earth, detection and tracking of multiple air targets near the surface of the earth.

The detection performance of an SBR system against air targets near the surface of the earth in the context of a multi-mission operation such as fence surveillance and area tracking has been analyzed in [6 - 9]. Some aspects of SBR tracking performance have been discussed in [10], primarily in the area of track initiation. However, further work on automatic tracking with a SBR system is required.

Automatic tracking systems currently used in surveillance radars are usually designed for tracking targets in relatively clean environments characterized by a low false alarm rate. When extraneous measurements originated from clutter or radar noise are present in the validation gate of a given track, errors in the measurement-to-track association process can occur, and as a result tracking can fail.

Advanced tracking techniques which achieve adequate performance in dense clutter environments have been discussed in the scientific literature over the past ten years [11 - 19]. These techniques incorporate the means to solve ambiguities in the measurement-to-track association process by taking account, in some way, of the measurements origin uncertainty. An overview of some of these techniques is provided in Section 2.0 of this thesis.

For space-based radar systems, the development and implementation of adequate tracking techniques require further theoretical investigations and experimental trials.

Automatic tracking of air-targets with a SBR presents challenges that are considerably more complex than those faced by most operational tracking systems currently in use.

SBR geometries and satellite motion can lead to large position and velocity errors in target state estimates. Incomplete coverage or low probability of detection due to the particular SBR / target geometry can cause intermittent updates on the target track, making tracking of a manoeuvring target a difficult task to handle. Adaptive estimation techniques must be incorporated in the tracking system to deal with such situations.

The clutter environment can be such that many false detections are likely to occur. The presence of these false detections will introduce an additional uncertainty associated with the measurements provided by the radar in addition to their inaccuracy. This additional uncertainty is related to the origin of the measurements and the tracking system must be able to accommodate such conditions in order to achieve adequate performance.

The purpose of this thesis is to provide quantitative performance measures of candidate SBR tracking techniques. Of particular interest is the impact of the SBR clutter environment on the tracking performance against air targets near the surface of the earth. This thesis deals primarily with the problem of tracking a single target in a clutter environment with a satellite-based radar.

1.2 Scope

The main objective of this thesis is to analyze and evaluate the performance of two tracking techniques in the context of a space-based radar application. These two tracking techniques are based upon the same state estimation approach, namely the Kalman filter algorithm, but differ in the way the measurement-to-track association process is performed. The first technique corresponds to the Nearest-Neighbour Data Association approach whereas the second technique is based upon the Probabilistic Data Association approach. Both techniques are analyzed in detail, and results of computer simulations are provided to quantify the performance of each technique in various scenarios. Of particular interest is the study of the effect of the presence of false measurement in the validation gate of a track on SBR tracking performance. Recommendations on the most promising approach for SBR tracking are also provided.

1.3 Outline

Section 2 presents the space-based radar tracking problem. An outline of the system under consideration is provided along with the basic functions of target tracking. In addition, an overview of various techniques dealing with the problem of tracking targets in a cluttered environment is presented.

Section 3 provides a detailed analysis of the two candidate tracking techniques under investigation in the context of this thesis.

These two techniques consist of the Nearest Neighbour Standard Filter and the Probabilistic Data Association Filter. Along with the main assumptions and constraints of each technique, detailed algorithms are described.

A computer simulation model of the techniques described in Section 3 applied to the SBR tracking problem is presented in Section 4. The model addresses both the SBR environment (e.g. target flight profile, satellite orbital flight, clutter environment) and the SBR tracking process (e.g. target dynamics model, measurement model).

The main results of simulation runs are provided in Section 5 along with a comparative evaluation of the two tracking techniques under investigation.

Section 6 summarizes the results and findings of the work that has been carried out and outlines areas for which further research is required.

Chapter 2

Space-Based Radar Tracking

2.1 General

This section provides a description of a typical SBR system environment including the satellite-based radar characteristics, the clutter environment, and the target environment. It also provides a description of target tracking, in terms of its main constituent functions. In addition, an overview of some advanced techniques dealing with the problem of tracking targets in a clutter environment discussed in the scientific literature is presented.

2.2 SBR System Environment

The system under consideration consists of a satellite-based radar whose mission is to detect targets flying near the surface of the earth within a given surveillance area and to subsequently track the detected targets. This multi-mission operation requires the SBR system to: [20]

- i) Monitor the entire surveillance area and detect all targets present in the area.
- ii) Initiate tracks on detected targets within the surveillance area in order to obtain an initial target state estimate.
- iii) Maintain track on initiated tracks within the surveillance area with the accuracy required to achieve the objectives of the mission.

A generic SBR system configuration is depicted in Figure 2.2-1. The main functions of the system are as follows :

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- Track Initiation : this function processes the radar measurements in order to generate new tracks along with their initial estimates;
- Track Maintenance : this function maintains the system track base by correlating new radar measurements with existing tracks;

Figure 2.2-1 SBR System Configuration

- **Resource Management** : this function processes radar beam / power requests from the Surveillance, Track Initiation, and Track Maintenance functions in order to optimize the use of the SBR resources.

This thesis will focus exclusively on the SBR Track Maintenance function. Although the SBR system is required to track several targets within the tracking area, the thesis will evaluate the performance of the tracking filter against a single target flying near the surface of the earth. The presence of the other targets, and the requirement to provide area surveillance in conjunction with tracking, will be reflected in the tracking update interval characteristic of a phased array radar operation.

It will be assumed that the target track has been initiated and that an initial estimate of the target dynamic parameters is available to start the operations of the tracking filter. The procedure used to generate the initial target estimate in simulations is described in Section 4.2.9.

Section 2.2.1 defines the SBR configuration in terms of satellite and radar parameters. Section 2.2.2 presents the clutter environment from which a model for the generation of false measurements in the SBR tracking simulation will be derived, while Section 2.2.3 describes a typical target flight profile.

2.2.1 SBR Configuration

The following is a description of the spaceborne radar parameters that constitute the baseline configuration for which the analysis of tracking performance will be carried out.

The definition of this baseline configuration is based upon [20].

- a) Satellite parameters :
 - Circular polar orbit
 - Altitude of 2,000 km
 - Usable grazing angles: 3° to 60°
- b) Radar parameters :
 - Monostatic pulse Doppler radar
 - Phased array antenna
 - Antenna beamwidths : $0.3^\circ \times 0.3^\circ$
 - Beam steering: fully agile
 - Nominal probability of detection : 0.9
 - Range cell size : 300 m
 - Measurement error standard deviations :
 - * Range : 300 m
 - * Azimuth : 0.1°
 - * Elevation : 0.1°

2.2.2 Clutter Environment

For the SBR system, the clutter environment influences the following parameters:

- i) It is the primary cause of the Minimum Detectable Velocity (MDV) of Doppler processing
- ii) It creates false alarms and therefore introduces false measurements in the tracking process.

The SBR uses Doppler discrimination to separate targets of interest from the earth clutter. The Doppler shift of a given target return is proportional to the relative velocity between the satellite and the target. Detection performance of a SBR in the presence of earth clutter has been analyzed in [7]. It was shown that the signal / interference ratio computed as a function of Doppler velocity can be used to quantify the detection performance of a SBR. In this ratio, the signal represents the target signal power received by the radar, whereas the interference is defined as the sum of receiver noise power and earth clutter power for the Doppler velocity of interest. In order to achieve a specified probability of detection under a given system false alarm rate, the signal / interference ratio must exceed a minimum value. This minimum signal / interference ratio defines upper and lower minimum detection velocities (MDV). For Doppler velocities outside these boundaries, the detection performance of the SBR will not be met. Within the context of this thesis, it is assumed that the SBR signal processing subsystem imposes a minimum value of Doppler velocity in order for the target return to be detected. A single value for the MDV will be used to characterize the detection performance of the SBR. Based upon [20], it is assumed that with Doppler processing, the MDV limit is equal to 100 meters/second.

In addition to its effect on the detection performance of the SBR, the presence of earth clutter influences the number of false measurements introduced in the tracking filter. False measurements are the results of false detections due to receiver noise and clutter returns. The presence of false measurements in the tracking process introduces an additional uncertainty factor, namely about the origin of the measurements.

The clutter model used in the analysis of the SBR tracking performance is described in section 4.2.5. The false alarm rate will be considered as a variable parameter within the context of this thesis. The sensitivity of both NNSF and PDAF to the false alarm rate will be analyzed in various simulation experiments.

2.2.3 Target Environment

It is assumed that the SBR system will be required to detect the presence of several targets in the surveillance area, and to track simultaneously several targets. This multi-mission mode of operation is made possible by the ability of the phased array antenna to direct the radar beam without inertia in any direction. Although a single target will be modelled in the simulation, the presence of the other targets and the surveillance duties will be taken into account in the tracking update interval. A typical value of the update interval is assumed to be 10 seconds [20].

A typical target scenario will be considered in the analysis and evaluation of SBR tracking performance.

The scenario consists of a single target flying in a southerly direction from the North Pole and is defined by the following parameters: [20]

- a) Flight profile : the target flies a constant direction profile starting from an initial latitude of 90 degrees with respect to the Earth Centred (EC) coordinate system and at an initial altitude of 10,000 m.

- b) Speed / Acceleration : the target flies at a speed of 300 m/s with random accelerations (i.e. process noise) ranging from 0 to 1.0 m/s².
- c) Manoeuvres : None

The following section provides a description of the target tracking function along with an overview of some advanced techniques described in the literature.

2.3 Target Tracking

Target tracking can be defined as the process whereby measurements obtained from the target are used to maintain an estimate of its current state trajectory.

Within the context of this thesis, the target state or track consists of the following kinematics components:

- Position
- Velocity

Hence, the two main components of the SBR tracking system are measurement-to-track association and state estimation as described below.

2.3.1 Measurement-to-track Association

The objective of the measurement-to-track association process is to select the most probable measurement(s) given by the radar for an update of the target track.

The major challenge that the SBR tracking system must face is the fact that the origin of the measurements is uncertain, due to the presence of false measurements such as clutter returns.

The first component of the measurement-to-track association process is the validation of the measurements provided by the radar. Validation is a coarse test used to determine whether a measurement belongs to the target track of interest. The validation of radar measurements is carried out according to the following as discussed in [21]:

- 1) An estimate of the predicted target track at the time of the measurement is created.
- 2) A validation region for the target track is computed based on the predicted estimate and measurement accuracy statistics. If the measurement falls within the validation region of the track, the measurement is validated for further correlation tests.

The second component of the measurement-to-track association process is the correlation of the validated measurements to the target track of interest. If there is a single validated measurement only, the association to the track can be made immediately.

Measurement-to-track association conflict situations arise when several radar measurements fall within the validation gate of the track. There are two fundamentally different approaches to the problem of measurement-to-track association, namely Bayesian and non-Bayesian.

The non-Bayesian approach generates an association decision for each new measurement based on the existing data.

Algorithms for tracking a single target in the presence of false measurements that follow this approach include the Nearest Neighbour Standard Filter (NNSF) and the Track - Split Filter (TSF). The NNSF is the simplest approach for tracking a target in a clutter environment and uses the measurement nearest to the predicted measurement in the filter as if it were the correct one [21 - 23]. The TSF splits the track every time more than one measurement falls in the validation gate of the track [24].

A "pruning" technique based on the likelihood function is used to reduce the number of false tracks [22]. An extension of the TSF to the problem of tracking multiple targets in a clutter environment is to consider simultaneously all the feasible partitions of the measurements into tracks, via the joint likelihood function of all the tracks [22].

The Bayesian approach evaluates probabilities of association for each measurement at each update. These association probabilities are then used in the estimation process. Algorithms for tracking a single target in a clutter environment that follow this approach include the Probabilistic Data Association Filter (PDAF) and the Optimal Bayesian Filter (OBF). The PDAF deals only with the latest set of validated measurements. The computed association probabilities are used in a suitably modified filter that accounts for the measurement origin uncertainty [25 - 32]. The OBF follows the same approach except that it uses all combinations of measurements from the initial to the current time rather than only the latest set of measurements. Sequences of measurements can be limited in size by using a sliding window that covers the current and the previous N update intervals [22, 33].

Algorithms dealing with the problem of tracking multiple targets in a clutter environment which falls under the Bayesian approach include the Joint Probabilistic Data Association Filter (JPDAF) and the Multiple Hypotheses Tracker (MHT). The JPDAF is an extension of the PDAF which assumes a known number of targets in the clutter environment. As with the PDAF, the algorithm computes the probabilities of association of only the latest set of measurements but the probabilities are computed differently, namely jointly across the targets [29, 34, 35, 36].

The MHT deals with the association of sequences of measurements and evaluates the probability that there is a target from which a sequence of measurements originated. Track "pruning" techniques are used to limit the number of hypotheses generated by the algorithm [37, 38].

2.3.2 Target State Estimation

Target state estimation is the process of maintaining the current state trajectory or track of the target. Prediction of the track to the time of the next update is also carried out in order to determine the centre of the validation region of the track for the next set of measurements. The size of the validation region of the track is determined by the prediction uncertainty.

The following provides an overview of some advanced state estimate techniques discussed in the literature.

For linear dynamic systems, the Kalman filter is the minimum mean square error estimation. It assumes specific models for the target dynamics and the measurement noise.

The Kalman filter provides a convenient measure of the estimation error through the use of the covariance matrix. In particular, the covariance of the predicted target position is used to compute the size of the validation gate of the track. For non linear dynamic systems, an adaptation of the Kalman filter called the Extended Kalman Filter (EKF) provides the simplest state estimation filter. The estimation is obtained by a Taylor series expansion of the nonlinear target dynamics and measurement equations.

When the target under consideration executes manoeuvres, adaptive estimation techniques must be selected in order to provide adequate tracking performance [39]. A number of filtering techniques have been designed to adapt to target manoeuvres. These techniques fall under two main approaches as follows:

- i) The manoeuvre is modelled as a random process.
- ii) The manoeuvre is estimated in real time.

When the manoeuvre is modelled as a random process, some techniques assume a white noise with either fixed level, variable level, or with several discrete levels. Some other techniques assume a "coloured" noise (i.e. auto-correlated process) taking into account the maximum target acceleration. When the manoeuvre is estimated in real time, the state estimate can be corrected with the estimated input [40], or can be augmented leading to a variable state dimension filter [41, 42]. Finally, some algorithms combine the two approaches, and assume that the system behaves according to one of a finite number of models which can differ in noise levels or structure. This category of filtering algorithms is usually referred to as Multiple Model Filtering [43 - 45].

Within the scope of this thesis, the following target tracking techniques will be investigated:

- i) Measurement-to-track Association:
 - Nearest Neighbour Standard Filter
 - Probabilistic Data Association Filter
- ii) Target State Estimation:
 - Kalman Filter
 - Extended Kalman Filter

Chapter 3

Tracking Techniques Analysis

3.1 General

This section provides a detailed description of the two target tracking techniques that will be evaluated in the context of the SBR application. As indicated in chapters 1 and 2, these techniques are based upon the Nearest Neighbour Standard Filter (NNSF) and the Probabilistic Data Association Filter (PDAF). The framework and the main assumptions used in the derivation of these techniques are based upon [21, 22, 46].

Within the context of this thesis, tracking is performed in the NED (i.e. North-East-Down) coordinate system.

The NED coordinate system is a reference frame attached to the satellite with its origin at the satellite location, the N axis pointing to the north, the D axis pointing to the centre of the earth, and the E axis perpendicular to the N and D axes.

The target state vector used in the tracking filter consists of target position and velocity in the NED coordinate system. As a result, the target equation of motion is linear.

An SBR measurement consists of target range, azimuth, and elevation in the NED coordinate system.

Because of the non linear relationship between the target state and the radar measurement, it is necessary to use a nonlinear state estimation technique. The Extended Kalman Filter is a widely used filtering technique applicable to non linear systems which will be considered in the context of this thesis.

The discrete-time model for the target dynamics is given by the following equation:

$$x(k+1) = F(k) x(k) + v(k) + G(k) u(k)$$

where:

$x(k)$ is the state of the target at time k ;

$F(k)$ is the state transition matrix at time k ;

$v(k)$ is the process noise sample at time k from a zero-mean white Gaussian noise sequence with covariance $Q(k)$.

The discrete-time model for the space-based radar measurement is given by the following equation:

$$z(k) = h[k, x(k)] + w(k)$$

where:

$z(k)$ is the radar measurement at time k ;

$h[k, x(k)]$ is the measurement vector-valued function at time k ;

$w(k)$ is the measurement noise sample at time k from a zero-mean white Gaussian noise sequence with covariance $R(k)$.

The initial target state $x(0)$ is assumed to be a random variable, Gaussian distributed with mean $\hat{x}(0 | 0)$ and covariance $P(0 | 0)$.

The process noise sequence v , the measurement noise sequence w , and the initial target state $x(0)$ are assumed to be mutually independent.

3.2 Nearest Neighbour Standard Filter

The Nearest Neighbour Standard Filter is the simplest approach for tracking a single target in a cluttered environment. The basic idea behind it is to select the measurement closest to the predicted target track position and use it as if it were the correct one. Once the measurement to be used is selected, the target state estimation is carried out with an extended Kalman filter. The following presents a detailed description of the Extended Kalman filter as well as the criteria for measurement selection in the Nearest Neighbour Standard Filter.

3.2.1 Extended Kalman Filter

The Extended Kalman filter uses the Kalman filter equations in a system that is linearized around the latest state estimate and has to be viewed as a suboptimal estimator for non linear systems.

The Extended Kalman filter recursive algorithm is presented in the following.

Let us denote the set of radar measurements available at time k as:

$$Z^k = \{ z(j), j \leq k \}$$

The estimate of the target state at time k is given by :

$$\hat{x}(k | k) = E [x(k) | Z^k]$$

The estimation error is defined as :

$$\tilde{x}(k | k) = x(k) - \hat{x}(k | k)$$

The covariance of the estimate at time k is given by :

$$\begin{aligned} P(k | k) &= E \{ [x(k) - \hat{x}(k | k)] [x(k) - \hat{x}(k | k)]' | Z^k \} \\ &= E [\tilde{x}(k | k) \tilde{x}(k | k)' | Z^k] \end{aligned}$$

The first-order Extended Kalman filter is obtained by using a series expansion of the non linear measurement equation around the latest estimate $\hat{x}(k | k)$ in order to obtain the predicted measurement.

The state of the target is computed recursively through the following steps :

1) State Prediction :

From the current updated target state estimate at time k , the predicted value of the state at time $k+1$ is computed according to:

$$\hat{x}(k+1 | k) = E [x(k+1) | Z^k] = F(k) \hat{x}(k | k) + G(k) u(k)$$

2) Measurement Prediction :

The first order expansion of the measurement equation leads to :

$$z(k+1) = h[k+1, \hat{x}(k+1 | k)] + h_x(k+1) [x(k+1) - \hat{x}(k+1 | k)] + w(k)$$

where $h_x(k+1)$ is the Jacobian of the vector-valued h , evaluated at the latest predicted state estimate.

From the predicted target state estimate at time $k+1$, the predicted value of the measurement at time $k+1$ is computed according to :

$$\hat{z}(k+1 | k) = E [z(k+1) | Z^k] = h[k+1, \hat{x}(k+1 | k)]$$

3) Innovation evaluation :

When the new measurement is obtained from the radar, the innovation or difference between the new measurement and its predicted value is computed according to :

$$v(k+1) = z(k+1) - \hat{z}(k+1 | k) = \tilde{z}(k+1 | k)$$

4) State Update :

The target state at time $k+1$ is computed as the sum of the predicted state and the innovation multiplied by the filter gain at time $k+1$ according to :

$$\hat{x}(k+1 | k+1) = \hat{x}(k+1 | k) + W(k+1) v(k+1)$$

The covariance of the state of the target and the filter gain are computed recursively through the following steps :

1) State Prediction Covariance :

From the current updated target state covariance at time k , the state prediction covariance at time $k+1$ is computed according to :

$$\begin{aligned}
P(k+1 | k) &= \\
E \{ [x(k+1) - \hat{x}(k+1 | k)] [x(k+1) - \hat{x}(k+1 | k)]' | Z^k \} \\
&= F(k) P(k | k) F(k)' + Q(k)
\end{aligned}$$

2) Measurement Prediction Covariance :

From the state prediction covariance at time $k+1$, the measurement prediction covariance at time $k+1$ is computed according to :

$$\begin{aligned}
S(k+1 | k) &= \\
E \{ [z(k+1) - \hat{z}(k+1 | k)] [z(k+1) - \hat{z}(k+1 | k)]' | Z^k \} \\
&= h_x(k+1) P(k+1 | k) h_x(k+1)' + R(k+1)
\end{aligned}$$

3) Filter Gain :

The filter gain is computed from the state prediction covariance and the measurement prediction covariance according to:

$$W(k+1) = P(k+1 | k) h_x(k+1)' S(k+1)^{-1}$$

4) Updated State Covariance :

The updated covariance of the target state at time $k+1$ is computed according to :

$$\begin{aligned}
P(k+1 | k+1) &= \\
E \{ [x(k+1) - \hat{x}(k+1 | k+1)] [x(k+1) - \hat{x}(k+1 | k+1)]' | Z^k \} \\
&= P(k+1 | k) - W(k+1) S(k+1) W(k+1)'
\end{aligned}$$

3.2.2 Nearest Neighbour Data Association

Prior to their incorporation into the state estimator, the radar measurements are validated using a validation gate as described below.

The validation gate represents the minimum-volume region in the measurement space where the true radar measurement will be with some probability.

Using the notations defined in Section 3.2.1 above, the norm of the innovation or normalized squared innovation $d^2(k+1)$ is defined as :

$$d^2(k+1) = [z(k+1) - \hat{z}(k+1 | k)]' S(k+1)^{-1} [z(k+1) - \hat{z}(k+1 | k)]$$

The validation gate at time $k+1$ is defined as :

$$V(k+1, g^2) = \{ z(k+1) : d^2(k+1) < g^2 \}$$

where g is the number of sigma (standard deviation) of the gate.

Under the assumption that the true measurement at time $k+1$, conditioned upon Z^k , follows a Gaussian distribution with mean $\hat{z}(k+1 | k)$ and covariance $S(k+1)$, the norm of the innovation is chi-squared distributed with number of degrees of freedom equal to the dimension of the measurement. For a given probability that the true measurement will fall in the validation gate, the value of g can be obtained from the tables of the chi-squared distribution.

In the case where more than one measurements from the radar fall in the validation gate, the Nearest Neighbour data association method consists of selecting the measurement with the smallest norm of innovation $d^2(k+1)$. This measurement will then be used to update the target state estimate as if it were the correct one.

3.3 Probabilistic Data Association Filter

The Probabilistic Data Association Filter is a Bayesian approach to the problem of tracking a single target in a cluttered environment. This approach evaluates probabilities of association for each measurement at each update. These association probabilities are used in the estimation process, thus taking into account the measurement origin uncertainty. The estimation process is based on a suitably modified extended Kalman filter.

The following presents a detailed description of the Probabilistic Data Association Filter.

3.3.1 General

In the Probabilistic Data Association Filter (PDAF) approach, it is assumed that there is only one target of interest, whose track has been initialized.

Hence, among the several validated measurements (i.e. radar measurements falling into the current validation gate), at most one of them is assumed to be from the target, while the remaining measurements are assumed to be false measurements. The algorithm's capability of distinguishing (probabilistically) between the correct measurement and the false measurements resides in the former having a Gaussian probability density function (pdf), while the latter are assumed to be uniformly distributed.

The main assumptions used in the derivation of the PDAF are as follows:

- There is a single target of interest whose track has been initialized.
- A validation gate about the predicted measurement is computed at each sampling interval.
- At most one of the validated measurements can originate from the target.
- The measurements which did not originate from the target (i.e. false measurements) are assumed to be independently and uniformly distributed in the measurement space.
- The conditional probability density function (pdf) of the target state given the past measurements is assumed Gaussian with given mean and covariance.

The procedures for data association and state estimation are presented in section 3.3.2 and 3.3.3 respectively.

3.3.2 Probabilistic Data Association

The set of validated measurements at time k is denoted by:

$$Z(k) = \{z_i(k)\}, \quad i=1, m_k$$

where:

$z_i(k)$ is the i -th validated measurement at time k

m_k is the number of measurements in the validation gate

The cumulative set of measurements from time 1 to time k is denoted by:

$$Z^k = \{Z(j)\}, \quad j=1, k$$

The PDAF uses a decomposition of the estimation with respect to the origin of each measurement of the latest set of measurements only. In order to perform this decomposition, the past is summarized approximately by assuming that the conditional probability of the target state at time k given the cumulative set of measurements up to time $k-1$ is given by:

$$P\{x(k) \mid Z^{k-1}\} = N[\hat{x}(k \mid k-1), P(k \mid k-1)]$$

where:

$N[\hat{x}, P]$ is a Gaussian distribution with mean $\hat{x}$ and covariance P .

$\hat{x}(k \mid k-1)$ is the predicted state at time k

$P(k \mid k-1)$ is the predicted state covariance at time k

The association events at time k are denoted by:

$\Theta_i(k) = \{z_i(k) \text{ is the target originated measurement}\}, \quad i=1, m_k$

$\Theta_0(k) = \{\text{none of the measurements originated from the target}\}$

The conditional probability of the event $\Theta_i(k)$ given the cumulative set of measurements up to time k is denoted by:

$$P\{\Theta_i(k) \mid Z^k\} = \beta_i(k), \quad i=0, m_k$$

The derivation of the above probability is provided in Appendix A.

3.3.3 State Estimation

The association events $\Theta_i(k)$ are mutually exclusive and exhaustive given the assumptions of the PDAF.

Using the total probability theorem for a set of exclusive and exhaustive events, the conditional mean of the state at time k can be written as:

$$\hat{x}(k | k) = E [x(k) | Z^k] =$$

$$\sum_{i=0}^{m_k} E[x(k) | \Theta_i(k), Z^k] P\{\Theta_i(k) | Z^k\} = \sum_{i=0}^{m_k} \hat{x}_i(k | k) \beta_i(k)$$

where:

$\hat{x}_i(k | k)$ is the updated state conditioned on the event $\Theta_i(k)$

$\beta_i(k)$ is the conditional probability of event $\Theta_i(k)$ given the cumulative set of measurements up to time k .

Following the extended Kalman filter derivation, the estimate conditioned on event $\Theta_i(k)$ is:

$$\hat{x}_i(k | k) = \hat{x}(k | k-1) + W(k) [z_i(k) - \hat{z}(k | k-1)] \quad i=1, m_k$$

where:

$\hat{x}(k | k-1)$ is the predicted state at time k

$\hat{z}(k | k-1)$ is the predicted measurement at time k

$W(k)$ is the Kalman filter gain at time k .

As per the extended Kalman filter, the gain $W(k)$ is given by:

$$W(k) = P(k | k-1) h_x(k)' S(k)^{-1}$$

where:

$P(k | k-1)$ is the covariance of the predicted state at time k

$h_x(k)$ is the Jacobian of the vector-valued h , evaluated at time k

$S(k)$ is the covariance of the predicted measurement at time k

For $i = 0$, since none of the measurements is correct:

$$x_0(k | k) = \hat{x}(k | k-1)$$

The conditional mean of the state can be rewritten as:

$$\hat{x}(k | k) = \beta_0 \hat{x}(k | k-1) + \sum_{i=1}^{m_k} \beta_i(k) [\hat{x}(k | k-1) + W(k) v_i(k)]$$

where:

$v_i(k) = z_i(k) - \hat{z}(k | k-1)$ is the innovation conditioned on event $\Theta_i(k)$

The updated state estimate at time k can be written as:

$$\hat{x}(k | k) = \hat{x}(k | k-1) + W(k) v(k) \text{ where:}$$

$$v(k) = \sum_{i=1}^{m_k} \beta_i(k) v_i(k) \text{ is the combined innovation.}$$

The covariance of the updated state is given by:

$$P(k | k) = \beta_0(k) P(k | k-1) + (1 - \beta_0(k)) P^c(k | k) + P^s(k)$$

where:

$$P^c(k | k) = P(k | k-1) - W(k) S(k)^{-1} W(k)'$$

$$P^s(k) = W(k) \left[\sum_{i=0}^{m_k} B_i(k) v_i(k) v_i(k)' - v(k) v(k)' \right] W(k)'$$

$P^c(k | k)$ is the covariance of the updated state in the absence of measurement origin uncertainty

$P^s(k)$ is the "spread of the means", and represents an increase in the covariance of the state due to the uncertainty of the measurement origin.

The derivation of the covariance associated with the updated state is provided in Appendix B.

Chapter 4

SBR Tracking Simulation

4.1 General

The evaluation of the various target tracking techniques described in Section 3.0 in the context of the SBR application is a complex task which requires the use of computer simulation methods. Due to the nature of the problem to be studied, i.e. a complex problem containing many random elements, the method which has been used is to examine the statistics of a random process by performing a large number of computer experiments and compiling statistical results. The random processes are simulated by random number generators and deterministic elements are simulated exactly.

Section 4.2 contains a detailed description of the computer model developed to study SBR tracking performance. The model includes modules simulating the SBR environment (e.g. satellite trajectory, target flight profile, false measurement generation), as well as modules simulating the SBR tracking function (e.g. target dynamics equation, measurement equation, measurement covariance). These modules are common to the different tracking techniques under consideration.

4.2 Model Description

The SBR tracking computer model consists of several components, as illustrated in Figure 4.2-0. A description of the main components is provided in the following sections.

4.2.1 Satellite Dynamics

This module simulates the motion of the satellite in its orbit around the earth. It is assumed that the satellite follows a circular polar orbit. The satellite's orbit is characterized by the following input parameters:

- ALT*: Satellite altitude above the surface of the Earth
- LONG0*: Initial satellite longitude with respect to the EC coordinate system
- LAT0*: Initial satellite latitude with respect to the EC coordinate system

Figure 4.2-0 SBR Tracking Computer Model Block Diagram

The EC coordinate system (O,X,Y,Z) has its origin at the centre of the earth, the X axis pointing to the Greenwich meridian, the X-Y plane as the Equatorial plane, and the Z axis pointing to the North Pole.

The equations of motion of the satellite are given by:

$$\begin{aligned} R_s &= ALT + R_e \\ \dot{\Theta}_s &= (C / (R_s)^3)^{1/2} \\ LONG &= LONG0 \\ LAT &= LAT0 + \dot{\Theta}_s T \end{aligned}$$

where:

R_s is the radial distance of the satellite from the centre of the earth

R_e is the earth radius (i.e. $R_e = 6378$ km)

$\dot{\Theta}_s$ is the satellite angular velocity along its orbit

C is the gravitational constant of the earth (i.e. $C = 3.986008 \cdot 10^{14}$ m³/sec²)

$LONG$ is the current satellite longitude with respect to the EC coordinate system

LAT is the current satellite latitude with respect to the EC coordinate system

T is the current time of the simulation

The computation of the satellite position $(OS)_{RHV}$, velocity $(V_s)_{RHV}$, and acceleration $(A_s)_{RHV}$ vectors with respect to the RHV (i.e. Radial, Horizontal, Vertical) coordinate system (O,R, H, V) as illustrated in Figure 4.2-1 is as follows:

$$\begin{aligned} (OS)_{RHV} &= (R_s, 0, 0)' \\ (V_s)_{RHV} &= (0, 0, R_s \dot{\Theta}_s)' \\ (A_s)_{RHV} &= (-R_s (\dot{\Theta}_s)^2, 0, 0)' \end{aligned}$$

The computation of the satellite position $(OS)_{EC}$, velocity $(V_J)_{EC}$, and acceleration $(A_J)_{EC}$ vectors with respect to the EC coordinate system is given by:

$$\begin{aligned} (OS)_{EC} &= S_3(-LON) S_2(LAT) (OS)_{RHH} \\ (V_J)_{EC} &= S_3(-LON) S_2(LAT) (V_J)_{RHH} \\ (A_J)_{EC} &= S_3(-LON) S_2(LAT) (A_J)_{RHH} \end{aligned}$$

where:

$S_k(theta)$, $k = 1, 2, 3$ represents an orthogonal matrix describing a counter-clockwise rotation about the k^{th} axis by the angle theta, as viewed from the positive end of the k^{th} axis.

4.2.2 Target Dynamics

This module simulates the motion of the target of interest near the surface of the earth. It is assumed that the target flies a constant velocity profile with respect to the EC coordinate system.

The EC coordinate system (O,X,Y,Z) is defined as per paragraph 4.2.1 above. The motion of the target is described by a state vector representing the target position, and velocity vectors with respect to the EC coordinate system.

The target state vector in the EC coordinate system is:

$$X_T = (x, \dot{x}, y, \dot{y}, z, \dot{z})'$$

The equation of motion of the target is given by:

$$X_T(k+1) = F X_T(k) + G W(k)$$

Figure 4.2-1 Satellite Motion in the EC Coordinate System

where:

F is the state transition matrix

G is the process noise transition matrix

W is the process noise assumed to be white, Gaussian.

The process noise represents disturbances with respect to the hypothesis of constant velocity flight.

In the case where the target flies a constant velocity segment, the matrices F and G are given by [22]:

$$F = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} T^2/2 & 0 & 0 \\ T & 0 & 0 \\ 0 & T^2/2 & 0 \\ 0 & T & 0 \\ 0 & 0 & T^2/2 \\ 0 & 0 & T \end{bmatrix}$$

where T is the sampling interval of the simulation.

The process noise is zero-mean with covariance Q :

$$Q = \sigma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where σ^2 is the process noise variance assumed to be the same for the three target position coordinates x, y, z .

The target motion in the EC coordinate system is characterized by the following input parameters as illustrated in Figure 4.2-2.

LONG0: Initial target longitude with respect to the EC coordinate system

LAT0: Initial target latitude with respect to the EC coordinate system

ALT0: Initial target altitude above the surface of the earth

VEL0: Initial target velocity vector with respect to the EC coordinate system

SIGMA: Process noise variance

The initial target position vector with respect to the EC coordinate system is computed as follows:

$$(OT_0)_{EC} = S_3(-LONG0) S_2(LAT0) (OT_0)_{RHY}$$

where:

$$(OT_0)_{RHY} = (R_{10}, 0, 0)'$$

R_{10} is the initial radial distance of the target from the centre of the Earth, given by:

$$R_{10} = ALT0 + R_e \text{ where:}$$

R_e is the earth radius ($R_e = 6378 \text{ km}$)

$S_k(theta)$ is defined as per paragraph 4.2.1 above.

Figure 4.2-2 Target Motion in the EC Coordinate System

4.2.3 Radar Detection

This module simulates the SBR capability to detect a moving target in a cluttered environment.

Detection of the target of interest is based upon three conditions enumerated as follows:

- i. The target must be in the field of view of the satellite.
- ii. The grazing angle at the target location must be within the limits (i.e. between 3° and 60°).
- iii. The radial component of the target velocity with respect to the satellite must be above the Minimum Detectable Velocity (MDV) limit (i.e. 100 m/s). If the above three conditions are satisfied, then the radar is susceptible of detecting the target with a probability of detection equal to 0.9.

The derivation of the equations representing the three conditions is presented next.

Using the following notations as illustrated in Figure 4.2-3:

R_s is the radial distance of the satellite from the centre of the earth

R_t is the radial distance of the target from the centre of the earth

R_{st} is the radial distance of the target from the satellite

R_e is the earth's radius

The target is within the field of view of the satellite if:

$$R_{st} < (R_s^2 - R_e^2)^{1/2} + (R_t^2 - R_e^2)^{1/2}$$

Using the following notations as illustrated in Figure 4.2-4 :

GRAZ is the grazing angle at the target location (i.e. the angle between the SBR line of sight and the horizontal at the target location)

ELEV is the target elevation with respect to the NED coordinate system

The grazing angle is given by:

$$GRAZ = \arccos ((R_s / R_t) \cos(ELEV))$$

The grazing angle is within limits if:

$$3^\circ \leq GRAZ \leq 60^\circ$$

Using the following notations:

$(OS)_{EC}$ is the satellite position vector with respect to the EC coordinate system

$(OT)_{EC}$ is the target position vector with respect to the EC coordinate system

$(ST)_{NED}$ is the target position vector with respect to the NED coordinate system

$(V)_{EC}$ is the target velocity vector with respect to the EC coordinate system expressed in the EC coordinate system

$(V)_{NED}$ is the target velocity vector with respect to the EC coordinate system expressed in the NED coordinate system

The radial component of the target velocity with respect to the satellite is given by:

$$V_{RAD} = ((ST)_{NED} \cdot (V)_{NED}) / R_{st}$$

This radial component is above the MDV limit if:

$$| V_{\text{RAD}} | \geq 100 \text{ m/s}$$

The velocity vector $(V.)_{\text{NED}}$ is given by:

$$(V.)_{\text{NED}} = S_2(-LAT - \pi/2) S_3(LONG) (V.)_{\text{EC}}$$

where:

LAT is the current satellite latitude with respect to the EC coordinate system

$LONG$ is the current satellite longitude with respect to the EC coordinate system

$S_k(\theta)$ is defined as per section 4.2.1 above.

4.2.4 Target Measurement Generation

This module simulates the generation of target measurements to be used in the SBR tracking function. A target measurement consists of the following parameters:

- i) $TRG-RANGE$: target range with respect to the NED coordinate system
- ii) $TRG-AZIM$: target azimuth with respect to the NED coordinate system
- iii) $TRG-ELEV$: target elevation with respect to the NED coordinate system

Figure 4.2-5 illustrates the target geometry in the NED coordinate system.

Figure 4.2-4 Grazing Angle Geometry

The target position vector with respect to the NED coordinate system is given by:

$$(ST)_{\text{NED}} = S_2(-LAT - \pi/2) S_3(LONG) (ST)_{\text{EC}}$$

where:

LAT is the current satellite latitude with respect to the EC coordinate system

LONG is the current satellite longitude with respect to the EC coordinate system

$S_k(\theta)$ is defined as per section 4.2.1 above.

$(ST)_{\text{EC}}$ is the difference between the target position vector and the satellite position vector with respect to the EC coordinate system respectively.

$$\text{Denoting } (ST)_{\text{NED}} = (x_N, x_E, x_D)'$$

$$\text{TRG-RANGE} = ((x_N)^2 + (x_E)^2 + (x_D)^2)^{1/2}$$

$$\text{TRG-AZIM} = \arctg(x_E / x_N)$$

$$\text{TRG-ELEV} = \arcsin(x_D / ((x_N)^2 + (x_E)^2 + (x_D)^2)^{1/2})$$

Figure 4.2-5 Target Geometry in the NED Coordinate System

Given the true target position in the NED coordinate system, the target measurement vector is derived by adding to the true position a measurement noise assumed to be white, Gaussian, zero-mean and of covariance R given by:

$$R = \begin{bmatrix} (\sigma_R)^2 & 0 & 0 \\ 0 & (\sigma_\phi)^2 & 0 \\ 0 & 0 & (\sigma_\theta)^2 \end{bmatrix}$$

where:

$(\sigma_R)^2$ is the measurement noise variance of the range component

$(\sigma_\phi)^2$ is the measurement noise variance of the azimuth component

$(\sigma_\theta)^2$ is the measurement noise variance of the elevation component

4.2.5 False Measurements Generation

This module simulates the generation of false measurements to be used in the SBR tracking function. False measurements are the result of clutter returns or receiver noise. The False Measurement Generation module determines the number of false measurements present at each radar scan and generates the components (i.e. Range, Azimuth, Elevation) of each false measurement with respect to the NED coordinate system.

The number of false measurements within the radar footprint is derived as follows.

The size and shape of the radar footprint is dependent upon the satellite / target geometry and the antenna beamwidth in both azimuth and elevation as illustrated in Figure 4.2-6. Given azimuth and elevation angles, the number of false measurements will be determined by the number of range cells within the footprint. The number of range cells in a footprint is dependent upon the length of the major axis of the footprint and the grazing angle. Considering the satellite / target geometry as illustrated in Figure 4.2-7, the number of range cells in the footprint is given by:

$$N = (d \cos(\beta)) / r$$

where:

d is the length of the major axis of the footprint

β is the grazing angle at the target location

r is the size of the range cell

The derivation of d is provided next, using the following notation as illustrated in Figure 4.2-8:

R_s is the radial distance of the satellite from the centre of the earth

R_t is the radial distance of the target from the centre of the earth

ϕ is the target elevation angle in the NED coordinate system

Θ_{EL} is the antenna elevation beamwidth

The grazing angle at the heel of the footprint is given by:

$$\beta_1 = \arccos \left((R_s / R_t) \cos(\phi + \Theta_{EL} / 2) \right)$$

The grazing angle at the toe of the footprint is given by:

$$\beta_2 = \arccos \left((R_s / R_t) \cos(\phi - \Theta_{EL} / 2) \right)$$

The respective values of the angle α (i.e. α_1 and α_2) are given by:

$$\alpha_1 = \phi + \Theta_{EL} / 2 - \beta_1$$

$$\alpha_2 = \phi - \Theta_{EL} / 2 - \beta_2$$

The respective values of the arc D (i.e. D_1 and D_2) are:

$$D_1 = R_t \alpha_1$$

$$D_2 = R_t \alpha_2$$

The major axis d of the footprint is then given by:

$$d = D_2 - D_1 = R_t (\beta_1 - \beta_2 - \Theta_{EL})$$

Given a false alarm rate per range cell pfa , the average number of false measurements within the footprint becomes:

$$N_F = N pfa$$

The probability mass function (pmf) of the number of false measurements is assumed to follow a Poisson distribution with average value N_F [22].

The false measurement vector with respect to the NED Coordinate system is derived as follows.

Each false measurement will be assigned a measurement vector with respect to the NED coordinate system, based on range, azimuth and elevation as measured by the radar.

The azimuth and elevation angles will be the same as the target azimuth and elevation angles.

The range component will be assumed to be uniformly distributed between the range values at the heel and the toe of the footprint respectively.

Referring to Figure 4.2-8, these two range values, R_1 and R_2 respectively are given by:

$$\begin{aligned} R_1 &= (R_t \sin(\alpha_1)) / (\cos (\phi + \Theta_{EL} / 2)) \\ R_2 &= (R_t \sin(\alpha_2)) / (\cos (\phi - \Theta_{EL} / 2)) \end{aligned}$$

4.2.6 Filter Target Model

This module implements the discrete-time target dynamics equation based upon the derivation of the equation of motion of the target with respect to the satellite-based NED coordinate system (Cartesian coordinates).

Assumptions

- i) The rotation of the earth is not taken into account: this assumption is based upon the fact that the angular velocity of the earth ($7.3 \cdot 10^{-5}$ radians/sec) is much smaller than the SBR angular velocity ($8.3 \cdot 10^{-4}$ radians/sec) and can be neglected in this analysis. As a result, the EC coordinate system is considered as an inertial reference frame.
- ii) The satellite is in a circular polar orbit around the earth with 0° longitude

Figure 4.2-6 Radar Footprint Geometry

Figure 4.2-7 Range Cell Projection onto the Footprint

The EC coordinate system $R0 = (0, i_x, i_y, i_z)$ is an inertial reference frame with its origin at the centre of the earth, the X axis pointing to the Greenwich meridian, the X-Y plane as the Equatorial plane, and the Z axis pointing to the North Pole.

The NED coordinate system $R1 = (S, i_N, i_E, i_D)$ is a reference frame attached to the satellite with its origin at the satellite location, the N axis pointing to the north, the D axis pointing to the centre of the earth, and the E axis perpendicular to the N and D axes.

The angular rotation vector characterizing the motion of R1 with respect to R0 is given by :

$$\Omega = -\dot{\Theta}_s i_y = -\dot{\Theta}_s i_E$$

The rotation matrix transforming a vector from R0 to R1 is $S_2(-\Theta_s - \pi/2)$ where $S_2(\Theta)$ is given by :

$$S_2(\Theta) = \begin{bmatrix} \cos(\Theta) & 0 & -\sin(\Theta) \\ 0 & 1 & 0 \\ \sin(\Theta) & 0 & \cos(\Theta) \end{bmatrix}$$

$$\begin{aligned} \text{Given } (x)_{R1} &= (x_N, x_E, x_D)' & \text{and} \\ (x)_{R0} &= (x_X, x_Y, x_Z)' & \text{then :} \\ (x)_{R1} &= S_2(-\Theta_s - \pi/2) (x)_{R0} \end{aligned}$$

Denoting by x the target position vector with respect to the NED coordinate system, the equation of motion of the target is derived as follows :

$$\frac{(dx)}{(dt)R0} = \frac{(dx)}{(dt)R1} + \Omega \wedge x$$

where $\wedge$ denotes the vector product operator

$$\Omega \wedge x = (-\dot{\Theta}_s x_D, 0, \dot{\Theta}_s x_N)'$$

$$\frac{(dx)}{(dt)R1} = (\dot{x}_N, \dot{x}_E, \dot{x}_D)'$$

$$\frac{(dx)}{(dt)R0} = (v_N)_{R0} - (v_s)_{R0} = v$$

$$\frac{(dv)}{(dt)R0} = \frac{(dv)}{(dt)R1} + \Omega \wedge v$$

$$v = (\dot{x}_N - \dot{\Theta}_s x_D, \dot{x}_E, \dot{x}_D + \dot{\Theta}_s x_N)'$$

$$\frac{(dv)}{(dt)R1} = (\dot{x}_N - \dot{\Theta}_s \dot{x}_D, \dot{x}_E, \dot{x}_D + \dot{\Theta}_s \dot{x}_N)'$$

$$\Omega \wedge v = (-\dot{\Theta}_s \dot{x}_D - (\dot{\Theta}_s)^2 x_N, 0, \dot{\Theta}_s \dot{x}_N - (\dot{\Theta}_s)^2 x_D)'$$

$$\frac{(dv)}{(dt)R0} = (a_v)_{R0} - (a_s)_{R0} = a$$

$$(a_s)_{R0} = R_s (\dot{\Theta}_s)^2 i_D$$

$$\frac{(dv)}{(dt)R0} = (a_1, a_2, a_3) \quad \text{where :}$$

$$a_1 = x_N - 2 \dot{\Theta}_s \dot{x}_D - (\dot{\Theta}_s)^2 x_N$$

$$a_2 = x_E$$

$$a_3 = x_D + 2 \dot{\Theta}_s \dot{x}_N - (\dot{\Theta}_s)^2 x_D$$

$$\frac{(da)}{(dt)R0} = \frac{(da)}{(dt)R1} + \Omega \wedge a$$

$$= (\dot{a}_v)_{R0} - (\dot{a}_s)_{R0}$$

$$\frac{(da)}{(dt)R1} = (b_1, b_2, b_3) \quad \text{where :}$$

$$b_1 = \bar{x}_N - 2 \dot{\Theta}_s x_D - (\dot{\Theta}_s)^2 \dot{x}_N$$

$$b_2 = \bar{x}_E$$

$$b_3 = \bar{x}_D + 2 \dot{\Theta}_s x_N - (\dot{\Theta}_s)^2 \dot{x}_D$$

$$\Omega \wedge a = (\Omega_1, \Omega_2, \Omega_3) \quad \text{where :}$$

$$\Omega_1 = -\dot{\Theta}_s \dot{x}_D - 2(\dot{\Theta}_s)^2 \dot{x}_N + (\dot{\Theta}_s)^3 x_D$$

$$\Omega_2 = 0$$

$$\Omega_3 = \dot{\Theta}_s \dot{x}_N - 2(\dot{\Theta}_s)^2 \dot{x}_D - (\dot{\Theta}_s)^3 x_N$$

$$\frac{(da)}{(dt)R0} = (c_1, c_2, c_3) \quad \text{where :}$$

$$c_1 = \bar{x}_N - 3\dot{\Theta}_s x_D - 3(\dot{\Theta}_s)^2 \dot{x}_N + (\dot{\Theta}_s)^3 x_D$$

$$c_2 = \bar{x}_E$$

$$c_3 = \bar{x}_D + 3\dot{\Theta}_s x_N - 3(\dot{\Theta}_s)^2 \dot{x}_D - (\dot{\Theta}_s)^3 x_N$$

For a 2nd order model: $(a_{\mathcal{V}})_{R0} = V$, white noise

The target state vector is given by:

$$X = (x_N, \dot{x}_N, x_E, \dot{x}_E, x_D, \dot{x}_D)'$$

The target acceleration process noise is given by:

$$\begin{aligned} (a_{\mathcal{V}})_{R0} &= (V_x, V_y, V_z)' && \text{in } R0 \\ &= (V_N, V_E, V_D)' && \text{in } R1 \quad \text{where:} \end{aligned}$$

$$\begin{aligned}
 V_N &= -\sin(\Theta_J) V_x + \cos(\Theta_J) V_z \\
 V_E &= V_y \\
 V_D &= -\cos(\Theta_J) V_x - \sin(\Theta_J) V_z
 \end{aligned}$$

Since: $\langle \sigma_x \rangle^2 = \langle \sigma_y \rangle^2 = \langle \sigma_z \rangle^2$ then:

$$\begin{aligned}
 E[(a_t)_{R0} \cdot (a_t)_{R0}] &= \sigma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= E[(a_t)_{R1} (a_t)_{R1}]
 \end{aligned}$$

The target dynamics equation is given by:

$$\dot{X}(t) = A(t) X(t) + B(t) U(t) + V(t)$$

$$(a_v)_{R0} - (a_J)_{R0} = \frac{(dv)}{(dt)R1} + \Omega \wedge v$$

$$\dot{x}_N = 2 \dot{\Theta}_s \dot{x}_D + (\dot{\Theta}_J)^2 x_N + V_N$$

$$x_E = V_E$$

$$x_D = -2 \dot{\Theta}_s \dot{x}_N + (\dot{\Theta}_J)^2 x_D - R_s (\dot{\Theta}_J)^2 + V_D$$

The state transition matrix A is as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ (\dot{\Theta}_s)^2 & 0 & 0 & 0 & 0 & 2\dot{\Theta}_s \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2\dot{\Theta}_s & 0 & 0 & (\dot{\Theta}_s)^2 & 0 \end{bmatrix}$$

$$B U = (0, 0, 0, 0, 0, -R_s (\dot{\Theta}_s)^2)'$$

$$V = (0, V_N, 0, V_E, 0, V_D)'$$

The discrete-time dynamics equation is given by:

$$X(k+1) = F(k) X(k) + G(k) U(k) + V(k)$$

Given the sampling interval T , F is given by:

$$F = e^{(A T)} \approx I + A T + (A^2 T^2) / 2$$

The derivation of F leads to the following matrix:

$$\begin{bmatrix} 1+(\dot{\Theta}_s)^2 (T^2/2) & T & 0 & 0 & 0 & \dot{\Theta}_s T^2 \\ (\dot{\Theta}_s)^2 T & 1-3(\dot{\Theta}_s)^2 (T^2/2) & 0 & 0 & (\dot{\Theta}_s)^3 T^2 & 2 \dot{\Theta}_s T \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -(\dot{\Theta}_s)^2 T^2 & 0 & 0 & 1+(\dot{\Theta}_s)^2 (T^2/2) & T \\ -(\dot{\Theta}_s)^3 T^2 & -2 \dot{\Theta}_s T & 0 & 0 & (\dot{\Theta}_s)^2 T & 1-3(\dot{\Theta}_s)^2 (T^2/2) \end{bmatrix}$$

$$\begin{aligned} G(k) U(k) &= \int_0^T F(T-\tau) B U d\tau \\ &= A^{-1} [e^{(A T)} - I] B U = (I T + (A T^2) / 2) B U \end{aligned}$$

$$\begin{aligned} V(k) &= \int_0^T F(T-\tau) V d\tau \\ &= A^{-1} [e^{(A T)} - I] V = (I T + (A T^2) / 2) V \end{aligned}$$

4.2.7 Filter Measurement Model

This module implements the discrete time measurement equation including the measurement vector and its associated covariance matrix used in the tracking filter.

As described in section 4.2.1 above, tracking is performed in the NED coordinate system in Cartesian coordinates.

The target state vector is given by: (2nd Order Model)

$$X = (x_N, \dot{x}_N, x_E, \dot{x}_E, x_D, \dot{x}_D)'$$

The measurement equation can be written as:

$$Z = H X + W$$

where:

Z is the filter measurement vector

H is the filter measurement matrix

W is the filter measurement noise vector, assumed to be white, Gaussian, zero-mean of covariance R_w

Denoting $Z = (z_N, z_E, z_D)'$ with:

z_N the measured x_N coordinate

z_E the measured x_E coordinate

z_D the measured x_D coordinate

The measurement matrix H becomes:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Given a noisy measurement (R, ϕ, Θ) obtained from the radar where:

R is the range component with respect to the NED coordinate system

ϕ is the azimuth component with respect to the NED coordinate system

Θ is the elevation component with respect to the NED coordinate system

The filter measurement vector becomes:

$$z_N = R \cos(\Theta) \cos(\phi)$$

$$z_E = R \cos(\Theta) \sin(\phi)$$

$$z_D = R \sin(\Theta)$$

The filter measurement noise covariance is given by:

$$R_w = \begin{bmatrix} (\sigma_N)^2 & (\sigma_{NE})^2 & (\sigma_{ND})^2 \\ (\sigma_{NE})^2 & (\sigma_E)^2 & (\sigma_{ED})^2 \\ (\sigma_{ND})^2 & (\sigma_{ED})^2 & (\sigma_D)^2 \end{bmatrix}$$

Based on the first order Extended Kalman filter derivation, the covariance R_w is given by [47 - 49]:

$$R_w = F_w \begin{bmatrix} (\sigma_R)^2 & 0 & 0 \\ 0 & (\sigma_\phi)^2 & 0 \\ 0 & 0 & (\sigma_\theta)^2 \end{bmatrix} F_w'$$

where:

$(\sigma_R)^2$ is the radar measurement noise variance of the range component

$(\sigma_\phi)^2$ is the radar measurement noise variance of the azimuth component

$(\sigma_\theta)^2$ is the radar measurement noise variance of the elevation component

F_w is the Jacobian of the function f_w defined by:

$$(z_N, z_E, z_D)' = f_w(R, \phi, \Theta)$$

$$f_{w1}(R, \phi, \Theta) = R \cos(\Theta) \cos(\phi)$$

$$f_{w2}(R, \phi, \Theta) = R \cos(\Theta) \sin(\phi)$$

$$f_{w3}(R, \phi, \Theta) = R \sin(\Theta)$$

The Jacobian of f_w , F_w , is as follows:

$$\begin{bmatrix} \cos(\Theta) \cos(\phi) & -R \cos(\Theta) \sin(\phi) & -R \sin(\Theta) \cos(\phi) \\ \cos(\Theta) \sin(\phi) & R \cos(\Theta) \cos(\phi) & -R \sin(\Theta) \sin(\phi) \\ \sin(\Theta) & 0 & R \cos(\Theta) \end{bmatrix}$$

The elements of the covariance matrix R_w are then given by:

$$\begin{aligned}
 (\sigma_N)^2 &= (\sigma_R)^2 \cos^2(\Theta) \cos^2(\phi) \\
 &+ R^2 (\sigma_\phi)^2 \cos^2(\Theta) \sin^2(\phi) \\
 &+ R^2 (\sigma_\Theta)^2 \sin^2(\Theta) \cos^2(\phi) \\
 (\sigma_{NE})^2 &= (\sin(2\phi)/2) ((\sigma_R)^2 \cos^2(\Theta) - R^2 (\sigma_\phi)^2 \cos^2(\Theta) \\
 &\quad + R^2 (\sigma_\Theta)^2 \sin^2(\Theta)) \\
 (\sigma_{ND})^2 &= \cos(\phi) (\sin(2\Theta)/2) ((\sigma_R)^2 - R^2 (\sigma_\Theta)^2) \\
 (\sigma_E)^2 &= (\sigma_R)^2 \cos^2(\Theta) \sin^2(\phi) \\
 &+ R^2 (\sigma_\phi)^2 \cos^2(\Theta) \cos^2(\phi) \\
 &+ R^2 (\sigma_\Theta)^2 \sin^2(\Theta) \sin^2(\phi) \\
 (\sigma_{ED})^2 &= \sin(\phi) (\sin(2\Theta)/2) ((\sigma_R)^2 - R^2 (\sigma_\Theta)^2) \\
 (\sigma_D)^2 &= (\sigma_R)^2 \sin^2(\Theta) + R^2 (\sigma_\Theta)^2 \cos^2(\Theta)
 \end{aligned}$$

The measurement noise covariance matrix R_w is evaluated at time $k+1$ based upon the value of the predicted measurement vector $\hat{z}(k+1 | k)$.

4.2.8 True Target State Generation

This module computes the true target state derived from the equation of motion of the target with respect to the NED coordinate system.

The true target state vector is used to evaluate the performance of the tracking filter by comparing the filter-derived state estimate and its associated covariance to the true target state and the true RMS error respectively.

The computation of the true target state is based upon the current satellite position, velocity, and acceleration vectors with respect to the EC coordinate system, as well as upon the current target position, velocity, and acceleration vectors with respect to the EC coordinate system.

Following the derivation of the target dynamics equation presented in Section 4.2.6, one has:

In the case of a 2nd order model:

$$X = (x_N, \dot{x}_N, x_E, \dot{x}_E, x_D, \dot{x}_D)'$$

$$(ST)_{NED} = (x_N, x_E, x_D)'$$

$$(ST)_{NED} = S_2(-LAT - \pi/2) S_3(LONG) (ST)_{EC}$$

where:

LAT is the current satellite latitude with respect to the EC coordinate system

LONG is the current satellite longitude with respect to the EC coordinate system

$S_k(\theta)$ is defined as per Section 4.2.1 above.

$(\vec{ST})_{EC} = (OT)_{EC} - (OS)_{EC}$ is the difference vector between the target and the satellite position vectors in the EC coordinate system. Denoting by x the position vector $(ST)_{NED}$, one has:

$$\begin{aligned} \frac{(dx)}{(dt)_{EC}} &= (V_t)_{NED} - (V_s)_{NED} \\ &= (\dot{x}_N - \dot{\Theta}_s x_D, \dot{x}_E, \dot{x}_D + \dot{\Theta}_s x_N)' \end{aligned}$$

where:

$(V_t)_{NED}$ is the target velocity vector with respect to the EC coordinate system expressed in the NED coordinate system.

$(V_s)_{NED}$ is the satellite velocity vector with respect to the EC coordinate system expressed in the NED coordinate.

The above equation is used to derive the velocity components of the true state vector.

4.2.9 Filter Initialization

This module computes the value of the initial target state estimate and the value of the initial state covariance matrix.

Within the framework of the Kalman filter, the initial state is assumed normally distributed with a mean equal to the initial estimate, and a covariance equal to the initial estimate covariance. According to the Bayesian assumptions in the Kalman filter, the true initial state is a random variable and the initial estimate is a fixed known quantity.

Following [22], in simulations one can reverse this point of view by fixing the initial state and generating the initial estimate with a Gaussian random number generator. The initial estimate is generated with a mean equal to the true initial state and with a covariance equal to the initial state covariance. This method relies on the fact that, switching the roles of the initial state $x(0)$ and the initial estimate $\hat{x}(0|0)$, one has under the Gaussian assumption, the algebraic identity:

$$P[x(0) | \hat{x}(0|0)] = P[\hat{x}(0|0) | x(0)]$$

where $P[Z | Y]$ represents the conditional probability of Z given the value of Y .

The true initial state $x(0)$ is computed as per Section 4.2.7 above. The initial state covariance is computed as follows:

Following [50,51], a polynomial fit can be used to derive the initial state covariance.

For a 2nd order model, the measurement $z(k)$ of the state in a given coordinate $x(k)$ at time k is given by:

$$z(k) = x(0) + \dot{x}(0) k T + w(k),$$

where:

T is the sampling interval

w is the measurement noise with covariance R

$x(0)$ is the initial position component of the state

$\dot{x}(0)$ is the initial velocity component of the state

Given that the measurements $z(-1)$ and $z(0)$ at time -1 and time 0 respectively are available from the track initiation function, one has:

$$\dot{x}(0 | 0) = z(0)$$

$$\dot{x}(0 | 0) = (z(0) - z(-1)) / T$$

The associated covariance matrix $P(0 | 0)$ is then given by:

$$P(0 | 0) = E [(\dot{x}(0 | 0), \dot{x}(0 | 0)) (\dot{x}(0 | 0), \dot{x}(0 | 0))']$$

$$= \begin{bmatrix} R & R / T \\ R / T & 2R / T^2 \end{bmatrix}$$

The covariance R , is computed for each coordinate of the state as per Section 4.2.4.

Chapter 5

Simulation Results

5.1 General

This section provides the results of the simulation exercises performed using the model described in section 4.0. This model simulates the tracking performance of a Space-Based Radar system in specified scenarios.

The objective of the simulation is to determine the performance of two tracking techniques in the context of a Space-Based Radar application.

More specifically, the purpose of the simulation is to quantify the performance of the Nearest Neighbour Standard Filter (NNSF) and of the Probabilistic Data Association Filter (PDAF) in the case where a space-based radar system is tracking a single target in a clutter environment. A detailed description of the algorithm used in each of these two techniques is provided in section 3.0.

To this end, this section presents relevant results and analysis describing the performance of each tracking technique within the context of a set of test cases. Two main scenarios were considered for the simulation of SBR tracking performance. These scenarios are defined in terms of a given target flight profile and satellite trajectory. The first scenario corresponds to the case where the SBR initiates its tracking at low grazing angles while the second scenario corresponds to the case where tracking is initiated at high grazing angles. The influence of the satellite / target geometry on the tracking performance was analyzed through a set of experiments. The differences in the geometry between the two scenarios were characterized primarily by the initial value of the filter measurement covariance, the size of the footprint and the number of false measurements within the footprint at each sampling interval.

The methodology followed for simulating the tracking performance of a space-based radar system is to perform a large number of simulation runs and to compile statistical results. This approach is necessitated by the fact that many of the processes being modelled are probabilistic in nature. Multiple runs of the tracking sequence generate statistically meaningful performance measures. For each experiment conducted in this study, a series of 50 runs was used. The outputs of the simulations provide the information to conduct the analysis of the SBR tracking performance.

These outputs include the following:

- o Target position, velocity, acceleration vectors, at each simulation time step, in both Earth Centred and North-East-Down coordinate systems for each run.
- o Satellite position, velocity, acceleration vectors, at each simulation time step, with respect to the Earth Centred coordinate system for each run.
- o Radar detection information (i.e. target detected / not detected), at each simulation time step for each run.
- o Initial filter state estimate and associated covariance for each run.
- o Filter state estimate and associated covariance at each simulation time step for each run.
- o Filter state covariance and true RMS error averaged over all runs.

A detailed description of the two scenarios along with the definition of each experiment is presented in the following sections.

5.2 Scenario 1

As described in section 5.1, the scenarios under consideration for the evaluation of SBR tracking performance are defined in terms of satellite trajectory and target flight profile. Scenario 1 corresponds to the case where the SBR initiates tracking on the target at low grazing angle. The target flight profile and the satellite trajectory are as follows:

Figure 5.2-4 Footprint Length Variation -Scenario 1

Within the context of Scenario 1, the satellite / target geometry at the start of the simulation is characterized by:

Satellite range (EC coordinate system):	8378 km
Target range (EC coordinate system):	6388 km
Target range (NED coordinate system):	4892 km
Target azimuth (NED coordinate system):	0 degrees
Target elevation (NED coordinate system):	41 degrees

The initial grazing angle value is:

$$GRAZ = 6 \text{ degrees}$$

Figure 5.2-3 illustrates the variation of grazing angle over the duration of the simulation.

The initial footprint dimensions are:

Footprint length: 490 kilometres

Footprint width: 43 kilometres

Figure 5.2-4 illustrates the variation of the footprint length over the duration of the simulation.

The initial number of range cells illuminated by the radar is:

$$NCELL = 1627$$

Figure 5.2-5 illustrates the variation of the number of range cells in the footprint over the duration of the simulation.

Figure 5.2-1 Target Trajectory - Scenario 1

Figure 5.2-2 Satellite Trajectory - Scenario 1

The initial target true state vector components are:

- i) Position:
 - North Axis: 3716 km
 - East Axis: 0 km
 - Down Axis: 3182 km
- ii) Velocity:
 - North Axis: -4522 m/s
 - East Axis: 0 m/s
 - Down Axis: -3234 m/s
- iii) Acceleration:
 - North Axis: -2.8 m/s²
 - East Axis: 0 m/s²
 - Down Axis: 4 m/s²

The initial radar measurement error standard deviations are:

- North axis: 5516 meters
- East axis: 6436 meters
- Down axis: 6439 meters

The initial filter state estimate standard deviations are:

- i) Position:
 - North axis: 5516 meters
 - East axis: 6436 meters
 - Down axis: 6439 meters
- ii) Velocity:
 - North axis: 780 meters/second
 - East axis: 910 meters/second
 - Down axis: 911 meters/second

Figure 5.2-3 Grazing Angle Variation - Scenario 1

Figure 5.2-4 Footprint Length Variation -Scenario 1

Figure 5.2-5 Variation of Number of Range Cells - Scenario 1

In the following sections, the results and analysis of various experiments conducted within the context of Scenario 1 are presented.

The tracking performance of the NNSF is evaluated through experiments 1-1 to 1-5, whereas the performance of the PDAF is evaluated through experiments 1-6 to 1-8.

Experiments 1-1 and 1-6 corresponds to an ideal case where no process noise affects the target dynamics and where no false measurements are present.

Experiments 1-2 and 1-3 corresponds to the case where some process noise acceleration is present in the target dynamics.

Experiments 1-4, 1-7 and 1-5, 1-8 corresponds to a low false alarm and a high false alarm environment respectively.

5.2.1 Experiment 1-1

For this experiment, the tracking technique simulated corresponds to the Nearest Neighbour Standard Filter as implemented for SRR tracking.

The main parameters characterizing each experiment include the filter sampling interval, the target dynamics model process noise, and the probability of false alarm per range cell as defined in section 4.2.5. For this experiment, the values of these parameters are as follows:

Sampling Interval: $T = 10 \text{ seconds}$

Process Noise Standard Deviation: $SIGMA = 0.0 \text{ meters/second}^2$

Probability of False Alarm: $PFA = 0.0$

As can be seen from the above parameters, experiment 1-1 corresponds to an ideal case where the target flies at a perfectly constant speed (i.e. no acceleration process noise) and where no false measurements are introduced into the tracking filter. As such, experiment 1-1 constitutes a baseline test which establishes the ideal performance of the Nearest Neighbour Standard Filter, as applied to the SBR tracking problem.

A batch of 50 simulation runs was carried out. For 3 out of the 50 runs, the initial state estimate and the measurements introduced into the filter were such that the target measurements did not fall within the validation gate of the filter. As a result, the filter was not capable of tracking the target and the results obtained from these runs were not representative of the filter performance. Average performance measures were then derived from the remaining 47 runs where the filter did not lose track on the target.

Figures 5.2.1-1 to 5.2.1-3 display the filter-derived target position RMS error along with the true target position RMS error for each component in the NED coordinate system. The filter-derived RMS error is based upon the computed covariance of the estimate whereas the true RMS error is derived by computing the difference between the real target position (known in the simulation) and the target position estimate as computed by the filter. These RMS errors are plotted for each sampling interval and are averaged over the 47 runs.

As can be noticed from the figures, there is a good match between the two curves.

This is due to the fact that the filter target dynamics model and measurement model are reflective of the true target dynamics and measurement characteristics. The target follows a constant speed profile, hence the 2nd order model implemented in the filter is appropriate. In addition, the measurement noise statistics used in the filter is representative of that used in the generation of the measurement vector in the simulation.

As described in section 4.2.9, the initial state covariance is based upon the measurement covariance. As can be seen in the figures, the initial target position standard deviations are large (i.e. of the order of 5 to 6 km for each coordinate). However, the estimate covariance as computed by the filter decreases rapidly as more measurements are processed. (After the first 10 scans, the covariance is reduced by a factor of 2). The transient response of the filter is then followed by a more steady response leading to an RMS error of the order of 500 meters at scan 56 as indicated below:

X Position (North axis): 591 meters

Y Position (East axis): 622 meters

Z Position (Down axis): 315 meters

As more measurements are processed, the confidence in the state estimate increases, and as a result the covariance of the estimate decreases.

By comparison, the true RMS error at scan 56 is given by:

X Position (North axis): 469 meters

Y Position (East axis): 616 meters

Z Position (Down axis): 232 meters

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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Figure 5.2.1-1 Filter/True RMS Error - Experiment 1-1

Figure 5.2.1-2 Filter/True RMS error - Experiment 1-1

Figure 5.2.1-3 Filter/True RMS error - Experiment 1-1

It is important to note that the differences between the filter-derived RMS errors and the true RMS errors can be attributed to the fact that the filter is not perfectly matched to the target dynamics. High order terms have been omitted in the derivation of the discrete-time dynamics equation. Even with perfect knowledge of the state, it can be seen from Figures 5.2.1-4 and 5.2.1-5 that extrapolation of the state based on the discrete-time model leads to some error as compared with the true state.

In addition, the derivation of the measurement covariance is based upon the use of the 1st order Extended Kalman Filter which constitutes also an approximation of the non-linear relationship between the target state and the measurements.

Figures 5.2.1-1 to 5.2.1-3 also indicate that the RMS error increases from scan 57 to scan 70. As can be seen from Figure 5.2-3, the grazing angle at the target location becomes greater than the 60° limit of radar performance at scan 57. As a result, the radar does not detect the target, and no measurements are processed by the filter. The filter extrapolates the target state based on its target dynamics model, and the covariance is also extrapolated. This results in an increase of the covariance consistent with the fact that the accuracy of the state estimate decreases as a result of the extrapolation.

The analysis of a representative simulation run is provided in Appendix C.

Figure 5.2.1-4 Filter Extrapolation RMS error

Figure 5.2.1-5 Filter Extrapolation RMS error

5.2.2 Experiments 1-2 / 1-3

The purpose of these two experiments is to quantify the effect of target process noise upon the Nearest Neighbour Standard Filter as implemented for SBR tracking. It is anticipated that the presence of target process noise will increase the track estimate covariance since an additional uncertainty regarding the target dynamic behaviour is introduced.

The difference between these two experiments and experiment 1-1 consists of introducing process noise into the target flight dynamics. A random acceleration characterized by a white Gaussian process of zero mean and standard deviation σ equal to $0.5 \text{ meters / second}^2$ and $1.0 \text{ meters / second}^2$ respectively is applied to both the true target dynamics and the filter target dynamics models. This process noise represents disturbances in the target flight with regard to the hypothesis of constant speed. As per the assumptions of the 2nd order model described in section 4.2.6, the random acceleration is assumed to be constant within a sampling interval. As a result, the changes in velocity over a sampling interval T are of the order of σT as indicated by the process noise covariance matrix (refer to section 4.2.6).

Figures 5.2.2-1 and 5.2.2-2 display the filter-derived target position RMS error along with the true target position RMS error when σ equals $0.5 \text{ meters / second}^2$ and $1.0 \text{ meters / second}^2$ respectively. As expected, it can be noted that the presence of process noise increases the covariance of the estimate. The RMS error value obtained at scan # 56 is now of the order of 2 km and 3 km respectively.

In addition, the lack of radar measurements between scan # 57 and 70 leads to a significant degradation in target track accuracy : the RMS error value is about 5.5 km and 10.0 km respectively. Depending upon the track accuracy requirements of the SBR system operation, the extrapolation of the target estimate over a long coverage gap (e.g. 10 to 15 min), as may be the case for SBR, may lead to the loss of the target track.

Figure 5.2.2-1 Filter / True RMS error - Experiment 1-2

Figure 5.2.2-2 Filter / True RMS error - Experiment 1-3

5.2.3 Experiments 1-4 / 1-5

The purpose of these two experiments is to quantify the effect of clutter upon the Nearest Neighbour Standard Filter as implemented for SBR tracking. It is anticipated that the presence of false measurements in the filter due to clutter will affect the performance of the filter since no mechanism to take account of the measurement origin uncertainty is provided within the framework of the NNSF.

The difference between these two experiments and experiment 1-1 consists of introducing false measurements into the tracking filter. As described in section 4.2.5, the number of false measurements generated in the simulation varies as a function of the SBR / target geometry. For these experiments, the false alarm rate per range cell PFA is equal to 10^{-3} and 10^{-2} respectively. The generation of false measurements over all range cells in a footprint follows a Poisson distribution with average $N PFA$, where N is the number of range cells in the beam footprint.

Experiment 1-4 corresponds to a very low false alarm environment where the presence of false measurements is only observed in the early stages of the simulation and should not significantly affect the performance of the filter. On the other hand, Experiment 1-5 corresponds to a dense false alarm environment in the early stages of the simulation and it is anticipated that the presence of a significant number of false measurement will affect the performance of the filter.

From the results of 50 simulation runs, it was observed that the filter succeeds in tracking the target 90% of the time (i.e. 45 runs out of 50) and 60% of the time (i.e. 30 runs out of 50) respectively.

As expected, the presence of few false measurements for the first scans of the simulation did not significantly affect the capability of the filter to track the target. However, a significant degradation in the tracking performance was observed when the environment is characterized by the presence of a large number of false measurements in the early stages of the tracking. It is noted that, as discussed in section 5.2.1, the area of the validation gate of the track is comparable with the size of the beam footprint, at least for the first few scans of the simulation. Thus a significant number of the false measurements will fall within the validation gate.

Because of the large uncertainty associated with the initial target estimate, these measurements may be selected to update the filter, regardless of their origin.

The Nearest Neighbour Standard Filter does not provide adequate performance under high false alarm conditions for Scenario 1 (i.e. when tracking is initiated at low grazing angle). Alternatives to the NNSF algorithm need to be investigated. Subsequent Scenario 1 experiments evaluate the capability of the Probabilistic Data Association Filter (PDAF) algorithm to provide improved performance.

5.2.4 Experiment 1-6

For this experiment, the tracking technique simulated corresponds to the Probabilistic Data Association Filter as implemented for SBR tracking.

The main parameters characterizing this experiment are as follows:

Sampling Interval: $T = 10 \text{ seconds}$

Process Noise Standard Deviation: $\Sigma GMA = 0.0 \text{ meters/second}^2$

Probability of False Alarm: $PFA = 0.0$

As can be seen from the above parameters, experiment 1-6 corresponds to an ideal case where the target flies at a perfectly constant speed (i.e. no acceleration process noise), and where no false measurements are introduced in the tracking filter. As such, experiment 1-6 constituted a baseline test which establishes the ideal performance of the Probabilistic Data Association Filter, as applied to the SBR tracking problem. This experiment has to be compared with experiment 1-1 where the ideal performance of the Nearest Neighbour Standard Filter was analyzed.

Since no false measurements are present in the experiment, it is expected that the performance of the filter will be very similar to the performance of the NNSF.

A batch of 50 runs was simulated. For each run, the filter was capable of tracking the target. As a result, average performance measures were derived from all 50 runs.

Figures 5.2.4-1 to 5.2.4-3 display the filter-derived target position RMS errors along with the true target position RMS error for each component in the NED coordinate system. As can be seen from the figures, the behaviour of the filter is very similar to the behaviour of the NNSF in experiment 1-1.

Since the initial state covariance is based upon the measurement covariance, the initial target position standard deviations are large (i.e. of the order of 6 to 7 km) for each coordinate.

Figure 5.2.4-1 Filter / True RMS error - Experiment 1-6

Figure 5.2.4-2 Filter / True RMS error - Experiment 1-6

Figure 5.2.4-3 Filter / True RMS error - Experiment 1-6

The transient response of the filter is followed by a more steady response leading to an RMS error of the order of 550 meters at scan 56 as indicated below:

X Position (North axis): 639 meters

Y Position (East axis): 676 meters

Z Position (Down axis): 347 meters

By comparison, the true RMS error at scan 56 is given by:

X Position (North axis): 519 meters

Y Position (East axis): 668 meters

Z Position (Down axis): 248 meters

It is noted that the RMS error values are higher than the values obtained with the NNSF in experiment 1-1. This is related to the computation of the updated covariance. As described in section 3.3.3, the derivation of the covariance of the updated state estimate leads to:

$$P(k | k) = \beta_0(k) P(k | k-1) + (1-\beta_0(k)) P^c(k | k) + P^s(k)$$

where:

$\beta_0(k)$ is the conditional probability of the event {none of the measurements originated from the target} given the cumulative set of measurements up to time k .

$P^c(k | k) = P(k | k-1) - W(k) S(k)^{-1} W(k)'$ (i.e. the covariance of the updated state in the absence of measurement origin uncertainty)

$P^s(k)$ is the "spread of the means", and represents an increase in the covariance due to the uncertainty of the measurement origin.

$P(k | k)$ can be rewritten as:

$$P(k | k) = P(k | k-1) - (1-\beta_0(k)) W(k) S(k)^{-1} W(k)' + P^s(k)$$

For the case of the NNSF, the covariance of the updated state estimate is simply given by:

$$P(k | k) = P(k | k-1) - W(k) S(k) W(k)'$$

This covariance is smaller than the covariance of the PDAF.

The analysis of a representative simulation run is provided in Appendix D.

5.2.5 Experiments 1-7 / 1-8

The purpose of these two experiments is to quantify the effect of clutter upon the Probabilistic Data Association Filter as implemented for SBR tracking. The presence of false measurements in the filter due to clutter should not significantly affect the performance of the filter since the PDAF design provides the means to discriminate between the target measurement and false measurements.

The difference between these two experiments and experiment 1-6 consists of introducing false measurements into the tracking filter. As in the case of Experiments 1-4 / 1-5, the false alarm rate per range cell *PFA* is equal to 10^{-3} and 10^{-2} respectively.

From the results of 50 simulation runs, it was observed that the filter succeeds in tracking the target 94% of the time (i.e. 47 runs out of 50) and 90% of the time (i.e. 45 runs out of 50) respectively.

The performance of the PDAF in a dense clutter environment is superior to the performance of the NNSF as observed in experiment 1-5 where the filter was successful in tracking the target only 60% of the time.

As expected, the PDAF is less sensitive to the presence of a large number of false measurements.

Figures 5.2.5-1 and 5.2.5-2 display the filter-derived target position RMS error along with the true target position RMS error for Experiments 1-7 and 1-8 respectively.

As expected, the performance of the filter is similar to the performance observed in experiment 1-6. There are significant differences, however, in the behaviour of the filter in the early stages of the simulation.

The presence of false measurements in the early stages of the simulation leads to a larger value of the target position standard deviation as compared with the case where no false measurements are present (refer to experiment 1-6). This is consistent with the design of the PDAF algorithm, since the basic concept of the PDAF is to incorporate all the measurements within the validation gate of the track so that the updated estimate is a weighted sum of all validated measurements.

However, by contrast with the NNSF algorithm, the target measurement is not rejected by the filter, and its persistence inside the validation gate over time prevents the loss of the target track most of the time.

Figure 5.2.5-1 Filter / True RMS error - Experiment 1-7

Figure 5.2.5-2 Filter / True RMS error - Experiment 1-8

5.3 Scenario 2

Scenario 2 corresponds to the case where the SBR initiates tracking on the target at high grazing angle. The target flight profile and the satellite trajectory are as follows:

- 1) Target flight profile with respect to the EC coordinate system:

The target flies a constant direction profile along the X axis, at a constant speed, for a duration of 600 seconds. The initial target flight parameters at time 0 in the simulation are as follows:

- o Initial Longitude: 0 degrees
- o Initial Latitude: 90 degrees
- o Initial Altitude: 10 kilometres
- o Initial Speed: 300 meters/second

Figure 5.3-1 illustrates the target flight profile for the duration of the scenario.

- 2) Satellite trajectory with respect to the EC coordinate system:

The satellite flies a circular polar orbit in the X-Z plane for a duration of 600 seconds with a counter-clockwise angular rotation as viewed from the extremity of the Y axis. The initial satellite flight parameters at time 0 in the simulation are as follows:

- o Initial Longitude: 0 degrees
- o Initial Latitude: 82 degrees
- o Initial Altitude: 2000 kilometres

Figure 5.3-1 Target Trajectory - Scenario 2

Figure 5.3-2 illustrates the satellite trajectory for the duration of the scenario.

As indicated in section 5.1, the satellite / target geometry of Scenario 2 influences several parameters which may have an effect on the SBR tracking performance. These parameters include grazing angle, footprint size, and measurement covariance. Within the context of Scenario 2, the satellite / target geometry at the start of the simulation is characterized by:

Satellite range (EC coordinate system):	8378 km
Target range (EC coordinate system):	6388 km
Target range (NED coordinate system):	2240 km
Target azimuth (NED coordinate system):	0 degrees
Target elevation (NED coordinate system):	66 degrees
The corresponding grazing angle value is:	
<i>GRAZ</i>	= 57 degrees

Figure 5.3-3 illustrates the variation of grazing angle over the duration of the simulation.

The corresponding footprint dimensions are:

Footprint length:	24 km
Footprint width:	20 km

Figure 5.3-4 illustrates the variation of the footprint length over the duration of the simulation.

The initial number of range cells illuminated by the radar is:

$$NCELL = 43$$

Figure 5.3-5 illustrates the variation of the number of range cells in the footprint over the duration of the simulation.

Figure 5.3-2 Satellite Trajectory - Scenario 2

The initial true state vector components are:

- i) Position:
 - North axis: 896 km
 - East axis: 0 km
 - Down axis: 2053 km
- ii) Velocity:
 - North axis: 4910 m/s
 - East axis: 0 m/s
 - Down axis: 696 m/s
- iii) Acceleration;
 - North axis: -0.5 m/s^2
 - East axis: 0 m/s^2
 - Down axis: 3.8 m/s^2

The initial radar measurement error standard deviation are:

- North axis: 3558 meters
- East axis: 1553 meters
- Down axis: 1577 meters

The initial filter state estimate standard deviations are:

- i) Position:
 - North axis: 3558 meters
 - East axis: 1553 meters
 - Down axis: 1577 meters
- ii) Velocity:
 - North axis: 503 meters/second
 - East axis: 220 meters/second
 - Down axis: 223 meters/second

Figure 5.3-3 Grazing Angle Variation - Scenario 2

Figure 5.3-4 Footprint Length Variation - Scenario 2

Figure 5.3-5 Variation of Number of Range Cells - Scenario 2

In the following sections, the results and analysis of various experiments conducted within the context of Scenario 2 are presented.

The tracking performance of the NNSF is evaluated in experiments 2-1 and 2-2, whereas the performance of the PDAF is analyzed in experiment 2-3.

Experiment 2-1 corresponds to an ideal case where no process noise affects the target dynamics and where no false measurements are present.

Experiments 2-2 and 2-3 correspond to a high false alarm environment.

5.3.1 Experiment 2-1

For this experiment, the tracking technique simulated corresponds to the Nearest Neighbour Standard Filter as implemented for SBR tracking.

The main parameters characterizing each experiment include the filter sampling interval, the target dynamics model process noise, and the probability of false alarm per range cell as defined in section 4.2.5. For this experiment, the values of these parameters are as follows:

Sampling Interval: $T = 10 \text{ seconds}$

Process Noise Standard Deviation: $SIGMA = 0.0 \text{ meters/second}^2$

Probability of False Alarm: $PFA = 0.0$

As can be seen from the above parameters, experiment 2-1 corresponds to an ideal case where the target flies at a perfectly constant speed (i.e. no acceleration process noise) and where no false measurements are introduced in the tracking filter.

As such, experiment 2-1 constitutes a baseline test which establishes the ideal performance of the Nearest Neighbour Standard Filter, as applied to the SBR tracking problem. This experiment is to be compared with experiment 1-1 where the ideal performance of the NNSF was analyzed in the context of Scenario 1.

A batch of 50 runs was simulated. For 2 out of the 50 runs, the initial state estimate and the measurements introduced in the filter were such that the filter was not successful in tracking the target. Figure 5.3.1-1 displays the filter-derived target position RMS error along with the true target position RMS error averaged over the 48 runs of the batch where the filter was successfully tracking the target.

As indicated in section 5.3, the initial radar measurement covariance is smaller than the value observed in Scenario 1 experiments, since the range of the target from the satellite is smaller at high grazing angle. As a result, the initial RMS error illustrated in Figure 5.3.1-1 is now of the order of 3.5 km, instead of 9 km, as observed in experiment 1-1.

As more measurements are processed, the covariance decreases, since the confidence of the filter in the state estimate increases. The transient response of the filter is followed by a more steady response leading to an RMS error of the order of 1.6 km at scan # 60.

It is important to note that this value is larger than the value obtained in experiment 1-1 (i.e. 900 m). By contrast with Scenario 1, Scenario 2 is such that the filter measurement covariance increases with time, as illustrated in Figures 5.3.1-2 to 5.3.1-4, obtained from a single run of the batch. This is due to the fact that, as the grazing angle becomes smaller, the target range in the NED coordinate system increases.

Figure 5.3.1-1 Filter / True RMS error - Experiment 2-1

As can be seen from the derivation of the filter measurement covariance in section 4.2.7, an increase in target range leads to an increase in measurement covariance. This increase in measurement covariance causes the Kalman gain to decrease more rapidly in order to render the state estimate less susceptible to new measurement errors. As a result, the covariance of the estimate is larger than it would normally be, were the measurement covariance constant.

As described in experiment 1-1 within the context of Scenario 1, a comparison between the size of the validation gate and the size of the beam footprint can be made. Figure 5.3.1-5 displays the validation gate and footprint areas for each scan of the run under consideration, while Figure 5.3.1-6 provides the ratio of the validation gate area to the footprint area. As can be seen from Figure 5.3.1-6, the size of the validation gate is greater than the size of the footprint in the early stages of the simulation. After about 5 scans, the ratio drops to a value below 0.5 and keeps decreasing until the end of the simulation. These results indicate that the false measurements may be validated by the filter early on in the simulation. However, by contrast with Scenario 1, Scenario 2 is such that the beam footprint area is relatively small in the early stages of the simulation. Hence even with a high false alarm rate per range cell such as 10^{-2} , the number of false measurements present in the footprint at the beginning of the simulation will not be very large. This number will increase, however, to reach a high value by the end of the simulation. As a result, one can expect that the filter will achieve better performance than in the context of Scenario 1, where the number of false measurements was large in the early stages of the tracking process.

Figure 5.3.1-2 Filter Measurement Variance - Experiment 2-1

Figure 5.3.1-3 Filter Measurement Variance - Experiment 2-1

Figure 5.3.1-4 Filter Measurement Variance - Experiment 2-1

Figure 5.3.1-5 Validation Gate/Footprint Area - Experiment 2-1

Figure 5.3.1-6 Ratio of Validation Gate /Footprint Area - Experiment 2-1

5.3.2 Experiments 2-2 / 2-3

The purpose of these two experiments is to compare the performance of the NNSF with that of the PDAF in a high false alarm rate environment within the context of Scenario 2. As indicated in Section 5.3.1, it is anticipated that, due to the SBR / target geometry of Scenario 2, the NNSF will achieve better performance than that observed in the case of Scenario 1.

For both experiments, the false alarm rate per radar range cell *PFA* is equal to 10^{-2} . Due to the geometry of the scenario, false measurements only appear after scan #15 in the simulation.

A batch of 50 simulation runs was carried out in each experiment. The NNSF was successful in tracking the target 96% of the time (i.e. 48 runs out of 50). This is comparable with the results obtained in experiment 2-1 where no false measurements were present. The PDAF was successful in tracking the target 100% of the time.

Figure 5.3.2-1 and 5.3.2-2 display the filter-derived target position RMS error along with the true target position RMS error for the NNSF and the PDAF respectively. As noticed in experiment 1-6 where the performance of the PDAF was compared with that of the NNSF, the RMS error values obtained for the PDAF are larger than the values obtained for the NNSF.

By comparing the results of these experiments with the results obtained within the context of Scenario 1, it can be seen that the SBR / target geometry plays a significant role in the performance of the tracking filter.

Figure 5.3.2-1 Filter / True RMS error - Experiment 2-2

Figure 5.3.2-2 Filter / True RMS error - Experiment 2-3

Initiating tracking on a target at high grazing angle offers the following advantages:

- i) The target range in the NED coordinate system is smaller, hence the filter measurement covariance is smaller.
- ii) The beam footprint size is smaller, hence the number of false measurements introduced in the tracking process is smaller.

As a result of the above conditions, tracking can be performed in a relatively "clean" environment, as compared with the case where tracking is initiated at low grazing angle. Subsequently, the behaviour of the filter is such that the presence of a larger number of false measurements at lower grazing angles does not affect its performance. Because of the size of the validation gate as computed by the filter, very few of these false measurements will be validated, and therefore will have any effect on the track.

Chapter 6

Summary and Suggestions For Future Research

6.1 Summary and Conclusions

A theoretical study of the performance of two different tracking techniques as applied to the SBR application was conducted. These techniques, namely the Nearest Neighbour Standard Filter and the Probabilistic Data Association Filter were tested through the use of a computer simulation model in a set of experiments where the SBR / target geometry, the target process noise variance, and the clutter environment were allowed to vary.

From the simulation results presented in Chapter 5, the following conclusions can be drawn :

- i) The SBR / target geometry plays a significant role in the performance of the tracking filter.

When tracking is initiated at low grazing angle (e.g. 6°), the initial size of the radar footprint is very large: of the order of $16,000 \text{ km}^2$. As a result, the number of false measurements generated by the radar in a high false alarm rate environment is significant. In addition, the initial value of the filter covariance is large since the target range from the satellite is of the order of $5,000 \text{ km}$. Thus the validation gate of the track is such that false measurements present within the footprint are likely to be validated by the filter.

The situation is different when tracking is initiated at high grazing angle (e.g. 57°). The size of the radar footprint is considerably smaller: of the order of 380 km^2 . For a given false alarm rate environment, the number of false measurements generated by the radar will be much smaller than that in the case where tracking is initiated at low grazing angle. The initial target range from the satellite is also reduced: of the order of $2,000 \text{ km}$. As a result, the initial value of the filter covariance is smaller and less false measurements are likely to be validated by the filter.

From a pure tracking standpoint, it can be concluded that initiation of tracking at high grazing angle provides a more favourable set of conditions with respect to the presence of false measurements in the filter as compared with the case where tracking is initiated at low grazing angle.

However, from an SBR system perspective, initiating tracking at high grazing angles presents some limitations. First, the MDV limitation may be such that detection of the target is marginal, hence the target measurement may only be available a small fraction of the time for new updates. Second, because the field of view of the radar is much broader at low grazing angles, it is likely that the surveillance function will be carried out at low grazing angles in order to take advantage of the geometry. As a result, the initiation of tracking will likely take place when the radar illuminates the target at a low grazing angle. The tracking system must therefore be designed with the capability to accommodate the presence of a large number of false measurements as well as large errors in the initial target state estimates.

- ii) The clutter environment characterized by the false alarm rate per radar range cell plays a significant role in the performance of the tracking filter.

In a low false alarm environment where the maximum number of false measurements present within a given footprint is under 5, both NNSF and PDAF achieve adequate performance. On average, both filters are tracking the target with a success rate greater than 90%.

In a dense false alarm environment where the maximum number of false measurements present within a given footprint reaches a value of 20, differences in performance between the NNSF and the PDAF are observed.

When tracking is initiated at low grazing angle, the NNSF does not achieve adequate performance: the filter loses track on the target 40% of the time on average. The presence of a large number of false measurements in the validation gate of the track in the early stages of tracking affects the capability of the NNSF to track the real target. By contrast, the performance of the PDAF is not significantly affected: the filter successfully tracks the target 90% of the time on average. The capability of the PDAF to discriminate between true and false measurements based upon the different nature of their respective statistical distribution provides the means to achieve good tracking performance in a dense clutter environment.

When tracking is initiated at high grazing angle, the results observed from simulation runs are somewhat different. The SBR / target geometry is such that only a small number of false alarms is present in the early stages of tracking. As a result, tracking is initiated in a relatively "clean" environment, and both NNSF and PDAF achieve adequate performance. Because the size of the validation gate of the track becomes rapidly smaller than the radar footprint area, the presence of an increasing number of false measurements within the footprint does not significantly affect the NNSF.

- iii) The SBR / target geometry characterized by target range values of the order of several thousand kilometres largely influences the accuracy of the track estimates.

Typical track position estimate RMS errors are of the order of one km, once the filter has reached a steady phase.

These errors increase as more process noise is introduced in the target dynamics model. The presence of process noise in the target flight dynamics can lead to the loss of a track in the case where a large detection gap occurs. Extrapolation of the track in the absence of radar measurements significantly affects the tracking accuracy of both the NNSF and PDAF: RMS error can reach values from 5 to 10 km after a 2 min gap. This is a serious problem for the SBR application since large coverage gaps are likely to occur during normal operation. It has been shown in [20] that for a satellite orbit of 2,000 km, the nadir gap time which is that section of the orbit between the 60 degrees grazing angle extremes, is of the order of 5 minutes. During that period of time, the satellite cannot detect the target, hence extrapolation of the track is required. Depending upon the track accuracy requirements of the SBR system operation, and depending upon the behaviour of the target (i.e. manoeuvring capability), extrapolation of the state estimate over a long period of time may lead to the loss of the target track. It may then be necessary to resort to track initiation techniques in order to reacquire the target and initiate a new track on it.

6.2 Suggestions For Future Research

This study constitutes a preliminary attempt to quantify the performance of different tracking techniques in the context of an SBR system application. The results obtained must be expanded to the more complex cases where targets are allowed to manoeuvre and/or where several targets are in the same neighbourhood.

In this case, other techniques such as the Joint Probabilistic Data Association Filter (JPDAF), the Multiple Model Probabilistic Data Association Filter (MMPDAF), or the Multiple Hypothesis tracking (MHT) must be investigated. The analysis of tracking performance through the use of computer simulation techniques should also address the processes of track initiation and track correlation, in the case of a multi-satellite configuration. These theoretical investigations are required to thoroughly understand the complex issues involved in SBR tracking; they constitute the necessary steps leading to the design, development and deployment of an operational system.

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Appendix A

Association Probabilities in the PDAF

The conditioning of the association probability $P\{\Theta_i(k) \mid Z^k\}$ can be broken down into the past data Z^{k-1} , the latest data $Z(k)$ and the number of measurements m_k at time k :

$$\beta_i(k) = P\{\Theta_i(k) \mid Z(k), m_k, Z^{k-1}\}$$

Using Baye's rule, the above probability can be written as :

$$\beta_i(k) = P\{Z(k) \mid \Theta_i(k), m_k, Z^{k-1}\} P\{\Theta_i(k) \mid m_k, Z^{k-1}\} / C$$

where C is a normalization constant.

For $i \neq 0$, the joint density of the validated measurements conditioned on $\Theta_i(k)$ is the product of the assumed normal pdf of the correct measurement and the assumed uniform pdf of the false measurements.

For $i = 0$, the above joint density becomes the product of the uniform pdf of the false measurements.

The pdf of the correct measurement is given by:

$$P\{z_i(k) \mid \Theta_i(k), m_k, Z^{k-1}\} = N[\hat{z}(k \mid k-1), S(k)] / P_G$$

where:

$N[\hat{z}, S]$ is a Gaussian distribution with mean $\hat{z}$ and covariance S

$\hat{z}(k \mid k-1)$ is the predicted measurement at time k

$S(k)$ is the predicted measurement covariance at time k .

P_G is a factor that accounts for restricting the normal density to the validation gate (i.e. P_G is the probability that the measurement fall into the validation gate).

The pdf of the false measurements is $(1 / V_k)$

where V_k is the volume of the validation gate at time k .

The probability $P\{Z(k) \mid \Theta_i(k), m_k, Z^{k-1}\}$ can then be written as:

$$P\{Z(k) \mid \Theta_i(k), m_k, Z^{k-1}\} =$$

$$\begin{aligned} & (V_k)^{-m_k+1} N[\hat{z}(k \mid k-1), S(k)] / P_G & i=1, m_k \\ & (V_k)^{-m_k} & i=0 \end{aligned}$$

The probability of events $\Theta_i(k)$ conditioned on the number of validated measurements is given by:

$$P\{\Theta_i(k) \mid m_k, Z^{k-1}\} = P\{\Theta_i(k) \mid m_k\} = \alpha_i(m_k)$$

Denoting by $\phi(k)$ the number of false measurements at time k , one has:

$$\begin{aligned} \alpha_i(m_k) = & P\{\Theta_i(k) \mid \phi(k) = m_k - 1, m_k\} P\{\phi(k) = m_k - 1 \mid m_k\} \\ & + P\{\Theta_i(k) \mid \phi(k) = m_k, m_k\} P\{\phi(k) = m_k \mid m_k\} \end{aligned}$$

since $\phi(k)$ must be either $m_k - 1$ or m_k .

$$\alpha_i(m_k) = 1/m_k P\{\phi(k) = m_k - 1 \mid m_k\} \quad i=1, m_k$$

$$\alpha_i(m_k) = P\{\phi(k) = m_k \mid m_k\} \quad i=0$$

Using Baye's rule, the above probabilities can be rewritten as:

$$\begin{aligned} P\{\phi(k) = m_k - 1 \mid m_k\} &= \\ P\{m_k \mid \phi(k) = m_k - 1\} P\{\phi(k) = m_k - 1\} / P\{m_k\} \\ &= P_D P_G \mu_F(m_k - 1) / P\{m_k\} \end{aligned}$$

where:

P_D is the probability of detection

P_G is the probability that the correct measurement falls in the validation gate.

μ_F is the probability mass function (pmf) for the number of false measurements in the validation gate.

Similarly, one has:

$$\begin{aligned} P\{\phi(k) = m_k | m_k\} &= \\ P\{m_k | \phi(k) = m_k\} P\{\phi(k) = m_k\} / P\{m_k\} \\ &= (1 - P_D P_G) \mu_F(m_k) / P\{m_k\} \end{aligned}$$

$$P\{m_k\} = P_D P_G \mu_F(m_k - 1) + (1 - P_D P_G) \mu_F(m_k)$$

Given the above, the probabilities $\alpha_i(m_k)$ can be written as:

$$\text{For } i = 1, m_k, \quad \alpha_i(k) =$$

$$1/m_k P_D P_G [P_D P_G + (1 - P_D P_G) \mu_F(m_k) / \mu_F(m_k - 1)]^{-1}$$

$$\text{For } i = 0, \quad \alpha_i(k) =$$

$$\begin{aligned} (1 - P_D P_G) \mu_F(m_k) / \mu_F(m_k - 1) \\ [P_D P_G + (1 - P_D P_G) \mu_F(m_k) / \mu_F(m_k - 1)]^{-1} \end{aligned}$$

Two models can be used for the pmf μ_F :

- i) a Poisson model with a given spatial density λ :

$$\mu_F(m) = e^{-\lambda V} (\lambda V)^m / m!$$

where V is the volume of interest.

- ii) a diffuse prior:

$$\mu_F(m) = \delta, \text{ where } \delta \text{ is a constant}$$

Appendix B

Updated State Covariance in the PDAF

As per the Kalman filter, the expression of the covariance of the updated state is:

$$\begin{aligned} P(k | k) &= E\{[x(k) - \hat{x}(k | k)] [x(k) - \hat{x}(k | k)]' | Z^k\} \\ &= \sum_{i=0}^{m_k} E\{[x(k) - \hat{x}(k | k)] [x(k) - \hat{x}(k | k)]' | \Theta_i(k), Z^k\} \beta_i(k) \end{aligned}$$

since the association events $\Theta_i(k)$ are mutually exclusive and exhaustive.

$$x(k) - \dot{x}(k | k) = [x(k) - \dot{x}_i(k | k)] + [\dot{x}_i(k | k) - \dot{x}(k | k)]$$

$$E\{[x(k) - \dot{x}(k | k)] [x(k) - \dot{x}(k | k)]' | \Theta_i(k), Z^k\}$$

$$= E\{[\dot{x}(k | k) - \dot{x}_i(k | k)] [\dot{x}(k | k) - \dot{x}_i(k | k)]' | \Theta_i(k), Z^k\} \\ + [\dot{x}_i(k | k) - \dot{x}(k | k)] [\dot{x}_i(k | k) - \dot{x}(k | k)]'$$

$$\text{since } E\{[x(k) - \dot{x}_i(k | k)] | \Theta_i(k), Z^k\} = 0$$

By definition,

$$E\{[x(k) - \dot{x}_i(k | k)] [\dot{x}(k | k) - \dot{x}_i(k | k)]' | \Theta_i(k), Z^k\} \\ = P_i(k | k), \text{ the covariance of the updated state conditioned} \\ \text{on the event } \Theta_i(k)$$

$$P(k | k) = \sum_{i=0}^{m_k} \beta_i(k) P_i(k | k) \\ + \sum_{i=0}^{m_k} \beta_i(k) [\dot{x}_i(k | k) - \dot{x}(k | k)] [\dot{x}_i(k | k) - \dot{x}(k | k)]'$$

$$\text{Since } \dot{x}(k | k) = \sum_{i=0}^{m_k} \beta_i(k) \dot{x}_i(k | k),$$

$$\sum_{i=0}^{m_k} [\dot{x}_i(k | k) - \dot{x}(k | k)] [\dot{x}_i(k | k) - \dot{x}(k | k)]' \beta_i(k)$$

=

$$\sum_{i=0}^{m_k} \beta_i(k) \dot{x}_i(k | k) \dot{x}_i(k | k)' - \dot{x}(k | k) \dot{x}(k | k)'$$

$P(k | k)$ can then be written as:

$$P(k | k) = \dot{P}(k | k) + P^s(k), \quad \text{where:}$$

$$\dot{P}(k | k) = \sum_{i=0}^{m_k} \beta_i(k) P_i(k | k)$$

$$P^s(k) = \sum_{i=0}^{m_k} \beta_i(k) \dot{x}_i(k | k) \dot{x}_i(k | k)' - \dot{x}(k | k) \dot{x}(k | k)'$$

As per the standard Kalman filter, one has:

$$P_i(k | k) = P^c(k | k) = P(k | k-1) - W(k) S(k)^{-1} W(k)', \quad i=1, m_k$$

$$P_0(k | k) = P(k | k-1)$$

$$\text{Using the identity: } \beta_0(k) = 1 - \sum_{i=1}^{m_k} \beta_i(k),$$

$\dot{P}(k | k)$ becomes:

$$\dot{P}(k | k) = \beta_0(k) P(k | k-1) + (1 - \beta_0(k)) P^c(k | k)$$

$P^s(k)$ can be rewritten as follows:

$$\begin{aligned} P^s(k) &= \sum_{i=0}^{m_k} \beta_i(k) [\dot{x}(k | k-1) + W(k) v_i(k)] [\dot{x}(k | k-1) \\ &\quad + W(k) v_i(k)]' \\ &\quad - [\dot{x}(k | k-1) + W(k) v(k)] [\dot{x}(k | k-1) + W(k) v(k)]' \end{aligned}$$

$$\text{where: } v(k) = \sum_{i=0}^{m_k} \beta_i(k) v_i(k) = \sum_{i=1}^{m_k} \beta_i(k) v_i(k)$$

since $v_0(k) = 0$

$$\begin{aligned}
 P^s(k) &= \dot{\hat{x}}(k | k-1) \dot{\hat{x}}(k | k-1)' + \dot{\hat{x}}(k | k-1) [W(k) v(k)]' \\
 &+ W(k) v(k) \dot{\hat{x}}(k | k-1)' \\
 &+ \sum_{i=0}^{m_k} \beta_i(k) W(k) [W(k) v_i(k)]' \\
 &- \dot{\hat{x}}(k | k-1) \dot{\hat{x}}(k | k-1)' - \dot{\hat{x}}(k | k-1) [W(k) v(k)]' \\
 &- W(k) v(k) \dot{\hat{x}}(k | k-1)' - W(k) v(k) [W(k) v(k)]'
 \end{aligned}$$

$$P^s(k) = W(k) \left[\sum_{i=0}^{m_k} \beta_i(k) v_i(k) v_i(k)' - v(k) v(k)' \right] W(k)'$$

Appendix C

Analysis of a representative simulation run of the NNSF

The analysis of a single simulation run provides additional information regarding the performance of the filter. First, table C-1 provides the size of the validation gate at each scan of the run. The volume of the validation gate is given by:[22]

$$V(k) = (4\pi/3) g^{3\vee} | S(k) |$$

where:

$S(k)$ is the covariance of the predicted measurement at time k

$| S(k) |$ is the determinant of $S(k)$

g is the number of standard deviations of the gate ($g = 4 \sigma$).

As can be seen in table C-1, the size of the validation gate decreases very rapidly with time. This result is directly correlated with the behaviour of the filter covariance since the covariance of the predicted measurement is based upon the covariance of the predicted state as per the following equation:

$$S(k) = H(k) P(k | k-1) H(k)' + R(k)$$

where:

$P(k | k-1)$ is the covariance of the predicted state

$R(k)$ is the measurement covariance.

The result is also in accordance with the fact that, as the grazing angle increases, the target range from the satellite decreases, and as a result the covariance of the measurement decreases (refer to section 4.2.7), as illustrated in Figures C-1 to C-3.

883888.1
120813.9
24408.4
15255.1
26587.0
11838.0
16710.0
23552.4
32727.3
9408.5
6966.6
5978.6
5385.3
4988.7
4676.7
4420.5
4199.5
4537.0
4100.0
3804.7
3574.0
3381.0
3210.6
3058.7
2918.6
2787.9
2664.8
2733.2
2554.4
2408.0
2280.1
2164.8
2058.8
1959.9
1866.6

Table C-1

Validation Gate Size (km³) - Experiment 1-1

1777.8
1693.1
1612.0
1534.0
1458.8
1447.2
1359.8
1280.3
1206.7
1137.6
1072.4
1010.4
951.2
894.6
840.3
788.1
737.8
689.4
642.7
597.7
554.2
512.2
488.3
466.3
446.4
428.6
413.1
400.1
389.9
382.8
379.4
380.1
385.2
395.4
411.1

Table C-1

Validation Gate Size (km³) - Experiment 1-1
Cont'd

Table C-2 provides the normalized square innovation for each scan of the run. This represents the "statistical distance" of each new measurement from the predicted measurement and is given by:

$$[z(k+1) - \hat{z}(k+1 | k)]' S(k+1)^{-1} [z(k+1) - \hat{z}(k+1 | k)]$$

where:

$z(k+1)$ is the new measurement at time $k+1$

$\hat{z}(k+1 | k)$ is the predicted measurement at time $k+1$

$S(k+1)$ is the covariance of the predicted measurement at time $k+1$.

As can be seen from table C-2, each and every new measurement provided by the radar falls into the validation gate, since the normalized square innovation values are smaller than the gate size (i.e. 16). In addition, due to the value of 0.9 for the probability of detection, no measurements are available for scans 4, 6, 7, 8, 17, 27, 40. After scan 56, the grazing angle is such that detection of the target is not possible, hence no more measurements are available to the filter.

It is also of interest to compare the size of the validation gate with the size of the footprint for each scan of the run. However, these two parameters cannot be compared directly, since the validation gate size is a volume of space around the predicted measurement, defined by the following equation:

$$[z(k+1) - \hat{z}(k+1 | k)]' S(k+1)^{-1} [z(k+1) - \hat{z}(k+1 | k)] \leq g^2$$

where $g = 4$ (i.e. 4 sigma gate)

Figure C-1 Filter Measurement Variance - Experiment 1-1

Figure C-2 Filter Measurement Variance - Experiment 1-1

Figure C-3 Filter Measurement Variance - Experiment 1-1

Scan #	Measurement #	Chi-Square Value
1	1	0.8
2	1	7.9
3	1	0.8
5	1	0.3
9	1	12.3
10	1	3.7
11	1	3.7
12	1	3.2
13	1	5.1
14	1	4.3
15	1	1.1
16	1	3.3
18	1	8.2
19	1	3.2
20	1	1.5
21	1	3.3
22	1	3.1
23	1	4.8
24	1	3.2
25	1	4.6
26	1	0.2
28	1	1.9
29	1	1.4
30	1	1.3
31	1	1.4
32	1	4.6
33	1	2.3
34	1	0.2
35	1	0.1
36	1	2.2
37	1	2.4
38	1	2.7
39	1	4.7
41	1	3.3
42	1	1.4
43	1	1.3
44	1	0.04
45	1	2.9
46	1	0.7
47	1	0.9
48	1	10.6
49	1	3.8
50	1	2.9
51	1	2.2
52	1	0.9
53	1	0.6
54	1	1.1
55	1	2.0
56	1	2.4

Table C-2

Normalized Square Innovation - Experiment 1-1

One approach consists of deriving an "equivalent validation gate area" as follows:

Given the volume of the gate:

$$V_G = (4\pi/3) g^3 \sqrt{|S|}$$

the equivalent area S can be defined as:

$$S = \pi R^2$$

hence:

$$S_G = \pi(3V/4\pi)^{2/3}$$

The area of the footprint is given by:

$$S_F = (\pi/4) L W$$

where:

L is the footprint length

W is the footprint width

Figure C-4 displays the validation gate and footprint areas for each scan, while Figure C-5 provides the ratio of the validation gate area to the footprint area for each scan of the run.

As can be seen from Figure C-5, both areas are equivalent at the start of the simulation (i.e. ratio equal to 0.7), but the validation gate area decreases much more rapidly than the area of the footprint. After 5 scans, the ratio has dropped to 0.1; it then increases slowly to a value 0.3 at the end of the simulation. Thus one would forecast that, for this scenario, the presence of clutter returns (i.e. false measurements) in the footprint may degrade the performance of the filter early on in the simulation since a significant percentage of these false measurements may be validated by the filter.

Figure C-4 Validation Gate/Footprint Area - Experiment 1-1

Figure C-5 Ratio of Validation Gate/Footprint Area - Experiment 1-1

However, if the filter succeeds in tracking the target in the early stages of the simulation, the impact of the presence of false measurements during the following scans will be reduced, since on average, only a small fraction (i.e. 2 or 3 out of 10) of these measurements will fall into the validation gate.

Appendix D

Analysis of a representative simulation run of the PDAF

The analysis of a single simulation run provides additional information regarding the performance of the filter. First, table D-1 provides the normalized square innovation for each scan of the run. This represents the "statistical distance" of each new measurement from the predicted measurement. As can be seen from table D-1, each and every new measurement provided by the radar falls within the validation gate of the track, since the chi-square values are smaller than the gate size (i.e. 16).

In addition, due to the value of 0.9 for the probability of detection, no measurements are available for scans 4, 6, 7, 8, 17, 27, 40. After scan 56, the grazing angle is such that detection of the target is not possible, hence no more measurements are available to the filter.

Table D-2 provides the size of the validation gate at each scan of the run. By comparing the volume of the validation gate as given by table D-2 with the volume observed in experiment 1-1 for the NNSF, it can be noticed that the same general trend applies to both cases. The size of the validation gate decreases very rapidly with time in accordance with the filter covariance. The values obtained in this experiment are also higher than the values corresponding to the NNSF, since it has been observed that the covariance computed by the PDAF is always larger than the covariance computed by the NNSF.

Scan #	Measurement #	Chi-Square Value
1	1	0.8
2	1	7.6
3	1	0.8
5	1	0.3
9	1	11.0
10	1	2.9
11	1	2.3
12	1	3.1
13	1	4.1
14	1	4.3
15	1	0.7
16	1	3.4
18	1	7.8
19	1	2.8
20	1	1.1
21	1	2.8
22	1	3.1
23	1	4.4
24	1	2.8
25	1	4.4
26	1	0.2
28	1	2.7
29	1	1.3
30	1	1.2
31	1	1.9
32	1	5.1
33	1	2.1
34	1	0.1
35	1	0.2
36	1	2.1
37	1	2.6
38	1	2.6
39	1	4.7
41	1	3.4
42	1	1.4
43	1	1.1
44	1	0.03
45	1	3.4
46	1	0.8
47	1	1.1
48	1	11.2
49	1	4.3
50	1	3.5
51	1	1.7
52	1	1.2
53	1	0.3
54	1	1.4
55	1	2.3
56	1	2.2

Table D-1

Normalized Square Innovation - Experiment 1-6

883888.2
159078.5
219351.7
30388.8
64409.1
15020.5
23542.9
35695.5
52049.4
79464.6
14556.4
8044.6
6413.5
5712.0
5205.5
4761.9
4468.6
4876.2
4865.3
4241.9
3849.2
3592.1
3398.8
3238.8
3070.2
2935.7
2780.6
2880.2
2680.2
2504.7
2358.5
2233.8
2147.1
2033.4

Table D-2

Validation Gate Size (km³) - Experiment 1-6

1927.4
1829.4
1739.5
1656.3
1576.5
1505.0
1505.2
1410.6
1320.9
1239.9
1164.9
1098.6
1032.9
970.9
996.9
917.4
846.5
781.2
722.4
667.9
617.7
570.8
526.1
504.2
484.5
467.2
452.2
439.8
430.2
423.8
420.8
421.8
427.3
437.7
453.5
475.2

Table D-2

Validation Gate Size (km³) - Experiment 1-6
Cont'd

END

1|2|0|4|9|1

FIN