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Analytical Algorithms for Macromodeling and Sensitivity Analysis of High-Speed Interconnects

by

Natalie M. Nakhla

**A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Applied Sciences**

Ottawa-Carleton Institute for Electrical and Computer Engineering

Department of Electronics

Faculty of Engineering

Carleton University

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Abstract

With the continually increasing operating frequencies, signal integrity and interconnect analysis in high-speed designs is becoming increasingly important. Recently, several algorithms were proposed for macromodeling and transient analysis of distributed transmission line interconnect networks. Techniques such as the method of characteristics yield fast transient results for long delay lines. However, they do not guarantee the passivity of the macromodel. It has been demonstrated that preserving passivity of the macromodel is essential to guarantee a stable global transient simulation. On the other hand, methods such as matrix rational approximation provide efficient macromodels for lossy coupled lines, while preserving the passivity. However, for long delay lines, these methods require higher order approximations, making the macromodel inefficient.

To address the above difficulties, this thesis presents a new efficient algorithm for passive and compact macromodeling of distributed transmission lines. The new method employs delay extraction prior to approximating the exponential stamp of the line, while guaranteeing the passivity. Closed-form delay extraction-based equivalent circuits are developed, which are obtained analytically in terms of the per-unit-length parameters of the line and pre-determined coefficients. The new method enables fast transient simulation of both on-chip and off-chip interconnects. In addition, the delay extraction-based macromodeling technique has been extended to sensitivity analysis. Several practical test cases are presented to validate the accuracy and efficiency of the new algorithm.

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List of Symbols

$\mathbf{0}$	Zero matrix
Υ	Propagation constant of transmission line using MoC
γ	Propagation constant of lossless section of DEFACT cell
$\hat{\gamma}_i$	Propagation constant for i^{th} mode
Γ	Diagonal modal propagation matrix
Λ^2	Diagonal matrix of eigenvalues defined in section 4.3.1
λ	Interconnect parameter of interest
Ψ_k	Product term in modified Lie formulae (MLF)
α	Diagonal normalizing matrix
a_k	Predetermined coefficients in MRA approximation
A	Matrix formed by R and G in (2.2)
B	Matrix formed by L and C in (2.2)
$\hat{B}$	Extracted delay portion
$B_a(s)$	Remaining B matrix after delay extraction
C	Per-unit-length capacitance matrix of transmission line

C_m	Per-unit-length capacitance matrix of lossless section of DEFACT cell
$\hat{C}$	Diagonal capacitance matrix for lossless DEFACT sections
C_π	Matrix of memory elements
C_∞	Infinite frequency asymptotic value of capacitance in MoC formulation for lossy lines
d	Length of the transmission line
D	Selector matrix with entries 0 or 1
ε_m	MLF error
E_I, E_V	Modal transformation matrices for lossless sections
E_{A1}, E_{B1}	Voltage controlled voltage sources for decoupling lines
E_1, E_2	Independent voltage sources used in sensitivity analysis
f	Frequency
f_{max}	Maximum frequency of interest
F^2	Diagonal matrix of eigenvalues of C_m
F_{A1}, F_{B1}	Current controlled current sources used for decoupling lines
G	Per-unit-length conductance matrix of transmission line
G_m	Per-unit-length conductance matrix of lossy DEFACT section

$\hat{G}$	Diagonal conductance matrix of lossy section of DEPACT cell
G_{π}	Matrix of memoryless elements
G_{ϕ}	Matrix composed of memoryless elements of subnetwork ϕ
$i(x, t)$	Current vector of distributed transmission line
i_1, i_2	Near and far-end current vectors of DEPACT sections
I_0, I_d	Near and far-end current vectors of transmission line in MoC formulations
$I_{\pi}(s)$	Laplace terminal current vector of interconnect subnetwork π in sensitivity analysis description
$\hat{i}(x, t)$	Modal current vector
L	Per-unit-length inductance matrix of transmission line
$L_e(s)$	Portion of inductance matrix which is due to the magnetic field external to the conductor
$L_i(s)$	Portion of inductance matrix which is due to the internal magnetic field
L_{∞}	Infinite frequency asymptotic value of inductance in MoC formulation for lossy lines
L_m	Per-unit-length inductance matrix of lossless section of DEPACT cell
$\hat{L}$	Diagonal inductance matrix for lossless DEPACT sections
m	DEPACT approximation order

M	Number of segments used to represent transmission line in lumped segmentation
N	Number of coupled lines
$P_{n,n}(Z)$	Polynomial matrix in MRA approximations
P_I, P_V	Transformation matrices for lossy sections
p_i, q_i	Coefficients for polynomial matrices in MRA approximation
$P(s)$	Rational function corresponding to delayless propagation operator in MoC formulation
Q	Matrix of right-eigenvectors of C_m
$Q_{n,n}(Z)$	Polynomial matrix in MRA approximations
R	Per-unit-length resistance matrix of transmission line
R_m	Per-unit-length resistance matrix of lossy DEFACT section
$\hat{R}$	Diagonal resistance matrix of lossy section of DEFACT cell
r_a, r_b	Resistors used to realize frequency-independent lossy sections
s	Laplace variable ($j\omega$)
s_{max}	Laplace variable corresponding to f_{max}
S	Matrix of right-eigenvectors of product defined in section 4.3.1
t	Time
τ	Delay of transmission line

τ_i	Delay for i^{th} mode
t_r	Rise time of propagating signal
U	Unity matrix
$v(x, t)$	Voltage vector of distributed transmission line
v_1, v_2	Near and far-end voltage vectors of DEPACT sections
V_0, V_d	Near and far-end voltage vectors of transmission line in MoC formulations
$V_\pi(s)$	Laplace terminal voltage vector of interconnect subnetwork π in sensitivity analysis description
v	Velocity
w	Wavelength
x	Position variable
Δx	Length of segment used in conventional lumped segmentation
X_k	Matrix term in BCH series
x_π	Vector of unknown voltages and currents in description of unified sensitivity equations
$Y(s)$	Admittance matrix of transmission lines
$\hat{Y}(s)$	Diagonal matrix containing admittance matrices of decoupled lines
$Y_\pi(s)$	Y-parameter matrix representing interconnect subnetwork π in sensitivity analysis description
Z	Matrix of transmission line parameters defined in (2.2)

Z_c	Characteristic impedance of transmission line in MoC formulations
$\hat{Z}_i$	Characteristic impedance for the i^{th} mode
Z_0	Impedance of lossy section of DEFACT cell (4.36)

Abbreviations

BCH	Baker-Campbell-Hausdorff
CPU.....	Central Processing Unit
DEPACT.....	Delay Extraction-Based Passive Compact Transmission Line Macromodeling Algorithm
MCM.....	Multichip Module
MLF	Modified Lie Formula
MNA.....	Modified Nodal Analysis
MoC	Method of Characteristics
MRA.....	Matrix Rational Approximation
MTL	Multiconductor Transmission Line
PCB.....	Printed Circuit Board
PR	Positive Real
p.u.l.	Per-unit-length
SPICE	Simulation Program with Integrated Circuits Emphasis
TL	Transmission Line

CHAPTER 1

Introduction

1.1 Background and Motivation

The rapid increase in operating speeds, density and complexity of modern integrated circuits has made interconnect analysis a requirement for all state-of-the-art circuit simulators. Interconnect effects such as ringing, signal delay, distortion and crosstalk can severely degrade signal integrity. Interconnects are present at various levels of design hierarchy, such as on-chip, packaging structures, MCMs, PCBs, backplanes and cables. This warrants different modeling strategies based on the hierarchy and the operating frequency.

At relatively lower frequencies, the interconnect can be modeled using lumped elements, such as resistors, capacitors and inductors. However, as the frequency of operation increases, the interconnect lengths become a significant fraction of the operating wavelength, and conventional lumped models become inadequate in describing the interconnect performance and transmission line models become

necessary. Skin and proximity effects also become prominent at higher frequencies and distributed models with frequency-dependent parameters may be needed [1].

The major difficulty usually encountered while linking distributed transmission lines and nonlinear simulators is the problem of mixed frequency/time [1]-[3]. This is because distributed elements are best characterized in the frequency-domain, whereas nonlinear components such as drivers and receivers are represented generally in the time-domain. Macromodeling techniques attempt at solving this problem by representing the interconnect in the form of a set of ordinary differential equations, which can then be directly linked to nonlinear circuit simulators (such as SPICE). The goal of these techniques is to develop a general model (referred to here as the macromodel) suitable for nonlinear simulation, which enables the circuit simulator to view the distributed interconnect as a regular circuit component.

Several publications can be found in the literature which try to address the issues described above [4]-[17]. One approach, referred to as conventional lumped segmentation provides a brute force solution to the problem of mixed frequency/time simulation. However, these type of methods lead to large circuit matrices, rendering the simulation inefficient [2].

There are other more advanced methods in the literature developed for interconnect analysis. These approaches can be classified into two main groups: method of characteristics (MoC) based algorithms [2], [3], [6], [18], [19] and matrix rational approximations (MRA) [10], [20], [21]. Algorithms based on MoC were first developed in the 1960's and are commonly used in today's commercial simulators, such as HSPICE [22]. These methods provide fast solutions for long low-loss lines,

and ensure the transmission line causality [3] (which implies that an excitation signal entering one end of a transmission line segment will appear at the other end only after the time-of-flight delay). However, although MoC based algorithms are ideal for lossless lines, they have difficulty modeling lossy interconnects. In addition, one of the main disadvantages of MoC approaches is that they do not guarantee the passivity of the resulting macromodels. Passivity of the macromodel is essential, since non-passive (even if they are stable) models when coupled with arbitrary nonlinear elements, can lead to unstable systems [20].

On the other hand, algorithms based on matrix rational approximations (MRA) [10], [20], [21], [23] guarantee the macromodel passivity. The MRA technique is based on approximating the exponential stamp of the transmission line with closed-form rational functions using pre-determined coefficients, which are computed as a function of the per-unit-length (p.u.l.) parameters of the line. Using these coefficients, a transmission line macromodel is derived in the form of ordinary differential equations that can be easily linked to nonlinear circuit simulators. MRA derives its accuracy and efficiency from the closed-form nature of the pre-determined coefficients (which avoids computationally expensive and sometimes unreliable numerical fitting algorithms associated with the MoC, particularly for MTLs with a large number of coupled lines). However, for long lines, MRA requires high-order approximations (since it tries to accurately capture the flat portion corresponding to the delay of the line), which leads to inefficient transient simulation [18]. This limits its usefulness to short lines (on-chip interconnects).

Moreover, accurate and efficient sensitivity analysis with respect to transmission line parameters is often required by designers in order to make the proper trade-

offs, generally between conflicting design requirements, to obtain the best possible performance. Sensitivity analysis is of significant importance in identifying critical design components, in tolerance assignments and in optimizing the overall interconnect network performance. Current sensitivity analysis methods are in general, inefficient, since they either suffer from accuracy issues or require excessive CPU time.

The goal of this thesis is to develop a new algorithm for passive macromodeling and sensitivity analysis of transmission line subnetworks which addresses the shortcomings of the current techniques. In specific, the contributions of the thesis are listed below.

1.2 Contributions

The subject of this thesis is passive macromodeling and sensitivity analysis of high-speed interconnect networks. The algorithm presented here provides an accurate and efficient approach for developing delay extraction-based passive compact transmission line macromodels (henceforth referred to as DEFACT macromodels). The main contributions of this thesis are as follows.

- 1) An accurate and efficient algorithm is developed for the macromodeling of high-speed interconnects [24]-[27]. The new method combines the merits of both the MoC and MRA algorithms. It employs an advanced delay extraction mechanism with a solid mathematical basis (chapter 3). The new time-domain macromodel (chapter 4) consists of a cascade of lossless and lossy sections which can be directly linked to nonlinear circuit simulators. In addition, equivalent circuit representations for these sections are derived. The new method leads to fast transient simulation

for both on-chip and off-chip interconnects.

- 2) A theoretical relationship between the realization order of DEPACT and the approximation error is given (chapter 3). This helps in selecting the appropriate order for the DEPACT macromodel for a given frequency bandwidth of interest [25].
- 3) A simplified passive macromodeling algorithm for the case of transmission lines with frequency-independent p.u.l. parameters is developed (chapter 4) [27].
- 4) A proof of passivity is derived (chapter 5) which demonstrates that the delay extraction-based macromodel is guaranteed to be passive by construction [25].
- 5) A new sensitivity analysis method is developed (chapter 6) which is based on the proposed delay extraction macromodeling technique [28]-[31]. A major advantage of the new sensitivity analysis algorithm is that the sensitivity of the circuit matrices is obtained analytically in terms of the p.u.l. parameters of the line and pre-determined coefficients, leading to significant computational cost advantages. In addition, sensitivity analysis can be performed with respect to both electrical and physical parameters. Equivalent circuit representations are derived, which can be directly interfaced with SPICE-like circuit simulators.
- 6) The proposed method has been extensively evaluated by the advanced circuit simulation group at the IBM T. J. Watson Research Center. Due to the positive benchmark assessment, the method was implemented by the IBM development group in their internal circuit simulator 'Power-Spice'.

1.3 Organization of the Thesis

The thesis is organized as follows. Chapter 2 presents the problem formulation and reviews some of the relevant techniques for macromodeling. Chapter 3 describes the mathematical basis behind delay extraction and its application for accurately approximating the transmission line. Chapter 4 provides a detailed description of the circuit realization of the new delay extraction-based macromodel. Various numerical examples which validate the accuracy and efficiency of the new method are also presented in chapter 4. In chapter 5, an overview of passivity is given, as well as a proof of passivity for the developed macromodel. A new sensitivity analysis method which uses the proposed delay extraction-based macromodel is introduced in chapter 6. Numerical examples which validate the new sensitivity analysis method are also found in chapter 6. Finally, chapter 7 presents conclusions and proposed future research.

CHAPTER 2

Overview of Previous Macromodeling Techniques

The simulation of high-speed interconnects requires selecting a suitable model and solution algorithm. The type of model and algorithm used depends on the application. Moreover, various techniques offer different challenges to circuit simulators. This chapter reviews some of the relevant methods used in the literature for interconnect analysis. The chapter is organized as follows. Section 2.1 presents the problem formulation. Sections 2.2, 2.3 and 2.4 describe three relevant techniques: conventional lumped segmentation, the method of characteristics (MoC) and matrix rational approximation (MRA), respectively. Finally, section 2.5 provides a comparison between MoC and MRA, which serves as a basis for the new method described in this thesis.

2.1 Problem Formulation

Consider the distributed transmission lines shown in Fig. 2.1. The transmission lines can be represented by Telegrapher's equations [2] as

$$\begin{aligned}\frac{\partial}{\partial x} \mathbf{v}(x, t) &= -\mathbf{R}\mathbf{i}(x, t) - \mathbf{L}\frac{\partial}{\partial t} \mathbf{i}(x, t) \\ \frac{\partial}{\partial x} \mathbf{i}(x, t) &= -\mathbf{G}\mathbf{v}(x, t) - \mathbf{C}\frac{\partial}{\partial t} \mathbf{v}(x, t)\end{aligned}\tag{2.1}$$

where $\mathbf{R} \in \mathbb{R}^{N \times N}$, $\mathbf{L} \in \mathbb{R}^{N \times N}$, $\mathbf{C} \in \mathbb{R}^{N \times N}$ and $\mathbf{G} \in \mathbb{R}^{N \times N}$ are the per-unit-length (p.u.l) parameter matrices of the transmission line, $\mathbf{v}(x, t) \in \mathbb{R}^N$, $\mathbf{i}(x, t) \in \mathbb{R}^N$ represent the voltage and current vectors of the transmission line as a function of position x and time t , respectively, and N is the number of coupled lines.

Fig. 2.1. Multiconductor transmission line

Next, the solution of (2.1) can be expressed in the matrix-exponential form, in the frequency-domain as

$$\begin{aligned} \begin{bmatrix} V(d, s) \\ -I(d, s) \end{bmatrix} &= e^Z \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix}; \\ Z &= (A + sB); \quad A(s) = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix} d; \quad B(s) = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix} d \end{aligned} \quad (2.2)$$

In order to include transmission lines in nonlinear circuit simulators, (2.2) must be expressed in the time-domain. However, the challenge here is that, the exponential matrix e^{A+sB} in (2.2) does not have an easy direct representation in the time-domain, and is best characterized in the frequency-domain, thus making it difficult to interface with nonlinear circuit simulators. This problem is referred to in the literature as the ‘mixed frequency/time’ problem.

To overcome this problem, the Telegrapher’s equations, which are in the form of partial differential equations, must be transformed into ordinary differential equations, which can then be linked to nonlinear simulators [4]. Approximating the exponential stamp of the line, in the form of a set of ordinary differential equations is referred to as ‘macromodeling’, and the new circuit which represents the transmission line is referred to as the ‘macromodel’. Several publications can be found in the literature which try to address the problem of mixed frequency/time [4]-[17]. Among the most relevant techniques are: conventional lumped segmentation [2], the method of characteristics (MoC) [6], [19], [32] and Matrix Rational Approximation (MRA) [9], [10], [20], [21], [23]. The following sections give a brief overview of these methods.

2.2 Conventional Uniform Lumped Segmentation

In this conventional approach, lumped equivalent circuits are used to represent the transmission line. In this method, the partial differential equations in (2.1) are approximated by the following difference equations [2]

$$\begin{aligned} v_{k+1}(t) - v_k(t) &\cong -R\Delta x i_k(t) - L\Delta x \frac{\partial i_k(t)}{\partial t} \\ i_{k+1}(t) - i_k(t) &\cong -G\Delta x v_k(t) - C\Delta x \frac{\partial v_k(t)}{\partial t} \end{aligned} \quad (2.3)$$

where the line is divided into M segments of length Δx , and where Δx is defined as

$$\Delta x = d/M \quad (2.4)$$

and $k=1, 2, \dots, M$. If each of these segments is electrically small at the frequencies of interest, (i.e. $\Delta x \ll w$, where $w = v/f$ is the wavelength, v is the velocity and f is the frequency of interest), then each segment can be replaced by lumped models. Fig. 2.2 shows a general lumped model used to discretize a single transmission line, where R , L , C and G are the p.u.l. parameters.

It is often of practical interest to know how many lumped segments (M) are required to accurately represent the distributed transmission line. A general rule of thumb is to use the following relation [4]

$$M \geq 10\tau/t_r \quad (2.5)$$

where $\tau = d\sqrt{LC}$ is the delay of the line, and t_r is the rise time of the propagating signal.

This can also be extended for the case of coupled transmission lines [2], where lumped equivalent circuits are used to approximate the partial differential equations represented by (2.1).

Fig. 2.2. Lumped transmission line model

Although conventional lumped segmentation provides a direct approach to solving the problem of mixed frequency/time, the approximation is only valid if Δx is chosen to be a small fraction of the wavelength w . If the frequency of interest is high, or if the interconnect is electrically long (i.e. $d > 0.1w$), many lumped segments are required. This leads to large circuit matrices, rendering the simulation inefficient [2]. In addition, conventional lumped segmentation has difficulty when

trying to handle a large number of coupled lines, or lines with frequency-dependent parameters.

2.3 Method of Characteristics (MoC)

The method of characteristics (MoC) transforms the partial differential equations of the transmission line into ordinary differential equations. Consider the Telegrapher's equations for a single transmission line in the frequency-domain as

$$\begin{aligned}\frac{\partial}{\partial x} V(x, s) &= -(R + sL)I(x, s) \\ \frac{\partial}{\partial x} I(x, s) &= -(G + sC)V(x, s)\end{aligned}\tag{2.6}$$

An analytical solution, in terms of the y -parameters for (2.6) can be derived as [4]

$$\begin{bmatrix} I_0 \\ I_d \end{bmatrix} = \frac{1}{Z_c(1 - e^{-2\Upsilon d})} \begin{bmatrix} 1 + e^{-2\Upsilon d} & -2e^{-\Upsilon d} \\ -2e^{-\Upsilon d} & 1 + e^{-2\Upsilon d} \end{bmatrix} \begin{bmatrix} V_0 \\ V_d \end{bmatrix}\tag{2.7}$$

where the propagation constant Υ and the characteristic impedance Z_c are defined as

$$\Upsilon = \sqrt{(R + sL)(G + sC)} \quad Z_c = \sqrt{\frac{R + sL}{G + sC}}\tag{2.8}$$

and where V_0 , I_0 and V_d , I_d represent the near and far-end voltages and currents of the line, respectively.

The y -parameters in (2.7) are a complex function of frequency, and in most cases, cannot be directly transformed into ordinary differential equations in the time-

domain. However, MoC succeeded in doing such a transformation, but only for lossless lines [19]. The terms in (2.7) can be re-arranged as follows

$$\begin{aligned} V_0 &= Z_c I_0 + e^{-\Upsilon d} [2V_d - e^{-\Upsilon d} (Z_c I_0 + V_0)] \\ V_d &= Z_c I_d + e^{-\Upsilon d} [2V_0 - e^{-\Upsilon d} (Z_c I_d + V_d)] \end{aligned} \quad (2.9)$$

Next, (2.9) can be re-written as

$$\begin{aligned} V_0 - Z_c I_0 &= W_1 \\ V_d - Z_c I_d &= W_2 \end{aligned} \quad (2.10)$$

where

$$\begin{aligned} W_1 &= e^{-\Upsilon d} [2V_d - e^{-\Upsilon d} (Z_c I_0 + V_0)] \\ W_2 &= e^{-\Upsilon d} [2V_0 - e^{-\Upsilon d} (Z_c I_d + V_d)] \end{aligned} \quad (2.11)$$

Using (2.9)-(2.11), a recursive relationship for W_1 and W_2 can be expressed as

$$\begin{aligned} W_1 &= e^{-\Upsilon d} [2V_d - W_2] \\ W_2 &= e^{-\Upsilon d} [2V_0 - W_1] \end{aligned} \quad (2.12)$$

If the transmission line is lossless, the propagation constant and characteristic impedance are given by

$$\Upsilon = s\sqrt{LC} \quad Z_c = \sqrt{\frac{L}{C}} \quad (2.13)$$

This results in Υ being purely imaginary and Z_c a real constant. Thus, (2.12) can be written in the time-domain by replacing $e^{-\Upsilon d}$ with a time delay as

$$\begin{aligned}
 w_1(t + \tau) &= 2v_d(t) - w_2(t) \\
 w_2(t + \tau) &= 2v_0(t) - w_1(t)
 \end{aligned}
 \tag{2.14}$$

where $\tau = d\sqrt{LC}$. In this case, the transmission line can be modeled using lumped impedances and time delay voltage controlled voltage sources. An equivalent circuit using MoC for a lossless single line is shown in Fig. 2.3. Thus, for lossless lines, the transmission line model can be directly linked to nonlinear circuit simulators.

Fig. 2.3. MoC interconnect model for single lossless line

For lossy lines, the propagation constant is not purely imaginary and thus, cannot be replaced with a time delay. In this case, analytical expressions for $w_1(t)$ and $w_2(t)$ cannot be found in the time-domain. To address this issue, several numerical techniques have been proposed in the literature to approximate Υ and Z_c (in (2.8)) in terms of rational functions in the frequency-domain [3], [6], [32]. To illustrate, consider the function $H(s) = e^{-\Upsilon d}$ in (2.12). MoC represents $H(s)$ by extracting the delay given by $\tau = d\sqrt{LC}$, and approximating the remaining portion with a rational function [32], i.e.

$$\begin{aligned}
 H(s) &= e^{-\Upsilon d} \\
 &\cong e^{-s\tau} P(s)
 \end{aligned}
 \tag{2.15}$$

where the rational function $P(s)$ in (2.15) corresponds to the delayless propagation operator.

In the case of lines with frequency-dependent parameters, the line delay in (2.15) is defined by the infinite frequency asymptotic values of inductance and capacitance as [32]

$$\tau = d\sqrt{L_\infty C_\infty} \tag{2.16}$$

The MoC formulations described above for single lines can be easily extended for the case of coupled transmission lines [2], [32].

One of the main disadvantages of MoC is that the resulting macromodels are not guaranteed to be passive. Passivity is essential, since nonpassive but stable macromodels when coupled with arbitrary nonlinear elements can lead to unstable systems (details of passivity will be described in chapter 5). In addition, the numerical fitting algorithms used by MoC can be computationally expensive and sometimes unreliable, particularly for a large number of coupled lines.

2.4 Matrix Rational Approximation (MRA)

In order to address the issue of passivity, passive matrix rational approximation (MRA) was recently suggested [20], [21], [23]. MRA approximates the exponential stamp of the line in (2.2) (also referred to as the hybrid parameters) with closed-form rational functions using pre-determined coefficients. Using MRA, the exponential matrix in (2.2) is expressed as

$$P_{N_1, N_2}(Z) e^Z \approx Q_{N_1, N_2}(Z) \quad (2.17)$$

where $P_{N_1, N_2}(Z)$, $Q_{N_1, N_2}(Z)$ are polynomial matrices expressed in terms of closed-form Padé rational functions [9], defined as

$$P_{N_1, N_2}(Z) = \sum_{j=0}^{N_1} \frac{(N_1 + N_2 - j)!(N_1)!}{(N_1 + N_2)!j!(N_1 - j)!} (-Z)^j \quad (2.18)$$

$$Q_{N_1, N_2}(Z) = \sum_{j=0}^{N_2} \frac{(N_1 + N_2 - j)!N_2!}{(N_1 + N_2)!j!(N_2 - j)!} (Z)^j \quad (2.19)$$

Using $Z = (A + sB)$, and letting $N_1 = N_2 = n$ (in order to preserve the macromodel passivity [9]), the exponential matrix in (2.2) can be expressed as

$$P_{n, n}(Z) e^{(A + sB)} \approx Q_{n, n}(Z) \quad (2.20)$$

where $P_{n, n}(s)$, $Q_{n, n}(s)$ are polynomial matrices, given by

$$\begin{aligned} P_{n, n}(Z) &= \sum_{i=0}^n p_i s^i \\ Q_{n, n}(Z) &= \sum_{i=0}^n q_i s^i \end{aligned} \quad (2.21)$$

The Padé rational functions in (2.20), can also be represented in an alternative form in terms of subsections, described by pole-zero pairs as follows

$$\begin{aligned}
[P_{n,n}(Z)]^{-1}Q_{n,n}(Z) &= \prod_{i=1}^{n/2} [P_{n,n}(Z)_i]^{-1}[Q_{n,n}(Z)_i] \quad (2.22) \\
&= \prod_{i=1}^{n/2} [(a_i U - Z)(a_i^* U - Z)]^{-1}[(a_i U + Z)(a_i^* U + Z)]
\end{aligned}$$

for even values of n and

$$\begin{aligned}
[P_{n,n}(Z)]^{-1}Q_{n,n}(Z) &= \prod_{i=0}^{(n+1)/2} [P_{n,n}(Z)_i]^{-1}[Q_{n,n}(Z)_i] \\
&= [a_0 U - Z]^{-1}[a_0 U + Z] \left[\prod_{i=1}^{(n-1)/2} [(a_i U - Z)(a_i^* U - Z)]^{-1} \right] \quad (2.23) \\
&\quad \times [(a_i U + Z)(a_i^* U + Z)]
\end{aligned}$$

for odd values of n . Here, U represents the unity matrix, and a_k ; $k \geq 0$ are pre-determined coefficients [10]. The symbol $*$ represents the complex conjugate operation.

The above approximations are formulated analytically in terms of p.u.l. parameters and constants (i.e. p_i and q_i). These constants are obtained *a priori*, using methods such as Padé approximation or minimax optimization [20], while ensuring macromodel passivity. Using (2.20)-(2.23), (2.2) can be translated into a set of ordinary differential equations, which can be directly linked to nonlinear simulators [4], [33]. One of the main advantages of MRA, is that it is passive by construction. As previously mentioned, passivity is essential for macromodeling of high speed interconnects. In addition, due to its closed-form nature, MRA avoids computationally expensive numerical fitting algorithms associated with the MoC.

2.5 MoC vs. MRA

The accuracy of the MRA approximation depends on the norm of the term $A + sB$, which is a function of the length d . In particular, the smaller the value of d , i.e. the shorter the line, the better the approximation. For large delay lines (e.g. long lines with small losses), high-order approximations are required, which lead to inefficient transient simulation [18]. To illustrate this point, consider the following single line circuit [34].

Fig. 2.4. Single line circuit

The circuit in Fig. 2.4 was simulated using both MRA and MoC. The results from both methods were in good agreement, however, the CPU time varied depending on the length of the line, as shown in Table 2.1.

Table 2.1: CPU Time Comparison

	MoC (sec)	MRA (sec)	Speed-up of MRA
$d=0.5$ cm	2.1	0.19	11x faster
$d=10$ m	4	463	100x slower

Computer: INTEL P4 2GHz CPU

Observing Table 2.1, we can see that for short lines, MRA is fast and efficient. However, as the length of the line is increased, for example $d=10$ m, MRA was more than 100 times *slower* than MoC. The reason for this can be seen in the time-domain response shown below.

Fig. 2.5. Time-domain response (far-end) for single line circuit

The plot in Fig. 2.5 shows that there is a long delay present in the time-domain response. Since MRA tries to accurately capture this flat delay portion, it requires a rational function approximation with a high order, which in turn, results in inefficient transient simulation.

To demonstrate the effect of the presence of the delay, Fig. 2.6 shows the real and imaginary parts of the propagation operator $H(s)$, (defined in (2.15)), for the line shown in Fig. 2.4. It is obvious from the plot, that approximating this function would require a very high order (due to the rapid oscillatory nature of the response). On the other hand, Fig. 2.7 shows the real and imaginary parts of the propagation operator after delay extraction. As can be seen, approximating such a waveform would require a rational function of a much lower order.

Fig. 2.6. Real and imaginary parts of $H(s)$ for the single TL in Fig. 2.4

Fig. 2.7. Real and imaginary parts of $H(s)$ after delay extraction

Hence, it is evident that, although MRA guarantees the passivity of the macromodel, and derives its accuracy and efficiency from the closed-form nature of the pre-determined coefficients, it does not employ any type of delay extraction prior to performing the rational function approximation. Thus, in the case of long lines, MRA requires high-order approximations, due to the presence of the delay. This limits its usefulness to short lines (on-chip interconnects). On the other hand, MoC does employ a delay extraction mechanism, however, it does not guarantee the passivity of the resulting macromodel. In addition, the numerical fitting techniques used by MoC can be computationally expensive and in some cases unreliable.

To address these issues, the delay extraction-based method proposed in this thesis combines the merits of both the MoC and MRA algorithms. It employs an efficient mechanism for delay extraction prior to performing MRA. The new method results in significantly lower order macromodels for long as well as short lossy coupled lines, which leads to fast transient simulation for both on-chip and off-chip interconnects [24]-[27].

2.6 Summary

In summary, this chapter presented an overview of some of the relevant techniques used for macromodeling of high-speed interconnects. Conventional lumped segmentation, the method of characteristics (MoC) and matrix rational approximation (MRA) were discussed. Examining the merits of these techniques emphasized the difficulties of current algorithms in the literature and the need for a new method that satisfies the accuracy, passivity and efficiency requirements. This new technique is the subject of the following chapters.

CHAPTER 3

Development of the Proposed Macromodel

3.1 Introduction

As discussed in the previous chapter, in the presence of long delay lines, matrix rational approximation (MRA) based approaches require high-order approximations in order to accurately capture the flat delay portion. Hence it is essential to extract the delay from the exponential matrix prior to performing MRA. However, the challenging task here, is that this has to be achieved while preserving the passivity of the macromodel. This chapter provides efficient solutions to the above problem and introduces the concept of passive delay extraction. The chapter is organized as follows. Section 3.2 presents the main concepts of delay extraction, including a detailed description of the mathematical basis involved. Section 3.3 derives an improved approximation for long lines. Finally, sections 3.4 and 3.5 describe other implementation aspects of the proposed delay extraction-based method, such as the application of the new method to lines with frequency-dependent parameters, and estimating the order of approximation, respectively.

3.2 Delay Extraction

Examining the terms in (2.2), it can be noted that the matrix B includes the inductance and capacitance parameters, hence, the delay portion is essentially contributed by B . However, extracting the delay from $e^{(A+sB)}$ is not a simple task, due to the fact that the matrices A and sB do not commute, i.e. $e^{(A+sB)} \neq e^A e^{sB}$. To illustrate this fact, consider a single transmission line with the following p.u.l. parameters (henceforth referred to as Line-1)

Fig. 3.1. An example transmission line (Line-1)

At a given frequency $f=2\text{GHz}$, we have

$$e^{(A+sB)} = \begin{bmatrix} -0.6715 - 0.9879i & 41.7748 + 63.878i \\ 0.0036 + 0.0262i & -0.6715 - 0.9879i \end{bmatrix}$$

$$\neq$$

$$e^A e^{sB} = \begin{bmatrix} -0.5445 - 1.6435i & 54.449 + 42.8072i \\ 0.0000 + 0.0164i & -0.5445 - 0.0000i \end{bmatrix} \quad (3.1)$$

As seen from (3.1), $e^{(A+sB)} \neq e^A e^{sB}$. This is due to the fact that the individual matrices A and sB do not commute, i.e.

$$\begin{aligned}
A \times sB &= \begin{bmatrix} 0 + 8.1053i & 0 \\ 0 & 0 + 5.2779 \times 10^{-6}i \end{bmatrix} \\
&\neq \\
sB \times A &= \begin{bmatrix} 0 + 5.2779 \times 10^{-6}i & 0 \\ 0 & 0 + 8.1053i \end{bmatrix}
\end{aligned} \tag{3.2}$$

From the example shown above, it was seen that, in order to extract the delay, which essentially implies separating the matrices A and sB , the product $e^A e^{sB}$ cannot be used to represent $e^{(A+sB)}$. Hence, it is required to find an accurate approximation of $e^{(A+sB)}$ in terms of another product of exponential matrices. The proposed delay extraction mechanism is based on the Baker-Campbell-Hausdorff series.

3.2.1 Baker-Campbell-Hausdorff Series

The Baker-Campbell-Hausdorff series (BCH) has a long history, as described in [35], [36], and can be applied to a broad range of problems, both theoretical and computational. The BCH series addresses the following problem which can be stated as follows:

Given two non-commuting matrices W and Y , it is desired to find a solution to the equation

$$e^X = e^W e^Y \tag{3.3}$$

The BCH series expresses the solution to (3.3) in the form of an infinite series

$$X = \sum_{k=1}^{\infty} X_k \quad (3.4)$$

where the matrix X_k is a function of W and Y .

The first few terms in the series are [36]

$$\begin{aligned} X_1 &= W + Y \\ X_2 &= \frac{1}{2}[WY - YW] \\ X_3 &= \frac{1}{12}[W^2Y + WY^2 - 2WYW + Y^2W + YW^2 - 2YWY] \end{aligned} \quad (3.5)$$

Applying the BCH series to delay extraction, we let

$$W = A + sB \quad Y = -sB \quad (3.6)$$

in (3.3), (where A and B are defined in (2.2)), which leads to the following

$$e^X = e^{A+sB} e^{-sB} \quad (3.7)$$

or

$$e^{A+sB} = e^X e^{sB} \quad (3.8)$$

Using (3.5) and (3.6), the first few terms of X in (3.8) can be expressed as

$$\begin{aligned}
X &= \sum_{k=1}^{\infty} X_k \\
X_1 &= A \\
X_2 &= \frac{s}{2}[BA - AB] \\
X_3 &= \frac{1}{12} [-s(A + sB)^2 B + s^2(A + sB)B^2 + 2s(A + sB)B(A + sB) \\
&\quad + s^2 B^2(A + sB) - sB(A + sB)^2 - 2s^2 B(A + sB)B]
\end{aligned} \tag{3.9}$$

Eq. (3.9) leads to the following series of approximations

$$\text{BCH-1:} \quad e^{A+sB} \cong e^{X_1} e^{sB} = e^A e^{sB} \tag{3.10}$$

$$\text{BCH-2:} \quad e^{A+sB} \cong e^{X_1+X_2} e^{sB} = e^{A+\frac{s}{2}(BA-AB)} e^{sB} \tag{3.11}$$

$$\text{BCH-3:} \quad e^{A+sB} \cong e^{X_1+X_2+X_3} e^{sB} = e^{\left[A+\frac{s}{2}(BA-AB)+\frac{1}{12}(-s(A+sB)^2 B+s^2(A+sB)B^2+\dots)\right]} e^{sB} \tag{3.12}$$

It is important to note that the relative error using the BCH series decreases as $\|A\|$ and $\|B\|$ decrease.

Next, examining (3.10), it can be seen that for our case of lossy transmission lines, the exponential matrix e^{X_1} essentially represents the stamp of the lossy component of the transmission line. On the other hand, the higher order approximations BCH-2 and BCH-3 represent the more general case of lossy subnetworks, but not in the form of transmission lines. In addition, there is no guarantee that the macromodel derived using the higher order BCH series would be passive. Therefore, the

proposed delay extraction method is derived using the first order BCH approximation (BCH-1).

3.2.2 Lie Product Formula

As mentioned before, the accuracy of the BCH approximations increases as $\|A\|$ and $\|B\|$ decrease. Taking advantage of this property, the Lie Product formula [37] modifies BCH-1 to achieve a higher accuracy by splitting the exponential stamp into multiple sections (i.e. implicitly decreasing the resulting $\|A\|$ and $\|B\|$ for each section). It provides a systematic approach to approximate e^{A+sB} with an error estimation (ε_m) in the form

$$e^{(A+sB)} \approx \prod_{k=1}^m \zeta_k + \varepsilon_m; \quad \zeta_k \equiv e^{\frac{sB}{m}} e^{\frac{A}{m}} \quad (3.13)$$

where the error $\|\varepsilon_m\| \cong O\left(\frac{1}{m}\right)$. The behaviour of the error as a function of the approximation order m is illustrated in Fig. 3.2 for Line-1 of Fig. 3.1.

An important point to note here is that the term ζ_k in (3.13) does not maintain the symmetry of the original transmission line circuit. In addition, since the error decays with $O\left(\frac{1}{m}\right)$, a relatively large value of m would be required to achieve a given accuracy, thus reducing the efficiency of the approximation [25].

Fig. 3.2. Lie Product error as a function of approximation order m

In order to address the above difficulties, in the next section we will introduce the concept of modified Lie formulae.

3.2.3 Modified Lie Product Formulae

In this section, a theorem is presented which maintains the required symmetry as well as provides more accurate delay extraction by modifying the Lie product formula given in (3.13).

Theorem 3.1: (*Modified Lie Formula-I [MLF-I]*)

The product

$$\prod_{k=1}^m \Psi_k; \quad \Psi_k \equiv e^{\frac{A}{2m}} e^{\frac{sB}{m}} e^{\frac{A}{2m}} \quad (3.14)$$

converges asymptotically to $e^{(A+sB)}$ as $m \rightarrow \infty$. The associated error (ε_m) is given

$$\text{by } \|\varepsilon_m\| \cong O\left(\frac{1}{m}\right).$$

Proof of Theorem 3.1

$$\text{Let } H = e^{\frac{A+sB}{m}} \text{ and } W = e^{\frac{A}{2m}} e^{\frac{sB}{m}} e^{\frac{A}{2m}}, m = 1, 2, \dots$$

In order to prove Theorem 3.1, the behavior of $H^m - W^m$ as $m \rightarrow \infty$ is needed. To obtain this behaviour, we introduce the following lemma.

Lemma 1: Given two matrices H and W , $H^m - W^m$ can be expressed as

$$H^m - W^m = \sum_{r=0}^{m-1} H^r (H - W) W^{m-r-1} \quad (3.15)$$

where m is a positive non-zero integer.

Proof of Lemma 1:

Starting with the identity

$$\sum_{k=0}^m H^k W^{m-k} = \sum_{k=0}^m H^{m-k} W^k \quad (3.16)$$

we can write,

$$W^m + \sum_{k=1}^m H^k W^{m-k} = H^m + \sum_{k=1}^m H^{m-k} W^k \quad (3.17)$$

Re-arranging (3.17) yields

$$H^m - W^m = \sum_{k=1}^m H^k W^{m-k} - \sum_{k=1}^m H^{m-k} W^k \quad (3.18)$$

From which we can prove that

$$H^m - W^m = \sum_{r=0}^{m-1} H^{r+1} W^{m-r-1} - \sum_{r=0}^{m-1} H^r W^{m-r} \quad (3.19)$$

After some mathematical manipulation, we get the following result

$$H^m - W^m = \sum_{r=0}^{m-1} H^r (H - W) W^{m-r-1} \quad (3.20)$$

□

From (3.20), it can be noted that

$$\|H^m - W^m\| \leq m \|H\|^r \|W\|^{m-r-1} \|H - W\| \quad (3.21)$$

If $\|H\|$ and $\|W\|$ are upper bounded by J , where J is $\max(\|H\|, \|W\|)$, (3.21)

simplifies to

$$\|H^m - W^m\| \leq m J^{m-1} \|H - W\| \quad (3.22)$$

Expanding $H - W$ in power series yields

$$\begin{aligned} H - W &= \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{A + sB}{m} \right)^k - \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{A}{2m} \right)^k \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{sB}{m} \right)^k \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{A}{2m} \right)^k \\ &= \Phi(s) \frac{1}{m^3} + [\dots] \frac{1}{m^4} + [\dots] \frac{1}{m^5} + \dots \\ &\cong O\left(\frac{1}{m^3}\right) \quad (\text{for large } m) \end{aligned} \quad (3.23)$$

where $\Phi(s)$ is given by

$$\Phi(s) = \frac{1}{24}[s(AB - BA)(-A - 2sB) + s(A + 2sB)(AB - BA)] \quad (3.24)$$

Substituting the above results into (3.22) and noting that $\|H\|$ and $\|W\|$ are bounded

above by $e^{\frac{\|A\| + s\|B\|}{m}}$ yields

$$\|H^m - W^m\| \leq \frac{1}{m^2} e^{(\|A\| + s\|B\|)} \|\Phi(s)\| \quad (3.25)$$

The expression in (3.25) goes to zero as $m \rightarrow \infty$. This result leads to

$$\lim_{m \rightarrow \infty} \left(e^{\frac{A}{2m}} e^{\frac{sB}{m}} e^{\frac{A}{2m}} \right) = e^{(A + sB)} \quad (3.26)$$

The associated error (ε_m) in this case is given by

$$\|\varepsilon_m\| = \max_{0 \leq s \leq s_{max}} \left\| e^{(A + sB)} - \prod_{k=1}^m \Psi_k \right\| \cong O\left(\frac{1}{m}\right) \quad (3.27)$$

Consider again the single line referred to as Line-1 in Fig. 3.1, Fig. 3.3 shows the error versus the approximation order m , using the Modified Lie Formula I (MLF-I).

Fig. 3.3. MLF-I error as a function of approximation order m

3.3 Improved Approximation for Long Lines

In the case of long low-loss lines, it can be noted that $\|A\| \ll \|sB\|$. This fact can be utilized to arrive at more accurate relationships based on the Modified Lie formula.

Theorem 3.2: (Modified Lie Formula-II [MLF-II])

The product

$$\prod_{k=1}^m \Psi_k; \quad \Psi_k \equiv e^{\frac{sB}{2m}} e^{\frac{A}{m}} e^{\frac{sB}{2m}} \quad (3.28)$$

converges asymptotically to $e^{(A+sB)}$ as $m \rightarrow \infty$. The associated error ε_m is given by

$$\|\varepsilon_m\| = \max_{0 \leq s \leq s_{max}} \left\| e^{(A+sB)} - \prod_{k=1}^m \Psi_k \right\| \leq \frac{1}{m^2} \left[\max_{0 \leq s \leq s_{max}} e^{\|A\| + s\|B\|} \|\mathfrak{G}(s)\| \right] \quad (3.29)$$

where $\|\mathfrak{G}(s)\| \cong \frac{1}{2} \|\Phi(s)\|$ for $\|A\| \ll \|sB\|$.

Proof of Theorem 3.2

Following a similar approach as in (3.14), it can be shown that the error ε_m in this case is given by

$$\|\varepsilon_m\| = \|H^m - W_2^m\| \leq mJ^{m-1} \|H - W_2\| \quad (3.30)$$

where

$$W_2 = e^{\frac{sB}{2m}} e^{\frac{A}{m}} e^{\frac{sB}{2m}} \quad (3.31)$$

$$H = e^{\frac{A+sB}{m}} \quad (3.32)$$

Next, it can be shown that

$$\|H - W_2\| = \mathfrak{G}(s) \frac{1}{m^3} + [\dots] \frac{1}{m^4} + [\dots] \frac{1}{m^5} + \dots \quad (3.33)$$

with $\mathfrak{G}(s)$ given by

$$\mathfrak{G}(s) = \frac{1}{24} [s(AB - BA)(2A + sB) - s(2A + sB)(AB - BA)] \quad (3.34)$$

If $\|A\| \ll \|sB\|$, then (3.34) can be written as

$$\begin{aligned}
 \|\mathcal{G}(s)\| &\cong \left\| \frac{s^2}{24} [(AB - BA)B - B(AB - BA)] \right\| \\
 &\cong \frac{1}{2} \|\Phi(s)\|
 \end{aligned}
 \tag{3.35}$$

Consider another single transmission line with the following p.u.l. parameters (henceforth referred to as Line-2)

Fig. 3.4. An example transmission line (Line-2)

Fig. 3.5 demonstrates the error for Line-2 using MLF-I and MLF-II, respectively.

Fig. 3.5. Improved approximation using MLF-II

Next, a further accurate formula can be obtained by taking the average of the approximations in (3.14) and (3.28).

Theorem 3.3: (*Modified Lie Formula-III [MLF-III]*)

$$e^{A+sB} \cong \prod_{k=1}^m \Psi_k; \quad \Psi_k \equiv \frac{1}{2} \left(e^{\frac{A}{2m}} e^{\frac{sB}{m}} e^{\frac{A}{2m}} + e^{\frac{sB}{2m}} e^{\frac{A}{m}} e^{\frac{sB}{2m}} \right) \quad (3.36)$$

Similarly, the error in this case is as given in (3.22), however, W is now replaced

with $W_3 = \frac{1}{2} \left(e^{\frac{A}{2m}} e^{\frac{sB}{m}} e^{\frac{A}{2m}} + e^{\frac{sB}{2m}} e^{\frac{A}{m}} e^{\frac{sB}{2m}} \right)$, and $\|H - W_3\|$ becomes

$$\|H - W_3\| = \varphi(s) \frac{1}{m^3} + [\dots] \frac{1}{m^4} + [\dots] \frac{1}{m^5} + \dots \quad (3.37)$$

where $\varphi(s)$ is given by

$$\varphi(s) = \frac{1}{48} [s(AB - BA)(A - sB) - s(A - sB)(AB - BA)] \quad (3.38)$$

Again, for the case when $\|A\| \ll \|sB\|$, it can be easily shown that for low-loss lines

$\|\varphi(s)\| \cong \frac{1}{2} \|\vartheta(s)\| \cong \frac{1}{4} \|\Phi(s)\|$. Fig. 3.6 demonstrates this fact numerically for Line-2.

Fig. 3.6. Error comparisons for MLF-I, MLF-II and MLF-III

As mentioned above, MLF-II and MLF-III provide more accurate approximations for low-loss lines ($\|A\| \ll \|sB\|$). To illustrate the behaviour of the error as a function of $\|A\|$ for Line-2 in Fig. 3.4, the relative error at $f=2\text{GHz}$ was calculated for varying R from $0.01\Omega/\text{cm}$ to $80\Omega/\text{cm}$, using $m=15$. The relative error is calculated as

$$\|\varepsilon_m\|_{rel} = \frac{\left\| e^{(A+sB)} - \prod_{k=1}^m \Psi_k \right\|}{\|e^{(A+sB)}\|} \quad (3.39)$$

Fig. 3.7 shows the MLF-II and MLF-III error versus $\|A\|$. As seen from the figure, the accuracy of the MLF approximations improves for decreasing $\|A\|$. This reiterates the usefulness of MLF-II and MLF-III for long low-loss lines.

Fig. 3.7. Behaviour of MLF-II and MLF-III error as a function of $\|A\|$

3.4 Frequency-Dependent Parameters

As the frequency increases, the current distribution of the conductors vary, which causes the p.u.l. resistance (R) and inductance (L) to change. The p.u.l. inductance matrix $L(s)$, can be separated into a portion $L_i(s)$, due to the internal magnetic field, and a portion $L_e(s)$, due to the magnetic field external to the conductor [2].

Thus, the total inductance $L(s)$ can be expressed as

$$L(s) = L_i(s) + L_e(s) \quad (3.40)$$

At lower frequencies, the resistance and internal inductance remain almost constant, since the current may be assumed to be uniformly distributed throughout the cross-section. On the other hand, at higher frequencies, the resistance increases approximately as $\sqrt{f}$, and the internal inductance $L_i(s)$ decreases approximately

with $\sqrt{f}$, because the current begins to concentrate in a thin layer at the outer edges of the conductor cross-section [2]. This phenomenon is referred to as the *skin effect*. Thus at higher frequencies, the internal inductance $L_i(s)$ becomes negligible, and the remaining inductance $L = L_e$ essentially remains constant, representing the inductance at high frequencies. In addition, for many practical applications, the p.u.l. capacitance C , and admittance G , do not vary significantly with frequency [2].

Getting back to the proposed delay extraction-based algorithm, in the case of lines with frequency-dependent parameters, the objective is to extract a maximum delay without affecting transmission line causality conditions. Thus, we can define the following [25]

$$B_a(s) = B(s) - \hat{B} = \begin{bmatrix} \mathbf{0} & -L_a(s) \\ -C_a(s) & \mathbf{0} \end{bmatrix} \quad (3.41)$$

such that $L_a(s)$, $C_a(s)$ are positive definite matrices [2]. From the above discussion, since L becomes constant at high frequencies, an appropriate choice for $\hat{B}$ would be $B(\infty)$.

The MLF relations developed in the previous sections can be easily adjusted to include the information from (3.41). For example, (3.28) can be modified to include frequency-dependent parameters as

$$e^{(A(s) + sB(s))} \cong \prod_{k=1}^m \Psi_k; \quad \Psi_k \equiv e^{\frac{s\hat{B}}{2m}} e^{\frac{A(s) + sB_a(s)}{m}} e^{\frac{s\hat{B}}{2m}} \quad (3.42)$$

For illustration purposes, consider a single transmission line of length 5cm, with the following frequency-dependent p.u.l. parameters [34].

Line-3

<i>Frequency(Hz)</i>	<i>R(Ω/cm)</i>	<i>L(nH/cm)</i>	<i>C(pF/cm)</i>	<i>G = 0</i>
0	0.11903	5.7964	1.362606	
1e5	0.11922	5.7373	1.362606	
2.15e5	0.11978	5.5629	1.362605	
4.64e5	0.12133	5.1299	1.362599	
1e6	0.12370	4.5856	1.362571	
2.15e6	0.12609	4.2299	1.362445	
4.64e6	0.12847	4.0633	1.361877	
1e7	0.1331	3.9975	1.353448	
2.15e7	0.14722	3.9385	1.345955	
4.64e7	0.18349	3.8339	1.340388	
1e8	0.24997	3.7130	1.333386	
2.15e8	0.36067	3.6095	1.323453	
4.64e8	0.5198	3.5293	1.310150	
1e9	0.75879	3.4744	1.291061	
2.15e9	1.1014	3.4372	1.290020	
4.64e9	1.6241	3.4120	1.289450	
1e11	7.6153	3.3678	1.287259	

Fig. 3.8. An example of a line with frequency-dependent parameters (Line-3)

The exponential stamp of the line was approximated using (3.42) for $f=1\text{GHz}$ and $m=30$ as

$$e^{(A+sB)} = \begin{bmatrix} -0.50824 + 3.1489 \times 10^{-2}i & 0.18633 - 44.759i \\ 6.4603 \times 10^{-4} - 1.6596e \times 10^{-2}i & -0.50824 + 3.1489 \times 10^{-2}i \end{bmatrix} \quad (3.43)$$

$$\prod_{k=1}^{30} \Psi_k = \begin{bmatrix} -0.50824 + 3.1490 \times 10^{-2}i & 0.18570 - 44.759i \\ 6.4628 \times 10^{-4} - 1.6596e \times 10^{-2}i & -0.50824 + 3.1490 \times 10^{-2}i \end{bmatrix}$$

with a relative error of 1.9582×10^{-5} .

3.5 Estimation of the Order of Approximation

As previously discussed, the error using the MLF formulations decreases as the approximation order $m \rightarrow \infty$. Thus, it is of interest to arrive at a relationship between the error and the order m . Note that (3.42) provides an estimation of the maximum allowable approximation error for a desired bandwidth (which is dependent on the maximum frequency of interest, f_{max}). In general, the maximum relative error happens at f_{max} . In this case, (3.39) can be explicitly written as

$$\|\epsilon_m\|_{rel} = \frac{\left\| e^{(A(s_{max}) + s_{max}B(s_{max}))} - \prod_{k=1}^m \Psi_k \right\|}{\left\| e^{(A(s_{max}) + s_{max}B(s_{max}))} \right\|} \quad (3.44)$$

For most practical cases, f_{max} can be obtained from the rise/fall time of the input signal as [4]

$$f_{max} \cong \frac{0.35}{\text{rise time}} \quad (3.45)$$

For a given value of $\|\epsilon_m\|_{rel}$, the order of approximation m can be determined using (3.44). Fig. 3.10 shows the approximation order versus $\|\epsilon_m\|_{rel}$ using MLF-II for the following line (henceforth referred to as Line-4)

Fig. 3.9. Example of transmission line (Line-4)

Fig. 3.10. Relative approximation error $\|\epsilon_m\|_{rel}$ versus realization order m for varying bandwidths

As expected, for a given maximum allowable error, the order m increases with the increase in the bandwidth of approximation. For illustration purposes, consider the rise time of an input signal to be 0.2ns. Using (3.45), the corresponding maximum frequency is 1.75GHz. From Fig. 3.10, it can be seen that, for example, in order to achieve a maximum relative error of $\|\epsilon_m\|_{rel} = 5 \times 10^{-4}$, a realization order of $m=8$ or $m=9$ should be sufficient. This fact was tested out by approximating the exponential stamp of Line-4 using the MLF-II formula for $m=8$. In this case, the relative error was 4.808×10^{-4} , which falls below $\|\epsilon_m\|_{rel}$.

3.6 Summary

In summary, this chapter presented the underlying concepts behind delay extraction, and how it used to accurately approximate the exponential stamp of the line. The Baker-Campbell-Hausdorff series was described, which lead to the derivation of the Modified Lie Product formulae, respectively. A new theorem was introduced which preserves the symmetric nature of the original transmission line circuit, and more accurate formulae were derived to achieve improved approximations for long lines. In addition, the applicability of the proposed technique to lines with frequency-dependent parameters was discussed and an estimation of the order of approximation was given. In the next chapter, circuit realization of the Modified Lie formulae for passive macromodeling of coupled transmission lines will be described.

CHAPTER 4

Realization of the Delay Extraction-Based Macromodel

4.1 Introduction

In chapter 3, the Lie Product formula was introduced, and Modified Lie formulae (MLF) were derived in order to accurately represent the transmission line behaviour. Using the MLF equations, the exponential stamp of the line is approximated with a product of exponential matrices. In order to simulate circuits which include transmission lines, it is necessary to realize the MLF equations in the form of a circuit. This chapter describes a new macromodel which represents the MLF equations in the form of linear passive circuits. The new macromodel consists of a cascade of lossless and lossy subnetworks which can be directly implemented in a circuit simulator and are guaranteed to be passive. For simplicity, the proposed technique will be henceforth referred to as 'DEPACT', which stands for Delay Extraction Based Passive Compact Transmission Line Macromodeling Algorithm

[25]. The chapter is organized as follows. Section 4.2 describes the overall structure of the DEPACT macromodel. Section 4.3 presents the realization of the lossless sections, which includes a derivation of the circuit equations and equivalent circuit representations. Section 4.4 discusses the special case of lines with frequency-independent parameters and derives the corresponding macromodels. Finally, section 4.5 provides several numerical examples which validate the accuracy and efficiency of the DEPACT algorithm.

4.2 Structure of the DEPACT Macromodel

As discussed in the previous chapter, the MLF-II and MLF-III equations provide more accurate approximations for the exponential stamp of the line. However, although the MLF-III equations achieve a lower error compared to MLF-II, the form of the approximation (shown in (3.36), which is the average of MLF-I and MLF-II) does not suitably represent the stamp of a regular transmission line and is rather difficult to implement in circuit simulators. This is because the hybrid stamp corresponding to the MLF-III approximation is given as the sum of two transmission line stamps. Therefore, for the rest of this thesis, the proposed DEPACT algorithm will be based on the MLF-II approximations given in (3.42), which provides a reasonably low error and an easy implementation in circuit simulators.

The product terms of the MLF-II equations in (3.42) can be viewed as a cascade of m transmission line subnetworks as shown in Fig. 4.1(a). These subnetworks are referred to as DEPACT cells. The DEPACT cells, which realize each Ψ_k in (3.42), can be represented by a cascade of lossy and lossless transmission lines. Fig. 4.1(b)

depicts the configuration for the k^{th} DEFACT cell. Here, the lossy terms are macromodeled using the passive matrix rational approximation (MRA) (described in chapter 2). It is important to note, that the resulting macromodels are of significant lower orders (since a significant delay portion is already extracted from these terms). They are later combined with the lossless sections using the MoC approach, along with an algorithm to decouple the multiconductor transmission line equations. To avoid repetition, since a review of MRA has been presented in chapter 2, and details can be found in [20], [21], [23], this chapter focuses on the analysis and realization of the lossless sections in Fig. 4.1(b).

Fig. 4.1. Macromodel realization of product terms in (3.42)

4.3 Realization of Lossless Sections

4.3.1 Derivation of Circuit Equations

Consider the lossless module of a particular MLF cell (N -port) as defined in Fig. 4.1. The distributed voltages and currents for the lossless section (as shown in Fig. 4.2) can be represented using Telegrapher's equations as

$$\frac{\partial}{\partial x} v(x, t) = -L_m \frac{\partial}{\partial t} i(x, t) \quad (4.1)$$

$$\frac{\partial}{\partial x} i(x, t) = -C_m \frac{\partial}{\partial t} v(x, t)$$

where

$$L_m = \frac{L(\infty)}{2m} \quad (4.2)$$

$$C_m = \frac{C(\infty)}{2m}$$

Fig. 4.2. N -port lossless section of MLF cell

Next, (4.1) can be expressed in terms of propagating modal voltages $\hat{v}(x, t)$ and currents $\hat{i}(x, t)$ as [2]

$$v(x, t) = E_V \hat{v}(x, t) \quad i(x, t) = E_I \hat{i}(x, t) \quad (4.3)$$

Since L_m and C_m are real, symmetric, and positive definite [2], transformation matrices E_V and E_I can be found such that they simultaneously diagonalize both L_m and C_m as follows

$$E_V^{-1} L_m E_I = \hat{L} \quad (4.4)$$

$$E_I^{-1} C_m E_V = \hat{C} \quad (4.5)$$

where $\hat{L}$ and $\hat{C}$ are diagonal matrices of the form

$$\hat{L} = \text{diag}\{\hat{l}_1, \hat{l}_2, \dots, \hat{l}_N\} \quad (4.6)$$

$$\hat{C} = \text{diag}\{\hat{c}_1, \hat{c}_2, \dots, \hat{c}_N\} \quad (4.7)$$

Using (4.3)-(4.5), (2.1) can be expressed for lossless lines, in terms of modal parameters as

$$\begin{aligned} \frac{\partial}{\partial x} \hat{v}(x, t) &= -\hat{L} \frac{\partial}{\partial t} \hat{i}(x, t) \\ \frac{\partial}{\partial x} \hat{i}(x, t) &= -\hat{C} \frac{\partial}{\partial t} \hat{v}(x, t) \end{aligned} \quad (4.8)$$

It is to be noted that in the case of coupled transmission lines, (4.8) represents a set of decoupled modal equations (since $\hat{L}$ and $\hat{C}$ are diagonal matrices). This implies

that the solution for the modal voltages and currents have a general form similar to that of the single line case. Hence, the modal propagation constant ($\hat{\gamma}_i$) and the characteristic impedance ($\hat{Z}_i$) of the i^{th} mode can be computed as

$$\hat{\gamma}_i = s\sqrt{\hat{L}_i\hat{C}_i} \quad \hat{Z}_i = \sqrt{\frac{\hat{L}_i}{\hat{C}_i}} \quad (4.9)$$

The actual voltages and currents ($v(x, t)$ and $i(x, t)$) can be obtained using the solution of the modal voltages and currents along with (4.3). Using (4.9), the solution to (4.8) can be written in the form [25]

$$\hat{V}_1 - \hat{Z}\hat{I}_1 = W_1 \quad (4.10)$$

$$\hat{V}_2 - \hat{Z}\hat{I}_2 = W_2 \quad (4.11)$$

$$W_1 = \Gamma[2\hat{V}_2 - W_2] \quad (4.12)$$

$$W_2 = \Gamma[2\hat{V}_1 - W_1] \quad (4.13)$$

where $\hat{V}_1$, $\hat{I}_1$, $\hat{V}_2$ and $\hat{I}_2$ represent the near and far-end modal voltage and current

vectors of the lossless section, respectively, and where $\hat{Z}$ and Γ are given by

$$\begin{aligned} \hat{Z} &= \text{diag}(\hat{Z}_1, \hat{Z}_2, \dots, \hat{Z}_N) \\ \Gamma &= \text{diag}(e^{-\hat{\gamma}_1 d}, e^{-\hat{\gamma}_2 d}, \dots, e^{-\hat{\gamma}_N d}) \end{aligned} \quad (4.14)$$

Using (4.10)-(4.13), the solution to (4.8) can be written in the time-domain as

$$\hat{v}_1(t) - \hat{Z}\hat{i}_1(t) = w_1(t) \quad (4.15)$$

$$\hat{v}_2(t) - \hat{Z}\hat{i}_2(t) = w_2(t) \quad (4.16)$$

$$w_1(t) = 2\hat{v}_2(t - \tau) - w_2(t - \tau) \quad (4.17)$$

$$w_2(t) = 2\hat{v}_1(t - \tau) - w_1(t - \tau) \quad (4.18)$$

where the delay $\tau_i = d\sqrt{\hat{l}_i\hat{c}_i}$ and the vectors which are functions of $(t - \tau)$ are defined as

$$f(t - \tau) = [f_1(t - \tau_1) \dots f_N(t - \tau_N)]^T \quad (4.19)$$

where superscript ' T ' denotes the transpose.

The actual line voltages $v(x, t)$ and currents $i(x, t)$ can be determined in terms of the modal voltages $\hat{v}(x, t)$ and currents $\hat{i}(x, t)$ using (4.3). Next, it can be shown that E_I and E_V can be computed as follows [2], [25]

$$E_I = QFS\alpha \quad (4.20)$$

$$E_V = QF^{-1}S\alpha^{-1} = (E_I^T)^{-1} \quad (4.21)$$

where

- Q is a matrix of the right eigenvectors of C_m and F^2 is a diagonal matrix of the eigenvalues of C_m , i.e. $Q^T C_m Q = F^2$.
- The matrix S consists of the right eigenvectors of the product $FQ^T L_m QF$, i.e. $S^T (FQ^T L_m QF) S = \Lambda^2$.

- The diagonal matrix α is a normalizing factor such that the Euclidean norm of the columns of E_I is unity. The diagonal elements α_{ii} are given by

$$\alpha_{ii} = 1 / \left(\sqrt{\sum_{k=1}^n E_{ki}^2} \right) \quad (4.22)$$

where

$$\begin{aligned} E &= QFS \\ &= \{E_{ki} ; k = 1, 2, \dots, N; i = 1, 2, \dots, N\} \end{aligned} \quad (4.23)$$

Substituting (4.20) and (4.21) into (4.4) and (4.5) results in

$$\hat{L} = \alpha \Lambda^2 \alpha \quad (4.24)$$

$$\hat{C} = \alpha^{-2} \quad (4.25)$$

Note that since the equations in (4.15)-(4.18) are represented in the time-domain, they can be solved simultaneously with the time-domain modified nodal analysis (MNA) equations [38] describing the rest of the circuit.

4.3.2 Equivalent Circuit Representation for Lossless Sections

Using (4.15)-(4.18), the solution to the modal equations can be represented with equivalent circuits consisting of time-domain dependent delay sources and lumped impedances. It is important to note here that, these equivalent circuits can be directly embedded within the circuit simulator netlist. For the case of single lines, the lossless sections in Fig. 4.1 can be represented by the equivalent circuit shown in Fig. 4.3. In this case (since there is no decoupling required), the characteristic

impedance (scalar Z) of the single line is given by $Z = \sqrt{L_m/C_m}$, and the dependent sources $w_1(t)$ and $w_2(t)$ are defined as

$$w_1(t) = 2v_2(t-\tau) - w_2(t-\tau) \quad (4.26)$$

$$w_2(t) = 2v_1(t-\tau) - w_1(t-\tau) \quad (4.27)$$

with $\tau = d\sqrt{L_m C_m}$.

Fig. 4.3. Equivalent circuit representation for the analysis of a lossless single line section

For the case of lossless coupled lines, the lines are decoupled into single line subcircuits of the form shown in Fig. 4.3. Fig. 4.4 depicts the overall decoupling process using the modal transformation matrices E_I and E_V , where the sources $w_1(t)$ and $w_2(t)$ are defined in (4.17) and (4.18), and the subscript ' i ' denotes the i^{th} component of the associated vector. Note that, implementing the decoupling terminal relations described in (4.3) can be realized by using controlled sources.

Fig. 4.4. Equivalent circuit representation for the analysis of lossless coupled sections

4.3.3 SPICE Implementation

In Fig. 4.4, the lumped impedances ($\hat{Z}_i$) and time-domain dependent sources (w_1 and w_2) can be directly implemented using a conventional circuit simulator such as HSPICE [22]. However, since the transformation matrices E_I and E_V relate the actual voltages and currents to the modal voltages and currents, we must also obtain an equivalent circuit representation for the decoupling relations. For illustration

purposes, we will consider here the case of two coupled lossless lines shown in Fig. 4.5.

Fig. 4.5. Two coupled lossless lines

Consider (4.3) and define the related matrices as follows

$$E_I = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \quad E_V^{-1} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \quad (4.28)$$

Using (4.3) and (4.28), we can constitute current controlled current sources F_{A1} and F_{B1} for the near-end as

$$\begin{aligned} F_{A1} &= k_{11} \hat{i}_{1_1} + k_{12} \hat{i}_{1_2} \\ F_{B1} &= k_{21} \hat{i}_{1_1} + k_{22} \hat{i}_{1_2} \end{aligned} \quad (4.29)$$

and the near-end voltage controlled voltage sources are obtained as

$$\begin{aligned} E_{A1} &= n_{11} v_{1_1} + n_{12} v_{1_2} \\ E_{B1} &= n_{21} v_{1_1} + n_{22} v_{1_2} \end{aligned} \quad (4.30)$$

To avoid repetition, the same approach as in (4.29) and (4.30) can be used to obtain the sources which decouple the far-end voltages and currents. Fig. 4.6 shows the

decoupling implementation using controlled sources, where the embedded single line subcircuits are shown in Fig. 4.3.

Fig. 4.6. Implementation of decoupling algorithm via controlled sources

4.4 Special Case: Frequency-Independent Parameters

Up until now, the preceding chapters have described the proposed macromodel for lines with frequency-dependent p.u.l. parameters. However, in the case when the line parameters are frequency-independent, further simplifications can be made. One of the main advantages of this special case is that, MRA based approximations can be avoided. Moreover, the resulting macromodel consist of only delay and lumped elements, which are computed analytically based on the p.u.l. parameters, leading to an easy implementation in SPICE-like circuit simulators. This section describes this special case and its corresponding macromodels.

For the case of distributed transmission lines with frequency-independent **RLGC** parameters, the delay extracted exponential matrix representing a lossy section can be expressed as [26], [27]

$$e^{\mathbf{Z}} = e^{\frac{\mathbf{A}}{m}} \quad (4.31)$$

which represents the middle term in (3.42), and where the constant matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix} d \quad (4.32)$$

In the following subsections, it will be shown that the macromodel realization of (3.42) can be implemented without any further numerical approximations (in other words, an exact representation for (4.31)), using only resistive networks. To simplify the explanation, we will start with the simple case of single lines.

4.4.1 Realization of Single Lossy Sections

Extracting the delay corresponding to $\hat{\mathbf{B}}$ (as described in chapter 3), the Telegrapher's equations for the single lossy section of the MLF cell in Fig. 4.1 can be written in the frequency-domain as

$$\begin{aligned} \frac{\partial}{\partial x} V(x, s) &= -R_m I(x, s) \\ \frac{\partial}{\partial x} I(x, s) &= -G_m V(x, s) \end{aligned} \quad (4.33)$$

where

$$R_m = \frac{R}{m} \quad G_m = \frac{G}{m} \quad (4.34)$$

An analytical solution, in terms of the y -parameters for (4.33) can be derived as [4]

$$I = YV$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{Z_0(1 - e^{-2\gamma d})} \begin{bmatrix} 1 + e^{-2\gamma d} & -2e^{-\gamma d} \\ -2e^{-\gamma d} & 1 + e^{-2\gamma d} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (4.35)$$

where I_1 , V_1 , I_2 and V_2 represent the near and far-end currents and voltages of the lossy section, respectively, and where

$$\gamma = \sqrt{R_m G_m} \quad Z_0 = \sqrt{\frac{R_m}{G_m}} \quad (4.36)$$

Eq. (4.35) can be re-written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} g_a + g_b & -g_b \\ -g_b & g_a + g_b \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (4.37)$$

where the admittances g_a and g_b are obtained by [27]

$$\begin{aligned} g_a + g_b &= \frac{1 + e^{-2\gamma d}}{Z_0(1 - e^{-2\gamma d})} \\ g_b &= \frac{2e^{-\gamma d}}{Z_0(1 - e^{-2\gamma d})} \\ g_a &= \frac{1 + e^{-2\gamma d} - 2e^{-\gamma d}}{Z_0(1 - e^{-2\gamma d})} = \frac{(1 - e^{-\gamma d})^2}{Z_0(1 - e^{-2\gamma d})} \end{aligned} \quad (4.38)$$

It is important to note that since $\gamma > 0$, this implies that $e^{-2\gamma d} < 1$, which results in all the resistor values being positive.

Next, using (4.37) and (4.38), (4.31) can be implemented in terms of a purely resistive network as shown in Fig. 4.7, where $r_a = 1/g_a$ and $r_b = 1/g_b$. For illustrative purposes, Fig. 4.8 shows the structure of the DEFACT cells for the case of frequency-independent parameters, which consist of a cascade of lossless single lines and purely resistive networks.

Fig. 4.7. Resistive network realization for single lossy sections

Fig. 4.8. Macromodel realization for single lines with frequency-independent parameters

It is important to note here that, (4.33)-(4.38) represent an exact implementation of (4.31), without any numerical approximations. In addition, the macromodel is realized in terms of purely resistive elements, whose values are computed analytically in terms of the p.u.l. line parameters. This aspect of the proposed method greatly simplifies its implementation in SPICE-like simulators

4.4.2 Realization of Coupled Lossy Sections

For coupled lossy sections, a decoupling method is employed which is similar to the technique used for representing lossless coupled lines (described in section 4.3). Using this technique, the lossy lines are decoupled into single line subcircuits represented by the resistive network shown in Fig. 4.7.

The distributed voltages and currents of the coupled lossy section are related by the Telegrapher's equations as

$$\begin{aligned}\frac{\partial}{\partial x}v(x, t) &= -R_m i(x, t) \\ \frac{\partial}{\partial x}i(x, t) &= -G_m v(x, t)\end{aligned}\tag{4.39}$$

Next, (4.39) can be expressed in terms of auxiliary voltages $\hat{v}(x, t)$ and currents $\hat{i}(x, t)$ as [2]

$$v(x, t) = P_V \hat{v}(x, t) \quad i(x, t) = P_I \hat{i}(x, t)\tag{4.40}$$

The transformation matrices P_V and P_I can be found such that they simultaneously diagonalize both R_m and G_m as

$$\begin{aligned}P_V^{-1} R_m P_I &= \hat{R} = \text{diag}\{\hat{r}_1, \hat{r}_2, \dots, \hat{r}_N\} \\ P_I^{-1} G_m P_V &= \hat{G} = \text{diag}\{\hat{g}_1, \hat{g}_2, \dots, \hat{g}_N\}\end{aligned}\tag{4.41}$$

To avoid repetition, P_V and P_I can be computed using the same method described for decoupling lossless lines in (4.20)-(4.25), except C_m and L_m are replaced with the matrices G_m and R_m , respectively.

Next, using (4.40) and (4.41), (4.39) can be expressed in terms of $\hat{R}$ and $\hat{G}$ as follows

$$\begin{aligned}
\frac{\partial}{\partial x} \hat{v}(x, t) &= -\hat{R} \hat{i}(x, t) \\
\frac{\partial}{\partial x} \hat{i}(x, t) &= -\hat{G} \hat{v}(x, t)
\end{aligned}
\tag{4.42}$$

Since $\hat{R}$ and $\hat{G}$ are diagonal matrices, (4.42) represents a set of decoupled equations. This implies that the solutions for the auxiliary voltages and currents have the same general form as those for the single line case. Fig. 4.9 depicts the circuit realization for the lossy coupled sections, where the resistor values $r_{a_i} = 1/g_{a_i}$ and $r_{b_i} = 1/g_{b_i}$ for the i^{th} resistive network are computed as

$$\begin{aligned}
g_{a_i} &= \frac{(1 - e^{-\gamma_i d})^2}{Z_{0i}(1 - e^{-2\gamma_i d})} \\
g_{b_i} &= \frac{2e^{-\gamma_i d}}{Z_{0i}(1 - e^{-2\gamma_i d})}
\end{aligned}
\tag{4.43}$$

and where

$$\gamma_i = \sqrt{\hat{r}_i \hat{g}_i} \quad Z_{0i} = \sqrt{\frac{\hat{r}_i}{\hat{g}_i}}
\tag{4.44}$$

Fig. 4.9. Equivalent circuit representation for the analysis of lossy coupled sections

4.4.3 Alternative Realization of Lossy Coupled Sections

In the previous section, an equivalent circuit was derived which decouples the coupled lossy lines into single resistive networks using transformation matrices. However, an alternate approach has been developed for the analysis of the lossy coupled sections which does not require any decoupling algorithm. The resulting macromodel is a resistive network derived from the exponential stamp of the lossy

coupled line. For illustrative purposes, we will first consider the case of two coupled lines, and then present the general formulation.

The exponential stamp of the lossy section in Fig. 4.1 can be written as

$$\begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = e^{\frac{A}{m}} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} \quad (4.45)$$

where v_1 , i_1 , v_2 , and i_2 represent the near and far-end voltage and current vectors of the lossy section, respectively, and the matrix A is defined in (4.31). As described in [4], (4.45) can be written in terms of y-parameters as

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (4.46)$$

Next, using the MNA approach [38], Fig. 4.10 shows the equivalent circuit representation which realizes the relationship in (4.46).

Fig. 4.10. Equivalent circuit representation for (4.46)

As an example, we will consider the value $Y(1,1)$ of the matrix Y . From the circuit in Fig. 4.10, the presence of the resistor between node 1 and ground requires the value $Y_{11} + Y_{12} + Y_{13} + Y_{14}$ to be placed in position (1,1) of the admittance matrix. Next, the resistors between nodes 1 and 2, nodes 1 and 3, and nodes 1 and 4 would require adding the terms $-Y_{12}$, $-Y_{13}$ and $-Y_{14}$, respectively, to the value in position (1,1). Thus, element (1,1) of the admittance matrix becomes $Y_{11} + Y_{12} + Y_{13} + Y_{14} - Y_{12} - Y_{13} - Y_{14} = Y_{11}$, which is the same as in (4.46). Continuing this process for all the resistors in Fig. 4.10, results in the same

equations as those in (4.46). Hence, for the general case of N -coupled lossy lines, the relationship given by

$$I = YV$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} \\ Y_{21} & Y_{22} & \dots & Y_{2N} \\ \vdots & \vdots & \dots & \vdots \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (4.47)$$

can be realized with a resistive network using the following algorithm

To represent diagonal elements: Y_{jj} ($j=1,2,\dots,N$)

Between node j and ground

$$r_{jj} = \frac{1}{\left(\sum_{k=1}^N Y_{jk} \right)}$$

To represent off-diagonal elements: $Y_{jk} = Y_{kj}$ ($j=1,2,\dots,N; k=1,2,\dots,N$)

Between nodes j and k

$$r_{jk} = \frac{-1}{Y_{jk}} \quad j \neq k$$

It is important to note, that in general, the off-diagonal elements (Y_{jk}) of the admittance matrix Y are negative. This results in a resistive network with all positive resistor values.

Using the developed technique described in this section, the frequency-independent coupled lossy sections of the DEFACT macromodel can be realized in terms of purely resistive elements (Fig. 4.10), whose values are computed analytically in terms of the line parameters. In addition, dependent voltage and current sources are not required in the realization since a decoupling scheme is not performed.

4.5 Numerical Results

This section presents several numerical examples which validate the accuracy and efficiency of the proposed DEFACT method. Seven examples are considered. Examples 1-4 use various linear and nonlinear circuits which consist of lines with frequency-independent parameters. Examples 5-6 consider transmission line networks with frequency-dependent parameters. Finally, example 7 presents a numerical comparison between the DEFACT order of approximation m , and the bandwidth required. Comparison of the CPU time was done using an INTEL P4 2GHZ CPU.

4.5.1 Example 1

In this example, we consider two coupled lines with frequency-independent parameters. The transmission line circuit is shown in Fig. 4.11. and the line parameters are given by [25]

$$\begin{aligned}
 R &= \begin{bmatrix} 1.24 & 0 \\ 0 & 1.24 \end{bmatrix} \Omega/\text{cm} & L &= \begin{bmatrix} 3.361 & 0.865 \\ 0.865 & 3.361 \end{bmatrix} \text{nH}/\text{cm} \\
 C &= \begin{bmatrix} 1.29 & -0.197 \\ -0.197 & 1.29 \end{bmatrix} \text{pF}/\text{cm} & G &= \begin{bmatrix} 10 & -1 \\ -1 & 10 \end{bmatrix} \text{mMho}/\text{cm} \\
 d &= 0.5\text{cm}
 \end{aligned} \tag{4.48}$$

The input voltage source is a trapezoidal pulse with rise/fall time of 0.5ns and a pulse width of 5ns. The circuit was simulated using the proposed DEPACT method (for $m=2$) and conventional lumped segmentation (U-element in HSPICE). Fig. 4.12 - Fig. 4.14 show the transient response for both the proposed DEPACT method and conventional uniform lumped segmentation [2]. As seen from the figures, the results are in excellent agreement. In addition, simulating the circuit using conventional lumped segmentation took 2.95 sec, while DEPACT took only 0.55 sec, providing a speed-up of 5.36.

Fig. 4.11. Two coupled line circuit (Example 1)

Fig. 4.12. Time-domain response of active line far-end (Example 1)

Fig. 4.13. Time-domain response of victim line near-end (Example 1)

Fig. 4.14. Time-domain response of victim line far-end (Example 1)

4.5.2 Example 2

In this example, we consider a nonlinear circuit consisting of two longer coupled lines with frequency-independent parameters, and an inverter (Fig. 4.15). The input in this case is a trapezoidal pulse with rise/fall time of 0.1ns and a pulse width of 5ns. The line parameters are

$$\begin{aligned}
 R &= \begin{bmatrix} 1.24 & 0 \\ 0 & 1.24 \end{bmatrix} \Omega/\text{cm} & L &= \begin{bmatrix} 3.361 & 0.865 \\ 0.865 & 3.361 \end{bmatrix} \text{nH}/\text{cm} \\
 C &= \begin{bmatrix} 1.29 & -0.197 \\ -0.197 & 1.29 \end{bmatrix} \text{pF}/\text{cm} & G &= \begin{bmatrix} 2 \times 10^{-5} & 0 \\ 0 & 2 \times 10^{-5} \end{bmatrix} \text{Mho}/\text{cm} \\
 d &= 0.1\text{m}
 \end{aligned} \tag{4.49}$$

Fig. 4.16 - Fig. 4.18 show a sample of the transient response for DEPACT using $m=6$. As seen, the results from both methods are in excellent agreement. Simulation

using conventional lumped segmentation took 288.6 sec, whereas simulating using DEPACT took only 3.44 sec, providing a speed-up of 84.

Fig. 4.15. Nonlinear circuit (Example 2)

Fig. 4.16. Transient response of active line far-end (Example 2)

Fig. 4.17. Transient response of victim line near-end (Example 2)

Fig. 4.18. Transient response of victim line far-end (Example 2)

4.5.3 Example 3

Next, we consider here three coupled interconnect networks [25] with nonlinear inverters shown in Fig. 4.19. The line parameters for MTL subnetwork #1 are

$$\begin{aligned} R &= \begin{bmatrix} 0.252 & 0 \\ 0 & 0.252 \end{bmatrix} \Omega/\text{cm} & L &= \begin{bmatrix} 3.36 & 0.865 \\ 0.865 & 3.36 \end{bmatrix} \text{nH}/\text{cm} \\ C &= \begin{bmatrix} 1.29 & -0.197 \\ -0.197 & 1.29 \end{bmatrix} \text{pF}/\text{cm} & G &= 0 \\ d &= 5\text{cm} \end{aligned} \quad (4.50)$$

and

$$\begin{aligned} R &= \begin{bmatrix} 0.34 & 0 \\ 0 & 0.34 \end{bmatrix} \Omega/\text{cm} & L &= \begin{bmatrix} 4.76 & 0.974 \\ 0.974 & 4.76 \end{bmatrix} \text{nH}/\text{cm} \\ C &= \begin{bmatrix} 1.02 & -0.178 \\ -0.178 & 1.02 \end{bmatrix} \text{pF}/\text{cm} & G &= 0 \\ d &= 4\text{cm} \end{aligned} \quad (4.51)$$

for MTL subnetworks #2 and #3.

The input voltage is a unit step response with a rise time of 0.1ns. Fig. 4.20 and Fig. 4.21 show the transient response of the far-end voltages of subnetwork #2 (labeled active and victim line in Fig. 4.19) using an approximation order of $m=6$ for DEFACT. As seen from the plots, the results from DEFACT are in very good agreement with the results from the conventional lumped segmentation. The transient simulation using the conventional lumped model required 23.4 sec, while DEFACT required a CPU time of 0.75 sec, thus providing a speed-up of 31.

Fig. 4.19. Coupled interconnect system with nonlinear terminations (Example 3)

Fig. 4.20. Transient response for active line far-end (Example 3)

Fig. 4.21. Transient response for victim line far-end (Example 3)

4.5.4 Example 4

We consider here four coupled lines (shown in Fig. 4.22) with the following p.u.l. parameters (based on the data in [15]) as

$$\begin{aligned}
 R &= \begin{bmatrix} 1.24 & 1e-4 & 1e-4 & 1e-4 \\ 1e-4 & 1.24 & 1e-4 & 1e-4 \\ 1e-4 & 1e-4 & 1.24 & 1e-4 \\ 1e-4 & 1e-4 & 1e-4 & 1.24 \end{bmatrix} \Omega/\text{cm} & L &= \begin{bmatrix} 3.361 & 0.865 & 0.152 & 0.865 \\ 0.865 & 3.361 & 0.865 & 0.865 \\ 0.152 & 0.865 & 3.361 & 0.865 \\ 0.865 & 0.865 & 0.865 & 3.361 \end{bmatrix} \text{nH}/\text{cm} \\
 C &= \begin{bmatrix} 1.29 & -0.197 & -0.006 & -0.197 \\ -0.197 & 1.29 & -0.197 & -0.197 \\ -0.006 & -0.197 & 1.29 & -0.197 \\ -0.197 & -0.197 & -0.197 & 1.29 \end{bmatrix} \text{pF}/\text{cm} \\
 G &= \begin{bmatrix} 1e-4 & -1e-5 & -1e-5 & -1e-5 \\ -1e-5 & 1e-4 & -1e-5 & -1e-5 \\ -1e-5 & -1e-5 & 1e-4 & -1e-5 \\ -1e-5 & -1e-5 & -1e-5 & 1e-4 \end{bmatrix} \text{mho}/\text{cm} \\
 d &= 5\text{cm}
 \end{aligned} \tag{4.52}$$

The input voltage is a trapezoidal pulse with rise/fall time of 0.5ns and a pulse width of 5ns. Fig. 4.23-Fig. 4.25 shows a sample of the transient response using $m=4$ DEPACT cells. As seen from the plots, the results are in excellent agreement. In addition, the conventional lumped segmentation took 84 sec, while the DEPACT method took only 0.73 sec, thus providing a speed-up of 115.

Fig. 4.22. Four coupled line circuit (Example 4)

Fig. 4.23. Transient response of active line far-end (node 2) in Example 4

Fig. 4.24. Transient response of victim line far-end (node 4) in Example 4

Fig. 4.25. Transient response of victim line near-end (node 5) in Example 4

4.5.5 Example 5

In this example, a long 10m cable with frequency-dependent parameters is considered. This example is based on one of three benchmark problems published in [34], and is referred to as Line-4 in [34]. The circuit and frequency-dependent line parameters [34] are shown in Fig. 4.26. The input voltage is a unit step with a rise time of 0.1ns. Fig. 4.27 shows the transient response at the far-end of the transmission line using $m=15$ for DEPACT. As seen, the results from both techniques are in excellent agreement. In addition, MRA produced the same response as shown in Fig. 4.27, however, DEPACT is more than 100 times faster than MRA, and more than 300 times faster than uniform lumped segmentation (Table 4.1).

Frequency (GHz)	R (kΩ / cm)	L (μH / cm)	C (pF/cm)	G (mho/cm)
0	1.3490e-006	2.7700e-003	8.8344e-001	0
1.0000e-004	1.8640e-006	2.7700e-003	8.8344e-001	4.0472e-006
2.1500e-004	1.9760e-006	2.7570e-003	8.8208e-001	8.7015e-006
4.6400e-004	2.3800e-006	2.6840e-003	8.8071e-001	1.8779e-005
1.0000e-003	3.2900e-006	2.5390e-003	8.7934e-001	4.0472e-005
2.1500e-003	4.6000e-006	2.4040e-003	8.7798e-001	8.7015e-005
4.6400e-003	6.5000e-006	2.3090e-003	8.7661e-001	1.8779e-004
1.0000e-002	9.3179e-006	2.2445e-003	8.7524e-001	4.0472e-004
2.1500e-002	1.3440e-005	2.1990e-003	8.7388e-001	8.7015e-004
4.6400e-002	1.9500e-005	2.1690e-003	8.7251e-001	1.8779e-003
1.0000e-001	2.8420e-005	2.1480e-003	8.7114e-001	4.0472e-003
2.1500e-001	4.1500e-005	2.1344e-003	8.6978e-001	8.7015e-003
4.6400e-001	6.0700e-005	2.1247e-003	8.6841e-001	1.8779e-002
1.0000e+000	8.8800e-005	2.1182e-003	8.6704e-001	4.0472e-002
2.1500e+000	1.3024e-004	2.1137e-003	8.6568e-001	8.7015e-002
4.6400e+000	1.9000e-004	2.1107e-003	8.6431e-001	1.8779e-001
1.0000e+001	2.8000e-004	2.1085e-003	8.6294e-001	4.0472e-001
2.1500e+001	4.1000e-004	2.1070e-003	8.6158e-001	8.7015e-001
4.6400e+001	6.0000e-004	2.1060e-003	8.6021e-001	1.8779
1.0000e+002	8.8400e-004	2.1050e-003	8.5884e-001	4.0472

Fig. 4.26. Long 10m cable circuit (Example 5)

Table 4.1: CPU Time Comparison (Example 5)

Simulations	DEPACT (sec)	MRA (sec)	Conventional Lumped Model (sec)
Example 5	4	463	1272

Fig. 4.27. Transient response of far-end (Example 5)

4.5.6 Example 6

In this example, we consider a PCB interconnect network (referred to as Line-6 in [34]), which consists of three lossless lines and two frequency-dependent coupled lossy lines (Fig. 4.28). As seen from the figure, the lossless lines are 5cm long, and have a delay $T_D=1\text{ns}$ and a characteristic impedance $Z_0=50\Omega$. The network is analyzed for different lengths of the lossy lines (5cm, 20cm and 40 cm) and the frequency-dependent p.u.l. parameters can be found in [34]. The input voltage is a unit step response with a rise time of 0.035ns. Fig. 4.29 - Fig. 4.31 show the transient response using the proposed DEPACT method for a line length of 40cm and $m=12$. Table 4.2 gives the comparison of the CPU cost for DEPACT versus conventional uniform lumped segmentation and MRA for the various line lengths. As seen from the table, the proposed method provides 1-2 orders of magnitude speed-ups compared to MRA and 2-3 orders of magnitude speed-ups compared to

uniform segmentation. Also, it is important to note that the speed-up of DEPACT improves as the line length increases (due to the fact that the delay extraction mechanism is employed prior to performing MRA). This is one of the main advantages of DEPACT, in contrast to MRA, which suffers for the case of long lines.

Fig. 4.28. PCB interconnect network (Example 6)

Table 4.2: CPU Time Comparison (Example 6)

Simulations	Conventional Lumped Model (sec)	MRA (sec)	DEPACT (sec)
Example 6 (5cm)	32	4.2	0.89
Example 6 (20 cm)	292	25	2.5
Example 6 (40 cm)	4641	74	4.3

Fig. 4.29. Transient response of active line near-end (Example 6)

Fig. 4.30. Transient response of active line far-end (Example 6)

Fig. 4.31. Transient response of victim line far-end (Example 6)

4.5.7 Example 7

As discussed in section 3.5, the sharper the rise time, the higher the maximum frequency of interest (i.e. larger bandwidth), which in turn, results in the requirement of more DEPACT cells. This example illustrates the impact of the frequency bandwidth on the approximation order (m) of the DEPACT macromodel. The example (Fig. 4.32) consists of two 10cm coupled lines with the following parameters [25]

$$\begin{aligned}
 R &= \begin{bmatrix} 1.24 & 0 \\ 0 & 1.24 \end{bmatrix} \Omega/\text{cm} & L &= \begin{bmatrix} 3.361 & 0.865 \\ 0.865 & 3.361 \end{bmatrix} \text{nH}/\text{cm} \\
 C &= \begin{bmatrix} 1.29 & -0.197 \\ -0.197 & 1.29 \end{bmatrix} \text{pF}/\text{cm} & G &= 0 \\
 d &= 10\text{cm}
 \end{aligned} \tag{4.53}$$

Several values for the rise time of the unit step input voltage source were considered. For a maximum allowable absolute error of 0.1%, the order of the realization was determined using the method described in section 3.5. Table 4.3 shows the order of the realization used by DEFACT for different rise times. As expected, the order m increases, as the rise time decreases. For verification, samples of the corresponding time-domain responses are shown in Fig. 4.33 - Fig. 4.36.

Fig. 4.32. Coupled interconnect circuit (Example 7)

Table 4.3: DEFACT Order m versus Rise Time (Example 7)

Rise Time	f_{max} (GHz)	DEFACT (order m)
0.5ns	2	3
0.1ns	10	6
0.02ns	50	12

Fig. 4.33. Transient response of active line far-end (rise time=0.5ns)

Fig. 4.34. Transient response of victim line far-end (rise time=0.5ns)

Fig. 4.35. Transient response of active line far-end (rise time=0.1ns)

Fig. 4.36. Transient response of victim line far-end (rise time=0.02ns)

4.6 Summary

In summary, this chapter described how the MLF-II equations derived in chapter 3 can be realized by the proposed DEPACT macromodel. The macromodel consists of a cascade of lossy and lossless subnetworks (referred to as DEPACT cells), and each of these cells can be represented by a cascade of lossy and lossless lines. MRA is used for the realization of the lossy lines, and a new approach based on MoC is used for the lossless lines. The equations for the lossless sections were derived, and equivalent circuit representations were presented. In the special case of lines with frequency-independent parameters, simplified macromodels were derived. All of the derived macromodels can be directly linked to circuit simulators. Finally, numerical results were presented which validate the accuracy and efficiency of the proposed DEPACT method. The next chapter reviews the concepts of passivity, and provides a proof of passivity for the DEPACT macromodel.

CHAPTER 5

Passivity of the DEPACT Macromodel

5.1 Overview of Passivity

Passivity implies that a network cannot generate more energy than it absorbs [39]-[41]. Transmission lines, by nature, are passive structures. Thus, when macromodeling, the macromodel *must* be passive, in order to accurately and efficiently represent the interconnect. It is important to note, that if a system is stable, it is not guaranteed to be passive. A network is stable if, for all bounded inputs, the output is bounded. If a network is not stable, the output response will grow as time approaches infinity and spurious oscillations occur. For a network to be stable, the poles of the transfer function must be in the left half plane. Passivity, on the other hand, automatically guarantees that the system is stable. Preserving the passivity is crucial, because stable, but non-passive macromodels can produce unstable networks when connected to other stable, even passive loads [16]. This is demonstrated in Fig. 5.1, which shows the transient response of a stable, but non-passive macromodel, when connected to an external resistor. In addition to having

spurious oscillations in the time-domain response, simulating non-passive macromodels can lead to numerous errors from the circuit simulator such as ‘time step too small’ or ‘failure to converge’ etc., and these errors can make circuit simulation very challenging and tedious.

Fig. 5.1. Transient response of a stable but non-passive macromodel with passive terminations

To illustrate the importance of passivity, consider a single port network shown in Fig. 5.2(a). As seen from the transfer function, since the poles are in the left half plane, the network is stable, however, it is not guaranteed to be passive. Next, the macromodel is terminated with a passive load as shown in Fig. 5.2(b). Although the termination is passive, this leads to an unstable overall network (since the poles of the total transfer function are now in the right half plane). This emphasizes the fact that the macromodel must be passive, in order to have an overall stable network. Next, we will review the important conditions that are necessary to preserve the passivity of the macromodel.

Fig. 5.2. Example of importance of passivity

5.2 Review of Passivity Conditions

The necessary and sufficient conditions for a linear N -port system admittance matrix $Y(s)$ to be passive are the following [41]

- 1) Each element of $Y(s)$ is analytic in $\text{Re}(s) > 0$.
- 2) $Y(s^*) = Y^*(s)$ for all s , where $*$ is the complex conjugate operator.

3) $Y(s)$ is a positive-real (PR) matrix. This means that the product $\mu^*T[Y^T(s^*) + Y(s)]\mu \geq 0$ for all possible values of s satisfying $\text{Re}(s) > 0$ and for any arbitrary matrix μ . For the practical case of networks with symmetric admittance matrices, the matrix $\text{Real}(Y(s)) = [Y^T(s^*) + Y(s)]/2$ must be non-negative definite for all s with $\text{Re}(s) > 0$ (i.e. the eigenvalues of $\text{Real}(Y(s))$ must be non-negative for all values of frequency).

As mentioned in chapter 2, the main disadvantage with the MoC approach is that it does not guarantee the passivity of the macromodel. To illustrate the passivity violation associated with MoC, consider two coupled lossy lines with frequency-dependent parameters referred to as Line-2 in [34]. In order to check that the MoC macromodel is passive, the eigenvalues of $\text{Real}(Y(s))$ were computed (condition (3)). Fig. 5.3 shows the frequency spectrum of one of the eigenvalues. As seen from the plot, passivity is clearly violated since the eigenvalue is negative at higher frequencies.

In contrast to MoC, the proposed DEPACT macromodel is guaranteed to be passive. To prove this fact, we need to prove the passivity of both the lossy sections, realized by (2.22) and (2.23), and the lossless sections realized by (4.1)-(4.25). This will be proven in the next sections.

Fig. 5.3. Illustration of passivity violation using MoC

5.3 Passivity of the DEPACT Cell

5.3.1 Passivity of Lossy Sections

It is to be noted that in (2.22) and (2.23), the matrix Z is defined in terms of the p.u.l. parameters and the delayless (lossy) portion of the transmission line stamp as

$Z(s) = (A(s) + sB_a(s))/m$, where

$$A(s) = \begin{bmatrix} 0 & -R(s) \\ -G(s) & 0 \end{bmatrix}; \quad B_a(s) = \begin{bmatrix} 0 & -L_a(s) \\ -C_a(s) & 0 \end{bmatrix} \quad (5.1)$$

In [9] and [21], it was shown that the passivity of the MRA realization given by (2.22) and (2.23) is guaranteed if $Z_L(s) = (R(s) + sL_a(s))$ and $Y_L(s) = (G(s) + sC_a(s))$ are positive-real (PR) rational function matrices. Several

techniques for approximating Z_L and Y_L in the form of PR rational functions can be found in [10] and [23].

5.3.2 Passivity of Lossless Sections

To prove the passivity of the lossless realization given by (4.1)-(4.25), the admittance matrix $Y(s)$ of the multiport lossless section is written in the form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Y(s) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (5.2)$$

where I_1 , V_1 , I_2 , V_2 are the near and far-end current and voltage vectors, respectively. Using (4.3), (5.2) can be re-written in the form

$$\begin{bmatrix} E_I & 0 \\ 0 & E_I \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = Y(s) \begin{bmatrix} E_V & 0 \\ 0 & E_V \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} \quad (5.3)$$

or

$$\begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} E_I^{-1} & 0 \\ 0 & E_I^{-1} \end{bmatrix} Y(s) \begin{bmatrix} E_V & 0 \\ 0 & E_V \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} \quad (5.4)$$

However, the modal currents and voltages are related through the solution of (4.8) as

$$\begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = M \hat{Y}(s) M^T \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} \quad (5.5)$$

where $\hat{Y}(s)$ is a block diagonal matrix of the form

$$\hat{Y}(s) = \text{diag}[\hat{Y}_1(s), \dots, \hat{Y}_k(s), \dots, \hat{Y}_n(s)] \quad (5.6)$$

with $\hat{Y}_k(s)$ being the input admittance matrix of the k^{th} lossless line with p.u.l. parameters $\hat{l}_k > 0$ and $\hat{c}_k > 0$, given by the decoupled set of equations (4.24) and (4.25), respectively. In (5.5), the matrix M represents a real permutation matrix.

Using (5.4) and (5.5), we can write

$$\begin{bmatrix} E_I^{-1} & 0 \\ 0 & E_I^{-1} \end{bmatrix} Y(s) \begin{bmatrix} E_V & 0 \\ 0 & E_V \end{bmatrix} = M \hat{Y}(s) M^T \quad (5.7)$$

Re-arranging (5.7) yields

$$\begin{aligned} Y(s) &= \begin{bmatrix} E_I & 0 \\ 0 & E_I \end{bmatrix} M \hat{Y}(s) M^T \begin{bmatrix} E_V^{-1} & 0 \\ 0 & E_V^{-1} \end{bmatrix} \\ &= \begin{bmatrix} E_I & 0 \\ 0 & E_I \end{bmatrix} M \hat{Y}(s) M^T \begin{bmatrix} E_I & 0 \\ 0 & E_I \end{bmatrix}^T \\ &= \chi [\hat{Y}(s)] \chi^T \end{aligned} \quad (5.8)$$

where the matrix χ is defined as

$$\chi = \begin{bmatrix} E_I & 0 \\ 0 & E_I \end{bmatrix} M \quad (5.9)$$

Passivity conditions (1) and (2) are satisfied since $\hat{Y}(s) = \text{diag}\{\hat{Y}_k(s)\}$ is analytic in $\text{Re}(s) > 0$ and χ is a real matrix. In addition, $\hat{Y}(s)$ is a positive-real matrix since

$\hat{Y}_k(s)$ is positive-real. Using the fact that χ is a real matrix, it follows from (5.8)

that $Y(s) = \chi[\hat{Y}(s)]\chi^T$ is positive-real [41], thus satisfying the third condition of passivity.

5.4 Summary

In summary, this chapter demonstrated that passivity is vital to ensure accurate and efficient macromodeling. In order for the proposed macromodel to be passive, both the lossy and lossless sections must be passive. In this chapter, a proof of passivity was given for both the lossy and lossless sections, which demonstrates that the overall DEFACT macromodel is guaranteed to be passive by construction.

CHAPTER 6

Sensitivity Analysis

6.1 Introduction

Accurate and efficient sensitivity analysis with respect to transmission line parameters is often required by designers in order to make the proper trade-offs, generally between conflicting design requirements, to obtain the best possible performance. Sensitivity analysis is of significant importance in identifying critical design components, in tolerance assignments and in optimizing the overall interconnect network performance [31], [42]-[45].

Several techniques have been presented in the literature for modeling and sensitivity analysis of distributed interconnects [4]-[17], [42]-[45], [46], [47]. Among the most commonly used algorithms are those based on the method of characteristics (MoC) [2], [19] (a review of MoC has been given in chapter 2). Also, a fast algorithm for sensitivity analysis based on MoC was proposed in [43]. Although MoC provides fast solutions for long low-loss lines, as mentioned before, it does not guarantee the passivity of the resulting macromodels. Recall that passivity is essential since non-

passive but stable models when coupled with arbitrary nonlinear elements can lead to unstable systems (chapter 5). To address this issue, algorithms based on MRA were introduced (chapter 2) and extension to sensitivity analysis was described in [46]. However, in the presence of large delay lines, MRA requires high-order approximations [18]. On the other hand, the proposed delay extraction-based method (DEPACT) (chapters 3-5) combines the merits of both MoC and MRA and is based on delay extraction prior to performing MRA.

In this chapter, extension of the DEPACT algorithm to sensitivity analysis is described [28]-[31]. A major advantage of the new sensitivity analysis algorithm is that, the sensitivity of the circuit matrices is obtained analytically in terms of the p.u.l. parameters of the line and pre-determined coefficients, leading to significant computational cost advantages. In addition, the proposed method enables sensitivity analysis with respect to both electrical and physical parameters and can handle transmission lines with lossy as well as frequency-dependent parameters.

For illustrative purposes, recall the configuration of the proposed DEPACT macromodel shown in Fig. 6.1. To aid in the description of the sensitivity analysis algorithm, the cascade of m DEPACT cells (which replaces the transmission line) is referred to as subnetwork π , while the lossy subnetwork realizing the middle term in the DEPACT cell is referred to as subnetwork ϕ .

Fig. 6.1. DEFACT Macromodel realization using MLF-II

In order to calculate the network sensitivity with respect to any interconnect parameter λ , the sensitivity of the DEFACT macromodel with respect to λ is required. This in turn, requires the sensitivity of the MRA based lossy sections (middle section in Fig. 6.1) and the delay based lossless sections (outer sections in Fig. 6.1) with respect to λ . Once these sensitivities are obtained, they can be combined with the sensitivity equations describing the rest of the circuit.

This chapter is organized as follows. Section 6.2 describes the sensitivity analysis of the lossy sections. Section 6.3 derives the sensitivity equations for the lossless sections and presents an equivalent circuit representation for the analysis. Section 6.4 discusses the special case of lines with frequency-independent parameters and

the corresponding sensitivity equations. In section 6.5, the unified sensitivity equations for the overall network are given, and section 6.6 describes sensitivity with respect to physical parameters. Section 6.7 presents an overview of the delay extraction-based sensitivity analysis approach and how it is interfaced with conventional circuit simulators. Finally, section 6.8 presents several numerical results which demonstrate the accuracy and efficiency of the new sensitivity analysis method.

6.2 Sensitivity Analysis of Lossy Sections

Eqs. (2.22) and (2.23) (chapter 2) for the lossy section (subnetwork ϕ) represented in Fig. 6.1 can be translated into a set of ordinary differential equations in the following form [10]

$$\begin{aligned} C_\phi \dot{x}_\phi + G_\phi x_\phi &= B_\phi i_\phi \\ v_\phi &= (B_\phi)^T x_\phi \end{aligned} \tag{6.1}$$

where

- $C_\phi, G_\phi \in \mathbb{R}^{N_\phi \times N_\phi}$ contain the stamps of the lumped memory and memoryless elements of subnetwork ϕ , and can be obtained in a closed form in terms of the p.u.l. parameters of the line.
- $\mathbb{R}^{N_\phi}$ is the node space of the subnetwork ϕ .
- $B_\phi \in \mathbb{R}^{N_\phi \times 2N}$ is a selector matrix, (with entries 0 or 1).

- i_ϕ and $v_\phi \in \mathfrak{R}^{2N}$ represent the terminal currents and voltages of subnetwork ϕ , respectively.
- Superscript ' T ' denotes the transpose

Using (6.1), the sensitivity equations can be written as [31]

$$\begin{aligned} C_\phi \frac{\partial \dot{x}_\phi}{\partial \lambda} + G_\phi \frac{\partial x_\phi}{\partial \lambda} &= B_\phi \frac{\partial i_\phi}{\partial \lambda} - \frac{\partial C_\phi}{\partial \lambda} \dot{x}_\phi - \frac{\partial G_\phi}{\partial \lambda} x_\phi \\ \frac{\partial v_\phi}{\partial \lambda} &= (B_\phi)^T \frac{\partial i_\phi}{\partial \lambda} \end{aligned} \quad (6.2)$$

where λ represents any interconnect parameter of interest.

A detailed computation of $\frac{\partial C_\phi}{\partial \lambda}$ and $\frac{\partial G_\phi}{\partial \lambda}$ in terms of the pre-determined coefficients and the sensitivity of the p.u.l. parameters was described in [46]. We will later combine these equations with the sensitivity equations of the lossless sections and the rest of the circuit.

6.3 Sensitivity Analysis of Lossless Sections

Recall from chapter 4 that the terminal voltages and currents of the lossless sections can be expressed as

$$v(x, t) = E_V \hat{v}(x, t); \quad i(x, t) = E_I \hat{i}(x, t) \quad (6.3)$$

where E_I and E_V are modal transformation matrices (given by (4.20)-(4.23)).

Differentiating (6.3) with respect to λ , the sensitivity of the actual voltages and currents can be expressed in terms of the sensitivity of the modal voltages and currents by the following relation

$$\begin{aligned}\frac{\partial v(x, t)}{\partial \lambda} &= E_V \frac{\partial \hat{v}(x, t)}{\partial \lambda} + \frac{\partial E_V}{\partial \lambda} \hat{v}(x, t) \\ \frac{\partial i(x, t)}{\partial \lambda} &= E_I \frac{\partial \hat{i}(x, t)}{\partial \lambda} + \frac{\partial E_I}{\partial \lambda} \hat{i}(x, t)\end{aligned}\tag{6.4}$$

Examining (6.4), it can be seen that $\frac{\partial i(x, t)}{\partial \lambda}$ and $\frac{\partial v(x, t)}{\partial \lambda}$ can be determined based on the sensitivity of the modal voltages and currents $\hat{i}(x, t)$ and $\hat{v}(x, t)$, and the sensitivity of the modal transformation matrices E_I and E_V . The details involved in determining these sensitivities are presented in the following sections.

6.3.1 Sensitivity of the Modal Voltages and Currents

It can be proved that the sensitivity equations for the lossless sections can be written as follows [31]

$$\begin{aligned}\frac{\partial}{\partial \lambda} \hat{v}_1(t) - \hat{Z} \frac{\partial \hat{i}_1}{\partial \lambda} - E_1 &= \tilde{w}_1(t) \\ \frac{\partial}{\partial \lambda} \hat{v}_2(t) - \hat{Z} \frac{\partial \hat{i}_2}{\partial \lambda} - E_2 &= \tilde{w}_2(t) \\ \tilde{w}_1(t) &= 2 \frac{\partial}{\partial \lambda} \hat{v}_2(t - \tau) - \tilde{w}_2(t - \tau) \\ \tilde{w}_2(t) &= 2 \frac{\partial}{\partial \lambda} \hat{v}_1(t - \tau) - \tilde{w}_1(t - \tau)\end{aligned}\tag{6.5}$$

where the independent sources E_1 and E_2 are obtained from the solution of (4.15)-(4.18) and are given by

$$\begin{aligned}
E_1 &= \frac{\partial \hat{Z}}{\partial \lambda} \hat{i}_1(t) + \frac{\partial \tau}{\partial \lambda} \left[\frac{\partial}{\partial t} w_2(t-\tau) - 2 \frac{\partial}{\partial t} \hat{v}_2(t-\tau) \right] \\
E_2 &= \frac{\partial \hat{Z}}{\partial \lambda} \hat{i}_2(t) + \frac{\partial \tau}{\partial \lambda} \left[\frac{\partial}{\partial t} w_1(t-\tau) - 2 \frac{\partial}{\partial t} \hat{v}_1(t-\tau) \right]
\end{aligned} \tag{6.6}$$

Proof

In order to obtain the sensitivity equations in (6.5), we first consider the solution to the modal equations in the Laplace domain

$$\hat{V}_1 - \hat{Z} \hat{I}_1 = W_1 \tag{6.7}$$

$$\hat{V}_2 - \hat{Z} \hat{I}_2 = W_2 \tag{6.8}$$

$$W_1 = \Gamma [2\hat{V}_2 - W_2] \tag{6.9}$$

$$W_2 = \Gamma [2\hat{V}_1 - W_1] \tag{6.10}$$

where $\Gamma = \text{diag} \left\{ e^{-\hat{\gamma}_i d} \right\}$, and the modal propagation constant $\hat{\gamma}_i$ is defined as

$$\hat{\gamma}_i = s \sqrt{\hat{l}_i \hat{c}_i} \tag{6.11}$$

Differentiating (6.9) with respect to λ , we get

$$\frac{\partial W_1}{\partial \lambda} = \Gamma \left[2 \frac{\partial \hat{V}_2}{\partial \lambda} - \frac{\partial W_2}{\partial \lambda} \right] - \frac{\partial \Gamma}{\partial \lambda} [2\hat{V}_2 - W_2] \tag{6.12}$$

where $\frac{\partial \Gamma}{\partial \lambda} = \text{diag} \left\{ d e^{-\hat{\gamma}_i d} \frac{\partial \hat{\gamma}_i}{\partial \lambda} \right\}$.

Using the relation $d \frac{\partial \hat{\gamma}_i}{\partial \lambda} = s \frac{\partial (d \sqrt{l_i \hat{c}_i})}{\partial \lambda} = s \frac{\partial \tau_i}{\partial \lambda}$, (6.12) can be re-written as

$$\frac{\partial W_1}{\partial \lambda} = \text{diag} \left\{ e^{-s\tau_i} \right\} \left[2 \frac{\partial \hat{V}_2}{\partial \lambda} - \frac{\partial W_2}{\partial \lambda} \right] - \text{diag} \left\{ s e^{-s\tau_i} \frac{\partial \tau_i}{\partial \lambda} \right\} [2 \hat{V}_2 - W_2] \quad (6.13)$$

Converting (6.13) to the time-domain yields

$$\frac{\partial w_1(t)}{\partial \lambda} = \left[2 \frac{\partial}{\partial \lambda} \hat{v}_2(t-\tau) - \frac{\partial}{\partial \lambda} w_2(t-\tau) \right] \frac{\partial \tau}{\partial \lambda} \left[2 \frac{\partial}{\partial t} \hat{v}_2(t-\tau) - \frac{\partial}{\partial t} w_2(t-\tau) \right] \quad (6.14)$$

Similarly, differentiating (6.10) with respect to λ , we obtain

$$\frac{\partial w_2(t)}{\partial \lambda} = \left[2 \frac{\partial}{\partial \lambda} \hat{v}_1(t-\tau) - \frac{\partial}{\partial \lambda} w_1(t-\tau) \right] \frac{\partial \tau}{\partial \lambda} \left[2 \frac{\partial}{\partial t} \hat{v}_1(t-\tau) - \frac{\partial}{\partial t} w_1(t-\tau) \right] \quad (6.15)$$

Next, differentiating (6.7) and (6.8), we can write

$$\frac{\partial \hat{v}_1(t)}{\partial \lambda} - \frac{\partial \hat{Z}}{\partial \lambda} \hat{i}_1(t) - \hat{Z} \frac{\partial \hat{i}_1(t)}{\partial \lambda} = \frac{\partial w_1(t)}{\partial \lambda} \quad (6.16)$$

$$\frac{\partial \hat{v}_2(t)}{\partial \lambda} - \frac{\partial \hat{Z}}{\partial \lambda} \hat{i}_2(t) - \hat{Z} \frac{\partial \hat{i}_2(t)}{\partial \lambda} = \frac{\partial w_2(t)}{\partial \lambda} \quad (6.17)$$

Combining (6.14)-(6.17), and with some simple mathematical manipulation, the sensitivity equations in (6.5) can be obtained. In section 6.3.3, an equivalent circuit representation for the sensitivity equations in (6.5) will be presented.

6.3.2 Sensitivity of the Modal Transformation Matrices

The next step is to calculate the derivatives of the modal transformation matrices

with respect to the parameter λ . In order to obtain $\frac{\partial E_I}{\partial \lambda}$ and $\frac{\partial E_V}{\partial \lambda}$, we use (4.20) and

(4.21) to get

$$\begin{aligned}\frac{\partial E_I}{\partial \lambda} &= \frac{\partial Q}{\partial \lambda} F S \alpha + Q \frac{\partial F}{\partial \lambda} S \alpha + Q F \frac{\partial S}{\partial \lambda} \alpha + Q F S \frac{\partial \alpha}{\partial \lambda} \\ \frac{\partial E_V}{\partial \lambda} &= -E_V \left(\frac{\partial E_I^T}{\partial \lambda} \right) E_I^{-1}\end{aligned}\tag{6.18}$$

Recall that $Q = \{Q_j\}$ is a matrix of the right eigenvectors of C_m and

$F^2 = \text{diag}\{F_j^2\}$ is a diagonal matrix with the eigenvalues of C_m . The derivatives

$\frac{\partial Q}{\partial \lambda}$ and $\frac{\partial F^2}{\partial \lambda}$ can be calculated by solving the following system of equations [47]

$$\begin{bmatrix} F_j^2 U_N - C_m & Q_j \\ Q_j^T & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial Q_j}{\partial \lambda} \\ \frac{\partial F_j^2}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \frac{\partial C_m}{\partial \lambda} Q_j \\ 0 \end{bmatrix}\tag{6.19}$$

where $U_N \in \mathbb{R}^{N \times N}$ is the identity matrix, and F_j^2 and Q_j represent the j^{th} eigenvalue and its corresponding eigenvector, respectively. Once the solution for

(6.19) is obtained, $\frac{\partial F_j^2}{\partial \lambda}$ can be computed from the following relationship

$$\frac{\partial F_j}{\partial \lambda} = \frac{1}{2} F_j^{-1} \frac{\partial F_j^2}{\partial \lambda}\tag{6.20}$$

where $\frac{\partial \mathbf{F}}{\partial \lambda} = \text{diag}\left\{\frac{\partial F_j}{\partial \lambda}\right\}$.

Similarly, since $\mathbf{S} = \{\mathbf{S}_j\}$ contains the right eigenvectors of the product

$\mathbf{FQ}^T \mathbf{L}_m \mathbf{QF}$, $\frac{\partial \mathbf{S}}{\partial \lambda}$ can be determined by solving

$$\begin{bmatrix} \Lambda_j^2 \mathbf{U}_N - (\mathbf{FQ}^T \mathbf{L}_m \mathbf{QF}) & \mathbf{S}_j \\ \mathbf{S}_j^T & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{S}_j}{\partial \lambda} \\ \frac{\partial \Lambda_j^2}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \frac{\partial (\mathbf{FQ}^T \mathbf{L}_m \mathbf{QF})}{\partial \lambda} \mathbf{S}_j \\ 0 \end{bmatrix} \quad (6.21)$$

Note that $\frac{\partial (\mathbf{FQ}^T \mathbf{L}_m \mathbf{QF})}{\partial \lambda}$ in (6.21) can be obtained in terms of $\frac{\partial \mathbf{L}_m}{\partial \lambda}$ and using (6.19)

and (6.20).

Next, the derivative of the normalizing diagonal matrix α can be computed using

(4.22) as

$$\frac{\partial \alpha_{ii}}{\partial \lambda} = \frac{-2\alpha_{ii}^2 \left(\sum_{k=1}^N E_{ki} \frac{\partial E_{ki}}{\partial \lambda} \right)}{2\alpha_{ii} \left(\sum_{k=1}^N E_{ki}^2 \right)} \quad (6.22)$$

where

$$\begin{aligned} \mathbf{E} &= \mathbf{QFS} \\ &= \{E_{ki} ; k = 1, 2, \dots, N; i = 1, 2, \dots, N\} \end{aligned} \quad (6.23)$$

Differentiating (6.23) with respect to λ and combining with (6.19)-(6.22), the sensitivity of the modal transformation matrices ((6.18)) can be determined.

Next, knowing the sensitivity of $\hat{i}(t)$, $\hat{v}(t)$, E_I and E_V with respect to λ , the sensitivity of the actual terminal voltages $(\frac{\partial v(t)}{\partial \lambda})$ and currents $(\frac{\partial i(t)}{\partial \lambda})$ of the lossless sections can be computed using (6.4).

6.3.3 Equivalent Circuit Representation for Sensitivity Analysis of Lossless Sections

Using the proposed method, an equivalent circuit representing the sensitivity analysis of the lossless sections is shown in Fig. 6.2. The sensitivities of the actual terminal voltages and currents are obtained in terms of the sensitivities of the modal voltages and currents through the transformation matrices E_I and E_V . The sensitivities of the modal voltages and currents are related through smaller subcircuits which consist of dependent delay sources in the time-domain, lumped impedances, and independent time-domain sources (E_1 and E_2), which are obtained from the solution of the original circuit (shown in Fig. 4.4).

Fig. 6.2. Equivalent circuit for sensitivity analysis of lossless sections

6.4 Special Case: Frequency-Independent Parameters

As discussed in chapter 4, in the case of lines with frequency-independent parameters, the delay extracted exponential matrix representing a lossy section can be expressed as [27]

$$e^{\frac{A}{m}} \quad (6.24)$$

For the case of single lines, it was shown that the exponential matrix above can be realized using the equivalent resistive circuit of the form shown in Fig. 4.7. In order to calculate the sensitivity of the resistive network in Fig. 4.7, the nodal equations for the circuit can be written as

$$\mathbf{G}_\phi \mathbf{v}_\phi = \mathbf{i}_\phi \quad (6.25)$$

where $\mathbf{G}_\phi \in \mathbb{R}^{2 \times 2}$, and $\mathbf{i}_\phi$ and $\mathbf{v}_\phi \in \mathbb{R}^{2 \times 1}$ represent the terminal voltages and currents of the lossy section, respectively. The matrix $\mathbf{G}_\phi$ can be written as

$$\mathbf{G}_\phi = \begin{bmatrix} \frac{1}{r_a} + \frac{1}{r_b} & -\frac{1}{r_b} \\ -\frac{1}{r_b} & \frac{1}{r_a} + \frac{1}{r_b} \end{bmatrix} \quad (6.26)$$

Differentiating (6.25) with respect to λ , the sensitivity equations for the resistive network can be written as [31]

$$\mathbf{G}_\phi \frac{\partial \mathbf{v}_\phi}{\partial \lambda} = \frac{\partial \mathbf{i}_\phi}{\partial \lambda} - \frac{\partial \mathbf{G}_\phi}{\partial \lambda} \mathbf{v}_\phi \quad (6.27)$$

where $\frac{\partial \mathbf{G}_\phi}{\partial \lambda}$ is expressed as

$$\frac{\partial \mathbf{G}_\phi}{\partial \lambda} = \begin{bmatrix} \left(\frac{-1}{r_a^2} \right) \frac{\partial r_a}{\partial \lambda} - \left(\frac{1}{r_b^2} \right) \frac{\partial r_b}{\partial \lambda} & \left(\frac{1}{r_b^2} \right) \frac{\partial r_b}{\partial \lambda} \\ \left(\frac{1}{r_b^2} \right) \frac{\partial r_b}{\partial \lambda} & \left(\frac{-1}{r_a^2} \right) \frac{\partial r_a}{\partial \lambda} - \left(\frac{1}{r_b^2} \right) \frac{\partial r_b}{\partial \lambda} \end{bmatrix} \quad (6.28)$$

Using the definition of r_a and r_b given in (4.38), the derivatives $\frac{\partial r_a}{\partial \lambda}$ and $\frac{\partial r_b}{\partial \lambda}$ can be obtained in terms of the sensitivities of the p.u.l. parameters with respect to λ .

For coupled lossy lines, recall that transformation matrices were used to decouple the lossy sections into resistive networks of the form depicted in Fig. 4.7. In this case, the sensitivity of the terminal voltages and currents of the lossy sections can be determined using P_I , P_V , $\frac{\partial P_I}{\partial \lambda}$, $\frac{\partial P_V}{\partial \lambda}$ and the sensitivity equations for each individual resistive network described by (6.27). The sensitivity of the transformation matrices P_I and P_V can be computed following a similar approach as in section 6.3.2 (for lossless sections), except by replacing C_m and L_m , with G_m and R_m , respectively.

6.5 Unified Sensitivity Equations

In general, distributed networks in the presence of nonlinear elements can be expressed as [4]

$$C_\pi \dot{\mathbf{x}}_\pi(t) + G_\pi \mathbf{x}_\pi(t) + D \mathbf{i}_\pi(t) + \mathbf{f}_\pi(\mathbf{x}_\pi(t)) = \mathbf{b}_\pi(t) \quad (6.29)$$

$$\mathbf{I}_\pi(s) = \mathbf{Y}_\pi(s) \mathbf{V}_\pi(s) \quad (6.30)$$

where

- $\mathbf{x}_\pi(t)$ is a vector of the unknown voltages and currents, G_π , C_π , $\mathbf{b}_\pi(t)$ and $\mathbf{f}_\pi(t)$ describe the lumped components, and $D = [d_{i,j} \in \{0, 1\}]$ is a selector matrix.

- $Y_\pi(s)$ is a $2N \times 2N$ y -parameter matrix representing the interconnect subnetwork π , where N is the number of coupled conductors.
- $V_\pi(s)$ and $I_\pi(s)$ represent the Laplace terminal voltages and currents of the interconnect subnetwork π , respectively.

To simplify notation, (6.29) and (6.30) assume only one set of coupled lines, however, extension to multiple sets of coupled lines is straightforward. To illustrate the formulation scheme described in (6.29) and (6.30), consider the circuit shown in Fig. 6.3. The network equations can be written as in (6.29) where

$$\begin{aligned}
 \mathbf{x}_\pi &= \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} & \mathbf{C}_\pi &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 & C_1 \end{bmatrix} & \mathbf{G}_\pi &= \begin{bmatrix} G_1 + G_2 & -G_2 & 0 & 0 & 0 \\ -G_2 & G_2 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{b}_\pi(t) &= \begin{bmatrix} i_{src}(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \mathbf{f}_\pi(\mathbf{x}_\pi(t)) &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ f(v_1 - v_4) \\ 0 \end{bmatrix} & \mathbf{D} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \tag{6.31}$$

and the equations describing the distributed transmission lines can be written as shown in (6.30) with

$$\mathbf{I}_\pi(s) = \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} \quad \mathbf{V}_\pi(s) = \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} \quad \mathbf{Y}_\pi(s) = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) & Y_{13}(s) & Y_{14}(s) \\ Y_{21}(s) & Y_{22}(s) & Y_{23}(s) & Y_{24}(s) \\ Y_{31}(s) & Y_{32}(s) & Y_{33}(s) & Y_{34}(s) \\ Y_{41}(s) & Y_{42}(s) & Y_{43}(s) & Y_{44}(s) \end{bmatrix} \tag{6.32}$$

Fig. 6.3. Example circuit with lumped, distributed and nonlinear devices

Using the proposed DEPACT macromodel, (6.30) can be replaced with an equivalent time-domain representation using m DEPACT cells as described in chapter 4.

Next, the sensitivity of the nonlinear network with respect to the parameter λ can be obtained using (6.29) as

$$C_{\pi} \frac{dz_{\pi}(t)}{dt} + \left(G_{\pi} + \frac{\partial f(x_{\pi}(t))}{\partial x_{\pi}(t)} \right) z_{\pi}(t) + E(t) + D \frac{\partial i_{\pi}(t)}{\partial \lambda} = 0 \quad (6.33)$$

where

$$\begin{aligned} E(t) &= \frac{\partial C_{\pi}}{\partial \lambda} \frac{dx_{\pi}(t)}{dt} + \frac{\partial G_{\pi}}{\partial \lambda} x_{\pi}(t) + \frac{\partial f(x_{\pi}(t))}{\partial \lambda} \\ z_{\pi} &= \frac{\partial x_{\pi}(t)}{\partial \lambda} \end{aligned} \quad (6.34)$$

Eqs. (6.2), and (6.4)-(6.6) represent an implicit relationship between the sensitivities of the terminal currents $\frac{\partial i_k(t)}{\partial \lambda}$ and terminal voltages $\frac{\partial v_k(t)}{\partial \lambda}$ of the k^{th}

DEPACT cell (for $k=1,2,\dots,m$). Combining these equations with (6.33), we get a consistent set of ordinary differential equations that can be solved to compute the required sensitivity information. These ordinary differential equations can be easily interfaced with commercial circuit simulators (such as HSPICE) by converting them to an equivalent circuit representation which can be directly embedded in the simulator netlist. Several efficient techniques have been previously proposed to perform this conversion [4], [33].

6.6 Sensitivity with respect to Physical Parameters

When studying the sensitivity of distributed networks, the design parameters of interconnects are usually required with respect to physical parameters (such as width and spacing of conductors). The sensitivities of electrical parameters are often intermediate steps to the calculation of sensitivities of physical parameters. In the case where λ represents a physical parameter of an interconnect, the sensitivity of the output nodes can be obtained as follows

$$\frac{\partial \mathbf{x}_\pi}{\partial \lambda} = \sum_{i=1}^N \sum_{j=1}^N \left(\frac{\partial \mathbf{x}_\pi}{\partial R_{i,j}} \frac{\partial R_{i,j}}{\partial \lambda} + \frac{\partial \mathbf{x}_\pi}{\partial L_{i,j}} \frac{\partial L_{i,j}}{\partial \lambda} + \frac{\partial \mathbf{x}_\pi}{\partial G_{i,j}} \frac{\partial G_{i,j}}{\partial \lambda} + \frac{\partial \mathbf{x}_\pi}{\partial C_{i,j}} \frac{\partial C_{i,j}}{\partial \lambda} \right) \quad (6.35)$$

where $R_{i,j}$, $L_{i,j}$, $G_{i,j}$, and $C_{i,j}$ are the per-unit-length parameters and the subscripts i and j are matrix indices.

6.7 Interface to Circuit Simulator

As previously mentioned, one of the main advantages of the proposed DEPACT macromodel is that, it can be directly linked to the nonlinear circuit simulator. In

Fig. 6.4, we present an overview of the method in terms of a series of steps. Using the DEPACT method, the transmission lines can be replaced with a cascade of m DEPACT cells, where each cell consists of lumped elements and delay sources. Next, these cells can be directly embedded in the circuit simulator netlist. Using the circuit simulator, all the terminal voltages and currents of the network can be obtained. Based on this information, we can perform sensitivity analysis using the equivalent macromodels developed in this chapter. The macromodels, which consist of lumped elements, delay sources and constant sources can be combined with the sensitivity circuits of the lumped components and directly included in the circuit simulator netlist. Finally, using the circuit simulator, we can obtain the sensitivity information of the overall network.

Fig. 6.4. Interface to circuit simulator

6.8 Numerical Results

In this section, we present several numerical results which validate the accuracy and efficiency of the new delay extraction-based sensitivity analysis method. Two examples are considered. The first example deals with a nonlinear circuit with three MTL subnetworks, and the second example considers a linear circuit with eight coupled transmission lines.

6.8.1 Example 1

In this example, a nonlinear circuit with three MTL subnetworks is considered (Fig. 6.5). The p.u.l. parameters of all three MTL subnetworks are identical. Fig. 6.6 and Fig. 6.7 show a sample comparison of the transient response corresponding to a 3V step input with rise time of 0.2ns using DEFACT (order $m=8$) and conventional lumped segmentation. Fig. 6.8 and Fig. 6.9 show the sensitivity of the active and victim lines of subnetwork-2, with respect to L_{11} , (of subnetwork-2) respectively. The sensitivity results using new DEFACT sensitivity analysis method are compared with the perturbation of the conventional lumped model [2] (referred to as perturbation). As seen, the results are in very good agreement. Table 6.1 shows a comparison of the savings in the main computational cost (in terms of the total number of LU decompositions) using the new method versus the perturbation approach.

$$R = \begin{bmatrix} 0.14 & 0 \\ 0 & 0.14 \end{bmatrix} \Omega/\text{cm} \quad L = \begin{bmatrix} 7.08 & 1.77 \\ 1.77 & 7.08 \end{bmatrix} \text{nH}/\text{cm}$$

$$C = \begin{bmatrix} 0.499 & -0.886 \\ -0.886 & 0.499 \end{bmatrix} \text{pF}/\text{cm}$$

$$G = 0; \quad d=20\text{cm}$$

Fig. 6.5. Transmission line network with nonlinear terminations (Example 1)

Table 6.1: Computational Cost Comparison (Example 1)

# of Parameters	Perturbation (# of LU decompositions)	DEPACT (# of LU decompositions)
27	49224	1758

Fig. 6.6. Transient response of active line far-end (Example 1)

Fig. 6.7. Transient response of victim line far-end (Example 1)

Fig. 6.8. Sensitivity of active line far-end with respect to L_{11} of subnetwork-2

Fig. 6.9. Sensitivity of victim line far-end with respect to L_{11} of subnetwork-2

6.8.2 Example 2

In this example, a MTL with eight coupled transmission lines is considered. The circuit and the p.u.l. parameters [48] are shown in Fig. 6.10, with $G=0$. The voltage source is a 1V step input with rise time of 0.1ns. Fig. 6.11 and Fig. 6.12 show the transient response at nodes V_1 (active line) and V_2 (victim line) using the DEPACT sensitivity analysis algorithm (order $m=6$) and conventional lumped segmentation, respectively. The sensitivities with respect to C_{11} for nodes V_1 and V_2 are shown in Fig. 6.13 and Fig. 6.14, respectively. As in example 1, the results are compared to the perturbation of the conventional lumped model. As seen, the results from both techniques are in excellent agreement.

$$R = \text{diag}\{0.323 \text{ } \Omega/\text{cm}\}$$

$$L = \begin{bmatrix} 5.10 & 2.20 & 1.50 & 1.10 & 0.80 & 0.051 & 0.033 & 0.012 \\ 2.20 & 5.35 & 2.20 & 1.50 & 1.10 & 0.80 & 0.051 & 0.033 \\ 1.50 & 2.20 & 5.35 & 2.20 & 1.50 & 1.10 & 0.80 & 0.051 \\ 1.10 & 1.50 & 2.20 & 5.35 & 2.20 & 1.50 & 1.10 & 0.80 \\ 0.80 & 1.10 & 1.50 & 2.20 & 5.35 & 2.20 & 1.50 & 1.10 \\ 0.051 & 0.80 & 1.10 & 1.50 & 2.20 & 5.35 & 2.20 & 1.50 \\ 0.033 & 0.051 & 0.80 & 1.10 & 1.50 & 2.20 & 5.35 & 2.20 \\ 0.012 & 0.033 & 0.051 & 0.80 & 1.10 & 1.50 & 2.20 & 5.10 \end{bmatrix} \text{ nH/cm}$$

$$C = \begin{bmatrix} 1.97 & -0.531 & -0.227 & -0.131 & -0.122 & -0.109 & -0.097 & -0.081 \\ -0.531 & 2.11 & -0.531 & -0.227 & -0.131 & -0.122 & -0.109 & -0.097 \\ -0.227 & -0.531 & 2.11 & -0.531 & -0.227 & -0.131 & -0.122 & -0.109 \\ -0.131 & -0.227 & -0.531 & 2.11 & -0.531 & -0.227 & -0.131 & -0.122 \\ -0.122 & -0.131 & -0.227 & -0.531 & 2.11 & -0.531 & -0.227 & -0.131 \\ -0.109 & -0.122 & -0.131 & -0.227 & -0.531 & 2.11 & -0.531 & -0.227 \\ -0.097 & -0.109 & -0.122 & -0.131 & -0.227 & -0.531 & 2.11 & -0.531 \\ -0.081 & -0.097 & -0.109 & -0.122 & -0.131 & -0.227 & -0.531 & 1.97 \end{bmatrix} \text{ pF/cm}$$

Fig. 6.10. Eight coupled transmission line network and p.u.l. parameters (Example 2)

Fig. 6.11. Transient response of active line far-end (Example 2)

Fig. 6.12. Transient response of victim line far-end (Example 2)

Fig. 6.13. Sensitivity of active line far-end with respect to C_{11}

Fig. 6.14. Sensitivity of victim line far-end with respect to C_{11}

6.9 Summary

In summary, this chapter described a new method for sensitivity analysis based on the proposed DEPACT macromodel. In order to obtain the sensitivity information of the overall macromodel, the sensitivities of the lossy and lossless sections are required. In this chapter, we derived the sensitivity equations for the lossy and lossless sections, as well as presented equivalent circuit representations for the analysis. Moreover, for the case of lines with frequency-independent parameters, simplified macromodels are achieved, since MRA based sensitivity analysis is not required. One of the major advantages of the new approach is that the sensitivity information is obtained using closed-form macromodels which can be directly linked to the circuit simulator. In addition, the method enables sensitivity analysis with respect to both electrical and physical parameters. Finally, numerical results were presented which validate the accuracy and efficiency of the proposed DEPACT based sensitivity analysis method.

CHAPTER 7

Conclusions and Future Work

7.1 Summary

A new algorithm (DEPACT) has been developed for passive macromodeling of high-speed interconnects. The method provides an accurate and compact approach to handling distributed interconnects within the framework of nonlinear SPICE-like circuit simulators. The DEPACT method combines the merits of both the MoC and MRA algorithms. It employs an efficient delay extraction mechanism prior to performing MRA. Simplified models were presented for the case of lines with frequency-independent parameters. The concept of passivity was discussed, and a proof of passivity was derived, which shows that the overall DEPACT macromodel is guaranteed to be passive by construction. In addition, the new method has been extended to time-domain sensitivity analysis. Computational results based on industrial examples were presented which validate the accuracy and efficiency of the DEPACT macromodel, as well as the developed sensitivity analysis algorithm. The major advantages of the new method are as follows.

- 1) The developed algorithm tackles the shortcomings associated with previous techniques, since it employs an advanced delay extraction mechanism, prior to performing rational function approximations. This leads to fast transient simulation for both on-chip and off-chip interconnects.
- 2) Closed-form macromodels were derived, which can be directly linked to nonlinear circuit simulators. This allows the circuit simulator to view the distributed interconnect as a regular circuit component.
- 3) Simplified models were developed for the case of lines with frequency-independent parameters. These models consist of resistive networks and lossless lines, providing for easy implementation in SPICE-like circuit simulators.
- 4) The DEFACT macromodel is guaranteed to be passive by construction.
- 5) A new sensitivity analysis algorithm has been developed based on the DEFACT macromodel. This method enables sensitivity analysis with respect to both electrical and physical parameters. Closed-form equivalent circuits are derived, which are obtained analytically in terms of the p.u.l. parameters of the line, and pre-determined coefficients. The new sensitivity analysis algorithm provides significant computational cost advantages compared to previous techniques.
- 6) The developed macromodel is general in nature and can be extended to other applications. For example, the macromodel was used as part of a

parallel work to develop high-speed interconnect macromodels in the presence of incident electromagnetic fields [49]-[51].

7.2 Future Research

Several suggestions for further research based on the results of this thesis are described below.

1) Large number of coupled interconnects:

It has been studied that the computational cost of simulating coupled lines increases rapidly as the number of lines is increased. To address this issue, a new technique based on waveform relaxation and transverse partitioning has been recently suggested [52]. In this approach, the N coupled line circuit is reduced to N single line subcircuits, where the coupling effects between the lines are represented by independent sources. Preliminary results indicate that the computational cost of the new approach grows linearly with the number of lines. In addition, the new method is so general, that any macromodel could be used for the analysis of the transmission lines. In this case, it would be of significant interest to make use of the DEFACT macromodel developed in this thesis for modeling of the interconnects. Moreover, the DEFACT macromodel itself would be simplified since only the analysis of single lines is required.

2) Special case of interconnects with frequency-independent conductance and capacitance:

In this thesis, the DEFACT method was developed for the general case of frequency-dependent lines, as well as the simplified case of lines with frequency-independent parameters. However, another practically important case can arise,

where only the p.u.l. resistance and inductance are functions of frequency, i.e. the variation of C and G with frequency is negligible. Preliminary studies show that further simplifications can be made which can lead to more efficient macromodels.

3) Nonuniform interconnects:

The method described in this thesis assumes uniform transmission lines. Extension of the new method to handle nonuniform lines (where the p.u.l. parameters vary with the spatial variable x) would be of significant research interest.

4) Alternative realization for lossy sections:

In this thesis, the frequency-dependent lossy sections were analyzed using passive matrix rational approximation. However, an alternative approach would be to analyze the lossy sections using a recently proposed method based on Integrated Congruence Transform [53]. This method can efficiently handle lines with frequency-dependent parameters, as well as nonuniform interconnects.

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