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Gauge-Boson Self-Interactions and
 $t\bar{t}$ Production as Probes of
New Physics
at High Energy e^+e^- Colliders

by

Mikuláš Gintner, RNDr.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Physics
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Ottawa, Ontario, Canada
June, 1997

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**“Gauge-Boson Self-Interactions and $t\bar{t}$ Production as Probes
of New Physics at High Energy e^+e^- Colliders”**

submitted by **Mikulas Gintner, RNDr**

in partial fulfillment of the requirements for

the degree of Doctor of Philosophy

Chair, Department of Physics

Thesis Supervisor

External Examiner (Ph.D.)

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Date: August 12/97

Abstract

We address two phenomenological problems, both related to the search for new physics beyond the SM with potential sensitivity to the mechanism of electroweak symmetry breaking (ESB). In the first problem, we analyze the sensitivity of the process $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$ to anomalous triple gauge boson couplings at energies appropriate to LEP2 and the NLC, considering all tree level diagrams and finite widths of the gauge bosons. While it is unlikely that LEP2 measurements would reveal anomalous couplings, the 500 GeV NLC measurements should be sensitive to loop contributions to the triple gauge boson vertices (TGV) while the 1 TeV NLC will be able to measure such effects. The different polarizations of initial states give different correlations between bounds on anomalous couplings. Thus the use of polarized beams at the NLC energies can help disentangle the nature of anomalous TGV's. We also examine the sensitivity of the off W -resonance production to the anomalous couplings and find that useful information could be extracted from this region of phase space. In the second problem, we investigate $t\bar{t}$ production via vector boson fusion at e^+e^- colliders as a means to study the mechanism of ESB. We calculate the cross sections for four different channels of the reaction $e^+e^- \rightarrow \ell\bar{\ell}V_1V_2 \rightarrow \ell\bar{\ell}t\bar{t}$ using the effective vector boson approximation (EVA). We find that the WW mode is sufficiently sensitive to distinguish different Higgs boson masses. We examine the error introduced to the $\sigma(e^+e^- \rightarrow t\bar{t})$ calculation by using high-energy approximations of the $V_1V_2 \rightarrow t\bar{t}$ cross sections rather than the full expressions and find it comparable to the error expected from the use of the EVA.

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Chapter 1

Introduction

1.1 The Standard Model — its strengths and weaknesses

The *Standard Model of electroweak and strong interactions* (SM) [1, 2] is a gauge-field theory based on the $SU(3)\times SU(2)\times U(1)$ gauge symmetry group. It describes the electromagnetic, weak and strong interactions of quarks and leptons, the fundamental constituents of matter. The formulation of the SM has represented a significant step in the quest for a theory unifying all the fundamental forces of nature. It has had a profound influence on the developments in high energy physics in the last two decades. The SM has provided a common platform for a quantum description of three out of the four known forces in nature. It was successful in predicting new phenomena such as the existence of the W and Z bosons, the mediators of the weak interactions, and the existence of the t quark, the heaviest elementary particle known to us at present. All measurements designed to test the SM have been found to be in good agreement with its predictions. Hence, the SM is the frontier of our understanding of the laws of the microcosmos; it is the

deepest level of the small-scale structure of the world into which our intellect and technology have penetrated.

Despite all its successes, the SM is not a fundamental theory. First of all, it is incomplete because it does not say anything about gravitational forces on the quantum scale, where the classical theory of gravity fails. There are other deficiencies of the SM. Although they might sound “less noble” than the missing description of quantum gravity, they are no less serious in preventing the SM from being the fundamental theory. For example, there are too many free parameters in the SM that must be supplied from experiment. There is no reason provided for the existing family replication. There is no explanation for the origin of fermion masses, nor for flavour mixings. Finally, the huge difference between the electroweak Fermi scale ($\mathcal{O}(10^2)$ GeV), and the Planck mass scale ($\mathcal{O}(10^{19})$ GeV), at which quantum gravitational effects are expected to become non-negligible, introduces complications if there is no new physics lying in between, not too far from the weak scale.

Hence, despite all the impressive experimental confirmations of the SM we have witnessed so far, we have good reasons to search for a more fundamental theory even below the Planck scale. We expect that the new theory should give us answers to at least some of the above questions. We also expect that it will contain the SM as its low-energy approximation. How can we learn about new physics beyond the SM? The direct way would be by discovering associated new particles. That would require building an accelerator able to produce and detect such particles. However, the effects of new physics may be observable even if the new particles are too heavy, beyond the reach of existing facilities. In such a case, new physics can reveal itself indirectly through small deviations between measurements and predictions of the SM, introduced through effective vertices, loop corrections to the SM couplings and higher order processes. However, below the new physics

scale. the contributions to observables would be suppressed by inverse powers of this scale, relative to the SM contributions. Thus, to search for the indirect effects of new physics below its scale, high precision measurements are required.

At present, the SM has been tested by high precision measurements at the energy scale of $\mathcal{O}(100)$ GeV. These measurements tested some parts of the SM beyond the lowest order of the perturbative expansion of the S -matrix. In particular, interactions of fermions to gauge bosons have been measured very well and found to be in good agreement with the SM predictions. Because of the insufficient beam energy of the existing e^+e^- colliders on the one hand and large backgrounds and other uncertainties at hadron colliders on the other hand, other parts of the SM have been probed much less stringently. In the following sections, we will focus on some of these weakly tested sectors of the SM.

1.2 Search for the mechanism of electroweak symmetry breaking

In order that the SM Lagrangian remains gauge invariant, there are no explicit mass terms allowed in it and thus all fields of the SM have to be introduced as massless. However, in reality, most of the particles have their masses different from zero. One of the great successes of the SM is that it can solve this discrepancy, saving the gauge principle while giving masses to particles. The fields that are introduced as massless acquire their masses through the so-called spontaneous breaking of the gauge symmetry. This spontaneous breaking is realized by the ground state having a lower symmetry than the Lagrangian itself¹. In the case

¹Actually, the symmetry of the Lagrangian is hidden rather than broken. After the fields are reparameterized according to the ground state, the symmetry of the Lagrangian is no longer obvious.

of the SM, the weak gauge bosons have non-zero masses while the photon and gluons are massless. Thus, only the electroweak part of the SM gauge symmetry group, the $SU(2) \times U(1)$, needs to be broken; and so we call this symmetry breaking “*electroweak symmetry breaking*” (ESB). The question is how can we achieve the symmetry of the ground state to be lower than the symmetry of the Lagrangian. In the basic SM scenario, the electroweak symmetry is spontaneously broken through the introduction of an elementary complex scalar doublet. The direct consequence of this scenario is the appearance of an elementary scalar particle, the Higgs boson, in the particle spectrum. The Higgs boson mass is not predicted by the theory and the Higgs boson has not yet been discovered by experiment. However, the mass must remain below $\mathcal{O}(1)$ TeV in order for the SM to remain a perturbative theory and to maintain unitarity. This becomes a problem if the scale of new physics is too large, as for example the Planck scale. This is because the mass of the elementary Higgs scalar is driven by the contribution of new physics through higher-order corrections to be as high as the scale itself. In order to stop the SM Higgs boson mass from becoming too large, one would have to introduce an unnaturally fine-tuned parameter that would cancel the huge radiative corrections.

The electroweak symmetry breaking sector of the SM is only weakly constrained by the present experimental data. Even though ESB is a necessary part of the SM, responsible for introducing masses to particles, the actual mechanism of the symmetry breaking remains unclear. The SM scalar doublet is only one of the possible solutions. The question of the actual mechanism of ESB constitutes one of the most pressing problems in particle physics today. Quite generally, there are two scenarios for possible solutions.

The first scenario is a *weakly-coupled electroweak symmetry breaking sector*. Its simplest version is the elementary complex doublet of scalar fields discussed above. More complicated alternatives of this scenario include extending the num-

ber of scalar doublets. They all suffer from the instability of the low Higgs mass against the influence of the Planck scale through radiative corrections. The solution to this problem lies in unnatural fine-tuning of parameters. The weakly-coupled ESB sector also includes supersymmetric theories. They are free from the fine-tuning problem because here, the radiative corrections to the masses of fundamental scalars are cancelled by the radiative corrections of the corresponding super-partners. Thus the Higgs scalar spectrum can remain light in these theories without the need for unnatural fine-tuning. If supersymmetry is behind ESB there would be a new spectrum of particles and interactions at the TeV-scale or below.

The second scenario is a *strongly-coupled electroweak symmetry breaking sector*. In this scenario the symmetry breaking is triggered by non-perturbative strong forces. Elementary Higgs bosons do not exist. One alternative is to introduce a Higgs-like composite scalar state. It's composite nature would be revealed at the TeV scale. Typical representatives of this scenario are technicolor models built in analogy to chiral symmetry breaking in QCD. In general, no matter what the nature of ESB dynamics, it should be revealed at or below the TeV energy scale.

1.3 On two interesting sectors of the Standard Model

There are two sectors of the SM, poorly measured to date, which may be fruitful in the search for the nature of ESB. They are the *electroweak gauge-boson self-interactions* and the *top quark interactions*. In the SM, the *gauge-boson self-interactions* are uniquely determined by the non-abelian character of the gauge symmetry of the SM. Measuring these interactions represents a direct test of the basic principles of the construction of gauge theories. The gauge-boson self-interactions are so deeply rooted in the internal structure of the SM that any

deviation from their predicted SM form would have damaging consequences for the renormalizability and high energy behaviour of the SM. In addition, since the longitudinal components of the W and Z bosons are “eaten-up” degrees of freedom of the Higgs sector their self-interactions can reveal information about the mechanism of ESB. Constraints from direct measurements of these couplings at current hadron colliders are weak. With the upgrade of the LEP collider² at CERN, sufficiently high energies have been achieved to produce real W pairs in e^+e^- collisions. Crossing the threshold of real W -pair production enables tests of the $WW\gamma$ and WWZ vertices directly. Roughly comparable precision can be achieved at the Tevatron $p\bar{p}$ collider by studying gauge boson production [3]. Still, to reach the level of precision of the loop contributions to the triple gauge-boson vertices would require building the next generation of the e^+e^- colliders³ with CM energy of at least 500 GeV. Significant improvements of precision measurements of the couplings can also be expected from experiments at the LHC⁴.

In 1995 the existence of the *top quark* was established conclusively. Its discovery has been confirmed by two independent groups, CDF and D0, at the Fermilab 1.8 TeV Tevatron proton-antiproton collider. Its mass has been measured directly and found to be 175 GeV with an uncertainty of ± 6 GeV [4, 5]. In addition to this direct evidence of the top quark there are indirect measurements of the top quark mass obtained from high precision measurements at LEP [4]. Still, there is a long way to go before we can claim to know top quark properties like its couplings, with accuracy comparable to that of other quarks and leptons in the SM. So the present weak constraints on the parameters of the top quark leave considerable

²The upgraded version is known as LEP2 or LEP200. It is expected to achieve energies up to 192 GeV and integrated luminosity up to 500 pb⁻¹.

³Such a project is often referred to as the Next Linear Collider (NLC). Its energy is considered to be anywhere from 500 GeV to several TeV.

⁴Large Hadron Collider project at CERN with expected CM pp collision energy 14 TeV and integrated luminosity up to 100 fb⁻¹.

room for deviations from SM expectations. Its very large mass makes the top quark a very suitable probe for investigation of new physics and the electroweak symmetry breaking mechanism in particular. As the heaviest known particle it lies nearest to the scale of new physics. Therefore, we expect that new physics is most likely to have the strongest impact on the effective interactions of the top quark compared to other light particles. In addition, because fermion couplings to the Higgs boson are proportional to the fermion mass, the top quark is the most strongly coupled fermion to the mechanism of ESB.

In our work we address two phenomenological problems, both related to the search for new physics beyond the SM with potential sensitivity to the mechanism of ESB. In the first one, we analyze the sensitivity of the process $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$ to the anomalous triple gauge boson couplings at energies appropriate to LEP2 (175 - 220 GeV) and the NLC (500 GeV, 1 TeV). We include all tree level diagrams and finite widths of the gauge bosons. At NLC energies we examine how the sensitivity depends on the polarization of the electron beam. We also examine the sensitivity of the off W-resonance production to the anomalous couplings.

In the second problem we investigate $t\bar{t}$ production via vector boson fusion at e^+e^- colliders as a means to study the mechanism of ESB. We calculate the cross sections for four different channels of the reaction $e^+e^- \rightarrow \ell\bar{\ell}V_1V_2 \rightarrow \ell\bar{\ell}t\bar{t}$ using the effective vector boson approximation. For the WW and ZZ channels we find how the corresponding cross sections depend on the mass of the Higgs boson. We examine the error introduced to the $\sigma(e^+e^- \rightarrow t\bar{t})$ calculation by using high-energy approximations of the $V_1V_2 \rightarrow t\bar{t}$ cross sections rather than the full expressions.

This thesis is organized as follows. In Chapter 2 we introduce the SM and its Lagrangian. In Chapter 3, after a brief introduction of the method of effective Lagrangians, we focus on the effective parameterization of triple gauge boson self-interactions. At the end of this Chapter we present the current and expected

experimental limits on anomalous triple gauge boson couplings. In Chapter 4 we present our calculations of the sensitivity of the process $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$ to the anomalous triple gauge boson couplings. Investigation of the $t\bar{t}$ production via vector boson fusion is presented in Chapter 5. Finally, our conclusions can be found in Chapter 6.

Chapter 2

The Standard Model of electroweak interactions

In this chapter we introduce the electroweak part of the SM. Since the strong interactions are not important to our calculations we will leave them out of our discussion. We are mainly interested in three electroweak sectors: interactions of fermions with gauge-bosons, gauge-boson self-interactions, and the Higgs sector.

2.1 Electroweak fields and symmetries

The electroweak sector of the SM Lagrangian represents a unified description of the electromagnetic and weak interactions [1, 2]. This description is based on the $SU(2)_L \times U(1)_Y$ group which is a local gauge symmetry of the electroweak Lagrangian. The spectrum of the particles in the SM can be divided into three categories. One category is the *fermions*. Fermions include *leptons*, particles subject to electroweak interactions, and *quarks* which also experience strong interactions. The strong interactions are not the subject of this study, so we will not discuss

them further¹. The leptons and quarks are organized into three families:

$$\begin{bmatrix} \nu_e & u \\ e^- & d \end{bmatrix}, \begin{bmatrix} \nu_\mu & c \\ \mu^- & s \end{bmatrix}, \begin{bmatrix} \nu_\tau & t \\ \tau^- & b \end{bmatrix} \quad (2.1)$$

plus the corresponding antiparticles. The $\nu_e, \nu_\mu, \nu_\tau, e^-, \mu^-, \tau^-$ are the leptons and the u, c, t, d, s, b are the quarks. The three families have identical gauge interactions. They differ only by their masses and the CKM matrix couplings.

The fermion fields are described through their left- and right-handed components

$$\psi_{L,R} = [(1 \mp \gamma_5)/2]\psi, \quad \bar{\psi}_{L,R} = \bar{\psi}[(1 \pm \gamma_5)/2] \quad (2.2)$$

In the SM the left-handed fields are $SU(2)_L$ doublets, while their right-handed partners transform as $SU(2)_L$ singlets. The general transformation pattern of each family can be broken down as follows:

leptonic sector:

$$\psi_1(x) = \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, \quad \psi_2(x) = l_R^- \quad (2.3)$$

quark sector:

$$\psi_3(x) = \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, \quad \psi_4(x) = (q_u)_R, \quad \psi_5(x) = (q_d)_R \quad (2.4)$$

where ν_l stands for the neutrino, l^- is one of the e^-, μ^-, τ^- , q_u is the up-type quark, and q_d is the down-type quark. The masses of the left-handed particles are identical to the masses of their right-handed partners. The neutrino appears to be massless with only a left-handed component.

The *gauge bosons* represent the second category of SM particles. The very presence of the gauge bosons in the SM Lagrangian is a direct consequence of the gauge principle applied to the construction of the theory. This also fixes

¹The only thing we have to keep in mind is that each quark has three different strong charges (called colours), which increases the number of degrees of freedom that must be taken into account.

their interactions. Thus, the $SU(2)_L \times U(1)_Y$ gauge symmetry introduces four electroweak gauge bosons; one photon γ , and three weak vector bosons $W^\pm$, Z^0 . These particles have all been verified experimentally.

The third part of the SM is the *Higgs sector*. In this sector the electroweak gauge group $SU(2)_L \times U(1)_Y$ is broken to the $U(1)_{EM}$ of electromagnetism. The electroweak Lagrangian is locally symmetric under $SU(2)_L \times U(1)_Y$ group. However, the ground state of the theory has a lower symmetry, much like a ferromagnet has a lower symmetry than the basic interactions. This phenomenon is called electroweak symmetry breaking (ESB) and it generates the masses of the weak gauge bosons and fermions. The actual mechanism responsible for ESB remains unknown. The simplest hypothesis introduces new fields in the form of a complex scalar $SU(2)_L$ doublet. This new scalar doublet has a non-zero vacuum expectation value and the vacuum itself has lower symmetry than $SU(2)_L \times U(1)_Y$. As a result, a physical scalar particle, the so-called *Higgs boson*, appears in the SM particle spectrum. It has not yet been discovered experimentally and its mass is not predicted by the SM scenario of ESB. As a matter of fact, the Higgs particle is not at all necessary for explaining all the current experimental data. Several alternative scenarios for ESB have been suggested, some introducing several fundamental scalars, others replacing the fundamental Higgs boson with composite particles or even removing it completely. Still, the search for the Higgs boson remains the primary goal of experimental activities regarding the mechanism of ESB.

2.2 Electroweak Lagrangian and interactions

2.2.1 Fermions and gauge bosons

The SM electroweak Lagrangian is invariant under the local $SU(2)_L \times U(1)_Y$ gauge transformation

$$\psi_j(x) \longrightarrow \psi'_j(x) = U(x)V(x)\psi_j(x) \quad (2.5)$$

where

$$U(x) = \exp[i\vec{\tau} \cdot \vec{\alpha}(x)], \quad V(x) = \exp[i\frac{Y}{2}\beta(x)] \quad (2.6)$$

and $\vec{\tau}$ are the three $SU(2)_L$ generators² that act on the doublet fields and $\alpha_i(x)$, $\beta(x)$ are real functions of x^μ . Y is a quantum number operator called the weak hypercharge. The relation of the Y to the electric charge and weak isospin of a particle will be shown later. To build the invariant Lagrangian we need to introduce the covariant derivative

$$D_\mu \psi_j(x) \equiv [\partial_\mu - ig\vec{\tau} \cdot \vec{W}_\mu - ig'\frac{Y_j}{2}B_\mu]\psi_j(x) \quad (2.7)$$

where g, g' are dimensionless electroweak coupling constants and $\vec{W}_\mu, B_\mu$ are four electroweak gauge fields with the following transformation properties

$$\vec{\tau} \cdot \vec{W}'_\mu(x) = U(x)\vec{\tau} \cdot \vec{W}_\mu(x)U^\dagger(x) + \frac{i}{g}U(x)\partial_\mu U^\dagger(x) \quad (2.8)$$

$$B'_\mu(x) = B_\mu(x) + \frac{1}{g'}\partial_\mu\beta(x) \quad (2.9)$$

Then the covariant derivative transforms as

$$[D_\mu \psi_j(x)]' = U(x)V(x)D_\mu \psi_j(x) \quad (2.10)$$

While the non-linearity of the $SU(2)$ commutation relations requires that there is a unique $SU(2)_L$ coupling for all the fermions, the abelian group structure of

²The τ_i 's have the following relation to the usual Pauli matrices σ_i : $\tau_i = \sigma_i/2$. Then $[\tau_i, \tau_j] = i\varepsilon_{ijk}\tau_k$ and $\text{Tr}(\tau_i\tau_j) = \frac{1}{2}\delta_{ij}$.

the weak hypercharge provides no restrictions for assigning the weak hypercharge quantum number. In order to build the gauge-invariant kinetic terms for the gauge fields, the field strengths are defined as:

$$\vec{W}_{\mu\nu} \equiv \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g \vec{W}_\mu \times \vec{W}_\nu \quad (2.11)$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \quad (2.12)$$

Their transformation properties are

$$\vec{\tau} \cdot \vec{W}'_{\mu\nu} = U(x) \vec{\tau} \cdot \vec{W}_{\mu\nu} U^\dagger(x) \quad (2.13)$$

$$B'_{\mu\nu} = B_{\mu\nu} \quad (2.14)$$

Now, the part of the SM Lagrangian that describes fermions, gauge bosons and their mutual interactions can be written down:

$$\mathcal{L}_{F-GB} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu} + \sum_j i \bar{\psi}_j(x) \gamma_\mu D^\mu \psi_j(x) \quad (2.15)$$

The gauge symmetry forbids mass terms for the gauge bosons as well as the fermions because they would explicitly break the gauge symmetry. Instead, masses for these particles are introduced by the mechanism of ESB.

The Lagrangian (2.15) contains the so-called charged current (CC) interactions of the left-handed fermions with the charged boson fields $W_\mu^+ \equiv (W_\mu^1 + iW_\mu^2)/\sqrt{2}$ and $W_\mu^- \equiv (W_\mu^1 - iW_\mu^2)/\sqrt{2} = (W_\mu^+)^{\dagger}$:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \{ W_\mu^- [(\bar{q}_u)_L \gamma^\mu (q_d)_L + \bar{\nu}_l \gamma^\mu l_L] + \text{h.c.} \} \quad (2.16)$$

Because the quark fields in (2.16) are not identical with the mass eigenstates, there are interactions between quarks of different generations. However, these are relatively small and we neglect them in our work.

Both gauge fields W_3^μ and B^μ are neutral and massless. In principle, the physical states, let them be called Z^μ and A^μ , could be any linear combination of

W_3^μ and B^μ :

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \quad (2.17)$$

Then, the neutral-current (NC) interactions of the Lagrangian (2.15) between the neutral physical bosons and fermions have the form

$$\mathcal{L}_{NC} = \bar{\psi}_j \gamma_\mu \{ A^\mu [g\tau_3 \sin \theta_W + g' \frac{Y_j}{2} \cos \theta_W] + Z^\mu [g\tau_3 \cos \theta_W - g' \frac{Y_j}{2} \sin \theta_W] \} \psi_j \quad (2.18)$$

One of the physical bosonic fields should correspond to the QED photon. Identifying A_μ with the photon results in the relations:

$$g \sin \theta_W = g' \cos \theta_W = e \quad (2.19)$$

$$Q_f = T_f^3 + \frac{Y_f}{2} \quad (2.20)$$

where Q_f is the electromagnetic charge of the fermion, T_f^3 is the z -component of its weak isospin, and Y_f is the weak hypercharge. The NC interactions can now be split into two separate parts

$$\mathcal{L}_{NC} = \mathcal{L}_{QED} + \mathcal{L}_{NC}^Z \quad (2.21)$$

where

$$\mathcal{L}_{QED} = e A_\mu \bar{f} \gamma^\mu Q_f f \quad (2.22)$$

is the electromagnetic interaction of the photon with charged fermions and

$$\mathcal{L}_{NC}^Z = \frac{e}{2 \sin \theta_W \cos \theta_W} Z_\mu \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f. \quad (2.23)$$

is the electroweak interaction of the Z boson with the fermions. Here, f denotes individual fermion fields and v_f and a_f are defined as

$$v_f = T_f^3 - 2Q_f \sin^2 \theta_W, \quad a_f = T_f^3 \quad (2.24)$$

The v_f 's and a_f 's for individual fermions are given in Table 2.1.

Table 2.1: The vector and axial-vector couplings for individual fermions.

FERMION (f)	v_f	a_f
ν_l	$+1/2$	$+1/2$
l	$-\frac{1}{2} + 2 \sin^2 \theta_W$	$-1/2$
q_u	$\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W$	$+1/2$
q_d	$-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W$	$-1/2$

The gauge symmetry of the electroweak Lagrangian is non-abelian. Consequently, in addition to interactions between fermions and gauge bosons, which are already familiar from QED, interactions among gauge bosons themselves are also present. The strength of these interactions is given by the $SU(2)_L$ coupling g . The gauge-boson self-interactions can be recognized in the kinetic term of the Lagrangian (2.15). There are two kinds of self-interactions to be found there: cubic ones and quartic ones. The cubic self-interactions are given by the piece

$$\begin{aligned} \mathcal{L}_{cub} = & -ie \cot \theta_W (Z_{\mu\nu} W^{+\mu} W^{-\nu} + W_{\mu\nu}^+ W^{-\mu} Z^\nu - W_{\mu\nu}^- W^{+\mu} Z^\nu) \\ & -ie (A_{\mu\nu} W^{+\mu} W^{-\nu} + W_{\mu\nu}^+ W^{-\mu} A^\nu - W_{\mu\nu}^- W^{+\mu} A^\nu) \end{aligned} \quad (2.25)$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ with $V = W^\pm, Z, A$. The quartic self-interactions are given by

$$\begin{aligned} \mathcal{L}_{quar} = & -\frac{e^2}{2 \sin^2 \theta_W} [(W_\mu^+ W^{-\mu})^2 - W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu}] \\ & -e^2 \cot^2 \theta_W (W_\mu^+ W^{-\mu} Z_\nu Z^\nu - W_\mu^+ Z^\mu W_\nu^- Z^\nu) \\ & -e^2 \cot \theta_W (2W_\mu^+ W^{-\mu} Z_\nu A^\nu - W_\mu^+ Z^\mu W_\nu^- A^\nu - W_\mu^+ A^\mu W_\nu^- Z^\nu) \\ & -e^2 (W_\mu^+ W^{-\mu} A_\nu A^\nu - W_\mu^+ A^\mu W_\nu^- A^\nu) \end{aligned} \quad (2.26)$$

2.2.2 Higgs sector

Insisting on the gauge symmetry of the electroweak Lagrangian guarantees its renormalizability. On the other hand, there is no way to put explicit mass terms

into the Lagrangian and maintain its gauge invariance. The solution of this dilemma is to construct a theory with a symmetric Lagrangian and a less-symmetric ground state. The so-called *Goldstone theorem* applies to this situation: Let G be a continuous symmetry group of the Lagrangian and H be a continuous symmetry group of the ground state, $H \subset G$. Let $g \equiv \dim G$ and $h \equiv \dim H$ be the numbers of generators of each of the groups. Then the particle spectrum of the theory is enriched by $g - h$ massless spin-0 particles called *Goldstone bosons*.

The simplest model for generating the required masses for the SM particles is to introduce an $SU(2)_L$ doublet of scalar fields

$$\phi(x) \equiv \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \quad (2.27)$$

where $\phi^+ = (\phi_1 + i\phi_2)/\sqrt{2}$ and $\phi^0 = (\phi_3 + i\phi_4)/\sqrt{2}$ with ϕ_i 's being real fields. The original massless Lagrangian (2.15) is then supplemented by the gauged scalar Lagrangian

$$\mathcal{L}_{S-GB} = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \quad (2.28)$$

with

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \quad (2.29)$$

where $\mu^2 > 0$ and $\lambda > 0$. The covariant derivative has the standard form

$$D_\mu \phi(x) \equiv (\partial_\mu - ig\vec{\tau} \cdot \vec{W}_\mu - ig' \frac{Y_\phi}{2} B_\mu) \phi(x) \quad (2.30)$$

where $\tau_\phi = 1/2$, $Y_\phi = 1$ for the scalar doublet (2.27). The Lagrangian (2.28) is invariant under local $SU(2)_L \times U(1)_Y$ transformations. There are an infinite number of degenerate ground states. The ϕ has the same expectation value in all these states:

$$|\langle 0 | \phi | 0 \rangle|^2 = \frac{\mu^2}{\lambda} \equiv \frac{v^2}{2} \quad (2.31)$$

These states are not invariant under the $SU(2)_L \times U(1)_Y$ transformations and only one of them becomes the true physical vacuum. Because the electric charge

remains a conserved quantity, the physical vacuum must remain invariant under the electromagnetic $U(1)$ group. Following this requirement, a proper choice is the vacuum for which

$$\langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (2.32)$$

where v is a real number. The particle spectrum is given by field fluctuations from this vacuum. It leads to the natural reparameterization of the scalar doublet:

$$\phi(x) = \exp[i\vec{\tau} \cdot \vec{\theta}(x)/v] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad (2.33)$$

where $\vec{\theta}(x)$ (Goldstone bosons) and $H(x)$ (Higgs boson) are real fields with zero vacuum expectation values. The exponential factor in (2.33) has the same form as the $SU(2)_L$ gauge transformation (2.6). Thus, the $SU(2)_L$ transformation can be chosen such that it cancels the $\vec{\theta}(x)$ fields. The gauge, in which $\vec{\theta}(x)$ disappear from the Lagrangian, is called the *physical* or *unitary* gauge. In this gauge the scalar Lagrangian (2.28) becomes

$$\mathcal{L}_{S-GB} = \frac{\lambda v^4}{8} + \mathcal{L}_H + \mathcal{L}_{HG^2} \quad (2.34)$$

where

$$\mathcal{L}_H = \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{1}{2} \lambda v^2 H^2 - \frac{\lambda v}{2} H^3 - \frac{\lambda}{8} H^4 \quad (2.35)$$

and

$$\begin{aligned} \mathcal{L}_{HG^2} = & \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2 v}{2} W_\mu^+ W^{-\mu} H + \frac{g^2}{4} W_\mu^+ W^{-\mu} H^2 \\ & + \frac{1}{2} \frac{g^2 v^2}{4 \cos^2 \theta_W} Z_\mu Z^\mu + \frac{g^2 v}{4 \cos^2 \theta_W} Z_\mu Z^\mu H + \frac{g^2}{8 \cos^2 \theta_W} Z_\mu Z^\mu H^2 \end{aligned} \quad (2.36)$$

Thus, the $W^\pm$ and Z bosons have acquired masses

$$M_Z \cos \theta_W = M_W = vg/2 \quad (2.37)$$

where $\sin^2 \theta_W \approx 0.23$, and a new physical particle — the *Higgs boson* — appeared in the spectrum. Its mass is

$$M_H = \sqrt{2} \mu = \sqrt{\lambda} v \quad (2.38)$$

where $v = 246$ GeV. Photons remain massless as expected. The sacrificed Goldstone bosons have supplied the longitudinal degrees of freedom to the now massive W and Z bosons.

The same mechanism also generates fermion masses when the following gauge-invariant fermion-scalar piece is added to the SM Lagrangian for every generation

$$\begin{aligned} \mathcal{L}_{S-F} = & -f_d (\bar{q}_u, \bar{q}_d)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} (q_d)_R - f_u (\bar{q}_u, \bar{q}_d)_L \begin{pmatrix} \phi^{0\dagger} \\ -\phi^{++} \end{pmatrix} (q_u)_R \\ & -f_l (\bar{\nu}_{l_u}, \bar{l})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} l_R + \text{h.c.} \end{aligned} \quad (2.39)$$

where the f_i are the so-called Yukawa couplings. After ESB this Lagrangian transforms to

$$\mathcal{L}_{S-F} = -\frac{1}{\sqrt{2}}(v + H)(f_d \bar{q}_d q_d + f_u \bar{q}_u q_u + f_l \bar{l} l) \quad (2.40)$$

implying the fermion masses

$$m_d = f_d v / \sqrt{2}; \quad m_u = f_u v / \sqrt{2}; \quad m_l = f_l v / \sqrt{2}. \quad (2.41)$$

The parameters f_i remain unpredicted and the fermion masses must be taken from experiment. The interactions of fermions with the Higgs boson are fixed by (2.40) and (2.41) in terms of fermion masses.

In its electroweak sector the SM contains 17 free parameters. Four of them are g , g' , μ^2 and λ . Usually, they are expressed through four other parameters which are combinations of the former ones. These are α , M_Z , G_F and M_H ; the fine-structure constant, the mass of Z boson, the Fermi constant and the mass of the Higgs boson, respectively. The advantage of this set of parameters lies in the fact that the first three variables of the set are the best measured parameters of the SM, and M_H , although unknown, is the object of intense interest. A multitude of free parameters appears in the Yukawa sector. For three generations and massless neutrinos, there are 13 free parameters: 9 fermion masses and 3 angles and 1

phase parameterizing the Cabibbo-Kobayashi-Maskawa (CKM) matrix which describes the inter-generational mixing of the quark states. We do not discuss this phenomenon here. Thus, $4 + 13$ gives us 17 free parameters of the electroweak sector as advertised. For completeness we mention that the strong $SU(3)_C$ sector supplies another one, possibly two free parameters: the QCD coupling constant α_s and a possible CP-violating $\bar{\theta}$ term in the strong Lagrangian.

Chapter 3

Effective Lagrangians

In this Chapter we briefly introduce Effective Lagrangians as a model-independent parameterization of new physics beyond the SM. Two different scenarios are considered: a linearly and non-linearly realized gauge symmetry. After this introduction we discuss the effective parameterization of triple gauge boson self-interactions. The three most common parameterizations of these interactions are introduced. In the last part we discuss the current and future experimental limits on the anomalous triple gauge boson couplings.

3.1 General notes about the formalism of Effective Lagrangians

The relationship between a particular quantum field theory and its low-energy approximation is treated formally in the language of the *Effective Quantum Field Theory*¹ (EFT). This formalism suggests that at energies below a scale Λ we do not have to use the full theory if some of its fields are heavier than this scale. Instead,

¹For more profound and complete discussion of the formalism of EFT. see e.g. references [71, 72, 75, 76, 77].

we can use the so-called *effective* theory that can be derived from the full theory by “integrating out” all heavy fields ($M_i > \Lambda$) which cannot be observed at low energy ($E < \Lambda$). The resulting effective Lagrangian can be written down in the form of an infinite sum

$$\mathcal{L}_{eff} = \sum_i \alpha_i(\Lambda) \mathcal{O}_i \quad (3.1)$$

where α_i are generally dimensional Λ -dependent coefficients and $\mathcal{O}_i$ are operators built from the light fields ($m_i < \Lambda$). The $\mathcal{O}_i$ are only constrained by the symmetries of the low-energy physics. In this sense the form of the effective Lagrangian (3.1) is independent of the theory from which it is derived. All information on virtual heavy physics effects is “frozen” into the coefficients α_i . Within its realm of applicability, i.e., below Λ , it provides a consistent and complete description of all low energy phenomena.

Based on these properties, the formalism could also be used the other way around: when we know only the low-energy physics and the more general theory is unknown². The corresponding effective Lagrangian can be written down using the knowledge of the *light particle spectrum* and the *low-energy symmetries*. The task is to construct the most general set of local interactions containing the given light fields and respecting these low-energy symmetries. Our ignorance of the full theory will be reflected in our inability to calculate values of the coefficients of these operators. They will have to be obtained from experiment. Thus, the coefficients remain free parameters, parameterizing all possible new physics which respects the given low-energy approximation.

Since the Lagrangian (3.1) contains an infinite number of terms it may appear that it has no predictive power. It seems that we would need to supply (through calculation from the full theory or by measurement) an infinite number of parameters in order to be able to calculate anything. Fortunately, this is not the case.

²The famous early example is Fermi’s four-fermion theory of weak interactions.

The terms in the effective Lagrangian can be arranged according to a hierarchy such that the lower order terms generate the dominant contribution to any observable. To any order in this hierarchy there is only a finite number of operators which need to be considered. We can also estimate the contribution produced by the remaining terms of higher orders. Then, due to its finite accuracy, any measurement at energy $E < \Lambda$ can be described by a limited number of terms in (3.1). The dominant terms of the effective Lagrangian can usually be identified with the low-energy theory and they are built of operators of energy dimension ≤ 4 . As the energy E increases, more and more terms become important for achieving the given accuracy, until the whole process breaks down at the scale Λ . For $E > \Lambda$ the effective Lagrangian violates S-matrix unitarity. Unitarity is then restored in the full theory by heavy field effects. The energy at which unitarity is violated also gives an upper bound on the masses of the heavy degrees of freedom. In a weakly coupled theory (like the SM) this upper bound substantially overestimates the masses of the heavy degrees of freedom.

The effective Lagrangians can be used in perturbative calculations like “ordinary” Lagrangians which implies we have to also deal with infinities in loop calculations. However, since the effective Lagrangians contain operators with dimensions > 4 as well, they are not renormalizable. The problem with non-renormalizable theories is that they can become finite only with an infinite number of counterterms. The effective Lagrangian already contains all such operators and thus all the counterterms it needs. This would be of no help though, if we had to renormalize an infinite number of parameters. Fortunately, due to the hierarchy argument combined with the finite precision of our measurements there are only a finite number of operators that are needed to handle the loop divergences.

We denote as $\mathcal{L}^{(4)}$ that part of the effective Lagrangian that consists of operators with dimensionality ≤ 4 . Based on the renormalizability of the $\mathcal{L}^{(4)}$ we

can distinguish two different classes of the effective Lagrangians. If the $\mathcal{L}^{(4)}$ is renormalizable then the conditions for the decoupling theorem [74] are satisfied and we are dealing with the so-called *decoupling scenario*. This means that the contributions of the terms with dimension > 4 to any observable vanish as $\Lambda \rightarrow \infty$. In other words, if we deal with energies much less than Λ , all these higher-term contributions are suppressed. The coefficients α_i in the sum (3.1) are proportional to inverse powers of Λ and this becomes the basic element defining the hierarchy in the effective Lagrangians. The operators with higher dimensions are suppressed by the higher powers of the $1/\Lambda$. This scenario typically occurs when a dimensional parameter in the full Lagrangian (e.g. mass of a field) becomes very large.

If the $\mathcal{L}^{(4)}$ is not renormalizable then the decoupling theorem does not hold and we have the *non-decoupling scenario*. In this case there are heavy physics effects that do not vanish as $\Lambda \rightarrow \infty$. Even though the coefficients α_i are still proportional to inverse powers of Λ , the question of hierarchy among the operators is more complicated. In the case of the effective Lagrangian with non-linearly realized gauge symmetry which is also considered in our work, the operators can be organized according to the value of the expression $d_n + f_n/2 - 2$ [75, 76, 77], where d_n is the number of derivatives (and gauge fields if the model is gauged) and f_n is the number of fermion fields. For a purely scalar sector ($f_n = 0$) this leads to organizing operators according to the number of derivatives — the derivative expansion. The non-decoupling scenario is typically realized when $\mathcal{L}_{eff}$ is obtained by letting a dimensionless parameter become large (e.g. the scalar self-coupling in the Higgs sector of the SM). Taking the dimensionless parameter to infinity will generate non-linear interactions among the fields.

3.2 Extending the Standard Model

There are good theoretical arguments that the SM is only valid over a limited range of energies so that we expect the SM to be a low-energy approximation of a more complete theory. There have been numerous attempts to construct renormalizable extensions of the SM by enlarging its symmetry group or particle content. However, due to the lack of any significant discrepancy between SM predictions and experiment we have not yet been able to discover a more fundamental theory. The multitude of viable alternatives leads people to realize the need for a general analysis of the extensions of the SM.

EFT is an appropriate approach for such an analysis. The potential influence of new physics on low-energy measurements can be analyzed by extending the SM Lagrangian with effective operators built from the fields of the SM, in accordance with (3.1). It is natural to require that the new operators respect the standard $SU(2) \times U(1)$ gauge symmetry. There are two standard scenarios on how to deal with ESB [61, 62, 72]. In the first, the gauge symmetry is *linearly realized*. In this case the effective Lagrangian is built with the same particle content as that of the SM; in particular, the Higgs boson is present in the particle spectrum and is relatively light. The leading term in the effective Lagrangian is the Lagrangian of the SM itself. Since the SM is renormalizable new physics decouples as its scale goes to infinity. Other terms are non-renormalizable but their effects are suppressed by the scale of new physics. There is no theoretical (nor experimental) restriction on how large the scale of new physics should be³. The larger it is, the more the non-standard effects are suppressed by inverse powers of Λ . Only experiment can give us more information about this scale.

The existence of the Higgs boson has not yet been confirmed. In fact, the Higgs boson is not a necessary ingredient of electroweak symmetry breaking. This

³Below the Planck scale, of course.

leads to the second scenario with the gauge symmetry *non-linearly realized*. The effective Lagrangian is built from the SM fields without the Higgs boson which means that $\mathcal{L}^{(4)}$ is non-renormalizable. Thus, the heavy physics does not decouple as its scale approaches infinity. This scenario imitates the situation when the Higgs boson is either too heavy or not present at all. In either case the low-energy Lagrangian will have bad high-energy behaviour, particularly for the scattering of longitudinal gauge bosons, which must be fixed by some new physics. Since the longitudinal gauge bosons are supplied by the ESB sector, along with new physics the mechanism of ESB must also be revealed. The scale of new physics cannot be too large in order to be able to take over the role of the Higgs boson in cutting off the growing amplitudes. Without the Higgs boson, the low-energy Lagrangian violates unitarity at a scale of roughly $4\pi v \sim 3$ TeV so that new physics must appear below this scale. In this scenario, the leading term of the effective Lagrangian, the $\mathcal{L}_{S-GB}$ of the SM (2.28), is replaced by the non-linear sigma model [61]

$$\mathcal{L}_{NL\sigma} = \frac{v^2}{4} \text{Tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) \quad (3.2)$$

where

$$\Sigma = \exp\left(\frac{2i\vec{\omega} \cdot \vec{\tau}}{v}\right), \quad (3.3)$$

$\vec{\tau}$ are the $SU(2)$ generators, and $v = 246$ GeV is defined by (2.37). The covariant derivative D_μ is

$$D_\mu \Sigma = \partial_\mu \Sigma - igW_\mu^i \tau^i \Sigma - ig' B_\mu \Sigma \tau^3 \quad (3.4)$$

The $\omega_{1,2,3}$ are the Goldstone Bosons that merge with the weak gauge bosons to make them massive.

It is obvious that the effective Lagrangian technique can be used to search for indirect effects of new physics in both scenarios. However, this approach seems to be more effective in the non-linear case. This is particularly true in the quest for the mechanism of ESB. In the first — light Higgs — scenario, the Higgs boson

would be found and studied directly at the next generation of colliders. In the second scenario there is no Higgs in the spectrum of the effective theory, either because it is supposed to be very heavy or even not exist at all, as some alternative models suggest. As we already mentioned, in this case there must be new physics at or below $\mathcal{O}(1)$ TeV and must provide an answer for the mechanism of ESB. Before we build experimental facilities that can reach this scale the only way to learn about it is through studies of its indirect effects at lower energies.

3.3 Anomalous triple gauge boson couplings

3.3.1 Gauge-boson self-interactions

The gauge boson self-interactions are introduced by the non-abelian character of the gauge symmetry upon which the SM is based. This was discussed in Chapter 2 where the self-interactions were shown to be described by the interaction Lagrangians (2.25) and (2.26). In contrast to the measurements of the couplings between fermions and gauge-bosons that have reached very high precision⁴ [4], the self-interactions of gauge-bosons have been tested rather poorly⁵. With the next generation of new colliders this situation should change. Starting with LEP2, future e^+e^- colliders can be considered to be W factories. At energies 200 GeV and higher the cross sections for the production of W and Z bosons will strongly dominate over those for production of fermions and scalars [73]. Measurements at future e^+e^- machines along with those at the Large Hadron Collider (LHC) will significantly improve the current limits on the gauge-boson self-interactions.

The weak gauge bosons represent a very interesting system that embodies two fundamental principles: the gauge principle and symmetry breaking. The pre-

⁴The SM has been verified at an accuracy of up to $\mathcal{O}(10^{-5})$.

⁵More details about the current experimental limits will be given in Subsection 3.3.3.

Figure 3.1: Tree level Feynman diagrams contributing to the process $e^+e^- \rightarrow W^+W^-$ in the limit of massless electrons.

cise form of the weak boson self-couplings is uniquely given by the requirements of gauge invariance and unitarity [73]. Without contributions from the Feynman diagrams with the self-couplings the cross sections for the production of gauge bosons would keep increasing with energy, eventually violating unitarity. This is illustrated in Fig. 3.2 for the process $e^+e^- \rightarrow W^+W^-$. Three diagrams contribute to this process in the SM (Fig. 3.1). Two of them contain triple gauge boson vertices (TGV). Any disruption of the SM form of the self-couplings would spoil this delicate cancellation of the contributions of the individual diagrams introducing discrepancies between the expected and measured cross sections which increases with collision energy. Therefore, at a fixed luminosity, the sensitivity of experiments to the deviation would be higher at higher energies.

The connection between weak gauge bosons and symmetry breaking is due to the fact that the longitudinal components of W and Z bosons are the Goldstone bosons of the ESB sector. Therefore, we expect that the mutual interactions of W_L 's and Z_L 's must carry information on the mechanism of ESB. These interactions become particularly important as a means to study the mechanism of ESB if there is no light Higgs boson. Otherwise, our efforts would be better invested in

Figure 3.2: Cancellation of the contribution of the Feynman diagrams containing the WWV vertices with that of the t -channel diagram to the $e^+e^- \rightarrow W^+W^-$ cross section. The long-dashed line is for the $(Z + \gamma)$ s -channel cross section, the short-dashed line is for the neutrino t -channel cross section. The solid line represents the total cross section combining all three diagrams together.

direct observations of the Higgs particle. In any case, measuring the gauge-boson self-couplings is one of the major tasks in testing the SM.

3.3.2 Effective parameterization of the triple gauge boson vertices

Potential deviations of the gauge-boson self-interactions from their SM forms can be parameterized using the formalism of EFT. In our work we investigate the triple gauge boson vertices $WW\gamma$ and WWZ . There are three main parameterizations of these gauge boson couplings that appear in the literature. The characteristic distinguishing the approaches is the degree to which constraints are imposed in terms of the symmetry and particle content of the low energy theory. The most commonly used parameterizations [16, 53] are described below.

General Form Factor Approach

The first approach is to describe the WWV vertices appealing to established symmetry principles — Lorentz invariance and electromagnetic gauge invariance. For the purpose of studying the WWV vertex in $e^+e^- \rightarrow W^+W^-$ there are seven independent WWZ form-factors and six independent $WW\gamma$ form-factors besides the electric charge of the W [7, 55, 56, 73]. The effective Lagrangian has the form⁶

$$\begin{aligned}
\mathcal{L}_{WWV}/g_V = & ig_1^V (W_{\mu\nu}^+ W^{-\mu} - W^{+\mu} W_{\mu\nu}^-) V^\nu + i\kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} \\
& + i\frac{\lambda_V}{M_W^2} W_{\lambda\mu}^+ W_\nu^{-\mu} V^{\nu\lambda} - g_4^V W_\mu^+ W_\nu^- (\partial^\mu V^\nu + \partial^\nu V^\mu) \\
& + g_5^V \varepsilon^{\mu\nu\rho\sigma} (W_\mu^+ \partial_\rho W_\nu^- - \partial_\rho W_\mu^+ W_\nu^-) V_\sigma \\
& + i\tilde{\kappa}_V W_\mu^+ W_\nu^- \tilde{V}^{\mu\nu} + i\frac{\tilde{\lambda}_V}{M_W^2} W_{\lambda\mu}^+ W_{\mu\nu}^- \tilde{V}^{\nu\lambda}
\end{aligned} \tag{3.5}$$

⁶ $\varepsilon_{0123} = -\varepsilon^{0123} = +1$

Table 3.1: Transformation properties of the individual terms of the Lagrangian (3.5) under discrete transformations.

	$g_1^V, \kappa_V, \lambda_V$	g_4^V	g_5^V	$\tilde{\kappa}_V, \tilde{\lambda}_V$
P	+	+	-	-
C	+	-	-	+
CP	+	-	+	-

where the V denotes either a photon or a Z boson, $g_\gamma = -e$ and $g_Z = -e \cot \theta_W$. Further, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\tilde{V}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}V_{\rho\sigma}$ and M_W is the W boson mass. The parameter g_1^γ is constrained by electromagnetic gauge invariance to be equal to 1. The given parameterization assumes the vector boson to be either on-shell or associated with a conserved current. In Table 3.1 we summarize transformation properties of the individual terms of the Lagrangian (3.5) under discrete transformations. In the static limit, some of the parameters of (3.5) can be related to more familiar physical quantities. Thus the κ_γ and λ_γ are related to the anomalous magnetic dipole moment, μ_W , and the electric quadrupole moment, Q_W , of the W^+ by

$$\mu_W = \frac{e}{2M_W}(1 + \kappa_\gamma + \lambda_\gamma) \quad (3.6)$$

$$Q_W = -\frac{e}{M_W^2}(\kappa_\gamma - \lambda_\gamma) \quad (3.7)$$

Similarly, the $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$ are related to the electric dipole moment, d_W , and the magnetic quadrupole moment, $\tilde{Q}_W$, of the W^+ by

$$d_W = \frac{e}{2M_W}(\tilde{\kappa}_\gamma + \tilde{\lambda}_\gamma) \quad (3.8)$$

$$\tilde{Q}_W = -\frac{e}{M_W^2}(\tilde{\kappa}_\gamma - \tilde{\lambda}_\gamma) \quad (3.9)$$

It would be a formidable task to analyze all these parameters at the same time. Fortunately, we can restrict their number by appealing to other measurements.

Thus, we can ignore the CP violating operators as they are tightly constrained by measurement of the electric dipole moment of fermions (the electric dipole moment of the neutron constrains the two CP violating parameters to $|\tilde{\kappa}|, |\tilde{\lambda}| < \mathcal{O}(10^{-4})$ [57]) and other CP violating observables. The same argument can be applied to the C violating $WW\gamma$ couplings. So if we do not consider C, P, and CP violating operators the number of terms in the Lagrangian (3.5) reduces to five

$$\begin{aligned} \mathcal{L}_{WWV}/g_V = & ig_1^V (W_{\mu\nu}^+ W^{-\mu} - W^{+\mu} W_{\mu\nu}^-) V^\nu + i\kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} \\ & + i \frac{\lambda_V}{M_W^2} W_{\lambda\mu}^+ W_\nu^{-\mu} V^{\nu\lambda} \end{aligned} \quad (3.10)$$

The remaining free parameters are g_1^Z , $\kappa_{\gamma,Z}$ and $\lambda_{\gamma,Z}$, which we will consider in this work. Note that this parameterization does not respect the electroweak gauge symmetry. This was criticized by some authors [62, 76] in the light of the fact that the electroweak gauge symmetry has been well established. In this context we would like to stress two points: First, using the parameterization (3.10) maintains the generality of the analysis. If we want to use the measurement of gauge boson self-couplings as a test of the electroweak gauge symmetry then we cannot use a parameterization where this symmetry is built in as an assumption. In addition, various models with lower symmetries can be easily related to the general parameterization (3.10). Secondly, if so desired, the Lagrangian (3.10) can be considered at tree level to be a unitary gauge realization of a gauge invariant Lagrangian [16, 73, 76].

The Lagrangian of eq. (3.10) has become the standard phenomenological parameterization of the $WW\gamma$ and WWZ vertices. The first four terms correspond to dimension 4 operators and the fifth and sixth terms correspond to dimension 6 operators. The mass in the denominator of the dimension 6 term would correspond to the scale of new physics, typically of order 1 TeV. However, it has become the convention to use M_W so that the W magnetic dipole and electric quadrupole can be written in a form similar to that of the muon. Nevertheless, one expects the

dimension 6 operator to be suppressed with respect to the dimension 4 operators by a factor of $M_W^2/(\Lambda = 1 \text{ TeV})^2 \simeq 10^{-2}$. At tree level the standard model requires $g_1^Z = \kappa_V = 1$ and $\lambda_V = 0$. Typically, radiative corrections from heavy particles will change κ_V by about $\sim 10^{-2}$ and λ_V by about $\sim 10^{-3}$ [59]. In particular, the contributions from a 200 GeV top quark and a 150 GeV Higgs boson to κ_V and λ_V are of order 10^{-3} .

Anomalous TGV couplings in (3.10) appear as constants. However, higher-order terms, similar to those in (3.10), introduce momentum dependence into the anomalous couplings [16, 58]. For example, the operator

$$\frac{1}{\Lambda^2} W_\mu^+ W_\nu^- \square V^{\mu\nu} \quad (3.11)$$

modifies the coupling κ_V by a factor proportional to q^2/Λ^2 . Therefore, it would be more appropriate to talk about form-factors rather than constants. At hadron colliders, where the TGV couplings are probed via gauge boson pair production over a large range of q^2 values, this fact cannot be ignored. Fortunately, when studying W^+W^- production at an e^+e^- collider at fixed $q^2 = s$, which is our case, the form factor behaviour is not so important.

The requirement of the $SU(2)_L \times U(1)_Y$ gauge symmetry leads to additional restrictions on the effective Lagrangian describing the anomalous TGV couplings. There are two standard alternative extensions of the SM based on effective Lagrangians with the gauge symmetry either linearly or non-linearly realized. These Lagrangians have already been introduced in Section 3.2. Below we describe how these approaches parameterize the WWV vertex.

Linearly Realized Higgs Sector

We recall that in this case the effective Lagrangian contains the same fields as the SM including the Higgs doublet field ϕ . Its leading operators are given by the

renormalizable SM Lagrangian and all heavy physics effects decouple. Considering next-to-leading operators without fermionic fields and conserving CP, there are 12 operators up to dimension 6; they are all of dimension 6 and separately conserve C and P. The corresponding part of the effective Lagrangian can be written as

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{i=1}^{12} \frac{f_i}{\Lambda^2} \mathcal{O}_i \quad (3.12)$$

where the full list of the operators $\mathcal{O}_i$ can be found in [20, 62, 78]. There are seven operators in this list that contribute to the process $e^+e^- \rightarrow W^+W^-$. Only five of these operators contribute to $WW\gamma$ and WWZ vertices. However, two of them, along with the remaining two operators which do not influence TGV's, contribute also to gauge-boson two-point functions at tree level and their respective coefficients are stringently constrained by high precision low energy and Z boson data [20, 78]. Thus, we are left with three anomalous TGV parameters to consider. In the standard notation, the linearly realized TGV Lagrangian is given by [62, 73]

$$\begin{aligned} \mathcal{L}_{WWV,linear} = & ig' \frac{\varepsilon_B}{\Lambda^2} (D_\mu \phi)^\dagger B^{\mu\nu} (D_\nu \phi) + ig \frac{\varepsilon_W}{\Lambda^2} (D_\mu \phi)^\dagger W^{\mu\nu} (D_\nu \phi) \\ & + \frac{2i}{3} \frac{L_\lambda}{\Lambda^2} g^3 \text{Tr}[W_{\mu\nu} W^{\nu\rho} W_\rho^\mu] \end{aligned} \quad (3.13)$$

where the anomalous couplings, f_i 's, are denoted here as ε_B , ε_W and L_λ . $W_{\mu\nu}$ and $B_{\mu\nu}$ are the $SU(2)$ and $U(1)$ field strength tensors given in terms of $W_\mu \equiv W_\mu^i \tau_i$ by

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu] \quad (3.14)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (3.15)$$

The parameters from this approach can be rewritten in terms of deviations of the parameters of the Lagrangian (3.10) from their SM values as

$$\Delta g_1^Z \equiv g_1^Z - 1 = \frac{\epsilon^2}{s_w^2} \frac{v^2}{4\Lambda^2} \left(\frac{\varepsilon_W}{c_w^2} \right) \quad (3.16)$$

$$\Delta\kappa_Z \equiv \kappa_Z - 1 = \frac{e^2}{s_w^2} \frac{v^2}{4\Lambda^2} (\varepsilon_W - \frac{s_w^2}{c_w^2} \varepsilon_B) \quad (3.17)$$

$$\Delta\kappa_\gamma \equiv \kappa_\gamma - 1 = \frac{e^2}{s_w^2} \frac{v^2}{4\Lambda^2} (\varepsilon_W + \varepsilon_B) \quad (3.18)$$

$$\lambda_\gamma = \lambda_Z = \left(\frac{e^2}{s_w^2}\right) L_\lambda \frac{M_W^2}{\Lambda^2} \quad (3.19)$$

In general, the coefficients f_i are expected to be numbers of order unity. All the (3.10) anomalous deviations are then suppressed by the factor $(e^2/s_w^2)(v^2/4\Lambda^2) = M_W^2/\Lambda^2$. Hence, taking $\Lambda \sim 1$ TeV, we might expect them to be of $\mathcal{O}(10^{-2})$. However, it was also pointed out in [79] that the three operators of the effective Lagrangian (3.13) cannot be generated at tree level by any renormalizable underlying theory. Thus, the expected size of the corresponding anomalous coupling might be further suppressed by $\sim 1/(16\pi)^2$. Consequently, it might even be possible that dimension 8 operators generated at tree level could dominate over the dimension 6 operators.

Non-Linearly Realized Higgs Sector

In this case, there is no Higgs boson in the low-energy spectrum. The effective Lagrangian includes only the would-be Goldstone bosons which give masses to weak gauge bosons. The non-linear realization [54, 61] is known as a *chiral Lagrangian*, due to its similarity to low-energy QCD which possesses a chiral symmetry. The effective Lagrangian conserves the custodial $SU(2)_C$ symmetry in the limit $g' \rightarrow 0$. If we consider next-to-leading operators without fermionic fields such that they conserve C and P symmetries, the corresponding part of the effective Lagrangian has the form

$$\mathcal{L}_{eff} = \mathcal{L}_{NL\sigma} + \sum_{i=1}^{11} \alpha_i \mathcal{O}_i \quad (3.20)$$

where $\mathcal{L}_{NL\sigma}$ is given by (3.2) and the list of the operators $\mathcal{O}_i$ can be found in [61, 78]. There are only two operators which contribute to anomalous TGV couplings and

not to two-point functions. In the standard notation the effective Lagrangian of the TGV is given by [61, 73]

$$\mathcal{L}_{WWV,non-linear} = -ig \frac{L_{9L}}{16\pi^2} \text{Tr}[W^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger] - ig' \frac{L_{9R}}{16\pi^2} \text{Tr}[B^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma] \quad (3.21)$$

where the anomalous couplings, α_i 's, are denoted here as L_{9L} and L_{9R} . The covariant derivative is given by (3.4). The field strength tensor $W_{\mu\nu}$ is given by (3.14) and $B_{\mu\nu}$ is given by

$$B_{\mu\nu} = (\partial_\mu B_\nu - \partial_\nu B_\mu) \tau_3 \quad (3.22)$$

Note that often in the literature the coefficient $1/16\pi^2$ is replaced with v^2/Λ^2 . New physics contributions are expected to result in values of $L_{9L,9R}$ of order 1 [54]. The parameters L_{9L} and L_{9R} are related to Δg_1^Z and $\Delta \kappa_V$ of (3.10) by

$$\Delta g_1^Z \equiv g_1^Z - 1 = \frac{e^2}{32\pi^2 s_w^2 c_w^2} L_{9L} \quad (3.23)$$

$$\Delta \kappa_Z \equiv \kappa_Z - 1 = \frac{e^2}{32\pi^2 s_w^2 c_w^2} (L_{9L} c_w^2 - L_{9R} s_w^2) \quad (3.24)$$

$$\Delta \kappa_\gamma \equiv \kappa_\gamma - 1 = \frac{1}{32\pi^2} \frac{e^2}{s_w^2} (L_{9L} + L_{9R}) \quad (3.25)$$

We see that Δg_1^Z and $\Delta \kappa_V$ are again of order $(e^2/s_w^2)(v^2/4\Lambda^2) = M_W^2/\Lambda^2$ and corresponding terms in (3.10) of dimension 6. On the other hand, in the non-linear realization, λ_V 's are expected to be of order M_W^4/Λ^4 and the corresponding term is effectively of dimension 8. The linear and non-linear realizations are obtained from each other by identifying $L_{9L} = 2\varepsilon_W$ and $L_{9R} = 2\varepsilon_B$. In the non-linear realization, the counterpart of L_λ is of higher dimension.

3.3.3 Current and future experimental limits on the anomalous TGV couplings

The measurements of the gauge-boson self-interactions still have a long way to go before reaching precision comparable to that obtained on the gauge-boson-fermion

couplings. The LEP1 data has reached such high accuracy that the fermionic loops alone are no longer enough to reproduce the data [80]. Bosonic loop corrections with the gauge-boson self-interactions are needed to reach agreement with experiment. This can be interpreted as strong support for the non-Abelian gauge structure of the SM [73].

The bounds on nonstandard WWV couplings can be extracted from the existing low-energy data on measurements like neutrino-nucleon scattering, polarized electron-deuterium asymmetry, and the $(g-2)$ factor of the muon. The TGV's occur in loops and thus contribute to the observed processes. These indirect bounds turn out to be only a little bit better than those given by the unitarity requirements, which, using the parameterization of (3.10), restrict the deviations to be less than ~ 1 [9]. The indirect measurements of TGV's via radiative corrections to precision electroweak measurements at the Z peak [18, 19, 20] give the following limits [18]: $\delta g_1^Z = -0.033 \pm 0.031$, $\delta \kappa_\gamma = 0.056 \pm 0.056$, $\delta \kappa_Z = -0.0019 \pm 0.044$, $\lambda_\gamma = -0.036 \pm 0.034$, and $\lambda_Z = 0.049 \pm 0.045$. However, there are ambiguities in these calculations associated with running the couplings down from the scale of new physics to low energy and from possible fortuitous cancellations among operators so that these limits are not particularly rigorous and it is necessary to use direct measurements for more reliable bounds.

Direct measurements of the WWV couplings have been performed by the CDF and D0 collaborations at the Tevatron $p\bar{p}$ collider ($\sqrt{s} = 1.8$ TeV) at Fermilab, using the processes $p\bar{p} \rightarrow W\gamma$, WW , WZ . They have obtained the following direct 95% CL limits from $W\gamma$ production varying one parameter at a time [3]:

$$\begin{aligned} \text{D0:} \quad & -0.9 < \Delta \kappa_\gamma < 0.9 \\ & -0.3 < \Delta \lambda_\gamma < 0.3 \end{aligned}$$

$$\text{CDF:} \quad -1.8 < \Delta \kappa_\gamma < 2.0$$

$$-0.7 < \Delta\lambda_\gamma < 0.6$$

Here, the possibility of a minimal $U(1)$ -only coupling ($\kappa = \lambda = 0$) is ruled out at the 88 % CL by the D0 measurement. That means that the photon couples not only to the W electric charge but also to its weak isospin. From WW and WZ production, varying one parameter at a time and assuming that $\kappa_Z = \kappa_\gamma \equiv \kappa$ and $\lambda_Z = \lambda_\gamma \equiv \lambda$, they obtained the following 95 % CL limits:

$$\begin{aligned} \text{D0:} \quad & -0.4 < \Delta\kappa < 0.6 \\ & -0.3 < \Delta\lambda < 0.4 \end{aligned}$$

$$\begin{aligned} \text{CDF:} \quad & -0.5 < \Delta\kappa < 0.6 \\ & -0.4 < \Delta\lambda < 0.3 \end{aligned}$$

Assuming $\Delta\kappa_\gamma = \Delta\lambda_\gamma = 0$ and $\Delta\lambda_Z = 0$, both experiments exclude the point $\kappa_Z = g_1^Z = 0$ at 99 % CL. This is direct evidence for a nonzero WWZ coupling. Therefore the present limits from the Tevatron have confirmed the existence of the weak boson self-interactions but the limits only slightly improved the bounds from unitarity considerations. Further improvement can be expected from the upgraded D0 and CDF detectors that should begin running at the Tevatron in 1999. It is expected that the Tevatron will collect $\mathcal{O}(1-10) \text{ fb}^{-1}$ data and its limits ($\sim \mathcal{O}(0.1)$) will be competitive with those from LEP2. In the longer term, measurements at the Large Hadron Collider (LHC: pp collider with $\sqrt{s} = 14 \text{ TeV}$) at CERN will improve these limits considerably [22]. It is estimated that with 100 fb^{-1} , one can obtain 95 % CL limits on $\Delta\kappa_\gamma$ and $\Delta\lambda_\gamma$ in the range $(5 - 10) \times 10^{-3}$ [81].

With the beginning of operation of LEP2 at CM energy 161 GeV in 1996, the threshold for real W pair production at an e^+e^- collider was crossed and thus direct testing of the WWV vertex has started. So far LEP2 has collected, per experiment, about 11 pb^{-1} at 161 GeV, about 0.4 pb^{-1} and 1 pb^{-1} during two

short runs at 164 and 170 GeV, respectively, and finally, about 11 pb^{-1} at 172 GeV. The so-far achieved limits are still too low to improve the constraints on anomalous TGV's obtained at other machines; they are roughly a factor of two greater than the CDF and D0 results [100, 101]. They are expected to be improved with further operation of the machine at higher energies (184 GeV in 1997 and 188 GeV in 1998) and with the collection of more luminosity ($\sim 100 - 150 \text{ pb}^{-1}/\text{year}$ per experiment, in three years). The expected limits are of the order of $\mathcal{O}(0.1)$ and are comparable with those expected from the upgraded Tevatron. These limits will be surpassed significantly by the LHC. However, in the long term, the measurements at high energy e^+e^- colliders (NLC; $\sqrt{s} \geq 500 \text{ GeV}$) will surpass those at the hadron colliders, reaching limits $\mathcal{O}(10^{-3})$ and better, thus becoming sensitive to the SM and non-SM loop contributions to the TGV's.

The effective parameterization of the TGV introduces several anomalous couplings — free parameters — each of them might assume non-SM values. In an actual experiment we observe the effects of all these parameters added up together. In some situations this “adding” can even lead to cancellation of individual contributions. In any case, we would eventually like to disentangle this mixture and extract values of the individual couplings. The general philosophy to achieve this is to measure processes and observables in which the effects of individual anomalous couplings contribute in different linear combinations. Thus, different observables will have different sensitivities to particular parameters. Combining these measurements can help us disentangle contributions of the individual parameters.

For example, in the measurements of TGV couplings, the hadron colliders may have a partly complementary role to that of e^+e^- colliders [73]. To understand this we have to take a closer look at the effective interaction Lagrangian (3.10) and realize that in the leading approximation the longitudinal polarization vector ϵ_μ of the gauge boson is equal to k_μ/M_V and therefore gives zero contribution

to the antisymmetric tensor $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. The λ terms involve only the field strengths and thus this coupling will predominantly affect the production of transverse gauge bosons. The terms with g_1^Z and κ also involve individual gauge fields where the contribution of their longitudinal components is enhanced by the factor proportional to k_μ/M_V . Now, if we consider the reaction $e^+e^- \rightarrow W^+W^-$, the κ term can be contributed to by two longitudinal fields in the final state, while in the g_1^Z term at least one of them would be transverse. Therefore, we expect this reaction to be more sensitive to κ 's. On the other hand, the typical reaction used to test the TGV couplings at hadron colliders is $p\bar{p} \rightarrow W^\pm Z, W^\pm\gamma$. There, the κ term could contain only one longitudinal field in the final state, while in the g_1^Z term there could be two of them for $W^\pm Z$ production. So the situation is reversed and there should be a greater sensitivity to the g_1^Z coupling.

Another example could be a use of polarized beams at e^+e^- colliders. The same observable has, in general, different dependences on the anomalous couplings for different polarizations of the initial state of a given reaction. The simultaneous study of polarized and unpolarized collisions can therefore give much better bounds on the anomalous couplings than either one of them separately. The same applies to the use of different reactions, as was already illustrated above on the complementarity of e^+e^- and hadron colliders. As a result, many processes have been studied to determine their usefulness for measuring TGV's. Probably the most studied and potentially the most useful among the processes at LEP2 and the NLC is $e^+e^- \rightarrow W^+W^-$ [7, 16, 34, 35, 36, 37, 38]. This also includes studies of various four fermion final states in the process $e^+e^- \rightarrow W^+W^- \rightarrow f_1\bar{f}_2f_3\bar{f}_4$ [39, 29, 40, 41]. Other TGV sensitive processes include $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ [11, 32, 33], $e^+e^- \rightarrow Z\nu\bar{\nu}$ [29, 30, 31], $e^+e^- \rightarrow W^+W^-\gamma$ [83], $e^+e^- \rightarrow W^+W^-Z$ [83]. For an e^-e^- collider the process $e^-e^- \rightarrow e^-W^-\nu_e$ [27] has been considered. In general, it has been found that the limits from these reactions are not competitive to those

obtainable from $e^+e^- \rightarrow W^+W^-$. The NLC is often considered to be able to work as a $\gamma\gamma$ or $e\gamma$ collider. The process $\gamma\gamma \rightarrow W^+W^-$ [23, 28, 84] has a huge cross section even relative to $e^+e^- \rightarrow W^+W^-$. It contains only the $WW\gamma$ vertex and thus would nicely complement measurements of TGV's in the $e^+e^- \rightarrow W^+W^-$ reaction. Another reaction with a relatively large rate and sensitive to WWZ couplings is $\gamma\gamma \rightarrow W^+W^-Z$ [85]. In the $e\gamma$ mode, there have been studies of $e\gamma \rightarrow \nu W$ [23, 24, 25, 26] and $e\gamma \rightarrow \nu WZ$ processes [26]. Some of the expected limits on L_{9L} and L_{9R} parameters from processes at LEP2, NLC and LHC colliders have been summarized in [73, 86]. We show these summaries in Figs. 3.3 and 3.4 borrowed from [73, 86]. Fig. 3.4 also includes some results of this work.

Figure 3.3: Comparison between the expected bounds on the two-parameter space $(L_{9L}, L_{9R}) \equiv (\epsilon_W, \epsilon_B) \equiv (\Delta g_1^Z, \Delta\kappa_\gamma)$ (see text for conversions) at the NLC500 (with no initial polarization) and LEP2. The NLC bounds are from $e^+e^- \rightarrow W^+W^-$, $W^+W^-\gamma$, W^+W^-Z (for the latter these are one-parameter fits), $\gamma\gamma \rightarrow W^+W^-$ and $e^+e^- \rightarrow W^- \nu e^+$. Limits from a single parameter fit are also shown (“bars”).

Figure 3.4: Comparison between the expected bounds on the two-parameter space (L_{9L}, L_{9R}) at the NLC and LHC colliders. Limits are from $e^+e^- \rightarrow W^+W^-$ using either only resonant diagrams with ISR and beam polarization, or all $\ell\nu q\bar{q}'$ diagrams without ISR and beam polarization (limits obtained in this work). Limits from $\gamma\gamma \rightarrow W^+W^-$ and the LHC limits from $pp \rightarrow WZ, W^+W^-$ are also included.

Chapter 4

Measurement of anomalous TGV couplings in the process

$$e^+e^- \rightarrow \ell\nu q\bar{q}'$$

In this Chapter we present our calculations of the sensitivity of the process $e^+e^- \rightarrow \ell\nu q\bar{q}'$ to the anomalous triple gauge boson couplings. We begin with a general discussion of the process $e^+e^- \rightarrow W^+W^- \rightarrow 4f$ as a means to study the TGV's. Later, we present our analysis of the semileptonic channel of this process; calculation of the total and differential cross sections with all tree level Feynman diagrams included. After a brief discussion of the Maximum Likelihood method and its application to our analysis, we present and discuss limits on the anomalous couplings that we obtained. These limits are based on a five-dimensional angular differential cross section and were obtained at three CM energy regimes; (175 - 220) GeV, 500 GeV, and 1 TeV.

4.1 The $e^+e^- \rightarrow W^+W^-$ process as a tool for studying TGV vertices

There are a host of W and Z production processes that, in principle, could be used to study TGV's. Some of them are more suitable for the task than others. To determine the usefulness of a process we can start with a simple criterion like the size of its cross section and subsequently proceed towards more sophisticated analysis, taking into account potential backgrounds, higher order corrections, experimental setup and so on. We can also search for the most sensitive observables and the most effective statistical techniques to extract the information on TGV from the given process.

It appears [73] that one of the most promising processes to measure TGV is $e^+e^- \rightarrow W^+W^-$ which is currently employed at LEP2. It has a relatively large cross section (see Fig. 3.2) compared to other W/Z production processes [73]. However, the cross section is dominated by the production of transverse states of W bosons. This is also often the case with other processes. As we already discussed in Subsection 3.3.3 the longitudinal W 's each contribute to the amplitudes a factor proportional to k_μ/M_W . Consequently, the individual diagrams of $e^+e^- \rightarrow W_L^+W_L^-$ give divergent amplitudes, each breaking unitarity as $\sqrt{s}$ rises. However, by virtue of the Ward identities of the gauge theory, these divergences cancel when we add all the amplitudes together, resulting in good high-energy behaviour. In addition, because of the cancellation, the production of the longitudinal W 's turns into only a minor contribution to the total cross section, the rest of which comes predominantly from W_T production from the neutrino t-channel diagram. Due to their t-channel diagram origin, most of the transverse W 's are produced in the forward region with respect to the colliding beam of the same electric charge as the W .

Potential deviations of TGV couplings from their SM values disrupt the delicate

cancellation. The consequent deviations of observables are amplified by a factor of s/M_W^2 for each W_L in the final state. Hence, we expect that the production of longitudinal W 's is the channel most sensitive to anomalous TGV couplings and that this sensitivity rises with the energy, $\sqrt{s}$. Recall also that longitudinal W 's are interesting objects in their own right because they originate in the ESB sector and thus carry information about the mechanism of ESB.

The analysis of the $e^+e^- \rightarrow W^+W^-$ process has been performed to the extent of the full radiative corrections by several authors [46, 47, 49, 50, 51, 52]. The radiative corrections have appeared to be quite large, mostly due to initial state radiation (ISR). ISR affects some distributions, e.g., it can shift the W 's from the forward region towards the backward region which can interfere with our attempts to isolate the longitudinal W 's from the transverse ones. Regarding the genuine weak corrections, a large part can be taken care of by the use of the Fermi constant G_F and a running electromagnetic coupling constant, α .

The W bosons are short-lived unstable particles. Therefore, in a real experiment, we observe their decay products rather than the W 's themselves. It means that a more realistic analysis must look at the process $e^+e^- \rightarrow W^+W^- \rightarrow f_1\bar{f}_2f_3\bar{f}_4$. The decay products of the W boson enable us to determine its polarization. Because of the $V - A$ structure of the W interactions to fermions, the polar and azimuthal angular distributions of the final fermions constitute an excellent polarimeter. However, four fermions in the final state bring another complication. Besides the three W -pair production diagrams there is a multitude of other diagrams which have the same final state, $e^+e^- \rightarrow f_1\bar{f}_2f_3\bar{f}_4$, but do not proceed via W -pairs. These constitute a potential background to our original diagrams. This background can be reduced by restricting the invariant masses of the decay products to be close to the W mass, thus isolating the resonant diagrams. On the other hand, even among the background diagrams, there are some that contain

TGV's and thus are sensitive to anomalous couplings. In addition, the real W boson is a resonance and has a finite width. Thus, by reducing the “background”, we are losing not only some statistics but also potential sensitivity to the TGV's. To find a proper balance between these counter-acting factors requires performing a detailed analysis of a given process. There have been many programs developed to calculate $4f$ processes at tree level [47], [50], [51], [69], [70], especially for LEP2 [82], but there are no complete calculation of radiative corrections. However, it is expected that the radiative corrections are dominated by ISR, which is not difficult to implement. Another problem that remains open is how to handle the width of the W boson in these calculations. Several schemes have been suggested [35, 43, 64].

There are three classes of the $e^+e^- \rightarrow 4f$ channels: the *leptonic* channels, where both W 's decay into leptons, the *semi-leptonic* channels, where one W decays into a pair of leptons and the other into a pair of quarks, and finally, the *hadronic* channels, where both W 's decay into quarks. The branching ratio of the decays of the W into hadrons dominates with about 68%. The probability of the W boson decaying into one of the three leptonic channels — $e^+\nu_e$, $\mu^+\nu_\mu$, $\tau^+\nu_\tau$ — is slightly above 10%. Therefore, it is expected that the hadronic modes will yield the highest statistics. On the other hand, there is a significant QCD background that will necessarily reduce our ability to extract useful information on the TGV couplings. In addition, when only hadronic jets are observed, there are ambiguities in identifying the charge of the originating W . The leptonic channels give much cleaner signals, but they have lower statistics. Also, there are two unobservable neutrinos and therefore the kinematics of the event cannot be fully reconstructed. The semi-leptonic mode seems to be the most promising one. Regarding the statistics and backgrounds, it is a compromise between the hadronic channels and the leptonic channels. The semi-leptonic mode has higher statistics than the leptonic

channels and cleaner signals than the hadronic channels. With only one unobserved neutrino, using the constraint of the initial beam energies, the kinematics of the event can be fully reconstructed and the W^+ and W^- can be discriminated by using lepton charge identification.

The process $e^+e^- \rightarrow W^+W^-$ includes two TGV vertices, $WW\gamma$ and WWZ . In the parameterization (3.10), these two vertices are parameterized by independent anomalous couplings. In such a case, it will not be easy to disentangle their contributions to measurements. For this purpose we can employ polarized $e^\pm$ beams and supplement information from other processes which do not contain both vertices, or which have different sensitivities to a given anomalous coupling, as was discussed in Subsection 3.3.3. In some approaches, like the chiral Lagrangian, the anomalous couplings of these two vertices are related by additional constraints and thus the problem is reduced or not present at all.

4.2 Introductory remarks on our study of the process $e^+e^- \rightarrow \ell\nu q\bar{q}'$

In our work we examine in detail the sensitivity of the semi-leptonic channel of the $e^+e^- \rightarrow W^+W^-$ reaction to anomalous $WW\gamma$ and WWZ couplings. We calculate the cross section and distributions of the $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$ process, where ℓ is either $e^\pm$ or $\mu^\pm$ and $q\bar{q}'$ can be either (ud) or (cs) , as functions of the anomalous couplings. We study the sensitivity of this process at various centre of mass energies appropriate to LEP2, namely we consider $175 \text{ GeV} \leq \sqrt{s} \leq 220 \text{ GeV}$ ¹. We also consider energies of 500 GeV and 1 TeV appropriate to the NLC. At the NLC energies we study the usefulness of initial state polarization. The sensitivity of the

¹At the time when this analysis was performed the CM energies at which LEP2 would be running were not yet decided.

process is evaluated by a maximum likelihood analysis applied to a five-dimensional differential cross section based on angular distributions of the W 's and the W -decay products. We looked at the question of how important the non-resonant diagrams are regarding the sensitivity to the anomalous couplings and how much information about the TGV's can be found in off W -resonance production.

To study the process $e^+e^- \rightarrow \ell^\pm \nu q \bar{q}'$ we included all tree level diagrams to the four fermion final states. Because of the typical energies under consideration we neglect masses of all fermions in the process. Consequently, we also neglect any interactions with the Higgs boson. Thus, there are 10 diagrams contributing to the $e^+e^- \rightarrow \mu^\pm \nu_\mu q \bar{q}'$ final state which are shown in Fig. 4.1. The TGV couplings we are studying are present in diagram (1a). This, along with diagram (1b) are the diagrams responsible for real W pair production. For the $e^\pm \nu_e q \bar{q}'$ final state the 10 diagrams shown in Fig. 4.2 must also be included with those of Fig. 4.1 for a total of 20 diagrams. Diagram (2a) of the second set also includes the TGV couplings. This diagram makes large contributions to single W production due to the pole in the t-channel photon propagator. This indicates the potential importance of including the complete set of the $4f$ diagrams when doing a more realistic analysis of the sensitivity of the $e^+e^- \rightarrow W^+W^-$ process. It also can be used to isolate the $WW\gamma$ vertex from the WWZ vertex [45]. The contribution of diagram (2a) with the $WW\gamma$ vertex has a peak for the $e^\pm$ scattered into very small angles. Thus, by isolating events where $e^\pm$ emerges almost aligned with the incoming beam of the same electric charge, the $WW\gamma$ vertex can, in principle, be isolated from the WWZ vertex.

We include final width effects by using vector boson propagators of the form $(s - M_V^2 + i\Gamma_V M_V)^{-1}$ which yields a gauge invariant result. Strictly speaking we should have included a momentum dependent vector boson width but this leads to problems with gauge invariance [35, 43, 63, 64]. Although a number of solutions to

Figure 4.1: The Feynman diagrams contributing to the process $e^+ e^- \rightarrow \mu^+ \nu_\mu q \bar{q}'$.

Figure 4.2: The Feynman diagrams that contribute to the process $e^+e^- \rightarrow e^+\nu_e q\bar{q}'$ in addition to those of Fig. 4.1.

this problem have been discussed [35, 43, 64] the difference between our treatment and more rigorous ones has a totally negligible effect on the TGV sensitivities we obtain from our analysis. A more rigorous treatment must of course be included in Monte Carlo simulations that will be used to analyze real experimental data. We will find that the non-resonant diagrams make non-negligible contributions to cross sections which are dependent on the kinematic cuts used in the analysis. These contributions are at least as important as electroweak radiative corrections.

To evaluate the cross-sections and different distributions, we used the CALKUL helicity amplitude technique [65] to obtain expressions for the matrix elements and performed the phase space integration using Monte Carlo techniques [66]. The basic definitions and relations of the CALKUL formalism are given in Appendix B. The Feynman rules for the anomalous TGV's that have been used to calculate the helicity amplitudes are given in Appendix A. The expressions for the helicity amplitudes are given in Appendix C. To obtain numerical results we used the values $\alpha = 1/128$, $\sin^2 \theta = 0.23$, $M_Z = 91.187$ GeV, $\Gamma_Z = 2.49$ GeV, $M_W = 80.22$ GeV, and $\Gamma_W = 2.08$ GeV. In our results we included two generations of quarks. In order to take into account finite detector acceptance we require that the charged lepton and quarks are at least 10 degrees away from the beam and have at least 10 GeV energy unless otherwise noted. The angular cut also helps us avoid the t-channel singularities connected with the forward production of massless electrons/positrons in the process $e^+e^- \rightarrow e^\pm \nu_e q \bar{q}'$.

In principle we should include QED radiative corrections from soft photon emission and the backgrounds due to a photon that is lost down the beam pipe [46, 47, 50]. These backgrounds are well understood and detector dependent. We assume the approach taken at LEP, that these effects can best be taken into account by the experimental collaborations. In any case, although initial state radiation must be taken into account their inclusion does not substantially effect the bounds

we obtain and therefore our conclusions.

4.3 Total cross sections

We calculate the total cross section as a function of $\sqrt{s}$ for both lepton modes: $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$, where $\ell = e/\mu$. The cross section is invariant with respect to the electric charge of the final lepton. To recognize contributions of the resonant and non-resonant diagrams to the cross section we apply various combinations of cuts to the invariant mass of the lepton-neutrino pair and the $q\bar{q}'$ pair. Namely, the cuts are $|M_{\ell\nu, q\bar{q}'} - M_W| < 5 \text{ GeV}$, where $M_{\ell\nu, q\bar{q}'}$ are the invariant masses of the $\ell\nu$ and $q\bar{q}'$ pair respectively. In addition, a 10° cut away from the beam is imposed on charged final state fermions. In Fig. 4.3 we show the cross sections obtained for different combinations of the invariant mass cuts. The corresponding values of the cross sections at energies of 175 GeV, 500 GeV, and 1 TeV are given in Table 4.1. Imposing the invariant mass cut on one fermion pair gives the single W cross section and imposing the cut on both fermion pairs gives the W -pair production cross section. In both cases, the single W and W pair thresholds are clearly seen. Although the single W production cross section is nonzero below the W -pair production threshold, it is still too small to obtain adequate statistics to perform studies of W boson properties.

For the muon mode the invariant mass cuts reduce the cross-section by 10% to 20% depending on $\sqrt{s}$, being between 175 GeV and 1 TeV, irrespective of whether the cut is on $M_{\ell\nu}$ or $M_{q\bar{q}'}$. The combined cuts reduce the muon cross section by 20% to 30%. The relatively small effect of these cuts verifies the dominance of the resonant diagrams on the total cross section. For the electron mode, the cross section without the cuts is larger than for the muon mode. The difference is small at 175 GeV but becomes increasingly larger at higher energy, due to the

Figure 4.3: $\sigma(e^+e^- \rightarrow e^+ \nu_e q\bar{q}')$ and $\sigma(e^+e^- \rightarrow \mu^+ \nu_\mu q\bar{q}')$ as a function of $\sqrt{s}$. A 10° cut away from the beam is imposed on charged final state fermions and no cut on their energy. In both cases the solid curve is the total cross section without any cuts on the $\ell\nu$ and $q\bar{q}'$ invariant masses. The dashed curves are for the cut $|M_{q\bar{q}'} - M_W| < 5$ GeV, the dotted curves for $|M_{\ell\nu} - M_W| < 5$ GeV and the dot-dashed curves for both $|M_{q\bar{q}'} - M_W| < 5$ GeV and $|M_{\ell\nu} - M_W| < 5$ GeV.

Table 4.1: Cross-sections for $e^+e^- \rightarrow \mu^+\nu_\mu q\bar{q}'$ and $e^+e^- \rightarrow e^+\nu_e q\bar{q}'$ including cuts on the invariant masses of the outgoing fermion pairs, $M_{\ell\nu}$ and $M_{q\bar{q}'}$. A 10° cut away from the beam is imposed on charged final state fermions and no cut on their energy. The cross-sections are given in pb.

$\sqrt{s}$ (GeV)	ℓ	no cut	$ M_{q\bar{q}'} - M_W < 5 \text{ GeV}$	$ M_{\ell\nu} - M_W < 5 \text{ GeV}$	both cuts
175	μ	1.10	1.00	1.00	0.91
	e	1.15	1.04	1.01	0.91
500	μ	0.39	0.34	0.34	0.29
	e	0.62	0.53	0.34	0.29
1000	μ	0.077	0.063	0.064	0.052
	e	0.44	0.39	0.064	0.052

enhancement arising from the non-resonant t-channel photon exchange diagrams of Fig. 4.2, reaching a factor of 6 at 1 TeV. Imposing the cut on $M_{q\bar{q}'}$, $|M_{q\bar{q}'} - M_W| < 5 \text{ GeV}$, affects the t-channel photon enhancement of the cross section only slightly. It reduces the cross section by 10% to 15%. On the other hand, when the $|M_{\ell\nu} - M_W| < 5 \text{ GeV}$ cut is imposed, which constrains the $\ell\nu$ pair to be on the W mass-shell, the contributions of all the diagrams of Fig. 4.2 seem to be wiped out and the electron mode cross section becomes practically identical to that of the muon mode. The reduction of the electron mode cross section ranges from 12% at 175 GeV to 85% at 1 TeV. When both cuts are applied at the same time, the reduction is 20% at 175 GeV and almost 90% at 1 TeV.

Despite the relative smallness of the off-resonance contributions to the muon mode they still contribute up to 30% of the cross section at 1 TeV. Clearly, they must be properly included when making high precision tests of standard model processes. For the electron mode they are even more important and are interesting in the context of single W production.

4.4 Distributions

As was already suggested in earlier discussion, different kinematic regions of the process $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$ might have different sensitivities to anomalous TGV couplings. As we will see later in this Section, the phase space regions with the highest statistics are least sensitive to anomalous couplings, and tend to overwhelm deviations, while the regions most sensitive to TGV's have poor statistics. Hence, it is reasonable to expect that a differential cross section is a more efficient means of extracting the information on the TGV's than the total cross section. The question is which distributions are most sensitive to anomalous couplings. Also, a good choice of distributions can help to further understand the contributions and sensitivity of the non-resonant diagrams. In our work we do not extract bounds on anomalous couplings from the total cross section.

These points about the sensitivity of the non-resonant diagrams can be amplified by examining $M_{\ell\nu, q\bar{q}'}$ invariant mass distributions. We show these distributions for $\sqrt{s} = 500$ GeV in Fig. 4.4. In addition, to examine the question of the impact of the initial state polarization on the sensitivity, we choose the initial electron to be either left-handed or right-handed. As the unpolarized cross sections are dominated by the left-handed electrons they are quite similar to the left-handed electron cross sections, so we do not include them separately. Also, the $q\bar{q}'$ invariant mass distributions for left-handed initial electrons are similar to the $M_{\mu\nu}$ distributions. The $M_{e\nu}$ distribution is identical to the $M_{\mu\nu}$ distribution for the right-handed electron case because there are no right-handed electron amplitudes in the Fig. 4.2 set of diagrams. The differences in these cross sections reflect the differences and relative importance in the Feynman diagrams that contribute to a process. In Fig. 4.4 we show the SM distribution and then three non-SM cases, varying one parameter at a time: $\kappa_Z = 1.1$, $\lambda_Z = 0.1$ and $\lambda_\gamma = 0.5$. Although the cross sections and the sensitivities to the TGV's are dominated by the production

of real W 's one can see that off-resonance production of the $\ell\nu q\bar{q}'$ final state can be quite sensitive to anomalous couplings. We will explore this in a later section. When comparing the $M_{\ell\nu}$ distributions for $e^+e^- \rightarrow \ell^+\nu_\ell q\bar{q}'$, the effects are especially pronounced for the $e\nu q\bar{q}'$ final state where there is the possibility of single W production [45].

We examined numerous distributions with the purpose of finding the distributions and isolating the regions of phase space most sensitive to anomalous couplings:

$$\frac{d\sigma}{dp_{T_W}}, \quad \frac{d\sigma}{dp_{T_\mu}}, \quad \frac{d\sigma}{dE_\mu}, \quad \frac{d\sigma}{d\cos\theta_{eW}}, \quad \frac{d\sigma}{d\cos\theta_{e\mu}}, \quad \text{etc.} \quad (4.1)$$

There is, of course, overlap among the kinematic regions of interest in these distributions. To gauge the sensitivity of these distributions to the TGV's we typically divided them into 4-bins and performed a χ^2 analysis. For $\sqrt{s} = 500$ GeV and integrated luminosity of 50 fb^{-1} we found, for example, that the κ 's could be measured to a couple of percent at 95% confidence level. It turns out that this is not competitive with the more sophisticated analysis of angular distributions we will describe below.

For the purpose of understanding W -boson properties the most interesting distributions are the various angular distributions. For the first intuitive hint in this direction, recall that longitudinal W production is the most sensitive to anomalous couplings, while the total cross section is dominated by transverse W 's. When we sum up these two contributions, the longitudinal part will be overwhelmed by the transverse production. We therefore have to find ways to separate these two contributions to the cross section. Because a large portion of the W_T 's originate in the forward production from the t-channel neutrino diagram, Fig 4.1 (b), unfolding the cross section in the W boson scattering angle enables us to separate the more sensitive regions with smaller cross sections from the less sensitive regions with larger cross sections. We can similarly use other variables that are sensitive to the

Figure 4.4: Invariant mass distributions ($M_{\mu\nu}$, $M_{e\nu}$, and $M_{qq'}$) of final state fermions in the processes $e^+e^- \rightarrow \mu^+\nu_\mu q\bar{q}'$ and $e^+e^- \rightarrow e^+\nu_e q\bar{q}'$ for $\sqrt{s} = 500$ GeV. Note the polarization of the initial electron. In all cases the solid line is the standard model cross section, the long-dashed line is for $\kappa_Z = 1.1$, the dotted line is for $\lambda_Z = 0.1$ and the dot-dashed line for $\kappa_\gamma = 0.5$.

polarization state of the W boson; the polar and azimuthal angular distributions of the W decay products.

To understand these points better we plotted and analyzed several of these angular distributions. We begin with the W -pair production cross section without decays to fermions [67, 68]. To leading order, the amplitude for W pair-production is given by three diagrams; via an s-channel photon, an s-channel Z^0 and a t-channel neutrino exchange. The cross sections at $\sqrt{s} = 200, 500$, and 1000 GeV, for $W_L W_L$, $W_L W_T$, and $W_T W_T$, different initial state polarizations, and as a function of the W scattering angle are shown in Fig. 4.5. They were calculated using the analytic expressions from ref. [7]. For the initial state $e_R^- e_L^+$, only the first two diagrams contribute, which at high energy is dominated by longitudinal W production. Because of the delicate cancellations between the diagrams, it is W_L production that is most sensitive to anomalous couplings. In contrast, the cross section for the $e_L^- e_R^+$ initial state produces both transverse and longitudinal W bosons with comparable rates. The e_L^- cross section is dominated by a peak in the forward direction with respect to the incoming e^- associated with the t-channel neutrino exchange, which is made up entirely of transverse W production. This contribution is relatively insensitive to new physics. The cross sections in the backward direction includes sizable longitudinal W production accounting for about 25% of the total cross section in the backward hemisphere. However, in the backward direction where the s-channel diagrams contribute substantially, the cross section for e_R^- is always quite small. For e_R^- there is a large change in the magnitude of the cross-section but only a small change in its shape.

In Fig. 4.6 we show the angular distribution of the outgoing W plotted for several values of anomalous couplings at $\sqrt{s} = 500$ GeV and for different polarizations of the electron beam in the process $e^+ e^- \rightarrow \mu^+ \nu_\mu q \bar{q}'$. Disruptions of the delicate gauge theory cancellations lead to large changes to the standard model

Figure 4.5: The angular distribution of the outgoing W^- with respect to the incoming electron for $\sqrt{s} = 200$ GeV, $\sqrt{s} = 500$ GeV, and $\sqrt{s} = 1$ TeV. The cross-section is given in units of $R = 4\pi\alpha^2/3s$. In all cases the top solid line is for $e_L^-e^+ \rightarrow W_TW_T$, the long-dashed line is for $e_L^-e^+ \rightarrow W_LW_T$, the medium-dashed line is for $e_L^-e^+ \rightarrow W_LW_L$, the short-dashed line is for the total of these three, the dotted line is for $e_R^-e^+ \rightarrow W_TW_T$, the dot-dashed line is for $e_R^-e^+ \rightarrow W_TW_L$, the double dot-dashed line is for $e_R^-e^+ \rightarrow W_LW_L$, and the bottom solid line is for the total of these last three. Note that there is no bottom-solid line for $\sqrt{s} = 500$ GeV.

results which are particularly pronounced, due to a factor of (s/M_W^2) , in the region where W_L production contributes.

Another important means of disentangling the W_L from the W_T background is the use of the W decay products angular distributions. Defining θ_ℓ and θ_q as the angle between the ℓ or q and the W momentum measured in the W rest frame, the angular distribution in θ peaks about $\cos \theta = 0$ for longitudinally polarized W bosons and at forward or backward angles for transversely polarized bosons. In addition, the parity violation of the W couplings distinguishes the two polarization states adding to the effectiveness of the decay as a polarimeter. In Fig. 4.7 we show the angular distributions for the outgoing quark with respect to the W direction (θ_q) for the three bins in the W scattering angle, $\cos \Theta_W < -0.9$, $-0.05 < \cos \Theta_W < 0.05$ and $\cos \Theta_W > 0.9$, (where we take Θ_W to be the W^- angle with respect to the incoming e^-) for the process $e^+e^- \rightarrow \mu^+\nu_\mu q\bar{q}'$ at $\sqrt{s} = 500$ GeV with the initial electron unpolarized. Several values of κ_V and λ_V are included to demonstrate the sensitivity of the distributions to anomalous couplings. The figure shows the dominance of the transverse W polarization at forward angles and the increasing importance of the W longitudinal polarization at $\cos \Theta_W = 0$. Note also the relative lack of sensitivity to anomalous couplings for the forward, dominantly transverse W 's and how the sensitivity increases as the scattering angle increases and longitudinal W 's contribute a larger fraction of the cross section.

We have shown the $d^2\sigma/d\cos \Theta_W d\cos \theta_q$ distribution as it displays the most dramatic change in the shape of the distributions. However, interference between the transverse and longitudinal W 's also depends on the azimuthal angle so that the azimuthal angular distribution also changes its shape, albeit to a lesser extent. One finds similar effects in the angular distributions for the decay $W \rightarrow \ell\nu$.

Figure 4.6: Angular distributions of W for $\sqrt{s} = 500$ GeV and (a) e_L^- and (b) e_R^- in $e^+e^- \rightarrow \mu^+\nu_\mu q\bar{q}'$. In both cases the solid line is the SM result, the dashed line is for $\kappa_Z = 1.1$, the dotted line for $\lambda_\gamma = -0.1$ and the dot-dashed line for $\kappa_\gamma = 0.5$. The distributions were obtained from the full Monte Carlo by imposing the cut $|M_{q\bar{q}} - M_W| < 10$ GeV.

Figure 4.7: Angular distributions of the outgoing quark with respect to the W^- direction in the W rest frame for the process $e^+e^- \rightarrow \mu^+\nu_\mu q\bar{q}'$. In all cases the solid line is the SM result, the dashed line is for $\kappa_Z = 1.1$, the dotted line for $\lambda_Z = 0.1$ and the dot-dashed line for $\lambda_\gamma = -0.1$. The distributions were obtained from the full Monte Carlo by imposing the cut $|M_{q\bar{q}} - M_W| < 10$ GeV.

4.5 Maximum Likelihood fit of 5-dimensional angular distribution

Following the previous discussion, in order to quantify the sensitivity of the $e^+e^- \rightarrow \ell^\pm \nu_\ell q \bar{q}'$ process to the anomalous TGV couplings, as suggested by Barklow [34] we use a combined distribution of five angular observables [16, 36, 41, 42]: Θ , θ_{qq} , ϕ_{qq} , $\theta_{\ell\nu}$, and $\phi_{\ell\nu}$, where Θ is the W^- scattering angle, θ_{qq} is the polar decay angle of the quark in the W^- rest frame using the W^- direction as the quantization axis, ϕ_{qq} is the azimuthal decay angle of the q in the W^- rest frame, and $\theta_{\ell\nu}$ and $\phi_{\ell\nu}$ are the analogous angles for the lepton in the W^+ rest frame. The azimuthal angles are defined as the angle between the normal to the reaction plane, $n_1 = p_e \times p_W$ and the plane defined by the W decay products, $n_2 = p_q \times p_{\bar{q}}$. The individual angles are shown in Fig. 4.8. There is an ambiguity in measurement of the $q\bar{q}$ observables since we do not expect to be able to distinguish which hadronic jet corresponds to the quark and which to the antiquark. We therefore include both cases in our analysis: we sum them and take an average. Unfortunately, this ambiguity has a negative impact on the sensitivity of the process.

The statistical approach which makes the most complete use of the information in an event is the maximum likelihood method [16, 34, 36]. To implement the maximum likelihood analysis we divided each of Θ , θ_{qq} , ϕ_{qq} , $\theta_{\ell\nu}$, and $\phi_{\ell\nu}$ into four bins so that the entire phase space was divided into $4^5 = 1024$ bins. With this many bins, some will not be very populated with events so that it is more appropriate to use Poisson statistics rather than Gaussian statistics. This leads naturally to the maximum likelihood method. The likelihood function is defined as

$$\mathcal{L}(p) = \prod_{i=1}^N \mathcal{P}(n_i; \tau_i(p)) \quad (4.2)$$

Figure 4.8: Angle definitions used in our 5-dimensional angular distribution analysis. Θ is the W scattering angle, θ_{qq} and $\theta_{l\nu}$ are the decay angles in the W rest frames and ϕ_{qq} and $\phi_{l\nu}$ are the azimuthal angles, again in the W rest frames.

where the $\mathcal{P}$ is the Poisson distribution given by

$$\mathcal{P}(n; \tau) = \exp(-\tau) \frac{\tau^n}{n!} \quad (4.3)$$

with τ as its mean value. The τ is a function of a parameter (or a set of parameters) p which we are trying to measure. The most likely value of $p = p_0$ describing the given measurement, $\{n_i\}_{i=1}^N$, is the one that maximizes the function $\mathcal{L}(p)$. If we approximate $\mathcal{L}(p)$ in the neighborhood of p_0 by the Gaussian function with a half-width of σ then for any p that is $k\sigma$ from p_0 ; $p = p_0 \pm k\sigma$:

$$\ell(p = p_0 \pm k\sigma) = \ell(p_0) - \frac{k^2}{2} \quad (4.4)$$

where $\ell(p) = \ln \mathcal{L}(p)$. If we want to quantify the sensitivity of a given experiment to the parameter p , we want to know “how many sigmas” a given value of the parameter p lies from the value p_0 given by our measurement. In our case, if n_i is the measured number of events corresponding to some non-SM values of anomalous couplings, and if n_i^{SM} is the value expected from the prediction of the SM, then the corresponding change in the log of the Likelihood function is given by

$$\delta \ln \mathcal{L} \equiv \ell(p_0) - \ell(p_{SM}) = \sum_i [(n_i^{SM} - n_i) + n_i(\ln n_i - \ln n_i^{SM})] \quad (4.5)$$

where the sum extends over all the bins. The number of events $n_i(p)$ is

$$\begin{aligned} n_i(p) &= L \int_{\Delta\Theta} d\cos\Theta \int_{\Delta\theta_{qq}} d\cos\theta_{qq} \int_{\Delta\phi_{qq}} d\phi_{qq} \int_{\Delta\theta_{\ell\nu}} d\cos\theta_{\ell\nu} \int_{\Delta\phi_{\ell\nu}} d\phi_{\ell\nu} \\ &\quad \times \frac{d^5\sigma(p)}{d\cos\Theta d\cos\theta_{qq} d\phi_{qq} d\cos\theta_{\ell\nu} d\phi_{\ell\nu}} \end{aligned} \quad (4.6)$$

where L is the expected integrated luminosity. From equations (4.4) and (4.5) the SM value lies $\sqrt{2 \cdot \delta \ln \mathcal{L}}$ sigmas from the measured value. When varying a single parameter at a time, the change in $\ln \mathcal{L}$ of 0.5 or 2.0 means that the SM was excluded with 68% and 95% probability, respectively. Generalization to multi-dimensional parameter spaces is straightforward.

Even more efficient in extracting the information on anomalous TGV couplings than a five dimensional angular distribution would be the analysis based on an event-by-event evaluation. In a few test runs for special cases of kinematic cuts we calculated the likelihood function on an event-by-event basis and found that the sensitivities improved a small amount over the five-dimensional distribution case described above. However, looking for a balance between complexity and efficiency of the calculations we used the five-dimensional angular distribution. To check the sensitivity to binning we varied the number of dimensions and bins used in our fits. For this binning approach we found that the results converged to the tightest bounds using the five-dimensional distribution and four bins per dimension.

The results we obtained are based solely on the statistical errors based on the integrated luminosity we assume for the various cases. To include the effects of systematic errors using the maximum likelihood approach requires an unweighted Monte Carlo simulation through a realistic detector. Since we did not have the facilities to do this, we attempted a simplified estimate of systematic errors using a χ^2 analysis to make our estimates. We assumed a systematic error of 5% of a measurement, which we combined in quadrature with the statistical error. In general, the systematic errors are negligible compared to the statistical errors. The only times they made a measurable difference was for the high luminosity cases of the 500 GeV and 1 TeV NLC, and even there, the effect was quite small. It is straightforward to see why this is so; with so many bins the number of events per bin is quite small resulting in a large statistical error. Thus, it appears that the total errors will be dominated by the statistical errors, but, clearly, a full detector Monte Carlo must be performed to properly understand the situation.

A thorough analysis of gauge boson couplings would allow all five parameters in the Lagrangian (3.10) — g_1^Z , κ_Z , κ_γ , λ_Z , λ_γ — to vary simultaneously to take into account cancellations (and correlations) among the various contributions.

This approach is impractical, however, due to the large amount of computer time that would be required to search the parameter space. Instead we found two-dimensional 87% CL exclusion contours for a selection of parameter pairs to give a sense of the correlations. In tables we show 95% CL exclusion bounds, varying only one parameter at a time while the rest remain at their SM values. For the case of the Chiral Lagrangian, the part responsible for TGV's is given by (3.21) (where we restricted ourselves to dimension four operators). Here, the parameter space reduces to two dimensions — L_{9L} and L_{9R} . In this case, the two-dimensional contours represent an analysis of the full parameter space.

4.6 LEP2

4.6.1 Introductory discussion

One of the primary physics goals of LEP2 is to make precision measurements of W boson properties, including precision measurements of W -boson couplings with the photon and Z^0 . An important question for LEP2 is the sensitivity of the TGV measurements to the centre of mass energy and luminosity² [82]. This issue has been addressed in a number of papers. The classic paper by Hagiwara, Peccei, Zeppenfeld, and Hikasa [7] examined the sensitivity of anomalous TGV's to the process $e^+e^- \rightarrow W^+W^-$ and how the sensitivity varied with centre of mass energies relevant to LEP2. Although this paper did point out the importance of separating longitudinally polarized W 's from transversely polarized W 's, the analysis was restricted to specific angular distributions and specific W boson polarizations and varied only one parameter at a time. In addition, it did not include the contributions from the background contributions: other tree level diagrams that contribute

²The values of energies and luminosities, at which LEP2 is expected to operate, are discussed in Subsection 3.3.3.

to the same four fermion final state.

Sekulin [36] and Aihara *et al.*, [16] included the information about the outgoing W polarizations by using the angular distributions of the W boson decay products and used a binned maximum log likelihood fit to a five dimensional differential cross section with respect to the W scattering angle and the polar and azimuthal decay angles of the W^+ and W^- bosons. Both of these analysis looked at different center of mass energies relevant to LEP2 but neither included the background contributions. The analysis of Sekulin assumed the narrow width approximation for the W decays. The analysis by Aihara *et al.*, was more sophisticated and included initial state radiation, detector smearing, and various kinematic cuts introduced to reduce backgrounds. Aihara *et al.*, also assumed relations among the parameters, which, in the language of the non-linearly realized Chiral Lagrangian, takes $L_{9L} = L_{9R}$. Their general conclusion that the sensitivity to the TGV parameters increased by a factor of 1.5 going from 176 GeV to 190 GeV is consistent with what we find.

The recent studies by Berends and van Sighem [48] and by Papadopoulos [35] are the closest in spirit to ours. Both of these studies included full tree level background processes and finite width effects. Berends and van Sighem also included initial state radiation but did not consider the variations with center of mass energy and did not quantify the sensitivities to TGV's. Papadopoulos looked at a range of centre of mass energies but restricted his study to specific angular distributions and varied only one parameter at a time.

In our work we tie all the various pieces of previous analysis together (except ISR) and apply them to the analysis of the process $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$: Recall that we include all tree level background diagrams and finite width effects and we perform a binned log likelihood fit to a five dimensional differential cross section. We vary the different parameters simultaneously so that correlations between them show

up, and we vary both the center of mass energy and luminosities relevant to LEP2.

4.6.2 Results for LEP2 energies

In this Subsection we present and discuss our results relevant to LEP2. We calculated the sensitivities to anomalous TGV couplings for the centre of mass energies $\sqrt{s} = 175, 192, 205$, and 220 GeV, assuming the integrated luminosities of 300 pb^{-1} and 500 pb^{-1} . We did not include a cut on $M_{\ell\nu}$ or $M_{q\bar{q}}$ as these cuts, in general, have virtually no effect on the sensitivities, except for the electron mode involving the $WW\gamma$ vertex, where the effect is still quite small. Unpolarized beams are considered for all limits.

We begin with a more detailed presentation of the exclusion contours for $\sqrt{s} = 175$ GeV. The 87% CL exclusion contours for the $g_1^Z - \kappa_Z$, $\kappa_\gamma - \kappa_Z$, $\kappa_\gamma - \lambda_\gamma$, and $\kappa_Z - \lambda_Z$ planes are shown in Fig. 4.9 and for the $L_{9L} - L_{9R}$ plane is shown in Fig. 4.10. The sensitivities of the couplings, varying one parameter at a time, are summarized in Table 4.3 along with the limits from NLC. In addition, although in less detailed fashion, these limits are also present in Table 4.2, where they are shown along with the limits obtained for other LEP2 energies. In each of these figures, contours are shown for the muon mode alone and then for the combined results of the e and μ modes with both charge possibilities. We also show contours for a reduced integrated luminosity of 300 fb^{-1} . For the L_{9L} vs L_{9R} plot we show contours for both the electron and muon modes since there is a visible difference for the two modes. By combining the four lepton modes the couplings can be measured to $\delta g_1^Z = \pm 0.22$, $\delta \kappa_Z = \pm 0.2$, $\delta \kappa_\gamma = \pm 0.27$, $\lambda_Z = \pm 0.18$, $\lambda_\gamma = \pm 0.3$, $\delta L_{9L} = \pm 55$, and $\delta L_{9R} = \sim \pm 300$. If the results of the four LEP experiments could be combined, these limits could be reduced further.

In order to investigate how the LEP2 sensitivity to anomalous couplings changes with the energy of collision, we calculated the exclusion limits for $\sqrt{s} = 192, 205$,

Figure 4.9: 87% C.L. contours for sensitivity to anomalous couplings for $\sqrt{s} = 175$ GeV. In all cases the inner solid contour is obtained from combining all 4 lepton charge states for $L = 500 \text{ pb}^{-1}$, the heavy outer solid line is for the μ^+ mode alone for $L = 500 \text{ pb}^{-1}$, and the dotted contour is for the reduced luminosity case of $L = 300 \text{ pb}^{-1}$ with all 4 modes combined.

Figure 4.10: 87% C.L. contours for sensitivity to L_{9L} and L_{9R} for $\sqrt{s} = 175$ GeV. (a) The heavy solid line is for the μ^+ mode, the dotted line is for the e^+ mode and the inner solid line is for combining all four lepton charge states; all for $L = 500 \text{ pb}^{-1}$. (b) Both curves are from combining all four lepton charge states. The solid line is for $L = 500 \text{ pb}^{-1}$ and the dotted line is for $L = 300 \text{ pb}^{-1}$.

Table 4.2: Sensitivities to anomalous couplings for the various parameters, varying one parameter at a time, for $\sqrt{s} = 175, 192, 205$, and 220 GeV. The values are obtained by combining the four lepton modes (e^- , e^+ , μ^- , and μ^+) and two generations of light quarks (ud , cs). The results are 95% C.L. exclusion limits for the given integrated luminosities.

$\sqrt{s}$ (GeV)	L (pb $^{-1}$)	g_1^Z	κ_Z	κ_γ	L_{9L}	L_{9R}
175	500	± 0.22	$^{+0.19}_{-0.20}$	$^{+0.27}_{-0.26}$	± 55	$^{+330}_{-230}$
	300	$^{+0.28}_{-0.29}$	± 0.25	$^{+0.36}_{-0.33}$	± 70	$^{+440}_{-300}$
192	500	± 0.15	± 0.12	$^{+0.16}_{-0.14}$	$^{+35}_{-34}$	$^{+170}_{-120}$
	300	± 0.20	± 0.16	$^{+0.21}_{-0.18}$	$^{+46}_{-44}$	$^{+240}_{-150}$
205	500	± 0.14	± 0.11	$^{+0.12}_{-0.11}$	$^{+31}_{-29}$	$^{+120}_{-100}$
	300	± 0.18	± 0.13	$^{+0.17}_{-0.14}$	$^{+40}_{-38}$	$^{+190}_{-110}$
220	500	$^{+0.10}_{-0.08}$	± 0.09	$^{+0.10}_{-0.08}$	$^{+28}_{-26}$	$^{+100}_{-60}$
	300	$^{+0.13}_{-0.12}$	± 0.12	$^{+0.13}_{-0.10}$	$^{+36}_{-33}$	$^{+135}_{-90}$

and 220 GeV, in addition to 175 GeV. For the purpose of comparison of the corresponding sensitivities, we put them all in the same graph. We show only the sensitivities that can be obtained by combining the e^+ , e^- , μ^+ , and μ^- modes. The 87% confidence limit contours for the $g_1^Z - \kappa_Z$, $\kappa_\gamma - \kappa_Z$, and $L_{9L} - L_{9R}$ planes for different LEP2 energies and the integrated luminosity of 500 pb^{-1} are shown in Fig. 4.11. The sensitivities of the couplings, varying one parameter at a time, are summarized in Table 4.2.

At threshold, anomalous couplings are quite sensitive to energy with improvements in sensitivities going from $\sqrt{s} = 175$ GeV to $\sqrt{s} = 192$ GeV ranging from about 1.7 for κ_Z to ~ 2 for L_{9R} . The corresponding changes going from $\sqrt{s} = 205$ GeV to $\sqrt{s} = 220$ GeV are 1.2 for κ_Z and ~ 1.4 for L_{9R} . This occurs even though the cross section only varies from 1.10 pb (1.15 pb) at 175 GeV to its maximum value of 1.28 pb (1.31 pb) at 200 GeV for the μ (e) mode.

Figure 4.11: 87% C.L. exclusion contours for sensitivity to anomalous couplings for $\sqrt{s} = 175, 192, 205$, and 220 GeV. In all cases the contours are obtained from combining all four lepton charge states for $L = 500 \text{ pb}^{-1}$. The contours correspond to increasing energy going from the outer contour to the inner with the outermost contour corresponding to $\sqrt{s} = 175$ GeV and the innermost corresponding to $\sqrt{s} = 220$ GeV.

The sensitivity can be understood by examining the helicity amplitudes in detail which are given by Hagiwara *et al.*, in Ref. [7]. At threshold, the ν -exchange diagram dominates so that at low energy the γ and Z diagrams are down by a factor of $\beta = \sqrt{1 - 4M_W^2/s}$ compared to the ν -exchange diagram. Thus, at threshold the cancellation between the various diagrams are less important than at higher energies. Near threshold the contributions from anomalous couplings go roughly like β with a further enhancement of roughly $\sqrt{s}/M_W$ for each longitudinal W boson in the final state. The net result is that in the threshold region the reaction $e^+e^- \rightarrow W^+W^-$ is not very sensitive to the TGV couplings. The coupling measurements are also limited by statistics, which are in turn related to the number of events, so that the achievable limits are inversely proportional to the square root of the integrated luminosity; $\delta \propto L^{-1/2}$. Reducing the luminosity from 500 pb^{-1} to 300 pb^{-1} decreases the sensitivity by a factor of 1.3.

In any case, these limits are at the very least an order of magnitude less sensitive than would be required to see the effects of new physics through radiative corrections and are comparable to the sensitivities that could be achieved at a high luminosity Tevatron upgrade. It is therefore unlikely that new physics will reveal itself at LEP2 through precision measurements of the TGV's.

4.7 Next Linear Collider

4.7.1 $\sqrt{s} = 500 \text{ GeV}$

For the $\sqrt{s} = 500 \text{ GeV}$ NLC option we assume an integrated luminosity of 50 fb^{-1} . At 500 GeV the results are most sensitive when we impose the W 's to be on mass-shell; i.e., $|M_{\ell\nu} - M_W| < 10 \text{ GeV}$ and $|M_{q\bar{q}} - M_W| < 10 \text{ GeV}$. This is slightly more pronounced for the electron mode. After these cuts are imposed the electron and muon modes are essentially identical. The 87% confidence limits for the $g_1^Z - \kappa_Z$,

$\kappa_\gamma - \kappa_Z$, $\kappa_\gamma - \lambda_\gamma$, and $\kappa_Z - \lambda_Z$ planes are shown in Fig. 4.12 and for the $L_{9L} - L_{9R}$ planes in Fig. 4.13. In each of these plots we show results for combining the four final states and the μ^+ mode alone. We also present contours for 10 fb^{-1} of integrated luminosity to show the effects of reducing the collider luminosity. The sensitivities, varying one parameter at a time, are included in Table 4.3.

Using the maximum likelihood analysis we find that κ_γ , κ_Z , λ_γ and λ_Z can be measured to better than ± 0.005 and g_1^Z to roughly ± 0.01 at 95 % C.L. including the invariant mass cut and assuming 50 fb^{-1} integrated luminosity. If the integrated luminosity were reduced to 10 fb^{-1} , the bounds become weaker by roughly a factor of two, while combining all four modes improves the single mode bounds by roughly a factor of two. These measurements of g_1^Z , κ_γ , and κ_Z should be precise enough to probe loop radiative corrections to the couplings. On the other hand, the measurements of λ_γ and λ_Z are still an order of magnitude too large to see expected deviations from tree level values due to radiative corrections.

The $L_{9L} - L_{9R}$ contours are shown in Fig. 4.13. L_{9L} can be measured to ± 3 and L_{9R} to $\sim \pm 8$ using a single mode and to ± 1.5 and ± 4 , respectively, combining all four $\ell\nu q\bar{q}$ final states.

In the earlier discussion of angular distributions we pointed out that reactions with different initial electron polarizations have different dependences on anomalous couplings [38]. In the following, we explore the consequences of this behaviour. We restrict our results to the dimension 4 operators, where deviations are most likely to show up. For the 500 GeV e^+e^- collider we took 50 fb^{-1} of integrated luminosity per polarization. We only include results for two combined lepton modes, μ^+ and e^+ . With the cuts applied, $|M_{\ell\nu,qq} - M_W| < 10 \text{ GeV}$, the contributions of the background diagrams of Fig. 4.2 are very small. Thus the bounds obtained with the combination of the two ℓ^+ modes are similar to those that would be obtained if all the four lepton modes are combined at a half of the luminosity. The

Figure 4.12: 87% C.L. contours for sensitivity to anomalous couplings for $\sqrt{s} = 500$ GeV. In all cases the inner solid contour is obtained from combining all 4 lepton charge states for $L = 50 \text{ fb}^{-1}$, the heavy solid line is for the μ^+ mode alone for $L = 50 \text{ fb}^{-1}$, and the dotted contour is for the reduced luminosity case of $L = 10 \text{ fb}^{-1}$ with all 4 modes combined. These results were obtained by imposing that the W 's are on mass shell; $|M_{\ell\nu} - M_W| < 10 \text{ GeV}$ and $|M_{q\bar{q}} - M_W| < 10 \text{ GeV}$.

Figure 4.13: 87% C.L. contours for sensitivity to L_{9L} and L_{9R} for $\sqrt{s} = 500$ GeV. The inner solid line is obtained by combining all four lepton charge states and is for $L = 50 \text{ fb}^{-1}$, the heavy solid line is for all 4 modes and $L = 10 \text{ fb}^{-1}$ and the dotted line is for the μ^+ mode alone for $L = 50 \text{ fb}^{-1}$. These results were obtained by imposing that the W 's are on mass shell; $|M_{\ell\nu} - M_W| < 10 \text{ GeV}$ and $|M_{q\bar{q}} - M_W| < 10 \text{ GeV}$.

87% C.L. contours for the $g_1^Z - \kappa_Z$, $\kappa_\gamma - \kappa_Z$, and $L_{9L} - L_{9R}$ planes are shown in Fig. 4.14. Shown are contours for $e_{L,R}^- e^+ \rightarrow \ell^+ \nu_\ell q \bar{q}'$ and unpolarized initial states. For the $g_1^Z - \kappa_Z$ plane there is not much difference in the shape of the contour for the different polarizations, although the bounds improve slightly. On the other hand, the different polarizations give much different dependences for the $\kappa_\gamma - \kappa_Z$ and $L_{9L} - L_{9R}$ contours. In the $\kappa_\gamma - \kappa_Z$ plane the right-handed electron polarizations give constraints orthogonal to the left-handed polarizations and unpolarized results. The unpolarized contours are aligned along the e_L^- contours, which is not too surprising, considering that $\sigma(e_L^-)$ dominates the right-handed contribution in the unpolarized cross-section. For the $L_{9L} - L_{9R}$ plane the left and right handed polarizations also give different dependences that would further constrain L_{9L} .

4.7.2 $\sqrt{s} = 1$ TeV

For the 1 TeV NLC collider we assume an integrated luminosity of 200 fb^{-1} . The 87% C.L. sensitivity contours for the $g_1^Z - \kappa_Z$ and $\kappa_\gamma - \kappa_Z$ are shown in Fig. 4.15. These results were obtained by imposing the W 's to be on mass shell: $|M_{\ell\nu} - M_W| < 10 \text{ GeV}$ and $|M_{q\bar{q}} - M_W| < 10 \text{ GeV}$. We do not bother showing the $\lambda_V - \kappa_V$ contours, since they are uncorrelated (one parameter is least sensitive when the other is taken to be equal to its SM value) so it is sufficient to give the sensitivity when all other parameters are set to their SM values. The $L_{9L} - L_{9R}$ contours are shown in Fig. 4.16. With these collider parameters g_1^Z can be measured to about ± 0.005 , while κ_Z and κ_γ can be measured to about 10^{-3} . These measurements will be sensitive enough to test the standard model at the level of radiative corrections. The limits, varying one parameter at the time, are included in Table 4.3.

Regarding the initial state polarization, for the 1 TeV e^+e^- collider we took 200 fb^{-1} of integrated luminosity per polarization. The sensitivities are shown in

Figure 4.14: 87% C.L. contours for sensitivity to anomalous couplings for polarized initial state electrons for $\sqrt{s} = 500$ GeV and $L = 50 \text{ fb}^{-1}$ per polarization and combining μ^+ and e^+ final states. In all cases the solid curves are for e_L^- , the dashed curves for e_R^- , and the heavy solid curve for unpolarized electrons (for $L = 50 \text{ fb}^{-1}$).

Figure 4.15: 87% C.L. contours for sensitivity to anomalous couplings for $\sqrt{s} = 1$ TeV. In both cases the inner solid contour is obtained from combining all 4 lepton charge states for $L = 200 \text{ fb}^{-1}$ and the dotted contour is for the reduced luminosity case of $L = 50 \text{ fb}^{-1}$ with all 4 modes combined. The μ^+ contour for $L = 200 \text{ fb}^{-1}$ lies on top of the dotted curves. These results were obtained by imposing that the W 's are on mass shell; $|M_{\ell\nu} - M_W| < 10 \text{ GeV}$ and $|M_{q\bar{q}} - M_W| < 10 \text{ GeV}$.

Figure 4.16: 87% C.L. contours for sensitivity to L_{9L} and L_{9R} for $\sqrt{s} = 1$ TeV. The inner solid line is obtained by combining all four lepton charge states and is for $L = 200 \text{ fb}^{-1}$ and the dotted line is for all 4 modes and $L = 50 \text{ fb}^{-1}$. The μ^+ mode alone for $L = 200 \text{ fb}^{-1}$ sits on top of the dotted contour. These results were obtained by imposing that the W 's are on mass shell; $|M_{\ell\nu} - M_W| < 10 \text{ GeV}$ and $|M_{q\bar{q}} - M_W| < 10 \text{ GeV}$.

Fig. 4.17. They are similar to, but more constraining than, the 500 GeV case so we do not comment further. More important than the improvement in sensitivity is the usefulness of polarization for disentangling the nature of anomalous TGV's if deviations are observed.

An important difference between the two polarizations, which can also be seen in Fig. 4.4, is that the cross section with left-handed electrons is about an order of magnitude larger than for right-handed electrons. At the same time the right-handed cross section is significantly more sensitive to anomalous couplings than the left-handed cross section. There is therefore a tradeoff between sensitivity and statistics, so that in most cases the bounds obtainable for the two polarizations are comparable. The unpolarized results offer no improvement over the polarized results since the right-handed cross-section is overwhelmed by the left-handed contribution. One exception to these comments is when we consider L_{9L} or L_{9R} , where the left-handed electrons are more constraining for L_{9L} , while the right-handed electrons are more constraining for L_{9R} . For some combinations of the parameters the different polarizations give much different correlations which could be useful for disentangling the nature of anomalous TGV's if deviations were observed.

4.8 Exploring the sensitivity of the off-resonance production

In this section we explore the information potential available from $\ell\nu q\bar{q}$ final states off the W resonance. Referring to the invariant mass distributions shown in Fig. 4.4, one sees that there is considerable sensitivity to anomalous gauge boson couplings even when the fermion pairs do not originate from real W production. We do not perform a rigorous analysis here, but demonstrate that there is consider-

Figure 4.17: 87% C.L. exclusion contours for sensitivity to anomalous couplings for polarized initial state electrons for $\sqrt{s} = 1$ TeV and $L = 200 \text{ fb}^{-1}$ per polarization and combining μ^+ and e^+ final states. In all cases the solid curves are for e_L^- , the dashed curves for e_R^- , and the heavy solid curve for unpolarized electrons (for $L = 200 \text{ fb}^{-1}$).

Table 4.3: Sensitivities to anomalous couplings for the various parameters varying one parameter at a time. The combined values are obtained by combining the four lepton modes (e^- , e^+ , μ^- , and μ^+) and two generations of light quarks (ud , cs). The results are 95% confidence level limits.

mode	δg_1^Z	$\delta \kappa_Z$	$\delta \kappa_\gamma$	λ_Z	λ_γ	L_{9L}	L_{9R}
$\sqrt{s} = 175 \text{ GeV}$, $L=500 \text{ pb}^{-1}$, no cuts on $M_{\ell\nu(q\bar{q})}$							
μ	+0.45 -0.44	+0.39 -0.38	+0.58 -0.48	+0.36 -0.35	+0.61 -0.50	± 110	+920 -420
e	+0.44 -0.43	+0.40 -0.38	+0.58 -0.52	+0.37 -0.35	+0.62 -0.49	+120 -110	+620 -440
combined	± 0.22	+0.19 -0.20	+0.27 -0.26	± 0.18	+0.29 -0.26	± 55	+330 -230
$\sqrt{s} = 500 \text{ GeV}$, $L=50 \text{ fb}^{-1}$, $ M_{\ell\nu(q\bar{q})} - M_W < 10 \text{ GeV}$							
μ	± 0.020	± 0.007	± 0.005	± 0.005	± 0.006	+2.2 -2.1	+4.6 -4.2
e	+0.019 -0.020	± 0.007	± 0.005	± 0.005	± 0.006	+2.2 -2.1	+4.6 -4.2
combined	± 0.0095	± 0.0035	± 0.0025	± 0.0025	± 0.0025	+1.1 -1.0	+2.2 -2.1
$\sqrt{s} = 1 \text{ TeV}$, $L=200 \text{ fb}^{-1}$, $ M_{\ell\nu(q\bar{q})} - M_W < 10 \text{ GeV}$							
μ	± 0.01	± 0.002	± 0.001	± 0.002	± 0.002	+0.61 -0.62	+1.3 -1.1
e	± 0.01	± 0.002	± 0.001	± 0.002	± 0.002	+0.61 -0.62	+1.3 -1.1
combined	± 0.0054	± 0.001	± 0.0006	± 0.0008	± 0.0008	± 0.28	+0.62 -0.56

able information in the non-resonant production. In particular, we do not consider possible backgrounds to non-resonant events and do not make any effort to optimize our cuts to enhance deviations from SM results. We consider $\sqrt{s} = 200$ GeV, 500 GeV, and 1 TeV for both the $e\nu q\bar{q}$ and $\mu\nu q\bar{q}$ final states and include initial state polarization when appropriate. We base our results on the total cross-section upon imposing the cuts $M_{ff'} < M_W - 15$ GeV and $M_{ff'} > M_W + 15$ GeV, where $M_{ff'}$ is the invariant mass of the final state fermion pairs and ff' stands for either $\ell\nu$ or $q\bar{q}$. These give rise to a large number of possibilities, so we only present the “best” case when the four possible final states are combined for each energy.

For $\sqrt{s} = 200$ GeV we only considered unpolarized initial state electrons and positrons. The results here should not be taken too seriously because of the low number of events expected in these kinematic regions. For example, the standard model predicts, for an integrated luminosity of 500 fb^{-1} and combining all 4 final states, only 40 events, when either $M_{\ell\nu} < M_W - 15$ GeV or $M_{\ell\nu} > M_W + 15$ GeV. With this warning, the optimum results occur for $M_{\ell\nu} < M_W - 15$ GeV and are given in Table 4.4. The results are slightly weaker for the case $M_{q\bar{q}} < M_W - 15$ GeV. Although for a specific case sensitivities may differ between the μ and e final states, they are generally quite similar. The case $M_{ff'} > M_W + 15$ GeV is not nearly as sensitive to anomalous couplings, except for L_{9R} and κ_γ . These results, along with the previous ones, which concentrated on the real W production, indicate that the results are dominated by real W production. The off-resonance limits are roughly a factor of two to three weaker than those given previously for real W production and are not likely to contribute much to bounds on TGV’s at LEP2.

For $\sqrt{s} = 500$ GeV, and combining the four final states, the sensitivity is greatest when $M_{\ell\nu} > M_W + 15$ GeV except for a few cases. The results for $M_{q\bar{q}}$ cuts are slightly less sensitive. Considering either the $e^\pm$ or $\mu^\pm$ final states separately we find that for the $e^\pm$ final states the $M_{\ell\nu} > M_W + 15$ GeV case is more

sensitive than the $M_{q\bar{q}} > M_W + 15$ GeV case, while for the $\mu^\pm$ final states they are comparable. With $M_{\ell\nu} > M_W + 15$ GeV the $e^\pm$ final states are more sensitive to couplings involving photons than the $\mu^\pm$ final states. In both cases, when we take $M_{\ell\nu} > M_W + 15$ GeV, the cross section is dominated by the $q\bar{q}$ pair originating from an on-shell W . This gives us the case of single- W production, which receives large contributions from t -channel photon exchange and hence it is more sensitive to the $WW\gamma$ coupling.

The results at 1 TeV are qualitatively similar to those at 500 GeV, so we do not repeat the discussion of the preceding paragraph, but only point out the few points that differ. Again, the highest sensitivity is for the constraint $M_{\ell\nu} > M_W + 15$ GeV. The achievable bounds for this case are included in Table 4.4. They are typically 4 to 5 times more constraining than those obtainable at 500 GeV; less than 1% for κ_γ and κ_Z , which is at the level of loop contributions from new physics. One interesting difference is that the muon mode for right-handed initial electrons provides the most stringent constraints for many of the TGV couplings. As before, the electron final state offers the best measurements of κ_γ .

From the above results it is clear that, although the constraints that could be obtained from off-resonance production are not as tight as those obtained from on-shell W production, there is nevertheless considerable information contained in these events. It appears to us that the method that makes optimal use of each event is to calculate the probability of each event, irrespective of where it appears in phase space, and compute a likelihood function for the combined probabilities. The only experimental cuts that should be included are those that represent detector acceptance and that are introduced to eliminate backgrounds.

Table 4.4: Sensitivities to anomalous couplings based on off-resonance cross sections varying one parameter at a time. The values are obtained by combining the four lepton modes (e^- , e^+ , μ^- , and μ^+) and two generations of light quarks (ud , cs). The results are 95% confidence level limits. A dash signifies that the bound is significantly weaker than the others.

Initial State	cut	δg_1^Z	$\delta \kappa_Z$	$\delta \kappa_\gamma$	λ_Z	λ_γ	L_{9L}	L_{9R}
$\sqrt{s} = 200 \text{ GeV}, L=500 \text{ pb}^{-1}$								
e^-	$M_{\ell\nu} < M_W - 15 \text{ GeV}$	+1.5 -1.1	+1.1 -0.9	+0.8 -1.0	+1.0 -1.0	+1.1 -0.7	+400 -320	+520 -780
$\sqrt{s} = 500 \text{ GeV}, L=50 \text{ fb}^{-1}$								
e_L^-	$M_{\ell\nu} > M_W + 15 \text{ GeV}$	+0.13 -0.18	+0.056 -0.074	+0.012 -0.012	+0.026 -0.034	+0.026 -0.041	+6.9 -7.2	+8.9 -9.2
e_R^-	$M_{\ell\nu} > M_W + 15 \text{ GeV}$	+0.08 -0.24	+0.03 -0.10	--- -0.018	+0.023 -0.033	+0.034 -0.023	--- ---	--- ---
e^-	$M_{\ell\nu} > M_W + 15 \text{ GeV}$	+0.15 -0.21	+0.07 -0.10	+0.016 -0.017	+0.031 -0.038	+0.033 -0.043	+10 -10	+13 -13
$\sqrt{s} = 1 \text{ TeV}, L=200 \text{ fb}^{-1}$								
e_L^-	$M_{\ell\nu} > M_W + 15 \text{ GeV}$	+0.038 -0.085	+0.013 -0.014	+0.005 -0.005	+0.002 -0.002	+0.003 -0.003	+2.4 -2.5	+3.8 -3.9
e_R^-	$M_{\ell\nu} > M_W + 15 \text{ GeV}$	+0.062 -0.102	+0.010 ---	--- -0.010	+0.007 -0.007	+0.007 -0.007	--- ---	--- -5.9
e^-	$M_{\ell\nu} > M_W + 15 \text{ GeV}$	+0.047 -0.096	+0.018 -0.021	+0.007 -0.007	+0.003 -0.003	+0.003 -0.003	+3.4 -3.5	+5.4 -5.6

Chapter 5

$t\bar{t}$ production via vector boson fusion at high energy e^+e^- colliders

Its great mass endows the top quark with special properties which make it a suitable probe for an investigation of the mechanism of ESB. In this Chapter, after a brief introduction to top quark physics, we examine the process $e^+e^- \rightarrow \ell\bar{\ell}t\bar{t}$ which is considered to be a good candidate for such an investigation. We calculate the cross section of the process using the effective vector boson approximation. We investigate the sensitivity of this cross section to the mass of the Higgs boson. We also evaluate the error on $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$ caused by using a high-energy approximation for $\sigma(WW/ZZ \rightarrow t\bar{t})$.

5.1 Introduction to top quark physics

Discovery of the top quark was a long anticipated event. Even before its direct observation at Fermilab, the top quark was pointed to by a wealth of indirect ex-

perimental evidence, like that from high precision measurements at LEP and SLAC (Stanford Linear Accelerator Center). Through higher order Feynman diagrams, even below its production threshold, the top quark makes important corrections to many observables in e^+e^- collisions, such as the width of the Z^0 boson or the bottom quark neutral-weak-current interactions. The comparison of these SM predictions with the high precision measurements favoured a top mass of 177 GeV with uncertainty of about ± 25 GeV [4]. The subsequent direct observation of the top quark at Fermilab's Tevatron confirmed this prediction, with the current value of the top quark mass being close to 175 GeV [4, 5].

The discovery of the top quark marked a new period in particle physics — the physics of the top quark. The top quark is unique among the other known fermions; its mass far exceeds their masses, and even the masses of weak bosons. (The second heaviest fermion - the bottom quark - has a mass of roughly 5 GeV.) Because of its great mass, its decay products may include new particles and thus reveal new physics. But more certainly, its great mass makes the top quark a very good probe for studying the mechanism of ESB. In particular, as the heaviest fermion available, the top quark would couple far more strongly to the Higgs boson than any other known fermion. Another interesting feature of the top quark is its rapid weak decay. Because of that, the top spin is transferred directly to the final state with negligible hadronization uncertainties, therefore allowing the helicity-dependent information to be propagated to the final state. Besides these “extras” there is an understandable desire to measure the mass and interactions of the top quark with the same level of accuracy as was done for other fermions. For example, the improved accuracy of the top quark mass, in combination with the precision measurement of the W mass, will give us an indirect limit on the mass of the Higgs boson. The present level of accuracy of these observables does not provide restrictions much better than those obtainable from theoretical considerations and

Figure 5.1: Tree level Feynman diagrams contributing to the process $e^+e^- \rightarrow t\bar{t}$.

leaves us with the wide interval of possible values of the Higgs boson mass from about 100 to 1000 GeV [103]. For the moment, the Fermilab Tevatron has a monopoly on the direct study of the top quark. The next steps will be the upgrade of the Tevatron and building the LHC at CERN, with both projects running in the next decade. An e^+e^- collider with sufficient energy can also play an important role in measuring top quark properties [81]. It provides the usual advantages; a relatively clean final state, precise vertex detection which is important to study the decays of the top quark, and a reasonable efficiency for the reconstruction of various decay modes.

The main process for the study of the top quark at the NLC is $e^+e^- \rightarrow t\bar{t}$ (see Fig. 5.1). This process will be particularly useful at energies close to the $t\bar{t}$ production threshold for the measurement of the basic properties of the top quark such as its mass, width and interactions with weak bosons [81]. However, despite the large Higgs-top coupling, the contribution of the Higgs s-channel is suppressed by the interaction of the Higgs boson with the e^+e^- pair by the factor of $m_e/M_W \approx 6 \times 10^{-6}$, relative to the contributions of the first two diagrams. The tree level cross section as a function of $\sqrt{s}$, ignoring the Higgs boson exchange diagram, is shown in Fig. 5.2. The light to moderate Higgs mass could be measurable in

Figure 5.2: The tree level cross section of the process $e^+e^- \rightarrow t\bar{t}$ as a function of $\sqrt{s}$ neglecting the Higgs boson exchange diagram.

this process at the $t\bar{t}$ threshold, where the cross section is sensitive to the Higgs boson contribution to the $t\bar{t}$ potential; an additional attractive short range force arising from Higgs boson exchange increases the modulus of the toponium wave function at the origin, and thereby enhances the cross section [81, 88].

In the case of a heavy or no-Higgs scenario, when the electroweak symmetry is broken strongly, we need to look for other reactions that would be sensitive to this case. Processes with longitudinal W bosons are natural candidates for the study of the mechanism of ESB, since these bosons are direct participants in the ESB sector. Because of that, considerable work has been devoted to understanding the heavy

Higgs boson scenario using vector boson scattering at high energy colliders. For example, at the NLC the scattering processes $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ and $W_L^+ W_L^- \rightarrow Z_L Z_L$ have been studied using the reactions $e^+ e^- \rightarrow \nu \bar{\nu} W^+ W^-$ and $e^+ e^- \rightarrow \nu \bar{\nu} Z Z$ [87]. However, the sensitivity of these processes appeared to be quite modest [81].

Recently, the production of top quarks in the vector boson fusion processes has attracted growing interest [90, 91, 92]. The first studies suggest that it is a potentially powerful tool for understanding the mechanism of ESB and that the sensitivity of these processes to the heavy Higgs boson could significantly overcome the sensitivity of vector boson scattering [81]. $t\bar{t}$ production from vector boson fusion can be observed at high energy $e^+ e^-$ colliders using the same mechanism as vector boson scattering; interactions of the vector bosons emitted from the $e^\pm$ beams. (It would be probably very difficult to study $V_1 V_2 \rightarrow t\bar{t}$ at a hadron collider due to the large background from $gg \rightarrow t\bar{t}$.) The generic diagram for these processes at an $e^+ e^-$ collider is shown in Fig. 5.3. The cross section for $e^+ e^- \rightarrow \ell \bar{\ell} t\bar{t}$ contains an additional suppression factor of α^2 relative to the cross section for $e^+ e^- \rightarrow t\bar{t}$. On the other hand, the fusion cross section $\sigma(V_1 V_2 \rightarrow t\bar{t})$ is enhanced by a factor of m_i^2/M_W^2 for every longitudinal gauge boson in the initial state. This is a manifestation of the equivalence theorem whereby longitudinal vector bosons take on the couplings of the scalar Goldstone bosons from which they acquire mass. Since $\sigma(e^+ e^- \rightarrow \ell \bar{\ell} t\bar{t})$ generally grows with $\sqrt{s}$, while $\sigma(e^+ e^- \rightarrow t\bar{t})$ decreases as $1/s$, vector boson fusion becomes more competitive at higher energy.

5.2 Calculation of the cross section for $e^+ e^- \rightarrow \ell \bar{\ell} t\bar{t}$

The algebra and expressions involved in exact calculations of $t\bar{t}$ production via vector boson fusion are tremendously complex. To the best of our knowledge

Figure 5.3: The generic diagram for the process $e^+e^- \rightarrow \ell\bar{\ell}V_1V_2 \rightarrow \ell\bar{\ell}t\bar{t}$. The shaded bulb represents all possible interactions of the subprocess $V_1V_2 \rightarrow t\bar{t}$.

no full calculations of these processes exist in the literature. An efficient way to simplify the complexity of the calculations is to use the *effective vector boson approximation* (EVA) [93, 99]. In this approximation, vector bosons are treated as constituents of colliding particles and the calculational requirements are reduced to finding the cross section for the subprocess $V_1V_2 \rightarrow t\bar{t}$, with V_i being a real particle — W , Z or photon, and convoluting this cross section with the appropriate $W/Z/\gamma$ distribution functions. The EVA was used for the investigation of several processes in the past, including heavy Higgs boson production, vector boson pair scattering, and the production of heavy leptons at both hadron and e^+e^- colliders. In all of these studies the EVA has been shown to be quite accurate ($\leq 10\%$) in the region of its expected validity when compared to the full calculations (see for example [94, 98, 99]). Although some calculations have also been performed for vector boson fusion $t\bar{t}$ production, the picture is far from complete. The total

cross section for $e^+e^- \rightarrow \ell\bar{\ell}t\bar{t}$, using the EVA, has been calculated by Kauffman at fixed CM energy of 2 TeV [90]. Closely related to this problem are calculations of heavy fermion production at pp colliders by Dawson and Willenbrock [94], and heavy lepton production by Yuan (pp and e^+e^- colliders) [96], and Éboli *et al.*, (pp collider) [97]. Approximate expressions for $V_1V_2 \rightarrow t\bar{t}$ cross sections can be found in Kauffman [90] and can also be adapted from expressions for $V_1V_2 \rightarrow L\bar{L}$ in Éboli *et al.*, [97]. Unfortunately, some of the published expressions in [90] and [97] contain errors. The exact helicity amplitudes for $V_1V_2 \rightarrow L_1\bar{L}_2$ have been published in the Appendix of Yuan's paper [96].

In our work [102] we calculate $e^+e^- \rightarrow \ell\bar{\ell}V_1V_2 \rightarrow \ell\bar{\ell}t\bar{t}$ cross sections as a function of $\sqrt{s}$ for various Higgs boson masses using the EVA with exact expressions for $V_1V_2 \rightarrow t\bar{t}$ cross sections. We consider all V_1V_2 channels for the $e^+e^- \rightarrow \ell\bar{\ell}t\bar{t}$ process:

- $W_{L/T}^+W_{L/T}^- \rightarrow t\bar{t}$
- $Z_{L/T}Z_{L/T} \rightarrow t\bar{t}$
- $\gamma Z_{L/T} \rightarrow t\bar{t}$
- $\gamma\gamma \rightarrow t\bar{t}$

Because full expressions for the $W_{L/T}^+W_{L/T}^- \rightarrow t\bar{t}$ and $Z_{L/T}Z_{L/T} \rightarrow t\bar{t}$ cross sections are still extremely lengthy and their calculation is sufficiently complex to counter-act the simplifying motivation for using the EVA, one usually considers neglecting terms proportional to $2M_{W,Z}/\sqrt{\hat{s}}$ in these expressions¹. This additional approximation is based on the hope that at high energy the error introduced by neglecting these terms will be small, at least when compared to the error introduced by the EVA. However, it does not have to be true since the convolution of

¹The $\sqrt{\hat{s}}$ is the centre of mass energy of the V_1V_2 pair.

$\sigma(WW/ZZ \rightarrow t\bar{t})$ with the W/Z distribution functions, which have large contributions from low energy $V_1 V_2$ collisions even at high $\sqrt{s}$, slows down the convergence of the approximate $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$. We address this question in our calculations and evaluate the actual error in $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$ caused by using the approximate expressions for $\sigma(WW/ZZ \rightarrow t\bar{t})$.

The effect of using the EVA for the process in Fig. 5.3 is equivalent to considering the four-momenta of the vector bosons in the limit $k_i^2 \rightarrow M_{V_i}^2$ with the V_i 's emitted in the forward direction, $\theta_i \rightarrow 0$. In other words, the electron/positron beams are treated as sources of on-shell vector bosons. In the high-energy limit, when the energy E of the $e^\pm$ beam is much greater than the vector boson mass, $E \gg M_V$, the probability distributions of vector bosons in the $e^\pm$ are given by the following expressions [93]: for longitudinal weak bosons:

$$f_{V_L/e}(x) = \frac{c_v^2 + c_a^2}{4\pi^2} \frac{1-x}{x} \quad (5.1)$$

for transverse weak bosons:

$$f_{V_T/e}(x) = \frac{c_v^2 + c_a^2}{8\pi^2} \frac{x^2 + 2(1-x)}{x} \ln \left(\frac{4E^2}{M_V^2} \right) \quad (5.2)$$

where $x = (E - E')/E$ is the momentum fraction of the electron carried by the vector boson. If the vector boson is the $W^\pm$ boson then

$$c_v = -c_a = \frac{g}{2\sqrt{2}} \quad (5.3)$$

If the vector boson is the Z boson then

$$c_v = -\frac{g}{4 \cos \theta_W} (1 - 4 \sin \theta_W) \quad (5.4)$$

$$c_a = \frac{g}{4 \cos \theta_W} \quad (5.5)$$

where $g = e/\sin \theta_W$ and $e = \sqrt{4\pi\alpha}$. In the case of the photon, the distribution function has the form

$$f_{\gamma/e}(x) = \frac{\alpha}{2\pi} \frac{x^2 + 2(1-x)}{x} \ln \left(\frac{4E^2}{m_e^2} \right) \quad (5.6)$$

where m_e is a mass of the electron.

The total cross section of the process in Fig. 5.3 is given by the integral

$$\sigma_{e^+e^- \rightarrow \ell \bar{\ell} t \bar{t}}(s) = \int_{\tau_0}^1 d\tau \hat{\sigma}_{V_1 V_2 \rightarrow t \bar{t}}(\tau s) \left. \frac{dL}{d\tau} \right|_{V_1 V_2 / e^+ e^-} \quad (5.7)$$

where the $\left. \frac{dL}{d\tau} \right|_{V_1 V_2 / e^+ e^-}$ is the differential luminosity of the interacting vector bosons

$$\left. \frac{dL}{d\tau} \right|_{V_1 V_2 / e^+ e^-} = \int_{\tau}^1 \frac{dx}{x} f_{V_1/e}(x) f_{V_2/e}(\tau/x) \quad (5.8)$$

Since the $V_1 V_2$ pair must have enough energy to produce a real $t \bar{t}$ pair, the lower limit in the integral (5.7) must fulfil the condition $\tau_0 \geq 4m_t^2/s$, where m_t is the top quark mass. Substituting the expressions (5.1), (5.2), and (5.6) into the integral (5.8) we obtain analytic expressions for differential luminosities of the subprocesses under consideration. For $W_{L/T}^+ W_{L/T}^- \rightarrow t \bar{t}$ and $Z_{L/T} Z_{L/T} \rightarrow t \bar{t}$ we get [93]

$$\left. \frac{dL}{d\tau} \right|_{V_L V_L / e^+ e^-} = \left(\frac{c_v^2 + c_a^2}{4\pi^2} \right)^2 \frac{1}{\tau} [2(\tau - 1) - (1 + \tau) \ln \tau] \quad (5.9)$$

$$\left. \frac{dL}{d\tau} \right|_{V_L V_T / e^+ e^-} = \frac{1}{2} \left(\frac{c_v^2 + c_a^2}{4\pi^2} \right)^2 \ln \left(\frac{4E^2}{M_V^2} \right) \frac{1}{\tau} \left[\frac{1}{2}(\tau - 1)(\tau + 1) - 2(1 + \tau) \ln \tau \right] \quad (5.10)$$

$$\left. \frac{dL}{d\tau} \right|_{V_T V_T / e^+ e^-} = \left(\frac{c_v^2 + c_a^2}{8\pi^2} \right)^2 \left[\ln \left(\frac{4E^2}{M_V^2} \right) \right]^2 \frac{1}{\tau} [2(\tau - 1)(3 + \tau) - (2 + \tau)^2 \ln \tau] \quad (5.11)$$

For $\gamma Z_{L/T} \rightarrow t \bar{t}$ we get

$$\left. \frac{dL}{d\tau} \right|_{Z_L \gamma / e^+ e^-} = \frac{c_v^2 + c_a^2}{4\pi^2} \frac{\alpha}{2\pi} \ln \left(\frac{4E^2}{m_e^2} \right) \frac{1}{\tau} \left[\frac{1}{2}(\tau - 1)(\tau + 1) - 2(1 + \tau) \ln \tau \right] \quad (5.12)$$

$$\left. \frac{dL}{d\tau} \right|_{Z_T \gamma / e^+ e^-} = \frac{c_v^2 + c_a^2}{8\pi^2} \frac{\alpha}{2\pi} \ln \left(\frac{4E^2}{m_e^2} \right) \ln \left(\frac{4E^2}{M_Z^2} \right) \frac{1}{\tau} [2(\tau - 1)(3 + \tau) - (2 + \tau)^2 \ln \tau] \quad (5.13)$$

Figure 5.4: The differential luminosities for interactions between W^+ 's and W^- 's emitted by the $e^\pm$ beams at energy $\sqrt{s} = 1$ TeV. The solid line is for $W_L W_L$ polarization, the dashed line is for $W_L W_T$ and the dotted line is for $W_T W_T$.

For $\gamma\gamma \rightarrow t\bar{t}$ we get

$$\left. \frac{dL}{d\tau} \right|_{\gamma\gamma/e^+e^-} = \left(\frac{\alpha}{2\pi} \right)^2 \left[\ln \left(\frac{4E^2}{m_e^2} \right) \right]^2 \frac{1}{\tau} [2(\tau - 1)(3 + \tau) - (2 + \tau)^2 \ln \tau] \quad (5.14)$$

In Fig. 5.4 we illustrate general behaviour of the differential luminosities by plotting the functions (5.9), (5.10) and (5.11) for W^+W^- mode. The luminosities for other modes differ from these only by multiplicative constants.

As the second step in the calculation of the total cross section (5.7) we need to find cross sections of the subprocesses $V_1 V_2 \rightarrow t\bar{t}$. The full sets of Feynman

diagrams for individual subprocesses at tree level are given in Fig. 5.5. We have calculated these cross sections and they are given in Appendix D. Since the full expressions for $\sigma(WW \rightarrow t\bar{t})$ and $\sigma(ZZ \rightarrow t\bar{t})$ are very lengthy and not illuminating, they are only given in the approximation $2M_{W,Z}/\sqrt{\hat{s}} = 0$. In Fig. 5.6 we show plots of $\sigma(WW \rightarrow t\bar{t})$ as a function of $\sqrt{\hat{s}}$ for different polarizations of W bosons. The mass of the Higgs boson considered for this graph is 1 TeV. The full expressions have been used to plot this graph.

5.3 Results

To obtain the cross section $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$, we take the invariant mass of the $t\bar{t}$ -pair to be at least 500 GeV. This cut translates into a lower limit on the integral (5.7) of $\tau_0 = (500 \text{ GeV})^2/s$. In our calculations we use the full expressions for all subprocesses, i.e. we include all orders in $2M_W/\sqrt{\hat{s}}$ and $2M_Z/\sqrt{\hat{s}}$. We will discuss the effect of taking $M_{W,Z}/\sqrt{\hat{s}} \rightarrow 0$ below. Of these processes, it is the subprocesses involving longitudinal vector bosons which are of interest since the V_L 's represent the ESB sector. The subprocesses with transverse gauge bosons are backgrounds. We also note that the effective vector boson approximation is less reliable for transverse vector bosons and those results should be viewed with caution [92].

In Fig. 5.7 we show the cross sections for $e^+e^- \rightarrow \ell\bar{\ell}t\bar{t}$ for the various subprocesses where the lepton ℓ is either an e or ν as appropriate. Given that it is the processes $V_L V_L \rightarrow t\bar{t}$ and their sensitivity to the Higgs boson mass that are of primary interest, in Fig. 5.8 we show the cross sections for $e^+e^- \rightarrow \nu\bar{\nu}W_L W_L \rightarrow \nu\bar{\nu}t\bar{t}$ and $e^+e^- \rightarrow e^+e^- Z_L Z_L \rightarrow e^+e^- t\bar{t}$ for $M_H = 100 \text{ GeV}$, $M_H = 500 \text{ GeV}$, $M_H = 1 \text{ TeV}$, and $M_H = \infty$ (which corresponds to the low energy theorem (LET)). In Fig. 5.9 we show the same cross sections as a function of the Higgs mass at fixed CM en-

Figure 5.5: Tree level Feynman diagrams contributing to the subprocesses $V_1 V_2 \rightarrow t \bar{t}$: (a) $\gamma\gamma(\gamma Z) \rightarrow t \bar{t}$; (b) $WW \rightarrow t \bar{t}$; (c) $ZZ \rightarrow t \bar{t}$.

Figure 5.6: The cross sections of $WW \rightarrow t\bar{t}$ for different polarizations of W bosons. The solid line is for $W_L W_L$ polarization, the dashed line is for $W_L W_T$ and the dotted line is for $W_T W_T$. The Higgs boson mass is 1 TeV.

ergy $\sqrt{s} = 1$ TeV. The steep decrease of the cross section for Higgs masses below 500 GeV is due to the cut on the $t\bar{t}$ invariant mass, $M_{t\bar{t}} \geq 500$ GeV. Since the cross section of the relevant subprocess peaks at $\sqrt{\hat{s}} = M_H$, a significant part of the cross section is thrown away by the cut if $M_H < 500$ GeV. For the $W_L W_L$ case the cross sections at $\sqrt{s} = 1$ TeV are ~ 0.1 fb, ~ 0.4 fb, and ~ 0.3 fb for $M_H = 100$ GeV, 1 TeV, and ∞ , respectively. For the expected yearly integrated luminosity of 200 fb^{-1} these should be distinguishable. However, once t -quark detection efficiencies and kinematic cuts to reduce backgrounds are included, the

Figure 5.7: Cross sections $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$ via vector boson fusion using the effective $W/Z/\gamma$ approximation with $M_H = 100$ GeV. The solid line is for $\gamma\gamma \rightarrow t\bar{t}$, the dot-dashed line for $\gamma Z_L \rightarrow t\bar{t}$, the dot-dot-dashed line for $\gamma Z_T \rightarrow t\bar{t}$, the dashed line for $W_L W_L \rightarrow t\bar{t}$, the short-dashed line for $W_T W_T \rightarrow t\bar{t}$, the dotted line for $Z_L Z_L \rightarrow t\bar{t}$, and the densely-dotted line for $Z_T Z_T \rightarrow t\bar{t}$.

situation is not so clear. We remind the reader that we already included a cut of $M_{t\bar{t}} > 500$ GeV and reducing this may increase the cross section enough to distinguish the cases. Although in Fig. 5.9 the cross section is the same for certain values less than and greater than $M_H \simeq 500$ GeV this does not concern us here since if $M_H < 500$ GeV the Higgs boson should be observed directly at the LHC. As the centre of mass energy increases, the cross section increases to 6 fb and 3 fb for $M_H = 1$ TeV and $M_H = \infty$ respectively at $\sqrt{s} = 2$ TeV and ~ 30 fb

and ~ 15 fb respectively at $\sqrt{s} = 5$ TeV. Thus, the different scenarios should be distinguishable at these energies.

From Figs. 5.8 and 5.9 it is obvious that the ZZ fusion is more sensitive to the Higgs boson mass than the WW mode. Unfortunately, the cross section $\sigma(ZZ \rightarrow t\bar{t})$ is at least an order of magnitude lower than the cross section for the WW mode. At $\sqrt{s} = 1$ TeV the corresponding ZZ mode cross sections are $\sim 2 \times 10^{-3}$ fb, $\sim 2 \times 10^{-2}$ fb, and $\sim 1 \times 10^{-2}$ fb for $M_H = 100$ GeV, 1 TeV, and ∞ , respectively, which at the luminosity of $200 \text{ fb}^{-1}/\text{year}$ would produce at best only a few events. With the increase of the CM energy to 2 TeV (5 TeV) the cross section increases to ~ 0.4 fb (~ 2 fb) and ~ 0.12 fb (~ 0.8 fb) for $M_H = 1$ TeV and ∞ , respectively. An additional complication for the ZZ mode arises from the fact that its final state, $e^+e^-t\bar{t}$, is identical to the final states of the $\gamma\gamma$ and γZ modes. These are not sensitive to M_H and represent a hardly reducible background to the ZZ mode.

Although our figures are obtained using the complete expressions for the subprocess cross sections, for brevity, we have given in Appendix D expressions in the limit that $2M_{W,Z}/\sqrt{s} = 0$ for $WW \rightarrow t\bar{t}$ and $ZZ \rightarrow t\bar{t}$. The complete expressions are not only lengthy but their calculation is sufficiently complex to diminish the simplifying motivation for using the EVA. To help decide whether it is necessary to use the full expressions rather than the approximate ones shown in the appendix and to know how large an error is introduced by neglecting terms proportional to $2M_{W,Z}/\sqrt{s}$, the differences between the exact and approximate expressions for $e^+e^- \rightarrow \ell\bar{\ell}t\bar{t}$ via WW and ZZ fusion are shown in Fig. 5.10 as functions of the CM energy and the Higgs mass. For CM energies between 0.6 TeV and 5 TeV and Higgs masses between 100 GeV and ∞ , deviations for W_LW_L and Z_LZ_L range up to about 10%. This is comparable to the precision of the EVA which is claimed to be better than 10%. (Of course, the precision of EVA for $e^+e^- \rightarrow \ell\bar{\ell}t\bar{t}$ in particular has not yet been established.) Errors for W_TW_T , W_LW_T , Z_TZ_T , and Z_LZ_T

Figure 5.8: $\sigma(e^+e^- \rightarrow \nu\bar{\nu}W_LW_L \rightarrow \nu\bar{\nu}t\bar{t})$ (upper figure) and $\sigma(e^+e^- \rightarrow e^+e^-Z_LZ_L \rightarrow e^+e^-t\bar{t})$ (lower figure) vs. $\sqrt{s}$ for several values of M_H . The solid lines are for $M_H = 100$ GeV, the dashed lines for $M_H = 500$ GeV, the dotted lines for $M_H = 1$ TeV, and the dot-dashed lines for $M_H = \infty$ (LET).

Figure 5.9: $\sigma(e^+e^- \rightarrow \nu\bar{\nu}WW \rightarrow \nu\bar{\nu}t\bar{t})$ and $\sigma(e^+e^- \rightarrow e^+e^-ZZ \rightarrow e^+e^-t\bar{t})$ for $\sqrt{s} = 1$ TeV as a function of M_H . The upper set of curves is for WW fusion and the lower set of curves for ZZ fusion. The solid curves are for $W_L W_L$ ($Z_L Z_L$) fusion, the dashed curves for $W_L W_T$ ($Z_L Z_T$) fusion, and the dotted curves for $W_T W_T$ ($Z_T Z_T$) fusion.

range up to 24%, 13%, 15%, and 3% respectively. However, as has been mentioned earlier, the EVA is expected to be less reliable for the LT and TT modes than the LL modes. The errors tend to diminish with energy but not dramatically and not always uniformly. The convergence of the approximation is slowed down by the convolution of $\sigma(WW/ZZ \rightarrow t\bar{t})$ with the distribution functions which have large contributions from low energy $V_i V_j$ collisions (as can be seen from Fig. 5.4) where the approximate expressions for $\sigma(WW/ZZ \rightarrow t\bar{t})$ are less accurate.

Figure 5.10: Errors introduced by taking $2M_{W,Z}/\sqrt{s} \rightarrow 0$ for the $W_L W_L$ mode (upper set of curves) and the $Z_L Z_L$ mode (lower set of curves) of $e^+e^- \rightarrow \ell\bar{\ell}t\bar{t}$. In both cases the solid curves are for $M_H = 100$ GeV, the dashed curves for $M_H = 500$ GeV, the dotted curves for $M_H = 1$ TeV, and the dot-dashed curves for $M_H = \infty$.

5.4 Summary

We calculated the cross section of the process $e^+e^- \rightarrow \ell\bar{\ell}V_1V_2 \rightarrow \ell\bar{\ell}t\bar{t}$, where $V_1V_2 = \gamma\gamma/\gamma Z/W^+W^-/ZZ$ and ℓ is either an electron or neutrino, as a function of $\sqrt{s}$ for various values of the Higgs boson mass. The calculation was performed in the effective vector boson approximation by convoluting the cross section of the subprocess $V_1V_2 \rightarrow t\bar{t}$ with the appropriate $W/Z/\gamma$ distribution functions. We found that the cross section for the WW mode at $\sqrt{s} = 1$ TeV is sufficiently large and sensitive to distinguish different Higgs boson masses. The sensitivity improves as $\sqrt{s}$ increases. The ZZ fusion is not very promising due to its small cross section and the background from $\gamma\gamma$ and γZ modes.

We examined the size of the error introduced in $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$ by neglecting terms proportional to $2M_{W,Z}/\sqrt{s}$ in the subprocess cross sections. We found that for CM energies between 0.6 TeV and 5 TeV and Higgs masses between 100 GeV and ∞ , deviations for W_LW_L and Z_LZ_L range up to about 10% which is comparable to the error expected from the EVA itself. To fully assess the reliability of the EVA in these calculations will eventually require comparison with the full calculations.

Chapter 6

Conclusions

Two phenomenological problems were addressed in this work, both related to the search for new physics beyond the SM with a potential sensitivity to the mechanism of ESB. In the first one (Chapter 4), we analyzed the sensitivity of the process $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$ to the anomalous triple gauge boson couplings at energies appropriate for LEP2 (175 - 220 GeV) and NLC (500 GeV, 1 TeV). In the second problem (Chapter 5), we investigated the $t\bar{t}$ production via vector boson fusion at e^+e^- colliders as a means to study the mechanism of ESB. In the following two sections we present conclusions for each of the two parts.

6.1 Sensitivity of the $e^+e^- \rightarrow \ell\nu q\bar{q}'$ process to the anomalous TGV couplings

We performed a detailed analysis of the measurement of the tri-linear gauge boson couplings in the process $e^+e^- \rightarrow \ell^\pm\nu q\bar{q}$. We included all tree level contributions to this final state and included finite gauge boson width effects. We found that the off-shell W contributions for the μ mode contribute from about 20% of the cross section at LEP2 to 30% at the 1 TeV NLC with the kinematic cuts we used.

For the electron mode, the corresponding off-shell W contributions are about 20% and 90%, respectively. Clearly, the non-resonant contributions must be included to properly account for the experimental situation.

To gauge the sensitivity of this process to anomalous gauge boson couplings we used the W decay distributions as a polarimeter to distinguish the longitudinal W modes, which are more sensitive to anomalous couplings, from the transverse modes. We implemented this through the use of a quintic differential cross section, with each angular variable divided into 4 bins, and then calculated the likelihood function of non-standard model couplings as compared to the Standard model. Using this approach we found that at LEP2, operating at 175 GeV, and assuming an integrated luminosity of 500 pb^{-1} , g_1^Z , κ_Z , κ_γ , λ_Z , and λ_γ could be measured to roughly ± 0.2 and L_{9L} and L_{9R} to ± 50 and ± 400 respectively. As W pair production is relatively insensitive to TGV's near threshold a modest increase in energy from $\sqrt{s} = 175 \text{ GeV}$ to $\sqrt{s} = 192 \text{ GeV}$ would yield sizeable improvements in the measurements, ranging from about a factor of 1.7 for κ_Z to ~ 2 for L_{9R} . Subsequent increases in energy to 205 GeV and 220 GeV would not yield the same improvement. As was mentioned in Section 3.3.3, most LEP2 data will be collected at 184 GeV and 188 GeV. It is extremely unlikely that measurements of this precision would reveal anomalous couplings. At a 500 GeV NLC with an integrated luminosity of 50 fb^{-1} , g_1^Z , κ_V and λ_V , could be measured to roughly ± 0.01 , ± 0.005 and ± 0.0025 respectively and L_{9L} and L_{9R} to ± 1 and ± 4 respectively. At the 1 TeV NLC with 200 fb^{-1} the corresponding numbers are $\delta g_1^Z \sim \pm 0.05$, $\delta \kappa_{Z,\gamma} \sim \pm 10^{-3}$, $\delta L_{9L} \sim \pm 0.5$ and $\delta L_{9R} \sim \pm 1$. The 500 GeV NLC measurements should be sensitive to loop contributions to the TGV's while the 1 TeV will be able to measure such effects.

In the NLC case, we also studied the influence of initial state polarization. The cross section with left-handed electrons dominates over the right-handed cross

section. On the other hand, the latter is more sensitive to the anomalous couplings than the former. Due to a trade-off between sensitivity and statistics, in most cases the left-handed and right-handed bounds are comparable. More importantly, for some combinations of the anomalous parameters the different polarizations give different correlations. This could be useful for disentangling the nature of anomalous TGV's.

We studied the sensitivity of the off-mass shell cross sections to anomalous couplings by imposing kinematic cuts on the invariant mass distributions of the outgoing fermion pairs. A cursory analysis found that the off-resonance cross section has some sensitivity to anomalous couplings and that useful information could be extracted from this region of phase space.

Although the inclusion of W decays to fermions and the non-resonant diagrams does not alter the precision to which the TGV's can be measured they do change the cross sections and kinematic distributions at the same level of radiative corrections and must be taken into account for an accurate comparison between experiment and theory.

The optimal strategy to maximize the information contained in each event is to construct a likelihood function based on the four vector of each of the outgoing fermions on an event-by-event basis, putting them through a realistic detector simulation. This would make the best use of the information whether it be on the W resonance or not. Kinematic cuts should only be introduced to reduce backgrounds. Since the precision of these measurements is beyond the level of loop induced radiative corrections it is crucial that radiative corrections are well understood and be included in event generators used in the study of these processes.

6.2 $t\bar{t}$ production via vector boson fusion at e^+e^- colliders

It appears that $t\bar{t}$ production via vector boson fusion at e^+e^- colliders provides a promising tool for probing the mechanism of ESB and that the sensitivity of these processes to the heavy Higgs boson mass significantly overcome the sensitivity of vector boson scattering [81, 90, 91, 92]. Due to the tremendous complexity of the algebra and expressions involved in the full calculations of the $t\bar{t}$ production, it is very useful to have and understand approximate methods for analysis of this process. In our work, besides the analysis of the process itself, we attempted to better understand how a standard approximation influences the results obtained.

We calculated the cross section of the process $e^+e^- \rightarrow \ell\bar{\ell}V_1V_2 \rightarrow \ell\bar{\ell}t\bar{t}$, where $V_1V_2 = \gamma\gamma/\gamma Z/W^+W^-/ZZ$ and ℓ is either an electron or neutrino, as a function of $\sqrt{s}$ for various values of the Higgs boson masses. The calculation was performed in the effective vector boson approximation, i.e. vector bosons emitted from the $e^\pm$ beams were considered to be on-shell and collinear with the beams. The cross section was then obtained by convoluting the cross section of the subprocess $V_1V_2 \rightarrow t\bar{t}$ with the appropriate $W/Z/\gamma$ distribution functions.

For the calculation of $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$ we found and used full analytical expressions for $\sigma(V_1V_2 \rightarrow t\bar{t})$. With the cut $M_{t\bar{t}} \geq 500$ GeV, $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$ through W_LW_L fusion at $\sqrt{s} = 1$ TeV is ~ 0.1 fb, ~ 0.4 fb, and ~ 0.3 fb for $M_H = 100$ GeV, 1 TeV, and ∞ , respectively. With the expected luminosity of $200 \text{ fb}^{-1}/\text{year}$ these cases should be distinguishable. With the increase of the CM energy to 2 TeV (5 TeV) the cross section increases to ~ 6 fb (~ 30 fb) and ~ 3 fb (~ 15 fb) for $M_H = 1$ TeV and ∞ , respectively. The ZZ fusion is also sensitive to the Higgs boson mass but it has at least an order of magnitude lower cross section than the WW mode. In addition, this process has $\gamma\gamma$ and γZ modes as its backgrounds.

Therefore this mode is not very promising.

Since the full expressions for the $\sigma(WW/ZZ \rightarrow t\bar{t})$ are extremely lengthy, one usually neglects terms proportional to $2M_{W,Z}/\sqrt{\hat{s}}$ when using these expressions with the EVA for calculation of cross sections at high energy colliders. We examined the size of the error introduced by neglecting these terms in $\sigma(e^+e^- \rightarrow \ell\bar{\ell}t\bar{t})$. We found that for CM energies between 0.6 TeV and 5 TeV and Higgs masses between 100 GeV and ∞ , deviations for $W_L W_L$ and $Z_L Z_L$ range up to about 10%. This is comparable to the error expected from the EVA itself. The errors for LT and TT modes have been found to be even bigger. However, in those cases the EVA is also claimed to be less reliable. The application of the EVA in these calculations is based on the previous successful experience with other fusion processes. Its reliability in this case is not yet fully assessed and will eventually require comparison with the full calculations.

Appendix A

Feynman rules for the anomalous TGV's

In this Appendix, the Feynman rules for the anomalous TGV's based on the effective Lagrangian (3.10) are given in the unitary gauge:

$$\begin{aligned}
 & ig_V \left\{ g_{\nu\lambda} [(g_1^V - \tilde{\lambda}_V k_2 \cdot k_3) k_1 - (g_1^V - \tilde{\lambda}_V k_1 \cdot k_3) k_2]_\mu \right. \\
 & - g_{\nu\mu} [(g_1^V - \tilde{\lambda}_V k_2 \cdot k_3) k_1 - (\kappa_V - \tilde{\lambda}_V k_1 \cdot k_2) k_3]_\lambda \\
 & - g_{\lambda\mu} (\kappa_V - \tilde{\lambda}_V k_1 \cdot k_2) k_3 - (g_1^V - \tilde{\lambda}_V k_1 \cdot k_3) k_2]_\nu \\
 & \left. + \tilde{\lambda}_V (k_{1\mu} k_{2\nu} k_{3\lambda} - k_{2\mu} k_{3\nu} k_{1\lambda}) \right\}
 \end{aligned}$$

where V is either Z or γ , $\tilde{\lambda}_V = \lambda_V/M_W^2$, $g_\gamma = e$, $g_Z = e \cot \theta_W$, $e = \sqrt{4\pi\alpha}$ and $g_1^\gamma = 1$.

Appendix B

CALKUL - helicity amplitude technique

The CALKUL technique [65] is an efficient computational method for processes involving massless external fermions. In this appendix we briefly summarize basic relations of this helicity amplitude technique which we used in the cross section calculations of Chapter 4.

Let k_0^μ, k_1^μ be fixed 4-vectors satisfying the following relations

$$k_0^2 = 0, \quad k_1^2 = -1, \quad k_0 \cdot k_1 = 0 \quad (\text{B.1})$$

Let u_{L0} be the so-called *basic* spinor for a left-handed fermion with momentum k_0 . Then

$$u_{R0} = \not{k}_1 u_{L0} \quad (\text{B.2})$$

is the right-handed spinor for a fermion with momentum k_0 . Spinors for fermions with any lightlike p ($p^2 = 0$) can be defined as

$$u_{L,R}(p) = \frac{1}{\sqrt{2p \cdot k_0}} \not{p} u_{R0,L0} \quad (\text{B.3})$$

This set of conventions defines the phases of spinors unambiguously except when p is parallel to k_0 . For p, q lightlike we define the *spinor products* $s(p, q)$ and $t(p, q)$

by

$$s(p, q) \equiv \bar{u}_R(p)u_L(q), \quad t(p, q) \equiv \bar{u}_L(p)u_R(q) \quad (\text{B.4})$$

The spinor products satisfy the following relations:

$$[s(p, q)]^* = t(q, p) \quad (\text{B.5})$$

$$s(p, q) = -s(q, p) \quad \text{and} \quad t(p, q) = -t(q, p) \quad (\text{B.6})$$

$$|s(p, q)|^2 = 2p \cdot q \quad (\text{B.7})$$

The following are the relations that help us to rewrite amplitudes in terms of the spinor products:

$$\bar{u}_\lambda(p)\Gamma u_\rho(q) = \lambda\rho\bar{u}_{-\rho}(q)\Gamma^R u_{-\lambda}(p) \quad (\text{B.8})$$

where Γ is an arbitrary string of γ matrices and Γ^R is the same string in reversed order; $\lambda, \rho = \pm 1$ for right/left-handed fermions. The equation

$$\bar{u}_\lambda(p)\gamma_\mu u_\lambda(q)[\gamma^\mu]_{ab} \equiv 2[u_\lambda(q)\bar{u}_\lambda(p) + u_{-\lambda}(p)\bar{u}_{-\lambda}(q)]_{ab} \quad (\text{B.9})$$

is called the *Chisholm identity*. Further it can be shown that

$$\not{p} = u_R(p)\bar{u}_R(p) + u_L(p)\bar{u}_L(p) \quad (\text{B.10})$$

$$\bar{u}_R(p)\not{p}u_R(q) = s(p, r)t(r, q) \quad (\text{B.11})$$

$$\bar{u}_L(p)\not{p}u_L(q) = t(p, r)s(r, q) \quad (\text{B.12})$$

$$\bar{u}_R(p)\not{p}u_L(q) = \bar{u}_L(p)\not{p}u_R(q) = 0 \quad (\text{B.13})$$

where p, q, r are lightlike momenta. If we write the 4-momentum conservation law in the form

$$\sum_i c_i p_i = 0, \quad \text{where } c_i = \begin{cases} +1 & \text{for initial particles} \\ -1 & \text{for final particles} \end{cases} \quad (\text{B.14})$$

then

$$\sum_i c_i s(q, p_i)t(p_i, r) = \sum_i c_i t(q, p_i)s(p_i, r) = 0 \quad (\text{B.15})$$

where p_i, q, r are lightlike momenta and we sum over all the external particles in the process.

In our calculations we use the following definitions of four-vectors k_0^μ, k_1^μ :

$$k_0^\mu = (1, 1, 0, 0) \quad (\text{B.16})$$

$$k_1^\mu = (0, 0, 1, 0) \quad (\text{B.17})$$

where a general four-vector p^μ is defined as

$$p^\mu = (p^0, p^x, p^y, p^z) \quad (\text{B.18})$$

Then

$$s(p, q) = (p^y + ip^z) \left(\frac{q^0 - q^x}{p^0 - p^x} \right)^{1/2} - (q^y + iq^z) \left(\frac{p^0 - p^x}{q^0 - q^x} \right)^{1/2} \quad (\text{B.19})$$

and $t(p, q)$ is obtained by complex conjugation.

Appendix C

Helicity amplitudes for

$$e^+e^- \rightarrow l^+\nu_l q\bar{q}'$$

In this Appendix we summarize the helicity amplitudes for the process $e^+e^- \rightarrow l^+\nu_l q\bar{q}'$. The corresponding Feynman diagrams are given in Fig. 4.1 and Fig. 4.2. Anomalous couplings have been considered for WWZ and $WW\gamma$ vertices. The SM amplitudes can be recovered by substituting $g_V = \kappa_V = 1$ and $\lambda_V = 0$. The labels L and R in the amplitudes $M_{e^+e^-}$ denote left-handed and right-handed polarization of particles in the initial state. The four-momenta of individual particles are denoted by their symbols. Only amplitudes with non-zero contributions are given here.

Fig. 4.1 a:

$$M_{LR}^Z = -C_Z P_1^Z P_1^W P_2^W D_{LR}^Z(v_l + a_l) \quad (C.1)$$

$$M_{RL}^Z = -C_Z P_1^Z P_1^W P_2^W D_{RL}^Z(v_l - a_l) \quad (C.2)$$

$$M_{LR}^\gamma = -C_\gamma P_1^\gamma P_1^W P_2^W D_{LR}^\gamma \quad (C.3)$$

$$M_{RL}^\gamma = -C_\gamma P_1^\gamma P_1^W P_2^W D_{RL}^\gamma \quad (C.4)$$

where ($V = Z, \gamma$)

$$\begin{aligned}
C_Z &= -\frac{\pi^2 \alpha^2}{\sin^4 \theta_W} \\
C_\gamma &= \frac{2\pi^2 \alpha^2}{\sin^2 \theta_W} \\
D_{LR}^V &= 8\left\{-t(\nu, q)s(\bar{q}', l)\left(2g_V + \frac{\lambda_V}{M_W^2}s\right)[t(e^+, l)s(l, e^-) + t(e^+, \nu)s(\nu, e^-)] \right. \\
&\quad + t(\nu, e^+)s(e^-, l)(g_V + \kappa_V + \frac{\lambda_V}{M_W^2}M_{qq}^2)[t(q, e^+)s(e^+, \bar{q}') + t(q, e^-)s(e^-, \bar{q}')] \\
&\quad - t(q, e^+)s(e^-, \bar{q}')(g_V + \kappa_V + \frac{\lambda_V}{M_W^2}M_{l\mu}^2)[t(\nu, q)s(q, l) + t(\nu, \bar{q}')s(\bar{q}', l)] \\
&\quad + \frac{1}{2}\frac{\lambda_V}{M_W^2}[t(\nu, q)s(q, l) + t(\nu, \bar{q}')s(\bar{q}', l)][t(q, e^+)s(e^+, \bar{q}') + t(q, e^-)s(e^-, \bar{q}')] \\
&\quad \times [t(e^+, l)s(l, e^-) + t(e^+, \nu)s(\nu, e^-)] \\
&\quad - \frac{1}{2}\frac{\lambda_V}{M_W^2}[t(\nu, e^+)s(e^+, l) + t(\nu, e^-)s(e^-, l)][t(q, l)s(l, \bar{q}') + t(q, \nu)s(\nu, \bar{q}')] \\
&\quad \times [t(e^+, q)s(q, e^-) + t(e^+, \bar{q}')s(\bar{q}', e^-)]\} \\
D_{RL}^V &= 8\left\{-t(q, \nu)s(l, \bar{q}')\left(2g_V + \frac{\lambda_V}{M_W^2}s\right)[s(e^+, l)t(l, e^-) + s(e^+, \nu)t(\nu, e^-)] \right. \\
&\quad + t(\nu, e^-)s(e^+, l)(g_V + \kappa_V + \frac{\lambda_V}{M_W^2}M_{qq}^2)[t(q, e^+)s(e^+, \bar{q}') + t(q, e^-)s(e^-, \bar{q}')] \\
&\quad - t(q, e^-)s(e^+, \bar{q}')(g_V + \kappa_V + \frac{\lambda_V}{M_W^2}M_{l\mu}^2)[t(\nu, q)s(q, l) + t(\nu, \bar{q}')s(\bar{q}', l)] \\
&\quad + \frac{1}{2}\frac{\lambda_V}{M_W^2}[t(\nu, q)s(q, l) + t(\nu, \bar{q}')s(\bar{q}', l)][t(q, e^+)s(e^+, \bar{q}') + t(q, e^-)s(e^-, \bar{q}')] \\
&\quad \times [s(e^+, l)t(l, e^-) + s(e^+, \nu)t(\nu, e^-)] \\
&\quad - \frac{1}{2}\frac{\lambda_V}{M_W^2}[t(\nu, e^+)s(e^+, l) + t(\nu, e^-)s(e^-, l)][t(q, l)s(l, \bar{q}') + t(q, \nu)s(\nu, \bar{q}')] \\
&\quad \times [s(e^+, q)t(q, e^-) + s(e^+, \bar{q}')t(\bar{q}', e^-)]\}
\end{aligned}$$

Fig. 4.1 b:

$$M_{LR} = C P_2^\nu P_1^W P_2^W D_{LR} \quad (\text{C.5})$$

where

$$C = -\frac{\pi^2 \alpha^2}{4 \sin^4 \theta_W}$$

$$D_{LR} = 64t(e^+, \nu)s(\vec{q}', e^-)[s(l, e^-)t(e^-, q) - s(l, \vec{q}')t(\vec{q}', q)]$$

Fig. 4.1 c:

$$M_{LR}^Z = C_Z P_1^Z P^e P_1^W D_{LR}(v_l + a_l)^2 \quad (C.6)$$

$$M_{RL}^Z = C_Z P_1^Z P^e P_1^W D_{RL}(v_l^2 - a_l^2) \quad (C.7)$$

$$M_{LR}^\gamma = C_\gamma P_1^\gamma P^e P_1^W D_{LR} \quad (C.8)$$

$$M_{RL}^\gamma = C_\gamma P_1^\gamma P^e P_1^W D_{RL} \quad (C.9)$$

where

$$C_Z = -\frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W}$$

$$C_\gamma = -\frac{2\pi^2 \alpha^2}{\sin^2 \theta_W}$$

$$D_{LR} = 16t(\nu, q)s(e^-, l)[s(\vec{q}', e^-)t(e^-, e^+) - s(\vec{q}', l)t(l, e^+)]$$

$$D_{RL} = 16t(\nu, q)s(e^+, l)[s(\vec{q}', e^+)t(e^+, e^-) - s(\vec{q}', l)t(l, e^-)]$$

Fig. 4.1 d:

$$M_{LR} = C P_1^Z P_1^\nu P_1^W D_{LR}(v_\nu + a_\nu)(v_l + a_l) \quad (C.10)$$

$$M_{RL} = C P_1^Z P_1^\nu P_1^W D_{RL}(v_\nu + a_\nu)(v_l - a_l) \quad (C.11)$$

where

$$C = \frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W}$$

$$D_{LR} = 16t(\nu, e^+)s(\vec{q}', l)[s(e^-, l)t(l, q) + s(e^-, \vec{q}')t(\vec{q}', q)]$$

$$D_{RL} = 16t(\nu, e^-)s(\vec{q}', l)[s(e^+, l)t(l, q) + s(e^+, \vec{q}')t(\vec{q}', q)]$$

Fig. 4.1 e:

$$M_{LR}^Z = C_Z P_1^Z P_1^q P_2^W D_{LR}(v_{\bar{q}'} + a_{\bar{q}'})(v_l + a_l) \quad (C.12)$$

$$M_{RL}^Z = C_Z P_1^Z P_1^q P_2^W D_{RL}(v_{\bar{q}'} + a_{\bar{q}'})(v_l - a_l) \quad (C.13)$$

$$M_{LR}^\gamma = C_\gamma P_1^\gamma P_1^q P_2^W D_{LR} \quad (C.14)$$

$$M_{RL}^\gamma = C_\gamma P_1^\gamma P_1^q P_2^W D_{RL} \quad (C.15)$$

where

$$\begin{aligned} C_Z &= -\frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W} \\ C_\gamma &= -\frac{2\pi^2 \alpha^2 Q_u}{\sin^2 \theta_W} \\ D_{LR} &= 16t(q, \nu)s(e^-, \bar{q}') [s(l, q)t(q, e^+) + s(l, \nu)t(\nu, e^+)] \\ D_{RL} &= 16t(q, \nu)s(e^+, \bar{q}') [s(l, q)t(q, e^-) + s(l, \nu)t(\nu, e^-)] \end{aligned}$$

Fig. 4.1 f:

$$M_{LR}^Z = C_Z P_1^Z P_3^q P_2^W D_{LR}(v_q + a_q)(v_l + a_l) \quad (C.16)$$

$$M_{RL}^Z = C_Z P_1^Z P_3^q P_2^W D_{RL}(v_q + a_q)(v_l - a_l) \quad (C.17)$$

$$M_{LR}^\gamma = C_\gamma P_1^\gamma P_3^q P_2^W D_{LR} \quad (C.18)$$

$$M_{RL}^\gamma = C_\gamma P_1^\gamma P_3^q P_2^W D_{RL} \quad (C.19)$$

where

$$\begin{aligned} C_Z &= \frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W} \\ C_\gamma &= \frac{2\pi^2 \alpha^2 Q_d}{\sin^2 \theta_W} \\ D_{LR} &= 16t(q, e^+)s(l, \bar{q}') [s(e^-, \bar{q}')t(\bar{q}', \nu) + s(e^-, l)t(l, \nu)] \\ D_{RL} &= 16t(q, e^-)s(l, \bar{q}') [s(e^+, \bar{q}')t(\bar{q}', \nu) + s(e^+, l)t(l, \nu)] \end{aligned}$$

Fig. 4.2 a:

$$M_{LR}^Z = C_Z P_2^Z P_3^W P_1^W D_{LR}^Z (v_l + a_l) \quad (C.20)$$

$$M_{LL}^Z = C_Z P_2^Z P_3^W P_1^W D_{LL}^Z (v_l - a_l) \quad (C.21)$$

$$M_{LR}^\gamma = C_\gamma P_2^\gamma P_3^W P_1^W D_{LR}^\gamma \quad (C.22)$$

$$M_{LL}^\gamma = C_\gamma P_2^\gamma P_3^W P_1^W D_{LL}^\gamma \quad (C.23)$$

where ($V = Z, \gamma$)

$$C_Z = -\frac{\pi^2 \alpha^2}{\sin^4 \theta_W}$$

$$C_\gamma = \frac{2\pi^2 \alpha^2}{\sin^2 \theta_W}$$

$$\begin{aligned} D_{LR}^V &= 8 \left\{ -t(q, \nu) s(e^-, \bar{q}') [2g_V + \frac{\lambda_V}{M_W^2} (e^+ - l)^2] [t(e^+, q) s(q, l) + t(e^+, \bar{q}') s(\bar{q}', l)] \right. \\ &\quad - t(\nu, e^+) s(l, e^-) (g_V + \kappa_V + \frac{\lambda_V}{M_W^2} M_{qq}^2) [t(q, e^+) s(e^+, \bar{q}') - t(q, l) s(l, \bar{q}')] \\ &\quad + t(q, e^+) s(l, \bar{q}') [g_V + \kappa_V + \frac{\lambda_V}{M_W^2} (e^- - \nu)^2] [t(\nu, q) s(q, e^-) + t(\nu, \bar{q}') s(\bar{q}', e^-)] \\ &\quad - \frac{1}{2} \frac{\lambda_V}{M_W^2} [t(q, e^-) s(e^-, \bar{q}') - t(q, \nu) s(\nu, \bar{q}')] [t(\nu, e^+) s(e^+, e^-) - t(\nu, l) s(l, e^-)] \\ &\quad \times [t(e^+, q) s(q, l) + t(e^+, \bar{q}') s(\bar{q}', l)] \\ &\quad + \frac{1}{2} \frac{\lambda_V}{M_W^2} [t(q, e^+) s(e^+, \bar{q}') - t(q, l) s(l, \bar{q}')] [t(\nu, q) s(q, e^-) + t(\nu, \bar{q}') s(\bar{q}', e^-)] \\ &\quad \left. \times [t(e^+, e^-) s(e^-, l) - t(e^+, \nu) s(\nu, l)] \right\} \\ D_{LL}^V &= 8 \left\{ -t(q, \nu) s(e^-, \bar{q}') [2g_V + \frac{\lambda_V}{M_W^2} (e^+ - l)^2] [s(e^+, q) t(q, l) + s(e^+, \bar{q}') t(\bar{q}', l)] \right. \\ &\quad - t(\nu, l) s(e^+, e^-) (g_V + \kappa_V + \frac{\lambda_V}{M_W^2} M_{qq}^2) [t(q, e^+) s(e^+, \bar{q}') - t(q, l) s(l, \bar{q}')] \\ &\quad + t(q, l) s(e^+, \bar{q}') [g_V + \kappa_V + \frac{\lambda_V}{M_W^2} (e^- - \nu)^2] [t(\nu, q) s(q, e^-) + t(\nu, \bar{q}') s(\bar{q}', e^-)] \\ &\quad - \frac{1}{2} \frac{\lambda_V}{M_W^2} [t(q, e^-) s(e^-, \bar{q}') - t(q, \nu) s(\nu, \bar{q}')] [t(\nu, e^+) s(e^+, e^-) - t(\nu, l) s(l, e^-)] \\ &\quad \left. \times [s(e^+, q) t(q, l) + s(e^+, \bar{q}') t(\bar{q}', l)] \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \frac{\lambda_V}{M_W^2} [t(q, e^+) s(e^+, \bar{q}') - t(q, l) s(l, \bar{q}')] [t(\nu, q) s(q, e^-) + t(\nu, \bar{q}') s(\bar{q}', e^-)] \\
& \times [s(e^+, e^-) t(e^-, l) - s(e^+, \nu) t(\nu, l)] \}
\end{aligned}$$

Fig. 4.2 b:

$$M_{LR}^Z = C_Z P_2^Z P^e P_1^W D_{LR} (v_l + a_l)^2 \quad (\text{C.24})$$

$$M_{LL}^Z = C_Z P_2^Z P^e P_1^W D_{LL} (v_l^2 - a_l^2) \quad (\text{C.25})$$

$$M_{LR}^\gamma = C_\gamma P_2^\gamma P^e P_1^W D_{LR} \quad (\text{C.26})$$

$$M_{LL}^\gamma = C_\gamma P_2^\gamma P^e P_1^W D_{LL} \quad (\text{C.27})$$

where

$$\begin{aligned}
C_Z &= -\frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W} \\
C_\gamma &= -\frac{2\pi^2 \alpha^2}{\sin^2 \theta_W} \\
D_{LR} &= 16t(\nu, q) s(l, e^-) [s(\bar{q}', \nu) t(\nu, e^+) + s(\bar{q}', q) t(q, e^+)] \\
D_{LL} &= 16t(\nu, q) s(e^+, e^-) [s(\bar{q}', \nu) t(\nu, l) + s(\bar{q}', q) t(q, l)]
\end{aligned}$$

Fig. 4.2 c:

$$M_{LR} = C P_2^Z P_2^\nu P_1^W D_{LR} (v_\nu + a_\nu)(v_l + a_l) \quad (\text{C.28})$$

$$M_{LL} = C P_2^Z P_2^\nu P_1^W D_{LL} (v_\nu + a_\nu)(v_l - a_l) \quad (\text{C.29})$$

where

$$\begin{aligned}
C &= -\frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W} \\
D_{LR} &= 16t(\nu, e^+) s(\bar{q}', e^-) [s(l, e^-) t(e^-, q) - s(l, \bar{q}') t(\bar{q}', q)] \\
D_{LL} &= 16t(\nu, l) s(\bar{q}', e^-) [s(e^+, e^-) t(e^-, q) - s(e^+, \bar{q}') t(\bar{q}', q)]
\end{aligned}$$

Fig. 4.2 d:

$$M_{LR} = C P_1^\nu P_1^W P_3^W D_{LR} \quad (C.30)$$

where

$$C = \frac{\pi^2 \alpha^2}{4 \sin^4 \theta_W}$$

$$D_{LR} = 64t(e^+, \nu)s(\bar{q}', l)[s(e^-, l)t(l, q) + s(e^-, \bar{q}')t(\bar{q}', q)]$$

Fig. 4.2 e:

$$M_{LR}^Z = C_Z P_2^Z P_4^q P_3^W D_{LR}(v_q + a_q)(v_l + a_l) \quad (C.31)$$

$$M_{LL}^Z = C_Z P_2^Z P_4^q P_3^W D_{LL}(v_q + a_q)(v_l - a_l) \quad (C.32)$$

$$M_{LR}^\gamma = C_\gamma P_2^\gamma P_4^q P_3^W D_{LR} \quad (C.33)$$

$$M_{LL}^\gamma = C_\gamma P_2^\gamma P_4^q P_3^W D_{LL} \quad (C.34)$$

where

$$C_Z = \frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W}$$

$$C_\gamma = \frac{2\pi^2 \alpha^2 Q_d}{\sin^2 \theta_W}$$

$$D_{LR} = 16t(q, e^+)s(e^-, \bar{q}')[s(l, e^+)t(e^+, \nu) - s(l, q)t(q, \nu)]$$

$$D_{LL} = -16t(q, l)s(e^-, \bar{q}')[s(e^+, l)t(l, \nu) + s(e^+, q)t(q, \nu)]$$

Fig. 4.2 f:

$$M_{LR}^Z = C_Z P_2^Z P_2^q P_3^W D_{LR}(v_{\bar{q}'} + a_{\bar{q}'})(v_l + a_l) \quad (C.35)$$

$$M_{LL}^Z = C_Z P_2^Z P_2^q P_3^W D_{LL}(v_{\bar{q}'} + a_{\bar{q}'})(v_l - a_l) \quad (C.36)$$

$$M_{LR}^\gamma = C_\gamma P_2^\gamma P_2^q P_3^W D_{LR} \quad (C.37)$$

$$M_{LL}^\gamma = C_\gamma P_2^\gamma P_2^q P_3^W D_{LL} \quad (C.38)$$

where

$$\begin{aligned}
C_Z &= \frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W} \\
C_\gamma &= \frac{2\pi^2 \alpha^2 Q_u}{\sin^2 \theta_W} \\
D_{LR} &= -16t(q, \nu) s(l, \vec{q}') [s(e^-, \nu) t(\nu, e^+) + s(e^-, q) t(q, e^+)] \\
D_{LL} &= -16t(q, \nu) s(e^+, \vec{q}') [s(e^-, \nu) t(\nu, l) + s(e^-, q) t(q, l)]
\end{aligned}$$

The vector and axial couplings are defined as follows

$$\begin{aligned}
v_l &= -1/2 + 2 \sin^2 \theta_W \\
a_l &= -1/2 \\
v_\nu &= 1/2 \\
a_\nu &= 1/2 \\
v_q &= -1/2 + 2/3 \sin^2 \theta_W \\
a_q &= -1/2 \\
v_{\vec{q}'} &= 1/2 - 4/3 \sin^2 \theta_W \\
a_{\vec{q}'} &= 1/2
\end{aligned}$$

The propagators in the amplitudes are considered in the limit when all the external particles are massless:

$$\begin{aligned}
P_1^\gamma &= s^{-1} \\
P_2^\gamma &= (-2l \cdot e^+)^{-1} \\
P^e &= (s - 2\sqrt{s}E_l)^{-1} \\
P_1^\nu &= (s - 2\sqrt{s}E_\nu)^{-1} \\
P_2^\nu &= [2(q \cdot \vec{q}' - q \cdot e^- - e^- \cdot \vec{q}')]^{-1} \\
P_1^q &= (s - 2\sqrt{s}E_{\vec{q}'})^{-1} \\
P_2^q &= [2(\nu \cdot q - e^- \cdot \nu - e^- \cdot q)]^{-1}
\end{aligned}$$

$$\begin{aligned}
P_3^q &= (s - 2\sqrt{s}E_q)^{-1} \\
P_4^q &= [2(l \cdot q - e^+ \cdot l - e^+ \cdot q)]^{-1} \\
P_1^Z &= (s - M_Z^2 + i\Gamma_Z M_Z)^{-1} \\
P_2^Z &= (-2l \cdot e^+ - M_Z^2 + i\Gamma_Z M_Z)^{-1} \\
P_1^W &= (M_{qq}^2 - M_W^2 + i\Gamma_W M_W)^{-1} \\
P_2^W &= (M_{l\nu}^2 - M_W^2 + i\Gamma_W M_W)^{-1} \\
P_3^W &= (-2e^- \cdot \nu - M_W^2 + i\Gamma_W M_W)^{-1}
\end{aligned}$$

where $M_{qq}^2 \equiv (q + \bar{q}')^2$ and $M_{l\nu}^2 \equiv (l + \nu)^2$.

The s and t functions we used in our calculations are defined by equation (B.19) in the Appendix B.

Appendix D

Cross section formulae for

$$V_1 V_2 \rightarrow t \bar{t}$$

In this appendix we give the formulas for the tree-level cross sections for $V_1 V_2 \rightarrow t \bar{t}$. The cross section for $WW \rightarrow t \bar{t}$ and $ZZ \rightarrow t \bar{t}$ are given in the approximation $x_{W,Z} = 0$. In all the expressions below we use the following substitutions:

$$\mathcal{L} = \ln \left(\frac{1 + \beta_t}{1 - \beta_t} \right) \quad (\text{D.1})$$

$$\chi_H = \frac{1}{(s - M_H^2)^2 + \Gamma_H^2 M_H^2} \quad (\text{D.2})$$

$$\beta_{t,z,w} = \sqrt{1 - x_{t,z,w}^2} \quad (\text{D.3})$$

$$x_t = 2m_t/\sqrt{s}, x_Z = 2M_Z/\sqrt{s}, x_W = 2M_W/\sqrt{s} \quad (\text{D.4})$$

$$s_W = \sin \theta_W, c_W = \cos \theta_W \quad (\text{D.5})$$

$$a_t = \frac{1}{2}, v_t = \frac{1}{2} - \frac{4}{3}s_W^2, \quad (\text{D.6})$$

Q_t is the electric charge of the top quark in terms of $|e|$, and N_c is the number of colours. Contributions from both transverse degrees of freedom are summed over.

$\gamma\gamma \rightarrow t\bar{t}$:

$$\sigma(\gamma\gamma \rightarrow t\bar{t}) = \frac{4\pi\alpha^2 Q_t^4 N_c}{s} [(1 + x_t^2 - \frac{x_t^4}{2})\mathcal{L} - \beta_t(1 + x_t^2)] \quad (\text{D.7})$$

$\gamma Z_L \rightarrow t\bar{t}$:

$$\begin{aligned} \sigma(\gamma Z_L \rightarrow t\bar{t}) &= \frac{4\pi\alpha^2 Q_t^2 N_c}{s_W^2 c_W^2 s(1 - M_Z^2/s)^3} \frac{m_t^2}{M_Z^2} \\ &\times \{ [a_t^2(1 - x_Z^2 + \frac{x_t^2 x_Z^2}{4}) - (a_t^2 + 2v_t^2) \frac{x_t^2 x_Z^2}{16} \frac{M_Z^2}{m_t^2}] \mathcal{L} \\ &+ \frac{1}{4} \beta_t x_Z^2 [2a_t^2 + (a_t^2 + v_t^2) \frac{M_Z^2}{m_t^2}] \} \end{aligned} \quad (\text{D.8})$$

$\gamma Z_T \rightarrow t\bar{t}$:

$$\begin{aligned} \sigma(\gamma Z_T \rightarrow t\bar{t}) &= \frac{\pi\alpha^2 Q_t^2 N_c}{s_W^2 c_W^2 s(1 - M_Z^2/s)^3} \\ &\times \{ (a_t^2 + v_t^2) [(\mathcal{L} - \beta_t)(1 + \frac{1}{16} x_Z^4) - \frac{1}{2} \beta_t x_Z^2] \\ &+ (a_t^2 - v_t^2) [\beta_t - (1 - \frac{1}{2} x_t^2) \mathcal{L}] x_t^2 \} \end{aligned} \quad (\text{D.9})$$

$W_L W_L \rightarrow t\bar{t}$: ($x_W = 0$ approximation)

$$\sigma(W_L W_L \rightarrow t\bar{t}) = \sum_i \sigma_i^{LL} \quad \text{where} \quad (\text{D.10})$$

$$\sigma_{\gamma\gamma}^{LL} = \frac{\pi\alpha^2 Q_t^2 N_c}{3} \frac{s}{M_W^4} \beta_t (1 + \frac{1}{2} x_t^2) \quad (\text{D.11})$$

$$\begin{aligned} \sigma_{ZZ}^{LL} &= \frac{\pi\alpha^2 N_c}{12s_W^4(1 - M_Z^2/s)^2} \frac{s}{M_W^4} \beta_t \\ &\times [v_t^2(1 + \frac{1}{2} x_t^2) + a_t^2(1 - x_t^2)] \end{aligned} \quad (\text{D.12})$$

$$\sigma_{HH}^{LL} = \frac{\pi\alpha^2 N_c}{8s_W^4} \frac{m_t^2}{M_W^4} \beta_t^3 s^2 \chi_H \quad (\text{D.13})$$

$$\sigma_{bb}^{LL} = \frac{\pi\alpha^2 N_c}{24s_W^4} \frac{s}{M_W^4} \beta_t^2 [\beta_t(1 + \frac{3}{2}x_t^2) + \frac{3}{4}x_t^4 \mathcal{L}] \quad (\text{D.14})$$

$$\sigma_{\gamma Z}^{LL} = \frac{\pi\alpha^2 Q_t N_c v_t}{3s_W^2(1 - M_Z^2/s)} \frac{s}{M_W^4} \beta_t(1 + \frac{1}{2}x_t^2) \quad (\text{D.15})$$

$$\sigma_{Hb}^{LL} = -\frac{\pi\alpha^2 N_c}{4s_W^4} \frac{m_t^2}{M_W^4} [\beta_t(1 - \frac{x_t^2}{2}) - \frac{x_t^4}{4} \mathcal{L}] s(s - M_H^2) \lambda_H \quad (\text{D.16})$$

$$\sigma_{\gamma b}^{LL} = -\frac{\pi\alpha^2 Q_t N_c}{6s_W^2} \frac{s}{M_W^4} [\beta_t(1 - \frac{x_t^2}{4} - \frac{3}{8}x_t^4) - \frac{3}{16}x_t^6 \mathcal{L}] \quad (\text{D.17})$$

$$\begin{aligned} \sigma_{Zb}^{LL} = & -\frac{\pi\alpha^2 N_c(a_t + v_t)}{12s_W^4(1 - M_Z^2/s)} \frac{s}{M_W^4} \\ & \times [\beta_t(1 - \frac{1}{4}x_t^2 - \frac{3}{8}x_t^4) - \frac{3}{16}x_t^6 \mathcal{L}] \end{aligned} \quad (\text{D.18})$$

$$\sigma_{\gamma H}^{LL} = \sigma_{ZH}^{LL} = 0 \quad (\text{D.19})$$

$W_T W_T \rightarrow t\bar{t}$: ($x_W = 0$ approximation)

$$\sigma(W_T W_T \rightarrow t\bar{t}) = \sum_i \sigma_i^{TT} \quad \text{where} \quad (\text{D.20})$$

$$\sigma_{\gamma\gamma}^{TT} = \frac{2\pi\alpha^2 Q_t^2 N_c}{3s} \beta_t(1 + \frac{1}{2}x_t^2) \quad (\text{D.21})$$

$$\begin{aligned} \sigma_{ZZ}^{TT} = & \frac{\pi\alpha^2 N_c}{6s_W^4 s(1 - M_Z^2/s)^2} \beta_t \\ & \times [v_t^2(1 + \frac{1}{2}x_t^2) + a_t^2(1 - x_t^2)] \end{aligned} \quad (\text{D.22})$$

$$\sigma_{HH}^{TT} = \frac{\pi\alpha^2 N_c}{4s_W^4} m_t^2 \beta_t^3 \chi_H \quad (\text{D.23})$$

$$\begin{aligned} \sigma_{bb}^{TT} = & \frac{\pi\alpha^2 N_c}{2s_W^4 s} [\beta_t (-\frac{7}{6} + \frac{17}{12}x_t^2 - \frac{1}{2}x_t^4) \\ & + (1 - x_t^2 + \frac{7}{8}x_t^4 - \frac{1}{4}x_t^6)\mathcal{L}] \end{aligned} \quad (\text{D.24})$$

$$\sigma_{\gamma Z}^{TT} = \frac{2\pi\alpha^2 Q_t N_c v_t}{3s_W^2 s(1 - M_Z^2/s)} \beta_t (1 + \frac{1}{2}x_t^2) \quad (\text{D.25})$$

$$\sigma_{Hb}^{TT} = \frac{\pi\alpha^2 N_c}{s_W^4} \frac{m_t^4}{s^2} [(1 - \frac{x_t^2}{2})\mathcal{L} - \beta_t](s - M_H^2)\chi_H \quad (\text{D.26})$$

$$\begin{aligned} \sigma_{\gamma b}^{TT} = & -\frac{\pi\alpha^2 Q_t N_c}{3s_W^2 s} [\beta_t (1 + \frac{5}{4}x_t^2 - \frac{3}{8}x_t^4) \\ & + (\frac{3}{4} - \frac{3}{16}x_t^2)x_t^4\mathcal{L}] \end{aligned} \quad (\text{D.27})$$

$$\begin{aligned} \sigma_{Zb}^{TT} = & -\frac{\pi\alpha^2 N_c}{6s_W^4 s(1 - M_Z^2/s)} \\ & \times \{ \beta_t [a_t (1 - \frac{1}{4}x_t^2 - \frac{3}{8}x_t^4) + \\ & v_t (1 + \frac{5}{4}x_t^2 - \frac{3}{8}x_t^4)] \\ & + \frac{3}{4}x_t^4 [v_t (1 - \frac{1}{4}x_t^2) - \frac{1}{4}a_t x_t^2]\mathcal{L} \} \end{aligned} \quad (\text{D.28})$$

$$\sigma_{\gamma H}^{TT} = \sigma_{ZH}^{TT} = 0 \quad (\text{D.29})$$

$W_L W_T \rightarrow t\bar{t}$: ($x_W = 0$ approximation)

$$\sigma(W_L W_T \rightarrow t\bar{t}) = \sum_i \sigma_i^{LT} \quad \text{where} \quad (\text{D.30})$$

$$\sigma_{\gamma\gamma}^{LT} = \frac{4\pi\alpha^2 Q_t^2 N_c}{3M_W^2} \beta_t (1 + \frac{1}{2}x_t^2) \quad (\text{D.31})$$

$$\begin{aligned} \sigma_{ZZ}^{LT} = & \frac{\pi\alpha^2 N_c}{3s_W^4 (1 - M_Z^2/s)^2 M_W^2} \beta_t \\ & \times [v_t^2 (1 + \frac{1}{2}x_t^2) + a_t^2 (1 - x_t^2)] \end{aligned} \quad (\text{D.32})$$

$$\begin{aligned} \sigma_{bb}^{LT} = & \frac{\pi\alpha^2 N_c}{6s_W^4 M_W^2} [\beta_t (1 - x_t^2 + \frac{3}{4}x_t^4) \\ & + \frac{3}{4}x_t^2 (1 - x_t^2 + \frac{1}{2}x_t^4) \mathcal{L}] \end{aligned} \quad (\text{D.33})$$

$$\sigma_{\gamma Z}^{LT} = \frac{4\pi\alpha^2 Q_t N_c v_t}{3s_W^2 (1 - M_Z^2/s) M_W^2} \beta_t (1 + \frac{1}{2}x_t^2) \quad (\text{D.34})$$

$$\sigma_{\gamma b}^{LT} = \frac{2\pi\alpha^2 Q_t N_c}{3s_W^2 M_W^2} [\beta_t (1 + \frac{x_t^2}{8} + \frac{3}{16}x_t^4) + \frac{3}{32}x_t^6 \mathcal{L}] \quad (\text{D.35})$$

$$\begin{aligned} \sigma_{Zb}^{LT} = & -\frac{\pi\alpha^2 N_c}{3s_W^4 (1 - M_Z^2/s) M_W^2} \\ & \times \{ \beta_t [a_t (1 - \frac{5}{8}x_t^2 + \frac{3}{16}x_t^4) + \\ & v_t (1 + \frac{1}{8}x_t^2 + \frac{3}{16}x_t^4)] \\ & - \frac{3}{8}x_t^4 [a_t (1 - \frac{1}{4}x_t^2) - \frac{1}{4}v_t x_t^2] \mathcal{L} \} \end{aligned} \quad (\text{D.36})$$

$$\sigma_{HH}^{LT} = \sigma_{Hb}^{LT} = \sigma_{\gamma H}^{LT} = \sigma_{ZH}^{LT} = 0 \quad (\text{D.37})$$

$Z_L Z_L \rightarrow t\bar{t}$: ($x_Z = 0$ approximation)

$$\sigma(Z_L Z_L \rightarrow t\bar{t}) = \sum_i \sigma_i^{LL} \quad \text{where} \quad (\text{D.38})$$

$$\sigma_{ss}^{LL} = \frac{\pi\alpha^2 N_c}{8s_W^4 c_W^4} \frac{m_t^2}{M_Z^4} \beta_t^3 s^2 \chi_H \quad (\text{D.39})$$

$$\begin{aligned} \sigma_{tt}^{LL} = \sigma_{uu}^{LL} &= \frac{\pi\alpha^2 N_c}{48s_W^4 c_W^4} \frac{s}{M_Z^4} \beta_t [(a_t^4 + v_t^4)(1 + \frac{1}{2}x_t^2) \\ &\quad + 6a_t^2 v_t^2(1 - \frac{3}{2}x_t^2)] \end{aligned} \quad (\text{D.40})$$

$$\sigma_{st}^{LL} = \sigma_{su}^{LL} = \frac{\pi\alpha^2 N_c a_t^2}{2s_W^4 c_W^4} \frac{m_t^2}{M_Z^4} (\beta_t - \frac{x_t^2}{2} \mathcal{L}) s(s - M_H^2) \chi_H \quad (\text{D.41})$$

$$\begin{aligned} \sigma_{tu}^{LL} &= -\frac{\pi\alpha^2 N_c}{24s_W^4 c_W^4} \frac{s}{M_Z^4} \\ &\quad \times \{ \beta_t [a_t^4(1 - \frac{23}{2}x_t^2) + v_t^4(1 + \frac{1}{2}x_t^2) \\ &\quad + 6a_t^2 v_t^2(1 - \frac{3}{2}x_t^2)] + 6a_t^4 x_t^4 \mathcal{L} \} \end{aligned} \quad (\text{D.42})$$

$Z_T Z_T \rightarrow t\bar{t}$: ($x_Z = 0$ approximation)

$$\sigma(Z_T Z_T \rightarrow t\bar{t}) = \sum_i \sigma_i^{TT} \quad \text{where} \quad (\text{D.43})$$

$$\sigma_{ss}^{TT} = \frac{\pi\alpha^2 N_c}{4s_W^4 c_W^4} m_t^2 \beta_t^3 \chi_H \quad (\text{D.44})$$

$$\begin{aligned} \sigma_{tt}^{TT} = \sigma_{uu}^{TT} &= \frac{\pi\alpha^2 N_c}{8s_W^4 c_W^4 s} \\ &\quad \times \{ -\frac{4}{3} \beta_t [a_t^4(1 - x_t^2) + v_t^4(1 + 2x_t^2) + 12a_t^2 v_t^2] \\ &\quad + [a_t^4(1 - \frac{1}{2}x_t^2) + v_t^4(1 + \frac{3}{2}x_t^2) \\ &\quad + 6a_t^2 v_t^2(1 + \frac{1}{2}x_t^2)] \mathcal{L} \} \end{aligned} \quad (\text{D.45})$$

$$\sigma_{st}^{TT} = \sigma_{su}^{TT} = \frac{\pi\alpha^2 N_c}{4s_W^4 c_W^4} (v_t^2 - a_t^2) \frac{m_t^2}{s} \beta_t^2 \mathcal{L}(s - M_H^2) \chi_H \quad (\text{D.46})$$

$$\begin{aligned} \sigma_{tu}^{TT} = & \frac{\pi\alpha^2 N_c}{12s_W^4 c_W^4 s} \{ \beta_t [(a_t^4 + v_t^4)(1 + 5x_t^2) \\ & + 6a_t^2 v_t^2 (1 + x_t^2)] \\ & - \frac{3}{2} x_t^2 [(a_t^4 + v_t^4)(1 + x_t^2) \\ & + 6a_t^2 v_t^2 (1 - \frac{1}{3}x_t^2)] \mathcal{L} \} \end{aligned} \quad (\text{D.47})$$

$Z_L Z_T \rightarrow t\bar{t}$: ($x_Z = 0$ approximation)

$$\sigma(Z_L Z_T \rightarrow t\bar{t}) = \sum_i \sigma_i^{LT} \quad \text{where} \quad (\text{D.48})$$

$$\sigma_{ss}^{LT} = \sigma_{st}^{LT} = \sigma_{su}^{LT} = 0 \quad (\text{D.49})$$

$$\begin{aligned} \sigma_{tt}^{LT} = \sigma_{uu}^{LT} = & \frac{\pi\alpha^2 N_c}{12s_W^4 c_W^4 M_Z^2} \\ & \times \{ \beta_t [a_t^4 (1 - x_t^2) + v_t^4 (1 + \frac{1}{2}x_t^2) \\ & + 6a_t^2 v_t^2 (1 + \frac{1}{4}x_t^2)] - 3a_t^2 v_t^2 x_t^2 \mathcal{L} \} \end{aligned} \quad (\text{D.50})$$

$$\begin{aligned} \sigma_{tu}^{LT} = & -\frac{\pi\alpha^2 N_c}{6s_W^4 c_W^4 M_Z^2} \{ \beta_t [a_t^4 (1 + 2x_t^2) \\ & + v_t^4 (1 + \frac{1}{2}x_t^2) + 6a_t^2 v_t^2 (1 + \frac{1}{4}x_t^2)] \\ & - \frac{3}{2} x_t^2 (a_t^4 + 3a_t^2 v_t^2) \mathcal{L} \} \end{aligned} \quad (\text{D.51})$$

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