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TEST-BASED HYBRID COMPONENT MODE SYNTHESIS

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**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy**

July 1998

**Department of Civil and Environmental Engineering
Faculty of Engineering
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Ottawa, Canada**

**The Doctor of Philosophy Program
in Civil Engineering
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The Undersigned recommend to the Faculty of Graduate Studies
and Research acceptance of the thesis

**TEST-BASED HYBRID
COMPONENT MODE SYNTHESIS**

submitted by
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in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

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ABSTRACT

The class of methods known as component mode synthesis (CMS) provides a means of obtaining efficient mathematical models of very large and/or complex systems. System parameters are derived by coupling models of individual components, obtained either analytically or from testing. One important application of test-based CMS is model-building for large space structures, for which testing of the entire assembly is not possible and component testing provides the only effective alternative.

In modal testing procedures normally used, the structure is supported either on a suitable number of rigid supports to simulate fixed-boundary conditions or from very flexible cables to simulate a free-free configuration. In either case, practical difficulties arise in ensuring that the supports do represent the assumed boundary condition, especially for large flexible space structures.

The present research develops a new approach of CMS in which the characteristics of individual components are determined experimentally through modal and static tests. The proposed technique is referred to as the experimental hybrid CMS approach. The component characteristics used by the approach are the mass matrix, constrained or rigid support normal modes and frequencies, residual flexibility attachment modes and constraint modes. Only a reduced mass matrix needs to be obtained analytically.

A novelty of the approach is that it allows the determination of rigid-support normal modes, residual flexibility attachment modes and constraint modes from tests on components attached to supports of arbitrary stiffness. This procedure obviates the need for providing fully rigid or highly flexible supports. In addition, reaction forces at the supports need not to be measured during modal testing.

The theoretical background of the experimental hybrid CMS approach is derived. Computer simulation studies are presented. They show that the approach is feasible and is not very sensitive to measurement errors. Experimental verification of the approach is carried out through tests on two beam systems and a grid system. Details of the experimental work are presented, along with test data and the CMS results. Good agreement is obtained between the test-based CMS results and their reference values in all cases.

*To Ruan-Isabelle, my twenty-two month old daughter,
who expresses daily the joy of learning and discovering*

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Table 6.11b Estimation of constraint mode elements for unit displacement at d.o.f. 17 and 18,
for component 2 of grid system

Table 6.12 Estimation of $\mathbf{k}^*$ matrix, for component 2 of grid system

NOTATIONS

a, b	Distance between the ends of a fixed-supported beam and a concentrated force applied along its length
$B_{\rho a}$	Residual damping matrix of a component
c	Component viscous damping matrix
C	Matrix containing the constraints arising from the compatibility equations between the components at their interfaces
e_{ω}	Error measure for frequency at system level for CMS estimation
e_{ϕ}	Error measure for mode shape at system level for CMS estimation
$e^{i\omega t}$	Unit harmonic excitation or response
EI	Flexural rigidity of a beam
f	External force vector of a component
f_a	Force matrix, made up of zeros and ones, used in the derivation of Γ_a and ϕ_a
f_f, f_s	Sub-vectors of f partitioned with respect to the free and fixed d.o.f.
F	Amplitude of sinusoidal external force vector
F_f, F_s	Sub-vectors of F partitioned with respect to the free and fixed d.o.f.
F_s	Matrix of static external forces applied along the supported d.o.f. of a component
$F_a(\omega)$	Vector of applied forces, used in the estimation of residual flexibility
G_{ff}	Component flexibility matrix
$G_d, G_{\rho a}$	Component residual flexibility matrix
G_{di}, G_{da}	Sub-matrices of G_d partitioned column-wise with respect to the interior and interface free d.o.f.
$G_{ii}, G_{ia}, G_{ai}, G_{aa}$	Sub-matrices of G_d partitioned column-wise and row-wise with respect to the interior free d.o.f. and interface free d.o.f.

$\mathbf{H}$	Receptance frequency response function matrix of a component, in test configuration
$\mathbf{H}_{ff}$, $\mathbf{H}_{ss}$, $\mathbf{H}_{fs}$, $\mathbf{H}_{sf}$	Sub-matrices of $\mathbf{H}$ partitioned with respect to the free and fixed d.o.f.
$\mathbf{H}^*$	Frequency response function matrix of a component, in fixed-support configuration
$\mathbf{H}^R$	Matrix of reference frequency response functions in fixed-support configuration, used in simulation
H_{ij} , $H_{i,j}$	Single FRF relating response at d.o.f. i with the force at d.o.f. j , in test flexible-support configuration
$H_{\theta_1, f}$	Frequency response function, in flexible-support configuration, relating the rotation at the end of the component (point 1) to a force at any point j
H_{w_1, m_1}	Frequency response function, in flexible-support configuration, relating the displacement at any point i and the moment at the end of the component (point 1)
H_{θ_1, m_1}	Drive point FRF, in flexible-support configuration, relating the rotation and the moment at the end of the component (point 1)
H_{pa}	Residual mass matrix of a component
H_{ij}^* , $H_{i,j}^*$	Single FRF relating response at d.o.f. i with the force at d.o.f. j , in fixed-support configuration
i	$= \sqrt{-1}$, the imaginary unit
$\mathbf{k}$	Component stiffness matrix
$\mathbf{k}_{ff}$, $\mathbf{k}_{ss}$, $\mathbf{k}_{fs}$, $\mathbf{k}_{sf}$	Sub-matrices of $\mathbf{k}$ partitioned with respect to the free and fixed d.o.f.
$\mathbf{k}_s$	Stiffness matrix of supports of arbitrary properties
$\bar{\mathbf{k}}$	Component generalized stiffness matrix with respect to $\mathbf{p}$ coordinates
$\bar{\mathbf{k}}_{ij}$	Sub-matrices of $\bar{\mathbf{k}}$, with $i = k, a$ or s , and $j = k, a$ or s
$\bar{\mathbf{k}}^\alpha$, $\bar{\mathbf{k}}^\beta$	Generalized stiffness matrices with respect to $\mathbf{p}$, for components α and β
$\mathbf{k}^*$	Effective constrained coordinate stiffness matrix of a component
$\hat{\mathbf{k}}$	Component stiffness matrix attached to support of arbitrary properties
$\mathbf{K}$	Generalized stiffness matrix of the system with respect to the independent coordinates $\mathbf{q}$

$\mathbf{K}_p$	Generalized stiffness matrix of the system with respect to the dependent coordinates p
$\mathbf{I}, I$	Identity matrix
L	length of a beam
m	Moment applied at the end or interface of a component
$\mathbf{m}$	Component mass matrix
$\mathbf{m}_f, \mathbf{m}_s, \mathbf{m}_f, \mathbf{m}_f$	Sub-matrices of $\mathbf{m}$ partitioned with respect to the free and fixed d.o.f.
$\bar{\mathbf{m}}$	Component generalized mass matrix with respect to p coordinates
$\bar{\mathbf{m}}_{ij}$	Sub-matrices of $\bar{\mathbf{m}}$, with $i = k, a$ or s , and $j = k, a$ or s
$\bar{\mathbf{m}}^\alpha, \bar{\mathbf{m}}^\beta$	Generalized mass matrices with respect to p coordinates, for components α and β
M	Component mass matrix associated with the free d.o.f., used the mode shape orthogonality checks
$\mathbf{M}$	Generalized mass matrix of the system with respect to the independent coordinates q
$\mathbf{M}_p$	Generalized mass matrix of the system with respect to the dependent coordinates p
n	Number of adjacent data points used in the estimation of $W(j)$
N_a	Number of free interface d.o.f. of a component
N_b	Number of interface d.o.f. of a component
N_c	Number of fixed interface d.o.f. of a component
N_f	Number of free d.o.f. of a component
N_i	Number of free interior d.o.f. of a component
N_m	Number of rows selected in the estimation of $\mathbf{H}^*$ of a component
N_n	Number of columns selected in the estimation of $\mathbf{H}^*$ of a component
N_r	Number of fixed interior d.o.f. of a component
N_s	Number of fixed d.o.f. of a component
$PORTH$	Pseudo orthogonality check matrix

P	Concentrated force applied along the length of a beam
$\mathbf{p}$	Vector of generalized or Ritz coordinates of a component
$\mathbf{p}$	Vector of dependent generalized coordinates at the system level
$\mathbf{p}_f$	Sub-vector of $\mathbf{p}$ partitioned with respect to the free d.o.f.
$\mathbf{p}_k, \mathbf{p}_a, \mathbf{p}_s$	Sub-vectors of $\mathbf{p}$ partitioned with respect to Φ_k, Γ_a and Ψ_s
$\mathbf{p}_r, \mathbf{p}_c$	Sub-vectors of $\mathbf{p}_s$ partitioned with respect to the interior and interface fixed d.o.f.
$\mathbf{p}_k^\alpha, \mathbf{p}_a^\alpha, \mathbf{p}_r^\alpha, \mathbf{p}_c^\alpha$	Sub-vectors of $\mathbf{p}$ associated with component α
$\mathbf{p}_k^\beta, \mathbf{p}_a^\beta, \mathbf{p}_r^\beta, \mathbf{p}_c^\beta$	Sub-vectors of $\mathbf{p}$ associated with component β
$\mathbf{q}$	Vector of independent generalized coordinates at the system level
R	Reaction at the supported end of a fixed-supported beam due to a force P
Re	Real part of a complex function
$\mathbf{R}_s$	Matrix of static support reaction forces of a component
$s^2(j)$	Variance of adjacent data points for data point j , used in the estimation of $W(j)$
$\mathbf{S}$	Transformation matrix at system level from the dependent generalized coordinates $\mathbf{p}$ to the independent generalized coordinates $\mathbf{q}$
$\mathbf{T}$	Transformation matrix at component level from $\mathbf{x}$ to $\mathbf{p}$ coordinates
$U_a(\omega)$	Vector of response displacements, used in the estimation of residual flexibility
$W(j)$	Weighting function for data point j , used in the estimation of residual flexibility
$\mathbf{x}$	Displacement vector of a component; vector of physical coordinates
$\mathbf{x}_f, \mathbf{x}_s$	Sub-vectors of $\mathbf{x}$ partitioned with respect to the free and fixed d.o.f.
$\mathbf{x}_i, \mathbf{x}_a$	Sub-vectors of $\mathbf{x}_f$ partitioned with respect to the interior and interface free d.o.f.
$\mathbf{x}_r, \mathbf{x}_c$	Sub-vectors of $\mathbf{x}_s$ partitioned with respect to the interior and interface fixed d.o.f.
$\mathbf{x}_a^\alpha, \mathbf{x}_a^\beta$	Vector of physical coordinates associated with the free interface d.o.f., for components α and β
$\mathbf{x}_b^\alpha, \mathbf{x}_b^\beta$	Vector of physical coordinates associated with the interface d.o.f., for components α and β
$\mathbf{x}_c^\alpha, \mathbf{x}_c^\beta$	Vector of physical coordinates associated with the fixed interface d.o.f.,

	for components α and β
x_i	Value of $\text{Re}(Y_{pa})$ for data point i , used in the estimation of $W(j)$
$\bar{x}$	Means of the values x_i corresponding to the n data points adjacent to data point j , used in the estimation of $W(j)$
<i>XORTH</i>	Cross orthogonality check matrix
<i>X</i>	Amplitude of sinusoidal displacement response vector
<i>X_f</i> , <i>X_s</i>	Sub-vectors of <i>X</i> partitioned with respect to the free and fixed d.o.f.
<i>Y_a(ω)</i>	Receptance FRF matrix, used in the estimation of residual flexibility
<i>Y_{pa}</i>	Matrix of residual functions, used in the estimation of residual flexibility
Δ	Distance between two adjacent points i and $i+1$ used in the finite difference approximations of rotational quantities; $\Delta = n\Delta_x$ with n an integer
Δ_f	Matrix containing as columns the static displacement patterns associated with the free d.o.f. of a component
Δ_s	Matrix containing as columns the static displacement patterns associated with the supported d.o.f. of a component
Δ_x	Smallest distance between two adjacent point i and $i+1$ used in the finite difference approximations of rotational quantities
Δ_{x1}	Distance between the two points of translational measurement used to estimate the rotation with respect to y axis at d.o.f. 1, for a grid component
Δ_x	Transverse displacement at a distance x from the free end of a cantilever beam caused by a concentrated force applied at the free end
$\Delta_1, \Delta_2, \Delta_3$	Measured static vertical displacements at points 1, 2 and 3, used in the estimation of the interface rotation of beam components through finite difference
ϕ_a	Approximation of system mode shape obtained from CMS
ϕ_r	Reference system mode shape to which ϕ_a is compared
ϕ^0	Elements of Φ_k associated with the free interface rotational d.o.f.
$\phi_1, \phi_2, \dots, \phi_{N_f}$	Orthonormal mode shapes forming the columns of Φ_k

Φ	Matrix of fixed-support normal modes of a component
Φ_k, Φ_d	Sub-matrices of Φ of retained and deleted normal modes of a component
Φ_{ak}	Sub-matrix of Φ_k for the rows corresponding to the free interface d.o.f.
Φ_{exp}	Matrix comprised of columns of mass normalized experimental mode shapes
Φ_{fem}	Matrix comprised of columns of mass normalized analytical mode shapes
Γ_a	Matrix of residual flexibility attachment modes of a component
Γ_{aa}	Sub-matrix of Γ_a for the rows corresponding to the free interface d.o.f.
φ_a	Matrix of attachment modes of a component
Λ	Diagonal matrix of the square of the natural frequencies ω
$\Lambda_{kk}, \Lambda_{dd}$	Sub-matrices of Λ corresponding to the retained and deleted fixed-support normal modes
θ	Rotation at the end or interface of a component
σ	Vector of Lagrange multipliers related to the interface constraints
ω	Natural frequency in radian per second
ω_a	Approximation of system natural frequency obtained from CMS
ω_r	Reference system natural frequency to which ω_a is compared
ω_{kk}	Natural frequencies corresponding to the retained fixed-support normal modes
Ψ_s	Matrix of constraint modes of a component
Ψ_r, Ψ_c	Sub-matrices of Ψ_s partitioned column-wise with respect to the interior and interface fixed d.o.f.
Ψ_{ar}, Ψ_{ac}	Sub-matrices of Ψ_r and Ψ_c for the rows corresponding to the free interface d.o.f.
$\mathbf{0}$	Matrix made up of zeros

1

INTRODUCTION

1.1 General Remarks

The dynamic analysis of large and complex systems is a formidable computational task. A class of methods known as component mode synthesis (CMS) has been efficiently used in such situations to improve the efficiency of the solution.

As its name implies, CMS involves the division of the system into several components (or substructures), development of component models analytically or through testing, and coupling of the components to obtain a representation of the complete system. When component parameters are to be derived experimentally, tests must be carried out on the component supported by truly rigid or completely flexible supports.

The main objective of this project is to develop a new approach of test-based component mode synthesis method which does not rely on the construction of truly rigid or truly flexible supports for the components in their test configuration. This new technique is referred to as the Experimental Hybrid CMS approach. The technique is particularly effective in the modeling and analysis of lightweight structures. This makes it attractive for application in spacecraft design. In fact, the approach is expected to significantly reduce the complexity and the cost of the tests for validating the analytical model of deployed spacecraft to a point where the validation process would be considered an acceptable cost-effective proposition.

1.2 Component Mode Synthesis

The first paper on CMS was published by Hurty (1965) in 1965. Since then, several CMS methods have been developed and CMS has been employed extensively and successfully for structural dynamic analysis of large and complex structural models, particularly in the aerospace industry. When compared to the analysis or testing of the full system, CMS offers the following advantages: 1) reduction in the total number of system degrees-of-freedom (d.o.f.) resulting in a faster and more economical computer solution; 2) feasibility of independent design, analysis and testing of the various components by different contractors (especially considering the fact they are often far from each others and even on different continents); 3) need for sharing only a limited amount of information (generally consisting of frequencies, normal modes and static shapes, as opposed to the complete mass and stiffness matrices of the components) among the various contractors; 4) the ability to revise the design of a particular component without requiring re-analysis and/or re-testing of the rest of the components.

The design of a spacecraft can be cited as one example of the use of CMS. Accurate analytical (Finite Element - FE) modeling of spacecraft is critical for assessing its structural integrity during the launch, and its performance in orbit. Modal testing has become a standard tool for correlating, updating and validating FE models of space structures (Chen, 1982; Chung et al., 1993). Present spacecraft model validation procedures consist of:

- 1) Validating the FE model of the spacecraft by performing a modal survey on the whole structure;
- 2) When the former procedure is not feasible or practical, validating the FE models of the spacecraft components through modal and static tests. A CMS method is used in this case to derive a spacecraft dynamic model, based on component test-validated models.

These validated system models are derived for spacecraft launch configuration, i.e. with the appendages (such as solar arrays and radar arrays) in their retracted or stowed position. Such models are then used to confirm that 1) the spacecraft will sustain the extreme loading conditions to which it will be subjected during the launch, 2) the spacecraft dynamic properties are such that no resonance is going to be induced in the coupled spacecraft/launch vehicle system which could jeopardize the critical launching phase of the mission, and 3) the notching criteria (decrease of excitation level at predetermined frequencies for preventing damages to appendages) used in vibration qualification tests are reliable.

A finite-element model of the spacecraft in on-orbit deployed configuration is needed to predict stress and deformation of the appendages when subjected to dynamic and thermal disturbances. An accurate FE model of the deployed spacecraft is important for assessing the performance of the various subsystems (e.g. radar subsystem such as for RADARSAT) and the overall performance of the spacecraft on orbit.

Presently, the FE model of the deployed spacecraft is seldom, if ever, validated with test data. One of the main reasons for this situation is the complexity and high cost associated with such model validation. The complexity arises because the appendages of the spacecraft are very flexible and cannot support their own weight in a 1 g environment. They must therefore be supported during testing. However, as discussed later, provision of truly rigid or truly flexible supports is very difficult.

Although spacecraft are presently launched without a test-validated FE model in its on-orbit deployed configuration, it is clear that such a model validation process would be most desirable if it could be performed with an acceptable amount of effort and time, and at a reasonable cost. This process would significantly increase the confidence in the accuracy of the model. Furthermore, in the event that unplanned emergency or critical attitude control maneuvers are envisioned once the spacecraft is in its on-orbit deployed configuration, the added confidence of knowing that a *validated* FE model is used in the simulation for verifying any possible adverse

consequences of these maneuvers on the integrity and future operation of the spacecraft would be even more significant.

Any of the CMS methods can be divided into two distinct phases: 1) definition of the model representation of each component, and 2) coupling of all components to form the system (or structure). The CMS methods are based on the Rayleigh-Ritz procedure in which the component physical displacements are represented by a linear combination of independent shape vectors. The weighting factors applied to the shape vectors are known as generalized coordinates. The coordinates serve as the amplitudes of the shape vectors. More specifically, CMS methods consist of the following steps:

- (1) Dividing the system into a number of components. The component d.o.f. are divided into interface d.o.f. (where the component is to be attached to adjacent components), and interior d.o.f.
- (2) Representing the dynamic characteristics of the individual components by their normal modes and frequencies. These characteristics can be evaluated analytically from FE models or determined experimentally. Each component is represented dynamically with a reduced number of normal modes. A reduced number is used, first because it is possible to obtain only a limited number of modes with reasonable accuracy from analysis or from tests, and second because good analytical results can be achieved with a limited number of mode shapes. The associated Ritz transformation leads to component generalized mass and stiffness matrices of reduced order.
- (3) Supplementing the normal modes with appropriate static shapes. These are the constraint and/or the attachment modes.
- (4) Coupling the components by using the conditions of displacement compatibility and force equilibrium at the interfaces, which provides the equations of motion for the coupled system.

- (5) Solving the system equations of motion to obtain estimates of the frequencies and mode shapes of the assembled structure, as well as of its response to a specific load (Craig, 1987).

Hintz (1975) provides a thorough discussion of the mode sets available as Ritz vectors, their possible combination, and the approximation involved in the truncation of mode sets. These mode sets are:

- (1) *Normal modes*: Component normal modes are dynamic modes obtained from a solution of the undamped eigenvalue problem involving all free d.o.f. of the component (interior d.o.f. and interface d.o.f. if any).

Normal modes may be classified according to the boundary conditions imposed at the interface d.o.f. as free-interface modes, fixed-interface modes, loaded interface modes (mass and stiffness coefficients are added to elements of the component mass and stiffness matrices corresponding to the interface d.o.f. to account for the effects of adjacent components), and hybrid-interface modes (where the interface is composed of both free and fixed d.o.f.).

- (2) *Constraint modes*: A constraint mode is defined as the displaced shape obtained by imposing a unit displacement on one physical coordinate and zero displacements on the remainder of a specified subset of the component physical coordinates. Constraint modes are static modes used to supplement the fixed-interface normal modes.
- (3) *Attachment modes*: An attachment mode is defined as the displaced shape obtained by imposing a unit force on one physical coordinate and zero force on the remainder of a specified subset of the component physical coordinates. Attachment modes are static modes used by many authors to supplement the free-interface normal modes, in order to improve the convergence of the CMS method.
- (4) *Rigid body modes*: These modes arise when a component is not sufficiently restrained to prevent its rigid body motion. Up to six (3 rotations and 3 translations) rigid body modes

may be present. They form a subset of the free-interface normal modes and, as such, may be considered as dynamic modes. However, rigid body modes may also be treated as static modes since they form a subset of the constraint modes (Craig and Bampton, 1968).

1.3 Review of Literature

A general review of literature is presented in this section. Since the subject and approach of the thesis encompass several topics, only the key references related to various aspects of this research are reviewed.

1.3.1 Substructure Methods

The CMS methods represent the class of substructuring approach most widely and effectively used for dynamic analysis of large and complex structures. CMS has been used extensively in the aerospace industry during the last three decades and a large number of papers related to its application are available in the literature. As mentioned by Nelson (1979), other segments of industry adopted CMS in the late seventies with considerable profit. These industries include the nuclear industry, and the construction industry specializing in design of building and machine foundations.

Although several CMS methods have been developed and implemented, there is no clear indication that any particular method is superior to all others, all the time. The accuracy of a specific method is partly problem dependent. The success of a method also depends on the ease with which the information needed to implement it can be obtained. The practical difficulty in acquiring the information varies and is to some extent dependent on the method and on the type of structure being analyzed (Baker, 1986).

The fixed-interface normal mode CMS methods, of which the methods of Hurty (1965), and Craig and Bampton (1968) are the best known examples, are expected to be most accurate when the final system allows the component of interest to have little motion near the interface d.o.f.

When the interfaces do undergo displacement, fixed-interface normal modes must be supplemented by static constraint modes.

The free-interface normal mode methods can take into account interface displacements. The technique of residual flexibility (MacNeal, 1971; Rubin, 1975) was developed to improve the truncated free-interface normal mode representation of a component (Goldman, 1969; Hou, 1969), by including the residual effects of modes with frequencies higher than the cut-off frequency for the normal modes included in the component representation. For the sample problems reported in the literature (Rubin, 1975; Smith and Hutton, 1994), these methods appear to be at least as accurate as, or even more accurate than, the fixed-interface normal mode methods. When the CMS method is to be used with test data, the free-interface with residual flexibility approach is considered to be more appropriate than the fixed-interface approach because it is easier and simpler to obtain the input data required for the former. However, as discussed in Section 1.3.2, this is not necessarily so in the case of large flexible appendages of spacecraft.

Another group of CMS methods of particular interest for the present project is comprised of methods based on the use of hybrid-interface modes for which the component interface consists of a combination of free and fixed d.o.f. MacNeal (1971) was the first author to propose such a method. His motivation for introducing the hybrid approach was twofold: 1) if test data or the results of a previous analysis are used, the analyst may have no option left regarding the selection of the interface, and 2) better results will be obtained if the interface for the component modes is representative of the true conditions of the system. Craig and Chang (1977) included the hybrid method as one of four different procedures that illustrate the use of attachment modes in CMS.

It should be noted that substructuring and synthesis approaches other than CMS have been developed, e.g. Crowley et al. (1984), and Hale and Meirovitch (1980). One example of an alternative to CMS is the impedance coupling method which consists of synthesizing test-based

frequency response functions (FRF's) of the components, instead of the derived analytical models as in the case of CMS. Su and Juang (1993) have compared the merits of impedance coupling method and CMS. They state that, although both the CMS and impedance approaches are feasible, CMS is preferable. One of the justifications they provide is the fact that it is always easier to identify an analytical model for a component than for the integrated system because 1) all the existing system identification methods encounter certain degree of numerical difficulty when the system to be identified has a large number of modes and/or has densely-clustered frequencies, and 2) components have simpler geometry, smaller number of modes and fewer closely-spaced modes. Also, as mentioned in Bregant et al. (1995), despite a straightforward formulation, impedance coupling still fails to yield acceptable results.

1.3.2 Modal Testing and Boundary Conditions

The dynamic parameters, such as the frequencies and mode shapes, of the components required in CMS are often obtained through modal testing. Modal testing can be defined as the process by which a mathematical description of the modal or dynamic behavior of a structure is obtained experimentally. The mathematical description usually consists of the following parameters: natural frequencies, damping ratios, mode shapes and modal masses.

A rapid expansion and recognition of modal testing has occurred in the last fifteen years as a result of the availability of low-cost, high-performance computers and test equipment, as well as the development of reliable parameter identification techniques. Modal testing is a very broad field which encompasses several topics and finds use in numerous applications in various disciplines (including aerospace, automotive and civil industries).

An extensive amount of literature has been published covering all aspects of modal testing, e.g. Ewins (1984).

The selection of the boundary condition under which modal testing is performed is a key element in selecting the type of CMS approach to be used.

A modal test is conducted on a test article (component in the present case) which is normally either in a free-free or fixed boundary condition. In a free-free condition, the test article is supported by a suspension system which does not restrain the structure at any point, so that the test article is "freely suspended" in space and exhibits rigid body motion. The theoretical frequencies of the rigid body modes of the structure is zero. In practice, it is not possible to provide a truly free-free condition for the test article, since the suspension system always provides some minimal restraint. Dynamic coupling between the suspension system and the test article can be considered as being negligible as long as the highest of the six rigid body frequencies of the suspension is at least an order of magnitude smaller than the lowest flexural frequency of the structure. This is due to the fact that when resonance frequencies of the mass of the structure on the stiffness of the suspension are far away from the frequency range of interest, they have no significant influence on the flexural modes of interest (Ewins, 1984; Maia et al., 1997).

For relatively rigid test articles, including spacecraft of previous generations, having all flexural frequencies above 15 Hz, this frequency separation is relatively easy to achieve by suspending the test article through very soft springs (long cables or bungee cords). More specifically, long suspension cables are needed to achieve low frequency pendulum motion in the horizontal plane, while the in-line springs are there to create the low frequency suspension in the vertical direction. Also, the suspension system must have low mass and low damping, relative to those of the test article, to avoid mass and damping coupling, respectively (Cooley, 1993).

For the present and the next generation of spacecraft having low fundamental frequencies (below or much below 10 Hz) in deployed configuration, the conventional suspension systems referred to in the preceding paragraph are not acceptable. This is because the low mass, low damping and especially low frequency test articles dictate that these parameters of the suspension systems to be even lower. Advanced suspension systems, both active and passive, have been designed and built to remedy this situation by providing supports which have close to zero

stiffness (Cooley and Giunta, 1992; Kienholz, 1994). However, they are expensive and require infrastructure with very high ceiling for the attachment of cables.

In view of these problems that arise with very flexible structures, some alternative approaches have been considered in order to derive the free-free modal parameters of a structure based on modal testing of the structure in restrained configuration. A first approach consists in applying additional excitation forces at the restrained locations. By inputting linearly independent forces into the restrained locations and taking several sets of data, standard FRF estimation techniques may be used to obtain the free-free response functions (Barney et al. 1990). The approach requires the measurement of reaction forces at the restrained locations. However, as discussed by Barney et al. (1990), this method adds significant costs since a large shaker must be placed at each restrained location to sufficiently uncouple forces. A second approach proposed by these researchers consists in measuring the passive reaction forces at the restrained locations and in using these forces in a computation to eliminate the influence of the supported points. This is done through a direct parameter estimation algorithm which computes the structure free-free mass, stiffness and damping matrices. The method requires perturbing the boundary conditions several times during testing since singularities exist in a single boundary condition testing. As mentioned, both approaches require measuring the reaction forces at the constraints and performing several tests sets.

The alternative boundary condition of a structure for a modal test is fixed or grounded, in which case an attempt is made to fix selected points of the test article to ground. This approach, while conceptually simple and easy to treat analytically, represents a considerable challenge in practical testing because construction of truly rigid supports is impractical and expensive. As mentioned in Ewins (1984), approximation of grounded condition requires extraordinary precaution in designing the support(s). Sometimes, for large and heavy space structures, the floor/ground beneath the supports must be reinforced so that floor deformations do not influence the test article modes (Blair and Vadlamudi, 1988).

As shown by Salvini and Sestieri (1993), it is possible to predict the frequency response functions of a constrained test article from experimental FRF's of the same structure in a less constrained configuration. The only condition imposed on the more constrained structure is that the added constraints (translational and/or rotational) be ideal, i.e. having zero displacement. The special requirements on the modal test are 1) to excite the structure along all added constrained d.o.f. (in order to get the experimental FRF's with these d.o.f. as reference) and at some free (unsupported) points, and 2) to measure the responses along these constrained d.o.f. and at the free points of interest. The fact that there is no need to measure the reaction force along the added constrained d.o.f. is quite significant since these forces could be difficult to measure accurately due to cross-sensitivity of the force transducer between the transverse and longitudinal directions. The relationship between the two sets of FRF's (test and constrained configurations) involves only simple matrix operations, including a matrix inversion of order equal to the number of d.o.f. of added constraints.

1.3.3 Estimation of Rotational Input and Response during Modal Testing

At present, the experimental evaluation of rotational FRF's is not very accurate because 1) at least until recently, acceptable rotational modal transducers had not been developed (Li et al., 1993), and 2) lumped moments are not easily applied (Salvini and Sestieri, 1993).

The interface d.o.f. of a component usually include rotational coordinates. Consequently, in the context of the experimental hybrid CMS approach, the need to estimate FRF's involving rotational quantities, referred to as rotational FRF's, still arise. More specifically, rotational/translational FRF's are needed in the normal modes; while translational/rotational and rotational/rotational FRF's are required for the evaluation of residual flexibility attachment modes which are described in Chapter 2.

Several researchers have investigated the problem of experimentally estimating rotational data and have developed different approaches. Ewins and Gleeson (1975) used special fixtures

attached to the test article in order to estimate rotational FRF's. Other researchers, such as Chung and Craig (1985), have used a cubic spline curve fitting algorithm to estimate the rotational displacements from the corresponding translational modes shapes.

Sattinger (1980) adapted the method of finite difference to derive the rotational FRF's (involving either rotational velocity and/or moment) from sets of conventionally measured translational FRF's (translational velocity/force). Using Sattinger's finite difference equations, Martinez et al. (1984) successfully applied the method in implementing a free-interface CMS method from a combined experimental/analytical model for a simple beam structure. Sestieri et al. (1991) analyzed the main sources of error and suggested some solutions for applying Sattinger's method on measured translational FRF's. Because of its simplicity and its documented successful applications, the finite difference approach has been retained for estimating rotational FRF's in the present project.

It is worth mentioning that recent developments have occurred in the design and utilization of rotational transducers. Li et al. (1993) reported on the development of a six-axis arrayed accelerometer to be used with the Perturbed Boundary Condition (PBC) Testing Method. This device consists of a rigid arm attached to the test article at a desired location. An array of transducers is mounted on this arm. The translational and rotational movements of the point of attachment are derived from the translational measurements of the array. Based on similar principles and the use of a total of 32 quartz crystal sensing elements, the prototype of a force transducer capable of measuring the six possible force and moment inputs to a structure at a given point has been developed (Stebbins et al., 1996). Techniques for calibrating such multi-axis force sensor and a recently commercially-available single force and two moment transducer have been investigated (Stebbins et al., 1997; McConnell et al., 1997).

1.3.4 Experimental Determination of Static Parameters

The component static parameters required by the hybrid CMS method are related to the presence of constraint modes in the Ritz transformation. They appear in both the generalized mass and stiffness matrices. For redundantly constrained components, it has been shown that these parameters could be deduced from 1) the modal stiffness and modal reaction forces obtained from modal test of the structure constrained (zero displacement) along the supported d.o.f. and 2) the static flexibility matrix measured from a static test on the component constrained at a non-redundant subset of the original set of constrained d.o.f. (Klosterrman, 1971; Allemang et al., 1987).

The method proposed in the present work includes a comparatively simpler approach in which these static parameters are obtained from a static flexibility test on the component attached to flexible supports providing non-zero displacements along the supported d.o.f. During the test, forces are applied and measured only at a few locations (supported d.o.f.), and displacements are measured only along these d.o.f. as well as along the free d.o.f. of interest.

It is worth mentioning that static tests are commonly performed as part of the qualification and acceptance tests of large space structures. The main objective of such static tests is normally to verify the strength of the structure (Sarafin and Larson, 1995). Sometimes, a static test may be performed in order to verify the FE stiffness matrix. This is rarely done because of the complexity of such a test, due to the very high number of d.o.f. involved, and the inaccessibility of some of these d.o.f. However, the verification of a small portion of the full stiffness matrix through such a test may be feasible.

1.3.5 Estimation of Component Mass Matrix

Another parameter required in the hybrid CMS method is the component physical mass matrix. Various approaches are available for obtaining this information:

- (1) In the case of a simple structure for which a lumped (diagonal) mass matrix is required, it is possible to derive a fairly accurate mass matrix simply from the knowledge of material density and geometric distribution of mass.
- (2) For more complex structures and when a consistent mass representation is needed, a mass matrix may be obtained analytically from the FE model using measured values of material density and mass distribution.
- (3) In order to eliminate the need for relying on analytical modeling, several techniques have been developed in recent years which are aimed at identifying the physical lumped mass, damping and stiffness matrices based on modal or vibration testing (e.g. Fritzen, 1986; Roemer and Mook, 1992).

A technique has been developed for using test data to obtain component parameters related to mass, stiffness and damping matrices that provide all the information necessary for coupling the components together through CMS (Craig, 1995; Craig and Blades, 1996). Although the approach has been verified through numerical simulations, including simulated noisy data, it has yet to be applied to real test data.

Physical parameter identification is still an area of on-going research. However, most existing methods seem to exhibit some problems when dealing with imperfect test data, especially in the presence of noise.

1.4 The Experimental Hybrid CMS Approach

The objective of the experimental hybrid CMS approach developed in this work is to obtain the modal parameters of the system from the modal and static parameters of its components. The component parameters are derived from tests on the components held by supports of arbitrary stiffness (implying non-zero displacement along the supported d.o.f.). Such supports may be located at interfaces and at interior points. The hybrid CMS method is based on a representation of the displacements of physical d.o.f. by a combination of three different mode

sets: constrained (or rigid support) normal modes, constraint modes and residual flexibility attachment modes. The constraint modes arise from the presence of the constrained d.o.f. (both at the interface and interior points). The residual flexibility attachment modes are associated with the free interface d.o.f. The component modal and static parameters (including the mode sets) are derived from test data as explained later in this section.

The novelty of the proposed approach is that the required component modal and static parameters in rigid support configuration are derived from experimental data obtained from tests on components which are supported by flexible supports having arbitrary stiffness. The fact that this approach eliminates the test requirements of having fully rigid supports (or very flexible suspension) results in considerable cost savings due to the simple design of the required supports. Although these supported d.o.f. could be any one or all of the six d.o.f. available at a point, they are limited to the minimum number of vertical d.o.f. These supported d.o.f. are located at the interface (normally at one extremity of the test article) and also, if necessary, at some interior points.

The proposed approach is divided into three distinct but interrelated steps.

- Step 1:** Estimation of FRF's and estimation of component modal parameters (frequencies, mode shapes and residual flexibility terms) in constrained or rigid support configuration, based on experimental data obtained from modal testing of the component attached to flexible supports, at the same locations as the "theoretical" rigid supports.
- Step 2:** Estimation of component static parameters (constraint modes and effective constrained coordinate stiffness matrix), based on experimental data obtained from static tests with the component attached to flexible supports, at the same locations as in Step 1.
- Step 3:** Derivation of the system generalized mass and stiffness matrices and estimation of the modal parameters (frequencies and mode shapes), using test-based modal and static parameters derived in Steps 1 and 2 and applying the hybrid CMS method.

1.5 Scope of Present Research

The work performed in this project consists of the following main tasks:

- (1) Derivation of theoretical background for the three steps of the approach.
- (2) Development of computer software modules for the implementation of the proposed approach. These modules include routines required for data transfer between programs, for data analysis and for the hybrid CMS method.
- (3) Computer simulation and sensitivity study designed to verify the robustness of the hybrid CMS method as well as of the overall approach.
- (4) Procurement and analysis of existing test articles. Design, analysis and manufacture of new test articles.
- (5) Design, analysis and manufacture of any hardware required for the tests described under Task 6.
- (6) Verification of the proposed approach with test-based data for three systems of increasing complexity.

In the development of theoretical background, it is assumed that all components are linear and that the connections between the components are rigid. The latter assumption is reasonable for the large majority of systems for which the proposed approach would be most suitable. Most deployed spacecraft are either designed or controlled in-orbit to exhibit small deformation under the low loads to which they are subjected. Consequently, these structures stay within their linear range. Also, the linearity assumption is consistent with the fact that the majority of FE models and analysis, as well as the conventional modal testing procedures, are based on linear behavior. It is further assumed that classical modes exist for the structure, implying no damping or proportional damping.

Because the experimental determination of mass matrix is a research topic by itself and test procedures for determining mass matrices have not been fully developed as yet, and because it

is generally accepted that mass matrix is much easier to accurately predict analytically than the stiffness matrix and its related parameters, analytical mass matrices are used for the present project.

The theoretical background is developed for a structure composed of only two components. Extension to the case of multiple components is straightforward and only implies a generalization of the equations involved in the coupling of the components.

1.6 Layout of the Thesis

The present chapter contains a general introduction, an overview of CMS including a description of the problem addressed by the present research, a review of relevant literature, an overview of the proposed experimental hybrid CMS approach and the scope of the research.

The theoretical background of the experimental hybrid CMS approach is developed in Chapter 2. This includes the methods of estimating the required component parameters from modal and static tests on a component attached to supports of arbitrary stiffness. Representative results of computer simulation studies are presented and discussed in Chapter 3.

Chapter 4 is the first of three chapters dealing with the experimental verification of the proposed approach. It concentrates on the common elements of the test program. The first part of the chapter describes key elements of the flexible supports that represent critical hardware for all tests. Subsequent parts of Chapter 4 describe the other test equipment used in the modal and static tests as well as the procedures used for carrying out the tests. A description of modal testing of a flexible support prototype and presentation of key test results conclude the chapter.

The results of the application of the experimental hybrid CMS approach to two beam systems of increasing complexity are presented in Chapter 5. Each system is composed of two components. For each of the two systems, the test articles and the boundary conditions are described first. Details of the finite element models for both the system and the components are presented next. Also included is a discussion of the system modal parameters used as a

reference in the CMS and the manner in which these parameters are obtained. Details of modal and static tests are presented for each of the two components. The system modal parameters are obtained with the hybrid CMS approach using the test-derived component parameters and the parameter values so obtained are compared with corresponding reference values.

The results obtained from the application of the experimental hybrid CMS approach to the evaluation of modal parameters of a free-free grid system are presented in Chapter 6. This grid is the most complex of the three systems tested in this project. The format used for the presentation of material in this chapter is similar to that used in Chapter 5. A summary and conclusions of the current research, and suggestions for future work are contained in Chapter 7.

2 THEORETICAL BACKGROUND

2.1 Introduction

The theoretical background for the experimental hybrid CMS approach is presented. This includes the methods of estimating the required component parameters from modal and static tests on a component attached to supports of arbitrary stiffness, and the method of synthesizing the results to obtain the system modal properties.

2.2 Hybrid CMS Method

The theoretical basis for the use in component mode synthesis of fixed-interface normal modes along with constraint modes (fixed-interface methods), or the use of free-interface normal modes along with residual flexibility attachment modes (free-interface methods), has been well described in the literature (Craig 1981). The hybrid CMS method makes use of normal modes, constraint modes and residual flexibility attachment modes by considering the case of the interface consisting of both fixed and free coordinates. Although, as mentioned earlier, a few researchers have discussed the use of hybrid CMS methods, none of them has presented a complete step-by-step derivation of the final generalized system mass and stiffness matrices; perhaps on account of the limited application the method has seen so far.

The theory of the hybrid CMS method is presented here for the case of a system composed of two linear components connected along several d.o.f. The method may be easily extended to the case when more than two components are present. For illustrative purposes, consider the cantilever beam shown in Fig. 2.1. The physical coordinates consist of two d.o.f. at each node

(vertical translation and in-plane rotation). The beam vibrates in the plane of the paper. The dynamic characteristics of interest are the free vibration frequencies and the mode shapes. The beam is used in the computer simulation studies presented later in this report. The cantilever beam is divided into two components α and β connected along the two d.o.f. as indicated.

It is assumed that for the purpose of modal testing of individual components, a support is provided along the interface d.o.f. 28; additional vertical supports are located at d.o.f. 14 for component α , and d.o.f. 48 for component β . Consequently, the interface consists of a fixed and a free d.o.f. However, the mathematical development presented here applies to the general case of several fixed and free interface d.o.f.

2.2.1 Definition of Degree of Freedom Sets and Subsets

The physical coordinates of each component are divided into the several sets and subsets of d.o.f. as defined in Table 2.1. The fixed d.o.f. correspond to the ones which are supported in the test configuration. As an example, the d.o.f. under various sets for component β are presented in the last column of Table 2.1. The definition of the various sets and subsets is useful in describing the various steps in the mathematical development of the method.

2.2.2 Component Modes

When damping is neglected, the equation of motion of a component can be expressed as

$$\mathbf{m} \ddot{\mathbf{x}} + \mathbf{k} \mathbf{x} = \mathbf{f} \quad (2.1)$$

in which $\mathbf{m}$ and $\mathbf{k}$ are the mass and stiffness matrices respectively, $\mathbf{x}$ is the displacement vector, $\ddot{\mathbf{x}}$ is the acceleration vector and $\mathbf{f}$ is the force vector. The $\mathbf{m}$ and $\mathbf{k}$ matrices are symmetric.

The rationale for neglecting damping in the equation of motion requires some discussion. Because it is extremely difficult to construct a truly representative physical damping matrix for a structure, a typical dynamic FE model either neglects damping or uses a proportional damping matrix (Bathe, 1982); both cases have the advantage of producing a normal mode solution

(Caughey, 1960). For structures exhibiting light non-proportional damping, the damping effects are usually neglected and a set of normal modes is extracted from test data. According to Ibrahim and Sestieri (1995), this approximation is acceptable for several applications including CMS. In fact, most CMS methods developed so far are based on undamped models (Craig, 1990). Furthermore, CMS methods based on complex modes do not necessarily perform significantly better than real normal mode synthesis (Huston et al., 1991; Wang and Kirkhope, 1995).

Partitioning Eq. 2.1 with respect to the free and fixed d.o.f. gives

$$\begin{bmatrix} \mathbf{m}_{ff} & \mathbf{m}_{fs} \\ \mathbf{m}_{sf} & \mathbf{m}_{ss} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_f \\ \ddot{\mathbf{x}}_s \end{bmatrix} + \begin{bmatrix} \mathbf{k}_{ff} & \mathbf{k}_{fs} \\ \mathbf{k}_{sf} & \mathbf{k}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{x}_f \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} \mathbf{f}_f \\ \mathbf{f}_s \end{bmatrix} \quad (2.2)$$

The various CMS methods are based on a transformation involving selected Ritz vectors in order to reduce the size of matrix equation 2.1. As mentioned earlier, the hybrid CMS method involves three sets of such Ritz vectors: constrained or rigid support normal modes Φ_k , residual flexibility attachment modes Γ_a , and constraint modes Ψ_s . The following subsections describe these three sets of vectors.

Rigid Support Normal Modes

These modes are obtained from the solution of free vibration problem involving all free d.o.f. (f set). The problem definition is obtained from the first row of Eq. 2.2, by setting each $\ddot{\mathbf{x}}_s$, $\mathbf{x}_s$ and $\mathbf{f}_s$ as zero.

$$\mathbf{m}_{ff} \ddot{\mathbf{x}}_f + \mathbf{k}_{ff} \mathbf{x}_f = \mathbf{0} \quad (2.3a)$$

The corresponding eigenvalue problem is

$$(\mathbf{k}_{ff} - \omega^2 \mathbf{m}_{ff}) \phi = \mathbf{0} \quad (2.3b)$$

Solution of Eq. 2.3b leads to the natural frequencies $\omega_1, \omega_2, \dots, \omega_{N_f}$, and the corresponding orthonormal mode shapes $\phi_1, \phi_2, \dots, \phi_{N_f}$. The orthonormality property implies that

$$\Phi^T \mathbf{m}_{ff} \Phi = \mathbf{I}_f \quad (2.4a)$$

$$\Phi^T \mathbf{k}_{ff} \Phi = \Lambda \equiv \text{diag}(\omega_r^2) \quad (2.4b)$$

where Φ is the matrix whose columns are the component normal modes ϕ_r , $\mathbf{I}_f$ is the identity matrix of order N_f and Λ is a diagonal matrix of order N_f containing the eigenvalues ω_r^2 .

Of the N_f normal modes, only k ($k < N_f$) modes are kept in the subsequent Ritz transformation. The corresponding shape matrix Φ_k is of dimensions $N_f \times k$, and the eigenvalue matrix Λ_{kk} is of dimensions $k \times k$.

It should be noted that the component is assumed to be restrained sufficiently to prevent any rigid body motion. This is a reasonable assumption since the motivation for developing the experimental hybrid CMS approach is for application to large flexible structures which need several supports in test configuration, and these supported d.o.f. become fixed d.o.f. for the CMS method. If the restrained component could undergo rigid body motion, the set of normal modes would include one or more rigid body (zero frequency) modes.

Constraint Modes

The constraint modes are static displacement shapes of the component along its free d.o.f. which result from the application of a unit displacement, in turn, at each of the fixed d.o.f., while the displacements along the remaining fixed d.o.f. are zero. There are N_s constraint modes which together form the constraint mode matrix denoted by Ψ_s . The static equation of motion of a component in a partition form can be expressed as

$$\begin{bmatrix} \mathbf{k}_{ff} & \mathbf{k}_{fs} \\ \mathbf{k}_{sf} & \mathbf{k}_{ss} \end{bmatrix} \begin{bmatrix} \Psi_s \\ \mathbf{I}_s \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{f}_s \end{bmatrix} \quad (2.5)$$

where $\mathbf{I}_s$ is an identity matrix of dimension N_s . The first row of the above equation gives

$$\Psi_s = -\mathbf{k}_{ff}^{-1} \mathbf{k}_{fs} \quad (2.6)$$

It should be noted that $\mathbf{k}_{ff}$ is nonsingular since the component is assumed to be restrained sufficiently to prevent any rigid body motion.

Constraint modes are required to supplement the rigid support normal modes and the residual flexibility attachment modes. The latter two sets specify zero displacement along those d.o.f. which have been fixed in the process of deriving them and consequently cannot by themselves describe a displaced shape that involves displacement along such d.o.f.

Residual Flexibility Attachment Modes

In order to explain the concept of residual flexibility attachment modes, an attachment mode need to be defined first. Historically, attachment modes preceded their residual flexibility version and were introduced to supplement the free-interface normal modes. An attachment mode is defined as the static displacement shape of the component along its free d.o.f. caused by a unit force applied along one of the interface free d.o.f., while displacements along the constrained d.o.f. are zero. There exists N_a attachment modes forming columns of the matrix Φ_a ($N_f \times N_a$). The set of attachment modes is expressed mathematically by

$$K_{ff} \Phi_a = F_a \quad (2.7a)$$

or

$$\Phi_a = G_{ff} F_a \quad (2.7b)$$

where G_{ff} is the component flexibility matrix, and F_a is given by

$$F_a = \begin{bmatrix} 0_{ia} \\ I_a \end{bmatrix} \quad (2.8)$$

in which 0_{ia} is a matrix of dimensions $N_i \times N_a$ having only zeros as its elements, and I_a is an identity matrix of dimension N_a .

For a component having no rigid body motion, the flexibility matrix may be expressed as a product of the modal parameters (Craig and Chang, 1977):

$$G_{ff} = \Phi \Lambda^{-1} \Phi^T \quad (2.9)$$

Equation 2.9 can be derived as follows. The free physical coordinates of the restrained component may be represented in terms of generalized coordinates, $\mathbf{p}_f$, by the coordinate transformation

$$\mathbf{x}_f = \Phi \mathbf{p}_f \quad (2.10)$$

where Φ is obtained from the eigenvalue problem defined by Eq. 2.3b. On setting $\ddot{\mathbf{x}}_r$ and $\mathbf{x}_r$ to zero, the first row of Eq. 2.2 gives

$$\mathbf{m}_{ff} \ddot{\mathbf{x}}_f + \mathbf{k}_{ff} \mathbf{x}_f = \mathbf{f}_f \quad (2.11)$$

Substituting Eq. 2.10 into Eq. 2.11, pre-multiplying both sides of the equation by Φ^T and applying relations 2.4 leads to

$$\ddot{\mathbf{p}}_f + \Lambda \mathbf{p}_f = \Phi^T \mathbf{f}_f \quad (2.12)$$

Neglecting the inertia of the modes (i.e. setting $\ddot{\mathbf{p}}_f$ as a static force),

$$\mathbf{p}_f = \Lambda^{-1} \Phi^T \mathbf{f}_f \quad (2.13)$$

and, from Eq. 2.10, the corresponding static deflection is given by

$$\mathbf{x}_f = \Phi \Lambda^{-1} \Phi^T \mathbf{f}_f \quad (2.14)$$

Comparison of Eqs. 2.7b and 2.14 shows that Eq. 2.9 is true.

Because matrix Λ is diagonal, the right-hand-side (R.H.S.) of Eq. 2.9 may be expressed as a function of kept and deleted modes (denoted by subscripts k and d , respectively), i.e.

$$\mathbf{G}_{ff} = \Phi_k \Lambda_{kk}^{-1} \Phi_k^T + \Phi_d \Lambda_{dd}^{-1} \Phi_d^T \quad (2.15)$$

The residual flexibility matrix $\mathbf{G}_d$ corresponds to the component flexibility associated with the deleted modes and is given by

$$\mathbf{G}_d = \Phi_d \Lambda_{dd}^{-1} \Phi_d^T \quad (2.16)$$

By using Eqs. 2.15 and 2.16, $\mathbf{G}_d$ may be obtained without any direct knowledge of deleted modes since

$$\mathbf{G}_d = \mathbf{G}_{ff} - \Phi_k \Lambda_{kk}^{-1} \Phi_k^T \quad (2.17)$$

From Eq. 2.7b, the residual flexibility attachment modes (or residual attachment modes) are given by

$$\Gamma_a = G_d \mathbf{f}_a \quad (2.18)$$

where $\mathbf{f}_a$ is obtained from Eq. 2.8. and G_d is computed from Eq. 2.17. or derived from measured data as discussed in Section 5.3.3.

From an experimental point of view, individual terms of G_d may be obtained by subtracting from the corresponding measured FRF, the FRF regenerated using the estimated modal parameters included in the frequency range of interest, i.e. of the kept modes (Lamontia, 1982; Duarte and Ewins, 1995). The best estimation of the residual terms may be obtained by using a least-square solution to the difference between the two FRF's, since this approach minimizes the effects of the noise normally present in measured data. Most of presently-available commercial parameter estimation software calculate the residual flexibility terms as part of the estimation phase of the mode shape coefficients.

The presence of residual flexibility attachment modes in the hybrid CMS method arises from the free-interface d.o.f. These modes account for the static flexibility of the deleted (higher frequency) normal modes. Their inclusion in the analysis improves the convergence of the analysis procedure.

2.2.3 Transformation to Component Generalized Coordinates

For each component, the Ritz transformation relating the physical coordinates $\mathbf{x}$ to the generalized or Ritz coordinates $\mathbf{p}$ is given by

$$\mathbf{x} = \mathbf{T} \mathbf{p} \quad (2.19)$$

where the transformation matrix $\mathbf{T}$ contains the complete Ritz vector set:

$$\mathbf{T} = \begin{bmatrix} \Phi_k & \Gamma_a & \Psi_r \\ 0 & 0 & \mathbf{I}_r \end{bmatrix} \quad (2.20)$$

Expansion of Eq. 2.19 gives

$$\begin{bmatrix} \mathbf{x}_f \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} \Phi_k & \Gamma_a & \Psi_s \\ 0 & 0 & \mathbf{I}_s \end{bmatrix} \begin{bmatrix} \mathbf{p}_k \\ \mathbf{p}_a \\ \mathbf{p}_s \end{bmatrix} \quad (2.21)$$

Using the principle of virtual work, it can be shown that the transformed stiffness and mass matrices are obtained from (Humar, 1990)

$$\bar{\mathbf{k}} = \mathbf{T}^T \mathbf{k} \mathbf{T} \quad (2.22a)$$

and

$$\bar{\mathbf{m}} = \mathbf{T}^T \mathbf{m} \mathbf{T} \quad (2.22b)$$

On substituting Eq. 2.20 in Eq. 2.22a and using the partitioned form of matrix $\mathbf{k}$, we obtain

$$\bar{\mathbf{k}} = \begin{bmatrix} \Phi_k^T \mathbf{k}_{ff} \Phi_k & \Phi_k^T \mathbf{k}_{ff} \Gamma_a & \Phi_k^T \mathbf{k}_{ff} \Psi_s + \Phi_k^T \mathbf{k}_{fs} \\ \Gamma_a^T \mathbf{k}_{ff} \Phi_k & \Gamma_a^T \mathbf{k}_{ff} \Gamma_a & \Gamma_a^T \mathbf{k}_{ff} \Psi_s + \Gamma_a^T \mathbf{k}_{fs} \\ \Psi_s^T \mathbf{k}_{ff} \Phi_k & \Psi_s^T \mathbf{k}_{ff} \Gamma_a & \Psi_s^T \mathbf{k}_{ff} \Psi_s + \Psi_s^T \mathbf{k}_{fs} \\ +\mathbf{k}_{sf} \Phi_k & +\mathbf{k}_{sf} \Gamma_a & +\mathbf{k}_{sf} \Psi_s + \mathbf{k}_{ss} \end{bmatrix} \quad (2.23a)$$

or

$$\bar{\mathbf{k}} = \begin{bmatrix} \bar{\mathbf{k}}_{kk} & \bar{\mathbf{k}}_{ka} & \bar{\mathbf{k}}_{ks} \\ \bar{\mathbf{k}}_{ak} & \bar{\mathbf{k}}_{aa} & \bar{\mathbf{k}}_{as} \\ \bar{\mathbf{k}}_{sk} & \bar{\mathbf{k}}_{sa} & \bar{\mathbf{k}}_{ss} \end{bmatrix} \quad (2.23b)$$

As shown below, some of the submatrices of $\bar{\mathbf{k}}$ are null matrices. The other sub-matrices may be expressed in terms of parameters obtained experimentally or derived from experimental data.

The derivation of a simplified expression for $\bar{\mathbf{k}}_{aa}$ follows. From Eqs. 2.23 and 2.18,

$$\bar{\mathbf{k}}_{aa} = \Gamma_a^T \mathbf{k}_{ff} \Gamma_a = \mathbf{f}_a^T \mathbf{G}_d^T \mathbf{k}_{ff} \mathbf{G}_d \mathbf{f}_a \quad (2.24)$$

Substituting for $\mathbf{G}_d$ from Eq. 2.16,

$$\begin{aligned}\bar{\mathbf{k}}_{aa} &= \mathbf{f}_a^T \left(\Phi_d \Lambda_{dd}^{-1} \Phi_d^T \right) \mathbf{k}_{ff} \left(\Phi_d \Lambda_{dd}^{-1} \Phi_d^T \right) \mathbf{f}_a \\ &= \mathbf{f}_a^T \Phi_d \Lambda_{dd}^{-1} \left(\Phi_d^T \mathbf{k}_{ff} \Phi_d \right) \Lambda_{dd}^{-1} \Phi_d^T \mathbf{f}_a\end{aligned}\quad (2.25)$$

From Eq. 2.4b,

$$\Lambda_{dd} = \Phi_d^T \mathbf{k}_{ff} \Phi_d \quad (2.26)$$

On substituting Eq. 2.26 in Eq. 2.25, and using Eq. 2.16,

$$\bar{\mathbf{k}}_{aa} = \mathbf{f}_a^T \mathbf{G}_d \mathbf{f}_a \quad (2.27)$$

The residual flexibility matrix $\mathbf{G}_d$ may be expanded as

$$\mathbf{G}_d = \begin{bmatrix} \mathbf{G}_{ii} & \mathbf{G}_{ia} \\ \mathbf{G}_{ai} & \mathbf{G}_{aa} \end{bmatrix} \quad (2.28a)$$

or

$$\mathbf{G}_d = [\mathbf{G}_{di} \quad \mathbf{G}_{da}] \quad (2.28b)$$

where $\mathbf{G}_{di}$ and $\mathbf{G}_{da}$ are the columns of the residual flexibility matrix involving the i and a d.o.f. subsets, respectively. Substitution of Eqs. 2.8 and 2.28a into Eq. 2.27 gives the expression sought

$$\bar{\mathbf{k}}_{aa} = \mathbf{G}_{aa} \quad (2.29)$$

On using the orthogonality property given by Eq. 2.4b along with Eqs. 2.6, 2.16, 2.18 and 2.29, Eq. 2.23a reduces to

$$\bar{\mathbf{k}} = \begin{bmatrix} \Lambda_{kk} & 0 & 0 \\ 0 & \mathbf{G}_{aa} & 0 \\ 0 & 0 & \mathbf{k}^* \end{bmatrix} \quad (2.30)$$

where

$$\mathbf{k}^* = \mathbf{k}_{ss} - \mathbf{k}_{sf} \mathbf{k}_{ff}^{-1} \mathbf{k}_{fs} \quad (2.31)$$

As mentioned earlier, $\mathbf{k}^*$ is referred to as the effective constrained coordinate stiffness.

The transformed mass matrix is obtained from the matrix product given by Eq. 2.22b

$$\bar{\mathbf{m}} = \begin{bmatrix} \Phi_k^T \mathbf{m}_{ff} \Phi_k & \Phi_k^T \mathbf{m}_{ff} \Gamma_a & \Phi_k^T \mathbf{m}_{ff} \Psi_s + \Phi_k^T \mathbf{m}_{fs} \\ \Gamma_a^T \mathbf{m}_{ff} \Phi_k & \Gamma_a^T \mathbf{m}_{ff} \Gamma_a & \Gamma_a^T \mathbf{m}_{ff} \Psi_s + \Gamma_a^T \mathbf{m}_{fs} \\ \Psi_s^T \mathbf{m}_{ff} \Phi_k & \Psi_s^T \mathbf{m}_{ff} \Gamma_a & \Psi_s^T \mathbf{m}_{ff} \Psi_s + \Psi_s^T \mathbf{m}_{fs} \\ + \mathbf{m}_{sf} \Phi_k & + \mathbf{m}_{sf} \Gamma_a & + \mathbf{m}_{sf} \Psi_s + \mathbf{m}_{ss} \end{bmatrix} \quad (2.32)$$

Using Eqs. 2.8, 2.18 and 2.28b, matrix Γ_a may be expressed as

$$\Gamma_a = \mathbf{G}_{da} \quad (2.33)$$

Using the orthogonality property of Eq. 2.4a along with Eqs. 2.16, 2.18 and 2.33,

Eq. 2.32 reduces to

$$\bar{\mathbf{m}} = \begin{bmatrix} \mathbf{I}_k & 0 & \Phi_k^T (\mathbf{m}_{ff} \Psi_s + \mathbf{m}_{fs}) \\ 0 & \mathbf{G}_{da}^T \mathbf{m}_{ff} \mathbf{G}_{da} & \mathbf{G}_{da}^T (\mathbf{m}_{ff} \Psi_s + \mathbf{m}_{fs}) \\ (\Psi_s^T \mathbf{m}_{ff} + \mathbf{m}_{sf}) \Phi_k & (\Psi_s^T \mathbf{m}_{ff} + \mathbf{m}_{sf}) \mathbf{G}_{da} & \Psi_s^T \mathbf{m}_{ff} \Psi_s + \Psi_s^T \mathbf{m}_{fs} \\ & & + \mathbf{m}_{sf} \Psi_s + \mathbf{m}_{ss} \end{bmatrix} \quad (2.34a)$$

or

$$\bar{\mathbf{m}} = \begin{bmatrix} \bar{\mathbf{m}}_{kk} & 0 & \bar{\mathbf{m}}_{ks} \\ 0 & \bar{\mathbf{m}}_{aa} & \bar{\mathbf{m}}_{as} \\ \bar{\mathbf{m}}_{sk} & \bar{\mathbf{m}}_{sa} & \bar{\mathbf{m}}_{ss} \end{bmatrix} \quad (2.34b)$$

It will be noted that if a diagonal mass matrix is used, Eq. 2.34a simplifies to

$$\bar{\mathbf{m}} = \begin{bmatrix} \mathbf{I}_k & 0 & \Phi_k^T \mathbf{m}_{ff} \Psi_s \\ 0 & \mathbf{G}_{da}^T \mathbf{m}_{ff} \mathbf{G}_{da} & \mathbf{G}_{da}^T \mathbf{m}_{ff} \Psi_s \\ \Psi_s^T \mathbf{m}_{ff} \Phi_k & \Psi_s^T \mathbf{m}_{ff} \mathbf{G}_{da} & \Psi_s^T \mathbf{m}_{ff} \Psi_s + \mathbf{m}_{ss} \end{bmatrix} \quad (2.35)$$

As shown subsequently in Sections 2.3 and 2.4, all the terms forming the component matrices $\bar{\mathbf{k}}$ and $\bar{\mathbf{m}}$ (Eqs. 2.30 and 2.34a) may be extracted from experimental data, except for the elements of the mass matrix. There is a strong possibility that it will be possible in the future to derive accurate lumped mass matrices from modal tests; however, for reasons given under Section 1.5, mass matrices for the present work are obtained analytically.

2.2.4 Transformation to System Generalized Coordinates

The transformed stiffness and mass matrices of the two individual components α and β are now assembled to obtain the global (or system) matrices $\mathbf{K}_p$ and $\mathbf{M}_p$ given by

$$\mathbf{K}_p = \begin{bmatrix} \bar{\mathbf{k}}^\alpha & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{k}}^\beta \end{bmatrix} \quad (2.36a)$$

and

$$\mathbf{M}_p = \begin{bmatrix} \bar{\mathbf{m}}^\alpha & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{m}}^\beta \end{bmatrix} \quad (2.36b)$$

where the superscript identifies the components. Subscript p indicates that these two matrices correspond to the global generalized coordinates $\mathbf{p}$ vector:

$$\mathbf{p} = [\mathbf{p}_k^\alpha \quad \mathbf{p}_a^\alpha \quad \mathbf{p}_r^\alpha \quad \mathbf{p}_c^\alpha \quad \mathbf{p}_k^\beta \quad \mathbf{p}_a^\beta \quad \mathbf{p}_r^\beta \quad \mathbf{p}_c^\beta]^T \quad (2.37)$$

where the vector $\mathbf{p}_r$ has been partitioned into its subsets, i.e. $\mathbf{p}_r = [\mathbf{p}_r \quad \mathbf{p}_c]^T$.

The condition of compatibility implies that the interface d.o.f. of the two components are dependent. Consequently, some of the generalized coordinates in $\mathbf{p}$ are dependent on each other. The following coordinate transformation is introduced to relate a set $\mathbf{q}$ of independent system generalized coordinates to the set $\mathbf{p}$

$$\mathbf{p} = \mathbf{S} \mathbf{q} \quad (2.38)$$

The transformation matrix $\mathbf{S}$ is derived from condition of physical displacement compatibility at the interface. This compatibility condition requires that

$$\mathbf{x}_b^\alpha = \mathbf{x}_b^\beta \quad (2.39)$$

By partitioning the d.o.f. b set into its subsets, Eq. 2.39 yields

$$\mathbf{x}_c^\alpha = \mathbf{x}_c^\beta \quad (2.40a)$$

$$\mathbf{x}_a^\alpha = \mathbf{x}_a^\beta \quad (2.40b)$$

Let $\mathbf{x}_a^\beta$ and $\mathbf{x}_c^\beta$ be the dependent coordinates. Correspondingly, the generalized coordinates $\mathbf{p}_a^\beta$ and $\mathbf{p}_c^\beta$ are selected as the dependent coordinates, and do not appear in $\mathbf{q}$ defined as

$$\mathbf{q} = [\mathbf{p}_k^\alpha \quad \mathbf{p}_a^\alpha \quad \mathbf{p}_r^\alpha \quad \mathbf{p}_c^\alpha \quad \mathbf{p}_k^\beta \quad \mathbf{p}_r^\beta]^T \quad (2.41)$$

Now, using the second row of Eq. 2.21 and splitting the s set into its subsets, we have

$$\begin{bmatrix} \mathbf{x}_r \\ \mathbf{x}_c \end{bmatrix} = \begin{bmatrix} \mathbf{I}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_c \end{bmatrix} \begin{bmatrix} \mathbf{p}_r \\ \mathbf{p}_c \end{bmatrix} \quad (2.42)$$

Therefore,

$$\mathbf{x}_c^\alpha = \mathbf{p}_c^\alpha \quad (2.43a)$$

$$\mathbf{x}_c^\beta = \mathbf{p}_c^\beta \quad (2.43b)$$

Combining Eqs. 2.40a and 2.43 yields

$$\mathbf{p}_c^\alpha = \mathbf{p}_c^\beta \quad (2.44)$$

From the first row of Eq. 2.21, we have for each component

$$\mathbf{x}_f = \Phi_k \mathbf{p}_k + \Gamma_a \mathbf{p}_a + \Psi_s \mathbf{p}_s \quad (2.45)$$

The third term in Eq. 2.45 may be partitioned as

$$\Psi_s \mathbf{p}_s = [\Psi_r \quad \Psi_c] \begin{bmatrix} \mathbf{p}_r \\ \mathbf{p}_c \end{bmatrix} \quad (2.46a)$$

$$\Psi_s \mathbf{p}_s = \Psi_r \mathbf{p}_r + \Psi_c \mathbf{p}_c \quad (2.46b)$$

Substitution of Eq. 2.46b in Eq. 2.45 gives

$$\mathbf{x}_f = \Phi_k \mathbf{p}_k + \Gamma_a \mathbf{p}_a + \Psi_r \mathbf{p}_r + \Psi_c \mathbf{p}_c \quad (2.47)$$

Since $\mathbf{x}_f = [\mathbf{x}_i \quad \mathbf{x}_a]^T$, one partition of Eq. 2.47 is given by

$$\mathbf{x}_a = \Phi_{ak} \mathbf{p}_k + \Gamma_{aa} \mathbf{p}_a + \Psi_{ar} \mathbf{p}_r + \Psi_{ac} \mathbf{p}_c \quad (2.48)$$

where the first of the two subscripts indicates that only the rows corresponding to the a set of d.o.f. are used.

From Eqs. 2.40b and 2.48,

$$\Phi_{ak}^{\alpha} p_k^{\alpha} + \Gamma_{aa}^{\alpha} p_a^{\alpha} + \Psi_{ar}^{\alpha} p_r^{\alpha} + \Psi_{ac}^{\alpha} p_c^{\alpha} = \Phi_{ak}^{\beta} p_k^{\beta} + \Gamma_{aa}^{\beta} p_a^{\beta} + \Psi_{ar}^{\beta} p_r^{\beta} + \Psi_{ac}^{\beta} p_c^{\beta} \quad (2.49)$$

The compatibility condition, given by Eqs. 2.44 and 2.49, may be expressed as a matrix constraint equation of the form

$$C p = 0 \quad (2.50)$$

where

$$C = \begin{bmatrix} 0 & 0 & 0 & I_c & 0 & 0 & 0 & -I_c \\ \Phi_{ak}^{\alpha} & \Gamma_{aa}^{\alpha} & \Psi_{ar}^{\alpha} & \Psi_{ac}^{\alpha} & -\Phi_{ak}^{\beta} & -\Gamma_{aa}^{\beta} & -\Psi_{ar}^{\beta} & -\Psi_{ac}^{\beta} \end{bmatrix} \quad (2.51)$$

and p is given by Eq. 2.37.

Solving for p_a^{β} , we have from Eq. 2.49

$$p_a^{\beta} = (\Gamma_{aa}^{\beta})^{-1} [\Phi_{ak}^{\alpha} p_k^{\alpha} + \Gamma_{aa}^{\alpha} p_a^{\alpha} + \Psi_{ar}^{\alpha} p_r^{\alpha} + \Psi_{ac}^{\alpha} p_c^{\alpha} - \Phi_{ak}^{\beta} p_k^{\beta} - \Psi_{ar}^{\beta} p_r^{\beta} - \Psi_{ac}^{\beta} p_c^{\beta}] \quad (2.52)$$

Using Eq. 2.44 and grouping the terms of the previous equation with respect to the generalized coordinates leads to

$$p_a^{\beta} = \Pi \Phi_{ak}^{\alpha} p_k^{\alpha} + \Pi \Gamma_{aa}^{\alpha} p_a^{\alpha} + \Pi \Psi_{ar}^{\alpha} p_r^{\alpha} + \Pi (\Psi_{ac}^{\alpha} - \Psi_{ac}^{\beta}) p_c^{\alpha} - \Pi \Phi_{ak}^{\beta} p_k^{\beta} - \Pi \Psi_{ar}^{\beta} p_r^{\beta} \quad (2.53)$$

where $\Pi = (\Gamma_{aa}^{\beta})^{-1}$.

Combining Eqs. 2.37, 2.38, 2.41, 2.44 and 2.53, the S matrix is derived as

$$S = \begin{bmatrix} I_k^{\alpha} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_a^{\alpha} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_r^{\alpha} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_c & 0 & 0 \\ 0 & 0 & 0 & 0 & I_k^{\beta} & 0 \\ \Pi \Phi_{ak}^{\alpha} & \Pi \Gamma_{aa}^{\alpha} & \Pi \Psi_{ar}^{\alpha} & \Pi (\Psi_{ac}^{\alpha} - \Psi_{ac}^{\beta}) & -\Pi \Phi_{ak}^{\beta} & -\Pi \Psi_{ar}^{\beta} \\ 0 & 0 & 0 & 0 & 0 & I_r^{\beta} \\ 0 & 0 & 0 & I_c & 0 & 0 \end{bmatrix} \quad (2.54)$$

The system stiffness and mass matrices corresponding to the fully independent generalized coordinates $\mathbf{q}$ are obtained from the transformation

$$\mathbf{K} = \mathbf{S}^T \mathbf{K}_p \mathbf{S} \quad (2.55a)$$

$$\mathbf{M} = \mathbf{S}^T \mathbf{M}_p \mathbf{S} \quad (2.55b)$$

where $\mathbf{K}_p$ and $\mathbf{M}_p$ are defined in Eqs 2.36.

A general CMS procedure, based on enforcement of compatibility at component interfaces using the Lagrange multiplier method, is shown to yield the system equation of motion (Craig and Chang, 1976; Craig, 1981)

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{S}^T \mathbf{C}^T \boldsymbol{\sigma} \quad (2.56)$$

where $\boldsymbol{\sigma}$ is a vector of Lagrange multipliers related to the interface constraints. As shown in Craig (1981), the R.H.S. of Eq. 2.56 vanishes as long as

$$\mathbf{C}\mathbf{S} = \mathbf{0} \quad (2.57)$$

As may be verified from Eqs. 2.51 and 2.54, the hybrid CMS method meets this condition. Consequently, the desired system frequencies and mode shapes are obtained by solving the eigenvalue problem corresponding to

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \quad (2.58)$$

The mode shapes of the system in physical coordinates are derived from the eigenvector solution of Eq. 2.58 by back-transformation using Eqs. 2.19 and 2.38.

2.3 Experimental Determination of Component Modal Parameters

In order to evaluate the transformed stiffness and mass matrices given by Eqs. 2.30 and 2.34a, we need to know the frequencies ω_k of the kept normal modes and the corresponding mode shapes ϕ_k , as well as the columns of the residual flexibility matrix associated with the free interface d.o.f., $\mathbf{G}_{da}$. These modal parameters are required for a configuration in which the

components are fully constrained (zero displacement) along supported d.o.f., including some interface d.o.f. The present section shows how these modal parameters may be derived from modal tests on components supported by supports of arbitrary stiffness (flexible supports) along the supported d.o.f.

Assuming the presence of viscous damping, the equation of motion of a structure (or component) may be expressed as

$$\mathbf{m} \ddot{\mathbf{x}} + \mathbf{c} \dot{\mathbf{x}} + \mathbf{k} \mathbf{x} = \mathbf{f} \quad (2.59)$$

where $\mathbf{c}$ is the damping matrix. For a sinusoidal force vector $\mathbf{f} = \mathbf{F} e^{i\omega t}$, the response vector is $\mathbf{x} = \mathbf{X} e^{i\omega t}$ and the equation of motion may be re-written as

$$(-\omega^2 \mathbf{m} + i\omega \mathbf{c} + \mathbf{k}) \mathbf{X} = \mathbf{F} \quad (2.60)$$

This equation may be expressed in terms of input and output as

$$\mathbf{H} \mathbf{F} = \mathbf{X} \quad (2.61)$$

where the complex matrix $\mathbf{H}$ is the frequency response function matrix, or receptance matrix, given by

$$\mathbf{H} = (-\omega^2 \mathbf{m} + i\omega \mathbf{c} + \mathbf{k})^{-1} \quad (2.62)$$

On partitioning along the free d.o.f. and supported d.o.f., Eq. 2.61 can be expressed by

$$\begin{bmatrix} \mathbf{H}_{ff} & \mathbf{H}_{fs} \\ \mathbf{H}_{sf} & \mathbf{H}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{F}_f \\ \mathbf{F}_s \end{bmatrix} = \begin{bmatrix} \mathbf{X}_f \\ \mathbf{X}_s \end{bmatrix} \quad (2.63)$$

where the first subscripts refer to the response d.o.f. and the second subscripts refer to the excitation d.o.f. The above equation is valid for any supports of arbitrary stiffness.

For the case of fully rigid supports, $\mathbf{X}_s = 0$. The constraint forces $\mathbf{F}_s$ are then obtained from the second row of Eq. 2.63 as

$$\mathbf{F}_s = -\mathbf{H}_{ss}^{-1} \mathbf{H}_{sf} \mathbf{F}_f \quad (2.64)$$

By substituting Eq. 2.64 in the first row of Eq. 2.63, we get

$$(\mathbf{H}_{ff} - \mathbf{H}_{fs} \mathbf{H}_{ss}^{-1} \mathbf{H}_{sf}) \mathbf{F}_f = \mathbf{X}_f \quad (2.65)$$

Therefore, the new FRF matrix for the structure on fully rigid supports can be calculated from

$$\mathbf{H}^* = \mathbf{H}_{ff} - \mathbf{H}_{fs} \mathbf{H}_{ss}^{-1} \mathbf{H}_{sf} \quad (2.66)$$

The estimation of $\mathbf{H}^*$ involves only simple matrix operations, including inversion of a matrix of order equal to the number of supported d.o.f. Although the above derivation was related to displacement response $\mathbf{x}$, the expression for $\mathbf{H}^*$ would still be the same in the standard case of acceleration being the measured response of the structure (the FRF matrix would then consist of inertances instead of receptances). The above derivation of $\mathbf{H}^*$ was originally presented in Salvini and Sestieri (1993) to predict the frequency response functions of a constrained test article from experimental FRF's of the same structure in a less constrained configuration.

As may be observed from Eq. 2.66, one of the requirements on the modal tests for deriving $\mathbf{H}^*$ is that the test article be excited along the supported d.o.f. and that response be measured also along these supported d.o.f. These measurements provide the $\mathbf{H}_{ss}$ matrix. The other requirements on modal tests depend on which elements of $\mathbf{H}^*$ are of interest. Theoretically, for a linear test article, for which the reciprocity principle applies, one can get the required frequencies ω_k and the shapes ϕ_k of the kept normal modes by obtaining either a single row or column of $\mathbf{H}^*$. However, one will probably want to obtain at least a few rows or columns of the FRF matrix $\mathbf{H}^*$. This is desirable for several practical considerations such as the verification of the reciprocity principle for the structure being tested, the estimation of closely-spaced or repeated frequencies, and a more uniform distribution of applied vibrating energy (when several forces are applied simultaneously). The choice as to whether the rows or columns of $\mathbf{H}^*$ are obtained depends on various considerations such as the characteristics of the test article and its supports (linear or not), robustness of the test article, availability of test equipment, amount of time available, and objectives of the test. This selection will define which elements of the FRF matrices $\mathbf{H}_{ff}$, $\mathbf{H}_{fs}$ and $\mathbf{H}_{sf}$ are to be measured.

The implications of the two approaches mentioned in the preceding paragraph are as follows.

- (1) In the first approach, selected rows of $\mathbf{H}^*$ (N_m in number) are obtained. This requires that excitation must be applied, in turn, at all free d.o.f. as well as at all supported d.o.f. Response is measured only at those free d.o.f. that correspond to the desired rows of $\mathbf{H}^*$, and also at all supported d.o.f. As seen from Fig. 2.2, this approach provides a partial $\mathbf{H}_{ff}$, a partial $\mathbf{H}_{fs}$, a full $\mathbf{H}_{sf}$ and a full $\mathbf{H}_{ss}$.

With this approach, excitation is normally applied by hitting sequentially the test article at all N_f free d.o.f. with an instrumented impact hammer. The approach is normally not practical when an exciter is used to induce vibrations in the structure, because of the effort and time required in moving the exciter(s) around.

If response measurements are made at all free interface d.o.f. (N_a d.o.f.), the procedure can provide the residual flexibility matrix $\mathbf{G}_{aa}$ as well as $\mathbf{G}_{ai}$. However, only some of the rows of $\mathbf{G}_{ia}$ are obtained. Other rows of $\mathbf{G}_{ia}$ can be obtained indirectly by using the reciprocity relationship.

The implication of applying excitation at all d.o.f. is that a moment excitation must be applied at all those 'a' d.o.f. which are rotational in nature. A direct application of moment excitation is not practical; however, alternative indirect procedures are available for obtaining moment induced responses.

It should also be noted that in order to obtain the full $\mathbf{G}_{ai}$ matrix, response measurements must be made at all 'a' d.o.f. including the rotational d.o.f. Direct measurement of rotational response is not very practical. However, as discussed later, measurement of translational responses combined with the use of finite difference approximation provides the rotational responses.

One problem with the present approach is that it is not applicable if the test article or the supports exhibit a behavior for which the linear assumption is not valid. This is so for

two reasons: (1) the impact hammer technique is not appropriate for nonlinear structures (SDRL 1992), and (2) the reciprocity principle is not valid for nonlinear structures.

- (2) In the second approach, selected columns of $\mathbf{H}^*$ (N_H in number) are obtained. This requires that excitation be applied only along those N_H free d.o.f. and also along all supported d.o.f. As seen from Fig. 2.3, this approach provides a partial $\mathbf{H}_{ff}$, a partial $\mathbf{H}_{sf}$, a full $\mathbf{H}_{fs}$ and a full $\mathbf{H}_{ss}$.

Again, to obtain the full $\mathbf{G}_{aa}$ matrix, excitation must be applied at all 'a' d.o.f. with attendant difficulty in applying moments. The procedure can directly provide the full $\mathbf{G}_{aa}$ matrix and the full $\mathbf{G}_{ia}$ matrix.

One advantage of the approach is that, by using appropriate excitation, it is possible to average out the nonlinear behavior of many nonlinear structures, and thus to generate test data that are more or less linear (SDRL 1992). Ewins (1984) mentions that true random excitation applies a linearisation procedure to the structure's behavior and is believed to provide an optimized linear model for a nonlinear structure. The burst random excitation used in the experimental phase of this work is a special case of true random excitation.

By applying standard curve fitting procedures, available in commercial modal analysis software, to the derived frequency response functions of $\mathbf{H}^*$, the required modal parameters of the modes of interest (kept modes) may be estimated. These modal parameters are the frequencies, mode shapes and residual flexibility terms of $\mathbf{G}_d$. The parameter estimation process also generates an estimate of damping values of the modes.

Most structures exhibit complex modes, rather than real normal modes, except in the rare case when the structures possess proportional damping. However, most of these non-proportionally damped systems exhibit modes that are almost real, the magnitudes of the complex parts being dependent on the system characteristics rather than the actual damping values (Imregun and Ewins, 1995). According to the same authors, it may be difficult to find systems with significant complex modes simply because of a higher damping unless close or

double modes exist. But, they also mention that several others factors may give rise to complex modes, such as analysis and/or measurement limitations (e.g. measurement noise, nonlinearities) and asymmetric system matrices (e.g. gyroscopic effects).

For lightly damped structures, the damping effects are usually neglected and a set of real modes is extracted from the measured data, using one of the several existing techniques for converting complex mode shapes into real ones (Ahmadian et al., 1995). In fact, as mentioned by Ibrahim and Sestieri (1995), normal mode identification of non-proportionally damped systems remains the basis for the majority of modal analysis applications such as modeling, structure modification, component mode synthesis, sensitivity analysis, modeling error localization and model updating, and many other applications. These two authors mention that such transformation (from complex to normal modes) becomes an acceptable approximation when the applications, in which the results are intended to be used, are relatively insensitive to the errors introduced, and that in modal analysis such applications are numerous. On the other hand, they warn that such approximation can lead to erroneous and misleading results when the measured mode shapes are used 1) to update analytical models, and 2) for structural integrity monitoring.

Based on the above discussion and since real normal modes are required in the hybrid CMS method presented herein, estimation of real normal modes are obtained from measured data in the present work. This is readily done since real normal mode estimation is normally provided as an option by commercially-available modal analysis software.

2.4 Experimental Determination of Component Static Parameters

For the hybrid CMS method proposed here, the constraint modes, Ψ_c , and the effective constrained coordinate stiffness, k^* , must also be obtained from experimental data. The present section demonstrates how these parameters may be derived from static tests on the

components supported by supports of arbitrary stiffness. These supports may have stiffnesses that are different from those of the supports used for modal testing, as long as the two sets of supported d.o.f. are the same.

The partitioned stiffness matrix of a component that has been modified by the addition of supports of arbitrary stiffness may be expressed as

$$\tilde{\mathbf{k}} = \begin{bmatrix} \mathbf{k}_{ff} & \mathbf{k}_{fs} \\ \mathbf{k}_{sf} & \mathbf{k}_{ss} + \mathbf{k}_s \end{bmatrix} \quad (2.67)$$

where $\mathbf{k}_s$ represents the stiffness matrix of the supports.

Now, apply a set of N_s arbitrary, but different, static displacement patterns, Δ_s , at the supports. Let $\mathbf{F}_s$ be the corresponding external forces applied along the supported d.o.f. and Δ_f the displacements produced at the free d.o.f. Since the forces are applied only along the supported d.o.f., we have

$$\begin{bmatrix} \mathbf{k}_{ff} & \mathbf{k}_{fs} \\ \mathbf{k}_{sf} & \mathbf{k}_{ss} + \mathbf{k}_s \end{bmatrix} \begin{bmatrix} \Delta_f \\ \Delta_s \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{F}_s \end{bmatrix} \quad (2.68)$$

Expanding the first row gives

$$\mathbf{k}_{ff} \Delta_f + \mathbf{k}_{fs} \Delta_s = \mathbf{0} \quad (2.69)$$

Equation 2.69 yields

$$\Delta_f \Delta_s^{-1} = -\mathbf{k}_{ff}^{-1} \mathbf{k}_{fs} = \Psi_s \quad (2.70)$$

The derivation of Ψ_s thus requires only the measurement of displacements of the supported structure under a series of static loads.

The effective constrained coordinate stiffness $\mathbf{k}^*$ is obtained from the same set of static tests as used for deriving Ψ_s . Equation 2.68 can be expressed in the form

$$\begin{bmatrix} \mathbf{k}_{ff} & \mathbf{k}_{fs} \\ \mathbf{k}_{sf} & \mathbf{k}_{ss} \end{bmatrix} \begin{bmatrix} \Delta_f \\ \Delta_s \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{F}_s \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{R}_s \end{bmatrix} \quad (2.71)$$

where $\mathbf{R}_s = -\mathbf{k}_s \Delta_s$ represents support reactions.

Expanding the second row of Eq. 2.71 gives

$$\mathbf{k}_{sf}\Delta_f + \mathbf{k}_{ss}\Delta_s = \mathbf{F}_s + \mathbf{R}_s \quad (2.72)$$

On substituting for Δ_f from Eq. 2.69 into Eq. 2.72, we get

$$\left(\mathbf{k}_{ss} - \mathbf{k}_{sf}\mathbf{k}_{ff}^{-1}\mathbf{k}_{fs}\right)\Delta_s = \mathbf{F}_s + \mathbf{R}_s \quad (2.73)$$

Comparison of Eqs. 2.31 and 2.73 yields

$$\mathbf{k}^* = (\mathbf{F}_s + \mathbf{R}_s)\Delta_s^{-1} \quad (2.74)$$

During the static tests on the supported structure, whenever a load is applied along a supported d.o.f., the reactions induced at the supports are measured in addition to the displacements and the applied load. Equation 2.74 then provides the value for $\mathbf{k}^*$.

For the application of Eqs. 2.70 and 2.74, Δ_s must be nonsingular in order to have an inverse. A necessary condition for the process to work is that the N_s static support displacement patterns must be linearly independent. This is easily achieved.

Table 2.1 Sets and subsets of component d.o.f.

Name	Description	Size	D.o.f. nos for component β
a	Subset of free interface d.o.f.	N_a	27
c	Subset of fixed interface d.o.f.	N_c	28
i	Subset of free interior d.o.f.	N_i	29 to 47
r	Subset of fixed interior d.o.f.	N_r	48
b	Set of interface d.o.f. (made up of a and c)	N_b	27 and 28
s	Set of fixed d.o.f. (made up of c and r)	N_s	28 and 48
f	Set of free d.o.f. (made up of a and i)	N_f	27, 29 to 47

Fig. 2.1 Cantilever beam system and its components

Fig. 2.2 Excitation at all free and supported d.o.f. Responses measured at selected free d.o.f. and all supported d.o.f.

Fig. 2.3 Excitation at selected free d.o.f. and all supported d.o.f. Responses measured at all free and supported d.o.f.

3

COMPUTER SIMULATION AND SENSITIVITY STUDY

3.1 Introduction

In order to assess the feasibility of the experimental hybrid CMS approach for which the theoretical background was presented in Chapter 2, a series of computer simulation and sensitivity studies have been carried out. Representative results of computer simulation studies which demonstrate the feasibility of different steps of the proposed approach are presented in this chapter.

3.2 Error Measures

In order to assess the sensitivity of the results of hybrid CMS analysis to the possible measurement errors, simulation studies are carried out on systems for which reasonably accurate finite element models can be developed. Error measures are defined and are based on the difference between system modal parameters obtained from CMS and finite element analysis.

An error measure for the frequencies is defined as

$$e_{\omega} = \frac{\omega_a - \omega_r}{\omega_r} \times 100 \quad (3.1)$$

where ω_a is the frequency approximation obtained from the CMS, and ω_r is the reference "exact" frequency obtained from finite element analysis.

The error measure for the mode shapes is defined as

$$e_{\phi} = \left[\frac{(\phi_a - \phi_r)^T (\phi_a - \phi_r)}{\phi_r^T \phi_r} \right]^{1/2} \times 100 \quad (3.2)$$

where ϕ_a is the mode shape approximation and ϕ_r is the reference mode shape.

3.3 Simulation Results

Simulation results, including results of sensitivity study with respect to errors in some input component parameters, are now presented for three different systems, namely a cantilever beam, a free-free beam and a free-free truss.. These results are consistent and confirm the feasibility of the hybrid CMS method.

3.3.1 Cantilever Beam System

The first system to which the hybrid CMS method is applied is the cantilever beam shown in Figure 2.1. The cross-section and modulus of elasticity E are constant for the whole length of the system. Each nodal point has two d.o.f. (in-plane rotation and vertical translation). A consistent mass matrix formulation is used.

The beam system has a total of 48 d.o.f. It consists of components α and β having respectively 28 and 22 d.o.f. Rotational displacement (d.o.f. 27) represents the free d.o.f. at the interface, as is likely to be the case in a real test situation. Vertical displacement (d.o.f. 28) is constrained and represents the fixed d.o.f. at the interface. Additional constraints are added at d.o.f. 14 and 48 of components α and β , respectively.

The frequencies and mode shapes of the system derived from Eq. 2.58 are compared with their exact or reference values. The latter are obtained by analyzing the FE model of the beam

system with the full 48 d.o.f. The errors in frequencies and mode shapes are obtained from Eqs. 3.1 and 3.2 respectively.

The system modal parameters are first obtained for four cases of different number of retained normal modes, with no error in the input parameters. The errors in the estimated system frequencies and mode shapes are shown in Figs. 3.1a and 3.1b. The two numbers identifying each curve indicate the number of retained normal modes for each component (no of modes for component α + no of modes for component β). As expected, the errors in both the frequencies and mode shapes increase with the order of the system mode, as well as with the reduction in the number of kept modes. It is observed from Fig. 3.1a that the frequency errors are negligible when 11+10 are kept. When 8+7 modes, about a third of the available number of normal modes, are kept, the frequency errors are still below 0.1%. Even when only 5 (3+2) normal modes are kept, the frequency errors are very small, about 1% or less for the first 5 modes, and less than 3% for the first 7 modes. Figure 3.1b shows that the trends for the mode shape errors are the same as for the frequencies. The errors are somewhat higher than in the frequencies but still quite acceptable.

The results of a study to determine the relative sensitivity of the system modes to errors in the component parameters is now discussed. The investigated parameters are: the modal frequencies of the kept modes (denoted by ω), the constraint modes (denoted by ψ), the columns of the residual flexibility matrix associated with the interface d.o.f. (denoted by G_{da}), and the elements of the kept normal mode shapes associated with the interface rotational d.o.f. (denoted by ϕ^θ). Some of the results of the study for the cantilever beam system are shown in Figs. 3.2a and 3.2b. The input errors applied systematically to a single parameter of both components are: +15% to all modal frequencies, +50% to all constraint modes, +200% to G_{da} , -98% to G_{da} (denoted by w/o G_{da}), and +100% to ϕ^θ . The negative error is applied to G_{da} in order to simulate the elimination of this parameter as input to the hybrid CMS method, by leaving only 2% of its original value. All input errors are applied in the case of a total number of kept normal modes of 15 (8+7). It is apparent from Figs. 3.1 that with these numbers of

retained normal modes, mode truncation causes negligible errors in the system parameters, especially in the frequencies.

The error levels applied to the various parameters were selected by considering the expected level of errors in deriving these parameters either from the modal or static tests. Modal frequencies are the dynamic parameters that are easiest to obtain accurately (SDRL, 1992). The process of obtaining $\mathbf{H}^*$ through Eq. 2.66 is likely to increase the errors in the estimated frequencies, but not to a great extent if the experimental FRF's in test configuration are of good quality (with the appropriate characteristics as discussed later in this chapter). The constraint modes, and $\mathbf{k}^*$, are static parameters which depend on measurements of static displacements and forces. These static measurements should be fairly precise when appropriate sensors are used, especially since the test articles are quite flexible. Consequently, the estimated ψ , and $\mathbf{k}^*$ should be fairly accurate (compared to some other parameters). In fact, the 50% error level on ψ , could be extremely conservative.

Normally, the accuracy in the mode shapes (relative displacement between translational d.o.f.) is good provided appropriate testing procedures (such as calibration) are followed (SDRL, 1992). In fact, next to the modal frequencies, mode shapes are the parameters estimated with most precision. There will obviously be an increase of errors in the mode shapes due to the transformation involved in deriving $\mathbf{H}^*$. However, as in the case of frequencies, it is possible to minimize this error. On the other hand, the error in the rotational terms of the kept normal mode shapes might be fairly high due to the transformation involved in obtaining rotational FRF's from translational FRF's. Sensitivity studies related to the interface parameters are carried out to assess the impact of aforementioned error.

Even in a standard modal test, the residual flexibility terms are the parameters that are most difficult to obtain and therefore likely to have the least precision (Smith and Peng, 1994). The transformation to obtain $\mathbf{H}^*$ will certainly increase the errors in G_{da} . The errors in the rotational terms could be expected to be worse than for the translational terms due to the additional transformation to get the rotational FRF's. This is so since the rotational FRF's are

obtained from the finite difference method which involves an approximation. However, according to Martinez et al. (1984), the error in residual flexibility terms of rotational d.o.f. is smaller than for the translational terms; this could be due to the fact that the former residual flexibility terms are larger than the latter ones (Duarte and Ewins, 1995). The high (+200%) error level applied to the G_{da} terms is representative of the uncertainty involved in estimating them. In fact, considering this high expected error level, the errors in the system modes are also evaluated for the case in which the residual flexibility terms are completely ignored.

The errors in system frequencies and mode shapes shown in Figs. 3.2 are presented in the form of absolute values, so that a log scale could be used. In reality, some of these values are positive while some others are negative. It should also be mentioned that errors shown in the figures are for systematic errors in the component parameters. In a real test situation, the input errors are more likely to be random than systematic. Some example simulation with random input errors on a similar system have been carried out. The results not presented here show that the system errors are significantly smaller in the case of random input errors.

The results of Fig. 3.2a show that, for the cantilever beam, the system modal frequencies are sensitive to errors in the component description in the following order of decreasing importance: modal frequencies, constraint modes, interface rotational d.o.f. and residual flexibility. The fact that the two component parameters (ϕ^0 and G_{da}) likely to be the least accurate also happen to be the parameters to which the system frequencies are least sensitive, is certainly encouraging. It is interesting to note that even when all relevant residual flexibility terms are neglected, the errors in the system frequencies are no more than 3%.

The results presented in Fig. 3.2b show that, for the cantilever beam, the system mode shapes are sensitive to errors in the component description in the following order of decreasing importance: constraint modes, interface rotational d.o.f., modal frequencies and residual flexibility. The encouraging information here is that the system mode shapes are also fairly insensitive to errors in the residual flexibility terms. The fact that the system mode shapes are most sensitive to the two parameters that directly involve displacements at the interface d.o.f.

(ψ , and ϕ^0) is to be expected. This does indicate, however, that if one is interested in obtaining good estimate of system mode shapes (as will probably be the case for validating FE models), special attention should be paid to obtaining as good an estimate as possible of the rotational elements of the component mode shapes.

3.3.2 Free-Free Beam System

The second system investigated in the simulation with the hybrid CMS method is the free-free beam shown in Fig. 3.3. The interface has two d.o.f. as for the cantilever beam, while the interior points only have one d.o.f. (vertical translation). Reduction of the stiffness matrix in order to eliminate the rotational d.o.f. at the interior points is done through static condensation (Humar, 1990). Reduction of the mass matrix with respect to the same d.o.f. is done through Guyan's reduction (Guyan, 1965).

Each component has a total of 28 elements and 30 d.o.f. The beam system consists of 58 d.o.f. The vertical constraints are located at the two ends and at mid-span of each component, namely at d.o.f. 1, 15 and 30 for component α , and at d.o.f. 2, 16 and 30 for component β . The interface thus consists of one fixed d.o.f. and one free d.o.f.

The errors in system frequencies and mode shapes are presented in Figs. 3.4a and 3.4b, for four different cases of number of kept normal modes when Eq. 2.34a is used in obtaining the transformed mass matrix. The results correspond to null errors in the parameters describing the components. Two further cases are presented where an approximation is used in deriving the transformed mass matrix. As for the cantilever beam system, the errors in both the frequencies and mode shapes increase with the order of the system mode, as well as with a reduction in the number of kept modes. The cases for which Eq. 2.34a is applied directly without any approximation are first discussed. From Fig. 3.4a, it is seen that the errors in system frequencies are negligible when 13+13 modes are kept. Even when only 9+9 modes are kept, the errors are below 0.1%. When only 6 (3+3) modes are kept, the results are still very good,

the frequency errors being less than 3% for the first 8 modes and 7.4% for the 9th mode. As shown in Fig. 3.4b, the level of errors in the mode shapes follows the same trends as do the frequencies.

As stated earlier, it is difficult to obtain a good estimate of the residual flexibilities. In view of this, the flexibility terms in matrix G_{ia} are often ignored, simplifying the transformation given by Eq. 2.34a. In order to verify the effect of neglecting this submatrix in applying this equation, the two cases with the lowest number of kept normal modes were run with a zero value for G_{ia} . These two cases were selected because they are the ones for which the additional errors due to the approximation in obtaining the mass matrix are likely to be more critical. The corresponding two curves are labeled 5+5 (G_{aa}) and 3+3 (G_{aa}). From Fig. 3.4a, it is seen that although there is a definite error increase in the estimated system frequency, the additional error is still acceptable for practical applications. In fact, for the case when 10 (5+5) normal modes are kept, the frequency error is no more than 1% for the first 6 modes, about 3% for the 7th mode, and less than 2% for the other two modes. For the extreme case when only 6 (3+3) normal modes are used, the frequency error is still about 2% or less for the first 6 modes, and less than 5% for modes 7 and 8. As shown in Fig. 3.4b, although the errors on the system mode shapes have increased as expected, they are still at an acceptable level for the lower modes. It is interesting to note that Martinez et al. (1984) came to the same conclusion about the acceptability of the additional error level caused by neglecting G_{ia} , in the case of a free-interface CMS method applied to a beam.

Figures 3.5a and 3.5b present the errors in system frequencies and modes shapes caused by errors in component parameters. These figures and the subsequent ones for the truss are obtained by applying Eq. 2.34a without any approximation. These results are obtained with a total of 18 (9+9) kept normal modes. As shown in Fig. 3.5a, the system modal frequencies are sensitive to errors in the component description in the same order of decreasing importance as for the cantilever beam system, namely: modal frequencies, constraint modes, interface rotational d.o.f. of normal modes and residual flexibility. Figure 3.5b shows that, for the free-free

beam, the system mode shapes are sensitive to errors in the component parameters in the following order of decreasing importance: constraint modes, modal frequencies, interface rotational d.o.f. and residual flexibility.

3.3.3 Free-Free Truss System

The third system used in the simulation of the hybrid CMS method is the planar truss shown in Fig. 3.6. This free-free truss has 110 equal bays and is 67 meter long. The truss is made of aluminum and consists of two components. All members have identical cross-sectional and mass properties, which are also uniform along their length. One component consists of 60 equal bays and has a total of 183 joints. The other component consists of 50 equal bays and has a total of 153 joints. Each joint has two d.o.f., vertical and in-plane horizontal translations. The relative lengths of the members are indicated in the figure.

The interface consists of 6 translational d.o.f, of which the upper vertical one is constrained and the other 5 d.o.f. are free. These fixed interface d.o.f. are d.o.f. 366 for component 1 and d.o.f. 6 for component 2. Additional equidistant vertical d.o.f. are constrained for each of the components: d.o.f. 6, 126 and 246 for component 1, and d.o.f. 156 and 306 for component 2. A horizontal constrained d.o.f. is included in each component to prevent rigid body motion in that direction; these are d.o.f. 1 for component 1 and d.o.f. 301 for component 2. During modal testing, the components would be supported along the constrained d.o.f. by supports of arbitrary stiffness.

Errors in system frequencies and mode shapes caused by the truncation in component modes are shown in Figs. 3.7a and 3.7b. The accuracy of system frequencies is almost as good as for the beam systems, with an error no more than about 2% when only 12 (6+6) normal modes are kept. For the case when only 6 (3+3) normal modes are kept, the frequency error is still no more than 3% for the first 7 modes, and no more than 7% for the 8th and 9th modes.

The results for the system mode shapes, presented in Fig. 3.7b, show that although the errors are higher than for the system frequencies, they are still quite low.

The errors in system frequencies and mode shapes caused by errors in component parameters are presented in Figs. 3.8a and 3.8b. These results are obtained with a total of 30 (15+15) kept normal modes. The sensitivity of system modal frequencies to errors in the component description is in the same order of decreasing importance as for the beams: modal frequencies, constraint modes and residual flexibility (Fig. 3.8a). It is to be noted that the error introduced in the component constraint modes has been reduced from 50% to 15%. Even this latter error value is probably conservative (i.e. too high). Figure 3.8a shows that the sensitivity to the errors in constraint modes has increased for the lower modes and decreased for the higher modes, compared to the beam simulations. The sensitivity to the errors in residual flexibility terms has increased to a maximum of about 10%, which is still quite acceptable considering the level of input error. As shown in Fig. 3.8b, the sensitivity of the system mode shapes to errors in the component parameters is in the following order of decreasing importance: constraint modes, modal frequencies and residual flexibility terms.

3.4 Derivation of $\mathbf{H}^*$ Through Modal Testing

A simulation study is carried out to verify the derivation of the FRF's (denoted as $\mathbf{H}^*$) of a component supported on rigid supports from experimental FRF's (denoted as $\mathbf{H}$) of the same component supported along the same d.o.f. on flexible supports. The development of the relation between $\mathbf{H}^*$ and $\mathbf{H}$, and some discussion concerning its application, are presented in Chapter 2. One objective of this study is to confirm that the mass of the supports does not perturb the derived FRF's by "mass loading" of the test article.

One of the component used to verify the theory is shown in its test and constrained (rigid support) configurations in Figs. 3.9a and 3.9b. It consists of a uniform beam attached to three

equidistant flexible vertical supports, located at the two extremities and at mid-span (at nodes 1, 19 and 37). The elements of $\mathbf{H}^*$ for two interior point d.o.f. (at nodes 7 and 25) are sought.

A FE model of the beam and the supports in the test configuration is developed using the MSC/Pal 2 finite element package (MSC. 1990) which runs on PC computers. The beam is modeled by 36 beam elements of equal length, with two d.o.f. at each node (vertical and in-plane rotation) for a total of 74 d.o.f. A model reduction is performed to reduce the number of active d.o.f. to 38 (two d.o.f. every second node). The vertical supports are modeled with spring elements having a stiffness $k_s = 147$ kN/m. The mass of each support, $m_s = 5.44$ kg, is included in the model as a lumped mass attached to the beam at its connections to the support. The parameter k_s represents the stiffness of the flexible supports, and m_s is the mass of each support transferred to the attachment points. The total mass of the three supports is about the same as the mass of the beam, so any potential mass loading effects will appear in the estimated $\mathbf{H}^*$.

In order to create reference FRF's to which the $\mathbf{H}^*$ matrix may be compared, a FE model of the beam in its constrained configuration is developed. This model is the same as the one in the test configuration, except that the vertical displacements at the attachment nodes 1, 19 and 37 are set to zero. This results in 35 active d.o.f. after model reduction.

A normal mode solution is performed, using the FE software, with beam models of the two configurations. On the basis of estimated modal frequencies, the following parameters are selected for the subsequent frequency response analysis (FRA): frequency range of 0-60 Hz, frequency resolution of 0.02 Hz. In order to avoid numerical instabilities, constant damping ratios of 1% is added to all modes within the frequency bandwidth of interest.

Using the FRA capability of Pal 2, all the required FRF's of the beam in its test configuration, namely all the elements of matrices $\mathbf{H}_{ff}$, $\mathbf{H}_{fs}$, $\mathbf{H}_{sf}$ and $\mathbf{H}_{ss}$ in Eq. 2.66 are computed. Obviously, some of these FRF's could be obtained without direct computation by using the reciprocity principle. In a similar manner, all the FRF's of the reference $\mathbf{H}^R$ matrix are com-

puted for the beam in its constrained configuration. Equation 2.66 is now used to derive $\mathbf{H}^*$ which is then compared with $\mathbf{H}^R$.

The receptance FRF's $H_{7,7}$, $H_{7,25}$ and $H_{25,25}$ of the beam in test configuration are shown in Figs. 3.10a, 3.11a and 3.12a respectively. The corresponding elements of $\mathbf{H}^*$ derived from Eq. 2.66 are shown by solid lines in Figs. 3.10b, 3.11b and 3.12b respectively. For comparison, the corresponding reference FRF's of $\mathbf{H}^R$ are also shown (dotted lines) in Figs. 3.10b, 3.11b and 3.12b. The two sets of curves are indistinguishable. It is apparent that for each of the selected interior point d.o.f., a perfect match is observed between the corresponding elements of $\mathbf{H}^*$ and the reference $\mathbf{H}^R$. This confirms the validity of Eq. 2.66 for deriving $\mathbf{H}^*$, and also shows that no mass loading of the test article is caused by the mass of the supports.

Fig. 3.1a Frequency errors for a cantilever beam system, for varying number of retained normal modes

Fig. 3.1b Mode shape errors for a cantilever beam system, for varying number of retained normal modes

Fig. 3.2a Frequency errors for a cantilever beam system, for various component parameter errors

Fig. 3.2b Mode shape errors for a cantilever beam system, for various component parameter errors

Fig. 3.3 Free-free beam system used in simulation of hybrid CMS method

Fig. 3.4a Frequency errors for a free-free beam system, for varying number of retained normal modes

Fig. 3.4b Mode shape errors for a free-free beam system, for varying number of retained normal modes

Fig. 3.5a Frequency errors for a free-free beam system, for various component parameter errors

Fig. 3.5b Mode shape errors for a free-free beam system, for various component parameter errors

Component 1: 60 equal bays

Component 2: 50 equal bays

Fig. 3.6 Free-free truss components

Fig. 3.7a Frequency errors for the free-free truss system, for varying number of retained normal modes

Fig. 3.7b Mode shape errors for the free-free truss system, for varying number of retained normal modes

Fig. 3.8a Frequency errors for the free-free truss system, for various component parameter errors

Fig. 3.8b Mode shape errors for the free-free truss system, for various component parameter errors

Fig. 3.9a Beam component in test configuration, attached to vertical flexible supports

Fig. 3.9b Beam component in constrained configuration attached to vertical rigid supports

Fig. 3.10a Receptance $H_{7,7}$ for beam in test configuration

Fig. 3.10b Receptance $H_{7,7}^*$ for beam (continuous line)

Reference $H_{7,7}^R$ (dotted line)

Fig. 3.11a Receptance $H_{7,25}$ for beam in test configuration

Fig. 3.11b Receptance $H_{7,25}^*$ for beam (continuous line)
Reference $H_{7,25}^R$ (dotted line)

Fig. 3.12a Receptance $H_{25,25}$ for beam in test configuration

Fig. 3.12b Receptance $H_{25,25}^*$ for beam (continuous line)

Reference $H_{25,25}^R$ (dotted line)

4 TEST EQUIPMENT AND PROCEDURES

4.1 Introduction

The results of computer simulation presented in Chapter 3 show that the proposed experimental hybrid CMS method is quite feasible and is not very sensitive to the expected measurement errors. The next phase in validating the approach is to verify it by testing system components of increasing complexity. The results of the test-based verifications of the new method are presented in Chapters 5 and 6. The present chapter concentrates on the common elements of the test program. The first part of the chapter presents an overview of the flexible supports that represent a critical component in all of the tests carried out under this project. This is followed by a description of the key elements of the flexible supports.

Subsequent parts of the chapter describe the other test equipment used in the modal and static tests as well as the procedures for the performance of these tests. Aspects of these procedures that are more specific to a particular test are described in subsequent chapters as appropriate. A description of modal testing of a flexible support prototype and presentation of key test results conclude the present chapter. These results show that the behavior of the flexible support is linear, a property that is critical for the subsequent testing of the various system components.

4.2 Flexible Supports

The flexible supports represent key hardware items required for the modal and static tests of the components carried out to determine the parameters required in the experimental hybrid CMS approach. In the context in which the approach is developed, these supports serve only to create a sufficient number of restrained d.o.f. so as to prevent large flexible structures, such as spacecraft appendages, from deforming too much and breaking under their own weight. Consequently, as discussed previously, a flexible support should be designed to provide vertical restraint to the test article at a single location, while allowing freedom of motion in the horizontal plane and about three orthogonal rotation axes. Since an excitation must be applied and measured along any supported or restrained d.o.f. in order to derive the $\mathbf{H}^*$ matrix, limiting the restrained d.o.f. to only vertical translation significantly reduces the complexity of modal testing (and static tests to a certain extent). Restrained rotation at a flexible support would imply the need of either 1) applying and measuring a pure dynamic torque with respect to that rotational d.o.f., or 2) deriving precisely rotational FRF's (including rotation/moment) from translational measurements using a technique such as finite difference. Both of these approaches can be very challenging and time-consuming, especially for complex structures.

The number of flexible supports required depends mainly on the geometry and flexibility of the component being tested. For the components of a free-free beam system, two or three supports will be required depending of their flexibility; while for 2-D components such as spacecraft appendages, a minimum of four and often a larger number of supports would be needed.

4.2.1 Units of the Flexible Support

Each flexible support is made up of four different types of units as shown in Fig. 4.1. A flexible support consists of a connection, an upper section, two upper section fixtures and two pedestals, as labeled in the photograph. It is to be noted that in the figure, which shows a static

test configuration, a load cell is incorporated between the upper section and the connection. In a modal test, these two units are directly bolted together without the intervening load cell. The purpose and a description of each of these units are presented in the following paragraphs.

Connection

This is the key unit of the flexible support. The role of the connection unit is to restrain the test article vertically at the attachment point, while allowing free motion (subject to practical limitations) along the other five d.o.f. The connection is made of various elements described in Section 4.2.2.

Upper Section

The upper section has the dual role of 1) supporting the connection to which the test article is attached, and 2) providing (through bending) most of the flexibility of the flexible support. The upper section consists of an aluminum beam of rectangular cross-section clamped on each side to an upper section fixture.

The requirement of achieving a different support flexibility depending on the type of test (static versus modal) and on the test article led to the adoption of an interchangeable upper section. The selected design permitted a change in upper section flexibility by changing the cross-section of the beam and/or its free span (distance between the pedestals). This characteristics of easy changeability of the upper section turned out to be extremely useful during the test phase of the project. The main criteria in selecting a support flexibility is the need to generate enough vertical motion of the test article along the supported d.o.f. under the external forces, so that this motion may be measured with adequate precision and without unacceptable noise level.

Pedestals

The two pedestals serve as columns that hold the upper section and its fixtures at a suitable height so as to allow the installation of the exciters, the connection and the test article underneath it. The pedestals provide negligible contribution to the overall vertical flexibility of the flexible support.

A pedestal is made up of a series of modules bolted on top of each others. The pedestals are the only units of the flexible supports that are not designed and built as part of this project. These units were available in the laboratory of the Canadian Space Agency (CSA), where the present tests were carried out. As many extension frame modules (Fig. 4.2) are stacked and bolted on top of each others as necessary to achieve the required height. A tapered frame module (Fig. 4.3) is bolted on top of these extension frames. Both the extension and the tapered frames are made of welded steel square tubes (McTavish, 1995). Each extension frame weighs about 40 kg and each tapered frame weighs about 34 kg.

The CSA laboratory also has several base rail elements made of aluminum alloy rectangular members that allow direct attachment of the pedestals to the rails of the seismic floor in the laboratory facility. These base rail elements were used in the testing of beam component.

Upper Section Fixtures

These two units serve to connect the upper section to the pedestals. Each fixture is made of an assembly of four elements (excluding the nuts and bolts):

- The top two elements act as jaws between which the upper section is clamped by bolting the jaws together (Fig. 4.1). Both elements are made of steel bars. This design allows easy assembly/disassembly of the upper section facilitating the modification of the characteristics of the flexible support.

- The third element has a U-shape and is also made of steel. This element is bolted underneath the lower jaw and serves to attach the threaded rod that constitutes the fourth element.
- The vertical threaded rod is the last element of the upper section fixture. The rod is made of steel and has a diameter of 19.8 mm (25/32"). It passes through the hole in the top plate of the tapered frame of the pedestal unit and through the lower plate of the U-shape element. Four nuts are used to allow some adjustment of the height of the assembly, and consequently of the upper section.

4.2.2 Design of Connection Unit

The connection unit is designed to provide restraint in only the vertical direction with free motion along the other five d.o.f. This requirement must be satisfied for accurate determination of the component modal and static parameters in a rigid support configuration. Considerable care was therefore taken in designing and manufacturing this unit.

Design Requirements

In order to minimize the length of the design cycle, a list of requirements was defined before starting to develop some concepts for the connection. Since a review of these requirements helps to appreciate the various points which had to be considered in the design of this device, a list of the requirements with their explanations follows.

R.1 *The connection unit should allow free motion along all d.o.f., except vertical translation (subject to practical limitation).*

This requirement is self-evident since it follows directly from the role of the connection unit and of the flexible support itself.

- R.2** *The connection unit should be rigid in the vertical direction, in the frequency range of interest. In the event that the connection is not rigid, it should exhibit a linear behavior.*

Vertical rigidity of the connection unit in the frequency range of interest is the ideal situation since it reduces the number of modes, and thus the dynamic behavior, of the component in its test configuration. In the likelihood that this ideal behavior is not realistic, the linearity requirement must be satisfied since it forms a basic assumption in modal analysis. In the limit, if the linearity requirement is too difficult to meet due to the presence of nonlinear hardware such as bearings, small dynamic nonlinear behavior which can be linearized with random excitation would still be acceptable.

- R.3** *The design of the connection unit should prevent the structure in its test configuration from having any non-elastic mode with a frequency which is either within an order of magnitude of the lowest frequency of the test article in its fixed-support configuration or within the frequency range of interest.*

The purpose of this requirement is to prevent the creation of any non-elastic mode which could interact with the elastic modes of the test article. As an example, such an interaction could be created by a simple connection unit made of a strut having spherical bearings at both ends and for which the top end would be fixed laterally. Any lateral motion of the test article caused for example by a small misalignment of the vertical excitation force would also generate a pendulum motion which could not be acceptable if its natural frequency does not respect the present requirement.

A non-elastic mode which does not meet the present requirement may still be acceptable if it is shown that it does not affect the response of the test article along the d.o.f. of interest. This would be the case if for all FRF's related to $\mathbf{H}^*$, the non-elastic mode appears as a negligible peak and if its presence does not affect the estimation of the shape coefficients of the flexible modes.

- R.4** *The attachment of the connection unit to the test article should allow the accelerometer and the force sensor to be placed on each side of the test article, as well as the alignment of the two with the centerline of the connection.*

Such a set-up arrangement allows both the force and the response to be measured along the restrained d.o.f. and thus reduces the measurement error occurring from misalignments. Such error could occur if the force sensor and the accelerometer were beside each others, and if one or both of them were not aligned with the centerline of the connection unit.

- R.5** *The contact area between the connection unit and the test article should be small enough for the support to be idealized as a point-restraint on the structure.*

This requirement relates to the fact that in the FE model of the component required for the derivation of a mass matrix, the structure is assumed to be fixed along specific d.o.f. at some nodes which are represented by points having zero dimensions.

- R.6** *The connection unit should be strong enough to carry the maximum load to be supported.*

This requirement is particularly important for the selection of any bearings included in the design of the connection.

- R.7** *The connection unit should be designed so that it may be readily attached to, and disconnected from, the upper section unit, and so that the attachment is rigid.*

The attachment and disconnection of the connection unit to the upper section must be readily possible since several changes of upper sections might be required during the modal and static testing of a test article.

- R.8** *The connection unit should be designed so that it may be readily attached to, and disconnected from, the test article at either an interior point or at the edge of the component, and so that such attachment is rigid.*

The attachment and disconnection of the connection unit to the test article must be readily possible since several of these operations are likely to be required during the testing of a component.

Description of the Connection Unit

A number of conceptual designs were carried out for the unit. They led to a simple version of the unit presented in this section. Figures 4.4 and 4.5 show two views of the connection unit taken from two different angles. For the purpose of description, the connection unit may be divided into the following four parts: Upper section attachment element - USAE, linear bearing element - LBE, radial thrust bearing element - RTBE, test article attachment element - TAAE.

1. Upper section attachment element (USAЕ)

This aluminum piece is shaped like an inverted U and serves to attach the connection unit to the upper section (Fig. 4.6). The attachment is made by means of two bolts passing through 8 mm (5/16") holes drilled along the centerline of the attachment element and along the cross-section of the upper section. One of these bolts and its nut are seen at the top of Fig. 4.5. A smaller threaded hole (6.35 mm or .25" diameter) is drilled between the two bolt holes in order to suspend the connection from a load cell during static test, as shown in Fig. 4.1. Four additional holes are drilled, two on each side of the element, to allow two parallel shafts to pass through and be glued to the element. These four holes were positioned as precisely as possible in order to ensure parallelism of the two shafts so that no friction would be created because of misalignment.

2. Linear bearing element (LBE)

This is the part of the connection unit which allows free translation of the test article in the horizontal plane, that is, perpendicular to the restrained d.o.f. The element is comprised of three

precision linear bearings sliding on hardened steel shafts each having a diameter of 9.5 mm (.375"). The linear bearings are fixed inside an aluminum block which is machined to reduce its weight. The upper two holes inside the block are drilled precisely so that the two linear bearings that they accommodate are parallel to each other (Fig. 4.5). These two linear bearings are perpendicular to the lower linear bearing and its shaft which allow translation in the other direction (Fig. 4.4).

3. Radial thrust bearing element (RTBE)

This element is built around two machined pieces of aluminum. The top one has a narrow U shape (Fig. 4.6). A hole is drilled at the top of the two vertical sides of the U to allow the insertion of the lower hardened shaft which is held in place with set-screws. The lower part of this piece is extruded to allow the positioning of a thrust bearing above and below, as shown in Figs. 4.4 and 4.6. The other machined part is a block with a large horizontal hole and with a threaded vertical cylinder at the top of it. The cylinder of the machined block, which goes through a hole drilled in the bottom of the U shaped piece, serves as a support for a bolt that attaches the two thrust bearings to the U piece. The bottom horizontal hole contains a radial bearing positioned vertically. A 6.35 mm (0.25") machined steel shaft passes through the radial bearing.

4. Test article attachment element (TAAE)

This element is made of two identical legs that attach to the test article (Fig. 4.5). The shaft of the radial bearing passes through the holes drilled near the top of the legs and is held in place by tightening set-screws. Attachment to the test article is accomplished by passing a bolt through the test article and through an 8 mm (5/16") threaded hole drilled in the bottom portion of each leg (Fig. 4.6). The two holes in the test articles must be separated by 38 mm (center-to-center). As shown in Fig. 4.5, this separation is sufficient to install an accelerometer between

the legs right underneath the connection. Also, the force sensor may be fixed underneath the test article and aligned with the accelerometer.

A prototype of the connection unit with the above design was built and tested to verify its dynamic behavior. The first series of tests involved the use of lumped masses as test article in order to eliminate any modes caused by the deformation of the test article, thus concentrating the study on the dynamics of the flexible support, especially the connection unit. A discussion of this series of tests with results and conclusions is presented in Section 4.5. The first set of tests was followed by tests on a six-feet long cantilever beam held at its free end by a flexible support. For the latter test configuration, the frequencies obtained from test-based FRF's related to $\mathbf{H}^*$ were compared with their analytical counterparts. The results from this series are not presented here, since they are similar to those obtained from tests on a component of the fixed-fixed beam system described in Chapter 5. Following the successful completion of the first two series of tests, three other connection units similar to the prototype were manufactured.

It may be noted that another connection prototype with a somewhat simpler design was also manufactured and subjected to the tests described in the previous paragraph. That prototype had a self-aligning bearing instead of the combination of thrust and radial bearings. The connection unit with radial/thrust bearings gave somewhat better performance especially with heavier test articles, and was therefore selected for use in the main test program.

Verification of the Requirements

The selected design of the connection unit meets all of the requirements stated earlier. It provides restraint in only the vertical direction. The combination of linear, thrust and radial bearings allows free motion along the remaining five d.o.f. The connection unit exhibited linear behavior, in part because during testing the load on the unit did not reverse, and any slacks in the bearings did not give rise to a nonlinearity. The connection unit also did not give rise to any non-elastic extraneous mode shapes affecting the flexible modes of the components, for the

tests described in Chapters 5 and 6. The dimensions of the unit were small enough in comparison to the test articles that the support provided by the unit could be idealized as a point support. The design permitted the force sensor and the accelerometer at the support location to be aligned with the centerline of the support. Finally, the unit was strong enough to support the loads applied on it during testing, and was easily attached or disconnected from the test article as well as the rest of the flexible support.

4.3 Modal Testing

A modal test may be divided into two successive phases: data acquisition and analysis. The data acquisition phase consists of exciting dynamically the test article and of measuring the excitation force(s) as well as the structural response at various locations on the test article. Frequently, this phase includes as a final step the estimation of the frequency response functions and the coherence functions. The analysis phase consists of processing the test data, usually through some form of curve fitting, in order to extract the estimated dynamic or modal parameters of the test articles. The most commonly sought modal parameters are the natural frequencies, damping values and mode shapes.

4.3.1 Test Equipment

This section first describes the equipment (hardware and software) typically encountered in a modal test and then presents the specific equipment used for the present research.

Typical Test Set-up

The main elements typically encountered in a conventional modal test set-up are shown in Fig. 4.7. All of them, except the analysis software, relate to the data acquisition phase of the process. A brief description of these items follows:

- **Exciter:** This device is used to physically vibrate the structure. The use of an exciter allows the application of a variety of dynamic disturbances to the test article. Present technology permits the simultaneous application of two or more exciters for certain types of excitation.
- **Transducers:** These usually comprise force sensors for measuring the force applied to the structure and accelerometers for measuring the structural responses at various locations. Both devices are manufactured in a wide variety of size, weight, sensitivity and other characteristics.
- **Generator:** This device creates the electric signal which drives the exciter. Various types of excitation (including sine, sine sweep, random and burst random) may be produced. Each type of excitation has its own advantages and disadvantages (SDRL, 1992). Although frequently occurring as a separate device, it can be incorporated into the analyser as shown in the figure.
- **Power Amplifier:** This component amplifies the electric signal created by the generator before it is fed to the exciter.
- **Analyser:** The main function of this device is to collect and measure the various signals developed by the transducers in order to assess the magnitude of the force and the responses, as well as the phase between the two. In the case of multiple input excitation, advanced signal processing algorithms separate the responses produced by individual exciters. As mentioned above, the analyser often provides the excitation signal. It also triggers the data collection, and filters and zooms the transducer signals. Finally, it produces various time-domain and frequency-domain functions (including FRF's, coherence, auto-spectrum, and cross-spectrum functions).
- **Analysis Software:** This item is an essential part of the equipment. Its purpose is to estimate the modal parameters of the test article by curve fitting appropriate mathematical equations to the acquired test-based FRF's (or other frequency or time-domain functions). It may be located inside the analyser or on a separate computer. Several curve fitting

techniques have been and continue to be developed (e.g. SDRL, 1994). Some procedures (such as the single-degree-of-freedom methods) are very simple and can be implemented on low cost analyzers while some others are very complex and require very sophisticated analyzers or computers. No one technique is best for all tests. In some cases, the simple procedures will give as good results as the more complex methods; while in other cases (e.g. for closely-spaced modes), only the most advanced procedures will be able to produce satisfactory estimation of modal parameters.

Test Equipment Used for the Present Research

The entire testing phase of this research was performed at the Modal Analysis Laboratory of the Canadian Space Agency, in Saint-Hubert, Quebec, which also supplied the equipment described below.

Modal System

The modal test facility used in this project runs under the control of the LMS International (Leuven Measurement & Systems) CADA-X program on an HP 715/50 Unix workstation. An HP 3565 measurement hardware is added as a front-end unit to the workstation. These items constitute an integrated modal system having the following relevant characteristics:

- 32 measurement channels with sampling rates up to 12.8 kHz per channel;
- 2 source modules for driving independently 2 exciters with random type excitation;
- Data acquisition and averaging software for MIMO (multiple-input-multiple-output) testing, limited to two inputs because of the source modules. The software also generates the time and frequency domain functions (including auto-power spectra, cross-power spectra, coherence functions and frequency response functions)
- Modal analysis software for parameter estimation. The software includes a variety of curve fitting algorithms for SDOF (single-degree-of-freedom), MDOF (multiple-degree-of-

freedom), SIMO (single-input-multiple-output) and MIMO; facility for animation of the estimated mode shapes, as well as validation tools such as the MAC (Modal Assurance Criteria) technique;

- Ability to create various types of Universal Files which are industry standard well-documented ASCII files for transfer of data or modal parameters to other software programs or systems;
- Command mode allowing some mathematical manipulation of time and frequency domain functions

Excitation and its Measurement

Two MODAL 50A exciters from MB Dynamics provided the dynamic excitation to the test articles. These exciters are especially designed for modal testing in order to eliminate problems that may be encountered when using environmental shakers. The main characteristics of this exciter model are: Minimum set-up time and simplicity in set-up procedure, portability with a weight of only 25 kg for a unit providing 222 N (50 lb.) dynamic force, 25.4 mm (1 in) peak-to-peak stroke, and broad usable frequency range 1-4000 Hz. These features make the exciters completely suited for the modal tests of this project. The exciters come with their own solid-state power amplifiers (model SS250VCF).

The dynamic force from each exciter was measured with a quartz force sensors manufactured by PCB Piezotronics. Two sensors in each of two different models (208A and 208B), for a total of four, were available. In operation, each force sensor is fixed to the test article and is connected through a coaxial cable to a signal conditioner which powers, with a direct current, the microelectronic amplifier incorporated within the force sensor. The sensor has a small mass of 24 g and overall dimensions of about 16 x 23 mm. As much as possible, the 2 force sensors of model 208B were used. These sensors have a compression/tension range of 44.5 N (10 lb.) with an excellent nominal sensitivity of 112.4 mV/N (500 mV/lb). When the

model 208B sensors were not available, for example when they were fixed under flexible supports, the excitation force was measured with one of the two sensors of model 208A. This model has a compression/tension range of 445 N (100 lb) with a corresponding sensitivity 10 times lower. For the force levels encountered during testing for this project, visual observation of the FRF's obtained from model 208A sensors did not show any significant noise increase due to their lower sensitivity.

It is important that at a point of excitation, the force is applied along only one direction without any other secondary excitation forces. This is achieved by connecting the exciter to the force sensor attached to the test article through a stinger. Stingers are designed so that they are axially stiff and flexurally weak in order to minimize the transmission of any bending moments to the sensor. For the modal tests in the project, the stingers consisted of stainless steel rods with a diameter of 1.59 mm (1/16 in). The length of the stingers varied between 75 to 125 mm.

Response Measurement

The responses of the test articles were measured simultaneously with Series 336C Flexcel accelerometers manufactured by PCB Piezotronics. The 32-channel modal system allowed the simultaneous measurement of up to 30 accelerations since two of the 32 channels were used up by the two force sensors. The Flexcel accelerometers are structured with a ceramic crystal sensing element and have a built-in microelectronic charge-to-voltage converter which conditions the signals. The accelerometers were connected to a 16-channel programmable signal conditioner. The two signal conditioners were then connected to the front-end of the modal system. The Flexcel accelerometers have very good characteristics for the present application: low mass of 4.5 g, small dimensions of 16 x 13 mm, excellent sensitivity 1000 mV/g and an acceptable measurement range of +/- 5 g. Their usable frequency ranges are: 1-2000 Hz for a maximum of 5% sensitivity deviation, and 4-2000 Hz for a maximum of 5% error on the phase shift. The 4-2000 Hz frequency range was sufficient for the present application.

Horizontal Restraints

One role of the flexible supports is to allow translation of the test article in the horizontal plane. As discussed earlier, this is done through three precision linear bearings sliding on hardened shafts resulting in very smooth motion with negligible friction. The allowable sliding distance of the bearings is 25 mm in one direction and 19 mm in the other perpendicular direction. These distances are much larger than what is required for their motion during testing, since any displacement of the bearings due to deformation of the test article is very small. Longer shafts were incorporated in the design in order to simplify the set-up of the test article under the flexible supports.

A test article without any fixed end which is supported by several flexible supports tends naturally to be in an unstable equilibrium. If left to itself, it will move sideways until the bearing(s) of at least one support will reach the end of the shaft in one direction. Ideally, the test set-up should be arranged so that the bearings of each flexible support should be at or near the center of their respective shaft in order to 1) ensure as much as possible the assumption of one restrained and five free d.o.f. at a *single* location, and 2) simplify the motion within the connections. Since a modal or a static test consists of several runs, it is also important to minimize any difference in the position of the test article with respect to its supports between the runs in order to keep the set-up stationary and the test data as consistent as possible.

In view of the foregoing considerations, it became apparent that some type of horizontal restraints must be used. The main concern in designing these horizontal restraints was to eliminate or at least minimize any potential disturbance to the dynamics characteristics of the test article. Several actions were taken accordingly. First of all, the structural part of the horizontal restraint was made of the same flexible 1.59 mm (1/16 in) stainless steel rod as used in the stinger. Each rod was fixed at its end away from the test article by a simple clamping mechanism to an available supporting frame. In order to minimize the bending and rotational stiffness of the horizontal restraints, they were made as long as possible without inducing

buckling in the rod under the axial force experienced by it during the tests. Their length was about 250 mm.

In order to damp out any potential axial vibration of the horizontal restraint, a damper was incorporated between the restraining rod and the test article. The damper, selected following several trials, is shown in Fig. 4.8. It consists of a pad of polyurethane soft foam sandwiched between two thin sheets of aluminum, and attached to them with double sided tape. The pad has an area of about 50 x 50 mm and a thickness of 25 mm. This yellow foam of relatively high compressibility is sometime used as packing material. The aluminum sheet away from the test article is fixed to the rod of the horizontal restraint with a set-screw and a nut on each side of the sheet; the set-crew is welded to the rod, as partly shown in the figure. In order to create a point connection between the horizontal restraint and the test article, a small bolt is fixed to the other aluminum sheet with its head on the side of the foam; the end of the bolt is machined in order to present a smooth curved surface touching the test article.

It was observed that at the end of a beam test article supported by three supports, the horizontal restraint experienced a large thrust compressing the pad completely. For these locations, the damper was slightly modified by replacing the yellow foam with a black soft foam which is stiffer and has a larger thickness (30 mm).

As discussed later, results of several sensitivity tests showed that the horizontal restraints had little impact on the FRF's related to H^* or the natural frequencies of the test article.

4.3.2 Test Procedures

The procedures described in this section are common to all of the modal tests presented in Chapters 5 and 6. These procedures lead to the determination of translational FRF's of the component in its test configuration incorporating flexible supports. The steps related to post-processing of the data in order to obtain the translational and rotational FRF's in the rigid

support configuration vary slightly depending of the components and are thus discussed in subsequent chapters.

The selection of the test procedures was guided by the following principles.

- 1) In order to allow for reciprocity check as defined below at the H^* level, FRF's with respect to at least two interior d.o.f. were required. Therefore, exciters had to be applied at two such locations. In addition, as discussion in Section 2.3, forces also had to be applied along all of supported d.o.f. This implied that a vertical excitation must be applied underneath each flexible support, since only this d.o.f. was restrained by the support as described in Section 4.2.1. In order to minimize the number of runs required for generating the data corresponding to all excitation locations, or driving points, each test run was performed with two exciters acting simultaneously. In general, during each run, one exciter was placed at a support and another one at an interior d.o.f.
- 2) In order to minimize the time required for the data acquisition phase, measurement of all translational responses with respect to a driving point had to be done in a single run. Since 2 channels of the modal system were required for force measurement, a total of 30 accelerometer locations could be specified for each component. This implied that less than 30 interior and interface d.o.f. could be defined on the structure, since a few additional accelerometers were required for deriving rotational FRF's at the component interface. Considering that for all test articles in this project, measurements were taken only along a single direction (vertical out-of-plane), this requirement was not a limiting factor.

The following general procedures were implemented for each test article.

(1) Selection of support locations

Support locations were selected to be reasonably well distributed along or throughout the test article. One of the supports for the beams, and two for the grids, always had to be located at

the interface of the component in order to implement the hybrid CMS approach combining fixed and free interface d.o.f.

(2) Selection of frequency bandwidth for the FRF's

The selection of this parameter is directly related to the selection of the number of normal modes that must be obtained. In order to have a meaningful assessment of the impact of truncation of modes, we need to start with a reasonably large number of retained modes, and then compare the values of such selection to those obtained from a smaller number of retained modes (e.g. 3 or 4). Baseband FRF measurements (in which the lower limit of frequency bandwidth is 0 Hz) were always obtained, as opposed to zoomed FRF which concentrates on frequency lines around the desired frequencies in order to increase the frequency resolution. This decision was taken in order to minimize the number of runs required to capture all of the desired modes, considering that the modal frequencies were fairly well distributed in the frequency range of interest. In the case when the upper limit of the bandwidth was too high, as for the components of the two beam systems, a second narrower baseband frequency bandwidth was selected in order to obtain enough frequency resolution for estimating precisely the first few modes of these lightly damped test articles. The front-end hardware of the modal system accepts only power of 2 as maximum frequency. Consequently, for each test article, the selected upper limit of frequency bandwidth was the minimum power of 2 for which the bandwidth still contained at least seven modes.

(3) Selection of response locations

The next step in preparation for testing a structure was to select the location of the accelerometers. As discussed earlier, a response measurement had to be made along each supported d.o.f. and some extra accelerometers were needed to obtain rotational FRF's at the component interface. Accelerometer locations for interior point d.o.f. were selected for providing a reasonably uniform distribution along the whole component. Only those positions that corresponded

to a node in the related FE model were considered as candidate accelerometer locations so as to allow subsequent integration of test data with modeling information in the CMS software.

(4) Selection of excitation locations

As discussed earlier, a driving point had to be located along each supported d.o.f. In addition to these, excitation had to be provided at additional locations close to the component interface in order to derive rotational FRF's (H^r response/moment). Selection of excitation locations at the interior point d.o.f. was based on the FE model of the component. Ideally, one of the two interior d.o.f. driving point, referred to as the main interior driving point, was at a location that exhibited significant displacement for all the modes of interest. The criteria for selecting the other driving point was that 1) it had to show reasonable displacements for most of the modes of interest and 2) its location had to be fairly far away from the main interior driving point in order to allow for a meaningful reciprocity check. When possible, all modal parameters used for CMS were estimated from the FRF's with respect to the main interior driving point only, with the other driving point data being used only for a reciprocity check.

(5) Installation of test article

For the components of the fixed-fixed beam system, the beam was securely attached to a fixed-end arrangement and cantilevered from it. A flexible support was then positioned above the free end of the beam so that the linear bearings were at the center of their shaft. It was then attached to the test article.

For the components of the free-free beam and grid systems, the flexible supports were positioned with respect to each others so that they were above the points of the test article they were specified to support. The test article was then attached to the supports which were subsequently slightly repositioned to have their linear bearings at the center of the shafts.

(6) *Installation of horizontal restraints*

For the components of the free-free beam and grid systems, when the test article was properly installed, three horizontal restraints were installed at acceptable locations and the connection units were slightly inclined so that the test article was resting against the horizontal restraints. At times, this process required several trials.

(7) *Installation of accelerometers on test article*

All accelerometers, including the additional ones for rotation, were fixed to the specified response locations before starting any test. The installation consisted in attaching a piece of Kapton tape to the base of the accelerometer and then gluing the tape to the test article. This installation technique minimized the risk of damaging the surface of the accelerometer while being effective for the frequency range in which the structures were vibrated.

(8) *Attachment of exciters to the test article*

Two force sensors were fixed at two of the specified excitation locations by first screwing a small disk to the transducer base and then gluing the disk to the test article. This installation technique prevents damage to the sensors from successive installation actions. In order to eliminate measurement errors due to misalignment, the force sensors were always fixed to the bottom face of the test articles, right underneath an accelerometer. Each exciter was then moved underneath the force sensor and oriented so that the stinger joining the exciter to the force sensor would be straight and axially in line with the force sensor.

(9) *Data acquisition*

Vibration of the test article was done through burst random excitation forcing functions applied independently by both exciters. With burst random excitation, the force is applied for only a certain initial percentage of the time window during which data sampling is performed. In the present case, the burst signal was applied for the first 50 % of the time. For the second half of the sampling period, no excitation was applied, and the measured force and acceleration

signals quickly died down to almost zero before the end of the acquisition window. Burst random excitation has significant advantages over other types of excitation, since it minimizes leakage error that may occur during signal processing and removes signal distortion that may result from the application of weighting functions. A total of 8192 samples were taken within each time window leading to 4096 frequency lines for the FRF's. For each run, a total of 40 averages was taken. This was found to be sufficient to average out the uncorrelated noise in most FRF's, except for those corresponding to locations exhibiting small motion.

(10) Data verification

At the end of a data acquisition run, the following frequency-domain functions were available for verifying the quality of data: frequency response functions, coherence functions, force auto-power spectra. Using this available data set, the following verifications were performed.

a) Reciprocity check

Reciprocity is one of the fundamental assumptions of modal analysis. Reciprocity means that the FRF relating response at d.o.f. i with the force at d.o.f. j (H_{ij}) is equal to the FRF relating the responses at d.o.f. j with the force at d.o.f. i (H_{ji}). Full reciprocity implies perfectly linear structure. In practice, one could expect and tolerate small differences in comparing the two FRF's H_{ij} and H_{ji} . The differences could be caused by different factors such as slight non linearity in the test articles or boundary conditions, error in transducer calibration, misalignment of accelerometer and force sensor at one of the d.o.f. After each run, reciprocity check was performed for verifying the acquired data and to detect any unacceptable discrepancy.

b) Drop in force auto-power spectrum

The force auto-power spectrum provides the value of excitation measured by a force sensor as a function of frequency. For open-loop excitation such as burst random, one may expect some drops in the force auto-power spectrum especially at the structure resonance, at which

frequency the test article is much easier to deform. However, a significant drop in only one of the two power spectra, combined with a corresponding lack of reciprocity of the FRF's at the same frequency, can be an indication of problems in the test set-up, such as resonance of one of the stingers. Consequently, the force auto-power spectra were carefully scrutinized in combination with reciprocity check, to make sure that no suspicious drops occurred in the power spectra.

c) Quality of frequency response functions

Each measured FRF was promptly examined to detect any obvious anomalies in its estimation. The main factor considered here was the noise level in the data set. One would expect some amount of noise at the antiresonances of a FRF or through some parts of the frequency bandwidth of an FRF corresponding to a response location at or near a flexible support. On the other hand, a very noisy FRF throughout the whole frequency bandwidth could be an indication of cable or connection problems, or the lack of rigidity at the attachment of an accelerometer to the test article.

d) Coherence function

A coherence function is a frequency-domain function measuring how well a response relates to the input. In the case of dual input, as in the present situation, the multiple-coherence function defines how much of the response is related to both forces. The coherence can be shown to be always between 0 and 1. In a perfect test situation, the coherence function corresponding to every response d.o.f. should be unity. Coherence functions that are at or near unity provide reassurance that the measurements were well taken. Two of the most common reasons that cause coherence to be less than unity are 1) non linear structural response and 2) uncorrelated noise in the measurement of the force or response (McConnell, 1995). Drop at an antiresonance could be expected, while drops at a resonance should be looked at more seriously in order to define and eliminate its cause as much as possible. Multiple-coherence

functions were observed together with the corresponding FRF's in an attempt to detect any measurement problems.

When suspicious measurement problems showed up during the four types of data verification just described, possible causes were examined and appropriate corrective actions were taken. This was followed by a repetition of the data acquisition run and of data verification (steps 9 and 10).

Steps 8 to 10 of the data acquisition procedures were repeated until measurements with respect to all required excitation locations were obtained. In the case of an uneven number of force locations, measurement runs were always conducted with two exciters leading to redundant data which could subsequently serve for a cross-check of the derived FRF's related to $\mathbf{H}^*$.

4.4 Static Tests

As discussed previously in Section 2.4, the static parameters of a component (Ψ , and $\mathbf{k}^*$) may be derived from static tests on the component attached to flexible supports. The test requirement is to subject the component to N_s linearly independent static displacement shapes along the supported d.o.f. The applied force producing the displaced shapes and the reaction force at the supports must be measured. Measurement must also be made of translational displacements at all supported d.o.f. and interior point d.o.f. Rotation at the interface are derived from translational displacements.

4.4.1 Test Equipment

This section describes the equipment used in the static tests performed on the six test articles. Except for a dial indicator, all equipment required for measuring displacements and forces was purchased since none of it was available in the CSA laboratories in Saint-Hubert. The mechanism for producing the displaced shapes was specially designed and manufactured for these tests.

Displacement Mechanism

The device used for applying the displacements is shown in Fig. 4.9. The element that produces the displacement is a turnbuckle the upper threaded rod of which is fixed to a rigid plate. The lower threaded rod is fixed in rotation but is free to translate along the axis of the turnbuckle. The latter boundary condition is achieved by bolting the threaded rod to an extremity of a spacer; the other extremity of the spacer is bolted to a T-nut sliding inside a T-slot. A hook is attached through a hole to the bottom end of the lower threaded rod.

Force Measurement

The single applied force and the reaction force at the supports were measured with S-beam tension load cells (model 6001) manufactured by Sensortronics. Four sensors were procured. The rated capacity was 222.4 N (50 lb.) for two of them and 444.8 N (100 lb.) for the other two. For the present application, only tensile forces needed to be measured since the weight of the test article transferred to a support was always larger than any compressive force produced by the applied displacement.

Each load cell was connected to a digital panel indicator (Tracker 130) which converted the output voltage of the load cell directly into force (lb.). The panel indicator also generated the excitation voltage (10 VDC) required to operate the load cell under its full-bridge resistance.

Displacement Measurement

Displacement indicators procured for the tests belonged to the Logic family of electronic digital indicators manufactured by Chicago Dial (CDI). The particular model used, BG3110, has a measurement range of 0-25.4 mm (0-1") with a resolution of 0.0013 mm (0.00005"). Three indicators were procured. Two of them were standard with an internal spring, while in the third one the internal spring was removed, and a spindle weight added instead to assure a constant pressure or reaction force throughout the full travel of the gauge.

A dial indicator was also available from the CSA model shop. This device, model 2923-10 from Mitutoyo, had a resolution of 0.0025 mm (0.0001") with a limited displacement range of 0-1.27 mm (0-.050"). Even with its range limitation, this indicator could be used for some test articles to measure displacement under a support where the motion was very small.

4.4.2 Test Procedures

The procedures described in this section are fairly general and cover elements that are common to all tests. Some variations in these procedures were needed for individual tests depending on the number of supports involved. The description of general procedures is preceded by a brief discussion of several issues relevant to these procedures.

As mentioned at the beginning of Section 4.4, one of the static test requirements is the measurement of deformed shapes, and support reactions produced by the imposition of selected displacement patterns along the supported d.o.f. The number of independent displacement patterns to be imposed is equal to the number of supported d.o.f. (N_s). In the present work, each of the N_s displaced shapes is produced by disconnecting in turn one of the flexible support from the test article and by applying a controlled displacement (which corresponds to an applied force) along that supported d.o.f., while the structure is still attached to the ($N_s - 1$) other supports.

In order to obtain precise estimates of all displacements and reaction forces with respect to the same reference applied force, several incremental displacements are applied. At the end of each increment, the applied force as well as the displacements and/or the reaction forces are measured. For each of the response parameters (displacement and reaction force), a linear regression is used to fit the line that best describes the relationship between the applied force and the response parameter. The slope of the line gives the least square evaluation of the parameter per unit applied force. This approach has the advantage of minimizing the effect of measurement errors. Also, the fact that the estimate of each displacement is an average value

with respect to exactly the same reference (unit force) is useful in the determination of interface rotations since this is done through finite difference approximation which is sensitive to errors in each of the three input displacements, especially for small rotation.

As mentioned earlier, two of the digital indicators each has an internal spring which forces the indicator probe to regain its initial deployed position once the contact with the surface they touch is removed. The indicator spring applies a reaction force at the point of contact which is proportional to the displacement from the resting position. Each of these two indicators was calibrated by measuring its reaction force as a function of its reading, i.e. distance traveled. Figure 4.10 shows the data obtained for one indicator. The equation of a line that best fits the data is also shown in the figure. For a maximum displacement of 25 mm, this reaction force is seen to be about 2.5 N. The calibration of the other digital indicator with an internal spring produced similar results. Depending on the magnitude of the applied external force with which this reaction force is compared, the latter may be negligible or significant. For the present application, the effect of the indicator reaction force was most significant when the displacement measurement was at the location of the applied force (or displacement to be more precise), and the error on the estimated displacement per unit applied force was of the order of a few percent.

For the components of the free-free beam and grid systems, the two digital indicators described above were used only for measuring displacements under the applied force and under flexible supports. It was thus possible to correct the force reading of the load cells to account for the reaction force of the indicator since both were acting along the same d.o.f.

The digital indicator without an internal spring applies a constant reaction force that is independent of the displacement of the probe. As explained earlier, the static test results involve only the measurement of the slope of various displacement versus applied force curves. A constant reaction force causes only a translation of the curve without affecting its slope. Consequently, this indicator could be used, without adding any error, for all interior d.o.f. It is to be noted that this indicator can work only if it is positioned above the surface it contacts. It

could not therefore be used at a supported location and the two digital indicators with springs were used for those locations.

The load cells used in the tests are calibrated with high precision for an excitation voltage of 10 volt dc. A different voltage leads to some calibration error which is practically constant for the full measurement range. This error was found to be no more than 1%. This calibration offset was small enough so that it could have been neglected. However, it was corrected through a re-calibration using a simple test procedure in which masses of known magnitudes were suspended from the load cell, and the difference between the load cell reading and the known weight was ascertained.

The procedures used in the static tests involve the following steps.

(1) Installation of load cells and loading mechanism

Select one supported d.o.f. as being the location of the applied force. For the $(N_s - 1)$ other supports, incorporate a load cell between the upper section and the connection units as shown in Fig. 4.1. For the selected support, remove the upper section and the connection unit. Attach a load cell and its adapter to the test article. The adapter is a small narrow plate which is fixed to the component through the same two holes used to attach the connection of the flexible support and which has a hole on top in its center for bolting the load cell through that single hole. Clamp the displacement mechanism to a fairly rigid support and position it so that the hook of the mechanism is about a foot above the load cell. Depending on the maximum load to be applied, attach the hook of the displacement mechanism through either fishing lines or a nylon rope to the hook fixed on top of the load cell.

(2) Installation of static horizontal restraints

For the components of the free-free beam and grid systems, install two foam rings around perpendicular horizontal shafts of a connection unit, making sure not to prevent any rotation (Fig. 4.11). In order to maintain stability of the grid component as it deformed during the

applied displacements, another ring was required at the flexible support opposite to the location of the applied displacement. These rings are made of black soft polyurethane foam of fairly low compressibility. Unlike the case of external horizontal restraints used for modal testing, these rings can be in direct contact with the connection unit since there is no risk of perturbing any dynamic motion. This eliminates the need to have any direct contact with the test article in order to restrain it horizontally. Through proper positioning of the displacement mechanism, it is possible to ensure that the test article is always resting against the static horizontal restraints in a stable position. At times, this process may require several trials.

(3) Installation of the indicator(s)

Select the displacement(s) to be measured during the tests. Clamp each displacement indicator to a rigid support and position the indicators so that they touch the test article and are ready to measure the displacement along their specified d.o.f.

(4) Data acquisition

Apply a set of incremented displacements and measure both the applied force and the response(s). During each run, one or several responses may be measured simultaneously. The reaction force at a support and the translational displacement along the same d.o.f. must be acquired during the same run, in order to be able to correct the reaction force of the support for the reaction force of the digital indicator at that support. In general, 9 or 10 increments of displacement are applied. Displacements are applied starting with a test article which is roughly horizontal and lifting it in reasonably equal steps to a total displacement of no more than 25 mm.

(5) Data processing and verification

The measured data provide the deformed shapes and the reaction forces for a unit applied force. When needed, processing of the data through finite difference approximation also provides the rotation at interfaces.

Steps 3 to 5 are repeated until all reaction forces and all required displacements are acquired. The required displacements consist of the translational displacements along the supported d.o.f. and along all interior d.o.f. with, in some cases, additional displacements required for the estimation of interface rotation(s).

Steps 1 to 5 are repeated N_s times in order to acquire all displacements and reaction forces with respect to the N_s different deformed shapes, one corresponding to each supported d.o.f.

4.5 Modal Testing of a Flexible Support with Lumped Mass Test Articles

4.5.1 Test Objective and Description

As soon as a prototype of the flexible support was manufactured, a series of modal tests was performed on it in order to understand some of its dynamic characteristics. The main objective of these tests was to establish if the connection unit of the flexible support exhibited a linear response when excited with burst random excitation. Such an excitation was chosen since this would be the type of excitation for the subsequent modal tests on components of various systems for verification of the experimental hybrid CMS approach. The other units of the flexible support were of no concern since there was no moving part and no element in them which could behave nonlinearly.

For these tests, the upper section of the flexible support consisted of an aluminum bar having a 13 x 102 mm (.5 x 4") cross-section. The free span of the upper section between the two pedestals was 1828 mm and the connection unit was located at mid-span. Two different rigid masses, one at a time, were suspended from the connection. These rigid masses were chosen as being representative of the loading imposed on supports by test articles attached to them. The rigid masses had mass values of 3.6 and 9.9 kg (8 and 22 lb.). The advantage of using a lumped mass rather than a flexible structure as a test article was the elimination of any

possible dynamic interaction between the test article and the flexible support, thus ensuring that any observed dynamic behavior was associated with the flexible support only.

Several test runs were performed using different test configurations. Some parameters were changed between these runs. These parameters included: mass of the rigid test article, locations of excitation, length of stingers connecting the force sensors to the exciters, tightening of the thrust bearing, excitation force level, and length of the stingers serving as horizontal restraints. The significant test results obtained during these runs are presented in this section.

Figure 4.12 shows a sketch of the set-up for the two exciters driving simultaneously the flexible support and the bigger rigid mass. Both excitation forces are along the vertical direction. One of the exciter is attached to a force sensor fixed underneath the upper section at 165 mm from the center of the connection unit. An accelerometer is fixed on top of the upper section beam at the same location. The second exciter drives the system from underneath the rigid mass along a diagonal. An accelerometer is fixed right beside the force sensor. By having this driving point off-centered with respect to the connection unit, motion is induced in the bearings.

Figure 4.13 shows a close-up of the rigid mass and its attachment to the connection unit. As shown in the figure, the connection unit is instrumented with several accelerometers. However, the FRF's obtained from these accelerometers were found to be quite noisy and were not of much use in the present test. Three horizontal restraints used in the test are shown in the figure and are located at about mid-height of the rigid mass. These restraints were not provided with any damper so as to eliminate any possible interaction with the dynamics of the flexible support. In any case, even if a disturbance is caused by these horizontal restraints, it is not important here since all data comparisons made to judge the linearity of connection behavior were carried out with horizontal restraints in identical configuration.

4.5.2 Test Results and Discussion

Reciprocity checks were performed by comparing the FRF's $H_{1,2}$ and $H_{2,1}$ where 1 refers to the force location on the upper section and 2 refers to the location underneath the rigid mass. One can see from Fig. 4.14 that the reciprocity is general very good throughout the whole frequency bandwidth of 0-512 Hz. More specifically, reciprocity is excellent from 0 to about 220 Hz. The lack of reciprocity in the 220-250 Hz region occurs near a major antiresonance and is thus of no concern. The $H_{1,2}$ curve is noisy above 280 Hz while $H_{2,1}$ is very smooth in that region. The noise in the former curve could be explained by the fact that the excitation energy input from underneath the rigid mass was largely being absorbed by the bearings of the connection unit, leaving not enough energy for moving the upper section.

The set-up for the smaller mass was basically the same as for the larger one. The force levels at driving points 1 and 2 were respectively 1/2 and 1/4 of those in the test with larger mass. The curves for the reciprocity check are shown in Fig. 4.15. The reciprocity is generally very good. It is excellent below 290 Hz. Above that frequency, $H_{1,2}$ becomes a bit noisy, but not as much as for the larger mass.

Considering that in the tests to be performed for verification of CMS, we are only concerned with the FRF's for which both the force and the response are on the test article, as opposed to one of them being on the upper section, it is of interest to compare the $H_{2,2}$ curves for two different force levels. For a system behaving perfectly linearly, these two curves should overlap completely. The curves for two different force levels, one twice the other, are shown in Fig 4.16 for the smaller rigid mass. Except for two small region (180-195 Hz and 280-295 Hz) for which the two curves differ slightly, the overlap is very good. It is interesting to note that the curves are very smooth for the region above 290 Hz; this tends to support the reasoning that energy dissipation in the bearings causes the noise present in portions of the $H_{1,2}$ FRF's. The overlap between $H_{2,2}$ curves obtained for two different force levels in a test on the bigger rigid

mass test article is shown in Fig. 4.17. The higher force level is 1.3 times the lower one. The overlap is very good, even better than that for the smaller rigid mass.

The data obtained from modal tests on flexible support prototype show that the connection unit does behave linearly in the frequency 0-512 Hz, when subjected to burst random excitation. The observed linear behavior of the connection could be attributed to the fact that possible nonlinear response of the bearings, due to a slack or gaps, was not mobilized or to the possibility that a slight nonlinear behavior was linearized by the random excitation. In any case, the tests did confirm that the connection unit would not cause any significant nonlinearity when used to support a test article in a modal test.

Fig. 4.1 Flexible support in static test configuration

Fig. 4.2 Extension frame module of pedestal unit
(Source: Dynacon Enterprises Ltd)

Fig. 4.3 Tapered frame module of pedestal unit
(Source: Dynacon Enterprises Ltd)

Fig. 4.4 Side view of connection

Fig. 4.5 Front view of connection

Fig. 4.6 Drawing of the connection unit

Fig. 4.7 Typical modal testing set-up

Fig. 4.8 Horizontal restraint with high compressibility damper

Fig. 4.9 Displacement mechanism for static tests

Fig. 4.10 Indicator calibration curve - reaction force versus displacement

Fig. 4.11 Horizontal restraints for static tests

Fig. 4.12 Schematic of the set-up for modal testing of a flexible support prototype

Fig. 4.13 Close-up of larger rigid mass
for testing of flexible support

Fig. 4.14 Reciprocity check between $H_{1,2}$ and $H_{2,1}$, for the larger rigid mass test article

Fig. 4.15 Reciprocity check between $H_{1,2}$ and $H_{2,1}$, for the smaller rigid mass test article

Fig. 4.16 $H_{2,2}$ for two different force levels, for the smaller rigid mass test article

Fig. 4.17 $H_{3,2}$ for two different force levels, for the larger rigid mass test article

5 VERIFICATION OF TEST-BASED CMS FOR BEAM SYSTEMS

5.1 Introduction

The experimental verification of the proposed hybrid CMS approach was performed by testing three different systems of increasing complexity, namely a fixed-fixed beam system, a free-free beam system and a grid system. Each system was composed of two similar components. The complexity of a system is related to the challenges involved in testing the components and in potential numerical problems encountered in estimating the static and dynamic parameters of the components in rigid support configuration based on data obtained from tests in a flexible support configuration. The complexity increases with the number of flexible supports to which the component test article is attached. It also increases with the number of free interface d.o.f. for which the rotational FRF's and the static rotations are derived from translational measurements.

This chapter presents the results of the application of experimental hybrid CMS approach to the first two systems which are made up of beam components. The third system, made up of grid components, is discussed in Chapter 6. For each system included in this chapter, a description of the test articles and the boundary conditions is presented first. Details of the finite element models for both the system and the components are presented next. Also included is a discussion of the system modal parameters used as a reference in the CMS and the manner in which these parameters are obtained. Details of modal testing are presented for each of the two components. The data processing required for obtaining the FRF's related to $\mathbf{H}^*$ and the modal parameters in the rigid support configuration is described. The derived modal

parameters of the components are then compared with their analytical counterparts. This is followed by a discussion of the static tests and the data processing involved in deriving the component Ψ_i and k^* . For both modal and static tests, the approach taken in obtaining the parameters related to rotational d.o.f. is discussed and the corresponding results are presented. Since the equipment and procedures used in the tests have already been described in Chapter 4, only the aspects specific to the tests reported here are presented in the chapter.

The system modal parameters are obtained by the CMS approach using the test-derived component parameters, and the parameter values so obtained are compared with corresponding reference values. The system parameters are derived again using the CMS approach but with analytically derived component parameters as input in the CMS. This comparison of these values with the test derived values provides a better appreciation of the reliability of test-based CMS.

5.2 Fixed-Fixed Beam System

5.2.1 Test Articles, Configurations and Boundary Conditions

The first system to which the hybrid CMS approach is experimentally applied is the fixed-fixed beam shown in Fig. 5.1. The beam system is clamped or built-in at both ends. The beam consists of an aluminum bar 3.96 m (13 ft) long and having a nominal rectangular cross-section of 19 x 152 mm (0.75 x 6"). The large width/thickness ratio is selected so that the lower modes are mainly bending modes in the vertical plane. The system is composed of components 1 and 2 having lengths of 2.13 m (7 ft) and 1.83 m (6 ft) respectively.

Component 1 is the cantilever beam shown on the left side of Fig. 5.1. The component interface is at its free end and consist of two d.o.f.: vertical translation (d.o.f. 1) and in-plane rotation (d.o.f. 1001). The component fixed interface d.o.f. in CMS corresponds to d.o.f. 1, while the free interface d.o.f. corresponds to d.o.f. 1001. The hybrid CMS method needs, as

input, the following three sets of modes: 1) the constrained or rigid support normal modes, of which the first two modes are depicted in Fig. 5.2, 2) the constraint mode depicted in Fig. 5.3, and 3) the residual flexibility attachment mode.

In order to obtain the required modal parameters (normal modes and residual flexibility term) in the rigid support or H^* configuration depicted in the schematic of Fig. 5.4a, a modal test is performed on the component in a test set-up with a flexible support supporting the beam along the fixed interface d.o.f., as depicted in the schematic of Fig. 5.4b. The flexible support is represented by a spring since it restrains the component only in the vertical direction.

In this simple case, in order to obtain the static constraint mode, a force is applied and measured at the supported d.o.f. (d.o.f. 1) in the absence of the flexible support, i.e. at the free end of component 1 in a cantilever configuration.

Component 2 is the cantilever beam shown on the right side of Fig. 5.1. It has the same interface d.o.f. numbers and types as those of component 1. The mode sets of component 2 required for CMS are the same as those for component 1, and both components have the same test set-up.

The CMS procedure presented in this work is most useful when applied to complex systems where finite element modeling involves considerable uncertainty. Another advantage of the procedure is that it does not rely on the provision of fully rigid or fully flexible supports. There is little advantage in using the CMS procedure to analyze a fixed-fixed beam. However, the simple nature of a fixed-fixed beam system permits the construction of built-in ends that are nearly rigid, as well as the development of a fairly accurate finite element model. It is possible through modal test on such simple system to validate the CMS approach by comparing the test-based CMS results with experimental modal parameters of the system used as reference values.

One drawback of a fixed-fixed beam system or its components is that the fixed boundary condition can only be approached in a laboratory. However, if the system modal parameters, used as reference for the CMS results, are also obtained experimentally, the effects of the "imperfect" boundary conditions would more or less cancel out.

In the laboratory, the built-in ends for both the system and the components were built by clamping the beam test article between three layers (2 below and one above) of two 102x102 mm H beams. The two H beams on top of the test article were made of steel and the four underneath it were made of aluminum. As shown in Fig. 5.5, all these beams were clamped to two rigid pedestals. Each of these steel pedestals was assembled with one extension frame module and a tapered frame module, and was bolted to two base rail elements. These base rails were shimmed with respect to the seismic floor and clamped to the rails of the floor.

During testing, the three test configurations (i.e. two components and one system) were obtained by using the same 5.2 m beam and by changing the number and position of the fixed-end assemblies.

5.2.2 Analytical Modeling

Description of FE Models and Their Role

In the finite element models of the system and the two components, the nodes are equidistant and spaced at 152 mm (6 "). Thus, the system model has a total of 25 nodes, and components 1 and 2 respectively have 14 and 12 nodes.

As shown in Fig. 5.1, in the present analysis, only in-plane motion is of interest. The finite element models of the system and the components are initially constructed with two d.o.f. per node, a vertical translation and a rotation. The rotational degrees are condensed out by performing static condensation and Guyan's reduction. The reduced system model has only vertical translation d.o.f. at each node. However, for each of the components, the node located at the free end has two d.o.f., vertical translation and rotation, while the interior points have only one d.o.f, vertical translation.

The stiffness and mass matrices obtained from the finite element models are used in the following manner:

- 1) The experimental hybrid CMS approach requires the analytical mass matrices of the two components, in addition to the test-based component modal and static parameters.
- 2) The analytical values of component mass and stiffness matrices are used to derive error-free component parameters (ω_{kk} , Φ_k , Ψ_s , k^* , G_{da}) that can be input to the CMS, as an alternate to test-derived values. As stated earlier, a comparison of the system parameter values obtained by two alternative approaches (i) with the input of test-based component parameters to CMS, and (ii) with the input of analytical component parameters to CMS, provides a useful measure of the reliability of test-based CMS.

Experimental Estimation of the Physical Properties of the Beam

In the generation of the FE models of the system and of the components, accurate estimates are required of the cross-section dimensions and elastic properties of the beam. The required parameters, that is the width and thickness of the cross-section and the Young's modulus of the material of the beam, were obtained through measurement.

Cross-section measurements were taken at the end of the beam corresponding to the free-end for the components and at 17 other locations each separated by 305 mm (1 foot). These 18 locations covered the free span of all three test articles. At each of these cross-sections, one measurement of the width and two measurements of the thickness were taken. An average value was computed for each of these dimensions and served as input parameter for the FE models. The averaged width was 152.8 mm (6.017") and the averaged thickness was 18.81 mm (0.7406").

Estimates of the Young's modulus were obtained using Dynamic Mechanical Analysis (DMA). The DMA instrument, (model 983 from Dupont), may be used to measure the Young's and shear moduli of specimen as a function of temperature between 25 and 500°C. In the

determination of Young's modulus through DMA, the following procedure is used (Elomari et al, 1995). The specimen is clamped between two parallel arms and then subjected to a uniform sinusoidal displacement producing constant maximum strain. The oscillation frequency is fixed (e.g. 1 Hz) and sample deformation is monitored by an LVDT. The measured lag between the drive signal and the LVDT provides the phase angle. The phase angle and the drive signal are used to calculate the Young's modulus and the damping capacity of the specimen.

In order to estimate the Young's modulus of the test articles, three blocks of 2 x 10 x 40 mm were cut from the two ends and the middle of a cross-section of the 6061-T6 aluminum beam. From these three specimens, thirty estimates of the modulus were obtained for a temperature of 25 degrees C. The average of these estimates gave $E = 70.56$ GPa. This value was used in the construction of finite element models.

5.2.3 Experimental Estimation of System Modal Parameters

Test Description, Instrumentation

The reference parameters for comparison with the estimated system parameters obtained from CMS based on test data were obtained by performing a modal test on the 3.96 m fixed-fixed beam. A total of 29 accelerometers were fixed on the upper face of the beam so as to measure the vertical acceleration at these locations (Fig. 5.6). Twenty-five of the 29 accelerometers were located on the centerline of the beam at every 152 mm (6 inches). These locations correspond to d.o.f. numbered from 1 to 25, where d.o.f. 1 and 25 are the closest to the fixed ends. Since only in-plane motion is of interest, only the bending modes need to be estimated precisely. The accelerometer locations were selected to be identical to those used in the two components, so as to permit the comparison of mode shape elements.

In addition to the accelerometers located on the centerline of the test article, four others were fixed, one on each side of the beam at two different cross-sections corresponding to d.o.f. 5 and 21. These four side accelerometers were added in order to properly identify any torsion

modes, so that such modes were not confused with the bending modes sought in the test. Normally, the sensors on the centerline should not pick up any torsion modes, but a slight misalignment in the position of the accelerometers and/or the force sensors could produce small, spurious peaks in the FRF's.

Two force sensors were fixed to the lower face of the test article at d.o.f. 11 and 21. The two exciters were positioned underneath the beam and attached to the force sensor through stingers, as shown in Fig. 5.7. The accelerometers and their cables are clearly visible in the figure. Data acquisition was performed with two different frequency bandwidths: 0-256 Hz and 0-512 Hz. Because of its higher resolution, only the former bandwidth was finally used, particularly since it contained enough bending modes for the purpose of these tests.

Two other data acquisition runs were performed, using the same frequency bandwidths, but with the exciter positions moved to d.o.f. 9 and 221 (on one side of the beam adjacent to d.o.f. 21). Curve fitting of the FRF's with respect to the force at d.o.f. 221 and subsequent mode shape animation resulted in the identification of torsion modes at 91.9 and 181.9 Hz. For the FRF's obtained with driving points at d.o.f. 11 and 21, only small glitches appeared at 91.9 Hz, while smooth curves without any glitch were observed around the second torsion mode frequency.

Test Data

All FRF's with respect to the three driving points on the centerline of the beam were of good quality with each of the first eight bending modes being well excited by at least one of the three forces. In fact, most modes appeared as well defined peaks on FRF's with respect to all three driving points. The coherence functions did not show any sharp drop, so that no further investigation of the data was required.

The overlap of FRF's $H_{11,21}$ and $H_{21,11}$ is shown in Fig. 5.8. An excellent reciprocity is observed throughout the whole frequency bandwidth, except around 116 Hz which corresponds

to the 6th bending mode. In the figure, a "B" refers to a bending mode while a "T" refers to a torsion mode. The reason for the lack of reciprocity around 116 Hz becomes obvious by looking at the point FRF's (for which the force and response are at the same location) $H_{11,11}$ and $H_{21,21}$ shown in Fig. 5.9. While a clear peak appears when the force and response are at d.o.f. 11, there is no peak at all when they are at d.o.f. 21. It is obvious that the lack of reciprocity in that frequency range is due to the fact that d.o.f. 21 is at a node for the 6th bending mode. The extraneous peaks, such as the one at about 100 Hz, are likely to correspond to modes within the fixed support assemblies or the floor to which the supports are clamped.

Estimation of System Modal Parameters and Comparison with FE Results

The LMS CADA-X modal analysis software offers several methods for estimating the modal parameters, from the simplest to the most advanced. However, the use of only one of these methods was found to be adequate for the entire experimental phase of the work. Modal parameters are in fact estimated in two steps: 1) frequencies and damping values, 2) residues or mode shape coefficients. For Step 1, the method that was used in the work is called the multiple input Least Square Complex Exponential (LSCE), also referred to as the polyreference technique. It is an advanced MIMO (multiple-input-multiple-output) estimation technique (Vold et al., 1982). The polyreference technique uses all data measured on a structure to obtain global estimates of modal frequency and damping of several modes, as well as modal participation factors which express the participation of modes in the structure response as a function of the reference d.o.f.. The fact that these data FRF's may be with respect to several input (or references) makes the technique very powerful, especially for difficult cases such as closely-spaced modes. An overview of the mathematical background of the technique and how it is implemented in the LMS software are given in LMS International (1992).

Step 2 of the parameter estimation process was performed with the Least Square Frequency Domain method. This technique allows simultaneous estimation of all residues, which are

related to the mode shape coefficients, within a specific frequency bandwidth, one FRF at the time. The implementation of the method in CADA-X allows the estimation of either real or complex mode shape coefficients, as well as the lower and upper residuals. The upper residual corresponds to the residual flexibility of the structure.

Parameter estimation for the fixed-fixed beam was performed using the 25 FRF's with respect to the excitation applied at d.o.f. 11 and associated with the accelerometers on the centerline of the test article. The response with respect to only one driving point was used since the modes are well separated and this driving point excites all the modes of interest. The highest mode which appeared in the FRF's at 246 Hz was not used, since it was fairly close to the upper limit of the frequency bandwidth and the corresponding peak was a bit noisy. Also, it was decided that the eight previous bending modes were enough to verify the results of the CMS approach for this beam system.

Table 5.1 presents the test-derived residues for the eight bending modes obtained from complex mode analysis. These results serve to verify the quality of the estimated modes. For perfect normal modes and perfect measurements, the phase of all residue coefficients would be either 90 or -90 degrees. As shown in the table, the phases are normally fairly close to these values except for coefficients with lower magnitude. The phase shifts of the 7th mode, at 154 Hz, are not as good as for the other modes because the mode is not that well excited with the force at d.o.f. 11 as shown in Fig. 5.9. However, these phase shifts are still quite acceptable, especially since this is a higher order mode. Estimation of residues based on real mode analysis was also performed and served as reference for comparison with the CMS results.

Table 5.2 shows the modal frequencies obtained from the above analysis of the test data. For comparison, the frequencies obtained from a FE analysis of the beam, modeled with 39 beam elements of equal length and two d.o.f. at each nodes (vertical translation and in-plane rotation), are also presented. These frequencies were verified with another FE model which had twice as many elements but gave essentially the same frequency values for these modes. The frequencies also matched the closed-form solution available for this case.

The last column of Table 5.2 gives the percentage error between test-based and analytical frequencies taking the latter as the reference (i.e. $(f_{\text{test}} - f_{\text{fem}})/f_{\text{fem}} \times 100$). The errors for all the modes are less than 3%. All experimental values are lower than their analytical counterpart. The most likely source of discrepancies is the difficulty in creating in the lab a truly fixed boundary condition. Flexible boundary would effectively bring down the modal frequencies of the test article, as observed in the present situation. The lack of perfect rigidity in the boundary could be due to the flexibility of the fixed end assembly and/or of the seismic floor on which the test set-up was mounted and/or of the floor of the lab to which the seismic floor was partly attached.

5.2.4 Estimation of H' for Component 2 Through Modal Testing

Component 2 is the first test article on which modal testing was performed in order to derive the H' related FRF's and the associated modal parameters. Details of the tests are discussed in this section.

Test Description, Instrumentation

In the test configuration, a flexible support is attached at the free end of the 1.83 m cantilever beam. An overview of the beam test article with its boundary conditions is shown in Fig. 5.10. A total of 22 accelerometers were fixed vertically on the upper face of the beam (Fig. 5.11). These accelerometers may be divided into the following three groups:

- (1) Twelve equally-spaced accelerometers on the centerline of the beam. These sensors are located at d.o.f. 1 to 12. Degree-of-freedom 1 is right at the end of the beam where the flexible support is attached, the accelerometer being fixed between the two legs of the connection unit. Degree-of-freedom 12 is the instrumented location closest to the fixed end of the test article.

- (2) Six accelerometers on the two sides of the beam at three different cross-sections corresponding to d.o.f. 2, 6 and 11. These sensors were added to identify the torsional modes of the component to separate them from the bending modes required for CMS. These six accelerometers also served to identify any coupling between bending and torsional modes through mode shape animation. Figure 5.12 presents an overview of the eighteen accelerometers in the first two groups.
- (3) Additional accelerometers fixed along the centerline of the beam at distances of 50.8 mm (2"), 101.6 mm (4"), 203.2 mm (8") and 406.4 mm (16") from d.o.f. 1 (Fig. 5.13). The corresponding d.o.f. are 15 to 18 respectively. These sensors are used for the estimation of rotational FRF's as discussed later in this section.

The two interior driving points were selected to be at d.o.f. 6 and 9. In addition, d.o.f. 1 must also be a driving point since it is along a supported d.o.f. Since two exciters could be run simultaneously, each test run was performed with one exciter underneath the flexible support and the other one at one of the two interior driving points. A force sensor (without cable) was fixed for each run at the interior driving point at which no exciter was installed. This eliminated any possible inconsistency in the test data due to the mass of the force sensor being absent from one of d.o.f. 6 and 9.

Data acquisition was performed with two different bandwidth: 0-1024 Hz and 0-256 Hz. The upper limit of the first bandwidth was high enough to include eight bending modes in the rigid support configuration. However, its frequency resolution was such that it was difficult to obtain a good curve fit of the low damping fundamental mode; this led to the addition of the second frequency bandwidth providing a frequency resolution which was four times higher.

In addition to the test runs described in the previous paragraph, two other test configurations were investigated in order to improve the understanding of the $\mathbf{H}^*$ related FRF's:

- (1) A run with the larger frequency bandwidth was performed with the exciters at d.o.f. 1 and 206 (one side of d.o.f. 6). This permitted a proper identification of 4 torsional modes with frequencies below 1000 Hz.
- (2) Test runs were also carried out on the cantilever beam (i.e. with flexible support disconnected) with exciters at d.o.f. 1 and 9. The estimated test-based modal frequencies were then compared with their analytical counterparts. The purpose of these runs was to get an error level on the measured modal frequencies for this simple test configuration for which most of the error is likely to arise from the flexibility of the boundary condition. As discussed later, this estimation confirmed that a major part of the difference between the modal frequencies as estimated from H^* related FRF's and their analytical values was due to the lack of rigidity of the fixed end of the test article.

Test data

Observation of individual FRF data did not show any anomaly except in $H_{1,6}$ and $H_{1,9}$ as discussed below. Coherence functions behaved as expected, in the sense that their values were close to unity except at an antiresonance of the FRF's or at frequencies for which significant noise level appeared in the FRF's.

The result of a reciprocity check for the run with forces at d.o.f. 1 and 9 performed by comparing the superimposed curves $H_{1,9}$ and $H_{9,1}$ is shown in Fig. 5.14. The reciprocity is seen to be generally very good except within the frequency bandwidth of 760-860 Hz, where $H_{1,9}$ is smooth while $H_{9,1}$ exhibits two small peaks. Figure 5.15 presents curves of $H_{1,6}$ and $H_{6,1}$ for another check on reciprocity. Again, an anomaly is detected in the bandwidth 760-860 Hz. On the other hand, as shown in Fig. 5.16, a reciprocity check using $H_{6,9}$ and $H_{9,6}$ is quite good in the same frequency bandwidth with smooth curves, despite the fact that the two FRF's are obtained from different test runs. One may get a better understanding of what is happening here by observing the auto power spectrum of the force at d.o.f. 1 shown in Fig. 5.17. The power

spectrum was obtained from the run carried out with excitations at d.o.f. 1 and 9. The deepest drop in power spectrum occurs at 809 Hz and another large one occurs at about 839 Hz. These two frequencies happen to be the ones at which the two small peaks are observed for $H_{9,1}$ in Fig. 5.14. The same observation could be made for the auto power spectrum of the run carried out with excitations at d.o.f. 1 and 6. From the previous observations and considering that the deepest drop in power spectrum is observed at a fairly low peak in the FRF, this anomaly could be attributed to the stinger at d.o.f. 1 getting into resonance. This discussion shows one example of the types of verification carried out with the test data.

Referring to Figure 5.14, it is observed that severe noise level appears near the antiresonance in the frequency range of 150-180 Hz. In fact, as seen from the two curves in Fig. 5.18, $H_{1,9}$ is much noisier than $H_{9,1}$. This phenomenon may be explained by the fact that the accelerometer at d.o.f. 1 is fixed to the test article right underneath the connection unit and therefore experiences all small nonlinear motion occurring in the bearings. In the frequency range 150-180 Hz where the accelerometer response is itself very low, the small nonlinear motions give rise to considerable noise. The accelerometer at d.o.f. 9 is located in a much more "quiet" physical region. This phenomenon was observed throughout the test program.

Estimation of H^ Involving Translational d.o.f.*

The translational (i.e. translation/force) H^* related FRF's were obtained for 17 response d.o.f. (11 on centerline and 6 on the side of the beam) with respect to 3 interior driving points (d.o.f. 6, 9 and 206). Location 6 was selected as the main interior driving point since all modes were well excited from the force at this d.o.f. The driving point at d.o.f. 9 served mainly to provide data for reciprocity checks.

The H^* related FRF's with respect to the force at d.o.f. 206 were derived to identify the torsion modes. They also served to improve the estimation of the elements of the 6th bending mode which is strongly coupled with the 3rd torsion mode. This is because the simultaneous

use of FRF's from two driving points permits the curve fitting algorithm to decouple two closely-spaced modes. Curve fitting and mode shape animation led to the identification of the first four torsion modes at 97.3, 293.1, 491.8 and 704.2 Hz. In order to confirm these modes, their frequencies were compared with the ones from an analytical model made up of 36 beam elements of equal length. The differences between the experimental and analytical torsion frequencies were respectively -3.3, -3.0, -2.5 and -0.4 percent. Although not required for the CMS analysis, the identification of the torsion modes is useful in explaining glitches or secondary peaks which may appear in the FRF's associated with sensors on the centerline of the test article.

A superposition of the FRF's for the force and response at d.o.f. 6 in both the flexible support and rigid support configurations is shown in Fig. 5.19. The continuous curve represents $\mathbf{H}'$ while the dotted one represents $\mathbf{H}$. Besides the fact that the peaks have shifted, one observes that more modes are present in $\mathbf{H}$ due to the dynamics of the flexible support, which is eliminated in the transformation to the $\mathbf{H}'$ rigid support configuration.

The $\mathbf{H}'$ related FRF for the force and response at d.o.f. 6 is shown in Fig. 5.20. The eight bending modes and the third torsion mode are identified. The curve looks pretty smooth with all peaks corresponding to bending modes clearly visible. In addition to the peaks corresponding to the identifiable modes, one may observe several secondary peaks and glitches such as the ones at about 185, 394, 671 and 758 Hz. It is not surprising that such peaks exist in the derived FRF's when extraneous peaks were also found in the direct testing of the fixed-fixed beam system for which these anomalies were attributed to the boundary conditions. It is thus likely that some of these extraneous peaks are due to the imperfect fixed end in the set-up.

A number of observations were employed to rule out the extraneous peaks as being potential physical modes of the test article, even before starting the parameter estimation phase. These observations can form a practical basis for minimizing the possibility of confusing pseudo modes with true physical modes. They are summarized in the following paragraphs.

(1) *Change of amplitude scale*

Because of the wide range of values which must be encompassed in plotting a FRF in term of amplitude of the response/force versus frequency, it is customary to use a logarithmic scale for the vertical axis, as has been done in this work. This has the effect of amplifying the peaks of the modes with lower amplitude. However, the logarithmic scale also amplifies the importance of glitches or small extraneous secondary peaks. As an example, the plot of Fig. 5.20 is shown in a linear scale in Fig. 5.21. In the new scale, which is in fact the one "seen" by the parameter estimation software, the small peaks at 185 and 758 Hz have basically disappeared while the peak at 394 Hz has been reduced to its real proportion. Consequently, in order to have a better perspective on the quality of the derived FRF's, one should also observe them in a linear scale for the amplitude.

(2) *Modification of flexible support stiffness*

Another method used to detect spurious peaks in the FRF's was to repeat the modal tests with flexible supports of different stiffnesses. The stiffness change could be achieved by modifying the cross-section dimensions of the upper section and/or by changing the distance between the pedestals. Another way was to change the stiffness of the two legs of the connection unit by modifying their design. Both methods of stiffness modification were implemented. In theory, the change of flexible support stiffness should have no effect on $\mathbf{H}^*$. However, in practice, it was found by superposing the corresponding $\mathbf{H}^*$ related FRF's from different flexible supports that the peaks associated with real modes did not change while extraneous peaks, which are normally quite small in the first place, were very often affected.

(3) *Phase shift*

Normally, it was observed that the extraneous peaks had a phase shift of no more than 30 degrees. Since the shift at a true mode should be around 90 degrees, peaks associated with small phase shift could be recognized as spurious peaks. One has to be careful here not to

ignore true modes which are either not well excited due to the force location or not picked up due to the accelerometer location. On the other hand, if such a peak has a small phase shift for more than one driving point and for all responses, it is reasonable to assume that it does not correspond to a true mode.

(4) *Parameter estimation*

If after applying any or all of the above observations, there was still some doubt as to whether or not some peaks represented true modes, reliance was placed on the parameter estimation techniques to eliminate pseudo modes.

A reciprocity check at the $\mathbf{H}^*$ level is obtained by comparing the two curves of Fig. 5.22. One may observe that the reciprocity is excellent for most frequencies and all peaks corresponding to the modes of the component. The only place where the reciprocity is really not good is for a narrow bandwidth at about 700 Hz which surround an antiresonance and is consequently of not that much interest. It is interesting to note a reasonable reciprocity for the bandwidth 760-860 Hz for which the reciprocity at the $\mathbf{H}$ level, between d.o.f. 1 and the two interior d.o.f. 6 and 9 (Figs. 5.14 and 5.15), was non-existent.

For the frequency bandwidth 150-180 Hz, one observes only a few extraneous peaks with good reciprocity while the curve $H_{1,6}$ and especially $H_{1,9}$ were very noisy in that region. An explanation for this improvement is probably that the noisy region at the $\mathbf{H}$ level occurs at a deep antiresonance where the amplitude is very small, thus minimizing its effects in the implementation of Eq. 2.66. It is apparent that noise in the test data related to $\mathbf{H}$ does not necessarily transfer to the derived $\mathbf{H}^*$ related FRF's.

Estimation of $\mathbf{H}^$ Involving Rotational d.o.f.*

As discussed earlier, the interface rotation elements of the mode shapes are obtained by curve fitting the $\mathbf{H}^*$ related FRF's involving the response along the rotational d.o.f. Since

measurement of rotational response is difficult, its value must be derived from the measurements of several translational responses near the point of interest. Sattinger (1980) shows that $H_{\theta f}$, where the rotation θ is at an end location of the test article and the force f is at any location, may be approximated by the following forward finite difference expression:

$$H_{\theta f, j} = \frac{-3H_{1j} + 4H_{2j} - H_{3j}}{2\Delta} \quad (5.1)$$

where H_{ij} is the FRF relating the translational response at point i to the force at point j , and Δ is the distance between two adjacent points i and $i+1$. Point 1 is at the component interface (d.o.f. 1) while points 2 and 3 are respectively one and two Δ away from the interface.

One question which arises is whether $H_{\theta f}$ should be obtained with the translational FRF's H_{ij} in the test configuration and then the $H_{\theta f}^*$ derived using Eq. 2.66, or should H_{ij}^* be derived first and then Eq. 5.1 used to get $H_{\theta f}^*$. In other words, which of these two paths would generate a smoother $H_{\theta f}^*$. Several trials with the experimental data showed that both approaches gave exactly the same FRF. In fact, it can be analytically shown that the two methods should indeed give the same results.

In order to obtain the residual terms required for the residual flexibility attachment modes in the hybrid CMS method, we need the FRF's H_{wm} and $H_{\theta m}$ where w is the translational displacement at any point, θ is the rotation at an interface d.o.f. and m is the moment applied at that d.o.f. Based on Sattinger's work (1980), these two FRF's may be approximated by the following expressions, for the special case where the rotation and moment are measured along the same d.o.f.

$$H_{w, m_1} = \frac{-3H_{11} + 4H_{12} - H_{13}}{2\Delta} \quad (5.2)$$

and

$$H_{\theta, m_1} = \frac{9H_{11} - 12H_{12} + 3H_{13} - 12H_{21} + 16H_{22} - 4H_{23} + 3H_{31} - 4H_{32} + H_{33}}{4\Delta^2} \quad (5.3)$$

Sestieri et al. (1991) have made several useful suggestions to improve the accuracy and smoothness of the derived rotational FRF's. The following suggestions were implemented in the present work:

- (1) *Mass correction:* Although the mass of a transducer (accelerometer and/or force sensor) might represent a small percentage of the mass of the test article, the effect of transducer masses might be quite significant, in the finite difference computation if the transducers are moved around between the measurement of the various H_{ij} . Sestieri et al. have derived various expressions for correcting the translational FRF's prior to their use in deriving the rotational quantities.
- (2) *Selection of the finite difference spacing:* Sestieri et al. discuss how an appropriate selection of the spacing between measurement locations is crucial in ensuring the accuracy of derived rotational FRF's. The authors mention that, as would be expected, a small spacing (with respect to the nodal distance of the highest modes considered) makes the rotational FRF's more precise. The authors, however, add that the smaller the spacing, the bigger the influence of noise in the data on the finite difference computations. For the present work, the transformation from $\mathbf{H}$ to $\mathbf{H}^*$ seems to result in a smoothing of the data leading to a reduction in noise level in the $\mathbf{H}^*$. This permits the selection of a smaller spacing between the points at which translational responses are measured.

For an appreciation of the effect of spacing between points of translational measurement, the frequency function $H_{0,6}^*$ was estimated for different spacing between the points. Figure 5.23 presents the curves corresponding to three different spacings: 50.8 mm (2"), 101.6 mm (4") and 203.2 mm (8"). As shown in Fig. 5.24, for a spacing of 50.8 mm (Δ_x), points 2 and 3 correspond to d.o.f. 15 and 16; for $2\Delta_x$, they correspond to d.o.f. 16 and 17; for $4\Delta_x$, they correspond to 17 and 18. The plots include the first four bending modes of the test article which are contained in the frequency bandwidth of 0-256 Hz. It is observed that the two smaller spacings give FRF's that are fairly close to each other, but the largest spacing leads to a higher

curve. The noisiest plot in the low frequency end corresponds, as expected, to the smallest spacing. Based on these comparisons, the $\mathbf{H}^*$ related FRF obtained with a spacing of $2\Delta_x$ was used in the curve fitting operation for estimating the first four bending modes.

The FRF $\mathbf{H}_{g,6}^*$ is shown in Fig 5.25 for spacings of Δ_x and $2\Delta_x$ for the complete frequency bandwidth of 0-1024 Hz. It is observed that beyond the fourth peak a significant difference in the FRF's occurs. This implies that a spacing of $2\Delta_x$ is probably too large. Since the noise in the FRF corresponding to the smallest spacing Δ_x is quite small, the smaller spacing produces a FRF with an acceptable noise level. In view of these observations, the FRF $\mathbf{H}_{g,6}^*$ obtained with a spacing of Δ_x was used in the parameter estimation of the 5th to the 8th bending modes.

No specific unit is provided for the vertical axis of Figs. 5.23 and 5.25. This is also the case for all subsequent rotational FRF's. The unit of the vertical axis is proportional to $(\text{rad/s}^2)/N$ or $(\text{rad/s}^2)/(N\cdot m)$ depending on whether the plot represents 'rotational acceleration/force' FRF's, labeled rotation/force, or 'rotational acceleration/moment' FRF's, labeled rotation/moment. The exact values for the ordinate is not important in this context since these plots are included to provide an indication of the quality of these derived data.

Estimation of Modal Parameters in Rigid Support Configuration

The modal parameters of the first four bending modes of component 2 were obtained by curve fitting the $\mathbf{H}^*$ related FRF's with a frequency bandwidth of 0-256 Hz. Bending modes 5 to 8 were derived from the FRF's with the other frequency bandwidth, i.e. 0-1024 Hz. All the corresponding $\mathbf{H}$ FRF's are from test runs for which the accelerometers at d.o.f. 15 to 18 were fixed to the beam; meaning that all translational and rotational $\mathbf{H}$ FRF's constitute a consistent data set.

The residues of the first four bending modes were estimated by curve fitting the $\mathbf{H}^*$ related FRF's with respect to the force at d.o.f. 6 and responses at d.o.f. 2 to 12, and d.o.f. 1001 (rotation), as well as at the six side accelerometers. Table 5.3 presents the residues obtained from complex mode analysis. The phase shifts for modes 2 to 4 are very good, being generally within 2 degrees from 90 or -90 degrees. The phase shifts for the first mode are not as good, although they are still fairly good with a maximum deviation of only about 4 degrees from 90 degrees. This implies an error of about 1% in the mode shape. The error may be even less than this since all the elements are off by about the same phase and the mode shape is to be mass normalized. The residues from real mode analysis were also computed using the same poles (frequency and damping). The real modes were used in the CMS.

The poles of the 5th, 7th and 8th bending modes were obtained by curve fitting the $\mathbf{H}^*$ related FRF's associated with the same response and force d.o.f. as above. The poles of the 6th bending mode and of the 3rd torsion mode were obtained using simultaneously the FRF's with respect to the forces at d.o.f. 6 and 206 and responses at the same d.o.f. as mentioned in the preceding paragraph. The simultaneous curve fitting of these FRF's was required on account of the strong coupling between the bending and torsion modes. After the estimation of poles, the complex and real mode residues were derived. Table 5.4 presents the complex mode residues. The residues for the 5th mode at 347 Hz are very good being normally within 1 degrees from +/- 90 degrees. The residues for the other modes are also fairly good, with the deviation in phase angle from 90 being generally less than 5 degrees.

A comparison of the experimental frequencies with their analytical counterparts is presented in Table 5.5. The analytical frequencies were obtained from finite element analysis of the beam modeled with 36 beam elements of equal length and with two d.o.f. at each node. These analytical frequencies were verified with another FE model which had twice as many elements and yet gave the same frequency values for these modes. The last column of Table 5.5 gives the percentage error between test-based and analytical frequencies taking the latter as the reference. The errors for all the modes are acceptable being around 3%, except for the 7th

mode for which the error is about 6%. All experimental values are lower than their analytical counterparts.

For an understanding of the significance of the frequency errors shown in Table 5.5, it is useful to examine the numbers shown in Table 5.6. This table presents the errors in the experimental frequencies for the first five bending modes of same structure in its cantilever configuration, that is with no flexible support at the free end of the beam. The FE model used for the comparison is very precise and has 72 beam elements. For these first five modes of this "simple" test configuration, the experimental frequencies are about 4 to 5% lower than their analytical counterpart. It is logical to reason that the major cause of the discrepancy is the lack of perfect rigidity of the fixed-end assembly and/or the floors.

In light of the information provided by the test results from the cantilever beam configuration, it is reasonable to conclude that a large part of the errors shown in Table 5.5 also arises from the flexibility of the fixed end and cannot be attributed to the transformation from the $\mathbf{H}$ to the $\mathbf{H}^*$ configuration.

Orthogonality check is performed as another means of verifying the estimated modal parameters. A lot of research has been done in the area of modal vector correlation and several orthogonality tools have been developed. Two of them will be used here to verify the experimental mode shapes. First, we define the following three orthogonality equations (Mains and Vold, 1995):

(1) Mass weighted orthogonality

$$\Phi_{fem} M \Phi_{fem} = I \quad (5.4)$$

where Φ_{fem} is comprised of columns of mass normalized analytical mode shapes, M is the mass matrix of the free d.o.f. and I is the identity matrix.

(2) Pseudo orthogonality check

$$\Phi_{exp} M \Phi_{exp} = PORTH \quad (5.5)$$

The pseudo-orthogonality check replaces the analytical modal vectors with mass normalized experimental modal vectors.

(3) Cross orthogonality check

$$\Phi_{jem} M \Phi_{exp} = XORTH \quad (5.6)$$

As evident from Eq. 5.6, the cross orthogonality check is between analytical and experimental modal matrices. If *XORTH* is an identity matrix, it means that the experimental modal vectors are orthogonal to the analytical modal vectors weighted by the analytical mass matrix.

As mentioned by Mains and Vold (1995), it is common practice to place the requirement on the modal testing that elements of the orthogonality matrices cannot differ by more than 0.1 of the corresponding elements of an identity matrix. This means that the diagonal terms must be between 0.9 and 1.1 and the off-diagonal terms must be between -0.1 and 0.1.

A pseudo orthogonality check was performed with the normalized experimental mode shapes and the reduced mass matrix of component 2. The reduced mass matrix was derived using the procedure outlined in Section 5.2.2. The elements of the resulting *PORTH* matrix are given in Table 5.7. It would be observed that the absolute values of all off-diagonal terms are less than 0.1. This means that the experimental mode shapes are consistent with each others.

The results of a cross orthogonality check are shown in Table 5.8. The requirement on the diagonal terms is respected for all modes, since all these terms are between 0.9 and 1. For the first three modes, the requirement placed on the off-diagonal terms are also respected. However, this requirement is not respected for some of the higher modes, the maximum off-diagonal term being 0.25. Considering that these mode shapes are consistent with each others as shown by the *PORTH* matrix and the fact the errors in the experimental frequencies

compared to their counterparts is most likely caused by the flexibility of the fixed end of the set-up, as opposed to errors in the test data, these experimental mode shapes are still acceptable.

5.2.5 Estimation of Ψ_i and k' for Component 2 Through Static Tests

The general procedures to be used for performing the static tests have already been described in Section 4.4.2. Hence, only those details of the procedures that are specific to the testing of the two components of the fixed-fixed beam system are presented here.

The flexible support was disconnected and removed. In order to attach the load cell to the test article, a third hole was drilled at the free end of the beam, right along the centerline. Since in a cantilever position, there is no bending moment at the free end and since the static tests were performed after the modal testing had been completed for both components, it was concluded that the addition of a hole at the free end would have negligible effect on the test results. This additional hole allowed the direct attachment of the load cell to the beam, instead of through an adapter.

The vertical displacements at all points, including the interior d.o.f., were measured with a digital indicator having an internal spring, since this was the only device available at the time of the tests. These measurements were made one location at a time. The picture in Fig. 5.26 shows both the load cell at the free end of the beam and the indicator positioned at an interior point close to that end. For d.o.f. 1 located at the free end of the beam, the reaction force of the spring obtained through its calibration equation was directly added to the measured applied force to get its corrected value. This direct addition could not be done for the interior points. Since the effect of this spring reaction force is not negligible, especially for the interior d.o.f. closest to the free end of the beam, it has been accounted for by computing the corresponding force it would create at the supported end of a fixed-supported beam. The equation for the reaction at the supported end due to a concentrated force along the length of the beam is given by (CISC, 1989)

$$R = \frac{P b^2(a+2L)}{2 L^3} \quad (5.7)$$

where R is the reaction force at the supported end, P is the concentrated reaction force of the spring, L is the length of the beam, a and b are the distance of the indicator to the supported and fixed ends respectively. Force R is added to the measured applied force for each sample taken. This addition of forces is valid as long as the supports, i.e. the loading set-up in this case, behaves linearly during the additional displacement caused by the spring reaction force. This linearity assumption is reasonable considering that this additional displacement is very small.

For displacement measurement at each d.o.f., ten different force levels were applied at the supported d.o.f. The relationship between measured displacement and applied load was consistent throughout the acquired data sets and was linear. Figure 5.27 shows the test data points identified by circles as well as the associated best fit line for the displacement at d.o.f. 7. The quality of the plot is typical of all the points except for d.o.f. 11 and 12. The data set for displacement at d.o.f. 12 is plotted in Fig. 5.28. Although this plot is the worst in terms of the data points deviating from a straight line, it is still extremely good. The slight fluctuation about the line is easily understandable, considering that the largest applied force displaced the beam at this location by only about 0.3 mm.

The rotation at the interface is estimated through the forward finite difference expression which may be written, for the static situation, as

$$\theta_i = \frac{-3\Delta_1 + 4\Delta_2 - \Delta_3}{2 \Delta} \quad (5.8)$$

where Δ_i is the measured vertical displacement at the d.o.f. selected to correspond to point i , $i=1, 2$ or 3 ; and Δ is the distance between two adjacent points. Point 1 always corresponds to d.o.f. 1 at the beam interface. All selected points must be equidistant. For the static tests where the radius of curvature of the beam is very large, it was found to be acceptable to use the already instrumented d.o.f. 2, 3, 4, etc. as the points of the finite difference expression. This

eliminated the need for additional measurements at equidistant locations closer to the interface of the beam.

The analytical reference displacement values were computed from the following "exact" formulation of the displacement along a cantilever beam for a force applied at the free end

$$\Delta_x = \frac{P}{6EI}(2L^3 - 3L^2x + x^3) \quad (5.9)$$

where Δ_x is the displacement caused at a distance x from the free end, P is the applied force (equal to unity in our application), L is the length of the beam, EI is the flexural rigidity of the beam computed from measured beam properties.

By definition, for this simple configuration, the coefficient of the constraint mode Ψ_i for the d.o.f. at a distance x from the free end is given by Δ_x divided by Δ_0 . Applying Eq. 2.74 to the present configuration and considering an applied force equal to unity, $\mathbf{k}^* = \Delta_0^{-1}$.

The estimated test-based static parameters and their reference values are given in Table 5.9. The upper portion of the table shows the coefficients of Ψ_i for the translational d.o.f. 2 to 12. One observes that the error increases with the d.o.f. number and as the corresponding measured displacement decreases. The errors for the first 8 locations are quite small, being less than 5%. The errors at d.o.f. 9 to 11 are higher, but still fairly low. The error at d.o.f. 12 is quite large, but still acceptable considering that the measured displacement is very small. In fact, since this coefficient is very small, a large error in it should not make a significant difference in the subsequent computations involved in the CMS.

Four different estimates are shown for the rotational coefficient of Ψ_i . The first one uses d.o.f. 2 and 3 as points 2 and 3 in Eq. 5.8; the second one uses d.o.f. 3 and 5; the third one uses d.o.f. 4 and 7; the 4th one uses d.o.f. 5 and 9. All cases produce very good estimates of the rotational coefficient. Since in a real test situation, one would not know the exact value in order to decide which estimate to select for the CMS computation, the following selection criteria is defined: the selected rotational estimate should correspond to the Δ for which the difference

with the adjacent estimates is the smallest. This criteria leads to the selection of the third estimate. The error on k^* is very small, being less than 1%.

5.2.6 Estimation of H^* for Component 1 Through Modal Testing

Component 1 of the fixed-fixed beam system was the second test article on which modal testing was performed in order to derive the modal parameters in the H^* configuration.

Test Description, Instrumentation

The test set-up and the instrumentation are basically the same as for component 2 with the following modifications. The length of the beam is 305 mm (1 foot) longer than component 2, so the test article was pulled out by that distance from the fixed-end assembly in order to get its total length of 2.13 m. In addition to the already installed 22 accelerometers, two extra ones were fixed on the centerline of the beam at the same spacing of 152 mm as before. These accelerometers were at d.o.f. 13 and 14, with the latter one being closest to the fixed end.

Observation of the analytical mode shapes did not clearly indicate which location would serve as the best driving point for properly exciting all bending modes of interest. Consequently, modal surveys were performed with an exciter located sequentially at d.o.f. 7, 9, 10 and 11. Figure 5.29 shows the test set-up with the exciter at d.o.f. 11. As before, d.o.f. 1 was also a driving point since it is along a supported d.o.f. Each test run was performed with an exciter underneath the flexible support and the other one at an interior driving point. Data acquisition was performed with the same two different bandwidths: 0-1024 Hz and 0-256 Hz. The upper limit of the first bandwidth was high enough to include nine bending modes in the rigid support configuration. The selection of the main interior driving point was based on how well each of the bending modes in H^* configuration was excited. The main interior driving point was selected to be at d.o.f. 9. The other three interior driving points could serve for reciprocity checks.

Data Obtained from Flexible and Rigid Support Configurations

Observation of individual FRF from the test configuration and coherence functions did not show any anomaly. An example of reciprocity checks between two interior driving points, d.o.f. 7 and 9, is shown in Fig. 5.30. The reciprocity is excellent up to about 850 Hz. It is of interest to note that these two FRF's are obtained from two different test runs.

A reciprocity check between the same two driving points is performed at the $\mathbf{H}^*$ level, the results are presented in Fig. 5.31. The reciprocity is excellent throughout the whole frequency range, except for a small bandwidth on the left side of the antiresonance at 474 Hz. The peaks for the first nine bending modes are clearly visible. The curves are very smooth without any significant glitches, except a few on the right side of the 8th mode. The very broad shape of the peak would imply a large damping if the peak were to be related to a single mode. On the other hand, the phase of the FRF's in that region seems to indicate the presence of only one mode. The curve fitting phase of the process should clarify the situation.

The rotational FRF $\mathbf{H}_{\theta,9}^*$ is shown in Fig 5.32 for the bandwidth 0-1024 Hz. The spacing of the points used in the finite difference is Δ_x . The smoothness is fairly good after the 4th bending mode. The noise level for the bandwidth of 0-250 Hz is of no concern since these first four modes are estimated from the FRF's with a bandwidth of 0-256 Hz and the $\mathbf{H}_{\theta,9}^*$ obtained with a spacing of $2\Delta_x$. The resulting $\mathbf{H}_{\theta,9}^*$ is shown in Fig. 5.33 and is seen to be quite smooth.

Estimation of Modal Parameters in Rigid Support Configuration

The modal parameters of the first four bending modes were obtained by curve fitting the $\mathbf{H}^*$ related FRF's with a frequency bandwidth of 0-256 Hz, while the higher modes were derived from the FRF's with the other frequency bandwidth, i.e. 0-1024 Hz. Data from all accelerometers, except those at d.o.f. 15 to 18, with respect to the force at d.o.f. 9 were used for the curve fit. Table 5.10 presents the residues of the first 4 bending modes while Table 5.11

shows the residues for modes 5 to 7, as obtained from complex mode analysis. The observed phase shifts show that the residues are excellent for all the modes, except for modes 4 and 7. Even for those modes, the residues are quite good, except for those shape elements of mode 7 which have a small amplitude. All attempts to get a reasonable curve fit to the suspicious 8th mode failed. The curve fitting algorithm did not seem to be able to stabilize on that mode, indicating that something was probably wrong with the data in that frequency range. Limiting the number of component modes to 7 in the input to CMS algorithm was found to be adequate for providing reasonably accurate estimates of the system modes. No additional attempt was therefore made to derive the 8th mode.

A comparison of the experimental frequencies with their analytical counterparts is presented in Table 5.12. The analytical frequencies are obtained from a FE analysis for the beam modeled with 36 beam elements of equal length and with six d.o.f. at each node. The errors are at about the same level as for component 2, none of them being more than 3.5%. Again, the experimental frequencies are systematically lower than the analytical values, and the discrepancy may reasonably be attributed to the flexibility in the fixed-end assembly.

The orthogonality checks for the experimental mode shapes are presented next. Table 5.13 shows the elements of the pseudo orthogonality check matrix. The absolute values of all off-diagonal terms are significantly less than 0.1. This means that these mode shapes are consistent with each others.

The results of the cross orthogonality check are shown in Table 5.14. The requirement on the diagonal terms is satisfied for all modes, since all these terms are between 0.9 and 1. For the first three modes, the requirement placed on the off-diagonal terms is also satisfied. However, this requirement is not satisfied for some of the higher modes, with the maximum off-diagonal term being 0.19. For the same reasons as for component 2, these experimental mode shapes are still considered to be acceptable.

5.2.7 Estimation of Ψ_r and k^* for Component 1 Through Static Tests

The procedures for the static tests of component 1 were the same as for component 2, with displacements measured at the two additional points, d.o.f. 13 and 14.

The estimated test-based static parameters and their reference values are given in Table 5.15. The error level is generally a bit smaller than for component 2. The errors for translational d.o.f. 2 to 10 are quite small, being less than 5%. The errors at d.o.f. 11 to 13 are higher, but still fairly low. The error at d.o.f. 14 is quite large, but still acceptable considering that the measured displacement is very small.

Five different estimates are shown for the rotational coefficient of Ψ_r . The first four use displacement measurements at the same d.o.f as for component 2. The 5th estimate has d.o.f. 6 and 11 as points 2 and 3 of the finite difference expression. The application of the criteria, defined previously, leads to the selection of the third estimate of the rotation which has an error of less than 2%. The error in k^* is again very small, being less than 1%.

5.2.8 Component Mode Synthesis

This section presents the results of the experimental verification of the hybrid CMS method for the fixed-fixed beam system using the test-based component modal and static parameters discussed in the previous sections. The only analytical parameter required by the approach is the reduced component mass matrices.

In the error estimates presented in this section, the following two sets of estimated system parameters are compared with reference values and error measures are obtained by using Eqs. 3.1 and 3.2.

- (1) A set of estimated system parameter values was obtained by applying the CMS approach with component parameters determined from modal and static tests. The system parameters obtained directly from modal testing of the system are used as the reference values. The corresponding errors are shown in Figs. 34a and 34b.

- (2) An alternative set of estimated system parameter values was also obtained from CMS, but in this case the component parameters were determined analytically. The CMS configuration used in this case was identical to that used with the first set. The reference modal parameters of the system are obtained from a finite element analysis. The computed error estimates are shown in Figs. 35a and 35b, and are referred to as simulation errors. The alternative set of parameters is not required by the experimental hybrid CMS approach. It is added only in order to get a better appreciation of the reliability of the test-based CMS.

Errors in the estimated system frequencies are shown in Fig. 5.34a for two cases of different number of retained normal modes for each component. The CMS system frequencies are obtained by completely ignoring the residual flexibility terms. As mentioned previously, this is implemented by replacing the elements of vector G_{aa} by very small numbers ($10e-15$).

When 7 and 8 modes are retained for components 1 and 2 respectively, the system frequency error for the first mode is only 2.7%. This value is very good, especially since it is obtained by completely neglecting residual flexibility effects. For the next 7 modes, the errors are again very small, being less than 2%. When only 4 + 4 modes are retained, the frequency error for the first mode is 4.5%. As could be expected, this number is somewhat higher than when more normal modes are retained. However, this number is still quite acceptable, especially when considering that the error in the simulation carried for the same case was still 3.3% (Fig. 5.35a). The frequency errors for the next 6 modes are again quite small, less than 2.5%. The frequency error in the 8th mode is nearly 6%, but is still acceptable. In fact, this error is smaller than the corresponding simulation value computed at 8.3%.

Errors in the estimated system mode shapes shown in Fig. 5.34b have the same trend as the frequencies, but are somewhat higher as expected. For the case of 7 + 8 retained modes, the errors are fairly small for all the system modes, although it becomes close to 18% for the 8th mode. When only 4 + 4 modes are kept, the errors are still acceptable for the lower modes. By comparing these errors with the corresponding values obtained from the simulation shown in

Fig. 5.35b, one observes that both sets of errors are basically in the same range, even for the higher numbers obtained for the case of 4 + 4 kept modes.

5.3 Free-Free Beam System

5.3.1 Test Articles, Configurations and Boundary Conditions

Following the successful demonstration of the application of proposed experimental hybrid CMS approach to the fixed-fixed beam system, the logical next step in the verification process is to apply the approach to simple components involving several flexible supports. Consequently, the next system tested is the free-free aluminum beam shown in Fig. 5.36. The structure has a total length of 9.75 m (32 ft) and a nominal rectangular cross-section of 25.4 x 152 mm (1" x 6"). Here again, the large width/thickness ratio ensures that the lower modes are mainly bending modes in the vertical plane. The system is composed of components 1 and 2 having lengths of 5.18 m (17 ft) and 4.57 m (15 ft) respectively.

Each component is supported at the following locations: at the interface (d.o.f. 1), at 2.44 m (8 ft) away from the interface (d.o.f. 9) and at 0.3 m (1 ft) from the other extremity of the test articles (d.o.f. 17 and 15 respectively). The interface of each component consists of two d.o.f.: vertical translation (d.o.f. 1) and in-plane rotation (d.o.f. 1001). The component fixed interface d.o.f. in CMS corresponds to d.o.f. 1, while the free interface d.o.f. corresponds to d.o.f. 1001.

The hybrid CMS method needs, as input, the following three sets of modes: 1) the constrained or rigid support normal modes, 2) the constraint modes, and 3) the residual flexibility attachment modes. The first three constrained normal modes of component 1 are depicted in Fig. 5.37, while its three static constraint modes are depicted in Fig. 5.38.

In order to obtain the required modal parameters (normal modes and residual flexibility terms) in the rigid support or H^* configuration depicted in the schematic of Fig. 5.39a, modal

testing is performed on the component in a test set-up with a flexible support supporting the beam along the three supported d.o.f., as depicted in the schematic of Fig. 5.39b.

In order to obtain the static constraint modes, each of the flexible supports is disconnected in turn and a force is applied along the supported d.o.f. for which the support is disconnected.

The mode sets of component 2 required for CMS are the same as those for component 1, and both components have similar test set-up.

Due to practical problems in procurement of a 9.75 m long beam, the reference system modal parameters were estimated through finite element modeling, instead of through modal testing as for the fixed-fixed beam system.

A second reason for not carrying out a modal test on a free-free system was the practical difficulty involved in such a test. As discussed in Chapter 1, free-free modal testing of flexible structures is not a simple matter due to the required frequency separation between the highest suspension frequency and the fundamental mode of the test article. The simplest way of testing such a long beam would be to suspend it from its upper edge by at least two long cables so that the beam would be horizontal with its bending modes of interest vibrating in the horizontal plane. Since the lowest frequency of the beam is around 1.4 Hz, such testing would require the highest suspension frequency to be about 0.14 Hz. Even if one is able to satisfy this requirement, other problems such as double pendulum motion could arise and complicate the measurement of the lowest modes.

Besides the procurement and testing difficulties just discussed, another point favoring the analytical estimation of the system modal parameters as the reference values was the fact that these parameters may be estimated quite precisely in the case of a free-free beam, especially if the model is based on measured physical and static properties as was the case here.

5.3.2 Analytical modeling

Description of FE Models and Their Role

The analytical mass and stiffness matrices for the free-free beam system and the two components were obtained from FE models with equidistant nodes at a spacing of 305 mm (1 ft). The system model thus has a total of 33 nodes. Components 1 and 2 have respectively 18 and 16 nodes.

As shown in Fig. 5.36, only in-plane motion is of interest. All three models were created by performing the same types of reduction, as described for the fixed-fixed beam system, from initial models consisting of two d.o.f. per node. This is again done in order to obtain final models for which all d.o.f. correspond to those at which the responses were measured (or derived from measurements as in the case of rotations) during component tests. Each node of the system has one d.o.f, vertical translation. For each of the components, the node located at the interface has two d.o.f, vertical translation and rotation; the interior points have only one d.o.f, vertical translation.

The matrices of the FE models play the following roles during the experimental work on the free-free beam system:

- (1) The mass matrices of the two components are required in order to implement the experimental hybrid CMS approach;
- (2) The mass and stiffness matrices of the system are required to compute the reference modal parameters to which the approximate CMS parameters are compared;
- (3) The mass and stiffness matrices of the two components are needed to perform a simulation of CMS, in which the exact component parameters are used to obtain the system parameters.

Experimental Estimation of the Physical Properties of the Beam

The physical and elastic properties, that is the width and thickness of the cross-section, the material density and the Young's modulus, required in the generation of the FE models were obtained through measurement.

Before describing the estimation of these physical parameters, it is appropriate to discuss the sequence in which the two components were manufactured and tested. First, component 1 was manufactured by cutting a 5.18 m long beam from the 6.1 m nominal length of the purchased aluminum bar. A hole was then drilled through the thickness of the beam on both sides of the centerline, at the three cross-sections corresponding to d.o.f. 1, 9 and 17. These holes were required to attach the connection units of the flexible supports. The next step consisted in performing all modal and static tests on component 1, and in confirming the acceptability of the test results. Next, the 5.18 m beam was cut at 0.61 m (2 ft) from the end corresponding to d.o.f. 18, in order to create component 2. A hole was drilled on each side of the centerline at the cross-section of d.o.f. 15. Finally, modal and static tests were performed on that component.

Cross-section measurements were taken at 33 locations along the longer beam component. These locations started at the extremity of the beam corresponding to d.o.f. 1 and were equally spaced at 152 mm (6 "). They spanned most of the length of component 1 and the full length of component 2. At each of these cross-sections, a measurement of the width and thickness were taken. At different locations, the thickness measurement was made at different points along the cross-section, in order to obtain an average value across the width of the beam. An average of each of these dimensions was estimated and served as input parameter for the FE models. The averaged width was 152.5 mm (6.005") and the averaged thickness was 26.01 mm (1.024").

The density of the material was estimated from two different samples cut from the extra length of the purchased bar. Several measurements were taken of the lengths, widths and thicknesses of these samples to obtain their averaged dimensions. From these dimensions and the

weight of the samples, estimates of the density were obtained. The density of both samples was estimated to be 2668 kg/m^3 (0.0964 lb/in^3). This value was input as the material density to the FE models.

In order to estimate the Young's modulus of the test articles, four small blocks were cut from different locations along a cross-section of one of the two samples used for the density. From these four specimens, 400 estimates of the modulus were obtained. The average value obtained from these estimates was 70.4 GPa. This value was input as the Young's modulus to the FE models.

5.3.3 Estimation of H^* for Component 1 Through Modal Testing

Test Description, Instrumentation

In test configuration, flexible supports were attached at three d.o.f., 1, 9 and 17. A total of 28 accelerometers were fixed in a vertical direction on the upper face of the beam (Fig. 5.40). These accelerometers may be divided into the following three groups:

- (1) Eighteen accelerometers on the centerline of the beam at every 305 mm (1 foot). These sensors are located at d.o.f. 1 to 18. The accelerometers at d.o.f. 1, 9 and 17 are fixed between the two legs of the connection units.
- (2) Six accelerometers on each side of the beam at three different cross-sections corresponding to d.o.f. 2, 8 and 16. These sensors are added to identify the torsional modes of the component. These six accelerometers also serve to identify any coupling between the bending and torsion modes through mode shape animation.
- (3) Additional accelerometers along the centerline of the beam at distances of 50.8 mm, 101.6 mm, 203.2 mm and 406.4 mm from d.o.f. 1; the corresponding d.o.f. are 19 to 22 respectively. These sensors are used for the estimation of rotational FRF's.

Figure 5.41 presents a close-up of the flexible support at d.o.f. 17. Also visible in the picture are the accelerometers at d.o.f. 16, 116 (side of 16), 17 and 18, as well as the exciter located underneath the flexible support. A horizontal restraint located right at the end of the beam can be seen in the background.

The two interior driving points were selected to be at d.o.f. 6 and 11, with the latter one being the main interior driving point. In fact, almost all $\mathbf{H}^*$ related FRF's were obtained with respect to d.o.f. 11. In addition to the above, driving points must be located at d.o.f. 1, 9 and 17 since they are supported d.o.f. Each test run was performed with an exciter underneath a flexible support and the other one at the interior driving point of interest. It may be noted that in order to obtain one set of $\mathbf{H}^*$ related FRF's, three different runs would have to be performed. For example, in tests related to the determination of $\mathbf{H}_{x,6}^*$, the first run would give $H_{x,1}$ and $H_{x,6}$ where x are all interior d.o.f., the second run would give $H_{x,9}$ and $H_{x,6}$ and the third run would give $H_{x,17}$ and $H_{x,6}$. Only one out of the three estimates of $H_{x,6}$ is utilized in calculating $\mathbf{H}_{x,6}^*$. In fact, if extraneous peaks or glitches appear in the estimation of the $\mathbf{H}^*$ related FRF's, this extra data can be used to identify the real dynamic characteristics of the test article, since the peaks of the $\mathbf{H}^*$ related FRF's corresponding to real modes are expected to be independent of the selection of $H_{x,6}$, while extraneous peaks or glitches will often vary slightly depending of which estimate of $H_{x,6}$ is used.

In addition to the above two interior driving points, forces were also applied at d.o.f. 19, 20, 21 and 22. Each of the additional test runs was performed with an exciter at one of these four interior d.o.f. and the other exciter at d.o.f. 11. These runs were made in order to estimate, through the finite difference expressions given by Eqs. 5.2 and 5.3, the $\mathbf{H}^*$ related rotational FRF's (response/moment). These response functions are required for the estimation of residual flexibility terms used for the residual flexibility attachment modes.

An overview of the test set-up is shown in Fig. 5.42. All units of the flexible supports attached to the test article at d.o.f. 1 and 9 are clearly visible. Also seen are the two exciters

located at d.o.f. 11 and 17. The accelerometer cables are visible, although the sensors are not that discernible.

Data acquisition was performed with two different bandwidths: 0-256 Hz and 0-64 Hz. The upper limit of the first bandwidth was high enough to include nine bending modes in the rigid support configuration. However, its frequency resolution was such that it was difficult to obtain a good curve fit to the first two modes. In view of this, the second frequency bandwidth was added, providing a frequency resolution which was four times higher.

Three horizontal restraints stabilized the test article with respect to the center of the shafts for the linear bearings of the flexible supports. The two restraints preventing lateral motion were located between d.o.f. 3 and 4, and between d.o.f. 14 and 15. The one restraint preventing longitudinal motion was located at the end of the beam, d.o.f. 18, along the centerline. All horizontal restraints were oriented horizontally and touched the beam at mid-thickness. They all had a length of approximately 240 mm, unless otherwise specified.

During testing, it was found that the quality of test data could at times be improved by changing the flexibility of the supports. Following are examples of the type of adjustments needed.

During the first set of runs, the curve fit of the H^* related FRF's gave very good results with credible phase shifts for eight of the nine bending modes. For the second mode, the curve fitting algorithm could not stabilize and gave unacceptable phase shifts. In fact, the FRF's showed a secondary peak which appeared just beside that mode. It was found that by simply increasing the flexibility of the supports, through an increase in the free span of their upper sections, the secondary mode disappeared and a very good curve fit could be achieved. The second mode is the only one for which almost all the mass of the test article is moving in phase (Fig. 5.37). A possible explanation for the initial problem in estimating this mode and the success of the solution found for it is that inertia moving in one direction caused significant nonlinear displacement in the bearings, most likely in the thrust bearings. An increase in the flexibility of the supports resulted in higher displacements at the points at which they were

connected to the beam, thus making any nonlinear displacement negligible in comparison to the overall motion.

It was also found that the phase shifts for the 8th mode obtained from the curve fit were not as good with the supports of lower flexibility as they were with the supports of higher flexibility. The quality of the other modes was found to be independent of the flexibility of supports.

The experience gained with this and other components showed that the problems which appeared in a few occasions in curve fitting $\mathbf{H}^*$ related FRF's could easily be solved by changing the stiffness of the flexible supports.

Data Obtained from Flexible and Rigid Support Configurations

Observation of individual FRF data from test configuration and coherence functions did not show any anomaly. The coherence functions were close to unity except at or near antiresonances of the corresponding FRF or at frequencies for which significant noise level appears in the FRF.

The FRF curves for excitation at d.o.f. 6 and response at d.o.f. 11, and excitation at d.o.f. 11 and response at d.o.f. 6 are plotted in Fig. 5.43 to verify the reciprocity of data. The data was obtained during one of a set of tests carried out on the test article. The reciprocity is fairly good with the major peaks overlapping nicely, except in the frequency bandwidth of 15-30 Hz where an extra peak occurs at 17.6 Hz for $H_{11,6}$ and two of the smaller peaks are slightly shifted.

The corresponding plots for the reciprocity check at the $\mathbf{H}^*$ level are given in Fig. 5.44. The reciprocity is excellent throughout the whole frequency bandwidth with the nine higher peaks, corresponding to the bending modes, overlapping exactly. The only frequencies for which the two curves are not overlapping completely are related to glitches or extraneous peaks such as at 121 Hz.

A superposition of the FRF's for the force and response at d.o.f. 11 in both the flexible and rigid support configurations is presented in Fig. 5.45. Here again, as expected, one observes that the peaks between the two configurations have shifted in addition to the fact that there are more modes present in the test configuration due to the dynamics of the flexible supports. These peaks are eliminated in the process of transformation to the $\mathbf{H}^*$ rigid support configuration.

The $\mathbf{H}^*$ related FRF for the force and response at d.o.f. 11 is shown in Fig. 5.46. The nine bending modes are identified. One may observe that the derived curve is very smooth with all the modes well defined. The only minor exceptions are the glitches and the extraneous peak around 30 Hz. In fact, this curve and the FRF's corresponding to the other translational d.o.f. are of a better quality than the plots at the $\mathbf{H}^*$ related FRF's for the components for the fixed-fixed beam system, in the sense that they have fewer glitches and extraneous peaks.

The rotational FRF $H_{\theta,11}^*$ is shown in Fig 5.47. This curve was derived from the finite difference expression applied to translational d.o.f. 1, 19 and 20. The spacing between these points was $\Delta_x = 50.8$ mm. Except for the first two modes, all peaks are well defined without significant noise. As discussed later, the first four modes are estimated from the curves with the higher frequency resolution, i.e. with the bandwidth of 0-64 Hz. Figure 5.48 presents the corresponding $H_{\theta,11}^*$ FRF with a spacing between the points of $2\Delta_x$. One may observe that the noise level between the first three modes is lower than in the previous plot. In fact, only the first mode is a bit noisy, although the level of noise is quite acceptable. Extraneous peaks occur at 14 and 26 Hz. However, from an observation of the translational $\mathbf{H}^*$ related FRF's, it is apparent that these extraneous peaks do not correspond to any bending modes of the test article. In fact, for these two frequencies, the translational FRF's exhibit either a smooth curve or, at worse, a small glitch.

The rotational FRF's $H_{\theta,11}^*$ and $H_{11,m}^*$ are plotted in Fig. 5.49 to verify the reciprocity of data. Both curves were derived from the finite difference expressions applied to translational d.o.f. 1, 20 and 21 (i.e. with a spacing between the points of $2\Delta_x$). Although the curve $H_{11,m}^*$ is

noisy up to about 25 Hz, the reciprocity is quite good with all peaks corresponding to the first four bending modes overlapping.

The rotational FRF $H_{\theta,m}^*$ obtained with a higher resolution frequency bandwidth (0-64 Hz) and for a spacing between the points of $2\Delta_x$ is shown in Fig. 5.50. The curve exhibits drive point characteristics with resonance and antiresonance alternating as the frequency increases. The fact that the response function is quite noisy around the first two modes is not that much of concern since this curve is needed only for the estimation of residual flexibility term, as opposed to identifying the characteristics of the individual modes, and it is not therefore necessary for the whole frequency bandwidth of the curve to be usable. In fact, it turns out that the use of only the upper half of the frequency bandwidth was sufficient for obtaining a good estimate of the residual flexibility term between rotation and moment for the case when the first four bending modes were retained.

Estimation of Modal Parameters in Rigid Support Configuration

The modal parameters of the first four bending modes were obtained by curve fitting the H^* related FRF's with respect to the force at d.o.f. 11 and with the frequency bandwidth of 0-64 Hz. Data from all accelerometers, except those at d.o.f. 19 to 22 were used. The selected rotational FRF was derived from translational measurements at three points with separation of 101.6 mm, i.e. at d.o.f. 1, 20 and 21. The estimated complex residues for these modes are shown in Table 5.16. The phase shifts of the residues are very good, being normally within about two degrees of ± 90 .

The modal parameters of bending modes 5 to 9 were obtained by curve fitting the H^* related FRF's with the same excitation point, that is d.o.f. 11, but with a frequency bandwidth 0-256 Hz. The selected rotational FRF was derived from translational measurements at three points separated by 50.8 mm, that is at d.o.f. 1, 19 and 20. The estimated complex residues for these modes are shown in Table 5.17. The phase shifts for modes 5, 6 and 8 are very good,

being normally within two degrees of ± 90 . The phase shifts for modes 7 and 9 are still fairly good since they are within about three degrees of ± 90 . In preparation for CMS analysis, real mode residues were also estimated from the same poles for these nine modes.

A comparison of the experimental frequencies of component 1 and their analytical counterparts is presented in Table 5.18. The analytical frequencies are obtained from FE analysis for the beam modeled with 34 beam elements of equal length and with two d.o.f. at each node. The physical parameters of the model, obtained experimentally, are given in Section 5.3.2. The differences between the experimental and analytical frequencies are very small, being no more than 1% for all nine bending modes.

The orthogonality checks for the experimental mode shapes are presented next. Table 5.19 contains the values of the pseudo orthogonality check matrix. The absolute values of all off-diagonal terms are significantly less than 0.1, thus satisfying the standard requirement commonly placed on modal testing. This means that these mode shapes are consistent with each others.

The results of the cross orthogonality check are shown in Table 5.20. The requirement on the diagonal terms is satisfied for all modes since all these coefficients are between 0.9 and 1. The requirement placed on the off-diagonal terms are also respected, since their absolute value are significantly less than 0.1.

These results clearly demonstrate that FRF's for a rigid support configuration can be derived quite effectively from tests on a configuration in which the test article is held by supports of arbitrary flexibility. The test articles were purposely chosen to be simple, because in a complex structure it may not be possible to develop accurate analytical values for comparison.

Repeatability and Effect of Horizontal Restraints

Several additional tests were performed in order to confirm that the three horizontal restraints did not have any significant effect on the dynamic characteristics of the test article.

The approach was to repeat the tests in identical configurations but with changed stiffness of the horizontal restraints, and to look for possible shifts in the frequencies.

The test results, not presented here, showed that the horizontal restraints did not affect the dynamic characteristics of the test article.

Estimation of Residual flexibility

An estimation of the columns of the residual flexibility matrix associated with the free interface d.o.f., G_{da} , is required for computing the transformed component mass matrix given by Eq. 2.34a. As mentioned earlier, the residual flexibility terms arise from the presence of the residual flexibility attachment modes as Ritz vectors in the hybrid CMS method and they correspond to the flexibility of the component associated with the neglected higher modes.

The residual flexibility terms may be extracted simultaneously with the mode shape residues in Step 2 of modal parameter estimation as implemented in the LMS software. However in this project, they are estimated separately in a post-analysis phase following the derivation of the residues, in order to get better values. These residual flexibility terms are obtained by implementing the method detailed in the report by Bookout (1995) and developed in the context of deriving constrained modal parameters based on free-free modal testing. This experimental method is based on the development by Rubin (1975) for expressing the residual terms. Only an outline and an example of the method is presented in this section. Details of the technique are available in the two references cited above.

Let the FRF matrix, Y_a , serves as a transfer function to obtain the response to a given set of applied forces, so that

$$U_a(\omega) = Y_a(\omega) F_a(\omega) \quad (5.10)$$

where U_a is the vector of response displacements and F_a is the vector of applied forces.

Application of standard parameter estimation technique to the FRF's in the frequency bandwidth of interest provides the modal parameters (frequencies, damping values and mode shapes)

of the retained modes as well as the lower residuals (related to rigid body motions). Synthesized analytical FRF's are then computed from these extracted parameters. On subtracting the synthesized FRF's from their corresponding measured FRF's, we obtain a remainder function matrix that is referred to as the residual function matrix $Y_{\rho\sigma}$.

Rubin (1975) has derived an expression for the residual function matrix, using the contribution of the residual flexibility, as well as the inertial and damping effects of the neglected modes. He showed that the complex matrix $Y_{\rho\sigma}$ may be expressed as

$$Y_{\rho\sigma}(\omega) = (\omega^2 H_{\rho\sigma} + G_{\rho\sigma}) - i\omega B_{\rho\sigma} \quad (5.11)$$

where $G_{\rho\sigma}$ is the residual flexibility matrix, and $H_{\rho\sigma}$ and $B_{\rho\sigma}$ are the residual mass and residual damping matrices respectively. From Eq. 5.11, residual flexibilities may be defined as the value of the residual functions evaluated at zero frequency.

The residual mass and flexibility terms are obtained from the real part of the residual functions

$$\text{Re}(Y_{\rho\sigma}) = \omega^2 H_{\rho\sigma} + G_{\rho\sigma} \quad (5.12)$$

As stated by Bookout (1995), determination of the residual mass and flexibility terms associated with a *theoretically perfect* residual function requires only a second-order polynomial curve fit of Eq. 5.12. However, due to noise in the measured FRF and to errors in the synthesized FRF attributed to the limitation in the evaluation of modal parameters, a typical residual function associated with real test has regions of ragged data. Consequently, such direct curve fit would lead to wrong estimates of the residual terms.

Bookout also mentioned that a theoretical residual function as a function of frequency has the characteristics of a relatively flat line for the lower frequencies, i.e. the difference (variance) between adjacent points is small. On the other hand, in the regions of ragged data, the variance between adjacent points is large. Consequently, the influence on the curve fit of the regions in which the variance between data points is large should be minimized, while the influence of the regions with small variance should be increased. Bookout suggests that this could be achieved

by generating a weighting function W defined as the inverse of the variance of adjacent data points

$$W(j) = \frac{1}{s^2(j)} = \frac{n-1}{\sum (x_i - \bar{x})^2} \quad (5.13)$$

where $s^2(j)$ is the variance, n is the number of data points for which the variance is computed (typically between 2 to 4), x_i is the residual function value of data point i adjacent to data point j , and $\bar{x}$ is the mean of the n adjacent data points for each j .

Bookout mentioned with justification that, when calculating the weighting function, the first and last few points will not have a weighting value assigned to them and they may be assigned a value of zero. In fact, a non-zero weighting value may be assigned to the data points of only a certain portion of the frequency bandwidth.

Bookout's report contains the derivation of a mathematical expression, based on the weighting function, for the estimation of the residual mass and flexibility values.

As an example of the above concepts with real data, Fig. 5.51 presents the measured $H_{\theta,m}^*$, in terms of displacement rather than acceleration, for the frequency bandwidth 0-256 Hz. It also contains the synthesized FRF obtained from the extracted modal parameters of the first nine bending modes and the lower residual. Subtraction of these two curves gives the residual function, for which the real part is shown in Fig. 5.52 along with its second-order polynomial curve fit. As may be observed in the figure, some data points in the test-based residual function are not shown on account of them having a negative value and the choice of logarithmic scale for the ordinate.

The weighting function is presented in Fig. 5.53 along with the residual function. The data points below 45 Hz and above 218 Hz were not assigned any weighting value, due to the high level of noise in the measured $H_{\theta,m}^*$. Based on the examples presented in the report, three adjacent points were used in the present project for the estimation of the weighting functions.

5.3.4 Estimation of Ψ_i and k' for Component 1 Through Static Tests

The general procedures used in the static tests have already been described in Section 4.4.2. Details which are specific to the free-free beam system are presented here.

During the static tests, it was observed that because of the large deformation imposed on the beam test article, the structure tended to slightly rotate with respect to its longitudinal axis, on account of a rotation of the single linear bearing of each of the two support connection units. This tendency to rotate could be and was reduced by increasing the flexibility of the flexible supports. Also, all accelerometers were removed to allow the displacement measurements to be made right along the centerline of the test article, since off-center measurements may be slightly in error on account of the rotation of the beam. Figure 5.54 shows a typical test set-up in which the load cell measuring the applied load as well as the digital indicator are located at d.o.f. 9.

The external force was applied in turn at each of the three supported d.o.f., after removing the flexible support at the corresponding d.o.f. For each of these three locations, several runs were performed in order to measure the displacement at all interior and supported d.o.f. Each run involved the application of 9 incremented displacements. Since three indicators with internal springs were available, it was possible to measure simultaneously the displacement and the force at the three supported d.o.f. during a single run. Two of the indicators were fixed right underneath the two flexible supports, while the third one was located underneath the load cell that measured the external force. During each of the other runs, displacement measurements were taken at a single interior d.o.f. with the digital indicator without an internal spring fixed above the d.o.f. of interest. Figure 5.55 shows the set-up for such measurements with the indicator located at d.o.f. 8. The upper section and the connection units of the flexible support located at d.o.f. 9 are also visible in the picture.

Measured displacement versus applied load relationships, examples of which are shown in Figs. 5.56 and 5.57, were very consistent and linear throughout, except for those locations for which the motion is very small. This indicates that the connection units of the flexible supports

have a linear behavior, or at least are not behaving in a nonlinear manner in the tests. Figure 5.56 presents the plot of displacement at d.o.f. 9 versus the applied force at d.o.f. 1; the quality of the data is typical of almost all measurements. The plot for the case in which the deviation of data from a straight line is most prominent is shown in Fig. 5.57 and represents the displacement at d.o.f. 18 for the applied load at d.o.f. 1. The data are still quite acceptable considering that the maximum displacement is no more than 0.12 mm.

The reaction force versus applied load relationships were also consistent and linear. The plot of Fig. 5.58 representing the reaction force at d.o.f. 1 for the load at d.o.f. 17 is representative of the quality of data for this type of measurement.

As for the components of the fixed-fixed beam system, the rotation at the interface was estimated through the finite difference expression given in Eq. 5.8. The points used in these expressions were the already instrumented interior d.o.f.

The estimated test-based static parameters and their reference values are given in Tables 5.21 and 5.22. The reference analytical values are computed by a finite element analysis. The elements of the constraint modes with unit displacement at d.o.f. 1, 9 and 17 are shown in Tables 5.21a, 5.21b and 5.21c respectively. The first fifteen rows show the elements corresponding to the translational d.o.f. 2 through 8, 10 through 16, and 18. The test-based mode shape elements for the translational d.o.f. are very good, a majority of them being in error by no more than 1%. In fact, all translational elements have an error smaller than 2%, except for the element corresponding to d.o.f. 18 which has a small magnitude and has an error of 5.5%.

For each constraint mode, four different estimates are shown for the rotational coefficient associated with d.o.f. 1001. The first estimate uses d.o.f. 2 and 3 as points 2 and 3 in the finite difference expression; the second one uses d.o.f. 3 and 5, the third one uses d.o.f. 4 and 7, and the fourth one uses d.o.f. 5 and 9. The differences in rotation estimates obtained with two different point spacings taken in sequence are much smaller than those in the components of the fixed-fixed beam system. Therefore it is considered quite adequate to use the estimates of

rotation obtained with the smallest point separation for use in the CMS algorithm. With this selection criteria, the maximum error in the rotational elements is 5.3%.

The values of the 3×3 $\mathbf{k}'$ matrix are given column by column in Table 5.22. The reference values computed from a finite element analysis are also shown. All errors are very small, being less than 2%. It is observed that all experimental values are lower than their analytical counterpart.

5.3.5 Estimation of $\mathbf{H}'$ for Component 2 Through Modal Testing

Test Description, Instrumentation

The test set-up and the instrumentation are basically the same as for component 1 with the following modifications. Since the length of component 2 is smaller than that of component 1 by 610 mm, accelerometers at d.o.f. 18 and 17 are removed (Fig. 5.59). Also, the two side accelerometers at the cross-section of d.o.f. 16 are moved to the cross-section at d.o.f. 14. The flexible support at d.o.f. 17 in component 1 is moved to d.o.f. 15.

The frequency bandwidths used in the tests were the same as for component 1. Preliminary tests with different driving points indicated that the best results for the fundamental mode were obtained with the force at d.o.f. 12 and with support flexibility selected to be comparatively high. The higher modes, above 64 Hz, were again well estimated with the driving point at d.o.f. 12 but with more stiff supports, although the support flexibility did not seem to be as important as for the fundamental mode. Two other locations at d.o.f. 2 and 8 served for reciprocity checks.

Figure 5.60 shows an overview of the test set-up in which the three flexible supports restraining the test article are fully visible. In this picture, the exciters are located at d.o.f. 12 and 17. The three supports to which the horizontal restraints are fixed are also visible on the side and at the end of the test article.

Several combinations of exciter locations were used during the runs of the preliminary tests. However, the most interesting combination is the one with which the final tests were performed for the bandwidth 0-64 Hz, since it minimized the number of test runs required in order to get the $\mathbf{H}^*$ related FRF's. A first run was carried out with exciters at d.o.f. 8 and 15, a second run with exciters at d.o.f. 2 and 9, and a third run with exciters at d.o.f. 1 and 12. This combination resulted in the derivation of the $\mathbf{H}^*$ related FRF's with respect to the three interior d.o.f. 2, 8 and 12 from only three test runs. In order to ensure consistency of test data, a mass equivalent to a force sensor was always added at d.o.f. 2, 8 and 12 whenever these locations did not serve as a driving point.

In addition to the above three interior driving points, forces were also applied at d.o.f. 19, 20, 21 and 22. Each of the additional test runs was performed with an exciter at one of these four interior d.o.f. and the other exciter at d.o.f. 8. These runs were made in order to estimate the $\mathbf{H}^*$ related rotational FRF's (response/moment).

Data Obtained from Flexible and Rigid Support Configurations

The following comments relate to the data obtained from excitation at d.o.f. 12. The only anomaly observed in the data obtained from the tests occurred in some FRF's which exhibited high level of noise in the frequency bandwidth of 190-215 Hz as seen in Fig. 5.61 for $H_{6,12}$. In fact, this phenomenon was observed for all FRF's corresponding to response d.o.f. 1 to 9, 19 to 22, 102, 202, 108 and 208. However, the corresponding coherence functions showed only a small drop in that frequency range, lying normally between 0.9 to 1. The FRF's corresponding to all of the other locations did not show this anomaly. It was seen that all data measured at the middle flexible support location, d.o.f. 9, and at a point symmetrically located on the opposite side with respect to the driving point were somewhat noisy. A possible explanation could be related to the excitation of vibration in a bearing of the flexible support at d.o.f. 9. However,

the corresponding H^* related FRF's were very smooth without any significant noise in the frequency bandwidth under question.

For all data, the value of the coherence functions was close to unity except at or near antiresonances of the corresponding FRF. where large drop could sometime be observed, or at frequencies for which significant noise level appeared in the FRF.

Two FRF curves at the H^* level are shown in Fig. 5.62 for the purpose of reciprocity check. One of these curves refers to excitation at d.o.f. 2 and response at d.o.f. 12, while the other refers to excitation at d.o.f. 12 and response at d.o.f. 2. Except for some glitches, the reciprocity is very good throughout the whole frequency bandwidth with all the peaks corresponding to the bending modes overlapping exactly.

The rotational FRF $H_{\theta,12}^*$ is shown in Fig 5.63. This curve was derived from the finite difference expression applied to translational d.o.f. 1, 19 and 20. These d.o.f. are located at a spacing of $\Delta_x = 50.8$ mm. Except for the first mode, all peaks are well defined without significant noise. As discussed later, the first three modes are estimated from the curves with the higher frequency resolution, i.e. with the bandwidth of 0-64 Hz. Figure 5.64 presents the $H_{\theta,12}^*$ FRF derived from translational measurements at points with a spacing of $2\Delta_x$. The noise level between the first three modes is significantly reduced. An extra peak is observed at 10.7 Hz, but it is found to be insignificant on a linear scale. In the curve fitting phase, it had no effect on the residues of the first bending mode.

The rotational FRF's $H_{\theta,12}^*$ and $H_{12,m}^*$ are plotted in Fig. 5.65 to verify the reciprocity of data. Both curves were derived from the finite difference expressions applied to translational d.o.f. 1, 20 and 21 (i.e. with a spacing between the points of $2\Delta_x$). The reciprocity is quite good with all peaks corresponding to the first three bending modes overlapping nicely.

The rotational FRF $H_{\theta,m}^*$ is shown in Fig. 5.66 for the higher resolution frequency bandwidth (0-64 Hz) and for a spacing between the points of $2\Delta_x$. As expected, the curve exhibits drive point characteristics with resonance and antiresonance alternating as the frequency increases. The fact that the response function is quite noisy around the first two modes is not that

much of concern since, here again, this curve is needed only for the estimation of residual flexibility term, and not for identifying the characteristics of individual modes.

Estimation of Modal Parameters in Rigid Support Configuration

The modal parameters of the first three bending modes were obtained by curve fitting the $\mathbf{H}^*$ related FRF's with respect to the force at d.o.f. 12 and with the frequency bandwidth of 0-64 Hz. Data from all accelerometers, except those at d.o.f. 19 to 22 were used. The selected rotational FRF was derived from translational measurements at three points separated by 101.6 mm, i.e. with d.o.f. 1, 20 and 21. The estimated complex residues for these modes are shown in Table 5.23. The phase shifts of the residues for modes 2 and 3 are very good, being normally within two and one degree respectively of ± 90 . The phase shifts for the first mode are not as good, although they are still acceptable with a maximum difference of 7 degrees, which implies an error of less than 1% during conversion to normal mode. In fact, the error is likely to be less than that value since all the elements are off by about the same phase and the mode shape is to be mass normalized.

The modal parameters of bending modes 4 to 8 were obtained by curve fitting the $\mathbf{H}^*$ related FRF's with the same response and excitation points as before, but with the other frequency bandwidth, 0-256 Hz. The selected rotational FRF was derived from translational measurements at d.o.f. 1, 19 and 20 which were separated by a distance of 50.8 mm. The estimated complex residues for these modes are shown in Table 5.24. The phase shifts for modes 4, 6 and 8 are very good, being within two degrees of ± 90 . The phase shifts for modes 5 and 7 are within about six degrees of ± 90 , except for two residue coefficients with very small amplitudes. It should be noted that the 8th bending mode at 209.7 Hz is highly coupled with the torsion mode at 208.5 Hz. In preparation for CMS analysis, real mode residues were also estimated for these eight modes using the same poles.

A comparison of the experimental frequencies of component 2 and their analytical counterparts is presented in Table 5.25. The analytical frequencies are obtained from FE analysis for the beam modeled with 30 beam elements of equal length and with two d.o.f. at each node. The physical parameters of the model, obtained experimentally, are given in Section 5.3.2. The differences between the experimental and analytical frequencies are very small, being no more than 1.2% for all eight bending modes.

The orthogonality checks for the experimental mode shapes are presented next. The results are basically the same as for component 1. Table 5.26 contains the values of the pseudo orthogonality check matrix. The absolute values of all off-diagonal terms are significantly less than 0.1, thus satisfying the standard requirement commonly placed on modal testing. This means that these mode shapes are consistent with each others.

The results of the cross orthogonality check are shown in Table 5.27. The requirement on the diagonal terms is satisfied for all modes since all these coefficients are between 0.9 and 1. The requirement placed on the off-diagonal terms are also satisfied, since their absolute values are significantly less than 0.1.

5.3.6 Estimation of Ψ_i and k' for Component 2 Through Static Tests

The procedures for the static tests of component 2 were the same as for component 1, except that 10 incremental loads were applied instead of 9.

The measured displacement versus applied load relationships are quite consistent and are linear, except for those locations for which the motion is very small. The quality of the plots is not quite as good as for component 1, but still very good. Figure 5.67 presents the plot of displacement at d.o.f. 7 versus the applied force at d.o.f. 1; the quality of the data is typical of the measurements for large amplitude displacements. Figure 5.68 presents a typical plot for displacement of medium amplitude; the quality of the plot is still quite good. The plot for the worst case in terms of deviation from a straight line is given in Fig. 5.69 and represents the

displacement at d.o.f. 8 for the applied load at d.o.f. 15. Although the data set is not very good, it is acceptable considering that the maximum displacement is no more than 0.25 mm.

The measured reaction force versus applied load relationships are also consistent and linear. The plot of Fig. 5.70 between the reaction force at d.o.f. 1 for the load at d.o.f. 15 represents the worst case of data quality for this type of measurement.

The estimated test-based static parameters and their reference values are given in Tables 5.28 and 5.29. The elements of the constraint modes with unit displacement at d.o.f. 1, 9 and 15 are shown in Tables 5.28a, 5.28b and 5.28c respectively. The test-based elements for the translational d.o.f. are very good, most of them being in error by no more than 1%. The largest error is only 3.2% in the case of an element with small amplitude.

For each constraint mode, four different estimates are shown for the rotational coefficient. The first estimate uses d.o.f. 2 and 3 as points 2 and 3 in the finite difference expression; the second one uses d.o.f. 3 and 5, the third one uses d.o.f. 4 and 7, the fourth one uses d.o.f. 5 and 9. As for component 1, the first estimate of rotation based on translational displacements at points with the closest spacing was used in the CMS algorithm. This selection criteria leads to very small errors with a maximum of just over 1%.

The coefficients of the $\mathbf{k}^*$ matrix are given column by column in Table 5.29. The test-based values are very close to analytical values, the difference being no more than about 2%. It is observed that the experimental values are normally lower than their analytical counterparts.

5.3.7 Component Mode Synthesis

This section presents the results of the verification of the experimental hybrid CMS approach for estimating the modal parameters of a free-free beam system. As in the case of a fixed-fixed beam system, CMS results are obtained by using two different sets of parameters and compared with a reference value. The two sets of parameters are as follows.

- (1) A set of component parameters obtained from modal and static testing.

- (2) An alternative set of component parameters obtained from finite element models of the components. The CMS results obtained with this set are referred to as simulated values.

The reference modal parameters of the system are obtained from a finite element analysis involving an eigenvalue solution of the equations of motion derived from reduced mass and stiffness matrices of the free-free beam system. These system modal parameters serve as reference for assessing the CMS values obtained both with test-based component parameters and with analytically obtained component parameters.

Errors in the estimated system frequencies are shown in Fig. 5.71a for two cases of different numbers of retained component normal modes. The CMS frequencies are obtained by completely ignoring the residual flexibility terms. This is done by replacing the vector G_{du} by very small numbers ($10e-14$). When 9 and 8 modes are retained for components 1 and 2 respectively, curve labeled 9 + 8, the errors in system frequencies for all the modes, except the first and third modes, are very small being less than 1%. The error for the third mode is also quite small at 2%. The error for the first mode is larger being 4%, but still quite acceptable, considering that the residual flexibility has been neglected and the fact that the corresponding error in the simulated CMS values obtained with precise values of component parameters is comparable at 2.7% (Fig. 5.72a). When only 4 + 3 modes are retained, the frequency errors are somewhat higher, as would be expected. However, these frequencies are still quite good, as one may observe by comparing them with their simulated values obtained from CMS with analytically obtained component parameters.

The third curve in Fig. 5.71a, labeled 4 + 3 (w G_{du}), correspond to the case when the test-derived interface residual flexibility term associated with $H_{\theta,m}^*$ is used while all other residual flexibility terms (G_{du}) are neglected as previously. The error in the first system frequency is reduced from 6.6% to 2.2%. The error for the third mode is reduced to less than 1%.

The errors in the estimated system mode shapes are shown in Fig. 5.71b. As expected, they tend to increase with the order of the system modes and with a smaller number for the retained

normal modes. For the case of $9 + 8$ retained modes, the errors are fairly small, being no more than about 10%. When only $4 + 4$ modes are kept, the errors are still acceptable for the lower system modes. By comparing the errors with the corresponding simulation based values shown in Fig. 5.72b, one observes that both sets are basically the same, especially for the three higher system modes for which the errors measured with reference to finite element values are comparatively large. Evidently these large errors arise because the number of retained modes is small.

It is observed from Fig. 5.71b that when the interface residual flexibility term is included, significant reduction in the errors for system modes occurs, especially for modes 2 to 8. Comparison of the errors for the three higher modes with the corresponding simulation values shows that both sets again behave in a similar manner.

Table 5.1 Complex residues for fixed-fixed beam system

Displacement residues (Amplitude/Phase)									
Response DOF		6.16586 Hz beam:11:+Z		16.8637 Hz beam:11:+Z		33.2085 Hz beam:11:+Z			
1	beam:1:+Z	3.65e-05	92.0	1.44e-05	91.0	1.44e-05	-91.0		
2	beam:2:+Z	9.11e-05	88.0	3.79e-05	91.0	3.93e-05	-90.9		
3	beam:3:+Z	1.81e-04	88.3	6.82e-05	91.0	6.64e-05	-90.8		
4	beam:4:+Z	2.76e-04	87.5	1.01e-04	90.8	8.99e-05	-90.9		
5	beam:5:+Z	3.71e-04	88.0	1.29e-04	91.8	1.01e-04	-90.1		
6	beam:6:+Z	4.98e-04	87.4	1.51e-04	91.0	9.75e-05	-90.7		
7	beam:7:+Z	6.12e-04	87.0	1.63e-04	90.8	7.97e-05	-90.7		
8	beam:8:+Z	7.09e-04	87.2	1.61e-04	91.0	4.85e-05	-90.4		
9	beam:9:+Z	8.11e-04	88.1	1.49e-04	91.9	1.08e-05	-88.5		
10	beam:10:+Z	9.00e-04	86.9	1.26e-04	90.8	2.87e-05	88.5		
11	beam:11:+Z	9.80e-04	87.3	9.15e-05	91.0	6.43e-05	88.8		
12	beam:12:+Z	1.00e-03	86.8	4.81e-05	91.0	8.66e-05	88.8		
13	beam:13:+Z	1.03e-03	86.8	1.73e-06	99.5	9.66e-05	88.9		
14	beam:14:+Z	9.86e-04	87.1	4.40e-05	-89.5	8.60e-05	89.0		
15	beam:15:+Z	9.54e-04	86.7	8.63e-05	-89.5	6.35e-05	88.8		
16	beam:16:+Z	9.06e-04	86.8	1.23e-04	-89.5	3.00e-05	88.7		
17	beam:17:+Z	8.27e-04	86.9	1.48e-04	-89.5	9.35e-06	-90.5		
18	beam:18:+Z	7.19e-04	87.2	1.60e-04	-88.8	4.71e-05	-90.3		
19	beam:19:+Z	6.22e-04	86.9	1.63e-04	-89.3	7.85e-05	-90.8		
20	beam:20:+Z	5.16e-04	87.2	1.55e-04	-89.0	9.95e-05	-90.6		
21	beam:21:+Z	3.88e-04	87.0	1.29e-04	-89.2	1.01e-04	-90.8		
22	beam:22:+Z	2.81e-04	87.1	9.95e-05	-89.4	8.86e-05	-91.0		
23	beam:23:+Z	1.82e-04	87.3	6.94e-05	-89.5	6.84e-05	-91.1		
24	beam:24:+Z	8.28e-05	86.1	3.67e-05	-89.5	3.90e-05	-90.9		
25	beam:25:+Z	1.45e-05	73.3	1.27e-05	-90.7	1.49e-05	-91.3		

55.0415 Hz beam:11:+Z		81.9442 Hz beam:11:+Z		115.961 Hz beam:11:+Z		154.094 Hz beam:11:+Z		197.363 Hz beam:11:+Z	
1.72e-05	-91.1	4.53e-06	88.3	1.63e-05	85.7	2.97e-06	87.4	1.26e-05	-91.7
4.24e-05	-90.9	1.07e-05	89.0	3.44e-05	88.5	5.89e-06	87.9	2.24e-05	-91.4
6.39e-05	-90.9	1.44e-05	89.1	3.86e-05	89.6	5.34e-06	87.8	1.45e-05	-91.1
7.40e-05	-90.9	1.35e-05	89.0	2.40e-05	90.8	1.27e-06	84.5	5.78e-06	87.7
6.56e-05	-90.3	7.58e-06	89.2	3.08e-06	-102.1	3.57e-06	-89.6	2.02e-05	88.7
4.03e-05	-90.7	1.14e-06	-82.9	2.77e-05	-91.1	5.59e-06	-89.5	1.55e-05	88.6
5.23e-06	-89.6	9.01e-06	-89.1	3.58e-05	-90.3	3.30e-06	-86.6	3.79e-06	-90.5
3.09e-05	89.0	1.29e-05	-89.3	2.25e-05	-89.2	1.64e-06	76.7	1.94e-05	-91.0
5.86e-05	89.7	1.15e-05	-88.4	4.00e-06	81.5	5.42e-06	85.4	1.68e-05	-90.3
6.90e-05	88.8	4.71e-06	-85.8	2.90e-05	88.8	5.36e-06	83.9	2.06e-06	80.3
6.06e-05	88.7	4.27e-06	83.9	3.82e-05	89.4	1.40e-06	70.9	1.92e-05	87.7
3.42e-05	88.3	1.16e-05	87.1	2.55e-05	89.8	3.41e-06	-84.0	1.79e-05	87.8
1.31e-06	-71.4	1.48e-05	87.7	8.86e-07	-103.7	5.55e-06	-86.6	2.29e-07	21.4
3.58e-05	-90.2	1.16e-05	87.2	2.57e-05	-90.5	3.23e-06	-83.8	1.75e-05	-90.7
6.03e-05	-90.7	4.41e-06	83.3	3.51e-05	-90.3	1.65e-06	73.3	1.85e-05	-90.8
6.84e-05	-90.9	4.47e-06	-85.0	2.39e-05	-90.2	5.44e-06	84.0	1.92e-06	-84.9
5.65e-05	-91.0	1.12e-05	-88.9	1.92e-06	88.2	5.34e-06	84.4	1.67e-05	87.8
2.90e-05	-90.6	1.31e-05	-89.3	2.72e-05	90.0	1.51e-06	75.9	1.99e-05	88.3
7.67e-06	90.1	9.31e-06	-90.0	3.74e-05	89.7	3.44e-06	-87.1	4.31e-06	86.7
4.37e-05	89.6	1.54e-06	-89.1	2.69e-05	89.8	5.76e-06	-89.3	1.55e-05	-90.7
6.65e-05	89.2	7.21e-06	89.2	2.63e-07	36.7	3.43e-06	-89.5	2.01e-05	-91.2
7.27e-05	89.0	1.32e-05	89.0	2.60e-05	-90.0	1.48e-06	84.7	5.66e-06	-91.0
6.37e-05	88.7	1.48e-05	88.6	3.91e-05	-90.5	5.59e-06	87.3	1.47e-05	88.3
3.94e-05	88.5	1.08e-05	88.2	3.05e-05	-90.9	5.73e-06	87.4	2.19e-05	88.4
1.53e-05	87.3	5.03e-06	86.4	1.16e-05	-92.8	2.79e-06	86.7	1.27e-05	88.5

Table 5.2 Comparison between experimental and analytical frequencies for the fixed-fixed beam system

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	6.17	6.30	-2.1
2	16.86	17.37	-2.9
3	33.21	34.06	-2.5
4	55.04	56.29	-2.2
5	81.94	84.09	-2.6
6	116.0	117.4	-1.2
7	154.1	156.3	-1.4
8	197.4	200.8	-1.7

Table 5.3 Complex residues of first 4 modes, for component 2
of fixed-fixed beam system

Acceleration residues (Amplitude/Phase)		19.7133 Hz		63.9828 Hz		133.816 Hz		228.266 Hz	
Response DOF		beam:6:+z		beam:6:+z		beam:6:+z		beam:6:+z	
1	beam:2:+z	2.74e+00	94.2	3.72e+00	90.5	3.08e+01	-90.1	5.90e+01	-90.2
2	beam:3:+z	5.48e+00	93.4	6.49e+00	89.6	4.29e+01	-91.0	5.77e+01	-90.9
3	beam:4:+z	7.63e+00	93.5	7.13e+00	89.7	2.74e+01	-91.0	4.41e+00	93.4
4	beam:5:+z	9.24e+00	93.6	5.51e+00	89.9	6.19e+00	90.2	6.24e+01	89.7
5	beam:6:+z	9.76e+00	93.6	1.96e+00	90.7	3.54e+01	89.2	5.21e+01	89.6
6	beam:7:+z	9.47e+00	93.6	2.27e+00	-91.5	4.10e+01	89.1	1.49e+01	-91.2
7	beam:8:+z	8.42e+00	93.8	5.86e+00	-90.8	1.91e+01	88.6	6.62e+01	-90.9
8	beam:9:+z	6.68e+00	93.8	7.65e+00	-90.6	1.57e+01	-90.1	4.61e+01	-91.5
9	beam:10:+z	4.72e+00	93.7	7.37e+00	-90.6	4.18e+01	-90.8	2.44e+01	91.2
10	beam:11:+z	2.71e+00	93.7	5.22e+00	-90.7	4.34e+01	-91.2	7.23e+01	89.6
11	beam:12:+z	1.02e+00	94.4	2.27e+00	-90.9	2.28e+01	-92.0	5.24e+01	88.8
12	beam:1001:+y	5.06e-01	-86.6	7.23e-01	-90.6	6.53e+00	89.0	1.42e+01	89.2

Table 5.4 Complex residues of modes 5 to 8, for component 2
of fixed-fixed beam system

Acceleration residues (Amplitude/Phase)		347.018 Hz		496.281 Hz		646.564 Hz		858.319 Hz	
Response DOF		beam:6:+z		beam:6:+z		beam:6:+z		beam:6:+z	
1	beam:2:+z	4.77e+01	90.2	7.97e+01	91.3	2.90e+01	86.0	2.10e+02	-94.7
2	beam:3:+z	2.40e+01	89.8	1.11e+00	-74.6	1.45e+01	-94.5	2.03e+02	85.7
3	beam:4:+z	3.63e+01	-90.1	8.14e+01	-88.3	2.20e+01	-94.3	1.32e+00	-92.4
4	beam:5:+z	4.12e+01	-90.2	4.73e+00	92.7	2.59e+01	87.2	2.04e+02	-94.5
5	beam:6:+z	1.79e+01	90.4	7.92e+01	91.6	9.33e+00	85.8	2.07e+02	85.2
6	beam:7:+z	4.83e+01	89.8	1.51e+01	-85.0	2.92e+01	-92.6	2.28e+01	-89.9
7	beam:8:+z	3.40e+00	85.8	7.98e+01	-88.2	6.52e+00	91.2	1.88e+02	-95.5
8	beam:9:+z	4.63e+01	-90.1	1.97e+01	94.9	2.57e+01	88.4	2.12e+02	85.7
9	beam:10:+z	2.39e+01	-90.9	8.04e+01	91.9	2.03e+01	-90.0	3.71e+01	-88.4
10	beam:11:+z	3.74e+01	90.2	2.61e+01	-85.6	1.50e+01	-91.2	1.78e+02	-95.5
11	beam:12:+z	4.71e+01	89.5	8.54e+01	-89.1	3.14e+01	90.0	2.33e+02	85.6
12	beam:1001:+y	1.17e+01	-90.3	2.40e+01	-88.4	1.06e+01	-94.5	9.87e+01	85.3

Table 5.5 Comparison between experimental and analytical frequencies, for component 2 of the fixed-fixed beam system

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	19.71	20.40	-3.4
2	63.98	66.10	-3.2
3	133.8	137.9	-3.0
4	228.3	235.7	-3.1
5	347.0	359.6	-3.5
6	496.3	509.3	-2.6
7	646.6	685.0	-5.7
8	858.3	886.5	-3.2

Table 5.6 Comparison between experimental and analytical frequencies, for the cantilever configuration of component 2

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	4.42	4.65	-4.9
2	27.9	29.15	-4.3
3	78.0	81.6	-4.4
4	153.7	159.9	-3.9
5	253.8	264.2	-3.9

Table 5.7 Pseudo orthogonality check for component 2 of fixed-fixed beam system

1.0000	0.0004	0.0036	0.0023	0.0028	-0.0108	0.0049	0.0137
0.0004	1.0000	-0.0078	-0.0065	0.0070	-0.0076	0.0097	0.0003
0.0036	-0.0078	1.0000	0.0036	-0.0038	0.0068	-0.0184	0.0072
0.0023	-0.0065	0.0036	1.0000	0.0162	-0.0119	-0.0096	0.0003
0.0028	0.0070	-0.0038	0.0162	1.0000	0.0368	0.0360	-0.0097
-0.0108	-0.0076	0.0068	-0.0119	0.0368	1.0000	0.0868	-0.0084
0.0049	0.0097	-0.0184	-0.0096	0.0360	0.0868	1.0000	-0.0371
0.0137	0.0003	0.0072	0.0003	-0.0097	-0.0084	-0.0371	1.0000

Table 5.8 Cross orthogonality check for component 2 of fixed-fixed beam system

0.9989	0.0428	-0.0054	-0.0142	0.0057	-0.0058	0.0039	0.0140
-0.0410	0.9952	0.0724	0.0172	-0.0190	-0.0180	0.0137	-0.0029
0.0099	-0.0758	0.9912	-0.0982	0.0284	0.0384	-0.0279	0.0150
0.0172	-0.0258	0.0932	0.9850	0.1491	0.0338	-0.0395	0.0165
-0.0052	0.0286	-0.0379	-0.1186	0.9738	-0.1233	0.0858	-0.0515
-0.0068	0.0158	-0.0344	-0.0484	0.1402	0.9788	0.2501	-0.0426
0.0042	-0.0117	0.0199	0.0422	-0.0629	-0.1373	0.9287	0.2214
-0.0022	0.0077	-0.0149	-0.0288	0.0522	0.0499	-0.2116	0.9568

Table 5.9 Estimation of static parameters for component 2 of fixed-fixed beam system

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	8.803e-001	8.754e-001	0.6	
3	7.565e-001	7.524e-001	0.5	
4	6.399e-001	6.328e-001	1.1	
5	5.239e-001	5.186e-001	1.0	
6	4.189e-001	4.111e-001	1.9	
7	3.202e-001	3.126e-001	2.4	
8	2.323e-001	2.243e-001	3.6	
9	1.553e-001	1.482e-001	4.8	
10	9.217e-002	8.595e-002	7.2	
11	4.473e-002	3.935e-002	13.7	
12	1.367e-002	1.005e-002	36.0	
1001	-1.961e-002	-2.084e-002	-5.9	152
1001	-2.075e-002	-2.084e-002	-0.4	305
1001	-2.113e-002	-2.084e-002	1.4	457
1001	-2.208e-002	-2.084e-002	6.0	610

kstar column corresponding to dof 1 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	2.910e+003	2.933e+003	-0.8

Table 5.10 Complex residues of first 4 modes, for component I
of fixed-fixed beam system

Acceleration residues (Amplitude/Phase)									
Response		14.5692 Hz		47.1969 Hz		98.2357 Hz		167.279 Hz	
DOF		beam:9:+Z		beam:9:+Z		beam:9:+Z		beam:9:+Z	
1	beam:2:+Z	1.27e+00	91.6	5.54e+00	-89.9	1.18e+01	-89.6	3.96e+01	89.6
2	beam:3:+Z	2.59e+00	90.9	1.02e+01	-90.9	1.86e+01	-90.4	4.90e+01	88.9
3	beam:4:+Z	3.67e+00	91.0	1.23e+01	-90.7	1.62e+01	-90.2	1.91e+01	89.1
4	beam:5:+Z	4.62e+00	91.0	1.16e+01	-90.7	6.01e+00	-89.8	2.67e+01	-91.4
5	beam:6:+Z	5.11e+00	91.0	7.85e+00	-90.8	7.28e+00	89.5	5.07e+01	-91.6
6	beam:7:+Z	5.29e+00	91.0	2.20e+00	-90.9	1.69e+01	89.7	3.34e+01	-92.3
7	beam:8:+Z	5.13e+00	91.2	4.01e+00	89.5	1.82e+01	89.8	1.09e+01	92.6
8	beam:9:+Z	4.60e+00	91.2	9.26e+00	89.5	1.04e+01	89.9	4.60e+01	89.3
9	beam:10:+Z	3.93e+00	91.1	1.27e+01	89.3	2.64e+00	-90.7	4.42e+01	88.3
10	beam:11:+Z	2.98e+00	92.1	1.31e+01	90.0	1.46e+01	-90.0	6.34e+00	83.1
11	beam:12:+Z	2.06e+00	91.3	1.12e+01	89.4	2.03e+01	-90.3	3.86e+01	-90.5
12	beam:13:+Z	1.16e+00	91.4	7.24e+00	89.3	1.73e+01	-90.3	5.49e+01	-91.7
13	beam:14:+Z	4.40e-01	92.0	2.95e+00	89.3	8.43e+00	-90.6	3.44e+01	-93.5
14	beam:1001:+Y	2.32e-01	-89.3	1.05e+00	89.1	2.38e+00	89.4	8.77e+00	-91.1

Table 5.11 Complex residues of modes 5 to 7, for component 1
of fixed-fixed beam system

Acceleration residues (Amplitude/Phase)

	Response DOF	257.629 Hz beam:9:+Z		362.149 Hz beam:9:+Z		488.051 beam:9:+Z	
1	beam:2:+Z	1.58e+01	88.9	1.15e+02	-90.4	2.17e+01	90.4
2	beam:3:+Z	1.32e+01	88.3	5.02e+01	-90.8	3.27e-01	78.3
3	beam:4:+Z	4.97e+00	-91.7	9.39e+01	89.3	2.13e+01	-89.3
4	beam:5:+Z	1.75e+01	-91.6	8.81e+01	89.4	6.76e-01	82.1
5	beam:6:+Z	8.89e+00	-91.6	5.99e+01	-90.8	2.12e+01	90.9
6	beam:7:+Z	1.03e+01	88.1	1.10e+02	-90.8	2.16e+00	-97.2
7	beam:8:+Z	1.72e+01	88.4	1.75e+01	90.0	2.15e+01	-89.6
8	beam:9:+Z	3.56e+00	89.1	1.16e+02	89.2	2.89e+00	85.1
9	beam:10:+Z	1.45e+01	-91.8	2.88e+01	88.7	2.17e+01	89.8
10	beam:11:+Z	1.52e+01	-91.4	1.04e+02	-90.5	3.32e+00	-90.3
11	beam:12:+Z	2.58e+00	90.2	7.02e+01	-91.1	2.17e+01	-90.5
12	beam:13:+Z	1.80e+01	88.4	8.15e+01	89.5	4.88e+00	92.5
13	beam:14:+Z	1.42e+01	87.7	1.15e+02	88.8	2.32e+01	89.8
14	beam:1001:+Y	3.48e+00	-92.0	2.87e+01	89.2	6.27e+00	-89.8

Table 5.12 Comparison between experimental and analytical frequencies,
for component 1 of the fixed-fixed beam system

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	14.57	14.99	-2.8
2	47.20	48.56	-2.8
3	98.24	101.3	-3.0
4	167.3	173.2	-3.4
5	257.6	264.2	-2.5
6	362.1	374.2	-3.2
7	489.0	503.3	-2.8

Table 5.13 Pseudo orthogonality check for component 1
of fixed-fixed beam system

1.0000	-0.0027	0.0020	0.0046	-0.0007	0.0030	-0.0001
-0.0027	1.0000	-0.0023	0.0022	-0.0052	0.0030	0.0011
0.0020	-0.0023	1.0000	0.0026	0.0049	0.0006	0.0031
0.0046	0.0022	0.0026	1.0000	0.0302	0.0086	-0.0127
-0.0007	-0.0052	0.0049	0.0302	1.0000	-0.0231	-0.0147
0.0030	0.0030	0.0006	0.0086	-0.0231	1.0000	-0.0101
-0.0001	0.0011	0.0031	-0.0127	-0.0147	-0.0101	1.0000

Table 5.14 Cross orthogonality check for component 1
of fixed-fixed beam system

0.9991	0.0351	-0.0073	-0.0111	-0.0031	-0.0030	-0.0022
-0.0367	0.9966	0.0655	0.0216	0.0182	0.0081	0.0088
0.0096	-0.0641	0.9916	-0.0995	-0.0302	-0.0307	-0.0082
0.0165	-0.0212	0.0928	0.9839	-0.1034	-0.0416	-0.0508
0.0038	-0.0254	0.0372	0.1189	0.9840	-0.1590	-0.0595
0.0070	-0.0094	0.0359	0.0554	0.1183	0.9696	-0.1868
0.0041	-0.0106	0.0203	0.0472	0.0511	0.1447	0.9579

Table 5.15 Estimation of static parameters for component 1 of fixed-fixed beam system

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	8.978e-001	8.930e-001	0.5	
3	7.867e-001	7.872e-001	-0.1	
4	6.873e-001	6.835e-001	0.6	
5	5.853e-001	5.831e-001	0.4	
6	4.953e-001	4.871e-001	1.7	
7	4.039e-001	3.965e-001	1.9	
8	3.153e-001	3.125e-001	0.9	
9	2.418e-001	2.362e-001	2.4	
10	1.748e-001	1.685e-001	3.7	
11	1.179e-001	1.107e-001	6.4	
12	6.921e-002	6.392e-002	8.3	
13	3.402e-002	2.911e-002	16.9	
14	9.853e-003	7.468e-003	31.9	
1001	-1.630e-002	-1.786e-002	-8.7	152
1001	-1.827e-002	-1.786e-002	2.3	305
1001	-1.818e-002	-1.786e-002	1.8	457
1001	-1.877e-002	-1.786e-002	5.1	610
1001	-1.895e-002	-1.786e-002	6.1	762

kstar column corresponding to dof 1 (in N/m)

dof	Test-based value	Reference value	Error (%)
1	1.841e+003	1.847e+003	-0.3

Table 5.16 Complex residues of first 4 modes, for component 1 of free-free beam system

Acceleration residues (Amplitude/Phase)									
Response		10.0855 Hz		15.7051 Hz		39.827 Hz		50.3073 Hz	
DOF		beam:ll:z		beam:ll:z		beam:ll:z		beam:ll:z	
1	beam:2:z	2.81e-01	-87.7	3.22e-01	91.4	3.00e+00	90.3	3.71e+00	-88.4
2	beam:3:z	5.40e-01	-88.3	5.95e-01	90.9	4.37e+00	89.5	4.88e+00	-89.2
3	beam:4:z	7.09e-01	-88.1	7.28e-01	91.1	3.21e+00	89.6	2.47e+00	-89.0
4	beam:5:z	7.81e-01	-88.1	7.22e-01	91.0	2.02e-01	91.4	1.79e+00	90.6
5	beam:6:z	7.18e-01	-88.2	5.62e-01	91.0	2.94e+00	-90.6	4.91e+00	90.7
6	beam:7:z	5.53e-01	-88.1	3.29e-01	90.9	4.32e+00	-90.6	4.83e+00	90.7
7	beam:8:z	3.00e-01	-87.6	1.05e-01	91.4	3.14e+00	-90.4	2.15e+00	90.9
8	beam:10:z	2.96e-01	91.8	9.26e-02	91.2	3.35e+00	89.6	1.40e+00	90.7
9	beam:11:z	5.57e-01	92.1	3.06e-01	90.9	4.94e+00	89.5	3.90e+00	90.7
10	beam:12:z	7.24e-01	91.9	5.29e-01	90.9	3.66e+00	89.5	4.41e+00	90.7
11	beam:13:z	7.89e-01	91.8	6.89e-01	90.8	3.71e-01	89.3	2.18e+00	90.7
12	beam:14:z	7.13e-01	92.5	6.99e-01	91.5	3.11e+00	-89.9	1.47e+00	-88.8
13	beam:15:z	5.51e-01	91.8	5.82e-01	90.9	4.72e+00	-90.5	3.90e+00	-89.4
14	beam:16:z	3.03e-01	92.0	3.33e-01	90.9	3.55e+00	-90.3	3.40e+00	-89.1
15	beam:18:z	3.09e-01	-86.8	3.39e-01	-88.1	4.17e+00	90.2	4.35e+00	91.3
16	beam:1001:y	2.55e-02	91.5	2.81e-02	-89.2	2.81e-01	-90.5	3.58e-01	90.7

Table 5.17 Complex residues of modes 5 to 9, for component I
of free-free beam system

Acceleration residues (Amplitude/Phase)		86.2287 Hz		102.168 Hz		137.576 Hz		170.522 Hz		200.475 Hz	
Response		beam:11:+Z		beam:11:+Z		beam:11:+Z		beam:11:+Z		beam:11:+Z	
DQF											
1	beam:2:+Z	6.08e+00	-89.3	8.70e+00	89.0	4.22e+00	87.8	1.17e+01	-89.7	2.28e+00	-92.8
2	beam:3:+Z	5.12e+00	-90.3	5.66e+00	88.2	1.10e+00	87.4	7.46e-01	80.8	7.43e-01	87.2
3	beam:4:+Z	1.86e+00	89.8	5.06e+00	-91.9	3.93e+00	-93.1	1.15e+01	89.5	2.00e+00	86.8
4	beam:5:+Z	6.77e+00	89.8	8.96e+00	-91.8	2.07e+00	-92.9	1.75e+00	-92.9	1.47e+00	-93.1
5	beam:6:+Z	3.63e+00	89.8	3.53e-01	-91.1	3.52e+00	86.9	1.13e+01	-90.5	1.45e+00	-93.1
6	beam:7:+Z	3.56e+00	-90.2	8.98e+00	88.2	3.07e+00	87.0	2.93e+00	88.4	2.05e+00	86.7
7	beam:8:+Z	6.12e+00	-90.1	7.31e+00	88.2	1.96e+00	-93.0	1.19e+01	89.5	1.13e+00	86.8
8	beam:10:+Z	8.82e+00	89.8	1.66e+00	88.5	9.61e+00	87.0	2.27e+00	-89.2	4.82e+00	87.0
9	beam:11:+Z	8.67e+00	89.8	5.46e+00	88.2	5.79e+00	86.9	4.32e+00	89.1	7.06e-01	87.0
10	beam:12:+Z	1.38e+00	-90.0	2.99e+00	88.1	7.44e+00	-93.0	2.70e+00	90.1	4.62e+00	-92.8
11	beam:13:+Z	9.67e+00	-90.2	3.19e+00	-91.6	7.40e+00	-93.1	4.36e+00	-90.6	1.01e+00	86.9
12	beam:14:+Z	6.61e+00	-89.8	4.98e+00	-91.4	5.82e+00	87.4	2.27e+00	-89.6	4.26e+00	87.4
13	beam:15:+Z	3.80e+00	90.0	2.00e-01	-96.1	9.25e+00	86.9	4.70e+00	89.5	2.63e+00	-93.1
14	beam:16:+Z	8.86e+00	90.2	4.14e+00	88.7	1.30e+00	-92.1	2.92e+00	90.4	4.13e+00	-92.6
15	beam:18:+Z	1.76e+01	-89.8	1.15e+01	-91.5	2.80e+01	87.3	1.15e+01	89.9	8.05e+00	-92.5
16	beam:1001:+Y	6.53e-01	90.1	9.76e-01	-91.6	5.28e-01	-93.1	1.61e+00	89.3	3.49e-01	87.0

Table 5.18 Comparison between experimental and analytical frequencies, for component 1 of free-free beam system

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	10.09	10.14	-0.5
2	15.71	15.81	-0.6
3	39.83	40.10	-0.7
4	50.31	50.62	-0.6
5	86.23	87.08	-1.0
6	102.2	102.9	-0.7
7	137.6	139.0	-1.0
8	170.5	171.8	-0.8
9	200.5	202.1	-0.8

Table 5.19 Pseudo orthogonality check for component 1 of free-free beam system

1.0000	-0.0070	0.0058	0.0005	0.0051	-0.0020	-0.0009	-0.0014	0.0044
-0.0070	1.0000	0.0034	0.0080	-0.0005	0.0047	0.0018	-0.0017	0.0064
0.0058	0.0034	1.0000	-0.0011	-0.0049	-0.0023	-0.0079	0.0020	-0.0088
0.0005	0.0080	-0.0011	1.0000	-0.0042	-0.0057	-0.0060	0.0038	-0.0104
0.0051	-0.0005	-0.0049	-0.0042	1.0000	-0.0153	-0.0165	0.0101	-0.0141
-0.0020	0.0047	-0.0023	-0.0057	-0.0153	1.0000	-0.0139	0.0007	-0.0099
-0.0009	0.0018	-0.0079	-0.0060	-0.0165	-0.0139	1.0000	0.0171	-0.0181
-0.0014	-0.0017	0.0020	0.0038	0.0101	0.0007	0.0171	1.0000	0.0169
0.0044	0.0064	-0.0088	-0.0104	-0.0141	-0.0099	-0.0181	0.0169	1.0000

Table 5.20 Cross orthogonality check for component 1 of free-free beam system

0.9999	-0.0070	-0.0014	0.0035	0.0031	0.0023	0.0009	-0.0027	0.0014
0.0001	0.9999	0.0049	-0.0002	0.0032	0.0016	0.0030	0.0007	0.0045
0.0073	-0.0016	0.9999	-0.0128	-0.0001	-0.0069	-0.0063	0.0042	-0.0047
-0.0030	0.0082	0.0118	0.9999	-0.0076	0.0000	-0.0081	-0.0012	-0.0094
0.0020	-0.0037	-0.0048	0.0035	0.9998	-0.0294	-0.0144	0.0132	-0.0107
-0.0042	0.0030	0.0045	-0.0057	0.0142	0.9995	-0.0221	-0.0047	-0.0122
-0.0018	-0.0011	-0.0015	0.0019	-0.0020	0.0077	0.9996	0.0198	-0.0162
0.0012	-0.0023	-0.0022	0.0049	-0.0030	0.0058	-0.0026	0.9996	0.0252
0.0029	0.0020	-0.0040	-0.0012	-0.0032	0.0020	-0.0024	-0.0079	0.9993

Table 5.21a Estimation of constraint mode elements for unit displacement
at d.o.f. 1, for component 1 of free-free beam

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	8.406e-001	8.442e-001	-0.4	
3	6.909e-001	6.914e-001	-0.1	
4	5.445e-001	5.444e-001	0.0	
5	4.083e-001	4.063e-001	0.5	
6	2.799e-001	2.798e-001	0.0	
7	1.678e-001	1.680e-001	-0.1	
8	7.308e-002	7.370e-002	-0.8	
10	-5.128e-002	-5.130e-002	0.0	
11	-8.154e-002	-8.200e-002	-0.6	
12	-9.570e-002	-9.520e-002	0.5	
13	-9.342e-002	-9.380e-002	-0.4	
14	-7.998e-002	-8.060e-002	-0.8	
15	-5.827e-002	-5.860e-002	-0.6	
16	-3.105e-002	-3.080e-002	0.8	
18	2.958e-002	3.130e-002	-5.5	
1001	1.369e-002	1.300e-002	5.3	305
1001	1.343e-002	1.300e-002	3.3	610
1001	1.375e-002	1.300e-002	5.8	914
1001	1.424e-002	1.300e-002	9.5	1219

Table 5.21b Estimation of constraint mode elements for unit displacement
at d.o.f. 9, for component 1 of free-free beam

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	1.850e-001	1.865e-001	-0.8	
3	3.672e-001	3.672e-001	0.0	
4	5.364e-001	5.361e-001	0.1	
5	6.882e-001	6.875e-001	0.1	
6	8.175e-001	8.154e-001	0.3	
7	9.174e-001	9.141e-001	0.4	
8	9.779e-001	9.775e-001	0.0	
10	9.793e-001	9.775e-001	0.2	
11	9.116e-001	9.141e-001	-0.3	
12	8.154e-001	8.154e-001	0.0	
13	6.857e-001	6.875e-001	-0.3	
14	5.369e-001	5.361e-001	0.2	
15	3.687e-001	3.672e-001	0.4	
16	1.866e-001	1.865e-001	0.0	
18	-1.842e-001	-1.875e-001	-1.8	
1001	-1.554e-002	-1.560e-002	-0.4	305
1001	-1.626e-002	-1.560e-002	4.2	610
1001	-1.706e-002	-1.560e-002	9.3	914
1001	-1.826e-002	-1.560e-002	17.1	1219

Table 5.21c Estimation of constraint mode elements for unit displacement at d.o.f. 17, for component 1 of free-free beam

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	-3.098e-002	-3.080e-002	0.6	
3	-5.841e-002	-5.860e-002	-0.3	
4	-7.981e-002	-8.060e-002	-1.0	
5	-9.374e-002	-9.380e-002	-0.1	
6	-9.401e-002	-9.520e-002	-1.2	
7	-8.206e-002	-8.200e-002	0.1	
8	-5.110e-002	-5.130e-002	-0.4	
10	7.320e-002	7.370e-002	-0.7	
11	1.667e-001	1.680e-001	-0.8	
12	2.783e-001	2.798e-001	-0.5	
13	4.032e-001	4.063e-001	-0.8	
14	5.409e-001	5.444e-001	-0.6	
15	6.877e-001	6.914e-001	-0.5	
16	8.425e-001	8.442e-001	-0.2	
18	1.150e+000	1.156e+000	-0.6	
1001	2.729e-003	2.600e-003	5.0	305
1001	2.914e-003	2.600e-003	12.1	610
1001	3.294e-003	2.600e-003	26.7	914
1001	3.906e-003	2.600e-003	50.2	1219

Table 5.22 Estimation of k^* matrix for component 1 of free-free beam

kstar column corresponding to dof 1 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	1.612e+003	1.627e+003	-0.9
9	-3.210e+003	-3.254e+003	-1.4
17	1.600e+003	1.627e+003	-1.6

kstar column corresponding to dof 9 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	-3.224e+003	-3.254e+003	-0.9
9	6.428e+003	6.508e+003	-1.2
17	-3.207e+003	-3.254e+003	-1.5

kstar column corresponding to dof 17 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	1.607e+003	1.627e+003	-1.2
9	-3.207e+003	-3.254e+003	-1.4
17	1.598e+003	1.627e+003	-1.8

Table 5.23 Complex residues of first 3 modes, for component 2
of free-free beam system

Acceleration residues (Amplitude/Phase)		11.9669 Hz		22.9813 Hz		45.6609 Hz	
Response DOF		beam:12:+Z		beam:12:+Z		beam:12:+Z	
1	beam:2:+Z	3.72e-01	-96.0	1.27e+00	92.0	1.56e+00	-89.0
2	beam:3:+Z	7.01e-01	-96.5	2.19e+00	91.0	2.15e+00	-89.9
3	beam:4:+Z	9.02e-01	-96.4	2.39e+00	91.1	1.30e+00	-89.8
4	beam:5:+Z	9.66e-01	-96.5	1.87e+00	91.1	4.24e-01	90.1
5	beam:6:+Z	8.52e-01	-96.4	8.24e-01	91.2	1.92e+00	90.1
6	beam:7:+Z	6.17e-01	-96.3	2.11e-01	-87.8	2.24e+00	90.2
7	beam:8:+Z	3.05e-01	-96.2	6.71e-01	-88.7	1.30e+00	90.2
8	beam:10:+Z	2.26e-01	83.5	1.98e+00	91.1	4.64e-01	-89.9
9	beam:11:+Z	3.57e-01	83.6	4.21e+00	91.0	1.30e-01	-90.1
10	beam:12:+Z	3.81e-01	83.7	5.42e+00	91.1	5.05e-01	90.2
11	beam:13:+Z	3.16e-01	83.4	5.10e+00	91.0	8.91e-01	90.1
12	beam:14:+Z	1.76e-01	84.1	3.04e+00	91.8	6.95e-01	90.8
13	beam:16:+Z	1.83e-01	-96.2	3.34e+00	-88.7	8.83e-01	-89.6
14	beam:1001:+Y	3.19e-02	83.6	1.13e-01	-89.0	1.49e-01	90.4

Table 5.24 Complex residues of modes 4 to 8, for component 2 of free-free beam system

Acceleration residues (Amplitude/Phase)		73.3646 Hz		99.8953 Hz		133.478 Hz		170.947 Hz		209.678 Hz	
Response		beam:12:+Z		beam:12:+Z		beam:12:+Z		beam:12:+Z		beam:12:+Z	
DOF											
1	beam:2:+Z	1.85e+00	-91.3	7.34e+00	85.2	4.73e+00	-87.5	6.71e+00	-93.7	1.38e+01	90.7
2	beam:3:+Z	1.85e+00	-92.2	4.98e+00	84.2	1.46e+00	-88.2	4.18e-01	84.9	5.77e+00	-89.2
3	beam:4:+Z	3.58e-02	83.0	4.01e+00	-95.4	4.27e+00	91.7	6.62e+00	85.6	1.10e+01	-89.8
4	beam:5:+Z	1.94e+00	87.9	7.71e+00	-95.6	2.69e+00	91.8	1.04e+00	-94.7	1.05e+01	90.6
5	beam:6:+Z	1.90e+00	87.9	8.78e-01	-97.9	3.63e+00	-88.3	6.53e+00	-94.5	6.53e+00	89.5
6	beam:7:+Z	1.03e-01	88.4	7.30e+00	84.5	3.93e+00	-88.2	1.74e+00	85.4	1.32e+01	-89.4
7	beam:8:+Z	1.32e+00	-92.1	6.58e+00	84.3	1.48e+00	91.7	6.86e+00	85.6	2.28e+00	-91.2
8	beam:10:+Z	3.89e+00	87.9	4.81e-01	-108.2	1.08e+01	-88.3	1.13e+00	-94.4	3.25e+01	-89.5
9	beam:11:+Z	5.22e+00	87.8	2.72e+00	86.5	7.37e+00	-88.3	2.55e+00	85.4	7.40e-01	-101.8
10	beam:12:+Z	1.69e+00	87.6	3.16e+00	83.7	7.87e+00	91.6	1.34e+00	85.6	3.00e+01	90.6
11	beam:13:+Z	3.22e+00	-92.1	2.39e-01	-66.1	9.98e+00	91.8	2.69e+00	-94.6	1.39e+01	-89.0
12	beam:14:+Z	4.42e+00	-91.6	2.76e+00	-94.0	2.47e+00	-88.0	1.69e+00	-94.2	2.91e+01	-89.2
13	beam:16:+Z	7.49e+00	88.2	7.19e+00	83.8	3.24e+01	92.2	6.52e+00	-94.1	4.96e+01	-88.4
14	beam:1001:+Y	1.90e-01	88.3	8.16e-01	-95.5	5.75e-01	92.2	9.24e-01	85.6	2.43e+00	-89.9

Table 5.25 Comparison between experimental and analytical frequencies, for component 2 of free-free beam system

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	11.97	12.02	-0.4
2	22.98	23.23	-1.1
3	45.66	45.92	-0.6
4	73.36	74.13	-1.0
5	99.90	100.4	-0.5
6	133.5	135.1	-1.2
7	170.9	172.1	-0.7
8	209.7	211.3	-0.8

Table 5.26 Pseudo orthogonality check for component 2 of free-free beam system

1.0000	0.0009	-0.0068	0.0024	0.0047	0.0029	0.0011	-0.0082
0.0009	1.0000	0.0026	-0.0053	-0.0002	0.0084	-0.0041	-0.0113
-0.0068	0.0026	1.0000	-0.0015	0.0030	0.0036	-0.0022	-0.0003
0.0024	-0.0053	-0.0015	1.0000	-0.0015	0.0191	-0.0077	-0.0076
0.0047	-0.0002	0.0030	-0.0015	1.0000	-0.0217	0.0004	0.0097
0.0029	0.0084	0.0036	0.0191	-0.0217	1.0000	0.0116	0.0163
0.0011	-0.0041	-0.0022	-0.0077	0.0004	0.0116	1.0000	-0.0151
-0.0082	-0.0113	-0.0003	-0.0076	0.0097	0.0163	-0.0151	1.0000

Table 5.27 Cross orthogonality check for component 2 of free-free beam system

0.9999	-0.0041	0.0018	-0.0001	-0.0004	0.0033	0.0023	-0.0087
0.0051	1.0000	-0.0019	-0.0015	0.0026	0.0069	-0.0058	-0.0110
-0.0087	0.0046	0.9999	-0.0071	-0.0054	0.0059	0.0025	-0.0017
0.0025	-0.0038	0.0056	0.9999	0.0050	0.0184	-0.0112	-0.0081
0.0050	-0.0028	0.0084	-0.0066	0.9999	-0.0232	-0.0071	0.0128
-0.0003	0.0015	-0.0023	0.0007	0.0015	0.9995	0.0195	0.0189
-0.0010	0.0016	-0.0046	0.0035	0.0076	-0.0079	0.9996	-0.0266
0.0005	-0.0003	0.0012	0.0006	-0.0029	-0.0022	0.0110	0.9991

Table 5.28a Estimation of constraint mode elements for unit displacement
at d.o.f. 1, for component 2 of free-free beam

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	8.425e-001	8.398e-001	0.3	
3	6.877e-001	6.830e-001	0.7	
4	5.355e-001	5.329e-001	0.5	
5	3.936e-001	3.929e-001	0.2	
6	2.671e-001	2.662e-001	0.4	
7	1.567e-001	1.562e-001	0.3	
8	6.684e-002	6.640e-002	0.7	
10	-4.080e-002	-4.090e-002	-0.2	
11	-5.963e-002	-5.950e-002	0.2	
12	-5.997e-002	-6.030e-002	-0.6	
13	-4.722e-002	-4.760e-002	-0.8	
14	-2.572e-002	-2.600e-002	-1.1	
16	2.765e-002	2.680e-002	3.2	
1001	-1.324e-002	-1.339e-002	-1.1	305
1001	-1.339e-002	-1.339e-002	0.0	610
1001	-1.409e-002	-1.339e-002	5.2	914
1001	-1.485e-002	-1.339e-002	10.9	1219

Table 5.28b Estimation of constraint mode elements for unit displacement
at d.o.f. 9, for component 2 of free-free beam

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	2.069e-001	2.070e-001	0.0	
3	4.096e-001	4.063e-001	0.8	
4	5.948e-001	5.898e-001	0.8	
5	7.532e-001	7.500e-001	0.4	
6	8.870e-001	8.789e-001	0.9	
7	9.759e-001	9.688e-001	0.7	
8	1.021e+000	1.012e+000	0.9	
10	9.340e-001	9.288e-001	0.6	
11	8.134e-001	8.056e-001	1.0	
12	6.444e-001	6.406e-001	0.6	
13	4.492e-001	4.444e-001	1.1	
14	2.320e-001	2.274e-001	2.0	
16	-2.322e-001	-2.292e-001	1.3	
1001	1.742e-002	1.736e-002	0.3	305
1001	1.844e-002	1.736e-002	6.2	610
1001	1.949e-002	1.736e-002	12.3	914
1001	2.097e-002	1.736e-002	20.8	1219

Table 5.28c Estimation of constraint mode elements for unit displacement at d.o.f. 15, for component 2 of free-free beam

d.o.f.	Test-based value	Reference value	Error (%)	Delta (mm)
2	-4.608e-002	-4.690e-002	-1.7	
3	-8.940e-002	-8.930e-002	0.1	
4	-1.232e-001	-1.228e-001	0.3	
5	-1.449e-001	-1.429e-001	1.4	
6	-1.471e-001	-1.451e-001	1.4	
7	-1.274e-001	-1.250e-001	1.9	
8	-7.998e-002	-7.810e-002	2.4	
10	1.106e-001	1.121e-001	-1.3	
11	2.520e-001	2.540e-001	-0.8	
12	4.205e-001	4.196e-001	0.2	
13	5.989e-001	6.032e-001	-0.7	
14	7.985e-001	7.986e-001	0.0	
16	1.206e+000	1.202e+000	0.3	
1001	-3.956e-003	-3.968e-003	-0.3	305
1001	-4.431e-003	-3.968e-003	11.7	610
1001	-5.076e-003	-3.968e-003	27.9	914
1001	-6.037e-003	-3.968e-003	52.1	1219

Table 5.29 Estimation of k^* matrix for component 2 of free-free beam

kstar column corresponding to dof 1 (in N/m)				
dof	Test-based value	Reference value	Error (%)	
1	1.853e+003	1.859e+003	-0.3	
9	-4.278e+003	-4.339e+003	-1.4	
15	2.456e+003	2.479e+003	-0.9	
kstar column corresponding to dof 9 (in N/m)				
dof	Test-based value	Reference value	Error (%)	
1	-4.354e+003	-4.339e+003	0.4	
9	9.997e+003	1.012e+004	-1.3	
15	-5.750e+003	-5.785e+003	-0.6	
kstar column corresponding to dof 15 (in N/m)				
dof	Test-based value	Reference value	Error (%)	
1	2.452e+003	2.479e+003	-1.1	
9	-5.661e+003	-5.785e+003	-2.1	
15	3.275e+003	3.306e+003	-0.9

Fig. 5.1 Fixed-fixed beam system and components

Fig. 5.2 Schematic of first two rigid support normal modes, for component 1 of fixed-fixed beam system

Fig. 5.3 Schematic of constraint mode, for component 1 of fixed-fixed beam system

(a) Component 1 of fixed-fixed beam system,
in rigid support configuration

(b) Component 1 of fixed-fixed beam system,
in flexible support configuration

Fig. 5.4 Schematics of component 1 of fixed-fixed beam system,
in rigid support and flexible support configurations

Fig. 5.5 Set-up for fixed boundary condition

Fig. 5.6 Location of response d.o.f. for modal testing of fixed-fixed beam system

Fig. 5.7 Modal testing of fixed-fixed beam system

Fig. 5.8 Reciprocity check for fixed-fixed beam system

Fig. 5.9 Point FRF's ($H_{11,11}$ and $H_{21,21}$) for fixed-fixed beam system

Fig. 5.10 Overview of test configuration for component 2 of fixed-fixed beam system

Fig. 5.11 Location of response d.o.f. for modal testing of component 2 of fixed-fixed beam system

Fig. 5.12 Centerline and side accelerometers

Fig. 5.13 Additional accelerometers for rotational FRF's

Fig. 5.14 Reciprocity check between $H_{1,9}$ and $H_{9,1}$, for component 2 of fixed-fixed beam system

Fig. 5.15 Reciprocity check between $H_{1,6}$ and $H_{6,1}$, for component 2 of fixed-fixed beam system

Fig. 5.16 Reciprocity check between $H_{6,9}$ and $H_{9,6}$, for component 2 of fixed-fixed beam system

Fig. 5.17 Auto power spectrum of force at d.o.f. 1, for component 2 of fixed-fixed beam system

Fig. 5.18a $H_{1,9}$ for component 2 of fixed-fixed beam

Fig. 5.18b $H_{9,1}$ for component 2 of fixed-fixed beam

Fig. 5.19 Superposition of $H_{6,6}^*$ and $H_{6,6}$ for component 2 of fixed-fixed beam

Fig. 5.20 $H'_{\omega, \omega}$ for component 2 of fixed-fixed beam - log scale

Fig. 5.21 $H_{6,6}^*$ for component 2 of fixed-fixed beam - linear scale

Fig. 5.22 Reciprocity check between $H_{9,6}^*$ and $H_{6,9}^*$, for component 2 of fixed-fixed beam system

Fig. 5.23 $H_{0,0}^*$ for three different spacings for frequency bandwidth 0-256 Hz, for component 2 of fixed-fixed beam system

Fig. 5.24 Degrees of freedom used for estimation of rotational FRF's for both components of fixed-fixed beam system

Fig. 5.25 $H_{\theta,6}^*$ for two different spacings for frequency bandwidth 0-1024 Hz, for component 2 of fixed-fixed beam system

Fig. 5.26 Instrumentation for static tests of fixed-fixed beam components

Fig. 5.27 Displacement at d.o.f. 7 for static test of component 2 of fixed-fixed beam system

Fig. 5.28 Displacement at d.o.f. 12 for static test of component 2 of fixed-fixed beam system

Fig. 5.29 Test set-up with exciter at d.o.f. 11 for component 1 of fixed-fixed beam

Fig. 5.30 Reciprocity check between $H_{7,9}$ and $H_{9,7}$, for component 1 of fixed-fixed beam system

Fig. 5.31 Reciprocity check between $H_{7,9}^*$ and $H_{9,7}^*$, for component 1 of fixed-fixed beam system

Fig. 5.32 $H_{0,9}^*$ for a spacing of Δ_x and frequency bandwidth 0-1024 Hz

Fig. 5.33 $H_{0,9}^*$ for a spacing of $2\Delta_x$ and frequency bandwidth 0-256 Hz

Fig. 5.34a Frequency errors for fixed-fixed beam system, from CMS with test-based component parameters

Fig. 5.34b Mode shape errors for fixed-fixed beam system, from CMS with test-based component parameters

Fig. 5.35a Frequency errors for fixed-fixed beam system, from CMS with simulation

Fig. 5.35b Mode shape errors for fixed-fixed beam system, from CMS with simulation

Fig. 5.36 Free-free beam system and its components

Fig. 5.37 Schematics of first three rigid support normal modes, for component 1 of free-free beam system

Fig. 5.38 Schematics of static constraint modes, for component 1 of free-free beam system

(a) Component 1 of free-free beam system, in rigid support configuration

(b) Component 1 of free-free beam system, in flexible support configuration

Fig. 5.39 Schematics of component 1 of free-free beam system, in rigid support and flexible support configurations

Fig. 5.40 Location of response d.o.f. for modal testing of component 1 of free-free beam system

Fig. 5.41 Close-up on flexible support
at d.o.f. 17

Fig. 5.42 Overview of modal testing set-up
for component 1 of free-free beam

Fig. 5.43 Reciprocity check between $H_{6,11}$ and $H_{11,6}$, for component 1 of free-free beam system

Fig. 5.44 Reciprocity check between $H_{6,11}^*$ and $H_{11,6}^*$, for component 1 of free-free beam system

Fig. 5.45 Superposition of $H'_{1,1,1}$ and $H_{1,1,1}$, for component 1 of free-free beam system

Fig. 5.46 Drive point FRF $H_{11,11}^*$ for component 1 of free-free beam system

Fig. 5.47 $H_{0,11}^*$ for a spacing of Δ_x and frequency bandwidth 0-256 Hz

Fig. 5.48 $H_{0,11}^*$ for a spacing of $2\Delta_x$ and frequency bandwidth 0-64 Hz

Fig. 5.49 Reciprocity check between $H_{11,m}^*$ and $H_{0,11}^*$, for component 1 of free-free beam system

Fig. 5.50 $H_{0,m}^*$ for a spacing of $2\Delta_x$ and frequency bandwidth 0-64 Hz

Fig. 5.51 Measured (dotted) FRF and synthesized (continuous) FRF's for $H_{\theta,m}^*$,
for component 1 of free-free beam system

Fig. 5.52 Real part of residual function (continuous)
and curve fit polynomial (dotted)

Fig. 5.53 Weighting function (dotted) and residual function (continuous)

Fig. 5.54 Applied force and displacement measurement at a supported d.o.f. location during preliminary static test of component 1 of free-free beam system

Fig. 5.55 Displacement measurement at an interior d.o.f. during static test of component 1 of free-free beam system

Fig. 5.56 Displacement at d.o.f. 9 for load applied at d.o.f. 1, for static tests of component 1 of free-free beam system

Fig. 5.57 Displacement at d.o.f. 18 for load applied at d.o.f. 1, for static tests of component 1 of free-free beam system

Fig. 5.58 Reaction force at d.o.f. 1 for load applied at d.o.f. 17, for static tests of component 1 of free-free beam system

Fig. 5.59 Location of response d.o.f. for modal testing of component 2 of free-free beam system

Fig. 5.60 Overview of test set-up for modal testing of component 2 of free-free beam system

Fig. 5.61 Example of FRF in test configuration with noisy curve in frequency bandwidth 190-215 Hz

Fig. 5.62 Reciprocity check between $H_{2,12}^*$ and $H_{12,2}^*$, for component 2 of free-free beam system

Fig. 5.63 $H_{0,12}^*$ for a spacing of Δ_x and frequency bandwidth 0-256 Hz

Fig. 5.64 $H_{0,12}^*$ for a spacing of $2 \Delta_x$ and frequency bandwidth 0-64 Hz

Fig. 5.65 Reciprocity check between $H_{12,m}^*$ and $H_{0,12}^*$, for component 2 of free-free beam system

Fig. 5.66 $H_{0,m}^*$ for a spacing of $2\Delta_x$ and frequency bandwidth 0-64 Hz

Fig. 5.67 Displacement at d.o.f. 7 for load applied at d.o.f. 1, for static tests of component 2 of free-free beam system

Fig. 5.68 Displacement at d.o.f. 15 for load applied at d.o.f. 9, for static tests of component 2 of free-free beam system

Fig. 5.69 Displacement at d.o.f. 8 for load applied at d.o.f. 15, for static tests of component 2 of free-free beam system

Fig. 5.70 Reaction force at d.o.f. 1 for load applied at d.o.f. 15, for static tests of component 2 of free-free beam system

Fig. 5.71a Frequency errors for free-free beam system,
from CMS with test-based component parameters

Fig. 5.71b Mode shape errors for free-free beam system,
from CMS with test-based component parameters

Fig. 5.72a Frequency errors for free-free beam system, from CMS with simulation

Fig. 5.72b Mode shape errors for free-free beam system, from CMS with simulation

6 VERIFICATION OF TEST-BASED CMS FOR GRID SYSTEM

6.1 Introduction

This chapter presents the results of the application of experimental hybrid CMS approach to the evaluation of frequencies and mode shapes of a free-free grid system. This free-free grid is the most complex of the three systems tested in this project. This increased complexity arises from: 1) the greater number of flexible supports to which the component test articles are attached, four instead of one and three for the beam components described in Chapter 5; 2) the greater number of free interface d.o.f., four instead of one, for which the rotational FRF's and the static rotations are derived from additional translational measurements; and 3) the greater uncertainty in obtaining analytical values for the mass and stiffness matrices.

The format used for the material presented in this chapter is similar to that used in Chapter 5. First, the test articles and the boundary conditions are described. This is followed by a presentation of the finite element models for the system and the two components. For each of the two components, the subsequent section presents a description of the modal testing including the data processing involved in obtaining the $\mathbf{H}^*$ related FRF's and the modal parameters in the rigid support configuration. The presentation includes a comparison of the derived natural frequencies of the components with their analytical counterparts. This is followed by a discussion of the static tests on components, derivation of the component Ψ_s and $\mathbf{k}^*$ as well as a comparison of these matrices with their analytical counterparts. For both modal and static tests, the approaches used for obtaining the parameters related to rotational d.o.f. and the results obtained are discussed in detail. The discussion of test equipment and general

procedures presented in Chapter 4 is applicable also to the grid system, hence only those details that are specific to the testing of the grid components are presented here.

The next section in the chapter describes the estimation of system modal parameters by the CMS approach using the test-derived component parameters and presents a comparison of the CMS values of these parameters with their reference counterparts derived from a finite element analysis. That section includes the estimation of system parameters by the CMS approach but using as input error-free analytical component parameters. This allows a better appreciation of the accuracy of system parameter values obtained from component test data. The section also presents a discussion on the differences between system reference parameters and the ones derived through CMS, including possible causes and solutions.

6.2 Test Articles and Boundary Conditions

The grid system with the overall dimensions of its components is shown in Fig. 6.1. The numbers, from 1 to 58, denote the nodes of the system FE model discussed in the next section. The grid system has been built for the Department of National Defence and transferred to the Canadian Space Agency for use in one of its ongoing research and development project. The grid was built in two pieces to facilitate transportation and storage. Physically, the grid system represents a stiffening support-structure which could form the backbone of a radar array appendage for surveillance satellites.

The grid system is designed as a flat structure with 2 main longerons, 10 cross members at right angle and 8 diagonals. The overall length is 5432 mm. The spacing of the longerons is 1023 mm between outer edges. All elements of the grid system are made of aluminum. The lengths of components 1 and 2 are 2751 mm and 2681 mm, respectively. As described in Piggott et al. (1994), the longerons are made of T-sections with dimensions 38.10 mm x 38.10 mm x 4.8 mm. The stem of the T lies in the plane of the grid so that the most flexible direction dominates the overall bending of the system. The cross-pieces and diagonals are made of solid

rectangular bars with dimensions of 12.7 mm x 25.4 mm (0.5" x 1"). The material for all these members is 6061-T6 aluminum.

Connections between the longerons, cross members and diagonals are made through an aluminum solid block to which the individual members are welded. All blocks, except those at nodes 55 and 58, have horizontal dimensions of 63.5 mm (2.5") by 63.5 mm and a height of 41.3 mm (1.625"). Each block has a hole drilled through its center. This hole is tapped, and a steel threaded insert screwed in and fixed. The inserts have internal 19.05 mm (0.75") coarse threads. These inserts are meant to provide a durable interface for attaching the test article to the pedestals. The blocks at nodes 55 and 58 have the same horizontal dimensions as others but a different height of 25.4 mm (1"). These two blocks are not threaded. Figure 6.2 shows a block, such as at node 17, connecting a cross member with the longeron. Figure 6.3 shows the junction of the longeron, two diagonals and one cross member, as for node 10. An accelerometer glued on an added small plate also appears in the figure. The details of welding are clearly visible in both pictures.

The grid system is in a free-free configuration. The reference modal parameters of the system for CMS are obtained analytically. For reasons discussed in Section 1.3.2, reference values could not be obtained through testing of the entire grid system because of its considerable flexibility.

Each component is one half of the grid system, as shown in Fig. 6.1. In test configuration, each component is supported by four flexible supports attached to the solid blocks of the components at the following locations: the two blocks at the component interfaces (nodes 28 and 31 in Fig. 6.1); the two blocks located one bay before the opposite end of the test articles, i.e. at nodes 8 and 10 for component 1, and at nodes 49 and 51 for component 2.

As for the two beam systems, the input to the hybrid CMS method consists of the following three sets of modes: 1) the constrained or rigid support normal modes, 2) the constraint modes, and 3) the residual flexibility attachment modes. The first two constrained normal modes of component 1 are depicted in Fig. 6.4. Two of the four static constraint modes are depicted in

Fig. 6.5. The other two constraint modes are mirror images with respect to the longitudinal centerline of the component of the modes shown in the figure. The numbering of the supported d.o.f. in these two figures corresponds to the one used for component measurement d.o.f. defined in the next section. The mode shapes of component 2 are basically the same as those of component 1.

6.3 Analytical Modeling

The mass and stiffness matrices of the three structures, the system and two components, are derived using a standard finite element program. The full model of the system consists of 58 nodes located as shown in Fig. 6.1. The longerons, diagonals and cross members are modeled as beam members. The nodes on these flexible members are located on the neutral axis of the members, either half way or one third and two third of the distance between the edge of the blocks they connect. Each solid block is modeled as a lumped mass at a node located at the center of the block (except for the blocks at the two interface nodes). The members of the grid are assumed as being attached to the center of the blocks, except the longerons which have an outward offset of 25.4 mm in the y direction to account for the true position of their neutral axis. The two grid components are taken as being assembled with the matching blocks of the two components at the interface located side by side as shown in the figure. For consistency with component modeling, the interface blocks at nodes 28 and 31 have their node offset by 31.75 mm from the center of the block towards the edge common to adjacent blocks. For these interface blocks, the attachment points for the grid members and the lumped mass are taken as being rigidly connected to the offset nodes. The lumped mass at an interface node is twice as large as that at other nodes. The contribution of welds to the mass or stiffness is neglected.

The component models are first derived with reference to the 6 d.o.f. at each node. A static condensation and a Guyan's reduction are performed on these full models in order to obtain reduced models for which the active d.o.f. correspond to the d.o.f. for which the structural

responses are either measured or derived. As shown in Fig. 6.6, the d.o.f. of a component along which measurements are made comprise out-of-plane translations at a smaller number of locations, 26 in the figure. In addition, rotations with respect to the two horizontal axes are measured at the interface nodes (nodes 1 and 2) for a total of four rotations. The numbering for the measurement nodes is exactly the same for both components and is as shown in Fig. 6.6. With the reduction in the component d.o.f., the node number in the figure also refers to the vertical translation d.o.f. number at that node.

The FE model of the system is also derived first with the 6 d.o.f. at each node. A static condensation and a Guyan's reduction are performed to derive a reduced model. The reduced model of the system consists of active d.o.f. corresponding to the same locations as for the components but without any rotational d.o.f.

The physical parameters input to the FE models are as follows. The cross section dimensions of the members of the grid are the nominal values shown on construction drawings. Their lengths as well as the dimensions of the solid blocks are also as shown on drawings, but have been partly verified through measurements on the test articles. The density and Young's modulus of the material are taken as the nominal values for aluminum, i.e. respectively 2700 kg/m^3 and 68.95 GPa . Poisson's ratio is taken to be 0.33. The lumped mass consists of the mass of the drilled solid block and of the insert, except for two end blocks.

The cross-section parameters of the T-section were computed on the basis of their physical dimensions shown on the construction drawings. The effective torsional moment of inertia was computed with an approximate formula taken from Young (1989). According to Young, errors in the results obtained from the formula, which is based on membrane analogy, are rarely greater than 10%.

6.4 Estimation of H^* for Component 1 Through Modal Testing

6.4.1 Test Description, Instrumentation

In test configuration, component 1 is attached to four flexible supports as shown in Fig. 6.7. The first two supports are at the interface d.o.f. 1 and 2 (Fig. 6.6). In order to be as consistent as possible with the hybrid CMS procedure which assumes that the supports are along interface d.o.f. specified to be right at the edge of the blocks, the legs of the connection units are attached as close as practical to the front of the component. Figure 6.8 contains a close-up of the support attachment at d.o.f. 2. The picture shows the adapter plate to which the legs of the support are bolted. The block of the grid is clamped between the adapter plate and another thinner plate underneath the grid using a bolt. The other two flexible supports are attached to the solid blocks located at d.o.f. 17 and 18. Details of the attachment of the supports are the same as at d.o.f. 1 and 2, except that the dimension of the adapter plate in the x direction is just as large as the legs of the supports since the component is supported right at the center of the blocks.

An accelerometer is fixed vertically on the upper surface of the component at each of the 26 d.o.f. locations shown in Fig. 6.6. The accelerometers at the supported d.o.f. are fixed right in between the legs of the connection units. The transducers located on the other blocks are glued to a thin plate fixed at the center of the blocks, as shown in Fig. 6.3. The accelerometers on the longerons are fixed on the stem as close as possible to the flange, in order to minimize their distance from the neutral axis.

For the present tests, the rotational FRF's are not derived through finite difference. This approach was ruled out because of the stiffness discontinuity between the blocks and the flexible members (longeron and cross-piece) which would have caused excessive errors in the derived FRF's. Since the solid blocks and the adapter plates retaining the support legs can be considered as being rigid in the frequency range of interest, the method of simple difference was applied.

As an example, with reference to Fig. 6.9 which depicts the position of additional accelerometers on the interface blocks, the $\mathbf{H}^*$ related FRF for the force f at an interior d.o.f. and the response being the rotation with respect to y at d.o.f. 1 is given by the expression

$$H_{\theta y, f}^* = (H_{101, f}^* - H_{1, f}^*) / \Delta x_1 \quad (6.1)$$

Since there is no motion along d.o.f. 1 in the $\mathbf{H}^*$ configuration, the expression simplifies to

$$H_{\theta y, f}^* = H_{101, f}^* / \Delta x_1 \quad (6.2)$$

Also the drive point FRF for rotation and moment with respect to y at d.o.f. 1 is obtained from

$$H_{\theta y, my}^* = H_{101, f}^* / \Delta x_1^2 \quad (6.3)$$

Similar expressions are derived for the other rotational FRF's. Consequently, the rotational $\mathbf{H}^*$ related FRF's are simply obtained by dividing a translational FRF by a scalar, thus avoiding the possible problems of subtracting very similar FRF's as in a finite difference expression. Since the test article is quite flexible, the rotational motion is large enough to make this approach quite practical as shown in the next section. Thus in order to measure the rotation, four accelerometers are fixed to the test articles, in addition to the 26 previously mentioned.

Observation of the analytical mode shapes showed that no single interior point could excite properly all possible modes of interest. Consequently, two of them were selected as the main interior driving points, namely d.o.f. 4 and 15. At least one of these two driving points excites well each mode of interest. In addition, these d.o.f. locations are well separated, which is a good characteristics for reciprocity checks. The four supported d.o.f. obviously constitute additional driving points, as required for the derivation of the $\mathbf{H}^*$ related FRF's. Also, forces had to be input at d.o.f. 101, 102, 201 and 202 in order to derive the FRF's with respect to the four interface moments.

Data acquisition with a single frequency bandwidth of 0-128 Hz was sufficient to obtain the first fifteen global modes of the component in the rigid support configuration. The excitation at the ten driving points necessitates a total of five runs with the two exciters at different locations for each run. Figure 6.10 shows an overall view of the test set-up with the exciters at d.o.f. 2

and 15. A close-up of the exciter at d.o.f. 15 is given in Fig. 6.11 where the accelerometer, force sensor, stinger and exciter are clearly visible.

6.4.2 Data Obtained from Flexible and Rigid Support Configurations

The $\mathbf{H}$ related FRF's and the coherence functions did not exhibit any unusual characteristics. The FRF's with excitations and responses at d.o.f. 4 and 15 are plotted in Fig. 6.12 for the purpose of reciprocity check. The reciprocity is excellent, except for the frequency regions of 54-56 Hz, 64-66 Hz and especially 70-80 Hz.

The corresponding reciprocity check for FRF's at the $\mathbf{H}^*$ level is presented in Fig. 6.13. The reciprocity is quite good with the major peaks generally overlapping quite well. In particular, a close overlap of peaks occurs in the problematic region of 70-80 Hz. The deviation from reciprocity in the frequency bandwidth of 54-56 Hz is no longer visible with no dynamic activity occurring in this region.

The plots of point FRF's $H_{4,4}^*$ and $H_{15,15}^*$ are shown in Figs. 6.14a and 6.14b. The first fifteen global modes of the component are identified with the corresponding peaks. As stated previously, each mode is well excited by at least one of the driving points. More specifically, the force at d.o.f. 4 excites well all modes except the 2nd and 8th, while the force at d.o.f. 15 excites well all modes except the 4th, 7th and 14th. The curves are very smooth with a few small glitches and extraneous peaks. In fact, some of these anomalies may correspond to local modes corresponding to in-plane vibration of diagonals and cross-members. However, such in-plane modes are not easily identified from the present data since all response measurements are made in the out-of-plane direction.

Eight rotational $\mathbf{H}^*$ related FRF's, corresponding to the four interface rotations and the two driving points, were derived. The next two plots represent typical samples for the two rotational directions. The plot for rotation at d.o.f. 1 with respect to the y axis and the force at d.o.f. 4 is given in Fig. 6.15. The noise level is either low or negligible throughout the frequency

bandwidth of interest. The plot for rotation at d.o.f. 1 with respect to the x axis and the force at d.o.f. 4 is given in Fig. 6.16. The noise level is very high for the frequency bandwidth encompassing the first four modes. This is expected since the corresponding mode shapes contain hardly any deformation in the cross member joining d.o.f. 1 and 2. The noise level is quite low for the rest of the frequency bandwidth.

The matrix of residual flexibility associated with the interface d.o.f., G_{aa} , has dimensions 4 by 4. These terms are estimated from the sixteen H^* related FRF's with interface rotation as response and interface moment as input. As discussed in Section 6.8, only the diagonal coefficients of G_{aa} , which correspond to the drive point rotational FRF's, are used for the CMS method. Two typical FRF's of this type are shown in Figs. 6.17 and 6.18. Both curves exhibit drive point characteristics with resonance and antiresonance alternating as the frequency increases. The curves are quite smooth except for the lower range of the frequency bandwidth.

Figures 6.19 and 6.20 represent examples of plots used for reciprocity checks between these rotational FRF's. The reciprocities, although not as good as the one obtained between translational FRF's as shown in Fig. 6.13, are still quite acceptable considering the purpose for which these FRF's are used.

6.4.3 Estimation of Modal Parameters in Rigid Support Configuration

The poles corresponding to all fifteen modes were obtained by curving fitting simultaneously the 44 H^* related FRF's associated with the response at the interior d.o.f. and the force at d.o.f. 4 and 15. The eight rotational FRF's were added for residue estimation. The FRF's of d.o.f. 1001, 1002, 2001 and 2002 correspond to the FRF's at the response location divided by the appropriate scalar. For example, $H_{1001,4}^* = H_{101,4}^* / \Delta x_1$. The estimated complex residues are shown in Tables 6.1a, 6.1b and 6.1c. For each mode, these residues are selected from the two available corresponding to the two driving points. For a majority of modes, the phase shifts of residues are quite good, except for those shape elements which have a small

amplitude. Possibly on account of coupling between them, the residues of the 13th and 14th modes are not as good as the others, but they are still acceptable.

A comparison of the experimental frequencies of component 1 and their analytical counterparts is presented in Table 6.2. The analytical values are obtained from the full FE model of component 1. The errors are computed using the analytical values as reference. All errors are less than 5%, with most of them less than 2%. The modes with the highest error (modes 2, 4 and 6) consist mainly of overall torsion of the component. Considering the complexity of the test article and that no model updating has been done based on test data, the agreement between the experimental and analytical frequencies is quite good.

Table 6.3 contains the values of the pseudo orthogonality check matrix. The standard 10% requirement placed on the off-diagonal terms is satisfied for the first six modes. Beyond this mode, this requirement is not satisfied for several values.

The results of the cross orthogonality check are shown in Table 6.4. The requirement on the diagonal terms is satisfied, here again, only for the first six modes since all these terms are between 0.9 and 1. Except in two cases, the requirement placed on the off-diagonal terms is also satisfied for the first six modes. However, this requirement is not satisfied for the higher modes.

The problem with these two orthogonality checks for the seventh and higher modes is believed to be mostly related to the reduction of analytical mass matrix of the component. This observation is based on the fact that a comparison of the analytical frequencies and mode shapes of the full and reduced models of the component in its rigid support configuration shows that these two types of parameters are quite similar for the first six modes. For the higher modes, significant differences appear for both the frequencies and the mode shapes. This in fact should not be surprising considering that, as stated by Mains and Vold (1995), much research has been done on analytical mass matrix reduction techniques. Mass matrix reduction is shown to have a major effect on the pseudo and cross orthogonality computations. Based on these observations

and on the small difference in frequency comparison between test and analysis for the higher modes, the experimental mode shapes are considered to be acceptable for all fifteen modes.

6.5 Estimation of Ψ_j and k^* for Component 1 Through Static Tests

The general procedures described Section 4.4.2 were followed for these tests. Specific details pertinent to the two components of the grid system are described in the following paragraphs.

The external force was applied in turn at each of the four supported d.o.f., after removing the flexible support at that d.o.f. For each of these four locations, several runs were performed in order to measure the displacement at all interior, rotational and supported d.o.f. Figure 6.21 shows the displacement mechanism located at d.o.f. 1. Each run consisted in applying 10 incremented displacements and in measuring the response displacement at a single location. At the three supported d.o.f., the reaction force was measured simultaneously with the displacement. For these points, as well as at the location of the applied force, an indicator with internal spring was positioned underneath the test article. Displacement measurements at interior d.o.f. were taken with the digital indicator without internal spring positioned above the test article and touching the later at the same point where the corresponding accelerometer was fixed. Figure 6.22 shows the set-up for such measurements with the indicator located at d.o.f. 16.

In order to obtain accurate translational measurements required for derivation of rotation, an extension aluminum bar with rectangular cross-section was fixed in succession in two orthogonal directions at each of d.o.f. 1 and 2. Vertical displacements were measured at three equidistant points on the bar separated by 279.4 mm (11"). These measurements were taken with the digital indicator without internal spring. From these measurements and considering that the bar was not deformed since no load was applied to it (except the constant reaction force

of the indicator), four different estimates of rotation per unit applied force could be derived and compared for cross-checking the results. One of these estimates was obtained using the measurements at three points and a finite difference expression. Three other estimates were obtained by simple difference using combination of measurements from two points. The extension bar, in position for measuring displacements for derivation of rotation at d.o.f. 2 with respect to the y axis, is shown in Fig. 6.23. The indicator located at the point closest to the flexible support is visible. Figure 6.24 presents another view of the same set-up showing the fixture arrangement of the indicator.

The data sets of relationship between the measured displacement and applied load are very consistent and linear, except for those locations for which the motion is very small. The data sets of the relationship between the reaction force and applied load are very good and linear. The plot of Fig. 6.25 representing the reaction force at d.o.f. 2 for the load at d.o.f. 1 is typical of data quality for this type of measured parameter.

The estimated test-based static parameters and their reference values are given in Tables 6.5 and 6.6. The reference analytical values are computed from a finite element model. In the reference model of both components, nodes 1 and 2 at the interface are assumed to be located at the center of the block, instead of at the edge. The elements of the constraint modes with unit displacement at d.o.f. 1 and 2 are shown in Table 6.5a, while those with unit displacement at d.o.f. 17 and 18 are shown in Table 6.5b. The test-based coefficients for the translational d.o.f. are very good, most of them being within 2% of the corresponding reference values. The larger differences correspond to small or very small displacements.

For each constraint mode, the four selected rotational estimates were obtained from simple difference equations using the two points on the extension bar closest to the interface. It would be observed that the computed value is close to the reference value when the rotation is large. On the other hand, the difference between the two sets of values can be quite large for very small rotations. Part of the difference can be attributed to the inaccuracy of the FE model. Also, the measurement set-up and approach chosen for rotation estimation are probably

inadequate for measuring small rotations with precision. This is of no concern since even a large error on very small rotational coefficient should not have much effect on the implementation of the CMS algorithm.

The values of the 4×4 $\mathbf{k}^*$ matrix are given column by column in Table 6.6. The reference values were computed along with the constraint modes. The differences between the derived and reference values are quite small, being no more than about 3%.

6.6 Estimation of $\mathbf{H}^*$ for Component 2 Through Modal Testing

6.6.1 Data Obtained from Flexible and Rigid Support Configurations

The test set-up, instrumentation and data acquisition procedures were exactly the same as for component 1 of the grid system. The $\mathbf{H}$ related FRF's and the coherence functions did not exhibit any unusual characteristics. The FRF curves for reciprocity check related to the two interior driving points are presented in Fig. 6.26. The reciprocity is excellent for the frequency bandwidth 0-55 Hz. For the higher frequencies, reciprocity is again fairly good except in the narrow frequency bandwidths of 76-78 Hz, 86-88 Hz, 95-96 Hz, 100-102 Hz and 106-107 Hz where the peaks do not match exactly.

The results of corresponding reciprocity check at the $\mathbf{H}^*$ level is shown in Fig. 6.27. The reciprocity is excellent up to 66 Hz. Above that frequency, it is still reasonably good except at some regions where the curves are not reciprocal. Thus reciprocity is not fully satisfied in the frequency bandwidth of 66-73 Hz where an extraneous peak occurs. Also, in the bandwidth of 86-88 Hz, the FRF's are not reciprocal at a peak corresponding to the 10th mode. Also, the last four peaks have somewhat different amplitudes. This lack of reciprocity at the peaks is not of real concern since it occurs at higher order modes. It could be caused by the nonlinearity in the thrust bearings of the connection units becoming more significant relative to the small displacements occurring at the restrained d.o.f for these higher order modes.

The plots of point FRF's $H_{4,4}^*$ and $H_{15,15}^*$ are shown in Figs. 6.28a and 6.28b. The first fifteen global modes of the component are identified with the corresponding peaks. Each mode is well excited by at least one of the driving points. More specifically, the force at d.o.f. 4 excites well all the modes except the 13th, while the force at d.o.f. 15 excites well all the modes except the 14th. The curves are very smooth with a few small glitches and extraneous peaks. In fact, here again, some of them may correspond to local modes related to in-plane vibration of diagonals and cross-members.

Eight rotational H^* related FRF's corresponding to the four interface rotations and the two interior driving points, as well as the sixteen H^* related FRF's with rotation as response and moment as input, were derived. No example of the curves is provided since the results are similar to those shown previously for component 1.

6.6.2 Estimation of Modal Parameters in Rigid Support Configuration

The poles corresponding to all fifteen modes were obtained by curving fitting simultaneously the 44 H^* related FRF's associated with the response at the interior d.o.f. and the force at d.o.f. 4 and 15. Residues were estimated also for the eight rotational FRF's. The estimated complex residues are shown in Tables 6.7a, 6.7b and 6.7c. For each mode, these residues correspond to one of the two driving points. For a majority of modes, the phase shifts of residues are quite good, except for those shape elements which have a small amplitude. Some of the phase shifts for the 11th, 12th and 13th modes are not very good. However, the discrepancy is related to the higher order modes for which errors in mode shape do not have as significant an effect on the CMS process as those for the lower order modes.

A comparison of the experimental frequencies of component 2 and their analytical counterparts is presented in Table 6.8. The analytical values are obtained from the full FE model of component 2. Except for the second mode which shows a difference of 6.3%, all errors are less

than 5%, most of them being less than 2.5%. The modes with the highest error (modes 2, 4 and 6) consist mainly of overall torsion of the component.

Table 6.9 contains the values of the pseudo orthogonality check matrix. The standard 10% requirement placed on the off-diagonal terms is satisfied for the first six modes. Beyond this mode, this requirement is not satisfied for several values.

The results of the cross orthogonality check are shown in Table 6.10. The requirement on the diagonal terms is satisfied only for the first nine modes since all these terms are between 0.9 and 1. The requirement placed on the off-diagonal terms is satisfied for the first six modes. However, this requirement is not satisfied for the higher modes.

As for component 1, the problem with these two orthogonality checks for the higher modes is believed to be related mostly to the reduction performed on the analytical mass matrix of the component. The experimental mode shapes are thus considered to be acceptable for all fifteen modes.

6.7 Estimation of Ψ_i and k^* for Component 2 Through Static Tests

The instrumentation and procedures used for component 1 were also used for the present component. For both measured displacements and reaction forces, the quality of data sets is generally similar to that of component 1.

The estimated test-based static parameters and their reference values are given in Tables 6.11 and 6.12. The reference analytical values are computed from a finite element analysis. The elements of the constraint modes with a unit displacement at d.o.f. 1 and 2 are shown in Table 6.11a, while those with a unit displacement at d.o.f. 17 and 18 are shown in Table 6.11b. The test-based coefficients for the translational d.o.f. are very good with most of them being within 2% of the corresponding reference value. The larger differences correspond to small or very small displacements.

For each constraint mode, the four selected rotational estimates were obtained from simple difference equations using the two points on the extension bar closest to the interface. It would be observed that the computed value is close to the reference value when the rotation is large. On the other hand, the difference between the two sets of values can be quite large for very small rotations. As for component 1, part of the difference can be attributed to the inaccuracy of the FE model.

The values of the 4×4 k^* matrix are given column by column in Table 6.12. The reference values were computed along with the constraint modes. Although the differences between experimental and analytical results are higher than for component 1, they are still acceptable, being no more than about 6%.

6.8 Component Mode Synthesis

This section examines the effectiveness of the experimental hybrid CMS approach in evaluating the dynamic parameters of the free-free grid system. The CMS results are obtained by using two different sets of component parameters and compared with a reference value. The two sets of parameters are as follows.

- (1) A set of component parameters obtained from modal and static testing.
- (2) An alternative set of component parameters obtained from finite element models of the components. The CMS results obtained with this set are referred to as simulated values.

The reference system parameters are based on analysis and are obtained from the eigenvalue solution of the equation of motion with the reduced mass and stiffness matrices of the free-free grid system. These system modal parameters serve as reference for the CMS implementation both with test-based component parameters and analytical component parameters.

The errors in the estimated system frequencies are shown in Fig. 6.29a for two cases of different numbers of retained component normal modes. The CMS frequencies are obtained by completely ignoring the residual flexibility terms. This is done by replacing the G_{dr} matrix by very small numbers (less than $1e-10$) selected by a random number generator to avoid ill-conditioning in the CMS computation. When 15 modes are retained for each component, curve labeled 15 + 15, the errors on system frequencies for all the modes, except the first one, are reasonably small being no more than 6.5%. The error for the first mode looks a bit large at 11.2%. However, considering that the residual flexibility has been completely neglected and the fact that the error in corresponding simulation value in Fig 6.30a is at 6.6%, the test-based results might be considered acceptable. When only 6 + 6 modes are retained, the frequency errors becomes quite high, with only two of the nine modes having an error less than 10% and five modes having an error higher than 20%. However, except for the first mode, the levels of error are fairly similar to their counterpart values obtained with simulation results.

The two curves of Fig. 6.30a identified with the additional 'w G_{ar} ' represent the case for which the residual terms of G_{ar} are accounted for. In fact, only the diagonal terms (drive points) of the two 4 x 4 matrices are used, the off-diagonal terms are still being neglected as well as all terms of G_{dr} . For both of these curves, the errors in system frequency drop below 2% for all nine modes. This indicates that, at least in the simulation, most of the error is caused because the residual flexibility has not been accounted for. It is important to emphasize that only four out of the sixteen coefficients of G_{ar} are required to produce the significant error reduction observed here. The off-diagonal terms could also be added. However, such addition does not result in significant improvement. Besides, experimental determination of reasonably accurate values of these off-diagonal terms is quite difficult since they are associated to the transfer FRF's (i.e. different d.o.f. for input and response) as opposed to drive point FRF's.

From the curve labeled '15 + 15 (w G_{aa})' of Fig. 6.29a, it is seen that the frequency error for the first mode using test-based component parameters has been significantly reduced from 11.2% to 1.1% simply by the addition of the four residual coefficients. In fact, the errors for modes 1, 3, 5 and 7, which correspond to the first four bending modes of the system, are all less than 2%. The frequency errors for modes 2, 4, 6 and 8, which are mainly torsion modes, vary between 2.5% to 4.4%. From Tables 6.2 and 6.8, it is seen that these values are no more than the frequency errors observed at the component level for the torsion modes. For the case with only 6 + 6 modes retained but the four residual terms are included, the frequency errors increase as expected compared to the case when 15 + 15 modes are retained and residual flexibility is included. However, a significant drop is observed compared to the case for which the 6 + 6 modes are used without residual flexibility. The errors for six of the first nine modes are seen to be less than 5%, and errors for the other three modes are less than 7.5%.

The errors in the estimated system mode shapes obtained from test-based CMS are shown in Fig. 6.29b. When 15 + 15 modes are retained, the errors in the first 4 modes for both cases, that is with and without residual flexibilities, are quite small. In fact, 6 of the 8 values have an error of less than 5%, the other two have errors below 9%. The errors for modes 5 and 6 are still quite acceptable varying between about 13% to 18%. The error for modes 7 and 8 for the case without residuals are quite large, being about 40%. However, they are not much larger than their simulation counterpart values at about 33% (Fig. 6.30b). The shape errors for mode 9 are also quite large at 26% and 34%. However, they are lower than the simulation counterpart value at 39% for the case when the residual flexibility terms are neglected. For a majority of modes, inclusion of residual flexibility terms leads to a significant reduction in the errors.

From Fig. 6.29b, it is seen that for the case when 6 + 6 modes are kept without accounting for residual flexibility, the shape errors are quite large for all modes, except for modes 3 and 4 for which the errors are acceptable at about 13%. For modes 5 to 9, these error levels are similar to those shown by their counterpart values from simulation. When the estimated residual terms are taken into account, the errors drop significantly for all modes. The errors for

the first four modes are quite small, being no more than about 4%. For modes 5 and 6, the errors are still quite acceptable, being between 8% to 12%. The errors for the last three modes are quite high. Evidently, these large errors arise, at least partly, because the number of retained modes is small.

Table 6.1a Complex residues of first 5 modes for component 1 of grid system

Acceleration residues (Amplitude/Phase)											
Response		7.06431 Hz		15.4657 Hz		16.6553 Hz		24.4513 Hz		39.3191 Hz	
DQF		grid:15:+2		grid:15:+2		grid:4:+2		grid:4:+2		grid:4:+2	
1	grid:3:+2	5.29e-01	87.6	1.78e+00	93.7	8.95e-01	-97.0	3.09e+00	-88.0	9.59e+00	91.3
2	grid:4:+2	5.11e-01	87.8	3.13e-01	90.0	3.86e+00	89.9	2.79e+00	92.3	1.60e+01	91.0
3	grid:5:+2	8.94e-01	87.6	2.71e+00	93.8	1.43e+00	-96.4	4.26e+00	-87.6	7.23e+00	91.5
4	grid:6:+2	8.82e-01	87.6	1.96e+00	93.5	1.40e+00	93.8	1.27e+00	-86.5	1.42e+01	91.3
5	grid:7:+2	8.81e-01	87.7	1.24e+00	92.7	3.83e+00	90.5	1.47e+00	90.5	1.80e+01	91.1
6	grid:8:+2	8.81e-01	87.9	3.72e-01	89.5	6.27e+00	89.8	4.27e+00	91.8	1.65e+01	91.2
7	grid:9:+2	1.04e+00	87.9	2.72e+00	94.1	2.25e+00	-94.0	3.73e+00	-87.2	6.60e+00	-88.6
8	grid:10:+2	1.07e+00	87.9	8.29e-02	72.1	7.08e+00	89.5	4.31e+00	91.4	1.44e+00	94.2
9	grid:11:+2	9.51e-01	87.8	1.86e+00	94.2	2.50e+00	-92.8	1.69e+00	-86.3	1.61e+01	-88.7
10	grid:12:+2	9.43e-01	87.8	1.22e+00	93.7	2.98e-01	101.0	5.85e-02	9.4	2.23e+01	-89.0
11	grid:13:+2	9.57e-01	87.6	5.76e-01	92.7	2.78e+00	89.7	1.39e+00	89.2	2.06e+01	-89.1
12	grid:14:+2	9.50e-01	88.2	2.71e-01	-81.3	5.40e+00	89.7	2.63e+00	91.2	1.27e+01	-88.7
13	grid:15:+2	5.89e-01	87.6	7.00e-01	94.5	2.06e+00	-91.7	1.18e-01	78.0	1.03e+01	-88.7
14	grid:16:+2	6.53e-01	88.4	5.87e-01	-84.4	3.00e+00	89.0	5.21e-01	87.9	1.04e+01	-89.0
15	grid:19:+2	7.55e-01	-92.6	5.59e-02	71.6	4.00e+00	89.1	3.73e+00	-88.3	1.90e+00	90.4
16	grid:20:+2	7.48e-01	-92.3	1.35e+00	93.7	1.27e+00	-94.4	3.15e+00	92.1	2.41e+00	91.1
17	grid:21:+2	1.56e+00	-92.3	5.81e-01	88.7	9.23e+00	89.6	9.31e+00	-87.7	5.27e+00	-88.0
18	grid:22:+2	1.62e+00	-92.4	1.72e+00	92.5	5.89e+00	90.3	2.62e+00	-86.9	7.93e+00	-88.8
19	grid:23:+2	1.64e+00	-92.2	2.56e+00	93.3	2.58e+00	92.6	3.08e+00	91.5	6.96e+00	-89.0
20	grid:24:+2	1.64e+00	-92.0	3.23e+00	93.7	9.79e-01	-100.8	7.86e+00	92.1	2.72e+00	-89.4
21	grid:25:+2	3.27e-02	88.1	1.99e-01	94.1	2.49e-01	-93.6	6.81e-01	-82.8	2.55e+00	93.3
22	grid:26:+2	9.68e-03	-100.7	8.30e-02	90.9	1.38e-01	91.3	2.30e-02	-46.2	3.55e+00	-89.8
23	grid:1001:+2	3.90e-02	88.3	1.35e-01	94.8	6.47e-02	-96.9	2.33e-01	-86.6	8.74e-01	92.2
24	grid:1002:+2	3.75e-02	87.9	3.18e-02	92.6	2.70e-01	92.5	1.88e-01	93.6	1.38e+00	90.7
25	grid:2001:+2	4.48e-03	-77.6	1.21e-03	107.2	1.36e-02	-97.0	1.06e-02	-31.1	8.32e-02	94.7
26	grid:2002:+2	2.39e-03	109.0	2.23e-02	98.8	5.11e-02	-108.5	9.87e-02	-86.1	2.15e-01	92.7

Table 6.1b Complex residues of modes 6 to 10 for component 1 of grid system

Acceleration residues (Amplitude/Phase)											
Response		50.2771 Hz		62.0638 Hz		63.2264 Hz		74.4045 Hz		83.0962 Hz	
DOF		grid:4:+z		grid:4:+z		grid:15:+z		grid:15:+z		grid:4:+z	
1	grid:3:+z	1.58e+01	-89.2	7.20e+00	-91.0	1.10e+01	-88.2	6.46e+00	90.5	2.01e+01	85.8
2	grid:4:+z	2.34e+01	91.3	9.94e+00	88.8	1.78e+00	-79.5	1.52e+00	84.0	1.64e+01	85.6
3	grid:5:+z	1.16e+01	-88.2	7.11e+00	-91.3	1.22e+01	-88.2	3.97e+00	89.6	7.54e+00	86.8
4	grid:6:+z	9.18e+00	-89.3	4.89e+00	89.8	4.31e+00	92.8	1.14e+01	-88.7	1.24e+01	-97.9
5	grid:7:+z	5.10e+00	92.8	9.89e+00	89.4	6.22e+00	91.9	1.16e+01	-88.9	1.29e+01	-98.8
6	grid:8:+z	2.44e+01	91.5	6.30e+00	89.0	2.18e+00	-82.5	3.23e+00	89.4	5.86e+00	86.2
7	grid:9:+z	1.48e+01	90.2	2.54e+00	-94.0	5.85e+00	-88.2	5.62e+00	91.4	7.52e+00	-92.9
8	grid:10:+z	4.17e+00	89.9	8.69e+00	-90.7	3.55e+00	-89.0	6.63e+00	91.5	7.86e+00	-91.9
9	grid:11:+z	3.23e+01	90.7	3.68e+00	91.4	7.19e+00	90.9	6.15e+00	93.3	5.31e+00	-92.9
10	grid:12:+z	1.18e+01	90.5	9.82e+00	88.9	1.07e+01	88.8	8.22e+00	-85.7	1.72e+00	55.5
11	grid:13:+z	6.83e+00	-88.4	4.52e+00	88.1	6.13e+00	88.2	9.14e+00	-85.9	3.99e+00	76.2
12	grid:14:+z	1.59e+01	-87.6	1.39e+01	-90.1	3.50e+00	-90.3	6.88e+00	92.8	1.38e+00	-89.8
13	grid:15:+z	2.16e+01	90.7	1.88e+00	91.7	8.00e+00	91.7	3.59e+00	95.3	4.75e+00	90.3
14	grid:16:+z	1.36e+01	-87.9	9.70e+00	-90.4	3.03e+00	-90.2	7.67e+00	93.1	4.51e+00	90.1
15	grid:19:+z	3.12e-01	-65.7	3.76e+00	-91.1	6.42e-01	-96.0	2.15e+00	-85.4	3.65e+00	-85.0
16	grid:20:+z	3.24e+00	89.0	3.10e+00	-92.9	2.27e+00	95.1	5.07e+00	-87.0	5.57e+00	81.8
17	grid:21:+z	1.08e+01	90.4	2.63e+00	-91.9	4.50e+00	95.0	3.90e+00	-88.7	1.55e+01	85.1
18	grid:22:+z	8.17e-01	103.5	7.08e+00	90.9	2.02e+00	-86.8	8.51e+00	95.1	2.13e+01	-97.1
19	grid:23:+z	4.49e+00	-91.4	5.13e+00	92.1	3.43e+00	-87.9	8.63e+00	94.8	2.16e+01	-96.4
20	grid:24:+z	6.44e-01	-70.9	1.27e+01	-91.6	3.57e+00	101.0	4.05e+00	-89.4	1.94e+01	86.3
21	grid:25:+z	2.05e+01	-91.1	2.72e+00	83.3	3.54e+01	91.0	1.55e+01	94.1	1.01e+01	-92.1
22	grid:26:+z	1.02e+01	-98.5	3.25e+01	88.3	9.15e+00	-85.4	2.49e+00	94.9	1.71e+01	86.5
23	grid:1001:+z	1.54e+00	-89.3	5.89e-01	-89.9	6.05e-01	-88.0	7.75e-01	91.7	1.89e+00	85.6
24	grid:1002:+z	1.62e+00	91.2	9.08e-01	88.6	7.48e-02	72.8	8.47e-02	80.6	1.75e+00	85.1
25	grid:2001:+z	1.41e+00	-91.9	2.91e-01	84.3	3.03e+00	91.2	1.26e+00	93.7	1.24e+00	-92.3
26	grid:2002:+z	2.18e+00	-90.4	1.72e-01	83.7	2.90e+00	91.1	9.95e-01	94.8	4.77e-01	-91.4

Table 6.1c Complex residues of modes 11 to 15 for component 1 of grid system

Acceleration residues (Amplitude/Phase)											
Response		88.6275 Hz		93.2073 Hz		98.7072 Hz		99.6711 Hz		104.299 Hz	
DQF		grid:15:+z		grid:4:+z		grid:4:+z		grid:4:+z		grid:4:+z	
1	grid:3:+z	9.29e-01	24.0	1.88e+01	91.5	1.31e+01	75.8	8.03e-01	160.7	1.27e+01	-93.1
2	grid:4:+z	3.99e+00	69.8	2.44e+01	91.3	1.57e+01	80.1	9.11e+00	96.3	1.25e+01	91.6
3	grid:5:+z	4.31e+00	-92.0	3.26e+00	93.6	1.42e+01	80.6	5.75e-01	-179.3	2.67e+00	-90.1
4	grid:6:+z	1.27e+01	94.5	3.61e+01	-91.5	2.56e+01	-98.7	3.04e+01	-87.5	2.15e+01	-88.9
5	grid:7:+z	1.40e+01	96.4	4.78e+01	-91.5	2.69e+01	-97.9	4.49e+01	-88.1	3.20e+01	-89.2
6	grid:8:+z	4.33e+00	-92.3	2.20e+00	87.4	1.63e+01	80.1	5.35e-01	160.9	1.48e+00	88.8
7	grid:9:+z	9.40e+00	-95.5	9.85e+00	-87.3	2.69e+00	102.9	4.29e-01	-86.4	1.05e+01	86.9
8	grid:10:+z	1.11e+01	-94.6	8.31e+00	-81.7	3.83e+00	-105.2	6.54e+00	84.7	9.01e-01	-58.6
9	grid:11:+z	2.16e-01	82.5	4.72e+00	-87.6	1.88e+01	-100.9	1.34e+00	-56.9	7.03e+00	87.2
10	grid:12:+z	3.52e+00	7.2	6.95e+01	91.6	1.83e+01	72.4	3.16e+00	-123.4	5.51e+01	-92.3
11	grid:13:+z	2.67e+00	2.5	5.32e+01	91.6	2.01e+01	74.8	1.91e+00	-140.1	3.89e+01	-92.2
12	grid:14:+z	7.01e-01	-93.8	6.70e+00	-85.7	1.92e+01	-99.8	1.28e+00	-40.5	6.38e-01	-47.0
13	grid:15:+z	8.33e+00	90.9	1.52e+01	-89.4	1.34e+01	-100.9	1.07e+00	51.2	1.03e+01	87.8
14	grid:16:+z	9.77e+00	87.5	8.12e+00	-92.3	1.57e+01	-99.3	7.85e+00	-84.9	2.51e-01	5.5
15	grid:19:+z	1.15e+01	-90.2	1.64e+01	89.9	7.19e+00	-96.8	4.95e+00	-89.2	2.97e+00	-95.3
16	grid:20:+z	1.43e+01	-92.0	1.31e+01	89.8	4.96e+00	-94.5	6.75e+00	-88.2	7.94e+00	90.5
17	grid:21:+z	1.02e+01	-96.0	9.44e-01	-71.9	1.01e+01	-98.7	4.27e+00	-84.2	5.55e+00	89.6
18	grid:22:+z	3.83e+01	88.3	2.29e+01	-91.3	3.51e+01	81.5	1.79e+01	94.9	1.03e+01	-91.4
19	grid:23:+z	3.29e+01	88.3	1.74e+01	-91.9	2.72e+01	81.8	1.42e+01	95.6	1.02e+01	-92.2
20	grid:24:+z	9.45e+00	-98.4	7.19e+00	-89.7	9.18e+00	-101.1	1.97e-01	72.4	3.01e+00	-96.0
21	grid:25:+z	2.50e+01	-95.3	2.76e+01	-86.6	4.51e+01	82.4	1.36e+01	-96.6	7.08e+00	-89.9
22	grid:26:+z	2.18e+01	-99.4	4.53e+01	-86.8	2.53e+01	-104.6	3.70e+01	89.2	3.76e+01	-90.0
23	grid:10Q1:+z	2.10e-01	-71.7	1.63e+00	90.9	1.82e+00	77.6	1.56e-01	-129.1	1.38e+00	-93.3
24	grid:10Q2:+z	8.93e-01	75.8	3.25e+00	90.6	1.66e-01	86.6	1.43e+00	91.8	1.61e+00	91.0
25	grid:20Q1:+z	2.42e+00	-96.3	3.15e+00	-87.2	4.60e+00	82.4	1.48e+00	-96.4	6.17e-01	-90.3
26	grid:20Q2:+z	8.80e-01	-91.6	6.95e-01	-80.5	4.36e-01	94.5	1.50e-01	-90.9	8.88e-02	-78.1

Table 6.2 Comparison between experimental and analytical frequencies for component 1 of grid system

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	7.06	7.01	0.7
2	15.47	14.94	3.5
3	16.66	16.23	2.6
4	24.45	23.35	4.7
5	39.32	38.70	1.6
6	50.28	48.78	3.1
7	62.06	62.08	0.0
8	63.23	64.11	-1.4
9	74.40	74.29	0.1
10	83.10	81.90	1.5
11	88.63	89.01	-0.4
12	93.21	92.05	1.3
13	98.71	96.03	2.8
14	99.67	96.67	3.1
15	104.3	100.5	3.8

Table 6.3 Pseudo orthogonality check for component 1 of grid system

Columns 1 through 7

1.0000	0.0241	0.0079	-0.0094	-0.0042	0.0263	0.0084
0.0241	1.0000	0.0060	-0.0102	-0.0054	0.0527	-0.1092
0.0079	0.0060	1.0000	-0.0038	-0.0053	0.0227	-0.0660
-0.0094	-0.0102	-0.0038	1.0000	-0.0145	0.0124	-0.0639
-0.0042	-0.0054	-0.0053	-0.0145	1.0000	0.0143	-0.1286
0.0263	0.0527	0.0227	0.0124	0.0143	1.0000	0.2613
0.0084	-0.1092	-0.0660	-0.0639	-0.1286	0.2613	1.0000
0.0857	0.0490	0.0114	-0.0474	-0.1114	-0.1055	-0.1335
0.2140	0.0891	0.0814	-0.0152	0.1089	-0.0755	-0.1039
0.1097	-0.1879	-0.0709	-0.0310	0.1409	0.0079	0.0543
-0.0546	-0.0130	0.0110	0.0274	0.0569	0.0443	0.0400
-0.1035	0.0985	0.0364	0.0343	-0.1248	-0.1217	0.1887
0.0955	-0.0864	0.0232	0.0448	0.2975	0.0844	-0.0951
0.2213	0.0842	0.0414	-0.1052	-0.2512	-0.0059	-0.3587
-0.1671	-0.2493	-0.1600	-0.0790	-0.1947	0.3369	-0.4280

Columns 8 through 14

0.0857	0.2140	0.1097	-0.0546	-0.1035	0.0955	0.2213
0.0490	0.0891	-0.1879	-0.0130	0.0985	-0.0864	0.0842
0.0114	0.0814	-0.0709	0.0110	0.0364	0.0232	0.0414
-0.0474	-0.0152	-0.0310	0.0274	0.0343	0.0448	-0.1052
-0.1114	0.1089	0.1409	0.0569	-0.1248	0.2975	-0.2512
-0.1055	-0.0755	0.0079	0.0443	-0.1217	0.0844	-0.0059
-0.1335	-0.1039	0.0543	0.0400	0.1887	-0.0951	-0.3587
1.0000	-0.0657	-0.1787	0.1644	-0.0316	0.1394	-0.3708
-0.0657	1.0000	0.1176	-0.0414	0.2446	-0.2891	-0.3238
-0.1787	0.1176	1.0000	0.4575	0.2492	-0.1210	0.2480
0.1644	-0.0414	0.4575	1.0000	0.0359	-0.0704	0.0646
-0.0316	0.2446	0.2492	0.0359	1.0000	0.3165	-0.0165
0.1394	-0.2891	-0.1210	-0.0704	0.3165	1.0000	0.0406
-0.3708	-0.3238	0.2480	0.0646	-0.0165	0.0406	1.0000
0.0340	0.2496	0.0185	-0.0104	0.0209	0.0417	0.0215

Column 15

-0.1671
 -0.2493
 -0.1600
 -0.0790
 -0.1947
 0.3369
 -0.4280
 0.0340
 0.2496
 0.0185
 -0.0104
 0.0209
 0.0417
 0.0215
 1.0000

Table 6.4 Cross orthogonality check for component 1 of grid system

Columns 1 through 7

0.9999	0.0252	0.0145	-0.0084	-0.0069	0.0238	0.0047
0.0003	0.9785	-0.1984	-0.0179	-0.0044	0.0405	-0.0958
-0.0067	0.2040	0.9797	0.0115	0.0020	0.0365	-0.0898
-0.0008	0.0050	-0.0186	0.9996	-0.0061	0.0073	-0.0671
0.0026	-0.0014	-0.0081	-0.0086	0.9990	0.0008	-0.1596
0.0032	0.0051	-0.0048	0.0058	0.0160	0.9879	0.3772
0.0011	0.0000	0.0016	0.0002	0.0002	-0.1297	0.4636
0.0020	0.0017	0.0018	-0.0009	0.0318	-0.0563	0.7694
0.0000	-0.0027	0.0003	-0.0038	0.0013	0.0024	-0.0033
-0.0002	0.0017	0.0039	0.0047	0.0011	-0.0100	0.0041
-0.0002	0.0016	0.0008	0.0014	0.0070	-0.0053	0.0212
-0.0028	0.0037	-0.0016	-0.0001	0.0091	-0.0037	-0.0130
0.0002	0.0024	0.0006	-0.0011	-0.0083	0.0062	0.0271
0.0011	0.0025	0.0017	0.0025	-0.0160	0.0028	0.0079
-0.0007	0.0014	-0.0012	-0.0014	0.0089	0.0112	-0.0042

Columns 8 through 14

0.0868	0.2143	0.1099	-0.0556	-0.1026	0.0932	0.2233
0.0450	0.0700	-0.1748	-0.0144	0.0928	-0.0911	0.0715
0.0172	0.0939	-0.1110	0.0058	0.0562	0.0076	0.0501
-0.0449	-0.0082	-0.0344	0.0221	0.0353	0.0462	-0.1063
-0.1217	0.1152	0.1410	0.0522	-0.1301	0.3084	-0.2338
-0.2000	-0.0794	0.0452	0.0405	-0.0929	0.0539	-0.0201
-0.8636	0.1293	0.1762	-0.1677	0.1938	-0.2315	0.1711
0.4225	-0.1303	-0.0648	0.1317	0.2008	0.0572	-0.6031
0.0800	0.9343	0.0958	-0.0254	0.3177	-0.3170	-0.5075
-0.0020	0.0139	0.7439	0.9073	0.2296	-0.1137	0.2792
0.0050	0.0296	-0.0980	0.1604	-0.8129	-0.7563	-0.1502
0.0371	-0.0505	0.5275	-0.3083	0.0861	-0.2794	0.1116
0.0033	-0.0370	0.1099	0.0039	-0.1132	0.0853	-0.1466
-0.0162	-0.0130	0.1186	-0.0032	-0.1288	0.1675	-0.2252
-0.0224	-0.0894	-0.0924	0.0210	0.1407	-0.1491	-0.1298

Column 15

-0.1680
 -0.2072
 -0.2028
 -0.0825
 -0.1797
 0.2764
 -0.3608
 -0.5774
 0.3837
 -0.0052
 -0.1607
 -0.1051
 0.2686
 0.0662
 0.1799

Table 6.5a Estimation of constraint mode elements for unit displacement at d.o.f. 1 and 2, for component 1 of grid system

psi_s for unit displacement applied at point 1			
d.o.f.	Test-based value	Reference value	Error (%)
3	8.161e-001	8.400e-001	-2.9
4	-1.485e-002	-1.636e-002	-9.2
5	6.377e-001	6.494e-001	-1.8
6	4.198e-001	4.305e-001	-2.5
7	2.327e-001	2.351e-001	-1.0
8	2.105e-002	2.021e-002	4.1
9	4.838e-001	4.964e-001	-2.5
10	2.705e-002	2.098e-002	28.9
11	3.037e-001	3.095e-001	-1.9
12	2.050e-001	2.114e-001	-3.0
13	1.215e-001	1.239e-001	-1.9
14	2.863e-002	2.864e-002	0.0
15	1.442e-001	1.460e-001	-1.2
16	1.266e-002	1.291e-002	-2.0
19	-1.176e-001	-1.223e-001	-3.9
20	-3.673e-002	-3.647e-002	0.7
21	-2.290e-001	-2.346e-001	-2.4
22	-1.785e-001	-1.841e-001	-3.1
23	-1.377e-001	-1.393e-001	-1.2
24	-9.304e-002	-9.307e-002	0.0
25	4.844e-001	4.880e-001	-0.7
26	3.614e-003	2.095e-003	72.5
1001	1.386e-002	1.401e-002	-1.1
2001	-2.800e-002	-2.731e-002	2.5
1002	5.480e-005	3.104e-004	-82.3
2002	-2.381e-002	-2.474e-002	-3.8

psi_s for unit displacement applied at point 2			
d.o.f.	Test-based value	Reference value	Error (%)
3	3.807e-003	-6.692e-003	-156.9
4	8.451e-001	8.497e-001	-0.5
5	2.009e-002	1.729e-002	16.2
6	2.375e-001	2.361e-001	0.6
7	4.293e-001	4.316e-001	-0.5
8	6.431e-001	6.465e-001	-0.5
9	6.023e-003	3.571e-003	68.7
10	4.721e-001	4.790e-001	-1.4
11	2.309e-002	2.380e-002	-3.0
12	1.238e-001	1.219e-001	1.6
13	2.092e-001	2.094e-001	-0.1
14	3.024e-001	3.047e-001	-0.7
15	1.878e-002	2.067e-002	-9.1
16	1.531e-001	1.538e-001	-0.4
19	-4.659e-002	-4.433e-002	5.1
20	-1.269e-001	-1.302e-001	-2.5
21	-9.985e-002	-9.878e-002	1.1
22	-1.494e-001	-1.493e-001	0.1
23	-1.930e-001	-1.940e-001	-0.5
24	-2.360e-001	-2.403e-001	-1.8
25	5.034e-001	5.120e-001	-1.7
26	-1.987e-003	-2.095e-003	-5.2
1001	-1.668e-003	-1.410e-003	18.3
2001	2.705e-002	2.731e-002	-1.0
1002	1.303e-002	1.229e-002	6.0
2002	2.435e-002	2.474e-002	-1.6

Table 6.5b Estimation of constraint mode elements for unit displacement at d.o.f. 17 and 18, for component 1 of grid system

psi_s for unit displacement applied at point 17			
d.o.f.	Test-based value	Reference value	Error (%)
3	1.985e-001	1.867e-001	6.3
4	-9.434e-003	-1.038e-002	-9.1
5	3.525e-001	3.506e-001	0.5
6	2.276e-001	2.250e-001	1.1
7	1.119e-001	1.094e-001	2.2
8	-2.327e-002	-2.021e-002	15.2
9	5.266e-001	5.303e-001	-0.7
10	-4.846e-002	-4.772e-002	1.6
11	6.887e-001	6.905e-001	-0.3
12	4.451e-001	4.441e-001	0.2
13	2.202e-001	2.205e-001	-0.2
14	-3.053e-002	-2.864e-002	6.6
15	8.729e-001	8.807e-001	-0.9
16	-3.254e-002	-3.965e-002	-17.9
19	1.145e+000	1.149e+000	-0.3
20	1.873e-002	9.729e-003	92.5
21	1.232e+000	1.235e+000	-0.2
22	8.270e-001	8.396e-001	-1.5
23	4.786e-001	4.838e-001	-1.1
24	8.786e-002	9.307e-002	-5.6
25	1.787e-002	1.202e-002	48.7
26	4.951e-001	4.979e-001	-0.6
1001	-1.397e-002	-1.401e-002	-0.3
2001	3.460e-004	5.760e-004	-39.9
1002	-1.089e-004	-3.104e-004	-64.9
2002	-2.368e-003	-1.995e-003	18.7

psi_s for unit displacement applied at point 18			
d.o.f.	Test-based value	Reference value	Error (%)
3	-2.165e-002	-2.005e-002	8.0
4	1.832e-001	1.771e-001	3.5
5	-1.591e-002	-1.729e-002	-8.0
6	1.108e-001	1.083e-001	2.3
7	2.287e-001	2.240e-001	2.1
8	3.628e-001	3.535e-001	2.6
9	-2.392e-002	-3.031e-002	-21.1
10	5.523e-001	5.477e-001	0.8
11	-2.213e-002	-2.380e-002	-7.0
12	2.257e-001	2.226e-001	1.4
13	4.464e-001	4.461e-001	0.1
14	6.999e-001	6.953e-001	0.7
15	-3.881e-002	-4.740e-002	-18.1
16	8.689e-001	8.730e-001	-0.5
19	2.189e-002	1.759e-002	24.4
20	1.148e+000	1.157e+000	-0.8
21	9.497e-002	9.878e-002	-3.9
22	4.923e-001	4.937e-001	-0.3
23	8.437e-001	8.496e-001	-0.7
24	1.236e+000	1.240e+000	-0.3
25	-7.909e-003	-1.202e-002	-34.2
26	5.023e-001	5.021e-001	0.0
1001	1.594e-003	1.410e-003	13.1
2001	-3.634e-004	-5.760e-004	-36.9
1002	-1.233e-002	-1.229e-002	0.3
2002	2.429e-003	1.995e-003	21.7

Table 6.6 Estimation of k^* matrix, for component 1 of grid system

kstar column corresponding to dof 1 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	1.537e+003	1.525e+003	0.8
2	-1.538e+003	-1.525e+003	0.8
17	-1.527e+003	-1.525e+003	0.1
18	1.530e+003	1.525e+003	0.3

kstar column corresponding to dof 2 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	-1.569e+003	-1.525e+003	2.9
2	1.570e+003	1.525e+003	2.9
17	1.557e+003	1.525e+003	2.1
18	-1.564e+003	-1.525e+003	2.6

kstar column corresponding to dof 17 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	-1.551e+003	-1.525e+003	1.7
2	1.550e+003	1.525e+003	1.6
17	1.572e+003	1.525e+003	3.1
18	-1.571e+003	-1.525e+003	3.0

kstar column corresponding to dof 18 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	1.552e+003	1.525e+003	1.7
2	-1.555e+003	-1.525e+003	1.9
17	-1.566e+003	-1.525e+003	2.7
18	1.573e+003	1.525e+003	3.1

Table 6.7a Complex residues of first 5 modes for component 2 of grid system

Acceleration residues (Amplitude/Phase)											
Response		7.78341 Hz		16.8353 Hz		18.525 Hz		29.3115 Hz		40.09 Hz	
DOF		grid:15:+z		grid:15:+z		grid:4:+z		grid:4:+z		grid:4:+z	
1	grid:3:+z	6.75e-01	91.5	2.32e+00	85.4	4.72e-01	87.8	2.98e+00	-86.6	9.72e+00	90.4
2	grid:4:+z	6.52e-01	92.1	6.98e-01	-93.0	4.41e+00	92.0	2.27e+00	93.1	1.79e+01	90.4
3	grid:5:+z	1.13e+00	91.7	3.49e+00	85.7	5.28e-01	86.9	3.75e+00	-86.4	6.90e+00	90.7
4	grid:6:+z	1.12e+00	91.7	1.87e+00	85.5	2.97e+00	91.3	1.35e+00	-85.8	1.47e+01	90.5
5	grid:7:+z	1.12e+00	91.8	3.67e-01	83.7	5.02e+00	91.7	9.36e-01	92.4	1.94e+01	90.4
6	grid:8:+z	1.11e+00	92.1	1.32e+00	-92.9	6.87e+00	92.1	3.31e+00	93.4	1.81e+01	90.4
7	grid:9:+z	1.31e+00	92.0	3.62e+00	86.2	5.42e-01	-83.1	2.51e+00	-86.3	7.33e+00	-89.7
8	grid:10:+z	1.34e+00	92.0	1.92e+00	-93.4	7.11e+00	92.1	2.98e+00	93.5	1.44e+00	88.8
9	grid:11:+z	1.16e+00	91.8	2.57e+00	86.3	1.67e+00	-87.0	5.90e-02	-94.0	1.63e+01	-89.6
10	grid:12:+z	1.16e+00	91.7	1.13e+00	86.3	7.34e-01	90.3	7.87e-01	92.7	2.41e+01	-89.5
11	grid:13:+z	1.18e+00	91.7	2.49e-01	-94.4	2.75e+00	91.6	1.24e+00	93.0	2.27e+01	-89.5
12	grid:14:+z	1.17e+00	92.3	1.87e+00	-93.2	4.68e+00	92.7	1.32e+00	94.1	1.41e+01	-88.8
13	grid:15:+z	7.04e-01	91.7	1.10e+00	86.6	2.07e+00	-87.8	1.42e+00	94.2	9.59e+00	-89.5
14	grid:16:+z	7.78e-01	92.3	1.53e+00	-93.5	1.78e+00	92.5	5.43e-01	-86.4	1.15e+01	-89.1
15	grid:19:+z	7.69e-01	-88.6	4.26e-01	-90.5	4.25e+00	91.6	5.09e+00	-86.2	1.37e-01	-74.6
16	grid:20:+z	7.70e-01	-88.5	1.75e+00	85.7	1.29e+00	90.7	4.20e+00	93.5	2.34e+00	91.5
17	grid:21:+z	1.56e+00	-88.4	4.61e-01	-86.5	9.87e+00	91.8	1.17e+01	-86.0	9.21e+00	-88.9
18	grid:22:+z	1.63e+00	-88.4	1.21e+00	83.6	8.96e+00	91.8	3.49e+00	-85.2	1.19e+01	-89.3
19	grid:23:+z	1.64e+00	-88.3	2.53e+00	85.4	7.21e+00	91.6	3.73e+00	93.2	9.45e+00	-89.6
20	grid:24:+z	1.67e+00	-88.4	3.73e+00	85.8	4.84e+00	91.2	9.85e+00	93.6	2.68e+00	-91.4
21	grid:25:+z	2.95e-02	88.1	3.13e-01	86.4	1.82e-01	-85.5	7.08e-01	-84.9	2.24e+00	91.0
22	grid:26:+z	1.97e-02	-90.6	9.15e-02	86.9	2.64e-01	90.3	7.80e-02	89.6	3.72e+00	-89.7
23	grid:1001:+z	5.09e-02	92.0	1.75e-01	86.5	4.42e-02	90.1	2.41e-01	-85.9	9.02e-01	91.2
24	grid:1002:+z	4.93e-02	91.5	3.72e-02	-92.8	3.22e-01	91.7	1.51e-01	92.6	1.54e+00	90.2
25	grid:2001:+z	4.29e-03	-79.3	2.87e-03	179.0	2.16e-02	-84.6	1.99e-02	-78.3	4.05e-02	95.7
26	grid:2002:+z	5.09e-04	8.8	4.79e-02	85.6	5.52e-02	-86.0	1.06e-01	-85.7	1.38e-01	91.5

Table 6.7b Complex residues of modes 6 to 10 for component 2 of grid system

Acceleration residues (Amplitude/Phase)											
Response		51.4034 Hz		63.2663 Hz		64.7873 Hz		75.309 Hz		86.897 Hz	
DOF		grid:4:+Z		grid:15:+Z		grid:4:+Z		grid:15:+Z		grid:4:+Z	
1	grid:3:+Z	1.45e+01	-86.8	1.27e+01	-90.9	1.42e+00	94.7	7.45e+00	90.8	2.78e+01	82.1
2	grid:4:+Z	2.11e+01	93.9	3.57e+00	94.8	8.75e+00	89.0	2.44e+00	87.3	2.64e+01	81.2
3	grid:5:+Z	9.98e+00	-86.3	1.33e+01	-90.3	1.85e+00	95.5	4.72e+00	90.2	5.05e+00	83.1
4	grid:6:+Z	9.85e+00	-88.1	5.46e+00	90.6	2.50e+00	87.8	1.22e+01	-90.5	1.30e+01	-105.8
5	grid:7:+Z	3.04e+00	99.9	9.69e+00	91.0	4.97e+00	88.2	1.26e+01	-91.3	1.56e+01	-108.3
6	grid:8:+Z	2.19e+01	93.9	1.42e+00	97.9	5.34e+00	90.2	3.78e+00	88.8	3.27e+00	92.6
7	grid:9:+Z	1.57e+01	92.6	5.17e+00	-89.2	2.88e-01	115.6	5.46e+00	90.2	1.98e+01	-97.1
8	grid:10:+Z	4.07e+00	91.2	7.23e+00	-88.9	6.07e+00	-92.2	6.35e+00	89.4	1.99e+01	-97.9
9	grid:11:+Z	3.11e+01	92.8	8.07e+00	89.1	3.05e+00	-89.2	5.53e+00	91.6	7.02e+00	-97.3
10	grid:12:+Z	1.16e+01	93.2	1.39e+01	89.5	1.47e+00	78.1	8.86e+00	-88.2	2.45e+01	77.9
11	grid:13:+Z	6.21e+00	-87.0	7.00e+00	89.5	7.01e-01	72.8	9.32e+00	-88.4	2.22e+01	78.6
12	grid:14:+Z	1.40e+01	-84.8	9.79e+00	-88.4	9.26e+00	-90.8	6.33e+00	92.2	5.15e+00	-97.3
13	grid:15:+Z	1.97e+01	92.7	7.20e+00	89.7	4.04e+00	-89.4	3.46e+00	94.4	7.78e+00	87.2
14	grid:16:+Z	1.18e+01	-84.8	7.12e+00	-89.2	5.77e+00	-91.2	7.24e+00	93.2	7.21e+00	86.3
15	grid:19:+Z	2.90e+00	89.7	1.16e+00	-85.3	2.89e+00	-91.3	2.51e+00	-86.9	3.86e+00	-91.1
16	grid:20:+Z	1.42e+00	80.0	6.11e-01	-80.5	4.77e+00	-91.7	4.36e+00	-88.5	5.42e+00	81.6
17	grid:21:+Z	1.54e+01	92.5	4.15e+00	89.6	3.96e+00	-90.1	3.60e+00	-90.5	1.61e+01	84.9
18	grid:22:+Z	4.58e+00	96.0	2.94e+00	93.1	3.77e+00	90.4	6.33e+00	92.7	5.02e+00	-103.7
19	grid:23:+Z	3.23e+00	-90.1	3.01e-01	-108.9	2.44e+00	92.0	6.50e+00	92.2	8.10e+00	-99.6
20	grid:24:+Z	3.02e+00	-78.1	4.52e+00	-87.6	1.29e+01	-91.2	3.93e+00	-91.3	2.03e+01	85.3
21	grid:25:+Z	2.19e+01	-87.6	2.99e+01	88.0	2.07e+01	-90.1	1.51e+01	90.7	3.39e+01	-96.8
22	grid:26:+Z	1.12e+01	-101.3	8.19e+00	94.5	3.27e+01	88.2	6.43e+00	89.3	1.41e+01	-99.4
23	grid:1001:+Z	1.41e+00	-86.2	9.09e-01	-91.1	8.27e-03	-135.2	8.48e-01	91.1	2.74e+00	81.8
24	grid:1002:+Z	1.39e+00	93.8	5.40e-01	92.2	6.61e-01	87.8	1.80e-01	86.9	3.22e+00	78.8
25	grid:2001:+Z	1.69e+00	-87.7	3.01e+00	87.9	1.97e+00	-90.1	1.35e+00	91.0	4.41e+00	-98.2
26	grid:2002:+Z	2.42e+00	-87.4	2.48e+00	87.9	1.77e+00	-90.0	9.68e-01	90.6	1.37e+00	-98.1

Table 6.7c Complex residues of modes 11 to 15 for component 2 of grid system

Acceleration residues (Amplitude/Phase)											
Response		91.4054 Hz		94.9655 Hz		100.53 Hz		101.255 Hz		105.379 Hz	
DOF		grid:15:+z		grid:4:+z		grid:15:+z		grid:4:+z		grid:4:+z	
1	grid:3:+z	8.04e+00	-81.2	1.03e+01	85.5	7.28e+00	-102.3	1.37e+00	-60.0	1.18e+01	-85.6
2	grid:4:+z	5.53e+00	-72.3	1.55e+01	83.2	4.61e+00	-64.3	2.19e+01	91.3	9.38e+00	97.1
3	grid:5:+z	9.33e+00	-82.7	1.10e+00	97.5	7.56e+00	-86.4	4.52e+00	83.0	2.96e+00	-66.8
4	grid:6:+z	2.42e+01	99.4	3.12e+01	-105.4	3.85e+00	-161.7	4.76e+01	-88.6	1.13e+01	-83.8
5	grid:7:+z	2.79e+01	100.6	4.41e+01	-105.6	9.50e+00	-127.4	6.85e+01	-88.4	1.74e+01	-84.4
6	grid:8:+z	9.13e+00	-85.5	5.08e-01	149.5	8.50e+00	-88.7	7.60e+00	88.9	3.93e-01	87.1
7	grid:9:+z	6.79e+00	-85.5	6.34e+00	-88.2	1.63e+00	-11.4	3.60e+00	90.8	1.05e+01	91.3
8	grid:10:+z	8.31e+00	-87.8	2.62e+00	-43.4	5.56e+00	90.5	2.73e+00	98.6	3.09e+00	-89.5
9	grid:11:+z	6.73e+00	97.4	3.39e+00	-82.7	9.72e+00	90.7	7.35e+00	-95.4	7.52e+00	98.2
10	grid:12:+z	1.13e+01	-76.4	5.48e+01	79.7	1.47e+01	-121.6	2.80e+01	-88.0	3.83e+01	-87.1
11	grid:13:+z	1.13e+01	-78.2	3.96e+01	80.2	1.30e+01	-112.9	1.54e+01	-86.5	2.71e+01	-86.5
12	grid:14:+z	4.89e+00	96.6	4.80e+00	-89.4	1.01e+01	91.3	1.05e+01	-92.3	3.45e-01	123.8
13	grid:15:+z	1.27e+01	98.1	1.39e+01	-103.3	7.22e+00	81.9	4.16e+00	98.2	4.75e+00	99.1
14	grid:16:+z	1.58e+01	95.3	8.02e+00	-108.6	3.47e+00	92.6	1.30e+01	-90.2	2.60e+00	89.2
15	grid:19:+z	1.32e+01	-83.3	8.48e+00	68.3	3.71e+00	112.1	1.71e+01	-90.4	5.42e+00	96.6
16	grid:20:+z	1.82e+01	-84.5	1.18e+00	-5.9	2.94e+00	99.4	1.28e+01	-89.6	9.44e+00	94.2
17	grid:21:+z	1.93e+01	-85.1	1.20e+01	-96.9	7.41e+00	87.5	1.53e+01	-89.9	1.04e+01	93.4
18	grid:22:+z	4.45e+01	95.7	5.24e+00	-160.9	1.58e+01	-79.3	5.08e+01	90.1	2.47e+01	-85.5
19	grid:23:+z	3.90e+01	95.4	3.78e+00	152.9	1.23e+01	-80.6	3.88e+01	90.2	2.11e+01	-86.1
20	grid:24:+z	2.05e+01	-84.9	1.71e+01	-99.2	9.84e+00	83.6	1.09e+01	-92.0	1.60e+00	-74.4
21	grid:25:+z	2.52e+01	-86.4	1.95e+01	-94.0	2.94e+01	-88.7	3.38e+00	56.8	6.22e+00	-82.0
22	grid:26:+z	2.28e+01	-87.5	2.55e+01	-91.3	3.35e+01	86.5	1.22e+01	101.3	4.33e+01	-86.8
23	grid:1001:+z	9.92e-01	-84.6	1.02e+00	84.8	9.62e-01	-105.1	1.32e-01	-32.8	1.40e+00	-91.2
24	grid:1002:+z	2.08e-01	29.7	2.22e+00	79.1	6.23e-01	62.9	2.24e+00	92.0	1.24e+00	96.6
25	grid:2001:+z	2.77e+00	-86.8	2.59e+00	-96.2	3.42e+00	-88.2	3.96e-01	51.5	5.16e-01	-78.3
26	grid:2002:+z	5.80e-01	-84.0	4.36e-01	-67.0	3.00e-01	-81.3	1.81e-01	-85.9	9.06e-02	-57.7

Table 6.8 Comparison between experimental and analytical frequencies for component 2 of grid system

Mode	Experimental frequency (Hz)	Analytical frequency (Hz)	% error
1	7.78	7.61	2.2
2	16.84	15.84	6.3
3	18.53	18.10	2.4
4	29.31	28.04	4.5
5	40.09	39.15	2.4
6	51.40	49.40	4.0
7	63.27	63.24	0.0
8	64.79	64.99	-0.3
9	75.31	74.75	0.7
10	86.90	85.07	2.2
11	91.41	91.36	0.1
12	94.97	93.54	1.5
13	100.5	97.10	3.5
14	101.3	98.48	2.9
15	105.4	101.9	3.4

Table 6.9 Pseudo orthogonality check for component 2 of grid system

Columns 1 through 7

1.0000	0.0111	0.0033	0.0248	0.0077	0.0333	-0.0784
0.0111	1.0000	-0.0106	-0.0034	0.0020	0.0323	-0.0592
0.0033	-0.0106	1.0000	-0.0222	-0.0009	0.0436	-0.0657
0.0248	-0.0034	-0.0222	1.0000	-0.0158	-0.0109	-0.0057
0.0077	0.0020	-0.0009	-0.0158	1.0000	-0.0306	-0.0201
0.0333	0.0323	0.0436	-0.0109	-0.0306	1.0000	0.1865
-0.0784	-0.0592	-0.0657	-0.0057	-0.0201	0.1865	1.0000
0.0244	-0.0294	-0.0826	0.0970	0.1923	0.1271	-0.0400
0.2208	0.0471	0.0827	0.0340	-0.0722	-0.0811	0.0079
0.1035	-0.1094	-0.1295	0.0415	-0.0256	-0.0930	0.2319
-0.0081	0.0626	0.0185	0.0085	0.1138	-0.0557	0.0624
0.1108	-0.1279	-0.1442	0.0214	-0.2009	0.0969	-0.0842
0.0532	0.1064	0.0429	0.1121	0.3538	-0.0913	0.2028
0.3204	0.0138	0.0538	0.0467	-0.0562	-0.0699	0.0722
0.0865	0.1448	0.2387	-0.1163	-0.2122	-0.3597	0.2038

Columns 8 through 14

0.0244	0.2208	0.1035	-0.0081	0.1108	0.0532	0.3204
-0.0294	0.0471	-0.1094	0.0626	-0.1279	0.1064	0.0138
-0.0826	0.0827	-0.1295	0.0185	-0.1442	0.0429	0.0538
0.0970	0.0340	0.0415	0.0085	0.0214	0.1121	0.0467
0.1923	-0.0722	-0.0256	0.1138	-0.2009	0.3538	-0.0562
0.1271	-0.0811	-0.0930	-0.0557	0.0969	-0.0913	-0.0699
-0.0400	0.0079	0.2319	0.0624	-0.0842	0.2028	0.0722
1.0000	-0.1230	-0.0363	-0.0471	0.0187	-0.2956	-0.2537
-0.1230	1.0000	0.2737	0.0777	-0.1283	0.0303	-0.5381
-0.0363	0.2737	1.0000	-0.1686	-0.4948	0.0210	-0.0180
-0.0471	0.0777	-0.1686	1.0000	0.1304	-0.0848	0.0823
0.0187	-0.1283	-0.4948	0.1304	1.0000	0.2680	-0.1955
-0.2956	0.0303	0.0210	-0.0848	0.2680	1.0000	-0.1128
-0.2537	-0.5381	-0.0180	0.0823	-0.1955	-0.1128	1.0000
0.4870	-0.0217	0.0101	-0.0327	0.0699	0.0034	-0.0065

Column 15

0.0865
 0.1448
 0.2387
 -0.1163
 -0.2122
 -0.3597
 0.2038
 0.4870
 -0.0217
 0.0101
 -0.0327
 0.0699
 0.0034
 -0.0065
 1.0000

Table 6.10 Cross orthogonality check for component 2 of grid system

Columns 1 through 7

0.9999	0.0108	0.0090	0.0244	0.0092	0.0298	-0.0789
0.0008	0.9972	-0.0818	0.0158	0.0047	0.0254	-0.0560
-0.0057	0.0714	0.9964	-0.0356	-0.0067	0.0443	-0.0671
0.0001	-0.0167	0.0144	0.9988	-0.0216	-0.0053	-0.0028
-0.0014	-0.0027	0.0065	0.0051	0.9988	-0.0257	-0.0111
0.0037	0.0034	0.0012	-0.0052	-0.0080	0.9801	0.3638
-0.0016	0.0008	-0.0036	-0.0003	-0.0038	-0.1711	0.9170
-0.0016	-0.0016	-0.0030	0.0009	0.0279	0.0739	-0.0901
-0.0009	-0.0016	-0.0024	0.0050	-0.0003	-0.0015	-0.0498
0.0007	-0.0023	0.0037	-0.0009	0.0012	-0.0039	-0.0037
0.0014	-0.0007	0.0038	-0.0002	-0.0131	-0.0039	0.0121
0.0020	-0.0015	-0.0027	0.0004	0.0152	-0.0081	0.0299
-0.0002	-0.0040	0.0002	-0.0015	-0.0025	-0.0060	-0.0177
0.0004	0.0035	0.0034	-0.0038	0.0138	-0.0007	0.0146
0.0006	-0.0004	0.0026	0.0039	0.0098	-0.0088	-0.0116

Columns 8 through 14

-0.0214	0.2223	0.1050	0.0057	-0.1081	-0.0551	-0.3211
0.0226	0.0414	-0.1005	-0.0590	0.1201	-0.1073	-0.0078
0.0912	0.0875	-0.1401	-0.0278	0.1574	-0.0473	-0.0533
-0.0936	0.0258	0.0344	-0.0064	-0.0165	-0.1083	-0.0422
-0.2207	-0.0759	-0.0229	-0.1131	0.1973	-0.3640	0.0701
-0.1741	-0.0784	-0.0393	0.0464	-0.0745	0.0637	0.0763
0.2060	0.1265	0.3003	-0.1097	0.1542	-0.3029	-0.1341
0.9256	0.0973	-0.0212	-0.0370	-0.0190	-0.3508	-0.1921
-0.0242	0.9403	0.3032	-0.1269	0.1724	0.0795	0.7028
0.0010	0.0262	0.4853	0.9082	0.4827	-0.0483	0.0179
-0.0246	0.0683	-0.3435	0.3049	-0.7333	-0.6472	0.4731
-0.0040	0.0738	-0.6280	0.1772	0.0033	0.3286	-0.0346
0.0165	0.0548	-0.0911	-0.0103	0.0934	-0.1075	-0.1809
0.0227	-0.0377	0.0672	0.0217	-0.1707	0.2491	0.1420
-0.0054	0.0533	0.0948	0.0234	-0.2000	0.0856	-0.2211

Column 15

-0.0882
-0.1241
-0.2489
0.1096
0.1900
0.2821
-0.2947
0.7331
0.0970
0.0164
-0.2128
0.1075
-0.2761
0.0135
-0.0414

Table 6.11a Estimation of constraint mode elements for unit displacement at d.o.f. 1 and 2, for component 2 of grid system

psi_s for unit displacement applied at point 1			
d.o.f.	Test-based value	Reference value	Error (%)
3	8.204e-001	8.402e-001	-2.4
4	-1.382e-002	-1.654e-002	-16.4
5	6.423e-001	6.497e-001	-1.1
6	4.239e-001	4.306e-001	-1.6
7	2.315e-001	2.350e-001	-1.5
8	1.783e-002	1.986e-002	-10.2
9	4.860e-001	4.969e-001	-2.2
10	2.219e-002	2.048e-002	8.4
11	3.062e-001	3.100e-001	-1.2
12	2.067e-001	2.116e-001	-2.3
13	1.207e-001	1.238e-001	-2.5
14	2.721e-002	2.812e-002	-3.2
15	1.443e-001	1.464e-001	-1.4
16	1.299e-002	1.248e-002	4.1
19	-1.094e-001	-1.103e-001	-0.8
20	-2.996e-002	-3.095e-002	-3.2
21	-2.089e-001	-2.117e-001	-1.3
22	-1.628e-001	-1.639e-001	-0.7
23	-1.213e-001	-1.217e-001	-0.4
24	-8.090e-002	-7.856e-002	3.0
25	4.797e-001	4.880e-001	-1.7
26	1.504e-003	2.068e-003	-27.3
1001	1.471e-002	1.400e-002	5.1
2001	-2.705e-002	-2.731e-002	-1.0
1002	2.225e-004	3.223e-004	-31.0
2002	-2.500e-002	-2.475e-002	1.0

psi_s for unit displacement applied at point 2			
d.o.f.	Test-based value	Reference value	Error (%)
3	2.643e-003	-6.881e-003	-138.4
4	8.287e-001	8.499e-001	-2.5
5	1.971e-002	1.696e-002	16.2
6	2.339e-001	2.360e-001	-0.9
7	4.248e-001	4.317e-001	-1.6
8	6.379e-001	6.468e-001	-1.4
9	7.302e-003	3.129e-003	133.4
10	4.683e-001	4.795e-001	-2.3
11	2.332e-002	2.335e-002	-0.1
12	1.199e-001	1.218e-001	-1.5
13	2.059e-001	2.096e-001	-1.7
14	3.015e-001	3.052e-001	-1.2
15	2.063e-002	2.031e-002	1.6
16	1.532e-001	1.542e-001	-0.7
19	-4.146e-002	-3.901e-002	6.3
20	-1.134e-001	-1.184e-001	-4.2
21	-8.961e-002	-8.695e-002	3.1
22	-1.337e-001	-1.348e-001	-0.8
23	-1.732e-001	-1.770e-001	-2.1
24	-2.122e-001	-2.201e-001	-3.6
25	5.048e-001	5.120e-001	-1.4
26	-2.491e-003	-2.068e-003	20.5
1001	-1.587e-003	-1.395e-003	13.8
2001	2.667e-002	2.731e-002	-2.3
1002	1.162e-002	1.228e-002	-5.4
2002	2.460e-002	2.475e-002	-0.6

Table 6.11b Estimation of constraint mode elements for unit displacement at d.o.f. 17 and 18, for component 2 of grid system

psi_s for unit displacement applied at point 17			
d.o.f.	Test-based value	Reference value	Error (%)
3	1.974e-001	1.865e-001	5.9
4	-2.608e-003	-1.020e-002	-74.4
5	3.580e-001	3.503e-001	2.2
6	2.333e-001	2.249e-001	3.8
7	1.159e-001	1.095e-001	5.8
8	-1.571e-002	-1.986e-002	-20.9
9	5.310e-001	5.299e-001	0.2
10	-4.050e-002	-4.722e-002	-14.2
11	6.946e-001	6.900e-001	0.7
12	4.473e-001	4.440e-001	0.7
13	2.241e-001	2.207e-001	1.6
14	-2.572e-002	-2.812e-002	-8.5
15	8.771e-001	8.804e-001	-0.4
16	-2.731e-002	-3.922e-002	-30.4
19	1.135e+000	1.137e+000	-0.2
20	1.204e-002	4.210e-003	185.9
21	1.206e+000	1.212e+000	-0.5
22	8.179e-001	8.195e-001	-0.2
23	4.644e-001	4.662e-001	-0.4
24	7.960e-002	7.856e-002	1.3
25	2.140e-002	1.199e-002	78.5
26	4.958e-001	4.979e-001	-0.4
1001	-1.422e-002	-1.400e-002	1.6
2001	5.194e-004	5.748e-004	-9.6
1002	-1.151e-004	-3.223e-004	-64.3
2002	-1.910e-003	-1.991e-003	-4.1

psi_s for unit displacement applied at point 18			
d.o.f.	Test-based value	Reference value	Error (%)
3	-2.168e-002	-1.986e-002	9.1
4	1.843e-001	1.769e-001	4.2
5	-2.163e-002	-1.696e-002	27.5
6	1.090e-001	1.084e-001	0.5
7	2.264e-001	2.239e-001	1.1
8	3.599e-001	3.532e-001	1.9
9	-2.871e-002	-2.987e-002	-3.9
10	5.459e-001	5.472e-001	-0.2
11	-2.415e-002	-2.335e-002	3.4
12	2.215e-001	2.227e-001	-0.5
13	4.443e-001	4.460e-001	-0.4
14	6.961e-001	6.948e-001	0.2
15	-4.033e-002	-4.705e-002	-14.3
16	8.682e-001	8.726e-001	-0.5
19	1.622e-002	1.227e-002	32.2
20	1.137e+000	1.145e+000	-0.7
21	8.431e-002	8.695e-002	-3.0
22	4.707e-001	4.792e-001	-1.8
23	8.245e-001	8.325e-001	-1.0
24	1.209e+000	1.220e+000	-0.9
25	-8.764e-003	-1.199e-002	-26.9
26	4.982e-001	5.021e-001	-0.8
1001	1.797e-003	1.395e-003	28.8
2001	-1.525e-004	-5.748e-004	-73.5
1002	-1.206e-002	-1.228e-002	-1.8
2002	2.218e-003	1.991e-003	11.4

Table 6.12 Estimation of k^* matrix, for component 2 of grid system

kstar column corresponding to dof 1 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	1.604e+003	1.522e+003	5.4
2	-1.601e+003	-1.522e+003	5.2
17	-1.591e+003	-1.522e+003	4.6
18	1.589e+003	1.522e+003	4.4

kstar column corresponding to dof 2 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	-1.578e+003	-1.522e+003	3.7
2	1.577e+003	1.522e+003	3.6
17	1.560e+003	1.522e+003	2.5
18	-1.569e+003	-1.522e+003	3.1

kstar column corresponding to dof 17 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	-1.602e+003	-1.522e+003	5.3
2	1.594e+003	1.522e+003	4.7
17	1.608e+003	1.522e+003	5.7
18	-1.615e+003	-1.522e+003	6.1

kstar column corresponding to dof 18 (in N/m)			
dof	Test-based value	Reference value	Error (%)
1	1.592e+003	1.522e+003	4.6
2	-1.573e+003	-1.522e+003	3.3
17	-1.579e+003	-1.522e+003	3.8
18	1.588e+003	1.522e+003	4.4

Fig. 6.1 Grid system with FE node numbering

Fig. 6.2 Block joining a longeron with a cross member

Fig. 6.3 Block joining a longeron with a cross member and two diagonals

Fig. 6.4 Schematics of first two rigid support normal modes, for component 1 of grid system

unit displacement along d.o.f. 2

unit displacement along d.o.f. 18

Fig. 6.5 Schematics of two constraint modes, for component 1 of grid system

Fig. 6.6 Location and numbering of d.o.f. for translational measurements of both components of grid system

Fig. 6.7 Grid component attached to four flexible supports

Fig. 6.8 Close-up of interface support attachment at d.o.f. 2

Fig. 6.9 Schematic of accelerometer locations for derivation of rotational FRF's

Fig. 6.10 Overview of modal test set-up for grid components

Fig. 6.11 Close-up of interior driving point d.o.f. 15

Fig. 6.12 Reciprocity check between $H_{4,15}$ and $H_{15,4}$, for component 1 of grid system

Fig. 6.13 Reciprocity check between $H_{4,15}^*$ and $H_{15,4}^*$, for component 1 of grid system

Fig. 6.14a Point H' FRF at d.o.f. 4, for component 1 of grid system

Fig. 6.14b Point H' FRF at d.o.f. 15, for component 1 of grid system

Fig. 6.15 H' FRF for rotation with respect to y axis at d.o.f. 1 and the force at d.o.f. 4, for component 1 of grid system

Fig. 6.16 H' FRF for rotation with respect to x axis at d.o.f. 1 and the force at d.o.f. 4, for component 1 of grid system

Fig. 6.17 Drive point rotational FRF H_{0,y_1,m,y_1}^* , for component 1 of grid system

Fig. 6.18 Drive point rotational FRF $H_{\theta_{x2}, m_{x2}}^*$, for component 1 of grid system

Fig. 6.19 Reciprocity check between $H_{\theta_{y2},m_{y1}}^*$ and $H_{\theta_{y1},m_{y2}}^*$, for component 1 of grid system

Fig. 6.20 Reciprocity check between $H_{\theta_{1,m_{x2}}}$ and $H_{\theta_{2,m_{x1}}}$, for component 1 of grid system

Fig. 6.21 Displacement mechanism set-up
at d.o.f. 1, for grid component

Fig. 6.22 Digital indicator located at d.o.f. 16,
for component 1 of grid system

Fig. 6.23 Front view of extension bar and digital indicator at d.o.f. 2, for grid component

Fig. 6.24 Side view of extension bar and indicator at d.o.f. 2, for grid component

Fig. 6.25 Reaction force at d.o.f. 2 for load applied at d.o.f. 1, for static tests of component 1 of grid system

Fig. 6.26 Reciprocity check between $H_{4,15}$ and $H_{15,4}$, for component 2 of grid system

Fig. 6.27 Reciprocity check between $H_{4,15}^*$ and $H_{15,4}^*$, for component 2 of grid system

Fig. 6.28a Point H' FRF at d.o.f. 4, for component 2 of grid system

Fig. 6.28b Point H' FRF at d.o.f. 15, for component 2 of grid system

Fig. 6.29a Frequency errors for grid system, from CMS with test-based component parameters

Fig. 6.29b Mode shape errors for grid system, from CMS with test-based component parameters

Fig. 6.30a Frequency errors for grid system,
from CMS with simulation

Fig. 6.30b Mode shape errors for grid system,
from CMS with simulation

7

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary

In the modal testing procedures normally used, the structure is supported either on a suitable number of rigid supports to simulate fixed-boundary conditions or from a very flexible suspension system to simulate a free-free configuration. In either case, practical difficulties arise in ensuring that the supports do represent the assumed boundary condition, especially for large flexible space structures which require additional supports in order to sustain their own weight. Because of the difficulty in testing very flexible structures, large spacecraft are fully tested only in their retracted launch configuration, but rarely in their operational deployed configuration. The prohibitive cost and difficulties involved in testing are considered to outweigh the gains obtained from such tests.

The present research develops a new approach to CMS in which the modal parameters of the system are derived from the test-based modal and static parameters of its components. The proposed technique is referred to as the experimental hybrid CMS approach. The component parameters are derived from tests on the components held by supports of arbitrary stiffness. Such supports may be located at the interfaces and at any required number of interior points. The hybrid CMS method is based on a representation of the displacements along the physical d.o.f. by a combination of three different mode sets: constrained (or rigid support) normal modes, constraint modes and residual flexibility attachment modes. The static constraint modes arise from the presence of the constrained d.o.f. while the residual flexibility attachment modes are associated with the free interface d.o.f.

The novelty of the approach presented in this project is that the component modal and static parameters are derived from tests on the components attached to supports of arbitrary stiffness. This procedure obviates the need for providing fully rigid or highly flexible supports, and may lead to considerable savings in costs associated with testing. The expected savings in costs make testing of spacecraft in their deployed configuration quite attractive.

The work includes derivation of the mathematical formulation of the experimental hybrid CMS approach and the results of a series of computer simulation studies performed on three different systems.

An experimental verification program is carried out in order to confirm the practicality of the proposed hybrid CMS approach. A key component of the overall experimental program is the flexible support which restrains the component at its attachment point only in the vertical direction. A prototype of the flexible support is designed, built and tested in order to understand some of its dynamic characteristics.

Experimental verification of the CMS approach is performed by applying the approach to three different systems of increasing complexity. The first system consists of a fixed-fixed beam which is made up of two cantilever beams. A single flexible support is attached at the free end interface of the component. The second system consists of a free-free beam made up of two beams. Each component is attached to three flexible supports including one at an end of the structure, i.e. the interface. Each component of these two beam systems has only a single free rotational interface d.o.f. The third system consists of a free-free grid which is subdivided into two components. The interface of each of the two grid components is made up of two nodes. A component is retained by four flexible supports, of which two are located at the interface rigid blocks. The component interface has four free rotational d.o.f.

7.2 Conclusions of the Present Research

The following important conclusions can be drawn from the analytical and experimental studies undertaken in this research:

- (1) The proposed experimental hybrid CMS approach is demonstrated to be an attractive cost-effective alternative to presently available methods for validating analytical models of large and complex structures such as spacecraft in their operational deployed configuration. The approach obviates the need for providing fully rigid or highly flexible supports, as well as the need for infrastructure with very high ceiling required for free-free test configuration.
- (2) Computer simulation studies show that the hybrid CMS method is not very sensitive to component measurement errors. In fact, the level of errors used in the simulation turned out to be very conservative (i.e. too high) compared to errors observed in subsequent testing. For all three systems simulated, the modal frequencies of the system are sensitive to errors in the component description in the following order of decreasing importance: modal frequencies, constraint modes, interface rotational d.o.f. of retained normal modes and residual flexibility. It is thus observed that the component parameters to which the system frequencies are less sensitive are the ones likely to be more in error. The system mode shapes are sensitive to errors in the component parameters in the following order of decreasing importance: constraint modes, modal frequencies, interface rotational d.o.f. of retained normal modes and residual flexibility.
- (3) The experimental verification of the hybrid CMS approach shows that the procedure is quite feasible. The system modal parameters (natural frequencies and mode shapes) derived from test-based CMS are in good agreement with their reference values for all three systems. For example, the frequency errors for the two beam systems are less than 4% for the first 8 modes when between 7 to 9 component normal modes are retained and all residual flexibility terms are neglected; for the more complex grid system, the frequency errors for

the first 9 modes are less than 5% when 15 normal modes are retained for each component and only part of the residual flexibility terms are accounted for.

- (4) The agreement between the natural frequencies of the components for the free-free beam obtained by curve fitting the derived FRF's in rigid support configuration and their analytical counterpart frequencies is excellent and clearly demonstrates that they can be derived quite effectively from tests in a configuration in which the test article is held by supports of arbitrary stiffness. All frequency errors for the first 8 modes are less than about 1%. The beam test articles used in the work were purposely chosen to be simple, because in a complex structure, it is not possible to easily develop accurate analytical values for comparison.
- (5) The test results confirm the simulation prediction that the inclusion of residual flexibility terms always reduces the errors in derived system parameters. The level of improvement is found to be structure dependent. For the free-free beam, the system parameters without residual flexibility are already very good and can be considered quite acceptable, so the improvement resulting from the inclusion of residual flexibility, although not negligible, is not that significant. On the other hand, in the case of the grid system which represents the most complex structure tested, the use of residual flexibility terms leads to a dramatic improvement in the accuracy of some of the estimated system parameters. For the grid system, when sufficient normal modes are retained (e.g. 15), only the fundamental frequency is in relatively large error (11.2%) when ignoring the residual flexibilities, with the other frequencies having an error of less than 7%. For the case when some residual flexibility terms are used, all frequency errors drop to less than 5%, with the first one being reduced to 1.1%. An explanation for the structure dependency seems to be the fact that for structures such as the grid for which modal density is fairly high, compared to beam structures for which the frequencies are well separated, the residual flexibilities are higher. As a result, when residual flexibilities are ignored, larger errors are caused in the estimates of system parameters.

- (6) Based on the previous discussion on residual flexibility, it is recommended that residual flexibility terms must be estimated and included in the CMS method. It is important to note that only the residual flexibility values from the drive point FRF's associated with the free interface rotational d.o.f., i.e. the diagonal coefficients of G_{nn} , are required. In fact, for both the free-free beam and grid systems, it is observed that the inclusion of the few required residual flexibility terms with a smaller number of retained normal modes results in more precise estimates of system parameters than if more than twice as many modes are kept but the residual flexibilities are ignored. All experimental residual terms in this work are estimated with the method presented by Bookout (1995) which is found to be very effective.
- (7) In order to improve the understanding of the significance of the error level in system modal parameters derived from test-based component parameters, simulated system parameters were computed with the same CMS configuration (e.g. number, type and location of component d.o.f., number of retained normal modes), but with analytically determined precise component parameters. This strategy was found to be very useful in the present work related to the validation of CMS approach.
- (8) The derived FRF's in rigid support configuration for the free-free beam and grid components which are attached only to flexible supports are smooth with few glitches and extraneous peaks. Also, as shown in the tests on component 2 of the fixed-fixed beam system, a number of simple modifications in the flexible supports may be employed, if necessary, to ascertain whether the observed anomalies are potential physical modes of the test article, even before starting the parameter estimation phase.
- (9) Successful approximation of rotational FRF's (rotation/force or rotation/moment) in rigid support configuration may be obtained from translational FRF's using (1) finite difference expressions in the case of uniform structure such as the beam test articles, or (2) simple difference expressions (leading to single FRF divided by a scalar) in the case of sufficiently large rigid elements of the test article at the interface, as for the grid structures.

7.3 Recommendations for Future Work

- (1) To increase the credibility of the experimental hybrid CMS approach and flexible support testing method within the space community where the main applications are found, the approach should be demonstrated by applying it to other representative test articles. Although not easily available, engineering models of spacecraft would represent such ideal test articles. The system modal parameters derived from test-based CMS could be compared with the results of FE models of the spacecraft.
- (2) Considering the prime motive for developing the experimental hybrid CMS approach, its ultimate validation would be its application to ground testing of spacecraft components and for which the spacecraft modal parameters derived from test-based CMS would be compared to the parameters obtained from measurements in on-orbit deployed configuration. In this case, an obvious additional unknown would be the potential differences between the spacecraft dynamic behavior in a 0-g on-orbit environment as compared to the 1-g environment in which the components would be tested.
- (3) Simulation sensitivity studies of the errors in system modal parameters with respect to the number of selected interior free d.o.f. should be carried out. Significant reduction in the number of such measurement points could imply some cost saving in test performance. These studies should also be carried out with available test data in order to confirm the simulation results.
- (4) Reduced mass matrices of the components are the only analytical parameters required by the experimental hybrid CMS approach. Investigation of the techniques aimed at deriving a mass matrix from modal testing should be performed. As mentioned in Chapter 1, physical parameter identification is still an area of on-going research. In fact, such an investigation could constitute a complete research project by itself. Successful implementation of such a technique would then make the CMS approach completely experimental.

- (5) Simulation sensitivity studies related to the analytical mass matrices of the components could be carried out. Such studies could evaluate the effects on the derived system parameters of the type of mass matrices used (e.g. lumped or consistent) and the type of matrix reduction algorithm employed when the matrices for the full model are reduced before being used in CMS.

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IMAGE EVALUATION TEST TARGET (QA-3)

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