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**The TETHER LABORatory Demonstration System (TE-LAB); Design,
Operation and Mathematical Model Validation Using an
Eigenvalue Approach**

by

Bruce Alexander McClure

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of
Master of Engineering

Department of Mechanical and Aerospace Engineering
Ottawa - Carleton Institute
for Mechanical and Aerospace Engineering

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**“The TEther LABoratory Demonstration System (TE-LAB); Design, Operation
and Mathematical Model Validation Using an Eigenvalue Approach”**

submitted by

Bruce Alexander McClure

in partial fulfilment of the requirements for the degree of
Master of Engineering

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ABSTRACT

The Canadian Space Agency (CSA) is currently developing a tethered sounding rocket mission called OEDIPUS-C (a follow-on to the OEDIPUS-A mission). These OEDIPUS missions employ a unique configuration consisting of two spinning end-bodies attached by an approximately one kilometre long tether. During the original OEDIPUS-A flight, unexpected dynamic behaviour was experienced by one of the subpayloads. To study these dynamic anomalies, the TEther LABoratory demonstration system (TE-LAB) was developed in parallel with a linearized mathematical model particular to the TE-LAB facility.

This thesis first describes the design of the TE-LAB facility as well as the complete set of experimental tests. A linear mathematical model is then summarized and the eigenvalue problem solved (including a rigorous study of the damped gyroscopic natural modes and animation of the basic mode shapes). A stability analysis based on the solution of the system eigenvalues is also performed. Finally the experimental results are compared to the analytics for validation of the mathematical model. The results of the ground-based exercise are then extrapolated to provide insight into the stability of the OEDIPUS configurations.

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Finally, the author wishes to thank Mr. J. Bergmans, a colleague in the design, operation and analytical study of the TEther LABoratory Demonstration System

Table of Contents

1. INTRODUCTION AND BACKGROUND	1
1.1. OEDIPUS-A	1
1.2. OEDIPUS-C AND THE TETHER DYNAMICS EXPERIMENT	4
1.3. SCOPE OF THESIS	7
2. DESIGN AND MANUFACTURE OF TE-LAB.....	8
2.1. BASIC SCALING.....	8
2.1.1. Considering Gravitational Effects	9
2.1.2. Observations and Conclusions Regarding Scaling.....	11
2.1.3. Scaling of a 1000 metre Tether.....	12
2.2. MECHANICAL SYSTEM.....	13
2.2.1. Lower Rotating Table and End-body.....	18
2.2.2. Constant Spin Drive Dynamics.....	19
2.2.3. Upper Rotating Table and Tether Spool Assembly	20
2.2.4. Calibration of Tether Spool Assembly.....	21
2.2.5. TE-LAB Tether	24
2.3. ROTATION RATE CONTROL SYSTEM.....	24
2.4. PAYLOAD ORIENTATION DETERMINATION SYSTEM (PODS).....	24
3. DETERMINATION OF TE-LAB PARAMETERS AND PERFORMANCE CHARACTERISTICS	26
3.1. DETERMINATION OF PAYLOAD INERTIA.....	26
3.1.1. Longitudinal and Transverse Inertias.....	27
3.1.2. Products of Inertia: Dynamic and Static Balancing	30
3.2. MEASUREMENT OF DAMPING OF THE GIMBALS	32
3.3. ANALYSIS OF THE TE-LAB END-BODY: A SIMPLE GYRO.....	38
4. DEVELOPMENT AND ANALYSIS OF THE TE-LAB LINEAR MODEL	44
4.1. LINEAR MODEL.....	45
4.1.1. The Spinning End-Body - No Tether	50
4.1.2. The Spinning Tether - No End-body.....	51
4.2. EIGENVALUE ANALYSIS.....	54
4.2.1. Determination of Natural Modal Parameters for the Damped Gyroscopic System.....	54
4.3. STABILITY OF THE LINEAR MODEL.....	56
4.4. MODAL PARAMETERS AS A FUNCTION OF SPIN-RATE.....	57
4.4.1. Assessment of Required Number of Assumed Tether Mode Shapes.....	65
4.5. MODE SHAPES AND ANIMATION.....	67
4.6. FORMATION OF LINEAR MODEL STABILITY BOUNDARIES	77
4.6.1. Stability Boundaries by the Eigenvalue Approach.....	77
4.6.2. Stability Boundaries by the Stiffness Matrix	78
5. TE-LAB TEST PROCEDURE WITH TYPICAL EXPERIMENTAL RESULTS.....	82
5.1. CALIBRATION OF THE TE-LAB PHOTOGRAMMETRIC SYSTEM	83
5.2. TE-LAB FREE-SPINNING END-BODY TESTS	84
5.3. TE-LAB SPINNING TETHER TESTS	84
5.4. TE-LAB SPINNING TETHER AND END-BODY TESTS	88
5.5. THE FAST FOURIER TRANSFORM ANALYSIS	92

6. COMPARISON OF EIGENVALUE CALCULATIONS TO EXPERIMENTAL RESULTS	103
6.1. EVALUATION OF TE-LAB FREE-SPINNING END-BODY - NO TETHER.....	103
6.2. EVALUATION OF THE TE-LAB SPINNING TETHER - NO END-BODY	106
6.3. EVALUATION OF TE-LAB SPINNING TETHER AND END-BODY	109
6.3.1. Damped Gyroscopic Natural Modes for the Spinning Tether and End-body	109
6.3.2. Stability Boundaries for the Spinning Tether and End-body	117
6.3.3. Specific Test Parameters Affecting the TE-LAB Dynamics.....	121
7. APPLICATION OF THE TE-LAB MODEL TO OEDIPUS	124
7.1. OEDIPUS-A STABILITY	124
7.2. OEDIPUS-C STABILITY	125
8. CONCLUSIONS, CONTRIBUTIONS AND FUTURE RESEARCH	132
8.1. SUMMARY OF CONTRIBUTIONS	132
8.2. CONCLUSIONS	133
8.3. FUTURE RESEARCH	134
9. REFERENCES.....	139
10. APPENDICES	141
10.1. APPENDIX A: OEDIPUS-A AND C FLIGHT PARAMETERS	141
10.2. APPENDIX B: TABLE OF TE-LAB PARAMETERS	143
10.3. APPENDIX C: FORMATION OF COEFFICIENT MATRICES FOR LINEAR MODEL	145
10.4. APPENDIX D: TE-LAB EXPERIMENTAL RESULTS	149

Chapter 1

1. Introduction and Background

1.1. OEDIPUS-A

On January 30, 1989 the OEDIPUS-A tethered payload was launched on a three stage Black Brant X sounding rocket from the Andøya launch site in northern Norway. Sponsored by the Space Science Division of the Canadian Space Agency (CSA), the project involved scientists from Canada and the United States. The sounding rocket mission utilized a unique payload configuration consisting of two subpayloads connected by an approximately one kilometer tether. The main OEDIPUS (Observations of Electric field Distributions in the Ionspheric Plasma - a Unique Strategy) scientific objectives were to; make passive observations of the auroral ionosphere, measure response of the large probe (two subpayloads connected by the long tether) in the ionospheric plasma and seek new insights in the plane and sheath-wave propagations in plasmas. The mission, as depicted in Fig. 1.1, lasted approximately 12 minutes, reaching an apogee of 500 km with 958 m of tether deployed.

The OEDIPUS-A payload (shown in Fig. 1.2), developed by Bristol Aerospace Limited under contract to CSA, consisted of fore and aft payloads - mass of 84 kg and 131 kg

Figure 1.1: OEDIPUS-A Flight Mission Showing Separation and Free-Fall States

respectively. Each payload was fitted with its own set of scientific instruments, control, power and telemetry systems. Due to nature of scientific experiments, active control was not permitted - rather passive spin stabilization was used. Therefore, prior to separation, the fore and aft payload were despun to 0.7 cps and aligned to within 1° of the Earth's magnetic field. Once aligned, a pressurized gas separation system was activated causing the two end-bodies to slowly separate. Situated on the fore payload was approximately 1300 m of Teflon coated copper wire wound on a spool-type reel. A magnetic hysteresis brake attached to the spool provided a small constant torque to achieve a smooth deceleration of the relative motion of the payloads.

Figure 1.2 OEDIPUS-A Payload Configuration

It was required that payload coning be minimized during flight; but, in so much as both the fore and aft payloads had sources of internal energy dissipation (flexible radial booms), payload coning was inevitable. Consequently, each payload would naturally approach a flat spin about its major axis - a dynamically stable configuration. An analysis done by NASA of the aft payload, predicted a maximum initial coning angle of 5° due to the separation propulsion system, and that the coning angle should remain within 10° over the duration of the flight. In their analysis, it was assumed that the effect of the tether could be neglected due to a relatively light mass as compared to the payloads and the low

tether tension experienced during flight. Therefore, the energy dissipation rate predicted prior to flight was based solely on the experimentally measured damping factor of the booms. However, a post-flight analysis of the OEDIPUS mission revealed some unexpected dynamic behaviour of the aft-payload. From the flight data, the forward payload experienced an initial tip-off of 7° which remained relatively constant throughout flight. More interestingly, the aft payload underwent an initial tip-off of 3.5° but rapidly increased to approximately 40° by end of the flight. An initial investigation was done by Tyc and Berry at the University of Manitoba in an attempt to understand what may have caused this unexpected attitude dynamics of OEDIPUS-A¹. Their analysis supported the theory that the interaction of the tether connecting the fore and aft payload was the cause of the rapid increase in coning experienced by the aft payload. It was decided that two mechanisms associated with the tether might influence the payload dynamics:

- (i) an additional source of energy dissipation (i.e. tether damping),
- (ii) an external force acting on payload.

1.2. OEDIPUS-C and the Tether Dynamics Experiment

OEDIPUS-C, a similar sounding rocket mission to OEDIPUS-A planned for the fall of 1995, is scheduled to be launched on a four stage Black Brant XII with a projected apogee of 800 km. The expected flight time is approximately 18 minutes. Due to the unanticipated findings of OEDIPUS-A, the Tether Dynamics Experiment (TDE) was initiated, a collaboration between the Canadian Space Agency, Bristol Aerospace, the University of Manitoba, the University of British Columbia, McGill University and Carleton University. The purpose of the TDE is to create a better understanding of a spinning tether - flexible body system and provide support for this follow-on mission.

The TDE includes a detailed analysis to investigate passive stabilization for the spinning tethered configuration. Four stabilization approaches have been considered: use of an OEDIPUS-A configuration with modified parameters such as tension; use of a boom system to change the payload inertias; despin of the tether; and use of a spin axis aligned momentum wheel. To this end, a mathematical model, described in detail by Tyc et al.², was developed from which the closed-form expressions for asymptotic stability were derived. The investigation by Tyc identified the use of a momentum wheel or using four long radial booms to be the only viable solutions for stabilization of the OEDIPUS system. Although the momentum wheel was technically feasible, the cost was found to be

prohibitive. Therefore, a passive stabilization method for the OEDIPUS-C payload that uses two pairs of relatively long radial booms on each of the subpayloads, to achieve the desired mass properties, has been chosen. The booms also satisfied the scientific objectives, being implemented as dipole antennas.

A ground test program was undertaken in support of the TDE, to validate the mathematical modeling and the chosen stabilization approach. Two different ground tests have been developed:

- (i) *"Hanging Spin Tests"*
- (ii) *"TEther LABoratory Demonstration System - TE-LAB"*

The Hanging Spin Test was developed at the University of British Columbia, to obtain early qualitative information of the spinning tethered system. This set-up³ (as shown schematically in Fig. 1.3) consists of a cylindrical mass suspended vertically by a tether from a spin motor. By spinning up the tether and end-body slowly, the speed at which the tether or end-body oscillations begin to suddenly grow can be observed. The critical speed at which the system became unstable could then be compared to that predicted by the analytical stability analysis. The tests were performed with four different end-bodies, including two with the inertia ratios of OEDIPUS-A and C aft payloads. The nominal tether tension is imparted by the weight of the end-body. The experimental critical speeds were found to closely match those predicted by the stability criteria. In addition, the tests indicated that the simulated OEDIPUS-A end-body developed large cone angles, while the OEDIPUS-C body remained relatively stable.

Figure 1.3 Schematic of Hanging Spin Tests Performed at UBC

The Tether Laboratory Demonstration System (TE-LAB) was developed in parallel with the qualitative tests performed at UBC. TE-LAB is aimed at achieving quantitative information about the end-body orientation and tether dynamics. It was conceived of at the CSA, and developed jointly by the author and another collaborating Masters student (John Bergmans).

Figure 1.4 Schematic of Tether Laboratory Demonstration System at CSA

The basic system consists of a gimballed end-body, permitting 3 DOF rotational motion, attached to a tether and upper rotating table. A photogrammetric system, developed at the University of Toronto by Laurin⁴ adopted and modified for TE-LAB by Bergmans⁵, permits targets on the end-body to be tracked and cartesian coordinates in the camera frame returned real-time. Based on the cartesian coordinates, the Euler angles can be

calculated using inverse kinematics. The tether tension is maintained by a spool mechanism on the upper table and is adjustable.

A linear mathematical model was developed at the CSA in conjunction with the TE-LAB set-up. This model accounts for the particular features of the ground test set-up such as gravity, damping in the gimbal mechanism and a relatively short test tether.

1.3. Scope of Thesis

The objectives of this thesis are as follows:

- (i) describe the TETHER LABORatory Demonstration System (TE-LAB), with emphasis on the design philosophy and the mechanical elements;
- (ii) obtain quantitative data on the behaviour of the spinning tethered system for comparison to the linear mathematical model developed at the CSA;
- (iii) extrapolate the results of the ground based exercise to provide insight into the stability of the OEDIPUS-C payload configuration.

Chapter 2 describes the design and manufacture of TE-LAB focusing mostly on the mechanical aspects with a brief overview of the Payload Orientation Determination System (PODS) and Rotation Rate Control System (the latter are developed in more depth in the thesis by Bergmans⁵). An overview of the linear mathematical model developed at the Canadian Space Agency by Vigneron et al.¹⁶ is presented in Chapter 3. The basis for an eigenvalue problem analysis performed by the author is also developed. Chapter 4 describes the basic laboratory parameters giving insight into the end-body being studied. The basic test procedure and a description of test results are set out in Chapter 5. A comparison between the linear model and experimental results is made in Chapter 6. Finally, Chapters 7 and 8 provide discussion with application to the OEDIPUS-C mission and the conclusions respectively.

This thesis is written in parallel with that of J. Bergmans⁵ and certain sections necessarily overlap.

Chapter 2

2. Design and Manufacture of TE-LAB

During the initial stages of TE-LAB, preliminary studies were done to determine the feasible laboratory parameters that would best emulate the dynamics of the OEDIPUS system in space⁶. From these studies, scaling laws were derived. The following describes the TE-LAB set-up with an emphasis on the mechanical design based on the scaling laws developed. Finally the data acquisition and control systems, as developed by Bergmans⁵, are briefly outlined.

2.1. Basic Scaling

A formal approach of developing the basic scaling parameters for TE-LAB (based on a simplified mathematical model by Tyc⁷) was done as a joint effort of the CSA and Bristol Aerospace⁸. This method was conducted in two steps: initially without gravity and then with gravitational effects included. Generally, the approach adopted required developing the equations of motion in a non-dimensional form. The non-dimensional parameters in the equations were then used to establish the scaling laws with the principal being that both the scaled laboratory experiment and flight configuration have the same parameter values.

The basic scaling is summarized for a single payload supported at its centre of mass. The end-body is assumed to be axisymmetric ($B = A$, $D = E = F = 0$, i.e. no products of inertia) without any external disturbance torques acting on the system. It is also assumed that the length of the tether is much greater than the tether attachment distance ($l \gg b$). Longitudinal oscillations of the tether are considered negligible. As derived by Tyc et al.^{7,8} the scaling laws for TE-LAB without gravitational effects are as follows:

$$T_2 = T_1 \left(\frac{\rho_2}{\rho_1} \right) \left(\frac{l_1}{l_2} \right)^2 \left(\frac{\omega_{s2}}{\omega_{s1}} \right)^2 \quad (2.1)$$

$$b_2 = \left(\frac{l_2}{l_1} \right) b_1 \quad (2.2)$$

$$A_2 = A_1 \left(\frac{\rho_2}{\rho_1} \right) \left(\frac{l_2}{l_1} \right)^3 \quad (2.3)$$

$$C_2 = C_1 \left(\frac{\rho_2}{\rho_1} \right) \left(\frac{l_2}{l_1} \right)^3 \quad (2.4)$$

$$\eta_2 = \eta_1 \quad (2.5)$$

where T represents the tether tension, ρ the tether density, l the length of tether, ω_s the end-body spin-rate, A the transverse inertia, C the longitudinal or spin inertia, b the distance between tether attachment point and body centre of mass, and finally, η the damping factor. Subscript "1" designates the in-space configuration and subscript "2" denotes the laboratory experiment system.

2.1.1. Considering Gravitational Effects

The in-flight configuration is dynamically different from the ground-based model (TE-LAB) due to the manner in which the gravitational forces act. In space, the spinning tethered system, subject to very low differential forces of gravity, is essentially in a state of free-fall. On the other hand, the ground-based model has a constant force of gravity acting on it (differential force of gravity is considered to be zero). TE-LAB minimizes the effect of gravity on the end-body by supporting it at its mass centre; however, the gravitational forces will affect the tether vibration, and in turn the end-body through

dynamic interaction. Therefore an attempt was made to include the gravity term in the development of the scaling laws.

To determine the dimensionless parameters, non-dimensional equations were derived for a one dimensional vertical string subject to gravity and a distributed external force. This derivation was done for both the ground-based and in-space configuration⁸. The difference in two models arises from the tension caused by the distributed mass of the tether. The tension in ground-based model at a distance x along the tether length is defined by a linear relationship as follows:

$$T = \rho g x \quad (2.6)$$

However, for the in-space configuration, assuming that the vertical string is in a free-fall state, it can be shown that the tension is given approximately by:

$$T_t = \rho g_h l \left(\frac{l}{R+h} \right) \left(\frac{x}{l} \right) \left(1 - \frac{x}{l} \right) \quad (2.7)$$

where R is radius of the earth, h the height above the earth's surface, g the gravitational acceleration at the earth's surface and finally g_h the gravitational acceleration at a height h above the earth's surface. Figure 2.1 demonstrates the tether tension distribution for both the space and ground configurations.

Figure 2.1 Tether Tension Distribution Curves for Ground-based and In-flight Configuration

This difference in tension distribution results in non-dimensional equations for the two cases that have different functional forms. Consequently, it is concluded that complete similarity cannot be achieved with the ground-based experiment.

Instead an approximate scaling law that includes gravity was developed by equating the maximum magnitudes of the gravity related terms which occur at $x/l = 1/2$ for the free fall case and $x/l = 1$ for the ground experiment. The subsequent scaling law that achieves similarity in the maximum magnitude of the gravity related term is

$$\omega_2 = 2 \beta \sqrt{\frac{l_1}{l_2}} \omega_1 \quad (2.8a)$$

where,
$$\beta = \left(\frac{R + h}{R} \right)^{\frac{3}{2}} \sqrt{\frac{R}{l_1}} \quad (2.8b)$$

2.1.2. Observations and Conclusions Regarding Scaling

From scaling laws (2.3) and (2.4), it is evident that for similar tethers (i.e. $\rho_2 = \rho_1$) in the in-flight and ground-based configurations, the inertias of the ground-based end-body must scale by $(l_2/l_1)^3$. The ceiling height of available test sites on the ground, limits the ground tether length to 10-15 metres. Therefore, to model an OEDIPUS-like system, the practically achievable ratio of tether lengths is $(l_2/l_1) = (10/1000)$. This dictates that the ground-based end-body inertia would have to be 10^{-6} that of the in-flight payload inertia, thus making fabrication of the ground-based model very difficult. The effect of the $(l_2/l_1)^3$ term on the scaled tether can be partially offset by using a tether with a higher density per unit length; however, care must be taken not to choose too heavy a tether so that the bending stiffness (not included in present modeling) becomes an important factor in the system dynamics.

Finally scaling law (2.8a) indicates that the effects of gravity can be minimized by using very high spin-rates in the ground-based TE-LAB set-up. As TE-LAB is to provide a quantitative simulation of the OEDIPUS configuration, photogrammetric targets on the end-body are required to permit tracking of its orientation. This imposes a restriction on spin-rate, limited by the processing rate of the PODS system.

2.1.3. Scaling of a 1000 metre Tether

Using the developed scaling laws (Eq. (2.1) through (2.5) and (2.8)), the required laboratory parameters, to rigorously scale the OEDIPUS configuration, can be calculated. Table 2.1 summarizes a 1000 m space tether scaled to a 15 m ground-based laboratory. A more thorough examination of scaling the OEDIPUS configuration was performed^{6,8} for which three cases are presented below: two that ignore the effects of gravity and the third scaling gravity. A complete set of OEDIPUS- A and C parameters can be found in Appendix A.

In all three cases, small payload inertias are required, and fabrication of the ground-based models proves to be impossible. By scaling the tether tension and neglecting gravity (as in case 1) the required ground-test tension is well below that achievable in the experimental set-up. That is, the minimum tether tension achievable is equal to $\rho g l$, the weight of the hanging tether. Conversely, for case 3 when gravity is scaled, the required tension and spin-rate to scale a 1000 m tether is not feasible in a laboratory setting. Case 2 demonstrates that for a heavier tether, the inertias of the mock-up are more reasonable. The tension for this case however, remains well below the nominal $\rho g l$.

Table 2.1 Scaling of 1000 m In-Flight Configuration to 15m Laboratory

Column No:	0	Required Laboratory Parameters		
		1	2	3
Case:	In-Space	Ignore g $\rho_2 = \rho_1$ $\omega_2 = \omega_1$	Ignore g $\rho_2 = 10\rho_1$ $\omega_2 = \omega_1$	Scale g $\rho_2 = \rho_1$
l (m)	1000	15	15	15
ρ (kg/m)	0.003	0.003	.03	.003
A (kg-m ²)	36.6	1.2×10^{-4}	1.2×10^{-3}	1.2×10^{-4}
B (kg-m ²)	39.2	1.3×10^{-4}	1.3×10^{-3}	1.3×10^{-4}
C (kg-m ²)	5.9	0.2×10^{-4}	0.2×10^{-3}	0.2×10^{-4}
b (m)	0.83	1.2×10^{-2}	1.2×10^{-2}	1.2×10^{-2}
T_s (N)	1.24	2.8×10^{-4}	0.025	547
T_{free} (N)	0.14	0.3×10^{-4}	0.003	61.8
ω (cps)	0.7	0.7	0.7	1086

Therefore, a compromise was made to design TE-LAB with a tether length in the 15 m range, and recognize that it emulated a spinning tethered system of 100 to 150 m in length. The results then would be used to validate a linear mathematical model in those ranges. By extending the mathematical model to 1000 m, it could be used to give insight into the stability of the OEDIPUS configuration. The final target parameters that were established to guide the design of TE-LAB are summarized in table 2.2.

Table 2.2 Scaling of 150m In-Flight Configuration to 15m Laboratory used as guidelines in design of TE-LAB

Parameter	In-Space Configuration	TE-LAB	
		Nominal	Range
l (m)	150	15	
ρ (kg/m)	0.003	0.030	.003 - .03
A (kg-m ²)	36.6	.37	low as possible
B (kg-m ²)	39.2	.39	low as possible
C (kg-m ²)	5.9	.059	low as possible
b (m)	0.83	.083	0-.25
T_s (N)	1.24	1.24	$\rho g l - 5.0$
T_{free} (N)	0.14	4.41	$\rho g l - 5.0$
ω (cps)	0.7	0.7	0-3

2.2. Mechanical System

The TE-LAB mechanical components as shown in Fig. 2.2 consists of the lower rotating table and end-body; the upper rotating table seating the tether spool assembly; and the tether itself. The following describes these basic components with a rationalization for their design.

Figure 2.2: Photograph Showing TE-LAB Set-up in David Florida Laboratory, Ottawa Ontario

Figure 2.3: Photograph Illustrating TE-LAB Lower Rotating Table and End-body

Figure 2.4: Photograph of Gimballs Mounted Internally to the TE-LAB End-Body Shell

Figure 2.5: Photograph of Upper Rotating Table and Tether Spool Assembly

2.2.1. Lower Rotating Table and End-body

The main purpose of the lower rotating table and end-body is to emulate a free-spinning body. The design was dictated by numerous requirements⁶. In addition to restrictions imposed by the scaling laws, the following were necessary in the design of the OEDIPUS payload mock-up: an adjustable tether attachment point; provision for attachment of various tethers; provision for future attachment of appendages; photogrammetric targets fixed to payload shell to be used in tracking payload orientation; ability to function in both spinning and rocking body mode of operation; and permit a 30 degree angle of nutation.

TE-LAB End-Body: The scaling laws demonstrate that an important aspect of the TE-LAB end-body design was to minimize the body's mass moments of inertia. To model the in-flight OEDIPUS-A aft payload, an inertia ratio (A/C) of approximately six is required. Considering the ground based end-body to be a simple hollow cylinder supported at its mass centre, it can be shown that maintaining an inertia ratio similar to that of OEDIPUS permits a coning angle of only 14° (well below the recommended 30°). Therefore, after consideration of various designs and manufacturing methods, a configuration as shown in Fig. 2.3 was adopted. The basic shell, machined of delrin, has a conical shape below its centre of mass to maximize the obtainable coning angles (approximately 30-35 degrees). Above the mass centre a slender cylindrical shape is realized to minimize the spin inertia. A maximum (A/C) of three was feasible for the TE-LAB end-body. The addition of adjustable weights (rings) above and below the centre of mass allow for various payload inertia arrangements. Further, a massive ring added at the centre of mass of the body permits a major axis spinner, more like the OEDIPUS-C configuration, to be demonstrated.

A set of rotating gimbals is mounted internally at the mass centre of the payload to permit the body a 3 degree of rotational freedom (see Fig. 2.4). The gimbals are free to move relative to the body along the longitudinal axis, guided by slots in the shell wall. Once located at the mass centre of the body, the gimbals are fixed (with set screws) such that the bearing axes and transverse axes are coincident. The gimbals provide a natural rocking body mode of operation by oscillating the end-body about either of the bearing axes. High precision bearings were used in the gimbals to maintain minimal internal energy dissipation so as to best approximate the free-spinning body. This energy dissipation is further evaluated in section 3.2.

The tether is fastened to the end-body along the spin axis by way of the tether attachment table, a disk that translates along the cylindrical portion of the payload shell (guided by slots). This provides a mechanism to adjust the tether attachment distance (i.e. distance between tether attachment point and mass centre of the body). Once positioned the adjustable table is locked in place by set screws located on either side of the end-body shell.

Lower Rotating Table Drive System: The drive system for the TE-LAB end-body consists of a small geared DC motor (with shaft encoder) incorporated with a hysteresis clutch, both inertially fixed to the ground (see Fig. 2.3). Spin-rates between 0 and 5 Hz are achievable. The hysteresis clutch provides the capability to run at constant speed (i.e. with clutch locked up) or deliver a constant torque to the spinning body if desired. As the TE-LAB tests were performed in a constant speed mode of operation, a rationalization of the constant spin drive versus a constant torque drive is provided in section 2.3.2. To reduce transfer of mechanical vibrations to end-body due to the drive system, flexible shaft couplings are used between all components. A set of rigidly fixed bearing blocks through which the drive shaft passes, further minimizes the amount of vibration experienced by the gimbals and payload mock-up.

2.2.2. Constant Spin Drive Dynamics

For a freely-spinning symmetric body with a given nutation angle ν , the precession and spin-rate, $\dot{\psi}$ and $\dot{\sigma}$ respectively, can be characterized by the following expressions:

$$\dot{\psi} = \frac{H}{A} \quad (2.9)$$

$$\dot{\sigma} = \left(\frac{A - C}{AC} \right) H \cos \nu \quad (2.10)$$

In the above equations, H is the angular momentum about the vertical axis while A and C are the transverse and longitudinal inertias respectively.

Euler's equation, $\dot{M} = H$, states that if no external moment M is applied to the body, the angular momentum remains constant. Consequently, the precession rate also remains constant. For a body with no source of internal energy dissipation (i.e. no damping), the spin-rate will remain constant as well. However, with the introduction of damping, the

body's nutation angle begins to slowly change. Rimrott⁹ further derives an expression for the change in terms of the precession and spin-rate,

$$\dot{\nu} = \beta \frac{H^2}{2AC^2} \dot{\sigma} \dot{\psi} \cos \nu \sin 2\nu \quad (2.11)$$

where β is an internal energy dissipation coefficient for the total system.

The above equations imply that for a minor axis spinner, the spin-rate decreases as the nutation angle approaches 90° . In fact, the spin-rate will be zero at a 90° nutation angle. Therefore a constant spin drive would not be appropriate for large nutation angles. However, to investigate the stability of the end-body, where small angles are expected, a constant spin drive provides a good approximation for a free-spinning body. That is for a small angle approximation $\cos \nu \approx 1$ and the spin-rate reduces to,

$$\dot{\sigma} \approx \frac{A-C}{CA} H = \text{constant} \quad (2.12)$$

2.2.3. Upper Rotating Table and Tether Spool Assembly

The upper rotating table and spool assembly perform two main tasks: synchronize the spin-rate with the lower table to limit twist in the tether during the spin-up and steady state mode of operation; and maintain a nominal tension in the tether during operation of TE-LAB.

Upper Rotating Table: Figure 2.5 displays the basic components of the upper table. It consists of an inertially fixed geared DC motor (with shaft encoder) to drive the table; a rotating platform to seat the tether spool assembly; and a set of slip rings supplying power to the spool assembly. Also incorporated in the upper drive system are a bearing block and flexible shaft coupling to reduce any mechanical vibrations that might affect the tether dynamics. The large weight secured to the upper platform serves as a counterbalance to overcome the dynamic effects of the offset tether spool assembly.

The upper table was then mounted on a 25 ft theodolite stand (i.e. a tall vertical stand normally used to align spacecraft components) thus making the height of the upper

rotating table more accessible and adjustable to permit various lengths of tethers to be tested.

Tether Spool Assembly: During the OEDIPUS-A flight, the tether tension was maintained during both the "separation" and "free-fall" states using a tether spool assembly located on the forward payload.¹⁰ This subsystem consisted of an aluminum drum with a width of approximately 10 cm and a diameter of 12 cm. The spool held approximately 1300 m of Teflon coated conductive wire. Cantilevered from the spool shaft was a hysteresis brake (MAGTROL HB100-3) to provide an adjustable torque setting for the entire spool assembly. The fixed spool break-away torque was pre-set such that a separation distance of $1000 \text{ m} \pm 300 \text{ m}$ between the forward and aft payload could be achieved. The amount of tether deployed was monitored using optical sensors located on the tether assembly.

While the OEDIPUS spool design (hysteresis brake and spool) provided a predictable break-away torque, no control over a slack tether was offered. Therefore, the tether tension device ultimately adopted for TE-LAB comprises a torque motor, hysteresis clutch and spool assembly (Fig. 2.5) as it provides superior tension control. The clutch coil (input shaft), coupled to a geared DC motor, is driven at a constant speed. The output shaft is rigidly fastened to the spool and is free to spin, having only the magnetic friction induced by the coil to overcome. By then varying the DC current to the coil, the magnetic resistance between stator and coil can be varied. The operational range of the TE-LAB spool assembly is 0 to 5N. Unlike the OEDIPUS-A brake and spool, the TE-LAB clutch spool assembly allows the tether to be both released and captured. To ensure that the tether release point is on the spin axis of the upper table, an eyelet aligned with the spin axis is secured to the spool assembly. This eyelet can also be adjusted to provide non-aligned operation if desired.

2.2.4. Calibration of Tether Spool Assembly

Illustrated in Fig. 2.6 is a schematic of the basic calibration set-up for the TE-LAB tether spool assembly. Calibration was performed using the TE-LAB configuration with the upper table stationary. Because the general performance pattern is a hysteresis curve, only the portion of plot where current is decreasing is considered. Consequently to obtain repeatable results in both calibration and TE-LAB operation, the clutch was always

saturated at its rated voltage of 12 volts and then the current decreased to the desired level. The torque motor was operated at its rated voltage of 27.5 volts.

Various weights, ranging from 0.1 N to 5 N, were suspended from the spool (a thread used to suspend masses was assumed to be massless) and the corresponding current was recorded. Tests were then repeated and the current levels for the two tests averaged. Experimental data was found to be repeatable. A fourth order polynomial was fit to the data using MATLAB to produce a typical nominal tether tension versus DC current plot. During TE-LAB operation this polynomial was used to give a direct correspondence between tether tension and clutch current. A typical calibration curve with its corresponding fourth order polynomial fit is presented in Fig. 2.7.

These calibration tests were repeated whenever any changes to the upper table were made (e.g. lubrication of slip rings, installation of new torque motor, etc.).

Figure 2.6: Schematic of Tether Spool Assembly Calibration Set-up

Figure 2.7: Typical Calibration Curve and Corresponding Fourth Order Polynomial Fit for Tether Spool Assembly

2.2.5. TE-LAB Tether

The tether used in the TE-LAB tests is the same as the actual flight tether used for the OEDIPUS-A mission. The tether wire is a composite material having a 0.61 mm diameter copper core and a Teflon coating. The outer diameter of the tether is approximately 0.95 mm. The mass density per unit length is 0.002976 kg/m. To study the effects of a heavier gauge tether on the end-body dynamics, small leaded weights are added to the OEDIPUS tether at even intervals. For this configuration, the tether mass density becomes 0.0275 kg/m. Figure 2.2 shows the heavy tether suspended from the spool assembly.

2.3. Rotation Rate Control System

The Rotation Rate Control System controls the synchronization of the upper and lower rotating tables, ensuring that minimal twist is experienced in the tether during operation. Two modes of operation exist: the spin-up mode and the constant speed mode. During spin-up, both tables are accelerated up at the same rate to the same rotational speed. The two motors (fitted with shaft encoders) are controlled such as to follow a reference parabolic position profile. This parabolic profile produces a ramped angular speed and constant acceleration profile. Once the motors reach a final desired speed, a ramped position profile is required to produce a constant spin mode. Since in both modes, each motor tracks the same reference position profile, the maximum angular difference between each motor will be the sum of the maximum individual motor tracking errors. This maximum error corresponds to the maximum twist in the tether. A typical TE-LAB test run at 3 Hz for approximately 100 seconds (payload cone angle up to 20°) will experience a tether twist of about $\pm 2.6^\circ$.

A more in depth description of the TE-LAB Rotation Rate Control System is given in the thesis by Bergmans⁵.

2.4. Payload Orientation Determination System (PODS)

As the TE-LAB set-up was required to provide a quantitative study of the OEDIPUS configuration dynamics, the Payload Orientation Determination System (PODS) was developed. The PODS system is based on the Digital Electro-Optical Position Sensor

(DEOPS), designed at the University of Toronto Institute for Aerospace Studies for study of flexible structures. Bergman's describes the PODS and DEOPS systems in depth⁵.

The basic system consists of a CCD camera tracking two symmetrically mounted reflective targets on the top edge of the end-body. A PC-Vision Plus imaging board permits transfer of the video image to the data acquisition computer, where the position of the targets are recorded as Cartesian coordinates in the camera frame. The targets are tracked at 60 Hz. From the known position of the targets relative to the centre of mass of the body, the camera Cartesian coordinates of the targets permit an off-line calculation of the Euler angles of the payload. An error of $\pm 0.5^\circ$ for a maximum payload cone angle of 10° is attainable. The system can successfully track and provide data for TE-LAB spin-rates of 0-3 cps.

Chapter 3

3. Determination of TE-LAB Parameters and Performance Characteristics

An evaluation of the basic system set-up was performed prior to operational test use, that included a determination of the basic TE-LAB parameters. These include; the tether properties (i.e. length, density per unit length); the end-body geometric and inertial properties; and the internal damping due to the gimbal mechanism. An analysis of the free-spinning gimballed body (no tether) was also performed.

3.1. Determination of Payload Inertia

An important parameter affecting the dynamics of the spinning tethered body system is the inertial properties of the end-body. In fact, the analysis by Tyc² determined that a viable approach to stabilize the OEDIPUS-C mission was to change the inertia of the subpayloads with the addition of long radial booms. Therefore, a reliable method of determining the TE-LAB end-body transverse and longitudinal inertias was required. The method of determining the principal axes inertias is described herein as well as a rationalization for assuming that the products of inertia of the payload can be neglected.

3.1.1. Longitudinal and Transverse Inertias

The general method of determining the end-body's longitudinal and transverse inertias is based on the torsional pendulum¹¹. The payload is suspended from two wires of length L that are separated by a distance of $2D$.

Figure 3.1 The Torsional Pendulum

The payload is then rotated about its centre of mass, making an angle θ in the horizontal. At the same time, the wires make an angle ϕ with the vertical. Considering only small oscillations, the angles can be related by:

$$\phi \approx \frac{D}{L} \theta \quad (3.1)$$

This results in a change in height dh of the payload given by:

$$dh \approx \frac{D^2}{2L} \theta^2 = a\theta^2 \quad (3.2a)$$

where,

$$a = \frac{D^2}{2L} \quad (3.2b)$$

The kinetic and potential energies of the system are:

$$T = \frac{1}{2} I \dot{\theta}^2 + 2ma^2 \theta^2 \dot{\theta}^2 \quad (3.3a)$$

$$U = mga\theta^2 \quad (3.3b)$$

The Lagrangian function; $L = T - U$ therefore is,

$$L = \frac{1}{2} I \dot{\theta}^2 + 2ma^2 \theta^2 \dot{\theta}^2 - mga\theta^2 \quad (3.4)$$

The approximate Euler-Lagrange equation is,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0 \quad (3.5)$$

This leads to the following equation of motion describing the torsional pendulum:

$$I\ddot{\theta} + 4ma\theta^2 \dot{\theta}^2 + 2mga\theta = 0 \quad (3.6a)$$

Assuming only small amplitude oscillations, the equation of motion can be linearized to,

$$\ddot{\theta} + \omega^2 \theta = 0 \quad (3.6b)$$

where,

$$\omega^2 = \frac{2mga}{I}$$

This leads to the parameter determination formula for the inertia of the payload; by physically measuring ω , m and a , the payload inertia is calculated using the following equation:

$$I = \frac{2mga}{\omega^2} \quad (3.7)$$

To best approximate the inertias of the payload, care was taken to minimize the $4ma^2\theta\dot{\theta}^2$ term of the non-linearized equation of motion. The wire separation distance to length ratio was minimized, therefore making a^2 very small. The small angle approximation was also observed.

The torsional pendulum test was used to approximate both the longitudinal and transverse inertia properties of the two payload configurations (major and minor axis spinner) used for the TE-LAB tests. For each case, a set of ten oscillations were timed to determine the period of oscillation. This was repeated several times and the average period P recorded. The natural frequency of the system was then calculated by:

$$\omega = \frac{2\pi}{P} \quad (3.8)$$

Table 3.1 presents the inertias about the principal axes of the shell for both configurations. A and B are transverse inertias and C is the spin inertia. The inertia of the gimbals was neglected, as it only accounts for a small percentage of the total end-body inertia.

Table 3.1 Inertia of TE-LAB Payload Mock-ups

	A (kg-m ²)	B (kg-m ²)	C (kg-m ²)	A/C
Major Axis				
Spinner	0.00542	0.00538	0.00625	0.86
Minor Axis				
Spinner	0.00607	0.00611	0.00249	2.45

3.1.2. Products of Inertia: Dynamic and Static Balancing

Dynamic Balancing: As in any rotating machinery, the problem of vibration due to dynamic imbalance exists with the TE-LAB spinning end-body. This existence of products of inertia - ($D, E, F \neq 0$) causes an induced moment on the rotating body. Therefore, to minimize the effects of the end-body products of inertia on the free-spinning body dynamics, steps were taken to dynamically balance the mock-up payload shell (without the gimbals) at the Canadian National Research Council's (NRC) Institute for Machinery Research.

The NRC precision balancing equipment (shown schematically in Fig. 3.2) consists of a spinning shaft fitted with plugs to seat the end-body shell. Using light springs, the spinning system is free to oscillate along the y-axis while held rigid in the x and z-axes. The shaft is driven by a belt so as not to introduce any extraneous vibrations into the system. A pair of non-contact linear displacement transducers located at the support bearings then measure the vibration of the rotating body in the free translational axis. The imbalance of the payload is obtained indirectly by measuring the amplitude and phase of the steady-state vibration in two planes along the spin axis.

Two test runs with the payload in different conditions were performed: without any additions to the payload; and with an arbitrary weight added at an arbitrary point on the payload. The vibrations as a result of the inertia imbalance can be represented by a pair of rotating vectors. By then observing the changes in the vibration vectors when a known imbalance is added to the system, it is possible to compute the magnitude and location of the correction masses required to annul the original imbalance. The NRC precision balancing equipment included computer software that automatically calculated the spinning rotor imbalance given the required phase and amplitude information. The steps listed above, were repeated until the rotor imbalance was within the sensitivity of the balancing instrument.

Prior to balancing the payload, the shaft and payload plugs (without the payload installed) was balanced. The payload was then mounted on the shaft and the balancing procedure repeated, noting that any vibration would be the result of end-body shell. The vibrations observed were found to be within the noise level of the balancing equipment. Therefore, it was concluded that the products of inertia of the machined end-body shell were sufficiently small and could be neglected.

Figure 3.2: Schematic of NRC Precision Balancing Machine Located at the Montreal Road Campus, Ottawa

Static Balancing: Once dynamically balanced, the payload shell is secured to the gimbals near the system (gimbals and end-body) mass centre. As the payload shell inertia products are approximately zero, any imbalance that may exist can be attributed to a misalignment of the gimbal pivot point and the system centre of mass. Therefore, the end-body and gimbal mechanism are "statically balanced" prior to attaching the tether. Two mechanisms exist in the TE-LAB end-body design to bring the body mass centre coincident with the gimbal pivot point: the adjustable tether attachment table above the gimbal mechanism; and four threaded rods aligned with the bearing axes of the gimbals.

Raising and lowering the tether attachment table translates the system mass centre along the longitudinal axis of the body. Furthermore, the threaded rod and adjustable mass system allow the body mass centre and gimbal pivot point to be aligned in the transverse plane. The balancing masses and tether table are positioned such that the end-body remains stationary, unless perturbed, at any given nutation or cone angle (angle from the vertical).

3.2. Measurement of Damping of the Gimbals

A free-spinning body (i.e. no external torques acting on it) with no internal energy dissipation can be shown to be stable for both a major ($C > A$) and minor ($A > C$) axis spinner. However, with the consideration of internal damping, this result is not valid. In fact with damping, only a major axis spinner will be stable (as stated by the major axis rule)¹². Thus the internal energy dissipation plays a significant role in the stability of the free-spinning body and cannot be neglected. For this reason an estimation of the damping due to the gimbal bearing friction is required.

To estimate damping, measurements were taken of TE-LAB in the rocking angular motion state (i.e. non-spinning) and with the tether absent. Viscous damping has been assumed to model the internal energy dissipation due to the gimbals, and thus a method of free vibration decay was used to estimate the respective damping ratio. Damping due to air friction and deformation of the end-body were neglected. In general, the damping resistance can be obtained by experimental measurement of the response of the end-body to a given excitation. With the payload balanced at its centre of mass, an aluminum ring was added to the bottom of the body. An eccentricity was created between the point of rotation and the system centre of mass of the mock-up and the added ring. The end-body was then oscillated about each gimbal axis (inner and outer gimbal bearings) as a simple pendulum. PODS was used to measure the damped response.

For free vibration with viscous damping¹³ (see Fig. 3.3)

$$\frac{u_i}{u_{n+i}} = e^{2\pi n\zeta\sqrt{1-\zeta^2}} \quad (3.9)$$

where u_i denotes displacement at any peak, u_{n+i} the displacement n cycles later, n the number of cycles considered, and ζ the viscous damping ratio.

Figure 3.3 Free Vibration Decay for Viscous Damping

Solving for ζ ,

$$\zeta = \frac{\delta_n}{\sqrt{4\pi^2 n^2 + \delta_n^2}} \quad (3.10a)$$

where the logarithmic decrement is given by,

$$\delta_n = \ln \left(\frac{u_1}{u_{n+1}} \right) \quad (3.10b)$$

For light damping, ζ reduces to,

$$\zeta = \frac{\delta_n}{2\pi n} \quad (3.11)$$

The free vibration decay was performed twice for rotation about both gimbal axes and the typical time histories as measured by PODS are shown in Fig. 3.4a and 3.5a. The experimental damping ratios and coefficients, presented in the Table 3.2, reveals light damping about each gimbal axis. Higher damping ratios are seen to exist in the inner shaft mounted bearings as opposed to the outer gimbal bearings. This higher damping ratio can be attributed to a greater axial pre-load on the inner bearings. The damping tests were found to produce repeatable results.

Table 3.2 Damping Ratios and Coefficients as determined by Free Vibration Decay

		ζ	C
Inner Bearings	test 1	0.00494	0.00042
	test 2	0.00510	0.00045
Outer Bearings	test 1	0.00299	0.00025
	test 2	0.00265	0.00023

In parallel with the experimental damped vibration, an analytical model was developed for comparison purposes. The general equation of motion of the simple pendulum is:

$$\left(A + \frac{1}{2} m_{ring} r^2 + \frac{1}{12} m_{ring} l^2 + m_{ring} d^2 \right) \ddot{\theta} + C \dot{\theta} + m_{ring} g d \sin \theta = 0 \quad (3.12a)$$

where m_{ring} is the mass of the aluminum ring, r the ring radius, l the ring thickness, d the distance from the gimbal pivot point to the aluminum ring mass centre, and C the bearing damping coefficient. Assuming small angle displacements Eq. (3.12a) reduces to,

$$\left(A + \frac{1}{2} m_{ring} r^2 + \frac{1}{12} m_{ring} l^2 + m_{ring} d^2 \right) \ddot{\theta} + C \dot{\theta} + m_{ring} g d \theta = 0 \quad (3.12b)$$

Let,

$$z = \left(A + \frac{1}{2} m_{ring} r^2 + \frac{1}{12} m_{ring} l^2 + m_{ring} d^2 \right) \quad (3.12c)$$

This equation of motion was solved using an ordinary differential equation solver in MATLAB with:

$$\bar{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} \theta \\ \omega \end{Bmatrix} \quad (3.13a)$$

$$\bar{X} = \begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} = \begin{Bmatrix} \dot{\theta} \\ \dot{\omega} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-m_{ring} g d}{z} & \frac{-C}{z} \end{bmatrix} \begin{Bmatrix} \theta \\ \omega \end{Bmatrix} \quad (3.13b)$$

where the damping coefficient C is determined using the following relation

$$\frac{C}{z} = 2\zeta\omega_d. \quad (3.14)$$

ω_d is the damped frequency of the system given by the following:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (3.15)$$

In the case of light damping, the damped frequency ω_d will approximately equal the natural frequency ω_n of the system.

In Fig. 3.4b and 3.5b, the results of the analytical model are given for the parameters determined in the experimental free vibration decay. The analytical model is seen to compare well with the free decay time history, having similar exponential decay and frequency of oscillation. Therefore, it is believed that viscous damping provides a good approximation to the energy dissipation due to the gimbal mechanism.

Figure 3.4 Free Vibration Test for Inner Gimbal Bearings Mounted on Shaft

- (a) Experimental End-Body Response as per PODS
- (b) Analytical Response for Damping Coefficient of 0.00042

Figure 3.5: Free Vibration Test for Outer Gimbal Bearings

- (a) Experimental End-Body Response as per PODS
- (b) Analytical Response for Damping Coefficient of 0.00025

3.3. Analysis of the TE-LAB End-body: A Simple Gyro

As a final assessment of the TE-LAB system, a simple comparison is made between the free-spinning body and the TE-LAB gimbaled end-body. If one models the gimbals precisely, the TE-LAB mock-up of the aft payload is similar to a simple gyro, shown schematically below. The drive shaft is assumed to be inertially fixed and is driven at a constant speed. ϕ represents the angle of rotation about the x axis and θ the angle of rotation of the rotor (payload) with respect to the gimbal about the y-axis.

Figure 3.6 Schematic of Simple Gyro

Based on Staley's¹⁴ derivation of the Tuned Rotor Gyro Dynamics, and Hartl's analysis¹⁵ to include heuristic damping, the equations of motion using small angle assumptions are derived as

$$\ddot{\phi} + \frac{C_{\phi}}{A} \dot{\phi} + \frac{(B\omega_s^2 + J\omega_s^2)}{A} \phi - \frac{(A - B)}{A} \omega_s \theta = 0 \quad (3.16)$$

$$\ddot{\theta} + \frac{C_{\theta}}{D} \dot{\theta} + \frac{E\omega_s^2}{D} \theta + \frac{(D\omega_s - E\omega_s)}{D} \phi = 0 \quad (3.17)$$

where, I_{rr} is the transverse inertia of the rotor; I_{gt} the transverse inertia of the gimbal; I_{rs} the spin inertia of the rotor; I_{gs} the spin inertia of the gimbal; C_{ϕ} the coefficient of damping about the x-axis; and C_{θ} the coefficient of damping about the y-axis.

As well,

$$\begin{aligned} A &= I_{rr} + I_{gt} \\ B &= I_{gt} + I_{rs} - I_{rr} \\ D &= I_{rr} \\ E &= I_{rs} - I_{rr} \\ J &= I_{gt} - \frac{1}{2} I_{gs} \end{aligned} \quad (3.18)$$

Assuming the gimbal ring of the gyro to be light with respect to the rotor (i.e. $I_{gt} = I_{gs} = 0$) the equations of motion reduce to

$$\begin{aligned} I_{rr}\ddot{\phi} + C_{\phi}\dot{\phi} + (I_{rr} - I_{rr})\omega_s^2\phi - (2I_{rr} - I_{rs})\omega_s\theta &= 0 \\ I_{rr}\ddot{\theta} + C_{\theta}\dot{\theta} + (I_{rr} - I_{rr})\omega_s^2\theta - (2I_{rr} - I_{rs})\omega_s\phi &= 0 \end{aligned} \quad (3.19)$$

For comparison purposes, the Euler equations for a free-spinning body (with the gimbals ignored and a heuristic rotation damping $C_{\psi}\omega_1$ and $C_{\phi}\omega_2$) are:

$$\begin{aligned} A\omega_1 - (A - C)\omega_2\omega_3 + C_{\psi}\omega_1 &= 0 \\ A\omega_2 - (A - C)\omega_1\omega_3 + C_{\phi}\omega_2 &= 0 \\ C\omega_3 &= 0 \end{aligned} \quad (3.20)$$

The third Euler equation implies the spin-rate is constant, $\omega_3 = \omega_s$. For a 3-2-1 Euler scheme, the angular rates as a function of the Euler angle rates¹⁶ are given by

$$\begin{aligned} \omega_1 &= \dot{\psi} - \omega_s\theta \\ \omega_2 &= \dot{\theta} + \omega_s\psi \end{aligned} \quad (3.21)$$

Differentiating ω_1, ω_2 and substitution back into the Eq. (3.20) yield

$$\begin{aligned} A\ddot{\psi} + C_\psi\dot{\psi} + (C - A)\omega_s^2\psi - (2C - A)\omega_s\dot{\theta} &= C_\psi\omega_s\theta \\ A\ddot{\theta} + C_\theta\dot{\theta} + (C - A)\omega_s^2\theta - (2C - A)\omega_s\dot{\psi} &= C_\theta\omega_s\psi \end{aligned} \quad (3.22)$$

Considering small angle displacements and light damping, the right hand side of Eq. (3.22) approach zero. Therefore the above Euler equations for the free-spinning body with the gimbals ignored closely match those for the free-spinning gimbaled body (Eq. (3.19)). For TE-LAB parameters, the gimbals are small and light, and the two sets of equations have very similar numerical coefficients. Thus it is acceptable to ignore the details of the gimbals and add gimbal damping in a physical heuristic manner.

This can be further demonstrated by solving for the system eigenvalues and corresponding natural frequencies of the two systems. For the free-spinning body with no gimbals or damping assume trial functions of the form

$$\begin{Bmatrix} \psi \\ \theta \end{Bmatrix} = \begin{Bmatrix} \psi_0 \\ \theta_0 \end{Bmatrix} e^{\lambda t} \quad (3.23)$$

The eigenvalues are solved as

$$\begin{aligned} \lambda_{1,2} &= -s^2 \\ \lambda_{3,4} &= -\frac{(C-A)^2}{A^2}s^2 \end{aligned} \quad (3.24)$$

The natural frequencies become the spin and nutation frequencies

$$\begin{aligned} \omega_{1,2} &= \pm is \\ \omega_{3,4} &= \pm i \frac{C-A}{A} s \end{aligned} \quad (3.25)$$

To solve for the eigenvalues of the general gyro the system equations are reduced to a linear form by letting

$$\bar{X} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} \phi \\ \theta \\ \dot{\phi} \\ \dot{\theta} \end{Bmatrix} \quad (3.26)$$

Therefore, the equations of motion become,

$$\dot{\bar{X}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{B\omega_s^2 + J\omega_s^2}{A} & 0 & -\frac{C_\phi}{A} & \frac{A-B}{A}\omega_s \\ 0 & -\frac{E\omega_s^2}{D} & -\frac{D\omega_s - E\omega_f}{D} & -\frac{C_\theta}{D} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} \quad (3.27)$$

For the above system of linear equations, the modal natural frequencies are then solved (as to be described in section 4.2) using MATLAB and plotted as a function of spin-rate. Fig. 3.7 and 3.8 show these *damped natural frequency plots* for a gyro with a major and minor axis configuration. In each plot, the actual TE-LAB end-body (with actual gimbal inertias included) have been evaluated as well an end-body with gimbal inertias 0, 5, 10, and 25 % of the payload inertia. The TE-LAB end-body and free-spinning body (i.e. without gimbals) are observed to be in close correspondence (i.e. almost perfectly overlaid).

With TE-LAB, where the gimbal's inertia is negligible as compared to the payload, the modal frequencies closely match those of a free-spinning body. However, for gimbals with appreciable inertia, analysis indicates that the gimbals could not be ignored.

Figure 3.7: Natural Frequencies as a Function of Spin-Rate for a Gyro with Maximum Moment of Inertia Evaluated with a Gimbal Inertia of 0, 5, 10 & 25 % that of the End-body Inertia

Figure 3.8: Natural Frequencies as a Function of Spin-Rate for a Gyro with Minimum Moment of Inertia Evaluated with a Gimbal Inertia of 0, 5 10 & 25 % that of the End-body Inertia

Chapter 4

4. Development and Analysis of the TE-LAB Linear Model

To understand the complex dynamics of the ground-based TE-LAB, a mathematical model was developed by Vigneron et al.¹⁶ This model was derived independently from that of the in-space dynamics of OEDIPUS by G. Tyc of the University of Manitoba. The two models use similar methods and assumptions, and the areas of correspondence are well understood. Validation of the TE-LAB model would also constitute validation of the OEDIPUS model. The following first summarizes the basic linearized model for TE-LAB as derived by Vigneron et al. Then, an eigenvalue analysis that was performed by the author is described. As an initial check for the linear model two special case eigenvalue problems were formulated. The first is the so-called TE-LAB Free-spinning (undamped) End-body mode where only the payload without the effects of the tether is considered. The second case involves solely the tether dynamics (again undamped) without any interaction of the payload - the TE-LAB Spinning Tether Mode. The general eigenvalue problem is then introduced, to be used for a numerical solution of the aforementioned linear mathematical model. The analytics are later compared to the experimental results in Chapter 8.

4.1. Linear Model

The configuration considered consists of a single rigid body, four attached flexible booms and a rotating tether (Fig. 4.1). The present TE-LAB hardware does not include flexible booms; however, provision has been made for their addition if desired. The model presented is rigorously linearized in both end-body attitude and boom deformation variables.

An inertial coordinate system $OX_1X_2X_3$ is defined, to which a body fixed coordinate $Ox_1x_2x_3$ system is referenced by three Euler angles ψ, θ, ϕ as depicted in Fig. 4.2. A 3-2-1 rotation scheme is used to define the Euler angles such that for unit vectors $[e_x]$ and $[e_X]$ of the $Ox_1x_2x_3$ and $OX_1X_2X_3$ frames, the following relationship holds

$$\begin{bmatrix} e_{x_1} \\ e_{x_2} \\ e_{x_3} \end{bmatrix} = (\psi)_1(\theta)_2(\phi)_3 \begin{bmatrix} e_{X_1} \\ e_{X_2} \\ e_{X_3} \end{bmatrix} \quad (4.1)$$

where,

- $(\phi)_3$ rotation about the inertial OX_3 axis
- $(\theta)_2$ rotation about the intermediate OX'_2 axis
- $(\psi)_1$ rotation about the final OX''_1 axis

Euler angles ψ and θ are considered to be small for small nutational behaviour of the spinning rigid body.

The reference frame $PX_1X_2X_3$ is an inertial frame associated with the tether in its non-rotational state. The point P is fixed in the reference frame $PY_1Y_2Y_3$ and associated unit vectors e_u, e_v, e_w rotate with the tether as depicted in Fig. 4.3. The tether displacement $u(s, t)$ has components in the rotating frame

$$u(s, t) = [e_u]^T \begin{bmatrix} u(s, t) \\ v(s, t) \\ w(s, t) \end{bmatrix} \quad (4.2)$$

Constraints are imposed on the rigid body Euler angles and the tether related variables $u(s, t)$. In general, the tether is assumed rigidly attached to the end-body (i.e. a "universal joint" type of connection) and thus spinning at the same rate.

Figure 4.1: Mathematical Model of a Spinning Tethered Body with Flexible Appendages Shown in a Reference Equilibrium State

Figure 4.2: Schematic Showing 3-2-1 Euler Angle Scheme Defined by a Right Hand Rotation Coordinate System

Figure 4.3: Tether Coordinate Frame

The equations are derived using an energy method, where the continuous displacement variables for the tether and booms are discretized. For the booms,

$$u(x, t) = \Phi^T(x) q_i(t) \quad (4.3)$$

where i ranges from 1 to 8, in correspondence with in-plane and out-of-plane motions of the four booms. $\Phi_i(x)$ are column matrices of assumed mode shapes in accordance with the Raleigh-Ritz procedure, and $q_i(t)$ are column matrices of time-variable functions.

For the tether,

$$\begin{aligned} u(s, t) &\equiv u_9(s, t) = \Phi^T(s) q_9(t) \\ v(s, t) &\equiv u_{10}(s, t) = \Phi^T(s) q_{10}(t) \end{aligned} \quad (4.4)$$

The linearized equations, derived using a classical Lagrangian method, are written as

$$[M]\{\ddot{x}\} + ([G] + [D])\{\dot{x}\} + [K]\{x\} = 0 \quad (4.5)$$

where the state vector x is given by

$$\{x\}^T = \{\psi \quad \theta \quad q_{9,1} \quad q_{10,1} \quad q_{9,2} \quad q_{10,2} \quad \cdots \quad q_{9,n} \quad q_{10,n}\} \quad (4.6)$$

In the above, $[M]$ is defined as the mass matrix, $[K]$ the stiffness matrix, $[D]$ the damping matrix and $[G]$ is a matrix corresponding to the gyroscopic effects. $[M]$, $[K]$, and $[D]$ are symmetric. $[G]$ is skew symmetric. For TE-LAB, the booms are considered rigid and $q_1(t)$ through $q_8(t)$ subsequently suppressed. Therefore considering n assumed tether modes, the state vector $\{x\}$ will be of order $(2n+2)$. The coefficient matrices $[M]$, $[G]$, $[D]$, $[K]$ will be of dimension $(2n+2)$ by $(2n+2)$. These coefficient matrices are further detailed in Appendix C.

It is further convenient to use a first order canonical matrix equation that is arranged from the second order form. It is

$$[A]\{\dot{Z}\} + [B]\{Z\} = 0 \quad (4.7)$$

where $[A]$, $[B]$ and $\{Z\}$ are defined by

$$[A] = \begin{bmatrix} [M] & 0 \\ 0 & [K] \end{bmatrix} \quad (4.8)$$

$$[B] = \begin{bmatrix} [D] & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} [G] & [K] \\ -[K] & 0 \end{bmatrix} \quad (4.9)$$

and state vector $\{Z\}$

$$\{Z\} = \left\{ \{\dot{x}\}^T, \{x\}^T \right\}^T \quad (4.10)$$

Note that $[A] = [A]^T$ (i.e. a symmetric and also positive definite matrix), and $[B]$ is expressed as the sum of a (positive definite) symmetric and skew-symmetric part.

4.1.1. The Spinning End-Body - No Tether

As an initial study of the TE-LAB linear model, the spinning end-body without the effects of the tether is considered. To solve for the system eigenvalues, the linear mathematical model is simplified by the following assumptions:

- Dynamic effects of the tether are neglected by suppressing q_p and q_{10} .
- The static force effect of the tether is dropped to obtain free-spinning body dynamics.
- Damping is assumed light and therefore neglected.

The equations of motions reduce to the following form:

$$\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{Bmatrix} \ddot{\Psi} \\ \ddot{\Theta} \end{Bmatrix} + \begin{bmatrix} 0 & -(A+B-C)\omega_s \\ (A+B-C)\omega_s & 0 \end{bmatrix} \begin{Bmatrix} \dot{\Psi} \\ \dot{\Theta} \end{Bmatrix} + \begin{bmatrix} (C-B)\omega_s^2 & 0 \\ 0 & (C-A)\omega_s^2 \end{bmatrix} \begin{Bmatrix} \Psi \\ \Theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (4.11)$$

To solve for the characteristic values of Eq. (4.11), assume a solution of the form

$$\begin{Bmatrix} \Psi \\ \Theta \end{Bmatrix} = \begin{Bmatrix} \Psi_0 \\ \Theta_0 \end{Bmatrix} e^{\lambda t} \quad (4.12)$$

By differentiating the general solution and substituting into Eq. (4.11), the equations as solved by Jablonski¹⁷ for an axisymmetric body (i.e. A=B) reduce to

$$A^2\lambda^4 + \omega_s^2\lambda^2[A^2 + (C-A)^2] + (C-A)^2\omega_s^4 = 0 \quad (4.13)$$

The solution to the above is given by

$$\begin{aligned} \lambda_{1,2} &= \pm i\omega_s \\ \lambda_{3,4} &= \pm i\left(\frac{C-A}{A}\right)\omega_s \end{aligned} \quad (4.14)$$

Therefore the undamped natural frequencies become

$$\omega_{1,2} = \omega_s \quad (4.15)$$

$$\omega_{3,4} = \left(\frac{C-A}{A}\right)\omega_s \quad (4.16)$$

These frequencies correspond with those determined by the free-spinning body analysis in section 3.3.1.

4.1.2. The Spinning Tether - No End-body

The second stage of analysis for the linear model is to study the tether dynamics without the effect of the end-body. The linear model is simplified by the following assumptions¹⁷

- Dynamic effects of the payload are neglected as coupling between tether and payload is weak. Therefore to model the tether, payload variables, ψ and θ , are suppressed.
- The tether damping is assumed small and can therefore be neglected.

The equations of motion reduce to the form

$$\begin{bmatrix} M_t & 0 \\ 0 & M_t \end{bmatrix} \begin{Bmatrix} \ddot{q}_9 \\ \ddot{q}_{10} \end{Bmatrix} + \begin{bmatrix} 0 & -2\omega_s M_t \\ 2\omega_s M_t & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_9 \\ \dot{q}_{10} \end{Bmatrix} + \begin{bmatrix} K - \omega_s^2 M_t & 0 \\ 0 & K - \omega_s^2 M_t \end{bmatrix} \begin{Bmatrix} q_9 \\ q_{10} \end{Bmatrix} = 0 \quad (4.17)$$

Solving as before, with a solution of the form

$$\begin{Bmatrix} q_9 \\ q_{10} \end{Bmatrix} = \begin{Bmatrix} q_{9,0} \\ q_{10,0} \end{Bmatrix} e^{\lambda t} \quad (4.18)$$

yields

$$\lambda^4 + 2\lambda^2(\omega^2 + \omega_s^2) + (\omega^2 - \omega_s^2) = 0 \quad (4.19)$$

where

$$K = K_t - K_s \quad (4.20a)$$

$$K_t = Tl\mathcal{K}_t = Tl(n^2\pi^2) \quad (4.20b)$$

$$K_s = \rho g l^2 \mathcal{K}_s = \rho g l^2 \left(\frac{n^2\pi^2}{2} \right) \quad (4.20c)$$

and,
$$\omega^2 = \frac{K}{M_t} \quad (4.21)$$

Full expressions for $\mathcal{K}_t$ and $\mathcal{K}_s$ are detailed in Appendix C.

Again solving the equation in λ , the natural frequencies are found to be

$$\omega_{1,2} = \omega + \omega_s \quad (4.22)$$

$$\omega_{3,4} = \omega - \omega_s \quad (4.23)$$

Figure 4.4 demonstrates these undamped natural frequencies plotted as a function of spin-rate for four tether modes and a nominal tether tension of 0.89 N. At zero spin-rate, a double root exists for each considered tether mode. As spin is introduced, each double root branches off into an ascending and descending pair of modal frequencies. The descending modal branches (modes 1, 3, 5 etc.) are observed to become zero at approximately $\omega_s = 1.25$ Hz, $\omega_s = 2.55$ Hz etc. These critical speeds correspond to ω being equal to ω_s such that $\omega_{3,4} = 0$.

Figure 4.4: Undamped Natural Frequencies as a Function of Spin-Rate for a Pure Spinning Tether - No End-Body: Tether Tension 0.89N

4.2. Eigenvalue Analysis

To analyze the complete linear system of dynamic equations, the eigenvalue problem (or characteristic value problem) has been developed. That is for a set of system parameters the system natural frequencies, damping ratios and corresponding natural mode shapes are determined. The following presents the analytics for a rigorous determination of the modal parameters that include both the damping and gyroscopic effects. It is these critical values that will later be compared to experimental TE-LAB results to validate the linear model.

4.2.1. Determination of Natural Modal Parameters for the Damped Gyroscopic System

As with the special case eigenvalue problems (sections 4.1.1 and 4.1.2) a solution of Eq. (4.5) of the following form is assumed:

$$Z = Y_k e^{\lambda_k t} \quad (4.24)$$

Substitution of Z results in the eigenproblem

$$(\lambda_k A + B) Y_k = 0 \quad (4.25)$$

The eigenvalues, λ_k , are solutions of the following¹⁸

$$\det(\lambda_k A + B) = 0 \quad (4.26)$$

As $[A], [B]$ are of dimension $4n+4$ the above equation is a $4n+4$ order polynomial yielding $4n+4$ solutions which can be real or complex. The rank of $[A], [B]$ being $4n+4$ as well, requires all the eigenvalues be non-zero. With the presence of damping, the complex eigenvalues occur in complex conjugate pairs (* denotes the complex conjugate).

$$\lambda_k = -\sigma_k + i\nu_k \quad (4.27a)$$

$$\lambda_k^* = -\sigma_k - i\nu_k \quad (4.27b)$$

These complex eigenvalues can be converted to the conventional natural modal frequencies and modal damping ratios, ω and ζ by the formula¹⁹

$$\omega_k^2 = \nu_k^2 + \sigma_k^2 \quad (4.28)$$

$$\zeta_k = \frac{\sigma_k}{(\nu_k^2 + \sigma_k^2)^{\frac{1}{2}}} \quad (4.29)$$

It is seen that the real part of the eigenvalue relates to the damping, and the imaginary part to the damped natural frequency. When solved for the linearized model, the above equations strictly describe the damped gyroscopic natural modal frequency and damping ratios of the system. However, the author chooses to refer to these parameters as damped natural modes where the gyroscopic effects are implied.

$4n+4$ eigenvectors exist corresponding to each of the complex eigenvalues. These eigenvectors have the following form;

$$Y_k = \begin{bmatrix} \lambda_k Q_k \\ Q_k \end{bmatrix} \quad (4.30)$$

where λ_k relate to the eigenvalues with positive imaginary parts (λ_k and Q_k are of order $2n+2$). Q_k further turns out to be complex in structure

$$Q_k = U_k + iV_k \quad (4.31)$$

Considering the second order canonical form of the linear equations, λ_k and Q_k are solutions of

$$[\lambda_k M + \lambda_k (D + G) + K] Q_k = 0 \quad (4.32)$$

The basic exponential trial function that is used to arrive at the eigenvalue problem is complex and its introduction converts a real-valued situation to one that is complex. To study the real-valued eigenvalue problem, the pairing of complex and complex-conjugate parameters suggest a real-valued trial function of the form

$$x_k(t) = \frac{1}{2} (Q_k e^{\lambda_k t} + Q_k^* e^{\lambda_k^* t}) \quad (4.33)$$

where * denotes complex-conjugate. Substitution of λ_k and Q_k into the above function reduces $x_k(t)$ to

$$x_k(t) = e^{-\sigma_k t} \{U_k \cos(v_k t) - V_k \sin(v_k t)\} \quad (4.34)$$

thus defining the real-valued time varying function for the kth damped gyroscopic mode.

4.3. Stability of the Linear Model

Two states of stability exist for a linear mechanical system: dynamic stability and static stability²⁰. For dynamic stability to exist, a system must have damping that opposes the direction of motion, hence diminishing the amplitudes of free vibration. Damping that acts in the same direction as the motion, thus causing the vibration amplitudes to increase, is said to be dynamically unstable. While dynamic stability considers the damping for a given system, static stability is dependent on the stiffness matrix K. A mechanical system is statically stable if a displacement from the equilibrium position sets up a force or couple tending to drive the system back to the equilibrium position. Static instability occurs if the force causes the displacement to increase. Therefore static instability implies a negative spring constant k (stiffness matrix that is not positive definite), or more generally, a negative value for one of the natural frequencies. It is noted that dynamic stability always infers static stability, but that the converse is not true: a statically stable system may be dynamically unstable.

For TE-LAB, the damping mechanisms (damping in the tether and gimbals) pervade the entire system due to the coupling between the tether and end-body. Further, no additional energy is fed into the system as a result of this damping. Hughes²¹ explains that the asymptotic stability of such a system is dependent solely on the positive definiteness of the stiffness matrix. Therefore, it can be concluded that with TE-LAB, dynamic and static stability are one and the same.

From an eigenvalue standpoint, the general solutions of the TE-LAB linear model (given by Eq. 4.34) are exponential functions that decay or grow at a rate related to the real part of their corresponding eigenvalues. Roots lying on the right side of the complex plane, (i.e. implying a positive real term $\lambda_1 = \sigma + iv$, $\lambda_2 = \sigma - iv$), produce a positive exponential term in the general solution. The result is an exponentially growing function.

However, an eigenvalue with a negative real term result in a damped sine wave solution and is thus considered stable.

Consequently, a necessary condition for stability of a constant linear mechanical system is that all the roots have negative real parts; but if the real part of any one of the eigenvalues is positive, the system is said to be dynamically unstable. For a system in which the damping is pervasive, dynamic stability and static stability are found to be one and the same.

4.4. Modal Parameters as a Function of Spin-rate

The damped natural frequencies and damping ratios are evaluated for a set of TE-LAB system parameters. These modal parameters are further plotted as a function of spin-rate over the operational range of the TE-LAB set-up, thus yielding two basic tools for evaluation and validation of the linear mathematical model: the damped modal frequency versus spin-rate plots, and the modal damping ratio versus spin-rate graphs. Figures 4.5 and 4.6 demonstrate the damped modal parameter graphs for both a major and minor axis spinner with no tether. Figure 4.7 then presents an approximately six metre tether with nominal tether tension of 0.89 N. Here, the system parameters are chosen such that no dynamic interaction exists between the tether and end-body. A third set of modal parameter plots illustrate the full TE-LAB test configuration for a minor axis spinner and heavy tether arrangement. For this case a nominal tension of 3.5 N is realized. Finally, Fig. 4.9a and b illustrate the major axis spinner/light OEDIPUS tether configuration. In all cases, the absolute magnitude of the damped gyroscopic natural modal frequency is calculated.

For the nutational modes of the rigid end-body alone, the linear model collapses at $\omega_s = 0$ suggesting no natural frequencies exist (or rigid body motion). However, as the body is spun up, Fig. 4.5a and 4.6a shows that two natural modes develop, $\omega_{1,2} \approx \omega_s$ and $\omega_{3,4} \approx \left(\frac{C-A}{A}\right)\omega_s$. These damped natural frequencies approximately equal the undamped natural frequencies predicted by the free-spinning body eigenvalue analysis. It can be noted from the modal damping plot for the major axis spinner, that the damping ratios are negative. This implies a stable spinner. That is with the presence of damping, the solutions $\begin{Bmatrix} \Psi \\ \theta \end{Bmatrix} = \begin{Bmatrix} \Psi_0 \\ \theta_0 \end{Bmatrix} e^{\lambda t}$ are exponentially decreasing harmonic functions. Thus given

any initial perturbation, the motion of the end-body will decay to pure spin about the major axis. On the other hand, the minor axis spinner with damping has positive damping ratios associated with its nutational frequencies suggesting an exponential growth in the payload cone angle. The damping ratio plots for the free-spinning bodies also demonstrate the reasoning behind spin stabilization of sub-orbital payloads. Although always unstable, increasing the spin-rate of the minor axis spinner causes the damping ratio and subsequently the nutation growth rate to decrease. Therefore, for short flights, the payload nutation is often maintained within a small limit cone angle.

Considering the tether dynamics alone, the damped modal frequencies of Fig. 4.7 are in close comparison to those frequencies (of Fig. 4.4) solved by the undamped eigenvalue problem (due to light damping). For a zero spin-rate the frequencies have double roots, all of which are stable. Similar to the undamped spinning tether analysis (section 4.1.2), an introduction of spin causes the tether modal frequencies to branch off into an ascending and descending pair of modes. As the spin-rate increases, the lower branch (mode 1) approaches zero corresponding to the point at which the stiffness k equals zero. In this case the tether pops out into its first mode or skipping rope mode. The zero k condition also reflects a change in sign (negative to positive) of the damping ratio thus implying a growing exponential function in the general solution. Once above this critical spin-rate, the linear model predicts that this first mode and consequently the system remains unstable. For a given tension, critical speeds exist when each of the tether modes (modes 1, 2, 3 etc.) become unstable. In Fig. 4.7, mode 1 of a 6.4 metre tether with 0.89 N tension becomes unstable at 1.25 Hz; mode 2 at 2.55 Hz. At these critical instability points the corresponding unstable mode dominates the basic tether shape.

Figure 4.8 illustrates the modal parameter graphs for a minor axis spinner with a heavy tether. For this configuration, the system frequencies become a non-trivial combination of the spinning tether and end-body modes. At $\omega_s = 0$, the coupled 'tether modes' and 'nutation modes' have double roots all of which are stable (from the modal damping ratio versus spin-rate graphs it is seen that all the modal damping ratios of the system are negative at $\omega_s = 0$). As the system spin-rate is increased from 0 Hz, each of these double roots split into an ascending and descending modal branch (labeled mode 1,2,3, etc.). The descending frequency branches (mode 1, 3, 5, etc.) approach zero as their respective eigenvalues move to the right half of the complex plane. This zero frequency state corresponds to the point at which the modal damping ratio becomes positive, thus implying an unstable mode (i.e. a growing exponential function in the general solution).

(a)

(b)

Figure 4.5: Damped Gyroscopic Modal Parameters as a Function of Spin-Rate for the Major Axis Spinner - No Tether

- (a) Damped Gyroscopic Natural Modal Frequencies
- (b) Modal Damping Ratios

Figure 4.6: Damped Gyroscopic Modal Parameters as a Function of Spin-Rate for the Minor Axis Spinner - No Tether

- (a) Damped Gyroscopic Natural Modal Frequencies
- (b) Modal Damping Ratios

(a)

(b)

Figure 4.7: Damped Modal Parameters as a Function of Spin-Rate for the Pure Spinning Tether - No End-body: Tether Tension 0.89 N with 10 % Tether Damping

(a) Damped Natural Modal Frequencies

(b) Modal Damping Ratios

(a)

(b)

Figure 4.8: Damped Gyroscopic Modal Parameters as a Function of Spin-Rate for the Minor Axis Spinner with a Heavy Tether: Tether Tension 3.51 N

(a) Damped Gyroscopic Natural Modal Frequencies

(b) Modal Damping Ratios

Figure 4.9a: Damped Gyroscopic Modal Frequencies as a Function of Spin-Rate for the Major Axis Spinner / Light Tether Configuration with a Tether Tension of 1.41 N (evaluated using 4 assumed tether modes)

Figure 4.9b: Damped Gyroscopic Modal Frequencies as a Function of Spin-Rate for the Major Axis Spinner / Light Tether Configuration with a Tether Tension of 1.41 N (evaluated using 5 assumed tether modes)

Finally Fig. 4.9a shows the analytical modal parameter plots for a maximum moment of inertia end-body with a light OEDIPUS tether. As with the minor axis spinner/heavy tether, the major axis spinner/light tether configuration exhibits pairs of damped modal frequencies that split off into ascending and descending branches. The descending modal branches, modes 1, 3, 5, 7 are seen to approach zero, the point at which the corresponding mode becomes unstable. Mode 9 does not cross the zero frequency axis, thus suggesting that this mode never becomes unstable. However, if one increases the number of assumed tether modes from four to five in the linear model (Fig. 4.9b), mode 9, having been diverted by mode 11, becomes unstable at a spin-rate of approximately 8.25 Hz. Mode 11 is now seen to remain stable. It is further observed that the descending modal branches approach one another but do not intersect. Rather Fig. 4.9a show modes 1 and 3 being deflected. It is also noted that the deflected portion of mode 1 becomes an extension of the undeflected portion of mode 3. Likewise, the deflected portion of 3 appears as an extension of the undeflected portion of mode 1. The same holds true for modes 3 and 5, modes 5 and 7 and so on. For a more complete description of these modal frequency versus spin-rate graphs, a physical animation of the real-valued mode shapes is performed in section 4.5.

4.4.1. Assessment of Required Number of Assumed Tether Mode Shapes

In light of solving the eigenvalue problem for the TE-LAB linear mathematical model, the question arises of how many assumed tether mode shapes (tether shape functions) are required to accurately model the ground-based spinning tethered end-body. By way of the Raleigh-Ritz method, the basic tether shape function is replaced by a series of shape functions multiplied by constant coefficients. The solution of these equations then gives the natural frequencies and mode shapes of the system. Figures 4.10a and b show the damped modal frequency versus spin-rate plots for the minor axis spinner/light OEDIPUS tether configuration with a nominal tether tension of 1.41 N. The plots are evaluated for 1 and 4 assumed tether mode shapes respectively. In both cases, effectively the same modal values are calculated for mode 1 (the fundamental frequency) regardless of the number of tether mode shapes included. It is further noted that within the TE-LAB operational range (i.e. 0 - 3 Hz), the higher frequency modes, modes 2, 3, and 4 of Fig. 4.10a, provide a good approximation when compared with the same modes of Fig. 4.10b. However, at greater spin-rates, deflection of the modal frequencies becomes evident, thus yielding less confidence in those modal frequencies solved using fewer assumed tether mode shapes.

Figure 4.10: Damped Gyroscopic Modal Frequencies as a Function of Spin-Rate for the Minor Axis Spinner / Light Tether Configuration with a Tether Tension of 1.41 N evaluated with:

- (a)** one assumed tether mode shape
- (b)** four assumed tether mode shapes

From a stability standpoint, a necessary condition for stability of a near mechanical system is that *all* the system roots have negative real parts. For 1 and 4 tether mode shapes assumed in the general equations of motion, the fundamental frequency becomes unstable at approximately 1 Hz (i.e. the point at which the modal frequency becomes zero also noted by eigenvalues in the right half of the complex plane). As this is the first unstable mode, the system stability and mode 1 stability become one and the same. Thus, it can be inferred that with one assumed tether mode shape, an accurate prediction of system instability can be made. The critical instability point for mode 3 of Fig. 4.10a is observed to be approximately 1.75 Hz agreeing with that of Fig. 4.10b. It can therefore be reasoned that the number of assumed tether mode shapes has minimal effect on the calculated value of the mode instability points.

Although, it is observed that more assumed tether mode shapes implies greater accuracy in calculation of both the fundamental and higher frequency modes, computational power limits the number of assumed tether mode shapes that can be included in the mathematical modeling. However, even with a limited number of assumed tether mode shapes, this method of analysis for the vibration of a system with many degrees of freedom provides a means of obtaining an accurate value for the fundamental or lowest natural frequency. As well a good approximation of the higher frequencies and mode shapes is obtained. Therefore, in most cases, four assumed tether mode shapes is considered sufficient to model the TE-LAB set-up with the exception of the heavy tether configuration where higher frequency modes are experienced.

4.5. Mode Shapes and Animation

From the eigenvalue problem one can also reproduce a physical visualization of these damped natural modes. The following provides a summary of the mathematical relationships for the visualization and animation of the TE-LAB system mode shapes based on communications with Vigneron²².

Considering Eq. (4.34), $x_k(t)$ (the 'kth real-valued damped gyroscopic mode') oscillates at the natural modal frequency ν_k and decays at the modal decay rate as related to $-\sigma_k$. Further $x_k(t)$ is a subset of the function $x(t)$ of Eq. (4.6).

Therefore,

$$\{x_k(t)\}^T = \{\psi(t)_k \quad \theta(t)_k \quad q_{9,1}(t)_k \quad q_{10,1}(t)_k \quad q_{9,2}(t)_k \quad q_{10,2}(t)_k \quad \cdots \quad q_{9,n}(t)_k \quad q_{10,n}(t)_k\} \quad (4.35)$$

From the linear model, the basic shape of the tether for the k th natural mode is given by

$$\begin{aligned} u_k(s,t) &= \left(1 - \frac{s}{l}\right) b \theta_k(t) + l \sum_n \Phi_n^T\left(\frac{s}{l}\right) q_{9,n}(t) \\ v_k(s,t) &= -\left(1 - \frac{s}{l}\right) b \psi_k(t) + l \sum_n \Phi_n^T\left(\frac{s}{l}\right) q_{10,n}(t) \end{aligned} \quad (4.36)$$

where the matrix of assumed tether modes shapes $\Phi^T\left(\frac{s}{l}\right)$ for n tether modes is of the form

$$\Phi^T\left(\frac{s}{l}\right) = \begin{bmatrix} \sqrt{2} \sin\left(\pi \frac{s}{l}\right) \\ \sqrt{2} \sin\left(2\pi \frac{s}{l}\right) \\ \vdots \\ \sqrt{2} \sin\left(n\pi \frac{s}{l}\right) \end{bmatrix} \quad (4.37)$$

Equations (4.34), (4.35) and (4.36) allow for a computer-generated animation of the k th mode, for $k = 1$ to $2n+2$. As the real-valued time-varying function x_k contains an exponential term, the corresponding modes shapes will either decay or grow with time, depending whether the particular mode shape is stable or unstable. However, to animate the tether mode shapes, this exponential term is equated to one. The basic mode shape animation is performed in a body-fixed reference frame as defined by the 3-2-1 Euler rotation sequence. An independent mode shape study was also performed by Chandrashaker of the CSA, but in an inertial reference frame. The results from the two studies are in general agreement.

A static approximation to the mode shape can be plotted at time zero. The following plots show these time-invariant tether mode shapes for three typical TE-LAB configurations:

- (i) tether dynamics alone - no interaction with payload (Fig. 4.11)
- (ii) minor axis spinner with a heavy tether (Fig. 4.12)
- (iii) major axis spinner with a light OEDIPUS tether (Fig. 4.13 - 4.16)

Figure 4.11: Real-Valued Mode Shapes for Pure Spinning Tether - No End-body with a Tether Tension of 0.89 N and Spin-Rate of 1.5 Hz

Figure 4.12: Real-Valued Mode Shapes for Minor Axis Spinner / Heavy Tether with a Tether Tension of 3.51 N and Spin-Rate of 1.0 Hz

Figure 4.13: Real-Valued Mode Shapes for Major Axis Spinner / Light OEDIPUS Tether with a Tether Tension of 1.41 N and Spin-Rate of 0.5 Hz

Figure 4.14: Real-Valued Mode Shapes for Major Axis Spinner / Light OEDIPUS Tether with a Tether Tension of 1.41 N and Spin-Rate of 1.15 Hz

Figure 4.15: Real-Valued Mode Shapes for Major Axis Spinner / Light OEDIPUS Tether with a Tether Tension of 1.41 N and Spin-Rate of 1.5 Hz

Figure 4.16: Real-Valued Mode Shapes for Major Axis Spinner / Light OEDIPUS Tether with a Tether Tension of 1.41 N and Spin-Rate of 3.0 Hz

The basic shapes, evaluated using MATLAB for ten tether modes, pertain to the modal parameters versus spin-rate graphs illustrated in Fig. 4.7, 4.8 and 4.9 respectively.

Figures 4.11a and b show a static approximation of the tether mode shapes for a 6.4 metre tether without the effects of the end-body. The tether tension is 0.89 N at a spin-rate of 1.5 Hz. The plots reveal that the two modes for a given pair of damped modal frequencies (paired as 1&2, 3&4, 5&6 and 7&8 of Fig. 4.7) exhibit similar tether shapes. These shapes can be likened to the characteristic mode shapes of a spinning string (i.e. a skipping rope mode, 2nd, 3rd, and 4th mode corresponding to the number of anti-nodes). However, by modeling a vertical string with the effects of gravity (as opposed to a horizontal spinning tether) the basic tether shapes exhibit marginally larger maximum displacements, q_9 and q_{10} , in the lower portion of the tether. As the end-body dynamics are negated, the displacements q_9 and q_{10} at $s=0$ are observed to be zero. At $s=l$ the tether displacement is always zero as the tether is assumed to be aligned with the spin axis. Mode 1 (of Fig. 4.7) is seen to be slightly beyond its critical instability point and consequently exhibits relatively large tether amplitudes. Conversely the higher frequency modes exhibit much smaller amplitude oscillation. Animation of the pure spinning tether mode shapes further reveals that the mode shape pertaining to the ascending damped modal frequency branches (mode 2, 4, 6, etc.) rotate in the opposite direction (negative spin vector in Right Hand Coordinate System) to the body-fixed reference frame at a rate equal to its respective natural frequency. Conversely the mode shapes related to the lower branches (mode 1, 3, 5, etc.) spin in the same direction (positive spin vector in Right Hand Coordinate System) as the body frame. At the onset of system instability, the mode shape corresponding to the unstable damped modal frequency undergoes a change in the relative spin direction (change from positive to negative spin vector). The spin direction of the various tether mode shapes are illustrated in the presented figures.

The second set of time-invariant mode shapes, presented in Fig. 4.12, illustrate a minimum moment of inertia end-body with a heavy tether. In this case, the shapes are evaluated for a tether tension of 3.51 N and a spin-rate of 1 Hz. While no longer a purely spinning tether due to interaction with the end-body, modal pairs 1 & 2, 5 & 6, and 7 & 8 (of Fig. 4.8) still demonstrate basic shapes characterized by the spinning string. However, these so-called 'tether modes' experience small tether displacements q_9 and q_{10} at $s=0$. The basic shapes of modes 3 & 4 are seen to be somewhat different from the 'tether modes' discussed above. Instead modes 3 & 4 exhibit large amplitude tether

displacements (relative to the maximum tether amplitude) at $s = 0$ as a result of the end-body nutations. Consequently, these modes are referred to as the system 'nutational modes'. As with the pure spinning tether mode shapes (Fig. 4.11), the animation reveals that the mode corresponding to the ascending modal branches (modes 2, 4, 6, etc.) rotate in the opposite direction to the body-fixed frame; whereas the modes related to the descending modal branches (modes 1, 3, 5, etc.) rotate in the same direction. As before, the spin direction of a mode changes, relative to the body-fixed frame, when it becomes unstable.

Finally, Fig. 4.13 through 4.16 illustrate the major axis spinner with a light OEDIPUS tether. The modes are evaluated for a nominal tether tension of 1.41 N at spin-rates of 0.5, 1.15, 1.5 and 3 Hz respectively. These mode shapes correspond to the damped modal frequencies of Fig. 4.9 and are so labeled. At 0.5 Hz, modes 1 & 2 (Fig. 4.13) demonstrate the characteristic 'nutational' shape as observed by large tether displacements at $s = 0$. The so-called 'tether' modes, modes 3 & 4, 5 & 6, 7 & 8 etc., have shapes similar to that of the pure spinning string, exhibiting with very little tether displacement at $s = 0$. Animation of the tether mode shapes show that the spin direction of a particular mode follows those patterns discussed above (i.e. for the pure spinning tether and the minor axis spinner/heavy tether configuration). By increasing the spin-rate to 1.15 Hz, a change in the mode shapes is observed (Fig. 4.14). Mode 1 is seen to assume a shape much like the first so-called 'tether' mode. Conversely, mode 3 begins exhibiting greater tether displacements (q_9 and q_{10}) at $s = 0$ comparable with that of the 'nutational modes'. It is seen from the corresponding damped modal frequency versus spin-rate plot (Fig. 4.9) that at 1.15 Hz, mode 1 and 3 have very similar frequency values. Furthermore, as the two modes rotate in the same relative direction, it is intuitive that the modes have similar tether shapes. If one further increases the spin-rate to 1.5 Hz (Fig. 4.15), mode 1 is seen to demonstrate the properties of the so-called 'nutational mode' and mode 3 that of the first 'tether mode'. This suggests that the 'nutational mode' has switched from mode 1 to mode 3, and similarly mode 1 has replaced mode 3 as the first 'tether mode'. For the three nominal spin-rate cases (0.5, 1.15, 1.5 Hz), system modes 5 & 6, 7 & 8, 9 & 10 exhibit the characteristic shapes of the higher order 'tether modes'.

To complete the description of the real-valued modal shapes for the maximum moment of inertia end-body/light OEDIPUS tether case, the modes are evaluated at 3 Hz (Fig. 4.16). Modes 2 and 5 are now considered to be the 'nutational modes' as both tether shapes exhibit large tether displacements at $s = 0$. This implies that the 'nutational mode' has

again switched, this time from mode 3 to mode 5. The basic tether shapes of modal pairs 1 & 4, 3 & 6, 7 & 8 and 9 & 10 are observed to be related to the 2nd, 3rd, and 4th so-called 'tether modes' respectively. This pattern of *switching modal shapes* is evident in the higher order system modes as their critical unstable spin-rates are approached, thus suggesting that the 'nutational mode' for the major axis spinner/light OEDIPUS tether never becomes unstable.

4.6. Formation of Linear Model Stability Boundaries

To develop a better understanding of the observed payload dynamic instability and provide a basic guide in the design of the TE-LAB test configurations, two approaches are used to determine the linear model stability boundaries for TE-LAB:

1. Resolution of the system eigenvalues
2. Formation of analytical stability equations based on the stiffness matrix [K]

4.6.1. Stability Boundaries by the Eigenvalue Approach

The TE-LAB stability boundaries can be determined by resolving the position of the system eigenvalues. As discussed in 4.3, for dynamic stability to exist all the system eigenvalues must lie on the left side of the complex plane. Therefore, by varying the tether tension and spin-rate in the eigenvalue problem and subsequently determining when the real part of each of the system eigenvalues changes sign, one can create the TE-LAB stability boundaries as a function of these crucial parameters. Using MATLAB, the eigenvalue problem is solved for a complete sweep of tether tension and spin-rate over the TE-LAB operational range. The critical points (reflecting where the natural frequencies equals zero) are then plotted as a function of tension and spin-rate. Figure 4.17 shows the stability boundaries for an approximately six metre tether without the payload attached. These stability boundaries are presented for four assumed tether modes. The stability boundaries for the minor axis spinner/heavy tether configuration are further illustrated in Fig. 4.18. Again the linear model is solved for four assumed tether modes.

For the spinning tether configuration, the linear model predicts that the system is stable to the left hand side of the first mode shape stability boundary (Fig. 4.17). At this first critical boundary, the first mode becomes unstable causing the spinning tether to pop into a 'skipping rope' oscillation. To the right of this first stability boundary, the mode (and

consequently the system) remains unstable as per the linear model. At the second stability limit, the corresponding second mode becomes unstable, causing the tether shape to be dominated by this second tether mode. Similarly, the stability of subsequent modes is dependent on their respective stability boundaries.

For the heavy tether/minor axis spinner configuration, similar stability boundaries are formed, again by evaluation of the system eigenvalues over a range of tensions and spin-rates. As discovered by the mode shape animation, one stability boundary can be related to the so-called 'nutational mode', the remainder of the system stability boundaries to the 'tether modes' (as labeled in Fig. 4.18). The linear model predicts that the system instability is dependent on this first so-called tether mode. Therefore, to the left of this first instability limit, the system is considered stable. Once crossed, system stability no longer exists and the system variables increase exponentially.

4.6.2. Stability Boundaries by the Stiffness Matrix

As previously discussed, the asymptotic stability of a linear mechanical system, is dependent on the $[K]$ matrix. Instability arises when a negative k occurs. Therefore, the necessary and sufficient conditions for stability require $[K]$ be positive definite, that is, each of its principal minors be greater than zero. The following stability equations of an axisymmetric tethered body with three tether modes were derived by Tyc and Jablonski²³ based on the principal minors of the $\det[K]$:

$$\lambda_n > 0; \quad n = 1, 2, 3 \quad (4.38)$$

$$C_o - A_o > \rho l b^2 \left[\frac{1}{3} + \frac{1}{\rho l b^2 \omega_s^2} \sum_{n=1}^3 \frac{k_n^2}{\lambda_n} \right] - \frac{bT}{\omega_s} \left(1 + \frac{b}{l} \right) + \frac{\rho g b^2 l}{\omega_s^2} \left(1 + \frac{b}{2l} \right) \quad (4.39)$$

where,

$$k_n = \rho g b l \left(\frac{l}{b} \frac{k_{n-1}}{\lambda_{n-1}} K_{g,n-1,n} - Q_{g,n} + \frac{l \omega_s^2}{g} R_n \right) \quad (4.40)$$

$$\lambda_n = T l K_{t,nm} - \rho g l^2 K_{g,nm} - \omega_s^2 \rho l^3 M_{t,nm} - \rho^2 g^2 l^4 \frac{K_{g,n-1,n}^2}{\lambda_{n-1}} \quad (4.41)$$

and $k_0 = 0$, $\lambda_0 = 1$, and $K_{g,0,1} = 0$.

The flexural constants $K_{g,nn}, Q_{g,n}, R_n, K_{t,nn}, M_{t,nn}$ are given in Appendix C.

The linear model predicts dynamic stability for the TE-LAB ground simulation if all of the above stability criteria are satisfied. A violation of any one of the criteria, implies system instability. The thesis by Bergmans⁵ provides a more in depth view of the TE-LAB stability criteria as derived by Tyc and Jablonski²². The equations are rearranged as a function of tension and spin-rate to provide a better tool for analysis of the TE-LAB set-up.

Figure 4.17: Stability Boundaries for the Spinning Tether - No End-body of an Approximately 6 m Tether evaluated with Four Assumed Tether Mode Shapes and plotted as a Function of Tension and Spin-Rate

Figure 4.18: Stability Boundaries of Minor Axis Spinner / Heavy Tether Configuration evaluated with Four Assumed Tether Mode Shapes and plotted as a Function of Tension and Spin-Rate

Chapter 5

5. TE-LAB Test Procedure with Typical Experimental Results

The following presents the basic TE-LAB tests procedure as well as a description of the results obtained. The technique of analyzing the data is further outlined. These results are later used in Chapter 6 to validate the linear mathematical model as solved by the eigenvalue problem.

The basic TE-LAB experimental procedure was completed for three configurations:

- (i) maximum moment of inertia spinner with light (OEDIPUS) tether,
- (ii) minimum moment of inertia spinner with light (OEDIPUS) tether,
- (iii) minimum moment of inertia spinner with heavy tether.

For the major axis spinner/light tether and minor axis spinner/light tether configurations, three types of tests were performed: the free-spinning end-body tests, the spinning tether tests, and the spinning tether and end-body tests. These two series of tests were performed twice, to prove the repeatability of the system. Only the spinning tether and end-body tests were performed for the minor axis spinner body/heavy tether configuration. In all cases the tether length was approximately 6.4 metres. The following

table presents the complete set of TE-LAB tests. The TE-LAB test parameters for the three configurations are further documented in Appendix B.

Table 5.1 Complete Set of TE LAB Experimental Tests

TE-LAB Test Configuration	Type of Test	Tension (N)	Spin-Rate (Hz)
(i) Major Axis Spinner/Light (OEDIPUS) Tether	Free-spinning End-body	NA	1.0, 2.0, 3.0
	Spinning Tether	minimum - 5.0	.5, .75, 1.0, 1.5, 2.0, 2.5, 3.0
	Spinning Tether and End-body	0.49, 1.41, 4.54	0, .5, .75, 1.0, 1.5, 2.0, 2.5, 3.0
(ii) Minor Axis Spinner/Light (OEDIPUS) Tether	Free-spinning End-body	NA	1.0, 2.0, 3.0
	Spinning Tether	minimum - 5.0	.5, .75, 1.0, 1.5, 2.0, 2.5, 3.0
	Spinning Tether and End-body	0.50, 0.89, 1.42, 2.76, 4.54	0, .5, .75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0
(iii) Minor Axis Spinner/Heavy Tether	Spinning Tether and End-body	2.02, 3.51, 4.54	0, .5, .75, 1.0, 1.5, 2.0, 2.5, 3.0

5.1. Calibration of the TE-LAB Photogrammetric System

Prior to any TE-LAB testing, initial tests to calibrate the photogrammetric system are performed. As discussed by Bergmans⁵, the calibration tests involve determining the camera coordinates of the end-body when in a known orientation. The *vertical test*, a spin of the payload about the inertial vertical axis with the gimbals locked, is first performed to determine the payload radius (distance between the targets) and the camera offset angle (angle between camera line of sight and the vertical). The second test, called the *nutation angle test*, requires the payload be held stationary at a known cone angle. This test is used to determine the payload half-height, or more generally, the distance along the spin axis between the centre of mass of the body and the photogrammetric targets. These values are later used in determination of the Euler angle orientation of the end-body.

5.2. TE-LAB Free-spinning End-Body Tests

The first phase of the TE-LAB test series is referred to as the *TE-LAB free-spinning end-body tests*. In this case the spinning body without the tether is considered. The response of the payload (spinning at a range of constant speeds) to an initial perturbation is measured using PODS and the end-body Euler angles evaluated. To invoke a response, a torque free impulse or *kick* (light taps in rapid succession on either side of the body) is imparted to the body.

Figures 5.1 demonstrates the full-time history responses (the Euler angle θ versus time) of the major axis spinner, to an imparted perturbation. The tests were performed for constant spin-rates of 1, 2 and 3 Hz. As expected, in all three cases the end-body vibrations damp out to pure spin about the major axis. The plots also suggest an inverse relationship between the rate of damping and the rotational speed of the end-body; that is, given an initial perturbation, the nutations of a slow spinning end-body will damp out more quickly than that of the same body with a higher rotational speed. Conversely, the minor axis spinner exhibits a slowly increasing nutation from pure spin (Figure 5.2). Again, it is realized the faster spinning body cones out more slowly than the body with less rotational speed.

In general, this response is as predicted by the linear model free-spinning body where the damping ratio quickly approaches zero as the spin-rate of the payload was increased. For both bodies, this decay or damping rate is related to the system stiffness [K]. In general, the faster spinning body is a dynamically stiffer system with subsequently larger natural frequencies. As the damping ratios are inversely proportional to these natural frequencies, one would expect a more slowly growing exponential function in the general solution for the free-spinning body with energy dissipation.

5.3. TE-LAB Spinning Tether Tests

The *TE-LAB Spinning tether tests* comprise a series of tests in which the tether dynamics are isolated from the payload dynamics. The coning motion of the payload is restricted by locking the gimbals - in effect reducing the end body to a simple rotating table. For a constant spin-rate, the tether tension is dropped until the onset of the tether instability

Figure 5.1: TE-LAB Free-spinning End-body - No Tether: Major Axis Spinner

(a) Spin-rate: 1 Hz; Initial Condition: Kick

(b) Spin-rate: 2 Hz; Initial Condition: Kick

(c) Spin-rate: 3 Hz; Initial Condition: Kick

Figure 5.2: TE-LAB Free-spinning End-body - No Tether: Minor Axis Spinner

(a) Spin-rate: 1 Hz; Initial Condition: Kick

(b) Spin-rate: 2 Hz; Initial Condition: Kick

((b) Spin-rate: 3 Hz; Initial Condition: Kick

Figure 5.3: Experimental Stability Boundaries for the TE-LAB Spinning Tether - No End-Body

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

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(tether mode) occurs. When unstable, the tether assumes a specific shape characterized by the number of anti-nodes (known as the mode number of the equilibrium state). For each mode, the spool assembly clutch current and corresponding tension was recorded. This was then repeated for a range of spin-rates creating the stability boundaries for each mode shape as a function of spin-rate and tension. The *TE-LAB tether mode* tests were performed for the light OEDIPUS tether only, with a nominal length of 6.4 m and repeated four times. The experimental results for four tether modes are presented in Fig. 5.3. These experimental mode stability boundaries are later plotted against the stability boundaries predicted by the linear model (Fig. 6.4).

5.4. TE-LAB Spinning Tether and End-body Tests

The final sequence of tests involves the full TE-LAB ground-based simulation where the interaction between the tether dynamics and payload is observed. For a nominal tension and spin-rate, the spinning tethered body dynamics are examined and full-time histories recorded. These tests are then repeated for a range of tensions and spin-rates, spanning the entire operating limits of the TE-LAB set-up (Tension: 0.5 - 5 N, Spin-rate: 0 - 3 Hz). A zero spin-rate corresponds to a rocking body configuration where the end-body is only allowed to oscillate about one gimbal axis. In the case of a limit cycle (i.e. payload precesses at a constant nutation angle) or stable configuration (i.e. no notable growth in payload nutation), an initial perturbation is imparted to the system in the form of either a *payload kick* or *tether pluck*.

Figure 5.4 shows a typical TE-LAB plot of the Euler angle (ψ, θ, ϕ) and nutation angle (ν) time histories for a major axis spinner/light tether configuration with a tether tension of 0.49 N. At time approximately equal to 30 seconds, a perturbation in the form of a payload kick is imparted on the system. This impulse causes the end-body nutation angle (angle of end-body spin axis to the vertical) to jump to 17 degrees as seen in Fig. 5.4d. With time however this initial end-body nutation damps out to pure spin about its major axis. Both the payload and tether exhibited stable dynamic behaviour. Figure 5.4c demonstrates the Euler angle ϕ , the angle of rotation about the spin axis. A constant spin speed is observed. For analysis of the TE-LAB dynamics, either of the Euler angles θ or ψ can be considered. To minimize the amount of post-processing, the remainder of the Euler angle time histories illustrate only the theta (θ) angle data.

Figure 5.4: Euler Angle and Nutation Angle Time Response for the Major Axis Spinner / Light OEDIPUS Tether with a Tether Tension of 0.49 N

(a) Euler Angle Theta

(b) Euler Angle Psi

(c) Euler Angle Phi

(d) Nutation Angle Nu

Figure 5.5: Euler Angle (Theta) Time Histories for Minor Axis Spinner / Light Tether Configuration with a Tether Tension of 4.54 N

(a) Rocking Body Mode: Spin-rate = 0 Hz

(b) Stable End-body: Spin-rate = 1.75 Hz

(b) Unstable End-body: Spin-rate = 2.0 Hz

Figure 5.6: Euler Angle (Theta) Time Histories for Minor Axis Spinner / Heavy Tether Configuration with a Tether Tension of 4.54 N

(a) Rocking Body Mode: Spin-rate = 0 Hz

(b) Stable End-body: Spin-rate = 1.0 Hz

(b) Unstable End-body: Spin-rate = 3.0 Hz

Figure 5.5 illustrates three representative Euler angle responses for the minor axis spinner/light tether set-up at a given tension of 4.54 N. For a zero spin-rate or rocking-body mode (Figure 5.5a), a tether pluck is imparted on the system such that the end-body and tether oscillate in a single plane. With time the system natural frequencies (i.e. tether and end-body) eventually damp out and the system returns to a static state; thus indicating stability. Figure 5.5b and c show this same configuration, but with spin-rates of 1.75 and 2.0 Hz respectively. At 1.75 Hz, the system exhibits stable behaviour; the end-body nutations damping out to pure spin about its minor axis. However, with an increase in spin-rate to 2.0 Hz, the end-body slowly cones out, suggesting an unstable configuration. Similar results are obtained for the minor axis spinner/heavy tether tests as shown in Figure 5.6a, b and c. Due to the heavy tether, more dynamic coupling is seen to exist between the tether and end-body.

5.5. The Fast Fourier Transform Analysis

For comparison to the linear model eigenvalue problem, a spectral analysis is performed on the TE-LAB time-histories. The MATLAB Fast Fourier Transform function is used to transform the experimental data from the time spectrum to the frequency domain, specified by an amplitude and phase spectrum. By identifying the amplitude peaks from the spectral plot, the TE-LAB system frequencies are readily determined. For small amplitude peaks, the phase spectrum is also referenced for existence of a corresponding phase shift. The MATLAB FFT function returns frequencies to one-half the sampling frequency (TE-LAB sampling rate: 60 Hz).

Due to the coupling between the tether and end-body dynamics, both the end-body and the tether frequencies can be measured solely by observing the end-body responses. Therefore to assess the TE-LAB system frequencies, the FFT was performed on the Euler angular displacement, θ ; the same results can be obtained by processing the Euler angle response, ψ . Figures 5.7 through 5.14 illustrate a sample set of minor axis spinner/heavy tether configuration tests and corresponding FFT analyses for a nominal tension of 3.5 N. For each figure, the first of the three plots presents the full time history response of the end-body. Dashed lines represent the partial time history over which the FFT analysis was performed. The second and 3rd plots then illustrate the amplitude and phase spectrum as per the FFT. Although the frequencies values for a given test were found to be independent of the initial conditions, for the tether pluck initial condition, the

frequencies related to the tether were more readily observed. Similarly, with an end-body disturbance (payload kick), the frequencies associated with end-body are found to be more prevalent.

Generally speaking, the following observations are made in the example data of Figure 5.7 through 5.14:

- (i) at spin rates below 1.5 Hz, the end-body is 'stable' (decaying amplitude of θ , Figures 5.7 - 5.11);
- (ii) at spin-rates above 1.5 Hz, the end-body is 'unstable' (growing amplitude of θ , Figures 5.12 - 5.14);
- (iii) the FFTs clearly distinguish three to seven system frequencies depending on the input parameters.

The experimentally determined natural frequencies are then extracted from the FFTs, and plotted as a function of spin-rate, as is illustrated in Figure 5.15. The test results consistently show a small peak in the amplitude spectrum at the test spin-rate. Further interpretation of the data is most efficiently described in conjunction with results of the linear model, and thus will be addressed fully in Chapter 6.

Figure 5.7: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 0 Hz

Figure 5.8: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 0.5 Hz

Figure 5.9: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 0.75 Hz

Figure 5.10: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 1.0 Hz

Figure 5.11: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 1.5 Hz

Figure 5.12: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 2.0 Hz

Figure 5.13: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 2.5 Hz

Figure 5.14: Euler Angle Time History (Theta) and Magnitude and Phase Spectrum evaluated using a FFT analysis for the Minor Axis Spinner with Heavy Tether: Tension = 3.51 N and Spin-rate = 3.0

Figure 5.15: Experimental Natural Frequencies Plotted as a Function of Spin-rate for the Minor Axis Spinner / Heavy Tether Configuration with a Tether Tension of 3.51 N

Chapter 6

6. Comparison of Eigenvalue Calculations to Experimental Results

For validation of the mathematical model, a comparison is made between the experimentally determined natural frequencies (by FFT analysis) and the damped gyroscopic natural modal frequencies calculated by the eigenvalue problem. The natural modes of the free-spinning body without interaction of the tether is first considered. An analysis of the spinning vertical tether based on the stability boundaries as predicted by the linear model is then presented. Finally, the full TE-LAB set-up is assessed. This includes a comparison of the natural frequencies and stability boundaries determined by experimental and analytical methods.

6.1. Evaluation of TE-LAB Free-spinning End-body - No Tether

As a first step to validate the linear model, the TE-LAB end-body without the tether is assessed. Fig. 6.1a and 6.1b present the experimental and analytical damped natural frequencies versus spin-rate plots for both the major and minor axis free-spinning body. The experimentally determined frequencies represented by the cross marks are in general agreement with the dark dotted lines, depicting the damped gyroscopic modes predicted

Figure 6.1a: Experimental and Analytical Damped Modal Frequencies as a Function of Spin-Rate for the Major Axis Spinner - no Tether

+	Experimental Natural Frequencies
....	Analytical Damped Natural Frequencies
....	5% error envelope on transverse inertias

Figure 6.1b: Experimental and Analytical Damped Modal Frequencies as a Function of Spin-Rate for the Minor Axis Spinner - no tether

by the linear mathematical model. The major axis spinner showed some discrepancy in the predicted and measured natural frequencies corresponding to the term $\left(\frac{C-A}{A}\right)s$. This may be attributed to the fact that the inertia ratio of the end-body is relatively near one (0.86 for major axis spinner) and thus the accuracy of the predicted natural frequencies is quite sensitive to the precision with which the end-body inertial properties are evaluated. To confirm this, the modal frequency plots also illustrate the damped natural frequencies when a $\pm 5\%$ error (corresponding to the light dotted lines) on the transverse inertias is considered. The major axis spinner exhibits a much large error envelope than experienced by the minor axis body.

A second source of error in the predicted modal frequencies is the spin-rate measurement of the end-body. The free-spinning body tests are performed at nominal spin-rates of 1, 2 and 3 Hz as predicted by the motor controller. However, for both the major and minor spinners, the experimental natural frequencies reveal a spin-rate slightly greater than the nominal rates. To evaluate the discrepancy between the spin-rate predicted by the shaft encoder of the motor and that determined by the photogrammetric system, a simple study of the end-body with the gimbals locked is made. Figure 6.2 illustrates the spin angle, ϕ , time history for the major axis spinner. Although, the body is spun at a nominal angular speed of 1 Hz, the FFT analysis reveals a spin-rate as determined by PODS of slightly greater than nominal rate (approximately 4% higher). This systematic error, found consistently throughout TE-LAB test results, is further detailed in the thesis by Bergmans⁵.

Finally, Fig. 6.3a and b present the results with the analytical modal frequencies having the spin-rate corrected by + 4%. The agreement between the experimental and analytical results is found to be much better. Therefore, the remainder of the results are presented as related to the corrected spin-rate (as per PODS) rather than the nominal spin-rates specified to the motor controller.

6.2. Evaluation of the TE-LAB Spinning Tether - No End-body

A comparison of the experimental and analytical stability boundaries is made for evaluation of the spinning tether - no end-body in Fig. 6.4. As described in section 3.1.3, the observable tether modes recorded for a range of tensions and spin-rates are plotted as a function of these crucial parameters. Figure 6.4 illustrates four tether modes

Figure 6.2: Fast Fourier Transform analysis for Major Axis Spinner with Gimbals locked

- (a) Full time history of rotation angle about spin axis
- (b) Amplitude spectrum as determined by FFT
- (b) Phase spectrum as determined by FFT

Figure 6.3: Experimental and Analytical Damped Modal Frequencies as a Function of the Corrected Spin-Rate for both Major and Minor Axis Spinner - no tether

(a) major axis spinner

(b) minor axis spinner

superimposed on the instability boundaries predicted by the analytical model. These graphs are for the light OEDIPUS tether with a nominal length of 6.4 metres. The solid line represents the stability boundary predicted by the eigenvalue problem of the linear mathematical model as set out in section 4.6.1. The eigenvalue problem is solved for a range of tensions between 0.25 N and 5.0 N and spin-rates of 0.0 Hz through 5 Hz. For comparison purposes, the sample mean and standard deviation are then calculated from the four experimental spinning tether tests depicted in Figure 5.3. These experimental instability points for the 1st, 2nd, 3rd and 4th tether modes are depicted as error bars, representing the sample mean and ± 3 sample standard deviations (σ).

The plots show that the experimentally determined boundaries closely match those predicted analytically. The analytical stability boundaries are found to consistently lie within $\pm 3 \sigma$ of the experimental results. The increase in scatter of the experimental data (i.e. greater standard deviation) with spin-rate is inherent in the method of determining the tether modes. For a given constant spin-rate, a particular tether mode shape is observable over a small range of tether tension. With increased spin-rates, this tension range becomes larger. As the recorded tension for a given mode is based on a qualitative observation of the maximum tether amplitude, the margin of error also becomes larger.

6.3. Evaluation of TE-LAB Spinning Tether and End-body

As a final step in validation of the linear model, data from the full TE-LAB set-up is considered. Two methods of comparison between analytical and experimental results can be used:

- (i) damped gyroscopic natural frequencies versus spin-rate plots;
- (ii) stability boundaries as predicted by the eigenvalue problem.

Presented herein are the experimental ground test natural frequencies as related to the modal parameter versus spin-rate plots. The following then provides insight into the stability observed in the ground-based tests as compared to these analytically predicted stability boundaries. An in depth presentation of stability boundaries is detailed in the thesis by Bergmans as well.

6.3.1. Damped Gyroscopic Natural Modes for the Spinning Tether and End-body

As an initial evaluation of the full TE-LAB set-up, the system damped gyroscopic frequencies are assessed. Figures 6.5a through 6.5c illustrate the experimentally

Figure 6.4: Experimental and Analytical Stability Boundaries for Four Tether Modes as a function of Tension and Spin-Rate

determined natural frequencies (by way of the FFT analyses) plotted with those predicted by the analytical model for the minor axis spinner/heavy tether configuration with nominal tensions of 2.0, 3.5 and 4.5 N respectively. In the modal plots, the experimental frequencies are represented by cross marks whereas the dotted lines symbolize the mathematical damped gyroscopic modes. The analytical frequencies are evaluated considering six assumed tether mode shapes for the heavy tether case and only four assumed tether mode shapes with the light OEDIPUS tether. Finally, the system spin-rate is demonstrated by a solid linear line. The latter is not predicted by the linear model but is observed in the FFT analyses. In general, the experimental results closely match the calculated natural frequencies with the exception of the frequencies corresponding to the spin-rate. For the TE-LAB rocking body mode of operation, the experimental frequencies support the existence of double roots as predicted by the linear model. These experimental roots then separate into two branches as the system is spun up. With the increase in spin-rate, the experimental roots from the lower branches approach 0 Hz at the same rate as the linear model damped natural frequencies. The small amplitude peaks that occur at the system spin-rate are attributed to two factors not accounted for in the linear model: a small imbalance (existence of products of inertia) in the end-body; and the effect of the small inner gimbal ring.

Results for tests with a light tether and minimum moment of inertia end-body are illustrated in Fig. 6.6. Typically the FFT analysis reveals that the lower TE-LAB system frequencies appear as larger amplitude peaks, thus giving high confidence in their existence. For the light tether, where the coupling is small, these larger amplitude peak frequencies often correspond to the so-called "nutational modes", which are related to the end-body dynamics. As the natural frequencies of the "tether modes" are measured indirectly through the end-body dynamics using PODS, their corresponding amplitude peaks are often much smaller or non-existent. Consequently, fewer system modes are evident in the FFT analyses for the light OEDIPUS tether tests, as illustrated in Fig. 6.6.

Finally, Fig. 6.7 presents the TE-LAB tests for the major axis spinner/light OEDIPUS tether configuration. As with the previous case (minor axis spinner/light tether), the major axis spinner/light OEDIPUS tether test results closely compare with that predicted by the linear model. Again, high confidence is placed in the existence of the lower system frequencies due to the large amplitude peaks observed in the FFT analyses.

Figure 6.5a: Damped Modal Frequencies for the Minor Axis Spinner/Heavy Tether Configuration with a Nominal Tension of 2.02 N

Figure 6.5b: Damped Modal Frequencies for the Minor Axis Spinner/Heavy Tether Configuration with a Nominal Tension of 3.51 N

Figure 6.5c: Damped Modal Frequencies for the Minor Axis Spinner/Heavy Tether Configuration with a Nominal Tension of 4.54 N

Figure 6.6: Damped Modal Frequencies for the Minor Axis Spinner/Light OEDIPUS Tether Configuration for Nominal Tether Tensions of

(a) 0.5 N

(b) 4.54 N

Figure 6.7: Damped Modal Frequencies for the Major Axis Spinner/Light OEDIPUS Tether Configuration for Nominal Tether Tensions of

(a) 0.5 N

(b) 4.54 N

From the above presented plots (as well as the complete set of modal frequency versus spin-rate plots included in Appendix D), it is evident that there is excellent agreement between experiment and theory, thus validating the basic premises of the theoretical model.

6.3.2. Stability Boundaries for the Spinning Tether and End-body

As a second evaluation of the linear model, the theoretical TE-LAB stability boundaries for each of the individual modes can be obtained from the eigenvalue calculations as set out in section 4.6.1. These boundaries, plotted as a function of tension and spin-rate for the three TE-LAB test configurations, are presented in Fig. 6.8, 6.9, and 6.10. The eigenvalue problem stability boundaries match those obtained from the closed form stability solutions as described by Bergmans⁵ and are so labeled. The experimental results of the time history responses are then compared to the analytics by observing the growth or decay of the end-body θ amplitude for a ground test with a given tension and spin-rate. If the end-body nutation damps out to pure rotation about its spin axis, 's' indicating stable end-body dynamics is superimposed on the stability boundary plots. For the case where the end-body's cone angle increases (to either an observed constant limit cycle cone angle or to the limits of TE-LAB set-up) a 'u' is added.

In the results of the minor axis spinner and light tether (presented in Fig. 6.8), the calculated stability boundaries of the various modes are shown by solid lines. For this case, mode 1, the lowest frequency mode, is essentially an 'end-body nutational' mode, whereas the higher frequency modes (2-5) can be considered 'tether' modes. The theory states that everything to the right of the mode 1 (or nutational mode) line is unstable in the solution that is the weighted superposition of the responses of all of the modes. The measurements clearly agree and support the projection of the linear model for this category of configuration (light tether/minimum moment of inertia spinner).

Results typical of the major axis spinner and light tether are shown in Fig. 6.9. Although the test results demonstrated good agreement in the measured frequencies (e.g. Fig. 6.7), there is clearly no simple and evident agreement for the nutation growth/decay parameter. The explanation for this disagreement is believed, with good confidence, to be associated with non-linearities that are absent in the linear model. Basically the nutational mode of the end-body is stable for all parameter values, and so there is no corresponding boundary

Figure 6.8: Stability Boundaries for the Major Axis Spinner/Light OEDIPUS Tether Configuration as a function of Tension and Spin-Rate

Figure 6.9: Stability Boundaries for the Minor Axis Spinner/Light OEDIPUS Tether Configuration as a function of Tension and Spin-Rate

s	stable end-body dynamics
u	unstable end-body dynamics
—	analytical stability boundary

Figure 6.10: Stability Boundaries for the Minor Axis Spinner/Heavy Tether Configuration as a function of Tension and Spin-Rate

lines on Figure 6.9. The lines of this figure correspond to the 'tether modes' which are technically unstable from the viewpoint of the rigorous linear model. In the practical experiment condition however, the tether modes are believed to be stable at equilibrium states that have small amplitudes from the $\theta = \psi = q_9 = q_{10} = 0$ state (i.e. small amplitude stable limit cycles exist which are not evident in the model because of the discarded non-linear terms). In this hypothesis, there would be accompanying small non-zero motions in θ and ψ of the end-body. However, due to the sensitivity of the instrumentation, the TE-LAB set-up is not able to observe these small non-zero limit cycles. Thus many of the experimental 's' of Fig. 6.8 correspond to stable limit cycles (as opposed to stable $q = 0$ states). To further explain the experimental data would require non-linear theoretical modeling which is yet to be completed by the TDE team.

Finally results for the minor axis spinner and heavy tether are shown in Fig. 6.10. It is observed that there is good agreement, with the exception of the boundary corresponding to mode 1. This graph is fully consistent with the non-linearity hypotheses of the previous paragraph. In this case mode 2 is the primary 'nutational mode' of the end-body and because it is a minor axis spinner, everything to the right of it is unstable. Mode 1 happens to be a tether mode and because the tether is 'heavy' its frequency and stability line is to the left of the 'nutational mode' (mode 2). It is believed thus to cause a stable limit cycle behaviour as opposed to an instability.

6.3.3. Specific Test Parameters Affecting the TE-LAB Dynamics

Two important parameters in the TE-LAB linear model, affecting the tether and subsequently the end-body dynamics, are tension and gravity. An increase in tether tension causes all the system frequencies to shift up, thus implying a stiffer and consequently more stable system. This is apparent in the earlier presented minor axis spinner/heavy tether modal frequency plots and is consistent with the results included in Appendix D. Gravity on the other hand has a destabilizing effect on the end-body dynamics. While not affecting the spinning body directly (because mounted at centre of mass), the gravitational term causes a linear gravity gradient over the length of the tether and subsequently a lower force acting on the body. Figure 6.11 compares the stability boundaries as per the linear model eigenvalue problem, with and without gravity included. The solid line represents the TE-LAB minor axis spinner/heavy tether configuration with gravity, whereas the dashed line is with the gravity effects negated. As with increased tension, the system frequencies without gravity shift up, resulting again in a stiffer, more

stable system. This causes the stability boundaries to shift to the right as in the aforementioned figure. From an analytical viewpoint, the gravity term appears as a destabilizing term in both the end-body closed form stability solution of Eq. (4.39) as well as the tether stability criteria, Eq. (4.38). On the other hand, tension is seen as a stabilizing term in both cases.

The heavy tether and minor axis spinner configuration clearly shows that it is imperative to include the effects of gravity in modeling the ground-based set-up. While not as obvious, the same results are achieved when modeling the light tether with and without gravity.

Figure 6.11: Stability Boundaries for the Minor Axis Spinner/Heavy Tether Configuration Evaluated with and without the Effects of Gravity

Chapter 7

7. Application of the TE-LAB Model to OEDIPUS

Having validated the TE-LAB linear mathematical model and successfully modeled a 100 to 150 m spinning tethered end-body system, the model is extended to describe an OEDIPUS-like configuration. The following discusses the stability of both the OEDIPUS-A and C flights based on an evaluation of the linear model damped gyroscopic modal parameters. The OEDIPUS configuration, assumed to be in a free-fall state (micro gravity environment with nominal tether tension of approximately 0.14 N), includes a rigid end-body (i.e. no damping) and a 1000 m tether with 10 % damping (given as a percentage of the hysteretic energy loss per stress cycle). The details of the OEDIPUS-A and C flight parameters are given in Appendix A.

7.1. OEDIPUS-A Stability

Figures 7.1a and b illustrate the damped gyroscopic modal frequencies vs. spin-rate graph and stability boundary graph respectively, for the OEDIPUS-A configuration evaluated with 10 assumed tether modes. These plots show that all the modes ('tether modes' and 'nutational modes') become unstable well-below the nominal 0.7 Hz spin-rate of OEDIPUS-A. In fact, with a nominal tension of 0.14 N, the first mode can be considered

unstable for spin-rates greater than approximately 0.004 Hz. Therefore, for the OEDIPUS-A configuration to be stable (as per the linear model), the system must be spun at rates less than 0.004 Hz. At these spin-rates, the end-bodies are not dynamically stiff, and thus prone to tip-off perturbations caused during separation. Even for a tether tension of 1.25 N (similar to the tension experienced during flight separation), the system modes are seen in Fig. 7.1b to be unstable at very low spin-rates. The TE-LAB linear model, providing valuable insight into the unstable dynamic behaviour of the OEDIPUS-A flight, supports the initiative to change the OEDIPUS flight parameters for the follow-on mission.

The analysis by Tyc² identified tether tension as a possible passive stabilizing approach for the spinning tether/end-body system dynamics. To achieve stability of all the system modes without changing the end-body inertia properties or nominal system spin-rate, a tension of approximately 5550 N is required as seen in Fig. 7.1c. For this set of flight parameters, the first mode (a 'tether mode') is observed to become unstable at 0.7 Hz, the spin-rate of the OEDIPUS-A flight. However, a tension of this magnitude is obviously not feasible as the only source of tether tension is due to the gravity gradient. Two 'tether modes' are noted to become unstable prior to the 'nutational mode' instability. By extending the non-linearities hypothesis as discussed in section 6.3.2 (that the unstable tether modes produce equilibrium states that have small amplitudes from the $\theta = \psi = q_9 = q_{10} = 0$ state) it is believed only small non-zero motions in θ and ψ of the end-body will result. If the "stability" of the OEDIPUS end-body dynamics is then based on the nutation growth/decay parameter more governed by the nutational mode stability boundary, a tension of approximately 825 N is required (still well above that due to the gravity gradient). Therefore, the modal analysis suggests that this method of passive stabilization of OEDIPUS-A is not practical as prohibitively high tether tensions are required.

7.2. OEDIPUS-C Stability

The approach ultimately adopted for stabilizing the OEDIPUS configuration is to change the payload inertia properties. With the addition of long radial booms, the OEDIPUS-A aft and forward payloads can be transformed into maximum moment of inertia spinners, much like the major axis spinner configuration observed to be "stable" in the ground-based tests (TE-LAB and the UBC tests). Figures 7.2 illustrate both the damped modal

Figure 7.1a: Damped Modal Frequencies as a Function of Spin-Rate for the OEDIPUS-A Free-Fall State (Tension = 0.14 N)

Figure 7.1b: Stability Boundaries Predicted by the TE-LAB Linear Mathematical Model for an OEDIPUS-A Configuration (Plotted as a Function of Tension and Spin-Rate)

Figure 7.1c: Damped Modal Frequencies as a Function of Spin-Rate for the OEDIPUS-A Configuration with a Tether Tension of 5550 N

frequencies vs. spin-rate plot and stability plot for this OEDIPUS-C configuration. Again, the 'tether modes' are observed to become unstable at very low spin-rates. These 'tether modes' match precisely those predicted for the OEDIPUS-A configuration. As discussed previously, it is believed that these unstable tether modes produce only small deflections of the end-body (small amplitudes of θ and ψ).

The stability boundary corresponding to the so-called 'nutational mode' is not present in the stability plot of Fig. 7.2b. The 'nutational modes' of the OEDIPUS-C configuration are seen to follow the same patterns as the so-called 'nutational modes' of the ground-based major axis spinner/light tether configuration (of Fig. 4.9). That is, in Fig. 7.2 a mode 5 (a 'nutational mode') is deflected by mode 7 (a 'tether mode') at approximately 0.004 Hz. At this so-called 'deflection point', the mode shape study reveals that mode 7 assumes the shape of the 'nutational mode' and mode 5 a 'tether mode'. This *deflecting and switching* of modes is further observed as the higher modes come in close proximity to the 'nutational mode' for increased values of spin-rate. The 'nutational mode', observed to switch from mode 5 to 7 to 9 to 11 etc., is found to be stable for all spin-rates, thus explaining the absence of a 'nutational mode' stability boundary in Fig. 7.2b. Therefore, with the maximum moment of inertia end-bodies, it is believed that the OEDIPUS-C configuration will demonstrate "stable" end-body dynamic behaviour in which only small angle nutations will be observed. Although, stable limit cycles are believed to exist for the end-body motions, the tether vibrations are clearly unstable.

Figure 7.2a: Damped Modal Frequencies as a Function of Spin-Rate for the OEDIPUS-C Free-Fall State (Tension = 0.14 N)

Figure 7.2b: Stability Boundaries as Predicted by the TE-LAB Linear Mathematical Model for the OEDIPUS-C Configuration (Plotted as a Function of Tension and Spin-Rate)

Chapter 8

8. Conclusions, Contributions and Future Research

The following chapter discusses the conclusions of this dissertation with a summary of contributions in both the areas of tether research and modal analysis. Based on this work recommendations for future research are made.

8.1. Summary of Contributions

In the research done for this thesis, contributions were made in the following areas:

- (1) A tether laboratory was developed where the complex interaction between the tether and end-body dynamics could be quantitatively studied. This is extremely valuable as the application of tethers in space assumes a greater importance.
- (2) The quantitative results were used to validate a linearized mathematical model of the spinning tether/end-body system, giving a high degree of confidence in the modeling techniques used.

- (3) Two unique modal analysis tools are developed to study the system dynamics and provide insight into the stability of the spinning tether/end-body system, are introduced: the Modal Parameters Vs. Spin-Rate graphs, and the tether mode shape animation.
- (4) The study provides further confidence in the stabilization approaches adopted for the OEDIPUS-C configuration scheduled for launch in 1995.
- (5) Finally, the thesis demonstrates, for the first time, that a spinning gimbal mechanism provides for a good approximation of the free-spinning body for small angle motions. The gimbal is a viable alternative to the earlier use of air-bearing technology.

8.2. Conclusions

The Tether Laboratory Demonstration System (TE-LAB) has been developed to support the Tether Dynamics Experiment for the OEDIPUS-C tethered sounding rocket mission scheduled to fly in the winter of 1996. TE-LAB has been described herein with an emphasis on the mechanical portions comprising the lower table and gimbaled body; the upper table and tether reel assembly; the tether itself; and a brief description of the data acquisition and control system. This description includes a determination of the basic laboratory parameters as well as an analysis of the TE-LAB gimbaled body. The gimbal mechanism was found to provide a relatively simple method of supporting the end-body at its mass centre while still permitting a 3 DOF motion. The analysis further revealed that if the gimbal's inertia is negligible relative to the end-body inertia (as with TE-LAB) this simple gyro provides a good approximation of the free-spinning body for small angle deflections.

In conjunction with the TE-LAB set-up, a linear mathematical model has been summarized and a rigorous evaluation of the eigenvalue problem conducted. The eigenvalue problem embodies a calculation of the system's damped gyroscopic natural modal parameters for which the Modal Parameters Vs. Spin-Rate graphs are introduced as the basic analysis tool. To further understand the intricacies of the TE-LAB analytical model, the particular system mode shapes were animated. Finally a stability analysis was performed. The stability analysis, based on a system eigenvalue approach, aided in

planning the specific TE-LAB experimental tests as well as providing a greater understanding of the spinning tethered system⁵.

A methodical approach to the TE-LAB experimental test procedure was taken, studying the simpler free-spinning end-body and spinning tether separately (i.e. no interaction) and then expanding the tests to include the full TE-LAB set-up. The ground-based tests were performed for a major axis spinner with an OEDIPUS tether; a minor axis spinner with an OEDIPUS tether; and a minor axis spinner with a "heavy" tether. A Fast Fourier Transform analysis was then performed on the complete set of test results and the experimentally determined natural frequencies were compared to the damped gyroscopic modal frequencies of the analytical model. The experimental and analytical results were found in close comparison, thus giving a high degree of confidence in the linear modeling techniques. The stability analysis also showed good agreement for the minor axis spinner/light tether configuration; however, there was no simple agreement of the nutation growth/decay parameter for the major axis spinner/light tether or minor axis spinner/heavy tether configurations. These differences are believed to be associated with non-linearities that are absent in the present linear model.

With this better understanding of the basic spinning tether/end-body system, the results of the ground-based exercise were extended to include the OEDIPUS configuration. The linear model clearly demonstrates that the OEDIPUS-A configuration was unstable and steps were needed to stabilize the system. The modal analysis further supports the addition of long radial booms as a suitable and adequate stabilization approach for the end-body dynamics. With this approach however, the tether vibrations are still clearly non-zero.

8.3. Future Research

During the course of the TE-LAB design, development and operation, various modifications have been identified worthwhile for future research. These items can be broken down into two categories: modification and adaptation of the TE-LAB set-up, and further studies in the analytical modeling of spinning tethered systems.

Several modifications to the TE-LAB set-up have been identified that not only improve the quantitative results, but also further its application. Such a modification would

Figure 8.1a: Complete Set of Possible Solutions to the Eigenvalue Problem for the Minor Axis Spinner / Heavy Tether Configuration. Highlighted are the Positive-Valued Damped Modal Frequencies Representing Stable Modes and Negative-Valued Frequencies Identifying Unstable Modes

Figure 8.1b: Complete Set of Possible Solutions to the Eigenvalue Problem for the Minor Axis Spinner / Heavy Tether Configuration. Highlighted are the Damped Modal Frequencies Illustrating the Spin Direction of the Corresponding Mode Shapes

include a tether force sensor on the end-body, to provide a more accurate analysis of the tether tension as well as a more direct measurement of the tether modal frequencies. In the present experimental set-up, the system modal frequencies resulting from the tether vibrations are measured indirectly through the end-body dynamics. Consequently, these frequencies often appear as small amplitude peaks (if existent at all) in the FFT analysis. A second TE-LAB modification identified is that of a tether deployment system. The results of the TE-LAB experimental tests give insight into the free-fall state of the OEDIPUS flight where the nominal distance between the subpayloads can be considered constant. With the addition of a deployment system, it would allow one to study the effect of end-body separation on the tether/end-body dynamic behaviour. Finally, the addition of flexible booms to the TE-LAB end-body would provide valuable insight into the effect of this additional form of damping on both the free-spinning body and spinning tether/end-body system. The TE-LAB system with flexible booms could be used to validate both the full linear mathematical model presented herein and the equations of motion developed by Vigneron to describe the dynamics of flexible satellites (i.e. spin stabilized crossed-dipole configurations)²⁴.

From a modeling standpoint, three possible improvements are recognized: study of the gimbal mechanism, a more rigorous assessment of the Modal Parameters Vs. Spin-Rate graphs, and finally a need for non-linear modeling.

While the gimbal mechanism has proved to be invaluable in modeling the free-spinning body, for the sake of completeness, the gimbal dynamics should be included in the linear model in a rigorous manner. Furthermore, the study of the simple gyro is performed assuming angular motions of the end-body. Therefore, for large angle deflections to be considered in the TE-LAB system, a study of large angle flexures of the gimbal mechanism is required.

A second area of study involves the Modal Parameter Vs. Spin-rate graphs. In evaluation of the damped gyroscopic natural modal frequencies by Eq. (4.28), there are two implied solutions (positive and negative valued). Although mathematically feasible, in engineering it is assumed that a negative frequency is not real and therefore is discarded. However, the question arises of whether a negative frequency is feasible in a rotating coordinate frame. Figure 8.1 a and b show the complete set of possible solutions to the eigenvalue problem for a minor axis spinner/heavy tether configuration with four assumed tether modes. Figure 8.1 a illustrates the case where the damped gyroscopic modal frequencies

of stable modes are considered to be positive valued while unstable modes are seen to be negative. An alternative approach of visualizing these natural modal frequencies is demonstrated in Fig. 8.1b. Here, the modes are presented in such a way as to give insight into the spin direction of a particular mode shapes as observed by the mode shape animation. The mode shapes corresponding to the positive valued frequencies are seen to rotate in the same direction of the body fixed frame. Conversely, the negative valued frequencies have mode shapes, spinning opposite to the body fixed frame. Once a mode becomes unstable, the corresponding damped modal frequency switches sign (negative to positive) and thus a change in spin direction. The author does not wish to conjecture which method of visualizing the damped gyroscopic modal frequencies is correct, but rather offers it as a possible future study.

As a final word on modeling of the spinning tethered system, a need for non-linear modeling has been recognized. The TE-LAB system showed that for certain test parameters, stable limit cycles of the end-body could be observed. These limit cycles can not be explained by way of the linear modeling, but instead only hypothesized on. Therefore, for a complete understanding of the OEDIPUS configuration these non-linearities must be included in the analytics.

9. References

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10. Appendices

10.1. Appendix A: OEDIPUS-A and C Flight Parameters

Table A1: OEDIPUS-A Flight Parameters

Forward Payload (one pair of booms)	Transverse Inertia (booms deployed)	A	24.26 kg-m ²
	Transverse Inertia (booms deployed)	B	24.39 kg-m ²
	Longitudinal Inertia (booms deployed)	C	1.84 kg-m ²
	Dist. of C.G. to Tether Spool Axis	b	0.79 m
	Approximate length		1.76 m
	Approximate Diameter		0.43 m
	Mass	m_r	84 kg
Aft Payload (one pair of booms)	Transverse Inertia (booms deployed)	A	36.59 kg-m ²
	Transverse Inertia (booms deployed)	B	39.24 kg-m ²
	Longitudinal Inertia (booms deployed)	C	5.88 kg-m ²
	Dist. of C.G. to Tether Attachment Point	b	0.78 m
	Approximate Length		1.67 m
	Approximate Diameter		0.43 m
	Mass	m_a	131 kg
Tether	Length	l	958 m
	Mass per Unit Length	ρ	0.003 kg/m
	Tension During Deployment	T	1.1 N
	Tension After Deployment (free-fall)	T	0.14 N
Other	Spin-rate	ω_r	0.71 cps

Table A2: Proposed OEDIPUS-C Flight Parameters

Forward Payload (two pairs of booms)	Transverse Inertia (booms deployed)	A	111.87 kg-m ²
	Transverse Inertia (booms deployed)	B	111.80 kg-m ²
	Longitudinal Inertia (booms deployed)	C	155.44 kg-m ²
	Dist. of C.G. to Tether Spool Axis	b	0.74 m
	Approximate Length		2.40 m
	Approximate Diameter		0.43 m
	Mass	m _f	122 kg
Aft Payload (two pairs of booms)	Transverse Inertia (booms deployed)	A	39.56 kg-m ²
	Transverse Inertia (booms deployed)	B	39.27 kg-m ²
	Longitudinal Inertia (booms deployed)	C	51.43 kg-m ²
	Dist. of C.G. to Tether Attachment Point	b	0.61 m
	Approximate Length		1.26 m
	Approximate Diameter		0.43 m
	Mass	m _a	104 kg
Tether	Length	l	1000 m
	Mass per Unit Length	ρ	0.003 kg/m
	Tension During Deployment	T	1.1 N
	Tension After Deployment (free-fall)	T	0.14 N
Other	Spin-rate	ω _c	0.08 cps

10.2. Appendix B: Table of TE-LAB Parameters

Table B1: Laboratory Test Parameters for Major Axis Spinner/Light OEDIPUS Tether Configuration

End-Body	Transverse Inertia (booms deployed)	A	0.00542 kg-m ²
	Transverse Inertia (booms deployed)	B	0.00538 kg-m ²
	Longitudinal Inertia (booms deployed)	C	0.00625 kg-m ²
	Dist. of C.G. to Tether Spool Axis	b	0.094 m
	Mass	m	1.66 kg
Tether	Length	<i>l</i>	6.42 m
	Mass per Unit Length	ρ	0.003 kg/m
	Tension Range	T	0 - 5.0 N
Other	Spin-rate Range	ω_s	0 - 3.0 cps

Table B2: Laboratory Test Parameters for Minor Axis Spinner/Light OEDIPUS Tether Configuration

End-Body	Transverse Inertia (booms deployed)	A	0.00607 kg-m ²
	Transverse Inertia (booms deployed)	B	0.00611 kg-m ²
	Longitudinal Inertia (booms deployed)	C	0.00249 kg-m ²
	Dist. of C.G. to Tether Spool Axis	b	0.129 m
	Mass	m	0.688 kg
Tether	Length	<i>l</i>	6.38 m
	Mass per Unit Length	ρ	0.003 kg/m
	Tension Range	T	0 - 5.0 N
Other	Spin-rate Range	ω_s	0 - 3.0 cps

Table B3: Laboratory Test Parameters for Minor Axis Spinner/heavy Tether Configuration

End-Body	Transverse Inertia (booms deployed)	A	0.00607 kg-m ²
	Transverse Inertia (booms deployed)	B	0.00611 kg-m ²
	Longitudinal Inertia (booms deployed)	C	0.00249 kg-m ²
	Dist. of C.G. to Tether Spool Axis	b	0.129 m
	Mass	m	0.688 kg
Tether	Length	<i>l</i>	6.38 m
	Mass per Unit Length	ρ	0.0275 kg/m
	Tension Range	T	0 - 5.0 N
Other	Spin-rate Range	ω_s	0 - 3.0 cps

10.3. Appendix C: Formation of Coefficient Matrices for Linear Model

The following develops the coefficient matrices considering n assumed tether modes. The second order canonical equations of motion for a rigid payload (no booms) with tether are given by

$$A\ddot{\psi} + \psi[(C - B)\omega_s^2 + (T_t - T_g)b] - \dot{\theta}[A + (B - C)]\omega_s - 2\omega_s R^T \dot{q}_9 - R^T \ddot{q}_{10} + (\omega_s^2 R^T - Q_g^T)q_{10} = -C_\psi \dot{\psi} \quad (C-1)$$

$$\dot{\psi}\omega_s[B - (C - A)] + B\ddot{\theta} + \dot{\theta}[(C - A)\omega_s^2 + (T_t - T_g)b] + R^T \ddot{q}_9 - (\omega_s^2 R^T - Q_g^T)q_9 - 2\omega_s R^T \dot{q}_{10} = -C_\theta \dot{\theta} \quad (C-2)$$

$$2\omega_s R\dot{\psi} + R\ddot{\theta} - (\omega_s^2 R - Q_g) \dot{\theta} + M_t \ddot{q}_9 + C_t \dot{q}_9 + (K_t - K_g - \omega_s^2 M_t)q_9 - 2\omega_s M_t \dot{q}_{10} = 0 \quad (C-3)$$

$$-R\ddot{\psi} + (\omega_s^2 R - Q_g)\dot{\psi} + 2\omega_s R\dot{\theta} + M_t \ddot{q}_{10} + C_t \dot{q}_{10} + (K_t - K_g - \omega_s^2 M_t)q_{10} - 2\omega_s M_t \dot{q}_9 = 0 \quad (C-4)$$

Rearranged in matrix form

$$[M]\{\ddot{x}\} + ([G] + [D])\{\dot{x}\} + [K]\{x\} = 0 \quad (C-5)$$

where the state vector $\{x\}$ is rearranged in the following manner

$$\{x\}^T = \{\psi \quad \theta \quad q_{10,1} \quad q_{9,1} \quad q_{10,2} \quad q_{9,2} \quad \dots \quad q_{10,n} \quad q_{9,n}\} \quad (C-6)$$

The four coefficient matrices are defined as

Mass Matrix [M]:

$$[M] = \begin{bmatrix} A & 0 & -R_1 & 0 & -R_2 & 0 & \dots & -R_n & 0 \\ 0 & B & 0 & R_1 & 0 & R_2 & \dots & 0 & R_n \\ -R_1 & 0 & M_t & & & & & & \\ 0 & R_1 & & M_t & & & & & \\ -R_2 & 0 & & & \ddots & & & & \\ 0 & R_2 & & & & \ddots & & & \\ \vdots & \vdots & & & & & \ddots & & \\ -R_n & 0 & & & & & & \ddots & \\ 0 & R_n & & & & & & & M_t \end{bmatrix} \quad (C-7)$$

Gyroscopic Matrix [G];

$$[G] = \begin{bmatrix} 0 & -(A + B - C)\omega_s & 0 & -2\omega_s R_1 & 0 & -2\omega_s R_2 & \dots & 0 & -2\omega_s R_n \\ (A + B - C)\omega_s & 0 & -2\omega_s R_1 & 0 & -2\omega_s R_2 & 0 & \dots & -2\omega_s R_n & 0 \\ 0 & 2\omega_s R_1 & 0 & 2\omega_s M_I & & & & & \\ 2\omega_s R_1 & 0 & -2\omega_s M_I & 0 & 2\omega_s M_I & & & & \\ 0 & 2\omega_s R_2 & & -2\omega_s M_I & \ddots & & & & \\ 2\omega_s R_2 & 0 & & & & & & & \\ \vdots & \vdots & & & & & & & \\ 0 & 2\omega_s R_n & & & & & & & \\ 2\omega_s R_n & 0 & & & & & & & 0 \end{bmatrix} \quad (C-8)$$

Viscous Damping Matrix [D];

$$[D] = \begin{bmatrix} C_\psi & & & & & & & & \\ & C_\theta & & & & & & & \\ & & C_{\phi 1} & & & & & & \\ & & & C_{\phi 1} & & & & & \\ & & & & C_{\phi 2} & & & & \\ & & & & & C_{\phi 2} & & & \\ & & & & & & \ddots & & \\ & & & & & & & C_{\phi n} & \\ & & & & & & & & C_{\phi n} \end{bmatrix} \quad (C-9)$$

Stiffness Matrix [K];

$$[K] = \begin{bmatrix} \mathcal{B} & 0 & E_1 & 0 & E_2 & 0 & \dots & E_n & 0 \\ 0 & \mathcal{B} & 0 & -E_1 & 0 & -E_2 & \dots & 0 & -E_n \\ E_1 & 0 & \mathcal{A}_{11} & 0 & -K_{12} & 0 & & -K_{1n} & 0 \\ 0 & -E_1 & 0 & \mathcal{A}_{11} & 0 & -K_{12} & & 0 & -K_{1n} \\ E_2 & 0 & -K_{12} & 0 & \mathcal{A}_{22} & 0 & & & \\ 0 & -E_2 & 0 & -K_{12} & 0 & \mathcal{A}_{22} & & & \\ \vdots & \vdots & & & & & \ddots & & \\ E_n & 0 & -K_{1n} & 0 & & & & \mathcal{A}_{nn} & \\ 0 & -E_n & 0 & -K_{1n} & & & & & \mathcal{A}_{nn} \end{bmatrix} \quad (C-10)$$

In the above, the basic physical parameters that can be readily changed in the TE-LAB set-up are given by,

A_0, B_0, C_0	Inertias of the rigid body about principal axes
b	distance from tether attachment point to centre of mass of end-body
l	length of tether
T	Tether tension at the Tether Spool Assembly
ρ	tether density
ω_s	constant spin-rate of the rotating system

To render the formulation more compact, the following further definitions of parameters were introduced as per the complete derivation presented in Reference 15

$$A = A_0 + \frac{1}{3} \rho l b^2 \quad (\text{C-11})$$

$$B = B_0 + \frac{1}{3} \rho l b^2 \quad (\text{C-12})$$

$$C = C_0 \quad (\text{C-13})$$

$$\mathcal{A}_{nn} = \mathcal{X}_{t,nn} - \mathcal{X}_{g,nn} - \omega_s^2 M_t \quad (\text{C-14})$$

$$\mathcal{B} = (C - A) \omega_s^2 + (T_t - T_g) b \quad (\text{C-15})$$

C_ψ damping coefficient related to ψ angle of rotation

C_θ damping coefficient related to θ angle of rotation

C_{in} tether damping coefficient given as a percentage e of the hysteretic energy loss per stress cycle

$$C_{in} = \frac{e}{2\pi} \left(n\pi \sqrt{\frac{T - \rho g l}{\rho l^2}} \right) M_t \quad (\text{C-16})$$

$$E_n = s^2 R_n - \rho l b g Q_{g,n} \quad (\text{C-17})$$

$$K_{g,nm} = \rho g l^2 \mathcal{X}_{g,nm} \quad (\text{C-18})$$

$$K_{t,nn} = \mathcal{X}_{t,nn} T l \quad (\text{C-19})$$

$$M_{t,nm} = \rho l^3 \mathcal{M}_{t,nm} \quad (\text{C-20})$$

$$R_n = \rho l^2 b \frac{\sqrt{2}}{n\pi} \quad (\text{C-21})$$

$$T_t = T \left(1 + \frac{b}{l} \right) \quad (\text{C-22})$$

$$T_t \quad T_t = \rho g l \left(1 + \frac{b}{2l} \right) \quad (C-23)$$

In the above, the dimensionless coefficient matrices are as follows

$$\mathcal{K}_{g,nm} \quad \mathcal{K}_{g,nm} = \begin{cases} \frac{n^2 \pi^2}{2}, n = m \\ mn \left[\frac{\cos(n+m)\pi}{(m+n)^2} - \frac{\cos(m-n)\pi}{(m-n)^2} - \frac{1}{(m+n)^2} + \frac{1}{(m-n)^2} \right], m \neq n \end{cases} \quad (C-24)$$

$$\mathcal{K}_{t,nm} \quad \mathcal{K}_{t,nm} = \begin{cases} n^2 \pi^2, n = m \\ 0, n \neq m \end{cases} \quad (C-25)$$

$$\mathcal{M}_{t,nm} \quad M_{t,nm} = \begin{cases} 1, n = m \\ 0, n \neq m \end{cases} \quad (C-26)$$

$$Q_{g,n} \quad Q_{g,n} = \begin{cases} \frac{2\sqrt{2}}{n\pi}, n: \text{even} \\ 0, n: \text{odd} \end{cases} \quad (C-27)$$

10.4. Appendix D: TE-LAB Experimental Results

The following presents the experimental natural frequencies superimposed on the analytical damped modal frequencies (as a function of spin-rate) for the complete set of TE-LAB Tests

Figure D2: Damped Gyroscopic Modal Frequencies for the Minor Axis Spinner / Light OEDIPUS Tether Configuration for Tether Tensions of 0.50 N, 0.89 N, 1.42 N, 2.76 N and 4.54 N (1st set of results)

Figure D3: Damped Gyroscopic Modal Frequencies for the Minor Axis Spinner / Light OEDIPUS Tether Configuration for Tether Tensions of 0.50 N, 0.89 N, 1.42 N, 2.76 N and 4.54 N (2nd set of results)

Figure D4: Damped Gyroscopic Modal Frequencies for the Minor Axis Spinner / Heavy Tether Configuration for Tether Tensions of 2.02 N, 3.51 N, and 4.54 N

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(tether mode) occurs. When unstable, the tether assumes a specific shape characterized by the number of anti-nodes (known as the mode number of the equilibrium state). For each mode, the spool assembly clutch current and corresponding tension was recorded. This was then repeated for a range of spin-rates creating the stability boundaries for each mode shape as a function of spin-rate and tension. The *TE-LAB tether mode* tests were performed for the light OEDIPUS tether only, with a nominal length of 6.4 m and repeated four times. The experimental results for four tether modes are presented in Fig. 5.3. These experimental mode stability boundaries are later plotted against the stability boundaries predicted by the linear model (Fig. 6.4).

5.4. TE-LAB Spinning Tether and End-body Tests

The final sequence of tests involves the full TE-LAB ground-based simulation where the interaction between the tether dynamics and payload is observed. For a nominal tension and spin-rate, the spinning tethered body dynamics are examined and full-time histories recorded. These tests are then repeated for a range of tensions and spin-rates, spanning the entire operating limits of the TE-LAB set-up (Tension: 0.5 - 5 N, Spin-rate: 0 - 3 Hz). A zero spin-rate corresponds to a rocking body configuration where the end-body is only allowed to oscillate about one gimbal axis. In the case of a limit cycle (i.e. payload precesses at a constant nutation angle) or stable configuration (i.e. no notable growth in payload nutation), an initial perturbation is imparted to the system in the form of either a *payload kick* or *tether pluck*.

Figure 5.4 shows a typical TE-LAB plot of the Euler angle (ψ, θ, ϕ) and nutation angle (ν) time histories for a major axis spinner/light tether configuration with a tether tension of 0.49 N. At time approximately equal to 30 seconds, a perturbation in the form of a payload kick is imparted on the system. This impulse causes the end-body nutation angle (angle of end-body spin axis to the vertical) to jump to 17 degrees as seen in Fig. 5.4d. With time however this initial end-body nutation damps out to pure spin about its major axis. Both the payload and tether exhibited stable dynamic behaviour. Figure 5.4c demonstrates the Euler angle ϕ , the angle of rotation about the spin axis. A constant spin speed is observed. For analysis of the TE-LAB dynamics, either of the Euler angles θ or ψ can be considered. To minimize the amount of post-processing, the remainder of the Euler angle time histories illustrate only the theta (θ) angle data.