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Decision Feedback Equalization with Cyclostationary Interference for DSL

by
Majeed Abdulrahman; B.A. Sc.

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering**

**Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University**

June, 1989

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**The undersigned hereby recommends to the Faculty of Graduate
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**submitted by Majeed Abdulrahman, B.A. SC., in partial fulfilment of
the requirements for the degree of Master of Engineering.**

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Abstract

Interference from digital signals in multipair cables has been shown to be cyclostationary. This work evaluates the performance of a decision feedback equalizer (DFE) that has forward and feedback filters in the presence of cyclostationary interference (CI) and additive white Gaussian noise (AWGN). Mathematical equations for the DFE with CI and AWGN were derived. A comparison was made between a DFE with CI and one with stationary interference (SI) to show the better of the two cases. Fractionally-spaced (FR-SP) and baud-rate DFE equalizers were compared in the presence of CI and AWGN. We have proposed the use of FR-SP DFE since it had much better performance than the baud-rate DFE. If the FR-SP DFE cannot be used because of its complexity, then we propose the use of a baud-rate DFE using the adaptive technique that searches for the receiver sampling phase which gives the best system performance in the presence of CI and AWGN. We also proposed that the transmitters' phases be the same at the central office, which results in an overall CI. This proves to be beneficial if we use a FR-SP DFE or baud-rate DFE, with an adaptive technique, for equalization at the receiver.

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Chapter 1 Introduction

1.1 Problem Description

The increasing demand for simultaneous transmission of voice, data and image information has led to the research on *Integrated Services Digital Networks (ISDN)* where those sources of information could be transmitted digitally on the same medium. Field trials have been conducted recently by many companies on the use of optical fibers for digital transmission. It has shown that there is great advantage in using the new technology; however, it is not economical to throw away the present loop plant immediately and replace it with fiber optics technology. Therefore, ISDN research has concentrated on using the current subscriber loop plant for digital transmission. This loop plant uses twisted pair lines for transmission. It is called a *digital subscriber loop (DSL)* when it carries digital information for ISDN.

The subscriber loop has a number of imperfections such as cable loss, distortion, bridged taps, crosstalk and impulse noise. The frequency-dependent loss of the cable depends on the length of the cable, gauge mix of the cable and bridged taps (open-circuited cable pairs that connect to the main cable). This loss could be compensated for at the receiver by increasing the high-frequency gain. But this gain will increase the noise, crosstalk and any interference signal that may be present, causing a degradation in the system performance. The most

important imperfections of the DSL are impulse noise that is caused by dial-pulse signalling, ringing and others in the central office, and crosstalk.

Crosstalk is one of the major impairments that will increase with transmission rate. There are two kinds of crosstalk that affect the performance when transmitting on the DSL, *far-end crosstalk (FEXT)* and *near-end crosstalk (NEXT)*, whose paths of entry are shown in Fig.1.

FEXT represents a crosstalk of a local transmitter into a remote receiver with a theoretical average attenuation given by [1]

$$|H_{fext}(f)|^2 = K_{fext} |C(f)|^2 |f|^2, \quad (1)$$

where $H(f)$ is the transfer function experienced by the crosstalk and $C(f)$ is the transfer function of the cable. NEXT represents a crosstalk of a local transmitter into a local receiver, and experiences a theoretical average attenuation of

$$|H_{next}(f)|^2 = K_{next} |f|^{3/2}. \quad (2)$$

By looking at the transfer functions, we find that FEXT will experience the loss of the cable (in addition to the crosstalk coupling loss), while NEXT would not. This shows that NEXT will be the dominant source of crosstalk. Therefore; in this thesis we will only be concerned with NEXT as the major noise source for the DSL. The characteristics of NEXT will be studied to help find a technique that will minimize the effect of NEXT at the decision instant of the receiver.

Figure 1 Forms of Crosstalk

The basic ISDN transmission rate that the *CCITT* committee suggested was 144 Kbit/sec. Several techniques were also investigated to achieve full duplex data transmission at 144 Kbit/sec. The methods investigated were *Frequency Separation*, *Time compression Multiplexing (TCM)* and *Echo Cancellation (EC)*. Frequency Separation has not been considered because it has a very strict filtering requirements. TCM, used in Japan, requires synchronization of the bursts of data; thus, it eliminates NEXT. But the EC method, shown in Fig. 2, was chosen in most parts of the world because it had narrow bandwidth, less attenuation, less transmission delay and less crosstalk[1] than the other techniques.

The EC method allows the transmission of the data signals in both directions at the same time within the same bandwidth. The transmit filter is used to minimize the radio frequency interference (RFI) and crosstalk to other services, while the receive filter will prevent aliasing and reduce the additive white Gaussian noise (AWGN) bandwidth. But one of the major impairments of this method is echoes that are generated due to the imperfectly matched hybrids and the effects of bridged taps. These echoes that add to the data, which the receiver is expecting to receive, are estimated by an *echo canceller(EC)*, shown in Fig. 2, and subtracted from the received signal at the input of the equalizer.

Line coding is the mapping between the transmitted data stream and the actual pulses that were transmitted on the DSL. A lot of research was done to find the best line code for ISDN. Recent work has found that a five-level line code, in a NEXT-limited environment, is optimum[1]. We also see that recent implementations of an ISDN transceiver have used the 2B1Q as the line code.

Figure 2 EC transceiver at one end

2B1Q means that every two bits are represented by a symbol; i.e., we have four symbols $\{a, -a, 3a, -3a\}$ used for transmission.

This introduction showed the many difficulties that are facing ISDN applications. In this thesis, we will only be concerned with simultaneously minimizing the effect of NEXT and Intersymbol Interference (ISI) on the system performance. We will assume that the echo will be cancelled by the echo canceller, and impulse noise does not exist in the DSL. FEXT is also considered negligible compared to NEXT, as we discussed previously.

1.2 Proposed Approach

Recent research on crosstalk showed that crosstalk interference from digital signals in a multipair cable has cyclostationary statistics[2,3]. This means that the statistics of the interference are periodic with a period of one baud interval. A short explanation for this characteristic of crosstalk is that the transmitters are transmitting every baud period; this will cause a periodic interference on the neighboring receiver(s). Therefore, the crosstalk interference has cyclostationary statistics. This result is of importance because the effect of interference is only relevant at the decision instants in the receiver¹. This suggested the use of a *Fractionally-Spaced Decision Feedback Equalizer (FR-SP DFE)* to take advantage of the cyclostationary nature of crosstalk. FR-SP DFE, as will be shown in the results of this thesis, will sample and weight the crosstalk differently at different points in a baud period. An example to explain the above at this point is the

¹ Residual echo(after echo canceller) will also be cyclostationary since it comes from data at the same baud rate.

variance of a cyclostationary process, shown in Figure 3, where the FR-SP DFE (2 samples/ baud period in this case) samples and weights the crosstalk differently at different points.

A DFE was chosen over a linear equalizer since it had advantages with regards to VLSI implementation and noise enhancement. The *minimum mean squared error (MMSE)* criterion was used to minimize the error without necessarily forcing all ISI to zero.

First, mathematical analysis was done in which equations were derived to calculate the MMSE of the FR-SP DFE assuming NEXT and additive white Gaussian noise (AWGN) as the only noise sources available. Then computer simulation was done to verify the derived equations and find the advantages of using this technique to reduce the effect of NEXT at the decision instant in the receiver.

1.3 Thesis contribution

The research that we have done has shown that a FR-SP DFE gives better performance than a baud-rate sampling DFE. It also showed that better performance is achieved if the interference is cyclostationary compared to that if the interference is stationary. It was also found that if the transmit phases of the interferers are uniformly distributed within a baud period then the statistics of the overall interference are more stationary as the number of interferers increases, while if all the interferers had the same transmit phase then the overall interfer-

Figure 3 An example of variance (instantaneous power) of a cyclostationary process

ence has cyclostationary statistics and the system performance is better in this case than the previous one.

The DFE that we studied here was not different from other baud-spaced or FR-SP DFE's. The new contribution is the analysis of its performance in cyclostationary crosstalk, how it varies with sampling and crosstalk phases, and with numbers of tap coefficients on subscriber loops.

The research also showed that if FR-SP DFE is not used because it makes the system more complex (the echo canceller design will change with changes in the equalizer design), then a baud-rate DFE with adaptive sampling phase (the receiver has the control of adjusting the sampling phase) performs better than one without adaptive sampling phase, since the former uses the cyclostationary nature of NEXT to its advantage. This aspect of adaptively choosing the receiver sampling phase to minimize the combination of cyclostationary NEXT, ISI and noise, is a point of departure from the conventional DFE receiver structure.

The results of this research could also be applied to non-ISDN systems with co-channel interference(crosstalk) like terrestrial digital microwave systems, satellite communications systems, digital cellular and indoor wireless systems, and digital exchange carrier systems in twisted pair cables.

1.4 Thesis Structure

The remaining chapters of this thesis are organized as follows:

In Chapter 2, the system that was used to study the performance of the DSL, with NEXT as the major impairment and FR-SP DFE as the equalizer at the

receiver, is analysed in more detail. Figures will show the different stages of the system and the elements that affect the system performance. Equations will also be derived to find the MMSE of FR-SP DFE.

In Chapter 3, simulation results are shown and analysed where we used the impulse response of *Bellcore Loop 9* for the basic transmission rate, 80 Kbaud, and the impulse response of *Bellcore loop 15* for a higher transmission rate, 400 Kbaud.

Finally, Chapter 4 summarizes the major conclusions from this work and provides recommendations for future research.

Chapter 2 System Description

2.1 Introduction

In this chapter, the FR-SP DFE system, with cyclostationary interference, will be explained in more detail. First, the transmission and reception of data will be explained. Then, NEXT will be studied in more detail. AWGN was assumed to exist and will be studied too. Then, we will talk about the transmit and receive filters that were used in this system, and their effect on the transmitted and received data. After that, we will study in more detail the FR-SP DFE. Finally, we will show the equations of the MMSE in general, where the details are given in Appendix A. Figure 4 shows the overall system that was used to study the performance of FR-SP DFE in the presence of cyclostationary interference, NEXT, and AWGN.

2.2 Transmission and Reception of Data

For a DSL system using 2B1Q line coding, a symbol is transmitted every baud interval. This symbol is transmitted using pulse amplitude modulation(PAM) signal. The transmitted sequence before filtering is

$$S(t) = \sum_{k=-\infty}^{\infty} a_k \pi(t - kT), \quad (3)$$

where a_k is the transmitted symbol at time kT , and $\pi(t)$ is the PAM pulse shape. This data sequence that the receiver is expecting to receive will be filtered by

Figure 4 Overall system block diagram

a transmit filter, as shown in Fig. 4. It will then suffer the impairments of the loop plant, such as line attenuation and distortion, before being filtered again by a receive filter. After that, the data bearing signal received will have the form:

$$D(t) = \sum_{k=-\infty}^{\infty} a_k \Phi_0(t - kT), \quad (4)$$

where $\Phi_0(t - kT)$ is the pulse shape including the effects of the channel and both the receive and transmit filters.

2.3 NEXT

Since we have duplex transmission in ISDN, we usually have a system receiving a data sequence, like the one in previous section, while other systems neighboring that system are transmitting in the opposite direction. These transmitting systems will cause NEXT to the receiving system. Fig. 4 includes the interference from "neighboring transmitters" on the received data. As was explained in Chapter 1, $K_{next}f^{3/2}$ represents the magnitude squared frequency response of the idealized crosstalk path of NEXT(actual path will depart from this). Therefore, the contribution of NEXT at the receiver end will be of this form:

$$X(t) = \sum_{l=1}^N \sum_{k=-\infty}^{\infty} b_{kl} \Phi_l(t - kT), \quad (5)$$

where

N = number of interferers,

b_{kI} = symbol transmitted by interferer I at time kT , and

$\Phi_I(t - kT)$ = impulse response after attenuation by the crosstalk path, and both the transmit and receive filters.

The magnitude response of NEXT, $K_{next}f^{3/2}$, could be used to find the impulse response of NEXT, $\Phi_I(t - kT)$, at the receiver. But measurements have been made to find this impulse response of NEXT[4]. Therefore, the measured impulse response was used in the evaluations done in this thesis. Two impulse responses of NEXT are shown in appendices C and D for 80 and 400 Kbaud transmission rates. These impulse responses are the measured ones after filtering by the transmit and receive filters. These filters are introduced in section 2.5.

2.4 AWGN

In the analysis it is assumed that NEXT was the major impairment of the system. But we also assumed that the the system has another impairment which is Additive White Gaussian Noise (AWGN), which exists in every system. However, the effects of AWGN on the decision point of the receiver have been assumed to be minor compared to that of NEXT on the decision point of the receiver. The AWGN will be filtered by the receive filter, so its effect will be represented in this analysis by

$$PSD_{out} = \frac{N_0}{2} |H_{receiver}(f)|^2 \quad (6)$$

where

$\frac{N_0}{2}$ = input power spectral density (PSD) of AWGN,

$H_{receiver}(f)$ = transfer function of the receive filter, and

PSD_{out} = power spectral density at the input of the FR-SP DFE.

2.5 Transmit and Receive Filter

In this system, we used 6th order Butterworth filters for each of the transmit and receive filters. It was also assumed that

$$|H_{transmit}(f)| = |H_{receiver}(f)| = |B(f)|^{1/2} \quad (7)$$

where $B(f)$ is the frequency response of the 6th order Butterworth filter. The cutoff frequency of the Butterworth filter was 40 and 200 KHZ for the 80 and 400 Kbaud transmission rates, respectively, that were used for testing the system.

2.6 FR-SP DFE

At this point, the sequence that will be received by the FR-SP DFE will be

$$r(t) = \sum_{k=-\infty}^{\infty} a_k \Phi_0(t - kT) + \sum_{l=1}^N \sum_{k=-\infty}^{\infty} b_{kl} \Phi_l(t - kT) + n(t), \quad (8)$$

where $n(t)$ is the AWGN, after filtering by the receive filter, at the input of the FR-SP DFE. The first term of $r(t)$ is the sequence that is needed to be detected, while the other two terms are the interference that should be cancelled. The FR-SP DFE, shown in Fig. 5, has a forward filter and feedback filter that are working

to minimize the effect of ISI and AWGN using the minimum mean-square error (MMSE) criteria.

Forward Filter

The forward filter is a fractionally-spaced filter, which means that there is a $\Delta = T/M$ delay between samples where T is the baud period and M is the number of samples/baud interval. The forward filter will work on minimizing the effect of NEXT, AWGN and precursor ISI. If more than one sample is taken per baud, then all the outputs/baud of the forward filter, except one, will be ignored since the slicer makes a decision every baud interval. The output of the forward filter is of this form:

$$FF = \sum_{m=0}^{N_w} W_m r(nT - m\Delta) \quad (9)$$

where $N_w + 1$ is the number of tap-coefficients in the forward filter, and W_m represents the setting of the m th tap at time nT .

Feedback Filter

This filter will try to minimize the effect of post-cursor ISI. The feedback filter has a T -spacing delay between samples since we make our decision every baud interval. The output,

$$FB = \sum_{m=1}^{N_f} f_m a_{n-m-D}, \quad (10)$$

Figure 5 Fractionally-Spaced Decision Feedback Equalizer(FR-SP DFE).

will be added to that of the forward filter to give the input of the slicer, where N_f is the number of tap-coefficients of the feedback filter, f_m is the setting of the m th tap at time nT and D is the delay of the FR-SP DFE.

2.7 Slicer

The input to the slicer will be,

$$Y_n = FF + FB. \quad (11)$$

The slicer will then decide, depending on this input, on which one of the 4 possible symbols that was transmitted. The error between the output and the input of the slicer will be used to find the setting of the tap-coefficients for the MMSE criteria.

2.8 Calculation of MMSE

As we mentioned before, the signal received at the input of the FR-SP DFE is

$$r(t) = \sum_{k=-\infty}^{\infty} a_k \Phi_0(t - kT) + \sum_{l=1}^N \sum_{k=-\infty}^{\infty} b_{kl} \Phi_l(t - kT - \Omega_l) + n(t) \quad (12)$$

The only difference between equation 12 and equation 8 is the variable Ω_l . This variable means that the l 'th NEXT phase is Ω_l at the input of the FR-SP

DFE, where $0 \leq \Omega_l < T$. The importance of this variable will be shown in the results of the simulation.

At the input of the FR-SP DFE, we could sample once or more in a baud period with a time shift, Θ . Therefore, when the received signal is sampled, the input to the slicer will be

$$Y_n = \sum_{m=0}^{N_w} w_m r(nT - (m\Delta + \Theta)) + \sum_{m=1}^{N_f} f_m a_{n-m-D} \quad (13)$$

where D is the delay in the system, as it was explained previously. Using the criteria of MSE, it was found(details are in Appendix A) that

$$MMSE = 1 - \mathbf{V}^T \mathbf{A}^{-1} \mathbf{V} = 1 - \mathbf{V}^T \mathbf{U}_{opt} \quad (14)$$

where $\mathbf{U}_{opt}$ is the vector of optimum setting for the forward and feedback tap-coefficients.

Chapter 3 Results and Analysis for Basic and High Transmission Rates(80 and 400 Kbaud)

3.1 Introduction

In this chapter, we will explain the different methods that were used to study the performance of the FR-SP DFE in the presence of cyclostationary interference, and analyze the results of the simulation. Two transmission rates used for the simulation were: 80 Kbaud or 160 Kbit/sec which includes the basic rate (144 Kbit/sec) that was suggested by the *CCITT* committee and control information (16 Kbit/sec), and 400 Kbaud transmission rate in order to study the system performance at a higher rate than the basic one.

The analysis presented in Appendix A showed that we need to know the impulse responses of both the channel and the NEXT path, and the autocorrelation function of the AWGN. The impulse responses and autocorrelation functions of AWGN that were used for the 80 and 400 Kbaud transmission rates are shown in Appendices C and D respectively, where the impulse responses are filtered by the transmit and receive filters, and the AWGN is bandlimited by the receive filter. The results of both transmission rates are shown and explained; however, the results of the 80 Kbaud transmission rate are analyzed in more detail than the results of the 400 Kbaud case to avoid repetitions in the analysis which is similar for both rates.

3.2 Effect of the Number of Filter Taps on Performance

80 Kbaud transmission rate

We first determined the effect on system performance when we change the number of taps of the forward filter (FF) and feedback filter (FB) . In this section, we have assumed the existence of one interferer, and the transmitted data and NEXT are in phase ($\Omega_1 = 0$). The sampling phase Θ was chosen to be the optimum one in the absence of NEXT. It was found, as Figure 6 shows, that $\Theta = \frac{5}{8}T$ for a baud-rate DFE, where T is the baud period. Then we found (see Figure 7) for the case of more than 10 FF taps that the FR-SP DFE with more FB taps and less FF taps performs better than having less FB taps and more FF taps; in this Figure FSE-1 means that the FR-SP DFE is sampling once/baud period. In other words, the FR-SP DFE is a baud-rate DFE when we take 1 sample in a baud period. But we also found that the system performance will degrade if the FF taps are less than 10 even though the FB taps have been increased. We concluded from this result that $N_w=10$ and $N_f=80$ for FSE-1 is the best combination to use for the evaluations in the upcoming sections.

For the case of FSE-2, or 2 samples/baud period, we found (see Figure 8) the same result as for the previous case where the FB taps have more effect on system performance than the FF taps have when we use more than 10 FF taps. It also performs the same way as FSE-1 when we have less than 10 FF taps. Even though the system performance is almost the same at $N_w=8$ and 10, we will use $N_w=10$ and $N_f=80$ as we chose for FSE-1 in order to compare the two cases

Figure 6 System performance with no NEXT (80 Kbaud)

Figure 7 Changing number of taps for FSE-1 (80 Kbaud)

using the same number of taps. Note that in the best FSE-2 case, the forward filter spans only half as many symbol intervals (5) as in the best FSE-1 case. Looking at Figures 7 and 8, we also found that, for a particular number of FF and FB taps, the FR-SP DFE performs much better than baud-rate DFE. This agrees with the theory that the more samples you take the better the estimate of the transmitted symbols.

400 Kbaud transmission rate

Using the same assumptions as for 80 Kbaud, we found, as Figure 9 shows, that the sampling phase to use is $\Theta = \frac{7}{8}T$ for the FSE-1 case.

For FSE-1, Figure 10 shows that $N_w=20$ and $N_f=70$ gives the best system performance; however, we will use $N_w=15$ and $N_f=75$ since it is less complex to build (lower FF taps) and has almost the same system performance as for the best case. For FSE-2, Figure 11 shows that the system performance is almost the same for N_w between 20 and 60, and it starts to degrade outside this range. We tested the FR-SP DFE with $N_w=20$ using the tests described in the following sections. This showed that the FR-SP DFE is sometimes worse than the baud-rate DFE; therefore, we decided that the forward filters of the DFE equalizers to cover the same time span. This means that $N_w=30$ for FSE-2. For this case, as Figure 11 shows, $N_f=60$, but we will use $N_f=75$ for the FSE-1 case because we will compare the two cases with both having the same number of FB taps.

Figure 8 Changing number of taps for FSE-2 (80 Kbaud)

Figure 9 System performance with no NEXT (400 Kbaud)

Figure 10 Changing number of taps for FSE-1 (400 Kbaud)

Figure 11 Changing number of taps for FSE-2 (400 Kbaud)

3.3 Cyclostationary vs. Stationary Interference

Previous studies and implementations of a digital receiver have always assumed that the interference from digital signals is stationary. In this section, we will use both *cyclostationary interference(CI)* and *stationary interference(SI)* in order to find whether it is advantageous to design a receiver with the assumption that the interference from digital signals is cyclostationary. There are two cases to look at: fixed arbitrary sampling phase, and variable sampling phase. Note, for both cases, that the interferers and the received data have been chosen to be in phase ($\Omega_i = 0$).

80 Kbaud transmission rate

Fixed Sampling Phase ($\Theta \approx \frac{5}{8}T$)

For the case of FSE-1, we found, as Fig. 12 shows, that the baud-rate DFE with SI performs almost the same as that with CI at this sampling phase. Another point we have showed in this figure is the degradation of the system performance with the increase of the number of interferers. All interferers are assumed to have the same power, and the maximum amplitude of one interferer is 1/50 of the maximum amplitude of the received symbol.

The result is different for the case of FSE-2, as Fig. 13 shows. The equalizer attempts to minimize the ISI and NEXT jointly at the decision instants at T -seconds intervals. Figure 14 shows the settings of the forward filter coefficients for FSE-1 and FSE-2 cases, while Figure 15 shows the combined response of the

Figure 12 Stationary vs. cyclostationary for FSE-1 and $\Theta = \frac{5}{3}T$ (80 Kbaud)

forward filter and the NEXT pulse for the baud-rate and FR-SP DFE equalizers. This figure tells us that the FR-SP DFE reduces the effect of NEXT more than the baud-rate DFE does. It also shows that there are approximately zero crossings at baud intervals which means that the interference has been approximately eliminated at baud intervals apart.

Figures 12 and 13 are displayed together in Figure 16 to show the big advantage of using a FR-SP DFE instead of a baud-rate DFE with the assumption that the interference is cyclostationary. Here, we see that if NEXT is stationary then a baud-rate and FR-SP DFE equalizers have the same system performance, but FR-SP DFE performs much better than the other one if the interference is cyclostationary.

Variable Sampling Phase ($0 \leq \Theta < T$)

In this analysis, we will change the sampling phase and study the system performance for both kinds of interference, stationary and cyclostationary. Here, we assume that we have one interferer that is in phase with the received data ($\Omega_1 = 0$). By looking at Figure 17 we find that the baud-rate DFE with CI performs better, at half of the sampling phases, than one with SI. With CI, as the sampling phase changes the system performance changes because the statistics of the interference varies within a baud period. This tells us that the performance of a baud-rate DFE with CI could be improved by judiciously selecting the receiver sampling phase.

Figure 13 Stationary vs. cyclostationary for FSE-2 and $\Theta = \frac{5}{8}T$ (80 Kbaud)

Figure 14 Forward tap coefficients for FSE-1 and FSE-2(80 Kbaud)

Figure 15 Combined Response of Forward filter and NEXT(80 Kbaud)

Figure 16 Stationary vs. cyclostationary for FSE-1 and FSE-2 (80 Kbaud)

Figure 17 Stationary vs. cyclostationary
for FSE-1 and $0 \leq \Theta < T$ (80 Kbaud)

For the case of FSE-2, Figure 18 shows that the system performance of FR-SP DFE with CI varies as the sampling phase changes, but it is always better than a FR-SP DFE with SI. This also shows that a FR-SP DFE performs better than a baud-rate DFE with both of them having CI. From Figures 17 and 18 we find that the SNR will depend on Θ , but variability with Θ will diminish to zero as the sampling rate is increased.

400 Kbaud transmission rate

Fixed Sampling Phase ($\Theta = \frac{7}{8}T$)

Figure 19 shows that the system performance with CI is a little better than a system with SI. However, as Figure 20 shows, a FR-SP DFE with CI gives much better system performance than a one with SI, where the gain is about 12 dB for the case of 1 interferer. Figure 21 shows that a FR-SP DFE performs better than a baud-rate DFE, assuming that we have CI.

Variable Sampling Phase ($0 \leq \Theta < T$)

Here we found, as Figure 22 shows, that at slightly more than 50 % of the range of sampling phases a baud-rate DFE with SI is better than one with CI. However, as Figure 23 shows, a FR-SP DFE with CI is always better than a one with SI.

3.4 In-phase vs. Out-of-phase

In this case we assume that we have one or more interferers where all the interferers have the same power level. *In-phase* means that the interferers are

Figure 18 Stationary vs. cyclostationary
for FSE-2 and $0 \leq \Theta < T$ (80 Kbaud)

Figure 19 Stationary vs. cyclostationary
for FSE-1 and $\Theta = \frac{7}{8}T$ (400 Kbaud)

Figure 20 Stationary vs. cyclostationary
for FSE-2 and $\Theta = \frac{7}{8}T$ (400 Kbaud)

Figure 21 Stationary vs. cyclostationary for FSE-1 and FSE-2 (400 Kbaud)

Figure 22 Stationary vs. cyclostationary
for FSE-1 and $0 \leq \Theta < T$ (400 Kbaud)

**Figure 23 Stationary vs. cyclostationary
for FSE-2 and $0 \leq \Theta < T$ (400 Kbaud)**

all in phase ($\Omega_i = 0$), while *Out-of-phase* means that the interferers' phases are distributed uniformly. As an example Figure 24 shows the case of 4 interferers that are out of phase ($\Omega_1 = 0$; $\Omega_2 = \frac{T}{4}$; $\Omega_3 = \frac{T}{2}$; $\Omega_4 = 3\frac{T}{4}$) where T is the baud period. In practice, the transmitters may be either by chance or intentionally out of phase. In this analysis, we will have two extreme cases : interferers in phase and interferers out of phase, and show the one that gives better system performance. The *out-of-phase* case is an extreme case of random phases (least cyclostationary), while the *in-phase* case represents the situation where all transmitting clocks at the central office are deliberately put in phase (most cyclostationary).

80 Kbaud transmission rate

For the out-of-phase case, we have changed the number of interferers and the sampling phase (Θ) to study the system performance as Figure 25 shows where OUT-OF-PHASE_N(X) means that there are N out-of-phase interferers and X is the difference in dB between the best and the worst sampling phase read from Figure 25. Figure 25 tells us that as the number of interferers increase, the variation between the best and the worst sampling phase decreases; thus, for a large number of out-of-phase interferers, a search for the best sampling phase would not be fruitful. This behaviour of the out-of-phase interferers is the same as that of the stationary interference; this leads to the conclusion that the more out-of-phase interferers exist the more stationary are their statistics.

For the in-phase case, Figure 26 shows that the difference between the best and worst sampling phase is almost the same as the number of interferers increases.

Figure 24 Four interferers that are out of phase

Figure 25 Interferers out-of-phase for FSE-1 (80 Kbaud)

Figure 26 Interferers in-phase for FSE-1 (80 Kbaud)

This means that the overall interference is cyclostationary and it is very important to search for the sampling phase that gives the best system performance. In Figure 27, we find that we will have better system performance if we have in-phase interferers and search for the best sampling phase.

In Figure 27, we also found that at some sampling phases the out-of-phase system is better than the in-phase system. However, this is not the case when we sample twice per baud period where the in-phase system is always better than the out-of-phase system, as shown in Figure 28. The reason is that the overall interference in the in-phase system is cyclostationary, and it is stationary in the out-of-phase system. Therefore, as shown previously in the cyclostationary vs. stationary case, the FR-SP DFE performs better when the interference is cyclostationary.

400 Kbaud transmission rate

As in the 80 Kbaud case, all interferers have the same power level. For the *out-of-phase* case, the difference in system performance between the best and the worst sampling phase decreases as the number of interferers decreases, as Figure 29 shows. Notice that the change of this number from out-of-phase_1 to out-of-phase_2 case is only 0.63 dB, but it is 4.89 dB for the basic transmission rate as shown in Figure 25. This agrees with previous research results [2,3] that cyclostationary peak-to-peak noise variation decreases as the baud rate increases.

For the *in-phase* case, Figure 30 shows that the cyclostationary statistics are preserved even though the number of interferers has increased.

Figure 27 Inphase vs. outphase for FSE-1 (80 Kbaud)

Figure 28 Inphase vs. outphase for FSE-2 (80 Kbaud)

Figure 29 Interferers out-of-phase for FSE-1 (400 Kbaud)

Figure 30 Interferers in-phase for FSE-1 (400 Kbaud)

Figure 31 Inphase vs. outphase for FSE-1 (400 Kbaud)

Figure 32 Inphase vs. outphase for FSE-2 (400 Kbaud)

Figure 31 shows that at some sampling phases the *out-of-phase* case is better than the *in-phase* case; however, this is not the case for the FSE-2 case where the *in-phase* case is always better as shown in Figure 32.

3.5 Adaptive and Conventional Techniques

In the previous sections, except for out-of-phase analysis, the received data and NEXT were assumed to be in phase ($\Omega_I = 0$). In this section, the reception of NEXT will be delayed with respect to the reception of the received data, because in reality the phase delay between them may vary from cable to cable; thus, the analysis in this section will help to study the system performance for this condition. Two techniques have been used to study the performance of the FR-SP DFE with CI: conventional and adaptive which are explained in the following subsections. For both cases, we assumed that there is only one dominant interferer.

80 Kbaud transmission rate

Conventional technique

In this case, the receiver started out with a given sampling phase which did not change when we delayed the reception of the NEXT with respect to the reception of the required data symbols. In other words, the receiver sampling phase was picked without regard to the phase of the NEXT. As Figure 33 shows, the system performance varies as the NEXT phase Ω_1 is changed. The simulation has showed that the system performance will vary by about 5 dB. For the FR-SP DFE, Figure

34 shows that the system performance is much better than that for the baud-rate DFE. This shows that the FR-SP DFE is better to use for equalization.

Adaptive technique

In the conventional case we had a particular sampling phase that did not change as the NEXT phase Ω_1 varied. But we have showed previously that for CI we have a change in system performance as the sampling phase changes. This led us to the idea of changing the sampling phase until we found the one that had the best system performance for a particular shift of NEXT. When this phase was found, we set our receiver sampling phase to it. Figure 35 shows that the system performance for the adaptive case is always better or equal to that of the conventional method, because we always search and set the receiver of the adaptive method to the best sampling phase.

Figure 36 shows the SNR obtained using the adaptive and conventional techniques for both FSE-1 and FSE-2. It shows that the FR-SP DFE is always better (conventional or adaptive) to use than the adaptive technique of baud-rate DFE. However, we have provided the adaptive technique analysis to give the system designer some flexibility for his choice of equalization technique. The existence of FR-SP DFE will double the complexity of the circuitry of the system blocks, especially the EC, that cooperate with the equalization process to improve the system performance. If the increased complexity is manageable, then the FR-SP DFE is better to use; otherwise, the adaptive technique for baud-rate DFE is

Figure 33 Conventional method for FSE-1 (80 Kbaud)

Figure 34 Conventional method for FSE-1 and FSE-2 (80 Kbaud)

Figure 35 Conventional and adaptive methods for FSE-1 (80 Kbaud)

**Figure 36 Conventional and adaptive
methods for FSE-1 and FSE-2 (80 Kbaud)**

better to use than the conventional technique. The only problem with the adaptive technique is the time it takes to find the best sampling phase. This sampling phase is found by starting with an arbitrary sampling phase, calculating the MSE, then moving the sampling phase up or down and calculating the MSE again to find the better of these two phases. The better sampling phase is compared with the next sampling phase we choose in order to keep the better of the two phases. The same procedure is repeated until the best sampling phase has been found.

400 Kbaud transmission rate

Conventional technique

Figure 37 shows the variation of the system performance with changes in the NEXT phase Ω_1 for the FSE-1 case. In Figure 38, we see that the FR-SP DFE is always better than the baud-rate DFE as the NEXT phase Ω_1 is shifted. Looking at Figures 34 and 38, we see that there is less variation in system performance for the 400 Kbaud case than that for the 80 Kbaud case. This again shows similarities with the results of previous work by Campbell et al [2,3].

Adaptive technique

Figure 39 shows that a gain of 2.5 dB could be achieved at some points if we use the adaptive technique instead of the conventional one for the FSE-1 case. The adaptive and conventional results for the FSE-1 and FSE-2 cases are shown in Figure 40. This Figure shows that a FR-SP DFE gives always better system performance than a baud-rate DFE. If FR-SP DFE is not possible to use then a

baud-rate DFE with an adaptation technique for the sampling phase is the second choice to use for equalization.

Figure 37 Conventional method for FSE-1 (400 Kbaud)

Figure 38 Conventional method for FSE-1 and FSE-2 (400 Kbaud)

Figure 39 Conventional and adaptive methods for FSE-1 (400 Kbaud)

Figure 40 Conventional and adaptive
methods for FSE-1 and FSE-2 (400 Kbaud)

Chapter 4 Conclusions and Recommendations

4.1 Conclusions

In this thesis, we have found that a DFE, baud-rate or FR-SP DFE, requires more feedback(FB) taps than feedforward(FF) taps in the presence of NEXT, AWGN and ISI. This applies for the two transmission rates that were used, 80 and 400 Kbaud. A baud-rate DFE with cyclostationary interference(CI) performed better, at some receiver sampling phases, than a baud-rate DFE with stationary interference(SI). But a FR-SP DFE with CI was always much better than one with SI. We have also found that the overall interference of a number of interferers that are out of phase is more stationary as this number increases. However, the overall interference is cyclostationary if those interferers are in phase.

At high baud rates, we found that the difference in performance between cyclostationary and stationary or almost-stationary NEXT decreases, and there is less variation in system performance compared to that of the 80 Kbaud case. For example, Figure 18 shows a gain of about 16 dB for a FR-SP DFE with CI over one with SI for the 80 Kbaud rate, while for the 400 Kbaud case, as Figure 23 shows, 11 dB has been gained for the same situation. This agrees with previous research results by Campbell et al [2,3] that cyclostationary peak-to-peak noise variation decreases as the baud-rate increases.

Two equalization techniques, conventional and adaptive, were also studied. The first one assumes that the receiver sampling phase is arbitrary and fixed, regardless of NEXT characteristics, while the other technique always has the sampling phase that gives the best system performance. It was found that a baud-rate DFE with adaptive technique always performs better than one with conventional technique, because the former always has the best sampling phase while the latter has a fixed sampling phase that gives either the same or worse system performance than the previous method. The only problem with the adaptive method is the time that it takes to find the best sampling phase, since it is going to test few sampling phases before finding the one that gives the best system performance.

We also found that a FR-SP DFE has much better system performance than a baud-rate DFE with adaptive technique. The fractionally-spaced forward filter takes advantage of the cyclostationary nature of the interference by sampling more than once in a baud period where the variance of the interference is variant; thus, it provides much better system performance than a baud-rate DFE with adaptive technique. Therefore, a better system performance is achieved by replacing the adaptive technique of the baud-rate DFE with a fractionally-spaced forward filter. This applies for both transmission rates that we have used, but we have found less variation in system performance for the 400 Kbaud case than that for the 80 Kbaud case. This could be seen in Figures 34 and 38 where we see more variation for the 80 Kbaud case, 5 dB for FSE-1 and 6 dB for FSE-2, than the 400 Kbaud case, 2 dB for FSE-1 and 4 dB for FSE-2. This result is also similar

to that of Campbell et al [2,3].

4.2 Recommendations

The different methods we have used to test a DFE showed that a FR-SP DFE gives a large SNR advantage over baud-rate DFE. If FR-SP DFE is not used because of its complexity, then a baud-rate DFE with adaptive sampling phase is recommended to use because it will take advantage of the cyclostationary statistics of the interference and gives better system performance than a baud-rate DFE with a fixed sampling phase. Conventional LMS coefficient adaptation can be used for the adaptation of the DFE. Note that cyclostationary NEXT would likely not have a major effect on adaptation properties as long as the far end signal power is well above that of the NEXT.

We also recommend that the transmitters' clocks be generated from one master clock so that they will all be transmitting at the same phase. This will cause their overall interference to be cyclostationary which helps the system performance to improve if we use an adaptive baud-rate DFE or FR-SP DFE for equalization.

Appendix A MMSE Analysis for FR-SP DFE with Cyclostationary Interference

The received signal at the input of the FR-SP DFE is:

$$r(t) = \sum_{k=-\infty}^{\infty} a_k \Phi_0(t - kT) + \sum_{l=1}^N \sum_{k=-\infty}^{\infty} b_{kl} \Phi_l(t - kT - \Omega_l) + n(t), \quad (15)$$

where a_k is the transmitted symbol, b_{kl} is the interfering symbol, and $n(t)$ is the AWGN.

The following assumptions have been made for finding the MMSE of the FR-SP DFE:

- 1) All a_k and b_{kl} are uncorrelated
- 2) $E(a_k) = 0$; $E(a_k^2) = 1$.
- 3) $E(b_{kl}) = 0$; $E(b_{kl}^2) = 1$.
- 4) There are no decision errors.

As it was shown previously, the input to the slicer will be

$$Y_n = \sum_{m=0}^{N_w} w_m r(nT - (m\Delta + \Theta)) + \sum_{m=1}^{N_f} f_m a_{n-m-D}, \quad (16)$$

where

$N_w + 1$ is the number of the forward filter's taps,

N_f is the number of the feedback filter's taps,

w_m is the setting of the m th tap of the forward filter,

f_m is the setting of the m th tap of the feedback filter,

Δ is the time between two samples,

Θ is the sampling phase,

and D is the delay in the FR-SP DFE.

This equation can be written in this form,

$$Y_n = U^T X_n, \quad (17)$$

where T is the transpose operator,

$$U^T = [w_0 \ w_1 \dots w_{N_w} \ f_1 \ f_2 \dots f_{N_f}], \quad (18)$$

and

$$X_n^T = [R_n^T \ a_{n-1-D}^T], \quad (19)$$

with

$$R_n^T = [r(nT) \ r(nT - \Delta) \ \dots \ r(nT - N_w \Delta)] \quad (20)$$

and

$$a_{n-1-D}^T = [a_{n-1-D} \ a_{n-2-D} \ \dots \ a_{n-N_f-D}] \quad (21)$$

The mean squared error (MSE) we need to minimize is

$$\begin{aligned}
 MSE &= E \left[(Y_n - a_{n-D})^2 \right] \\
 &= \mathbf{U}^T \mathbf{A} \mathbf{U} - 2 \mathbf{U}^T \mathbf{V} + 1
 \end{aligned}
 \tag{22}$$

where $\mathbf{A} = E [\mathbf{X}_n \mathbf{X}_n^T]$, and $\mathbf{V} = E [\mathbf{X}_n a_{n-D}]$.

Setting the derivative of the MSE to zero, we find the optimum weight of the forward and feedback coefficients, which is

$$\mathbf{U}_{\text{opt}} = \mathbf{A}^{-1} \mathbf{V} \tag{23}$$

Substituting this back in the equation of the MSE, we will get the minimum MSE, which is

$$\begin{aligned}
 MMSE &= 1 - \mathbf{V}^T \mathbf{A}^{-1} \mathbf{V} \\
 &= 1 - \mathbf{V}^T \mathbf{U}_{\text{opt}}
 \end{aligned}
 \tag{24}$$

The next thing to do is to find expressions for $\mathbf{V}$ and $\mathbf{A}$ in terms of the received data and the decisions of the slicer.

$$\begin{aligned}
\mathbf{V} &= E[\mathbf{X}_n a_{n-D}] \\
&= E \begin{bmatrix} r(nT) a_{n-D} \\ r(nT - \Delta) a_{n-D} \\ \cdot \\ \cdot \\ \cdot \\ r(nT - N_w \Delta) a_{n-D} \\ a_{n-1-D} a_{n-D} \\ a_{n-2-D} a_{n-D} \\ \cdot \\ \cdot \\ \cdot \\ a_{n-N_f-D} a_{n-D} \end{bmatrix} = \begin{bmatrix} \Phi_0(DT) \\ \Phi_0(DT - \Delta) \\ \cdot \\ \cdot \\ \cdot \\ \Phi_0(DT - N_w \Delta) \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad (25)
\end{aligned}$$

$$\begin{aligned}
\mathbf{A} &= E[\mathbf{X}_n \mathbf{X}_n^T] \\
&= E \left[\begin{pmatrix} \mathbf{R}_n \\ \mathbf{a}_{n-1-D} \end{pmatrix} (\mathbf{R}_n^T \quad \mathbf{a}_{n-1-D}^T) \right] \quad (26)
\end{aligned}$$

This matrix $\mathbf{A}$ could be written as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_2^T & \mathbf{I} \end{bmatrix}, \quad (27)$$

where $\mathbf{A}_1 = E(\mathbf{R}_n \mathbf{R}_n^T)$, $\mathbf{A}_2 = E(\mathbf{R}_n \mathbf{a}_{n-1-D}^T)$, and $\mathbf{I}$ is the identity matrix.

Here, we will evaluate $\mathbf{A}_1$ and $\mathbf{A}_2$.

$$\mathbf{A}_2 = E \left[\begin{pmatrix} r(nT) \\ r(nT - \Delta) \\ \vdots \\ r(nT - N_w \Delta) \end{pmatrix} (a_{n-1-D} \ a_{n-2-D} \ \cdot \ \cdot \ \cdot \ a_{n-N_f-D}) \right] \quad (28)$$

$$\mathbf{A}_2 = \begin{bmatrix} \Phi_0((D+1)T) & \Phi_0((D+2)T) & \cdot & \cdot & \cdot & \Phi_0((D+N_f)T) \\ \Phi_0((D+1)T - \Delta) & \Phi_0((D+2)T - \Delta) & \cdot & \cdot & \cdot & \Phi_0((D+N_f)T - \Delta) \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \Phi_0((D+1)T - N_w \Delta) & \Phi_0((D+2)T - N_w \Delta) & \cdot & \cdot & \cdot & \Phi_0((D+N_f)T - N_w \Delta) \end{bmatrix} \quad (29)$$

$$\mathbf{A}_1 = E \left[\begin{pmatrix} r(nT) \\ r(nT - \Delta) \\ \vdots \\ r(nT - N_w \Delta) \end{pmatrix} (r(nT) \ r(nT - \Delta) \ \cdot \ \cdot \ \cdot \ r(nT - N_w \Delta)) \right] \quad (30)$$

$$\mathbf{A}_1 = \begin{bmatrix} q(0,0) & q(0,1) & . & . & . & q(0,N_w) \\ q(0,1) & q(1,1) & . & . & . & q(1,N_w) \\ q(0,2) & q(1,2) & q(2,2) & . & . & q(2,N_w) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ q(0,N_w) & q(1,N_w) & . & . & . & q(N_w,N_w) \end{bmatrix} \quad (31)$$

where

$$\begin{aligned} q(i,j) &= E[r(nT - i\Delta)r(nT - j\Delta)] \\ &= q(j,i) \\ &= \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} E[a_{k_1}a_{k_2}] \Phi_0(nT - i\Delta - k_1T) \Phi_0(nT - j\Delta - k_2T) \\ &\quad + \sum_{l_1} \sum_{l_2} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} E[b_{l_1k_1}b_{l_2k_2}] \Phi_{l_1}(nT - i\Delta - k_1T - \Omega_{l_1}) \\ &\quad \times \Phi_{l_2}(nT - j\Delta - k_2T - \Omega_{l_2}) + E[n(nT - i\Delta)n(nT - j\Delta)] \end{aligned} \quad (32)$$

Here, we used the assumption that $E[a_k b_{lk}] = 0$. We have also assumed that $E[a_k^2] = E[b_{lk}^2] = 1$; therefore,

$$\begin{aligned} q(i,j) &= \sum_{k=-\infty}^{\infty} \Phi_0(kT - i\Delta) \Phi_0(kT - j\Delta) \\ &\quad + \sum_l \sum_{k=-\infty}^{\infty} \Phi_l(kT - i\Delta - \Omega_l) \Phi_l(kT - j\Delta - \Omega_l) \\ &\quad + R((i-j)\Delta) \end{aligned} \quad (33)$$

where $R(t)$ is the autocorrelation function of the power spectral density of AWGN at the output of the receiver filter.

Appendix B MMSE Analysis for FR-SP DFE with Stationary Interference

In this analysis, we assume that we have stationary interference, but it has the same power spectrum as the cyclostationary interference. Looking at the analysis for the cyclostationary interference, we find that this interference will only affect the $q(i, j)$ terms of the A_1 matrix. Therefore, we will only find the $q(i, j)$ terms for stationary interference. The remaining equations will not be derived since they are the same as the ones for the cyclostationary case except replacing the $q(i, j)$ terms of the cyclostationary case with that of the stationary case.

The interference is

$$I(t) = \sum_l \sum_{k=-\infty}^{\infty} b_{lk} \Phi_l(t - kT) \quad (34)$$

Then we will find the autocorrelation function of the interference, call it $A(T)$, averaged over T . Averaging over the symbol interval T essentially eliminates the cyclostationary behaviour of the interference. The autocorrelation function is

equal to

$$\begin{aligned}
 A(T) &= \frac{1}{T} \int_0^T \sum_l \sum_{k=-\infty}^{\infty} \Phi_l(t - kT) \Phi_l(t - T - kT) dt \\
 &= \frac{1}{T} \sum_l \sum_{k=-\infty}^{\infty} \int_0^T \Phi_l(t - kT) \Phi_l(t - T - kT) dt \\
 &= \frac{1}{T} \sum_l \int_{-\infty}^{\infty} \Phi_l(t) \Phi_l(t - T) dt
 \end{aligned} \tag{35}$$

Therefore the $q(i, j)$ of the stationary case will be:

$$\begin{aligned}
 q(i, j) &= \sum_{k=-\infty}^{\infty} \Phi_0(kT - i\Delta) \Phi_0(kT - j\Delta) \\
 &\quad + \frac{1}{T} \sum_l \int_{-\infty}^{\infty} \Phi_l(t) \Phi_l(t - (i - j)\Delta) \\
 &\quad + R((i - j)\Delta)
 \end{aligned} \tag{36}$$

Appendix C Basic Rate(80 Kbaud) Data

We chose $N_0 = 1 \times 10^{-6}$ for the AWGN, and the maximum amplitude of NEXT to be 1/50 of the maximum amplitude of the channel response. The channel and NEXT impulse responses, filtered by the transmit and receive butterworth filters with a cutoff frequency at 40 Khz, are shown in figures 41 and 42. The autocorrelation function of AWGN is shown in figure 43.

Figure 41 Impulse response of transmitted data(80 Kbaud)

Figure 42 Impulse response of NEXT(80 Kbaud)

Figure 43 Autocorrelation function of AWGN(80 Kbaud)

Appendix D High rate(400 kbaud) data

We chose $N_0 = 1 \times 10^{-6}$ for the AWGN, and the maximum amplitude of NEXT to be 1/50 of the maximum amplitude of the channel response. The channel and NEXT impulse responses, filtered by the transmit and receive butterworth filters with a cutoff frequency at 200 Khz, are shown in figures 44 and 45. The autocorrelation function of AWGN is shown in figure 46.

Figure 44 Impulse response of transmitted data(400 Kbaud)

Figure 45 Impulse response of NEXT(400 Kbaud)

Figure 46 Autocorrelation function of AWGN(400 Kbaud)

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Bibliography

1. D. G. Messerschmitt, "Design Issues in the ISDN U-Interface Transceiver," *IEEE J. SAC.*, VOL. SAC-4, NO.8, November 1986.
2. J. C. Campbell, A. J. Gibbs, and B. M. Smith, "The Cyclostationary Nature of Crosstalk Interference from Digital Signals in Multipair Cable-Part I: Fundamentals," *IEEE Trans. Commun.*, Vol. COM-31, NO. 5, May 1983.
3. J. C. Campbell, A. J. Gibbs, and B. M. Smith, "The Cyclostationary Nature of Crosstalk Interference from Digital Signals in Multipair Cable-Part II: Applications and Further Results," *IEEE Trans. Commun.*, Vol. COM-31, NO. 5, May 1983.
4. A. Fung, L. S. Lee and D. D. Falconer, "A Facility for Near End Crosstalk Measurements on ISDN Subscriber Loops". Submitted to Globecom '89.
5. B. R. Petersen and D. D. Falconer, "Linear Equalization in Cyclostationary Crosstalk". Submitted to Globecom '89.
6. Vilas Joshi, "Sequence Estimation Techniques for Enhanced Digital Subscriber Loop Transmission Capability". Ph.D. thesis, Carleton university, June 1988.
7. J. W. Lechleider, "Loop Transmission Aspects of ISDN Basic Access," *IEEE J. SAC.*, VOL. SAC-4, NO.8, November 1986.
8. J. Salz, "On Mean-Square Decision Feedback Equalization and Timing Phase," *IEEE Trans. Commun.*, Vol. COM-25, NO. 12, December 1977.
9. S. U. Qureshi, "Adaptive Equalization," *Proceedings of the IEEE*, Vol. 73, No. 9, September 1985.

10. S. V. Ahamed, P.P. Bohn, and N. L. Gottfried, "A Tutorial on Two-Wire Digital Transmission in the Loop Plant," *IEEE Trans. Commun.*, Vol. COM-29, November 1981.
11. David W. Lin, "Minimum Mean-Squared Error Decision-Feedback Equalization for Digital Subscriber Line Transmission: with Possibly Correlated Line Codes,". Submitted to *IEEE Trans. Commun.* in March, 1988.

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