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**Finite Difference Model of A Spiral Ground
Heat Exchanger for Ground-Source Heat Pumps**

by

Andrew Morrison

B.A.Sc. In Mechanical Engineering, University of Waterloo

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering

**Department of Mechanical and Aerospace Engineering
Ottawa-Carleton Institute
for Mechanical and Aerospace Engineering**

Carleton University
Ottawa, Ontario
January, 1999

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**Finite Difference Model of A Spiral Ground
Heat Exchanger for Ground-Source Heat Pumps**

submitted by

Andrew Morrison, B.A.Sc., P.Eng.

In partial fulfilment of the requirements for
the degree of Master of Engineering

Chair, Department of Mechanical and Aerospace Engineering

Thesis Supervisor

Carleton University

December, 1998

Summary

Closed loop ground-source heat pumps use electrical energy to drive a refrigeration cycle which extracts heat from the ground during the heating season and rejects heat from the conditioned space during the cooling season. On a seasonal basis, residential ground-source heat pump (GSHP) systems provide three to four times the energy, in the form of heat, to the space as it requires to drive the compressor and fans. This can represent a significant energy reduction over conventional space heating systems.

A horizontal heat exchanger design called the spiral, or slinky, heat exchanger developed at Oklahoma State University optimizes the soil contact area with respect to the trench length. The typical slinky heat exchanger is constructed from 3/4 inch (1.88 cm) polyethylene pipes. Instead of positioning the pipe in parallel straight lengths, separated by 0.6 to 0.9 metres, as is done for conventional horizontal ground heat exchangers, the slinky heat exchanger is constructed from overlapping spirals of pipe as seen in Figure 2.1. The slinky coil is fabricated off site in a rig then backfilled with sand or excavated soil at the site. Because of the increased contact area with the soil, less trench length is required, reducing the heat exchanger installation costs.

The dense packing of the ground heat exchanger piping makes modelling this configuration more difficult than the conventional parallel pipe horizontal ground heat exchanger. As yet, no simulation tools have been developed to model a spiral heat exchanger.

A finite difference program was developed to model a slinky ground heat exchanger for a residential ground-source heat pump system. Slinky ground heat exchangers are constructed from overlapping spirals of pipe layed flat in the bottom of a trench. To simplify the model, the heat exchanger was simulated as an equivalent length of equally spaced straight pipes. A simulation of an existing residential GSHP system located near Eau Claire, Wisconsin was performed from the beginning of December to the end of March. The results of the simulation were compared with monitored entering water temperatures and undisturbed ground temperatures from the ground-source heat pump system in Wisconsin.

The finite difference predictions of the heat pump entering water temperature (EWT) were compared with the monitored EWTs. There was extremely good agreement between the average EWTs over the simulation periods. The finite difference program slightly under predicted the magnitude of short term fluctuations in the heat pump EWT.

Additional work is required to include summer operation in the heat pump simulation. To simulate the heat transfer during the summer, moisture migration away from the pipes needs to be included into the ground heat transfer model. An adjustment is required to modify the monitored heat pump loads for variations in the predicted EWT if further parametric studies are to be done. Further validation should also be considered to improve the prediction of the heat transfer around the pipes.

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Nomenclature

a	Coefficient in Equation 3.22
$A_{i,j}$	Coefficient for temperature in current control volume - Equation 3.13 (W/m °C)
$A_{i-1,j}, A_{i+1,j}$	Coefficient for temperature in adjacent X-direction control volumes - Equation 3.13 (W/m °C)
$A_{i,j-1}, A_{i,j+1}$	Coefficient for temperature in adjacent Y-direction control volumes - Equation 3.13 (W/m °C)
$A_{i,j}^o$	Coefficient for temperature from previous time step - Equation 3.13 (W/m °C)
A_s	Surface temperature amplitude (°C)
b (Eqn 3.13)	Coefficient in Equation 3.13 (W/m)
b (eqn 3.22)	Coefficient in Equation 3.22
BC_x	Size of variable grid domain in X-direction (metres)
C_{fusion}	Heat of fusion of water per unit mass (kJ/kg)
C_p	Heat capacity (J/kg°C)
$C_{p,\text{ground}}, C_{p,\text{soil}}$	Heat capacity of ground/soil in control volume (J/kg°C)
depth	Depth below ground surface (metres)
$D_{i,j}$	Coefficient in Equation 3.14 incorporating known terms
D_{snow}	Depth of snow cover (mm)
EWT_{mon}	Monitored heat pump entering water temperature (°C)
EWT_{Pred}	Predicted heat pump entering water temperature (°C)
g	Gravitational constant (m/s ²)
h_{air}	Convection heat transfer coefficient between air and ground surface (W/m ² °C)
h_{fluid}	Convection heat transfer coefficient between fluid and inside pipe (W/m ² °C)
I_{solar}	Incident solar radiation on ground surface (W/m ²)
k_e	Weighted thermal conductivity between control volumes i and $i+1$ (W/m°C)
k_w	Weighted thermal conductivity between control volumes i and $i-1$ (W/m°C)
k_n	Weighted thermal conductivity between control volumes j and $j+1$ (W/m°C)
k_s	Weighted thermal conductivity between control volumes j and $j-1$ (W/m°C)
k_{fluid}	Thermal conductivity of fluid inside pipe (W/m°C)

Nomenclature (Cont'd)

k_{fill}	Thermal conductivity of gap between pipe and surrounding soil (W/m°C)
k_{freeze}	Thermal conductivity of frozen soil (W/m°C)
$k_{\text{ground}}, K_{\text{soil}}$	Thermal conductivity of ground/soil (W/m°C)
k_{freeze}	Thermal conductivity of ice (W/m°C)
k_{pipe}	Thermal conductivity of heat exchanger pipe (W/m°C)
L (eqn 3.34)	Characteristic length of pipe cross sectional area (metres)
ΔL	length of pipe control volume (metres)
m	Mass of fluid in pipe control volume (kg)
$\dot{m}$	Mass flow rate through the pipe control volume (kg/s)
MC	Mass fraction of water in control volume
Nu_{ID}	Nusselt number for flow inside pipe
NV_x	Number of control volumes in variable grid spacing region, X-direction
OD	Outer diameter of heat exchanger pipe (metres)
p	Non-dimensional distance from pipe centre relative to pipe radius
P_j	Coefficient in Equation 3.16
Pr	Prandlt number for fluid inside pipe
$Q_{\text{extracted}}$	Energy extracted from loop by heat pump (MJ)
Q_{fusion}	Energy required to completely freeze ground control volume (kJ)
Q_j	Coefficient in Equation 3.16
Q_{pipe}	Heat flux through the pipe wall (W)
Q_{sensible}	Energy associated with control volume's temperature rise above (or drop below) freezing - 0°C (kJ)
Q_{sol}	incident solar radiation on the ground (W/m ²)
Q_{sur}	Surface heat flux into the ground (W/m ²)
r_i	Inside pipe radius (metres)
r_o	Outside pipe radius (metres)
R_{source}	Thermal resistance between fluid in the pipe and the ground (°C/W)
R_{fluid}	Thermal resistance between fluid and pipe (°C/W)
R_{pipe}	Thermal resistance of pipe wall (°C/W)
R_{pipe} (eqn 5.2)	Outside pipe radius (metres)
R_{contact}, R_c	Contact resistance between pipe and surrounding soil (°C/W)

Nomenclature (Cont'd)

Ra_L	Raleigh number for natural convection inside pipe - no flow
Re_{ID}	Reynold's number for flow inside pipe
RS	Root square temperature difference ($^{\circ}C$)
S	Linearized volumetric source term (W/m^3)
Sc	Term in Linearized volumetric source term representing (W/m^3)
Sp	Term in Linearized volumetric source term representing ($W/m^3\ ^{\circ}C$)
Δt	Time increment (s)
t	Time (s)
t (eqn 3.36)	Thcikness of gap between pipe ans surrounding soil (metres)
t_{days}	Day for which ground temperature is being evaluated (from January 1 st)
t_o	Day from January 1 st at which minimum surface temperature occurs (days)
t_{run}	Run time of heat pump as percentage of total hours in a period (%)
T	Temperature ($^{\circ}C$)
T_{air}, T_{db}	Ambient air temperature or dry bulb temperature ($^{\circ}C$)
$T_{depth,time}$	Ground temperature predicted by Kusuda and Auchenbach (eqn 3.22) ($^{\circ}C$)
T_{fluid}	Temperature of fluid in pipe ($^{\circ}C$)
$T_{i,j,k}$	3D ground control volume temperature ($^{\circ}C$)
T_{in}	Temperature fluid entering pipe control volume ($^{\circ}C$)
T_{out}	Temperature fluid exiting pipe control volume ($^{\circ}C$)
T_i^n	Temperature of fluid in current control volume at previous time step ($^{\circ}C$)
T_i^{n+1}	Temperature of fluid in current control volume at current time step ($^{\circ}C$)
T_{i-1}^n	Temperature of fluid in previous control volume at previous time step ($^{\circ}C$)
T_{i+1}^n	Temperature of fluid in next control volume at previous time step ($^{\circ}C$)
T_{ground}	Temperature of ground ($^{\circ}C$)
$T_{ground, mon}$	Monitored ground temperature ($^{\circ}C$)
$T_{ground, pred}$	Temperature of ground ($^{\circ}C$)
T_{mean}	Mean ground temperature ($^{\circ}C$)
$T_{modified}$	Modified temperature of control volume by freezing algorithm ($^{\circ}C$)
ΔT	Difference between bottom and top inside pipe surface temperature ($^{\circ}C$)
ΔT (eqn 3.38)	Temperature of control volume above or below freezing ($^{\circ}C$)
V_{fluid}	Velocity of fluid in pipe (m/s)

Nomenclature (Cont'd)

V_{wind}	Wind speed (m/s)
x	Position in X-direction (metres)
X_1, X_2	X-location of sides of control volume for variable grid spacing (metres)
ΔX	Length of ground control volume in X-direction (metres)
y	Position in Y-direction (metres)
ΔY	Length of ground control volume in Y-direction (metres)
z	Non-dimensional variable used in Equation 5.2
ΔZ	Length of ground control volume in Z-direction (metres)
α	Thermal diffusivity of soil (m ² /s)
α_{fluid}	Thermal diffusivity of fluid (m ² /s)
α_{sur}	Solar absorptivity of ground surface
β	Expansion coefficient of fluid (1/°C)
β_x	Clustering parameter in X-direction for variable grid spacing
η_x	Transformation metric for variable grid spacing
ρ	Density (kg/m ³)
$\rho_{\text{ground}}, \rho_{\text{soil}}$	Density of ground/soil (kg/m ³)
μ	Dynamic viscosity of fluid inside pipe (kg/m s)
ν	Kinematic viscosity of fluid inside pipe (m ² /s)

Chapter 1 Introduction

Closed loop ground-source heat pumps use electrical energy to drive a refrigeration cycle which extracts heat from the ground during the heating season and rejects heat from the conditioned space during the cooling season. On a seasonal basis, residential ground-source heat pump (GSHP) systems provide three to four times the energy, in the form of heat, to the space as it requires to drive the compressor and fans. This can represent a significant energy reduction over conventional space heating systems.

The typical ground-source heat pump extracts heat from the ground using a heat exchanger constructed from 3/4 inch to 1-1/4 inch polyethylene pipe buried in the ground. Water is circulated through the polyethylene pipe as the heat transfer fluid. In cold climates where the heat exchanger fluid is near or below the freezing temperature of water, a water/anti-freeze solution such as ethanol, methanol or propylene glycol is used to prevent the fluid from freezing. A coaxial refrigerant to water heat exchanger, usually constructed from copper and steel piping, transfers the heat between the heat pump refrigeration cycle and the ground heat exchanger fluid. The ground pipes are either buried horizontally in trenches 1.5 to 1.8 metres (five to six feet) below the surface or vertically in drilled wells. The ground heat exchanger represents a substantial portion of the cost for a closed loop GSHP system. Reducing the cost of ground heat exchanger is essential if the economics of these systems is to be improved. As a result, simulation tools for ground heat exchangers have become extremely popular in the design of these systems to ensure accurate sizing.

1.1 Identification of Problem

A horizontal heat exchanger design called the spiral, or slinky, heat exchanger developed at Oklahoma State University optimizes the soil contact area with respect to the trench length. The typical slinky heat exchanger is constructed from 3/4 inch (1.88 cm) polyethylene pipes. Instead of positioning the pipe in parallel straight lengths, separated by 0.6 to 0.9 metres, as is done for conventional horizontal ground heat exchangers, the slinky heat exchanger is constructed from overlapping spirals of pipe as seen in Figure 2.1. The slinky coil is fabricated off site in a rig then backfilled with sand or excavated soil at the site. Because of the increased contact area with the soil, less trench length is required, reducing the heat exchanger installation costs.

The dense packing of the ground heat exchanger piping makes modelling this configuration more difficult than the conventional parallel pipe horizontal ground heat exchanger. As yet, no simulation tools have been developed to model a spiral heat exchanger.

1.2 Research Objective

This thesis will develop and validate a finite difference model of a spiral heat exchanger represented by an equivalent length of straight horizontal pipes. The model will predict the heat pump entering water temperature and the soil temperature in the surrounding soil. The resulting program will also be used to examine the impact of heat pump loads on fluid

temperatures leaving the ground heat exchangers. The heat pump unit itself was not examined in this analysis since sufficient programs are currently available to calculate the load on the heat pump and the energy it extracts from the ground heat exchanger.

The finite difference model could be used as a tool to determine optimum spiral/slinky heat exchanger configuration and develop design guidelines for contractors and engineers.

1.3 Approach

The work to develop and implement a numerical model of a spiral ground-source heat pump ground heat exchanger was accomplished by:

- 1) The development of finite difference equations from first principals to evaluate the heat transfer in the ground.
- 2) Development of a FORTRAN program to implement the numerical algorithms.
- 3) Validation of the computer program using monitored data and transient heat transfer equations
- 5) A parametric study of the heat pump loads using the finite difference program.

Chapter 2 Background

2.1 Wisconsin Monitoring Site

The ground-source heat pump (GSHP) site which will be used to validate the finite difference model was one of eight ground-source heat pump systems monitored as part of the Enhanced Heat Pumps Applications project. The monitoring study was funded by the Energy Center of Wisconsin to promote technical and market improvements of GSHP systems in the state ^[1].

The ground heat exchanger at this site is a slinky layed flat on the bottom of two, 2.1 metre (7 foot) wide trenches each 30.5 metres (100 ft) long. The slinky coil is 1.05 m (3.4 feet) in diameter and buried 2.1 metres (6.9 feet) below the surface. As seen in Figure 2.1, each of the trenches has two slinky coils layed side by side. Because of the limited property size, the two trenches were angled as seen in Figure 3.2.

A single speed pump is used to circulate 0.68 l/s of 20 percent methanol solution through the heat pump and the slinky heat exchanger. The methanol solution is circulated only when the heat pump is operating. The fluid leaving the heat pump is divided into four parallel flows by a manifold located in the basement of the house.

A propane furnace was installed to provide auxiliary heating, and the heat pump was

equipped with a radio controlled over-ride . This was a requirement of the project sponsor, so the heat pump could be shut down during the electric utility's peak load periods, which rarely occurred. Approximately three percent of the total annual heating was provided by the furnace ^[1].

The environmental parameters monitored at this site included; dry-bulb temperature and undisturbed ground temperature at 0.9 and 1.8 metres. The system parameters that were monitored were the heat pump/circulation pump run time, heat pump entering water temperature, and heat pump leaving water temperature. The energy rejected and extracted by the heat pump was calculated by the monitoring system using the fluid temperatures entering and leaving the heat pump and the circulation pump flow rate. The monitoring system recorded the environmental and system parameters at fifteen minute intervals.

Other weather information such as solar radiation, wind speed and snow depth were obtained from hourly typical meteorological year (TMY) weather data ^[2] based on the years 1960-1990.

2.2 Literature Search

This section will review earlier work involving finite difference computer models for a variety of ground-source heat pump heat exchanger designs. The numerical models of ground-source heat pump systems have progressed since the early eighties as computer

power and heat exchanger designs have progressed.

Andrews and Metz ^[3] and Metz ^[4] developed a finite difference analysis to study a ground storage tank and a horizontal ground heat exchanger coupled to a solar assisted heat pump system. The program was incorporated into the transient building simulation program TRNSYS, which is used to model the mechanical system in the house. The boundary conditions in the soil and at the surface are based on measured undisturbed soil temperatures at various depths and times of year. Due to computer constraints at the time this work was performed, the finite difference grid spacing is relatively large. The storage tank was modelled as a well mixed tank and freezing of the soil is not modelled. The horizontal heat exchanger field was modelled as a thin plate with a surface area equal to the heat exchanger field. To account for the smaller pipe surface area of the heat exchanger, an adjustment factor was added for the reduced surface area of the actual heat exchanger system. The model was validated against monitored data of a solar assisted ground-source heat pump system located at the Brookhaven National Labs in New York State.

Mei and Emerson ^[5] and Mei ^[6] developed a finite difference model to analyse two dimensional heat transfer in cylindrical coordinates from a single pipe horizontal ground heat exchanger. The numerical analysis in the soil was performed from the pipe's outer radius to the far field radius which is the distance at which the soil temperature is unaffected by the ground heat exchanger. This model accounted for the impact of ice formation around the buried pipe during the heating season, but did not account for the impact of surface effects,

contact resistance, moisture migration or backfill material. The boundary conditions at the far field radius are the soil's undisturbed temperature as a function of time and depth. The far field radius was assumed to be 1.22 metres (4 feet). The program was validated using experimental results taken from a residential test site located in Upton, New York. A parametric study was also performed to analyse the impact of soil properties (conductivity, etc.), heat exchanger fluid inlet temperature, pipe material and dimensions, and heat pump cyclic operation schedule.

Mei ^[6,7] developed two separate finite difference models to analyse two dimensional heat transfer in cylindrical coordinates for a single pipe horizontal ground heat exchanger with trench backfill thermal properties different from those of the native soil, and a two pipe (over/under) horizontal ground heat exchanger. The numerical analysis in the soil was performed from the pipe's outer radius to the far field radius. In the case of the two pipe model, the cylindrical coordinates were centred around the lower of the two pipes. Unlike the earlier single pipe horizontal model developed by Mei and Emerson, neither of these models accounted for ice formation around the pipe during the heating season. The models also did not account for surface effects, contact resistance or moisture migration, and the two pipe model did not account for different backfill materials. The boundary conditions at the far field radius are the soil's undisturbed temperature estimated by the Kusuda and Achenbach equation for both depth and time of year. The far field radius was assumed to be 1.85 metres (6 feet). The program was validated using experimental results taken from a test site located at the University of Tennessee. A parametric study was also performed to

analyse the impact of soil properties, heat exchanger fluid inlet temperature, pipe material and dimensions, and heat pump cyclic operation.

Safemazandarani, Edwards, Johnson and Mohammad-zadeh ^[8] developed a mathematical model of a direct expansion ground heat exchanger system. A direct-expansion system is similar to conventional ground heat exchangers, except that instead of circulating water or an anti-freeze solution such as ethanol through the ground loop, the heat pump's refrigerant is circulated directly. In the case of this system being modelled, the heat exchanger pipes were enclosed in a metal culvert filled with water. This model was based on two dimensional transient heat transfer in Cartesian coordinates. The boundary conditions for the model are zero heat transfer at the bottom and horizontal edge of the analysis region. The heat transfer at the surface is set equal to the incident solar radiation, convection to the air and net radiation between the ground and the sky. The heat transfer of the refrigerant in the heat exchanger pipes was predicted using two-phase equations. The model was validated with experimental data from a direct expansion system.

Couvillion and Cotton ^[9] developed a finite element model to analyse the impact of different backfill materials on the performance of horizontal ground heat exchangers. The numerical model was based on two dimensional heat and mass transfer in Cartesian coordinates with a constant line source used to represent the ground heat exchanger. The boundary conditions in the ground assume zero heat transfer at the edge of the analysis region and a defined temperature at the ground surface which varies with the time of year. The initial moisture

content of the soil was based on measured values. The model was validated with laboratory experiments where the heat flux from the pipe was known and the moisture interaction at the surface could be eliminated. The soil temperature was measured around the heat exchanger pipe. The soil moisture levels were only measured at the beginning of the experiment. Without the ability to control the moisture transfer at the ground surface and initial moisture levels it would be extremely difficult to properly model moisture migration around a ground heat exchanger.

Mukerji , Murphy and Tagavi ^[10] performed a numerical analysis of a slinky ground heat exchanger (cylindrical spiral flattened into a single plane). The slinky heat exchanger was idealized in this study as a flat plate buried vertically in a two dimensional semi infinite domain. The model accounts for the impact of the ambient air temperature at the ground surface. The numerical evaluation of the ground heat transfer was carried out to a depth equal to five times the mean burial depth of the heat exchanger and with a width (perpendicular to the heat exchanger trench) of seven times the mean burial depth. The boundary conditions at the edge and the bottom of the analysis region are given by the undisturbed ground temperature estimated by Kusuda and Auchenbach ^[18]. No validation was performed with monitored data, however, the model was checked using a grid function convergence test.

Mukerji , Tagavi and Murphy ^[11] developed a steady state model of a slinky ground heat exchanger based on a line source approximation and the principal of superposition. It was

determined in this study that the grid requirements of a numerical solution of this model would make a numerical solution of this heat exchanger extremely difficult. In this analysis, the heat exchanger was broken into infinitely small elements. Each element was treated as a point source in a semi-infinite medium. The temperature distribution around the pipe was determined by integrating the impact of the point sources. An analysis was performed for both a conventional single horizontal straight pipe heat exchanger and a slinky heat exchanger. The results of the slinky analysis were presented in the form of the ratio for the steady state heat transfer of a slinky heat exchanger relative to the steady state performance of a single horizontal pipe. Although these results would give an approximation of slinky performance under operating conditions, it cannot account for the transient effects the soil between pipes will experience during the on/off cycles of a ground-source heat pump.

Tarnawski and Leong ^[12,13] developed a two dimensional finite element model for horizontal ground heat exchangers. This model was based on the equations governing heat and mass flow in soils developed by Philip and deVries. Freezing and thawing of the soil moisture was modelled on a volume by volume basis as an isothermal process. Climatological data such as ambient temperature, solar radiation, wind velocity, rainfall, and snow were used to simulate the ground surface conditions, but the impact of the heat exchanger on the ground surface was not taken into account. The heat exchanger fluid is coupled to the soil heat transfer equation through the known heat flux boundary condition at the pipe/soil interface. The pipe/soil heat flux is calculated from the temperature difference and the thermal resistance between the heat exchanger fluid and the soil. The thermal resistance is the sum

of the internal pipe convective resistance and the conduction resistance of the pipe wall. The contact resistance between the pipe and the soil is neglected in this model. The numerical model was developed into a complete GSHP system analysis package called GHeads.

In 1995, Dobson, O'Neal and Aldred ^[14] developed a model to simulate vertical U-tube ground heat exchangers. The numerical model utilized a constant heat flux cylindrical source in an infinite medium to model the soil surrounding the heat exchanger pipe wall. The pipe was discretized into smaller elements to allow an accurate assumption of constant heat flux along its length. A fully implicit finite difference method was used to solve for the fluid temperatures inside the pipe.

Sun and Murphy ^[15] utilized a numerical model to analyse a Geo-Bag heat exchanger. Instead of plastic pipe, this heat exchanger is constructed from polyethylene sheets. The bag heat exchanger was modelled as a flat plate in a semi-infinite medium. A packaged finite element program was used to solve the two dimensional numerical equations. The thermal and physical properties of the soil were assumed to be homogeneous. The temperature at the heat exchanger/soil interface was assumed to be the fluid temperature in the heat exchanger. Convective heat transfer was assumed at the ground surface, and the boundary conditions in the soil were the undisturbed ground temperature calculated by analytical equations. The mass transfer in the soil was accounted for by modifying the soil properties at each time step to account for the impact of moisture migration. The results of the numerical model were compared to monitored data of a GeoBag heat exchanger. A parametric study was also

performed to evaluate the impact of the heat exchanger design and external conditions on the heat exchanger performance.

Piechowski ^[16] developed a finite difference model for a two pipe horizontal heat exchanger which includes soil moisture migration around the pipes. The model is based on the work by Mei ^[6,7] which was discussed earlier. The soil mass transfer was based on algorithms developed by Philip and deVries. Unlike Mei's model, Cartesian coordinates are used to simulate the heat conduction in the soil. The ground surface is simulated by assuming convection heat transfer between the ambient air and the ground. The soil/pipe interface temperature is calculated from the average temperature in the control volume. The heat flux boundary condition at the pipe/soil interface is calculated from the temperature difference between the fluid and the soil. To minimize computational time, the moisture diffusion equations are limited to a radial region around the pipes equal to half the control volume dimensions where the moisture and temperature gradients are assumed to be largest. The moisture content of the soil outside this region was assumed to be constant.

Figure 2.1: Installation of Slinky Ground Heat Exchanger

Figure 2.2: Layout of Slinky Ground Heat Exchanger

Chapter 3 Model Development

This section discusses the equations developed to model the transient heat transfer inside the pipes and in the surrounding soil. The heat exchanger pipes are subdivided into equally sized control volumes along their length. The fluid temperatures are calculated at each node from finite difference equations using the ground temperature calculated at the previous time step.

The ground heat transfer is modelled as two-dimensional conduction in Cartesian coordinates. The heat transfer from the pipes is treated as a volumetric source term. The dimensions of the control volumes around the pipes are constant. Variable control volume dimensions are used in the ground away from the pipes. A separate algorithm is used to model the freezing and thawing of the moisture in the soil.

3.1 Horizontal Spiral Heat Exchanger

To model a spiral heat exchanger exactly as it is constructed would require simulation of three dimensional heat transfer around the pipes from which it is constructed. Due to the geometry of the spiral heat exchanger, the distance between the pipes will vary along its length, resulting in conduction along the axial direction. The computational time required to simulate this system would be excessive.

To simplify the heat transfer equations, the spiral pipe configuration was treated as a series

of equally spaced straight pipes located in the same trench as the spiral heat exchanger. The number of equivalent straight pipes is based on the amount of pipe contained in the spiral heat exchanger being modelled. This ensures an equivalent pipe to trench ratio between the model and the spiral heat exchanger. The heat transfer becomes two dimensional without axial conduction. The simulation of the two dimensional heat conduction equations will significantly reduce the computational time of the program.

Replacing a spiral pipe configuration with straight pipes for modelling purposes will result in some inaccuracies due to a constant pipe spacing assumption as opposed to the variable spacing which occurs in a slinky configuration. Although the inaccuracies were not quantified, because the same pipe length is used in the simulation the pipe/soil surface area is consistent between the simulated and actual heat exchangers. The trench length and width are also consistent. This maintains an equivalent mass of soil in close proximity to the pipes. For these reasons it is not expected that the error of assuming parallel pipes will be significant.

3.2 Pipe Heat Transfer Equations

The temperature of the heat transfer fluid in the pipe increases as it moves along the length of the ground heat exchanger during winter operation. Heat from the warmer ground away from the pipes is transferred to the surrounding soil and then into the heat exchanger fluid through the pipe walls. Figure 3.1 illustrates the subdivision of the pipe into equally sized

control volumes and the energy balance for an individual control volume. The temperature drop across each pipe control volume (ΔL) will depend on the heat flux leaving the fluid through the pipe wall, expressed as Q_{pipe} . This value will be negative during winter when the soil is warmer than the fluid in the pipe.

Applying the law of conservation of energy to an individual control volume as shown in Figure 3.1 yields the partial differential equation

$$m C_{p,fluid} \frac{\partial T}{\partial t} = \dot{m} C_{p,fluid} (T_{in} - T_{out}) - Q_{pipe} . \quad (3.1)$$

Equation 3.1 states that the rate of change in the energy of the pipe control volume fluid is equal to the net rate of energy entering the control volume by advection less the heat transferred to the ground through the heat exchanger walls. Converting this partial differential equation to a discretization equation gives,

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{\dot{m}}{m} (T_{in} - T_{out}) - \frac{Q_{pipe}}{m C_{p,fluid}} . \quad (3.2)$$

The control volume cells are assumed to be well mixed, and flow leaving is assumed to be at the control volume temperature. The inlet and outlet fluid temperature boundary conditions and the heat loss through the heat exchanger pipe walls are defined by,

$$T_{in} = T_{i-1}^n, \quad T_{out} = T_i^n, \quad (3.3a)$$

$$Q_{pipe} = \frac{T_i^n - T_{ground}}{R_{source}}. \quad (3.3b)$$

Where R_{source} represents the total thermal resistance between the heat exchanger fluid in the pipe and the surrounding soil. It includes the internal convection resistance, the pipe conduction resistance and the contact resistance between the pipe and the soil. A more detailed description is given in Section 3.5. Equations 3.3a and 3.3b state that the temperature of fluid entering the control volume is assumed to be the fluid temperature in the upstream control volume, and that the temperature of the fluid leaving the control volume is the control volume fluid temperature. Substituting for the inlet and outlet temperatures and the heat loss through the pipe the differential equation becomes,

$$T_i^{n+1} = T_i^n + \frac{\dot{m}}{m}(T_{i-1}^n - T_i^n)\Delta t - \frac{(T_i^n - T_{ground})}{m C_p R_{source}} \Delta t. \quad (3.4)$$

Re-arranging the equation and substituting for the ground temperature gives,

$$T_i^{n+1} = (1 - \frac{\dot{m}}{m}\Delta t - \frac{\Delta t}{m C_p R_{source}})T_i^n + (\frac{\dot{m}}{m}\Delta t)T_{i-1}^n + \frac{\Delta t}{m C_p R_{source}}T_{ground}(ij). \quad (3.5)$$

The temperature for the ground adjacent to the heat exchanger pipe is based on the ground temperatures from the previous time step. This does create some error in the procedure due to the time lag, however, ground temperatures tend to change slowly and there is not expected to be a large change in the ground temperature between time steps.

Because the inlet and outlet control volume temperatures are defined by fluid temperatures at the previous time step, the finite differential equation becomes fully explicit. The explicit discretization equation is conditionally stable and a limit is required on the length of the maximum allowable time step. The stability criteria for the explicit finite difference equation for the heat exchanger fluid temperature is given by equation 3.6.

$$\frac{\Delta t}{m} \left(\dot{m} + \frac{1}{C_p R_{source}} \right) \leq 1. \quad (3.6)$$

Re-arranging equation 3.6 results in

$$\Delta t \leq \frac{m}{\left(\dot{m} + \frac{1}{C_p R_{source}} \right)}. \quad (3.7)$$

During no flow periods, the mass flow will become zero and the maximum time step (Δt) will become large since $m \gg 1/(C_p \cdot R_{source})$ for a typical heat exchanger design at no flow conditions. The maximum Δt from equation 3.7 will always be larger than the no flow time steps used for the explicit analysis in the finite difference model, ensuring stability. For the heat exchanger simulation outlined in Table 4.2, the maximum allowable no flow time step is approximately 30 minutes.

During explicit time periods with flow, the maximum time step to ensure stability will be dependant on the thermal resistance (R_{source}) between the fluid in the pipe and the ground. As the resistance becomes infinite and there is no longer heat transfer between the fluid and

the ground the maximum time step simplifies into

$$\Delta t \leq \frac{m}{\dot{m}} = \frac{\Delta L}{V_{fluid}}. \quad (3.8)$$

This states that the maximum time step to ensure stability must be less than or equal to the time for the fluid to completely pass through the pipe control volume. Since R_{source} is a finite value, the maximum allowable time step of the explicit equation for the pipe fluid temperature will be smaller. However, since

$$\dot{m} \gg \frac{1}{C_p R_{source}}, \quad (3.9)$$

for typical ground-source heat pump applications, the maximum time step (Δt) will only be slightly less than the limit given in equation 3.8. For the purpose of the implementation of this model, the maximum allowable time step (Δt) for periods with flow is,

$$\Delta t \leq 0.95 \frac{\Delta L}{V_{fluid}}. \quad (3.10)$$

3.3 Ground Heat Conduction Equations

A finite difference approach was selected to model the ground conduction in order to predict the interaction between the heat exchanger pipes. Due to the small temperature gradient along the length of the pipes, axial conduction in the ground was neglected, thereby

simplifying the system into a two dimensional Cartesian heat conduction in the ground surrounding the pipes. The ground was divided into axial grid lengths parallel to the heat exchanger pipes. The axial ground grid lengths were selected to be the same length as the pipe grid lengths. A two dimensional finite difference analysis is performed for each axial position in the ground. The two dimensional partial differential equation for transient conduction in the ground is

$$\frac{\partial T}{\partial t} = \frac{k_{ground}}{\rho C_p} \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{q_{pipe}}{\rho C_p}. \quad (3.11)$$

where q_{pipe} is a volumetric source term representing the heat transferred to the ground by the heat exchanger pipe. The horizontal direction perpendicular to the pipes is denoted as the X-direction and the depth is denoted as the Y-direction. The Z-direction represents the axial direction of the pipes. The volumetric source term created by the pipe can also be expressed in linear form as,

$$q_{pipe} = S_c + S_p T_{ground}. \quad (3.12)$$

The two dimensional discretization equation for transient heat transfer given

by Patankar ^[17] is

$$A_{ij} T_{ij} = A_{i+1,j} T_{i+1,j} + A_{i-1,j} T_{i-1,j} + A_{i,j+1} T_{i,j+1} + A_{i,j-1} T_{i,j-1} + b. \quad (3.13)$$

where;

$$A_{i+1,j} = k_e \frac{\Delta Y}{(\Delta X_i + \Delta X_{i+1})/2}, \quad (3.13 \text{ a})$$

$$A_{i-1,j} = k_w \frac{\Delta Y}{(\Delta X_{i-1} + \Delta X_i)/2}, \quad (3.13 \text{ b})$$

$$A_{i,j+1} = k_n \frac{\Delta X}{(\Delta Y_i + \Delta Y_{j+1})/2}, \quad (3.13 \text{ c})$$

$$A_{i,j-1} = k_s \frac{\Delta X}{(\Delta Y_{i-1} + \Delta Y_j)/2}, \quad (3.13 \text{ d})$$

$$A_{i,j} = A_{i+1,j} + A_{i-1,j} + A_{i,j+1} + A_{i,j-1} + A_{i,j}^o - S_p \Delta X \Delta Y. \quad (3.13 \text{ e})$$

Where the coefficient for the temperature from the previous time step ($A_{i,j}^o$) is given by

$$A_{i,j}^o = \frac{\rho C_p \Delta X \Delta Y}{\Delta t}. \quad (3.13 \text{ f})$$

If a pipe is located in the ground control volume, to account for the soil displaced by the pipe

the coefficient $A_{i,j}^o$ becomes

$$A_{i,j}^o = \frac{\rho C_p (\Delta X \Delta Y - \frac{\pi OD^2}{4})}{\Delta t}. \quad (3.13 \text{ g})$$

The term b is defined by

$$b = S_c \Delta X \Delta Y + A_{i,j}^o T_{i,j}^o. \quad (3.13 \text{ h})$$

The soil conductivities used in equations 3.13-a to 3.13-d are the weighted conductivities of the soil between the two grids. The conductivity is calculated by weighting the conductivities of the soil based on their length as a fraction of the distance between control volume centres. A weighted conductivity allows the analysis of interfaces where the soil conductivity changes, such as backfilled trenches and ice/soil interfaces. The terms Sc and Sp are the linearized source terms which are discussed in Section 3.5.

The finite difference equation is solved using the line-by-line variation of the tridiagonal-matrix algorithm (TDMA) as outlined by Patankar^[17]. This method examines a line of grids and assumes that the temperatures along neighbouring grid lines are at the last calculated values. The grid temperatures along the chosen line are solved using the TDMA method. For the two dimensional discretization equation described in equation 3.13, the TDMA method is applied along the Y-direction. The geometry of an XY plane perpendicular to the axial direction is illustrated in Figure 3.2. The neighbouring grids in the X-direction will be represented using the last calculated values. The neighbouring heat transfer terms are treated as known and combined with 'b' into the term D_{ij} as shown in equations 3.14 and 3.15.

The discretized equation can now be re-written as;

$$A_{ij}T_{ij} = A_{ij+1}T_{ij+1} + A_{ij-1}T_{ij-1} + D_{ij}. \quad (3.14)$$

where

$$D_{i,j} = A_{i+1,j} T_{i+1,j} + A_{i-1,j} T_{i-1,j} + b. \quad (3.15)$$

As indicated earlier, the neighbouring temperatures in the X-direction ($T_{i+1,j}$ and $T_{i-1,j}$) are the most recently calculated values and are treated as known.

The temperature of any ground grid can also be expressed in the form of a forward substitution equation,

$$T_{i,j} = P_j T_{i,j+1} + Q_j \quad (3.16)$$

where;

$$P_j = \frac{A_{i,j+1}}{A_{i,j} - A_{i,j-1} P_{j-1}} \quad (3.16 \text{ a})$$

$$Q_j = \frac{D_{i,j} + A_{i,j-1} Q_{j-1}}{A_{i,j} - A_{i,j-1} P_{j-1}} \quad (3.16 \text{ b})$$

The A and D terms were defined earlier in equations 3.14 and 3.15. To account for the adiabatic boundary condition assumption at the bottom boundary ($j = 1$)

$$A_{i,0} = 0 \quad (3.17)$$

which results in,

$$P_{j=1} = \frac{A_{i,2}}{A_{i,1}}. \quad (3.18)$$

At the surface boundary ($j = n$) the coefficient

$$A_{i,n+1} = 0 \quad (3.19)$$

which results in

$$Q_{n+1} = \frac{D_{i,n+1}}{A_{i,n+1}}. \quad (3.20)$$

The value for P and Q are calculated for each grid along the line of Y-grids from the bottom boundary to the surface. Equation 3.16, used to calculate the grid temperature is then applied for each grid from the surface ($j = n$) to the bottom boundary ($j = 1$), back substituting values of P_j and Q_j .

Once the new ground temperatures have been calculated for the current line of Y-grids, the X-position is increased and the line by line method is repeated. The sweep was only performed in the X-direction to reduce computational time. Once the new ground temperatures have been calculated in XY plane, the analysis is moved to the next Z-location along the axial direction of the pipe.

3.4 Boundary Conditions

The boundary conditions in the direction perpendicular to the pipes, were assumed to be sufficiently far away from the heat exchanger that there was no heat flux, resulting in a temperature gradient of zero. This can be expressed mathematically as

$$\frac{\partial T}{\partial x} = 0. \quad (3.21)$$

In the vertical direction, the temperature of undisturbed ground will fluctuate seasonally about a mean ground temperature which is dependant on local weather conditions. The amplitude of the fluctuations will be a maximum at the ground surface, where the ground is most exposed to the ambient air. The amplitude of the seasonal temperature fluctuation decreases with depth below the ground surface until the fluctuations approach zero and the temperature remains at the mean ground temperature throughout the year. This phenomenon was expressed mathematically by Kusuda and Auchenbach ^[18] using the sinusoidal relationship

$$T_{depth,time} = T_{mean} - A_s \cdot e^{a \cdot depth} \cdot \cos(b + a \cdot depth). \quad (3.22)$$

Where

$$a = - \sqrt{\frac{\pi}{365 \cdot 24 \cdot 3600 \alpha}}, \quad (3.22 a)$$

Where t_0 is the day from January first at which the minimum surface temperature occurs, t_{days} is the day being evaluated (from January first), α is the ground diffusivity, A_s is the surface temperature amplitude and T_{mean} is the mean ground temperature. The seasonal ground

$$b = \frac{2 \cdot \pi}{365 \cdot (t_{days} - t_0)}. \quad (3.22 \text{ b})$$

temperature is also expressed graphically in Figure 3.3.

The depth of the bottom boundary for the ground analysis region was selected as the temperature at which the temperature seasonal fluctuations would be negligible. The required depth of the bottom boundary was calculated using the Kusuda and Auchenbach^[18] equation.

The boundary at the ground-air interface is specified by the net heat transfer from the surroundings into the ground. The net surface heat transfer is illustrated in Figure 3.4 and is the sum of the surface/air heat transfer and the incident solar radiation. The surface heat transfer is calculated for each surface control volume. The heat transfer between the surface grid and the ambient surroundings is converted to a volumetric source term and multiplied by the surface area of the control volume ($\Delta X \cdot \Delta Z$) which replaces the $Sc \cdot \Delta X \cdot \Delta Y$ term in equation 3.13h. The surface heat flux into the ground in watts per m^2 is given by

$$Q_{sur} = \alpha_{sur} \cdot Q_{sol} - h_{air} \cdot (T_{i,j,k} - T_{air}). \quad (3.23)$$

Where α_{sur} is the solar absorptivity of the ground, Q_{sol} is the incident solar radiation on the ground, h_{air} is the convection heat transfer coefficient, T_{air} is the air temperature, $T_{i,j,k}$ is the ground control volume temperature.

3.5 Source Terms From Heat Exchanger Pipe

When a portion of the heat exchanger pipe is located in a ground control volume, the heat exchanger will act as a sink when the heat pump is in heating mode and the pipes are extracting energy from the ground. Figure 3.5 illustrates the components of the heat transfer between the pipe and the ground. According to Patankar^[17], the linearized volumetric source term used in the finite difference equations for the ground conduction is written as;

$$S = S_c + S_p T_{i,j,k}. \quad (3.24)$$

The volumetric source term in the ground must also equal the heat transferred from the heat exchanger fluid to the ground control volume through the walls of the pipe. The heat transfer from a pipe into the ground grid is determined using the equation for steady state conduction, and is expressed as

$$Q_{pipe} = \frac{T_{fluid} - T_{i,j,k}}{R_{source}}. \quad (3.25)$$

In heating mode, the heat passes from the ground into the fluid and the term Q_{pipe} is negative. To convert the heat conduction through the pipe wall to a volumetric source term which can

be applied in the finite difference equation 3.13, the heat "generated" by the equivalent volumetric source term must be equal to the heat transfer from the pipe. This can be expressed mathematically by equating 3.24 and 3.25.

$$S_c + S_p T_{ij,k} = \frac{T_{fluid}}{R_{source}} - \frac{T_{ij,k}}{R_{source}}. \quad (3.26)$$

The right hand terms that are not multiplied by the control volume temperature $T_{ij,k}$ are set equal to S_c , and the terms multiplied by the ground temperature are set equal to S_p . The volumetric source term constants can now be defined as,

$$S_c = \frac{T_{fluid}}{R_{source}}, \quad S_p = -\frac{1}{R_{source}}. \quad (3.27)$$

As stated earlier, the term R_{source} is the total thermal resistance between the fluid in the pipe and the ground control volume in which the pipe is located. The total thermal resistance can be broken down into three components; convection resistance between the fluid and the interior of the pipe (R_{fluid}), the conduction resistance of the pipe wall (R_{pipe}), and the contact resistance between the outside of the pipe and the surrounding soil ($R_{contact}$). Because these three resistance components are in series, the total thermal resistance can be written as,

$$R_{source} = R_{fluid} + R_{pipe} + R_{contact}. \quad (3.28)$$

where;

$$R_{fluid} = \frac{1}{2\pi h_{fluid} r_i \Delta Z}, \quad (3.28 \text{ a})$$

$$R_{pipe} = \frac{\ln(r_o/r_i)}{2\pi \Delta Z k_{pipe}}, \quad (3.28 \text{ b})$$

$$R_{contact} = Constant. \quad (3.28 \text{ c})$$

The terms r_o and r_i are the interior and exterior pipe radii respectively. ΔZ is the length of the pipe control volume and also the ground control volume in the axial direction. The term k_{pipe} is the conductivity of the high density polyethylene pipe. $R_{contact}$ is a defined constant which is discussed in more detail in Section 3.6 and h_{fluid} is the convection coefficient of the fluid. The value of h_{fluid} will depend on the flow rate inside the pipe.

The operation of the circulation pump is controlled by the compressor inside the heat pump. The flow is zero when the heat pump is off and the flow is the design flow when the heat pump compressor is running.

The equation for h_{fluid} inside the pipes at design flow rates is given by the Dittus-Boelter equation in terms of Nusselt number for turbulent flow. This correlation, summarized by Incropera and DeWitt ^[19], applies to fully developed turbulent flow in circular smooth pipes with pipe walls at a higher temperature than the fluid (heating). The Dittus-Boelter equation

for heating is

$$Nu_{ID} = 0.023 \cdot Re_{ID}^{0.8} \cdot Pr^{0.4}. \quad (3.29)$$

Where Re_{ID} is the pipe flow Reynolds number, Pr is the Prandtl number of the heat transfer fluid. The equation is valid for Reynold's numbers greater than 10,000 for typical horizontal and slinky heat exchangers and the Prandtl number in the range $0.7 < Pr < 160$. If the heat pump system is cooling, the pipe surface temperatures will be lower than the fluid temperature, and the exponent for the Prandtl number in equation 3.29 changes to 0.3. The Nusselt number for flow inside a pipe is defined as,

$$Nu_{ID} = h_{fluid} \cdot \frac{ID}{k_{fluid}}. \quad (3.30)$$

k_{fluid} is the thermal conductivity of the fluid and ID is the inner diameter of the heat exchanger pipe. Combining equations 3.29 and 3.30 yields,

$$h_{fluid} = 0.023 \cdot Re_{ID}^{0.8} \cdot Pr^{0.4} \cdot \frac{k_{fluid}}{ID}. \quad (3.31)$$

The Reynold's number for fluid flow in a circular pipe is given by

$$Re_{ID} = 4 \cdot \frac{\dot{m}}{\pi \cdot ID \cdot \mu}, \quad (3.32)$$

where $\dot{m}$ is the mass flow rate of the fluid and μ is the dynamic viscosity. The value of the

Reynolds number is significantly greater than 10,000 for the case being considered. As a result, the flow is turbulent and equation 3.29 is applicable.

A constant h_{fluid} has been assumed if there is no flow in the pipes. When the flow through the heat exchanger stops, a convection current is created inside the pipe perpendicular to its length. This is due to the colder ground temperatures above the heat exchanger pipes caused by its closer proximity to the surface. An illustration of the no flow convection currents is provided in Figure 3.6.

The constant heat transfer coefficient for zero flow in the heat exchanger pipes is based on the Globe and Dropkin correlation presented by Incropera and DeWitt ^[19] for rectangular ducts. This correlation gives the Nusselt number for natural convection in rectangular cavities with warmer bottom surfaces than top surfaces. The Globe and Dropkin equation is

$$\overline{Nu}_L = 0.069 Ra_L^{1/3} Pr^{0.074} . \quad (3.33)$$

Where the Raleigh number (Ra) is defined as

$$Ra_L = \frac{g \beta (\Delta T) L^3}{\alpha_{\text{fluid}} \nu_{\text{fluid}}} . \quad (3.34)$$

The circular heat exchanger pipes were represented in these equations by defining the characteristic length (L) as the square root of the pipe cross sectional area. The fluid

properties, diffusivity (α_{fluid}) and viscosity (ν_{fluid}) were determined at $-3.8\text{ }^{\circ}\text{C}$, which is assumed to represent the average of the inside top and bottom surface temperatures for the entire simulation period. The term g represents the gravitational constant (9.81 m/s^2), β is the expansion coefficient of the fluid and ΔT is the difference between the bottom and top surface temperatures.

The term ΔT is the temperature difference between the bottom and top of the pipe wall, which creates the convection loops. ΔT was estimated from preliminary ground temperature calculation to be $0.01\text{ }^{\circ}\text{C}$ to account for the small vertical temperature gradient in the soil. In reality, this value will change throughout the simulation depending on the operation of the heat pump, prior to the no flow periods. While the heat pump is operating, the temperature difference between the inside pipe walls will be small due to the high heat transfer coefficient of the turbulent flow. Once the flow stops, the temperature difference begins to increase as the surrounding ground temperature has a larger impact on the wall temperatures. A constant temperature difference was assumed for all periods without flow.

3.6 Contact Resistance

The heat transfer at the interface between the heat exchanger pipe and the ground surrounding the pipe must pass through the physical gap between the two surfaces called the thermal contact resistance. The thermal resistance of this gap will depend on the material filling the gap and the size of the gap. If it is assumed that the gap around the pipe can be

represented as a cylinder on the outside of the pipe, the thermal contact resistance can be represented by the formula

$$R_c = \frac{1}{2\pi k_{fill}} \ln\left(\frac{r_o + t}{r_o}\right). \quad (3.35)$$

In most cases, the thickness of the gap (t) will be significantly smaller than the outer pipe radius (r_o), and equation 3.35 can be simplified to

$$R_c = \frac{1}{2\pi k_{fill}} \frac{t}{r_o}. \quad (3.36)$$

During fall and early winter operation, the soil surrounding the pipe will be saturated with moisture, and the gap between the pipe and the soil will be filled with water. Once the ground surrounding the pipe begins to freeze, the fluid next to the pipe freezes and becomes an inflexible solid. The expansion and contraction of the ice surrounding the pipe due to the temperature variations can cause a small space to form between the pipe and the ice.

In the summer the magnitude of the contact resistance can become significantly larger, particularly in warmer climates. Heat being rejected from the heat exchanger pipes causes moisture in the soil surrounding the pipes to migrate away from the heat exchanger ^[21]. This is often called "thermal dry-out". The water in the gap between the pipes and the soil is

replaced with air, which has a thermal conductivity approximately one twentieth that of water at 20 °C.

3.7 Ground Freezing

When the temperature of the anti-freeze solution flowing through the ground heat exchanger is below zero Celsius, the moisture in the ground around the pipes will freeze. A cylinder of ice forms around the pipes, and increases in thickness as the freezing in the ground continues. The ground freezing enhances the thermal performance of the ground heat exchangers as the phase change of the moisture slows the temperature drop in the adjacent soil and consequently the fluid returning from the heat exchangers will be warmer. A small error will be encountered due to freezing analysis being applied in Cartesian co-ordinates instead of cylindrical co-ordinates. The error is reduced by using a small grid spacing around the pipes.

The ground freezing was modelled by assigning a latent heat of fusion to each ground control volume, which represented the energy that must be removed to completely freeze the moisture in the ground control volume. The value for the latent heat of a control volume is updated at each time period. The ground is completely frozen when no additional energy extraction is required, and the heat of fusion is set to zero. If the ground is completely thawed the heat of fusion (Q_{fusion}) is set to its maximum value defined as

$$Q_{fusion,max} = \Delta X \Delta Y \Delta Z \cdot \rho_{ground} \cdot C_{fusion} \cdot MC. \quad (3.37)$$

Where ρ_{ground} is the density of the ground in the control volume, C_{fusion} is the heat of fusion of water per unit mass, and MC is the mass fraction of water in the control volume. The ground is assumed completely thawed, and Q_{fusion} is set to its maximum for all control volumes at the start of the simulation.

At each time step the program calculates the temperature of each control volume using the procedure outlined in Section 3.3 to determine if it has dropped below or climbed above zero degrees Celsius, indicating that a phase change is occurring. If a phase change is occurring, the ground temperature will end up above (thawing) or drop below (freezing) the freezing point when calculated by the finite difference algorithm in Section 3.3. This temperature difference above or below the freezing point can also be expressed as a sensible heat called $Q_{sensible}$. The sensible heat is defined as

$$Q_{sensible} = \Delta X \Delta Y \Delta Z \cdot \rho_{ground} \cdot C_{p,ground} \cdot \Delta T. \quad (3.38)$$

Where $C_{p,ground}$ is the heat capacity of the ground control volume and ΔT is the temperature of the control volume above or below freezing. Except at the end of a freezing or thawing cycle, the latent heat Q_{fusion} will be significantly larger than the sensible heat $Q_{sensible}$.

In the case that $Q_{fusion} > Q_{sensible}$, the grid temperature is set to zero, and Q_{fusion} is reduced by the magnitude of the sensible energy if the control volume is in the process of freezing. The

value of Q_{fusion} is increased by the magnitude of the sensible energy if the control volume is thawing. The updated value of the latent energy is expressed as

$$Q_{\text{fusion,new}} = Q_{\text{fusion,old}} - Q_{\text{sensible}} \quad (\text{freezing}), \quad (3.39 \text{ a})$$

$$Q_{\text{fusion,new}} = Q_{\text{fusion,old}} + Q_{\text{sensible}} \quad (\text{thawing}). \quad (3.39 \text{ b})$$

If the control volume is at the end of a freezing/thawing cycle, and Q_{sensible} is greater than the energy required to complete the freezing or thawing process then Q_{fusion} is either set to zero (fully frozen) or to its maximum value (fully thawed). The grid temperature is then adjusted to account for the amount of energy required to complete the thawing or freezing cycle. The modified control volume temperature is defined as

$$T_{\text{modified}} = \frac{(Q_{\text{sensible}} - Q_{\text{fusion}})}{\Delta X \Delta Y \Delta Z \cdot \rho_{\text{ground}} \cdot C_{p,\text{ground}}} \quad (\text{freezing}), \quad (3.40 \text{ a})$$

$$T_{\text{modified}} = \frac{(Q_{\text{sensible}} - (\Delta X \Delta Y \Delta Z \cdot \rho_{\text{ground}} \cdot C_{\text{fusion}} \cdot MC) - Q_{\text{fusion}})}{\Delta X \Delta Y \Delta Z \cdot \rho_{\text{ground}} \cdot C_{p,\text{ground}}} \quad (\text{thawing}). \quad (3.40 \text{ b})$$

If a control volume is completely frozen, its conductivity is modified to be a mass weighted

average of the conductivity of the unfrozen soil and the conductivity of ice based on the mass fraction of moisture in the soil. The conductivity of the frozen soil is expressed as

$$K_{freeze} = K_{ice} \cdot MC + K_{soil} \cdot (1 - MC) \quad (3.41a)$$

where MC is the moisture content of the soil being examined. Equation 3.41a assumes that heat passes through the soil and moisture phases of the control volume in series and the resistance of the two materials is summed. If it were assumed that the heat travels through the two materials in parallel the conductivity of the frozen soil is expressed as

$$K_{freeze} = \frac{K_{ice} \cdot K_{soil}}{MC \cdot K_{soil} + (1 - MC) \cdot K_{ice}} \quad (3.41b)$$

For the ground heat exchanger examined in this thesis the moisture content is 9.4%, K_{ice} is 2.2 W/m² and K_{soil} is 1.9 W/m². The difference in values of K_{freeze} calculated using equation 3.41a and 3.41b is less than one percent.

The conductivity of the frozen soil could also have been calculated using the volume fraction of the moisture. Because the conductivity of the soil and ice is small, the difference between the volume fraction and mass fraction assumptions will be small.

The process described above to modify the calculated control volume temperatures from section 3.3 is summarized in Table 3.1. In this table, “Initial T_{ground} ” represents the temperature calculated by the finite difference model discussed in Section 3.3.

3.8 Grid Dimensions

A combination of constant and variable ground control volume dimensions were used to subdivide the plane perpendicular to the heat exchanger pipes. The control volume dimensions around the heat exchanger pipes were constant. Away from the pipes, the dimensions become larger since the temperature gradients are significantly smaller. The control volume lengths along the axial direction of the pipes are constant due to the small axial temperature gradients.

An algebraic grid generation technique outlined by Hoffmann ^[22] was used as the basis to generate the variable control volume dimensions in both the X-direction (horizontal) and Y-direction (vertical). These equations created smaller grid dimensions next to the constant grid region around the pipes. The equations used to determine the X-location of both sides of a control volume in the variable dimension regions are given by

$$X_1 = BC_x \frac{(\beta_x + 1) - (\beta_x - 1) \cdot [(\beta_x + 1)/(\beta_x - 1)]^{1-\eta_x}}{[(\beta_x + 1)/(\beta_x - 1)]^{1-\eta_x} + 1}, \quad (3.42 \text{ a})$$

$$X_2 = BC_x \frac{(\beta_x + 1) - (\beta_x - 1) \cdot [(\beta_x + 1)/(\beta_x - 1)]^{1-(\eta_x + \Delta\eta_x)}}{[(\beta_x + 1)/(\beta_x - 1)]^{1-(\eta_x + \Delta\eta_x)} + 1}. \quad (3.42 \text{ b})$$

The size of the grid in the X-direction is given by

$$\Delta X = X_2 - X_1. \quad (3.43)$$

The term BC_x is the size of the variable grid domain in the X-direction. This is also the distance between the constant grid region and the boundary of the analysis region. β_x is the clustering parameter which defines the magnitude of the variation between the smallest and largest grid spacings. It must be defined between 1 and infinity. The transformation metric η is used to transform the computational domain (constant grid spacing) dimensions to the physical domain (variable grid spacing). Where η_x is defined by

$$\eta_x = \frac{1}{NV_x - 1} \quad (3.44)$$

where NV_x is the number of control volumes in the variable grid spacing region.

The ground is divided into five regions. The constant volume region is located around the heat exchanger pipes with constant grid dimensions in both X- and Y-directions. The X-direction (horizontal) contains two, mirror image, variable grid regions on either side of the constant volume region. The Y-direction (vertical) has two different variable grid regions. One extends from the top of the constant volume region to the ground surface. The second region extends from the bottom of the constant volume region down to the bottom of the analysis region at D_{mean} where the ground temperature is assumed to be constant at the mean ground temperature. The grid dimensions are illustrated in Figure 3.7.

Table 3.1: Ground Freezing Logic Table

Initial T_{ground} (Sect. 3.3)	Ground Already Frozen?	$Q_{\text{sensible}} > E_{\text{freeze/thaw}}$	Freezing Calculation Procedure
> 0	Yes	Yes	After thawing, the ground control volume will rise a certain temperature above 0°C, although less than the initial calculation. Modify T_{ground} using equation 3.40 b.
> 0	Yes	No	The ground will only thaw partially and remain at 0°C. Calculate the increase in Q_{fusion} using equation 3.39 b.
> 0	No	Yes/No	None
> 0	Partially	Yes	After thawing the remainder of the way, the ground control volume will rise a certain temperature above 0°C less than the initial calculation. Modify T_{ground} using equation 3.40 b.
> 0	Partially	No	The ground will only thaw partially and remain at 0°C. Calculate the increase in Q_{fusion} using equation 3.39 b.
< 0	Yes	Yes/No	None
< 0	No	Yes	After freezing, the ground control volume will drop a certain temperature below 0°C, although less than the initial calculation. Modify T_{ground} using equation 3.40 a.
< 0	No	No	The ground will only freeze partially and remain at 0°C. Calculate the decrease in Q_{fusion} using equation 3.39 a.
< 0	Partially	Yes	After freezing the remainder of the way, the ground control volume will drop a certain temperature below 0°C less than the initial calculation. Modify T_{ground} using equation 3.40 a.
< 0	Partially	No	The ground will only freeze partially and remain at 0°C. Calculate the decrease in Q_{fusion} using equation 3.39 a.

“Initial T_{ground} ” is the control volume temperature calculated using the equations from Section 3.3. The column “Ground Already Frozen?” indicates whether the ground was determined to be frozen at the previous time step. For example, if the initial T_{ground} is >0 and the ground is already frozen then the ground was frozen at the previous time step, and the temperature calculated by the finite difference equations has climbed above 0 °C. This would indicate that the ground has begun thawing.

$E_{\text{freeze/thaw}}$ is the energy required to complete the freezing or thawing process.

Figure 3.1: Discretization of Heat Exchanger

Figure 3.2: Geometry of Line By Line/TDMA Analysis

Figure 3.3: Undisturbed Ground Temperature Profile

Figure 3.4: Heat Transfer At Ground Surface Grids

Figure 3.5: Components of Source Term in Ground

Figure 3.6: Idealized Natural Convection Circuits in Pipes

Figure 3.7: Ground Control Volume Grid Sizing

Chapter 4 Simulation of Ground Heat Exchanger

The simulation of a slinky ground heat exchanger was performed for a four month winter period from December to March inclusive. The heat exchanger which was simulated is an actual heat exchanger of a residential ground-source heat pump system located in Eau Claire, Wisconsin. The monitoring site was described earlier in Section 2.1.

4.1 System/Environment Input Parameters

All available system and environment parameters known at the heat pump system monitoring site in Eau Claire were used as input parameters to the finite difference model. Weather data that were not available directly from the monitoring site were obtained from typical meteorological year (TMY) data compiled from 1960-1990 hourly weather data for Eau Claire, Wisconsin. The TMY data file was downloaded from the U.S. Department of Energy's Web Site. Additional system design data, such as soil thermal properties and anti-freeze properties were based on known values for materials reported at the monitoring site..

4.1.1 Weather Data

Ambient dry bulb temperatures were collected for the monitored ground-source heat pump system on fifteen minute intervals. The ambient dry bulb temperatures were recorded along with the fluid temperatures entering the heat pump (EWT) and leaving the heat pump (LWT),

the undisturbed ground temperatures at 0.9 and 1.8 metres (three and six feet) below the surface, the heat pump/circulation pump run time, and the energy extracted from the loop by the heat pump calculated by the data logging equipment ^[1]. Very little information is available about the temperature measuring devices for the ground temperature and heat pump entering water temperature. No information was reported about the accuracy of these devices ^[1]. It was assumed for the purpose of this study, that thermocouples with an accuracy of ± 0.5 °C were used to monitor the ground temperatures and the heat pump entering water temperature.

The wind speed used to calculate the ground surface heat transfer, the incident horizontal solar radiation, and snow depth were obtained from the hourly TMY data file for Eau Claire ^[2]. The hourly wind speed and the incident solar radiation data was subdivided into fifteen minute time steps for consistency with the finite difference input file. The snow depth was averaged on a bi-monthly basis from the hourly data.

Monthly summaries of the weather data used for the simulation of the slinky ground heat exchanger are presented in Table 4.1. January was the coldest month of the simulation, with an average temperature of -12.6 °C. It was also the month with the longest heat pump run time, with the heat pump operating 75 percent of the month. In January, 15,700 MJ of heat was extracted from the ground loop by the heat pump. During the entire four month simulation 50,200 MJ were extracted by the heat pump. As expected, March was the warmest month with the lowest heating requirement.

4.1.2 System Design and Simulation Input Data

Most of the heat exchanger system design parameters were either reported as measured values for the heat exchanger system or indirectly based on known data for observed materials (eg: soil and fluid thermal property data). The burial depth of the pipe, heat exchanger length, width of the heat exchanger trench, equivalent pipe/trench ratio, and the design flow rate of circulation pump are all known values for the monitored heat exchanger system.

The SDR 16.0 high density polyethylene pipe dimensions and conductivity were obtained from an ASTM pipe standard ^[23] and GS2000 heat exchanger sizing software ^[24] respectively. The heat exchanger ethanol solution properties information were obtained from the ASHRAE Commercial/Institutional Ground-Source Heat Pump Design Manual ^[25]. The ground description was reported to be a clay soil, which was assumed to be saturated during the winter. The thermal properties of the clay soil are based on reported properties for typical clay soils ^[26]. The thermal properties of the snow were obtained from Incropera and DeWitt ^[19].

The contact resistance between the soil and the pipes were determined by assuming a 0.03 millimetre air gap between the pipe and the soil and using Equation 3.34. The moisture content of the clay soil was assumed to be 9.4 percent by mass. The moisture content of the soil was taken from measured data at a ground-source heat pump system in upper New York

State ^[6]. Both the observed and estimated values used for system design and inputs to the simulation program are summarized in Table 4.2.

4.2 Predicted Undisturbed Ground Temperature

Monthly summaries of the undisturbed ground temperatures predicted by the finite difference model are presented in Tables 4.3 for 0.9 and 1.8 metres (three and six feet) below the surface. As shown in this table, the average monthly temperature ranged between 0.4 to 2.4 degrees Celsius at 0.9 metres (3 ft) below the surface and 1.9 to 4.8 degrees Celsius at 1.8 metres (6 ft) below the surface. As expected, the average monthly temperature at 1.8 metres (6 ft) below the surface is warmer than the temperature at 0.9 metres (3 ft) due to the insulating effect of the soil. The average monthly ground temperatures at both depths were always above freezing during the simulation period.

The minimum average monthly ground temperatures occur during March, even though the average air temperature for March is the warmest of the simulation period. This thermal lag is due to the insulating effect of the ground. The soil's thermal diffusivity is a measure of its ability to transfer heat over its ability to store heat. Thermal diffusivity is an indication of the time lag that will be experienced by heat conducting through a material and is defined as

$$\alpha = \frac{k_{soil}}{\rho_{soil} \cdot C p_{soil}} \quad 4.1$$

A higher thermal diffusivity will result in a shorter lag between the minimum ground temperatures and the minimum monthly air temperature.

The undisturbed ground temperatures predicted by the finite difference model are illustrated in Figures 4.1 to 4.4 for the simulation period.

Rapid changes in the ground temperature at 0.9 metres (3 ft) can be observed in some of these figures. These are caused by the freezing algorithm which assumes finite ground control volumes with uniform temperatures. Although the control volumes at 0.9 metres (3 ft) below the surface do not freeze during the period being simulated, they are affected by control volumes closer to the surface that do freeze. As these control volumes freeze and thaw, the temperatures remain at zero Celsius, reducing the temperature variations in the lower control volumes. After the phase change is complete in the control volumes closer to the surface, the temperature begins changing, and the temperature of the adjacent grids underneath begin to change more rapidly than during the phase change. At 1.8 metres (6 ft) ground temperatures are more consistent, since they are less exposed to the surface effects.

The predicted ground temperature never fell below zero at the depth of three and 1.8 metres (6 ft), although the temperature continued to drop at the end of the simulation at March 31st. The minimum undisturbed ground temperature at 0.9 metres (3 ft) was predicted to be 0.33 °C and 1.77 °C at 1.8 metres (6 ft).

The isotherms in Figures 4.5 to 4.8 illustrate the ground temperatures through a cross section of the analysis region down to a depth of 7 metres. The temperature gradients below this depth are small and were not plotted in order to better examine the ground temperatures closer to the surface. These isotherm plots are the undisturbed ground temperatures predicted by the finite difference program at the end of each month of the simulation.

As expected, the surface temperatures were lowest at the end of January, since this month had the lowest ambient air temperature. The warmest surface temperatures are at the end of March. For the end-of the month plots in Figures 4.5 to 4.8, the ground temperatures at lower depths are at a maximum at the end of December, and continue to drop through to the end of March. This is due to the lag required for the surface effects to be observed at lower depths.

Figure 4.9 is a profile of the predicted undisturbed ground temperature at the centre of the analysis region. Each of the four curves illustrates how the temperature changes with depth at the end of each month of the simulation period. This plot illustrates the point made earlier regarding thermal lag in the soil. At lower depths (below two metres), the ground temperature continues to drop through the entire simulation period, even as the air temperature begins to rise at the end of the winter.

The frost depth predicted by the program reaches down to a depth of 0.73 metres at the end of January, and remains at this depth for the remainder of the simulation period.

4.3 Predicted Heat Pump Entering Water Temperature

The finite difference simulation was performed for the winter months of December to March. Monthly summaries of the predicted entering water temperatures and the heat pump characteristics are presented in Table 4.4.

The average heat pump entering water temperature (EWT) for the four month simulation period was $-2.18\text{ }^{\circ}\text{C}$. December had the warmest EWT and March the coldest, despite the fact that December had the second highest load on the heat pump and March the lowest load. This is due to the temperature of the soil surrounding the heat exchanger being reduced over the heating season. December is near the beginning of the heating season, and energy has been rejected to the ground for the entire summer. In contrast, March is near the end of the heating season when the undisturbed ground temperature is at its lowest after heat has been extracted from the surrounding soil for the several months.

Figures 4.11 to 4.14 illustrate the predicted heat pump entering water temperature for each month of the simulation period. The continuous drop of the heat pump EWT throughout the simulation period can be clearly seen in these figures. The highest EWT occurs December 2nd and is $1.8\text{ }^{\circ}\text{C}$. The minimum EWT occurs March 8th and is $-4.9\text{ }^{\circ}\text{C}$. Although freezing of the moisture in ground allows shorter heat exchanger lengths, the spiral heat exchanger for this system is shorter than ideal design practice. The heat pump manufacturer's literature recommends that the minimum entering water temperature not be lower than $-6.7\text{ }^{\circ}\text{C}$ ^[27]. The

manufacturer's minimum recommended operating temperature is only 1.8 °C below the minimum heat pump EWT predicted by the finite difference program.

During periods of continuous heating by the ground-source heat pump, the EWT drops rapidly. An example of these concave curves in which the predicted heat pump EWT drops rapidly is seen in Figure 4.10. At the beginning of a typical continuous run period, the fluid in the pipes and the soil surrounding the pipes are relatively warm. This is because a period of part load operation, in which the heat pump operates only a fraction of the time, proceeds the full load period. During a part load period, the ground has had a chance to regenerate and transfer heat to the soil around the pipes and warm the heat transfer fluid inside the pipes. The amount of regeneration that takes place in the surrounding soil depends on the length and fraction run time of the part load period. As the full load run-time occurs, heat is removed from the loop, and in turn is removed from the ground faster than the adjacent ground can be regenerated. The longest continuous run period experienced during the simulation period is 23 hours during January 25th and 26th.

The periods where no heat pump EWT is shown in the graphs, the circulation pump and the heat pump are not operating. This is either due to no load required by the house, the heat pump being shut down by the electric utility to reduce their system load, or insufficient heat capacity available from the heat pump. When the utility shuts off the heat pump or back-up heat is required, a propane furnace is used to provide the space heating.

The majority of the no flow periods occurred during the end of January and beginning of February. These periods are characterized by sharp increases in the EWT after start up. This is due to the fluid sitting in the heat exchanger pipes being warmed by the soil surrounding the pipes. The surrounding soil is also regenerated as heat is transferred from the undisturbed ground away from the pipes.

4.4 Impact of Heat Exchanger On Ground Temperatures

As the heat exchanger pipes remove heat from the ground, the surrounding soil is cooled which affects the ability of the heat exchanger to extract heat from the ground. The isotherms plots in Figures 4.15 to 4.18, are cross sections of the analysis region perpendicular to the heat exchanger at its midpoint. These plots are similar to the undisturbed ground temperature plots in Figures 4.5 to 4.8. They illustrate the predicted ground temperature at the end of each month of the simulation and the impact of the heat exchanger pipes on the ground temperature. The location of the heat exchanger pipes are shown as black circles in the isotherm plots.

As with the undisturbed ground temperature isotherms, the temperature at greater depths drops continuously throughout the simulation period as the surface effects slowly penetrate into the lower depths.

Early in the heating season, the heat exchanger pipes have only a moderate impact on the

surrounding ground temperature. At the end of December (Figure 4.15), the cold temperature region around the pipes appears to be an extension of the lower temperatures near the surface. Later in the winter, there is a distinct cold region around the pipes as seen in Figures 4.16 and 4.17. At the end of March, the surface temperatures have increased and the ground around the pipes has warmed as the loads on the heat exchanger have decreased. But since some heat is still being extracted at the end of March, there is still a distinct cold region around the pipes at the end of March. If the simulation were continued beyond the end of March, the amount of heat being extracted would continue to decrease into the spring and the warmer spring air temperatures would penetrate deeper below the surface.

The horizontal cross sections of the ground temperature in the analysis region are also shown in Figures 4.19 to 4.22. These cross sections taken halfway along the heat exchanger length illustrate the impact of the heat exchanger pipes on the surrounding ground at specific depths. The temperature at a depth of 0.08 metres represents the ground surface. The ground surface is at its coldest during January when the air and heat exchanger pipes are also the coldest. The surface temperature is at its warmest at the end of March when it is in the process of thawing.

These profiles illustrate that the heat exchanger pipes have significant impact on the ground temperatures. Even at the ground surface, there is a significant difference in temperature between the ground directly above the pipes and the ground away from the pipes near the edge of the analysis region. This is illustrated in Figure 4.17.

The profiles at the depth of 2.1 metres (blue) represents a horizontal cross section through the ground heat exchanger. The jagged pattern at the centre of the cross section are due to the lower temperatures of the control volumes containing the pipes.

The profile at a depth of 2.21 metres (red) represents the control volumes directly below the pipes. As expected, the ground is warmer at this depth than at the depth of the pipes, and the jagged pattern observed at the depth of the pipes are not evident.

Table 4.1: Summary of Weather Variables

Month	Avg. T_{db} (°C)	Avg. t_{run} (%)	$Q_{extracted}$ (MJ)	Avg. V_{wind} (m/s)	I_{solar} (W/m ²)	D_{snow} (mm)
December	-9.0	63	13,178	3.3	54	55
January	-12.6	75	15,698	4.0	70	115
February	-8.3	62	10,801	3.3	118	470
March	-5.2	57	10,486	4.3	163	215
Average	-8.8	64		3.7	101	214
Total			50,164			

Table 4.2: Summary of Design and System Input Variables

Design/Simulation Parameter	Value used in Simulation	Design/Simulation Parameter	Value used in Simulation
Heat exchangers		Backfill soil	
Parameters		Soil material	Damp clay
Heat exchanger length	4 x 30.5 m	Conductivity	1.90 W/m °C
Burial depth	2.1 m	Heat capacity	962 J/kg °C
Heat exchanger trench width	1.05 m	Soil diffusivity	9.42 10 ⁻⁷ m ² /s
Equivalent pipe/trench ratio	6	Initial Ground Temperature	
Equivalent pipe spacing	.21 m	T _{mean}	8.5 °C
Contact resistance	.015 W/m ² °C	A _{surface} (surf. amplitude)	15.5 °C
Pipe Dimensions		t _{min} (min surf. temp.)	4 °C
Pipe construction	3/4" series 160	Heat Exchanger Fluid	
Interior diameter	.0219 m	Anti-freeze fluid	25% ethanol
Exterior diameter	.0267 m	Thermal conductivity	0.46 W/m °C
Thermal conductivity	.319 W/m°C	Heat capacity	4,250 J/kg °C
Snow Cover		Density	967 kg/m ³
Absorptivity	0.4	Dynamic viscosity	2.8 10 ⁻³ kg/m•s
Conductivity	0.2 W/m °C	Prandlt number	41.6
Native soil		No flow heat transfer coefficient	30.0 W/m ² °C
Soil material	Damp clay	Design flow rate	0.68 l/s
Conductivity	1.90 W/m °C	Simulation parameters	
Heat capacity	962 J/kg °C	No. pipe control volumes	20
Soil diffusivity	9.42 10 ⁻⁷ m ² /s	No. ground C.V's (X/Y/Z)	29/26/20
Moisture content of soil	9.4 %	Depth of analysis region	13.0 m
Latent heat of fusion for water	334 J/kg	Width of analysis region	6.0 m
Conductivity of ice	2.24 W/m °C		

Table 4.3: Predicted Undisturbed Ground Temperatures

Month	Average Monthly Air Temperature (°C)	Predicted EWT At 0.9 Metres Below Surface (°C)	Predicted EWT At 1.8 Metres Below Surface (°C)
December	-9.0	2.38	4.83
January	-12.6	1.30	3.44
February	-8.3	0.45	2.39
March	-5.2	0.36	1.92

Table 4.4: Monthly Summary of Simulation Results

Month	Average Predicted EWT (°C)	Total Month HP Run Time (Hours)	Energy Extracted From Ground by Heat Exchanger (MJ)	Heat Pump Load (MJ)	COP
December	-0.47	470	13,179	19,441	3.10
January	-2.38	560	15,698	23,158	3.03
February	-2.78	431	10,801	15,934	3.01
March	-3.10	422	10,486	15,469	3.00
Total/Average	-2.18	1,883	50,164	74,002	3.04

Figure 4.1: Predicted Undisturbed Ground Temperature For December - 0.9 and 1.8 Metres Below Surface

Figure 4.2: Predicted Undisturbed Ground Temperature For January - 0.9 and 1.8 Metres Below Surface

Figure 4.3: Predicted Undisturbed Ground Temperature For February - 0.9 and 1.8 Metres Below Surface

Figure 4.4: Predicted Undisturbed Ground Temperature For March - 0.9 and 1.8 Metres Below Surface

Figure 4.5: Undisturbed Ground Temperature Plot - December 31st

Figure 4.6: Undisturbed Ground Temperature Plot - January 31st

Figure 4.7: Undisturbed Ground Temperature Plot - February 29th

Figure 4.8: Undisturbed Ground Temperature Plot - March 31st

Figure 4.9: Seasonal Profiles of Predicted Undisturbed Ground Temperatures

Figure 4.10: Full and Partial Run-Time Periods

Figure 4.11: Predicted Heat Pump Entering Water Temperature For December, 1994

Figure 4.12: Predicted Heat Pump Entering Water Temperature For January, 1995

Figure 4.13: Predicted Heat Pump Entering Water Temperature For February, 1995

Figure 4.14 Predicted Heat Pump Entering Water Temperature For March, 1995

Figure 4.15 Ground Temperature Plot in Plane Perpendicular to Heat Exchanger - December 31st

Figure 4.16 Ground Temperature Plot in Plane Perpendicular to Heat Exchanger - January 31st

Figure 4.17 Ground Temperature Plot in Plane Perpendicular to Heat Exchanger - February 29th

Figure 4.18 Ground Temperature Plot in Plane Perpendicular to Heat Exchanger - March 31st

Figure 4.19: Ground Temperature Profile at Heat Exchanger Midpoint for December 31st

Figure 4.20: Ground Temperature Profile at Heat Exchanger Midpoint for January 31st

Figure 4.21: Ground Temperature Profile at Heat Exchanger Midpoint for February 29th

Figure 4.22: Ground Temperature Profile at Heat Exchanger Midpoint for March 31st

Chapter 5 Validation of Heat Exchanger Model

To ascertain the finite difference model's ability to predict the fluid temperature entering the ground-source heat pumps, three different validation methods were performed. An equation was developed by Ingersoll et al. ^[27] to predict the ground temperature at various radial positions from the outside of a single pipe buried in an infinite medium. This was used to evaluate the short term transient performance of the finite difference model. The program algorithms were modified so that it could be compared with Ingersoll's predictions.

A monitored ground-source heat pump site located in Wisconsin was also used to validate the finite difference algorithms. The undisturbed ground temperatures were monitored at this site as discussed in Section 2.1. The finite difference model was modified to predict the undisturbed ground temperatures. To do this, the source terms discussed in Section 3.5 were set to zero, to simulate undisturbed ground. The finite difference prediction of undisturbed ground temperature was compared to the monitored data.

The heat pump entering fluid temperature was also monitored at the Wisconsin site. The entering fluid temperature predicted by the finite difference model was compared to the monitored data to further validate the model.

5.1 Comparison of Transient Equation With Constant Pipe Heat Flux

Ingersoll et al. ^[28] developed a procedure to predict the temperature drop of soil around a single buried pipe with a constant surface heat flux. Given an initial and uniform soil temperature around a buried pipe and the heat flux at the pipe surface, Ingersoll's procedure predicts the change in ground temperature with time at various radial distances from the pipe. Ingersoll's analysis assumes an infinite medium (no surface effects) and no freezing of the moisture in the soil.

The transient prediction for the ground temperature at any time and distance from the pipe is given by:

$$T_{ground} = \frac{2\pi R_{pipe} q_{pipe}}{k_{soil}} \times G(z, p) \quad (5.1)$$

where,

$$G(z, p) = \frac{1}{\pi^2} \int_0^\infty \frac{e^{-\beta^2 z} - 1}{J_1^2(\beta) + Y_1^2(\beta)} (J_0(p\beta)Y_1(\beta) - J_1(\beta)Y_0(p\beta)) \frac{d\beta}{\beta^2}, \quad (5.2)$$

and

$$z = \frac{\alpha_{soil} t}{R_{pipe}^2}, \quad (5.3)$$

$$p = \frac{r}{R_{pipe}}. \quad (5.4)$$

Ingersoll tabulated results of the Bessel function integral $G(z,p)$ for given values of z and p . These results are reproduced in Table 5.1.

Modifications were made to the finite difference model to make it comparable to the Ingersoll equations. The spiral heat exchanger, represented by parallel pipes, was simplified to a single buried pipe. The freezing algorithm was removed from the finite difference model, creating a conduction only analysis. The ground surface heat flux was set to zero, to prevent the air temperature and incident solar radiation from affecting the soil temperatures near the pipe. Because the transient analysis only simulated 3,000 minutes (50 hours), there was insufficient time for the far field radius (radius from pipe at which temperature effects are experienced) to expand out to the ground surface. Consequently no additional surface modification were required to simulate an infinite medium.

The entire ground field was initially set to a constant temperature of 5 °C. A constant heat flux at the pipe surface from the ground into the pipe of 206 W/m² was chosen. This is equivalent to a 2 °C temperature difference and a thermal resistance between the fluid and the ground (R_{source}) of 10 °C/W.

The Ingersoll transient analysis was compared with the finite difference model. The Ingersoll equation was developed in two dimensional cylindrical co-ordinates and the finite difference model is in Cartesian co-ordinates. In order to compare the two results, the grid size of the finite difference model was adjusted so that the first and second control volume centres from the pipe control volume were 5 and 10 times the pipe outer radius, or in terms of the Ingersoll equations, $p = 5$ and 10 . The pipe outer diameter was chosen to be 26.7 mm (3/4 " Series 160 HDPE pipe). This is an outer pipe radius of 13.4 mm. The control volume dimensions in the constant grid size region surrounding the pipe was set to 67 mm in both the X and Y directions.

In the finite difference model, the heat exchanger pipes are treated as volumetric source terms in the control volume in which they are located. The temperatures of the grids are calculated at their centre. If a pipe is located in a control volume, the control volume centre will also be the centre of the pipe. In order to properly compare ground temperature prediction at the pipe surface of the finite difference model with Ingersoll's ground temperature prediction, the pipe surface temperature was interpolated by assuming a linear temperature gradient between the centre of the grid containing the pipe and the centre of the next grid.

Ground temperatures calculated using the finite difference model and the Ingersoll equations were compared at the pipe surface, five and ten times the pipe outer radius. The comparisons are illustrated in Figures 5.1, 5.2 and 5.3 respectively.

At the pipe surface the finite difference model predicted a slightly higher temperature than the transient conduction equations. At 1,500 minutes (25 hours) there is a 0.29°C difference (24% difference) between the two predictions. The temperature difference at the pipe surface is most likely due to the assumption that the heat transfer through the pipe wall is a volumetric source acting over the entire control volume. A linear temperature gradient was assumed between the pipe control volume and the adjacent control volume to estimate the temperature of the pipe surface. At 5 and 10 pipe radii from the centre of the pipe, the finite difference model predicts a slightly lower ground temperature. At 1,500 minutes (25 hours) there is a $.08^{\circ}\text{C}$ difference between the two predictions at five pipe radii and $.08^{\circ}\text{C}$ at 10 pipe radii. This a 3 percent and 2 percent difference respectively.

There is good agreement between the finite difference model and Ingersoll's transient equations away from the pipe surface. At the pipe surface the agreement is not as good because of the volumetric source term assumption in the finite difference model. This effect becomes less pronounced away from the pipe.

At smaller control volume sizes the difference between the finite difference and Ingersoll predictions at the pipe surface is reduced. There is little change at 5 and 10 pipe radii. This indicates that the models are less compatible at the pipe surface

5.2 Comparison Of Finite Difference Model With Monitored Undisturbed Ground Temperature At Site In Wisconsin

The monitored undisturbed ground temperature is used to validate the ground conduction and surface heat transfer algorithms in the finite difference model without the effects of the ground heat exchanger pipes. The finite difference equations developed in Chapter 3 are used to model the undisturbed ground temperature. The pipe source terms in the soil are set to zero to predict the undisturbed ground temperature, effectively shutting off the ground heat exchanger.

Monthly plots of the predicted and monitored undisturbed ground temperature comparison are illustrated in Figures 5.4 to 5.11 for the simulation period. Monthly summaries of the comparisons are presented in Tables 5.2a and 5.2b for both 0.9 and 1.8 metres (3 and 6 ft) below the surface respectively. The “Efficiency Difference” column expresses the impact the difference between the monitored and predicted ground temperature would have on the heat pump efficiency. This percentage efficiency difference is based on the theoretical heat pump efficiency, and is discussed in more detail in Appendix C.

As seen in Table 5.2a, there is some difference between the predicted and monitored ground temperature at 0.9 metres (3 ft) below the surface. The predicted undisturbed ground temperature at 1.8 metres (6 ft), the depth at which the heat exchanger is buried, is reasonably accurate. The percentage efficiency difference of the average monthly undisturbed ground temperature at 1.8 metres (6 ft) is between 0.1 and 2.1 percent. The

predicted temperature at the shallower depth is more susceptible to surface conditions such as air temperature, wind speed and incident solar radiation. The latter two items were not measured at the monitoring site and were based on typical meteorological data ^[2]. At 1.8 metres (6 ft), the ground is more sheltered from these ambient conditions.

The large difference between the monitored and predicted ground temperatures 0.9 metres (3 ft) below the surface at the beginning of December indicate that a longer preliminary simulation period (the simulation was run for 10 days prior to December 1st) would have been more appropriate.

With the exception of the month of December at a depth of 0.9 metres (3 ft), the differences between the predicted and monitored undisturbed ground temperature less than the ± 0.5 °C accuracy of the typical thermocouple. Since no information was available regarding the monitoring devices used to collect data at the Wisconsin site, it was assumed that thermocouples were used for all temperature measurements for the purposes of this study.

In the undisturbed ground temperature plots of Figures 5.4 to 5.7, rapid changes in the ground temperature at 0.9 metres (3 ft) are observed throughout the simulation period which were discussed earlier in Section 4.2. These rapid changes are due to the inaccuracies in the freezing algorithm which assumes the ground control volumes have finite dimensions and are isothermal. Although the control volumes at 0.9 metres (3 ft) below the surface do not freeze during the period being simulated, they are affected by control volumes closer to the

surface and more exposed to the elements. As the control volumes within 0.9 metres (3 ft) of the surface freeze and thaw, the temperatures remain at zero degrees Celsius. After the phase change is complete in the control volumes closer to the surface, the temperature begins changing again, and the temperature of the lower grids begin to change more rapidly than when the freezing was taking place. At 1.8 metres (6 ft) below the surface ground temperatures are more stable, since they are less exposed to the surface effects.

The percentage differences between the monitored and predicted average monthly undisturbed ground temperatures was not presented because the absolute monthly temperature are small in magnitude, 0.7 to 1.8 °C for 0.9 metres (3 ft) below surface and 2.0 to 4.6 °C for 1.8 metres (6 ft) below the surface, making small differences appear excessively large.

The root square (RS) difference was determined by squaring the difference between the predicted and monitored ground temperatures at each fifteen minute time step and summing for the entire month. This is expressed mathematically by

$$RS = \sqrt{(T_{ground,mon} - T_{ground,pred})^2} . \quad (5.5)$$

The average root square difference is the average of all the fifteen minute root square differences for the month. As seen in Table 5.2a and 5.2b, there is good agreement between the monthly ground temperature difference and the difference. The difference is only a measure of the magnitude of the difference between the predicted and actual undisturbed

ground temperatures, and ignores whether the difference is positive or negative. Since in most months the ground temperatures are consistently above or below the actual temperatures, positive and negative differences are not cancelled by each other when the RS values are summed. Consequently, if the predicted and actual temperatures cross each other, such as January at 0.9 metres (3 ft) below the surface in Figure 5.5, the average difference and the RS difference will be different.

5.3 Comparison With Monitored EWTs From Slinky Heat Exchanger in Wisconsin

The monitored heat pump entering water temperature (EWT) data was used to validate the finite difference ground heat exchanger model and its interaction on surrounding environment. Monthly plots of the predicted and monitored heat pump entering water temperature comparison are illustrated in Figures 5.12 to 5.15 for the simulation period.

Table 5.3 presents the monitored and predicted average monthly heat pump entering water temperatures. The “Efficiency Difference” column expresses the impact the difference between the monitored and predicted heat pump EWT would have on the heat pump efficiency. This percentage efficiency difference is based on the theoretical heat pump efficiency, and is discussed in more detail in Appendix C.

Only small differences were observed in the average monthly EWTs, ranging from 0.0 to 0.2

degrees Celsius, well below the ± 0.5 °C accuracy of the typical thermocouple which were assumed to be used for monitoring temperatures at the Wisconsin site. The largest difference occurred in the month of March. This is also illustrated in the EWT comparison plot for March in Figure 5.15. The predicted heat pump EWT was visibly lower for this month, particularly during periods of partial flow rate. The cause of the under prediction in March is over-cooling of the ground in the simulation during the peak heating months (ie: January and February). Over-cooling of the soil surrounding the pipes is likely caused by an over-prediction of the energy extracted from the soil by the heat exchanger pipes in the finite difference model, or under-prediction of the soil's ability to transfer heat into the soil around the pipes.

Over-prediction of the energy extracted could be caused by the separate calculation performed to calculate the energy transferred into the heat exchanger fluid and energy extracted from the ground. The energy transferred into the fluid from the ground is calculated for each pipe control volume based on the temperature difference between the ground control volume in which the pipe is located and the fluid, and the thermal resistance between the fluid and the ground (R_{source}). This calculation was done for each explicit (3.2 second) time step in the pipe finite difference calculation. In contrast, the energy extracted from the ground was calculated for each implicit (15 minute) time step. This calculation was based on the difference between the average pipe control volume fluid temperature and the ground control volume temperature being calculated. The thermal resistance of the implicit time step was a time weighted average of the resistance with and without flow during the

implicit time step. To reduce the difference between energy transferred into the fluid and the energy extracted from the ground by the pipes, the energy transferred to the pipes during the explicit time steps could have been summed over the implicit time step then used for the implicit source terms in the ground finite difference equations.

Under-prediction of the soil's ability to transfer heat to the pipes could also cause under-prediction of the heat pump EWT in March. Since no measured soil properties at the monitoring site were available, typical properties for the observed soil type were used in the finite difference model ^[26]. There can, however, be large variations in thermal properties of a given type of soil, depending on a number of factors such as density, granular size and moisture content.

More detailed information on the thermal properties of the soil and monitored ground temperatures around the heat exchanger pipes would have allowed a more accurate prediction of the soil temperatures around the heat exchanger pipes and consequently better heat pump EWT predictions.

The difference between the predicted and monitored average monthly heat pump EWT is estimated to result in a 0.4 percent lower efficiency for the entire simulation period.

The root square (RS) difference was determined by squaring the difference between the predicted and monitored EWT at each fifteen minute time step and summing for the entire

month. This is expressed mathematically by the following equation.

$$RS = \sqrt{(EWT_{mon} - EWT_{pred})^2} . \quad (5.6)$$

The average root square difference is the average of all the fifteen minute root square differences for the month. The RS difference was larger than the average EWT difference calculated for each month. This is because the RS difference is a measure of the magnitude of the difference between the predicted and actual EWT, and neglects the sign (positive or negative). The average of the EWT difference is the sum of all the positive and negative differences divided by the number of observations. Summing the positive and negative values reduces the magnitude of the average difference. Figures 5.14 and 5.15 illustrate that the EWT fluctuations in the monitored entering water temperatures are significantly larger than the predicted fluctuations.

The finite difference model appears to predict the average heat pump entering water temperature very well, but slightly under-predicts short term fluctuations in the heat pump EWTs. Observing the EWT comparison plots in Figures 5.12 to 5.15 confirms that the actual short term fluctuations for the heat pump EWT are much larger than the fluctuations predicted by the program.

Table 5.1: Solutions to Ingersoll Equation $G(z,p)$ Functions

z	$G(z,p=1)$	$G(z,p=2)$	$G(z,p=5)$	$G(z,p=10)$
0.1	0.049			
0.2	0.067			
0.3	0.080			
0.4	0.090			
0.5	0.099			
0.6	0.107			
0.7	0.113			
0.8	0.118			
1.0	0.128	0.035	0.001	0.000
1.2	0.137			
1.5	0.148			
2.0	0.163			
2.5	0.175			
3.0	0.186			
4.0	0.203			
5.0	0.217	0.112	0.015	0.000
6.0	0.228			
8.0	0.247			
10.0	0.263	0.155	0.039	0.002
12.0	0.275	0.167	0.047	0.004
15.0	0.291	0.182	0.058	0.007
20.0	0.312	0.203	0.074	0.013
25.0	0.328	0.219	0.087	0.019
30.0	0.342	0.232	0.098	0.025
50.0	0.380	0.271	0.132	0.046
100.0	0.433	0.323	0.181	0.084
500.0	0.560	0.449	0.304	0.197
1000.0	0.614	0.504	0.359	0.250

Table 5.2a: Comparison of Monitored and Predicted Undisturbed Ground Temperatures At 0.9 metres (3 Feet)

Month	Monitored Undisturbed T _g (°C)	Predicted Undisturbed T _g (°C)	Undisturbed T _g Difference (°C)	Efficiency Difference (Appendix C) (%)	Average RS Undisturbed T _g Difference (°C)
December	1.8	2.4	-0.6	-3.2	0.6
January	1.1	1.3	-0.2	-1.3	0.3
February	0.8	0.5	0.4	1.9	0.4
March	0.7	0.4	0.4	1.8	0.4

Table 5.2b: Comparison of Monitored and Predicted Undisturbed Ground Temperatures At 1.8 metres (6 Feet)

Month	Monitored Undisturbed T _g (°C)	Predicted Undisturbed T _g (°C)	Undisturbed T _g Difference (°C)	Efficiency Difference (Appendix C) (%)	Average RS Undisturbed T _g Difference (°C)
December	4.6	4.8	-0.2	-1.6	0.2
January	3.1	3.4	-0.4	-2.1	0.4
February	2.4	2.4	0.0	0.1	0.1
March	2.0	1.9	0.1	0.4	0.1

Table 5.3: Comparison of Monitored and Predicted Average Monthly EWT

Month	Monitored EWT (°C)	Predicted EWT (°C)	EWT Difference (°C)	Efficiency Difference (Appendix C) (%)	Average RS EWT Difference (°C)
December	-0.4	-0.5	-0.1	-0.5	0.2
January	-2.4	-2.4	0.0	-0.2	0.2
February	-2.8	-2.9	-0.1	-0.3	0.2
March	-3.0	-3.2	-0.2	-0.9	0.2
Average	-2.2	-2.2	0.0	-0.4	0.2

Figure 5.1: Finite Difference and Ingersoll Transient Prediction of Ground Temperature at Pipe Surface

Figure 5.2: Finite Difference and Ingersoll Transient Prediction of Ground Temperature at 5 Pipe Radii

Figure 5.3: Finite Difference and Ingersoll Transient Prediction of Ground Temperature at 10 Pipe Radii

Figure 5.4: Monitored and Predicted Undisturbed Ground Temperature For December -0.9 Metre Depth

Figure 5.5: Monitored and Predicted Undisturbed Ground Temperature For January - 0.9 Metre Depth

Figure 5.6: Monitored and Predicted Undisturbed Ground Temperature For February - 0.9 Metre Depth

Figure 5.7: Monitored and Predicted Undisturbed Ground Temperature For March - 0.9 Metre Depth

Figure 5.8: Monitored and Predicted Undisturbed Ground Temperature For December - 1.8 Metre Depth

Figure 5.9: Monitored and Predicted Undisturbed Ground Temperature For January - 1.8 Metre Depth

Figure 5.10: Monitored and Predicted Undisturbed Ground Temperature For February - 1.8 Metre Depth

Figure 5.11: Monitored and Predicted Undisturbed Ground Temperature For March - 1.8 Metre Depth

Figure 5.12: Monitored and Predicted Heat Pump Entering Water Temperature For December, 1995

Figure 5.13: Monitored and Predicted Heat Pump Entering Water Temperature For January, 1996

Figure 5.14: Monitored and Predicted Heat Pump Entering Water Temperature For February, 1996

Figure 5.15: Monitored and Predicted Heat Pump Entering Water Temperature For March, 1996

Chapter 6 Parametric Study

6.1 Impact of Heat Pump Loads

A parametric study was undertaken to examine the impact of heat pump loads on the entering water temperatures. The data input file along with the monitored heat pump loads (energy extracted from the ground) used earlier in the model development were also used in this parametric study. To vary the heat pump loads for this parametric study, an adjustment factor was applied to the energy loads as they were read in by the program. This will illustrate the impact of using the same ground heat exchanger for a house with larger heating loads. For the monitored loads case, the adjustment factor is 1.0. In the low loads case, the monitored loads were reduced by fifty percent, or multiplied by the factor 0.67. The monitored loads were increased by a 50 percent, or multiplied by a factor of 1.5 for the high loads case.

Table 6.1 presents the monthly results of the heat pump loads parametric study. The average monthly heat pump EWT, the monthly maximum EWT and monthly minimum EWT have been calculated. The average, maximum and minimum values were also determined for the entire simulation period.

As expected, a fifty percent reduction in heat pump loads (low loads) increases the heat pump entering water temperatures (EWTs) experienced by the system. The EWT for the low

loads case never falls below $-2.2\text{ }^{\circ}\text{C}$ during the entire simulation period, and the average EWT for the simulation period is $-0.7\text{ }^{\circ}\text{C}$.

A fifty percent increase in heat pump loads (high loads) significantly decreases the temperature of the fluid returning from the ground heat exchanger. The average heat pump EWT for the high loads case over the entire simulation period is $-4.9\text{ }^{\circ}\text{C}$. The minimum EWT for the winter operation, experienced in February, is $-9.7\text{ }^{\circ}\text{C}$. As stated earlier, the minimum heat pump EWT recommended by the manufacturer of the ground-source heat pump is $-6.7\text{ }^{\circ}\text{C}$. Only December and January have an average EWT above the manufacturers lower limit, and clearly the heat pump could not operate adequately at these conditions.

The maximum heat pump EWTs occurred during December for the three different heat pump load scenarios. As expected the lowest maximum value was experienced for the high loads case, while the low loads case experienced the highest maximum return fluid temperatures.

Figures 6.2 to 6.5 are plots of the entering water temperatures for the three heat pump loads. The low loads curve (red) does not drop below freezing until early January, then remains between 0 and $2\text{ }^{\circ}\text{C}$ for the remainder of the simulation. The heat extraction by the heat exchanger pipes is now sufficiently low that the ground can transfer an adequate amount of heat to the pipes to maintain the temperature in the pipes. The phase change of the moisture around the pipes also helps to maintain the soil around the pipes near the freezing point,

which is why the return temperature is just below zero Celsius for most of the winter season. The high load curve (pink) illustrates what was observed in the monthly values in Table 6.1. The heat exchanger fluid temperatures are significantly lower than the EWTs observed for the monitored heat pump loads (blue). The most severe deviations from the monitored loads curve occur during full load periods, particularly during the end of January and beginning of February. These are the periods with the highest amounts of heat being extracted by the heat pump, which is increased by fifty percent for the high loads analysis. This is far beyond the heat exchangers ability to extract heat from the ground, and the temperature in the loop drops sharply during periods of high loads.

For the same reasons, the temperature recovery during periods of partial or no heat pump operation are larger for the high load case. Because the temperature in the loop is very low after a full load period, the temperature difference between the fluid and the ground is large and more heat is transferred through the pipe walls, resulting in a larger jump in the pipe fluid temperature.

For this parametric study, a single multiplier is used to adjust the heat pump loads. The impact of the EWT on the heat pump's performance was ignored. In reality, the heat pump's heating capacity and the amount of energy it extracts from the ground both drop at lower EWTs. Figure 6.1 illustrates the EWT's impact on the heat pump system used at the monitoring site. Although the performance data is not available at entering water temperatures experienced in the high loads case, it is clear that the heat extracted by the heat

pump would be lower at the temperatures predicted by the model. Assuming the heat pump could in fact operate at the temperatures experienced in the high heat pump loads simulation, the extraction rate would be lower than the monitored values used in the simulation. This would reduce the temperature drop of the fluid through the simulation period. Supplemental heating would be required due to the heat pump's lower capacity.

In a similar manner, the heat pump low loads case would have a higher extraction rate and higher COP (coefficient of performance). As a result, less compressor energy is required and more heat is extracted from the ground, reducing the predicted EWT.

Since the house loads and the heat pump performance were not modelled in this study, the impact of the predicted EWT on the heat pump performance was not examined.

The isotherm plots in Figures 6.6 and 6.7 show the ground temperatures at the end of February for the low loads and high loads simulations respectively. The ground temperatures for the monitored loads at the end of February were shown earlier in Figure 5.12. As expected, the ground temperatures around the pipes for the low loads case are significantly warmer. The ground temperatures around the pipes for the high loads are extremely low.

6.2 Impact of Heat Pump Loads and Heat Exchanger Length

An additional parametric study was performed to examine the impact of modifying both the

heat pump loads and the heat exchanger length by the same adjustment factor. As with the previous parametric study, adjustment factors of 1.0, 0.67 and 1.5 were used on both the loads and heat exchanger length.

Table 6.2 presents the monthly results of the heat pump loads and heat exchanger length parametric study. The average monthly heat pump EWT, the monthly maximum EWT and monthly minimum EWT have been calculated. The average, maximum and minimum values were also determined for the entire simulation period.

A fifty percent reduction in heat pump loads (low loads) and heat exchanger length decreases the heat pump entering water temperatures (EWTs) experienced by the system. The EWT for the low loads case never falls below -5.4°C during the entire simulation period, and the average EWT for the simulation period is -2.7°C .

A fifty percent increase in heat pump loads (high loads) and heat exchanger length significantly increases the temperature of the fluid returning from the ground heat exchanger. The average heat pump EWT for the high loads case over the entire simulation period is -2.0°C . The minimum EWT for the winter operation, experienced in February, is -3.9°C .

This would indicate that the heat exchanger length has a larger impact on the heat pump EWT than the heat pump loads. This is probably due to the non-linear nature of the heat exchanger efficacy. An infinitely long ground heat exchanger with an efficacy of 100

percent would produce a heat pump EWT of the undisturbed ground at the depth of the heat exchanger. Figures 6.8 to 6.11 are plots of the entering water temperatures for the three adjustment factors.

Table 6.1: Impact of Heat Pump Loads On Monthly Heat Pump Entering Water Temperatures

Month	Average Monthly Heat Pump EWT (°C)		Minimum Monthly Heat Pump EWT (°C)		Maximum Monthly Heat Pump EWT (°C)		
	Low Loads	Monitored High Loads	Low Loads	Monitored High Loads	Low Loads	Monitored Loads	High Loads
December	0.4	-0.5	-2.0	-0.4	-2.0	1.6	1.2
January	-0.8	-2.4	-5.5	-1.8	0.9	0.0	-0.6
February	-1.1	-2.9	-6.2	-1.9	-0.1	-1.0	-3.8
March	-1.3	-3.2	-6.3	-2.2	-0.2	-1.4	-3.5
Sim. Period	-0.7	-2.2	-4.9	-2.2	-2.0	1.6	1.2

Table 6.2: Impact of Heat Pump Loads and Heat Exchanger Length On Monthly

Heat Pump Entering Water Temperatures

Month	Average Monthly Heat Pump EWT (°C)			Minimum Monthly Heat Pump EWT (°C)			Maximum Monthly Heat Pump EWT (°C)		
	Low Loads /Short Length	Monitored Loads /Actual Length	High Loads /Long Length	Low Loads /Short Length	Monitored Loads /Actual Length	High Loads /Long Length	Low Loads /Short Length	Monitored Loads /Actual Length	High Loads /Long Length
December	-1.1	-0.5	0.1	-2.6	-2.0	-1.1	0.9	1.6	2.5
January	-2.9	-2.4	-1.8	-4.7	-4.1	-3.4	-0.1	0.0	0.2
February	-3.3	-2.9	-2.2	-5.3	-4.7	-3.8	-1.3	-1.0	-0.6
March	-3.6	-3.2	-2.5	-5.4	-4.9	-4.1	-1.7	-1.4	-0.8
Sim. Period	-2.7	-2.2	-1.6	-5.4	-4.9	-4.1	0.9	1.6	2.5

Figure 6.1: Impact of Heat Pump Entering Water Temperature (EWT) On Performance

Figure 6-2: Impact Of Heat Pump Loads On Predicted EWT For December -95

Figure 6-3: Impact Of Heat Pump Loads On Predicted EWT For January -96

Figure 6-4: Impact Of Heat Pump Loads On Predicted EWT For February -96

Figure 6-5: Impact Of Heat Pump Loads On Predicted EWT For March -96

Figure 6-6: Undisturbed Ground Temperature Plot With Low Heat Pump Loads - February 29th

Figure 6-7: Undisturbed Ground Temperature Plot With High Heat Pump Loads - February 29th

Figure 6-8: Impact Of Heat Pump Loads and HX Length On Predicted EWT For December -95

Figure 6-9: Impact Of Heat Pump Loads and HX Length On Predicted EWT For January -96

Figure 6-10: Impact Of Heat Pump Loads and HX Length On Predicted EWT For February -96

Figure 6-11: Impact Of Heat Pump Loads and HX Length On Predicted EWT For March -96

Chapter 7 Future Work

Some additional work is required to refine the finite difference model presented in this thesis. A practical ground heat exchanger model must be able to simulate an entire year of operation, and the simulation developed in this study is a winter only model. The simulation of summer operation should include moisture migration. Moisture migration is the term used to describe the phenomenon of moisture being driven away from the soil surrounding warm buried pipes, resulting in the drying out of the soil. Soil thermal properties are heavily dependant on the moisture content of the soil, and consequently moisture migration can have a significant impact on the heat exchanger performance. A simple method of simulating moisture migration in summer, would be to modify the soil properties near the pipes. Alternatively, more complex diffusion equations such as those used by Tomawski and Leong [12,13] could be used. The disadvantage of a complex model such as this, is that the moisture diffusion properties of the soil are required. These properties are not as well documented for different soils as the thermal properties.

An adjustment to the heat pump load should be added to the program to account for the impact of the heat pump EWT. The EWT will affect the heat pump COP (coefficient of performance) and the energy extraction while the heat pump is heating. If variations in the heat exchanger and system design (heat exchanger length, house loads, etc.) and environmental factors (eg: air temperature, ground temperature, etc.) are to be studied, the heat pump EWTs will change. Consequently, the impact of the heat pump EWT changes

should be included in the model to better reflect the system performance.

Further investigation of the program's under-prediction of short term fluctuations is required. The long term peaks caused by prolonged full load periods, which are more important from a heat exchanger sizing perspective, are well predicted by the program. But good prediction of the short term fluctuations is important for analysing the energy consumption of the heat pump system.

Further validation with data from monitored systems that have measured information about the soil thermal properties (conductivity, moisture content, thermal diffusivity, etc), intermediate fluid temperatures measurements and ground temperature data near the heat exchanger would improve the model's prediction capabilities. A number of approximations were required to model the Wisconsin slinky heat exchanger, such as soil thermal properties, contact resistance and soil moisture content. When a large number of assumptions are made in a computer simulation, it is possible that multiple errors compensate for each other, masking errors in the model. The more variables that are measured the less chance there is for these types of errors to go undetected.

Chapter 8 Conclusions

A finite difference program was developed to model a slinky ground heat exchanger for a residential ground-source heat pump system. Slinky ground heat exchangers are constructed from overlapping spirals of pipe layed flat in the bottom of a trench. To simplify the model, the heat exchanger is simulated as an equivalent length of equally spaced straight pipes. A simulation of an actual residential GSHP system located near Eau Claire, Wisconsin was performed from the beginning of December to the end of March. The results of the simulation were compared with monitored entering water temperatures and undisturbed ground temperatures from the ground-source heat pump system in Wisconsin.

The fluid flowing through the pipes was modelled using a one dimensional explicit time step, finite difference algorithm. The ground heat transfer was modelled with a two dimensional, implicit time step finite difference algorithm at each pipe control volume along its length. The ground heat transfer along the axial direction of the pipes was neglected. The ground was subdivided along the length of the pipe into control volumes equal to the pipe control volumes.

The heat exchanger pipes caused significant cooling in the surrounding ground during the peak heating months, which extends up to the ground surface. The impact on the ground below the pipes is much lower, and there is negligible impact beyond one metre below the pipes. Horizontally, at the depth of the heat exchanger, there is a large jump in ground

temperature between the control volumes in which the pipes are located, and the adjacent control volumes. During peak months, the difference is approximately $0.7\text{ }^{\circ}\text{C}$ for control volumes with a spacing of 10 centimetres ($0.07\text{ }^{\circ}\text{C}/\text{cm}$).

The finite difference model was compared with a transient solution for a single buried pipe located in an infinite medium without ground freezing. Given an initial and uniform soil temperature around a buried pipe and the heat flux at the pipe surface, the transient equations predict the change in ground temperature with time at various radial distances from the pipe. The finite difference program was modified to make the results comparable. After 25 hours, the finite difference program over predicted the temperature at the pipe surface by $0.3\text{ }^{\circ}\text{C}$ and under predicted the ground temperature by $0.1\text{ }^{\circ}\text{C}$ at 5 and 10 pipe radii.

The finite difference predictions of the undisturbed ground temperature were compared with monitored temperatures. The finite difference algorithm produced good estimates of undisturbed ground temperature at 1.8 metres (six feet) below the surface. The average root square difference over the simulation period was $0.2\text{ }^{\circ}\text{C}$. The results were not as good at 0.9 metres (3 feet) below the surface, with an average root square difference over the simulation period of $0.4\text{ }^{\circ}\text{C}$. Data for surface effects such as solar radiation, wind speed and snow cover were not available at the monitoring site and had to be taken from typical meteorological year weather files. The surface effects have more of an impact at the three foot depth because it is more exposed to the surface.

The finite difference predictions of heat pump entering water temperature (EWT) were compared with the monitored EWTs. There was extremely good agreement between the average EWTs over the simulation period. The difference between the predicted and measured EWT was less than 0.1 °C for the entire simulation period. The average root square difference was greater at 0.2 °C. The estimated efficiency difference due to the difference between the predicted and monitored heat pump EWT is 0.4 percent. The finite difference program under predicted the magnitude of short term fluctuations in the heat pump EWT. Over the simulation period, the errors in predicting the peaks and valleys in the EWT cancelled each other out, resulting in good average temperature results. The finite difference program still gave a reasonable prediction of the short term fluctuations.

The heat pump loads were modified to examine the impact they had on the entering water temperatures. Simulations were run with a fifty percent reduction and a fifty percent increase in the heat pump loads on the ground. There was a significant deviation from the EWTs predicted with the base case monitored loads. The average EWTs for the simulation period were -0.7 °C, -2.2 °C and -4.9 °C for the low loads, base case loads and the high loads respectively. The heat pump would likely not be able to operate for the full heating season in the case of the high heat pump loads because of the extremely low entering water temperatures. The loads were not adjusted to account for the impact of EWT on the heat pump performance.

Both heat pump loads and heat exchanger length were also modified by the same factor to

examine the impact on the entering water temperatures. Lower loads and a shorter length resulted in lower heat pump entering water temperature. Higher loads and a longer length resulted in higher heat pump entering water temperature. This indicates that the heat exchanger length and heat pump loads are not linear, and the heat exchanger length has a larger impact on the heat pump EWT than the heat pump loads.

Additional work is required to include summer operation in the heat pump simulation. To simulate the heat transfer during the summer, moisture migration in the soil around the pipes needs to be included in the ground heat transfer model. This could either be done with a simple adjustment of the soil's thermal properties during the summer, or by the use of moisture diffusion equations. An adjustment is required to modify the monitored heat pump loads on the ground for variations in the predicted EWT if further parametric studies are to be done. Further validation should also be considered to improve the prediction of the heat transfer around the pipes.

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APPENDIX A

FORTRAN Implementation of The Finite Difference Algorithms

C ** Finite Difference Program to Model Slinky Horizontal Ground Heat Exchanger **
C ** For Ground-Source Heat Pump System
C ** Filename: DIFFER-3.FOR **

INCLUDE 'FGRAPH.FT'
INCLUDE 'FGRAPH.FD'

C ----- Definition of Data Types -----

REAL*4 PI, dt, time, tstart, BD ! General variables
REAL*4 ID, OD, L, Kpipe ! Pipe variables
REAL*4 Rho, Cp_f, Mu, Kfluid, Pr, Qflow, Mflow, Red, M ! Fluid variables
REAL*4 hfluidf, hfluidnf, Rfluidf, Rfluidnf, kice
REAL*4 Ksoil, alpha, RHO_g, Cp_g, Cp_w, moisture, Dmean, depth ! Soil variables
REAL*4 Kback, alpha_b, RHO_b, Dback ! Backfill variables
REAL*4 ke(32,35), kw(32,35), ks(32,35), kn(32,35) ! Interface conductivity
REAL*4 Tm, As, to, Ksnow, Dsnow ! Kusuda & Achenback variables
REAL*4 BCX, X, Y, Z, dX(32), dY(35), dZ, dL ! Cartesian coordinate variables
REAL*4 Rcontact, Rpipe, Rsourcef, Rsourcecf, Rsource ! Resistance variables
REAL*4 a, b, c, KUSUDA, EWT, LWT, LWTa ! Fluid Temp variables
REAL*4 Tfluid(25), Tpsum(25), Tfold(25,10), P(35), Q(35), Tf, Qhp ! Fluid Temp variables
REAL*4 Qsol, hair, ABSg, ABSs, flow, Vwind ! Surface temp. variables
REAL*4 T(32,35,25), Qfreeze(32,35,25), Sc, Sp ! Ground conduction var's
REAL*4 Ayp1, Aym1, Axp1, Axm1, area, Ao, Atot, Bo, d ! Ground conduction var's
REAL*4 trun

INTEGER*4 i, j, k, Itime
INTEGER*4 NZ, Nequiv, Nt, NgridX, NgridY, Iday, Icount
INTEGER*4 ipipe(10), jpipe(10), Pflag
INTEGER*2 ihr1, imin1, isec1, i100th1, ihr2, imin2, isec2, i100th2

C ----- Open I/O Files -----

OPEN(1, file = 'ground.out')
OPEN(2, file = 'fluid.out')
OPEN(3, file = 'grid.out')
OPEN(4, file = 'WI-15min.inp')
fact = 1.0

C ----- Variable Definitions -----

C ** General Definitions **
PI = ACOS(-1.0) ! Calculate PI
tstart = -1*(10+31)*24*60 ! Start of simulation from Jan. 1st in minutes
dt = 5.0 ! Time step in minutes
Nt = (1*24*60)/dt ! Number of time steps
BD = 2.1 ! Burial depth of pipe below surface, metres

C ** Pipe Properties **

ID = 0.0219 ! Pipe inner diameter in metres
OD = 0.0267 ! Pipe outer diameter in metres
L = 400/3.281/4.0*Flength ! Trench length in metres
Kpipe = 0.391 ! Conductance of pipe, W/mK

C ** Fluid Properties **

C 25% Ethanol solution

Rho = 967 ! Fluid density in kg/m³
Cp_f = 4.25E3 ! Heat capacity of fluid, J/kg K
Mu = 2.8E-3 ! Dynamic viscosity, kg/m s
Kfluid = 0.46 ! Fluid conductivity, W/m K
Pr = 41.6 ! Prandtl number, non-dimensional

C ** Ground Properties **

C ** Assume soil is damp clay soil **

Ksoil = 1.90 ! Soil conductivity in W/mK
kice = 2.24 ! Conductivity of ice W/mK
RHO_g = 2096.0 ! Soil density in kg/m³
Cp_g = 0.962E3 ! Soil heat capacity in J/(kg*K)
alpha = Ksoil/(RHO_g*Cp_g) ! Calculated soil diffusivity
Cp_w = 334.0E3 ! Latent heat of fusion for water (J/kg)
moisture = 0.094 ! % moisture content of ground - based on Mei

C ** Surface Properties **

ABSg = 0.8 ! Absorptivity of ground
ABSs = 0.4 ! Absorptivity of snow
ABS = ABSs
Ksnow = 0.20

C ** Trench Backfill Properties **

C ** Assume backfill is saturated sand **

Kback = Ksoil ! Backfill conductivity in W/mK
alpha_B = alpha ! Backfill diffusivity in m/s²
RHO_B = rho_G ! Backfill density in kg/m³
Cp_B = Cp_g ! Backfill heat capacity in J/(kg*K)
Dback = 2.1 ! Depth of backfilled trench (m)
Wback = 1.1 ! Width of backfill trench (m)

C ** Ground Temperature Properties **

Tm = 8.5 ! Mean ground temperature (deg. C)
As = 15.5 ! Amplitude of ground surface temperature (deg. C)
to = 4 ! Time of minimum ground temp. from January 1st (days)

C ** Grid Spacing of Ground **

BCX = 6.0 ! Distance between left and right boundaries in X-dir., metres

```

C ** Number of Grids in Ground - Note, # Y & Z grids must be odd so that pipe
C ** is located in middle of centre grid. **
NZ = 25 ! Number of grids in Z-direction
Npipe = NZ ! No. of grids in pipe same as in ground
write(*,*) 'hi'
dZ = (L)/NZ ! Size of grids in Z-direction, metres
! This assumes negligible edge effects @ ends of pipe
write(*,*) 'hi'
dL = dZ ! Grid size in pipe = grid size in Z-direction of ground
Dmean = 13 ! Depth at which ground temperature is Tmean continuously

C ** Fluid analysis time step calculation **
Dflow = 10.8*3.7854/60 ! Design flow rate, litres/s
Vflow = (Dflow/1000)/(PI*ID**2/4.0)/4.0 ! Fluid velocity through indiv. HXs
dtpipe = dL/Vflow*.95 ! time step for fluid analysis in seconds
Dmass = (Dflow/1000)*Rho/4.0 ! Mass flow rate per pipe @Design flow, kg/s
Red = 4*Dmass/(PI*ID*Mu) ! Reynold's number, non-dimensional
M = Rho*(PI/4*ID**2*dL) ! Mass of fluid in pipe control volume

C ** Thermal Resistance Equations **
Rcontact = 0.015 ! Contact resistance between ground and pipe, K/W
Rpipe = LOG(OD/ID)/(2*PI*dL*Kpipe) ! Cond. resistance of pipe, K/W
hfluidnf = 22.0 ! Modified due to error in Beta assumption
hfluidf = 0.023*Red**(0.8)*Pr**(0.4)*Kfluid/ID !Heat trans. coef. of fluid, W/m^2 K
Rfluidf = 1/(hfluidf*PI*ID*dL) !Conv res. of fluid, K/W
Rfluidnf = 1/(hfluidnf*PI*ID*dL) !Conv res. of fluid, K/W
Rsourcef = (Rcontact + Rpipe + Rfluidf) ! Total thermal res. between fluid & ground (k/W)
Rsourcenf = (Rcontact + Rpipe + Rfluidnf) ! Total thermal res. between fluid & ground
(k/W)

C ** HX spiral properties **
Nequiv = 6 ! No. of straight Horiz. pipes to represent spiral (incl. return)
Spipe = 0.210 ! Horizontal spacing between pipes (metres)
Xspiral = 0.0 ! X-location of spiral centre @ midpoint of area
Yspiral = BD ! Y-loc. of spiral (Always on single plane @ HX BD)

CALL VARIABLE(BCx,ispiral,dx,BD,Dmean,jspiral,dY,NgridX,NgridY) ! Call
subroutine to calculate variable grid spacing
CALL PIPELOC(Xspiral,Yspiral,Nequiv,Spipe,BCX,dX,dY,Dmean ! Call subroutine to
determine grids with pipes
& ,ipipe,jpipe,NgridX, NgridY)

C ----- Begin Program -----

CALL CLEARSCREEN($GCLEARSCREEN) ! Clear screen
CALL GETTIM(ihr1,imin1,isecl,i100th1) ! Read time program starts (to calc. run time)
CALL GETDAT(iyr1,imon1,iday1) ! Read date when program starts (to calc run time)

```

```

C ** Initial Ground Temperatures and energy content**
Depth = 0 ! Start @ ground surface and move down
DO 0160 j = Ngridy, 1, -1 ! Start @ Ngridy (surface) and work down
  IF (j .EQ. 1) THEN ! If CV is @ surface
    Depth = depth + dY(j)*0.5 ! Depth at centre of C.V.
  ELSE ! CV is not @ surface
    Depth = depth + dY(j)*0.5 + dY(j+1)*0.5 ! Depth at centre of C.V.
  ENDIF
  DO 0140 k = 1, NZ, 1 ! Count through each CV in Z-dir
    DO 0120 i = 1, Ngridx, 1 ! Count through each CV in X-dir
      T(i,j,k) = KUSUDA(depth,tstart,alpha,As,Tm,to)*1.0 ! Undisturbed ground Temp
      vol = dX(i)*dY(j)*dZ ! Calculate CV volume (m^3)
      Qfreeze(i,j,k) = Cp_w*RHO_g*vol*moisture ! Energy req'd to freeze ground (Joules)
0120    CONTINUE
      DO 0110 Pcount = 1,Nequiv ! Set Tfluid from prev. time step for each pipe
        Tfold(k,Pcount) = T(1,1,k) ! Set prev. old Tfluid to undist. Tg @ same depth (j)
0110      CONTINUE
0140    CONTINUE
0160  CONTINUE

C ** Initial Fluid Temperature Leaving Heat Pump/Entering HX
LWT = KUSUDA(BD,tstart,alpha,As,Tm,to) ! Set initial LWT to undist. Tg

DO 2000 Icount = 1, (10+31+31+29+31)*96*3 ! Time counter (Dec.-March + start-up)
  time = dt*(Icount-1) + tstart ! Time in minutes from Jan. 1st (neg -> earlier)

  IF (MOD(Icount,3) .EQ. 1) THEN ! Only read data every third time step (input file 15
min. data)
    C ** The following data is read from input file WI-15MIN.INP
    C ** Iday -> The integer value representing the simulation day (neg -> start-up)
    C ** Itime -> The time step for each day (1-96)
    C ** Tair -> The ambient air temp. from Wisconsin site monitored data (deg C)
    C ** energy -> The monitored energy rejected to loop by HP - neg. is energy extracted (kJ)
    C ** Qsol -> Horizontal solar energy from typical meteorological year (TMY2) (W/m^2)
    C ** flow -> Flow rate through circ. pump, calc. from design flow & monit. trun (litres)
    C ** Vwind -> Wind speed from typical meteorological year (TMY2) for surface calcs (m/s)
    C ** LWTa -> Monitored heat pump leaving water temp., also heat exchanger entering temp.
(deg C)
    C ** trun -> Monitored heat pump/circ. pump run time for each 15 min time period (minutes)
    READ(4,1200) Iday,Itime,Tair,energy,Qsol,flow,Vwind,LWTa,trun ! Monitored data
    energy = kJ
    energy = energy/3.*fact ! Convert 15 min. energy data to 5 min (kJ)
    flow = flow/3. ! Convert 15 min. flow data to 5 min (litres)
    trun = trun/3. ! Convert 15 min. run time data to 5 min (minutes)
    hair = 5.70 + 3.81*Vwind

```

```

C ** Depth of snow for various times of year based on TMY data
  IF (icount .LE. 10*96*3) THEN ! November
    Dsnow = 0.04 ! Depth in metres
  ElseIf (icount .LE. (10+15)*96*3) THEN ! 1st half December
    Dsnow = 0.04 ! Depth in metres
  ElseIf (icount .LE. (10+31)*96*3) THEN ! 2nd half December
    Dsnow = 0.07 ! Depth in metres
  ElseIf (icount .LE. (10+31+15)*96*3) THEN ! 1st half January
    Dsnow = 0.05 ! Depth in metres
  ElseIf (icount .LE. (10+31+31)*96*3) THEN ! 2nd half January
    Dsnow = 0.18 ! Depth in metres
  ElseIf (icount .LE. (10+31+31+14)*96*3) THEN ! 1st half February
    Dsnow = 0.60 ! Depth in metres
  ElseIf (icount .LE. (10+31+31+29)*96*3) THEN ! 2nd half February
    Dsnow = 0.34 ! Depth in metres
  ElseIf (icount .LE. (10+31+31+29+15)*96*3) THEN ! 1st half March
    Dsnow = 0.31 ! Depth in metres
  ElseIf (icount .LE. (10+31+31+29+31)*96*3) THEN ! 2nd half March
    Dsnow = 0.12 ! Depth in metres
  ENDIF

  ENDIF ! Finished check for every third time step
1200  FORMAT(2I5,F7.2,2F9.0,F7.1,F7.1,F8.1,F7.2) ! Read data variable format

C ** Calculation of Interface Conductivities **
C ** Since backfill can be different material than ground **
  Y = -Dmean + 0.5*dY(1) ! Start @ bottom BC and move upwards to surface
  DO 0240 j = 1, NgridY, 1 ! Move to next Y grid
    X = -BCX/2.0 + 0.5*dX(1) ! Start X sweep @ left BC
    DO 0220 i = 1, NgridX, 1 ! Move to next X grid
      CALL GROUND(Dback,ke,kw,ks,kn,dX,dY,Ksoil,Kback,Wback
    &    ,X,Y,i,j) ! Call subroutine to calc. interface conduct.
      IF (Qfreeze(i,j,NZ/2+1) .EQ. 0) THEN ! Check if grid is frozen
C If adjacent grids also frozen adjust interface conduct. to account for ice conductivity
        IF (Qfreeze(i,j+1,NZ/2+1) .EQ. 0) kn(i,j)=kice*.2+kn(i,j)*.8
        IF (Qfreeze(i,j-1,NZ/2+1) .EQ. 0) ks(i,j)=kice*.2+ks(i,j)*.8
        IF (Qfreeze(i+1,j,NZ/2+1) .EQ. 0) ke(i,j)=kice*.2+ke(i,j)*.8
        IF (Qfreeze(i-1,j,NZ/2+1) .EQ. 0) kw(i,j)=kice*.2+kw(i,j)*.8
      ENDIF
      X = X + (dx(i) + dx(i+1))/2.0 ! X-location of grid centre
0220  CONTINUE
      Y = Y + dY(j)*0.5 + dY(j+1)*0.5 ! Y-location of grid centre
0240  CONTINUE

C ** Flow Properties **
  Qflow = flow/(dt*60) ! Volumetric flow rate through heat pump, l/s
  Mflow = (Qflow/1000)*Rho/4.0 ! Mass flow rate per pipe, kg/s

```

```

C ** Boundary Temperatures of Ground - Updated for Each Time Step **
  DO 400 k = 1, NZ, 1
    DO 0500 i = 1, Ngridx, 1
      T(i,1,k) = Tm ! Bottom boundary - Assume deep enough that temp. is constant
0500    CONTINUE
0400  CONTINUE

C ** Calculation of Heat Transfer in Pipes **
C ** Implicit ground time step dt broken down into explicit time steps for **
C ** pipe, which are ~ time required for flow to pass completely from one pipe **
C ** grid to the next. **
  Ntime = INT((trun*60)/dtpipe+0.9) ! number of sub time steps for pipe fluid analysis
  a = 1 - Dmass/M*dtpipe - dtpipe/(M*Cp_f*Rsourcef)
  b = Dmass/M*dtpipe
  c = dtpipe/(M*Cp_f*Rsourcef)

  EWTsum = 0 ! Reset sum of EWTs at each pipe analysis time step
  tsum = 0 ! Reset summ of explicit time substeps

  IF(trun .EQ. dt) THEN ! Check if operation continuous for time step
    Nflow = 0 ! If continuous, a no flow time step is not required.
  ELSE
    Nflow = 1 ! If not contin., a no flow time step is required.
  ENDIF

  IF (trun .GT. 0) THEN ! trun is not zero
    Qhp = energy/(trun*60)*1000 ! Determine the rate heat is added to loop (Watts)
  ELSE
    Qhp = 0 ! trun is zero, cannot divide by zero and Qhp is zero anyway.
  ENDIF

  DO 1450 It = 1, Ntime + Nflow, 1 ! Run through explicit calc. of fluid in pipe
c    IF (time .LE. 10*24*60) THEN ! If it is start-up time step, then use monitored LWT
      IF (Icount .LE. 10*96*3) THEN ! If it is start-up time step, then use monitored LWT
        LWT = LWTa ! Set LWT to monitored LWT
      ENDIF

      tsum = tsum + dtpipe ! Sum of explicit time substeps
      IF (It .EQ. Ntime + 1) THEN ! It is the no flow time step (last)
        Mflow = 0 ! No flow occurs during this time step
        dtp_nf = 60*(dt - trun) ! The size of the no flow time step in seconds
        a = 1 - Mflow/M*dtp_nf - dtp_nf/(M*Cp_f*Rsourcef)
        b = Mflow/M*dtp_nf
        c = dtp_nf/(M*Cp_f*Rsourcef)
      ENDIF
    
```

```

DO 1400 k = 1,Npipe,1 ! Step through analysis for each pipe control volume
  IF (It .EQ. 1) THEN ! If 1st explicit time step, reset sums to calc. avgs
    Tpsum(k) = 0
  ENDIF
  DO 1300 Pcount = 1,Nequiv ! Run fluid temp. calculation for each pipe
    IF (k .EQ. 1 .AND. Pcount .EQ. 1) THEN ! 1st grid of pipe, fluid enters from HP
      Tfluid(k) = a*Tfold(k,Pcount) + b*LWT
    &      + c*T(ipipe(Pcount),jpipe(Pcount),k)
    ELSE
      Tfluid(k) = a*Tfold(k,Pcount) + b*Tprev !
    &      + c*T(ipipe(Pcount),jpipe(Pcount),k)
    ENDIF
  C ** Fluid entering next grid will be fluid from this grid @ last time step
    Tprev = Tfold(k,Pcount) ! Set Tprev to old fluid T
    Tfold(k,Pcount) = Tfluid(k) ! Set old Tfluid for next time step
  1300 CONTINUE ! Go to next equivalent pipe
    IF (It .EQ. Ntime + 1) THEN ! No flow (last) explicit time step
      Tpsum(k) = Tpsum(k) + Tfluid(k)*dtp_nf ! Time weighted Tfluid sum for no flow
    ELSE ! Fluid flow explicit time step
      Tpsum(k) = Tpsum(k) + Tfluid(k)*dtpipe ! Time weighted Tfluid sum for flow
    ENDIF

  1400 CONTINUE
    IF (It .LT. Ntime + 1) THEN ! Only sum EWT for explicit time steps with flow
      EWTsum = EWTsum + Tfluid(Npipe) ! Sum of Tfluid as it leaves last pipe CV
    ENDIF
    IF (Mflow .GT. 0) THEN ! If flow thru' HP, calc. LWT
      LWT = Tfluid(Npipe) + Qhp/(Dflow*RHO/1000*Cp_f) ! The HX fluid leaving the heat
      pump (deg C)
    ENDIF
  1450 CONTINUE

  C ** Weight Rsource between flow and no flow time steps
    Rsource = dt/(trun/Rsourcef + (dt-trun)/Rsourcef)

    IF (Qflow .EQ. 0) THEN ! When there is no flow, set EWT to -999.0
      EWT = -999.0 ! Default EWT for no flow
    ELSE ! When fluid flowing, set EWT to average over time step dt
      EWTold = EWT
      EWT = EWTsum/Ntime ! Avg. temp. of fluid leaving heat exchanger
    ENDIF

    Qhp = energy/(dt*60)*1000 ! The rate heat is added to loop (Watts)
    IF (Qflow .EQ. 0) THEN ! If flow thru' HP, calc. LWT
      LWT = EWTold ! Set LWT to EWT from last time step when no flow
    ELSE
      LWT = EWT + 1000*energy/(flow*RHO/1000*Cp_f) ! The HX fluid leaving the heat

```

```

        pump (deg C)
    ENDIF

    IF (MOD(Icount,3) .EQ. 1 .AND. icount .GT. 10*96*3) THEN ! Write data every 3rd time
        step
        write(2,1500) Iday,Itime,time,EWT,LWT,LWTa,trun,T(14,12,10)
    ENDIF

1500  FORMAT(I6,',',I5,',',F9.2,',',F8.2,',',F9.2,',',F9.2,',',F9.2
    & ',',F9.2)
        write(*,1510) Iday,Itime,time,EWT,LWT,LWTa,LWT-LWTa,trun,Dsnow

1510  FORMAT(2I6,F8.0,4F9.2,3F9.2)

C ** Ground Conduction **
    Z = 0 ! Start simulation @ one end of heat exchanger
    DO 2100 k = 1, NZ, 1
        Tf = Tpsum(k)/(dt*60) ! Avg. fluid temp for time step - for Sp & Sc
        X = -BCX/2.0 ! Start X sweep @ left BC
        DO 2200 i = 1, Ngridx, 1
            Y = -Dmean ! Start TDMA @ bottom BC and move upwards to surface
            DO 2300 j = 1, Ngridy, 1
                CALL SOURCE(i,j,ipipe,jpipe,Tf,Rsource,Sc,Sp,Pflag
    &         ,Nequiv,dX,dY,dZ) ! Subroutine to calc source terms
                IF (j .EQ. 1) THEN ! Y TDMA coefficients @ bottom BC
                    Aym1 = 0
                    Ayp1 = Kn(i,j)*dx(i)/(dY(j+1)/2.0 + dY(j)/2.0)
                ELSEIF (j .EQ. Ngridy) THEN ! Y TDMA coefficients @ ground surface
                    Tsnow = (Dsnow/Ksnow*(Tair*hair + Qsol*ABS)+T(i,j,k))
    &         /(Dsnow/Ksnow*hair + 1)
                    Qsur = (Tsnow - T(i,j,k))/(Dsnow/Ksnow)
                    Ayp1 = 0
                    Aym1 = Ks(i,j)*dx(i)/(dY(j-1)/2.0 + dY(j)/2.0)
                ELSE ! Y TDMA coefficients away from BCs
                    Ayp1 = Kn(i,j)*dx(i)/(dY(j)/2.0 + dY(j+1)/2.0)
                    Aym1 = Ks(i,j)*dx(i)/(dY(j-1)/2.0 + dY(j)/2.0)
                ENDIF
                IF (i .EQ. 1) THEN ! X TDMA coefficients
                    Axp1 = Ke(i,j)*dY(j)/(dx(i)/2.0 + dx(i+1)/2.0)
                    Axm1 = 0 ! No heat transfer between i=1 and i=0 nodes
                ELSEIF (i .EQ. Ngridx) THEN
                    Axp1 = Ke(i,j)*dY(j)/(dx(i)/2.0 + dx(i+1)/2.0) ! No heat transfer between i=NZ and
i=NZ+1 nodes
                    Axm1 = Kw(i,j)*dY(j)/(dx(i-1)/2.0 + dx(i)/2.0)
                ELSE
                    Axp1 = Ke(i,j)*dY(j)/(dx(i)/2.0 + dx(i+1)/2.0) !X TDMA coefficients away from
adiabatic boundaries

```



```

      Axml = Kw(i,j)*dY(j)/(dx(i-1)/2.0 + dx(i)/2.0)
    ENDIF
C ** Calculate cross-sectional area for grid spacing in XY plane. This is used
C ** to determine the heat capacity of the grid in the finite equation. Subtract
C ** the pipe area where the pipe is in the grid.
      IF (Pflag.EQ. 1) THEN ! Pipe located in ground grid
        area = dX(i)*dY(j) - PI*OD**2/4 ! Reduce ground vol. by pipe vol.
      ELSE ! Pipe is not located in grid --> No source term
        area = dX(i)*dY(j) ! Ground grid volume
      ENDIF
      Ao = Rho_g*Cp_g*area/(dt*60) ! TDMA coefficient for prev. time step
      Atot = Ayp1 + Aym1 + Axp1 + Axml + Ao - Sp*dX(i)*dY(j)
      IF (j.EQ. NgridY) THEN ! If ground surface include surface HT
        Bo = Ao*T(i,j,k) + Qsur/dZ ! surface term
      ELSE
        Bo = Sc*dX(i)*dY(j) + Ao*T(i,j,k)
      ENDIF
C ** Use most recent temperatures in Z sides of control volume **
      IF (i.EQ. 1) THEN
        d = Axp1*T(i+1,j,k) + Bo
      ELSEIF (i.EQ. Ngridx) THEN
        d = Axml*T(i-1,j,k) + Axp1*T(i,j,k) + Bo
      ELSE
        d = Axp1*T(i+1,j,k) + Axml*T(i-1,j,k) + Bo
      ENDIF
      IF (j.EQ. 1) THEN
        P(j) = Ayp1/Atot
      ELSE
        P(j) = Ayp1/(Atot - Aym1*P(j-1))
      ENDIF
      Q(j) = (d + Aym1*Q(j-1))/(Atot - Aym1*P(j-1))
      Y = Y + dY(j)*0.5 + dY(j+1)*0.5 ! Move CV Y-location
2300   CONTINUE

      DO 2400 j = Ngridy,2,-1 ! Move from surface to bottom BC calculating temperatures
        T(i,j,k) = P(j)*T(i,j+1,k) + Q(j) ! Calc. grid temperature
        CALL FREEZE(dX,dY,dZ,T,Qfreeze,Cp_g,i,j,k,RHO_g,Cp_w
          & ,moisture) ! Subroutine to adjust for ground freezing
2400   CONTINUE
      X = X + (dx(i) + dx(i+1))/2.0 ! Move CV X-location
2200   CONTINUE ! Move to next X grid (Horizontally Perp. to pipe)
      Z = Z+dZ ! Move CV Z-location
2100   CONTINUE ! Move to next Z grid (Along pipe)

      IF (icount.EQ. 41*96*3 .OR. icount.EQ. 72*96*3
        & .OR. icount.EQ. 101*96*3 .OR. icount.EQ. 132*96*3) THEN
C ** Calculate grid spacing and write to file GRID.OUT **

```

```

C ** Note: Not used for calculation purposes **
depth = Dmean - dY(1)*.5
DO 3400 j = 1, Ngridy, 1
  Xloc = dX(1)*.5
  DO 3500 i = 1, Ngridx, 1
    write(1,3300) i,j,depth,Xloc,T(i,j,NZ/2+1)
    & ,Qfreeze(i,j,NZ/2+1), kn(i,j), ks(i,j), kw(i,j), ke(i,j)
3300  FORMAT(I3,',',I3,',',F8.2,',',F8.2,',',F8.2,',',F14.2,4F8.2)
    Xloc = Xloc + dX(i)*0.5 + dX(i+1)*0.5
3500  CONTINUE
    depth = depth - dY(j)*0.5 - dY(j+1)*0.5
    write(1,*)
3400  CONTINUE

  ENDIF
2000  CONTINUE ! Move to next time step

C ** Claculate run time of program and print out to FLUID.OUT **
CALL GETTIM(ihr2,imin2,isec2,i100th2)
CALL GETDAT(iyr2,imon2,iday2)
IF (iday1 .EQ. iday2) THEN
  ihrtot = ihr2-ihr1
ELSE
  ihrtot = 24 + ihr2-ihr1
ENDIF

IF (imin2-imin1 .LT. 0) THEN
  imintot = 60 + imin2 - imin1
  ihrtot = ihrtot - 1
ELSE
  imintot = imin2-imin1
ENDIF

IF (isec2-isec1 .LT. 0) THEN
  isectot = 60 + isec2 - isec1
  imintot = imintot - 1
ELSE
  isectot = isec2-isec1
ENDIF
write(2,3600) ihrtot, imintot, isectot
write(*,3600) ihrtot, imintot, isectot
3600  FORMAT(6X,'run time(h:m:s)',2X,I3,',',I2,',',I2)

CLOSE (1) ! Close file GROUND.OUT
CLOSE (2) ! Close file FLUID.OUT
CLOSE (3) ! Close file GRID.OUT
CLOSE (4) ! Close file WI-15MIN.INP

```

3999 CONTINUE

END

C ----- Program Subroutines -----

C ** Subroutine to determine variable grid spacing in ground **
SUBROUTINE VARIABLE(BCx,ispiral,dx,BD,Dmean,jspiral,dY,NgridX
& ,NgridY)

REAL*4 dx(32), dY(35), BD, Dmean ! General variables
REAL*4 BCx, zeta, bX, BCxc, BCXv, dzeta, Xtot ! X grid variables
REAL*4 BY1, BY2, BCYc, BCtop,BCYv1, BCYv2 ! Y grid variables
REAL*4 deta1, deta2, eta, Y1, Ytot ! Y grid variables
INTEGER*4 ispiral,jspirale,NXc,NXv,NgridX,NgridY,NYc,NYv1,NYv2

C ** Algorithm to create variable grid spacing **

C ** X-grids **

zeta = 0.0 !
bX = 1.1 !
BCxc = 1.10 ! Size of constant grid area
NXc = 11 ! Number of grids in constant grid space
BCxv = (BCx - BCxc)/2.0 ! Size of variable grid area on each side of pipe
NXv = 9 ! Number of grids in variable grid space on each side of pipe
Ngridx = NXc + 2*(NXv-1) ! Total number of grids in X direction
dzeta = 1.0/(NXv - 1) !
ispiral = (NXc + 1)/2.0 + NXv - 1 ! X-grid in which pipe is located

C ** Calculate size of grids for one side of pipe then apply to opposite side **

DO 0030 i = ispiral, (NXc + 2*NXv - 2)
IF (i .EQ. ispiral) THEN
dx(i) = BCxc/NXc ! If grid with pipe (centre) then use const. grid size
ELSEIF (i .LE. (ispiral + (NXc-1)/2.0)) THEN ! Grid size constant
dx(i) = BCxc/NXc ! Calculate constant grid size
dx(2*ispiral - i) = dx(i) ! Apply same size to mirror image grid
ELSE ! Calculate variable grid size
X1 = BCxv*((bX+1)-(bX-1)*(((bX+1)/(bX-1))**(1-zeta)))
& /(((bX+1)/(bX-1))**(1-zeta) + 1) ! Left grid boundary
zeta = zeta + dzeta ! Transformed coordinate location
X2 = BCxv*((bX+1)-(bX-1)*(((bX+1)/(bX-1))**(1-zeta)))
& /(((bX+1)/(bX-1))**(1-zeta) + 1) ! Right grid boundary
dx(i) = X2-X1 ! Grid size is right boundary - left boundary
dx(2*ispiral - i) = dx(i) ! Apply same size to mirror image grid
ENDIF

0030 CONTINUE

C ** Y-grids: Divide up Y coordinate into 3 seperate sections. Variable grid **

C ** from surface to const. grid region, constant grid region and different **

```

C ** variable grid to bottom boundary. **
  bY1 = 1.02 ! Variable grid coeff. for variable region 1 (bottom)
  bY2 = 1.30 ! Variable grid coeff. for variable region 2 (top)
  BCyc = 1.0 ! Size of constant grid size region
  BCtop = .15 ! Size of surface grid
  NYc = 9 ! Number of grids in constant grid size region
  BCyv1 = Dmean - (BD + BCyc/2.0) ! Size of variable grid region 1 (bottom)
  BCyv2 = BD - BCyc/2.0 - BCtop ! Size of variable grid region 2 (top)
  NYv1 = 8 ! Number of grids in variable grid size region 1 (bottom)
  NYv2 = 9 ! Number of grids in variable grid size region 2 (top)
  Ngridy = NYc + (NYv1 + NYv2 - 2) + 1 ! Total number of Y grids (+ 1 for srface grid)
  deta1 = 1.0/(NYv1 - 1) ! Transformed coordinate grid size - region 1
  deta2 = 1.0/(NYv2 - 1) ! Transformed coordinate grid size - region 2
  jspiral = (NYc + 1)/2.0 + (NYv1 - 1) ! Y grid where pipe located
  DO 0040 j = jspiral, (NYv1 - 1 + NYc) ! Calculate grid size in const. region
    IF (j .EQ. jspiral) THEN ! Grid where pipe located
      dY(j) = BCyc/NYc
    ELSE ! Other constant area grid calculate for grid then apply to mirror image
      dY(j) = BCyc/NYc
      dY(2*jspiral - j) = dY(j)
    ENDIF
  0040 CONTINUE

  eta = 0.0
  DO 0050 j = 1, (NYv1-1) ! Calculate grid sizes in Region 1
    Y1 = BCYv1*((bY1+1)-(bY1-1)*(((bY1+1)/(bY1-1))**(1-eta)))
    & /(((bY1+1)/(bY1-1))**(1-eta) + 1)
    eta = eta + deta1
    Y2 = BCYv1*((bY1+1)-(bY1-1)*(((bY1+1)/(bY1-1))**(1-eta)))
    & /(((bY1+1)/(bY1-1))**(1-eta) + 1)
    dY(NYv1-j) = Y2-Y1
  0050 CONTINUE

  eta = 0.0
  DO 0060 j = 1, (NYv2-1) ! Calculate grid sizes in Region 2
    Y1 = BCYv2*((bY2+1)-(bY2-1)*(((bY2+1)/(bY2-1))**(1-eta)))
    & /(((bY2+1)/(bY2-1))**(1-eta) + 1)
    eta = eta + deta2
    Y2 = BCYv2*((bY2+1)-(bY2-1)*(((bY2+1)/(bY2-1))**(1-eta)))
    & /(((bY2+1)/(bY2-1))**(1-eta) + 1)
    dY(NYv1-1+NYc+j) = Y2-Y1
  0060 CONTINUE
  dY(NgridY) = BCtop
C ** Write grid sizes to file GRID.OUT for reference purposes **
  DO 0070 i = 1, (NXc + 2*NXv - 2)
    Xtot = Xtot + dx(i)
    write(3,*) i,dx(i), Xtot

```

0070 CONTINUE

Ytot = - Dmean + dY(1)*0.5

DO 0080 j = 1, NgridY

Ytot = Ytot + (dY(j) + dY(j+1))*0.5

write(3,*) j,dY(j), Ytot

0080 CONTINUE

END

C ** to determine location of heat exchanger pipes **

SUBROUTINE PIPELOC(Xspiral,Yspiral,Nequiv,Spipe,BCX,dX,dY
& ,Dmean,ipipe,jpipe,NgridX, NgridY)

REAL*4 Xspiral,Yspiral,BCX,Y,Dmean

REAL*4 Xpipe(10),Ypipe(10), dX(32), dY(35), Xcount

INTEGER*4 Nequiv, Pcount, ipipe(Nequiv), jpipe(Nequiv)

INTEGER*4 i, j, NgridX, NgridY, icount, jcount, Nflag

Xcount = (-Nequiv/2.0 + 0.5)*Spipe ! Location of first pipe

DO 3400 Pcount = 1,Nequiv ! Count thru' pipes except return pipe (#1)

Xpipe(Pcount) = Xspiral + Xcount ! Pipe-1 is the return pipe located @ spiral centre

Ypipe(Pcount) = Yspiral ! Pipe-1 is the return pipe located @ spiral centre

c write(*,*) Pcount, Xpipe(Pcount), Xcount, Spipe

Xcount = Xcount + Spipe ! Move X-location counter over one pipe spacing

C ** Move through X and Y directions and determine grids in which pipes are located **

C ** Treat pipe as a line with no thickness, and assume entire pipe is in same grid **

Nflag = 0 ! Flag to see if ipipe located

X = -BCX/2.0 ! Start X sweep @ left BC

icount = 1 ! Set X-grid location counter to 1

DO 3100 i = 1, NgridX, 1

X = X + dX(i)

IF (X .GT. Xpipe(Pcount) .AND. Nflag .EQ. 0) THEN ! Pipe located in this grid

ipipe(Pcount) = icount ! Set pipe location to current X-grid

Nflag = 1 ! End check through field

ENDIF

icount = icount + 1

3100 CONTINUE

Nflag = 0

Y = -Dmean ! Start @ bottom BC and move upwards to surface

jcount = 1 ! Set Y-grid location counter to 1

DO 3200 j = 1, NgridY, 1

Y = Y + dY(j)

IF (Y .GT. -Ypipe(Pcount) .AND. Nflag .EQ. 0) THEN ! Pipe located in this Y-grid

jpipe(Pcount) = j ! Set pipe location to current Y-grid

```

        Nflag = 1 ! End check through field
    ENDIF
    jcount = jcount + 1
3200  CONTINUE
3400  CONTINUE

    DO 3600 icount = 1,Nequiv
c      write(*,*) icount,ipipe(icount),jpipe(icount)

3600  CONTINUE

    END

C  ** Subroutine to determine source term for ground calculations **
    SUBROUTINE SOURCE(i,j,ipipe,jpipe,Tf,Rsource,Sc,Sp,Pflag,Nequiv
& ,dX,dY,dZ)

    REAL*4 Tf,Rsource,Sc,Sp,dX(32),dY(35),dZ
    INTEGER*4 i,j,ipipe(8),jpipe(8), Pflag, count, Nequiv

    Pflag = 0
    DO 5100 count = 1, Nequiv
        IF(i .EQ. ipipe(count) .AND. j .EQ. jpipe(count)) THEN
            Pflag = 1
        ENDIF
5100  CONTINUE

    IF (Pflag .EQ. 1) THEN ! A pipe located in ground grid
        Sc = (Tf/Rsource)/(dX(i)*dY(j)*dZ) ! Calc source terms on per vol. base
        Sp = (-1.0/Rsource)/(dX(i)*dY(j)*dZ)
    ELSE ! No pipe is located in grid --> No source term
        Sc = 0.
        Sp = 0.
    ENDIF
    END

C  ** Subroutine to determine the grid thermal properties **
    SUBROUTINE GROUND(Dback,ke,kw,ks,kn,dX,dY,Ksoil,Kback,Wback
& ,X,Y,i,j)

    REAL*4 Dback,ke(32,35),kw(32,35),ks(32,35),kn(32,35)
    REAL*4 fe, fw, fs, fn ! Grid ratios
    REAL*4 dX(32),dY(35),Ksoil,Kback,Wback,X,Y
    INTEGER*4 i,j

```

```

IF (-Y .GT. Dback) THEN ! Grid below bottom of trench
  Kw(i,j)= Ksoil
  Ke(i,j)= Ksoil
  Ks(i,j)= Ksoil
  IF (-1*(Y + dY(j)*0.5 + dY(j+1)*0.5) .GT. Dback) THEN ! Above grid below depth of
    trench
    Kn(i,j)= Ksoil
  ELSE
    fn = (-Y - Dback)/(0.5*dY(j) + 0.5*dY(j+1))
    Kn(i,j)= 1.0/((1-fn)/kback + fn/ksoil)
  ENDIF

ELSE ! Grid above bottom of trench
  Kn(i,j)= Ksoil
  IF (X .LT. (0 - Wback/2.0)) THEN ! Grid to left of trench
    Kw(i,j) = Ksoil
    Ks(i,j) = Ksoil
    IF ((X+dX(i)*0.5+dX(i+1)*0.5) .LT. (0-Wback/2.0)) THEN ! Next grid left of trench
      Ke(i,j)= Ksoil
    ELSE ! Next grid is in trench
      fe = (-Wback/2.0 - X)/(0.5*dX(i) + 0.5*dX(i+1))
      Ke(i,j)= 1.0/((1-fe)/kback + fe/ksoil)
    ENDIF
  ELSEIF (X .GT. (0 + Wback/2.0)) THEN ! Grid to right of trench
    Ke(i,j)= Ksoil
    Ks(i,j) = Ksoil
    IF ((X-dX(i)*0.5-dX(i+1)*0.5) .GT. (0+Wback/2.0)) THEN ! Previous grid right of
      trench
      Kw(i,j)= Ksoil
    ELSE ! Previous grid is in trench - Interface between two grids
      fw = (X - Wback/2.0)/(0.5*dX(i) + 0.5*dX(i-1))
      Kw(i,j)= 1.0/((1-fw)/kback + fw/ksoil)
    ENDIF

  ELSE ! Grid is inside trench
    Kn(i,j) = Kback ! Assume trench is backfilled to surface
    IF ((X-dX(i)*0.5-dX(i-1)*0.5) .LT. (0-Wback/2.0)) THEN ! Previous grid left of trench
      Ke(i,j) = Kback
      fw = (X - (-Wback/2.0))/(0.5*dX(i) + 0.5*dX(i-1))
      Kw(i,j)= 1.0/((1-fw)/ksoil + fw/kback)
    ELSEIF ((X+dX(i)*0.5+dX(i+1)*0.5) .GT. (0+Wback/2.0)) THEN ! Next grid right of
      trench
      Kw(i,j)= Kback
      fe = (Wback/2.0 - X)/(0.5*dX(i) + 0.5*dX(i+1))
      Ke(i,j)= 1.0/((1-fe)/ksoil + fe/kback)
    ELSE ! Previous and next grids are in trench
      Ke(i,j) = Kback

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      Kw(i,j) = Kback
    ENDIF

    IF (-1*(Y - dY(j)*0.5 - dY(j-1)*0.5) .GT. Dback) THEN ! Lower grid below depth of
      trench
      fs = (Dback - (-Y))/(.5*dY(j) + .5*dY(j-1))
      Ks(i,j) = 1.0/((1-fs)/ksoil + fs/kback)
    ELSE
      Ks(i,j) = Kback
    ENDIF
  ENDIF
ENDIF

ENDIF
END

C ** Subroutine to account for ground freezing **
SUBROUTINE FREEZE(dX,dY,dZ,T,Qfreeze,Cp_g,i,j,k,RHO_g,Cp_w
& ,moisture)

  REAL*4 Qlat, Qsens, Cp_w, moisture, vol, Cp_g, freeze_flag
  REAL*4 RHO_g,T(32,35,25),Qfreeze(32,35,25),dX(32),dY(35)

  INTEGER*4 i,j,k

  vol = dX(i)*dY(j)*dZ ! volume of grid
  IF (Qfreeze(i,j,k) .EQ. 0.0) THEN
    freeze_flag = 1 ! ground completely frozen
  ELSEIF (Cp_w*RHO_g*vol*moisture - Qfreeze(i,j,k) .LT. 100.) THEN
    freeze_flag = 0 ! ground completely thawed
  ELSE
    freeze_flag = 0.5 ! ground partially frozen
  ENDIF
  Qsens = ABS(vol*RHO_g*Cp_g*(T(i,j,k) - 0))
  Qlat = ABS(vol*Rho_g*moisture*Cp_w)

  IF (T(i,j,k) .GE. 0) THEN ! Calculated temperature above freezing
    IF (freeze_flag .EQ. 1) THEN ! At last time step, ground was completely frozen
      IF (Qsens .GE. Qlat) THEN
C ** Sensible heat of ground above freezing is greater than latent
C ** heat of fusion. The modified temp. will still be above freezing since
C ** since (Qsens - Qlat) is still > 0.
        T(i,j,k) = (Qsens-Qlat)/(vol*RHO_g*Cp_g) ! Modified temperature
        Qfreeze(i,j,k) = Cp_w*RHO_g*vol*moisture ! Set to completely thawed
        freeze_flag = 0 ! Ground completely thawed
      ELSEIF (Qsens .LT. Qlat) THEN
C ** Sensilbe heat of ground above freezing is less than latent heat of

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C ** fusion. Not enough heat is available for the ground to completely thaw,
C ** and the modified temp. is 0. A new value for Qfreeze must be cal'd
      T(i,j,k) = 0 ! Ground still partially frozen
      Qfreeze(i,j,k) = Qsens ! Remaining energy req'd to freeze ground
      freeze_flag = 0.5 ! Ground partially thawed
      ENDIF

      ELSEIF (freeze_flag .EQ. 0.5) THEN ! At last time step, ground was partially thawed
        IF (Qsens .GE. (Qlat-Qfreeze(i,j,k))) THEN
C ** Sensible heat of ground above freezing is greater than remaining latent
C ** heat of fusion. The modified temp. will still be above freezing since
C ** since (Qsens - Qlat) is still > 0.
          T(i,j,k) = (Qsens-(Qlat-Qfreeze(i,j,k)))/(vol*RHO_g*Cp_g) ! Modified temperature
          Qfreeze(i,j,k) = Cp_w*RHO_g*vol*moisture ! Set to completely thawed
          freeze_flag = 0 ! Ground completely thawed
        ELSEIF (Qsens .LT. (Qlat-Qfreeze(i,j,k))) THEN
C ** Sensible heat of ground above freezing is less than remaining latent heat of
C ** fusion. Not enough heat is available for the ground to completely thaw,
C ** and the modified temp. is 0. A new value for Qfreeze must be cal'd
          T(i,j,k) = 0 ! Ground still partially frozen
          Qfreeze(i,j,k) = Qfreeze(i,j,k) + Qsens ! Calc remaining energy req'd to freeze ground
          freeze_flag = 0.5 ! Ground partially thawed
        ENDIF
      ENDIF

      ELSE ! Calculated temperature is below freezing

        IF (freeze_flag .EQ. 0) THEN ! At last time step, ground was completely thawed
          IF (Qsens .GE. Qlat) THEN
C ** Sensible heat reduction of ground below freezing is greater than latent
C ** heat of fusion. The modified temp. will still be below freezing since
C ** since (Qsens - Qlat) is still > 0.
            T(i,j,k) = -(Qsens-Qlat)/(vol*RHO_g*Cp_g) ! Modified temperature
            Qfreeze(i,j,k) = 0 ! Set to completely frozen
            freeze_flag = 1 ! Ground completely frozen
          ELSEIF (Qsens .LT. Qlat) THEN
C ** Sensible heat reduction of ground below freezing is less than latent heat of
C ** fusion. Not enough heat is removed for the ground to completely freeze,
C ** and the modified temp. is 0. A new value for Qfreeze must be cal'd
            Tlast = T(i,j,k)
            T(i,j,k) = 0 ! Ground still partially frozen
            Qfreeze(i,j,k) = Qlat-Qsens ! Calc remaining energy req'd to freeze ground
            freeze_flag = 0.5 ! Ground partially thawed
          ENDIF

        ELSEIF (freeze_flag .EQ. 0.5) THEN ! At last time step, ground was partially thawed
          IF (Qsens .GE. Qfreeze(i,j,k)) THEN

```

```

C ** Sensible heat of ground below freezing is greater than remaining latent
C ** heat of fusion. The modified temp. will still be below freezing
C ** since (Qsens - Qfreeze(i,j,k)) is still > 0.
      T(i,j,k) = -(Qsens-Qfreeze(i,j,k))/(vol*RHO_g*Cp_g) ! Modified temperature
      Qfreeze(i,j,k) = 0 ! Set to completely frozen
      freeze_flag = 1 ! Ground completely frozen
      ELSEIF (Qsens .LT. Qfreeze(i,j,k)) THEN
C ** Sensible heat of ground below freezing is less than remaining latent heat of
C ** fusion. Not enough heat is removed for the ground to completely freeze,
C ** and the modified temp. remains @ 0. A new value for Qfreeze must be cal'd
      Tlast = T(i,j,k)
      T(i,j,k) = 0 ! Ground still partially frozen
      Qfreeze(i,j,k) = Qfreeze(i,j,k) - Qsens ! Calc remaining energy req'd to freeze ground
      freeze_flag = 0.5 ! Ground partially frozen
      ENDIF

      ENDIF

      ENDIF
      END

```

C ----- Program Functions -----

```

C  ** Function to Calculate Temperature of Undisturbed Ground Temperature **
      REAL*4 FUNCTION KUSUDA(depth,time,alpha,As,Tm,to)

      REAL*4 depth, alpha, tdays, a, b

      PI = ACOS(-1.0)
      tdays = time/(60*24)
      a = -SQRT(PI/(365*24*3600*alpha))
      b = 2*PI/365*(tdays-to)
      KUSUDA = Tm - As * exp(depth*a) * COS(b + a*depth)
      END

```

APPENDIX B

Visual Basic Code For Ground Isotherm Plots

Dim MaxT, MinT, dX(30), dY(30), T(30, 30) As Single

Dim i, j As Integer

Sub cmdIsotherm_Click ()

' This subroutine reads in data for the grid dimensions and C.V. temperatures
' from input files (Modified DIFFER-3.FOR output files), and creates an colour isotherm
' plot to represent the ground temperatures in the analysis region.
' Selecting the Isotherm button in the form frmGrid initializes this subroutine

' Set variable names for input data files

GridFile = FreeFile

TempFile = FreeFile

fact = 900 ' Y direction dimension factor to fit screen

factX = 1500 ' X direction dimension factor to fit screen

' Set max, min and mid-range (not exact middle) control volume temperatures and
corresponding

' Colours. Program will interpolate colours from this information.

Tmax = 10 ' Max C.V. temperature

Rmax = 255 ' RGB (Red, green, blue) colour settings for max temp. (approx. red)

Gmax = 75

Bmax = 75

Tmin = -15 ' Min C.V temperature

Rmin = 0 ' RGB (Red, green, blue) colour settings for min temp. (approx. blue)

Gmin = 150

Bmin = 255

Tmid = Tmin * .37 + Tmax * .63 ' Mid range temperature

Rmid = 220 ' RGB (Red, green, blue) colour settings for max temp. (faint blue)

Gmid = 220

Bmid = 255

Open "c:\lang\vb\grid\grid.dat" For Input As GridFile ' Open the grid dimension data file

'Set control volume dimensions from grid.dat input file - FORTRAN output file with
modified format

For i = 1 To 27 ' Step through X-direction control volumes

dX(i) = Input(4, GridFile) ' Set X-dimension of grid

dummy = Input(2, GridFile) ' Read carriage return in input file

Next i

For j = 1 To 25 ' Step through Y-direction control volumes

```

dY(j) = Input(4, GridFile) ' Set Y-dimension of grid
dummy = Input(2, GridFile) ' Read carriage return in input file
Next j

Close GridFile ' Close grid dimensions input file
Open "c:\lang\vb\grid\ground2.dat" For Input As TempFile ' Open the 'End of Month'
ground temperature data file

Y = 12500 - 2.55 * fact ' Starting Y-location in Twips
For j = 1 To 25 ' Step through Y grids
    Y = Y - dY(j) * fact ' Move from bottom to top

    X = 500 ' Starting X-location in Twips
    For i = 1 To 27 ' Step through X grids
        T(i, j) = Val(Input(6, TempFile)) ' Read temperature from 'end of month' file
        dummy = Input(2, TempFile) ' Read carriage return in input file

        ' Determine control volume colour by linear interpolation between mid-range
        temperature/colour
        ' and min. temperature (if T(i,j) < mid-range) or max. temperature (if T(i,j) > mid-range).
        If T(i, j) <= Tmid Then ' T(i,j) <= mid-range temperature
            R = (T(i, j) - Tmin) / (Tmid - Tmin) * (Rmid - Rmin) + Rmin
            G = (T(i, j) - Tmin) / (Tmid - Tmin) * (Gmid - Gmin) + Gmin
            B = (T(i, j) - Tmin) / (Tmid - Tmin) * (Bmid - Bmin) + Bmin
        Else ' T(i,j) > mid-range temperature
            R = Rmax - (Tmax - T(i, j)) / (Tmax - Tmid) * (Rmax - Rmid)
            G = Gmax - (Tmax - T(i, j)) / (Tmax - Tmid) * (Gmax - Gmid)
            B = Bmax - (Tmax - T(i, j)) / (Tmax - Tmid) * (Bmax - Bmid)
        End If

        If j >= 2 Then ' Do not draw bottom C.V. to maintain proportions
            Line (X, Y)-Step(dX(i) * factX, dY(j) * fact), RGB(R, G, B), BF ' Draw box with
            RGB fill
        End If
        X = X + dX(i) * factX ' Move to next X-location
    Next i
Next j

Xmid = (6120 - 100) / 2 * 1.5 + 500 ' Centre of pipes
Ypipe = 2.1 * fact + 800 ' Position of bottom of pipes
Spipe = .19 * 1000 ' pipe spacing in twips
Xpipe = Xmid - Spipe * 2.5 ' Left position of pipes

```

```

' Draw Pipes for reference
For ipipe = 1 To 6 ' There are 6 pipes
    frmGrid.Circle (Xpipe, Ypipe), 15, RGB(0, 0, 0) ' draw black circle
    Xpipe = Xpipe + Spipe ' Move to next pipe
Next ipipe

' Draw temperature scale to right of isotherm
X = 9800 ' X-position of scale
Y = 900 ' Y-position of scale
For Tscale = Tmax To Tmin Step -2 ' 2 °C jump in temp scale
' Do same calc. as C.V.'s above for temp. scale
    If Tscale <= Tmid Then
        R = (Tscale - Tmin) / (Tmid - Tmin) * (Rmid - Rmin) + Rmin
        G = (Tscale - Tmin) / (Tmid - Tmin) * (Gmid - Gmin) + Gmin
        B = (Tscale - Tmin) / (Tmid - Tmin) * (Bmid - Bmin) + Bmin
    Else
        R = Rmax - (Tmax - Tscale) / (Tmax - Tmid) * (Rmax - Rmid)
        G = Gmax - (Tmax - Tscale) / (Tmax - Tmid) * (Gmax - Gmid)
        B = Bmax - (Tmax - Tscale) / (Tmax - Tmid) * (Bmax - Bmid)
    End If

    Line (X, Y)-Step(200, 200), RGB(R, G, B), BF ' Draw temp. scale box
    CurrentX = X + 240 ' Add Temp scale text
    CurrentY = Y
    If Tscale = Tmax Then
        Print Tscale + " °C"
    Else
        Print Tscale
    End If
    Y = Y + 200
Next Tscale

' Add text for X and Y axis
frmGrid.FontSize = 10 ' text font
CurrentX = 4400 ' Position of X axis test
CurrentY = 400
Print "Ground Surface" ' X axis test

CurrentY = 675 ' Initial Y position for Y axis
For depth = 0 To 7 ' Move from 0 to 7 metres below surface

    Print depth ' Y axis scale

```

```
    CurrentY = (depth + 1) * fact + 675 ' Print Y axis scale  
    Next depth ' Move to next axis point
```

```
End Sub
```

```
Sub cmdExit_Click ()
```

```
    End
```

```
End Sub
```

APPENDIX C

Calculation of Percent Error Using Carnot Efficiency

In order to compare differences between predicted and monitored ground temperature and heat pump EWT in a form that is meaningful to the ground-source heat pump performance, the Carnot efficiency for heat pumps was used.

The Carnot efficiency for a heat pump cycle is given by

$$\beta_{\alpha} = \frac{T_c}{T_c - T_u} \quad (C.1)$$

where β_{HP} is the theoretical heat pump efficiency, T_H is the condenser temperature represented by the room temperature of the house and T_L is the evaporator temperature represented by either the undisturbed ground temperature of the heat pump EWT.

The Carnot efficiency for the monitored temperatures is given by

$$\beta_{\alpha \in T_{WP}} = \frac{T_c}{T_c - T_{WP}} \quad (C.2)$$

The Carnot efficiency for the predicted temperatures is given by

$$\beta_{\alpha \in T_{\hat{u}}} = \frac{T_c}{T_c - T_{\hat{u}}} \quad (C.3)$$

The percentage difference between the predicted and monitored efficiency is given by

$$\% \Delta \frac{\beta}{\beta_{\alpha \in T_{WP}}} = \frac{\beta_{\alpha \in T_{\hat{u}}} - \beta_{\alpha \in T_{WP}}}{\beta_{\alpha \in T_{WP}}} \times 100 \quad (C.4)$$

Substituting for $\beta_{HP,mon}$ and $\beta_{HP,mon}$, the equation becomes

$$\% \Delta \frac{\beta}{\beta_{\alpha \in T_{\omega \varphi}}} = \frac{\frac{T_{\zeta}}{T_{\zeta} - T_{;\omega \bar{v}}} - \frac{T_{\zeta}}{T_{\zeta} - T_{T_{\omega \varphi}}}}{\frac{T_{\zeta}}{T_{\zeta} - T_{T_{\omega \varphi}}}} \quad (C.5)$$

Simplifying equation C.5 results in

$$\% \Delta \frac{\beta}{\beta_{\alpha \in T_{\omega \varphi}}} = \frac{T_{T_{\omega \varphi}} - T_{;\omega \bar{v}}}{T_{\zeta} - T_{;\omega \bar{v}}} \quad (C.6)$$