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**THE TRANSACTION ROLE OF REAL MONEY BALANCES
IN THE THEORY OF FIRMS**

by

KAMAL K. SHARAN. B.A., M.A.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy
Department of Economics

Carleton University
Ottawa, Ontario

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IN THE THEORY OF FIRMS"**

submitted by

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in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

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Abstract

This thesis investigates the role that money plays in relation to the "transactions technology" of the firm where money holdings are viewed as an integral part of the firm's exchange activities. Holding buffer stocks of money allows the firm to lower the costs of bunching some types of transactions while permitting the separation of others over time. This approach allows the integration of real and money sectors through the theory of the firm.

Chapter I motivates the analysis by reviewing the evidence on the importance of transactions costs in the theory of the firm. Chapter II sets up a dynamic model of the firm and derives the Euler equations that emphasize the interrelationship between the firm's money holding and capital accumulation decisions.

Chapter III derives the stationary state characteristics of the model and shows the stability properties of the model through an analysis of the conditions for convergence to the stationary state.

Chapters IV and V describe the technique used to estimate the model (the generalised method of moments) and the sources and transformation features of the data set.

Chapter VI presents the empirical findings of the thesis. The results show that while the money-augmented model is consistent with the data, the data does not allow us to conclude that the money-augmented model is superior to its traditional alternatives. A money-augmented model may be preferred over a simple model, however, because it permits comparative static analysis showing the channels through which money can affect the real variables in the model. It also gives insight to the way that stable monetary policy may promote income and employment growth in the economy.

Chapter VII concludes the thesis by summarizing the main argument and bringing together the major conclusions.

DEDICATION

**To my parents Ramji Sharan
and the late Sukumari Sharan**

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CHAPTER I: Introduction and Overview

1.1 Introduction:

In neo-classical macroeconomics the role of money in the economy is typically modelled by using either consumer theory or producer theory. From the perspective of consumer theory, real money balances are treated as generating a stream of services just like any other durable good and thus become an argument in the utility function representing monetary services.¹ Accordingly, the demand for real money balances is derived from a choice-theoretic framework where the consumer maximizes the present value of a flow of utility subject to an intertemporal budget constraint. The solution to this consumer choice problem provides the demand for both consumer goods and real money balances.

From the perspective of producer theory, the production function is most often used to explain why firms hold money. In this framework, real money balances are treated as a third factor (in addition to labour and capital) in the production function.² Most often money is not viewed as increasing productivity directly, but as providing indirect production services by releasing other factors for more intensive use in the production process.³ Money increases

¹ See Hicks (1935), Patinkin (1955,1965), Friedman (1959), Barnett, Fisher and Serletis (1992), Cuthbertson (1991), Cuthbertson and Taylor (1987), Laidler (1993).

² Nadiri (1969, 1970), Friedman (1959).

³ Fischer (1974).

output by promoting increased specialization and permitting greater efficiency in the allocation of resources.

The production approach to money has been popular among empirical researchers testing productivity theory. Based on a productivity interpretation of the role of money, Sinai and Stokes (1972) estimated a three factor production function for the U.S. (using annual aggregate data for the period 1929-1967). They found that the estimated coefficient for real money balances was significantly positive and concluded that real money balances were an omitted variable from the usual estimation of the production process. They also found that real money balances were complementary with labour and a substitute for capital. Overall, Sinai and Stokes argue that increased productivity, initially attributed to disembodied technical change, is really due to the productive contribution of real money balances. Their pioneering work stimulated a large number of empirical studies that exploited different data-sets across different countries, or used different types of production or cost functions.⁴ In almost all of these cases the basic findings of Sinai and Stokes (1972) have been confirmed.

⁴ Sinai and Stokes (1972, 1975, 1979, 1981), Niccoli (1975), Khan and Kouri (1975), Prais (1975a, 1975b), Short (1979), Subramanyam (1980), Ben-Zion and Ruttan (1975), Boyes and Kavanaugh (1979), Smith (1980), Khan (1985), Khan and Ahmed (1985), Nguyen (1986), Bennet, Kamath and Jensen (1987), You (1981), Hasan and Mahmud (1992), Hasan (1993).

This thesis follows a third interpretation of the role of real money balances in the economy and adopts an alternative way of integrating money into the analysis of the firm. To model money, I place money in relation to the firm's payments arrangements through the concept of a transaction technology. By using a transaction technology, I am arguing that real money balances are held as a part of the firm's strategy to reduce overall transaction costs. Holding money allows the firm to reduce cost by permitting greater flexibility in the firm's contracting and distribution activities. Essentially, the transaction approach focuses on money's role as medium of exchange and derives a value for money from the gains associated with circumventing the double coincidence of wants.⁵ Instead of treating real money balances as a part of the physical production process, this thesis treats real money balances as a part of the firm's exchange activities. Holding real balances allows the firm to reduce some of these trading costs and thus provides the firm with an additional way to maximize profits.

The view that holding money will help the firm to minimize cost or maximize profit is also supported by the

⁵ The double coincidence of wants means that bartering requires not merely the exchange of goods against any other goods but also the exchange of reciprocally desired goods. The double coincidence of wants, then, has two parts. The first part is that all trades in a barter economy must satisfy some ultimate want. The second part is that the only compensation a trader can receive for supplying a second barter's want is received from the second trader. See Starr (1972) p. 291.

finance literature. Models in this literature assume that internally generated funds for investment are available to the firm at lower cost than are external funds.⁶

Empirically, Hayashi (1985) has shown that investment expenditures are sensitive to measures of internal finance.⁷ Similarly, Bond and Meghir (1994) show that investment displays excessive sensitivity to a measure of cash flow and that investment is positively related to lagged cash flows.

In this thesis real money balances appear in three different parts of the firm's profit maximization problem. First, I use a transaction technology function to stand for how a firm can, by holding money, economize on the costs incurred when transacting with clients, workers, and other suppliers in the production process. Second, real money balances enter into the investment decision of the firm through its adjustment cost. Holding real money balances allows the firm to economize by reducing the expected cost of altering the capital stock. Third, real money balances are assumed to have their own adjustment cost. Holding a higher average level of real money balances allows the firm to economize on the cost of adjusting quickly to variations

⁶ See Bond and Meghir (1994) and the articles listed below in footnote 7 below.

⁷ Additional panel evidence on this issue can be found in Pazzari, Hubbard and Petersen (1988), Hayashi and Inoue (1991) and Blundell, Bond, Devereux and Schiatarelli (1992).

between actual and expected money flows. The benefit to the firm of holding higher average level of real money balances can be seen as a way of either avoiding the financial embarrassment of not having on hand the means of final settlement or avoiding the costs of liquidating illiquid assets on short notice.

1.2 Transaction Cost Technology:

Transaction costs arise from the activities associated with transfer of ownership, or, more generally, property rights. "They are a concomitant of decentralized ownership rights, property and exchange".¹ Transaction costs, like production costs, are a catch-all term for a heterogeneous assortments of inputs used in trade. A transaction technology specifies what resource inputs are required to achieve a given transfer. It may be formalized in a transaction function analogous to a production function. A transaction function may exhibit constant, decreasing and increasing returns to scale. These scale economies are often independent of the quantity transferred and may depend on the size of the participating firms, the size of the individual transaction or the size of the market as a whole. In principle, each transaction function relates to a pair or group of economic agents where transaction costs are incurred to reduce or eliminate the uncertainties arising in trade.

Following the work of Coase (1937), there has been an increased recognition of the need to invoke transaction costs to explain the role of economic organizations or institutions such as firms. As Coase has put it, "the main reason why it is profitable to establish a firm would seem

¹ Eatwell, Millgate and Newman (1988), p. 677.

to be that there is a cost of using the price mechanism".⁹ Although Coase did not use the term "transaction cost", the literature which follows this line of thought is called "transaction cost economics". Underlying Coase's "Theory of the Firm" (1937) and more fundamentally his "Problem of Social Cost" (1960) is the observation that all economic interactions create incentives to redistribute income as well as enhance efficiency. Hence institutions are designed and resources spent to channel social interaction away from redistribution towards mutually advantageous "productive" activity. Transaction cost economics then revolves around the control of the allocation process, i.e., how resources are used to monitor, reward and police the behaviour of economic agents in order to enhance the joint profitability of economic activity. In the context of my thesis, a transaction technology function is a shorthand way of describing the firm's institutionalized activity of trade and the role of resources in reducing the costs of its exchange activities.

In monetary theory, the origin of the transaction approach may be thought of as arising from an attempt to explain the first of the three Keynesian motives for holding money, namely the transaction motive for holding money.¹⁰ Given the institutionalized way that money payments and

⁹ Coase (1937) p 390.

¹⁰ Keynes (1936).

receipts flow through the firm, the costs of changing these money flows (in terms of either brokerage fees, the cost of agency, the cost of information, or time costs) would lead the firm to adopt an optimal money holding strategy.

Initially, the firm's optimal money holding strategy was treated as a problem in optimal inventory holdings.¹¹ Baumol (1952), for example, derived a transactions demand for money where the non-synchronized timing of cash inflows and outflows was perfectly known to the agent. The agent could alter the timing of these flows by using a bond market, and he/she would choose to do so if the cost of using the bond market was lower than the interest return permitted by investing. At the optimal level of money holdings, the familiar square root formula was obtained.¹²

Following Baumol (1952), Tobin (1956) extended the inventory theoretic approach by incorporating money holdings within a more complex maximization problem. In Tobin's (1956) model, the firm maximizes the net revenue from total transaction balances assumed to consist of money and bonds. Transaction balances are held as a means of bridging the gap between receipts that occur at discrete periods of time. The main objective of his approach was to show that interest rates affect the quantity of transaction balances held in the form of money.

¹¹ Baumol (1952), Tobin (1956), Miller and Orr (1966).

¹² Oliviera (1971).

Miller and Orr (1966) and Whalen (1966) further extended the approach by introducing specific forms of uncertainty into the model. They suggested that even if the firm's pattern of money inflows and outflows is not as discrete as had been assumed in the earlier Baumol/Tobin framework, a model could be developed that results in a similar form to the demand for money function. The addition of uncertainty did not change the essential character of Baumol's (1952) results.¹¹

The Baumol-Tobin model remains the most popular microeconomic model of the demand for money under transaction costs. Moreover by generalizing the structure, Feenstra (1986) has shown how the Baumol-Tobin model can be interpreted as implying a transaction technology function that is increasing in the flow of expenditure and decreasing in the level of real money balances. The transaction technology function generated by the Baumol-Tobin approach has the following characteristics:

- (a) the relevant scale variable is the flow of consumption expenditures. The number of transaction increases with the scale of expenditure;
- (b) the transaction cost is an increasing function of the number of transactions. However, by holding

¹¹ That is, both Whalen and Miller and Orr derive money demand functions with the characteristic form of economies of scale in the level of trading activity.

more money, the time and the expense of transacting can be economized upon by transacting less frequently;¹⁴

- (c) the transaction technology function is homogeneous of degree zero in prices (i.e, there is no money illusion).¹⁵

Although the Baumol/Tobin transaction approach allows for the derivation of a transaction technology function, the primary money flows through the firm in that analysis are assumed to be independent of the firm's money holding solution. The level of average money holdings implied by non-synchronized discreteness in the firm's receipts and payments could be economized upon by using a secondary market (such as the bond market), but the primary money flows were unalterable due to the separation of the process of trade from the holding of money. Implicitly, the interaction between how the trade is transacted and how money is used was ignored by assuming a fixed institutional payment setting.

Saving (1971) began the integration of money holding with the volume of trade by introducing time as a transaction cost. In the absence of money, Saving argued

¹⁴ Jovanovic (1982), Williamson (1987) and Hess (1991) have recognized the importance of time as a determinant of transaction cost technology. Hess (1991) has shown that for the U.S. economy even the deregulation of deposits and technological improvement in electronic fund transfer did not eliminate the transaction costs.

¹⁵ A person is said to suffer from 'money illusion' if there is a failure to distinguish between monetary and real changes.

that additional time must be spent in exchange to achieve the double coincidence of individual trading wants required under barter exchange. A money economy reduced the time spent in transaction by permitting the separation of a final sale from a final purchase. Money intermediated exchange and permitted individuals to realize the gains associated with specialized productive activity. Thus, money holdings lowered transaction costs and their savings could be measured as the value of time saved in exchange. By lowering the cost of trade, money also increased the number of transactions that could be engaged in economically.

In 1972 Saving extended his argument by showing how a transaction demand for money could be generated by present value minimizing behaviour utilizing a general transaction cost function.¹⁶ The value of money was derived from its capacity to reduce the resources required to contract with and control the physical factors used in the production and sale of the finished product. Saving then postulated that transaction costs would depend on both the number and the size of transactions. Given these considerations, the transaction costs of the firm increased with an increase in the costs of trading in money alternatives (such as bonds) and decreased with the average holdings of inventories of both output and factors of production, including money. The fact that transaction costs decrease with an increase in

¹⁶ See also Feige and Parkin (1971).

average real money holdings implies a real return that can be equated to the productivity of more traditional inputs.¹⁷

Karni (1973) also incorporated the value of time into the inventory theoretic approach of Baumol (1952), where the value of time spent transacting was the value of income foregone by not working. He then investigated the interdependence that existed among the variables entering the money demand function, specifically the relationship between the volume of planned transactions and the time cost of cash withdrawals. Karni found that a change in the marginal value of time would affect the cost of converting assets into cash and the volume of planned transactions (assumed to be a function of total income).¹⁸

Clower and Howitt (1978) developed a general class of models that incorporate the Baumol-Tobin model of demand for money as a special case. In their model, economic agents choose the frequency of both sales and purchases as well as the time phasing of their transactions.¹⁹ They found that

¹⁷ Saving (1976) also showed that the simple linear specification of the inventory theoretic approach is a special case of a more general transactions cost approach. The transactions function approach has the additional intuitive appeal of having a directly obtainable expression for marginal revenue and marginal costs.

¹⁸ While time has been the most obvious way of incorporating money into a transactions process, it is not the only way. See, for example, Ferris (1981) who shows how money interacts with other trading instruments (e.g. trade credit) to allow trading plans to become more predictable and hence to lower the cost of trade.

¹⁹ In a similar model Parkin and Feige (1971) deal with trading frequencies for more than one commodity, but in each case trading frequencies were assumed to satisfy integer constraints so that each frequency was an exact multiple or divisor of every other.

the demand for traded inventories (i.e., both goods and money), were discontinuous functions of the relative frequency of sales and purchases. In their model, people hold real money balances because money is less costly to hold than other traded goods. Were this not true, traders would trade equilibrium quantities of goods directly and hold no money (because there is no independent advantage to holding money as a store of value or for consumption). Clower and Howitt (1978) then support the view that since money is almost costless to store, it should be paid competitive interest.²⁰ By reducing the cost of holding money, the community gains on net from the lower cost of choosing more optimal trading frequencies.

In the partial equilibrium framework of Baumol (1952) and Tobin (1956), economic agents hold money that is dominated in rate of return by other assets because money is required to finalize transactions and because it is costly to deal in alternatives to holding money. In the general equilibrium versions of these models, such as Jovanovic (1982), transaction costs are also used to explain the rate of return dominance of money.²¹ In addition, however, a general equilibrium setting is more appropriate for discussing the effects of aggregate phenomenon such as

²⁰ Other supporters of this view include Samuelson (1968), Friedman (1969), Feige and Parkin (1971).

²¹ See also Grossman and Weiss (1983) and Romer (1986).

inflation. Jovanovic's (1982) model, for example, predicts that a higher inflation rate increases the cost of holding money, leading to a reduction in average money balances and an increase in the transaction cost associated with additional trips to the bank.²² Alternatively, Williamson (1987) constructs a simple tractable model where there are fixed costs of financial intermediation in addition to the costs of transacting with intermediaries. He showed that, in such a model, welfare is adversely affected by both inflation and reserve requirements.

Kimborough (1986) was among the first to model money as a means of reducing transaction costs in a dynamic intertemporal optimization context. In his model, money economizes on shopping time. Hence, the optimal inflation tax will always equal zero because of the scarce resource (time) used up in the transaction process. His analysis then showed that the steady state analysis of Jovanovic (1982), Grossman and Weiss (1983) and Romer (1986) could be generalized into the intertemporal setting.

Dowd (1990) tested whether the value of time should be modelled explicitly in a transaction cost equation. To do so he constructed a transaction technology function in which the transaction cost was the product of the nominal wage and

²² Hess (1991) broadens transaction costs to include service charges, commissions, bid/ask spread and time required to effect a transaction.

real expenditures.²³ He called this the pure value-of-time transaction cost model. His analysis showed that time costs were a significant determinant of money holdings, thus giving empirical support to this strand of the transaction cost money literature.²⁴

Wang and Yip (1992) examine the role of the money growth rate on per capita real variables and economic welfare in the steady state under three different approaches to money, i.e., money in the utility function approach, money in the cash-in-advance approach, and, money in transaction costs approach. In the transaction cost approach, Wang and Yip introduced money into the optimization problem by assuming a shopping time technology following Saving (1971) and Kimborough (1986). The resulting transaction function had money facilitate trade by reducing transaction time. Wang and Yip showed that shopping time increased with the number of consumption goods purchased. Also because the benefit of using money had diminishing returns, successive reductions in real money balances would produce more than proportionate increases in shopping time and therefore result in increases in marginal transaction costs.

²³ The nominal wage was the proxy for the money value of time.

²⁴ Using quarterly data for the U.K. between 1975:3-1984:4, Dowd found Wage coefficient to be statistically significant at the 2.5%.

Finally, Guioditti and Végh (1993) examine the claim that if money plays a transaction role analogous to an intermediate good, the optimal inflation tax should again be zero. In their analysis the optimality of a zero inflation tax is shown to depend on the properties of the transaction technology function.²⁵ In particular, they show that if the transaction technology exhibits increasing returns to scale, it will be optimal to resort to an inflation tax.²⁶ Because most of the transaction cost technologies used in the literature exhibit increasing returns to scale, they conclude that a money rule with a zero inflation rate would not be optimal.²⁷

After briefly summarizing the huge literature on transaction costs and the demand for money, it is clear that there is a well established tradition that links money holdings to the trading process and the "transaction costs" of both undertaking trade and using the alternatives to money. Throughout the literature the types of transaction costs have included trips to the bank, shopping time, brokerage fees, agency costs, information costs, menu costs, and computing and networking difficulties. But whatever the

²⁵ The standard transactions models of the demand for money (such as Baumol (1952) and Tobin (1956)), exhibit increasing returns to scale rather than the required constant returns to scale.

²⁶ They show that with a non-constant returns to scale transaction technology, transaction costs are not linearly homogeneous and therefore do not satisfy the conditions for zero inflation tax.

²⁷ See also Friedman (1969).

type of transaction cost considered, holding larger quantities of real money balances always tends to introduce greater flexibility in exchange and so to reduce the impact of the relevant transaction cost. To formalize this in my thesis, I use a transaction technology function with real money balances as an argument. While the assumption of a specific functional form is left until Chapter II; the general form of the cost function falls directly out of this survey. The firm's transaction costs should increase with the scale of the firm's activities and decrease with the holding of real money balances.

1.3 Adjustment Costs for Capital: The Investment Function

The second way that I have modified the theory of the firm is to add money to the firm's investment adjustment cost, i.e., the cost of changing the capital stock. Historically, the discussion of the cost of adjustment problem in the theory of the firm began with the recognition that without a cost of adjustment there would be no basis for the investment function.²⁸ Without adjustment costs the dynamic theory of the firm reduces to a static theory where both capital and non-capital factors become choice variables with the same time dimension. Eisner and Strotz's (1963) generalization of Jorgenson's (1963) work on the optimal stock of capital led to the development of flexible-accelerator analysis, where the crucial feature was an endogenous speed of adjustment dependent on parameters characterizing the firm's technology.²⁹ Jorgenson (1972) then fully modified the neo-classical investment function to accommodate adjustment costs of investment. Similarly, Tobin's (1969) q-theory of investment received its solid analytical foundation from the use of adjustment costs

²⁸ Eatwell, Millgate, Newman (1988), p.2..

²⁹ Treadway (1969).

(Abel, 1979).³⁰ Current empirical work on investment theory now takes as its standard form an adjustment cost that is well-behaved, i.e., a cost that increases more than in proportion to the rate of investment. This implies marginal adjustment costs increase with scale of investment. In the current investment literature, it is widely accepted that financial variables do affect investment spending decisions.³¹ Finance models typically assume that internally generated funds for investment are available to the firm at lower cost than is external finance. Further it has been shown empirically that investment and borrowing decisions are inseparable and that investment variation is sensitive to a measure of cash flow.³² Several reasons for why liquidity considerations can affect the firm's investment decisions are often given.³³

First, firms are sometimes viewed as financially constrained by the high cost of external finance and hence prefer to finance investment activity from their holdings of internally generated funds.³⁴ Second, because of

³⁰ In the framework of positive adjustment costs Tobin's q-model can be seen as a consequence of choice-theoretic decision making.

³¹ For a detail discussion see Fazzari, Hubbard and Petersen (1988), Gertler and Hubbard (1988), Oliner and Rudebusch (1992), Whited (1992), Carpenter (1992).

³² Bond and Meghir (1994).

³³ Chirinko (1987), Chirinko and Schaller (1993), Chirinko and Fazzari (1992).

³⁴ Fazzari, Hubbard, and Petersen (1988), Gertler and Hubbard (1988), Ciccolo (1982), Devereux and Schiantarelli (1990), Hoshi, Kashyap and Schafstein (1991), Hayashi and Inoue (1991).

information costs, in particular, the cost of asymmetric information, liquidity may play a special role in the investment decision. Asymmetric knowledge of the firm's investment opportunities between the firm's managers and their suppliers of their financial capital results in a premium (over the cost of internal funds) reflected in higher borrowing rates and sometimes in a lemons premium when a firm issues new equity.³⁵ Capital market research has tended to show that real investment does depend on financial factors suggesting the significance of different transaction costs, agency costs or asymmetric information.³⁶ In addition Stiglitz and Weiss (1981) have argued that asymmetric information may cause lenders to ration credit. By holding larger quantities of liquid assets, including real money balances, the firm can avoid these costs or at least reduce their impact. Third, money holdings are sometimes viewed as a proxy for the additional revenue that enters the investment equation due to imperfect competition or scale economies.³⁷ Fourth, given that investment projects take time to complete, higher money holdings allow the firm to avoid the dislocation costs of having to abandon

³⁵ Myers and Majluf (1984).

³⁶ Gertler (1988).

³⁷ Mahdavi, S., et. al. (1994) have shown that investment and cash flow are cointegrated for the U.S. economy. They found a unidirectional "causal" effect between cash flow and investment. See also Domovitz, Hubbard and Petersen (1988), Hall (1988).

or postpone an ongoing project because of a shortfall of cash or a sudden change in financing requirements.³⁸ Firms choose to smooth their investment path because of the rising cost of acquiring and installing capital as the rate of investment increases.³⁹ In a world of uncertain borrowing costs and random shocks, money holding may lower the cost of achieving this

For all these reasons, when the marginal cost of external finance exceeds the opportunity cost of internal funds, the firm can benefit by holding larger quantities of liquid assets (including money). Holding money balances for the purpose of initially financing investment allows the firm to relax the financial cost of rapidly increasing its capital stock.⁴⁰ It should be noted, however, that while the transactions approach to money emphasizes money's use in exchange and hence directs attention to a narrower definition of money, the liquidity arguments of this section focus on money's role as a store of value and hence appeal to a broader definition of money. In the empirical tests of Chapter VI a test for the transactions demand for money leads to the adoption of M1 as the relevant money measure. However, should all measures of money not move together,

³⁸ Meltzer (1960).

³⁹ Eisner and Stortz (1963), Lucas (1967).

⁴⁰ Chirinko and Schaller (1993).

this difference should be born in mind when interpreting the estimation results.

Fazzari and Petersen (1993) are only the latest to emphasize that fluctuations in investment account for a substantial portion of the movement in GNP. Since money flows are volatile over the business cycle, the link between money, internal finance, and investment may help to explain an important feature of output fluctuations. As Fazzari and Petersen (1993) show, the magnitude of investment fluctuations induced by cyclical changes in internal finance are tied to the strength of the firm's balance sheet. Firms that are highly liquid are able to smooth investment despite fluctuations in their cash flow, without the need of costly external funds.

My second adjustment to theory of the firm then incorporates real money balances into the investment adjustment cost of the firm as a measure of these liquidity effects. Holding real money balances is assumed to reduce the severity of the financial constraints on the investment decision by reducing the firm's need to raise costly external funds (on short notice).⁴¹ A statement of the form of the investment function used in the analysis is left until chapter II. Here I discuss only its general form.

⁴¹ This means that if the firm had to obtain external financing on short notice then the firm will typically pay higher interest rate and a premium for short notice.

The adjustment cost of investment must be convex to represent increasing costs when changing the capital stock. For empirical work a quadratic adjustment cost has most often been used, in part because it provides a simple positively sloped linear marginal cost curve for empirical estimation. With a convex adjustment cost, the equilibrium rate of investment is determined by equating the falling value of the marginal product of capital with the increasing marginal cost of adjustment. When the rate of investment exceeds the required rate of replacement investment, the capital stock grows. This reduces the value of marginal product of capital and the succeeding level of investment. In this situation $\dot{K}$ ($= dK/dt$) declines with the growth in the capital stock, where $\dot{K}$ is the equilibrium stock of capital, K is the level of capital stock and t is time. A stationary state is approached when marginal cost of replacement investment equals marginal value product of capital $\dot{K} = 0$ and net investment falls to zero. The use of a convex adjustment cost then justifies the *ad hoc* distributed lag flexible accelerator function for net investment.⁴² Moreover, convex investment adjustment costs

⁴² Söndström (1976) shows that with capital adjustment costs the flexible accelerator function is defined as $\dot{K} = \delta(K^* - K)$, where δ is the rate of adjustment of capital, K^* is the equilibrium level of the capital stock, and K is the actual level of capital stock at any point in time.

explain both the observed pattern of adjustment over time and the high rate of investment that initiates a cycle.⁴

Real money balances enter to reduce the investment adjustment cost. Higher levels of real money holdings should then encourage higher levels of investment and speed the adjustment to a new steady state. However, successive increases should increase the benefit at a decreasing rate.

It follows that the adjustment cost function should be convex, increasing in the level of investment and decreasing with the increase in both capital stock and the holding of real money balances.

⁴ Rothschild (1971).

1.4 The Cost of Adjusting Real Money Holdings:

The third type of transaction cost consideration introduced into the theory of the firm is an adjustment cost for changing real money balances.⁴⁴ Money is given an adjustment cost of its own for two reasons. First, the economic reason is that there may be a cost of quickly bridging the gap between actual and desired money holdings. Second, there is an analytical wish to treat the firm's two stocks -- money and capital -- analogously. As Laidler (1993) emphasizes, empirical estimations of the demand for money frequently find that a lagged money term is one of the more significant explanatory variables. This suggests the empirical relevance of cost in adjusting money holdings.

Larger levels of money are held to reduce the cost of adjusting quickly to unanticipated variations in money flows and thus quickly closing the gap between actual and desired real money balances at any particular point in time. The higher the initial stock of real money balances, the less costly it is for the firm to offset negative shocks to its cash flow by running down existing money holdings (rather than having to raise required money balance from external sources). On the other hand, higher real balance holdings cause the firm to lose the opportunity cost of that money

⁴⁴ The adjustment cost of changing real balances follows the same pattern as the adjustment cost of changing capital stock (investment).

if a positive money shock is realized. The firm then trades off the two opposing types of costs leading to the use of real money balances as stocks of precautionary liquidity, providing insurance against the uncertainty of future shortfalls in cash flow.

The money adjustment cost can be thought of as arising from the second of the three Keynesian motives for holding money, namely the precautionary motive.⁴⁵ Money balances are held for precautionary reasons because firms face uncertainty over input and output prices.⁴⁶ If the firm's money holdings fall short of its net expenditures, a penalty cost given by the cost of obtaining instant credit must be absorbed. When the penalty cost is proportional to the size of the cash shortfall, then the adjustment cost produces an optimal level of money holdings that depends directly upon the degree of uncertainty in the trading process.

Uncertainty can arise for reasons other than uncertain prices. Ferris (1981) uses uncertain delivery time to generate a demand for both money and goods to be held as inventories by the firm under the precautionary motive. He then uses trade credit as a mechanism that separates the exchange of money from the uncertainty present in the exchange of goods (due to uncertain delivery time). He concludes that trade credit allows trading partners to

⁴⁵ Keynes (1936).

⁴⁶ Whalen (1966), Tsiang (1969).

receive advance information about the timing of their money flows, allowing them to reduce their holdings of money for precautionary reasons. More efficient management of information interacts with net money holdings and reduces the net cost of trade.

The unpredictable nature of receipts and expenditures of cash leads the firm to keep money balances on hand to minimize the cost of bridging the gap. The size of the money adjustment cost then determines the amount of real money balances that the firm will want to hold. If the firm holds too large a quantity of money, it will forego more in opportunity cost than it will benefit in extra protection. If it holds too little money, the expected penalty cost of illiquidity will exceed the opportunity cost of foregone interest.

There are then three specific considerations which influence the optimal level of real money balances held for the adjustment reasons. The first of these considerations is the penalty cost of illiquidity. This refers to the unpleasant consequences that arise if the firm does not accurately predict its cash requirements at any given point in time. If the firm does not have cash on hand or have access to a low cost line of credit, then the cost of illiquidity may be as high as abandoning an ongoing profitable project or even bankruptcy. However, by holding assets or acquiring access to a line of credit, the cost of

illiquidity is lowered to the cost of raising cash funds on short notice. When the firm has assets which can be quickly converted into cash, the cost of illiquidity falls further to the lower transaction cost of their immediate conversion. Should this cost differential disappear, precautionary money holdings will fall to zero. The level of real balances will then be determined by the transaction technology alone.

The second consideration is the opportunity cost of holding real balances. This is the same as in the two previous cases. The opportunity cost is the money rate of interest. The firm's cost It increases directly and proportionally with the increases in the holdings of real money balances.

The third consideration is volume and variability of money receipts and expenditures. The pattern of potential differences between the receipt and expenditure of funds defines a probability distribution of expected net disbursements. If net disbursements are positive then a penalty is to be paid and real money balances are held to minimize the adverse effects of such disbursements. In general, the probability distribution of net disbursement has a mean that reflects the trend in net accumulation over time and a standard deviation which reflects the degree of uncertainty in the net money flows through the firm. Moreover, the two tend to be connected. As the average volume of receipts and expenditures increase, the standard

deviation will also tend to increase. This increases the expected penalty cost associated with the now higher probability of having insufficient cash on-hand and leads the profit maximizing firm to hold higher levels of real money balances.

Finally, adding an adjustment cost to money allows money to be treated analogously to capital from a modelling perspective. The firm will face a money accumulation constraint and an equation of motion for money in the same way as it does for capital. In my formal model, the adjustment cost of changing real money balances follows the investment adjustment cost in being quadratic and convex. The quadratic form then provides a linear positively sloped marginal cost curve for empirical estimation. The specific form of this cost function is presented in Chapter II.

1.5 Recent Canadian Studies on The Demand for Money:

In this Section I briefly survey some of the more recent Canadian studies of the demand for money.

You (1981) followed the earlier work of Sinai and Stokes (1972) by estimating a Cobb-Douglas production function using aggregate Canadian data between 1946 and 1977 to test whether real balances were a part of an aggregate Canadian production function. His results strongly suggest that real balances do affect the aggregate supply of the economy quite apart from any demand side effect. He also found that when real balances were introduced into the production function, the unexplained portion of output variation virtually vanished. He followed Sinai and Stokes in interpreting this finding to mean that what had previously been attributed to disembodied technical change was really due to the role of real balances in the production process.

Khan (1985) generalized the work of You by using a translog cost function to study the production elasticities of various inputs (including real balances) on annual Canadian data for the period between 1947-1973. He found that capital and real balances were substitutes over the entire sample period. He also found that the Allen partial elasticities (AES) for real balances and labour were negative, indicating that real balances and labour were

complementary.⁴⁷ He also found that the own price elasticities of capital, labour and real balances were inelastic. The demand for real balances was the most inelastic, followed by labour and then capital services.

Khan also tested for separabilities and found that neither linear nor non-linear separability existed between labour and capital and real balances. His results implied that neither the Cobb-Douglas nor the CES production function were appropriate for estimating a multi-factor input production function. For our purposes, his results suggest that it would be inappropriate to estimate a demand for money solely as a function of its own service-price and the value of output. Rather as a factor demand, the demand for money should incorporate the prices of other primary resources.⁴⁸ Similarly, it would be inappropriate to estimate a production process solely as a function of the quantities of primary inputs independent of level of real balances.

These results also have important stabilization and welfare implications. Since real balances appear to be complementary to labour and substitute for capital, an increase in the opportunity cost of money would affect not

⁴⁷ You (1981) also found the same relationship, for Canada, between real balances and capital and between real balances and labour as Sinai and Stokes did for the U.S. over approximately the same period of time.

⁴⁸ Since labour and real balances appear to be complements and capital and real balances appear to be substitutes, it would be reasonable to expect that wages and capital service costs exert negative and positive influences on the demand for money.

only the demand for money but also the demands for labour and capital--impacting, in turn, on unemployment, capital utilization rates and growth.

Most recently, Hasan and Mahmud (1992) used Canadian annual aggregate data to test the role of money in Canadian manufacturing industries. In their 1992 article they used a translog cost function (as in Khan (1985)) for the period 1965-84 with the following four arguments: capital, real balances, production workers and non-production workers. They found that real balances were inseparable from the other factors of production, namely, capital and labour even at the disaggregated level.⁴⁹ In addition the own and cross price elasticities showed that the demand for capital and production workers were highly inelastic and the demand for production workers and money were complementary.

Hasan and Mahmud also used annual Canadian manufacturing data over the period 1965-1987 to test whether money belonged in a translog cost function representation of the Canadian manufacturing sector. The four inputs were again capital, real balances, production workers and non-production workers. Their results suggested that all inputs were significant cost determinants: all the estimates of own price, cross and substitution elasticities were significant at the one percent level of significance and all of the own

⁴⁹ These findings confirm those of Khan (1985) who found that neither linear nor non-linear separability existed between capital, labour and real money balances in the translog cost function for Canada.

price elasticities were negative as expected. In addition the own price elasticities for all inputs except non-production workers were low.⁵⁰ Finally, Hasan and Mahmud found that real balances and non-production workers were substitutes and real balances and production workers were complementary.⁵¹

An important aspect of their article was to stress the macroeconomic implications of their work. Since money and production workers are complementary, an increase in the cost of holding money (i.e. the interest rate) will decrease the demand for production workers as well as money. This will shift the aggregate supply curve upwards and to the left. At the same time higher interest rates may suppress private investment and consequently shift the aggregate demand curve downward and to the left. This can result in lower employment and output even at the higher prices.

The importance of this brief summary of recent Canadian findings is to underline the fact that most of the recent work on the demand for money highlights the simultaneous nature of the money holding choice for the firm. The firm's money holdings are determined jointly with the decision to employ labour, utilize capital and capital and/or add to the capital stock. While my work emphasizes a transaction

⁵⁰ The own price elasticity of real balances in this study is very close to Denis and Smith (1978) study of the U.S. manufacturing sector data.

⁵¹ These results are also similar to Denis and Smith's (1978) study of the U.S. manufacturing sector data.

rather than a production approach to money, it shares with these studies the conviction that the analysis of the money holding decision is a part of a larger firm optimization problem.

1.6 Preview of the Thesis:

In the following chapters I analyze the dynamic behaviour of the firm in terms of the evolution of two stocks: capital and money. These two stocks are integrated into the analysis of the optimal behaviour of the firm so that the dynamic adjustment path of the firm depends on their interaction. Chapter II develops the formal intertemporal optimization problem leading to the derivation of Euler equations to be later estimated.

Because both capital and money play an active role in firm's decision making process, the stationary state levels of the capital stock and the money stock become mutually dependent. The simultaneous nature of the stationary state solution is discussed in Chapter III (under simplified conditions). Chapter III also discusses stability and the conditions for convergence to the stationary state. Chapter IV discusses the appropriateness of the technique of Generalized Method of Moments used to test the predictions of the model.

Chapter V describes the data-set used to test the empirical predictions of the model. It provides the sources of the data set, analyzes their statistical properties, trends, and describes their co-movement. Because a direct measure of some of the variables were not readily available,

Chapter V also discusses how these variables were constructed.

Chapter VI presents the empirical findings of the thesis. The empirical results for the Euler equations of our model are presented first as single equations and later as a simultaneous system. The implications of the findings are discussed and used to evaluate the theory proposed in the thesis. Chapter VI also discusses the short run comparative static analysis of the model and performs some policy simulations. The thesis concludes in Chapter VII by summarizing the findings and highlighting areas for future research.

Chapter II: The Model

2.1 Introduction:

Satisfactory modelling of a money economy presupposes the identification of an appropriate role for money and the motives that govern the behaviour of its holders. To rationalize its use, most writers have regarded money balances as an argument in either the utility or the production function. As discussed earlier in Chapter I, this thesis endogenizes money within the firm's transaction technology and analyzes the effects of money holdings on the firm's behaviour. In essence the firm must use money in its external dealings with both customers and input suppliers and so holds money as one part of strategy for minimizing the types of transaction costs that arise in these settings. Given that the objective of the firm is to maximize the present value of the flow of profits, money holdings assist by reducing the transaction costs that the firm might otherwise incur.

This chapter formally develops a model of the intertemporal profit maximizing firm, where the firm uses both capital and money stocks to determine the optimal money accumulation and investment paths. The chapter derives the Euler equations for both investment and money, and sets the form of the equations that will be estimated later. The chapter also discusses in brief the estimation methodology.

2.2 The Model:

In the model presented below, real money balances economize on transaction costs. Rather than including real balances simply as an argument in the production function, the model presented here provides a more explicit treatment of real balances in the transaction process and allows real balances to provide liquidity in relation to the investment decision. Formally, the competitive firm is modelled as solving a dynamic optimization problem in order to maximize profit given several cost conditions that depend on the level of money holdings.

2.2.1 Formulation of The Transaction Cost Function:

I begin the modelling process by formalizing the first set of reasons for why the firm may want to hold real money balances, that is, its incentive to minimize transaction costs. It is assumed that the firm faces a technology for transacting that can be represented by a (transaction cost) function which is increasing in the volume of its sales and decreasing in its level of money holdings. In general terms we could write the transaction cost function as

$$TC_t = \gamma(y_t, m_t)$$

with partial derivatives $\gamma_y > 0$, $\gamma_m < 0$, and $\gamma_{yy} > 0$, $\gamma_{mm} > 0$, and where y is the income level and m is the level of real money balances. Saving (1971) used a

transaction cost technology of the form $T(t) = T(c(t), m(t))$ with the restrictions $T_c > 0$, $T_m < 0$, $T_{cc} > 0$ and $T_{mm} > 0$. Here T was assumed to be a continuous, twice differentiable function with c representing consumption and m representing real money balances. Saving showed how money's use facilitated transactions and how transaction time was reduced by holding real money balances. Kimborough (1986) assumed a specific linear homogeneous transaction cost function of the form $T = s(m/c)c$, where again c was consumption and m was real money balances. This formulation of transaction cost function appears to be a simple linear version of Drazen's (1979) shopping time technology function, where T is homogeneous of degree zero function of the expenditure and money, i.e., $T = T(m/c)$. Similarly Guidotti and Végh (1993) assume a transaction cost function of the form $S_t = l(c_t, m_t)$ with the restrictions $l_c \geq 0$, $l_m \leq 0$, $l_{cc} > 0$, $l_{mm} > 0$ and $l_{mm}l_{cc} - l_{cm}^2 \geq 0$. All of the above transaction cost functions assume convexity (implied by the term, e.g., $l_{cc} \geq 0$ and $l_{mm} > 0$) to ensure the existence of a unique interior solution.

In my analysis I follow Baumol (1952) and Tobin (1956) by choosing a specific functional form that is simple (amenable to empirical testing) and yet captures the desired features discussed above. Since the Generalized Method of Moments estimation technique (for J-test) requires the model to be overidentified, I chose a specific functional form for

the transaction cost technology (and for all other cost functions) which minimizes the number of parameters that need to be estimated.⁵² The firm's transaction costs are assumed to increase linearly in the average rate of money's turnover within the firm. For Baumol/Tobin this turnover rate represents the average number of trips to the bank implied by how the firm arranged its transactions. For Whalen/Miller and Orr it represents (the inverse of) the expected duration between trips to the bank implied by stochastic money flows and an $s - S$ optimal inventory holding strategy. Following this tradition, I adopt as a specific form the nominal transaction cost function:

$$TC_t = \frac{\gamma}{2} \frac{P_t F(K_t, N_t)}{\left(\frac{M_t}{P_t}\right)} = \frac{\gamma}{2} \frac{P_t F(K_t, N_t)}{m_t}, \quad (2.1)$$

where TC_t represents the transaction cost in period t , γ is the real transaction cost coefficient, $F(K_t, N_t)$ is the production function of the firm as a function of capital (K_t) and non-capital factors (N_t) in period t , and $M_t/P_t = m_t$ is the level of the firm's real money balance holdings in period t . P_t is the firm's output price (used to convert

⁵² Dowd (1990) has used a Cobb-Douglas type of transaction cost function of the form $TC = PK^0 X^{k1} m^{-k2} w^A$, where m was real money balances, x was real expenditure, P was the price of x , w was the real wage, A was the index of wage rate and k 's were the constants. Dowd showed that with some restrictions imposed on k 's this transaction function was equivalent to Baumol's (1952) transaction cost.

the real cost coefficient into nominal terms), and ϕ_t is the general price level in period t .

The transaction costs modelled in equation (2.1) are then Baumol-type transaction costs, such as the time-cost required to match output with customers, or the money-costs paid to agents to intermediate financial transactions (through brokerage fees). The lumpy nature of transaction costs provides an incentive for transactions to be bunched in order to minimize their impact.⁵³ By holding inventories of different goods including money, the firm can avoid these costs or at least minimize their impact on profits.

By choosing a transaction function of this type we satisfy the set of minimum requirements. The transaction cost function must be convex and homogeneous of degree one in nominal magnitudes (i.e., exhibit no money illusion). The marginal cost of transacting must increase with real expenditures and decrease with increases in the real money balances.

⁵³ A fixed transaction cost tends to result in bunching of transactions. This effect has played a major role in explaining the transactions demand for money such as Baumol (1952) and Tobin (1956). Cash balances are held because for short holding periods the cost of buying and selling assets are too high compared to their yields. See Hicks (1935).

2.2.2 Formulation of Investment Adjustment or Liquidity

Costs:

A second reason why the firm may wish to hold money is because it may lower the adjustment or liquidity cost of investment. In particular, by holding real balances the firm can lower the financing cost of its investment programme by avoiding the higher cost of external finance on short notice. Given the case for including money as discussed at length in Chapter I, the investment adjustment cost function can be written in general terms as

$$IC_t = \alpha(I_t, K_t, m_t) \quad \text{with } \alpha_I > 0, \alpha_K < 0, \alpha_m < 0, \alpha_{II} > 0, \alpha_{KK} > 0, \alpha_{mm} > 0,$$

where I is investment, K is the capital stock, and m is real money. The restrictions imply that the investment adjustment cost is an increasing convex function of the level of investment. The specific form of the investment adjustment cost function used in this thesis adopts the standard form used in the investment literature.⁵⁴ Treating m_t symmetrically with K_t , the nominal value of the liquidity cost of investment becomes:

⁵⁴ For similar types of investment adjustment cost function formulation see Schaller (1993b), Bond and Meghir (1994), Pezzari and Petersen (1993), Hubbard and Kashyap (1992).

$$IC_t = \frac{\alpha}{2} \frac{P_t^I I_t^2}{\left(\frac{M_t}{\rho_t}\right) K_t} = \frac{\alpha P_t^I}{2 m_t} \left(\frac{I_t}{K_t}\right)^2 K_t, \quad (2.2)$$

where IC_t is the investment adjustment or liquidity cost in period t , α is the real liquidity cost coefficient, I_t is the level of real investment in period t , P_t^I is the price of investment goods in period t (used to convert the real cost into nominal units). This investment cost function is twice differentiable, convex and homogeneous of degree one in nominal magnitudes. This implies that higher rates of investment increase dislocation within the firm and raise the effective cost of adding capital goods ($IC_t > 0$). Larger sized firms, however, can undertake a given size of investment at lower cost ($IC_K < 0$).

2.2.3 Formulation of Money Adjustment cost:

A third reason for holding larger average quantities of real money balances is that there may arise adjustment costs when changing the level of real money balances. We assume that there is a higher cost of acquiring money on short notice and hence a cost associated with changing the level of real balances. The functional form of money adjustment cost is based on the form of the investment adjustment cost adopted earlier. In most general terms the money adjustment

cost could be written as $AC_t = \eta(u_t, m_t)$, where m_t represents the level of real balances and u_t represents the change in that level. The restrictions imposed on the functional form are that $\eta_u > 0, \eta_m < 0, \eta_{uu} > 0, \eta_{mm} > 0$. These imply that the money adjustment cost is increasing and convex in u . Following the specific functional form chosen for the investment adjustment cost equation in (2.2) above, I represent the nominal value of the money adjustment cost as:

$$AC_t = \frac{\eta}{2} \frac{\rho_t (u_t / \rho_t)^2}{m_t} = \frac{\eta}{2} \rho_t \left[\frac{(u_t / \rho_t)}{m_t} \right]^2 m_t, \quad (2.3)$$

where AC_t is the nominal adjustment cost of real balances in period t , η is the money adjustment cost coefficient, and u_t is the change in the nominal stock of money balances (i.e., $M_t - M_{t-1}$). The chosen form means that the adjustment cost function for money balances (like investment) will be twice differentiable, convex and homogeneous of degree one in nominal magnitudes. This cost function leads the firm to wish to smooth variations in its actual money holdings over time.

2.3 The Firm's Objective Function

Given the cost specifications, in Sections 2.2.1 through 2.2.3, the firm's dynamic optimization problem is to maximize the present value of the net monetary flows passing through the firm subject to two additional constraints (equations of motion). The first is the capital accumulation identity which reflects the way that the firm's investment decision alters the stock of capital over time. The level of capital in any period t , K_t , is equal to the depreciation adjusted capital stock from the previous period plus the purchase of new capital, i.e., $K_t = I_t + (1 - \delta)K_{t-1}$, where δ is the one period depreciation rate on capital goods.⁵⁵ The second constraint is the equation of motion used to reflect the way that money balances accumulate over time. In my dynamic optimization problem, money balances are treated analogously to investment. The stock of money balances at any period of time t is equal to the previous level of money balances plus the flow acquisition of money, u_t , i.e., $M_t = u_t + M_{t-1}$.

⁵⁵ When the capital stock is stationary ($K_t = K_{t-1}$), gross investment just offsets depreciation, so that $I_t = \delta K_t$.

The firm has two sources for financing accumulation. It can use either its own internal resources or it can borrow at the market interest rate. I assume that the firm faces an external bond market that is perfectly competitive so that its subjective rate of discount will be equated to the market rate of interest.⁵⁶ Assuming that the firm is subject to a No-Ponzi-Game-Condition, bond holdings reallocate money flows over time rather than change their present value.⁵⁷ Therefore, I abstract from the bond market in what follows.

With this background, the firm's decision problem at time $t = 0$ over a time horizon of n years can be written as choosing the levels of N_t , I_t , K_t , u_t , and M_t that maximize the present value of the monetary flows through the firm. More formally,

⁵⁶ That is, $\beta_t = \frac{1}{1+r_t}$, where β_t is the subjective rate of discount in period t and r_t is the market rate of interest in period t .

⁵⁷ The No-Ponzi-Game-Condition means that the borrowing can not increase asymptotically faster than the interest rate. This implies that the firm can not go on borrowing without repaying its debt.

$$\begin{aligned}
\mathcal{L} = & \text{Max } E_0 \sum_{t=0}^{t=n} \mathfrak{R}_t \left\{ [P_t F(K_t, N_t) - W_t N_t - \frac{\alpha P_t^I \left(\frac{I_t}{K_t} \right)^2}{2m_t} K_t \right. \\
& - P_t^I I_t - \frac{\gamma P_t F_t(N_t, K_t)}{2m_t} - \frac{\eta}{2} \varphi_t \left(\frac{u_t / \varphi_t}{m_t} \right)^2 m_t - u_t \\
& + \lambda_t^K [I_t - K_t + (1-\delta) K_{t-1}] \\
& \left. + \lambda_t^M [u_t - M_t + M_{t-1}] \right\}
\end{aligned} \tag{2.4}$$

with $K(t=0) = K_0$, $M(t=0) = M_0$ and where,

$$\mathfrak{R}_t = \prod_{i=1}^{t-1} \beta_i,$$

$\frac{\alpha P_t^I \left(\frac{I_t}{K_t} \right)^2}{2m_t} K_t$ is the liquidity or investment adjustment cost in period t ,

$\frac{\gamma P_t F_t(N_t, K_t)}{2m_t}$ is the transaction cost in period t ,

$\frac{\eta}{2} \varphi_t \left(\frac{u_t / \varphi_t}{m_t} \right)^2 m_t$ is the adjustment cost of real balances in period t ,

β_i is the rate of discount in period i , i.e., $\beta_i = \frac{1}{1+r_{i-1}}$,

N_t is non-capital factors of production in period t ,

K_t is capital stock in period t ,

I_t is new investment in period t ,

W_t is the cost of non-capital factors in period t ,

r_t is the rate of interest in period t ,

P_t^I is the price of investment goods in period t

(acquisition price of investment goods),

P_t is the output price in period t ,

φ_t is the general price level in period t ,

δ is the rate of depreciation of capital,

M_t is the stock of money in period t ,

u_t is the change in the money stock in period t ,

$E_0(\cdot)$ is the expectations operator in period 0,

λ_t^K is the shadow price of capital in period t ,

λ_t^M is the shadow price of money balances in period t .

$P_I^t I_t$ is the money cost of accumulating capital and,

u_t is the money cost of accumulating nominal money balances.

2.4 Profit Maximisation and First-Order Conditions:

Given the dynamic optimization problem in equation (2.4), the first-order conditions for profit maximization are first derived and then discussed in sections that follow.

2.4.1 First-Order Conditions for The Non-Capital Factors of Production:

It has been assumed that the non-capital factors of production in the model are variable within the period and can be changed without incurring an adjustment cost.⁵⁴ Optimal input use then requires the firm to set input use by equating the value of marginal product of non-capital factors of production to their unit cost as in static theory. Because the most significant non-capital factor of production is labour, I denote the unit cost at W_t , the wage rate.

In this model the first-order condition for the non-capital factors N_t in period t is:

⁵⁴ Salop (1973), Sargent (1979), and Martenson (1970) have developed models where non-capital factors of production such as labour do have adjustment costs in the form of hiring and firing costs.

$$\frac{\partial \mathcal{L}}{\partial N_t} = E_0 \left[P_t F_N - W_t - \frac{\gamma P_t F_N}{2m_t} \right] = 0$$

$$\text{i.e., } E_0 \left[F_N \left(P_t - \frac{P_t \gamma}{2m_t} \right) \right] = E_0 (W_t) \quad (2.5)$$

$$E_0 (F_N) = E_0 \left[\frac{W_t}{P_t} \left(\frac{2m_t}{2m_t - \gamma} \right) \right] = E_0 (w'_t)$$

where $w'_t = \frac{W_t}{P_t} \left(\frac{2m_t}{2m_t - \gamma} \right)$ is the real wage adjusted for the presence of transaction costs. Equation (2.5) equates the expected marginal product of the non-capital factors of production with the expected full marginal cost, i.e., the expected marginal product of non-capital factors of production adjusted for the effect of additional N_t on expected output and hence transaction costs.⁵⁹

Optimal input use implies that the traditional marginal product of labour will not equal the real wage. Rather, the marginal product must exceed the real wage by a proportion that depends directly on the firm's real money holdings. In (2.5) γ is the transaction cost coefficient, and because

$\gamma > 0$, $\left(\frac{2m_t}{2m_t - \gamma} \right) > 1$. The presence of expected transaction costs therefore decreases the firm's use of its non-capital inputs (for the same level of capital, K_t) relative to the zero transaction cost case.

⁵⁹ The full marginal cost in this context means expected marginal cost of hiring one unit of the factor of production adjusted for its marginal transaction cost.

2.4.2 First-Order Condition for the Capital Factors of Production:

In our model we have assumed that capital is a quasi-fixed factor, i.e., it can be changed only if the firm is prepared to bear the adjustment cost.

The first-order condition for capital stock in period t is:

$$\frac{\partial \mathcal{L}}{\partial K_t} = E_0 \left[P_t F_K \left(1 - \frac{\gamma}{2m_t} \right) + \frac{\alpha P_t^I}{2m_t} \left(\frac{I_t}{K_t} \right)^2 - \lambda_t^K + \beta_{t+1} (1-\delta) \lambda_{t+1}^K \right] = 0, \quad (2.6)$$

$$i.e. E_0 \left[P_t F_K \left(\frac{2m_t - \gamma}{2m_t} \right) + \frac{\alpha P_t^I}{2m_t} \left(\frac{I_t}{K_t} \right)^2 \right] = E_0 [\lambda_t^K - \beta_{t+1} (1-\delta) \lambda_{t+1}^K].$$

As was true of the non-capital factors of production, equation (2.6) shows that the expected marginal product of capital must be adjusted for the effects of expected transaction costs (negative). Since $\gamma > 0$, $\left(\frac{2m_t - \gamma}{2m_t} \right) < 1$, which implies that the presence of transaction costs will decrease the firm's use of capital (for the same level of non-capital inputs N_t) relative to the zero transaction cost. In addition, however, holding additional capital lowers the firm's investment adjustment cost, indirectly increasing the productivity of capital. In this formulation, then, the expected marginal product of capital adjusted for its marginal effects on both transaction and liquidity costs is

equated to the expected difference between the present and future shadow prices of capital.

2.4.3 First-Order Condition for Investment:

The first order-condition for investment in period t is:

$$\frac{\partial V}{\partial I_t} = E_0 \left[-P_t^I - \left(\frac{\alpha P_t^I}{m_t} \right) \left(\frac{I_t}{K_t} \right) + \lambda_t^K \right] = 0, \quad (2.7)$$

$$E_0(\lambda_t^K) = E_0 \left[P_t^I \left(1 + \frac{\alpha}{m_t} \left(\frac{I_t}{K_t} \right) \right) \right].$$

In equation (2.7) the expected shadow price of capital equipment, $E_0(\lambda_t^K)$, is the expected marginal increase in the firm's value that results from adding one unit of capital to the firm. The market price of a new unit of capital is P_t^I . Hence, equation (2.7) shows that the expected shadow price of investment, $E_0(\lambda_t^K)$, is equated to the expected acquisition price of a new capital good plus its effect on the expected marginal adjustment cost of investment. Equation (2.7) implies that the firm would invest up to the point at which the shadow price of an additional unit of capital is equal to the marginal cost of investment, where the full cost of

investment includes both purchase price of capital and the marginal effect on the firm's marginal adjustment cost.⁶⁰

Tobin (1969) argued that the level of investment should be positively related to the market value of an additional unit of capital relative to its replacement cost (often referred to as Tobin's marginal q). In terms of my analysis Tobin's q -theory of investment suggests that the firm's investment decision depends upon the ratio of the marginal value of capital, λ_t^K , to the price of new capital goods,

P_t^I , i.e., $\left(\frac{\lambda_t^K}{P_t^I} \right)$. For Tobin, however, the firm's capital goods holdings are in equilibrium if and only if $q = 1$.⁶¹

In my analysis, equilibrium requires the expected ratio of

shadow to acquisition price of capital, $E_0 \left(\frac{\lambda_t^K}{P_t^I} \right)$, to equal one plus the expected marginal liquidity cost of investment. (now incorporating real money balances), i.e.,

⁶⁰ Note that the investment adjustment cost is written in terms of gross rather than net investment. Hubbard and Kashyap (1992) and Watkins (1993) are some examples of those who use gross investment to write their investment adjustment cost.

⁶¹ $q = 1$ implies that $\dot{K} = 0$. This means that the ratio of shadow value of the firm of having an additional unit of capital in place to the price of acquiring new capital, $\left(\frac{\lambda_t^K}{P_t^I} \right)$, is equal to $q = 1$ if capital holdings are in equilibrium.

$$E_0 \left(\frac{\lambda_t^K}{P_t^I} \right) = E_0 \left[1 + \frac{\alpha}{m_t} \left(\frac{I_t}{K_t} \right) \right]. \quad (2.8)$$

In equilibrium the firm must replenish capital by an amount that just offsets the depreciation rate each period. This results in an expected marginal cost of $\left(\frac{\alpha \delta}{m_t} \right)$ that is in addition to the unit acquisition cost in the stationary state. Thus when $K_t = K^*$, and $I_t = \delta K^*$, equation (2.8) becomes,

$$E_0 \left(\frac{\lambda_t^K}{P_t^I} \right) = E_0 \left(1 + \frac{\alpha \delta}{m_t} \right). \quad (2.9)$$

It follows that my analysis is directly equivalent to Tobin's only if the level of investment is zero in the steady state, i.e., $\delta = 0$ so that $I_t = 0$ when $K_t = K^*$.

To solve for the Euler equation the expected shadow price one period in the future is needed. This is found by advancing (2.7) one period forward to find:

$$E_0 (\lambda_{t+1}^K) = E_0 \left[P_{t+1}^I \left(1 + \frac{\alpha}{m_{t+1}} \left(\frac{I_{t+1}}{K_{t+1}} \right) \right) \right]. \quad (2.10)$$

2.4.4 First-Order Condition for the Money Stock:

The first-order condition for the money stock in period t is:

$$\frac{\partial \mathcal{L}}{\partial M_t} = E_0 \left[\frac{\alpha P_t^I / \varphi_t \left(\frac{I_t^2}{K_t} \right)}{2m_t^2} + \frac{\gamma P_t / \varphi_t F(K_t, N_t)}{2m_t^2} + \frac{\eta \left(\frac{u_t / \varphi_t}{m_t} \right)^2}{2} \right] \quad (2.11)$$

$$- E_0 \left[\lambda_t^M - \left(\frac{1}{1+r_t} \right) \lambda_{t+1}^M \right] = 0.$$

where, α is the cost coefficient for the investment adjustment cost function, γ is the cost coefficient for the transaction cost function and η is the cost coefficient for the money adjustment cost function. Equation (2.11) shows that the firm's optimal holding of money will be determined by equating the sum of the expected values of the marginal investment adjustment cost, the marginal transaction cost and the marginal money adjustment cost to the expected difference between the present and future shadow prices of money balances.

2.4.5 First-Order Condition for the Change in the Money

Stock:

The first-order condition for the change in the money stock in period t is:

$$\frac{\partial \mathcal{L}}{\partial u_t} = E_0 \left(-1 - \eta \frac{u_t / \phi_t}{m_t} + \lambda_t^M \right) = 0, \quad (2.12)$$

$$E_0 (\lambda_t^M) = E_0 \left(1 + \eta \frac{u_t / \phi_t}{m_t} \right),$$

where, $E_0 (\lambda_t^M)$ is the expected shadow price of the stock of new real balances. Equation (2.12) implies that the firm would hold an additional unit of real money balances up to the point at which the shadow price of an additional unit of money is equal to the marginal cost of changing the money stock. The marginal cost of changing money stock is equal to acquisition price of money, \$1, plus the change in the marginal adjustment cost of money, $\phi_t \eta u_t / m_t$. The formulation of the optimal money flow condition can again be seen as analogous to Tobin's q form of the optimal capital flow equation. Since the money price of money is one and $u_t = 0$ in equilibrium, the shadow price of money relative to its acquisition cost equal one in the stationary state.

Taking the expected shadow price of money balances one period into the future to solve for $E_0 (\lambda_{t+1}^M)$ gives,

$$E_0 (\lambda_{t+1}^M) = E_0 \left(1 + \eta \frac{u_{t+1} / \phi_{t+1}}{m_{t+1}} \right). \quad (2.13)$$

2.4.6 Adoption of Cobb-Douglas Form:

To solve for the optimal investment and money behaviour of the firm in a form that will permit empirical testing it is helpful to make assumptions about the functional form of the relevant functions. The transaction, liquidity and adjustment costs have already been given specific forms. To tie down the two marginal products we now assume a Cobb-Douglas production function that is homogeneous of degree one (i.e., the firm is operating under constant returns to scale). This implies:

$$F_K K_t + F_N N_t = F$$

$$F_K = \frac{F}{K_t} - \frac{F_N N_t}{K_t}, \quad = \frac{F}{K_t} - \frac{C_t}{K_t} \quad (2.14)$$

where, $F_N N_t = w_t' N_t = C_t$, and $w_t' = \frac{W_t}{P_t} \left(\frac{2m_t - \gamma}{2m_t} \right)$.

2.5 Solving For The Euler Equations:

2.5.1 The Euler Equation For Investment:

To solve for the optimal investment behaviour of the firm, we substitute the first-order condition of the flow equation into the stock equation.⁶² That is, we substitute the solutions for $E_0(\lambda_t^K)$ and $E_0(\lambda_{t+1}^K)$ from (2.7) and (2.10) into the first-order condition for capital in (2.6). Using $E_0(\beta_{t+1}) = E_0\left(\frac{1}{1+r_t}\right)$ and (2.14), we obtain following

investment equation:

$$E_0 \left\{ P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \left(1 - \frac{\gamma}{2m_t} \right) + \frac{\alpha P_t^I}{2m_t} \left(\frac{I_t}{K_t} \right)^2 - P_t^I \left[1 + \frac{\alpha}{m_t} \left(\frac{I_t}{K_t} \right) \right] \right. \quad (2.15)$$

$$\left. + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left[1 + \frac{\alpha}{m_{t+1}} \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] \right\} = 0.$$

Rearranging (2.15) and collecting all of the expected marginal benefit and expected marginal cost terms we have:

⁶² Bond and Meghir (1994) and Schaller (1993b) are some of the examples who use this method to write Euler equations in terms of observable variables.

$$\begin{aligned}
& E_0 \left[P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \left(1 - \frac{\gamma}{2m_t} \right) + \alpha \left[\frac{P_t^I}{2m_t} \left(\frac{I_t}{K_t} \right)^2 - \frac{P_t^I}{m_t} \left(\frac{I_t}{K_t} \right) + \left(\frac{1-\delta}{1+r_t} \right) \left(\frac{P_{t+1}^I}{m_{t+1}} \right) \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] \right] \\
& = E_0 \left[P_t^I - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \right]
\end{aligned}
\tag{2.16}$$

where γ is the cost coefficient for transaction costs and α is the cost coefficient for the adjustment/liquidity cost function across the period.

The time dating of the variables in (2.16) indicates that the optimal decision involves trade offs over time so that the capital decision will be dynamic. The left hand side of the equation (2.16) can be interpreted as the full expected marginal benefit of holding one additional unit of capital in period t . This includes the expected marginal product of that capital adjusted for its expected effect (negative) on transaction costs and its expected effect (positive) on the change in liquidity costs across periods. The right hand side of the equation can be interpreted as the difference in the expected price of that unit of capital between present and the future time period. Along the optimal path, then, the expected benefit of holding one additional unit of capital must be equal to the expected marginal cost.

2.5.2 The Euler Equation For Money:

The optimal money behaviour of the firm can be solved for by substituting the flow equation solutions for $E_0(\lambda_t^M)$ and $E_0(\lambda_{t+1}^M)$ from (2.12) and (2.13) into the stock equation (2.11). Using $E_0(\beta_{t+1}) = E_0\left(\frac{1}{1+r_t}\right)$, we find,

$$E_0 \left\{ \frac{\alpha}{2} \left(\frac{P_t^I / \varphi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) + \frac{\gamma}{2} \left(\frac{(P_t / \varphi_t) F(K_t, N_t)}{m_t^2} \right) + \frac{\eta}{2} \left(\frac{u_t / \varphi_t}{m_t} \right)^2 \right. \\ \left. - \left(\eta \frac{u_t / \varphi_t}{m_t} + 1 \right) + \frac{1}{1+r_t} \left(\eta \frac{u_{t+1} / \varphi_{t+1}}{m_{t+1}} + 1 \right) \right\} = 0 \quad (2.17)$$

which after rearranging and collecting all of the marginal benefit and marginal cost terms becomes:

$$E_0 \left\{ \frac{\alpha}{2} \left(\frac{P_t^I / \varphi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) + \frac{\gamma}{2} \left(\frac{(P_t / \varphi_t) F(K_t, N_t)}{m_t^2} \right) \right. \\ \left. + \eta \left[\frac{1}{2} \left(\frac{u_t / \varphi_t}{m_t} \right)^2 - \left(\frac{u_t / \varphi_t}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1} / \varphi_{t+1}}{m_{t+1}} \right) \right] \right\} \quad (2.18) \\ = E_0 \left(- \frac{r_t}{1+r_t} \right).$$

Where γ is again the cost coefficient for transaction function, α is the cost coefficient for the liquidity function and η is the cost coefficient for the money adjustment function across the periods. Equation (2.18)

again indicates choices over time so that the money holding decision is dynamic. The left hand side of the equation (2.18) can be interpreted as the expected marginal benefit of holding an additional real money balance, in terms of reducing both the liquidity and transaction costs of the firm and net of its net effect on the money adjustment cost. The right hand side of (2.18) represents the present value of the market opportunity cost of holding that real dollar inside the firm. Along the optimal path, then, the expected marginal benefit of holding money is equal to the expected marginal opportunity cost.

2.6 Hypothesis Testing Approach:

In this section I present a brief statement of how I undertook the empirical estimation of the Euler equations for investment and real balances. To do so I utilized a relatively new approach to try and determine whether real money balances belonged in the firm's optimization problem. This test used the Generalized Method of Moments (GMM). The GMM approach uses a set of moment conditions to estimate the parameters of a model and to undertake a model specification test. Under GMM, the overidentifying restrictions placed on the model are that the instruments are orthogonal to the error terms under rational expectations. If this orthogonality condition is satisfied, the given model will have been appropriately specified. The errors in estimation process would be purely expectational errors and would contain no information about the model's specification.

GMM provides a J-statistic as a criterion for deciding whether a model is appropriately specified. Hansen (1982) lemma 4.2 has shown that in a test for model selection if the model is overidentified then the J-statistic converges asymptotically to a χ^2 distribution.⁶³ This J-statistic has a χ^2 distribution with $r-q$ degrees of freedom.⁶⁴ Thus if the

⁶³ A model is said to be overidentified when there are more orthogonality conditions than the number of parameters to be estimated.

⁶⁴ Where r is the number of instruments and q is the number of parameters to be estimated.

J-statistic generated when the model is estimated is less than the critical value of χ^2 distribution, the null hypothesis of no specification error can not be rejected. Not being able to reject the null hypothesis is interpreted as meaning that the instruments in the estimation are orthogonal to the error terms and the model is appropriately specified.⁶⁵ However, if the J-statistic is greater than the appropriate critical value of χ^2 distribution then null hypothesis of no specification error is rejected. This is interpreted to mean that the instruments are not orthogonal to the error terms and the model is inappropriately specified.

In my case, the Euler equations are the first set of moment conditions and the combination of the Euler equation with the rational expectations hypothesis will form the second set of moment conditions. Under rational expectations, the error term in the Euler equation should reflect only expectational errors and should therefore be orthogonal to information available in previous periods. Hence if the Euler equation is misspecified, the error term will contain more than just the expectational error and this will be indicated by a correlation of the error term with either the explanatory variables themselves or with information available from previous periods.

⁶⁵ Schaller (1993b).

GMM estimation of the firm's Euler equations thus allows us to test whether real money balances should be included in the firm's optimization problem.⁶⁶ This is potentially useful because of the way that the Euler equation minimizes expectations information. The Euler equations in (2.15) and (2.17) represent period-to-period arbitrage conditions, so that by estimating each period separately, only the expectations of the next period, (not the entire future stream of expectations) need be included.⁶⁷ Imposing rational expectations then means that expected values are replaced with the actual values plus an expectational error. Thus, by estimating across adjacent periods and simplifying the modelling of expectations, the Euler equation approach reduces the complications of estimating the entire dynamic path of adjustment to a stationary or steady state.

When comparing the model developed above with and without real balances, we expect that the J-test will not reject the money augmented model. This would indicate that the way this thesis has included money in the theory of the

⁶⁶ Schaller (1993b) uses a similar approach to estimate an Euler investment equation.

⁶⁷ Since the Euler equations represent period to period arbitrage conditions, the firm will choose the same time path for investment (money holdings) in both periods t and $t+1$ as long as it makes the optimal trade off between the two periods. This is the same as if the firm made its decision based on the entire future stream of marginal products of capital (marginal benefits of money holdings). The Euler equations then make the decision making process easier because the information requirements of predicting the next period are much less demanding than those for predicting the entire future stream of profitability.

firm has not been rejected by the data. This result would in turn support the method in which this thesis integrates the real and financial sectors in the theory of the firm.

In our context this implies that real balances are a significant part of the firm's optimization problem indicated by the estimated J-statistic not being able to reject null hypothesis of no specification error for the money augmented model.

2.7 Derivation of Estimating Equations:

2.7.1 The Investment Equation:

Using the reasoning of section 2.6 we reduce the general multi-period problem of section 2.5 to the easier problem of estimating the sequential arbitrage problem represented by the Euler equations. Because of the additive separability of objective function in (2.4) and the difference equation characterization of equations of motion, the choices made at date t will affect the returns in periods t and beyond but not earlier. Hence our problem is recursive.⁵⁵ Assuming then that the firm chooses sequentially so that the current and past values of the endogenous variables are observable, the firm forms an expectation only over the $t+1$ period values of the endogenous variables. Assuming also that the firm adopts rational expectations, the firm will predict accurately only on average. This implies that we need only add a random expectational error term, ϵ_t^K , to equation (2.15) to obtain an appropriate estimation equation of the following form:⁵⁶

⁵⁵ Sargent (1987) p.30.

⁵⁶ Bond and Meghir (1994) show that Euler equations can be estimated by evaluating the expectations at realized values. Therefore, this introduces an expectational error and under rational expectations an appropriate method of moments estimation can be used to obtain consistent parameter estimates. Abel (1980) and Shapiro (1986) have also used this form for modelling investment.

$$\begin{aligned}
& P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \left(1 - \frac{\gamma}{2m_t} \right) + \frac{\alpha P_t^I \left(\frac{I_t}{K_t} \right)^2}{2m_t} - P_t^I \left[1 + \frac{\alpha}{m_t} \left(\frac{I_t}{K_t} \right) \right] \\
& + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left[1 + \frac{\alpha}{m_{t+1}} \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] = e_t^A.
\end{aligned} \tag{2.19}$$

Collecting the terms for estimation purposes, we obtain the following equation:

$$\begin{aligned}
& \left[P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - P_t^I + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \right] \\
& + \alpha \left[\frac{P_t^I \left(\frac{I_t}{K_t} \right)^2}{2m_t} - \frac{P_t^I \left(\frac{I_t}{K_t} \right)}{m_t} + \left(\frac{1-\delta}{1+r_t} \right) \frac{P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right)}{m_{t+1}} \right] \\
& - \gamma \left[\left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \frac{P_t}{2m_t} \right] = e_t^K.
\end{aligned} \tag{2.20}$$

With its expectational error, equation (2.20) describes the predicted investment behaviour of the firm under my version of the firm's liquidity and transaction constraints.

If real money balances had no effect on the investment behaviour of the firm then equation (2.20) would reduce to:

$$P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - P_t^I + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \quad (2.21)$$

$$+ \alpha \left[\frac{P_t^I}{2} \left(\frac{I_t}{K_t} \right)^2 - P_t^I \left(\frac{I_t}{K_t} \right) + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] = \epsilon_t^K.$$

This is the traditional version of the investment equation.⁷⁰ It follows that if real money balances did belong to the investment equation then all of the (currently omitted) money terms from (2.20) would appear on the right hand side of the equation with the error term in equation (2.21). That is from (2.20) and (2.21) we get:

$$P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - P_t^I + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \quad (2.22)$$

$$+ \alpha \left[\frac{P_t^I}{2} \left(\frac{I_t}{K_t} \right)^2 - P_t^I \left(\frac{I_t}{K_t} \right) + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] = \epsilon_t^K.$$

$$\text{where, } \epsilon_t^K = \epsilon_t^K + \frac{\gamma}{2} \left[\frac{P_t}{m_t} \left(\frac{F_t}{K_t} - \frac{C_t}{K_t} \right) \right] + \frac{\alpha}{2} \left[P_t^I \left(\frac{I_t}{K_t} \right)^2 - \frac{P_t^I}{m_t} \left(\frac{I_t}{K_t} \right)^2 \right] \\ - \alpha \left[P_t^I \left(\frac{I_t}{K_t} \right) - \frac{P_t^I}{m_t} \left(\frac{I_t}{K_t} \right) \right] + \alpha \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left[\left(\frac{I_{t+1}}{K_{t+1}} \right) - \left(\frac{I_{t+1}}{K_{t+1} m_{t+1}} \right) \right]$$

The hypothesis that money plays an important role in the investment decision of the firm asserts that what appears to

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For example, a similar investment function with adjustment cost in the Euler equation can be found in Chirinko (1993),

$\frac{I_t}{K_t} = \frac{\rho I_{t+1}}{K_{t+1}} - \left(\frac{1}{\alpha} \right) (P_t^I - P_{t+1}^I) + \left(\frac{1}{\alpha} \right) \lambda + u_t$ where λ is the functional form for marginal product of capital and the adjustment cost technology.

be an error term in (2.22) is not just a random expectational error. That term should contain some unutilized information about the investment behaviour of the firm. This means that the form of the investment equation in (2.21) is misspecified (for not including transaction and money liquidity costs). When this misspecification is corrected, we should then find that the money augmented model is not rejected by the data. That is, the prediction that follows from my analysis is that if we compare the investment behaviour of the firm with and without real balances, [i.e., (2.20) and (2.22) respectively], the investment function with real balances (i.e., (2.20)) will not be rejected by the data indicated by the J-statistic not being able to reject the null hypothesis of no specification error.

2.7.2 The Money Equation:

Repeating the same argument as in 2.7.1 I write equation (2.17) to represent choice in period t and add a random expectational error term, ϵ_t^M to produce the following equation,

$$\begin{aligned}
& \frac{\alpha}{2} \left(\frac{P_t^I / \varphi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) + \frac{\gamma}{2} \left(\frac{(P_t / \varphi_t) F(K_t, N_t)}{m_t^2} \right) + \frac{\eta}{2} \left(\frac{u_t / \varphi_t}{m_t} \right)^2 \\
& - \left(\eta \frac{(u_t / \varphi_t)}{m_t} + 1 \right) + \frac{1}{1+r_t} \left(\frac{\eta u_{t+1} / \varphi_{t+1}}{m_{t+1}} + 1 \right) = \epsilon_t^M,
\end{aligned} \tag{2.23}$$

Collecting all common terms for estimation, (2.23) can be written as:

$$\begin{aligned}
& \frac{\alpha}{2} \left(\frac{P_t^I / \varphi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) + \frac{\gamma}{2} \left(\frac{(P_t / \varphi_t) F(K_t, N_t)}{m_t^2} \right) \\
& + \eta \left[\frac{1}{2} \left(\frac{u_t / \varphi_t}{m_t} \right)^2 - \left(\frac{u_t / \varphi_t}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1} / \varphi_{t+1}}{m_{t+1}} \right) \right] \\
& - \frac{r_t}{1+r_t} = \epsilon_t^M
\end{aligned} \tag{2.24}$$

Equation (2.24) then describes the estimating equation for money holding behaviour of the firm under my assumptions of the liquidity, transaction and money adjustment costs. The hypothesis to be tested is that all the three real money balance costs (the transaction cost, the liquidity cost and the real balance adjustment cost) are significant determinants of the money holding behaviour of the firm. On the other hand, if liquidity and adjustment costs did not belong in the money equation, then (2.24) would be written solely with the transaction cost considerations as:

$$\frac{\gamma}{2} \left(\frac{(P_t/\phi_t) F(K_t, N_t)}{m_t^2} \right) - \frac{r_t}{1+r_t} = \epsilon_t^M \quad (2.25)$$

Equation (2.25) then describes the money holding behaviour of the firm under transaction costs alone and this variation results in one of the traditional versions of the money holding equation.⁷¹ Under our full hypothesis, however, what would appear to be the error term in (2.25) includes more than a random expectational error. It should contain information from the liquidity and money adjustment cost considerations of the optimizing firm. From (2.24) it follows that if liquidity and adjustment costs did matter then these terms will appear in the right hand side of equation (2.25) as:

$$\begin{aligned} \frac{\gamma}{2} \left(\frac{(P_t/\phi_t) F(K_t, N_t)}{m_t^2} \right) - \frac{r_t}{1+r_t} &= \epsilon_t^M - \frac{\alpha}{2} \left(\frac{P_t^I/\phi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) \\ &- \eta \left[\frac{1}{2} \left(\frac{u_t/\phi_t}{m_t} \right)^2 - \left(\frac{(u_t/\phi_t)}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\phi_{t+1}}{m_{t+1}} \right) \right] = \phi_t^a \end{aligned} \quad (2.26)$$

Alternatively, if an analysis wrongly assumed that money had no liquidity role in investment but faces investment like costs of adjustments, then our analysis would suggest that (2.24) would be estimated as:

⁷¹ The demand for money depends only on transactions cost and exhibits economies of scale in money holding.

$$\begin{aligned}
& \frac{Y}{2} \left(\frac{(P_t/\phi_t) F(K_t, N_t)}{m_t^2} \right) - \frac{I_t}{1+r_t} \\
& + \eta \left[\frac{1}{2} \left(\frac{u_t/\phi_t}{m_t} \right)^2 - \left(\frac{(u_t/\phi_t)}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\phi_{t+1}}{m_{t+1}} \right) \right] \quad (2.27) \\
& = \epsilon_t^M - \frac{\alpha}{2} \left(\frac{P_t^I/\phi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) = \phi_t^b
\end{aligned}$$

In both cases, the hypothesis states that the error terms in both (2.26) and (2.27) are not random expectational errors but contain information about adjustment and liquidity costs. By introducing them explicitly into the model, we derive the fully specified money equation (2.24).

The prediction that follows from this analysis is that when the misspecification of the money equation is corrected a better fit will be found with the data indicated by a J-statistic estimated from equation (2.24) which does not reject the null hypothesis of no specification error.

Chapter III: The Stationary State and Convergence
Conditions Under Perfect Foresight

3.1 Introduction:

Although the model derived in Chapter II is tested in its Euler equation form, I would like to show the circumstances under which the quantities implied by these equations of motion will converge to a stationary or steady state. To illustrate the stability conditions of the model, I will discuss the necessary conditions for convergence to a stationary state under the special case of perfect foresight where present and future prices are both known and constant.

Given a set of unchanging prices ($P_t = P_{t+1} = P^*$, $P_t^I = P_{t+1}^I = P^{I*}$, $\phi_t = \phi_{t+1} = \phi^*$, $W_t = W_{t+1} = W^*$ and $r_t = r_{t+1} = r^*$) the model will have a stationary state if there exist values of the endogenous variables that do not change over time. This means that in the stationary state $K_t = K_{t+1} = K_{t+2} = K^*$, $M_t = M_{t+1} = M_{t+2} = M^*$, and $N_t = N^*$. This in turn implies that $I_t = I_{t+1} = I^*$, $I^* = \delta K^*$, and $u^* = 0$. Formally, N^* is solved from the first-order condition in (2.5), given the values of K_t and M_t , and the values of the two stocks (K^* , M^*) are solved by having (2.5), (2.15) and (2.17) hold simultaneously. This is done in the following sections.

3.2 Stationary State Solution for Investment Behaviour:

Given the above characteristics of a perfect foresight stationary state, I can solve for the values of capital and real money holdings that would arise in a stationary state. First for capital, the optimal time path as calculated by the Euler equation in (2.15) can be written under perfect foresight as:

$$P_t \left(\frac{F_t}{K_t} - \frac{C_t}{K_t} \right) \left(1 - \frac{\gamma}{2m_t} \right) + \frac{\alpha P_t^I}{2m_t} \left(\frac{I_t}{K_t} \right)^2 - P_t^I \left[1 + \frac{\alpha}{m_t} \left(\frac{I_t}{K_t} \right) \right] \\ + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left[1 + \frac{\alpha}{m_{t+1}} \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] = 0. \quad (3.1)$$

Then utilizing the characteristics that quantities will be unchanging in a stationary state and the assumption of unchanging prices, equation (3.1) can be written as:

$$P^* \left(\frac{F^* - C^*}{K^*} \right) \left(1 - \frac{\gamma}{2m^*} \right) + \frac{\alpha P^{I*}}{2m^*} \left(\frac{I^*}{K^*} \right)^2 - P^{I*} \left[1 + \frac{\alpha}{m^*} \left(\frac{I^*}{K^*} \right) \right] \\ + P^{I*} \left(\frac{1-\delta}{1+r^*} \right) \left[1 + \frac{\alpha}{m^*} \left(\frac{I^*}{K^*} \right) \right] = 0. \quad (3.2)$$

When I collect terms and recognize that $I^* = \delta K^*$ in a stationary state, the stationary state value of K^* can be found as:

$$K^* = \frac{(P^*/P^{I^*}) (F^* - C^*) (2m^* - \gamma)}{\left(\frac{r^* + \delta}{1 + r^*}\right) (2m^* + 2\alpha\delta) - \alpha\delta^2} . \quad (3.3)$$

Since $I^* = \delta K^*$, the stationary state value of I^* can be determined as:

$$I^* = \frac{\delta (P^*/P^{I^*}) (F - C) (2m^* - \gamma)}{\left(\frac{r^* + \delta}{1 + r^*}\right) (2m^* + 2\alpha\delta) - \alpha\delta^2} . \quad (3.4)$$

Note that the optimal solution for K^* and I^* in (3.3) and (3.4) are functions of both M^* and N^* .⁷²

⁷² N^* is implicit in the value of $(F - C)$ in (3.4).

3.3 Stationary State Solution for Money Behaviour:

Again using the characteristics of the stationary state and the assumption of perfect foresight, I can solve for the stationary state value of real money holdings. Under perfect foresight the optimum path for real money holdings by the firm can be found from equation (2.17) as,

$$\begin{aligned} \frac{\alpha}{2} \left(\frac{P_t^I / \varphi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) + \frac{\gamma}{2} \left(\frac{(P_t / \varphi_t) F(K_t, N_t)}{m_t^2} \right) + \frac{\eta}{2} \left(\frac{u_t / \varphi_t}{m_t} \right)^2 \\ - \left(\eta \frac{(u_t / \varphi_t)}{m_t} + 1 \right) + \frac{1}{1+r_t} \left(\frac{\eta u_{t+1} / \varphi_{t+1}}{m_{t+1}} + 1 \right) = 0 . \end{aligned} \quad (3.5)$$

Collecting terms, setting $u_t = 0$ (because in the stationary state $M_t = M_{t+1}$) and imposing the other assumptions of stationary state, the money holding equation can be reduced to:

$$\frac{\alpha}{2} \left(\frac{P^{I*} / \varphi^*}{m^{*2}} \right) \left(\frac{I^{*2}}{K^*} \right) + \frac{\gamma}{2} \left(\frac{(P^* / \varphi^*) F(K^*, N^*)}{m^{*2}} \right) = \left(\frac{r^*}{1+r^*} \right) . \quad (3.6)$$

Again recognizing that in the stationary state $I^* = \delta K^*$ the stationary state value of money holding, M^* , and hence m^* , can be solved as:

$$m^* = \frac{M^*}{P^*} = \sqrt{\left(\frac{1+r^*}{2r^*}\right) (\alpha \delta^2 (P^{I^*}/P^*) K^* + \gamma (P^*/P^*) F(K^*, N^*))}. \quad (3.7)$$

Equation (3.7) suggests that the stationary state holdings of money will depend on the size of the transaction costs facing the firm (the coefficient for which is γ), the liquidity cost of investment (the coefficient for which is α), as well as on the opportunity cost of holding money, r^* . It will also be recognized that (3.7) maintains the traditional square root form of the money function that is typical of transaction money demand models. Hence money holdings exhibit economies of scale in the level of real transaction. Finally, as was true of (3.3) and (3.4), the optimal solution for M^* in (3.7) depends on the level of K^* and N^* .

3.4 Simultaneous Solution of M^* and K^* :

To solve for the reduced form of the stationary state values of K^* , m^* and N^* , I solve (3.3), (3.7) and (2.5) simultaneously.⁷¹ Substituting m^* from (3.7) and N^* from (2.5) into equation (3.3), a reduced form for K^* is found as:

$$K^* = \frac{(P^*/P^{I*}) (F-C) \left(\sqrt{2 \left(\frac{1+I^*}{I^*} \right) (\delta^2 \alpha P^{I*} / \phi^* K^* + \gamma P^* / \phi^* F(K^*, N^*) - \gamma)} \right)}{\left(\frac{I^* + \delta}{1+I^*} \right) \left(\sqrt{2 \left(\frac{1+I^*}{I^*} \right) (\delta^2 \alpha P^{I*} / \phi^* K^* + \gamma P^* / \phi^* F(K^*, N^*) + 2\alpha\delta)} - \alpha\delta^2 \right)} \quad (3.8)$$

$$\text{where, } N^*(K, M/\phi, W/P) \rightarrow F_N^{-1} \left(\frac{W^*}{P^*} \left(\frac{2m^*}{2m^* - \gamma} \right) \right) = \psi \left(\frac{W^*}{P^*} \left(\frac{2m^*}{2m^* - \gamma} \right) \right).$$

Having solved for K^* , the actual value of m^* can be found by substituting (3.8) back into (3.7) (along with (2.5)).

⁷¹ From equation (2.5) in Chapter II

$$N^* \left(K^*, \frac{M^*}{\phi}, \frac{W^*}{P^*} \right) \rightarrow F_N^{-1} \left(\frac{W}{P} \left(\frac{2m^*}{2m^* - \gamma} \right) \right) = \psi \left(\frac{W^*}{P^*} \left(\frac{2m^*}{2m^* - \gamma} \right) \right).$$

3.5 Solving for Stability Conditions:

Given the existence of a stationary state, I can now solve for the conditions under which this dynamic system will be stable about the stationary state (once again under perfect foresight). To do so I must analyze optimal investment and money holding behaviour through equations (3.1), (3.5) and (2.5) in the neighbourhood of the stationary state K^* , M^* , and N^* .

Equations (3.1) and (3.5), however, are nonlinear second order difference equations. Hence, I first expand each second order equation as two first-order equations then linearize each equation, using a Taylor series approximation, around the equilibrium stationary state values of the endogenous variables. Our system will then consist of five equations in the five endogenous variables K_t, M_t, I_t, u_t, N_t . These five equations are the two Euler equations in capital and money, the non-capital factor first-order condition equation, and the two equations of motion. I proceed by taking Taylor series expansions of each of these equations.

3.5.1 The Capital Equation:

I begin by linearizing the Euler equation for capital where (3.1) is rewritten as:

$$\begin{aligned} \phi(K_t, I_t, M_t, N_t, u_t) = & P_t F_K \left(1 - \frac{\gamma}{2m_t}\right) + \frac{\alpha P_t^I}{2m_t} \left(\frac{I_t}{K_t}\right)^2 \\ & - P_t^I \left[1 + \frac{\alpha}{m_t} \left(\frac{I_t}{K_t}\right)\right] + \left(\frac{1-\delta}{1+I_t}\right) P_{t+1}^I \left[1 + \frac{\alpha}{m_{t+1}} \left(\frac{I_{t+1}}{K_{t+1}}\right)\right] = 0, \end{aligned} \quad (3.1')$$

$$\text{where, } F_K = \left(\frac{F-C}{K}\right).$$

In the steady state (3.1') can be written as:¹⁴

$$\phi(K^*, I^*, M^*, N^*, u^*) = 0 \quad (3.9)$$

where, X^* is the value of $X = K, I, M, N$, and u in the stationary state. The general form of the Taylor series approximation of (3.9) around the equilibrium stationary state values is,

¹⁴ For estimation purposes and to illustrate the solution for the stationary state it was convenient to write the first term of equation (3.1) as $P_t \left(\frac{F-C}{K_t}\right) \left(1 - \frac{\gamma}{2m_t}\right)$. For linearizing the Euler equation it is more convenient to use $\left(\frac{F-C}{K_t}\right) = F_K$.

$$\begin{aligned}
 b_{27} &= \sigma \left[\left(-\frac{\alpha P^* \delta^* K^*}{m^{*1}} \right) - \frac{\gamma P^* F(K^*, N^*)}{m^{*1}} \right], \quad b_{28} = \frac{\sigma \eta^* \varphi}{m^*} \\
 b_{26} &= \sigma \left[\left(-\frac{\alpha}{2} \frac{P^*}{\varphi^*} \frac{\delta^*}{m^{*1}} \right) + \left(\frac{\gamma}{2} \left(\frac{P^*}{\varphi^*} \right) \frac{F_K(K^*, N^*)}{2m^{*1}} \right) \right], \quad b_{29} = \sigma \left[\frac{\alpha P^* \delta^* \varphi^*}{m^{*1}} \right].
 \end{aligned}
 \tag{3.22}$$

3.5.3 The Non-Capital Factors Equation:

The third equation involves the first order condition for the non-capital factors of production. In its most general form, equation (2.5) can be written as:

$$\theta(K^*, I^*, M^*, N^*, u^*) = 0,$$

where, (X^*) is the value of $X = K, I, M, N$ and u in the stationary state equilibrium. Taking a Taylor series approximation of the above equation around the equilibrium values I find,

$$\begin{aligned}
\Phi(K_t, I_t, M_t, N_t, u_t) &= \Phi(K^*, I^*, M^*, N^*, u^*) \\
&= \frac{\partial \Phi}{\partial K_t} (K_t - K^*) + \frac{\partial \Phi}{\partial K_{t+1}} (K_{t+1} - K^*) \\
&\quad + \frac{\partial \Phi}{\partial M_t} (M_t - M^*) + \frac{\partial \Phi}{\partial M_{t+1}} (M_{t+1} - M^*) + \frac{\partial \Phi}{\partial I_t} (I_t - I^*) + \frac{\partial \Phi}{\partial I_{t+1}} (I_{t+1} - I^*) \\
&\quad + \frac{\partial \Phi}{\partial N_t} (N_t - N^*) + \frac{\partial \Phi}{\partial u_t} (u_t - u^*) + \frac{\partial \Phi}{\partial u_{t+1}} (u_{t+1} - u^*) = 0
\end{aligned} \tag{3.10}$$

where the partial derivatives are evaluated at X^* . Now calculating from (3.1') all the relevant partial derivatives I find:

$$\begin{aligned}
\frac{\partial \Phi}{\partial K_t} &= P_t F_{KK} \left(1 - \frac{\gamma}{2m_t} \right) + \frac{\alpha P_t^I}{m_t} \frac{I_t}{K_t^2} \left(1 - \frac{I_t}{K_t} \right) \\
\frac{\partial \Phi}{\partial K_{t+1}} &= -\alpha \left(\frac{1-\delta}{1+r} \right) \left(\frac{P_{t+1}^I}{m_{t+1}} \frac{I_{t+1}}{(K_{t+1})^2} \right) \\
\frac{\partial \Phi}{\partial M_t} &= \frac{P_t}{\theta_t} \left(\frac{F_K}{2m_t^2} \right) - \frac{P_t^I}{\theta_t} \frac{\alpha}{m_t^2} \left(\frac{1}{2} \left(\frac{I_t}{K_t} \right)^2 + \frac{I_t}{K_t} \right) \\
\frac{\partial \Phi}{\partial M_{t+1}} &= - \left(\frac{1-\delta}{1+r} \right) \frac{P_{t+1}^I}{\theta_{t+1}} \frac{\alpha}{M_{t+1}^2} \frac{I_{t+1}}{K_{t+1}}, \quad \frac{\partial \Phi}{\partial I_t} = \frac{\alpha P_t^I}{m_t} \frac{I_t}{K_t^2} - \frac{\alpha P_t^I}{m_t} \frac{1}{K_t}, \\
\frac{\partial \Phi}{\partial I_{t+1}} &= \alpha \left(\frac{1-\delta}{1+r} \right) \frac{P_{t+1}^I}{m_{t+1}} \frac{1}{K_{t+1}}, \quad \frac{\partial \Phi}{\partial N_t} = P_t F_{KN} \left(1 - \frac{\gamma}{2m_t} \right) \frac{\partial \Phi}{\partial u_t} = 0, \quad \frac{\partial \Phi}{\partial u_{t+1}} = 0
\end{aligned} \tag{3.11}$$

where all the partial derivatives are to be evaluated at their stationary state values. Imposing the characteristics of stationary state equilibrium, and using the definition of capital accumulation, i.e., $[K_{t+1} = I_{t+1} + (1-\delta)K_t]$ and the

constancy of nominal prices so that $M_{t+1} = u_{t+1} + M_t$, then eliminating common terms and recognizing that at the stationary state $u^* = 0$, allows us to write (3.10) as:

$$\begin{aligned}
 & \left[P^* F_{KK} \left(1 - \frac{\gamma}{2m^*} \right) + \frac{\alpha P^{*I}}{m^*} \frac{I^*}{K^*} (1 - \delta) \right] (K_t - K^*) \\
 & - \left[\alpha \left(\frac{1 - \delta}{1 + r^*} \right) \left(\frac{P^{*I}}{m^*} \frac{\delta}{K^*} \right) \right] (K_{t+1} - K^*) \\
 & + \left[\frac{P^*}{\phi^*} \left(\frac{F_K}{2m^{*2}} \right) - \frac{P^{*I}}{\phi^*} \frac{\alpha \delta}{m^{*2}} \left(1 + \frac{\delta}{2} \right) \right] (M_t - M^*) \\
 & - \left[\left(\frac{1 - \delta}{1 + r^*} \right) \frac{P^{*I}}{\phi^*} \frac{\alpha \delta}{m^{*2}} \right] (M_{t+1} - M^*) + \left[\frac{\alpha P^{*I}}{m^* K^{*2}} \left(\frac{\delta}{K^*} - 1 \right) \right] (I_t - I^*) \\
 & + \left[\alpha \left(\frac{1 - \delta}{1 + r^*} \right) \frac{P^{*I}}{m^* K^*} \right] (I_{t+1} - I^*) + \left[P^* F_{KN} \left(1 - \frac{\gamma}{2m^*} \right) \right] (N_t - N^*) = 0 ,
 \end{aligned} \tag{3.12}$$

Using for convenience the notations

$$\bar{K}_t \equiv K_t - K^*, \bar{K}_{t+1} \equiv K_{t+1} - K^*, \bar{M}_t \equiv M_t - M^*, \bar{M}_{t+1} \equiv M_{t+1} - M^*,$$

$$\bar{I}_t \equiv I_t - I^*, \bar{I}_{t+1} \equiv I_{t+1} - I^* \text{ and } \bar{N}_t \equiv N_t - N^*, \text{ I now rewrite (3.12)}$$

so that the coefficient for $\bar{I}_{t+1}$ is one [as in equation (3.13) below]. This establishes a form that will allow easier representation as part of a matrix of difference equations in what follows. (3.12) can be written as:

$$\begin{aligned}
& \bar{I}_{t+1} + \beta \left[P^* F_{KK} \left(1 - \frac{\gamma}{2m^*} \right) + \frac{\alpha P^{*I}}{m^*} \frac{I^*}{K^*} (1 - \delta) \right] \bar{K}_t \\
& + \delta \bar{K}_{t+1} + \beta \left(\frac{P^*}{\phi^*} \left(\frac{F_K}{2m^{*2}} \right) - \frac{P^{*I}}{\phi^*} \frac{\alpha \delta}{m^{*2}} \left(1 + \frac{\delta}{2} \right) \right) \bar{M}_t \\
& + \beta \left(\alpha \frac{P^{*I}}{m^* K^{*2}} \left(\frac{\delta}{K^*} - 1 \right) \right) \bar{I}_t \\
& + \beta \left(P^* F_{KN} \left(1 - \frac{\gamma}{2m^*} \right) \right) \bar{N}_t = 0
\end{aligned} \tag{3.13}$$

Here, $\beta = \frac{1}{\frac{\alpha P^{*I}}{m^* K^*} \left(\frac{1-\delta}{1+r^*} \right)}$ is the reciprocal of the coefficient

of $\bar{I}_{t+1}$ in (3.12) above. To further simplify presentation, equation (3.13) can be written as a linear difference equation of the following form:

$$a_{11} \bar{I}_{t+1} + a_{12} \bar{K}_{t+1} + a_{13} \bar{M}_{t+1} + a_{14} \bar{N}_{t+1} + b_{15} \bar{I}_t + b_{16} \bar{K}_t + b_{17} \bar{M}_t + b_{18} \bar{N}_t = 0 \tag{3.14}$$

where, $a_{11} = 1, a_{12} = \delta, a_{13} = 0, a_{14} = 0$ and

$$\begin{aligned}
b_{16} &= \beta \left[P^* F_{PK} \left(1 - \frac{Y^*}{2m^*} \right) + \frac{\alpha P^{*I}}{m^*} \frac{I^*}{K^*} (1 - \delta) \right] \\
b_{17} &= \beta \left[\frac{P^*}{\phi^*} \left(\frac{F_K}{2m^{*2}} \right) - \frac{P^{*I}}{\phi^*} \frac{\alpha \delta}{m^{*2}} \left(1 - \frac{\delta}{2} \right) \right] \\
b_{15} &= \beta \left[\frac{\alpha P^{*I}}{m^* K^{*2}} \left(\frac{\delta}{K^*} - 1 \right) \right], \quad b_{18} = 0 .
\end{aligned} \tag{3.15}$$

3.5.2 The Money Equation:

I now turn to linearize the Euler equation describing optimum money holding behaviour about the stationary state. Following our earlier derivation, in the steady state equation (3.5) can be written as:

$$\psi(K^*, I^*, M^*, N^*, u^*) \tag{3.16}$$

where, X^* is the value of $X = K, I, M, N$, and u in the stationary state equilibrium. Taking the Taylor series approximation of (3.16) around the stationary state I find:

$$\begin{aligned}
\Psi(K_t, M_t, N_t, u_t) &= \Psi(K^*, M^*, N^*, u^*) \\
&= \frac{\partial \Psi}{\partial M_t} (M_t - M^*) + \frac{\partial \Psi}{\partial M_{t+1}} (M_{t+1} - M^*) + \frac{\partial \Psi}{\partial K_t} (K_t - K^*) + \frac{\partial \Psi}{\partial K_{t+1}} (K_{t+1} - K^*) \\
&\quad + \frac{\partial \Psi}{\partial I_t} (I_t - I^*) + \frac{\partial \Psi}{\partial I_{t+1}} (I_{t+1} - I^*) + \frac{\partial \Psi}{\partial N_t} (N_t - N^*) \\
&\quad + \frac{\partial \Psi}{\partial u_t} (u_t - u^*) + \frac{\partial \Psi}{\partial u_{t+1}} (u_{t+1} - u^*) = 0.
\end{aligned} \tag{3.17}$$

Again the values of the partial derivatives from (3.5) can be solved as:

$$\begin{aligned}
\frac{\partial \Psi}{\partial M_t} &= \left(-\frac{\alpha P_t^I}{m_t^3} \frac{I_t^2}{K_t} \right) - \frac{\gamma P_t F(K_t, N_t)}{m_t^3}, \quad \frac{\partial \Psi}{\partial M_{t+1}} = 0, \\
\frac{\partial \Psi}{\partial K_t} &= \left(-\frac{\alpha}{2} \frac{P_t^I}{\phi_t} \frac{1}{m_t^2} \right) \left(\frac{I_t}{K_t} \right)^2 + \left(\frac{\gamma}{2} \left(\frac{P_t}{\phi_t} \right) \frac{F_K(K_t, N_t)}{2m_t^2} \right) \\
\frac{\partial \Psi}{\partial K_{t+1}} &= 0, \quad \frac{\partial \Psi}{\partial I_t} = \frac{\alpha P_t^I / \phi_t}{m_t^2} \frac{I_t}{K_t}, \quad \frac{\partial \Psi}{\partial I_{t+1}} = 0 \\
\frac{\partial \Psi}{\partial u_t} &= \left(\frac{\eta / \phi_t}{m_t} \right), \quad \frac{\partial \Psi}{\partial u_{t+1}} = \left(\eta \frac{1}{1 + I_t} \right) \frac{1 / \phi_{t+1}}{m_{t+1}} \\
\frac{\partial \Psi}{\partial N_t} &= \frac{\gamma}{2} \frac{P_t}{\phi_t} \frac{F_N(K_t, N_t)}{m_t^2} = 0.
\end{aligned} \tag{3.18}$$

I again evaluate these partial derivatives at their stationary state equilibrium values. Using the money accumulation relation (under constant prices) of $M_{t+1} = u_{t+1} + M_t$, recognizing that $u^* = 0$, and using previous notation for deviations, equation (3.17) can be written as:

$$\begin{aligned}
& \left[\left(-\frac{\alpha P^{*I} \delta^2 K^*}{m^{*3}} \right) - \frac{\gamma P^* F(K^*, N^*)}{m^{*3}} \right] \bar{M}_t \\
& + \left[\left(\frac{\alpha}{2} \frac{P^{*I}}{\varphi^*} \frac{\delta}{m^{*2}} \right) + \left(\frac{\gamma}{2} \left(\frac{P^*}{\varphi^*} \right) \frac{F_K(K^*, N^*)}{2m^{*2}} \right) \right] \bar{K}_t + \left[\frac{\alpha P^{*I} / \varphi^* \delta}{m^{*2}} \right] \bar{I}_t \\
& - \left[\left(\frac{\eta / \varphi^*}{m^*} \right) \right] \bar{u}_t + \left[\left(\eta \frac{1}{1+r^*} \right) \frac{1/\varphi^*}{m^*} \right] \bar{u}_{t+1} + \left[\frac{\gamma}{2} \frac{P^*}{\varphi^*} \frac{F_N(K^*, N^*)}{m^{*2}} \right] \bar{N}_t = 0
\end{aligned} \quad (3.19)$$

To rearrange this equation for simple matrix representation I rewrite (3.19) in such a way that the coefficient for $\bar{u}_{t+1}$ is now equal to one. That is, (3.19) becomes,

$$\begin{aligned}
& \bar{u}_{t+1} + \sigma \left[\left(-\frac{\alpha P^{*I} \delta^2 K^*}{m^{*3}} \right) - \frac{\gamma P^* F(K^*, N^*)}{m^{*3}} \right] \bar{M}_t \\
& + \sigma \left[\left(-\frac{\alpha}{2} \frac{P^{*I}}{\varphi^*} \frac{\delta}{m^{*2}} \right) + \left(\frac{\gamma}{2} \left(\frac{P^*}{\varphi^*} \right) \frac{F_K(K^*, N^*)}{2m^{*2}} \right) \right] \bar{K}_t \\
& + \sigma \left[\frac{\alpha P^{*I} / \varphi^* \delta}{m^{*2}} \right] \bar{I}_t - \sigma \left[\left(\frac{\eta / \varphi^*}{m^*} \right) \right] \bar{u}_t + \sigma \left[\frac{\gamma}{2} \frac{P^*}{\varphi^*} \frac{F_N(K^*, N^*)}{m^{*2}} \right] \bar{N}_t = 0,
\end{aligned} \quad (3.20)$$

where $\sigma = \frac{m \varphi^* (1+r^*)}{\eta}$ is the reciprocal of the coefficient

of $\bar{u}_{t+1}$ in (3.19).

In general coefficient form, equation (3.20) can be written as a linear difference equation of the following form:

$$a_{21} \bar{I}_{t+1} + a_{22} \bar{K}_{t+1} + a_{23} \bar{M}_{t+1} + a_{24} \bar{u}_{t+1} + b_{25} \bar{I}_t + b_{26} \bar{K}_t + b_{27} \bar{M}_t + b_{28} \bar{u}_t = 0 \quad (3.21)$$

where, $a_{21} = 0, a_{22} = 0, a_{23} = 0, a_{24} = 1$. and

$$b_{27} = \sigma \left[\left(- \frac{\alpha F^{*1} \delta^* K^*}{m^{*1}} \right) - \frac{\gamma F^* F(K^*, N^*)}{m^{*1}} \right], \quad b_{28} = \frac{\sigma \eta^* \varphi}{m^*} \quad (3.22)$$

$$b_{26} = \sigma \left[\left(- \frac{\alpha}{2} \frac{P^{*1}}{\varphi^*} \frac{\delta}{m^{*2}} \right) + \left(\frac{\gamma}{2} \left(\frac{P^*}{\varphi^*} \right) \frac{F_K(K^*, N^*)}{2m^{*2}} \right) \right], \quad b_{25} = \sigma \left[\frac{\alpha F^{*1} \varphi^* \delta}{m^{*2}} \right].$$

3.5.3 The Non-Capital Factors Equation:

The third equation involves the first order condition for the non-capital factors of production. In its most general form, equation (2.5) can be written as:

$$\theta(K^*, I^*, M^*, N^*, u^*) = 0,$$

where, (X^*) is the value of $X = K, I, M, N$ and u in the stationary state equilibrium. Taking a Taylor series approximation of the above equation around the equilibrium values I find,

$$\begin{aligned}
\theta(K_t, M_t, N_t, I_t, u_t) &\equiv \theta(K^*, M^*, N^*, I^*, u^*) = \frac{\partial \theta}{\partial K_t} (K_t - K^*) \\
&+ \frac{\partial \theta}{\partial K_{t+1}} (K_{t+1} - K^*) + \frac{\partial \theta}{\partial M_t} (M_t - M^*) + \frac{\partial \theta}{\partial M_{t+1}} (M_{t+1} - M^*) + \frac{\partial \theta}{\partial I_t} (I_t - I^*) \\
&+ \frac{\partial \theta}{\partial I_{t+1}} (I_{t+1} - I^*) + \frac{\partial \theta}{\partial N_t} (N_t - N^*) + \frac{\partial \theta}{\partial u_t} (u_t - u^*) \\
&+ \frac{\partial \theta}{\partial u_{t+1}} (u_{t+1} - u^*) = 0.
\end{aligned} \tag{3.23}$$

Now, calculating from (2.5) all the relevant partial derivatives I find:¹⁸

$$\begin{aligned}
\frac{\partial \theta}{\partial K_t} &= F_{NK} \left(1 - \frac{\gamma}{2m_t}\right), \quad \frac{\partial \theta}{\partial K_{t+1}} = 0, \quad \frac{\partial \theta}{\partial M_t} = F_N \frac{\gamma}{2m_t^2} \frac{1}{\phi_t} \\
\frac{\partial \theta}{\partial M_{t+1}} &= 0, \quad \frac{\partial \theta}{\partial I_t} = 0, \quad \frac{\partial \theta}{\partial I_{t+1}} = 0, \quad \frac{\partial \theta}{\partial N_t} = F_{NN} \left(1 - \frac{\gamma}{2m_t}\right), \\
\frac{\partial \theta}{\partial u_t} &= 0, \quad \frac{\partial \theta}{\partial u_{t+1}} = 0.
\end{aligned} \tag{3.24}$$

where all the partial derivatives are evaluated at their stationary state values. Using again the notation for deviations $K_t - K^* \equiv \bar{K}_t$, $M_t - M^* \equiv \bar{M}_t$, and $N_t - N^* \equiv \bar{N}_t$, (3.23) can be reduced to:

¹⁸ $\theta(K_t, N_t, M_t / \phi_t) \equiv F_N \left(1 - \frac{\gamma}{2m_t}\right) - \frac{W}{P} = 0$

$$\left[F_{NK} \left(1 - \frac{Y}{2m_t} \right) \right] \bar{K}_t + \left[F_N \frac{Y}{2m_t^2} \frac{1}{\theta_t} \right] \bar{M}_t + \left[F_{NN} \left(1 - \frac{Y}{2m_t} \right) \right] \bar{N}_t = 0 \quad (3.25)$$

Equation (3.25) can now be used to solve for $\bar{N}_t$ as a function of $\bar{K}_t$ and $\bar{M}_t$. That is:

$$\bar{N}_t = - \frac{F_{NK}}{F_{NN}} (\bar{K}_t) - \frac{Y F_N}{2m_t^2 F_{NN} \left(1 - \frac{Y}{2m_t} \right)} \frac{1}{\theta_t} (\bar{M}_t) \quad (3.26)$$

Equation (3.26) will be used later to eliminate $\bar{N}_t$ from the Euler equations for capital and money (i.e., in (3.13) and (3.20)).

3.5.4 The Investment Equation (The Capital Equation of Motion):

The equation defining the investment or capital accumulation equation for period $t+1$ can be written in general functional form as:

$$\varphi = I_{t+1} - K_{t+1} - (1-\delta)K_t = 0 \quad (3.27)$$

Its Taylor series approximation around the stationary state equilibrium values is:

$$\frac{\partial \varphi}{\partial I_{t+1}} (I_{t+1} - I_t) - \frac{\partial \varphi}{\partial K_{t+1}} (K_{t+1} - K^*) - \frac{\partial \varphi}{\partial I_t} (K_t - K^*) = 0 . \quad (3.28)$$

Here partial derivatives are straight forward. Using the deviation notation, (3.28) can be then written as:

$$[1] \bar{I}_{t+1} - [1] \bar{K}_{t+1} - [1-\delta] \bar{K}_t = 0 . \quad (3.29)$$

Equation (3.29) written for general presentation becomes:

$$a_{31} \bar{I}_{t+1} + a_{32} \bar{K}_{t+1} + a_{33} \bar{M}_{t+1} + a_{34} \bar{U}_{t+1} + b_{35} \bar{I}_t + b_{36} \bar{K}_t + b_{37} \bar{M}_t + b_{38} \bar{U}_t = 0 , \quad (3.30)$$

where

$$a_{31} = 1 , a_{32} = -1 , a_{33} = 0 , a_{34} = 0 , \\ b_{35} = 0 , b_{36} = (1-\delta) , b_{37} = 0 , b_{38} = 0 .$$

3.5.5 The Money Accumulation Equation (The Equation of Motion for Money Holding):

The money accumulation equation for period $t+1$ defines $u_{t+1} = M_{t+1} - M_t$. Written in general form this is:

$$\chi = u_{t+1} - M_{t+1} + M_t = 0 . \quad (3.31)$$

Taking a Taylor series approximation of (3.31) around the stationary state gives us:

$$\frac{\partial \chi}{\partial u_{t+1}} (u_{t+1} - u^*) - \frac{\partial \chi}{\partial M_{t+1}} (M_{t+1} - M^*) + \frac{\partial \chi}{\partial M_t} (M_t - M^*) = 0. \quad (3.32)$$

Once again using the deviation notation (3.32) can be written as:

$$[1] \bar{u}_{t+1} - [1] \bar{M}_{t+1} + [1] \bar{M}_t = 0. \quad (3.33)$$

Equation (3.33) can then be written in general matrix notation as:

$$a_{41} \bar{I}_{t+1} + a_{42} \bar{K}_{t+1} + a_{43} \bar{M}_{t+1} + a_{44} \bar{u}_{t+1} + b_{45} \bar{I}_t + b_{46} \bar{K}_t + b_{47} \bar{M}_t + b_{48} \bar{u}_t = 0. \quad (3.34)$$

where,

$$a_{41} = 0, \quad a_{42} = 0, \quad a_{43} = -1, \quad a_{44} = 1,$$

$$b_{45} = 0, \quad b_{46} = 0, \quad b_{47} = 1, \quad b_{48} = 0.$$

3.6 Reducing the 5x5 System to a 4x4 System:

I started with a system of five equations and five endogenous variables. This system can be reduced to a system of four equations and four endogenous variables by substituting $\bar{N}_t$ from (3.26) into the other four equations. Since $\bar{N}_t$ does not appear in (3.30) and (3.34), this means substituting (3.26) into only the capital and money equations, i.e., (3.13) and (3.20) respectively. When I do so, equation (3.13) becomes:

$$\begin{aligned} \bar{I}_{t+1} + \beta & \left[P^* F_{KK} \left(1 - \frac{\gamma}{2m^*} \right) + \frac{\alpha P^{*I}}{m^*} \frac{I^*}{K^*} (1 - \delta) - \frac{P^* F_{NK} F_{KV} \left(1 - \frac{\gamma}{m_t} \right)}{F_{NN}} \right] \bar{K}_t \\ & + \beta \left(\frac{P^*}{\varphi^*} \left(\frac{F_K}{2m^{*2}} \right) - \frac{P^{*I}}{\varphi^*} \frac{\alpha \delta}{m^{*2}} \left(1 - \frac{\delta}{2} \right) - \frac{P^*}{\varphi^*} \frac{\gamma F_N F_{KV}}{2m^{*2} F_{NN}} \right) \bar{M}_t \quad (3.35) \\ & + \beta \left(\alpha \frac{P^{*I}}{m^* K^{*2}} \left(\frac{\delta}{K^*} - 1 \right) \right) \bar{I}_t + \delta \bar{K}_{t+1} = 0 . \end{aligned}$$

Similarly, equation (3.20) becomes:

$$\begin{aligned}
& \bar{u}_{t+1} + \sigma \left[\left(-\frac{\alpha P^{*1} \delta^2 K^*}{m^{*2}} \right) - \frac{\gamma P^* F(K^*, N^*)}{m^{*1}} - \frac{\gamma}{2} \frac{P^*}{\theta^*} \frac{F_N(K^*, N^*)}{m^{*2}} - \frac{\gamma F_N}{\theta^* m^{*2} F_{NN}} \right] \bar{M}_t \\
& + \sigma \left[\left(-\frac{\alpha}{2} \frac{P^{*1}}{\theta^*} \frac{\delta}{m^{*2}} \right) + \left(\frac{\gamma}{2} \left(\frac{P^*}{\theta^*} \right) \frac{F_K(K^*, N^*)}{2m^{*2}} - \frac{\gamma}{2} \frac{P^*}{\theta^*} \frac{F_N(K^*, N^*)}{m^{*2}} - \frac{F_{NN}}{F_{NN}} \right) \right] \bar{K}_t \\
& + \sigma \left[\frac{\alpha P^{*1} / \theta^* \delta}{m^{*2}} \right] \bar{I}_t - \sigma \left[\left(\frac{\eta / \theta^*}{m^*} \right) \right] \bar{u}_t = 0 .
\end{aligned}
\tag{3.36}$$

Now putting equations (3.14), (3.21), (3.30) and (3.34) into the form of a first order vector system leads to the following:⁷⁶

$$\begin{bmatrix} 1 & \delta & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_{t+1} \\ \bar{u}_{t+1} \\ \bar{K}_{t+1} \\ \bar{M}_{t+1} \end{bmatrix} + \begin{bmatrix} b_{15} & b_{16} & b_{17} & b_{18} \\ b_{25} & b_{26} & b_{27} & b_{28} \\ 0 & (1 \ \delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{I}_t \\ \bar{u}_t \\ \bar{K}_t \\ \bar{M}_t \end{bmatrix} = 0
\tag{3.37}$$

Defining the first coefficient matrix as A and the second as B, the above system of difference equations can be written as:

⁷⁶ Farmer (1992) p. 42.

$$A \begin{bmatrix} \bar{I}_{t+1} \\ \bar{U}_{t+1} \\ \bar{K}_{t+1} \\ \bar{M}_{t+1} \end{bmatrix} + B \begin{bmatrix} \bar{I}_t \\ \bar{U}_t \\ \bar{K}_t \\ \bar{M}_t \end{bmatrix} = 0 . \quad (3.38)$$

The values of the coefficients in matrix A are repeated for convenience below as:

$$\begin{aligned} a_{11} &= 1 & a_{12} &= \delta & a_{13} &= 0 & a_{14} &= 0 \\ a_{21} &= 0 & a_{22} &= 0 & a_{23} &= 0 & a_{24} &= 1 \\ a_{31} &= -1 & a_{32} &= 1 & a_{33} &= 0 & a_{34} &= 0 \\ a_{41} &= 0 & a_{42} &= 0 & a_{43} &= -1 & a_{44} &= 1, \end{aligned} \quad (3.39)$$

and the values of the coefficients of matrix B are:

$$\begin{aligned} b_{16} &= \beta \left[P^* F_{KK} \left(1 - \frac{\gamma}{2m^*} \right) + \frac{\alpha P^{*I}}{m^*} \frac{I^*}{K^*} (1 - \delta) - \frac{P^* F_{KK} F_{KN} \left(1 - \frac{\gamma}{m^*} \right)}{F_{NN}} \right] \\ b_{17} &= \beta \left[\frac{P^*}{\phi^*} \left(\frac{F_K}{2m^{*2}} \right) - \frac{P^{*I}}{\phi^*} \frac{\alpha \delta}{m^{*2}} \left(1 - \frac{\delta}{2} \right) - \frac{P^*}{\phi^*} \frac{\gamma F_N F_{KN}}{2m^2 F_{NN}} \right] \\ b_{15} &= \beta \left[\frac{\alpha P^{*I}}{m^* K^{*2}} \left(\frac{\delta}{K^*} - 1 \right) \right], \quad b_{18} = 0 . \\ b_{35} &= 0 \quad b_{36} = (1 - \delta) \quad b_{37} = 0 \quad b_{38} = 0 \\ b_{45} &= 0 \quad b_{46} = 0 \quad b_{47} = 1 \quad b_{48} = 0 \end{aligned} \quad (3.40)$$

$$b_{27} = \sigma \left[\left(-\frac{\alpha P^{*I} \delta^2 K^*}{m^{*3}} \right) - \frac{\gamma P^* F(K^*, N^*)}{m^{*3}} - \frac{\gamma}{2} \frac{P^*}{\phi} \frac{F_N(K^* N^*)}{m^2} \frac{\gamma F_N}{\phi^* m^{*2} F_{NN}} \right]$$

$$b_{26} = \sigma \left[\left(-\frac{\alpha}{2} \frac{P^{*I}}{\phi^*} \frac{\delta}{m^{*2}} \right) + \left(\frac{\gamma}{2} \left(\frac{P^*}{\phi^*} \right) \frac{F_K(K^*, N^*)}{2m^{*2}} \right) - \frac{\gamma}{2} \frac{P^*}{\phi^2} \frac{F_N(K^* N^*)}{m^{*2}} \frac{F_{NK}}{F_{NN}} \right]$$

$$b_{25} = \sigma \left[\frac{\alpha P^{*I} / \phi^* \delta}{m^{*2}} \right], \quad b_{28} = \sigma \left(\frac{\eta / \phi^*}{m^*} \right).$$

To discuss the conditions under which this four variable system is stable it is desirable to transform the matrix system in (3.37) into a form where the current matrix A becomes the identity matrix. To establish this form, I must premultiply equation (3.38) by A^{-1} that would result in linear first order system. However, for A^{-1} to exist, the determinant of matrix A must be non zero. When this is calculated, the determinant of coefficient matrix A is, $|A| = (1+\delta)$. Since $\delta > 0$, the determinant of matrix A will be non-zero, and A^{-1} will exist.

When calculated, A^{-1} takes the following form:

$$A^{-1} = \begin{bmatrix} \frac{1}{1+\delta} & 0 & \frac{\delta}{1+\delta} & 0 \\ \frac{1}{1+\delta} & 0 & -\frac{1}{1+\delta} & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (3.41)$$

Pre-multiplying equation (3.38) with A^{-1} , I obtain the following transformed system of first-order linear difference equations. Writing out the result gives us the following system:

$$\begin{bmatrix} \bar{I}_{t+1} \\ \bar{U}_{t+1} \\ \bar{K}_{t+1} \\ \bar{m}_{t+1} \end{bmatrix} + A^{-1}B \begin{bmatrix} \bar{I}_t \\ \bar{U}_t \\ \bar{K}_t \\ \bar{m}_t \end{bmatrix} = 0, \quad (3.42)$$

where the characteristic matrix which results from the pre-multiplication of matrix B by matrix A^{-1} is:

$$A^{-1}B = \begin{bmatrix} \frac{b_{15}}{1+\delta} & -\frac{b_{16}-\delta+\delta^2}{1+\delta} & \frac{b_{17}}{1+\delta} & 0 \\ \frac{b_{15}}{1+\delta} & \frac{b_{16}-1+\delta}{1+\delta} & \frac{b_{17}}{1+\delta} & 0 \\ b_{25} & b_{26} & b_{27}-1 & b_{28} \\ b_{25} & b_{26} & b_{27} & b_{28} \end{bmatrix}. \quad (3.43)$$

Matrix (3.42) solves into a polynomial equation of the fourth degree. The characteristic polynomial equation calculated from equation (3.42) takes the following form: The solution to this equation will have four roots.⁷⁷

⁷⁷ Norman (1986), Proposition 4.29, P. 123.

$$\begin{aligned}
& \lambda^4 + \left[\frac{2 - b_{16} - b_{28}(1+\delta) - b_{27}(1+\delta) - b_{15}}{(1+\delta)} \right] \lambda^3 + \\
& \left[\frac{1 - \delta - 2b_{28} - (1-\delta)b_{27} - (2-\delta-b_{27}-b_{28})b_{15} - (1-b_{28}-b_{27})b_{16} - (b_{26}+b_{25})b_{17}}{1+\delta} \right] \lambda^2 + \\
& \left[\frac{b_{16}b_{28} - (1-\delta)b_{25}b_{17} - (1-\delta)b_{15} - (1-\delta)b_{28} + (1-\delta)b_{15}b_{27} + (2-\delta)b_{15}b_{28}}{(1+\delta)} \right] \lambda + \\
& \left[\frac{(1-\delta)b_{15}b_{28}}{(1+\delta)} \right] = 0.
\end{aligned} \tag{3.44}$$

Such higher-order-difference-equations are not easily solved.⁷⁸ However, the convergence conditions for the relevant time path can be determined qualitatively. The qualitative requirement for global convergence is that each of the roots be less than one in absolute value. In such a situation the Schur Theorem becomes directly applicable.⁷⁹ According to this theorem the roots of n-th degree polynomial equation will be less than one if and only if the n-determinants of the n-partitioned matrices are all positive. Since the condition given in the theorem is "if and only if" this is both a necessary and sufficient condition for the convergence of the characteristic equation of n-th degree.

A very simple procedure is used in the construction of these n-partitioned determinants. Each of the n-

⁷⁸ Chiang (1967) P.551.

⁷⁹ Chiang (1967) p. 551-552.

determinants is partitioned in four areas. Each area of the t -th determinant, Δ_t , consists of a $t \times t$ sub-determinant. The upper-left area has a_0 alone on the diagonal, zeros above the diagonal, and progressively larger subscripts for the successive coefficients in each column below the diagonal elements. When the elements of upper left area is transposed lower right-area is obtained. In the upper right area we place the a_n elements on the diagonal, with zeros below the diagonal, and progressively smaller subscripts for the successive coefficients as we go up each column from the diagonal. When the elements of this area is transposed we obtain the lower right area.

The application of this theorem to my problem is straight forward. Since the coefficients of the characteristic equation are same as those appearing on the left hand side of the difference equation (i.e., coefficients of x_{t+1} , x_t ,) while the right hand side has only the constant, I can apply the Schur theorem directly. It is worth emphasizing that a_0 is always equal to 1. Following the Schur theorem, equation (3.44) will have four partitioned matrices and four sub-determinants. If all of these determinants are positive then the system will globally converge. However, if two of the determinants are positive, then the system may have a saddle path convergence. Before writing the partitioned matrices, I

redefine the coefficients and the constant of (3.45) as follows:

$$\begin{aligned}
 a &= \left[\frac{2 - b_{16} - b_{28}(1+\delta) - b_{27}(1+\delta) - b_{15}}{(1+\delta)} \right] \\
 b &= \left[\frac{1 - \delta - 2b_{28} - (1-\delta)b_{27} - (2-\delta-b_{27}-b_{28})b_{15} - (1-b_{28}-b_{27})b_{16} - (b_{26}+b_{25})b_{17}}{1+\delta} \right] \\
 c &= \left[\frac{b_{16}b_{28} - (1-\delta)b_{25}b_{17} - (1-\delta)b_{15} - (1-\delta)b_{28} + (1-\delta)b_{15}b_{27} + (2-\delta)b_{15}b_{28}}{(1+\delta)} \right] \\
 d &= \left[\frac{(1-\delta)b_{15}b_{28}}{(1+\delta)} \right].
 \end{aligned} \tag{3.45}$$

With the above definition of coefficients the characteristic polynomial equation (3.44) can be rewritten as:

$$\lambda^4 + a\lambda^3 + b\lambda^2 + c\lambda + d \tag{3.46}$$

These coefficients then simplify the presentation of four partitioned matrices and their determinants. The first partitioned matrix and determinant are:

$$\begin{bmatrix} 1 & d \\ d & 1 \end{bmatrix} \tag{3.47}$$

$$\Delta_1 = 1 - d^2.$$

The second partitioned matrix and determinant are:

$$\begin{bmatrix} 1 & 0 & d & c \\ a & 1 & 0 & d \\ d & 0 & 1 & a \\ c & d & 0 & 1 \end{bmatrix} \quad (3.48)$$

$$\begin{aligned} \Delta_2 &= 1 - 2d^2 - d^2a^2 + 2dac + d^4 - c^2 \\ &= (1 - d^2)^2 - (da - c)^2. \end{aligned}$$

The third partitioned matrix and determinant are

$$\begin{bmatrix} 1 & 0 & 0 & d & c & b \\ a & 1 & 0 & 0 & d & a \\ b & a & 1 & 0 & 0 & d \\ d & 0 & 0 & 1 & a & b \\ c & d & 0 & 0 & 1 & a \\ b & a & d & 0 & 0 & 1 \end{bmatrix} \quad (3.49)$$

$$\begin{aligned} \Delta_3 &= -d^6 + (2a^2 + b^2 + 3)d^4 + (-2ca - acb - 2b^2 - 2a^2 - ba^2)d^3 \\ &\quad + (3cab - a^2 - a^4 - 3ca^3 + 3ba^2 + c^2)d^2 \\ &\quad + (2b^2 - 3ba^2 + 2ca^2a^2 + 2a^2 + a^4 - 3cab)d \\ &\quad + 1 + c^2a^2 - a^2 + acb - c^2 + ba^2 - ca^3 - b^2. \end{aligned}$$

The fourth partitioned matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & d & c & b & a \\ a & 1 & 0 & 0 & 0 & d & c & b \\ b & a & 1 & 0 & 0 & 0 & d & c \\ c & b & a & 1 & 0 & 0 & 0 & d \\ d & 0 & 0 & 0 & 1 & a & b & c \\ c & d & 0 & 0 & 0 & 1 & a & b \\ b & c & d & 0 & 0 & 0 & 1 & a \\ a & b & c & d & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.50)$$

The determinant of fourth partitioned matrix is a polynomial of eighth degree.⁸⁰

If all Δ_i determinants are positive, then the Schur theorem ensures that each of the λ_i characteristic roots are less than one in absolute value so that the dynamic system will converge globally. However, if we can show that two of the Δ_i determinants are positive, then it may be sufficient to ensure a saddle path convergence of the system (3.44).

From (3.47) we get:

From (3.48) we get a positive determinant if:

under these conditions, then, Δ_1 and Δ_2 are positive. Thus with Δ_1 and Δ_2 positive, even if Δ_3 and Δ_4 are non-positive, I can conclude that the system (3.44) will at minimum be saddle path convergent. Should, all of the Δ_n determinants

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$$\begin{aligned} \Delta_4 = & d^8 + (-3a^2 - c^2 - 4 - 2b^2) d^6 + (2bc^2 + 8ca + 6acb + 4b^2) d^5 \\ & + (2b^2 + 6c^2 + 5a^2 - 4acb^2 - 8bc^2 + b^4 - 6ba^2 - 10acb - c^2a^2 + 3a^4 - 2c^3a \\ & + b^2a^2 - c^2b^2) d^4 \\ & + (2c^3a + 8ba^2 + 2b^3a^2 - 16ca - 6a^3cb + 8acb^2 + 6c^2a^2 + 4bc^2 + 2b^2a^2 - 2b^3ac \\ & + 6c^2b^2 + 4c^2ba^2 - 10ca^3 - 8b^2 - 4b^4 + 2c^3ab + 2c^4 + 4acb) d^3 \\ & + (-4 + 5c^2 - a^2 + 2b^2 + 4ba^2 + 8c^2ba^2 + 4acb - 8c^2b^2 + 14c^2a^2 - 6ca^3 + 8bc^2 - a^4 \\ & + 2b^3ac - 2b^2ca^3 - 10c^3ab - 2a^3cb + a^2c^2b^2 - 8b^2a^2 - 4b^3a^2 + 2c^2b^3 - 6c^3a + 6b^4 \\ & - c^4 - 8acb^2 + 2c^2a^4 - c^4a^2 + 6ba^4 + b^2a^4 - 2c^4b - a^6) d^2 \\ & + (8ca + 4b^2 - 8ba^2 + 8c^2ba^2 - 10acb + 2c^2b^2 + 6c^2a^2 + 2ca^3 - 6bc^2 \\ & + 2a^4 + 2b^3ac - 2b^2ca^3 - 2c^3ab - 10a^3cb + 4a^2c^2b^2 + 6b^2a^2 + 2b^3a^2 - 4c^3a^3 \\ & - 4c^4a^3 - 10c^3a - 4b^4 + 8acb^2 - 2c^3ab^2 - 2ba^4 + 2ca^5 + 2c^5a + 6c^4b) d \\ & + (1 - 3c^2 - a^2 - 2ba^2 + 4c^2ba^2 + 6acb + c^2b^2 - c^2a^2 - 2ca^3 - 2b^3ac - 6c^3ab \\ & + 2a^3cb + a^2c^2b^2 - b^2a^2 + 2c^2b^3 + c^4b^2 + b^4 + 3c^4 - 4acb^2 - 2c^3ab^2 \\ & - c^6 - c^2a^4 + 2c^4a^2) \end{aligned}$$

$$\Delta_1 = 1-d^2$$

$$= \left(\frac{1-\delta}{1+\delta} \right) \left(1 - \frac{(1+r)^4 \left(\frac{\delta}{K} - 1 \right)^2}{K^2 (1-\delta)^2} \right) > 0,$$

PROOF:

Since $\left(\frac{1-\delta}{1+\delta} \right)$ is positive. It can also be shown that

$$1 - \frac{(1+r)^4 \left(\frac{\delta}{K} - 1 \right)^2}{K^2 (1-\delta)^2} > 0 \quad \text{This condition} \Rightarrow$$

$$K^2 (1-\delta)^2 > (1+r)^4 \left(\frac{\delta}{K} - 1 \right)^2 \quad \therefore K \text{ is relatively larger than } \delta,$$

$$(1-\delta) > \left(1 - \frac{\delta}{K} \right). \text{ Finally with } K \text{ large,}$$

$K^2 > (1+r)^2$. Together these findings confirm that

$$\Delta_1 = 1-d^2 > 0.$$

$$\Delta_2 = 1-2d^2-d^2a^2+2dac+d^4-c^2 > 0.$$

After rearranging this $\Rightarrow$

$$(1-d^2)^2 - (da-c)^2 > 0$$

$$(1-d^2) > (da-c).$$

$\therefore (1-d^2)$ is positive, (above)

$\Delta_2 > 0$, if $c > da$.

be positive, the system would be globally convergent.⁴¹

⁴¹ However, most of the dynamic macroeconomic systems are saddle path convergent.

3.7 Stability Condition for the Estimated model:

In Chapter VI of the thesis , I present the empirical estimates of Euler equations derived in Chapter II. Here I use the coefficient estimates from that work to illustrate the stability properties of my system.⁸² Using the parameter estimates from my system estimation, and substituting the calculated values of the b's and δ into equation (3.43), my specific characteristic equation results in the following A'B matrix:⁸³

$$A^{-1}B = \begin{bmatrix} -0.182 & 109.656 & -0.002 & 0 \\ -0.0182 & 108.695 & -0.002 & 0 \\ 57435.53 & 62.818 & 122223.523 & 1.062 \\ 57435.53 & 62.818 & 62.818 & 1.062 \end{bmatrix} \quad (3.51)$$

The stability condition can now be stated quite simply. The stability of this system of difference equations requires λ_i , characteristic roots of matrix A'B above, to be less than one in absolute value.⁸⁴ Equation (3.51) is a polynomial equation of fourth degree. The polynomial

⁸² We have used the estimated coefficients from the full simultaneous system estimation i.e. equation (6.2.3.b) and (6.2.3.b'), i.e., the results summarized in Table 6.3 (3).

⁸³ The values of b's and δ have been calculated from the average value of the series between 1946 and 1992.

⁸⁴ Stokey and Lucas (1994) in Theorem 6.8 show that if the n-roots are smaller than one in absolute value then the linear quadratic model is globally stable. They also show that the largest of the n characteristic roots determines the speed of convergence of stationary state.

equation calculated from equation (3.51) takes the following form:

$$\lambda^4 - 12333.26\lambda^3 + (0.132) 10^7\lambda^2 - 13.971\lambda - 0.0185 . \quad (3.52)$$

There will be four distinct roots (eigenvalues) as solutions to this polynomial equation.⁴⁵ The four eigenvalues as calculated from equation (3.52) are:

$$\lambda_1 = 1224.58$$

$$\lambda_2 = 108.69$$

$$\lambda_3 = -0.0001$$

$$\lambda_4 = 0.0001$$

(3.53)

It is clear from the above vector of eigenvalues that only two eigenvalues λ_3, λ_4 are less than one in absolute value. Whereas the other two eigenvalues λ_1, λ_2 are greater than one in absolute value. The requirement for global convergence of a system of polynomial equation of degree n is that all of the n -eigenvalues be less than one in absolute value. Thus our system will not be globally convergent but rather a *saddle path* convergent. This means that the system will converge only if it begins on the saddle path (due to two converging roots with absolute value less than one).

⁴⁵ If the polynomial equation is of degree n , there will be n distinct roots including complex numbers.

To find the saddle path of this system I plot the equation (3.52) with the λ values between -1000 and 1000. The two dimensional plot of equation (3.45) is given on figure 3.1 below. The fitted curve shows the saddlepath along which the system will converge.

Figure 3.1

Figure 3.1 illustrates the saddle path for λ values between 1000 and -1000, along which the system will converge towards the stationary state. Given the initial choice of saddle path quantities, the economy will converge to the stationary state. This property means that we can take the estimation of the strictly short run (annual) adjustment path more seriously, knowing that it is consistent with a long run equilibrium relationship. With this background I proceed to describe in more detail the estimation procedure used to test my analysis.

Chapter IV: The Generalized Method of Moments

4.1 Introduction:

In this chapter I discuss the technique of Generalized Method of Moments that is used to estimate the Euler equations derived in Chapter II. Generalized Method of Moments (GMM) was first introduced by Hansen in 1982. GMM has since been applied to a wide variety of models such as probit models and non-linear rational expectational models.⁸⁶ This chapter describes how the GMM estimation technique applies to my problem. This survey relies heavily on the excellent survey articles of Ogaki (1993) and Hall (1993).

GMM is considered to be a very general estimating technique in the sense that the estimators arising from those of other techniques such as ordinary least squares, two-stage least squares, systems of non-linear simultaneous equations, and the estimators of dynamic rational expectations models may be viewed as special cases of GMM. In some cases the maximum likelihood estimator may also be viewed as a special case of GMM.⁸⁷

⁸⁶ See, for example, Avery, Hansen and Hoitz (1983) and Hansen and Singleton (1982).

⁸⁷ Hamilton (1994).

4.2 Advantages of GMM:

The advantages of GMM over other estimation procedures arise because GMM requires only a set of moment conditions rather than a full density function.⁸⁸ For example, if we were to estimate an Euler equation by using maximum likelihood, we would need to know the exact joint likelihood function of the variables to be estimated. To use maximum likelihood, then, it becomes necessary to specify the conditional distribution of the ratios of the variables that describe the trade-off between the two time periods. The estimates of the model's parameters are then obtained by maximizing that likelihood function.

There are two drawbacks in this approach: first, finding the joint distribution is difficult, and second, computing it is cumbersome. From the first point, if the joint distribution of the likelihood function is not defined accurately, maximum likelihood is likely to give biased inferences if the conditional distribution is misspecified.⁸⁹ The second advantage of GMM is that it does not require the strong distributional assumption of normally distributed error terms. The econometric estimation

⁸⁸ Hamilton (1994).

⁸⁹ Estimation of a correctly specified maximum likelihood function gives more efficient estimators than GMM but the procedure of maximum likelihood has a greater probability of misspecification (maximum likelihood fails to be consistent if the distribution of observable variables is misspecified).

strategy implied by GMM circumvents the theoretical requirement of an explicit representation of the stochastic equilibrium and yet permits identification and estimation of parameters of agent's objective functions as well as tests of the overidentification restrictions implied by the model.⁹⁰ Overall GMM does not require a complete explicit representation of the economic environment and also does not require strong assumptions about the nature of the forcing variables. Therefore, estimation can be conducted with only a subset of economic environment specified.

⁹⁰ Hansen and Singleton (1982).

4.3 The GMM Estimation:

The basic idea underlying the GMM estimation strategy is to use the theoretical model to generate a family of orthogonality conditions. These orthogonality conditions are then used to construct a criterion function whose minimization yields an estimate of the model's parameters, β . The criterion function is constructed in a manner that guarantees that the parameter estimate is consistent, asymptotically normal, and has an asymptotic covariance matrix that is consistent. The orthogonality conditions are also used to construct a test of the overidentifying restrictions implied by the theoretical model.

The dynamic optimization problem of economic agents imply a set of stochastic Euler equations which are satisfied in equilibrium. These Euler equations imply a set of population orthogonality conditions that depend in a non-linear way on observable variables and on unknown parameters characterizing the profit/preference function.⁹¹ The idea under GMM is to construct a non-linear instrumental variables estimate of these parameters by making the sample versions of the orthogonality conditions as close as possible to zero. It is worth noting that these estimators are consistent and have a limiting distribution under some

⁹¹ The population orthogonality conditions permit the disturbance terms used in construction of the orthogonality conditions to be serially correlated and conditionally heteroscedastic.

very weak assumptions about the stochastic process generating the observable time series. With Euler equation estimation there are typically more orthogonality conditions available than number of parameters to be estimated. In this sense the problem is 'overidentified' but can be estimated by using Hansen's (1982) J-test to examine how close the sample versions of the population orthogonality conditions approach to zero.

Under GMM the estimator is obtained by minimizing a quadratic form in a vector of sample moments which are functions of parameter vector β and the data set X_i . To illustrate the GMM estimation procedure, we assume a general functional form to be estimated such as:

$$y_i = f(X_i, \beta_0, \epsilon_i), \quad (4.1)$$

where, y_i is the vector of dependent variables, X_i is a set of explanatory variables, β_0 is the vector of true parameters, and ϵ_i is the vector of error terms.

Equation (4.1) can be rewritten as:

$$\epsilon_i = f(y_i, X_i, \beta_0) \quad (4.2)$$

Here ϵ_i in equation (4.2) represents the disturbance of GMM, where β_0 is the true value of β , the parameter to be estimated. When the law of large numbers is applied to $f(y_i, X_i, \beta)$ for all admissible β s, the sample mean of $f(y_i, X_i, \beta)$ converges to its population mean. The idea then is to choose the $\hat{\beta}$ that minimizes the moment restriction

condition, $E[f(y_i, X_i, \beta_0)] = 0$ by minimizing a quadratic form of the sample mean with respect to β .

The requirement for minimizing the moment restrictions is finding a set of valid instruments (Z) that is orthogonal to the error term. This would imply:

$$Plim \frac{Z'\epsilon}{T} = 0, \quad (4.3)$$

where T is the number of observations. We can then estimate (4.1) with the help of GMM. Since ϵ_i is unobservable, we can find a sample analogous to (4.2) which should be as close to zero as possible at the probability limit. Thus substituting (4.2) into (4.3) we obtain:

$$Plim \frac{Z'f(y_t, X_t, \beta_0)}{T} = 0 \quad (4.4)$$

Since y_i and X_i are observable we have to find a vector β such that population moment equals zero when evaluated at β_0 . Under GMM this can be done by taking the square of (4.4) and minimizing the sum of squares assuming a criterion function u , such that:

$$u = \frac{Z'f(y_t, X_t, \beta_0)}{T}. \quad (4.5)$$

Given the choice of u , the problem is to minimize $u'wu$.⁹² Here u is the criterion function and w is the optimal weighting matrix. The weighting matrix is chosen in

⁹²

where w is the optimal weighting matrix which is inverse of covariance matrix u . The minimization function therefore is $\{(Z'f/T)'w_1(Z'f/T)\}$.

such a way that it results in asymptotically efficient GMM estimator. This can be done by choosing an estimate of β , $\hat{\beta}$, such that it will make $u'wu$ as close to zero as possible. The choice of the weighting matrix is very important because it sets the orthogonality conditions. The optimal weighting matrix is a symmetric, positive definite matrix that depends on sample information. The GMM estimator $\hat{\beta}$ then becomes the solution of this minimization problem.

Because the estimation procedure used by GMM is less demanding in terms of information than maximum likelihood, it is less restrictive and therefore easier to use by a researcher. For example, one can use GMM without making any explicit assumption about the data generation process of X_i . This becomes a very attractive feature of GMM for two reasons:

- (i) the data generation process for X_i may involve many (possibly non-linear) equations. GMM offers the ability to estimate β without having to estimate the complete system,
- (ii) economic theory often implies a set of moment conditions without suggesting a unique model for X_i .

The estimation procedure used by GMM is thus based on moment conditions. To use GMM one needs to find at least as

many sets of moment conditions as there are parameters to be estimated. The moment conditions are the restrictions imposed by this estimation procedure on the model. These restrictions are called orthogonality conditions. This means that the expectational errors and the vector of instruments to be used in the estimation procedure are orthogonal, i.e., the expectational errors and the vector of instruments are uncorrelated.⁹¹

⁹¹ For example, the Euler equations in a rational expectations model.

4.4 The GMM Conditions:

Given that the set of moment conditions are satisfied, GMM uses the sample moments to produce an estimate of the parameter vector β that most closely approximates the population moment condition. For the resulting parameter estimates $\hat{\beta}$ to be the most efficient estimates of true β_0 (under GMM), three conditions must be satisfied.

(i) the population moment conditions must be valid. That is, $E[\epsilon_i(\beta_0) Z_i] = 0$.⁴⁴ This condition suggests that for the true parameter β_0 , the orthogonality condition must hold;

(ii) there must be at least as many moment conditions, r , as parameters to be estimated q , i.e., $r \geq q$. This requirement is for identification purposes and is similar to that imposed by any estimation procedure.⁴⁵

When these conditions are met $T^{1/2}$ times the sample moment evaluated at β_0 converges to a normal distribution as T (the sample size) approaches towards infinity.

⁴⁴ Where β_0 is the true parameter.

⁴⁵ For example, in simultaneous equation estimation, it is necessary that the number of equations be at least equal to the number of unknowns.

4.5 Important assumptions:

There are two important assumptions under which the large sample properties of GMM estimators are derived.

(i) **Stationarity:** The variables must be stationary.

This assumption rules out deterministic trends, autoregressive unit roots, and unconditional heteroscedasticity. Since many macroeconomic variables show nonstationarity there are chances that this assumption may be easily violated. To make sure that the macro variables used in our estimation are stationary we use the first differences of our variables.

(ii) **Identification:** The identification assumption is that β_0 is the unique minimizer $J_0(\beta)$, where $J_0(\beta)$ is the minimized value of the objective function. Since $J_0(\beta) \geq 0$ and $J_0(\beta_0) = 0$, β_0 does minimize the objective function. This assumption then requires $J_0(\beta)$ to be strictly positive for all other β .

4.6 The Overidentifying Restriction in GMM:

$\hat{\beta}$ is obtained by minimizing a quadratic loss function in a sample moment. The resulting estimates are consistent because the population moment condition equals zero when evaluated at β_0 , i.e., $E[\epsilon_i(\beta_0) Z_i] = 0$.

When there are exactly the same number of moment conditions, r , as parameters to be estimated, q , then β is 'just identified' because there is just enough information (i.e. moment conditions) to estimate β . However, when $r \neq q$ the problem becomes more difficult.

When $r < q$, the number of moment conditions are less than the number of parameters to be estimated and β is "underidentified". This means that there is not enough information from which one can estimate β .

When $r > q$, the number of moment conditions exceed the number of parameters to be estimated and β is "overidentified". There is now more information available than is needed to estimate β . When this happens different GMM estimators can be obtained from different weighting matrices. In these circumstances one must choose such a weighting matrix which will provide an asymptotically efficient GMM estimator, $\hat{\beta}$. That is, when overidentifying restrictions exist, the choice of W_T (the weighting or distance matrix) will affect the $\hat{\beta}$. It can be shown that

the optimal choice of W_T is proportional to the inverse of asymptotic covariance matrix u^* . The optimal weighting matrix is the one that takes account of this covariance structure in the construction of the quadratic form.

The asymptotic normality of $\hat{\beta}$ facilitates inference about β . Such inference is based on the assumption that the model is correctly specified. It is important to test whether the data is consistent with the model before focusing on inference within the context of that model, e.g. tests for serial correlation and heteroscedasticity in ϵ_t . In such a situation, Hansen's (1982) overidentifying restriction test is used as a test principle to be applied to non-linear dynamic models. Hansen (1982) has shown that the best W_T is the covariance matrix of the error terms. It should be noted that the choice of optimal W_T is based on a given functional form, $f(y_t, x_t, \beta)$.⁹⁷

⁹⁶ Hall (1993), Ogaki (1993).

⁹⁷ In the case of instrumental variables estimators this means that instrumental variables are given.

4.7 The Chi-square (χ^2) Test of the Overidentifying Restrictions: The J-test

The desirable statistical properties of $\hat{\beta}$ rely crucially on the validity of the moment condition $E[Z_i, \epsilon_i(\beta_0)] = 0$. It is then important to test whether the data are consistent with that condition. If β is overidentified then only q linear combinations of $Z'u(\hat{\beta})$ can be set equal to zero and $Z'u(\hat{\beta}) \neq 0$ by construction. However, if the population moment condition is true then one would expect $Z'u(\hat{\beta}) \approx 0$. This provides the basis for a test of model specification.

Hansen (1982) has shown that T times the minimized value of objective function has an asymptotic χ^2 distribution with $r-q$ degrees of freedom in large samples.⁴⁴ This is sometimes called Hansen's J-test. According to Hansen's (1982) lemma 4.2, if $E[Z_i, \epsilon_i(\beta_0)] = 0$ then the test statistic converges asymptotically to a χ^2 distributed random variable with appropriate degrees of freedom.⁴⁵ There are $r-q$ degrees of freedom because only $r-q$ of the

⁴⁴ That is number of instruments (r) minus number of parameters (q) to be estimated.

⁴⁵ When the model is overidentified (more orthogonality conditions than the number of parameters to be estimated) the J-statistic converges asymptotically to a χ^2 distribution.

sample moment conditions of $Z'u(\hat{\beta}')$ are free on account of the q restrictions implied by the first order condition for $\hat{\beta}'$. The estimated J-statistic must be larger than the critical χ^2 value (for the appropriate degrees of freedom) before the null hypothesis of no specification error can be rejected. However, if the estimated J-statistic is smaller than the critical χ^2 value then the null hypothesis of no specification error cannot be rejected.

4.8 GMM and Euler Equation Estimation:

The Lucas critique (1976) is the observation that policy evaluations based on traditional simultaneous equation simulation are often wrong because those analyses typically assume that behavioural parameters will remain constant across policy regimes. Theory suggests that rational agents will take policy changes into account when making their decisions so that the behavioural parameters will change. The Lucas critique is then associated with the hypothesis of policy irrelevance and subsequently, models were built to test this latter hypothesis. These models were primarily dynamic non-linear models that were cumbersome to estimate by traditional methods such as maximum likelihood.

From this setting, Hansen and Singleton (1982) showed that if one used GMM, estimation of these models would be relatively easy. To use GMM, all that was needed was two sets of moment conditions. The Euler equations provide the first set of moment conditions directly when the first order condition is satisfied. The second set of moment conditions are obtained when the Euler equation is combined with rational expectations. Together these conditions satisfy the orthogonality restriction.

The advantage of using Euler equation estimation is that the Euler equation uses information in adjacent time

periods to make decisions and does not require knowledge of the entire future period. This is a great advantage over other types of estimations (such as maximum likelihood) which require a more complete specification of the future. In addition, using GMM to estimate a non-linear dynamic Euler equation has two other advantages over other estimation procedures:

- (i) it is computationally more convenient, and
- (ii) it avoids a bias due to potential misspecification of a fully specified model.

In my model the first order conditions of the firm's intertemporal optimizing problem provide the first set of moment conditions. To find the additional set of moment conditions I combine the first order conditions with rational expectations. Given rational expectations, any error in the estimated model should be uncorrelated with the information available at that period. The combination of rational expectations and optimal decision making satisfies the orthogonality requirement that estimated error and the vector of instruments be orthogonal to each other. That is, $E[\epsilon_i(\beta_0) Z_i] = 0$, where ϵ_i is again the expectational error conditioned on β_0 (the true value of β), and Z_i instrument-set.

The J-statistic then tests whether the model with real balances can indeed better describe the data in the sense that inclusion of real balances in the model specification

is not rejected by the data. The null hypothesis is that my model with money acting as a 'transaction technology' has no specification error. If the estimated J-statistic is smaller than the critical value of $\chi^2_{r,q}$ distribution, one can not reject the null hypothesis of no specification error. However, when the model is rejected the χ^2 statistic gives little guidance as to why that is the case. Hansen and Jagannathan (1991) have developed a diagnostic procedure that provides some guidance. Because the idea behind the test is to bring the estimated objective function as close to zero as possible, any specification that decreases the J-statistic reduces the chances of rejecting the model and thus provides a basis for comparing alternative specifications. In context of my model, this means that the inclusion of real balances produces such a J-statistic that the null hypothesis of no specification error cannot be rejected by the data.

4.9 Some Small and Large Sample Studies Using GMM:

So far we have discussed only the asymptotic properties of GMM. Tauchen (1986) simulated a small artificial economy and examined the finite sample behaviour of GMM estimates of parametrized parameters in a consumption based asset pricing model. The sample sizes he used were between 50 and 75. He concluded that the estimator performed well but was sensitive to the choice of instruments (Z_i). He advised in favour of using a small number of instruments. Kocherlakota (1990) conducted a simulation similar to Tauchen (1986) and used the iterated GMM estimate in which one re-estimates with updated version of the weighting matrix until the estimates converge. His simulation was size 90. He found that GMM performed worse when a larger set of instruments was used. Both studies used a highly non-linear but small system of equations. Monte Carlo simulations by Tauchen (1986) and Kocherlakota (1990) strongly suggest that one should be quite parsimonious in the choice of the instrument set.

Ferson and Forester (1991) used GMM in large equation systems. They used real financial data in a seemingly unrelated regression model with cross equation restrictions. They controlled the size of the model, the number of instruments and the sample size and considered two-step and iterated GMM estimators. They found a

relatively small bias even in sample size of 60. The behaviour of the overidentifying restrictions appeared to be sensitive to the choice of estimator.

Nelson and Startz (1990) provide simulated evidence that the finite sample behaviour of the test statistic for model specification can be very sensitive to the quality of the instruments, (i.e., the correlation between the regressor and the instrument). If the instrument set is of poor quality, the statistic tends to reject more often. Nelson and Startz (1990) emphasize that a good instrument should not only be uncorrelated with ϵ_i , but must also be correlated with X_i .

All these studies suggest that even if the estimated model is correctly specified, the finite sample performance of GMM will be sensitive to both the number of orthogonality conditions and the sample size. In addition, Ferson and Forester (1991) argue that the iterated GMM estimator will generate better finite sample behaviour than will the two-step estimator and should be used in practice.

4.10 Computer Package used for Estimation:

To estimate the Euler equations I used the GMM estimation procedure in TSP (version 4.2B). The weighting matrix used in GMM estimation is adjusted for first order moving average serial correlation introduced by first differencing.

**Chapter V: Time Series Data Sources
and Their Statistical Properties**

5.1 Introduction

This chapter describes the time series data used in the estimates presented in the next chapter and the sources of this data. Most of the data was taken directly from the Statistics Canada economic database, Canadian Statistical Information Matrix (CANSIM). For some of the variables, however, direct measures were not available and had to be computed from the data series that were available. This will be discussed in detail later. The frequency of the data used in the tests was annual.¹⁰⁰ Whenever the frequency was not annual, the monthly or quarterly data were converted to annual values by using the CANSIM conversion process.

Data was collected for the entire post World War II period 1946-1992 (i.e., 47 annual observations). The following sections describe each of the data series in detail, the method used in its computation (if applicable), the original source of the data, the applicable conversion process and the symbol used to represent the variables in the estimating equations.

¹⁰⁰ The year is the conventional time period used to estimate investment Euler equations of the type appearing in my thesis. See, for example, Schaller (1993a), Hubbard and Kashyap (1992), Fazzari and Petersen (1993), Watkins (1993), and Bond and Meghir (1994) to name a few. The focus on adjustment costs, however, suggests that higher frequency might be more appropriate. Unfortunately, I could not find an independent capital series for any other frequency than annual. In future work I plan to test the significance of frequency by simulating a capital series from quarterly investment data.

5.1.1 Real Capital Stock (K_t) : The data for the capital stock was obtained from matrix number 6571, series number D816089 of CANSIM. The title of the matrix was Flows and Stocks of Fixed Non-Residential Capital in Canada, 1970 SIC annually from 1926, total. The title of the series chosen was end of year gross capital stock total in 1986 constant dollars and covers the capital stock of both manufacturing and non-manufacturing industries. The "perpetual inventory method" was used to calculate this data series. The original source of this data is "Fixed Capital Flows and Stock, (13-568) STC".

5.1.2 Real Investment (I_t): The data for gross investment was obtained from matrix number 6571, series number D816079 of CANSIM. The data is the flow counterpart to the gross fixed capital stock described above. The title of the matrix was Flows and Stocks of Fixed Non-Residential Capital in Canada, 1970 SIC annually from 1926, total, in 1986 constant dollars. The title of the investment series is gross fixed capital formation, total, for manufacturing and non-manufacturing industries which includes the adjustments and historical revisions made in the national accounting system. The "perpetual inventory method" was used to calculate this data series. The original source of this data is "Fixed Capital Flows and Stock, (13-568) STC".

5.1.3 Rate of Capital Depreciation (δ): Data for the capital depreciation rate is not readily available. Therefore, this series had to be calculated from available data. The rate of capital depreciation was calculated by dividing the straight line capital consumption allowance (1986 constant dollars, CANSIM matrix number 6571, series number D816104) by the previous year's end of year real gross capital stock (CANSIM Matrix number 6571, Series number D816089). This gives an annual rate of capital depreciation. The constant depreciation rate used in our work was the average depreciation rate between 1946 and 1992. The original source is "Fixed Capital Flows and Stock, (13-568) STC".

5.1.4 Real Gross Domestic Product (RGDP): The data for real gross domestic product, RGDP, was obtained from CANSIM matrix number 6840, series number D114442. The title of the matrix was Gross Domestic Product Expenditure Based in 1986 prices, and was available annually from 1926 measured in millions of dollars. The original source of this data is the Statistics Canada publication SDDS 2501 STC (13-531-13-201).

5.1.5 General Price index (ϕ_t): The general price level, ϕ_t , is the GDP implicit price deflator with the base year 1986 = 100. The data was obtained from CANSIM matrix number 6841,

series number D6841 of CANSIM entitled Implicit Price Index, Gross Domestic Product annually from 1926. The original source of this matrix was the Statistic Canada publication SDDS 2501 STC (13-531 and 13-201). The data for the most recent years was updated from "Canadian Economic Observer : Historical Supplement 1993/1994".

5.1.6 Price Index of Investment (P_t^I): Investment price

index data is not explicitly available but could be inferred from other CANSIM series. That is, an implicit investment price index was generated by dividing gross capital formation in current dollars (matrix number 6520 series number D814000 of CANSIM) by gross capital formation at 1986 constant dollars (matrix number 6571 series number D816079 of CANSIM). The investment price index then had the base year 1986 = 100.

5.1.7 Price Index of Industrial Output (P_t) : The price index of output P_t , is the GDP implicit price deflator of industrial production with the base year 1986 = 100. This series as an explicit price index is available only until 1971 at 1971 = 100 base (matrix number 0389, series number D102184). For the years between 1961-1992 the implicit industrial production price index was obtained by dividing industrial production at current dollars (matrix number

4663, series number I28035) from industrial production at 1986 constant dollars (matrix number 4670, series number I34035). Then price index between 1946-1961 was converted to 1986 =100 base.

5.1.8 Rate of interest (i_t): The rate of interest used was the three month discount rate yield on Canadian Government Treasury Bills. While the use of a twelve month corporate rate would be preferable for my study such a rate is not available for the entire 1946-1992 time period. Use of the treasury bill then implies that while the average level of the shorter rate will be too low, changes in the average rate should be highly correlated with changes in the longer duration instrument.¹⁰¹ The treasury bill rate was taken from CANSIM matrix number 2560 series number B1400. This series was available as monthly data and converted to an annual series by taking the average of the twelve months.

5.1.9 Real Interest Rate (r_t): A real interest rate series was generated for use in the estimation of the investment equation. This section shows how it was generated.

The real (ex ante) interest rate is defined as:

$$r_t = i_t - \pi_t^e, \quad (5.1)$$

¹⁰¹ However, there was a period in 1970s when a small supply of treasury bills in conjunction with an artificial demand for bills arising from the chartered banks' secondary reserve requirement, led to an abnormally low yield on treasury bills.

where r_t is real interest rate in year t , i_t is nominal interest rate, and π_t^e is expected rate of inflation. The problem is that the expected rate of inflation is not observable and must be constructed.

Inflationary expectations are assumed to be formed rationally on the basis of a subset of desirable macroeconomic variables. While the subset of past variables necessarily restricts the information set available, theory suggests that the variables used can be expected to be among the more significant determinants of inflation and hence its expectations. To estimate the expected rate of inflation I regressed the actual rate of inflation π_t against the lagged values of the inflation rate π_{t-1} , rates of change of the money supply $\dot{m}_{t-1}$ and rates of change of real gross domestic product $\dot{y}_{t-1}$, where each of these variables were lagged for up to four years ($i=1,2,3,4$). That is, the ordinary least squares regression equation was:

$$\begin{aligned} \pi_t = & \alpha_0 + \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \pi_{t-3} + \beta_4 \pi_{t-4} \\ & + \gamma_1 \dot{m}_{t-1} + \gamma_2 \dot{m}_{t-2} + \gamma_3 \dot{m}_{t-3} + \gamma_4 \dot{m}_{t-4} \\ & + \delta_1 \dot{y}_{t-1} + \delta_2 \dot{y}_{t-2} + \delta_3 \dot{y}_{t-3} + \delta_4 \dot{y}_{t-4} . \end{aligned} \quad (5.2)$$

Where, α_0 is the constant, β_i are coefficient for actual inflation rate, γ_i are coefficient for change in the money supply and δ_i are the coefficient for change in the real gross domestic product. After estimation the coefficients that were not significantly different from zero were dropped

from the equation in order of their insignificance. That is, the least significant variable was dropped first, where least significant was determined by the size of t-statistic. Every time a variable was dropped, the OLS regression was rerun with the remaining variables. This process was stopped when each of the remaining variables stayed significantly different from zero.

Using the GDP price deflator as the price level and after dropping the insignificant lagged variables, in the manner described above, the best fitting equation for the inflation rate between 1947 and 1992 was found to be

$$\pi_t = 0.032 + 0.655\pi_{t-1} + 0.321\dot{m}_{t-1} \quad (5.3)$$

From this regression equation, an expected inflation rate series was generated by substituting the actual rates into the estimated equation.

Once the expected inflation series was generated, that rate was subtracted from the nominal interest rate to generate a real interest rate. That is, using (5.1) the generated expected rate of inflation in (5.3) was subtracted from the three month treasury bill rate.¹⁰² This formed the basis for the real rate of interest used in the investment equation of the model.

¹⁰² As discussed in relation to the treasury bill rate above, the short term nature of the interest rate meant that some values of the real interest rate were negative over this time period.

As a check on the inflation rate implicit in the series, I also used the Consumer Price Index (CPI) for the general price level. The computed rate of expected inflation rate from both CPI and GDP price deflator was consistent.

5.1.10 Real Labour Compensation (C_t) : Current labour compensation is obtained from matrix number 6721 series number D20002 of CANSIM. The title of the matrix was Gross Domestic Product, Income Based by quarters in millions of current dollars adjusted for seasonal variation. The title of the series was wages salaries and supplementary labour income. The series was converted to annual series by taking the average of four quarters. The series was verified with Ms. Salvia Reid of Labour Division of Statistics Canada. To maintain consistency with the capital series used, the relevant labour compensation series is labour compensation across all industries. The data, however, include only salaries paid to the employees. The imputed value of the earnings of self employed individuals were not included.¹⁰¹

¹⁰¹ In the calculation of this series following industries were included;

- (1) food and beverage industries,
- (2) tobacco product industries,
- (3) rubber product industries,
- (4) plastic product industries,
- (5) clothing product industries,
- (6) leather and allied products industries,
- (7) primary textile industries,
- (8) textile products industries,
- (9) wood industries,
- (10) furniture and fixture industries,
- (11) paper and allied product industries,

(continued...)

The major primary sources for the calculation of this series at Statistics Canada are:

- (1) Revenue Canada information (T4 slips),
- (2) CFDB (central frame data base),¹⁰⁴
- (3) Survey of employment, payroll and hours¹⁰⁵,
- (4) Census of manufacturing survey¹⁰⁶,
- (5) Labour force survey¹⁰⁷.

5.1.11 Non-Capital Factors of Production (N₁) : This series was obtained directly from the Labour Force Survey because the series was revised in 1991, employment is not available on CANSIM for the entire sample period. The figures are in thousands of persons and are rounded to the nearest thousands so that the components may not add up to total. The series is an annual average and includes

¹⁰³(...continued)

- (12) printing, publishing and allied industries,
- (13) primary metal industries,
- (14) fabricated metal product industries (excludes machinery and transportation equipment),
- (15) machinery industries (excludes electrical machinery),
- (16) transportation equipment,
- (17) electrical and electronic product industry,
- (18) non-metallic mineral product industry,
- (19) refined petroleum and coal products industries,
- (20) chemical and chemical products industries,
- (21) other manufacturing industries.

¹⁰⁴ CFDB is the data base for the whole of Canada and is categorised on the basis of geography and industry.

¹⁰⁵ This data is also CFDB based and is available on the monthly frequency.

¹⁰⁶ This census had its own frame but recently they moved to the CFDB base.

¹⁰⁷ This series is also available at the monthly frequencies and is strictly the data for employment.

individuals of both sexes. The Labour force survey excludes residents of Yukon, North Western Territory, full time members of Canadian Armed Forces, and inmates of penal and chronic health care institutions.¹⁰⁸ The definition of persons employed is taken from the International Labour Organization (ILO), i.e., persons who did any work for pay or for profit. This includes paid work in the context of an employer/employee relationship or self employment. It also includes unpaid work that contributes directly to the operation of a farm, business or professional practice owned and operated by the related members of the same household.

5.1.12 Real Money Balances (m_t): This series was generated by dividing the M1 money series by the GDP implicit price index, P_t .¹⁰⁹

An explicit M1 data series was not available on CANSIM for the entire period and was calculated as sum of two separate series: total currency in circulation (matrix number 921 series number B2010) and net demand deposits at the chartered banks (matrix number 913 series number B478).¹¹⁰ The title of the series B2010 was Currency

¹⁰⁸ Guide to the Labour Force Survey, Catalogue no. 71-528, Statistics Canada Publication.

¹⁰⁹ The M1 series is aggregate so that it includes money held by both firms and households.

¹¹⁰ The matrix title for the two series were "Currency outside banks and chartered banks, unadjusted, millions of dollars, by the month end" and "Chartered banks assets and liabilities, unadjusted, millions of dollars, by the month end".

outside bank, total, month end. The title of the series B478 was demand deposits (less private sector floats). Both series were unadjusted. The original source for these series is the Bank of Canada.¹¹¹ In addition both components are available only on a monthly frequency. This series was then converted to an annual series by the CANSIM method of average of twelve months.¹¹² Once the annual M1 series was generated, real balances were generated by dividing M1 from the general price level P_t with base 1986=100.

My model compares the Euler equations with and without real balances. To make the coefficients of the variables with and without money comparable, the real money stock was normalized to one by dividing the real money stock by its average value for the period 1946 and 1992.

¹¹¹ This definition of M1 was confirmed by Ms. Therese Laflèche of the Research Department of the Bank of Canada.

¹¹² For estimation purposes I also converted the monthly series of M1 (and also of rate of interest) by picking the middle month of the year (i.e. June), but the results of the estimation did not change significantly.

5.2 Statistical Summary of the Data Set:

5.2.1 Summary of the Levels of the Time Series:

Table 5.1 presents the summary statistics of the data set used in the estimations of Chapter VI. The table provides for each variable the mean, the standard deviation, the minimum and the maximum values taken by the variable between 1946-1992.

Table 5.1
Summary Statistics of Variables (1946-1992)
Real values in millions of dollars (Price Index 1986 = 100)
Nominal values in current Dollars

	Mean	Standard deviation	Minimum value	Maximum value
Capital	716.12	393.88	221.82	1515.65
Investment	45.51	24.82	9.33	94.26
Real wage	293.08	158.07	87.18	565.78
Labour supply*	15.43	13.04	3.18	44.99
Labour participation*	156.59	89.33	41.62	313.90
Real price	49.77	35.17	14.00	123.10
Producer's price index**	48.87	35.84	11.83	112.00
Investment price index**	50.47	34.00	12.90	121.00
Depreciation rate	0.038	0.004	0.03	0.04
Real interest rate	0.059	0.041	0.003	0.17

Note: * Billions of dollars,

** Price index 1986 = 100.

5.2.2 Graphical Representation of Some of the Time Series

Data:

While an impression of absolute size and variability is interesting, I am more interested in this thesis in the relative size of variables and their comovement over time. For this reason, I present, in this section a group of charts to illustrate how some of the important data series have varied relative to each other over the time period 1946-1992. The charts compared are:

- (i) real money balances and real output,
- (ii) real money balances and real investment,
- (iii) real investment and real output, and finally
- (iv) real output and real labour compensation.

What is immediately apparent by looking at the charts is that all of the real quantities in the analysis have grown through time. The charts then present a visual representation of the almost overwhelming influence of growth in these series as is true of most macroeconomic time series. Before looking at the time series properties of the variables, however, it is interesting to see how these variables have grown relative to each other over time.

Figure 5.1
Comparison of Four Key Variables
in Their Levels

Graph 5.1 shows a set of four pair-wise comparisons of the key variables in my analysis in levels. The first comparison is between real money balances and real output. Here except for a short period at the very beginning, the movement of the two variables is highly synchronized, with

there being some tendency for real balances to lead real output. The second comparison is between real money balances and real investment. Once again the variables move together with real money balances seeming to lead both the fall and rise in real investment. The third comparison is between real investment and real output. Once again the two series exhibit comovement with real investment showing greater variation than real output, and, like the case of real money balances, showing a tendency to lead real output over the business cycle. The fourth comparison is between real output and real labour compensation. As might be expected, two series are the most synchronized.

The simple fact that is made apparent by these diagrams is that our three key variables real output, real investment and real money balances are closely correlated, both over the longer run period and over the business cycle. This gives preliminary support to the notion that a theory seeking to causally tie these variables together might be on the right track.

5.2.3 Coefficient of Correlation of the Data Series:

More formally, the comovement of our primary variables can be compared by calculating the correlation among the time series variables for real capital, real investment, real output and real balances. Because the correlation among all variables in levels was high, I present the

correlation of the first differences of the data series in Table 5.2.¹³ The reason for taking the first differences of the time series will become more prominent shortly. Our variables are not stationary in levels so that the hypothesis will be tested in first difference form.

The following table provides the correlation results.¹⁴

Table 5.2
Correlation Matrix for Primary Time Series
Variables in First Differences:
Sample Period 1947-1992

Variables	REAL CAPITAL	REAL INVESTMENT	REAL OUTPUT	REAL MONEY BALANCES
REAL CAPITAL	1.00	0.28	0.22	-0.005
REAL INVESTMENT	0.28	1.00	0.52	-0.04
REAL OUTPUT	0.22	0.52	1.00	0.39
REAL MONEY BALANCES	-0.005	-0.04	0.39	1.00

¹³ The correlation coefficients in levels were over .95 for most of the variables in my study.

¹⁴ Note that while both money and investment are positively correlated with real output, the correlation is somewhat stronger for investment.

5.2.4 Graphical Representation of Rate of Change of Data:

One reason for looking at the first difference of a data series is that it provides a basis for calculating rates of change in the data over time. Using rates of change makes the comparison between any two series easier since the two series have a common base. Here, I present the previous set of comparisons in terms of their rates of change. Comparing the rates of change gives better insight into the scale of the comovement of the series being considered. The series compared graphically are again:

- (i) real money balances (RRB) and real output (RGDP),
- (ii) real money balances (RRB) and real investment (RINV),
- (iii) real investment (RINV) and real output (RGDP),
- (iv) real output (RGDP) and real labour compensation (RRLC).

Figure 5.2

Comparison of Four Key Variables in their Rate of Change Over Time

Graph 5.II provides a comparison of the rates of change of the different series. The first set of comparisons is between the rates of change of real balances and real output. By inspection the comovement of the two rates of change can be seen to be very pronounced (giving visual

representation to the correlation findings of Table 5.2). The chart also illustrates that real money balances tend to lead changes in real output. The second comparison is between real balances and real investment. The correlation coefficient in Table 5.2 shows that these two variables are negatively correlated in their first differences. This relationship is featured more prominently in the graph. The third comparison is between real investment and real output. As the correlation table also suggested, these two time series are also highly synchronized. However, there is less visual evidence of investment leading real output (compared to real money). The final comparison is between the rates of change in real output and real labour compensation. Again these two series are very highly synchronized.

5.3. Statistical Properties of Time Series Used:

5.3.1 Stationarity Test:

Generally, macroeconomic time series are nonstationary. A typical time series may have a time trend, a fixed seasonal pattern and a time varying variance. A series is called weakly stationary if it has a finite mean, finite variance and finite covariances all of which are independent of time. Stationarity is an important property for time series data, without which there could be no guarantee that there were no fundamental change in the structure of the process under investigation. This, in turn, makes standard empirical tests unreliable and makes prediction difficult if not impossible. By and large, most macroeconomic time series become stationary upon first differencing. This means that the rate of change in the data fluctuates around some fixed positive mean.

The Augmented Dickey-Fuller Test procedure, also known as the unit root test, is a test for stationarity in a time series.¹¹⁵ The existence of a unit root in time series data indicates that the data series is generated by a process that is nonstationary. The null hypothesis of the Augmented Dickey-Fuller test is that the series has a unit

¹¹⁵ When $AR(p)$ process (where $p \geq 2$) is used to regress Δx_t , it is called an Augmented Dickey-Fuller test since the right hand side contains lagged first differences of the test variable.

root (i.e., the series is nonstationary) and the test produces a Dickey-Fuller test statistic (an adjusted t-statistic) on the coefficient of the lagged test variable.¹¹⁶ It also reports the critical value of a test of the hypothesis that coefficient is zero. If the calculated t-statistic is larger than the critical value in the Dickey-Fuller adjusted t-statistic table (the MacKinnon test statistic), then we must reject the null hypothesis of unit root in the series and accept the alternative hypothesis that the series is stationary.¹¹⁷

The representative Augmented Dickey-Fuller test equation used to test a time series can be written as:

$$\Delta x_t = \beta_1 x_{t-1} + \beta_2 \Delta x_{t-1} + \beta_3 + \beta_4 t .$$

where β_1 is the coefficient on the lagged test variable,

β_2 is the coefficient on the lagged first difference of the test variable,

β_3 is the constant,

β_4 is the coefficient on the time trend.

In this form, the null hypothesis is that $\beta_1 = 0$. There are three choices for the form of Dickey-Fuller test. The Augmented Dickey-Fuller test may be run;

(i) with neither constant nor time trend ($\beta_3 = \beta_4 = 0$),

¹¹⁶ The t-statistic table has been adjusted by Dickey-Fuller (MacKinnon) because the left hand side of the Augmented Dickey-Fuller regression equation is nonstationary.

¹¹⁷ The critical adjusted t-statistic is chosen on the basis of number of observations and the estimation option chosen.

- (ii) with just a constant ($\beta_4=0$),
- (iii) with both constant and a linear time trend (β_3 , $\beta_4>0$).

In my test I chose the third option. Table 5.3 provides the Augmented Dickey-Fuller t-statistic for the variables used in our Euler equation estimations first in levels and then in first differences.

Table 5.3
Augmented Dickey-Fuller Test Results
Using Constant and Time Trend
Sample period 1946-1992 (Annual)

VARIABLES	AUGMENTED DICKEY-FULLER STATISTICS FOR LEVELS OF VARIABLES ¹¹⁸	AUGMENTED DICKEY-FULLER STATISTICS FOR FIRST DIFFERENCE OF VARIABLES ¹¹⁹
REAL OUTPUT	-2.45	-5.81
REAL CAPITAL	-0.76	-5.81
REAL INVESTMENT	-3.04	-5.92
REAL BALANCES	-2.43	-4.70
REAL LABOUR COMPENSATION	-1.27	-3.69*
NOMINAL MONEY SUPPLY (M1)	-0.26	-5.35
NOMINAL INTEREST RATE	-2.99	-5.30
INVESTMENT- CAPITAL RATIO	-4.74	-6.96
OUTPUT-REAL BALANCE RATIO	-1.94	-4.80

Note: 1. The one percent MacKinnon critical value for rejection of a unit root in levels is -4.17.

2. The one percent MacKinnon critical value for rejection of a unit root in first differences is -4.18.

3. The value for rejection at 5% is -3.51 in first difference.

* Real labour compensation is significantly different from zero only at five percent.

¹¹⁸ 45 observations after adjusting endpoints (1947-1992).

¹¹⁹ 44 Observations after adjusting endpoints (1949-1992).

It is clear from the above table that for the level values of the primary series we cannot reject the null hypothesis of the existence of a unit root at the one percent MacKinnon critical level. The primary time series are nonstationary in their levels. In first difference form, the test results indicate that we can reject the null hypothesis of the existence of unit roots for real output, real capital, real investment, real balances, nominal money supply, nominal interest rate, investment-capital ratio and the output-real balances ratio, all at the one percent MacKinnon significance level. For the real labour compensation series, however, the null hypothesis of existence of unit roots can be rejected only at the five percent significance level. Utilizing this significance level, all of the series tested are stationary in first differences. Because none of the data series has a second unit root, all of our data series are individually integrated of order one (called an $I(1)$ process). These findings suggest that our Euler equation tests should be performed on the first differences of the variables. These findings could be overturned only if the set of variables, were found to be fully cointegrated.

My estimation equations also use several of the primary variables in combination as part of the estimating framework. For example in the investment equation, I use the constructed variable X_1 where,

$$X1 = P_t^I - P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I . \quad (5.4)$$

I then ran augmented Dickey-Fuller tests on the full set of compound variables. The following table presents these results.

Table 5.4
Augmented Dickey-Fuller Test Results
With Constant and Time Trend for Calculated Variables
Sample period 1946-1992 (Annual)

Name of Variables ¹²¹	AUGMENTED DICKY-FULLER STATISTICS FOR LEVELS OF VARIABLES ¹²²	AUGMENTED DICKY-FULLER STATISTICS FOR FIRST DIFFERENCE OF VARIABLES ¹²²
	(-4.17)	(-4.18)
X1	-5.84*	-7.26
X2	-6.19*	-8.16
X3	-2.79	-6.82
X4	-2.08	-4.17
X6	-3.61	-8.58
X7	-4.52*	-7.32

120

$$X2 = \left[\frac{P_t^I}{m_t} \left(\frac{I_t}{K_t} \right) - \frac{P_t^I}{2m_t} \left(\frac{I_t}{K_t} \right)^2 - \left(\frac{1-\delta}{1+r_t} \right) \frac{P_{t+1}^I}{m_{t+1}} \left(\frac{I_{t+1}}{K_{t+1}} \right) \right],$$

$$X3 = \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \frac{P_t}{2m_t},$$

$$X4 = \frac{(P_t/\phi_t) F(K_t, N_t)}{2m_t^2},$$

$$X6 = \left[\frac{1}{2} \left(\frac{u_t/\phi_t}{m_t} \right)^2 - \left(\frac{u_t/\phi_t}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\phi_{t+1}}{m_{t+1}} \right) \right],$$

$$X7 = \frac{1}{2} \left(\frac{P_t^I/\phi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right),$$

¹²¹ 45 observations after adjusting endpoints (1947-1992).

¹²² 44 Observations after adjusting endpoints (1949-1992).

In the above Table, the significance levels are the same as in Table 5.3. The MacKinnon critical values are included in brackets. The starred variables do not have unit roots in their levels because their construction necessitates the taking of a first difference of the relevant time series.

It is clear from the above table that in their levels many of the combined variables contain a unit root process. In addition, like their primary counterparts, they become stationary in their first difference form. These findings, reinforce the idea that the appropriate form for hypothesis testing is in first differences.

5.3.2 Cointegration Test:

While the Augmented Dickey-Fuller equation tests the stationarity of individual time series, there is a related test for groups of data. Under Johansen's cointegration test, a group of nonstationary time series is called cointegrated if there exists a linear combination of the series that is stationary in levels, although the series individually might be nonstationary in their levels. The linear combination of these time series is then called the *cointegration equation*. The finding of cointegration means that the linear combination of two or more series does not have a stochastic trend. Hence, a linear combination of the levels of two or more time series can be stationary even if these time series are individually nonstationary.

If there are n variables then there may be as few as zero and as many as $n-1$ cointegration equations. The number of cointegrating equations is called the rank of the cointegration test. If the rank of cointegration test is n then it means that none of the series are cointegrated. The normal interpretation of finding of cointegration is that there is existence of a long run equilibrium relationship between the variables in the data. Such a finding allows for short run deviations from the equilibrium relationship.

Johansen's cointegration test determines the number of cointegrating equations among a group of variables and thus

the cointegration rank. The Johansen-test procedure computes a likelihood ratio statistic for each added cointegrating equation. The test statistic for significance does not have the usual χ^2 distribution but follows an adjusted χ^2 distribution.¹²³ The null hypothesis for the Johansen test is that there is no cointegration between two or more series. If the computed likelihood ratio is larger than the critical value of the adjusted χ^2 distribution, we reject the null hypothesis of no cointegration and accept the alternative that the time series may be cointegrated in levels.

There can be as many as five different sets of assumptions about the characteristics of the time series and the form of the cointegrating equations for a Johansen's test. For my tests I chose the assumption that the series have means and linear trends and that the cointegrating equations has only intercepts. The resulting Johansen cointegration test equation can be written as follows. Assuming a two vector error correction model (VEC) with one cointegrating equation and no lagged difference terms,

$$x_{2,t} = \beta x_{1,t}, \quad (5.5)$$

and the VEC becomes:

¹²³ In the same way as the Dickey-Fuller (MacKinnon) t-statistic is an adjusted t-statistic.

$$\begin{aligned}\Delta x_{1,t} &= \delta_1 + \gamma_1(x_{2,t-1} - \mu - \beta x_{1,t-1}) + \epsilon_{1,t} , \\ \Delta x_{2,t} &= \delta_2 + \gamma_2(x_{2,t-1} - \mu - \beta x_{1,t-1}) + \epsilon_{2,t} .\end{aligned}\tag{5.6}$$

Where, the δ 's correspond to the linear time trend in the level of series and μ 's correspond to the intercepts.

This Johansen cointegration test was then run on the group of five real variables: real capital, real investment, real output, real labour compensation and real money balances. Because five variables were used in the cointegration test, there could be at most four cointegrating equations present. The results are summarized in Table 5.5 below.

Table 5.5
Cointegration Test Results on 44 observations (1949-1992)
Test Assumption : Linear Deterministic Trend in Data
Lag intervals : 1 to 2
Variables considered : Real Capital, Real Investment,
Real Balances, Real Output, Real Labour Compensation.

Eigen values	Likelihood Ratio	5 % Critical Value	1 % Critical Value	Hypothesized no. of CE(s)
0.65	117.27	68.52	76.07	None**
0.55	70.60	47.21	54.46	At most 1**
0.39	34.48	29.68	35.65	At most 2*
0.20	12.13	15.41	20.04	At most 3
0.04	1.89	3.76	6.65	At most 4

Note: * denotes rejection at 5% significance level.

** denotes rejection at 1% significance level.

CE denotes the number of cointegrating equations.

The null hypothesis of no cointegration, is the hypothesis that the variables have no long run equilibrium

relationship that keeps them proportional to each other in the long run. The table also provides the hypothesized number of cointegrating equations for these five variables.

The first row in the table tests the null hypothesis that there is no cointegrating equation among the five real variables. This hypothesis is rejected at the one percent level of significance. By rejecting the null hypothesis, the expectation is that there will be at least one cointegrating equation among the five variables. The second row tests the hypothesis that there is at the most one cointegrating equation among these five real variables. This was rejected at one percent level of significance, suggesting that there are two or more cointegrating equations among the five variables. The third row tests the null hypothesis that there are at most two cointegrating equations among the five real variables. This could be rejected only at the five percent level of significance suggesting that there are likely more than two cointegrating equations among the five real variables. The fourth and fifth rows test the null hypotheses that there might be at most three or four cointegration equations respectively among the five real variables. However, the likelihood ratio indicates that we can not reject the null hypotheses of three or more cointegrating vectors at either the one or five percent level of significance. This suggests that the five time series variables considered are cointegrated in

their levels and there are likely to be either three or four cointegrating equations among these five variables. In addition, it is worth noting that this group of real variables becomes cointegrated once real balances are introduced into the group of test variables.

The likelihood ratio indicates that the hypothesis that there are more than two cointegrating equations can be accepted within a five percent margin of error. This is typically interpreted as implying that there are most likely three cointegarting equations in the group of variables being tested, but there is some probability that there may be as many as four cointegrating equations in this group. This finding is consistent with the Johansen test requirement that the cointegration rank be less than the number of variables tested for the cointegartion.

Given that the Johansen tests have established that the null hypothesis of no cointegration can not be rejected, I conclude that some subset of these variables is stationary in their levels as a cointegration equation. This implies that there may exist a long run equilibrium relationship among the variables.

Chapter VI : Empirical Results and Analysis

6.1 Introduction

This chapter presents the empirical results of this thesis. As described in Chapter II, the estimation exercise uses Generalized Method of Moments (GMM) to estimate the Euler equations describing the firm's optimal investment and money holding behaviour - both as single equations and as a simultaneous system. These results test whether and in what way real money balances are part of the firm's intertemporal optimization problem. To the extent that real balances are a significant part of the dynamic optimization problem, the introduction of real balances into the more traditional specification of the problem should allow rejection of the traditional specification by the data.

Under GMM estimation, the J-statistic tests whether the proposed hypothesis is or is not consistent with the maintained hypothesis of no specification error.¹²⁴ Hence the J-statistic can be used to distinguish between competing hypotheses only when one hypothesis is clearly rejected while the other is not. In my case, however, the J-tests of section 6.2 fall in the middle grey area. Hence while I can show that the data is consistent with my preferred

¹²⁴ In my examples, the relevant J-statistic has a χ^2 distribution with either seven, eight or nine degrees of freedom (degrees of freedom depend on the number of parameters estimated and the number of instruments used). If the estimated J-statistic is less than the chosen critical value of the χ^2 distribution then the null hypothesis of a correctly specified model cannot be rejected.

specification, I cannot show that my money-augmented form clearly dominates the more traditional specification.

Since neither model (traditional or money-augmented) gives evidence of specification error, the question arises why would one choose a more complicated model over a simpler model? One reason why a more complicated analysis may be preferred is if the complexity allows greater insight into the direct and indirect ways that government policy instruments can affect real behaviour. In my model, the greater complexity arises from the interrelation of money with the transactions and investment decisions of the firm. Moreover, the variables interact endogenously over time. To illustrate the new channels this provides for the effect of policy on firm performance, I solve for the comparative static effects of changes in the model's parameters.

Holding the stocks of capital and money constant at their stationary state values, changes in investment, flow changes in money and the non-capital factors of production are solved for changes in the model's exogenous prices (including interest rate) and the estimated cost parameters in section 6.3. Given that policy can affect any one of these variables, the comparative static analysis quantifies the various direct and indirect ways that policy matters. It is through these channels that monetary policy has a potential role in influencing employment and output. The implications of the model for a monetary policy of price

stability form the basis of section 6.4. Here money is not superneutral and a reduction in money growth can increase real employment and output.

Related estimation results for alternative specifications are reported as part of the sensitivity analysis in section 6.5. Here the sensitivity analysis shows that the money-augmented model is robust over different time periods and different definitions of some key variables. However, some of the findings are sensitive to the form of instrument specification. For example, when the instruments are used as ratios, the investment adjustment cost coefficient is reduced in size by one half (and also becomes statistically insignificant). The use of ratios also makes some of the estimated coefficients insignificantly different from zero. Similarly, when the instruments are lagged for three periods, the transaction cost coefficient and the money adjustment cost coefficient become insignificantly different from zero and the money adjustment cost coefficient changes sign.

Section 6.6 concludes this chapter by presenting a brief recapitulation of the key empirical results.

6.2 Empirical Estimation Results:

6.2.1 Single Equation Estimation of Investment Equation:

In the standard model of the firm, real money balances do not influence the optimal investment path of the firm. Hence, I begin by estimating the Euler equation for investment with and without real money balances. This involves the estimation of firm's investment equations, described earlier in Chapter II. According to my hypothesis, the introduction of real money balances into the cost specification of the optimizing firm should result in improved empirical specification.

The estimation equations for the traditional and money-augmented specifications appear below as equation (6.2.1.a) and equation (6.2.1.b), respectively.¹²⁵ These equations are:

¹²⁵ These equations are derived in section 2.7.1 of Chapter II. The error terms are fully defined there. In 6.2.1.b the error term, e_t^K , is a pure forecast error. In 6.2.1.a the e_t^K error term for the investment equation is the forecast error plus the errors attributable to the omission of a number of money terms which are predicted to contain some unutilized information.

$$P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - P_t^I + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \quad (6.2.1.a)$$

$$- \alpha \left[\frac{P_t^I}{2} \left(\frac{I_t}{K_t} \right)^2 + P_t^I \left(\frac{I_t}{K_t} \right) - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] = \epsilon_t^K.$$

and,

$$P_t^I - P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \quad (6.2.1.b)$$

$$- \alpha \left[P_t^I \left(\frac{I_t}{K_t} \right) + P_t^I \left(\frac{I_t}{K_t} \right)^2 - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right]$$

$$- \gamma \left[\left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \frac{P_t}{2m_t} \right] = \epsilon_t^\gamma.$$

The coefficient α appears in both equations and represents the coefficient for the investment adjustment cost function, whereas γ appears only in equation (6.2.1.b) and represents the coefficient for the transaction cost technology of the firm.

An important methodological issue in estimating these equations is the appropriate specification for the rate of interest. Theoretically correct rate of interest for the investment decision is the real rate of interest, where the real interest rate is the nominal rate of interest minus the expected rate of inflation. Because expected inflation rates are unobservable, the generation of an expected inflation rate series is controversial. In section 5.1.9 of

Chapter V, I discuss in detail how I estimate expected inflation in order to generate real interest rates.

The Euler investment equations in (6.2.1.a) and (6.2.1.b) are overidentified in the sense that there are more orthogonality conditions present than number of parameters to be estimated. Generalised Method of Moments uses non-linear two stage least squares method of estimation for single equations. In the first stage, a reduced form is estimated and the fitted values of the explanatory variables are calculated. In the second stage, the fitted values are used in place of the explanatory variables. Because the single investment equations (with and without money terms) are linear in their parameters the GMM estimates are solved for analytically without the need for iteration.

The estimation results for the alternative investment equations are presented in Table 6.1.¹²⁶

¹²⁶ These estimates are based on a narrow definition of money (i.e., total currency and demand deposits in the economy) and the three month treasury bill rate was used to generate the real interest rate. In work not reported in this thesis I also tried a sectoral measure of money use, i.e., total currency and demand deposits held by non financial private corporations as the money measure but without much success (the source for this series was National Balance Sheet Accounts: Annual Estimates). This might be because of the much shorter duration of the sectoral money series.

Table 6.1

**GMM Estimates of the Aggregate Investment
Euler Equation with and without Real Money Balances.
1950 - 1991 (41 observations)**

Variable coefficients	Euler investment equation without real balances. (6.2.1.a)	Euler investment equation with real balances. (6.2.1.b)
(1)	(2)	(3)
Investment adjustment cost α (t-ratio)	11.97* (6.95)	11.67* (6.65)
Transaction cost γ (t-ratio)		3.60 (0.85)
Durbin - Watson statistic	2.35	2.29
J-statistic (Critical χ^2 at 5% significance level)	8.67 (16.91)	7.92 (15.50)

*Significantly different from zero at the one percent significance level.

NOTE: The variables have been first-differenced to ensure stationarity. The weighting matrix used in GMM estimation is adjusted for the MA(1) serial correlation introduced by the first differencing. The instruments set are constant and second lagged values of Investment (I), Capital (K), Real Rate of interest (RR), Money Supply (MA), General Price Level (p), Investment Price (P^I), Output Price (PP), Labour Compensation (LC) and Income (GDP). The J-test, has a χ^2 distribution with nine degrees of freedom in column (2) and eight degrees of freedom in column (3).

The GMM estimation results reported in Table 6.1 are derived on the basis of ten orthogonality conditions, whereas only one (two) orthogonality condition(s) was(were) needed to identify model (6.2.1.a) ((6.2.1.b)). In Table 6.1 the statistic in the parenthesis below the J-statistic is the 5% critical value of the χ^2 distribution with degree of freedom equal to the number of overidentifying restrictions ($10-1 = 9$ for model 6.2.1.a and $10-2 = 8$ for model 6.2.1.b).

Hubbard and Kashyap (1992) compare the empirical performance of a constant depreciation rate for capital to a time varying depreciation rate for investment analysis and find that the time varying rate performs better in practice. The empirical work presented in this Chapter is based on a constant depreciation rate (the average rate for the whole period). However, I have also estimated my models using the time varying depreciation rate and found qualitatively similar results.

Column (2) of Table 6.1 presents the results of estimating the traditional investment equation. The coefficient for the investment adjustment cost in this specification ($\alpha = 11.97$) is positive and significantly different from zero at the one percent significance level.¹²⁷ The estimated size of an investment adjustment

¹²⁷ Two tailed test for significance level.

cost parameter ($\alpha = 11.97$) can be interpreted in several ways.¹²⁸ First, one can look at how the marginal adjustment cost changes if investment is increased on an average by one dollar. From the definition of the investment adjustment cost [in equation (2.2) of Chapter II] the marginal increase in the investment adjustment cost can be calculated as¹²⁹:

$$LC_t = \frac{\alpha}{2} P_t^I \left(\frac{I_t}{K_t} \right)^2 K_t,$$

$$\frac{\partial LC_t}{\partial I_t} = \alpha P_t^I \left(\frac{I_t}{K_t} \right), \text{ substituting the corresponding values}$$

$$\frac{\partial LC_t}{\partial I_t} = (11.97) (0.57) \left(\frac{47.47}{742.98} \right),$$

$$\frac{\partial LC_t}{\partial I_t} = 0.44.$$

where, LC_t is the investment adjustment cost, I_t is the level of investment, K_t is the capital stock and P_t^I is the investment price. With $\alpha = 11.97$, a one dollar increase in investment expenditure will result in a marginal adjustment cost of 44 cents (evaluated at the average investment level of \$47.5 billion over our time period). On the other hand, with $\alpha = 11.97$, the impact of investment on the average adjustment cost can be calculated as:

¹²⁸ $\alpha = 11.97$ is comparable to some of the other estimates of investment adjustment cost in the literature. For example Gordon (1992) found following adjustment cost estimates for aggregate Canadian data. 6.02(food), 4.21 (textiles), 3.69 (wood), 4.30 (metal), 2.07 (chemical), 2.04 (miscellaneous) 6.60 (total).

¹²⁹ The values for real investment, real capital, and investment price index is taken from the average values taken by these variables between 1950-1991.

$$\frac{LC_t}{I_t} = \frac{\alpha}{2} P_t^I \left(\frac{I_t}{K_t} \right), \text{ substituting the corresponding values}$$

$$\frac{LC_t}{I_t} = \left(\frac{11.97}{2} \right) (0.57) \left(\frac{47.47}{742.98} \right)$$

$$\frac{LC_t}{I_t} = 0.22.$$

Thus $\alpha = 11.97$ implies that the average investment adjustment cost over the same period is 22 cents per dollar invested. As would be expected the average investment adjustment cost is lower than the marginal for the positive levels of investment activity over our time period.

Second, one can calculate the effect of an increase in capital stock on the marginal and average investment adjustment cost. Doing so yields,

$$LC_t = \frac{\alpha}{2} P_t^I \left(\frac{I_t}{K_t} \right)^2 K_t$$

$$\frac{\partial LC_t}{\partial K_t} = - \frac{\alpha P_t^I}{2} \left(\frac{I_t}{K_t} \right)^2$$

$$\frac{\partial LC_t}{\partial K_t} = \left(- \frac{(11.97)(0.57)}{2} \right) \left(\frac{47.47}{742.98} \right)^2$$

$$\frac{\partial LC_t}{\partial K_t} = - 0.014.$$

Holding the level of investment constant at its mean, and a given value of $\alpha = 11.97$, a one dollar increase in the real capital stock decreases the investment adjustment costs by 1.4 cents (evaluated at the period mean size of the capital stock). Over the same period the investment adjustment cost per unit of the capital stock is calculated as 1.4 cents, as shown below.

$$\begin{aligned}
 LC_t &= \frac{\alpha}{2} P_t^I \left(\frac{I_t}{K_t} \right)^2 K_t \\
 \frac{LC_t}{K_t} &= \frac{\alpha P_t^I}{2} \left(\frac{I_t}{K_t} \right)^2 \\
 \frac{LC_t}{K_t} &= \left(\frac{(11.97)(0.57)}{2} \right) \left(\frac{47.47}{742.98} \right)^2 \\
 \frac{LC_t}{K_t} &= 0.014.
 \end{aligned}$$

As a general statement describing investment behaviour, the traditional Euler investment equation performs well. The Durbin-Watson statistic is 2.35 and falls within the range of acceptance of no autocorrelation at the five percent significance level.¹⁰ The estimated J-statistic for the equation is 8.67 which is less than the critical χ^2 value with nine degrees of freedom at the five percent significance level. Therefore, we cannot reject the null hypothesis of no specification error.

Before I discuss the results of my estimation of the money-augmented investment equation, it is important to mention that the original money-augmented investment equation performed poorly and gave clear evidence of model misspecification.¹¹ For this reason I present the results of the redefined money-augmented investment equation, where real money balances affect investment decisions only

¹⁰ The Durbin-Watson regions for $n = 40$ with one explanatory variable are $d_L = 1.442$ and $d_U = 1.544$.

¹¹ The original estimated equation was equation (2.20) in Chapter 11.

indirectly through the transactions cost function. The direct channel has been eliminated by excluding the money term from the investment adjustment cost function presented in (6.2.1.b). The results for the redefined equation are reported in column (3) of Table 6.1. The estimated coefficient of the investment adjustment cost, α , is 11.67 which has the expected positive sign and is significantly different from zero at the one percent significance level.¹² The estimated value of $\alpha = 11.67$ is almost identical to the previous traditional equation estimate ($\alpha = 11.97$). The new estimate of α implies a similar marginal investment adjustment cost as the previously discussed traditional case, i.e., on average a one dollar increase in the investment expenditure will increase the adjustment cost of investing by 42 cents per annum. The estimated value of α further implies that the investment adjustment cost per unit of real investment is 21 cents per annum. The coefficient of the transaction cost function $\gamma = 3.60$, however, has the expected positive sign but is insignificantly different from zero.

For the newly estimated equation, the Durbin-Watson statistic is 2.28. As such the estimated statistic is consistent with the hypothesis of no autocorrelation at the

¹² Two tailed test.

five percent significance level.¹¹ In addition, the J-statistic is 7.92, which is less than the critical χ^2 value with eight degrees of freedom at the five percent significance level. The equation is therefore consistent with the hypothesis of no specification error.

It should be noted here that the estimated adjustment cost coefficient, α , is almost the same size as its counterpart in the traditional model and is significantly different from zero at the one percent significance level. On the other hand, the estimated transaction cost coefficient, γ , in the money augmented model is statistically insignificant.

¹¹ In a two tailed test $d_1 = 1.6 < d = 2.28 < 4 - d_1 = 2.4$, therefore one can not reject the null hypothesis of zero autocorrelation with $k' = 2$ and $n = 40$.

6.2.2 Single Equation Estimation of Money Equation:

In this section the results of the analysis for the different potential roles for money within a single Euler equation are reported. This involved estimating equations (2.24), (2.25) and (2.27) of Chapter II. That is, beginning from the assumption of only a simple transaction role for money, I ask whether the successive addition of first a money adjustment cost and then an investment adjustment cost will improve the specification of the Euler equation.¹⁴ These three equations are presented below as (6.2.2.a), (6.2.2.b) and (6.2.2.c).¹⁵

$$\frac{y}{2} \left(\frac{(P_t/\phi_t) F(K_t, N_t)}{m} \right) - \frac{I_t}{1+r_t} = \epsilon_t^M \quad (6.2.2.a)$$

¹⁴ The optimum money holding path of the firm cannot be compared with and without the presence of real balances. Here the equations present alternatives to test for the significance of different possible roles for money.

¹⁵ These equations are defined in Section 2.2.2 of Chapter II. Note that the three error terms are different.

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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PRECISIONSM RESOLUTION TARGETS

$$\begin{aligned}
& \frac{\gamma}{2} \left(\frac{(P_t/\phi_t) F(K_t, N_t)}{m_t^2} \right) - \frac{r_t}{1+r_t} \\
& + \eta \left[\frac{1}{2} \left(\frac{u_t/\phi_t}{m_t} \right)^2 - \left(\frac{(u_t/\phi_t)}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\phi_{t+1}}{m_{t+1}} \right) \right] \quad (6.2.2.b) \\
& = \epsilon_t^M
\end{aligned}$$

and,

$$\begin{aligned}
& \frac{\gamma}{2} \left(\frac{(P_t/\phi_t) F(K_t, N_t)}{m_t^2} \right) \\
& + \eta \left[\frac{1}{2} \left(\frac{u_t/\phi_t}{m_t} \right)^2 - \left(\frac{(u_t/\phi_t)}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\phi_{t+1}}{m_{t+1}} \right) \right] \quad (6.2.2.c) \\
& + \frac{\alpha}{2} \left(\frac{P_t^I/\phi_t}{m_t^2} \right) \left(\frac{I_t^2}{K_t} \right) - \frac{r_t}{1+r_t} = \epsilon_t^M,
\end{aligned}$$

Equation (6.2.2.a) presents the most basic version of my model where there is only a transaction cost motive for holding money and where γ is the cost coefficient on the transaction technology function. In equation (6.2.2.b), the money holding equation is broadened to incorporate a short run money adjustment cost in addition to a transaction cost function, where η is the coefficient for the money adjustment cost function. In (6.2.2.c) the Euler money holding equation adds an effect of money on the firm's investment adjustment costs in addition to the transaction and money adjustment costs modelled previously. Here α is again the cost coefficient for the investment adjustment

cost function. These equations are estimated by using the non-linear two stage least squares method discussed earlier. All of the three versions of the single money equation are linear in their parameters, therefore the GMM estimates are solved for analytically without the need for iteration. The results are summarized in Table 6.2.

Table 6.2
GMM Estimates of the Alternative
Aggregate Money Euler Equation 1950-1991 (41 observations)

Variable coefficients (1)	Euler money equation with only a transaction cost coefficient (2)	Euler money equation with transaction and an adjustment cost coefficient (3)	Euler money equation with all the three cost coefficients (4)
Transaction cost γ (t-ratio)	0.0008 (1.31)	0.0015* (2.01)	0.001 (1.53)
Money adjustment cost α (t-ratio)		0.186 (1.75)	0.171 (1.45)
Investment adjustment cost β (t-ratio)			0.029 (0.53)
Durbin-Watson statistic	1.96	2.62	2.59
J-statistic (Critical χ^2 at 5% significance level)	7.34 (15.50)	4.44 (14.06)	5.31 (12.59)

* Significantly different from zero at the five percent significance level.

NOTE: Same instruments set and details as in Table 6.1. Except that labour compensation is not an instrument and the second lag of nominal interest rate has been used as an instrument. Thus there are only nine instruments are being used here.

When the money equation was first estimated with just the transaction cost specification (results summarized in

column two of Table 6.2) the coefficient for transaction cost, γ , was positive as predicted, but both small and insignificantly different from zero. For the overall equation however, the Durbin-Watson statistic of 1.96 allows rejection of the null hypothesis of autocorrelation at the five percent significance level.¹³⁶ Similarly, the estimated J-statistic (7.34) is less than the critical value of the χ^2 distribution with eight degrees of freedom at the five percent significance level (15.5). One cannot reject the null hypothesis of no specification error.

The introduction of a money adjustment cost in column three of Table 6.2 better confirms to the predictions of the model. Incorporation of a short run money adjustment cost improves both size and significance level of the transaction cost coefficient. The coefficient for transaction cost, γ , is positive as expected and now significantly different from zero at the five percent significance level.¹³⁷ The money adjustment cost coefficient, η , also is positive as expected and significantly different from zero at the ten percent significance level.¹³⁸ The inclusion of money adjustment cost then increases both the magnitude and the significance of the transaction cost coefficient, indicating greater sensitivity of the firm's transaction costs to money

¹³⁶ $d_U = 1.54 < d = 1.96 < 4 - d_U = 2.46$.

¹³⁷ Two tailed test.

¹³⁸ Two tailed test.

holdings than did the money equation just with a transaction cost function. With γ equal to 0.0015, a one dollar increase in real money holdings will decrease marginal transaction costs by 7.3 cents (these calculations have been done by taking the average value of the variables between 1950-1991).¹³⁹

With a Durbin-Watson statistic of 2.62, the results fall in the inconclusive range at the one percent level of significance.¹⁴⁰ The J-statistic is 4.44, consistent with the hypothesis of no specification error at the five percent level of significance.¹⁴¹

The introduction of an investment adjustment cost in column four of Table 6.2 did not improve the results. Its inclusion worsened the results both with respect to the significance of the previously estimated parameters and with

$$\begin{aligned}
 & \text{139} \\
 TC_t &= \frac{\gamma P_t F(K_t, N_t)}{2m_t} \\
 \frac{\partial TC_t}{\partial m_t} &= \frac{-\gamma P_t F(K_t, N_t)}{2m_t^2} \frac{1}{\phi_t} \\
 \frac{\partial TC_t}{\partial m_t} &= \frac{-0.0015(0.50)(305.91)}{(0.50)2(1.77)^2} \\
 \frac{\partial TC_t}{\partial m_t} &= -0.0732.
 \end{aligned}$$

¹⁴⁰ If $4-d_u < d < 4-d_l$ the results are inconclusive. With $n = 40$ and $K' = 2$ at the one percent significance level $4-d_u = 2.60 < d = 2.62 < 4-d_l = 2.80$.

¹⁴¹ It is worth noting again that the models being compared all involve money holdings. I am considering which alternate specification of money costs is most applicable to the firm.

respect to the equation's conformity to the data. Now both the estimated transaction cost coefficient and the money adjustment cost coefficient become statistically insignificant. Furthermore, the estimate of the newly introduced investment adjustment cost coefficient, α , is very close to zero in both size and significance (compared to an α estimate from investment equation in Table 6.1). The reduction in the size and the precision of the estimated parameter, α , together with the loss in precision in the estimation of transaction and money adjustment cost coefficients suggests that an investment adjustment cost does not belong in the optimal money holding equation. Even more revealing, the addition of the new parameter results in the J statistic increasing in size to $J = 5.31$ from $J = 4.44$ (with fewer degrees of freedom) whereas it should have decreased. Based on these facts it seemed that a more appropriate specification of the money equation would include only transaction and money adjustment costs.

6.2.3 Simultaneous System Estimation:

Although the two stage least square results reported above give consistent estimators, two stage estimates are not asymptotically efficient because they do not take into account the correlation of structural disturbances across equations. Because of this, two stage least squares does not use all the available information about the system. This can be an advantage because it omits the transfer of incorrect information if one of the system equations is misspecified. However, since our single equation tests have suggested the desirable forms of the investment and the money equation, three stage least squares can be used by the Generalized Method of Moments to fully utilize the available information.

Three stage least squares coincides with the GMM estimator of Hansen (1982) when the errors are serially independent and the same instruments are used for each equation. In Hansen's (1982) notation, the GMM estimator sets the orthogonality conditions $u_i (b, y, X) \otimes z_i$ as close to zero as possible using the estimated variance of this vector as the metric.¹⁴² The nonlinear three stage least squares estimates thus obtained are consistent and asymptotically efficient, as well as numerically identical to those

¹⁴² In the above notation u_i is the vector of error terms, b is the vector of parameters to be estimated, X is the matrix of explanatory variables and z_i is the vector of instrument set.

obtained by the corresponding GMM estimator.

My system then consists of two equations: the full investment and the money Euler equations. The simplified system that is compared to a money-augmented system consists of the traditional Euler equation for investment and a money equation that considers only the transactions motive for holding money. The simplified version of the system consists of the following equations:¹⁴

$$P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - P_t^I + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \quad (6.2.3.a)$$

$$- \alpha \left[\frac{P_t^I}{2} \left(\frac{I_t}{K_t} \right)' + P_t^I \left(\frac{I_t}{K_t} \right) - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] = \epsilon_t^K,$$

and

$$\frac{\gamma}{2} \left(\frac{(P_t/\phi_t) F(K_t, N_t)}{m_t^2} \right) - \frac{r_t}{1+r_t} = e_t^M. \quad (6.2.3.a')$$

The money-augmented system consists of the following equations:

¹⁴ The simplified system used as a benchmark is already an improvement over the traditional statement of the firm's dynamic optimization problem. That analysis typically has only an investment Euler equation. Generally, the investment equation is not combined with a money equation to create a system of equations.

$$\begin{aligned}
P_t^I &= P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \\
&= \alpha \left[P_t^I \left(\frac{I_t}{K_t} \right) + P_t^I \left(\frac{I_t}{K_t} \right)^2 - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] \\
&= \gamma \left[\left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \frac{P_t}{2m_t} \right] = \epsilon_t^K,
\end{aligned} \tag{6.2.3.b}$$

and,

$$\begin{aligned}
&\frac{\gamma}{2} \left(\frac{(P_t/\Phi_t) F(K_t, N_t)}{m_t} \right) - \frac{r_t}{1+r_t} \\
&+ \eta \left[\frac{1}{2} \left(\frac{u_t/\Phi_t}{m_t} \right)^2 - \left(\frac{(u_t/\Phi_t)}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\Phi_{t+1}}{m_{t+1}} \right) \right] \\
&= \epsilon_t^M.
\end{aligned} \tag{6.2.3.b'}$$

The results of system estimation are summarized in Table 6.3.

Table 6.3
GMM Estimates of the Simultaneous Estimation
of Investment and Money Euler Equations.
1950-1992 (41 observations)

Variable coefficients	Euler investment equation with adjustment costs and money equation with transaction costs	Euler investment equation with adjustment and transaction costs and money equation with transaction and money adjustment costs
(1)	(2)	(3)
Investment adjustment cost α (t-ratio)	13.64* (11.73)	13.21* (9.48)
Transaction cost β (t-ratio)	0.00035 (1.31)	0.0010** (1.97)
Money adjustment cost γ (t-ratio)		0.168** (2.07)
Durbin-Watson statistic $\hat{u}$	2.22	2.25
Durbin-Watson statistic $\hat{u}_M$	1.96	2.58
J-statistic (critical χ^2 at 5% significance level)	10.63 (15.50)	10.07 (14.06)

*Significantly different from zero at the one percent significance level.

** Significantly different from zero at the five percent significance level.

NOTE: Same instrument set (ten instruments) as in Table 6.1.

Column (2) of Table 6.3 presents the estimation results for the simplified system. The estimated coefficient of the investment adjustment cost ($\alpha = 13.64$) is positive and significantly different from zero at the one percent significance level. This size is consistent with the size found in most of the other recent studies of investment.¹⁴⁴ Since the size of the estimated adjustment cost coefficient, α , is very close to that found in the single investment equation estimates, it has a very similar effect on cost when either investment or the capital stock is changed.¹⁴⁵ With the estimated adjustment cost coefficient $\alpha = 13.64$, a one dollar increase in investment expenditure causes an additional increase in adjustment costs by 50 cents (evaluated at the \$47.47 billion average size of investment). Similarly, the investment adjustment cost per unit of real investment over the whole period is calculated to be 25 cents. The estimated coefficient of the transaction cost ($\gamma = 0.00035$) is positive, but insignificantly different from zero. This estimate of γ is also close in size to the γ estimate of the single money equation estimation.¹⁴⁶

The Durbin-Watson statistics are 2.22 and 1.96 for the

¹⁴⁴ For example Gordon (1992) found following adjustment cost estimates for aggregate Canadian data. 6.02(food), 4.21 (textiles), 3.69 (wood), 4.30 (metal), 2.07 (chemical), 2.04 (miscellaneous) 6.60 (total).

¹⁴⁵ The α estimate in column (2) of Table 6.1 is 11.97.

¹⁴⁶ The γ estimate in column (2) of Table 6.2 is 0.0008.

investment and money equations, suggesting the absence of serial correlation.¹⁴⁷

For the estimated system the J-statistic is 10.63 which is smaller than the critical value of the χ^2 distribution (with eight degrees of freedom) at the five percent significance level (15.50). Therefore one cannot reject the null hypothesis of no specification error.

Column three of Table 6.3 presents the results for the full system with real balances included. The estimated coefficient of investment adjustment cost ($\alpha = 13.21$) is positive and significantly different from zero at the one percent significance level. Because the size of α is close to those estimated earlier, the marginal and the average adjustment costs also will be close to those found earlier. Using the average value of variables between 1950 and 1991 the estimate of $\alpha = 13.21$ implies that a one dollar increase in investment expenditure causes an additional increase in adjustment cost by 48 cents per annum while the average adjustment cost per unit of real investment over the period has been 24 cents.

Similarly, one can calculate the marginal adjustment cost for capital. Holding the level of investment constant at its mean, a one dollar increase in the capital stock

¹⁴⁷ With $n = 40$ and two explanatory variables at five percent significance level.

decreases investment adjustment by 1.5 cents per annum.¹⁴⁸

Whereas, the adjustment cost over this time period per unit of real capital is calculated to be 1.5 cents per annum.

The estimated transaction cost coefficient ($\gamma = 0.001$ in column (3)) is positive and significantly different from zero at the five percent significance level.¹⁴⁹ The size of the estimated transaction cost coefficient, $\gamma = 0.001$, means that the transaction costs of the firm decrease by 5 cents per annum for an additional dollar held by the firm.¹⁵⁰ The money adjustment cost coefficient estimate, $\eta = 0.168$, is both positive and significantly

$$\begin{aligned}\frac{\partial LC_t}{\partial K_t} &= -\frac{\alpha P_t^I}{2} \left(\frac{I_t}{K_t} \right)^2 \\ &= \left(\frac{-(13.21)(0.57)}{2} \right) \left(\frac{47.47}{742.98} \right)^2 = -0.0153.\end{aligned}$$

¹⁴⁸ Using an Euler equation specification in a currency substitution model Imrohoglu (1994) found that the estimated coefficient for transaction cost γ was equal to 0.001 and also that money services provided significant transaction cost-reducing services.

$$\begin{aligned}TC_t &= \frac{\gamma P_t F(K_t, N_t)}{2m_t} \\ \frac{\partial TC_t}{\partial m_t} &= -\frac{\gamma P_t F(K_t, N_t)}{2m_t^2} \frac{1}{\theta_t} \\ \frac{\partial TC_t}{\partial m_t} &= -\frac{0.001(0.50)(305.91)}{(0.50)2(1.77)^2} \\ \frac{\partial TC_t}{\partial m_t} &= -0.0488.\end{aligned}$$

different from zero at the five percent significance level. The significance of the estimated money adjustment cost coefficient indicates that short run money holdings will differ from their long run value and that there is a significant transition phenomenon in altering real balances. The estimated coefficient for money adjustment, $\eta = 0.168$, implies that ten dollars held in money balances decreases money adjustment costs by .03 cents per annum.¹⁵¹

For the money-augmented investment equation the Durbin-Watson statistic is 2.25, again suggesting the absence of serial correlation at the five percent significance level.¹⁵² For the money equation however, the Durbin-Watson statistic is 2.58 which falls in the inconclusive region at the five percent significance level.¹⁵³ For the money-

¹⁵¹

$$AC_t = \eta \rho_t \left(\frac{u_t / \rho_t}{m_t} \right)^2 m_t$$

$$\frac{\partial AC_t}{\partial m_t} = - \frac{\eta \rho_t}{2} \left(\frac{u_t / \rho_t}{m_t} \right)^2 \frac{1}{\rho_t}$$

$$\frac{\partial AC_t}{\partial m_t} = - \frac{0.168}{2} \left(\frac{0.05 / 0.50}{1.77} \right)^2$$

$$\frac{\partial AC_t}{\partial m_t} = -0.000268.$$

¹⁵² With $n = 40$ and three explanatory variables at five percent significance level. $d_U = 1.65 < d = 2.257 < 4 - d_U = 2.35$.

¹⁵³ With $n = 40$ with $k' = 3$ at the five percent significance level $4 - d_U = 2.341 < d = 2.589 < 4 - d_L = 2.662$ the results are inconclusive at best.

augmented system the J-statistic is equal to 10.07 which is smaller than the critical value of the χ^2 distribution with seven degrees of freedom at the five percent significance level. At this level one cannot reject the null hypothesis of no specification error.

On the basis of the J-statistic one cannot discriminate between a model with and without real balances because in both cases the J-test is unable to reject the null hypothesis of no specification error. However, while the test results are ambiguous, the money-augmented specification offers some important advantages over the traditional investment model. First, the general characteristics of the system estimation results indicate that there may be some integration of real and monetary decision making. This was indicated by a more precise, statistically significant estimate of the transaction cost coefficient, γ , in the system estimation (thus underlining the importance of transaction cost considerations in the optimal decision making process of the firm). Second, the money-augmented specification allows several channels through which monetary variables can affect the real variables. This is more completely described by the short run comparative static analysis that follows in section 6.3.

6.3 Comparative Static Features of the Money-Augmented Model in the Short Run:

An advantage of the money-augmented specification is that it shows the channels through which money has an effect on the real behaviour of the firm and stages through which these effects, arise. It also shows the expansionary effects produced on the firm through monetary policy. To illustrate these effects a short run comparative static analysis about the model's stationary state is developed to show how the set of simultaneously determined endogenous variables interrelate and respond to changes in the external constraints on the system.¹⁵⁴

Before I present the quantitative results it is important to emphasize that the variables used to calculate the comparative statics have different units of measurement. For example, the investment figures (I) are in billions of dollars whereas the flow changes in money (u) are very small, typically less than one. The latter scaling arose because, money holdings were normalized to one to allow easy comparison across the two systems. The flow changes in money between any two periods then appear to be small and, because of this, so does the estimate of the transaction cost coefficient. To aid in the interpretation of the

¹⁵⁴ Detailed calculation and results are reported in Appendix A.1 and A.2. The comparative statics are short run in the sense that the stock values of both capital and money are held constant.

results I work with elasticities, which are invariant to the scale chosen for the variables of interest.

The parameter estimates used to calculate the specific elasticity numbers are taken from the system estimation of Table 6.3 (3) weighted by the mean value of the relevant variable over the whole period. Moreover, the elasticities come from the complete general equilibrium adjustment of the full system of equations. This is shown in detail in Appendix A.1. Thus the elasticity of investment expenditures with respect to the transaction cost coefficient, γ , incorporates the whole series of changes in the set of endogenous variables to that exogenous change in the transaction cost parameter. To further interpret the meaning of a change in the exogenous coefficient, note that because the transaction and adjustment costs are linear in their coefficients, the full costs will change proportionally with a change in their coefficients (in the immediate run).¹⁵⁵ Thus, it is equivalent to consider a one percent change in a cost coefficient and a one percent

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$$\text{Given that } TC_t = \frac{\gamma P_t F(K_t, N_t)}{2m_t}, \quad \left. \frac{dTC_t}{d\gamma} \right|_{\bar{P}, \bar{K}, \bar{m}} = \frac{P_t F(K_t, N_t)}{2m_t} + \frac{\gamma P_t F_N}{2m_t} \frac{\partial N_t}{\partial \gamma},$$

$$\text{the elasticity of } TC_t \text{ with } \gamma \text{ is } \frac{\partial TC_t}{\partial \gamma} \frac{\gamma}{TC_t} = 1 + \frac{F_N N}{N} E_\gamma^N.$$

Assuming that $E_\gamma^N = 0$ for the immediate run the elasticity of TC_t with γ will be one.

change in the associated cost.

I begin by calculating the elasticity of investment expenditures with respect to the transaction cost

coefficient.¹⁵⁶ This elasticity is $\frac{\partial I}{\partial \gamma} \frac{\gamma}{I} = -.94$, indicating

that a one percent increase in the transaction cost coefficient reduces investment expenditures by 0.9 percent in the immediate period. This elasticity is just below unity indicating that in the immediate period a reduction in the transaction cost will reduce investment expenditures just less than proportionately. This elasticity illustrates how flow related transaction costs can affect the real variables influencing growth in the model. As the transaction cost parameter increases, the cost of the firm doing business increases and this tends to decrease the scale of the firm and hence the demand for capital. Because money holdings substitute for higher transaction costs we would also expect to find real balances affected - illustrating the complicated interplay of real and money factors in the analysis.

The elasticity of flow changes in money demand with respect to the transaction cost coefficient is estimated to

be $\frac{\partial u}{\partial \gamma} \frac{\gamma}{u} = 2.6$ As such, the flow money elasticity with

¹⁵⁶ For this calculation $\gamma = .001$ has been used from column (3) of Table 6.3 and $I = \$47.47$ billion, the average value of investment over the period has been used.

respect to transaction cost is larger than the investment elasticity. A one percent increase in the transaction cost causes a 2.6 percent increase in the flow change in money holdings in the very short run. Holding higher average levels of money balances appears to be a low cost substitute for increasing transactions costs. Although higher transaction cost again lower the scale of the firm, the substitution effect of higher real balances dominates the scale effect of lower output.

The elasticity of non-capital factors of production with respect to the transaction cost coefficient is very small-almost equal to zero. This indicates that transaction costs have almost no effect on the level of non-capital factors in the immediate period. The virtual absence of a response in non-capital factors to a change in transaction costs comes from the fixed capital stock in the short run and relative inflexibility of production plans on a short notice. However, over time the non-capital factors will be affected by changes in the transaction cost. These effects increase over time as γ reduces the steady state values of K^* and m^* .

Next, I calculated the responses of the endogenous variables to a change in the investment adjustment cost coefficient, α . Again the elasticity responses to a change in α incorporates the general equilibrium response of the endogenous variables to the investment adjustment cost and I

use the linearity of the investment cost function to describe the proportional change in α as an equal proportionate change in total adjustment cost.¹⁵⁷

First, I calculated the elasticity of investment expenditure with respect to the investment adjustment cost coefficient, α , to be $\frac{\partial I}{\partial \alpha} \frac{\alpha}{I} = -.09$. A ten percent increase

in the investment adjustment cost reduces investment expenditures by about one percent in the immediate period. This elasticity is relatively small indicating that the rate at which the capital stock approaches its long run level is not dramatically changed by the short term changes in the adjustment cost.

The short run comparative static effect on the flow change in both money holdings and non-capital factors of production is zero. This is because investment adjustment costs effect investment and future output but have no immediate effect on today's production plan. However, changes in the investment adjustment cost will produce an effect in the longer run, feeding back into both the money

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Given that $LC_t = \frac{\alpha P^I_t \left(\frac{I_t}{K_t} \right)^2}{2} K_t$, $\frac{dLC_t}{d\alpha} \Big|_{\bar{P}^I_t, \bar{K}_t, \bar{m}} = \frac{P^I_t \left(\frac{I_t}{K_t} \right)^2}{2} K_t + \frac{\alpha P^I_t I_t}{K_t} \frac{dI_t}{d\alpha}$,
the elasticity of LC_t with α is $\frac{dLC_t}{d\alpha} \frac{\alpha}{LC_t} = 1 + 2 \cdot E^I_\alpha$, where E^I_α
is the elasticity of I_t with α which equals zero for
the immediate run.

stock and level of use of the non-capital factors of production.

I also calculated the short run comparative static effect of a change in the money adjustment cost coefficient, η , on flow money demand.¹⁵⁸ The elasticity calculation shows that the response is inelastic in the short run.¹⁵⁹ The increase in the money adjustment cost does adversely affect the firm's flow demand for money and leads the firm to wish to adjust its money flows less frequently. This result accords with the analogous treatment of capital and its adjustment cost in the model.

The investment expenditures and the non-capital factors are unaffected by a change in the money adjustment cost coefficient in the short run. This is similar to the absence of an initial effect on u of change in α . But over the longer run the investment spending is affected by changing the long run stock of money.

The short run comparative static effect of changes in the (nominal) rate of interest, r , are also calculated and presented in detail in Appendix A.2. A one percent increase in the rate of interest decreases investment spending by 0.8 percent by the firm, as expected, because the interest rate

¹⁵⁸ Just like the transaction cost and the investment adjustment cost, the money adjustment cost also will have a proportionate change in the cost when the cost coefficient, η , changes.

¹⁵⁹ $\frac{\partial u}{\partial \eta} \frac{\eta}{u} = -0.013.$

rise reduces the present value of the net returns to the firm. On the other hand a one percent increase in the interest rate decreases flow money demand by 0.6 percent. This is because a rise in the nominal interest rate increases the opportunity cost of holding money. The flow changes in money demand are very sensitive to changes in the nominal interest rate.

The non-capital factors of production remain unaffected by the changes in the interest rate in the short run. Over the long run they will be affected through the effect of interest rate on the stationary state values of the stocks of both capital and money.

Lastly, as expected, an increase in the price of capital will decrease investment spending because capital goods are more expensive. The elasticity calculation for changes in investment expenditure due to changes in the price of capital shows that the response of investment expenditures is more than proportionate to a change in the price of capital goods. In contrast, an increase in the price of capital leaves the flow demand for money and the non-capital factors unaffected in the short run. However, over the long run the change in the stock of capital (in response to the higher price of capital) will feed back on money holdings and the use of the other non-capital factors of production.

6.4 Policy Implications

I now turn to use the results of this comparative static analysis to explore the implications for monetary policy over time. To do so I will focus primarily on one particular case, the effects produced on the firm when the central bank adopts a policy of price stability. This will be interpreted to mean that the central bank engineers successfully a fall in the inflation rate from some small but positive rate down to zero. From the point of view of the firm, the impact of this policy on the new stationary state is felt primarily through the concomitant fall in the money rate of interest (with the underlying real rate of interest assumed to be unaffected by the price stabilizing plan).

Using π to represent the inflation rate, the comparative static exercise corresponding to this regime change can then be represented as:¹⁶⁰

¹⁶⁰ The detail analysis and computation is presented in Appendix A.1.

$$\begin{bmatrix} \frac{\alpha P_t^I}{K_t} \left(\frac{I_t}{K_t} - 1 \right) & 0 & P_t F_{KN} \left(1 - \frac{\gamma}{2m_t} \right) \\ 0 & \frac{\eta/\phi_t}{m_t} \left(\frac{u_t/\phi_t}{m_t} - 1 \right) & \frac{\gamma}{2} \left(\frac{P_t}{\phi_t} \frac{F_N(K_t, N_t)}{m_t^2} \right) \\ 0 & 0 & F_{NN} \end{bmatrix} \begin{bmatrix} dI \\ du \\ dN \end{bmatrix} = \begin{bmatrix} 0 \\ d\pi \\ 0 \end{bmatrix}.$$

In the above equation the first row in the right hand side is the derivative of first order condition of investment equation. Similarly the second and third rows are the derivatives of first order conditions of flow money demand and non-capital factors of production. Substituting the estimated parameters and average value of the variables over the period 1950-91 in the above equation we get the following

$$\begin{bmatrix} -.0095 & 0 & .000091 \\ 0 & -.177 & .00005 \\ 0 & 0 & -.00002 \end{bmatrix} \begin{bmatrix} dI \\ du \\ dN \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ d\pi \\ 0 \end{bmatrix}. \quad (6.4.1)$$

Note that from the problem setup I have assumed that since the investment decision is a real decision over time, a change in the steady state rate of inflation has no immediate effect on the real rate of interest and hence on the investment decision. With this interpretation, it can be seen by inspecting (6.4.1) that the impact of a change in the inflation rate on both investment and non-capital factors of production will be zero. Only the flow demand for money is affected initially. Cramer's rule solves for

$\partial u / \partial \pi$ as $-.56$, and using this result to solve for the elasticity of the flow demand for money following a fall in the rate of inflation, I find that this elasticity is extremely high, i.e., $\frac{\partial u}{\partial \pi} \frac{\pi}{u} = -9.4$. The program of price stability initiall, has a large effect on the flow demand for money alone.

This finding remains true only for the immediate period. Through time, the flow accumulation of real money will increase the money stock and this enables the firm to economize on its transactions costs by allowing a greater degree of nonsynchronization in the timing of its transactions. This, in turn, increases the profitability of the firm at current levels of output. In addition, however, larger money holdings will also increase the marginal products of both capital and labour (net of transactions costs). This leads the profit maximizing firm to expand its scale of production which, in turn, spreads into further increases in the demand for complementary factors of production. The actual values of these long run levels can be solved by doing similar comparative statics for the stocks about the long run stationary state.

It follows that while money is neutral in this analysis, money is not superneutral. Over the longer run, monetary policy in the form of a lower trend rate of money growth will produce positive real effects. A slower rate of

money growth translates into greater real money holdings and lower transactions costs. This, in turn, implies higher levels of real output and factor employment and use.

The case is sometimes made that in addition to its effect on expected inflation and the money rate of interest, a program of price stability will also lower absolute and relative price variability and hence lower the cost of borrowing. This then results in a lower real rate of interest (or a reduction in the Canadian risk premium relative to the U.S. interest rates). In this case we would add a fall in the real interest rate (as analyzed in Section 6.3) to the more traditional case examined above.

Finally, this analysis can be used more creatively (but in much the same way) to explore the effects produced by such policy initiatives as the introduction of a technical innovation in the payments system (e.g. new forms of credit or debit cards) or other institutional innovations designed to lower the cost of the cheque clearing system. These can be seen as ways of lowering the γ facing the firm that will spread from money to the other factors of production. Other policy initiatives leading to the setup, encouragement and/or deepening of the treasury bill market or the strengthening of the market for overnight funds might more appropriately be handled as a lowering of the real rate. In all these ways both the impact and long run effect of policy on the three endogenous variables can be found. Policy can

be expected to influence both the steady state and the transition path to that steady state.

6.5 Sensitivity Analysis:

In this section a set of sensitivity analyses are undertaken to determine whether the main estimation results presented in Table 6.3 are robust across a number of dimensions. The sensitivity analysis is performed in two stages. In the first stage I look at different instrument sets and investigate the effects of different lags in the instrument sets. In the second stage, I investigate the effect of using different variables and different sample periods. These results are summarized in Tables 6.5 and 6.6.

Before presenting the results of sensitivity analysis, I first present the correlation coefficients that arise between the variables to be estimated and the instrument set (with two period lags). These correlations are presented in Table 6.5.

Table 6.4
Correlation Coefficient Between Estimated
Variables and the Instruments set

	X1D	X2D	X3D	X4D	X5D	X6D
$X(1)$	0.12	-0.13	0.18	0.006	-0.07	-0.03
$X(2)$	0.13	-0.16	0.21	-0.002	-0.008	-0.04
$QD(1)$	0.02	-0.08	0.11	-0.004	0.02	-0.04
$QD(2)$	0.03	-0.07	0.12	0.01	0.03	-0.04
$PD(1)$	-0.02	-0.04	0.06	0.04	0.01	-0.06
$PD(2)$	0.003	-0.06	0.08	0.01	-0.01	-0.05
$PP(1)$	0.009	-0.09	0.09	-0.02	-0.03	-0.05
$PP(2)$	0.01	-0.06	0.09	0.03	-0.01	-0.04
$RP(1)$	-0.02	-0.18	0.14	0.15	0.19	-0.23

$$\text{where, } X1 = P_t^I - P_t \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) - \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I ,$$

$$X2 = \left[\frac{P_t^I \left(\frac{I_t}{K_t} \right)}{m_t} - \frac{P_t^I \left(\frac{I_t}{K_t} \right)^2}{2m_t} - \left(\frac{1-\delta}{1+r_t} \right) \frac{P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right)}{m_{t+1}} \right] ,$$

$$X3 = \left(\frac{F}{K_t} - \frac{C_t}{K_t} \right) \frac{P_t}{2m_t} ,$$

$$X4 = \frac{(P_t/\varphi_t) F(K_t, N_t)}{2m_t^2} , \quad X5 = \frac{r_t}{1+r_t} ,$$

$$X6 = \left[\frac{1}{2} \left(\frac{u_t/\varphi_t}{m_t} \right)^2 - \left(\frac{u_t/\varphi_t}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\varphi_{t+1}}{m_{t+1}} \right) \right] .$$

Table 6.5 presents the correlation coefficients between

the instrument set and the explanatory variables $X1D$, $X2D$, $X3D$, $X4D$, $X5D$ and $X6D$. The suffix D in each of the variable shows that these variables have been first differenced. The instrument set includes second lags of capital (K), investment (I), Income (GDP), Labour Compensation (LC), General price level (P), Producer's price (PP), Investment price (PI) and Real interest rate (RR). It should be noted that the investment adjustment cost variable $X2D$ and the money adjustment cost variable $X6D$ are both negatively correlated with all of the instruments. This means that smaller values of estimated variables are associated with higher values of instruments. These are also the two variables that incorporate the adjustment cost across the periods and thus link the present values with discounted future values. Table 6.5 shows that none of the instruments are individually highly correlated with the explanatory variables. However, separate OLS regression of each of the explanatory variables on the instrument sets indicate that estimated variables as a group have a reasonably good fit with the instrument set as indicated by a high R^2 in each of the regressions.

Table 6.5 presents four different alternative versions of the simultaneous equation model discussed in the 6.2.3, for the money-augmented model. The four alternatives refer to four different instrument sets, differing in the use of ratio or level forms.

Table 6.5
Sensitivity analysis Using Different
Sets of Instruments For the period 1950-1991

	α	γ	π	Durbin Watson Statistic	J- Statistic
Model A	13.21* (9.48)	0.001** (1.97)	0.16** (2.07)	2.25 ^I 2.58 ^M	10.07 (14.06)
Model AA	7.39 (1.15)	0.002 (0.79)	0.43 (1.2)	2.52 ^I 3.07 ^M	3.72 (5.991)
Model B	13.24* (15.81)	0.0004 (0.59)	-0.20 (-1.86)	2.26 ^I 2.81 ^M	7.74 (14.06)
Model BB	10.95* (4.78)	0.0002 (1.18)	0.09 (0.47)	2.41 ^I 1.95 ^M	6.63 (5.99)

* Significantly different from zero at the one percent significance level.

** Significantly different from zero at the five percent significance level.

I = Investment Equation, M = Money Equation.

Model A uses ten exogenous variables in levels that are lagged two periods as the instrument set. This is the final money-augmented system equation reported in Table 6.3 column three and discussed in section 6.2. They are reproduced here for comparison with the results when the instrument set is changed. The instruments are the constant, and the two period lags of the capital stock, investment, income, labour compensation, money supply, real interest rate, general price level, output price level and investment price level. These results were discussed earlier.

Model AA uses five ratios of exogenous variables as instruments. These are the constant, and second period lags

of the ratios of investment to capital, income to capital, labour compensation to capital and money supply to general price level (real balances). The results here are weaker than those from model A. In particular the coefficient of investment adjustment cost, α , is reduced almost by one half in size and at the same time becomes insignificantly different from zero.¹⁶¹ The transaction cost coefficient increases somewhat in size but loses its significance. Finally, the money adjustment cost coefficient, η , is almost two and a half times larger than in model A, but it also becomes insignificantly different from zero. For the equation as a whole the Durbin-Watson statistic now indicates that one must reject the null hypothesis of no autocorrelation at the five percent level. This suggests that the ratio of instruments may not be a good set of instrument for such a test. Finally, while there is a significant drop in the J-statistic compared to model A, that drop fundamentally is because fewer instruments are used in estimation.

Model B uses the same ten instruments as in model A but differs in that instruments are now lagged three periods. Here the investment cost coefficient, α , remains the same in size and has the same level of significance as in model A. On the other hand, the transaction cost coefficient, γ , is one half the size of that in model A and is insignificantly

¹⁶¹ However, still close to the α estimates of Gordon (1992).

different from zero. The major anomaly involves the money adjustment cost coefficient, η , which though insignificant at the five percent is now negative (when the predicted sign is positive). The Durbin-Watson statistic shows evidence of negative autocorrelation. The results indicate that three period lags in the instruments set may not be an appropriate instrument set for such a test. Finally, the J-statistic is smaller than the critical value of the χ^2 distribution. One cannot reject the null hypothesis of no specification error.

Model BB uses five instruments in ratio form as in model AA but lagged for three periods. Here the size of investment cost coefficient, α , is close to that in model A and B and also significantly different from zero at the one percent. The size of transaction cost coefficient, γ , is one quarter of that in model A and is insignificantly different from zero. The money adjustment cost coefficient drops down to 0.099 and is insignificantly different from zero. Here the Durbin-Watson statistic shows no evidence of serial correlation. Finally, the J-statistic equal to 6.63 is smaller than the critical value of the χ^2 distribution. This does not allow the rejection of the null hypothesis of no specification error at the five percent significance level.

Overall, the results of my instrument sensitivity tests show that the size of the investment cost coefficient, α , is

fairly stable across specifications.¹⁶² On the other hand, the transaction cost coefficient, γ , and the money adjustment cost coefficient, η , do vary with changes in the different instrument sets. While the general ranking of parameter size and significance remain the same across instrument sets, the particular results are quite sensitive to the instrument set chosen.

Table 6.6 presents the results of the sensitivity analysis performed on different definitions of some of the key variables used in the analysis and the different sample periods considered. The different variables included a new capital series, a money (rather than a real) interest rate in the investment equation and a different way of normalizing real stock of money. Finally, the final equation was run over the 1950-71 and 1950-81 sub-periods.

¹⁶² The results seem to be robust in the sense that the estimated adjustment cost coefficient does not vary significantly when different combinations of the instrument sets are used with different lags.

Table 6.6
Sensitivity Analysis with Different
Definitions of Variables and Different Sample Period.

	R^2	F	t	Durbin Watson Statistic	J-Statistic
1. Final Model	13.21 (9.48)	0.001** (1.97)	0.16** (2.07)	2.25 2.58	10.07 (14.06)
2. New X	11.13* (10.48)	0.0007** (2.32)	0.042 (0.96)	2.63 ^{IE} 2.07 ^{ME}	10.23 (14.06)
3. New X and New Y	11.96* (11.01)	0.0002** (2.19)	0.02 (1.09)	2.50 ^{IE} 2.10 ^{ME}	10.08 (14.06)
4. New X and New Y	11.13* (10.45)	0.002** (2.33)	0.024 (1.00)	2.63 ^{IE} 2.08 ^{ME}	10.23 (14.06)
5. 1950-71	14.03* (21.13)	0.002* (14.61)	0.07* (5.64)	1.95 ^{IE} 2.31 ^{ME}	7.79 (14.06)
6. 1950-51	6.674* (3.36)	0.002* (23.96)	0.02 (0.84)	2.44 ^{IE} 2.23 ^{ME}	9.72 (14.06)
7. Final Model with nominal rate	15.78* (8.93)	0.0007 (1.36)	0.10 (1.46)	2.09 ^{IE} 2.38 ^{ME}	10.43 (14.06)

* Significantly different from zero at the one percent significance level.

** Significantly different from zero at the five percent significance level.

IE: Investment Equation, ME: Money Equation.

Note: All of the seven models reported in this table use the same instruments set as used in Table 6.3 (3). The figures in the parenthesis under the J-statistic is the value of critical χ^2 distribution with seven degrees of freedom at the five percent level of significance.

Row (1) records my final result from the money-augmented simultaneous system (i.e., results summarized in

column three of Table 6.4). These are presented here for comparison purposes. The results were discussed earlier in section 6.2.

In row (2) a new capital series is generated using the investment figures reported in the GDP accounts. I used a constant depreciation rate of 0.048 and the initial capital stock value was taken from Statistics Canada publication *Fixed Capital Flows and Stock* (all industries).¹⁶³ The estimate of the investment cost coefficient, α , is very close to that in row (1) and also significantly different from zero at the one percent. Similarly, the transaction cost coefficient, γ , is still very close in size to that in row (1), and while smaller it is more precise, being also significantly different from zero at the five percent significance level. The money adjustment cost coefficient, η , is quite small and now insignificantly different from zero. The J-statistic does not reject the null hypothesis of no specification error.

In row (3) a new normalization of real money balances was generated by dividing the money supply by the general price level. Then the average value of that real balance series between 1948-92 was used to normalize the average

¹⁶³ The method used to generate the new series for the capital stock was taken from econometric package TSP 4.0. The capital stock was generated according to the following formula $K_t = (1-\delta)K_{t-1} + I_t$.

level of real balances to one.¹⁶⁴ These results are not very far off from the results of the model in row (1). The estimated investment adjustment cost parameter is almost the same size as that in row (1) and is also significantly positive at the one percent significance level. The estimated transaction cost parameter is one quarter the size of that in row (1) but still significantly different from zero at the five percent. The estimated money adjustment cost parameter is now insignificantly different from zero. The J-statistic does not allow to reject the null hypothesis of no specification error at the five percent.

In row (4) the newly generated series for both capital and real balances were used. The results are qualitatively and quantitatively similar to the models (2) and (3). Once again the investment and transaction cost coefficients are significantly different from zero and the money adjustment cost coefficient is not. One cannot reject the null hypothesis of no specification error.

In row (5) the sample size has been reduced to 1950-71, all other specifications remaining the same as in the original equation (results reported in row (1)). In this case the results for the size and significance of the investment adjustment cost parameter are roughly same as in

¹⁶⁴ Whereas, in the original estimations reported in Table 6.1 - Table 6.5 real balance series was created by normalizing money supply series to one and then dividing it by general price level which has a value of one in 1986.

model (1). The estimated transaction cost coefficient is twice the size of that in model (1) but now significantly different from zero at the one percent. Although the estimated coefficient for the money adjustment cost is smaller than that in model (1) it is now significantly different from zero at the one percent. Again the J-statistic does not allow rejection of the null hypothesis of no specification error.

In row (6) the sample period becomes 1950-81, and all other features remain the same as in row (1). The results were a little different in this estimation. The size of investment adjustment cost parameter is reduced by one half from 13.21 to 6.67, although still significantly different from zero at the one percent significance level. As was true of the earlier period, the estimated transaction cost parameter is twice the size of that in model (1) and significantly different from zero at the one percent significance level. Now, however, the estimated money adjustment cost parameter is insignificantly different from zero. Still the J-statistic does not allow to reject the null hypothesis of no specification error.

For completeness I present in row (7) a model where all the variables and instruments are same as in model (1) except for the fact that the nominal interest rate is used

in the investment equation.¹⁶⁵ The effect of using nominal interest is that the investment adjustment cost coefficient is slightly larger and transaction and money adjustment cost coefficients are slightly smaller. Now, the estimated transaction and money adjustment cost parameters are insignificantly different from zero. However, even in this case one cannot reject the null hypothesis of no specification error.

The general conclusion that arises from the sensitivity tests is that the estimated coefficients are reasonably robust to different measures of capital, interest rates and time period. The estimated coefficient for investment adjustment cost is almost always robust in size and significance.¹⁶⁶ The estimated coefficient for the transaction cost parameter is mostly in the range of 0.001 to 0.002 and is either significant at the one or the five percent in various models.¹⁶⁷ The size of estimated coefficient for money adjustment cost most of the time is between 0.16 to 0.04 and is significant in our model (1) and in model (5). Finally, the J-statistic findings never led to the rejection of the null hypothesis of no specification error.

¹⁶⁵ This also means that lagged nominal interest rate is used as an instrument.

¹⁶⁶ Except for the exceptional case of 1950-71 sample period.

¹⁶⁷ Except for model (2).

6.6 Conclusion:

In this chapter the model developed earlier is empirically estimated using GMM under two different approaches. In the first approach, the separate investment and money equations were estimated using two stage least squares. In the second approach the two equations were estimated as a system using three stage least squares.

First, consider the estimation results based on the single equation approach. Here the traditional investment equation performed well in the sense that estimated coefficients were statistically significant, the equation's J-statistic was consistent with no specification error. When the money-augmented investment equation was estimated, the investment cost coefficient had the same size and significance as in the traditional model. However, the transaction cost coefficient was found to be statistically insignificant.

When a single money equation included just a transaction and money adjustment cost term, both of these coefficients were found to be positive and significantly different from zero. In addition the model was consistent with the hypothesis of no specification error. The introduction of an investment adjustment cost, however, did not improve the results and produced coefficients that were statistically insignificant. This suggested that the money

equation should be specified to include only transaction and money adjustment costs.

One of the hypotheses proposed in the thesis is that capital and money stocks interact on the investment decisions of the firm. The implication then is that investment decisions are influenced by the decision to hold money and money decisions are influenced by investment activity. This proposition called for system estimation. It was expected that system estimation would improve the fit with the data and result in significant estimates for each of the cost function coefficients.

When system estimation was performed, the transaction cost coefficient in the traditional system turned out to be statistically insignificant whereas in the money-augmented system all of the cost coefficients were statistically significant. The significance of transaction cost coefficient in the money-augmented system emphasized the importance of the interrelation of the investment and money decisions of the firm. However, the J-tests did not discriminate between traditional and money-augmented models. That is, the J-test would not reject the null hypothesis of no specification error in either case.

An advantage of the money augmented system is that it can demonstrate the channels through which money can affect the real variables in the system. The money-augmented model is also useful for performing some policy simulations. The

money-augmented model is neutral in money but not superneutral. Thus the transaction setting provides a channel where both the level and the variation in inflation rate interact negatively on the rate of economic growth through their effect on the firms incentive to invest.¹⁶⁸ The model predicts that a monetary policy of price stability can achieve output and employment expansions.

The last section in this chapter performs a sensitivity analysis on the main system estimation. The results were robust over some specifications and time periods considered but sensitive to choice of other instruments and time period.

¹⁶⁸ Barro (1995) has shown an inverse relation between inflation and economic growth.

Chapter VII: Conclusions

7.1 Summary and Conclusion:

This thesis builds and tests empirically a forward looking profit maximizing model of the firm that allows the interplay of two stocks (money and capital) in optimal decision making. To model this dynamic interaction, the thesis defines three different ways that money may interact with the different transactions activities of the firm. First, there is the direct role of money in the firm's transaction technology, where holding money can reduce Baumol-type transaction costs. Second, real money balances may affect investment decisions through its effects on transaction costs and the optimal scale of the firm's activities. Third, money may have its own adjustment cost, so that holding larger money stocks saves the firm the cost of encountering sudden disruptions in its money flows.

An important contribution of this thesis is the attention given to the long run convergence and stability conditions of the model. First, the optimal levels of capital and money stocks under stationary state conditions are derived. Here the optimal money stock exhibits the familiar square root formula. Second, general convergence and stability conditions of the firm about the stationary state are established. I used the particular coefficients from my later empirical work to show that the model is

saddle path convergent about the stationary state and to graph its particular form.

The Euler equations derived in Chapter II were then estimated on aggregate Canadian annual data (1950-1991). Tests were performed of the hypothesis that the inclusion of real balances in the cost specification of the firm provides a more appropriate specification of the firm's adjustment path than the traditional specification without real balances.

The model was estimated first in terms of single equations under two stage least squares and second as a simultaneous system under three stage least squares.

The single equation results indicate that adding a liquidity role to the transaction role of money does not increase the explanatory power of the equation and provides greater evidence of model misspecification. These findings were used to establish the form of the augmented money system that was to be compared to the more traditional alternative. This augmented system includes only transaction and money adjustment costs, allowing money to interact with the firm's investment decision only through the influence of these costs on the optimal scale of the firm.

Estimation of the traditional system in isolation found that the system results were not quantitatively different from their single equation counterparts. As was true of the

single equation estimates of the traditional system, the transaction cost coefficient was not significantly different from zero. The estimates of the money-augmented system, however, were more consistent with the proposed theory. When estimated as a part of a system, all of the estimated coefficients were found to have the predicted sign and to be significantly different from zero. In particular, the sign and significance of the transaction cost coefficient in the money-augmented system is consistent with money playing the hypothesized role in the transaction technology setting. The coefficient estimates also yield comparative statics effects that seem to be reasonable in terms of the predicted size of those effects. This suggests the economic (as well as statistical) importance of including real money holdings in this modelling exercise.

The J-test did not discriminate between the traditional and the money-augmented system. Nonetheless, the money-augmented system has certain advantages over the traditional system. First, it shows the channels through which money may affect the real decisions. Second, the model can be used for some policy simulations that cannot be performed using the traditional model. Some simulations of monetary policy were performed using the model's comparative statics about the stationary state.

Finally, my sensitivity analysis show that the results are somewhat sensitive to the choice of instruments. The

investment adjustment cost coefficient is fairly robust across specifications. However, both the transaction cost and the money adjustment cost coefficients are fairly sensitive to the choice of instruments and lag structure.

7.2 Future Research Prospects:

There are some questions raised by this thesis that constitute an agenda for future research. One direction for future work is suggested by the inconclusiveness of the tests using aggregate data and this suggests that one might profitably explore sectoral data in more detail.

Unfortunately, some preliminary experimentation that I did with sectoral data did not perform well. One reason may be that the time period for which the sectoral data was readily available was too short. Another reason may be that the M1 definition of the money used in this study includes money holdings by both households and firms. Deposits held primarily by households might be excluded to further refine the definition of money. Finally, the use of firm-level panel data might be explored.

Another potentially fruitful direction for further research would be to explore a broader definition of money (such as M2 and M2+) that may better capture the effect of liquid assets on investment behaviour. Similarly, the thesis has developed a quasi-dynamic model where only money and capital have adjustment costs. Since many economists believe that labour is a quasi-fixed factor, it would be interesting to see that how the model would change when non-capital factors of production are also given an adjustment cost. Given that significant structural change has arisen

in the technology of transferring funds and other forms of cash management, some accounting for this type of structural change might also be attempted.

In my thesis I used the longest available time period for which consistent data was available. This involved annual data. My findings, however, suggest that the adjustment costs are significant determinants of firm behaviour and this, in turn, suggest that observations over shorter periods may be more appropriate as a test of these hypotheses. In addition, GMM estimation requires a large number of observations (rather than long time period), because the J-statistic converges to a χ^2 distribution only asymptotically. Finally, the sensitivity analysis also suggests that money adjustment cost changes in significance for different time periods. Considering these facts, a more frequent data set, say a quarterly data set, might estimate more precisely the dynamic adjustment process.

Finally, it would be an interesting exercise to see how this model works in a developing country. Widely different trading practices suggest that the role of money in transactions would be quite different. It is not clear whether one would expect more or less conformity to the predictions of this thesis in less developed countries as compared to Canada.

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Appendix A.1

Short Run Comparative Static Analysis of the Money-Augmented Model

Before performing the short run Comparative Static analysis it is important to identify the specific problem being described. In this section I hold the stocks of capital and money constant at their stationary state values (K^* , m^*) and allow investment (I_t), flow changes in money (u_t) and the non-capital factors of production (N_t) to change about the stationary state when one of the model's parameters change. This formally solves for the new point on the saddle path in the neighbourhood of the old stationary state. Most generally, I am looking at the short run problem of how a change in a model parameter will affect the endogenous flow variables I_t (investment), u_t (flow changes in money) and N_t (the non-capital factors of production) given the stocks of capital and money and the estimated parameters of the system. How the endogenous stocks adjust to the model's parameters is implicit in the long run stationary state equilibrium and the convergence conditions discussed at length in Chapter III.

The short run system is characterized by the model's first order conditions for optimization: the Euler equations for investment and money together with the first order condition for the non-capital factors of production. These three equations are:

$$\begin{aligned}
& \left[P_t F_K - P_t^I + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \right] \\
& + \alpha \left[\frac{P_t^I}{2} \left(\frac{I_t}{K_t} \right)^2 - P_t^I \left(\frac{I_t}{K_t} \right) + \left(\frac{1-\delta}{1+r_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right] \\
& - \gamma \left[\frac{F_K P_t}{2m_t} \right] = 0,
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
& \frac{\gamma}{2} \left(\frac{(P_t/\vartheta_t) F(K_t, N_t)}{m_t^2} \right) \\
& + \eta \left[\frac{1}{2} \left(\frac{u_t/\vartheta_t}{m_t} \right)^2 - \left(\frac{u_t/\vartheta_t}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1}/\vartheta_{t+1}}{m_{t+1}} \right) \right] \\
& - \frac{r_t}{1+r_t} = 0,
\end{aligned} \tag{A.2}$$

and,

$$F_N(K_t, N_t) - \frac{W_t}{P_t} \left(\frac{2m_t}{2m_t - \gamma} \right) = 0. \tag{A.3}$$

Where (A.1) is the first order condition of investment equation and (A.2) and (A.3) are first order conditions of money and non-capital factors respectively. Given these first order conditions, I allow all the variables to change in order to derive the model's constraints on the variation of the system about the stationary state. Taking the full differential of the optimal investment equation, equation (A.1) becomes:

$$\begin{aligned}
& P_t F_K \left(1 - \frac{\gamma}{2m_t}\right) dN_t + \alpha \left(\frac{P_t^I I_t}{K_t^2} - \frac{P_t^I}{K_t} \right) dI_t \\
& - P_{t+1}^I \left(1 + \frac{\alpha I_{t+1}}{K_{t+1}}\right) \frac{1-\delta}{(1+r_t)^2} dr_t - \left(1 + \frac{\alpha I_t}{K_t} - \frac{\alpha}{2} \left(\frac{I_t}{K_t}\right)^2\right) dP_t^I \quad (A.4) \\
& + \left[\frac{P_t^I}{2} \left(\frac{I_t}{K_t}\right)^2 - P_t^I \left(\frac{I_t}{K_t}\right) + \left(\frac{1-\delta}{1+r_t}\right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}}\right) \right] d\alpha - \left[\frac{F_K P_t}{2m_t} \right] d\gamma = 0.
\end{aligned}$$

Collecting the common terms, equation (A.4) becomes:

$$\begin{aligned}
& P_t F_K \left(1 - \frac{\gamma}{2m_t}\right) dN_t + \frac{\alpha P_t^I}{K_t} \left(\frac{I_t}{K_t} - 1\right) dI_t - P_{t+1}^I \left(1 + \frac{\alpha I_{t+1}}{K_{t+1}}\right) \frac{1-\delta}{(1+r_t)^2} dr_t \\
& - \left(1 + \frac{\alpha I_t}{K_t} - \frac{\alpha}{2} \left(\frac{I_t}{K_t}\right)^2\right) dP_t^I + \left[\frac{P_t^I}{2} \left(\frac{I_t}{K_t}\right)^2 - P_t^I \left(\frac{I_t}{K_t}\right) + \left(\frac{1-\delta}{1+r_t}\right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}}\right) \right] d\alpha \quad (A.4.a) \\
& - \left[\frac{F_K P_t}{2m_t} \right] d\gamma = 0.
\end{aligned}$$

Next, the total differential of the optimal money equation is:

$$\begin{aligned}
& \left(\frac{\gamma}{2} \frac{P_t / \varphi_t F_N(K_t, N_t)}{m_t^2} \right) dN_t + \eta \left(\frac{u_t / \varphi_t^2}{m_t^2} - \frac{1 / \varphi_t}{m_t} \right) du_t \\
& - \left(\frac{\eta u_{t+1} / \varphi_{t+1}}{m_{t+1}} \frac{1}{(1+r_t)^2} - \frac{1}{(1+r_t)^2} \right) dr_t \quad (A.5) \\
& + \left[\frac{1}{2} \left(\frac{u_t / \varphi_t}{m_t} \right)^2 - \left(\frac{u_t / \varphi_t}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1} / \varphi_{t+1}}{m_{t+1}} \right) \right] d\eta \\
& + \frac{1}{2} \left(\frac{(P_t / \varphi_t) F(K_t, N_t)}{m_t^2} \right) d\gamma = 0.
\end{aligned}$$

Collecting the common terms, equation (A.5) becomes:

$$\begin{aligned}
& \left(\frac{\gamma}{2} \frac{P_t / \varphi_t F_N(K_t, N_t)}{m_t^2} \right) dN_t + \frac{\eta / \varphi_t}{m_t} \left(\frac{u_t / \varphi_t}{m_t} - 1 \right) du_t \\
& - \frac{1}{(1+r_t)^2} \left(1 + \frac{\eta u_{t+1} / \varphi_{t+1}}{m_{t+1}} \right) dr_t + \frac{1}{2} \left(\frac{P_t / \varphi_t}{m_t^2} F(K_t, N_t) \right) d\gamma \quad (\text{A.5.a}) \\
& + \left[\frac{1}{2} \left(\frac{u_t / \varphi_t}{m_t} \right)^2 - \left(\frac{u_t / \varphi_t}{m_t} \right) + \frac{1}{1+r_t} \left(\frac{u_{t+1} / \varphi_{t+1}}{m_{t+1}} \right) \right] d\eta = 0 .
\end{aligned}$$

Finally, the total differential of the first order condition for the non-capital factors is:

$$\begin{aligned}
& F_{NN} dN_t - \frac{1}{P_t} \frac{2\gamma}{(2m_t - \gamma)^2} dW_t \\
& + \frac{W_t}{P_t^2} \frac{2\gamma}{(2m_t - \gamma)} dP_t + \frac{W_t}{P_t} \frac{2m_t}{(2m_t - \gamma)^2} d\gamma = 0 . \quad (\text{A.6})
\end{aligned}$$

Given that the stocks of capital and money are held constant at their stationary state values, the short run question becomes how the corresponding flows (i.e., investment and flow changes in money holdings) adjust when one of the exogenous variables is perturbed. This problem can be represented in matrix form, where investment, the flow change in the money and non-capital factor flows become endogenous left hand side variables and all the prices (including interest rates) and estimated parameters (α , γ , η) become the exogenous right hand side variables. The

general problem can then be written as:¹⁶⁸

$$\begin{bmatrix} I_I & I_u & I_N \\ u_I & u_u & u_N \\ N_I & N_u & N_N \end{bmatrix} \begin{bmatrix} dI \\ du \\ dN \end{bmatrix} = \begin{bmatrix} I_P^I & I_\alpha & I_\gamma & I_\eta & I_P & I_r & I_W \\ u_P^I & u_\alpha & u_\gamma & u_\eta & u_P & u_r & u_W \\ N_P^I & N_\alpha & N_\gamma & N_\eta & N_P & N_r & N_W \end{bmatrix} \begin{bmatrix} dP_t^I \\ d\alpha \\ d\gamma \\ d\eta \\ dP_t \\ dr_t \\ dW_t \end{bmatrix}, \quad (\text{A.7})$$

or even more succinctly as:

$$A_1 \begin{bmatrix} dI \\ du \\ dN \end{bmatrix} = B_1 \begin{bmatrix} dK \\ dm \\ dP_t^I \\ d\alpha \\ d\gamma \\ d\eta \\ dP_t \\ dr_t \\ dW_t \end{bmatrix}. \quad (\text{A.8})$$

When Matrix A_1 is expanded it takes the following form:

¹⁶⁸ I have simplified presentation by utilizing the symbols I_i to present the derivative of the investment equation (A.4.a) with respect to the variable x where $x = I, u, N, P^I, \alpha, \gamma, \eta, P, r$, and W . Similarly I have used u_i and N_i to represent the corresponding derivatives of equations (A.5.a) and (A.6) with respect to variable x .

$$A_1 = \begin{bmatrix} \frac{\alpha P_t^I}{K_t} \left(\frac{I_t}{K_t} - 1 \right) & 0 & P_t F_{KN} \left(1 - \frac{\gamma}{2m_t} \right) \\ 0 & \frac{\eta/\vartheta_t}{m_t} \left(\frac{u_t/\vartheta_t}{m_t} - 1 \right) & \frac{\gamma}{2} \left(\frac{P_t}{\vartheta_t} \frac{F_N(K_t, N_t)}{m_t^2} \right) \\ 0 & 0 & F_{NN} \end{bmatrix}, \quad (\text{A.9})$$

so that the determinant of matrix A_1 can be solved for as:¹⁶⁹

$$|A_1| = F_{NN} \left[\frac{\alpha P_t^I}{K_t} \left(\frac{I_t}{K_t} - 1 \right) \frac{\eta/\vartheta_t}{m_t} \left(\frac{u_t/\vartheta_t}{m_t} - 1 \right) \right] < 0. \quad (\text{A.10})$$

Using the parameter estimates from Table 6.3 (3) and the average values of the variables between 1950 and 1991, the values of each of the terms in (A.9) and (A.10) can be calculated. Substituting these values into the matrix and its determinant, matrix (A.9) and equation (A.10) take the following specific form:

¹⁶⁹ The determinant can be signed negative because three of the five terms in (A.10) are negative; that is F_{NN} , $[(I_t/K_t) - 1]$ and $[(u_t/\vartheta_t)/m - 1]$ are all negative.

$$A_1 = \begin{bmatrix} -0.0095 & 0 & 0.000091 \\ 0 & -0.1771 & 0.00005 \\ 0 & 0 & -0.00002 \end{bmatrix}$$

$$|A_1| = -0.0000000337.$$

Turning next to determine the values of the coefficients of matrix of exogenous variables, the coefficients in the first row of matrix B_1 are:

$$\begin{aligned} I_P^I &= \frac{\alpha I_t}{K_t} \left(1 - \frac{1}{2} \left(\frac{I_t}{K_t} \right) + 1 \right) \\ I_a &= \frac{P_t^I I_t}{K_t} \left(1 - \frac{1}{2} \frac{I_t}{K_t} - \left(\frac{1-\delta}{1+I_t} \right) P_{t+1}^I \left(\frac{I_{t+1}}{K_{t+1}} \right) \right) \\ I_\gamma &= \frac{P_t}{2m_t} \left(\frac{F-C_t}{K_t} \right), \quad I_P = \left(1 - \frac{\gamma}{2m_t} \right) \left(\frac{F-C_t}{K_t} \right) d^P_t \\ I_\eta &= 0, \quad I_W = 0, \quad I_r = P_{t+1}^I \left(1 + \frac{\alpha I_{t+1}}{K_{t+1}} \right) \frac{1-\delta}{(1+I_t)^2} \end{aligned} \tag{A.11}$$

The terms in the second and third rows of matrix B_1 are:

$$\begin{aligned}
u_w &= 0, \quad u_p^I = 0, \quad u_a = 0, \quad u_\gamma = -\frac{1}{2} \frac{(P_t/\phi_t) F(K_t, N_t)}{m_t^2} \\
u_\eta &= \frac{u_t/\phi_t}{m_t} \left(1 - \frac{1}{2} \frac{u_t/\phi_t}{m_t} \right) - \frac{1}{1+r_t} \frac{u_{t+1}/\phi_{t+1}}{m_{t+1}} \\
u_p &= \frac{\gamma}{2} \frac{(1/\phi_t) F(K_t, N_t)}{m_t^2}, \quad u_r = \frac{1}{(1+r_t)^2} \left(1 + \frac{\eta u_{t+1}/\phi_{t+1}}{m_{t+1}} \right)
\end{aligned} \tag{A.12}$$

and,

$$\begin{aligned}
N_p^I &= 0, \quad N_a = 0, \quad N_\eta = 0, \quad N_\gamma = -\frac{W_t}{P_t} \frac{2m_t}{(2m_t - \gamma)^2} \\
N_p &= \frac{W_t}{P_t^2} \frac{2m_t}{(2m_t - \gamma)}, \quad N_r = 0, \quad N_w = -\frac{1}{P_t} \frac{2\gamma}{(2m_t - \gamma)^2}
\end{aligned} \tag{A.13}$$

Substituting in the estimated and sample values for some of the coefficients, exogenous prices and the rate of interest, the right hand matrix of (A.7) becomes:

$$A_1 \begin{bmatrix} dI \\ du \\ dN \end{bmatrix} = \begin{bmatrix} 1.8 & 0.0003 & 0.00003 & 0 & I_p & 0.59 & I_w \\ 0 & 0 & 48.55 & -0.0008 & u_p & 1 & u_w \\ 0 & 0 & 92.92 & 0 & N_p & 0 & N_w \end{bmatrix} \begin{bmatrix} dP_t^I \\ da \\ d\gamma \\ d\eta \\ dP_t \\ dr_t \\ dW_t \end{bmatrix}.$$

Using Cramer's rule to solve for the comparative static effects of a change in the exogenous variables, the sign and the specific values of any parameter change can now be calculated. Once the comparative static results are calculated they are used to calculate the unit free

measurement of elasticity for the specific flow variables and the parameters. The results of elasticity calculation are reported in appendix A.2.

Appendix A.2
Short Run Elasticity Calculations

Table A.1

	Investment	Flow changes in Money	Non-Capital Factors
Adjustment Cost	$\frac{\partial I}{\partial \alpha} \frac{\alpha}{I} = -0.09$	$\frac{\partial u}{\partial \alpha} \frac{\alpha}{u} = 0$	$\frac{\partial N}{\partial \alpha} \frac{\alpha}{N} = 0$
Money Adjustment cost	$\frac{\partial I}{\partial \eta} \frac{\eta}{I} = 0$	$\frac{\partial u}{\partial \eta} \frac{\eta}{u} = -0.01$	$\frac{\partial N}{\partial \eta} \frac{\eta}{N} = 0$
Interest rate	$\frac{\partial I}{\partial r} \frac{r}{I} = -0.08$	$\frac{\partial u}{\partial r} \frac{r}{u} = -0.56$	$\frac{\partial N}{\partial r} \frac{r}{N} = 0$
Investment Price	$\frac{\partial I}{\partial P^I} \frac{P^I}{I} = -5.31$	$\frac{\partial u}{\partial P^I} \frac{P^I}{u} = 0$	$\frac{\partial N}{\partial P^I} \frac{P^I}{N} = 0$
Transaction cost	$\frac{\partial I}{\partial \gamma} \frac{\gamma}{I} = -.94$	$\frac{\partial u}{\partial \gamma} \frac{\gamma}{u} = 2.60$	$\frac{\partial N}{\partial \gamma} \frac{\gamma}{N} = 0$

END

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FIN