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**BOUNDARY ELEMENT ANALYSIS
OF HYDRODYNAMIC PRESSURE
ON ARCH DAMS**

BY

© MOHAMED AIMAN ELKHODARY

**A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of**

MASTER OF ENGINEERING

**DEPARTMENT OF CIVIL ENGINEERING
CARLETON UNIVERSITY
OTTAWA , CANADA
SEPTEMBER , 1984.**

The undersigned recommend to the Faculty of Graduate Studies acceptance of the thesis

**BOUNDARY ELEMENT ANALYSIS
OF HYDRODYNAMIC PRESSURE
ON ARCH DAMS**

Submitted by M.A. ELKHODARY, in partial fulfillment of the requirements for the degree
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ABSTRACT

The analytical problem of the hydrodynamic pressures arising on an arch dam subjected to ground motion is reviewed. The possibility of modelling the water reservoir by boundary elements is discussed. A formulation of the boundary element method of analysis for the hydrodynamic pressures arising from upstream-downstream and cross-stream ground motion is presented.

The method is applied to the problem of hydrodynamic pressure distribution on a rigid arch dam at one end of a finite water reservoir when the system is subjected to a horizontal ground motion. The effect of the boundary condition at the far end is studied. The straight gravity dam is also studied as a limiting case and the results are compared to the available classical 2-D solutions.

The method is then applied to the problem of hydrodynamic pressure generated by a ground motion on the face of a rigid arch dam at the downstream end of an infinite water reservoir. The results obtained by the boundary element method are compared with the known classical solutions.

The advantages and disadvantages of the method as applied to the above mentioned problems are discussed.

It is concluded that the boundary element method can be an effective tool in the analysis of the hydrodynamic pressures on an arch dam but that the case of infinite reservoir is not yet completely solved.

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Chapter 1

INTRODUCTION

1.1 General

With the increase in the demand for electricity, development of hydropower resources is being rigorously pursued in many countries, and more and more high dams with large capacity of water storage are being constructed. The dams are of several types: gravity dams, arch dams, buttress dams and others. The choice of a particular type is decided by the topography and geology of the dam site. An arch dam is generally suitable for a site where the river is narrow and the foundation rock strata are solid. When these two conditions are satisfied, the arch dam is very economical since it requires less amount of concrete than any other type of dam. This is because an arch dam resists external forces through the compressive strength of concrete, resulting in a most effective utilization of the material.

In terms of consequences of failure, dams are quite different from most other structures. If a normal structure should be damaged by some unexpected causes, the damage is normally confined to the structure itself. On the other hand, if a large dam were to break, the stored water would dash along in rapids, sweeping away the inhabitants, houses, fields, roads, bridges and all other things downstream, and the disaster would be immeasurable. Accordingly, a dam should be absolutely safe against all external forces that it is likely to experience.

It is therefore essential that arch dams built in zones of seismic activity are carefully designed for earthquake forces.

The dynamic behaviour of a concrete arch dam is complicated because of its three dimensional geometry and the effect of impounded water. Improved understanding of the effect of the

impounded water on the dynamics of concrete arch dams is essential in developing reliable procedures for computing the dynamic deformations and stresses in a dam subjected to earthquake ground motion.

The analysis of many engineering problems including that of the dynamic response of dams can be reduced to the solution of a set of differential equations. It is possible to solve these equations exactly in only a limited number of cases and that too under very restricted conditions. Therefore, in the solution of many engineering problems, approximate numerical solutions play an important role. Several numerical methods have been developed to obtain such approximate solutions.

The finite element method and the finite difference method have been applied successfully and extensively to many engineering problems. In the finite element method, a continuum with infinite degrees of freedom is approximated by an assemblage of elements so that the problem reduces to one having a finite number of unknowns. Each component element interconnects with others at common nodal points. A detailed discussion of the finite element method will not be presented here because the method has received extensive treatment in the literature [1,2].¹

The basic concept of the finite difference method of discretisation is to replace the continuous variables of the governing differential equation by discrete variables. The relationship between these discrete variables is expressed in the form of finite difference equations. The finite difference method has been extensively used in the solution of engineering problems and is well described in the literature [3].

The finite element and finite difference methods are called 'Domain methods' since they deal with the domain under consideration. In a certain class of problems, the domain methods are either inefficient or fail to give an accurate solution. For example, until very recently problems involving infinite domains could not be dealt with efficiently by using the domain methods.

The finite element method requires special care in data preparation and errors are frequent. Domains with complicated geometry pose special problems when the finite difference method is used. These problems have resulted in a renewal of interest in other numerical methods, including

---

¹ Numbers in square brackets refer to references given at the end of this study.

the boundary solution techniques such as the 'Boundary Element Method'.

1.2 Boundary Element Method

The term 'Boundary Element Method' is used to indicate a method wherein the external surface of the domain is divided into a series of elements, called boundary elements, over which the functions under consideration can vary in different ways in much the same manner as the domain is divided into elements in the finite element method. If the values of the unknown variables can be determined on the boundary, it is possible to obtain the values of the variables at any point of interest within the domain. The points on the boundary where the unknown values of the variable or variables are sought are called 'nodes'. For the constant element, a node is taken to be at the center of each segment. One can also choose linear elements (i.e. elements for which the nodes are at the corners of elements) and curved elements such as the quadratic elements (Fig. 1.1). The boundary elements need not be of equal sizes. They can be chosen in any convenient and suitable way.

In certain class of problems, the boundary element method offers important advantages over the 'domain' type methods, such as the finite element and the finite difference methods. One of the important features of the boundary element method is the much smaller resulting system of equations and the considerable reduction in the data input required in the solution of a problem.

Also, the boundary element method is well suited to problems with infinite domain such as those occurring in soil mechanics, hydraulics, and hydrodynamics, for which the classical 'domain' methods are unsuitable. Another important feature of the boundary element method is that the values of the functions under consideration are calculated for internal points only where they are needed.

The boundary element method may be coupled with other methods such as the finite element method to deal with situations where neither the boundary element method nor the other methods are appropriate by themselves. This implies that the weaknesses of one approach may be offset by the strengths of the other to form a combination more powerful than either approach taken

CONSTANT BOUNDARY
ELEMENTS

LINEAR BOUNDARY
ELEMENTS

QUADRATIC BOUNDARY
ELEMENTS

FIG.1.1 DIFFERENT TYPES OF ELEMENTS

individually. This is indeed the case for a class of problems which involves large or infinite domains while simultaneously containing a subregion with non-homogeneous material properties.

The boundary element method has some drawbacks which are worthy of further research effort. Following are some of them:

- 1) Boundary element matrices are usually fully populated which means that the method may be less efficient computationally than other methods in certain applications.
- 2) The boundary element method is not well suited to problems involving regions with inhomogeneous material properties.

1.3 Review of the literature

A brief review of the literature related to the problems discussed in the present study is presented in this section.

Early results of Westergaard's and Zanger's analyses of earthquake-induced hydrodynamic pressures on rigid, straight dams with vertical upstream face [4] and sloping upstream face [5] provided a basis for added mass representation of hydrodynamic effects in analysis of dams. Because corresponding results for arch dams have not been available in appropriate form, the results for straight gravity dams have been extrapolated and adapted in analysis of arch dams [6].

Under the assumption that the arch dam is a segment of a rigid cylinder ; thus having a constant radius ; bounded by vertical, radial banks of the river valley enclosing a central angle of 90 degrees, the wave equation governing the hydrodynamic pressures was solved mathematically for prescribed harmonic motions of the boundaries [7]. This result along with Fourier transform procedures permitted evaluation of hydrodynamic pressures due to arbitrary earthquake motions [8]. Although the dam was assumed to be rigid and only one direction of ground motion was considered in these studies, it was shown that water compressibility as well as curvature of the arch dam in plan has significant influence on hydrodynamic pressures.

Electrical analogue [9] and finite difference [10,11] procedures have also been employed to model arbitrary, three-dimensional geometry of the reservoir and dam. Neglecting water compressibility,

hydrodynamic pressures due to prescribed accelerations of rigid dams [10] or flexible dams [11] were determined by these procedures.

A finite element or finite difference analysis of the complete dam - water system can account for interaction effects between the reservoir and the dam [12,13,14]. It should however be noted that all these methods require a large computational effort for problems of practical size.

Problems involving time dependent variables have been solved by the boundary element method either by numerically integrating over time, Chang, Kang and Chen [15] or by removing the time variation by the use of Laplace transform, Cruse and Rizzo [16], Rizzo and Shippy [17,18] and Shippy [19].

James R. Hutchinson [20] has applied the boundary element method to find the eigenvalues and eigenvectors of stretched membranes. In his work, the time dependence is removed by the assumption of a sinusoidal motion. The process of finding the eigenvalues and eigenvectors of the considered membranes was confused because of the false frequencies which correspond to solutions for a membrane external to the boundary of the membrane under consideration.

The boundary element method has been applied to the problem of hydrodynamic pressure distribution on gravity dams subject to ground motion [21] by Hanna. To the knowledge of the author, the method has not been applied to the problem of hydrodynamic pressure distribution on arch dams subject to ground motion.

1.4 Scope of Present Study

The main objectives of the present study are as follows:

- i) To review the mathematical basis and formulation of the boundary element method.
- ii) To study the feasibility of applying the boundary element method to the problem of hydrodynamic pressure distribution on rigid arch dams storing finite water reservoirs when they are subjected to horizontal ground motions.

- iii) To investigate the possibility of applying the boundary element method to the problem of hydrodynamic pressure distribution on rigid arch dams storing infinite water reservoirs.
- iv) To present the advantages and disadvantages of the boundary element method in its application to the field of structural dynamics.

1.5 Format of Thesis

The thesis has been divided into five chapters and four appendices.

Chapter 1 introduces the importance of arch dams and of the study of their dynamic behaviour. This chapter also introduces the boundary element method as a numerical technique and briefly reviews its advantages over the domain type methods such as the finite element and the finite difference methods. In addition, this chapter surveys the literature associated with the problems studied in this thesis and presents the scope of this study and the format of the thesis.

In Chapter 2, the system and the ground motion are described and the equations of motion are developed showing that for steady state, the Helmholtz equation can be used to represent the motion. Details of the classical procedures for the analysis of an arch dam subjected to harmonic upstream-downstream ground motion as well to harmonic cross-stream ground motion are given stating the boundary conditions and the classical solutions. This chapter presents the mathematical basis of the boundary element method and identifies the method as an application of the Green's theorem. Since the governing differential equations of all the problems discussed in the present work can be reduced to the Helmholtz equation, the application of the boundary element method to the solution of this equation is presented. The choice of an appropriate fundamental solution is discussed. Solution techniques are described for the constant and infinite boundary elements.

In chapter 3, the application of the boundary element method to the solution of the vibration problem of the finite reservoir in both directions of ground motion is presented. Boundary conditions are given. Results of investigation for different values of excitation velocity are shown. Different sizes of elements are used to compare results. A comparison with the two dimensional case is given as a check for the calculation procedures.

In Chapter 4, the application of the boundary element method to the solution of the vibration problem of the infinite reservoir in both directions of motion is presented. Different approaches that may be taken in representing the boundary at infinity are discussed. The results are compared with known classical solutions.

A summary of the work done in the present study and the conclusions arrived at are contained in Chapter 5.

Finally, the appendices present the list of symbols, the user's guide to the computer programs, listings of the programs and tabulated results of the solved cases.

Chapter 2

FORMULATION OF THE ARCH DAM PROBLEM

2.1 Introduction

The arch dam-reservoir system and the exciting ground motion are described in this chapter. The equations of motion governing the reservoir vibration are also developed. It is shown that these equations reduce to a Helmholtz equation when the time variable is eliminated. Details of the classical procedures for the analysis of an arch dam subjected to harmonic upstream-downstream ground motion as well to harmonic cross-stream ground motion are given and the applicable boundary conditions are outlined. The classical solutions derived by C.S. Porter and A.K. Chopra [22] are presented here in brief for the sake of completeness.

The mathematical formulation of the boundary element method as applied to a steady state vibration problem is developed in this chapter. In many problems of structural dynamics, the time variable can be eliminated from the governing differential equation provided that the exciting function is assumed to be harmonic so that the resulting response is also harmonic. When the time variable is eliminated this way, the governing wave equation of reservoir vibration reduces to the Helmholtz equation. The formulation of the 'Boundary Element Method' presented here takes the Helmholtz equation as the starting point. This equation is first multiplied by an arbitrary function of the coordinates and then converted to an integral equation by integrating over the domain. It is shown by using the Green's theorem and by an appropriate choice of the arbitrary function, that the integration over the domain can be converted to an integral over the boundary. The appropriately selected multiplying function must itself satisfy a certain differential equation. The solution of this equation is called the fundamental solution and plays an important role in the development of the method.

FIG.21 ARCH DAM-RESERVOIR SYSTEM

This chapter then describes how the resulting integral equations are discretized so that the problem reduces to the solution of a set of algebraic simultaneous equations.

2.2 System and Ground Motion

The arch dam - water system investigated is shown in Fig. 2.1. The upstream face of the arch dam is a segment of a circular cylinder, radius R , and height H_d , contained within radially extending banks enclosing a central angle of 90 degrees. In addition, the geometry and properties of the system are assumed to be symmetrical about the $x - z$ ($\theta = 0$) plane.

The reservoir, which has a horizontal bottom, is filled to a height H with water extending to infinity in the upstream (x) direction.

The earthquake ground motion is defined by the upstream - downstream (x), and cross stream (y) translational components of ground motion. The upstream - downstream (x) component of ground motion is along the plane of symmetry of the dam. The cross stream (y) component is perpendicular to the plane of symmetry (Fig. 2.1). At any instant of time, the ground motion is identical throughout the reservoir bottom and banks, but the motion varies with time.

2.3 Equations of motion

The following assumptions are made in determining the distribution of the hydrodynamic pressure on the upstream face of the considered arch dam:

- 1) Water is assumed to be compressible but its internal viscosity is neglected.
- 2) The effects of waves at the free surface of water are ignored. The associated errors were found to be small [8,22].
- 3) The dam is assumed to be rigid and fixed at the base and at the banks.
- 4) The motion of water is limited to small amplitudes.

With the assumptions listed above, the motion of water in the reservoir is governed by the following equations:

$$\frac{\partial^2 u}{\partial t^2} = -\frac{1}{\rho} \frac{\partial P}{\partial x} \quad (2.1a)$$

$$\frac{\partial^2 v}{\partial t^2} = -\frac{1}{\rho} \frac{\partial P}{\partial y} \quad (2.1b)$$

$$\frac{\partial^2 w}{\partial t^2} = -\frac{1}{\rho} \frac{\partial P}{\partial z} \quad (2.1c)$$

where:

x, y and z = The cartesian coordinates.

u, v and w = Components of the displacement of a particle along the cartesian coordinates.

t = The time variable.

ρ = The mass of water per unit volume.

P = The hydrodynamic pressure (in excess of hydrostatic pressure).

The pressures are governed by the wave equation in cartesian coordinates (Fig. 2.1).

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{C^2} \frac{\partial^2 P}{\partial t^2} \quad (2.2)$$

where:

C = The velocity of sound in water (4720 ft/sec. $\approx$ 1440 m/sec.).

The time dependance can be removed by assuming that P is of the form:

$$P = \bar{P}(x, y, z)e^{i\omega t} \quad (2.3)$$

Substitution of Eqn. (2.3) into Eqn. (2.2) gives:

$$\frac{\partial^2 \bar{P}}{\partial x^2} + \frac{\partial^2 \bar{P}}{\partial y^2} + \frac{\partial^2 \bar{P}}{\partial z^2} + \frac{\omega^2}{C^2} \bar{P} = 0 \quad (2.4)$$

Putting $\omega^2/C^2 = k^2$, Eqn. (2.4) becomes:

$$\frac{\partial^2 \bar{P}}{\partial x^2} + \frac{\partial^2 \bar{P}}{\partial y^2} + \frac{\partial^2 \bar{P}}{\partial z^2} + k^2 \bar{P} = 0 \quad (2.5)$$

which is the familiar Helmholtz equation. The formulation of the 'Boundary Element Method' will be given later by treating Eqn. (2.5) as the governing differential equation.

Eqns. (2.1) and (2.2) together with appropriate boundary conditions at the reservoir boundaries – the upstream face of the dam, the reservoir bottom, the free surface of the water, and the reservoir banks – define the problem for the fluid domain.

Eqn. (2.2) can be represented in cylindrical coordinates as:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} + \frac{1}{r^2} \frac{\partial^2 P}{\partial \theta^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{C^2} \frac{\partial^2 P}{\partial t^2} \quad (2.6)$$

in which $P(r, \theta, z, t)$ is the hydrodynamic pressure.

The following expressions relate the hydrodynamic pressure and displacements of any particle of water:

$$\frac{\partial^2 v^r}{\partial t^2} = -\frac{1}{\rho} \frac{\partial P}{\partial r} \quad (2.7a)$$

$$\frac{\partial^2 v^\theta}{\partial t^2} = -\frac{1}{r} \frac{1}{\rho} \frac{\partial P}{\partial \theta} \quad (2.7b)$$

$$\frac{\partial^2 v^z}{\partial t^2} = -\frac{1}{\rho} \frac{\partial P}{\partial z} \quad (2.7c)$$

where v^r , v^θ , and v^z are, respectively, the radial, tangential, and vertical components of water particle displacement.

2.4 Analysis due to Harmonic Upstream-Downstream Ground Motion

2.4.1 Boundary Conditions

The hydrodynamic pressures, acting in the radial directions (normal to the upstream face) are governed by the wave equation (2.6) together with the following boundary conditions:

- 1) The radial component of fluid motion at the upstream face of the dam (boundary $r = R$) and the radial motion of the upstream face of the dam are the same.

- 2) The fluid motion normal to the banks (boundaries $\theta = \pm\pi/4$) and the normal component of motion of the banks are the same.
- 3) There is no vertical motion of the fluid at the bottom of the reservoir.
- 4) Fluid pressure at the free surface is zero.
- 5) Since the system, symmetrical about the $z - z(\theta = 0)$ plane, is excited by the z -component of ground motion, the hydrodynamic pressures must be symmetric about the same plane.
- 6) The radiation boundary condition not permitting any reflected waves applies at the upstream end ($r = \infty$) of the reservoir.

It is a property of linear time invariant systems that when the excitation is a simple harmonic motion, the steady state response is also a simple harmonic motion at the same frequency.

In studying the effects of structure - fluid interaction on the dynamics of arch dams, it is most appropriate and convenient to consider harmonic ground motion and to develop procedures to determine the complex frequency response functions for various response quantities of interest.

Let the excitation in the upstream - downstream direction be represented by a ground acceleration equal to $e^{i\omega t}$. Then, using Eqn. (2.7), the boundary conditions can be expressed analytically as follows:

$$\frac{\partial P}{\partial r}(R, \theta, z, t) = -\frac{W}{g} \cos \theta e^{i\omega t} \quad (2.8a)$$

$$\frac{\partial P}{r \partial \theta}(r, \frac{\pi}{4}, z, t) = \frac{W}{g} \sin \frac{\pi}{4} e^{i\omega t} \quad (2.8b)$$

$$\frac{\partial P}{\partial z}(r, \theta, 0, t) = 0 \quad (2.8c)$$

$$P(r, \theta, H, t) = 0 \quad (2.8d)$$

$$\frac{\partial P}{r \partial \theta}(r, 0, z, t) = 0 \quad (2.8e)$$

In addition, no wave reflections are permitted at the upstream end of the reservoir ($r = \infty$).

Because the governing wave equation as well as the boundary conditions are linear, the principle of superposition applies. The complex frequency function for the hydrodynamic pressures on the dam face $\bar{P}_e^*(\theta, z, \omega)$ can therefore be expressed as:

$$\bar{P}_e^*(\theta, z, \omega) = \bar{P}_{OD}^*(\theta, z, \omega) + \bar{P}_{OB}^*(\theta, z, \omega) \quad (2.9)$$

The complex frequency functions $\bar{P}_{OD}^*$ and $\bar{P}_{OB}^*$ in Eqn. (2.9) are defined as follows:

$P_{OD}^*(\theta, z, t) = \bar{P}_{OD}^*(\theta, z, \omega)e^{i\omega t}$ is the solution of the wave equation (2.6) at $r = R$ (upstream face of the dam) for the following boundary conditions:

$$\frac{\partial P}{\partial r}(R, \theta, z, t) = -\frac{W}{g} \cos \theta e^{i\omega t} \quad (2.10a)$$

$$\frac{\partial P}{r \partial \theta}(r, \frac{\pi}{4}, z, t) = 0 \quad (2.10b)$$

and those specified by Eqns. (2.8c) to (2.8e).

$P_{OB}^*(\theta, z, t) = \bar{P}_{OB}^*(\theta, z, \omega)e^{i\omega t}$ is the solution of the wave equation at $r = R$ for the following boundary conditions:

$$\frac{\partial P}{\partial r}(R, \theta, z, t) = 0 \quad (2.11a)$$

$$\frac{\partial P}{r \partial \theta}(r, \frac{\pi}{4}, z, t) = \frac{1}{\sqrt{2}} \frac{W}{g} e^{i\omega t} \quad (2.11b)$$

and those specified by Eqns. (2.8c) to (2.8e).

The complex frequency functions $\bar{P}_{OD}^*(\theta, z, \omega)$ and $\bar{P}_{OB}^*(\theta, z, \omega)$ are for the hydrodynamic pressures on the upstream face of the dam for the following excitations, respectively:

- i) Acceleration of rigid dam in the upstream - downstream direction but with the reservoir banks stationary.
- ii) Acceleration of only the reservoir banks in the upstream - downstream direction.

2.4.2 Classical Solutions

The solutions of the wave equation for the two sets of boundary conditions presented in Section 2.4.1 are derived by C.S. Porter and A.K. Chopra [22]. The final expressions for $P_{OD}^*(\theta, z, \omega)$ and $P_{OB}^*(\theta, z, \omega)$ are as follows:

$$P_{OD}^*(\theta, z, \omega) = \frac{16\sqrt{2}WR}{g\pi^2} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^m}{(2m-1)(1-16n^2)} \frac{\epsilon_n(-1)^n}{[C_n(\lambda_m R) + iD_n(\lambda_m R)]} \cos 4n\theta \cos \alpha_m z \quad (2.12)$$

$$P_{OB}^*(\theta, z, \omega) = \frac{2\sqrt{2}WR}{g\pi} \left\{ \sum_{m=1}^{\infty} [E_m(\lambda_m R) + iF_m(\lambda_m R)] \cos \alpha_m z \right. \\ \left. + \frac{8}{\pi} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \epsilon_n [U_{mn}(\lambda_m R) + iV_{mn}(\lambda_m R)] \cos 4n\theta \cos \alpha_m z \right\} \quad (2.13)$$

where:

ω = excitation frequency.

W = unit weight of water.

g = acceleration due to gravity.

$\epsilon_n = 1$ when $n = 0$.

$\epsilon_n = 2$ when $n \neq 0$.

$\omega_1^* = \pi C/2H$ = the first eigen-frequency of the water in the reservoir.

$\alpha_m = (2m-1)\pi/2H$

$\lambda_m R = R\sqrt{|\alpha_m^2 - \omega^2/C^2|} = (\pi R/2H)\sqrt{|(2m-1)^2 - (\omega/\omega_1^*)^2|}$

m_l = the largest integer 'm' satisfying the inequality $\omega/\omega_1^* > (2m-1)$

Expressions for functions C_n, D_n, E_m, U_{mn} and V_{mn} differ depending on whether m is smaller or larger than m_l .

* For $m \leq m_l$ they are as follows:

$$C_n(\lambda_m R) = \frac{[A_n(\lambda_m R)J_{4n}(\lambda_m R) + B_n(\lambda_m R)Y_{4n}(\lambda_m R)]}{\lambda_m R[A_n^2(\lambda_m R) + B_n^2(\lambda_m R)]} \quad (2.14a)$$

$$D_n(\lambda_m R) = \frac{[B_n(\lambda_m R)J_{4n}(\lambda_m R) - A_n(\lambda_m R)Y_{4n}(\lambda_m R)]}{\lambda_m R[A_n^2(\lambda_m R) + B_n^2(\lambda_m R)]} \quad (2.14b)$$

$$E_m(\lambda_m R) = \frac{(-1)^m}{(2m-1)\lambda_m R} \{ \sin[\lambda_m R \sin(\frac{\pi}{4} - \theta)] + \sin[\lambda_m R \sin(\frac{\pi}{4} + \theta)] \} \quad (2.14c)$$

$$F_m(\lambda_m R) = \frac{(-1)^m}{(2m-1)\lambda_m R} \{ \cos[\lambda_m R \sin(\frac{\pi}{4} - \theta)] + \cos[\lambda_m R \sin(\frac{\pi}{4} + \theta)] \} \quad (2.14d)$$

$$U_{mn}(\lambda_m R) = \frac{(-1)^{m+n}}{(2m-1)} \{ T_n(\lambda_m R)C_n(\lambda_m R) + \frac{\pi}{4} A_n(\lambda_m R)D_n(\lambda_m R) \} \quad (2.14e)$$

$$V_{mn}(\lambda_m R) = \frac{(-1)^{m+n}}{(2m-1)} \{ T_n(\lambda_m R)D_n(\lambda_m R) - \frac{\pi}{4} A_n(\lambda_m R)C_n(\lambda_m R) \} \quad (2.14f)$$

where:

$$A_n(\lambda_m R) = J_{4n-1}(\lambda_m R) - J_{4n+1}(\lambda_m R)$$

$$B_n(\lambda_m R) = Y_{4n-1}(\lambda_m R) - Y_{4n+1}(\lambda_m R)$$

$$T_n(\lambda_m R) = \sum_{k=0}^{\infty} \epsilon_{2k} J_{2k}(\lambda_m R) (16n^2 + 4k^2 - 1) / (16n^2 - 4k^2 - 4k - 1)(16n^2 - 4k^2 + 4k + 1)$$

$J_n(x)$ = Bessel function of the first kind of order n .

$Y_n(x)$ = Bessel function of the second kind of order n .

* For $m > m_l$ the above listed functions are as follows:

$$C_n(\lambda_m R) = \frac{-K_{4n}(\lambda_m R)}{\lambda_m R[K_{4n-1}(\lambda_m R) + K_{4n+1}(\lambda_m R)]} \quad (2.14g)$$

$$D_n(\lambda_m R) = 0 \quad (2.14h)$$

$$E_m(\lambda_m R) = \frac{(-1)^{m+1}}{(2m-1)\lambda_m R} [e^{-\lambda_m R \sin(\pi/4 - \theta)} + e^{-\lambda_m R \sin(\pi/4 + \theta)}] \quad (2.14i)$$

$$F_m(\lambda_m R) = 0 \quad (2.14j)$$

$$U_{mn}(\lambda_m R) = \frac{(-1)^{m+1}}{(2m-1)} G_n(\lambda_m R) C_n(\lambda_m R) \quad (2.14k)$$

$$V_{mn}(\lambda_m R) = 0 \quad (2.14l)$$

where:

$K_n(z)$ = modified Bessel function of the second kind of order n .

$$G_n(\lambda_m R) = \int_0^{\pi/4} \left[\sin\left(\frac{\pi}{4} - \theta\right) e^{-\lambda_m R \sin(\pi/4 - \theta)} + \sin\left(\frac{\pi}{4} + \theta\right) e^{-\lambda_m R \sin(\pi/4 + \theta)} \right] \cos 4n\theta d\theta$$

In the above equations the Bessel functions are grouped so that the expressions for $C_n(\lambda_m R)$ and $D_n(\lambda_m R)$, Eqns. 2.14a, b and g, are well behaved functions [22].

The above solutions of the wave equation for the two sets of boundary conditions presented in Section 2.4.1 are for fluid domains described by Fig. 2.1. The fluid is bounded by the upstream face of the dam, which is a segment of a circular cylinder of radius R , and by radially extending banks enclosing a central angle of 90 degrees. For arbitrary central angles, standard analytical procedures are able to provide solutions of the wave equation for $P_{OD}^s(\theta, z, \omega)$ but not for $P_{OB}^s(\theta, z, \omega)$ [7].

As the central angle approaches 180 degrees and $R \rightarrow \infty$, representing a straight dam in the limit, $P_{OD}^s(\theta, z, \omega)$ approaches the previously obtained [25] two-dimensional solution of the equation for a straight gravity dam.

An eigen - frequency of the wave equation in the particular fluid domain under consideration corresponds to each pair of functions $\cos 4n\theta$, $n=0,1,2,\dots$ and $\cos \alpha_m z$, $m=1,2,3,\dots$

The hydrodynamic pressures $P_{OD}^s(\theta, z, \omega)$ and $P_{OB}^s(\theta, z, \omega)$ are unbounded at the eigen frequencies corresponding to $n = 0$ with $m = 1, 2, 3, \dots$. At all the other eigen - frequencies, the pressures are bounded and do not resonate. The resonant frequencies and the eigen - frequencies corresponding to $n = 0$ and $m = 1, 2, 3, \dots$ are $\omega_m' = (2m-1)\pi C/2H$. They are the same as the resonant frequencies obtained for two-dimensional fluid domains [25].

2.5 Analysis due to Harmonic Cross-Stream Ground Motion

The analytical procedures and results developed in this section for the hydrodynamic pressure due to the cross-stream component of ground motion closely parallel those for Section 2.4.

2.5.1 Boundary Conditions

As defined in Section 2.3, the pressures acting in the radial direction (normal to the upstream face) are governed by the wave Eqn. (2.6) together with the following boundary conditions:

- 1) The radial component of fluid motion at the upstream face of the dam (boundary $r = R$) and the radial motion of the upstream face of the dam are the same.
- 2) The fluid motion normal to the banks (boundaries $\theta = \pm\pi/4$) and the normal component of motion of the banks are the same.
- 3) There is no vertical motion of the fluid at the bottom of the reservoir.
- 4) Fluid pressure at the free surface is zero.
- 5) Since the system, symmetrical about the $x - z(\theta = 0)$ plane, is excited by the y -component of ground motion, the hydrodynamic pressures are antisymmetric about the same plane.
- 6) The radiation boundary condition not permitting any reflected waves applies at the upstream end ($r = \infty$) of the reservoir.

Using Eqn. (2.7), the boundary conditions can be expressed analytically as follows:

$$\frac{\partial P}{\partial r}(R, \theta, z, t) = -\frac{W}{g} \sin \theta e^{i\omega t} \quad (2.15a)$$

$$\frac{\partial P}{r \partial \theta}(r, \frac{\pi}{4}, z, t) = -\frac{W}{g} \cos \frac{\pi}{4} e^{i\omega t} \quad (2.15b)$$

$$\frac{\partial P}{\partial z}(r, \theta, 0, t) = 0 \quad (2.15c)$$

$$P(r, \theta, H, t) = 0 \quad (2.15d)$$

$$P(r, 0, z, t) = 0 \quad (2.15c)$$

In addition to these boundary conditions, no wave reflections are permitted at the upstream end of the reservoir ($r = \infty$)

Because the governing wave equation as well as the boundary conditions are linear, the principle of superposition applies. The complex frequency function for the hydrodynamic pressures on the dam face $\bar{P}_c^y(\theta, z, \omega)$ can therefore be expressed as:

$$\bar{P}_c^y(\theta, z, \omega) = \bar{P}_{OD}^y(\theta, z, \omega) + \bar{P}_{OB}^y(\theta, z, \omega) \quad (2.16)$$

The complex frequency functions $\bar{P}_{OD}^y$ and $\bar{P}_{OB}^y$ in Eqn. (2.16) are defined as follows:

$P_{OD}^y(\theta, z, t) = \bar{P}_{OD}^y(\theta, z, \omega)e^{i\omega t}$ is the solution of the wave equation (2.6) at $r = R$ (upstream face of the dam) for the following boundary conditions:

$$\frac{\partial P}{\partial r}(R, \theta, z, t) = -\frac{W}{g} \sin \theta e^{i\omega t} \quad (2.17a)$$

$$\frac{\partial P}{r \partial \theta}(r, \frac{\pi}{4}, z, t) = 0 \quad (2.17b)$$

and those specified by Eqns. (2.15c) to (2.15e).

$P_{OB}^y(\theta, z, t) = \bar{P}_{OB}^y(\theta, z, \omega)e^{i\omega t}$ is the solution of the wave equation at $r = R$ for the following boundary conditions:

$$\frac{\partial P}{\partial r}(R, \theta, z, t) = 0 \quad (2.18a)$$

$$\frac{\partial P}{r \partial \theta}(r, \frac{\pi}{4}, z, t) = -\frac{1}{\sqrt{2}} \frac{W}{g} e^{i\omega t} \quad (2.18b)$$

and those specified by Eqns. (2.15c) to (2.15e).

The complex frequency functions $\bar{P}_{OD}^y(\theta, z, \omega)$ and $\bar{P}_{OB}^y(\theta, z, \omega)$ are for the hydrodynamic pressures on the upstream face of the dam for the following excitations, respectively:

- i) Acceleration of rigid dam in the y -direction but with the reservoir banks stationary.
- ii) Acceleration of only the reservoir banks in the y -direction.

2.5.2 Classical Solutions

The solutions of the wave equation for the two sets of boundary conditions presented in Section 2.5.1 are derived by C.S. Porter and A.K. Chopra [22]. The final expressions for $\bar{P}_{OD}^y(\theta, z, \omega)$ and $\bar{P}_{OB}^y(\theta, z, \omega)$ are as follows:

$$\bar{P}_{OD}^y(\theta, z, \omega) = \frac{32\sqrt{2}WR}{g\pi^2} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^m}{(2m-1)} \frac{(-1)^n}{(\mu_n^2 - 1)} [C_n(\lambda_m R) + iD_n(\lambda_m R)] \sin \mu_n \theta \cos \alpha_m z \quad (2.19)$$

$$\begin{aligned} \bar{P}_{OB}^y(\theta, z, \omega) = & \frac{2\sqrt{2}WR}{g\pi} \left\{ \sum_{m=1}^{\infty} [E_m(\lambda_m R) + iF_m(\lambda_m R)] \cos \alpha_m z \right. \\ & \left. + \frac{16}{\pi} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} [U_{mn}(\lambda_m R) + iV_{mn}(\lambda_m R)] \cos \mu_n \theta \cos \alpha_m z \right\} \quad (2.20) \end{aligned}$$

where:

$$c_{2k} = 1 \text{ when } k = 0.$$

$$c_{2k} = 2 \text{ when } k \neq 0.$$

$$\mu_n = 4n + 2$$

and the other terms are as defined earlier.

Expressions for functions C_n, D_n, E_m, U_{mn} , and V_{mn} differ depending on whether m is smaller or larger than m_l .

* For $m \leq m_l$ they are as follows:

$$C_n(\lambda_m R) = \frac{[A_n(\lambda_m R)J_{\mu_n}(\lambda_m R) + B_n(\lambda_m R)Y_{\mu_n}(\lambda_m R)]}{\lambda_m R[A_n^2(\lambda_m R) + B_n^2(\lambda_m R)]} \quad (2.21a)$$

$$D_n(\lambda_m R) = \frac{[B_n(\lambda_m R)J_{\mu_n}(\lambda_m R) - A_n(\lambda_m R)Y_{\mu_n}(\lambda_m R)]}{\lambda_m R[A_n^2(\lambda_m R) + B_n^2(\lambda_m R)]} \quad (2.21b)$$

$$E_m(\lambda_m R) = \frac{(-1)^m}{(2m-1)\lambda_m R} \{ \sin[\lambda_m R \sin(\pi/4 + \theta)] - \sin[\lambda_m R \sin(\pi/4 - \theta)] \} \quad (2.21c)$$

$$F_m(\lambda_m R) = \frac{(-1)^m}{(2m-1)\lambda_m R} \{ \cos[\lambda_m R \sin(\pi/4 + \theta)] - \cos[\lambda_m R \sin(\pi/4 - \theta)] \} \quad (2.21d)$$

$$U_{mA}(\lambda_m R) = \frac{(-1)^{m+n}}{(2m-1)} \{ T_n(\lambda_m R) C_n(\lambda_m R) + \frac{\pi}{4} A_n(\lambda_m R) D_n(\lambda_m R) \} \quad (2.21e)$$

$$V_{mB}(\lambda_m R) = \frac{(-1)^{m+n+1}}{(2m-1)} \{ T_n(\lambda_m R) D_n(\lambda_m R) - \frac{\pi}{4} A_n(\lambda_m R) C_n(\lambda_m R) \} \quad (2.21f)$$

where:

$$A_n(\lambda_m R) = J_{\mu_n-1}(\lambda_m R) - J_{\mu_n+1}(\lambda_m R)$$

$$B_n(\lambda_m R) = Y_{\mu_n-1}(\lambda_m R) - Y_{\mu_n+1}(\lambda_m R)$$

$$T_n(\lambda_m R) = \sum_{k=0}^{\infty} \epsilon_{2k} J_{2k}(\lambda_m R) (\mu_n^2 + 4k^2 - 1) / (\mu_n^2 - 4k^2 - 4k - 1)(\mu_n^2 - 4k^2 + 4k - 1)$$

$$J_{\mu_n}(z) = \text{Bessel function of the first kind of order } \mu_n.$$

$$Y_{\mu_n}(z) = \text{Bessel function of the second kind of order } \mu_n.$$

* For $m > m_l$ the above listed functions are as follows:

$$C_n(\lambda_m R) = \frac{-K_{\mu_n}(\lambda_m R)}{\lambda_m R [K_{\mu_n-1}(\lambda_m R) + K_{\mu_n+1}(\lambda_m R)]} \quad (2.21g)$$

$$D_n(\lambda_m R) = 0 \quad (2.21h)$$

$$E_m(\lambda_m R) = \frac{(-1)^{m+1}}{(2m-1)\lambda_m R} [e^{-\lambda_m R \sin(\pi/4 + \theta)} - e^{-\lambda_m R \sin(\pi/4 - \theta)}] \quad (2.21i)$$

$$F_m(\lambda_m R) = 0 \quad (2.21j)$$

$$U_{mn}(\lambda_m R) = \frac{(-1)^{m+1}}{(2m-1)} G_n(\lambda_m R) C_n(\lambda_m R) \quad (2.21k)$$

$$V_{mn}(\lambda_m R) = 0 \quad (2.21l)$$

where:

$K_{\mu_n}(z)$ = modified Bessel function of the second kind of order μ_n .

$$G_n(\lambda_m R) = \int_0^{\pi/4} [\sin(\pi/4 + \theta) e^{-\lambda_m R \sin(\pi/4 + \theta)} - \sin(\pi/4 - \theta) e^{-\lambda_m R \sin(\pi/4 - \theta)}] \sin \mu_n \theta d\theta$$

For ground motion in the 'y' direction and for the particular fluid domain under consideration an eigen-frequency of the wave equation corresponds to each pair of functions $\sin \mu_n \theta$, $n=0,1,2,\dots$ and $\cos \alpha_m z$, $m=1,2,3,\dots$. The hydrodynamic pressures $P_{OD}^y(\theta, z, \omega)$ and $P_{OB}^y(\theta, z, \omega)$ are bounded at all eigen - frequencies. This contrasts with the results for ground motion in the 'z' direction in which case the pressure functions are unbounded at eigen-frequencies corresponding to $n=0$, $m=1,2,3,\dots$.

The eigenfunctions $\cos 4n\theta$ and $\cos \alpha_m z$ define the distribution of hydrodynamic pressures on the face of the dam due to the x-component of ground motion. For $n=0$ and $m=1,2,3,\dots$, the pressures are unbounded and independent of angular coordinate. The eigenfunctions $\sin \mu_n \theta$ define the angular distribution of pressures due to the y-component of ground motion. Because this antisymmetric excitation produces antisymmetric eigenfunctions there are no eigenfunctions that are independent of angular coordinate and resonance (unbounded response) does not occur at any eigen - frequency. Furthermore, the antisymmetric excitation of the banks causes a canceling of pressures due to motion of the banks $\theta = \pm \pi/4$. Because of this canceling effect $P_{OB}^y(\theta, z, \omega)$ remains bounded at all eigen - frequencies.

2.6 Boundary Element Formulation

2.6.1 Green's theorem

The boundary element method is based on the application of Green's theorem which converts

an integral over an entire domain to one over the boundary of that domain. It follows from Green's theorem that if both P and P^* are functions of x , y and z then:

$$\int_V \frac{\partial P}{\partial z} P^* dx dy dz = - \int_V P \frac{\partial P^*}{\partial z} dx dy dz + \int_S PP^* l_z dS \quad (2.22a)$$

Similarly

$$\int_V \frac{\partial P}{\partial y} P^* dx dy dz = - \int_V P \frac{\partial P^*}{\partial y} dx dy dz + \int_S PP^* l_y dS \quad (2.22b)$$

and

$$\int_V \frac{\partial P}{\partial x} P^* dx dy dz = - \int_V P \frac{\partial P^*}{\partial x} dx dy dz + \int_S PP^* l_x dS \quad (2.22c)$$

where:

V represents the domain under consideration and

S represents the boundary of the domain V .

l_x , l_y and l_z are the direction cosines of the outward normal to the x , y and z axes respectively.

Also it is known that:

$$\int_S P \frac{\partial P^*}{\partial x} l_x dS + \int_S P \frac{\partial P^*}{\partial y} l_y dS + \int_S P \frac{\partial P^*}{\partial z} l_z dS = \int_S P \frac{\partial P^*}{\partial n} dS \quad (2.23a)$$

$$\int_S P^* \frac{\partial P}{\partial x} l_x dS + \int_S P^* \frac{\partial P}{\partial y} l_y dS + \int_S P^* \frac{\partial P}{\partial z} l_z dS = \int_S P^* \frac{\partial P}{\partial n} dS \quad (2.23b)$$

where $\frac{\partial P}{\partial n}$ represents differentiation with respect to the outward normal to the boundary.

Eqns. (2.22) and (2.23) form the starting point in the formulation of the boundary element method.

2.6.2 Helmholtz equation

As mentioned in Section 2.3 the reduced Helmholtz equation is a partial differential equation which governs many practical problems in structural dynamics. Its general form is:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} + k^2 P = 0 \quad (2.24)$$

where:

P is a function of x , y and z , and

k is a constant $= \omega/C$.

Since the governing differential equations of all problems discussed in the present study can be reduced to a Helmholtz equation, the formulation of the boundary element method is explained by treating Eqn. (2.24) as the governing differential equation.

Multiplying Eqn. (2.24) by P^* ; a function of x , y and z ; the properties of which will be discussed later and integrating over the domain under consideration, V , one gets:

$$\int_V \left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} + k^2 P \right) P^* dx dy dz = 0 \quad (2.25)$$

By the application of Green's theorem, Eqn. (2.25) can be reduced as follows:

$$\begin{aligned} \int_V \frac{\partial^2 P}{\partial x^2} P^* dx dy dz &= - \int_V \frac{\partial P}{\partial x} \frac{\partial P^*}{\partial x} dx dy dz + \int_S \frac{\partial P}{\partial x} P^* l_x dS \\ &= \int_V P \frac{\partial^2 P^*}{\partial x^2} dx dy dz - \int_S P \frac{\partial P^*}{\partial x} l_x dS + \int_S P^* \frac{\partial P}{\partial x} l_x dS \end{aligned} \quad (2.26a)$$

Similarly

$$\int_V \frac{\partial^2 P}{\partial y^2} P^* dx dy dz = \int_V P \frac{\partial^2 P^*}{\partial y^2} dx dy dz - \int_S P \frac{\partial P^*}{\partial y} l_y dS + \int_S P^* \frac{\partial P}{\partial y} l_y dS \quad (2.26b)$$

and

$$\int_V \frac{\partial^2 P}{\partial z^2} P^* dx dy dz = \int_V P \frac{\partial^2 P^*}{\partial z^2} dx dy dz - \int_S P \frac{\partial P^*}{\partial z} l_z dS + \int_S P^* \frac{\partial P}{\partial z} l_z dS \quad (2.26c)$$

Substitution of Eqns. (2.26) into Eqn. (2.25) gives:

$$\int_V \left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} + k^2 P \right) P^* dx dy dz = \int_V \left(\frac{\partial^2 P^*}{\partial x^2} + \frac{\partial^2 P^*}{\partial y^2} + \frac{\partial^2 P^*}{\partial z^2} + k^2 P^* \right) P dx dy dz$$

$$\begin{aligned}
& - \left[\int_S P \frac{\partial P^*}{\partial z} l_x dS + \int_S P \frac{\partial P^*}{\partial y} l_y dS + \int_S P \frac{\partial P^*}{\partial z} l_z dS \right] \\
& + \left[\int_S P^* \frac{\partial P}{\partial x} l_x dS + \int_S P^* \frac{\partial P}{\partial y} l_y dS + \int_S P^* \frac{\partial P}{\partial z} l_z dS \right] = 0
\end{aligned} \quad (2.27)$$

or

$$\int_V \left(\frac{\partial^2 P^*}{\partial x^2} + \frac{\partial^2 P^*}{\partial y^2} + \frac{\partial^2 P^*}{\partial z^2} + k^2 P^* \right) P dx dy dz = \int_S P \frac{\partial P^*}{\partial n} dS - \int_S \frac{\partial P}{\partial n} P^* dS \quad (2.28)$$

It can be noticed that the right-hand side of Eqn. (2.28) contains integration over the boundary S of the domain only. By an appropriate choice of P^* , the integration over the domain V appearing on the left-hand side of Eqn. (2.28) can be avoided. This is discussed in some detail in the following section.

2.6.3 Fundamental solution

In order to reduce the problem being considered to a boundary problem, the function P^* is chosen so as to satisfy the following equation:

$$\frac{\partial^2 P^*}{\partial x^2} + \frac{\partial^2 P^*}{\partial y^2} + \frac{\partial^2 P^*}{\partial z^2} + k^2 P^* = \Delta^i \quad (2.29)$$

where

Δ^i is a Dirac Delta function centered at point 'i' having coordinates (x_i, y_i, z_i) .

The solution of Eqn. (2.29) is called the fundamental solution.

Substitution of Eqn. (2.29) into Eqn. (2.28) gives :

$$\int_V \Delta^i P dx dy dz = \int_S P \frac{\partial P^*}{\partial n} dS - \int_S P^* \frac{\partial P}{\partial n} dS \quad (2.30)$$

Using the properties of the Dirac Delta function [24] it can be shown that Eqn. (2.30) reduces to:

$$c_i(P)_i = \int_S P q^* dS - \int_S P^* q dS \quad (2.31)$$

where:

$c_i = 1$ for a point inside the domain V ,

$c_i = \frac{1}{2}$ for a point on the smooth boundary of the domain V ,

$c_i = 0$ for a point outside the domain V ,

$q^* = \partial P^* / \partial n$ and

$q = \partial P / \partial n$

Eqn. (2.31) relates the value of the function under consideration, P , at any point 'i' to its values on the boundary of the domain because the integrations contained in this equation are to be carried out only on the boundary of the domain.

2.6.4 Boundary element discretisation

In order to evaluate the integrals contained in Eqn. (2.31), it is assumed that the boundary of the domain is divided into n boundary elements as shown in Figs. 1.1 and 3.2. The points where the unknown values of the variable are sought, are called 'nodes'. The position of the nodes differs according to the type of boundary elements used. For constant elements, the nodes are taken to be at the center of each element and for linear elements, they are taken to be at the corners of the elements. The constant boundary element type will be considered in the present work.

2.6.4.1 Constant boundary elements

For the constant boundary elements, the values of P and q are assumed to be constant over each element and equal to their values at the mid-node of the element. Eqn. (2.31) for a given point, i , on the boundary becomes in its discretized form:

$$-\frac{1}{2}P_i + \sum_{j=1}^n P_j \int_{S_j} q^* ds = \sum_{j=1}^n q_j \int_{S_j} P^* dS \quad (2.32)$$

where P_j and q_j are the values of these variables over the j^{th} element and S_j represents the surface of that element.

Eqn. (2.32) applies for a particular node 'i'. The terms $\int_{S_j} q^* dS$ relate the 'i' node with the

segment 'j' over which the integral is carried out. These integral values will be denoted by h_{ij} . The integral values $\int_S P^* dS$ will be denoted by G_{ij} .

Eqn. (2.32) then becomes:

$$-\frac{1}{2}P_i + \sum_{j=1}^n P_j h_{ij} = \sum_{j=1}^n q_j G_{ij} \quad (2.33)$$

Putting

$$H_{ij} = h_{ij} \text{ when } i \neq j$$

$$H_{ij} = h_{ij} - \frac{1}{2} \text{ when } i = j$$

Eqn. (2.33) becomes:

$$\sum_{j=1}^n H_{ij} P_j = \sum_{j=1}^n G_{ij} q_j \quad (2.34)$$

The whole set of equations for the n nodes can now be expressed in matrix form as:

$$[H]\{P\} = [G]\{q\} \quad (2.35)$$

where:

$[H]$ is a matrix containing the elements H_{ij} ,

$[G]$ is a matrix containing the elements G_{ij} ,

$\{P\}$ is a vector containing the known and unknown values of the considered function P , and

$\{q\}$ is a vector containing the known and unknown values of the derivative of P .

Now if it is assumed that ' n_1 ' values of P and ' n_2 ' values of q are known so that $n_1 + n_2 = n$, there are n unknowns in Eqn. (2.35). Re-ordering the equations in such a way that all the unknowns are transferred to the left-hand side, one can write:

$$[B]\{x\} = \{F\} \quad (2.36)$$

where $\{x\}$ is the vector of unknowns P_S and q_S .

3.6.4.3 Infinite boundary elements

In order to represent an infinite reservoir, it is necessary to extend the mesh to a sufficiently large distance from the dam. This will need an extremely large computer memory. At such a boundary, a suitable radiation condition should be applied to ensure that no waves are reflected. The use of specially developed elements extending to infinity may be a reasonable alternative for representing an infinite domain. One such element is used in this study to test the applicability of this procedure.

The infinite element used is extending from points 2 and 3, through points 1 and 4, to points 6 and 5 which lie at infinity. It is to be mapped on to an infinite domain $0 \leq \zeta_1 \leq \infty$ in one direction and a finite domain $-1 \leq \zeta_2 \leq +1$ in the perpendicular direction (Fig. 2.2).

This is achieved by using the expression:

$$X = \phi_1 X_1 + \phi_2 X_2 + \phi_3 X_3 + \phi_4 X_4$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are the shape functions. Points 1 and 4 are chosen arbitrary. They were chosen here so as to lie both at $\zeta_1 = 10$. Also, $\zeta_2 = -1$ for point 1 and $\zeta_2 = +1$ for point 4. By this choice the shape functions are:

$$\phi_1 = \frac{\zeta_1}{10} \left(\frac{1 + \zeta_2}{2} \right) \quad , \quad \phi_2 = \left(1 - \frac{\zeta_1}{10} \right) \left(\frac{1 + \zeta_2}{2} \right)$$

$$\phi_3 = \left(1 - \frac{\zeta_1}{10} \right) \left(\frac{1 - \zeta_2}{2} \right) \quad , \quad \phi_4 = \frac{\zeta_1}{10} \left(\frac{1 - \zeta_2}{2} \right)$$

In this mapping, some of the points which can be identified are:

* Point 1 : ($\zeta_1 = 10, \zeta_2 = +1$)

$\phi_1 = 1, \phi_2 = 0, \phi_3 = 0$ and $\phi_4 = 0$

$X = X_1$

* Point 2 : ($\zeta_1 = 0, \zeta_2 = +1$)

$\phi_1 = 0, \phi_2 = 1, \phi_3 = 0$ and $\phi_4 = 0$

$$X = X_2$$

* Point 5 : ($\zeta_1 = \infty$, $\zeta_2 = -1$)

$$\phi_1 = 0, \phi_2 = 0, \phi_3 = 1 - \frac{f_1}{f_0} \text{ and } \phi_4 = \frac{f_1}{f_0}$$

$$X = X_3 \left(1 - \frac{f_1}{f_0}\right) + X_4 \frac{f_1}{f_0} = X_3 + (X_4 - X_3) \frac{f_1}{f_0} = \infty \quad X_4 > X_3$$

In a similar manner, it can be shown that point 3 corresponds to $\zeta_1 = 0$ and $\zeta_2 = -1$, point 4 corresponds to $\zeta_1 = 10$ and $\zeta_2 = -1$ and point 6 corresponds to $\zeta_1 = \infty$ and $\zeta_2 = +1$.

As shown in Fig. 2.3, the representation of the hydrodynamic pressure function (P) over the infinite element is chosen [27] to be

$$P(\zeta_1, \zeta_2) = P(0, \zeta_2) \frac{r(z, z_0)}{r(x, x_0)}$$

and the derivative (q) is chosen to be

$$q(\zeta_1, \zeta_2) = q(0, \zeta_2) \left[\frac{r(z, z_0)}{r(x, x_0)} \right]^2$$

where z_0 is a point on the upstream surface of the arch dam lying on the line that passes through the point of origin of the cartesian coordinates, point z and point x (Fig. 2.3).

The type of decay in the (P) and (q) expressions matches that for (P^*) and (q^*) fundamental solution, respectively.

A detailed discussion and formulation of the infinite elements are presented in Chapter 4.

FIG. 2.2 TWO DIMENSIONAL MAPPING USED IN THE INFINITE ELEMENT FORMULATION

FIG. 2.3 ASYMPTOTIC FUNCTIONS OVER INFINITE ELEMENTS

Chapter 3

HYDRODYNAMIC PRESSURE DISTRIBUTION IN A FINITE WATER RESERVOIR

3.1 Introduction

In this chapter, the boundary element method is applied to find the distribution of the hydrodynamic pressure acting on the upstream face of the arch dam shown in Fig. 3.1. The upstream face of the arch dam is a segment of a circular cylinder, radius R , and height H_d , storing a finite reservoir of depth H and length L . The solution of the vibration problem of the finite reservoir subject to a ground motion in either of the two horizontal directions is presented. Two different conditions are considered for the far-end of the reservoir. The first condition specifies that the far end is stationary and the second that it is moving with the same acceleration as the upstream face of the dam. The constant boundary element over which the pressure and pressure gradient are assumed to remain constant is used. The idealization of the boundary elements and comparison between the results obtained with different meshes are shown. Also, a comparison with the two dimensional case is given and the results of investigation for the three dimensional case are presented. No classical solutions were found for the 3-D case.

3.2 Boundary Conditions

Boundary conditions along the dam face and the banks are presented in Section 2.4.1 for the case of a harmonic upstream-downstream ground motion and in Section 2.5.1 for the case of a harmonic cross-stream ground motion. As for the boundary conditions at the far end of the reservoir, the following two cases arise :

FIG. 31 ARCH DAM-RESERVOIR SYSTEM

- i) Assuming that the far end of the reservoir is stationary, there will be no horizontal motion of fluid. The following boundary condition should be satisfied

$$\frac{\partial^2 u}{\partial t^2}(L+R, \theta, z, t) = 0$$

which gives

$$\frac{\partial \bar{P}}{\partial n}(L+R, \theta, z) = 0$$

- ii) Assuming that the far end is moving with the same acceleration as the upstream face of the dam, the horizontal acceleration of the particles will be equal to the horizontal ground acceleration. For the upstream-downstream ground motion, the following boundary condition should be satisfied

$$\frac{\partial^2 u}{\partial t^2}(L+R, \theta, z, t) = \frac{\partial^2 u_g}{\partial t^2} \cos \theta e^{i\omega t}$$

which gives

$$\frac{\partial \bar{P}}{\partial n} = -\frac{W}{g} \cos \theta$$

For the cross-stream ground motion, the following boundary condition should be satisfied

$$\frac{\partial^2 u}{\partial t^2}(L+R, \theta, z, t) = \frac{\partial^2 u_g}{\partial t^2} \sin \theta e^{i\omega t}$$

which gives:

$$\frac{\partial \bar{P}}{\partial n} = \frac{W}{g} \sin \theta$$

3.3 Boundary Element Idealization

Following the steps of Section 2.6 for the boundary element formulation, the equation for a point on the boundary can be written as :

$$\sum_{j=1}^n H_{ij}(\bar{P})_j = \sum_{j=1}^n G_{ij} \left(\frac{\partial \bar{P}}{\partial n} \right)_j \quad (3.1)$$

where.

$$\begin{aligned} H_{ij} &= \int_{S_j} \frac{\partial P^*}{\partial n} dS \quad \text{when } i \neq j \\ H_{ij} &= -\frac{1}{2} + \int_{S_j} \frac{\partial P^*}{\partial n} dS \quad \text{when } i = j \\ G_{ij} &= \int_{S_j} P^* dS \end{aligned}$$

The fundamental solution P^* ; defined in Section 2.6.3 ; is given by:

$$\begin{aligned} P^* &= \frac{-1}{4\pi r} e^{-ikr} \\ &= \frac{-1}{4\pi r} [\cos kr - i \sin kr] \\ &= -\frac{\cos kr}{4\pi r} + \frac{\sin kr}{4\pi r} i \end{aligned} \quad (3.2)$$

$$\frac{\partial P^*}{\partial n} = \frac{\partial P^*}{\partial r} \cos \theta \quad (3.3)$$

where (θ) is the angle between (r) ; the vector from the boundary node (i) to the point (j) under consideration ; and (n) ; the outward normal at the point (j) under consideration (see Fig. 3.2).

$$\begin{aligned} \frac{\partial P^*}{\partial n} &= \frac{\cos \theta}{4\pi r^2} (1 + ikr) e^{-ikr} \\ &= \frac{\cos \theta}{4\pi r^2} (1 + ikr) (\cos kr - i \sin kr) \\ &= \frac{\cos \theta}{4\pi r^2} (\cos kr + kr \sin kr) + \frac{\cos \theta}{4\pi r^2} (-\sin kr + kr \cos kr) i \end{aligned} \quad (3.4)$$

Substitution of Eqn. (3.4) into the expression for (H_{ij}) gives:

$$H_{ij} = \int_{S_j} \frac{\cos \theta}{4\pi r^2} (\cos kr + kr \sin kr) dS + i \int_{S_j} \frac{\cos \theta}{4\pi r^2} (-\sin kr + kr \cos kr) dS \quad (3.5)$$

Transforming into (ζ) local coordinates ; shown in Fig. 2.2 for infinite elements and in Fig. 4.2 for finite and mapped infinite elements:

$$\begin{aligned} H_{ij} &= \int \frac{\cos \theta}{4\pi r^2} (\cos kr + kr \sin kr) |G| d\zeta_1 d\zeta_2 \\ &+ i \int \frac{\cos \theta}{4\pi r^2} (-\sin kr + kr \cos kr) |G| d\zeta_1 d\zeta_2 \end{aligned} \quad (3.6)$$

FIG. 12 BOUNDARY ELEMENT IDEALIZATION OF HALF THE RESERVOIR

where $|G|$ is the jacobian of transformation between $(x, y$ and $z)$ coordinates system and $(\zeta_1$ and $\zeta_2)$ coordinates system.

Using Gauss Quadrature for calculations of integrals:

$$H_{ij} = (H_{ij})_R + i(H_{ij})_I \quad (3.7)$$

where:

$$(H_{ij})_R = \frac{1}{4\pi} \sum_{k=1}^m \left(\frac{\cos \theta}{r^2} \right)_k (\cos kr + kr \sin kr)_k |G| \omega_k$$

$$(H_{ij})_I = \frac{1}{4\pi} \sum_{k=1}^m \left(\frac{\cos \theta}{r^2} \right)_k (-\sin kr + kr \cos kr)_k |G| \omega_k$$

and

m = the number of Gauss points.

ω_k = the weighting factor corresponding to the k^{th} Gauss point.

In a similar way, the term G_{ij} is obtained as follows:

$$\begin{aligned} G_{ij} &= \frac{-1}{4\pi} \int_{S_j} \frac{\cos kr - i \sin kr}{r} dS \\ &= \frac{-1}{4\pi} \int_{S_j} \frac{\cos kr}{r} dS + \frac{i}{4\pi} \int_{S_j} \frac{\sin kr}{r} dS \end{aligned} \quad (3.8)$$

Transforming into (ζ) local coordinates:

$$G_{ij} = - \int \frac{\cos kr}{4\pi r} |G| d\zeta_1 d\zeta_2 + i \int \frac{\sin kr}{4\pi r} |G| d\zeta_1 d\zeta_2 \quad (3.9)$$

$$G_{ij} = (G_{ij})_R + i(G_{ij})_I \quad (3.10)$$

where:

$$(G_{ij})_R = \frac{-1}{4\pi} \sum_{k=1}^m \left(\frac{\cos kr}{r} \right)_k |G| \omega_k$$

and

$$(G_{ij})_I = \frac{1}{4\pi} \sum_{k=1}^m \left(\frac{\sin kr}{r} \right)_k |G| \omega_k$$

FIG. 3.3 COLLAPSED QUADRILATERAL ELEMENTS

For the special case $i=j$

$$\begin{aligned}
 H_{ii} &= H_{ii} = -\frac{1}{2} + \int_{S_i} \frac{\partial P^*}{\partial n} dS \\
 &= -\frac{1}{2} + \int_{S_i} \frac{\partial P^*}{\partial r} \cos \theta dS \\
 &= -\frac{1}{2}
 \end{aligned} \tag{3.11}$$

because (r) and (n) are perpendicular to each other.

Eqn. (3.11) can be written in the same form as Eqn. (3.7), i.e.

$$H_{ii} = (H_{ii})_R + i(H_{ii})_I \tag{3.12}$$

where $(H_{ii})_R = -\frac{1}{2}$ and $(H_{ii})_I = 0$

As for (G_{ii}) , a special procedure was used to avoid the singularity over the boundary node. Excluding the singular point and an infinitesimal circle around it, the value of (G_{ii}) was found to equal zero. The evaluation of (G_{ii}) over the rest of the element was done by the use of collapsed quadrilateral elements (see Fig. 3.3).

If (N_i) is the shape function, $\partial N_i / \partial \eta_1$ can be expressed as:

$$\frac{\partial N_i}{\partial \eta_1} = \frac{\partial N_i}{\partial \zeta_1} \frac{\partial \zeta_1}{\partial \eta_1} + \frac{\partial N_i}{\partial \zeta_2} \frac{\partial \zeta_2}{\partial \eta_1} + \frac{\partial N_i}{\partial \zeta_3} \frac{\partial \zeta_3}{\partial \eta_1}$$

and $\partial N_i / \partial \eta_2$ will have a similar expression.

where:

$$\frac{\partial N_i}{\partial \zeta_1} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial \zeta_1} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial \zeta_1} + \frac{\partial N_i}{\partial z} \frac{\partial z}{\partial \zeta_1}$$

Similarly $\partial N_i / \partial \zeta_2$ and $\partial N_i / \partial \zeta_3$ could be written.

Two Jacobians have to be calculated. $|G_1|$ is the Jacobian relating the global system of coordinates (x , y and z) and the local system of coordinates (ζ_1 , ζ_2 and ζ_3). $|G_2|$ is the Jacobian relating the two local systems of coordinates (ζ_1 , ζ_2 and ζ_3) and (η_1 and η_2).

The shape functions to transfer from η coordinates system to ζ coordinates system are:

$$\zeta_1 = \psi_1 \zeta_{1(1)} + \psi_2 \zeta_{1(2)} \quad \text{and} \quad \zeta_2 = \psi_1 \zeta_{2(1)} + \psi_2 \zeta_{2(2)}$$

where:

$$\psi_1 = \frac{1}{4}(1 + \eta_1)(1 - \eta_2) \quad , \quad \psi_2 = \frac{1}{4}(1 + \eta_1)(1 + \eta_2)$$

$\zeta_{1(1)}$ and $\zeta_{1(2)}$ are the ζ_1 coordinate for points (1) and (2) respectively. Points (3) and (4) are chosen to lay always in the origin of $\zeta_1 \zeta_2$ system of coordinates, i.e. $\zeta_{3(3)} = \zeta_{4(4)} = 0$.

The shape functions to transfer from ζ system of coordinates to the global system of coordinates are:

$$x = \phi_1 x_{(1)} + \phi_2 x_{(2)} + \phi_3 x_{(3)} + \phi_4 x_{(4)}$$

$$y = \phi_1 y_{(1)} + \phi_2 y_{(2)} + \phi_3 y_{(3)} + \phi_4 y_{(4)}$$

and

$$z = \phi_1 z_{(1)} + \phi_2 z_{(2)} + \phi_3 z_{(3)} + \phi_4 z_{(4)}$$

where

$$\phi_1 = \frac{1}{4}(1 + \zeta_1)(1 - \zeta_2) \quad , \quad \phi_2 = \frac{1}{4}(1 + \zeta_1)(1 + \zeta_2)$$

$$\phi_3 = \frac{1}{4}(1 - \zeta_1)(1 + \zeta_2) \quad , \quad \phi_4 = \frac{1}{4}(1 - \zeta_1)(1 - \zeta_2)$$

The complete system of equations can be written in matrix form as:

$$[H]\{P\} = [G]\{q\} \quad (3.13)$$

where P and $\bar{q}$ are now complex and can be written in the form :

$$P = (P)_R + i(P)_I \quad \text{and} \quad \bar{q} = (\bar{q})_R + i(\bar{q})_I$$

Eqn. (3.13) can be written as:

$$\{ [H]_R + i[H]_I \} \{ [P]_R + i[P]_I \} = \{ [G]_R + i[G]_I \} \{ [\bar{q}]_R + i[\bar{q}]_I \} \quad (3.14)$$

where:

$[H]_R$ = A matrix containing the elements $(H_{ij})_R$

$[H]_I$ = A matrix containing the elements $(H_{ij})_I$

$[G]_R$ = A matrix containing the elements $(G_{ij})_R$

$[G]_I$ = A matrix containing the elements $(G_{ij})_I$

$[P]_R$ = A vector containing the elements $(P)_R$

$[P]_I$ = A vector containing the elements $(P)_I$

$[\bar{q}]_R$ = A vector containing the elements $(\bar{q})_R$

$[\bar{q}]_I$ = A vector containing the elements $(\bar{q})_I$

On carrying out the matrix multiplication contained in Eqn. (3.14) and separating the real and the imaginary parts, the following matrix equations are obtained :

$$[H]_R[P]_R - [H]_I[P]_I = [G]_R[\bar{q}]_R - [G]_I[\bar{q}]_I \quad (3.15)$$

and

$$[H]_I[P]_R + [H]_R[P]_I = [G]_I[\bar{q}]_R + [G]_R[\bar{q}]_I \quad (3.16)$$

The two systems of Eqns. (3.15) and (3.16) can be written as:

$$\begin{bmatrix} [H]_R & -[H]_I \\ [H]_I & [H]_R \end{bmatrix} \begin{Bmatrix} [P]_R \\ [P]_I \end{Bmatrix} = \begin{bmatrix} [G]_R & -[G]_I \\ [G]_I & [G]_R \end{bmatrix} \begin{Bmatrix} [\bar{q}]_R \\ [\bar{q}]_I \end{Bmatrix} \quad (3.17)$$

The system of Eqns. (3.17) can be written in a more compact form as:

$$[H'] \{ P' \} = [G'] \{ q' \} \quad (3.18)$$

where:

$$[H'] = \begin{bmatrix} [H]_R & -[H]_I \\ [H]_I & [H]_R \end{bmatrix}, \quad \{P'\} = \begin{Bmatrix} [P]_R \\ [P]_I \end{Bmatrix}$$

$$[G'] = \begin{bmatrix} [G]_R & -[G]_I \\ [G]_I & [G]_R \end{bmatrix}, \quad \{q'\} = \begin{Bmatrix} [\bar{q}]_R \\ [\bar{q}]_I \end{Bmatrix}$$

On solving the system of Eqns. (3.18), the unknown values of $[P]_R$ and $[P]_I$ at points on the upstream face of the dam can be obtained. The hydrodynamic pressure (P) at any point on the upstream face of the dam can thus be expressed as follows :

$$\begin{aligned} P &= P e^{i\omega t} \\ &= \{ [P]_R + i[P]_I \} \{ \cos \omega t + i \sin \omega t \} \end{aligned} \quad (3.19)$$

A computer program called HYPRFIRE (Hydrodynamic Pressure in Finite Reservoirs) was written to determine the hydrodynamic pressure distribution for the finite reservoir case. The description of the program and the user's manual are presented in Appendix B and the listing of the program is in Appendix C.

A number of example problems were solved. The results are presented in the following sections.

3.4 Results of Investigation

The following cases have been investigated using the computer program HYPRFIRE. In all the cases studied, the radius of the upstream face of the arch dam (R) was taken as equal to 90 feet and the depth of water in the reservoir (H) equal to 60 feet. These values were chosen so as to be able to compare the results with the available classical solutions [22] for the case of an infinite reservoir.

3.4.1 Exciting frequency in the upstream-downstream direction

a) Exciting frequency less than the fundamental natural frequency:

The response of the finite water reservoir shown in Fig. 3.1 to ground motion is calculated using the boundary element method. To test the convergence of the method, the problem is

solved twice, first with 142 constant boundary elements (see Fig. 3.4a) and then with 173 constant boundary elements (see Fig. 3.4b). The first mesh has six rings of boundary elements. The lengths of the rings perpendicular to the upstream face of the dam are 12, 15, 18, 21, 25 and 29 feet starting from the ring next to the upstream face of the dam. The second mesh has seven rings of boundary elements. The lengths of the rings are 10, 12, 15, 18, 21, 22 and 22 feet in the same order as the first mesh. The overall length (L) of the reservoir is 120 feet ($L = 2H$). Two different values of exciting frequency were considered, 0.4 and 0.8 of the fundamental natural frequency, ω_1^* which is equal to 123.6 rad./sec. or 19.67 cycles/sec. For $\omega/\omega_1^* = 0.4$ the wave length is equal to 600 feet. For $\omega/\omega_1^* = 0.8$ the wave length is 300 feet. The two meshes used are fine enough to represent these cases. The results are presented in Figs. 3.5 and 3.6. for the 142-element mesh. From Section D.3.1 (Appendix D), it is found that the convergence is very good. Theoretically, for finite reservoir the response should be in phase with the excitation ; i.e. the imaginary part of the result should be zero. The numerical results show that the imaginary part of the result does not represent more than 0.26 percent of the real part for $\omega/\omega_1^* = 0.4$ or 0.79 percent of the real part for $\omega/\omega_1^* = 0.8$ (see Appendix D).

Solutions were obtained (see Appendix D) for two different values of L ($L=2H$ and $3H$). It is found that if the length of the reservoir is equal to or greater than twice it's depth and the far end of the reservoir is stationary, the hydrodynamic pressure distribution is the same as that given by the classical solution for an infinite reservoir of the same depth.

b) Exciting frequency greater than the fundamental natural frequency:

The two meshes mentioned above were used to calculate the hydrodynamic pressure distribution and to check convergence for different values of exciting frequency. The frequency values used were 1.2, 1.6, 2.0, 2.4, 2.8 and 3.2 of the fundamental natural frequency, ω_1^* . The results are obtained for these cases using the 142-element mesh. They are presented in Figs. 3.7 to 3.12 inclusive. It is found that for this case, the hydrodynamic pressure distribution for an infinite reservoir can not be deduced from that for a finite reservoir. The hydrodynamic pressure distribution for the infinite reservoir case could not be plotted in the same figures. Tabulated results of the hydrodynamic pressure on the upstream face of the dam are given in Appendix D.

From the tables in Appendix D, it is found that the imaginary part starts to appear and increases as the ratio of ω/ω_1' increases. The maximum value of the imaginary part is equal to about 4 percent of the real part for $\omega/\omega_1' = 1.2$ and 1.6. The average value of this ratio is about 30 percent for $\omega/\omega_1' = 2.0$ and 10 percent for $\omega/\omega_1' = 2.4$. Then it increases rapidly to about 70 percent for $\omega/\omega_1' = 2.8$ and about 140 percent for $\omega/\omega_1' = 3.2$.

It should be noticed that the wave length for the case of $\omega/\omega_1' = 3.2$ is 75 feet. This means that the whole wave length is covered by 4 elements of the mesh or less. This representation is inaccurate. Using a finer mesh reduces the imaginary part in most cases (see Appendix D). Due to the restrictions on the computer memory size available, a finer mesh could not be used. Computer memory size restrictions also precluded the use of double precision numbers in the solution.

3.4.2 Exciting frequency in the cross-stream direction

a) The exciting frequency less than the fundamental natural frequency:

The response of the finite water reservoir is calculated as mentioned in the last section. The convergence in this case does not seem to be as good as that for the case where the exciting frequency is in the upstream-downstream direction. Results for this case are presented in Figs. 3.13 and 3.14. From the tables in Appendix D, it is found that the imaginary part average does not exceed 5 percent of the real part. It is found also that the hydrodynamic pressure distribution for the infinite reservoir can not be obtained from that for a finite reservoir. The hydrodynamic pressure distribution for the infinite reservoir could not be plotted in the same figures.

b) Exciting frequency greater than the fundamental natural frequency:

Results are presented in Figs. 3.15 to 3.20 inclusive for the same values of exciting frequencies as for the case of upstream-downstream direction using the same 142-element mesh. From the tables in Section D.3.2 (Appendix D), the convergence seems to be poor. For ω/ω_1' greater than or equal to 2.0, the method seems to break down. One of the reasons of this poor convergence might be that the mesh is not fine enough especially with the very high frequencies such as $\omega/\omega_1' = 2.8$

and 3.2 (see Appendix D). Due to the restrictions on the computer memory size available, a finer mesh could not be used.

It is clear from the results of the cases studied and the results given in Appendix D that the use of a finer mesh gives better results and reduces the imaginary part which appears in the output.

3.4.3 Response of a straight gravity dam

As a check on the boundary element computation procedure, a straight gravity dam was analysed treating it as a limiting case of an arch dam with $R = \infty$. The 3-D program HYPRFIRE was used. The length of the reservoir parallel to the dam is arbitrary and was chosen to be of the same order of magnitude as the height of the dam. The results obtained from the 3-D boundary element program were compared with the classical solutions for the 2-D problem (see Figs. 3.22 and 3.23).

The mesh used has 102 elements (see Fig. 3.21). Analyses were carried out for a range of excitation frequencies ranging from $\omega / \omega_1^* = 0.4$ to 3.2. The pressure distribution obtained from the boundary element method was nearly the same as that obtained by the classical solution except for $\omega / \omega_1^* = 3.2$. The match was better for smaller values of ω / ω_1^* .

For the case of $\omega / \omega_1^* = 3.2$, the results of the boundary element method did not correspond to the classical solution. Obviously for such high frequency, the mesh must be much finer or more refined elements must be used. In fact, fineness of the mesh must be increased as the exciting frequency increases and hence the wave length decreases.

Classical solutions for the finite reservoir case contain no imaginary part. The imaginary part obtained from the boundary element method was also quite small to be almost negligible up to the frequency ratio $\omega / \omega_1^* = 2.0$ (see Appendix D). For higher frequency ratios the mesh became inadequate and the imaginary part became substantial. In the case of $\omega / \omega_1^* = 3.2$ the inadequacy of the mesh resulted in a break down of the boundary element method.

5	10	15	20	25
4	9	14	19	24
3	8	13	18	23
2	7	12	17	22
1	6	11	16	21

DAM

35	54	71	87	103	119
34	53				
33	52	70	86	102	118
32	51				
31		69	85	101	117

PLANE OF SYMMETRY

41	60	77	93	109	125
42	61				
43	62	78	94	110	126
44					
45	63	79	95	111	127

BANK

142	139	136	133	130
141	138	135	132	129
140	137	134	131	128

RESERVOIR END

FIG. 3.4a 142-ELEMENT MESH

FIG. 3.4 a (cont'd) 142-ELEMENT MESH

6	12	18	24	30
5	11	17	23	29
4	10	16	22	28
3	9	15	21	27
2	8	14	20	26
1	7	13	19	25

DAM

41	63	83	101	118	134	150
40	62					
39	61	82	100	117	133	149
38	60					
37	59	81	99	116	132	148
36	58	80	98			

PLANE OF SYMMETRY

47	69	89	107	124	140	156
48	70					
49	71	90	108	125	141	157
50	72					
51	73	91	109	126	142	158
52	74	92	110			

BANK

173	170	167	164	161
172	169	166	163	160
171	168	165	162	159

RESERVOIR END

FIG. 3-4 b 173-ELEMENT MESH

BOTTOM

TOP

FIG. 3.4b (cont'd) 173-ELEMENT MESH

FIG. 3.5 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.6 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.7 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3-8 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3-9 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.10 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3-12 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.13 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3-14 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.15 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.16 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.17 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.18 HYDRODYNAMIC PRESSURE IN FINITE RESERVOIR

FIG. 3.19 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3-20 HYDRODYNAMIC PRESSURE IN FINITE WATER RESERVOIR

FIG. 3.21 GRAVITY DAM-RESERVOIR MESH

FIG. 3-22 COMPARISON OF SOLUTIONS, STRAIGHT GRAVITY DAM-RESERVOIR SYSTEM
(STATIONARY FAR END)

— CLASSICAL SOLUTION

--- 3-D B.E.M. PROGRAM

FIG. 3.22 (Cont'd) COMPARISON OF SOLUTIONS, STRAIGHT GRAVITY DAM-RESERVOIR SYSTEM
(STATIONARY FAR END)

— CLASSICAL SOLUTION

--- 3-D B.E.M. PROGRAM

FIG. 3.23 COMPARISON OF RESULTS, STRAIGHT GRAVITY DAM-RESERVOIR SYSTEM
(MOVING FAR END)

— CLASSICAL SOLUTION

--- 3-D B.E.M. PROGRAM

FIG. 3.23 (Cont'd) COMPARISON OF RESULTS, STRAIGHT GRAVITY DAM RESERVOIR SYSTEM (MOVING FAR END)

— CLASSICAL SOLUTION

--- 3-D B.E.M. PROGRAM

Chapter 4

HYDRODYNAMIC PRESSURE DISTRIBUTION IN AN INFINITE WATER RESERVOIR

4.1 Introduction

In this chapter, the boundary element method is applied to find the hydrodynamic pressure distribution on the upstream face of the arch dam shown in Fig. 2.1. The system and the ground motion are described in Section 2.2. Equations of motion are given in Section 2.3. Boundary conditions and the equations of the classical solutions are given in Section 2.4 for the upstream-downstream ground motion and in Section 2.5 for the cross-stream ground motion. The boundary element idealization is shown in Section 3.3. The infinite boundary elements are discussed in this chapter and their implement in the computer program is explained. The results of investigation are presented and compared with the available classical solutions.

4.2 Representation of Infinite Boundary Elements

Different types of infinite elements were surveyed to choose a reasonable type for the problem under consideration. These types of elements could be divided into two main categories. The first category maintains the infinite length of the element. The second category maps the element into a finite element. The main difficulty in the formulation of the element is the selection of a method for the integration of the functions over the element. If feasible an exact integration method would be the best method to use. However, an exact integration is not feasible in the boundary element method. The boundary node, which is the integration origin, changes its place over all boundary elements to formulate the matrices (see Fig. 4.1). This fact made it difficult to use the schemes of integration which are commonly employed with the infinite elements in the finite element method.

FIG.4.1 ILLUSTRATION OF INTEGRATION SURFACES AND NODES FOR INFINITE ELEMENTS

FIG.4.2 TWO-DIMENSIONAL MAPPING USED IN MAPPED INFINITE ELEMENT FORMULATION

From the available elements, two different types were studied and included in computer programs to solve the problem under consideration. The results are compared with the classical solutions. One formulation from each category mentioned above was tried.

The first element used is introduced in Section 2.6.4.2. This element is similar to the infinite element discussed by P. Bettess and O.C. Zienkiewicz [28]. However, the scheme of integration and the shape functions used are different. Gauss-Laguerre numerical integration is used in the infinite direction (ζ_1) and the standard Gauss-Legendre Integration is used in the perpendicular direction (ζ_2). Fifteen points were chosen in the direction of (ζ_1) and three in the direction of (ζ_2). The results obtained by using this element are comparable to the classical solutions. The comparison is given in the next section. This element formulation was included in a computer program called HYPINRE (Hydrodynamic Pressure in Infinite Reservoirs).

The second infinite element used in this study is chosen from the category which maps the infinite element on to a finite domain $-1 \leq \zeta_1 \leq \zeta_2$ (see Fig. 4.2) [26].

The expression used to achieve this is:

$$X = N_1(\zeta_2)[(2X_3 - X_4)\bar{N}_0 + X_4\bar{N}_2] + N_2(\zeta_2)[(2X_2 - X_1)\bar{N}_0 + X_1\bar{N}_2]$$

where:

$$\bar{N}_0 = \frac{-\zeta_1}{1 - \zeta_1} \quad \text{and} \quad \bar{N}_2 = \frac{1}{1 - \zeta_1}$$

$N_1(\zeta_2)$ and $N_2(\zeta_2)$ are standard Lagrange shape functions:

$$N_1(\zeta_2) = \frac{\zeta_2 - \zeta_2(2)}{\zeta_2(3) - \zeta_2(2)} = \frac{\zeta_2 - 1}{-2} = \frac{1}{2}(1 - \zeta_2)$$

and

$$N_2(\zeta_2) = \frac{\zeta_2 - \zeta_2(3)}{\zeta_2(2) - \zeta_2(3)} = \frac{\zeta_2 + 1}{2} = \frac{1}{2}(1 + \zeta_2)$$

In this mapping, some of the points which can be identified are:

* Point 1: ($\zeta_1 = 0$, $\zeta_2 = +1$)

$$\bar{N}_0 = 0, \bar{N}_2 = 1, N_1(\zeta_2) = 0 \text{ and } N_2(\zeta_2) = 1$$

$$X = 0[(2X_3 - X_4)0 + (X_4)1] + 1[(2X_2 - X_1)0 + (X_1)1] = X_1$$

* Point 2: ($\zeta_1 = -1$, $\zeta_2 = +1$)

$$\bar{N}_0 = \frac{1}{2}, \bar{N}_2 = \frac{1}{2}, N_1(\zeta_2) = 0 \text{ and } N_2(\zeta_2) = 1$$

$$X = 0[(2X_3 - X_4)\frac{1}{2} + (X_4)\frac{1}{2}] + 1[(2X_2 - X_1)\frac{1}{2} + (X_1)\frac{1}{2}] = X_2$$

* Point 6: ($\zeta_1 = +1$, $\zeta_2 = +1$)

$$\bar{N}_0 = \infty, \bar{N}_2 = \infty, N_1(\zeta_2) = 0 \text{ and } N_2(\zeta_2) = 1$$

$$X = 0[(2X_3 - X_4)\infty + (X_4)\infty] + 1[(2X_2 - X_1)\infty + (X_1)\infty] = (2X_3)\infty = \infty$$

In a similar manner, it can be shown that point 3 corresponds to $\zeta_1 = -1$ and $\zeta_2 = -1$, point 4 corresponds to $\zeta_1 = 0$ and $\zeta_2 = -1$ and point 5 corresponds to $\zeta_1 = +1$ and $\zeta_2 = -1$. The Jacobian can be calculated and the numerical integration using the standard Gauss-Legendre can be done. This scheme was also used in a computer program with 16 points in (ζ_1) direction and 4 points in (ζ_2) direction. The results of this scheme are far off from the classical solutions. The reason for these inaccurate results might be the contraction of the wave in the infinite direction and hence the inapplicability of the standard Gauss - Legendre integration scheme. The detailed results for this type of infinite element have not therefore been presented.

4.3 Results of Investigation

The following cases have been investigated using the computer programs incorporating the infinite elements discussed above. In all the cases studied, the radius of the upstream face of the arch dam (R) was taken as equal to 90 feet and the depth of water in the reservoir (H) equal to 60 feet. These values were chosen so as to be able to compare the results with the available classical solutions [22]. The description given next is applicable to both directions of exciting frequency except where otherwise noted.

4.3.1 Exciting frequency less than the fundamental natural frequency

The 142 element mesh shown in Fig. 3.4a was used after modification for the infinite case. The reservoir end is removed and new 16 infinite elements ; which do not contain boundary nodes

are added next to the last ring of elements at the far end of the reservoir. Two values of exciting frequency were considered, 0.4 and 0.8 of the fundamental natural frequency, ω_1^* . The results are presented in Figs. 4.3, 4.4, 4.11 and 4.12. It is found that there is excellent match between the boundary element solution and the classical solutions. Also, it was noticed that the use of HYPRFIRE program, prepared for finite reservoirs, with the 142 element mesh after removing the 15 elements at the reservoir end and adding one more ring of 16 elements to make the reservoir length equal to 150 feet gave the same results as those obtained by the infinite reservoir program. For the cross-stream ground motion, a 120 feet reservoir length was adequate. It appears that for ω/ω_1^* less than one, the use of infinite elements is not necessary.

4.3.2 Exciting frequency more than the fundamental natural frequency

The same modified mesh with infinite elements was used to calculate the hydrodynamic pressure distribution for six different frequency values. These values are 1.2, 1.6, 2.0, 2.4, 2.8 and 3.2 of the fundamental natural frequency, ω_1^* . The results are presented for the upstream-downstream ground motion in Figs. 4.5 to 4.10 inclusive and for the cross-stream ground motion in Figs. 4.13 to 4.18 inclusive. It is found that although the boundary element solutions are similar to the classical solutions, the match is not completely satisfactory. It is apparent that more elaborate work is required in the formulation of the infinite element. Also, it is noticed that the results obtained by using the HYPRFIRE program, which is prepared for finite reservoirs, are sensitive to the mesh lengths. After some trials using the HYPRFIRE program, it is found that for the upstream-downstream ground motion, better results were obtained when the length of the mesh was equal to a multiple of the wave length or an integer multiple plus half wave length. In Figs. 4.5 to 4.10 only one trial is shown. This did not work for the case of $\omega/\omega_1^* = 1.2$. It is also found that for the cross-stream ground motion better results were obtained when the length of the mesh was equal to 120 feet, $L=2H$. Again, only one trial is shown in Figs. 4.13 to 4.18. An exception occurred for the case of $\omega/\omega_1^* = 1.6$ (see Fig. 4.14). As in the finite reservoir case, the mesh gradation is not adequate for the very high frequency values such as $\omega/\omega_1^* = 2.8$ and 3.2.

According to the results obtained for the cases of $\omega/\omega_1^* > 1$, it is concluded that they are not satisfactorily solved. The use of infinite elements should give better results but more work is required.

— CLASSICAL SOLUTION
 $\frac{0^\circ}{45^\circ}$
 Δ ○ HYPRFIRE PROGRAM (L=150')
 ○ HYPINRE PROGRAM

FIG. 4.3 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

— CLASSICAL SOLUTION

$\sigma/45^\circ$

■ • HYPRFIRE PROGRAM ($L=150'$)

▲ ○ HYPINRE PROGRAM

FIG. 4. 4 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

— CLASSICAL SOLUTION

$0^\circ, 45^\circ$

● • HYPRFIRE PROGRAM ($L=200$)

△ ○ HYPINRE PROGRAM

FIG. 4.5 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

— CLASSICAL SOLUTION

0° 45°

■ ● HYPRFIRE PROGRAM (L=150)

▲ ○ HYPINRE PROGRAM

FIG. 4.6 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

— CLASSICAL SOLUTION

0° 45°
 ● ● HYPRFIRE PROGRAM ($L=120$)
 ○ ○ HYPINRE PROGRAM

FIG. 4.7 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

FIG. 4.8 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

— CLASSICAL SOLUTION
 $\frac{0^\circ}{45^\circ}$
 ■ ● HYPRFIRE PROGRAM ($L=150'$)
 ▲ ○ HYPINRE PROGRAM

FIG. 4. 9 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

FIG. 4.10 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO UPSTREAM-DOWNSTREAM GROUND MOTION.

FIG. 4-11 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

— CLASSICAL SOLUTION

22.5° 45°
 ■ ● HYPRFIRE PROGRAM (L=120)
 △ ○ HYPINRE PROGRAM

FIG. 4.12 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

FIG.4-13 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

FIG. 4-14 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

FIG. 4.15 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

FIG. 4-16 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

22

OF / DE

— CLASSICAL SOLUTION

$\frac{22.5^\circ}{45^\circ}$
 $\blacksquare \bullet$ HYPFIRE PROGRAM (L=120)
 $\Delta \circ$ HYPINRE PROGRAM

FIG. 4.17 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

FIG. 4-18 COMPLEX FREQUENCY RESPONSE FOR HYDRODYNAMIC PRESSURE DUE TO CROSS STREAM GROUND MOTION.

Chapter 5

SUMMARY AND CONCLUSIONS

5.1 Summary

A boundary element method for the calculation of the hydrodynamic pressure on arch dams is presented. The boundary element method is introduced as a numerical technique. Then a review is made of the literature related to the boundary element method and the arch dam analysis problem. The problem of hydrodynamic pressure on arch dam is described. A model for the system is described and the equations of motion are presented. This is followed by a description of the analysis method for a harmonic upstream-downstream as well to cross-stream ground motion. In each case, the boundary conditions and the classical solutions are presented.

The boundary element method is then described. The reduction of the governing differential equation to the well known Helmholtz equation and the choice of an appropriate fundamental solution to convert the domain problem to a boundary problem are discussed.

The method is applied to the problem of hydrodynamic pressure distribution. It is applied first to the case of an arch dam storing a finite reservoir. The boundary conditions and the idealization of the method are given. Results of investigation are presented for both directions of the horizontal exciting frequency. The straight gravity dam case is studied as a limiting case of the arch dam problem. The results are compared with the known classical solutions.

The method is then applied to the case of an arch dam storing an infinite reservoir. The infinite elements are formulated and included in the computer program. Results of investigation are presented for both directions of horizontal exciting frequency and compared with the classical solutions.

5.2 Conclusions

As a result of the present study, the following conclusions are drawn:

- 1) The boundary element method offers certain advantages over the domain methods. The main advantage is the reduction in size of the problem because instead of the domain, only the surface enclosing the domain needs to be defined. However, unlike in the case of the finite element method, the coefficient matrices in the resulting simultaneous equations are unsymmetrical and fully populated. This may, at times, offset the advantage obtained from a reduction in the size of the problem.
- 2) An interesting feature of the boundary element method is the simplicity of the input data required to run a problem compared to the data required to run a finite element program. The availability of mesh generation programs can reduce the importance of this feature.
- 3) The boundary element method is an effective tool for the analyses of hydrodynamic forces on arch dams storing reservoirs of finite or infinite length provided the exciting frequency ω is less than the first natural frequency ω_1^* . For infinite reservoirs and ω/ω_1^* greater than 1, the boundary element solutions are not quite satisfactory. Use of infinite elements shows promise but more work is required in this area.

5.3 Recommendations for future research

The following recommendations are made for future research:

- 1) Work is required on the development of efficient methods for solving large fully populated non-symmetrical matrices; most of the existing development has been confined to the methods of solving symmetrical banded system of equations which arise in the finite element method.
- 2) The effect of water viscosity has always been neglected in calculating the hydrodynamic pressure on dams because the problem becomes complicated if the viscosity is taken into account. If suitable methods can be developed, it would be of interest to include the effect of water viscosity in the solution.

- 3) The dam was assumed to be rigid and fixed at the base and banks in this study. Inclusion of the effects of the flexibility of the dam and the different types of fixity will be of great interest and should be straightforward.
- 4) More development is needed in the field of infinite element formulation specially for use in the boundary element formulation presented in this study.
- 5) The study of the effects of changing the different parameters affecting the hydrodynamic pressure distribution such as the dam geometry and the geometry of banks needs more research.

REFERENCES

- 1) O.C. Zienkiewicz, "The Finite Element Method in Engineering Science", McGraw-Hill, London, U.K., 3rd Ed., 1977.
- 2) W.H. Bowes and L.T. Russel, "Stress Analysis by the Finite Element Method for Practicing Engineers", D.C. Heath and Company, U.S.A., 1975.
- 3) R. Bellman, "Methods for the Numerical Solutions of the Partial Differential Equations", Modern Analytic and Computational Methods in Science and Mathematics.
- 4) H.M. Westergaard, "Water Pressures on Dams During Earthquakes", Transactions, ASCE, Vol. 98, pp. 418-433, 1933.
- 5) C.N. Zangar, "Hydrodynamic Pressures on Dams Due to Horizontal Earthquake Effects", Engineering Monograph No. 11, U.S. Bureau of Reclamation, May 1952.
- 6) R.W. Clough and A.K. Chopra, "Earthquake Response Analysis of Concrete Dams", in Structural and Geotechnical Mechanics, Editor W.J. Hall, Prentice-Hall, 1977.
- 7) S. Kotsubo, "External Forces on Arch Dams During Earthquakes", Memoirs Faculty of Engineering, Kyushu University, Fukuoka, Japan, Vol. 20, No. 4, 1961.
- 8) P.R. Perumulswami and L. Kar, "Earthquake Hydrodynamic Forces on Arch Dams", Journal of Engineering Mechanics Division, ASCE, Vol. 99, No. EM5, Oct. 1973.
- 9) O.C. Zienkiewicz and B. Nath, "Earthquake Pressures on Arch Dams - An Electric Analogue Solution", Proceedings, Institute of Civil Engineers, Vol. 25, pp. 165-175, Proc. Paper 6668, 1963.
- 10) R. Priscu, A. Popovici, L. Ilie and D. Stenatiu, "New Aspects in the Earthquake Analysis of Arch Dams", in Criteria and Assumptions for Numerical Analysis of Dams, Ed. by D.N. Naylor, et al., Proceedings of an International Symposium, Swansea, U.K., pp. 709-725, Sept. 1975.

- 11) T. Hatano and K. Hiroshi, "Numerical Solution of Hydrodynamic Pressures During Earthquakes on Arch Dams", Technical Report: C-65002, Technical Laboratory, Central Research Institute of Electric Power Industry, Tokyo, Japan, Jan. 1966.
- 12) A.K. Chopra, E.L. Wilson and I. Farhoomand, "Earthquake Analysis of Reservoir - Dam Systems", Proceedings, Fourth World Conference on Earthquake Engineering, Santiago, Chile, Jan. 1969.
- 13) O.C. Zienkiewicz and R.E. Newton, "Coupled Vibrations of a Structure Submerged in a compressible Fluid", International Symposium on Finite Element Techniques, Stuttgart, West Germany, May 1969.
- 14) T. Hatano and T. Nakagama, "Seismic Analysis of Arch Dams - Coupled Vibrations of Dam Body and Reservoir Water", Technical Report, Technical Laboratory No. 1, Central Research Institute of Electric Power Industry, Tokyo, Japan, Nov. 1972.
- 15) Y.P. Chang, C.S. Kang and D.J. Chen, "The Use of Fundamental Green's Functions for the Solution of Problems of Heat Conduction in Anisotropic Media", Journal of Heat and Mass Transfer, Vol. 16, pp. 1905-1918, 1973.
- 16) T.A. Cruse and F.J. Rizzo, "A Direct Formulation and Numerical Solution of the General Transient Elastodynamic Problem .I", Journal of Mathematical Analysis and Applications, Vol. 22, No. 1, pp. 244-259, 1968.
- 17) F.J. Rizzo and D.J. Shippy, "A Method of Solution for Certain Problems of Transient Heat Conduction", AIAA Journal, Vol. 8, No. 11, pp. 2004-2009, 1970.
- 18) F.J. Rizzo and D.J. Shippy, "An Application of the Correspondance Principle of Linear Viscoelasticity Theory", SIAM Journal on Applied Mathematics, Vol. 21, No. 2, pp. 321-330, 1971.
- 19) D.J. Shippy, "Application of the Boundary Integral Equation Method to Transient Phenomena in Solids", Boundary Integral Equation Method : Computational Applications in Applied Mechanics, AMD, Vol. 11, AMSE, 1975.

- 20) J.R. Hutchinson, "Determination of Membrane Vibrational Characteristics by the Boundary Integral Equation Method", in *Recent Advances in Boundary Element Methods*, pp. 301-316, Ed. by C.A. Brebbia, Pentech Press, London, U.K., 1978.
- 21) Y.G. Hanna, "Application of Boundary Element Method to Certain Problems in Structural Dynamics", M.Eng. Thesis, Carleton University, Ottawa, Canada, Apr. 1980.
- 22) C.S. Porter and A.K. Chopra, "Dynamic Response of Simple Arch Dams Including Hydrodynamic Interaction", Report No. EERC-80/17, Earthquake Engineering Research Center, University of California, Berkeley, U.S.A., July 1980.
- 23) C.A. Brebbia, "The Boundary Element Method for Engineers", Pentech Press, Devon, U.K., 1978.
- 24) C.A. Brebbia, "Introduction to Boundary Element Methods" in "Recent Advances in Boundary Element Methods", Pentech Press, London, U.K., pp. 36-41.
- 25) A.K. Chopra, "Hydrodynamic Pressures on Dams During Earthquakes", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 93, No. EM6, Proc. Paper 5695, pp. 205-223, Dec. 1967.
- 26) O.C. Zienkiewicz, C. Emson and P. Bettess, "A Novel Boundary Infinite Element", *International Journal for Numerical Methods in Engineering*, Vol. 19, pp. 393-404, 1983.
- 27) J.O. Watson, "Advanced Implementation of the Boundary Element Method for Two- and Three-Dimensional Elastostatics", in "Developments in Boundary Element Methods-1", pp. 31-63, Ed. by P.K. Banerjee and R. Butterfield, Applied Science Publishers Ltd., London, U.K., 1979.
- 28) P. Bettess and O.C. Zienkiewicz, "Diffraction and Refraction of Surface Waves using Finite and Infinite Elements", *International Journal for Numerical Methods in Engineering*, Vol. 11, pp. 1271-1290, 1977.

Appendix A

NOTATION

$[B]$	matrix used in calculations for hydrodynamic pressure P .
C	velocity of sound in water. (4720 ft/sec. $\approx$ 1440 m/sec.)
c_i	numerical multiplier defined in Section 2.6.3.
$\{F\}$	vector used in calculations for hydrodynamic pressure P .
$[G]$	matrix containing the elements G_{ij} .
$[G]$	Jacobian of transformation between $(x, y$ and $z)$ coordinates system and $(\zeta_1$ and $\zeta_2)$ coordinates system.
g	acceleration due to gravity.
$[H]$	matrix containing the elements H_{ij} .
H	depth of water in the reservoir. (Fig. 2.1)
H_d	height of the arch dam. (Fig. 2.1)
i	$\sqrt{-1}$
$J_n(z)$	Bessel function of first kind of order (n) .
k	wave constant $= \omega/C$.
$K_n(z)$	modified Bessel function of the second kind of order (n) .
l_x	the direction cosine of the outward normal to the x axis.
l_y	the direction cosine of the outward normal to the y axis.
l_z	the direction cosine of the outward normal to the z axis.
m_i	the largest integer (m) satisfying the inequality $ \omega/\omega_i > (2m - 1)$.
$\underline{n}$	the outward normal at the point under consideration.

$\{P\}$	vector containing the values of hydrodynamic pressure P .
P	hydrodynamic pressure in excess of the hydrostatic pressure.
P_{OB}^x	hydrodynamic pressure on the upstream face of the dam due to acceleration of only the reservoir banks in the upstream-downstream direction.
P_{OB}^*	complex frequency response function for P_{OB}^x .
P_{OB}^y	hydrodynamic pressure on the upstream face of the dam due to acceleration of only the reservoir banks in the cross-stream direction.
P_{OB}^{y*}	complex frequency response function for P_{OB}^y .
P_{OD}^x	hydrodynamic pressure on the upstream face of the dam due to acceleration of the rigid dam in the upstream-downstream direction with the banks remaining stationary.
P_{OD}^*	complex frequency response function for P_{OD}^x .
P_{OD}^y	hydrodynamic pressure on the upstream face of the dam due to acceleration of the rigid dam in the cross-stream direction with the banks remaining stationary.
P_{OD}^{y*}	complex frequency response function for P_{OD}^y .
P^*	fundamental solution for the Helmholtz equation.
$\{q\}$	vector containing the values of the derivative of hydrodynamic pressure P .
q^*	derivative of P^* with respect to the outward normal $\underline{n}$.
R	radius of upstream face of the dam.
$\underline{r}$	vector from the boundary node to the point under consideration.
S	boundary of the domain V .
t	time variable.
u, v and w	components of the displacement of a particle in the cartesian coordinates.
V	domain under consideration.

v^r	the radial component of water particle displacements.
v^θ	the tangential component of water particle displacements.
v^z	the vertical component of water particle displacements.
W	unit weight of water.
$\{z\}$	the vector of unknowns hydrodynamic pressure P and derivative of pressure q .
x, y and z	orthogonal cartesian coordinates. (Fig. 2.1)
$Y_n(x)$	Bessel function of the second kind of order (n) .
Δ^i	Dirac Delta function centered at point i .
c_n	numerical multiplier equal to 1 when $n=0$ but equal to 2 when $n \neq 0$.
θ	angular coordinate of reservoir-dam system (Fig. 2.1); also, the angle between $\underline{r}$ and $\underline{n}$ (Fig. 3.2).
ρ	the mass of water per unit volume.
ω	circular frequency of harmonic ground motion.
ω_m^r	$= \frac{\pi C}{2H}(2m-1)$, $m=1,2,3,\dots$; the m^{th} eigen-frequency of the reservoir.
μ_n	summation index $= 4n+2$.
ξ_1 and ξ_2	local system of coordinates.

Appendix B

USER'S GUIDE TO COMPUTER PROGRAMS

B.1 Description of the HYPRFIRE Program

This FORTRAN computer program can be used to calculate the hydrodynamic pressure on a rigid dam storing a finite water reservoir and subjected to a horizontal excitation frequency. The program uses the boundary element method with constant boundary elements, i.e. elements with mid-element nodes only. The functions are assumed to have constant values over the element equal to their values at the mid-element node.

There are no limitations on the shape of the dam or the reservoir. The dam could have any shape with a vertical or inclined upstream face. The reservoir bottom or banks could be also of any shape. The upstream surface of the dam, the banks, the surface, the bottom and the upstream end of the reservoir are to be covered by a boundary element mesh.

The main program defines the limiting number of boundary nodes that it can handle. The number is set to 210 as shown in the listing (see Appendix C). The program calls the four following subroutines:

INPUT	To read all data and print the first part of the output.
FMAT	To form the two matrices [G] and [H] and rearrange them according to the boundary conditions to form the matrix [B] of Eqn. (2.36).
SLNPD	To solve the system of equations by Gauss elimination.
OUTPUT	To reorder the unknowns vector and print out the rest of the output file.

The subroutine FMAT calls the two following subroutines:

INTE	To compute the [G] and [H] matrix elements except those on the diagonal by means of numerical integration along the boundary elements.

INLO

To calculate the diagonal elements of matrix [G]. It calls the subroutine INTL four times and sum up the values calculated. Subroutine INTL uses collapsed rectangular elements to eliminate the singularity.

B.2 Input Data

Card Group	No. of Cards	Format	FORTTRAN variables
A	1	18A4	TITLE
B	1	General Format	N=No. of coordinate nodes. NE=No. of boundary nodes. CONST=Wave constant= ω/C
C	N	General Format	X, Y, Z coordinates of each node.
D	NE	General Format	K(1,I), K(2,I), K(3,I) and K(4,I) (Nodes' numbers at the corners of each element)
E	NE	General Format	KODE(I)=0 when pressure is known. and =1 when pressure derivative is known. FI(I)=Pressure value or pressure derivative value

* Note that nodes' numbers for each element have to be given in counterclockwise direction looking into the reservoir.

B.3 Control Cards

The program has been implemented on the Honeywell CP VI at Carleton University. The following Job control cards are required to execute the program.

```
!JOB
!RES MEM=210 , TIME=25
!FORTRAN HYPRFIRE
```



```

!LINK
!SET 5 DATA
!SET 6 OUT
!*L
!END

```

The input data must be on file DATA, while the output will be directed to file named OUT.

B.4 Output

The program prints out the input data as well as the hydrodynamic pressure value and the pressure derivative value at each boundary node. The real and the imaginary parts of both the pressure and the pressure derivative are given. The absolute maximum of the pressure is also given.

B.5 Example

For the purpose of illustrating the use of the program, a very simple example is presented here.

A cubic reservoir is solved using 24-element mesh (see Fig. B.1). The reservoir has a length and a height each of 60 feet. The system is subjected to a harmonic upstream - downstream excitation with a frequency equal to 49.44 rad/sec. ($\omega/\omega_1 = 0.4$). The input data and the output are shown in the following sections.

B.6 Input data file

```

EXAMPLE
26,24,0.010471975
0 60 0
0 60 30
0 60 60
0 30 0
0 30 30
0 30 60
0 0 0
0 0 30
0 0 60

```

FIG. B.1 24-ELEMENT MESH EXAMPLE

30 60 0
30 60 30
30 60 60
30 30 0
30 30 60
30 0 0
30 0 30
30 0 60
60 60 0
60 60 30
60 60 60
60 30 0
60 30 30
60 30 60
60 0 0
60 0 30
60 0 60

4,5,2,1
5,6,3,2
7,8,5,4
8,9,6,5
1,10,13,4
4,13,15,7
7,15,16,8
8,16,17,9
9,17,14,6
6,14,12,3
2,3,12,11
1,2,11,10
10,18,21,13
13,21,24,15
15,24,25,16
16,25,26,17
17,26,23,14
14,23,20,12
11,12,20,19
10,11,19,18
18,19,22,21
19,20,23,22
21,22,25,24
22,23,26,25
1,1.937888
1,1.937888
1,1.937888
1,1.937888

1,0
1,0
1,0
1,0
0,0
0,0
1,0
1,0
1,0
1,0
1,0

1.0
0.0
0.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0

B.7 Output file

*****

CONSTANT QUADRILATERAL BOUNDARY ELEMENTS

EXAMPLE

DATA

NUMBER OF NODES = 26
NUMBER OF ELEMENTS = 24
THE CONSTANT = .010471975

THE COORDINATES OF NODES

NODE	X	Y	Z
1	.000	60.000	.000
2	.000	60.000	30.000
3	.000	60.000	60.000
4	.000	30.000	.000
5	.000	30.000	30.000
6	.000	30.000	60.000
7	.000	.000	.000
8	.000	.000	30.000
9	.000	.000	60.000
10	30.000	60.000	.000
11	30.000	60.000	30.000
12	30.000	60.000	60.000
13	30.000	30.000	.000
14	30.000	30.000	60.000
15	30.000	.000	.000
16	30.000	.000	30.000
17	30.000	.000	60.000
18	60.000	60.000	.000
19	60.000	60.000	30.000
20	60.000	60.000	60.000
21	60.000	30.000	.000
22	60.000	30.000	30.000

23	60.000	30.000	60.000
24	60.000	.000	.000
25	60.000	.000	30.000
26	60.000	.000	60.000

CONNECTIVITY				BOUNDARY CONDITIONS		
ELEMENT	NODES			CODE	VALUE	
1	4	5	2	1	1	.1937888E+01
2	5	6	3	2	1	.1937888E+01
3	7	8	5	4	1	.1937888E+01
4	8	9	6	5	1	.1937888E+01
5	1	10	13	4	1	.0000000E+00
6	4	13	15	7	1	.0000000E+00
7	7	15	16	8	1	.0000000E+00
8	8	16	17	9	1	.0000000E+00
9	9	17	14	6	0	.0000000E+00
10	6	14	12	3	0	.0000000E+00
11	2	3	12	11	1	.0000000E+00
12	1	2	11	10	1	.0000000E+00
13	10	18	21	13	1	.0000000E+00
14	13	21	24	15	1	.0000000E+00
15	15	24	25	16	1	.0000000E+00
16	16	25	26	17	1	.0000000E+00
17	17	26	23	14	0	.0000000E+00
18	14	23	20	12	0	.0000000E+00
19	11	12	20	19	1	.0000000E+00
20	10	11	19	18	1	.0000000E+00
21	18	19	22	21	1	.0000000E+00
22	19	20	23	22	1	.0000000E+00
23	21	22	25	24	1	.0000000E+00
24	22	23	26	25	1	.0000000E+00

RESULTS

BOUNDARY ELEMENTS	REAL PART		IMAGINARY PART		P. TOTAL
	PRESSURE	P. DERIVATIVE	PRESSURE	P. DERIVATIVE	
1	.90750E+02	.19379E+01	-.80507E-01	.00000E+00	.90750E+02
2	.53107E+02	.19379E+01	-.14301E-01	.00000E+00	.53107E+02
3	.90750E+02	.19379E+01	-.80508E-01	.00000E+00	.90750E+02
4	.53107E+02	.19379E+01	-.14301E-01	.00000E+00	.53107E+02
5	.72967E+02	.00000E+00	-.86828E-01	.00000E+00	.72967E+02
6	.72967E+02	.00000E+00	-.86829E-01	.00000E+00	.72967E+02
7	.69164E+02	.00000E+00	-.69105E-01	.00000E+00	.69164E+02
8	.32835E+02	.00000E+00	-.56190E-02	.00000E+00	.32835E+02
9	.00000E+00	-.30384E+01	.00000E+00	-.10467E-02	.00000E+00
10	.00000E+00	-.30384E+01	.00000E+00	-.10467E-02	.00000E+00
11	.32835E+02	.00000E+00	-.56193E-02	.00000E+00	.32835E+02
12	.69164E+02	.00000E+00	-.69105E-01	.00000E+00	.69164E+02
13	.47982E+02	.00000E+00	-.37728E-01	.00000E+00	.47982E+02
14	.47982E+02	.00000E+00	-.37728E-01	.00000E+00	.47982E+02

15	.44839E+02	.00000E+00	- .21467E-01	.00000E+00	.44839E+02
16	.17839E+02	.00000E+00	.23358E-01	.00000E+00	.17839E+02
17	.00000E+00	- .13923E+01	.00000E+00	- .42058E-02	.00000E+00
18	.00000E+00	- .13923E+01	.00000E+00	- .42058E-02	.00000E+00
19	.17839E+02	.00000E+00	.23358E-01	.00000E+00	.17839E+02
20	.44839E+02	.00000E+00	- .21468E-01	.00000E+00	.44839E+02
21	.42378E+02	.00000E+00	- .10117E-01	.00000E+00	.42378E+02
22	.16693E+02	.00000E+00	.31989E-01	.00000E+00	.16693E+02
23	.42378E+02	.00000E+00	- .10117E-01	.00000E+00	.42378E+02
24	.16693E+02	.00000E+00	.31989E-01	.00000E+00	.16693E+02

DETERMINANT = .878124E-05

*****

B.8 Description of the HYPINRE Program

This FORTRAN computer program is a modified version of the HYPRFIRE program mentioned in Section B.1. The modification was done in order to represent the infinite reservoir. A new subroutine (INT2) is added. This subroutine is called by subroutine FMAT to calculate the integrals of [G] and [H] matrix elements over the infinite elements at the far end of the reservoir. These integrals are added in the matrices to the corresponding parts of the boundary elements in the last ring of the mesh. These new infinite boundary elements do not include boundary nodes. This helps in two aspects:

- 1) The problem of singularity in these elements is avoided.
- 2) The matrices do not become larger.

The program defines the limiting dimensions of the system of equations. In this case, the max. number of nodes is set to be 210, the max. number of infinite strips is 20, and the max. number of geometric corner nodes to define the elements is 250.

B.9 Input Data

Card Group	No. of Cards	Format	FORTTRAN variables
A	1	18A4	TITLE
B	1	General Format	N=No. of coordinate nodes. NE=No. of boundary nodes. NF=No. of infinite elements. CONST=wave constant= ω/C .
C	N	General Format	X, Y, Z coordinates of each node.
D	NE+NF	General Format	K(1,I), K(2,I), K(3,I) and K(4,I) (Nodes' numbers at the corners of each element)
E	NE	General Format	KODE(I)=0 when pressure is known and =1 when pressure derivative is known. FI(I)=Pressure value or pressure derivative value

* Note that nodes' numbers for each element have to be given in counterclockwise direction looking into the reservoir. Also, nodes for infinite elements have to be given in the same direction starting and ending by far nodes. Referring to Fig. 2.2, the nodes have to be given in this order 1,2,3,4.

B.10 Control Cards

The program has been implemented on the Honeywell CP VI computer at Carleton University. The following Job control cards are required to execute the program.

```
!JOB
!RES MEM=250 , TIME=30
!FORTRAN HYPINRE
!LINK
!SET 5 DATA
```

```

!SET 6 OUT
!*L
!END

```

The input data must be on file DATA, while the output will be directed to file named OUT.

B.11 Output

The program prints out the input data as well as the hydrodynamic pressure value and the pressure derivative value at each boundary node. The real and the imaginary parts of both the pressure and the pressure derivative are given. The absolute maximum of the pressure is also given. It should be noticed that no results are given for the infinite elements because there are no nodes in these elements.

B.12 Example

The same cubic reservoir mentioned in Section B.5 will be solved again on the assumption that the reservoir is infinite this time. (see Fig. B.2). Twenty boundary elements plus eight infinite elements are used.

B.13 Input data file

```

EXAMPLE
33,20,8,0.010471975
0 60 0
0 60 30
0 60 60
0 30 0
0 30 30
0 30 60
0 0 0
0 0 30
0 0 60
30 60 0
30 60 30
30 60 60
30 30 0
30 30 60
30 0 0

```

FIG. 8.2 20-ELEMENT, 8-NODE ELEMENTS MESH EXAMPLE

30 0 30
30 0 60
60 60 0
60 60 30
60 60 60
60 30 0
60 30 60
60 0 0
60 0 30
60 0 60
90 60 0
90 60 30
90 60 60
90 30 0
90 30 60
90 0 0
90 0 30
90 0 60
4,5,2,1
5,6,3,2
7,8,5,4
8,9,6,5
1,10,13,4
4,13,15,7
7,15,16,8
8,16,17,9
9,17,14,6
6,14,12,3
3,12,11,2
2,11,10,1
10,18,21,13
13,21,23,15
15,23,24,16
16,24,25,17
17,25,22,14
14,22,20,12
12,20,19,11
11,19,18,10
29,21,18,26
31,23,21,29
32,24,23,31
33,25,24,32
30,22,25,33
28,20,22,30
27,19,20,28
26,18,19,27
1,1.937888
1,1.937888
1,1.937888
1,1.937888
1,0
1,0
1,0
1,0
0,0
0,0

1.0
1.0
1.0
1.0
1.0
1.0
0.0
0.0
1.0
1.0

B.14 Output file

*****

CONSTANT QUADRILATERAL BOUNDARY ELEMENTS

EXAMPLE

DATA

NUMBER OF NODES = 33
NUMBER OF ELEMENTS = 20
INFINITE ELEMENTS = 8
THE CONSTANT = .010471975

THE COORDINATES OF NODES

NODE	X	Y	Z
1	.000	60.000	.000
2	.000	60.000	30.000
3	.000	60.000	60.000
4	.000	30.000	.000
5	.000	30.000	30.000
6	.000	30.000	60.000
7	.000	.000	.000
8	.000	.000	30.000
9	.000	.000	60.000
10	30.000	60.000	.000
11	30.000	60.000	30.000
12	30.000	60.000	60.000
13	30.000	30.000	.000
14	30.000	30.000	60.000
15	30.000	.000	.000
16	30.000	.000	30.000
17	30.000	.000	60.000
18	60.000	60.000	.000
19	60.000	60.000	30.000
20	60.000	60.000	60.000

21	60.000	30.000	.000
22	60.000	30.000	60.000
23	60.000	.000	.000
24	60.000	.000	30.000
25	60.000	.000	60.000
26	90.000	60.000	.000
27	90.000	60.000	30.000
28	90.000	60.000	60.000
29	90.000	30.000	.000
30	90.000	30.000	60.000
31	90.000	.000	.000
32	90.000	.000	30.000
33	90.000	.000	60.000

CONNECTIVITY
ELEMENT   NODES

BOUNDARY CONDITIONS
CODE   VALUE

1	4	5	2	1	1	.1937888E+01
2	5	6	3	2	1	.1937888E+01
3	7	8	5	4	1	.1937888E+01
4	8	9	6	5	1	.1937888E+01
5	1	10	13	4	1	.0000000E+00
6	4	13	15	7	1	.0000000E+00
7	7	15	16	8	1	.0000000E+00
8	8	16	17	9	1	.0000000E+00
9	9	17	14	6	0	.0000000E+00
10	6	14	12	3	0	.0000000E+00
11	3	12	11	2	1	.0000000E+00
12	2	11	10	1	1	.0000000E+00
13	10	18	21	13	1	.0000000E+00
14	13	21	23	15	1	.0000000E+00
15	15	23	24	16	1	.0000000E+00
16	16	24	25	17	1	.0000000E+00
17	17	25	22	14	0	.0000000E+00
18	14	22	20	12	0	.0000000E+00
19	12	20	19	11	1	.0000000E+00
20	11	19	18	10	1	.0000000E+00

INFINITE ELEMENTS CONNECTIVITY
ELEMENT   NODES

21	29	21	18	26
22	31	23	21	29
23	32	24	23	31
24	33	25	24	32
25	30	22	25	33
26	28	20	22	30
27	27	19	20	28
28	26	18	19	27

RESULTS

BOUNDARY ELEMENTS	REAL PART		IMAGINARY PART		P. TOTAL
	PRESSURE	P. DERIVATIVE	PRESSURE	P. DERIVATIVE	
1	.79571E+02	.19379E+01	.44176E+01	.00000E+00	.79693E+02
2	.48823E+02	.19379E+01	.17250E+01	.00000E+00	.48854E+02
3	.79571E+02	.19379E+01	.44176E+01	.00000E+00	.79693E+02
4	.48823E+02	.19379E+01	.17250E+01	.00000E+00	.48854E+02
5	.60145E+02	.00000E+00	.48823E+01	.00000E+00	.60343E+02
6	.60145E+02	.00000E+00	.48823E+01	.00000E+00	.60343E+02
7	.57227E+02	.00000E+00	.44956E+01	.00000E+00	.57404E+02
8	.28177E+02	.00000E+00	.17205E+01	.00000E+00	.28229E+02
9	.00000E+00	-.28591E+01	.00000E+00	-.12985E+00	.00000E+00
10	.00000E+00	-.28591E+01	.00000E+00	-.12985E+00	.00000E+00
11	.28177E+02	.00000E+00	.17205E+01	.00000E+00	.28229E+02
12	.57227E+02	.00000E+00	.44956E+01	.00000E+00	.57404E+02
13	.29132E+02	.00000E+00	.51123E+01	.00000E+00	.29577E+02
14	.29132E+02	.00000E+00	.51123E+01	.00000E+00	.29577E+02
15	.27254E+02	.00000E+00	.48239E+01	.00000E+00	.27643E+02
16	.11305E+02	.00000E+00	.15432E+01	.00000E+00	.11410E+02
17	.00000E+00	-.93923E+00	.00000E+00	-.59997E-01	.00000E+00
18	.00000E+00	-.93923E+00	.00000E+00	-.59997E-01	.00000E+00
19	.11305E+02	.00000E+00	.15432E+01	.00000E+00	.11410E+02
20	.27254E+02	.00000E+00	.48239E+01	.00000E+00	.27643E+02

DETERMINANT = .644661E-02

*****

Appendix C

COMPUTER PROGRAMS LISTING

Program HYPRFIRE (Hydrodynamic Pressure in Finite Reservoirs)

```
C  PROGRAM * HYPRFIRE *
C  HYDRODYNAMIC PRESSURE IN FINITE RESERVOIRS
C  ** CONSTANT QUADRILATERAL ELEMENTS **
C
C  COMMON N,NE,NX,CONST
C  DIMENSION X(240),Y(240),Z(240),XM(210),YM(210),ZM(210)
C  DIMENSION KODE(420),K(4,210),FI(420)
C  DIMENSION DFI(420),G(420,420),H(2,420)
C  N=NUMBER OF CORNER NODES
C  NE=NUMBER OF BOUNDARY ELEMENTS
C  NX=MAX. DIM. OF THE SYSTEM OF EQUATIONS
C  K=CORNER NODES FOR EACH ELEMENT
C
C  CALL INPUT(X,Y,Z,K,KODE,FI)
C
C  CALL FMAT(X,Y,Z,XM,YM,ZM,K,FI,DFI,KODE,G,H)
C
C  CALL SLMPD(G,DFI,D)
C
C  CALL OUTPUT(FI,DFI,KODE,D)
C
C  STOP
C  END
C-----
C  SUBROUTINE INPUT(X,Y,Z,K,KODE,FI)
C
C  TO INPUT DATA
C
C  COMMON N,NE,NX,CONST
C  DIMENSION X(1),Y(1),Z(1),KODE(1),K(4,1),FI(1),TITLE(18)
C
C  READ NAME OF THE JOB
C  READ(5,100)TITLE
C  WRITE(6,200)TITLE
C
C  READ BASIC PARAMETERS
C  READ(5,*)N,NE,CONST
C  WRITE(6,300)N,NE,CONST
C  NX=2*NE
C
C  READ COORDINATES OF CORNER NODES IN X,Y AND Z
C  WRITE(6,400)
C  DO 20 I=1,N
C    READ(5,*)X(I),Y(I),Z(I)
C  20 WRITE(6,500)I,X(I),Y(I),Z(I)
C
```

```

C   READ NODES FOR EACH ELEMENT
      WRITE(6,600)
      DO 40 I=1,NE
40    READ(5,*)(K(J,I),J=1,4)
C
C   READ BOUNDARY CONDITIONS
C   FI(I)=VALUE OF POTENTIAL IN THE ELEMENT I IF KODE(I)=0,
C   VALUE OF POTENTIAL DERIVATIVE IF KODE(I)=1
      DO 60 I=1,NE
      READ(5,*)KODE(I),FI(I)
      KODE(I+NE)=KODE(I)
      FI(I+NE)=0
60    WRITE(6,700)I,(K(J,I),J=1,4),KODE(I),FI(I)
      RETURN
100  FORMAT(18A4)
200  FORMAT(120(' '),//20X,'CONSTANT QUADRILATERAL BOUNDARY ',
      *'ELEMENTS'//20X,18A4)
300  FORMAT(// 'DATA'// 'NUMBER OF NODES =',I4/'NUMBER OF ELEMENTS =',
      *I4/'THE CONSTANT =',F12.9/)
400  FORMAT(// 'THE COORDINATES OF NODES'//6X,'NODE',14X,'X',18X,'Y'
      *,18X,'Z'//)
500  FORMAT(5X,I4,3(10X,F9.3))
600  FORMAT(// 'CONNECTIVITY',45X,'BOUNDARY CONDITIONS',// 'ELEMENT',
      *5X,'NODES',40X,'CODE',9X,'VALUE'//)
700  FORMAT(15,2X,4(4X,I4),19X,12,5X,E14.7)
      END
-----
C   SUBROUTINE FMAT(X,Y,Z,XM,YM,ZM,K,FI,DFI,KODE,G,H)
C
C   TO COMPUTE G & H MATRICES & FORM THE SYSTEM AX=F
C
      COMMON N,NE,NX,CONST
      DIMENSION X(1),Y(1),Z(1),XM(1),YM(1),ZM(1),FI(1),DFI(1)
      DIMENSION G(NX,1),H(2,1),K(4,1),KODE(1)
C
      DO 10 I=1,NX
      DO 10 J=1,NX
      G(I,J)=0
10    H(I,J)=0
C
C   COMPUTE MID POINT COORDINATES
      DO 90 I=1,NE
      N1=K(1,I)
      N2=K(2,I)
      N3=K(3,I)
      N4=K(4,I)
      XM(I)=(X(N1)+X(N2)+X(N3)+X(N4))/4
      YM(I)=(Y(N1)+Y(N2)+Y(N3)+Y(N4))/4
      ZM(I)=(Z(N1)+Z(N2)+Z(N3)+Z(N4))/4
C
C   COMPUTE G AND H MATRICES
      DO 60 J=1,NE
      N1=K(1,J)
      N2=K(2,J)
      N3=K(3,J)
      N4=K(4,J)

```

```

      IF (I-J) 20,30,20
20  CALL INTE(XM(I),YM(I),ZM(I),X(N1),Y(N1),Z(N1),X(N2),Y(N2),Z(N2)
    *,X(N3),Y(N3),Z(N3),X(N4),Y(N4),Z(N4),H(1,J),H(2,J),
    *G(I,J),G(I+NE,J))
      H(1,J+NE)=-H(2,J)
      H(2,J+NE)=H(1,J)
      GO TO 40
30  CALL IMLO(XM(I),YM(I),ZM(I),X(N1),Y(N1),Z(N1),X(N2),Y(N2),Z(N2)
    *,X(N3),Y(N3),Z(N3),X(N4),Y(N4),Z(N4),H(1,J),H(2,J),
    *G(I,J),G(I+NE,J))
      H(2,J+NE)=-.5
      H(1,J+NE)=0
40  G(I,J+NE)=-G(I+NE,J)
      G(I+NE,J+NE)=G(I,J)
50  CONTINUE

C
C   ARRANGE THE SYSTEM OF EQUATIONS
      DO 70 J=1,NE
      IF(KODE(J))70,70,60
60  CH=G(I,J)
      G(I,J)=-H(1,J)
      H(1,J)=-CH
      CH=G(I+NE,J)
      G(I+NE,J)=-H(2,J)
      H(2,J)=-CH
      G(I,J+NE)=-H(1,J+NE)
      G(I+NE,J+NE)=-H(2,J+NE)
70  CONTINUE

C
C   DFI CONTAINS THE INDEPENDANT COEFFICIENTS
C   AFTER SLN WILL CONTAIN VALUES OF SYSTEM UNKNOWNNS
      DFI(I)=0
      DFI(I+NE)=0
      DO 80 J=1,NE
      DFI(I)=DFI(I)+H(1,J)*FI(J)
80  DFI(I+NE)=DFI(I+NE)+H(2,J)*FI(J)
90  CONTINUE
      RETURN
      END

-----
C   SUBROUTINE INTE(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
    *XD,YD,ZD,A,B,C,D)

C
C   TO COMPUTE G & H MATRICES' OFF-DIAGONAL ELEMENTS
C
C   COMMON N,NE,MX,CONST

      DIMENSION E(4),F(4)
      DATA E/- .861130,- .339981, .339981, .861136/
      DATA F/.347855, .652145, .652145, .347855/
      A=0
      B=0
      C=0
      D=0
      PI=3.141592654

```



```

DO 100 IN=1,4
DO 100 IM=1,4
FO1=(E(IM)-1)/4
FO3=(1+E(IM))/4
FO5=(E(IM)-1)/4
FO7=(1+E(IM))/4
F11=4*FO1*FO5
F12=-4*FO1*FO7
F13=4*FO3*FO7
F14=-4*FO3*FO5
X=F11*XA+F12*XB+F13*XC+F14*XD
Y=F11*YA+F12*YB+F13*YC+F14*YD
Z=F11*ZA+F12*ZB+F13*ZC+F14*ZD
X1=FO1*(XA-XB)+FO3*(XC-XD)
Y1=FO1*(YA-YB)+FO3*(YC-YD)
Z1=FO1*(ZA-ZB)+FO3*(ZC-ZD)
X2=FO5*(XA-XD)+FO7*(XC-XB)
Y2=FO5*(YA-YD)+FO7*(YC-YB)
Z2=FO5*(ZA-ZD)+FO7*(ZC-ZB)
G1=Y1*Z2-Z1*Y2
G2=Z1*X2-X1*Z2
G3=X1*Y2-Y1*X2
G=SQRT(G1*G1+G2*G2+G3*G3)
R=SQRT((X-XP)**2+(Y-YP)**2+(Z-ZP)**2)
DRN=(G1*(X-XP)+G2*(Y-YP)+G3*(Z-ZP))/(R*G)
RB=CONST*R
RB1=COS(RB)
RB2=SIN(RB)
A=A+DRN*G*F(IM)*F(IM)*(RB1+RB*RB2)/(4*R*R*PI)
B=B+DRN*G*F(IM)*F(IM)*(RB*RB1-RB2)/(4*R*R*PI)
C=C-RB1*G*F(IM)*F(IM)/(4*R*PI)
D=D+RB2*G*F(IM)*F(IM)/(4*R*PI)
100 CONTINUE
RETURN
END

```

```

C-----
SUBROUTINE INLO(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,A,B,C,D)

```

```

C
C   TO COMPUTE G & H MATRICES' DIAGONAL ELEMENTS
C
COMMON N,WE,MX,CONST
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,1.,-1.,1.,1.,C,D)
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,1.,1.,-1.,1.,C1,D1)
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,-1.,1.,-1.,-1.,C2,D2)
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,-1.,-1.,1.,-1.,C3,D3)
A=-.5
B=0
C=C+C1+C2+C3
D=D+D1+D2+D3
RETURN
END

```

```

C-----
      SUBROUTINE INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,ZE11,ZE21,ZE12,ZE22,A,B)
C
C      TO COMPUTE G MATRIX'S DIAGONAL ELEMENTS
C
      COMMON N,NE,MX,CONST
      DIMENSION E(4),F(4)
      DATA E/- .861136,-.339981,.339981,.861136/
      DATA F/.347855,.652145,.652145,.347855/
      A=0
      B=0
      PI=3.141592654
      DO 100 IM=1,4
      DO 100 IN=1,4
      C1=1+E(IM)
      D1=1+E(IN)
      D2=1-E(IN)
      X21=(D2*ZE11+D1*ZE12)/4
      Y21=(D2*ZE21+D1*ZE22)/4
      X22=C1*(ZE12-ZE11)/4
      Y22=C1*(ZE22-ZE21)/4
      G20=ABS(X21*Y22-X22*Y21)
      ZE1=C1*(D2*ZE11+D1*ZE12)/4
      ZE2=C1*(D2*ZE21+D1*ZE22)/4
      A1=1+ZE1
      A2=1-ZE1
      B1=1+ZE2
      B2=1-ZE2
      X=(A1*B2*XA+A1*B1*XB+A2*B1*XC+A2*B2*XD)/4
      Y=(A1*B2*YA+A1*B1*YB+A2*B1*YC+A2*B2*YD)/4
      Z=(A1*B2*ZA+A1*B1*ZB+A2*B1*ZC+A2*B2*ZD)/4
      X1=(B1*(XB-XC)+B2*(XA-XD))/4
      Y1=(B1*(YB-YC)+B2*(YA-YD))/4
      Z1=(B1*(ZB-ZC)+B2*(ZA-ZD))/4
      X2=(A1*(XB-XA)+A2*(XC-XD))/4
      Y2=(A1*(YB-YA)+A2*(YC-YD))/4
      Z2=(A1*(ZB-ZA)+A2*(ZC-ZD))/4
      G1=Y1*Z2-Y2*Z1
      G2=Z1*X2-Z2*X1
      G3=X1*Y2-X2*Y1
      G10=SQRT(G1*G1+G2*G2+G3*G3)
      XO=X-XP
      YO=Y-YP
      ZO=Z-ZP
      R=SQRT(XO*XO+YO*YO+ZO*ZO)
      RB=CONST*R
      A=A-COS(RB)*G10*G20*F(IM)*F(IN)/(4*R*PI)
100  B=B+SIN(RB)*G10*G20*F(IM)*F(IN)/(4*R*PI)
      RETURN
      END
C-----
      SUBROUTINE SLNPD(A,B,D)
C
C      TO SOLVE LINEAR SYSTEM OF EQUATIONS BY GAUSS ELIMINATION
C      PROVIDING INTERCHANGING ROWS WHEN ENCOUNTERING

```

```

C      A ZERO DIAGONAL COEFFICIENT
C
      DIMENSION A(NX,1),B(1)
      COMMON N,NE,NX,CONST
      N1=NX-1
      DO 100 K=1,N1
      K1=K+1
      C=A(K,K)
      IF(ABS(C)-.00001)10,10,60
C
C      INTERCHANGE ROWS TO GET NON ZERO DIAGONAL COEFFICIENT
10 DO 40 J=K1,NX
   IF(ABS(A(J,K))- .00001)40,40,20
20 DO 30 L=K,NX
   C=A(K,L)
   A(K,L)=A(J,L)
30 A(J,L)=C
   C=B(K)
   B(K)=B(J)
   B(J)=C
   WRITE(IMP,1)J,K
1  FORMAT(/,'ROWS',I4,'AND',I4,'INTERCHANGED')
   GOTO 60
40 CONTINUE
50 WRITE(IMP,200)K
200 FORMAT(/,'*** SINGULARITY IN ROW',I5,' ***/')
   GOTO 150
C
C      DIVIDE ROW BY DIAGONAL COEFFICIENT
60 C=A(K,K)
   DO 70 J=K1,NX
70 A(K,J)=A(K,J)/C
   B(K)=B(K)/C
C
C      ELIMINATE UNKNOWN X(K) FROM ROW I
C      DO 90 I=K1,NX
      C=A(I,K)
      DO 80 J=K1,NX
80 A(I,J)=A(I,J)-C*A(K,J)
90 B(I)=B(I)-C*B(K)
100 CONTINUE
C
C      COMPUTE LAST UNKNOWN
C      IF(ABS(A(NX,NX))- .00001)110,110,120
110 WRITE(IMP,200)NX
   GOTO 150
120 B(NX)=B(NX)/A(NX,NX)
C
C      BACK-SUBSTITUTION PROCESS
C      DO 130 L=1,N1
      K=NX-L
      K1=K+1
      DO 130 J=K1,NX
130 B(K)=B(K)-A(K,J)*B(J)
C
C      COMPUTE VALUE OF DETERMINANT

```

```

      D=1
      DO 140 I=1,NX
140   D=D*A(I,I)
150   RETURN
      END
C-----
      SUBROUTINE OUTPUT(FI,DFI,KODE,D)
C
      COMMON N,NE,NX,CONST
      DIMENSION FI(1),DFI(1),KODE(1)
      WRITE(6,100)
      DO 40 I=1,NX
      IF(KODE(I))40,40,30
30   CH=FI(I)
      FI(I)=DFI(I)
      DFI(I)=CH
40   CONTINUE
      DO 50 I=1,NE
      A=FI(I)
      B=FI(NE+I)
      C=SQRT(A*A+B*B)
50   WRITE(6,200)I,A,DFI(I),B,DFI(NE+I),C
      WRITE(6,300)D
      RETURN
100  FORMAT(/120('='),// 'RESULTS'/'=====/'/'BOUNDARY',15X,
      * 'REAL PART',26X,'IMAGINARY PART'/'ELEMENTS',6X,'PRESSURE',9X,
      * 'P.DERIVATIVE',9X,'PRESSURE',9X,'P.DERIVATIVE',9X,'P.TOTAL'//)
200  FORMAT(I5,5(5X,E14.7))
300  FORMAT('/DETERMINANT = ',E15.6/120('*'))
      END
C-----

```

Program HYPINRE (Hydrodynamic Pressure in Infinite Reservoirs)

```

C  PROGRAM * HYPINRE *
C  HYDRODYNAMIC PRESSURE IN INFINITE RESERVOIRS
C  ** CONSTANT QUADRILATERAL ELEMENTS **
C
COMMON N,NE,NX,CONST,NF
DIMENSION X(250),Y(250),Z(250),XM(210),YM(210),ZM(210)
DIMENSION KODE(420),K(4,230),FI(420)
DIMENSION DFI(420),G(420,420),H(2,420)
C  N=NUMBER OF CORNER NODES
C  NE=NUMBER OF BOUNDARY ELEMENTS
C  NF=NUMBER OF INFINITE ELEMENTS ( WITHOUT BOUNDARY NODES )
C  NX=MAX. DIM. OF THE SYSTEM OF EQUATIONS
C  K=CORNER NODES FOR EACH ELEMENT
C
CALL INPUT(X,Y,Z,K,KODE,FI)
C
CALL FMAT(X,Y,Z,XM,YM,ZM,K,FI,DFI,KODE,G,H)
C
CALL SLNPD(G,DFI,D)
C
CALL OUTPUT(FI,DFI,KODE,D)
C
STOP
END
-----
C  SUBROUTINE INPUT(X,Y,Z,K,KODE,FI)
C
C  TO INPUT DATA
C
COMMON N,NE,NX,CONST,NF
DIMENSION X(1),Y(1),Z(1),KODE(1),K(4,1),FI(1),TITLE(18)
C
C  READ NAME OF THE JOB
C  READ(5,100)TITLE
C  WRITE(6,200)TITLE
C
C  READ BASIC PARAMETERS
C  READ(5,*)N,NE,NF,CONST
C  WRITE(6,300)N,NE,NF,CONST
C  NX=2*NE
C
C  READ COORDINATES OF CORNER NODES IN X,Y AND Z
C  WRITE(6,400)
C  DO 20 I=1,N
C  READ(5,*)X(I),Y(I),Z(I)
20  WRITE(6,500)I,X(I),Y(I),Z(I)
C
C  READ NODES FOR EACH ELEMENT AND INFINITE ELEMENTS
C  WRITE(6,600)
C  DO 40 I=1,NE+NF
40  READ(5,*)(K(J,I),J=1,4)
C
C  READ BOUNDARY CONDITIONS

```

```

C      FI(I)=VALUE OF POTENTIAL IN THE ELEMENT I IF KODE(I)=0.
C      VALUE OF POTENTIAL DERIVATIVE IF KODE(I)=1
      DO 60 I=1,NE
      READ(5,*)KODE(I),FI(I)
      KODE(I+NE)=KODE(I)
      FI(I+NE)=0
60    WRITE(6,700)I,(K(J,I),J=1,4),KODE(I),FI(I)
      WRITE(6,800)
      DO 70 I=NE+1,NE+NF
70    WRITE(6,700)I,(K(J,I),J=1,4)
      RETURN
100   FORMAT(18A4)
200   FORMAT(120(' '),//20X,'CONSTANT QUADRILATERAL BOUNDARY ',
      *'ELEMENTS'//20X,18A4)
300   FORMAT(//'DATA'//'NUMBER OF NODES =' ,I4/'NUMBER OF ELEMENTS =' ,
      *I4/'INFINITE ELEMENTS =' ,I4/'THE CONSTANT =' ,F12.9/)
400   FORMAT(//'THE COORDINATES OF NODES'//6X,'NODE',14X,'X',18X,'Y'
      *,18X,'Z'//)
500   FORMAT(5X,I4,3(10X,F9.3))
600   FORMAT(//'CONNECTIVITY',45X,'BOUNDARY CONDITIONS'//,'ELEMENT',
      *5X,'NODES',40X,'CODE',9X,'VALUE'//)
700   FORMAT(15,2X,4(4X,I4),19X,I2,5X,E14.7)
800   FORMAT(//'INFINITE ELEMENTS CONNECTIVITY'/'ELEMENT',5X,'NODES'
      */)
      END
-----
C      SUBROUTINE FMAT(X,Y,Z,XM,YM,ZM,K,FI,DFI,KODE,G,H)
C
C      TO COMPUTE G & H MATRICES & FORM THE SYSTEM AX=F
C
      COMMON N,NE,NX,CONST,NF
      DIMENSION X(1),Y(1),Z(1),XM(1),YM(1),ZM(1),FI(1),DFI(1)
      DIMENSION G(NX,1),H(2,1),K(4,1),KODE(1)
C
      DO 10 I=1,NX
      DO 10 J=1,NX
      G(I,J)=0
10    H(I,J)=0
C
      DO 90 I=1,NE
C      COMPUTE MID POINT COORDINATES
      N1=K(1,I)
      N2=K(2,I)
      N3=K(3,I)
      N4=K(4,I)
      XM(I)=(X(N1)+X(N2)+X(N3)+X(N4))/4
      YM(I)=(Y(N1)+Y(N2)+Y(N3)+Y(N4))/4
      ZM(I)=(Z(N1)+Z(N2)+Z(N3)+Z(N4))/4
C
C      COMPUTE G AND H MATRICES
      DO 50 J=1,NE
      N1=K(1,J)
      N2=K(2,J)
      N3=K(3,J)
      N4=K(4,J)
      IF (I-J) 20,30,20

```

```

20 CALL INTE(XM(I),YM(I),ZM(I),X(N1),Y(N1),Z(N1),X(N2),Y(N2),Z(N2)
  *,X(N3),Y(N3),Z(N3),X(N4),Y(N4),Z(N4),H(1,J),H(2,J),
  *G(I,J),G(I+NE,J))
  H(1,J+NE)=-H(2,J)
  H(2,J+NE)=H(1,J)
  GO TO 40
30 CALL INLO(XM(I),YM(I),ZM(I),X(N1),Y(N1),Z(N1),X(N2),Y(N2),Z(N2)
  *,X(N3),Y(N3),Z(N3),X(N4),Y(N4),Z(N4),G(I,J),G(I+NE,J))
  H(1,J)=-.5
  H(2,J)=0.
  H(2,J+NE)=-.5
  H(1,J+NE)=0
40 G(I,J+NE)=-G(I+NE,J)
  G(I+NE,J+NE)=G(I,J)
50 CONTINUE
  DO 130 J=NE-NF+1,NE
    N1=K(1,J+NF)
    N2=K(2,J+NF)
    N3=K(3,J+NF)
    N4=K(4,J+NF)
    CALL INT2(XM(I),YM(I),ZM(I),X(N1),Y(N1),Z(N1),X(N2),Y(N2),Z(N2)
  *,X(N3),Y(N3),Z(N3),X(N4),Y(N4),Z(N4),A1,B1,C1,D1)
    H(1,J)=H(1,J)+A1
    H(2,J)=H(2,J)+B1
    G(I,J)=G(I,J)+C1
    G(I+NE,J)=G(I+NE,J)+D1
    H(1,J+NE)=-H(2,J)
    H(2,J+NE)=H(1,J)
    G(I,J+NE)=-G(I+NE,J)
    G(I+NE,J+NE)=G(I,J)
130 CONTINUE
C
C   ARRANGE THE SYSTEM OF EQUATIONS
  DO 70 J=1,NE
    IF(KODE(J))70,70,60
60  CH=G(I,J)
    G(I,J)=-H(1,J)
    H(1,J)=-CH
    CH=G(I+NE,J)
    G(I+NE,J)=-H(2,J)
    H(2,J)=-CH
    G(I,J+NE)=-H(1,J+NE)
    G(I+NE,J+NE)=-H(2,J+NE)
70  CONTINUE
C
C   DFI CONTAINS THE INDEPENDANT COEFFICIENTS
C   AFTER SLN WILL CONTAIN VALUES OF SYSTEM UNKNOWNNS
  DFI(I)=0
  DFI(I+NE)=0
  DO 80 J=1,NE
    DFI(I)=DFI(I)+H(1,J)*FI(J)
80  DFI(I+NE)=DFI(I+NE)+H(2,J)*FI(J)
90  CONTINUE
  RETURN
  END

```

C-----

```
SUBROUTINE INTE(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,A,B,C,D)
```

C
C
C

TO COMPUTE G & H MATRICES' OFF-DIAGONAL ELEMENTS

```
COMMON N,NE,NX,CONST,MF
DIMENSION E(4),F(4)
DATA E/-.861136,-.339981,.339981,.861136/
DATA F/.347855,.652145,.652145,.347855/
A=0
B=0
C=0
D=0
PI=3.141592654
DO 100 IM=1,4
DO 100 IN=1,4
F01=(E(IM)-1)/4
F03=(1+E(IM))/4
F05=(E(IN)-1)/4
F07=(1+E(IN))/4
F11=4*F01*F05
F12=-4*F01*F07
F13=4*F03*F07
F14=-4*F03*F05
X=F11*XA+F12*XB+F13*XC+F14*XD
Y=F11*YA+F12*YB+F13*YC+F14*YD
Z=F11*ZA+F12*ZB+F13*ZC+F14*ZD
X1=F01*(XA-XB)+F03*(XC-XD)
Y1=F01*(YA-YB)+F03*(YC-YD)
Z1=F01*(ZA-ZB)+F03*(ZC-ZD)
X2=F05*(XA-XD)+F07*(XC-XB)
Y2=F05*(YA-YD)+F07*(YC-YB)
Z2=F05*(ZA-ZD)+F07*(ZC-ZB)
G1=Y1*Z2-Z1*Y2
G2=Z1*X2-X1*Z2
G3=X1*Y2-Y1*X2
G=SQRT(G1*G1+G2*G2+G3*G3)
R=SQRT((X-XP)**2+(Y-YP)**2+(Z-ZP)**2)
DRN=(G1*(X-XP)+G2*(Y-YP)+G3*(Z-ZP))/(R*G)
RB=CONST*R
RB1=COS(RB)
RB2=SIN(RB)
A=A+DRN*G*F(IM)*F(IN)*(RB1+RB*RB2)/(4*R*R*PI)
B=B+DRN*G*F(IM)*F(IN)*(RB*RB1-RB2)/(4*R*R*PI)
C=C-RB1*G*F(IM)*F(IN)/(4*R*PI)
D=D+RB2*G*F(IM)*F(IN)/(4*R*PI)
100 CONTINUE
RETURN
END
```

C-----

```
SUBROUTINE INLO(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,C,D)
```

C
C

TO COMPUTE G & H MATRICES' DIAGONAL ELEMENTS

C

```

COMMON N,NE,MX,CONST,NF
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,1.,-1.,1.,1.,C,D)
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,1.,1.,-1.,1.,C1,D1)
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,-1.,1.,-1.,-1.,C2,D2)
CALL INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,-1.,-1.,1.,-1.,C3,D3)
C=C+C1+C2+C3
D=D+D1+D2+D3
RETURN
END

```

C

```

SUBROUTINE INTL(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
*XD,YD,ZD,ZE11,ZE21,ZE12,ZE22,A,B)

```

C

C

C

TO COMPUTE G MATRIX' DIAGONAL ELEMENTS

```

COMMON N,NE,MX,CONST,NF
DIMENSION E(3),F(3)
DATA E/-.774598889,0,.774598889/
DATA F/.555556,.888889,.555556/
A=0
B=0
PI=3.141592654
DO 100 IM=1,3
DO 100 IN=1,3
C1=1+E(IM)
D1=1+E(IN)
D2=1-E(IN)
X21=(D2*ZE11+D1*ZE12)/4
Y21=(D2*ZE21+D1*ZE22)/4
X22=C1*(ZE12-ZE11)/4
Y22=C1*(ZE22-ZE21)/4
G20=ABS(X21*Y22-X22*Y21)
ZE1=C1*(D2*ZE11+D1*ZE12)/4
ZE2=C1*(D2*ZE21+D1*ZE22)/4
A1=1+ZE1
A2=1-ZE1
B1=1+ZE2
B2=1-ZE2
X=(A1*B2*XA+A1*B1*XB+A2*B1*XC+A2*B2*XD)/4
Y=(A1*B2*YA+A1*B1*YB+A2*B1*YC+A2*B2*YD)/4
Z=(A1*B2*ZA+A1*B1*ZB+A2*B1*ZC+A2*B2*ZD)/4
X1=(B1*(XB-XC)+B2*(XA-XD))/4
Y1=(B1*(YB-YC)+B2*(YA-YD))/4
Z1=(B1*(ZB-ZC)+B2*(ZA-ZD))/4
X2=(A1*(XB-XA)+A2*(XC-XD))/4
Y2=(A1*(YB-YA)+A2*(YC-YD))/4
Z2=(A1*(ZB-ZA)+A2*(ZC-ZD))/4
G1=Y1*Z2-Y2*Z1
G2=Z1*X2-Z2*X1
G3=X1*Y2-X2*Y1
G10=SQRT(G1*G1+G2*G2+G3*G3)

```

```

XO=X-XP
YO=Y-YP
ZO=Z-ZP
R=SQRT(XO*XO+YO*YO+ZO*ZO)
RB=CONST*R
A=A-COS(RB)*G10*G20*F(IN)*F(IM)/(4*R*PI)
100 B=B+SIN(RB)*G10*G20*F(IN)*F(IM)/(4*R*PI)
RETURN
END

```

```

C-----
C SUBROUTINE INT2(XP,YP,ZP,XA,YA,ZA,XB,YB,ZB,XC,YC,ZC,
C *XD,YD,ZD,A1,B1,C1,D1)
C
C TO COMPUTE INFINITE ELEMENTS
C

```

```

COMMON N,NE,NX,CONST,NF
DIMENSION E(3),C(15),F(3),D(15)
DATA E /-.77459667,0,.77459667/
DATA F /.555556,.888889,.555556/
DATA C /.093308,.492692,1.215595,2.26995,3.667623,5.425337,
*7.565916,10.120229,13.130282,16.654408,20.776479,25.623894,
*31.407519,38.530683,48.028086/
DATA D /.239578,.576468,.887009,1.223667,1.574454,1.94478,
*2.340659,2.782131,3.255679,3.806318,4.458478,5.27002,6.359583,
*8.031801,11.527741/
A1=0
B1=0
C1=0
D1=0
PI=3.141592654
DO 100 IM=1,15
DO 100 IN=1,3
F1=1-C(IN)/10.0
F2=(1+E(IM))/2.0
F3=(1-E(IM))/2.0
X=C(IN)*(F2*XA+F3*XD)/10.0+F1*(F2*XB+F3*XC)
Y=C(IN)*(F2*YA+F3*YD)/10.0+F1*(F2*YB+F3*YC)
Z=C(IN)*(F2*ZA+F3*ZD)/10.0+F1*(F2*ZB+F3*ZC)
X1=(F2*(XA-XB)+F3*(XD-XC))/10.0
Y1=(F2*(YA-YB)+F3*(YD-YC))/10.0
Z1=(F2*(ZA-ZB)+F3*(ZD-ZC))/10.0
X2=C(IN)*(XA-XD)/20.0+F1*(XB-XC)/2.0
Y2=C(IN)*(YA-YD)/20.0+F1*(YB-YC)/2.0
Z2=C(IN)*(ZA-ZD)/20.0+F1*(ZB-ZC)/2.0
G1=Y1*Z2-Z1*Y2
G2=Z1*X2-X1*Z2
G3=X1*Y2-Y1*X2
G=SQRT(G1*G1+G2*G2+G3*G3)
R=SQRT((X-XP)**2+(Y-YP)**2+(Z-ZP)**2)
XO=F2*XB+F3*XC
YO=F2*YB+F3*YC
ZO=F2*ZB+F3*ZC
RO=SQRT((XO-XP)**2+(YO-YP)**2+(ZO-ZP)**2)
DRN=(G1*(X-XP)+G2*(Y-YP)+G3*(Z-ZP))/(R*G)
RB=CONST*R
RB1=COS(RB)

```

```

      RB2=SIN(RB)
      RP=4*R**3*PI
      A1=A1+DRN*G*D(IN)*F(IM)*(RB1+RB*RB2)*RO/RP
      B1=B1+DRN*G*D(IN)*F(IM)*(RB*RB1-RB2)*RO/RP
      C1=C1-RB1*G*D(IN)*F(IM)*RO**2/RP
      D1=D1+RB2*G*D(IN)*F(IM)*RO**2/RP
100 CONTINUE
      RETURN
      END
-----
C      SUBROUTINE SLNPD(A,B,D)
C
C      TO SOLVE LINEAR SYSTEM OF EQUATIONS BY GAUSS ELIMINATION
C      PROVIDING INTERCHANGING ROWS WHEN ENCOUNTERING
C      A ZERO DIAGONAL COEFFICIENT
C
      DIMENSION A(NX,1),B(1)
      COMMON N,NE,NX,CONST,NF
      N1=NX-1
      DO 100 K=1,N1
      K1=K+1
      C=A(K,K)
      IF(ABS(C)-.00001)10,10,60
C
C      INTERCHANGE ROWS TO GET NON ZERO DIAGONAL COEFFICIENT
10 DO 40 J=K1,NX
      IF(ABS(A(J,K))- .00001)40,40,20
20 DO 30 L=K,NX
      C=A(K,L)
      A(K,L)=A(J,L)
30 A(J,L)=C
      C=B(K)
      B(K)=B(J)
      B(J)=C
      WRITE(IMP,1)J,K
1  FORMAT(/,'ROWS',I4,'AND',I4,'INTERCHANGED')
      GOTO 60
40 CONTINUE
50 WRITE(IMP,200)K
200 FORMAT(/'*** SINGULARITY IN ROW',I5,' ***'/)
      GOTO 150
C
C      DIVIDE ROW BY DIAGONAL COEFFICIENT
60 C=A(K,K)
      DO 70 J=K1,NX
70 A(K,J)=A(K,J)/C
      B(K)=B(K)/C
C
C      ELIMINATE UNKNOWN X(K) FROM ROW I
      DO 90 I=K1,NX
      C=A(I,K)
      DO 80 J=K1,NX
80 A(I,J)=A(I,J)-C*A(K,J)
90 B(I)=B(I)-C*B(K)
100 CONTINUE
C

```

```

C      COMPUTE LAST UNKNOWN
      IF(ABS(A(NX,NX))- .00001) 110, 110, 120
110  WRITE(IMP,200)NX
      GOTO 150
120  B(NX)=B(NX)/A(NX,NX)
C
C      BACK-SUBSTITUTION PROCESS
      DO 130 L=1,M1
      K=NX-L
      K1=K+1
      DO 130 J=K1,NX
130  B(K)=B(K)-A(K,J)*B(J)
C
C      COMPUTE VALUE OF DETERMINANT
      D=1
      DO 140 I=1,NX
140  D=D*A(I,I)
150  RETURN
      END
C-----
      SUBROUTINE OUTPUT(FI,DFI,KODE,D)
C
      COMMON N,NE,NX,CONST,NF
      DIMENSION FI(1),DFI(1),KODE(1)
      WRITE(6,100)
      DO 40 I=1,NX
      IF(KODE(I))40,40,30
30  CH=FI(I)
      FI(I)=DFI(I)
      DFI(I)=CH
40  CONTINUE
      DO 50 I=1,NE
      A=FI(I)
      B=FI(NE+I)
      C=SQRT(A*A+B*B)
50  WRITE(6,200)I,A,DFI(I),B,DFI(NE+I),C
      WRITE(6,300)D
      RETURN
100  FORMAT(/120('='),// 'RESULTS'/'=====/'/'BOUNDARY',15X,
      * 'REAL PART',26X, 'IMAGINARY PART'/'ELEMENTS',6X, 'PRESSURE',9X,
      * 'P. DERIVATIVE',9X, 'PRESSURE',9X, 'P. DERIVATIVE',9X, 'P. TOTAL'/' )
200  FORMAT(15,5(5X,E14.7))
300  FORMAT(/ 'DETERMINANT = ',E15.6/120('='))
      END
C-----

```

Appendix D

TABLES OF HYDRODYNAMIC PRESSURE

D.1 Introduction

This appendix contains the tables of results for the hydrodynamic pressure distribution on upstream faces of the dams which are studied in Chapter 3.

The first group of tables (Section D.2) gives the hydrodynamic pressure distribution on the upstream face of an arch dam for different excitation frequencies. The arch dam - reservoir system is shown in Fig. 3.1 and the mesh used is shown in Fig. 3.3.

The second group of tables (Section D.3) gives the results of the convergence tests. The finite reservoir system was represented by two meshes. The first mesh has 142 elements (see Fig. 3.3) and the second mesh has 173 elements (see Fig. 3.4).

In the third group of tables (Section D.4), the results for two different lengths of reservoirs are given. This shows that for a stationary far end, if the excitation frequency is in the upstream - downstream direction and is less than the fundamental natural frequency ω_1^* , the length of the reservoir beyond $2H$ has negligible effect.

The fourth group of tables (Section D.5) shows the comparison between the classical solutions of the 2-D straight gravity dam and the results of the boundary element method using 3-D mesh. The mesh has 102 elements (see Fig. 3.21).

D.2 Hydrodynamic Pressure Distribution on Arch Dams

D.2.1 Exciting frequency in the upstream-downstream direction

NODE	$\omega/\omega_1^* = 0.4$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	113.530	-0.313	99.386	0.064
2	107.510	-0.255	94.749	0.058
3	92.993	-0.195	82.924	0.042
4	68.818	-0.129	62.446	0.022
5	30.884	-0.052	28.823	0.002
6	101.100	-0.260	86.826	0.091
7	95.365	-0.213	82.483	0.080
8	81.912	-0.161	71.726	0.061
9	60.141	-0.103	53.663	0.037
10	27.039	-0.038	24.927	0.010
11	92.828	-0.227	78.395	0.119
12	87.772	-0.186	74.743	0.104
13	75.663	-0.140	65.354	0.080
14	56.160	-0.089	49.599	0.049
15	26.202	-0.032	24.059	0.016
16	88.001	-0.212	73.435	0.148
17	83.482	-0.173	70.336	0.128
18	72.351	-0.130	61.956	0.099
19	54.273	-0.082	47.663	0.062
20	25.939	-0.029	23.784	0.021
21	85.115	-0.218	70.493	0.177
22	80.831	-0.177	67.646	0.152
23	70.163	-0.132	59.758	0.118
24	52.722	-0.082	46.137	0.078
25	25.452	-0.292	23.321	0.030

NODE	$\omega/\omega_1^r = 0.8$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	184.410	-1.459	127.720	-0.235
2	171.630	-1.183	120.450	-0.122
3	143.960	-0.853	103.510	-0.073
4	101.420	-0.488	75.755	-0.061
5	41.620	-0.118	33.256	-0.049
6	170.130	-1.285	113.110	-0.008
7	157.810	-1.058	106.320	0.059
8	131.560	-0.775	90.803	0.068
9	91.950	-0.455	66.000	0.041
10	37.520	-0.118	29.025	0.002
11	159.270	-1.142	101.910	0.164
12	147.880	-0.944	96.067	0.200
13	123.450	-0.694	82.420	0.176
14	86.870	-0.407	60.638	0.111
15	36.300	-0.102	27.730	0.027
16	152.140	-1.039	94.557	0.268
17	141.500	-0.855	89.491	0.287
18	118.450	-0.621	77.291	0.242
19	83.800	-0.356	57.590	0.152
20	35.670	-0.083	27.087	0.041
21	147.870	-0.997	90.263	0.281
22	137.580	-0.806	85.573	0.300
23	115.210	-0.564	74.107	0.249
24	81.490	-0.296	55.416	0.151
25	34.920	-0.040	26.420	0.036

NODE	$\omega/\omega_1 = 1.2$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	70.009	-1.651	262.500	-10.149
2	68.821	-1.266	242.370	-8.637
3	63.303	-0.925	200.430	-6.525
4	50.692	-0.601	137.760	-3.974
5	25.201	-0.248	53.702	-1.196
6	47.465	-0.887	239.300	-9.224
7	47.432	-0.637	220.450	-7.928
8	44.696	-0.442	181.550	-6.046
9	37.010	-0.279	124.140	-3.728
10	19.631	-0.118	48.175	-1.148
11	24.492	-0.098	214.940	-8.320
12	26.506	0.032	198.290	-7.173
13	27.774	0.066	163.690	-5.490
14	26.136	0.037	112.720	-3.398
15	16.494	-0.017	44.888	-1.050
16	6.172	0.496	194.970	-7.668
17	10.051	0.534	180.320	-6.606
18	14.828	0.435	149.500	-5.056
19	18.118	0.251	103.860	-3.131
20	14.225	0.043	42.312	-0.970
21	-4.608	0.708	182.980	-7.518
22	0.340	0.695	169.470	-6.444
23	7.141	0.515	140.770	-4.917
24	13.208	0.240	98.060	-3.042
25	12.688	-0.020	40.465	-0.954

NODE	$\omega/\omega_1^* = 1.6$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-105.140	-5.800	-275.650	-3.555
2	-88.398	-5.501	-242.200	-3.949
3	-59.967	-4.647	-181.680	-3.758
4	-26.815	-3.242	-104.200	-2.918
5	0.041	-1.317	-25.360	-1.397
6	-116.720	-5.160	-284.370	-3.387
7	-100.010	-4.893	-251.290	-3.684
8	-71.116	-4.094	-190.840	-3.406
9	-36.094	-2.793	-112.360	-2.536
10	-4.146	-1.069	-29.150	-1.122
11	-119.400	-4.273	-281.830	-2.832
12	-102.730	-4.104	-249.300	-3.167
13	-73.760	-3.451	-189.780	-2.956
14	-37.980	-2.359	-111.910	-2.209
15	-4.341	-0.912	-28.590	-0.996
16	-115.390	-3.404	-272.150	-2.091
17	-99.115	-3.348	-240.580	-2.528
18	-70.851	-2.866	-182.810	-2.467
19	-35.888	-1.988	-107.210	-1.903
20	-3.249	-0.784	-26.610	-0.887
21	-111.480	-2.733	-264.380	-1.274
22	-95.640	-2.812	-233.650	-1.907
23	-68.102	-2.489	-177.280	-2.067
24	-34.096	-1.785	-103.510	-1.726
25	-2.623	-0.749	-25.410	-0.895

NODE	$\omega/\omega_1^r = 2.0$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-59.594	-13.612	-126.520	-4.963
2	-46.014	-12.483	-106.400	-4.920
3	-24.748	-10.110	-72.603	-4.264
4	-3.114	-6.640	-33.592	-3.003
5	8.167	-2.373	-1.871	-1.234
6	-45.524	-7.104	-121.250	1.105
7	-34.694	-6.530	-103.030	0.627
8	-18.042	-5.301	-72.139	0.214
9	-1.276	-3.482	-35.741	-0.067
10	7.446	-1.224	-3.836	-0.181
11	2.464	0.664	-86.468	8.143
12	8.139	0.603	-72.091	7.065
13	15.201	0.465	-48.321	5.404
14	19.607	0.275	-20.879	3.308
15	14.752	0.068	1.473	0.974
16	67.218	4.731	-33.800	11.327
17	66.459	4.491	-24.693	10.106
18	61.368	3.700	-10.813	7.935
19	49.238	2.429	3.227	4.997
20	24.957	0.817	9.854	1.566
21	113.180	5.033	5.370	11.239
22	107.820	5.054	10.500	10.234
23	94.020	4.334	16.956	8.176
24	69.875	2.950	20.802	5.241
25	31.858	1.052	15.676	1.709

NODE	$\omega/\omega_1^* = 2.4$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-105.010	-10.892	36.693	-5.333
2	-84.464	-10.492	43.626	-4.660
3	-51.636	-8.807	49.778	-3.719
4	-17.713	-5.964	46.826	-2.446
5	4.011	-2.204	25.289	-0.857
6	-114.370	-10.062	23.315	-2.729
7	-94.461	-9.700	29.819	-2.370
8	-62.157	-8.109	36.160	-1.873
9	-27.101	-5.451	35.460	-1.194
10	-0.376	-1.971	20.101	-0.353
11	-105.300	-9.125	24.535	-0.718
12	-86.997	-8.829	30.127	0.004
13	-57.312	-7.377	-35.289	0.028
14	-24.586	-4.950	34.305	0.053
15	0.822	-1.789	20.079	0.087
16	-83.227	-8.826	37.837	1.384
17	-67.400	-8.518	41.853	1.338
18	-42.097	-7.098	44.281	1.104
19	-14.870	-4.744	40.055	0.752
20	4.413	-1.699	22.362	0.323
21	-65.828	-8.809	49.073	1.585
22	-51.985	-8.497	51.851	1.556
23	-30.153	-7.073	51.996	1.289
24	-7.464	-4.703	44.778	0.873
25	6.812	-1.664	24.017	0.359

NODE	$\omega/\omega_1 = 2.8$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-71.514	-43.560	-25.456	-43.441
2	-45.194	-40.557	-5.041	-40.312
3	-8.958	-33.644	20.941	-33.380
4	17.331	-22.397	35.183	-22.180
5	17.305	-7.825	23.023	-7.692
6	-19.579	15.694	25.127	10.305
7	-1.463	13.629	37.475	8.866
8	21.843	10.232	50.736	6.481
9	33.462	6.226	50.622	3.868
10	21.077	2.083	26.472	1.379
11	27.874	52.108	74.836	43.326
12	39.430	46.864	80.412	39.038
13	52.049	37.110	82.538	30.913
14	51.472	23.740	69.623	19.813
15	27.369	8.090	33.073	6.868
16	12.212	17.308	67.206	12.182
17	24.805	15.303	73.034	10.822
18	39.447	11.792	75.691	8.285
19	42.933	7.376	64.770	5.184
20	24.722	2.511	31.661	1.866
21	-32.146	-37.955	30.529	-37.503
22	-15.022	-35.146	40.149	-34.509
23	7.453	-28.903	49.235	-28.289
24	22.174	-19.061	47.579	-18.603
25	17.840	-6.605	26.062	-6.383

NODE	$\omega/\omega_1^* = 3.2$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	351.750	-116.290	809.990	-286.970
2	49.363	-210.710	83.070	-511.410
3	-300.850	-306.590	-759.520	-738.210
4	-399.000	-286.640	-996.560	-687.870
5	-164.560	-115.810	-421.010	-277.330
6	332.570	-95.585	808.530	-239.630
7	34.949	-189.310	83.369	-468.690
8	-310.390	-285.170	-760.500	-702.710
9	-406.280	-269.490	-1001.800	-662.920
10	-167.540	-107.940	-422.850	-265.360
11	322.130	-92.122	806.460	-221.460
12	35.782	-178.480	93.089	-444.700
13	-296.860	-266.360	-738.410	-673.410
14	-389.630	-250.790	-978.980	-637.400
15	-159.940	-100.350	-413.050	-255.550
16	318.840	-123.490	794.750	-273.600
17	47.482	-197.170	102.580	-479.370
18	-268.740	-268.910	-705.000	-685.200
19	-359.350	-244.620	-940.620	-635.020
20	-147.030	-96.669	-396.530	-252.580
21	312.140	-159.590	770.320	-340.040
22	52.491	-222.270	97.059	-527.560
23	-250.020	-279.680	-686.280	-709.380
24	-337.370	-246.100	-911.980	-642.460
25	-138.510	-97.252	-385.620	-256.350

D.2.2 Exciting frequency in the cross-stream direction

NODE	$\omega/\omega_1' = 0.4$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-13.104	0.216	-18.130	0.348
2	-11.195	0.178	-15.725	0.287
3	-8.006	0.139	-11.580	0.222
4	-3.306	0.095	-5.568	0.150
5	2.165	0.042	1.432	0.064
6	-1.218	0.165	-5.735	0.270
7	0.466	0.136	-3.612	0.222
8	2.738	0.104	-0.487	0.170
9	5.307	0.069	3.255	0.110
10	5.946	0.027	5.277	0.042
11	4.002	0.121	0.452	0.197
12	5.028	0.099	1.818	0.161
13	6.184	0.075	3.640	0.122
14	6.969	0.049	5.349	0.078
15	5.468	0.020	4.939	0.030
16	4.331	0.080	2.099	0.127
17	4.830	0.063	2.803	0.101
18	5.263	0.047	3.654	0.075
19	5.275	0.031	4.250	0.048
20	3.688	0.013	3.356	0.019
21	1.787	0.034	1.080	0.051
22	1.923	0.024	1.270	0.037
23	2.017	0.018	1.498	0.027
24	1.948	0.013	1.621	0.018
25	1.302	0.007	1.206	0.008

NODE	$\omega/\omega_1^r = 0.8$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-29.232	0.745	-43.565	1.058
2	-25.758	0.622	-38.699	0.891
3	-19.540	0.468	-29.766	0.664
4	-10.634	0.290	-17.115	0.395
5	-0.220	0.094	-2.326	0.109
6	-14.908	0.589	-27.841	0.861
7	-11.881	0.499	-23.568	0.738
8	-7.008	0.377	-16.265	0.557
9	-0.864	0.232	-6.760	0.336
10	3.949	0.071	2.017	0.094
11	-6.004	0.416	-16.211	0.612
12	-3.993	0.335	-13.231	0.532
13	-0.919	0.269	-8.248	0.403
14	2.488	0.164	-2.188	0.241
15	4.023	0.048	2.487	0.062
16	-1.589	0.238	-8.037	0.338
17	-0.524	0.205	-6.385	0.301
18	1.048	0.155	-3.611	0.228
19	2.622	0.092	-0.353	0.131
20	2.838	0.023	1.863	0.024
21	-0.354	0.065	-2.096	0.071
22	0.250	0.057	-1.653	0.072
23	0.697	0.041	-0.822	0.050
24	1.121	0.020	0.154	0.019
25	1.051	-0.004	0.750	-0.018

NODE	$\omega/\omega_1^r = 1.2$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	172.670	-3.816	344.090	-9.343
2	156.230	-3.384	310.750	-8.215
3	124.240	-2.537	246.380	-6.193
4	80.650	-1.423	158.220	-3.579
5	29.720	-0.272	55.161	-0.822
6	169.160	-3.476	323.770	-8.310
7	154.220	-3.145	293.680	-7.442
8	124.520	-2.437	234.910	-5.757
9	83.014	-1.488	153.360	-3.499
10	31.562	-0.386	54.685	-0.968
11	141.650	-2.635	263.980	-6.262
12	129.410	-2.445	239.880	-5.723
13	104.890	-1.929	192.420	-4.493
14	70.081	-1.180	125.920	-2.759
15	26.306	-0.307	44.678	-0.753
16	93.159	-1.449	170.780	-3.530
17	85.354	-1.429	155.660	-3.375
18	69.277	-1.149	125.070	-2.686
19	46.233	-0.693	81.833	-1.628
20	17.179	-0.150	28.859	-0.385
21	30.700	-0.222	55.725	-0.692
22	28.414	-0.332	51.322	-0.848
23	23.133	-0.267	41.355	-0.670
24	15.421	-0.126	27.013	-0.338
25	55.351	0.052	9.157	0.068

NODE	$\omega/\omega_1^* = 1.6$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-41.189	-7.467	-502.110	-63.417
2	-36.676	-6.981	-452.250	-58.625
3	-28.378	-5.687	-356.980	-47.365
4	-16.375	-3.712	-225.220	-30.789
5	-2.139	-1.268	-70.730	-10.593
6	-14.738	-7.522	-429.970	-59.490
7	-11.717	-6.940	-388.250	-54.603
8	-6.801	-5.601	-303.260	-43.911
9	-0.600	-3.626	-189.520	-28.449
10	4.126	-1.213	-57.937	-9.683
11	6.772	-6.710	-320.990	-48.921
12	7.624	-6.166	-288.250	-44.762
13	8.508	-4.958	-225.880	-35.937
14	8.730	-3.199	-140.760	-23.284
15	6.196	-1.066	-42.929	-7.969
16	13.956	-4.632	-193.410	-32.154
17	13.632	-4.287	-174.090	-29.429
18	12.484	-3.455	-136.410	-23.662
19	10.109	-2.228	-84.846	-15.389
20	5.393	-0.738	-25.703	-5.352
21	6.404	-1.499	-60.254	-10.884
22	6.180	-1.445	-54.763	-10.020
23	5.495	-1.174	-42.940	-8.137
24	4.248	-0.755	-26.521	-5.396
25	2.070	-0.235	-7.492	-2.013

NODE	$\omega/\omega_1^* = 2.0$			
	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	20.015	-0.519	-31.424	3.132
2	18.299	-0.551	-28.130	2.570
3	14.834	-0.467	-21.944	1.904
4	10.926	-0.283	-12.510	1.171
5	6.823	-0.060	-0.920	0.389
6	37.019	-1.020	-10.128	2.031
7	34.940	-0.940	-7.603	1.690
8	30.135	-0.751	-3.574	1.254
9	23.004	-0.466	1.497	0.764
10	11.959	-0.129	4.880	0.240
11	41.805	-1.320	3.597	0.941
12	39.343	-1.168	4.858	0.790
13	33.768	-0.907	6.434	0.584
14	24.985	-0.556	7.538	0.351
15	11.609	-0.156	5.863	0.104
16	31.740	-1.095	6.943	0.242
17	29.827	-0.950	7.399	0.211
18	25.442	-0.721	7.649	0.156
19	18.481	-0.428	7.124	0.090
20	8.171	-0.107	4.439	0.021
21	11.173	-0.359	3.061	0.008
22	10.557	-0.302	3.167	0.018
23	9.008	-0.212	3.135	0.015
24	6.514	-0.102	2.777	0.009
25	2.783	0.012	1.609	0.003

$$\omega/\omega_1^* = 2.4$$

NODE	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-13.297	2.485	151.800	38.523
2	-12.285	2.085	136.740	35.560
3	-10.011	1.852	107.950	28.862
4	-5.407	1.459	69.687	18.893
5	1.453	0.701	28.213	6.577
6	-67.957	-14.357	89.083	22.192
7	-59.754	-13.267	81.857	20.459
8	-45.015	-10.586	67.074	16.552
9	-24.984	-6.785	46.427	10.774
10	-3.692	-2.313	19.741	3.645
11	-142.670	-26.023	-8.564	7.726
12	-126.880	-23.895	-5.945	7.131
13	-98.044	-19.189	-2.283	5.769
14	-59.183	-12.436	1.852	3.751
15	-16.212	-4.303	3.838	1.245
16	-153.290	-21.202	-62.187	3.707
17	-137.420	-19.430	-55.088	3.473
18	-107.490	-15.621	-42.217	2.846
19	-66.544	-10.179	-24.938	1.855
20	-20.043	-3.592	-6.399	0.597
21	-62.410	-7.798	-31.937	1.219
22	-56.711	-6.953	-28.957	1.408
23	-44.668	-5.620	-22.623	1.197
24	-27.802	-3.756	-13.795	0.764
25	-8.180	-1.475	-3.804	0.173

$$\omega/\omega_1^* = 2.8$$

NODE	STATIONARY END		MOVING END	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-5.777	6.008	21.819	4.897
2	-7.805	5.211	16.594	4.278
3	-9.385	4.060	9.220	3.318
4	-6.733	2.591	4.673	2.111
5	0.579	0.882	4.288	0.738
6	15.875	-0.804	44.768	-2.383
7	14.135	-0.748	39.718	-2.096
8	11.588	-0.610	31.187	-1.668
9	10.063	-0.395	22.137	-1.061
10	7.657	-0.137	11.558	-0.327
11	22.007	-7.737	49.806	-9.780
12	20.791	-6.905	45.481	-8.677
13	18.271	-5.465	37.313	-6.847
14	14.785	-3.499	26.602	-4.357
15	8.331	-1.175	12.179	-1.419
16	14.248	-9.168	34.969	-10.928
17	13.859	-8.174	32.340	-9.716
18	12.595	-6.463	26.929	-7.658
19	10.289	-4.131	19.235	-4.863
20	5.554	-1.379	8.484	-1.578
21	4.235	-3.848	11.547	-4.439
22	4.196	-3.393	10.759	-3.910
23	3.940	-2.669	9.054	-3.055
24	3.325	-1.692	6.525	-1.909
25	1.814	-0.535	2.835	-0.563

$$\omega/\omega_1^* = 3.2$$

NODE	STATIONARY END		MOVING END	
	REAL PART	IMAG PART	REAL PART	IMAG PART
1	86.729	-23.889	109.360	-67.994
2	78.101	-31.875	169.630	-92.756
3	84.296	-38.098	224.200	-112.690
4	69.008	-32.167	200.900	-95.861
5	28.969	-11.900	81.332	-35.773
6	35.577	-55.266	74.314	-95.122
7	52.855	-59.670	136.380	-114.370
8	68.344	-59.955	196.460	-126.980
9	62.832	-46.173	183.940	-103.580
10	28.501	-16.741	76.434	-38.172
11	7.085	-40.972	39.492	-72.740
12	26.093	-45.377	94.590	-88.712
13	46.102	-46.673	150.520	-99.675
14	47.657	-36.441	146.240	-81.807
15	22.306	-13.279	61.414	-30.156
16	7.852	11.376	31.139	-8.826
17	21.183	4.636	67.710	-22.716
18	35.279	-3.022	104.77	-36.441
19	35.624	-5.986	100.800	-34.546
20	16.252	-2.492	42.104	-13.009
21	5.650	16.746	13.952	10.271
22	10.369	13.502	26.442	4.884
23	14.888	8.708	38.659	-1.886
24	14.185	4.412	36.393	-4.588
25	6.099	1.497	14.673	-1.499

D.3 Tests of Convergence

D.3.1 Exciting frequency in the upstream-downstream direction

NODE	$\omega/\omega_1' = 0.4$ <i>MOVING END</i>			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	99.712	-0.126	99.386	0.064
2	94.839	-0.105	94.749	0.058
3	82.746	-0.088	82.924	0.042
4	62.051	-0.068	62.446	0.022
5	28.265	-0.036	28.823	0.002
6	87.058	-0.080	86.826	0.091
7	82.457	-0.067	82.483	0.080
8	71.443	-0.055	71.726	0.061
9	53.190	-0.041	53.663	0.037
10	24.360	-0.020	24.927	0.010
11	78.646	-0.047	78.395	0.119
12	74.729	-0.039	74.743	0.104
13	65.095	-0.032	65.354	0.080
14	49.132	-0.024	49.599	0.049
15	23.418	-0.014	24.059	0.016
16	73.758	-0.023	73.435	0.148
17	70.395	-0.019	70.336	0.128
18	61.769	-0.017	61.956	0.099
19	47.237	-0.015	47.663	0.062
20	23.116	-0.010	23.784	0.021
21	71.056	-0.011	70.493	0.177
22	67.974	-0.010	67.646	0.152
23	59.841	-0.011	59.758	0.118
24	45.967	-0.012	46.137	0.078
25	22.793	-0.010	23.321	0.030

NODE	$\omega/\omega_1 = 1.2$ <i>MOVING END</i>			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG PART	REAL PART	IMAG. PART
1	261.970	-7.629	262.500	-10.149
2	241.360	-6.468	242.370	-8.637
3	199.410	-4.897	200.430	-6.525
4	137.100	-3.017	137.760	-3.974
5	53.450	-0.952	53.700	-1.196
6	238.640	-6.685	239.300	-9.224
7	219.340	-5.724	220.450	-7.928
8	180.310	-4.365	181.550	-6.046
9	123.240	-2.706	124.140	-3.728
10	47.770	-0.856	48.180	-1.148
11	214.300	-5.717	214.940	-8.320
12	197.210	-4.908	198.290	-7.173
13	162.480	-3.753	163.690	-5.490
14	111.780	-2.331	112.720	-3.398
15	44.350	-0.736	44.890	-1.050
16	194.460	-4.971	194.970	-7.668
17	179.400	-4.263	180.320	-6.606
18	148.450	-3.259	149.500	-5.056
19	103.010	-2.024	103.860	-3.131
20	41.740	-0.638	42.310	-0.970
21	182.850	-4.700	182.980	-7.518
22	168.970	-4.010	169.470	-6.444
23	140.140	-3.058	140.770	-4.917
24	97.590	-1.900	98.060	-3.042
25	40.080	-0.607	40.470	-0.954

NODE	$\omega/\omega_1' = 2.4$ MOVING END			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	38.023	-3.311	36.693	-5.333
2	44.552	-2.811	43.626	-4.660
3	49.858	-2.224	49.778	-3.719
4	46.193	-1.488	46.826	-2.446
5	24.492	-0.559	25.289	-0.857
6	23.914	-1.092	23.315	-2.729
7	30.094	-0.889	29.819	-2.370
8	35.773	-0.676	36.160	-1.873
9	34.583	-0.422	35.460	-1.194
10	19.275	-0.116	20.101	-0.353
11	24.447	0.991	24.535	-0.718
12	29.774	0.953	30.127	0.004
13	34.458	0.795	35.289	0.028
14	33.173	0.548	34.305	0.053
15	19.097	0.236	20.079	0.087
16	37.350	1.980	37.837	1.384
17	41.104	1.859	41.853	1.338
18	43.162	1.528	44.281	1.104
19	38.761	1.030	40.055	0.752
20	21.299	0.403	22.362	0.323
21	48.727	1.967	49.073	1.585
22	51.237	1.891	51.851	1.556
23	51.046	1.569	51.996	1.289
24	43.704	1.060	44.778	0.873
25	23.094	0.411	24.017	0.359

NODE	$\omega/\omega_1^* = 2.8$ <i>MOVING END</i>			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG PART	REAL PART	IMAG. PART
1	-28.639	-36.102	-25.456	-43.441
2	-8.055	-33.491	-5.041	-40.312
3	17.381	-27.808	20.941	-33.380
4	31.749	-18.635	35.183	-22.180
5	21.158	-6.570	23.023	-7.692
6	29.131	9.928	25.127	10.305
7	40.856	8.621	37.475	8.866
8	52.428	6.359	50.736	6.481
9	50.858	3.805	50.622	3.868
10	26.119	1.334	26.472	1.379
11	83.431	38.130	74.836	43.326
12	87.800	34.362	80.412	39.038
13	87.527	27.230	82.538	30.913
14	72.122	17.487	69.623	19.813
15	33.490	6.083	33.073	6.868
16	71.126	11.238	67.206	12.182
17	76.154	10.024	73.034	10.822
18	77.276	7.711	75.691	8.285
19	65.035	4.834	64.770	5.184
20	31.232	1.726	31.661	1.866
21	27.324	-31.120	30.529	-37.503
22	36.876	-28.630	40.149	-34.509
23	45.703	-23.482	49.235	-28.289
24	44.469	-15.511	47.579	-18.603
25	24.351	-5.375	26.062	-6.383

NODE	$\omega/\omega_1 = 3.2$ <i>MOVING END</i>			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	406.970	-681.150	809.990	-286.970
2	-257.610	-849.510	83.070	-511.410
3	-968.800	-966.860	-759.520	-738.210
4	-1103.740	-811.530	-996.560	-687.870
5	-468.310	-318.930	-421.010	-277.330
6	379.420	-645.290	808.530	-239.630
7	-255.520	-813.440	83.369	-468.690
8	-964.960	-933.390	-760.500	-702.710
9	-1101.850	-786.310	-1001.800	-662.920
10	-465.340	-306.950	-422.850	-265.360
11	405.500	-633.760	806.460	-221.460
12	-243.060	-793.070	93.089	-444.700
13	-939.340	-904.850	-738.410	-673.410
14	-1075.250	-759.920	-978.980	-637.400
15	-453.540	-296.390	-413.050	-255.550
16	393.820	-681.770	794.750	-273.600
17	-235.060	-824.100	102.580	-479.370
18	-910.300	-914.320	-705.000	-685.200
19	-1041.790	-756.150	-940.620	-635.020
20	-438.880	-292.760	-396.530	-252.580
21	370.100	-738.270	770.320	-340.040
22	-241.700	-865.640	97.059	-527.560
23	-896.950	-935.500	-686.280	-709.380
24	-1020.650	-762.560	-911.980	-642.460
25	-432.160	-295.700	-385.620	-256.350

D.3.2 Exciting frequency in the cross-stream direction

NODE	$\omega/\omega_1^* = 0.4$ <i>MOVING END</i>			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-19.417	0.393	-18.130	0.348
2	-16.904	0.325	-15.725	0.287
3	-12.506	0.251	-11.580	0.222
4	-6.220	0.168	-5.568	0.150
5	0.927	0.069	1.432	0.064
6	-7.540	0.328	-5.735	0.270
7	-4.276	0.273	-3.612	0.222
8	-1.858	0.208	-0.487	0.170
9	2.215	0.134	3.255	0.110
10	4.546	0.049	5.277	0.042
11	-2.013	0.284	0.452	0.197
12	-0.428	0.237	1.818	0.161
13	1.820	0.180	3.640	0.122
14	4.049	0.114	5.349	0.078
15	4.235	0.041	4.939	0.030
16	-1.140	0.262	2.099	0.127
17	-0.082	0.217	2.803	0.101
18	1.402	0.165	3.654	0.075
19	2.771	0.105	4.250	0.048
20	2.711	0.038	3.356	0.019
21	-2.595	0.263	1.080	0.051
22	-1.827	0.217	1.270	0.037
23	-0.668	0.164	1.498	0.027
24	0.122	0.104	1.621	0.018
25	0.868	0.038	1.206	0.008

NODE	$\omega/\omega_1' = 0.4$ STATIONARY END			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-12.922	0.288	-13.104	0.216
2	-11.060	0.239	-11.195	0.178
3	-7.895	0.187	-8.006	0.139
4	-3.288	0.128	-3.306	0.095
5	1.894	0.055	2.165	0.042
6	-1.251	0.236	-1.218	0.165
7	1.048	0.197	0.466	0.136
8	2.617	0.152	2.738	0.104
9	5.068	0.099	5.307	0.069
10	5.491	0.039	5.946	0.027
11	3.930	0.202	4.002	0.121
12	4.925	0.169	5.028	0.099
13	6.052	0.130	6.184	0.075
14	6.750	0.084	6.969	0.049
15	5.130	0.032	5.468	0.020
16	4.450	0.185	4.331	0.080
17	4.953	0.154	4.830	0.063
18	5.382	0.118	5.263	0.047
19	5.309	0.077	5.275	0.031
20	3.550	0.029	3.688	0.013
21	2.767	0.186	1.787	0.034
22	3.002	0.155	1.923	0.024
23	3.145	0.119	2.017	0.018
24	2.893	0.079	1.948	0.013
25	1.667	0.031	1.302	0.007

$\omega/\omega_1 = 0.8$ *MOVING END*

NODE	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG PART	REAL PART	IMAG. PART
1	-60.347	1.599	-43.565	1.058
2	-53.807	1.352	-38.699	0.891
3	-41.704	1.019	-29.766	0.664
4	-24.850	0.623	-17.115	0.395
5	-5.264	0.191	-2.326	0.109
6	-46.587	1.423	-27.841	0.861
7	-40.448	1.218	-23.568	0.738
8	-29.652	0.926	-16.265	0.557
9	-15.509	0.570	-6.760	0.336
10	-1.323	0.175	2.017	0.094
11	-38.440	1.251	-16.211	0.612
12	-33.241	1.078	-13.231	0.532
13	-24.101	0.822	-8.248	0.403
14	-12.472	0.505	-2.188	0.241
15	-1.236	0.152	2.487	0.062
16	-35.329	1.103	-8.037	0.338
17	-30.881	0.953	-6.385	0.301
18	-22.926	0.724	-3.611	0.228
19	-12.735	0.441	-0.353	0.131
20	-2.424	0.128	1.863	0.024
21	-35.504	1.001	-2.096	0.071
22	-31.483	0.863	-1.653	0.072
23	-24.088	0.645	-0.822	0.050
24	-14.445	0.375	0.154	0.019
25	-4.085	0.092	0.750	-0.018

NODE	$\omega/\omega_1' = 0.8$ <i>STATIONARY END</i>			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	-35.556	1.152	-29.232	0.745
2	-31.469	0.985	-25.758	0.622
3	-24.053	0.754	-19.540	0.468
4	-13.602	0.478	-10.634	0.290
5	-1.531	0.165	-0.220	0.094
6	-22.149	0.999	-14.908	0.589
7	-18.443	0.856	-11.881	0.499
8	-12.246	0.656	-7.008	0.377
9	-4.392	0.412	-0.864	0.232
10	2.370	0.136	3.949	0.071
11	-14.697	0.851	-6.004	0.416
12	-11.840	0.734	-3.993	0.355
13	-7.168	0.564	-0.919	0.269
14	-1.652	0.352	2.488	0.164
15	2.133	0.113	4.023	0.048
16	-12.293	0.735	-1.589	0.238
17	-10.118	0.636	-0.524	0.205
18	-6.502	0.488	1.048	0.155
19	-2.243	0.303	2.622	0.092
20	1.061	0.095	2.838	0.023
21	-12.937	0.663	-0.354	0.065
22	-11.143	0.574	0.250	0.057
23	-8.010	0.436	0.697	0.041
24	-4.193	0.264	1.121	0.020
25	-3.884	0.076	1.051	-0.004

NODE	$\omega/\omega_1 = 2.0$ MOVING END			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	1.861	4.561	-31.424	3.132
2	1.863	3.987	-28.130	2.570
3	1.822	3.111	-21.944	1.904
4	2.578	1.998	-12.510	1.171
5	3.784	0.720	-0.920	0.389
6	13.088	0.510	-10.128	2.031
7	13.269	0.441	-7.603	1.890
8	12.934	0.337	-3.574	1.254
9	11.887	0.219	1.497	0.764
10	7.955	0.092	4.880	0.240
11	4.391	2.797	3.597	0.941
12	5.545	2.502	4.858	0.790
13	6.963	1.992	6.434	0.584
14	7.753	1.282	7.538	0.351
15	5.683	0.427	5.863	0.104
16	-22.142	-2.272	6.943	0.242
17	-18.729	-2.096	7.399	0.211
18	-12.971	-1.716	7.649	0.156
19	-6.077	-1.135	7.124	0.090
20	-0.112	-0.388	4.439	0.021
21	-46.416	0.013	3.061	0.008
22	-41.060	-0.175	3.167	0.018
23	-31.452	-0.288	3.135	0.015
24	-18.994	-0.285	2.777	0.009
25	-5.624	-0.136	1.609	0.003

NODE	$\omega/\omega_1^* = 2.4$ <i>MOVING END</i>			
	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	17.986	2.923	151.800	38.523
2	15.849	2.545	136.740	35.560
3	12.232	2.029	107.950	28.862
4	8.821	1.379	69.687	18.893
5	5.799	0.560	26.213	6.577
6	37.470	0.603	89.083	22.192
7	34.970	0.588	81.857	20.459
8	29.844	0.508	67.074	16.552
9	22.594	0.375	46.427	10.774
10	11.525	0.174	19.741	3.645
11	42.709	-1.874	-8.564	7.726
12	40.044	-1.567	-5.945	7.131
13	34.318	-1.176	-2.283	5.769
14	25.341	-0.707	1.852	3.751
15	11.580	-0.200	3.838	1.245
16	32.969	-3.392	-62.187	3.707
17	31.024	-2.929	-55.088	3.473
18	26.613	-2.260	-42.217	2.846
19	19.407	-1.411	-24.938	1.855
20	8.427	-0.445	-6.399	0.597
21	20.889	-3.934	-31.937	1.219
22	19.762	-3.451	-28.957	1.408
23	16.953	-2.697	-22.623	1.197
24	12.139	-1.712	-13.795	0.764
25	4.864	-0.568	-3.804	0.173

$\omega/\omega_1^* = 2.8$ MOVING END

NODE	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG. PART	REAL PART	IMAG. PART
1	102.950	35.311	21.819	4.897
2	89.912	32.779	~~16.594~~	~~4.278~~
3	68.046	27.193	9.220	3.318
4	42.623	18.249	4.673	2.111
5	16.830	6.519	4.288	0.736
6	36.370	-16.838	44.768	-2.383
7	32.892	-14.863	39.718	-2.096
8	26.352	-11.362	31.187	-1.668
9	19.113	-8.976	22.137	-1.061
10	10.086	-2.272	11.558	-0.327
11	-25.994	-48.876	49.806	-9.780
12	-21.841	-44.066	45.481	-8.677
13	-15.698	-34.979	37.313	-6.847
14	-7.422	-22.410	26.602	-4.357
15	0.323	-7.598	12.179	-1.419
16	-12.993	-17.860	34.969	-10.928
17	-10.323	-15.983	32.340	-9.716
18	-6.600	-12.455	26.929	-7.658
19	-2.212	-7.817	19.235	-4.863
20	1.091	-2.596	8.484	-1.578
21	34.213	30.800	11.547	-4.439
22	31.301	28.390	10.759	-3.910
23	25.760	23.310	9.054	-3.055
24	17.548	15.460	6.525	-1.909
25	6.736	5.460	2.835	-0.563

$\omega/\omega_1 = 3.2$ MOVING END

NODE	173 ELEMENTS MESH		142 ELEMENTS MESH	
	REAL PART	IMAG PART	REAL PART	IMAG PART
1	103.380	-326.660	109.360	-67.994
2	-90.890	-340.350	169.630	-92.756
3	-298.720	-326.840	224.200	-112.690
4	-335.150	-246.840	200.900	-95.861
5	-140.710	-92.240	81.332	-35.773
6	152.520	-309.200	74.314	-95.122
7	-68.460	-332.800	136.380	-114.370
8	-286.860	-331.530	196.460	-126.980
9	-335.410	-256.790	183.940	-103.580
10	-140.590	-96.472	76.434	-38.172
11	188.880	-283.020	39.492	-72.740
12	37.246	-322.290	94.590	-88.712
13	-312.960	-339.590	150.520	-99.675
14	-369.020	-272.630	146.240	-81.807
15	-156.370	-104.300	61.414	-30.156
16	156.918	-264.270	31.139	-8.826
17	-96.562	-318.020	67.710	-22.716
18	-368.860	-351.710	104.770	-36.441
19	-422.42	-290.480	100.800	-34.546
20	-179.250	-112.500	42.104	-13.009
21	126.970	-255.790	13.952	10.271
22	-136.190	-317.420	26.442	4.884
23	-415.610	-359.760	38.659	-1.886
24	-462.520	-301.370	36.393	-4.588
25	-197.230	-118.390	14.673	-1.499

D.4 Different Reservoir Lengths (upstream-downstream excitation)

NODE	$\omega/\omega_1' = 0.4$ STATIONARY END			
	$L = 3 H$		$L = 2 H$	
	REAL PART	IMAG PART	REAL PART	IMAG PART
1	112.110	-0.538	113.530	-0.313
2	106.530	-0.416	107.510	-0.255
3	92.325	-0.348	92.993	-0.195
4	68.562	-0.238	68.818	-0.129
5	31.378	-0.106	30.884	-0.052
6	99.816	-0.445	101.100	-0.260
7	94.574	-0.369	95.385	-0.213
8	81.423	-0.284	81.912	-0.161
9	60.033	-0.187	60.141	-0.103
10	27.658	-0.075	27.039	-0.038
11	91.490	-0.387	92.828	-0.227
12	86.955	-0.321	87.772	-0.186
13	75.156	-0.246	75.663	-0.140
14	56.044	-0.160	56.160	-0.089
15	26.838	-0.063	26.202	-0.032
16	86.541	-0.364	88.001	-0.212
17	82.569	-0.302	83.482	-0.173
18	71.766	-0.231	72.351	-0.130
19	54.104	-0.150	54.273	-0.082
20	26.570	-0.059	25.939	-0.029
21	83.280	-0.378	85.115	-0.218
22	79.517	-0.313	80.831	-0.177
23	69.206	-0.240	70.163	-0.132
24	52.239	-0.159	52.722	-0.082
25	25.868	-0.066	25.452	-0.029

NODE	$\omega/\omega_1' = 0.8$ <i>STATIONARY END</i>			
	$L = 3 H$		$L = 2 H$	
	REAL PART	IMAG PART	REAL PART	IMAG PART
1	177.000	-2.031	184.410	-1.459
2	165.300	-1.695	171.630	-1.183
3	139.110	-1.272	143.960	-0.853
4	98.563	-0.782	101.420	-0.488
5	41.374	-0.248	41.620	-0.118
6	162.870	-1.797	170.130	-1.285
7	151.670	-1.517	157.810	-1.058
8	126.870	-1.147	131.560	-0.775
9	89.218	-0.707	91.950	-0.455
10	37.380	-0.221	37.520	-0.118
11	151.930	-1.612	159.270	-1.142
12	141.680	-1.367	147.880	-0.944
13	118.710	-1.035	123.450	-0.694
14	84.014	-0.638	86.870	-0.407
15	36.165	-0.197	36.300	-0.102
16	144.630	-1.492	152.140	-1.039
17	135.160	-1.265	141.500	-0.855
18	113.610	-0.955	118.450	-0.621
19	80.967	-0.585	83.800	-0.356
20	35.523	-0.178	35.670	-0.083
21	139.930	-1.462	147.870	-0.997
22	130.810	-1.231	137.580	-0.806
23	109.980	-0.918	115.210	-0.564
24	78.354	-0.550	81.490	-0.296
25	34.574	-0.156	34.920	-0.040

NODE	$\omega/\omega_1^* = 0.4$ <i>MOVING END</i>			
	$L = 3H$		$L = 2H$	
	REAL PART	IMAG PART	REAL PART	IMAG PART
1	108.440	-0.321	99.386	0.064
2	103.210	-0.269	94.749	0.058
3	89.709	-0.218	82.924	0.042
4	66.912	-0.160	62.446	0.022
5	30.850	-0.082	28.823	0.002
6	96.100	-0.240	86.826	0.091
7	91.219	-0.201	82.483	0.080
8	78.770	-0.159	71.726	0.061
9	58.349	-0.111	53.663	0.037
10	27.111	-0.052	24.927	0.010
11	87.736	-0.184	78.395	0.119
12	83.565	-0.154	74.743	0.104
13	72.474	-0.121	65.354	0.080
14	54.339	-0.084	49.599	0.049
15	26.283	-0.040	24.059	0.016
16	82.764	-0.152	73.435	0.148
17	79.158	-0.127	70.336	0.128
18	69.070	-0.101	61.956	0.099
19	52.392	-0.072	47.663	0.062
20	26.013	-0.035	23.784	0.021
21	79.505	-0.146	70.493	0.177
22	76.110	-0.124	67.646	0.152
23	66.520	-0.101	59.758	0.118
24	50.545	-0.074	46.137	0.078
25	25.326	-0.039	23.321	0.030

NODE	$\omega/\omega_1^* = 0.8$ <i>MOVING END</i>			
	$L = 3H$		$L = 2H$	
	REAL PART	IMAG PART	REAL PART	IMAG PART
1	153.410	-1.478	127.720	-0.235
2	143.980	-1.194	120.450	-0.122
3	122.240	-0.892	103.510	-0.073
4	87.828	-0.569	75.755	-0.061
5	37.826	-0.213	33.256	-0.049
6	139.170	-1.196	113.110	-0.008
7	130.250	-0.972	106.320	0.059
8	109.900	-0.724	90.803	0.068
9	78.391	-0.453	66.000	0.041
10	33.796	-0.157	29.025	0.002
11	128.120	-0.990	101.910	0.164
12	120.160	-0.804	96.067	0.200
13	101.650	-0.597	82.420	0.176
14	73.127	-0.372	60.638	0.111
15	32.558	-0.128	27.730	0.027
16	120.760	-0.882	94.557	0.268
17	113.590	-0.713	89.491	0.287
18	96.522	-0.528	77.291	0.242
19	70.065	-0.329	57.590	0.152
20	31.915	-0.114	27.087	0.041
21	116.080	-0.897	90.263	0.281
22	109.250	-0.720	85.573	0.300
23	92.920	-0.533	74.107	0.249
24	67.501	-0.336	55.416	0.151
25	30.990	-0.122	26.420	0.036

D.5 Comparison for 2-D Straight Gravity Dam

Z/H	$\omega/\omega_1' = 0.4$ <u>STATIONARY END</u>		
	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	94.790	90.030	-0.091
0.3	89.201	85.772	-0.072
0.5	77.544	74.784	-0.052
0.7	58.465	56.499	-0.031
0.9	27.725	27.215	-0.009

Z/H	$\omega/\omega_1' = 0.8$ <u>STATIONARY END</u>		
	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	154.800	143.010	-0.663
0.3	143.500	133.700	-0.487
0.5	120.870	112.880	-0.324
0.7	86.470	80.904	-0.162
0.9	37.443	35.276	-0.020

Z/H	$\omega/\omega_1' = 1.2$ <u>STATIONARY END</u>		
	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	70.946	50.876	-0.195
0.3	68.144	50.919	-0.082
0.5	61.504	47.939	-0.061
0.7	48.729	40.014	-0.081
0.9	24.571	21.998	-0.077

$\omega/\omega_1^r = 1.6$ <u>STATIONARY END</u>			
Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG PART
0.1	-83.914	-98.598	-4.093
0.3	-71.042	-83.391	-4.019
0.5	-48.200	-57.501	-3.475
0.7	-21.091	-26.439	-2.442
0.9	0.698	0.438	-0.971

$\omega/\omega_1^r = 2.0$ <u>STATIONARY END</u>			
Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG PART
0.1	37.681	27.658	-4.021
0.3	39.587	31.510	-3.549
0.5	40.931	34.695	-2.852
0.7	37.175	33.221	-1.891
0.9	21.077	20.275	-0.676

$\omega/\omega_1^r = 2.4$ <u>STATIONARY END</u>			
Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	-79.664	-96.684	-14.754
0.3	-64.267	-78.957	-14.147
0.5	-38.729	-50.261	-11.827
0.7	-11.952	-19.031	-7.927
0.9	4.690	3.691	-2.818

$\omega/\omega_1^* = 2.8$ <u>STATIONARY END</u>			
Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	-10.086	-14.447	-2.759
0.3	5.846	1.771	-2.992
0.5	26.617	22.113	-2.905
0.7	36.572	32.736	-2.166
0.9	22.957	21.951	-0.786

$\omega/\omega_1^* = 3.2$ <u>STATIONARY END</u>			
Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	-173.890	4.230	-70.033
0.3	-210.260	39.540	-165.520
0.5	-232.230	79.159	-261.090
0.7	-185.240	87.571	-248.310
0.9	-65.146	44.934	-97.685

$\omega/\omega_1^* = 0.4$ <u>MOVING END</u>			
Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	83.345	79.643	0.244
0.3	78.875	76.409	0.213
0.5	69.348	67.400	0.166
0.7	53.201	51.824	0.106
0.9	25.911	25.698	0.036

$\omega/\omega_1^* = 0.8$ MOVING END

Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	106.580	102.950	0.234
0.3	99.994	97.547	0.257
0.5	86.338	84.323	0.216
0.7	64.311	62.776	0.130
0.9	29.806	29.354	0.028

 $\omega/\omega_1^* = 1.2$ MOVING END

Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	232.030	201.380	-7.779
0.3	213.450	186.620	-6.678
0.5	176.820	155.160	-5.088
0.7	122.770	108.160	-3.143
0.9	50.085	44.316	-0.989

 $\omega/\omega_1^* = 1.6$ MOVING END

Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	-189.640	-211.090	-1.912
0.3	-166.430	-184.920	-2.365
0.5	-123.900	-137.820	-2.388
0.7	-69.703	-77.557	-1.893
0.9	-16.054	-16.321	-0.886

$\omega/\omega_1^* = 2.0$ MOVING END

Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	-34.356	-34.984	2.873
0.3	-25.415	-25.086	2.400
0.5	-10.675	-10.151	1.690
0.7	4.028	4.635	0.910
0.9	9.654	10.899	0.192

 $\omega/\omega_1^* = 2.4$ MOVING END

Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	-0.608	-6.754	-7.507
0.3	6.970	2.383	-7.001
0.5	17.705	14.216	-5.794
0.7	24.216	22.057	-3.864
0.9	17.144	17.152	-1.343

 $\omega/\omega_1^* = 2.8$ MOVING END

Z/H	CLASSICAL	BOUNDARY ELEMENT SOLUTION	
	SOLUTION	REAL PART	IMAG. PART
0.1	29.744	20.346	-6.540
0.3	40.495	32.509	-6.221
0.5	52.467	45.497	-5.371
0.7	52.106	47.018	-3.683
0.9	28.076	26.527	-1.240

Z/H	$\omega/\omega_1^* = 3.2$ <i>MOVING END</i>		
	CLASSICAL SOLUTION	BOUNDARY ELEMENT SOLUTION	
		REAL PART	IMAG PART
0.1	-347.850	-36.964	-121.470
0.3	-419.500	56.894	-322.900
0.5	-465.440	163.140	-526.370
0.7	-378.090	186.490	-505.800
0.9	-140.790	87.599	-199.520

END

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FIN