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Determination of α_s using Jet Rates at LEP

by

Michael A. Donkers,

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Physics
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Ottawa, Ontario, Canada
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Determination of α_s using Jet Rates at LEP

Submitted by
Michael Donkers, B.Sc., M.Sc.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Physics

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Abstract

Jets are produced in any high energy collision of particles in which quarks are produced in the final state. Using the OPAL detector to measure particles produced in e^+e^- collisions at the LEP accelerator, the rate of jet formation has been measured at 91 GeV as well as each of the LEP2 energies, ranging from 161 GeV to 207 GeV. The jet rate observables, in particular the differential 2-jet rate and the average jet rate can be used to determine a value of the strong coupling constant, α_s , by fitting to various theoretical predictions. The value of α_s has been determined using data at 91 GeV and a combined sample comprising all of the LEP2 energies with a luminosity weighted centre-of-mass energy of 195.8 GeV for 10 theoretical predictions and two jet clustering algorithms. A fit of the 91 GeV and LEP2 values of α_s determined using the $\ln R$ matching prediction is also performed on the D_2 and $\langle N \rangle$ distributions to the 3-loop α_s prediction to produce 4 values of $\alpha_s(M_{Z^0})$.

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Chapter 1

Introduction

The Standard Model of particle physics is currently the most established model describing the interactions of the fundamental particles within the universe. Three of the four fundamental forces, the electromagnetic, the weak and the strong force (gravity is often neglected, as it has no quantum field theoretical description and its strength is very weak at these scales) are described within the Standard Model. The electromagnetic and weak forces are described by a single unified field theory, known as the “electroweak theory”. The strong force is described by a separate field theory known as Quantum Chromodynamics (QCD). This theory describes the interactions of quarks, the only fundamental fermions to experience the strong force, and gluons, the bosons which mediate (and also experience) the strong interaction.

The predictions based on QCD rely on one fundamental parameter, the strength of the coupling between partons (quarks and gluons) under the strong interaction, often called the strong coupling constant, denoted by α_s . The value of α_s can be measured using interactions that involve quarks and gluons, for example hadronic decays of the τ lepton or the Z^0 line shape [1]. The most common method of measuring α_s is through collision experiments in which two particles are accelerated to high energies and brought into collision producing quarks in the final state.

The LEP collider at CERN brings intense beams of electrons and positrons into collision at extremely high energies. In these collisions the electrons annihilate with the

positrons often producing quarks, which go on to produce composite particles called hadrons through a process, called hadronization, which is not theoretically well understood. These hadrons tend to be produced in collimated streams travelling in the same general direction as their parent quark or gluon. These streams of particles are referred to as jets. Observables based on counting the number of jets in $e^+e^- \rightarrow \text{hadrons}$ events, such as the rate of jet formation and the average number of jets per event, can be compared to QCD expectations to determine a value of the strong coupling constant.

Values of the strong coupling constant will be determined in this analysis using jet multiplicity distributions for the different centre-of-mass energy points explored by LEP. A comparison of the high energy points (with $\sqrt{s} \geq 161$ GeV) to those produced at the Z^0 pole (with $\sqrt{s} = 91.2$ GeV) as well as values from lower energies will test the prediction of the running of the strong coupling constant. The running of α_s refers to the expectation that the value of α_s will change (in fact decrease) as the scale (in this case centre-of-mass energy) of the interaction increases.

The field theory behind the Standard Model (and QCD in particular) will be outlined in Chapter 2. Also discussed will be models involved in the formation of hadrons from quarks and gluons and a method for defining jets observed in the detector based on the geometric and kinematic properties of the hadrons. Finally a description of jet multiplicity observables along with their QCD predictions will be presented.

A description of the LEP collider is given in Chapter 3. The OPAL detector, which is one of four detectors at LEP used to detect and measure properties of particles produced following the annihilation of an electron - positron pair, is also described in some detail.

The data sample collected by OPAL and used in this analysis is presented in Chapter 4 along with a description of the Monte Carlo samples which are used to compare and correct the data. A method for selecting the desired multihadronic events from the full data sample is described. Chapter 4 also includes details regarding the treatment of expected background and a description of the method to determine the size of expected systematic errors.

The results of the analysis on the jet rate distributions, including plots and tables

of values are given in Chapter 5. Fits of the 3-jet rate, the differential 2-jet rate and the average jet rate to various QCD expectations to determine the value α_s , are also presented. Finally, a comparison of the values of α_s determined using the differential 2-jet rate and the average jet rate to previously determined values from lower energies is shown to demonstrate the running of the strong coupling constant. A determination of the value of $\alpha_s(M_{Z^0})$ is also made based on the observed running. These results are summarized with conclusions in Chapter 6.

Chapter 2

Theoretical Framework

2.1 Standard Model

The interactions between fundamental particles are a result of four fundamental forces of nature: the electromagnetic, the strong, the weak, and the gravitational forces. The last force, gravity, is very weak and can be neglected. The fundamental particles that are the building blocks of matter are all half integer spin particles, and therefore are fermions. These fundamental particles are grouped into categories depending on the forces that affect them. Particles which experience all of the fundamental forces are placed in a category known as quarks. Quarks are the basic constituents of protons and neutrons. Particles which do not experience the strong force are grouped together and are called leptons. The leptons are further subdivided into a group which experiences both the weak and the electromagnetic force (electrons, muons, and taus) and those that are electrically neutral and hence only experience the weak force, the neutrinos. Quarks and leptons are grouped into three successively heavier isospin pairs or doublets called generations (see Table 2.1). The interactions between fundamental particles are characterized by the exchange between quarks and/or leptons of quanta, which have integer spin and are therefore bosons.

The electromagnetic and weak interactions have been unified, through quantum field theory, into a force called the electroweak force. Although this theory describes

completely the interactions which involve leptons, it does not describe all of the quark interactions. The description of quark interactions proceeding through the strong interaction are described by the quantum theory called quantum chromodynamics (QCD), which together with the theory of the electroweak interaction, forms the basis for the Standard Model of particle physics.

Type	Particle		Spin	Charge	Mass
quark	u	up	$\frac{1}{2}$	$\frac{2}{3}$	~ 5 MeV
	d	down	$\frac{1}{2}$	$-\frac{1}{3}$	~ 10 MeV
	s	strange	$\frac{1}{2}$	$-\frac{1}{3}$	~ 200 MeV
	c	charm	$\frac{1}{2}$	$\frac{2}{3}$	~ 1.3 GeV
	b	bottom	$\frac{1}{2}$	$-\frac{1}{3}$	~ 4.3 GeV
	t	top	$\frac{1}{2}$	$\frac{2}{3}$	174.3 ± 5.1 GeV
lepton	e	electron	$\frac{1}{2}$	-1	511 keV
	ν_e	e neutrino	$\frac{1}{2}$	0	~ 0
	μ	muon	$\frac{1}{2}$	-1	105.7 MeV
	ν_μ	μ neutrino	$\frac{1}{2}$	0	~ 0
	τ	tau	$\frac{1}{2}$	-1	1777 MeV
	ν_τ	τ neutrino	$\frac{1}{2}$	0	~ 0
gauge boson	γ	photon	1	0	0
	Z^0		1	0	91.1876 ± 0.0021 GeV
	W^-, W^+		1	± 1	$80.451 \pm .033$ GeV
	g	gluon	1	0	0
	G	graviton	2	0	0

Table 2.1: List of fundamental particles of the Standard Model. Particles are grouped into successively heavier generations (masses taken from [2])

2.2 Gauge Theory and Local Symmetry

The Standard Model is a gauge theory based on relativistic quantum mechanics describing particle interactions as interactions between quantized fields. Interactions

take place between fermionic (matter) fields and are mediated by bosonic (gauge) fields and are characterized by a Lagrangian density $\mathcal{L}$, which is a function of the fields $\psi_i(x)$ and their gradients $\partial_\mu \psi_i(x)$. The equations of motion can then be determined by the use of the Euler–Lagrange equation,

$$\frac{\delta \mathcal{L}}{\delta \psi_i} = \frac{\partial}{\partial x_\mu} \frac{\delta \mathcal{L}}{\delta (\partial_\mu \psi_i)}. \quad (2.1)$$

A typical classical Lagrangian density for non-interacting free fermions is [3]

$$\mathcal{L} = \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i \quad (2.2)$$

where ψ_i corresponds to a fermion with mass m_i , the γ 's are a set of four 3×3 matrices known as the Dirac γ matrices and are related to the metric tensor through an anti-commutation relation, $\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu}$, with $\mu, \nu = 1, 2, 3, 4$ [3]. The bar over the first ψ_i indicates that it is a Hermitian conjugate of that field multiplied by the Dirac matrix γ^0 . Invariance of $\mathcal{L}$ under a symmetry (gauge) transformation is an important property since it corresponds to a conservation law, through Noether's theorem [4]. Consider a gauge transformation of the field $\psi_i(x)$ in Eqn 2.2, such that,

$$\psi_i(x) \rightarrow \psi'_i(x) \equiv e^{iq_i \theta(x)} \psi_i(x) \quad (2.3)$$

where q_i is an arbitrary parameter dependent on the type of fermion, ψ_i , and $\theta(x)$ is a continuous arbitrary function of the position variable x . This transformation would correspond to a rotation, and is known as a local gauge transformation since it is dependent on the local position x of the field.

Obviously the mass term in Eqn 2.2 will remain unchanged by this rotation. Unfortunately the terms involving the gradients of the fields will not transform as easily,

$$\partial_\mu \psi_i(x) \rightarrow \partial_\mu \psi'_i(x) = e^{iq_i \theta(x)} \partial_\mu \psi_i(x) - iq_i [\partial_\mu \theta(x)] e^{iq_i \theta(x)} \psi_i(x). \quad (2.4)$$

Clearly, the last term removes the invariance of $\mathcal{L}$ under the gauge transformation. It is thus necessary to introduce a transformation of the derivative with the introduction of a gauge field A_μ ,

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - iq_i A_\mu(x). \quad (2.5)$$

D_μ is called a covariant derivative, and A_μ is the vector potential of the gauge field. This gauge field will have a corresponding transformation of the form,

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \partial_\mu \theta(x) \quad (2.6)$$

which will restore the gauge invariance of $\mathcal{L}$. In the case where the parameter q_i is associated with the electric charge, this field will represent the electromagnetic field and hence, the photon. The invariance of the Lagrangian in this case, through Noether's theorem, leads to the conservation of the electromagnetic current.

The arbitrariness associated with the gauge field due to this transformation is lifted by putting a constraint on the fields, which will fix the gauge function, $\theta(x)$, for example, by working in the Lorentz gauge, $\partial_\mu A^\mu = 0$. The choice of constraint will not affect the underlying physics. The introduction of the vector potential, A_μ , necessitates the introduction of a mass term in the Lagrangian of the form $\mathcal{L}_\gamma = M_\gamma A_\mu A^\mu$, where M_γ represents the mass of the gauge boson associated with the gauge field [5]. The addition of this mass term breaks the invariance of the Lagrangian, which will only be restored if $M_\gamma = 0$, that is, if the gauge boson is massless.

The infinite set of simple transformations corresponding to rotations such as those in Eqn 2.3 form a unitary group called U(1). Quantum electrodynamics, the quantum theory of the electromagnetic interaction, is invariant under these U(1) transformations. In this theory Maxwell's equations are represented through the derivative of the field strength tensor,

$$\partial_\mu F^{\mu\nu} = 0 \quad (2.7)$$

where $F^{\mu\nu}$ is the field strength tensor which is defined through the gauge fields to be

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (2.8)$$

This allows for a construction of the Fermi Lagrangian for the electromagnetic interaction,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}_i(i\gamma_\mu D^\mu - m_i)\psi_i - \frac{1}{2a}(\partial_\nu A_\mu)^2 \quad (2.9)$$

where a is an arbitrary constant. The last term in Eqn 2.9 is no longer strictly gauge invariant and is referred to as a gauge fixing term [6].

More complicated transformations are also possible such as transformations in isospin space, corresponding to the special unitary group $SU(2)$ of symmetry transformations, which involves the non-commuting Pauli matrices τ as generators, making it non-Abelian. It is this symmetry group which gives rise to the theoretical basis for the weak interaction. These transformations have the form,

$$\psi_i(x) \rightarrow \psi'_i(x) = e^{ig\tau \cdot \Theta} \psi_i(x) \quad (2.10)$$

where g is a coupling constant and Θ is an arbitrary vector. This is a global gauge transformation since Θ is a global constant over all of position space x . The covariant derivative associated with this transformation will then be defined as,

$$D_\mu = \partial_\mu - ig\tau \cdot W_\mu \quad (2.11)$$

where W_μ is a vector gauge field analogous to A_μ for the electromagnetic interaction.

2.3 Quantum Chromodynamics

2.3.1 Colour and $SU(3)$

Quantum chromodynamics (QCD) is the quantum theory which describes the strong interaction, that is the interaction between quarks. Quarks are observed to exist in triplets, that is individual quarks can exist in one of three colour states, referred to as red, green and blue.

$$\psi = \begin{pmatrix} q_{red} \\ q_{green} \\ q_{blue} \end{pmatrix} \quad (2.12)$$

Consider a transformation of this field, similar to the global $SU(2)$ gauge transformation previously, only here the rotation is in colour space,

$$q_a \rightarrow q'_a = (e^{it \cdot \Theta})_{ab} q^b \quad (2.13)$$

where $a, b = 1, 2, 3$ representing each of the possible colours. This set of rotations represents a group of transformations, through the generator $\mathbf{t}$, which is a 3×3 matrix

that gives rise to SU(3) colour symmetry. The SU(3) theory for the strong interaction results in an octet of massless vector bosons, called gluons, that interact with colour triplets of massless spin- $\frac{1}{2}$ quarks.

The covariant derivative under the colour gauge transformation, is similar to that coming from an SU(2) transformation. It has the following configuration,

$$\mathcal{D}_\mu = \partial_\mu + \frac{ig\lambda}{2}B_\mu \quad (2.14)$$

where g is a coupling constant which will determine the strength of the strong interaction. The λ 's in Eqn 2.14 represent a set of eight 3×3 hermitian and traceless matrices, known as the Gell-Mann matrices, (analogous to the Pauli matrices of SU(2)). The Gell-Mann matrices are related to the generators of the SU(3) colour group, $\mathbf{t}^A = \frac{1}{2}\lambda^A$. The term B_μ^A represents eight gauge colour fields ($A = 1, \dots, 8$), which give rise to the eight colour carrying vector bosons (gluons).

One can determine a field strength tensor analogous to that for the electromagnetic interaction, derived from the gauge (gluon) fields, B_μ ,

$$F_{\alpha\beta}^A = [\partial_\alpha B_\beta^A - \partial_\beta B_\alpha^A - gf^{ABC}B_\alpha^B B_\beta^C] \quad (2.15)$$

where the indices A,B and C, run over the eight colour degrees of freedom of the gluon field. The last term in Eqn 2.15 is added as a result of the transformation of the gauge field, B_μ , thereby maintaining the gauge invariance of the kinetic term in the QCD Lagrangian. This extra term is a non-Abelian term which gives rise to gluon self interactions, meaning that the gluon will also carry a colour charge (actually two colour charges) and will therefore, interact with other gluons. This is in contrast to the photon (the force carrier of the electromagnetic force) which does not carry an electric charge and therefore, will not interact with other photons. The QCD Lagrangian can then be written as,

$$\mathcal{L}_{QCD} = -\frac{1}{4}F_{\alpha\beta}^A F_A^{\alpha\beta} + \sum_{flavours} \bar{q}^a (i\gamma_\mu D^\mu - m)_{ab} q^b - \sum_A \frac{1}{2\rho} (\partial^\alpha B_\alpha^A)^2 \quad (2.16)$$

where the last term is used to fix the gauge with an appropriate choice of value for ρ . Usually the Feynman gauge, $\rho = 1$, is chosen for convenience.

2.3.2 Running Coupling Constant

The strength of the coupling constant in QCD and hence of the strong interaction is based on the value of g in the covariant derivative, Eqn 2.14, and is denoted as $\alpha_s = g^2/4\pi$. It is analogous to the fine structure constant of the electromagnetic force. All observables within QCD, like cross-sections or decay widths, will depend on the value of this strong coupling constant. Generally, the calculation of these observables will require taking a perturbative expansion using α_s as the perturbation parameter.

All of the possible processes that can contribute to a given observable must be taken into account when calculating an expected value of that observable. This includes loops and vertex corrections which can be applied to an infinite order. These loops and vertex corrections (see Fig 2.1) appear as corrections in the base calculation of the observable of interest. The inclusion of these higher order processes will lead to infinities in theoretical calculations of observables, since integration over momentum transfers from the higher order processes often turn out to be divergent, which makes theoretical calculations irreconcilable with experimental observation.

Figure 2.1: Feynman diagram of a gluon and quark QCD loop and a vertex correction that contribute to the need for renormalized constants.

The divergences in the calculations are removed through a mechanism known as renormalization. As a consequence of this mechanism, constants within the theory are redefined in the effort to compensate for the infinities that arise from the momentum integration. There are several schemes which will renormalize the theory, the most common being the modified minimal subtraction scheme ($\overline{\text{MS}}$) [7]. In this scheme, the dimension of the momentum integrals are changed from $n = 4$ to $n = 4 - 2\varepsilon$, so that $\varepsilon = 2 - \frac{n}{2}$. This transformation allows the integration to be split into two functions.

One that is completely convergent and a Γ function, which will contain a pole when $n = 4$, thereby effectively removing the divergence arising from the initial integration. Calculations can then be carried out with dimension $n < 4$. In the final stage of the calculation the limit as $\varepsilon \rightarrow 0$ is taken, at which point the pole and its residue can safely be subtracted away.

In natural units, the coupling constant is dimensionless in the non-renormalized theory, and must remain so after renormalization, therefore it needs to be modified by the change in the integral of the dimension of the momentum, that is, $g \rightarrow \mu^\varepsilon g$. This redefinition of the coupling constant introduces a dependence on the arbitrary scale μ . The exact dependence of α_s on μ is governed by the renormalization group equation via the definition [6],

$$\frac{\partial \alpha_s}{\partial \tau} = \beta(\alpha_s) \quad \tau = \ln \frac{Q^2}{\mu^2} \quad (2.17)$$

The QCD β function is defined by the following expansion [2]

$$\beta(\alpha_s) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \beta_2 \alpha_s^4 - \dots \quad (2.18)$$

where

$$\begin{aligned} \beta_0 &= 11 - \frac{2}{3} N_f \\ \beta_1 &= 51 - \frac{19}{3} N_f \\ \beta_2 &= 2857 - \frac{5033}{9} N_f + \frac{325}{27} N_f^2 \end{aligned}$$

and N_f is the number of active flavours, that is the number of flavours of quarks with mass less than half of the momentum transfer scale $Q/2$.

The final theoretical determination of α_s will, through the solution to Eqn 2.17, have a dependence on two scales, the first denoted by Q represents the energy at which a particular process is occurring and the other being the renormalization scale, μ . Therefore to second order (that is only taking the first two terms in the β function expansion) the expression for the strong coupling constant, now running with the energy scale Q is,

$$\frac{1}{\alpha_s(Q)} + \left[\frac{\beta_1}{\beta_0} \ln \frac{\alpha_s(Q)}{\beta_0 + \beta_1 \alpha_s(Q)} \right] = \frac{1}{\alpha_s(\mu)} + \left[\frac{\beta_1}{\beta_0} \ln \frac{\alpha_s(\mu)}{\beta_0 + \beta_1 \alpha_s(\mu)} \right] + \beta_0 \ln \frac{Q^2}{\mu^2} \quad (2.19)$$

The arbitrariness of the parameter μ makes it difficult to use this determination. By convention μ is chosen to be the scale of interactions at the mass of the Z^0 boson, $\mu = M_{Z^0}$, thereby allowing the determination of the coupling constant at any other scale.

It is often convenient to choose a parameter, Λ , which provides a parameterization of the scale dependence of α_s on μ . With the choice of this parameter the coupling constant can be expressed as an expansion of powers in terms of the scale, Q , and the QCD cutoff parameter, Λ (here to third order) [2].

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(Q^2/\Lambda^2)]}{\ln(Q^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(Q^2/\Lambda^2)} \right. \\ \left. \times \left(\left(\ln[\ln(Q^2/\Lambda^2)] - \frac{1}{2} \right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4} \right) \right]. \quad (2.20)$$

so that, to first order [8],

$$\alpha_s(\mu) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2}} \quad (2.21)$$

The value of Λ corresponds to the scale at which the value of the coupling constant would diverge to infinity, that is $\alpha_s(Q) \rightarrow \infty$ as $Q \rightarrow \Lambda$. The value of Λ will depend on the renormalization scheme that is chosen. Using the minimal subtraction scheme, $\Lambda \sim 200$ MeV for scales in which there are 5 active quark flavours.

The dependence of α_s on the scale, as seen in Fig 2.2, is known as the running of α_s . Fig 2.2 shows values of $\alpha_s(Q)$ determined using several different methods. Any process which produces quarks in the final state can be used to determine a value for α_s . For instance, in the hadronic decays of tau leptons, heavy $q\bar{q}$ states like the Υ (which is a $b\bar{b}$ meson) and the Z^0 boson, the hadronic decay widths can be expressed in terms of a perturbative expansion around $\alpha_s(Q)$. A fit of the data to these expansions will provide values for $\alpha_s(m_\tau)$, $\alpha_s(m_q)$ and $\alpha_s(M_{Z^0})$ respectively. Variables that characterize the nature of $q\bar{q}$ events that are produced in e^+e^- and hadron-hadron collisions are also used for the determination of $\alpha_s(Q)$, where Q corresponds to the centre-of-mass energy at which the collision took place. These variable include what are known as event shape variables and jet rate observables. The theoretical description for these variables

Figure 2.2: Scale dependence of α_s taken from [8], comparing theoretical expectation (lines) to several different experimental observations (points).

involves a combination of the perturbative expansion and a resummed calculation.

A feature of the running of the coupling constant comes when looking at small energies, which corresponds to relatively large distances. In this regime, the coupling between two coloured objects gets stronger as the distance between them gets larger. There is an important consequence of this effect, known as confinement. Confinement states that no coloured object, like a quark or gluon, can exist as a free particle. As a pair of quarks separate the energy required to further increase the distance between them also increases, thereby binding those quarks together into a colourless composite object. The converse of this, known as asymptotic freedom, states that at very large energies (and hence smaller distances) the coupling between any two coloured objects gets smaller, meaning that the objects can be treated as being essentially free.

2.4 Partons

The high energy collision and subsequent annihilation of e^+e^- pairs at LEP results in the production of quark – antiquark pairs ($e^+e^- \rightarrow q\bar{q}$), approximately 70% of the time [2]. These quark–antiquark pairs will have the same (anti-) flavour and the same (anti-) colour so that there is no change in the total colour or flavour after the annihilation. Soon after the production of the quark–antiquark pair, they will begin to radiate gluons, typically soft. However, radiation of hard (very energetic) gluons, to which a quark will transfer a large fraction of its momentum, can also occur. The quarks, antiquarks and gluons (collectively called partons) will only exist in a “free” state for a short time. Confinement tells us that as these partons continue to lose energy they will eventually be collected into colourless bound states. These occur as one of two types, mesons which are bound states of a quark and an antiquark, and baryons, which are bound states of three quarks or three antiquarks. It will be these hadrons (mesons and baryons) that will eventually be observed in a detector.

2.4.1 Parton Production

Parton creation through e^+e^- annihilation is an electroweak process, mediated by either a Z^0 boson or a photon, that is $e^+e^- \rightarrow Z^0/\gamma^* \rightarrow q\bar{q}$, as shown in Fig 2.3. The

Figure 2.3: Feynman diagrams for the production of a quark-antiquark pair through the exchange of a photon or a Z^0 boson.

fact that there are two processes will give rise to an interference term appearing in the production cross-section, that is,

$$\begin{aligned}
 \sigma_{e^+e^- \rightarrow q\bar{q}}(s) &= \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} \\
 &= \frac{s}{M_Z^2} \frac{12\pi\Gamma_{ee}\Gamma_{q\bar{q}}}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \\
 &\quad + \frac{4\pi\alpha(s)^2}{3s} Q_q^2 N_c \\
 &\quad - \frac{2\sqrt{2}\alpha(s)}{3} \frac{(s - M_Z^2)M_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} (Q_q G_F N_c g_V^e g_V^q) \quad (2.22)
 \end{aligned}$$

where, from the first term in Eqn 2.22 representing the Z^0 exchange, Γ_{ee} and $\Gamma_{q\bar{q}}$ are the partial decay widths for $Z^0 \rightarrow e^+e^-$ and $Z^0 \rightarrow q\bar{q}$ respectively. Γ_Z corresponds to the full Z^0 decay width and s represents the centre-of-mass energy squared. The second

term, which corresponds to the photon exchange has a dependence on the value of the fine structure constant, $\alpha(s)$, as well as the charge of the quark, Q_q , and the number of colours, N_c . In the last term, corresponding to the γ - Z^0 interference, g_V^e and g_V^q are the neutral vector couplings of the e^+e^- and $q\bar{q}$ pair to the Z^0 and G_F is the Fermi constant.

The quark production cross-section is obviously influenced by the centre-of-mass energy of the e^+e^- annihilation. The trend, as can be seen from Eqn 2.22 is for the cross-section to decrease as $\sqrt{s}$ increases, in particular $\sigma \propto 1/s$, as seen in Fig 2.4. The inclusive production of quarks will depend also on the total number of quark flavours that are kinematically accessible. In this case the cross-section in Eqn 2.22 must be modified slightly so that $Q_q^2 \rightarrow \sum Q_{q_i}^2$. This means that as an energy threshold for the production of a new flavour of $q\bar{q}$ pair is crossed, the total production cross-section will have a slight stepwise increase. When a threshold is crossed the new $q\bar{q}$ forms a

Figure 2.4: Total cross-section for the production of quarks as a function of centre-of-mass energy showing several resonance peaks as various production thresholds are crossed, taken from [9].

resonance structure meaning that it becomes highly favourable for the production of quark pairs of that specific flavour. This results in a strong increase in the production cross-section for this narrow resonance seen in Fig 2.4.

At relatively small values of $\sqrt{s}$ ($\lesssim 3$ GeV), only three active flavours are kinematically accessible, the up, down and strange quarks. However above this threshold, production of charm quark pairs becomes possible and the overall cross-section for the production of quarks increases, marked by the strong resonance which corresponds to the first $c\bar{c}$ bound state, the J/ψ meson (see Fig 2.4). A similar increase in the production cross-section is seen when a threshold for the production of bottom quark pairs is crossed near 10 GeV, at which time the production of $b\bar{b}$ bound states, the Υ mesons, becomes possible. Top-antitop quark production will not occur until a threshold energy of at least 350 GeV is reached, so for centre-of-mass energies below this threshold only 5 active quark flavours are accessible.

Near a centre-of-mass energy that is of the order of the mass of the Z^0 boson ($M_{Z^0} = 91.2$ GeV), the Z^0 exchange starts to dominate over the photon exchange. As a result the overall cross-section for quark production increases by several orders of magnitude (see Fig 2.4). The exact energy dependence (neglecting any radiative corrections) of the cross-section for quark production in this energy range is

$$\sigma_{e^+e^- \rightarrow q\bar{q}}(s) = \frac{s}{M_Z^2} \frac{12\pi\Gamma_{ee}\Gamma_{qq}}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}, \quad (2.23)$$

where s corresponds to the square of the centre-of-mass energy and Γ_{ee} and Γ_{qq} are the partial widths for $Z^0 \rightarrow e^+e^-$ and $Z^0 \rightarrow q\bar{q}$ respectively [5]. A comparison of the relation in Eqn 2.23 to experimental results is seen in Fig 2.5.

2.4.2 Parton Cascade

Immediately after their creation the quark and antiquark pair will begin to communicate with each other through the strong force by emitting soft (very low energy) gluons. Since these gluons are very soft they will be emitted at relatively large angles with respect to the direction of the parent quark momentum. The radiation of a hard

Figure 2.5: Cross-section for the production of hadrons at centre-of-mass energies near the Z^0 peak for events with a relatively loose cut (full circles) and tight cut (open circles) against initial state radiation as a function of $\sqrt{s}$, taken from [10]. The data points are compared to a prediction from the ZFITTER program [11]. The differences between data and prediction are shown in the inset.

gluon in which one of the quarks transfers a significant portion of its momentum to the gluon can also occur,

$$\sigma_{q\bar{q}g} \propto \int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \quad (2.24)$$

where $x_i = \frac{2E_i}{\sqrt{s}}$, (E_i being the parton energy), so that $x_1 + x_2 + x_3 = 2$. Gluons, like quarks, are coloured objects and so will start emitting soft gluons immediately upon their creation. Following the production of a hard gluon, there are two possibilities. The gluon can split into two new gluons (with new colour charge combinations), or it could split into another quark–antiquark pair, the flavour of which will be influenced by the energy of the gluon, in that the higher the gluon energy the more quark flavours will be kinematically accessible. This cascade, seen schematically in Fig 2.6, of quarks emitting gluons which then go on to split into new quark or gluon pairs will continue

Figure 2.6: Schematic diagram of a parton cascade followed by a hadronization phase that leads to the production of hadrons

until the energy of the parton becomes low enough (or the distance between them becomes large enough) that the coupling between the partons becomes large and the partons are no longer asymptotically free. At this point, the perturbative nature of QCD is no longer valid, and confinement forces partons to form collections of colourless multi-quark states (hadrons). Since this process of hadron production from previously free partons, known as hadronization, is purely non-perturbative there is currently no analytical description of it.

2.5 Hadrons

Several models have been proposed which describe the formation of hadrons during the hadronization process. These models are optimized to accurately model the observed hadron distributions at some known energy and then used to predict the same distributions at other, typically higher, energies. The models are incorporated into Monte Carlo simulations which are then tested against experimental observations. The two most popular models describing the hadronization process are the string and the cluster model

String Model

The Lund string model [12] begins with the quark–antiquark pair moving in opposite directions. As the colour charges (quark and antiquark) begin to separate, the QCD colour potential (given by $V(r) \sim \frac{a}{r} + br$) between them begins to increase. The colour field between the $q\bar{q}$ pair can be seen as a colour flux tube, which will begin to stretch into a string as the quark and the antiquark continue to move away from each other. As the string is stretched by the separating $q\bar{q}$ the potential energy stored in the string will continue to increase. Eventually the stretching string will break, creating a new quark pair ($q'\bar{q}'$), leading two separate strings, $q\bar{q}'$ and $q'\bar{q}$. These separate strings may continue to stretch and further break into new $q\bar{q}$ pairs, until all the invariant mass of all the strings reaches that of typical hadron masses, such that each string fragment will

represent a hadron. The emission of hard gluons in this model is represented by the formation of a kink in the string connecting a $q\bar{q}$ pair, in essence this creates separate regions along the string that can fragment into new $q\bar{q}$ pairs, see Fig 2.7. This model is the basis for hadron production in the JETSET/PYTHIA Monte Carlo generators [13].

Cluster Model

The cluster model which is implemented within the HERWIG [14] Monte Carlo generator, allows for the generation of partons according to standard perturbative QCD, complete with the emission of gluons and their subsequent splitting into new parton pairs. However after the perturbative phase is over, all gluons are forced to split non-perturbatively into $q\bar{q}$ pairs. Neighbouring quarks and antiquarks are then combined to form colourless clusters, see Fig 2.8. Clusters whose invariant mass is above some limit are further divided into subclusters. The remaining clusters then are identified with hadrons by the flavours of their constituent quarks.

Local Parton-Hadron Duality

After the hadronization phase is complete, the only particles remaining to be detected will be hadrons, with no “direct” connection to the underlying partons that were produced. The hadrons, however, in a general sense maintain the underlying properties of their parton parents (a property known as Local Parton-Hadron Duality [15]). For instance, quark flavour is maintained throughout the hadronization phase. This means that following hadronization, a baryon or meson must be found which matches the flavour of one of the original quarks produced prior to the parton cascade. Similarly, hadrons will be expected to be moving in the same general direction as their parent partons. This means that the collection of hadrons originating from a parent quark will be collimated around the momentum direction of that parent quark (or antiquark). This collimated stream of particles is known as a jet. In any $q\bar{q}$ event, jet formation is a natural consequence, so by analyzing the structure of these jets one can get information

Figure 2.7: Schematic diagram of the string fragmentation model of hadronization. The string is represented by the thick hatched line.

Figure 2.8: Schematic diagram of the cluster model of hadronization. Hatched circles indicate formation of clusters.

about the underlying partons.

2.6 Jet Clustering

Jets are not specific objects that can be detected with some array of instruments. They must be derived from the distribution of hadrons that one can observe in a detector. Jets are found by applying a jet clustering algorithm to the collection of hadrons in the event. These algorithms use the kinematic and/or spatial (geometric) properties of the individual hadrons in order to classify them as belonging to a specific jet within the event. This is generally done by comparing the hadrons in the event, using some parameter to organize the hadrons into collections of jets. The number of jets found in any given event generally depends, therefore, on the value of the parameter that is chosen before running the clustering algorithm.

Cone Algorithm

The Cone algorithm [16] clusters particles into jets by creating cones of size R (where R is half the opening angle of the cone measured in radians) around the direction of flight of each particle in the event. The momenta of all of the particles contained within each cone is then summed and the direction of the resultant vector is compared with the direction of the cone axis. If the two directions are similar, the set of particles within the cone are defined to be a proto-jet, otherwise the cone is removed. To find additional proto-jets new cones of opening angle $2R$ are created with their axes along directions bisecting the directions of previous pairs of proto-jets. Again the momentum of the constituent particles is summed with the resultant momentum direction compared to that of the cone axes, thereby identifying as many proto-jets as possible. The total number of proto-jets is reduced by subjecting them to an energy cut requiring the total energy of the particles in a proto-jet to be greater than some cutoff value ϵ . If the proto-jet does not meet the requirement it is removed from the list of potential jets. This still leaves a significant number of proto-jets, many of which overlap with each

other, potentially sharing particles. A criteria for dealing with the cases of overlapping proto-jet cones are as follows:

- if the cones of two proto-jets overlap but no particles are contained in the overlapping region, the proto-jets are unchanged.
- if all of the particles of one proto-jet are a subset of another proto-jet cone, the lower energy proto-jet is eliminated
- if the overlap is finite, the energy of the particles common to both proto-jets is summed and divided by the total energy of the least energetic proto-jet. If this fraction is above a certain cutoff value then the particles in the overlap region will be assigned to the proto-jet with the highest energy. The lower energy proto-jet is then removed from the list of potential jets
- if the fraction is below the cutoff value, particles in the overlap region are assigned to the proto-jet which is nearest to them.

Figure 2.9: Comparison of the variation of the number of jets in an event as a function of R and ϵ . Figures show n jet fraction or the fraction of events in a sample that have n jets as a function of R and ϵ .

Following this the list of proto-jets is once again subjected to an energy test, with the surviving proto-jets being named as full jets. The number of jets in an event will vary with the size of the cone defined by the parameter R and the cutoff value for the total energy, ε , contained within a cone. The values of $R=0.7$ and $\varepsilon=7$ GeV [17] generally require the fewest number of trial cones to identify a proto-jet and hence to determine the total number of jets in an event. To observe the dependence of the number of jets on R and ε , one of these parameters is held at its optimal value while the other is varied, as shown in Fig 2.9.

JADE Algorithm

The JADE [18] algorithm takes pairs of hadrons and uses them to define a jet resolution parameter, y_{ij} , which is then compared to a cutoff value (y_{cut}) to determine the number of jets in an event. If the jet resolution parameter is less than the cutoff value, particles i and j are merged into a new pseudo-particle k (by summing the four-momenta of the two particles), which can then be used to form another y_{ij} and again compared to the cutoff value. If the value of y_{ij} is greater than y_{cut} then, all of the particles that were merged into i and j are assigned to a jet and removed from the list of eligible particles for jet formation. This process continues until all of the remaining particles have thus been assigned to a jet. Within the JADE algorithm the jet resolution parameter is related to the invariant mass of the two particles being compared (in particular in the case where particles are considered to be massless: $m_{ij}^2 = m_i^2 + m_j^2 + 2(E_i E_j - |\mathbf{p}_i||\mathbf{p}_j| \cos \theta_{ij})$), and is defined as

$$y_{ij} = \frac{2E_i E_j (1 - \cos \theta_{ij})}{E_{\text{vis}}^2} \quad (2.25)$$

where E_{vis} is the total energy observed within the detector. The dependence of the number of jets on the value of y_{cut} is seen in Fig 2.10.

Figure 2.10: Dependence of the number of jets in an event on the cutoff value of the jet resolution parameter, y_{cut} , for the JADE algorithm. Shown are the fractions for two- and three-jet events only, for a data sample at 183 GeV

Durham and Cambridge Algorithms

The Cambridge [19] and Durham [20] algorithms are similar to the JADE algorithm in that they use hadrons to construct a jet resolution parameter, which is then compared to some cutoff value. The main difference between these algorithms and the JADE algorithm comes in the definition of y_{ij} . Durham and Cambridge use a definition that is more closely related to the transverse momenta of particles being compared ($k_{\perp} = E \sin \theta$, where for small angles $\sin^2 \theta \approx 2(1 - \cos \theta)$), hence these algorithms are often referred to as $k_{\perp}$ algorithms. The jet resolution parameter is defined as,

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta)}{E_{\text{vis}}^2}. \quad (2.26)$$

The difference between the algorithms comes in their implementation. In the Durham scheme, the softest particle is considered first and compared with the particle nearest in angle in order to minimize the transverse momentum. Consequently, this particle is generally clustered not with the hardest hadron in the jet, but with the softest hadrons,

Figure 2.11: Two- and three-jet rates as a function of y_{cut} for the Durham (left) and Cambridge (right) algorithms

leading to the formation of soft “junk” jets, especially for small values of the jet resolution parameter. As a result these jets do not necessarily correspond to an initiating parton.

The Cambridge algorithm avoids the onset of “junk” jet formation [21] by handling the test variable y_{ij} in two stages,

$$y_{ij} = v_{ij} \frac{\min(E_i^2, E_j^2)}{E_{\text{vis}}^2} \quad (2.27)$$

where

$$v_{ij} = 2(1 - \cos \theta_{ij}). \quad (2.28)$$

The particles in the event are arranged into a table of energies E_i and a matrix of quantities v_{ij} . The algorithm then begins by choosing the two particles with the smallest v_{ij} (closest together in angle). The jet resolution parameter is then computed for these two particles and compared to the cutoff value. If $y_{ij} < y_{\text{cut}}$ particles i and j are merged and the table of energies and v_{ij} matrix are recomputed. The algorithm then starts again with the particles that minimize v_{ij} . If $y_{ij} > y_{\text{cut}}$ the particle (or pseudo-particle)

with the smaller energy is taken as a jet and deleted from the table, while the higher energy particle is returned to the table. Resolving the lower energy particle only as the jet is called soft freezing, in that it prevents softer jets from gaining further particles in the clustering process. This then reduces improper clustering, and hence the number of jets resolved in an event, as seen in Fig 2.12. This procedure continues until all particles in the event have been assigned to a jet.

Figure 2.12: Average jet multiplicity as a function of y_{cut} for data at 183 GeV. The Durham algorithm reconstructs more jets on average as a result of soft “junk” jet formation than does the Cambridge algorithm for small values of y_{cut}

2.7 Jet Multiplicity Observables

2.7.1 Jet Rates

When looking at jets one of the simplest quantities to work with is the jet fraction or jet rate. This is defined as the fraction of events in a given sample in which n jets can be

reconstructed as a function of the resolution parameter (R or ε for the CONE algorithm or y_{cut} for the JADE, Durham and Cambridge algorithms). The exact definition for jet rate is,

$$R_n(y_{\text{cut}}) = \frac{\sigma_n(y_{\text{cut}})}{\sigma_{\text{tot}}} \equiv \frac{N_n(y_{\text{cut}})}{N_{\text{tot}}}, \quad (2.29)$$

where σ_n is the cross-section for the production of n jets in a hadronic event and σ_{tot} is the total hadronic cross-section. This relates directly to the ratio of the number of events observed with n -jets, N_n to the total number of events, N_{tot} . Obviously the dependence of R_n on the resolution parameter comes through the cross-section σ_n . The perturbative expansion for the fractions of events with 2, 3 and 4 jets as a function of y_{cut} can be written as follows [22],

$$\begin{aligned} R_2(y_{\text{cut}}) &= 1 - A(y_{\text{cut}})\bar{\alpha}_s + [2A(y_{\text{cut}}) - B(y_{\text{cut}}) - C(y_{\text{cut}})]\bar{\alpha}_s^2 + \dots \\ R_3(y_{\text{cut}}) &= A(y_{\text{cut}})\bar{\alpha}_s + [B(y_{\text{cut}}) - 2A(y_{\text{cut}})]\bar{\alpha}_s^2 + \dots \\ R_4(y_{\text{cut}}) &= C(y_{\text{cut}})\bar{\alpha}_s^2 \end{aligned} \quad (2.30)$$

where $\bar{\alpha}_s = \alpha_s/2\pi$. The series is truncated after the second-order term due to the relatively small value of the strong coupling and hence will only be an approximation of the true prediction. The functions A, B and C are calculated through the integration of the $\mathcal{O}(\alpha_s^2)$ matrix elements [22]. However a parametrization [22], which uses fits to $\mathcal{O}(\alpha_s^2)$ matrix elements, is often used,

$$F(y_{\text{cut}}) = \sum_{n=0}^4 k_n \left(\ln \frac{1}{y_{\text{cut}}} \right)^n \quad (2.31)$$

where $F = A, B, C$ and k_n are numerical coefficients which depend on the specific jet algorithm being employed. The $\mathcal{O}(\alpha_s^2)$ prediction for the 2-jet rate can be written as

$$R_2(y_{\text{cut}}) = 1 + \mathcal{A}(y_{\text{cut}})\bar{\alpha}_s + \mathcal{B}(y_{\text{cut}})\bar{\alpha}_s^2 \quad (2.32)$$

where $\mathcal{A}(y_{\text{cut}})$ and $\mathcal{B}(y_{\text{cut}})$ are the cumulative coefficients for the $\bar{\alpha}_s$ and $\bar{\alpha}_s^2$ terms respectively from Eqn 2.30.

The truncation of the series in Eqn 2.30 to $\mathcal{O}(\alpha_s^2)$ is justified when the higher order terms are negligible, which is only true for relatively large values of y_{cut} . Due to the

logarithmic dependence on y_{cut} as smaller values are used, the logarithms will become larger, especially for the higher order terms [23]. Taking this logarithmic nature into account the 2-jet fraction can be written as

$$R_2(y_{\text{cut}}) = K(\alpha_s) \exp G(\alpha_s, L) + D(\alpha_s, L) \quad (2.33)$$

where

$$K(\alpha_s) = 1 + \sum_{n=1}^{\infty} K_n \bar{\alpha}_s^n \quad (2.34)$$

and

$$\begin{aligned} G(\alpha_s, L) &= \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \bar{\alpha}_s^n L^m \\ &= Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) \\ &\quad + \alpha_s^2 g_4(\alpha_s L) + \dots \end{aligned} \quad (2.35)$$

where $L = \ln(1/y_{\text{cut}})$ and $D(\alpha_s, L)$ is a remainder function which has a non-logarithmic dependence on y_{cut} and should vanish as $y_{\text{cut}} \rightarrow 0$. The exponentiation of $G(\alpha_s, L)$ implies that terms with $m > n+1$ are not present in the sum in Eqn 2.34. The function $Lg_1(\alpha_s L)$ is the sum of the leading logarithm terms, that is, terms with $\alpha_s^n L^{n+1}$ and the function $g_2(\alpha_s L)$ is the sum of the next leading log terms, $\alpha_s^n L^n$, to all orders of α_s . Clearly these two functions provide the dominant contributions to any prediction and so remaining subdominant terms can be neglected, leading to the Next-to-Leading Log Approximation (NLLA). The process of splitting the summation into functions that contain increasingly subdominant terms is known as resummation.

The NLLA prediction for the 2-jet fraction is then given by

$$R_2^{NLLA}(y_{\text{cut}}) = (1 + K_1 \bar{\alpha}_s + K_2 \bar{\alpha}_s^2) \exp[Lg_1(\alpha_s L) + g_2(\alpha_s L)] \quad (2.36)$$

where the first coefficient K_1 is known exactly from the $\mathcal{O}(\alpha_s)$ matrix elements and the second coefficient K_2 is known from an integration of the $\mathcal{O}(\alpha_s^2)$ matrix elements [24]. The NLLA predictions can be modified by including the term, $G_{21} \bar{\alpha}_s^2 L$, which is a sub-leading next-to-next-to-leading log term, as it is of the form $\bar{\alpha}_s^n L^{n-1}$. It is implicitly included in the $\mathcal{O}(\alpha_s^2)$ expectation in the $\mathcal{B}(y_{\text{cut}})$ term. G_{21} is not determined analytically but must be determined through a numerical integration of the $\mathcal{O}(\alpha_s^2)$ matrix elements.

A resummed expression also exists for the 3-jet fraction found in [25]. It is given explicitly as

$$\begin{aligned}
R_3(y_{\text{cut}}) = & -\frac{4}{3}\bar{\alpha}_s L \exp\left[-\frac{4}{3}\bar{\alpha}_s L^2\right] \left[\frac{1}{2} \sqrt{\frac{2\pi}{3\bar{\alpha}_s L^2}} \operatorname{erf}\left(\sqrt{\frac{3}{2}\bar{\alpha}_s L^2}\right) \left[3 - 2L + \right. \right. \\
& \left. \left. \beta_0 \bar{\alpha}_s L^2 \left(\frac{8}{9}\bar{\alpha}_s L^2 - \frac{1}{2} \right) - 8\bar{\alpha}_s L^2 + \frac{\beta_0}{6} \right] + \right. \\
& \left. \frac{1}{3} \left[\exp\left(\frac{3}{2}\bar{\alpha}_s L^2\right) \right] \left[\frac{8\beta_0 \bar{\alpha}_s L^2}{9} - \frac{\beta_0}{6} - \frac{2}{\bar{\alpha}_s L} - 12 \right] - \frac{\beta_0}{6} \right]. \quad (2.37)
\end{aligned}$$

For small values of y_{cut} , that is for large values of L , the NLLA expressions should provide a good prediction, but as L gets smaller, contributions from the neglected subleading terms can become important. In this smaller L regime however $\mathcal{O}(\alpha_s^2)$ predictions should be sufficient, so to improve predictions for a wider range of y_{cut} values these two predictions can be matched, that is the function from Eqn 2.30 can be added to Eqn 2.36 with overlapping terms being subtracted away.

There are several schemes which allow a matching of terms between the $\mathcal{O}(\alpha_s^2)$ and the NLLA predictions. The R -matching scheme directly compares Eqn 2.30 to Eqn 2.36,

$$\begin{aligned}
R_2(y_{\text{cut}}) = & R_2^{\text{NLLA}}(y_{\text{cut}}) - (K_1 + G_{11}L + G_{12}L^2)\bar{\alpha}_s - [K_2 + G_{22}L^2 + G_{23}L^3 \\
& + (G_{11}L + G_{12}L^2)(K_1 + \frac{1}{2}(G_{11}L + G_{12}L^2))]\bar{\alpha}_s^2 - R_2^{\mathcal{O}(\alpha_s^2)}(y_{\text{cut}}). \quad (2.38)
\end{aligned}$$

The overlapping terms were determined by expanding the exponential in Eqn 2.36 and keeping only the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ terms. The coefficients G_{ij} are given as follows [26],

$$\begin{aligned}
G_{12} &= -C_F & G_{23} &= -\frac{11}{9}C_F C_A + \frac{2}{9}C_F N_f \\
G_{11} &= 3C_F & G_{22} &= \frac{11}{4}C_F C_A - \frac{1}{2}C_F N_f
\end{aligned}$$

where $C_F = 4/3$, $C_A = 3$, and N_f corresponds to the number of active quarks, for most high energy interactions $N_f = 5$. An additional term, $G_{21}\bar{\alpha}_s^2 L$, may be included in the R matching prediction, through the modification of the $R_2^{\text{NLLA}}(y_{\text{cut}})$ term. Under this modification the G_{21} term is added into the exponential and then must be subtracted after exponentiation, by also including the subleading term in the overlap terms.

A second matching scheme comes about when comparing the logarithms of both the $\mathcal{O}(\alpha_s^2)$ and the NLLA predictions. This matching scheme, called $\ln R$ -matching, has the following form [27],

$$\begin{aligned} \ln R(y_{\text{cut}}) = & Lg_1(\alpha_s L) + g_2(\alpha_s L) - (G_{11}L + G_{12}L^2)\bar{\alpha}_s - (G_{22}L^2 + G_{23}L^3)\bar{\alpha}_s^2 \\ & + \mathcal{A}(y_{\text{cut}})\bar{\alpha}_s + [\mathcal{B}(y_{\text{cut}}) - \tfrac{1}{2}\mathcal{A}(y_{\text{cut}})^2] \bar{\alpha}_s^2 \end{aligned} \quad (2.39)$$

where $\mathcal{A}(y_{\text{cut}})$ and $\mathcal{B}(y_{\text{cut}})$ correspond to the merged coefficients of the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ terms in $R_2(y_{\text{cut}})$ of Eqn 2.30, respectively. In the case of the $\ln R$ -matching prediction the term containing the G_{21} coefficient is neglected as it is implicitly included in the $\mathcal{B}(y_{\text{cut}})$ coefficient, and hence included in the exponentiation.

In the NLLA and matched prediction there is nothing which guarantees that $R_2 \rightarrow 1$ and $dR/dy \rightarrow 0$ as $y_{\text{cut}} \rightarrow y_{\text{max}}$. Consequently the matched predictions may not perform as well as the $\mathcal{O}(\alpha_s^2)$ predictions in this region. A modification can be introduced for the $\ln(R)$ matching scheme by replacing L with a modified variable,

$$L \rightarrow L' \equiv \ln \left(\frac{1}{y_{\text{cut}}} - \frac{1}{y_{\text{max}}} + 1 \right). \quad (2.40)$$

2.7.2 Differential Jet Rate

The differential jet rates define the rate of change of the jet fractions as the value of the jet resolution parameter (y_{cut}) changes, that is

$$D_n(y_{\text{cut}}) = \frac{dR_n(y_{\text{cut}})}{dy_{\text{cut}}} \quad (2.41)$$

For the case when $n = 2$, the differential 2-jet rate reduces to $D_2 = y_2$, where y_2 is the value of the jet resolution parameter where the event flips from a 2- to a 3-jet event. This is technically only true for the Durham algorithm since there is no true definition of y_n in the Cambridge algorithm. All of the predictions described above that hold for R_2 also hold for the D_2 distribution as it is defined as the difference between two R_2 values that are nearby in y_{cut} divided by the difference in their respective y_{cut} values, that is

$$D_2(y_{\text{cut}}) = \frac{R_2(y_{\text{cut}}) - R_2(y_{\text{cut}} - \Delta y)}{\Delta y}. \quad (2.42)$$

Using predictions based on D_2 rather than the on R_2 is more desirable as the differential 2-jet rates are expected to have smaller statistical bin-to-bin correlations. In a data sample the value of R_2 is determined using every event in the sample for each bin of y_{cut} so each event can appear in multiple bins, giving rise to potential bin-to-bin correlations. The determination of D_2 avoids the problem by using the fact that D_2 can be determined from y_2 . Each event will only have one y_2 , so the event will only appear in the distribution once, reducing potential correlations.

2.7.3 Average Jet Multiplicity

The average jet multiplicity is another observable that is sensitive to the value of the strong coupling constant. It is defined to be the average number of jets per event in a given sample at some particular value of y_{cut} ,

$$\langle N \rangle(y_{\text{cut}}) = \frac{1}{\sigma_{\text{tot}}} \sum_n n \sigma_n(y_{\text{cut}}) \equiv \frac{1}{N_{\text{tot}}} \sum_n n N_n(y_{\text{cut}}). \quad (2.43)$$

As was the case for the jet rates, the average jet multiplicity can be expressed in a perturbative $\mathcal{O}(\alpha_s^2)$ prediction [28],

$$\begin{aligned} \langle N \rangle(y_{\text{cut}}) &= 2 + A(y_{\text{cut}}) \bar{\alpha}_s + [B(y_{\text{cut}}) + 2C(y_{\text{cut}}) - 2A(y_{\text{cut}})] \bar{\alpha}_s^2 + \dots \\ &= 2 + \mathcal{A}_N(y_{\text{cut}}) \bar{\alpha}_s + \mathcal{B}_N(y_{\text{cut}}) \bar{\alpha}_s^2 + \dots \end{aligned} \quad (2.44)$$

where A, B and C have the same definition as in Eqns 2.30 and 2.31, which are then combined to a single coefficient for the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ terms through $\mathcal{A}_N(y_{\text{cut}})$ and $\mathcal{B}_N(y_{\text{cut}})$ respectively, with higher order terms neglected.

The NLLA prediction for the average jet multiplicity is as follows,

$$\begin{aligned} \langle N \rangle^{NLLA}(y_{\text{cut}}) &= 2 + \sum_{n=1}^{\infty} \sum_{m=0}^{2n} H_{nm} \bar{\alpha}_s^n L^m \\ &= 2 + h_1(L\alpha_s^2) + h_2(L\alpha_s^2)/L + h_3(L\alpha_s^2)/L^2 + \dots \end{aligned} \quad (2.45)$$

where $h_i(L\alpha_s^2)$ corresponds to the sum of an infinite series of powers of terms consisting of $L\alpha_s^2$. Subleading terms that are implicitly included in the $\mathcal{O}(\alpha_s^2)$ predictions can be

included in the NLLA prediction, as was done for R_2 . This modification has the form

$$\langle N \rangle^{mod}(y_{cut}) = \langle N \rangle^{NLLA}(y_{cut}) + H_{22}L^2 + H_{21}L \quad (2.46)$$

The value for H_{21} is determined from a fit to $\mathcal{B}_N$. An analytic expression for H_{22} is given by [27]

$$H_{22} = \frac{2}{9}C_F N_f \left(1 - \frac{C_F}{C_A}\right) \left[\left(\frac{4C_A}{C_A} - 1\right) \frac{N_f}{C_A} - 1 \right] \quad (2.47)$$

With the full NLLA prediction, the expressions for the R -matching and $\ln R$ -matching schemes are also available. The R -matching prediction is

$$\begin{aligned} \langle N \rangle(y_{cut}) &= \langle N \rangle^{NLLA}(y_{cut}) - (H_{12}L^2 + H_{11}L)\bar{\alpha}_s - (H_{24}L^4 + H_{23}L^3)\bar{\alpha}_s^2 \\ &\quad + \mathcal{A}_N(y_{cut})\bar{\alpha}_s + \mathcal{B}_N(y_{cut})\bar{\alpha}_s^2. \end{aligned} \quad (2.48)$$

Unlike the R_2 case, the inclusion of the subleading terms in the NLLA prediction are directly subtracted away in the overlap terms. For the average jet rate there is no modification for the R matching prediction.

The $\ln R$ -matching prediction is given by

$$\begin{aligned} \ln[\langle N \rangle(y_{cut}) - 1] &= \ln[\langle N \rangle^{NLLA}(y_{cut}) - 1] - (H_{11}L + H_{12}L^2)\bar{\alpha}_s \\ &\quad - [H_{24}L^4 + H_{23}L^3 + H_{22}L^2 - \frac{1}{2}(H_{11}L + H_{12}L^2)^2]\bar{\alpha}_s^2 \\ &\quad + \mathcal{A}_N(y_{cut})\bar{\alpha}_s + [\mathcal{B}_N(y_{cut}) - \frac{1}{2}\mathcal{A}_N(y_{cut})^2]\bar{\alpha}_s^2 \end{aligned} \quad (2.49)$$

where the H_{nm} can be determined from [28],

$$\begin{aligned} H_{12} &= C_F & H_{24} &= \frac{1}{12}C_F C_A \\ H_{11} &= -3C_F & H_{23} &= \frac{1}{9}(C_F C_A - C_F N_f) \end{aligned}$$

The inclusion of the subleading terms in the NLLA prediction cannot be directly subtracted away as was the case for the R matching prediction. Here the inclusion of subleading terms in the NLLA prediction are inside a logarithm and the overlap terms are outside.

2.7.4 Renormalization Scale Dependence

A dependence on the renormalization scale, μ , is not expected in any exact determination of jet rates or average jet multiplicities. However due to the omission of higher order terms, in particular in the $\mathcal{O}(\alpha_s^2)$ approximation, a dependence on renormalization can occur. There are fewer terms neglected in the NLLA, so its dependence on μ is not expected to be as large. In order to account for this dependence on the renormalization scale, several terms require modification, in particular [27],

$$\begin{aligned} \mathcal{B}(y_{\text{cut}}) &\rightarrow \mathcal{B}(y_{\text{cut}}) + \mathcal{A}(y_{\text{cut}})\beta_0 \ln x_\mu \\ g_2(\alpha_s L) &\rightarrow g_2(\alpha_s L) + \beta_0 \alpha_s^2 \frac{dg_1(\alpha_s L)}{d(\alpha_s L)} \ln x_\mu \\ G_{22} &\rightarrow G_{22} + \beta_0 G_{12} \ln x_\mu \end{aligned} \tag{2.50}$$

as well as a modification to α_s ,

$$\bar{\alpha}_s(\mu) \rightarrow \bar{\alpha}_s(Q) + \bar{\alpha}_s^2(Q)\beta_0 \ln x_\mu \tag{2.51}$$

where β_0 is defined in Eqn 2.18 and [2]. The renormalization scale dependence comes through the renormalization scale factor, which is defined as

$$\mu = x_\mu Q \tag{2.52}$$

thereby also relating the renormalization scale to the centre-of-mass energy, Q . The matching schemes are expected to give better predictions and so it is expected they will have little dependence on μ , that is $x_\mu \sim 1$. Thus the matched predictions can be applied without the modifications introduced in Eqns 2.50 and Eqn 2.51.

Chapter 3

LEP and OPAL

The large electron–positron (LEP) collider accelerates intense beams of electrons and positrons to relatively high energies and brings them into collision so that subsequent interactions can be measured and studied. LEP is located at CERN outside of Geneva, Switzerland, straddling the French and Swiss border. It began operation in 1989, with a nominal centre-of-mass collision energy of ~ 91.2 GeV, allowing for a detailed study of the Z^0 boson. After six years of running LEP had produced several million Z^0 events.

In 1995 LEP underwent an upgrade allowing it to achieve higher centre-of-mass energies, beginning with 130 and 136 GeV (known as the LEP1.5 phase) increasing incrementally each year from 1995 until 2000 (collectively known as the LEP2 phase), to allow for the study of $W^\pm$ bosons and for a search for physics beyond the Standard Model. LEP ceased operation in November 2000 to make room for CERN’s next generation particle collider, the Large Hadron Collider (LHC).

Counter-rotating beams of electrons and positrons are accelerated and brought into collision at four of eight possible points around the 26.7 km circumference of LEP. These four interaction points contain general purpose particle detectors, ALEPH [29], DELPHI [30], L3 [31] and OPAL [32](see Fig 3.1). Each detector operates independently, incorporating particle tracking, identification and calorimetric methods which allow a detailed experimental study of all facets of the Standard Model and its potential extensions.

3.1 Large Electron-Positron Collider

Electrons are initially produced from a thermionic source and are injected into the first of two linacs (linear accelerator). The first linac accelerates the electrons to 200 MeV and directs them to collide with a tungsten target at which point positrons are produced through pair creation from bremsstrahlung photons. The electrons and positrons produced from the target are collected, magnetically separated, and then injected into a second linac where they are accelerated to 600 MeV. This 2-linac system forms the LEP Injector Linacs (LIL) seen in Fig 3.1. Electrons and positrons are continually being collected in the Electron Positron Accumulator (EPA) until a sufficient current is produced at which point they are injected into the Proton Synchrotron (PS) which further accelerates them to 3.5 GeV. Next, the beams are directed to the Super Proton Synchrotron (SPS) where they are further accelerated to an energy of 22 GeV at which point they are finally injected into the LEP ring.

During the LEP1 phase of operation each beam of positrons and electrons was accelerated to an energy of 45 GeV through the use of 128 copper RF cavities powered by sixteen 1 MW klystrons. Each cavity had an operating frequency of 352.2 MHz and an operating RF voltage of 400 MV. In addition to accelerating the electron and positron beams to their maximum energies, the RF cavities help maintain a constant centre-of-mass energy by replacing beam energy lost to synchrotron radiation every orbit. The beams circulate around the LEP ring in bunches, typically four per beam, with each bunch containing on the order of 10^{11} particles (either electrons or positrons). Each bunch orbits the ring approximately every $89\mu\text{s}$ with dimensions $150\mu\text{m} \times 10\mu\text{m} \times 10000\mu\text{m}$. The total luminosity is then determined by

$$\mathcal{L} = \frac{I^2 n_b}{4\pi e^2 f_0 \sigma_x \sigma_y} \quad (3.1)$$

Figure 3.1: Overview of the acceleration stages of the LEP collider.

where I (in A) is the bunch current, n_b is the number of bunches, f_0 is the revolution frequency of the bunches (in Hz) and σ_x and σ_y (in cm) are the vertical and horizontal dimensions of each bunch at the interaction points. The operational luminosity during the first half of this LEP1 phase was approximately $11 \times 10^{30} \text{cm}^{-2} \text{s}^{-1}$ and $23 \times 10^{30} \text{cm}^{-2} \text{s}^{-1}$ during the second half when LEP ran with 8 bunches per beam. Over the six years of data taking during the LEP1 phase an integrated luminosity of 169.5pb^{-1} was achieved.

During the LEP1.5 and LEP2 phases many of the copper cavities were replaced with superconducting niobium-plated copper cavities allowing for much larger accelerating gradients leading to larger centre-of-mass energies as well as higher luminosities, up to $1 \times 10^{32} \text{cm}^{-2} \text{s}^{-1}$. Over the six years that comprised the LEP1.5 and LEP2 phase LEP delivered 708.22pb^{-1} to the OPAL detector. The breakdown of the integrated luminosity as a function of the centre-of-mass energy over the lifetime of OPAL is seen in Fig 3.2. Periodically during the high energy phase of LEP, the centre-of-mass energy was reset to the Z^0 pole, to allow experiments to collect additional Z^0 data. This data is used to recalibrate the detectors using a data sample with precisely known characteristics, free of any high energy backgrounds. The total integrated luminosity collected during all of the calibration runs is 14.5pb^{-1} .

3.2 OPAL

The purpose of the OPAL detector is to identify particles produced following an electron-positron annihilation and then as accurately as possible to measure the trajectories and kinetic energies of those particles. OPAL [32] is a standard multi-purpose particle detector, with an inner particle tracking system, surrounded by two layers of calorimeters which themselves are enclosed by a final layer of muon detectors. A cut-away diagram showing the position of the various components of the OPAL detector can be seen in Fig 3.3. The barrel and endcap regions of these various sub-detector systems enclose nearly all of the solid angle surrounding the e^+e^- interaction region.

Figure 3.2: Total integrated luminosity delivered to the OPAL detector as a function of centre-of-mass energy. The grey bars indicate data that was collected during the LEP1.5 and LEP2 phase while the white bar corresponds to data that was collected during LEP1.

3.2.1 Central Tracker

The central tracker in OPAL is composed of four sub-detectors: the silicon microvertex detector, the vertex chamber, the Jet chamber and the Z chamber, all inside

Figure 3.3: Cutaway diagram showing the layout of the OPAL detector

a solenoidal magnet which supplies a uniform magnetic field of 0.435 T. The central tracker is devoted to detecting and measuring tracks created by the passage of charged particles through silicon wafers as in the case of the microvertex detector or through ionizing gas (held at a pressure of 4 bar) in the case of the vertex, Jet and Z chambers.

Silicon Microvertex Detector

The innermost of the OPAL sub-detectors is the silicon microvertex detector [33, 34, 35]. The microvertex detector was installed in 1991 inside the previous LEP beam pipe, which is composed of a 0.13 cm thick carbon fibre composite to provide strength against the large vacuum, of up to 10^{-11} torr. The inside of the beam pipe has a $100\text{ }\mu\text{m}$ layer of aluminum to provide stability for the e^+e^- beams. A second beam pipe composed

of beryllium was installed inside the silicon microvertex detector. The detector itself is composed of two cylinders of silicon micro-strip detectors with radii of 6.1 and 7.5 cm respectively. Both the inner and outer cylinders are constructed of overlapping ladders, 12 in the inner cylinder and 15 in the outer cylinder. Each ladder consists of 3 silicon wafers daisy-chained together along its length of 18 cm. A resolution of $5\text{ }\mu\text{m}$ is achieved in the position measurement of a track in the $r - \phi$ direction and a resolution of $10\text{ }\mu\text{m}$ in the $r - z$ plane. These measurements, along with further track information from the gas tracking chambers, allows for the determination of the position of any vertices (typically arising from the decay of heavy flavour hadrons) that are displaced from the primary vertex (i.e. interaction point).

Vertex Chamber

The vertex chamber [36] is located just outside of the silicon microvertex detector. It is a cylindrical drift chamber measuring 1 m in length and 0.47 m in diameter. The chamber consists of two layers, each segmented into 36 sectors. The sectors in the inner layer each contain a plane of 12 sense wires strung parallel to the beam line. Sectors in the outer layer contain a plane of 6 sense wires inclined at a stereo angle of $\sim 4^\circ$, with respect to the beam line. Precise measurements of the drift time to the axial wires allows a precision measurement of position in the $r - \phi$ plane, providing an average resolution of $55\text{ }\mu\text{m}$. A measurement of the difference in timing signals between the ends of the sense wires allows for a coarse determination of the z position with a precision of approximately 5 cm. A far more refined z measurement is produced by comparing the axial and stereo drift time information offline, providing a resolution of $700\text{ }\mu\text{m}$.

Jet Chamber

The Jet chamber [37, 38] is also a cylindrical drift chamber located just outside the vertex chamber. It has a length of 4 m and an inner diameter of .5 m and an outer diameter of 3.7 m. The chamber is divided into 24 sectors with each sector containing

a plane of 159 sense wires. The $r - \phi$ position of a track is determined by measuring drift time with a spatial resolution of $135 \mu\text{m}$. The z position is determined by a charge division technique providing a resolution of 6 cm.

The curvature of the track due to the magnetic field is used to determine the momentum of the particle. The momenta of tracks in the $r - \phi$ plane can be measured with a precision,

$$\frac{\sigma_p}{p} = \sqrt{0.02^2 + (0.0015 \cdot p[\text{GeV}/c])^2}$$

The measure of the charge along a wire also enables a determination of the particle's energy loss, dE/dx , through the gas contained within the chamber. This measured energy loss along with the determination of particle momentum provides a powerful particle identification tool for a given track [39]. Measurement of minimum ionizing particles (e.g. from dimuon events) which have at least 130 separate measured points yields a precision of 3.8% in the dE/dx determination.

Z Chamber

The final sub-detector in the series that makes up the central detector is the Z chamber [40], designed to improve the measurement of the track position in the z direction as the track leaves the Jet chamber. It consists of 24 drift chambers 4 m long, .5 m wide and .059 m thick which are arranged to form a cylindrical layer around the Jet chamber. Each chamber is divided into $8 .5 \times .5$ m cells with 6 sense wires in each cell. The Z chamber provides a resolution of $300 \mu\text{m}$ for tracking in the Z direction.

Combined Performance of Central Detectors

Information from all of the tracking subdetectors can be combined to give improved tracking precision. Primary and secondary vertices are determined by the distance of closest approach to the interaction point. This value, known as the impact parameter, is measured in the $r - \phi$ plane with a resolution of $18 \mu\text{m}$ and in the $r - z$ plane with a resolution of $24 \mu\text{m}$. These resolutions are obtained using tracks from $\mu^+\mu^-$ pairs produced at the Z^0 peak.

3.2.2 Time-of-Flight

Outside of the magnet coil at a mean radius of 2.36 m is OPAL's time-of-flight (TOF) system, comprised of 160 scintillation counters, each 6.84 m long. The TOF system performs three important duties, first it generates trigger signals and second, it provides particle identification by determining the time of flight from the interaction region and lastly it provides a veto against events that arise because of interactions of cosmic rays with the detector. The timing resolution has been determined to be 460 ps.

3.2.3 Calorimeters

OPAL contains two calorimeters for measuring the total energy deposited by particles created within the detector. The first calorimeter is an electromagnetic calorimeter measuring energy and position for electrons, positrons and photons. The second calorimeter measures the energy of hadrons emerging from the electromagnetic calorimeter as they have more penetrating power than either electrons or photons. As particles interact with the material of the calorimeters they will begin to shower, through bremsstrahlung and pair creation in the electromagnetic calorimeter or through a hadron cascade in the hadron calorimeter. As this shower progresses the initiating particles deposit their energy in the calorimeter. The lateral width of showers initiated by high energy particles will often exceed the size of the individual calorimeter segment, so that the energy of the shower will be deposited in several segments. The combined signal from these adjacent segments are referred to as a "cluster."

Electromagnetic Calorimeter

The electromagnetic calorimeter consists of two parts, a presampler and a lead glass calorimeter. The presampler is a cylinder comprised of limited streamer mode chambers located between the TOF and the lead glass calorimeter. The purpose of the presampler is to identify electromagnetic showers that begin in the ~ 2 radiation lengths of material in front of the lead glass (due to the magnet coil and the pressure vessel containing the central tracking detectors). Outside of the presampler is the lead

glass calorimeter which consists of a cylindrical array of 9440 lead glass blocks. The lead glass blocks, each representing 24.6 radiation lengths, are located at a radius of 2.455 m and are arrayed in a pointing geometry to the interaction point, allowing coarse position determination, and minimizing the probability of a particle passing through more than one block.

In the forward (endcap) regions, the presampler is umbrella shaped with 32 chambers in 16 sectors of multiwire proportional chambers, located between the pressure vessel and the electromagnetic endcap calorimeter. The endcap calorimeters are dome-shaped each containing 1132 lead glass blocks aligned coaxially with the beam line. The endcap calorimeters give the electromagnetic calorimeter system complete solid angle coverage down to $|\cos\theta| < 0.98$. The typical energy resolution of the electromagnetic calorimeter is found to be

$$\begin{aligned} \text{Barrel } \frac{\sigma_E}{E} &= 0.2\% + \frac{6.3\%}{\sqrt{E}} \\ \text{Endcap } \frac{\sigma_E}{E} &\sim \frac{5\%}{\sqrt{E}} \text{ for low energy} \end{aligned}$$

where E is the energy in GeV of a particle deposited in the calorimeter.

Hadronic Calorimeter

The hadron calorimeter measures the energy of hadrons which will escape the lead glass blocks of the electromagnetic calorimeter. The calorimeter is composed of 9 layers of chambers based upon limited streamer tube design [41]. Each layer is placed between layers in the iron flux return of OPAL's solenoid magnet. Each layer of the flux return provides 10 cm of absorber (at least four interaction lengths of material) to particles that emerge from the electromagnetic calorimeter. It is this material which provides the stopping power to contain hadron showers. Typically, hadrons will begin showering in the 2.2 interaction lengths of the electromagnetic calorimeter, so any measure of a hadron's energy must include information from the electromagnetic calorimeter as well. The solid angle coverage of the hadron calorimeter is extended into the forward region by two sub-detectors, a donut-shaped endcap calorimeter with 8 layers of chambers

alternating with 7 slabs of iron, and the hadron pole tip calorimeter which fits inside the opening in the endcap, extending the solid angle coverage to $|\cos\theta| < 0.99$. The hadron calorimeter in the barrel and endcap have typical energy resolutions of

$$\frac{\sigma_E}{E} = \frac{120\%}{\sqrt{E}}$$

3.2.4 Muon Detector

The muon detector lies outside the hadron calorimeter and is designed to detect high energy muons ($E_\mu > 3$ GeV) which will easily traverse the detector without being stopped. In the barrel region the muon detector [42] consists of 110 large area drift chambers, arranged in four cylindrical layers outside the hadron calorimeter. In the endcap region the detector consists of four layers of limited streamer tubes perpendicular to the beam axis, giving a solid angle coverage down to $|\cos\theta| < 0.98$. Muons are identified by matching tracks from the Central Tracking chambers to specific hits occurring in the muon chambers. This guards against the rare case where an isolated neutral pion or kaon punches through the hadron calorimeter and causes a signal in the muon chambers.

3.2.5 The Forward Detectors

The forward region of the OPAL detector contains a set of detectors surrounding the beamline just outside the pressure vessel containing the central tracking chambers. These detectors extend the acceptance down to ~ 25 mrad. There are two detectors housed in this region of OPAL, the Forward Detector, which consists of three lead-scintillator modules and one proportional tube chamber, and the Silicon Tungsten [43] detector which is a sampling calorimeter. The primary purpose of these two detectors is to measure low angle Bhabha scattered electrons in order to determine the luminosity of the collected data samples and to tag forward electrons from $\gamma\gamma$ interactions.

3.2.6 Trigger and Data Acquisition Systems

The OPAL trigger system [44] provides the decision making process which determines whether information collected in the various sub-detectors represents interesting physics events and so should be assembled within the data acquisition system [45]. Independent conditions are applied to the signals from the various sub-detectors to produce a flag (or trigger) on an event as arising from a physics reaction.

For 4×4 bunch operation, bunch crossings occur every $22.2 \mu\text{s}$ corresponding to a potential trigger rate of 45 kHz. Fast information from the sub-detectors is used to trigger possible physics reactions thereby reducing the rate to 1-5 Hz, which is more manageable by the data acquisition system.

The trigger system divides the detector into 144 overlapping bins covering the 4π solid angle, 6 bins in θ and 24 bins in ϕ . Sub-detector trigger signals can be either “stand-alone” signals coming from variables like total energy sums or track multiplicities, or they can be signals from sectors in the $\theta - \phi$ matrix. All of the trigger signals from the sub-detectors are then combined within the central trigger logic where decisions are made by looking for coincidences between sub-detectors in $\theta - \phi$ along with the stand-alone signals.

Following the triggering of an event, data from the 14 sub-detectors and the trigger system are collected and processed by the data acquisition system. Data from each sub-detector is read out by its own front-end electronics into a local system crate (LSC). The LSCs acquire the data, format and prepare it for the software filter, and when possible pack the raw data in order to reduce the size of the sub-event. The sub-events from each LSC are then sent to the event builder (EVB) where the data is synchronized and merged into a complete event. The events are then sent to a filtering processor which performs a fast analysis of each event. The event is checked, analyzed, monitored and compressed before being written to disk. Events are classified into various physics categories, at which point events classified as backgrounds can be optionally accepted or rejected.

The raw data from the filter is then transferred to a farm of computer processors

where the data is fully reconstructed and converted into physical quantities like energy or momenta. Data files containing the reconstructed events are then concatenated and stored on disk and backed up to cartridge for offline analysis.

Chapter 4

Data and Monte Carlo Samples

The data used in this analysis covers a range of centre-of-mass energies provided by the LEP collider between the years 1996-2000 for the centre-of-mass energies of 161 GeV to 207 GeV. Data collected during calibration runs from 1996 to 2000 each with a centre-of-mass energy of 91 GeV are also used. The breakdown of the centre-of-mass energies, along with the respective luminosities and number of events to be analyzed is given in Table 4.1. It is expected that α_s variations will be small across all the LEP2 high energy data points, so that a useful measurement can be made using a sample composed of the combined LEP2 datasets. This combined sample will have a total integrated luminosity of 706.4pb^{-1} and a luminosity weighted centre-of-mass energy of 195.7 GeV, and will be referred to as the LEP2 dataset.

The data is compared and corrected with Monte Carlo data which has undergone a full detector simulation. A Monte Carlo generator produces four-vectors (corresponding to four-momenta) that represent particles that are produced by a specific interaction. These particles are allowed to interact with material, propagate through detector space and decay. In the case of quarks they initially hadronize according to some specific hadronization model. The new hadrons will then propagate through space and decay. A table (or tree) of all the produced partons' (quarks and gluons, collectively) and hadrons' four-momenta and flavour is saved for each event generated. The hadrons are then sent through GOPAL [46], a program used to simulate the detector response

Energy (GeV)		Luminosity (pb^{-1})	Number of Events			
Nominal	Measured		Data	PYTHIA	HERWIG	ARIADNE
91	91.5	14.5	431349	200k	100k	—
161	161.3	10.1	1932	100k	80k	—
172	172.1	10.4	1824	100k	80k	—
183	182.7	57.7	8918	100k	100k	100k
189	188.7	185.2	26936	500k	100k	100k
192	191.6	29.5	4229	100k	100k	125k
196	195.5	76.7	10706	200k	100k	100k
200	199.5	79.2	10226	200k	100k	100k
202	201.6	37.8	5009	100k	100k	100k
205	204.8	81.0	10857	200k	100k	100k
207	206.6	138.8	17576	375k	100k	100k

Table 4.1: Integrated luminosity and number of events, after preselection, of all samples. Included in this table are the sizes of the Monte Carlo samples used to correct, and make comparisons with, the data. The measured energy corresponds to twice the mean of the recorded beam energy of each event. The data for the 91 GeV dataset corresponds only to the data collected during the OPAL Z calibration runs.

to these particles, transforming the known hadrons into simulated signals within the sub-detectors of OPAL. The data is corrected using a Monte Carlo generated dataset since the detector has finite resolution and acceptance. The Monte Carlo data that has gone through GOPAL is compared to the tree level hadron expectation and a correction factor can be determined and applied to the data. It is expected that this corrected dataset will be more representative of the underlying physics of hadrons.

The standard hadronization model that is used for comparison and correction of the data is the string model. PYTHIA [13], which uses the string model of hadronization provides the default Monte Carlo data samples used to correct the high energy datasets. The Z calibration dataset is corrected using JETSET [47] which is a progenitor of PYTHIA and hence very similar. Hadronization models can be tested by comparing results with Monte Carlo samples that employ different models. The HERWIG [14] Monte Carlo, which uses the cluster model of hadronization, is also used to correct the datasets. This allows for a comparison to the string model of hadronization.

The generation of the initial quark–antiquark pair is implemented using the $\mathcal{KK}$ event generator [48] to better model photon production in the initial and final states. The quark–antiquark pair is then allowed to cascade forming other partons until finally all of the remaining partons hadronize through PYTHIA or HERWIG. Parameters involved in the Monte Carlo simulation are tuned to OPAL data collected at the Z^0 peak using global event shape variables [49].

Yet another Monte Carlo, ARIADNE [50], is employed to compare distributions against measured values. ARIADNE follows the same hadronization model (the string model) implemented in PYTHIA, however it differs in the treatment of the parton cascade which is initiated following the creation of the initial quark–antiquark pair. In PYTHIA and HERWIG a matrix element calculation (actually a leading logarithmic approximation) decides the branching between the available fundamental processes $q \rightarrow qg$, $g \rightarrow gg$ and $g \rightarrow qq$. ARIADNE follows the colour dipole model [51] to generate a succession of partonic states. In this model an initial gluon is emitted from the colour dipole formed by the initial $q\bar{q}$ pair. The next, softer, gluon will be emitted from the two colour dipoles ($g-q$ and $g-\bar{q}$). The next, even softer, gluon will be radiated from the three independent dipoles, and so on. In previous analyzes the Monte Carlo COJETS [52] was also used. However, it was shown that COJETS did not accurately represent the data and so will not be included here.

The size of contributions from background processes that can mimic the desired physics signals and potentially skew any final results can also be estimated using Monte Carlo. KORALW [53] is such a Monte Carlo and is used to simulate 4-fermion events such as those that arise from $e^+e^- \rightarrow W^+W^- \rightarrow qqqq$, $q\bar{q}\ell\nu$ and from $e^+e^- \rightarrow Z^0Z^0 \rightarrow qqqq$. A second background Monte Carlo, grc4f [54], is used to simulate events in which hadrons are produced from $e^+e^-q\bar{q}$ final states. The data collected during the Z calibration runs have a centre-of-mass energy very much below W^+W^- or Z^0Z^0 production, and hence four-fermion backgrounds are not expected to be a concern for these samples.

4.1 Event Selection

A hadronic event must pass a strict set of cuts in order to be included in any sample meant for future analysis. In fact each event must pass three stages of cuts, each meant to steadily reduce levels of potential background and to provide the highest data quality possible.

4.1.1 Preselection

Events must first pass a trigger and a detector condition. First the event must have two of the track trigger, the electromagnetic barrel and endcap trigger and the TOF trigger set. Second the Jet chamber and the electromagnetic calorimeter (both the barrel and endcap) must be on and in proper working order.

4.1.2 Stage 1

Multihadronic Event Tag

The Stage 1 cut begins by requiring that an event be classified as one of two multihadron types, a Tokyo multihadron event for the Z calibration data or a LEP2 multihadron event for the high energy data. An event will be classified as a Tokyo or LEP2 multihadron if it has a sufficient number of good quality clusters and tracks observed in the detector. A good quality cluster (see Table 4.2) is defined by the number of blocks that make up the cluster and a minimum energy deposited in that cluster. At least 1 block per cluster in the barrel or 2 blocks per cluster in the endcap region is required. The energy deposited in the cluster must reach a minimum value of 100 MeV for a cluster in the barrel region or a minimum of 200 MeV in the endcap region.

Parameter	Barrel	Endcap
E_{clus}	≥ 0.100 GeV	≥ 0.200 GeV
N_{blocks}	≥ 1	≥ 2

Table 4.2: Parameters that define good clusters in the barrel and endcap regions

Good quality tracks are those reconstructed tracks that have a minimum of 20 hits within the Jet chamber. The first hit within the Jet chamber must be no more than 60.0 cm away from the e^+e^- collision point. The track is required to have an impact parameter value, $|d_0|$, of at most 2.0 cm, that is the point of closest approach of the track to the e^+e^- collision point in the $x - y$ plane must be less than 2.0 cm. Similarly, the point of closest approach in the z direction, $|z_0|$, must be less than 40.0 cm. This ensures that the tracks are consistent with coming from the primary vertex. The track must also be fully contained within the tracking chambers, so that it is not travelling along the beam line. This is ensured by requiring that $|\cos \theta| \leq 0.995$, where θ is the polar angle of the track with respect to the z direction. A momentum constraint is also imposed upon each track by requiring that the momentum in the $x - y$ plane be greater than or equal to 0.050 GeV/ c . Finally, in order to guard against pathological cases in which the fits of reconstructed tracks to the hit pattern within the central track chamber returns improbably large χ^2 values, a cut is placed upon the returned χ^2 in both the $r - \phi$ plane and the $s - z$ plane. These requirements are summarized in Table 4.3.

Parameter	Requirement
$N_{CJ \text{ Hits}}$	≥ 20
$R_{1^{st} \text{ hit}}$	$\leq 60.0 \text{ cm}$
$ d_0 $	$\leq 2.0 \text{ cm}$
$ z_0 $	$\leq 40.0 \text{ cm}$
$ \cos \theta $	≤ 0.995
p_{xy}	$\geq 0.050 \text{ GeV}/c$
$\chi^2_{r-\phi}$	$\leq 999.$
χ^2_{s-z}	$\leq 999.$

Table 4.3: Parameters which define good reconstructed tracks

Two further conditions are required for the classification of a multihadronic event. The conditions concern the energy distribution of clusters within the detector. The first condition requires that the total visible energy deposited in the calorimeter be

some defined fraction of the total centre-of-mass energy. Secondly, an energy balance requirement is imposed so as to remove beam halo events (events in which particles that are orbiting with the beams enter the detector without collision, thereby causing non-physics signals in the endcap region) as well as beam-gas and beam-wall events (events in which a particle from the electron or positron beam interacts with gas in the vacuum pipe or the wall of the beam pipe thereby producing a spray of particles entering the detector along the beam line into the endcap region). The exact requirements for the classification of an event to be a Tokyo or LEP2 multihadron based on these conditions are seen in Table 4.4. These conditions are determined during the event reconstruction at which time a header bit is set in the “data tree” signalling whether the event has been classified as a Tokyo multihadron or a LEP2 multihadron (or both) event. An example of a typical multihadronic event observed within OPAL can be seen in Fig 4.1.

Criteria	Tokyo MH	LEP2 MH
N_{trk}	≥ 7	≥ 7
N_{clus}	≥ 5	≥ 5
$R_{\text{vis}} = \frac{\sum E_{\text{clus}}}{2E_{\text{beam}}}$	≥ 0.10	≥ 0.14
$R_{\text{bal}} = \frac{\sum E_{\text{clus}} \cos \theta}{\sum E_{\text{clus}}}$	≥ 0.65	≥ 0.75

Table 4.4: Criteria for an event to be classified as a Tokyo or LEP2 MH event. The θ in the R_{bal} term corresponds to the polar angle of the cluster with respect to the beam direction. The summations are taken over all good clusters.

Tracking Requirements

Once an event has been determined to be a multihadronic event, all the tracks and clusters in the event are grouped into “objects” using the MT package [55], developed for use with the OPAL detector. The MT package uses tracks from the central tracking chambers and extrapolates their direction to the electromagnetic and hadronic clusters. The position of the track at the calorimeter as well as the position of the centre of a cluster and its boundaries are used to determine if there is a true association. If an

Figure 4.1: Graphical representation of a multihadronic event seen in the OPAL detector. The lines in the centre represent charged tracks reconstructed in the central tracking chambers. The blocks outside of the tracks represent clusters recorded in the electromagnetic (inner blocks) and the hadronic (outer blocks) calorimeters. The small arrow represents a hit in the muon chambers.

association between a track and cluster is established, the energy between the track and cluster is compared. In the case where the energy of the cluster is less than that expected from the tracking information, with some tolerance, the energy of the

cluster is omitted in favour of the tracking, since the tracking detectors typically provide better resolution than the calorimeters. This avoids double counting of energy. For the case when the energy of the cluster is larger than that expected within the expected resolution, the energy of the cluster is reduced by the expected values, based on tracking information, with the remaining energy being interpreted to be due to photon or neutral hadrons. Clusters which are not matched to a track are used to determine energy and momentum of “neutral” particles (particles which leave no tracks in the central detector). This process thereby produces an array of objects each with a unique energy and/or momentum.

The stage 1 cut also requires that there be at least 7 tracks reconstructed, using the MT package, in an event. The purpose of this requirement is to reduce the potential backgrounds due to the production of τ leptons ($e^+e^- \rightarrow \tau^+\tau^-$) which can decay hadronically. The hadronic τ decays though do not typically produce a high multiplicity of particles, so this cut should effectively remove this background.

The event is also required to be fully contained within the detector volume by requiring that the thrust vector point to a position well away from the beam direction, that is $|\cos\theta_T| < 0.9$. The thrust vector, $\vec{n}_T$ is defined as the unit vector which maximizes the expression [56, 57],

$$T = \max_{\vec{n}} \left(\frac{\sum_i |p_i \cdot \vec{n}|}{\sum_i |p_i|} \right) \quad (4.1)$$

where p_i corresponds to the momentum of MT object i . This helps to ensure that events which may have particles that are lost down the beam line are not accepted.

Initial State Radiation

Events which radiate a photon in the initial state, are a major concern. The result of radiating an ISR photon is to reduce the effective centre-of-mass energy of the e^+e^- collision and hence the invariant mass of the created $q\bar{q}$ pair. The tendency of these events is to reduce this invariant mass to that of the Z^0 mass (see Fig 2.4). These radiative return events make up approximately three quarters of all our LPE2 multihadronic

events. The effective centre-of-mass energy ($\sqrt{s'}$) is directly related to the energy of the radiated photon via

$$s' = s - 2E_\gamma\sqrt{s} \quad (4.2)$$

The removal of these events from a multihadronic sample is accomplished, first by identifying all isolated photon candidates in the detector with energies greater than 10 GeV, which would appear as clusters in the electromagnetic calorimeter with no associated track in the tracking chambers. The Durham algorithm is then used to cluster the remaining particles into jets using a resolution parameter value of $y_{cut} = 0.2$. The possibility that one or more photons have travelled close to the beam pipe and hence gone undetected is addressed by performing kinematic fits for three cases, first: two photons escape detection, second: one photon escapes detection, and lastly: no photons escape detection. The case which returns the best fit is selected, and the effective centre-of-mass energy, $\sqrt{s'}$, is calculated from the invariant mass of the jets, excluding any photons.

However, if none of the probability requirements are met the event is forced into a two jet configuration, using the Durham algorithm, and the total photon energy is determined using the polar angles of these two jets,

$$E_\gamma = \sqrt{s} \left(\frac{|\sin(\theta_1 + \theta_2)|}{|\sin(\theta_1 + \theta_2)| + \sin\theta_1 + \sin\theta_2} \right) \quad (4.3)$$

and $\sqrt{s'}$ can be determined from Eqn 4.2.

The last step is to make sure that the value of the effective centre-of-mass energy is close to the true centre-of-mass energy, thereby ensuring little effect from any initial state radiation. A cut is placed on the difference of $\sqrt{s}$ and $\sqrt{s'}$, requiring it to be less than 10 GeV, for an event to be admitted to a sample of multihadronic events. The effect of this cut can be seen in Fig 4.2. The full effect of the Stage 1 cuts on the data at each centre-of-mass energy is shown in Table 4.5.

Figure 4.2: $(\sqrt{s} - \sqrt{s'})$ distribution for several centre-of-mass energies showing the region rejected by the ISR cut to remove events which contain photons radiated in the initial state.

4.1.3 Stage 2

The dominant background to $e^+e^- \rightarrow Z/\gamma^* \rightarrow q\bar{q} \rightarrow \text{hadrons}$ comes from $e^+e^- \rightarrow W^+W^-$ production in which one or both of the bosons decays hadronically, thereby producing two or four quarks in the final state. The Stage 2 cuts are meant to address these backgrounds by reducing the amount of background events that may make their way into any signal sample. Two likelihood values are constructed which are used to indicate how likely an event is a non-QCD four-quark event or a semi-leptonic W^+W^- event.

The four-quark likelihood value [58], $\mathcal{L}_{qqqq}$, is estimated using seven kinematic variables, which use characteristics of hadronic W^+W^- decays, like their four-jet nature or angular structure to distinguish them from events with similar signatures. These

variables are used to construct probabilities based on two hypotheses, first the event is due to a hadronically decaying W^+W^- pair ($W^+W^- \rightarrow q\bar{q}q\bar{q}$) and second that the event is due to a hadronically decaying Z^0 ($e^+e^- \rightarrow Z/\gamma^* \rightarrow q\bar{q}$). The probabilities are combined to produce two separate likelihoods, one for each hypothesis, which are then themselves combined to produce the final discriminating likelihood value, $\mathcal{L}_{qqqq}$. A cut is placed on the value of this likelihood requiring it to be less than 0.25 for an event to be accepted as a $e^+e^- \rightarrow Z/\gamma^* \rightarrow q\bar{q}$ event. A typically event $W^+W^- \rightarrow q\bar{q}q\bar{q}$ observed within OPAL is seen in Fig 4.3. The effect of this cut on the data and the expected backgrounds can be seen in Fig 4.4.

The semi-leptonic likelihood [58, 59], $\mathcal{L}_{qq\ell\nu}$, is based upon three separate likelihoods, one for each lepton species (e , μ and τ). Each of these likelihoods are based on 10 variables, which describe properties of the lepton, the jets produced by the $q\bar{q}$ pair and the missing energy carried away by the neutrino. A cut requiring the semi-leptonic likelihood to be less than 0.5 is placed on an event for it to be accepted as $e^+e^- \rightarrow Z/\gamma^* \rightarrow q\bar{q}$ event. The effect of the cut on $\mathcal{L}_{qq\ell\nu}$ can be seen in Fig 4.5.

These likelihood cuts also reduce the backgrounds arising from $e^+e^- \rightarrow Z^0Z^0$, in which one or both of the Z^0 bosons decays hadronically. This background is less problematic than the W^+W^- backgrounds since the cross-section for Z^0Z^0 production is much lower and hence will produce far fewer potential background events.

The expected size of the total background contribution to data can be determined by Monte Carlo. Identical cuts that were used on the data are applied to the background Monte Carlo samples, the events that remain are scaled to the luminosity of the data, giving the expected size of the background contamination within the data sample. The effect of the Stage 1 and Stage 2 cuts on the data and the expected four-fermion backgrounds for each centre-of-mass energy dataset can be seen in Table 4.5. The Stage 1 cuts produce a significant reduction in the number of observed events. This is a result of the ISR constraint, as seen in Fig 4.2, where much of the original sample is removed by the imposed cut. The Stage 2 cuts remove a large amount of the expected four-fermion backgrounds. Before the Stage 2 cuts the backgrounds accounted for $\sim 35\%$ of the data sample, for all of the high energy datasets. However, following the Stage 2 cuts

Figure 4.3: Graphical representation of a $W^+W^- \rightarrow q\bar{q} q\bar{q}$ background event observed in the OPAL detector.

this contribution is reduced to a more manageable $\sim 6\%$, which can then be subtracted from the data using background Monte Carlo.

Figure 4.4: Stage 2 cut on the likelihood for an event to be a $q\bar{q}q'\bar{q}'$ event arising from non-QCD production. The expected contribution of the 4-quark non-QCD background to the total sample is shown by the dotted line, and the expected $q\bar{q}\ell\bar{\ell}$ non-QCD backgrounds are represented by the dashed line. The vertical line indicates the value of the parameter where the cut is applied. Expected signal is determined from Monte Carlo (PYTHIA + total four-fermion) normalized to the luminosity of the 189 GeV data sample.

4.2 Correction Procedure

The Stage 2 cut clearly does not remove all of the expected backgrounds from W^+W^- and Z^0Z^0 production. The remaining backgrounds must be taken into account by estimating their expected contribution to each data sample. Distributions reflecting this expected background are produced from Monte Carlo simulations. The luminosity

Figure 4.5: Stage 2 cut on the likelihood for an event to be a $qql\nu$ event. The expected contribution of the 4-quark non-QCD background to the total sample is shown by the dotted line, and the expected $q\bar{q}l\bar{l}$ non-QCD backgrounds are represented by the dashed line. The vertical line indicates the value of the parameter where the cut is applied. Expected signal is determined from Monte Carlo (PYTHIA + total four-fermion) normalized to the luminosity of the 189 GeV data sample.

of each Monte Carlo background sample is determined from the number of events in the sample and the cross-section of the simulated background process which comes from the standard model theoretical expectation. Using this luminosity and the integrated luminosity of the data sample the number of expected background events passing the selection cuts can be normalized to the size of the data sample. The normalized background distribution can then be subtracted from the corresponding data distribution on a bin-by-bin basis.

Dataset		Preselection	Stage 1 Cuts	Stage 2 Cuts
91 GeV	Data	431349	395696	–
	Background	–	–	–
161 GeV	Data	1932	304	283
	Background	99.3	18.5	5.9
172 GeV	Data	1824	281	225
	Background	177.2	57.0	10.1
183 GeV	Data	8918	1456	1077
	Background	2649.3	404.3	60.1
189 GeV	Data	26936	4457	3255
	Background	7944.0	1349.1	170.7
192 GeV	Data	4229	717	530
	Background	1747.2	218.1	24.9
196 GeV	Data	10706	1736	1191
	Background	4492.9	570.2	67.5
200 GeV	Data	10226	1640	1149
	Background	4583.7	579.9	69.2
202 GeV	Data	5009	810	541
	Background	2169.9	276.5	33.3
205 GeV	Data	10857	1690	1193
	Background	4946.0	590.0	67.5
207 GeV	Data	17576	2721	1814
	Background	8325.2	976.1	112.3

Table 4.5: Effect of selection cuts on the number of observed data events and the expected four-fermion backgrounds.

Corrections must also be made for effects that arise from finite detector resolution and a limited detector acceptance (recall that the Stage 1 cut on $|\cos\theta_T|$ will reduce the fiducial volume available for the data sample). There are also residual effects of ISR photons that must be corrected. This correction is accomplished by comparing the data using two separate Monte Carlo samples, one which has gone through a full detector simulation, so it will include effects of detector resolution and acceptance and initial state radiation, called the “detector” level. The other sample uses the tree level hadrons and has not gone through a detector simulation. This is expected to produce

distributions arising solely from the properties of the underlying hadrons and so is called the “hadron” level, which is free of any detector biases or detector mismodelling. The data can also be corrected using Monte Carlo information from the generated partons, producing distributions that would arise solely from the underlying partons, called the “parton level”.

Each bin of the data distribution is multiplied by the ratio of the two Monte Carlo distributions (hadron/parton level to detector level). This then will correct the data distribution to either the hadron or the parton level, thereby removing detector and residual ISR effects in the data. Thus, any bin, i , of a data distribution is corrected as follows,

$$\mathcal{H}_i^{data} = \left(\frac{\mathcal{H}_i^{MC}}{\mathcal{D}_i^{MC}} \right) (\mathcal{D}_i^{data} - \mathcal{D}_i^{bkgd}) \quad (4.4)$$

where $\mathcal{H}$ and $\mathcal{D}$ represent distributions at the hadron level and the detector level respectively, and $\mathcal{D}_i^{bkgd}$ corresponds to the expected size of the total background in bin i .

4.3 Systematic Uncertainties

Systematic uncertainties are estimated for several detector, Monte Carlo and analysis conditions. The influence of detector variations, especially in the responses of the tracking chambers and the calorimeters, are estimated by comparing new results to that of the standard result, where standard result refers to results obtained using normal Stage 1 and Stage 2 cut values (see Table 4.6).

Systematic errors concerned with the definition of objects defined within the tracking chambers and calorimeters, and hence used for jet clustering, is estimated by comparing the results using the MT package to an alternate method of defining tracks. The alternate method uses a simple matching of tracks in the central detector and with clusters measured by the calorimeters. In the case where clusters are matched with tracks, the cluster information is ignored to avoid double counting of energy. The difference in results between the distribution using MT and the distribution using simple track and cluster matching is taken as the systematic uncertainty.

The constraint on the direction of the thrust axis is varied so that $|\cos\theta_T| < 0.7$. This variation tests the response of the detectors in the endcap region by restricting events to be contained within the barrel region. The difference between this variation and the default value is taken as the containment contribution to the total systematic uncertainty.

There are two systematic effects to be accounted for due to the backgrounds. The first effect is an uncertainty that arises from the value of the cut placed on the W^+W^- hadronic and semi-leptonic likelihoods. To account for this systematic uncertainty the value of the likelihood cuts are varied to 0.1 and 0.4 for $\mathcal{L}_{qqqq}$ and to 0.25 and 0.75 for $\mathcal{L}_{qq\ell\nu}$. The second effect is an uncertainty introduced during background subtraction. This uncertainty is estimated by varying the size of the background sample that is subtracted. The cross-section for the background processes are varied by $\pm 5\%$. The largest deviation from the default value for each of these variations is kept and used to determine an overall systematic error. The Z calibration data is not expected to contain any background from W^+W^- or Z^0Z^0 processes, so these variations are applied only to the high energy datasets.

The systematic effect introduced through the selection of events with little or no initial state radiation is estimated by repeating the selection using a second method of ISR determination [60]. This alternate method always assumes there is one photon while performing the kinematic fits, where the photon either escapes undetected down the beam line or is detected in the electromagnetic calorimeter. This variation is only applied to the high energy datasets.

Systematic uncertainties that arise from the uncertainty in the hadronization procedure are determined by using HERWIG Monte Carlo samples that employs a different hadronization model (the cluster model) to correct data samples to the “hadron” or “parton” levels. Differences that arise in the results to those that use the standard hadronization model, the string model, are taken to be the systematic uncertainty from hadronization.

Two further systematic variations must be considered when fitting for a determination of α_s . The first is to account for an uncertainty introduced due to the range of

bins that is used in performing the fit of the data to the theoretical prediction. The fit range is first varied by increasing the range by two bins, by adding one bin to each endpoint of the fit range. In the case where one of the endpoints is already at the maximum allowed bin value only the other point is extended by one bin. A second variation decreases the fit range by two bins, by removing a bin from each endpoint of the fit range. The largest deviation from the central fit value is taken as the contribution to the systematic errors.

The second fit related systematic variation takes into account the uncertainty due to renormalization scale (x_μ) dependence of the resummed predictions. During the fits on the resummed predictions the value of x_μ is set to the default value of 1.0. It is then varied to 0.5 and 2.0 respectively. The largest deviation from the central fit value is then taken as the contribution to the total systematic error coming from renormalization scale uncertainty.

All of the differences between the standard results and those due to the variations are added in quadrature, and taken as the total systematic uncertainty on the standard result. A summary of all of the systematic variations is shown in Table 4.6.

Variation	Normal Cut	Variation 1	Variation 2
Tracking*	MT objects	Tracks + Clusters	
qqqq ($\mathcal{L}_{qqqq}$)	< 0.25	< 0.1	< 0.4
qq $\ell\nu$ ($\mathcal{L}_{qq\ell\nu}$)	< 0.5	< 0.75	< 0.25
Backgrounds (σ_{bkgd})	1.0	+5%	-5%
Hadronization*	PYTHIA	HERWIG	–
ISR Algorithm	Default [61]	Alternate [60]	–
Containment ($ \cos\theta_T $)*	< 0.9	< 0.7	–
Fit Range*	min 6 bins	+2(1) bins	-2 bins
Renorm. Scale Dep. (x_μ)*	1	0.5	2.0

Table 4.6: Summary of systematic variations applied to the high energy datasets. The * indicates those systematic variations used for the 91 GeV dataset.

Chapter 5

Results

The measurements of the jet multiplicity observables will be presented here. These measurements are performed using several jet clustering algorithms, five for the n-jet rate distributions and two for the differential and average jet rate distributions. The analysis was performed on each dataset first individually, then a sample which combines all of the LEP2 data into one was analyzed as a whole. This LEP2 sample has vastly superior statistics to each individual data sample. For each distribution and energy the data is compared with the PYTHIA and HERWIG (and ARIADNE for datasets with $\sqrt{s} > 172$ GeV) Monte Carlo expectations.

These observables are then used to determine a value of α_s at each of the LEP2 energy points as well as at the Z^0 pole using the Z^0 calibration sample. The α_s values will be determined by applying a fit for various theoretical predictions to the differential 2-jet rate (D_2) as well as the average jet rate. A fit of the values of the 3-jet rate, R_3 , using a constant value of y_{cut} , for the JADE, Durham and Cambridge algorithms, as a function of centre-of-mass energy will also be used to determine values $\alpha_s(M_Z)$.

A direct comparison is also made between the value of α_s determined from the fits of the D_2 and average jet rate distributions for the 91 GeV and LEP2 data samples to previously determined values of α_s . These previous measurements include lower energy samples and use the same distributions for the α_s determination as those that are used in this analysis. This comparison will demonstrate the phenomenon of the running of

the strong coupling constant.

5.1 Jet Rates

The n -jet fractions for the 91 GeV sample and those for the LEP2 sample (161 to 207 GeV) with a luminosity weighted centre-of-mass energy of 195.7 GeV are shown in Figs 5.1 to 5.3 and Figs 5.4 to 5.6 respectively. A table containing all the R_n values as a function of the jet resolution parameter for each of these two samples as well as plots and tables of values for each of the individual energy points are given in Appendix A. Each plot shows the fraction of events in a given sample that are defined to be n -jets for a given value of jet resolution parameter at the hadron level. The jet fractions were calculated using five separate clustering algorithms. For the CONE R and CONE ϵ variants, the displayed fractions are for the case of $n \leq 2$, $n=3$ and $n \geq 4$. The JADE, Durham and Cambridge algorithms each show the n -jet fractions for $n=2,3,4,5$. The error bars on each of the points represent the total statistical and systematic errors (see Section 4.3 for a full description of the systematic error sources considered) added in quadrature. The Monte Carlo expectations corresponding to PYTHIA, HERWIG and ARIADNE¹ are also displayed on each plot, for each algorithm and energy. The Monte Carlo expectations reasonably match the data points within the total statistical and systematic errors, although in some cases the agreement is not perfect.

5.2 R_3 Fits

A plot of the 3-jet rate as a function of the centre-of-mass energy using the JADE algorithm at a constant y_{cut} value of 0.08 is shown in Fig 5.7. A fit to the data, corrected to the parton level, with the $\mathcal{O}(\alpha_s^2)$ prediction from Eqn 2.30 is performed to extract a value of $\alpha_s(Q)$ with $Q = M_Z$. Similarly, the 3-jet rates using the Durham and

¹The was no ARIADNE sample available for the 91 GeV, 161 GeV or 172 GeV data samples. Therefore no ARIADNE values are plotted in Figs 5.1 to 5.6. See Appendix A for those energies where ARIADNE is available.

Figure 5.1: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm with $\sqrt{s} = 91$ GeV. The PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

Figure 5.2: The hadron level n -jet rates for the JADE algorithm with $\sqrt{s} = 91$ GeV. The PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

Figure 5.3: The hadron level n -jet rates for the Cambridge and Durham algorithms with $\sqrt{s} = 91$ GeV. The PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

Figure 5.4: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm with the LEP2 data sample. The PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

Figure 5.5: The hadron level n -jet rates for the JADE algorithm with the LEP2 data sample. The PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

Figure 5.6: The hadron level n -jet rates for the Cambridge and Durham algorithms for the data with the LEP2 data sample. The PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

Cambridge algorithms with $y_{\text{cut}}=0.01$ can be fit to an $\mathcal{O}(\alpha_s^2)$ prediction (Eqn 2.30) as seen in Fig 5.8. The Durham and Cambridge algorithms can also be used to perform fits using a Next-to-Leading-Logarithmic Approximation (NLLA) prediction (see Eqn 2.37) as seen in Fig 5.9. The fits are all performed using a χ^2 minimization with the MINUIT [62] package. The fit returns a value for Λ_{QCD} , which is then used to determine $\alpha_s(M_Z)$ using a 3-loop expansion (see Eqn 2.20). The values of y_{cut} were chosen so that the fits would be performed in the regions of applicability for the $\mathcal{O}(\alpha_s^2)$ and NLLA predictions. Also with these values of y_{cut} a large fraction of 3-jet events are selected from the data sample while at the same time minimizing the amount of potential 4-jet background. The results of the fits are summarized in Table 5.1.

Figure 5.7: Three-jet rate as a function of $\sqrt{s}$ using the JADE algorithm with $y_{\text{cut}} = 0.08$. The solid line represents the best fit line according to the α_s^2 prediction and the dashed lines represent the size of the error on the best fit line. Previous JADE and OPAL data have been taken from [63]. Also shown are R_3 values as measured by AMY [64], MARK II [65] and TASSO [66]. Error bars indicate total errors. The TASSO value at 44 GeV and the new OPAL value at 91 GeV have both been shifted slightly to the left and right, respectively, for clarity.

Figure 5.8: Three-jet rate as a function of $\sqrt{s}$ for the Durham and Cambridge algorithms with $y_{\text{cut}} = 0.01$. The solid line represents the fit according to the α_s^2 prediction and the dashed lines represent the error on the best fit line. Previous JADE and OPAL data are taken from [63]. Error bars indicate total errors. The new OPAL value at 91 GeV has been shifted slightly to the right for clarity.

Figure 5.9: Three-jet rate as a function of $\sqrt{s}$ for the Durham and Cambridge algorithms with $y_{\text{cut}} = 0.01$. The solid line represents the fit according to the prediction of the Next to Leading Log Approximation and the dashed lines represent the error on the best fit line. Previous JADE and OPAL data are taken from [63]. Error bars indicate total errors. The new OPAL value at 91 GeV has been shifted slightly to the right for clarity.

Figure 5.10: Comparison of Λ_{QCD} values returned from the fit of R_3 to the JADE, Durham and Cambridge $\mathcal{O}(\alpha_s^2)$ and the Durham and Cambridge NLLA predictions to the current world average value. The solid line corresponds to the world average value of Λ_{QCD} and the dashed lines indicate the error range in this value.

Prediction	Algorithm	Λ_{QCD} (MeV)	$\alpha_s(M_Z)$	χ^2 (dof)
$\mathcal{O}(\alpha_s^2)$	Jade	286.7 ± 17.0	0.12537 ± 0.00104	18.0 (10)
	Durham	171.6 ± 20.6	0.11463 ± 0.00194	1.78 (5)
	Cambridge	146.8 ± 21.3	0.11208 ± 0.00212	2.37 (5)
NLLA	Durham	802.9 ± 119.8	0.14840 ± 0.00408	1.49 (5)
	Cambridge	459.1 ± 73.9	0.13403 ± 0.00355	2.17 (5)

Table 5.1: Results of the $\mathcal{O}(\alpha_s^2)$ and NLLA fits to R_3 as a function of $\sqrt{s}$ for the JADE, Durham and Cambridge algorithms.

Unfortunately low energy measurements of R_3 using Durham and Cambridge do not exist, so the lever arm in the $\mathcal{O}(\alpha_s^2)$ and NLLA fits is very limited. Clearly the NLLA R_3 fits are inconsistent with the $\mathcal{O}(\alpha_s^2)$ predictions. A comparison of the value of the Λ_{QCD} parameter determined in the fits with the accepted value of $\Lambda_{QCD} = 216^{+25}_{-24}$ MeV [2] is shown in Fig 5.10. The NLLA predictions are inconsistent with the expectations from the $\mathcal{O}(\alpha_s^2)$ fits and the world average value of Λ_{QCD} .

5.3 Differential Jet Rates

The differential 2- 3- and 4-jet rates, D_n , are plotted as a function of y_{cut} for the Durham and Cambridge algorithms for the 91 GeV and the LEP2 data sample in Figs 5.11 to 5.12 and Figs 5.13 to 5.14 respectively. The plots and tables of values for the individual high energy points and the 91 GeV sample can be seen Appendix B. The differential 3- and 4-jet rates have a negative slope in the region of large y_{cut} , and so the negative values of D_3 and D_4 are plotted. The values are also scaled down by one and two orders of magnitude, respectively, for clarity. The curves on the plots represent the expected Monte Carlo differential jet rate distributions. There is good agreement between the data and the expectation from PYTHIA and HERWIG (and ARIADNE for the individual high energy plots in Appendix B).

The results of the fits of the various NLLA, $\mathcal{O}(\alpha_s^2)$ and matching schemes are shown in Tables 5.2 and 5.3 for the 91 GeV data sample and Tables 5.6 and 5.7 for the LEP2 data sample. Only the $k_{\perp}$ algorithms are used as they have the most complete theoretical calculations, and these will be used to fit to a value of the strong coupling constant. The fits are performed using the ASCFIT [67] package which is based upon the MINUIT and HBOOK [68] packages. ASCFIT was developed to determine the best fit value of α_s using one of several possible theoretical predictions for different jet multiplicity and event shape observables. ASCFIT also allows the renormalization scale parameter, x_{μ} to vary. However, since this is not expected to be important in the NLLA and matched predictions it is set to unity for these predictions. In the $\mathcal{O}(\alpha_s^2)$ fit the renormalization scale parameter is allowed to vary as a free parameter and the tables of returned x_{μ} values for the $\mathcal{O}(\alpha_s^2)$ predictions are provided in Appendix D. The fitting routine determines the theoretical values of R_2 , the 2-jet rate, at the maximum and the minimum values of a particular y_{cut} bin and divides the difference of these two values by the bin width thereby producing a theoretical expectation for D_2 , the differential 2-jet rate, based on the R_2 equations, as given in Section 2.7.1.

A plot of the fits of the 91 GeV and LEP2 data samples to the $\ln R$ matching prediction can be seen in Figs 5.16 and 5.17 and Figs 5.18 and 5.19, respectively, where

Figure 5.11: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge algorithm for data with $\sqrt{s} = 91$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

Figure 5.12: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Durham algorithm for data with $\sqrt{s} = 91$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

Figure 5.13: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge algorithm for the LEP2 data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

Figure 5.14: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Durham algorithm for the LEP2 data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

the error bars indicate the size of the statistical errors only. The systematic error is determined separately, by comparing the value of this fit with those using the systematic variations outlined in Section 4.3. The $\ln R$ matching prediction is favoured since it does not require an ad hoc modification to correct for subleading terms appearing outside of the exponentiation, and therefore is expected to produce a more robust fit.

Fits to the parton level differential 2-jet rates were performed using 10 different schemes based on the predictions from NLLA, $\mathcal{O}(\alpha_s^2)$, R matching and $\ln R$ matching. The 10 schemes used are as follows,

1. NLLA
2. $\mathcal{O}(\alpha_s^2)$
3. R matching
4. $\ln R$ matching
5. R matching with the G_{21} subleading term included
6. NLLA with the G_{21} term
7. $\ln R$ with modified y_{cut} (such that $L \rightarrow L'$ from Eqn 2.40)
8. NLLA with modified y_{cut}
9. NLLA with the G_{21} term and modified y_{cut}
10. R matching with the G_{21} term and modified y_{cut}

Fit Range Determination

The range over which the D_2 distributions and the subsequent average jet rates were fitted was determined by splitting the 189 GeV PYTHIA Monte Carlo into 100 statistically independent samples, each with approximately 8000 events. Each sample will have comparable statistics to the 189 GeV dataset (3255 events) and to the combined LEP2 dataset (11,358). Each of these samples was run through the full analysis separately (with the exception of background subtraction), providing 100 statistically independent distributions. A fit was performed on each distribution for a range of

possible end-points of the fit range (requiring that the fit range be at least five bins). A reduced χ^2 value (the χ^2 per degree of freedom) was determined for each fit range variation and each theoretical prediction. The reduced χ^2 values were averaged over the 100 different samples and plotted against the overall fit range size (see Fig 5.15). The fit range that produced the smallest reduced χ^2 for a given theoretical prediction was chosen as the default fit range for that prediction for 189 and all other energy datasets. In the case that no clear χ^2 minimum was found, as in Fig 5.15, the largest reasonable range allowed was chosen. This range is then used as the default fit range for all other LEP2 energies. A similar procedure was applied to the 91 GeV PYTHIA Monte Carlo, using 12000 events per subsample, to determine the fit range for the 91 GeV dataset.

Figure 5.15: Reduced χ^2 averaged over 100 subsamples from the 189 GeV PYTHIA Monte Carlo sample for the Durham and Cambridge algorithms, for the $\ln R$ matching prediction.

Figure 5.16: Fit to the parton level differential 2-jet rate, D_2 , for the 91 GeV sample, as a function of y_{cut} for the $\ln R$ matching prediction using the Cambridge algorithm. The inset plots show the pull of each data point to the line of best fit and the two vertical lines delineate the fit range. Error bars represent statistical errors only.

Figure 5.17: Fit to the parton level differential 2-jet rate, D_2 , for the 91 GeV sample, as a function of y_{cut} for the $\ln R$ matching prediction using the Durham algorithm. The inset plots show the pull of each data point to the line of best fit and the two vertical lines delineate the fit range. Error bars represent statistical errors only.

Figure 5.18: Fit to the parton level differential 2-jet rate, D_2 , for the LEP2 data sample, as a function of y_{cut} for the $\ln R$ matching prediction using the Cambridge algorithm. The inset plots show the pull of each data point to the line of best fit and the two vertical lines delineate the fit range. Error bars represent statistical errors only.

Figure 5.19: Fit to the parton level differential 2-jet rate, D_2 , for the LEP2 data sample, as a function of y_{cut} for the $\ln R$ matching prediction using the Durham algorithm. The inset plots show the pull of each data point to the line of best fit and the two vertical lines delineate the fit range. Error bars represent statistical errors only.

Statistically Weighted Averaging

The values in Tables 5.4 and 5.5 for the combined sample were determined by taking a statistically weighted average of the fit results for the individual energy points

$$\tilde{\alpha}_s = \frac{1}{w} \sum_{i=1}^N \frac{\alpha_s^i}{\sigma_i^2} \quad (5.1)$$

where

$$w = \sum_{i=1}^N \frac{1}{\sigma_i^2}$$

and σ_i is the statistical error on each α_s^i value. The total statistical error on the $\tilde{\alpha}_s$ value is $\sigma = 1/\sqrt{w}$. These sums are run over all ten of the high energy α_s fit values. The α_s values of the various predictions obtained from the statistically weighted average of the fit values from each of the individual high energy data samples are shown in Table 5.4 and 5.5. These values can be compared to the same values when the fit is applied to the full combined LEP2 sample, shown in Tables 5.6 and 5.7. The fit values along with the determined values of the statistical and systematic errors from the LEP2 data set are very similar to those determined using the statistically weighted average, suggesting that these two methods for determining the value of α_s for the entire LEP2 data set are equivalent.

Fits for α_s values are also performed using each of the systematic variations (see Section 4.3) over the same fit range as that for the standard cuts. A weighted statistical mean is then determined for each variation and compared to the statistically weighted mean determined from the standard cuts. The difference between the weighted mean of the standard cut and that of the variations determine the size of the contribution to the systematic error for the particular systematic variation.

D_2 Fit Results

The results of the α_s fits using the various theoretical D_2 predictions are plotted in Fig 5.20 for the 91 GeV dataset. The returned α_s values for the statistically weighted average of the individual high energy fits are shown in Fig 5.21 and the α_s from the

Figure 5.20: Comparison of fit results for the 91 GeV Cambridge and Durham D_2 distributions for all of the theoretical predictions. Inner error bars correspond to statistical error while outer error bars indicate total error.

Figure 5.21: Comparison of fit results from the statistically weighted average Cambridge and Durham D_2 distributions for all of the theoretical predictions. Inner error bars correspond to statistical error while outer error bars indicate total error.

Figure 5.22: Comparison of fit results for the LEP2 data Cambridge and Durham D_2 distributions for all of the theoretical predictions. Inner error bars correspond to statistical error while outer error bars indicate total error.

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-2.25 – -1.38	0.12526	0.00069	0.00435	15.86 (7.0)
$\mathcal{O}(\alpha_s^2)$	-2.62 – -0.88	0.11997	0.00054	0.00261	14.08 (14.0)
NLLA + G_{21}	-2.62 – -1.38	0.13172	0.00071	0.00406	57.42 (10.0)
R match	-2.25 – -1.00	0.12041	0.00060	0.00677	29.21 (10.0)
lnR match	-2.25 – -1.00	0.11875	0.00057	0.00431	15.61 (10.0)
mod R match	-2.62 – -0.88	0.12296	0.00057	0.00651	49.33 (14.0)
mod lnR match	-2.25 – -1.00	0.11859	0.00057	0.00422	14.73 (10.0)
NLLA + mod y	-2.25 – -1.38	0.12249	0.00067	0.00383	13.31 (7.0)
NLLA + G_{21} + mod y	-2.25 – -1.38	0.12232	0.00068	0.00382	12.35 (7.0)
mod R + mod y	-2.25 – -0.88	0.12191	0.00060	0.00677	40.29 (11.0)

Table 5.2: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 91 GeV dataset using the Cambridge algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-2.25 – -1.38	0.13577	0.00071	0.00573	11.67 (7.0)
$\mathcal{O}(\alpha_s^2)$	-2.62 – -0.88	0.12195	0.00053	0.00184	11.98 (14.0)
NLLA + G_{21}	-2.62 – -1.38	0.14261	0.00075	0.00511	21.50 (10.0)
R match	-2.25 – -1.00	0.12541	0.00057	0.00828	7.26 (10.0)
lnR match	-2.25 – -1.00	0.12585	0.00057	0.00518	14.59 (10.0)
mod R match	-2.62 – -0.88	0.12722	0.00056	0.00853	18.39 (14.0)
mod lnR match	-2.25 – -1.00	0.12566	0.00057	0.00505	12.59 (10.0)
NLLA + mod y	-2.25 – -1.38	0.13266	0.00068	0.00497	8.59 (7.0)
NLLA + G_{21} + mod y	-2.25 – -1.38	0.13266	0.00069	0.00493	6.83 (7.0)
mod R + mod y	-2.25 – -0.88	0.12685	0.00058	0.00824	9.45 (11.0)

Table 5.3: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 91 GeV dataset using the Durham algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (9 dof)
NLLA	-3.25 – -1.25	0.09461	0.00154	0.00393	3.81
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.09935	0.00156	0.00351	5.37
NLLA + G_{21}	-3.25 – -1.25	0.09795	0.00166	0.00396	3.72
R match	-3.25 – -0.75	0.09275	0.00147	0.00601	4.65
$\ln R$ match	-3.25 – -0.75	0.09132	0.00139	0.00318	4.09
mod R match	-3.00 – -0.75	0.09436	0.00154	0.00605	4.35
mod $\ln R$ match	-3.50 – -0.75	0.09120	0.00138	0.03475	4.09
NLLA + mod y	-3.25 – -1.25	0.09299	0.00148	0.00373	3.96
NLLA + G_{21} + mod y	-3.25 – -1.25	0.09314	0.00150	0.00373	3.86
mod R + mod y	-3.25 – -0.75	0.09421	0.00153	0.08442	4.39

Table 5.4: Statistically weighted average of α_s fit values from the individual LEP2 data sample fits, with statistical and systematic errors for several QCD parameterizations of the differential 2-jet rate for the LEP2 dataset using the Cambridge algorithm over the stated fit range. The χ^2 compares the statistically weighted average to the fit value from the individual LEP2 datasets.

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (9 dof)
NLLA	-3.25 – -1.25	0.11153	0.00135	0.00226	1.76
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10274	0.00118	0.00200	1.23
NLLA + G_{21}	-3.25 – -1.25	0.11580	0.00146	0.00279	1.69
R match	-3.00 – -0.75	0.10657	0.00141	0.00578	1.16
$\ln R$ match	-3.25 – -0.75	0.10655	0.00130	0.00343	1.17
mod R match	-3.00 – -0.75	0.10778	0.00143	0.00550	1.33
mod $\ln R$ match	-3.50 – -0.75	0.10741	0.00112	0.01525	4.01
NLLA + mod y	-3.25 – -1.25	0.11015	0.00135	0.00229	1.64
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10978	0.00135	0.00221	1.57
mod R + mod y	-3.25 – -0.75	0.10693	0.00132	0.09701	1.24

Table 5.5: Statistically weighted average of α_s fit values from the individual LEP2 data sample fits, with statistical and systematic errors for several QCD parameterizations of the differential 2-jet rate for the LEP2 dataset using the Durham algorithm over the fit range. The χ^2 compares the statistically weighted average to the fit value from the individual LEP2 datasets.

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10447	0.00145	0.00278	34.93 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10360	0.00160	0.00222	7.07 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10831	0.00156	0.00315	36.96 (8.0)
R match	-3.25 – -0.75	0.10428	0.00152	0.00567	13.65 (10.0)
lnR match	-3.25 – -0.75	0.10175	0.00138	0.00300	11.22 (10.0)
mod R match	-3.25 – -0.75	0.10531	0.00153	0.00485	12.58 (10.0)
mod lnR match	-3.25 – -0.75	0.10164	0.00137	0.02240	11.66 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10304	0.00143	0.00240	30.18 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10280	0.00143	0.00242	31.27 (8.0)
mod R + mod y	-3.25 – -0.75	0.10518	0.00152	0.09527	13.38 (10.0)

Table 5.6: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the LEP2 dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11179	0.00136	0.00211	32.00 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10335	0.00120	0.00145	12.34 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11608	0.00147	0.00262	34.92 (8.0)
R match	-3.00 – -0.75	0.10728	0.00142	0.00547	7.59 (9.0)
lnR match	-3.25 – -0.75	0.10701	0.00131	0.00318	13.35 (10.0)
mod R match	-3.00 – -0.75	0.10843	0.00144	0.00512	9.61 (9.0)
mod lnR match	-3.50 – -0.75	0.10769	0.00112	0.01428	14.50 (11.0)
NLLA + mod y	-3.25 – -1.25	0.11044	0.00136	0.00204	25.49 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.11007	0.00136	0.00197	28.07 (8.0)
mod R + mod y	-3.25 – -0.75	0.10740	0.00132	0.09747	13.24 (10.0)

Table 5.7: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the LEP2 dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

fits applied directly to the LEP2 data sample are shown in Fig 5.22. The values of the returned values along with the χ^2 quality of fit and the number of degrees of freedom, are presented in Tables 5.3 to 5.6. The $\ln R$ fit results are highlighted with bold font. The χ^2 values represent a comparison of return fit values of α_s from the 10 separate energy points with the statistically weight mean value. The small χ^2 values indicate that there is little variation between the α_s values determined at the individual energies, justifying the approach of a combining the individual samples into one.

Tables and plots of the fits to the individual high energy points are provided in Appendix B. The results of the $\ln R$ matching prediction fit to the 91 GeV and LEP2 data samples are summarized for the Durham and Cambridge algorithms in Table 5.8. The LEP2 values are those determined by taking the statistically weighted average of the fits for each individual LEP2 energy point. The table also includes the contribution of each systematic variation to the total systematic error. The largest contribution to the total systematic error for both energy ranges and jet algorithms is from the variation of the renormalization scale parameter, x_μ . It has the largest relative effect on the 91 GeV data sample, leading to larger total errors despite fewer systematic variations and smaller statistical errors.

The choice of values to vary the renormalization parameter is largely based on convention. With the renormalization scale parameter now being the largest contribution to the systematic error it may be necessary to study this systematic in more detail and re-evaluate the way in which it is determined. The variation using $x_\mu=2$ always leads to the largest systematic variation with respect to the default value.

	Durham		Cambridge	
	91 GeV	LEP2	91 GeV	LEP2
α_s	0.12585	0.10655	0.11875	0.10193
Tracking	0.00061	0.00117	0.00178	0.00045
Containment	0.00051	0.00123	0.00045	0.00143
ISR	–	0.00032	–	0.00001
$\mathcal{L}_{qq\ell\nu}$	–	0.00005	–	0.00006
$\mathcal{L}_{qqqq}$	–	0.00055	–	0.00098
Backgrounds	–	0.00016	–	0.00015
Hadronization	0.00185	0.00024	0.00153	0.00130
Fit Range	0.00016	0.00080	0.00054	0.00054
Theory (x_μ)	0.00477	0.00276	0.00355	0.00214
Statistical	0.00057	0.00130	0.00057	0.00137
Total Syst.	0.00518	0.00343	0.00431	0.00312
Total Error	0.00521	0.00367	0.00435	0.00341

Table 5.8: Breakdown of systematic errors on the D_2 distribution $\ln R$ fit values of α_s for the 91 GeV and LEP2 data samples.

The difference between the returned α_s value between the 91 GeV sample and LEP2 sample shows clear evidence for the running of the strong coupling constant, for both the Durham and Cambridge algorithms. In the case of the Durham algorithm there is a difference of 3.03 standard deviations between the two data samples. For the Cambridge algorithm difference 3.04 standard deviations. In both cases any correlations between errors were neglected.

5.4 Average Jet Rates

The average number of jets (at the hadron level) per event in a sample, $\langle N \rangle$, for a given value of y_{cut} is plotted for the 91 GeV dataset in Figs 5.23 and 5.24 and for the LEP2 dataset in Figs 5.25 and 5.26. These average jet rates are plotted separately for the Durham and Cambridge jet clustering algorithms and compared with the PYTHIA and HERWIG expectations. The Monte Carlo predictions agree extremely well with the data distributions. The error bars on the plots represent the total (statistical + systematic) errors.

The plots and tables of values of the average jet rate distributions for the 91 GeV and the individual high energy data points are provided in Appendix C. The most complete theoretical calculations of the average jet rate exist only for the $k_\perp$ jet algorithms, and hence only the Durham and Cambridge average jet rates are presented here.

The fits were performed on the parton level average jet rate distributions using 10 different schemes based on the predictions from NLLA, $\mathcal{O}(\alpha_s^2)$, R matching and $\ln R$ matching. The 10 schemes used are as follows,

1. NLLA
2. $\mathcal{O}(\alpha_s^2)$
3. R matching
4. $\ln R$ matching
5. $\ln R$ matching with the subleading terms included
6. NLLA with the subleading terms included
7. $\ln R$ with modified y_{cut} (such that $L \rightarrow L'$ from Eqn 2.40)
8. NLLA with modified y_{cut}
9. NLLA with the subleading terms included and modified y_{cut}
10. $\ln R$ matching with the subleading terms included and modified y_{cut}

This list is slightly different from the one presented for D_2 . For the average jet rates subleading terms are included for the $\ln R$ matched predictions as opposed to the R matched predictions for the differential 2-jet rates. So in this list the predictions involving modifications of the matched predictions are applied to the $\ln R$ matching.

Figure 5.23: Average jet rates at the hadron level as a function of y_{cut} for the Cambridge algorithm for data with $\sqrt{s} = 91$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total (statistical + systematic) errors.

Figure 5.24: Average jet rates at the hadron level as a function of y_{cut} for the Durham algorithm for data with $\sqrt{s} = 91$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total (statistical + systematic) errors.

Figure 5.25: Average jet rates at the hadron level as a function of y_{cut} for the Cambridge algorithm and the LEP2 dataset. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total (statistical + systematic) errors.

Figure 5.26: Average jet rates at LEP2 dataset the hadron level as a function of y_{cut} for the Durham algorithm and the LEP2 dataset. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total (statistical + systematic) errors.

Results of the fits to the average jet rates using the 10 fitting schemes mentioned above are shown in Tables 5.9 and 5.10 for the 91 GeV dataset using the Durham and Cambridge algorithms. Similarly, statistically weighted average fits are shown for the LEP2 dataset in Tables 5.11 and 5.12. Since all of the data are present in each bin of the average jet rate distribution there are expected to be large bin-to-bin correlations. Consequently, the fit must take into account the covariance matrix for the distribution. The LEP2 combined data sample does not correspond to a single energy point described by a single Monte Carlo, hence the determination of a covariance matrix (see below for a description of the method) for this sample is complicated and non-trivial. Since the statistically weighted average values are expected to be equivalent to those values determined from the combined LEP2 sample only the statistically weighted average values are determined. Plots of the fit for the 91 GeV data sample to the $\ln R$ matching prediction using the Cambridge and Durham algorithms are shown in Figs 5.27 and 5.28 respectively. Again, as in for the differential 2-jet rate, systematic errors are determined by comparing the fit values using the standard cut to those using the systematic variations outlined in Section 4.3.

Inter-bin Correlations

The statistical errors are strongly correlated between points, since the same events are used to determine $\langle N \rangle$ at each value of y_{cut} . The fit is performed using a correlated χ^2 fit in which the covariance matrix was determined from the PYTHIA Monte Carlo sample divided into many detector level subsamples. Each subsample is then corrected to the parton level using a second PYTHIA sample that was statistically independent of the subsamples. 1500 subsamples were created for the Z^0 calibration dataset and 1000 subsamples were created for the high energy datasets. A random number was generated for each event in the full sample and compared to a predetermined cutoff number. If the random number for the event was less than the cutoff value the event is added to the subsample otherwise the event is rejected. A new set of random numbers is generated for each subsample before being applied to the events of the full Monte

Figure 5.27: Fit of the parton level average jet rate of the 91 GeV data sample, as a function of y_{cut} for the $\ln R$ matching prediction using the Cambridge algorithm. Error bars represent statistical errors only.

Figure 5.28: Fit of the parton level average jet rate of the 91 GeV data sample, as a function of y_{cut} for the $\ln R$ matching prediction using the Durham algorithm. Error bars represent statistical errors only.

Carlo sample. In this way a large number of statistically independent subsamples (each with statistics roughly equivalent to the size of the 189 GeV data sample) are created.

The average jet rate distribution was determined for each subsample. The distributions from the individual subsamples were then used to determine an estimated covariance matrix as follows:

$$W_{ij} = \frac{1}{n-1} \left[\sum_{k=1}^n x_i^k x_j^k - \frac{1}{n} \sum_{k=1}^n x_i^k \sum_{k=1}^n x_j^k \right]. \quad (5.2)$$

where x_i^k is the average jet rate for a given y_{cut} bin i and sample k , and n is the number of subsamples. The estimated covariance matrix is then used to determine the correlation matrix, $\rho_{ij} = W_{ij}/\sigma'_i\sigma'_j$ (where $\sigma'_m = \sqrt{W_{mm}}$). Finally the true covariance matrix is determined using the correlation matrix and the statistical errors from the data sample, $V_{ij} = \rho_{ij}\sigma_i\sigma_j$, where σ_i and σ_j are the statistical errors on bins i and j , respectively. A graphical representation of the covariance and correlation matrices for the 91 and 189 GeV data samples is seen in Figs 5.29 and 5.30. The covariance matrices of the other high energy datasets should be very similar to that of the 189 GeV dataset. The covariance matrix is then used in the determination of the χ^2 as follows

$$\chi^2 = \sum_{i,j}^n (x_i - \bar{x}_i)(V^{-1})_{ij}(x_j - \bar{x}_j) \quad (5.3)$$

where $\bar{x}_n$ corresponds to the theoretical value for bin n .

The random number cutoff value is chosen so that there is a statistically significant number of events in each subsample. Each Z^0 calibration subsample contained approximately 12000 events, before Stage 1 cuts, whereas the subsamples for each of the high energy datasets contained approximately 8000 events before Stage 1 and Stage 2 cuts. The effect of varying the number of events in each subsample is seen in Fig 5.31. There is little variation in the elements of the covariance matrix as the number of events per subsamples varies.

Figure 5.29: Graphical representation of the covariance matrices determined for the data samples at 91 GeV and 189 GeV for the Durham and Cambridge algorithms. The covariance matrices were determined using 1500 and 1000 subsamples for the 91 GeV and 189 GeV datasets respectively. The z-axis represents the value of the individual elements of the covariance matrix.

Figure 5.30: Graphical representation of the correlation matrices determined for the data samples at 91 GeV and 189 GeV for the Durham and Cambridge algorithms. The top plots correspond to the 91 GeV Durham(left) and Cambridge(right) algorithms and the bottom plots correspond to the 189 GeV Durham(left) and Cambridge(right) algorithms. The correlation matrices were determined using 1500 and 1000 subsamples for the 91 GeV and 189 GeV datasets respectively. The z-axis represents the value of the individual elements of the correlation matrix.

Figure 5.31: Effect of varying the number of events analyzed for each subsample. The four plots correspond to the value of four elements of the covariance matrix, $V_{3\ 5}$, $V_{4\ 17}$, $V_{7\ 12}$ and $V_{15\ 18}$, determined for the 189 GeV dataset. The covariance matrix was evaluated using 1000 subsamples.

The number of subsamples was chosen so that the elements of the covariance matrix would be stable and free of fluctuations. The values of four elements, distributed about the covariance matrix, were evaluated as the number of subsamples was varied, as seen in Fig 5.32 for the 91 GeV data sample and Fig 5.33 for the 189 GeV data sample, both using the Cambridge algorithm. The 91 GeV data sample runs over more subsamples since it has double the number of bins of the high energy data samples. It was determined that 1500 subsamples for the 91 GeV dataset and 1000 subsamples for

LEP2 energy datasets would be sufficient to ensure a stable covariance matrix for the fits.

Figure 5.32: Effect of varying the number of subsamples for the determination of the covariance matrix. The four plots correspond to the value of four elements of the covariance matrix, V_{35} , V_{417} , V_{712} and V_{1518} , determined for the 91 GeV dataset.

Figure 5.33: Effect of varying the number of events analyzed for each subsample. The four plots correspond to the value of four elements of the covariance matrix, $V_{3\ 5}$, $V_{4\ 17}$, $V_{7\ 12}$ and $V_{15\ 18}$, determined for the 189 GeV dataset.

Average Jet Rate Fit Results

The results for the 10 average jet rate predictions are shown in Fig 5.34 for the 91 GeV dataset and Fig 5.35 for the statistically weighted average of the individual LEP2 high energy fits. The plots show a comparison of the returned α_s values as a function of prediction. These values along with the χ^2 quality of fit parameter and the number of degrees of freedom are shown in Tables 5.9 and 5.10 for the 91 GeV data sample and Tables 5.11 and 5.12 for the statistically weighted average LEP2 values. The $\ln R$ fit results are highlighted with bold font.

The 91 GeV sample provides a stringent test of the QCD predictions due to the small statistical error on the individual bins, which are typically at the per mille level. Within the determined fit range these small statistical errors give rise to very large χ^2 values. This may be due to the fact that the size of the subsamples used to determine the fit range was significantly smaller than the size of the data sample, due to the lack of available Monte Carlo statistics. A check was performed by systematically reducing the size of the fit range without a significant reduction in the observed χ^2 per degree of freedom. The returned fit value of α_s underwent little variation over the varying fit ranges, indicating that the value of α_s determined using the fit ranges in Tables 5.9 and 5.10 are an adequate representation of the true value of α_s .

Tables for the individual high energy fits are provided in Appendix C. A breakdown of the contribution from each systematic variation to the total systematic error on the fit value of α_s is shown in Table 5.13.

Figure 5.34: Comparison of fit results for the 91 GeV average jet rate distributions for all of the theoretical predictions for the Durham and Cambridge algorithms. Inner error bars correspond to statistical error while outer error bars indicate total error .

Figure 5.35: Comparison of fit results from the statistically weighted average LEP2 average jet rate distributions for all of the theoretical predictions for the Durham and Cambridge algorithms. Inner error bars correspond to statistical error while outer error bars indicate total error.

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-2.88 – -1.38	0.14496	0.00044	0.00215	355.85 (12)
$\mathcal{O}(\alpha_s^2)$	-2.25 – -1.25	0.11974	0.00125	0.00476	12.31 (8)
NLLA + G_{21}	-2.25 – -0.88	0.13955	0.00093	0.00068	37.52 (11)
R match	-3.00 – -1.00	0.13079	0.00037	0.00132	134.74 (16)
lnR match	-3.00 – -0.88	0.12448	0.00033	0.00121	97.00 (17)
mod ln R match	-3.00 – -0.88	0.13197	0.00038	0.00135	143.64 (17)
ln R match +mod y	-3.00 – -1.00	0.12450	0.00033	0.00123	90.35 (16)
NLLA + mod y	-3.00 – -1.38	0.14351	0.00037	0.00220	364.13 (13)
NLLA + G_{21} + mod y	-2.25 – -1.38	0.13971	0.00093	0.00070	18.96 (7)
mod ln R + mod y	-3.00 – -0.88	0.13196	0.00038	0.00134	142.83 (17)

Table 5.9: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 91 GeV dataset using the Cambridge algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-2.88 – -1.38	0.14925	0.00041	0.00313	609.10 (12)
$\mathcal{O}(\alpha_s^2)$	-2.25 – -1.25	0.11938	0.00101	0.00280	10.52 (8)
NLLA + G_{21}	-2.12 – -0.88	0.14967	0.00108	0.00166	32.37 (10)
R match	-3.00 – -0.88	0.13229	0.00033	0.00086	100.75 (17)
lnR match	-3.00 – -1.00	0.12354	0.00028	0.00094	78.06 (16)
mod ln R match	-3.00 – -0.88	0.13038	0.00032	0.00098	106.60 (17)
ln R match +mod y	-3.00 – -1.12	0.12355	0.00028	0.00095	73.52 (15)
NLLA + mod y	-3.00 – -1.38	0.14636	0.00034	0.00302	733.95 (13)
NLLA + G_{21} + mod y	-2.12 – -0.88	0.14964	0.00107	0.00173	16.76 (10)
mod ln R + mod y	-3.00 – -1.00	0.13039	0.00032	0.00095	99.36 (16)

Table 5.10: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 91 GeV dataset using the Durham algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	$\chi^2/9$ dof
NLLA	-3.25 – -1.25	0.1215	0.00058	0.00232	10.07
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10413	0.00054	0.00151	9.38
NLLA + G_{21}	-3.25 – -1.25	0.12149	0.00057	0.00287	7.84
R match	-3.50 – -0.50	0.11414	0.00047	0.00310	16.75
lnR match	-3.50 – -0.50	0.10958	0.00043	0.00226	14.44
mod ln R match	-3.50 – -0.50	0.11493	0.00048	0.00262	17.26
ln R match + mod y	-3.50 – -0.50	0.10957	0.00043	0.00226	14.40
NLLA + mod y	-3.50 – -1.25	0.12501	0.00047	0.00281	17.65
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12179	0.00047	0.00287	8.10
mod ln R + mod y	-3.50 – -0.50	0.11494	0.00048	0.00263	17.16

Table 5.11: Statistically weighted average of α_s fit values from the individual LEP2 data sample fits, with statistical and systematic errors for several QCD parameterizations of the average jet rate, using the Cambridge algorithm over the stated fit range. The χ^2 compares the statistically weighted average to the fit value from the individual LEP2 datasets.

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	$\chi^2/9$ dof
NLLA	-3.25 – -1.25	0.13379	0.00049	0.00369	71.61
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10555	0.00043	0.00104	14.73
NLLA + G_{21}	-3.25 – -1.25	0.12882	0.00048	0.00359	23.50
R match	-3.50 – -0.50	0.11805	0.00037	0.00297	25.99
lnR match	-3.25 – -0.50	0.11151	0.00041	0.00260	19.00
mod ln R match	-3.25 – -0.50	0.11657	0.00046	0.00232	21.69
ln R match + mod y	-3.25 – -0.50	0.11150	0.00041	0.00260	19.07
NLLA + mod y	-3.50 – -1.25	0.13114	0.00037	0.00379	159.678
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12704	0.00037	0.00364	54.50
mod ln R + mod y	-3.25 – -0.50	0.11658	0.00046	0.00232	21.56

Table 5.12: Statistically weighted average of α_s fit values from the individual LEP2 data sample fits, with statistical and systematic errors for several QCD parameterizations of the average jet rate, using the Durham algorithm over the fit range. The χ^2 compares the statistically weighted average to the fit value from the individual LEP2 datasets.

The χ^2 values in Tables 5.11 and 5.12 compare the values of the statistically weighted average α_s value to that of the α_s value determined for the individual LEP2 energies. The χ^2 are somewhat higher than observed for the D_2 distribution but in general are

acceptable values. The exceptional cases being the NLLA predictions using the Durham algorithm. A study of the individual LEP2 α_s values showed the values distributed as expected about the statistically weighted mean, with much of the contribution to the large χ^2 attributable to the high statistical sample at 189 GeV.

	Durham		Cambridge	
	91 GeV	LEP2	91 GeV	LEP2
α_s	.12354	.11151	.12448	.10958
Tracking	0.00025	0.00048	0.00009	0.00029
Containment	0.00006	0.00017	0.00000	0.00040
ISR	—	0.00002	—	0.00002
$\mathcal{L}_{qq\ell\nu}$	—	0.00003	—	0.00003
$\mathcal{L}_{qqqq}$	—	0.00034	—	0.00038
Backgrounds	—	0.00019	—	0.00017
Hadronization	0.00003	0.00084	0.00112	0.00118
Fit Range	0.00090	0.00049	0.00045	0.00089
Theory (x_μ)	0.00306	0.00232	0.00247	0.00158
Statistical	0.00028	0.00041	0.00033	0.00043
Total Syst.	0.00320	0.00260	0.00275	0.00226
Total Error	0.00321	0.00263	0.00277	0.00230

Table 5.13: Breakdown of systematic errors on the average jet rate distribution $\ln R$ fit values of α_s for the 91 GeV and LEP2 data samples.

As in the D_2 fits the largest source of systematic error is due to the variation of the renormalization scale parameter. However, in the case of the average jet rates, the relative size of the x_μ variation is similar between the 91 GeV data sample and the LEP2 data sample. The difference in the fitted α_s values between the 91 GeV data sample and the LEP2 data sample corresponds to a 2.89 sigma difference for the Durham algorithm and a 4.61 sigma difference for the Cambridge algorithm, assuming no correlations between any of the systematic errors. This difference indicates that the value of α_s changes as the centre-of-mass energy is increased from the Z^0 peak to LEP2 energies.

5.5 Running of α_s

The dependence of α_s on centre-of-mass energy can be further demonstrated by comparing the values of α_s determined for the 91 GeV and the LEP2 datasets with values determined using similar techniques at lower energies. Fig 5.36 and 5.37 shows the 91 GeV and the LEP2 α_s values determined using the $\ln R$ matching scheme from the D_2 distribution, with previously determined OPAL and JADE [63] values. The fit values for the average jet rates using the $\ln R$ matching scheme compared to lower energy values (also from [63]) are shown in Fig 5.38 and 5.39 for Durham and Cambridge, respectively. In the figures the fits from the 91 GeV and LEP2 data samples as well as the lower energy points are compared to the 3-loop determination of α_s based on the world average value for $\Lambda = 216^{+25}_{-24}$ MeV.

Using the values of α_s determined for the 91 GeV and LEP2 data samples, along with the previously determined lower energy values, a fit to the 3-loop α_s (Eqn 2.20) expectation can be performed to determine the value of $\alpha_s(M_{Z^0})$. The results of the fit are summarized in Table 5.14

	D_2^C	D_2^D	$\langle N \rangle^C$	$\langle N \rangle^D$
$\alpha_s(M_{Z^0})$	0.11541	0.12078	0.12601	0.12575
total error	0.00277	0.00232	0.00289	0.00302
$\chi^2 / (5 \text{ dof})$	4.36	2.82	4.78	10.44

Table 5.14: Results of the fits of a 3-loop α_s expectation to determine $\alpha_s(M_{Z^0})$ from the α_s values determined from the Durham and Cambridge D_2 and $\langle N \rangle$ distributions using the 91 GeV and LEP2 datasets as well as the values from [63].

Figure 5.36: Running of α_s for the D_2 distribution and the $\ln R$ matching prediction with the Cambridge algorithm. Open points correspond to α_s values determined in [63], and closed points correspond to the new determinations of α_s . The solid line represents the 3-loop α_s expectation based on the value of $\Lambda = 216^{+25}_{-24}$ MeV from [2]. The dashed lines correspond to the expected error on the 3-loop α_s expectation based on the error on Λ .

Figure 5.37: Running of α_s for the D_2 distribution and the $\ln R$ matching prediction with the Durham algorithm. Open points correspond to α_s values determined in [63], and closed points correspond to the new determinations of α_s . The solid line represents the 3-loop α_s expectation based on the value of $\Lambda = 216^{+25}_{-24}$ MeV from [2]. The dashed lines correspond to the expected error on the 3-loop α_s expectation based on the error on Λ .

Figure 5.38: Running of α_s for the average jet rate distribution and the $\ln R$ matching prediction with the Cambridge algorithm. Open points correspond to α_s values determined in [63], and closed points correspond to the new determinations of α_s . The solid line represents the 3-loop α_s expectation based on the value of $\Lambda = 216^{+25}_{-24}$ MeV from [2]. The dashed lines correspond to the expected error on the 3-loop α_s expectation based on the error on Λ .

Figure 5.39: Running of α_s for the average jet rate distribution and the $\ln R$ matching prediction with the Durham algorithm. Open points correspond to α_s values determined in [63], and closed points correspond to the new determinations of α_s . The solid line represents the 3-loop α_s expectation based on the value of $\Lambda = 216^{+25}_{-24}$ MeV from [2]. The dashed lines correspond to the expected error on the 3-loop α_s expectation based on the error on Λ .

Chapter 6

Summary and Conclusion

The data from 10 LEP2 high energy points, ranging from 161 GeV to 207 GeV, and one energy point at 91 GeV from several Z^0 calibration runs have been analyzed using the OPAL detector. The analysis also investigated the cumulative sample of the 10 LEP2 energy points. This combined sample provided a large statistical data sample corresponding to 706.4pb^{-1} with a luminosity weighted centre-of-mass energy of 195.7 GeV.

This data has been used to determine the n -jet fractions, the differential n -jet rates and the average jet rates for each of the energies and the LEP2 combined sample. The different jet multiplicity distributions have been compared to PYTHIA, HERWIG, and ARIADNE Monte Carlo expectations. This allows a comparison of the different underlying models of hadronization.

Hadron level n -jet fractions have been determined using five separate jet clustering algorithms, the R and ε variants of the CONE algorithm, the JADE algorithm and two $k_{\perp}$ algorithms, Durham and Cambridge. In the case of the CONE algorithms, measurements of the n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$ were presented. In the case of the JADE, Durham and Cambridge algorithms measurements for the 2-, 3-, 4-, and 5-jet fractions as a function of the jet resolution parameter were presented. In all cases there is generally good agreement between the measured jet fractions and the Monte Carlo expectations. However, at all energies there is a slight deficit in the

3-jet fraction with respect to the PYTHIA expectation accompanied by a slight excess in the 2-jet fraction.

The 3-jet fraction using the JADE, Durham and Cambridge algorithms at a constant value y_{cut} ($y_{\text{cut}}=0.08$ for JADE and $y_{\text{cut}}=0.01$ for Durham and Cambridge) as a function of centre-of-mass energy was used to determine a value of Λ_{QCD} , which in turn was used to determine a value of $\alpha_s(M_{Z^0})$. The value of Λ_{QCD} was obtained from fits of the JADE, Durham and Cambridge R_3 values to an $\mathcal{O}(\alpha_s^2)$ prediction and the Durham and Cambridge R_3 values to a Next-to-Leading-Logarithmic Approximation. The predictions fit the data extremely well with reduced $\chi^2 < 1$ for the Durham and Cambridge fits and a $\chi^2/dof = 18/10$ for the JADE fit. Due to the lack of low energy determinations of R_3 using Durham and Cambridge the $\mathcal{O}(\alpha_s^2)$ and NLLA fits have a relatively small lever arm. This, together with the relatively large errors on the existing lower energy measurements, makes making any definitive statement about the Durham and Cambridge fits difficult. As it stands, there is a clear inconsistency between the NLLA determinations of Λ_{QCD} and the determinations using the $\mathcal{O}(\alpha_s^2)$ prediction and the expectation from the world average value. The influence of the renormalization scale dependence on the NLLA and $\mathcal{O}(\alpha_s^2)$ predictions was not investigated and hence could lead to an underestimation of the total error.

Hadron level determinations of the differential n -jet rates have been made for the Durham and Cambridge algorithms. There is generally good agreement between the measurement D_n values and the expectation from PYTHIA and HERWIG (and ARIADNE where available). The differential 2-jet rates, D_2 , have been used to determine the value of $\alpha_s(Q)$ for each LEP2 energy point and the 91 GeV sample. The determinations were carried out by fitting the parton level data distributions to NLLA, $\mathcal{O}(\alpha_s^2)$ and R and $\ln R$ matching predictions, as well as their modifications, over an appropriate range of y_{cut} , with $\alpha_s(Q)$ as the fit parameter. In the case of the $\mathcal{O}(\alpha_s^2)$ fits, the renormalization scale parameter was also allowed to vary. The $\mathcal{O}(\alpha_s^2)$ predictions tend to favour small values of x_μ . All fits returned reasonable values for χ^2/dof , indicating the validity of the QCD predictions within the given fit ranges.

It is expected that there is only a small variation in $\alpha_s(Q)$ between each of the

LEP2 datasets. The LEP2 data points were combined to produce a fit value of α_s at a scale corresponding to a luminosity weighted centre-of-mass energy of 195.7 GeV. This was done in two separate ways, first the data was combined into a single dataset, upon which the α_s fits were performed. Second, the fit α_s values from each energy dataset were used to produce a statistically weighted average α_s value. The α_s values returned from each method were very similar, indicating that the methods were equivalent in producing a value of α_s for the combined LEP2 samples. Further, the small χ^2 values determined by comparing the statistically weighted average of α_s to the individually determined values indicates that $\alpha_s(Q)$ is constant, within statistical errors across the LEP2 energy range.

There is fairly good agreement of the fit values of α_s between the various predictions for both the 91 GeV and LEP2 datasets. In all cases the NLLA predictions (NLLA + modifications) return somewhat larger values of α_s than the other predictions. A comparison between the $\ln R$ matching values, which implicitly include subleading terms, and the modified R matching values, which include subleading terms not present in the R matching prediction, show excellent agreement within errors.

The running of α_s is demonstrated in the comparison of the 91 GeV fit value using the $\ln R$ matching scheme to that of the statistically weighted average of all the individual LEP2 values. In the case of the Durham algorithm there is a separation of 3.03 standard deviations between the two values of α_s . The separation between values when using the Cambridge algorithm is 3.04 standard deviations. The determination of the separation was calculated assuming no correlations between errors. If correlations were taken into account, the separation would certainly become more significant.

The largest source of systematic error in the determination of α_s for the 91 GeV and LEP2 data samples was the variation of the renormalization scale parameter, x_μ . In particular, the size of this contribution is always due to the upward variation of x_μ to a value of 2. With the variation of x_μ becoming the largest systematic, further study may be required in the choice of these variations, which is based currently on convention.

Hadron level determinations of the average jet rates have also been made for the 91 GeV, the 10 high energy, and the combined LEP2 data samples using the Cambridge

and Durham algorithms. The average jet rate in all cases show good agreement with expectations from the PYTHIA and HERWIG (and ARIADNE for the high energy samples with $\sqrt{s} > 172$ GeV) Monte Carlo samples. The average jet rates, at the parton level, have also been used to determine values of α_s by performing fits to NLLA, $\mathcal{O}(\alpha_s^2)$ and matching predictions.

The determination of α_s for the combined LEP2 sample was not made due to the complexity required in a determination of the bin-to-bin correlations for this sample. However, the comparison of the returned α_s values between the LEP2 sample and the values determined from the statistically weighted average of the individual LEP2 energy points for the D_2 distributions indicates the two determinations are equivalent. As a consequence only the statistically weighted α_s values are determined for the average jet rate distributions.

There is a clear discrepancy between values of α_s for the 91 GeV and the LEP2 sample determined using the NLLA predictions and other predictions. This is not unexpected as previous analyses [63] have noted difficulties with the NLLA fits. The matching predictions, however, are generally consistent with each other within total errors over the given fit ranges.

The χ^2 values for the fits to the 91 GeV are quite large due in part to the small statistical error on each bin. It was determined that there was no significant decrease on the overall fit quality by decreasing the size of the fit range, and it is expected that this value of α_s will be representative of the true α_s value.

As for the D_2 distribution, the running of $\alpha_s(Q)$ is demonstrated by comparing the values of α_s returned from the $\ln R$ fit for the 91 GeV and the LEP2 data samples. The two values of α_s are separated by 2.89 standard deviations using the Durham algorithm and 4.61 standard deviations when using the Cambridge algorithm. This difference indicates that the value of $\alpha_s(Q)$ does indeed decrease with increasing centre-of-mass energy.

The running of α_s was further demonstrated by comparing values determined from the $\ln R$ matching fit for the D_2 and average jet rate distributions along with lower energy $\ln R$ predictions with a 3-loop expectation for $\alpha_s(Q)$. In general there is good

agreement between the fitted points and the $\alpha_s(Q)$ expectation within the given total errors. A fit using these values to the 3-loop expectation was performed to determine values of $\alpha_s(M_{Z^0})$

$$D_2^D : \alpha_s(M_{Z^0}) = 0.11541 \pm 0.00277$$

$$D_2^C : \alpha_s(M_{Z^0}) = 0.12078 \pm 0.00232$$

$$\langle N \rangle^D : \alpha_s(M_{Z^0}) = 0.12601 \pm 0.00289$$

$$\langle N \rangle^C : \alpha_s(M_{Z^0}) = 0.12575 \pm 0.00302$$

The values of $\alpha_s(M_{Z^0})$ from the D_2 distributions agree very well with the world average value of $0.11790^{+0.00126}_{-0.00204}$. However the values determined using the average jet rates are about 2.5 standard deviations above the world average. At present the reason for the larger values of α_s for the average jet rates is not fully understood. As noted in Section 2.7.3, the $\ln R$ matching prediction for the average jet rates does not include subleading terms; the larger returned value of α_s may indicate the need for further study of these subleading terms.

Appendix A

Jet Rates

In this section tables of values and plots of jet fractions for the CONE R and ε variants, JADE, Durham and Cambridge algorithms are presented. The plots and tables represent the 10 LEP2 energy points. The last set of tables correspond to the jet fraction values for the 91 GeV and LEP2 datasets, whose plots are found in Section 5.1. A table of parton level values for R_3 for the 91 GeV and LEP2 datasets using JADE with $y_{\text{cut}}=0.08$ and Durham and Cambridge with $y_{\text{cut}}=0.01$ used in Section 5.2 is also presented.

A.1 Numerical values for R_n

161 GeV Data Sample

$\varepsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4919 \pm 0.0303 \pm 0.0361$	$0.3311 \pm 0.0286 \pm 0.0358$	$0.1807 \pm 0.0254 \pm 0.0293$
6.00	$0.6661 \pm 0.0278 \pm 0.0296$	$0.2482 \pm 0.0270 \pm 0.0326$	$0.0971 \pm 0.0222 \pm 0.0276$
10.00	$0.7499 \pm 0.0251 \pm 0.0291$	$0.2049 \pm 0.0257 \pm 0.0339$	$0.0529 \pm 0.0198 \pm 0.0313$
14.00	$0.7800 \pm 0.0239 \pm 0.0288$	$0.1933 \pm 0.0255 \pm 0.0325$	$0.0370 \pm 0.0199 \pm 0.0360$
18.00	$0.7995 \pm 0.0231 \pm 0.0269$	$0.1789 \pm 0.0251 \pm 0.0308$	$0.0555 \pm 0.0295 \pm 0.0602$
22.00	$0.8260 \pm 0.0219 \pm 0.0318$	$0.1728 \pm 0.0253 \pm 0.0379$	$-0.0015 \pm 0.0006 \pm 0.0015$
25.50	$0.8671 \pm 0.0196 \pm 0.0249$	$0.1318 \pm 0.0228 \pm 0.0311$	$-0.0006 \pm 0.0003 \pm 0.0005$

Table A.1: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 161 GeV, as a function of the energy cutoff, ε , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.5299 \pm 0.0303 \pm 0.0375$	$0.2814 \pm 0.0268 \pm 0.0288$	$0.1921 \pm 0.0266 \pm 0.0346$
0.50	$0.6236 \pm 0.0288 \pm 0.0337$	$0.2613 \pm 0.0266 \pm 0.0324$	$0.1219 \pm 0.0244 \pm 0.0299$
0.70	$0.6963 \pm 0.0269 \pm 0.0290$	$0.2311 \pm 0.0265 \pm 0.0302$	$0.0832 \pm 0.0215 \pm 0.0251$
0.90	$0.7485 \pm 0.0255 \pm 0.0293$	$0.2337 \pm 0.0273 \pm 0.0323$	$0.0211 \pm 0.0112 \pm 0.0168$
1.10	$0.8565 \pm 0.0204 \pm 0.0296$	$0.1390 \pm 0.0229 \pm 0.0351$	$0.0046 \pm 0.0049 \pm 0.0073$
1.30	$0.9335 \pm 0.0140 \pm 0.0194$	$0.0678 \pm 0.0173 \pm 0.0241$	$0.0000 \pm 0.0000 \pm 0.0000$
1.50	$0.9935 \pm 0.0036 \pm 0.0057$	$0.0067 \pm 0.0073 \pm 0.0122$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.2: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 161 GeV, as a function of the cone radius, R , with a constant energy cutoff $\varepsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.1881 \pm 0.0237 \pm 0.0293$	$0.4894 \pm 0.0300 \pm 0.0355$	$0.2535 \pm 0.0268 \pm 0.0417$	$0.0565 \pm 0.0171 \pm 0.0306$
-2.06	$0.3032 \pm 0.0276 \pm 0.0350$	$0.4966 \pm 0.0297 \pm 0.0499$	$0.1711 \pm 0.0249 \pm 0.0400$	$0.0401 \pm 0.0180 \pm 0.0287$
-1.90	$0.3965 \pm 0.0291 \pm 0.0403$	$0.4482 \pm 0.0297 \pm 0.0563$	$0.1548 \pm 0.0258 \pm 0.0425$	$0.0111 \pm 0.0141 \pm 0.0209$
-1.70	$0.4917 \pm 0.0295 \pm 0.0364$	$0.4213 \pm 0.0300 \pm 0.0441$	$0.1051 \pm 0.0258 \pm 0.0366$	$0.0006 \pm 0.0003 \pm 0.0011$
-1.52	$0.6067 \pm 0.0286 \pm 0.0401$	$0.3431 \pm 0.0299 \pm 0.0429$	$0.0769 \pm 0.0257 \pm 0.0385$	$0.0000 \pm 0.0000 \pm 0.0002$
-1.40	$0.6793 \pm 0.0274 \pm 0.0338$	$0.2933 \pm 0.0291 \pm 0.0356$	$0.0540 \pm 0.0258 \pm 0.0351$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7115 \pm 0.0265 \pm 0.0375$	$0.2828 \pm 0.0293 \pm 0.0457$	$0.0108 \pm 0.0120 \pm 0.0251$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7637 \pm 0.0247 \pm 0.0281$	$0.2346 \pm 0.0280 \pm 0.0337$	$0.0014 \pm 0.0006 \pm 0.0229$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8010 \pm 0.0232 \pm 0.0282$	$0.1982 \pm 0.0265 \pm 0.0330$	$0.0007 \pm 0.0004 \pm 0.0016$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8360 \pm 0.0216 \pm 0.0260$	$0.1634 \pm 0.0246 \pm 0.0300$	$0.0004 \pm 0.0004 \pm 0.0008$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8603 \pm 0.0198 \pm 0.0265$	$0.1414 \pm 0.0241 \pm 0.0322$	$0.0000 \pm 0.0000 \pm 0.0001$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.9053 \pm 0.0165 \pm 0.0207$	$0.0942 \pm 0.0205 \pm 0.0252$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9331 \pm 0.0136 \pm 0.0253$	$0.0650 \pm 0.0181 \pm 0.0338$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9585 \pm 0.0107 \pm 0.0181$	$0.0388 \pm 0.0145 \pm 0.0248$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.3: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 161 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0239 $\pm$ 0.0093 $\pm$ 0.0264	0.0257 $\pm$ 0.0107 $\pm$ 0.0153	0.0144 $\pm$ 0.0074 $\pm$ 0.0235	0.0193 $\pm$ 0.0089 $\pm$ 0.0217
-4.75	0.0244 $\pm$ 0.0095 $\pm$ 0.0269	0.0253 $\pm$ 0.0105 $\pm$ 0.0149	0.0144 $\pm$ 0.0074 $\pm$ 0.0254	0.0378 $\pm$ 0.0121 $\pm$ 0.0186
-4.50	0.0260 $\pm$ 0.0101 $\pm$ 0.0285	0.0291 $\pm$ 0.0111 $\pm$ 0.0135	0.0497 $\pm$ 0.0138 $\pm$ 0.0403	0.0685 $\pm$ 0.0160 $\pm$ 0.0306
-4.25	0.0414 $\pm$ 0.0132 $\pm$ 0.0305	0.0509 $\pm$ 0.0147 $\pm$ 0.0317	0.1476 $\pm$ 0.0233 $\pm$ 0.0315	0.1499 $\pm$ 0.0229 $\pm$ 0.0635
-4.00	0.0719 $\pm$ 0.0178 $\pm$ 0.0451	0.1419 $\pm$ 0.0233 $\pm$ 0.0429	0.2381 $\pm$ 0.0278 $\pm$ 0.0346	0.1780 $\pm$ 0.0232 $\pm$ 0.0292
-3.75	0.1399 $\pm$ 0.0237 $\pm$ 0.0407	0.2219 $\pm$ 0.0271 $\pm$ 0.0549	0.2648 $\pm$ 0.0272 $\pm$ 0.0368	0.1491 $\pm$ 0.0206 $\pm$ 0.0310
-3.50	0.2136 $\pm$ 0.0273 $\pm$ 0.0360	0.2967 $\pm$ 0.0283 $\pm$ 0.0300	0.2545 $\pm$ 0.0263 $\pm$ 0.0295	0.1150 $\pm$ 0.0188 $\pm$ 0.0288
-3.25	0.3038 $\pm$ 0.0294 $\pm$ 0.0549	0.2908 $\pm$ 0.0275 $\pm$ 0.0669	0.2546 $\pm$ 0.0260 $\pm$ 0.0336	0.0965 $\pm$ 0.0181 $\pm$ 0.0233
-3.00	0.3683 $\pm$ 0.0300 $\pm$ 0.0445	0.3266 $\pm$ 0.0279 $\pm$ 0.0460	0.2162 $\pm$ 0.0252 $\pm$ 0.0285	0.0644 $\pm$ 0.0170 $\pm$ 0.0193
-2.75	0.4617 $\pm$ 0.0302 $\pm$ 0.0470	0.3442 $\pm$ 0.0284 $\pm$ 0.0465	0.1512 $\pm$ 0.0229 $\pm$ 0.0260	0.0430 $\pm$ 0.0167 $\pm$ 0.0256
-2.50	0.5445 $\pm$ 0.0299 $\pm$ 0.0340	0.3253 $\pm$ 0.0279 $\pm$ 0.0385	0.1222 $\pm$ 0.0232 $\pm$ 0.0360	0.0058 $\pm$ 0.0093 $\pm$ 0.0270
-2.25	0.6159 $\pm$ 0.0290 $\pm$ 0.0336	0.2956 $\pm$ 0.0275 $\pm$ 0.0387	0.0942 $\pm$ 0.0233 $\pm$ 0.0324	0.0024 $\pm$ 0.0007 $\pm$ 0.0123
-2.00	0.7038 $\pm$ 0.0269 $\pm$ 0.0285	0.2364 $\pm$ 0.0263 $\pm$ 0.0326	0.0854 $\pm$ 0.0273 $\pm$ 0.0379	0.0020 $\pm$ 0.0015 $\pm$ 0.0035
-1.75	0.7782 $\pm$ 0.0244 $\pm$ 0.0255	0.1949 $\pm$ 0.0253 $\pm$ 0.0276	0.0618 $\pm$ 0.0333 $\pm$ 0.0488	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.50	0.8364 $\pm$ 0.0216 $\pm$ 0.0253	0.1601 $\pm$ 0.0242 $\pm$ 0.0327	0.0019 $\pm$ 0.0009 $\pm$ 0.0605	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9103 $\pm$ 0.0162 $\pm$ 0.0200	0.0874 $\pm$ 0.0196 $\pm$ 0.0236	0.0007 $\pm$ 0.0008 $\pm$ 0.0012	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9583 $\pm$ 0.0107 $\pm$ 0.0168	0.0392 $\pm$ 0.0147 $\pm$ 0.0228	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9897 $\pm$ 0.0052 $\pm$ 0.0108	0.0082 $\pm$ 0.0075 $\pm$ 0.0144	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	0.9999 $\pm$ 0.0004 $\pm$ 0.0004	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.4: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 161 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-4.75	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0024 $\pm$ 0.0025 $\pm$ 0.0027
-4.50	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0076 $\pm$ 0.0056 $\pm$ 0.0109	0.0133 $\pm$ 0.0060 $\pm$ 0.0137
-4.25	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0096 $\pm$ 0.0057 $\pm$ 0.0062	0.0274 $\pm$ 0.0098 $\pm$ 0.0106	0.0828 $\pm$ 0.0170 $\pm$ 0.0209
-4.00	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0408 $\pm$ 0.0130 $\pm$ 0.0186	0.1017 $\pm$ 0.0191 $\pm$ 0.0197	0.2032 $\pm$ 0.0256 $\pm$ 0.0382
-3.75	0.0217 $\pm$ 0.0098 $\pm$ 0.0123	0.1164 $\pm$ 0.0206 $\pm$ 0.0272	0.2022 $\pm$ 0.0256 $\pm$ 0.0323	0.2295 $\pm$ 0.0262 $\pm$ 0.0379
-3.50	0.0765 $\pm$ 0.0178 $\pm$ 0.0348	0.2028 $\pm$ 0.0260 $\pm$ 0.0379	0.2888 $\pm$ 0.0278 $\pm$ 0.0439	0.2099 $\pm$ 0.0243 $\pm$ 0.0401
-3.25	0.1528 $\pm$ 0.0239 $\pm$ 0.0346	0.3111 $\pm$ 0.0294 $\pm$ 0.0382	0.2838 $\pm$ 0.0265 $\pm$ 0.0515	0.1634 $\pm$ 0.0219 $\pm$ 0.0312
-3.00	0.2725 $\pm$ 0.0294 $\pm$ 0.0345	0.3655 $\pm$ 0.0288 $\pm$ 0.0346	0.2124 $\pm$ 0.0241 $\pm$ 0.0287	0.1108 $\pm$ 0.0196 $\pm$ 0.0266
-2.75	0.4010 $\pm$ 0.0310 $\pm$ 0.0563	0.3622 $\pm$ 0.0284 $\pm$ 0.0454	0.1906 $\pm$ 0.0238 $\pm$ 0.0330	0.0360 $\pm$ 0.0133 $\pm$ 0.0228
-2.50	0.4871 $\pm$ 0.0305 $\pm$ 0.0338	0.3406 $\pm$ 0.0279 $\pm$ 0.0437	0.1617 $\pm$ 0.0246 $\pm$ 0.0520	0.0102 $\pm$ 0.0101 $\pm$ 0.0198
-2.25	0.5773 $\pm$ 0.0296 $\pm$ 0.0374	0.3196 $\pm$ 0.0277 $\pm$ 0.0334	0.1069 $\pm$ 0.0239 $\pm$ 0.0278	0.0044 $\pm$ 0.0012 $\pm$ 0.0027
-2.00	0.6799 $\pm$ 0.0276 $\pm$ 0.0296	0.2477 $\pm$ 0.0265 $\pm$ 0.0310	0.1055 $\pm$ 0.0299 $\pm$ 0.0352	0.0024 $\pm$ 0.0018 $\pm$ 0.0036
-1.75	0.7544 $\pm$ 0.0253 $\pm$ 0.0264	0.2136 $\pm$ 0.0260 $\pm$ 0.0286	0.0786 $\pm$ 0.0378 $\pm$ 0.0478	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.50	0.8224 $\pm$ 0.0224 $\pm$ 0.0248	0.1741 $\pm$ 0.0249 $\pm$ 0.0304	0.0017 $\pm$ 0.0008 $\pm$ 0.0570	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9054 $\pm$ 0.0165 $\pm$ 0.0213	0.0926 $\pm$ 0.0203 $\pm$ 0.0257	0.0001 $\pm$ 0.0001 $\pm$ 0.0002	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9605 $\pm$ 0.0101 $\pm$ 0.0160	0.0350 $\pm$ 0.0142 $\pm$ 0.0215	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9895 $\pm$ 0.0052 $\pm$ 0.0104	0.0083 $\pm$ 0.0076 $\pm$ 0.0145	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	1.0000 $\pm$ 0.0004 $\pm$ 0.0004	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.5: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 161 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.1: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 161$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

172 GeV Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.5088 \pm 0.0342 \pm 0.0379$	$0.3229 \pm 0.0325 \pm 0.0375$	$0.1707 \pm 0.0298 \pm 0.0352$
6.00	$0.6492 \pm 0.0316 \pm 0.0350$	$0.2894 \pm 0.0334 \pm 0.0361$	$0.0621 \pm 0.0217 \pm 0.0255$
10.00	$0.7400 \pm 0.0284 \pm 0.0357$	$0.2134 \pm 0.0311 \pm 0.0406$	$0.0457 \pm 0.0224 \pm 0.0349$
14.00	$0.7801 \pm 0.0266 \pm 0.0338$	$0.1919 \pm 0.0304 \pm 0.0377$	$0.0210 \pm 0.0211 \pm 0.0297$
18.00	$0.8173 \pm 0.0251 \pm 0.0355$	$0.1590 \pm 0.0287 \pm 0.0411$	$0.0307 \pm 0.0259 \pm 0.0374$
22.00	$0.8393 \pm 0.0237 \pm 0.0333$	$0.1472 \pm 0.0288 \pm 0.0414$	$0.0234 \pm 0.0284 \pm 0.0519$
25.50	$0.8753 \pm 0.0215 \pm 0.0277$	$0.1122 \pm 0.0260 \pm 0.0355$	$0.0351 \pm 0.0403 \pm 0.0840$

Table A.6: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 172 GeV, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.5245 \pm 0.0342 \pm 0.0473$	$0.2918 \pm 0.0311 \pm 0.0420$	$0.1828 \pm 0.0312 \pm 0.0342$
0.50	$0.6199 \pm 0.0321 \pm 0.0391$	$0.2691 \pm 0.0319 \pm 0.0435$	$0.1087 \pm 0.0275 \pm 0.0361$
0.70	$0.6621 \pm 0.0313 \pm 0.0446$	$0.2895 \pm 0.0334 \pm 0.0498$	$0.0473 \pm 0.0202 \pm 0.0253$
0.90	$0.7262 \pm 0.0294 \pm 0.0355$	$0.2492 \pm 0.0331 \pm 0.0369$	$0.0332 \pm 0.0166 \pm 0.0333$
1.10	$0.8367 \pm 0.0242 \pm 0.0302$	$0.1540 \pm 0.0282 \pm 0.0333$	$0.0150 \pm 0.0116 \pm 0.0183$
1.30	$0.9224 \pm 0.0173 \pm 0.0188$	$0.0811 \pm 0.0217 \pm 0.0247$	$0.0001 \pm 0.0001 \pm 0.0001$
1.50	$0.9941 \pm 0.0046 \pm 0.0118$	$0.0061 \pm 0.0078 \pm 0.0188$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.7: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 172 GeV, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.2215 \pm 0.0274 \pm 0.0389$	$0.4696 \pm 0.0341 \pm 0.0377$	$0.2524 \pm 0.0319 \pm 0.0404$	$0.0550 \pm 0.0217 \pm 0.0256$
-2.06	$0.3338 \pm 0.0314 \pm 0.0331$	$0.4603 \pm 0.0341 \pm 0.0361$	$0.1802 \pm 0.0305 \pm 0.0409$	$0.0375 \pm 0.0230 \pm 0.0324$
-1.90	$0.4090 \pm 0.0328 \pm 0.0364$	$0.4626 \pm 0.0345 \pm 0.0495$	$0.1221 \pm 0.0283 \pm 0.0518$	$0.0064 \pm 0.0167 \pm 0.0240$
-1.70	$0.5305 \pm 0.0332 \pm 0.0396$	$0.4145 \pm 0.0349 \pm 0.0396$	$0.0463 \pm 0.0226 \pm 0.0381$	$0.0019 \pm 0.0008 \pm 0.0019$
-1.52	$0.6250 \pm 0.0324 \pm 0.0407$	$0.3533 \pm 0.0349 \pm 0.0476$	$0.0100 \pm 0.0151 \pm 0.0233$	$0.0002 \pm 0.0002 \pm 0.0003$
-1.40	$0.6785 \pm 0.0307 \pm 0.0355$	$0.3075 \pm 0.0351 \pm 0.0414$	$0.0084 \pm 0.0160 \pm 0.0238$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7288 \pm 0.0292 \pm 0.0329$	$0.2593 \pm 0.0339 \pm 0.0387$	$0.0228 \pm 0.0301 \pm 0.0485$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7690 \pm 0.0277 \pm 0.0324$	$0.2227 \pm 0.0328 \pm 0.0372$	$0.0389 \pm 0.0476 \pm 0.0860$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.7968 \pm 0.0264 \pm 0.0293$	$0.2032 \pm 0.0321 \pm 0.0356$	$0.0024 \pm 0.0012 \pm 0.0040$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8372 \pm 0.0241 \pm 0.0260$	$0.1602 \pm 0.0297 \pm 0.0315$	$0.0028 \pm 0.0030 \pm 0.0045$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8720 \pm 0.0215 \pm 0.0242$	$0.1249 \pm 0.0275 \pm 0.0296$	$0.0000 \pm 0.0000 \pm 0.0005$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.9146 \pm 0.0177 \pm 0.0329$	$0.0801 \pm 0.0234 \pm 0.0432$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9297 \pm 0.0161 \pm 0.0207$	$0.0683 \pm 0.0223 \pm 0.0283$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9419 \pm 0.0149 \pm 0.0175$	$0.0606 \pm 0.0212 \pm 0.0255$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.8: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 172 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-5.00	0.0176 $\pm$ 0.0091 $\pm$ 0.0206	0.0190 $\pm$ 0.0098 $\pm$ 0.0205	0.0281 $\pm$ 0.0119 $\pm$ 0.0144	0.0473 $\pm$ 0.0152 $\pm$ 0.0194
-4.75	0.0176 $\pm$ 0.0090 $\pm$ 0.0206	0.0186 $\pm$ 0.0096 $\pm$ 0.0195	0.0327 $\pm$ 0.0127 $\pm$ 0.0149	0.0609 $\pm$ 0.0170 $\pm$ 0.0252
-4.50	0.0241 $\pm$ 0.0110 $\pm$ 0.0233	0.0365 $\pm$ 0.0131 $\pm$ 0.0192	0.0854 $\pm$ 0.0206 $\pm$ 0.0373	0.1178 $\pm$ 0.0234 $\pm$ 0.0328
-4.25	0.0505 $\pm$ 0.0170 $\pm$ 0.0305	0.0981 $\pm$ 0.0221 $\pm$ 0.0316	0.1063 $\pm$ 0.0227 $\pm$ 0.0263	0.1707 $\pm$ 0.0268 $\pm$ 0.0305
-4.00	0.1034 $\pm$ 0.0239 $\pm$ 0.0364	0.1181 $\pm$ 0.0238 $\pm$ 0.0278	0.2045 $\pm$ 0.0288 $\pm$ 0.0339	0.1984 $\pm$ 0.0274 $\pm$ 0.0359
-3.75	0.1443 $\pm$ 0.0265 $\pm$ 0.0368	0.2605 $\pm$ 0.0324 $\pm$ 0.0497	0.2275 $\pm$ 0.0286 $\pm$ 0.0392	0.1626 $\pm$ 0.0251 $\pm$ 0.0681
-3.50	0.2319 $\pm$ 0.0306 $\pm$ 0.0520	0.3246 $\pm$ 0.0333 $\pm$ 0.0557	0.2280 $\pm$ 0.0287 $\pm$ 0.0344	0.1016 $\pm$ 0.0205 $\pm$ 0.0257
-3.25	0.3145 $\pm$ 0.0328 $\pm$ 0.0496	0.3542 $\pm$ 0.0330 $\pm$ 0.0366	0.1933 $\pm$ 0.0273 $\pm$ 0.0336	0.0913 $\pm$ 0.0215 $\pm$ 0.0246
-3.00	0.3829 $\pm$ 0.0334 $\pm$ 0.0444	0.3602 $\pm$ 0.0328 $\pm$ 0.0339	0.1609 $\pm$ 0.0266 $\pm$ 0.0575	0.0799 $\pm$ 0.0229 $\pm$ 0.0406
-2.75	0.4529 $\pm$ 0.0338 $\pm$ 0.0436	0.3580 $\pm$ 0.0326 $\pm$ 0.0439	0.1558 $\pm$ 0.0283 $\pm$ 0.0326	0.0333 $\pm$ 0.0186 $\pm$ 0.0287
-2.50	0.5449 $\pm$ 0.0334 $\pm$ 0.0411	0.3333 $\pm$ 0.0325 $\pm$ 0.0438	0.1114 $\pm$ 0.0274 $\pm$ 0.0348	0.0018 $\pm$ 0.0120 $\pm$ 0.0178
-2.25	0.6223 $\pm$ 0.0324 $\pm$ 0.0433	0.3018 $\pm$ 0.0325 $\pm$ 0.0463	0.0732 $\pm$ 0.0265 $\pm$ 0.0327	0.0078 $\pm$ 0.0022 $\pm$ 0.0203
-2.00	0.6909 $\pm$ 0.0307 $\pm$ 0.0371	0.2857 $\pm$ 0.0331 $\pm$ 0.0424	0.0034 $\pm$ 0.0120 $\pm$ 0.0309	0.0039 $\pm$ 0.0029 $\pm$ 0.0055
-1.75	0.7889 $\pm$ 0.0271 $\pm$ 0.0376	0.1913 $\pm$ 0.0303 $\pm$ 0.0422	0.0133 $\pm$ 0.0233 $\pm$ 0.0376	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.50	0.8584 $\pm$ 0.0229 $\pm$ 0.0328	0.1287 $\pm$ 0.0274 $\pm$ 0.0392	0.0259 $\pm$ 0.0315 $\pm$ 0.0579	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9212 $\pm$ 0.0172 $\pm$ 0.0294	0.0717 $\pm$ 0.0226 $\pm$ 0.0375	0.0000 $\pm$ 0.0000 $\pm$ 0.0013	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9511 $\pm$ 0.0136 $\pm$ 0.0176	0.0483 $\pm$ 0.0194 $\pm$ 0.0245	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9828 $\pm$ 0.0080 $\pm$ 0.0131	0.0186 $\pm$ 0.0134 $\pm$ 0.0233	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	0.9999 $\pm$ 0.0006 $\pm$ 0.0006	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.9: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 172 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-5.00	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-4.75	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0041
-4.50	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0052 $\pm$ 0.0038 $\pm$ 0.0056	0.0260 $\pm$ 0.0099 $\pm$ 0.0127
-4.25	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0035 $\pm$ 0.0036 $\pm$ 0.0066	0.0433 $\pm$ 0.0137 $\pm$ 0.0176	0.0829 $\pm$ 0.0192 $\pm$ 0.0291
-4.00	0.0000 $\pm$ 0.0000 $\pm$ 0.0071	0.0450 $\pm$ 0.0143 $\pm$ 0.0221	0.1211 $\pm$ 0.0233 $\pm$ 0.0359	0.1454 $\pm$ 0.0248 $\pm$ 0.0337
-3.75	0.0174 $\pm$ 0.0088 $\pm$ 0.0108	0.1276 $\pm$ 0.0246 $\pm$ 0.0451	0.1698 $\pm$ 0.0261 $\pm$ 0.0282	0.2458 $\pm$ 0.0304 $\pm$ 0.0307
-3.50	0.0853 $\pm$ 0.0203 $\pm$ 0.0422	0.2217 $\pm$ 0.0298 $\pm$ 0.0352	0.2076 $\pm$ 0.0281 $\pm$ 0.0405	0.2622 $\pm$ 0.0303 $\pm$ 0.0399
-3.25	0.1729 $\pm$ 0.0273 $\pm$ 0.0557	0.2994 $\pm$ 0.0323 $\pm$ 0.0480	0.2636 $\pm$ 0.0300 $\pm$ 0.0536	0.1450 $\pm$ 0.0243 $\pm$ 0.0482
-3.00	0.2650 $\pm$ 0.0316 $\pm$ 0.0620	0.3916 $\pm$ 0.0337 $\pm$ 0.0584	0.2121 $\pm$ 0.0281 $\pm$ 0.0300	0.0841 $\pm$ 0.0206 $\pm$ 0.0399
-2.75	0.3894 $\pm$ 0.0342 $\pm$ 0.0372	0.3767 $\pm$ 0.0327 $\pm$ 0.0422	0.1760 $\pm$ 0.0275 $\pm$ 0.0352	0.0574 $\pm$ 0.0215 $\pm$ 0.0291
-2.50	0.4815 $\pm$ 0.0340 $\pm$ 0.0420	0.3541 $\pm$ 0.0324 $\pm$ 0.0432	0.1530 $\pm$ 0.0293 $\pm$ 0.0515	0.0086 $\pm$ 0.0166 $\pm$ 0.0224
-2.25	0.5980 $\pm$ 0.0332 $\pm$ 0.0402	0.3117 $\pm$ 0.0321 $\pm$ 0.0393	0.0832 $\pm$ 0.0268 $\pm$ 0.0329	0.0138 $\pm$ 0.0255 $\pm$ 0.0386
-2.00	0.6689 $\pm$ 0.0316 $\pm$ 0.0343	0.2883 $\pm$ 0.0326 $\pm$ 0.0361	0.0371 $\pm$ 0.0252 $\pm$ 0.0410	0.0065 $\pm$ 0.0049 $\pm$ 0.0087
-1.75	0.7756 $\pm$ 0.0281 $\pm$ 0.0352	0.2054 $\pm$ 0.0307 $\pm$ 0.0407	0.0101 $\pm$ 0.0205 $\pm$ 0.0334	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.50	0.8326 $\pm$ 0.0247 $\pm$ 0.0268	0.1569 $\pm$ 0.0293 $\pm$ 0.0324	0.0376 $\pm$ 0.0463 $\pm$ 0.0745	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9113 $\pm$ 0.0182 $\pm$ 0.0209	0.0828 $\pm$ 0.0241 $\pm$ 0.0260	0.0000 $\pm$ 0.0000 $\pm$ 0.0012	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9508 $\pm$ 0.0136 $\pm$ 0.0184	0.0481 $\pm$ 0.0195 $\pm$ 0.0265	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9825 $\pm$ 0.0080 $\pm$ 0.0132	0.0188 $\pm$ 0.0135 $\pm$ 0.0240	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	1.0000 $\pm$ 0.0006 $\pm$ 0.0006	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.10: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 172 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.2: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 172$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

183 GeV Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4587 \pm 0.0165 \pm 0.0184$	$0.3748 \pm 0.0163 \pm 0.0227$	$0.1686 \pm 0.0147 \pm 0.0225$
6.00	$0.6444 \pm 0.0158 \pm 0.0208$	$0.2976 \pm 0.0162 \pm 0.0186$	$0.0576 \pm 0.0107 \pm 0.0215$
10.00	$0.7118 \pm 0.0148 \pm 0.0189$	$0.2545 \pm 0.0159 \pm 0.0208$	$0.0302 \pm 0.0098 \pm 0.0165$
14.00	$0.7596 \pm 0.0140 \pm 0.0180$	$0.2142 \pm 0.0153 \pm 0.0187$	$0.0287 \pm 0.0114 \pm 0.0184$
18.00	$0.7974 \pm 0.0133 \pm 0.0186$	$0.1824 \pm 0.0147 \pm 0.0194$	$0.0349 \pm 0.0159 \pm 0.0278$
22.00	$0.8247 \pm 0.0125 \pm 0.0185$	$0.1657 \pm 0.0147 \pm 0.0203$	$0.0208 \pm 0.0155 \pm 0.0303$
25.50	$0.8495 \pm 0.0117 \pm 0.0196$	$0.1465 \pm 0.0144 \pm 0.0222$	$0.0117 \pm 0.0128 \pm 0.0292$

Table A.11: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 183 GeV, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4703 \pm 0.0164 \pm 0.0301$	$0.3273 \pm 0.0158 \pm 0.0270$	$0.2036 \pm 0.0159 \pm 0.0210$
0.50	$0.5736 \pm 0.0162 \pm 0.0224$	$0.3240 \pm 0.0161 \pm 0.0215$	$0.1010 \pm 0.0132 \pm 0.0165$
0.70	$0.6672 \pm 0.0156 \pm 0.0207$	$0.2829 \pm 0.0160 \pm 0.0216$	$0.0490 \pm 0.0103 \pm 0.0154$
0.90	$0.7551 \pm 0.0143 \pm 0.0163$	$0.2287 \pm 0.0155 \pm 0.0182$	$0.0160 \pm 0.0062 \pm 0.0084$
1.10	$0.8304 \pm 0.0124 \pm 0.0172$	$0.1683 \pm 0.0142 \pm 0.0196$	$0.0046 \pm 0.0039 \pm 0.0059$
1.30	$0.9382 \pm 0.0078 \pm 0.0105$	$0.0610 \pm 0.0094 \pm 0.0124$	$0.0020 \pm 0.0033 \pm 0.0039$
1.50	$0.9938 \pm 0.0024 \pm 0.0034$	$0.0069 \pm 0.0047 \pm 0.0072$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.12: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 183 GeV, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.1955 \pm 0.0128 \pm 0.0177$	$0.4799 \pm 0.0164 \pm 0.0186$	$0.2676 \pm 0.0159 \pm 0.0238$	$0.0549 \pm 0.0113 \pm 0.0159$
-2.06	$0.2975 \pm 0.0148 \pm 0.0178$	$0.5161 \pm 0.0166 \pm 0.0193$	$0.1784 \pm 0.0151 \pm 0.0281$	$0.0009 \pm 0.0066 \pm 0.0140$
-1.90	$0.3711 \pm 0.0158 \pm 0.0185$	$0.5116 \pm 0.0167 \pm 0.0174$	$0.1127 \pm 0.0138 \pm 0.0192$	$0.0002 \pm 0.0057 \pm 0.0090$
-1.70	$0.4911 \pm 0.0163 \pm 0.0219$	$0.4505 \pm 0.0170 \pm 0.0232$	$0.0554 \pm 0.0123 \pm 0.0157$	$0.0045 \pm 0.0120 \pm 0.0233$
-1.52	$0.5989 \pm 0.0160 \pm 0.0253$	$0.3666 \pm 0.0171 \pm 0.0276$	$0.0494 \pm 0.0147 \pm 0.0237$	$0.0000 \pm 0.0000 \pm 0.0001$
-1.40	$0.6767 \pm 0.0153 \pm 0.0195$	$0.3092 \pm 0.0170 \pm 0.0208$	$0.0200 \pm 0.0130 \pm 0.0198$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7420 \pm 0.0143 \pm 0.0227$	$0.2498 \pm 0.0163 \pm 0.0232$	$0.0034 \pm 0.0107 \pm 0.0255$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7737 \pm 0.0137 \pm 0.0187$	$0.2223 \pm 0.0161 \pm 0.0209$	$0.0084 \pm 0.0117 \pm 0.0227$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8005 \pm 0.0131 \pm 0.0179$	$0.1979 \pm 0.0157 \pm 0.0203$	$0.0132 \pm 0.0145 \pm 0.0274$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8357 \pm 0.0121 \pm 0.0180$	$0.1619 \pm 0.0149 \pm 0.0210$	$0.0161 \pm 0.0180 \pm 0.0311$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8714 \pm 0.0110 \pm 0.0162$	$0.1286 \pm 0.0139 \pm 0.0185$	$0.0000 \pm 0.0000 \pm 0.0014$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.8998 \pm 0.0098 \pm 0.0151$	$0.1018 \pm 0.0130 \pm 0.0189$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9250 \pm 0.0086 \pm 0.0137$	$0.0771 \pm 0.0121 \pm 0.0178$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9445 \pm 0.0075 \pm 0.0143$	$0.0595 \pm 0.0111 \pm 0.0208$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.13: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 183 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0101±0.0033±0.0037	0.0231±0.0051±0.0073	0.0344±0.0065±0.0176	0.0357±0.0066±0.0103
-4.75	0.0101±0.0033±0.0036	0.0249±0.0052±0.0063	0.0354±0.0063±0.0073	0.0583±0.0082±0.0143
-4.50	0.0136±0.0039±0.0040	0.0393±0.0067±0.0093	0.0760±0.0095±0.0165	0.1130±0.0113±0.0199
-4.25	0.0291±0.0061±0.0071	0.0875±0.0103±0.0199	0.1390±0.0125±0.0259	0.1709±0.0127±0.0350
-4.00	0.0809±0.0102±0.0106	0.1470±0.0126±0.0154	0.2089±0.0142±0.0220	0.1808±0.0125±0.0180
-3.75	0.1255±0.0119±0.0150	0.2256±0.0142±0.0168	0.2453±0.0145±0.0189	0.1927±0.0132±0.0259
-3.50	0.2072±0.0141±0.0196	0.2822±0.0152±0.0203	0.2583±0.0146±0.0216	0.1606±0.0123±0.0165
-3.25	0.2833±0.0155±0.0182	0.3412±0.0157±0.0164	0.2329±0.0142±0.0237	0.1016±0.0111±0.0241
-3.00	0.3638±0.0163±0.0251	0.3794±0.0159±0.0332	0.1779±0.0136±0.0328	0.0686±0.0105±0.0184
-2.75	0.4456±0.0165±0.0299	0.3753±0.0159±0.0225	0.1416±0.0135±0.0181	0.0296±0.0094±0.0163
-2.50	0.5398±0.0166±0.0250	0.3427±0.0157±0.0201	0.1033±0.0133±0.0210	0.0071±0.0074±0.0102
-2.25	0.6159±0.0160±0.0230	0.3150±0.0159±0.0243	0.0605±0.0123±0.0234	0.0099±0.0118±0.0238
-2.00	0.6971±0.0152±0.0205	0.2664±0.0157±0.0216	0.0348±0.0124±0.0210	0.0032±0.0026±0.0103
-1.75	0.7685±0.0138±0.0216	0.2153±0.0156±0.0214	0.0246±0.0158±0.0357	0.0000±0.0000±0.0000
-1.50	0.8513±0.0116±0.0200	0.1422±0.0142±0.0219	0.0104±0.0124±0.0266	0.0000±0.0000±0.0000
-1.25	0.9056±0.0095±0.0157	0.0935±0.0127±0.0182	0.0119±0.0200±0.0262	0.0000±0.0000±0.0000
-1.00	0.9551±0.0068±0.0122	0.0446±0.0100±0.0159	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9876±0.0038±0.0080	0.0111±0.0061±0.0108	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	0.9999±0.0012±0.0012	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.14: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 183 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0001±0.0001±0.0002	0.0009±0.0007±0.0017
-4.75	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0005±0.0006±0.0012	0.0052±0.0020±0.0024
-4.50	0.0000±0.0000±0.0000	0.0010±0.0011±0.0020	0.0080±0.0029±0.0051	0.0226±0.0044±0.0056
-4.25	0.0000±0.0000±0.0016	0.0041±0.0019±0.0029	0.0410±0.0067±0.0135	0.0848±0.0093±0.0164
-4.00	0.0025±0.0015±0.0029	0.0328±0.0058±0.0063	0.1139±0.0111±0.0125	0.1710±0.0127±0.0163
-3.75	0.0267±0.0055±0.0066	0.0938±0.0101±0.0138	0.1967±0.0136±0.0168	0.2220±0.0138±0.0163
-3.50	0.0731±0.0091±0.0105	0.1961±0.0137±0.0157	0.2719±0.0150±0.0234	0.2153±0.0134±0.0195
-3.25	0.1398±0.0121±0.0192	0.3086±0.0157±0.0272	0.2836±0.0147±0.0237	0.1738±0.0128±0.0158
-3.00	0.2767±0.0154±0.0212	0.3678±0.0163±0.0293	0.2389±0.0139±0.0221	0.0837±0.0105±0.0211
-2.75	0.3670±0.0163±0.0246	0.3908±0.0159±0.0176	0.1865±0.0140±0.0171	0.0460±0.0097±0.0171
-2.50	0.4701±0.0166±0.0295	0.3779±0.0159±0.0224	0.1377±0.0142±0.0199	0.0125±0.0075±0.0145
-2.25	0.5712±0.0163±0.0290	0.3480±0.0161±0.0270	0.0694±0.0128±0.0176	0.0137±0.0116±0.0142
-2.00	0.6660±0.0156±0.0219	0.2959±0.0161±0.0214	0.0328±0.0122±0.0192	0.0055±0.0043±0.0116
-1.75	0.7546±0.0141±0.0245	0.2268±0.0159±0.0266	0.0318±0.0176±0.0475	0.0000±0.0000±0.0000
-1.50	0.8391±0.0120±0.0197	0.1548±0.0147±0.0221	0.0124±0.0151±0.0370	0.0000±0.0000±0.0000
-1.25	0.9032±0.0097±0.0147	0.0954±0.0127±0.0173	0.0115±0.0190±0.0250	0.0000±0.0000±0.0000
-1.00	0.9492±0.0071±0.0123	0.0527±0.0107±0.0165	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9884±0.0036±0.0078	0.0095±0.0059±0.0106	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	0.9999±0.0012±0.0012	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.15: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 183 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.3: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 183$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

189 GeV Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4534 \pm 0.0095 \pm 0.0132$	$0.3737 \pm 0.0095 \pm 0.0147$	$0.1777 \pm 0.0087 \pm 0.0115$
6.00	$0.6332 \pm 0.0091 \pm 0.0125$	$0.3062 \pm 0.0096 \pm 0.0144$	$0.0625 \pm 0.0066 \pm 0.0088$
10.00	$0.6951 \pm 0.0087 \pm 0.0117$	$0.2704 \pm 0.0095 \pm 0.0146$	$0.0340 \pm 0.0062 \pm 0.0101$
14.00	$0.7463 \pm 0.0082 \pm 0.0113$	$0.2287 \pm 0.0092 \pm 0.0131$	$0.0271 \pm 0.0070 \pm 0.0080$
18.00	$0.7838 \pm 0.0078 \pm 0.0114$	$0.2025 \pm 0.0091 \pm 0.0132$	$0.0097 \pm 0.0068 \pm 0.0144$
22.00	$0.8210 \pm 0.0073 \pm 0.0110$	$0.1690 \pm 0.0088 \pm 0.0118$	$0.0129 \pm 0.0095 \pm 0.0162$
25.50	$0.8423 \pm 0.0070 \pm 0.0124$	$0.1532 \pm 0.0087 \pm 0.0146$	$0.0089 \pm 0.0121 \pm 0.0226$

Table A.16: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 189 GeV, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4627 \pm 0.0095 \pm 0.0137$	$0.3231 \pm 0.0090 \pm 0.0119$	$0.2175 \pm 0.0095 \pm 0.0123$
0.50	$0.5614 \pm 0.0093 \pm 0.0131$	$0.3229 \pm 0.0094 \pm 0.0127$	$0.1191 \pm 0.0085 \pm 0.0136$
0.70	$0.6506 \pm 0.0090 \pm 0.0142$	$0.3017 \pm 0.0096 \pm 0.0180$	$0.0471 \pm 0.0061 \pm 0.0110$
0.90	$0.7424 \pm 0.0083 \pm 0.0173$	$0.2429 \pm 0.0094 \pm 0.0216$	$0.0147 \pm 0.0039 \pm 0.0059$
1.10	$0.8422 \pm 0.0071 \pm 0.0117$	$0.1565 \pm 0.0082 \pm 0.0156$	$0.0017 \pm 0.0014 \pm 0.0032$
1.30	$0.9330 \pm 0.0051 \pm 0.0073$	$0.0686 \pm 0.0059 \pm 0.0084$	$0.0001 \pm 0.0001 \pm 0.0002$
1.50	$0.9941 \pm 0.0025 \pm 0.0026$	$0.0057 \pm 0.0030 \pm 0.0037$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.17: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 189 GeV, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.1883 \pm 0.0073 \pm 0.0121$	$0.4972 \pm 0.0095 \pm 0.0125$	$0.2564 \pm 0.0091 \pm 0.0164$	$0.0528 \pm 0.0064 \pm 0.0133$
-2.06	$0.3074 \pm 0.0086 \pm 0.0096$	$0.5092 \pm 0.0096 \pm 0.0121$	$0.1644 \pm 0.0086 \pm 0.0148$	$0.0228 \pm 0.0063 \pm 0.0176$
-1.90	$0.3885 \pm 0.0091 \pm 0.0142$	$0.4968 \pm 0.0098 \pm 0.0158$	$0.1028 \pm 0.0079 \pm 0.0151$	$0.0229 \pm 0.0081 \pm 0.0165$
-1.70	$0.4969 \pm 0.0093 \pm 0.0204$	$0.4438 \pm 0.0100 \pm 0.0235$	$0.0599 \pm 0.0076 \pm 0.0101$	$0.0025 \pm 0.0060 \pm 0.0094$
-1.52	$0.5952 \pm 0.0091 \pm 0.0210$	$0.3830 \pm 0.0103 \pm 0.0260$	$0.0185 \pm 0.0060 \pm 0.0123$	$0.0015 \pm 0.0016 \pm 0.0028$
-1.40	$0.6750 \pm 0.0088 \pm 0.0139$	$0.3137 \pm 0.0101 \pm 0.0168$	$0.0087 \pm 0.0060 \pm 0.0131$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7300 \pm 0.0084 \pm 0.0127$	$0.2666 \pm 0.0099 \pm 0.0143$	$0.0037 \pm 0.0044 \pm 0.0150$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7647 \pm 0.0080 \pm 0.0117$	$0.2357 \pm 0.0098 \pm 0.0134$	$0.0018 \pm 0.0066 \pm 0.0077$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8010 \pm 0.0076 \pm 0.0108$	$0.2004 \pm 0.0095 \pm 0.0131$	$0.0006 \pm 0.0066 \pm 0.0113$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8298 \pm 0.0072 \pm 0.0119$	$0.1727 \pm 0.0091 \pm 0.0140$	$0.0360 \pm 0.0367 \pm 0.0873$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8658 \pm 0.0066 \pm 0.0111$	$0.1362 \pm 0.0086 \pm 0.0137$	$0.0000 \pm 0.0000 \pm 0.0076$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.8974 \pm 0.0059 \pm 0.0097$	$0.1054 \pm 0.0080 \pm 0.0112$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9255 \pm 0.0052 \pm 0.0077$	$0.0762 \pm 0.0074 \pm 0.0090$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9455 \pm 0.0046 \pm 0.0070$	$0.0584 \pm 0.0069 \pm 0.0093$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.18: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 189 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-5.00	0.0119 $\pm$ 0.0020 $\pm$ 0.0034	0.0196 $\pm$ 0.0027 $\pm$ 0.0042	0.0271 $\pm$ 0.0032 $\pm$ 0.0074	0.0298 $\pm$ 0.0034 $\pm$ 0.0058
-4.75	0.0121 $\pm$ 0.0021 $\pm$ 0.0033	0.0236 $\pm$ 0.0029 $\pm$ 0.0052	0.0385 $\pm$ 0.0038 $\pm$ 0.0063	0.0486 $\pm$ 0.0043 $\pm$ 0.0081
-4.50	0.0167 $\pm$ 0.0025 $\pm$ 0.0056	0.0384 $\pm$ 0.0038 $\pm$ 0.0058	0.0724 $\pm$ 0.0053 $\pm$ 0.0087	0.0939 $\pm$ 0.0059 $\pm$ 0.0079
-4.25	0.0350 $\pm$ 0.0039 $\pm$ 0.0056	0.0817 $\pm$ 0.0056 $\pm$ 0.0091	0.1407 $\pm$ 0.0072 $\pm$ 0.0133	0.1608 $\pm$ 0.0073 $\pm$ 0.0085
-4.00	0.0738 $\pm$ 0.0056 $\pm$ 0.0082	0.1467 $\pm$ 0.0073 $\pm$ 0.0100	0.2098 $\pm$ 0.0081 $\pm$ 0.0094	0.1986 $\pm$ 0.0076 $\pm$ 0.0113
-3.75	0.1297 $\pm$ 0.0069 $\pm$ 0.0106	0.2304 $\pm$ 0.0084 $\pm$ 0.0121	0.2491 $\pm$ 0.0084 $\pm$ 0.0140	0.1906 $\pm$ 0.0075 $\pm$ 0.0131
-3.50	0.1969 $\pm$ 0.0080 $\pm$ 0.0116	0.3046 $\pm$ 0.0089 $\pm$ 0.0139	0.2536 $\pm$ 0.0084 $\pm$ 0.0168	0.1456 $\pm$ 0.0070 $\pm$ 0.0195
-3.25	0.2767 $\pm$ 0.0087 $\pm$ 0.0103	0.3424 $\pm$ 0.0090 $\pm$ 0.0128	0.2449 $\pm$ 0.0086 $\pm$ 0.0163	0.0946 $\pm$ 0.0064 $\pm$ 0.0178
-3.00	0.3593 $\pm$ 0.0092 $\pm$ 0.0113	0.3665 $\pm$ 0.0091 $\pm$ 0.0141	0.2000 $\pm$ 0.0085 $\pm$ 0.0183	0.0615 $\pm$ 0.0061 $\pm$ 0.0116
-2.75	0.4424 $\pm$ 0.0094 $\pm$ 0.0137	0.3686 $\pm$ 0.0092 $\pm$ 0.0133	0.1517 $\pm$ 0.0082 $\pm$ 0.0168	0.0321 $\pm$ 0.0057 $\pm$ 0.0094
-2.50	0.5191 $\pm$ 0.0094 $\pm$ 0.0142	0.3627 $\pm$ 0.0093 $\pm$ 0.0122	0.1009 $\pm$ 0.0077 $\pm$ 0.0154	0.0149 $\pm$ 0.0055 $\pm$ 0.0098
-2.25	0.6094 $\pm$ 0.0092 $\pm$ 0.0137	0.3208 $\pm$ 0.0094 $\pm$ 0.0117	0.0597 $\pm$ 0.0074 $\pm$ 0.0092	0.0111 $\pm$ 0.0071 $\pm$ 0.0133
-2.00	0.6914 $\pm$ 0.0087 $\pm$ 0.0159	0.2691 $\pm$ 0.0093 $\pm$ 0.0155	0.0392 $\pm$ 0.0081 $\pm$ 0.0117	0.0317 $\pm$ 0.0359 $\pm$ 0.0749
-1.75	0.7723 $\pm$ 0.0080 $\pm$ 0.0128	0.2113 $\pm$ 0.0090 $\pm$ 0.0128	0.0154 $\pm$ 0.0084 $\pm$ 0.0182	0.0000 $\pm$ 0.0000 $\pm$ 0.0012
-1.50	0.8385 $\pm$ 0.0071 $\pm$ 0.0095	0.1581 $\pm$ 0.0087 $\pm$ 0.0117	0.0073 $\pm$ 0.0147 $\pm$ 0.0225	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9037 $\pm$ 0.0057 $\pm$ 0.0077	0.0974 $\pm$ 0.0078 $\pm$ 0.0096	0.0294 $\pm$ 0.0300 $\pm$ 0.0609	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9532 $\pm$ 0.0043 $\pm$ 0.0073	0.0468 $\pm$ 0.0063 $\pm$ 0.0086	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9841 $\pm$ 0.0030 $\pm$ 0.0061	0.0179 $\pm$ 0.0048 $\pm$ 0.0074	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	0.9999 $\pm$ 0.0022 $\pm$ 0.0022	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.19: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 189 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-5.00	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0004 $\pm$ 0.0003 $\pm$ 0.0005	0.0007 $\pm$ 0.0004 $\pm$ 0.0006
-4.75	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0002 $\pm$ 0.0002 $\pm$ 0.0003	0.0010 $\pm$ 0.0005 $\pm$ 0.0006	0.0053 $\pm$ 0.0012 $\pm$ 0.0019
-4.50	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0012 $\pm$ 0.0005 $\pm$ 0.0006	0.0089 $\pm$ 0.0017 $\pm$ 0.0017	0.0252 $\pm$ 0.0029 $\pm$ 0.0034
-4.25	0.0003 $\pm$ 0.0003 $\pm$ 0.0005	0.0118 $\pm$ 0.0020 $\pm$ 0.0030	0.0335 $\pm$ 0.0034 $\pm$ 0.0038	0.0841 $\pm$ 0.0054 $\pm$ 0.0067
-4.00	0.0051 $\pm$ 0.0013 $\pm$ 0.0020	0.0386 $\pm$ 0.0038 $\pm$ 0.0062	0.1060 $\pm$ 0.0062 $\pm$ 0.0099	0.1892 $\pm$ 0.0077 $\pm$ 0.0171
-3.75	0.0255 $\pm$ 0.0031 $\pm$ 0.0058	0.0983 $\pm$ 0.0059 $\pm$ 0.0092	0.2001 $\pm$ 0.0079 $\pm$ 0.0093	0.2533 $\pm$ 0.0084 $\pm$ 0.0107
-3.50	0.0761 $\pm$ 0.0055 $\pm$ 0.0105	0.2108 $\pm$ 0.0082 $\pm$ 0.0101	0.2800 $\pm$ 0.0086 $\pm$ 0.0141	0.2240 $\pm$ 0.0079 $\pm$ 0.0130
-3.25	0.1633 $\pm$ 0.0075 $\pm$ 0.0109	0.3015 $\pm$ 0.0089 $\pm$ 0.0106	0.2886 $\pm$ 0.0086 $\pm$ 0.0110	0.1680 $\pm$ 0.0074 $\pm$ 0.0095
-3.00	0.2681 $\pm$ 0.0088 $\pm$ 0.0115	0.3607 $\pm$ 0.0091 $\pm$ 0.0132	0.2526 $\pm$ 0.0085 $\pm$ 0.0110	0.0930 $\pm$ 0.0064 $\pm$ 0.0105
-2.75	0.3685 $\pm$ 0.0093 $\pm$ 0.0149	0.3934 $\pm$ 0.0092 $\pm$ 0.0196	0.1838 $\pm$ 0.0082 $\pm$ 0.0174	0.0504 $\pm$ 0.0061 $\pm$ 0.0125
-2.50	0.4721 $\pm$ 0.0095 $\pm$ 0.0143	0.3818 $\pm$ 0.0093 $\pm$ 0.0146	0.1251 $\pm$ 0.0080 $\pm$ 0.0150	0.0208 $\pm$ 0.0061 $\pm$ 0.0123
-2.25	0.5669 $\pm$ 0.0094 $\pm$ 0.0136	0.3499 $\pm$ 0.0094 $\pm$ 0.0120	0.0771 $\pm$ 0.0080 $\pm$ 0.0133	0.0037 $\pm$ 0.0060 $\pm$ 0.0095
-2.00	0.6629 $\pm$ 0.0089 $\pm$ 0.0172	0.2919 $\pm$ 0.0095 $\pm$ 0.0183	0.0497 $\pm$ 0.0085 $\pm$ 0.0157	0.0019 $\pm$ 0.0073 $\pm$ 0.0194
-1.75	0.7557 $\pm$ 0.0082 $\pm$ 0.0134	0.2268 $\pm$ 0.0092 $\pm$ 0.0143	0.0162 $\pm$ 0.0090 $\pm$ 0.0154	0.0000 $\pm$ 0.0000 $\pm$ 0.0007
-1.50	0.8314 $\pm$ 0.0072 $\pm$ 0.0100	0.1638 $\pm$ 0.0087 $\pm$ 0.0118	0.0110 $\pm$ 0.0169 $\pm$ 0.0285	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.8990 $\pm$ 0.0058 $\pm$ 0.0089	0.1022 $\pm$ 0.0079 $\pm$ 0.0110	0.0000 $\pm$ 0.0000 $\pm$ 0.0313	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9513 $\pm$ 0.0043 $\pm$ 0.0083	0.0489 $\pm$ 0.0065 $\pm$ 0.0088	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9838 $\pm$ 0.0030 $\pm$ 0.0065	0.0185 $\pm$ 0.0049 $\pm$ 0.0088	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	1.0000 $\pm$ 0.0022 $\pm$ 0.0022	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.20: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 189 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.4: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 189$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

192 GeV Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4614 \pm 0.0229 \pm 0.0262$	$0.3774 \pm 0.0231 \pm 0.0292$	$0.1643 \pm 0.0201 \pm 0.0263$
6.00	$0.6364 \pm 0.0215 \pm 0.0288$	$0.2923 \pm 0.0230 \pm 0.0328$	$0.0775 \pm 0.0172 \pm 0.0252$
10.00	$0.7113 \pm 0.0202 \pm 0.0282$	$0.2347 \pm 0.0220 \pm 0.0308$	$0.0659 \pm 0.0185 \pm 0.0289$
14.00	$0.7560 \pm 0.0190 \pm 0.0255$	$0.2059 \pm 0.0215 \pm 0.0264$	$0.0603 \pm 0.0235 \pm 0.0358$
18.00	$0.7972 \pm 0.0179 \pm 0.0280$	$0.1765 \pm 0.0208 \pm 0.0324$	$0.0562 \pm 0.0297 \pm 0.0469$
22.00	$0.8264 \pm 0.0168 \pm 0.0289$	$0.1555 \pm 0.0203 \pm 0.0341$	$0.0709 \pm 0.0528 \pm 0.0889$
25.50	$0.8558 \pm 0.0155 \pm 0.0248$	$0.1320 \pm 0.0196 \pm 0.0293$	$0.0365 \pm 0.0413 \pm 0.0587$

Table A.21: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 192 GeV, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4903 \pm 0.0231 \pm 0.0316$	$0.2817 \pm 0.0206 \pm 0.0276$	$0.2354 \pm 0.0238 \pm 0.0327$
0.50	$0.5411 \pm 0.0223 \pm 0.0407$	$0.3207 \pm 0.0228 \pm 0.0427$	$0.1515 \pm 0.0218 \pm 0.0294$
0.70	$0.6504 \pm 0.0213 \pm 0.0331$	$0.2849 \pm 0.0229 \pm 0.0364$	$0.0731 \pm 0.0172 \pm 0.0267$
0.90	$0.7237 \pm 0.0199 \pm 0.0262$	$0.2504 \pm 0.0231 \pm 0.0333$	$0.0383 \pm 0.0132 \pm 0.0185$
1.10	$0.8084 \pm 0.0176 \pm 0.0290$	$0.1911 \pm 0.0217 \pm 0.0365$	$0.0112 \pm 0.0068 \pm 0.0118$
1.30	$0.9267 \pm 0.0115 \pm 0.0217$	$0.0739 \pm 0.0150 \pm 0.0275$	$0.0021 \pm 0.0026 \pm 0.0040$
1.50	$0.9960 \pm 0.0020 \pm 0.0070$	$0.0013 \pm 0.0045 \pm 0.0156$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.22: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 192 GeV, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.2146 \pm 0.0184 \pm 0.0291$	$0.4602 \pm 0.0226 \pm 0.0387$	$0.2426 \pm 0.0216 \pm 0.0325$	$0.0893 \pm 0.0201 \pm 0.0339$
-2.06	$0.3264 \pm 0.0210 \pm 0.0470$	$0.4666 \pm 0.0228 \pm 0.0381$	$0.1814 \pm 0.0220 \pm 0.0277$	$0.0493 \pm 0.0220 \pm 0.0357$
-1.90	$0.3881 \pm 0.0217 \pm 0.0344$	$0.4600 \pm 0.0232 \pm 0.0373$	$0.1645 \pm 0.0238 \pm 0.0406$	$0.0005 \pm 0.0121 \pm 0.0372$
-1.70	$0.5098 \pm 0.0221 \pm 0.0273$	$0.4212 \pm 0.0241 \pm 0.0280$	$0.0754 \pm 0.0205 \pm 0.0274$	$0.0108 \pm 0.0158 \pm 0.0595$
-1.52	$0.5959 \pm 0.0218 \pm 0.0269$	$0.3738 \pm 0.0245 \pm 0.0292$	$0.0485 \pm 0.0224 \pm 0.0355$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.40	$0.6648 \pm 0.0209 \pm 0.0320$	$0.3153 \pm 0.0244 \pm 0.0344$	$0.0770 \pm 0.0405 \pm 0.0720$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7253 \pm 0.0199 \pm 0.0286$	$0.2661 \pm 0.0238 \pm 0.0347$	$0.0312 \pm 0.0271 \pm 0.0411$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7708 \pm 0.0186 \pm 0.0298$	$0.2292 \pm 0.0233 \pm 0.0378$	$0.0120 \pm 0.0058 \pm 0.0363$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8026 \pm 0.0176 \pm 0.0286$	$0.1998 \pm 0.0228 \pm 0.0370$	$0.0052 \pm 0.0033 \pm 0.0161$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8452 \pm 0.0161 \pm 0.0231$	$0.1530 \pm 0.0208 \pm 0.0289$	$0.0055 \pm 0.0058 \pm 0.0098$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8674 \pm 0.0150 \pm 0.0227$	$0.1353 \pm 0.0205 \pm 0.0301$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.8961 \pm 0.0133 \pm 0.0213$	$0.1082 \pm 0.0196 \pm 0.0297$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9236 \pm 0.0115 \pm 0.0192$	$0.0791 \pm 0.0177 \pm 0.0279$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9520 \pm 0.0090 \pm 0.0142$	$0.0482 \pm 0.0156 \pm 0.0215$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.23: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 192 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0036±0.0027±0.0118	0.0165±0.0058±0.0078	0.0377±0.0091±0.0257	0.0389±0.0097±0.0150
-4.75	0.0055±0.0033±0.0099	0.0227±0.0069±0.0094	0.0418±0.0095±0.0262	0.0466±0.0103±0.0107
-4.50	0.0104±0.0049±0.0088	0.0307±0.0079±0.0114	0.0723±0.0130±0.0297	0.1221±0.0162±0.0254
-4.25	0.0271±0.0084±0.0147	0.0916±0.0149±0.0228	0.1066±0.0149±0.0246	0.1686±0.0178±0.0289
-4.00	0.0837±0.0146±0.0221	0.1392±0.0174±0.0289	0.1812±0.0180±0.0194	0.1989±0.0186±0.0268
-3.75	0.1222±0.0164±0.0306	0.2062±0.0195±0.0351	0.2563±0.0200±0.0431	0.1748±0.0173±0.0388
-3.50	0.2133±0.0201±0.0336	0.2433±0.0196±0.0274	0.2793±0.0208±0.0282	0.1363±0.0164±0.0237
-3.25	0.2851±0.0211±0.0424	0.3158±0.0212±0.0325	0.2322±0.0200±0.0258	0.1234±0.0178±0.0415
-3.00	0.3620±0.0218±0.0393	0.3573±0.0219±0.0253	0.1777±0.0193±0.0400	0.1064±0.0199±0.0384
-2.75	0.4526±0.0224±0.0398	0.3326±0.0215±0.0278	0.1722±0.0211±0.0496	0.0525±0.0177±0.0259
-2.50	0.5262±0.0224±0.0312	0.3441±0.0221±0.0330	0.0906±0.0187±0.0263	0.0635±0.0234±0.0327
-2.25	0.6173±0.0217±0.0272	0.3027±0.0222±0.0279	0.0604±0.0184±0.0289	0.1019±0.0637±0.0910
-2.00	0.6845±0.0207±0.0314	0.2720±0.0225±0.0305	0.0544±0.0233±0.0340	0.0000±0.0000±0.0000
-1.75	0.7924±0.0180±0.0284	0.1835±0.0209±0.0294	0.0434±0.0354±0.0619	0.0000±0.0000±0.0000
-1.50	0.8375±0.0162±0.0248	0.1620±0.0212±0.0322	0.0232±0.0122±0.0352	0.0000±0.0000±0.0000
-1.25	0.9007±0.0129±0.0209	0.1012±0.0187±0.0287	0.0000±0.0000±0.0044	0.0000±0.0000±0.0000
-1.00	0.9514±0.0090±0.0147	0.0522±0.0161±0.0272	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9864±0.0041±0.0084	0.0137±0.0126±0.0255	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	1.0000±0.0008±0.0008	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.24: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 192 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0005	0.0000±0.0000±0.0005
-4.75	0.0000±0.0000±0.0000	0.0000±0.0000±0.0004	0.0022±0.0017±0.0023	0.0054±0.0029±0.0032
-4.50	0.0000±0.0000±0.0000	0.0030±0.0024±0.0048	0.0032±0.0024±0.0026	0.0287±0.0074±0.0117
-4.25	0.0011±0.0012±0.0020	0.0066±0.0035±0.0052	0.0297±0.0081±0.0150	0.1095±0.0147±0.0373
-4.00	0.0070±0.0036±0.0047	0.0459±0.0102±0.0140	0.1061±0.0151±0.0286	0.1688±0.0174±0.0222
-3.75	0.0245±0.0075±0.0104	0.1108±0.0153±0.0255	0.1849±0.0184±0.0252	0.2130±0.0188±0.0249
-3.50	0.0803±0.0136±0.0215	0.1982±0.0190±0.0275	0.2363±0.0197±0.0227	0.2270±0.0187±0.0196
-3.25	0.1472±0.0176±0.0339	0.3145±0.0218±0.0279	0.2403±0.0191±0.0250	0.1905±0.0187±0.0231
-3.00	0.2693±0.0213±0.0389	0.3616±0.0219±0.0273	0.2121±0.0190±0.0419	0.1325±0.0188±0.0323
-2.75	0.3675±0.0223±0.0406	0.3771±0.0217±0.0294	0.1776±0.0198±0.0292	0.0788±0.0186±0.0267
-2.50	0.4821±0.0227±0.0351	0.3553±0.0219±0.0290	0.1232±0.0194±0.0281	0.0775±0.0267±0.0440
-2.25	0.5843±0.0220±0.0299	0.3204±0.0223±0.0338	0.0894±0.0207±0.0331	0.0404±0.0325±0.0568
-2.00	0.6693±0.0210±0.0318	0.2862±0.0226±0.0303	0.0434±0.0216±0.0300	0.0000±0.0000±0.0378
-1.75	0.7702±0.0187±0.0296	0.2034±0.0215±0.0301	0.0554±0.0374±0.0592	0.0000±0.0000±0.0000
-1.50	0.8263±0.0167±0.0233	0.1708±0.0215±0.0300	0.0155±0.0421±0.0555	0.0000±0.0000±0.0000
-1.25	0.9011±0.0130±0.0205	0.0991±0.0185±0.0277	0.0000±0.0000±0.0053	0.0000±0.0000±0.0000
-1.00	0.9510±0.0090±0.0139	0.0517±0.0161±0.0251	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9841±0.0046±0.0090	0.0202±0.0144±0.0243	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	1.0000±0.0008±0.0008	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.25: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 192 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.5: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 192$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

196 GeV Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4676 \pm 0.0156 \pm 0.0206$	$0.3471 \pm 0.0154 \pm 0.0209$	$0.1919 \pm 0.0150 \pm 0.0219$
6.00	$0.6448 \pm 0.0146 \pm 0.0231$	$0.2754 \pm 0.0154 \pm 0.0254$	$0.0869 \pm 0.0129 \pm 0.0185$
10.00	$0.7064 \pm 0.0138 \pm 0.0210$	$0.2470 \pm 0.0154 \pm 0.0221$	$0.0502 \pm 0.0127 \pm 0.0235$
14.00	$0.7547 \pm 0.0129 \pm 0.0220$	$0.2115 \pm 0.0151 \pm 0.0255$	$0.0412 \pm 0.0150 \pm 0.0321$
18.00	$0.7863 \pm 0.0124 \pm 0.0218$	$0.1933 \pm 0.0150 \pm 0.0237$	$0.0252 \pm 0.0184 \pm 0.0508$
22.00	$0.8134 \pm 0.0118 \pm 0.0234$	$0.1765 \pm 0.0150 \pm 0.0311$	$-0.0013 \pm 0.0154 \pm 0.0325$
25.50	$0.8386 \pm 0.0111 \pm 0.0229$	$0.1573 \pm 0.0147 \pm 0.0318$	$-0.0151 \pm 0.0133 \pm 0.0278$

Table A.26: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 196 GeV, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4577 \pm 0.0154 \pm 0.0206$	$0.3205 \pm 0.0147 \pm 0.0181$	$0.2246 \pm 0.0162 \pm 0.0225$
0.50	$0.5530 \pm 0.0151 \pm 0.0242$	$0.3219 \pm 0.0155 \pm 0.0248$	$0.1312 \pm 0.0149 \pm 0.0206$
0.70	$0.6618 \pm 0.0144 \pm 0.0211$	$0.2663 \pm 0.0154 \pm 0.0196$	$0.0805 \pm 0.0131 \pm 0.0194$
0.90	$0.7435 \pm 0.0134 \pm 0.0206$	$0.2288 \pm 0.0154 \pm 0.0230$	$0.0345 \pm 0.0090 \pm 0.0133$
1.10	$0.8324 \pm 0.0115 \pm 0.0151$	$0.1647 \pm 0.0141 \pm 0.0169$	$0.0068 \pm 0.0051 \pm 0.0106$
1.30	$0.9444 \pm 0.0071 \pm 0.0138$	$0.0541 \pm 0.0088 \pm 0.0181$	$0.0002 \pm 0.0002 \pm 0.0003$
1.50	$0.9977 \pm 0.0013 \pm 0.0019$	$0.0025 \pm 0.0024 \pm 0.0035$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.27: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 196 GeV, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.1811 \pm 0.0117 \pm 0.0169$	$0.4966 \pm 0.0155 \pm 0.0239$	$0.2696 \pm 0.0154 \pm 0.0191$	$0.0486 \pm 0.0113 \pm 0.0252$
-2.06	$0.3000 \pm 0.0139 \pm 0.0167$	$0.5137 \pm 0.0157 \pm 0.0185$	$0.1679 \pm 0.0148 \pm 0.0227$	$0.0291 \pm 0.0121 \pm 0.0199$
-1.90	$0.3831 \pm 0.0148 \pm 0.0176$	$0.4888 \pm 0.0160 \pm 0.0175$	$0.1269 \pm 0.0146 \pm 0.0191$	$0.0121 \pm 0.0135 \pm 0.0340$
-1.70	$0.5031 \pm 0.0151 \pm 0.0208$	$0.4329 \pm 0.0165 \pm 0.0214$	$0.0706 \pm 0.0144 \pm 0.0206$	$0.0157 \pm 0.0116 \pm 0.0205$
-1.52	$0.6004 \pm 0.0148 \pm 0.0204$	$0.3740 \pm 0.0170 \pm 0.0241$	$0.0267 \pm 0.0130 \pm 0.0320$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.40	$0.6842 \pm 0.0140 \pm 0.0222$	$0.3015 \pm 0.0167 \pm 0.0283$	$0.0135 \pm 0.0147 \pm 0.0309$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7389 \pm 0.0133 \pm 0.0212$	$0.2536 \pm 0.0164 \pm 0.0273$	$0.0038 \pm 0.0140 \pm 0.0291$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7765 \pm 0.0126 \pm 0.0203$	$0.2198 \pm 0.0161 \pm 0.0263$	$0.0107 \pm 0.0352 \pm 0.1572$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8109 \pm 0.0119 \pm 0.0177$	$0.1859 \pm 0.0155 \pm 0.0240$	$0.0490 \pm 0.0787 \pm 0.1257$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8332 \pm 0.0113 \pm 0.0176$	$0.1677 \pm 0.0152 \pm 0.0249$	$0.0000 \pm 0.0000 \pm 0.0127$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8621 \pm 0.0104 \pm 0.0155$	$0.1408 \pm 0.0148 \pm 0.0208$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.8979 \pm 0.0091 \pm 0.0166$	$0.1043 \pm 0.0138 \pm 0.0239$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9187 \pm 0.0081 \pm 0.0141$	$0.0864 \pm 0.0136 \pm 0.0221$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9406 \pm 0.0070 \pm 0.0129$	$0.0671 \pm 0.0129 \pm 0.0231$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.28: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 196 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0088±0.0028±0.0080	0.0183±0.0044±0.0055	0.0205±0.0046±0.0084	0.0368±0.0062±0.0161
-4.75	0.0090±0.0028±0.0075	0.0212±0.0047±0.0052	0.0318±0.0058±0.0084	0.0520±0.0073±0.0155
-4.50	0.0109±0.0033±0.0114	0.0401±0.0066±0.0094	0.0604±0.0080±0.0153	0.1105±0.0106±0.0199
-4.25	0.0276±0.0057±0.0121	0.0768±0.0091±0.0094	0.1452±0.0118±0.0226	0.1627±0.0120±0.0178
-4.00	0.0639±0.0085±0.0153	0.1585±0.0122±0.0207	0.2100±0.0131±0.0202	0.1972±0.0127±0.0145
-3.75	0.1341±0.0115±0.0191	0.2312±0.0138±0.0186	0.2400±0.0135±0.0176	0.1861±0.0122±0.0178
-3.50	0.1891±0.0127±0.0232	0.3082±0.0148±0.0177	0.2677±0.0141±0.0169	0.1330±0.0110±0.0178
-3.25	0.2681±0.0140±0.0197	0.3353±0.0148±0.0179	0.2556±0.0143±0.0226	0.1037±0.0111±0.0166
-3.00	0.3603±0.0150±0.0279	0.3466±0.0147±0.0233	0.2270±0.0145±0.0277	0.0557±0.0102±0.0178
-2.75	0.4564±0.0154±0.0219	0.3374±0.0147±0.0170	0.1750±0.0145±0.0262	0.0310±0.0101±0.0126
-2.50	0.5356±0.0153±0.0193	0.3286±0.0149±0.0171	0.1307±0.0148±0.0234	0.0006±0.0069±0.0146
-2.25	0.6019±0.0148±0.0306	0.3264±0.0156±0.0332	0.0666±0.0134±0.0182	0.0103±0.0027±0.0089
-2.00	0.7099±0.0138±0.0204	0.2554±0.0154±0.0246	0.0194±0.0110±0.0218	0.0082±0.0062±0.0109
-1.75	0.7861±0.0125±0.0175	0.1988±0.0149±0.0242	0.0019±0.0107±0.0214	0.0000±0.0000±0.0000
-1.50	0.8523±0.0108±0.0164	0.1422±0.0140±0.0211	0.0177±0.0221±0.0821	0.0000±0.0000±0.0000
-1.25	0.8980±0.0090±0.0145	0.1064±0.0137±0.0212	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-1.00	0.9448±0.0066±0.0161	0.0621±0.0123±0.0286	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9796±0.0039±0.0082	0.0325±0.0117±0.0191	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	1.0000±0.0007±0.0007	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.29: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 196 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000±0.0000±0.0000	0.0000±0.0000±0.0002	0.0007±0.0008±0.0011	0.0009±0.0007±0.0015
-4.75	0.0000±0.0000±0.0000	0.0000±0.0000±0.0007	0.0006±0.0006±0.0008	0.0075±0.0024±0.0036
-4.50	0.0000±0.0000±0.0000	0.0018±0.0014±0.0019	0.0060±0.0022±0.0023	0.0326±0.0054±0.0087
-4.25	0.0000±0.0000±0.0000	0.0056±0.0022±0.0032	0.0374±0.0061±0.0092	0.0973±0.0094±0.0111
-4.00	0.0039±0.0018±0.0027	0.0399±0.0063±0.0096	0.1133±0.0104±0.0112	0.1888±0.0125±0.0226
-3.75	0.0233±0.0050±0.0072	0.1089±0.0104±0.0141	0.2214±0.0133±0.0192	0.2444±0.0138±0.0257
-3.50	0.0761±0.0089±0.0173	0.2149±0.0134±0.0152	0.2710±0.0140±0.0216	0.2209±0.0131±0.0156
-3.25	0.1598±0.0121±0.0203	0.3136±0.0149±0.0173	0.2855±0.0142±0.0157	0.1481±0.0116±0.0165
-3.00	0.2722±0.0145±0.0218	0.3401±0.0146±0.0233	0.2686±0.0143±0.0207	0.0953±0.0112±0.0173
-2.75	0.3728±0.0152±0.0192	0.3649±0.0148±0.0187	0.1984±0.0140±0.0213	0.0716±0.0126±0.0260
-2.50	0.4804±0.0155±0.0237	0.3537±0.0149±0.0163	0.1546±0.0149±0.0235	0.0166±0.0105±0.0248
-2.25	0.5722±0.0152±0.0219	0.3397±0.0153±0.0187	0.0788±0.0142±0.0243	0.0297±0.0217±0.0634
-2.00	0.6810±0.0142±0.0214	0.2822±0.0155±0.0276	0.0191±0.0117±0.0200	0.0126±0.0093±0.0168
-1.75	0.7688±0.0129±0.0201	0.2148±0.0151±0.0260	0.0008±0.0125±0.0258	0.0000±0.0000±0.0000
-1.50	0.8359±0.0113±0.0185	0.1617±0.0146±0.0244	0.0379±0.0151±0.0489	0.0000±0.0000±0.0000
-1.25	0.8941±0.0092±0.0161	0.1101±0.0138±0.0236	0.0000±0.0000±0.0091	0.0000±0.0000±0.0000
-1.00	0.9429±0.0067±0.0174	0.0643±0.0126±0.0320	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9804±0.0038±0.0084	0.0299±0.0114±0.0206	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	1.0000±0.0007±0.0007	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.30: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 196 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.6: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 196$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

200 GeV Data Sample

$\varepsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4636 \pm 0.0160 \pm 0.0205$	$0.3482 \pm 0.0156 \pm 0.0264$	$0.1962 \pm 0.0157 \pm 0.0225$
6.00	$0.6194 \pm 0.0152 \pm 0.0214$	$0.3081 \pm 0.0162 \pm 0.0294$	$0.0801 \pm 0.0129 \pm 0.0262$
10.00	$0.6916 \pm 0.0144 \pm 0.0197$	$0.2736 \pm 0.0163 \pm 0.0242$	$0.0328 \pm 0.0108 \pm 0.0216$
14.00	$0.7444 \pm 0.0136 \pm 0.0189$	$0.2260 \pm 0.0158 \pm 0.0208$	$0.0319 \pm 0.0137 \pm 0.0253$
18.00	$0.7769 \pm 0.0130 \pm 0.0194$	$0.1993 \pm 0.0155 \pm 0.0210$	$0.0441 \pm 0.0196 \pm 0.0322$
22.00	$0.8110 \pm 0.0122 \pm 0.0187$	$0.1785 \pm 0.0154 \pm 0.0211$	$0.0036 \pm 0.0177 \pm 0.0254$
25.50	$0.8298 \pm 0.0116 \pm 0.0216$	$0.1665 \pm 0.0155 \pm 0.0270$	$-0.0043 \pm 0.0201 \pm 0.0329$

Table A.31: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 200 GeV, as a function of the energy cutoff, ε , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4662 \pm 0.0159 \pm 0.0236$	$0.3060 \pm 0.0149 \pm 0.0199$	$0.2318 \pm 0.0167 \pm 0.0234$
0.50	$0.5573 \pm 0.0155 \pm 0.0258$	$0.3167 \pm 0.0157 \pm 0.0250$	$0.1303 \pm 0.0153 \pm 0.0326$
0.70	$0.6446 \pm 0.0150 \pm 0.0202$	$0.2905 \pm 0.0161 \pm 0.0257$	$0.0720 \pm 0.0129 \pm 0.0267$
0.90	$0.7298 \pm 0.0140 \pm 0.0178$	$0.2500 \pm 0.0162 \pm 0.0230$	$0.0272 \pm 0.0087 \pm 0.0156$
1.10	$0.8210 \pm 0.0121 \pm 0.0158$	$0.1808 \pm 0.0152 \pm 0.0216$	$0.0042 \pm 0.0036 \pm 0.0057$
1.30	$0.9324 \pm 0.0078 \pm 0.0144$	$0.0683 \pm 0.0106 \pm 0.0197$	$0.0002 \pm 0.0002 \pm 0.0004$
1.50	$0.9947 \pm 0.0021 \pm 0.0035$	$0.0048 \pm 0.0045 \pm 0.0084$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.32: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 200 GeV, as a function of the cone radius, R , with a constant energy cutoff $\varepsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.1992 \pm 0.0124 \pm 0.0176$	$0.4881 \pm 0.0159 \pm 0.0248$	$0.2497 \pm 0.0155 \pm 0.0300$	$0.0675 \pm 0.0131 \pm 0.0284$
-2.06	$0.3118 \pm 0.0143 \pm 0.0194$	$0.5027 \pm 0.0162 \pm 0.0225$	$0.1590 \pm 0.0148 \pm 0.0237$	$0.0472 \pm 0.0153 \pm 0.0232$
-1.90	$0.3926 \pm 0.0151 \pm 0.0198$	$0.4895 \pm 0.0164 \pm 0.0273$	$0.1105 \pm 0.0143 \pm 0.0272$	$0.0123 \pm 0.0117 \pm 0.0229$
-1.70	$0.4904 \pm 0.0154 \pm 0.0229$	$0.4442 \pm 0.0170 \pm 0.0282$	$0.0744 \pm 0.0147 \pm 0.0229$	$0.0000 \pm 0.0000 \pm 0.0331$
-1.52	$0.6016 \pm 0.0152 \pm 0.0220$	$0.3744 \pm 0.0173 \pm 0.0231$	$0.0227 \pm 0.0126 \pm 0.0238$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.40	$0.6639 \pm 0.0146 \pm 0.0174$	$0.3288 \pm 0.0175 \pm 0.0213$	$0.0046 \pm 0.0107 \pm 0.0171$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7084 \pm 0.0140 \pm 0.0207$	$0.2928 \pm 0.0177 \pm 0.0242$	$0.0011 \pm 0.0111 \pm 0.0180$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7468 \pm 0.0135 \pm 0.0184$	$0.2589 \pm 0.0175 \pm 0.0217$	$0.0022 \pm 0.0157 \pm 0.0257$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.7768 \pm 0.0130 \pm 0.0197$	$0.2332 \pm 0.0173 \pm 0.0245$	$0.0020 \pm 0.0141 \pm 0.0176$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8145 \pm 0.0121 \pm 0.0172$	$0.1942 \pm 0.0165 \pm 0.0212$	$0.0000 \pm 0.0000 \pm 0.0146$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8531 \pm 0.0110 \pm 0.0160$	$0.1549 \pm 0.0159 \pm 0.0214$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.8901 \pm 0.0097 \pm 0.0145$	$0.1176 \pm 0.0151 \pm 0.0215$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9199 \pm 0.0083 \pm 0.0143$	$0.0860 \pm 0.0145 \pm 0.0216$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9449 \pm 0.0068 \pm 0.0133$	$0.0598 \pm 0.0137 \pm 0.0221$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.33: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 200 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0146±0.0041±0.0085	0.0220±0.0047±0.0055	0.0347±0.0063±0.0103	0.0314±0.0058±0.0072
-4.75	0.0142±0.0040±0.0080	0.0248±0.0051±0.0058	0.0438±0.0070±0.0092	0.0676±0.0086±0.0125
-4.50	0.0246±0.0056±0.0130	0.0409±0.0068±0.0077	0.0998±0.0105±0.0238	0.1168±0.0109±0.0137
-4.25	0.0423±0.0075±0.0115	0.0981±0.0104±0.0158	0.1620±0.0129±0.0212	0.1621±0.0122±0.0164
-4.00	0.0852±0.0101±0.0188	0.1732±0.0132±0.0167	0.2131±0.0136±0.0185	0.1777±0.0120±0.0167
-3.75	0.1469±0.0124±0.0197	0.2549±0.0148±0.0204	0.2299±0.0134±0.0176	0.1622±0.0118±0.0242
-3.50	0.2248±0.0141±0.0174	0.2919±0.0148±0.0161	0.2409±0.0138±0.0200	0.1386±0.0115±0.0140
-3.25	0.2927±0.0148±0.0242	0.3373±0.0152±0.0180	0.2219±0.0138±0.0221	0.1101±0.0117±0.0150
-3.00	0.3611±0.0154±0.0201	0.3652±0.0152±0.0247	0.1931±0.0142±0.0250	0.0687±0.0113±0.0236
-2.75	0.4324±0.0156±0.0232	0.3778±0.0156±0.0211	0.1482±0.0140±0.0191	0.0441±0.0113±0.0271
-2.50	0.5157±0.0157±0.0224	0.3567±0.0158±0.0220	0.1130±0.0138±0.0240	0.0165±0.0108±0.0161
-2.25	0.6013±0.0153±0.0214	0.3173±0.0158±0.0207	0.0823±0.0150±0.0320	0.0162±0.0162±0.0250
-2.00	0.6778±0.0145±0.0197	0.2846±0.0162±0.0230	0.0410±0.0150±0.0212	0.0058±0.0036±0.0158
-1.75	0.7615±0.0133±0.0209	0.2216±0.0159±0.0229	0.0296±0.0209±0.0306	0.0000±0.0000±0.0000
-1.50	0.8340±0.0116±0.0185	0.1677±0.0154±0.0225	0.0590±0.0305±0.0663	0.0000±0.0000±0.0000
-1.25	0.8948±0.0093±0.0174	0.1115±0.0146±0.0252	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-1.00	0.9568±0.0060±0.0130	0.0401±0.0114±0.0177	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9873±0.0031±0.0071	0.0100±0.0089±0.0126	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	0.9998±0.0007±0.0007	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.34: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 200 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000±0.0000±0.0000	0.0000±0.0000±0.0003	0.0000±0.0000±0.0000	0.0004±0.0004±0.0007
-4.75	0.0000±0.0000±0.0000	0.0000±0.0000±0.0005	0.0007±0.0007±0.0010	0.0048±0.0019±0.0026
-4.50	0.0000±0.0000±0.0004	0.0000±0.0000±0.0014	0.0080±0.0027±0.0032	0.0387±0.0059±0.0120
-4.25	0.0011±0.0011±0.0020	0.0074±0.0026±0.0045	0.0418±0.0065±0.0086	0.1126±0.0106±0.0258
-4.00	0.0066±0.0026±0.0037	0.0506±0.0073±0.0089	0.1085±0.0104±0.0125	0.1831±0.0127±0.0145
-3.75	0.0292±0.0058±0.0069	0.1346±0.0116±0.0169	0.2084±0.0135±0.0251	0.2032±0.0128±0.0330
-3.50	0.0779±0.0091±0.0121	0.2370±0.0143±0.0271	0.2551±0.0141±0.0248	0.2150±0.0130±0.0218
-3.25	0.1635±0.0124±0.0214	0.3339±0.0157±0.0237	0.2480±0.0136±0.0160	0.1669±0.0126±0.0219
-3.00	0.2669±0.0148±0.0165	0.3861±0.0154±0.0205	0.2128±0.0135±0.0273	0.0989±0.0115±0.0161
-2.75	0.3716±0.0156±0.0261	0.3899±0.0154±0.0314	0.1781±0.0138±0.0215	0.0625±0.0118±0.0209
-2.50	0.4673±0.0158±0.0252	0.3794±0.0156±0.0256	0.1311±0.0142±0.0269	0.0282±0.0125±0.0205
-2.25	0.5638±0.0156±0.0209	0.3444±0.0159±0.0214	0.0914±0.0146±0.0310	0.0130±0.0157±0.0290
-2.00	0.6521±0.0148±0.0219	0.3011±0.0163±0.0241	0.0600±0.0162±0.0269	0.0079±0.0049±0.0206
-1.75	0.7434±0.0136±0.0202	0.2375±0.0162±0.0217	0.0389±0.0220±0.0295	0.0000±0.0000±0.0000
-1.50	0.8246±0.0118±0.0188	0.1767±0.0156±0.0224	0.0708±0.0365±0.0845	0.0000±0.0000±0.0000
-1.25	0.8914±0.0095±0.0167	0.1147±0.0148±0.0242	0.0000±0.0000±0.0095	0.0000±0.0000±0.0000
-1.00	0.9505±0.0064±0.0134	0.0509±0.0124±0.0193	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9872±0.0031±0.0071	0.0102±0.0087±0.0122	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	0.9999±0.0007±0.0007	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.35: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 200 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.7: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 200$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

202 GeV Data Sample

$\varepsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4620 \pm 0.0232 \pm 0.0261$	$0.3309 \pm 0.0222 \pm 0.0339$	$0.2199 \pm 0.0236 \pm 0.0333$
6.00	$0.6268 \pm 0.0220 \pm 0.0269$	$0.2791 \pm 0.0228 \pm 0.0278$	$0.1115 \pm 0.0210 \pm 0.0254$
10.00	$0.6836 \pm 0.0209 \pm 0.0286$	$0.2687 \pm 0.0235 \pm 0.0328$	$0.0630 \pm 0.0203 \pm 0.0273$
14.00	$0.7401 \pm 0.0198 \pm 0.0293$	$0.2270 \pm 0.0229 \pm 0.0341$	$0.0529 \pm 0.0247 \pm 0.0329$
18.00	$0.7610 \pm 0.0194 \pm 0.0261$	$0.2159 \pm 0.0230 \pm 0.0351$	$0.0837 \pm 0.0426 \pm 0.1061$
22.00	$0.7915 \pm 0.0183 \pm 0.0277$	$0.2066 \pm 0.0236 \pm 0.0341$	$0.0010 \pm 0.0298 \pm 0.0385$
25.50	$0.8206 \pm 0.0173 \pm 0.0266$	$0.1819 \pm 0.0231 \pm 0.0337$	$-0.0167 \pm 0.0486 \pm 0.0597$

Table A.36: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 202 GeV, as a function of the energy cutoff, ε , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4362 \pm 0.0226 \pm 0.0312$	$0.3227 \pm 0.0221 \pm 0.0348$	$0.2503 \pm 0.0248 \pm 0.0316$
0.50	$0.5631 \pm 0.0226 \pm 0.0264$	$0.2866 \pm 0.0221 \pm 0.0278$	$0.1660 \pm 0.0236 \pm 0.0362$
0.70	$0.6436 \pm 0.0217 \pm 0.0278$	$0.2806 \pm 0.0232 \pm 0.0285$	$0.0900 \pm 0.0198 \pm 0.0238$
0.90	$0.7287 \pm 0.0203 \pm 0.0258$	$0.2551 \pm 0.0237 \pm 0.0301$	$0.0233 \pm 0.0116 \pm 0.0220$
1.10	$0.8180 \pm 0.0177 \pm 0.0236$	$0.1797 \pm 0.0218 \pm 0.0276$	$0.0136 \pm 0.0089 \pm 0.0099$
1.30	$0.9394 \pm 0.0111 \pm 0.0185$	$0.0606 \pm 0.0141 \pm 0.0240$	$0.0005 \pm 0.0006 \pm 0.0008$
1.50	$0.9911 \pm 0.0040 \pm 0.0059$	$0.0171 \pm 0.0122 \pm 0.0147$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.37: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 202 GeV, as a function of the cone radius, R , with a constant energy cutoff $\varepsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.2103 \pm 0.0185 \pm 0.0237$	$0.4502 \pm 0.0227 \pm 0.0291$	$0.2692 \pm 0.0230 \pm 0.0337$	$0.0896 \pm 0.0208 \pm 0.0313$
-2.06	$0.3195 \pm 0.0211 \pm 0.0225$	$0.4739 \pm 0.0231 \pm 0.0274$	$0.1999 \pm 0.0235 \pm 0.0302$	$0.0192 \pm 0.0136 \pm 0.0201$
-1.90	$0.3916 \pm 0.0221 \pm 0.0299$	$0.4618 \pm 0.0234 \pm 0.0341$	$0.1524 \pm 0.0237 \pm 0.0286$	$0.0246 \pm 0.0193 \pm 0.0297$
-1.70	$0.5179 \pm 0.0226 \pm 0.0307$	$0.4113 \pm 0.0241 \pm 0.0359$	$0.0866 \pm 0.0226 \pm 0.0342$	$0.0032 \pm 0.0016 \pm 0.0058$
-1.52	$0.6189 \pm 0.0219 \pm 0.0335$	$0.3420 \pm 0.0245 \pm 0.0422$	$0.0657 \pm 0.0257 \pm 0.0426$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.40	$0.6749 \pm 0.0212 \pm 0.0308$	$0.3139 \pm 0.0250 \pm 0.0368$	$0.0211 \pm 0.0242 \pm 0.0272$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7213 \pm 0.0202 \pm 0.0266$	$0.2743 \pm 0.0250 \pm 0.0320$	$0.0345 \pm 0.0395 \pm 0.0605$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7690 \pm 0.0189 \pm 0.0295$	$0.2291 \pm 0.0244 \pm 0.0373$	$0.0182 \pm 0.0243 \pm 0.0422$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.7961 \pm 0.0182 \pm 0.0283$	$0.2108 \pm 0.0243 \pm 0.0369$	$0.0073 \pm 0.0040 \pm 0.0406$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8295 \pm 0.0171 \pm 0.0270$	$0.1763 \pm 0.0232 \pm 0.0364$	$0.0088 \pm 0.0092 \pm 0.0178$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8507 \pm 0.0163 \pm 0.0300$	$0.1603 \pm 0.0229 \pm 0.0388$	$0.0015 \pm 0.0017 \pm 0.0032$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.8907 \pm 0.0142 \pm 0.0287$	$0.1198 \pm 0.0217 \pm 0.0422$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9109 \pm 0.0128 \pm 0.0250$	$0.1035 \pm 0.0215 \pm 0.0381$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9457 \pm 0.0102 \pm 0.0149$	$0.0619 \pm 0.0185 \pm 0.0251$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.38: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 202 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0179±0.0063±0.0107	0.0204±0.0072±0.0144	0.0346±0.0091±0.0144	0.0457±0.0102±0.0132
-4.75	0.0183±0.0064±0.0104	0.0277±0.0084±0.0167	0.0525±0.0112±0.0222	0.0737±0.0125±0.0150
-4.50	0.0223±0.0073±0.0105	0.0540±0.0117±0.0253	0.0659±0.0122±0.0227	0.1261±0.0164±0.0210
-4.25	0.0401±0.0105±0.0159	0.0930±0.0151±0.0259	0.1347±0.0166±0.0262	0.1836±0.0187±0.0290
-4.00	0.0717±0.0133±0.0190	0.1726±0.0193±0.0230	0.1797±0.0181±0.0206	0.2086±0.0187±0.0360
-3.75	0.1320±0.0169±0.0180	0.2608±0.0216±0.0496	0.1977±0.0184±0.0339	0.1885±0.0182±0.0262
-3.50	0.2277±0.0202±0.0261	0.2597±0.0209±0.0256	0.2273±0.0194±0.0251	0.1852±0.0191±0.0279
-3.25	0.2915±0.0214±0.0254	0.2816±0.0208±0.0230	0.2606±0.0209±0.0258	0.1416±0.0199±0.0244
-3.00	0.3687±0.0224±0.0302	0.3211±0.0215±0.0313	0.2236±0.0211±0.0296	0.0884±0.0189±0.0294
-2.75	0.4442±0.0228±0.0270	0.3534±0.0221±0.0289	0.1626±0.0207±0.0295	0.0542±0.0187±0.0237
-2.50	0.5373±0.0226±0.0303	0.3372±0.0225±0.0293	0.1088±0.0200±0.0254	0.0244±0.0172±0.0238
-2.25	0.6035±0.0221±0.0281	0.3277±0.0233±0.0292	0.0619±0.0192±0.0274	0.0180±0.0427±0.0721
-2.00	0.6791±0.0211±0.0296	0.2801±0.0234±0.0320	0.0618±0.0255±0.0595	0.0031±0.0020±0.0130
-1.75	0.7585±0.0196±0.0247	0.2348±0.0233±0.0285	0.0018±0.0181±0.0338	0.0000±0.0000±0.0004
-1.50	0.8168±0.0174±0.0257	0.1916±0.0236±0.0308	0.0001±0.0472±0.0717	0.0000±0.0000±0.0000
-1.25	0.8903±0.0139±0.0238	0.1209±0.0216±0.0326	0.0102±0.0106±0.0207	0.0000±0.0000±0.0000
-1.00	0.9538±0.0093±0.0203	0.0485±0.0166±0.0321	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9833±0.0050±0.0075	0.0267±0.0180±0.0235	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	1.0000±0.0010±0.0010	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.39: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 202 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0008±0.0010±0.0022
-4.75	0.0000±0.0000±0.0000	0.0000±0.0000±0.0007	0.0029±0.0022±0.0039	0.0094±0.0045±0.0057
-4.50	0.0000±0.0000±0.0000	0.0055±0.0035±0.0060	0.0095±0.0045±0.0053	0.0236±0.0068±0.0154
-4.25	0.0000±0.0000±0.0070	0.0208±0.0074±0.0118	0.0458±0.0100±0.0140	0.0875±0.0132±0.0152
-4.00	0.0036±0.0026±0.0047	0.0494±0.0105±0.0158	0.1010±0.0145±0.0334	0.1715±0.0179±0.0268
-3.75	0.0297±0.0084±0.0113	0.1067±0.0146±0.0190	0.2205±0.0203±0.0427	0.2074±0.0188±0.0212
-3.50	0.0834±0.0133±0.0237	0.1960±0.0193±0.0272	0.2898±0.0215±0.0329	0.1880±0.0178±0.0392
-3.25	0.1669±0.0180±0.0210	0.2757±0.0212±0.0298	0.2825±0.0209±0.0279	0.1807±0.0187±0.0204
-3.00	0.2704±0.0214±0.0236	0.3074±0.0209±0.0221	0.3009±0.0222±0.0385	0.0898±0.0163±0.0402
-2.75	0.3847±0.0230±0.0271	0.3245±0.0213±0.0268	0.2281±0.0217±0.0364	0.0781±0.0196±0.0368
-2.50	0.4745±0.0232±0.0269	0.3613±0.0222±0.0243	0.1483±0.0211±0.0287	0.0321±0.0192±0.0498
-2.25	0.5659±0.0224±0.0286	0.3478±0.0231±0.0270	0.0896±0.0219±0.0320	0.0153±0.0054±0.0286
-2.00	0.6566±0.0215±0.0314	0.2954±0.0233±0.0321	0.0716±0.0261±0.0528	0.0097±0.0101±0.0184
-1.75	0.7349±0.0201±0.0259	0.2578±0.0237±0.0298	0.0034±0.0190±0.0278	0.0000±0.0000±0.0004
-1.50	0.8053±0.0179±0.0247	0.2024±0.0238±0.0298	0.0022±0.0501±0.0729	0.0000±0.0000±0.0000
-1.25	0.8882±0.0139±0.0226	0.1188±0.0216±0.0311	0.0218±0.0395±0.0624	0.0000±0.0000±0.0000
-1.00	0.9558±0.0091±0.0204	0.0445±0.0161±0.0330	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9831±0.0050±0.0076	0.0277±0.0184±0.0228	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	1.0000±0.0010±0.0010	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.40: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 202 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.8: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 202$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

205 GeV Data Sample

$\varepsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4698 \pm 0.0157 \pm 0.0173$	$0.3313 \pm 0.0151 \pm 0.0203$	$0.2097 \pm 0.0159 \pm 0.0202$
6.00	$0.6269 \pm 0.0148 \pm 0.0174$	$0.2957 \pm 0.0157 \pm 0.0194$	$0.0861 \pm 0.0135 \pm 0.0220$
10.00	$0.7020 \pm 0.0139 \pm 0.0201$	$0.2516 \pm 0.0156 \pm 0.0225$	$0.0476 \pm 0.0129 \pm 0.0177$
14.00	$0.7509 \pm 0.0132 \pm 0.0165$	$0.2135 \pm 0.0151 \pm 0.0186$	$0.0427 \pm 0.0157 \pm 0.0211$
18.00	$0.7902 \pm 0.0124 \pm 0.0177$	$0.1838 \pm 0.0149 \pm 0.0192$	$0.0363 \pm 0.0202 \pm 0.0321$
22.00	$0.8178 \pm 0.0116 \pm 0.0192$	$0.1663 \pm 0.0149 \pm 0.0199$	$0.0105 \pm 0.0237 \pm 0.0486$
25.50	$0.8419 \pm 0.0110 \pm 0.0156$	$0.1461 \pm 0.0145 \pm 0.0176$	$0.0253 \pm 0.0441 \pm 0.0751$

Table A.41: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 205 GeV, as a function of the energy cutoff, ε , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4361 \pm 0.0154 \pm 0.0182$	$0.3592 \pm 0.0152 \pm 0.0218$	$0.2033 \pm 0.0157 \pm 0.0247$
0.50	$0.5402 \pm 0.0152 \pm 0.0181$	$0.3392 \pm 0.0156 \pm 0.0189$	$0.1251 \pm 0.0149 \pm 0.0179$
0.70	$0.6522 \pm 0.0146 \pm 0.0194$	$0.2825 \pm 0.0156 \pm 0.0215$	$0.0703 \pm 0.0130 \pm 0.0181$
0.90	$0.7349 \pm 0.0136 \pm 0.0168$	$0.2352 \pm 0.0156 \pm 0.0195$	$0.0439 \pm 0.0114 \pm 0.0198$
1.10	$0.8319 \pm 0.0117 \pm 0.0149$	$0.1673 \pm 0.0144 \pm 0.0182$	$0.0044 \pm 0.0043 \pm 0.0077$
1.30	$0.9373 \pm 0.0076 \pm 0.0104$	$0.0632 \pm 0.0101 \pm 0.0139$	$0.0004 \pm 0.0002 \pm 0.0005$
1.50	$0.9933 \pm 0.0026 \pm 0.0045$	$0.0080 \pm 0.0050 \pm 0.0089$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.42: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 205 GeV, as a function of the cone radius, R , with a constant energy cutoff $\varepsilon = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.1939 \pm 0.0122 \pm 0.0156$	$0.4939 \pm 0.0156 \pm 0.0270$	$0.2559 \pm 0.0152 \pm 0.0224$	$0.0545 \pm 0.0120 \pm 0.0175$
-2.06	$0.2925 \pm 0.0139 \pm 0.0160$	$0.5186 \pm 0.0158 \pm 0.0204$	$0.1726 \pm 0.0149 \pm 0.0229$	$0.0294 \pm 0.0129 \pm 0.0194$
-1.90	$0.3836 \pm 0.0148 \pm 0.0175$	$0.4871 \pm 0.0160 \pm 0.0207$	$0.1268 \pm 0.0149 \pm 0.0186$	$0.0353 \pm 0.0194 \pm 0.0254$
-1.70	$0.4990 \pm 0.0152 \pm 0.0194$	$0.4291 \pm 0.0165 \pm 0.0248$	$0.0874 \pm 0.0156 \pm 0.0269$	$0.0062 \pm 0.0170 \pm 0.0897$
-1.52	$0.6053 \pm 0.0149 \pm 0.0230$	$0.3570 \pm 0.0168 \pm 0.0289$	$0.0655 \pm 0.0178 \pm 0.0238$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.40	$0.6933 \pm 0.0141 \pm 0.0220$	$0.2846 \pm 0.0165 \pm 0.0235$	$0.0555 \pm 0.0242 \pm 0.0416$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7456 \pm 0.0133 \pm 0.0182$	$0.2423 \pm 0.0164 \pm 0.0219$	$0.0414 \pm 0.0282 \pm 0.0454$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7742 \pm 0.0127 \pm 0.0188$	$0.2243 \pm 0.0165 \pm 0.0243$	$0.0016 \pm 0.0247 \pm 0.0421$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8095 \pm 0.0119 \pm 0.0203$	$0.1905 \pm 0.0161 \pm 0.0264$	$0.0000 \pm 0.0000 \pm 0.1011$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8390 \pm 0.0112 \pm 0.0203$	$0.1622 \pm 0.0156 \pm 0.0265$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8707 \pm 0.0102 \pm 0.0179$	$0.1311 \pm 0.0150 \pm 0.0278$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.9024 \pm 0.0091 \pm 0.0147$	$0.1004 \pm 0.0143 \pm 0.0274$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9326 \pm 0.0076 \pm 0.0127$	$0.0659 \pm 0.0131 \pm 0.0250$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9535 \pm 0.0063 \pm 0.0119$	$0.0461 \pm 0.0119 \pm 0.0264$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.43: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 205 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0057 $\pm$ 0.0023 $\pm$ 0.0055	0.0179 $\pm$ 0.0041 $\pm$ 0.0113	0.0278 $\pm$ 0.0054 $\pm$ 0.0073	0.0392 $\pm$ 0.0064 $\pm$ 0.0160
-4.75	0.0067 $\pm$ 0.0025 $\pm$ 0.0060	0.0228 $\pm$ 0.0048 $\pm$ 0.0129	0.0467 $\pm$ 0.0069 $\pm$ 0.0168	0.0588 $\pm$ 0.0080 $\pm$ 0.0087
-4.50	0.0117 $\pm$ 0.0035 $\pm$ 0.0057	0.0485 $\pm$ 0.0072 $\pm$ 0.0143	0.0854 $\pm$ 0.0095 $\pm$ 0.0174	0.1232 $\pm$ 0.0112 $\pm$ 0.0161
-4.25	0.0306 $\pm$ 0.0063 $\pm$ 0.0122	0.1117 $\pm$ 0.0110 $\pm$ 0.0114	0.1415 $\pm$ 0.0118 $\pm$ 0.0184	0.1748 $\pm$ 0.0122 $\pm$ 0.0161
-4.00	0.0773 $\pm$ 0.0097 $\pm$ 0.0105	0.1756 $\pm$ 0.0126 $\pm$ 0.0282	0.2192 $\pm$ 0.0136 $\pm$ 0.0293	0.1951 $\pm$ 0.0126 $\pm$ 0.0138
-3.75	0.1335 $\pm$ 0.0117 $\pm$ 0.0196	0.2577 $\pm$ 0.0144 $\pm$ 0.0171	0.2437 $\pm$ 0.0135 $\pm$ 0.0345	0.1813 $\pm$ 0.0122 $\pm$ 0.0306
-3.50	0.2085 $\pm$ 0.0134 $\pm$ 0.0161	0.3052 $\pm$ 0.0146 $\pm$ 0.0167	0.2562 $\pm$ 0.0138 $\pm$ 0.0210	0.1496 $\pm$ 0.0119 $\pm$ 0.0284
-3.25	0.2748 $\pm$ 0.0143 $\pm$ 0.0190	0.3561 $\pm$ 0.0149 $\pm$ 0.0210	0.2361 $\pm$ 0.0139 $\pm$ 0.0189	0.0967 $\pm$ 0.0112 $\pm$ 0.0187
-3.00	0.3332 $\pm$ 0.0148 $\pm$ 0.0197	0.3955 $\pm$ 0.0152 $\pm$ 0.0295	0.1977 $\pm$ 0.0138 $\pm$ 0.0179	0.0570 $\pm$ 0.0108 $\pm$ 0.0204
-2.75	0.4234 $\pm$ 0.0153 $\pm$ 0.0204	0.3821 $\pm$ 0.0151 $\pm$ 0.0163	0.1452 $\pm$ 0.0137 $\pm$ 0.0205	0.0607 $\pm$ 0.0137 $\pm$ 0.0208
-2.50	0.5179 $\pm$ 0.0154 $\pm$ 0.0230	0.3565 $\pm$ 0.0153 $\pm$ 0.0228	0.1008 $\pm$ 0.0133 $\pm$ 0.0178	0.0397 $\pm$ 0.0166 $\pm$ 0.0233
-2.25	0.6089 $\pm$ 0.0150 $\pm$ 0.0211	0.3083 $\pm$ 0.0154 $\pm$ 0.0222	0.0798 $\pm$ 0.0143 $\pm$ 0.0184	0.0581 $\pm$ 0.0354 $\pm$ 0.0427
-2.00	0.6914 $\pm$ 0.0141 $\pm$ 0.0215	0.2580 $\pm$ 0.0155 $\pm$ 0.0207	0.0710 $\pm$ 0.0173 $\pm$ 0.0268	0.0000 $\pm$ 0.0000 $\pm$ 0.0520
-1.75	0.7798 $\pm$ 0.0127 $\pm$ 0.0227	0.1976 $\pm$ 0.0151 $\pm$ 0.0260	0.0508 $\pm$ 0.0247 $\pm$ 0.0423	0.0000 $\pm$ 0.0000 $\pm$ 0.0005
-1.50	0.8439 $\pm$ 0.0109 $\pm$ 0.0191	0.1513 $\pm$ 0.0150 $\pm$ 0.0240	0.0257 $\pm$ 0.0287 $\pm$ 0.0386	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9085 $\pm$ 0.0085 $\pm$ 0.0166	0.0913 $\pm$ 0.0137 $\pm$ 0.0234	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9527 $\pm$ 0.0061 $\pm$ 0.0131	0.0498 $\pm$ 0.0120 $\pm$ 0.0318	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9875 $\pm$ 0.0029 $\pm$ 0.0074	0.0098 $\pm$ 0.0080 $\pm$ 0.0128	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	0.9999 $\pm$ 0.0012 $\pm$ 0.0012	0.0000 $\pm$ 0.0000 $\pm$ 0.0001	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.44: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 205 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0007 $\pm$ 0.0008 $\pm$ 0.0013	0.0013 $\pm$ 0.0008 $\pm$ 0.0012
-4.75	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0018 $\pm$ 0.0011 $\pm$ 0.0016	0.0063 $\pm$ 0.0022 $\pm$ 0.0025
-4.50	0.0023 $\pm$ 0.0028 $\pm$ 0.0046	0.0007 $\pm$ 0.0007 $\pm$ 0.0012	0.0101 $\pm$ 0.0030 $\pm$ 0.0036	0.0320 $\pm$ 0.0053 $\pm$ 0.0075
-4.25	0.0032 $\pm$ 0.0020 $\pm$ 0.0022	0.0108 $\pm$ 0.0032 $\pm$ 0.0038	0.0442 $\pm$ 0.0066 $\pm$ 0.0101	0.1064 $\pm$ 0.0101 $\pm$ 0.0137
-4.00	0.0096 $\pm$ 0.0032 $\pm$ 0.0037	0.0536 $\pm$ 0.0073 $\pm$ 0.0082	0.1076 $\pm$ 0.0101 $\pm$ 0.0251	0.2195 $\pm$ 0.0135 $\pm$ 0.0365
-3.75	0.0293 $\pm$ 0.0058 $\pm$ 0.0065	0.1428 $\pm$ 0.0116 $\pm$ 0.0138	0.2173 $\pm$ 0.0135 $\pm$ 0.0179	0.2336 $\pm$ 0.0134 $\pm$ 0.0190
-3.50	0.0900 $\pm$ 0.0100 $\pm$ 0.0165	0.2234 $\pm$ 0.0135 $\pm$ 0.0301	0.2802 $\pm$ 0.0143 $\pm$ 0.0254	0.2062 $\pm$ 0.0125 $\pm$ 0.0173
-3.25	0.1746 $\pm$ 0.0127 $\pm$ 0.0191	0.3207 $\pm$ 0.0151 $\pm$ 0.0248	0.2828 $\pm$ 0.0140 $\pm$ 0.0214	0.1459 $\pm$ 0.0115 $\pm$ 0.0166
-3.00	0.2481 $\pm$ 0.0140 $\pm$ 0.0264	0.4053 $\pm$ 0.0154 $\pm$ 0.0260	0.2321 $\pm$ 0.0134 $\pm$ 0.0166	0.0876 $\pm$ 0.0112 $\pm$ 0.0180
-2.75	0.3664 $\pm$ 0.0153 $\pm$ 0.0181	0.3987 $\pm$ 0.0151 $\pm$ 0.0296	0.1755 $\pm$ 0.0135 $\pm$ 0.0252	0.0603 $\pm$ 0.0118 $\pm$ 0.0204
-2.50	0.4656 $\pm$ 0.0155 $\pm$ 0.0229	0.3799 $\pm$ 0.0153 $\pm$ 0.0200	0.1287 $\pm$ 0.0137 $\pm$ 0.0206	0.0394 $\pm$ 0.0154 $\pm$ 0.0295
-2.25	0.5752 $\pm$ 0.0153 $\pm$ 0.0263	0.3321 $\pm$ 0.0153 $\pm$ 0.0228	0.0885 $\pm$ 0.0145 $\pm$ 0.0214	0.0551 $\pm$ 0.0350 $\pm$ 0.0971
-2.00	0.6602 $\pm$ 0.0145 $\pm$ 0.0216	0.2867 $\pm$ 0.0157 $\pm$ 0.0234	0.0747 $\pm$ 0.0178 $\pm$ 0.0248	0.0219 $\pm$ 0.0549 $\pm$ 0.0646
-1.75	0.7608 $\pm$ 0.0130 $\pm$ 0.0222	0.2133 $\pm$ 0.0154 $\pm$ 0.0234	0.0632 $\pm$ 0.0248 $\pm$ 0.0404	0.0000 $\pm$ 0.0000 $\pm$ 0.0001
-1.50	0.8333 $\pm$ 0.0113 $\pm$ 0.0185	0.1595 $\pm$ 0.0150 $\pm$ 0.0223	0.0914 $\pm$ 0.0691 $\pm$ 0.2163	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9047 $\pm$ 0.0086 $\pm$ 0.0164	0.0947 $\pm$ 0.0137 $\pm$ 0.0240	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9521 $\pm$ 0.0061 $\pm$ 0.0132	0.0496 $\pm$ 0.0122 $\pm$ 0.0344	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9890 $\pm$ 0.0025 $\pm$ 0.0067	0.0036 $\pm$ 0.0069 $\pm$ 0.0086	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	0.9999 $\pm$ 0.0012 $\pm$ 0.0012	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.45: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 205 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.9: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 205$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

207 GeV Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4569 \pm 0.0127 \pm 0.0176$	$0.3655 \pm 0.0127 \pm 0.0169$	$0.1839 \pm 0.0122 \pm 0.0167$
6.00	$0.6375 \pm 0.0121 \pm 0.0146$	$0.3004 \pm 0.0129 \pm 0.0177$	$0.0615 \pm 0.0096 \pm 0.0150$
10.00	$0.6985 \pm 0.0115 \pm 0.0153$	$0.2613 \pm 0.0128 \pm 0.0171$	$0.0369 \pm 0.0102 \pm 0.0182$
14.00	$0.7446 \pm 0.0110 \pm 0.0147$	$0.2280 \pm 0.0127 \pm 0.0165$	$0.0210 \pm 0.0112 \pm 0.0194$
18.00	$0.7819 \pm 0.0104 \pm 0.0148$	$0.1986 \pm 0.0125 \pm 0.0165$	$0.0113 \pm 0.0145 \pm 0.0191$
22.00	$0.8143 \pm 0.0099 \pm 0.0155$	$0.1712 \pm 0.0123 \pm 0.0174$	$0.0129 \pm 0.0203 \pm 0.0278$
25.50	$0.8436 \pm 0.0092 \pm 0.0144$	$0.1460 \pm 0.0120 \pm 0.0171$	$-0.0112 \pm 0.0220 \pm 0.0399$

Table A.46: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 207 GeV, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4574 \pm 0.0126 \pm 0.0156$	$0.3124 \pm 0.0119 \pm 0.0143$	$0.2343 \pm 0.0136 \pm 0.0166$
0.50	$0.5580 \pm 0.0124 \pm 0.0142$	$0.3111 \pm 0.0124 \pm 0.0181$	$0.1369 \pm 0.0127 \pm 0.0184$
0.70	$0.6527 \pm 0.0120 \pm 0.0130$	$0.2928 \pm 0.0129 \pm 0.0150$	$0.0535 \pm 0.0096 \pm 0.0132$
0.90	$0.7515 \pm 0.0111 \pm 0.0129$	$0.2314 \pm 0.0127 \pm 0.0154$	$0.0143 \pm 0.0060 \pm 0.0084$
1.10	$0.8453 \pm 0.0095 \pm 0.0113$	$0.1535 \pm 0.0115 \pm 0.0139$	$0.0012 \pm 0.0018 \pm 0.0032$
1.30	$0.9403 \pm 0.0066 \pm 0.0072$	$0.0585 \pm 0.0078 \pm 0.0085$	$0.0010 \pm 0.0014 \pm 0.0024$
1.50	$0.9959 \pm 0.0030 \pm 0.0034$	$0.0015 \pm 0.0034 \pm 0.0054$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.47: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 207 GeV, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.2106 \pm 0.0101 \pm 0.0124$	$0.4701 \pm 0.0127 \pm 0.0147$	$0.2581 \pm 0.0125 \pm 0.0233$	$0.0658 \pm 0.0103 \pm 0.0196$
-2.06	$0.3157 \pm 0.0114 \pm 0.0158$	$0.4928 \pm 0.0129 \pm 0.0186$	$0.1757 \pm 0.0125 \pm 0.0177$	$0.0288 \pm 0.0106 \pm 0.0248$
-1.90	$0.3886 \pm 0.0120 \pm 0.0176$	$0.4836 \pm 0.0131 \pm 0.0180$	$0.1290 \pm 0.0125 \pm 0.0144$	$0.0041 \pm 0.0103 \pm 0.0185$
-1.70	$0.5211 \pm 0.0124 \pm 0.0151$	$0.4163 \pm 0.0134 \pm 0.0170$	$0.0662 \pm 0.0117 \pm 0.0147$	$0.0153 \pm 0.0084 \pm 0.0162$
-1.52	$0.6361 \pm 0.0121 \pm 0.0183$	$0.3354 \pm 0.0134 \pm 0.0188$	$0.0225 \pm 0.0107 \pm 0.0154$	$0.0012 \pm 0.0014 \pm 0.0028$
-1.40	$0.6900 \pm 0.0116 \pm 0.0154$	$0.2964 \pm 0.0137 \pm 0.0172$	$0.0107 \pm 0.0116 \pm 0.0152$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7423 \pm 0.0111 \pm 0.0143$	$0.2517 \pm 0.0134 \pm 0.0175$	$0.0007 \pm 0.0141 \pm 0.0164$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7792 \pm 0.0105 \pm 0.0149$	$0.2189 \pm 0.0133 \pm 0.0187$	$0.0120 \pm 0.0231 \pm 0.0304$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8087 \pm 0.0100 \pm 0.0149$	$0.1910 \pm 0.0131 \pm 0.0179$	$0.0230 \pm 0.0492 \pm 0.0603$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8400 \pm 0.0094 \pm 0.0145$	$0.1592 \pm 0.0126 \pm 0.0178$	$0.0158 \pm 0.0249 \pm 0.0387$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8807 \pm 0.0085 \pm 0.0123$	$0.1152 \pm 0.0117 \pm 0.0144$	$0.0000 \pm 0.0000 \pm 0.0073$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.9104 \pm 0.0076 \pm 0.0110$	$0.0872 \pm 0.0110 \pm 0.0129$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9326 \pm 0.0067 \pm 0.0106$	$0.0661 \pm 0.0107 \pm 0.0143$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9569 \pm 0.0056 \pm 0.0108$	$0.0381 \pm 0.0097 \pm 0.0133$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.48: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, at 207 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0116 $\pm$ 0.0027 $\pm$ 0.0049	0.0197 $\pm$ 0.0036 $\pm$ 0.0092	0.0232 $\pm$ 0.0041 $\pm$ 0.0104	0.0342 $\pm$ 0.0048 $\pm$ 0.0076
-4.75	0.0121 $\pm$ 0.0028 $\pm$ 0.0048	0.0257 $\pm$ 0.0041 $\pm$ 0.0094	0.0363 $\pm$ 0.0051 $\pm$ 0.0120	0.0660 $\pm$ 0.0067 $\pm$ 0.0110
-4.50	0.0170 $\pm$ 0.0035 $\pm$ 0.0038	0.0527 $\pm$ 0.0062 $\pm$ 0.0156	0.0834 $\pm$ 0.0077 $\pm$ 0.0158	0.1272 $\pm$ 0.0090 $\pm$ 0.0149
-4.25	0.0444 $\pm$ 0.0059 $\pm$ 0.0097	0.1072 $\pm$ 0.0087 $\pm$ 0.0163	0.1425 $\pm$ 0.0095 $\pm$ 0.0256	0.1721 $\pm$ 0.0099 $\pm$ 0.0175
-4.00	0.0896 $\pm$ 0.0081 $\pm$ 0.0115	0.1678 $\pm$ 0.0102 $\pm$ 0.0133	0.1916 $\pm$ 0.0104 $\pm$ 0.0154	0.2031 $\pm$ 0.0103 $\pm$ 0.0139
-3.75	0.1528 $\pm$ 0.0099 $\pm$ 0.0137	0.2293 $\pm$ 0.0111 $\pm$ 0.0139	0.2343 $\pm$ 0.0109 $\pm$ 0.0155	0.1898 $\pm$ 0.0102 $\pm$ 0.0208
-3.50	0.2279 $\pm$ 0.0111 $\pm$ 0.0142	0.2815 $\pm$ 0.0117 $\pm$ 0.0154	0.2484 $\pm$ 0.0111 $\pm$ 0.0224	0.1573 $\pm$ 0.0100 $\pm$ 0.0186
-3.25	0.3032 $\pm$ 0.0119 $\pm$ 0.0142	0.3303 $\pm$ 0.0120 $\pm$ 0.0139	0.2357 $\pm$ 0.0113 $\pm$ 0.0152	0.1028 $\pm$ 0.0094 $\pm$ 0.0129
-3.00	0.3777 $\pm$ 0.0124 $\pm$ 0.0161	0.3538 $\pm$ 0.0121 $\pm$ 0.0179	0.1922 $\pm$ 0.0112 $\pm$ 0.0187	0.0732 $\pm$ 0.0094 $\pm$ 0.0163
-2.75	0.4516 $\pm$ 0.0125 $\pm$ 0.0195	0.3640 $\pm$ 0.0123 $\pm$ 0.0209	0.1443 $\pm$ 0.0110 $\pm$ 0.0177	0.0411 $\pm$ 0.0094 $\pm$ 0.0191
-2.50	0.5352 $\pm$ 0.0125 $\pm$ 0.0180	0.3493 $\pm$ 0.0125 $\pm$ 0.0156	0.0949 $\pm$ 0.0106 $\pm$ 0.0177	0.0134 $\pm$ 0.0091 $\pm$ 0.0199
-2.25	0.6235 $\pm$ 0.0121 $\pm$ 0.0179	0.3037 $\pm$ 0.0125 $\pm$ 0.0168	0.0627 $\pm$ 0.0113 $\pm$ 0.0136	0.0109 $\pm$ 0.0145 $\pm$ 0.0267
-2.00	0.7034 $\pm$ 0.0115 $\pm$ 0.0174	0.2560 $\pm$ 0.0126 $\pm$ 0.0165	0.0408 $\pm$ 0.0119 $\pm$ 0.0175	0.0056 $\pm$ 0.0163 $\pm$ 0.0266
-1.75	0.7805 $\pm$ 0.0105 $\pm$ 0.0169	0.2043 $\pm$ 0.0125 $\pm$ 0.0182	0.0075 $\pm$ 0.0128 $\pm$ 0.0243	0.0015 $\pm$ 0.0017 $\pm$ 0.0033
-1.50	0.8533 $\pm$ 0.0090 $\pm$ 0.0139	0.1410 $\pm$ 0.0119 $\pm$ 0.0156	0.0118 $\pm$ 0.0182 $\pm$ 0.0320	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9140 $\pm$ 0.0073 $\pm$ 0.0123	0.0820 $\pm$ 0.0107 $\pm$ 0.0139	0.0000 $\pm$ 0.0000 $\pm$ 0.0276	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9603 $\pm$ 0.0052 $\pm$ 0.0121	0.0332 $\pm$ 0.0090 $\pm$ 0.0141	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9885 $\pm$ 0.0033 $\pm$ 0.0075	0.0047 $\pm$ 0.0064 $\pm$ 0.0114	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	1.0000 $\pm$ 0.0027 $\pm$ 0.0027	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.49: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 207 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0003 $\pm$ 0.0003 $\pm$ 0.0007
-4.75	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0001	0.0008 $\pm$ 0.0006 $\pm$ 0.0007	0.0055 $\pm$ 0.0016 $\pm$ 0.0017
-4.50	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0008 $\pm$ 0.0006 $\pm$ 0.0007	0.0140 $\pm$ 0.0029 $\pm$ 0.0049	0.0327 $\pm$ 0.0044 $\pm$ 0.0079
-4.25	0.0006 $\pm$ 0.0006 $\pm$ 0.0008	0.0122 $\pm$ 0.0028 $\pm$ 0.0037	0.0567 $\pm$ 0.0061 $\pm$ 0.0074	0.1065 $\pm$ 0.0080 $\pm$ 0.0114
-4.00	0.0076 $\pm$ 0.0022 $\pm$ 0.0029	0.0538 $\pm$ 0.0060 $\pm$ 0.0080	0.1278 $\pm$ 0.0090 $\pm$ 0.0152	0.1917 $\pm$ 0.0102 $\pm$ 0.0210
-3.75	0.0321 $\pm$ 0.0048 $\pm$ 0.0055	0.1226 $\pm$ 0.0087 $\pm$ 0.0097	0.2218 $\pm$ 0.0110 $\pm$ 0.0147	0.2183 $\pm$ 0.0106 $\pm$ 0.0128
-3.50	0.0942 $\pm$ 0.0079 $\pm$ 0.0085	0.2311 $\pm$ 0.0112 $\pm$ 0.0159	0.2598 $\pm$ 0.0113 $\pm$ 0.0148	0.2190 $\pm$ 0.0106 $\pm$ 0.0162
-3.25	0.1933 $\pm$ 0.0107 $\pm$ 0.0119	0.3052 $\pm$ 0.0119 $\pm$ 0.0184	0.2652 $\pm$ 0.0112 $\pm$ 0.0165	0.1666 $\pm$ 0.0101 $\pm$ 0.0151
-3.00	0.2888 $\pm$ 0.0120 $\pm$ 0.0127	0.3717 $\pm$ 0.0123 $\pm$ 0.0226	0.2290 $\pm$ 0.0111 $\pm$ 0.0176	0.0865 $\pm$ 0.0090 $\pm$ 0.0209
-2.75	0.3915 $\pm$ 0.0126 $\pm$ 0.0156	0.3799 $\pm$ 0.0122 $\pm$ 0.0152	0.1777 $\pm$ 0.0111 $\pm$ 0.0166	0.0511 $\pm$ 0.0092 $\pm$ 0.0128
-2.50	0.4925 $\pm$ 0.0126 $\pm$ 0.0198	0.3610 $\pm$ 0.0124 $\pm$ 0.0187	0.1214 $\pm$ 0.0111 $\pm$ 0.0186	0.0342 $\pm$ 0.0113 $\pm$ 0.0168
-2.25	0.5968 $\pm$ 0.0124 $\pm$ 0.0167	0.3147 $\pm$ 0.0123 $\pm$ 0.0161	0.0824 $\pm$ 0.0118 $\pm$ 0.0156	0.0164 $\pm$ 0.0143 $\pm$ 0.0411
-2.00	0.6823 $\pm$ 0.0117 $\pm$ 0.0181	0.2713 $\pm$ 0.0126 $\pm$ 0.0184	0.0465 $\pm$ 0.0129 $\pm$ 0.0170	0.0310 $\pm$ 0.0365 $\pm$ 0.0778
-1.75	0.7619 $\pm$ 0.0108 $\pm$ 0.0168	0.2202 $\pm$ 0.0127 $\pm$ 0.0170	0.0220 $\pm$ 0.0164 $\pm$ 0.0251	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.50	0.8456 $\pm$ 0.0092 $\pm$ 0.0126	0.1472 $\pm$ 0.0120 $\pm$ 0.0142	0.0036 $\pm$ 0.0215 $\pm$ 0.0349	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.25	0.9098 $\pm$ 0.0074 $\pm$ 0.0139	0.0855 $\pm$ 0.0108 $\pm$ 0.0167	0.0000 $\pm$ 0.0000 $\pm$ 0.0325	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.00	0.9599 $\pm$ 0.0052 $\pm$ 0.0120	0.0319 $\pm$ 0.0089 $\pm$ 0.0129	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.75	0.9885 $\pm$ 0.0033 $\pm$ 0.0077	0.0045 $\pm$ 0.0064 $\pm$ 0.0123	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.50	1.0000 $\pm$ 0.0027 $\pm$ 0.0027	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.50: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 207 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

Figure A.10: The hadron level n -jet rates for the R and ϵ variants of the CONE algorithm are shown in the top two plots. The 2-, 3-, 4- and 5-jet rates as a function of y_{cut} for the JADE, Durham and Cambridge algorithms for the data with $\sqrt{s} = 207$ GeV are shown in the lower three plots. In all plots the PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Outer error bars indicate total errors while the inner errors indicate statistical errors.

91 GeV Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	0.5201 $\pm$ 0.0018 $\pm$ 1.0402	0.3235 $\pm$ 0.0018 $\pm$ 0.6470	0.1570 $\pm$ 0.0015 $\pm$ 0.3141
6.00	0.7175 $\pm$ 0.0017 $\pm$ 1.4350	0.2396 $\pm$ 0.0016 $\pm$ 0.4793	0.0431 $\pm$ 0.0008 $\pm$ 0.0863
10.00	0.8007 $\pm$ 0.0015 $\pm$ 1.6015	0.1827 $\pm$ 0.0015 $\pm$ 0.3654	0.0165 $\pm$ 0.0005 $\pm$ 0.0330
14.00	0.8659 $\pm$ 0.0013 $\pm$ 1.7318	0.1294 $\pm$ 0.0013 $\pm$ 0.2589	0.0043 $\pm$ 0.0003 $\pm$ 0.0085
18.00	0.9214 $\pm$ 0.0010 $\pm$ 1.8428	0.0778 $\pm$ 0.0010 $\pm$ 0.1556	0.0003 $\pm$ 0.0001 $\pm$ 0.0007
22.00	0.9701 $\pm$ 0.0006 $\pm$ 1.9402	0.0295 $\pm$ 0.0006 $\pm$ 0.0591	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
25.50	0.9957 $\pm$ 0.0003 $\pm$ 1.9914	0.0044 $\pm$ 0.0002 $\pm$ 0.0088	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.51: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 91 GeV, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	0.6142 $\pm$ 0.0018 $\pm$ 1.2284	0.2870 $\pm$ 0.0017 $\pm$ 0.5740	0.0987 $\pm$ 0.0012 $\pm$ 0.1975
0.50	0.6794 $\pm$ 0.0017 $\pm$ 1.3587	0.2604 $\pm$ 0.0017 $\pm$ 0.5209	0.0601 $\pm$ 0.0010 $\pm$ 0.1201
0.70	0.7422 $\pm$ 0.0016 $\pm$ 1.4845	0.2237 $\pm$ 0.0016 $\pm$ 0.4475	0.0342 $\pm$ 0.0008 $\pm$ 0.0685
0.90	0.8049 $\pm$ 0.0015 $\pm$ 1.6097	0.1818 $\pm$ 0.0015 $\pm$ 0.3637	0.0134 $\pm$ 0.0005 $\pm$ 0.0268
1.10	0.8740 $\pm$ 0.0012 $\pm$ 1.7479	0.1233 $\pm$ 0.0013 $\pm$ 0.2466	0.0028 $\pm$ 0.0002 $\pm$ 0.0057
1.30	0.9474 $\pm$ 0.0008 $\pm$ 1.8947	0.0522 $\pm$ 0.0009 $\pm$ 0.1044	0.0004 $\pm$ 0.0001 $\pm$ 0.0009
1.50	0.9944 $\pm$ 0.0003 $\pm$ 1.9888	0.0055 $\pm$ 0.0003 $\pm$ 0.0110	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.52: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, at 91 GeV, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	0.1282 $\pm$ 0.0012 $\pm$ 0.2564	0.4385 $\pm$ 0.0018 $\pm$ 0.8769	0.3138 $\pm$ 0.0018 $\pm$ 0.6277	0.0996 $\pm$ 0.0012 $\pm$ 0.1991
-2.06	0.2298 $\pm$ 0.0015 $\pm$ 0.4597	0.5123 $\pm$ 0.0019 $\pm$ 1.0246	0.2192 $\pm$ 0.0017 $\pm$ 0.4385	0.0373 $\pm$ 0.0008 $\pm$ 0.0747
-1.90	0.3130 $\pm$ 0.0016 $\pm$ 0.6260	0.5173 $\pm$ 0.0019 $\pm$ 1.0345	0.1542 $\pm$ 0.0015 $\pm$ 0.3084	0.0168 $\pm$ 0.0006 $\pm$ 0.0335
-1.70	0.4386 $\pm$ 0.0018 $\pm$ 0.8772	0.4755 $\pm$ 0.0019 $\pm$ 0.9510	0.0834 $\pm$ 0.0011 $\pm$ 0.1669	0.0040 $\pm$ 0.0003 $\pm$ 0.0079
-1.52	0.5545 $\pm$ 0.0018 $\pm$ 1.1091	0.4048 $\pm$ 0.0019 $\pm$ 0.8097	0.0410 $\pm$ 0.0008 $\pm$ 0.0819	0.0008 $\pm$ 0.0002 $\pm$ 0.0017
-1.40	0.6371 $\pm$ 0.0018 $\pm$ 1.2741	0.3422 $\pm$ 0.0018 $\pm$ 0.6845	0.0213 $\pm$ 0.0006 $\pm$ 0.0426	0.0001 $\pm$ 0.0000 $\pm$ 0.0002
-1.30	0.6973 $\pm$ 0.0017 $\pm$ 1.3947	0.2920 $\pm$ 0.0018 $\pm$ 0.5841	0.0111 $\pm$ 0.0004 $\pm$ 0.0223	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.22	0.7441 $\pm$ 0.0016 $\pm$ 1.4882	0.2505 $\pm$ 0.0017 $\pm$ 0.5011	0.0057 $\pm$ 0.0003 $\pm$ 0.0115	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.15	0.7828 $\pm$ 0.0015 $\pm$ 1.5657	0.2142 $\pm$ 0.0016 $\pm$ 0.4285	0.0032 $\pm$ 0.0003 $\pm$ 0.0064	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.10	0.8148 $\pm$ 0.0014 $\pm$ 1.6296	0.1838 $\pm$ 0.0015 $\pm$ 0.3677	0.0015 $\pm$ 0.0002 $\pm$ 0.0030	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-1.02	0.8518 $\pm$ 0.0013 $\pm$ 1.7037	0.1478 $\pm$ 0.0014 $\pm$ 0.2956	0.0005 $\pm$ 0.0001 $\pm$ 0.0010	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.94	0.8889 $\pm$ 0.0012 $\pm$ 1.7778	0.1111 $\pm$ 0.0012 $\pm$ 0.2222	0.0000 $\pm$ 0.0000 $\pm$ 0.0001	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.87	0.9175 $\pm$ 0.0010 $\pm$ 1.8350	0.0825 $\pm$ 0.0011 $\pm$ 0.1651	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000
-0.80	0.9424 $\pm$ 0.0008 $\pm$ 1.8848	0.0577 $\pm$ 0.0009 $\pm$ 0.1153	0.0000 $\pm$ 0.0000 $\pm$ 0.0000	0.0000 $\pm$ 0.0000 $\pm$ 0.0000

Table A.53: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5, using the JADE algorithm, at 91 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-4.93	0.0129±0.0004±0.0257	0.0227±0.0006±0.0455	0.0285±0.0006±0.0569	0.0317±0.0007±0.0635
-4.81	0.0129±0.0004±0.0257	0.0228±0.0006±0.0455	0.0284±0.0006±0.0569	0.0318±0.0007±0.0635
-4.68	0.0129±0.0004±0.0257	0.0228±0.0006±0.0455	0.0284±0.0006±0.0568	0.0321±0.0006±0.0641
-4.56	0.0128±0.0004±0.0257	0.0227±0.0005±0.0454	0.0287±0.0006±0.0575	0.0335±0.0006±0.0670
-4.43	0.0129±0.0004±0.0257	0.0230±0.0006±0.0461	0.0300±0.0006±0.0601	0.0388±0.0007±0.0777
-4.31	0.0129±0.0004±0.0258	0.0238±0.0006±0.0476	0.0345±0.0007±0.0690	0.0522±0.0008±0.1044
-4.18	0.0133±0.0004±0.0265	0.0267±0.0006±0.0534	0.0463±0.0008±0.0927	0.0758±0.0010±0.1516
-4.06	0.0148±0.0004±0.0297	0.0352±0.0007±0.0705	0.0686±0.0009±0.1372	0.1098±0.0012±0.2196
-3.93	0.0194±0.0005±0.0388	0.0531±0.0008±0.1062	0.1011±0.0011±0.2022	0.1458±0.0013±0.2916
-3.81	0.0292±0.0006±0.0585	0.0816±0.0010±0.1631	0.1412±0.0013±0.2825	0.1789±0.0014±0.3578
-3.68	0.0468±0.0008±0.0935	0.1216±0.0012±0.2432	0.1805±0.0014±0.3610	0.2031±0.0015±0.4063
-3.56	0.0765±0.0010±0.1531	0.1635±0.0014±0.3270	0.2177±0.0015±0.4355	0.2087±0.0015±0.4174
-3.43	0.1132±0.0012±0.2264	0.2109±0.0015±0.4217	0.2424±0.0016±0.4848	0.1986±0.0015±0.3972
-3.31	0.1581±0.0014±0.3162	0.2562±0.0016±0.5124	0.2543±0.0016±0.5085	0.1758±0.0015±0.3517
-3.18	0.2095±0.0015±0.4189	0.2970±0.0017±0.5940	0.2499±0.0016±0.4997	0.1453±0.0014±0.2907
-3.06	0.2632±0.0017±0.5264	0.3269±0.0017±0.6539	0.2357±0.0016±0.4714	0.1141±0.0012±0.2282
-2.93	0.3175±0.0017±0.6350	0.3502±0.0017±0.7003	0.2118±0.0016±0.4236	0.0857±0.0011±0.1714
-2.81	0.3722±0.0018±0.7445	0.3625±0.0018±0.7251	0.1836±0.0015±0.3673	0.0621±0.0010±0.1243
-2.68	0.4245±0.0018±0.8491	0.3650±0.0018±0.7300	0.1585±0.0014±0.3171	0.0422±0.0008±0.0844
-2.56	0.4740±0.0019±0.9481	0.3619±0.0018±0.7239	0.1312±0.0013±0.2624	0.0281±0.0007±0.0562
-2.43	0.5220±0.0019±1.0440	0.3504±0.0018±0.7009	0.1071±0.0012±0.2142	0.0182±0.0006±0.0364
-2.31	0.5667±0.0019±1.1335	0.3353±0.0018±0.6706	0.0862±0.0011±0.1724	0.0108±0.0004±0.0217
-2.18	0.6122±0.0018±1.2245	0.3146±0.0017±0.6292	0.0662±0.0010±0.1324	0.0067±0.0004±0.0133
-2.06	0.6556±0.0018±1.3112	0.2920±0.0017±0.5840	0.0487±0.0009±0.0975	0.0034±0.0003±0.0068
-1.93	0.6986±0.0017±1.3972	0.2644±0.0017±0.5289	0.0349±0.0007±0.0698	0.0020±0.0002±0.0039
-1.81	0.7387±0.0017±1.4775	0.2370±0.0016±0.4740	0.0231±0.0006±0.0462	0.0008±0.0001±0.0017
-1.68	0.7782±0.0016±1.5564	0.2067±0.0015±0.4135	0.0144±0.0005±0.0289	0.0004±0.0001±0.0007
-1.56	0.8164±0.0015±1.6328	0.1749±0.0014±0.3499	0.0082±0.0004±0.0165	0.0001±0.0000±0.0003
-1.43	0.8511±0.0013±1.7023	0.1445±0.0013±0.2890	0.0040±0.0002±0.0080	0.0000±0.0000±0.0000
-1.31	0.8836±0.0012±1.7673	0.1142±0.0012±0.2285	0.0018±0.0002±0.0037	0.0000±0.0000±0.0000
-1.18	0.9130±0.0011±1.8260	0.0860±0.0011±0.1720	0.0007±0.0001±0.0015	0.0000±0.0000±0.0000
-1.06	0.9373±0.0009±1.8747	0.0624±0.0009±0.1249	0.0001±0.0000±0.0002	0.0000±0.0000±0.0000
-0.93	0.9589±0.0007±1.9179	0.0409±0.0008±0.0818	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.81	0.9766±0.0005±1.9532	0.0233±0.0006±0.0466	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.68	0.9905±0.0003±1.9810	0.0094±0.0004±0.0188	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.56	0.9986±0.0001±1.9973	0.0013±0.0002±0.0026	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.54: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, at 91 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-4.93	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0001±0.0001±0.0002	0.0000±0.0000±0.0000
-4.81	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0001	0.0001±0.0000±0.0001
-4.68	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000	0.0002±0.0000±0.0005
-4.56	0.0000±0.0000±0.0000	0.0000±0.0000±0.0001	0.0001±0.0000±0.0002	0.0006±0.0001±0.0011
-4.43	0.0000±0.0000±0.0000	0.0000±0.0000±0.0001	0.0003±0.0000±0.0006	0.0017±0.0001±0.0034
-4.31	0.0000±0.0000±0.0000	0.0001±0.0000±0.0002	0.0007±0.0001±0.0014	0.0052±0.0002±0.0103
-4.18	0.0000±0.0000±0.0000	0.0002±0.0000±0.0004	0.0023±0.0001±0.0047	0.0137±0.0004±0.0274
-4.06	0.0001±0.0000±0.0002	0.0006±0.0001±0.0013	0.0074±0.0003±0.0147	0.0307±0.0006±0.0615
-3.93	0.0002±0.0000±0.0004	0.0026±0.0002±0.0052	0.0186±0.0005±0.0372	0.0628±0.0009±0.1256
-3.81	0.0005±0.0001±0.0011	0.0077±0.0003±0.0155	0.0402±0.0007±0.0805	0.1072±0.0011±0.2145
-3.68	0.0018±0.0001±0.0036	0.0188±0.0005±0.0376	0.0772±0.0010±0.1545	0.1612±0.0014±0.3224
-3.56	0.0047±0.0002±0.0093	0.0406±0.0007±0.0811	0.1295±0.0012±0.2589	0.2109±0.0015±0.4218
-3.43	0.0120±0.0004±0.0241	0.0763±0.0010±0.1525	0.1871±0.0014±0.3741	0.2465±0.0016±0.4931
-3.31	0.0270±0.0006±0.0541	0.1249±0.0012±0.2498	0.2449±0.0016±0.4899	0.2551±0.0016±0.5102
-3.18	0.0527±0.0008±0.1054	0.1858±0.0014±0.3717	0.2859±0.0017±0.5719	0.2425±0.0016±0.4850
-3.06	0.0920±0.0011±0.1841	0.2498±0.0016±0.4996	0.3049±0.0017±0.6098	0.2073±0.0015±0.4146
-2.93	0.1448±0.0013±0.2896	0.3095±0.0017±0.6189	0.2972±0.0017±0.5944	0.1638±0.0014±0.3276
-2.81	0.2079±0.0015±0.4158	0.3573±0.0018±0.7147	0.2703±0.0017±0.5407	0.1196±0.0013±0.2391
-2.68	0.2783±0.0017±0.5566	0.3879±0.0018±0.7757	0.2320±0.0016±0.4640	0.0795±0.0011±0.1590
-2.56	0.3538±0.0018±0.7075	0.3959±0.0018±0.7918	0.1899±0.0015±0.3798	0.0503±0.0009±0.1007
-2.43	0.4255±0.0019±0.8510	0.3895±0.0018±0.7790	0.1507±0.0014±0.3014	0.0301±0.0007±0.0602
-2.31	0.4899±0.0019±0.9798	0.3748±0.0018±0.7495	0.1164±0.0013±0.2330	0.0172±0.0005±0.0345
-2.18	0.5500±0.0019±1.1000	0.3528±0.0018±0.7057	0.0868±0.0011±0.1737	0.0100±0.0004±0.0200
-2.06	0.6062±0.0018±1.2124	0.3266±0.0018±0.6532	0.0623±0.0010±0.1246	0.0049±0.0003±0.0099
-1.93	0.6592±0.0018±1.3184	0.2956±0.0017±0.5912	0.0429±0.0008±0.0859	0.0023±0.0002±0.0046
-1.81	0.7089±0.0017±1.4178	0.2623±0.0017±0.5246	0.0277±0.0007±0.0555	0.0009±0.0001±0.0019
-1.68	0.7552±0.0016±1.5105	0.2275±0.0016±0.4551	0.0167±0.0005±0.0335	0.0003±0.0001±0.0006
-1.56	0.7997±0.0015±1.5995	0.1906±0.0015±0.3812	0.0094±0.0004±0.0187	0.0002±0.0001±0.0003
-1.43	0.8384±0.0014±1.6768	0.1567±0.0014±0.3134	0.0047±0.0003±0.0094	0.0000±0.0000±0.0001
-1.31	0.8749±0.0012±1.7498	0.1230±0.0013±0.2460	0.0019±0.0002±0.0039	0.0000±0.0000±0.0000
-1.18	0.9072±0.0011±1.8145	0.0919±0.0011±0.1839	0.0006±0.0001±0.0013	0.0000±0.0000±0.0000
-1.06	0.9345±0.0009±1.8690	0.0653±0.0010±0.1306	0.0001±0.0000±0.0002	0.0000±0.0000±0.0000
-0.93	0.9576±0.0007±1.9152	0.0423±0.0008±0.0846	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.81	0.9761±0.0005±1.9521	0.0239±0.0006±0.0478	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.68	0.9901±0.0003±1.9802	0.0099±0.0004±0.0198	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.56	0.9985±0.0001±1.9970	0.0014±0.0002±0.0029	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.55: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, at 91 GeV, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

LEP2 Data Sample

$\epsilon(\text{GeV})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
2.00	$0.4614 \pm 0.0052 \pm 0.0080$	$0.3579 \pm 0.0051 \pm 0.0072$	$0.1865 \pm 0.0049 \pm 0.0072$
6.00	$0.6356 \pm 0.0050 \pm 0.0130$	$0.2958 \pm 0.0052 \pm 0.0115$	$0.0726 \pm 0.0039 \pm 0.0089$
10.00	$0.7024 \pm 0.0048 \pm 0.0136$	$0.2579 \pm 0.0051 \pm 0.0138$	$0.0403 \pm 0.0037 \pm 0.0088$
14.00	$0.7513 \pm 0.0045 \pm 0.0132$	$0.2202 \pm 0.0050 \pm 0.0136$	$0.0320 \pm 0.0042 \pm 0.0070$
18.00	$0.7870 \pm 0.0043 \pm 0.0132$	$0.1938 \pm 0.0049 \pm 0.0127$	$0.0271 \pm 0.0052 \pm 0.0102$
22.00	$0.8187 \pm 0.0041 \pm 0.0130$	$0.1709 \pm 0.0048 \pm 0.0132$	$0.0129 \pm 0.0060 \pm 0.0146$
25.50	$0.8437 \pm 0.0039 \pm 0.0133$	$0.1507 \pm 0.0047 \pm 0.0134$	$0.0055 \pm 0.0072 \pm 0.0149$

Table A.56: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, for the LEP2 dataset, as a function of the energy cutoff, ϵ , with a constant radius $R = 0.7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$R(\text{rad})$	$R(n \leq 2)\%$	$R(n=3)\%$	$R(n \geq 4)\%$
0.30	$0.4638 \pm 0.0052 \pm 0.0143$	$0.3193 \pm 0.0049 \pm 0.0090$	$0.2198 \pm 0.0052 \pm 0.0104$
0.50	$0.5614 \pm 0.0051 \pm 0.0133$	$0.3163 \pm 0.0050 \pm 0.0094$	$0.1270 \pm 0.0048 \pm 0.0139$
0.70	$0.6545 \pm 0.0049 \pm 0.0138$	$0.2874 \pm 0.0052 \pm 0.0134$	$0.0608 \pm 0.0038 \pm 0.0097$
0.90	$0.7416 \pm 0.0046 \pm 0.0137$	$0.2385 \pm 0.0051 \pm 0.0145$	$0.0233 \pm 0.0026 \pm 0.0052$
1.10	$0.8346 \pm 0.0041 \pm 0.0088$	$0.1638 \pm 0.0046 \pm 0.0101$	$0.0042 \pm 0.0012 \pm 0.0016$
1.30	$0.9357 \pm 0.0030 \pm 0.0038$	$0.0646 \pm 0.0032 \pm 0.0044$	$0.0004 \pm 0.0004 \pm 0.0008$
1.50	$0.9946 \pm 0.0019 \pm 0.0021$	$0.0049 \pm 0.0015 \pm 0.0030$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.57: Hadron level value of n -jet fractions for $n \leq 2$, $n = 3$ and $n \geq 4$, using the CONE algorithm, for the LEP2 dataset, as a function of the cone radius, R , with a constant energy cutoff $\epsilon = 7$ applied. In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	$R(n=2)\%$	$R(n=3)\%$	$R(n=4)\%$	$R(n=5)\%$
-2.30	$0.1973 \pm 0.0040 \pm 0.0078$	$0.4847 \pm 0.0052 \pm 0.0074$	$0.2569 \pm 0.0050 \pm 0.0108$	$0.0593 \pm 0.0038 \pm 0.0114$
-2.06	$0.3090 \pm 0.0047 \pm 0.0078$	$0.5018 \pm 0.0052 \pm 0.0088$	$0.1706 \pm 0.0048 \pm 0.0132$	$0.0263 \pm 0.0037 \pm 0.0117$
-1.90	$0.3873 \pm 0.0049 \pm 0.0112$	$0.4878 \pm 0.0053 \pm 0.0078$	$0.1202 \pm 0.0046 \pm 0.0112$	$0.0143 \pm 0.0039 \pm 0.0097$
-1.70	$0.5035 \pm 0.0051 \pm 0.0136$	$0.4326 \pm 0.0054 \pm 0.0123$	$0.0686 \pm 0.0044 \pm 0.0085$	$0.0007 \pm 0.0032 \pm 0.0044$
-1.52	$0.6075 \pm 0.0050 \pm 0.0149$	$0.3641 \pm 0.0055 \pm 0.0152$	$0.0331 \pm 0.0042 \pm 0.0104$	$0.0045 \pm 0.0037 \pm 0.0067$
-1.40	$0.6804 \pm 0.0048 \pm 0.0140$	$0.3058 \pm 0.0055 \pm 0.0144$	$0.0190 \pm 0.0045 \pm 0.0083$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.30	$0.7331 \pm 0.0046 \pm 0.0133$	$0.2612 \pm 0.0054 \pm 0.0138$	$0.0068 \pm 0.0043 \pm 0.0075$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.22	$0.7697 \pm 0.0044 \pm 0.0138$	$0.2296 \pm 0.0053 \pm 0.0136$	$0.0024 \pm 0.0050 \pm 0.0095$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.15	$0.8029 \pm 0.0042 \pm 0.0128$	$0.1986 \pm 0.0051 \pm 0.0120$	$0.0025 \pm 0.0046 \pm 0.0099$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.10	$0.8343 \pm 0.0040 \pm 0.0122$	$0.1672 \pm 0.0049 \pm 0.0117$	$0.0051 \pm 0.0076 \pm 0.0219$	$0.0000 \pm 0.0000 \pm 0.0000$
-1.02	$0.8681 \pm 0.0037 \pm 0.0115$	$0.1340 \pm 0.0047 \pm 0.0110$	$0.0059 \pm 0.0130 \pm 0.0190$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.94	$0.9009 \pm 0.0034 \pm 0.0090$	$0.1018 \pm 0.0044 \pm 0.0076$	$0.0004 \pm 0.0011 \pm 0.0017$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.87	$0.9267 \pm 0.0031 \pm 0.0092$	$0.0758 \pm 0.0041 \pm 0.0091$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$
-0.80	$0.9493 \pm 0.0027 \pm 0.0079$	$0.0536 \pm 0.0037 \pm 0.0077$	$0.0000 \pm 0.0000 \pm 0.0000$	$0.0000 \pm 0.0000 \pm 0.0000$

Table A.58: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the JADE algorithm, for the LEP2 dataset, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0112±0.0011±0.0019	0.0199±0.0015±0.0018	0.0278±0.0018±0.0048	0.0346±0.0020±0.0037
-4.75	0.0115±0.0011±0.0019	0.0237±0.0016±0.0020	0.0386±0.0021±0.0049	0.0573±0.0025±0.0057
-4.50	0.0164±0.0014±0.0027	0.0422±0.0022±0.0033	0.0760±0.0029±0.0042	0.1114±0.0035±0.0055
-4.25	0.0361±0.0022±0.0027	0.0906±0.0032±0.0057	0.1405±0.0038±0.0078	0.1669±0.0040±0.0063
-4.00	0.0786±0.0031±0.0053	0.1571±0.0040±0.0059	0.2058±0.0043±0.0066	0.1945±0.0041±0.0069
-3.75	0.1371±0.0039±0.0051	0.2349±0.0046±0.0062	0.2415±0.0045±0.0059	0.1816±0.0040±0.0063
-3.50	0.2106±0.0044±0.0084	0.2926±0.0048±0.0064	0.2531±0.0045±0.0075	0.1456±0.0038±0.0061
-3.25	0.2858±0.0048±0.0100	0.3347±0.0049±0.0064	0.2384±0.0046±0.0085	0.1019±0.0036±0.0090
-3.00	0.3628±0.0050±0.0103	0.3612±0.0049±0.0065	0.1976±0.0045±0.0127	0.0679±0.0035±0.0088
-2.75	0.4462±0.0052±0.0137	0.3621±0.0050±0.0066	0.1524±0.0045±0.0121	0.0390±0.0034±0.0085
-2.50	0.5293±0.0052±0.0142	0.3488±0.0050±0.0091	0.1046±0.0043±0.0116	0.0168±0.0033±0.0092
-2.25	0.6131±0.0050±0.0142	0.3133±0.0050±0.0103	0.0667±0.0043±0.0107	0.0152±0.0051±0.0099
-2.00	0.6957±0.0048±0.0151	0.2645±0.0050±0.0132	0.0427±0.0045±0.0101	0.0069±0.0059±0.0148
-1.75	0.7771±0.0044±0.0130	0.2064±0.0049±0.0120	0.0193±0.0052±0.0098	0.0000±0.0000±0.0083
-1.50	0.8444±0.0039±0.0109	0.1524±0.0047±0.0097	0.0016±0.0070±0.0123	0.0000±0.0000±0.0000
-1.25	0.9057±0.0033±0.0096	0.0957±0.0042±0.0080	0.0166±0.0122±0.0471	0.0000±0.0000±0.0000
-1.00	0.9556±0.0026±0.0070	0.0447±0.0034±0.0060	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9866±0.0020±0.0056	0.0142±0.0025±0.0055	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	0.9999±0.0018±0.0018	0.0000±0.0000±0.0005	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.59: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Durham algorithm, for the LEP2 dataset, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

$\log_{10}(y_{\text{cut}})$	R(n=2)%	R(n=3)%	R(n=4)%	R(n=5)%
-5.00	0.0000±0.0000±0.0000	0.0000±0.0000±0.0001	0.0002±0.0001±0.0002	0.0006±0.0002±0.0003
-4.75	0.0000±0.0000±0.0000	0.0001±0.0001±0.0001	0.0010±0.0003±0.0003	0.0056±0.0007±0.0009
-4.50	0.0001±0.0001±0.0001	0.0012±0.0003±0.0004	0.0089±0.0009±0.0012	0.0287±0.0017±0.0018
-4.25	0.0006±0.0002±0.0006	0.0095±0.0010±0.0012	0.0409±0.0021±0.0025	0.0961±0.0031±0.0038
-4.00	0.0055±0.0008±0.0009	0.0437±0.0022±0.0034	0.1117±0.0034±0.0062	0.1892±0.0042±0.0118
-3.75	0.0269±0.0018±0.0022	0.1136±0.0034±0.0052	0.2076±0.0043±0.0086	0.2308±0.0044±0.0065
-3.50	0.0810±0.0030±0.0038	0.2164±0.0045±0.0052	0.2684±0.0046±0.0066	0.2178±0.0042±0.0048
-3.25	0.1664±0.0041±0.0064	0.3095±0.0049±0.0058	0.2747±0.0046±0.0067	0.1643±0.0040±0.0065
-3.00	0.2709±0.0048±0.0089	0.3662±0.0050±0.0076	0.2401±0.0045±0.0078	0.0935±0.0036±0.0075
-2.75	0.3759±0.0051±0.0119	0.3820±0.0050±0.0056	0.1839±0.0044±0.0096	0.0562±0.0036±0.0126
-2.50	0.4784±0.0052±0.0136	0.3690±0.0050±0.0081	0.1323±0.0045±0.0109	0.0248±0.0036±0.0103
-2.25	0.5765±0.0051±0.0142	0.3362±0.0050±0.0087	0.0818±0.0045±0.0144	0.0150±0.0051±0.0145
-2.00	0.6692±0.0049±0.0157	0.2857±0.0051±0.0136	0.0504±0.0048±0.0105	0.0023±0.0064±0.0105
-1.75	0.7592±0.0045±0.0149	0.2221±0.0050±0.0154	0.0267±0.0058±0.0113	0.0000±0.0000±0.0047
-1.50	0.8339±0.0040±0.0114	0.1623±0.0048±0.0103	0.0081±0.0098±0.0150	0.0000±0.0000±0.0000
-1.25	0.9020±0.0033±0.0091	0.0988±0.0043±0.0076	0.0074±0.0139±0.0197	0.0000±0.0000±0.0000
-1.00	0.9536±0.0026±0.0072	0.0468±0.0035±0.0061	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.75	0.9867±0.0020±0.0054	0.0137±0.0025±0.0051	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000
-0.50	1.0000±0.0018±0.0018	0.0001±0.0001±0.0002	0.0000±0.0000±0.0000	0.0000±0.0000±0.0000

Table A.60: Hadron level value of n -jet fractions for $n = 2, 3, 4$, and 5 , using the Cambridge algorithm, for the LEP2 dataset, as a function of the jet resolution parameter, y_{cut} . In all cases the first quoted error indicates statistical error while the second quoted error corresponds to the total systematic error.

A.2 Parton Level R_3 Values

Dataset	JADE($y_{\text{cut}}=0.08$)	Durham($y_{\text{cut}}=0.01$)	Cambridge($y_{\text{cut}}=0.01$)
91 GeV	$19.49 \pm 0.16 \pm 0.42$	$31.13 \pm 0.18 \pm 0.88$	$28.91 \pm 0.18 \pm 0.88$
LEP2	$17.13 \pm 0.50 \pm 0.91$	$29.05 \pm 0.52 \pm 1.02$	$27.20 \pm 0.52 \pm 1.10$

Table A.61: Parton level R_3 values for 91 GeV and LEP2 datasets used to perform $\mathcal{O}(\alpha_s^2)$ and NLLA fits for α_s determination in Section 5.2. The first error corresponds to the statistical error and the last error corresponds to the systematic error.

Appendix B

Differential n -Jet Rates

In this section tables of values and plots of the differential n -jet rates for the Durham and Cambridge algorithm are presented. The tables and plot contain the D_n information at the hadron level for the 10 energies that are contained within the LEP2 data sample. Tables of the values used to generate Figs 5.11 to 5.14 are also presented.

The second section contains tables of values of the D_2 fits for the 10 LEP2 energies. The fits are performed using the NLLA, $\mathcal{O}(\alpha_s^2)$ and matching predictions.

B.1 Differential n -jet Rate Distributions

161 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$-0.686 \pm 0.840 \pm 1.289$	$-2.703 \pm 1.721 \pm 2.588$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 2.055$	$1271.874 \pm 750.581 \pm 781.644$
-4.50	$310.289 \pm 315.038 \pm 343.351$	$449.226 \pm 322.907 \pm 670.434$	$3259.781 \pm 895.278 \pm 1736.600$
-4.25	$588.034 \pm 298.972 \pm 396.130$	$1635.919 \pm 481.351 \pm 852.585$	$3412.286 \pm 702.233 \pm 1977.870$
-4.00	$808.263 \pm 259.489 \pm 367.994$	$2077.072 \pm 402.485 \pm 510.076$	$977.634 \pm 231.067 \pm 694.854$
-3.75	$567.563 \pm 159.684 \pm 410.178$	$816.357 \pm 164.365 \pm 382.961$	$248.890 \pm 87.154 \pm 268.293$
-3.50	$732.724 \pm 125.636 \pm 341.022$	$-5.881 \pm 0.515 \pm 423.994$	$-400.522 \pm 237.583 \pm 1072.058$
-3.25	$181.673 \pm 43.550 \pm 96.491$	$188.524 \pm 42.886 \pm 142.414$	$-133.162 \pm 35.571 \pm 81.890$
-3.00	$157.643 \pm 30.467 \pm 33.083$	$-2.210 \pm 0.250 \pm 10.114$	$-31.262 \pm 12.591 \pm 29.799$
-2.75	$97.108 \pm 18.777 \pm 21.125$	$57.490 \pm 58.357 \pm 308.317$	$-69.317 \pm 14.280 \pm 24.678$
-2.50	$41.405 \pm 9.024 \pm 11.567$	$-11.558 \pm 4.386 \pm 6.228$	$-14.853 \pm 5.208 \pm 7.559$
-2.25	$24.664 \pm 5.401 \pm 6.389$	$-13.334 \pm 3.500 \pm 4.197$	$-7.909 \pm 3.299 \pm 5.317$
-2.00	$17.765 \pm 3.487 \pm 5.178$	$-10.801 \pm 2.517 \pm 5.658$	$-6.365 \pm 2.301 \pm 3.129$
-1.75	$4.784 \pm 1.419 \pm 1.603$	$-2.146 \pm 0.874 \pm 1.453$	$-4.566 \pm 2.012 \pm 2.906$
-1.50	$3.997 \pm 0.947 \pm 1.181$	$-3.153 \pm 0.788 \pm 1.057$	$-2.017 \pm 1.535 \pm 2.497$
-1.25	$2.185 \pm 0.545 \pm 1.009$	$-2.043 \pm 0.509 \pm 0.918$	$0.086 \pm 0.092 \pm 0.122$
-1.00	$0.568 \pm 0.220 \pm 0.369$	$-0.545 \pm 0.211 \pm 0.368$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.334 \pm 0.139 \pm 0.229$	$-0.324 \pm 0.135 \pm 0.220$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$-0.007 \pm 0.001 \pm 0.039$	$0.007 \pm 0.001 \pm 0.038$	$0.000 \pm 0.000 \pm 0.000$

Table B.1: Differential n -jet rates, for $n=2$ to $n=4$ for the 161 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-1.175 \pm 1.161 \pm 1.612$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$-0.344 \pm 0.449 \pm 0.954$	$-5.375 \pm 3.191 \pm 8.997$
-4.50	$0.000 \pm 0.000 \pm 0.000$	$-0.783 \pm 0.594 \pm 133.599$	$1066.955 \pm 454.899 \pm 658.118$
-4.25	$0.000 \pm 0.000 \pm 0.177$	$347.094 \pm 205.213 \pm 215.775$	$1134.785 \pm 386.425 \pm 505.159$
-4.00	$-0.148 \pm 0.150 \pm 194.203$	$1013.276 \pm 278.517 \pm 337.640$	$1753.279 \pm 358.907 \pm 494.678$
-3.75	$330.594 \pm 119.660 \pm 140.097$	$875.094 \pm 190.633 \pm 204.708$	$1073.598 \pm 204.528 \pm 385.548$
-3.50	$435.681 \pm 104.966 \pm 170.890$	$567.921 \pm 111.736 \pm 231.943$	$132.608 \pm 46.518 \pm 146.982$
-3.25	$259.757 \pm 57.621 \pm 61.380$	$340.923 \pm 61.928 \pm 99.001$	$65.192 \pm 32.382 \pm 253.936$
-3.00	$269.229 \pm 42.096 \pm 61.345$	$23.097 \pm 10.267 \pm 19.013$	$-59.627 \pm 17.414 \pm 24.698$
-2.75	$96.444 \pm 17.918 \pm 45.997$	$8.584 \pm 4.478 \pm 21.058$	$-51.003 \pm 12.145 \pm 17.478$
-2.50	$50.336 \pm 9.386 \pm 14.052$	$-12.818 \pm 3.943 \pm 16.582$	$-20.046 \pm 5.966 \pm 9.249$
-2.25	$33.324 \pm 6.228 \pm 8.404$	$-15.729 \pm 4.205 \pm 5.356$	$-15.539 \pm 3.988 \pm 7.668$
-2.00	$13.062 \pm 2.877 \pm 7.274$	$-7.461 \pm 1.971 \pm 7.340$	$-5.918 \pm 2.328 \pm 3.527$
-1.75	$7.773 \pm 1.743 \pm 2.623$	$-5.003 \pm 1.301 \pm 2.462$	$-4.193 \pm 1.868 \pm 3.076$
-1.50	$4.393 \pm 0.970 \pm 1.270$	$-3.474 \pm 0.823 \pm 1.150$	$-3.323 \pm 2.098 \pm 3.442$
-1.25	$2.292 \pm 0.543 \pm 1.084$	$-2.209 \pm 0.523 \pm 1.052$	$0.033 \pm 0.023 \pm 0.080$
-1.00	$0.627 \pm 0.228 \pm 0.464$	$-0.625 \pm 0.228 \pm 0.457$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.324 \pm 0.135 \pm 0.253$	$-0.326 \pm 0.136 \pm 0.256$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$-0.007 \pm 0.001 \pm 0.040$	$0.007 \pm 0.001 \pm 0.040$	$0.000 \pm 0.000 \pm 0.000$

Table B.2: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 161 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.1: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 161$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

172 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.784$	$-2.625 \pm 1.965 \pm 580.018$
-4.75	$0.000 \pm 0.000 \pm 0.823$	$-1.407 \pm 1.024 \pm 336.461$	$1707.352 \pm 1004.664 \pm 1236.790$
-4.50	$464.522 \pm 470.568 \pm 592.027$	$2885.861 \pm 945.005 \pm 1486.633$	$3166.977 \pm 975.740 \pm 3111.252$
-4.25	$1020.138 \pm 463.953 \pm 611.540$	$1120.174 \pm 466.448 \pm 792.990$	$1902.639 \pm 539.390 \pm 1056.004$
-4.00	$1188.520 \pm 349.519 \pm 539.861$	$1447.124 \pm 347.825 \pm 766.158$	$940.145 \pm 282.414 \pm 747.350$
-3.75	$466.329 \pm 149.153 \pm 186.550$	$1147.817 \pm 235.522 \pm 313.828$	$148.539 \pm 76.642 \pm 156.389$
-3.50	$659.241 \pm 129.741 \pm 271.052$	$128.198 \pm 56.634 \pm 193.539$	$-35.496 \pm 4.658 \pm 69.233$
-3.25	$183.021 \pm 49.575 \pm 79.702$	$113.168 \pm 38.242 \pm 73.468$	$-26.312 \pm 13.558 \pm 72.848$
-3.00	$139.252 \pm 31.735 \pm 45.665$	$22.472 \pm 13.755 \pm 24.494$	$-72.091 \pm 20.774 \pm 42.965$
-2.75	$72.967 \pm 17.989 \pm 40.647$	$-14.162 \pm 6.363 \pm 46.572$	$-28.143 \pm 10.431 \pm 14.273$
-2.50	$65.825 \pm 13.215 \pm 30.237$	$-25.763 \pm 7.850 \pm 28.834$	$-29.405 \pm 8.180 \pm 9.683$
-2.25	$12.122 \pm 4.264 \pm 9.673$	$3.148 \pm 2.379 \pm 9.238$	$-15.841 \pm 5.227 \pm 5.844$
-2.00	$18.361 \pm 4.047 \pm 4.489$	$-14.146 \pm 3.167 \pm 3.860$	$-2.346 \pm 2.173 \pm 3.410$
-1.75	$8.775 \pm 2.154 \pm 3.142$	$-7.779 \pm 1.858 \pm 2.796$	$0.584 \pm 0.074 \pm 1.610$
-1.50	$2.594 \pm 0.937 \pm 1.859$	$-2.372 \pm 0.840 \pm 1.673$	$0.290 \pm 0.074 \pm 0.230$
-1.25	$2.135 \pm 0.649 \pm 0.721$	$-1.890 \pm 0.598 \pm 0.705$	$-1.116 \pm 1.334 \pm 2.222$
-1.00	$0.484 \pm 0.251 \pm 0.428$	$-0.472 \pm 0.244 \pm 0.415$	$0.000 \pm 0.000 \pm 0.003$
-0.75	$0.280 \pm 0.155 \pm 0.203$	$-0.276 \pm 0.153 \pm 0.206$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.044 \pm 0.058 \pm 0.076$	$-0.043 \pm 0.057 \pm 0.075$	$0.000 \pm 0.000 \pm 0.000$

Table B.3: Differential n -jet rates, for $n=2$ to $n=4$ for the 172 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-0.504 \pm 0.646 \pm 0.890$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.830$	$-1.014 \pm 0.773 \pm 1.292$
-4.50	$0.000 \pm 0.000 \pm 0.000$	$-0.390 \pm 0.403 \pm 0.510$	$601.772 \pm 355.834 \pm 469.773$
-4.25	$0.000 \pm 0.000 \pm 0.152$	$431.541 \pm 254.707 \pm 516.804$	$2255.145 \pm 640.239 \pm 1023.311$
-4.00	$71.279 \pm 72.246 \pm 245.605$	$1350.226 \pm 369.770 \pm 725.666$	$1352.848 \pm 345.614 \pm 371.151$
-3.75	$377.897 \pm 145.739 \pm 206.782$	$676.883 \pm 177.965 \pm 248.805$	$612.695 \pm 176.081 \pm 482.787$
-3.50	$414.620 \pm 108.997 \pm 228.633$	$771.232 \pm 149.202 \pm 525.596$	$-13.663 \pm 1.001 \pm 217.862$
-3.25	$381.032 \pm 79.124 \pm 163.479$	$86.677 \pm 34.766 \pm 233.574$	$738.372 \pm 211.065 \pm 1527.510$
-3.00	$215.117 \pm 41.389 \pm 52.691$	$95.109 \pm 26.978 \pm 46.580$	$-103.363 \pm 28.341 \pm 83.528$
-2.75	$79.224 \pm 17.952 \pm 46.272$	$0.135 \pm 2.044 \pm 29.376$	$-28.046 \pm 9.149 \pm 19.674$
-2.50	$53.036 \pm 11.260 \pm 13.205$	$-7.229 \pm 2.956 \pm 15.126$	$-40.901 \pm 9.742 \pm 12.347$
-2.25	$25.493 \pm 5.998 \pm 6.477$	$-4.509 \pm 1.712 \pm 4.513$	$-19.034 \pm 5.493 \pm 7.125$
-2.00	$16.672 \pm 3.726 \pm 4.352$	$-10.022 \pm 2.537 \pm 3.504$	$-7.296 \pm 3.214 \pm 4.581$
-1.75	$9.288 \pm 2.213 \pm 2.366$	$-8.157 \pm 1.920 \pm 2.134$	$-0.211 \pm 1.005 \pm 1.428$
-1.50	$3.902 \pm 1.079 \pm 2.204$	$-3.617 \pm 0.986 \pm 2.057$	$0.324 \pm 0.075 \pm 0.232$
-1.25	$2.364 \pm 0.672 \pm 1.143$	$-2.155 \pm 0.634 \pm 1.107$	$-1.827 \pm 2.250 \pm 3.269$
-1.00	$0.470 \pm 0.250 \pm 0.451$	$-0.471 \pm 0.250 \pm 0.453$	$0.000 \pm 0.000 \pm 0.002$
-0.75	$0.272 \pm 0.152 \pm 0.213$	$-0.275 \pm 0.153 \pm 0.212$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.046 \pm 0.060 \pm 0.076$	$-0.046 \pm 0.061 \pm 0.077$	$0.000 \pm 0.000 \pm 0.000$

Table B.4: Differential n -jet rates, for $n=2$ to $n=4$ for the 172 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.2: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 172$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

183 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$127.738 \pm 140.854 \pm 211.069$	$131.587 \pm 137.768 \pm 228.294$
-4.75	$74.430 \pm 80.393 \pm 121.221$	$368.639 \pm 223.479 \pm 296.781$	$1152.756 \pm 369.896 \pm 1181.449$
-4.50	$364.987 \pm 170.107 \pm 224.600$	$1611.225 \pm 336.961 \pm 674.648$	$2987.563 \pm 471.498 \pm 688.003$
-4.25	$870.307 \pm 209.838 \pm 383.086$	$1847.363 \pm 280.456 \pm 450.211$	$2182.323 \pm 281.252 \pm 543.456$
-4.00	$1005.181 \pm 150.218 \pm 215.318$	$1129.923 \pm 148.629 \pm 187.363$	$838.331 \pm 121.849 \pm 383.440$
-3.75	$497.455 \pm 72.174 \pm 124.454$	$942.767 \pm 102.426 \pm 301.988$	$283.648 \pm 52.768 \pm 229.879$
-3.50	$547.462 \pm 60.247 \pm 80.931$	$175.705 \pm 29.553 \pm 190.775$	$214.888 \pm 49.803 \pm 438.477$
-3.25	$225.304 \pm 25.996 \pm 31.582$	$195.520 \pm 26.320 \pm 97.069$	$-132.557 \pm 19.024 \pm 54.018$
-3.00	$147.927 \pm 16.619 \pm 25.994$	$22.979 \pm 5.835 \pm 19.615$	$-86.582 \pm 11.636 \pm 31.385$
-2.75	$87.049 \pm 9.459 \pm 13.720$	$-6.435 \pm 1.284 \pm 12.781$	$-43.126 \pm 6.062 \pm 11.259$
-2.50	$52.306 \pm 5.619 \pm 11.112$	$-16.034 \pm 2.507 \pm 12.005$	$-26.591 \pm 4.100 \pm 6.946$
-2.25	$21.507 \pm 2.737 \pm 3.542$	$-10.094 \pm 1.513 \pm 2.882$	$-9.678 \pm 2.158 \pm 3.507$
-2.00	$13.934 \pm 1.703 \pm 2.025$	$-9.657 \pm 1.259 \pm 1.582$	$-3.296 \pm 1.105 \pm 1.344$
-1.75	$7.716 \pm 0.981 \pm 1.053$	$-6.336 \pm 0.814 \pm 0.976$	$-0.938 \pm 0.736 \pm 1.011$
-1.50	$4.791 \pm 0.593 \pm 0.913$	$-4.156 \pm 0.521 \pm 0.866$	$-0.796 \pm 0.669 \pm 1.260$
-1.25	$1.421 \pm 0.273 \pm 0.516$	$-1.374 \pm 0.262 \pm 0.502$	$0.120 \pm 0.080 \pm 0.109$
-1.00	$0.908 \pm 0.168 \pm 0.263$	$-0.871 \pm 0.163 \pm 0.263$	$-0.081 \pm 0.129 \pm 0.173$
-0.75	$0.224 \pm 0.070 \pm 0.144$	$-0.221 \pm 0.069 \pm 0.143$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.020 \pm 0.027 \pm 0.036$	$-0.020 \pm 0.027 \pm 0.035$	$0.000 \pm 0.000 \pm 0.000$

Table B.5: Differential n -jet rates, for $n=2$ to $n=4$ for the 183 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-4.518 \pm 5.833 \pm 37.236$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 55.583$	$240.058 \pm 150.525 \pm 161.518$
-4.50	$0.000 \pm 0.000 \pm 56.606$	$163.824 \pm 102.798 \pm 142.909$	$583.841 \pm 179.933 \pm 246.170$
-4.25	$-1.252 \pm 1.312 \pm 23.320$	$306.092 \pm 97.586 \pm 179.150$	$1981.208 \pm 286.639 \pm 618.266$
-4.00	$192.559 \pm 58.318 \pm 146.133$	$755.393 \pm 127.147 \pm 146.290$	$1183.651 \pm 153.930 \pm 582.887$
-3.75	$267.207 \pm 56.715 \pm 123.410$	$851.334 \pm 101.132 \pm 124.073$	$1109.378 \pm 113.368 \pm 431.538$
-3.50	$355.969 \pm 50.738 \pm 76.881$	$571.357 \pm 60.678 \pm 72.705$	$204.766 \pm 32.659 \pm 97.412$
-3.25	$302.096 \pm 33.095 \pm 55.255$	$278.648 \pm 29.634 \pm 43.070$	$27.950 \pm 17.610 \pm 72.256$
-3.00	$224.020 \pm 20.700 \pm 31.787$	$84.072 \pm 12.122 \pm 18.677$	$-112.178 \pm 13.439 \pm 30.802$
-2.75	$94.483 \pm 9.780 \pm 15.047$	$2.725 \pm 0.969 \pm 29.809$	$-53.249 \pm 6.447 \pm 7.775$
-2.50	$54.613 \pm 5.460 \pm 9.428$	$-8.972 \pm 1.557 \pm 8.562$	$-33.591 \pm 4.308 \pm 5.833$
-2.25	$31.986 \pm 3.324 \pm 6.607$	$-15.048 \pm 1.984 \pm 5.961$	$-14.752 \pm 2.366 \pm 3.091$
-2.00	$15.795 \pm 1.762 \pm 2.302$	$-9.970 \pm 1.195 \pm 2.651$	$-4.688 \pm 1.387 \pm 2.314$
-1.75	$8.423 \pm 1.012 \pm 1.351$	$-6.583 \pm 0.819 \pm 1.128$	$-2.160 \pm 0.953 \pm 1.247$
-1.50	$4.860 \pm 0.598 \pm 0.657$	$-4.326 \pm 0.535 \pm 0.637$	$-0.464 \pm 0.652 \pm 1.357$
-1.25	$1.694 \pm 0.293 \pm 0.552$	$-1.655 \pm 0.284 \pm 0.547$	$0.153 \pm 0.100 \pm 0.246$
-1.00	$0.870 \pm 0.164 \pm 0.242$	$-0.846 \pm 0.162 \pm 0.242$	$-0.049 \pm 0.080 \pm 0.105$
-0.75	$0.226 \pm 0.071 \pm 0.112$	$-0.225 \pm 0.071 \pm 0.112$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.023 \pm 0.028 \pm 0.045$	$-0.023 \pm 0.028 \pm 0.045$	$0.000 \pm 0.000 \pm 0.000$

Table B.6: Differential n -jet rates, for $n=2$ to $n=4$ for the 183 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.3: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 183$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

189 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	0.000 $\pm$ 0.000 $\pm$ 0.000	141.634 $\pm$ 85.792 $\pm$ 187.185	637.286 $\pm$ 179.116 $\pm$ 244.055
-4.75	52.786 $\pm$ 55.123 $\pm$ 90.355	928.575 $\pm$ 201.480 $\pm$ 372.636	1469.311 $\pm$ 249.832 $\pm$ 498.522
-4.50	620.093 $\pm$ 133.131 $\pm$ 214.136	1461.817 $\pm$ 188.264 $\pm$ 279.290	2340.339 $\pm$ 235.922 $\pm$ 634.240
-4.25	796.687 $\pm$ 111.484 $\pm$ 230.386	1622.989 $\pm$ 150.467 $\pm$ 176.833	2714.177 $\pm$ 185.437 $\pm$ 607.571
-4.00	844.138 $\pm$ 79.547 $\pm$ 146.984	1413.106 $\pm$ 97.478 $\pm$ 145.693	914.518 $\pm$ 72.099 $\pm$ 241.951
-3.75	631.099 $\pm$ 48.445 $\pm$ 84.576	728.186 $\pm$ 49.713 $\pm$ 90.497	309.218 $\pm$ 29.984 $\pm$ 112.716
-3.50	420.125 $\pm$ 28.510 $\pm$ 31.111	312.517 $\pm$ 23.020 $\pm$ 29.163	-1.411 $\pm$ 5.715 $\pm$ 31.024
-3.25	251.551 $\pm$ 16.126 $\pm$ 17.455	118.112 $\pm$ 11.177 $\pm$ 28.733	-102.207 $\pm$ 9.026 $\pm$ 29.198
-3.00	144.168 $\pm$ 9.154 $\pm$ 14.481	35.762 $\pm$ 5.180 $\pm$ 20.389	-86.865 $\pm$ 6.555 $\pm$ 21.405
-2.75	85.299 $\pm$ 5.392 $\pm$ 9.882	-3.821 $\pm$ 0.442 $\pm$ 9.027	-51.953 $\pm$ 4.079 $\pm$ 14.509
-2.50	48.002 $\pm$ 3.122 $\pm$ 4.455	-15.142 $\pm$ 1.399 $\pm$ 5.510	-23.042 $\pm$ 2.200 $\pm$ 5.101
-2.25	26.024 $\pm$ 1.721 $\pm$ 3.512	-12.914 $\pm$ 1.014 $\pm$ 2.503	-10.377 $\pm$ 1.261 $\pm$ 2.011
-2.00	15.387 $\pm$ 1.053 $\pm$ 1.480	-9.887 $\pm$ 0.742 $\pm$ 1.008	-5.110 $\pm$ 0.829 $\pm$ 1.797
-1.75	7.723 $\pm$ 0.572 $\pm$ 0.811	-6.209 $\pm$ 0.466 $\pm$ 0.792	-1.095 $\pm$ 0.443 $\pm$ 1.152
-1.50	3.697 $\pm$ 0.310 $\pm$ 0.496	-3.244 $\pm$ 0.273 $\pm$ 0.434	-0.594 $\pm$ 0.413 $\pm$ 0.515
-1.25	2.009 $\pm$ 0.177 $\pm$ 0.300	-1.906 $\pm$ 0.167 $\pm$ 0.295	0.001 $\pm$ 0.386 $\pm$ 0.612
-1.00	0.670 $\pm$ 0.086 $\pm$ 0.119	-0.660 $\pm$ 0.085 $\pm$ 0.117	0.000 $\pm$ 0.000 $\pm$ 0.076
-0.75	0.288 $\pm$ 0.053 $\pm$ 0.065	-0.284 $\pm$ 0.052 $\pm$ 0.064	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.50	0.047 $\pm$ 0.021 $\pm$ 0.038	-0.047 $\pm$ 0.021 $\pm$ 0.037	0.000 $\pm$ 0.000 $\pm$ 0.000

Table B.7: Differential n -jet rates, for $n=2$ to $n=4$ for the 189 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	0.000 $\pm$ 0.000 $\pm$ 0.000	25.960 $\pm$ 31.541 $\pm$ 79.164	73.461 $\pm$ 46.879 $\pm$ 102.689
-4.75	0.000 $\pm$ 0.000 $\pm$ 0.000	17.606 $\pm$ 18.272 $\pm$ 21.892	326.721 $\pm$ 98.424 $\pm$ 157.597
-4.50	0.000 $\pm$ 0.000 $\pm$ 0.000	117.870 $\pm$ 42.525 $\pm$ 86.268	782.783 $\pm$ 121.218 $\pm$ 125.657
-4.25	79.553 $\pm$ 29.790 $\pm$ 50.532	538.030 $\pm$ 82.565 $\pm$ 177.066	1257.322 $\pm$ 128.645 $\pm$ 236.194
-4.00	153.927 $\pm$ 32.120 $\pm$ 63.390	704.045 $\pm$ 70.059 $\pm$ 102.723	1548.344 $\pm$ 103.616 $\pm$ 189.224
-3.75	309.328 $\pm$ 36.219 $\pm$ 45.259	839.030 $\pm$ 57.824 $\pm$ 78.463	962.290 $\pm$ 59.186 $\pm$ 141.098
-3.50	354.492 $\pm$ 28.809 $\pm$ 36.692	660.384 $\pm$ 37.156 $\pm$ 61.538	223.597 $\pm$ 19.417 $\pm$ 88.799
-3.25	333.141 $\pm$ 19.818 $\pm$ 27.210	217.718 $\pm$ 14.994 $\pm$ 39.830	-1.782 $\pm$ 4.709 $\pm$ 60.351
-3.00	167.567 $\pm$ 10.052 $\pm$ 24.682	94.805 $\pm$ 7.131 $\pm$ 13.902	-99.140 $\pm$ 6.894 $\pm$ 21.094
-2.75	111.520 $\pm$ 6.054 $\pm$ 6.533	5.364 $\pm$ 1.820 $\pm$ 22.807	-65.804 $\pm$ 4.358 $\pm$ 12.196
-2.50	56.544 $\pm$ 3.250 $\pm$ 5.013	-13.369 $\pm$ 1.238 $\pm$ 8.413	-30.439 $\pm$ 2.339 $\pm$ 5.555
-2.25	29.130 $\pm$ 1.779 $\pm$ 3.223	-12.057 $\pm$ 0.933 $\pm$ 2.044	-14.524 $\pm$ 1.440 $\pm$ 2.124
-2.00	16.661 $\pm$ 1.062 $\pm$ 1.403	-11.277 $\pm$ 0.772 $\pm$ 1.363	-4.566 $\pm$ 0.759 $\pm$ 1.443
-1.75	9.091 $\pm$ 0.607 $\pm$ 1.211	-7.044 $\pm$ 0.489 $\pm$ 0.900	-2.460 $\pm$ 0.581 $\pm$ 1.609
-1.50	3.849 $\pm$ 0.314 $\pm$ 0.520	-3.439 $\pm$ 0.280 $\pm$ 0.422	-0.445 $\pm$ 0.442 $\pm$ 0.778
-1.25	2.030 $\pm$ 0.176 $\pm$ 0.313	-1.971 $\pm$ 0.170 $\pm$ 0.291	0.076 $\pm$ 0.378 $\pm$ 0.639
-1.00	0.713 $\pm$ 0.088 $\pm$ 0.118	-0.714 $\pm$ 0.087 $\pm$ 0.117	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.75	0.259 $\pm$ 0.051 $\pm$ 0.072	-0.260 $\pm$ 0.051 $\pm$ 0.072	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.50	0.065 $\pm$ 0.024 $\pm$ 0.046	-0.065 $\pm$ 0.024 $\pm$ 0.046	0.000 $\pm$ 0.000 $\pm$ 0.000

Table B.8: Differential n -jet rates, for $n=2$ to $n=4$ for the 189 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.4: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 189$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

192 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$460.024 \pm 488.359 \pm 1218.050$	$1173.894 \pm 724.868 \pm 840.993$
-4.75	$338.693 \pm 355.897 \pm 397.808$	$1080.112 \pm 516.704 \pm 861.558$	$457.746 \pm 332.663 \pm 708.813$
-4.50	$620.253 \pm 324.083 \pm 496.299$	$1443.773 \pm 447.823 \pm 673.793$	$2261.320 \pm 563.647 \pm 639.727$
-4.25	$635.793 \pm 245.107 \pm 476.174$	$1995.142 \pm 420.046 \pm 668.057$	$1837.450 \pm 346.548 \pm 500.907$
-4.00	$1086.429 \pm 216.559 \pm 311.251$	$1002.937 \pm 202.428 \pm 244.309$	$1162.701 \pm 191.236 \pm 312.472$
-3.75	$586.934 \pm 114.707 \pm 141.317$	$588.407 \pm 100.016 \pm 212.249$	$689.919 \pm 130.431 \pm 597.290$
-3.50	$444.231 \pm 69.901 \pm 99.046$	$332.599 \pm 58.576 \pm 152.430$	$-150.990 \pm 39.582 \pm 252.231$
-3.25	$235.491 \pm 36.691 \pm 41.240$	$167.322 \pm 33.763 \pm 52.275$	$-109.749 \pm 22.752 \pm 71.932$
-3.00	$151.122 \pm 22.150 \pm 26.175$	$16.201 \pm 10.995 \pm 35.296$	$-82.468 \pm 14.245 \pm 20.740$
-2.75	$82.161 \pm 12.952 \pm 29.551$	$-6.668 \pm 1.851 \pm 28.440$	$-35.815 \pm 7.875 \pm 15.044$
-2.50	$49.926 \pm 7.725 \pm 13.120$	$-15.880 \pm 3.541 \pm 7.874$	$-24.003 \pm 5.423 \pm 10.795$
-2.25	$25.236 \pm 4.084 \pm 6.600$	$-10.385 \pm 2.117 \pm 5.629$	$-10.425 \pm 3.252 \pm 5.311$
-2.00	$16.355 \pm 2.610 \pm 3.569$	$-10.979 \pm 1.896 \pm 2.417$	$-4.571 \pm 1.932 \pm 2.819$
-1.75	$7.000 \pm 1.318 \pm 1.912$	$-5.167 \pm 1.035 \pm 1.767$	$-2.338 \pm 1.455 \pm 2.337$
-1.50	$3.916 \pm 0.776 \pm 1.079$	$-3.151 \pm 0.659 \pm 0.911$	$-4.724 \pm 3.247 \pm 5.081$
-1.25	$1.759 \pm 0.412 \pm 0.740$	$-1.713 \pm 0.396 \pm 0.690$	$0.302 \pm 0.225 \pm 0.415$
-1.00	$0.671 \pm 0.198 \pm 0.287$	$-0.665 \pm 0.195 \pm 0.284$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.219 \pm 0.112 \pm 0.164$	$-0.218 \pm 0.111 \pm 0.164$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.074 \pm 0.087 \pm 0.202$	$-0.074 \pm 0.086 \pm 0.204$	$0.000 \pm 0.000 \pm 0.000$

Table B.9: Differential n -jet rates, for $n=2$ to $n=4$ for the 192 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-0.576 \pm 0.688 \pm 33.853$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$-1.325 \pm 1.711 \pm 169.079$	$411.614 \pm 265.945 \pm 397.380$
-4.50	$0.000 \pm 0.000 \pm 0.000$	$153.443 \pm 116.707 \pm 187.929$	$350.194 \pm 206.172 \pm 867.639$
-4.25	$50.472 \pm 52.423 \pm 77.640$	$319.322 \pm 148.242 \pm 156.773$	$1896.985 \pm 393.171 \pm 673.927$
-4.00	$199.794 \pm 92.203 \pm 101.255$	$1165.238 \pm 237.253 \pm 422.960$	$877.108 \pm 179.200 \pm 706.756$
-3.75	$379.315 \pm 98.794 \pm 113.968$	$671.367 \pm 124.369 \pm 165.750$	$1064.116 \pm 154.399 \pm 402.071$
-3.50	$361.677 \pm 70.613 \pm 100.725$	$537.781 \pm 77.870 \pm 123.298$	$120.416 \pm 33.281 \pm 127.788$
-3.25	$239.834 \pm 40.662 \pm 118.739$	$338.274 \pm 46.456 \pm 110.957$	$42.855 \pm 26.292 \pm 162.415$
-3.00	$207.238 \pm 26.614 \pm 48.945$	$39.297 \pm 10.640 \pm 42.630$	$-63.121 \pm 12.059 \pm 26.433$
-2.75	$109.570 \pm 14.557 \pm 17.514$	$-0.252 \pm 0.192 \pm 23.647$	$-43.654 \pm 8.018 \pm 33.276$
-2.50	$63.558 \pm 8.380 \pm 10.669$	$-16.460 \pm 3.393 \pm 11.069$	$-29.422 \pm 5.684 \pm 14.230$
-2.25	$26.713 \pm 4.114 \pm 5.745$	$-5.902 \pm 1.430 \pm 6.381$	$-18.004 \pm 3.860 \pm 6.944$
-2.00	$19.095 \pm 2.775 \pm 4.993$	$-13.388 \pm 2.062 \pm 5.735$	$-4.768 \pm 1.965 \pm 4.155$
-1.75	$7.389 \pm 1.328 \pm 1.593$	$-5.542 \pm 1.042 \pm 1.391$	$-2.184 \pm 1.489 \pm 2.497$
-1.50	$3.885 \pm 0.771 \pm 1.210$	$-3.296 \pm 0.676 \pm 1.142$	$-2.768 \pm 2.240 \pm 3.818$
-1.25	$1.790 \pm 0.405 \pm 0.580$	$-1.669 \pm 0.382 \pm 0.571$	$-0.960 \pm 1.501 \pm 2.665$
-1.00	$0.743 \pm 0.206 \pm 0.335$	$-0.746 \pm 0.206 \pm 0.332$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.259 \pm 0.121 \pm 0.173$	$-0.260 \pm 0.121 \pm 0.174$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.082 \pm 0.093 \pm 0.215$	$-0.082 \pm 0.093 \pm 0.220$	$0.000 \pm 0.000 \pm 0.000$

Table B.10: Differential n -jet rates, for $n=2$ to $n=4$ for the 192 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.5: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 192$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

196 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$163.199 \pm 170.456 \pm 197.930$	$679.074 \pm 318.811 \pm 447.315$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$653.910 \pm 277.337 \pm 307.981$	$2499.260 \pm 564.631 \pm 1010.620$
-4.50	$369.974 \pm 169.462 \pm 265.769$	$1482.427 \pm 312.062 \pm 444.403$	$2650.017 \pm 410.649 \pm 477.558$
-4.25	$548.968 \pm 150.190 \pm 225.147$	$2014.665 \pm 273.698 \pm 293.333$	$2366.365 \pm 271.366 \pm 304.968$
-4.00	$1035.787 \pm 146.178 \pm 212.189$	$1370.894 \pm 152.311 \pm 176.469$	$1182.083 \pm 139.936 \pm 300.393$
-3.75	$592.853 \pm 74.258 \pm 145.507$	$792.636 \pm 88.984 \pm 121.153$	$174.351 \pm 37.469 \pm 92.407$
-3.50	$410.146 \pm 46.189 \pm 62.610$	$314.955 \pm 38.377 \pm 53.428$	$12.030 \pm 14.077 \pm 127.118$
-3.25	$252.633 \pm 26.781 \pm 32.238$	$67.255 \pm 13.855 \pm 37.801$	$-53.004 \pm 9.988 \pm 50.117$
-3.00	$178.596 \pm 17.183 \pm 37.761$	$-0.294 \pm 2.765 \pm 39.375$	$-70.663 \pm 9.475 \pm 34.630$
-2.75	$79.396 \pm 8.378 \pm 20.956$	$-6.530 \pm 1.269 \pm 13.667$	$-45.534 \pm 5.994 \pm 9.212$
-2.50	$47.384 \pm 5.030 \pm 6.077$	$-9.263 \pm 1.544 \pm 3.064$	$-32.005 \pm 4.259 \pm 4.556$
-2.25	$28.198 \pm 2.967 \pm 4.249$	$-10.491 \pm 1.453 \pm 3.249$	$-18.202 \pm 2.853 \pm 3.894$
-2.00	$16.324 \pm 1.782 \pm 1.941$	$-10.815 \pm 1.260 \pm 1.575$	$-4.921 \pm 1.416 \pm 2.052$
-1.75	$6.313 \pm 0.865 \pm 1.365$	$-4.974 \pm 0.693 \pm 1.225$	$-1.374 \pm 0.841 \pm 1.161$
-1.50	$3.477 \pm 0.496 \pm 0.798$	$-3.188 \pm 0.447 \pm 0.804$	$0.248 \pm 0.333 \pm 0.914$
-1.25	$1.425 \pm 0.252 \pm 0.448$	$-1.389 \pm 0.240 \pm 0.411$	$0.903 \pm 0.659 \pm 1.092$
-1.00	$0.823 \pm 0.163 \pm 0.265$	$-0.816 \pm 0.160 \pm 0.261$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.392 \pm 0.104 \pm 0.258$	$-0.387 \pm 0.103 \pm 0.260$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.049 \pm 0.047 \pm 0.068$	$-0.048 \pm 0.046 \pm 0.067$	$0.000 \pm 0.000 \pm 0.000$

Table B.11: Differential n -jet rates, for $n=2$ to $n=4$ for the 196 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-3.734 \pm 3.002 \pm 132.287$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$81.868 \pm 88.619 \pm 107.312$	$248.801 \pm 150.161 \pm 159.314$
-4.50	$0.000 \pm 0.000 \pm 0.000$	$90.158 \pm 66.984 \pm 89.222$	$681.414 \pm 183.099 \pm 299.016$
-4.25	$52.339 \pm 38.107 \pm 65.996$	$316.144 \pm 98.091 \pm 102.974$	$1629.306 \pm 243.083 \pm 384.975$
-4.00	$145.741 \pm 50.275 \pm 51.353$	$772.896 \pm 123.692 \pm 263.649$	$1742.187 \pm 176.325 \pm 327.971$
-3.75	$263.296 \pm 54.544 \pm 79.733$	$1062.980 \pm 108.410 \pm 163.902$	$712.571 \pm 81.805 \pm 312.802$
-3.50	$377.036 \pm 48.071 \pm 62.824$	$573.214 \pm 55.181 \pm 73.593$	$254.508 \pm 36.863 \pm 90.670$
-3.25	$319.979 \pm 32.088 \pm 46.882$	$214.493 \pm 24.557 \pm 29.025$	$29.809 \pm 12.260 \pm 65.205$
-3.00	$198.613 \pm 17.860 \pm 22.131$	$42.547 \pm 8.208 \pm 20.767$	$-94.790 \pm 11.093 \pm 17.970$
-2.75	$103.720 \pm 9.567 \pm 10.887$	$10.831 \pm 3.949 \pm 32.124$	$-64.230 \pm 6.803 \pm 13.420$
-2.50	$53.234 \pm 5.244 \pm 5.681$	$-7.189 \pm 1.143 \pm 5.463$	$-30.126 \pm 3.902 \pm 5.253$
-2.25	$32.163 \pm 3.042 \pm 5.935$	$-10.936 \pm 1.390 \pm 4.509$	$-20.701 \pm 2.857 \pm 4.805$
-2.00	$16.617 \pm 1.745 \pm 2.975$	$-10.429 \pm 1.204 \pm 3.139$	$-6.050 \pm 1.459 \pm 2.934$
-1.75	$7.816 \pm 0.935 \pm 1.152$	$-6.359 \pm 0.760 \pm 1.163$	$-0.900 \pm 0.807 \pm 1.659$
-1.50	$3.763 \pm 0.507 \pm 0.833$	$-3.516 \pm 0.464 \pm 0.820$	$0.399 \pm 0.370 \pm 1.114$
-1.25	$1.668 \pm 0.268 \pm 0.390$	$-1.650 \pm 0.260 \pm 0.363$	$0.661 \pm 0.399 \pm 1.460$
-1.00	$0.875 \pm 0.167 \pm 0.263$	$-0.880 \pm 0.167 \pm 0.263$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.385 \pm 0.102 \pm 0.224$	$-0.387 \pm 0.102 \pm 0.232$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.048 \pm 0.045 \pm 0.067$	$-0.048 \pm 0.045 \pm 0.068$	$0.000 \pm 0.000 \pm 0.000$

Table B.12: Differential n -jet rates, for $n=2$ to $n=4$ for the 196 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.6: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 196$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

200 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	0.000 $\pm$ 0.000 $\pm$ 0.000	226.639 $\pm$ 169.817 $\pm$ 274.291	279.556 $\pm$ 214.238 $\pm$ 528.861
-4.75	137.447 $\pm$ 140.349 $\pm$ 201.407	488.257 $\pm$ 261.969 $\pm$ 393.658	2277.797 $\pm$ 530.240 $\pm$ 548.437
-4.50	595.135 $\pm$ 231.862 $\pm$ 343.643	1811.352 $\pm$ 348.273 $\pm$ 638.100	4221.438 $\pm$ 534.101 $\pm$ 1381.458
-4.25	1167.353 $\pm$ 225.104 $\pm$ 313.152	1992.869 $\pm$ 286.351 $\pm$ 344.448	1834.772 $\pm$ 248.195 $\pm$ 495.538
-4.00	831.381 $\pm$ 130.778 $\pm$ 152.597	1447.281 $\pm$ 165.872 $\pm$ 235.107	616.577 $\pm$ 96.724 $\pm$ 331.253
-3.75	682.274 $\pm$ 84.293 $\pm$ 106.733	671.545 $\pm$ 78.194 $\pm$ 139.077	179.553 $\pm$ 41.352 $\pm$ 93.567
-3.50	426.209 $\pm$ 46.597 $\pm$ 77.877	267.042 $\pm$ 36.607 $\pm$ 64.784	-41.655 $\pm$ 7.224 $\pm$ 60.617
-3.25	221.502 $\pm$ 25.438 $\pm$ 34.835	75.744 $\pm$ 14.601 $\pm$ 30.151	-38.789 $\pm$ 7.892 $\pm$ 49.841
-3.00	121.004 $\pm$ 14.126 $\pm$ 29.100	57.675 $\pm$ 9.873 $\pm$ 31.922	-90.653 $\pm$ 10.964 $\pm$ 26.621
-2.75	80.156 $\pm$ 8.587 $\pm$ 14.882	-5.534 $\pm$ 1.078 $\pm$ 13.175	-41.795 $\pm$ 6.189 $\pm$ 11.698
-2.50	45.982 $\pm$ 5.146 $\pm$ 10.124	-15.520 $\pm$ 2.343 $\pm$ 6.839	-19.823 $\pm$ 3.527 $\pm$ 5.930
-2.25	28.531 $\pm$ 3.039 $\pm$ 4.159	-15.362 $\pm$ 1.902 $\pm$ 3.979	-10.595 $\pm$ 2.154 $\pm$ 3.277
-2.00	12.151 $\pm$ 1.552 $\pm$ 2.661	-6.595 $\pm$ 0.985 $\pm$ 2.363	-6.851 $\pm$ 1.614 $\pm$ 3.637
-1.75	9.921 $\pm$ 1.112 $\pm$ 1.950	-7.871 $\pm$ 0.898 $\pm$ 1.544	-1.935 $\pm$ 1.033 $\pm$ 1.471
-1.50	3.178 $\pm$ 0.483 $\pm$ 0.669	-2.696 $\pm$ 0.418 $\pm$ 0.627	-1.689 $\pm$ 1.228 $\pm$ 1.705
-1.25	2.187 $\pm$ 0.315 $\pm$ 0.421	-2.115 $\pm$ 0.301 $\pm$ 0.411	0.000 $\pm$ 0.000 $\pm$ 2.270
-1.00	0.745 $\pm$ 0.158 $\pm$ 0.251	-0.740 $\pm$ 0.156 $\pm$ 0.243	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.75	0.304 $\pm$ 0.102 $\pm$ 0.152	-0.302 $\pm$ 0.101 $\pm$ 0.149	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.50	0.047 $\pm$ 0.052 $\pm$ 0.084	-0.047 $\pm$ 0.051 $\pm$ 0.084	0.000 $\pm$ 0.000 $\pm$ 0.000

Table B.13: Differential n -jet rates, for $n=2$ to $n=4$ for the 200 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	0.000 $\pm$ 0.000 $\pm$ 0.000	0.000 $\pm$ 0.000 $\pm$ 0.000	-1.498 $\pm$ 1.582 $\pm$ 2.787
-4.75	0.000 $\pm$ 0.000 $\pm$ 0.000	0.000 $\pm$ 0.000 $\pm$ 73.653	136.052 $\pm$ 100.144 $\pm$ 191.225
-4.50	0.000 $\pm$ 0.000 $\pm$ 111.722	35.736 $\pm$ 43.901 $\pm$ 169.709	841.998 $\pm$ 212.868 $\pm$ 252.882
-4.25	32.555 $\pm$ 33.960 $\pm$ 48.709	625.809 $\pm$ 142.803 $\pm$ 153.866	1919.000 $\pm$ 272.284 $\pm$ 607.251
-4.00	194.159 $\pm$ 63.231 $\pm$ 73.215	1110.413 $\pm$ 152.224 $\pm$ 250.000	1569.830 $\pm$ 175.125 $\pm$ 297.085
-3.75	294.625 $\pm$ 58.751 $\pm$ 64.460	982.549 $\pm$ 106.228 $\pm$ 156.597	782.339 $\pm$ 88.967 $\pm$ 185.929
-3.50	376.996 $\pm$ 48.523 $\pm$ 63.631	640.987 $\pm$ 61.247 $\pm$ 135.245	79.079 $\pm$ 19.331 $\pm$ 104.305
-3.25	310.354 $\pm$ 32.332 $\pm$ 39.910	224.735 $\pm$ 25.450 $\pm$ 38.000	-61.435 $\pm$ 10.415 $\pm$ 54.465
-3.00	177.395 $\pm$ 17.324 $\pm$ 19.150	69.445 $\pm$ 10.063 $\pm$ 15.126	-71.715 $\pm$ 9.188 $\pm$ 31.595
-2.75	98.832 $\pm$ 9.366 $\pm$ 11.387	238.670 $\pm$ 184.370 $\pm$ 992.934	-39.582 $\pm$ 5.403 $\pm$ 16.591
-2.50	56.061 $\pm$ 5.496 $\pm$ 7.146	-17.554 $\pm$ 2.503 $\pm$ 4.822	-24.485 $\pm$ 3.576 $\pm$ 7.409
-2.25	31.134 $\pm$ 3.108 $\pm$ 5.251	-14.191 $\pm$ 1.715 $\pm$ 4.397	-14.050 $\pm$ 2.450 $\pm$ 4.309
-2.00	14.840 $\pm$ 1.675 $\pm$ 2.210	-8.311 $\pm$ 1.086 $\pm$ 2.368	-7.232 $\pm$ 1.577 $\pm$ 3.698
-1.75	9.715 $\pm$ 1.066 $\pm$ 1.753	-7.552 $\pm$ 0.853 $\pm$ 1.420	-2.669 $\pm$ 1.145 $\pm$ 2.039
-1.50	3.483 $\pm$ 0.501 $\pm$ 0.892	-2.964 $\pm$ 0.438 $\pm$ 0.860	-1.829 $\pm$ 1.135 $\pm$ 1.776
-1.25	2.283 $\pm$ 0.320 $\pm$ 0.367	-2.240 $\pm$ 0.310 $\pm$ 0.365	2.167 $\pm$ 2.198 $\pm$ 3.315
-1.00	0.929 $\pm$ 0.173 $\pm$ 0.263	-0.932 $\pm$ 0.173 $\pm$ 0.260	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.75	0.251 $\pm$ 0.095 $\pm$ 0.177	-0.252 $\pm$ 0.096 $\pm$ 0.178	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.50	0.047 $\pm$ 0.050 $\pm$ 0.078	-0.047 $\pm$ 0.050 $\pm$ 0.078	0.000 $\pm$ 0.000 $\pm$ 0.000

Table B.14: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 200 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.7: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 200$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

202 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$323.971 \pm 342.038 \pm 463.807$	$-36.337 \pm 25.980 \pm 244.603$
-4.75	$0.000 \pm 0.000 \pm 242.907$	$1428.133 \pm 633.051 \pm 1258.951$	$2454.929 \pm 839.270 \pm 1484.634$
-4.50	$523.366 \pm 308.732 \pm 432.368$	$1742.672 \pm 502.309 \pm 569.444$	$3101.993 \pm 627.878 \pm 1198.699$
-4.25	$482.897 \pm 219.584 \pm 424.325$	$1808.301 \pm 383.589 \pm 836.987$	$990.830 \pm 249.251 \pm 1034.244$
-4.00	$898.128 \pm 190.029 \pm 286.214$	$1376.098 \pm 230.171 \pm 446.148$	$886.708 \pm 183.750 \pm 595.700$
-3.75	$837.450 \pm 134.963 \pm 162.499$	$491.384 \pm 99.468 \pm 252.462$	$381.357 \pm 75.743 \pm 164.912$
-3.50	$470.803 \pm 72.760 \pm 81.998$	$108.938 \pm 35.097 \pm 123.419$	$18.158 \pm 26.183 \pm 113.095$
-3.25	$173.978 \pm 33.053 \pm 63.452$	$137.423 \pm 27.203 \pm 87.050$	$3.934 \pm 8.063 \pm 30.778$
-3.00	$134.934 \pm 21.747 \pm 34.082$	$61.486 \pm 18.447 \pm 101.237$	$-114.467 \pm 18.155 \pm 48.904$
-2.75	$83.847 \pm 12.858 \pm 17.807$	$1.254 \pm 2.385 \pm 11.344$	$-51.780 \pm 9.656 \pm 18.977$
-2.50	$55.930 \pm 8.185 \pm 11.123$	$-14.507 \pm 3.216 \pm 9.782$	$-28.972 \pm 5.861 \pm 8.690$
-2.25	$21.356 \pm 3.925 \pm 6.516$	$-9.845 \pm 2.186 \pm 4.811$	$-8.623 \pm 2.855 \pm 6.009$
-2.00	$15.191 \pm 2.498 \pm 3.760$	$-9.750 \pm 1.718 \pm 3.221$	$-5.236 \pm 2.225 \pm 2.612$
-1.75	$6.097 \pm 1.285 \pm 1.618$	$-4.319 \pm 0.975 \pm 1.858$	$-3.196 \pm 1.807 \pm 3.988$
-1.50	$4.038 \pm 0.781 \pm 0.927$	$-3.740 \pm 0.707 \pm 0.825$	$0.620 \pm 0.182 \pm 0.768$
-1.25	$2.565 \pm 0.524 \pm 0.803$	$-2.372 \pm 0.485 \pm 0.741$	$-1.126 \pm 2.671 \pm 3.246$
-1.00	$0.786 \pm 0.228 \pm 0.389$	$-0.777 \pm 0.223 \pm 0.385$	$0.006 \pm 0.008 \pm 0.013$
-0.75	$0.336 \pm 0.144 \pm 0.224$	$-0.330 \pm 0.141 \pm 0.225$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.012 \pm 0.049 \pm 0.107$	$-0.012 \pm 0.048 \pm 0.106$	$0.000 \pm 0.000 \pm 0.000$

Table B.15: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 202 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 1.163$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$145.543 \pm 174.220 \pm 248.219$	$403.555 \pm 298.691 \pm 392.986$
-4.50	$0.000 \pm 0.000 \pm 0.000$	$385.010 \pm 233.253 \pm 249.716$	$1356.882 \pm 435.083 \pm 513.512$
-4.25	$77.205 \pm 79.569 \pm 167.305$	$399.749 \pm 170.843 \pm 200.462$	$1242.230 \pm 304.449 \pm 343.844$
-4.00	$103.054 \pm 60.725 \pm 98.273$	$1235.305 \pm 228.262 \pm 508.805$	$977.618 \pm 192.880 \pm 574.511$
-3.75	$446.692 \pm 106.449 \pm 138.245$	$443.148 \pm 101.242 \pm 209.401$	$1445.888 \pm 186.099 \pm 535.720$
-3.50	$345.812 \pm 66.402 \pm 78.667$	$590.397 \pm 86.346 \pm 165.508$	$202.420 \pm 43.064 \pm 162.862$
-3.25	$320.053 \pm 47.517 \pm 49.711$	$190.741 \pm 32.909 \pm 37.241$	$-37.652 \pm 9.254 \pm 102.494$
-3.00	$183.903 \pm 25.837 \pm 39.423$	$29.528 \pm 9.860 \pm 69.833$	$-40.319 \pm 9.697 \pm 52.697$
-2.75	$98.621 \pm 13.758 \pm 15.704$	$72.177 \pm 33.180 \pm 338.537$	$-72.335 \pm 10.965 \pm 24.320$
-2.50	$58.513 \pm 7.887 \pm 9.365$	$3.064 \pm 2.290 \pm 8.523$	$-52.296 \pm 7.576 \pm 11.320$
-2.25	$26.781 \pm 4.179 \pm 4.873$	$-12.562 \pm 2.337 \pm 4.061$	$-9.903 \pm 2.997 \pm 5.345$
-2.00	$13.105 \pm 2.293 \pm 4.112$	$-8.067 \pm 1.541 \pm 2.127$	$-6.007 \pm 2.362 \pm 4.299$
-1.75	$8.632 \pm 1.480 \pm 2.883$	$-6.577 \pm 1.184 \pm 2.266$	$-3.134 \pm 1.672 \pm 3.437$
-1.50	$4.362 \pm 0.787 \pm 1.068$	$-3.990 \pm 0.715 \pm 0.974$	$-0.154 \pm 1.114 \pm 1.557$
-1.25	$2.371 \pm 0.488 \pm 0.770$	$-2.273 \pm 0.468 \pm 0.753$	$-0.951 \pm 2.536 \pm 3.007$
-1.00	$0.868 \pm 0.236 \pm 0.440$	$-0.885 \pm 0.238 \pm 0.449$	$0.006 \pm 0.009 \pm 0.014$
-0.75	$0.333 \pm 0.143 \pm 0.238$	$-0.336 \pm 0.145 \pm 0.243$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.016 \pm 0.053 \pm 0.115$	$-0.016 \pm 0.054 \pm 0.117$	$0.000 \pm 0.000 \pm 0.000$

Table B.16: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 202 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.8: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 202$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

205 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 82.743$	$201.146 \pm 207.127 \pm 489.864$	$1397.322 \pm 518.095 \pm 760.117$
-4.75	$176.773 \pm 180.421 \pm 323.350$	$797.535 \pm 311.300 \pm 546.286$	$3107.436 \pm 625.257 \pm 1332.561$
-4.50	$821.208 \pm 270.424 \pm 436.606$	$2309.451 \pm 400.160 \pm 659.759$	$2580.967 \pm 406.641 \pm 1764.736$
-4.25	$850.479 \pm 195.444 \pm 226.474$	$2111.123 \pm 278.771 \pm 560.174$	$2036.994 \pm 252.285 \pm 408.371$
-4.00	$943.572 \pm 138.285 \pm 149.838$	$1465.177 \pm 159.563 \pm 217.990$	$1002.861 \pm 124.794 \pm 152.528$
-3.75	$676.763 \pm 82.530 \pm 94.432$	$591.961 \pm 72.352 \pm 103.988$	$337.915 \pm 52.182 \pm 127.660$
-3.50	$368.670 \pm 42.062 \pm 55.835$	$373.101 \pm 44.071 \pm 127.731$	$-138.380 \pm 23.510 \pm 68.309$
-3.25	$216.705 \pm 24.486 \pm 34.304$	$91.273 \pm 15.732 \pm 42.182$	$-66.460 \pm 11.633 \pm 55.704$
-3.00	$129.401 \pm 14.654 \pm 20.643$	$32.900 \pm 6.796 \pm 18.159$	$-84.892 \pm 10.297 \pm 31.447$
-2.75	$88.854 \pm 9.038 \pm 16.849$	$-14.794 \pm 2.679 \pm 16.267$	$-48.430 \pm 6.273 \pm 8.522$
-2.50	$52.483 \pm 5.304 \pm 7.284$	$-17.914 \pm 2.430 \pm 4.175$	$-22.361 \pm 3.686 \pm 5.636$
-2.25	$29.498 \pm 3.055 \pm 3.121$	$-15.705 \pm 1.871 \pm 2.498$	$-9.259 \pm 2.056 \pm 3.725$
-2.00	$15.812 \pm 1.736 \pm 2.363$	$-10.883 \pm 1.265 \pm 1.862$	$-3.519 \pm 1.215 \pm 2.511$
-1.75	$7.297 \pm 0.898 \pm 0.985$	$-4.915 \pm 0.665 \pm 1.272$	$-4.333 \pm 1.312 \pm 2.959$
-1.50	$3.721 \pm 0.514 \pm 0.997$	$-3.183 \pm 0.444 \pm 0.924$	$-1.065 \pm 0.948 \pm 1.102$
-1.25	$2.020 \pm 0.307 \pm 0.451$	$-1.936 \pm 0.292 \pm 0.423$	$0.000 \pm 0.000 \pm 0.479$
-1.00	$0.715 \pm 0.162 \pm 0.335$	$-0.714 \pm 0.160 \pm 0.316$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.225 \pm 0.091 \pm 0.265$	$-0.223 \pm 0.090 \pm 0.267$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.015 \pm 0.031 \pm 0.035$	$-0.015 \pm 0.031 \pm 0.034$	$0.000 \pm 0.000 \pm 0.000$

Table B.17: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 205 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 32.506$	$0.000 \pm 0.000 \pm 56.455$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$202.294 \pm 162.449 \pm 348.719$	$184.444 \pm 134.542 \pm 504.978$
-4.50	$59.794 \pm 65.253 \pm 107.643$	$90.849 \pm 66.056 \pm 98.475$	$1113.695 \pm 253.183 \pm 431.930$
-4.25	$109.141 \pm 58.091 \pm 86.137$	$616.339 \pm 139.952 \pm 282.924$	$1627.843 \pm 244.553 \pm 307.707$
-4.00	$222.032 \pm 69.145 \pm 74.588$	$961.539 \pm 137.693 \pm 173.230$	$1541.391 \pm 167.035 \pm 238.800$
-3.75	$339.611 \pm 64.092 \pm 83.308$	$999.468 \pm 103.620 \pm 138.310$	$932.239 \pm 94.636 \pm 117.533$
-3.50	$390.756 \pm 49.873 \pm 77.675$	$578.403 \pm 55.882 \pm 153.046$	$161.531 \pm 27.782 \pm 67.226$
-3.25	$300.896 \pm 30.366 \pm 46.810$	$258.857 \pm 28.350 \pm 67.906$	$-91.750 \pm 16.265 \pm 77.921$
-3.00	$150.760 \pm 15.607 \pm 22.187$	$78.138 \pm 10.522 \pm 17.750$	$-76.093 \pm 9.667 \pm 15.770$
-2.75	$100.687 \pm 9.426 \pm 14.324$	$-4.398 \pm 0.861 \pm 61.053$	$-49.589 \pm 5.755 \pm 15.193$
-2.50	$61.919 \pm 5.722 \pm 10.279$	$-18.894 \pm 2.585 \pm 10.163$	$-30.964 \pm 3.956 \pm 9.969$
-2.25	$33.100 \pm 3.198 \pm 3.897$	$-15.157 \pm 1.822 \pm 2.396$	$-13.147 \pm 2.240 \pm 3.265$
-2.00	$17.969 \pm 1.799 \pm 2.359$	$-11.579 \pm 1.250 \pm 2.106$	$-5.455 \pm 1.438 \pm 2.559$
-1.75	$8.434 \pm 0.969 \pm 1.666$	$-5.805 \pm 0.721 \pm 1.892$	$-4.368 \pm 1.330 \pm 2.544$
-1.50	$3.677 \pm 0.503 \pm 0.884$	$-3.178 \pm 0.438 \pm 0.770$	$-1.160 \pm 1.085 \pm 1.787$
-1.25	$2.271 \pm 0.320 \pm 0.597$	$-2.143 \pm 0.303 \pm 0.589$	$0.000 \pm 0.000 \pm 0.585$
-1.00	$0.823 \pm 0.170 \pm 0.278$	$-0.822 \pm 0.168 \pm 0.283$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.202 \pm 0.091 \pm 0.297$	$-0.202 \pm 0.090 \pm 0.299$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$-0.007 \pm 0.021 \pm 0.053$	$0.007 \pm 0.021 \pm 0.053$	$0.000 \pm 0.000 \pm 0.000$

Table B.18: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 205 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.9: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 205$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

207 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$200.201 \pm 148.269 \pm 180.363$	$682.710 \pm 263.763 \pm 532.554$
-4.75	$0.000 \pm 0.000 \pm 344.010$	$1626.179 \pm 387.203 \pm 504.091$	$2575.944 \pm 457.738 \pm 596.196$
-4.50	$851.332 \pm 218.597 \pm 407.098$	$1894.295 \pm 294.563 \pm 390.329$	$3249.965 \pm 367.088 \pm 579.309$
-4.25	$1119.441 \pm 173.798 \pm 235.116$	$1902.938 \pm 213.420 \pm 255.390$	$2008.233 \pm 201.353 \pm 377.477$
-4.00	$1046.407 \pm 117.288 \pm 147.120$	$1282.865 \pm 120.616 \pm 211.452$	$629.223 \pm 81.419 \pm 160.594$
-3.75	$680.147 \pm 66.477 \pm 102.276$	$605.306 \pm 59.285 \pm 90.522$	$273.593 \pm 38.322 \pm 69.657$
-3.50	$425.889 \pm 37.833 \pm 38.704$	$288.638 \pm 31.162 \pm 44.311$	$0.157 \pm 11.682 \pm 27.357$
-3.25	$258.392 \pm 22.072 \pm 28.025$	$89.228 \pm 12.637 \pm 26.680$	$-67.509 \pm 9.052 \pm 22.231$
-3.00	$127.615 \pm 11.458 \pm 15.354$	$44.891 \pm 8.925 \pm 33.584$	$-84.442 \pm 8.626 \pm 18.703$
-2.75	$85.523 \pm 7.263 \pm 8.030$	$-10.761 \pm 1.504 \pm 5.297$	$-44.534 \pm 4.853 \pm 6.560$
-2.50	$48.631 \pm 4.140 \pm 5.694$	$-11.617 \pm 1.482 \pm 7.422$	$-26.374 \pm 3.105 \pm 6.989$
-2.25	$26.965 \pm 2.407 \pm 2.932$	$-15.124 \pm 1.504 \pm 3.165$	$-8.719 \pm 1.665 \pm 3.650$
-2.00	$14.184 \pm 1.337 \pm 1.696$	$-8.716 \pm 0.891 \pm 1.523$	$-5.278 \pm 1.225 \pm 1.518$
-1.75	$8.208 \pm 0.812 \pm 1.096$	$-6.459 \pm 0.646 \pm 0.907$	$-1.331 \pm 0.744 \pm 1.627$
-1.50	$3.821 \pm 0.418 \pm 0.694$	$-3.391 \pm 0.368 \pm 0.662$	$-0.299 \pm 0.549 \pm 1.240$
-1.25	$1.580 \pm 0.228 \pm 0.263$	$-1.522 \pm 0.214 \pm 0.246$	$1.388 \pm 0.838 \pm 1.454$
-1.00	$0.770 \pm 0.133 \pm 0.197$	$-0.737 \pm 0.128 \pm 0.188$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.154 \pm 0.068 \pm 0.086$	$-0.151 \pm 0.066 \pm 0.084$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.007 \pm 0.031 \pm 0.056$	$-0.007 \pm 0.030 \pm 0.055$	$0.000 \pm 0.000 \pm 0.000$

Table B.19: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 205 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 5.238$
-4.75	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 1.143$	$584.143 \pm 187.560 \pm 294.092$
-4.50	$31.234 \pm 32.730 \pm 54.839$	$192.628 \pm 81.125 \pm 164.425$	$1136.908 \pm 199.515 \pm 262.132$
-4.25	$22.298 \pm 22.457 \pm 30.645$	$726.078 \pm 127.470 \pm 204.643$	$1736.805 \pm 200.066 \pm 240.416$
-4.00	$208.031 \pm 51.765 \pm 61.360$	$1067.681 \pm 116.330 \pm 178.015$	$1399.890 \pm 130.072 \pm 307.011$
-3.75	$470.540 \pm 59.952 \pm 77.311$	$796.505 \pm 73.803 \pm 166.928$	$807.744 \pm 72.701 \pm 298.702$
-3.50	$478.370 \pm 44.361 \pm 120.434$	$496.084 \pm 42.612 \pm 110.801$	$161.586 \pm 21.022 \pm 58.220$
-3.25	$317.894 \pm 25.852 \pm 66.176$	$273.043 \pm 22.715 \pm 107.710$	$-70.994 \pm 9.211 \pm 73.291$
-3.00	$176.558 \pm 13.825 \pm 19.843$	$61.136 \pm 7.480 \pm 33.092$	$-78.037 \pm 7.761 \pm 22.210$
-2.75	$96.538 \pm 7.496 \pm 14.583$	$-2.504 \pm 1.389 \pm 10.463$	$-49.237 \pm 4.896 \pm 9.224$
-2.50	$60.928 \pm 4.500 \pm 4.881$	$-13.892 \pm 1.612 \pm 5.875$	$-33.647 \pm 3.290 \pm 7.354$
-2.25	$29.811 \pm 2.475 \pm 4.572$	$-14.475 \pm 1.425 \pm 4.010$	$-11.925 \pm 1.806 \pm 3.326$
-2.00	$15.593 \pm 1.355 \pm 2.082$	$-9.178 \pm 0.879 \pm 1.833$	$-6.361 \pm 1.272 \pm 1.707$
-1.75	$8.489 \pm 0.806 \pm 0.971$	$-6.604 \pm 0.638 \pm 0.740$	$-1.743 \pm 0.804 \pm 1.554$
-1.50	$4.179 \pm 0.433 \pm 0.591$	$-3.757 \pm 0.385 \pm 0.535$	$-0.213 \pm 0.618 \pm 1.243$
-1.25	$1.761 \pm 0.237 \pm 0.300$	$-1.721 \pm 0.226 \pm 0.290$	$2.238 \pm 1.629 \pm 3.993$
-1.00	$0.799 \pm 0.134 \pm 0.188$	$-0.760 \pm 0.130 \pm 0.179$	$0.000 \pm 0.000 \pm 0.000$
-0.75	$0.165 \pm 0.070 \pm 0.086$	$-0.164 \pm 0.069 \pm 0.085$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$-0.025 \pm 0.018 \pm 0.066$	$0.024 \pm 0.018 \pm 0.065$	$0.000 \pm 0.000 \pm 0.000$

Table B.20: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 207 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure B.10: The differential 2-, 3- and 4-jet rates at the hadron level as a function of y_{cut} for the Cambridge and Durham algorithms for data with $\sqrt{s} = 207$ GeV. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. Error bars indicate total (statistical + systematic) errors.

91 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-4.93	$0.000 \pm 0.000 \pm 0.000$	$0.813 \pm 1.150 \pm 2.151$	$5.253 \pm 3.215 \pm 12.221$
-4.81	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 5.427$	$13.556 \pm 4.389 \pm 28.246$
-4.68	$30.487 \pm 25.617 \pm 103.768$	$5.943 \pm 2.825 \pm 13.909$	$27.280 \pm 5.198 \pm 56.939$
-4.56	$182.484 \pm 153.190 \pm 455.279$	$15.387 \pm 3.923 \pm 31.702$	$88.994 \pm 9.543 \pm 178.529$
-4.43	$97.619 \pm 28.123 \pm 221.617$	$53.392 \pm 7.165 \pm 107.080$	$230.622 \pm 15.238 \pm 462.406$
-4.31	$90.889 \pm 15.634 \pm 201.603$	$105.353 \pm 8.446 \pm 212.710$	$516.462 \pm 21.868 \pm 1034.020$
-4.18	$113.004 \pm 11.998 \pm 250.026$	$285.131 \pm 14.226 \pm 570.903$	$882.932 \pm 26.116 \pm 1766.220$
-4.06	$105.392 \pm 7.420 \pm 225.915$	$486.188 \pm 16.461 \pm 972.840$	$1099.723 \pm 25.668 \pm 2200.877$
-3.93	$171.401 \pm 8.156 \pm 344.023$	$670.167 \pm 17.248 \pm 1341.010$	$1113.912 \pm 22.090 \pm 2227.965$
-3.81	$240.559 \pm 8.492 \pm 481.565$	$761.490 \pm 15.763 \pm 1523.503$	$905.208 \pm 16.731 \pm 1812.098$
-3.68	$233.133 \pm 6.680 \pm 467.216$	$698.348 \pm 12.862 \pm 1396.886$	$653.052 \pm 12.011 \pm 1307.179$
-3.56	$227.997 \pm 5.515 \pm 459.995$	$568.678 \pm 9.819 \pm 1138.224$	$415.881 \pm 8.662 \pm 832.085$
-3.43	$212.074 \pm 4.558 \pm 426.692$	$443.544 \pm 7.453 \pm 887.269$	$168.149 \pm 4.841 \pm 336.674$
-3.31	$197.067 \pm 3.832 \pm 394.787$	$315.371 \pm 5.530 \pm 630.897$	$-24.242 \pm 1.905 \pm 72.472$
-3.18	$172.916 \pm 3.159 \pm 346.521$	$187.489 \pm 3.623 \pm 375.015$	$-38.785 \pm 1.381 \pm 77.719$
-3.06	$156.324 \pm 2.679 \pm 313.192$	$111.008 \pm 2.531 \pm 222.059$	$-79.056 \pm 2.004 \pm 158.319$
-2.93	$135.225 \pm 2.265 \pm 270.527$	$47.964 \pm 1.385 \pm 96.167$	$-74.807 \pm 1.697 \pm 149.669$
-2.81	$113.827 \pm 1.864 \pm 227.672$	$21.471 \pm 0.998 \pm 42.995$	$-62.974 \pm 1.377 \pm 125.981$
-2.68	$95.802 \pm 1.570 \pm 191.706$	$-1.634 \pm 0.166 \pm 4.418$	$-43.687 \pm 0.966 \pm 87.415$
-2.56	$73.868 \pm 1.216 \pm 147.741$	$-7.930 \pm 0.318 \pm 15.894$	$-33.947 \pm 0.793 \pm 67.919$
-2.43	$56.903 \pm 0.962 \pm 113.846$	$-13.107 \pm 0.423 \pm 26.296$	$-21.070 \pm 0.526 \pm 42.151$
-2.31	$41.509 \pm 0.706 \pm 83.035$	$-12.396 \pm 0.336 \pm 24.821$	$-14.911 \pm 0.403 \pm 29.834$
-2.18	$31.477 \pm 0.542 \pm 62.967$	$-12.148 \pm 0.301 \pm 24.315$	$-9.613 \pm 0.281 \pm 19.232$
-2.06	$22.227 \pm 0.387 \pm 44.457$	$-9.445 \pm 0.219 \pm 18.907$	$-6.926 \pm 0.220 \pm 13.871$
-1.93	$15.989 \pm 0.282 \pm 31.998$	$-8.414 \pm 0.185 \pm 16.834$	$-3.745 \pm 0.134 \pm 7.492$
-1.81	$11.175 \pm 0.202 \pm 22.351$	$-6.537 \pm 0.141 \pm 13.080$	$-2.274 \pm 0.090 \pm 4.557$
-1.68	$7.838 \pm 0.144 \pm 15.676$	$-5.178 \pm 0.109 \pm 10.357$	$-1.275 \pm 0.059 \pm 2.556$
-1.56	$5.543 \pm 0.106 \pm 11.088$	$-4.018 \pm 0.085 \pm 8.040$	$-0.632 \pm 0.035 \pm 1.267$
-1.43	$3.727 \pm 0.074 \pm 7.456$	$-2.888 \pm 0.062 \pm 5.780$	$-0.328 \pm 0.023 \pm 0.659$
-1.31	$2.452 \pm 0.051 \pm 4.904$	$-2.077 \pm 0.046 \pm 4.154$	$-0.104 \pm 0.010 \pm 0.210$
-1.18	$1.595 \pm 0.036 \pm 3.191$	$-1.420 \pm 0.033 \pm 2.845$	$-0.050 \pm 0.007 \pm 0.101$
-1.06	$1.007 \pm 0.024 \pm 2.015$	$-0.884 \pm 0.022 \pm 1.770$	$-0.012 \pm 0.003 \pm 0.024$
-0.93	$0.635 \pm 0.017 \pm 1.271$	$-0.597 \pm 0.016 \pm 1.194$	$-0.003 \pm 0.002 \pm 0.006$
-0.81	$0.381 \pm 0.012 \pm 0.762$	$-0.361 \pm 0.011 \pm 0.724$	$0.000 \pm 0.000 \pm 0.000$
-0.68	$0.204 \pm 0.007 \pm 0.408$	$-0.190 \pm 0.007 \pm 0.381$	$0.000 \pm 0.000 \pm 0.000$
-0.56	$0.063 \pm 0.004 \pm 0.127$	$-0.061 \pm 0.004 \pm 0.122$	$0.000 \pm 0.000 \pm 0.000$

Table B.21: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 91 GeV data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-4.93	0.000 $\pm$ 0.000 $\pm$ 0.000	0.000 $\pm$ 0.000 $\pm$ 0.000	0.000 $\pm$ 0.000 $\pm$ 0.000
-4.81	0.000 $\pm$ 0.000 $\pm$ 0.000	0.000 $\pm$ 0.000 $\pm$ 0.000	0.000 $\pm$ 0.000 $\pm$ 2.894
-4.68	0.000 $\pm$ 0.000 $\pm$ 0.000	2.743 $\pm$ 2.963 $\pm$ 7.358	3.657 $\pm$ 1.755 $\pm$ 7.728
-4.56	0.000 $\pm$ 0.000 $\pm$ 0.000	1.029 $\pm$ 1.236 $\pm$ 3.013	14.037 $\pm$ 3.539 $\pm$ 28.712
-4.43	44.694 $\pm$ 54.792 $\pm$ 130.163	2.842 $\pm$ 1.272 $\pm$ 6.403	26.482 $\pm$ 4.118 $\pm$ 53.243
-4.31	38.627 $\pm$ 47.348 $\pm$ 98.094	3.933 $\pm$ 1.160 $\pm$ 8.542	57.921 $\pm$ 5.614 $\pm$ 116.667
-4.18	116.085 $\pm$ 47.661 $\pm$ 260.599	14.345 $\pm$ 2.267 $\pm$ 28.957	153.428 $\pm$ 9.150 $\pm$ 307.271
-4.06	96.392 $\pm$ 24.920 $\pm$ 211.002	33.336 $\pm$ 3.220 $\pm$ 67.179	315.213 $\pm$ 12.692 $\pm$ 630.662
-3.93	110.995 $\pm$ 16.835 $\pm$ 223.570	100.929 $\pm$ 5.999 $\pm$ 203.075	471.007 $\pm$ 13.704 $\pm$ 942.461
-3.81	202.646 $\pm$ 19.921 $\pm$ 408.731	157.993 $\pm$ 6.499 $\pm$ 317.088	649.227 $\pm$ 14.247 $\pm$ 1298.960
-3.68	161.893 $\pm$ 10.146 $\pm$ 326.452	268.641 $\pm$ 7.865 $\pm$ 537.617	746.370 $\pm$ 13.548 $\pm$ 1492.858
-3.56	168.951 $\pm$ 7.563 $\pm$ 343.216	361.667 $\pm$ 8.066 $\pm$ 723.499	696.714 $\pm$ 11.268 $\pm$ 1393.822
-3.43	176.529 $\pm$ 5.868 $\pm$ 354.151	396.574 $\pm$ 7.336 $\pm$ 793.437	568.165 $\pm$ 8.771 $\pm$ 1136.464
-3.31	172.257 $\pm$ 4.556 $\pm$ 344.874	391.254 $\pm$ 6.285 $\pm$ 782.540	366.244 $\pm$ 6.128 $\pm$ 732.762
-3.18	148.328 $\pm$ 3.318 $\pm$ 297.703	341.026 $\pm$ 5.073 $\pm$ 682.152	164.243 $\pm$ 3.466 $\pm$ 328.912
-3.06	146.502 $\pm$ 2.805 $\pm$ 293.195	251.382 $\pm$ 3.694 $\pm$ 502.884	8.821 $\pm$ 0.610 $\pm$ 23.409
-2.93	122.732 $\pm$ 2.104 $\pm$ 245.561	162.722 $\pm$ 2.563 $\pm$ 325.498	-42.690 $\pm$ 1.219 $\pm$ 85.501
-2.81	103.024 $\pm$ 1.641 $\pm$ 206.220	86.125 $\pm$ 1.613 $\pm$ 172.270	-72.433 $\pm$ 1.435 $\pm$ 144.917
-2.68	86.369 $\pm$ 1.306 $\pm$ 172.750	35.388 $\pm$ 0.944 $\pm$ 70.836	-68.602 $\pm$ 1.232 $\pm$ 137.323
-2.56	70.789 $\pm$ 1.046 $\pm$ 141.608	1.136 $\pm$ 0.168 $\pm$ 3.154	-54.246 $\pm$ 0.968 $\pm$ 108.498
-2.43	55.635 $\pm$ 0.828 $\pm$ 111.305	-10.287 $\pm$ 0.353 $\pm$ 20.607	-36.438 $\pm$ 0.695 $\pm$ 72.902
-2.31	43.418 $\pm$ 0.657 $\pm$ 86.874	-13.922 $\pm$ 0.359 $\pm$ 27.934	-22.509 $\pm$ 0.476 $\pm$ 45.026
-2.18	32.448 $\pm$ 0.499 $\pm$ 64.902	-12.748 $\pm$ 0.291 $\pm$ 25.536	-13.929 $\pm$ 0.327 $\pm$ 27.878
-2.06	24.511 $\pm$ 0.388 $\pm$ 49.026	-11.612 $\pm$ 0.246 $\pm$ 23.228	-9.095 $\pm$ 0.241 $\pm$ 18.202
-1.93	17.497 $\pm$ 0.284 $\pm$ 35.001	-9.561 $\pm$ 0.195 $\pm$ 19.125	-5.220 $\pm$ 0.157 $\pm$ 10.442
-1.81	12.464 $\pm$ 0.210 $\pm$ 24.929	-7.573 $\pm$ 0.152 $\pm$ 15.150	-2.907 $\pm$ 0.102 $\pm$ 5.821
-1.68	8.736 $\pm$ 0.152 $\pm$ 17.473	-6.101 $\pm$ 0.120 $\pm$ 12.205	-1.538 $\pm$ 0.063 $\pm$ 3.078
-1.56	6.051 $\pm$ 0.109 $\pm$ 12.102	-4.437 $\pm$ 0.088 $\pm$ 8.875	-0.760 $\pm$ 0.039 $\pm$ 1.521
-1.43	4.216 $\pm$ 0.080 $\pm$ 8.437	-3.297 $\pm$ 0.067 $\pm$ 6.599	-0.350 $\pm$ 0.023 $\pm$ 0.702
-1.31	2.738 $\pm$ 0.055 $\pm$ 5.476	-2.330 $\pm$ 0.049 $\pm$ 4.661	-0.136 $\pm$ 0.012 $\pm$ 0.272
-1.18	1.773 $\pm$ 0.038 $\pm$ 3.547	-1.570 $\pm$ 0.035 $\pm$ 3.142	-0.063 $\pm$ 0.009 $\pm$ 0.128
-1.06	1.119 $\pm$ 0.026 $\pm$ 2.239	-0.961 $\pm$ 0.023 $\pm$ 1.923	-0.009 $\pm$ 0.002 $\pm$ 0.018
-0.93	0.678 $\pm$ 0.018 $\pm$ 1.357	-0.646 $\pm$ 0.017 $\pm$ 1.294	-0.001 $\pm$ 0.001 $\pm$ 0.002
-0.81	0.396 $\pm$ 0.012 $\pm$ 0.792	-0.365 $\pm$ 0.011 $\pm$ 0.731	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.68	0.207 $\pm$ 0.008 $\pm$ 0.414	-0.192 $\pm$ 0.007 $\pm$ 0.384	0.000 $\pm$ 0.000 $\pm$ 0.000
-0.56	0.068 $\pm$ 0.004 $\pm$ 0.136	-0.065 $\pm$ 0.004 $\pm$ 0.131	0.000 $\pm$ 0.000 $\pm$ 0.000

Table B.22: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the 91 GeV data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

LEP2 Data Sample

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 16.818$	$158.218 \pm 48.917 \pm 73.511$	$624.815 \pm 104.671 \pm 124.692$
-4.75	$67.303 \pm 30.878 \pm 90.565$	$908.801 \pm 112.713 \pm 173.368$	$1937.742 \pm 157.587 \pm 206.421$
-4.50	$626.067 \pm 75.511 \pm 163.031$	$1662.111 \pm 108.301 \pm 134.475$	$2859.139 \pm 140.160 \pm 177.734$
-4.25	$858.604 \pm 63.022 \pm 160.728$	$1826.627 \pm 85.964 \pm 138.115$	$2212.484 \pm 87.792 \pm 114.951$
-4.00	$946.920 \pm 45.543 \pm 87.857$	$1371.479 \pm 51.509 \pm 80.151$	$884.711 \pm 38.659 \pm 98.873$
-3.75	$637.917 \pm 26.254 \pm 72.303$	$702.306 \pm 26.233 \pm 48.673$	$289.989 \pm 15.997 \pm 51.445$
-3.50	$442.287 \pm 15.802 \pm 25.618$	$274.249 \pm 11.849 \pm 24.816$	$-11.575 \pm 3.387 \pm 34.031$
-3.25	$235.163 \pm 8.434 \pm 9.514$	$111.659 \pm 5.778 \pm 9.808$	$-78.408 \pm 4.161 \pm 14.692$
-3.00	$143.179 \pm 5.005 \pm 5.960$	$29.056 \pm 2.294 \pm 6.863$	$-84.161 \pm 3.463 \pm 7.153$
-2.75	$84.480 \pm 2.899 \pm 6.071$	$-6.425 \pm 0.395 \pm 6.246$	$-47.087 \pm 2.055 \pm 4.471$
-2.50	$49.537 \pm 1.709 \pm 3.515$	$-14.740 \pm 0.740 \pm 2.801$	$-24.608 \pm 1.224 \pm 1.864$
-2.25	$25.895 \pm 0.935 \pm 1.479$	$-12.604 \pm 0.543 \pm 1.171$	$-10.718 \pm 0.706 \pm 1.328$
-2.00	$15.032 \pm 0.557 \pm 0.666$	$-9.731 \pm 0.393 \pm 0.627$	$-4.849 \pm 0.440 \pm 1.144$
-1.75	$7.596 \pm 0.307 \pm 0.457$	$-5.900 \pm 0.246 \pm 0.350$	$-1.779 \pm 0.293 \pm 0.584$
-1.50	$3.784 \pm 0.168 \pm 0.282$	$-3.311 \pm 0.148 \pm 0.245$	$-0.687 \pm 0.240 \pm 0.387$
-1.25	$1.858 \pm 0.094 \pm 0.161$	$-1.778 \pm 0.090 \pm 0.156$	$0.239 \pm 0.200 \pm 0.316$
-1.00	$0.729 \pm 0.049 \pm 0.067$	$-0.715 \pm 0.048 \pm 0.067$	$0.010 \pm 0.067 \pm 0.112$
-0.75	$0.267 \pm 0.028 \pm 0.056$	$-0.263 \pm 0.028 \pm 0.055$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.030 \pm 0.010 \pm 0.017$	$-0.030 \pm 0.010 \pm 0.017$	$0.000 \pm 0.000 \pm 0.000$

Table B.23: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the LEP2 data sample using the Cambridge jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	D_2	D_3	D_4
-5.00	$0.000 \pm 0.000 \pm 0.000$	$9.381 \pm 10.340 \pm 16.143$	$24.865 \pm 15.566 \pm 18.742$
-4.75	$0.000 \pm 0.001 \pm 0.001$	$34.581 \pm 16.802 \pm 27.591$	$298.030 \pm 51.836 \pm 59.532$
-4.50	$7.925 \pm 5.784 \pm 6.719$	$119.258 \pm 24.159 \pm 65.438$	$880.077 \pm 71.867 \pm 91.647$
-4.25	$52.318 \pm 12.982 \pm 19.050$	$516.696 \pm 42.845 \pm 52.685$	$1585.998 \pm 78.381 \pm 100.729$
-4.00	$172.178 \pm 18.633 \pm 28.110$	$907.868 \pm 43.834 \pm 85.290$	$1478.398 \pm 54.597 \pm 131.355$
-3.75	$338.024 \pm 20.486 \pm 21.169$	$868.793 \pm 31.930 \pm 48.442$	$897.389 \pm 30.539 \pm 89.350$
-3.50	$387.123 \pm 16.225 \pm 31.501$	$591.142 \pm 18.803 \pm 42.183$	$183.749 \pm 9.598 \pm 73.252$
-3.25	$316.742 \pm 10.526 \pm 16.902$	$237.943 \pm 8.474 \pm 12.332$	$-16.659 \pm 2.179 \pm 44.602$
-3.00	$183.574 \pm 5.713 \pm 7.645$	$70.668 \pm 3.317 \pm 9.914$	$-86.611 \pm 3.458 \pm 8.955$
-2.75	$102.612 \pm 3.139 \pm 7.453$	$2.470 \pm 0.595 \pm 6.522$	$-55.306 \pm 2.095 \pm 4.786$
-2.50	$57.524 \pm 1.780 \pm 2.304$	$-12.939 \pm 0.656 \pm 3.606$	$-31.129 \pm 1.281 \pm 2.575$
-2.25	$30.142 \pm 0.987 \pm 1.128$	$-12.632 \pm 0.526 \pm 1.068$	$-14.820 \pm 0.785 \pm 1.349$
-2.00	$16.216 \pm 0.563 \pm 0.800$	$-10.373 \pm 0.394 \pm 0.819$	$-5.434 \pm 0.449 \pm 1.203$
-1.75	$8.594 \pm 0.320 \pm 0.713$	$-6.610 \pm 0.256 \pm 0.476$	$-2.417 \pm 0.328 \pm 0.884$
-1.50	$4.000 \pm 0.171 \pm 0.283$	$-3.552 \pm 0.152 \pm 0.251$	$-0.698 \pm 0.271 \pm 0.507$
-1.25	$1.983 \pm 0.096 \pm 0.166$	$-1.921 \pm 0.093 \pm 0.160$	$0.136 \pm 0.248 \pm 0.388$
-1.00	$0.785 \pm 0.050 \pm 0.088$	$-0.780 \pm 0.050 \pm 0.089$	$-0.037 \pm 0.082 \pm 0.148$
-0.75	$0.253 \pm 0.028 \pm 0.055$	$-0.253 \pm 0.028 \pm 0.055$	$0.000 \pm 0.000 \pm 0.000$
-0.50	$0.032 \pm 0.011 \pm 0.018$	$-0.032 \pm 0.011 \pm 0.018$	$0.000 \pm 0.000 \pm 0.000$

Table B.24: Differential n -jet rates, for $n=2$, $n=3$ and $n=4$ for the LEP2 data sample using the Durham jet algorithm. The first error corresponds to the statistical error while the second error corresponds to the total error.

B.2 Fit Values from the Differential 2-jet Rates

161 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11241	0.00780	0.00909	7.09 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10345	0.00672	0.07474	6.28 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11680	0.00850	0.01001	7.17 (8.0)
R match	-3.00 – -0.75	0.10545	0.00765	0.08009	6.00 (9.0)
lnR match	-3.25 – -0.75	0.10740	0.00739	0.08059	6.48 (10.0)
mod R match	-3.00 – -0.75	0.10640	0.00778	0.08102	6.24 (9.0)
mod lnR match	-3.50 – -0.75	0.10510	0.00680	0.09532	7.04 (11.0)
NLLA + mod y	-3.25 – -1.25	0.11115	0.00779	0.00925	6.81 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.11088	0.00778	0.00903	6.76 (8.0)
mod R + mod y	-3.25 – -0.75	0.10774	0.00748	0.09821	6.58 (10.0)

Table B.25: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 161 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10477	0.00890	0.01071	10.20 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.09748	0.00835	0.08732	7.77 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10846	0.00954	0.01110	10.13 (8.0)
R match	-3.25 – -0.75	0.10213	0.00977	0.08859	13.06 (10.0)
lnR match	-3.25 – -0.75	0.10087	0.00856	0.08717	11.08 (10.0)
mod R match	-3.25 – -0.75	0.10381	0.00962	0.09026	12.53 (10.0)
mod lnR match	-3.25 – -0.75	0.10062	0.00855	0.09085	11.13 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10291	0.00896	0.01032	10.28 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10272	0.00894	0.01029	10.14 (8.0)
mod R + mod y	-3.25 – -0.75	0.10363	0.00963	0.09405	12.61 (10.0)

Table B.26: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 161 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

172 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10786	0.00951	0.00453	5.74 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.09642	0.00800	0.00522	5.68 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11206	0.01039	0.00441	5.80 (8.0)
R match	-3.00 – -0.75	0.10251	0.00951	0.01206	4.33 (9.0)
lnR match	-3.25 – -0.75	0.10144	0.00869	0.00890	5.23 (10.0)
mod R match	-3.00 – -0.75	0.10388	0.00974	0.01284	4.22 (9.0)
mod lnR match	-3.50 – -0.75	0.10124	0.00791	0.02369	5.22 (11.0)
NLLA + mod y	-3.25 – -1.25	0.10665	0.00940	0.00410	5.39 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10655	0.00940	0.00407	5.33 (8.0)
mod R + mod y	-3.25 – -0.75	0.10166	0.00880	0.09210	5.31 (10.0)

Table B.27: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 172 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.09098	0.01063	0.01447	18.68 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.08856	0.01076	0.01096	16.92 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.09449	0.01121	0.01515	18.59 (8.0)
R match	-3.25 – -0.75	0.08733	0.00985	0.01142	19.50 (10.0)
lnR match	-3.25 – -0.75	0.08730	0.00931	0.00965	18.73 (10.0)
mod R match	-3.25 – -0.75	0.08883	0.01038	0.01181	19.26 (10.0)
mod lnR match	-3.25 – -0.75	0.08724	0.00925	0.03483	18.74 (10.0)
NLLA + mod y	-3.25 – -1.25	0.08944	0.01009	0.01399	18.52 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.08953	0.01015	0.01406	18.38 (8.0)
mod R + mod y	-3.25 – -0.75	0.08873	0.01029	0.07958	19.27 (10.0)

Table B.28: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 172 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

183 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11153	0.00430	0.00672	9.91 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10384	0.00382	0.00613	7.98 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11584	0.00466	0.00719	10.33 (8.0)
R match	-3.00 – -0.75	0.10727	0.00438	0.00813	8.08 (9.0)
lnR match	-3.25 – -0.75	0.10733	0.00409	0.00705	8.83 (10.0)
mod R match	-3.00 – -0.75	0.10824	0.00445	0.00795	8.80 (9.0)
mod lnR match	-3.50 – -0.75	0.10937	0.00346	0.02488	9.77 (11.0)
NLLA + mod y	-3.25 – -1.25	0.11030	0.00428	0.00691	9.00 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10994	0.00428	0.00675	9.39 (8.0)
mod R + mod y	-3.25 – -0.75	0.10771	0.00413	0.09797	8.87 (10.0)

Table B.29: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 183 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10210	0.00458	0.00624	11.70 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10139	0.00509	0.00725	11.06 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10574	0.00496	0.00685	11.92 (8.0)
R match	-3.25 – -0.75	0.10237	0.00475	0.00787	11.89 (10.0)
lnR match	-3.25 – -0.75	0.10010	0.00434	0.00667	11.85 (10.0)
mod R match	-3.25 – -0.75	0.10342	0.00480	0.00757	11.81 (10.0)
mod lnR match	-3.25 – -0.75	0.10001	0.00433	0.04568	11.88 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10076	0.00453	0.00614	11.12 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10052	0.00453	0.00621	11.30 (8.0)
mod R + mod y	-3.25 – -0.75	0.10332	0.00478	0.09362	11.88 (10.0)

Table B.30: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 183 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

189 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11249	0.00253	0.00217	17.96 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10326	0.00220	0.00266	13.09 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11676	0.00271	0.00282	18.62 (8.0)
R match	-3.00 – -0.75	0.10772	0.00270	0.00571	10.33 (9.0)
lnR match	-3.25 – -0.75	0.10705	0.00244	0.00332	12.34 (10.0)
mod R match	-3.00 – -0.75	0.10906	0.00273	0.00539	9.89 (9.0)
mod lnR match	-3.50 – -0.75	0.10844	0.00208	0.01103	13.58 (11.0)
NLLA + mod y	-3.25 – -1.25	0.11110	0.00253	0.00188	16.21 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.11070	0.00253	0.00193	16.79 (8.0)
mod R + mod y	-3.25 – -0.75	0.10750	0.00247	0.09756	11.89 (10.0)

Table B.31: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2–rate for the 189 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10463	0.00263	0.00264	16.09 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10380	0.00295	0.00197	3.98 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10846	0.00284	0.00292	16.85 (8.0)
R match	-3.25 – -0.75	0.10443	0.00275	0.00625	5.09 (10.0)
lnR match	-3.25 – -0.75	0.10180	0.00250	0.00286	6.33 (10.0)
mod R match	-3.25 – -0.75	0.10535	0.00277	0.00546	5.54 (10.0)
mod lnR match	-3.25 – -0.75	0.10169	0.00250	0.02100	6.45 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10325	0.00260	0.00259	14.35 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10301	0.00260	0.00247	14.79 (8.0)
mod R + mod y	-3.25 – -0.75	0.10523	0.00276	0.09531	5.78 (10.0)

Table B.32: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2–rate for the 189 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

192 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11299	0.00593	0.00774	6.25 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10399	0.00517	0.00730	5.26 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11747	0.00637	0.00860	6.33 (8.0)
R match	-3.00 – -0.75	0.10431	0.00580	0.00828	4.67 (9.0)
lnR match	-3.25 – -0.75	0.10748	0.00575	0.00814	7.05 (10.0)
mod R match	-3.00 – -0.75	0.10511	0.00590	0.00806	5.21 (9.0)
mod lnR match	-3.50 – -0.75	0.10936	0.00482	0.00827	7.50 (11.0)
NLLA + mod y	-3.25 – -1.25	0.11149	0.00594	0.00771	6.38 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.11102	0.00593	0.00738	6.54 (8.0)
mod R + mod y	-3.25 – -0.75	0.10789	0.00582	0.09822	6.99 (10.0)

Table B.33: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2–rate for the 192 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10327	0.00635	0.00592	2.72 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10113	0.00707	0.00793	1.02 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10693	0.00687	0.00644	2.87 (8.0)
R match	-3.25 – -0.75	0.10210	0.00655	0.00887	1.16 (10.0)
lnR match	-3.25 – -0.75	0.09980	0.00602	0.00668	1.39 (10.0)
mod R match	-3.25 – -0.75	0.10302	0.00660	0.00830	1.27 (10.0)
mod lnR match	-3.25 – -0.75	0.09974	0.00601	0.01068	1.38 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10190	0.00629	0.00592	2.48 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10164	0.00629	0.00582	2.61 (8.0)
mod R + mod y	-3.25 – -0.75	0.10296	0.00659	0.09328	1.26 (10.0)

Table B.34: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2–rate for the 192 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

196 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10901	0.00406	0.00285	4.73 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10230	0.00364	0.00279	4.67 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11306	0.00439	0.00370	5.20 (8.0)
R match	-3.00 – -0.75	0.10529	0.00414	0.00558	5.71 (9.0)
lnR match	-3.25 – -0.75	0.10526	0.00390	0.00365	6.79 (10.0)
mod R match	-3.00 – -0.75	0.10625	0.00421	0.00531	6.51 (9.0)
mod lnR match	-3.50 – -0.75	0.10601	0.00341	0.01095	7.10 (11.0)
NLLA + mod y	-3.25 – -1.25	0.10769	0.00405	0.00295	4.35 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10730	0.00405	0.00285	4.99 (8.0)
mod R + mod y	-3.25 – -0.75	0.10556	0.00394	0.09564	6.86 (10.0)

Table B.35: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 196 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.09978	0.00421	0.00333	8.94 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.09703	0.00430	0.00421	7.84 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10321	0.00448	0.00412	9.30 (8.0)
R match	-3.25 – -0.75	0.09956	0.00435	0.00588	10.85 (10.0)
lnR match	-3.25 – -0.75	0.09768	0.00400	0.00393	11.00 (10.0)
mod R match	-3.25 – -0.75	0.10048	0.00440	0.00554	11.42 (10.0)
mod lnR match	-3.25 – -0.75	0.09757	0.00399	0.01151	11.10 (10.0)
NLLA + mod y	-3.25 – -1.25	0.09834	0.00414	0.00331	8.91 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.09802	0.00414	0.00331	9.53 (8.0)
mod R + mod y	-3.25 – -0.75	0.10030	0.00438	0.09042	11.60 (10.0)

Table B.36: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 196 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

200 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11126	0.00432	0.00310	10.14 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10319	0.00375	0.00267	8.89 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11546	0.00464	0.00333	10.39 (8.0)
R match	-3.00 – -0.75	0.10759	0.00454	0.00640	7.96 (9.0)
lnR match	-3.25 – -0.75	0.10717	0.00414	0.00381	8.46 (10.0)
mod R match	-3.00 – -0.75	0.10888	0.00461	0.00615	7.89 (9.0)
mod lnR match	-3.50 – -0.75	0.10847	0.00348	0.00843	8.86 (11.0)
NLLA + mod y	-3.25 – -1.25	0.10986	0.00431	0.00328	9.52 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10953	0.00430	0.00318	9.64 (8.0)
mod R + mod y	-3.25 – -0.75	0.10752	0.00419	0.09762	8.56 (10.0)

Table B.37: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 200 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10330	0.00486	0.00514	15.65 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10498	0.00501	0.00353	12.76 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10724	0.00520	0.00494	15.55 (8.0)
R match	-3.25 – -0.75	0.10298	0.00518	0.00824	15.53 (10.0)
lnR match	-3.25 – -0.75	0.10043	0.00456	0.00408	14.43 (10.0)
mod R match	-3.25 – -0.75	0.10465	0.00525	0.00735	14.42 (10.0)
mod lnR match	-3.25 – -0.75	0.10034	0.00456	0.01040	14.53 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10162	0.00476	0.00494	15.24 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10154	0.00476	0.00478	14.87 (8.0)
mod R + mod y	-3.25 – -0.75	0.10444	0.00523	0.09467	14.59 (10.0)

Table B.38: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 200 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

202 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10816	0.00634	0.00458	6.67 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10079	0.00542	0.00259	5.90 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11223	0.00685	0.00449	6.77 (8.0)
R match	-3.00 – -0.75	0.10472	0.00639	0.00655	5.48 (9.0)
lnR match	-3.25 – -0.75	0.10486	0.00600	0.00458	5.47 (10.0)
mod R match	-3.00 – -0.75	0.10601	0.00655	0.00622	5.45 (9.0)
mod lnR match	-3.50 – -0.75	0.10748	0.00508	0.01220	6.25 (11.0)
NLLA + mod y	-3.25 – -1.25	0.10685	0.00631	0.00490	6.21 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10667	0.00630	0.00471	6.13 (8.0)
mod R + mod y	-3.25 – -0.75	0.10502	0.00606	0.09514	5.72 (10.0)

Table B.39: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2–rate for the 202 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10354	0.00731	0.00884	12.80 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10115	0.00742	0.00583	9.53 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10727	0.00795	0.00954	12.66 (8.0)
R match	-3.25 – -0.75	0.10382	0.00814	0.01179	16.19 (10.0)
lnR match	-3.25 – -0.75	0.10198	0.00703	0.00728	13.42 (10.0)
mod R match	-3.25 – -0.75	0.10591	0.00806	0.01178	15.00 (10.0)
mod lnR match	-3.25 – -0.75	0.10176	0.00704	0.02320	13.53 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10180	0.00732	0.00852	12.79 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10181	0.00731	0.00848	12.42 (8.0)
mod R + mod y	-3.25 – -0.75	0.10557	0.00804	0.09613	15.17 (10.0)

Table B.40: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2–rate for the 202 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

205 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11483	0.00424	0.00328	6.82 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10359	0.00362	0.00515	8.30 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11926	0.00453	0.00384	6.83 (8.0)
R match	-3.00 – -0.75	0.10893	0.00471	0.00823	7.45 (9.0)
lnR match	-3.25 – -0.75	0.10878	0.00420	0.00490	6.87 (10.0)
mod R match	-3.00 – -0.75	0.11049	0.00477	0.00791	6.48 (9.0)
mod lnR match	-3.50 – -0.75	0.10949	0.00347	0.03209	6.93 (11.0)
NLLA + mod y	-3.25 – -1.25	0.11326	0.00426	0.00321	6.73 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.11284	0.00425	0.00322	6.66 (8.0)
mod R + mod y	-3.25 – -0.75	0.10927	0.00425	0.09942	6.76 (10.0)

Table B.41: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 205 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10864	0.00447	0.00379	3.37 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10747	0.00487	0.00561	2.32 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.11273	0.00479	0.00427	3.51 (8.0)
R match	-3.25 – -0.75	0.10805	0.00483	0.00785	4.04 (10.0)
lnR match	-3.25 – -0.75	0.10516	0.00432	0.00396	2.14 (10.0)
mod R match	-3.25 – -0.75	0.10911	0.00479	0.00688	3.52 (10.0)
mod lnR match	-3.25 – -0.75	0.10506	0.00431	0.02141	2.15 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10705	0.00444	0.00350	3.16 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10675	0.00444	0.00345	3.26 (8.0)
mod R + mod y	-3.25 – -0.75	0.10900	0.00478	0.09916	3.50 (10.0)

Table B.42: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 205 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

207 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.11036	0.00344	0.00411	5.63 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.75	0.10145	0.00302	0.00297	5.12 (10.0)
NLLA + G_{21}	-3.25 – -1.25	0.11460	0.00370	0.00457	6.11 (8.0)
R match	-3.00 – -0.75	0.10535	0.00355	0.00548	4.06 (9.0)
lnR match	-3.25 – -0.75	0.10503	0.00329	0.00419	5.18 (10.0)
mod R match	-3.00 – -0.75	0.10652	0.00362	0.00527	4.31 (9.0)
mod lnR match	-3.50 – -0.75	0.10336	0.00291	0.02795	6.24 (11.0)
NLLA + mod y	-3.25 – -1.25	0.10902	0.00342	0.00369	4.69 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10865	0.00342	0.00372	5.19 (8.0)
mod R + mod y	-3.25 – -0.75	0.10542	0.00333	0.09551	4.95 (10.0)

Table B.43: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 207 GeV dataset using the Durham algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.10302	0.00363	0.00296	8.66 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.00 – -0.75	0.10389	0.00406	0.00424	3.50 (9.0)
NLLA + G_{21}	-3.25 – -1.25	0.10684	0.00393	0.00328	9.04 (8.0)
R match	-3.25 – -0.75	0.10189	0.00376	0.00510	4.83 (10.0)
lnR match	-3.25 – -0.75	0.09954	0.00343	0.00311	6.67 (10.0)
mod R match	-3.25 – -0.75	0.10288	0.00381	0.00436	5.10 (10.0)
mod lnR match	-3.25 – -0.75	0.09948	0.00343	0.01545	6.56 (10.0)
NLLA + mod y	-3.25 – -1.25	0.10163	0.00357	0.00255	7.77 (8.0)
NLLA + G_{21} + mod y	-3.25 – -1.25	0.10140	0.00357	0.00275	8.09 (8.0)
mod R + mod y	-3.25 – -0.75	0.10282	0.00380	0.09290	4.94 (10.0)

Table B.44: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the differential 2-rate for the 207 GeV dataset using the Cambridge algorithm over the fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Appendix C

Average Jet Rates

In this section tables of values and plots of the average jet rates for the Durham and Cambridge algorithm are presented. The tables and plot contain the $\langle N \rangle$ information at the hadron level for the 10 energies that are contained within the LEP2 data sample. Tables of the values used to generate Figs 5.23 to 5.26 are also presented.

The second section contains tables of values of the average jet rate fits for the 10 LEP2 energies. The fits are performed using the NLLA, $\mathcal{O}(\alpha_s^2)$ and matching predictions

C.1 Average Jet Rate Distributions

161 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$11.750 \pm 0.2627 \pm 0.3231$	-5.00	$14.614 \pm 0.2380 \pm 0.2854$
-4.75	$9.942 \pm 0.2238 \pm 0.2689$	-4.75	$12.204 \pm 0.2035 \pm 0.2221$
-4.50	$8.154 \pm 0.1831 \pm 0.2519$	-4.50	$10.088 \pm 0.1775 \pm 0.1929$
-4.25	$6.573 \pm 0.1550 \pm 0.2379$	-4.25	$8.231 \pm 0.1524 \pm 0.1656$
-4.00	$5.186 \pm 0.1249 \pm 0.1463$	-4.00	$6.695 \pm 0.1311 \pm 0.1452$
-3.75	$4.261 \pm 0.0972 \pm 0.1175$	-3.75	$5.438 \pm 0.1100 \pm 0.1405$
-3.50	$3.689 \pm 0.0831 \pm 0.0946$	-3.50	$4.482 \pm 0.0921 \pm 0.1226$
-3.25	$3.331 \pm 0.0733 \pm 0.1038$	-3.25	$3.782 \pm 0.0774 \pm 0.1219$
-3.00	$3.064 \pm 0.0645 \pm 0.1042$	-3.00	$3.303 \pm 0.0679 \pm 0.0782$
-2.75	$2.783 \pm 0.0535 \pm 0.0655$	-2.75	$2.903 \pm 0.0553 \pm 0.0894$
-2.50	$2.598 \pm 0.0451 \pm 0.0474$	-2.50	$2.705 \pm 0.0484 \pm 0.0581$
-2.25	$2.477 \pm 0.0408 \pm 0.0427$	-2.25	$2.532 \pm 0.0423 \pm 0.0522$
-2.00	$2.356 \pm 0.0370 \pm 0.0375$	-2.00	$2.393 \pm 0.0387 \pm 0.0397$
-1.75	$2.247 \pm 0.0317 \pm 0.0332$	-1.75	$2.277 \pm 0.0330 \pm 0.0338$
-1.50	$2.168 \pm 0.0269 \pm 0.0292$	-1.50	$2.183 \pm 0.0276 \pm 0.0310$
-1.25	$2.091 \pm 0.0225 \pm 0.0254$	-1.25	$2.095 \pm 0.0227 \pm 0.0265$
-1.00	$2.042 \pm 0.0188 \pm 0.0230$	-1.00	$2.040 \pm 0.0184 \pm 0.0224$
-0.75	$2.011 \pm 0.0163 \pm 0.0188$	-0.75	$2.011 \pm 0.0163 \pm 0.0185$
-0.50	$2.000 \pm 0.0165 \pm 0.0195$	-0.50	$1.999 \pm 0.0165 \pm 0.0196$

Table C.1: Average jet rates for the 161 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.1: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 161 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

172 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$11.471 \pm 0.2954 \pm 0.3445$	-5.00	$14.711 \pm 0.2592 \pm 0.3442$
-4.75	$9.544 \pm 0.2370 \pm 0.2651$	-4.75	$12.175 \pm 0.2219 \pm 0.2882$
-4.50	$7.757 \pm 0.1962 \pm 0.2673$	-4.50	$9.927 \pm 0.1882 \pm 0.2274$
-4.25	$6.353 \pm 0.1698 \pm 0.2074$	-4.25	$8.105 \pm 0.1586 \pm 0.1906$
-4.00	$5.134 \pm 0.1375 \pm 0.1767$	-4.00	$6.641 \pm 0.1375 \pm 0.2094$
-3.75	$4.268 \pm 0.1185 \pm 0.1439$	-3.75	$5.446 \pm 0.1197 \pm 0.1648$
-3.50	$3.646 \pm 0.1025 \pm 0.1247$	-3.50	$4.490 \pm 0.1051 \pm 0.1517$
-3.25	$3.226 \pm 0.0809 \pm 0.1298$	-3.25	$3.806 \pm 0.0940 \pm 0.1349$
-3.00	$2.988 \pm 0.0693 \pm 0.0816$	-3.00	$3.269 \pm 0.0756 \pm 0.1166$
-2.75	$2.768 \pm 0.0566 \pm 0.0625$	-2.75	$2.910 \pm 0.0616 \pm 0.0698$
-2.50	$2.582 \pm 0.0487 \pm 0.0548$	-2.50	$2.698 \pm 0.0526 \pm 0.0697$
-2.25	$2.452 \pm 0.0430 \pm 0.0514$	-2.25	$2.497 \pm 0.0459 \pm 0.0519$
-2.00	$2.331 \pm 0.0352 \pm 0.0418$	-2.00	$2.372 \pm 0.0385 \pm 0.0435$
-1.75	$2.226 \pm 0.0319 \pm 0.0416$	-1.75	$2.239 \pm 0.0326 \pm 0.0388$
-1.50	$2.149 \pm 0.0283 \pm 0.0372$	-1.50	$2.176 \pm 0.0299 \pm 0.0317$
-1.25	$2.079 \pm 0.0225 \pm 0.0334$	-1.25	$2.089 \pm 0.0233 \pm 0.0256$
-1.00	$2.049 \pm 0.0203 \pm 0.0232$	-1.00	$2.049 \pm 0.0203 \pm 0.0239$
-0.75	$2.018 \pm 0.0175 \pm 0.0212$	-0.75	$2.018 \pm 0.0175 \pm 0.0212$
-0.50	$2.002 \pm 0.0165 \pm 0.0180$	-0.50	$2.001 \pm 0.0165 \pm 0.0181$

Table C.2: Average jet rates for the 172 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.2: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 172 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

183 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$11.193 \pm 0.1341 \pm 0.2650$	-5.00	$14.586 \pm 0.1234 \pm 0.1454$
-4.75	$9.422 \pm 0.1117 \pm 0.1765$	-4.75	$12.185 \pm 0.1060 \pm 0.1259$
-4.50	$7.713 \pm 0.0921 \pm 0.1397$	-4.50	$9.966 \pm 0.0919 \pm 0.1094$
-4.25	$6.267 \pm 0.0772 \pm 0.0952$	-4.25	$8.148 \pm 0.0802 \pm 0.0879$
-4.00	$5.070 \pm 0.0646 \pm 0.0804$	-4.00	$6.662 \pm 0.0696 \pm 0.0768$
-3.75	$4.260 \pm 0.0533 \pm 0.0720$	-3.75	$5.452 \pm 0.0598 \pm 0.0690$
-3.50	$3.687 \pm 0.0460 \pm 0.0600$	-3.50	$4.511 \pm 0.0522 \pm 0.0794$
-3.25	$3.288 \pm 0.0400 \pm 0.0486$	-3.25	$3.791 \pm 0.0430 \pm 0.0530$
-3.00	$2.987 \pm 0.0351 \pm 0.0472$	-3.00	$3.236 \pm 0.0379 \pm 0.0571$
-2.75	$2.775 \pm 0.0311 \pm 0.0551$	-2.75	$2.945 \pm 0.0335 \pm 0.0574$
-2.50	$2.590 \pm 0.0277 \pm 0.0440$	-2.50	$2.700 \pm 0.0292 \pm 0.0488$
-2.25	$2.458 \pm 0.0254 \pm 0.0369$	-2.25	$2.517 \pm 0.0261 \pm 0.0404$
-2.00	$2.339 \pm 0.0231 \pm 0.0280$	-2.00	$2.373 \pm 0.0235 \pm 0.0301$
-1.75	$2.248 \pm 0.0215 \pm 0.0297$	-1.75	$2.264 \pm 0.0218 \pm 0.0312$
-1.50	$2.153 \pm 0.0196 \pm 0.0255$	-1.50	$2.166 \pm 0.0199 \pm 0.0256$
-1.25	$2.096 \pm 0.0184 \pm 0.0222$	-1.25	$2.098 \pm 0.0184 \pm 0.0214$
-1.00	$2.045 \pm 0.0170 \pm 0.0199$	-1.00	$2.051 \pm 0.0172 \pm 0.0201$
-0.75	$2.013 \pm 0.0162 \pm 0.0179$	-0.75	$2.013 \pm 0.0162 \pm 0.0177$
-0.50	$2.003 \pm 0.0161 \pm 0.0165$	-0.50	$2.006 \pm 0.0160 \pm 0.0168$

Table C.3: Average jet rates for the 183 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.3: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 183 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

189 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$11.226 \pm 0.0774 \pm 0.1486$	-5.00	$14.466 \pm 0.0710 \pm 0.1065$
-4.75	$9.410 \pm 0.0645 \pm 0.1429$	-4.75	$11.989 \pm 0.0613 \pm 0.0957$
-4.50	$7.641 \pm 0.0525 \pm 0.1207$	-4.50	$9.799 \pm 0.0523 \pm 0.0992$
-4.25	$6.151 \pm 0.0426 \pm 0.0962$	-4.25	$8.015 \pm 0.0448 \pm 0.0809$
-4.00	$5.037 \pm 0.0356 \pm 0.0603$	-4.00	$6.529 \pm 0.0385 \pm 0.0711$
-3.75	$4.227 \pm 0.0300 \pm 0.0542$	-3.75	$5.349 \pm 0.0323 \pm 0.0440$
-3.50	$3.688 \pm 0.0259 \pm 0.0470$	-3.50	$4.398 \pm 0.0278 \pm 0.0505$
-3.25	$3.299 \pm 0.0224 \pm 0.0382$	-3.25	$3.726 \pm 0.0239 \pm 0.0351$
-3.00	$3.008 \pm 0.0194 \pm 0.0250$	-3.00	$3.259 \pm 0.0209 \pm 0.0257$
-2.75	$2.788 \pm 0.0171 \pm 0.0216$	-2.75	$2.932 \pm 0.0179 \pm 0.0239$
-2.50	$2.613 \pm 0.0149 \pm 0.0199$	-2.50	$2.696 \pm 0.0156 \pm 0.0193$
-2.25	$2.463 \pm 0.0133 \pm 0.0167$	-2.25	$2.521 \pm 0.0137 \pm 0.0181$
-2.00	$2.348 \pm 0.0121 \pm 0.0173$	-2.00	$2.382 \pm 0.0124 \pm 0.0183$
-1.75	$2.243 \pm 0.0108 \pm 0.0152$	-1.75	$2.261 \pm 0.0109 \pm 0.0151$
-1.50	$2.166 \pm 0.0098 \pm 0.0117$	-1.50	$2.174 \pm 0.0099 \pm 0.0126$
-1.25	$2.096 \pm 0.0087 \pm 0.0101$	-1.25	$2.101 \pm 0.0088 \pm 0.0111$
-1.00	$2.047 \pm 0.0079 \pm 0.0099$	-1.00	$2.048 \pm 0.0079 \pm 0.0107$
-0.75	$2.016 \pm 0.0074 \pm 0.0090$	-0.75	$2.016 \pm 0.0074 \pm 0.0093$
-0.50	$2.001 \pm 0.0073 \pm 0.0077$	-0.50	$2.000 \pm 0.0073 \pm 0.0078$

Table C.4: Average jet rates for the 189 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.4: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 189 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

192 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$11.435 \pm 0.1868 \pm 0.2801$	-5.00	$14.704 \pm 0.1799 \pm 0.3613$
-4.75	$9.666 \pm 0.1607 \pm 0.2652$	-4.75	$12.200 \pm 0.1547 \pm 0.2991$
-4.50	$7.895 \pm 0.1349 \pm 0.2110$	-4.50	$10.122 \pm 0.1354 \pm 0.2540$
-4.25	$6.396 \pm 0.1089 \pm 0.1652$	-4.25	$8.198 \pm 0.1125 \pm 0.1979$
-4.00	$5.180 \pm 0.0920 \pm 0.1542$	-4.00	$6.721 \pm 0.0993 \pm 0.1934$
-3.75	$4.348 \pm 0.0742 \pm 0.1317$	-3.75	$5.453 \pm 0.0806 \pm 0.1795$
-3.50	$3.780 \pm 0.0649 \pm 0.1261$	-3.50	$4.543 \pm 0.0711 \pm 0.1449$
-3.25	$3.351 \pm 0.0559 \pm 0.1021$	-3.25	$3.828 \pm 0.0599 \pm 0.1073$
-3.00	$3.038 \pm 0.0484 \pm 0.0911$	-3.00	$3.309 \pm 0.0531 \pm 0.1022$
-2.75	$2.813 \pm 0.0427 \pm 0.0760$	-2.75	$2.984 \pm 0.0461 \pm 0.0813$
-2.50	$2.636 \pm 0.0382 \pm 0.0500$	-2.50	$2.722 \pm 0.0401 \pm 0.0612$
-2.25	$2.472 \pm 0.0331 \pm 0.0424$	-2.25	$2.518 \pm 0.0337 \pm 0.0448$
-2.00	$2.362 \pm 0.0292 \pm 0.0427$	-2.00	$2.377 \pm 0.0291 \pm 0.0440$
-1.75	$2.227 \pm 0.0254 \pm 0.0375$	-1.75	$2.253 \pm 0.0263 \pm 0.0378$
-1.50	$2.166 \pm 0.0226 \pm 0.0293$	-1.50	$2.180 \pm 0.0232 \pm 0.0281$
-1.25	$2.100 \pm 0.0202 \pm 0.0267$	-1.25	$2.100 \pm 0.0202 \pm 0.0264$
-1.00	$2.049 \pm 0.0180 \pm 0.0217$	-1.00	$2.050 \pm 0.0180 \pm 0.0212$
-0.75	$2.016 \pm 0.0164 \pm 0.0183$	-0.75	$2.018 \pm 0.0165 \pm 0.0189$
-0.50	$2.001 \pm 0.0164 \pm 0.0202$	-0.50	$1.998 \pm 0.0166 \pm 0.0191$

Table C.5: Average jet rates for the 192 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.5: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 192 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

196 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$11.221 \pm 0.1243 \pm 0.1768$	-5.00	$14.465 \pm 0.1214 \pm 0.1846$
-4.75	$9.381 \pm 0.1025 \pm 0.1308$	-4.75	$11.980 \pm 0.1020 \pm 0.1517$
-4.50	$7.632 \pm 0.0841 \pm 0.0968$	-4.50	$9.797 \pm 0.0882 \pm 0.1559$
-4.25	$6.166 \pm 0.0699 \pm 0.0841$	-4.25	$8.035 \pm 0.0745 \pm 0.1155$
-4.00	$5.054 \pm 0.0597 \pm 0.0810$	-4.00	$6.541 \pm 0.0647 \pm 0.1127$
-3.75	$4.242 \pm 0.0501 \pm 0.0820$	-3.75	$5.302 \pm 0.0537 \pm 0.0954$
-3.50	$3.694 \pm 0.0424 \pm 0.0735$	-3.50	$4.405 \pm 0.0459 \pm 0.0783$
-3.25	$3.327 \pm 0.0367 \pm 0.0576$	-3.25	$3.729 \pm 0.0397 \pm 0.0648$
-3.00	$3.019 \pm 0.0314 \pm 0.0552$	-3.00	$3.272 \pm 0.0340 \pm 0.0507$
-2.75	$2.790 \pm 0.0282 \pm 0.0426$	-2.75	$2.968 \pm 0.0305 \pm 0.0554$
-2.50	$2.610 \pm 0.0246 \pm 0.0339$	-2.50	$2.708 \pm 0.0261 \pm 0.0470$
-2.25	$2.471 \pm 0.0214 \pm 0.0363$	-2.25	$2.523 \pm 0.0226 \pm 0.0401$
-2.00	$2.319 \pm 0.0186 \pm 0.0254$	-2.00	$2.352 \pm 0.0190 \pm 0.0255$
-1.75	$2.224 \pm 0.0168 \pm 0.0209$	-1.75	$2.243 \pm 0.0172 \pm 0.0233$
-1.50	$2.150 \pm 0.0153 \pm 0.0199$	-1.50	$2.166 \pm 0.0156 \pm 0.0211$
-1.25	$2.102 \pm 0.0141 \pm 0.0181$	-1.25	$2.106 \pm 0.0142 \pm 0.0195$
-1.00	$2.056 \pm 0.0129 \pm 0.0202$	-1.00	$2.057 \pm 0.0129 \pm 0.0213$
-0.75	$2.021 \pm 0.0119 \pm 0.0141$	-0.75	$2.020 \pm 0.0119 \pm 0.0142$
-0.50	$1.999 \pm 0.0117 \pm 0.0125$	-0.50	$1.999 \pm 0.0116 \pm 0.0124$

Table C.6: Average jet rates for the 196 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.6: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 196 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

200 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$10.830 \pm 0.1277 \pm 0.2527$	-5.00	$14.290 \pm 0.1202 \pm 0.1593$
-4.75	$9.114 \pm 0.1069 \pm 0.1520$	-4.75	$11.816 \pm 0.1039 \pm 0.1279$
-4.50	$7.390 \pm 0.0887 \pm 0.1956$	-4.50	$9.685 \pm 0.0885 \pm 0.1525$
-4.25	$5.966 \pm 0.0724 \pm 0.1558$	-4.25	$7.941 \pm 0.0765 \pm 0.1319$
-4.00	$4.909 \pm 0.0607 \pm 0.1219$	-4.00	$6.473 \pm 0.0660 \pm 0.0878$
-3.75	$4.164 \pm 0.0515 \pm 0.0918$	-3.75	$5.262 \pm 0.0558 \pm 0.0806$
-3.50	$3.651 \pm 0.0442 \pm 0.0589$	-3.50	$4.400 \pm 0.0490 \pm 0.0827$
-3.25	$3.279 \pm 0.0376 \pm 0.0648$	-3.25	$3.732 \pm 0.0423 \pm 0.0659$
-3.00	$3.014 \pm 0.0327 \pm 0.0468$	-3.00	$3.260 \pm 0.0355 \pm 0.0541$
-2.75	$2.803 \pm 0.0285 \pm 0.0403$	-2.75	$2.937 \pm 0.0303 \pm 0.0444$
-2.50	$2.629 \pm 0.0253 \pm 0.0391$	-2.50	$2.714 \pm 0.0265 \pm 0.0416$
-2.25	$2.486 \pm 0.0227 \pm 0.0359$	-2.25	$2.536 \pm 0.0233 \pm 0.0366$
-2.00	$2.362 \pm 0.0199 \pm 0.0229$	-2.00	$2.397 \pm 0.0207 \pm 0.0276$
-1.75	$2.257 \pm 0.0180 \pm 0.0252$	-1.75	$2.278 \pm 0.0184 \pm 0.0247$
-1.50	$2.169 \pm 0.0159 \pm 0.0221$	-1.50	$2.179 \pm 0.0161 \pm 0.0222$
-1.25	$2.106 \pm 0.0144 \pm 0.0210$	-1.25	$2.109 \pm 0.0145 \pm 0.0203$
-1.00	$2.044 \pm 0.0124 \pm 0.0172$	-1.00	$2.050 \pm 0.0127 \pm 0.0175$
-0.75	$2.012 \pm 0.0117 \pm 0.0141$	-0.75	$2.011 \pm 0.0118 \pm 0.0138$
-0.50	$1.998 \pm 0.0118 \pm 0.0121$	-0.50	$1.997 \pm 0.0118 \pm 0.0123$

Table C.7: Average jet rates for the 200 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.7: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 200 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

202 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$10.958 \pm 0.1915 \pm 0.5703$	-5.00	$14.426 \pm 0.1754 \pm 0.2486$
-4.75	$9.138 \pm 0.1603 \pm 0.4467$	-4.75	$12.050 \pm 0.1546 \pm 0.2153$
-4.50	$7.514 \pm 0.1296 \pm 0.2517$	-4.50	$9.918 \pm 0.1331 \pm 0.1643$
-4.25	$6.055 \pm 0.1020 \pm 0.2493$	-4.25	$8.090 \pm 0.1133 \pm 0.1392$
-4.00	$5.029 \pm 0.0868 \pm 0.1459$	-4.00	$6.603 \pm 0.0960 \pm 0.1296$
-3.75	$4.205 \pm 0.0712 \pm 0.1236$	-3.75	$5.346 \pm 0.0812 \pm 0.1305$
-3.50	$3.698 \pm 0.0631 \pm 0.0663$	-3.50	$4.426 \pm 0.0687 \pm 0.0911$
-3.25	$3.347 \pm 0.0542 \pm 0.0561$	-3.25	$3.777 \pm 0.0590 \pm 0.0759$
-3.00	$3.052 \pm 0.0483 \pm 0.0583$	-3.00	$3.325 \pm 0.0517 \pm 0.0564$
-2.75	$2.804 \pm 0.0414 \pm 0.0564$	-2.75	$2.993 \pm 0.0463 \pm 0.0637$
-2.50	$2.603 \pm 0.0362 \pm 0.0446$	-2.50	$2.716 \pm 0.0385 \pm 0.0485$
-2.25	$2.471 \pm 0.0316 \pm 0.0370$	-2.25	$2.526 \pm 0.0327 \pm 0.0406$
-2.00	$2.365 \pm 0.0291 \pm 0.0385$	-2.00	$2.395 \pm 0.0299 \pm 0.0420$
-1.75	$2.254 \pm 0.0255 \pm 0.0289$	-1.75	$2.280 \pm 0.0261 \pm 0.0299$
-1.50	$2.189 \pm 0.0237 \pm 0.0299$	-1.50	$2.201 \pm 0.0241 \pm 0.0295$
-1.25	$2.111 \pm 0.0208 \pm 0.0287$	-1.25	$2.115 \pm 0.0211 \pm 0.0279$
-1.00	$2.047 \pm 0.0180 \pm 0.0258$	-1.00	$2.045 \pm 0.0179 \pm 0.0262$
-0.75	$2.016 \pm 0.0168 \pm 0.0196$	-0.75	$2.016 \pm 0.0168 \pm 0.0194$
-0.50	$2.003 \pm 0.0164 \pm 0.0172$	-0.50	$2.002 \pm 0.0165 \pm 0.0171$

Table C.8: Average jet rates for the 202 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.8: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 202 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

205 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$11.058 \pm 0.1240 \pm 0.1572$	-5.00	$14.196 \pm 0.1173 \pm 0.1316$
-4.75	$9.205 \pm 0.1032 \pm 0.1610$	-4.75	$11.726 \pm 0.1008 \pm 0.1216$
-4.50	$7.404 \pm 0.0846 \pm 0.1122$	-4.50	$9.646 \pm 0.0867 \pm 0.1157$
-4.25	$6.001 \pm 0.0698 \pm 0.1015$	-4.25	$7.805 \pm 0.0744 \pm 0.0901$
-4.00	$4.861 \pm 0.0569 \pm 0.0657$	-4.00	$6.336 \pm 0.0632 \pm 0.0789$
-3.75	$4.129 \pm 0.0475 \pm 0.0956$	-3.75	$5.153 \pm 0.0534 \pm 0.0714$
-3.50	$3.623 \pm 0.0408 \pm 0.0725$	-3.50	$4.315 \pm 0.0454 \pm 0.0687$
-3.25	$3.275 \pm 0.0355 \pm 0.0477$	-3.25	$3.653 \pm 0.0386 \pm 0.0462$
-3.00	$3.035 \pm 0.0314 \pm 0.0477$	-3.00	$3.242 \pm 0.0327 \pm 0.0448$
-2.75	$2.827 \pm 0.0284 \pm 0.0411$	-2.75	$2.938 \pm 0.0295 \pm 0.0436$
-2.50	$2.633 \pm 0.0253 \pm 0.0337$	-2.50	$2.722 \pm 0.0265 \pm 0.0378$
-2.25	$2.483 \pm 0.0227 \pm 0.0296$	-2.25	$2.528 \pm 0.0232 \pm 0.0382$
-2.00	$2.360 \pm 0.0205 \pm 0.0306$	-2.00	$2.395 \pm 0.0208 \pm 0.0278$
-1.75	$2.240 \pm 0.0177 \pm 0.0270$	-1.75	$2.263 \pm 0.0182 \pm 0.0293$
-1.50	$2.162 \pm 0.0158 \pm 0.0231$	-1.50	$2.175 \pm 0.0163 \pm 0.0234$
-1.25	$2.091 \pm 0.0138 \pm 0.0201$	-1.25	$2.095 \pm 0.0139 \pm 0.0201$
-1.00	$2.048 \pm 0.0127 \pm 0.0175$	-1.00	$2.047 \pm 0.0127 \pm 0.0176$
-0.75	$2.013 \pm 0.0118 \pm 0.0139$	-0.75	$2.011 \pm 0.0117 \pm 0.0137$
-0.50	$2.001 \pm 0.0118 \pm 0.0132$	-0.50	$2.001 \pm 0.0117 \pm 0.0130$

Table C.9: Average jet rates for the 205 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.9: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 205 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

207 GeV Data Sample

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	$10.722 \pm 0.0970 \pm 0.2192$	-5.00	$14.051 \pm 0.0953 \pm 0.1474$
-4.75	$8.920 \pm 0.0789 \pm 0.2102$	-4.75	$11.628 \pm 0.0805 \pm 0.1336$
-4.50	$7.209 \pm 0.0644 \pm 0.1727$	-4.50	$9.467 \pm 0.0683 \pm 0.1076$
-4.25	$5.862 \pm 0.0545 \pm 0.1375$	-4.25	$7.704 \pm 0.0584 \pm 0.0836$
-4.00	$4.874 \pm 0.0460 \pm 0.0671$	-4.00	$6.314 \pm 0.0507 \pm 0.0700$
-3.75	$4.160 \pm 0.0398 \pm 0.0533$	-3.75	$5.213 \pm 0.0430 \pm 0.0596$
-3.50	$3.619 \pm 0.0333 \pm 0.0502$	-3.50	$4.297 \pm 0.0368 \pm 0.0611$
-3.25	$3.234 \pm 0.0285 \pm 0.0417$	-3.25	$3.643 \pm 0.0314 \pm 0.0508$
-3.00	$2.977 \pm 0.0250 \pm 0.0328$	-3.00	$3.195 \pm 0.0269 \pm 0.0385$
-2.75	$2.771 \pm 0.0218 \pm 0.0269$	-2.75	$2.894 \pm 0.0230 \pm 0.0303$
-2.50	$2.591 \pm 0.0189 \pm 0.0251$	-2.50	$2.676 \pm 0.0203 \pm 0.0297$
-2.25	$2.451 \pm 0.0169 \pm 0.0216$	-2.25	$2.496 \pm 0.0177 \pm 0.0230$
-2.00	$2.334 \pm 0.0152 \pm 0.0211$	-2.00	$2.362 \pm 0.0156 \pm 0.0206$
-1.75	$2.232 \pm 0.0133 \pm 0.0186$	-1.75	$2.254 \pm 0.0137 \pm 0.0193$
-1.50	$2.149 \pm 0.0117 \pm 0.0158$	-1.50	$2.157 \pm 0.0119 \pm 0.0145$
-1.25	$2.086 \pm 0.0103 \pm 0.0147$	-1.25	$2.091 \pm 0.0105 \pm 0.0159$
-1.00	$2.040 \pm 0.0091 \pm 0.0147$	-1.00	$2.041 \pm 0.0092 \pm 0.0145$
-0.75	$2.011 \pm 0.0086 \pm 0.0112$	-0.75	$2.011 \pm 0.0086 \pm 0.0113$
-0.50	$2.001 \pm 0.0086 \pm 0.0089$	-0.50	$2.001 \pm 0.0086 \pm 0.0088$

Table C.10: Average jet rates for the 207 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

Figure C.10: Average jet rates as a function of y_{cut} for the Durham and Cambridge algorithms for the 207 GeV data sample. PYTHIA and HERWIG Monte Carlo expectations are represented by the curves. The error bars indicate total error.

Values for the 91 GeV and LEP2 Data Samples

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-4.93	$12.883 \pm 0.0144 \pm 5.7671$	-4.93	$16.774 \pm 0.0154 \pm 3.5490$
-4.81	$12.224 \pm 0.0139 \pm 4.4482$	-4.81	$15.547 \pm 0.0148 \pm 1.0958$
-4.68	$11.484 \pm 0.0133 \pm 2.9694$	-4.68	$14.356 \pm 0.0142 \pm 8.7126$
-4.56	$10.691 \pm 0.0127 \pm 1.3836$	-4.56	$13.214 \pm 0.0135 \pm 6.4286$
-4.43	$9.871 \pm 0.0121 \pm 9.7439$	-4.43	$12.115 \pm 0.0129 \pm 4.2312$
-4.31	$9.049 \pm 0.0114 \pm 8.0982$	-4.31	$11.076 \pm 0.0123 \pm 2.1520$
-4.18	$8.239 \pm 0.0108 \pm 6.4799$	-4.18	$10.100 \pm 0.0117 \pm 0.2013$
-4.06	$7.454 \pm 0.0102 \pm 4.9092$	-4.06	$9.187 \pm 0.0111 \pm 8.3741$
-3.93	$6.713 \pm 0.0096 \pm 3.4271$	-3.93	$8.336 \pm 0.0106 \pm 6.6732$
-3.81	$6.026 \pm 0.0091 \pm 2.0535$	-3.81	$7.551 \pm 0.0100 \pm 5.1030$
-3.68	$5.404 \pm 0.0086 \pm 0.8089$	-3.68	$6.823 \pm 0.0095 \pm 3.6473$
-3.56	$4.855 \pm 0.0081 \pm 9.7100$	-3.56	$6.160 \pm 0.0090 \pm 2.3203$
-3.43	$4.386 \pm 0.0077 \pm 8.7737$	-3.43	$5.564 \pm 0.0085 \pm 1.1289$
-3.31	$3.988 \pm 0.0073 \pm 7.9777$	-3.31	$5.027 \pm 0.0081 \pm 0.0543$
-3.18	$3.656 \pm 0.0069 \pm 7.3139$	-3.18	$4.545 \pm 0.0077 \pm 9.0902$
-3.06	$3.391 \pm 0.0067 \pm 6.7837$	-3.06	$4.122 \pm 0.0073 \pm 8.2450$
-2.93	$3.172 \pm 0.0064 \pm 6.3450$	-2.93	$3.756 \pm 0.0069 \pm 7.5130$
-2.81	$2.993 \pm 0.0062 \pm 5.9872$	-2.81	$3.442 \pm 0.0066 \pm 6.8857$
-2.68	$2.846 \pm 0.0060 \pm 5.6939$	-2.68	$3.180 \pm 0.0063 \pm 6.3604$
-2.56	$2.727 \pm 0.0058 \pm 5.4543$	-2.56	$2.966 \pm 0.0061 \pm 5.9320$
-2.43	$2.628 \pm 0.0057 \pm 5.2563$	-2.43	$2.797 \pm 0.0059 \pm 5.5950$
-2.31	$2.543 \pm 0.0056 \pm 5.0877$	-2.31	$2.665 \pm 0.0057 \pm 5.3319$
-2.18	$2.467 \pm 0.0055 \pm 4.9356$	-2.18	$2.558 \pm 0.0056 \pm 5.1160$
-2.06	$2.399 \pm 0.0054 \pm 4.7998$	-2.06	$2.466 \pm 0.0055 \pm 4.9320$
-1.93	$2.339 \pm 0.0053 \pm 4.6794$	-1.93	$2.388 \pm 0.0054 \pm 4.7760$
-1.81	$2.285 \pm 0.0052 \pm 4.5713$	-1.81	$2.320 \pm 0.0053 \pm 4.6409$
-1.68	$2.236 \pm 0.0052 \pm 4.4734$	-1.68	$2.261 \pm 0.0052 \pm 4.5239$
-1.56	$2.191 \pm 0.0051 \pm 4.3838$	-1.56	$2.209 \pm 0.0051 \pm 4.4195$
-1.43	$2.152 \pm 0.0051 \pm 4.3057$	-1.43	$2.166 \pm 0.0051 \pm 4.3326$
-1.31	$2.118 \pm 0.0050 \pm 4.2364$	-1.31	$2.127 \pm 0.0050 \pm 4.2542$
-1.18	$2.087 \pm 0.0050 \pm 4.1755$	-1.18	$2.093 \pm 0.0050 \pm 4.1869$
-1.06	$2.062 \pm 0.0049 \pm 4.1257$	-1.06	$2.065 \pm 0.0049 \pm 4.1313$
-0.93	$2.041 \pm 0.0049 \pm 4.0827$	-0.93	$2.042 \pm 0.0049 \pm 4.0853$
-0.81	$2.023 \pm 0.0049 \pm 4.0475$	-0.81	$2.024 \pm 0.0049 \pm 4.0485$
-0.68	$2.009 \pm 0.0049 \pm 4.0197$	-0.68	$2.010 \pm 0.0049 \pm 4.0207$
-0.56	$2.002 \pm 0.0049 \pm 4.0044$	-0.56	$2.002 \pm 0.0049 \pm 4.0047$

Table C.11: Average jet rates for the 91 GeV data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

$\log_{10}(y_{\text{cut}})$	N	$\log_{10}(y_{\text{cut}})$	N
-5.00	11.094 $\pm$ 0.0412 $\pm$ 0.1217	-5.00	14.391 $\pm$ 0.0389 $\pm$ 0.0697
-4.75	9.293 $\pm$ 0.0344 $\pm$ 0.0981	-4.75	11.935 $\pm$ 0.0334 $\pm$ 0.0474
-4.50	7.556 $\pm$ 0.0283 $\pm$ 0.0834	-4.50	9.772 $\pm$ 0.0287 $\pm$ 0.0518
-4.25	6.109 $\pm$ 0.0233 $\pm$ 0.0563	-4.25	7.978 $\pm$ 0.0246 $\pm$ 0.0404
-4.00	4.998 $\pm$ 0.0196 $\pm$ 0.0355	-4.00	6.505 $\pm$ 0.0213 $\pm$ 0.0320
-3.75	4.210 $\pm$ 0.0165 $\pm$ 0.0336	-3.75	5.314 $\pm$ 0.0180 $\pm$ 0.0305
-3.50	3.669 $\pm$ 0.0142 $\pm$ 0.0323	-3.50	4.399 $\pm$ 0.0155 $\pm$ 0.0296
-3.25	3.290 $\pm$ 0.0123 $\pm$ 0.0292	-3.25	3.722 $\pm$ 0.0133 $\pm$ 0.0273
-3.00	3.012 $\pm$ 0.0108 $\pm$ 0.0275	-3.00	3.255 $\pm$ 0.0115 $\pm$ 0.0256
-2.75	2.790 $\pm$ 0.0095 $\pm$ 0.0243	-2.75	2.936 $\pm$ 0.0101 $\pm$ 0.0291
-2.50	2.608 $\pm$ 0.0084 $\pm$ 0.0207	-2.50	2.699 $\pm$ 0.0089 $\pm$ 0.0217
-2.25	2.465 $\pm$ 0.0076 $\pm$ 0.0199	-2.25	2.518 $\pm$ 0.0078 $\pm$ 0.0229
-2.00	2.344 $\pm$ 0.0069 $\pm$ 0.0175	-2.00	2.376 $\pm$ 0.0070 $\pm$ 0.0186
-1.75	2.238 $\pm$ 0.0062 $\pm$ 0.0142	-1.75	2.259 $\pm$ 0.0063 $\pm$ 0.0155
-1.50	2.160 $\pm$ 0.0057 $\pm$ 0.0115	-1.50	2.171 $\pm$ 0.0058 $\pm$ 0.0118
-1.25	2.095 $\pm$ 0.0052 $\pm$ 0.0102	-1.25	2.099 $\pm$ 0.0052 $\pm$ 0.0098
-1.00	2.045 $\pm$ 0.0048 $\pm$ 0.0081	-1.00	2.047 $\pm$ 0.0048 $\pm$ 0.0083
-0.75	2.013 $\pm$ 0.0045 $\pm$ 0.0071	-0.75	2.013 $\pm$ 0.0045 $\pm$ 0.0069
-0.50	2.001 $\pm$ 0.0045 $\pm$ 0.0049	-0.50	2.000 $\pm$ 0.0045 $\pm$ 0.0048

Table C.12: Average jet rates for the LEP2 data sample using the Cambridge(left) and Durham(right) algorithms. The first error corresponds to the statistical error while the second error corresponds to the total error.

C.2 Fit Values from the Average Jet Rates

161 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12566	0.00329	0.00678	30.89 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10593	0.00451	0.00286	6.91 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.12442	0.00321	0.00609	13.93 (8.0)
R match	-3.50 – -0.50	0.11569	0.00261	0.00409	7.17 (12.0)
lnR match	-3.25 – -0.50	0.11030	0.00274	0.00460	6.13 (11.0)
mod ln R match	-3.25 – -0.50	0.11564	0.00306	0.00478	6.14 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11029	0.00274	0.00461	6.13 (11.0)
NLLA + mod y	-3.50 – -1.25	0.12166	0.00266	0.00567	33.29 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12098	0.00262	0.00518	16.11 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11564	0.00306	0.00477	6.14 (11.0)

Table C.13: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 161 GeV data using the Durham algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12546	0.00359	0.00685	25.21 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10616	0.00307	0.00231	6.24 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12376	0.00351	0.00589	11.66 (8.0)
R match	-3.50 – -0.50	0.11465	0.00294	0.00374	8.61 (12.0)
lnR match	-3.50 – -0.50	0.10985	0.00267	0.00312	7.94 (12.0)
mod ln R match	-3.50 – -0.50	0.11556	0.00300	0.00363	8.22 (12.0)
ln R match +mod y	-3.50 – -0.50	0.10985	0.00267	0.00311	7.94 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12092	0.00297	0.00578	28.11 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12010	0.00292	0.00467	13.89 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11555	0.00300	0.00363	8.22 (12.0)

Table C.14: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 161 GeV data using the Cambridge algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

172 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12978	0.00390	0.00638	20.69 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10501	0.00305	0.00282	4.37 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.12815	0.00381	0.00602	7.16 (8.0)
R match	-3.50 – -0.50	0.11831	0.00299	0.00455	6.06 (12.0)
lnR match	-3.25 – -0.50	0.11267	0.00329	0.00473	4.38 (11.0)
mod ln R match	-3.25 – -0.50	0.11811	0.00370	0.00519	5.02 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11267	0.00329	0.00473	4.38 (11.0)
NLLA + mod y	-3.50 – -1.25	0.12582	0.00298	0.00645	21.66 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12459	0.00295	0.00584	8.38 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11811	0.00370	0.00519	5.02 (11.0)

Table C.15: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 172 GeV data using the Durham algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12059	0.00421	0.00699	16.93 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10248	0.00369	0.00315	1.64 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.11881	0.00408	0.00574	5.04 (8.0)
R match	-3.50 – -0.50	0.11292	0.00377	0.00434	3.81 (12.0)
lnR match	-3.50 – -0.50	0.10838	0.00341	0.00367	3.35 (12.0)
mod ln R match	-3.50 – -0.50	0.11367	0.00384	0.00409	3.90 (12.0)
ln R match +mod y	-3.50 – -0.50	0.10838	0.00341	0.00367	3.35 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12146	0.00379	0.00402	15.80 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.11996	0.00372	0.00374	4.72 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11367	0.00384	0.00410	3.89 (12.0)

Table C.16: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 172 GeV data using the Cambridge algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

183 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.13123	0.00174	0.00417	45.44 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10605	0.00155	0.00161	12.18 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.12880	0.00172	0.00389	17.30 (8.0)
R match	-3.50 – -0.50	0.12086	0.00142	0.00423	15.45 (12.0)
lnR match	-3.25 – -0.50	0.11368	0.00147	0.00443	11.54 (11.0)
mod ln R match	-3.25 – -0.50	0.11942	0.00166	0.00473	12.87 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11368	0.00147	0.00443	11.54 (11.0)
NLLA + mod y	-3.50 – -1.25	0.12797	0.00141	0.00572	52.16 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12653	0.00141	0.00524	20.76 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11942	0.00166	0.00473	12.86 (11.0)

Table C.17: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 183 GeV data using the Durham algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12415	0.00198	0.00234	35.65 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10351	0.00163	0.00193	7.56 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12138	0.00195	0.00321	14.86 (8.0)
R match	-3.50 – -0.50	0.11546	0.00164	0.00400	12.05 (12.0)
lnR match	-3.50 – -0.50	0.11056	0.00148	0.00300	10.25 (12.0)
mod ln R match	-3.50 – -0.50	0.11631	0.00167	0.00383	11.71 (12.0)
ln R match +mod y	-3.50 – -0.50	0.11056	0.00148	0.00300	10.26 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12391	0.00163	0.00312	33.40 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12177	0.00162	0.00292	13.48 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11631	0.00167	0.00382	11.71 (12.0)

Table C.18: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 183 GeV data using the Cambridge algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

189 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12876	0.00101	0.00483	71.70 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10561	0.00106	0.00072	6.94 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.12648	0.00099	0.00395	50.26 (8.0)
R match	-3.50 – -0.50	0.11806	0.00079	0.00307	12.54 (12.0)
lnR match	-3.25 – -0.50	0.11158	0.00085	0.00253	5.19 (11.0)
mod ln R match	-3.25 – -0.50	0.11700	0.00096	0.00236	7.73 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11158	0.00085	0.00253	5.20 (11.0)
NLLA + mod y	-3.50 – -1.25	0.12489	0.00080	0.00475	95.42 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12366	0.00079	0.00382	63.86 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11700	0.00096	0.00236	7.73 (11.0)

Table C.19: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 189 GeV data using the Durham algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12370	0.00112	0.00259	59.09 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10403	0.00097	0.00121	18.43 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12131	0.00109	0.00292	59.20 (8.0)
R match	-3.50 – -0.50	0.11550	0.00092	0.00355	26.60 (12.0)
lnR match	-3.50 – -0.50	0.11059	0.00084	0.00258	21.69 (12.0)
mod ln R match	-3.50 – -0.50	0.11637	0.00094	0.00336	25.50 (12.0)
ln R match +mod y	-3.50 – -0.50	0.11059	0.00084	0.00258	21.70 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12343	0.00092	0.00383	48.63 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12167	0.00092	0.00324	52.16 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11637	0.00094	0.00336	25.52 (12.0)

Table C.20: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 189 GeV data using the Cambridge algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

192 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.14077	0.00200	0.00555	26.87 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10883	0.00164	0.00375	7.23 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.13372	0.00195	0.00570	8.55 (8.0)
R match	-3.50 – -0.50	0.12106	0.00145	0.00523	7.54 (12.0)
lnR match	-3.25 – -0.50	0.11437	0.00170	0.00467	6.52 (11.0)
mod ln R match	-3.25 – -0.50	0.11957	0.00190	0.00491	6.53 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11436	0.00170	0.00467	6.49 (11.0)
NLLA + mod y	-3.50 – -1.25	0.13817	0.00143	0.00611	27.58 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.13208	0.00141	0.00586	8.48 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11958	0.00190	0.00491	6.62 (11.0)

Table C.21: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 192 GeV data using the Durham algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12987	0.00257	0.00506	29.81 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10717	0.00236	0.00375	9.71 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12475	0.00248	0.00544	14.88 (8.0)
R match	-3.50 – -0.50	0.11692	0.00200	0.00518	10.54 (12.0)
lnR match	-3.50 – -0.50	0.11230	0.00182	0.00439	9.89 (12.0)
mod ln R match	-3.50 – -0.50	0.11775	0.00205	0.00485	10.52 (12.0)
ln R match +mod y	-3.50 – -0.50	0.11229	0.00182	0.00437	9.88 (12.0)
NLLA + mod y	-3.50 – -1.25	0.13055	0.00203	0.00493	27.89 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12582	0.00200	0.00509	13.96 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11777	0.00205	0.00488	10.51 (12.0)

Table C.22: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 192 GeV data using the Cambridge algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

196 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.13141	0.00168	0.00489	69.70 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10585	0.00144	0.00208	8.11 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.12848	0.00164	0.00416	24.34 (8.0)
R match	-3.50 – -0.50	0.11950	0.00133	0.00352	15.02 (12.0)
lnR match	-3.25 – -0.50	0.11254	0.00142	0.00310	11.05 (11.0)
mod ln R match	-3.25 – -0.50	0.11796	0.00159	0.00307	13.19 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11254	0.00142	0.00310	11.04 (11.0)
NLLA + mod y	-3.50 – -1.25	0.12750	0.00132	0.00527	77.66 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12583	0.00131	0.00476	27.85 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11795	0.00159	0.00307	13.18 (11.0)

Table C.23: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 196 GeV data using the Durham algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12596	0.00180	0.00332	61.56 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10454	0.00141	0.00234	11.31 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12313	0.00176	0.00385	26.65 (8.0)
R match	-3.50 – -0.50	0.11586	0.00151	0.00434	19.68 (12.0)
lnR match	-3.50 – -0.50	0.11101	0.00137	0.00340	16.59 (12.0)
mod ln R match	-3.50 – -0.50	0.11673	0.00155	0.00418	19.19 (12.0)
ln R match +mod y	-3.50 – -0.50	0.11100	0.00137	0.00340	16.59 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12471	0.00151	0.00362	58.93 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12256	0.00149	0.00350	24.11 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11673	0.00155	0.00418	19.20 (12.0)

Table C.24: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 196 GeV data using the Cambridge algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

200 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.13808	0.00145	0.00434	61.51 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10910	0.00217	0.00373	7.14 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.13131	0.00140	0.00451	19.75 (8.0)
R match	-3.50 – -0.50	0.11901	0.00106	0.00400	4.41 (12.0)
lnR match	-3.25 – -0.50	0.11256	0.00121	0.00343	5.44 (11.0)
mod ln R match	-3.25 – -0.50	0.11762	0.00135	0.00338	4.16 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11256	0.00121	0.00343	5.42 (11.0)
NLLA + mod y	-3.50 – -1.25	0.13572	0.00106	0.00505	62.03 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12988	0.00104	0.00482	18.89 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11763	0.00135	0.00338	4.24 (11.0)

Table C.25: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 200 GeV data using the Durham algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12664	0.00180	0.00366	57.79 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10838	0.00361	0.00615	8.32 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12187	0.00173	0.00402	22.21 (8.0)
R match	-3.50 – -0.50	0.11395	0.00142	0.00369	7.57 (12.0)
lnR match	-3.50 – -0.50	0.10954	0.00129	0.00295	8.39 (12.0)
mod ln R match	-3.50 – -0.50	0.11473	0.00145	0.00325	7.58 (12.0)
ln R match +mod y	-3.50 – -0.50	0.10954	0.00129	0.00296	8.39 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12715	0.00146	0.00342	53.83 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12260	0.00142	0.00365	19.75 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11474	0.00145	0.00325	7.58 (12.0)

Table C.26: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 200 GeV data using the Cambridge algorithm over the stated it range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

202 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.14015	0.00198	0.00446	34.25 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10891	0.00225	0.00438	9.64 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.13320	0.00193	0.00443	13.78 (8.0)
R match	-3.50 – -0.50	0.12004	0.00145	0.00348	9.96 (12.0)
lnR match	-3.25 – -0.50	0.11393	0.00168	0.00307	9.18 (11.0)
mod ln R match	-3.25 – -0.50	0.11908	0.00188	0.00287	9.48 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11393	0.00168	0.00306	9.19 (11.0)
NLLA + mod y	-3.50 – -1.25	0.13681	0.00144	0.00428	37.14 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.13094	0.00142	0.00418	14.94 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11909	0.00188	0.00287	9.44 (11.0)

Table C.27: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 202 GeV data using the Durham algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12835	0.00258	0.00230	35.08 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10659	0.00244	0.00645	13.18 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12350	0.00249	0.00292	18.84 (8.0)
R match	-3.50 – -0.50	0.11458	0.00201	0.00305	12.28 (12.0)
lnR match	-3.50 – -0.50	0.11015	0.00182	0.00222	11.80 (12.0)
mod ln R match	-3.50 – -0.50	0.11538	0.00205	0.00245	11.95 (12.0)
ln R match +mod y	-3.50 – -0.50	0.11014	0.00182	0.00222	11.81 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12770	0.00205	0.00258	33.15 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12322	0.00201	0.00286	17.32 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11540	0.00205	0.00246	11.91 (12.0)

Table C.28: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 202 GeV data using the Cambridge algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

205 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.13621	0.00136	0.00375	55.01 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10625	0.00137	0.00285	6.98 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.12934	0.00132	0.00402	16.51 (8.0)
R match	-3.50 – -0.50	0.11715	0.00101	0.00348	7.39 (12.0)
lnR match	-3.25 – -0.50	0.11055	0.00114	0.00307	6.53 (11.0)
mod ln R match	-3.25 – -0.50	0.11528	0.00127	0.00292	7.33 (11.0)
ln R match +mod y	-3.25 – -0.50	0.11054	0.00114	0.00308	6.56 (11.0)
NLLA + mod y	-3.50 – -1.25	0.13402	0.00101	0.00463	55.97 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12809	0.00099	0.00435	16.05 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11530	0.00127	0.00292	7.17 (11.0)

Table C.29: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 205 GeV data using the Durham algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12620	0.00175	0.00337	48.94 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10605	0.00223	0.00425	5.20 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.12170	0.00169	0.00350	16.47 (8.0)
R match	-3.50 – -0.50	0.11292	0.00136	0.00358	4.94 (12.0)
lnR match	-3.50 – -0.50	0.10858	0.00124	0.00287	4.79 (12.0)
mod ln R match	-3.50 – -0.50	0.11367	0.00139	0.00316	5.12 (12.0)
ln R match +mod y	-3.50 – -0.50	0.10858	0.00124	0.00288	4.78 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12546	0.00140	0.00331	45.90 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12134	0.00137	0.00355	14.36 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11368	0.00139	0.00316	5.07 (12.0)

Table C.30: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 205 GeV data using the Cambridge algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

207 GeV Data Sample

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.13492	0.00110	0.00376	80.57 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.50 – -0.50	0.10395	0.00069	0.00153	5.28 (12.0)
NLLA + G_{21}	-3.25 – -1.25	0.12767	0.00106	0.00393	22.32 (8.0)
R match	-3.50 – -0.50	0.11518	0.00082	0.00338	12.18 (12.0)
lnR match	-3.25 – -0.50	0.10869	0.00091	0.00302	6.36 (11.0)
mod ln R match	-3.25 – -0.50	0.11313	0.00101	0.00295	9.75 (11.0)
ln R match +mod y	-3.25 – -0.50	0.10868	0.00091	0.00303	6.43 (11.0)
NLLA + mod y	-3.50 – -1.25	0.13272	0.00083	0.00352	81.22 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12662	0.00081	0.00361	20.01 (9.0)
mod ln R + mod y	-3.25 – -0.50	0.11315	0.00101	0.00294	9.39 (11.0)

Table C.31: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 207 GeV data using the Durham algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Scheme	Fit Range $\log_{10}(y_{\text{cut}})$	Fitted α_s	Stat. Error	Syst. Error	χ^2 (dof)
NLLA	-3.25 – -1.25	0.12390	0.00140	0.00263	75.42 (8.0)
$\mathcal{O}(\alpha_s^2)$	-3.25 – -0.50	0.10188	0.00117	0.00169	3.48 (11.0)
NLLA + G_{21}	-3.25 – -1.25	0.11891	0.00135	0.00320	25.06 (8.0)
R match	-3.50 – -0.50	0.11078	0.00108	0.00340	13.10 (12.0)
lnR match	-3.50 – -0.50	0.10672	0.00099	0.00271	10.27 (12.0)
mod ln R match	-3.50 – -0.50	0.11146	0.00110	0.00288	14.23 (12.0)
ln R match +mod y	-3.50 – -0.50	0.10672	0.00099	0.00271	10.33 (12.0)
NLLA + mod y	-3.50 – -1.25	0.12489	0.00112	0.00274	70.19 (9.0)
NLLA + G_{21} + mod y	-3.50 – -1.25	0.12016	0.00109	0.00333	23.52 (9.0)
mod ln R + mod y	-3.50 – -0.50	0.11148	0.00110	0.00289	13.87 (12.0)

Table C.32: Fitted values of α_s with statistical and systematic errors for several QCD parameterizations of the average jet rate for the 207 GeV data using the Cambridge algorithm over the stated fit range. The quality of the fit is characterized by the chi-square (χ^2) and the number of degrees of freedom (dof).

Appendix D

Renormalization Scale Dependence

In this section tables of values for the renormalization scale parameter, x_μ , returned from the $\mathcal{O}(\alpha_s^2)$ fits to the D_2 and the $\langle N \rangle$ distributions are presented. The values are determined for all 10 LEP2 energies as well as the 91 GeV data sample and the combined LEP2 data samples. As in the case of the fit for α_s a statistically weighted average is also determined and shown in the tables. The systematic errors are calculated in an identical manner as was done for the determination of α_s in Sections 5.3 and 5.4.

Data Sample	x_μ	Stat. Error	Syst Error
161 GeV	0.27687	0.30452	0.18939
172 GeV	0.17633	0.21871	0.78905
183 GeV	0.18317	0.06286	0.10247
189 GeV	0.17783	0.03003	0.03964
192 GeV	0.18748	0.08993	0.08039
196 GeV	0.34855	0.25365	0.41399
200 GeV	0.12537	0.02114	0.02499
202 GeV	0.14820	0.05484	0.06449
205 GeV	0.15378	0.02966	0.08535
207 GeV	0.14037	0.02272	0.06166
91 GeV	0.02360	0.02360	0.56816
LEP2	0.17080	0.01499	0.01980
Stat. Weighted Average	0.14705	0.01180	0.02332

Data Sample	x_μ	Stat. Error	Syst Error
161 GeV	0.16749	0.04583	0.83273
172 GeV	0.22058	0.12670	0.19929
183 GeV	0.19133	0.03243	0.02967
189 GeV	0.17231	0.01507	0.02025
192 GeV	0.15258	0.02775	0.05365
196 GeV	0.19361	0.03256	0.05550
200 GeV	0.17505	0.02683	0.02068
202 GeV	0.17868	0.04439	0.05144
205 GeV	0.15122	0.01915	0.03500
207 GeV	0.18028	0.02411	0.05628
91 GeV	0.00459	0.00459	0.27823
LEP2	0.017796	0.00878	0.01568
Stat. Weighted Average	0.17069	0.00831	0.03507

Table D.1: Renormalization scale parameter values returned from the $D_2 \mathcal{O}(\alpha_s^2)$ fit for the Cambridge(left) and Durham(right) algorithms.

Data Sample	x_μ	Stat. Error	Syst Error
161 GeV	0.04196	0.01926	0.02185
172 GeV	0.04871	0.03146	0.02648
183 GeV	0.03493	0.00949	0.01489
189 GeV	0.04150	0.00713	0.01156
192 GeV	0.06596	0.03334	0.17146
196 GeV	0.03298	0.00679	0.02026
200 GeV	0.15036	0.10322	0.14659
202 GeV	0.06994	0.03646	0.16914
205 GeV	0.09235	0.04486	0.81136
207 GeV	0.05815	0.01706	0.02596
91 GeV	0.00279	0.00279	0.12472
Stat. Weighted Average	0.05198	0.00514	0.02341

Data Sample	x_μ	Stat. Error	Syst Error
161 GeV	0.16749	0.04583	0.83273
172 GeV	0.03449	0.01492	0.01350
183 GeV	0.03214	0.00709	0.01308
189 GeV	0.04289	0.00701	0.01275
192 GeV	0.04453	0.01687	0.06246
196 GeV	0.03488	0.00750	0.03857
200 GeV	0.07238	0.03064	0.03555
202 GeV	0.05683	0.02654	0.10251
205 GeV	0.05046	0.01665	0.61982
207 GeV	0.03480	0.00659	0.00835
91 GeV	0.00224	0.00224	0.10401
Stat. Weighted Average	0.03825	0.00233	0.01177

Table D.2: Renormalization scale parameter values returned from the $\langle N \rangle \mathcal{O}(\alpha_s^2)$ fit for the Cambridge(left) and Durham(right) algorithms.

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