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Full Name of Author — Nom complet de l'auteur

IVANO ANTHONY PAGOTTO

Date of Birth — Date de naissance

OCT. 13, 1960

Country of Birth — Lieu de naissance

CANADA

Permanent Address — Residence fixe

2865 CEDARWOOD DR, APT 115
OTTAWA, ONTARIO, CANADA
K1V 0G8

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PROF. ROBERT BELL

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Ivano Pagotto

**STRESS INTENSITY FACTORS
FOR CORNER CRACKED TURBINE
DISCS UNDER ARBITRARY LOADING
USING FINITE ELEMENT METHODS**

Ivano A. Pagotto

*A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the
Degree of Master of Engineering*

**DEPARTMENT OF MECHANICAL AND AERONAUTICAL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, ONTARIO
CANADA.**

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The undersigned hereby recommend to the Faculty of Graduate Studies and Research, acceptance of the thesis. "Stress Intensity Factors For Corner Cracked Turbine Discs Under Arbitrary Loading Using Finite Element Methods", submitted by Ivano Pagotto, in partial fulfillment of the requirements for the degree of Master of Engineering

Thesis Supervisor

Chairman, Department of Mechanical
and Aeronautical Engineering

ABSTRACT

The purpose of this report is to present stress intensity factors for a disc with a corner crack at its bore. This information is valuable in the prediction of service lives for turbine discs. The finite element method is used to study several cases of the cracked disc. These were all for a disc with a ratio of outer to inner radius of 8 and whose thickness was equal to its bore diameter. The ratio of crack depth to disc thickness ranged from 0.05 to 0.5 and the elliptical aspect ratio of the crack ranged from 1 to 4. The occurrence of a single corner crack as well as that of 2, 3 and 4 equispaced corner cracks were studied for each case. The results are presented in the form of several non-dimensional magnification factors which can be superimposed to simulate any loading condition.

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NOMENCLATURE

a - semi-minor axis of elliptical crack

b - semi-major axis of elliptical crack

t - Disc thickness

R - Outer radius of disc

R_i - Inner radius of disc

ϕ - Elliptical angle

E - Elliptical integral of the second kind

K - Stress intensity factor

H_1 - Stress intensity magnification factor

G - Shear modulus

Y - Young's modulus

m - crack tip element length parameter

L - crack tip element length parameter

ν - Poisson's Ratio

σ - stress

P_o - External radial tensile pressure

r - radius

u - displacement in the x direction

v - displacement in the y direction

w - displacement in the z direction

Chapter 1

INTRODUCTION

1.1 Introduction

The design requirements of modern turbines entering service have resulted in higher and higher stress levels. This has resulted in the introduction into service of a number of low life components. Turbine discs are some of the most highly stressed components in the engine and it is important that a reliable life prediction of these components be made to avoid catastrophic service failures.

Historically, methods used for predicting the life of turbine discs have resulted in conservative estimates of useful life. These traditional design methods were based on a statistical crack initiation criterion where the life of the component was defined as the number of cycles required for a detectable crack of some finite size to appear. Improvements in utilizing the potential life of each disc however are only possible by adopting a fracture mechanics approach.

Linear elastic fracture mechanics can be used to predict the cyclic fatigue life of a component provided that the appropriate stress intensity factor, among other things, is known. Application of fracture mechanics to assess the fatigue life of turbine discs however has been hampered by the lack of three dimensional stress intensity factor solutions which can adequately model elliptically shaped flaws.

This report describes work which has been carried out using the finite element method to determine stress intensity factors for discs with radial, elliptical corner cracks at the inner bore. This configuration of cracked disc is a common occurrence in turbine engines but has not yet been analyzed in a three dimensional study. The following chapters include some background in the areas of life prediction of discs, a method of obtaining stress intensity factors under general

loading conditions, a detailed description of the finite element models used and a discussion of the results.

1.2 Objectives

The objectives of this thesis are to present stress intensity factors for a disc with a radial corner crack at the inner bore. The results will be presented in the form of non-dimensional magnification factors which can be used to obtain results for any arbitrary loading condition.

The following disc and combinations of crack geometries will be analyzed:

Disc Geometry:

$$R_o/R_i = 8$$

$$t/R_i = 2$$

Crack Geometries:

$$b/a = 1, 2, 4$$

$$b/t = 0.5, 0.25, 0.1, 0.05$$

The above configurations will be considered for the case of 1, 2, 3 and 4 equispaced cracks. Figure 1.1 shows a diagram of the geometry of the problem.

Disc Geometries: $R_o/R_i = 8$ and $t/R_i = 2$

Crack Geometries: $b/a = 1, 2, 4$ and $b/t = 0.5, 0.25, 0.1, 0.05$

FIGURE 1.1 Diagram of Corner Cracked Disc Problem

Chapter 2

BACKGROUND AND THEORY

2.1 Fracture Mechanics of Turbine Discs

A fracture mechanics analysis of turbine discs can be divided into two classes of problems. The first class is the problem of subsurface crack growth from unavoidable defects in the disc material. The second class of problems is the initiation and growth of surface cracks as a result of fatigue loading of initially defect-free material. Linear elastic fracture mechanics forms the basis of analyzing both problems.

Examples of failures of discs which ruptured due to cyclic crack growth of an internal material defect are available in the literature, for example (1,2). The processing of disc materials however now includes stringent controls which minimize the occurrence and size of these defects. In addition, fracture mechanics has reached the level where the fatigue life of components with buried cracks can be reliably predicted (2,3). The internal material defect is usually treated as a simple circular embedded crack for which the stress intensity factor equation is readily available. This class of problems will not be considered in this report.

The complexity of the stress fields and crack geometry for surface crack problems however, has precluded the use of fracture mechanics for such problems. Surface cracks occur at several locations in the disc, such as the rim slots, bolt holes and often at the bore of the disc. These cracks seldom take the shape of a simple straight through edge crack, rather they are usually only part through the disc, elliptical in shape, and require a three dimensional analysis. Stress intensity factor solutions, K_I , are needed for such geometries in order that a fracture mechanics analysis of crack growth can be performed. Of interest in this report is the determination of K_I solutions for a disc with elliptically shaped corner cracks at its bore as shown in fig 1.1. Further

discussion of the stress intensity factor, its importance and the methods used in obtaining it are presented in later sections.

2.2 Stress and Fatigue Conditions of Turbine Discs

A large increase in size and loading of rotating machinery during the last 30 years has resulted in a great increase in operating stresses in turbine discs. For example, Hurchalla et al (4) of Pratt and Whitney report that the average tangential stress level in turbine discs doubled from 55000 psi (380 MPa) in the J75 in 1955 to 110000 psi (760 MPa) in the F100 engine in 1975. A knowledge of the state of stress and the fatigue conditions under which the disc operates are required so that a fracture mechanics analysis and life prediction can be performed.

The stresses that are developed in the disc are primarily due to high centrifugal body forces, inertia forces due to the presence of blades at the rim of the disc and thermal forces caused by nonuniform disc temperature (1). These and any other stresses, such as vibratory or residual stresses can either combine detrimentally or tend to cancel each other out. In the work presented in this report the stresses will be treated as though they remained in the elastic response range of the material involved. This will allow for the superposition of the stresses and for implementation of Linear Elastic Fracture Mechanics (LEFM).

Of great importance in the fracture mechanics analysis of turbine discs are the fatigue conditions under which they operate. Each turbine disc has associated with it a characteristic duty cycle. This duty cycle is simply the stress history of the disc from start to stop of the engine. For aircraft, the term used is flight cycle and consists of the stress history from take-off to landing. A typical duty cycle may consist of subcycles of stress which occur as a result of such things as the varying demands of take-off, climb, cruise, descent and landing. This type of engine operation results in one, or a few stress cycles per duty cycle of the disc. Cyclic loading of this type is termed Low Cycle Fatigue (LCF) because of the relatively low number of stress cycles experienced as opposed to say, a piston rod in an internal combustion engine. High cycle fatigue may also occur as the result of vibration, flutter and buffetting, however the resulting

stresses at the bore are very low (2), and it is the usual design practice to attempt to eliminate the possibility of high cycle fatigue before entering service. This paper deals with bore cracks and LCF conditions are assumed to determine the design life of the disc.

2.3 Design Life of Turbine Discs

The use of fracture mechanics in the design and maintenance of turbine discs is becoming more important as the need to fully utilize the potential life of these components increases. The necessity of adopting a fracture mechanics approach in this endeavor as opposed to the traditional design methodology is discussed in this section.

2.3.1 Traditional Cycle Life Design

The dominant failure mode for turbine discs is low cycle fatigue. Traditional LCF designs are based on a crack initiation criterion where the life of the disc is defined as the number of cycles required for a detectable crack of some finite size to appear. The size of the crack is dependant on the Non-Destructive Evaluation limits (NDE). Life predictions are made based on material data of the type shown in figures 2.1 and 2.2. These are S-N type curves, or more commonly total strain range - cycles to failure curves obtained experimentally, where failure of the material is defined as crack initiation. Because of the statistical behaviour of engineering materials, there is a good deal of scatter associated with the number of cycles required to initiate a crack at some given stress or strain level. For design purposes this scatter is compensated for by lowering the failure curve to a conservative "design allowable" level where the probability of crack initiation becomes very low, as shown in fig 2.2. To illustrate, the LCF design criteria for turbine discs designed by Pratt and Whitney will be discussed. This design procedure is quite typical of the approach used by other engine manufacturers. At Pratt and Whitney the life of a turbine disc is defined as the number of cycles required to initiate a $1/32$ inch (0.79 mm) long crack on the surface, with a probability of 0.1% (2,4-8). When applied to a disc designed for 15000 cycles of life, it means that out of 1000 discs, one will develop a $1/32$ inch (0.79 mm) crack at 15000 cycles. After 15000 cycles, all of the discs would be retired or removed from service even though only one of them had a detectable crack.

This procedure has been very successful in preventing catastrophic failures in the field however it is very wasteful. Out of the 1000 discs which would be retired at the end of the design life, 999 would have some undefined residual life remaining. This remaining residual life can be significant as shown in fig 2.3 which is based on discs used in Pratt and Whitney's F100 engine.

Under the traditional design procedure discussed above, it is not possible to make use of the potential life of the remaining discs without accepting a higher possibility of failure of the minimum life disc. However a new method which can extend the safe life of the discs is being implemented at Pratt and Whitney (5-8), and other companies. This method is based upon the use of a fracture mechanics analysis of the crack propagation phase and is termed Retirement For Cause (RFC).

2.3.3 Retirement For Cause

The RFC concept is shown in fig 2.4. The number of cycles, N_c , required for a crack of initial, detectable size A_0 to grow to critical size, A_c is calculated. This requires the use of an appropriate material crack growth model and appropriate stress intensity factors for the given crack. Periodic inspection intervals for the component are then established. Each component which is found to have a detectable crack at one of the inspections is removed from service, or, "retired for cause". The inspection interval N_i is chosen so that no component with an initial crack equal to or smaller than A_0 could fail within the interval. By following this procedure, each component is allowed to operate for its own specific crack initiation life. At least three inspection intervals are provided so that a crack which is missed in one inspection can be safely detected in the next inspection thus ensuring that the maximum usable life of each disc is utilized while still maintaining a good degree of safety.

Inspection intervals must be established in a cost effective manner since each inspection requires a "tear down" of the engine to inspect its components. The degree of confidence in the NDE capabilities, the scatter in the crack growth rates between typical and minimum, and the required safety factor should also be considered in establishing N_i .

It should be noted that RFC can be used in the maintenance of existing components or can be implemented in the design stage where it is sometimes referred to as Damage Tolerant Design (DTD), see for example (5). The first engine to be considered for RFC was the F100 built for the United States Air Force by Pratt and Whitney. It was evaluated in a three year program from 1977 to 1980 (7). Extended service lives for 47 components including the turbine discs were evaluated and a cost analysis performed. It was found that for an investment of 16 million dollars, RFC would save 240 million dollars over the 15 year life of the F100 engine.

It should also be noted that while RFC ensures that the full crack initiation life of the component is utilized, it does not ensure that the full crack propagation life is utilized. The crack initiation life usually makes up the majority of the total life of the component, however further improvements could be achieved by making use of part of the crack propagation life. This would involve keeping the cracked disc in service until failure was about to occur, by setting the rejection crack size to some calculated value greater than the detectable size. Referring to fig 2.4 this would require setting A_0 to the largest possible value while still providing three inspection intervals to detect and reject discs with cracks equal to or larger than A_0 .

2.3.3 Life Prediction Systems

To successfully predict the life of the turbine disc a fracture mechanics life prediction system similar to the one shown in fig 2.5 is required. Cook et al (5) developed and implemented this system for the damage tolerant design of cold section turbine discs. The three key ingredients in this or any fracture mechanics life prediction system are:

- (i) stress analysis which provides information on the stress state in the disc for a typical duty cycle
- (ii) stress intensity factor solutions, K , for the crack and disc geometries of interest
- (iii) material properties and cyclic crack growth relation da/dN vs ΔK .

Of prime importance in the life prediction are the stress intensity factors for cracks in the critical areas of interest. As discussed previously, one of the most important of these areas is the disc bore which often develops elliptically shaped corner cracks. K solutions for cracks of this type are not readily available in the literature. It is for this reason that the purpose for this work was to present K solutions for discs with corner cracks at the bore under arbitrary loading.

The fatigue crack growth rate in metals generally follows the trend of the log-log sigmoidal curve shown in fig 2.6 where the cyclic crack growth rate, da/dN , is described as a function of the range in the stress intensity factor, dK . This behaviour is frequently described as having three regions. Region I is associated with very slow crack growth in the vicinity of the threshold stress intensity factor, ΔK_{th} . Region II describes stable cyclic crack growth and has been studied extensively. Region III describes the behaviour exhibited at very high crack growth rates where the stress intensity approaches the critical value and fracture occurs.

There are many semi-empirical crack growth models available in the literature, for example see (9). The most common of these is the well known Paris law which is included in the above mentioned reference. The Paris law takes the form

$$da/dN = C \Delta K^m \quad (2.1)$$

and models the stable crack growth of region II. There are also several laws which incorporate the curved portions of region I and III together with the straight portion of region II. In addition, as discussed in (9), there are many models which include the effect of stress ratio, R , environment, and other effects. Hill et al of Pratt and Whitney (6), in their fracture mechanics analysis of the TF33 disc, used the modified Walker equation

$$da/dN = \frac{C \Delta K^m}{(1-R)^{(1-M)m}} \quad (2.2)$$

In this equation, the necessary parameters C , m , M were obtained empirically from material

tests carried out at the temperatures of interest. This gave good results for this specific case.

In any case, the appropriate growth law is numerically integrated to determine crack size as a function of duty cycles. From this analysis the appropriate inspection intervals for RFC can be determined, or, the remaining life of the cracked disc calculated.

2.4 Stress Intensity Factors

There are three basic modes of crack surface displacement which can cause crack growth. These are illustrated in fig 2.7 and described below.

- I The opening mode - crack surfaces move directly apart

- II The edge sliding mode - crack surfaces move normal to the crack front and remain in the crack plane

- III The shear mode - crack surfaces move parallel to the crack front and remain in the crack plane

These three modes of crack surface displacement can be conveniently characterized by the stress intensity factor, K , with subscripts I, II, III to denote the mode. K has the dimensions of $(\text{stress}) \times (\text{length})^{1/2}$ and is a function of loading stress, crack size and shape, and geometry of the cracked specimen.

Mode I or the opening mode is of primary interest since it dominates most cases of engineering importance including the case of radial bore cracks in turbine discs, which are under axisymmetric loading. The stress intensity factor, K_I , has many practical uses such as in the calculation of critical crack size and fatigue life.

If it is assumed that the crack is perfectly sharp, the stress and displacement fields for

mode I are given by the following equations.

$$\begin{aligned}
 \sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + \dots \\
 \sigma_{yy} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + \dots \\
 \sigma_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} + \dots \\
 u &= \frac{K_I}{8G} \sqrt{\frac{2r}{\pi}} \left((2k-1) \cos \frac{\theta}{2} - \cos \frac{3\theta}{2} \right) + \dots \\
 v &= \frac{K_I}{8G} \sqrt{\frac{2r}{\pi}} \left((2k+1) \sin \frac{\theta}{2} - \sin \frac{3\theta}{2} \right) + \dots \quad (2.3)
 \end{aligned}$$

For plane strain conditions

$$\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{yy})$$

$$w = 0$$

$$k = 3 - 4\nu$$

For plane stress conditions

$$\sigma_{zz} = 0$$

$$w = \frac{\nu}{Y} \int (\sigma_{xx} + \sigma_{yy}) dz$$

$$k = \frac{3 - \nu}{1 + \nu}$$

These equations were developed by Westergaard using an Airy stress function approach. An inverse square root stress singularity which occurs at the crack tip ($r=0$) is evident in these equations. As shown in fig 2.8 the stress σ_{yy} approaches infinity at the crack tip. This results in a zone of plastic deformation at the crack tip, however, LEFM disregards this plastic zone. The

Westergaard equations shown above can be used to determine K_I from a numerical analysis which outputs the stresses or displacements near the crack tip.

For the case of a buried elliptical crack, a state of triaxial stress exists at the crack tip resulting in plane strain conditions. For surface cracks however, there can be no stress normal to the free surface. This results in a biaxial state of stress, or plane stress conditions at the free surfaces. The buried portion of the surface crack however remains in a state of plane strain. All calculations of K_I are therefore carried out assuming plane stress conditions at the free surfaces and plane strain conditions elsewhere.

The well known stress intensity factor solution for an elliptical crack embedded in a plate of infinite depth and surface dimensions published by Irwin (10) is as follows.

$$K_{Ic}(\phi) = \frac{\sigma_o}{E} \sqrt{\frac{\pi a}{b}} (a^2 \sin^2 \phi + b^2 \cos^2 \phi)^{\frac{1}{2}} \quad (2.4)$$

This solution will be used as a normalizing factor in the presentation of K_I results.

2.5 Previous Research on Stress Intensity Factors for Discs

The first published work in the literature related to cracks in turbine discs was that of Winne and Wundt (11) whose work was motivated by turbine failures in the late 1950's. That paper presented an application of the Griffith-Irwin theory of crack propagation to the bursting of discs and included experimental studies. Since then a number of researchers have presented mode I stress intensity factor solutions for edge cracks at the bore of a disc.

Owen and Griffiths (12) used finite element methods in two dimensions to present K_I values for one and two edge cracks at the bore of a disc. That work was associated with the investigation of a specific steam turbine disc burst and both centrifugal loads and radial tensile blade forces acting on the rim of the disc were considered. Delale and Erdogan (13) presented K_I results for a hollow cylinder or disc, containing a radial crack for various crack lengths and radius ratios. A plane elasticity formulation was used in this study. External edge cracks, bore edge cracks and embedded cracks were considered and various loading conditions including

(centrifugal body loads were studied. Tracy (14) used a modified mapping collocation method to obtain K_I for one to four equal length, equally spaced edge cracks at the inner and outer surfaces of a rotating disc. This method is based on the complex variable formulation of plane elasticity. Tracy's paper included results for two radius ratios. Tan (15) presented K_I results for a single edge crack at the bore of a rotating disc using the BIE method. A wide range of crack depth and radius ratio was considered in this paper and results for both plane stress and plane strain conditions were presented.

The two dimensional solutions for edge cracks at the bore of a disc presented by the above mentioned researchers can be considered to provide upper bound or conservative estimates of K_I . This is because the cracks which occur in a disc are rarely straight fronted through cracks. As discussed previously, a common occurrence is an elliptically shaped corner crack at the intersection of the bore and one of the flat surfaces of the disc as shown in fig 1.1. It is the purpose of this report to present K_I solutions for this configuration of cracked disc. This requires a three dimensional stress analysis which can usually only be carried out by numerical methods.

There has been very little published on the problem of elliptical corner cracks at the bore of a disc. Kobayashi et al (16) presented the K_I distribution along the crack front for elliptical corner cracks at the bore of a rotating disc for a range of crack sizes and aspect ratios. These results were obtained by the alternating method. The procedure used was to calculate the K_I distribution for a quarter elliptical crack in a quarter infinite solid subjected to a stress distribution equal to the hoop stress in an uncracked rotating disc and to multiply the result by a correction factor to account for the curvature of the bore. This curvature correction factor was derived from the stress intensity factors of a single edge cracked bore in a large plate and a single edge cracked semi-infinite plate.

Tan (17) presented K_I distributions for elliptical corner cracks at the bore of a rotating disc using the BIE method. Two different crack depths and two aspect ratios were considered under the condition of centrifugal body load and radial tensile stress at the outer rim (blade load). Both Kobayashi et al and Tan's work considered a disc with a radius ratio of eight.

The above mentioned studies are the only ones available which present stress intensity factors for corner cracks at the bore of a disc. Cruse and Desner (18) presented a method based on fracture mechanics for predicting the residual life of surface cracks in complex structural details. This method made use of a BIE analysis of a quarter elliptical crack in a rectangular solid. Crack growth was simulated using an influence function technique and the method was applied to a corner crack at a through bolt hole in a rotating disc. This method however did not account for the influence of the curved hole surface. As pointed out by Tan (17) other related works, such as studies of cracks occurring at fastener holes, are available but can not be satisfactorily applied to the cracked disc problem of interest in this report because of the difference in the stress distributions.

2.6 Numerical Solution Methods for Crack Problems

In practice there are three considerations which may demand numerical solutions in the analysis of cracked components, these are:

- a) if the cracks have irregular fronts in three dimensional situations,
- b) if the cracks are situated near stress concentrations,
- c) if a complicated stress distribution exists

The Finite Element Method (FEM) and the Boundary Integral Equation method (BIE), have become the most widely used forms of numerical methods for the stress analysis of cracked components. The BIE method is the most efficient in terms of computing time and costs as well as in data preparation. This is because only the boundary of the component need be modelled, thus reducing the problem to a two dimensional one. FEM has also been widely used in this area and is adopted in this study because of the excellent FE package ASAS which was available. The ASAS system is described further in a later section.

2.7 Superposition to Obtain K_I Under Arbitrary Loading

It is desirable to obtain K_I results for the cracked disc for a wide variety of loading conditions. This can be achieved by obtaining results for several specific loading conditions which can then be superimposed to give the desired loading of the disc. The method of superposition used in this study is described in the following section.

As shown in fig 2.9 the mode I stress intensity factor, K_I , for the crack in the disc subjected to remote loading is equal to the stress intensity factor, K_I , for the same crack with the crack surface subjected to a hoop stress profile $\sigma(r)$. This is because the stress intensity factor K_I is set equal to zero by applying a stress profile $\sigma(r)$ which is sufficient to "close" the crack. The stress profile $\sigma(r)$ is the profile developed over the section of the crack by the remote loading. Therefore, in order to determine the K_I distribution for the crack under an arbitrary loading situation, the stress profile $\sigma(r)$ which exists in the uncracked disc is fitted with a third order polynomial

$$\sigma(r) = A_0 + A_1 r + A_2 r^2 + A_3 r^3 \quad (2.5)$$

The crack face is loaded with the appropriate pressure loads to achieve this stress profile. This is done by applying four separate pressure load cases, $P(r)$, to the crack face.

- 1) $\sigma(r) = P(r) = -1$ (constant crack face pressure)
- 2) $\sigma(r) = P(r) = -r$ (linear crack face pressure)
- 3) $\sigma(r) = P(r) = -r^2$ (quadratic crack face pressure)
- 4) $\sigma(r) = P(r) = -r^3$ (cubic crack face pressure)

The K_I distribution is calculated for each of these load cases and superimposed according to the desired stress distribution defined by the polynomial in equation 2.5 as follows

$$K_I(\phi) = A_0 K_{I0}(\phi) + A_1 K_{I1}(\phi) + A_2 K_{I2}(\phi) + A_3 K_{I3}(\phi) \quad (2.6)$$

The subscripts 0,1,2,3 are associated with their respective load cases.

The stress profile in equation 2.5 can be rewritten so as to "non-dimensionalize" it and thus present the results in a more general form. To do this, equation 2.5 is written as shown

below.

$$\sigma(r) = B_0 + B_1 \left(\frac{r}{R_i} \right) + B_2 \left(\frac{r}{R_i} \right)^2 + B_3 \left(\frac{r}{R_i} \right)^3 \quad (2.7)$$

where

$$B_0 = A_0$$

$$B_1 = A_1 R_i$$

$$B_2 = A_2 R_i^2$$

$$B_3 = A_3 R_i^3$$

Equation 2.6 then becomes:

$$K_I(\phi) = B_0 K_{I0}(\phi) + \frac{B_1}{R_i} K_{I1}(\phi) + \frac{B_2}{R_i^2} K_{I2}(\phi) + \frac{B_3}{R_i^3} K_{I3}(\phi) \quad (2.8)$$

At this point it is convenient to define non-dimensional magnification factors, $H_j(\phi)$, for each load case denoted by the subscript j .

$$H_j(\phi) = \frac{K_{Ij}(\phi)}{K_{0j}(\phi)} \quad (2.9)$$

where

$$K_{0j}(\phi) = \frac{\sigma_{0j}}{E} \sqrt{\frac{\pi a}{b}} (a^2 \sin^2 \phi + b^2 \cos^2 \phi)^{\frac{1}{2}}$$

$$\sigma_{0j} = R_i^j$$

$$j = 0, 1, 2, 3$$

Here $K_{0j}(\phi)$ is the solution for an embedded elliptical crack as discussed in section 2.4. The nominal stress used in this function is taken as the stress applied at the inner bore, $r = R_i$, for the associated load case defined by the subscript j . From equation 2.9 we see that

$$K_{Ij}(\phi) = K_{0j}(\phi) H_j(\phi) = \frac{1}{E} \sqrt{\frac{\pi a}{b}} (a^2 \sin^2 \phi + b^2 \cos^2 \phi)^{\frac{1}{2}} R_i^j H_j(\phi) \quad (2.10)$$

Therefore, equation 2.8 becomes

$$K_I(\phi) = \frac{1}{E} \sqrt{\frac{\pi a}{b}} (a^2 \sin^2 \phi + b^2 \cos^2 \phi)^{\frac{1}{2}} (B_0 H_0(\phi) + B_1 H_1(\phi) + B_2 H_2(\phi) + B_3 H_3(\phi)) \quad (2.11)$$

Once the magnification factors $H_j(\phi)$ for any cracked disc have been determined, the K_I distribution resulting from any arbitrary stress profile defined by equation 2.7 can be obtained from equation 2.11. This method of superposition can be used to obtain the K_I distribution for any duty cycle provided its hoop stress profile can be closely approximated by a third order polynomial. It should be noted that the polynomial need only fit the stress profile over the radial range of the crack. It was found that very close fits for most stress states could be achieved. For example, the centrifugal stress profile for a spinning disc was approximated with a third order polynomial to a maximum error of less than 0.5% over the range of the crack.

This method of superposition has been used to advantage by many researchers. McGowan and Raymond (19), Heliot et al (20), and Raju and Newman (21), to mention a few, have used this method together with BIE or FEM results to obtain K_I results for pressurized cylinders.

FIGURE 2.1 Strain Range vs. Cycles For a Typical Material

Ref. (4)

FIGURE 2.2 S-N Curve For a Typical Material

Ref. (5)

FIGURE 2.3 Potential Residual Life of Retired Discs
Ref. (7)

FIGURE 2.4 Retirement For Cause Concept
Ref. (5)

FIGURE 2.5 A Disc Life Prediction System
Ref. (5)

FIGURE 2.6 Typical Crack Propagation Curve

FIGURE 2.7 Modes of Crack Surface Displacement

FIGURE 2.8 Stress Variation at the Crack Tip

FIGURE 2.9 Remote Loading To Crack Face Loading by Superposition

Chapter 3

FINITE ELEMENT ANALYSIS

3.1 Finite Element Analysis Applied to LEFM

The stress analysis of structures with cracks to obtain stress intensity factors is one of the most difficult and important problems in a fracture mechanics analysis. Since theoretical stress intensity factors are generally the product of a highly sophisticated mathematical analysis for idealized geometries and loading conditions it is not possible to find an exact solution for many real situations. The need for numerical methods to solve these problems is apparent.

The finite element method is one of the best numerical methods available for obtaining stress intensity factors whenever exact solutions are not available. Finite element methods are rapidly adapted in structural analysis and are applicable to large classes of geometries, materials and loading conditions in a realistic way. The technical details of the finite element method are described in the literature (22) and will not be covered in this report.

The most important problem in the use of finite element methods to model a cracked structure is the stress singularity which occurs at the crack tip. The difficulty lies in the fact that the singularity results in rapidly varying stresses near the crack tip. To accurately model the crack tip region therefore requires either a very large number of elements, or the use of special elements which can model this singularity. Chan et al (23) used constant strain elements to model a compact tension specimen, and a rotating disc with two cracks. It was found that many hundreds of elements were required to achieve even 5% accuracy for these problems. The use of special elements which can model the stress singularity is therefore necessary. A review of several of the most important formulations which model the singularity is given in (24). Among the approaches discussed in this reference, classical solution based singularity elements,

polynomial singularity function elements and modified isoparametric elements have received the most attention.

Isoparametric elements have been widely used to model crack problems. These elements can be modified to model the $\frac{1}{\sqrt{r}}$ stress singularity at the crack tip as required by the Westergaard solution for the stress distribution around a crack. This is the element used to model the crack tip behaviour in this work, therefore a short description follows.

3.1.1 Isoparametric Elements

Isoparametric elements are widely available in finite element programs. Their use in modelling crack problems was the result of the fact that a simple modification of the basic isoparametric element gives the required $\frac{1}{\sqrt{r}}$ stress singularity. This was first recognized by Henshall and Shaw (25) who found that by shifting the mid-side node to the quarter point in quadratic isoparametric triangular or quadrilateral two dimensional elements, the required singularity resulted at the nearest adjacent node.

Barsoum (26) investigated quadratic two dimensional and three dimensional isoparametric elements. He demonstrated the idea of "collapsing" nodes along one edge to a single point, and placing the adjacent nodes at the quarter point as shown in fig 3.1. This collapsed isoparametric element also contained the $\frac{1}{\sqrt{r}}$ singularity at the crack tip. In a later publication Barsoum (27) showed that the collapsed isoparametric elements contained the required singularity along any line emanating from the crack tip node while the simple modified isoparametric elements only exhibited this singularity along the boundaries of the element. He recommends the use of the collapsed formulation.

3.2 Finite Element Package ASAS

The finite element package used in this study is the Atkins Stress Analysis System, version H (ASAS). This package has gained wide acceptance as a reliable, easy to use, general purpose finite element system. ASAS has facilities for linear elastic stress analysis, natural frequency analysis and heat conduction analysis. An extensive library of element types and

a variety of loading and boundary condition types are included in the system. Also included are a comprehensive series of data checking procedures which check input data for errors and consistency.

The system is enhanced by a variety of pre and post processors which cover a wide range of capabilities such as graphic display, band-width optimisation and mesh generation. The graphic display capabilities are invaluable in the assembly of finite element meshes.

Included in the ASAS system are capabilities for multi-level sub-structuring and fracture mechanics analysis. These capabilities together with the ability for multiple load case analysis were of particular importance in this study.

3.2.1 Fracture Mechanics Facilities

The ASAS element library contains a number of special crack tip elements for modelling cracked structures. These elements are used around the tip of the crack and have been formulated to represent the singularity at that point. Among these are three elements based on the degenerate isoparametric elements described by Barsoum, as discussed in section 3.1.1.

CTM6 - 6 node triangular isoparametric membrane

CB15 - 15 node wedge shaped isoparametric solid

CTA6 - 6 node triangular isoparametric axisymmetric solid

The three dimensional CB15 element is used in the finite element models of this study. The isoparametric elements shown above do not give an output of the stresses because of their rapid variation across the element, however the displacements are provided. The Westergaard equations shown in section 2.4 can therefore be used to evaluate K_I from the nodal displacements. These equations can be evaluated at a series of nodes along radial lines emanating from the crack tip and the results extrapolated to give the value of K_I at the crack tip. Ingraffea et al (28) derived a closed form expression for the stress intensity factors K_I , K_{II} , and K_{III} .

in terms of the solution displacements for the isoparametric wedge element used in this study. These simple to use expressions are shown in fig 3.2 and are used in the calculations performed in this study.

3.2.2 Multi-Level Substructuring Capabilities

The multi-level substructuring capability of ASAS has greatly increased the power of this system to solve larger problems and to solve problems more efficiently and economically.

In substructured analysis, the structure to be analyzed is broken down into a number of substructures or components. The lowest level components are formed by the assembly of the available finite elements. Higher level components can consist of any number of low level components which have already been created and can also include the use of the basic finite element. Once a component has been created it can be thought of as a "super-element" and used as though it were an ordinary element. Any component may be used more than once in any given higher level component by using translation, rotation or mirroring. The final assembly of components and elements which represents the whole structure is known as the global structure.

The analysis is a three stage process. First, each component is partially solved up to its boundary with other components. Next, the boundaries of the components are assembled together to form the global structure and solved. Finally, the boundary displacements are sent back into each component and combined with the initial partial solution to form the total displacements and stresses in that component. The stresses and displacements for any or all of the components can then be recovered. An example of the use of the multi-level substructuring technique to model a box girder bridge deck taken from (29) is shown in fig 3.3.

The substructuring technique can be applied to any linear stress analysis problem but is clearly advantageous in a number of areas. For example, it may be impossible to analyze a very large structure because of the limitations of computer storage space and time. By using substructures however this large problem can be analyzed by a series of small computer runs. This was very advantageous in the large meshes used to model the turbine discs in this study.

FIGURE 3.1 Quarter-Point Crack Tip Element

$$K_I = \frac{E}{4(1-\nu^2)} \sqrt{\left(\frac{\pi}{2L_1}\right)} [2v_B - v_C + 2v_E - v_F + v_D - 2v_B + v_C - 2v_E + v_F - v_D \\ + \frac{1}{2}\eta(-4v_B + v_C + 4v_E - v_F + 4v_B - v_C - 4v_E + v_F) \\ + \frac{1}{2}\eta^2(v_F + v_C - 2v_D - v_F - v_C + 2v_D)]$$

$$K_{II} = \frac{E}{4(1-\nu^2)} \sqrt{\left(\frac{\pi}{2L_1}\right)} [2u_B - u_C + 2u_E - u_F + u_D - 2u_B + u_C - 2u_E + u_F - u_D \\ + \frac{1}{2}\eta(-4u_B + u_C + 4u_E - u_F + 4u_B - u_C - 4u_E + u_F) \\ + \frac{1}{2}\eta^2(u_F + u_C - 2u_D - u_F - u_C + 2u_D)]$$

and

$$K_{III} = \frac{E}{4(1+\nu)} \sqrt{\left(\frac{\pi}{2L_1}\right)} [2w_B - w_C + 2w_E - w_F + w_D - 2w_B + w_C - 2w_E + w_F - w_D \\ + \frac{1}{2}\eta(-4w_B + w_C + 4w_E - w_F + 4w_B - w_C - 4w_E + w_F) \\ + \frac{1}{2}\eta^2(w_F + w_C - 2w_D - w_F - w_C + 2w_D)]$$

FIGURE 3.2 SIF Equations From Nodal Displacements

Ref. (24)

FIGURE 3.3 Substructuring Of Complex Structures
Ref. (29)

Chapter 4

PRESENT WORK

4.1 Introduction

This chapter describes the finite element studies carried out to analyze the problem of discs with semi-elliptical corner cracks under arbitrary loading. The geometry of the problem, the cases considered, and the procedure of modeling the problem with finite element methods are presented. Finally the results for all the cases considered are presented.

4.2 Discs with Corner Cracks

As discussed in chapter 2, it is desired to obtain stress intensity factors for a variety of crack depths, b/t , and elliptical aspect ratios, b/a , under arbitrary loading conditions. The following subsections will present the geometry of the problem, the crack depths and shapes considered, the loading, and the boundary conditions required to model the problem.

4.2.1 Geometries and Loading Conditions Considered

Referring to fig 1.1, the cases considered were all for a disc with a semi-elliptical corner crack emanating from the intersection of the bore and one of the flat surfaces of the disc. The radius ratio $R_2/R_1 = 8$ and thickness $t = 2R_1$ was used for all cases while the semi-major axes, b , and the semi-minor axes, a , of the crack were varied to obtain a variety of crack depths and shapes. Three different elliptical aspect ratios, $b/a=1, 2$ and 4 were treated. For each of these crack shapes four different crack depths were analyzed, these were $b/t=0.05, 0.1, 0.25, 0.5$. In addition, the case of 1, 2, 3, and 4 equally spaced corner cracks, as shown in fig 1.1, were considered.

As discussed in chapter 2, it is desirable to obtain results which can be used to simulate any arbitrary loading condition which produces a radially varying hoop stress distribution. Referring to section 2.7, this requires the use of four loading cases from which the results can be appropriately superimposed. The four load cases; constant, linear, quadratic, and cubic crack face pressure are easily applied to the crack face using the variable pressure load capabilities of ASAS. The pressure acts normal to the crack face and thus acts in the same direction as the hoop stress.

4.2.2 Symmetry and Boundary Conditions

A disc with 1, 2, 3 and 4 equally spaced cracks is to be analyzed, however because of symmetry, the whole disc need not be modeled. As shown in fig 4.1, by making use of symmetry only a segment of angle $180^\circ/N$ where N is the number of cracks, needed to be considered. Thus, for the case of a disc with one crack, only half the disc is modeled; for a disc with two cracks, one quarter of the disc is modeled; for a disc with three cracks, one sixth of the disc is modeled; and for a disc with four cracks only one eighth of the disc need be modeled. By applying the displacement suppressions shown in the figure, the required symmetry is imposed and the problem can therefore be solved with considerable savings.

The finite element analysis requires that any structure to be analyzed be constrained from translation and rotation. The constraints imposed to achieve this are shown in fig 4.2 for a one crack model and a two crack model. The constraints for the three and four crack models are similar to those for the two crack model and are not shown. These boundary conditions were used in all of the finite element studies performed.

4.3 Modeling of Discs with Corner Cracks

In the following subsections, the procedure used for modelling the discs with finite elements will be briefly discussed. The use of a mesh generator to produce the meshes, the sub-structuring of the disc into a number of components, and the design of the finite element models with regard to degree of mesh refinement and accuracy will also be described.

4.3.1 Mesh Generation

The manual production of a large number of three dimensional finite element meshes can be an enormous task, extensive in time and effort required, and very prone to errors. Therefore the construction of the meshes used in this study was carried out with use of a mesh generator which produces input data in ASAS input format. This mesh generator was developed at Carleton University (30) and was designed specifically for the purpose of modeling structures which contain elliptical cracks. The advantages of the quick and easy production of meshes and error free input data for ASAS possible with this mesh generator can not be overemphasized.

The mesh generator operates in the following manner. A two dimensional mesh is input in ASAS format. This simple mesh, as shown in fig 4.3, is easily constructed by hand, to the desired dimensions and degree of mesh refinement. The mesh generator then takes this simple two dimensional mesh and rotates it around an elliptical path, of the required aspect ratio, in a stepwise manner to produce the three dimensional mesh as shown in fig 4.4. The desired number of elements along the projected elliptical path is input to the program and the path is divided into elements in such a way as to ensure that if it is a crack mesh, the length of each crack tip element along the crack front is equal. The mesh generator was modified so that rows of elements could be stacked on top of the basic mesh as shown in fig 4.5. The thickness of the mesh could also be specified as desired. This allowed for the efficient production of disc meshes of the desired radius ratio and thickness. In addition the mesh generator was modified to output the appropriate load data (4 load cases), nodal suppressions, and link nodes for linking with other meshes. This is discussed further in section 4.3.2.

4.3.2 Substructuring

The use of substructuring to model the disc was adopted for two main reasons. Firstly, to model a complete 180° mesh with a satisfactory degree of mesh refinement requires an analysis with a very large number of degrees of freedom to be solved, and is therefore too costly both in computer CPU time and money to be handled in just one single computer run. However, by breaking the total problem down into a number of substructures or components which are

linked together to form a global structure, the computer analysis is reduced to a series of small manageable computer runs. A diagram of the substructuring scheme used to model the discs is shown in fig 4.6. The meshes for each of these components were generated as discussed in the previous section. The other reason for adopting a substructuring procedure is evident from this diagram. To analyze any given disc for the case of 1, 2, 3, and 4 cracks, each substructure shown is analyzed by itself only once and then combined in four separate global analyses. For the case of a disc with a single crack, components 1 to 7 are linked together in a global structure. For a disc with two cracks, components 1 to 6 are linked together in another global analysis and so on. In this way the components which are common to all of the global structures need only be analyzed once but can be used in each global analysis.

The use of substructuring requires that the nodes to be linked together between components be specified in the input. The mesh generator was modified so that this information was provided automatically. The boundary conditions were also automatically imposed both at the component level and the global structure level by the mesh generator, so as to comply with the requirements discussed in section 4.2.2.

4.3.3 Mesh Refinement and Accuracy

Before constructing the finite element meshes it was necessary to establish the behaviour of the crack tip elements, and to determine the density of mesh required to give acceptable results. This was achieved primarily by drawing upon the experience gathered by other researchers who have used ASAS at Carleton University. Reference (31) includes an investigation of how crack tip element size and length affects the stress intensity factors calculated from ASAS finite element results. In this investigation, two benchmark problems for which known K_I solutions were available were performed. These were the case of a circular crack embedded in an infinite medium for which the exact solution is known and the case of a semi-circular surface crack for which finite element results by Raju and Newman (32) are available. The results of this investigation will be discussed in the following.

The mesh parameters of interest in the investigation were (i) the number of elements around the crack front, (ii) the length of the element perpendicular to the crack front compared to the

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crack depth, i.e. a/L ratio, and (iii) the length of the element along the crack front, i.e. the m/L ratio. These element dimensions are defined in fig 4.7. The investigation consisted of two series of analyses. In the first, three meshes with 2, 4, and 8 elements along the crack front were analyzed. These meshes had m/L ratios of 4, 2.07, and 1.03 and consisted of 38, 76, and 152 elements respectively. For the embedded crack, the accuracy of the results compared to the exact solution were 7.6% for the 38 element model, 4.6% for the 76 element model and 3.6% for the 152 element model. For the surface crack the accuracies were all within 2% of the results obtained by Raju and Newman as reported in (32). In the second series of tests, the number of elements along the crack front was kept at four and the length L of the element was varied. Three analyses of the surface crack were carried out for $a/L = 2.5, 5$, and 10 and the results compared with Raju and Newman.

From these benchmark tests, the authors of (31) concluded that in order to reach a compromise between computer costs and accuracy, the crack mesh should be constructed using a minimum of four crack tip elements around the crack and the dimensions of the crack tip elements should be less than $m/L = 2$ and $a/L = 5$. In the present study the guidelines for crack mesh design discussed above were followed as closely as possible. The number of crack tip elements around the crack was chosen as 5 for all meshes however, to avoid the problems of irregular element shape for elliptical cracks with high aspect ratios. It was not possible to maintain both the a/L and b/L ratios above 5 for a few of the shallowest cracks analyzed because the mesh generator maintains a constant value of L along the crack, however this criterion was easily met for most meshes. Having modeled the crack tip region to the desired degree of accuracy, the rest of the disc was modeled by a decreasing degree of mesh refinement in a direction away from the crack while still avoiding irregular element shapes.

4.4 Results

In this section the results for the stress intensity factor solutions obtained from the finite element results are presented. The graphical presentation of the non-dimensionalized results is described and the use of several post processing programs for the calculation of results is detailed. In addition the results for some specific load cases are presented.

4.4.1 Description of Results

The stress intensity factors are presented in the form of non-dimensional magnification factors, $H_j(\phi)$. These magnification factors give the magnitude of the stress intensity factor relative to the solution for an embedded elliptical crack, and are thoroughly detailed in section 2.7. A magnification factor is determined for each load case (constant, linear, quadratic, cubic crack face pressure) by calculating the K_I values from the nodal displacement output and normalizing with respect to the embedded crack solution under a nominal stress equal to the bore stress for that load case, as discussed in section 2.7.

Plots of $H_j(\phi)$ for each cracked disc case considered are shown in figures 4.8. The angle ϕ is the elliptical angle with $\phi = 90$ at the bore and $\phi = 0$ at the flat surface. From these plots, the stress intensity solution for any stress state which can be approximated by the third order polynomial shown in equation 2.7, is given by equation 2.11. The calculations are performed assuming conditions of plane stress at the surfaces and plane strain interior to the disc. This is because of the biaxial state of stress at the surface and the triaxial state of stress interior to the disc near the crack tip.

In the interests of saving time and avoiding errors, several post processor computer programs were used to analyze the results. The $H_j(\phi)$ were calculated by a program which searches the ASAS output for the appropriate nodal displacements to be used in the K_I equations shown in fig 3.2, and normalizes the results for each of the four load cases. Another program was then used to obtain K_I solutions for any specific load case defined by its polynomial stress distribution. This program reads the $H_j(\phi)$ values and prompts the user for the coefficients of the polynomial stress distribution to be modeled. The polynomial used need only fit the stress distribution over the radial range of the crack. In addition, a graphics program was then used to plot the results of these post processors. The use of these programs to analyze and present the results provided a fully automated procedure which was error free, easy to use, and greatly reduced the time and effort required in calculating results from the enormous ASAS finite element output.

4.4.2 Centrifugal Load Case

The case of centrifugal body load for a disc with $b/a=1$ and $b/t=0.5$ was simulated using a polynomial which fits the theoretical hoop stress distribution to within 0.5% over the entire radial range of the crack. The theoretical stress distribution under conditions of plane stress is given by

$$\sigma(r) = \frac{3+\nu}{8} \rho \omega^2 \left(R_i^2 + R_o^2 + \frac{R_i^2 R_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2 \right) \quad (4.1)$$

The coefficients for the polynomial, B_i , were obtained using a least squares approximation and were used in equation 2.11 together with the $H_j(\phi)$ results shown in fig 4.8a to obtain the normalized K_I results shown in fig 4.9. These results are normalized with respect to the solution for an embedded circular crack subjected to a tensile stress equal to the bore stress which the disc experiences under centrifugal load. The results of Tan (17) and Kobayashi et al (18) for the same configuration are plotted on the same graph for purposes of comparison.

4.4.3 Blade Load Case

The case of radial tensile pressure applied at the outer radius of the disc was also analyzed for a disc with $b/a=1$ and $b/t=0.5$. This loading simulates the effect of the inertial loads of blades or "blade load". The theoretical hoop stress distribution is given by

$$\sigma(r) = \frac{P_o R_o^2}{R_o^2 - R_i^2} \left(1 + \frac{R_i^2}{r^2} \right) \quad (4.2)$$

The polynomial used to approximate the theoretical stress distribution was once again calculated with a least squares approximation and was within 0.5% of the theoretical over the radial range of the crack. The resulting K_I distribution for this case was calculated using equation 2.11 and the $H_j(\phi)$ values from the curves in fig 4.8a. A plot of the results is shown in fig 4.10. These results are normalized with respect to the case of a circular embedded crack subjected to a tensile stress equal to the bore stress under the applied "blade load". The results of Tan (17) for the same configuration and loading are plotted on the same graph for purposes of comparison.

FIGURE 4.1 Simplification of Disc Models Due to Symmetry

Boundary conditions for one crack case

(cross-hatched areas suppressed from displacement in the x direction)

Boundary conditions for two crack case

FIGURE 4.2 Boundary Conditions For FEM Disc Models

FIGURE 4.3 Example Of 2D Input Mesh For Mesh Generator

FIGURE 4.4 Basic 3D Mesh Created By Mesh Generator

FIGURE 4.5 Modified 3D Mesh

FIGURE 4.6 Component Topology For Disc Models

FIGURE 4.7 Definition of m/L Ratio

FIGURE 4.8a Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=1$, $b/t=0.5$)

FIGURE 4.8b Stress Intensity Magnification Factors For
a Disc with 2 Corner Cracks ($b/a=1$, $b/t=0.5$)

FIGURE 4.8c Stress Intensity Magnification Factors For
a Disc with 3 Corner Cracks ($b/a=1$, $b/t=0.5$)

FIGURE 4.8d Stress Intensity Magnification Factors For
a Disc with 4 Corner Cracks ($b/a=1$, $b/t=0.5$)

FIGURE 4.8e Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=2$, $b/t=0.5$)

FIGURE 4.8f Stress Intensity Magnification Factors For
a Disc with 2 Corner Cracks ($b/a=2$, $b/t=0.5$)

FIGURE 4.8a Stress Intensity Magnification Factors For
a Disc with 3 Corner Cracks ($b/a=2$, $b/t=0.5$)

FIGURE 4.8h Stress Intensity Magnification Factors For
a Disc with 4 Corner Cracks ($b/a=2$, $b/t=0.5$)

FIGURE 4.8i Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=4$, $b/t=0.5$)

FIGURE 4.8j Stress Intensity Magnification Factors For
a Disc with 2 Corner Cracks ($b/a=4$, $b/t=0.5$)

FIGURE 4.8k Stress Intensity Magnification Factors For
a Disc with β Corner Cracks ($b/a=4$, $b/t=0.5$)

FIGURE 4.81 Stress Intensity Magnification Factors For
a Disc with 4 Corner Cracks ($b/a=4$, $b/t=0.5$)

Fig. 11-24. 4.8m Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=1$, $b/t=0.25$)

FIGURE 4.8n Stress Intensity Magnification Factors For
a Disc with 2 Corner Cracks ($b/a=1$, $b/t=0.25$)

FIGURE 4.80 Stress Intensity Magnification Factors For
a Disc with 3 Corner Cracks ($b/a=1$, $b/t=0.25$)

FIGURE 4.8p Stress Intensity Magnification Factors For
a Disc with 4 Corner Cracks ($b/a=1$, $b/t=0.25$)

FIGURE 4.8q Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=2$, $b/t=0.25$)

FIGURE 4.8r Stress Intensity Magnification Factors For
a Disc with 2 Corner Cracks ($b/a=2$, $b/t=0.25$)

FIGURE 4.8s Stress Intensity Magnification Factors for a Disc with 3 Corner Cracks ($b/a=2$, $b/t=0.25$)

FIGURE 4.8t Stress Intensity Magnification Factors For
a Disc with 4 Corner Cracks ($b/a=2$, $b/t=0.25$)

FIGURE 4.8u Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=4$, $b/t=0.25$)

FIGURE 4.8v Stress Intensity Magnification Factors For

a Disc with 2 Corner Cracks ($b/a=4$, $b/t=0.25$)

FIGURE 4.8w Stress Intensity Magnification Factors For
a Disc with 3 Corner Cracks ($b/a=4$, $b/t=0.25$)

FIGURE 4.8x Stress Intensity Magnification Factors For
a Disc with 4 Corner Cracks ($b/a=4$, $b/t=0.25$)

FIGURE 4.8y Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=1$, $b'/t=0.1$)

FIGURE 4.8z Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=2$, $b-t=0.1$)

FIGURE 4.8aa Stress Intensity Magnification Factors for
a Disc with 1 Corner Crack (b, a=4, b, t=0.1)

FIGURE 4.8bb Stress Intensity Magnification Factors for
a Disc with 1 Corner Crack ($b/a=1$, $b_1=0.001$)

FIGURE 4.8cc Stress Intensity Magnification Factors for a Disc with 1 Corner Crack ($b/a=2$, $b/t=0.05$)

FIGURE 4.8dd Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=4$, $b/t=0.05$)

FIGURE 4.8ee Stress Intensity Magnification Factors For
a Disc with 1 Corner Crack ($b/a=1.3333$, $b/t=0.5$)

FIGURE 4.9 Stress Intensity Factors For a Disc with 1 Corner Crack Under Centrifugal Load (b/a=1, b/t=0.5)

FIGURE 4.10 Stress Intensity Factors For a Disc with 1 Corner Crack Under Blade Load ($b/a=1$, $b/t=0.5$)

Chapter 5

DISCUSSION

5.1 Stress Intensity Magnification Factors

The $H_j(\phi)$ results shown in figures 4.8 represent the complete set of FEM results for all the cracked disc cases considered. As discussed in chapter 2, the results for any one of these cases can be superimposed to obtain normalized K_I results under any desired hoop stress condition which can be approximated by a polynomial. The accuracy of the superposition method was verified by including a centrifugal load case in the FE analysis of the disc with $b/t=0.5$ and $b/a=1$. Normalized K_I results were calculated from the output for this load case and compared with the results obtained by the superposition method. The results were nearly identical with deviations of less than 1%.

It is much more practical to discuss the stress intensity factors obtained in a physically realistic manner rather than in terms of the magnification factors, $H_j(\phi)$, which mean nothing by themselves. In the next section therefore, the results will be discussed by analyzing some practical load cases. The cases of centrifugal body load and that of blade load will be examined since known theoretical equations for the hoop stress under these conditions are readily available, and since these conditions are typical in the operation of turbines.

5.2 Centrifugal and Blade Load Cases

As discussed in section 4.4, the results for a cracked disc with $b/t=0.5$ and $b/a=1$ were superimposed to obtain the case of centrifugal load and these results are shown in fig 4.9. The K_I results are normalized with respect to the embedded crack solution under a tensile stress equal to the bore stress. The values of K_I were found to decrease from $\phi = 0$ to a minimum

value at about $\phi = 45^\circ$ and then increase again to a level at $\phi = 90^\circ$ just slightly higher than the value at $\phi = 0^\circ$. It is noteworthy here to mention that the results were lower than the embedded crack but not close to $K_I = 1.0$ but were still affected by the free surfaces which tended to magnify the stress intensity factor as expected. The validity of the K_I calculations at the free surfaces however is questionable and a great deal of uncertainty has been expressed over the meaning of K_I at the point of intersection of a crack front and a free surface (23).

The value of K_I at the bore $\phi = 0^\circ$ is likely lower than the embedded crack surface because of the effect of the added constraint of the curved bore surface to crack opening. Moving in a positive radial direction along the crack front from the bore ϕ decreases from 90° to 0° the hoop stress decreases which results in a decreasing K_I . However, the bore curvature constraint effect is also decreasing and finally an increase in K_I results, probably due to the magnifying effect of the free surface at $\phi = 0^\circ$.

Also shown in fig 4.2 are the results of Tan (17) using the BIE method and Kobayashi et al (16) using the alternating method. The FEM results of this work compare quite favorably with the BIE results of Tan, with the maximum difference between the two being about 6%. The results are higher than the BIE results as expected for a FEM analysis. The results of Kobayashi et al using the alternating method however show a large discrepancy compared to the FEM results. There are several reasons for this discrepancy, first, the flat back surface of the disc is not taken into account in the analysis. In addition, the curvature correction factor used by Kobayashi et al is obtained from two dimensional solutions of pressure edge cracks at a stress ratio of unity (24). The validity of this correction has been questioned by Grandt in his discussion of Kobayashi's paper (16). He suggests that the correction factor be obtained from a single edge cracked bore in a rotating disc and showed that the difference in correction factors would be about 20% for large cracks. The deficiencies discussed above however do not fully account for the discrepancies in results obtained by Kobayashi et al. As pointed out by Tan (17) however, the accuracy of the results obtained by the alternating method, in terms of solution accuracy, is inferior to the finite element and the BIE methods and therefore cannot be expected to correlate well with these results.

The results for the case of simulated blade load are also presented in fig 4.10 for a disc with

$b/t=0.5$ and $b/a=1$. The blade load case is simply the case of a stationary disc subjected to a uniform radial tensile stress at the outer rim, similar to the loads resulting from the inertia of turbine blades. The hoop stress distribution is given by Lamé's equation as shown in section 4.4.3. The K_I results in fig 4.10 are again normalized with respect to the embedded crack solution subjected to a normal stress equal to the bore hoop stress under the applied blade load. The results for this case were slightly higher than those for the centrifugal load case but the shape of the curve is essentially the same. The results of Tan (17) for this case are also plotted in fig 4.10 and can be seen to correlate quite well with a maximum difference of only about 8% less than the FEM results obtained.

In addition to the cracked disc geometries being considered in this work (see section 4.2.1), a disc with $b/t=0.5$ and $b/a=1.3333$ was also analyzed. This was primarily for the purpose of further comparison of results with the results of Tan (17) who also considered this particular geometry. The $H_I(\phi)$ results for this case were again superimposed using the previously mentioned procedure to obtain both the case of centrifugal load, fig 5.1, and blade load, fig 5.2. In each of these figures the results of Tan are also plotted for comparison. The values of K_I in these figures are again normalized with respect to the embedded elliptical crack solution subjected to the normal tensile stress which exists at the disc bore, for each case. Comparing the results for $b/t=0.5$ and $b/a=1$ (figures 4.9 and 4.10) to those for $b/t=0.5$ and $b/a=1.3333$ (figures 5.1 and 5.2) it appears that the results are qualitatively quite similar. However it should be noted that the elliptical angle ϕ in these figures is the auxiliary angle as defined in fig 1.1 and therefore the point on the crack front defined by a given angle ϕ depends on the aspect ratio of the ellipse. In addition, for elliptical cracks, the normalizing K_I solution is not a constant but varies with ϕ , therefore the K_I values in fig 5.1 and 5.2 do not indicate the relative magnitudes of K_I along the crack front in absolute terms. The normalized values of K_I in these figures indicate that K_I is higher at $\phi = 90$ (the bore), however when the absolute values of K_I are calculated, the results show that K_I is about 7% higher at $\phi = 0$ for both the centrifugal load case, fig 5.1, and the blade load case, fig 5.2. The results of Tan (17) for the same disc geometry and loading are also plotted on figures 5.1 and 5.2. As can be seen, the results are in good agreement with the BIE results of Tan, the maximum difference between the two being about 7%.

Figure 5.3 shows the results of K_I normalized with respect to the embedded crack solution for the case of centrifugal load, for different sizes of circular corner cracks. Note that once again the nominal stress used for normalizing was the bore hoop stress. From this figure we note that for large cracks the K_I values are lower than for the embedded crack. This was partly attributed to the effect of the bore curvature constraint in the above discussion. We see however, from fig 5.3 that for small cracks ($b/t=0.1, 0.05$), the K_I values approach and even exceed the embedded crack results. This is likely due to the fact that for these small cracks, the bore curvature constraint effect is no longer a big factor since the ratio of radial crack depth to bore radius, a/R_i , has become quite small (for circular cracks with $b/t=0.1$, $a/R_i = 0.2$ and for $b/t=0.05$, $a/R_i = 0.1$). At the same time the hoop stress only drops to a level equal to 85% of the bore hoop stress at the radially deepest point ($\phi = 0$) for cracks with $a/R_i = 0.2$, so the stress does not vary a great deal from the nominal bore hoop stress for radially shallow cracks. The results for radially shallow cracks are therefore expected to be close to the embedded crack solution and may be higher because of the magnifying effect of the free surfaces. This trend is also evident in fig 5.4 which shows centrifugal load case results for elliptical cracks with $b/a=2$ and in fig 5.5 which shows the results for elliptical cracks with $b/a=4$. In fig 5.4, the results for $b/a=2$, the K_I values only approach or exceed the embedded crack results for cracks with $b/t=0.1$ and 0.05 both of which have $a/R_i < 0.2$. Similarly in fig 5.5, the results for $b/a=4$, the K_I values only exceed the embedded crack results for cracks with $b/t=0.25, 0.1$, and 0.05 which all have $a/R_i < 0.2$.

5.3 Multiple Cracks

Figures 4.8 include results for the case of a cylinder with 1, 2, 3 and 4 equispaced corner cracks, for discs with $b/t=0.5$ and 0.25 . It was found that the results for multiple crack cases only differed significantly for radially deep cracks, i.e. for cracks with $a/R_i > 0.5$. The $H_i(\phi)$ curves do not clearly show this difference, therefore the results for a disc with 1, 2, 3 and 4 circular corner cracks with $b/t=0.5$, and subjected to centrifugal load were superimposed and plotted in fig 5.6. From this figure it can be seen that 2 or 3 equispaced cracks is the worst case, and that a single crack is the least severe of the cases considered. This trend has also been observed in

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studies of cracked cylinders (32). The maximum difference between the results is about 7.5%. In comparison, the results for a disc with $b/t=0.5$ and $b/a=2$, which has $a/R_1 = 0.5$, was found to have a maximum difference of only 2.5% between the one crack case and the three crack case, which was the most severe. The same was true for a disc with $b/t=0.25$, $b/a=1$ and $a/R_1 = 0.5$. Multiple crack cases were only considered for discs with $b/t=0.5$ and 0.25 since the shallower the crack (lower a/R_1), the less the effect of multiple cracks on the K_I results. In fact for a disc with $b/t=0.25$ and $b/a=4$, which has $a/R_1 = 0.125$, the maximum difference between the results for 1,2,3 and 4 cracks was only 0.5%.

In summary then, the effect of multiple cracks on the value of K_I is only significant for cracks with $a/R_1 > 0.5$, for which a disc with 2 or 3 equispaced cracks is the most severe case. Increasing numbers of equispaced cracks can be expected to result in lower and lower values of K_I as found in studies of cracked cylinders (32).

5.4 Areas For Future Work

The work carried out and presented in this report was presented in as general a form as possible allowing for the simple calculation of stress intensity factors for any practical loading condition. Further investigation to provide results for a range of radius ratios and thicknesses is required however to provide a more complete base of data for use in analyzing discs of different sizes. The additional work that would be required could be easily carried out by following the procedures used in this work, however this was not done because of the enormous computing cost required.

In addition it may be possible to adapt the results obtained for radially short cracks (those with $b/t=0.05, 0.1$) to the case of a corner cracked hole in a large plate. This is because for these short cracks the outer radius is relatively far from the crack tip and will therefore have no effect on the stress intensity factor, that is the crack will appear to be at the corner of a hole in a large plate.

Chapter 7

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FIGURE 5.2 Stress Intensity Factors For a Disc with 1 Corner Crack Under Blade Load ($b/a=1.3333$, $b/t=0.5$)

FIGURE 5.3 Stress Intensity Factors For a Disc with 1 Corner Crack Under Centrifugal Load ($b/a=1$, $b/t=0.05$ to 0.5)

FIGURE 5.4 Stress Intensity Factors For a Disc with 1 Corner Crack Under Centrifugal Load ($b/a=2$, $b/t=0.05$ to 0.5)

FIGURE 5.5 Stress Intensity Factors For a Disc with 1 Corner Crack Under Centrifugal Load ($b/a=4$, $b/t=0.05$ to 0.5)

FIGURE 5.6 Stress Intensity Factors For a Disc with 1 to 4 Corner Cracks Under Centrifugal Load ($b/a=1$, $b/t=0.5$)

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- 7) The FE results obtained were found to be in good agreement with some published BIE results for the case of centrifugal body load and inertial blade load.
- 8) The results obtained could be utilized in a crack growth analysis to calculate the potential life of turbine discs. The use of fracture mechanics in the design and maintenance of turbine discs is needed in order to employ a "retirement for cause" program.

Chapter 7

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