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EARTHQUAKE RESISTANT DESIGN OF LIQUID STORAGE TANKS

by

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B.Sc. (Civil Engineering)
University of Santo Tomas

A thesis submitted to

The Faculty of Graduate Studies and Research
In partial fulfillment of the requirements

For the degree of
Master of Engineering

Department of Civil and Environmental Engineering

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Earthquake Resistant Design of Liquid Storage Tanks

submitted by

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in partial fulfillment of the requirements for
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ABSTRACT

The seismic performance and safety of liquid storage tanks located in areas of high seismic activity are major concerns of tank designers and engineers. Experience with tank performance in recent earthquakes and laboratory tests indicates that the seismic design provisions in the current design standards and procedures may not be adequate. In this thesis, the current state of the earthquake resistant design of liquid storage tanks is reviewed. The methodologies and philosophies of methods for seismic design of liquid storage tanks commonly used in design practice are discussed.

The review of the different design procedures is carried out in three major parts: determination of design loadings, design actions and design criteria. Design loadings pertain to the hydrodynamic forces resulting from the induced hydrodynamic pressure acting on the tank wall and bottom due to ground motion excitations. Limits on the maximum lateral resultant force and overturning moment at the base are the seismic design loads generally prescribed in design standards and procedures. Design actions are the responses of the liquid-tank system to the design loadings. In most design procedures, the maximum axial compressive stress in the tank wall is considered as the most important parameter in the design of liquid storage tanks. Design criteria relate to the reserve capacities of the tank which control the failure modes of the tank system. The performance and safety of liquid storage tanks are greatly affected by their design criteria. The dynamic buckling capacity of the tank wall is an important design criterion for seismic design of liquid storage tanks.

The current seismic design standards and procedures for liquid storage tanks are

classified as either rigid or flexible tank procedures. Rigid tank procedures assume the tank wall as rigid, whereas flexible tank procedures consider the vibration characteristics of the tank wall and its interaction with the vibrating liquid during ground motions. The difference in the design requirements and current practice in design for anchored and unanchored tanks are discussed.

In a parameter study, the design responses determined by the different design procedures and results from laboratory tests of practical and model tanks are compared. The comparison results show that rigid tank procedures, such as the API (1988) and AWWA (1984) standards, in comparison with flexible tank procedures may significantly underestimate or overestimate the design loadings, depending on the characteristics of the earthquake input. The simplified uplift models adopted in the current design standards and procedures are found to be inadequate in predicting the maximum compressive stress in the tank wall of an unanchored tank during uplift, as indicated by the significant difference in the predicted and test results.

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1.2 Organization

The organization of this thesis follows the normal course of a typical design process of a civil engineering structure. Forces (design loadings) acting on the tank structure are first determined, then the resulting critical stresses (design actions) are evaluated and finally these stresses are checked with the allowable strength to satisfy design criteria. In Chapter 2, tank damages and performance in past earthquakes are discussed, as well as its response and typical behaviour when subjected to earthquake ground motions. Chapter 3 deals with the loads imposed on tank structures by seismic excitations as calculated by different design methods. These loads are the hydrodynamic pressure, lateral base shear and base overturning moment. The responses of tanks to these loads are discussed in Chapter 4. A tank is classified as either anchored or unanchored tank. This is important because an unanchored tank behaves differently from an anchored tank due to the tendency of its bottom plate to uplift. An important response parameter considered in the survey and comparison study is the shell axial compressive stress in the tank wall, which is the major cause of buckling failure observed in many tanks damaged during past earthquakes.

To ensure a satisfactory and safe performance of its intended function, the capacity of a tank to withstand the demand arising from the critical failure mode must be investigated. A discussion of this is presented in Chapter 5. Although tank damage may come in different forms, the present study focuses on the buckling resistance which is the most common cause of observed seismic tank damage. In Chapter 6, results obtained from a comparison study of the different surveyed design methods are

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Chapter 1

INTRODUCTION

1.1 Purpose and Objectives

Liquid storage tanks are very important components in lifeline and industrial facilities, hence their response and behaviour during earthquakes must be properly accounted for in the design process. This is not only because of their economic value, but more importantly because of the danger and hazards that failure of these facilities can bring to human lives and to the environment. Experiences from past earthquakes have shown that liquid storage tanks are very vulnerable to earthquake damage. In view of the performance of the damaged tanks, it seems that the procedure adopted in their design cannot fully capture the true response mechanism of liquid storage tanks during earthquakes in spite of the numerous theoretical investigations that have been conducted on the seismic behaviour of these structures in recent years. There is clearly a gap between understanding of the response mechanism and behaviour of liquid storage tanks during earthquakes and the design practice. This is because the

seismic behaviour of liquid storage tanks is a very complex problem. The seismic provisions in the design standards and guidelines currently available and used in practice are generally based on simplified models developed by different researchers over a period of time. These models involved different assumptions, many of them are based on intuition and some engineering judgement. As a result, designers presently face the difficulty of which procedures to follow in the seismic design of liquid storage tanks. In view of this, there is a need for a research to identify the differences and similarities between the different procedures, and their implications on the design.

One of the objectives of this thesis is to survey the methodologies and philosophies of methods for earthquake resistant design of liquid storage tanks commonly used in design practice. In this respect, the thesis can serve as a useful guide to engineers engaged in the seismic design and construction of liquid storage tanks, leading towards more objective and safer design. There are two objectives to this thesis:

1. To give a state-of-the-art review of the methodologies and philosophies of the methods currently available and used in design practice for the seismic design of ground supported thin-walled metal cylindrical liquid storage tanks through discussions of their basic principles and assumptions.
2. To compare and evaluate these methods by means of examples and experimental test results of model and actual size tanks, in order to gain some insight as to their reliability and restrictions.

1.2 Organization

The organization of this thesis follows the normal course of a typical design process of a civil engineering structure. Forces (design loadings) acting on the tank structure are first determined, then the resulting critical stresses (design actions) are evaluated and finally these stresses are checked with the allowable strength to satisfy design criteria. In Chapter 2, tank damages and performance in past earthquakes are discussed, as well as its response and typical behaviour when subjected to earthquake ground motions. Chapter 3 deals with the loads imposed on tank structures by seismic excitations as calculated by different design methods. These loads are the hydrodynamic pressure, lateral base shear and base overturning moment. The responses of tanks to these loads are discussed in Chapter 4. A tank is classified as either anchored or unanchored tank. This is important because an unanchored tank behaves differently from an anchored tank due to the tendency of its bottom plate to uplift. An important response parameter considered in the survey and comparison study is the shell axial compressive stress in the tank wall, which is the major cause of buckling failure observed in many tanks damaged during past earthquakes.

To ensure a satisfactory and safe performance of its intended function, the capacity of a tank to withstand the demand arising from the critical failure mode must be investigated. A discussion of this is presented in Chapter 5. Although tank damage may come in different forms, the present study focuses on the buckling resistance which is the most common cause of observed seismic tank damage. In Chapter 6, results obtained from a comparison study of the different surveyed design methods are

presented. Design calculations are presented on model and actual size tanks. These are correlated with results from various experiments. Chapter 7 contains conclusions and recommendations for future study.

Chapter 2

SEISMIC BEHAVIOUR OF LIQUID STORAGE TANKS

2.1 Typical Liquid Storage Tanks

In practice, liquid storage tanks may come in different forms, sizes, and can be made of different materials. They can be of cylindrical or rectangular shapes, broad or tall, and made of metal, concrete, or timber, or a combination of different materials. The present study concerns the design of ground supported thin-walled metal cylindrical liquid storage tanks. This is the most common type of tank used in industry due to its efficiency in carrying loads and its ease of construction.

Shown in Figure 2.1 is a typical cylindrical metal tank which basically consists of a thin-walled cylindrical tank shell to resist the outward liquid pressure, a thin flat bottom plate resting on the foundation to prevent the liquid from leaking out, and a fixed or floating roof. Different types of roof may be employed: cone shaped,

dome shaped, flat plate type, or the more flexible type of floating roof. A common roof structure is a system of trusses supporting a thin plate. For a large diameter tank, interior columns may be used to support the roof. The connection between the roof and the tank shell is normally designed to be a weak connection, such that, if the tank is over-filled, this connection will fail first before failure of the connection between the bottom plate and the cylindrical shell.

In the design of tanks, the tank wall is generally treated as a membrane that resists a purely tensile hoop stress. This provides an efficient design, because metal is a very economic material in resisting tension. Depending on the magnitude of the tensile stress induced by the hydrostatic pressure, the wall of a large tank may have different wall thicknesses along its height. To improve the stability of the tank shell, a ring girder at the top rim and additional stiffening ring girders along the height of the tank wall may also be employed.

The foundation which supports the tank structure can be of the concrete ring wall type, solid concrete slab, or concrete base supported by piles or caissons. In most cases, especially for medium to large capacity tanks of broad proportion with the tank height less than the diameter, the tanks are not anchored to the foundation, and thus may uplift or separate from the support foundation during strong ground shaking

2.2 Damages to Liquid Storage Tanks in Past Earthquakes

The seismic performance of liquid storage tanks is a matter of considerable importance. The cost of tank failure extends beyond its economic value. The cost should include the hazards associated with tank failure which can be very significant. Spillage of highly inflammable products has frequently led to extensive uncontrolled fires as during the Alaska and the Nigata earthquakes in 1964. Spilling or flushing out of dangerous liquids or gases from damaged tanks could result in polluting waterways as occurred during the Miyaki Oki earthquake in 1978. The need for a safe water supply after an earthquake is essential to prevent outbreak of disease, hence water supply reservoir is a critical lifeline facility that must remain functional after earthquakes.

Observations on the performance of ground-based liquid storage tanks during past earthquakes have shown that the most common modes of failure are: tank wall buckling, roof damages, and piping failures. Extensive examples of these tank failures were observed in many earthquakes, aside from those already mentioned, San Fernando (1971), San Juan, Argentina (1977), Imperial Valley (1979), Coalinga (1983), and Loma Prieta (1989). Foundation failures due to differential settlement have also been observed as one of the reasons for tank failure, especially for large tanks.

Tank wall or shell buckling usually occurs either in the form of "elephant's-foot-bulge" or "shell crippling". "Elephant's-foot-bulge", also known as elastic-plastic buckling failure mode of the tank wall, is characterized by the formation of a bulge at the base of the tank wall, as shown in Figure 2.2. At present, the cause of "elephant's-

foot-bulge" is not fully understood. However, current research and investigations, such as those by Scharf et al. (1989a) and Priestley et al. (1986), generally accept that "elephant's-foot-bulge" is caused by the plastification of the tank shell due to the combined action of the shell axial compressive stress generated by the overturning of the tank wall and the hoop stress resulting from the static and the dynamically activated pressures due to the horizontal and vertical components of ground motions. The total internal pressure in this form of shell buckling failure is very large, such that it results in a very high hoop stress that leads to yielding of the tank wall. This in turn decreases the effective stiffness of the tank shell, thus reducing the buckling capacity of the tank wall. In comparison, the other form of shell buckling failure, "shell crippling", also known as elastic buckling failure mode of the tank wall, is characterized by the formation of diamond-shaped buckles at some distance above the base of the tank, as shown in Figure 2.3. This phenomenon is mainly due to a large axial compressive stress in the tank wall. In this form of shell buckling failure, the internal pressure is not large enough for yielding of the tank wall to occur. However, it reduces the level of tank wall imperfections, hence it increases the buckling capacity of the tank wall.

For the other forms of tank failure, roof damage is caused by liquid sloshing, especially for tanks with insufficient free-board. Failures caused by rupture of piping connections and accessories are due to the inability of the system to accommodate motions of the tank during earthquakes. Failures of this type is the major cause of the loss of tank contents. Therefore the piping system must be designed to be flexible to accommodate the expected movements during earthquakes.

Design criteria have been developed based on the observed tank damages. As discussed in later chapters of the thesis, the seismic provisions in the design standards put emphasis on designing against the global overturning moment and the maximum axial compressive stress in the tank wall since experience shows that they are the main factors that have led to tank damage in the past.

2.3 Seismic Behaviour of Liquid Storage Tanks

When the base of the tank is subjected to horizontal ground motions, inertial forces are generated by the vibrating tank structure and liquid content. In response to the inertial forces from the vibrating flexible tank, the magnitude and distribution of the hydrodynamic pressure on the tank wall and bottom are further modified. The total response of the liquid-tank system is usually analyzed by separating it into two response modes: the impulsive mode and the convective mode, as shown in Figure 2.4. The impulsive response mode is associated with the liquid motion as a result of the tank wall motion. Depending on the analysis procedure used, the wall can be rigid or flexible as discussed in the next chapter. The convective response mode is associated with the free sloshing or oscillations of the liquid, particularly near the liquid free surface. The characteristics of the resulting hydrodynamic pressure depend on the characteristics of the ground motion, the properties of the liquid, the physical properties and configuration of the tank, and the foundation-soil characteristics. The hydrodynamic pressure induces a resultant lateral force on the tank wall which causes an overturning moment at the base. The overturning moment is subsequently resisted

by a distribution of longitudinal axial stresses around the tank wall.

Previous studies on the seismic behaviour of liquid storage tanks typically treated the tank wall as rigid and anchored at the base. The support foundation was assumed to be stiff, and only one horizontal component of the excitation was considered. The works by Housner (1957) and Epstein (1976) are examples of these. Later, tank damages observed in actual earthquakes and experimental tests confirmed that flexibility of the tank wall has a significant effect on the hydrodynamic pressure induced on the tank. This resulted in studies and procedures that take into account the tank wall or shell flexibility (Haroun and Housner 1981, Balendra et al. 1982, Veletsos 1984). The design provisions specified in the design standards commonly adopted for the seismic design of liquid storage tanks in North America (AWWA 1984, API 1988) consider shell flexibility in an "ad hoc" manner.

There are three common approaches adopted by researchers in the study of liquid storage tanks. The first approach is the finite element method where both the liquid domain and the tank wall are modelled by finite elements (Balendra et al. 1982, Liu and Ma 1982, Ma et al. 1982, Barton and Parker 1987). This approach is generally considered impractical due to the considerable amount of computation required in a finite element model, particularly in the analysis of the highly nonlinear and complex dynamic response of an unanchored tank. The second approach is the simplified approach in which the liquid response is modelled by boundary integral techniques or variational method while the tank system is modelled by finite elements or by using assumed deformed shapes of the tank shell (Haroun and Housner 1981, Veletsos 1984). This approach satisfies the boundary conditions at the liquid-tank interface

only in an average sense due to the difficulty in satisfying exactly these boundary conditions by the liquid response shape functions. The third approach is the Ritz-type approach where the liquid response is solved analytically while the tank structure is divided into substructure elements, each formulated by Ritz discretization using derived displacement shape functions (Lau and Clough 1990, Lau and Zeng 1991). The substructure elements are: the linear contact and the nonlinear uplifted regions of the bottom plate, and the linear cylindrical tank shell substructure elements. This approach is more efficient than the finite element method in the modelling of the complex and highly nonlinear behaviour of the uplifted bottom plate, in which both the membrane and flexural mechanisms must be considered. Furthermore, the Ritz approach discretizes the tank system into relatively few super elements with derived shape functions that span an entire region as compared to dividing the structure into many finite elements with simple shape functions that are defined within the element domain. However, the accuracy and efficiency of the Ritz approach depend largely on the correct choice of the Ritz shape functions which requires considerable insight into the problem. But if an efficient, complete and simplified Ritz functions can be derived, such as those derived by Lau and Zeng (1991), the Ritz approach can be a very effective method applicable for nonlinear dynamic analysis of unanchored tanks.

The importance of vertical acceleration on the responses of liquid storage tanks during earthquakes have also attracted the attention of researchers and design practitioners. The importance is acknowledged in the API (1988) and the AWWA (1984) standards, but there is no specific provision or recommendation on how to incorporate the vertical acceleration effects in the design process. The combined action of

horizontal and vertical accelerations was considered by Scharf et al. (1989a) as part of the Austrian recommendations for the seismic design of liquid storage tanks.

Foundation flexibility has a considerable effect on the vibration response as demonstrated by various experimental tests. The soil-structure interaction effect couples the vibration response between the soil and the tank. At present, a sound and simplified method that takes into account this interaction effect has not been developed. Other important factors currently being investigated are the effects of sliding or rocking and the influence of initial out-of-roundness or irregularities of the tank wall.

Recent works have focused more on the behaviour of unanchored tanks where the tank walls are supported by concrete ring-wall foundations and the interior part of the bottom plates are supported by compacted soil. The tank shell and bottom plate of an unanchored tank may uplift from the foundation during earthquakes in response to the overturning moment generated by the vibrating liquid of the tank system. The resulting rocking motion of the tank structure induces additional compressive stresses on the contact side and tensile stresses on the uplifted side of the tank wall. The preference for unanchored tanks over anchored tanks is mainly for economic reason. For a medium to large size tank, the cost of providing an adequate foundation and anchorage system is usually high compared to the cost of the tank structure itself, which is not required for an unanchored system. Furthermore, the difficulty involved in designing the anchorage system, and the possibility of losing all the liquid content in the event of a tear-off or rupture failure of the tank shell or bottom plate at the anchorage point due to inaccurate anchor forces determined from the uplift analysis and used in the design are other important reasons and concerns that lead to the

common practice of unanchored tanks.

The behaviour of unanchored tanks are typically studied either under the static tilt condition (Clough 1977, Wozniak and Mitchell 1978, Auli et al. 1985, Manos 1986, Haroun et al. 1987, Ishida and Kobayashi 1988, Peek 1988, Peek and Jennings 1988, Peek et al. 1988, Lau and Clough 1989) or full dynamic earthquake loading condition (Barton and Parker 1987, Natsiavas 1987, 1988, 1990, Ishida and Kobayashi 1988, Lau and Zeng 1992, Zeng 1993). In a static tilt test, a quasi-static uplift state of an unanchored tank is reached where the effect of uplift on the stresses in the tank wall can be observed and analyzed. The design provisions for unanchored tanks in the present design procedures are based on the study results from some of these earlier works (Clough 1977, Wozniak and Mitchell 1978).

Dynamic investigations of unanchored tanks were initially carried out through laboratory tests on the Earthquake Engineering Research Center shaking table in the University of California at Berkeley (Clough 1977, Niwa 1978, Cambra 1982). The test tank is mounted on a shake table which is subjected to simulated earthquake motions. The effects of different parameters were studied, including the effects of base support conditions and roof systems. Later, information obtained from these tests and those from static tilt tests were used to develop nonlinear uplift models and analysis procedures for the dynamic response of unanchored tanks which incorporate the relevant uplift resistant mechanisms in an uplifting tank. For example, Lau and Zeng (1992) presented a time history analysis procedure to study the dynamic uplift behaviour of unanchored cylindrical tanks using the large displacement theory in the modelling of the uplifted bottom plate, in which the membrane and flexural mech-

anisms are considered. This analysis procedure is based on Ritz-type substructure formulation of the liquid-tank system as previously stated.

Figure 2.1 Typical cylindrical liquid storage tank

Figure 2.2 ''Elephant's-foot-bulge'' wall buckling

Figure 2.3 ''Shell crippling'' wall buckling

Figure 2.4 Response of the liquid-tank system to a horizontal ground motion

Chapter 3

DESIGN LOADINGS

3.1 General

The design process of a typical civil engineering structure involves the following major steps: determinations of the design loadings, the design actions or responses to the design loadings, and checking of the structural capacity against design criteria. To ensure a safe design, the structural capacity (supply) must be greater than the design actions (demand) caused by the loadings. The presentation in the thesis follows a typical design approach.

Design loadings for seismic design of liquid storage tanks pertain to the hydrodynamic forces resulting from the induced hydrodynamic pressures acting on the tank wall and bottom due to ground motion excitations. The hydrodynamic forces occur as lateral forces acting on the tank wall which cause a lateral resultant base shear and an overturning moment at the tank base.

3.2 Rigid Anchored Tank Procedures

Early works on the seismic behaviour of liquid storage tanks are based on the assumptions that the tank is rigid and anchored to a rigid foundation. Thus when the base of a rigid tank is subjected to ground motions, the tank structure moves with the same acceleration as the ground. This section discusses the commonly available procedures for determining the design loadings of rigid anchored tanks.

3.2.1 Housner Method

A method for determining the hydrodynamic forces in an accelerating tank was developed by Housner (1957). The general assumptions of the Housner method are:

1. The tank structure is rigid and anchored to a rigid foundation.
2. The liquid is incompressible and inviscid.

As the tank is excited by the ground motion, the liquid vibrates in two different modes: the impulsive mode and the convective mode. The impulsive mode arises from the motion of the liquid mass (impulsive mass) in response to the rigid tank wall motion, while the convective mode is related to the free sloshing or oscillating motion of the liquid, particularly near the liquid free surface. The portion of the liquid mass that participates in the sloshing motion is known as the convective mass.

The hydrodynamic pressure loading of the vibrating liquid is obtained from the solution of the Laplace's equation of an ideal fluid that satisfies the boundary conditions at the liquid-tank interface. Simplifications and approximations are introduced

in the solution procedure, especially on the liquid flow. In the impulsive response, it is assumed that the liquid is restrained by thin, massless, vertical membranes free to move only in the direction of the excitation. This is equivalent to assuming that the impulsive pressure is generated by the vibration of laminae of liquid placed side by side. The surface of the liquid remains horizontal. For tall or slender tanks, with liquid height to tank radius ratio, H/R , greater than 1.5, Housner (1957) suggested that for design purpose the impulsive response component can be separated into two component parts. The first impulsive component part is due to the vibration of the upper part of the liquid up to a depth of 1.5 times the tank radius from the liquid free surface in response to the rigid tank wall motion. The second impulsive component part is due to the remaining liquid below which moves as a completely constrained liquid mass with the rigid tank at the same acceleration as the ground.

For the convective or sloshing response, the vibration motion of the oscillating liquid is assumed to occur under the constraints of thin, massless, horizontal rigid membranes. These rigid membranes are free to rotate about a diametral axis normal to the direction of excitation. Additional constraints are imposed on the liquid flow such that continuity of flow is preserved so that all the liquid particles at the same height and distance from the center of the tank move with the same vertical and horizontal velocities parallel to the excitation axis. In the calculation of the convective response, the approximation employed in the determination of the impulsive response for tanks with liquid height to tank radius ratio, H/R , greater than 1.5, as stated earlier, is not applied. Thus for all liquid height to tank radius ratios, the convective response is calculated from the motion of the full liquid depth.

The total response of the liquid can be modelled by a mechanical spring-mass system, as shown in Figure 3.1. An equivalent single-degree-of-freedom oscillator is used to model the convective response with m_c and k_c representing the convective liquid mass and stiffness of the equivalent oscillator respectively. The spring stiffness, k_c , is calculated such that the frequency of the spring-mass system is equal to the fundamental vibration frequency of the convective response. The convective mass, m_c , and its position, h_c , relative to the tank base are determined such that the lateral force and overturning moment exerted by the mechanical oscillator at the tank base is the same as that by the oscillating convective liquid. The rigidly attached impulsive mass, m_i , and its height, h_i , are determined such that they generate the same lateral force and overturning moment as the impulsive liquid pressure.

The maximum lateral base shear and overturning moment from the two response modes at the tank base is determined conservatively as the absolute sum of the maximum impulsive and convective components (US Atomic Energy Commission TID-7024 1963). The impulsive acceleration is taken as the maximum ground acceleration while the convective acceleration can be obtained from the response spectrum of the design earthquake using the calculated fundamental convective frequency.

It is noted here that the Housner method takes into account the effects generated by a one directional horizontal base excitation. It does not consider the simultaneous action of two horizontal and the vertical components of ground excitation.

3.2.2 Epstein Procedure

The procedure proposed by Epstein (1976) for the determination of design loadings of rigid anchored tanks is a modification of the Housner method. In the Epstein procedure, similar limitations as discussed in the Housner method are also adopted in the derivation of the liquid response. Tanks are classified specifically into two types depending on the liquid depth to tank radius ratio, H/R . Broad or shallow tanks are tanks with $H/R \leq 1.5$, while tall or slender tanks have $H/R > 1.5$, as shown in Figure 3.2 with their corresponding mechanical spring-mass models. Epstein (1976) also adopted similar assumptions as Housner (1957) in the calculation of the impulsive response for tall tanks. However, Epstein (1976) further applied these assumptions in the determination of the convective response. It is assumed then, that the generation of the convective response is mainly due to the upper part of the liquid up to a depth of $1.5R$ from the liquid free surface, as shown in Figure 3.2(b). Thus the convective response is calculated from the motion of the upper part of the liquid up to a depth of $1.5R$ from the liquid free surface instead of the full liquid depth.

With the above assumptions on the vibration motion of the liquid, the impulsive mass for tall tanks, as shown in Figure 3.2b, is divided into two parts: one from the upper part of the liquid, m_{i2} , and the other from the constrained liquid below, m_{i1} . The convective mass, m_c , the effective height, h_c , and the spring stiffness, k_c , of the equivalent oscillator in the convective response are determined in accordance to the ratio $H/R = 1.5$. For both broad and tall tanks, simplified expressions, design charts, and design parameters have been developed to determine the maximum

lateral base shear and overturning moment at the tank base. The total maximum response is obtained from the absolute sum of the maximum impulsive and convective components.

Investigation of other liquid heights that may give higher overturning moment than the full design depth was discussed by Epstein (1976). However, in most practical tanks which are generally large, the full tank condition will usually give the maximum response, thus the effect of the liquid depth parameter is not so critical in this case. Procedures for the design of rectangular tanks are also available. A procedure to determine the maximum free surface displacement of the liquid is developed from the convective response of the liquid. Importance for the consideration of the simultaneous action of two horizontal components of base excitation and the possible effect of the vertical acceleration are pointed out, but no specific procedure is suggested to account for the vertical effect.

3.2.3 API and AWWA Standards

The American Petroleum Institute (API 1988) and the American Waterworks Association (AWWA 1984) standards are currently the most commonly used design procedures in the seismic design of liquid storage tanks. The API standard is primarily intended for oil storage tanks, while the AWWA standard is specifically for water storage tanks. The design provisions in the two standards are very similar to each other with only a few differences. The basis and differences in the design procedures of these two standards are discussed in this section.

In determining the design loadings, both standards put particular emphasis on the

maximum total overturning moment, M , at the base of the tank as generated by a unidirectional horizontal ground motion, as the most critical design loading parameter. The overturning moment M is used to evaluate the stability of the tank against wall buckling and overturning. As in the previous two procedures, the hydrodynamic loading on the liquid-tank system is determined from the effective weights and heights of the liquid masses in an equivalent spring-mass system that participate in the two response modes. These are expressed by

Impulsive Mode

$$\text{for } \frac{D}{H} \geq 1.333 \quad \left(\frac{H}{R} \leq 1.5 \right)$$

$$\frac{W_i}{W_l} = \frac{\tanh(0.866 \frac{D}{H})}{0.866 \frac{D}{H}} \quad (3.1)$$

$$\frac{h_i}{H} = 0.375 \quad (3.2)$$

$$\frac{h'_i}{H} = 0.375 \left[1.0 + 1.333 \left(\frac{0.866 \frac{D}{H}}{\tanh(0.866 \frac{D}{H})} - 1.0 \right) \right] \quad (3.3)$$

$$\text{for } \frac{D}{H} < 1.333 \quad \left(\frac{H}{R} > 1.5 \right)$$

$$\frac{W_i}{W_l} = 1.0 - 0.218 \frac{D}{H} \quad (3.4)$$

$$\frac{h_i}{H} = 0.500 - 0.094 \frac{D}{H} \quad (3.5)$$

$$\frac{h'_i}{H} = 0.500 + 0.060 \frac{D}{H} \quad (3.6)$$

Convective Mode

$$\frac{W_c}{W_l} = 0.230 \frac{D}{H} \tanh\left(\frac{3.67}{\frac{D}{H}}\right) \quad (3.7)$$

$$\frac{h_c}{H} = 1.0 - \frac{\cosh\left(\frac{3.67}{\frac{D}{H}}\right) - 1.0}{\frac{3.67}{\frac{D}{H}} \sinh\left(\frac{3.67}{\frac{D}{H}}\right)} \quad (3.8)$$

$$\frac{h'_c}{H} = 1.0 - \frac{\cosh\left(\frac{3.67}{\frac{D}{H}}\right) - 1.937}{\frac{3.67}{\frac{D}{H}} \sinh\left(\frac{3.67}{\frac{D}{H}}\right)} \quad (3.9)$$

where

W_i, W_c = effective weights of the liquid in the impulsive and the convective modes respectively

W_l, H = total weight and height of the liquid respectively

h_i, h_c = effective heights of the liquid for tank base overturning moment calculation in the impulsive and the convective modes respectively

h'_i, h'_c = effective heights of the liquid for foundation overturning moment calculation in the impulsive and the convective modes respectively

D, R = diameter and radius of the tank respectively

Equations 3.1–3.9 are derived based on the rigid tank assumption of Housner (US Atomic Energy Commission TID-7024 1963). Only the first sloshing mode is considered in the derivation of the convective mode parameters as in the Housner (Housner 1957) and Epstein (Epstein 1976) procedures. The effective heights, h'_i and h'_c , determined by including the hydrodynamic pressure acting on the tank bottom plate, are necessary to obtain the maximum total overturning moment, M_f , for the design of foundation. Design charts equivalent to Equations 3.1–3.9 are presented in Figures 3.3 and 3.4.

For the design of the tank shell, the maximum total overturning moment, M , at

the tank base is given by the following expressions in the two design standards

API Standard

$$M = ZI(C_1W_sH_s + C_1W_rH_t + C_1W_ih_i + C_2W_ch_c) \quad (3.10)$$

AWWA Standard

$$M = ZK[0.14(W_sH_s + W_rH_t + W_ih_i) + SC_3W_ch_c] \quad (3.11)$$

where

- Z = seismic zone coefficient
- W_s, H_s = total weight and center of gravity of the tank shell referred from the base respectively
- W_r, H_t = total weight of the roof and total height of the tank shell respectively
- C_1 = lateral force coefficient associated with the impulsive mode, equals to 0.24 in API standard
- C_2, C_3 = lateral force coefficients associated with the convective mode in the API and AWWA standards respectively
- S = site amplification factor
- I = importance or essential facilities factor
- K = structural coefficient

The terms W_i , W_c , h_i , and h_c in the above equations have been defined earlier in Equations 3.1–3.9.

Equations 3.10 and 3.11 give the maximum overturning moment at the tank base due to the shaking of the ground motions as determined from the equivalent static lateral seismic load on the tank structure. In the API formula, as given by Equation 3.10, the combined effect of the coefficients Z , I , C_1 and C_2 , as given by the terms ZIC_1 and ZIC_2 , can be considered as the effective spectral accelerations of the impulsive and convective modes, respectively. Although the tank is assumed rigid in the derivation of the impulsive liquid mass and heights in Equations 3.1-3.9, the lateral force coefficient, C_1 , in Equation 3.10 endeavours to account for the effect of the tank wall flexibility on the maximum overturning moment at the tank base. It is set equal to 0.24 in unit of gravitational acceleration g , which represents the maximum amplified ground motion to which the tank may be subjected. This value for C_1 is consistent with the Uniform Building Code (UBC 1982) requirements for structures other than buildings. The coefficient C_1 is equivalent to the combined effect of the parameters KCS in the equivalent static lateral seismic load formula of the UBC (1982) by taking $K = 2$, $S = 1$, and $C = 0.12$. In such case, the value of $0.24g$ for C_1 yields seismic forces that are considerably higher than those considered for buildings in the UBC (1982). This is considered to be appropriate by the API standard, since liquid storage tanks have lower energy dissipation capacity than typical building structures, because in comparison liquid storage tanks have lower damping and few nonstructural elements. Also, dissipation of energy through ductile action of the tank shell is not considered by the API standard, since it assumes that the tank shell buckling, which is the most common cause of tank failure, occurs in the elastic range, as discussed in Chapter 5. However, since the size of the earthquake that may occur in a particular

site is not known and the effects of different parameters on the tank response are not well considered in the API seismic design provisions, such as tank flexibility and the partial uplift of the bottom plate in unanchored tanks during strong ground motions, C_1 may underestimate or overestimate the actual impulsive acceleration. It is noted here that in the determination of the impulsive response in the API standard, the effect of soil foundation-tank structure interaction is not considered.

The lateral force coefficient, C_2 , is related to the convective response which depends on the fundamental sloshing period, T , given as follows

$$T = k_p \sqrt{D} \quad (3.12)$$

$$k_p = \frac{0.578}{\sqrt{\tanh\left(\frac{3.67}{H}\right)}} \quad (3.13)$$

Equations 3.12-3.13 are similar to the fundamental sloshing period expressions in the Housner method and Epstein procedure. Alternatively, the coefficient k_p can be obtained from Figure 3.5. The expressions for C_2 are

if $T \leq 4.5$ seconds

$$C_2 = \frac{0.30S}{T} \quad (3.14)$$

if $T > 4.5$ seconds

$$C_2 = \frac{1.35S}{T^2} \quad (3.15)$$

The above equations for C_2 are based on the consideration that the fundamental sloshing period is relatively long so that the corresponding value of spectral acceleration falls in the region of maximum spectral velocity or displacement in the response spectrum. Accordingly, Equations 3.14-3.15 are derived from design response spectra

with maximum spectral velocities of 1.5 to 2.3 *ft/sec* (0.46 to 0.70 *m/s*) and maximum spectral displacements of 1.1 to 1.65 *feet* (0.34 to 0.50 *meters*).

The site amplification factor, S , as given in Table 3.1, depends on the type of soil foundation. According to Table 3.1 and Equations 3.14–3.15, softer soil foundations tend to amplify the convective response more than stiffer or rock-type soil foundations. The importance factor, I ranges from 1.0 to a maximum of 1.5 for essential tanks that must provide emergency post-earthquake service to the public.

The seismic zone coefficient, Z , reflects the seismicity risks or damage expectation at a particular site. It depends on the seismicity as determined from the seismological condition of the site. Table 3.2 gives the values of Z , which are the same as those specified in the UBC (1982), in conjunction with the seismic zone map shown in Figure 3.6 for the United States. The zoning in Figure 3.6 is defined according to the peak ground motion acceleration contour map developed by the Applied Technology Council (ATC 1977). The accelerations depicted on the contour map by the ATC (1977) are a measure of the effective peak velocity of regions of approximately equal seismic risks so as to be appropriate for the long period convective response as well as the short period impulsive response. The zoning map by the ATC (1977) is considered more reliable than the UBC (1982) zoning map which is largely based on past earthquake damage experience. For tanks located in other countries, seismic zoning maps with similar basis are required.

In the AWWA formula given in Equation 3.11, the terms $ZK(0.14)$ and $ZKSC_3$ represent the effective spectral accelerations for the impulsive and fundamental convective modes respectively. The structure coefficient, K , is closely related to the K

parameter in the UBC (1982) which is a measure of the lateral resistance capability of the structure. The coefficient K in AWWA standard, however, has already included the importance factor, I , by considering all water tanks as essential facilities. which is not the case in the UBC (1982). Furthermore, the coefficient K in AWWA standard has implicitly included an overturning amplification factor that depends on the overturning resistance capability of different types and configurations of tanks. A higher overturning amplification factor is given to tank structures that are more vulnerable to overturning, such as unanchored tanks, than those that are more capable of resisting overturning, such as anchored tanks. Thus by taking $K = 2.0$ in UBC and by considering that unanchored tanks are more vulnerable to overturning than anchored tanks by using an overturning resistance factor of 1.5 for unanchored tanks and 1.0 for anchored tanks, the values for the structural coefficient, K , as presented in Table 3.3, are 2.0 for anchored tanks and 2.5 for unanchored tanks. The numerical coefficient of 0.14 is equivalent to the combined effect of CS in the UBC (1982). Therefore with $C = 0.12$ and in contrast to the API standard, the site amplification factor, S , for the impulsive response has a value of 1.167 in the AWWA standard. The lateral force coefficient, C_3 , in the AWWA expression has the same function as the coefficient C_2 excluding the site amplification factor, S , in the API formula. It is given by

if $T < 4.5$ seconds

$$C_3 = \frac{1}{6T} \quad (3.16)$$

if $T \geq 4.5$ seconds

$$C_3 = \frac{0.75}{T^2} \quad (3.17)$$

The fundamental sloshing period, T , is evaluated in the same way as in the API standard. It is noted here that the value of C_2 in the API standard is 1.8 times greater than that of SC_3 in the AWWA standard. This is obtained by taking the ratio of Equations 3.14–3.15 to their corresponding expressions in Equations 3.16–3.17 including the soil amplification factor, S . This indicates that the convective response determined by the API standard is greater than that obtained by the AWWA standard for tanks under similar conditions.

It is interesting to compare the effective spectral accelerations ZIC_1 and ZIC_2 in the API standard to their equivalences, $ZK(0.14)$ and $ZKSC_3$, respectively in the AWWA standard. The comparison is shown in Table 3.4 considering the case of high seismicity, $Z = 1.0$, and high functional requirements, $I = 1.5$. The reason for using $I = 1.5$ is to be consistent with the assumption in the AWWA standard that water tanks are essential facilities. The comparison results indicate that in all cases the API standard gives greater values of spectral acceleration, hence seismic loads, than the AWWA standard, especially in the case of an anchored tank. Thus in determining the design loadings, the API standard is more conservative than the AWWA standard.

The importance of the need for consideration of vertical acceleration is stated in the AWWA standard, but it is not elaborated further on how to undertake the analysis. Furthermore, equations for computing the dynamic hoop force from the seismic hydrodynamic effect are presented. Also, both standards stress the importance of providing sufficient free-board allowance.

It is very important to note here that in both standards, the evaluation of the overturning moment is the same for anchored and unanchored tanks. The AWWA

standard, however, differentiates between the two cases through the structure coefficient K as previously mentioned. Also, both standards recommend the use of site specific response spectrum to determine a more appropriate spectral accelerations instead of using the seismic lateral coefficients ZIC_1 , ZIC_2 , $ZK(0.14)$ and $ZKSC_3$.

3.2.3.1 National Building Code of Canada

In designing tanks in accordance to the provisions specified in the API or AWWA standards in Canada, equivalent seismic lateral coefficients are required. The seismic design provisions of the National Building Code of Canada (NBCC 1990) can be utilized for this purpose. In the NBCC (1990), the equivalent static lateral seismic force, V_e , is expressed as follows

$$V_e = v\bar{S}IFW \quad (3.18)$$

where

v	= zonal velocity ratio
$\bar{S}$	= seismic response factor
I	= importance factor
F	= foundation factor
W	= total weight of the structure

The zonal velocity ratio, v , is derived from probabilistic studies of ground motions in the 1 second period range, normalized to a spectral velocity of 1.0m/s. It has the same function as the seismic zone coefficient, Z , in the API and the AWWA standards. Closely related is the zonal acceleration ratio, a , which is obtained from probabilistic

evaluations of ground motions in the 0.2 second period range, normalized to $1.0g$. These ratios have corresponding velocity-related seismic zone, Z_v , and acceleration-related seismic zone, Z_a , as listed in Table 3.5. The seismic zones are developed based on the seismic hazard maps presented in Figures 3.7 and 3.8 and obtained from statistical analysis of earthquakes that have been experienced in Canada and adjacent regions. The zonal acceleration ratio, a , is implicitly included in the expression for V_e through the seismic response factor, $\bar{S}$, as plotted in Figure 3.9. The factor $\bar{S}$ essentially represents the idealized elastic response of 5% damped multi-degree-of-freedom systems. It has similar function as the C_1 and C_2 coefficients in the API and the AWWA standards. The factor $\bar{S}$ is dependent on the fundamental period T . When applied to tank design, there are two values of $\bar{S}$ that are required, since there are two fundamental periods to be considered: the liquid sloshing mode and the interaction vibration of the tank wall and the liquid. If the design procedures of the API and the AWWA standards are followed, the tank wall is assumed to be rigid as previously discussed. Hence, if the effect of the tank wall vibration is to be included in the design, other procedures that consider the tank wall flexibility, such as the Haroun-Housner (1981) procedure, must be followed.

The importance factor, I , and the foundation factor, F , in NBCC (1990) have the same functions as the importance factor, I , and site-amplification factor, S , in the API and the AWWA standards. The products of $v\bar{S}IF$, i.e. the impulsive and convective components, can be interpreted as the effective spectral accelerations which can replace the seismic lateral coefficients ZIC_1 , ZIC_2 , $ZK(0.14)$ and $ZKSC_3$ in the API and AWWA standards. In the design of liquid storage tanks, the total weight

of the structure. W_i should be divided into three components: the impulsive liquid weight, the convective liquid weight, and the combined total weight of the tank wall, roof and other tank components as well as other special loads, such as load due to snow. The impulsive and convective weights may be determined using the API or AWWA standards, or other appropriate procedures if the effect of tank wall flexibility is to be included in the design. The weight components must be used in conjunction with their appropriate effective spectral accelerations given by the products of $v\bar{S}_i F$, i.e. the impulsive and convective components.

It is noted here that the above provisions for V_e is for elastic response only. However, a designed minimum base shear, V , which implicitly takes into account the inelastic behaviour of the structure in relation to its energy absorption capacity is further specified in NBCC (1990). The base shear, V , corresponds to that at the ultimate limit state, where the structure is assumed to be at the point of collapse. It is obtained by introducing a force modification factor, R , and a calibration factor, U , equal to 0.6, into the expression for V_e . These two factors have the effect of reducing the magnitude of V_e for structures with high energy dissipation capacities. The force modification factor, R , reflects the capability of the structure to dissipate energy through inelastic behaviour, and is based on design and construction experience, as well as the evaluation of the performance of different types of structural systems in major and moderate earthquakes. It ranges from a value of 1 to 4 with higher values representing higher energy absorption capacities of the structure. The purpose of the calibration factor, U , is to maintain the design base shears at the same level of protection for buildings with good to excellent capability of resisting seismic loads

consistent with the R factors used. In the design of liquid storage tanks, however, the combined effect of R and U may be approximately taken equal to 1.0, due to the lack of energy dissipation mechanisms in liquid storage tanks, such as they have low damping and lack of nonstructural bearing elements. To add to this, the uncertainties in the tank response, specially for uplifting tanks, render the justification of the above approximation. Also, if the NBCC (1990) seismic design provisions are followed, tank structures will fall under the category of "other lateral load-resisting systems", which is given a value of R equal to 1. In such cases, for tanks designed according to the NBCC (1990) seismic design provisions, the designed minimum base shear, V , is equivalent to the equivalent static lateral seismic force, V_e . As a result of this, NBCC(1990) provides higher values of seismic design base shear for tanks than for typical building structures. In general, since the NBCC (1990) seismic design provisions have been developed for the design of typical building structures, good judgement is required in applying these provisions on the design of liquid storage tanks.

3.3 Flexible Anchored Tank Procedures

In view of the poor performance of tanks during earthquakes (Alaska earthquake of 1964, Clough 1977, Niwa 1978), the focus of tank investigations in recent years has shifted to the development of analytical techniques and procedures that take into account the vibration characteristics of the flexible tank wall. The procedures are more complex than that for rigid tanks due to the coupling effect of the motions of

the tank wall and the dynamic liquid response.

3.3.1 Veletsos Procedure

Procedures for determining the hydrodynamic forces in an accelerating flexible anchored tank have been developed by Veletsos (1984). In the model, the tank wall is analyzed as a uniform cantilever beam. These analysis procedures are more accurate than the Housner method since no unnecessary constraints are imposed on the liquid flow and more importantly the contribution of the tank flexibility in the total response is also considered.

In the analysis, the liquid motion is determined by satisfying the Laplace's equation of the liquid flow in accordance with prescribed boundary conditions at the liquid-tank interface and at the liquid free surface. It is assumed that the liquid is incompressible, inviscid and irrotational, while the tank is anchored to a rigid foundation. Moreover, the liquid and tank motions are assumed to be within the linearly elastic range of response. The following are the boundary conditions that must be satisfied:

1. At the tank base, the velocity of the tank bottom and the liquid must be the same. Thus for horizontally excited tanks, the vertical component of liquid velocity at the bottom is zero.
2. At the tank wall, the radial velocities of the liquid must be the same as the tank wall. For rigid tanks, the motion of the wall and the liquid is the same as the ground. For flexible tanks, the coupling of the the motions of the deformable

wall and the liquid is considered.

3. At the liquid free surface, the state of zero pressure must be satisfied. In determining the impulsive pressure, the hydrodynamic pressure at the liquid free surface is assumed to be zero. Thus in the solution for the convective pressure at the free surface, the difference between the actual boundary condition and the zero pressure condition must be corrected.

As in previous procedures, the total response is expressed as the sum of the impulsive and convective components. The analysis procedures for flexible tanks are modified from the solution for rigid tanks.

Rigid Tank Solution

For a rigid tank, the impulsive response components are determined as follows

$$p_i(z, \theta, t) = c_o(z) \ddot{x}_g(t) \rho_l R \cos \theta \quad (3.19)$$

$$Q_i(t) = m_i \ddot{x}_g(t) \quad (3.20)$$

$$M_i(t) = m_i h_i \ddot{x}_g(t) \quad (3.21)$$

$$\Delta M_i(t) = m_i \Delta h_i \ddot{x}_g(t) \quad (3.22)$$

where

$p_i(z, \theta, t)$ = instantaneous impulsive pressure

$Q_i(t), M_i(t)$ = instantaneous impulsive components of the lateral base shear and overturning moment at the base respectively

$\Delta M_i(t)$ = instantaneous moment contribution to the overturning moment at the foundation due to the impulsive pressure

at the tank bottom

m_i = effective liquid mass that participates in the impulsive action

h_i = effective liquid height for calculation of the overturning moment at the base in the impulsive mode

Δh_i = effective height of m_i to generate the correct $\Delta M_i(t)$

$c_o(z)$ = dimensionless function of z defining the heightwise distribution of the impulsive pressure

$\ddot{x}_g(t)$ = instantaneous ground acceleration

ρ_l = mass density of liquid

R = radius of tank

The configuration of the liquid-tank system and the circumferential distribution of the hydrodynamic wall pressure, p (impulsive or convective), are shown in Figure 3.10 and 3.11 respectively .

In Equation 3.19, the impulsive pressure, p_i , is a function of the height. It has a $\cos \theta$ distribution in the circumferential direction. To get the maximum impulsive pressure, the peak values of each of the terms in Equation 3.19 must be determined. For the time function, $\ddot{x}_g(t)$, the peak value is the maximum ground acceleration. The peak value for the $\cos \theta$ function occurs at the diametral axis in the direction of the excitation. This is shown in Figure 3.11, where the zero values for the circumferential pressure occur along the axis normal to the axis of excitation. The peak value of $c_o(z)$ occurs at the base and decreases to zero at the liquid free surface. It is observed that

the value of $c_o(z)$ at the base depends on the liquid height to tank radius ratio, H/R , and increases rapidly with H/R . Thus tall tanks generate larger impulsive pressure than broad tanks with the maximum occurring at the base. The parameters needed in the calculation of the impulsive components of the hydrodynamic pressure, the lateral base shear, and the overturning moments at the tank base and at the foundation are listed in Table 3.6 in which m_l is the total mass of the liquid.

The convective response components are determined as follows

$$p_c(z, \theta, t) = c_{sj}(z) A_{cj}(t) \rho_l R \cos \theta \quad (3.23)$$

$$Q_c(t) = m_{cj} A_{cj}(t) \quad (3.24)$$

$$M_c(t) = m_{cj} h_{cj} A_{cj}(t) \quad (3.25)$$

$$\Delta M_c(t) = m_{cj} \Delta h_{cj} A_{cj}(t) \quad (3.26)$$

where

$p_c(z, \theta, t)$ = instantaneous convective pressure

$Q_c(t), M_c(t)$ = instantaneous convective components of the lateral base shear and overturning moment at the base respectively

$\Delta M_c(t)$ = instantaneous moment contribution to the overturning moment at the foundation due to the convective pressure at the tank bottom

m_{cj} = effective liquid mass that participates in the j th mode of sloshing vibration

h_{cj} = effective liquid height for calculation of the overturning moment at the base in the j th mode of sloshing vibration

- Δh_c = effective height of m_c , to generate the correct $\Delta M_c(t)$ in the j th mode of sloshing vibration
- $c_s(z)$ = dimensionless function of the heightwise distribution of the convective pressure in the j th mode of sloshing vibration
- $A_c(t)$ = instantaneous pseudoacceleration for a single-degree-of-freedom oscillator when subjected to the same ground motion and the same natural frequency and damping as the j th mode of liquid sloshing vibration

In Equation 3.23, the exact convective pressure, p_c , is the sum of the contributions from all the sloshing modes. In similar manner as in the derivation of the impulsive pressure, the maximum convective pressure is obtained by determining the peak values of $c_s(z)$, $A_c(t)$ and $\cos \theta$ functions. The maximum of the dimensionless function, $c_s(z)$, hence the convective pressure, occurs at the liquid free surface, and $c_s(z)$, decreases with depth. The convective pressure decays much more rapidly with depth for tall tanks than for broad tanks. The peak value of the instantaneous pseudoacceleration, $A_c(t)$, is obtained from a design response spectrum using the frequency and damping of the j th mode of sloshing vibration. The recommended damping is between 0.01% and 1.0% of the critical damping. In the study, Veletsos showed that the contributions from the higher sloshing modes are only a small fraction of that from the fundamental mode. Thus it is assumed that considering only the contribution from the fundamental mode, $j = 1$ in Equations 3.23–3.26, is sufficient for

engineering design purpose. The fundamental convective frequency, f_c , is given by

$$f_c = \frac{1}{2\pi} \sqrt{\frac{g}{R} \kappa_1 \tanh(\kappa_1 \frac{H}{R})} \quad (3.27)$$

where

$$\kappa_1 = 1.841$$

$$g = \text{gravitational acceleration}$$

$$H, R = \text{liquid height and tank radius respectively}$$

Equation 3.27 is similar in form to the fundamental convective frequency proposed in the rigid tank procedures (Haroun 1957, Epstein 1976, AWWA 1984, API 1988). The parameters needed in the calculation of the convective components of the hydrodynamic pressure, the lateral base shear and the overturning moments at the tank base and at the foundation are listed in Table 3.7 in which m_l is the total mass of the liquid.

Flexible Tank Solution

Since the convective mode is associated with oscillations of much longer periods than those characterizing the impulsive mode, it is assumed that the coupling between the two modes tends to be weak. Hence, the convective mode cannot be affected materially by the flexibility of the tank. Thus the influence of the tank wall flexibility is considered only in the impulsive response. Therefore for an engineering design purpose, the procedure for the calculation of the convective response in rigid tanks is considered applicable also to flexible tanks.

To determine the influence of tank wall flexibility on the impulsive hydrodynamic pressure, analytical techniques have been derived (Veletsos 1974, Veletsos and Yang

1976, 1977). In the earlier work, the liquid-tank system is assumed to behave as a single-degree-of-freedom system that vibrates in a prescribed shape along its height. The cross-section of the cylindrical tank is assumed to undergo no distortion and has no initial imperfections.

In the improved method, the liquid-tank system is analyzed as a uniform multi-degree-of-freedom cantilever beam. The deflection of the tank is assumed as a linear combination of terms, each of which is defined by a generalized time-dependent coordinate and the vibration mode shape of a uniform cantilever beam. It is noted here, however, that the analysis is limited to tanks of uniform wall thickness filled with water. The exact expression of the instantaneous impulsive hydrodynamic pressure, $p_i(z, \theta, t)$, exerted on the wall of a flexible tank is given by

$$p_i(z, \theta, t) = \left[\sum_{k=1}^{\infty} c_{ok}(z) A_{ik}(t) \right] \rho_l R \cos \theta \quad (3.28)$$

where

- $c_{ok}(z)$ = dimensionless function of z that defines the heightwise variation of the impulsive pressure component in the k th mode of vibration of the liquid-tank system
- $A_{ik}(t)$ = instantaneous pseudoacceleration for a single-degree-of-freedom oscillator when subjected to the same ground motion and the same natural frequency and damping as the k th mode of vibration of the liquid-tank system

Equation 3.28 is similar in form to Equation 3.19 for rigid tanks except that it involves the summation of the contributions from infinite number of terms instead from

a single term, and that each term is multiplied by the pseudoacceleration function, $A_{i_k}(t)$, instead of the ground acceleration, $\ddot{x}_g(t)$. The presence of more than one term in Equation 3.28 implies that the heightwise distribution of the impulsive pressure on the wall changes with time. The flexibility of the tank has no effect in the circumferential pressure distribution, and it is assumed to be proportional to $\cos\theta$, as for rigid tanks. Each individual terms in Equation 3.28 represents the contribution of the particular vibration mode of the liquid-tank system to the total impulsive response. If all the $A_{i_k}(t)$ functions in Equation 3.28 are taken equal to $\ddot{x}_g(t)$, Equation 3.28 will yield the impulsive hydrodynamic pressure exerted on a rigid tank. Thus, the following approximate relationship between the pressure distribution functions for rigid and flexible tanks can be obtained

$$c_o(z) = \sum_{k=1}^{\infty} c_{o_k}(z) \quad (3.29)$$

Therefore, $c_{o_k}(z)$ represents the k th component contribution in the expression of $c_o(z)$, and the wall pressure corresponding to the k th natural mode of vibration of the liquid-tank system. In an analogous manner, similar relationships can be obtain for the impulsive masses and heights for rigid and flexible tanks.

From parametric studies (Veletsos and Yang 1976, 1977, 1983), it is shown that some approximations can be made to the above procedures. For broad tanks with H/R between 0.25 and 1.0, the heightwise distribution of the impulsive pressure for rigid tanks is in good agreement with the heightwise distribution of the impulsive pressure in the fundamental mode for flexible tanks. Similarly for such cases, the parameters m_i , $m_i h_i$ and $m_i \Delta h_i$ for rigid tanks can be applied approximately for

flexible tanks when calculating the lateral base shear and overturning moments. Thus the impulsive response components of a flexible broad tank with H/R between 0.25 and 1.0 can be expressed approximately as follows

$$p_i(z, \theta, t) \simeq c_o(z) A_i(t) \rho_l R \cos \theta \quad (3.30)$$

$$Q_i(t) \simeq m_i A_i(t) \quad (3.31)$$

$$M_i(t) \simeq m_i h_i A_i(t) \quad (3.32)$$

$$\Delta M_i(t) \simeq m_i \Delta h_i A_i(t) \quad (3.33)$$

where

$A_i(t)$ = instantaneous pseudoacceleration corresponding to the
fundamental mode of vibration of the liquid-tank system

The other terms in the above equations have been defined earlier in Equations 3.19–3.22.

Two important points must be noted here, however, when implementing Equations 3.30–3.33. First, the use of the rigid impulsive mass, m_i , in the above flexible tank equations implies that the contributions from all the impulsive vibration modes are included, and they are all subjected to the pseudoacceleration of the fundamental mode. Second, although the distribution of the impulsive pressure appears to be independent of the tank flexibility, its magnitude is highly dependent on this factor as indicated by the presence of the pseudoacceleration function, $A_i(t)$, in Equation 3.30. The maximum values of the response quantities given in Equations 3.30–3.33 are obtained by replacing $A_i(t)$ with its peak value A_i . The value of A_i is the spectral

acceleration obtained from a design response spectrum using the fundamental frequency of the liquid-tank system, f_i , and an assumed damping. Damping is usually taken as 2% of critical damping.

For flexible tall tanks adjustment must be made to the values of $Q_i(t)$ and $\Delta M_i(t)$ which might be unduly conservative due to the approximations used in broad tanks. To correct the error, three different options are suggested. The most preferred option is determining $Q_i(t)$ from Equation 3.31 by replacing m_i with the sum of the impulsive masses (m_{i1} and m_{i2}) of the first two impulsive vibration mode of the liquid-tank system. Similarly in the determination of $\Delta M_i(t)$ by Equation 3.33 by replacing $m_i \Delta h_i$ by the sum of $m_{i1} \Delta h_{i1}$ and $m_{i2} \Delta h_{i2}$. The parameters m_{i1} , m_{i2} , $m_{i1} \Delta h_{i1}$ and $m_{i2} \Delta h_{i2}$ are presented in design charts, details of which can be found in Veletsos (1984).

The fundamental frequency, f_i , of the flexible liquid-tank system is determined as follows

$$f_i = \frac{C_l}{2\pi} \frac{1}{H} \sqrt{\frac{E}{\rho}} \quad (3.34)$$

$$C_l = C_w \sqrt{\frac{\rho_w}{\rho_l}} \quad (3.35)$$

where

E = Young's modulus of elasticity of the tank wall

ρ = mass density of the tank

H = depth of liquid

ρ_w = mass density of water

ρ_l = mass density of liquid under consideration

Equation 3.34 is based on the assumption that the fundamental or lowest natural frequency of a perfect circular cylindrical tank subjected to uni-directional horizontal ground motions involves a single wave in the circumferential direction. It must be noted, however, that due to the initial imperfections on the tank shell, the vibration modes of the flexible tank wall involving several waves in the circumferential direction are excited. These vibration modes of the flexible tank wall are generally associated with frequencies lower than that corresponding to the vibration mode involving a single wave in the circumferential direction. The values of the coefficient C_w , as listed in Table 3.8, are for roofless tanks of uniform wall thickness, with tank wall thickness to radius ratio, t_s/R , equal to 0.0005 and 0.001 and with the Poisson's ratio of the tank material equal to 0.3. The expression for C_l in Equation 3.35 is based on the assumption that the mass of the tank itself is negligible in comparison to that of the liquid, which is normally the case in practice.

The contribution of the tank inertia may also be taken into account, although generally it is very small compared to the hydrodynamic effects. If included, the tank mass, which consists of the tank wall and the roof, will be subjected to an acceleration that corresponds to the fundamental frequency of the liquid-tank system with the roof. To obtain the total response, the absolute sum of the maxima or the square root of sum of squares (SRSS) of maxima of the impulsive and convective components plus the tank inertia component if included can be adopted.

The effect of the vertical component of ground motion can be included, although it does not contribute to the overturning moment. The dynamically activated pressure, p_v , due to the vertical component of ground motion is uniformly distributed in the

circumferential direction and increases linearly from top to bottom. Its maximum value at the base, P_V , is given by

$$P_V = \rho_l H A_V \quad (3.36)$$

where

A_V = spectral acceleration associated with the fundamental natural frequency of the axisymmetric breathing mode of vibration of the liquid-tank system, f_V

$$f_V = \frac{1}{2\pi} \frac{C_V}{H} \sqrt{\frac{E}{\rho}}$$

C_V = coefficient given in Table 3.9

The other terms in the above equation have been defined earlier in Equations 3.34-3.35. Equation 3.36 is based on the assumption that the tank flexibility affects only the magnitude and temporal variation of the hydrodynamic wall pressure but not its spatial distribution (Veletsos and Kumar 1984). The fundamental natural frequency, f_V , of the axisymmetric liquid-tank coupled vibration given above considers the tank behaves like a membrane as an approximation. This leads to vibration modes that violate the condition of no radial displacement at the base and to frequency values that are lower than the exact values. Table 3.9 shows the results obtained using the membrane theory (Equation 3.36) and by the more accurate bending theory employed by Kumar (1981). These results are for steel tanks fully filled with water, with the tank wall thickness to radius ratio equal to 0.001, with the relative mass densities of the liquid and the tank wall equal to 0.127, and with the Poisson's ratio of the tank material equal to 0.3.

3.3.2 Haroun-Housner Procedure

The analysis procedure developed by Haroun and Housner (1981) modelled the tank wall by finite elements. The liquid region is analyzed as a continuum by the boundary solution technique (Haroun 1980) with the same boundary conditions and assumptions as stated earlier in the discussion of the Veletsos procedure (1984). At the tank wall-liquid interface, the shell has a displacement which depends on the magnitude and distribution of the hydrodynamic pressure which in turn depends on the shell displacement itself. However, this coupling of the shell and the liquid motions is shown to be important for the impulsive response mode only, as noted before in the Veletsos procedure. Thus, the effect of tank flexibility is included only in the impulsive response.

The impulsive response of the liquid-tank system is evaluated using Hamilton's Principle

$$\delta \int_{t_1}^{t_2} (T - U + W) dt = 0 \quad (3.37)$$

In Equation 3.37, T and U are the kinetic energy and the strain energy of the shell respectively, W is the work done by the hydrodynamic pressure, while the variational operator, δ , is taken during the time interval t_1 to t_2 . The kinetic energy, T , is derived in terms of the shell assembled mass matrix and nodal displacement vector, while the strain energy, U , is determined in terms of the shell assembled stiffness matrix and nodal displacement vector as discretized by ring shaped finite elements. The work done, W , is derived by the added mass concept, which represents the effects of the hydrodynamic pressure on the tank wall vibration. Equation 3.37 leads to the

matrix equation of motion of the shell, and consequently the estimation of the shell deformation and impulsive hydrodynamic pressure.

For design purpose, the dynamic response of the liquid-tank system is evaluated by reference to the mechanical spring-mass model shown in Figure 3.12. This model is derived such that the concentrated masses (m_c , m_f , m_r) with their corresponding heights (h_c , h_f , h_r), natural frequencies (ω_c , ω_f) and damping (ζ_c , ζ_f) generate the same response as the actual liquid-tank system. The impulsive mass is divided into two masses. The rigidly attached mass, m_r , is the effective mass that simulates the dynamic effects associated with the rigid body motion of the shell, while the elastically attached mass, m_f , is the effective mass that simulates the dynamic effects of the deformational component of the wall motion. In Veletsos model these two masses are combined into a single mass. The effective mass, m_c , simulates the hydrodynamic effects associated with the free sloshing or oscillations of the liquid. The effective masses and other relevant design parameters are presented in design charts for different liquid height to tank radius ratios, H/R , as shown in Figures 3.13-3.19, in which m_l is the total liquid mass, t_s is the thickness of the tank wall, g is the acceleration due to gravity and ρ and E are the mass density and Young's modulus of elasticity of the tank wall respectively. Figures 3.13-3.14 are similar to the convective equations for the fundamental sloshing mode in the Veletsos procedure and in the procedures that assume tanks are rigid (Haroun 1957, Epstein 1976, AWWA 1984, API 1988). Also, the liquid-tank coupled vibration frequency parameter, $\omega_f H \sqrt{\rho/E}$, as shown in Figure 3.15, is related to the frequency coefficient C_w shown in Table 3.8 from the Veletsos procedure. As can be noted from the curves in these figures, the

impulsive component dominates the response in tall tanks, whereas for broad tanks the convective component is the dominating mode.

The maximum values of the total lateral base shear and the overturning moment at the tank base are calculated as follows.

$$Q = \sqrt{(m_c A_c)^2 + (m_f A_f)^2 + [(m_r - m_f) \ddot{x}_g]^2} \quad (3.38)$$

$$M = \sqrt{(m_c A_c h_c)^2 + (m_f A_f h_f)^2 + [(m_r h_r - m_f h_f) \ddot{x}_g]^2} \quad (3.39)$$

where

$Q, M, \ddot{x}_g$ = maximum total lateral base shear, overturning moment at the tank base and ground acceleration respectively

A_c, A_f = spectral accelerations corresponding to the fundamental frequencies and damping of ω_c and ζ_c (for sloshing) and ω_f and ζ_f (for wall flexibility) respectively

m_r, m_f, m_c = effective masses associated with rigid tank wall motion, deformational component of wall motion and liquid sloshing motion respectively

h_c, h_f, h_r = effective heights corresponding to the effective masses m_r, m_f, m_c respectively

If the contribution of the roof is to be included, it must be considered separately. It is noted here that Equations 3.38–3.39 consider only the effects of the fundamental mode. As mentioned earlier in Veletsos procedure, the contributions of the higher modes may be significant for the lateral base shear of tall tanks.

3.3.3 Balendra et al. Procedure

This procedure is based on results obtained from analytical studies where both the liquid and the tank wall were modelled by finite elements. These analytical studies were first initiated by Balendra and Nash (1978) where the effect of liquid sloshing was neglected. Later, Balendra et al. (1982) included the sloshing effect which resulted in the development of this design procedure. In the analysis, the equations of motion of the liquid-tank system is subjected to the same boundary conditions and assumptions as stated in the Veletsos procedure.

Ring shaped finite elements are used to discretize the tank wall. Each ring element has four degrees of freedom at each nodal circle: the axial displacement u , the circumferential displacement v , the radial displacement w , and the slope of the generator w' which is the derivative of w with respect to the axial coordinate z . The liquid is modelled by annular ring shaped finite elements of rectangular cross-section with hydrodynamic pressure as the nodal degrees-of-freedom. The equations of motion in matrix form are given by

$$[M_s]\{\ddot{q}\} + [K_s]\{q\} - [S]^T\{p\} = \{0\} \quad (3.40)$$

$$[M_l]\{\ddot{p}\} + [K_l]\{p\} + [S]\{\ddot{q}\} = \{0\} \quad (3.41)$$

where $[M_s]$ and $[K_s]$ are the mass and stiffness matrices of the tank shell respectively, $[M_l]$ and $[K_l]$ are the mass and stiffness matrices of the liquid respectively, $[S]$ is the coupling force matrix due to interaction between the tank shell and the liquid, $[S]^T$ is the transpose matrix of $[S]$, $\{q\}$ is the generalized nodal displacement amplitudes, $\{p\}$ is the nodal dynamic pressure amplitudes and $\{\ddot{q}\}$ and $\{\ddot{p}\}$ are the second deriva-

tives with respect to time of $\{q\}$ and $\{p\}$ respectively. The combined solution of Equations 3.40 and 3.41 gives the tank response.

In free-vibration analysis of the coupled system, the lower frequencies are those associated with liquid sloshing, while the higher frequencies are those associated with the coupled vibrations of the tank wall and the liquid. The calculated sloshing frequencies are in good agreement with those determined by Veletsos and Yang (1976). The sloshing frequencies are given by

$$\omega_{c_j}^2 = \frac{g}{R} \kappa_j \tanh\left(\kappa_j \frac{H}{R}\right) \quad (3.42)$$

where

- ω_{c_j} = the sloshing natural frequency in the j th mode
- κ_j = 1.8412, 5.3314, and 8.5363 for the first three modes respectively
- R, H = tank radius and liquid depth respectively
- g = gravitational acceleration

The first two coupled vibration frequencies, ω_{i_1} and ω_{i_2} , of steel tank filled with water are shown in Figure 3.20 in which τ is the tank shell thickness to radius ratio, t_s/R , $\bar{\beta}$ is the liquid depth to tank height ratio, H/H_t , and α is the liquid height to tank radius ratio, H/R . These frequencies correspond to modes involving a single wave in the circumferential direction ($m = 1$) and are normalized with respect to $\omega_{o_k}^*$, which are the frequencies of a flexural steel beam of length, H , second moment of area,

$I = \pi R^3 t_s$, and mass per unit length, $\bar{m} = 2\pi R t_s \bar{\rho} + \pi R^2 \rho_l$. It is given by

$$\omega_{o_k}^* = \frac{a_k^2 R}{H^2} \sqrt{\frac{\bar{E}}{2\bar{\rho}[1 + (R/2t_s)(\rho_l/\bar{\rho})]}} \quad (3.43)$$

where $\bar{E}$ and $\bar{\rho}$ are the modulus of elasticity and mass density of steel respectively, ρ_l is the mass density of the liquid, t_s is the thickness of the shell and a_k has a value that depends on the support condition of the beam assumed to be identical to that of the tank. For tanks anchored at the base and free at the top, the values of a_k for the first three modes are 1.875, 4.694 and 7.855 respectively. The coupled vibration frequencies presented in Figure 3.20 are found to be in excellent agreement with those obtained by Veletsos and Yang (1976).

The maximum total hydrodynamic pressure, $P(z/H, \theta)$, on the tank wall is determined as follows

$$P(z/H, \theta) = [\sum_j C_{c_j}(z/H) A_{c_j} + \sum_k C_{i_k}(z/H) A_{i_k}] \rho_l H \cos \theta \quad (3.44)$$

The first series in Equation 3.44 represents the convective components, whereas the second series represents the impulsive components. The variables A_{c_j} and A_{i_k} are the spectral accelerations of the j th sloshing and k th coupled vibration frequencies respectively. The convective wall pressure coefficient, $C_{c_j}(z/H)$, is a function of z/H and α , whereas the impulsive pressure coefficient, C_{i_k} , is a function of z/H , α , $\bar{\beta}$, and τ , where

$$\tau = t_s/R, \quad t_s \text{ is the thickness of tank wall}$$

$$\bar{\beta} = H/H_t, \quad H_t \text{ is the total height of tank}$$

$$\alpha = H/R$$

Integrating $P(z/H)$ over the entire liquid height in the direction of the horizontal base excitation at $\theta = 0$, gives the maximum total lateral base shear, Q , which is given by

$$Q = \sum_j m_{c_j} A_{c_j} + \sum_k m_{i_k} A_{i_k} \quad (3.45)$$

$$m_{c_j} = m_l \alpha \int_0^{1.0} C_{c_j}(z/H) d(z/H) \quad (3.46)$$

$$m_{i_k} = m_l \alpha \int_0^{1.0} C_{i_k}(z/H) d(z/H) \quad (3.47)$$

where

m_{c_j} = j th convective component of liquid mass in the j th sloshing mode

m_{i_k} = k th impulsive component of liquid mass in the k th coupled vibration mode

m_l = total liquid mass

The values of m_{c_j} and m_{i_k} for the first two modes are shown in Figures 3.21 and 3.22, respectively, in which m_l is the total mass of the liquid. Also shown in Figure 3.21 are the first impulsive and convective mode masses as determined by the Veletsos procedure. The above figure shows that for tanks with liquid height to tank radius ratio, α , greater than 1.0, the values of m_{i_1} from Veletsos procedure are slightly smaller than those obtained from the Balendra et al. procedure. For practical design, the convective masses, m_{c_1} and m_{c_2} , can be obtained from Figure 3.21 for all values of $\bar{\beta}$ and τ . This is consistent with the assumption that the flexibility of the tank wall, as accounted for by the parameter τ , does not significantly affect the convective

response. However, the impulsive masses, m_{i1} and m_{i2} , are highly dependent on the values of $\bar{\beta}$ and τ , as shown in Figure 3.22.

The maximum total overturning moment at the foundation, M_f , is determined by

$$M_f = \left[\sum_j C_{mcj} A_{cj} + \sum_k C_{mik} A_{ik} \right] m_l H \quad (3.48)$$

where C_{mcj} and C_{mik} are the dimensionless foundation overturning moment coefficients of the convective and impulsive modes respectively. The values for the first mode are shown in Figure 3.23 for different values of α , $\bar{\beta}$ and τ .

It is interesting to note that the Balendra et al. procedure, like the Veletsos procedure, combines the response components due to rigid and deformable motions of the tank wall. The total overturning moment at the base corresponding to the maximum shell compressive stress was not determined explicitly by Balendra et al. (1982). Instead, the stress resultants of the shell elements are calculated by the finite element model.

3.3.4 Austrian Recommendations

Recommendations on liquid storage tank design have been developed by Scharf et al. (1989a). When a tank is subjected to a uni-directional horizontal ground excitation, the generated hydrodynamic pressure is the result of the superposition of three pressure components: the dynamically activated convective pressure due to liquid sloshing, p_c , the dynamically activated impulsive pressure due to horizontal rigid tank motion, p_r , and the dynamically activated impulsive pressure due to the interaction vibration of the tank wall and the liquid, p_f . In the development of the

design recommendations, similar assumptions as those in the Haroun-Housner procedure were adopted by Scharf et al. (1989a). The liquid is modelled analytically, while the tank is modelled by finite elements.

The dynamic pressure component p_f is determined by an iterative procedure, which is based on a numerical algorithm developed using the "added mass concept" and the solution of the potential equation

$$P_f(\xi) = 2R\rho_l g \sum_j \frac{I_1(\lambda_j)}{\lambda_j I_1'(\lambda_j)} \left[\int_0^1 f(\xi) \cos\left(\frac{j\pi\xi}{2}\right) d\xi \right] \cos\left(\frac{j\pi\xi}{2}\right) \quad (3.49)$$

where

$$\xi = z/H \quad \lambda_j = (j\pi)/(2\alpha) \quad \alpha = H/R \quad I_1' = dI_1/d\lambda_j \quad j = 1, 3, 5, \dots$$

P_f	= amplitude of p_f normalized to an excitation $\overline{A_f}$ of 1 g
R, H	= tank radius and liquid height respectively
z	= axial coordinate from the bottom
I_1	= modified Bessel function of the first kind and of order 1
$f(\xi)$	= amplitude of vibration mode with $\max f(\xi) = 1$
ρ_l	= mass density of liquid
g	= gravitational acceleration
$\overline{A_f}$	= effective spectral acceleration corresponding to the fundamental natural frequency of the liquid-tank coupled vibration mode

In the above equation, $f(\xi) = \sin(\pi\xi/2)$ is assumed. In determining the amplitude of the dynamically activated pressure due to rigid tank motion, P_r , Equation 3.49 is

used but with $f(\xi) = 1$. Details of the iterative procedure are given in the reference (Scharf et al. 1989b).

The amplitude of the dynamic pressure component p_c normalized to an acceleration of $\overline{A}_c$ is given by the following equation (Fischer and Rammerstorfer 1982)

$$P_c(\xi) = 0.84 R \rho_l g \frac{\cosh(1.84\alpha\xi)}{\cosh(1.84\alpha)} \quad (3.50)$$

where

$$\begin{aligned} P_c(\xi) &= \text{amplitude of dynamic pressure component } p_c \\ &\quad \text{normalized to an acceleration } \overline{A}_c \text{ of } 1g \\ \overline{A}_c &= \text{effective spectral acceleration corresponding to the} \\ &\quad \text{fundamental natural frequency of the liquid sloshing mode} \end{aligned}$$

Integration of the individual pressure components gives the effective liquid masses and heights for the simple single-degree-of-freedom oscillator model, as shown in Figure 3.24 in which α is the liquid height to tank radius ratio, m_l is the total liquid mass and H is the liquid height. The results given in Figure 3.24 are obtained from parametric studies by Rammerstorfer et al. (1988).

Multiplying the effective masses m_c , m_f , and m_r corresponding to sloshing motion, liquid-tank coupled vibration and rigid tank motion respectively with their corresponding effective spectral accelerations $\overline{A}_c$, $\overline{A}_f$, and $\overline{A}_r$ respectively yields the maximum lateral base shear. Furthermore, multiplying the base shear components by their corresponding effective heights h_c , h_f , and h_r gives the maximum base overturning moment.

The values of $\overline{A_c}$ and $\overline{A_f}$ are calculated from the products of the spectral accelerations A_c and A_f with their corresponding mode participation factors Γ_c and Γ_f respectively. The spectral accelerations A_c and A_f are obtained from the response spectrum of the design earthquake using the fundamental natural frequencies f_c for liquid sloshing and f_f for liquid-tank interaction vibration respectively. The value of $\overline{A_r}$ is taken as the maximum horizontal ground acceleration, $\ddot{x}_g$. The mode participation factors Γ_c and Γ_f can be obtained from Figure 3.24(c). Alternatively, Γ_c can be determined by

$$\Gamma_c = \frac{2 \sinh(1.84\alpha) [\cosh(1.84\alpha) - 1]}{\sinh(1.84\alpha) \cosh(1.84\alpha) - 1.84\alpha} \quad (3.51)$$

It is noted here that in the procedures by Veletsos (1984), Haroun and Housner (1981) and Balendra et al. (1982), the spectral accelerations A_c and A_f are used directly for determining the maximum lateral base shear and overturning moments at the tank base and at the foundation level. This, however, would give similar result for the impulsive response as to that determined from the Austrian recommendations, since the above three procedures have already incorporated the impulsive mode participation factor, Γ_f , in the impulsive mass parameter. However, in the determination of the convective response, the convective mode participation factor, Γ_c , is not considered by the above three procedures. Thus from the values of the convective mode participation factor shown in Figure 3.24(c), the convective response as determined by the Austrian recommendations is 1.5 to 2.5 times greater than those determined by the Veletsos, Haroun-Housner and Balendra et al. procedures.

The fundamental natural frequencies f_c for liquid sloshing and f_f for liquid-tank

interaction vibration are given by

$$f_c = \frac{1}{2\pi} \sqrt{1.84 \frac{g}{R} \tanh(1.84\alpha)} \quad (3.52)$$

$$f_f = \frac{1}{2F_S(\alpha)R} \sqrt{\frac{E}{\rho_l H} t_{s_{1/3}}} \quad (3.53)$$

where

$t_{s_{1/3}} = t_s(\xi = 1/3)$, thickness of tank wall at 1/3 depth from the bottom

F_S = determined from Figure 3.24(c)

The above expression for f_c is similar to Equation 3.27 in Veletsos procedure. Equation 3.53 is a modified form of the expression proposed by Sakai et al. (1984) for tanks with thickness steps in the tank wall. It is noted here that for broad tanks, the liquid-tank coupled vibration frequencies as determined by Equation 3.53 are significantly different from those determined by the Veletsos, Haroun-Housner and Balendra et al. procedures. However, for tall tanks the results from this equation are in good agreement with those from the above three procedures. The recommended damping values are $\zeta = 0\%$ for sloshing and $\zeta = 2\%$ for the liquid-tank coupled vibration.

The maximum resultant hydrodynamic pressure $p_H(\xi, \theta)$ at a given height and circumferential angle θ is given by

$$p_H(\xi, \theta) = \sqrt{(P_c A_c \Gamma_c)^2 + (P_r \bar{x}_g)^2 + (P_f A_f \Gamma_f)^2} \cos \theta \quad (3.54)$$

Equation 3.54 represents an approximation. Since the evaluation of P_f concerns the relative movement of the shell with respect to the rigid tank motion, the appropriate acceleration in the term with P_f in Equation 3.54 should be the maximum relative

acceleration response, A_f^{rel} , instead of the spectral or maximum absolute acceleration response, A_f . However, this approximation is considered acceptable for practical design purpose. This is verified by study results of acceleration response spectra of given earthquakes which showed that in a wide frequency range, in which the relevant natural frequencies of typical liquid storage tanks lie, A_f^{rel} is approximately equal to A_f . Also, this is very important for the practical application of the response spectrum method for the calculation of the maximum dynamic responses of the tank, because A_f is easily obtained from the displacement or velocity response spectrum of the design earthquake than A_f^{rel} .

Following the above considerations, the maximum overturning moment M at the tank base is given by

$$M = \sqrt{(m_c \bar{A}_c h_c)^2 + (m_r \bar{A}_r h_r)^2 + (m_f \bar{A}_f h_f)^2} \quad (3.55)$$

Two-Directional Horizontal Base Excitation

If two-directional horizontal base excitation is considered, a conservative superposition rule is suggested which assumes that the maximum value of dynamically activated pressure due to N-S (north-south) component of excitation, $p_{H,NS,max}$, occurs at exactly the same time as the maximum pressure resulting from the E-W (east-west) component of excitation, $p_{H,EW,max}$. Considering the simultaneous action of the N-S and E-W excitations, the maximum horizontal hydrodynamic pressure, $p_{H,max}$, and maximum base overturning moment, M_{max} , are

$$p_{H,max} = p_{H,NS,max} \left[1 + \left(\frac{p_{H,EW,max}}{p_{H,NS,max}} \right)^2 \right]^{\frac{1}{2}} + p_{H,EW,max} \left[1 + \left(\frac{p_{H,NS,max}}{p_{H,EW,max}} \right)^2 \right]^{\frac{1}{2}} \quad (3.56)$$

$$M_{max} = M_{NS,max} \left[1 + \left(\frac{M_{EW,max}}{M_{NS,max}} \right)^2 \right]^{\frac{1}{2}} + M_{EW,max} \left[1 + \left(\frac{M_{NS,max}}{M_{EW,max}} \right)^2 \right]^{\frac{1}{2}} \quad (3.57)$$

Vertical Excitation

The hydrodynamic pressure generated by the vertical component of excitation has a constant distribution around the circumference of the tank and its maximum can be computed from the following equations which are derived from relations for a rigidly-based tanks by Fischer and Seeber (1988).

$$P_V(\xi) = \sqrt{P_{rv}^2 + P_{fv}^2} \quad (3.58)$$

$$P_{rv} = H\rho_l(1 - \xi)x_{gv}'' \quad (3.59)$$

$$P_{fv} = 0.815\varphi H\rho_l \cos\left(\pi\frac{\xi}{2}\right) A_{fv} \quad (3.60)$$

where

P_V = total hydrodynamic pressure due to vertical excitation

P_{rv} = hydrodynamic pressure component due to vertical excitation of a rigid tank

P_{fv} = hydrodynamic pressure component due to the deformability of the tank wall when subjected to vertical excitation

x_{gv}'' = maximum vertical ground acceleration

φ = correction factor which takes into account the clamping condition of the tank wall at the bottom

if $0.0 < \alpha \leq 0.8$: $\varphi = 1.0$

if $0.8 < \alpha \leq 4.0$: $\varphi = 1.078 + 0.274 \ln(\alpha)$

$\alpha = H/R$

A_{fv} = spectral acceleration corresponding to the fundamental natural frequency of the axisymmetric liquid-tank interaction

$$\begin{aligned}
 & \text{vibration, } f_{fv} \\
 f_{fv} &= \frac{1}{4R} \sqrt{\frac{2EI_1(\lambda_1)t_{s1/3}}{\pi\rho_l H(1-\mu^2)I_0(\lambda_1)}} \\
 \mu &= \text{Poisson's ratio} \\
 \lambda_1 &= \frac{\pi}{2\alpha} \\
 I_0, I_1 &= \text{modified Bessel function of the first kind, of order zero} \\
 & \text{and of order one respectively}
 \end{aligned}$$

The damping under vertical excitation is dependent on the shear wave velocity of the soil and the shell thickness-to-radius ratio (Fischer and Seeber 1988).

Although vertical excitation does not contribute to the total base overturning moment, it has a considerable influence on the buckling stability of the tank. This point will be further discussed in Chapter 5.

3.3.5 New Zealand Recommendations

Priestley et al. (1986) have developed a set of recommendations for the seismic design of storage tanks in New Zealand. It covers tanks in general regardless of material type and configuration. This section focuses only on vertical ground supported cylindrical steel tanks.

The response analysis of the liquid-tank system follows that of the Veletsos and the Haroun-Housner procedures. Mechanical spring-mass models are used to simulate the impulsive and convective effects. Expressions and design charts are provided to calculate the effective liquid mass and heights, and the fundamental periods of vibrations

of the sloshing and the liquid-tank interaction modes. Modifications are introduced to include the effect of foundation flexibility. Equations for period shift, which is related to the relative stiffness of the tank and foundation, and increased damping associated with inertia modes are included in the recommendations. Moreover, the response due to the vertical excitation is also considered.

Since the recommendations are intended to serve as a design standard, like the API and the AWWA standards, one of the primary concerns is the realistic determination of the expected maximum acceleration to which the tank may be subjected. Accordingly, the maximum or design acceleration depends on the following factors: seismic zone-geographic coefficient, peak acceleration coefficient, acceleration spectra, and probability factor.

3.3.6 Design Guidelines by Adams

The guidelines developed by Adams (1990) are primarily intended for oil and refined product tanks with emphasis on refrigerated liquified gases. The guidelines are based mostly from Veletsos procedure, hence only important points and variations from Veletsos procedure are discussed here. For tanks subjected to horizontal base motion, the fundamental coupled vibration frequency, f_t , of the liquid-tank system can be calculated either by Equation 3.34 of the Veletsos procedure or by the following expression (Sakai et al. 1979)

$$f_t = \frac{\Delta}{2} \sqrt{\frac{9800\pi E t_{s,1/3}}{W_t}} \quad (3.61)$$

where

$$\Delta = 0.067\left(\frac{H}{D}\right)^2 - 0.3\frac{H}{D} + 0.46$$

$$W_l = \text{total weight of the liquid in kilogram - force (kgf)}$$

$$E = \text{Young's modulus of elasticity of the tank (kgf/mm}^2\text{)}$$

$$D, H = \text{tank diameter and liquid depth respectively (mm)}$$

$$t_{s,1/3} = \text{shell thickness at 1/3 height from the bottom (mm)}$$

Design tables reproduced from the Veletsos procedure, but with H/R ratio limited to the range between 0.5 and 1.5, are provided for the calculation of the impulsive and convective responses. Thus the guidelines cover broad tanks only. Moreover, there are no recommended adjustments when H/R exceeds 1.0 as specified in the Veletsos procedure. These adjustments are particularly significant when it comes to the evaluation of the maximum total lateral base shear and foundation overturning moment.

The contribution of the vertical component of ground motion in the total response is included. The vertical acceleration generates the hydrodynamic pressure p_v that is uniformly distributed in the circumferential direction and increases linearly from top to bottom. Its maximum value at the tank base, P_v , is given by

$$P_v = 0.66\rho_l H A_v \quad (3.62)$$

where ρ_l is the mass density of the liquid, H is the liquid height and A_v is the spectral acceleration associated with the fundamental natural frequency of the axisymmetric breathing mode of vibration of the liquid-tank system, f_v , as discussed in the Veletsos procedure. Equation 3.62 is similar to Equation 3.36 in the Veletsos procedure. It

differs only in the coefficient 0.66 which assumes that the vertical component of motion is two-thirds of the horizontal values.

3.4 Unanchored Tank Procedures

At present, reliable analytical techniques for the dynamic response of anchored tanks are available. Some of them are incorporated into design standards as discussed in the earlier sections. However, the majority of tanks encountered in the field today are unanchored mainly because of economic reason. As a consequence of the tank base not fixed to the foundation, the behaviour of unanchored tanks becomes nonlinear due to the complexity of bottom plate uplift mechanism. Because of this, the focus of present research efforts has been to develop a realistic model to simulate this nonlinear uplift behaviour.

As shown in experimental studies of unanchored tanks (Clough et al. 1979, Clough and Niwa 1979, Shih 1981, Manos and Clough 1982), the axial compressive stress developed at the shell bottom is substantially greater than that in the case of anchored tanks under similar loading conditions. This suggests that the present approach used in the design procedures such as the API and AWWA standards, of assuming the same loadings for anchored and unanchored tanks, is not realistic. Several studies have shown that the above assumption can be significantly unconservative. A comparison study by Natsiavas and Babcock (1988), in which numerical results from analysis are compared with experimental data, concludes that there are significant differences in hydrodynamic loadings between anchored and unanchored tanks. This is due to the

loss in stiffness of the tank system as the bottom plate uplifts, causing reduction in the vibration frequencies, and changes in mode shapes and damping.

Results of a comparative study by Lau and Zeng (1992) on the dynamic response behaviour of anchored and unanchored tanks show differences in the hydrodynamic pressures in the two cases. The study employs a nonlinear time-history analysis procedure based on a Ritz-type formulation of the liquid-tank system. The tank is modelled by substructure elements: the linear contact and the nonlinear uplifted regions of the bottom plate, and the linear cylindrical tank shell substructure elements. An important observation in the study is that the flexibility of the tank wall has also significant influence on the convective pressure in unanchored tanks. This is contrary to the commonly accepted notion that the tank flexibility does not affect substantially the convective response.

Of all the procedures discussed in the previous sections, only the Austrian recommendations have provisions that concern with the determination of design loadings for unanchored tanks. The proposed method is an iterative procedure where the fundamental frequency of the coupled vibration of the tank wall and the liquid for anchored tanks, f_f , is first calculated from Equation 3.53. It is then check for the given response spectrum whether an increase of the spectral acceleration, A_{UL} , caused by the increase of the natural period due to uplift is possible. If an increase of A_{UL} is not possible, the procedure for anchored tanks is used which gives conservative estimate of dynamic loads for unanchored tanks. If an increase of A_{UL} is possible, the proposed iterative procedure is followed. Details of the iterative procedure are given in Scharf et al. (1989a).

The iterative procedure is based on the concept that the dynamically activated pressure is given by the superposition of three pressure components. These pressure components are p_e due to liquid sloshing, p_r due to rigid body motion, and p_{UL} due to coupled vibration of the tank shell and the liquid of which nonlinear rocking motion is included. The pressure p_{UL} is obtained from the same expression for p_f on anchored tanks but with the vibration amplitude, $f(\xi)$, replaced by the "nonlinear mode shape", $f(\xi, M)$, where M is the base overturning moment. This nonlinear mode shape depends on an equivalent stiffness $c_{UL}(z)$ which assumes a mode shape obtained from the deformation of a quasi-static nonlinear finite element analysis, as discussed by Scharf (1989). Integration of the pressure distribution of p_{UL} gives the corresponding effective mass m_{UL} and the effective height m_{UL} .

The response spectrum method is used to determine the maximum total response. But since the response spectrum is generally for linear system, a modification is introduced that results in an equivalent natural period T_{UL} for the nonlinear coupled vibration mode. The mode participation factor Γ_f is modified by applying the relations given by Fischer and Rammerstorfer (1982) to account for nonlinearity. This resulted in a new mode participation factor, Γ_{UL} .

Table 3.1 Site amplification factor, S
(API 1988, AWWA 1984)

	Soil Profile Type		
	A	B	C
Site Amplification Factor S	1.0	1.2	1.5

Notes:

1. Soil Profile A comprises (a) rock of any characteristic, whether shale-like or crystalline in nature, characterized by a shear-wave velocity greater than 2500 feet per second, or (b) stiff soil less than 200 feet deep in which the soil that overlies rock consists of stable deposits of sands, gravels, or stiff clays.
2. Soil Profile B comprises deep cohesionless or stiff clay, including soil that is more than 200 feet deep in which the soil that overlies rock consists of stable deposits of sands, gravels, or stiff clays.
3. Soil Profile C comprises soft to medium-stiff clays and sands and is characterized by 30 feet or more of soft to medium-stiff clay with or without intervening layers of sand or other cohesionless soils.
4. In locations where the soil profile is not known in sufficient detail to determine the soil profile type, Soil Profile C shall be assumed.

Table 3.2 Seismic zone coefficient, Z
(API 1988, AWWA 1984)

Seismic Zone Coefficient	Seismic Zone (from Figure 3.6)
0.1875	1
0.375	2
0.75	3
1.0	4

Table 3.3 Structure coefficient, K, for type of tank
in AWWA (1984) standard

Structure	Structure Coefficient K
Cross-braced elevated tank	2.25
Pedestal-type elevated tank	3.00
Anchored flat-bottom tank	2.00 [*]
Unanchored flat-bottom tank	2.50 ^{**}

*The above values of K include a recognition that water tanks are essential facilities in that they must be functional for post-emergency operations. No additional importance factor need be applied.

**For structures not considered essential, structure coefficient K may be divided by a reduction factor ranging from 1.0 to 1.5 as specified by the purchaser.

Table 3.4 Comparison of effective spectral accelerations
between API (1988) and AWWA (1984) standards

	API (1988)	AWWA (1984)	
		anchored	unanchored
Impulsive Acceleration	ZIC_1	$ZK(0.14)$	
	0.36	0.28	0.35
Convective Acceleration	ZIC_2	$ZKSC_1$	
for $T \leq 4.5$ sec	$0.45 \frac{S}{T}$	$0.334 \frac{S}{T}$	$0.4175 \frac{S}{T}$
for $T > 4.5$ sec	$2.025 \frac{S}{T^2}$	$1.5 \frac{S}{T^2}$	$1.875 \frac{S}{T^2}$

Notes: Z = 1.0, I = 1.5

S = site amplification factor, T = fundamental convective period

Table 3.5 Definition of seismic zones in NBCC (1990)

Acceleration-Related or Velocity-Related Seismic Zone (from Figures 2.7 and 2.8) Z_a , Z_v	Range of Peak Horizontal Ground Acceleration, g, or Velocity, m/s for 10 percent probability of exceedance in 50 years	Zonal Acceleration Ratio, a Zonal Velocity Ratio, v
0	0.04	0.00
1	0.04 to <0.08	0.05
2	0.08 to <0.11	0.10
3	0.11 to <0.16	0.15
4	0.16 to <0.23	0.20
5	0.23 to <0.32	0.30
6	≥ 0.32	0.40

Table 3.6 Impulsive parameter coefficients for pressure, lateral base shear and overturning moments in rigid tanks (Veletsos 1984)

$\frac{H}{R}$	$c_c(0)$	$\frac{m_i}{m_t}$	$\frac{h_i}{H}$	$\frac{\Delta h_i}{H}$
0.0	.742(H/R)	.542(H/R)	0.40	--
0.1	0.076	0.056	0.40	8.33
0.2	0.158	0.115	0.40	3.76
0.3	0.243	0.176	0.40	2.24
0.4	0.329	0.239	0.40	1.49
0.5	0.414	0.300	0.40	1.06
0.6	0.494	0.359	0.40	0.792
0.7	0.568	0.414	0.40	0.609
0.8	0.537	0.464	0.401	0.480
0.9	0.691	0.508	0.403	0.337
1.0	0.740	0.548	0.404	0.317
1.1	0.722	0.583	0.406	0.264
1.2	0.818	0.614	0.407	0.222
1.3	0.848	0.641	0.409	0.190
1.4	0.873	0.665	0.411	0.163
1.5	0.894	0.686	0.413	0.142
1.6	0.912	0.705	0.415	0.124
1.7	0.927	0.722	0.417	0.109
1.8	0.939	0.737	0.419	0.097
1.9	0.949	0.750	0.421	0.087
2.0	0.957	0.763	0.422	0.076
2.1	0.965	0.774	0.424	0.070
2.2	0.970	0.784	0.426	0.064
2.3	0.975	0.793	0.428	0.059
2.4	0.979	0.802	0.430	0.053
2.5	0.982	0.810	0.431	0.048
3.0	0.992	0.842	0.439	0.033
3.5	0.996	0.864	0.445	0.023
4.0	0.998	0.881	0.450	0.018
4.5	0.998	0.894	0.454	0.014
5.0	0.999	0.905	0.458	0.011

Table 3.7 Convective parameter coefficients for pressure, lateral base shear and overturning moments in rigid tanks (Veletsos 1984)

$\frac{H}{R}$	$c_{s1}(0)$	$\frac{m_{c1}}{m_l}$	$\frac{m_{c2}}{m_l}$	$\frac{h_{c1}}{H}$	$\frac{\Delta h_{c1}}{H}$	$\frac{h_{c2}}{H}$	$\frac{\Delta h_{c2}}{H}$
0.0	0.837	0.837	0.073	0.500	$0.295(R/H)^2$	0.500	$0.35(R/H)^2$
0.1	0.823	0.827	0.067	0.501	29.33	0.500	3.356
0.2	0.783	0.801	0.054	0.506	7.211	0.543	0.732
0.3	0.724	0.761	0.042	0.512	3.007	0.585	0.263
0.4	0.652	0.712	0.033	0.521	0.687	0.630	0.113
0.5	0.575	0.660	0.027	0.533	1.028	0.674	0.524
0.6	0.500	0.608	0.023	0.545	0.674	0.712	0.0256
0.7	0.429	0.558	0.020	0.559	0.463	0.745	0.0128
0.8	0.365	0.511	0.017	0.574	0.328	0.772	0.0066
0.9	0.308	0.470	0.015	0.590	0.239	0.795	0.0034
1.0	0.259	0.432	0.014	0.606	0.177	0.814	0.0018
1.1	0.217	0.399	0.012	0.621	0.133	0.830	0.0010
1.2	0.182	0.370	0.011	0.637	0.101	0.844	0.0005
1.3	0.152	0.344	0.011	0.652	0.077	0.856	0.0003
1.4	0.126	0.321	0.010	0.667	0.059	0.866	0.0002
1.5	0.105	0.306	0.009	0.681	0.046	0.875	0.0001
1.6	0.088	0.282	0.009	0.694	0.036	0.883	
1.7	0.073	0.265	0.008	0.707	0.028	0.890	
1.8	0.061	0.252	0.008	0.719	0.022	0.896	
1.9	0.051	0.239	0.007	0.731	0.017	0.901	
2.0	0.042	0.227	0.007	0.742	0.014	0.906	
2.1	0.035	0.216	0.007	0.752	0.011	0.911	
2.2	0.029	0.206	0.006	0.762	0.009	0.915	
2.3	0.024	0.198	0.006	0.771	0.007	0.918	
2.4	0.020	0.189	0.006	0.779	0.005	0.922	
2.5	0.017	0.182	0.005	0.787	0.004	0.925	
3.0	0.0067	0.151	0.005	0.820	0.001	0.937	
3.5	0.0027	0.130	0.004	0.845		0.946	
4.0	0.0011	0.114	0.003	0.864		0.953	
4.5	0.0004	0.101	0.003	0.879		0.958	
5.0	0.0002	0.091	0.003	0.891		0.962	

Table 3.8 Frequency coefficient C_w for tanks full with water; $\mu = 0.3$, $\rho_1 \rho = 0.127$ (Veletsos and Tang 1983)

H/R	Value of C_w	
	$t_s/R=0.0005$	$t_s/R=0.001$
0.3	0.0421	0.0601
0.4	0.0468	0.0667
0.5	0.0506	0.0719
0.6	0.0538	0.0763
0.7	0.0564	0.0799
0.8	0.0587	0.0829
0.9	0.0605	0.0854
1.0	0.0620	0.0875
1.2	0.0641	0.0903
1.4	0.0650	0.0915
1.6	0.0651	0.0917
1.8	0.0646	0.0909
2.0	0.0637	0.0896
2.2	0.0625	0.0879
2.4	0.0611	0.0859
2.6	0.0596	0.0838
2.8	0.0580	0.0815
3.0	0.0563	0.0792

Table 3.9 Coefficient C_v in expression for fundamental frequency of axisymmetric, breathing mode of vibration of steel tanks full with water (Veletsos 1984)

H/R	Membrane Solution	Pending Solution
0.20	0.0467	0.0521
0.30	0.0567	0.0616
0.50	0.0705	0.0738
0.75	0.0808	0.0834
1.00	0.0868	0.0889
1.50	0.0925	0.0950
2.00	0.0949	0.0964
3.00	0.0969	0.0982
5.00	0.0979	0.0992

Figure 3.1 Housner's mechanical spring-mass model of the liquid-tank system

(a) broad or shallow tank ($H/R \leq 1.5$)

(b) tall or slender tank ($H/R > 1.5$)

Figure 3.2 Epstein's mechanical spring-mass models of the liquid-tank system

Figure 3.3 Effective liquid weights, W_i and W_c , in API (1988) and AWWA (1984) standards

Figure 3.4 Effective liquid heights, h_i and h_c , in API (1988) and AWWA (1984) standards

Figure 3.5 Factor k_p in API (1988) and AWWA (1984) standards

Figure 3.6 Seismic zone map of the United States
(API 1988, AWWA 1984)

Figure 3.6 Continued

Figure 3.7 Contours of peak horizontal ground accelerations, in units of g , having a probability of exceedance of 10 per cent in 50 years (NBCC 1990)

Figure 3.8 Contours of peak horizontal ground velocities, in m/s , having a probability of exceedance of 10 per cent in 50 years (NBCC 1990)

Figure 3.9 Seismic response factor, $\bar{S}$, for 1990 NBCC

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**PM-1 3 1/2" x 4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT**

PRECISIONSM RESOLUTION TARGETS

Figure 3.10 Liquid-tank system configuration

Figure 3.11 Circumferential distribution of hydrodynamic wall pressure, p

Figure 3.12 Haroun-Housner's mechanical spring-mass model of the liquid-tank system

Figure 3.13 Convective mass, m_c , and height, h_c
(Haroun and Housner 1981)

Figure 3.14 Convective frequency parameter, $\omega_c \sqrt{R/g}$
(Haroun and Housner 1981)

Figure 3.15 Liquid-tank coupled vibration frequency parameter,
 $\omega_f H \sqrt{\rho/E}$ (Haroun and Housner 1981)

Figure 3.16 Equivalent mass, m_f
(Haroun and Housner 1981)

Figure 3.17 Equivalent height, h_f
(Haroun and Housner 1981)

Figure 3.18 Equivalent mass, m_r
(Haroun and Housner 1981)

Figure 3.19 Equivalent height, h_r
(Haroun and Housner 1981)

Figure 3.20 First two coupled vibration frequencies of steel tank for $m = 1$
(Balendra et al. 1982)

Figure 3.21 Convective and impulsive liquid masses for $\bar{\beta} = 1.0$ and $\tau = 0.001$ (Balendra et al. 1982)

Figure 3.22 Impulsive liquid masses due to first and second modes (Balendra et. al. 1982)

Figure 3.23 Convective and impulsive foundation overturning moment coefficients due to first mode (Balendra et al. 1982)

Figure 3.24 Response parameters in Austrian recommendations

(a) effective liquid masses (b) heights

(c) participation factors and factor F_s

(d) mechanical model of liquid-tank system
(Scharf et al., 1989a)

Chapter 4

DESIGN ACTIONS

4.1 General

Design actions are the relevant stresses, deformations and other response parameters caused by the design loadings. These actions must not exceed the reserve capacity of the tank system to ensure the adequate performance and safety of the designed liquid storage tanks.

At present, the prevention of shell wall buckling is the primary consideration in the seismic design of liquid storage tanks. Experiences have shown that shell buckling is one of the common types of structural damage found in tanks during earthquakes. Consequently, one of the important actions to be considered in the design of thin walled liquid storage tanks is the maximum axial compressive stress in the shell which causes buckling at the bottom shell course of the tank wall.

For an anchored tank, the procedures from different standards and guidelines are generally similar on the methodology of determining the maximum axial compressive

stress in the shell. However in an unanchored tank, because of the complexity of the uplift behaviour, different analytical models and techniques have been developed and adopted by different design standards and guidelines. Two of these models are the quasi-static models by Wozniak and Mitchell (1978) and Clough (1977). These models have been adopted by current design standards and guidelines because they describe the uplift mechanism in simple forms suitable for design purpose.

Other design actions of importance are the hoop stress and liquid sloshing height. Hoop stress is the sum of the hydrostatic and the hydrodynamic hoop stresses in the circumferential direction of the tank wall. Excessive hoop stress may lead to yielding of the tank wall. The liquid sloshing height is the vertical displacement of the liquid free-surface. An estimate of the maximum liquid sloshing height is needed in the design in order to determine the minimum free-board between the roof and the height of the liquid free surface at full condition. Insufficient free-board may lead to damage of the roof, the upper part of the tank shell, and internal roof support columns.

4.2 Anchored Tanks

The maximum axial compressive stress in the shell of an anchored tank is determined by the simple beam theory with the tank modelled as a cantilever beam. This assumes that the axial compressive force varies directly with the distance from the centerline of the tank in the direction of the lateral loading. It is given approximately as follows

$$\sigma_s = \left[W; + \frac{4M}{D} \right] \frac{1}{\pi D t_s} \quad (4.1)$$

where

- σ_c = maximum axial compressive stress in the shell
- W_t = total weight of the tank (tank wall plus portion of the fixed roof)
- M = design overturning moment at the tank base
- t_s = thickness of shell
- D = diameter of the tank

Equation 4.1 is also applicable to unanchored tanks when the restoring moment due to gravity loads is greater than the overturning moment M at times when no uplift occurs. In an anchored tank, when the restoring moment is less than M the difference is taken up by the anchorage. In such case, the tank stresses due to anchor forces at the points of attachment must be properly investigated.

As stated in Chapter 3, procedures developed by Balendra et al. (1982) evaluate the stress resultants induced in the shell directly from the finite elements using the element stiffness matrix and the element coupling force matrix. The stress resultants that can be determined are N_s and $N_{s\theta}$, the axial and inplane shear stress resultants respectively, Q_s the transverse shear stress resultant, M_s the axial bending moment resultant, and N_θ the hoop stress resultant. The maxima of these stress resultants are obtained from the first impulsive mode and from the first convective mode. Hence, superposition of the contributions from these two modes gives the maximum values for design. The maximum stress resultants are calculated as follows

$$N_s = [n_{s1} A_{s1} + n_{s\omega_1} A_{s1}] \frac{m_l}{\pi R} \quad (4.2)$$

$$N_{\theta} = [n_{\theta_1} A_{c_1} + n_{\theta_{o_1}} A_{i_1}] \frac{m_l}{\pi R} \quad (4.3)$$

$$N_{s\theta} = [n_{s\theta_1} A_{c_1} + n_{s\theta_{o_1}} A_{i_1}] \frac{m_l}{\pi R} \quad (4.4)$$

$$Q_s = [q_{s_1} A_{c_1} + q_{s_{o_1}} A_{i_1}] \frac{m_l}{\pi R} \quad (4.5)$$

$$M_s = [m_{s_1} A_{c_1} + m_{s_{o_1}} A_{i_1}] \frac{m_l}{\pi R} \quad (4.6)$$

where

A_{c_1}, A_{i_1} = spectral accelerations corresponding to the fundamental natural frequencies of the convective and impulsive modes respectively

n, q, m = coefficients obtained from charts

m_l = total mass of liquid

R = radius of tank

The coefficients $n_{s_1}, n_{\theta_1}, n_{s\theta_1}, q_{s_1}$ and m_{s_1} are the maximum values obtained from the corresponding dimensionless coefficients $n(z/H), q(z/H)$ and $m(z/H)$ which define the heightwise variation of the corresponding maximum stress resultants due to the first convective mode. Similarly, $n_{s_{o_1}}, n_{\theta_{o_1}}, n_{s\theta_{o_1}}, q_{s_{o_1}}$ and $m_{s_{o_1}}$ are the maximum values obtained from the corresponding dimensionless coefficients $n(z/H), q(z/H)$ and $m(z/H)$ which define the heightwise variation of the corresponding maximum stress resultants due to the first impulsive mode. Details of the above equations are given in the reference (Balendra et al. 1982). It is important to note that the axial stress N_s and the hoop stress N_{θ} are the critical stresses that control the stability of the tank.

4.3 Unanchored Tanks

As stated previously, the dynamic behaviour of unanchored tanks is very different from that of anchored tanks. The possibility of partial uplift of the bottom plate may result in a highly nonlinear and complex response. Early studies on unanchored tanks were experimental studies which investigated the influence of uplift on the total response. Later on, different analytical models and techniques were developed to describe the uplift mechanism. Results from the analytical models were correlated with the experimental test results.

In this section, two commonly used approaches to model the uplift behaviour in the design of unanchored tanks are discussed. These are the Wozniak-Mitchell model, which is adopted by the API and AWWA standards, and the Clough model. Although these models have been found to be inadequate to predict the actual uplift behaviour by laboratory tests, analytical studies and observed tank damages, they are still widely used.

Uplift models are typically classified as either pseudo-static or dynamic models. Dynamic models describe the time history behaviour of unanchored tanks. However, pseudo-static models give important insights into the characteristics of the uplift mechanism which help in the development of the dynamic models. Some of the important analytical uplift models developed by researchers are discussed in this chapter.

4.3.1 Wozniak-Mitchell Model

Wozniak and Mitchell (1978) developed a quasi-static beam model of the bottom plate which forms the basis of the seismic design provisions for unanchored tanks in the API and AWWA standards. The model considers the bending behaviour of the bottom plate but neglects its membrane action. Furthermore, the restraining effect of the flexible tank wall on the bottom plate deformation is not considered. The assumptions of the uplift model by Wozniak and Mitchell, as shown in Figure 4.1 are:

1. The uplift displacement u is very small.
2. The uplifted length of the bottom plate L is very small compared to the radius of the tank. $D/2$.
3. Since $L \ll D/2$, the uplifted bottom plate is modelled as a series of beams with unit width and constant length L along the circumference with an angle of $2\pi - 2\beta$, where β is the half-angle contact arc of the shell with the foundation, as shown in Figure 4.2.
4. The beams rest on a rigid foundation and subjected to a uniform loading w , which is also the pressure, p , at the bottom due to the static height of the liquid, H , and lifted up by a force w_L in the tank shell, which is equivalent to the portion of the liquid that is uplifted and provides resistance to the applied overturning moment.
5. Two plastic hinges are assumed to form at the ultimate state, one at the tank shell and bottom plate junction, at C, and one at some distance inward from

the shell, at B.

6. The bottom plate membrane force, N , is assumed to be zero, whereas the shear force Q is neglected.
7. The value of w_L is the maximum weight of the liquid per unit length of shell circumference that can be utilized to resist overturning.

Applying the above assumptions and equating moments, about A and B, and shear at B, which is equal to zero, in the model shown in Figure 4.1, w_L can be derived as follows

$$M_p + \frac{wL^2}{2} - w_L L = 0 \quad (4.7)$$

$$2M_p + \frac{w(1-j)^2 L^2}{2} - w_L(1-j)L = 0 \quad (4.8)$$

$$w_L - (1-j)Lw = 0 \quad (4.9)$$

where

M_p = $F_{yb} \frac{t_b^2}{4}$, plastic moment capacity of the bottom plate
beam model

F_{yb} = yield strength of the bottom plate under the shell

t_b = thickness of bottom plate under the shell

Multiplying Equation 4.9 by L and adding the result to Equation 4.7 gives the following

$$M_p + \frac{wL^2}{2} - (1-j)L^2w = 0 \quad (4.10)$$

$$\frac{wL^2}{2} = \frac{M_p}{2J-1} \quad (4.11)$$

where

$$J = (1 - j)$$

Multiplying Equation 4.9 by JL and adding the result to Equation 4.8 gives the following

$$2M_p + \frac{wL^2}{2}J^2 - wJ^2L^2 = 0 \quad (4.12)$$

$$\frac{wL^2}{2} = \frac{2M_p}{J^2} \quad (4.13)$$

Solving Equations 4.11 and 4.13 simultaneously, results in

$$J = 2 \pm \sqrt{2} \quad (4.14)$$

Taking the negative sign in Equation 4.14 and substituting $1 - j$ for J gives $j = 0.414$.

Substituting the computed value of j into Equations 4.7 and 4.9 and solving for w_L gives the following relationships respectively

$$w_L = \frac{\frac{wL^2}{2} + M_p}{L} \quad (4.15)$$

$$w_L = wL(2 - \sqrt{2}) \quad (4.16)$$

Solving Equations 4.15 and 4.16 simultaneously and using the relation in Equation 4.13 gives the expression

$$L = 3.41 \sqrt{\frac{M_p}{w}} \quad (4.17)$$

Substituting $F_{yb}t_b^2/4$ for M_p yields

$$L = 1.707t_b \sqrt{\frac{F_{yb}}{w}} \quad (4.18)$$

Evaluating w_L in Equation 4.7 with proper substitutions of the quantities already determined and using $w = 62.4GH$ results in

$$w_L = 7.9t_b \sqrt{F_{yb}GH} \quad (4.19)$$

where w_L is in pounds per foot of shell circumference, t_b is in inches, F_{yb} is in pounds per square inch, H is the full liquid depth in feet and G is the specific gravity of the liquid. Equation 4.19 is the formula given in the API and AWWA standards for the maximum weight of the liquid that can be utilized in resisting the overturning moment at the tank base, M , in pounds per foot of shell circumference. By solving Equations 4.18 and 4.19, the following relationship between L and w_L is obtained

$$L = 1.707 \frac{w_L}{w} = 0.0274 \frac{w_L}{GH} \quad (4.20)$$

where L is the uplifted length of the bottom plate in feet. In the API standard, L has been limited to a value of 3.425% of the tank diameter D while in AWWA standard it is limited to about 3.5% of D . These limits for L set the value of w_L to a maximum of $1.25GHD$ and $1.28GHD$ respectively. These limitations are obtained in conjunction with the initial assumption that $L \ll R$, though in practice this is not always true. When w_L exceeds the limit, remedial measures must be taken such as anchoring the tank or changing its structural properties. When the bottom plate under the shell is thicker than the remainder, its total width measured radially inward from the shell must not be less than L . This is necessary to be consistent with the assumption in the derivation of a single uniform thickness for the entire bottom plate. Also, another reason is to provide for a more uniform distribution of the vertical axial stress from the tank wall to the foundation along the full width of the annular plate or the thicker part of the bottom plate.

In calculating the maximum axial compressive force, b , in the shell per unit length of shell circumference, additional assumptions are made.

1. During uplift, the tank rests on an arc along the shell circumference of an angle of 2β , and the contact area between the bottom plate and the foundation. The bottom plate-foundation contact area consists of the area of a sector with the angle 2β and radius $D/2$, and an area of another sector of the angle $(2\pi - 2\beta)$ and radius $(D/2 - L)$, as shown in Figure 4.2.
2. The distribution of the axial compressive force, b_x , in the shell per unit length of shell circumference is assumed to vary linearly from maximum at the point of the excitation axis to zero at the two ends of the shell-foundation contact arc. A linear distribution is assumed around the contact arc, as shown in Figure 4.3.
3. The maximum weight of uplifted liquid, w_L , is assumed to act uniformly along the uplifted part of the shell and acting downwards.
4. In the equilibrium condition, it is assumed that w_L is distributed along the whole circumference as with w_t , the weight of the shell per unit length of the shell circumference. To compensate for this, a reaction force, w_{L_s} , per unit length of shell circumference acting upward with a maximum value equal to w_L is assumed. It must be noted here that this reaction force is inaccurate, since it does not exactly compensate for the assumed distribution of w_L around the tank shell. This is because its assumed distribution is not uniform but similar to that of the shell axial compressive force, b_x , as shown by the dashed line in Figure 4.3.

5. The overturning moment M shown in Figure 4.3 is the base overturning moment that applies when the tank is not in uplift condition as in the same condition of an anchored tank.

Applying the above assumptions in the models shown in Figures 4.3 and 4.4, the following relationships can be derived

$$b_x = b \frac{\cos \phi - \cos \beta}{1 - \cos \beta} \quad (4.21)$$

$$b_T = 2 \int_0^\beta b_x \frac{D}{2} d\phi = bD \frac{\sin \beta - \beta \cos \beta}{1 - \cos \beta} \quad (4.22)$$

$$w_{LT} = w_L D \frac{\sin \beta - \beta \cos \beta}{1 - \cos \beta} \quad (4.23)$$

where

- b_x = axial compressive force in the shell per unit length
of shell circumference at an arbitrary point within
the contact arc of the shell with the foundation
- b_T = total axial compressive force in the shell
- w_{LT} = total reaction force acting upward in order to
compensate for the assumed uniform circumferential
distribution of w_L .
- β = half-angle contact arc of the shell with the foundation
- ϕ = angle of the point where b_x acts with respect to the
excitation axis and the center of the tank, as shown in
Figure 4.4

The equilibrium of vertical forces in Figure 4.3 results in the following

$$b_T + w_{LT} = w_t \pi D + w_L \pi D$$

$$\frac{b + w_L}{w_t + w_L} = \pi \frac{1 - \cos \beta}{\sin \beta - \beta \cos \beta} \quad (4.24)$$

By equating moments about the center of the tank, the moment arm $k \frac{D}{2}$ is determined as

$$\begin{aligned} k \frac{D}{2} b_T &= 2 \int_0^\beta b_r \left(\frac{D}{2} \right)^2 \cos \phi d\phi \\ k \frac{D}{2} &= \frac{D}{8} \frac{2\beta - \sin 2\beta}{\sin \beta - \beta \cos \beta} \end{aligned} \quad (4.25)$$

where

$$k \frac{D}{2} = \text{distance from the point where } b_T \text{ and } w_{LT} \text{ act to the center of the tank}$$

Equating moments about the point where b_T acts results in the following

$$\begin{aligned} M &= \pi D (w_t + w_L) k \frac{D}{2} \\ \frac{M}{D^2 (w_t + w_L)} &= \frac{\pi}{8} \frac{2\beta - \sin 2\beta}{\sin \beta - \beta \cos \beta} \end{aligned} \quad (4.26)$$

Equations 4.24 and 4.26 are the bases in determining the value of b in both the API and AWWA standards. By substituting different values of β in Equation 4.24, the corresponding values of b can be obtained. On the other hand, β depends on the moment parameter $M/[D^2(w_t + w_L)]$ in Equation 4.26. The moment parameter $M/[D^2(w_t + w_L)]$ represents the stability of the tank against overturning. It gives the limiting values for the validity of the relationship of Equations 4.24 and 4.26. The equations are valid for $M/[D^2(w_t + w_L)]$ between $\frac{\pi}{4}$ (uplift commences) and $\frac{\pi}{2}$

(shell becomes unstable). When uplift commences, β is equal to π and it follows that $M/[D^2(w_t + w_L)]$ gives a value of $\frac{\pi}{4}$ or 0.785. For the case when the shell becomes unstable, it is assumed that overturning occurs when the contact arc of the shell with the foundation becomes very small, such that the resultant compressive force b_T acts at a point on the shell bottom over the excitation axis. Thus, by taking the moment equilibrium about the point where b_T acts and taking note that the value of $k\frac{D}{2}$ is equal to $\frac{D}{2}$, $M/[D^2(w_t + w_L)]$ equals $\frac{\pi}{2}$.

The limiting values for $M/[D^2(w_t + w_L)]$ in API and AWWA standards are different. The AWWA standard sets its limit to 1.54 which corresponds to a half-angle contact arc of the tank shell with the foundation, β , equal to 25° . Furthermore, a formula is given for the parameter $(b + w_L)/(w_t + w_L)$. The formula is an approximation of the nonlinear relationship that exists between the parameters $M/[D^2(w_t + w_L)]$ and $(b + w_L)/(w_t + w_L)$ for different values of β . In the API standard, two uplift conditions are considered. The first condition is when uplift commences up to a value of $M/[D^2(w_t + w_L)]$ equal to 1.5, while the second condition is when $M/[D^2(w_t + w_L)]$ exceeds 1.5 but less than or equal to 1.57 when the tank becomes unstable. For values of $M/[D^2(w_t + w_L)]$ less than 1.5, the value of $(b + w_L)/(w_t + w_L)$ is slightly greater in the AWWA standard than in the API standard. Therefore for the same design overturning moment and tank conditions, the AWWA standard gives a slightly greater maximum axial compressive stress in the tank shell than the API standard. Hence, the former is more conservative in this case. When the tank becomes unstable by exceeding the above limits, a remedial measure is necessary. The remedial measure may be undertaken by anchoring the tank or changing its structural properties such

as increasing the thickness of the shell and the bottom plate or changing the liquid height to tank radius ratio.

4.3.2 Clough Model

The uplift analysis model developed by D.P. Clough (1977) has been adopted in the New Zealand recommendations (Priestley et al. 1986) and Adams (1992) design guidelines. The uplift model does not consider the deformation behaviour of the tank wall and the bottom plate. Instead, an iteration procedure is employed to achieve a global equilibrium of the liquid-tank system using an assumed linear distribution of the tank wall reaction forces and contact and uplift regions of the bottom plate. Furthermore, the bottom plate, which rests on a rigid foundation, is subjected to a pressure due to the static height of the liquid. As shown in Figure 4.5, the contact area of the bottom plate with the foundation during uplift is assumed to be circular with a radius r . It is assumed that the tank rests on an arc along the shell circumference with an angle of 2β . When uplift occurs, the portion of the liquid that is uplifted contributes to the resistance against the applied overturning moment, M , at the tank base. This results in an increase of the axial compressive stress in the shell that is in contact with the foundation. Referring to Figures 4.5 and 4.6, the following assumptions are made in the model:

1. The maximum overturning moment, M , at the tank base is calculated when the tank is not in uplift condition and is responding to the seismic action as an anchored tank.

2. The distribution of the axial compressive force, V_T , per unit length of shell circumference is linear along the shell-foundation contact arc with an angle 2β . It is maximum at the point on the excitation axis and decreases at both ends of the contact arc.
3. A relationship is established between the half-angle contact arc β , r , and tank radius R , which simplifies the bottom plate response during uplift.
4. The factors that contribute to the resistance of overturning are the resultant shell compressive force V_T , the reaction of the foundation in contact with the bottom plate W_f , the total weight of the liquid W_l , and the total weight of the tank W_t .

The weight of the tank may be ignored, since in most cases it is very small when compared to the weight of the liquid. In the following derivation, W_t is included for completeness as in the case of the New Zealand recommendations.

Applying Assumption 3 in Figure 4.5, the following relationships can be derived

$$\tan \beta = \frac{r}{R - r}$$

$$\beta = \tan^{-1}\left(\frac{e}{1 - e}\right) \quad (4.27)$$

$$W_l = \rho_l G \pi R^2 H \quad (4.28)$$

$$W_f = \rho_l G \pi r^2 H = W_l e^2 \quad (4.29)$$

where

$$e = \frac{r}{R}$$

$$\rho_l, G = \text{mass density and specific gravity of the liquid respectively}$$

$$H = \text{full liquid depth}$$

Then applying Assumption 2 in Figures 4.5 and 4.6, the axial compressive force V_x in the shell per unit length of shell circumference at an arbitrary point within the shell-foundation contact arc is determined by

$$V_x = V(1 - \frac{\phi}{\beta}) \quad (4.30)$$

Thus the resultant axial compressive force V_T is given as

$$V_T = 2 \int_0^\beta V_x R d\phi = V R \beta \quad (4.31)$$

where

$$\phi = \text{angle of the point where } V_x \text{ acts with respect to the excitation axis and the center of the tank, as shown in Figure 4.5}$$

$$V = \text{maximum value of } V_x \text{ at } \phi = 0$$

The ratio, k , of the moment arm of V_T from the center of the tank to the radius of the tank is expressed as follows

$$k R V_T = 2 \int_0^\beta V_x R^2 \cos \phi d\phi$$

$$k = \frac{2}{\beta^2} (1 - \cos \beta) \quad (4.32)$$

By taking equilibrium of forces and moments about the center of the tank in Figure 4.6, the following relationships can be obtained

$$V_T + W_f = W_l + W_i$$

$$V_T = W_l(1 - \epsilon^2) + W_i \quad (4.33)$$

$$M_R = W_f(R - r) + V_T k R$$

$$M_R = R W_l \left[k \left(1 + \frac{W_i}{W_l} \right) + (1 - k) \epsilon^2 - \epsilon^3 \right] \quad (4.34)$$

where

M_R = Resisting moment

Using the above equations, an iterative procedure is developed to arrive at a value of M_R equal to the applied base overturning moment M . The iterative procedure is as follows

1. Assume an initial value for ϵ .
2. Calculate β using Equation 4.27.
3. Calculate k using Equation 4.32.
4. Solve for M_R using Equation 4.34.
5. Repeat steps 1 to 4 until $M_R = M$.

In Adams (1992) design guidelines, it is proposed to limit the bottom plate lift-off to the width of the annular plate as given in the construction detail of the annular plate weld attachment. In this case the ratio ϵ is defined, thus M_R can be calculated

directly. However, if the calculated M_R is less than the applied base overturning moment M , no specific design rule is suggested. Upon achieving the condition in step 5, the total axial compressive force V_T is determined from Equation 4.33 using the correct value of e . The maximum axial compressive stress in the shell, σ_c , is then given as

$$\sigma_c = \frac{V_T}{R\beta t_s} \quad (4.35)$$

where t_s is the thickness of the tank wall. In the New Zealand recommendations, σ_c is determined as

$$\sigma_c = \frac{2.5cV_T}{R\beta t_s} \quad (4.36)$$

where c depends on the type of foundation.

$c = 1.0$ for rigid foundation

$c = 0.5$ for flexible foundation

The numerical coefficient of 2.5 is a calibration factor derived by comparing the results obtained by Clough (1977), which were found to be unconservative by Haroun (1983), with the results from experiments undertaken by Clough and Niwa (1979).

4.3.3 Other Approaches

More realistic and accurate analytical models and techniques than the Wozniak-Mitchell and Clough models have been developed to describe the behaviour of an unanchored tank during uplift. These models consider the relevant uplift resistance mechanisms developed in an uplifting tank, which are neglected in the Wozniak-Mitchell and Clough models. These uplift resistance mechanisms include the bottom

plate membrane action, the restraining effect of the flexible tank wall and the variation of dynamic pressure at the bottom. Uplift analytical models are usually categorized into pseudo-static and dynamic models. Dynamic models describe the time history response of unanchored tanks. On the other hand, pseudo-static models are designed to provide insight into uplift mechanism which can be used in the development of dynamic models. However, most of them are too complex and are thus not suitable for use as design tools. Some of the relevant uplift analytical models are briefly discussed in this section.

Pseudo-static Models

An improved equivalent beam uplift model was developed by Cambra (1982). An elemental strip of the bottom plate is assumed to be an elastic beam under transverse load due to hydrostatic pressure and a longitudinal membrane force. The contact region rests on a Winkler foundation with a constant stiffness. It is a significant improvement from Wozniak-Mitchell model because the model also considers the bottom plate membrane mechanism in addition to the bending mechanism in the uplifted region, and the bending moment at the junction of the uplift and contact regions of the bottom plate, which is otherwise zero for rigid foundations. However, some incompatibilities and assumptions are introduced in the analysis. For example, in the calculation of the membrane force, it is assumed that the membrane force carries by itself the total load on the the uplifted bottom plate strip beam which is considered as an extensible string with no longitudinal displacements allowed at its ends. The membrane force is then introduced as a given longitudinal force in the linear equation of an elastic beam. This is incompatible with the general equilibrium because

the total load, again, is now carried through the bending of the beam. In reality, a part of the load is carried by the membrane action and the remainder by the flexural action of the bottom plate. This is particularly important at region near the tank wall-bottom plate connection.

An empirical design approach based on experimental results of model tanks and observed performance of tanks in past major earthquakes was proposed by Manos (1986). The method is basically a methodology to determine the buckling stability of unanchored tanks. It involves determining the maximum impulsive acceleration that an uplifting tank can withstand. This is achieved by equating the earthquake induced overturning moment determined from a modified Housner model to the resisting overturning moment limit derived from experimental results. Correlating with some field observation data, the approach was found to be more successful in predicting observed tank damages than the API or AWWA standards. But since it is not based on rigorous analyses, the model can not accurately describe the complex nonlinear uplift mechanism.

The nonlinear behaviour of the bottom plate was analyzed using an energy approach by Haroun et al. (1987). The bottom plate is modelled as a circular plate with an uplifted, crescent-shaped region subjected to a static loading and vertical uplifting force along the outer edge of the plate. It was found that under a large static uplift force, the bottom plate membrane action increases the load carrying capacity of the plate considerably. Consequently, the maximum uplift displacement is significantly reduced. An important limitation of the model is that the restraining action of the tank wall is neglected due to the assumption that the the outer edge of the plate

model is free to rotate.

The uplift behaviour of the bottom plate was studied analytically by Auli et al. (1985). The bottom plate is modelled as a strip, with two springs representing the bending and extensional stiffness of the tank wall. The strip is subjected to the hydrostatic load, and its membrane action is considered. The model is based on simplified assumptions which include among others a strip model instead of a shell-plate model, the assumption of a rigid foundation and the consideration that uplift condition is due to the liquid pressure acting only on the bottom plate and not on the tank wall. To relax these restrictions, an axisymmetric uplift finite element model of the tank is presented by Auli et al. (1985). The axisymmetric uplift condition is achieved by uniformly uplifting the tank wall all around its circumference. Using these two models, effective springs which represent the retaining action of the bottom plate against uplifting of the lower edge of the tank shell are obtained. A comparison between results from these two models concludes that the stiffening effect due to membrane forces in the bottom plate caused by bending of the plate and by the wall pressure should be taken into account. Furthermore, the elastic-plastic material behaviour should be used instead of pure elastic material behaviour if higher uplift values occur. A finite element model of the tank, in which a cylindrical shell without a bottom plate but resting on nonlinear springs, of which their characteristics are taken from the above pseudo-static uplift models, was then developed. Implementation of this model in a dynamic response analysis of a full-scaled broad tank resulted in the observation that two maxima of the axial compressive shell stress lying symmetrically out of the earthquake direction developed. This might explain the occurrence of shell

buckling at wall positions lying not directly in the direction of the earthquake, as observed sometimes for very broad tanks.

Pseudo-static axisymmetric uplift analyses of the bottom plate on both rigid and elastic Winkler foundations were conducted by Lau and Zeng (1995). The bottom plate is modelled by one-dimensional beam and two-dimensional plate models. The analysis results show that aside from the deformation of the bottom plate, which include both the flexural and membrane actions, the interaction between the uplifted bottom and the tank shell contributes significantly to the uplift resistance. The results also indicate that there are significant differences in the uplift behaviour between rigid and elastic foundations particularly in the extent of uplift separation in the tank bottom. Based on the axisymmetric uplift analysis results, a simple approach using nonlinear springs is presented to model the seismic uplift behaviour of the bottom plate of an unanchored tank. This simplified uplift model can then be implemented for applications in dynamic earthquake response analyses.

The contact problem of the partially uplifted base plate and its interaction with the shell was analyzed by Peek (1986, 1988) and Peek and Jennings (1988) using two analysis methods. The first method is an approximate method in which the tank wall is idealized as supported from below by a circular foundation bed of Winkler nonlinear springs simulating the hold-down action of the base plate. The nonlinear springs represent the interaction between the restraining force at a point in the shell circumference and uplift at that point only by assuming that the variation in the circumferential direction of the uplift mechanism in a cylindrical tank is weak. The second method is a comprehensive method in which the two-dimensional nonlinear

contact problem of the base plate was analyzed using the finite difference energy method and a Fourier expansion of the displacement field in the circumferential direction. Both methods include nonlinearities due to contact, finite displacements and yield of steel. Analytical results from these methods (Peek 1988, Peek and Jennings 1988) show good agreements in some cases, but not so good in other cases when compared to experimental results. The discrepancies are attributed to the uncertainty in some of the experimental conditions and their effects, such as the initial stresses due to welding in the test tanks. Furthermore, it is found that the approximate method is acceptable for tall tanks but not always for broad, roofless tanks especially for calculating the vertical uplift.

An analytical procedure to predict the static tilt behaviour of a cylindrical tank that is free to uplift was proposed by Lau and Clough (1989, 1990). The static tilt condition is achieved by tilting the supporting platform of the tank model from its initial horizontal position resulting in the liquid filled tank being subjected to an effective static lateral acceleration component. The procedure is based on substructuring approach with each substructure element discretized by derived Ritz shape functions. The bottom plate, of which both the membrane and flexural mechanisms are considered, is modelled by large displacement theory and divided into the contact and the uplifted regions. The flexibility of the tank wall and stiffening effects of the top rim wind girder and bottom toe ring are also included. It was shown that the bottom plate membrane action has a significant contribution to the uplift resistance mechanism. Lau and Zeng (1991) developed a simplified uplift analysis of the above procedure by adopting a set of simplified displacement shape functions in the dis-

cretization process. This has the advantage over the more exact analysis due to the reduced amount of computation required, hence more suitable for nonlinear dynamic time history analysis.

Dynamic Model

To model the rocking motion of unanchored tanks, Ishikawa and Kobayashi (1988) developed a four-degree-of-freedom model, in which the tank shell and liquid are represented as a mass-spring system, and modelling the supporting soil as elastic springs, and the partially uplifting bottom plate as a bilinear rotational spring. Results from this model were found to be in good agreement when compared to the results obtained from static tilt test and shaking table test conducted on a small tank model. However, it is not verified if the model gives realistic estimates for the maximum responses for large full-scaled tanks.

Results of a three-dimensional finite element model of the liquid-tank system where the liquid is modelled by the effective added mass approach or by displacement-based fluid elements were presented by Barton and Parker (1987). The responses of both anchored and unanchored tanks were analyzed using time history analyses and modified response spectrum modal calculations. In the analysis of unanchored tanks, gap conditions between the floor and the tank are introduced to allow lift-off of the tank base. Geometric and material nonlinearities are not considered, thus effects of large deformations and yielding of the tank material are not accounted for. Results obtained using the model show the dependency of rocking frequency to the amplitude of base lift-off. But it is unrealistic that the results show that there are situations during an earthquake in which the whole tank has absolutely no contact with the

support.

Two simplified models for the seismic response determination of tall unanchored tanks that take into consideration the effects due to bottom plate uplift and foundation flexibility were developed by Natsiavas (1987, 1990). The first model employs the energy method in the geometrical discretization and derivation of the equations of motion of the liquid-tank system. The second model is a modified form of the Housner model (Figure 3.1) by including the effects from the rocking motion due to base uplift and ground flexibility. In both models, the complex nonlinear behaviour of base uplift is modelled by a nonlinear rotational spring placed between the tank base and the foundation. The characteristics of this spring were obtained from static tilt tests. An equivalent spring is used to simulate the effect of ground flexibility. It is noted here that in both models, the contribution from the tank shell flexibility is ignored in the formulation. Comparisons between the numerical results (Natsiavas and Babcock 1988) obtained using these models and experimental test results show that the hydrodynamic pressure of unanchored tanks is mainly caused by rigid body translation and by the rigid body rotation due to uplift. This observation is inconsistent with the accepted knowledge, as verified by different studies, such as Lau and Clough 1989, that the tank shell flexibility and deformation have very significant effect on the uplift behaviour of unanchored tanks.

To study the seismic response of unanchored tanks, a finite element model of the tank was developed by Yi and Natsiavas (1992). The tank shell is geometrically discretized by cylindrical finite elements while the base uplift effects is modelled using the previous analytical model (Natsiavas 1987), that is by nonlinear rotational spring.

The formulation and numerical solution of the problem become more complex than the previous model (Natsiavas 1987). In return, the model can be easily extended to model more accurately some aspects of the structural behaviour such as the effects of stiffening rings and roofs. In its present form, however, the effects due to liquid sloshing, stiffening rings and roof are excluded. It includes effects due to shell flexibility and soil-structure interaction. Numerical results for both anchored and unanchored tanks were obtained and found to be in good agreement with similar results of a previous investigation (Natsiavas 1987).

A modified response spectrum method for estimating the maximum dynamic response of unanchored tanks was proposed by Scharf (1989). It is adopted in the Austrian recommendations for earthquake resistant design of liquid storage tanks (Scharf et al. 1989a). In the method, the Housner model (Figure 3.1) is modified by replacing the contribution due to the coupled vibration of the tank shell and the liquid by the contribution due to the interaction vibration including rocking motion due to uplift. This is represented by a single-mass oscillator with nonlinear spring characteristics in the modified mechanical spring-mass model of the liquid-tank system. The parameters of the modified mechanical spring-mass model are obtained from results of the finite element analysis of the tank in which the tank wall, the roof and the retaining action of the bottom plate are modelled by shell, beam and truss elements respectively, as in the study by Auli et al. (1985). An equivalent period for the amplitude-dependent interaction vibration including uplift in combination with an iteration procedure and a design response spectrum is then determined to get an estimate of the maximum dynamic response.

With the aid of the simplified static uplift analysis procedure in Lau and Zeng (1991), a time history analysis procedure to evaluate the nonlinear dynamic uplift response of unanchored tanks with flexible bottom subjected to horizontal ground motion was developed by Zeng (1993). Detailed displacement and stress time histories at any point on the tank can be obtained. Further studies by Lau and Zeng (1992) based on this time history analysis procedure indicated that the base uplift in unanchored tanks may significantly affect the hydrodynamic loading, particularly the convective component. Moreover, in the evaluation of the convective component, the flexibility of the tank seems to be more important in unanchored tanks than in anchored tanks which is contrary to findings of previous studies.

In summary, the current status in the seismic response analysis of unanchored tanks does not provide for a refined yet efficient method of analysis that is feasible for design application. The present available analytical models are either oversimplified, such as those involving linear and nonlinear springs to capture the complex uplift behaviour, thus not reliable and accurate, or more detailed and complex such that the analysis becomes too complex and time-consuming, thus not suitable for design.

Figure 4.1 Wozniak-Mitchell beam model of uplifted bottom plate

Figure 4.2 Contact area of bottom plate with the foundation during uplift (Wozniak-Mitchell model)

Figure 4.3 Assumed load distribution around the tank shell circumference (Wozniak-Mitchell model)

Figure 4.4 Linear distribution of axial compressive stress in the tank shell (Wozniak-Mitchell model)

Figure 4.5 Clough model of contact area of bottom plate with the foundation during uplift

Figure 4.6 Forces acting on the uplifted tank
(Clough model)

Chapter 5

DESIGN CRITERIA

5.1 General

In the design of liquid storage tanks, design criteria are imposed to ensure that the reserve capacity of the tank system exceeds the demand from design actions. The objectives of the design criteria are to control the failure modes which may endanger the performance and safety of liquid storage tanks. The primary failure mode considered in the design of liquid storage tanks is instability failure related to buckling of the shell wall or overturning of the tank system. Consequently, the most important design criterion is the dynamic buckling capacity of the tank shell. Other common tank failures are damage to the roof due to liquid sloshing due to insufficient free-board, failure of bottom plate and foundation, sliding of the tank, and fracture of piping at the connection with the tank wall.

Numerous experiments, static and dynamic tests, have been performed to study the dynamic buckling behaviour of liquid storage tanks. The results have shown

that current methods do not adequately estimate the buckling capacity of the tank during strong ground motions. In the following sections, the commonly used methods in practice to predict the limit or critical buckling strength of the tank shell are discussed.

5.2 Dynamic Buckling Capacity

5.2.1 API Standard

The buckling design capacity of the tank shell in the API standard is based on the critical buckling stress value of $0.125Et_s/R$ proposed by Miller (1977), where E is the elastic modulus of the tank shell, t_s is the shell thickness, and R is the tank radius. This critical buckling stress value is obtained from experimental data for thin wall cylinders under uniform compression neglecting the effect of internal pressure. By introducing a safety factor of 1.5 and substituting E of steel equal to 29×10^6 psi in this critical buckling stress value, the following expression is obtained

$$\sigma_a = 400000 \frac{t_s}{D} \quad (5.1)$$

where

- σ_a = maximum allowable axial compressive stress in the shell,
neglecting internal pressure, in *psi*
- t_s = shell thickness, in inches
- D = tank diameter, in feet

Buckling test results show that there is an increase in the critical buckling stress as the internal pressure increases. To include the effect of internal pressure on the buckling strength of cylindrical tank shells, the results from theoretical studies and experimental tests by Lo et al. (1950) for thin wall cylinders under uniform compression and static pressure are adopted to modify the buckling strength expression given in Equation 5.1. The stabilizing effect of the internal pressure is characterized by a proportional increase in the critical buckling stress with the square root of the pressure parameter $GH \frac{D^2}{t_s^3}$, where G is the specific gravity of the liquid and H is the liquid height. Furthermore, it was observed from the above experimental test results that the critical buckling stress increases to $10^6 t_s/D$ psi as the value of $GH \frac{D^2}{t_s^3}$ increases to 1×10^6 and greater. Incorporating these observations into Equation 5.1 results in the following equations for the maximum allowable axial compressive stress in the shell, F_a , which includes the effect of internal pressure;

For $GH \frac{D^2}{t_s^3} < 1 \times 10^6$:

$$F_a = \frac{10^6}{2.5} \frac{t_s}{D} + 600\sqrt{GH} \quad (5.2)$$

For $GH \frac{D^2}{t_s^3} \geq 1 \times 10^6$:

$$F_a = 10^6 \frac{t_s}{D} \quad (5.3)$$

where F_a is in psi, t_s is in inches and D and H are in feet. The above equations are applicable up to the limits of the theoretical studies and experimental tests by Lo et al. (1950). They are normally applicable to very small tanks only, where the shell thickness is established by minimum values rather than by hoop stress. Consequently, for tanks with a relatively thick shell wall in proportion to its diameter, Equations 5.2

and 5.3 are not applicable. To cover and maintain an adequate safety factor for all shell thickness to diameter ratios, an upper limit of $F_a \leq 0.5F_y$, where F_y is the yield stress of shell plate, is recommended.

By taking the ratio of Equations 5.2 and 5.3 to the classical buckling stress for perfect cylinders under uniform compression, $F_{cl} = 0.6Et/R$, it can be shown that Equations 5.2 and 5.3 are expressions of F_{cl} but with a safety factor or knock-down factor between 2.9 to 7.3. This safety factor is assumed to include all the factors that contribute to the tank shell buckling mechanism, such as the initial geometric imperfections in the tank wall and the non-uniform distribution of axial compressive stress in the tank shell during dynamic response in earthquakes.

5.2.2 AWWA Standard

The AWWA Standard has adopted a similar approach in the design of liquid storage tanks against seismic buckling failure as the API standard. The maximum allowable axial compressive stress, F_a , in *psi*, in the AWWA Standard is given by

$$F_a = 1.33(\sigma_a + \frac{\Delta\sigma_{cr}}{2}) \quad (5.4)$$

The term σ_a in the above equation is the allowable compressive stress in the shell under uniform compression neglecting the stabilizing effect of the internal pressure. The increase in critical buckling stress due to internal pressure is represented in Equation 5.4 by the term $\Delta\sigma_{cr}$ with a safety factor of 2. The terms σ_a and $\Delta\sigma_{cr}$ are expressed as follows

$$\sigma_a = 10^6 \frac{t_s}{R} (2 - \frac{200}{3} \frac{t_s}{R}) \quad (5.5)$$

$$\Delta\sigma_{cr} = \frac{\Delta C_c E t_s}{R} \quad (5.6)$$

where

σ_a = allowable stress due to uniform compression neglecting
internal pressure, in *psi*

$\Delta\sigma_{cr}$ = increase in critical buckling stress due to
internal pressure, in *psi*

ΔC_c = pressure stabilizing buckling coefficient

E = modulus of elasticity of the tank shell, in *psi*

t_s = thickness of tank shell, in inches

R = radius of tank, in inches

It is noted here that the negative term inside the parenthesis in Equation 5.5 can be viewed as the reduction effect of tank shell imperfections on the buckling capacity of the tank wall. It may be recalled that in Equations 5.2 and 5.3 in the API standard, the effect of tank shell imperfections on the buckling capacity of the tank wall is not included directly in their formulations but only through safety factors. In Equation 5.5, as the level of tank shell imperfection decreases, that is as the shell thickness to tank radius ratio, t_s/R , decreases, the buckling capacity of the tank wall increases, whereas as the tank shell imperfection increases, that is t_s/R increases, the buckling capacity of the tank wall decreases. The pressure stabilizing buckling coefficient ΔC_c in Equation 5.6 is determined from Figure 5.1. It is noted from the figure that ΔC_c , which reflects the increase in the critical buckling stress due to internal pressure, is dependent not only on the internal pressure itself but also on the shell thickness to

tank radius ratio. It increases as the internal pressure increases or as the tank wall imperfection level decreases, that is as t_s/R decreases. The AWWA standard permits a one-third increase in the basic allowable shell stress for seismic loadings, as given in Equation 5.4. Similarly as in Equations 5.2 and 5.3 in the API standard, Equation 5.4 can be viewed as an expression of the classical buckling stress for perfect cylinders under uniform compression but with a safety factor between 2.4 to 6.8.

In comparing the buckling design criteria in the API standard with that in the AWWA standard, it is noted that:

1. In the AWWA standard, the stabilizing effect of internal pressure on the buckling capacity of the tank wall depends on the shell thickness to radius ratio, t_s/R . This is not the case in the API standard in which the effect of internal pressure on the critical buckling stress is constant for all t_s/R ratios.
2. The critical buckling stress considered by both the API and AWWA standards is based on uniform axial compression of the shell. However, the earthquake induced stresses in the tank shell during dynamic response are unsymmetric in nature, which gives rise to variation of the axial stress around the shell circumference.
3. The API and AWWA standards do not distinguish between the elastic buckling and elastic-plastic buckling failure modes of the tank wall. As it will be discussed later, these two buckling modes are caused by different buckling mechanisms, thus they should be treated separately. Also, it will be shown later in a comparison study that although the provisions for the allowable axial compressive

stress in the tank shell in the API and AWWA standards are generally conservative for elastic buckling, they may overestimate the elastic-plastic buckling strength of the tank shell, especially for high internal pressure.

4. The critical buckling stress adopted in the design standards is based on elastic thin shell theory. This may be unrealistic for large excitations. Under a severe earthquake, the tank may experience stresses beyond the elastic range, hence post-buckling behaviour may be an important factor.

5.2.3 Austrian Recommendations

There are two typical buckling failure patterns of the tank wall that are considered in the Austrian recommendations: the elastic buckling (shell crippling) and elastic-plastic buckling (elephant's-foot-bulge). At present, however, the difference between the mechanisms that cause these two buckling failure modes are not yet fully understood. In fact, as discussed earlier, the buckling provisions in the API and AWWA standards do not distinguish between elastic buckling and elastic-plastic buckling failure modes of the tank wall. According to the Austrian recommendations, however, the type of buckling failure mode that may occur depends on the combination and relative levels of the axial compressive stress and hoop stress in the tank shell. On the other hand, the magnitude of these stresses, particularly the hoop stress, depends on the different combinations of the pressure components due to the static and dynamic pressures generated by the horizontal and vertical components of the ground motion.

The Austrian recommendations consider three possible combinations of the max-

imum pressure components on the tank wall, as shown in Figure 5.2, that have to be considered with respect to different buckling failure modes of the tank wall. The resultant pressure in Case I is given by the combination $p = p_{stat} + p_H + p_V$, where p_{stat} is the hydrostatic pressure, and p_H and p_V are the dynamically activated pressures generated, respectively, by the horizontal and vertical components of the ground motions. The pressure distribution of Case I results in high circumferential or hoop tensile stress at the tank wall bottom course. Combining this with the high axial compressive stress due to the overturning moment may lead to elastic-plastic buckling of the tank shell. Niwa and Clough (1982) suggested that elastic-plastic buckling mechanism results from the combined action of axial compression exceeding the critical stress and hoop stress close to the yield limit. In Case II, the resultant pressure is given by the combination $p = p_{stat} + p_H - p_V$. The total internal pressure on the tank wall is reduced by the dynamic pressure generated by the vertical component of the ground motion which results in a considerably lower hoop stress at the tank wall bottom course than in Case I. This pressure distribution is believed to be the most critical one with respect to elastic buckling of the tank shell. This type of buckling failure mode is mainly due to the high axial compressive stress from the overturning moment and that the effect of hoop stress is not that significant as compared to that in the elastic-plastic buckling failure mode. In Case III, the resultant pressure is given by the combination $p = p_{stat} - p_H - p_V$. This pressure distribution may lead to regions of low pressures near the top of the tank where the wall is rather thin. Buckling at the top of the tank wall shell due to external pressure is a potential mode of failure in this case (Natsiavas and Babcock 1987).

For elastic-plastic buckling, an empirical relation for the critical axial load is derived by Rotter and Seide (1987)

$$N_{aep} = 0.605 \frac{Et_s^2}{R} \left(1 - \left[\frac{pR}{t_s F_{ys}}\right]^2\right) \left(1 - \frac{1}{1.12 + s^{1.15}}\right) \left(\frac{\frac{F_{ys}}{250} + s}{1 + s}\right) \quad (5.7)$$

where

- N_{aep} = critical axial membrane force for elastic-plastic buckling,
in N/mm
- t_s = shell wall thickness, in mm
- R = tank radius, in mm
- s = $\frac{R/t_s}{400}$
- F_{ys} = yield stress of the shell plate, in MPa
- p = internal pressure, in MPa
- E = modulus of elasticity of the tank wall, in MPa

In the above equation, the internal pressure p is the pressure at the bottom shell course according to the pressure combination of Case I stated above.

For the elastic buckling capacity of the tank shell, it is suggested that a good estimate would be to use the classical buckling stress for a perfectly thin cylindrical shell under uniform compression with a proper knockdown factor. This knockdown factor must take into account the shell imperfection and the stabilizing effect of the internal pressure. However, there is no recommendation given on how to determine the knockdown factor.

5.2.4 New Zealand Recommendations

The failure modes considered in the New Zealand recommendations are the elastic buckling (shell crippling), elastic-plastic buckling (elephant's-foot-bulge) of the tank shell, tank shell buckling by inplane membrane shear, and yielding of the material under the combined actions of bending moment and/or hoop tension. The first two failure modes are the most common causes of tank damage. The failure mode due to yielding of material represents an upper limit to the allowable membrane stress determined from the buckling failure modes. The critical type of shell wall buckling mode depends on the critical mode of pressure combination, as shown in Figure 5.2. The following provisions for allowable stresses are based on an extensive review of experimental results and theories of cylindrical shell buckling by Rotter (1985a).

Elastic Buckling

The axial or vertical compressive stress that causes elastic buckling in the shell is a function of the internal pressure, the circumferential variation of axial stress, and the relative amplitude of imperfections in the tank wall. Initial wall imperfections decrease the buckling capacity of the tank shell from the classical "perfect shell" limit. It is known that while internal pressure decreases the effective wall imperfection amplitude, the circumferential variation of the axial stress in the tank shell reduces the probability that the maximum axial stress and the maximum imperfection occur simultaneously at the same location. Hence, the combination of these two factors increases the buckling capacity of the tank system. In considering the effect of the internal pressure on the buckling stability of the tank shell, the contribution from the

vertical excitation which may reduce the total internal pressure on the tank wall, as in the case in the pressure distribution of Case II in Figure 5.2, must be included.

The following equations giving the allowable elastic buckling stress are adopted by the New Zealand recommendations on the seismic design of liquid storage tanks,

$$\frac{F_{ae}}{F_{cl}} \leq 0.19 + 0.81 \frac{F_p}{F_{cl}} \quad (5.8)$$

$$F_p = F_{cl} \sqrt{1 - (1 - \frac{\bar{p}}{5})^2 (1 - (\frac{F_o}{F_{cl}})^2)} \leq F_{cl} \quad (5.9)$$

$$F_{cl} = 0.6E \frac{t_s}{R} \quad (5.10)$$

$$\bar{p} = \frac{pR}{t_s F_{cl}} \leq 5 \quad (5.11)$$

$$\lambda^2 \leq 2; F_o = F_{ys} (1 - \frac{\lambda^2}{4}) \quad (5.12)$$

$$\lambda^2 \geq 2; F_o = \bar{\sigma} F_{cl} \quad (5.13)$$

$$\lambda^2 = \frac{F_{ys}}{\bar{\sigma} F_{cl}} \quad (5.14)$$

$$\bar{\sigma} = 1 - \psi(\frac{\delta}{t_s}) [(1 + \frac{2}{\psi(\frac{\delta}{t_s})})^{\frac{1}{2}} - 1] \quad (5.15)$$

where

F_{ae} = allowable elastic buckling stress

F_{cl} = classical buckling stress for perfect cylindrical shells
under uniform compression

F_p = buckling stress for tanks under uniform compression
and internal pressure considering wall imperfection

F_o = buckling stress for unpressurized tank under uniform
compression considering wall imperfection

$\bar{p}$ = nominal hoop stress ratio

p	= internal pressure at point of consideration
$\frac{\delta}{t_s}$	= ratio of maximum wall imperfection amplitude to wall thickness
ψ	= 1.24
t_s	= shell wall thickness
R	= tank radius
F_y	= yield strength of shell plate
E	= modulus of elasticity of the tank shell

The above equations have consistent units of length and mass. Equation 5.8 is obtained by modifying the buckling formula given in the European Convention for Constructional Steelwork (ECCS 1983). It represents the critical buckling stress due to axial stress being induced by bending rather than axial loading. This critical buckling stress is taken to be a weighted mean of the classical buckling stress, F_{cl} , as given by Equation 5.10, and the buckling stress due to uniform compression and internal pressure, F_p , as given by Equation 5.9, including the effect of tank wall imperfection. Equations 5.12–5.15 account for the effects due to initial wall imperfections in an unpressurized tank subjected to uniform compression. Equation 5.12 is applicable to relatively thick-walled tanks with critical buckling stress higher than $0.5F_y$, where buckling is essentially inelastic. Equation 5.13 is applicable to relatively thin-walled tanks.

The proposed method for measuring the imperfection level, δ/t_s , is the method recommended in the ECCS. Alternatively, the following formula (Rotter 1985a) may

be used.

$$\frac{\delta}{t_s} = \frac{0.06}{a} \sqrt{\frac{R}{t_s}} \quad (5.16)$$

where

$a = 1.0$ for normal construction

$= 1.5$ for quality construction

$= 2.5$ for very high quality construction

Elastic-Plastic Buckling

This type of shell buckling results from a combination of high hoop stress due to internal pressure and axial compressive stress due to overturning moment. Local bending stresses due to the restraint of the tank bottom also contribute to this effect. The internal pressure is the sum of the hydrostatic pressure and the hydrodynamic pressures due to the horizontal and vertical components of the ground motion, as shown in the pressure combination of Case I in Figure 5.2. The following expression developed by Rotter (1985b), which is similar to Equation 5.7 of the Austrian recommendations, estimates the critical stress, F_{aep} , to be

$$F_{aep} \leq F_{cl} \left[1 - \left(\frac{pR}{t_s F_{ys}} \right)^2 \right] \left(1 - \frac{1}{1.12 + s^{1.5}} \right) \left[\frac{s + \frac{F_{ys}}{250}}{s + 1} \right] \quad (5.17)$$

where

F_{aep} = critical stress for elastic-plastic buckling, in MPa

F_{cl} = classical buckling stress, in MPa

t_s = shell wall thickness, in mm

R = tank radius, in mm

$s = \frac{R/t_s}{400}$

p = internal pressure, in MPa

F_y = yield stress of the shell plate, in MPa

In the above equation, the internal pressure reduces the critical stress for elastic-plastic buckling. This is opposite to the effect of internal pressure on the elastic buckling stress, which is to increase the critical stress. This is because in elastic-plastic buckling, the internal pressure is very large such that it results to a high hoop stress that leads to yielding of the tank wall material. In such case, the effective stiffness of the tank wall is reduced, thereby reducing the buckling capacity of the tank wall. On the other hand, in the case of elastic buckling, the internal pressure is not large enough for yielding of the tank wall to occur. Instead, the internal pressure decreases the tank wall imperfections thus increasing the buckling capacity of the tank wall.

5.3 Resistance to Overturning

The decision whether anchorage is required depends mainly on the overturning resistance capacity of the tank. Although prevention of overturning is the main purpose of anchorage, it can also be used as a means to reduce the stresses and displacements experienced by the tank shell. Experiences have shown that very few tanks had actually failed by overturning. This is because most typical thin-walled metal tanks will fail first by buckling under high overturning moment before they overturn. Failure in liquid storage tanks due to overturning moment is closely related to the buckling stability of the tank shell, the behaviour of which is not well understood.

In the API and AWWA standards, the prevention of overturning is accounted for in the limitations imposed on the approximate relationship between maximum shell axial compressive force, b , and design base overturning moment, M , as discussed in Chapter 4. These limitations depend on the value of the half-angle contact arc β of the shell with the foundation. The value of β is equal to π when uplift commences and reduces to zero when the tank becomes unstable or about to overturn. In the assumed overturning mechanism, the latter condition occurs when the half-angle contact arc is so small that the resultant shell compressive force is almost acting at a point on the shell bottom along the excitation axis.

In both standards, the overturning stability is reflected by the moment parameter $\frac{M}{D^2}(w_t + w_L)$. The limits of this parameter range from a value of $\frac{\pi}{4}$ or $0.785(\beta = \pi)$ when uplift commences to $\frac{\pi}{2}$ or $1.57(\beta = 0)$ when the tank becomes unstable. The AWWA standard sets an upper limit of 1.54. If the moment parameter exceeds the upper limit, the tank is unstable. Remedial measures are required, such as modifying the tank geometrical or structural properties, or by anchoring the tank.

In the New Zealand recommendations, the tank must be anchored if the following equations are not satisfied

$$\frac{H}{R} > \frac{2}{\eta \gamma A_h(T) A_p} \quad (5.18)$$

$$R < (\eta \gamma A_p)^2 \frac{H}{R} \quad (5.19)$$

where

η = peak horizontal acceleration coefficient for annual
probability of exceedance of 0.01

γ = geographic coefficient representing regional seismicity

A_p = probability factor

$A_h(T)$ = normalized horizontal acceleration response for period T

H, R = liquid height and tank radius respectively

Details of the coefficients η , γ , A_p and $A_h(T)$ are given in the reference (Priestley et al. 1986). The above equations are based on the study results by Ishiyama (1984) concerning overturning of rigid bodies it was shown that peak ground velocity is the most important factor when it comes to overturning since it supplies the necessary input energy to overturn a body. The above equations normally apply to small tanks only, since except for small tanks there is a little chance of tanks overturning in an earthquake. In connection with this, it was further assumed that the base can uplift across the entire diameter without material failure.

- ΔC_c = pressure stabilizing buckling coefficient
 P = hydrostatic pressure at point under consideration, in *psi*
 E = Young's modulus of elasticity, in *psi*
 R = radius of tank, in inches
 t_s = thickness of the shell plate, in inches

Figure 5.1 Increase in axial-compressive buckling-stress coefficient of cylinders due to internal pressure (AWWA Standard, 1984)

Figure 5.2 Schematic presentation of different possible combinations of the maximum pressure components

Chapter 6

COMPARISON AND PARAMETRIC STUDIES

6.1 General

In a comparison study of the different design procedures specified in the various standards and recommendations for thin-walled metal liquid storage tanks, tanks of different dimensions are designed in accordance to the design provisions as stipulated in the different procedures. The design loadings and design actions are determined, and provisions of design criteria are checked for the designed tanks. For the design loadings, lateral base shear and overturning moments at the base of the tank and at the foundation level are calculated. The maximum axial compressive stress in the tank wall and the buckling capacity of the shell wall are determined according to the design provisions.

6.2 Design Load Calculations

6.2.1 Description of Tank and Ground Motions Considered

The liquid storage tank considered is a ground supported, flat bottom, open top, cylindrical steel tank, as shown in Figure 6.1. It is fully filled with water with a weight of $3500kN$. The tank bottom plate is $9.5mm$ thick and rests on a rigid foundation. The shell thickness to tank radius ratio is 0.001. In the parametric study, the liquid height to tank radius ratio is varied from 0.6 to 3.0, while the other tank parameters such as the volume of the liquid content, the shell thickness to tank radius ratio, and the bottom plate thickness are kept constant. The roof condition, whether the tank has a fixed or flexible top, and the presence of a wind girder can significantly affect the response of a tank during an earthquake. However, these effects are not considered in the present parametric study.

The selected input ground motions for the present study are the 1994 Northridge earthquake (Newhall station, N-S component), the 1940 El Centro earthquake (Imperial Valley record, N-S component), and the 1988 Saguenay earthquake (Chicoutimi-Nord station, W-E component), as shown in Figure 6.2. These selected ground motions are typical earthquakes representative of east and west coast seismic events, and motions of different peak acceleration to peak velocity ratios. The response spectra for the above selected ground motions are shown in Figures 6.3(a)–(c) with a damping ratio of 2% critical. The amount of damping exhibited in the response of the liquid-tank system depends on the response mode being considered, i.e. impulsive or convective mode. The general amount of 2% damping is believed to be appropriate,

considering the uncertainties involved in the response. It is important to note also that the response spectrum method is limited only to the elastic range, which may not be the case for tanks subjected to strong ground motions. In the present study, the tank response is assumed to remain in the elastic range as considered in most design procedures.

6.2.2 Discussion of Results

Figures 6.4–6.6 show the calculated design loadings of resultant lateral force and the overturning moments at the tank base and at the foundation, as obtained from the different design methods. The calculated results show that for the El Centro and Northridge earthquakes, the results obtained from the flexible tank procedures, i.e. procedures that consider tank wall flexibility (curves (b),(c),(d),(e)), are generally greater than those obtained from the rigid tank procedures, i.e. procedures which assume the tanks are rigid (curves (a),(f),(g),(h)). In fact, the difference between the design loadings as determined by the flexible tank procedures and rigid tank procedures is very significant for many cases of liquid height to tank radius ratio, H/R , that are considered here, especially for the El Centro earthquake. For example, the calculated lateral base shears from the Austrian recommendations (curve (e)) are greater than those from the anchored tank case of the AWWA standard (curve (g)) by as much as 6 times at H/R equal to 0.6 for the Northridge earthquake, as shown in Figure 6.4(a), and 3.5 times at H/R equal to 1.5 for the El Centro earthquake, as shown in Figure 6.4(b). It is noted here that in the present study, the predicted impulsive acceleration as determined by the API and AWWA standards, which is

equal to $0.36g$ and constant for all tank dimensions, is at the same level as the peak ground acceleration of the El Centro earthquake. However, the design loadings as determined by the API and AWWA standards for the El Centro earthquake, as shown in Figures 6.4(b)–6.6(b), are significantly less than those determined by the flexible tank procedures for all the H/R ratios that are considered. This is because of the assumption in rigid tank procedures that the impulsive acceleration is equal to the peak ground acceleration, as in Epstein procedure (curve (a)), or equal to a seismic coefficient, as in API and AWWA standards (curves (f),(g),(h)). However, due to the flexibility of the tank wall, the impulsive acceleration determined from the flexible tank procedures for the El Centro earthquake is substantially greater than the peak ground acceleration, hence, resulting in greater seismic loads than those obtained from rigid tank procedures.

The trend of the calculated results for the Saguenay earthquake, as shown in Figures 6.4(c)–6.6(c), is generally the opposite of those for the Northridge and El Centro earthquakes. The design loadings as determined by the rigid tank procedures, except the Epstein procedure (curve (a)), are consistently greater than those determined by the flexible tank procedures. For example, in Figure 6.4(c), the lateral base shear as determined from the API and AWWA standards (curves (f),(g),(h)) is 6 times that obtained from the Veletsos procedure (curve (b)) at the liquid height to tank radius ratio equal to 3.0. These results and those from the Northridge and El Centro earthquakes show the inadequacy of using the rigid tank wall assumption in the determination of the design loads, and that the flexibility of the tank wall significantly modifies the tank response.

The results also show that the design loadings generally increase with liquid height to tank radius ratio, H/R . This is not surprising, since an increase in the H/R ratio as the tank becomes taller, there is an increase in the proportion of the liquid mass participating in the impulsive mode, while the sloshing liquid mass decreases. Thus, the dominant response in a tall tank is the impulsive mode, whereas in a broad tank the dominant response is the convective mode. In connection with this, the contribution of the impulsive mode in the total response is generally greater than that from the convective mode, especially in tall tanks. This may be as the tank becomes taller, the coupled vibration period, which usually falls in the short period range, increases from 0.1 to 0.5 *seconds* in the present case, whereas the sloshing period, which is characterized by long period oscillations, decreases from 7 to 5 *seconds* in the present case. When these changes are applied to the acceleration response spectra, as shown in Figure 6.3, it results in the impulsive acceleration being much greater than the sloshing acceleration in the tank response.

For procedures that assume the tanks are rigid, the design loadings increase as the liquid height to tank radius ratio increases. This is because these procedures use an impulsive acceleration equal to either the input peak ground acceleration, as in the Epstein procedure (curve (a)), or a seismic coefficient which is the same for all input ground motions and tank dimensions, as in API and AWWA standards, in the determination of the impulsive response. By taking note of the above consideration and the fact that the impulsive response becomes the dominant response as the tank becomes taller as H/R increases, the main parameters that can significantly influence the magnitude of the total response in a tall tank are the impulsive mass and its

height. With this consideration, it follows that for rigid tank procedures the lateral base shear and overturning moments, at the tank base and at the foundation level, will consistently increase as liquid height to tank radius ratio increases.

For the case of procedures that consider tank wall flexibility (curves (b), (c), (d), (e)), the trend in the calculated design loadings is not obvious. For example, in Figure 6.4(a), the lateral base shears from curves (b), (c) and (d) increase from H/R of 0.6 to 2.0 and then decrease. This phenomenon is mainly due to the spectral accelerations obtained from the response spectrum of the input ground motion, as shown in Figure 6.3(a), which is characterized by a significant rise or drop in the spectral values as the period changes slightly. This variation in the spectral values over small period ranges, however, is typical characteristic of a response spectrum due to earthquakes. By taking note of the above consideration, as the H/R ratio increases from 2.0 to 2.5 in Figure 6.4(a) the impulsive period also increases which when applied to the response spectrum shown in Figure 6.3(a) results in an impulsive acceleration for the tank with H/R of 2.5 that is considerably smaller than that for the tank with H/R of 2.0. This decrease in the impulsive acceleration as H/R increases from 2.0 to 2.5 more than offset the increase in the impulsive mass and height. As a result, the combined effect of the impulsive mass, height and acceleration results in a significant reduction in the total tank response from H/R of 2.0 to 2.5, as shown in Figure 6.4(a).

The increase in the overturning moments, at the tank base and at the foundation level, as H/R increases, as shown in Figure 6.5 and Figure 6.6 respectively, are more noticeable than that in the lateral base shear shown in Figure 6.4. This is due to the

increase in the effective heights of the impulsive and convective masses as the tank becomes taller, hence increasing the overturning moments.

In general, the design loadings as determined by the Veletsos procedure (curve (b)), the Haroun-Housner procedure (curve (c)) and the procedure by Balendra et al. (curve (d)) are in good agreement. However, the results as determined by the Austrian recommendations (curve (e)) show some differences with the results determined by the above three procedures. Two factors contribute to these differences. The first is the convective mode participation factor included in the Austrian recommendations which results in a convective response that is 1.5 to 2.5 times greater than those determined by the Veletsos procedure, Haroun-Housner procedure and procedure by Balendra et al. The second factor is the significant differences in the results of the calculated coupled vibration period, especially for broad tanks, as determined by the Austrian recommendations and the above three procedures. When these coupled vibration periods are applied to the response spectra shown in Figure 6.3, considerable differences emerge in the spectral values and thus the impulsive responses from these procedures.

Figures 6.4-6.6 show that the calculated design loadings from the API standard (curve (f)) are always greater than those from the AWWA standard (curves (g),(h)). However, in the case of an unanchored tank, the results from the AWWA standard (curve (h)) are nearly the same as those from the API standard which does not distinguish between the design loads for anchored and unanchored tanks. These results are due to the differences in the seismic coefficients employed in these standards. These seismic coefficients, which are independent of the input ground motions in the present

study, reflect the ground effect and thus represent the impulsive and convective accelerations. As shown in Table 3.4, the API standard gives greater impulsive and convective accelerations, and hence seismic loads, than the AWWA standard for both anchored and unanchored tank cases.

The overturning moments calculated at the foundation level are shown in Figure 6.6. Although the foundation moment is not needed for the tank design, it is required for the design of the foundation. Similar observations can be noted for the foundation moment as for the lateral base shear and base overturning moment.

To summarize, depending on the characteristics and magnitude of the input ground motion, rigid tank procedures, especially the API and AWWA standards, in comparison with flexible tank procedures, may overestimate, as in the case of the Saguenay earthquake, or underestimate, as in the Northridge and El Centro earthquakes, the seismic design loads on the tank. This is due to the modification to the impulsive acceleration by the flexibility of the tank wall, resulting in a significantly greater or lower response than that obtained using the peak ground acceleration, as assumed in rigid tank procedures.

6.3 Design Action Calculations

6.3.1 Description of Tanks Considered

There are three tanks considered in this section. The first tank is a broad aluminum model tank (BT) tested by Manos and Clough (1982) using shaking table. The tank is 5 feet (1.524m) filled with water as shown in Figure 6.7(a). During the tests, the input

ground motions were scaled to peak acceleration of $0.5g$ and in time according to the similitude requirements of the model tank. The input ground motions used in the tests (column 2 in Table 6.1) are the N-S component of the 1940 El Centro earthquake (El), the Pacoima Dam record (S74W component) of the 1971 San Fernando earthquake (Pa), and the N65E component of the 1966 Parkfield earthquake (Pd). Three cases of the tank were tested. These are BT-1 for the free base or unanchored tank with rigid foundation, BT-2 for the fixed base or anchored tank with rigid foundation, and BT-3 for the free base with flexible foundation. In all the test cases, the tank has an open top. No significant structural damage was observed during the tests.

The second tank is a tall aluminum model tank (TT) tested by Niwa (1978) using the shaking table. The tank is 13 feet ($3.962m$) filled with water as shown in Figure 6.7(b). Two cases of the tank were tested. These are TT-1 for the unanchored tank case and TT-2 for the anchored tank case. In both test cases, the tank had an open top and rested on a rigid foundation. The TT tank was tested by the same procedure as the BT tank, except that the peak acceleration of the input ground motions for the TT-1 tank was scaled to $0.125g$. No significant structural damage was observed during the tests.

The third tank is a tall stainless steel tank (W-1) tested by Niwa and Clough (1982) using the shaking table. Unlike the first two tanks, this tank was tested by applying simultaneously the vertical component and the larger horizontal component of the 1980 Livermore, California earthquake (Li). In the test, the excitation intensity was gradually increased until damage was observed. The observed damage was shell wall buckling of the diamond-shaped type which developed near the base. Figure 6.7(c)

shows the dimensions of the tank. It is fully filled with water and has a conical roof. It rested on a rigid foundation and was free to uplift.

6.3.2 Discussion of Results

To examine the design actions considered in the different design procedures, in particular the maximum axial compressive stress in the tank shell, the experimental test results of the above tanks are compared with the predicted values obtained using the API and AWWA standards and the Clough model. The results are listed in Table 6.1. Columns 3-5 in Table 6.1 are the measured or observed experimental values. Columns 6-10 are the predicted values calculated using the provisions in the API/AWWA standards, which are based on the Wozniak-Mitchell model, while columns 11-14 are obtained using the Clough model. The experimental values of the base overturning moment (column 3) were used to determine the predicted quantities.

For the two anchored tanks (BT-2 and TT-2), there is no uplift and the predicted maximum axial compressive stress in the tank shell is calculated by the simple beam theory. In the BT-2 tank, the predicted compressive stresses are about one-third of the test values (column 4), whereas in the TT-2 tank the predicted values are all greater than the test values. This indicates that the use of simple beam theory in the stress prediction of anchored tanks is inadequate and may lead to unconservative design. Furthermore, the degree of base constraint of the tested tanks may be different from that assumed in the simple cantilever beam model of an anchored tank. This suggests that the degree of base constraint must be accurately considered in the simplified model.

The predicted responses of unanchored tanks by the API/AWWA standards generally do not agree with the test results. The calculated w_L (column 6 (cal)), which is the maximum weight of the uplifted liquid that can be utilized to resist overturning in the Wozniak-Mitchell model, is always greater than the design w_L (column 6 (des)). The design w_L is the limit specified in the API/AWWA standards for the calculated w_L such that the uplifted length of the bottom plate will not exceed 3.5% of the tank diameter. However, if the values of the design w_L of the unanchored tanks considered here are used in the calculations, the corresponding values of the uplift moment parameter, $M/[D^2(w_t + w_L)]$ (column 8), exceed the stability limit considered by the API/AWWA standards. Thus, according to the API/AWWA standards all the unanchored tanks considered here are unstable and will overturn when subjected to the measured base overturning moments from the input ground motions. The test results do not concur with this conclusion. None of the tanks toppled during the tests and the structural damage observed was only minor. The reason for these discrepancies in the predicted and test results is that the uplift mechanism, hence, the amount of uplifted liquid, is highly dependent on the earthquake input. It is not constant for a particular tank, and does not depend only on the dimensions and properties of the tank and its contained liquid, as assumed in the Wozniak-Mitchell model. If the calculated w_L is used instead of the design w_L , the values of the uplift moment parameter will be within the limits in which the tanks are considered stable in accordance with the provisions of the API/AWWA standards. The resulting predicted maximum axial compressive stresses in the tank shell are listed in column 10. For tanks BT-1 and TT-1, which rest on rigid foundations, the predicted compressive stresses are

considerably less than the test values, whereas for tank BT-3, which has a flexible foundation, the results are closer. It is important to note here that as the measured base overturning moments of the two tanks BT-1 and BT-3 are approximately of the same magnitude, the difference in the results of the measured axial stresses in the two tanks (column 4) indicate that flexibility in the support foundation tends to relieve the tank wall from high axial stresses when compared to rigid foundations. This is because a flexible foundation provides a larger area of contact between the tank wall base rim and the foundation than a rigid foundation, thus spreading the axial compressive stress over a larger portion of the tank wall circumference. In accordance to the API/AWWA standards, the W-1 tank is found unstable against overturning regardless of whether the calculated w_L or the design w_L is used. However in the actual test, the tank did not overturn but only suffered "diamond-shaped" buckles on the tank wall.

In comparison, the results obtained using the Clough model are closer to the test results than those from the API/AWWA standards. This is because in the Clough model, the response of an uplifting tank is directly influence by the input ground motion through the base overturning moment. This is not the case in the API/AWWA standards, which determine the maximum amount of uplifted liquid independent of the input ground motion. However, the trend of the comparison is not consistent. The predicted compressive stresses of the TT-1 tank are very close to the test values if the modification factor of 2.5 as suggested in the New Zealand recommendations is used. On the other hand, the predicted stresses for the BT-1 tank are significantly less than the test results regardless of whether the modification

factor of 2.5 is used or not. For tank BT-3, which rested on a flexible foundation, the predicted and measured stresses are in good agreement. The predicted stresses, however, are all greater than the test values with or without the modification factor. In the case of the W-1 tank, the predicted stress is a little less than the test value. But if the modification factor is included, it becomes about twice the test value. The significant difference in the above predicted and test results indicates the inadequacy of the current design procedures in predicting the tank shell stresses in an uplifting tank. One reason for this is that current design procedures do not take into account the effect of the support foundation flexibility in the tank response.

In both the API/AWWA standards and Clough model, the calculated extent of bottom plate uplift (columns 7 and 12) are considerably greater than the test values (column 5) for tanks BT-1 and BT-3. As mentioned previously, this is because the bottom plate uplift is highly dependent on the input ground motion and does not depend only on the dimensions and properties of the tank and its contained liquid, as assumed in the Wozniak-Mitchell model. Furthermore, the extent of bottom plate uplift is a highly complex phenomenon in which the nonlinear behaviour of the bottom plate, such as its membrane action, is very significant in resisting the uplift. The currently available design procedures do not take into account of this nonlinear behaviour of the bottom plate, and thus greatly overestimate the uplift extent.

To summarize, the Wozniak-Mitchell model, which is adopted by the API and AWWA standards, and the Clough model are inadequate in predicting the maximum axial stress in the tank wall of unanchored tanks during earthquakes. This is due to the simplifying assumptions in the uplift phenomenon considered in these models

which resulted in the neglect of the more relevant uplift resistance mechanisms, such as the membrane action of the bottom plate, and the dynamic nature of the response itself.

6.4 Design Criterion Calculations

To evaluate the design criteria of the different design methods, results from experimental tests are compared with the predicted buckling capacity of the tanks using the provisions of the API and AWWA standards and the New Zealand recommendations. The test results taken from the laboratory tests of Shih (1981), Peek (1986) and Niwa and Clough (1982) are plotted in Figure 6.8.

Shih (1981) performed static-tilt tests on anchored mylar tanks. In the test, the tank was inclined by tipping the bottom plate to a desired angle and slowly filling the tank with water until buckling occurred. The stress at the point of buckling was determined using the membrane theory. The observed structural damage is the typical elastic buckling. Peek (1986) also carried out static-tilt tests for unanchored mylar tanks. In the test, the tilt angle was held fixed and the tank was filled slowly with water until the first signs of a buckle could be detected visually on the tank wall. The tank wall stress at incipient buckling was determined using a specially developed method of analysis for unanchored tanks. In the actual tests two cases were examined: with and without pre-uplift of the base. The case of without pre-uplift is the one presented here. Also included in the present comparison study are the test results by Niwa and Clough (1982) on unanchored stainless steel tanks using

the shaking table. This is the same tank discussed in the previous section.

6.4.1 Discussion of Results

Elastic Buckling

The predicted and measured elastic buckling stresses, F_{ae} , normalized by the classical critical stress, $F_{cl} = 0.6Et_s/R$, for perfect cylindrical shells under uniform compression are shown in Figure 6.8 in which P is the internal pressure, E is the modulus of elasticity of the tank wall, t_s is the tank wall thickness and R is the radius of the tank. The results of the API (curve (a)), AWWA (curve (b)) and New Zealand recommendations (curve (c)) indicate that the elastic buckling resistance increases with internal pressure. This is due to the reduction of the wall imperfections resulting from the high internal pressure. However, the buckling capacity as determined by the New Zealand recommendations is significantly greater than that by the API and AWWA standards. This indicates that the buckling strength of the tank wall is greater when it is subjected to both bending and axial compression, which is the case considered in the New Zealand recommendations, than when it is subjected to axial compression alone, which is the case considered in the API and AWWA standards. Also, it must be noted here that the actual axial stresses that develop in the tank shell during an earthquake is due to bending of the tank wall and due to axial loads from the weight of the tank wall rather than due to uniform axial loads. Thus, the above considerations imply that the provisions for the elastic buckling capacity of the tank wall in the API and AWWA standards will underestimate the full buckling strength of tanks during earthquakes.

The predicted values of buckling capacity from the API and AWWA standards are generally the same. This is not surprising, since they are based on the same formulation. However, the AWWA standard does account for the effect of the tank radius to shell thickness ratio, R/t_s , which may implicitly include the effect of wall imperfections. Curves (b1) and (b2) in Figure 6.8 are obtained using the AWWA standard with R/t_s of 200 and 2000 respectively. Although, these curves may suggest that the elastic buckling resistance of thin-walled tanks is greater than thick-walled tanks, the absolute values of the elastic buckling stress, as determined by Equations 5.4 and 5.6, for thick-walled tanks are, however, greater than those for thin-walled tanks. Thus, the significance of these curves is that the elastic buckling capacity of the tank wall increases as the internal pressure increases and as the level of wall imperfection decreases as reflected by the increase in the R/t_s ratio.

When the predicted values in buckling capacity are compared to the test results, the predicted values are generally quite conservative. In fact, the results by Shih (1981) (points (d)) indicate that the elastic buckling stress is 1.5 to 2.0 times greater than the classical buckling stress. Also, the test results show that an increase in the internal pressure does not automatically corresponds to an increase in the buckling resistance. One reason for this observation is due to the level of wall imperfections at the actual tests which may offset the effect of the internal pressure. Since tests were repeated on the test tanks over the course of their studies by Shih (1981) and Peek (1986), different levels of wall imperfection might have resulted as the tests progressed.

Elastic-Plastic Buckling

The predicted elastic-plastic buckling stress, F_{aep} , normalized by the classical critical stress, F_{cl} , for perfect cylindrical shells under uniform compression is shown in Figure 6.9 in which P is the internal pressure. E is the modulus of elasticity of the tank wall, t_s is the tank wall thickness and R is the radius of the tank. The formula proposed by Rotter (1985b), which is included in the New Zealand recommendations, is plotted for different tank radius to shell thickness ratio. Also plotted is the API standard (curve (a)) for elastic buckling. It may be recalled that the API and AWWA standards treat elastic and elastic-plastic buckling similarly.

The curves calculated by Rotter formula show a pattern of decreasing buckling stress as internal pressure is increased. This is in contrast to the behaviour of elastic buckling in which the presence of internal pressure increases the critical stress. At present, there is no definite explanation yet for the above phenomenon, since current knowledge on the mechanisms that cause the different buckling failure modes of the tank wall are not fully established. However, it was suggested by Niwa and Clough (1982) that the bulge formation on the tank wall typical for "elephant's-foot-buckling" is a consequence of the plastification of some regions on the tank wall, particularly near the tank base, due to high hoop stress, as a result of high internal pressure, near the yield limit of the tank wall in combination with the high axial compressive stress from the overturning moment. Based on this, for elastic-plastic buckling, an increase in the internal pressure leads to a reduction of the effective stiffness of the tank wall due to yielding as caused by the hoop stress from the internal pressure which then decreases the buckling capacity of the tank wall.

It can be seen in Figure 6.9 that at some range, the API standard seems to be more conservative than the Rotter formula but as the internal pressure increases the API standard tends to be more unconservative. Also, the results from the Rotter formula indicate that as R/t , increases, which can be viewed as when the level of wall imperfection decreases, the elastic-plastic buckling capacity of the tank wall increases. Comparing Figures 6.8 and 6.9, it appears that although the API and AWWA standards are conservative for elastic buckling, they may become unconservative for elastic-plastic buckling especially for the case of higher internal pressure.

It is noted here that although the provisions of the API and AWWA standards result in a conservative estimation of the buckling strength of the tank wall, the overall design of the tank may still be unconservative. This is because in these standards the estimation of the design loads and the resulting stresses (design actions) may be very low which cannot be overcome by the conservatism in the estimation of the buckling strength (design criterion).

Table 6.1 Observed and predicted maximum shell axial compressive stresses in the tank wall
(See notes next page.)

Tank name (1)	E.Q. record (2)	M (N-m) (3)	σ_{cexp} (MPa) (4)	L_{exp} (mm) (5)	API/AWWA (Wozniak-Mitchell Model)					Clough Model			
					w_L cal/des (N/m) (6)	L_{pre} cal/des (mm) (7)	$\frac{M}{D^2(w_t + w_L)}$ (8)	β (deg) (9)	σ_{cpre} (MPa) (10)	e (11)	L_{pre} cal/des (mm) (12)	β (deg) (13)	σ_{cpre} Clough/NZ (MPa) (14)
BT-1	El	65103	25.72	12	4125/	472/	1.151/ 4.068/	104*	3.71*	0.908	337	84	5.24/13.11
	Pa	77181	36.12	22	1095	128	1.365/ 4.822/	71*	6.14*	0.899	369	84	5.75/14.37
	Pd	47147	20.06	16			0.834/ 2.946/	160*	2.25*	0.935	238	86	3.74/9.34
BT-2	El	27447	4.28	----	ANCHORED TANK					ANCHORED TANK			
	Pa	19728	3.44	----									
	Pd	27073	4.61	----									
BT-3	El	59272	3.75	31	4125/	472/	1.048/ 3.703/	120*	3.10*	0.917	304	85	4.74/5.92
	Pa	84533	5.38	45	1095	128	1.495/ 5.282/	44*	10.79*	0.876	454	82	7.07/8.84
	Pd	71299	4.27	30			1.261/ 4.46/	88*	4.66*	0.898	373	84	5.81/7.26
TT-1	El	40674	16.27		7483/	329/	0.938/ 3.431/	138*	4.30*	0.921	187	85	6.95/17.38
	Pa	20066	7.85	not measured	1838	81	0.463/ 1.693/	**	2.13**	0.964	85	88	3.41/8.53
	Pd	44199	17.40				1.020/ 3.729/	124*	4.87*	0.913	206	85	7.62/19.68
TT-2	El	277940	20.13	----	ANCHORED TANK					ANCHORED TANK			
	Pa	260315	23.55	----									
	Pd	208794	15.16	----									
W-1	Li	413521	102.05	not measured	8044/ 3466	230/99	5.508/ 11.267	***	***	0.621	1097	59	84.97/ 212.43

Notes for Table 6.1:

- M = measured maximum overturning moment at the tank base
 σ_{cexp} = measured maximum axial compressive stress in the tank shell bottom
 L_{exp} = measured maximum bottom plate uplift
 w_L = maximum liquid weight that contributes to the resistance to overturning according to API and AWWA standards (Wozniak Mitchell model)
 w_t = weight of the tank shell per unit length of shell circumference
 L_{pre} = predicted maximum bottom plate uplift
 $\frac{M}{D^2 (w_t + w_L)}$ = uplift-moment parameter according to API and AWWA standards
 β = half-angle contact arc of the tank shell with the foundation
 σ_{cpre} = predicted maximum axial compressive stress in the tank shell
 e = non-dimensional ratio of bottom plate contact area with the foundation as per Clough model
 cal = calculated value based on the Wozniak-Mitchell model
 $pred$ = predicted value based on the API and AWWA standards and the Clough model
 NZ = predicted value based on the New Zealand Recommendations
 $*$ = based on calculated values
 $**$ = based on calculated values but in this case uplift-moment parameter is less than the limit specified in API/AWWA standards
 $***$ = tank is unstable, half-angle contact arc of the tank shell with the foundation is unrealistic

Figure 6.1 Dimensions of tank considered in design load calculations

(a) 1994 Northridge Earthquake (Newhall station)

(b) 1940 El Centro Earthquake (Imperial Valley record)

(c) 1988 Saguenay Earthquake (Chicoutimi-Nord Station)

Figure 6.2 Input ground motions in design load calculations

(a) 1994 Northridge Earthquake (Newhall station)

(b) 1940 El Centro Earthquake (Imperial Valley record)

(c) 1988 Saguenay Earthquake (Chicoutimi-Nord station)

Figure 6.3 Acceleration response spectra

(a) 1994 Northridge Earthquake

(b) 1940 El Centro Earthquake

(c) 1988 Saguenay Earthquake

Figure 6.4 Normalized lateral base shear

(a) 1994 Northridge Earthquake

(b) 1940 El Centro Earthquake

(c) 1988 Saguenay Earthquake

Figure 6.5 Normalized base overturning moment

(a) 1994 Northridge Earthquake

(b) 1940 El Centro Earthquake

(c) 1988 Saguenay Earthquake

Figure 6.6 Normalized foundation overturning moment

(a) BT-Tank

(b) TT-Tank

(c) W-1 Tank

Figure 6.7 Dimensions of tanks considered in design action calculations (mm)

Figure 6.8 Elastic buckling stress normalized by classical critical stress for perfect cylindrical shells under uniform compression:

- (a) API standard (1988), all R/t_s
- (b) AWWA standard (1984), (1) $R/t_s = 200$, (2) $R/t_s = 2000$
- (c) New Zealand recommendation [Priestley et al. (1986)], (1) $R/t_s = 200$, (2) $R/t_s = 2000$
- (d) Anchored mylar tank [Shih (1981)], $R/t_s = 800$
- (e) Unanchored mylar tank [Peek (1986)], $R/t_s = 1250$
- (f) Unanchored stainless steel tank [Niwa & Clough (1982)], $R/t_s = 730$

Figure 6.9 Elastic-plastic buckling stress for a cylinder with a yield stress of 250MPa normalized by classical critical stress for perfect cylindrical shells under uniform compression.

Chapter 7

SUMMARY AND CONCLUSIONS

A brief summary and conclusion on the current state of the earthquake resistant design of liquid storage tanks are presented in this chapter.

7.1 Summary

A survey of the methodologies and philosophies of the design methods currently available and used in practice for ground-supported flat bottom thin-walled metal cylindrical liquid storage tanks is presented. The survey follows the normal course of a typical design process of a civil engineering structure, i. e., separating the analysis into design loadings, design actions and design criteria.

The response analysis of tanks, in terms of design loadings, are generally based on the assumptions of either rigid or flexible anchored tanks on rigid foundations. In rigid

tanks, the acceleration of the tank wall is the same as the ground, while in flexible tanks the interaction between the vibrating tank and liquid considerably changes the overall response. The total response of the system is generally considered to consist of two modes: the impulsive mode and the convective mode. The impulsive mode is related to the liquid motion as a result of the tank motion, while the convective mode is characterized by liquid sloshing.

The resultant lateral force and overturning moment at the base are two of the most important design loadings considered in the design of liquid storage tanks. To predict the maxima of these loadings, equivalent mechanical spring-mass models were developed and employed by design procedures such as the Housner method and the Epstein procedure. The API and AWWA standards, which are the two most complete procedures at present, adopted the Housner method but used seismic coefficients to estimate the effective accelerations to which the liquid-tank system may be subjected. In any case, these procedures are based on rigid anchored tank response.

Motivated by reported tank damages due to earthquakes in the field and laboratory experimental studies, which have shown that flexibility of the shell wall has a considerable effect on the overall response, procedures considering tank flexibility were developed by a number of researchers. The Veletsos procedure used a variational approach that included the tank wall flexibility effect on the impulsive pressure component by replacing the ground acceleration in the relevant equations with the pseudoacceleration function corresponding to the fundamental natural frequency of the liquid-tank system. A similar tank design model was also proposed in the Haroun-Housner procedure using the boundary integral techniques. This model, unlike the

Veletsos model, divides the impulsive response component into rigid mode, associated with the dynamic effects due to the rigid body motion of the tank, and liquid-tank interaction mode, associated with the dynamic effects resulting from the flexibility and deformation of the tank wall. The Balendra et al. procedure includes the tank wall flexibility effects by using the finite element approach. Design recommendations were developed based on the results obtained from these procedures (New Zealand recommendations, Adams design guidelines). The effect of tank wall flexibility on the convective pressure is generally ignored due to the fact that the short period vibration response of the tank wall is not significantly coupled to the relatively long period response of the sloshing liquid, and thus the convective pressure cannot significantly be affected by the tank wall flexibility. The design loads of an unanchored tank are generally assumed to be the same as that of an anchored tank. However, recent works, such as Manos and Clough (1982), Scharf et al. (1989a) and Lau and Zeng (1992), have indicated that the response of unanchored tanks may be different from anchored tanks.

The axial compressive stress in the tank wall is one of the most important design actions in the design of liquid storage tanks. Excessive axial stress in the shell may result in buckling of the tank wall, especially at the bottom. For anchored tanks, the distribution and magnitude of the shell axial stress around the tank circumference due to the overturning moment are generally considered acceptable to be described by a simple theory. For unanchored tanks, however, a number of static and dynamic models were developed by researchers to describe the uplift behaviour. Because of their simplicity and design applicability, static models are the ones adopted by cur-

rent design procedures. Two of these models are the Wozniak-Mitchell model and the Clough model. The Wozniak-Mitchell model is a quasi-static beam model of the bottom plate which involves the development of two plastic hinges, while the Clough model involves the assumption of a circular contact area between the partially uplifted bottom plate and the supporting foundation. Both models assumed a linear distribution of the axial compressive stress in the part of the shell that remains in contact with the foundation. The bottom plate membrane stresses, deformability of the tank wall and variation of the dynamic pressure at the bottom are neglected. Dynamic uplift models are more realistic and accurate in predicting the uplift behaviour of unanchored tanks than static models. However, these models need to be simplified in order to be suitable for design applications.

The design criterion that is particularly important in the design of liquid storage tanks is the dynamic buckling capacity of the tank shell. The dynamic buckling capacity is influenced by the presence of the internal pressure, the geometric imperfections of the shell, and the degree of concentration in the circumferential distribution of the axial compressive stress from the overturning moment. At present, there is no reliable method that can give an accurate estimate on the dynamic buckling strength of thin-walled liquid storage tanks during dynamic response. The design standards API and AWWA give a buckling capacity based mainly on test results of cylindrical thin shell subjected to uniform compression, which is clearly different from the case in the actual condition. The Austrian recommendations and New Zealand recommendations adopt improved estimates on the dynamic buckling capacity which take into consideration the effects of internal pressure, the circumferential variation

of axial stress, the geometric imperfections of the tank wall, and the difference in buckling capacity of the two buckling modes: the elastic buckling failure mode (shell crippling) and the elastic-plastic buckling failure mode (elephant's-foot-bulge). The different possible combinations in the superposition of the liquid pressure effects due to the horizontal and vertical components of the ground motion is believed to be a major factor that gives rise to these buckling modes.

Comparison of predicted responses obtained using the different design procedures and experimental test results of practical and model tanks is presented. In general, the results indicate the inadequacy of present design procedures, especially those that neglect tank flexibility and uplift resistance mechanisms in unanchored tanks, in predicting the tank response.

7.2 Conclusions

Based on the work presented in this thesis, several conclusions can be drawn on the current state of the earthquake resistant design of liquid storage tanks.

1. There is a substantial difference between the design response of tanks obtained from procedures considering tank flexibility and that obtained from procedures considering tanks as rigid. The vibration characteristics of the tank wall and its interaction with the vibrating liquid in relation to the characteristics of the earthquake input greatly influence this difference.
2. The seismic coefficients in the API and AWWA standards do not accurately reflect the actual accelerations of the liquid-tank system, especially the impulsive

acceleration, as indicated by the significant difference in the design loadings obtained from the API and AWWA standards and those obtained from procedures considering tank wall flexibility.

3. The hydrodynamic loads on an excited tank generally increase as the liquid height to tank radius ratio increases. In connection with this observation, the dominant response in tall tanks is the impulsive mode, while in broad tanks the convective mode is the dominant one.
4. The quasi-static models (Wozniak-Mitchell model and Clough model) adopted in current design procedures cannot predict the actual state of stress in the shell wall, as indicated by the significant difference in the predicted and test results. This may be attributed to the relevant uplift resistance mechanisms, such as the membrane action of the bottom plate, developed in an unanchored tank during uplift which are neglected in the design models.
5. For anchored tanks, the assumption of a simple beam distribution for the shell axial compressive stress may sometimes lead to underestimation of the maximum compressive stress, as shown in the comparison of predicted and test results. Therefore, further investigations are needed in this matter.
6. Dynamic buckling of the shell wall is mainly due to the axial compressive stress from the bending action of the tank wall as caused by the dynamically activated liquid pressure rather than uniform compression from axial loads. Also, the internal pressure increases the buckling capacity of the tank wall by reducing

the level of tank wall imperfections.

7. Contrary to the assumption in the API and AWWA standards, buckling failure modes are of two distinct types: the elastic buckling mode and the elastic-plastic buckling mode. The different possible combinations in the superposition of the liquid pressure effects due to the horizontal and vertical components of the ground motion is believed to be a major factor that gives rise to these buckling modes. As shown in the comparison of predicted and test results, the provisions for the dynamic buckling capacity of the tank wall in the API and AWWA standards lead to underestimation of the elastic buckling strength of the tank wall, however, they may also lead to overestimation of the elastic plastic buckling capacity, especially for higher internal pressure.

7.3 Recommendations for Future Work

In order to develop an optimum design procedure, further investigations and studies toward the development of simple methodologies reflecting the effects of the following are recommended.

1. Improvement is needed in the quantification of the seismicity input by the seismic coefficients that can better correlate the earthquake input and the vibration characteristics of the liquid-tank system.
2. Further study on the seismic loads generated in an unanchored tank during uplift, which might be substantially different from that of an anchored tank, is

needed.

3. Considerations of the bottom plate membrane stress, deformability of the tank wall, variation of dynamic pressure at the bottom, and the magnitude of vertical uplift and extent of bottom plate uplift in the development of an uplift model suitable for design purpose are required.
4. Further research on the buckling phenomena of shells subjected to dynamic internal pressure is needed, especially the establishment of a clear and proper distinction between elastic and elastic-plastic buckling.
5. Investigations to better and more accurately define the magnitude and distribution of wall imperfections of typical tanks and their contributing effects on the dynamic buckling behaviour of liquid storage tanks should be carried out.
6. The effects of the simultaneous action of the horizontal and vertical components of the ground motion should be studied.
7. The effects of soil-tank structure interaction should be determined through further studies, especially its effect on the coupled natural frequency and damping of the system.
8. The effects of the roof system, ring girders and other stiffening parts on the tank seismic performance should be quantified.

REFERENCES

- Adams, N. J. J. (1992). Seismic design rules for flat bottom cylindrical liquid storage tanks. *Int. J. of Pressure Vessel and Piping*, **49**, 61-96.
- American Petroleum Institute (1988). Seismic design of storage tanks. In: *Welded Steel Tanks for Oil Storage*, API 650, Appendix E, Washington, D. C.
- American Waterworks Association (1984). *AWWA Standard for Welded Steel Tanks for Water Storage*. AWWA-D100, Denver, Colorado.
- Applied Technology Council (1977). *Recommended Comprehensive Seismic Design Provisions for Buildings*, ATC-3-05, Final review draft, Palo Alto, California.
- Auli, W., F. D. Fischer and F. G. Rammerstorfer (1985). Uplifting of earthquake-loaded liquid-filled tanks. In: *Proceedings of the Pressure Vessels and Piping Conference*, ASME, Vol. 98(7), pp. 71-85.
- Balendra, T. and W. A. Nash (1978). *Earthquake Analysis of a Cylindrical Liquid Storage Tank with a Dome by Finite Element Method*. Research report, Department of Civil Engineering, University of Massachusetts.
- Balendra, T., K. K. Ang, P. Paramasivam and S. L. Lee (1982). Seismic design of flexible cylindrical liquid storage tanks. *Earthquake Eng. and Structural Dynamics*, **10**, 477-496.
- Barton, D. C. and J. V. Parker (1987). Finite element analysis of the seismic response of anchored and unanchored liquid storage tanks. *Earthquake Eng. and*

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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Structural Dynamics, **15**, 299-322.

Cambra, F. J. (1982). *Earthquake Response Considerations of Broad Liquid Storage Tanks*. Report No. 82/25, Earthquake Engineering Research Center, University of California at Berkeley.

Clough, D. P. (1977). *Experimental Evaluation of Seismic Design Methods for Broad Cylindrical Tanks*. Report No. 77/10, Earthquake Engineering Research Center, University of California at Berkeley.

Clough, R. W. and A. Niwa (1979). *Static Tilt Test of a Tall Cylindrical Liquid Storage Tank*. Report No. 79/06, Earthquake Engineering Research Center, University of California at Berkeley.

Clough, R. W., A. Niwa and D. P. Clough (1979). Experimental study of cylindrical tanks. *J. of the Structural Div., Proc. of ASCE*, **105**, 2565-2590.

Epstein, H. I. (1976). Seismic design of liquid storage tanks. *J. of the Structural Div. ASCE*, **102**, 1659-1673.

European Convention for Constructional Steelwork (ECCS) (1983). *Buckling of Steel Shells-European Recommendations*. 2nd Edition.

Fischer, F. D. and F. G. Rammerstorfer (1982). The stability of liquid-filled cylindrical shells under dynamic loading. In: *Buckling of Shells* (E. Raman, ed.), pp. 569-597. Springer, New York.

- Fischer, F. D. and R. Seeber (1988). Analysis of vertically excited liquid storage tanks considering liquid-soil interaction. *Earthquake Eng. and Structural Dynamics*, **16**, 329-342.
- Haroun, M. A. (1980). *Dynamic Analyses of Liquid Storage Tanks*. Report No. 80/01. Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California.
- Haroun, M. A. (1983). Behaviour of unanchored oil storage tanks: imperial valley earthquake. *J. of Technical Topics in Civil Eng., ASCE*, **109**, pp. 23-40.
- Haroun, M. A. and G. W. Housner (1981). Seismic design of liquid storage tanks. *J. of the Technical Councils of ASCE*, **107**, 191-207.
- Haroun, M. A., H. S. Badawi and C. B. Nanda (1987). Nonlinear uplift analysis of crescent-shaped plate. In: *Proceedings of the Pressure Vessels and Piping Conference, ASME*, pp. 317-324.
- Housner, G. W. (1957). Dynamic pressures on accelerated fluid containers. *Bulletin of the Seismological Soc. of America*, **53**, 381-387.
- Ishida, K. and N. Kobayashi (1988). An effective method of analyzing rocking motion for unanchored cylindrical tanks including uplift. *J. of Pressure Vessel Tech., Trans. of ASME*, **110**, 1087-1100.
- Ishiyama, Y. (1984). Motion of rigid bodies and criteria for overturning by earthquake excitation. *Bulletin of New Zealand Nat. Soc. for Earthquake Eng.*, **17**, 24-37.

- Kumar, A. (1981). *Studies of Dynamic and Static Response of Cylindrical Liquid-Storage Tanks*. Ph. D. Thesis, Rice University, Houston, Texas.
- Lau, D. T. and R. W. Clough (1989). *Static Tilt Behaviour of Unanchored Cylindrical Tanks*. Report No. 89/11, Earthquake Engineering Research Center, University of California at Berkeley.
- Lau, D. T. and R. W. Clough (1990). Use of ritz shape functions in analysis of uplifting cylindrical tanks. In: *Proceedings of the Fourth U. S. National Conference on Earthquake Engineering*, Vol. 3, pp. 167-176, California.
- Lau, D. T. and X. Zeng (1991). Simplified uplift analysis of unanchored cylindrical tanks. In: *Proceedings of the Third U. S. Conference on Lifeline Earthquake Engineering*, pp. 975-984.
- Lau, D. T. and X. Zeng (1992). Hydrodynamic forces in uplifting cylindrical tanks. *Fluid-Structure Vibration and Sloshing*, ASME, **232**, 39-44.
- Lau, D. T. and X. Zeng (1995). Nonlinear behaviour of bottom plate in cylindrical liquid storage tanks for seismic applications. *Canadian J. of Civil Engineering*, **22**, 180-189.
- Liu, K. W. and D. C. Ma (1982). Coupling effect between liquid sloshing and flexible fluid-filled systems. *Nuclear Engineering and Design*, **72**, 345-357.
- Lo, H., H. Crate and E. B. Schwartz (1950). *Buckling of Thin-Walled Cylinders under Axial Compression and Internal Pressure*. NACA TN 2021.

- Ma, D. C., Gvildys, J., Chang, Y. W. and W. K. Liu (1982). Seismic behaviour of liquid-filled shells. *Nuclear Engineering and Design*, **70**, 437-455.
- Manos, G. C. (1986). Earthquake tank-wall stability of unanchored tanks. *J. of Structural Eng. ASCE*, **112**, 1863-1880.
- Manos, G. C. and R. W. Clough (1982). *Further study of the Earthquake Response of a Broad Cylindrical Liquid Storage Tank Model*. Report No. 82/07, Earthquake Engineering Research Center, University of California at Berkeley.
- Miller, C. D. (1977). Buckling of axially compressed cylinders. *J. of the Structural Div., ASCE*, **103**, 695-721.
- National Building Code of Canada, NBCC (1990). National Research Council of Canada, Ottawa, Ontario, Canada.
- Natsiavas, S. (1987). *Response and Failure of Fluid-Filled Tanks under Base Excitation*. Report No. 86/26, California Institute of Technology, Pasadena, California.
- Natsiavas, S. (1988). An analytical model for unanchored fluid-filled tanks under base excitation. *J. of Applied Mechanics*, **55**, 648-653.
- Natsiavas, S. (1990). Simplified models for the dynamic response of tall unanchored liquid containers. *J. of Pressure Vessel Tech., Trans. of ASME*, **112**, 124-131.
- Natsiavas, S. and C. D. Babcock (1987). Buckling at the top of a fluid-filled tank during base excitation. *J. of Pressure Vessel Tech., Trans. of ASME*, **109**,

374-380.

Natsiavas, S. and C. D. Babcock (1988). Behaviour of unanchored fluid filled tanks subjected to ground excitation. *J. of Applied Mechanics, Trans. of ASME*, **55**, 654-659.

Niwa, A. (1978). *Seismic Behaviour of Tall Liquid-Storage Tanks*. Report No. 78/04. Earthquake Engineering Research Center, University of California at Berkeley.

Niwa, A. and R. W. Clough (1982). Buckling of cylindrical liquid storage tanks under earthquake loading. *Earthquake Eng. and Structural Dynamics*, **10**, 107-122.

Peek, R. (1986). *Analysis of Unanchored Liquid Storage Tanks under Seismic Loads*. Ph. D. Thesis, California Institute of Technology, Pasadena, California.

Peek, R. (1988). Analysis of unanchored liquid storage tanks under lateral loads. *Earthquake Eng. and Structural Dynamics*, **16**, 1087-1100.

Peek, R. and P. C. Jennings (1988). Simplified analysis of unanchored tanks. *Earthquake Eng. and Structural Dynamics*, **16**, 1073-1085.

Peek, R., P. C. Jennings and C. D. Babcock (1988). Preuplift method to 'anchor' liquid storage tank. *J. of Structural Eng., ASCE*, **114**, 475-487.

Priestley, M. J. N., B. J. Davidson, G. D. Honey, D. C. Hopkins, R. J. Martin, G. Ramsey, J. V. Vessey and J. H. Wood (1986). *Seismic Design of Storage Tanks*. New Zealand National Society for Earthquake Engineering, Wellington.

- Raumerstorfer, F. G., K. Scharf, F. D. Fischer and R. Seeber (1988). Collapse of earthquake excited tanks. *Res Mechanica J.*, **25**, 129-143.
- Rotter, J. M. (1985a). Buckling of ground-supported cylindrical steel bins under vertical compressive wall loads. In: *Proceedings of Metal Structures Conference*, pp. 112-127. Institution of Engineers, Australia.
- Rotter, J. M. (1985b). *Local Inelastic Collapse of Pressurised Thin Cylindrical Steel Shells under Axial Compression*. Research report, School of Civil and Mining Engineering, University of Sydney.
- Rotter, J. M. and P. Seide (1987). On the design of unstiffened shells subjected to an axial load and internal pressure. In: *Proceedings of the ECCS Colloquium on Stability of Plate and Shell Structures*, pp. 539-548. Ghent University, Belgium.
- Sakai, F., Ogawa, H. and A. Isoe (1984). Horizontal, vertical and rocking fluid-elastic response and design of cylindrical liquid storage tanks. In: *Proceedings of the Eight World Conference in Earthquake Engineering*, Vol. 5, pp. 263-270, San Francisco, California.
- Sakai, F., H. Sakata and H. Ogawa (1979). *Seismic Design of Oil Storage Tanks related with proposal of API*. Kawasaki Heavy Industry Engineering Review, **71**, (in Japanese).
- Scharf, K. (1989). *Contributions to the Problem of Earthquake Excited Liquid Storage Tanks*. Doctoral Thesis, Institute of Light Weight Structures, Technical University of Vienna.

- Scharf, K., F. G. Rammerstorfer and F. D. Fischer (1989a). The scientific background of the Austrian recommendation for earthquake resistant liquid storage tank design. In: *Structural Dynamics and Soil-Structure Interaction* (Çakmak, A. S. and I. Herrera, eds.), pp. 281-313.
- Scharf, K., F. G. Rammerstorfer and F. D. Fischer (1989b). Elastic plastic stability analysis of earthquake loaded tanks. In: *Nichtlineare Berechnungen im konstruktiven Ingenieursbau* (E. Stein, ed.), Springer, Berlin.
- Shih, C. F. (1981). *Failure of Liquid Storage Tanks due to Earthquake Excitation*. Report No. 81-04, California Institute of Technology, Pasadena, California.
- Uniform Building Code, UBC (1982). International Conference of Building Officials, Whittier, California.
- U. S. Atomic Energy Commission (1963). *Nuclear Reactors and Earthquakes*, TID 7024, pp. 183-209, pp. 367-390, Office of the Technical Service, Washington DC.
- Veletsos, A. S. (1974). Seismic effects in flexible liquid storage tanks. In: *Proceedings of the International Association for Earthquake Engineering, Fifth World Conference*, Rome, Vol. 1, pp. 630-639.
- Veletsos, A. S. (1984). Seismic response and design of liquid storage tanks. In: *Guidelines for the Seismic Design of Oil and Gas Pipeline Systems*, ASCE Technical Council on Lifeline Earthquake Engineering, pp. 255-365, New York.

- Veletsos, A. S. and A. Kumar (1984). Dynamic response analysis of vertically excited liquid storage tanks. In: *Proceedings of the Eight World Conference in Earthquake Engineering*, Vol. , pp. , San Francisco, California.
- Veletsos, A. S. and Y. Tang (1983). Private communications.
- Veletsos, A. S. and J. Y. Yang (1976). Dynamics of fixed-based liquid-storage tanks. In: *Proceedings of US-Japan Seminar for Earthquake Engineering Research with Emphasis on Lifeline Systems*, Tokyo, pp. 317-341.
- Veletsos, A. S. and J. Y. Yang (1977). Earthquake response of liquid storage tanks. In: *Advances in Civil Engineering through Engineering Mechanics Division Specialty Conference*, ASCE, pp. 1-24. North Carolina.
- Veletsos, A. S. and J. Y. Yang (1983). Private communications.
- Wozniak, R. S. and W. W. Mitchell (1978). Basis of seismic design provisions for welded steel oil storage tanks. In: *Proceedings-Refining Department of American Petroleum Institute*, Vol. 57, pp. 485-501. Washington, D. C.
- Yi, W. and S. Natsiavas (1992). Seismic response of unanchored fluid-filled tanks using finite elements. *J. of Pressure Vessel Tech., Trans. of ASME*, **114**, 74-79.
- Zeng, X. (1993). *Earthquake Response Analysis of Unanchored Cylindrical Liquid Storage Tanks*. Ph. D. Thesis. Carleton University, Ottawa, Canada.

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