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(Signed) *M. Kassem El Samny*

PERMANENT ADDRESS:

*1150 Meadowslands DR.
Apt. 307
Ottawa K2E-6J4
ONT.*

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RAPID DETERMINATION OF
THE PRECONSOLIDATION PRESSURE
AND ELASTIC MODULI OF A
COHESIVE SOIL

by

MOHAMED KASSEM AHMED ELSAMNY, P.Eng.

B.Sc. (Civil Engineering) Cairo University
Higher Dipl. (Soil Mechanics and Foundation) Cairo University
M.Sc. (Civil Engineering) Cairo University

A thesis submitted to the Faculty of
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the requirements for the degree of
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Canada

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The undersigned recommend to the Faculty of graduate studies, acceptance of the thesis: "Rapid Determination of the Preconsolidation Pressure and Elastic Moduli of a Cohesive Soil; submitted by Mohamed Kassem Ahmed Elsamny, P.Eng., B.Sc., Higher Diploma, M.Sc., in partial fulfillment of the requirements for the degree of Doctor of Philosophy,

Thesis Supervisor

Chairman, Division of
Solid Mechanics and Structural
Engineering

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ABSTRACT

This thesis is an experimental study of the relationships between the response to dynamic loading and the preconsolidation pressure and elastic moduli of kaolinite.

The experimental technique employed in this work was a modification of the Hopkinson pressure bar. Tests covering an appropriate range of stress levels were carried out on laboratory prepared specimens representing a variety of preconsolidation pressures.

The experimental results show that if the applied stress on the material tested is less than that consistent with its preconsolidation pressure, the material flows at an almost constant stress level. Also, if the applied stress on the clay exceeds that consistent with the preconsolidation pressure, the soil has a definite yield point, followed by strain-hardening characteristics, which, in turn, are followed by quasi-fluid behaviour.

The experiments also show that the magnitudes of the elastic moduli of a cohesive soil increase linearly with increasing preconsolidation pressure and increasing rates of deformation.

The technique employed appears to lend itself to application for the rapid determination of the preconsolidation pressure of cohesive soils as well as to the prediction of the elastic moduli of cohesive soils at any prescribed rate of deformation.

A correlation between the work reported on in this dissertation and certain concepts of work-hardening of materials is included.

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NOTATION

β	constant
C	condenser capacity
C_1, C_2, C_3	capacitances
C_0	elastic wave velocity
σ	stress
σ_{ij}	stress-increment vector
δ	displacement
E	Young's modulus
ξ	radial expansion
ξ_I	radial deformation due to incident pulse
ξ_R	radial deformation due to reflected pulse
e	coefficient of proportionality
G	shear modulus
γ	shear strain
J_1, J_2, J_3	invariant of stress tensor
λ	mutual distance
μ	Poisson's ratio
n	number of segments per unit area
p	pressure
ρ	unit weight

R_1, R_2	resistances
r	radius of bar
ϵ	strain
ϵ_{ij}^p	plastic strain-increment
S	applied force
t	time
τ	shear stress
U_x	displacement
w.c.	water content

CHAPTER I

Introduction

The behaviour of soils under the action of various loading phenomena is an important consideration in the design of foundations and earth structures. The problem of design is complicated by many factors, the most obvious of which are the physical characteristics of the natural soils themselves.

Two of the most important of these characteristics are the preconsolidation pressure of a soil mass and the elastic moduli of the soil mass under various rates of loading.

Briefly, the preconsolidation pressure of a soil mass is a function of its stress history, the knowledge of which constitutes an important step in the determination of the expected behaviour of the soil when subjected to loading phenomena. In the case of the application of static loads, current techniques involve the carrying out of standard consolidation tests which can be expensive and time consuming. The first part of the investigation was done to confirm and to conclude on the relationships between preconsolidation pressure and the response of cohesive soils to dynamic loading. This portion of the work is a continuation of that carried out by Fletcher (1967).

Progress in the development of reliable theoretical and experimental techniques to determine the elastic moduli of soils has been limited. Few, if any, valid conclusions have been reached with regard to these parameters. The second part of the investigation was

carried out to demonstrate and confirm a relationship between preconsolidation pressure, time-rate of loading, and the elastic moduli of cohesive soils.

The experimental technique used in this study was a modification of the Hopkinson pressure bar, introduced by Hopkinson (1914) and modified by Kolsky (1949), Lindholm (1964) and again by Fletcher (1967).

The thesis includes a discourse on the work hardening characteristics of soils and the relationship between these characteristics and the preconsolidation pressure of soils.

CHAPTER II

A History of Soil Dynamics and Theories of Consolidation

2.1 Introduction

The purpose of this Chapter is to account for previous work in the field of soil dynamics, those parts of the study of the consolidation of cohesive soils which relate to the work reported on in this thesis, and work pertaining to the determination of the elastic moduli of cohesive soils.

Quite complete reviews of the literature in the area of the dynamic response of solids, including soils, appropriate to the research carried out by the author, have been completed in previous works (Fletcher, 1967, Yeboa, 1969). Since 1969 there have been only a few publications related directly to this work. However, in the interests of completeness and continuity, a complete review of the pertinent research in the field, beginning with Hopkinson (1914), is included in this Chapter. This portion of the review is treated under two separate headings, one under which the related literature in solid mechanics in general is dealt with, and the second under which literature dealing specifically with soil mechanics is treated.

The literature which deals with the development of the

experimental apparatus used in this work is reviewed in Chapter III. It was felt that a complete treatment of this aspect of the work would be desirable, and, further, that it could be best done in that part of the thesis which deals with the apparatus.

With regard to the literature concerning theories of consolidation and the determination of the preconsolidation pressure of cohesive soils, no complete review was available; hence, an attempt at such a survey is made in this Chapter. The review begins with the work of Terzaghi (1923) and is concluded with that of Wissa et al. (1971).

Only a few publications exist which are related directly to the study of the elastic moduli of soils. Since this topic is one of the principal concerns of this thesis, the few works available are reported on in some detail.

This chapter is concluded with a discussion of present methods of testing soils under dynamic loading conditions with a view to more completely informing the reader of the state of this aspect of soil mechanics as related to the author's work.

2.2 Stress wave propagation in solids

2.2.1 Introduction

Most research in the field of stress wave propagation in

solids has been theoretical in nature and has led to a variety of theoretical and experimental attempts meant to describe the behaviour of soils subjected to dynamic loading conditions. Briefly, this section includes a summary of what can be termed classic works from which essentially all other work in the field has emanated. This is followed by a summary of what are considered to be the most important subsequent writings in terms of the work carried out by the author.

2.2.2 Classical Works

Hopkinson (1914) presented a description of the propagation of stress waves along a cylindrical steel rod. He described a method of analyzing impact, such as that due to a rifle bullet or the detonation of high explosives on a solid, as well as a method of measuring the impact duration and the pressures developed. Hopkinson's analysis, which took the form of dimensional analysis, has been quite widely used particularly in studies of the effects of explosions in soils.

Donnell (1930) introduced an important treatment of the problem of stress wave propagation in solids which has practical applications in many fields of engineering. He attempted a theoretical analysis of mechanical wave transmissions, such as those from earthquake shocks, by using the principle of superposition. He discussed in detail the propagation of longitudinal pressure waves in elastic bars that obey Hooke's law. Also, he briefly mentioned the behaviour of stress waves in materials which do not obey Hooke's law, that is, those materials exhibiting non linear stress-strain relationships.

No dynamical study of the propagation of waves in an inelastic material appears to have been made till 1940 when Sir G.I. Taylor published a paper intended to illustrate the connection between the plastic properties of a material and the transformation of a suddenly applied pressure pulse through the material. He outlined a one-dimensional plastic wave propagation theory in which he pointed out that it may be expected that the passage of a longitudinal wave down a bar of plastic material may exhibit the same kind of displacement-time and pressure-time characteristics as those observed in the earth near an explosion, provided that the wave is produced by a similar explosion and the bar has the same physical properties as the earth. It may be noted that the plastic bar would expand laterally during the passage of the wave just as the level of the earth rises during the passage of the earth shock from an explosion. He concluded that if the velocity-time curve is known at one point, it is known at all points of the bar, and at all times. He also showed how the plastic stress-strain relationship can be deduced from the velocity-time relationship observed at a fixed station in the bar.

A paper closely related to Taylor's work was published by T. von Karman (1941) which was an attempt to compute the stress and strain caused by impact beyond the elastic limit using a uniform wire having strain hardening properties. Both Taylor's and von Karman's theories considered the stress as a unique function of strain, that is, consideration of the time-rate of strain was neglected.

In agreement with the above mentioned work, Rakhmatulin (1945), presented a study which, though not associated with problems of soil dynamics, is considered of great importance. This work considered the propagation of longitudinal elasto-plasto waves in semi-infinite bars.

In 1950, von Karman and Duwez described a theory for computing the stress distribution along a cylindrical bar at any instant during a longitudinal impact. They showed that for a given material there exists a critical impact velocity. If the material is impacted with a velocity larger than this critical velocity, instantaneous breakdown of the material will occur.

The partial differential equations governing wave propagation which were independently derived by Taylor (1940), and von Karman (1942), assume the stress-strain relationship to be independent of strain rate. This assumption led to some discrepancies between theory and experiment. Malvern (1951) overcame this deficiency by considering stress as a function of strain rate as well as instantaneous plastic strain.

These two theories of strain rate independence, and dependence (due to Taylor, von Karman and Malvern, respectively) have provided the basis for almost all subsequent works in the field of non-elastic wave propagation in solids.

2.2.3 General Literature

White and Griffis (1947) presented a theoretical analysis of the behaviour of a long uniform ductile wire subjected to a longitudinal impact at one end. Their analysis was concerned with the way in which plastic strains and corresponding stresses are propagated along the specimen, and the distribution of the corresponding permanent strains. The stress strain relationship was assumed to be linear, becoming concave downward.

Lee (1952) presented a one-dimensional theory of plastic wave propagation which included the determination of physical plastic-elastic boundaries in a material. His method was based on the theory of characteristics of hyperbolic equations arising from the analysis of plastic wave boundary value problems. The equations were solved by numerical integration.

One of the most important aspects of the study of stress wave propagation of transient pulses of short duration which may be initiated when a solid is subjected to impulsive loading was discussed by R.M. Davies (1956). He studied the change of form of stress wave pulses as they recede from their origin, as well as the main features of the distortion when pulses are propagated in elastic and visco-elastic solids.

A number of theoretical analyses have dealt with the problem of the interaction between reflected stress waves and the initial plastic wave front. In most of these studies a bi-linear stress-strain

law was assumed to be valid. The amplitude of the elastic wave front was a significant fraction of the amplitude of the plastic wave front. It was shown independently by Riparbelli (1953) and Rubin (1954) that the elastic velocity of the incremental loading wave is consistent with the plastic wave theory of Malvern (1951) for a visco-elastic material. The paper presented by Bell (1961) has shown that there exists a close agreement between the experimental reflection behaviour and the theoretical predictions of Lee (1952). Bell's theory assumes that the velocity of a plastic wave front is independent of the strain rate, and the actual static stress-strain curve of the material is used rather than a bi-linear approximation as assumed by Riparbelli and Rubin.

The propagation of stress waves in elastic and anelastic solids has been carefully reviewed and discussed in detail by Kolsky (1964). The review included the elastic, visco elastic, plastic, and the strain-dependent stress wave propagation theories.

Bell (1968) introduced a parabolic stress-strain function into the finite amplitude wave theory of Taylor, von Karman, and Rakhmatulin, and into the non-linear unloading wave theory of Lee. His experimental studies provide a detailed description of all of the wave propagation phenomena on either side of a lubricated elastic-plastic interface.

Thus far a general coverage of studies in the area of solid dynamics as related to this work has been made with no specific mention of their application to soil dynamics. This aspect is treated in the following section.

2.3 Stress Wave Propagation in Soils

2.3.1 Introduction

Within the past few years a significant amount of theoretical work and experimental data concerned with stress wave propagation in soils have been documented in the literature. The data include tests conducted on both sand and clay with a variety of confining conditions ranging from no lateral restraint to one dimensional compression. This section of the literature survey is concerned with a description of those works in the field of soil dynamics appropriate to this thesis.

2.3.2 Literature Survey

The design of any structure subjected to dynamic loading requires the determination of appropriate stress-strain-time-strength parameters of soils. Numerous investigations have been conducted for the purpose of establishing the static and dynamic stress-strain-time response of soils. These studies have contributed much to present-day knowledge in the fields of soil dynamics and soil rheology.

Although the response properties of a soil may be a function of a large number of factors, once the environment, history, boundary conditions, and consistency are specified, the material behaviour with these many controlling factors becomes, in primitive form, one of

relating three quantities. These are stress, strain, and time.

Numerous experimental techniques have been used to study the relationships between these quantities by using constant rate of deformation, impact, shock, vibratory, and pulse methods.

R. Hill (1948) derived equations to investigate the formation of spherical cavities in soils under dynamic loading conditions with frictional strength and compressibility being neglected. D.W. Taylor in conjunction with R.V. Whitman (1951) carried out a series of tests to investigate strain rate effects in soils. Whitman (1953) reported that unconfined specimens of cohesive soils exhibit a higher strength dependency on strain rate than did confined specimens. In addition, he found that with unconfined tests, rapid load application results in the ultimate strength occurring at higher strains relative to tests utilizing slower rates of strain.

A more realistic approach, which took account of the frictional as well as the cohesive strength of soils, was applied by Chadwick, Cox and Hopkins (1964) in which they proposed a mathematical model of a deep underground explosion. This model is based on the concept of spherical cavity formation in an elastic-plastic, incompressible soil extending to infinity.

If the non-linear properties, hysteretic properties, and the particulate nature of soils is accounted for, the problem of predicting

the characteristics of the particle velocity front, the stress wave front, and their respective relationships with the stress-strain behaviour of the soil becomes considerably more complex. It is important to relate these three characteristics in order to predict ground motion, attenuation of peak stresses, and accelerations. According to Weidlinger (1965), a theoretical approach usually assumes a constitutive equation which describes the expected stress-strain behaviour of soil and employs the tools of continuum mechanics to define wave-propagation parameters. When the material behaves according to the specified constitutive law, the correlation between actual and predicted behaviours will be reasonably good. In general, the available theoretical formulations predict that stress and particle velocity pulses increase in rise time and decrease in magnitude as they propagate through a material which exhibits strain-softening behaviour (Kolsky, 1963, and von Karman, 1950). The rise time will decrease ("shock up") if the stress-strain behaviour is characterized by strain hardening (Kolsky (1963), White and Griffis (1948)).

The work initiated by D.W. Taylor (1951) and later continued by Whitman is limited to rise times of the order of 5 milliseconds or larger, since smaller rise-times would result in large lateral inertia effects. This shortcoming represents one of the most difficult problems which can be overcome by the use of the Hopkinson pressure bar technique for dynamic loading.

The unconfined compression tests carried out by Casagrande

and Shannon (1948) achieved rise times between 10 milliseconds and one second. They observed that the strength of clay increases with increasing rates of loading.

The experimental studies carried out by Lee and Seed (1967) showed that cyclic stress applications would induce liquefaction or partial liquefaction of saturated sands. The number of cycles required to induce failure was found to be proportional to the confining pressure. They concluded that the higher the cyclic stress, the smaller is the number of stress cycles required to induce liquefaction or failure.

With regard to the strain rate effect, Morgan and Moore (1968), stated that the strength of saturated sand or clay being tested increases with the strain rate. The angle of shearing resistance was not found to be influenced by the increase in strain-rate.

Hardin and Black (1968) and Humphries and Wahls (1968), studied the influence of principal stress ratio and the static stress level on the dynamic shear modulus of normally consolidated Kaolinite and the effect of static stress history on the dynamic response.

Some of the above investigations, which were carried out to study the effects of transient loads, have shown an appreciable divergence between the theory based on the assumptions that the soil is an ideal elastic body and the experimental results.

Smith and Newmark (1958) carried out an investigation of the characteristics of stress wave propagation in soils in which the assumption of linear visco-elastic behaviour was applied. They realized the limitation of the visco-elastic model in taking into account the effects of non-linear stress-strain characteristics and the variation of the yield stress with strain rate. To approximate these effects a yield stress which increases with strain rate was incorporated into the spring elements which were given trilinear characteristics. The results of wave propagation in a variety of hypothetical soils were evaluated by means of a computer program. The following observations were made:

- (1) The amount of stress attenuation can be increased by increasing the relaxation time or decreasing the spring stiffness ratio. Both tend to increase the hysteresis loop and spread the wave out.
- (2) In soils whose stiffness increases with depth the stress may increase with depth, however accelerations would decrease.

Parkin (1962) developed a strain-rate sensitive model for one-dimensional wave propagation and demonstrated that the initial sharp stress peak observed in experiments can be explained without requiring the presence of lateral inertia as suggested by Smith and Newmark (1958). The results of the analysis showed that it is possible to obtain good agreement between the theoretical and experimental values of peak impact stress in sands for impact velocities ranging from 20 to 100 ips without the use of lateral inertia effects. This does not, of course, disprove the lateral

inertia hypothesis, but it does indicate that there are other possible explanations of the observed phenomenon. Parkin pointed out that the strain-rate theory is not a physical theory and no hypothesis concerning the origin of strain-rate sensitivity in dry sands is proposed. In fact one might question the discard of the lateral inertia phenomena that are known to exist in reality, for a strain-rate mechanism whose significance has not been established. In Parkin's work there could be two reasons for the occurrence of the initial stress peak at the impact end of the sand column. First, the impacting mass applied a frictional restraint to lateral expansion of the soil column. Until the wave travels several diameters along the specimen, the sand acts as though it were confined by the friction. Second, the lateral inertia effectively increases the specimen confining pressure; thus, at high impact velocities the stress should rapidly rise to a value above the static strength and then decrease to the static value after the lateral expansion has had time to take place. Approximate calculations of the time required for the lateral inertia effects to dissipate gave a range of 0.1 to 0.4 milliseconds. The actual recorded decay time for the stress peak was around 0.4 milliseconds.

The concept of a "locking material" has been applied to plastic wave propagation in materials. Ideal locking materials were introduced by Prager (1957) who considered the material to have properties such that beyond a certain strain, stress increase occurs without further increase in strain.

Salvadori et al (1960) have developed equations for the propagation of waves and shocks in locking and dissipative media by using a linear hysteretic model. An extension of the above work has been made by Skalak and Meidlinger (1961) to the case of a bi-linear material subjected to an exponentially decaying pressure pulse on its free surface. The problem was solved by the method of characteristics.

Heierli (1961) has developed an approach for one-dimensional problems which may be used with arbitrarily applied pressure pulses, stress-strain characteristics, and boundary conditions. This approach, which may be called the "method of impulses", employs a linear stress-strain relationship and involves dividing the applied pressure pulse into a finite number of steps. The solution involved the prediction of the stress and particle velocities as functions of time and depth in a soil column when the pressure at the surface is known. Selig (1964) reviewed the basic characteristics of shock induced stress wave propagation theories with a discussion of the effect of elastic, visco-elastic, plastic, non linear and inelastic behaviour. By using long horizontal bars of dry sand confined under constant lateral pressure, Selig showed that the major features of the observed stress waves can be explained on the basis of the characteristic non-linear, inelastic stress-strain behaviour of the sand without including time dependent effects.

At the present time, it is a common practice to conceive the parallel combination of elasticity and viscosity as dynamical properties

of soils in order to evaluate their character. R.L. Konder and R.J. Krizek (1965) assumed the soil response to be represented in terms of visco-elastic parameters and showed non-linear behaviour even at small values of dynamic strain. They indicated that the elastic portion of the clay response is non-linear even at small strains.

From the above review it is concluded that the dynamic properties of soils influence other important considerations in engineering works.

2.4 Consolidation of Cohesive Soils

2.4.1 Introduction

The process of consolidation involves the flow of fluid from the soil voids. During consolidation the stress is transferred from the porewater to the soil skeleton. Terzaghi's classical one-dimensional consolidation theory is based on the principle of effective stress. In this theory the effective stress represents that part of the total stress which governs the compression of the soil skeleton.

The stress history of soils since deposition has a considerable influence on their static as well as their dynamic behaviour. This stress history can best be studied from the results of a compression test plotted in the form of void ratio (or strain) versus the log of effective stress as in figures 2.1.a and 2.1.b.

The behaviour of the soil on the first application of stress is referred to as "virgin", and this portion of the curve of void ratio versus effective stress is called the virgin compression curve or field consolidation curve as shown in figures 2.1.a and 2.1.b.

A solid deposit is said to be normally consolidated if the present effective overburden pressures are the maximum pressures to which the deposit has ever been subjected at any time in its history. Also, a soil deposit that has been fully consolidated under a pressure larger than the present overburden pressure is said to be preconsolidated or overconsolidated.

Most natural deposits of clays are preconsolidated as a result of the change in the effective stresses by one or a combination of the following:

- i) Pressures due to overburden that was subsequently eroded or the loads of glaciers of past ages.
- ii) Dessication stresses due to moisture changes in the upper strata of a soil mass by natural means.
- iii) Changes in static groundwater level.
- iv) The loads of buildings that have been demolished.
- v) Temporary overloading such as may arise from an overriding continental ice sheet.

Two of the causes which lead to the preconsolidation of clays are illustrated in figure 2.2.

If a clay specimen which at some time in the past has been subjected to a stress of unknown magnitude is placed in a standard oedometer and again loaded, a curve such as shown in figure 2.1.b is obtained. The initial branch of the loading curve has a relatively flat slope which is usually referred to as the elastic portion of the curve. At a pressure close to the value of the maximum past stress level, the curve "breaks" and undergoes a significant change. The second branch of the curve exhibits a much steeper slope. The value of the maximum level of past stressing is called the preconsolidation pressure.

It is of practical importance to determine the maximum past pressure or the preconsolidation pressure of a clay soil which is to support a proposed structure, since it can be seen from figure 2.1.b that much greater deformation is expected from the soil if the building load exceeds the maximum past pressure.

The following section of the literature survey is concerned with a description of those works in the area of consolidation and the determination of the preconsolidation pressure of cohesive soils appropriate to this thesis.

FIGURE 2-1-a VOID RATIO VERSUS EFFECTIVE STRESS
FOR NORMALLY CONSOLIDATED SOILS

FIGURE 2-1-b VOID RATIO VERSUS EFFECTIVE STRESS
FOR PRECONSOLIDATED SOILS.

FIGURE 2-2 DIAGRAM ILLUSTRATING TWO GEOLOGICAL PROCESSES LEADING TO PRECOMPRESSION OF CLAYS. (AFTER TERZAGHI 1936)

2.4.2 Consolidation Theories Review

Consolidation theory is one of the major branches of theoretical soil mechanics which can be applied to many important practical problems. The first rational treatment of the subject was provided by Terzaghi (1923) who considered the soil as a two phase system consisting of solid particles and water. The Terzaghi theory gives a unique shape of time settlement curves for all consolidating samples subjected to one-dimensional quasi-static compressive stress. In practice, it was found that this was not always the case, and that with some samples the motion was considerably retarded. Taylor and Merchant (1940) proposed a modification of the Terzaghi theory. They produced the first rheological model for studying clay behaviour and Taylor subsequently called it theory A. Their theoretical analysis can be simulated to a Hookean spring in series with a Kelvin body (Christie, 1964). Later on, D.W. Taylor (1942) introduced his theory B which was, in fact, a modification of theory A.

Tan (1954-1957) developed a theory of consolidation in which the soil skeleton is assumed to be visco-elastic. Subsequently, Gibson and Lo (1961) proposed a similar one-dimensional theory in which the compressibility of an element of the soil skeleton is represented by a rheological model comprised of a Hookean spring in series with a Kelvin body.

The classical theory of one-dimensional consolidation

(Terzaghi 1923) is based on the following assumptions:

- a - The soil is homogeneous and is fully saturated.
- b - Compression of the soil skeleton and flow of water occur in one direction only.
- c - Water and individual soil particles are incompressible.
- d - The flow of water through the soil takes place in accordance with Darcy's law, and the coefficient of permeability is assumed to be constant for any increment of stress.
- e - Only small strains, and the corresponding small changes in void ratio, are involved.
- f - The relationship between void ratio and effective stress is linear and independent of time.

A theory of non-linear consolidation for oedometer boundary conditions has been developed by Davis and Raymond (1965) assuming a constant coefficient of consolidation, while both the compressibility and permeability were allowed to decrease with increasing pressure.

Attention has recently been focused on the non-linear behaviour of soils during consolidation. McNabb (1960) has derived a one-dimensional consolidation equation in a very general form. Richart (1957) has solved the problem by means of the finite difference technique considering a variable void ratio. Schiffman (1958) considered the case of non-linear variation of the coefficient of permeability.

The conventional oedometer test, based on Terzaghi's work, has been used by practicing engineers for nearly 40 years without major modifications. Most laboratories use standard procedures to perform the test and to interpret the data. Usually, load-increment ratios of unity are applied, and each increment is left on for 24 hours to obtain characteristic time-settlement curves. Using these procedures a consolidation test may take several weeks to be completed.

In 1969 two new methods of performing these tests were reported. One is referred to as the controlled gradient test of Lowe, et al. (1969), and the other is the constant rate of strain test of Smith and Wahls (1969). To overcome some of the limitations of the conventional test and to incorporate some of the controls of modern soil tests, a consolidometer apparatus was developed at MIT by A.Z. Wissa et al. (1971). Specimens can be saturated at constant volume under a back pressure and loaded, with no lateral strain, by incremental loads, at a constant rate of strain, or at a constant rate of stress.

2.4.3 Determination of the Preconsolidation Pressure of Cohesive Soils

The amount of preconsolidation in clay soils can be estimated in a number of ways. However, the most common method is by conducting oedometer consolidation tests to determine the preconsolidation pressure. All important factor that affects test results is sample disturbance. The stress reduction and changes in principal stress ratio that

occur due to soil sampling and specimen preparation are probably responsible for most of the disagreements between laboratory results and field observations. One of the aims of the present research is to establish a reliable technique for determining the preconsolidation pressure in-situ to avoid sample disturbances.

Based on laboratory observations of compression, rebound, and recompression characteristics of soil specimens, Casagrande (1936) developed a graphical method for obtaining the preconsolidation pressure. Terzaghi (1944) questioned the merits of this procedure and cites cases of serious disagreement. In reply, Casagrande and Fadum (1944) suggested that there exists no alternate means of estimating the preconsolidation history of a natural clay deposit. Rutledge (1944) quotes evidence in support of the procedure, and Schmertmann (1955) has further developed the interpretation originated by Casagrande (1936). It is now a generally accepted treatment, although there is no doubt that large discrepancies may occur with certain types of soil especially if samples at great depths. The implications of stress changes due to sampling are discussed at length by Rutledge (1944) and Terzaghi (1944) in an attempt to visualize the pressure-void ratio curve in nature and its relationship to the laboratory curve. Rutledge (1944) concluded that sample disturbance tended to obscure the stress history and preconsolidation pressure of a soil. Van Zelst (1948) reported that most disturbance was due to remoulding caused by trimming the flat faces of test specimens.

The consolidation apparatus, testing, and sampling techniques are factors which clearly have an important influence on the validity of test results. D.W. Taylor (1942) investigated the effects of friction between the consolidation ring in the apparatus and the specimen being tested. He found that this side friction had a considerable effect on the shape of pressure-void ratio curve but that the rate of compression was affected in only a small way.

The above information indicates a requirement for great care in the field sampling procedure, and in the preparation of samples for testing in the laboratory in order that valid information regarding the compressibility of a particular soil can be made available.

2.5 Stress History Effects on the Elastic Moduli of Cohesive Soils

2.5.1 Introduction

In natural deposits, the rigidity generally increases with depth as a consequence of the increasing effective overburden pressure. The relationship between elastic moduli and amount of overburden has not been clearly shown. This section of the literature survey is concerned with a description of those few works related to the stress history effects on the elastic moduli of soils.

2.5.2 Literature Survey

Few laboratory studies have contributed to the understanding of factors that influence the dynamic moduli of soils, particularly with regard to their stress history. However, most of these studies (see the Summary in Hardin and Black, 1968) have concentrated on cohesionless soils, and only limited investigations have been published on cohesive soils.

A. Paun (1963) introduced a dynamic analogy for foundation-soil systems. In order to arrive at a suitable solution he assumed that the elastic moduli of cohesionless soils increased linearly with depth and that the elastic moduli of cohesive soils remained constant with increasing depth.

Gibson (1967) studied, theoretically, the influence of a variable Young's modulus with depth on the stress and displacements in an isotropic elastic half-space. He examined in some detail the linearly increasing magnitude of Young's modulus with depth.

Humphries and Wahls (1968) studied the effects of static stress history on the dynamic response of Kaolin and bentonite. They concluded that the shear modulus increases with increasing effective pressure and decreasing void ratio. They also indicated that the dynamic shear modulus is not significantly affected by the length of the test specimens.

Hardin and Black (1968) assumed that the shear modulus is a function of effective stress, void ratio, degree of saturation, grain shape, soil structure and temperature. They concluded that the shear modulus decreases with increasing void ratio, and increases with time of applied load. They also concluded that the shear modulus is independent of the amplitude of vibration.

From the above review it can be concluded that there is no reliably known relationship between the elastic moduli and the stress history of cohesive soils.

2.6 Discussion of Present Methods of Testing

Most of the past work in soil dynamics was done by using triaxial and direct shear devices with various modifications for dynamic loadings in which the tests were carried out only at stress levels up to those representing failure conditions for the soil being tested. However, tests short of failure are not completely satisfactory since they do not give a complete account of the material behaviour. Also, it is a fact that the triaxial compression test and direct shear test both cause nonhomogeneous stress and strain conditions in the test specimen. This makes the interpretation of these test results difficult, even in the case of quasi static tests. The geometry of the specimens and rigidity of their boundaries dictate, in advance, the zones in which the shear strains and sample dilation will be concentrated. Careful measurements

of the changes in geometry and strain are often made and provide useful information in static triaxial or direct shear tests. However, in dynamic triaxial and direct shear tests, the changes in stress and geometry are large and occur quickly; hence the ability to measure these dynamic effects is limited. Thus, experimental studies will require development before they can be relied upon to provide means of understanding the behaviour of soils under rapid loading and unloading conditions.

One of the objectives of this research is to study the dynamic response of cohesive soils by using a modification of the Hopkinson pressure Bar. This technique for the testing of cohesive soils was introduced by Fletcher (1967).

2.7 Conclusions

On the basis of the above literature review it seems reasonable to conclude that no clear understanding of the stress-strain-time behaviour of soils has been reached. One of the principal reasons for the lack of success in this area has been the lack of testing techniques and appropriate theoretical explanations which lend themselves to the study of soils subjected to dynamic loading conditions.

CHAPTER III

Dynamic Testing Apparatus and

Evolution of the Hopkinson Pressure Bar

3.1 Introduction

The purpose of this chapter is to review briefly various experimental techniques used in the testing of soils under dynamic loading conditions. Also, a review is done on the development and modification of the Hopkinson Pressure Bar technique for the dynamic testing of materials.

In the determination of the dynamic properties of soils, two principal groups of experimental techniques have been used. The first is basically a vibratory technique in which relatively small amplitudes and a large number of loading cycles are employed. The second group can be classified generally to include systems involving impulse or impact loading. The impact system of loading is usually applied to simulate the effects of explosions and earthquakes on a soil mass. In this work, apparatus consistent with the second method has been adopted. One aim of such a system is to achieve a transient load characterized by a short rise time and short duration. Impulse or impact systems have been used to investigate many problems, some of which are:

- a - The dynamic stress-strain relationships of various soil types.
- b - The dynamic load-carrying capacity of footings.
- c - The performance of earth structures under dynamic loading conditions.
- d - Wave propagation studies in soils.
- e - The dynamic interaction between buried structures and surrounding soils.

3.2 Dynamic Testing Systems

There are six general categories of apparatus available:

1. Gravity
2. Shock tube.
3. Hydraulic.
4. Pneumatic.
5. Explosively operated.
6. The Split Hopkinson Pressure Bar.

3.2.1 Gravity Apparatus

The earliest of the gravity apparatus were modified static testing apparatus. A type of such device was used by Casagrande and Shannon (1948). The apparatus consisted of a pendulum which was released from a selected height to strike a compressed spring which was connected to a piston. The advancing piston developed a pressure on the hydraulic system which was transmitted to the specimen through a second piston. This particular machine produced rise time in the order of 10 milliseconds.

Seed and Lundgren (1954) utilized a system consisting of a weight dropped from a selected height onto a spring which transmitted the pressure through a piston to the soil sample. The rise time achieved by this system was about 20 milliseconds. A recent use of the pendulum apparatus has been reported by Selig (1964). The apparatus basically consisted of a steel reaction pendulum with the soil specimen attached and a second steel pendulum for impacting the specimen.

Casagrande and Shannon (1948) have reported another gravity apparatus. The arrangement consists essentially of a beam, dash pot and yoke. The dash pot controls the velocity of the falling beam and the yoke transmits the load from the beam to the specimen.

3.2.2 Shock Tube Apparatus

This particular method of loading has not been used extensively in the testing of soils. In one such case though, a soil specimen was confined laterally in a shock tube and exposed to a shock wave which travelled over its surface. Selig (1964) used a shock tube to study the behaviour of soil columns. The equipment makes use of an air shock which induces a shock wave in the soil specimen. The propagation of the wave along the soil column was recorded through embedded pressure gauges. The pulse rise times recorded for this technique were in the order of 5 to 20 milliseconds.

3.2.3 Hydraulic Apparatus

This system was employed by Casagrande and Shannon (1948). It afforded a slower rate of loading. It consisted of a constant volume vane-type hydraulic pump connected to a hydraulic cylinder through valves with which either the pressure in the cylinder or the volume of liquid delivered to the cylinder could be controlled. The rise times of this system could be varied from 50 milliseconds to 10 minutes.

3.2.4 Pneumatic Apparatus

This apparatus had a wide application in the testing of soils under quasi-static conditions. Pneumatically operated systems have been used to study the dynamic load-carrying capacities of footings when subjected to single large impulse loads. The system utilizes the rapid expansion of high pressure gases. Sinnamon and Newmark (1961) utilized a series of chambers and valves to apply gas pressure to soil specimens and foundation footings.

The sophisticated type of the pneumatically operated equipment reported by Cunney and Sloan (1961) varied in its rise time from 5 to 150 milliseconds. The method is well suited to the study of the dynamic load carrying capacity of footings.

3.2.5 Explosively Operated Apparatus

The principle of these devices is that of the rapid generation of gas pressure by burning explosives. In some respects this type of apparatus is similar to the pneumatic type and in its simpler forms is less expensive; however, if a high degree of test control is required, the cost becomes very high.

The methods discussed so far have been applied in connection with some investigations into specific problems in soil dynamics. With regard to the work carried out by the author, the above systems have certain disadvantages. The rate of loading of the gravity apparatus is too slow to simulate the effects of very rapid dynamic loading and to study effectively the response of soils to dynamic loading. The pneumatically and explosively operated apparatus have satisfactorily high loading rates but they are bulky and expensive. However, they have been used in the study of the performance of footings subjected to dynamic loads. Very little is reported about their application to the study of wave propagation problems in soils. The shock tube technique has the fastest pulse rise time of the above techniques but its application appears to be limited by the difficulties involved in effectively controlling the force-time shape of the pressure pulse.

3.2.6 The Split Hopkinson Pressure Bar

The device known as the split Hopkinson Pressure Bar provides an indirect method of applying a short rise time, short duration, high amplitude pulse to a material specimen sandwiched in series between two lengths of a cylindrical steel bar. This apparatus has been used for a variety of purposes, including ballistic studies, studies of longitudinal wave transmission in various materials and the determination of dynamic stress-strain relationships in some materials.

The split Hopkinson pressure bar was first used by Kolsky (1949) to study the response of certain metals and synthetic materials to very rapid loading. Further modifications of the technique are due to Lindholm (1964). The principle of this technique was first applied in the area of soil mechanics by Fletcher (1967) to study the response of saturated clays to very rapid loading. The technique is based on the theory of one-dimensional wave propagation. Figure 3.1 shows the experimental configuration used in this work.

3.3 Development of Hopkinson Pressure Bar

The determination of the actual pressures produced by a blow or by the detonation of high explosives is a problem of much scientific and practical interest but is of considerable difficulty. Hopkinson (1914) developed a technique to measure transient impulsive

FIGURE 3-1 HOPKINSON PRESSURE BAR .

FIGURE 3-1 HOPKINSON PRESSURE BAR .

pressures in steel bars. The device is essentially an elastic bar with a large length to diameter ratio into which some pressure-time load is applied at one end and propagated along the bar.

According to a study made by R.M. Davies (1948) the original Hopkinson method suffers from two disadvantages. First, the inevitable adhesion between the detachable end-pieces and the bar makes it difficult to obtain accurate results when the pressure is less than about 3 tons/sq.in. Second, although the method can be used satisfactorily to measure maximum pressures and, with more difficulty, to measure the time during which the pressure exceeds any given value, the relation between pressure and time cannot be detected.

Davies further described a modified form of the apparatus in which the measurements are made electronically. He obtained complete stress-time relationships by measuring either the radial or longitudinal deformation of the bar with electrical condensers. These radial or longitudinal displacements produce a change in the capacity of a suitable condenser unit which is charged to a high potential and is connected through a feed circuit and an amplifier to a double-beam cathode-ray oscillograph.

Kolsky (1949) employed the split Hopkinson pressure bar technique for making stress-strain measurements. The specimens, which were in the form of thin discs, were placed between the flat faces of two cylindrical steel bars. The transient pressure was applied by firing

a detonator at the end of one of the bars and the displacement at the free end of the other bar was measured with a parallel-plate condenser microphone. A cylindrical condenser microphone was also fitted around the bar between the detonator and the specimen. This was used to measure the amplitude of the pressure pulse arriving at the specimen, and the deformation of the specimen could then be evaluated. The thickness of the specimens was about 0.03 cm., and the specimen diameters were made smaller than that of the elastic bar in order to allow for lateral expansion when compressed axially. Both surfaces of the specimen were lubricated before being placed in the apparatus in order to reduce friction effects.

In 1956 Davies discussed the problem of the transmission of transient stress pulses of short duration in solids, together with the associated problem of the measurement of pressures subject to very rapid changes. The split Hopkinson pressure bar method, which forms one of the basic experimental techniques in the field of stress pulses transmission, was described in some detail. An analysis of the limitations of the method leads to a discussion of the change of form of stress wave pulses as they propagate from the origin, the main features of the distortion were discussed when the pulses are propagated in (a) elastic solids, (b) visco-elastic solids, and (c) solids stressed beyond the elastic limit.

In a series of studies, Bell (1956, 1960, 1968) used an

elastic steel bar for transmission of stress waves into an adjacent specimen. He was able to measure the strain response of the specimen over gauge lengths directly. This measurement did not require the assumption in advance of either a theory of stress wave propagation or the definition of a constitutive equation for the material. Bell observed that under very high strain rates, there exist non-uniform finite strain distributions and non-linear wave propagation and reflection.

Chiddister and Malvern (1963) used short specimens in the split Hopkinson bar in testing aluminum at elevated temperatures. The impact was provided by a third steel striker bar.

The mechanical behaviour of some metals and polymers has been investigated by Davies and Hunter (1963) using the method of the split Hopkinson pressure bar. Cylindrical specimens of comparable length and diameter were sandwiched between the two elastic rods and deformed under the action of a compressive stress wave induced into the free end of one of the rods by the detonation of an explosive pellet. Measurements were made of the displacement-time relation for the free end of the other rod using a parallel plate condenser microphone. They proved the validity of the split Hopkinson pressure bar technique for evaluating the mechanical behaviour of soft metals and polymers for strain rates in the range 10^3 - 10^4 sec.⁻¹.

An application of the split Hopkinson pressure bar to the dynamic testing of materials was given by Lindholm (1964) whereby

continuous records of strain vs. time, stress vs. time, and stress vs. strain in times of the order of 200 μ secs. were obtained. He showed that the split Hopkinson pressure bar method is applicable to certain metallic materials.

Conn (1965) discussed the validity and limitations of a one-dimensional theory of bar impact particularly as it related to the split Hopkinson bar technique and the thickness of the samples being tested.

The dynamic response of concrete and cement-paste has been investigated by Goldsmith and Yang (1966) using the split Hopkinson pressure bar technique. Transient loading was accomplished by central longitudinal impact of a 0.5 in. diam. steel sphere on a ballistically suspended 0.75 in. diam. Hopkinson bar made of concrete at initial velocities ranging from 1650 ips. to 3260 ips.

Fletcher (1967) employed a modification of the split Hopkinson pressure bar to determine the stress-strain response of saturated Kaolin clay at very high rates of loadings. Plate-shaped samples were subjected to loads applied in a direction parallel to the axis of the plate. Variations of both load intensity and deformation of samples were recorded as functions of time and were subsequently correlated assuming the rationality of certain superposition principles.

3.4 Selection and Modification of Apparatus for Test Program

In most of the experiments which have been conducted dealing with soil dynamics or soil-structure interaction, the observed measurements have been restricted to the input loading. The stress and strain developed in the soil have been omitted from direct consideration because of the problems involved in measuring them. Techniques for measuring stress and strain are thus in demand. For example there are two inherent difficulties with the use of gauges in soil. First, the gauge does not behave in the same way as the soil around it; and second, the installation of the gauge usually causes disturbance of the soil in the immediate vicinity of the gauge. The first point is especially true for pressure transducers which, in order to be ideal, would require the matching of their stress-strain properties with those of the surrounding soil.

The author's work is essentially a continuation of that of Fletcher (1967). A further modification of the Hopkinson pressure bar is applied. The change made involves the use of only the "incident" bar with a cylindrical condenser mounted in such a way that only the incident and reflected stress pulses are recorded. The principal advantage of this system is that field samples can be treated without extrusion from the sampling tube and to be used in field after the development of practicable apparatus. This results in a reduction in sample disturbance and suggests the possibility of the availability of field application of the technique after further development.

A detailed account of the apparatus and its application in this work follows in Chapter IV.

CHAPTER IV

Description of Test Equipment

4.1 Introduction

The accurate measurement of pressures subjected to very rapid changes presents a number of difficulties. Ordinary mechanical gauges which can withstand high pressures are severely limited by their natural frequencies, and when dealing with changes of pressure which take place in times of the order of 10μ sec., their readings may be false. These false readings are due to the limited time response of the gauges. Piezo-electric gauges will faithfully record pressures which change in times of the order of 10μ sec., but their use is limited to the measurement of comparatively low pressures because of the fragility of the Piezo-electric crystal elements which they contain. The most satisfactory method so far developed for dealing with the problem seems to involve the use of the Hopkinson pressure-bar.

The experimental technique used in this work provides a method of obtaining strain vs. time information at speeds very much in excess of those which could be obtained using mechanical devices.

This chapter includes the description of the Hopkinson Pressure bar technique, performance of the apparatus, and the attendant theory of measurements.

4.2 Description of the Method

A schematic diagram of the Hopkinson pressure bar with all arrangements as used in this work is shown in Figure 4-1. A photograph of the bar and attendant apparatus is shown in Figure 4-2.

A compressive stress pulse is initiated at the anvil end of the incident bar by means of a steel ball rolling from a predetermined height over a smooth surface. This pulse, which has an intensity of stress that varies with the height from which the ball was rolled down the ramp, moves along the incident bar to provide incident compressive stress at the bar-specimen interface. This stress pulse is of short duration but of amplitude high enough to cause deformation in the saturated soil sample. This incident stress pulse decomposes into reflected and transmitted components at the soil-bar interface. The reflected component propagates back into the bar in the form of a tension wave. Variations of the diameter of the incident bar caused by the stress wave vary the capacitance of the condenser. This capacitance variation is converted into a proportional change in voltage, which after necessary amplification is recorded on a cathode ray oscilloscope. The radial displacement-time relations $\xi_I(t)$ and $\xi_R(t)$ in the bar are recorded through the condenser microphone, where $\xi_I(t)$ and $\xi_R(t)$ are radial deformations in the bar due to incident and reflected pulses respectively. Lateral deformations of the incident bar due to the stress

FIGURE 4-1

SCHEMATIC DIAGRAM OF APPARATUS.

FIGURE 4-2 SAMPLE AFTER PLACEMENT IN
TESTING DEVICE .

FIGURE 4-2 SAMPLE AFTER PLACEMENT IN
TESTING DEVICE .

pulse provide the information needed to evaluate stress versus displacement at the soil face.

With the experimental information, $\xi_I(t)$ and $\xi_R(t)$, it is possible to determine the stress-time and displacement-time relationships for the specimen under the test conditions, as will be shown in section 4.7 and appendix A.

4.3 The Pressure Bar

The incident pressure bar is constructed of one inch diameter steel drill rod. The pressures to be studied were well within the elastic range of the steel used so that elastic wave theories could be used. The end in contact with the specimen is ground flat. The incident bar is six feet long. The factors which determine the minimum length of the bar are based on consideration of dispersion of the stress wave which is related to the wave length of the pulses to be measured, and the assumption that the pressure due to the pulse is uniformly distributed over the cross section of the bar. Davies (1948) estimated that when a bar of two feet long and one inch in diameter is used to measure stress pulses of about 20 microseconds duration the distortion causes errors of between 2 and 3 per cent in the values obtained of the measured stress.

The assumption that the pressure is uniformly distributed over the cross section of the bar is acceptable at any point past about four diameter of the bar from the leading end (Davies (1948)).

The pressure end of the incident bar was protected by using a short hardened steel anvil bar of the same diameter as the incident bar. The front surface of the anvil was exposed to the striker, which was a steel ball, and the back surface was ground and wrung to the end of the incident bar.

Davies (1948) has stated that the minimum pressures which could be accurately recorded with a one inch diameter bar with the instrumentation then available was about 200 psi. Fletcher (1967) has been successful in recording pressures in the order of 25 psi. The author was successful in recording pressures less than 25 psi.

Certain properties of the bar had to be determined so that the experimental data could be processed.

From the manufacturer's specifications the following properties were available:

Young's Modulus $E = 30 \times 10^6$ psi
Shear Modulus $G = 11.55 \times 10^6$ psi
Poisson's Ratio $\mu = 0.3$
Unit Weight $\rho = 489$ pcf

The properties of the bar were determined experimentally by conducting a direct tensile strength test using electrical strain gages. From the experimental test data the following values were obtained:

Young's Modulus $E = 28.89 \times 10^6$ psi
Shear Modulus $G = 11.30 \times 10^6$ psi
Poisson's Ratio $\mu = 0.2779$
Unit weight $\rho = 485$ pcf

The values of the elastic wave velocity, C_o , needed to be known for interpretation of the experimental data. The calculated elastic wave velocity given by the equation $C_o = \sqrt{\frac{E}{\rho_m}}$ is found to be 200,484 in/sec. It was also determined experimentally by observing the time required for the passage of a wave between two condensers at a measured distance on the incident bar and was found to be 200,000 in./sec. The difference in the two values is considered to be within the limits of acceptable experimental error.

4.4 The Bar Conducer Unit

The construction of the cylindrical condenser is shown in Figure 4-3. The configuration is similar to the one used by Fletcher (1967) and Mathur (1969). The condenser unit consisted of a mild steel disc 0.126 inches thick with an opening of 1.0125 inch diameter. The condenser was carefully centered on the bar using a lathe so that the gap of 0.00625 inches between the bar and conductor was uniform around the circumference of the bar. The disc conductor is insulated from the guard rings by thin sheets of mylar C. The conductor is insulated from the bar D by perspex collars E which support it elastically relative to the bar. The disc conductor was charged to 300 V with respect to the pressure bar.

When the load is applied to the pressure end of the bar, a longitudinal wave of compression travels down the bar and causes a lateral expansion of the bar. When the wave is reflected, and a

FIGURE 4-3 CONDENSER UNIT CONSTRUCTION.

longitudinal wave of extension travels back along the bar, a lateral contraction of the bar takes place. The change in capacitance of the cylindrical condenser is caused only by the change in radius of the bar due to its lateral expansion and contraction. This change in capacitance was converted into a proportional variation in voltage by a feed unit circuit and fed to an oscilloscope after adequate amplification, and finally recorded on the screen.

As the pulse travels along the bar it gives rise to radial expansion ξ

$$\xi = \frac{\mu r P}{E} \quad (4.1)$$

where μ = Poisson's ratio

r = radius of bar

P = pressure

E = Young's Modulus.

Thus,

$$P = \frac{\xi E}{r \mu} \quad (4.2)$$

The cylindrical condenser capacity is given by the expression

$$C_1 = \frac{2 \pi E_o L}{\ln \left(\frac{r'}{r} \right)} \quad (4.3)$$

where E_o = Dielectric constant of air (8.85×10^{-12})
Coulomb/Newton-Meter)

L = length of condenser cylinder
= 0.126 inches

r = bar radius
= 0.500 inches

r' = Condenser cylinder radius
= 0.50625 inches

For the configuration used in this work, the values of C_1 and cable capacitance are found in page 53.

For good response of the condenser two conditions should be met:

- (1) The gap between the ring condenser and the bar ($r' - r$) should be large compared to the change in radius of the bar, ξ , so that the change in capacitance due to the change in radius will be small compared to C_1 . For the condenser used, ($r' - r$) was 0.00625 inches and the maximum change in the bar diameter was about 15×10^{-6} inches.
- (2) The length of the pressure pulse in terms of time should be long compared to the length L of the condenser. That is $\frac{L}{C_o}$ should be small compared to t . In this case $\frac{L}{C_o} = 0.63$ microseconds and $t = 100$ microseconds.

Thus the above two conditions for good response are met in the present experiment.

It is likely that some error will exist due to a small eccentricity between the baraxis and the condenser. Fletcher (1967) evaluated this error by estimating the maximum probable eccentricity to be 0.0015 inches and he showed that the error is negligible.

4.5 Condenser Feed Unit

The bar condenser is incorporated in a circuit as shown in Figure 4-4.

This configuration is similar to that used by Davies (1948), Kolsky (1949), Davies and Hunter (1963), and Fletcher (1967). The condenser C_1 represents the bar condenser whose initial capacity is existing 14.3 pF. The lower plate of C_1 is the grounded surface of the pressure bar, and the upper surface is the insulated conductor of the condensor unit. This insulated plate is connected through a resistance R_1 to a 300 V d.c. power supply. The insulated plate of C_1 is also connected through a large capacitance C_3 which isolates the amplifier from the power supply. From here, the condenser C_1 is shunted with C_2 and R_2 . The entire circuit, as well as the leads to the amplifier, are adequately shielded to prevent pick up from extraneous sources such as grounded conductors in the vicinity of the apparatus.

The effective time constant of this circuit is found from the following expression:

FIGURE 4-4 CONDENSER FEED UNIT.

TYPICAL VALUES :

R_1	56.8	M Ω	
R_2	2.0	M Ω	
C_1	14.3	P F	
C_2	33 0.0	P F	
C_3	0.1	μ F	
C_i	49.5	P F	(input cable)
C_{0t}	49.6	P F	(output cable)

$$\text{time constant} = R_2 \frac{1}{\frac{1}{C_i + C_1 + C_2 + C_{ot}} + \frac{1}{C_3}}$$

In the present case where $R_2 = 2.0 \text{ M}$, $C_1 = 14.3 \text{ pF}$, $C_2 = 330.0 \text{ pF}$, $C_3 = 0.1 \text{ }\mu\text{F}$, $C_i = 49.5 \text{ pF}$ and $C_{ot} = 49.5 \text{ pF}$, the time constant takes the value of 0.9 milliseconds.

This value is large compared to the maximum pressure pulse time of about 100 microseconds, so that the circuit operates essentially under constant charge conditions which is required for the proportionality in the capacitance variation in voltage fed to the amplifier.

The transient voltage due to the change in the capacitance of the condenser caused by the change in bar diameter is fed to a Hewlett-Packard 141A oscilloscope via an amplifier under single sweep conditions. The time base is triggered through the delayed sweep selector operated when the striker ball closes the triggering circuit. The sweep is delayed by the delayed sweep selector till the pressure pulse is nearly at the condenser.

4.6 Performance of the Apparatus

The lateral displacement in the bar can be calculated from the relation

$$\xi = \frac{Y}{\frac{C_1 V}{CND} + \frac{Y}{D}} \quad (4.4)$$

where Y = oscilloscope deflection
 C₁ = capacitance of the bar condenser
 V = voltage across the bar condenser
 N = sensitivity of the oscilloscope
 D = gap between the plates of the bar condenser
 C = equivalent capacity for the circuit.

For condenser calibration, the results obtained were compared with those results obtained from another condenser having different C₁ and D. The measured value of each condenser was checked against that of the other to be certain that any inaccuracies were minimized. This was done by placing both condensers on the incident bar as close together as possible and then impacting the bar. The displacements deduced from the maximum trace deflections should be the same for each condenser unit when detecting the same disturbance. The striker ball was given a vertical fall to cause deflection on the oscilloscope screen using different scope sensitivities. The lateral displacements for maximum trace deflections were calculated and the disagreement was negligible.

First sets of experimental data (figures 6.1 to 6.7) are based on the use of the following values:

$$C_1 = 13.5 \text{ pF}$$

$$D = 0.0075 \text{ in}$$

$$C_i + C_{OT} = 54 \text{ pF}$$

Also, figures 6.9 to 6.12 are based on the use of the manufacturer's specifications of bar properties. Due to a suspicion in the measured values of $C_i + C_{OT}$ that gives an error in the order of 4%, the writer verified the results obtained using the above values by carrying out tests on three groups of soil samples representing consolidation pressures of 25, 50 and 75 psi. Each group was subjected to a number of different stress levels. The information obtained (figures 6.8 and 6.17 to 6.21) are based on the use of the bar properties obtained experimentally and the following circuit values:

$$C_1 = 14.3 \text{ pF}$$

$$D = 0.00625 \text{ in}$$

$$C_i + C_{OT} = 99.1 \text{ pF}$$

Comparison between the obtained results is shown in figures 6.22 and 6.23.

4.7 Theory of Measurements

If an elastic bar is subjected to an impact at one end in such a way that the limit of proportionality of the bar is not exceeded, a longitudinal elastic wave will propagate along the bar. For the purposes of this study it is necessary to make the following assumptions with regard to the nature of the elastic wave front.

i - The whole cross sectional area of the bar has the same longitudinal displacement, i.e. the cross-section of the rod is considered to remain plane during the motion.

ii - Uniform distribution of stress exists over each cross-section.

Consider a small element PQ of length dx and let the cross-sectional area of the rod be A (see Figure 4-5). If the stress on the face passing through P is σ_x the stress on the other face will be given by $\sigma_x + \left(\frac{\partial \sigma_x}{\partial x}\right)dx$, and if the displacement of the element is given by U , we have, from Newton's second law of motion:

$$\rho A dx \frac{\partial^2 u}{\partial t^2} = A \frac{\partial \sigma_x}{\partial x} dx \quad (4.5-1)$$

(where ρ is the density of the rod material).

iii - The ratio between the stress σ_x and the strain $\frac{\partial u}{\partial x}$ in the element is Young's Modulus E , so that

$$\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} \quad (4.5-2)$$

FIGURE 4-5 FORCES ACTING ON AN ELEMENT OF BAR IN LONGITUDINAL MOTION.

Equation 4.5-2 is the well known wave equation in one dimension which has a solution of the form:

$$U = f(C_o \cdot t - x) + F(C_o \cdot t + x) \quad 4.5-3$$

where f and F are here arbitrary functions depending on the initial condition and $C_o = \sqrt{\frac{E}{\rho}}$. The function f corresponds to a wave travelling in the direction of increasing X , whilst the function F corresponds to a wave travelling in the opposite direction.

For simplicity consider a wave travelling in the direction of decreasing X only, then we have:

$$U = F(C_o \cdot t + x) \quad 4.5-4$$

Differentiating both sides with respect to x we have:

$$\frac{\partial u}{\partial x} = F'(C_o \cdot t + x) \quad 4.5-5$$

where F' denotes differentiation with respect to the argument $(C_o \cdot t + x)$.

Similarly, if we differentiate (4.5-4) with respect to t we have

$$\frac{\partial u}{\partial t} = C_o F'(C_o \cdot t + x) \quad 4.5-6$$

Thus from equations 4.5-5 and 4.5-6

$$\frac{\partial u}{\partial t} = C_o \frac{\partial u}{\partial x}$$

Now $\frac{\partial u}{\partial x}$ is equal to $\frac{\sigma_x}{E}$, so that

$$\sigma_x = \left(\frac{E}{C_o}\right) \frac{\partial u}{\partial t} = \rho C_o \frac{\partial u}{\partial t} \quad 4.5-7$$

Equation 4.5-7 shows that there is a linear relation between stress and particle velocity at any point.

The condenser provides information via the oscilloscope with regard to the incident and reflected pressure pulses. This information, when processed gives the lateral deformation in the bar due to these pulses and is denoted as ξ_I and ξ_R . These values are converted directly to longitudinal strain by the relationship:

$$\epsilon_x = \frac{\xi}{\mu r} \quad 4.5-8$$

where μ is Poisson's ratio of bar material.

and r is the bar radius.

From 4.5-7

$$\sigma_x = \epsilon_x E = \rho C_o \frac{\partial u}{\partial t} \quad 4.5-9$$

since $C_o = \sqrt{\frac{E}{\rho}}$

∴

$$U_x = c_o \int_0^t \epsilon_x dt$$

$$= \frac{c_o}{\mu_r} \int_0^t \xi dt$$

4.5-10

The displacement, U_x , of the face of the incident bar in contact with the specimen is the result of both the incident stress pulse and the reflected stress pulse.

Thus:

$$U_x = \frac{c_o}{\mu_r} \int_0^t \xi_I dt + \left(\frac{-c_o}{\mu_r} \right) \int_0^t \xi_R dt$$

$$= \frac{c_o}{\mu_r} \int_0^t (\xi_I - \xi_R) dt$$

4.5-11

The applied stress on the sample face is simply

$$\sigma_x = E (\epsilon_I + \epsilon_R)$$

4.5-12

This can be written as

$$\sigma_x = \frac{E}{\mu r} (\xi_I + \xi_R) \quad 4.5-13$$

In the above derivations the recorded signals ξ_I and ξ_R are always assumed to be shifted along the time axis so as to be coincident at the specimen face. Typical records showing the variations of pressure with time for the incident and reflected pulses at various stress levels are shown in Figure 4-6.

4.8 Determination of the Elastic Moduli of the Test Specimens

The determination of the dynamic modulus, E , of the soil specimens is based on the assumption that the particle velocities in the pressure bar and the soil specimen are identical at the interface between the bar and the sample. This relationship was shown by Donnell (1930) and leads to the expression

$$E_S = \frac{P_T^2}{(P_I - P_R)^2} \cdot \frac{M_B E_B A_B}{M_S A_S} \quad (4.5.14)$$

where -: E_S = modulus of elasticity of soil specimen.

E_B = modulus of elasticity of bar material.

P_T = magnitude of transmitted force through soil specimen.

P_I = magnitude of incident force in the bar.

FIGURE 4-6 TYPICAL OSCILLOSCOPE RECORDS SHOWING
THE VARIATION OF PRESSURE VS. TIME
FOR INCIDENT AND REFLECTED PULSES
AT VARIOUS STRESS LEVELS .

(FOR SAMPLES 1.0" DIAMETER CONSOLIDATED TO 75 PSI .)

FIGURE 4-6 TYPICAL OSCILLOSCOPE RECORDS SHOWING
THE VARIATION OF PRESSURE VS. TIME
FOR INCIDENT AND REFLECTED PULSES
AT VARIOUS STRESS LEVELS .

(FOR SAMPLES 1.0" DIAMETER CONSOLIDATED TO 75 PSI .)

P_R = magnitude of reflected force at the interface between the bar and soil specimen.

M_B = the inertia constant of the bar, equals to the mass of bar per unit length.

M_S = the inertia constant of the soil material.

A_B = area of bar.

A_S = area of soil specimen.

The dynamic shear modulus, G , consistent with the consolidation state of the sample and the rate of soil deformation being applied was determined from the relationship

$$G = \frac{E}{2(1+\mu)}$$

For the elastic range, this relationship is assumed to be valid and the value of Poisson's ratio μ was taken to be 0.5 for a saturated soil.

In case of soil specimens having an area bigger than that of the bar, it is necessary to know whether the area of the Hopkinson bar or the soil sample should be used for the terms M_S and A_S in equation 4.5.14. This will be done by testing samples of 2.0" diameter as well as 1.0" diameter.

CHAPTER V

Sample Preparation

5.1 Introduction

The information presented in this chapter concerns the preparation and treatment of the soil materials to be tested, as well as details concerning the consolidation apparatus.

5.2 Laboratory Manufactured Specimens

A commercially produced Kaolin from Edgar, Florida was used in this work. The material was received in dry, powdered form. It has a specific gravity of solids of 2.6, a liquid limit of 68% and a plastic limit of 37%. Samples were prepared under three different consolidation pressures (water contents) as follows:

- a) 25 psi $w/c = 53\%$
- b) 50 psi $w/c = 47\%$
- c) 75 psi $w/c = 45\%$

5.2.1 Preparation of Laboratory Manufactured Samples

In sample preparation, caution was used to avoid air entrapment and segregation. The Kaolin was mechanically mixed with

distilled water at a moisture content of about $2\frac{1}{2}$ times the liquid limit (150% water content). The slurry was de-aired under vacuum during the mixing process for about 30 minutes after which it was placed in the plastic cylinder of the modified consolidation apparatus described below.

5.2.2 Modified Apparatus for Consolidation

The ordinary consolidometer which was developed by Bishop (1954) and modified by Fletcher (1967), was further modified by the author. The standard ring which is 0.75 inches high and 2 inches in diameter for undisturbed samples has been replaced by a plastic cylinder 7 inches high and 2 inches in diameter, into which the prepared slurry was placed. This cylinder was constructed to accept the same head cap as the standard ring. The base has been modified by replacing the three 2.5 inch long bolts with three 8 inch bolts in order to accommodate the 7 inch high plastic cylinder. The standard consolidometer loading cap which is 1.97 inches in diameter has been replaced by a 7 inch long plastic piston with a perforated head 1.97 inches in diameter and 0.5 inch thick, to which is attached a thin porous wafer. This permits free drainage of water during the consolidation process. The loading apparatus has been replaced by a simple light aluminum hanger to load the prepared slurry directly. This apparatus, the Direct Load Consolidometer, is shown in Figure 5-1.

Figure 5-2 shows a comparison between the ordinary consolidometer developed by Bishop (1954) and the modified one.

As samples were required, the necessary amount of slurry was placed in the Direct Load Consolidometer and consolidated to the required water content or preconsolidation pressure as outlined in paragraph 5.2.

The process of consolidation of laboratory manufactured soil samples is shown in Figures 5-3 and 5-4.

5.2.3 Consolidation Process

Each sample was consolidated following the square root of time method (Lambe, 1960). Figures 5-5a) to 5-5g) show the plots of compression readings versus the square root of elapsed time. The straight line portion of the curve is extended back to intersect the zero time axis. Through this point of intersection, a straight line having an inverse slope of 1.15 times the tangent is drawn. This straight line cuts the compression-time curve at 90% ultimate compression and is denoted by t_{90} in figures 5.5.a to 5.5.g.

A comparison of the square root of time and the log-time methods of analysis of consolidation under given loading condition has been made. Figures 5.6.a to 5.6.d show the plots of compression dial readings versus log of elapsed time. The results of consolidation are presented by plotting the void ratio against log effective stress as shown in figure 5.7.

It can be seen from figure 5.7 that, under unloading conditions, as the effective stress is reduced to zero, there is no significant change in void ratio, which remains essentially constant.

By cutting each of the consolidated samples into 10 slices, uniformity of water content within the sample has been determined, and was found to be the same throughout ($\pm 0.1\%$) the sample. The degree of saturation has also been calculated and found to be 100%. This would appear to indicate that under any given consolidation load, the sample has been consolidated uniformly.

It has been found that a comparison of the log-time and root-time methods of analysis of a sample under a given loading condition has resulted in a 1% difference in water content.

To calculate the elastic moduli of the soil, it was necessary to determine whether the area of the Hopkinson pressure bar or the area of the soil sample itself was applicable. To do this, supplementary tests have been done using soil samples equal in area to the Hopkinson pressure bar.

After completion of the consolidation process each sample was extruded, trimmed by very thin wire, and placed in another of three plastic cylinders of 3 inches, 2 inches, and one inch lengths and 2 inches inside diameter. The sample, before extrusion from the consolidation apparatus, and after placement in the testing device is shown in figures 5.8 and 5.9

Volumetric change and clay structure:

After the remoulded sample is prepared as described before, it is mounted in the consolidation apparatus shown in figure 5.4. Load is applied in increments until the desired maximum pressure is reached after which the load is removed in increments in the same sequence as for loading. A typical stress-strain curve showing a cycle of loading and unloading is given in figure 5.7.

The same classical features, i.e. primary and secondary volumetric changes were also experienced during unloading. For each unloading increment, the time was large enough for the secondary swelling to be manifested. After all the load was removed, the sample was left for overnight after which the dynamic test was conducted. Therefore, the state of stress in the sample before performing the Hopkinson Bar test was such that the total stress as well as the water pressure were both equal to zero and hence the effective stress is also nil.

It is of interest to follow more closely the change in the state of stress during one cycle of loading and unloading. When the load is applied, it is carried instantaneously by the pore water and since the sample is fully saturated, the increase in pore water pressure equals the applied total stress increment. Dissipation of pore pressure takes place which results in a gradual redistribution of the neutral and effective stresses. Finally, the whole total load increment is transferred to the soil grains while the pore

pressure approaches zero. This picture is supported by the time settlement curves which show the consolidation to continue well into the secondary phase.

During this part of stress application, as a result of the expulsion of water, plastic volumetric changes occur in the sample. These volumetric changes tend to orient the clay particles in a parallel fashion which involves both particle rotation and translation. The particle orientation increases with applied load and it attains its maximum parallelism as maximum load is reached. In such a way, the structural type of clay is called the dispersed type which has a lower void ratio than the flocculated one (Scott 1963).

Due to load removal and subsequent swelling, the same process (i.e. particle rotation and translation) takes place but in a reverse order with particle rotation less pronounced.

The total volumetric change which occurs during one cycle of load application is composed of two components; recoverable (elastic) B_1C_1 and permanent (plastic) A_1C_1 as shown in figure 5.10. Unloading and reloading take place (theoretically) along line CB as long as maximum or yield stress (P_m) is not exceeded. If, however, (P_m) is exceeded during reloading, the compression would follow BD and a new yield stress (P_m') is reached. Each value of yield stress is associated with a particular density of random packing of solids. The line AD, therefore, represents the irrecoverable process, where as the swelling-compression line CB represents the recoverable

process. The flocculated structure has the greater swelling tendency than the dispersed one (remolded).

In general, the undisturbed clays have a more random particle orientation than do the remolded. Since the remolded clays (such as kaolin in our case) are more efficiently packed than the undisturbed clays, the amount of particle shifting during unloading is less than occurs in the undisturbed clays. In other words, due to the denser and more efficient packing in the remolded clay (kaolin), an increment of unloading pressure causes a small void ratio increase than occurs in the undisturbed sample under the same increment of unloading pressure. In other words, the tested Kaolin has lower swelling tendency than any other type of clay soils.

FIGURE 5-1 DIRECT LOADING CONSOLIDOMETER APPARATUS _
UNASSEMBLED .

FIGURE 5 - 1 DIRECT LOADING CONSOLIDOMETER APPARATUS _
UNASSEMBLED .

FIGURE 5-2 COMPARISON BETWEEN THE ORDINARY CONSOLIDOMETER APPARATUS DEVELOPED BY BISHOP (1954) AND THE MODIFIED ONE .

FIGURE 5-2 COMPARISON BETWEEN THE ORDINARY CONSOLIDOMETER APPARATUS DEVELOPED BY BISHOP (1954) AND THE MODIFIED ONE .

FIGURE 5-3 SAMPLE IN THE PROCESS OF
CONSOLIDATION .

FIGURE 5-3 SAMPLE IN THE PROCESS OF
CONSOLIDATION .

FIGURE 5-4 CONSOLIDATION PROCESS .

FIGURE 5 - 4 CONSOLIDATION PROCESS .

FIGURE 5.5.a COMPRESSION VS. ROOT TIME

 $P = 1.0 \text{ psi}$

FIGURE 5-5-b COMPRESSION VS. ROOT TIME
P = 2.0 psi

FIGURE 5.5-c COMPRESSION VS. ROOT TIME
 $P = 4.0$ psi

FIGURE 5.5.d COMPRESSION VS. ROOT TIME

P = 8.0 psi

FIGURE 5.5.e COMPRESSION VS. ROOT TIME
 $P = 16.0 \text{ psi}$

FIGURE 5-5-g COMPRESSION VS. ROOT TIME

P = 64.0 psi

FIGURE 5-6-a COMPRESSION VS TIME

FIGURE 5.6.b COMPRESSION VS. TIME

FIGURE 5.6.c COMPRESSION VS. TIME

FIGURE 5-6-d COMPRESSION VS TIME

FIGURE 5.7 VOID RATIO VS. EFFECTIVE STRESS

FIGURE 5-6 SAMPLE BEFORE EXTRUSION FROM
THE DIRECT LOAD CONSOLIDOMETER
APPARATUS .

FIGURE 5-8 SAMPLE BEFORE EXTRUSION FROM
THE DIRECT LOAD CONSOLIDOMETER
APPARATUS

FIGURE 5-9 SAMPLE AFTER PLACEMENT IN TESTING DEVICE .

FIGURE 5-9 SAMPLE AFTER PLACEMENT IN TESTING DEVICE .

FIGURE 5-10 PRESSURE VS VOLUMETRIC OR AXIAL STRAIN

CHAPTER VI

Presentation and Discussion

of Experimental Results

6.1 Introduction

The Hopkinson pressure bar was used to investigate the response of soil specimens to dynamic loading, and to determine certain of their dynamic properties. Dynamic tests were carried out on three groups of soil samples representing three different preconsolidation pressures and three different sample lengths. Each group was subjected to eight different stress levels. In addition, a number of quasi-static tests were done, using the unconfined testing technique, to obtain corresponding data for the elastic moduli for slower rates of loading.

The information which was made available by using this technique included force-time, displacement-time, and hence, force-displacement relationships for a loading pulse of the order of 100 microseconds in duration. An example for all data reduction is included in appendix A. On the basis of this information, conclusions with regard to the relationship between the preconsolidation pressure of the soil tested and its response to dynamic loading are drawn.

Also, the information made available is used to determine the elastic moduli of the specimens under the test conditions.

6.2 Experimental Results

6.2.1 Pressure-Displacement Relationships

Figures 6-1-a to 6-1-i show the variations of displacement, U_x , of the soil-bar interface, and pressure with time, as well as the variation of the pressure with displacement, for a pre-consolidation pressure $P_c = 25.0$ psi and thicknesses, H , of 1.0, 2.0, and 3.0 inches for 2.0" diameter samples.

From the pressure-displacement curves it is seen that as the pressure increases from zero, the displacement increases linearly, and that at the point of yield, the displacement increases with constant or decreasing pressure. As the pressure increases further, the displacement increases after yielding in a non-linear fashion till an ultimate value is reached. The pressure then decreases to meet that reached at failure. The same qualitative behaviour is demonstrated in Figures 6-2, 6-3, and 6-8 for values of preconsolidation pressures of 50.0 and 75.0 psi and varying thicknesses for the samples of 2.0" and 1.0" diameters.

6.2.2 Effect of Specimen Length

Figures 6-4 to 6-6 show that the specimen length has no significant influences on the pressure-displacement response of the material at its preconsolidation pressure to the type of loading to which it was subjected in these experiments.

A summary of the pressure-displacement response at the preconsolidation pressure levels is shown in Figure 6-7.

6.2.3 Elastic Moduli Values

For the calculation of the elastic moduli, tests have been done on samples of 2.0" and 1.0" diameters to determine whether the area of the Hopkinson bar or the area of the soil sample itself should be used for the terms M_S and A_S in equation 4.5.14.

The values calculated for the elastic moduli of the soil specimens, using the area of the bar for the terms M_S and A_S in equation 4.5.14, are shown in Tables 6.1 and 6.2 for the samples of 2.0" diameter. These values are for different preconsolidation pressures and different rates of deformation.

Figures 6.9 to 6.12 show the effect of the preconsolidation pressure on the elastic moduli for different rates of deformation for the samples of 2.0" diameter.

Figure 6.13 and 6.14 show the relationship between the

elastic moduli and the rate of deformation for the 1.0" diameter samples consolidated to 75.0 psi.

A comparison is shown in Tables 6.3 and 6.4 and Figures 6.15 and 6.16 for the calculated values of the elastic moduli for the samples of 2.0" and 1.0" diameter consolidated to 75.0 psi. These values were based on the use of the Hopkinson bar area for the terms M_S and A_S in Equation 4.5.14. These values have resulted in a 10% difference between them, which may be considered as an experimental margin. For the second set of experimental data, (which are based on the use of bar properties obtained experimentally, $C_1 = 14.3$ pF, $D = 0.00625$ in and $C_1 + C_{OT} = 99.1$ pF), the same qualitative behaviour is obtained and is demonstrated in figures 6.17 to 6.21. A comparison between all test results is shown in figures 6.22 and 6.23.

6.3 Observations

From the pressure-displacement curves for different preconsolidation pressures and different specimen lengths, Figures 6.1 and 6.3, and 6.17 to 6.19, it can be concluded that if the pulse amplitude does not exceed the preconsolidation pressure the soil flows, or yields, at an almost constant stress level, and that the resisting force may be due to internal friction or cohesion. This phenomenon is clearly shown in Figures 6.4 to 6.7. Also, it can be observed that if the pulse amplitude is greater than the preconsolidation pressure, the soil has a definite yield point, then exhibits

strain hardening characteristics followed by quasi-fluid behaviour during which it rapidly loses its strength after the peak stress is reached. Thus by applying successively greater stress levels to the clay samples, the stress level at which strain-hardening begins can be determined.

As known, the response of clay soil body to any external loading is dependant on its micro-structure which is a function of density of packing, shape and degree of preferred orientation of particles. In view of the kind of clay structure of the tested samples, as discussed in Chapter V, the writer is convinced that the Hopkinson Bar test reflects the structure of the material tested. In other words, the structure of the material tested has more influence on its behaviour. However, if the applied stress is less than the yield stress or the preconsolidation stress (P_m) figure 5.10, strain hardening will not occur. And if the applied stress exceeds (P_m), the material will go to a new yield surface or strain hardening will occur. This stress level is equivalent to the preconsolidation pressure.

It has been noted that a few of the stress-displacement curves do not follow the established pattern for determining the preconsolidation pressure, this deviation may be due to sample disturbance during placement and trimming as well as the errors in reading and interpreting the oscilloscope records for the value of Y .

From Figures 6.4 to 6.6 it may be observed that for a particular preconsolidation pressure, the pressure-displacement

behaviour of the soil is independent of specimen thickness.

The relationship between the preconsolidation pressure and the elastic moduli appears to be linear. This is shown in Figures 6-9 to 6-12, and Tables 6.1 and 6.2. This linear relationship has been observed for different deformation rates used in the tests. The observed relationship agrees well with the theoretical analysis presented by Gibson (1967) when he studied the linearly increasing magnitude of Young's modulus with depth. A complete theoretical analysis will be made in the following chapter in order to represent this relationship mathematically.

Figures 6-9 to 6-12 also show that the elastic moduli increase with increasing deformation rate.

From Figures 6-9 to 6-16 and Tables 6.1 to 6.4, it can be observed that for the calculation of the elastic moduli the area of the Hopkinson bar should be used for the terms M_S and A_S in Equation 4.5.14. The use of the sample area, which is considered to be incorrect, will give more than 150.0% difference between the values of E .

6.4 Concluding Remarks

1) If the applied stress on the clay is less than its preconsolidation pressure, the material flows under an almost constant stress after having an elastic response only. In other words, if the applied stress does not exceed the yielding stress, only elastic response is expected.

2) If the applied stress on the clay is greater than its preconsolidation pressure (yield stress), the material has a definite yield point; then exhibits strain hardening characteristics, followed by quasi-fluid behaviour during which it rapidly loses its strength.

3) The magnitude of the elastic moduli of the soil tested increases linearly with increasing preconsolidation pressure. This relationship corroborates with the theoretical analysis presented by Gibson (1967) and Lambe and Whitman (1969).

FIGURE 6-1-a DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED
TO : 25.0 psi , w.c.= 53.0 % & H= 1.0"

FIGURE 6-1-b STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO:
25.0 psi , w.c. = 53.0 % & H = 1 - 0"

FIGURE 6-1-c STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO 25.0 psi, w.c. = 53.0% & H = 1.0"

FIGURE 6-1-d DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED
TO : 25.0 psl , w.c.=53.0 % & H=2-0"

FIGURE 6-1-e STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO :
 25.0 psi , w. c. = 53.0 % & = 2-0"

FIGURE 6-1-f STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO 25.0 psi, w.c.=53.0% & H = 2.0"

FIGURE 6-1 g DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED
TO 25.0 psi , w.c. = 53.0 % & H = 3.0"

FIGURE 6-1-h STRESS VS TIME FOR SAMPLE PRECONSOLIDATED TO :
25.0 psi , w.c. = 53.0 % & H = 3 - 0"

FIGURE 6-1-1 STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO 25.0 psi , w.c. = 53.0% & H = 3"

FIGURE 6-2-a DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED
TO: 50.0 psi , w.c.= 47.0 % & H=1.0"

FIGURE 6-2-b STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO :
50.0 psi , w.c. = 47.0 % & H = 1 - 0"

FIGURE 6-2-c STRESS VS DISPLACEMENT FOR SAMPLE PRECONSOLIDATED TO 50.0psi, w.c.= 47.0% & H=1.0"

FIGURE 6-2-d DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED TO 50.0 psi , w.c. = 47.0% & H = 2-0".

FIGURE 6-2-e STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO :
 50.0 psi , w.c. = 47.0% & H = 2.0"

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FIGURE 6-2-f STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO 50.0 psi, w.c. = 47.0 % & H = 2.0"

FIGURE 6-2-g DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED
TO : 50.0 psi , w.c.= 47.0 % & H= 3.0"

FIGURE 6-2- h STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO:
 50.0 psi , w.c. = 47.0 % & H = 3-0"

FIGURE 6-2-i STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO 50.0 psi , w.c. = 47.0% & H = 3.0"

FIGURE 6-3-a DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED
TO: 75.0 psi , w.c.=45.0 % & H=1.0"

FIGURE 6-3-b STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO :
75.0 psi, w.c. = 45.0 % & H = 1 - 0"

FIGURE 6-3-c STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO 75.0 psi, w.c. = 45.0 % & H = 1-0"

FIGURE 6-3-d DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED TO : 75.0 = psi , w.c. = 45.0 % & H = 2-0"

FIGURE 6-3-e STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO:
75.0 psi , w.c. = 45.0% & H = 2-0"

FIGURE 6-3-f STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO : 75.0 psi , w.c. = 45.0 % & H = 2-(

FIGURE 6-3-g DISPLACEMENT VS TIME FOR SAMPLE PRECONSOLIDATED TO , 75.0 psl , w.c.=45.0 % & H=3.0" .

FIGURE 6-3-h STRESS VS TIME FOR A SAMPLE PRECONSOLIDATED TO:
75.0 psi, w.c. = 45.0 % & H = 3-0"

FIGURE 6-3-1 STRESS VS DISPLACEMENT FOR A SAMPLE PRECONSOLIDATED TO: 75.0 psi, w.c. = 45.0 % & H = 3-

FIGURE 6.4 EFFECT OF SPECIMEN LENGTH AT 25 psi PRECONSOLIDATION PRESSURE

FIGURE 6.5 EFFECT OF SPECIMEN LENGTH AT 50 psi PRECONSOLIDATION PRESSURE

FIGURE 6-6 EFFECT OF SPECIMEN LENGTH AT 75 psi PRECONSOLIDATION PRESSURE

FIGURE 6.7 STRESS VS. DISPLACEMENT AT DIFFERENT PRECONSOLIDATION PRESSURES

DISPLACEMENT, U_x IN. $\times 10^{-6}$

FIGURE 6-8 STRESS VS DISPLACEMENT FOR SAMPLES CONSOLIDATED TO 75.0 psi, w.c. = 44% & 1.0" DIAMETER.

Table 6.1 - Values of elastic moduli, E, with respect to deformation rate for different preconsolidation pressures and sample thicknesses. (Sample diam. = 2.0", Bar diam. = 1.0")

Deformation rate	* E			** E			*** E		
	$\times 10^5$ psi			$\times 10^5$ psi			$\times 10^5$ psi		
In/Sec	H=1.0"	H=2.0"	H=3.0"	H=1.0"	H=2.0"	H=3.0"	H=1.0"	H=2.0"	H=3.0"
0.25	3.87	3.48	4.08	5.95	6.56	6.25	8.96	9.47	9.2
0.30	4.40	4.80	4.99	7.12	7.52	7.39	10.64	10.96	10.8
0.35	4.65	4.99	5.36	7.76	8.03	8.32	11.92	12.24	12.52
0.40	5.2	5.00	5.68	9.12	9.6	9.37	13.76	14.4	14.08
0.45	5.95	6.3	6.72	10.88	10.16	10.58	15.39	15.69	15.12
0.50	6.59	7.00	7.23	11.92	11.36	11.71	16.5	16.96	17.31
0.55	7.44	7.71	8.08	12.51	12.83	13.15	18.48	18.59	18.01
0.60	8.4	8.88	9.15	14.75	14.0	14.32	20.5	20.8	20.08
0.0025		0.1			0.14			0.16	
0.00082		0.091			0.12			0.13	

* corresponding to $P_C = 25$ psi

** corresponding to $P_C = 50$ psi

*** corresponding to $P_C = 75$ psi

All values are based on manufacturer's specifications of bar properties.

Table 6.2 - Values of elastic moduli, G, with respect to deformation rate for different preconsolidation pressures and sample thicknesses. (Sample diam. = 2.0", Bar diam. = 1.0")

Deformation rate	* G			** G			*** G		
	x 10 ⁵ psi			x 10 ⁵ psi			x 10 ⁵ psi		
In/Sec	H=1.0"	H=2.0"	H=3.0"	H=1.0"	H=2.0"	H=3.0"	H=1.0"	H=2.0"	H=3.0"
0.25	1.29	1.16	1.35	1.98	2.18	2.08	2.98	3.15	3.06
0.3	1.46	1.59	1.66	2.37	2.5	2.46	3.54	3.65	3.60
0.35	1.55	1.66	1.78	2.58	2.67	2.77	3.97	4.07	4.17
0.4	1.73	1.66	1.89	3.03	3.19	3.12	4.58	4.79	4.69
0.45	1.98	2.10	2.23	3.62	3.38	3.49	5.13	5.23	5.04
0.5	2.19	2.33	2.41	3.97	3.78	3.90	5.50	5.65	5.77
0.55	2.48	2.57	2.69	4.17	4.27	4.38	6.15	6.19	6.00
0.6	2.80	2.96	3.09	4.90	4.66	4.77	6.83	6.93	6.69
0.0025		0.033			0.045			0.053	
0.00082		0.03			0.04			0.046	

* corresponding to $P_C = 25.0$ psi

** corresponding to $P_C = 50$ psi

*** corresponding to $P_C = 75$ psi

All values are based on manufacturer's specifications of bar properties.

FIGURE 6-9 EFFECT OF THE PRECONSOLIDATION PRESSURE ON THE ELASTIC MODULI, E , FOR DIFFERENT RATES OF DEFORMATION (2.0" DIAM. SAMPLES)

FIGURE 6-10 EFFECT OF THE PRECONSOLIDATION PRESSURE ON THE ELASTIC MODULI, G , FOR DIFFERENT RATES OF DEFORMATION
(2.0" DIAM. SAMPLES)

FIGURE 6-11 ELASTIC MODULUS VS PRECONSOLIDATION PRESSURE .

FIGURE 6.12 ELASTIC MODULUS G VS. PRECONSOLIDATION PRESSURE

FIGURE 6-13 ELASTIC MODULUS VS RATE OF DEFORMATION FOR SAMPLE OF 1.0" DIAMETER, CONSOLIDATED 75 psi

FIGURE 6-14 ELASTIC MODULUS G VS RATE OF DEFORMATION FOR SAMPLE CONSOLIDATED TO 75 psi & 1.0" DIAMETER.

Table 6.3 - Comparison between the values of the elastic modulus
E for samples consolidated to 75.0 psi and diameters
of 2.0" and 1.0"

Rate of Deformation In/Sec	E x 10 ⁵ psi	
	2.0" Diameter Sample	1.0" Diameter Sample
0.25	9.0	8.0
0.30	10.7	9.5
0.35	12.1	11.0
0.40	14.0	12.25
0.45	15.25	13.75
0.50	16.75	15.25
0.55	18.25	16.75
0.6	20.0	18.25

Values are based on manufacturer's specifications of bar properties.

Table 6.4 - Comparison between the values of the elastic modulus G for samples consolidated to 75.0 psi and diameters of 2.0" and 1.0"

Rate of Deformation In/Sec	$G \times 10^5$ psi	
	2.0" Diameter Sample	1.0" Diameter Sample
0.25	3.0	2.67
0.30	3.57	3.17
0.35	4.03	3.67
0.40	4.67	4.07
0.45	5.07	4.57
0.50	5.57	5.07
0.55	6.07	5.58
0.60	6.67	6.07

Values are based on manufacturer's specifications of bar properties

FIGURE 6.15 ELASTIC MODULUS E VS. RATE OF DEFORMATION FOR SAMPLES CONSOLIDATED TO 75.0 psi, 2.0" AND 1.0" DIAM.

FIGURE 6.16 ELASTIC MODULUS G VS. RATE OF DEFORMATION FOR SAMPLES CONSOLIDATED TO 75.0 psi, 2.0" AND 1.0" DIAM.

FIGURE 6-17 STRESS VS DISPLACEMENT FOR SAMPLES CONSOLIDATED TO 25 psi

FIGURE 6-18 STRESS VS DISPLACEMENT FOR SAMPLES CONSOLIDATED TO 50 psi

FIGURE 6-19 STRESS VS DISPLACEMENT FOR SAMPLES CONSOLIDATED TO 75 psi

FIGURE 6-20 EFFECT OF PRECONSOLIDATION ON ELASTIC MODULUS

FIGURE 6-21 G VS RATE OF DISPLACEMENT

Text complete; leaf 125f omitted in numbering.

CURVE 5 new experimental data
 " 1,2 based on manufacturer's specifications of bar properties
 " 3,4,5 " " experimentally determined bar properties

FIGURE 6-22 COMPARISON OF ELASTIC MODULUS E VALUES

FIGURE 6-23 COMPARISON OF ELASTIC MODULUS G VALUES

CHAPTER VII

Soils as Work-Hardening Materials

7.1 Introduction

The purpose of this chapter is to explain the mechanical behaviour of soil materials in terms of work hardening, and to relate this explanation to the experimental and theoretical work presented in the previous chapters.

The inelastic behaviour of real materials is enormously complicated. Any explicit description in phenomenological or mathematical terms is bound to be a drastic idealization of actual behaviour and should not be expected to be valid over a wide range of conditions. Yet, even for metals, which are less complex than soils in terms of their plastic behaviour, there often is confusion in the literature when it was rediscovered that the simple idealizations such as perfect plasticity and isotropic hardening fall so far short of accurate descriptions of real behaviour for general loading.

7.2 Stress-Strain Relationship for Metals

Perfect plasticity is an appropriate idealization for

structural metal because it describes the essential features of its behaviour. The tangent modulus, when loading in the plastic range, is small compared with the elastic modulus, and the unloading response is elastic in nature.

Perfect plasticity is not nearly as close an approximation for soils. Some of the disadvantages were discussed at the time that the idealization was proposed by Drucker and Prager (1952), and extended and improved upon by Shields (1955). The difference between frictional and plastic behaviour was explained by Drucker (1954).

Structural metals when examined under quasi-static loading may be idealized as time-independent with separable increments of elastic and plastic deformation. This means that in simple tension, for example, there is a single stress-strain curve with elastic unloading sharply differentiated from elastic-plastic loading as shown in Figure 7-1.

7.3 Yield Surfaces for Solids

More generally, at each stage of plastic deformation a yield surface separates those elastic and elastic-plastic behaviours of solids as shown in Figure 7-2. The term yield surface is used to emphasize the fact that the three components of stress may be independent variables.

FIGURE 7-1 LOADING AND UNLOADING PHENOMENA OF METALS

FIGURE 7-2 DIAGRAMATIC REPRESENTATION OF YIELD SURFACE (LOCUS) WHICH SEPARATES ELASTIC FROM ELASTIC-PLASTIC BEHAVIOR OF SOLIDS

In Figure 7-3, the initial yield locus $F = 0$ has been sketched. The vector σ_{ij} represents a combination of stress that brings the material to the point of yielding. The fan of small vectors σ_{ij} represent some of the many possible combinations of stress-increment components which would each result in the same hardening of the material thus achieving a new yield locus $F' = 0$ for the material. The normal at any point of a current yield surface gives the ratios of the components of the increment of plastic strain produced by further loading from the yield state of stress represented by the point.

Drucker (1959, 1964) has introduced the concept of "stability" which illuminates this matter. For all stress-increment vectors directed outwards from the tangent to the yield locus, the vector product of stress-increment vector $\dot{\sigma}_{ij}$ with the associated plastic strain-increment vector $\dot{\epsilon}_{ij}^P$ will be positive or zero.

$$\dot{\sigma}_{ij} \dot{\epsilon}_{ij}^P \geq 0 \quad 7.1$$

Plastic materials are stable in the sense that they only yield for stress increments that satisfy equation 8.1.

In general, when an appropriate combination of stress components is chosen as an equivalent stress σ_{eq} and a corresponding choice is made for equivalent strain ϵ_{eq} , or better, an equivalent

FIGURE 7.3 HARDENING AND ASSOCIATED PLASTIC FLOW

plastic strain ϵ_{eq}^P , the equivalent stress is a function of the three stress invariants of the stress tensor. One available choice of parameters for the mathematical requirements of the loading paths is the set of principal stresses, σ_1 , σ_2 , and σ_3 . A convenient choice is the set J_1 , J_2 and J_3 where J_1 is the first invariant of the stress tensor

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 = -3p \quad 7.2$$

defined here as three times the average tension or the negative of three times the average pressure. J_2 is the second invariant of the stress deviator.

$$\begin{aligned} J_2 &= \frac{1}{6} \{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \} \\ &= \frac{3}{2} \tau_o^2 \end{aligned} \quad 7.3$$

where τ_o is the octahedral shearing stress, and J_3 is the third invariant of the stress deviator

$$J_3 = \frac{1}{27} (2\sigma_1 - \sigma_2 - \sigma_3)(2\sigma_2 - \sigma_3 - \sigma_1)(2\sigma_3 - \sigma_1 - \sigma_2) \quad 7.4$$

The initial yield surface is then given by σ_{eq} or $F = 0$ or some function of the invariants $f(J_1, J_2, J_3)$, equal to a constant. The initial plastic strain increments for continued loading are normal to this surface as shown in Figure 7-3.

Due to those stress increments there will be successive yield surfaces established by any loading path, radial or otherwise, and this condition is termed isotropic strain-hardening. The current yield surface is determined by the maximum σ_{eq} reached in the past but is otherwise path independent. Isotropic strain-hardening satisfies all the mathematical requirements stated for the definition of valid stress-strain relationships and is often employed in achieving the solutions of problems when work-hardening must be given some consideration.

At this stage, it is helpful to draw attention to the perfectly plastic idealization. Figure 7-4.a shows the choice of stress level, σ_0 , which gives the limit yielding surface shown in Figure 7-4.b.

The perfect plastic idealization does not account for the work-hardening of the material beyond the arbitrarily chosen yield level.

7.4 Triaxial Test Simulation

The triaxial test, oftenly used for soil mechanics

(a)

(b)

FIGURE 7.4 PERFECTLY PLASTIC IDEALIZATIONS AND SUCCESSIVE YIELD SURFACES FOR VARIOUS LOADING PATHS

problems, can be simulated by simple compression test for metals (Drucker 1959). A schematic plot of triaxial test results for structural metals is shown in Figure 7-5.

The triaxial test has two independent stress variables σ_1 and $\sigma_2 = \sigma_3$. One set of coordinates on which to plot triaxial test results could be σ_1 vs $\sigma_2\sqrt{2}$. This represents the plane in principal stress space passing through the σ_1 - axis and bisecting the angle between the σ_1 - σ_2 plane and the σ_1 - σ_3 plane (Henkel, 1959). As $\sigma_2 = \sigma_3$ at all times in the triaxial test, all loading and unloading paths for such tests must lie in this plane. Another set of coordinates could be p vs q where $q = \sigma_1 - \sigma_2$, and $p = \frac{1}{3}\sigma_1 + \frac{2}{3}\sigma_2$. This relates the fact that an all-around cell pressure, p , and superposed additional axial stress, q , represent a set of independent stress components equally appropriate to those of σ_1 , σ_2 , and σ_3 .

With compression taken as positive, the invariants of the stress tensor and stress deviator for the triaxial test are:

$$J_1 = -3p = -(\sigma_1 + 2\sigma_2) \quad 7.5$$

$$J_2 = \frac{1}{3} (\sigma_1 - \sigma_2)^2 = \frac{3}{2} \tau_o^2 = \frac{4}{3} \tau_{\max}^2$$

$$= \frac{1}{3} q^2 \quad 7.6$$

FIGURE 7.5 TRIAXIAL TEST ON METAL IS EQUIVALENT TO SIMPLE COMPRESSION. (AFTER DRUCKER 1959). (ALL SCALES ARE LINEAR BUT UNEQUAL AS INDICATED)

$$J_3 = 0$$

7.7

The lines parallel to the p-axis, $\sigma_1 = \sigma_2 = \sigma_3$ are the successive yield surfaces.

The experimental curves suggest, on the basis of the above discussions in paragraphs 7.2, 7.3, and 7.4, that a soil-water system can be examined in terms of a work-hardening material.

The stress-strain relationship obtained during the consolidation process for soil-water systems is comparable to the simple tension or shear test for metals. Kenkel (1959, 1960, Drucker (1961), Roscoe and Poorooshab (1963), Calladine (1963), and Schofield (1963) discussed in details the soil as work-hardening material. It was concluded by the above investigators that the soil-water system can be represented by work-hardening material.

7.5 Experimental Data and Work-Hardening Soils

The curves in Figure 7-6 are representative of the experimental data obtained and demonstrate the relationship between the stress and displacement obtained by using the modified Hopkinson pressure bar in the testing of cohesive soils.

As discussed above, the initial yield surface, as shown in Figures 7-2 and 7-3, separates the elastic deformation from the elastic-plastic deformation. The preconsolidation pressure, P_c

FIGURE 7.6 TYPICAL STRESS-DISPLACEMENT RELATIONSHIP BELOW AND ABOVE THE PRECONSOLIDATION PRESSURE

Figure 7-6, is considered as the stress level that brings the materials or soils to the state of stress equivalent to the state of stress at the initial yielding surface shown in Figures 7-2 and 7-3. Thus, any stress level below that equivalent to the preconsolidation pressure will produce only deformations in the elastic range. This phenomenon is shown by the line oa_1 Figure 7-6. After a_1 , frictional effects begin and continue to a_2 , followed by unloading.

Once any stress level above the preconsolidation pressure is applied to the soil, the elastic response, ob_1 in Figure 7-6 will be followed by plastic deformation similar to work-hardening and could be represented by successive yielding, $b_1 b_2$, after which unloading or failure, $b_2 b_3$, will occur.

7.6 Conclusions

From the above discussions, it can be concluded that by applying successively greater stress levels to clay samples using the modified Hopkinson pressure bar, the stress level at which strain- or work-hardening begins can be determined. This stress level is equivalent to the field preconsolidation pressures.

CHAPTER VIII

Concluding Remarks

An experimental program was carried out to study the relationship between the response to dynamic loading and the pre-consolidation pressure and elastic-moduli of a cohesive soil. On the basis of experiment, the following principal conclusions are drawn:

1 - The behaviour of the cohesive soil under the action of various pulses amplitudes indicates that as long as the amplitude of stress does not exceed the preconsolidation pressure of the material, it flows under an almost constant stress level. On the other hand, when the pulse amplitude is larger than the preconsolidation pressure, the soil exhibits strain-hardening characteristics which are followed by quasi-fluid behaviour and a rapid loss of strength.

2 - The elastic moduli of the cohesive soil vary linearly with the preconsolidation pressure as well as with the rate of deformation of the material.

3 - The experimental technique employed in this work appears to lend itself well to application in the rapid determination of the preconsolidation pressure of the cohesive soil as well as to the determination of the elastic moduli of cohesive soils at any prescribed rate of deformation. Based on the analysis made in this work for the kind of structure of the material tested, the use of the

Hopkinson Bar test in testing highly swelling soils is questioned. This may be done as a future expansion of this work or would be the subject of another thesis.

With regard to the work required to bring the theory and techniques propounded in this thesis to the level of application, the following are required:

- 1 - The development of appropriate circuitry to be appended to the apparatus to perform the mathematical tasks which have had to be done by hand or computer for this work. This aspect is largely a question of financing since the required circuitry is, for the most part, commercially available.
- 2 - The carrying out of an extensive and comprehensive test program on field specimens in order to thoroughly confirm the validity of the technique.
- 3 - The development of a practicable apparatus for use in the laboratory and/or field.

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APPENDIX AStress-Displacement Calculations

The radial displacement is given by the equation

$$\xi = \frac{Y}{\frac{C_1 V}{CND} + \frac{Y}{D}}$$

where

Y = oscilloscope deflection in cm

C₁ = cylindrical condenser capacity
= 14.3

V = D.C. power supply
= 300 V

D = the gap between the plates of the bar condenser
= 0.00625

C = equivalent capacity of the feed circuit Figure 4-4
= 544.9 pF

N = sensitivity of the oscilloscope

For the purpose of data reduction

$$\xi = \frac{l.Y}{1260. + 160 N.Y}$$

The incident and reflected pulses have been enlarged five times for the purpose of measurements as shown in Figures A.1. The cylindrical condenser was put at a distance of 13 inches from the bar-specimen interface. That gives a difference in time of 130 microseconds between recording the incident and reflected pulses as shown in Figure A.1.

Equations 4.5.11 and 4.5.13 have been used for calculating the stress and displacement, U_x at the bar-soil interface.

The computer program enclosed shows all measurements and calculations of data for a sample of 2.0" diameter consolidated to 50.0 psi.

FIGURE A-1

FIGURE A - 2

FIGURE A - 3

$$\frac{(50)^3}{3}$$

INQUIRY		REFLECTED		TRANSLATED	
NAME	ADDRESS	NAME	ADDRESS	NAME	ADDRESS
1. J. A. B. C.	1234567890	1. J. A. B. C.	1234567890	1. J. A. B. C.	1234567890
2. D. E. F. G.	2345678901	2. D. E. F. G.	2345678901	2. D. E. F. G.	2345678901
3. H. I. J. K.	3456789012	3. H. I. J. K.	3456789012	3. H. I. J. K.	3456789012
4. L. M. N. O.	4567890123	4. L. M. N. O.	4567890123	4. L. M. N. O.	4567890123
5. P. Q. R. S.	5678901234	5. P. Q. R. S.	5678901234	5. P. Q. R. S.	5678901234
6. T. U. V. W.	6789012345	6. T. U. V. W.	6789012345	6. T. U. V. W.	6789012345
7. X. Y. Z. A.	7890123456	7. X. Y. Z. A.	7890123456	7. X. Y. Z. A.	7890123456
8. B. C. D. E.	8901234567	8. B. C. D. E.	8901234567	8. B. C. D. E.	8901234567
9. F. G. H. I.	9012345678	9. F. G. H. I.	9012345678	9. F. G. H. I.	9012345678
10. J. K. L. M.	0123456789	10. J. K. L. M.	0123456789	10. J. K. L. M.	0123456789
11. N. O. P. Q.	1234567890	11. N. O. P. Q.	1234567890	11. N. O. P. Q.	1234567890
12. R. S. T. U.	2345678901	12. R. S. T. U.	2345678901	12. R. S. T. U.	2345678901
13. V. W. X. Y.	3456789012	13. V. W. X. Y.	3456789012	13. V. W. X. Y.	3456789012
14. Z. A. B. C.	4567890123	14. Z. A. B. C.	4567890123	14. Z. A. B. C.	4567890123
15. D. E. F. G.	5678901234	15. D. E. F. G.	5678901234	15. D. E. F. G.	5678901234
16. H. I. J. K.	6789012345	16. H. I. J. K.	6789012345	16. H. I. J. K.	6789012345
17. L. M. N. O.	7890123456	17. L. M. N. O.	7890123456	17. L. M. N. O.	7890123456
18. P. Q. R. S.	8901234567	18. P. Q. R. S.	8901234567	18. P. Q. R. S.	8901234567
19. T. U. V. W.	9012345678	19. T. U. V. W.	9012345678	19. T. U. V. W.	9012345678
20. X. Y. Z. A.	0123456789	20. X. Y. Z. A.	0123456789	20. X. Y. Z. A.	0123456789

157c

(50)

APPENDIX B

Evaluation of Errors

The equations which are used to determine the final values of stress, displacement, and elastic moduli are:

$$U_x = \frac{C_o}{\mu r} \int_0^t (\xi_I - \xi_R) dt$$

$$\sigma_x = \frac{E_B}{\mu r} (\xi_I - \xi_R)$$

$$E_S = \frac{P_T^2}{(P_I + P_R)^2} \cdot \frac{M_B E_B A_B}{M_S A_S}$$

where, in general

$$\xi = \frac{Y}{\frac{c_1 v}{CND} + \frac{Y}{D}}$$

and

$$C_o = \sqrt{\frac{E_B g}{\rho_B}}$$

The errors in each component of these equations are estimated to be as follows:

E_B	=	1.0%
μ_B	=	2.0%
M_B	=	0%
ρ_B	$\approx$	0%
$r_B (0.5'' \pm 0.0005)$	$\approx$	0%
M_s	$\approx$	0%
$r_{\text{condenser}} (0.50625 \pm 0.0005)$	=	0%
D estimated	=	2.0%
g	=	0%
Y estimated	=	1.0%
$N \left(\frac{0.1}{100} \right)$	$\approx$	0.0%
C_1 estimated	=	2.0%
$V \left(\frac{0.5}{300} \times 100 \right)$	$\approx$	0.0%
$C_{(\text{total})} \left(\frac{0.5}{544} \times 100 \right)$	=	1.0%

The errors which may be expected in each equation are:

$$1) \quad C_o = \sqrt{\frac{E_B \cdot g}{\rho_B}} = \frac{E_B^{\frac{1}{2}} g^{\frac{1}{2}}}{\rho_B^{\frac{1}{2}}}$$

$$\frac{\partial C_o}{\partial E_B} = \frac{1}{2} E_B^{-\frac{1}{2}} \cdot \frac{g^{\frac{1}{2}}}{\rho_B^{\frac{1}{2}}}$$

$$\frac{\partial C_O}{\partial g} = \frac{1}{2} g^{-1/2} \frac{E_B^{1/2}}{\rho_B^{1/2}}$$

$$\frac{\partial C_O}{\partial \rho_B} = \frac{0 - E_B^{1/2} g^{1/2} \cdot (\frac{1}{2} \rho_B^{-1/2})}{(\rho_B^{1/2})^2} = \frac{1}{2} \cdot \frac{E_B^{1/2} \cdot g^{1/2}}{\rho_B \cdot \rho_B^{1/2}}$$

$$\begin{aligned} \% \text{ error} &= \frac{dC_O}{C_O} = \sqrt{\left(\frac{\partial C_O}{\partial E_B} \cdot \Delta E_B\right)^2 + \left(\frac{\partial C_O}{\partial g} \cdot \Delta g\right)^2 + \left(\frac{\partial C_O}{\partial \rho_B} \cdot \Delta \rho_B\right)^2} \\ &= \sqrt{\left(\frac{1}{2} \cdot \frac{1}{E_B} \cdot \Delta E_B\right)^2 + \left(\frac{1}{2} \cdot \frac{1}{g} \cdot \Delta g\right)^2 + \left(\frac{1}{2} \cdot \frac{1}{\rho_B} \cdot \Delta \rho_B\right)^2} \\ &\approx 1.0\% \end{aligned}$$

$$2) \quad \xi = \frac{Y}{\frac{C_1 V}{CND} + \frac{Y}{D}} = \frac{YCND}{C_1 V}$$

$$\frac{\partial \xi}{\partial Y} = \frac{CND}{C_1 V}$$

$$\frac{\partial \xi}{\partial C} = \frac{YND}{C_1 V}$$

$$\frac{\partial \xi}{\partial N} = \frac{YCD}{C_1 V}$$

$$\frac{\partial \xi}{\partial D} = \frac{YCN}{C_1 V}$$

$$\frac{\partial \xi}{\partial C_1} = \frac{-YCND \cdot V}{(C_1 V)^2}$$

$$\frac{\partial \xi}{\partial V} = \frac{-YCND \cdot C_1}{(C_1 V)^2}$$

$$\begin{aligned} \% \text{ error} = \frac{d\xi}{\xi} = & \sqrt{ \left(\frac{\partial \xi}{\partial Y} \Delta Y \right)^2 + \left(\frac{\partial \xi}{\partial C} \Delta C \right)^2 + \left(\frac{\partial \xi}{\partial N} \Delta N \right)^2 } \\ & + \left(\frac{\partial \xi}{\partial D} \Delta D \right)^2 + \left(\frac{\partial \xi}{\partial C_1} \Delta C_1 \right)^2 + \left(\frac{\partial \xi}{\partial V} \Delta V \right)^2 \end{aligned}$$

The error ΔY

$$Y_T = (Y_I - Y_R)$$

$$\frac{\partial Y_T}{\partial Y_I} = 1$$

$$\frac{\partial Y_T}{\partial Y_R} = -1$$

$$\frac{dY_T}{Y_T} = \Delta Y_T = \% \text{ error} = \sqrt{ \frac{ \left(\frac{\partial Y_T}{\partial Y_I} \Delta Y_I \right)^2 + \left(\frac{\partial Y_T}{\partial Y_R} \Delta Y_R \right)^2 }{Y_T^2} }$$

Ex. (1): $Y_I = 3.00$

$Y_R = 2.75$

$\Delta Y_I = 1\%$

$\Delta Y_R = 1\%$

$$\begin{aligned}\% \text{ error in } Y &= \sqrt{\left(\frac{1}{0.25} \times \frac{1}{100} \times 3.0\right)^2 + \left(\frac{1}{0.25} \times \frac{1}{100} \times 2.75\right)^2} \\ &= 16.4\%\end{aligned}$$

Ex.(2): $Y_I = 2.6$

$Y_R = 2.05$

$\% \text{ error in } Y_T = 6.0\%$

Ex.(3): $Y_I = 1.5$

$Y_R = 1.375$

$Y_T = 0.125$

$\% \text{ error in } Y_T = 16.2\%$

Ex.(4): $Y_I = 1.675$

$Y_R = 1.45$

$\% \text{ error in } Y_T = 10.0\%$

The error in ξ

$$\begin{aligned}\frac{d\xi}{\xi} &= \sqrt{\left(\frac{\partial \xi}{\partial Y} \cdot \Delta Y\right)^2 + \left(\frac{\partial \xi}{\partial C} \Delta C\right)^2 + \dots} \\ &= \sqrt{\left(\frac{1}{Y} \cdot \Delta Y\right)^2 + \left(\frac{1}{C} \Delta C\right)^2 + \dots}\end{aligned}$$

Ex.(1):

$\% \text{ error in } \xi = 16.6\%$

Ex.(2): $= 6.5\%$

$$\text{Ex. (3):} \quad = 16.5\%$$

$$\text{Ex. (4):} \quad = 10.4\%$$

$$3) \quad U_X = \frac{C_O}{\mu r} (\xi_I - \xi_R) = \frac{C_O}{\mu r} (Y_T)$$

$$\frac{\partial U_X}{\partial C_O} = \frac{1}{\mu r} (Y_T)$$

$$\frac{\partial U_X}{\partial \mu} = \frac{C_O r}{(\mu r)^2} (Y_T)$$

$$\frac{\partial U_X}{\partial r} = \frac{C_O \mu}{(\mu r)^2} (Y_T)$$

$$\frac{\partial U_X}{\partial Y_T} = \frac{C_O}{\mu r}$$

$$\% \text{ error} = \frac{dU_X}{U_X} = \sqrt{\left(\frac{1}{C_O} \cdot \Delta C_O \right)^2 + \left(\frac{1}{\mu} \Delta \mu \right)^2 + \left(\frac{1}{r} \Delta r \right)^2 + \left(\frac{1}{Y_T} \Delta Y_T \right)^2}$$

$$\text{Ex. (1):} \quad = 16.6\%$$

$$\text{Ex. (2):} \quad = 6.5\%$$

$$\text{Ex. (3):} \quad = 16.2\%$$

$$\text{Ex. (4):} \quad = 10.4\%$$

$$4) \quad \sigma_x = \frac{E_B}{\mu r} (Y_T)$$

$$\frac{\partial \sigma_x}{\partial E_B} = \frac{1}{\mu r} (Y_T)$$

$$\frac{\partial \sigma_x}{\partial \mu} = \frac{E_B \cdot Y}{(\mu r)^2} (Y_T)$$

$$\frac{\partial \sigma_x}{\partial r} = \frac{E_B \cdot \mu}{(\mu r)^2} (Y_T)$$

$$\frac{\partial \sigma_x}{\partial Y_T} = \frac{E_B}{\mu r}$$

$$\% \text{ error} = \frac{\partial \sigma_x}{\sigma_x} = \sqrt{\frac{\left(\frac{\partial \sigma_x}{\partial E_B} \cdot \Delta E_B\right)^2 + \left(\frac{\partial \sigma_x}{\partial \mu} \cdot \Delta \mu\right)^2}{+ \left(\frac{\partial \sigma_x}{\partial r} \cdot \Delta r\right)^2 + \left(\frac{\partial \sigma_x}{\partial Y} \cdot \Delta Y\right)^2}}$$

% error =

Ex. (1): = 16.6%

Ex. (2): = 6.5%

Ex. (3): = 16.5%

Ex. (4): = 10.4%

$$5) \quad E_S = \frac{Y_T^2}{(Y_I + Y_R)^2} \cdot K$$

$$\frac{\partial E_S}{\partial Y_T} = \frac{2Y_T}{(Y_I + Y_R)^2} \cdot K$$

$$\frac{\partial E_S}{\partial Y_I} = \frac{2Y_T^2}{(Y_I + Y_R)^2} \cdot K$$

$$\frac{\partial E_S}{\partial Y_R} = \frac{2Y_T^2}{(Y_I + Y_R)^2} \cdot K$$

$$\% \text{ error} = \frac{dE_S}{E_S} = \frac{\sqrt{\left(\frac{\partial E_S}{\partial Y_T} \cdot \Delta Y_T \right)^2 + \left(\frac{\partial E_S}{\partial Y_I} \cdot \Delta Y_I \right)^2 + \left(\frac{\partial E_S}{\partial Y_R} \cdot \Delta Y_R \right)^2}}{E_S}$$

$$\text{Ex. (1):} \quad \% \text{ error} = \frac{dE_S}{E_S}$$

$$= \frac{\sqrt{(2 \cdot \Delta Y_T)^2 + \left(\frac{2}{Y_I + Y_R} \cdot \Delta Y_I \right)^2 + \left(\frac{2}{Y_I + Y_R} \cdot \Delta Y_R \right)^2}}{E_S}$$

$$\text{Ex. (1):} \quad = 32.0\%$$

$$\text{Ex. (2):} \quad = 12.5\%$$

$$\text{Ex. (3):} \quad = 32.0\%$$

$$\text{Ex. (4):} \quad = 20.0\%$$