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Delay and Crosstalk Simulation of High-Speed VLSI Interconnects with Nonlinear Terminations

by

Dong Hui Xie, B. Eng.

**A Thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of**

Master of Engineering

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada
September 1992**

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ISBN 0-315-79822-X

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**“Delay and Crosstalk Simulation of High-Speed VLSI Inter-
connects with Non-linear Terminations”**

submitted by Dong Hui Xie, B. Eng.

in partial fulfillment of the requirements for the degree of
Master of Engineering

Thesis Supervisor

Chair, Department of Electronics

Carleton University
September 16, 1992

Abstract

A method is presented for analysis of VLSI interconnects which contain both lossy coupled transmission lines and **nonlinear** components. An equivalent time domain macromodel is derived for the linear subnetworks which contain lossy coupled transmission lines. The macromodel takes the form of a set of ordinary differential equations. The method takes full advantage of the Asymptotic Waveform Evaluation technique which offers two to three orders of magnitude speedup relative to other methods with comparable accuracy. In addition, it does not require explicit evaluation of the dominant poles of the network. The proposed technique has been implemented within the framework of an existing conventional circuit simulator, Hspice.

Acknowledgments

I would like to express my gratefulness to my supervisor Dr. Michel Nakhla. His advice in guiding me to complete this studies and to the right track of many things is most significant and invaluable. His endeavour and suggestions in reviewing this thesis are most essential for me to finish the writing of the thesis.

I am also very grateful to Dr. Q. J. Zhang who introduced me to this program. His encouragements and advice helped me make great progresses in my studies.

Richard Griffith, our research engineer, has given me many helps in my studies, And he improved and modified the AWE code for me. All the examples in this thesis are finished on the basis of this code.

I would also like to acknowledge the support given me by my wife, Zhuo Han, without her support I would not be able to finish this study.

My fellow graduate students, Yuji Wei, Sanjay L. Manney, Eli Chiprout and Nelson L. S. Liu gave me a lot of helps in my studies; and Sanaie Ramin helped with the input/output format of AWE code. I would like to thank them all.

Table of Contents

	Page
Acceptance Sheet	ii
Abstract	iii
Acknowledgments	iv
Contents	v
List of Figures	vii
List of Symbols	ix
 Chapter 1 Introduction	 1
 Chapter 2 Literature Survey	 5
2.1 Introduction	5
2.2 Quasi-Transverse Electromagnetic Model for Multiconductor Transmission Lines	6
2.3 Lumped Equivalent Circuit Method	8
2.4 Green's Function Method	11
2.5 Mixed-Domain Method	13
2.6 AWE Method	14
2.7 Summary	16
 Chapter 3 Asymptotic Waveform Evaluation Technique(AWE)	 17
3.1 Introduction	17
3.2 The AWE Algorithm	18
3.2.1 The particular solution algorithm	18
3.2.2 The impulse response algorithm	21
3.3 Numerical Examples	22
3.4 Comments on AWE Technique	26
 Chapter 4 Frequency Domain Transfer Function	 27
4.1 Formulation of The Network Equations	27
4.2 Equivalent Frequency Domain Representation of $Z_{\phi}(s)$	30

4.2.1	Formulation of The Linear Subnetwork Equations	30
4.2.2	The Generalized Asymptotic Waveform Evaluation Technique	35
4.2.3	Evaluation of The Moments	36
4.2.4	The Multiconductor Transmission Line Moments	37
4.2.5	The Evaluation of $Z_{\phi}(s)$	40
Chapter 5	Time Domain Equivalent Representation	42
5.1	Time Domain Equivalent Representation of $Z_{\phi}(s)$	42
5.2	Proof of The Time Domain Equivalent Representation	44
Chapter 6	Equivalent Circuit Representation in Hspice	49
Chapter 7	Computational Results And Comparisons	57
7.1	Examples	57
7.2	Computational Considerations	59
7.2.1	Stability Problems	60
7.2.2	Optimal Pole Selection Criteria	62
Chapter 8	Conclusions And Future Research	84
8.1	Conclusions	84
8.2	Suggestions for Future Research	85
Appendix		86
References		89

List of Figures

Figure	Page
Figure 2.1 Lumped-element equivalent circuit of a lossy transmission line with two signal conductors	10
Figure 3.1 Circuit for example 3.1	24
Figure 3.2 Transient response of the circuit shown in Figure 3.1	25
Figure 4.1 An example circuit contains coupled transmission lines and nonlinear terminations	34
Figure 6.1a Hspice representation of integration circuit	50
Figure 6.1b Hspice code of Figure 6.1a	50
Figure 6.2a Hspice equivalent circuit representation of the differential equation (6.1)	53
Figure 6.2b Hspice code of Figure 6.2a	55
Figure 7.1 Interconnect circuit with lossless transmission lines and nonlinear terminations	64
Figure 7.2a Transient response V_1 of the circuit shown in Figure 7.1	65
Figure 7.2b Transient response V_2 of the circuit shown in Figure 7.1	66
Figure 7.2c Transient response V_3 of the circuit shown in Figure 7.1	67
Figure 7.2d Transient response V_4 of the circuit shown in Figure 7.1	68
Figure 7.3 Interconnect circuit for example 2	69
Figure 7.4a Transient response V_1 of the circuit shown in Figure 7.3	70
Figure 7.4b Transient response V_2 of the circuit shown in Figure 7.3	71
Figure 7.4c Transient response V_5 of the circuit shown in Figure 7.3	72

Figure 7.4d	Transient response V_6 of the circuit shown in Figure 7.3	73
Figure 7.5	Interconnect circuit with lossy coupled transmission lines and nonlinear terminations	74
Figure 7.6a	Transient response V_1 of the circuit shown in Figure 7.5	75
Figure 7.6b	Transient response V_2 of the circuit shown in Figure 7.5	76
Figure 7.6c	Transient response V_3 of the circuit shown in Figure 7.5	77
Figure 7.6d	Transient response V_4 of the circuit shown in Figure 7.5	78
Figure 7.7	Multi-chip interconnect circuit with nonlinear terminations	79
Figure 7.8a	Transient response V_1 of the circuit with 35 transmission lines	80
Figure 7.8b	Transient response V_2 of the circuit with 35 transmission lines	81
Figure 7.8c	Transient response V_1 of the circuit with 70 transmission lines	82
Figure 7.8d	Transient response V_1 of the circuit with 98 transmission lines	83

List of Symbols

R_l	matrix of resistances per unit length of transmission line.
G_l	matrix of conductances per unit length of transmission line.
L_l	matrix of inductances per unit length of transmission line.
C_l	matrix of capacitances per unit length of transmission line.
$v_k(x, t), i_k(x, t)$	the vector of voltage and the vector of current.
Z_l, Y_l	the matrices of impedances and admittances of transmission line.
D	the total length of transmission line.
P	the number of lumped equivalent circuit sections of transmission line.
Δx	the length of each section of transmission line.
L_k	the inductive lumped element.
L_{mkl}	the mutual inductance.
R_k	the resistive element of each section.
R_{mkl}	the mutual resistance.
C_k	the capacitances of the lumped equivalent circuit.
C_{mkl}	the mutual capacitance.
$Y(s)$	Y-parameter matrix of a multiport linear subnetwork.
$Z(s)$	Z-parameter matrix of a multiport linear subnetwork.
$V(s)$	voltage vector in frequency domain.
$I(s)$	current vector in frequency domain.
$y_{ij}(s)$	the element of $Y(s)$.
p and k	complex poles and residues.
d_{ij}	direct coupling between the input and output in $y_{ij}(s)$.
q_{ij}	the order of poles in $y_{ij}(s)$.
$u(t)$	the unit step function.
S_p	the slope of a ramp function.
x	time-domain state vector.
u	time-domain excitation vector.
A	constant matrix.
B	constant matrix.
C	constant matrix.
m, M	moment vectors in Maclaurin's series.

X	frequency-domain state vector.
a_j	coefficients of approximate characteristic polynomial.
b_j	coefficients of approximate numerator polynomial.
$\delta(t)$	the impulse excitation.
π	a nonlinear network.
ϕ	linear subnetworks.
$v_\pi(t)$	vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms of network π .
$i_\phi(t)$	vector of currents entering the linear subnetwork ϕ .
n_ϕ	number of terminals of subnetwork ϕ .
$e_\pi(t)$	vector of source waveforms.
C_π, G_π	constant matrices.
$F(v)$	a function describing the nonlinear elements of the circuit.
D_ϕ, D_k	selector matrices.
s	the complex frequency.
$Z_\phi(s)$	complex frequency domain impedance representation of the subnetwork ϕ ,
$V_\phi(s)$	vector of terminal voltages of the nodes that connect subnetwork ϕ to the network π ,
$i_\phi(t)$	vector of currents entering the linear subnetwork ϕ nodes.
$I_\phi(s)$	Laplace transform of $i_\phi(t)$.
$v(t)$	vector of node voltage waveforms appended by independent voltage source current and linear inductor current waveforms of network ϕ .
W, H	constant matrices with entries determined by the lumped linear components.
e	a constant vector with entries determined by the independent voltage and current sources.
i_k	vector of currents entering the transmission line subnetwork k .
n_k	number of coupled transmission lines in the subnetwork k ,
N_s	number of transmission line subnetworks,
V_k, I_k	Laplace domain terminal voltages and currents of the subnetwork k , respectively.
γ_m^2	an eigenvalue of the matrix $Z_l Y_l$.
S_m	eigenvector associated with γ_m^2 .
E_1, E_2	diagonal matrices defined in terms of the eigenvalues.
S_v	a matrix with the eigenvectors S_m in the columns.
Ω	a diagonal matrix with $\Omega_{m,m} = \gamma_m$.

Chapter 1

Introduction

Rapid advances in the development of VLSI circuit technology and packaging techniques are yielding larger chips with smaller and faster devices. As a result, the interconnection delay time is often significantly longer than the device switching time. In addition, as interconnection densities and switching speeds increase, the electrical length of interconnects becomes a significant fraction of a wavelength and conventional lumped-impedance models can no longer be used for accurate simulation of delay and crosstalk. Instead a distributed transmission line model for the interconnect should be used [1]-[3]. For digital systems, if the signal propagation time, or interconnect delay, is greater than 10 percent of the signal rise/fall time, then coupled lossy transmission lines have to be used to model the interconnects accurately. Furthermore, simulation of high-speed VLSI circuit design requires that the interconnects and the non-linear electronic digital and analog terminations be simulated as an integrated system. A state-of-the-art circuit simulator that can handle multiple coupled lossy transmission lines, digital, and analog electronic circuits is needed to ensure the correct design for production and manufacturing.

The difficulty in developing such a simulator is because the electromagnetic propagation modes in lossy transmission lines are defined in the frequency domain only. In contrast to this, nonlinear terminations can be in the general case analyzed in the time domain only.

Several methods have been proposed for the analysis of networks which contain coupled transmission lines[4]-[9]. In most cases, the lossy transmission line has been analyzed either by using a series of lumped models in the time domain or by using frequency domain transformation. The later methods are based on the evaluation of the domain Green's functions of the transmission lines terminated in quasi-matched loads. The solution of the nonlinear problem is obtained in this case using time-stepping and convolution. In general, these techniques provide a detailed analysis of delay and crosstalk, but they require more computer time that the circuit designer can normally afford especially for VLSI designs where very large number of interconnects has to be considered. This created the need for a computationally less expensive method which can adequately approximate the circuit response.

Recently, an Asymptotic Waveform Evaluation (AWE) technique[10]-[11] has been published for approximating the waveform response of general linear-lumped circuits. In [12]-[13], a generalization of the AWE technique has been presented to handle linear interconnect models which contain lossy coupled transmission lines. It was shown that this technique is useful for both delay and crosstalk estimation and can be used to evaluate response of linear high-speed interconnect circuits with negligible error compared with conventional circuit simulators such as SPICE while two to three orders of magnitude faster. An alternative approach has been recently proposed in [14]. In [15], a method has been presented for the transient analysis of lossy coupled transmission lines with nonlinear terminations. It combines the AWE technique with a piecewise decomposition algorithm

where the nonlinear elements are replaced by time-dependent sources. The time domain response of the resulting linear network is obtained using the AWE technique. The parameters defining the time dependent sources are evaluated iteratively such that the topological and constitutive relations of the nonlinear network are satisfiable. The method is efficient. However, it requires a special stand-alone simulator to be developed. In addition, the algorithm as presented in [15] involves a implicit convolution step which could be computationally expensive especially for mismatched lines. In [16] a modification of the above algorithm is described which avoids the need for the numerical convolution step.

In this thesis, an alternative method [33] is proposed. It extends *the AWE method to handle VLSI interconnects which contain both lossy coupled transmission lines and nonlinear components*. In addition, it can be easily implemented within the framework of an existing circuit simulator.

The proposed method has the following main advantages:

1. It solves the problem of mixed frequency/time domain analysis by replacing the linear subnetwork which contains lossy coupled transmission lines with an equivalent set of ordinary linear differential equations. This equivalent presentation is efficiently derived using the AWE technique. The resulting linear differential equations can be solved simultaneously with the nonlinear differential equations describing the rest of the network using any of the conventional circuit simulators such as SPICE.
2. The method takes full advantage of the efficiency of the AWE technique in analyzing the linear subnetwork (two to three orders of magnitude faster compared to other techniques). However, it has the additional simplification that it does not require explicit evaluation of the dominant poles and corresponding residues. Only the coefficients of the Pade approximation are needed.

3. It can handle general transmission line networks with no topological or electrical constraints.

The organization of the thesis is as follows: in Chapter 2, brief introduction of the transmission line theory and the literature survey are presented. In Chapter 3, the basic theory of the Asymptotic Waveform Evaluation Technique is introduced. Chapter 4 describes the formulation of the network equations and the evaluation of the frequency domain transfer function of the transmission line subnetworks using the AWE technique. The time domain equivalent model is described in Chapter 5. The technique to solve the time domain equivalent model by using Hspice is presented in Chapter 6. Examples are provided in Chapter 7 and results are compared, where applicable, with previously published methods.

Chapter 2

Literature Survey

2.1 Introduction

As mentioned before, the fundamental difficulty encountered in transmission line simulation arises because circuits containing nonlinear devices or time-dependent characteristics must be characterized in the time domain while transmission lines with loss, dispersion are best simulated in the frequency domain.

The mathematical methods for solving the transmission line problems, both mature and under development, can be roughly classified into four categories, Lumped Equivalent Circuit methods, Green's Function Methods, Mixed-Domain Methods, and the Asymptotic Waveform Evaluation Technique.

In the following, we first present the most frequently used transmission line model. Next we review briefly the important papers in each category.

2.2 Quasi-Transverse Electromagnetic Model (TEM) for Multiconductor Transmission Lines

With the advent of new devices having clocking rate, or rise/fall times in the order of a few picoseconds and with the significant increase in the level of integration found in most VLSI systems, the problems of crosstalk between neighbor lines due to electromagnetic coupling and reflections due to mismatch and incorrect line terminations have become prominent in VLSI circuit system. It is required that in high-speed and high-performance VLSI system design, the interconnects be modeled as multiple coupled lossy transmission lines, so that the electrical effects of interconnect transmission lines can be properly evaluated.

Consider a transmission line which is assumed to be of an arbitrary cross section, but uniform along its length and which contains N signal conductors and one ground conductor. Also consider the line is generally embedded in inhomogeneous dielectrics. For the transmission line of which the maximal cross-sectional dimensions are sufficiently smaller than the wavelength of the highest frequency component of the signal propagating along the line, the longitudinal electromagnetic field components are much smaller than the transversal components. The waves of the signal are of a hybrid nature which can be approximated TEM waves, which are, more precisely, called quasi-TEM waves [17].

Assume following matrices of the quasi-TEM parameters of the line circuit theory are known,

R_l : matrix of resistances per unit length; G_l : matrix of conductances per unit length;

L_l : matrix of inductances per unit length; C_l : matrix of capacitances per unit length;

The elements of R_l are related to the self- and mutual resistances due to conductor losses, the elements of G_l are the self- and mutual conductances due to the losses in the dielectrics, the elements of L_l define the self- and mutual inductances determined by the induced electromagnetic field in the electromagnetic induction, and the elements of C_l reflect the self- and mutual coefficients due to the electrostatic effects. It is to be noted that the off-diagonal elements of C_l are always negative.

Let $v_k(x, t)$ and $i_k(x, t)$ represent the vector of line voltage and the vector of line current of the transmission line at a distance x from the near end of the line and at a particular time instant t . The voltages and currents for the quasi-TEM mode propagation are described by the telegrapher equations

$$\begin{aligned}\frac{\partial}{\partial x} v(x, t) &= -R_l i(x, t) - L_l \frac{\partial}{\partial t} i(x, t) \\ \frac{\partial}{\partial x} i(x, t) &= -G_l v(x, t) - C_l \frac{\partial}{\partial t} v(x, t)\end{aligned}\tag{2.1}$$

In the complex frequency domain, the equations are

$$\begin{aligned}\frac{d}{dx} V(x) &= -Z_l I(x) \\ \frac{d}{dx} I(x) &= -Y_l V(x)\end{aligned}\tag{2.2}$$

where Z_l and Y_l are the matrices of line impedances and admittances per unit length,

$$Z_l = R_l + sL_l$$

$$Y_l = G_l + sC_l$$

and $V(x)$ and $I(x)$ are the vectors of complex line voltages and currents, respectively.

It is to be noted that along a transmission line with N signal conductors and one ground conductor, generally N different quasi-TEM modes can propagate, and for a lossy line, in the general case, the quasi-TEM modes can be defined in the frequency domain only.

2.3 Lumped Equivalent Circuit Method

This method divides the transmission line into many sections and approximates each section with a lumped equivalent circuit [18]-[19]. Applying the Euler's method to the partial derivatives in (2.1) with respect to the space coordinate (x) leads to

$$\begin{aligned} v_{m+1}(t) - v_m(t) &= -\Delta x R_l i_m(t) - \Delta x L_l \frac{d}{dt} i_m(t) \\ m &= 1, \dots, P+1 \end{aligned} \quad (2.3)$$

$$\begin{aligned} i_{m+1}(t) - i_m(t) &= -\Delta x G_l v_{m+1}(t) - \Delta x C_l \frac{d}{dt} v_{m+1}(t) \\ m &= 1, \dots, P \end{aligned} \quad (2.4)$$

where $\Delta x = D/P$, and D is the length of the line and P is the number of sections.

Equation (2.3) and (2.4) can be implemented by a lumped equivalent circuit in the form of T-network or π -network. In the case of T-network, the signal conductor is replaced by a chain of T-networks. The mutual couplings between the conductors can be taken into account through the inductive, resistive, capacitive, and conductive coupling elements. Each T-network approximates a transmission line section of length Δx . Hence, each inductive lumped element of the section is given by $L_k/2 = (L_{kk}\Delta x)/2$, while mutual inductances are $L_{mkl}/2 = ((L_{kl}\Delta x)/2) \ (k \neq l)$. Since the inductive elements of two adjacent cells are connected in series, they can be replaced by a single element of inductance $L_{kk}\Delta x$. However, the inductance elements located at the ends of the line cannot be combined. Similarly, each resistive element of the cell is given by $R_k/2 = (R_{kk}\Delta x)/2$, and mutual resistances are $R_{mkl}/2 = ((R_{kl}\Delta x)/2) \ (k \neq l)$. Note that the mutual resistances are due to the finite conductivity of the ground conductor, as well as to the eddy current induced in one signal conductor due to the current in another signal conductor. The adjacent resistances can be combined into a single resistance, just

like the inductances. The self-capacitance of each section is simulated by the capacitance of the lumped elements connected between the signal conductors and ground which is

$$C_k = \sum_{l=1}^N C_{kl} \Delta x ,$$

while the capacitances of the elements connected between the signal conductors which simulate the mutual capacitance of each section are $C_{mkl} = -C_{kl} \Delta x$ ($k \neq l$). Similarly, conductances are given by

$$G_k = \sum_{l=1}^N G_{kl} \Delta x \text{ and } G_{mkl} = -G_{kl} \Delta x \text{ } (k \neq l) .$$

The lumped equivalent circuit of the transmission line which contains 2 signal conductors and one ground conductor is shown in Fig.2.1

This approach has certain advantages and suffers from some severe problems. It is simple to code, and can be directly applied with the existing circuit simulator. In addition, the nonlinear

transmission line termination problems can be easily solved. But commercial programs using the approach typically can handle only as many as three coupled lines.

In the implementation of the approach, a very large number of sections are required so that the line transit time over one section is much shorter than the rise/fall time of the waveforms considered. This will cause the simulation speed to be prohibitively slow. And this approach tends to generate spurious signal oscillations, which are artifacts of the model alone. A lossy line modeled in the approach often will exhibit signal ringing even if it is perfectly terminated.

Fig.2.1 Lumped-element equivalent circuit of a lossy transmission line with two signal conductors.

2.4 Green's Function Method

Green's function method has been used in several ways to handle the problems of multiple coupled lossy transmission line with nonlinear terminations.

In [20], the frequency domain admittance parameter description of the distributed network augmented with quasi-matched termination loads is converted to a time-domain description by using a Fourier transform. This time-domain description is the time-domain Green's function of the augmented distributed system. The properties of the original transmission line are restored by inserting complementary termination networks. The complete system response is found by convolving the Green's function with the transient response of the terminating nonlinear load.

However, there are certain difficulties with the application of the approach. When the response of the line with terminal network is to span a time interval greater than a few line transit times, the number of samples of Green's function would have to be very large and very long registers are needed to keep them so that the aliasing problems of using the inverse Fourier transform can be avoided. Even for moderately lossy lines, the response duration exceeds many line transit times. In addition, this approach requires evaluation of set of convolution integrals at each time point. Hence, it is a computationally expensive method.

An alternative approach was given in [6]. In this approach, by matching the transmission lines on both sides with a reference system consisting of lossless coupled transmission lines, the frequency domain modal scattering parameter description of the transmission lines is formulated. The time-domain scattering parameter Green's function

is obtained by using the inverse Fourier transform. The Green's function is related to the terminal voltage wave vectors and convolved with the nonlinear terminations to produce the system response.

Comparing to A. Djordjevic's approach [20], this technique offers good computational stability and accuracy. But because of lack of a simple error criteria and a clear rule for the proper choice of the reference system, it is still difficult to avoid the aliasing problem of inverse Fourier transform and establish confidence on its convergence and efficiency.

In [21], a more direct scattering parameter technique was presented. By using the similar technique as in [20], the transmission line network is terminated with quasi-matched reference impedance, the frequency domain scattering parameter Green's function is obtained. The time-domain scattering parameter Green's function is derived by using the inverse Fourier transform. This Green's function is directly convolved with port sources to produce the terminal transient responses of the transmission line. The effect of the reference impedance is removed by inserting the complementary reference impedance at each port. Converting the nonlinear terminations into nonlinear controlled voltage source and incorporating with the Green's function, the complete network responses are achieved by using circular convolution.

But this approach still suffers from aliasing problems and computational inefficiency. When the transients in the system cover several transmission line transit times, the time span of Green's function has to be long. This will result a large sampled data set of the Green's function and make the computation very inefficient. To avoid the aliasing errors and make the Green's function bandlimited, a low-pass filter has to be used otherwise the aliasing errors results in ripples in the time-domain Green's function and can lead to convergence problems. However, the incorporation of the filter will change the charac-

teristic impedance of the transmission line and lead to additional loss and dispersion in the simulated response.

2.5 Mixed-Domain Method

As mentioned, the analysis of arbitrary nonlinear terminal networks, in the general case, can be performed only in time domain, while the analysis of lossy coupled transmission lines is best performed in the frequency domain. To combine the two cases, the Mixed-Domain method uses one of two techniques, piecewise decomposition or convolution integral.

Since the convolution integral methods have been described in section 2.4, in this section, we mainly discuss the piecewise decomposition method.

The method is presented in [3] and [23]. Using this method, the transmission line network with arbitrary nonlinear terminations is partitioned into linear subnetworks and nonlinear subnetworks. The linear subnetworks contain all the frequency-dependent components. The nonlinear subnetworks are replaced by piecewise linear time-dependent sources. With the terminals of the linear subnetworks being excited by the piecewise time-dependent sources, the linear subnetworks are first analyzed in the frequency domain and then the results are converted to the time domain. The solution of the nonlinear subnetworks are obtained in the time domain. The complete responses of the network are obtained through the coordination of the two kinds of subnetworks by using the Newton-Raphson iteration. In particular, the parameters of the piecewise waveforms are adjusted in the N-R iterations so that the topological and constitution relations of each subnetworks are satisfied.

Generally, the frequency domain solutions of the linear subnetworks are converted to the time-domain solutions by using one of the two techniques, 1) inverse FFT; 2) numerical inverse Laplace Transform (NILT). The inverse FFT is generally precluded because of its inefficiency and aliasing errors in the conversion from the bandlimited frequency domain to the time domain. The NILT is applied to the analysis of linear networks containing lossy coupled transmission lines in [22]. Using NILT, the aliasing effect can be avoided and the waveforms with infinite duration can be evaluated with good accuracy. But it is still computationally inefficient and requires a particularly developed simulator.

The mixed-domain methods are general and not limited to the analysis of transmission lines. But they are all computer-intensive and the simulation speed is low.

2.6 AWE Method

The Asymptotic Waveform Evaluation technique is proposed by Pillage and Rohrer [10]-[11]. It is a variable refinement waveform estimator for linear RLC approximate interconnect models. In general, AWE can evaluate circuits with non-monotone detail, such as undershoot, overshoot and glitches due to charge sharing, by increasing the order of the approximation as required. Higher order approximations are obtained recursively, at an incremental CPU time. Detailed AWE technique will be presented in Chapter 3.

The AWE technique is a very efficient method for transient analysis of RLC circuits. Because of its efficiency, it has been extended to the analysis of networks containing lossy coupled transmission lines.

In [12]-[13], the AWE is generalized to the analysis of linear network containing lossy coupled transmission lines. The results have been shown that 2-3 order lower simu-

lation time of Generalized AWE than all the other methods is achieved with very good accuracy. And the GAWE is combined with the piecewise decomposition method to analyze the transmission lines with nonlinear terminations [15] and two to three orders of magnitude speedup has been achieved. However this method requires a special simulator and is difficult to accommodate all the existing circuit simulators. Based on GAWE, the time-domain analysis of multiple lossy coupled transmission line with nonlinear terminations is developed in Chapter 4 and Chapter 5.

In [24], a time-domain analysis method based on the GAWE is presented. In this method, based on the Modified Nodal Analysis(MNA) formulation, the approximate Y parameter matrix of a multiport linear subnetwork containing the transmission lines is obtained by using AWE approximation :

$$Y(s)V(s) = I(s)$$

where $V(s)$ and $I(s)$ correspond to the port voltage and current vectors respectively. The elements of $Y(s)$ have the form,

$$y_{ij}(s) = \sum_{r=1}^{q_{ij}} \frac{k_{rij}}{s - p_{rij}} + d_{ij}$$

where p_{rij} and k_{rij} are possibly complex poles and residues, and d_{ij} represents any direct coupling between the input and output.

At each time point, the port voltages are decomposed into piecewise linear ramp functions of time with slope S_p . The functions are convolved with the Y parameters in time domain and the port currents due to a single element of the Y matrix and a single ramp are calculated as,

$$i(t) = S_p \left(\sum_r \left(\frac{-k_r}{p_r^2} + \frac{(-k_r)t}{p_r} + \frac{k_r}{p_r^2} e^{p_r t} \right) \right) u(t)$$

Combining all the port current equations with the nonlinear termination equations which are formulated by applying the trapezoidal integration technique, the complete network solutions are produced step by step.

This approach is general and can be used for all kinds of interconnect problems. But because of the convolution and piecewise decomposition, the efficiency is still limited.

In [16], an alternative approach is presented. In this approach, the H parameters of the linear network containing lossy coupled transmission lines are formulated and approximated by the AWE. The time domain terminal voltages and currents are obtained by the numerical integration of the H parameter formulation with the nonlinear terminations, the desired output waveforms can be calculated recursively. Although the approach can avoid convolution, it cannot be easily implemented using current circuit simulators.

2.7 Summary

To facilitate the high-speed and high-performance VLSI circuit design, a computationally efficient, accurate and easy-to-use simulator which can handle arbitrary linear and nonlinear networks which contain multiconductor lossy interconnects is required. And the simulator should be easily implemented within an existing framework of a circuit simulator. However, from the approaches presented in this chapter, such a demand could be rarely met. This situation makes the efforts in the following chapters a necessity.

Chapter 3

Asymptotic Waveform Evaluation Technique

3.1 Introduction

Recently, a method, Asymptotic Waveform Evaluation(AWE) [3]-[4], was developed at Carnegie Mellon University. AWE is a variable refinement waveform estimator for generalized RLC approximate interconnect models and is not just limited to tree structures. It is based on deriving the moments of the circuit response. Higher order moments can be used to approximate the response, at an incremental cost in CPU time. This makes the method suitable to estimate the circuit response with non-monotonic detail such as undershoot, overshoot and glitches. In this chapter, some of the key concepts in Asymptotic Waveform Evaluation technique are reviewed.

3.2 The AWE Algorithm

There are two different methods to describe the AWE algorithm. One is particular solution algorithm, another is impulse response algorithm. In the following, we will give the detailed presentations of the two algorithms separately.

3.2.1 The particular solution algorithm

AWE method is most conveniently explained in general in terms of the differential state equations for a lumped, linear, time-invariant circuit:

$$\dot{x} = Ax + Bu \quad (3.1)$$

where x is the n -dimensional state vector and u is the m -dimensional excitation vector. For the investigation of the delay and rise time effects in the interconnects, the input excitations can be assumed to be of the particular form

$$u_p(t) = u_0 + u_1 t \quad \text{for } t > t_0 \quad (3.2)$$

where u_0 and u_1 are constant m -dimensional vectors.

For the excitation u_p , (3.2), the differential-state equation (3.1) has the particular solution

$$x_p(t) = -A^{-1}Bu_0 - A^{-2}Bu_1 - A^{-1}Bu_1 t \quad (3.3)$$

For the solution to be valid, the A -matrix may not be singular. This restriction is not unreasonable for most of the interconnect circuits in which delay estimation may be of interest. Actually, this condition is equivalent to specifying that the circuit in question

have a unique and well-defined dc solution when all of its capacitances are open-circuited and all of its inductances are short-circuited. Under this restriction, the solution of (3.1) can be completed by solving the homogeneous equation:

$$\dot{x}_h = A x_h \quad (3.4)$$

with the initial condition

$$x_p(0) = x_0 + A^{-1} B u_0 + A^{-2} B u_1 \quad (3.5)$$

where x_0 is the initial state at time zero. The Laplace transform solution of the homogeneous equation is

$$X_h(s) = (sI - A)^{-1} x_h(0) \quad (3.6)$$

To approximate this solution, $X_h(s)$ is first expanded in a Maclaurin's series

$$X_h(s) = -A^{-1} (I + A^{-1}s + A^{-2}s^2 + \dots) x_h(0) \quad (3.7)$$

and as many moments as necessary or desirable are matched in terms of lower order approximating functions. Focusing now on a specific component of $X_h(s)$, say the i th, its initial conditions and first $2q-1$ moments are characterized as

$$\begin{aligned} [m_{-1}]_i &= [x_h(0)]_i \\ [m_0]_i &= [-A^{-1}x_h(0)]_i \\ [m_1]_i &= [-A^{-2}x_h(0)]_i \\ &\dots \\ &\dots \\ [m_{2q-2}]_i &= [-A^{-2q+1}x_h(0)]_i \end{aligned} \quad (3.8)$$

Then, these moments are matched to a lower order frequency-domain rational function of the form

$$\begin{aligned}\hat{X}_i(s) &= \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \dots + \frac{k_q}{s-p_q} \\ &= \sum_{r=1}^q \frac{k_r}{s-p_r}\end{aligned}\quad (3.9)$$

where k_r and p_r , $r = 1, \dots, q$ are the complex approximating poles and residues of the frequency-domain approximation function. Correspondingly, in time-domain, the moments are matched to those of an approximating function of the form

$$\begin{aligned}\hat{x}_i(t) &= L^{-1}(\hat{X}_i(s)) \\ &= \sum_{r=1}^q k_r \exp(p_r t)\end{aligned}\quad (3.10)$$

where L^{-1} stands for the inverse Laplace transformation.

The reciprocals of the desired poles p_r , $r = 1, \dots, q$ can be found by solving a characteristic polynomial

$$a_0 + a_1 p^{-1} + a_2 p^{-2} + \dots + a_{q-1} p^{q-1} + p^{-q} = 0 \quad (3.11)$$

Using the general Pade approximation, the coefficients a_r , $r = 0, 1, \dots, q-1$ can be found by solving

$$\begin{bmatrix} m_{-1} & m_0 & m_1 & \dots & m_{q-2} \\ m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_4 & \dots & m_q \\ \dots & \dots & \dots & \dots & \dots \\ m_{q-2} & m_{q-1} & m_q & \dots & m_{2q-3} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \dots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_{q-1} \\ m_q \\ m_{q+1} \\ \dots \\ m_{2q-2} \end{bmatrix} \quad (3.12)$$

And the approximating residues k_r , $r = 1, \dots, q$ can be obtained by solving

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ p_1^{-1} & p_2^{-1} & \dots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & \dots & p_q^{-2} \\ \dots & \dots & \dots & \dots \\ p_1^{-q+1} & p_2^{-q+1} & \dots & p_q^{-q+1} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \dots \\ k_q \end{bmatrix} = \begin{bmatrix} m_{-1} \\ m_0 \\ m_1 \\ \dots \\ m_{q-2} \end{bmatrix} \quad (3.13)$$

Once the desired poles and residues of the approximating function are found, the approximation for the homogeneous solution can be completed using (3.10). The complete approximate solution for the response is equal to

$$x_i(t) = x_{p_i}(t) + \hat{x}_i(t) \quad (3.14)$$

3.2.2 The impulse response algorithm

An alternative way of explaining the AWE technique can be given in terms of the impulse response

$$\dot{x} = Ax + B\delta(t) \quad (3.15)$$

where $\delta(t)$ is the impulse excitation.

The Laplace transform solution of (3.15) is

$$X_h(s) = (I - A)^{-1}B \quad (3.16)$$

Following the derivation of (3.6), first (3.16) is expanded into a Maclaurin's series

$$X_h(s) = -A^{-1}(I + A^{-1}s + A^{-2}s^2 + \dots)B \quad (3.17)$$

Next take as many moments as necessary to approximate (3.17) by the lower order function $\hat{X}_i(t)$ as shown in (3.9).

$$\begin{aligned}
[m_0]_i &= [-A^{-1}B]_i \\
[m_1]_i &= [-A^{-2}B]_i \\
&\dots \\
&\dots \\
[m_{2q-1}]_i &= [-A^{-2q}B]_i
\end{aligned} \tag{3.18}$$

The coefficients of the characteristic polynomial of (3.11) are calculated by

$$\begin{bmatrix} m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_3 & \dots & m_q \\ m_2 & m_3 & m_4 & \dots & m_{q+1} \\ \dots & \dots & \dots & \dots & \dots \\ m_{q-1} & m_q & m_{q+1} & \dots & m_{2q-1} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \dots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_q \\ m_{q+1} \\ m_{q+2} \\ \dots \\ m_{2q-1} \end{bmatrix} \tag{3.19}$$

The roots of the characteristic polynomial are the reciprocal poles and the residues of $\hat{X}_i(t)$ are determined by

$$\begin{bmatrix} p_1^{-1} & p_2^{-1} & \dots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & \dots & p_q^{-2} \\ p_1^{-3} & p_2^{-3} & \dots & p_q^{-3} \\ \dots & \dots & \dots & \dots \\ p_1^{-q} & p_2^{-q} & \dots & p_q^{-q} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \dots \\ k_q \end{bmatrix} = \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ \dots \\ m_{q-1} \end{bmatrix} \tag{3.20}$$

It has been noted that the function $\hat{X}_i(t)$ is the approximating transfer function such that the frequency response corresponding to an input function $F(s)$ can be easily obtained as $\hat{X}_i(s) F(s)$.

3.3 Numerical Examples

AWE technique is an efficient approach to waveform estimation for linear RLC interconnect circuit models. Floating capacitors, inductors, linear controlled sources, finite input rise time, and charge sharing are all easily addressed in terms of AWE at any level of detail by merely increasing the order of the approximation. The efficiency of AWE technique can be demonstrated by some CPU comparisons with Hspice in Table 3.1. This table is extracted from [29].

Table 3.1 CPU Comparison between AWE and Hspice

circuit name	node count	element count	CPU seconds	
			AWE	Hspice
small RC tree	19	35	0.02	0.40
medium sized interconnect	75	121	0.04	0.63
backplane ⁽¹⁾	505	2294	3.54	(2)
clock distribution line ⁽³⁾	14,000	55,000	300	37271

(1) backplane is modelled by 2000 inductors and some controlled sources, resistors, and capacitors.

(2) Hspice could not simulate of presence of inductor loops.

(3) clock distribution line is modelled by 40,000 resistors and 15,000 capacitors.

Example 3.1

To illustrate the accuracy of the AWE technique, consider the RLC circuit shown in Figure 3.1. The input voltage is a 5-V step with a 0.1ns rise time. The comparison between Hspice simulation and the AWE approximation is shown in Figure 3.2. It is noted that at a second order approximation, AWE is unable to completely detect the overshoot, therefore, a higher order approximation is required. At fourth order, AWE response almost overlaps the Hspice waveform.

Figure 3.1 Circuit for example 3.1

Figure 3.2 Transient response of the circuit shown in Figure 3.1

3.4 Comments on AWE Technique

AWE is an efficient method to predict the signal delay and crosstalk in the interconnections. In addition, it can solve the circuit which may contain floating capacitors, inductors and have nonzero input rise times. High simulation accuracy can be achieved by considering the difference between two consecutive order of the approximate response waveform.

However, conventional AWE technique could generate some right half plane poles for a stable, linear circuit especially at higher orders of approximation. To overcome this problem, a new method to obtaining a large number of accurate network poles using complex frequency hopping has been proposed in [25]. This new method retains the efficiency of AWE technique and greatly increases the analysis accuracy of large RLC networks which include lossy, coupled transmission lines and nonlinear terminations. Therefore, based on this new method, good confidence to the accuracy of the results obtained by using AWE technique can be established.

In next chapter, the formulation of nonlinear network which contains multiconductor lossy transmission lines will be presented. And the Generalized Asymptotic Waveform Evaluation (AWE) [13] technique will be summarized and directly applied to the formulation.

Chapter 4

Frequency Domain Parameter Evaluation

In this chapter, the equations of the network consisting of lossy coupled transmission line and nonlinear components are formulated and the frequency domain responses of the linear subnetwork which contains the transmission lines are evaluated by using the AWE technique.

4.1 Formulation of The Network Equations

Consider a nonlinear network π which contains linear lumped components, nonlinear components and arbitrary linear subnetworks. The linear lumped components may be described by equations in either the time or frequency domain while the nonlinear components may only be described by time domain equations. The arbitrary linear subnetworks may contain distributed elements that are best described in the frequency domain. Let the linear subnetworks be grouped into a single subnetwork ϕ . The subnetwork ϕ is not constrained to be connected.

Without loss of generality the modified nodal admittance matrix equation[26] of the network π can be written as

$$C_{\pi} \frac{d}{dt} v_{\pi}(t) + G_{\pi} v_{\pi}(t) + D_{\phi} i_{\phi}(t) + F(v_{\pi}(t)) - e_{\pi}(t) = 0 ; \quad t \in [0, T] \quad (4.1)$$

where

$v_{\pi}(t) \in \mathfrak{R}^{N_{\pi}}$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,

$e_{\pi}(t) \in \mathfrak{R}^{N_{\pi}}$ is the vector of source waveforms,

$C_{\pi} \in \mathfrak{R}^{N_{\pi} \times N_{\pi}}, G_{\pi} \in \mathfrak{R}^{N_{\pi} \times N_{\pi}}$ are constant matrices,

$F(v) : \mathfrak{R}^{N_{\pi}} \rightarrow \mathfrak{R}^{N_{\pi}}$ is a function describing the nonlinear elements of the circuit and,

$D_{\phi} = [d_{i,j}]$, $d_{i,j} \in \{0, 1\}$, $i \in \{1, 2, \dots, N_{\pi}\}$, $j \in \{1, 2, \dots, n_{\phi}\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps, $i_{\phi}(t) \in \mathfrak{R}^{n_{\phi}}$ the vector of currents entering the linear subnetwork nodes, into the node space $\mathfrak{R}^{N_{\pi}}$ of the nonlinear network π .

The first, second and fourth terms in (4.1) cover the networks lumped components and independent sources. The third term describes currents at subnetwork terminals, and then maps them into the rest of the network through the D_k matrix. The linear multi-terminal subnetwork ϕ can be characterized in the frequency domain by any of the usual matrix sets such as Z , Y , H or S parameters. Without loss of generality, the relation between its terminal voltages and currents can be written in the form

$$Z_{\phi}(s) I_{\phi}(s) = v_{\phi}(s) \quad (4.2)$$

where

s is the complex frequency,

$z_{\phi}(s)$ is the complex frequency domain impedance representation of the subnetwork ϕ ,

$v_{\phi}(s)$ is the vector of terminal voltages of the nodes that connect the subnetwork to the network π ,

$I_{\phi}(s) = L[i_{\phi}(t)]$, where L denotes the Laplace transform.

EXAMPLE

To illustrate the formulation method, consider the network shown in Figure 4.1. The network can be described by (4.1) and (4.2) where

$$v_{\pi} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_E \\ q \end{bmatrix}, \quad F(v_{\pi}(t)) = \begin{bmatrix} -f_1(v_2 - v_1) \\ f_1(v_2 - v_1) \\ f_2(v_3) \\ 0 \\ 0 \\ -f_3(v_4) \end{bmatrix}, \quad e_{\pi}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E(t) \\ 0 \end{bmatrix}$$

$$C_{\pi} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C & -C & 0 \\ 0 & 0 & -C & C & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad G_{\pi} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$i_{\phi} = \begin{bmatrix} i_2 \\ i_3 \end{bmatrix}, \quad D_{\phi} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$$

The difficulty in solving (4.1) and (4.2) simultaneously is due to the fact that they implicitly contain a mixture of frequency and time domain representations. The analysis algorithm described in this paper overcomes this difficulty through three basic steps:

- 1) Using the AWE technique, $Z_{\phi}(s)$ in (4.2) is approximated by a q-order polynomial model.
- 2) Using the q-order polynomial model, an equivalent time domain representation is obtained. The time domain model is in the form of a set of linear ordinary differential equations.
- 3) The derived differential equations and (4.1) are solved simultaneously using standard numerical integration techniques or any of the conventional circuit simulators such as SPICE.

These steps are explained in more details in the following sections.

4.2 Equivalent Frequency Domain Representation of

$$Z_{\phi}(s)$$

Details of the AWE method and its application to linear networks which contain lossy coupled transmission lines are described in [10] - [13]. In this section, a summary of the basic principles and application to the determination of an equivalent representation of $Z_{\phi}(s)$ is given.

4.2.1 Formulation of The Linear Subnetwork Equations

Consider the linear subnetwork ϕ which contains linear lumped components and an

arbitrary number of subnetworks which consist of lossy coupled transmission lines. The Modified Nodal Admittance matrix equations of the subnetwork ϕ , can be written as

$$W \frac{d}{dt} \mathbf{v}(t) + H \mathbf{v}(t) + \sum_{k=1}^{N_s} D_k \mathbf{i}_k(t) - \mathbf{e} \delta(t) = 0 \quad (4.3)$$

Where

$\mathbf{v}(t) \in \mathfrak{R}^N$ is the vector of node voltage waveforms appended by independent voltage source current and linear inductor current waveforms,

$W \in \mathfrak{R}^{N \times N}$ and $H \in \mathfrak{R}^{N \times N}$ are constant matrices with entries determined by the lumped linear components,

$\mathbf{e} \in \mathfrak{R}^N$ is a constant vector with entries determined by the independent voltage and current sources,

$D_k = [d_{i,j}]$, $d_{i,j} \in \{0, 1\}$, $i \in \{1, 2, \dots, N\}$, $j \in \{1, 2, \dots, 2n_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $\mathbf{i}_k \in \mathfrak{R}^{2n_k}$, the vector of currents entering the transmission line subnetwork k , into the node space $\mathfrak{R}^N$ of the network ϕ ,

n_k is the number of coupled transmission lines in the subnetwork k ,

N_s is the number of transmission line subnetworks,

$\delta(t)$ is the unit impulse function.

Let the frequency domain equations of the transmission line subnetwork k be in the form

$$A_k V_k(s) + B_k I_k(s) = 0 \quad (4.4)$$

where V_k and I_k represent the Laplace domain terminal voltages and currents of the subnetwork k , respectively.

A_k and B_k can be described in terms of the line parameters [13]. Results are summarized in the Appendix.

Using (4.4), the Laplace transform of (4.3) is

$$\begin{bmatrix} sW + H & D_1 & D_2 & \dots & D_{N_s} \\ A_1 D_1^t & B_1 & 0 & \dots & 0 \\ A_2 D_2^t & 0 & B_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ A_{N_s} D_{N_s}^t & 0 & 0 & \dots & B_{N_s} \end{bmatrix} \begin{bmatrix} V(s) \\ I_1(s) \\ I_2(s) \\ \dots \\ \dots \\ I_{N_s}(s) \end{bmatrix} = \begin{bmatrix} e \\ 0 \\ 0 \\ \dots \\ \dots \\ 0 \end{bmatrix} \quad (4.5)$$

or

$$U(s) = [Y^{-1}(s)] E \quad (4.6)$$

where

$$U(s) = \begin{bmatrix} V(s) & I_1(s) & I_1(s) & \dots & I_{N_s}(s) \end{bmatrix}^t, \\ E = \begin{bmatrix} e & 0 & 0 & 0 & \dots & 0 \end{bmatrix}^t,$$

$$Y(s) = \begin{bmatrix} sW + H & D_1 & D_2 & \dots & D_{N_s} \\ A_1 D_1^t & B_1 & 0 & \dots & 0 \\ A_2 D_2^t & 0 & B_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ A_{N_s} D_{N_s}^t & 0 & 0 & \dots & B_{N_s} \end{bmatrix}$$

EXAMPLE:

Consider the linear subnetwork ϕ shown in Figure 4.1. Assuming an input current source $\delta(t)$ at node 2, the subnetwork can be replaced by (4.3) where:

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad H = \begin{bmatrix} G_3 & 0 & 0 & 0 \\ 0 & G_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_2 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$U(s) = \begin{bmatrix} V(s) & I_1(s) \end{bmatrix}^t$$

$$I_1(s) = \begin{bmatrix} I_a & I_b & I_c & I_d \end{bmatrix}^t$$

$$V(s) = \begin{bmatrix} V_2 & V_3 & V_5 & V_6 \end{bmatrix}^t$$

A_1 and B_1 are 4×4 matrices such that

$$A_1 \begin{bmatrix} V_3 \\ V_6 \\ V_2 \\ V_5 \end{bmatrix} + B_1 \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} = 0$$

Derivation of A_1 and B_1 is described in the Appendix.

Fig. 4.1 An example circuit contains coupled transmission lines and nonlinear components

4.2.2 The Generalized Asymptotic Waveform Evaluation Technique

To approximate the transient solution $u(t)$, (4.6) is expanded in Maclaurin's series in the form

$$U(s) = \sum_{n=0}^{\infty} M_n s^n \quad (4.7)$$

where

$$M_n = \frac{\frac{\partial^n}{\partial s^n} [Y^{-1}] |_{s=0}}{n!} E \quad (4.8)$$

The moments $n_n = [M_n]_{(i)}$; $n = 0, 1, 2, \dots, 2q-1$ of an output i is then matched to a lower order frequency domain function in the form

$$[U(s)]_{(i)} = \frac{\sum_{j=0}^{q-1} b_j s^{q-j-1}}{\sum_{j=0}^{q-1} a_j s^{q-j-1}} + b_q \quad (4.9)$$

where the coefficients in (4.9) are related to the moments by

$$\begin{bmatrix} m_1 & m_2 & m_3 & \dots & m_q \\ m_2 & m_3 & m_4 & \dots & m_{q+1} \\ m_3 & m_4 & m_5 & \dots & m_{q+2} \\ \dots & \dots & \dots & \dots & \dots \\ m_q & m_{q+1} & m_{q+2} & \dots & m_{2q-1} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \dots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_{q+1} \\ m_{q+2} \\ m_{q+3} \\ \dots \\ m_{2q} \end{bmatrix} \quad (4.10)$$

and

$$\begin{bmatrix} b_{q-1} + b_q \\ b_{q-2} + a_{q-1}b_q \\ b_{q-3} + a_{q-2}b_q \\ \dots \\ b_0 + a_1b_q \\ a_0b_q \end{bmatrix} = \begin{bmatrix} m_0 & 0 & 0 & \dots & 0 \\ m_1 & m_0 & 0 & \dots & 0 \\ m_2 & m_1 & m_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ m_{q-1} & m_{q-2} & m_{q-3} & \dots & 0 \\ m_q & m_{q-1} & m_{q-2} & \dots & m_0 \end{bmatrix} \begin{bmatrix} 1 \\ a_{q-1} \\ a_{q-2} \\ \dots \\ a_0 \end{bmatrix} \quad (4.11)$$

4.2.3 Evaluation of The Moments

The moments $[M_n]_i$ can be evaluated using the recursive equation [13]

$$[Y]^{(0)} M_n = - \sum_{r=1}^n \frac{[Y]^{(r)} M_{n-r}}{r!} \quad (4.12)$$

$$[Y]^{(0)} = Y(s=0)$$

$$[Y]^{(r)} = \left[\frac{\partial^r}{\partial s^r} Y(s) \right]_{s=0}$$

with

$$[Y]^{(0)} M_n = E \quad (4.13)$$

The derivatives of Y are obtained as

$$[Y]^{(1)} = \begin{bmatrix} C & 0 & 0 & \dots & 0 \\ A_1^{(1)} D_1' & B_1^{(1)} & 0 & \dots & 0 \\ A_2^{(1)} D_2' & 0 & B_2^{(1)} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ A_{N_s}^{(1)} D_{N_s}' & 0 & 0 & \dots & B_{N_s}^{(1)} \end{bmatrix} \quad (4.14)$$

and

$$[Y]^{(r)} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ A_1^{(r)} D_1^t & B_1^{(r)} & 0 & \dots & 0 \\ A_2^{(r)} D_2^t & 0 & B_2^{(r)} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ A_{N_r}^{(r)} D_{N_r}^t & 0 & 0 & \dots & B_{N_r}^{(r)} \end{bmatrix}; \quad (r \geq 2) \quad (4.15)$$

Equation (4.15) shows that, in the special case where the network contains lumped components only,

$$[Y]^{(r)} = 0 \quad (r \geq 2) \quad (4.16)$$

and (4.12) can be reduced to

$$[Y]^{(0)} M_n = -[Y]^{(1)} M_{n-1} \quad (4.17)$$

In the case where the subnetwork k consists of a multiconductor transmission line system, the derivatives $A_k^{(r)}$ and $B_k^{(r)}$ can be obtained in terms of the line parameters, as described in the next section.

4.2.4 The Multiconductor Transmission Line Moments

Details of the evaluation of the moments can be found in [13]. For the sake of completeness a summary is given here.

For a transmission line uniform along its length with an arbitrary cross section the derivatives $A_k^{(r)}$ and $B_k^{(r)}$ and hence the moments may be derived from parameters describing the line. The cross section with N signal conductors, can be represented by the following $N \times N$ matrices of line parameters: the inductance per unit length L_l , the resistance per unit length R_l , the capacitance per unit length C_l , and the conductance per unit

length G_l .

Let γ_m^2 be an eigenvalue of the matrix $Z_l Y_l$ with an associated eigenvector S_m , where

$$\begin{aligned} Z_l &= R_l + sL_l \\ Y_l &= G_l + sC_l \end{aligned} \quad (4.18)$$

It can be shown [13] that the terminal voltages and currents are related by (4.4) where

$$A_k = \begin{bmatrix} S_v E_1 S_v^{-1} & -U \\ S_i E_2 S_v^{-1} & 0 \end{bmatrix} \quad (4.19)$$

$$B_k = \begin{bmatrix} S_v E_2 S_i^{-1} & 0 \\ S_i E_1 S_i^{-1} & -U \end{bmatrix} \quad (4.20)$$

E_1 and E_2 are diagonal matrices defined in terms of the eigenvalues,

$$E_1 = \text{diagonal} \left\{ \frac{\exp(-\gamma_m D) + \exp(\gamma_m D)}{2} \right\}, \quad m = 1, \dots, N \quad (4.21)$$

$$E_2 = \text{diagonal} \left\{ \frac{\exp(-\gamma_m D) - \exp(\gamma_m D)}{2} \right\}, \quad m = 1, \dots, N \quad (4.22)$$

where D is the length of the line,

S_v is a matrix with the eigenvectors S_m in the columns,

$S_i = Z_l^{-1} S_v \Omega$, Ω is a diagonal matrix with $\Omega_{m,m} = \gamma_m$.

From (4.19) and (4.20) the moments required in (4.12) are

$$A_k^{(r)} = \begin{bmatrix} [S_v E_1 S_v^{-1}]^{(r)} & 0 \\ [S_i E_2 S_v^{-1}]^{(r)} & 0 \end{bmatrix} \quad (4.23)$$

$$B_k^{(r)} = \begin{bmatrix} [S_v E_2 S_i^{-1}]^{(r)} & 0 \\ [S_i E_1 S_i^{-1}]^{(r)} & 0 \end{bmatrix} \quad (4.24)$$

To find a closed-form expression for $A_k^{(r)}$ and $B_k^{(r)}$, Leibnitz's rule is used to expand the derivatives in (4.23) and (4.24) in terms of the derivatives of the eigenvectors S_v and the eigenvalues γ_m^2 .

Consider $S_v E_1 S_v^{-1} = \Phi$ or $S_v E_1 = \Phi S_v$, differentiation yields

$$\Phi^{(n)} S_v = - \sum_{r=1}^n \binom{n}{r} \Phi^{(n-r)} S_v^{(r)} + \sum_{r=1}^n \binom{n}{r} S_v^{(n-r)} E_1^{(r)} + S_v^{(n)} E_1 \quad (4.25)$$

Which gives derivatives for Φ in terms of lower order derivatives and the derivatives of S_v and E_1 . Differentiating E_1 gives

$$E_1^{(n+1)} = \text{diagonal} \left[\begin{array}{c} -\frac{D}{2} ((\exp(-\gamma_m D) - \exp(\gamma_m D))^{(n)}) \gamma_m^{(1)} - \\ \frac{D}{2} \sum_{r=1}^n ((\exp(-\gamma_m D) - \exp(\gamma_m D))^{(n-r)}) \gamma_m^{(r+1)} \end{array} \right], \quad (4.26)$$

$m = 1, \dots, N$

Next, the derivatives of the eigenvalues and eigenvectors are evaluated using the solution of the eigenvalue problem

$$[\gamma_m^2 U - Z_l Y_l] S_m = 0 \quad (4.27)$$

where U is the identity matrix. The first derivative can be obtained as

$$\left[\gamma_m^2 U - Z_l Y_l \quad 2\gamma_m S_m \right] \begin{bmatrix} \frac{dS_m}{ds} \\ \frac{d\gamma_m}{ds} \end{bmatrix} = \left[\frac{d}{ds} Z_l Y_l \right] S_m \quad (4.28)$$

Equation (4.28) represents a system of n equations with $(n+1)$ unknowns. To solve this system another equation normalizing the eigenvector S_m such that $S_m^T S_m = 1$ or

$$S_m' \frac{dS_m}{ds} = 0 \quad (4.29)$$

is added to (4.28) yielding

$$\begin{bmatrix} \gamma_m^2 U - Z_l Y_l & 2\gamma_m S_m \\ S_m' & 0 \end{bmatrix} \begin{bmatrix} \frac{dS_m}{ds} \\ \frac{d\gamma_m}{ds} \end{bmatrix} = \begin{bmatrix} \frac{d}{ds} Z_l Y_l \\ 0 \end{bmatrix} S_m \quad (4.30)$$

Higher order derivatives are obtained recursively as

$$T \begin{bmatrix} S_m^{(n+1)} \\ \gamma_m^{(n+1)} \end{bmatrix} + \sum_{r=1}^n \binom{n}{r} T^{(r)} \begin{bmatrix} S_m^{(n-r+1)} \\ \gamma_m^{(n-r+1)} \end{bmatrix} = \begin{bmatrix} -\sum_{r=1}^n \binom{n}{r} Z_l Y_l^{(n-r+1)} S_m^{(r)} \\ 0 \end{bmatrix} \quad (4.31)$$

where

$$T = \begin{bmatrix} \gamma_m^2 U - Z_l Y_l & 2\gamma_m S_m \\ S_m' & 0 \end{bmatrix} \quad (4.32)$$

A similar method can be used to calculate the derivatives of other terms in (4.23) and (4.24) with respect to the derivatives of S_m and γ_m (4.31).

4.2.5 The Evaluation of $Z_\phi(s)$

To evaluate $Z(s)$, we notice that $[Z(s)]_{i,j}$ can be obtained by connecting a unit impulse current source at terminal j of the subnetwork ϕ while keeping all other terminals open-circuited and computing the voltage at terminal i .

Using (4.7), $Z_\phi(s)$ can be approximated by

$$Z_{\phi}(s) = \begin{bmatrix} \frac{\sum_{m=0}^{q-1} b_m^{1,1} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{1,1} & \dots & \frac{\sum_{m=0}^{q-1} b_m^{1,n_{\phi}} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{1,n_{\phi}} \\ \dots & \dots & \dots \\ \frac{\sum_{m=0}^{q-1} b_m^{n_{\phi},1} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{n_{\phi},1} & \dots & \frac{\sum_{m=0}^{q-1} b_m^{n_{\phi},n_{\phi}} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{n_{\phi},n_{\phi}} \end{bmatrix} \quad (4.33)$$

It is to be noted that the same denominator is used for all outputs while corresponding numerators are calculated for each output/input transfer function.

EXAMPLE

Consider again the network shown in Figure 4.1, (4.2) can be approximated in the form:

$$\begin{bmatrix} \frac{\sum_{m=0}^{q-1} b_m^{1,1} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{1,1} & \frac{\sum_{m=0}^{q-1} b_m^{1,2} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{1,2} \\ \frac{\sum_{m=0}^{q-1} b_m^{2,1} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{2,1} & \frac{\sum_{m=0}^{q-1} b_m^{2,2} s^{q-m-1}}{\sum_{m=0}^q a_m s^{q-m}} + b_q^{2,2} \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$$

In next chapter, the equivalent time domain representation of $Z_{\phi}(s)$ will be derived.

Chapter 5

Time Domain Equivalent Representation

In this chapter, the time domain equivalent representation of $Z_\phi(s)$ is presented, and proved mathematically.

5.1 Time Domain Equivalent Representation of $Z_\phi(s)$

By introducing new states $\{x_{j,i} ; i = 1, 2, \dots, n_\phi ; j = 1, 2, \dots, q\}$, (4.2) can be represented in the time domain by a set of differential equations in the form

$$\frac{d}{dt}x_i(t) - A_i x_i(t) - B_i i_\phi = 0 \quad (5.1a)$$

$$v_{\phi,i}(t) - C_i x_i(t) - D_i i_\phi = 0 \quad (5.1b)$$

where

$$x_i(t) = [x_{1,i} \ x_{2,i} \ \dots \ x_{q,i}]^t, \quad v_{\phi,i}(t) = [v_{\phi}(t)]_i$$

$$A_i = \begin{bmatrix} 0 & 0 & \dots & 0 & -\frac{a_1}{a_0} \\ \frac{1}{a_{q-1}} & 0 & \dots & 0 & -\frac{a_1}{a_0} \\ 0 & \frac{a_{q-1}}{a_{q-2}} & \dots & 0 & -\frac{a_1}{a_0} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{a_2}{a_1} & -\frac{a_1}{a_0} \end{bmatrix} \quad B_i = \frac{a_1}{a_0} \begin{bmatrix} \frac{b_{q-1}^{i,1}}{a_{q-1}} & \dots & \frac{b_{q-1}^{i,n_{\phi}}}{a_{q-1}} \\ \frac{b_{q-2}^{i,1}}{a_{q-2}} & \dots & \frac{b_{q-2}^{i,n_{\phi}}}{a_{q-2}} \\ \frac{b_{q-3}^{i,1}}{a_{q-3}} & \dots & \frac{b_{q-3}^{i,n_{\phi}}}{a_{q-3}} \\ \dots & \dots & \dots \\ \frac{b_0^{i,1}}{a_1} & \dots & \frac{b_0^{i,n_{\phi}}}{a_1} \end{bmatrix}$$

$$C_i = [0 \ 0 \ \dots \ 0 \ 1] \quad D_i = [b_q^{i,1} \ b_q^{i,2} \ \dots \ b_q^{i,n_{\phi}}]$$

Using (4.1) and (5.1), the time domain equations for the whole network can be written in the form

$$C_{\pi} \frac{d}{dt} v_{\pi}(t) + G_{\pi} v_{\pi}(t) + D_{\phi} \dot{i}_{\phi} - F(v_{\pi}(t)) - e_{\pi}(t) = 0, t \in [0, T] \quad (5.2a)$$

$$\dot{X} - AX - Bi_{\phi} = 0 \quad (5.2b)$$

$$D_{\phi}^t v_{\pi} - CX - Di_{\phi} = 0 \quad (5.2c)$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_{n_{\phi}} \end{bmatrix}, \quad v_{\phi} = \begin{bmatrix} v_{\phi,1} \\ v_{\phi,2} \\ \dots \\ v_{\phi,n_{\phi}} \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & \emptyset & \dots & \emptyset \\ \emptyset & A_2 & \dots & \emptyset \\ \dots & \dots & \dots & \dots \\ \emptyset & \emptyset & \dots & A_{n_{\phi}} \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \dots \\ B_{n_{\phi}} \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & \emptyset & \dots & \emptyset \\ \emptyset & C_2 & \dots & \emptyset \\ \dots & \dots & \dots & \dots \\ \emptyset & \emptyset & \dots & C_{n_{\phi}} \end{bmatrix}, \quad D = \begin{bmatrix} D_1 \\ D_2 \\ \dots \\ D_{n_{\phi}} \end{bmatrix}$$

EXAMPLE

Consider the network shown in Figure 4.1 again. The time domain equivalent representation of (4.2) with $q = 3$ can be described by (5.2) where

$$X = \begin{bmatrix} x_{1,1} & x_{2,1} & x_{3,1} & x_{1,2} & x_{2,2} & x_{3,2} \end{bmatrix}^t$$

$$A = \begin{bmatrix} 0 & 0 & -\frac{a_1}{a_0} & 0 & 0 & 0 \\ \frac{1}{a_2} & 0 & -\frac{a_1}{a_0} & 0 & 0 & 0 \\ 0 & \frac{a_2}{a_1} & -\frac{a_1}{a_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{a_1}{a_0} \\ 0 & 0 & 0 & \frac{1}{a_2} & 0 & -\frac{a_1}{a_0} \\ 0 & 0 & 0 & 0 & \frac{a_2}{a_1} & -\frac{a_1}{a_0} \end{bmatrix}$$

$$B = \frac{a_1}{a_0} \begin{bmatrix} \frac{b_2^{1,1}}{a_2} & \frac{b_2^{1,2}}{a_2} \\ \frac{b_1^{1,1}}{a_2} & \frac{b_1^{1,2}}{a_2} \\ \frac{b_0^{1,1}}{a_1} & \frac{b_0^{1,2}}{a_1} \\ b_2^{2,1} & b_2^{2,2} \\ \frac{b_1^{2,1}}{a_2} & \frac{b_1^{2,2}}{a_2} \\ \frac{b_0^{2,1}}{a_1} & \frac{b_0^{2,2}}{a_1} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} b_3^{1,1} & b_3^{1,2} \\ b_3^{2,1} & b_3^{2,2} \end{bmatrix}$$

5.2 Proof of The Time Domain Equivalent Representation

Proof of (5.1) and (5.2):

Using (4.9), the element $[Z_\phi(s)]_{i,j}$ of $Z_\phi(s)$ can be written as,

$$[Z_\phi(s)]_{i,j} = [\hat{Z}_\phi(s)]_{i,j} + b_q^{i,j} \quad (5.3)$$

where

$$[\hat{Z}_\phi(s)]_{i,j} = \frac{b_{q-1}^{i,j} + b_{q-2}^{i,j}s + \dots + b_0^{i,j}s^{q-1}}{1 + a_{q-1}s + \dots + a_0s^q} \quad (5.4)$$

$$i \in \{1, 2, \dots, n_\phi\} ; j \in \{1, 2, \dots, n_\phi\} ;$$

Introduce $\psi_{i,j}(s)$ such that

$$\psi_{i,j}(s) = [\hat{Z}_\phi(s)]_{i,j} \times I_j(s) \quad (5.5)$$

where $I_j(s) = [I_\phi(s)]_j$

Substituting (5.5) into (4.2) gives,

$$V_{\phi,i}(s) = \sum_{j=1}^{n_\phi} (\psi_{i,j}(s) + b_q^{i,j}I_j(s)), \quad (5.6)$$

Substituting (5.4) into (5.5) and after some algebraic manipulation, we obtain,

$$\begin{aligned} & (1 + a_{q-1}s + \dots + a_0s^q) \times \psi_{i,j}(s) \\ &= (b_{q-1}^{i,j} + b_{q-2}^{i,j}s + \dots + b_0^{i,j}s^{q-1}) \times I_j(s) \end{aligned} \quad (5.7)$$

Assuming zero initial conditions and taking the inverse Laplace transformation of (5.7) yields,

$$\begin{aligned} & a_0 \frac{d^q}{dt^q} \psi_{i,j}(t) + a_1 \frac{d^{q-1}}{dt^{q-1}} \psi_{i,j}(t) + \dots + a_{q-1} \frac{d}{dt} \psi_{i,j}(t) + \psi_{i,j}(t) \\ &= b_0^{i,j} \frac{d^{q-1}}{dt^{q-1}} i_j(t) + \dots + b_{q-1}^{i,j} i_j(t) \end{aligned} \quad (5.8)$$

Introducing variables: $\omega_{1,i,j}(t), \omega_{2,i,j}(t), \dots, \omega_{q,i,j}(t)$, (5.8) can be equivalently rewritten as a set of 1st order differential equations in the form:

$$\left. \begin{aligned}
\dot{\omega}_{1,i,j}(t) &= -\frac{a_1}{a_0}\omega_{q,i,j}(t) + \frac{b_{q-1}^{i,j}a_1}{a_0}i_j(t) \\
\dot{\omega}_{2,i,j}(t) &= \frac{1}{a_{q-1}}\omega_{1,i,j}(t) - \frac{a_1}{a_0}\omega_{q,i,j}(t) + \frac{b_{q-2}^{i,j}a_1}{a_0a_{q-1}}i_j(t) \\
&\dots \\
\dot{\omega}_{q-1,i,j}(t) &= \frac{a_3}{a_2}\omega_{q-2,i,j}(t) - \frac{a_1}{a_0}\omega_{q,i,j}(t) + \frac{b_1^{i,j}a_1}{a_0a_2}i_j(t) \\
\dot{\omega}_{q,i,j}(t) &= \frac{a_2}{a_1}\omega_{q-1,i,j}(t) - \frac{a_1}{a_0}\omega_{q,i,j}(t) + \frac{b_0^{i,j}}{a_0}i_j(t) \\
\psi_{i,j}(t) &= \omega_{q,i,j}(t)
\end{aligned} \right\} \quad (5.9)$$

Writing above equation in matrix form, we have,

$$\begin{bmatrix} \dot{\omega}_{1,i,j}(t) \\ \dot{\omega}_{2,i,j}(t) \\ \dot{\omega}_{3,i,j}(t) \\ \dots \\ \dot{\omega}_{q,i,j}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 & -\frac{a_1}{a_0} \\ \frac{1}{a_{q-1}} & 0 & \dots & 0 & -\frac{a_1}{a_0} \\ 0 & \frac{a_{q-1}}{a_{q-2}} & \dots & 0 & -\frac{a_1}{a_0} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{a_2}{a_1} & -\frac{a_1}{a_0} \end{bmatrix} \begin{bmatrix} \omega_{1,i,j}(t) \\ \omega_{2,i,j}(t) \\ \omega_{3,i,j}(t) \\ \dots \\ \omega_{q,i,j}(t) \end{bmatrix} + \frac{a_1}{a_0} \begin{bmatrix} \frac{b_{q-1}^{i,j}}{a_{q-1}} \\ \frac{b_{q-2}^{i,j}}{a_{q-2}} \\ \frac{b_{q-3}^{i,j}}{a_{q-3}} \\ \dots \\ \frac{b_0^{i,j}}{a_1} \end{bmatrix} i_j(t) \quad (5.10a)$$

$$\psi_{i,j}(t) = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_{1,i,j}(t) \\ \omega_{2,i,j}(t) \\ \omega_{3,i,j}(t) \\ \dots \\ \omega_{q,i,j}(t) \end{bmatrix} \quad (5.10b)$$

Taking the inverse Laplace transformation of (5.6) leads to,

$$v_{\phi,i}(t) = \sum_{j=1}^{n_p} (\psi_{i,j}(t) + b_{q,i,j}i_j(t)) \quad (5.11)$$

Let

$$x_{1,i}(t) = \omega_{1,i,1}(t) + \omega_{1,i,2}(t) + \dots + \omega_{1,i,n_\phi}(t)$$

...

$$x_{q,i}(t) = \omega_{q,i,1}(t) + \omega_{q,i,2}(t) + \dots + \omega_{q,i,n_\phi}(t)$$

Using (5.10) and (5.11), we get,

$$\begin{bmatrix} \dot{x}_{1,i} \\ \dot{x}_{2,i} \\ \dot{x}_{3,i} \\ \dots \\ \dot{x}_{q,i} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 & -\frac{a_1}{a_0} \\ \frac{1}{a_{q-1}} & 0 & \dots & 0 & -\frac{a_1}{a_0} \\ 0 & \frac{a_{q-1}}{a_{q-2}} & \dots & 0 & -\frac{a_1}{a_0} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{a_2}{a_1} & -\frac{a_1}{a_0} \end{bmatrix} \begin{bmatrix} x_{1,i} \\ x_{2,i} \\ x_{3,i} \\ \dots \\ x_{q,i} \end{bmatrix} + \frac{a_1}{a_0} \begin{bmatrix} \frac{b_{q-1}^{i,1}}{a_{q-1}} & \dots & \frac{b_{q-1}^{i,n_\phi}}{a_{q-1}} \\ \frac{b_{q-2}^{i,1}}{a_{q-2}} & \dots & \frac{b_{q-2}^{i,n_\phi}}{a_{q-2}} \\ \frac{b_{q-3}^{i,1}}{a_{q-3}} & \dots & \frac{b_{q-3}^{i,n_\phi}}{a_{q-3}} \\ \dots & \dots & \dots \\ \frac{b_0^{i,1}}{a_1} & \dots & \frac{b_0^{i,n_\phi}}{a_1} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \dots \\ i_{n_\phi} \end{bmatrix} \quad (5.12a)$$

$$v_{\phi,i}(t) = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,i}(t) \\ x_{2,i}(t) \\ x_{3,i}(t) \\ \dots \\ x_{q,i}(t) \end{bmatrix} + \begin{bmatrix} b_q^{i,1} & b_q^{i,2} & \dots & b_q^{i,n_\phi} \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \\ \dots \\ i_{n_\phi}(t) \end{bmatrix} \quad (5.12b)$$

Eq.(5.1) can be directly derived from (5.12). To prove (5.2), we rewrite (5.1) for $1, 2, \dots, n_\phi$, we have,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_{n_\phi} \end{bmatrix} = \begin{bmatrix} A_1 & \emptyset & \dots & \emptyset \\ \emptyset & A_2 & \dots & \emptyset \\ \dots & \dots & \dots & \dots \\ \emptyset & \emptyset & \dots & A_{n_\phi} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_{n_\phi} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ \dots \\ B_{n_\phi} \end{bmatrix} i_\phi$$

$$\begin{bmatrix} v_{\phi,1} \\ v_{\phi,2} \\ \dots \\ v_{\phi,n_\phi} \end{bmatrix} = \begin{bmatrix} C_1 & \emptyset & \dots & \emptyset \\ \emptyset & C_2 & \dots & \emptyset \\ \dots & \dots & \dots & \dots \\ \emptyset & \emptyset & \dots & C_{n_\phi} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_{n_\phi} \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \\ \dots \\ D_{n_\phi} \end{bmatrix} i_\phi \quad (5.13)$$

where $\emptyset$ stands for null matrix.

Similarly, by using the notations X , v_ϕ , A , B , C , D defined in (5.2), the time domain equivalent equations of (4.2) can be written as,

$$\left. \begin{aligned} \dot{X} &= AX + Bi_\phi \\ v_\phi &= CX + Di_\phi \end{aligned} \right\} \quad (5.14)$$

By combining (5.14) with (4.1) and using $v_\phi = D_\phi^t v_\pi$, (5.2) can be obtained.

Equation (5.2) can be solved using standard algorithms for numerical solutions of ordinary differential equations or by using any of the conventional circuit simulators. For circuit simulators which do not directly accept as input a macromodel of differential equations, an equivalent circuit representation will be derived in next chapter.

Chapter 6

Equivalent Circuit Representation in Hspice

In order to solve (5.2) using an existing conventional circuit simulator such as SPICE, an equivalent circuit is derived for the macromodel of the linear subnetwork ϕ represented by the set of differential equations (5.12)-(5.14).

To build the equivalent circuit [27],[28], it is convenient to use the node voltages as the analogs of differential equation variables. Besides, in order to simulate these equations, it is apparent that in the circuit, functions to perform summation, amplification, integration, and multiplication of variables are required. With the differential equation variables being implemented by the node voltages, the summation, amplification and multiplication operations can be easily performed by routine application of Hspice's multidimensional polynomial voltage-controlled-voltage sources, where the polynomial control functions are specified by a set of coefficients $P_0, P_1, \dots, P_N$. The meaning of the coeffi-

coefficients depends on the dimension of the polynomial function. Both the numbers of dimensions and the number of coefficients are arbitrary.

Integration, which is required to solve the differential equations, can be performed with the aid of the equivalent circuit shown in Fig.6.1. In this circuit, a one dimensional voltage-controlled voltage source Eout is built, the voltage developed by Eout is dependent on the voltage v_2 with a gain of 2. And the two resistors have the same value which was selected to have minimal effect on the response of the integrator as well as to provide for good DC and transient solution convergence. The value of capacitor will be determined later. The output voltage across the Eout, therefore, is $v_{out} = \frac{2}{RC} \int v_{in}$. The node numbers are included in the circuit. The Hspice subcircuit coding of the integrator is presented in Fig.6.1b.

Figure 6.1a

```
.SUBCKT INT 1 3 R=100 C=1
*CONNECTIONS : IN OUT
*INTEGRATOR SUBCIRCUIT MODEL
*V(3)=2V(1)/SRC
R1 1 2 R
C 2 0 C
R2 3 0 R
EOUT 3 0 2 0 2
```

Figure 6.1b

Consider the network shown in Figure 4.1 as an example. The macromodel of the subnetwork ϕ with $q = 3$ is given in Section IV. The differential equations for terminal $i = 1$ of the macromodel are rewritten as,

$$\left. \begin{aligned} \dot{x}_{11} &= -\frac{a_1}{a_0}x_{31} + \frac{a_1}{a_0}(b_2^{1,1}i_1 + b_2^{1,2}i_2) \\ \dot{x}_{21} &= \frac{1}{a_2}x_{11} - \frac{a_1}{a_0}x_{31} + \frac{a_1}{a_0}\left(\frac{b_1^{1,1}}{a_2}i_1 + \frac{b_1^{1,2}}{a_2}i_2\right) \\ \dot{x}_{31} &= \frac{a_2}{a_1}x_{21} - \frac{a_1}{a_0}x_{31} + \frac{a_1}{a_0}\left(\frac{b_0^{1,1}}{a_1}i_1 + \frac{b_0^{1,2}}{a_1}i_2\right) \\ v_{\phi 1} &= x_{31} + b_3^{1,1}i_1 + b_3^{1,2}i_2 \end{aligned} \right\} \quad (6.1)$$

An equivalent circuit representation of the differential equations for terminal $i = 1$ in the subnetwork ϕ is shown in Figure 6.2a. Note, in this diagram, part of the circuit for terminal $i = 2$ is included. Also, terminal voltages v_2 and v_3 in Figure 4.1 are represented as $v_{\phi,1}$ and $v_{\phi,2}$ in Figure 6.2a, terminal currents i_2 and i_3 in Figure 4.1 as i_1 and i_2 in Figure 6.2a respectively. For simplicity, the circuit which produce the voltage ψ_2 is omitted. But it can be constructed in the same way as the circuit producing the voltage ψ_1 . This diagram makes use of the standard analog computer symbol of an integrator to represent the subcircuit model defined in Fig.6.1. Also, voltage source symbols are used to represent the multidimensional voltage-controlled dependent voltage sources. Note also that the network has been labeled with node numbers and pseudo-element (X) designators for Hspice coding and simulation.

Terminal voltages and currents, and solution variables as node voltages are displayed in Figure 6.2a at each node in the network. In addition, dashed lines and coefficient boxes serve to indicate control voltages for each source and the nature of each control voltage term. In the equivalent circuit of Figure 6.2a, it is noted that each terminal current term in the differential equations is simulated with the voltage produced by the terminal current

flowing through a 1Ω resistor which is connected between the terminal and the dependent voltage source ETER. For example, the current i_1 in Figure 6.2a is realized by the voltage $R_{v1}i_1 = i_1$. The sequential order of each control voltage in the associated dependent voltage source definition is also indicated by the letter designations, A, B, C, or D, at the "input" to each coefficient box. With the explicit inclusion of all this information, the Hspice input data can be generated directly from the diagram.

The formation of the equivalent circuit in Figure 6.2a utilized the basic approach for implementing Hspice differential equation simulations. That is, first the derivative of each variable is expressed as the algebraic summation of all relevant variables. In the circuit, the derivative of variable is simulated with the output voltage of the dependent voltage source, and the relevant variables are simulated as the control voltages. For example, $\dot{x}_{11}$ is the output voltage of E_1^1 which is actually equal to the algebraic summation of control voltages $R_{v1}i_1$, $R_{v2}i_2$, and $\frac{2}{R_3C_3}x_{31}$ together with their corresponding coefficients. Once the 1st derivative is obtained, it is integrated through an integrator to generate the variable itself. In the circuit, the variable x_1 is produced as the output of the integrator XINT1. It is noted that the output of XINT1 is actually $\frac{2}{R_1C_1}x_1$. The coefficient $\frac{2}{R_1C_1}$ in the simulation of the circuit has the effect of a scaling factor and will make all the parameters of the circuit in a very good appropriate range so that the network equations can be numerically solved accurately.

And, in the equivalent circuit, the polynomial voltage-controlled voltage sources are also used to express the terminal voltage equation in the differential network equations. For example, source ETER1 yields the terminal voltage $v_{\phi,1}$ in terms of the control voltages $R_{v1}i_1$, $R_{v2}i_2$, and $\frac{2}{R_3C_3}x_{31}$ through their corresponding coefficients. In all the situations, the polynomial voltage-controlled voltage sources realize the summations, multiplications and amplifications required to produce the derivatives and the terminal

Fig. 6.2a An equivalent Hspice circuit representation of the differential equation (6.1)

voltages.

From the circuit, the network equations for terminal 1 can be easily written as,

$$\left. \begin{aligned} \dot{x}_{11} &= -\frac{2}{R_3 C_3} x_{31} + P_{11}^1 i_1(t) + P_{11}^2 i_2(t) \\ \dot{x}_{21} &= \frac{2}{R_1 C_1} x_{11} - \frac{2}{R_3 C_3} x_{31} + P_{21}^1 i_1(t) + P_{21}^2 i_2(t) \\ \dot{x}_{31} &= \frac{2}{R_2 C_2} x_{21} - \frac{2}{R_3 C_3} x_{31} + P_{31}^1 i_1(t) + P_{31}^2 i_2(t) \\ v_{\phi,1} &= \frac{2}{R_3 C_3} x_{31} + \alpha_{11} i_1(t) + \alpha_{21} i_2(t) \end{aligned} \right\} \quad (6.2)$$

By comparing the coefficients in (6.2) with those in (6.1), the differential equations of the macromodel of the example subnetwork, all the circuit parameters can be determined. Later, a general systematic way to calculate the parameter values will be given.

With all the parameters of the circuit being known, using the information provided in Figure 6.2a, the Hspice coding for the circuit was developed. The coding is listed in Figure 6.2b; it primarily consists of the interconnection of the five E sources and three integrators. The integrators are coded as pseudo-elements with associated node connections, a subcircuit name, and component values of the integrators. For example, pseudo-element X1 references the integrator model called INT; the input is connected to node 5, the output node is connected to node 6, the resistance value is R1, and the capacitance value is C1. In the circuit, all initial conditions default to zero. If it is necessary, the initial conditions can be specified by statement .IC of Hspice.

Three of the five voltage-controlled voltage sources included in the coding are used to implement the derivative terms of the network equations. For instance, source E11, connected from node 5 to ground, is controlled by a three dimensional polynomial function, where the A input is node 1 to node 2, the B input is node 3 to node 4, and the C input is node 10 to ground. Obviously, the source implements:

$$V(5) = P_{11}^1 V(1, 2) + P_{11}^2 V(3, 4) - V(10)$$

Note, since $V(5)$, $V(1,2)$, $V(3,4)$, and $V(10)$ are the voltage analogs of the variables $\dot{x}_{11}$, i_1 , i_2 , and $\frac{2}{R_3 C_3} x_{31}$, respectively, source E11 implements the equation

$$\dot{x}_{11} = -\frac{2}{R_3 C_3} x_{31} + P_{11}^1 i_1 + P_{11}^2 i_2$$

In exactly the same way as above, for each terminal i ; $i = 1, \dots, n_\phi$, an equivalent network for (5.12) can be constructed, from the network, (6.2) and (6.3) can be directly derived.

```

X1 5 6 INT R=R1 C=C1
X2 7 8 INT R=R2 C=C2
X3 9 10 INT R=R3 C=C3
ETER1 2 0 POLY(4) 1 2 1 2 3 4 10 0 0 -1  $\alpha_{11}$   $\alpha_{21}$  1
ETER2 4 0 POLY(4) 3 4 1 2 3 4 11 0 0 -1  $\alpha_{12}$   $\alpha_{22}$  1
E11 5 0 POLY(3) 1 2 3 4 10 0  $P_{11}^1$   $P_{11}^2$  -1
E21 7 0 POLY(4) 1 2 3 4 10 0 6 0  $P_{21}^1$   $P_{21}^2$  -1 1
E31 9 0 POLY(4) 1 2 3 4 10 0 8 0  $P_{31}^1$   $P_{31}^2$  -1 1
RV1 1 2 1
RV2 3 4 1

```

Fig. 6.2b

$$\left. \begin{aligned} \dot{x}_{1i} &= -\frac{2}{R_q C_q} x_{qi} + P_{1i}^1 i_1(t) + \dots + P_{1i}^{n_\phi} i_{n_\phi}(t) \\ \dot{x}_{2i} &= \frac{2}{R_1 C_1} x_1 - \frac{2}{R_q C_q} x_{qi} + P_{2i}^1 i_1(t) + \dots + P_{2i}^{n_\phi} i_{n_\phi}(t) \\ &\dots \\ \dot{x}_{qi} &= \frac{2}{R_{q-1} C_{q-1}} x_{q-1,i} - \frac{2}{R_q C_q} x_{qi} + P_{qi}^1 i_1(t) + \dots + P_{qi}^{n_\phi} i_{n_\phi}(t) \\ \Psi_i &= \frac{2}{R_q C_q} x_{qi} \end{aligned} \right\} \quad (6.3)$$

$$v_{\phi,i} = \psi_i + (\alpha_{1i} R_{v1} i_1 + \dots + \alpha_{n_{\phi}i} R_{vn_{\phi}} i_{n_{\phi}}) \quad (6.4)$$

Then comparing the coefficients of (6.3) and (6.4) with those of (5.12) yields

$$\frac{2}{R_1 C_1} = \frac{1}{a_{q-1}} \quad \frac{2}{R_2 C_2} = \frac{a_{q-1}}{a_{q-2}} \quad \dots \quad \frac{2}{R_{q-1} C_{q-1}} = \frac{a_2}{a_1} \quad \frac{2}{R_q C_q} = \frac{a_1}{a_0} \quad (6.5)$$

$$\left. \begin{aligned} \frac{2}{R_q C_q} P_{1i}^1 &= \frac{a_1}{a_0} b_{q-1}^{i,1} & \dots & \quad \frac{2}{R_q C_q} P_{1i}^{n_{\phi}} = \frac{a_1}{a_0} b_{q-1}^{i,n_{\phi}} \\ \frac{2}{R_q C_q} P_{2i}^1 &= \frac{a_1}{a_0} \frac{b_{q-2}^{i,1}}{a_{q-1}} & \dots & \quad \frac{2}{R_q C_q} P_{2i}^{n_{\phi}} = \frac{a_1}{a_0} \frac{b_{q-2}^{i,n_{\phi}}}{a_{q-1}} \\ &\dots & & \quad \dots \\ \frac{2}{R_q C_q} P_{qi}^1 &= \frac{a_1}{a_0} \frac{b_0^{i,1}}{a_1} & \dots & \quad \frac{2}{R_q C_q} P_{qi}^{n_{\phi}} = \frac{a_1}{a_0} \frac{b_0^{i,n_{\phi}}}{a_1} \end{aligned} \right\} \quad (6.6)$$

$$\alpha_{1i} = b_q^{i,1} \quad \dots \quad \alpha_{n_{\phi}i} = b_q^{i,n_{\phi}} \quad (6.7)$$

In the equivalent network, let all the resistances $R_j = 10\Omega$; $j = 1, \dots, q$. By solving (6.5), (6.6) and (6.7), all the capacitances C_j ; $j = 1, \dots, q$ and the coefficients of voltage controlled voltage sources P_{ji}^k ; $i = 1, \dots, n_{\phi}$; $j = 1, \dots, q$; $k = 1, \dots, n_{\phi}$ and α_{ki} ; $i = 1, \dots, n_{\phi}$; $k = 1, \dots, n_{\phi}$ can be obtained.

With all the equivalent network parameters being determined, the equivalent circuit network for the macromodel of the subnetwork ϕ can be implemented in Hspice. Inserting the equivalent network of (5.12)-(5.14) into the original whole network π , the resulting circuit, which is mathematically represented in (5.2), can be analyzed by Hspice.

A program has been developed to calculate the equivalent circuit parameters and to write the Hspice code of the equivalent circuit. Negligible cpu time is needed for the running of the program.

Chapter 7

Computational Results and Comparisons

7.1 Examples

Example 1

An interconnect circuit which contains lossless transmission lines and nonlinear components is shown in Figure 7.1. The linear subnetwork is replaced by a lumped component model derived from the differential equations obtained using a 6th order AWE. The combined network is simulated using Hspice. In Figure 7.2, the time-domain results are compared with those obtained directly using Hspice and a distributed model for the lossless transmission lines. The applied voltage is a pulse with 10 picoseconds rise/fall time and 6 nanoseconds pulse width.

Example 2

An interconnect circuit which contains lossless transmission lines and four nonlinear components is shown in Figure 7.3. The output transient responses obtained using the proposed method with the AWE of order 7 are compared with Hspice in Figure 7.4. The applied voltage is a pulse with 10 picoseconds rise/fall time and 200 nanoseconds pulse width.

Example 3

Figure 7.5 shows a circuit consists of two lossy coupled transmission lines. Both lines are of the same length 0.1m. Line 1 has the following parameters:

$$L = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \text{ nH/m} \quad C = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \text{ pF/m}$$

$$R = \begin{bmatrix} 75 & 15 \\ 15 & 75 \end{bmatrix} \text{ } \Omega/\text{m} \quad G = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \text{ S/m}$$

and the parameters for line 2 are:

$$L = \begin{bmatrix} 494.6 & 63.3 & 7.8 & 0.0 \\ 63.3 & 494.6 & 63.3 & 7.8 \\ 7.8 & 63.3 & 494.6 & 63.3 \\ 0.0 & 7.8 & 63.3 & 494.6 \end{bmatrix} \text{ nH/m}$$

$$C = \begin{bmatrix} 62.8 & -4.9 & -0.3 & 0.0 \\ -4.9 & 62.8 & -4.9 & -0.3 \\ -0.3 & -4.9 & 62.8 & -4.9 \\ 0.0 & -0.3 & -4.9 & 62.8 \end{bmatrix} \text{ pF/m}$$

$$R = \begin{bmatrix} 50 & 10 & 1 & 0 \\ 10 & 50 & 10 & 1 \\ 1 & 10 & 50 & 10 \\ 0 & 1 & 10 & 50 \end{bmatrix} \text{ } \Omega/\text{m} \quad G = \begin{bmatrix} 0.1 & -0.01 & -0.001 & 0.0 \\ -0.01 & 0.1 & -0.01 & -0.001 \\ -0.001 & -0.01 & 0.1 & -0.01 \\ 0.0 & -0.001 & -0.01 & 0.1 \end{bmatrix} \text{ S/m}$$

The applied voltage is a pulse with 10 picoseconds rise/fall time and 6 nanoseconds pulse width. The output response obtained using the proposed method is shown in Figure 7.6. Note that this circuit can't be analyzed using SPICE.

Example 4

In this example, comparison between the CPU time of proposed method and that of Hspice is made with three multi-chip interconnect circuits on SPARC station 2. Each circuit is constructed by cascading number of subcircuits as shown in Figure 7.7. The comparison results are listed in Table 1. The transient output responses are shown in Figure 7.8.

TABLE 1
CPU Comparison of the Proposed Method and Hspice

Circuit No.	Number of Transmission Lines	Node Count	Lumped Element Count	CPU Seconds	
				Proposed Method	Hspice
1	35	102	147	8.83	86.94
2	70	202	292	8.03	162.05
3	98	282	408	8.55	232.03

7.2 Computational Considerations

In this section, we first discuss the stability problems associated with the time domain analysis technique presented in previous chapters, next we will introduce some new methods to improve the accuracy of AWE technique so that the unstable problems

can be avoided.

7.2.1 Stability problems

In the time-domain realization of the frequency domain representation of the transmission lines network obtained by using the AWE technique, there exist stability problems due to the approximation nature of the frequency domain representation.

In Chapter 4, the Z parameters are obtained by using the AWE technique and expressed as the rational function in (4.9). The roots of the denominator of (4.9) are the poles of the approximate Z parameters, and the roots of the numerator of (4.9) are the approximate zeros of the Z parameters. And it was generally observed that in higher order approximation, some of the dominate poles of a stable network obtained by using AWE may be positive. The same situation exists for the zeros. Some zeros of the Z parameter, which should be located in the left hand side of the complex plane, are given by the AWE to be in the right hand side.

The positive poles and zeros can be easily discovered by applying the Ruth-Hurwitz criterion to the denominator and numerator polynomials. Positive poles must be discarded for obvious unstable reasons. However, the problems with the positive zeros are complicated, as in some situations the positive zeros may lead to an unstable realization. Let us take a look at the following simple example in which a two-port-linear network is assumed.

The frequency domain Z parameter representation of the network can be obtained by using AWE as follows,

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} \frac{P_{11}(s)}{Q_{11}(s)} & \frac{P_{12}(s)}{Q_{12}(s)} \\ \frac{P_{21}(s)}{Q_{21}(s)} & \frac{P_{22}(s)}{Q_{22}(s)} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

where the $P(s)$ and $Q(s)$ are polynomials.

From linear system theory, we know that for a stable system, the roots of $P_{11}(s)$ and $P_{22}(s)$ must be negative and the roots of $P_{12}(s)$ and $P_{21}(s)$ can be either positive or negative. In the case that using AWE gives us some positive roots of $P_{11}(s)$ or $P_{22}(s)$, the system implemented by the Z parameters will be unstable. Assume some roots of $P_{11}(s)$ are positive, and port 2 of the network is open-circuit and the port 1 is connected with voltage source. Then the voltage at port 2 can be shown to be

$$V_2(s) = \frac{Q_{11}(s)P_{21}(s)}{P_{11}(s)Q_{21}(s)} V_1(s)$$

Obviously, the positive roots of $P_{11}(s)$ lead to positive poles in the resulted system, and the system is unstable.

Generally, for an n -port network, the Z-parameter formulation which is approximated by using AWE technique, the zeros of $P_{ii}(s)$, $i = 1, \dots, n$ must be in the left-hand side of the complex plane, otherwise the resulted time-domain equivalent representation will be unstable. In the case where the left-hand side zeros occur, the approximation order of the AWE may be reduced or a different set of moments has to be used to calculate the approximate denominator and numerator polynomials in (4.10) and (4.11). The same approach can be applied to judge the stability of the Y-parameter formulation obtained by using AWE.

In next, we will present new methods to apply the AWE technique so that good accuracy can be achieved and unstable problems can be avoided.

7.2.2 Optimal Pole Selection Criteria

From previous discussion, we know that for a stable linear network, AWE technique can produce unstable positive zeros, and this problem is mainly resulted by the approximate poles. Therefore, if we can find more accurate poles, the unstable positive zero problem should be, to some extent, able to avoided.

Fortunately, optimal pole selection criteria to address above problem have been developed in [30] and [31]. In the following, we will give the summary of the criteria.

1. The AWE is applied only up to the point of obtaining all the needed output moments of an order q moment matching approximation.
2. Given the set of all outputs, a subset of outputs is selected including the desired output and using moment matching, sets of approximate characteristic equations of (4.9) are derived for each subset output. The subset outputs may be selected incrementally as needed until a minimum moment skew or a maximum number of outputs is achieved. The q -1st moment skew ϵ is defined as

$$\epsilon_{q-1} = \frac{m_{q-1} - \hat{m}_{q-1}}{m_{q-1}}$$

where m_{q-1} is the q -1st moment given by (4.8); $\hat{m}_{q-1}$ is the q -1st moment in the Maclaurin's series expansion of (4.9).

3. Using the acquired sets of characteristic equation in (4.9) from each selected output, corresponding sets of numerator polynomials in (4.9) are obtained using the moments from the desired output.
4. For a given output, one set of characteristic equation and corresponding numerator polynomial in (4.9) is selected as the more accurate approximation based on the preceding error criterion which verifies the skew in one or more additional Maclaurin's series

moments not used in the approximation.

5. In multi-port characterization of a linear network, a set of characteristic equations for each output is extracted, and the set that yields minimum moment skew at all or most of the outputs is selected.

Fig. 7.1 Interconnect Circuit with lossless transmission lines and nonlinear terminations

Fig. 7.2a Transient response V_1 of the circuit shown in Figure 7.1

Fig. 7.2b Transient response V_2 of the circuit shown in Figure 7.1

Fig. 7.2c Transient response V_3 of the circuit shown in Figure 7.1

Fig. 7.2d Transient response V_4 of the circuit shown in Figure 7.1

Fig. 7.3 Interconnect circuit for Example 2

Fig. 7.4a Transient response v_1 of the circuit shown in Figure 7.3

Fig. 7.4b Transient response V_2 of the circuit shown in Figure 7.3

Fig. 7.4c Transient response V_s of the circuit shown in Figure 7.3

Fig. 7.4d Transient response V_6 of the circuit shown in Figure 7.3

Fig. 7.5 Interconnect Circuit with lossy coupled transmission lines and nonlinear terminations

Fig. 7.6a Transient response V_1 of the circuit shown in Figure 7.5

Fig. 7.6b Transient response V_2 of the circuit shown in Figure 7.5

Fig. 7.6c Transient response V_3 of the circuit shown in Figure 7.5

Fig. 7.6d Transient response V_4 of the circuit shown in Figure 7.5

Fig.7.7 Multi-chip interconnect circuit with nonlinear termination

Fig.7.8a Transient response V_1 of the circuit with 35 transmission lines

Fig.7.8b Transient response V_2 of the circuit with 35 transmission lines

Fig.7.8c Transient response V_1 of the circuit with 70 transmission lines

Fig.7.8d Transient response V_1 of the circuit with 98 transmission lines

Chapter 8

Conclusions And Future Research

8.1 Conclusions

This study has been concentrated on developing an efficient and easy-to-use numerical analysis technique to predict propagation signal delay and crosstalk in the interconnects of a high-speed VLSI system where interconnects are modeled as distributed transmission lines with nonlinear components. The product of the study is a method which extends the AWE method to the efficient numerical analysis of VLSI interconnects which contain both lossy coupled transmission lines and nonlinear components. This new method can be easily implemented within the framework of an existing circuit simulator. Using this method, the solution of the mixed frequency/time domain simulation problem is obtained by replacing the linear subnetworks which contain lossy coupled transmission lines with a set of ordinary differential equations which can be derived using the asymptotic waveform evaluation technique. The simultaneous solution of the resulting linear differential equations and the nonlinear differential equations describing the rest of the network has been

conveniently implemented within the framework of an existing circuit simulator. The method takes full advantage of the efficiency of the AWE technique in analyzing the linear subnetwork. Two to three orders of magnitude faster solution compared to other techniques has been achieved. Furthermore, it has the additional simplification that it does not require explicit evaluation of the dominant poles and corresponding residues. Only the coefficients of the Pade approximation are needed. And, it can handle general transmission line networks with no topological or electrical constraints. Several examples were given to demonstrate the accuracy and efficiency of the proposed technique.

8.2 Suggestions For Future Research

The method presented in this thesis is general and not limited to the uniform lossy coupled transmission lines. However, its accuracy depends on the accuracy of the frequency domain transfer function obtained by using AWE technique. To extend the application of the method to the analysis of non-uniform lossy coupled transmission lines and improve the accuracy of the method comprise the main tasks for future research.

- 1 Combine this method with the frequency domain stamp of non-uniform lossy coupled transmission lines[32] to solve the network containing non-uniform interconnects with nonlinear components. This work will be very significant for practical VLSI circuit design and analysis.
- 2 Using the complex frequency AWE hopping technique[25] to achieve accurate high order transfer functions of lossy coupled transmission line network, and then applying the proposed method to solve the complex networks containing many lossy coupled interconnects and nonlinear components.

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Appendix

For the sake of integrity of the thesis, a summary of the proof of equation (4.4) is extracted from [13] and presented in the following.

Proof of Equation (4.4)

Consider a multiconductor transmission line formed by (N+1) conductors, one of which is chosen as the reference ground. Let x be the length along the multiconductor transmission line, with $x=0$ corresponding to the near end and $x=D$ to the far end. In the frequency domain the multiconductor transmission line can be described by the following differential equations[9]:

$$\frac{d}{dx}V(x) = -Z_L I(x) \quad (A1)$$

$$\frac{d}{dx}I(x) = -Y_L V(x) \quad (A2)$$

Combining above two equations leads to a set of wave equations,

$$\frac{d^2}{dx^2}V(x) = Z_L Y_L V(x) \quad (A3)$$

$$\frac{d^2}{dx^2}I(x) = Y_L Z_L I(x) \quad (A4)$$

which will have a solution of the form

$$V_m(x) = V_m(0) \exp(\pm \gamma_m x) \quad (A5)$$

$$I_m(x) = I_m(0) \exp(\pm \gamma_m x) \quad (A6)$$

where γ_m is the complex propagation mode constant.

Substituting the solution form into the wave equation (A3) yields

$$[\gamma_m^2 U - Z_L Y_L] V_m(0) = 0 \quad (A7)$$

where U is an identity matrix. $V_m(0)$ will have nontrivial solutions if γ_m^2 satisfies the eigenvalue problem

$$\det \{ \gamma_m^2 U - Z_L Y_L \} = 0 \quad (A8)$$

For inhomogeneous dielectrics, there exist in general n distinct eigenvalues γ_m^2 , $m=1, N$. Each eigenvalue has its corresponding eigenvector S_m . Let Ω be a diagonal matrix whose elements are the complex propagation constants $\{\gamma_1, \gamma_2, \dots, \gamma_N\}$, and S_v be a matrix with the eigenvectors S_m in the column.

The solutions for terminal voltages and currents can be obtained from above wave-form equations and eigenvalue problems. Using these two solutions, the relationship between the near- and far-end voltages and currents can be obtained as

$$\begin{bmatrix} I(0) \\ I(D) \end{bmatrix} = \begin{bmatrix} S_i & -S_i \\ S_i E(D) & S_i [E(D)]^{-1} \end{bmatrix} \begin{bmatrix} S_v & S_v \\ S_v E(D) & S_v [E(D)]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} V(0) \\ V(D) \end{bmatrix} \quad (A9)$$

where

$$S_i = Z_L^{-1} S_v \Omega \quad (A10)$$

After some algebraic manipulation, (A9) can be written as

$$AV + BI = 0 \quad (A11)$$

where

$$E_1 = \text{diagonal} \left\{ \frac{\exp(-\gamma_m D) + \exp(\gamma_m D)}{2} \right\}, \quad m = 1, \dots, N \quad (A12)$$

$$E_2 = \text{diagonal} \left\{ \frac{\exp(-\gamma_m D) - \exp(\gamma_m D)}{2} \right\}, \quad m = 1, \dots, N \quad (A13)$$

$$A_k = \begin{bmatrix} S_v E_1 S_v^{-1} & -U \\ S_i E_2 S_v^{-1} & 0 \end{bmatrix} \quad (A14)$$

$$\mathbf{B}_k = \begin{bmatrix} \mathbf{S}_v \mathbf{E}_2 \mathbf{S}_i^{-1} & 0 \\ \mathbf{S}_i \mathbf{E}_1 \mathbf{S}_i^{-1} & -\mathbf{U} \end{bmatrix} \quad (\text{A15})$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}(0) \\ \mathbf{V}(D) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{near} \\ \mathbf{V}_{far} \end{bmatrix} \quad (\text{A16})$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{I}(0) \\ \mathbf{I}(D) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{near} \\ \mathbf{I}_{far} \end{bmatrix} \quad (\text{A17})$$

References

- [1] R. Sainati and T. Moravec, "Estimating high speed interconnect performance," *IEEE Trans. Circuits Syst.*, Vol. CAS-36, pp.533-541, Apr. 1989.
- [2] H. Hasegawa and S.Seki, "Analysis of interconnection delay on very high-speed LSI/VLSI chips using an MIS microstrip line model," *IEEE Trans. Electron Devices*, Vol. ED-31, pp.1954-1960, Dec. 1984.
- [3] M. Nakhla, "Analysis of pulse propagation on high-speed VLSI chips," *IEEE Journal of Solid-State Circuits*, pp.490-494, Apr. 1990.
- [4] M. Cases and D. Quinn, "Transient response of uniformly distributed RLC transmission lines," *IEEE Trans. Circuits Syst.*, Vol. CAS-27, pp. 200-207, Mar. 1980.
- [5] S. S. Gao, A.Y. Yang and S. M. Kang, "Modelling and simulation of interconnection delays and crosstalks in high-speed integrated circuits," *IEEE Trans. Circuits Syst.*, Vol. CAS-37, pp. 1-9, Jan. 1990.
- [6] J. E. Schutt-Aine and R. Mitra, "Nonlinear transient analysis of coupled transmission lines," *IEEE Trans. Circuits Syst.*, Vol. CAS-36, pp. 959-967, Jul. 1989.
- [7] A. J. Groudin, "Transient analysis of uniform resistive transmission lines in a homogeneous medium," *IBM J. Res. Develop.*, Vol. 23, pp. 675-681, Nov. 1979.
- [8] A. J. Groudin, and C. S. Chang, "Coupled lossy transmission line characterization and simulation," *IBM J. Res. Develop.*, Vol.25, pp. 25-41, Jan. 1981.
- [9] A. R. Djordjevic, T. K. Sarkar, and R. F. Harrington, "Time-domain response of multi-conductor transmission lines," *Proceedings IEEE*, Vol. 75, pp. 743-764, June 1987.
- [10] L. T. Pillage and R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis," *IEEE Trans. on Computer-Aided Design*, Vol. 9, pp. 352-366, April 1990.

-
- [11] L. T. Pillage, Asymptotic Evaluation for Timing Analysis, Carnegie-Mellon University, Research Report No. CMUCAD-89-34.
- [12] T. Tang and M. Nakhla, "Analysis of high-speed VLSI interconnects," *Proc. IEEE ICCAD-90*, pp. 542-545, Nov. 1990.
- [13] T. Tang and M. Nakhla, "Analysis of high-speed VLSI interconnects using the asymptotic waveform evaluation technique," *IEEE Trans. on Computer-Aided Design*, Vol. 11, No. 3, pp. 341-352, March 1992.
- [14] M. M. Alaybeyi, J. E. Bracken, J. Y. Lee, V. Raghavan, R. J. Trihy and R. A. Rohrer, "Analysis of MCMs Using Asymptotic Waveform Evaluation (AWE)," *Proceedings of IEEE Multi-Chip Module Conference 1992*, pp. 48-51, March 1992.
- [15] T. Tang, M. Nakhla and Richard Griffith, "Analysis of lossy multiconductor transmission lines using the asymptotic waveform evaluation technique," *IEEE Trans. on Microwave Theory and Techniques*, Vol. 39, No. 12, pp. 2107-2116, December 1991.
- [16] E. Chiprout and M. Nakhla, "Fast Nonlinear Waveform Estimation of Large Distributed Networks," *Proceedings of 1992 IEEE MTT-S International Microwave Symposium*, June 1992.
- [17] S. A. Schelkunoff, "Conversion of Maxwell's equations into generalized telegrapher's equations," *Bell Syst. Tech. J.*, vol. 34, pp. 995-1043, Sept. 1955.
- [18] W. S. Meyer and H. W. Dommel, "Numerical modelling of frequency-dependent transmission parameters in an electromagnetic transient program," *IEEE Trans. Power App. Syst.*, vol. PAS-93, pp. 389-395, Jan. 1974.
- [19] D. M. Triesenberg, "An efficient state variable transmission line model," *IEEE Trans. Power App. Syst.*, vol. PAS-98, pp. 484-492, Mar. 1979.

- [20] A. Djordjevic, T. Sarkar and R. F. Harrington, "Analysis of Lossy Transmission Lines with Arbitrary Nonlinear Terminal Networks," *IEEE Trans. on M.T.T.*, vol. MTT-34, No. 6, June 1986.
- [21] D. Winklestein, M. B. Steer, and R. Pomerleau, "Simulation of Arbitrary Transmission Line Networks with Nonlinear Terminations," *IEEE Trans. on Circuits and Systems*, vol. 38, No. 4, April 1991.
- [22] R. Griffith and M. Nakhla, "Time-domain analysis of lossy coupled transmission lines," *IEEE Trans. Microwave Theory and Techniques*, Vol. MTT-38, pp. 1480-1487, Oct. 1990.
- [23] R. Griffith and M. Nakhla, "Mixed Frequency/time domain analysis of nonlinear networks," *IEEE Trans. on Computer-aided Design of Integrated Circuits and Systems*, pp.1032-1043, August 1992.
- [24] V. Raghavan, J. E. Bracken and R. A. Rohrer, "AWESpice: A General Tool for the Accurate and Efficient Simulation of Interconnect Problems," *Proc. 29th ACM/IEEE Design Automation Conference*, June 1992.
- [25] E. Chiprout and M. Nakhla, "High Frequency Transient Response Estimation Using Complex Frequency Hopping," (*to be published*)
- [26] C. W. Ho, A. E. Ruehli and P. A. Brennan, "The modified nodal approach to network analysis," *IEEE Trans. Circuits Syst.*, Vol. CAS-22, pp. 504-509, June 1975.
- [27] D. B. Herbert, "Simulating Differential Equations with SPICE2," *IEEE Circuits & Devices*, Vol.8, No.1, pp.11-14, January 1992.
- [28] K. B. Kumar, "Novel Techniques to Solve Sets of Coupled Differential Equations with SPICE," *IEEE Circuits & Devices*, Vol.8, No.1, pp.11-14, January 1991.

- [29] X. Huang, V. Raghavan and R. A. Rohrer, "AWESim: A Program for the efficient analysis of linear(ized) circuits," *IEEE International Conference on Computer-Aided Design(ICCAD)*, Nov. 1990.
- [30] E. Chiprout and M. Nakhla, "Optimal pole selection in asymptotic waveform evaluation," *Proceedings of 1992 IEEE International Symposium on Circuits and Systems*, Vol. 4, pp.1962-1962, May 1992.
- [31] E. Chrpout and M. Nakhla, " Generalized moment-matching methods for transient analysis of interconnect networks," *Proc. ACM/IEEE Design Automation Conference (DAC)*, June 1992.
- [32] S. L. Manney, M. Nakhla and Q. J. Zhang, "Time Domain Analysis of Nonuniform Frequency Dependent High-Speed Interconnects," (*to be published in ICCAD-92*).
- [33] D. H. Xie and M. Nakhla, "Delay and Crosstalk Simulation of High-Speed VLSI Interconnects with Non-linear Terminations," *IEEE International Conference on Computer-Aided Design(ICCAD)*, Nov. 1990.

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