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TARIFF FINANCING
OF PUBLIC GOODS AND PUBLIC INPUTS

by

JAMES P. FEEHAN, B.A, M.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

Department of Economics

Carleton University

Ottawa, Ontario

January 18, 1989

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The undersigned hereby recommend to
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acceptance of the thesis,

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in partial fulfilment of the requirements

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ABSTRACT

Trade taxes are inferior, on efficiency grounds, to other means of taxation. This result is true whether the underlying motive is protection or the raising of revenue. Trade theorists have devoted a great deal of attention to the former motive but very little to the use of trade taxes for financing government activities. This thesis examines important normative issues associated with that second motive.

We start with reliance on an import tariff as a given and focus on the formation, in a second best sense, of optimal tariff policy. To do so, in a meaningful way, requires the specification of the use to which government revenue is to be put. One obvious possibility is to finance production of a public good. Another is to pay for production of a public input, an intermediate input which is collective to firms' production functions for private goods. The formation of tariff policy is considered for each of the public good and public input when produced by the private sector as well as when produced by the public sector.

The theoretical framework is a small open economy with two traded goods and two inelastically supplied primary factors of production. To this is added a third non-traded good, either the public good or the public input, which is to be financed by a specific tariff. Then rules for optimal provision are derived for

the public good/input under conditions of both private sector and public sector production. Implicit in each rule is the associated optimal revenue tariff.

These rules are investigated in some detail. Observations are made regarding the role of factor intensity rankings. For the case of public inputs, comparisons are made with the "conventional rules" of Atkinson and Stern (1974). When production is undertaken by the public sector, the resulting rules embody shadow prices for factors. These shadow prices are related to the literature. Moreover, a subsidiary aspect of this work is elaboration, generalization and clarification of the theory of public inputs.

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CHAPTER 1: INTRODUCTION

Trade taxes are sometimes imposed for protection, sometimes for retaliation against others' protectionist policies, and sometimes for revenue. Trade theorists have devoted very little attention to the revenue motive. Yet, for many countries, trade taxes are or, at some point in the past, have been a major revenue source. The purpose of this thesis is to establish the optimal policy rules for government expenditure when the source of funds is limited to a tariff on imports. These policy rules are derived under a variety of assumptions regarding the nature of the expenditure. The authorities may provide either Samuelson collective consumption goods or public inputs; the latter is elaborated upon in Chapter 3. Also, the production of either may be undertaken by the private sector or by the public sector. Thus, there are four situations to be investigated.

The framework for the analyses of those four cases is the familiar Heckscher-Ohlin-Samuelson trade model. In particular, the public good/input is introduced into a two-factor, two-good (2x2) model of a small open economy. From that starting point we proceed with the normative analysis of deriving optimality rules for the maximization of social welfare.

BACKGROUND

Despite the primary focus of the literature on the protective tariff, revenues from tariffs have been important for practically all countries until the post-World War II period.

Since that time, the more industrialized countries such as the United States, Canada and the United Kingdom have moved away from trade taxes; employing other methods of taxation for revenue and either liberalizing trade or using non-tariff devices for protection. However, many less developed and moderately developed countries continue to derive substantial portions of their total revenues from trade taxes, primarily tariffs. Table 1 presents a list of countries which derive more than 10% of their total tax revenues from trade taxes.¹ An examination of Table 1 reveals that a large number of countries raise 20% to 50% of their tax revenues from trade taxes with import duties being relied upon substantially more than export duties. As well, these countries are not limited, as many may expect, to the severely underdeveloped countries in Africa. Other less developed countries in Asia, Latin America and elsewhere together with some moderately developed countries also derive significant revenues from trade taxes.

The continued reliance of many nations on revenue-generating trade taxes has prompted a search for explanations of this apparently inefficient policy. It is well-established that trade taxes are inferior to other means of taxation. Per unit of revenue raised, the distortionary cost of trade taxes, due to the combined consumption and production distortions, exceeds that of most other taxes.

Corden (1974) has offered a reasonable theoretical basis for this behaviour. There are other costs, the most important of

which is the administration and compliance costs. Corden suggests that particularly in less developed countries these costs may be substantial enough to overwhelm other considerations and therefore trade taxes are the best feasible means for revenue-generation. On the other hand, political constraints may be the underlying cause of the choice of trade taxes; Gillespie (1988) uses political forces as the determinant of tax structure.

Related to Corden's theoretical suggestions, a number of empirical studies have examined the correlation between the extent to which a country is developed and its reliance on trade taxes for revenue. Lewis (1963) was an early work in this area. There have been subsequent contributions by Greenaway (1980, 1981 and 1984), Greenaway and Sapsford (1987), Hitiris and Weekes (1987) and Reizman and Slemrod (1987). Essentially, these studies estimate the correlation between a country's reliance on trade taxes, measured by the proportion of central government revenue arising from that source, and the level of development, normally measured by the GDP per capita. An additional explanatory variable frequently employed is the size of the trade sector, normally measured as the value of imports plus exports relative to GDP. These studies find that there is a negative correlation between reliance on trade taxes and the degree of development and a positive correlation with the size of the trade sector. However, the overall fits of the regression equations have recently been found to vary with the group of countries under

consideration. The strong correlations seem to hold only for middle level developing countries.

THEORETICAL WORK

Despite empirical confirmation that circumstances in many countries are such that reliance on tariffs for revenue may be rationalized, there has been little work on the appropriate formation of expenditure policy when the tariff is the mechanism for financing.² Important exceptions to this observation are Boadway, Maital and Prachowny (1973), Vanek (1971) and Blomqvist (1974).

Boadway et al. consider the case where tariff revenues are employed to finance a Samuelson public good. They identify the manner by which the Samuelson condition for provision of that good would have to be amended in light of the use of a distortionary tariff as the revenue generator. Vanek (1971), later extended by Blomqvist (1974), considers a different situation. The revenue no longer finances provision of a collective consumption good but, rather, it finances economic development by eliminating the "savings-investment gap". Vanek develops a rule for the "optimal revenue tariff". That rule reflects the trade-off between the costs associated with the distortionary effects of the tariff and the benefits arising from the capital formation financed by the tariff revenue.

Other than these insightful contributions there is little else in the literature. The purpose of this thesis is to add to this rather sparse treatment of a relatively important issue.

OVERVIEW

Chapter 2 deals with optimal provision of a public good where financing is via a tariff. That analysis is carried out in a framework different from that of Boadway et al. They employed a one-factor model in which the factor, labour, was in variable supply; here the framework is the Heckscher-Ohlin-Samuelson model. Next, the issue of financing development is related to the theory of public inputs. Since that theory has received very little attention by economists, chapter 3 is devoted to a review and assessment of developments in that area. Then in chapter 4, the implications of the open economy are investigated and a rule for the optimal revenue tariff is developed. Both the rule for tariff-financing a public good, from chapter 2, and the rule for tariff-financing a public input, from chapter 4, are derived under the condition that they are produced by competitive firms and simply paid for by government. However, it is generally possible to generate welfare gains if government has the added flexibility of hiring factor inputs. This is due to the fact that since factor prices are distorted, cost minimization, at market prices, is no longer generally socially optimal. In chapter 5, rules for the optimal public sector employment of factors are developed and these findings are related to the use of shadow pricing of factors. Chapter 6 constitutes the conclusion.

An important outcome of this theoretical study is the identification of parameters and relationships which are relevant in determining the optimal size of the public sector.

TABLE 1

TAX REVENUE FROM TAXES ON INTERNATIONAL TRADE

<u>Country</u>	<u>Year</u>	<u>Tax Revenue(TR) (in domestic currency)</u>	<u>Import Duties</u>	<u>Export Duties</u>	<u>Total Duties as % of TR</u>
WESTERN HEMISPHERE					
Argentina	1985	7,484,492	328,598	774	14.7
Bahamas	1986	373	242	4	65.9
Barbados	1985	683	97	3	14.6
Belize	1985	88,649	45,538	1,729	53.3
Boliva	1984	573	147	-	25.6
Cayman Islands	1985	44	20	1	48.1
Chile	1986	724	75	-	10.4
Columbia	1983	292,393	44,751	2,676	16.2
Costa Rica	1986	46,120	7,102	4,334	24.8
Dominican Rep.	1985	1,581	503	40	34.4
El Salvador	1986	2,744	204	964	42.6
Guatemala	1982	750	78	49	16.9
Haiti	1986	1,024	256	55	30.4
Honduras	1981	699	200	114	44.9
Netherlands (Antilles)	1982	353	105	-	29.8
Panama	1985	998	132	11	14.3
Paraguay	1985	114,042	13,336	13	11.7
Peru	1986	40,892	8,743	1,199	24.3
St. Lucia	1983	105	34	2	34.8
St. Vincent	1986	79	40	2	53.0
Surinam	1986	375	99	2	26.9
Uruguay	1986	209,275	22,153	1,674	11.4
MIDDLE EAST					
Egypt	1986	9,166	1,926	170	22.9
Iran	1985	1,352	266	-	19.7
Jordan	1985	305	136	-	44.7
Syrian Arab Rep.	1981	5,839	2,000	120	36.3
Yemen Arab Rep.	1986	5,656	3,101	-	54.8

<u>Country</u>	<u>Year</u>	<u>Tax Revenue (TR)</u>	<u>Import Duties</u>	<u>Export Duties</u>	<u>Total Duties as % of TR</u>
ASIA					
Bangladesh	1984	21,471	8,957	78	42.1
Burma	1985	4,622	1,210	-	26.2
Fiji	1985	279	42	-	15.2
India	1986	310	104	1	33.6
Korea	1987	15,219	2,602	-	17.1
Malaysia	1986	15,456	2,065	1,141	20.7
Maldives	1985	90	52	-	58.1
Nepal	1985	3,172	1,009	56	33.6
Pakistan	1985	58,215	22,881	490	40.2
Papua New Guinea	1985	431	104	13	27.2
Phillipines	1985	61,192	13,522	997	23.7
Solomon Islands	1987	51	27	4	59.4
Sri Lanka	1987	35,707	11,140	1,680	35.5
Thailand	1985	144,483	33,077	2,615	24.7
Western Somoa	1984	47,543	25,061	253	53.2
EUROPE					
Cyprus	1986	340	71	-	20.8
Iceland	1984	20,133	3,580	8	17.8
Malta	1985	123	34	-	27.7
Yugoslavia	1986	1,602	578	-	36.1
AFRICA					
Botswana	1985	481	149	1	31.1
Burkina Faso	1987	74,231	29,060	1,434	41.1
Burundi	1981	10,461	2,633	179	26.9
Cameroon	1985	810	112	20	16.4
Central African Republic	1981	28,264	9,387	2,955	43.7
Chad	1986	14,118	7,218	114	51.9
Comoros	1986	7,632	4,125	1,255	70.5
Congo	1980	97	16	-	16.8
Cote D'Ivoire	1984	541,974	155,544	62,325	40.2
Ethiopia	1981	1,411	278	229	35.9
Gabon	1985	450	96	10	23.6
Gambia	1982	79	55	4	75.2
Ghana	1986	61,923	14,236	14,231	46.0
Guinea	1983	6,150	921	2,328	52.8
Guinea-Bissau	1985	1,948	567	43	31.3
Kenya	1985	17,483	3,043	541	20.5
Lesotho	1985	213,020	161,244	259	75.8

<u>Country</u>	<u>Year</u>	<u>Tax Revenue (TR)</u>	<u>Import Duties</u>	<u>Export Duties</u>	<u>Total Duties as % of TR</u>
Liberia	1986	173	52	-	29.8
Madagascar	1982	126,567	24,120	5,070	23.0
Malawi	1985	367	74	-	20.2
Mali	1985	59	9	4	22.1
Mauritius	1987	3,592	1,740	460	61.2
Morocco	1985	25,478	4,466	333	18.8
Niger	1980	65,435	24,362	3,149	42.0
Rwanda	1980	11,874	3,216	2,636	48.5
Senegal	1983	168	63	1	37.8
Sierra Leone	1985	258	104	6	42.8
Sudan	1982	673	384	33	61.9
Swaziland	1986	223	120	1	54.3
Togo	1986	81,610	27,523	985	34.9
Tunisia	1984	1,652	633	14	39.2
Uganda	1986	284,374	17,640	191,483	73.5
Zaire	1985	36,550	11,384	4,520	43.5
Zambia	1985	1,427	202	130	23.2
Zimbabwe	1985	1,903	321	-	16.9

Source: International Monetary Fund (1987), Government Finance
Statistics Yearbook, Volume XI.

Footnotes

1. This list is derived from recent IMF data and is quite comprehensive. However, data on a few countries are not available. The countries omitted include some eastern European states such as the Soviet Union, East Germany and Czechoslovakia as well as a few other countries, e.g. Algeria.
2. Johnson (1951) is a related theoretical work. He develops the notion of the "maximum revenue tariff" and proves it exceeds the "optimal tariff". The latter is the well-known concept associated with a country's use of market power to improve its terms-of-trade. The maximum revenue tariff is nicely elaborated on by Caves and Jones (1985) and has been recently extended by Wagstaff (1984). Another related theoretical contribution is made by Ramaswami and Srinivasan (1968) in which trade in intermediate products is considered together with the design of a revenue-raising schedule of tariffs. As in the Johnson analysis, however, the use to which the tariff revenue is put is not specified.

CHAPTER 2. PUBLIC GOODS, THE OPEN ECONOMY AND TARIFF FINANCING

This chapter examines the implications of relying on a tariff to finance provision of a public good. The analysis utilizes the Heckscher-Ohlin-Samuelson (HOS) model and draws on procedures employed by Bhagwati and Srinivasan (1980) in their insightful analysis of revenue-seeking. Crucial to the analysis are the findings of Komiya (1967) and Melvin (1968) regarding the two factor, three good version of the HOS model. The result is an efficiency rule for public good provision. That rule is similar in form to the Atkinson and Stern (1974) rule but now reflects the implications of using a tariff to provide a public good in light of international trading opportunities and factor-intensity rankings. As in the optimal tax literature, attention is devoted to determining whether the social cost of the public good can be expected to exceed its price.

The remainder of this chapter is organized as follows. First, the model is presented and the rules for provision of a public good are derived under the assumption that non-distortionary taxation is available. Then the issue of tariff financing is addressed. This yields a rule for public good provision which reflects the trade-off between the benefits of the public good, on the one hand, and the costs of financing it, on the other. Next, the nature of the cost of the public good is investigated in some detail. Finally, two important limitations of the analysis presented in this chapter are noted. The resolution of those

limitations forms the basis for the remaining chapters.

THE MODEL

There are three goods, an exportable, an importable and a non-traded public good.¹ Each is produced by competitive firms. Aggregate production levels are denoted by X_1 , X_2 and X_3 , respectively.

The public good is of the Samuelson (1954) variety. That is to say, the public good is characterized by jointness of consumption and exclusion is either impossible or prohibitively costly.² It is also assumed, initially, that the public good is privately produced and simply purchased by government and distributed without charge.

Utility functions of individuals have consumption of these three goods as their only arguments. For notational simplicity, all individuals are assumed to be identical. The representative utility function is given by

$$U = U(C_1/H, C_2/H, X_3), \quad (1)$$

where C_1 and C_2 denote the total quantities consumed of the exportable and importable, respectively, and H is the number of individuals. A social welfare function of the utilitarian type is assumed to exist so that maximization of (1) is equivalent to maximization of welfare.

Capital and labour are the only factors of production and are in fixed supply. Competition ensures full employment of both.

Production functions have continuous first- and second-order partial derivatives and are each characterized by constant returns to scale. There are different factor intensities, no factor-intensity reversals and no joint production. Following from these assumptions, the transformation frontier, denoted by

$$F(X_1, X_2, X_3) = 0 \quad (2)$$

is concave to the origin and exhibits the "straight-line" properties noted by Samuelson (1953) and elegantly elaborated upon by Melvin (1968).

Finally, the balance-of-trade condition is given by

$$(C_1 - X_1) - P(X_2 - C_2) = 0, \quad (3)$$

where P denotes the world price ratio of good 2 to good 1. Thus, P reflects the foreign rate of transformation of good 2 into good 1, FRT_{12} . By the small-country assumption, P is taken as fixed.

EFFICIENCY WITH NON-DISTORTIONARY TAX

As a preliminary to deriving the rules for efficient provision where revenue must come from a tariff, it is first worthwhile to derive the conditions associated with Pareto-optimality when non-distortionary taxation is possible. One approach which can be used to derive these conditions is the Samuelson tradition of having a central planner allocate resources and outputs in a manner that maximizes social utility. The conditions for achieving

this are examined and compared to what would have occurred if optimizing firms and consumers had been left to interact through private markets.

The central planner would solve the following maximization problem:

$$\text{Maximize } HU(C_1/H, C_2/H, X_3), \quad (4)$$

subject to the balance of trade condition, (3), and the production possibilities, (2). Thus, the Lagrangean is

$$\begin{aligned} Z = & HU(C_1/H, C_2/H, X_3) - \mu F(X_1, X_2, X_3) \\ & - \delta [(C_1 - X_1) - P(X_2 - C_2)], \end{aligned} \quad (5)$$

where μ and δ are the Lagrangean multipliers and the choice variables are C_1 , C_2 , X_1 , X_2 and X_3 .

The first-order conditons for the maximization of (5) include the following:

$$\partial U / \partial C_1 - \delta = 0, \quad (6)$$

$$\partial U / \partial C_2 - \delta P = 0, \quad (7)$$

$$-\mu \partial F / \partial X_1 + \delta = 0, \quad (8)$$

$$-\mu \partial F / \partial X_2 + \delta P = 0, \quad (9)$$

and

$$H \partial U / \partial X_3 - \mu \partial F / \partial X_3 = 0. \quad (10)$$

Let MRS_{ij} denote the marginal rate of substitution between goods i and j in consumption, and MRT_{ij} denote the domestic marginal rate of transformation between goods i and j , for $i, j = 1, 2$

and 3. Then manipulation of these five first-order conditions implies that production and consumption must be such that

$$MRS_{12} = MRT_{12} = P, \quad (11)$$

and

$$HMRT_{13} = MRT_{13} = MRT_{12}MRS_{23} \quad (12)$$

and

$$HMRT_{23} = MRT_{23} = MRT_{21}MRS_{13}. \quad (13)$$

Optimizing behaviour on the part of firms and consumers ensures that the first of these will be achieved but the remaining two cannot be expected to hold. Thus, the Samuelson central planner may limit intervention in the economy to purchasing the public good from private producers and distributing it according to the two conditions, (12) and (13) given above. (Note that a fuller central planning problem could have been constructed; one in which factor allocations were also choice variables. The assumptions that factor markets are competitive and that there are no production externalities permit the use of the production possibilities function rather than explicitly considering the factors and the production functions.)

Adopting the terminology of Atkinson and Stern (1974), conditions (12) and (13) may be referred to as the "conventional rules" for efficient provision of a public good in a non-distorted open-economy. They have an obvious interpretation. Production of the public good using resources formerly devoted to production of good i , ($i = 1, 2$), can be undertaken either directly, as in the

closed economy, or, in a round-about fashion through international trade.

TARIFF FINANCING

With this background in mind, one may proceed to the central issue of this chapter; namely the implications for public good production in a situation where financing of the public good is via revenue which must be raised through the use of an import tariff. In order to facilitate the analysis, the approach will rely on duality techniques rather than the solution of the primal problem as carried out in the preceding section.

Figure 1, a device similar to that used by Bhagwati and Srinivasan (1980) in their analysis of revenue-seeking, is helpful in explaining the problem at hand. For a given world price ratio, the choice of the tariff, t , sets the domestic price ratio, $P+t$. If no production of the non-traded good takes place then labour and capital are fully employed in the traded goods' industries. Consequently, production occurs at a point on the "conventional" transformation frontier, TT' , such as R in Figure 1. The line through R , labeled NN' , with absolute slope of $P+t$ is the corresponding national income line.

The choice of tariff rate also determines the slope of the generalized Rybczynski locus, labeled RR' . Movement along RR' from R in the direction of R' indicates the locus of traded-goods' production points which result as labour and capital are withdrawn for employment in increasing production of the non-traded good.

Figure 1

Since $P+t$ implies a unique wage-rental ratio which in turn implies the capital-labour ratios in production, it follows that these ratios are unchanged along RR' . Therefore, the Rybczynski locus corresponding to a domestic price ratio is, in fact, linear.³ In summary, the tariff, in conjunction with the world price ratio, determines the slope and location of the Rybczynski line while the choice of X_3 determines a point on that line. These production relationships may be expressed as

$$X_i = X_i(P+t, X_3), \quad i = 1, 2. \quad (14)$$

Turn now to consideration of the conditions which a (t, X_3) pair must satisfy in an equilibrium situation. With the tariff set as in Figure 1, and in the absence of international transfers, consumption of the private goods must be at a point on NN' , the national income line. Suppose that it takes place at C . Then, since trading opportunities are given by P , private goods' production must be taking place at Q on the Rybczynski line; otherwise trade would not be balanced. Thus tariff revenue, expressed in units of good 1, is SN . As well, payments to factors engaged in production of private goods are given by OS while payments for the public good, SN , coincide with tariff revenue.

In short, three conditions must be met in a tariff-financing equilibrium. These are: national income equals national expenditure on private goods, both evaluated at domestic prices; tariff revenue equals expenditure on public good provision; and trade is balanced. Note, however, that a consequence of any two of

these conditions being met is that the third holds. Therefore in what follows only the balance-of-trade condition and the financing condition are explicitly considered.

The balance-of-trade requirement may be re-expressed more fully as

$$C_1(P+t, X_3, U) - X_1(P+t, X_3) = P[X_2(P+t, X_3) - C_2(P+t, X_3, U)], \quad (15)$$

while the financing condition is written as

$$v(P+t)X_3 = tC_2(P+t, X_3, U) - tX_2(P+t, X_3). \quad (16)$$

The $C_i()$ are the compensated demand functions for goods $i = 1$ and 2 . The price of the public good relative to good 1 is denoted by " v ". In the presence of incomplete specialization, $v()$ is the relationship between v and the domestic price ratio, $P+t$.⁴

The two equation system given by (15) and (16) contains three unknowns: U , t and X_3 . Thus, they imply that U is a function of simply either one of the other two. The intent of the analysis is to identify the characteristics of the optimal tariff i.e. the tariff at which $dU = 0$. To do so, the following procedure may be employed. First, note four results:

(i) that, as a result of individual utility maximizing behaviour,

$$(\partial C_1 / \partial t) + (P+t)(\partial C_2 / \partial t) = 0; \quad (17)$$

(ii) that the cost of producing one more unit of X_3 in terms of the other goods is given simply by its price so

$$(\partial X_1 / \partial X_3) + (P+t)(\partial X_2 / \partial X_3) = -v; \quad (18)$$

(iii) that, from the conditions associated with the tangency of the price plane to the three dimensional transformation frontier,⁵

$$(\partial X_1 / \partial t) + (P+t) (\partial X_2 / \partial t) = 0; \quad (19)$$

and also

(iv) that

$$(\partial C_1 / \partial X_3) + (P+t) (\partial C_2 / \partial X_3) = -HMRS_{13}, \quad (20)$$

which is obtained by substituting the compensated demand functions into the utility function, (1), and differentiating with respect to X_3 , holding utility constant.

Next, take the differential of (15) at the optimum. Note that, at that point, U is constant, and use (17), (18), (19) and (20) to obtain

$$HMRS_{13} = v - t(dM/dX_3), \quad (21)$$

where M denotes imports and

$$dM/dX_3 = \partial M / \partial X_3 + (\partial M / \partial t) (dt/dX_3). \quad (22)$$

Condition (21) is interesting. It illustrates that the manner by which the conventional Samuelson rule for public-good provision (i.e. $HMRS_{13} = v$) must be amended depends solely on whether the full impact of an increase in public-good provision results in a decrease or increase in imports. Specifically, if a balanced

budget increase in provision of the public good causes a rise in imports then the marginal cost of the public good is less than its observed market price.

An examination of (21) and (22) indicates that the relationship between t and X_3 is crucial and that a complete analysis requires explicit consideration of dt/dX_3 . This entails taking the differential of (16), the second equation of the two-equation system; again the differential is taken at the optimum (where $dU = 0$). Substituting the resulting expression for dt/dX_3 into (21) via (22) and writing the result in terms of elasticities yields:

$$HMRS_{13} = \frac{(v - t\partial M/\partial X_3)(1 - e^V)}{1 + e^M - e^V}, \quad (23)$$

where $e^V = (dv/dt)(t/v)$, denotes the elasticity of the price of the public good while $e^M = (\partial M/\partial t)(t/M)$, denotes the elasticity of imports, both with respect to the tariff. (See Appendix 2.A for a complete derivation.)

In the neighbourhood of the optimal tariff the right-hand-side of (23) must be positive valued. The issue to be addressed in the next section is whether that expression is larger or smaller than v .

THE COST MEASURE

The expression on the right-hand-side of (23) measures the economic cost of an additional unit of the public good. Its structure is quite similar to that of the cost measure associated with

the standard optimal commodity tax rule, e.g. see Atkinson and Stern (1974, p.122), Atkinson and Stiglitz (1980, p.492) or Usher (1982, p.18). The difference is the presence of e^v . This reflects the rigidity of the tax regime. Since only a tariff is available, the authorities are effectively constrained to imposing equal production and consumption taxes. The result of this restriction is a distortionary effect on the relative price of the public good. A greater range of tax instruments would have permitted the additional degrees of freedom required to avoid that problem.

To determine whether the magnitude of the cost measure exceeds or is less than v requires an examination of dv/dt , $\partial M/\partial X_3$ and $\partial M/\partial t$. That is to say, it is necessary to investigate the relationship between the tariff and the price of the public good; the relationship between the level of imports and the amount of the public good and the relationship between the tariff and the amount of imports. These investigations are done in the following three subsections.

-
- (i) The relationship between the tariff and the price of the public good.

By either drawing on Komiya (1967) or using Lerner-diagram analysis it can be readily shown that the sign of dv/dt depends solely on the ranking of factor intensities. Let k_i denote the capital-labour ratio for good $i = 1, 2$ and 3 . When the capital-labour ratio of the exportable lies between those of the importable and of the public good, i.e. $k_2 > k_1 > k_3$ or $k_3 > k_1 > k_2$, then

dv/dt is negative. For the four other possible rankings, dv/dt is positive.

(ii) The relationship between the amount of the public good and imports

Here we consider $\partial M/\partial X_3$. It can be decomposed as the difference, $\partial C_2/\partial X_3 - \partial X_2/\partial X_3$. The sign of $\partial X_2/\partial X_3$ depends solely on the factor-intensity ranking. From Bhagwati and Srinivasan (1980), if k_2 lies between k_3 and k_1 then the Rybczynski line points in a north-westerly direction from point R in Figure 1. That is to say, production of X_2 falls as resources are withdrawn for production of the public good. For the other possible rankings, where k_2 does not fall between k_1 and k_3 , production of X_2 rises along with X_3 .

Next consider the impact of increased provision of the public good on consumption of the importable. It may be noted from the preceding discussion that when k_2 lies between the other two ratios then increases in X_3 may improve trading opportunities depending on whether the Rybczynski line is steeper or flatter than the world-price line. For the other factor intensity rankings where k_2 does not lie between the k_1 and k_3 , trading possibilities are unambiguously lessened by increases in X_3 . However, an improvement (deterioration) in trading opportunities does not ensure a positive (negative) value for $\partial C_2/\partial X_3$. In general, a change in X_3 shifts the indifference map in private goods' space. This effect could well be in opposition to and dominate the influence of the change in trading possibilities. It could also be

large enough to overwhelm $\partial X_2/\partial X_3$ in the event that these effects are not reinforcing vis a vis the sign of $\partial M/\partial X_3$. In short, the sign of $\partial M/\partial X_3$ is ambiguous.

(iii) The relationship between the tariff and imports

The impact of a change in the tariff on the level of imports is the difference, $\partial C_2/\partial t - \partial X_2/\partial t$. In order to determine the sign of $\partial X_2/\partial t$, consider the initial production point on the Rybczynski line. It is determined by the equality of $P+t$ with the marginal rate of transformation between goods 1 and 2 (MRT_{12}) along the relevant "interior" transformation frontier. Such a frontier displays the traded goods' production possibilities after capital and labour are withdrawn for production of the third good; note, as well, that the domestic price ratio determines the proportions in which the factors are withdrawn for production of the third good.

With these characteristics in mind, it can be shown that the sign of $\partial X_2/\partial t$ is positive. In what follows the economic explanation is provided (see Appendix 2.B for mathematical proof). The impact of an increase in t on production of good 2 can be thought of as the sum of two components. First, the increase in the tariff causes the familiar movement along the interior transformation frontier to the new point of equality between MRT_{12} and the new domestic price ratio. This is illustrated in Figure 2 by the movement from A to B. So long as both traded goods are produced, the effect is to reduce production of good 1 and to

Figure 2

increase production of good 2.

In addition to this movement along the transformation frontier, there is a second effect, a Rybczynski effect, which is reinforcing. The increase in the domestic price of good 2 relative to good 1 causes the price of the factor in which good 2 is intensive to rise relative to the price of the other factor. Thus, while production of the third good is constant, k_3 changes. Less of the factor in which good 2 is intensive is used and, X_3 being held constant, more of the other factor is used in the production of the third good. The quantities of the factors available for private goods' production change; there is more of the factor in which good 2 is intensive and less of the other factor. The Rybczynski theorem predicts a further increase in production of good 2 and a decrease in production of good 1. This is illustrated in Figure 2 by the movement from B to C.

In summary, the two impacts of an increase in the tariff are reinforcing; both tend to give rise to an increase in production of the importable and a decrease in the production of the exportable. Also, $\partial C_2 / \partial t$ is necessarily non-positive; holding U and X_3 constant, an increase in the tariff causes the standard non-positive substitution effect. This result ensures that $\partial M / \partial t$ is negative.

Having established the preceding results, it is feasible to return to (23) and make a number of observations. First, as with the optimal commodity tax rule, the impact of public good provision on consumption introduces ambiguity in ascertaining the cost of an extra unit of the public good. Specifically, since the sign of $\partial C_2 / \partial X_3$ is ambiguous so, too, is the sign of $\partial M / \partial X_3$. Therefore, using the Usher (1982) terminology, the "net cost of an extra unit of the public good", $v - t \partial M / \partial X_3$, may be greater or less than v .

However, the full cost of a unit of the public good is not merely the net cost. That amount must be adjusted by the "marginal cost of revenue" which in (23) is the interpretation associated with $(1 - e^V) / (1 + e^M - e^V)$, hereafter denoted by α . If e^V , the elasticity of the price of the public good with respect to the tariff, is negative then it immediately follows that the marginal cost of a dollar of public funds exceeds one dollar. However, as already noted, e^V can be positive. In that case its magnitude relative to e^M must be taken into account in determining the size of α . Figure 3 displays the possibilities.

For illustration adopt the reasonable assumption that the elasticity of tariff revenue with respect to the tariff, $1 + e^M$, is a positive fraction. Then if k_1 lies between the other two capital-labour ratios, e^V is negative and so α exceeds unity. However, if the capital intensity of the exportable is either the largest or the smallest then it cannot be concluded that α is less than unity. Three possibilities arise. First, if e^V exceeds unity, indeed α is

Figure 3

a positive fraction. The remaining two possibilities arise when e^V is a positive fraction. When e^V is less than $1+e^M$ then it is again the case that α is greater than one. However, in the event that the positive fractional value of e^V is greater than $1+e^M$ then α actually takes a negative value.

TWO LIMITATIONS

One crucial assumption of the preceding analysis is that the public good is privately produced. An immediate implication is that it is produced with minimum cost. However, the tariff causes factor price distortions. Cost minimization at market prices of factors is no longer desirable since these prices diverge from shadow prices. Government can do better by hiring factors in proportions consistent with their shadow prices and producing the public good itself. Alternatively, government may regulate firms which produce the public good. A condition on the purchase of their output would have to be that they employ the "correct" mix of capital and labour.⁶

In addition, there is a second source of limitation. It is typically the case that much of the expenditure by governments is not on collective consumption goods but, rather, on social infrastructure and on other projects and programmes designed to enhance production possibilities. Hence, it may be argued that the analysis presented above is limited in its applicability as it does not consider government financing of "public inputs".

The following three chapters deal with these concerns. Chapter 3 delves into the theory of public inputs. This is felt necessary in light of the limited literature on the subject. The background contained in chapter 3 provides the basis for chapter 4 in which the derivation of rules for efficient tariff-financed provision of a public input is presented. Then, in chapter 5, rules for public production for both a public good and a public input are derived; this requires the identification of the shadow prices of the factors.

SUMMARY

In this chapter a rule for optimal tariff-financed production of a public good was derived. The analysis was carried out in the HOS context, a setting most familiar to international-trade economists. The rule is similar in form to the optimal commodity tax rule with the difference in structure arising from the limitation on the set of policy instruments. As well, the cost measure as derived here explicitly reflects the trading opportunities and the implications of the factor-intensity rankings as associated with the HOS model. The determinants of the magnitude of the cost measure were then investigated in some detail. Among the findings were that: (i) the impact of public-good provision on consumption of the importable introduces ambiguity in determining whether the public good's cost measure exceeds its price; (ii) this ambiguity is compounded by the uncertainty over whether the impact of the tariff on the price of the public good exceeds its impact on

tariff revenue; and (iii) the direction of these impacts is influenced by the ranking of factor intensities. Finally, two important limitations were identified. Both are addressed in the chapters which follow.

FOOTNOTES

1. The absence of international trade in the public good is crucial. Komiya (1967) and Melvin (1968) establish that production is indeterminate if all goods are tradeable.
2. While it is assumed throughout that the public good is not congestable, it is a simple matter to allow for congestability. In the utility function, X_3 can be replaced by X_3/H^b , $0 < b < 1$, where b is a measure of the "publicness" of the good. When b equals zero, we have a pure public good, when it is unity, X_3 is purely private.
3. These results follow from assuming constant returns to scale, cost-minimization by firms, and production of some of all three goods. Full elaboration can be found in Bhagwati and Srinivasan (1980).
4. For further elaboration see Melvin (1968).
5. At any equilibrium, production occurs on the three-dimensional transformation frontier at the point of tangency with the price plane. Holding X_3 fixed restricts production to the associated two-dimensional "slice" of the transformation frontier. In equilibrium it must also be that, at the production point on this slice, the price ratio $P+t$ coincides with the MRT_{12} along it. Hence the result is obtained.
6. If other policy instruments were available then even greater improvements would be possible if those instruments could be used to offset the factor-price distortions experienced by the other sectors of the economy.

APPENDIX 2.A

DERIVATION OF RESULT (23)

Recall from the text, that starting at the optimum, i.e. $dU = 0$, we have

$$HMRS_{13} = v - t(dM/dX_3) \quad (21)$$

and

$$dM/dX_3 = (\partial M/\partial X_3) + (\partial M/\partial t)(dt/dX_3) \quad (22)$$

and, as well, recall the financing condition

$$v(P+t)X_3 = tC_2(P+t, X_3, U) - tX_2(P+t, \lambda_3) \quad (16)$$

which may be simply expressed as

$$vX_3 = tM \quad (16a)$$

where M denotes imports.

Then, take the differential of (16a) at the optimum (noting that $dU = 0$ in the neighbourhood of the optimum) to obtain

$$[v - t(\partial M/\partial X_3)]dX_3 = [M + t(\partial M/\partial t) - (\partial v/\partial t)X_3]dt \quad (16b)$$

so

$$\frac{dt}{dX_3} = \frac{v - t(\partial M/\partial X_3)}{M + t(\partial M/\partial t) - (\partial v/\partial t)X_3} \quad (16c)$$

which, using the fact that, from (16a) $X_3/M = t/v$, may be re-expressed as

$$\frac{dt}{dX_3} = \frac{v [1 - (t/v)(\partial M/\partial X_3)]}{M (1 + e^M - e^V)} \quad (16d)$$

where e^M and e^V denote the elasticities as defined in the text.

Next, substitute (16d) into (22) and then into (21) and factor out the v to arrive at

$$HMRS_{13} = v \left[\frac{1 - (t/v) (\partial M / \partial X_3) - (t/M) (\partial M / \partial t) [1 - (t/v) (\partial M / \partial X_3)]}{1 + e^M - e^V} \right] \quad (21a)$$

$$= \frac{v(1 + e^M - e^V - (1 + e^M - e^V)(t/v) (\partial M / \partial X_3) - e^M [1 - (t/v) (\partial M / \partial X_3)])}{1 + e^M - e^V} \quad (21b)$$

$$= \frac{v[1 + e^M - e^V - (1 - e^V)(t/v) (\partial M / \partial X_3) - e^M]}{1 + e^M - e^V} \quad (21c)$$

$$= \frac{v(1 - e^V)[1 - (t/v) (\partial M / \partial X_3)]}{1 + e^M - e^V} \quad (21d)$$

from which (23) follows trivially.

APPENDIX 2.B

Derivation of $\partial X_2 / \partial t$ from the
Rybczynski Relationship: $X_2 = X_2(t, X_3)$

The expression for $\partial X_2 / \partial t$ can be conveniently thought of as consisting of the sum of two effects. The first effect is the change in X_2 due to a change in the tariff, holding the X_1 - X_2 transformation frontier fixed. The second effect is the change in X_2 due to the change in the X_1 - X_2 transformation frontier associated with the change in the factor supplies available for production of those two traded goods; the change in factor supplies being caused by the change in the capital-labour ratio employed in the production of the non-traded good as a result of the tariff changing domestic prices. Both effects can be shown to be positive.

The first effect is positive so long as the relevant transformation frontier is concave to the origin and there is not complete specialization in production of good 2. Therefore it is necessary only to show that the second effect is also positive. That is done below.

The second effect is described by the expression:

$$\left(\frac{\partial X_2}{\partial K_T} \frac{\partial K_T}{\partial k_3} + \frac{\partial X_2}{\partial L_T} \frac{\partial L_T}{\partial k_3} \right) \frac{dk_3}{dt} \quad (B1)$$

where K_T denotes the quantity of capital available for production of the traded goods, L_T is similarly defined for labour. Further,

since X_3 is being held constant,

$$\frac{\partial K_T}{\partial k_3} = \frac{-\partial K_3}{\partial k_3} < 0 \quad (B2)$$

and

$$\frac{\partial L_T}{\partial k_3} = \frac{-\partial L_3}{\partial k_3} > 0 \quad (B3)$$

where L_3 and K_3 denote the level of labour and capital used in the production of the third good, respectively.

Expressions for the remaining derivatives in (B1) are readily available from Kemp (1969, pp.109-110). They are:

$$\frac{\partial X_2}{\partial K_T} = \frac{f_2}{k_2 - k_1}, \quad (B4)$$

$$\frac{\partial X_2}{\partial L_T} = \frac{-k_1 f_2}{k_2 - k_1}, \quad (B5)$$

and

$$\frac{dk_3}{dt} = \frac{f_2}{v^2 (k_2 - k_1) f_3''}, \quad (B6)$$

where f_2 (f_3) is the output-labour ratio for the second (third) good and f_3'' denotes the second order derivative of f_3 with respect to k_3 . From the assumptions made regarding the production technologies f_3'' is negative.

Using (B2) to (B6), the expression in (B1) can be rewritten as

$$\left(\frac{\partial K_3}{\partial k_3} - k_1 \frac{\partial L_3}{\partial k_3} \right) \frac{-(f_2)^2}{v^2 (k_2 - k_1)^2 f_3''} , \quad (B7)$$

which, based on the information provided above, is positive.

CHAPTER 3: THE THEORY OF PUBLIC INPUTS

INTRODUCTION

As noted in the preceding chapter, countries that use trade taxes as a source of revenue often use some of that revenue to finance development projects. Such projects do not provide public goods in the Samuelson (1954) sense. That is to say, the quantity of the government-financed service does not directly enter individual utility functions. Rather, the good enhances production possibilities thereby increasing the availability of those private goods which individuals consume. Such intermediate goods are described as "public inputs".

In this chapter the theory of public inputs is elaborated upon in some detail. This is necessary for two reasons. First, there are at least three types of public input in the literature. It is necessary to determine which one is the most appropriate to incorporate into the analysis of tariff-financing. Secondly, the theory of public inputs is not nearly as well developed or known as the theory of public goods. Therefore, it worthwhile in itself to survey and assess the existing literature; something which has not been done to date. Doing so here leads to interesting insights and worthwhile clarifications.

BACKGROUND

Public inputs are intermediate goods and services which collectively enter production functions. Given their collective nature, the free-rider problem arises for reasons analogous to those

associated with the more widely known Samuelson (1954) collective consumption good. It is possible to argue that public budgets are directed towards public-input provision at least as much as public-good provision. In fact, much of the literature in local public economics and development economics deals with issues of provision of commodities which are public inputs, e.g. capital infrastructure. This is seldom explicitly recognized and neither the exact collective nature of the input nor the theory of public inputs is brought formally into analyses. Therefore, the varieties of public inputs and the associated Samuelson-type efficiency rules should be of considerable interest.

Kaizuka (1965) is the first to derive such rules. However, Boadway (1973) and Henderson (1974) question the validity of Kaizuka's derivations. In response, Hillman (1978) and McMillan (1979) defend the Kaizuka rules. The upshot of the Boadway-Henderson-Hillman-McMillan, BHHM, debate is the conclusion that the applicability of the Kaizuka rules depends on whether the public input is "firm-augmenting" or "factor-augmenting". As well, and independently of the BHHM debate, Negishi (1973) suggests a third variety of collective input, one which is similar to the "free-access resource" of Gould (1972) but with access available to more than one industry. It will be described as a "semi-public input".

This chapter employs a common framework to derive and examine the rules which describe the conditions required for economy-wide production efficiency associated with each of the three types of public input. There are a number of reasons for this investigation.

The BHHM debate is limited to a one-good framework; it is important to ascertain whether their findings remain valid in a more general setting and, as well, consideration of the more general case provides greater insight into the implications of public-input provision in relation to factor allocations within the private sector. By additionally incorporating the semi-public inputs into this analysis, the common framework makes it possible to more clearly identify the distinctions among the three. Finally, based on these differences it is possible to identify which of the three is of greatest practical relevance.

THE KAIZUKA RULES

Kaizuka (1965) identifies the circumstances under which production occurs on the transformation frontier in the presence of public inputs. He carries out his analysis for arbitrary numbers of final private goods, primary factors of production, and public inputs. Without loss of generality, his results can be stated in the context of a two-good, two-factor, single public input, (2x2x1) model.

Let the produced quantities of the two final goods be denoted by X_1 and X_2 , and let the fixed endowments of the two primary factors be denoted by K , for capital, and L , for labour. The quantities of the primary factors employed in each industry are to be given by K_i and L_i for $i = 1, 2$ and 3. Finally, let X_3 denote the quantity of the public input and let the production functions be denoted by F_i . Then consider the following relationships:

$$X_i = F_i(K_i, L_i, X_3), \quad i = 1, 2. \quad (1)$$

and

$$X_3 = F_3(K_3, L_3). \quad (2)$$

Given the fixed endowments and (1) and (2), Kaizuka concludes that for efficient production the factors must be allocated in a manner which jointly satisfy the following:

$$(\partial F_1 / \partial K_1) / (\partial F_1 / \partial L_1) = (\partial F_2 / \partial K_2) / (\partial F_2 / \partial L_2) = (\partial F_3 / \partial K_3) / (\partial F_3 / \partial L_3), \quad (3)$$

$$(\partial F_1 / \partial X_3) / (\partial F_1 / \partial L_1) + (\partial F_2 / \partial X_3) / (\partial F_2 / \partial L_2) = 1 / (\partial F_3 / \partial L_3), \quad (4)$$

and

$$(\partial F_1 / \partial X_3) / (\partial F_1 / \partial K_1) + (\partial F_2 / \partial X_3) / (\partial F_2 / \partial K_2) = 1 / (\partial F_3 / \partial K_3). \quad (5)$$

Condition (3) is the familiar requirement that the marginal rates of technical substitution between factors be equalized across industries. In the presence of competitive profit-maximizing firms, it can be expected to hold. However, (4) and (5) must also be met if production is to be efficient. As pointed out by Kaizuka, these conditions cannot be expected to be satisfied in a competitive equilibrium. Consequently, there is a basis for government intervention.¹

FACTOR-AUGMENTING PUBLIC INPUTS

The crux of the BHHM debate is the degree of homogeneity of the production functions for the private goods. Two possibilities are considered by those authors: linear homogeneity in capital and labour only, or linear homogeneity in all inputs. Throughout it is assumed that the public input is produced with constant returns to

scale.

Note that linear homogeneity in capital and labour may result in a non-convex production set. Others, e.g. Tawada (1980), Manning and McMillan (1982) and Abe, Okamoto and Tawada (1986) have dealt with that issue. In this analysis it is assumed that factor substitution possibilities are sufficient to overcome any tendency towards non-convexity.² More will be said about this issue of convexity in the next chapter.

The output of any industry is the summed output of all firms that comprise that industry. Assuming all firms have access to the same technology, each will produce an equal amount and use the same mix of capital and labour in the process. The relationship between industry output and per-firm output may therefore be described by

$$X_i = N_i F_i(\hat{k}_i, \hat{l}_i, X_3), \quad i = 1, 2 \quad (6)$$

where N_i denotes the number of firms/plants and $\hat{k}_i = K_i/N_i$ and $\hat{l}_i = L_i/N_i$ denote respectively the quantities of capital and labour employed by each firm in industry i . As will be shown, whether the Kaizuka rules are appropriate for achieving productive efficiency depends on the assumptions regarding (i) the properties of F_i for $i = 1$ and 2 , and (ii) whether the number of firms is fixed. While different assumptions will be made regarding these two production functions, note that the same symbol, " F ", is used to denote them. This method of representation is consistent with Hillman (1978) and McMillan (1979).

In the event that the functions F_i in (6) are characterized by

constant returns to capital and labour only, the expressions in (6) simplify to (1). The number of firms becomes irrelevant and the rules given by (3), (4) and (5) are directly applicable. In these circumstances the public input is said to be of the "factor-augmenting" variety. This descriptor is particularly apt since, in this case, provision of the public input has an impact equivalent to a combination of Harrod-neutral and Solow-neutral technological changes.³ As a result, the gains generated by provision of the public input are captured entirely by the private factors of production according to the manner by which their marginal productivities are altered.⁴

FIRM-AUGMENTING PUBLIC INPUTS

Complications arise when the production technologies in (6) are linearly homogeneous with respect to all inputs. This is the basis of the Boadway (1973) and Henderson (1974) criticisms. Use of this homogeneity property does not lead from (6) to (1) but to:

$$X_i = F_i(K_i, L_i, N_i X_3), \quad i = 1, 2. \quad (7)$$

An examination of (7) indicates that the number of firms in each industry becomes pertinent in determination of the conditions required for production efficiency. The public input is now a type of service to firms, not to factors per se. Thus, as provision of the public input enhances production possibilities, rents will, at least initially, accrue to firms. Public inputs of this type are therefore described as "firm-augmenting".

Three possibilities may be entertained when the public input is firm-augmenting. First, as noted by Henderson (1974), there may be no limit to firm divisibility in which case the optimal size of firms is infinitesimal, the number of firms in each industry infinite and production possibilities unbounded. This case has been adequately addressed by Henderson and will not be pursued further.

A second possibility is that the number of firms in each industry is fixed.⁵ This forces (7) into a mold similar to that of (1). As a result, the Kaizuka rules continue to define what is required for productive efficiency. Boadway (1973) considers this case where there is only one final good. He points that it is sufficient for government to provide the public input but notes that there are different distributional implications. He argues that now rents accrue to firms, not only initially, but in equilibrium. These rents may be interpreted as those associated with the right to be a firm. This observation generalizes to the many-good setting since it follows from the fixity of the number of firms.

The third and most realistic approach is to posit a limit to divisibility of firms due to a minimum input requirement for at least one of the private factors. This approach is suggested by Henderson (1974). He also suggests the issues which should be considered in determining efficiency conditions, but it is Hillman (1978) who develops and solves the formal optimization problem. His results represent a powerful defence of Kaizuka-type rules. He finds that the required number of firms is in fact the maximum number which the divisibility constraint permits, and that primary

inputs must be allocated according to rules which deviate from the standard Kaizuka rules only by the presence of an additional parameter, namely the Lagrangean multiplier on the indivisibility constraint. Also he notes that, since rents accrue to firms in the private industry, freedom of entry ensures that the actual number of firms is always the optimal number. Hillman concludes that government need not be concerned with the pattern of industrial organization; all the authorities need do is choose the appropriate level of the firm-augmenting public input according to the modified rules.

In Hillman's model, once an efficient mix of capital and labour is chosen for producing the public input, the remaining mix of the two factors are, residually, efficient for the private sector; and since the private sector consists of a single industry it is simply a truism that residual capital and labour will be allocated efficiently among private industries. However, if the private sector consists of more than one industry it may no longer be true that the residual amount of capital and labour will split among industries in a manner consistent with Pareto efficiency. In what follows, the conditions for such an efficient allocation are identified. This permits one to determine whether the optimizing behaviour of firms leads to satisfaction of those conditions or whether a second policy intervention is required.

The requirements for productive efficiency can be derived from the first-order conditions associated with the following problem:

$$\text{Maximize} \quad N_1 F_1(\hat{k}_1, \hat{l}_1, X_3) \quad (8)$$

$$\text{subject to } N_1 \hat{k}_1 + N_2 \hat{k}_2 + K_3 = K \quad (9)$$

$$N_1 \hat{l}_1 + N_2 \hat{l}_2 + L_3 = L \quad (10)$$

$$N_2 F_2(\hat{k}_2, \hat{l}_2, X_3) = X_2^* \quad (11)$$

$$F_3(K_3, L_3) = X_3 \quad (12)$$

and

$$\hat{k}_i - s_i \geq 0, \quad i = 1, 2. \quad (12')$$

where s_i denotes the minimum positive amount of capital required before a firm can produce, X_2^* is an arbitrary amount of good 2, and (9) and (10) are the resource endowment constraints. The choice variables are the number of firms, the amount of the public input as well as $\hat{k}_i$ and $\hat{l}_i$.

Let w and r denote the Kuhn-Tucker multipliers on the labour and capital constraints respectively; let a_i , $i = 1$ and 2 , denote the multipliers on (12) and let P denote the shadow price of good 2. Then (as derived in Appendix 3.A) the necessary conditions for efficiency are (4), as before, and

$$\frac{\partial F_i / \partial \hat{k}_i + (X_3 / s_i) (\partial F_i / \partial X_3)}{\partial F_i / \partial \hat{l}_i} = \frac{\partial F_3 / \partial K_3}{\partial F_3 / \partial L_3}, \quad i = 1, 2. \quad (13)$$

and

$$\begin{aligned} & \frac{N_1 (\partial F_1 / \partial X_3)}{(\partial F_1 / \partial \hat{k}_1) + (X_3 / s_1) (\partial F_1 / \partial X_3)} + \\ & \frac{N_2 (\partial F_2 / \partial X_3)}{\partial F_2 / \partial \hat{k}_2} \left[1 - \frac{P (X_3 / s_2) (\partial F_2 / \partial X_3)}{(\partial F_1 / \partial \hat{k}_1) + (X_3 / s_1) (\partial F_1 / \partial X_3)} \right] = \frac{1}{\partial F_3 / \partial K_3} \end{aligned} \quad (14)$$

which replace (3) and (5), respectively.

Conditions (13) and (14) differ from their respective counterparts (3) and (5). Consider first how (13) differs from (3). In (13) there are additional terms in the numerators of the left-hand-side expressions, namely $(X_3/s_i)(\partial F_i/\partial X_3)$. Their presence reflects the fact that making more capital available to one of the private industries causes not only an increase in production according to the marginal product of capital, but also a second effect, one which is due to the collective nature of the public input. With more capital there can be more firms, and an increase in the number of firms is equivalent to a gratuitous increase in X_3 at the industry level. Thus, the sum of the two terms in the left-hand-side numerators of (13) constitutes the social marginal product of capital.

Condition (14) differs from (5) also because of the need to take account of the fact that the social marginal products of capital in the private industries differ from their private marginal products. In fact, (14) reflects a rather peculiar type of trade-off. On the one hand, by hiring more capital, the authorities can increase the amount of X_3 according to $\partial F_3/\partial K_3$. However, on the other hand, by releasing capital from public sector employment, private production is increased by the marginal productivities in the private sector plus, through the creation of new firms, the increased amount of public input available to private industry. In short, more of the public input can be had by either producing more of it or by simply creating more firms.

The policy implications in this $2 \times 2 \times 1$ setting are quite dif-

ferent from those derived by Hillman (1978) for the one-good model. Hillman concludes that the authorities need be concerned solely with providing the public input and that the pattern of industrial organization is of no concern since rents would always attract firms into private industry until each firm operates with the minimum required amount of capital. This conclusion is indeed valid for the one-good world. And it is also true that for many goods, firms will still be of minimum size in each industry. However, in the case of more than one good, capital has the choice of moving to more than a single industry. In general, it cannot be expected that capital will allocate itself efficiently among industries since it moves according to its private rather than social marginal product. Therefore, while firm-size remains efficient, the number of firms is not at an efficient level. The authorities must do more than finance the public input, they must also use factor taxes/subsidies in order to ensure that condition (13) holds. For the case of more than one good, the pattern of industrial organization becomes a crucial concern.

SEMI-PUBLIC INPUTS

Another variety of public input has been introduced by Negishi (1973). Adopting the Meade (1952) terminology for classifying externalities, Negishi refers to this type of public input as an "unpaid factor". As with firm-augmenting public inputs the production functions for the private goods are assumed to be linearly homogeneous in all inputs. However, now the input has the peculiar feature of being collective across industries but

congestable within each industry. This characteristic has led others who explicitly acknowledge it, e.g. Tawada (1980) and Tawada and Okamoto (1983) to adopt the alternate term "semi-public input" to describe this intermediate good. In the special case where the input benefits a single industry, it becomes equivalent to the "free-access resource" of Gould (1972).

A semi-public input may best be thought of as one which has multiple uses. An amount of it generates a vector of characteristics in the sense of Lancaster (1966). Each characteristic benefits a different industry but is congested as the number of firms in that industry increases. An example might be a dam which benefits both farms (through the creation of a water reservoir) and an inland fisheries (through the control of downstream flooding). A second example may be a road system which is beneficial to two different industries which use the roads in different seasons.

For an input of this type, the relationships describing private-industry production become:

$$X_i = N_i F_i(K_i/N_i, L_i/N_i, X_3/N_j), \quad i=1,2; \quad (15)$$

where, following Negishi (1973), it is assumed that both F_1 and F_2 exhibit constant-returns-to-scale with respect to all three inputs. Observe now that since the public input is congestable within an industry, unlike a firm-augmenting public input, the number of firms is not relevant. Within an industry, the input is in effect a private input made available freely to firms.

Using (15) it can be readily verified that (3), (4) and (5)

still define the conditions required for production efficiency. However, Negishi (1973) claims that government must do more than provide the semi-public input according to (4) and (5). He argues that, in the presense of factor mobility, condition (3) cannot be expected to hold. His reasoning is as follows. Any provision of the semi-public input generates rents. With freedom of entry, he asserts that these rents are captured fully by capital so that in response capital moves between industries until the rates of return are equated between them. Capital is paid the value of its marginal product, $VMPK$, plus a share of the rent from the industry in which it locates. Therefore, the $VMPK$ in one industry is not equal to that in another industry except by chance. Failure to satisfy that equality means that marginal rates of substitution between capital and labour are not being equated accross industries, i.e. (3) does not hold.

It is not clear why Negishi permits all rents to be captured by capital. Both mobile factors can be expected to capture shares of the rents. However, dropping Negishi's extreme assumption does not change the conclusions. Input mixes will be inefficient. Government must not only provide X_3 using an efficient mix of L_3 and K_3 but must also launch a scheme to correct the associated factor price distortions.

The intuition behind this conclusion is as follows. Public provision solves only one of two problems associated with the existence of the semi-public input. It makes the input available. However, this availability is what generates the second problem.

That is the problem of misallocation of primary factors as they move to capture rents generated by the semi-public input. Lindahl pricing or a system of factor taxes/subsidies would be required to remedy this second problem.

Before concluding it is worthwhile to note that Negishi's introduction of the unpaid factor provides an impetus to define a type of public input not considered in the literature; one which is also public to industries and private to firms but for which production functions are linearly homogeneous in primary inputs only.⁶ In this environment, (15) may be re-expressed as

$$X_i = F_i(K_i, L_i, X_3/N_i), \quad i = 1, 2; \quad (16)$$

from which it is clear that the number of firms per industry is relevant. To achieve Pareto-efficiency it is required that the standard Kaizuka conditions be satisfied and, additionally, that there be a single firm in each industry. It is not sufficient that government merely fund provision of the public input. It must limit entry to just one firm per industry and, as a result, deal with the familiar problems associated with the monopolization of production.

SUMMARY AND CONCLUSION

In this chapter the debate over the applicability of the Kaizuka rules was reviewed. It was found that difficulties arise when production functions for private goods are linearly homogeneous with respect to both the primary and public inputs. Hillman (1978) and McMillan (1979), in a single-good framework, indicate the

Kaizuka rules continue to apply either directly or with slight modification. The analysis was extended to model with two final goods. This led to more general versions of the rules which permitted insight into factor allocation among private industries. Then policy implications were dealt with.

The case of "semi-public" inputs was also examined in the $2 \times 2 \times 1$ setting. It was established that whether production functions are linearly homogeneous in either the primary inputs or all inputs the Kaizuka rules remain applicable. However, if the homogeneity is in all inputs then government provision must be accompanied by factor market intervention. If, on the other hand, the homogeneity is with respect to the primary inputs only, then Pareto-efficiency requires not only the satisfaction of the Kaizuka rules but, in addition, there must be a sole producer in each industry; and policy must be adopted to deal with this pattern of industrial organization.

Finally, it is worthwhile to conclude on a practical note. McMillan (1979) points out that convincing examples of firm-augmenting public inputs are not easily found. This observation seems to apply equally to semi-public inputs. It may well be that emphasis on non-factor-augmenting public inputs is misplaced. Consequently, in the remaining discussions relating to tariff financing of public inputs we deal with the factor-augmenting variety only.

Footnotes

1. It is not essential that government do the intervening. Firms may agree among themselves to establish and fund an agency charged with the responsibility of producing the input.
2. For semi-public inputs, the production frontier remains strictly convex; see Tawada (1980) for a proof of this.
3. When the Harrod and Solow effects are equivalent, the combined effect amounts to a Hicks-neutral technological change.
4. Manning et al. (1985) prove that, since the factors receive the benefits of a factor-augmenting public input, Lindahl pricing is infeasible but that it is possible to devise a uniform factor income tax which leads to a first-best outcome.
5. Situations of this kind (with a fixed number of firms and a firm-augmenting public input) have been the basis for Sandmo (1972), Groves and Loeb (1975), Laffont (1976) and Pestieau (1976).
6. Tawada (1980) states that if the public input is private within industries and public only across industries then it must be that the production functions are linearly homogeneous with respect to all inputs including the public input. This appears to be an error. The degree of homogeneity and the extent to which additional firms may congest one another in usage of X_3 are independent.

APPENDIX 3.A

The first-order conditions for the problem described by (8), (9), (10), (11), (2) and (12) are:

$$F_1(\hat{k}_1, \hat{l}_1, F_3(K_3, L_3)) - r\hat{k}_1 - w\hat{l}_1 \leq 0, \quad N_1(\partial Z / \partial N_1) = 0, \quad (A1)$$

$$P_2 F_2(\hat{k}_2, \hat{l}_2, F_3(K_3, L_3)) - r\hat{k}_2 - w\hat{l}_2 \leq 0, \quad N_2(\partial Z / \partial N_2) = 0, \quad (A2)$$

$$N_1(\partial F_1 / \partial \hat{k}_1) - rN_1 + a_1 \leq 0, \quad \hat{k}_1(\partial Z / \partial \hat{k}_1) = 0, \quad (A3)$$

$$N_1(\partial F_1 / \partial \hat{l}_1) - wN_1 \leq 0, \quad \hat{l}_1(\partial Z / \partial \hat{l}_1) = 0, \quad (A4)$$

$$PN_2(\partial F_2 / \partial \hat{k}_2) - rN_2 + a_2 \leq 0, \quad \hat{k}_2(\partial Z / \partial \hat{k}_2) = 0, \quad (A5)$$

$$PN_2(\partial F_2 / \partial \hat{l}_2) - wN_2 \leq 0, \quad \hat{l}_2(\partial Z / \partial \hat{l}_2) = 0, \quad (A6)$$

$$[N_1(\partial F_1 / \partial X_3) + PN_2(\partial F_2 / \partial X_3)](\partial F_3 / \partial K_3) - r \leq 0, \quad K_3(\partial Z / \partial K_3) = 0, \quad (A7)$$

$$[N_1(\partial F_1 / \partial X_3) + PN_2(\partial F_2 / \partial X_3)](\partial F_3 / \partial L_3) - w \leq 0, \quad L_3(\partial Z / \partial L_3) = 0, \quad (A8)$$

$$K - N_1\hat{k}_1 - N_2\hat{k}_2 - K_3 \geq 0, \quad r(\partial Z / \partial r) = 0, \quad (A9)$$

$$L - N_1\hat{l}_1 - N_2\hat{l}_2 - L_3 \geq 0, \quad w(\partial Z / \partial w) = 0, \quad (A10)$$

$$\hat{k}_1 - s_1 \geq 0, \quad a_1(\partial Z / \partial a_1) = 0, \quad (A11)$$

$$\hat{k}_2 - s_2 \geq 0, \quad a_2(\partial Z / \partial a_2) = 0, \quad (A12)$$

where Z denotes the associated Lagrangean.

A number of observations can now be made which are helpful in deriving the conditions for Pareto-efficiency. These are:

Observation A: Since more can always be had through subdivision of firms then the optimal plant size is always s_i for $i = 1$ and 2 , i.e. (A11) and (A12) hold with strict equality. Moreover, because of the existence of rents, for any arbitrary amount of capital firms will enter until each has exactly s_i with which to operate. Thus the number of firms in each industry is determined solely by the availability of capital and actual firm size is always s_i .

Observation B: Assuming that allocations of both factors to each activity are all non-zero then (A1) to (A10) all hold with strict equality.

Observation C: With the production functions, (6), linearly homogeneous in all inputs, it is true that

$$\hat{k}_1(\partial F_1/\partial \hat{k}_1) + \hat{l}_1(\partial F_1/\partial \hat{l}_1) + X_3(\partial F_1/\partial X_3) = X_1 \quad (A13)$$

and

$$\hat{k}_2(\partial F_2/\partial \hat{k}_2) + \hat{l}_2(\partial F_2/\partial \hat{l}_2) + X_3(\partial F_2/\partial X_3) = X_2. \quad (A14)$$

Observation D: From (A1), (A3) and (A4) it follows that

$$\hat{k}_1[(\partial F_1/\partial \hat{k}_1) + a_1/N_1] + \hat{l}_1(\partial F_1/\partial \hat{l}_1) = X_1 \quad (A15)$$

so that using (A13), together with the observation that each firm operates with the minimum amount of capital, it follows that

$$a_1 = N_1(X_3/s_1)(\partial F_1/\partial X_3) \quad (A16)$$

and, in a similar fashion, but using (A2), (A5) and (A6) and then (A14), one obtains

$$a_2 = PN_2(X_3/s_2)(\partial F_2/\partial X_3). \quad (A17)$$

(A15) and (A16) indicate, respectively, the increase in good 1 and good 2 production associated with a small slackening of the constraints on minimum capital requirements.

Using these observations, the Pareto-efficiency conditions can be derived. First use (A3) to (A8) together with (A16) and (A17) to obtain (13). Substitution of (A4) and (A6) into (A8) yields (4). Finally, substitution of (A3) and (A5) into (A7), and using (A16) and (A17), gives (14).

CHAPTER 4: TARIFF FINANCING OF A PUBLIC INPUT

INTRODUCTION

Taking reliance on the tariff for revenue as a "fact of life", Vanek (1971) developed the "Optimal Revenue Tariff". It is defined as the tariff rate which maximizes the present value of the difference between the cost of the tariff and the benefits which it generates. The benefits occur in the future and arise from the use of the tariff revenue to finance capital formation.

This chapter extends the Vanek analysis by linking the tariff revenue directly to the provision of a public input. Such inputs, as noted earlier, are produced inputs which collectively enter firms' production functions. As with Samuelson collective consumption goods, their collective nature gives rise to a "free rider" problem. Hence there is a rationale for government intervention or for some other mechanism of collective action. In fact, it is possible to argue that a greater share of government expenditure is devoted to the provision of public inputs rather than to the extensively analysed collective consumption goods. Certainly, a substantial portion of LDCs' revenues appears to be devoted to items which may be characterized as public inputs.

This chapter considers the case where the public input is factor augmenting. This choice of a factor-augmenting public input is based on the conclusion of the preceding chapter that the other varieties of public input are of a rather more peculiar and unrealistic nature.

CONVEXITY OF THE PRODUCTION SET

In the preceding chapter it was demonstrated that the Kaizuka rules for public input provision remain valid when a public input is factor augmenting. These rules indicate that to achieve productive efficiency where there are two primary factors of production, factors must be allocated such that (i) marginal rates of substitution between the two factors are equalized across all industries including the industry which produces the public input, and (ii) each factor must be allocated such that the sum of marginal rates of substitution between that factor and the public input for all private goods be equal to the reciprocal of the marginal physical product of that factor in producing the public input.

The efficiency rules merely serve to ensure that allocation of factors is such that production takes place on the production possibilities frontier. There is a second issue which must be taken into account. When a public input is factor augmenting it is possible that the production set, defined over the private goods, may not be convex, i.e. that the production possibility frontier in private goods' space may not be everywhere strictly concave to the origin.

To illustrate this problem and for future reference consider the construction of the production possibilities frontier for the case in which there are two private goods and one public input. In Figure 1 the AB curve is the standard two dimensional PPF in private goods space. It is defined for a zero-level of

FIGURE 1

public input availability. Next, consider the AA'C curve. It displays the maximum production of good 1 which is possible for each amount of X_3 , when there is no production of good 2. The only Pareto-efficient point on AA'C is A'. The BB'C curve and the point B' have similar respective interpretations in relation to the second good. It follows that merely being on the three-dimensional surface, i.e. condition (i) being satisfied, is not sufficient for productive efficiency. For any arbitrary amount of one of the final goods, capital and labour must be allocated in a manner that maximizes production of the other good; this entails the allocation of the appropriate amounts of the two factors to production of X_3 according to (ii) given above. The locus of points, labelled A'EB' in Figure 1, on the three-dimensional surface is the set of points characterized by productive efficiency. When projected onto X_1 - X_2 space this locus forms a frontier lying above the AB curve and having endpoints A" and B".

While the three dimensional surface in Figure 1 is strictly concave to the origin, the crucial issue is whether the projected two dimensional PPF is everywhere concave to the origin; if it is not then a number of difficulties may arise, e.g., multiple equilibria and corner solutions.

Manning and McMillan (1979), Tawada (1980), Manning and McMillan (1982), Tawada and Abe (1984), Abe, Okamoto and Tawada (1986) and, most recently, Altenburg (1987) consider the conditions required for the PPF to be concave to the origin when there exists a factor-augmenting public input. It has been

established that a sufficient condition for concavity of the PPF to the origin is that all production functions, including that of the public input, be concave. However, even if the functions are not concave, there are circumstances in which the PPF retains the desirable shape. The authors cited above deal primarily with the search for forms of the production functions which deliver that shape. In essence, this amounts to identifying the mathematical form of production functions which lead to degrees of factor substitutability sufficient to overcome the tendency towards convexity due to the increasing returns to scale effects associated with the public input. For the purposes of this analysis, we simply adopt the working assumption that, at least in the neighbourhood of the optimum, there is sufficient factor substitutability to ensure a PPF which is strictly concave to the origin.

THE MODEL

There are two final consumption goods and the amounts of each that are produced domestically are denoted by X_1 and X_2 . The associated production functions are described by

$$X_i = F_i(K_i, L_i, X_3), \quad i=1,2, \quad (1)$$

where F_i are strictly quasi-concave, K_i and L_i are inputs of the primary inputs, capital and labour, respectively, and X_3 is the quantity of the non-traded public input. The F_i are homogeneous of degree one in capital and labour so that the public input is of

the "factor-augmenting" variety. The production function for the public input is given by

$$X_3 = F_3(K_3, L_3) \quad (2)$$

where F_3 exhibits constant returns to scale. The primary inputs are in fixed supply and factor markets are competitive, ensuring full employment. Product markets are also competitive.

There is a well-defined and "well-behaved" social utility function. It is written:

$$U = U(C_1, C_2) \quad (3)$$

where C_i represents the quantity of good i consumed, for $i = 1$ and 2 . Consumption may differ from domestic production because this economy is open to international trade in goods 1 and 2 . The trading opportunities are described by

$$X_1 + PX_2 = C_1 + PC_2 \quad (4)$$

where P is the world price ratio of good 2 to good 1 . The home country is assumed to be sufficiently small so that it is unable to influence P .

TARIFF FINANCING

If the appropriate quantity of the public input is to be provided then government can either purchase factors and make the public input itself or purchase the public input from private firms. It is assumed that the latter option is the only one available. This amounts to imposing an additional constraint on

government. The authorities choose X_3 but have no control over the factor mix employed to make it. Firms choose that mix in accordance with relative factor prices but in the presence of the tariff those prices are distorted, making private cost minimization socially undesirable. Despite this observation, the real world does appear to be characterized by governments choosing the lowest bids; or when this is not the case it is for reasons unrelated to the use of appropriate shadow prices. Therefore, the assumption is quite defensible and reflective of the policy environment in which the authorities may be operating. In the next chapter, the case of public sector production of the public input is considered.

To illustrate the problem at hand, consider Figure 2. The curve given by A^*B^* is the production possibility frontier as derived in Figure 1 but drawn in good 1-good 2 space. With a world price ratio of P , if lump-sum taxation were feasible then production would occur at point Q^* (with government purchasing the level of public input corresponding to that point and with the capital-labour ratio used in its production being that as determined by P and X_3) while consumption would occur at C^* . This means that the authorities should purchase the public good up to the point where the value of national income, in terms of private goods' production, evaluated at world prices, is maximized.

However, suppose that the only source of funds available is the tariff. Consider the implications of financing an arbitrary amount of the public input; say, X_3^* the amount associated with

FIGURE 2

Q^* . Fixing the level of production at that amount means the economy is restricted to producing along a slice of the three dimensional surface described by Figure 1. That slice, given by aQ^*b in Figure 2, is a restricted production possibility frontier which lies everywhere below A^*B^* except at Q^* where it is tangent. With product prices altered due to the imposition of the tariff, production would no longer occur at Q^* . Firms take the domestic prices and, given X_3^* , the consequent prices of the primary factors as their signals. The result is that production takes place at the tangency of the domestic price line with the restricted frontier. For tariff revenue to be just sufficient to finance X_3^* (taking into account the impact of the tariff on the price of the public input itself) suppose that the required per-unit tariff is t^* . Then production occurs at Q_t and consumption at C_t with tariff revenue given by R expressed in terms of the importable.

While (Q_t, C_t) represents a feasible equilibrium it is in general possible to do better. To derive the conditions which characterize optimal policy in the tariff-financing situation it is sufficient to consider two conditions which must hold in equilibrium. First, it must be that national income equal national expenditure, both evaluated at domestic prices. National income is the sum of all payments to labour and capital for their services in producing all three of the importable, the exportable and the public input, while domestic expenditure is the value of expenditures on the two consumption goods. This condition is

expressed as:

$$X_1(P+t, X_3) + (P+t)X_2(P+t, X_3) + v(P+t, X_3)X_3 = C_1(P+t, U) + (P+t)C_2(P+t, U) \quad (8)$$

where $X_i()$ are the supply functions for the final goods and the $C_i()$ are the compensated demand functions, for $i = 1$ and 2 . Also, $v()$ is the relationship between the relative price of a unit of the public input, denoted by v , and its determinants which are the domestic price ratio as well as the quantity of the public input. This relationship holds when there is incomplete specialization in private goods' production. It may be explained quite succinctly. For any amount of the public input, the production relationships between the private goods and the primary factors are set. Then, since the public input is not tradeable, Komiya's (1967) analysis shows that the world price ratio sets the factor prices which, in turn, pins down the price of the non-traded good.

The second condition which must hold at any feasible equilibrium is that government expenditure be financed by tariff revenue, the tariff being the only instrument available to government. This condition is described by:

$$t[C_2(P+t, U) - X_2(P+t, X_3)] = v(P+t, X_3)X_3. \quad (9)$$

Note that when (8) and (9) are simultaneously satisfied the balance of trade condition is also met so it does not have to be explicitly brought into the analysis.

Conditions (8) and (9) constitute a system of two equations in three unknowns: t , X_3 and U . This implies that we may solve for U as simply a function of the tariff rate.

To determine the characteristics of the tariff which maximizes social utility, one may start by totally differentiating the system of equations, (8) and (9). In the process of doing so, note that, from the familiar tangency conditions associated with optimizing behaviour,

$$(\partial X_1 / \partial t) + (P+t)(\partial X_2 / \partial t) = 0 \quad (10)$$

and

$$(\partial C_1 / \partial t) + (P+t)(\partial C_2 / \partial t) = 0 . \quad (11)$$

As well, there is no loss of generality if the value of the expression $[(\partial C_1 / \partial U) + (P+t)(\partial C_2 / \partial U)]$ is normalized to a value of unity. Using this together with (10) and (11), the result of the differentiation expressed in matrix notation is:

$$\begin{bmatrix} -1 & \frac{\partial X_1}{\partial X_3} + (P+t)\frac{\partial X_2}{\partial X_3} + X_3\frac{\partial v}{\partial X_3} + v \\ -t\frac{\partial C_2}{\partial U} & t\frac{\partial X_2}{\partial X_3} + X_3\frac{\partial v}{\partial X_3} + v \end{bmatrix} \begin{bmatrix} \frac{dU}{dt} \\ \frac{dX_3}{dt} \end{bmatrix} = \begin{bmatrix} \frac{C_2 - X_2 - X_3\frac{\partial v}{\partial t}}{\partial t} \\ \frac{C_2 - X_2 - X_3\frac{\partial v}{\partial t} + t\frac{\partial C_2}{\partial t} - t\frac{\partial X_2}{\partial t}}{\partial t} \end{bmatrix} \quad (12)$$

At the social-utility maximizing tariff, i.e. the optimal revenue tariff, dU/dt is zero. This implies that the numerator in the solution for dU/dt obtained from the application of Cramer's rule to (12) must be zero-valued at that tariff. Manipulation of

the expression for that numerator, and writing a number of components in terms of elasticities (see Appendix 4.A for details), yield an expression which reflects the optimal revenue tariff in terms of the associated optimal amount of X_3 . It is:

$$\frac{\partial X_1}{\partial X_3} + \frac{P \partial X_2}{\partial X_3} = \beta \quad (13)$$

where

$$\beta = \frac{-e^M [1 + E^V + (X_2/M)E^2]v}{1 - e^V + e^M}.$$

In the preceding expression $M \equiv C_2 - X_2$ is the quantity of imports; $e^M = (\partial M / \partial t)(t/M)$ is the elasticity of imports with respect to the tariff and $e^V = (\partial v / \partial t)/(t/v)$ is the elasticity of the price of the public input with respect to the tariff; $E^V = (\partial v / \partial X_3)/(X_3/v)$ is the elasticity of v with respect to the public input and $E^2 = (\partial X_2 / \partial X_3)(X_3/X_2)$ is the elasticity of output of good 2 with respect to the public input; and β reflects the interacting distortionary effects due to the tariff.

If lump-sum taxation is available then β is zero. That first-best result indicates that the quantity of the public input should be increased to the point where a further increase does not add to the value of national income evaluated at world prices. However, with reliance on tariff financing, provision no longer should be taken to the point where national income, evaluated at world prices, is maximized. Rather, that limit is now given by the expression on the right-hand-side of (13) and the relevant

issue is whether β is greater or less than zero.

DETERMINANTS OF β

The non-positive substitution effect and the configuration of the production set ensure that the sign of e^M is negative. Examining (13), this leaves three derivatives (elasticities) to sign. First, consider $\partial v / \partial t$. Komiya (1967) provides the basis for assessing this derivative. Komiya's analysis immediately leads to:

$$\frac{\partial v}{\partial t} = \frac{f_2 (k_1 - k_3)}{f_3 (k_1 - k_2)} \quad (14)$$

where k_i are the capital-labour ratios for $i = 1, 2$ and 3 , while f_j are the average product of labour functions for $j = 1, 2$ and 3 . From (14) it is clear that the sign depends solely on the rankings of the capital-labour ratios.

To determine the expression for $\partial X_2 / \partial X_3$, the following procedure may be employed. First, use the homogeneity properties of the production functions to re-express them as

$$X_1 = L_1 f_1(k_1, X_3), \quad (15)$$

$$X_2 = L_2 f_2(k_2, X_3) \quad (16)$$

and

$$X_3 = L_3 f_3(k_3). \quad (17)$$

Noting that the factor-intensities are functions of X_3 , differentiate each of (15) to (17) with respect to X_3 to obtain

$$\partial X_1 / \partial X_3 = f_1 (\partial L_1 / \partial X_3) + L_1 A_1, \quad (18)$$

$$\partial X_2 / \partial X_3 = f_2 (\partial L_2 / \partial X_3) + L_2 A_2 \quad (19)$$

and

$$1 = f_3 (\partial L_3 / \partial X_3) + L_3 A_3, \quad (20)$$

where A_i denotes $[(\partial f_i / \partial k_i) (\partial k_i / \partial X_3) + \partial f_i / \partial X_3]$ for $i = 1$ and 2 , and A_3 denotes $(\partial f_3 / \partial k_3) (\partial k_3 / \partial X_3)$.

Next, differentiate the full-employment conditions which are

$$k_1 L_1 + k_2 L_2 + k_3 L_3 = K \quad (21)$$

and

$$L_1 + L_2 + L_3 = L. \quad (22)$$

Then substitute (18), (19) and (20) into the resulting expressions to obtain

$$\begin{bmatrix} \frac{1}{f_1} & \frac{1}{f_2} \\ \frac{k_1}{f_1} & \frac{k_2}{f_2} \end{bmatrix} \begin{bmatrix} \frac{\partial X_1}{\partial X_3} \\ \frac{\partial X_2}{\partial X_3} \end{bmatrix} = \begin{bmatrix} \frac{-1}{f_3} + \frac{L_1 A_1}{f_1} + \frac{L_2 A_2}{f_2} + \frac{L_3 A_3}{f_3} \\ \frac{-k_3}{f_3} + \frac{k_1 L_1 A_1}{f_1} + \frac{k_2 L_2 A_2}{f_2} + \frac{k_3 L_3 A_3}{f_3} - S \end{bmatrix}, \quad (23)$$

where S denotes $L_1 (\partial k_1 / \partial X_3) + L_2 (\partial k_2 / \partial X_3) + L_3 (\partial k_3 / \partial X_3)$.

Finally, direct application of Cramer's rule to (23) yields the desired result, namely

$$\frac{\partial X_2}{\partial X_3} = \frac{f_2 (k_1 - k_3) (1 - L_3 A_3)}{f_3 (k_2 - k_1)} + L_2 A_2 - \frac{f_2 S}{(k_2 - k_1)}. \quad (24)$$

From (24) it follows that the manner by which X_2 varies due to changes in the amount of the public input depends only in part on the factor-intensity rankings. Without additional information regarding the production functions, the impact of a change in public input provision, at given output prices, on production of good 2 cannot be determined.

The third derivative which must be examined is $\partial v / \partial X_3$. Its sign depends on the manner by which public-input provision affects the real wage, the real rental rate on capital, and on the wage-rental ratio. In turn, these are determined by the nature of the production functions. To illustrate, consider the Lerner diagram displayed as Figure 3. The q_1 and q_2 curves are the unit-value isoquants for the given domestic price ratio (as determined by the world price ratio and the tariff) for goods 1 and 2, respectively. With incomplete specialization, the wage-rental ratio is determined by the (absolute value of the) slope of the line tangent to these two isoquants; the price of the public input being determined recursively. In equilibrium, with some of the public input being produced, its unit-value isoquant, q_3 in Figure 3, must also be tangent to the wage-rental line as established by the unit-value isoquants for the two final goods.

Consider now the impact of an increase in the quantity of public-input provision. With no change in the domestic price ratio, the unit-value isoquants of the final goods both shift inwards (how far and in what pattern depends on the production technologies). A new wage-rental line is established as the real

FIGURE 3

rewards to the factors change as a result of the change in X_3 . If both the real wage and rental increase then the new wage-rental line lies everywhere below the original line; the relative price of the public input must rise. However, if a change in X_3 provision has a proportionately different effect on the unit-value isoquants then it is possible that the real reward to one of the factors may fall. For instance, in Figure 3 the impact of an increase in public input provision is considered for the case where production of good 1 is affected in greater proportion. This makes the wage-rental line flatter and a decrease in the real wage cannot be ruled out. Suppose, as shown, that in fact the real wage does decline. Since in this illustration the public input is labour intensive, it is possible for its relative price to rise, fall or remain the same; in Figure 3 the borderline case of no change is presented.

The sign of $\partial v / \partial X_3$ depends on the rankings of the capital-labour ratios and on the elasticities of output with respect to the public input. The dependence on the sensitivity of output to the quantity of the public input introduces complications. In the case where the impact of the public input is equivalent to a Hicks-neutral technological change, if the elasticities on outputs vary in a manner such that for some levels of the public input one is greater than the other, and the reverse occurs for other levels of the public input then the relationship between v and X_3 is not monotonic. Further complications arise if provision of the public input causes pivoting of the isoquant maps. In these cir-

cumstances factor-intensity reversals are possible.

Having analysed the determinants of the signs and magnitudes of the components contained in the expression for β , an important observation is in order. Earlier, it was established that, when lump-sum taxation is available, the quantity of the public input should be increased up to the point where national income is maximized. With tariff-financing, since the tariff imposes distortionary losses whereas lump-sum taxation does not, one may have therefore expected that provision should not go "as far". In other words, tariff-financed production of the public input should stop at a point prior to attaining the maximum value of national income evaluated at world prices. However, nothing in the preceding analysis rules out negative values for β . In short, the one-to-one relationship between welfare gains and increases in national income at world prices is broken once the distortionary tariff is employed to generate the income gain.

SUMMARY

This chapter examined the optimal use of tariff revenue in financing the provision of a public input. Specifically, the analysis was undertaken for the case of factor-augmenting public inputs. The first-best (lump-sum tax) rule involved increasing the provision of the public input to the point where an additional unit does not give rise to a further increase in national income evaluated at world prices. Moving to the world in which only the tariff is available it was found that the new rule is rather more

complex. Depending on a number of considerations, it may be desirable to increase the level of provision beyond the point where national income is maximized. That is to say, at the optimal revenue tariff, it may be possible to increase national income evaluated at world prices but doing so would be undesirable.

Finally, it is worthwhile to note the symmetry of this analysis with that of tariff-financing of a public good. In both situations the approach for setting up the problem was identical. The balance of trade condition and the tariff-financing condition, with appropriate substitution for the supply and compensated demand functions, implied, in both cases, that social utility was determined by X_3 and t but was an independent function of only one of those two policy variables. Using the first-order condition for maximization of social utility, the properties of the optimal revenue tariff were represented in terms of the rule for public good/input production. Despite this symmetry in approach, the problem in either case was quite different. With a public good, the isoquant maps for the private goods were invariant with respect to the quantity of the public good, but that quantity caused indifference curves to pivot about in private goods' space. On the other hand, with the public input, indifference curves were invariant to the quantity of the public input, while isoquants shifted in response to changes in the level of the public input.

APPENDIX 4.A

Derivation of (13)

Differentiate (8) to obtain:

$$\begin{aligned} & \left[\frac{\partial X_1}{\partial t} + (P+t) \frac{\partial X_2}{\partial t} + X_2 + \frac{\partial v X_3}{\partial t} - \frac{\partial C_1}{\partial t} + (P+t) \frac{\partial C_2}{\partial t} + C_2 \right] dt \\ & + \left[\frac{\partial X_1}{\partial X_3} + (P+t) \frac{\partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \right] dX_3 = \left[\frac{\partial C_1}{\partial U} + (P+t) \frac{\partial C_2}{\partial U} \right] dU \end{aligned} \quad (A1)$$

Then, without loss of generality, normalize $[\partial C_1/\partial U + (P+t)\partial C_2/\partial U]$ to a value of unity and use (10) and (11) from the text to re-express (A1) as:

$$\left[\frac{\partial v X_3}{\partial t} + X_2 - C_2 \right] dt + \left[\frac{\partial X_1}{\partial X_3} + (P+t) \frac{\partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \right] dX_3 = dU \quad (A2)$$

Differentiate (9) to obtain:

$$\begin{aligned} & \left[C_2 - X_2 + t \left(\frac{\partial C_2}{\partial t} - \frac{\partial X_2}{\partial t} \right) - \frac{\partial v X_3}{\partial t} \right] dt + \left[\frac{-t \partial X_2}{\partial X_3} - v - \frac{\partial v X_3}{\partial X_3} \right] dX_3 \\ & = \left(\frac{-t \partial C_2}{\partial U} \right) dU \end{aligned} \quad (A3)$$

Put (A2) and (A3) in matrix format and let $M = C_2 - X_2$ to obtain:

$$\begin{bmatrix} -1 & \frac{\partial X_1}{\partial X_3} + (P+t) \frac{\partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \\ \frac{-t \partial C_2}{\partial U} & \frac{t \partial X_2}{\partial X_3} + v + \frac{\partial v X_3}{\partial X_3} \end{bmatrix} \begin{bmatrix} \frac{dU}{dt} \\ \frac{dX_3}{dt} \end{bmatrix} = \begin{bmatrix} M - \frac{\partial v X_3}{\partial t} \\ M + \frac{t \partial M}{\partial t} - \frac{\partial v X_3}{\partial t} \end{bmatrix} \quad (A4)$$

which is (12) in the text.

Next, to identify the characteristics of public input provision when the optimal revenue tariff is in place, use (A4) to solve for dU/dt and set it equal to 0. This gives:

$$\begin{aligned} & \left(M - \frac{\partial v X_3}{\partial t} \right) \left(\frac{t \partial X_2}{\partial X_3} + v + \frac{\partial v X_3}{\partial X_3} \right) - \\ & \left[\frac{\partial X_1}{\partial X_3} + (P+t) \frac{\partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \right] \left(M + \frac{\partial M}{\partial t} - \frac{\partial v X_3}{\partial t} \right) = 0, \end{aligned} \quad (A5)$$

which implies:

$$\begin{aligned} & \left(M - \frac{\partial v X_3}{\partial t} \right) \left(\frac{t \partial X_2}{\partial X_3} + v + \frac{\partial v X_3}{\partial X_3} \right) - \\ & \left(\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} + \frac{t \partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \right) \left(M - \frac{\partial v X_3}{\partial t} \right) - \\ & \left[\frac{\partial X_1}{\partial X_3} + (P+t) \frac{\partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \right] \left(\frac{t \partial M}{\partial t} \right) = 0, \end{aligned} \quad (A6)$$

or

$$\begin{aligned} & - \left(\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right) \left(M - \frac{\partial v X_3}{\partial t} \right) - \left(\frac{t \partial M}{\partial t} \right) \left(\frac{t \partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \right) \\ & - \left(\frac{t \partial M}{\partial t} \right) \left(\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right) = 0, \end{aligned} \quad (A7)$$

or

$$- \left(\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right) \left(M - \frac{\partial v X_3}{\partial t} + \frac{t \partial M}{\partial t} \right) = \left(\frac{t \partial M}{\partial t} \right) \left(\frac{t \partial X_2}{\partial X_3} + \frac{\partial v X_3}{\partial X_3} + v \right), \quad (A8)$$

Manipulation of (A8) results in:

$$\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} = \frac{\left(-\frac{t}{M} \frac{\partial M}{\partial t} \right) \left(\frac{t}{v} \frac{\partial X_2}{\partial X_3} + \frac{\partial v}{\partial X_3} \frac{X_3}{v} + 1 \right) v}{1 - \frac{\partial v}{\partial t} \frac{X_3}{M} + \frac{t}{M} \frac{\partial M}{\partial t}}, \quad (\text{A9})$$

which, noting $t/v = X_3/M$, gives (13) in the text.

CHAPTER 5: SHADOW PRICING FOR PUBLIC SECTOR PRODUCTION

INTRODUCTION

In Chapter 2 the optimal rule for production of a tariff-financed public good was developed and analysed. Two important limitations were then identified. First, it was acknowledged that revenue raised by a tariff may be devoted to paying for public inputs rather public goods. To remedy this first shortcoming, the theory of public inputs was elaborated upon in some detail in Chapter 3 and then, in Chapter 4, an optimal rule for production of a tariff-financed public input was derived and investigated.

The second limitation had to do with the assumption of private sector production of the public good and public input. While there are many examples of governments purchasing the services of private firms for public works, it is perhaps even more frequently the case that the production takes place within the public sector. In fact, as observed earlier, this may be a preferable practice since the use of a distortionary tax to raise revenue makes cost-minimization by firms inefficient.¹

In this chapter, efficiency rules for the employment of capital and labour in the public sector are derived. This is done for both a collective consumption good and a factor-augmenting public input. Derivation of these rules amounts to identification of the appropriate shadow prices for capital and

labour. Thus, in the section which immediately follows, there is a discussion of the existing literature dealing with shadow pricing of factors in a tariff-distorted economy. Following that discussion, the first-best rules for public sector production of a public good/input are discussed briefly. Next, the hiring rules are derived for the case of tariff-financing. These rules are then compared with their first-best counterparts as well as with those derived under the assumption of cost-minimizing private sector production.

SHADOW PRICING

The cost-benefit methodology formulated by Little and Mirrlees (1969) ignited trade theorists' interest in shadow pricing of factors in distorted open economies. A number of efforts have been made to clarify and extend the Little-Mirrlees suggestions for shadow pricing. Among the more important contributions to this process of clarification are Findlay and Wellisz (1976), Srinivasan and Bhagwati (1978) and Bhagwati and Wan (1979).

The model employed by Srinivasan and Bhagwati (S-B) with the extensions of it by Bhagwati and Wan (1979) provides a concise explanation of the issues. The S-B model contains the following characteristics. There are three traded goods, each of which is produced with constant-returns to scale by two factors, namely, capital and labour. The two factors are in perfectly inelastic

supply. Initially, there is incomplete specialization and a tariff is in place leading to distorted factor prices.

S-B proceed to determine the nature of the shadow prices for capital and labour when these factors are to be withdrawn for use in the production of the third good. They find that, for the withdrawal of factors for a "small" project, the shadow price of a factor is the value of the change in national income, evaluated at world prices, due to that factor's withdrawal from production of the other two goods.² Thus, the S-B shadow price of labour is $\hat{w} = -[(\partial X_1 / \partial L_3) + P(\partial X_2 / \partial L_3)]$ and for capital the shadow price is $\hat{r} = -[(\partial X_1 / \partial K_3) + P(\partial X_2 / \partial K_3)]$ where K_3 and L_3 denote the quantities of capital and labour, respectively, that are withdrawn for use in production of the third good. In turn, it is noted that these shadow prices are such that the following is necessarily true:

$$\hat{w}l_i' + \hat{r}k_i' = p_i, \quad i = 1 \text{ and } 2; \quad (1)$$

where l_i' (k_i') denotes the output-labour (output-capital) ratio used in the production of good i , at the tariff-distorted prices and p_i is the exogenous world price of the i -th good.³

S-B take the international price of the third good as an accurate measure of its social valuation. Thus, the criterion for acceptance of a project producing some of the third good is simply $p_3 > \hat{w}l_3' + \hat{r}k_3'$ where l_3' and k_3' denote the labour-output and capital-output ratios associated with the production

of the third good. (Note that the Rybczynski line properties of the production surface imply that these capital- and labour-output ratios remain constant so long as there is incomplete specialization and the tariff is not changed.)

In the S-B analysis the third good is a tradable good. Therefore, its world price reflects its true value. However, if the third good is non-tradable then there is no international price. The social valuation of the third good is determined at home. This is the situation which is to be investigated for the particular case in which the third good is of a collective nature. Further, the tariff is no longer taken to be arbitrary but is now the method of remunerating the factors which are employed in the production of the third good. Also, while S-B considered the case of a small project, the analysis presented here identifies the resource allocation conditions which occur at the social optimum. In the section which immediately follows, the first-best rules are identified. Then, in the subsequent two sections, the rules for tariff-financing of a public good and a public input, respectively, are derived.

FIRST-BEST RULES

If revenue can be raised in a non-distortionary manner then it makes no difference if a public good or a public input is produced by the private sector or the public sector. Cost-minimization is optimal since factor prices are not distorted.

As shown in Appendix 5.A, if capital and labour are employed by the public sector to produce a public good then they should be hired up to such that

$$HMRS_{13}MPK^3 = r \quad (2)$$

and

$$HMRS_{13}MPL^3 = w, \quad (3)$$

where r and w are the actual rental rate on capital and wage rate, respectively, while MPL^3 and MPK^3 denote the marginal products of labour and capital in the production of the public good, respectively.

Appendix 5.B shows that the rules for hiring capital and labour for public sector production of a factor-augmenting public input are

$$[(\partial X_1 / \partial X_3) + P(\partial X_2 / \partial X_3)]MPK^3 = r \quad (4)$$

and

$$[(\partial X_1 / \partial X_3) + P(\partial X_2 / \partial X_3)]MPL^3 = w. \quad (5)$$

The pairs (2) and (3), and (4) and (5) indicate that each factor should be hired up to where the marginal social gain of hiring one more unit of that factor is just equal to its wage. In (2) and (3) the marginal social gain is the sum of the marginal rates of substitution multiplied by the relevant factor's marginal product. In (4) and (5) the marginal social gain is the multiplicative product of the increase in national income and the relevant factor's marginal product. On the right-

hand-sides of (2) to (5) are the actual factor prices; they coincide with their shadow prices since there is no distortionary tax. It may also be noted that (2) and (3) can be readily manipulated to show that they are equivalent to the first-best rules associated with private sector production of the public good. The same holds for (4) and (5) regarding the public input.

TARIFF FINANCING: PUBLIC PRODUCTION OF THE PUBLIC GOOD

Appendixes 5.C and 5.D contain the derivation of efficiency rules for the public-sector employment of factors in the production of a public good and a public input, respectively. In either situation, the method of deriving these conditions is the same as that for tariff-financing when private production occurs. Recall from chapters 2 and 4 that the balance of trade condition and the tariff-financing condition must both hold in equilibrium. Substituting the compensated demand and supply functions as well as the production functions for the public good (input) into these two conditions reveals that social utility is an implicit and independent function of two variables. Those variables are the quantities of capital and labour employed in the public sector.⁴ Exactly how social utility varies with employment in the public sector depends on whether it is a public good or a public input which is being made. With a public good, the amount produced affects social utility directly (social indifference curves shift in private goods' space) while

in the case of a public input it is the production frontier which is directly affected (the PPF in private goods' space shifts).

For the public good, it is shown in Appendix 5.C that at maximal feasible social utility, capital and labour employed by the public sector must be such that

$$\text{HMRS}_{13}\text{MPK}^3 = \phi[\hat{r} - t(\partial C_2/\partial X_3)\text{MPK}^3] \quad (6)$$

and

$$\text{HMRS}_{13}\text{MPL}^3 = \phi[\hat{w} - t(\partial C_2/\partial X_3)\text{MPL}^3]. \quad (7)$$

In the two equations given above, $\phi = (1 - e^E)/(1 + e^M - e^E)$ where E denotes public sector expenditure i.e. $E = wL_3 + rK_3$, and $e^E = (\partial E/\partial t)(t/E)$. Also, recall that the terms $\hat{w}$ and $\hat{r}$ are the S-B shadow prices. Thus, since the right-hand-side expressions are now the shadow prices of the factors, the S-B shadow prices, while retaining a role, no longer tell the full story.

The conditions given by (6) and (7) are, not surprisingly, very similar in form to the rule that would be followed if the authorities had purchased the public good from a competitive cost-minimizing private sector. Continuing with the terminology adopted in the earlier chapters, ϕ is interpreted as the marginal cost of revenue and the term which follows it is the net cost of the respective factor.

The key difference from the private-sector-produced public good rule, derived in Chapter 2, is that in the latter case, capital and labour are hired in a manner such that the marginal

rates of technical substitution across all industries are equalized. Such an allocation would be inefficient since the factor prices are distorted; being on the efficiency locus is no longer optimal. Yet, that is what cost-minimizing firms would do. On the other hand, where the authorities are permitted to hire the factors and follow (6) and (7) above, the marginal rate of substitution between the factors in the public sector differs from that employed in private industry. Exactly how the public sector's marginal rate of substitution should differ is indicated by the ratio of the right-hand-side expressions in (6) and (7), i.e. the ratio of the shadow prices.

TARIFF FINANCING: PUBLIC PRODUCTION OF THE PUBLIC INPUT

When the public sector is engaged in the production of a public input then, as derived in Appendix 5.D, capital and labour should be employed in the production of the public input in such a combination that

$$sMPK^3 = \phi\{\hat{r} - a[t(\partial X_2/\partial X_3)MPK^3 + (\partial E/\partial K_3) - r]\} \quad (8)$$

and

$$sMPL^3 = \phi\{\hat{w} - a[t(\partial X_2/\partial X_3)MPL^3 + (\partial E/\partial L_3) - w]\} \quad (9)$$

where "s" denotes $[(\partial X_1/\partial X_3) + P(\partial X_2/\partial X_3)]$; ϕ and E are as defined above; and "a" denotes $e^M/(1 - e^E)$. Again, the basic method of deriving the preceding results is to go to the balance of trade

condition and the tariff-financing condition and, after substituting in the compensated demand functions, supply functions and production function for the public input, expressing social utility as an independent function of K_3 and L_3 . Then it is simply a matter of finding the maximal value for that function.

SOME OBSERVATIONS

In (6) and (7) and in (8) and (9) the S-B shadow prices do not indicate the true cost of employing factors in the public sector. This is due in part to the fact that S-B do not have the tariff revenue as the means of financing. Rather, they follow the traditional approach of assuming that tariff revenue, from a fixed tariff, is distributed in lump-sum fashion. The financing of their public sector project is via lump-sum taxation. Hence, in the conditions derived above, ϕ , the marginal cost of tax revenue, would be unity.

Also, the "net cost" of each factor, given by the expression in brackets on the right-hand-sides of (6), (7), (8) and (9), does not coincide with either its actual price or the S-B shadow price. The difference from the S-B prices reflects the impact of what the public sector is producing. S-B take the relative price of what the public sector produces as well as the tariff as given. Further, since the output of their public sector project does not affect relative factor prices (unlike a public input)

then shadow prices do not vary as factors are withdrawn by the public sector. In the present analysis, with public sector employment being financed by the tariff, any change in one is accompanied by the appropriate adjustment in the other. The extent to which the tariff may need to be adjusted due to a change in public sector employment depends on the manner by which that change, through its impact on domestic production and consumption, affects tariff revenue at the initial rate.

Footnotes

1. This social gain may not necessarily occur. The extra degree of freedom for policy could be offset partially or more than fully if public sector production is characterized by monopoly power on the part of a bureaucracy acting according to interests which diverge from that of a Samuelson central planner.
2. The project need not be small. The values of the shadow prices of the factors remain constant as long as both of the other goods continue to be produced.
3. An important implication of the two conditions in (1) is that it is possible for one of the shadow factor prices to be negative.
4. More generally, the independent variables are any two of: the tariff, public sector employment of capital and public sector employment of labour.

Appendix 5.A

First Best Rules for Government

Production of a Public Good

In equilibrium, trade must be balanced so we have

$$X_1(P, K_3, L_3) + PX_2(P, K_3, L_3) = C_1(P, F_3(K_3, L_3), U) + PC_2(P, F_3(K_3, L_3), U) \quad (1A)$$

where P is the given terms of trade,

$X_3 = F_3(K_3, L_3)$ is the production function for the public good, U denotes social utility and

$C_1(\cdot)$ and $C_2(\cdot)$ are compensated demand functions.

With government production, the choice variables are K_3 and L_3 since (1A) implies that U is a function of K_3 and L_3 .

Totally differentiate (1A) with respect to K_3 and L_3 and U to obtain

$$\begin{aligned} & \left[\frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} - \left[\frac{\partial C_1}{\partial X_3} + P \frac{\partial C_2}{\partial X_3} \right] \frac{\partial F_3}{\partial K_3} \right] dK_3 + \\ & \left[\frac{\partial X_1}{\partial L_3} + P \frac{\partial X_2}{\partial L_3} - \left[\frac{\partial C_1}{\partial X_3} + P \frac{\partial C_2}{\partial X_3} \right] \frac{\partial F_3}{\partial L_3} \right] dL_3 - \left[\frac{\partial C_1}{\partial U} + P \frac{\partial C_2}{\partial U} \right] dU = 0. \end{aligned} \quad (2A)$$

At the optimum it must be that $dU = 0$. This means that $\partial U / \partial L_3 = \partial U / \partial K_3 = 0$. For $\partial U / \partial K_3 = 0$, we have from (2A) that

$$\frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} = \left[\frac{\partial C_1}{\partial X_3} + P \frac{\partial C_2}{\partial X_3} \right] \frac{\partial F_3}{\partial K_3}. \quad (3A)$$

Define K_T as the quantity of capital available for production of private goods. Then, since

total capital is fixed at $\bar{K}$, we have $dK_T = -dK_3$. Noting this, the negative of the expression on the left-hand-side in (3A) is simply the rental rate on capital, denoted by r . Also note that, as shown elsewhere, the expression in brackets on the right-hand-side in (3A) is $-HMRS_{13}$.

Thus (3A) translates into

$$H \cdot MRS_{13} MP_K^3 = r \quad (4A)$$

where MP_K^3 denotes the marginal physical product of capital in production of good 3, the public good.

Following a similar procedure, it can also be shown that $\partial U / \partial L_3 = 0$ implies

$$H \cdot MRS_{13} MP_L^3 = w \quad (5A)$$

where w is the wage rate and MP_L^3 is the marginal physical product of labour in production of the public good.

2 of/de 2

MicroD

Appendix 5.B

First-Best Rules for Government

Production of a Public Input

Start again with the balance of trade condition, writing it to reflect the fact that the quantity of the public input affects production of the private goods. Thus, we have

$$X_1(P, F_3(K_3, L_3), K_3, L_3) + PX_2(P, F_3(K_3, L_3), K_3, L_3) = C_1(P, U) + PC_2(P, U), \quad (1B)$$

where $X_i(\cdot)$ and $C_i(\cdot)$, $i = 1, 2$, are the supply and compensated demand functions, respectively. Also, F_3 is the production function for the public input.

Taking the world price ratio, P , as fixed (1B) contains three variables: K_3 , L_3 and U . Thus (1B) establishes U as an implicit function of K_3 and L_3 . It follows that at the optimum ($dU = 0$) K_3 and L_3 must be such that $\partial U / \partial K_3$ and $\partial U / \partial L_3$ are both zero. To determine the characteristics of the optimum we then proceed by totally differentiating (1B) to obtain:

$$\begin{aligned} & \left[\frac{\partial X_1}{\partial X_3} \frac{\partial F_3}{\partial K_3} + \frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} \right] dK_3 \\ & + \left[\frac{\partial X_1}{\partial X_3} \frac{\partial F_3}{\partial L_3} + \frac{\partial X_1}{\partial L_3} + P \frac{\partial X_2}{\partial X_3} \frac{\partial F_3}{\partial L_3} + P \frac{\partial X_2}{\partial L_3} \right] dL_3 = \left[\frac{\partial C_1}{\partial U} + P \frac{\partial C_2}{\partial U} \right] dU \end{aligned} \quad (2B)$$

From (2B) it follows that

$$\frac{\partial U}{\partial K_3} = \left[\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right] \frac{\partial F_3}{\partial K_3} + \left[\frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} \right] \quad (3B)$$

where, for convenience, $\partial C_1 / \partial U + P \partial C_2 / \partial U$ has been normalized to a value of unity. Noting that the second expression on the right-hand-side of (3B) is equal to the negative of the rental rate on capital, it follows from (3B) that when $\partial U / \partial K_3 = 0 \dots$

$$\left[\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right] MP_K^3 = r \quad (4B)$$

where MP_K^3 denotes the marginal physical product of capital in the production of the public input, and the expression which precedes it is the change in national income per unit change in the supply of the public input.

By similar procedure, when $\partial U / \partial L_3 = 0$ we have

$$\left[\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right] MP_L^3 = w \quad (5B)$$

where w denotes the wage rate and MP_L^3 denotes the marginal physical product of labour in the production of the public input.

Appendix 5.C

Government production of a public good

where financing is via a tariff.

Government must choose t , K_3 and L_3 in the combination which maximizes social utility, U , while satisfying

$$X_1(t, K_3, L_3) + PX_2(t, K_3, L_3) = C_1(t, F_3(K_3, L_3), U) + PC_2(t, F_3(K_3, L_3), U) \quad (1C)$$

$$t[(C_2(t, F_3(K_3, L_3), U) - X_2(t, K_3, L_3))] = w(t)L_3 + r(t)K_3 \quad (2C)$$

where (1C) is the balance of trade condition and (2C) is the tariff financing condition. Note that F_3 is the production function for the public good. Also, P , the world price ratio, is assumed to be fixed. Thus, production, demands and factor prices are expressed as functions of t rather than $P + t$.

In this two equation system there are four variables: U , t , K_3 and L_3 . Jointly the two equations imply that U is implicitly a function of any two of the remaining three variables (the third would not be independent). We take U to be a function of K_3 and L_3 .

Thus, at the optimum (where $dU = 0$) K_3 and L_3 must be such that $\partial U / \partial K_3 = 0$ and $\partial U / \partial L_3 = 0$.

To start to solve the problem, totally differentiate (1C) and (2C). Differentiating (1C) yields

$$\begin{aligned} & \left[\frac{\partial X_1}{\partial t} + P \frac{\partial X_2}{\partial t} - \left[\frac{\partial C_1}{\partial t} + P \frac{\partial C_2}{\partial t} \right] \right] dt + \left[\frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} - \left[\frac{\partial C_1}{\partial K_3} + P \frac{\partial C_2}{\partial K_3} \right] \frac{\partial F_3}{\partial K_3} \right] dK_3 \\ & + \left[\frac{\partial X_1}{\partial L_3} + P \frac{\partial X_2}{\partial L_3} - \left[\frac{\partial C_1}{\partial L_3} + P \frac{\partial C_2}{\partial L_3} \right] \frac{\partial F_3}{\partial L_3} \right] dL_3 = \left[\frac{\partial C_1}{\partial U} + P \frac{\partial C_2}{\partial U} \right] dU \end{aligned} \quad (3C)$$

$$\text{which may be expressed as } a_1 dt + b_1 dK_3 + c_1 dL_3 = e_1 dU. \quad (4C)$$

Differentiating (2C) gives:

$$\left[t \left[\frac{\partial C_2}{\partial t} - \frac{\partial X_2}{\partial t} \right] + C_2 - X_2 - \left[\frac{\partial w}{\partial t} L_3 + \frac{\partial r}{\partial t} K_3 \right] \right] dt + \left[t \left[\frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} - \frac{\partial X_2}{\partial K_3} \right] - r \right] dK_3$$

$$+ \left[t \left[\frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial L_3} - \frac{\partial X_2}{\partial L_3} \right] - w \right] dL_3 = -t \frac{\partial C_2}{\partial U} dU \quad (5C)$$

$$\text{or} \quad a_2 dt + b_2 dK_3 + c_2 dL_3 = e_2 dU \quad (6C)$$

So we have

$$a_1 dt + b_1 dK_3 + c_1 dL_3 = e_1 dU \quad (4C)$$

$$a_2 dt + b_2 dK_3 + c_2 dL_3 = e_2 dU \quad (6C)$$

where, again, it is noted that t is not independent of K_3 and L_3 .

Next, observe that (6C) implies

$$dt = \frac{-b_2}{a_2} dK_3 - \frac{c_2}{a_2} dL_3 + \frac{e_2}{a_2} dU \quad (7C)$$

Then substitute (7C) into (4C) to obtain

$$\left[\frac{-a_1 b_2}{a_2} + b_1 \right] dK_3 + \left[\frac{-a_1 c_2}{a_2} + c_1 \right] dL_3 = \left[e_1 - \frac{a_1 e_2}{a_2} \right] dU \quad (8C)$$

Thus

$$\frac{\partial U}{\partial K_3} = \frac{-a_1 b_2}{a_2} + b_1 \quad (9C)$$

and

$$\frac{\partial U}{\partial L_3} = \frac{-a_1 c_2}{a_2} + c_1 \quad (10C)$$

which at maximum social utility must both be zero.

Results obtained by setting $\partial U / \partial K_3$ and $\partial U / \partial L_3$ equal to zero will be symmetric so we will focus on $\partial U / \partial K_3 = 0$.

Prior to doing so, note that the following observations will be used.

$$(i). \quad \partial X_1 / \partial t + (P+t) \partial X_2 / \partial t = 0 \text{ which implies that } \partial X_1 / \partial t + P \partial X_2 / \partial t = -t \partial X_2 / \partial t,$$

$$(ii). \quad \partial C_1 / \partial t + (P+t) \partial C_2 / \partial t = 0 \text{ which implies that } \partial C_1 / \partial t + P \partial C_2 / \partial t = -t \partial C_2 / \partial t,$$

and, recalling (20) from Chapter 2,

$$(iii). \quad \frac{\partial C_1}{\partial X_3} + P \frac{\partial C_2}{\partial X_3} = - \left[HMRS_{13} + t \frac{\partial C_2}{\partial X_3} \right].$$

Now, substitute for a_1 , b_1 and b_2 in (9C) to obtain

$$\begin{aligned} \frac{\partial U}{\partial K_3} = & \frac{- \left[\frac{\partial X_1}{\partial t} + P \frac{\partial X_2}{\partial t} - \left[\frac{\partial C_1}{\partial t} + P \frac{\partial C_2}{\partial t} \right] \right] \left[t \left[\frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} - \frac{\partial X_2}{\partial K_3} \right] - r \right]}{a_2} \\ & + \left[\frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} - \left[\frac{\partial C_1}{\partial X_3} + P \frac{\partial C_2}{\partial X_3} \right] \frac{\partial F_3}{\partial K_3} \right] \end{aligned} \quad (11C)$$

which, using i, ii, and iii may be re-written as

$$\begin{aligned} \frac{\partial U}{\partial K_3} = & \frac{- t \left[\frac{\partial C_2}{\partial t} - \frac{\partial X_2}{\partial t} \right] \left[t \left[\frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} - \frac{\partial X_2}{\partial K_3} \right] - r \right]}{a_2} \\ & + \left[\frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} + \left[HMRS_{13} + t \frac{\partial C_2}{\partial X_3} \right] \frac{\partial F_3}{\partial K_3} \right] \end{aligned} \quad (12C)$$

Setting $\partial U / \partial K_3 = 0$ implies

$$\frac{\partial X_1}{\partial K_3} + P \frac{\partial X_2}{\partial K_3} + \left[HMRS_{13} + t \frac{\partial C_2}{\partial X_3} \right] \frac{\partial F_3}{\partial K_3} = \frac{t \left[\frac{\partial C_2}{\partial t} - \frac{\partial X_2}{\partial t} \right] \left[t \left[\frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} - \frac{\partial X_2}{\partial K_3} \right] - r \right]}{a_2} \quad (13C)$$

Next, let $M = C_2 - X_2$, let $-[(\partial X_1 / \partial K_3) + P(\partial X_2 / \partial K_3)]$ be denoted by $\hat{r}$, substitute for a_2 and then re-arrange (13C) to obtain

$$HMRS_{13} \frac{\partial F_3}{\partial K_3} = \hat{r} - t \frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} + \frac{t \frac{\partial M}{\partial t} \left[t \left[\frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} - \frac{\partial X_2}{\partial K_3} \right] - r \right]}{t \frac{\partial M}{\partial t} + M - \left[\frac{\partial w}{\partial t} L_3 + \frac{\partial r}{\partial t} K_3 \right]} \quad (14C)$$

Letting $E = wL_3 + rK_3$ denote total expenditure on the public good, noting that in equilibrium $E = tM$, and expressing in terms of elasticities, (14C) becomes

$$\text{HMRS}_{13} \text{MP}_K^3 = \frac{\left[e_t^M + 1 - e_t^E \right] \left[\hat{r} - t \frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} \right] + e_t^M \left[t \left[\frac{\partial C_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} - \frac{\partial X_2}{\partial K_3} \right] - \hat{r} \right]}{1 + e_t^M - e_t^E} \quad (15C)$$

which simplifies to

$$\text{HMRS}_{13} \text{MP}_K^3 = \frac{1 - e_t^E}{1 + e_t^M - e_t^E} (\hat{r} - t(\partial C_2 / \partial X_3) \text{MP}_K^3). \quad (16C)$$

By similar reasoning $\partial U / \partial L_3 = 0$ implies

$$\text{HMPS}_{13} \text{MP}_L^3 = \frac{1 - e_t^E}{1 + e_t^M - e_t^E} (\hat{w} - t(\partial C_2 / \partial X_3) \text{MP}_L^3), \quad (17C)$$

where $\hat{w} \equiv -(\partial X_1 / \partial L_3) - P(\partial X_2 / \partial L_3)$,

$$e_t^M \equiv \frac{\partial M}{\partial t} \frac{t}{M}$$

and

$$e_t^E \equiv \frac{\partial E}{\partial t} \frac{t}{E}$$

Appendix 5.D

Government Production of a Public Input

financed by a Tariff

Start with the two equation system,

$$X_1(t, F_3(K_3, L_3), K_3, L_3) + P X_2(t, F_3(K_3, L_3), K_3, L_3) = C_1(t, U) + P C_2(t, U) \quad (1D)$$

and

$$t[C_2(t, U) - X_2(t, F_3(K_3, L_3), K_3, L_3)] = w(t, F_3(K_3, L_3))L_3 + r(t, F_3(K_3, L_3))K_3. \quad (2D)$$

Differentiating (1D) and (2D) yields, respectively,

$$\left[\frac{t \partial M}{\partial t} \right] dt + \left[-\dot{r} + \gamma \frac{\partial F_3}{\partial K_3} \right] dK_3 + \left[-\dot{w} + \gamma \frac{\partial F_3}{\partial L_3} \right] dL_3 = g_1 dU \quad (3D)$$

and

$$\begin{aligned} \left[\frac{t \partial M}{\partial t} + M - \frac{\partial E}{\partial t} \right] dt - \left[t \left[\frac{\partial X_2}{\partial K_3} + \frac{\partial X_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} \right] - \frac{\partial E}{\partial K_3} \right] dK_3 \\ - \left[t \left[\frac{\partial X_2}{\partial L_3} + \frac{\partial X_2}{\partial X_3} \frac{\partial F_3}{\partial L_3} \right] - \frac{\partial E}{\partial L_3} \right] dL_3 = g_2 dU \end{aligned} \quad (4D)$$

where M denotes imports; $-\dot{r} \equiv \partial X_1 / \partial K_3 + P \partial X_2 / \partial K_3$; $-\dot{w} \equiv \partial X_1 / \partial L_3 + P \partial X_2 / \partial L_3$; $\gamma \equiv \partial X_1 / \partial X_3 + P \partial X_2 / \partial X_3$, $E \equiv wL_3 + rK_3$, $g_1 \equiv \partial C_1 / \partial U + P \partial C_2 / \partial U$ and $g_2 \equiv -t \partial C_2 / \partial U$.

Combining (3D) and (4D) in a manner designed to eliminate dt yields

$$\frac{\partial U}{\partial K_3} dK_3 + \frac{\partial U}{\partial L_3} dL_3 = g_3 dU \quad (5D)$$

where

$$\frac{\partial U}{\partial K_3} \equiv \left[t \frac{\partial M}{\partial t} + M - \frac{\partial E}{\partial t} \right] \left[\dot{r} - \gamma \frac{\partial F_3}{\partial K_3} \right] / \left[t \frac{\partial M}{\partial t} \right] - t \left[\frac{\partial X_2}{\partial K_3} + \frac{\partial X_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} \right] - \frac{\partial E}{\partial K_3}, \quad (6D)$$

$$\frac{\partial U}{\partial L_3} \equiv \left[t \frac{\partial M}{\partial t} + M - \frac{\partial E}{\partial t} \right] \left[\dot{w} - \gamma \frac{\partial F_3}{\partial L_3} \right] / \left[t \frac{\partial M}{\partial t} \right] - t \left[\frac{\partial X_2}{\partial L_3} + \frac{\partial X_2}{\partial X_3} \frac{\partial F_3}{\partial L_3} \right] - \frac{\partial E}{\partial L_3}, \quad (7D)$$

and

$$g_3 \equiv g_2 - g_1 \left[t \frac{\partial M}{\partial t} + M - \frac{\partial E}{\partial t} \right] / \left[t \frac{\partial M}{\partial t} \right]. \quad (8D)$$

Setting $\partial U/\partial K_3 = 0$ implies

$$\gamma \frac{\partial f_3}{\partial K_3} = \dot{r} - \frac{e_t^M}{1 + e_t^M - e_t^E} \left[t \left[\frac{\partial X_2}{\partial K_3} + \frac{\partial X_2}{\partial X_3} \frac{\partial F_3}{\partial K_3} \right] + \frac{\partial E}{\partial K_3} \right], \quad (9D)$$

where $e_t^M \equiv (\partial M/\partial t)(t/M)$ and $e_t^E \equiv (\partial E/\partial t)(t/E)$.

Finally, obtaining a common denominator on the R–H–S of (9D), substituting for γ and re–arranging yields

$$\left[\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right] MP_K^3 = \frac{1 - e_t^E}{1 + e_t^M - e_t^E} \left[\dot{r} - \left[\frac{e_t^M}{1 - e_t^E} \right] \left[t \frac{\partial X_2}{\partial X_3} MP_K^3 + \frac{\partial E}{\partial K_3} - r \right] \right]. \quad (10D)$$

By a similar procedure, $\partial U/\partial L_3 = 0$ implies

$$\left[\frac{\partial X_1}{\partial X_3} + P \frac{\partial X_2}{\partial X_3} \right] MP_L^3 = \frac{1 - e_t^E}{1 + e_t^M - e_t^E} \left[\dot{w} - \left[\frac{e_t^M}{1 - e_t^E} \right] \left[t \frac{\partial X_2}{\partial X_3} MP_L^3 + \frac{\partial E}{\partial L_3} - w \right] \right]. \quad (11D)$$

CHAPTER 6: SUMMARY AND CONCLUSION

OVERVIEW

Motivated by the empirical reality that countries have used, and many continue to use, tariffs which generate substantial revenues, this thesis focussed on the derivation of Samuelson-type rules for public expenditure financed by tariff revenue. In particular, four situations were considered. These were where the expenditure is on either publicly or privately produced public goods, and where the expenditure is on either publicly or privately produced public inputs. Rules for efficient, tariff-financed, provision of the public good/input were derived as well as rules for the hiring of capital and labour for public sector production. In the latter circumstances the results were related to the shadow pricing rules developed by Srinivasan and Bhagwati (1978).

SOME POSSIBLE EXTENSIONS

The framework developed herein lends itself to some further applications and extensions. One possibility is that the model may be extended to consider the implications of using tariff revenue for redistributive purposes; that would simply entail introduction of different utility functions for individuals and a specification of a social welfare function in which distribution matters.

A second potential extension may be to build on the work of Findlay and Wilson (1984) in which the analysis switches from normative to positive. In their model, they allow for the state to play a productive role in society through its provision of production-enhancing public services, i.e. a public input. Then, depending on nature of preferences of those in control of the state, predictions are made regarding the nature of taxation, the supply of the public input and the extraction of rents by the state.

A third use to which the analysis may be put is as a background for thinking about the actual use of revenues by developing countries. Clearly, there are limits to which countries should go in providing public goods and inputs when financed by tariffs. This thesis identifies the key parameters which define these limits.

On a related matter, reference to the analysis contained here may be helpful for historical evaluation of tariff policy. For instance, prior to World War I, the Canadian federal government derived practically all of its revenue from tariffs. While the tariff structure was in part designed to be protectionist, as a component of the "National Policy", the revenue from the tariff was essential to the financing the completion of huge public works, particularly the national railroad, the other major component of the National Policy. There is an on-going debate as to the wisdom of the National

Policy, see e.g. Dales (1966). The framework provided here may prove useful to economic historians in thinking about the relative benefits and costs of that controversial policy.

A FINAL CAVEAT

This analysis has derived rules for "efficiency" when a tariff is employed to finance a public good or a public input. It has also noted that public sector production of those items may be preferable to private sector production. In doing so, there is a danger that the analysis may be construed as providing a theoretically valid argument in favour of using trade taxes. This is certainly not the case. The fundamental problem is the reliance on the tariff; taking that as a given, this analysis considers the best that can be done. If the tariff could be replaced by another tax instrument which is less expensive in terms of collection and related costs as well as the familiar production and consumption costs then "best" could be improved upon. In fact, it is possible to argue that the appropriate public service for tariff revenue to be spent on in the first place is the development of a superior tax system.

BIBLIOGRAPHY

- Abe, K., H. Okamoto and M. Tawada (1986), "A Note on the Production Possibilities Frontier with Pure Public Intermediate Goods", Canadian Journal of Economics, 19, pp. 351-356.
- Altenburg, L. (1987), "Production Possibilities with a Public Intermediate Good", Canadian Journal of Economics, 20, pp. 715-734.
- Atkinson, A., and J. Stiglitz (1980), Lectures on Public Economics, Maidenhead: McGraw-Hill.
- Atkinson, A., and N. Stern, (1974), "Pigou, Taxation and Public Goods", Review of Economic Studies, 41, pp. 119-128.
- Bhagwati, J. (1982), "Directly-unproductive Profit-seeking (DUP) Activities", Journal of Political Economy, 90, pp. 988-1002.
- Bhagwati, J., and T. Srinivasan (1980), "Revenue Seeking: A Generalization of the Theory of Tariffs," Journal of Political Economy, 88, pp. 1069-1087.
- Bhagwati, J., and T. Srinivasan (1983), Lectures on International Trade, Cambridge: The MIT Press.
- Bhagwati, J., and H. Wan (1979), "The Stationary of Shadow Prices in Project Evaluation, with and without Distortions", American Economic Review, 69, pp. 261-273.
- Blomqvist, A. (1974), "Tariff Revenue and Optimal Capital Accumulation in Less Developed Countries", Economic Journal, 84, pp. 70-89.
- Boadway, R., S. Maital and M. Prachowny (1973), "Optimal Tariffs, Optimal Taxes and Public Goods", Journal of Public Economics, 2, pp. 391-403.
- Boadway, R. (1973), "Similarities and Differences between Public Goods and Public Factors", Public Finance, 28, pp. 245-257.
- Caves, R., and R. Jones (1977), World Trade and Payments, second edition, Boston: Little, Brown and Company.
- Corden, W.M., (1974), Trade Policy and Economic Welfare, Oxford: The Clarendon Press.
- Dales, J. (1966), The Protective Tariff in Canada's Development, Toronto: University of Toronto Press.
- Feehan, J. (1988), "Efficient Tariff Financing of Public Goods", Journal of International Economics, 25, pp. 155-164.

- Feehan, J. (1985), "Public Goods in a Small Open Economy: Lump-Sum Taxation versus the Tariff for Revenue," Carleton Economics Papers 85-05, Carleton University, Ottawa.
- Findlay, R., and J. Wilson (1984), "The Political Economy of Leviathan", Institute for International Economic Studies Seminar Paper No. 285, Stockholm: University of Stockholm.
- Findlay, R., and S. Wellisz (1976), "Project Evaluation, Shadow Prices and Trade Policy", Journal of Political Economy, 84, pp. 543-552.
- Gillespie, W. I. (1988), "Financing Federal Spending in Canada: The Birth, Growth and Death of Taxes, 1867-1967", mimeo, Carleton University, Ottawa.
- Gould, J. (1972), "Externalities, Factor Proportions and the Level of Exploitation of Free Access Resources", Economica, 56, pp. 383-402.
- Greenaway, D. (1980), "Trade Taxes as a Source of Government Revenue: An International Comparison," Scottish Journal of Political Economy, 27, pp. 175-182.
- Greenaway, D. (1981), "Taxes on International Trade and Economic Development", in A.T. Peacock and F. Forte, (eds.), The Political Economy of Taxation, Oxford: Basil Blackwell.
- Greenaway, D. (1984), "A Statistical Analysis of Fiscal Dependence on Trade Taxes and Economic Development", Public Finance, 39, pp. 70-89.
- Greenaway, D., and D. Sapsford (1987), "Further Econometric Analysis of the Relationship between Fiscal Dependence on Trade Taxes and Economic Development", Public Finance, 42, pp. 309-319.
- Groves, T., and M. Loeb (1975), "Incentives and Public Inputs", Journal of Public Economics, 4, pp. 211-226.
- Henderson, J. (1974), "A Note on the Economics of Public Intermediate Goods", Economica, 41, pp. 322-327.
- Hillman, A. (1978), "Symmetries and Asymmetries between Public-Input and Public-Good Equilibria", Public Finance, 33, pp. 269-279.
- Hitiris, T., and A. Weekes (1987), "Fiscal Dependence on Trade Taxes and Economic Development: An Econometric Investigation", Public Finance, 42, pp. 297-308.
- International Monetary Fund (1987), Government Finance Statistics Yearbook, Volume XI.

- Johnson, H. (1951), "Optimum Welfare and Maximum Revenue Tariffs", Review of Economic Studies, 19, pp. 28-35.
- Kaizuka, K. (1965), "Public Goods and the Decentralization of Production", Review of Economics and Statistics, 47, pp. 118-120.
- Kemp, M. (1969), The Pure Theory of International Trade and Investment, Prentice-Hall: Englewood Cliffs.
- Kemp, M. (1982), editor, Production Sets, Academic Press: New York.
- Komiya, R. (1967), "Non-traded Goods and the Pure Theory of International Trade", International Economic Review, 8, pp. 132-152.
- Laffont, J. J. (1976), "Collective Factors of Production under Uncertainty", Journal of Public Economics, 5, pp. 313-324.
- Lancaster, K. (1966), "A New Approach to Consumer Theory", Journal of Political Economy, 74, pp. 132-137.
- Lewis, S. (1963), "Government Revenue from Foreign Trade: An International Comparison", Manchester School, 31, pp. 39-46.
- Little, I., and J. Mirrlees (1969), A Manual of Industrial Project Analysis in Developing Countries, OECD: Paris.
- Manning, R., and J. McMillan (1979), "Public Intermediate Goods, Production Possibilities and International Trade", Canadian Journal of Economics, 12, pp. 243-257.
- Manning, R., and J. McMillan (1982), "The Scale Effects of Public Goods on Production Possibility Sets", in M. Kemp, ed., Production Sets, Academic Press: New York.
- McMillan, J. (1979), "A Note on the Economics of Public Intermediate Goods", Public Finance, 34, pp. 293-299.
- McMillan, J., (1978), "A Dynamic Analysis of Public Intermediate Goods in an Open Economy", International Economic Review, 19, pp. 665-667.
- Meade, J. (1952), "External Economies and Diseconomies in a Competitive Situation", Economic Journal, 62, pp. 54-67.
- Melvin, J. (1968), "Production and Trade with Two Factors and Three Goods", American Economic Review, 58, pp. 1249-1267.
- Negishi, T. (1973), "The Excess of Public Expenditures on Industries", Journal of Public Economics, 2, pp. 231-240.

- Pestieau, P. (1976), "Public Intermediate Goods and Distortionary Taxation", European Economic Review, 7, pp. 351-357.
- Pestieau, P. (1976), "Public Intermediate Goods and Majority Voting", Public Finance, 31, pp. 209-217.
- Ramaswami, V., and T. Srinivasan (1968) "Optimal Subsidies and Taxes When Some Factors are Traded", Journal of Political Economy, 76, pp. 569-582.
- Reizman R. and J. Slemrod (1987), "Tariffs and Collection Costs" Weltwirtschaftliches Archiv, 123, pp. 545-549.
- Samuelson, P. (1953), "Prices of Factors and Goods in General Equilibrium," Review of Economic Studies 21, pp. 1-20.
- Samuelson, P. (1954), "Pure Theory of Public Expenditures", Review of Economics and Statistics, 36, pp. 387-389.
- Sandmo, A. (1972), "Optimality Rules for the Provision of Collective Factors of Production", Journal of Public Economics, 1, pp. 149-157.
- Srinivasan, T., and J. Bhagwati (1978), "Shadow Prices for Project Evaluation in the Presence of Distortions: Effective Rates of Protection and Domestic Resource Costs", Journal of Political Economy, 86, pp. 97-116.
- Tawada, M., and K. Abe (1984), "Production Possibilities and International Trade with a Public Intermediate Good", Canadian Journal of Economics, 17, pp. 232-248.
- Tawada, M. (1980), "The Production Possibility Set with Public Intermediate Goods", Econometrica, 48, pp. 1005-1012.
- Tawada, M., and H. Okamoto (1983), "International Trade with a Public Intermediate Good", Journal of International Economics, 15, pp. 101-115.
- Usher, D. (1982), "The Private Cost of Public Funds; Variations on Themes by Browning, Atkinson and Stern", unpublished.
- Vanek, J. (1971), "Tariffs, Economic Welfare and Development Potential", Economic Journal, 81, pp. 904-913.
- Wagstaff, P. (1984), "The Cost of Maximum Revenue Trade Taxes", Economica, 51, pp. 43-52.

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